INVESTIGATING MIDDLE SCHOOL STUDENTS' ACHIEVEMENT AND STRATEGIES IN PROPORTIONAL REASONING PROBLEMS

THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF SOCIAL SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

GAMZE ÖZEN YILMAZ

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
THE DEPARTMENT OF ELEMENTARY SCIENCE AND MATHEMATICS
EDUCATION

AUGUST 2019

Approval of the Graduate School of Social Sciences

Assoc. Prof. Dr. Sadettin KİRAZCI
Director (Acting)

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Elvan ŞAHİN
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Didem AKYÜZ
Supervisor

Examining Committee Members

Assist. Prof. Dr. Mesture Kayhan ALTAY	(Hacettepe Uni., İMO)	
Assoc. Prof. Dr. Didem AKYÜZ	(METU, MSE)	
Assist. Prof. Dr. Şerife SEVİNÇ	(METU, MSE)	

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : GAMZE ÖZEN YILMAZ

Signature :

ABSTRACT

INVESTIGATING MIDDLE SCHOOL STUDENTS' ACHIEVEMENT AND STRATEGIES IN PROPORTIONAL REASONING PROBLEMS

Özen Yılmaz, Gamze

M.S., Department of Elementary Science and Mathematics Education

Supervisor: Assoc. Prof. Dr. Didem Akyüz

August 2019, 167 pages

The aims of the study are to specify the academic achievement of the fifth, sixth, seventh and eighth grade students in proportional reasoning problems, to determine how the academic achievement of these students change according to problem types, to examine their solution strategies in these problems, and to reveal how these strategies diversify from fifth to eighth grade. To collect data in order to achieve these aims, a proportional reasoning test were prepared. The study was designed as a mixed study. It was carried out with a total of 858 students- 255 fifth, 209 sixth, 256 seventh and 138 eighth grade students- studying at a public school in Mamak district of Ankara in the fall semester of the 2018-2019 academic year. The data were obtained from all students in the quantitative part of the study. In the qualitative part of the study, the data were collected from 80 students in total. These students were 20 students with the highest score in proportional reasoning test at each grade level. The findings of the study revealed that the achievement of the students in proportional reasoning test increased depending on the grade levels and changed according to the types of problems. It was further observed that the

students frequently used certain solution strategies, and these strategies differed according to their grade levels. Additionally, it was concluded that students had the proportional reasoning skills although they were not taught these skills formally.

Keywords: Proportional reasoning, solution strategies, middle school students

ÖZ

ORTAOKUL ÖĞRENCİLERİNİN ORANTISAL AKIL YÜRÜTME PROBLEMLERİNDEKİ BAŞARILARINI VE KULLANDIKLARI STRATEJİLERİ İNCELEME

Özen Yılmaz, Gamze

Yüksek Lisans, İlköğretim Fen ve Matematik Alanları Eğitimi Bölümü

Tez Yöneticisi: Doç. Dr. Didem Akyüz

Ağustos 2019, 167 sayfa

Bu çalışmanın amacı beşinci, altıncı, yedinci ve sekizinci sınıf öğrencilerinin orantısal akıl yürütme problemlerindeki akademik başarılarını ve bu öğrencilerin akademik başarılarının problem türlerine göre nasıl değiştiğini belirlemek, bu problemlerde kullandıkları çözüm stratejilerini incelemek ve bu stratejilerin 5. sınıftan 8. sınıfa nasıl değiştiğini ortaya koymaktır. Bu amaçlarla veri toplamak için, iki orantısal akıl yürütme testi hazırlanmıştır. Araştırma nicel ve nitel araştırma tekniklerini kullanan bir çalışma olarak tasarlanmıştır. Çalışma 2018-2019 eğitim-öğretim yılının güz döneminde, Ankara'nın Mamak ilçesinde bir devlet okulunda öğrenim görmekte olan 255 beşinci sınıf, 209 altıncı sınıf, 256 yedinci sınıf ve 138 sekizinci sınıf öğrencisi olmak üzere toplam 858 öğrenci ile yürütülmüştür. Araştırmanın nicel teknikleri kullanan kısmında öğrencilerin tamamından veri elde edilmiştir. Nitel teknikleri kullanan kısmında ise her sınıf seviyesinden orantısal akıl yürütme testlerinden en yüksek puanı almış 20 öğrenci olmak üzere toplamda 80 öğrenciden veri elde edilmiştir. Çalışmanın sonucunda öğrencilerin orantısal akıl yürütme testlerindeki başarılarının sınıf seviyesine bağlı olarak arttığı ve problem çeşitlerine göre değiştiği tespit edilmiştir. Öğrencilerin belirli çözüm stratejilerini sıklıkla kullandıkları ve yine sınıf seviyelerine göre bu stratejilerinin farklılaşabildiği görülmüştür. Ayrıca, formal bir şekilde öğretilmese de öğrencilerin orantısal akıl yürütme becerisine sahip olduğu sonucuna ulaşılmıştır.

Anahtar Kelimeler: Orantısal akıl yürütme, çözüm stratejileri, ortaokul öğrencileri

To my all beloved ones…

ACKNOWLEDGMENTS

First of all, I extend my endless gratitude to my family. I dedicate this study to my mother, Nurgül Özen and my father, Adem Özen, who are the architects of my existence and the point I have reached today. They have raised me, made great sacrifices, and have never given up loving and supporting me.

I would also like to thank Neslihan Melike and İlayda, my precious sisters, who have always helped me feel strong during the most challenging periods with their endless support.

I would like to thank my respectable supervisor, Assoc. Prof. Dr. Didem Akyüz, who supported me throughout my journey by devoting her precious time despite her hectic work schedule and by offering tremendous help from the determination of the thesis subject to the final stage of the thesis.

I would like to express my gratitude to the invaluable thesis committee members, Assist. Prof. Dr. Şerife Sevinç and Assist. Prof. Dr. Mesture Kayhan Altay, who guided me with their positive and constructive feedback.

I extend my appreciation to my precious friends, Research Assistant Şahabettin Akşab and Research Assistant Birnaz Kanbur Tekerek, who shared their knowledge with me in the moments when I felt technically inadequate during the thesis writing process.

Finally and most importantly, I would like to thank my beloved husband, Emre Yılmaz, for his patience as I was not able to spend enough time with him during my hard work, for enduring me during this stressful process, and for supporting me all the time. Your support, patience, joy, smiling face that amused me all the time, and most importantly your love were all very important to me during this journey. Thanks a million.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

MoNE	Ministry of National Education
NCTM	National Council of Teachers of Mathematics
SPSS	Statistical Package for the Social Sciences
TIMSS	Trends in International Mathematics and Science Study
PRT	Proportional Reasoning Test

CHAPTER 1

INTRODUCTION

The aim of teachers in mathematics education is to keep up with today's technology, to provide the ability of thinking to survive in the world where life is constantly complicated, to create relationships among the events, to use reasoning, to estimate and to provide students with problem solving skills rather than skills of teaching and calculating numbers and operations (Umay, 2003). The aim of mathematics education is to educate individuals who can transfer the knowledge they have, solve problems, and produce solutions to the situations they face (MoNE, 2013). Reasoning is one of the significant skills for mathematics learning. It is indeed essential to the knowing and doing of mathematics (NCTM, 1989). It can be defined as the process of obtaining new information by using the specific tools of mathematics (symbols, definitions, relations, etc.) and thinking techniques (induction, deduction, comparison, generalization, etc.) (MoNE, 2013). Baykul (2014) defines reasoning which is a process and has evolved over time as the ability to make a decision by thinking about a subject. Reasoning is also described as a cognitive process in which people receive knowledge and make an inference beyond the original data (Kurtz, Gentner, & Gunn, 1999). At the same time, in mathematics, facts can only be reached by reasoning, and in mathematics there is reasoning on the basis of all rules and processes (Umay & Kaf, 2005). An individual who can reason mathematically can see the relationships between mathematical concepts, can distinguish geometric shapes, use proportional reasoning, use the spatial ability for three-dimensional shapes, show and represent different representations of data, and interpret the data (TIMSS, 2003). Problems can be solved more successfully with the help of reasoning. The higher the people reasoning ability, the more successful they may actually be. These people can evaluate events in different ways and express them in different ways, and then they can transfer these skills to new situations. Therefore, reasoning skills in mathematics teaching are at the top of the skills that should be gained by the students (İncebacak & Ersoy, 2016).

Mathematical reasoning forms the basis of mathematics, one of the areas in which reasoning is used intensively (Umay, 2003). Mathematical reasoning provides students with permanent and progressive mathematics. Mathematics requires exploring patterns, developing and evaluating arguments, to make logical assumptions, selecting and applying new solutions to a problem, and reaching and defending a conclusion as a course of its nature (Kramarski & Mevarech, 2003). People who mathematically reason are keen to note these patterns and structures in both the real world and mathematics, and question how they are formed (NCTM, 2000). For this reason, one the aims of the Turkish Education System is to equip students with the ability to express their thoughts and reasonings in the process of problem solving and to see the gaps in the mathematical reasoning of others (MoNE, 2018).

Reasoning is proportional when it is based upon multiplicative relationship, regardless of the method of representing a situation or solving a problem. Proportional reasoning consists of the ability of solving proportional reasoning problems, distinguishing proportional situation from nonproportional situations, and understanding the mathematical relationships of the multiplicative problem situations. (Cramer, Post, & Currier, 1993). Therefore, it is a quite difficult and complicated skill (Christou & Papageorgiou, 2002). Lesh, Post and Behr (p. 93) who consider proportional reasoning as a critical concept state “it is the capstone of children’s elementary school arithmetic; on the other hand, it is the cornerstone of all that is to follow”. In learning psychology, proportional reasoning is considered as an important step in the conceptual transition from the level of concrete transactions to the formal level of formalization (Skemp, 1987). Proportional reasoning serves as an important bridge between the arithmetic field of mathematics, which is concrete and numerical, and abstractions in algebra and advanced mathematics (Fuson ve Abrahamson, 2005; Lamon, 2007). Proportional reasoning is fundamental to algebraic thinking, and therefore, it is important and necessary to fully understand the nature and characteristics of proportional reasoning (Küpçü & Özdemir, 2012). Although most people define proportional reasoning through the use of the cross-multiplication strategy, studies show that correct proportional reasoning does not involve merely understanding fractions and rational numbers, but also the competence in other areas such as ratio sense, relative thinking, and partitioning, unitizing and changing quantities (Lamon, 1999). Most students have trouble with proportions because traditional instruction does not develop a comprehension of multiplicative relationships (Vanhille & Baroody, 2002). As an alternative, teachers have students memorize cross multiplication algorithm in order to solve proportional reasoning problems. Most students try to solve the problems which consist of multiplicative reasoning by reasoning additively because they never learn or they forget this nonmeaningful algorithm. Vanhille and Baroody (2002) also emphasize that even though students apply this algorithm successfully, multiplicative reasoning of students does not improve. Students should be able to achieve proportional reasoning skills intuitively. The way to do this is to enable students to deal with many problems on their own before any algorithm or solution is given. They need to make sense of the problems with their informal problem-solving strategies (Ben-Chaim et al., 1998).

In Turkish Education System, ratio, which is one of the basic concepts of proportional reasoning, is first taught in the 6th grade. The 6th graders are able to determine the ratio of the two multiplicities in the same or different units. They are able to determine the ratio of two parts to each other or each part to the whole in cases where a whole is divided into two parts, and they are able to find the other one when one of two ratios is given in a problem. Moreover, proportion, which is the other basic concept of proportional reasoning, is introduced in 7th grade. 7th graders are able to decide whether the two quantities are proportional or inversely proportional by examining real-life situations and thus, to solve the problems related to the proportion or inverse proportion (MoNE, 2013). The concepts of proportion and proportion are a part of many subjects such as mixing problems, rational numbers, percentage, fractions, data processing, similarity, the area of the circle and polygons. In fact, they constitute the basis of these subjects. When the interdisciplinary approaches are taken into consideration, it can be said that subjects such as map and scale in social studies lesson, movement and physical force in the science course, and perspective in the visual arts course need proportional reasoning (Kaplan and Öztürk, 2012).

Although the concepts of ratio and proportion are mentioned for the first time in the 6th and 7th grades, studies show that students can solve proportional reasoning problems without being taught ratio and proportion. The study by Ojose (2015) showed that all grade level students could have a conceptual understanding of the subject of proportion without the need to be taught the concept. The findings demonstrated that children already have proportional reasoning in their schemes before formal teaching. Moreover, most studies have shown that as the class level of the students increase, the competence of proportional thinking increase as well (Mersin, 2018; Hilton et al., 2016; Toluk Uçar &Bozkuş, 2016; Larson, 2013; Van Dooren et al., 2009). The development of the proportional thinking process requires time and experience and, in this respect, it is emphasized that studies should be spread over time and students should gain the ability of proportional reasoning by giving various examples (Baykul, 2009).

There are two types of proportional reasoning: quantitative and qualitative. Qualitative proportional reasoning includes verbal values, while quantitative proportional reasoning includes numerical values. According to Kadijevic (2002), although qualitative proportional reasoning has a significant effect on problem solving skills, it is rarely used in scientific research. In addition, qualitative proportional reasoning is thought to improve quantitative proportional reasoning. This may lead to the underestimation of qualitative reasoning and the perception that proportional reasoning merely involves numerical values. For this reason, qualitative proportional reasoning should precede quantitative proportional reasoning and it should be seen as a necessary element for proportional reasoning, not just complementary (Kadijevic, 2002). In addition to qualitative and quantitative proportional reasoning problems, another type of the proportional reasoning problems is missing value problems (Haller, Ahlgren, Post, Behr & Lesh 1989). In the missing value problems, three of the four values with a proportional situation are given and what is asked is to find the fourth value. However, it does not mean that the students who correctly solve missing value problems can reason proportionally (Tjoea & Torre, 2014).

Different solution strategies are identified in order to specify the proportional reasoning skills of students. Cramer and Post (1993) mention the unit rate strategy, the factor of change strategy, the equivalent fractions strategy, and the cross-product algorithm. In addition, Cramer and Post, Bart, Post, Behr and Lesh (1994) add the equivalence class strategy and Ben-Chaim, Fey, Fitzgerald, M., Benedetto and Miller (1998) and Parker (1999) add the build-up strategy. The majority of the studies show that the students who learn the cross-product strategy use this strategy widely in order to solve the proportional reasoning problems. For example, according to the study of Bal-İncebacak and Ersoy (2016), the 7th grade students mostly used the cross-product strategy in different kinds of the proportional reasoning problems. It was seen that students preferred the method of comparison between the quantities by making cross-product.

Kahraman, Kul and Aydoğdu-İskenderoğlu (2018) conducted a study in order to learn the strategies used by 7th and 8th graders in quantitative proportional reasoning problems. As a result of the study, it was seen that the 7th graders mostly used the unit rate strategy and the 8th graders mostly used the cross-product strategy. It was stated that the 7th graders mostly used the unit rate strategy because they did not yet learn the cross-product strategy. On the other hand, Artut and Pelen (2015) conducted a study in order to investigate the strategies used by 6th graders to solve proportional reasoning problems and whether or not these strategies vary with problem type and number structure. According to the results, 6th graders mostly used the factor of change strategy in both the missing value problems and numerical comparison problems. Moreover, they mostly used the factor of change strategy regardless of the number structures of the problems.

When previous studies are examined, it is seen that the studies do not cover all middle school grade levels; they are about the proportional reasoning of the students at specific grade levels such as only the 6th grade, only the 7th grade, or only the 7th and 8th grade. The number of longitudinal studies to determine students' achievement in proportional reasoning test and the strategies used by them in order to solve proportional reasoning problems from 5 to 8 grades is limited in Turkey. Unlike other studies, this study was conducted with 5th, 6th, 7th and 8th grade students in order to specify their academic achievement in proportional reasoning problems and to examine their solving strategies in these problems.

1.1.Aim of the Study

The aims of the study are to specify academic achievement of the students from 5th to 8th grade in proportional reasoning problems, to determine how academic achievement of these students change according to problem types and to examine their solving strategies in these problems.

1.2.Research Questions

1. Does the academic achievement of the students change from 5th to 8th grade in the test of proportional reasoning problems?
   - Does the academic achievement of these students change according to problem types?

2. What kind of strategies are mostly used by the students from 5th to 8th grade in proportional reasoning problems?

1.3.Significance of the Study

The first experience of students with mathematics in school life is with natural numbers. The first years of primary school include addition and subtraction based on the relationship between countable objects. Rational numbers and integers are introduced to students in addition to natural numbers in middle school years (MoNE, 2013). In these years, students have to make a number of important transitions in their mathematical thoughts. A fundamental change in thoughts is necessary in transition from natural numbers to rational numbers and from additive concepts to multiplicative concepts (McIntosh, 2013).

Mastery of many of the number concepts and number relationships in the middle grades appears to require a reconceptualization of number, a significant change from the primary grades in the way number is conceived. Multiplication is not simply repeated addition, and rational numbers are not simply ordered pairs of whole numbers. The new concepts are not the sums of previous ones. Competency with middle school number concepts requires a break with simpler concepts of the past and a reconceptualization of number itself. (Hiebert & Behr, 1988).

Proportional reasoning is a kind of reasoning also used in daily cases such as maps, scale models, medicine doses related to the weight of the patient, comparison-shopping, and economics (Valverde & Castro, 2012). It is crucial for students to be successful in many mathematical areas, including ratio and proportion, measurement and unit conversions, geometry and probability (Hilton & Hilton, 2018). In mathematics education, the concepts of ratio and proportion are considered necessary and important for teaching other concepts. To illustrate, proportional reasoning is a basis in order to learn algebra well because proportional relationships provide students with powerful meanings to improve algebraic thinking and function perception (Cai & Sun, 2002). In addition, the concept of proportional reasoning is an interdisciplinary concept as students encounter proportional reasoning problems in science, social science, statistics, etc. (Akatugba & Wallace, 2009; Nunes & Bryant, 2011). It consists of reasoning about percentages, temperatures, densities, concentrations, velocities, chemical compositions, and economic values (Sophian & Wood, 1997).

Mathematics education is significant in all grade levels, but it is most vital in middle grade levels because in these years, many students reinforce ideas about their competences, attitudes, interests and motivations as mathematics learners. These concepts affect how they approach mathematics in the following years (NCTM, 2010). In other words, proportional reasoning is quiet prognostic of later mathematical success (Siegler, Fazio, Bailey, & Zhou, 2013). In the same way, Smith (2003) reports the significance and complexity of the proportionality as:

> No area of elementary school mathematics is as mathematically rich, cognitively complicated, and difficult to teach as fractions, ratios, and proportionality. These ideas…are the first place in which students encounter numerals like “3/4” that represent relationships between two discrete or continuous quantities, rather than a single discrete (“three apples”) or continuous quantity (“4 inches of rope”) (p.3).

Although the secondary school mathematics curriculum includes many important concepts, one of the most common one is proportionality. To understand mathematics at high school and college level, it is essential to grasp the concept of proportion in middle school years (Johnson, 2010).

In spite of its significance, elementary school students and even adults have difficulties in reasoning proportionally (Bock, Dooren, Janssens & Verschaffe, 2002; Smith, Solomon & Carey, 2005). Therefore, it is significant to improve and evaluate ways to reinforce this sort of reasoning (Boyer & Levine, 2015). Comparing and analyzing students’ solutions in qualitative and quantitative proportional reasoning problems are primarily important in order to understand students’ proportional reasoning skills. The solution strategies used by the students to solve these problems give an idea about students’ proportional reasoning levels. As proportional reasoning is the capstone of elementary school arithmetic (Lesh et al., 1988), it is important to examine whether there is a relationship between students’ proportional reasoning levels and their academic success.

On the basis of proportional reasoning, there is the ability of comparing the quantities. Therefore, taking into account the relative changes of the quantities that determine the structure of the comparison, the ability to comment on the nature of the comparison and the development of decision-making skills are important in gaining the ability of proportional reasoning and in preventing the misconceptions of the ratio-proportion concepts (Akar, 2009).

In Turkey, in most of the studies related to the skills of the students in proportional reasoning problems, it was seen that students at the 7th or higher grades were selected as the sample, or these studies focused only on students from one or two grade levels (İncebacak & Ersoy, 2016; Kahraman, Kul & Aydoğdu-İskenderoğlu, 2018; Artut & Pelen, 2015; Küpçü & Özdemir, 2011; Poçan, Yaşaroğlu & İlhan, 2017). The number of longitudinal studies to determine the skills of proportional reasoning from 5 to 8 grades is limited in Turkey. Unlike other studies, this study was conducted with 5th, 6th, 7th and 8th graders in order to reveal their academic achievement in proportional reasoning problems, to investigate how their academic achievement change according to the problem types and to examine their solving strategies in these problems.

In the studies conducted with 8th graders, it was seen, as might be expected, that students use the cross-product strategy in their solutions mostly because they know or memorize this strategy (Kahraman, Kul & Aydoğdu-İskenderoğlu, 2018; Incebacak & Ersoy, 2016). Therefore, it is believed that the application of this study to the 5th, 6th and 7th graders who do not know the cross-product strategy will enable to make comparisons between the students who have been taught and not taught the concept of proportion.

For these reasons, the results of this study are believed to provide distinctive and valuable information about students' instinctive abilities and difficulties related to proportional reasoning.

1.4.Definitions of Important Terms

Ratio: Ratio is to compare with each other the quantities which have the same or different unities (MoNE, 2018).

Proportion: Proportion is the equality of two ratios (MoNE, 2018).

Proportional reasoning: Proportional reasoning is the skill to recognize a mathematical statement including a ratio, to express this statement symbolically, and to solve proportional problems (Cramer & Post, 1993).

CHAPTER 2

LITERATURE REVIEW

2.1.The Terms of Ratio, Proportion and Proportional Reasoning

Ratio and proportion are the components of proportional reasoning, and hence, their definition is quite important to be able to understand proportional reasoning. Ratio is to compare with each other the quantities which have the same or different unities (Babai, Cohen & Stavy, 2018; MoNE, 2018; Cai &Sun, 2002) or it is to join the quantities in a composed unit (Lobato & Ellis, 2010). However, in the literature, there is no consensus for definitions of proportion and proportional reasoning. According to Vernaud (1983), a proportion is the multiplicative relationship between the measured quantities of two physically measurable attributes that he called ‘measure spaces’, while it is the equality of two ratios as stated by the MoNE (2018), Lobato and Ellis (2010), and Lim (2009). Lamon (1995) describes students' understanding of a proportional relationship as the realization of both a scalar relationship within quantity types and a functional relationship between quantity types. The big idea underlying proportions is that “when two quantities are related proportionally, the ratio of one quantity to the other is invariant as the numerical values of both quantities change by the same factor” (Lobato & Ellis, 2010, p. 11). According to Levin (1999), proportion is an argument of equal ratios or fractions and written as a/b=b/c. On the basis of these, ratio is to compare the quantities with each other, and, proportion is the equality of the ratios. Moreover, Dole and Wright (n.d) express that ratio describes a situation in comparative terms, and proportion is when this comparison is used to describe a related situation in the same comparative terms. For instance, the meaning of the sentence ‘the ratio of boys to girls in a classroom is 2 to 3’ is the comparison of the number of boys to the number of girls.

In this classroom, if there are 30 students, the numbers of boys and girls are 12 and 18, respectively.

The ratio mentioned so far is referred to as direct proportion in the literature. In a direct proportion, one of the quantities increases, while the other increases or one of the quantities decreases and the other decreases in a multiplicative way. Another type of the proportion is inverse proportion. Inverse proportion occurs when the quantities change in a different direction. That is, one of the quantities increases, while the other decreases in a multiplicative way, or vice versa. To illustrate, the expression "if 6 people complete a job in 4 days, 12 people complete the same job in 2 days" includes inverse proportion between its quantities (Dole, Wright and Clarke, n.d).

Like proportion, a common definition of proportional reasoning is not found in the literature. According to the NCTM Curriculum and Evaluation Standards (1989), proportional reasoning is the ability to reason proportionally in students throughout the grades 5 and 8. According to Lesh, Post and Lehrer (1988), proportional reasoning is the ability to make a decision and an interpretation about comparing the quantities. Flowers (1998) defines proportional reasoning as the ability to understand and use the ratio. On the other hand, in many studies, proportional reasoning is described as the skill to recognize a mathematical statement including a ratio, to express this statement symbolically, and to solve proportional problems (Cramer & Post, 1993; Clark & Lesh, 2003; Cramer, Post & Currier, 1993). Lamon (2007) characterizes proportional reasoning as supplying:

> reasons in support of claims made about the structural relationships among four quantities (say a, b, c, d) in a context simultaneously involving covariance of quantities and invariance of ratios or products (pp. 637-638).

Rather than different descriptions of proportional reasoning, its relationship with other mathematical concepts and its importance for them are highlighted in most of the studies. As Lamon (2007) states:

> Of all the topics in the school curriculum, fractions, ratios, and proportions arguably hold the distinction of being the most protracted in terms of development, the most difficult to teach, the most mathematically complex,

> the most cognitively challenging, the most essential to success in higher mathematics and science, and one of the most compelling research sites (p.629).

Lesh, Post and Behr (1988) point out that proportional reasoning is the capstone of the elementary curriculum and the cornerstone of algebra. Similarly, according to Walle (2009), proportional reasoning is the important component of the concepts of fractions, algebra, similarity, data graphs and probability in mathematics. It can be symbolized by a fraction and then fractions' laws can be applied to ratios (Livy & Vale, 2011). Babai, Cohen and Stavy (2018) state that proportional reasoning is the skill of comparison of the amounts multiplicatively by using ratios and quantitative conceptions like fractions. Moreover, in addition to these mathematical concepts, proportional reasoning is crucial for other courses. In their study, Wollman and Lawson (1978) state that the concept of ratio and proportion is a necessary and fundamental mathematical tool to understand the concepts of velocity, momentum, pressure, density etc. in physics, chemistry, and genetics in biology. As Heinz and Sterba-Boatwright (2008) state:

> Proportional reasoning is at the core of so many important concepts in mathematics and science, including similarity, relative growth and size, dilations, scaling, pi, constant rate of change, slope, rates, percent, trig ratios, probability, relative frequency, density, and direct and inverse variations (p. 528).

Babai, Cohen and Stavy (2018) also express that proportional reasoning is required in calculations in science disciplines such as concentration, probability, density, velocity and current. In addition, Mitchell and Lawson (1988) theorize that the lack of proportional reasoning of the students in the department of the biology affects their achievement in genetics. Furthermore, Dole and Wright (n.d.) state that in addition to most science and mathematics topics, proportional reasoning is needed in real-world and everyday life situations. As an illustration, drawing a building's plan, mapping of the road between school and home, dividing four pizzas into 3 people, and deciding better purchase when the price of 1 kg is 3.5 TL and the price of 1.5 kg is 4.20 TL need reasoning proportionally. They also express that proportional reasoning is basic for rational number concepts such as fractions, ratios, percentages, decimals and proportions.

In spite of its importance, proportional relations are hard for students to understand. Modestou and Gagatsis (2010) claim that most of the students who graduated from high school do not have the ability of proportional reasoning. Middle school is accepted as the most critical period for learning ratio and proportional relations (Lobato, Ellis, Charles, & Zbiek, 2010). In addition, NCTM (1989) states that "the ability to reason proportionally develops in students throughout grades 5-8" (p. 82). It is important to investigate the development of proportional reasoning in these years in order to determine how they make sense of proportional and non-proportional situations. NCTM (1989) expresses that students need to see many kinds of problem situations that can be modeled and then resolved through proportional reasoning. Based on the solutions of these problem situations, the proportional reasoning ability of the students can be determined. In ratio and proportion problems, most of the middle school students use cross-multiplication to solve the proportion and then to find the missing value (Cramer & Post, 1993). Nevertheless, this method has been identified as a memorization method, and thus, it cannot be said that these students solve the proportion problem through proportional reasoning. The teaching of cross-multiplication algorithm is not approved by many mathematic educators (Dole & Wright, n.d.). The students who can reason proportionally solve the proportional problem situations, distinguish the proportional and non-proportional situations, and especially comprehend the mathematical relationships in the multiplicative proportional problems (Cramer, Post & Currier, 1993). In brief, it cannot be claimed that all the students who can solve the proportional problems can reason proportionally.

2.2.Proportional Reasoning from Early Grades to Middle School

When used correctly, informal knowledge may help learning, but under certain circumstances it may enforce restrictions and barriers that restrict understanding and cause systematic errors (Fischbein et al., 1985). Resnick and Singer (1993) state that when formal concepts are well integrated into the intuitive system, basic mathematical concepts such as proportion can be improved in the best way. For this reason, in order to achieve the highest level of proportional reasoning skills in elementary school students, students should understand the concept of proportional reasoning at a basic level (Lesh et al., 1988). Proportional reasoning skills starts formally with the teaching of fractions in primary education from an early age and then continues in secondary and university years. However, it is very difficult to achieve this skill in full (Singh, 2000). It is estimated that more than fifty percent of adults even do not have proportional reasoning (Lamon, 1999). In recent years, the shift in the interest of researchers in this area is thought to be due to the fact that the ability of proportional reasoning is important but the process of obtaining it is difficult. In general, proportional reasoning is necessary at school and out of school in every moment of life, so it is necessary to achieve this skill from the elementary school years and it should be continued during the following education years. Experiences from daily life and school life play an important role in the commitment to proportion. In the early years of childhood, children face proportional relationships in simple forms (Van den Brink & Streefland, 1979), e.g. if a car has four wheels, two cars have 8 wheels. However, proportional reasoning is not only important for the applicability and usefulness of everyday life situations, but also for the solution of most problems in mathematics and science (Van Dooren et al., 2005).

In spite of the significance and prevalence of proportional reasoning, there is a conflict over the developmental process. According to Piaget, at the beginning, proportional reasoning focuses on identifying, estimating and evaluating the relationship between the two relations, rather than examining the relationship between two concrete objects. Therefore, proportional reasoning examines the secondary level relationships, not the primary ones.

In proportional situations, students use multiplicative reasoning, which requires the understanding of the relative change (Baxter & Junker, 2001). Therefore, at the end of their study, Inhelder and Piaget (1958) argued that children are not capable of proportionate reasoning till about 11 years of age. Other studies also support this view. To illustrate, Noelting (1980) offered children between the ages of 6 and 12 with two proportions, each symbolizing a series of glasses of orange juice concentrate and a series of glasses of water, and asked children which ratio would make a more intensive orange juice. The results were consistent with what Inhelder and Piaget put forward because children under 12 years of age could not choose the correct answer. In contrast, there are studies which argue early development of proportional reasoning. For example, Christou and Philippou (2002) investigated the informal understanding of 120 fourth and fifth grade students in proportional problems and how students' intuitions affect their strategies of solving proportional problems. They conducted a test consisting of eight proportional reasoning problems. The students' solutions were not mathematically sophisticated for each problem, but they were strong enough to make the predictions about the nature of proportionate reasoning easier. All the students intuitively tended to use the unit rate method; thus, they could not produce multiplicative methods when their known methods could not provide sufficient solutions. When the numbers in the problems did not allow students to calculate the unit rate, they turned to other solution strategies such as the building-up method, which is one of the simplest methods to solve the problem. In addition, the fifth graders calculated the unit value more comfortably than the fourth graders, and therefore they needed other solutions less. This meant that school practices probably played a more decisive role in the improvement of rate logic than early mathematical improvement.

Van Dooren et al. (2005) argue that the importance of proportional reasoning starts at the second year of primary school when children learn about multiplication and division and when to use these operations simply and in one step in the problems such as “1 kg of apples costs 2 euro. How much do 3 kg of apples cost?” At the beginning of the 3rd or 4th grade, proportional reasoning is introduced to the students and they are faced with proportional reasoning problems with missing value. The main teaching of proportionality usually begins in primary (or lower secondary) classes, where students are given the missing value problems and are confronted with various contexts (prices, mixtures, ...) which consist of proportional reasoning (Kaput & West, 1994). For instance, “Grandma adds 2 spoonfuls of sugar to juice of 10 lemons to make lemonade. How many lemons are needed if 6 spoonfuls of sugar are used?" (Van Dooren, De Bock & Verschaffel 2010). In order to solve this problem, students usually use internal ratio 6 spoonfuls sugar to 2 spoonfuls sugar or external ratio of 10 lemons to 2 spoonfuls sugar (Vergnaud, 1988). In addition, these solving methods, there is an approach named as *unit factor* which finds firstly the unit value of the quantities, (that is, if 10 lemons need 2 spoonfuls sugar, 5 lemons will need spoonful of sugar.) (Vergnaud, 1988). Lastly, there is a more elementary method named as *building up*: if 2+2+2 spoonful of sugar is needed, 10+10+10 lemons will be need. Even if this method, which is the basis of the repeated addition, includes the properties of additive reasoning, it is accepted a multiplicative approach because it properly manages the multiplicative proper of the problem situation. (Van Dooren, De Bock & Verschaffel 2010). However, there is a mistaken method called as *additive method* which is used by subtracting the given values from each other in order to find the missing value. To illustrate, in the above lemonade problem, because two spoonfuls of sugar increase 4 spoonfuls of sugar, 4 lemons are needed more, that is 10+4=14 lemons. In order to examine the improvement of misusage of proportional reasoning by age and students' educational practices, De Bock et al. (2005) conducted a test with various sorts of proportional and nonproportional missing value problems to 1062 students from 2nd grade level to 8th grade level. According to the results, students tended to use proportional methods when proportional reasoning was not clearly applicable. Although there were some practical errors in the second grade, the number of students increased substantially up to 5th grade level in parallel to the increasing proportional reasoning capacities of the students. From the sixth grade on, students began to differentiate between situations where proportional reasoning was applied or not, but even in grade 8, students still made significantly proportional errors. In addition, the probability of error diversified according to the type of non-proportional reasoning problems.

Van Dooren, De Bock and Verschaffel (2010) conducted a study in order to investigate the usage of additive method in multiplicative problems and multiplicative method in additive problems. They examined the development of both forms of mistaken solution strategies as age progresses. They prepared a test consisting of 6 additive design and 6 multiplicative design of missing value problems for 325 3rd, 4th, 5th and 6th grade level students. The results of the study showed that while the inclination to use additive strategies in missing-value problems decreases with age, the inclination to use multiplicative strategies increases significantly. All third-grade level students solved the multiplicative problems additively, whereas nearly third of sixth graders used proportional strategies for all the problems. In addition to these results, all grade level students used a solution strategy depending on the numbers given in the problems. While students applied more multiplicative strategies when numbers in the problems generated integer ratios, they used additive strategies when the numbers did not generate integer ratios.

1996 National Assessment of Educational Progress (NAEP) gave two questions to fourth grade students in order to obtain information about how students in the elementary grades solve the multiplicative and proportional reasoning problems. The first question was solved correctly by 3 percent of students and the second question was solved correctly by 6 percent of students. Most students had errors and misconceptions about proportional reasoning because they thought proportion in additive way or could not identify a proportional situation. In addition, while both problems could be solved without multiplication or proportionality, the students tried to solve them using multiplication even though they had errors. Students should be presented with problems where multiplication is necessary or they can use multiplication correctly to solve proportions (Kenney, Lindquist & Heffernan, 2002). Time is needed for students to progress their proportional reasoning which is particularly dependent upon multiplication and division (Dole, Wright & Clarke, n.d) This progress is promoted by enabling students to explore, discuss and experience proportional situations and through ratio and proportion’s rich conceptual understanding.

Langrall and Swafford (2000) highlighted the drawbacks of giving certain rules to children in relation to proportional relations at an early age and stated that teaching about proportional reasoning should start with situations in which children can do modeling and visualization. In order to help students understand the situation of change between the two quantities related to each other, informal reasoning should be used by providing qualitative comparisons to the students before quantitative comparisons. Students can develop numerical reasoning strategies after solving proportion problems using informal reasoning skills. Thus, before students learn the rules for proportional reasoning, they can construct their own informal knowledge and develop concepts for proportional reasoning (Uçar & Bozkuş, 2016).

Dole, Wright, Clarke, and Hilton (2007) administered a 10-item test that measured proportional reasoning skills to approximately 700 students from grade 5 to grade 9. This test consisted of missing value problems, rate problems, and relative thinking problems, and the solutions were coded using a three-level code that indicated whether the answer was correct or incorrect, the quality of the answer, and the students' thinking. The most successful group was the 9th grade students with an average of 6.2. Then, 7th and 8th grade students followed up with an average of 4.8. The 5th grade students showed the lowest success. This study showed that students' proportional reasoning skills increased depending on their age and grade level.

Hilton et Al. (2016) conducted a study to investigate the development of proportional reasoning of middle school students aged between 9 and 14. They applied a test consisting of 12 two-tier items to the students. The first tier of each item was true-false argument and the second tier had four alternatives in order to understand the students’ proportional reasoning types and their common errors in the proportional and non-proportional conditions. One of the results of the research was that there is a decrease in the number of students who misuse the additive reasoning across the year level. A closer examination of the data for non-proportional items showed that the percentage of students using improper multiplicative reasoning in these items diminished. In addition, they emphasized that the development of the proportional reasoning skill takes quite remarkable time and many students need more instruction and exposure to these significant skills and concepts.

Ojose (2015) investigated the gaps and comprehension of 114 sixth, seventh and eighth grade students in proportional reasoning concepts and other related concepts such as decimal, percentage and ratio. The study consisted of two phases of qualitative and quantitative data collection and analysis. The students' solutions showed that the increase in the grade level did not mean that students would perform better in proportional reasoning problems. Based on the fact that the sixth and seventh grade students in this study were not taught the proportional reasoning concept, the analysis and interviews showed that all grade level students could have a conceptual understanding of the subject of proportion without the need to be taught the concept. These findings emphasized that children already had a mathematical feel of proportional reasoning in their schemes before formal teaching.

Mersin (2018) conducted a study with 146 sixth, seventh and eighth grade level students in order to identify the types of reasoning that students used in proportional and non-proportional situations in different types of the problems. As a result of the study, 7^{th} grade students were found to be more successful than 5^{th} and 6^{th} grade students. Even though it is thought that the reason for this is that the 7^{th} grade students formally learnt the subject of proportionality, it was seen that proportional reasoning levels of students increased as grade level increased.

Küpçü and Özdemir (2011) conducted a study to determine the relationship between individual characteristics (gender, cognitive style and proportional reasoning levels) and proportional reasoning success of the 134 seventh and eighth grade students in solving proportional problems. In order to determine the students' individual characteristics related with proportional reasoning, three different success tests were conducted. In order to determine the problem-solving success in the research, the achievement tests for solving the proportional problems, percent problems, and similarity problems in the triangles, proportional reasoning level test and cognitive styles test to determine individual differences were used. After the proportional reasoning level test, the levels of students were specified according to the levels of Langrall and Swafford (2000). As a result of the study, it was seen that most of the 7th grade students were distributed equally in level one and level two, but 8th grade students were more in level two. This situation suggested that the increase in the grade level and more study with the concepts of proportion increased the proportional thinking skills of elementary school students. This result indicates that the increase in cognitive levels, age and mathematics experiences in proportion also improve the proportional thinking skills.

Doğan and Çetin (2009) conducted a study to determine the misconceptions of 1085 7th and 9th grade students about ratio and proportion and to determine whether there was a decrease in these misconceptions as the grade level increased. In the application, a test with 20 multiple choice questions which were suitable for both grade levels was used. These questions were asked to determine whether the concepts of ratio and proportion are known correctly, to determine errors in the concepts of inverse proportion and direct proportion, to see the errors in the proportionality processes, and to determine whether the proportionality properties are used correctly in the processes. It was seen that 9th grade students had less misconceptions than 7th grade students, but the misconceptions continued mostly in 9th grade. In addition, they had wrong information that one of the quantities increases while the other decreases, or vice versa in an inverse proportion. Therefore, the students thought that the increase or decrease between the quantities should be in an additive way, not in a multiplicative way. For this reason, 66,3% of the 7th grade and 59,3% of the 9th grade students could not solve the inverse proportion problem in the test correctly. Although these percentages were quite high, it was seen that misconceptions of the students decreased when the grade level increased. Moreover, it was seen that the students applied the strategy of cross-product in a way by heart, and therefore they used this strategy in their solutions without questioning the problem.

2.3.Additive and Multiplicative Reasoning

The students in early years of school are firstly taught additive reasoning which is one of the types of the mathematical reasoning. It includes of the abilities associated with counting, adding, joining, subtracting, separating, and removing (Bright, Joyner, & Wallis, 2003; Lamon, 2007). Multiplicative reasoning refers to reasoning about multiplication, division, linear functions, ratios, rates, rational numbers, shrinking, enlarging, scaling, duplicating, exponentiating, and fair sharing (Lamon, 2007). In addition, Bright et al. (2003) provided the following contrasts:

> Proportional or multiplicative reasoning is in contrast to additive reasoning. Additive reasoning involves using counts – for example, sums or differences of numbers – as the critical factor in comparing quantities. Multiplicative or proportional reasoning involves using ratios as the critical factor in comparing quantities (p. 166).

Identifying whether the ratio or product of two quantities which are proportional or inversely proportional to each other in a given situation is constant is remarkably difficult for most students. Considering that sum or difference of these quantities is constant is a misconception that is made typically and frequently (Glaser & Riegler 2015). Glaser and Riegler (2015) referred to this situation as additive reasoning, in contrast to proportional or multiplicative reasoning. One of the most important stages in the development process of the proportional reasoning skill is the ability of the student to be able to switch from additive reasoning to multiplicative reasoning (Fernandez & Llinares, 2009). In the first years of primary school, additive reasoning strategies are used to solve proportional reasoning problems, but proportional or multiplicative reasoning strategies are used in the later years. (Van Dooren, De Bock & Verschaffel 2010). Additive reasoning consists of using counts such as sums or differences of the numbers, while ratios are used in multiplicative or proportional reasoning as the critical factor in comparing quantities. (Bright, Jeane & Charles, 2003). Whereas multiplicative situations are expressed as f(x)=ax, additive situations are expressed as f(x)=x+b (Fernandez & Llinares, 2009). For example, ‘When Ayse is 5 years old, Ali is 10 years old. How old is Ali when Ayşe is 15?’ is an additive situation (Uçar & Bozkuş, 2016). In order to find the age of Ali, the difference between the two quantities must be found and this difference must be added to the small number. However, ‘If a car goes 240 km in 4 hours, how many kilometers does it go in 2 hours?’ is a multiplicative situation. This problem has two different variables (time and path) and it is necessary to apply multiplication or division to solve the problem. Students must re-conceptualize the concept of unit in order to move from cumulative reasoning to multiplicative reasoning (Hiebert & Behr, 1988) because multiplication requires working with composite units instead of dealing with singleton units (Sowder et. al., 1998). For instance, the student should be able to think of 4 as a single unit in 3x4 process and find 3 out of 4. Therefore, the multiplicative reasoning is a more complex process than additive reasoning, as it requires a different flexible unitizing (Uçar & Bozkuş, 2016).

Proportional reasoning skills do not improve instinctively and most of the students are inclined to additive methods, have difficulty in making distinctions of situations of proportion from non-proportion and overuse the multiplicative methods in improper situations. (Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005). In addition to this, students use additive methods in proportional situations (Van Dooren et al., 2010). This over-generalization may be associated with students' initial habits of adding and counting (Boyer, Levine, Susan, & Huttenlocher, 2008).

Multiplicative thinking skills begin to develop from the second year, but it is very slow (Clark & Kamii, 1996). Depending on this situation, it is stated that students solve problems with additive thinking even though the problem situations which necessitate multiplicative thinking are shown in early ages. Another reason for this situation may be that these students tend to use additive methods due to their insufficiency in multiplication and division (Lo & Watanabe, 1997). In addition, when students encounter proportional reasoning problems involving numbers that are not easily divisible to each other, they use additive methods that are easier and more familiar to compute for them (Clark & Kamii, 1996). With the increase in the grade level, there is a slight decrease in the tendency of students to use additive strategies, while there is an increase in the tendency to use multiplicative strategies

(Fernandez, Llinares, Van Dooren, De Bock & Verschaffel, 2012., Uçar & Bozkuş, 2016).

One of the studies in this context was conducted with the 3rd, 4th, 5th and 6th graders by Van Dooren, De Bock, Gillard ve Verchaffel (2009). In this study which examined the strategies used by students in solving additive and multiplicative problems, it was determined that students could not use appropriate strategies in problem solving. In both types of problems, students used additive or multiplicative strategies without making discrimination. It was observed that the use of multiplicative strategies increased, and the use of additive strategy decreased as students' level of education increased. It was concluded that the tendency of students to use additive strategy at an early age changed with the tendency to use multiplicative strategy at later times. Meron and Utilizin (1999) examined the methods used by the 3rd and 4th graders as the smaller age group for the problems involving multiplicative situations. It was seen that many of the students used additive strategy in the case of problems where the multiplicative strategy was appropriate or answered the questions with the counting process. Peled and others (1999) stated that the students could do the multiplication process mentally but could not establish a relationship between multiplicative situations. They explained that the reason for these results is that multiplicative structures are difficult and complex, so this complexity leads to the difficulty of defining these structures and the application of multiplicative strategy.

Bright, Joyner and Wallis (2003) emphasized the importance of presenting students the situations in which multiplicative and additive reasoning could be applied correctly or incorrectly. They devised an instrument tool in order to evaluate the answers of 8th and 9th grade students to four multiple-choice questions involving proportional reasoning. Three of the four multiple-choice questions they used included proportional reasoning and one had additive reasoning. The first question asked which of the shape formed by a rectangle and the extension of this rectangle by 200% looked more like a square. 59.1% of the students answered this question correctly. The second question was a problem asking which of the four rectangular fields is more like a square. 67.4% of the students answered this question correctly. The third question included additive reasoning, and 59.1% of the students solved it correctly. The fourth problem included multiplicative reasoning, and 45.4% of the students answered it correctly. Uçar and Bozkuş (2016) conducted a study in order to determine the strategies used by 320 4th, 5th, 6th and 7th grade level students to solve proportional and non-proportional problems and to reveal how students' ability to distinguish proportional situations from non-proportional situations develop. The findings revealed that only 8 students solved all the problems correctly, and the students had difficulty in distinguishing the proportional problems from non-proportional problems and accordingly, they used inappropriate strategies to solve the problems. In particular, the majority of 4th, 5th and 6th grade students use the additive solution strategy in proportional and non-proportional problems, whereas 7th grade students generally use the multiplicative solution strategy in all problems. This result shows that even if the problem situations which require multiplicative thinking are shown to the students, students solve problems by additive thinking (Clark & Kamii 1996). Another result obtained from the study is that the type of strategies used by the students varies from additive to multiplicative as the grade level increases. In addition, the seventh-grade students were not able to distinguish between additive and multiplicative problem situations, even though they were educated about proportion concept.

On the other hand, there are also studies showing that additive strategies are not used unnecessarily in multiplicative situations. One of them belong to Atabaş and Öner (2016). They conducted a study with 5th and 6th grade students in order to determine their proportional reasoning skills in the proportional and non-proportional problems and to specify whether the proportional reasoning of these students was affected by whether the ratio in the problem was an integer or not. They conducted a 4-item test including a missing value problem, a proportional comparison problem, a constant problem and an additive problem. One of the results of the study was that 6th grade students were more successful in solving proportional problems than 5th grade students. Contrary to most studies in the literature, it was not seen that students used additive methods unnecessarily in proportional problems (missing value and comparison) in the study of Atabaş and Öner (2016). This result was thought to be due to the fact that the participants of the study were from only two grade levels.

2.4.Problem Solving

It is widely accepted that the main element of mathematics is problem solving and its process. This thinking process, which leads people to the problems they face, is used both in daily life and in all branches of science. The main aim of primary education is to prepare individuals for life and higher education. Mental skills needed to achieve both goals are effective reasoning, critical thinking and problem solving. In the development of these skills, all the courses in the primary education program are effective, but the mathematics lesson takes up more than all of the above skills (Özsoy, 2005).

Thinking starts with a problem, and the solution of the problem turns into a goal for the individual and this aim directs the individual's thinking. The thinking that arises with the problem constitutes a process. The human brain needs a lot of things to achieve its producer ability, but the brain first of all needs the method that can be applied to different areas. Without the scientific method, even if the human brain is equipped with all the information, it cannot produce; it only stores. Attitudes and skills towards scientific thinking are gained through the process of scientific method. Scientific method is used synonymously with problem solving process (Kalaycı, 2001).

In addition to the necessity of everyday life, problem-solving skills are necessary to be successful in the mathematics course. Problem solving can contribute to the development of cognitive strategy while learning mathematics (Yıldızlar, 1999). Students who are successful in problem solving are also expected to be successful in mathematics (Özsoy, 2005). In recent years, problem solving has been used as a tool to determine students' ways of thinking and comprehending in mathematics learning (Arıkan & Ünal, 2015). Problem solving is a skill that must be constantly developed to strengthen our survival, so it is a daily requirement (Skemp, 1987).

Problem solving is a learning continuum which is carried out both in everyday life and at school (Jonassen, 1997). When solving problems, students who use memorized solutions in the traditional approach do not have the chance to produce their own solutions (Hines, 2008). Even if the students do not know the solution clearly, they should try to solve the problem by using their experience and knowledge.

As students gain success in the process of problem solving and feel that their solutions are valued, their confidence in their ability to do mathematics increases. Thus, students are more patient and creative when solving problems (MoNE, 2013). Verbal problems help students to develop new mathematical models and help them gain experience in this area. In addition, it provides a suitable environment for students to develop language, reasoning, mathematical development and interaction (Reusser ve Stebler, 1997). Inoue (2005) emphasizes that in the problem-solving studies in mathematics courses, students should take into consideration the experiences they have acquired in real life outside the school. The ability to accurately reflect the real-life situations of the results obtained in the problem-solving studies in mathematics courses can be achieved by taking the problems that address real-life situations as much as possible in school mathematics and by encountering mathematical problems that allow students to take into account different perspectives (Cooper & Harries, 2002). In addition, it is important to use proportional situations and introduce different examples and solution strategies of problems in order to enable students to think proportionally, to develop different strategies and to think in depth (Capraro et al., 2009). In this context, Şen and Güler (2017) conducted a study to show the effect of the instruction on problem solving strategies on the proportional reasoning abilities of the sixth-grade students in order to solve the proportional reasoning problems. During 8 lesson hours, 16 students in the experimental group were taught problem solving strategies in order to be able to solve proportional reasoning problems, but 16 control group students did not receive any training other than usual training. According to the results of pre-test, before the instruction of problem-solving strategies, the proportional reasoning levels of the students in both groups were low. However, it was seen that the proportional reasoning levels of the students in the experimental group where the teaching of problem-solving strategies were applied were higher than the control group students after the instruction of problem-solving strategies. These results revealed that the instruction of problem-solving strategies had a positive impact on the abilities of proportional reasoning of students.

Artut and Aladağ (2012) conducted a study with middle school students to determine their skills to solve problems that require proportional reasoning problems and problems as well as proportional reasoning problems, but which require realistic responses. The research results revealed that students were more successful in solving proportional reasoning problems. As the grade levels of the students increased, the students' success in solving these problems increased. On the other hand, one of the reasons why students are more successful in solving proportional problems is that time and experience are necessary for the development of proportional thinking process. On the other hand, one of the reasons for the increase in the grade level of the students to increase the success of solving proportional problems is that time and experience are necessary for the development of the proportional thinking process. As the grade levels of the students increase, students' experiences with proportional reasoning also increase. Smith and Regan (1999) made a detailed classification by focusing on individual differences in order to reveal the causes of different levels of learning in the same learning environment. In this classification, students were classified according to their cognitive, affective, social and physiological characteristics, and it was emphasized that this distinction was effective in designing teaching and learning process and organizing activities. When the literature is examined, it is seen that the studies related to the determination of the factors affecting the problem-solving success of the individuals (especially mathematical problems) include distinctions corresponding to the classification given above. For example, Charles and Lester (1984) identified the factors that affect the problem-solving skills of the individual in three groups as cognitive, affective and experience factors. Cognitive factors include knowledge of mathematical concepts, logical thinking and reasoning, spatial reasoning in some problems, memory, computational ability, and estimation.

Factors such as willingness to solve problems, self-confidence, stress and anxiety, uncertainty, patience and perseverance, interest in problem solving or problem situations, motivation, desire to show success, desire to satisfy teacher constitute the group of affective factors. Experience factors include encountering problems in certain subjects, pre-use of certain problem-solving strategies. Proportional reasoning problems are one of the most common types of problems in which structural similarity forms are observed among the factors affecting the problem-solving success mentioned above (Küpçü & Özdemir, 2012). Most of the fields of mathematics or science are related to basic but deep concepts. In order to solve mathematics, science and daily life problems, it is often necessary to reveal similar patterns in two different situations or to recognize structural similarities. Some different problem types have been developed in order to evaluate the proportionality and different proportional reasoning situations related to the important concepts that can be encountered in the second-grade mathematics curriculum. These are explored in the following section.

2.5.The Types of the Proportional Reasoning Problems

In the literature, many problem types are identified in order to discover the proportional reasoning skills of students. Lamon (1993) describe four semantic problem types by investigating the problem situations typically structured. These problem types and their examples posed by Langrall and Swafford (2000) are below. The type of *Well-Chunked Measures* includes comparison of two extensive resulting in an intensive measure or rate such as speed, unit price. The following is an example of this type of problem: “Dr. Day drove 156 miles and used 6 gallons of gasoline. At this rate, can he drive 561 miles on a full tank of 21 gallons of gasoline?”. The second type is *Part-Part-Whole.* In this problem type, a subset of a whole is compared with its complement (e.g. boys with girls in a class) or with the whole itself (e.g. boys with all students in a class). For example, “Mrs. Jones put her students into groups of 5. Each group had 3 girls. If she has 25 students, how many girls and how many boys does she have in her class?”. The third type is *Associated Sets.* In this problem type, occasionally the relationship between two quantities is unknown unless their relationship is identified in the problem. That is, the connection of two sets cannot be known (e.g., people and pizzas, children and chocolate bars) until some expressions in the problem show that rate pairs should be formed. The following is an example of this problem type: “Ellen, Jim and Steve bought 3 helium-filled balloons and paid $2 for all 3 balloons. They decided to go to the store and buy enough balloons for everyone in the class. How much did pay for 24 balloons?”. The last type of the proportional reasoning problems according to Lamon (2007) is *Stretchers and Shrinkers.* These problems highlight the relationship between continuous quantities, such as circumference, length or height. They involve enlarging or stretching and reducing or shrinking. For example, “A 6"x8" photograph was enlarged so that the width changed from 8"x12". What is the height of the new photograph?”

On the other hand, Haller, Ahlgren, Post, Behr and Lesh (1989) describe four different types of proportional reasoning problems. In the *Missing-value problems,* one of the quantities proportionally related to each other is not given and students are required to find this quantity. The following problem is a typical example of missing value problem: “Steve and Mark were running equally fast around a track. It took Steve 20 minutes to run 4 laps. How long did it take Mark to run 12 laps?”. In the *Numerical comparison* problems, all four quantities are given, and students are required to compare the given ratios. For example, “Tom and Bob ran around a track after school. Tom ran 8 laps in 32 minutes. Bob ran 2 laps in 10 minutes. Who was the fastest runner? (a) Tom (b) Bob (c) They ran equally fast. (d) Not enough information to tell.” The other type is *Qualitative Ratio Change* problems which include any numerical comparison. To illustrate, “If Cathy ran less laps in more time than she did yesterday, her running speed would be (a) faster (b) slower (c) exactly the same (d) there is not enough information to tell”. The last type of the proportional reasoning problems according to Haller, Ahlgren, Post, Behr and Lesh (1989) is *Qualitative Comparison* problems. Like qualitative ratio change problems, this problem type includes any numerical comparison. The students are expected to comment on the proportional relationship in the problem situation. For example, “Bill ran the same number of laps as Greg. Bill ran for more time than

Greg. Who was the faster runner? (a) Bill. (b) Greg (c) They ran at exactly the same speed. (d) There is not enough information to tell.”

In addition, proportional problems are generally differentiated into their contexts (Lesh, Post & Behr, 1988). De La Cruz (2013) expresses that the solution strategies and achievement level of students are affected by the context of the problems. According to De La Cruz (2013), there are four common context of proportional reasoning problems: rates, similarity, mixture and part-part-whole. ‘A printing press takes exactly 12 minutes to print 14 dictionaries. How many dictionaries can it print in 30 minutes?’ is an example of the problems including *rates.* To illustrate similarity problems, ‘You gave your grandmother a 4 in by 6 in picture but she would like to enlarge it to match the other photos hanging on her wall. If she enlarges the length from 6 in to 8 in, what would the width of the enlarged photo be?’ can be given. ‘If Suzie uses a lemonade recipe that calls for 1 cup of lemon juice for every 2 cups of water, how many cups of lemon juice would she need to make lemonade if she was using 8 cups of water?’ is an example of the *mixture* problems. ‘Ms Levi’s class has 12 girls and 18 boys. If there is the same ratio of girls to boys in the school as there is in Ms. Levi’s class and there are 360 children in the school, how many boys are there?’ is an example of the problems consisting of *part-part-whole.*

In addition to the contextual structure, proportion problems are often distinguished by the sorts of the multiplicative relationships within and between ratios in the problems (Bezuk, 1988, Steinthorsdottir & Sriraman, 2009). Steinthorsdottir and Sriraman (2009) state that a ‘within’ relationship is the multiplicative relationship between elements in the same ratio, whereas a ‘between’ relationship is the multiplicative relationship between the corresponding parts of different ratios’(p.7) (Figure1).

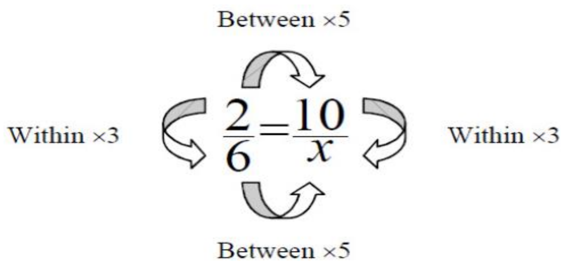


Figure 1 The within and between ratios

These multiplicative relationships can be integer and non-integer as described in Table 1.

Table 1 The examples of subcategories of missing value problems according to numerical structure.

Subcategories	**Examples**	**Numerical structure**
The factor of change between ratios is an integer.	If 10 pieces of gum costs 34 cents, how much will 5 pieces of gum cost?	$\frac{34c}{10\text{ pieces}}=\frac{?}{5\text{ pieces}}$ $\times 2$
The factor of change within ratios is an integer.	If 10 pieces of gum costs 50 cents, how much will 15 pieces of gum cost?	$\times 5$ $\frac{50c}{10\text{ pieces}}=\frac{?}{15\text{ pieces}}$
Both factors of change within and between ratios are integers.	If 10 pieces of gum costs 50 cents, how much will 5 pieces of gum cost?	$\times 5$ $\frac{50c}{10\text{ pieces}}=\frac{?}{5\text{ pieces}}$ $\times 2$
Neither factor of change is an integer.	If 10 pieces of gum costs 34 cents, how much will 15 pieces of gum cost?	$\frac{34c}{10\text{ pieces}}=\frac{?}{15\text{ pieces}}$

When the literature was reviewed, it was seen that different types of problems created different forms, and students had some difficulties with these different types of problems. These difficulties included not knowing the situations in which proportional reasoning will be used, not being able to distinguish the difficulties in determining multiplicative or relative relations, using unnecessary multiplicative approaches (Walle et. al. 2012), ignoring some data, associating multiplicities with qualitative strategies rather than applying quantitative strategies, and using unnecessary cross-product algorithms (Lesh et. al., 1988).

The study of Fernandez et al. (2011) was about the effects numerical structure of the proportional reasoning problems. They found that the magnitude of the numbers in the problems and the numerical relationship between the quantities greatly affected how the students solved the problems. Number structure characteristics are highly effective in solving students' proportional reasoning problems, and the effect of integer and non-integer factor of change has been shown in some studies. Riehl and Steinthorsdottir (2017) found that students were more successful in solving problems where the factor of change was an integer than non-integer. In addition, they showed that when problems included only one integer ratio, students were more successful when factor of change within ratios was an integer.

Many researchers found that when the missing value problems consist of integer ratios, students showed better performances and they unnecessarily used the proportional solution methods in the non-proportional problem types. On the contrary, when the problems have non-integer ratios, the students were extremely inclined to use additive solution methods in both additive and proportional problems. For example, Dooren, Bock and Verschaffel (2010) conducted a study with 325 fourth, fifth and sixth grade students in order to investigate the usage of additive methods in proportional problems and the usage of multiplicative methods in additive problems. While students applied more multiplicative strategies when numbers in the problems generated integer ratios, they used additive strategies when the numbers did not generate integer ratios. As a result, the students were more likely to look at the numerical characteristics of the problem than the additive or proportional situation of the problem in order to decide whether the solution method of the problem should be additive or proportional. Moreover, Jiang et. al. (2016) investigated the performance of both Spanish and Chinese students in additive proportional problems from a cross-sectional perspective. One of the considerations of the study was the effect of number structure in the problem sentences. 925 fifth to eight grade students did a test which included addition and proportional problems in missing value format. The study concluded that both Chinese and Spanish students tended to use proportional methods when the problem had integer ratio, regardless of the problem type.

Özgün-Koca and Kayhan-Altay (2009) conducted a study to 6^{th} and 7^{th} grade students in order to investigate the proportional reasoning skills in terms of these students' gender, their grade levels and problem types- missing value and numerical comparison. According to the results, students were more successful in solving the missing value problems than numerical comparison problems. Additionally, 7^{th} graders were more successful than 6^{th} graders.

Tjioe and Torre (2014) conducted a study in order to investigate the abilities of students from two different middle schools to realize proportionality. The first group referred to as low-proficiency group consisted of 242 8^{th} grade students in a state secondary school with a 46% success rate according to the state evaluation conducted in 2010. The second group referred to as high-proficiency group consisted of 151 8^{th} grade students in a state secondary school with 89% success rate according to the state evaluation conducted in 2010. The test instrument had two typical missing value problems and two recognizing proportionality problems with four alternatives. In these recognizing proportionality problems, the students were asked to decide whether they could use to a proportion in order to solve these given problems (one with a proportional relationship and the other with an additive relationship). With respect to student performance in the missing value problems, it was clear that there was a significant difference between the two groups: the students in the high-proficiency group performed much better than the students in the low-proficiency group. The students in the high-proficiency group had 94.0% and 93.4% success rate in the first and second missing value problems, while the students in the low-proficiency group had 86.0% and 73.1% success rate in the same problems. In both the low- and high-proficiency group, the majority of the students who answered the missing value problems incorrectly chose the alternatives that were the result of unproper usage of additive reasoning. This meant that students whose proportional thinking ability was not sufficiently developed tended to use additive strategies. On the other hand, in the recognizing proportionality problems, there was no meaningful difference between high and low-proficiency group. The success rate of the groups in these problems ranged from 30% to 45%. The study showed that to be able solve missing value problems did not mean being able to reason proportionally. It was concluded that teaching missing value problems to solve is not the same as teaching to reason proportionality.

Artut and Pelen (2015) explored the strategies used by 165 sixth grade students in the proportional and non-proportional problems and whether these strategies vary by types and number structure of the problems. The test instrument had 12 open-ended questions consisting of missing value, comparison and non-proportional problems. The findings showed that the mostly used strategy was factor of change strategy in missing value and comparison problems and multiplicative strategy in non-proportional problems. It was revealed that the students used additive strategy in the proportional problems and additive strategies in the non-proportional problems. This showed that the students had difficulty in distinguishing between proportional and non-proportional problem situations. Additionally, while students mostly used the factor of change strategies in the within, between and both within and between integer relation problems, they mostly used the unit rate strategy in the within non-integer relation problems. However, the students were inclined to use multiplicative strategies in the problems where the ratio was an integer and to use additive strategies in cases where the ratio was not an integer regardless of whether the problem was proportional or non-proportional. The study concluded that number structure of the problems affected the strategies used by the students in the problems.

Heller at al. (1989) investigated the effect of the types of the ratio and structure of the problems on the achievement of the seventh-grade students by using qualitative and quantitative proportional reasoning test. The study concluded that the types of the ratio had an important effect on the difficulty of proportional and qualitative reasoning.

2.6.The Strategies Used in Proportional Reasoning Problems

In the literature, different solution strategies are identified in order to specify the proportional reasoning skills of the students. Cramer and Post (1993) found that the students use four different strategies to solve proportional reasoning problems. These are unit rate strategy, factor of change strategy, equivalent fractions strategy and cross-product algorithm. In addition to Cramer and Post, Bart, Post, Behr, and Lesh (1994) proposed a micro-theory to analyze the problems including reasoning proportionally and to reveal students' cognitive processes and their errors. In the studies conducted as a part of the Rational Number Project, Bart et al. revealed the equivalence class strategy. Ben-Chaim, Fey, Fitzgerald, M., Benedetto and Miller (1998) and Parker (1999) added the build-up strategy to other strategies as a result of their research. Each strategy will be described by using it to solve the following problem:

> In a bookstore, if 4 books of the same kind cost 40 dollars, then find the total price of 12 books.

The unit rate strategy includes computing one unit of a quantity and then multiplying the result with another quantity to generate the wanted answer. In this problem, each book costs 40:4=10 dollars and 8 books cost 8x10=80 dollars.

The factor of change strategy includes comparing the quantities, deciding the factor of change between two quantities and multiplying the factor with the value of given quantity. In this problem, 12 books are 3 times as many as 4 books. Therefore, the answer is 3x40 dollars = 120 dollars.

The equivalent fractions strategy perceives the ratios in the problem as equivalent fractions. The students try to find an equivalence fraction to the given ratio. In this problem, $\frac{4}{40} = \frac{12}{?}$ $\quad \frac{4x3}{40x3} = \frac{12}{120}$

Therefore, 12 books cost 120 dollars.

The cross-product algorithm consists of establishing a proportion, making a cross product and solving the resulting equation by division.

4 books 40 dollars

12 books A dollars

4xA=12x40 A= $\frac{12x40}{4}$ =120

The equivalence class strategy includes generating equivalence fractions until finding the wanted fraction.

$\frac{4}{40} \equiv \frac{8}{80} \equiv \frac{12}{120}$

The build-up strategy consists of establishing a relationship within a ratio and then extending it to the second ratio by addition. It is also mentioned as the *addition and scaling* (Hart, 1988) strategy because of involving both a multiplicative and additive strategy.

4 books 40 dollars
8 books 80 dollars
12 books 120 dollars

When the literature was examined, it was seen that some studies have been done in order to find the strategies used by middle school students in the problems of proportional reasoning. Kahraman, Kul and İskenderoğlu (2018) analyzed the strategies used by 28 seventh grade students who had not learned the subject of proportion yet and 28 eighth grade students who had learned this subject in the previous academic year in the ten open-ended qualitative proportional reasoning problems. When the solutions are examined, it is seen that eighth-grade students used more solution strategies than the seventh-grade students. The strategy mostly used by seventh grade students was the unit rate strategy, and the strategy mostly used by the eighth-grade students was cross-multiplication. The most common erroneous solutions were solutions using additive relationship. Students who could not realize the multiplicative relation between quantities or could not realize it turned to additive relationship by changing the solution strategy. Moreover, the number of erroneous solution strategies decreases as the grade level increases. The researchers stated that as the grade level increased, the students' experience with proportional reasoning could be influential in the development of students' skills related to proportional reasoning. It can be said that students' proportional reasoning skills improved over time, and erroneous solution strategies decreased. In this case, the fact that the 8^{th} graders had learned the subject of proportion might have an effect. Similarly, Küpçü (2008) concluded that the eighth-grade students were more advanced in their proportional reasoning skills than 7^{th} grade students because of their age to encounter proportional situations. Küpçü (2008) conducted a study with 134 seventh and eighth grade students in order to find the effect of the activity-based teaching on the success of the solving of the proportional reasoning problems. One of the results of the study was that these students used cross-product algorithm for missing value problems and unit ratio strategy for quantitative comparison problems. However, Kayhan (2005) reached to a different result in his study conducted with 143 sixth and seventh grade students in order to investigate their solution strategies in proportional reasoning problems. According to the analysis, the students used fifteen different strategies mostly preferring the unit ratio strategy in the problems. The reasons for this were indicated as prior knowledge and personal preferences of the students and structure of the problems. In addition, the strategies preferred by the students changed based on the problem types.

Duatepe, Akkuş-Çıkla and Kayhan (2005) applied a proportional reasoning test to 295 middle school students in order to determine the strategies used by the students and how these strategies vary by problem types. The study concluded that the students used mostly the cross-product algorithm in the missing value problems, the unit ratio strategy in the numerical comparisons problems and giving clues about reasoning proportionally without any strategy in the qualitative comparison problems.

Avcu and Doğan (2014) and Avcu and Avcu (2010) investigated the solution strategies of 278 seventh grade students in the proportional reasoning problems. They revealed that the students mostly used cross product algorithm and factor of change strategy. Aladağ (2009), who conducted a study similar to the one above, applied a test to 590 6^{th}, 7^{th} and 8^{th} grade level students. He found that the sixth-grade students did not use a specific strategy, while seventh grade students mostly used the cross-product algorithm and eighth grade students mostly used the unit rate algorithm in solving the proportional reasoning problems. The reason why the sixth-grade students did not use a specific strategy may be that the subject of the strategies used in proportional reasoning belongs to the seventh-grade curriculum. Therefore, the seventh-grade students might have used mostly the cross-product algorithm. This finding is compatible with the above-mentioned studies' results. The eighth-grade students might have used unit ratio in order to solve the problem practically and find the result quickly.

Incebacak and Ersoy (2016) conducted a study in order to reveal the reasoning skills of the students. For this purpose, a total of 94 students in a middle school were asked to solve two real life problems prepared to reveal their high-level thoughts. It was seen that more than half of the students used proportional reasoning for both problems while solving problems. When the solution strategies used in general were examined, it was observed that students used different solution strategies for different types of problems, but it was seen that the use of cross-product algorithm was common. When students encounter problems with fractional expressions, they prefer to make a comparison between the numbers and use directly the cross-product algorithm. Slovin (2000) argues that the reason why this solution strategy is the first reference strategy is the context used in the proportional reasoning problems. While the students were expected to solve the question with the logic of this algorithm except proportional reasoning, the rate of reaching the correct solution was low due to the problems they were not familiar with. In order to improve students' proportional reasoning skills, the context in the problems should be different from the traditional approach and suitable for the use of different strategies (Duatepe et al, 2005).

The conceptual dimension of ratio and proportion bridges advanced mathematical thinking (Lesh, Post and Behr 1988). Since teachers who have flexible thinking paths about proportional reasoning and who have developed a wide variety of demonstrations will help students to develop their proportional reasoning skills (Parker,1999), it is important to determine the proportional reasoning levels of teacher candidates and determine the level of their thoughts on the subject. Based on this, Akkuş-Çıkla and Duatepe (2002) examined 12 first grade preservice teachers' reasoning on ratio and proportion problems. The aim of the study was to investigate their proportional reasoning skills and strategies used by them in the problems. As a consequence of the study, it was observed that the students who solved ratio-proportion problems using the cross-product method did not respond to the questions of conceptual knowledge adequately and correctly, and did not use a definite and correct language in this concept. Despite the fact that the ability to use the cross-product algorithm at the proportional reasoning levels defined by Langrall and Swanford (2000) is considered to be the highest achievable level (level 3) for proportional reasoning, it was considered that using the method of cross-product algorithm was acceptable in level 2 behavior at the end of this study. Therefore, existing markers for level 3 are added into level 2. In addition to demonstrating the behaviors of level 2, the behavior of definite and correct language usage which demonstrates that conceptual information is intact should be expected at level 3. While teacher candidates were able to consider quantitatively the proportional situations which were necessary to reach level 3, none of them used a definite and correct language. As a result, it was observed that pre-service teachers showed the operational skills required by the questions but did not have the conceptual knowledge required for the same question. Without the conceptual knowledge, the correct way the students do the operations is an indication that they are processing by heart. The reasons for this are the way of lecturing about the ratio and proportion in the current mathematics textbooks, the problem types and the problem solutions which require only the cross-product algorithm based on memorization.

On the other hand, the study by Arican (2016) ended in the opposite way. Arican (2016) investigated the strategies used by preservice middle and high school mathematics teachers in order to solve single and multiple proportion problems formed by three quantities and difficulties and conveniences of the preservice teachers in solving these proportional problems. Nine real world missing value word problems were used in the study. During the analysis, ratio table, unit ratio and double number line strategies were classified within proportional reasoning category because the preservice teachers used their proportional reasoning without using any proportion formula in these strategies. The strategies that the preservice teachers used were called the proportion formula strategy. Based on the responses of the preservice teachers, the ratio table strategy was the most frequently used and the most appropriate strategy for solving single and multiple ratio problems. It was showed that using the ratio table strategy helped preservice teachers realize the constant ratio and product relationships between quantities. The second strategy mostly used by the preservice teachers was the proportion formula strategy. In this strategy, preservice teachers created a direct or inverse ratio indicating the equality of the two ratios, and then calculated the missing value by cross-multiplication or multiplication (or division) within or between ratios. The preservice teachers' most common mistake in using this strategy was to establish a direct proportion to solve problems with inverse proportions. Especially, in multiple proportion problems, it was not easy to form a proportion formula or to use cross-multiplication in order to solve these problems, because multiple proportion problems had three quantities. Therefore, the results of this study exemplified how preservice teachers could reason about proportional relationships when they could not use calculation methods such as cross-multiplication.

These results show that the concept of ratio and proportion in Turkish schools heavily depends on the use of the cross-multiplication method. In addition, as evident in these studies, sixth, seventh and eighth level students and preservice teachers also have some hardships on proportional reasoning. As the concepts of ratio and proportion develop in middle grades, improving instruction in middle grades is essential (Sowder et al., 1998).

Dooley (2006) did a research with 107 high school students. The aim was to investigate their proportional reasoning abilities, explore students’ conceptual understanding of cross-multiplication and divide algorithm and evaluate the effect of the manipulatives on students thinking. Twenty-one students were interviewed. After the interviews, it was seen that only two of the 21 interviewees exhibited advanced proportional reasoning skills and nineteen of the interviewees could not use the cross-multiplication and division algorithm to solve proportional reasoning problems.

Cramer and Post (1993) investigated the strategies used by 913 seventh and eighth grade students in proportional reasoning problems within the Rational Number Project. The research concluded that the seventh-grade students used unit ratio strategy and the eighth-grade students used cross-multiplication algorithm mostly. Lewin-Beinberg (2002) specified the mistakes of the students rather than the solution strategies in the missing value problems consisting of proportional reasoning in a part of the fractions and division study. Christou and Philippou (2002) aimed to find informal understanding of the fourth and fifth grade students in solving proportional reasoning problems and to investigate how students' intuitions affect their strategies to solve proportional reasoning problems. According to the results, students intuitively used the unit rate strategy. Norton (2005) examined the effect of LEGO construction activities on the proportional reasoning skills of the 46 sixth grade students. The students had a 90 minutes lesson for the usage of LEGOs during ten weeks, and the pre and post-tests. An important difference was found between the pre and post-tests of the students. The reason for this difference was the usage of LEGO on the proportional reasoning, because the usage of LEGO enables to understand the relationship between part and whole.

Another example is that Pakmak (2014) investigated what kind of strategies the 106 sixth grade students used in the qualitative and quantitative proportional reasoning problems and how they used these strategies. After conducting the proportional reasoning test on the students, the lowest score that the students could get from the test was 0 and the highest score was defined as 56. The four level of proportional reasoning skills remaining in this score range is as follows: The range from 0 to 13 points is very low, the range from 14 to 27 points is low, the range from 28 to 41 points is medium and the range from 42 to 56 points is high. Accordingly, all the students were ranked from the highest score to the lowest score. The top 20 students who received the highest score in the ranking were designated as the study group. One of these 20 students was at high level, 13 students were at medium level and the others were at low level of proportional reasoning. There were no students at too low level. According to the findings, in spite of choosing the students with highest scores, the students' proportional reasoning levels were not at sufficient level. The other result of this study is that the most frequently used strategy in qualitative proportional reasoning problems was inverse ratio algorithm and the most commonly used strategy for quantitative proportional reasoning problems is the unit rate strategy. In the qualitative proportional reasoning questions of the study, the unit rate strategy, which is used in the form of conducting the related transactions on the numbers given in the quantitative proportional execution questions, was applied in the form of digitizing, symbolizing or drawing. The implementation of the unit rate strategy in the form of the methods developed by the student shows that this strategy is used with the correct interpretation of the relationship between variables, not by memorization.

CHAPTER 3

METHODOLOGY

This chapter will present information about the research design, the population and sample, the data collection instruments, validity and reliability, the data collection procedures, analysis of the data, assumptions and limitations, and the internal and external validity of the study.

3.1.Research Design

The aims of this study were to specify students' academic achievement in proportional reasoning problems, to determine the proportional reasoning levels of students and the relationship between academic achievements and levels of the students and to examine their solution strategies in these problems. Therefore, answers to the following research questions were investigated in this study:

1. Does the academic achievement of the students change from 5^{th} to 8^{th} grade in the test of proportional reasoning problems?
    - Does the academic achievement of these students change according to problem types?
2. What kind of strategies are mostly used by the students between 5^{th} and 8^{th} grade in proportional reasoning problems?

In this study, quantitative methodology was used to address the first research question, which investigates the academic achievement of the students from 5^{th} to 8^{th} grade in the test of proportional reasoning problems. Students' achievement scores were formed by scoring their solutions for each problem between 0 and 3. The mean scores of students' overall achievement scores were compared on the basis of grade levels. For each problem, the distribution of students across scores was calculated. Moreover, this distribution was expressed in percentages based on grade levels and points between 0 and 3 in a table using the SPSS program. In addition, the success of the students in problem types according to their scores from each problem in the tests was compared on the basis of grade levels. Moreover, qualitative methodology was used to answer the second research question of the study about the strategies mostly used in proportional reasoning problems. The solution strategies used by the students in each problem were examined in detail. Additionally, how the solution strategies used by students in each problem change according to the grade level was analyzed. Therefore, a mixed method research with both quantitative and qualitative methodology was carried out to address the two research questions. Mixed method research is defined as the researcher's combination of qualitative and quantitative methods, approaches and concepts in a study or consecutive studies (Creswell, 2003). Moreover, Creswell (2006) states that using quantitative and qualitative approaches together in a mixed approach leads to a better understanding of research problems than using both approaches separately.

3.2.Population and Sample

The target population of this study is fifth, sixth, seventh and eighth grade students in the public schools of Ankara. Since access to the entire target population is not possible, the accessible population is composed of fifth, sixth, seventh and eighth grade students at a public school in Mamak District of Ankara.

Table 2 The number of participants

		Male		Female		Total Frequency
		Frequency	Percent	Frequency	Percent	
	Grade 5	118	27,4	137	32,0	255
	Grade 6	116	27,0	93	21,7	209
	Grade 7	125	29,1	131	30,6	256
	Grade 8	71	16,5	67	15,7	138
	Total	430	100,0	428	100,0	858

As can be seen from the Table 2, the number of participants of the study was 858. The number of male participants was 430 (50.1%), and the number of the female participants was 428 (49.9%). In addition, 255 participants were at grade 5 (29.7%), 209 participants were at grade 6 (24.4%), 256 participants were at grade 7(29.8%), and 138 participants were at grade 8 (16.1%).

The convenience sampling means working with a group of individuals who are conveniently ready to work (Frankel, Wallen & Hyun, 2006). In view of this fact, convenience sampling method was used for the subjects of the quantitative part of the study because the researcher is a mathematics teacher at this school. Therefore, there was no problem in obtaining permission from the school administration, and the teachers and the students willingly participated in the study. For the qualitative part of the study, purposive sampling was used. In qualitative research, the number of participants in a sample usually ranges from 1 to 20 (Frankel, Wallen & Hyun, 2006). For this reason, it was decided to choose 20 students. The students were ranked from the highest to the lowest in the Excel program according to their achievement scores and the first 20 students from each grade level with the highest score were selected, because it was thought that the students who got the best scores from the achievement test may provide richer data in terms of the range of solution strategies.

3.3.Data Collection Instruments

3.3.1.Achievement test

A test was prepared to determine proportional reasoning achievement and solution strategies of the participants. This test was called the Proportional Reasoning Test (PRT), which included 20 problems related to proportional reasoning. The problems in the test were totally different and were independently built from each other. The 1st and the 11th problems, the 2nd and the 12th problems, the 3rd and the 13th problems, etc. had the same content because the test was created in split half form in order to ensure reliability. Most of the problems were adapted from the available literature and some of them were constructed by the researcher. The

8^{th}, 9^{th}, 10^{th}, 15^{th}, 17^{th}, 18^{th}, and 20^{th} problems were multiple choice, and others were open-ended. In order to help students easily understand the questions, images were added to the questions as much as possible. The Turkish adaptation of the test were added to Appendix A. The original versions of the problems in the test and the changes made in order to adapt and translate into Turkish language are below. The 6^{th}, 7^{th}, 8^{th}, 9^{th}, 15^{th}, 17^{th} and 18^{th} problems were adapted from the study of Hilton et al. (2016). The 1^{st}, 2^{nd}, 3^{rd}, 5^{th} and 14^{th} problems were adapted from the study of Misailidou and Williams (2003).

The first problem in Figure 2 was a type of missing-value problem. In this problem, the factor of change across ratios is an integer. Because this problem was suitable to solve for all grade level students, no changes were made while translating it into Turkish except for the currency.

At a fruit stand, 3 apples cost 90 pence. You want to buy 7 apples. How much will they cost?

Figure 2 The original version of the first problem

The second problem in Figure 3 was a missing-value problem. The factor of change within the given ratio is an integer. While translating it into Turkish, only the proper names were adapted into Turkish and the currency was changed.

A printing press takes exactly 12 min to print 14 dictionaries. How many dictionaries can it print in 30 min?

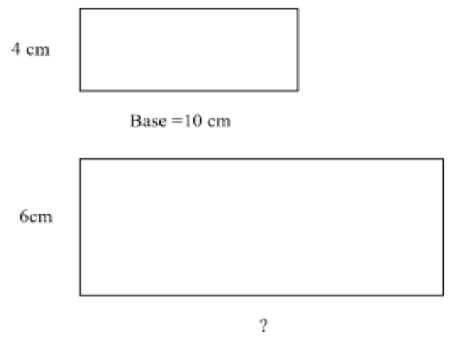
Figure 3 The original version of the second problem

There is a sale at a bookstore. Every book in this sale costs exactly the same. Mary bought 6 books from the sale and paid 4 pounds. Rosy bought 24 books from the sale. How much did Rosy pay?

Figure 4 The original version of third problem

The third problem in Figure 4 was a missing-value problem. Neither factor of change is an integer. This problem was translated into Turkish without any changes.

The fifth problem in Figure 5 was a missing-value problem. When this problem was translated into Turkish, it meant that these two rectangles have exactly the same shape with all the features. During the pilot study, it was seen that the expression of the two rectangles being both the same and one larger than the other caused confusion. For this reason, this statement was translated into Turkish in a way that the two rectangles are similar to each other, but one is larger than the other.



These two rectangles have exactly the same shape, but one is larger than the other. What is the length of the base of the larger rectangle?

Figure 5 The original version of fifth problem

Sandra decided to save to buy an iPod costing \$84. To help her buy the iPod, Sandra's mother agreed to give her \$5 for every \$2 that Sandra saved. Sandra has saved \$24, so she has enough for the iPod.
True or False
Because (choose the best reason)
A. Now Sandra and her mother have more than they need for the iPod.
B. Now Sandra and her mother still don't have enough for the iPod.
C. Sandra's mother will give Sandra \$60.
D. As long as Sandra's mother pays more, it's OK.

Figure 6 The original version of the sixth problem

The sixth problem in Figure 6 was an example of the type of part-part-whole. While translating it into Turkish, only the proper names were adapted into Turkish and the currency was changed. The alternatives of the problem were removed, and then it was asked as an open-ended question.

The seventh problem in Figure 7 was a numerical comparison problem. In this problem, the proper names were adapted into Turkish and the currency was changed. Decimal numbers that might be a problem in reaching the correct result were replaced with natural numbers so that students did not have to struggle with decimal numbers: 20 instead of 2 and 16 instead of 1,6 were written. In addition, the prices of two chocolate packs in different brands, which are more familiar to students were asked to compare instead of Gatorade and Cran-raspberry juice.

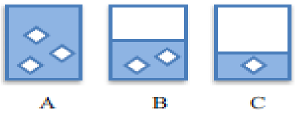
Max and Eliza bought supplies for snacks and reported the following expenses: Gatorade cost \$2.00 for 16 ounces. Cran-raspberry juice cost \$1.60 for 12 ounces. They bought Cran-raspberry juice. Did they make the most economical choice?

Figure 7 The original version of the seventh problem

The eighth problem in Figure 8 was related to the inverse proportion. There was no change in the problem statement except for the proper name, George. The D option was modified to prevent students from choosing option D even though the best option was to be selected. Therefore, the new option was that running faster does not affect the elapsed time.

George runs 100m in 20 seconds. If he runs the same distance at twice the speed, he will take twice as long. True or False **Because (choose the best reason)** A. Doubling the speed doubles the time. B. Doubling the speed halves the time. C. The distance doesn’t change. D. Running faster will take less time

Figure 8 The original version of the eighth problem



Three cups have different amounts of water and sugar. Cup A is full of water with 3 lumps of sugar. Cup B is half full of water with 2 lumps of sugar. Cup C is one third full of water with 1 lump of sugar. When the lumps of sugar have been stirred in, Cup B will be the sweetest.
True or False
Because (choose the best reason)
A. Cup A is the sweetest because it has the most sugar.
B. Cup C is the sweetest because it has the least water.
C. A full glass of B would need 4 lumps of sugar.
D. They are all the same sweetness.

Figure 9 The original version of the ninth problem

The ninth problem in Figure 9 was a numerical comparison problem. The problem was translated into Turkish without any changes.

The tenth problem in Figure 10 was adapted from the study of Bright et al. (2003). It was related to qualitative comparison. Firstly, the proper names were adapted into Turkish and centimeter was used as the unit of length measure. After the pilot study, it was seen that the students were unfamiliar with the phrase 'more square'. The answer of the most students was that the rectangle could not be a square because square was a square and rectangle was a rectangle. Therefore, the phrase ‘more similar to the square’ was used instead of the phrase ‘more square’.

Mrs. Allens took a 3 inch by 5-inch photo of the Cape Hateras Lighthouse and made an enlargement on a photocopier using the 200% option. Which is more square, the original photo or the enlargement?

A. The original photo is more square.
B. The enlargement is more square.
C. The photo and the enlargement are equally square.
D. There is not enough information to determine which is more square.

Figure 10 The original version of the tenth problem

The eleventh problem in Figure 11 was adapted from the study of Dole and Wright (n.d). The problem was translated into Turkish without any changes.

If 5 chocolates cost \$.75, how much do 13 cost?

Figure 11 The original version of the eleventh problem

The twelfth problem in Figure 12 was adapted from the study of Christou and Philippou (2002). This problem was a missing value problem. The factor of change within the given ratio is an integer. For this reason, for the students who want to find the unit rate, the numbers of 60 to 600, 20 to 200 and 9 to 12 were used in order not to make students struggle with decimal numbers.

George worked 9 weeks and earned £60. If he earns the same amount of money each week, how long does it take him to earn £20?

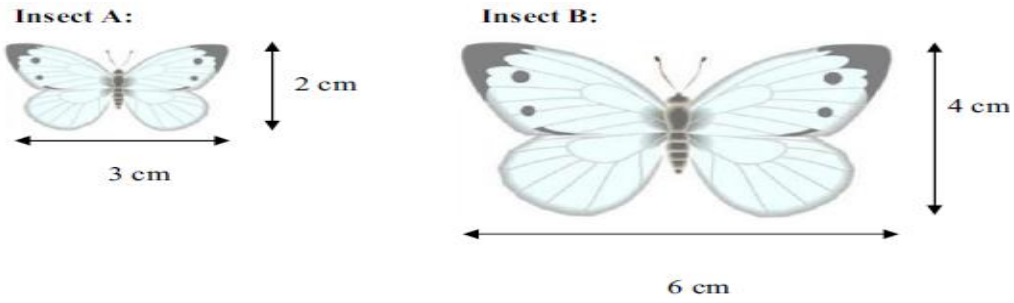
Figure 12 The original version of the twelfth

The fourteenth problem in Figure 13 was related to part-part-whole. No changes were made when translating it into Turkish.

Mrs. Green put her students into groups of 5, with 3 girls in each group. If Mrs. Green has 25 children in her class, how many boys and how many girls does she have?

Figure 13 The original version of the fourteenth problem

The fifteenth problem in Figure 14 was a stretchers and shrinkers problem. It was related to numerical comparison. During translation, no specific changes were made except adapting the proper name to Turkish.



Bill has drawn two diagrams. The area of insect B is twice that of insect A.
True or False
Because (choose the best reason)
A. The area of Insect B is 4 times greater.
B. Insect A is half the width of Insect B.
C. Insect B is twice as long as insect A.
D. Bill has only doubled one dimension

Figure 14 The original version of the fifteenth problem

The seventeenth problem in Figure 15 was related to numerical comparison. The problem was translated into Turkish as it was, because there was no situation in which the students would be forced or there would be confusion.

Washing powder A is the best value.
True or False
Because (choose the best reason)
A. Washing powder A costs the least.
B. Washing powder B costs a little bit more but you get 10 more loads of washing.
C. The cost per load of washing is less.
D. Both washing powders are the same value.

Figure 15 The original version of the seventeenth problem

The eighteenth problem in Figure 16 was related to inverse proportion. In this problem, all the numbers were changed so that the students could easily realize and find the ratios between the quantities. Accordingly, the numbers in the options of the problem were also changed. The D option was especially changed to prevent students from choosing the D option even though the best option was to be selected.

Six people can paint my fence in 3 days. If all people paint at the same rate, it would take 12 people to paint the fence in 2 days.
True or False
Because (choose the best reason)
A. The number of people multiplied by the number of days must stay the same, so you need 9 people.
B. If you decrease the time by 1 day, you must increase the people by 1, so you need 7 people.
C. If you decrease the time by 1 day, you must decrease the people by 2, so you need 4 people.
D. There is less time so more people are needed

Figure 16 The original version of the eighteenth problem

The twentieth problem in Figure 17 was adapted from the study of Bright et al. (2003). This problem was a stretchers and shrinkers problem. After the pilot study, it was seen that some students chose the A option because four was four times of one. To prevent students from choosing option A, the option was changed as '37 feet by 40 feet'.

The Science Club has four separate rectangular plots for experiments. With plants:
Which rectangle is most square?
A. 1 foot by 4 feet
B. 17 feet by 20 feet
C. 7 feet by 10 feet
D. 27 feet by 30 feet

Figure 17 The original version of the twentieth problem

In addition, the types of problems according to contextual and numerical in PRT are shown in Table 3.

Table 3 The types of problems in the PRT

Problems		According to contextual structure	According to numerical structure
1	Missing value	Rate	The factor of change between ratios is an integer.
2	Missing value	Rate	The factor of change within ratios is an integer.
3	Missing value	Rate	Neither factor of change is an integer.
4	Missing value	Part-part whole	The factor of change within ratios is an integer.
5	Missing value	Similarity	Both factors of change are non-integers.
6	Missing value	Part-part whole	The factor of change within ratios is an integer.
7	Numerical comparison		
8	Numerical comparison (Inverse ratio)		
9	Numerical comparison	Mixture	
10	Numerical comparison	Similarity	
11	Missing value	Rate	The factor of change between ratios is an integer.
12	Missing value	Rate	Both factors of change within and between ratios are integers.
13	Missing value	Rate	Neither factor of change is an integer.
14	Missing value	Part-part whole	The factor of change within ratios is an integer.
15	Numerical comparison	Similarity	
16	Missing value	Part-part whole	The factor of change within ratios is an integer.
17	Numerical comparison		
18	Numerical comparison (Inverse ratio)		
19	Numerical comparison	Mixture	
20	Numerical comparison		

3.4.Validity and Reliability

In recent years, validity is defined as ‘referring to the appropriateness, correctness, meaningfulness, and usefulness of the specific inferences researchers make based on the data they collect’ (Fraenkel, Wallen & Hyun, 2006, p. 148). In other words, it is the degree to which a measurement tool can directly measure the property intended to measure without involving any other proper (Ercan & Kan, 2004). Validity depends on the amount and type of available evidence to strengthen the researchers' comments after collecting the data (Fraenkel, Wallen & Hyun, 2006). The content-related evidence which is one of these evidences refers to the content and format of the instrument (Fraenkel, Wallen & Hyun, 2006, p. 148). To obtain the content-related validity of this study, the test was given to two elementary mathematics teachers, one of whom has a doctorate degree and two research assistants who study for a doctorate degree. According to these experts, in general, the problems of the test were explicit, suitable for all middle school grade levels of the study and consistent with the national objectives. After expert opinions, the test was made ready for the pilot study with required changes. The pilot study was conducted in a smaller school in Mamak District of Ankara with similar conditions to the school where the original study was conducted. The test was conducted in one class of each grade which was selected according to convenient sampling. 18 students from fifth grade, 19 students from sixth grade, 16 students from seventh grade and 26 students from eight grade took part in the pilot study. The test was conduct in two consecutive days. The purpose of the pilot study was to examine the explicitness and understandability of the items of test, suitably of the test for all grade levels and to determine the application time. The test duration of 40 minutes was adequate for all students at all grades. While analyzing the solutions of the students in the pilot study, it was seen that some students selected more than one option in some of the multiple-choice test items, because an option other than the best option was more meaningful for students. Therefore, some options were revised so that students could choose only one. In addition, because the problems which consisted of long sentences were left unanswered, these problems were visualized by showing the numbers of problem sentences in the figures in order to prevent the problem being left unanswered due to the long sentence. Finally, because some students were correctly not able to solve the problems with decimal numbers in the solution due to the struggle with decimal numbers, the quantities in these problems were changed appropriately.

Reliability is referred to as "the consistency of the scores obtained" (Fraenkel, Wallen & Hyun, 2006, p. 154). In other words, reliability that is one of the characteristics that the scale should carry is an indicator of the stability of the values obtained from repeated measurements of an instrument under the same conditions (Öncü, 1994).

The relationship between the scores obtained by the same people at two different times from the same instrument or from the two different parts of the same instrument is expressed by the reliability coefficient (Fraenkel, Wallen & Hyun, 2006). There are ways more than one to obtain the reliability coefficient. In this study, the split-half method was used in order to ensure reliability. This method involves scoring two halves of a test separately for each person and then calculating a correlation coefficient for the two sets of scores. The coefficient indicates the degree to which the two halves of the test provide the same results and hence describes the internal consistency of the test (Fraenkel, Wallen & Hyun, 2006, p.156).

Two halves of the PRT were the first 10 problem and the last ten problems. The reliability coefficient is calculated using what is known as the *Spearman-Brown prophecy formula* (Fraenkel, Wallen & Hyun, 2006, p.156). According to the results in Table 4 and Table 5, the reliability coefficient was .87 in the pilot study and .77 in the actual study. Fraenkel and Wallen (2006) stated that if the reliability value was above .70, the relationship could be considered as relatively high in educational sciences. Therefore, both analyses in the pilot and actual studies indicated that scores were reliable.

Table 4 The reliability coefficients in the pilot study

Cronbach's Alpha	Part 1	Value	,848
		N of Items	10[a]
	Part 2	Value	,820
		N of Items	10[b]
	Total N of Items		20
Correlation Between Forms			,870
Spearman-Brown Coefficient	Equal Length		,930
	Unequal Length		,930
Guttman Split-Half Coefficient			,930

a. The items are: Problem1, Problem2, Problem3, Problem4, Problem5, Problem6, Problem7, Problem8, Problem9, Problem10.

b. The items are: Problem11, Problem12, Problem13, Problem14, Problem15, Problem16, Problem17, Problem18, Problem19, Problem20.

Table 5 The reliability coefficients in the actual study

Cronbach's Alpha	Part 1	Value	,797
		N of Items	10[a]
	Part 2	Value	,791
		N of Items	10[b]
	Total N of Items		20
Correlation Between Forms			,766
Spearman-Brown Coefficient	Equal Length		,867
	Unequal Length		,867
Guttman Split-Half Coefficient			,867

a. The items are: Problem1, Problem2, Problem3, Problem4, Problem5, Problem6, Problem7, Problem8, Problem9, Problem10.

b. The items are: Problem11, Problem12, Problem13, Problem14, Problem15, Problem16, Problem17, Problem18, Problem19, Problem20.

In addition to the reliability coefficient, Cronbach alpha coefficients were calculated in order to check the internal consistency of the test. The Cronbach alpha coefficient method developed by Cronbach (1951) is an internal consistency prediction method that is suitable to be used when the items cannot be scored as true-false and can be scored as 1-3, 1-4, 1-5. Because the items in the test of this study were scored as 0, 1, 2 and 3, the application of Cronbach alpha coefficient method was deemed appropriate. In the pilot study, the Cronbach alpha coefficient was .92. In the actual study, the Cronbach alpha coefficient was .88. Because these coefficients also were higher than .70, it could be said that the scores were reliable.

In order to be sure about the reliability of the content analysis, a mathematics teacher working in a different school was informed about the purpose and procedure of the study. He was asked whether he could analyze student papers according to the rubric and codes. The researcher and the co-coder scored 10% of the student papers independently according to the rubric and encoded the students' solution strategies according to the codes determined by the researcher earlier. This process continued until 95% agreement between the coding of the researcher and the co-coder was achieved.

3.5.Data Collection Procedures

The instruments of the study were prepared by receiving the expert opinions and making the necessary revisions at the end of the fall of 2019. The essential permissions to gather the data were taken from Middle East Technical University, Human Subjects Ethics Committee and then the Ministry of National Education for the actual study. The pilot study was conducted in order to obtain validity and reliability of the instruments at the end of the fall semester of 2018-2019. The actual study was conducted in the spring semester of 2018-2019. Before the actual study, the students were informed about the implementation of the test. The students were asked to write their solutions clearly and explain why they chose a specific option in multiple choice questions. In addition, they were asked not to write any solution for the questions they did not know about, and not to make random markings. The other teachers in the classrooms during the application of the test did not answer students' questions about the test. In addition, the PRT was performed at the same time for all students for 80 minutes during the first two course on the application day. The students were given the first 10 problems of proportional reasoning test in the first lesson and the last 10 problems in the second lesson in separate papers. The students were allowed to rest during the10-minute break between two courses.

3.6.Role of Researcher

The researcher was a mathematics teacher at the school where the study was conducted. She only lectured to 6th grade students, but this did not have any effect on the results of the study. Since the study was applied to all the classes at the same time, the researcher could be present in only one class. For this reason, the researcher gave all the students the necessary information about the purpose of the test and how the students should solve the problems in the test one day before the application. In addition, because neither the researcher nor the other teachers in the classrooms answered students' questions about the test during the application of the test, it could be considered that test environment was equal and objective for all the students.

3.7.Analysis of Data

The aims of this study were to specify how the academic achievement of the students from 5th to 8th grade is in the proportional reasoning problems, to determine how the academic achievements of these students change according to problem types and to examine their solution strategies in these problems. To achieve these aims, the test of the study were analyzed.

Quantitative methodology was used in order to specify the academic achievement level of the students from 5th to 8th grade in proportional reasoning problems and to determine how the academic achievements of these students change according to problem types. The solutions of the students in the problems of the test were graded between 0 and 3 using a rubric adapted from the study of Akkuş and Duatepe (2006) in Appendix B. For each student, these points were entered into computer by using the SPSS Statistics 22 program. The mean scores of students' overall achievement scores were compared according to the grade levels for the first aim of the study. For the second aim, the distribution of student numbers to scores was calculated for each problem. Moreover, this distribution was also expressed in percentages based on grade levels and points between 0 and 3 in a table. In addition, the success of the students in problem types according to their scores from each problem in the test were compared according to the grade levels.

In order to examine the strategies mostly used by the students in these problems in the test and to reveal how these strategies diversify from 5th to 8th grade, students' solutions were examined with content analysis, which is one of the qualitative research techniques. Content analysis is a technique that allows researchers to investigate participants' behaviors indirectly by analyzing their communication Frankel, Wallen & Hyun, 2006). Based on this, the solution strategies used by the students in each problem were analyzed. These strategies were classified by the strategies mentioned in the related literature. The strategies were coded from 1 to 6 as unit rate, factor of changes, equivalent fractions, equivalent class, build up and cross product respectively. Moreover, the additive method was coded as 7 and the misusage of the additive method was coded as 8. On the other hand, other inaccurate solutions of the students were not considered. The suitable code corresponding to each student's solution to each problem was entered into the computer via the SPSS Statistics 22 program. In this way, frequency and percentage of usage of strategies were calculated according to the problems and grade levels. The mostly used strategies in each grade level was determined. Moreover, in order to illustrate the different strategies that students from different grade levels used in each problem, pictures were added from the students' papers.

3.8.Assumptions and Limitations

In this section, some assumptions and limitations of the study are mentioned. First of all, it was assumed that achievement and strategies of students in proportional reasoning could be determined by the Proportional Reasoning Test, the test instrument of the current study. It was also assumed that the students solved the problems in the test honestly and correctly. In addition to these assumptions, the findings of this study were limited because the sample of the study was not randomly selected. Convenience sampling was used. In addition, the researcher of the study was the teacher of some students of the sample. The teacher prejudices might have affected the results of the study. Finally, the achievement and strategies of the students in proportional reasoning were limited to the problems in the PRT.

3.9.Internal and External Validity of the Study

Fraenkel and Wallen (2011) mentioned that “internal validity enables to observe differences on the dependent variable are directly related to the independent variable and not due to some other unintended variable” (p. 166). Relying on this definition, there were some possible threats to internal validity which were tried to be controlled for the current study. These were subject characteristics, mortality, location, instrumentation and data collector bias. The subject characteristic threat was eliminated for this study because the classes were randomly created by the school administration at the beginning of the semester, and also all the students in the school participated in the study. Additionally, the subjects of the current study were from nearly the same socioeconomic level. Mortality is the loss of some of the subjects of the study as the study progresses (Fraenkel & Wallen, 2011). Loss of the subjects was not a problem in the current study, because the test instrument was applied to the same students who were present at the school on the application date. Moreover, the location was not seen as a threat because the test instruments of the current study were applied to the students in their own classes during the semester. In addition, evidence may be lacking for the validity of scores from the instruments used in a study. The absence of such validity does not necessarily threaten internal validity (Fraenkel & Wallen, 2011). Instrumentation was not seen as a threat in the current study. Besides, the day before the application of the test instrument, all the necessary explanations about how to solve the problems in the PRT and how to state their solutions of the problems were made by the researcher to all the students. Additionally, the teachers who would be in the classroom during the application were told that they should not answer any questions from the students about the test. Therefore, data collector bias was prevented from becoming a threat.

The results of this study were limited to the sample of this study. The participants of the study were the fifth, sixth, seventh and eighth grade students at a public school in Ankara. Since convenience sample was used in this study, the participants did not represent a larger sample of populations related to external validity.

CHAPTER 4

RESULTS

The aims of this study were to specify the academic achievement of the students from 5th to 8th grade in the proportional reasoning problems, to determine how the academic achievements of these students change according to problem types and to examine their solution strategies in these problems. Based on these aims, the results of the study were clearly and briefly represented in this section under different titles.

4.1. The Academic Achievement of the Students in the Proportional Reasoning Problems

To specify the academic achievement of the students in the test, the solutions of the students in the problems of the test were graded between 0 and 3 using a rubric adapted from the study of Akkuş and Duatepe (2006) and then, the total scores of the students were created by adding up the points they received from each question. These total points were named as achievement scores. The maximum point which a student would get from the test was 60. The means of student achievement scores for each grade level are given in Table 6.

Table 6 The descriptive statistics of the means of students' achievement scores

	Grade 5	Grade 6	Grade 7	Grade 8	Total
Number	255	209	256	138	858
Mean	13,63	16,79	21,29	29,20	19,19
Median	12	15	21,50	30	18
Mode	8	1	23	39	8

Table 6 (continued).

Std. Deviation	9,27	11,40	11,32	13,64	12,37
Minimum	0	0	0	0	0
Maximum	40	47	56	55	56
Skewness	,550	,426	,150	-,252	,443
Kurtosis	-,573	-,792	-,494	-,699	-,526
Std. error of skewness	,153	,168	,152	-,252	,083
Std. error of kurtosis	,304	,335	,303	,410	,167

As can be seen in Table 6, the mean score of the fifth-grade students was 13,63, the mean score of the sixth-grade students was 16,79, the mean score of the seventh-grade students was 21,29 and the mean score of the eighth-grade students was 29,20 in PRT. The general average was 19,19 in PR Test. In general, it is seen that the mean scores of the students increased according to the grade level. On the other hand, the mean scores of the fifth and sixth grade students in the test were below the general average, while the mean scores of the seventh and eighth grade students in the test were above the overall average.

In addition, Independent Sample t Test was used in order to determine whether there was a statistically significant difference between the means of achievement scores of the students according to their grade levels. Initially, assumptions of this test were checked. For normality assumption, the values of skewness and kurtosis in the Table 6 were checked. Because the values of skewness, kurtosis and standard error of skewness and kurtosis were within the appropriate range for all grade levels, it was accepted that the distribution was normal. In addition, independence of observations assumption was checked by all the teachers in the classrooms during the application of the test instrument. It was ensured that the students solved the tests themselves without looking at each other's papers.

Table 7 Independent Sample t Test on the means of achievement of the students

Grade		Levene's Test for Equality of Variances		t-test for Equality of Means		
		F	Sig.	t	df	Sig. (2-tailed)
5-6	Equal variances assumed	14,829	,000	-3,290	462	,001
	Equal variances not assumed			-3,224	398,688	,001
5-7	Equal variances assumed	10,464	,001	-8,358	509	,000
	Equal variances not assumed			-8,361	490,736	,000
5-8	Equal variances assumed	28,764	,000	-13,390	391	,000
	Equal variances not assumed			-11,992	207,110	,000
6-7	Equal variances assumed	,333	,564	-4,246	463	,000
	Equal variances not assumed			-4,242	443,276	,000
6-8	Equal variances assumed	4,195	,041	-9,170	345	,000
	Equal variances not assumed			-8,842	256,603	,000
7-8	Equal variances assumed	6,473	,011	-6,154	392	,000
	Equal variances not assumed			-5,822	239,827	,000

According to the results of the Independent Samples t test in Table 7, the p-value was less than 0.05, whether or not the variances of the samples are equal. Therefore, the difference between the means of the students according to grade levels was statistically significant.

As a result of these analyses, it could be said that there was a statistically significant increase in the proportional reasoning achievement of the students when their grade level increased.

4.1.1.The academic achievement of the students in the missing value problems

In the Table 8, it was given the percentages of the students according to the points they got from each missing value problem.

Table 8 The percentages of the students according to the points which they got from missing value problems in PRT

Missing value problems		1	2	3	4	5	6	11	12	13	14	16
		Percent										
Grade 5	0 point	28,6	51,4	79,2	44,7	98,0	68,6	43,1	54,9	63,9	38,8	59,2
	1 point	21,2	21,2	7,8	41,6	1,2	3,5	20,4	12,5	7,1	5,5	36,9
	2 point	3,1	3,1	9,8	4,7	,0	11,0	4,7	1,6	18,8	,0	2,4
	3 point	47,1	24,3	3,1	9,0	,8	16,9	31,8	31,0	10,2	55,7	1,6
	Total	100	100	100	100	100	100	100	100	100	100	100
Grade 6	0 point	23,9	41,1	70,3	37,3	95,2	57,4	38,3	45,9	57,4	30,6	58,9
	1 point	25,8	19,6	5,3	37,8	2,4	2,9	16,3	12,9	8,1	7,7	29,2
	2 point	3,3	5,7	17,7	1,4	1,4	8,6	5,3	5,3	13,9	3,8	1,9
	3 point	46,9	33,5	6,7	23,4	1,0	31,1	40,2	35,9	20,6	57,9	10,0
	Total	100	100	100	100	100	100	100	100	100	100	100
Grade 7	0 point	15,2	37,5	56,6	29,7	86,3	47,7	21,9	27,0	34,4	17,2	66,4
	1 point	12,1	18,4	14,8	45,3	7,8	4,3	14,5	10,2	9,8	8,2	16,8
	2 point	5,1	1,6	14,1	2,3	,8	17,6	8,2	3,1	14,5	1,6	3,9
	3 point	67,6	42,6	14,5	22,7	5,1	30,5	55,5	59,8	41,4	73,0	12,9
	Total	100	100	100	100	100	100	100	100	100	100	100

Table 8 (continued).

Grade 8	0 point	8,0	19,6	31,2	23,2	58,7	25,4	18,1	15,9	22,5	19,6	61,6
	1 point	8,0	7,2	13,0	38,4	3,6	5,8	9,4	9,4	8,0	10,1	9,4
	2 point	2,2	2,9	6,5	1,4	,7	20,3	4,3	2,2	5,1	1,4	2,9
	3 point	81,9	70,3	49,3	37,0	37,0	48,6	68,1	72,5	64,5	68,8	26,1
	Total	100	100	100	100	100	100	100	100	100	100	100
All grades	0 point	20,2	39,6	62,6	35,0	87,5	52,7	31,6	38,1	46,9	27,3	61,7
	1 point	17,5	17,7	10,1	41,3	3,8	4,0	15,9	11,4	8,3	7,6	24,6
	2 point	3,6	3,3	12,5	2,7	,7	13,9	5,8	3,0	14,1	1,6	2,8
	3 point	58,7	39,4	14,8	21,1	7,9	29,5	46,7	47,4	30,8	63,5	11,0
	Total	100	100	100	100	100	100	100	100	100	100	100

In the 1st problem, 3 apples were given 90 cents, and the students were asked to calculate how many cents would be 7 apples. It was a missing value problem with the context of rate, and the factor of change between ratios was an integer. The majority of the 5th (47.1%), 6th (46.9%), 7th (67.6%) and 8th (81.9%) grade level students got 3 points from the 1st problem. It is seen that the percentages of the 5th, 6th, 7th and 8th grade level students in the 1st problem were the highest. The reason for this might be that the solution to the 1st problem involved the unit rate strategy that the students were used to using in an internal way, and that most students could easily use this strategy. Moreover, the students could use the factor of change strategy in this problem, because the factor of change between ratios is an integer. On the other hand, it was seen that the percentage of the 5th (21.2%) and 6th (25.8%) grade students who got 1 point could not be underestimated. When the solutions of these students in the 1st problem were examined in general, it was seen that the solutions had clues about these students having proportional reasoning. For example, the students who considered the price of an apple as 90 Kr found the price of 7 apples multiplying 7 to 90 Kr or the students found the unit price of apple, but then they made irrelevant operations.

In the 2nd problem, the price of 24 books in a bookstore where 6 books were 4 TL was asked. The 2nd problem was a missing value problem with the context of rate, and the factor of change within ratios is an integer. 24.3% of the 5th grade, 33.5% of the 6th grade, 42.6% of the 7th grade and 70.3% of the 8th grade level students got 3 points from this problem. It was seen that the percentages of the students who got full point from the 2nd problem decreased compared to the 1st problem. It might be concluded that the unit price of a book was non-integer, and therefore, some students had difficulty in calculating the unit price. In order to find the unit price, these students tried to divide 6 into 4 instead of dividing 4 TL to 6 books. At this point, the students might have accepted that 4 books had a price of 6 TL by considering that a small number could not be divided into a large number. As in the 1st problem, the percentages of the 5th (21.2%), 6th (19.6%) and also 7th (18.4%) grade students who got 1 point were not low. The students who got 1 point had clues about reasoning in their solutions. The students who considered the price of a book as 4 TL found the price of 24 books by multiplying 24 to 4 TL. In addition, some students correctly or incorrectly calculated the factor of change within 6 books and 24 books, but then they solved the problem inconsequentially.

In the 3rd problem, how many dictionaries a printer which could print 14 dictionaries in 12 minutes could print in 30 minutes was asked. This problem was a missing value problem with the context of rate, and neither factor of change was an integer. It was observed that the students had difficulty in solving this problem compared to the first two problems. The percentages of the 5th (3.1%), 6th (6.7%) and 7th (14.5%) grade students who got 3 points from the 3rd problem were too low and almost half of the 8th (49.3%) grade students got 3 points. In this case, more than half of the 5th (79.2%), 6th (70.3%) and 7th (56.6%) grade students could not solve this problem correctly. The main reason for this was that the students could not use unit rate or factor of change strategy, because the factor of change between and within ratios was not an integer. Therefore, the students were unable to calculate how many dictionaries could be written in a minute or in how many minutes a dictionary could be written. It was seen that most of the students who could solve the problem correctly used the build-up strategy.

In the 4th problem, it was given that 30 students could be divided into groups of 6 students as there would be 4 boys in each group. The students were asked how many of 30 students were girls. This was a missing value problem consisting of the context of part-part-whole, and the factor of change within ratios was an integer. Only 9% of the 5th grade students, about 20% of the 6th, 7th grade students, and 37% of the 8th grade students could solve this problem correctly. These percentages were lower than those in the first two problems although this problem was a missing value problem and the factor of change was an integer as in the first two problems. However, the context of the 4th problem was different because it consisted of part-part-whole. The reason for low percentages of the students got full point might be that most of the students did not realize that the number of people in a group would be 6 and they thought that 6 was the number of groups. Therefore, they calculated the number of boys as 24 by multiplying 4 and 6 and found the number of the girls as 6 by subtracting from 30 students to 24 boys. If it was the way they thought, they would have reached the right solution. For this reason, 41.6% of 5th, 38.8% of 6th, 45.3% of 7th and 38.4% of 8th grade students got 1 point from the 4th problem.

In the 5th problem, a rectangle whose short side was 4 cm in length and tall side was 10 cm in length was given and the students were asked how many centimeters the tall side of another rectangle which was similar to the first rectangle and whose short side was 6 cm in length was. This was also a missing value problem and the factors of change within and between ratios were not an integer. Additionally, the 5th problem had the context of similarity which was accepted as one of the most difficult contexts for students. Both because of the fact that factor of change was not an integer and the context was hard, the majority of the 5th (98%), 6th (95.2%) and 7th (86.3%) grade students could not solve the problem and they got 0 point. The number of 5th, 6th and 7th grade students who could solve the problem correctly was almost none. On the other hand, more than half of the 8th grade students (58.7%) could not solve the problem, but 37% of them got the full score from the problem. When the students' solutions were examined in general, it was seen that the students could not notice the multiplicative relationship between the sides of two similar rectangles. For this reason, most of the students thought that the difference between the tall sides of the rectangles should be 2 cm because of the fact that difference between the short sides was 2 cm. Then, they found the tall side of the second rectangle as 12 cm. This showed that the students had a fully additive reasoning in this problem.

In the 6th problem, it was asked whether Emre, who wanted to buy a music player worth 84 TL saved 2 TL, and whose mom gave 5 TL, could buy that music player with the money given to him by his mother in addition to 24 TL which he saved. It was a missing value problem that allowed students to comment. Moreover, its context was part-part-whole like the 4th problem. Unlike the 4th problem, the factor of change within ratios in the 6th problem was an integer. For this reason, it was seen that the percentage of the students who got full point from this problem increased compared to the 4th problem. 16.9% of the 5th, 31.1% of the 6th, 30.5% of the 7th and 48.6% of the 8th grade students got 3 points from the 6th problem. Unfortunately, the percentages of the students except 8th grade students who could not solve the problem were still very high: 68.6% of the 5th, 57.4% of the 6th, 47.7% of the 7th and 25.4% of the 8th grade students got 0 point from this problem. Most of the students could not establish a relationship between the parts and wholes and they did meaningless additive operations. The reason why the percentage of the students who got the full score from the 6th problem was low compared to the first two problems could be that the context of this problem was part-part-whole. On the other hand, most of the students who could solve the problem correctly used the factor of change strategy. They could calculate the total money given to Emre by his mother by multiplying 12 and 5 TL because they realized the multiplicative relationship between 24 TL saved by Emre in total and 2 TL. They concluded that Emre would have enough money to be able to buy the music player worth 84 TL.

In the 11th problem, it was given that 5 chocolate bars were priced at 0.75 TL and the students were asked what the price of 13 bars of chocolate was. This was a missing value problem with the context of rate. It was accepted that the factor of change between ratios was an integer when 0.75 TL was converted to 75 Kr. The difference of this problem from the 1st problem was that this problem had decimal number. For this reason, the percentage of students who solved the problem correctly and got the full score from the 11th problem was lower than the 1st problem. The first and eleventh problems in the test confirmed that the number structures in the problems had an impact on students' ability to solve problems correctly. 31.8 % of 5th, 40.2 % of 6th, 55.5% of 7th and 68.1 of 8th grade students could solve the eleventh problem correctly. Most of these students calculated the unit price of a chocolate bar as 0.15 TL, and then they found the price of 13 bars of chocolate by multiplying 13 by 0.15TL. On the other hand, 43.1% of the 5th, 38.3% of the 6th, 21.9% of the 7th and 18.1 % of the 8th grade students got 0 point from this problem. In all grade levels, the percentage of students who got 0 points and the percentage of students who got 1 point were close to each other. When the solutions of the students whose score was 1 point were examined, it was seen that many of them took 0.75 TL as the price of one chocolate bar. This showed that they had proportional reasoning but did not read the problem carefully.

In the 12th problem, it was asked how many weeks Gamze who earned 600 TL in 12 weeks should work for to earn 200 TL. This problem was a missing value problem with the context of rate. Unlike the 2nd problem, both factors of change within and between ratios were integers. This allowed students to use both the unit rate and the factor of change strategy in the problem. However, this did not cause a significant increase in the percentage of students who got a full score compared to the 2nd problem. There was a slight increase. On the contrary, these percentages showed a decrease compared to the eleventh problem. The reason why the students were more unsuccessful in this problem compared to the 11th problem might be that the 12th problem contained large numbers. 31% of the 5th, 35.9% of the 6th, 59.8% of the 7th and 72.5% of the 8th grade students got 3 points from this problem. While the factor of change within 600 TL and 200 TL may be easier to use, the students mostly preferred to calculate the unit rate. On the other hand, almost half of the 5th (54.9%) and 6th (45.9%) grade students could not solve the problem and got 0 point.

Additionally, 27% of the 7th and 15.9% of the 8th grade students could not solve this problem. In general terms, it was seen that the 7th and 8th grade students were more successful in this problem.

In the 13th problem, it was given that a printer could print 18 books in 4 minutes. The students were asked how many books that printer could print in 10 minutes. This was a missing value problem with the context of rate. Both factors of change between and within ratios were not integers. Actually, this problem was very similar to the 3rd problem due to the context and due to the fact that the factor of change was non-integer, but the percentage of the students who got full point from this problem was more than the 3rd problem. The reason for this might be that even if the change factor was not an integer, calculating how many times 10 minutes were 4 minutes might be easier than calculating how many times 30 minutes were 12 minutes. Especially the 5th, 6th and 7th grade students solved the problem by using the build-up strategy mostly. They indicated that 10 minutes equaled to the sum of 4, 4 and 2 minutes and the number of the books which could be printed by that printer in 10 minutes equaled to the sum of 18, 18 and 9 books. In short, they concluded that the printer could print 45 books in 10 minutes. On the other hand, it was seen that the 8th grade students mostly used the cross-product strategy. As could be seen in the table, 10.2% of the 5th, 20.6% of the 6th, 41.4% of the 7th and 64.5% of the 8th grade students got full point from this problem. Like in the other problems, the big increase in the percentages of the students who got the full score from the 5th to the 8th grade was noteworthy. Unfortunately, more than half of the 5th and 6th grade students, 34.4% of the 7th and 22.5% of the 8th grade students could not solve this problem. These percentages were more than the 11th and 12th problems because of the factors in the percentages of the 3rd problem being higher than the 1st and 2nd problems.

In the 14th problem, it was given that 25 students could be divided into groups of 5 students as 3 girls in each group. The students were asked how many of 25 students were girls and boys. This was a missing value problem consisting of the context of part-part-whole, and the factor of change within ratios was an integer. This problem was very similar to the 4th problem due to the context and due to the fact that the factor of change was an integer, but the percentage of the students who got a full point from this problem significantly increased compared to the 4th problem. In the 4th problem, most of the students were mistaken because they thought the number of people in a group was as the total number of groups. Fortunately, in this problem, most of the students understood the givens of the problem correctly. They mostly used the factor of change strategy. 55.7% of the 5th, 57.9% of the 6th, 73% of the 7th and 68.8% of the 8th grade students got a full point from this problem. In the 4th problem, as the number of groups and the number of people in the group were different, these students could not reach the correct result. On the other hand, in the 14th problem, the number of groups and the number of people in the group were the same. For this reason, even if the students accepted the number of people in the group as the number of groups, they could reach the right result. This situation escaped the researcher's notice while creating the test. As a result of this, unlike the 4th problem, the percentages of the students with full score were the highest in the 14th problem. 55.7% of the 5th, 57.9% of the 6th, 73% of the 7th and 68.8% of the 8th grade students got full point from this problem. On the other hand, 38.8% of the 5th, 30.6% of the 6th, 17.2% of the 7th and 19.6% of the 8th grade students could not solve the problem.

In the 16th problem, Sıla wanted to buy a music player 210 TL in worth. If she saved 2 TL, her mom would give 5 TL to Sıla. It was asked how much money her mother would give to Sıla in total. This problem was similar to the 6th problem, because they were missing value problems, the factors of change were integers and their contexts were part-part-whole. Despite all these similarities, the percentage of the students who got a full point from this problem was lower than in the 6th problem. Only 1.6% of the 5th, 10% of the 6th, 12.9% of the 7th and 26.1% of the 8th grade students could correctly solve this problem. The reason for this decline in percentages might be that the students had to find the money saved by Sıla. In the 6th problem, the students easily calculated the factor of change between 24 TL and 2 TL, and then they expressed that the mom should have given 5 TL in 12 times. In this problem, when her mother gave her 5 TL for each 2 TL, Sıla would have 7 TL.

The students were expected that they found how many times her mom should have given 5 TL to Sıla by dividing 210 TL into 7 TL. Unfortunately, more than half of the all grade level students could not think of this way of solving the problem. 36.9% of the 5th, 29.2% of the 6th, 16.8% of the 7th and 9.4% of the 8th grade students got 1 point because they tried to find the factor of change by dividing 210 TL into 2 TL. This could be considered as a clue to the existence of proportional reasoning in these students.

4.1.2.The academic achievement of the students in the numerical comparison problems

In the Table 9, it was given the percentages of the students according to the points which they got from each numerical comparison problems.

Table 9 The percentages of the students according to the points which they got from each numerical comparison problem in PRT

Numerical comparison problems		7	8	9	10	15	17	18	19	20
		Percent								
Grade 5	0 point	94,5	74,5	71,8	98,4	78,6	96,5	57,3	68,2	96,9
	1 point	3,5	1,2	21,2	,4	20,6	2,4	,4	29,4	1,6
	2 point	1,6	,0	2,4	,0	,4	,4	,0	,8	1,6
	3 point	,4	24,3	4,7	1,2	,4	,8	42,4	1,6	,0
	Total	100	100	100	100	100	100	100	100	100
Grade 6	0 point	95,7	68,4	73,2	98,6	69,7	91,9	53,1	64,6	95,2
	1 point	1,9	,0	18,2	,0	23,2	2,4	,0	26,8	1,4
	2 point	1,4	1,4	1,4	,0	,0	2,4	,0	,5	1,4
	3 point	1,0	30,1	7,2	1,4	7,2	3,3	46,9	8,1	1,9
	Total	100	100	100	100	100	100	100	100	100
Grade 7	0 point	81,6	59,4	62,9	90,2	64,1	93,0	41,8	62,5	93,4
	1 point	9,4	,4	26,2	3,1	24,6	3,1	1,6	26,6	4,3
	2 point	3,1	,4	3,1	,4	,8	,4	,0	,8	1,2
	3 point	5,9	39,8	7,8	6,3	10,5	3,5	56,6	10,2	1,2

Table 9 (continued).

	Total	100	100	100	100	100	100	100	100	100
Grade 8	0 point	76,8	35,5	56,5	76,1	59,4	74,6	26,8	60,9	94,2
	1 point	6,5	,7	20,3	10,9	9,4	12,3	,7	18,1	2,9
	2 point	5,8	,7	8,0	,0	2,2	3,6	,0	6,5	,7
	3 point	10,9	63,0	15,2	13,0	29,0	9,4	72,5	14,5	2,2
	Total	100	100	100	100	100	100	100	100	100
All grades	0 point	88,1	62,2	67,0	92,4	44,4	90,8	46,7	64,5	95,0
	1 point	5,4	,6	21,8	2,8	45,2	4,2	,7	26,1	2,6
	2 point	2,7	,6	3,3	,1	,7	1,4	0	1,6	1,3
	3 point	3,8	36,6	7,9	4,7	9,7	3,6	52,6	7,8	1,2
	Total	100	100	100	100	100	100	100	100	100

In the 7^{th} problem, it was given that the package consisting of 16 A brand chocolates was priced at 20 TL and the package consisting of 12 B brand chocolates was priced at 16 TL. The students were asked whether their choice was economical in the case that Merve and Elif preferred to buy the package of 12 B brand chocolates. It was a numerical comparison problem encouraging the students to make comments. However, the majority of the students could not get the right results. 94.5% of the 5^{th}, 95.7% of the 6^{th}, 81.6% of the 7^{th} and 76.8% of the 8^{th} grade students got 0 point from this problem. This might be due to the fact that the students were not accustomed to solving such problems in their textbooks. In general, these students tried to solve the problem using the additive strategy regardless of the multiplicative relationship between number of chocolates in a package and the price of that package. For example, according to these students, 20 TL, the price of the package of A brand was 4 more than 16, the number of chocolates in the package and similarly 16 TL, the price of the package of B brand was 4 more than 12, the number of chocolates in the package. In short, these students concluded that both of the chocolate packages had the same economic value because the price is 4 more than the number of chocolates in both packages. Another reason for students to reach this conclusion was that the price of the package of A branded chocolates was 4 TL more than the price of the package of B branded chocolates, and the number of A branded chocolates was 4 more than the number of the B branded chocolates. These students thought that the 4 chocolate differences between the packages were equal to 4 TL without taking into account the unit prices of the A and B chocolates. All of these showed that the majority of students were not able to recognize the multiplicative relations in the comparison problem and then they used additive reasoning. On the other hand, there were very few students who solved the problem in the right way and make the correct comments. It was observed that most of these students decided that the package of A-branded chocolates was more economical by calculating the unit price of chocolates, by equalizing the number of chocolates in the packages, or by equalizing the price of the packages.

The 8th problem with two phases included inverse ratio. In the first phase, it was given that if Neslihan, who could run 100 meter in 20 second, runs the same distance twice as fast, the time would double and it was asked whether this statement was true or false. In the second phase, the students were asked to mark an option that was appropriate to their answers from 4 options. The options that the students marked could show whether they could recognize the inverse ratio without the need for student solutions. 24.3% of 5th, 30.1% of the 6th, 39.8% of 7th and 63% of the 8th grade students got full point from this problem. As could be seen, the percentage of the 8th grade students who could recognize the inverse ratio was quite higher than the other grade level students. These students with full point marked option B where the time was halved if the speed was doubled. On the other hand, 74.5% of the 5th, 68.4% of 6th, 59.4% of the 7th and 35.5% of the 8th grade students got 0 point from this problem. The majority of these students marked option A where the time was also doubled if the speed was doubled. This showed that these students could not recognize the inverse relationship between the time and speed when the distance remained constant.

The 9th problem had two phases like the 8th problem. In this problem, there were three containers consisting of water and sugar cubes. Container A was completely filled with water and sugar cubes thrown into it, container B was filled with water up to half and 2 sugar cubes were thrown into it, and 1/3 of container C was filled with water and 1 sugar cube was thrown into it. The students were asked whether the water in container B was the sweetest after the water in the containers were mixed. This was a numerical comparison problem with the context of mixture. This context was another context that was said to be difficult for students in literature. This situation was confirmed by the fact that most of the students could not make a comparison in this problem correctly. 71.8% of the 5th, 73.2% of the 6th, 62.9% of the 7th and 56.5% of the 8th grade students got 0 point from the 9th problem. Most of these students considered that the water in container A was the sweetest, because the number of sugar cubes thrown into container A was the highest. Then, they marked option A which included this statement, but they did not take into account the amount of water in the containers. In addition, some students considered that the water in container C was the sweetest, because the amount of water was the least in C. Then, they marked option B, but they did not consider the number of the sugar cubes in the containers. Furthermore, approximately 20 percent of students at each grade level received 1 point from the problem because they marked only the correct option without any explanation. On the other hand, there was also a small number of students from each grade level who checked the correct option by making some valid comments. These students used the strategy of equivalent fractions. They assumed that the whole container was able to receive 6 units of water. In this case, the ratio of number of the sugar cubes to water was 3/6 in container A, 2/3 in container B, and 1/2 in container C. The students compared the amount of sugar in the containers by equalizing the denominator of these ratios. In the new situation, the ratio of number of the sugar cubes to water was 3/6 in container A, 4/6 in container B, and 3/6 in container C. They concluded that the sweetest water-sugar mixture was in container B, because 4 cubes of sugar should be discarded in 6 units of water.

In the 10th problem, there were a rectangular photograph with a length of 3 cm and 5 cm and a new photograph which was created with a 200% extension of this photograph. The students were asked which photograph looked more like a square. This problem was a numerical comparison problem. Besides, its context was similarity like the 5th problem. Nevertheless, almost none of 5th (98.4%), 6th (98.6%) and 7th (90.2%) grade students could answer this problem correctly. Some of them left it unanswered without marking any options or marked the D option which indicated that the given information to decide which photo looked more like a square was not enough. On the other hand, only 13% of the 8th grade students got full point from the problem. As these students stated that all sides of the rectangular photo were enlarged at an equal ratio, they marked the C option indicating that both photos equally looked like a square. In addition, the percentage (10.9%) of the 8th grade students who got 1 point was very close to the percentage of the 8th grade students who got 3 points. They received 1 point because they just marked the right option without any explanation.

In the 15th problem, there were two rectangular pictures of butterflies. The picture of butterfly A had sides of 2 cm and 3 cm in length and the picture of butterfly B had sides of 4 cm and 6 cm in length. It was given that the area of the picture of butterfly B was 2 times of the picture of butterfly A. Because this problem had two phases, the students were asked whether that expression was true or false in the first phase. In the second phase, the students were asked to choose an option which was suitable for their decision in the first phase. In this problem, the students were expected to realize that the ratio between the side lengths of these rectangles should have been 4, but the ratio between their areas should have been 4. Few students received full point by explaining that they realized this situation. The context of this problem was similarity like the 5th problem, but this was a numerical comparison problem unlike the 5th problem which was a missing value problem. When the tables were examined, there was no significant difference between the percentages of the students who got a full score. In both of the 5th and 15th problems, the percentages of 5th, 6th and 7th grade students who got a full score was approximately below 10 percent. On the other hand, the percentage of the 8th grade students who got a full score from the 15th problem was 29%. This percentage was lower than that in the 5th problem. Almost 20% of the 5th, 6th and 7th grade students got 1 point from this problem, because they just marked the right option without any explanations. In addition, the percentages of the students who could not solve the problem were more than 50% for all grade levels like in the 5th problem. It might be concluded that the context of the problem was similarity.

In the 17th problem, there were two different brands of washing powder. 1 kg of A branded washing powder was priced at 5 TL and it could wash laundry for 20 times. On the other hand, 1.5 kg of B branded washing powder was priced at 6.5 TL and it could wash laundry for 30 times. This problem had two phases. The first phase asked the students whether the expression of that 'A branded washing powder was more economical' was true or false. In the second phase, the students were asked to mark an option that was appropriate to their answers to the 1st phase from 4 options. This problem was a numerical comparison problem like the 7th problem. In this problem, the students were expected to calculate the unit cost of these washing powder or the prices of 1 kilogram of them for comparison. Unfortunately, some of the students thought that A branded washing powder was more economical, because its cost was less than the other. On the other hand, some of the students thought that B branded washing powder was more economical because its weight and number of washes were more than the other. In addition to these students, some students thought that although the price of B branded washing powder was higher, its number of washes and weight were more, so both washing powders were of equal economic value. As can be seen, none of these students could make a valid comparison because they could not conceive of calculating the unit price of detergents. For this reason, 96.5% of the 5th, 91.9% of the 6th, 93.0% of the 7th and 74.6% of the 8th grade students got 0 point from this problem. These percentages were similar to the percentages in the 7th problem. In general terms, 3.6% of all students could make a correct comparison between the washing powders. They used factor of change, unit rate, equivalent fractions and build up strategies to solve the problem.

The 18th problem with two phases included inverse ratio. In the 1st phase of the problem, 6 painters with the same speed could paint the fence of a garden in 4 days. The expression that 10 painters with the same speed were needed in order to be able to paint these fences in 2 days was given. The students were asked whether this expression was true or false. In the second phase, the students were asked to mark an option that was appropriate to their answers from 4 options. These options that the students had marked could show whether they could recognize the inverse ratio without the need for students' solution. This problem was similar to the 8th problem. 42.4% of the 5th, 46.9% of the 6th, 56.6% of the 7th and 72.5% of the 8th grade students could realize that the number of painters should have been doubled in order to paint the same fences in 2 days. Then, they got a full point from this problem. It was seen that these percentages were a little higher than the 8th problem. On the other hand, some students realized that the number of painters should have increased in order to paint the same fences in less time. However, they erroneously had an additive reasoning because they thought that when the number of days was decreased by 2, the number of painters should have been increased by 2. Approximately half of the 5th and 6th grade students, 41.8% of the 7th and 26.8% of the 8th grade students could not realize the inverse ratio and they got 0 point from this problem.

In the 19th problem, there were two carafes filled with lemonade. The red carafe had two cups of lemon squash and 4 cups of water. The green carafe had 4 four cups of lemon squash and 6 cups of water. The students were asked which lemonade in the carafes tasted more lemon. This problem was a numerical comparison problem and its context was mixture like the 9th problem. The students were expected that they compared the numbers of cups of the lemon squash in the carafes by equalizing the number of cups of water or compared the number of cups of water in the carafes by equalizing the numbers of cups of the lemon squash. The green carafe had more 2 cups of lemon squash and 2 cups of water than the red carafe. Because of the equality of these increments in lemon squash and water, some students concluded that lemonades in the carafes tasted lemon equally. A group of students thought that the red carafe had more lemon taste, because it had less cups of water than the green carafe. Another group of students expressed that the green carafe had more lemon taste, because it had more cups of lemon squash. These inferences indicated that these students improperly had an additive reasoning. The percentages of these students were almost about 60% in all grade levels. On the other hand, only 7.8% of all students could solve the problem correctly. Most of these students could compare the taste of lemon in the carafes by using the factor of change strategy. In addition, the reason why the percentage of the students who got 1 point from this problem in all grade levels was about 30% was that they just wrote the correct answer without any operation or explanation.

In the 20th problem, there were 4 rectangular flowerpots in different lengths. The students were asked which of these rectangles looked more similar to the square shape. The lengths of the rectangular flowerpots were given in the options. They were 27 cm - 30 cm, 17 cm - 20 cm, 7 cm - 10 cm and 37 cm - 40 cm. This was a numerical comparison problem. Since all sides of a square were equal, the ratio between the sides was 1. For this reason, the students were expected to find in which rectangle the ratio between the lengths of different sides was closest to 1. More than 90% of the students in all grade levels got 0 point from this problem. Most of them did not mark an option because they thought that all the flowerpots looked like having a square shape equally due to the equality of the difference in the lengths of the sides of all rectangles. Some of the students did not mark any options by writing that they could not understand the problem. It was seen that the context of this problem was difficult to understand for the students like in the 10th problem, but the percentage of the 8th grade students was more in the 10th problem than this problem.

In general, more than half of all the students could not solve the 2nd, 5th, 6th, 7th, 8th, 9th and 10th problems. Considering all the students, the problem which 58.7% of the students got full point was just the 1st problem. Following the 1st problem, the problems that students solved best were the 2nd, 6th and 8th problems. In general terms, the percentage of students getting full points from the problems generally increased from grade 5 to grade 8.

When looking at the Table 4 and Table 5, in general, it was seen that all grade level students solved the missing value problems better than the comparison problems. Among the missing value problems, the ones with the integer factor of change were better solved than the ones with the non-integer factor of change.

4.2. The Strategies Mostly Used by the Middle School Students in the Proportional Reasoning Problems

In this section, the strategies mostly used by the middle school students in the proportional reasoning problems were mentioned according to the grade levels and problems. To specify the mostly used strategies, solutions of 20 students with the best score at each grade level on the test were examined in detail. The frequencies and percentages of the usage of strategies were calculated in SPSS. Only the right solutions of the students were taken into consideration when determining the strategies used in problem solving. Additionally, the solutions with incorrect use of additive reasoning instead of multiplicative reasoning were examined in the category named non-additive.

Firstly, the strategies mentioned in the literature were illustrated with the solutions of the students in Table 10. However, the equivalent class strategy did not have an illustration because this strategy was not used by any students.

Table 10 The strategies illustrated with sample student solutions

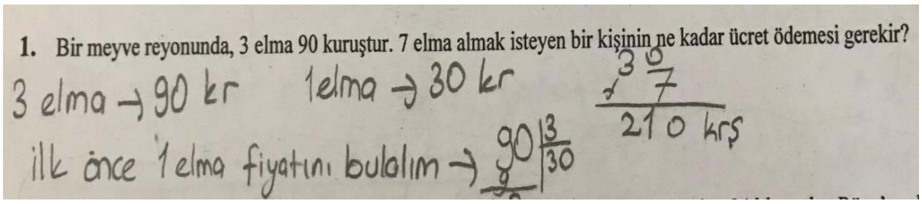
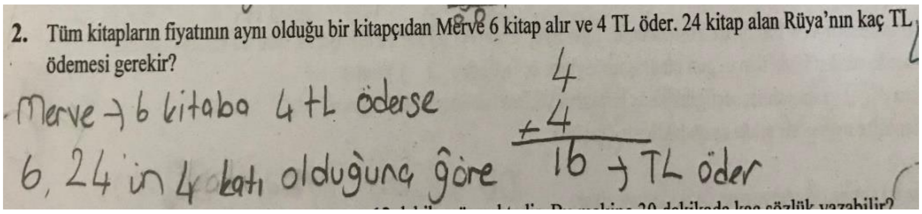
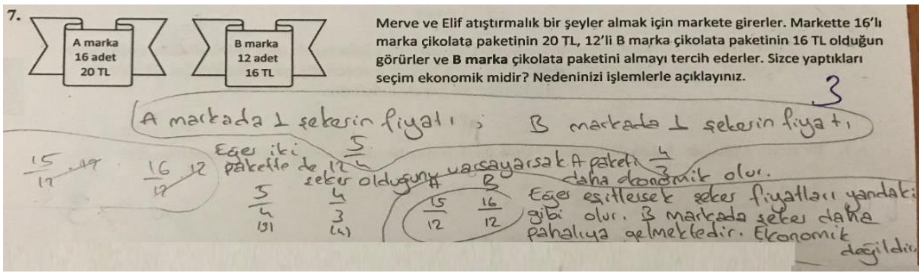
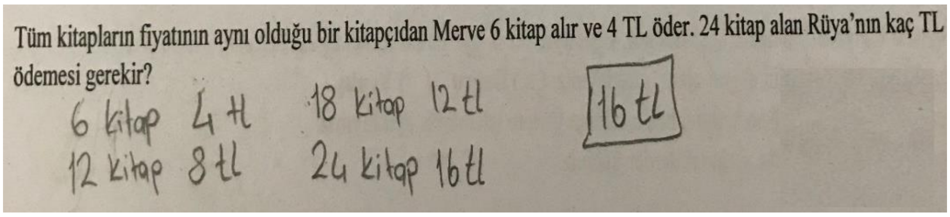
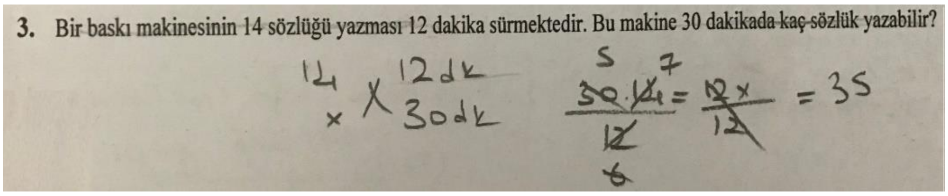
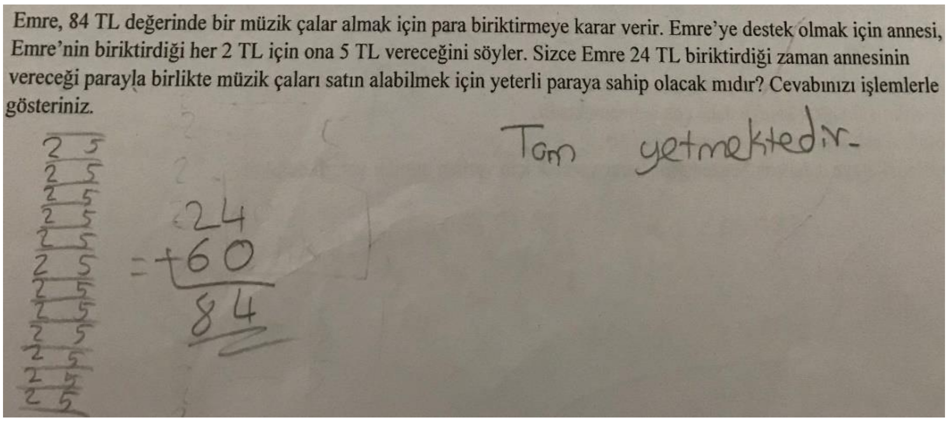
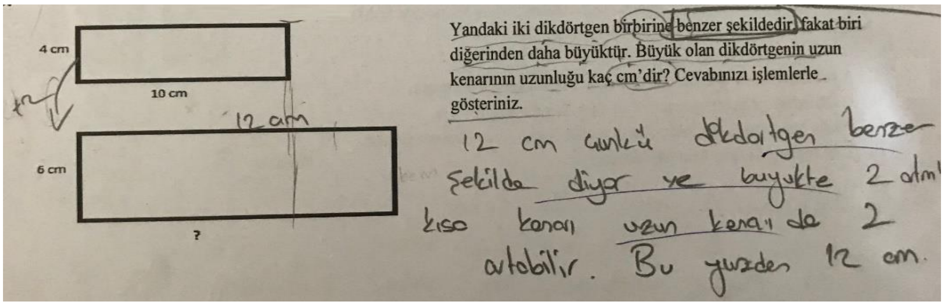
<table>
<tr><th>Strategies</th><th>Sample student solutions</th></tr>
<tr><td>Unit rate</td><td>
</td></tr>
<tr><td>Factor of change</td><td>
</td></tr>
<tr><td>Equivalent fractions</td><td>
</td></tr>
</table>

Table 10 (continued).

Equivalent class	This strategy was not used by any students.
Build-up	 Tüm kitapların fiyatının aynı olduğu bir kitapçıdan Merve 6 kitap alır ve 4 TL öder. 24 kitap alan Rüya'nın kaç TL ödemesi gerekir? 6 kitap 4 tl 18 kitap 12 tl 12 kitap 8 tl 24 kitap 16 tl $\boxed{16\ tl}$
Cross-product	 3. Bir baskı makinesinin 14 sözlüğü yazması 12 dakika sürmektedir. Bu makine 30 dakikada kaç sözlük yazabilir? 14 12dk x 30dk 30·14 = 12x = 35
Additive method	 Emre, 84 TL değerinde bir müzik çalar almak için para biriktirmeye karar verir. Emre'ye destek olmak için annesi, Emre'nin biriktirdiği her 2 TL için ona 5 TL vereceğini söyler. Sizce Emre 24 TL biriktirdiği zaman annesinin vereceği parayla birlikte müzik çaları satın alabilmek için yeterli paraya sahip olacak mıdır? Cevabınızı işlemlerle gösteriniz. Tam yetmektedir. 2 5 24 + 60 = 84
Non-additive	 Yandaki iki dikdörtgen birbirine benzer şekildedir fakat biri diğerinden daha büyüktür. Büyük olan dikdörtgenin uzun kenarının uzunluğu kaç cm'dir? Cevabınızı işlemlerle gösteriniz. 4 cm 10 cm 6 cm ? 12 cm 12 cm çünkü dikdörtgen benzer şekilde diyor ve büyükte 2 olan kısa kenarı uzun kenarı da 2 artabilir. Bu yüzden 12 cm.

After the students' solutions were analyzed according to the strategies mentioned above, the frequencies and percentages of usage of the strategies were shown in the Table 11 and Table 18.

4.2.1.The strategies mostly used by the students in the missing value problems

In the Table 11, it was given that the frequencies and percentages of usage of the strategies in the missing value problems.

Table 11 The frequencies and percentages of usage of the strategies in the missing value problems

Strategies	Grade 5		Grade 6		Grade 7		Grade 8	
	frequency	percentage	frequency	percentage	frequency	percentage	frequency	percentage
Unit rate	54	35 %	44	25,1 %	49	26,3 %	33	15,9 %
Factor of change	64	41,5%	74	42,3 %	100	53,8 %	69	33,2 %
Equivalent fractions	0	0 %	0	0 %	0	0 %	0	0 %
Equivalent class	0	0 %	0	0 %	0	0 %	0	0 %
Build-up	22	14,3 %	36	20,6 %	22	11,8 %	6	2,9 %
Cross-product	0	0 %	0	0 %	0	0 %	84	40,4 %
Additive	3	2 %	14	8 %	9	4,9 %	11	5,3 %
Non-additive	11	7,2 %	7	4 %	6	3,2 %	5	2,3 %
Total	154	100 %	175	100 %	186	100 %	208	100 %

As can be seen in the Table 11, the mostly used strategy by the 5th grade students in the missing value problems was factor of change (41.5 %) and then unit rate (35%). 22% of the solutions had build-up strategy. The strategies of equivalent fractions, equivalent class and cross-product were not used. In addition, 3 solutions had the additive method. On the other hand, 7.2 % of the solutions consisted of additive reasoning instead of multiplicative reasoning.

At 6th grade level, the mostly used strategy was factor of change (42.3 %) and then unit rate (25.1 %). The usage percentage of the unit rate strategy decreased compared to the 5th grade level. On the other hand, the percentage of the build-up strategy (20.6%) increased compared to the 5th grade level. As in grade 5, the strategies of equivalent fractions, equivalent class and cross-product were not used by any students. Additionally, 8 % of the solutions included the additive method. Because 4% of the solutions consisted of additive reasoning instead of multiplicative reasoning, it was expressed that the misusage of the additive reasoning at 6th grade level decreased compared to the 5th grade level.

At 7th grade level, like at 5th and 6th grade level, the mostly used strategy was factor of change with a percentage of 53.8. Then, the second mostly used strategy was unit rate with 26.3 %. While none of the solutions had the strategies of equivalent fractions, equivalent class and cross-product, the percentages of build-up (11.8%) strategies were too low. In addition, 4.9% of the solutions had the additive method and this percentage was lower than at 6th grade level. Moreover, the percentage (3.2%) of the solutions which consisted of misusage of the additive reasoning dropped compared to the 6th grade level.

At 8th grade level, the mostly used strategy was cross-product with a 40.4%. This was followed by the factor of change strategy which was included in the 33.2% of the solutions. However, the percentage (15.9%) of the usage of unit rate and build up (2.9%) strategy was lower than that at 5th, 6th and 7th grade level. Like other grade levels, the strategies of equivalent fractions and equivalent class were not used by any students. In addition, the percentage of the solutions which included the additive method was 5.3. Furthermore, the percentage (2.3%) of the solutions including misusage of additive method decreased compared to the other grade levels.

In general, the mostly used strategies in the missing value problems were respectively the strategies of factor of change, unit rate and build-up in the 5th, 6th and 7th grade level and the strategies of cross-product, factor of change and unit rate in the 8th grade level. On the other hand, equivalent fractions and equivalent class strategies were not used by any students in the missing value problems. In addition, the misusage of additive method decreased as the grade level increased.

4.2.1.1.The mostly used strategies in each missing value problem

In this section, the frequencies and percentages of usage of the strategies and the examples of the students' solutions in each missing value problem were given.

Table 12 The strategies used in the 1^{st} and 11^{th} problems

		The 1^{st} problem		**The 11^{th} problem**	
Grade		Frequency	Percent	Frequency	Percent
5	No Correct Solution	0	0	2	10
	Unit Rate	19	95	15	75
	Build-up	1	5	3	15
	Total	20	100	20	100
6	No Correct Solution	0	0	3	15
	Unit Rate	19	95	12	60
	Build up	0	0	5	25
	Additive Method	1	5	0	0
	Total	20	100	20	100
7	No Correct Solution	1	5	2	10
	Unit Rate	18	90	15	75
	Build up	0	0	3	15
	Factor of Change	1	5	0	0
	Total	20	100	20	100
8	No Correct Solution	1	5	0	0
	Unit Rate	13	65	10	50
	Cross-product	6	30	10	50
	Total	20	100	20	100

The 1^{st} and 11^{th} problem of the test were missing value problems with the context of rate, and the factor of change between ratios was integer in the problems. In the first problem, 13 apples were given 90 cents and the students were asked to calculate how many cents would be 7 cents. As can be seen in Table 12, 19 (95%) fifth grade students, 19 (95%) sixth grade students, 18 (90%) seventh grade students and 13 (65%) eighth grade students used the unit rate strategy in order to reach to the correct answer. In the 11^{th} problem, it was given that 5 chocolate bars were priced at 0,75 TL and the students were asked what the price of 13 bars of chocolate was. Here, as some students preferred the build-up strategy, the percentage of students using the unit rate strategy decreased slightly. As can be seen Table 12, 15 fifth grade students, 12 (60%) sixth grade students, 15 (75%) seventh grade students and 10 (50%) eighth grade students used the unit rate strategy in order to reach the correct answer in the eleventh problem. Following the unit rate strategy, the highest frequency and percentage belonged to the cross-product strategy because 6 (30%) 8th grade students in the 1st problem and 10 (50%) 8th grade students in the 11th problem solved the problem correctly with the cross-product strategy. As a result, the majority of the students correctly solved this missing value problem by using the unit rate strategy. The other used strategies in this problem were cross-product, factor of change and build up. Moreover, it was seen that all grade level students used two types of the strategies mostly.

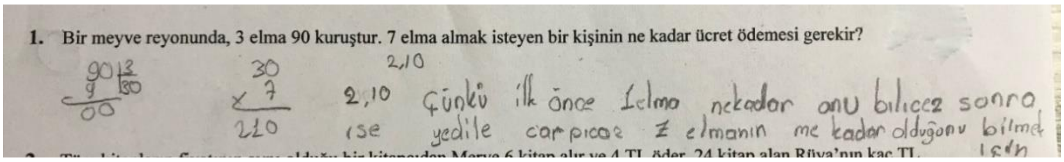


Figure 18 Unit rate strategy for the first problem

The example in Figure 18 belongs to the unit rate strategy used in the 1st problem. This student (Grade 5) calculated the price of one apple and then found the price of 7 apples by multiplying 30 Kr by 7.

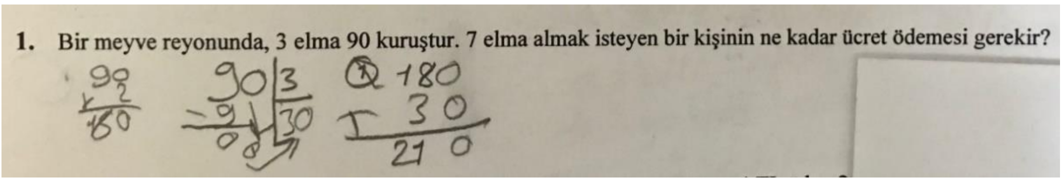


Figure 19 Build-up strategy for the first problem

The example in Figure 19 belongs to the build-up strategy used in the 1st problem. Because the build-up strategy included both of multiplicative and additive thinking, it was seen that this student (Grade 5) calculated the total price of 7 apples both by finding the price of an apple and by adding.

The example in Figure 20 belongs to the build-up strategy used in the 11th problem. Firstly, this student (Grade 5) calculated the price of one apple. Then, he just found the price of 3 apples by multiplying 15 Kr by 3. Because he already knew the price of 5 apples, he added 45 Kr to two 75 Kr.

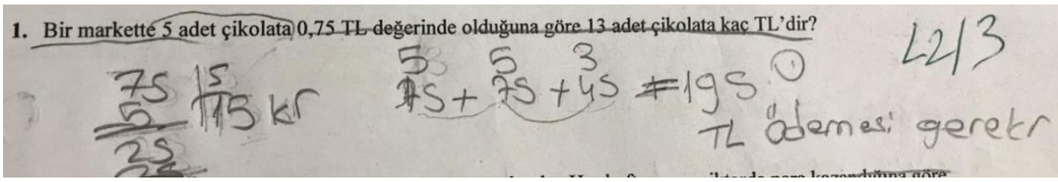


Figure 20 Build-up strategy for the eleventh problem

Table 13 The strategies used in the 2nd and 12th problem

		The 2nd problem		**The 12th problem**	
Grade		Frequency	Percent	Frequency	Percent
5	No Correct Solution	1	5	1	5
	Unit Rate	2	10	18	95
	Factor of Change	13	65	0	0
	Build-up	4	20	0	0
	Additive	0	0	1	5
	Total	20	100	20	100
6	No Correct Solution	2	10	1	5
	Unit Rate	1	5	12	60
	Factor of Change	12	60	6	30
	Build-up	4	20	1	5
	Additive Method	1	5	0	0
	Total	20	100	20	100
7	No correct solution	0	0	1	5
	Unit rate	0	0	15	75
	Factor of Change	18	90	4	20
	Build-up	2	10	0	0
	Total	20	100	20	100
8	Unit rate	0	0	10	50
	Factor of Change	7	35	3	15
	Build up	0	0	1	5
	Cross-product	13	65	6	30
	Total	20	100	20	100

The 2nd and 12th problems were missing value problems with the context of rate. The factor of change within ratios in the 2nd problem and both of within and between ratios in the 12th problem was an integer. In 2nd problem, it was asked what the price of 24 books was in a bookstore where 6 books were 4 TL. As can be seen in Table

13, 13 (65%) of the 5^{th} grade students, 12 (60%) of the 6^{th} grade students, 18 (90%) of the 7^{th} grade students and 7 (35%) of the eight grade students used the factor of change strategy. Although most of the 5^{th}, 6^{th} and 7^{th} grade students used factor of change, the majority of the 8^{th} grade students used cross-product. In the 12^{th} problem, it was asked how many weeks Gamze who earned 600 TL in 12 weeks should work for 200 TL. 18 (90%) 5^{th}, 12 (60%) 6^{th}, 15 (75%) 7^{th} and 10 (50%) 8^{th} grade students used the unit rate strategy. Most of the students used the unit rate strategy. This could have resulted from the fact that the factor of change between in addition to within ratio was an integer. In short, the majority of the students solved the 2^{nd} problem by using the factor of change strategy and the 12^{th} problem by the unit rate strategy.

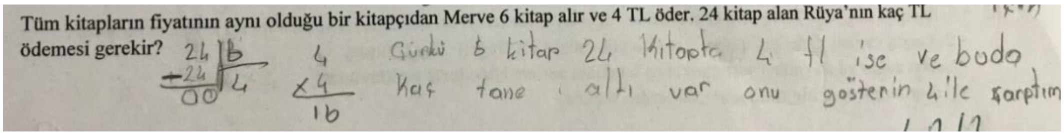


Figure 21 Factor of change strategy for the second problem

The example in Figure 21 is the factor of change strategy, which was the strategy used mostly by the 5^{th} grade students in the 2^{nd} problem. This student (Grade 5) firstly found the factor of change within 6 books and 24 books. Because while the number of books increased by 4 times, the total price of books also must be 4 times, they multiplied 4 TL by 4.

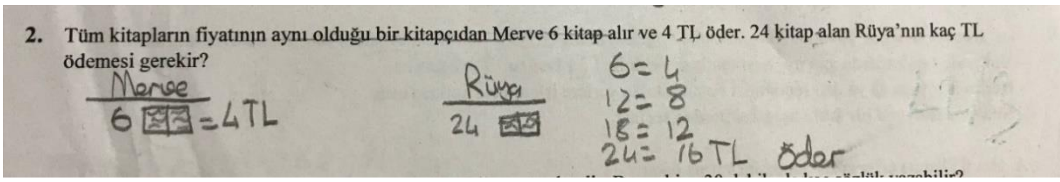


Figure 22 Build-up strategy for the second problem

The example in Figure 22 is related to the build-up strategy in the 2^{nd} problem. The students who used the build-up strategy established a relationship within a ratio and then extended it to the second ratio by addition. When 6 books were added to 6 books, the total was 12 books and when 4 TL was added to 4 TL, the total was 8 TL. When 6 books were added to 12 books, the total was 18 books and when 4 TL was added to 8 TL, the total was 12 TL. In this way, this student (Grade 6) found that 24 books were priced at 16 TL.

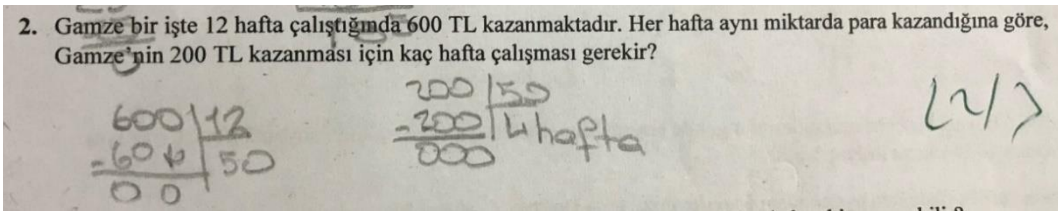


Figure 23 Unit rate strategy for the twelfth problem

The example in Figure 23 belongs to the unit rate strategy used in the 12th problem. Firstly, this student (Grade 8) calculated the money that Gamze earned in a week. Then, the student divided 200 TL into 50 TL in order to find out in how many weeks Gamze would earn 200 TL.

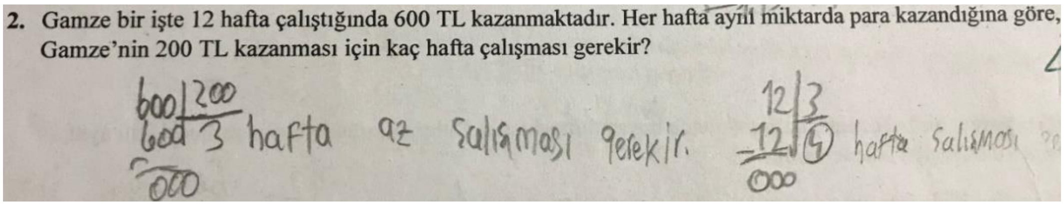


Figure 24 Factor of change strategy for the twelfth problem

In the example in Figure 24, the factor of change strategy was used. Firstly, the factor of change within 600 TL and 200 TL was calculated. Because the factor of change was 3, the student (Grade 6) also divided 12 weeks into 3.

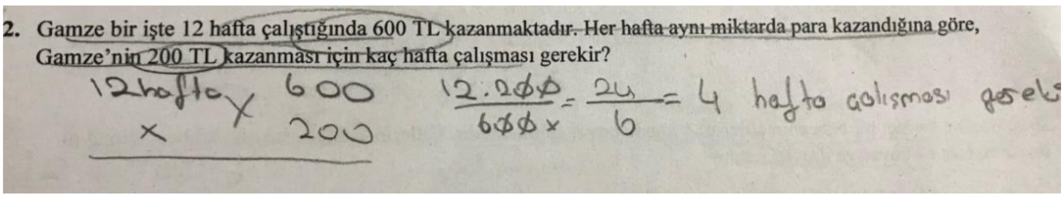


Figure 25 Cross-product strategy for the twelfth problem

In the example in Figure 25, this student (Grade 8) used the cross-product strategy. The student multiplied 12 weeks by 200 TL, and then he divided the product into 600 TL.

Table 14 The strategies used in the 3rd and 13th problems

		The 3rd problem		The 13th problem	
Grade		Frequency	Percent	Frequency	Percent
5	No Correct Solution	14	70	12	60
	Factor of Change	1	5	2	10
	Build-up	5	25	6	30
	Total	20	100	20	100
6	No Correct Solution	6	30	3	15
	Factor of Change	2	10	3	15
	Build-up	12	60	13	65
	Non-additive	0	0	1	5
	Total	20	100	20	100
7	No Correct Solution	5	25	3	15
	Unit Rate	1	5	0	0
	Factor of Change	6	30	8	40
	Build-up	6	30	9	45
	Additive Method	2	10	0	0
	Total	20	100	20	100
8	Factor of Change	1	5	1	5
	Build-up	2	10	3	15
	Cross-product	16	80	16	80
	Additive Method	1	5	0	0
	Total	20	100	20	100

The 3rd and 13th problems of the test were missing value problems with the context of rate, but unlike the 1st, 2nd, 11th and 12th problems, the factor of change in the 3rd and 13th problems were non-integer. In the 3rd problem, it was asked how many dictionaries a printer which could print 14 dictionaries in 12 minutes could print in 30 minutes. As can be seen in Table 14, most of the 5th grade students (25%) used the factor of change strategy, most of the 6th grade students (60%) used the build-up strategy, most of the 7th grade students (30%) equally used the factor of change and build up strategy, and the majority of the 8th grade students (80%) used the cross-product strategy. These percentages were very close to those in the 13th problem.

In the 13th problem, it was given that a printer could print 18 books in 4 minutes. The students were asked how many books that printer could print in 10 minutes. Most of the 5th and 6th grade students used the build-up strategy. Most of the 7th grade students (45%) used the build-up strategy, but the percentage (40%) of the students who used the factor of change was very close to the percentage of the students who used the build-up strategy. The 8th grade students mostly used the cross-product strategy.

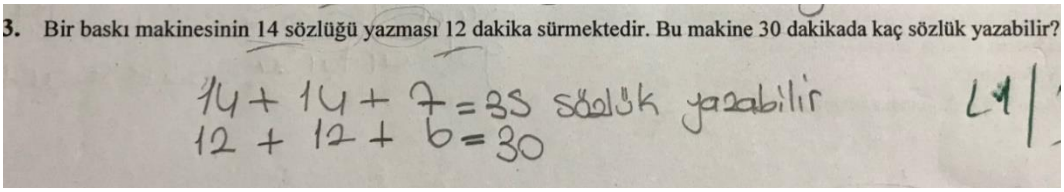


Figure 26 Build-up strategy for the third problem

In the solution in Figure 26, the student (Grade 5) used the build-up strategy. He firstly found in how many minutes 7 dictionaries could be printed. Then, he added 6 minutes to two 12 minutes.

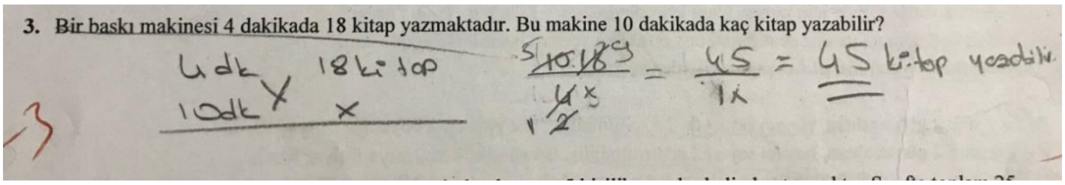


Figure 27 Cross-product strategy for the thirteenth problem

In the solution in Figure 27, the student (Grade 8) used the cross-product strategy. The student multiplied 10 minutes by 18 books, and then he divided the product into 4 minutes.

In the solution in Figure 28, the student (Grade 7) calculated the factor of change between 4 minutes and 18 books. Because 18 was 4.5 times of 4, he multiplied 10 minutes by 4.5.

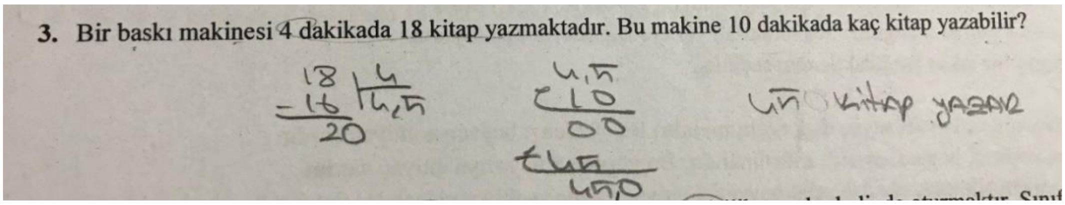


Figure 28 Factor of change strategy for the thirteenth problem

Table 15 The strategies used in the 4th and 14th problem

		The 4th problem		The 14th problem	
Grade		Frequency	Percent	Frequency	Percent
5	No Correct Solution	9	45	0	0
	Factor of Change	8	40	18	90
	Build-up	3	15	0	0
	Additive Method	0	0	2	10
	Total	20	100	20	100
6	No Correct Solution	5	25	0	0
	Factor of Change	10	50	16	80
	Build-up	1	5	0	0
	Additive Method	4	20	4	20
	Total	20	100	20	100
7	No Correct Solution	8	40	0	0
	Factor of Change	8	40	19	95
	Build-up	1	5	0	0
	Additive Method	3	15	1	5
	Total	20	100	20	100
8	No Correct Solution	4	20	0	0
	Factor of Change	13	65	15	75
	Additive Method	3	15	5	25
	Total	20	100	20	100

The 4th and 14th problems were missing value problems including part-part-whole context, and the factor of change within ratios was an integer. In the 4th problem, it was given that 30 students could be divided into groups of 6 students as 4 boys in each group. The students were asked how many of 30 students were girls. To begin with, as can be seen in Table 15, 9 students in the 5th grade (45%), 5 students in the 6th grade (25%), 8 students in the 7th grade (35%) and 4 students in the 8th grade (20%) could not solve this problem. Most of the students from all grade level used the factor of change strategy in order to solve this problem correctly. Whereas the

5^{th}, 6^{th} and 7^{th} grade students used the build-up strategy in addition to the factor of change strategy, the 8^{th} grade students did not use any different strategy than the factor of change.

Additionally, there were some 6^{th}, 7^{th} and 8^{th} grade students who reached the correct answer by using the additive method. For example, the solution of the 4^{th} problem in Figure 29 belongs to a 6^{th} grade student. He firstly found the number of groups, and then he subtracted the number of male students from the total number of students in each group one-by-one in order to find the number of girls in a group. He found out that there were 2 female students in each group. Finally, he added 2 five times.

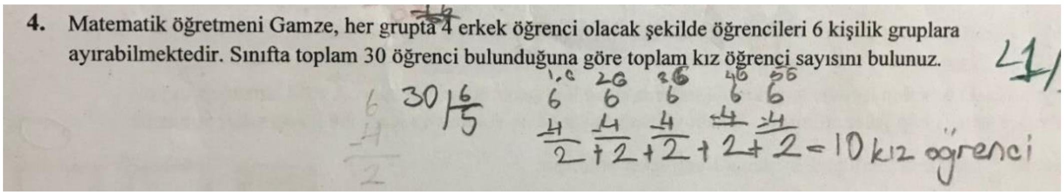


Figure 29 Additive method for the fourth problem

The 14^{th} problem was a missing value problem including part-part-whole context, and the factor of change within ratios was an integer. It was given that 25 students could be divided into groups of 5 students as 3 girls in each group. The students were asked how many of 25 students were girls and boys. In the 4^{th} problem, as the number of groups and the number of people in a group were different, these students could not reach the correct result. On the other hand, in the 14^{th} problem, the number of groups and the number of people in a group were the same. For this reason, even if the students accepted the number of people in the group as the number of groups, they could reach the right result. This situation escaped the researcher's notice while creating the test. As a result of this, there were no students who could not solve the 14^{th} problem. The majority of all grade level students could correctly solve this problem by using the factor of change strategy. 90% of the 5^{th}, 80% of the 6^{th}, 95% of the 7^{th} and 75% of the 8^{th} grade level students used the factor of change strategy. The solution in Figure 30 is an example of the factor of change strategy. This student (Grade 6) firstly found the number of groups in the class, because the number of groups was the factor of change in this problem. Because there would be 2 male students in each group, she calculated the total number of male students by multiplying the number of 2 male students by 5 and the total number of female students by multiplying the number of 3 female students by 5.

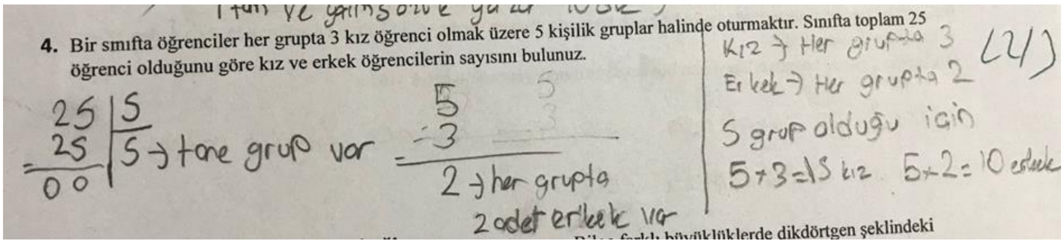


Figure 30 Factor of change strategy for the fourteenth problem

There were some students using the additive method in the 14th problem. The solution in Figure 31 from a 5th grade student is an example of the additive method. He drew a table for the solution. The first column belongs to the number of female students, the second column belongs to the number of male students, and each row belongs to the number of the students in a group. He placed five students in rows until the total number of students was 25. When the total number of students was 25, he realized that there were 15 female students and 10 male students in the class.

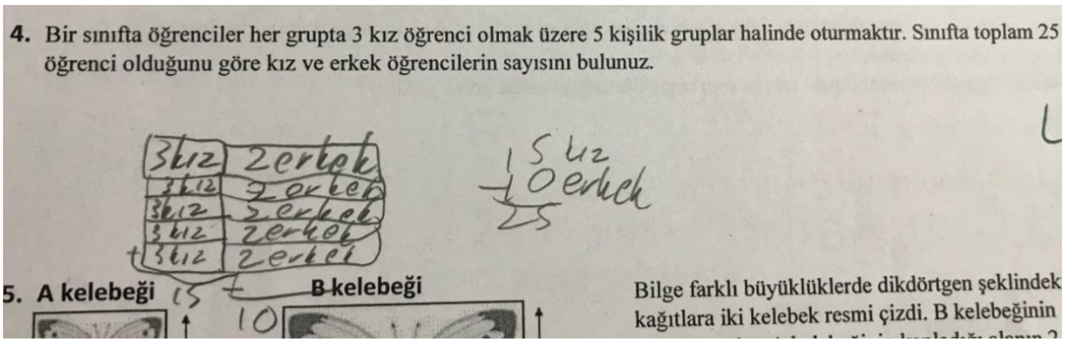


Figure 31 Additive method for the fourteenth problem

Table 16 The strategies used in the 5th problem

		The 5th problem	
Grade		Frequency	Percent
5	No Correct Solution	9	45
	Factor of Change	0	0
	Non-additive Method	11	55
	Total	20	100
6	No Correct Solution	11	55
	Factor of change	0	0
	Additive Method	4	20
	Non-additive Method	5	25
	Total	20	100
7	No Correct Solution	4	20
	Factor of Change	6	30
	Build-up	1	5
	Additive Method	3	15
	Non-additive Method	6	30
	Total	20	100
8	No Correct Solution	1	5
	Factor of Change	1	5
	Cross-product	14	70
	Additive Method	1	5
	Non-additive Method	3	15
	Total	20	100

The 5th problem was a missing value problem involving similarity, and both factors of change were non-integers. In the 5th problem, a rectangle whose short side was 4 cm in length and tall side was 10 cm in length was given, and the students were asked how many centimeters the tall side of another rectangle which was similar to the first rectangle and whose short side was 6 cm in length were. This problem could not be solved correctly by any 5th grade students. As can be seen Table 16, 55% of the 5th grade students improperly used the additive method instead of multiplicative methods because they could not realize the multiplicative relationship between the quantities. For example, the solution in Figure 32 belongs to a 5th grade student. He wrongly used additive reasoning instead of multiplicative reasoning. Because the short side of the rectangle increased 2 cm, this student increased the long side of the rectangle by 2 cm by ignoring the multiplicative relationship between the sides of similar shapes.

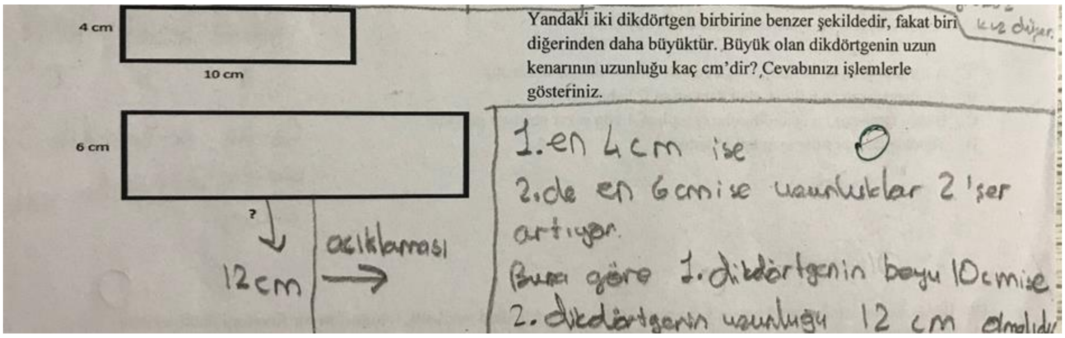


Figure 32 Incorrect use of additive reasoning for the fifth problem

Only 4 students from the 6th grade reached the correct answer of the problem, but without using any multiplicative strategies. These students solved the problem by reasoning additively. The percentage (25%) of the students who improperly used the additive method was less than that at the 5th grade level. For example, the correc solution in Figure 33 belongs to a 6th grade student. This student realized the multiplicative relationship between similar rectangles, but he calculated the long side of the larger rectangle by adding half of the length of long side of smaller rectangle to the length of long side of smaller rectangle.

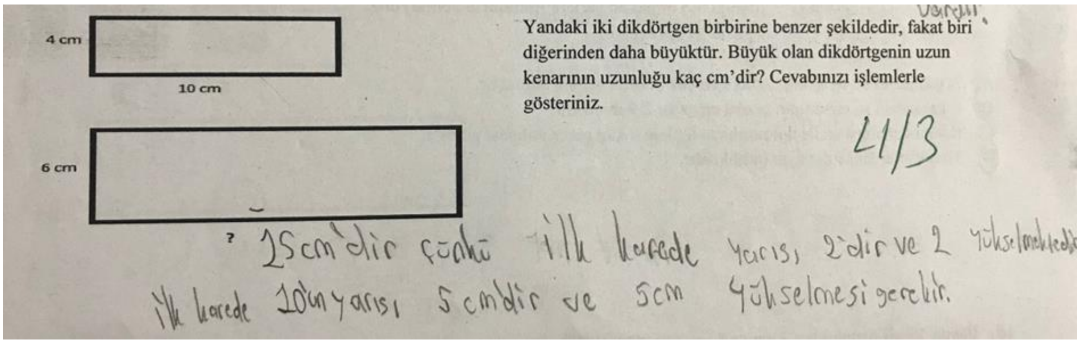


Figure 33 Additive method for the fifth problem

Although the percentage of the 7th grade students using the additive method wrongly instead of multiplicative relationship was 30%, it was seen that there wer some students who reached the right solution using a strategy. 6 students (30% used the factor of change strategy and 1 student used the build-up strategy Therefore, the mostly used strategy by the 7th grade students to solve the fifth problem was factor of change. For example, the solution in Figure 34 illustrating the use of the factor of change strategy belongs to a 7th grade student. This student calculated the ratio between the length of the short and long side of the smaller rectangle. This ratio was the factor of change in this problem. He thought that this ratio might be the same in the larger rectangle, because these rectangles were similar to each other. For this reason, he increased the length of the short side of the larger rectangle by 2.5 times.

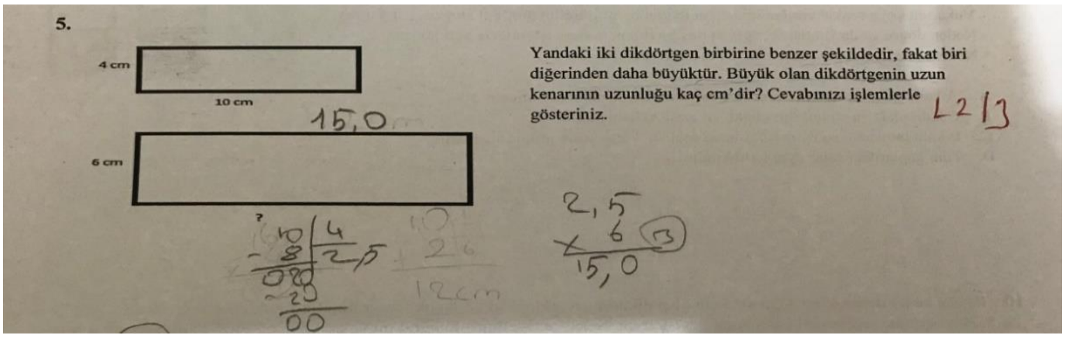


Figure 34 Factor of change strategy for the fifth problem

At the 8th grade level, the majority of the students (70%) reached the correct answer by using cross-product strategy. Only 1 student (5%) used the build-up strategy. Although there were three students using the additive method improperly, their percentage was the least compared to the other grade level students. The solution in Figure 35 belongs to an 8th grade student who used the cross-product strategy. After writing the proportion between the length of the short and long side of the rectangles, the student multiplied 4 by x and 6 by 10. Because the products should be equal to each other, he found the length of the long side of the larger rectangle (x) by dividing 60 into 4.

To conclude, the 5th and 6th grade students did not use any strategy, while the 7th grade students mostly used the factor of change strategy and the 8th grade students used the cross-product strategy in the 5th problem.

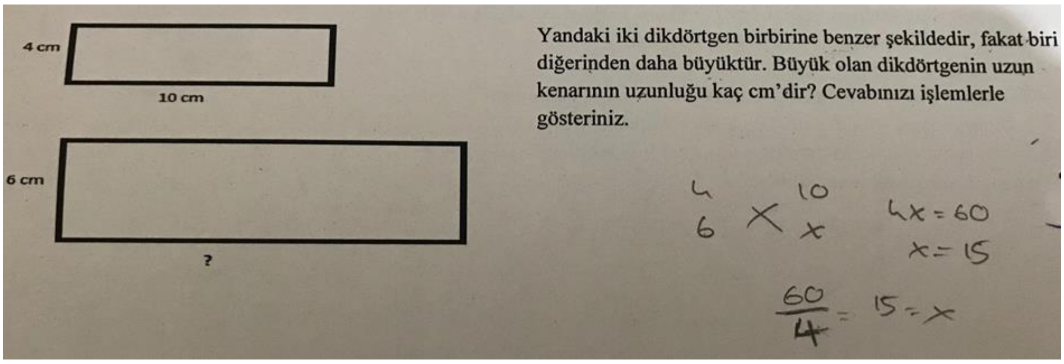


Figure 35 Cross product strategy for the fifth problem

Table 17 The strategies used in the 6th and 16th problem

		The 6th problem		**The 16th problem**	
Grade		Frequency	Percent	Frequency	Percent
5	No Correct Solution	0	0	18	90
	Factor of Change	20	100	2	10
	Total	20	100	20	100
6	No Correct Solution	0	0	13	65
	Factor of Change	19	95	6	30
	Additive Method	1	5	1	5
	Total	20	100	20	100
7	No Correct Solution	0	0	10	50
	Factor of Change	20	100	10	50
	Total	20	100	20	100
8	No Correct Solution	0	0	6	30
	Factor of Change	15	75	13	65
	Cross-product	3	15	0	0
	Additive Method	2	10	1	5
	Total	20	100	20	100

The 6th problem was a missing value problem consisting of part-part whole context and the factor of change within ratios was an integer. It was given that if Emre who wanted to buy a music player worth 84 TL saved 2 TL, his mom would give 5 TL to Emre. It was asked whether he could buy that music player with the money given to him by his mother in addition to 24 TL which he saved. As can be seen in Table

17, the majority of the 5th (100%), 6th (95%), 7th (100%) and 8th (75%) grade students used the factor of change strategy in this problem. In addition, three students (15%) from the 8th grade level used the cross-product strategy.

The solution from the 7th grade in Figure 37 is an example of the factor of change strategy which was the mostly used strategy. This student firstly divided 24 TL into 2 TL in order to find how many times Emre's mother would give 5 TL to him. This quotient was the factor of change in this problem. Then, he multiplied 5 TL by 12 in order to find the total money which Emre's mother would give to him. The student concluded that Emre could buy that music player when 60 TL was added to 24 TL.

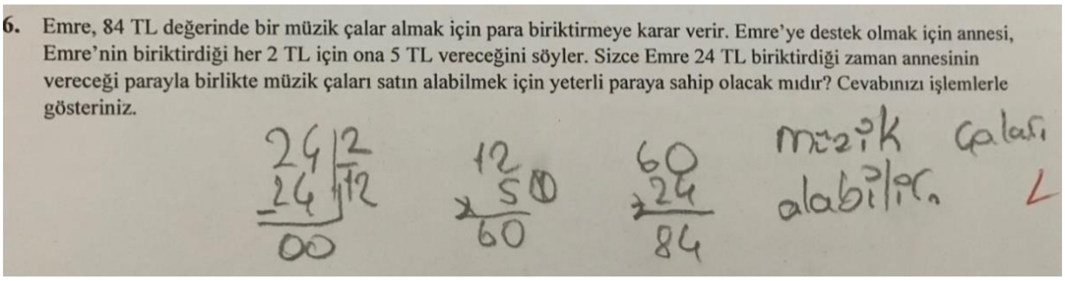


Figure 36 Factor of change strategy for the sixth problem

The solution from the 8th grade in Figure 37 is an example of the cross-product strategy. After writing the proportion between the amounts of the money saved by Emre and given by his mother, he multiplied 2 TL by x TL and 24 TL by 5 TL. Because these products were equal to each other, he found the money (x) which Emre's mother would give to him by dividing 120 into 2.

The solution from the 6th grade in Figure 38 is an example of the additive method. This student calculated the total money in order to buy that music player by writing the money saved by Emre and given by his mother one by one and separately. The student's way of thinking indicates that he could not notice the multiplicative relationship between these amounts of money.

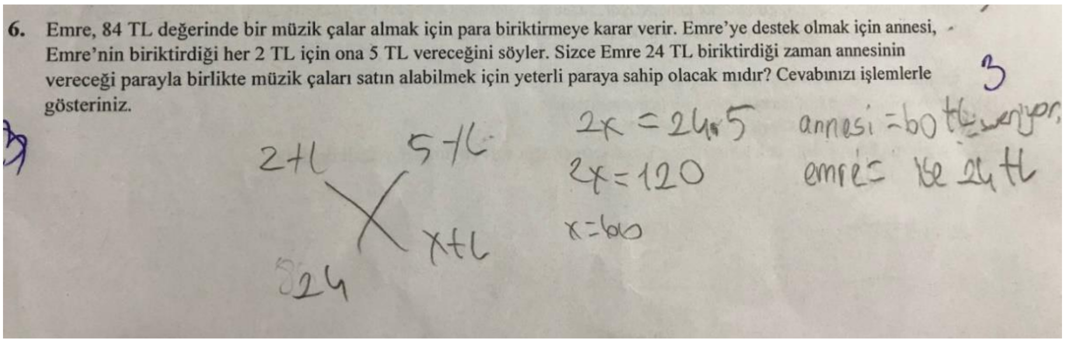


Figure 37 Cross product strategy for the sixth problem

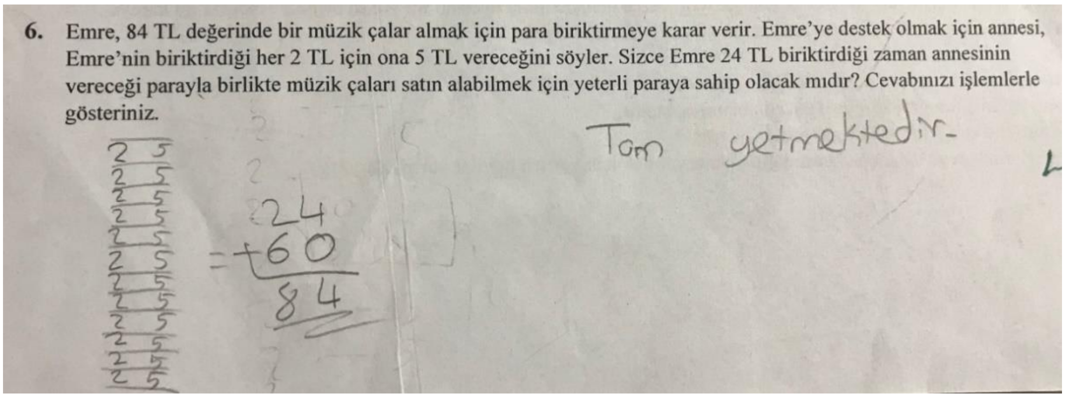


Figure 38 Additive method for the sixth problem

The 16th problem was a missing value problem consisting of the context of part-part-whole, and the factor of change within ratios was an integer like the 6th problem. In this problem, Sıla wanted to buy a music player 210 TL in worth. If she saved 2 TL, her mom would give 5 TL to Sıla. It was asked how much money her mother would give to Sıla in total. The mostly used strategy in this problem also was the factor of change strategy, but the percentages of the students who used this strategy at each grade were significantly lower than the 6th problem of . The reason for this decrease in percentages might be that students had to find the money saved by Sıla. Only 2 students from the 5th grade, 6 students from the 6th grade, 10 students from the 7th grade, and 13 students from the 8th grade could solve this problem using the factor of change strategy.

The solution from the 6th grade level in Figure 39 illustrated to the factor of change strategy. This student firstly found out the money which Sıla would have at once by adding 7 TL to 2 TL. Then, she calculated how many times Sıla’s mother would give 5 TL to her by dividing 210 TL into 7 TL. This quotient was the factor of change in this problem. Finally, she found out the money which Sıla’s mother would give to her by multiplying 5 TL by 30.

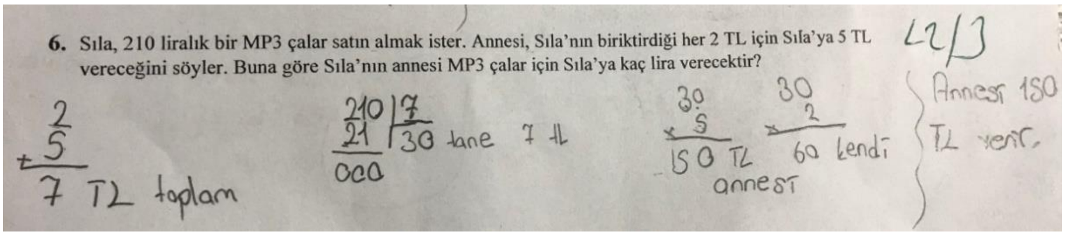


Figure 39 Factor of change strategy for the sixteenth problem

The solution from the 8th grade in Figure 40 illustrates the additive method. This student calculated the total money in order to buy that music player by writing the money saved by Sıla and given by her mother one by one and separately.

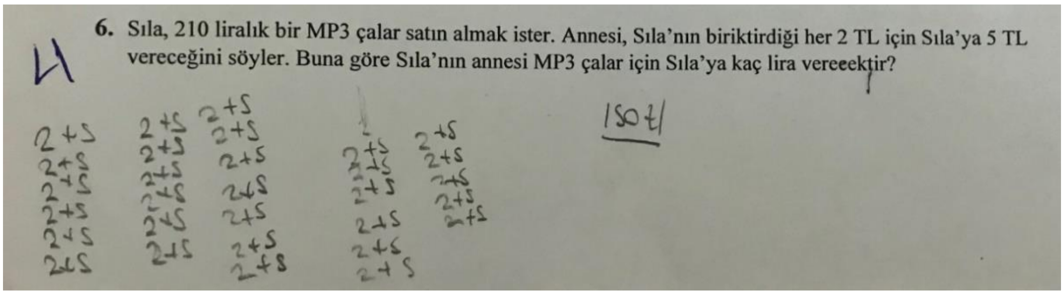


Figure 40 Additive method for the sixteenth problem

4.2.2.The strategies mostly used by the students in the numerical comparison problems

In Table 18, the frequencies and percentages of usage of the strategies in numerical comparison problems were given.

As can be seen in Table 18, the mostly used strategy by the 5th grade students in the numerical comparison problems was factor of change (37,5%). 1,8% of the solutions was solved by the unit rate strategy. The other strategies were not used.

In the 60,7 % of the solutions, the additive method was incorrectly used as far as multiplicative reasoning was concerned.

Table 18 The frequencies and percentages of usage of the strategies in numerical comparison problems

Strategies	**Grade 5**		**Grade 6**		**Grade 7**		**Grade 8**	
	frequency	percentage	frequency	percentage	frequency	percentage	frequency	percentage
Unit rate	1	1,8 %	5	7,6 %	15	24,2 %	5	7,7 %
Factor of change	21	37,5 %	23	34,8 %	23	37,1 %	29	44,6 %
Equivalent fractions	0	0 %	11	16,7 %	14	22,6 %	16	24,6 %
Equivalent class	0	0 %	0	0 %	0	0	0	0 %
Build-up	0	0 %	1	1,5 %	0	0	0	0 %
Cross-product	0	0 %	0	0 %	0	0	11	0 %
Additive	0	0 %	1	1,5 %	0	0	0	16,9 %
Non-additive	34	60,7 %	25	37,9 %	10	16,1 %	4	6,2 %
Total	56	100 %	66	100 %	62	100 %	65	100 %

In the 6th grade level, the mostly used strategy was factor of change strategy (34,8%), and then equivalent fractions strategy (16,7%). Like in the 5th grade, the strategies of equivalent class and cross-product were not used. While one solution had the additive method, 37,9 % of the solutions had wrong usage of the additive method in the situations of multiplicative reasoning.

In the 7th grade level, the mostly used strategy was the factor of change strategy (37,1%), and the percentage of the usage of the unit rate (24,2%) and equivalent fractions (22,6%) strategy were close to each other. The other strategies were not used. The percentage (16,1 %) of the students who wrongly used the additive method in the situations of multiplicative reasoning decreased compared to the 5th and 6th grade levels.

In the 8th grade level, the mostly used strategy in the numerical comparison problems was the factor of change strategy with the percentage of 44,6 % while it was the cross-product strategy in the missing value problems. This strategy was followed by the strategy of equivalent fractions with the percentage of 24,6%. However, the equivalent class, build-up and cross product strategies was never used. While the percentage (16,9) of the usage of the additive method was the highest, the percentage (6,2%) of the solutions which had the additive method in the situations of multiplicative reasoning was the lowest in this grade level.

In general, it was seen that in the numerical comparison problems the mostly used strategy by the all graders was the factor of change strategy. At the second order, there was the equivalent fractions strategy at the 6th and 8th grade levels and the unit rate strategy at the 7th grade level. The other strategies were almost never used by the students. The percentage of the wrong usage of the additive method in multiplicative situations decreased as the grade level increased. In addition, these percentages were significantly higher than those for the missing value problems. This showed that the students were more successful in solving missing value problems and had more difficulty in solving numerical comparison problems.

4.2.2.1.The mostly used strategies in each numerical comparison problem

In this part, the frequencies and percentages of usage of the strategies and the examples of the students' solutions in each numerical comparison problem were given.

The 15th problem was a numerical comparison problem with context of the similarity. In this problem, there were two rectangular pictures of butterflies. The picture of butterfly A had sides of 2 cm and 3 cm in length and the picture of butterfly B had sides of 4 cm and 6 cm in length. The expression that the area of the picture of butterfly B was 2 times of the picture of butterfly A was given to the students. Because this problem had two phases, the students were asked whether that expression was true or false in the first phase. In the second phase, the students were asked to choose an option which was suitable for their decision in the first phase. The aim of this problem was to understand whether the students could realize the ratio between the areas of similar rectangles. In fact, this problem was not a problem in which students would use a wide variety of strategies. As a result of this, as in the Table 19, 17 students (85%) from the 5th, 6th and 7th grades and 19 students (95%) from the 8th grade used the factor of change strategy in order to express the change of the areas of the similar rectangles in the problem. The solution in Figure 41 that belongs to an 8th grade level student illustrates this strategy. This student showed the ratio between the lengths of the short sides of these rectangles and the ratio of the lengths of the long sides of the rectangles as 2. He calculated the areas of the rectangles, and then he concluded that the area covered by butterfly B was 4 times larger than the area covered by butterfly B.

Table 19 The strategies used in the 15th problem

		The 15th problem	
Grade		Frequency	Percent
5	No Correct Solution	3	15
	Factor of Change	17	85
	Non-additive Method	0	0
	Total	20	100
6	No Correct Solution	3	15
	Factor of change	17	85
	Additive Method	0	0
	Non-additive Method	0	0
	Total	20	100
7	No Correct Solution	3	15
	Factor of Change	17	85
	Build-up	0	0
	Additive Method	0	0
	Non-additive Method	0	0
	Total	20	100
8	No Correct Solution	1	5
	Factor of Change	19	95
	Cross-product	0	0
	Additive Method	0	0
	Non-additive Method	0	0
	Total	20	100

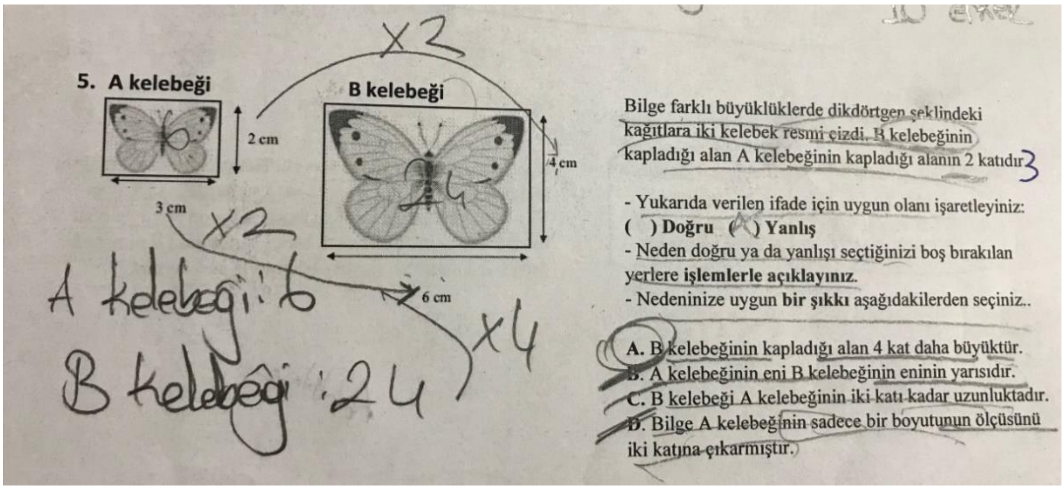


Figure 41 Factor of change strategy for the fifteenth problem

Table 20 The strategies used in the 7th and 17th problem

		The 7th problem		**The 17th problem**	
Grade		Frequency	Percent	Frequency	Percent
	No Correct Solution	9	45	8	40
	Unit rate	0	0	1	5
5	Factor of change	0	0	2	10
	Non-additive Method	11	55	9	45
	Total	20	100	20	100
	No Correct Solution	7	35	10	50
	Unit Rate	3	15	2	10
	Equivalent Fractions	0	0	1	5
6	Build Up	0	0	1	5
	Additive Method	1	5	0	0
	Non-additive Method	9	45	6	30
	Total	20	100	20	100
	No Correct Solution	3	15	15	75
	Unit Rate	12	60	3	15
7	Equivalent Fractions	0	0	1	5
	Non-additive Method	5	25	1	5
	Total	20	100	20	100
	No Correct Solution	9	45	10	50
	Unit Rate	4	20	1	5
	Factor of Change	0	0	4	20
8	Equivalent Fractions	3	15	0	0
	Cross-product	3	15	4	20
	Non-additive Method	1	5	1	5
	Total	20	100	20	100

In the 7th problem, it was given that the package consisting of 16 A brand chocolates was priced at 20 TL and the package consisting of 12 B brand chocolates was priced at 16 TL. The students were asked whether their choice was economical in the case that Merve and Elif preferred to buy the package of 12 B brand chocolates. None of the 5th grade students could solve this problem. As can be seen in Table 20, 55% of them did not realize the multiplicative relationship, and they made an erroneous comparison between the prices of the chocolate packages in an additive way. For example, in the solution from the 5th grade in Figure 42, the student tried to establish a relationship between ratios in an additive way, and she ignored the unit prices of chocolates in both packages. Because this student thought that the difference between 16 TL and 12 chocolates and the difference between 20 TL and 16 chocolates equal to 4 TL, she concluded that both of the chocolate packages were equally economical.

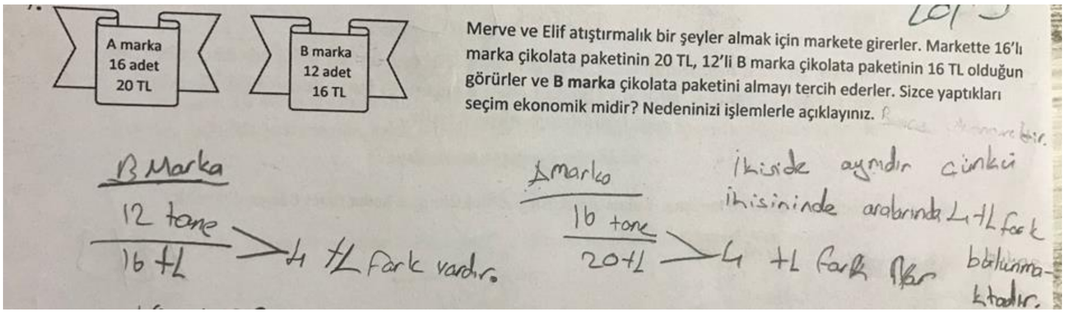


Figure 42 Incorrect use of additive method for the seventh problem

The solution in Figure 43 was another example of erroneous additive method in the 5th grade. In this solution, the student compared only the prices, and thought that the low-cost chocolate package would be more economical by neglecting the number of chocolates in the packages.

The percentage of the students who made an erroneous comparison between the prices of the chocolate packages in an additive way dropped to 45% at the 6th grade. Only 15% of the 6th grade students reached the correct solution by using a strategy (unit rate). The solution from the 6th grade in Figure 44 is an example of the usage of the unit rate strategy. This student found the unit prices of chocolate bars in two packages by dividing the total price of a package into the number of chocolate bars in that package. He concluded that the B branded chocolate package was not more economical, because the unit price of the A branded chocolate bars was lower than the unit price of the B branded chocolate bars.

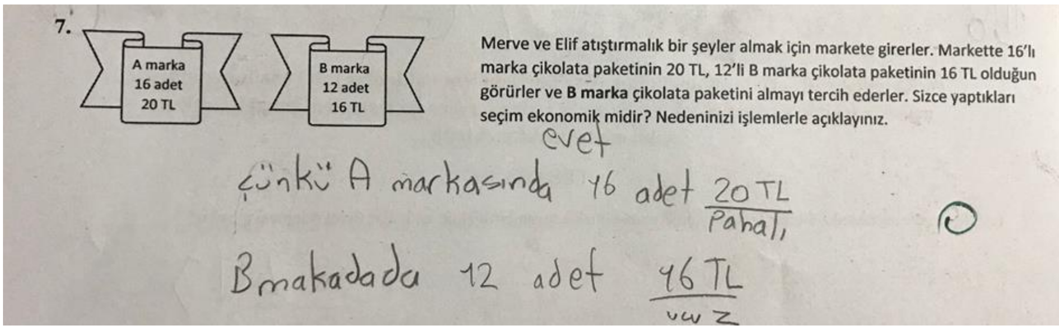


Figure 43 Incorrect use of additive method for the seventh problem

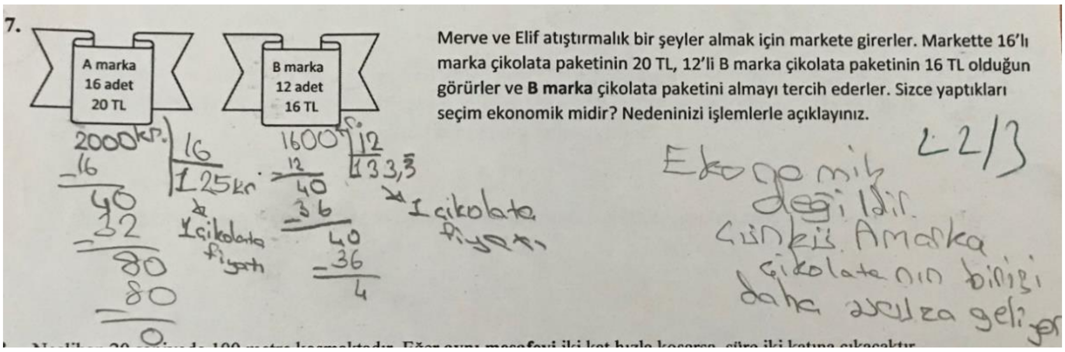


Figure 44 Unit rate strategy for the seventh problem

In addition, the majority of the 7th grade students (60%) reached the correct answer using the unit rate strategy. However, the 7th grade students did not use another strategy.

The 8th grade students reached the correct result using different kinds of strategies in this problem. They used the strategies of unit rate (20%), equivalent fractions (15%) and cross-product (15%). The example in Figure 45 is related to the unit rate strategy. This student naturally made similar mathematical operations to the solution in the example in Figure 45.

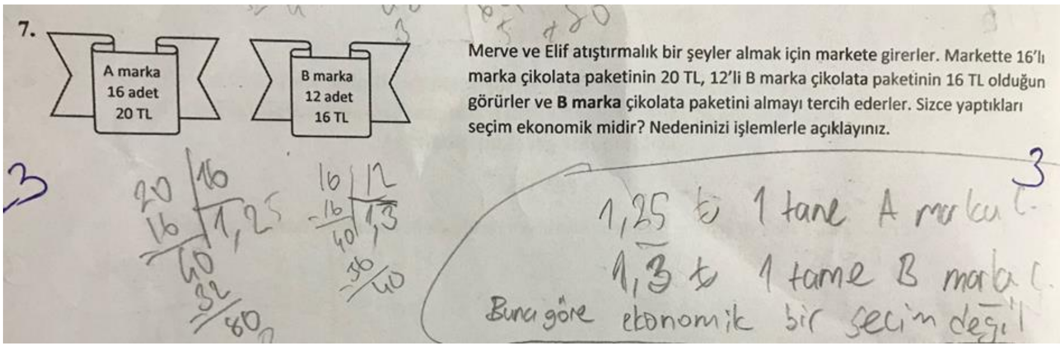


Figure 45 Unit rate strategy for the seventh problem

The solution in Figure 46 is an example of the cross-product strategy in the 8th grade. This student used the cross-product strategy in order to calculate the price of 100 chocolate bars for each chocolate brand. He found that the price of 100 A branded chocolate bars was 125 TL, and the price of 100 B branded chocolate bars was about 133 TL. Thus, he concluded that the B branded chocolate package was not more economical.

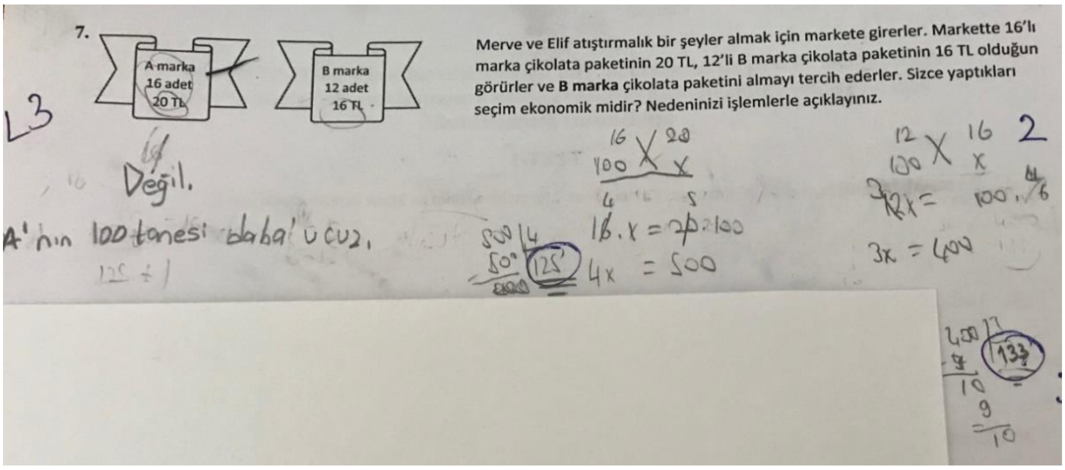


Figure 46 Cross-product strategy for the seventh problem

The solution in Figure 47 belongs to the strategy of equivalent fractions. This strategy was used by only the 8th grade students. This student found the ratios of the price of each package to the number of chocolate bars in both packages. These fractional ratios were the unit prices of the chocolate bars. To compare the unit prices, the student equated the denominators of the fractions. He concluded that the B branded chocolate package was not more economical.

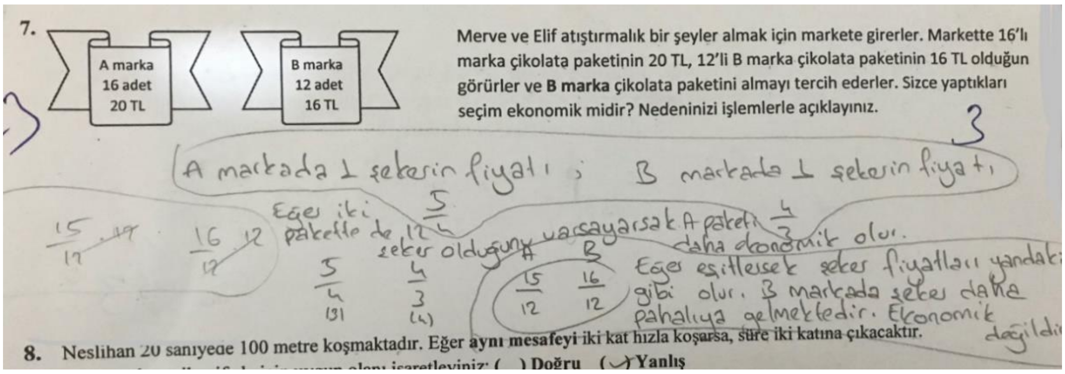


Figure 47 Equivalent fractions strategy for the seventh problem

In the 17th problem, there were two different brands of washing powder. 1 kg of A branded washing powder was priced at 5 TL and it could wash laundry for 20 times. On the other hand, 1.5 kg of B branded washing powder was priced at 6.5 TL and it could wash laundry for 30 times. This problem had two phases. The first phase asked the students whether the expression ‘The A branded washing powder is more economical’ was true or false. In the second phase, the students were asked to mark an option that was appropriate to their answers in the 1st phase from 4 options. Whereas none of the 5th grade students could solve the 7th problem, three students could solve the 17th problem. They used the factor of change and unit rate strategies. The solution from the 5th grade in Figure 48 is an example of the factor of change strategy. Here, the student firstly found that the price of half kilogram of A branded washing powder was 2.5 TL. He showed that if both washing powders had equal economical value, the price of 1.5 kg of the B branded washing powder should be 7.5 TL. He reached this solution by multiplying 2.5 TL by 3, because the weight of the B branded washing powder had three halves kilogram.

At the 6th grade, 4 students could solve the problem. Two of them used the unit rate strategy and others used the equivalent fractions and build-up strategies. The solution from the 6th grade in Figure 49 is an example of the build-up strategy. This student firstly found the weight and price of the A branded washing powder that could be used in 10 washes. Then, he added these weight and price to the same weight and price in order to find the weight and price of the A branded washing powder that could be used in 20 washes. Finally, he added the weight and price of the A branded washing powder that could be used in 20 washes to the weight and price of the A branded washing powder that could be used in 10 washes in order to find the weight and price of the A branded washing powder that could be used in 30 washes. He showed that if both washing powders had equal economical value, the price of the B branded washing powder that could be used in 30 washes should be 7.5 TL. Therefore, he concluded that the B branded washing powder was more economical, because its price for 30 washes was 6.5 TL.

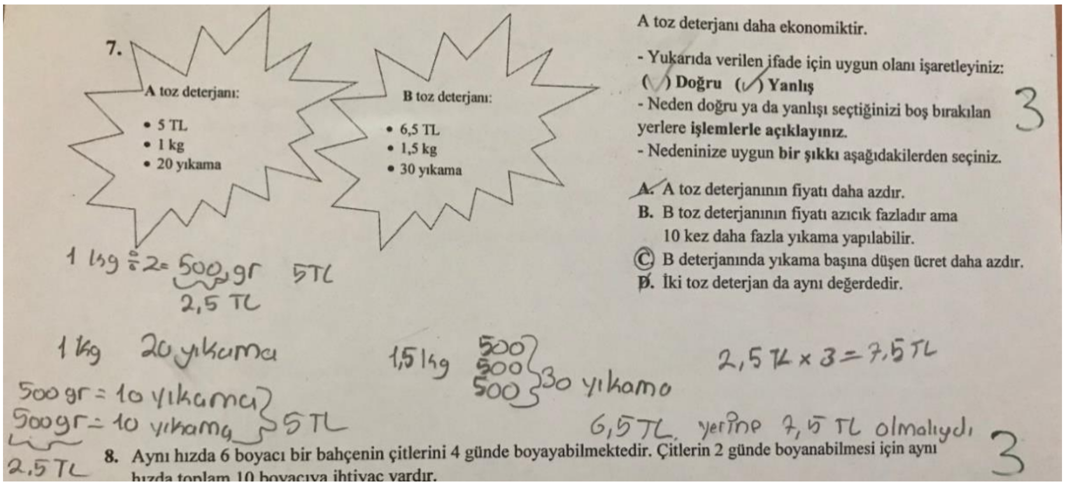


Figure 48 Factor of change strategy for the seventeenth problem

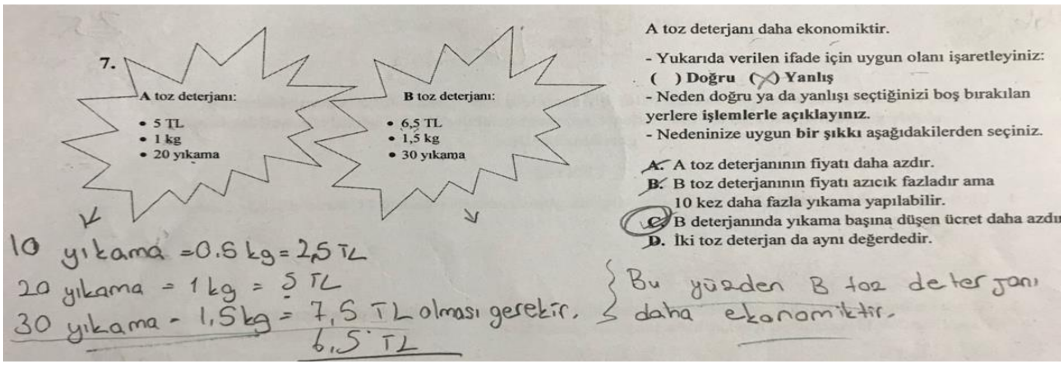


Figure 49 Build-up strategy for the seventeenth problem

The 7th grade students used the strategy of unit rate (15%) and equivalent fractions (5%). The 8th grade students used the strategy of unit rate (5%), factor of change (20%) and cross-product (20%). The rest of these students could not solve the problem or erroneously used the additive method. The solution from the 7th grade in Figure 50 is an example of the erroneous additive method. Because the price and weight of the B branded washing powder was higher than the other, this student thought that two detergents were of equal economic value. He ignored that the unit price of these washing powders could be different.

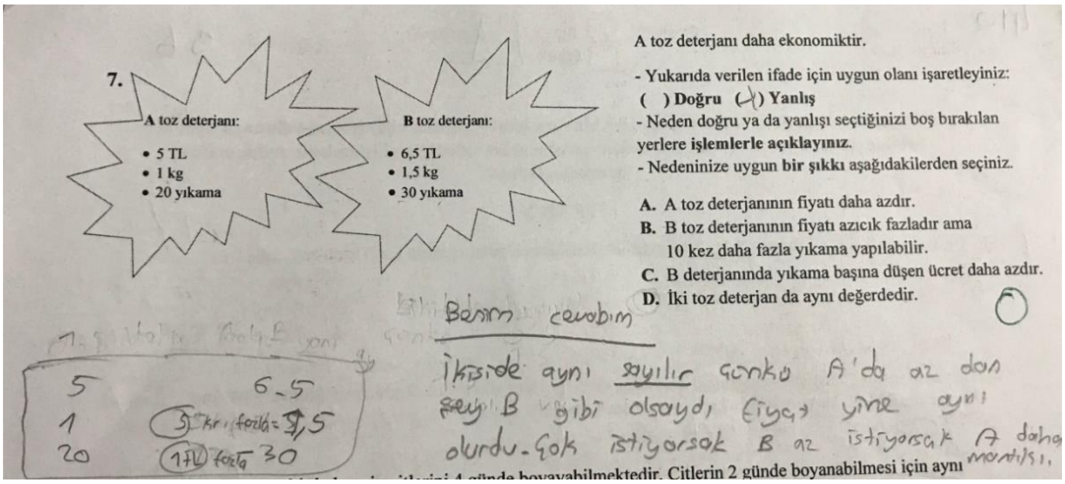


Figure 50 Incorrect use of additive reasoning for the seventeenth problem

The 8th problem with two phases included inverse ratio. In the first phase, the expression "If Neslihan, who could run 100 meter in 20 seconds, ran the same distance twice as fast, the time would double" was given and it was asked whether this statement was true or false. In the second phase, the students were asked to mark an option that was appropriate to their answers from 4 options. These options that the students had marked could show whether they could recognize the inverse ratio without the need for solutions. Therefore, the students were not expected to use any strategy. Similarly, the 18th problem with two phases included inverse ratio. In the 1st phase of the problem, 6 painters with the same speed could paint the fence of a garden in 4 days. The expression "10 painters with the same speed are needed in order to be able to paint these fences in 2 days" was given. The students were asked whether this expression was true or false. In the second phase, the students were asked to mark an option that was appropriate to their answers from 4 options. These options that the students had marked could show whether they could recognize the inverse ratio without the need for any solution. Therefore, the students were not expected to use any strategy.

Table 21 The strategies used in the 9th and 19th problems

		The 9th problem		The 19th problem	
Grade		Frequency	Percent	Frequency	Percent
5	No Correct Solution	17	85	7	35
	Factor of Change	0	0	2	10
	Equivalent Fractions	1	5	0	0
	Non-additive Method	2	10	11	55
	Total	20	100	20	100
6	No Correct Solution	10	50	4	20
	Factor of Change	0	0	6	30
	Equivalent Fractions	9	45	1	5
	Non-additive Method	1	5	9	45
	Total	20	100	20	100
7	No Correct Solution	10	50	7	35
	Factor of Change	2	10	4	20
	Equivalent Fractions	8	40	5	25
	Non-additive Method	0	0	4	20
	Total	20	100	20	100
8	No Correct Solution	7	35	8	40
	Factor of Change	0	0	6	30
	Equivalent Fractions	11	55	2	10
	Cross-product	2	10	2	10
	Non-additive	0	0	2	10
	Total	20	100	20	100

The 9th problem was a numerical comparison problem involving the context of mixture. In this problem, there were three containers consisting of water and sugar cubes. Container A was completely filled with water, and sugar cubes were thrown into it; Container B was filled with water up to half and 2 sugar cubes were thrown into it, and 1/3 of Container C was filled with water and 1 sugar cube was thrown into it. The students were asked whether the water in Container B was the sweetest after the water in the containers were mixed. This problem could be solved by only

1 student from the 5th grade, and by only 9 students from the 6th grade. These students used the strategy of equivalent fractions.

The solution from the 6th grade in Figure 51 is an example of the strategy of equivalent fractions in this problem. This student showed that 3/3 of Container A, 1.5/3 of Container B and 1/3 of Container C was filled with water. She said that if all of Containers B and C were filled with water, 4 cubes of sugar should be put into Container B and 3 cubes of sugar into Container C. In this case, Container B had the sweetest sugar-water mixture.

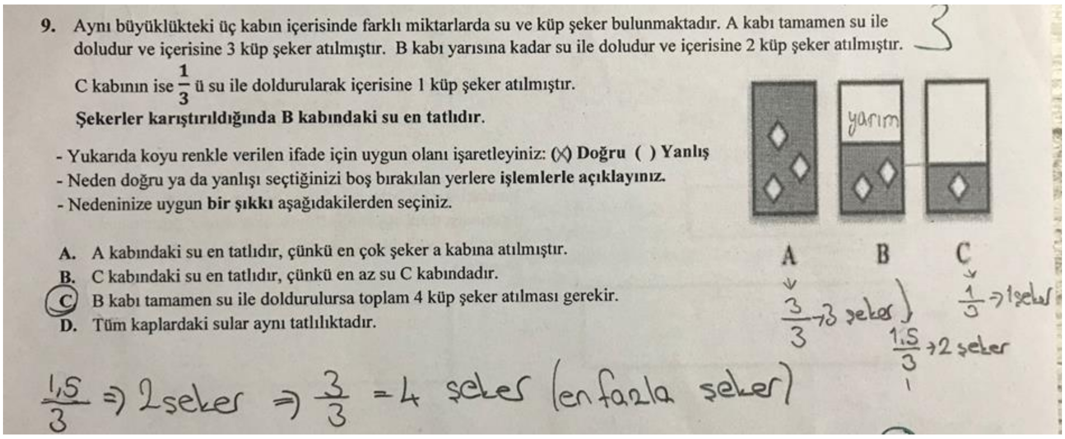


Figure 51 Equivalent fractions strategy for the ninth problem

The 7th grade students used two different strategies to solve this problem: the strategies of factor of change (10%) and equivalent fractions (40%). Like the 7th grade students, the 8th grade students also used two different strategies in order to find the correct solution: the strategies of equivalent fractions (55%) and cross product (10%).

The solution from the 7th grade in Figure 52 is an example of the factor of change strategy. This student thought that Containers B and C were also completely filled with water. In this case, if the amount of water in Container B doubled, the number of cubes should be doubled. If the amount of water in Container C tripled, the number of cubes in the container should be tripled. For these reasons, the student found that the total number of cubes in Container B would be 4, and the total number of cubes in Container C would be 3. Then, she concluded that Container B had the sweetest sugar-water mixture.

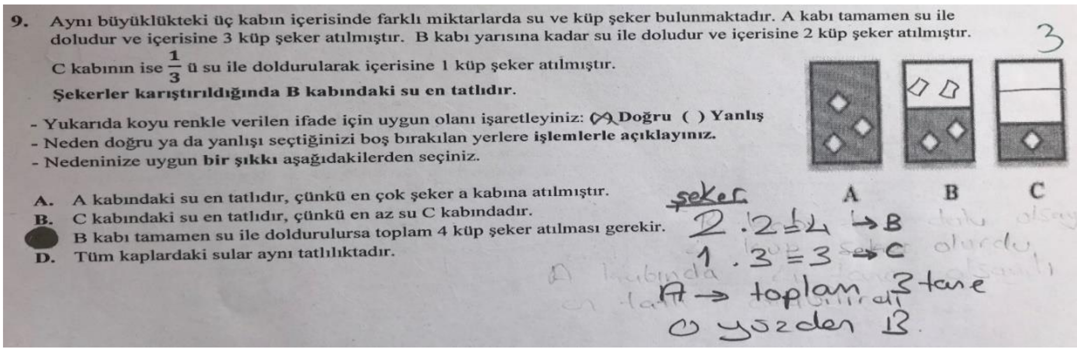


Figure 52 Factor of change strategy for the ninth problem

The 19th problem was a numerical comparison problem involving the context of mixture. In this problem, there were two carafes filled with lemonade. The red carafe had two cups of lemon squash and 4 cups of water. The green carafe had 4 four cups of lemon squash and 6 cups of water. The students were asked which lemonade in the carafes tasted more lemon. Only 2 students from the 5th grade could solve this problem correctly, and these students used the factor of change strategy. Most of the 6th grade students (30%) used the factor of change strategy, and only 1 student reached the correct answer using the strategy of equivalent fractions. Most of the 7th grade students (25%) used the strategy of equivalent fractions, and 20% of the 7th grade students used the factor of change strategy. On the other hand, it was observed that the number of the types of strategies used by the 8th grade students increased to three. 6 students used the factor of change strategy, 2 students used the strategy of equivalent fractions, and 2 students used the cross-product strategy.

The solution from the 6th grade in Figure 53 is an example of the factor of change strategy. This student stated that the number of cups of water was 2 times of the number of cups of lemon squash in the red carafe, but the number of cups of water was less than 2 times of the number of cups of lemon squash in the green carafe. For this reason, the lemonade in the green carafe tasted more lemon.

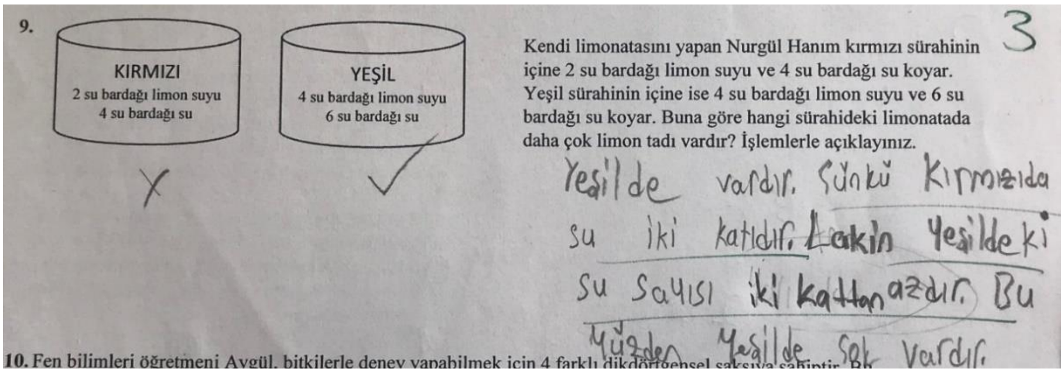


Figure 53 Factor of change strategy for the nineteenth problem

The solution from the 7th grade in Figure 54 is an example of the strategy of equivalent fractions. This student firstly wrote the ratios of the number of cups of lemon squash to the number of cups of water for both carafes. To compare these fractional ratios, he equalized the numerators of the ratios by multiplying the numerator and denominator of the ratio in the red carafe by 2. Therefore, the ratio in the red carafe was 4/8, and the ratio in the green carafe was 4/6. To conclude, the student wrote that when the numbers of cups of the lemon squash were equal each other, the taste of lemon in the lemonade with less water was felt more.

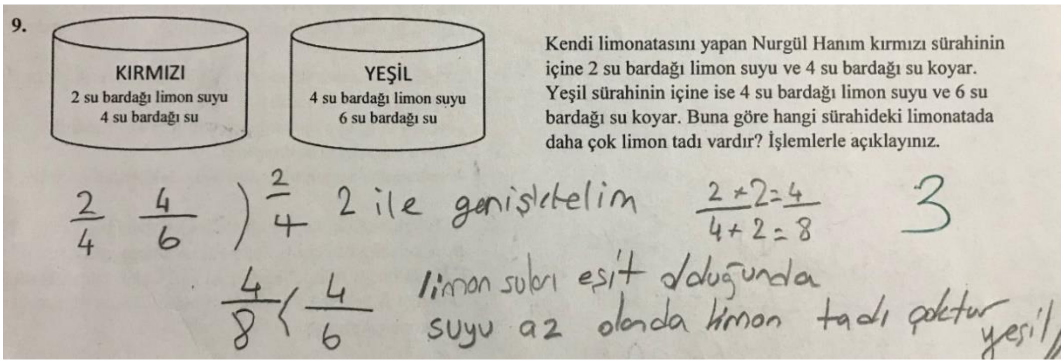


Figure 54 Equivalent fractions strategy for the nineteenth problem

The solution from the 8th grade in Figure 55 is an example of the cross-product strategy. This student firstly calculated the number of cups of lemon squash to be put in a cup of water for both carafes using the cross-product strategy. The number of cups of lemon squash to be put in a cup of water was 0.5 of a cup in the red carafe and about 2/3 of a cup in the red carafe. For this reason, the student concluded that the taste of lemon was more felt in the green carafe, because the number of cups of lemon squash to be put in a cup of water was more than in the red carafe.

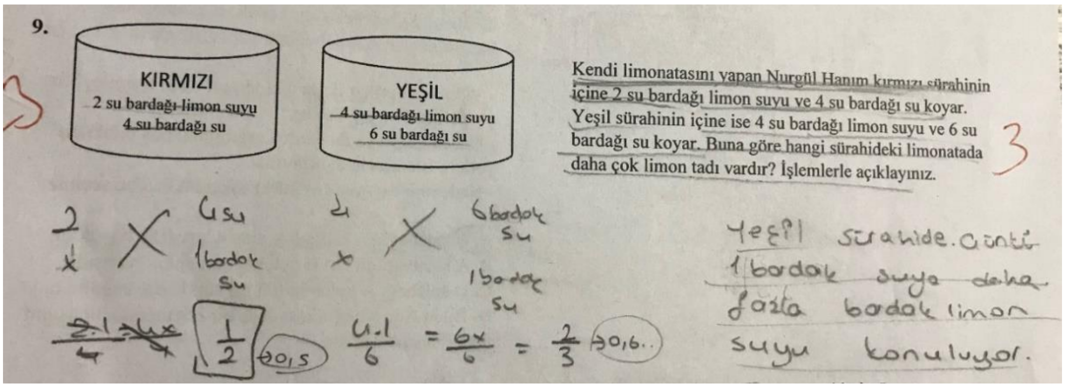


Figure 55 Cross-product strategy for the nineteenth problem

The solution from the 5th grade level in Figure 56 is an example of erroneous additive method. This student stated that the number of cups of water was two more than the number of cups of lemon squash in both carafes. For this reason, he concluded that lemon taste was equally felt in both carafes. This student could not think of the multiplicative relationship between the numbers of the cups of lemon squash and water.

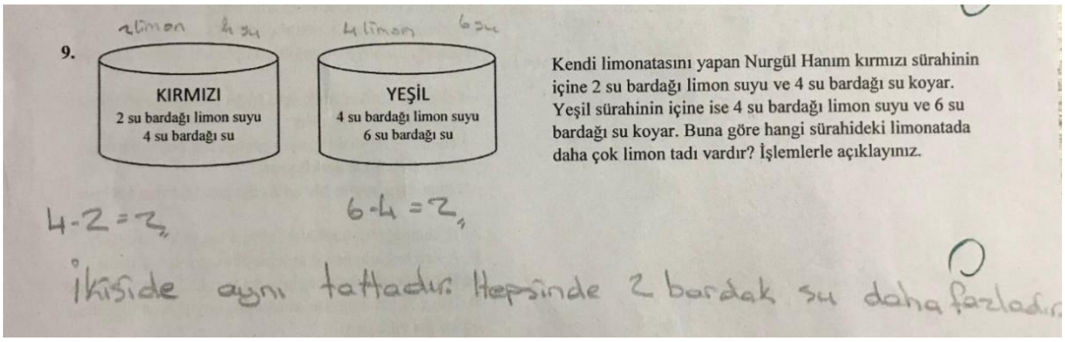


Figure 56 Incorrect use of additive reasoning for the nineteenth problem

In the 10th problem, there were a rectangular photograph with a length of 3 cm and 5 cm and a new photograph which was created with a 200% extension of this photograph. The students were asked which photograph looked more like a square. This problem was a numerical comparison problem. In the solution, the students were expected to write that since all sides of the photograph were enlarged at the same ratio, the enlarged photograph and the original photograph looked like a square equally. Therefore, the students were not expected to use any strategy.

Similarly, in the 20th problem, there were 4 rectangular flowerpots at different lengths. The students were asked which of these rectangles looked more similar to the square shape. The lengths of the rectangular flowerpots were given in the options. They were 27 cm-30 cm, 17 cm-20 cm, 7 cm-10 cm and 37 cm-40 cm in length. This was a numerical comparison problem. Since all sides of a square were equal, the ratio between the sides was 1. For this reason, the students were expected to find in which rectangle the ratio between the lengths of different sides was closest to 1. Since the difference between the lengths of the given sides was 3 in all options, the students were expected to select the rectangle with the largest edge length. Therefore, the students were not expected to use any strategy.

As a summary, it was seen that the students used different strategies according to types and contexts of the proportional reasoning problems and even according to whether the factors of change within and between ratios in the missing value problems were an integer. In general, the 5th, 6th and 7th grade students mostly used the factor of change, unit rate and build-up strategies in the missing value problems, but the 8th grade students used the cross-product strategy in these problems. Moreover, all the students mostly used the strategies of factor of change, equivalent fractions and unit rate in the numerical comparison problems. On the other hand, equivalent class strategy was not used in any problem.

CHAPTER 5

DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

The aims of this study were to specify the academic achievement of the students from the 5^{th} to 8^{th} grade in proportional reasoning problems, to determine whether academic achievement of these students changes according to problem types and to examine their solution strategies in these problems. Based on these aims, in Chapter 1, the importance of proportional reasoning and the necessity of analyzing solution strategies of the students were mentioned. In Chapter 2, the terms related to proportional reasoning, improvement of proportional reasoning skills depending on grade level, additive and multiplicative reasoning, types of proportional reasoning problems and solution strategies, and the results of various studies related to proportional reasoning were mentioned. Next, Chapter 3 focused on the development of the achievement test, methodology and research design. In Chapter 4, both qualitative and quantitative findings were represented. This final chapter focuses on the research questions in line with the qualitative and quantitative findings represented in Chapter IV. In addition, some implications of the study and recommendations for further research are given.

5.1 Discussion of the Findings

5.1.1. Discussion of academic achievements of the students between the 5^{th} and 8^{th} grade in the proportional reasoning test

In the quantitative part of the current study, the first research question, which searches for the academic achievement of 858 students from the 5^{th} to 8^{th} grade in the test of proportional reasoning problems was addressed. After the achievement scores of the students were given, the means of achievement scores were calculated according to the grade levels. The averages of students' achievement scores in PRT increased as the grade level increased. In this case, the mean scores of the 5^{th} grade students were the lowest and the average scores of the 8^{th} grade students were the highest. This finding may be considered to be consistent with the previous studies which concluded that the success of the students increased as the grade level increased (Mersin, 2018; Hilton et al., 2016; Toluk-Uçar & Bozkuş, 2016; Larson, 2013; Van Dooren et al., 2009; Dole et al., 2007). The development of the proportional thinking process requires time and experience and, in this respect, it is emphasized that the studies should be spread over time and students should gain the ability of proportional reasoning through various examples (Baykul, 2009). On the other hand, this finding of the current study did not correspond to the results of the study by Ojose (2015). In the study of Ojose (2015), because the lower grade students were more successful than higher grade students in some types of problems, the students' solutions showed that the increase in the grade level did not mean that students would perform better in proportional reasoning problems. Another result of the study of the Ojose (2015) was that the sixth and seventh grade students in the study were not taught the proportional reasoning concept. The analysis and the interviews showed that all grade level students could have a conceptual understanding of the subject of proportion without the need to be taught the concept. This result was consistent with another finding of the current study. In the Turkish Education System, ratio, which is one of the components of proportional reasoning, is first taught in the 6^{th} grade, while the proportion topic is introduced in the 7^{th} grade. However, at the time of conducting the test of this study, the concept of proportion was not taught to the 7^{th} grade students yet. For this reason, in the current study, the 5^{th}, 6^{th} and 7^{th} graders were not formally taught the concept of proportional reasoning, but they already had a mathematical feel of proportional reasoning in their schemes before formal teaching. These students were able to reach the correct solutions to the problems, proving the existence of their proportional reasoning. On the other hand, even though the fifth, sixth and seventh grade students had proportional reasoning skills, the percentage of the students at these grade levels who correctly solved the problems in the test was quite low compared to the percentage at the eighth grade. At this point, it can be said that both formal learning of proportionality in the lesson and gaining experience in proportional reasoning problems improve students' proportional reasoning skills.

The test instrument of the study (PRT) consisted of missing value, numerical comparison and inverse ratio problems. The percentage of students who could not solve especially numerical comparison problems and scored zero points was quite high. For all the grades, the percentage of the students who got 3 points from the missing value problems was higher than from the numerical comparison problems. For this reason, it could be said that the types of the problems that the students were most successful in solving were missing value problems. Similar findings related to students' better solving of missing value problems than numerical comparison problems were reported in the study by Ben-Chaim et al. (1998) and Özgün-Koca and Kayhan-Altay (2009). Ben-Chaim et al. concluded that the percentage of the students who could correctly solve the missing value problems was higher than the percentage of the students who could correctly solve the numerical comparison problems. It might be related to the fact that students did not encounter comparison problems very much in the classroom environment. Additionally, it might be related to the fact that missing value problems were simpler than numerical comparison problems. In missing value problems, three values are given, and the students are asked to find the fourth one. However, in numerical comparison problems, all four values are given, and students are asked to compare ratios. This was difficult for students. Most students might not know what kind of operations they needed to do for comparison, because all four values are given in the problem. As far as inverse ratio problems were concerned, the 8th and 18th problems were of this kind of proportional reasoning problems. These problems were aimed at understanding whether students knew the concept of inverse proportion, and they did not require any mathematical solution. 24.3% of the 5th, 30.1% of the 6th, 39.8% of the 7th and 63% of the 8th grade students got full point from the 8th problem, and in the 18th problem, 42.4% of the 5th, 46.9% of the 6th, 56.6% of the 7th and 72.5% of the 8th grade students could realize the inverse relationship between the quantities. These percentages may not be considered very low compared to other problems. Taking into account the fact that the fifth, sixth and seventh grade students have not been taught these subjects yet, it might be said that the concept of inverse proportion has started to develop in these students. This finding might be considered as compatible with the results of the study by Mersin (2017). In that study, the fifth and sixth grade students often found it difficult to answer problems with an inverse proportion. The difficulty of these students in these problems showed that the concept of proportion was still developing. In such problems, the seventh-grade students were more successful because they had already encountered the concept of proportion, as 8th grade students were more successful in the current study. In addition, it was seen that the percentages in the 18th problem was a little higher than the 8th problem. This might be due to the context of the problems. The 8th problem was related to speed and time, and the 18th problem was related to how the number of painters changed the dyeing time. This might be due to the fact that the students were more able to relate the 18th problem to daily life.

In addition to problem types, numerical structures of the problems affected the achievement of the students in the current study. The students in all grade levels showed better performances when the factor of change within and between ratio in the missing value problems was an integer. The percentages of the students who got 3 points from the missing value problems were 58.7% in the 1st problem where factor of change between ratios was an integer, 39.4% in the 2nd problem where factor of change within ratios was an integer, 21.1% in the 4th problem where factor of change within ratios was an integer and 29.5% in the 6th problem where factor of change within ratios was an integer. On the other hand, the percentage of the students who got 3 points from the 3rd problem where neither factor of change was an integer was 14.8. They had difficulty solving the missing value problems when the factor of change between and within ratio was a non-integer. In this case, students could find a multiplicative relationship neither between nor within ratios from their mind, and it might be difficult for students to make operations with decimal numbers. This often leads students to use additive methods incorrectly or not to try to solve the problem. This finding corresponded to the findings of previous studies in which number structure of missing value problems was reported (Riehl & Steinthorsdottir, 2017; Artut & Pelen, 2015; Dooren, Bock and

Verschaffel, 2010; Tjioe & Torre, 2014; Heller et al., 1989). Moreover, the students were able to solve problems where the factor of change between ratios was an integer better than the problems where the factor of change within ratios was an integer. This might be due to the fact that problems where the factor of change between ratios was an integer lead to students to find the unit rate. For example, in the 1st problem, three apples were given 90 cents and students were asked to calculate how many cents would be given for 7 apples. Here, the ratio of three apples to 90 cents was expressed as the factor of change, and additionally, it was easy to find the price of one apple for most students. The percentage of these students was 58.7%. On the other hand, in the 2nd problem, the price of 24 books in a bookstore where 6 books were 4 TL was asked. Here, the price of a book was a decimal number, because the factor of change between ratios was a non-integer. Hence, most students had difficulty finding the unit rate. The students who realized that the factor of change within ratios was an integer were able to solve the problem correctly. The percentage of these students was 39.4%; this was quite low compared to the 1st problem. Therefore, the students were able to solve problems where the factor of change between ratios was an integer better than the problems where the factor of change within ratios was an integer. A similar result to the current study was not found in the literature, but a study with the opposite result was found. This finding did not correspond to the result of the study by Riehl and Steinthorsdottir (2017). They showed that when problems included only one integer ratio, students were more successful when factor of change within ratios was an integer. They concluded that easiest problems had an integer factor of change within ratios. On the other hand, students had the most difficulty in solving the problems where both factor of change between and within ratios were not integers. The 3rd and 13th problems were the examples to this type of problem. The percentage of the students who got 3 points was 14.8% in the 3rd problem and 30.8% in the 13th problem. These percentages were higher in the 13th problem. The reason for this might be that even if the change factor was not an integer, calculating how many times 10 minutes were 4 minutes might be easier than calculating how many times the 30 minutes were 12 minutes.

5.1.2. Discussion of strategies mostly used by the students between the 5th and 8th grade in the proportional reasoning test

In qualitative part of the current study, the second research question of the study about the strategies mostly used by these strategies in the proportional reasoning problems was addressed. The solution strategies used by the students in each problem were examined in detail.

In the current study, the mostly used strategy was factor of change at the 5th, 6th and 7th grades. This finding might be considered different from the results of most studies in the literature. The unit rate strategy was mostly preferred by the students to solve the proportional reasoning problems according to the result of these studies (Kahraman, Kul & İskenderoğlu, 2018; Küpçü, 2008; Kayhan, 2005; Özgün-Koca & Kayhan-Altay, 2009; Cramer & Post,1993; Christou & Philippou, 2002; Pakmak, 2014). There were a few studies in the literature which had similar results with the current study. According to the study of Avcu and Doğan (2014) and Avcu and Avcu (2010), the students mostly used the factor of change strategy to solve the proportional reasoning problems. Since the subject of proportion was not included in the curriculum, it might be thought that the fifth, sixth and seventh grade students intuitively solved the proportional reasoning problems by using the factor of change strategy. Moreover, the most commonly used strategy after the factor of change strategy was the unit rate strategy as in most studies in the literature. On the other hand, the 8th grade students mostly used the cross-product strategy to solve the proportional reasoning problems in the current study. This finding might be considered as consistent with most of the previous studies which reported the mostly used strategies by 8th grade students (Kahraman, Kul & İskenderoğlu, 2018; Küpçü, 2008; Duatepe, Akkuş-Çıkla & Kayhan, 2005; Incebacak & Ersoy, 2016; Cramer & Post, 1993). This might be due to the fact that the 8th grade students had already been taught the subject of proportion and especially the cross-product strategy to solve proportional reasoning problems. For this reason, it was a predictable result. Although most people define proportional reasoning with the usage of cross-product strategy, researches show that the correct proportional reasoning does not involve merely understanding fractions and rational numbers, but also competence in other areas such as ratio sense, relative thinking, partitioning, unitizing and changing quantities (Lamon, 1999). Therefore, teaching of cross-product algorithm is not approved by many mathematic educators (Dole, Wright & Clarke, n.d). When the students who reached the correct results of the problems with the cross-product strategy were examined, it was seen that most of them had difficulty in solving the problems except the missing value problems. Numerical comparison problems in PRT of the current study could be solved by very few students. Students can develop numerical comparison strategies after solving proportion problems with informal reasoning skills. Thus, before students learn the rules for proportional reasoning, they can construct their own informal knowledge and develop concepts for proportional reasoning (Uçar & Bozkuş, 2016).

The 1st and 11th problems of was missing value problems in which the factor of change was an integer and which was suitable for finding the unit rate to be solved. Therefore, most of the students used the unit rate strategy in order to solve the problem. According to the study of Christou and Philippou (2002), when the numbers in the problems did not allow students to calculate the unit rate, they turned to other solution strategies such as the build-up strategy, which is one of the simplest methods to solve the problem. However, in the current study, when the numbers in the problems did not allow students to calculate the unit rate, most of the students firstly preferred the factor of change strategy. The 2nd problem illustrates this situation very well. Most of the students used the factor of change strategy to solve this problem. On the other hand, the students could not even use the factor of change strategy to solve the 3rd and 13th problems, because these problems were missing value problems in which the factor of change was a non-integer. Therefore, they mostly used the build-up strategy. Most of the students, including even the 8th grade students, used the factor of change strategy to solve the 4th and 14th problems. The fact that the context of the problem was part-part-whole might have been effective for the 8th grade students to use the factor of change strategy instead of the cross-product strategy. The 5th and 15th problems had the context of similarity which was accepted as one of the most difficult contexts for students. In the 5th problem, none of the 5th and 6th grade students could use a strategy, while most of the 7th grade students used the factor of change strategy even though the factor of change was non-integer, and most of the 8th grade students used the cross-product strategy. These finding corresponded to the study related to the fact that the solution strategies and achievement level of students were affected by the context of the problem (De La Cruz, 2013). Although the 7th, 9th, 17th and 19th problems were numerical comparison problems, the mostly used strategies for solving these problems were different. The mostly used strategy was equivalent fractions in the 7th and 9th problems, while the mostly used strategy was factor of change strategy in the 17th and 19th problems. The reason for using different strategies in the same type of problems might be the numerical structure of the problems. To be more specific, in the 7th problem, it was given that the package consisting of 16 A brand chocolates was priced at 20 TL and the package consisting of 12 B brand chocolates was priced at 16 TL. In short, the students were expected to determine which chocolate package was more economical. Similarly, in the 17th problem, there were two different brands of washing powder. 1 kg of A branded washing powder was priced at 5 TL and it could wash laundry for 20 times. On the other hand, 1.5 kg of B branded washing powder was priced at 6.5 TL and it could wash laundry for 30 times. In short, the problem asked which detergent brand was more economical. Although the factor of change between and within ratios in both problems was not an integer, it was easy to find the factor of change in the 17th problem. Because 1.5 kg was 1.5 times 1 kg, multiplying 5 TL by 1.5 was easy for most students in the 17th problem. Therefore, the factor of change strategy was mostly used in the 17th. On the other hand, it was not easy to figure out how many times 20 was 16 or how many times 16 was 12 in the 7th problem. For this reason, most students could not calculate the factor of change in any way. Therefore, the students found which package was more economical by calculating the unit price of chocolates, by equalizing the number of chocolates in the packages, or by equalizing the price of the packages. The mostly used strategy was equivalent fractions in the 7th problem. Shortly, the reason for using different strategies in the same type of problems might be the numerical structure of the problems. This finding corresponds to the study of Fernandez et al. (2011) which was related the effects of the numerical structure of the proportional reasoning problems. They found that the magnitude of the numbers in the problems and the numerical relationship between the quantities greatly affected how the students solved the problems. Additionally, the problems in which the additive method was mostly used and the additive method was used incorrectly were the 7^{th}, 9^{th}, 17^{th} and 19^{th} problems. This might be explained by the fact that students did not know how to develop a solution because they did not encounter numerical comparison problems very often. In addition, incorrect use of the additive method to solve the multiplicative reasoning problems decreased from the 5^{th} to the 8^{th} grade level. This might mean that school practices and age probably played a decisive role in the improvement of logic of ratio and proportion. This finding might be considered as consistent with most of previous studies in the literature (Van Dooren, De Bock & Verschaffel, 2010; Dole, Wright & Clarke, n.d; Dole et Al., 2007; Hilton et Al., 2016; Mersin, 2018; Küpçü & Özdemir, 2011; Doğan & Çetin, 2009). In order to overcome these difficulties such as using the additive method incorrectly to solve the multiplicative reasoning problems, it is stated that teachers should support the development of students. From this point of view, it is emphasized that it is a useful first step for teachers to identify the difficulties that students experience in proportional reasoning in order to support students to develop appropriate additive and multiplicative reasoning strategies (Van Dooren et al., 2005; Bright et al., 2003).

5.2. Implications for Mathematics Education

The findings of the current study showed that the academic achievements of the students in proportional reasoning increased as the grade level increased. This increase indicates that school practices and age probably played a decisive role in the improvement of logic of ratio and proportion. Although the achievement level increased with the grade level, the average achievement scores of the students were low in general. In addition to this, it was observed that students used a limited number of strategies, left some types of problems unanswered, and used strategies incorrectly in some types of problems. This might be an indication that students generally have low proportional reasoning skills. Even though the low averages of the fifth, sixth and seventh grade students could be considered normal because they had not yet encountered the content of proportionality, the average of 8th grade students below 30 out of 60 was an indicator of low success. Although the secondary school mathematics curriculum includes many important concepts, one of the most common one is proportionality. To understand mathematics at high school and college level, it is essential to grasp proportion in middle school years (Johnson, 2010). At this point, students need to be experienced more often with situations aimed at improving their proportional reasoning skills in these middle school years. In addition, proportional reasoning is necessary not only for mathematics courses but also for many other disciplinary areas, which is a topic that needs to be emphasized (Lesh, Post, & Behr, 1989). Therefore, it may be useful for teachers to prepare lesson plans by identifying students' wrong strategies and conceptual and operational deficiencies. Instead of solving certain types of proportional reasoning problems, it may be more beneficial to have students solve all kinds of problems mentioned in the current study. To be more specific, students are used to solving missing value problems, but they have difficulty solving numerical comparison problems. Students are successful in solving the problems where the factors of change within or between ratios are integers, but they have difficulty when the factors of change are non-integers. Additionally, they have difficulties in solving problems whose contexts are similarity or mixture. Therefore, these types of problems should be included frequently in textbooks, and teachers should ensure that their students solve these kinds of problems. Enabling students to solve these types of problems frequently will be effective in terms of using different strategies in their solutions.

Experiences from daily life and school life play an important role in proportional reasoning skills. In the early years of childhood, children face proportional relationships in simple forms (Van den Brink & Streefland, 1979). Before students learn the rules for proportional reasoning, they can construct their own informal knowledge and develop concepts for proportional reasoning (Uçar & Bozkuş, 2016). To verify these facts, in the current study, although the 5th, 6th and 7th grade students were not taught about the subject of proportionality, they were able to reason proportionally in the proportional reasoning problems. However, their solutions of the problems showed that their proportional reasoning skills were very limited. The test of the current study consisted of missing value, numerical comparison and inverse ratio problems. All the students, including the 8th grade students, were able to partially solve the missing value problems better, while most of the students had difficulty solving the numerical comparison problems. Because missing value problems were classified according to their context and numerical structure, it was observed that most of the students could not solve some types of the missing value problems. The reason why students had difficulty in solving these kinds of problems might be that they were not experienced in solving a wide range of proportional reasoning problems in the classroom environment. In order to enable students to solve different types of proportional reasoning problems, teachers should direct students to reflect on them by presenting different problems in addition to the problems in the textbook.

Another finding of the current study was that the students mostly used the unit rate, factor of change and cross-product strategies to solve the proportional reasoning problems. While equivalent fractions and build-up strategies were rarely used, the equivalent class strategy was never used. In order to prevent the students from using certain strategies, the problems posed by the teachers to the students in the classroom should force students to use different strategies. In the usage of different strategies by the students, teachers need to take an active role in teaching proportion ratio concepts. Teachers need to solve a lot of kind of problems in the classroom that will encourage students to use different strategies. In this way, students will be looking for a different solution since they cannot use strategies like the unit rate and factor of change that they are used to. Another reason for this situation might stem from the fact that teachers might not be aware of these strategies. Thus, it would be beneficial if teachers thoroughly learn these different strategies and share them with their students. It is important for teachers to understand proportional reasoning with different viewpoints, to apply specific teaching strategies in order to improve students' proportional reasoning and to develop basic concepts related to proportional reasoning (Hilton et al., 2016). At this point, it may be more effective to attach importance to identifying and developing the proportional reasoning skills of prospective teachers. For this reason, different solution strategies should be discussed more in the lessons.

Additionally, the 8th grade students used mostly the cross-product strategy, because they had been already taught the subject of ratio and proportionality and cross-product strategy. However, the students who used this strategy also had difficulties in finding the right answer as they could not remember the strategy. In the ratio and proportion problems, most of the middle school students use the cross-multiplication to solve the proportion and then, find the missing value (Cramer & Post, 1993). Nevertheless, this method is identified as memorization such that it cannot be said that these students solve the proportion through proportional reasoning. Thus, the teaching of cross-multiplication algorithm is not approved by many mathematic educators (Dole & Wright, n.d). The students who can reason proportionally solve the proportional problem situations, distinguish the proportional and non-proportional situations and especially comprehend the mathematical relationships in the multiplicative proportional problems (Cramer, Post & Currier, 1993). In brief, it cannot be claimed that all the students who can solve the proportional problems can reason proportionally. Thus, it is important to support conceptual understanding before moving on to procedural strategies. In this way, students might have a chance to solve problems correctly when they have difficulty in remembering the formulas.

5.3. Recommendations for Further Research Studies

The participants of the current study were selected based on convenience sampling from the accessible population which included public schools in Mamak District of Ankara. The first recommendation can be made regarding the sample of the study. The same study can be conducted with a larger sample randomly selected from nationwide schools in a way to represent the fifth, sixth, seventh and eighth grade students in Turkey. Thus, the results of the study can contribute to a larger number of students. Another recommendation may be to include primary and high school students in the sample of the study in a longitudinal study. Thus, the development of students' proportional reasoning skills can be observed more comprehensively from primary to high school years. On the other hand, by applying the study to a smaller number of students instead of applying to more students, the proportional reasoning skills of the students can be examined in depth through interviews. Thus, the strategies students use, the origin of the misuse of the strategies, or the reason why students have difficulty in some types of problems may be better understood. In this study, students' academic success in solving proportional reasoning problems and the strategies they used to solve the problems were mostly focused on. In the qualitative part of the current study, even though some attention was paid to the misuse of the additive solution methods in multiplicative cases, there was not much focus on why students had these misuses. Therefore, further research studies can focus on why students might have had these misuses to prevent them.

The test instruments of the current study consisted of missing value, numerical comparison and inverse ratio problems. Qualitative proportional reasoning should precede quantitative proportional reasoning and it should be seen as a necessary element for proportional reasoning, not just complementary (Kadijevic, 2002). For this reason, future studies can conduct a test including qualitative comparison problems in addition to the missing value, numerical comparison and inverse ratio problems.

Additionally, whether the fact that the students use certain strategies to solve the problems is related to how textbooks or teachers teach the subject of ratio and proportion can be investigated. For this purpose, a study can be conducted to examine how the subject of proportion is taught in the textbooks used by middle school students. Moreover, with a sample of mathematics teachers working in different middle schools, a qualitative study can be conducted to understand the level of proportional reasoning skills of these teachers and how they teach their students the subject of proportion.

REFERENCES

Akar, G. K. (2009). Oran konusunun kavramsal öğreniminde karşılaşılan zorluklar ve çözüm önerileri. *İlköğretimde Karşılaşılan Matematiksel Zorluklar ve Çözüm Önerileri*, 263-285. Pegem Akademi.

Akatugba, A. H., & Wallace, J. (2009). An integrative perspective on students' proportional reasoning in high school physics in a West African context. *International Journal of Science Education, 31*(11), 1473–1493.

Akkus, O. ve Duatepe-Paksu, A. (2006). Orantısal akıl yurutme becerisi testi ve teste yonelik dereceli puanlama anahtarı gelistirilmesi, *Eğitim Arastırmaları*, 6(25),1-10.

Aladağ, A. (2009). *İlköğretim öğrencilerinin orantısal akıl yürütmeye dayalı sözel problemler ile gerçekçi cevap gerektiren problemleri çözme becerilerinin incelenmesi.* Yüksek lisans tezi, Çukurova Üniversitesi, 2009.

Arıkan, E. E. Ünal, H. (2015). Investigation of problem-solving and problem-posing abilities of seventh-grade students. *Educational Sciences: Theory & Practice*, 15(5), 1403-1416.

Arican, M. (2018). Preservice middle and high school mathematics teachers' strategies when solving proportion problems. *International Journal of Science and Mathematics Education*, 16, 315–335. doi 10.1007/s10763-016-9775-1.

Artut, P. Aladağ, A. (2012). Öğrencilerin orantısal akıl yürütme ve gerçekçi problem çözme becerilerinin incelenmesi. *İlköğretim Online*, 11(4), 995-1009.

Artut, P. Pelen, M. (2015). 6th grade students' solution strategies on proportional reasoning problems. *Procedia - Social and Behavioral Sciences,* 197, 113 – 119.

Atabaş, Ş , Öner, D . (2016). An examination of Turkish middle school students' proportional reasoning. *Boğaziçi University Journal of Education*, 33 (1), 63-85. Retrieved from http://dergipark.org.tr/buje/issue/29688/319298.

Avcu, R. Avcu, S. (2010). 6th grade students' use of different strategies in solving ratio and proportion problems. *Procedia Social and Behavioral Sciences*, 9, 1277–1281.

Avcu, R. Doğan, M. (2014). What are the strategies used by seventh grade students while solving proportional reasoning problems? *International Journal of Educational Studies in Mathematics*, 1(2), 34-55.

Babai, R., Cohen, E., & Stavy, R. (2018). Proportional reasoning: Reducing the interference of natural numbers through an intervention based on the problem-solving framework of executive functions. *Neuroeducation*, 5(2), 109-118.

Baxter G. P., Junker, B. A (2001) *Case study in proportional reasoning.* Paper presented at the annual meeting of the National Council for Measurement in Education Seattle, Washington.

Baykul, Y. (2014*). İlkokulda Matematik Öğretimi* (12. Baskı). Ankara: Pegem Akademi.

Ben-Chaim, D., Fey, J. T., Fitzgerald, W. M., Benedetto, C., & Miller, J. (1998). Proportional reasoning among 7th grade students with different curricular experiences. *Educational Studies in Mathematics*, 36, 247-273.

Bezuk, N. (1988). *Effects of the numeric ratio on strategies used by preservice and inservice elementary teachers on proportional reasoning word problems.* Proceedings of the 10th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, 72–78. Dekalb, IL.

Bright, G. W., *Jeane* M. *J.*, & *Charles* W., (*2003).* Assessing Proportional. Thinking. *Mathematics Teaching in Middle School*, 9, 166–72.

Bock, D., Dooren, W., Janssens & D., Verschaffel, L. (2002). Improrer use of linear reasoning: an in-depth study of the nature and the irresistibility of secondary school students‟ errors. *Educational Studies in Mathematics,* 50, 311-334.

Boyer, T.W., Levine, S.C. & Huttenlocher, J. (2008). Development of proportional reasoning: Where young children go wrong. *Developmental Psychology*, 44(5), 1478-1490.

Boyer, T. W., & Levine, S. C. (2015). Prompting Children to Reason Proportionally: Processing Discrete Units as Continuous Amounts. *Developmental Psychology*. 2015, 51(5), 615–620.

Cai, J. & Sun, W. (2002). Developing students' proportional reasoning: A Chinese perspective. B. Litweller, G. Bright (Ed.), *2002 Yearbook, Making sense of fractionsi ratios, and proportions*, 87-99. Reston, Virginia: National Council of Teachers of Mathematics.

Christou C., Philippou, G. (2002). Mapping and development of intuitive proportional thinking. *Journal of Mathematical Behavior*, 20, 321–336.

Clark, F. B., & Kamii, C. (1996). Identification of multiplicative thinking in children in grades 1-5. *Journal for Research in Mathematics Education*, 41-51.

Clark, K. ve Lesh, R. (2003). Whodunit? Exploring proportional reasoning through the footprint problem. *School Science and Mathematics*, 103(2), 92-98.

Cooper, B., Harries, T. (2002). Children's responses to contrasting 'realistic' mathematics problems: Just how realistic are children ready to be? *Educational Studies in Mathematics*, 49, 1- 23.

Cramer, K. ve Post, T. (1993). Connecting research to teaching proportional reasoning. Mathematics Teacher, 86(5), 404-407.

Cramer, K. A., Post, T. R. & Currier, S. (1993). Learning and teaching ratio and proportion: Research implications. In D. Owens (Ed.), *Research ideas for the classroom* (pp.159–178). New York: MacMillen Publishing Company.

Creswell, J. W. (2003). *Research design: Qualitative, quantitative, and mixed methods approaches* (2nd ed.). Thousand Oaks, CA: Sage.

Creswell, J.W. (2006). *Understanding Mixed Methods Research*, (Chapter 1). Available at: http://www.sagepub.com/upm-data/10981_Chapter_1.pdf.

Çelik, A., Özdemir, E. Y. (2011). İlköğretim öğrencilerinin orantısal akıl yürütme becerileri ile oran-orantı problemi kurma becerileri arasındaki ilişki. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi*, 30, 1-11

Çıkla, O. A., Duatepe, A., Kayhan, M. (2005). An investıgation on students' solution strategies for different proportional reasoning items. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, 28, 73-81.

De la Cruz, J. A. (2008). *A Look at seventh grade mathematics teachers' instructional decisions related to proportional reasoning: A Cognitively guided perspective*. Doctoral dissertation, Assumption College.

De la Cruz, J. A. (2013). Selecting Proportional Reasoning Tasks.

De Bock, D. (2002). *The illusion of linearity: An empirical analysis of secondary school students' improper proportional reasoning in geometry problems.* Unpublished doctoral dissertation, University of Leuven, Leuven, Belgium.

Dole, S., Wright, T. (nd). Proportional Reasoning. School of Education.

Dole, S., Wright, T., Clarke, D. and Hilton, G. (2007). Making connections science and mathematics: The MCSAM Project. In U. Cheah, Y. Wahyudi, R. Devadason, K. Ng, J. Chavez, and D. Mangao (Eds.), *Redefining learning culture for sustainability* (pp.184–194). Second International Conference on Science and Mathematics Education, Penang, Malaysia, 13–16 November 2007.

Dooley, B. (2006). *An Investigation Of Proportional Thinking Among High School Students*. A Dissertation Presented to the Graduate School of Clemson 85 University, In Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy Curriculum and Instruction, December 2006.

Duatepe, A., Akkuş-Çıkla, O. & Kayhan, M. (2005). Orantısal akıl yürütme gerektiren sorularda öğrencilerin kullandıkları çözüm stratejilerinin soru türlerine göre değişiminin incelenmesi. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, 28, 73-81.

Ercan, İ., Kan, İ. (2004). Ölçeklerde Güvenirlik ve Geçerlik. *Uludağ Üniversitesi Tıp Fakültesi Dergisi* 30(3), 211-216.

Fischbein, E., Deri, M., Nello, M. S., & Marino, M. S. (1985). The role of implicit models in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education*, *16*, 3–17.

Fernández, C., Llinares, S., Van Dooren, W., De Bock, D., & Verschaffel, L. (2012). The development of students' use of additive and proportional methods along primary and secondary school. *European Journal of Psychology of Education*, 27(3), 421-438.

Fernandez, C., Llinares, S., Van Dooren, W., De Bock, D. & Verschaffel, L. (2011). Effect of number structure and nature of quantities on secondary school students' proportional reasoning. *Studia Psychologica 53*(1), 69–81.

Frankel, J. R., Wallen, N. E., Hyun, H.H. (2006). How to design and evaluate research in education. Mcgrow Hill, Inc.

Fuson, K. C., & Abrahamson, D. (2005). Understanding ratio and proportion as an example of the apprehending zone and conceptual-phase problem-solving models. In J. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 213–234). New York: Psychology Press.

Glaser, K., Riegler, P. (2015). Beginning students may be less capable of proportional reasoning than they appear to be. *Teaching Mathematics and Its Applications*, 34, 26-34

Hart, K. (1988). Ratio and proportion. In J. Hiebert, & M. Behr (Eds.). *Number concepts and operations in the middle grades* (pp. 198–219). Reston, VA: National Council of Teachers of Mathematics.

Heinz, K., & Sterba-Boatwright, B. The when and why of using proportions. Mathematics Teacher, 2008, 101(7), 528-533.

Heller, P., Ahlgren, A., Post, T., Behr, M., and Lesh, R. (1989). Proportional reasoning: The effect of two context variables, rate type and problem setting. *Journal for Research in Science Teaching*, 26(1), 205-220.

Hilton, A., Hilton, G. (2018). A string number-line lesson sequence to promote students' relative thinking and understanding of scale, key elements of proportional reasoning. *Australian Primary Mathematics Classroom*, 23(1), 13-18.

Hilton, A., Hilton, G., Dole, S. and Goos, M. (2016). Promoting middle school students' proportional reasoning skills through an ongoing professional development programme for teachers. *Educational Studies in Mathematics,* 92, 193–219.

Hiebert, J. & Behr, M. (1988). Introduction: Capturing the major themes. In Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 1–18). Hillsdale, NJ: Erlbaum and Reston, VA: National Council of Teachers of Mathematics.

Hines, A. (2008). Global trends in culture, infrastructure and values. *Futurist*, *42*(5), 18–23.

Inoue, N. (2005). The realistic reasons behind unrealistic solutions: the role of interpretive activity in word problem solving. *Learning and Instruction*, 15, 69-83.

İncebacak, B. B., Ersoy, E. (2016). Analysis of mathematical reasoning ability of the grade 7 students according to TIMSS. *The Journal of International Social Research*, 9(46), 474-481.

Jiang, R., Li, X., Fernandez, C. & Fu, X. (2016). Students' performance on missing-value word problems: A cross-national developmental study. Eur J Psychol Educ (2017) 32:551–Published online: 21 October 2016/570

Jonassen, D. H. (1997). Instructional design models for well-structured and ill structured problem-solving learning outcomes. *Educational Technology Research and Development*, *45*(1), 65–94.

Johnson, G. J. (2010), *Proportionality in middle-school mathematics textbooks*. Doctor of Philosophy, Department of Secondary Education College of Education, University of South Florida..

Kahraman, H., Kul, E. ve Aydoğdu-İskenderoğlu, T. (2018). 7. ve 8. sınıf öğrencilerinin niceliksel orantısal akıl yürütme problemlerinde kullandıkları stratejiler. *Türk Bilgisayar ve Matematik Eğitimi Dergisi*. doi:10.16949/turkbilmat.333046.

Kalaycı, Nurdan. (2001). *Sosyal Bilimlerde Problem Çözme*. Ankara: Gazi Kitabevi

Kayhan, M. (2005). *6. ve 7. Sınıf öğrencilerinin oran-orantı konusuna yönelik çözüm stratejilerinin; sınıf düzeyine, cinsiyete ve soru tipine göre değişiminin incelenmesi.* Yüksek Lisans Tezi, Hacettepe Üniversitesi Fen Bilimleri Enstitüsü, Ankara.

Kadijevic, D. (2002). Are Quantitative and Qualitative Reasoning Related? *The Teaching of Mathemetics*, 2, 91-98.

Kenney, P., Linquist, M., & Heffernan, C. (2002). Butterflies and Catterpillars: Multiplicative and Proportional Reasoning in the Early Grade. B. Litweller, G. Bright (Ed.), *2002 Yearbook, Making sense of fractionsi ratios, and proportions* (pp.87-*99).* Reston, Virginia: National Council of Teachers of Mathematics.

Kurtz, K. J., Gentner, D., & Gunn, V. (1999). Reasoning. In B. M. Bly & D. E. Rumelhart (Eds.), *Handbook of perception and cognition (2nd ed.). Cognitive science* (pp. 145-200). San Diego, CA, US: Academic Press.

Küpçü, A. R. (2008). *Etkinlik temelli öğretim yaklaşımının orantısal akıl yürütmeye dayalı problem çözme başarısına etkisi*. Doktora tezi, Marmara Üniversitesi Eğitim Bilimleri Enstitüsü, İstanbul.

Küpçü, A. R., Özdemir A. Ş. (2012). İlköğretim öğrencilerinin bilişsel stil, cinsiyet ve orantısal düşünme seviyelerine göre orantı ilişkili problem çözme başarıları. *Kastamonu Eğitim Dergisi*, 20-2, 451-472

Lamon, S. J. (1999). *Teaching fractions and ratios for understanding: Essentianl Content Knowledge and Instructional Strategies for Teachers*. Mahwah, N.J.: Lawrence Erlbaum Associates.

Lamon, Susan J. (1993). Ratio and Proportion: Children's Cognitive and Metacognitive Processes. Thomas P. Carpenter, Fennema E., and Romberg, T., A. (Ed.), In Rational Numbers: An Integration of Research (pp131–56). Hillsdale, N.J.: Lawrence Erlbaum Associates.

Lamon, S. J. (2007). Rational numbers and proportional reasoning. In F. K. Lester (Ed.), Second handbook of research on mathematics teaching and learning, 1, 629-667. Charlotte, NC: Information Age Publishing.

Langrall, C. W., Swafford, J. (2000). Three balloons for two dollars. *Mathematics Teaching in the Middle School*, 6, 254- 261.

Lesh, R., Post, T., & Behr, M. (1988). *Proportional Reasoning*. Retrieved on 20 November-2018 from http://www.cehd.umn.edu/ci/rationalnumberproject/88_8.html.

Levin-Weinberg, S. (2002). Proportional reasoning: One problem, many solutions, making sense of fractions, ratios and proportions. B. Litweller, G. Bright (Ed.), *2002 Yearbook, Making sense of fractionsi ratios, and proportions* (p138-144). Reston, Virginia: National Council of Teachers of Mathematics.

Levin, Suzanne W. (1999). *Fractions and division: Research conceptualizations, textbook presentations, and students performances*. Doctoral dissertation, University of Chicago, 1999.

Lim, K. H. (2009). Burning the candle at just one end: Using nonproportional examples helps students determine when proportional strategies apply. *Mathematics Teaching in the Middle School*, 14, 492–500.

Livy, S. & Herbert, S. (2013). Second-year pre-service teachers' responses to proportional reasoning test items. *Australian Journal of Teacher Education*, 38(11), article 2.

Lo, J., & Watanabe, T. (1997). Developing ratio and proportion schemes: A story of a fifth grader. *Journal for Research in Mathematics Education*, 28(2), 216-236.

Lobato, J., & Ellis, A. B. (2010). Developing essential understanding of ratios, proportions, and proportional reasoning for teaching mathematics in Grades 6-8. Reston, VA: National Council of Teachers of Mathematics.

McIntosh, M. B. (2013), *Developing Proportional Reasoning in Middle School Students*. Masters of Mathematics, College of Science, The University of Utah.

Ministry of National Education of Turkey [MoNE] (2013). Ortaokul Matematik Dersi Öğretim Programı, Ankara.

Ministry of National Education [MoNE], (2018). Ortaokul Matematik 6. Sınıf Ders Kitabı. Ankara: MEB Devlet Kitapları.

Ministry of National Education [MoNE], (2018). Ortaokul Matematik 5. Sınıf Ders Kitabı. Ankara: MEB Devlet Kitapları.

Misailidou, C. and Williams, J. (2003). Diagnostic assessment of children's proportional reasoning. *Journal of Mathematical Behavior*, 22, 335–368.

Mitchell, A. ve Lawson, A. E. (1988). Predicting genetics achievement in nonmajors college biology. *Journal of Research in Science Teaching*, 25(1), 23-37.

Modestou, M., & Gagatsis, A. (2010). Cognitive and metacognitive aspects of proportional reasoning. *Mathematical Thinking and Learning*, 12(1), 36-53.

Morgan, G. A., Leech, N. L., Gloeckner, G., W., & Barret, K. C., (2004). *Spss for Introductory Statistics: Use and Interpretation.* Second Edition. London, Lawrance Erlbaum Associates

National Council of Teachers of Mathematics [NCTM]. (1989*). Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM Publications

Noelting G. (1980). The development of proportional reasoning and the ratio concept, part I – Differentiation of stages. *Educational studies in Mathematics*, 11, 217–254.

Ojose, B. (2015). Proportional reasoning and related concepts: Analysis of gaps and understandings of middle grade students. *Universal Journal of Educational Research* 3(2), 104-112, 2015. doi 10.13189/ujer.2015.030206.

Öncü H. (1994). Eğitimde Ölçme ve Değerlendirme. Ankara: Matser Basım.

Özgün-Koca S. A. & Kayhan-Altay M.(2009). An Investigation of Proportional Reasoning Skills of Middle School Students. *Investigations in Mathematics Learning,* 2:1, 26-48. doi 10.1080/24727466.2009.11790289.

Özsoy, G. (2005). Problem çözme becerisi ile matematik başarısı arasındaki ilişki. *Gazi Eğitim Fakültesi Dergisi, Cilt 25, Sayı 3, 179-190.*

Pakmak, G. S. (2014). 6. sınıf öğrencilerinin niceliksel ve niteliksel orantısal akıl yürütme problemlerinin çözümündeki anlayışlarının incelenmesi, Yüksek lisans tezi, Pamukkale Üniversitesi, Denizli.

Parker, M. (1999). Building on " building up": Proportional-reasoning activities for future teachers. *Mathematics Teaching in The Middle School*, 4(5), 286- 289

Peled, I., Levenberg, L., Mekhmandarov, I., Meron R., & Ulitsin A. (1999). Obstacles in applying a mathematical model: The Case of the Multiplicative Structure. In O. Zaslavsky (Ed.), *Proceedings of the 23rd International Conference for the Psychology of Mathematics Education*. Haifa, Israel: Technion – Israel Institute of Technology.

Piaget, J.; Inhelder, B. (1958). The Origin of the Idea of Chance In Children. New York, NY, US.

Reusser, K. Ve Stebler, R. (1997). Every word problem has a solution – the social rationality of mathematical modeling in schools. *Learning and Instruction*, 7, 309– 327.

Resnick, L., & Singer, J. A. (1993). Protoquantitative origins of ratio reasoning. In: T. Carpenter, E. Fennema, & T. Romberg (Eds.), *Rational numbers: an integration of research* (pp. 107–130). New Jersey: Lawrence Erlbaum.

Riehl S. M. &. Steinthorsdottir O. B. (2017). Missing-value proportion problems: The effects of number structure characteristics. Investigations in Mathematics Learning, 11(1), 56-68.

Siegler, R. S., Fazio, L. K., Bailey, D. H., & Zhou, X. (2013). Fractions: The new frontier for theories of numerical development. *Trends in Cognitive Sciences, 17,* 13–19.

Singh, P. (2000). Understanding the concepts of proportion and ratio constructed by two grade six students. *Educational Studies in Mathematics, 43*(3), 271-292.

Skemp, R. R. (1987). *The Psychology of Learning Mathematics*. Lawrance Erlbaum Associates, New Jersey.

Slovin, H. (2000). Moving to Proportional Reasoning. *Mathematics Teaching in the Middle School*, 6(1), s. 58-60.

TIMSS & PIRLS (2003). *International Study Lynch School of Education.* Boston: College.

Smith, C. L., Solomon, G. E. A., & Carey, S. (2005). Never getting to zero: Elementary school students' understanding of the infinite divisibility of number and matter. *Cognitive Psychology, 51,* 101–140.

Sophian, C., & Wood, A. (1997). Proportional reasoning in young children: The parts and the whole of it. *Journal of Educational Psychology, 89,* 309–317.

Steinthorsdottir, O. B., Sriraman, B. (2009). Icelandic fifth grade girls' developmental trajectories in proportional reasoning. *Mathematics Education Research Journal*, 21(1), 6-30.

Sowder, J., Armstrong, B., Lamon, S., Simon, M., Sowder, L., & Thompson, A. (1998). Education teachers to teach multiplicative structures in the middle grades. *Journal of Mathematics Teacher Education*, 1, 127-155.

Şen, C., Güler, G. (2017). Effect of Strategy Teaching For the Solution of Ratio Problems on Students' Proportional Reasoning Skills. *Malaysian Online Journal of Educational Sciences 5(2), 1-15.*

Thompson, P. W. (1996). Imagery and the development of mathematical reasoning. *Theories of mathematical learning*, 267–283.

Uçar-Toluk Z., Bozkuş F. (2016). İlkokul ve ortaokul öğrencilerinin orantısal durumları orantısal olmayan durumlardan ayırt edebilme becerileri. *Ahi Evran Üniversitesi Kırşehir Eğitim Fakültesi Dergisi*, 17(3), 281-299.

Umay, A. (2003). Matematiksel Muhakeme Yeteneği. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, 24, 234-243.

Umay, A., & Kaf,Y. (2005). Matematikte kusurlu akıl yürütme üzerine bir çalışma. *Hacettepe Üniversitesi Egitim Fakültesi Dergisi* S.28, 188-195.

Valverde, G., & Castro, E. (2012). Prospective elementary school teachers' proportional reasoning. *PNA*, *7*(1), 1-19.

Van Dooren, W., De Bock, D., Gillard, E., & Verschaffel, L. (2009). Add? Or multiply? A study on the develpment of primary school students' proportional reasoning skills. In M. Tzekaki, M. Kaldrimidou, & C. Sakonidis (Eds.). *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (pp. 281- 288). Thessaloniki, Greece: PME.

Van Dooren, W., De Bock, D., & Verschaffel, L. (2010). From addition to multiplication … and back: The development of students' additive and multiplicative reasoning skills. *Cognition and Instruction*, 28(3), 360-381.

Van Dooren, W., De Bock, D., Hessels, A., Janssens, D., & Verschaffel, L. (2005). Not everything is proportional: Effects of age and problem type on propensities for overgeneralization. *Cognition and Instruction*, 23(1), 57–86.

Van de Walle, J. A. (2009). Chapter 19: Proportional reasoning. *In elementary and middle school mathematics: teaching developmentally*, 353-373. US: Pearson Education, Inc.

Vanhille, L. S., Baroody, A. J. (2002). Fraction Instruction thet fosters multiplicative reasoning. B. Litweller, G. Bright (Ed.), *2002 Yearbook, Making sense of fractionsi ratios, and proportions* (pp224-236*).* Reston, Virginia: National Council of Teachers of Mathematics.

Vergnaud, G. (1988). Multiplicative structures. In J. Hiebert & M. Behr (Eds.*), Number concepts and operations in the middle grades* (pp.141–161). Reston, VA: Lawrence Erlbaum & National Council of Teachers of Mathematics.

Wollman, W. ve Lawson, A. (1978). The influence of instruction on proportional reasoning in seventh graders. *Journal of Research in Science Teaching*, 15(3), 227-232.

Yıldızlar, Mehmet. (1999). *İlkokul 1., 2. ve 3. Sınıf Öğrencilerinde Problem Çözme Davranışlarının Öğretiminin Problem Çözmedeki Başarıya ve Matematiğe Olan Tutuma Etkisi*. Yayımlanmamış Doktora Tezi, Hacettepe Üniversitesi Sosyal Bilimler Enstitüsü. Ankara.

APPENDICES

APPENDIX A. PROPORTIONAL REASONING TEST

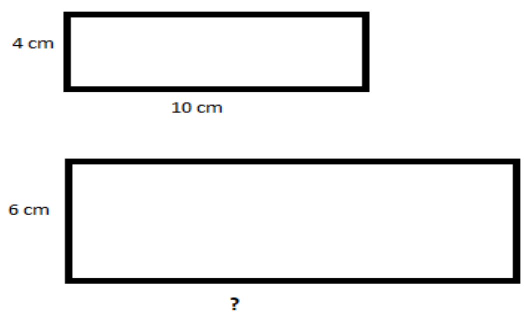
1. Bir meyve reyonunda, 3 elma 90 kuruştur. 7 elma almak isteyen bir kişinin ne kadar ücret ödemesi gerekir?

2. Tüm kitapların fiyatının aynı olduğu bir kitapçıdan Merve 6 kitap alır ve 4 TL öder. 24 kitap alan Rüya'nın kaç TL ödemesi gerekir?

3. Bir baskı makinesinin 14 sözlüğü yazması 12 dakika sürmektedir. Bu makine 30 dakikada kaç sözlük yazabilir?

4. Matematik öğretmeni Gamze, her grupta 4 erkek öğrenci olacak şekilde öğrencileri 6 kişilik gruplara ayırabilmektedir. Sınıfta toplam 30 öğrenci bulunduğuna göre toplam kız öğrenci sayısını bulunuz.

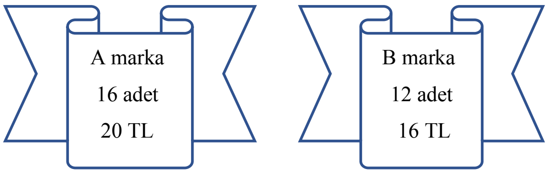
5.



Yandaki iki dikdörtgen birbirine benzer şekildedir, fakat biri diğerinden daha büyüktür. Büyük olan dikdörtgenin uzun kenarının uzunluğu kaç cm'dir? Cevabınızı işlemlerle gösteriniz.

6. Emre, 84 TL değerinde bir müzik çalar almak için para biriktirmeye karar verir. Emre'ye destek olmak için annesi, Emre'nin biriktirdiği her 2 TL için ona 5 TL vereceğini söyler. Sizce Emre 24 TL biriktirdiği zaman annesinin vereceği parayla birlikte müzik çaları satın alabilmek için yeterli paraya sahip olacak mıdır? Cevabınızı işlemlerle gösteriniz.

7.



Merve ve Elif atıştırmalık bir şeyler almak için markete girerler. Markette 16'lı A marka çikolata paketinin 20 TL, 12'li B marka çikolata paketinin 16 TL olduğunu görürler ve **B marka** çikolata paketini almayı tercih ederler. Sizce yaptıkları seçim ekonomik midir? Nedeninizi işlemlerle açıklayınız.

8. Neslihan 20 saniyede 100 metre koşmaktadır. Eğer **aynı mesafeyi** iki kat hızla koşarsa, süre iki katına çıkacaktır.

- Yukarıda verilen ifade için uygun olanı işaretleyiniz: **( ) Doğru ( ) Yanlış**

- Neden doğru ya da yanlışı seçtiğinizi boş bırakılan yerlere **işlemlerle açıklayınız**.

- Nedeninize uygun **bir şıkkı** aşağıdakilerden seçiniz.

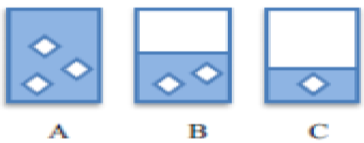
A. Hız iki katına çıkarsa zaman da iki katına çıkar.

B. Hız iki katına çıkarsa zaman yarıya düşer.

C. Mesafe değişmez.

D. Daha hızlı koşmak geçen zamanı etkilemez.

9. Aynı büyüklükteki üç kabın içerisinde farklı miktarlarda su ve küp şeker bulunmaktadır. A kabı tamamen su ile doludur ve içerisine 3 küp şeker atılmıştır. B kabı yarısına kadar su ile doludur ve içerisine 2 küp şeker atılmıştır. C kabının ise $\frac{1}{3}$ ü su ile doldurularak içerisine 1 küp şeker atılmıştır.



Şekerler karıştırıldığında B kabındaki su en tatlıdır.

- Yukarıda koyu renkle verilen ifade için uygun olanı işaretleyiniz:

( ) Doğru ( ) Yanlış

- Neden doğru ya da yanlışı seçtiğinizi boş bırakılan yerlere **işlemlerle açıklayınız.**

- Nedeninize uygun **bir şıkkı** aşağıdakilerden seçiniz.

A.A kabındaki su en tatlıdır, çünkü en çok şeker a kabına atılmıştır.
B.C kabındaki su en tatlıdır, çünkü en az su C kabındadır.
C.B kabı tamamen su ile doldurulursa toplam 4 küp şeker atılması gerekir.
D.Tüm kaplardaki sular aynı tatlılıktadır.

10. İlayda, kenar uzunlukları 3 cm ve 5 cm olan dikdörtgen şeklindeki vesikalık fotoğrafını bir fotokopi makinesinde %200 büyütme seçeneğini kullanarak genişletiyor. Sizce genişletilmiş fotoğraf mı yoksa orijinal fotoğraf mı kareye daha çok benzemektedir? Cevabınızı boş bırakılan yerlere açıklayınız.

A.Orijinal fotoğraf kareye daha çok benzemektedir.
B.Genişletilmiş fotoğraf kareye daha çok benzemektedir.
C.İki fotoğraf eşit şekilde kareye benzemektedir.
D.Hangisinin kareye daha çok benzediğine karar vermek için verilen bilgiler yeterli değildir.

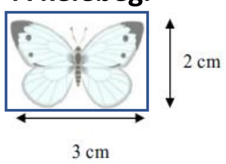
11. Bir markette 5 adet çikolata 0,75 TL değerinde olduğuna göre 13 adet çikolata kaç TL'dir?

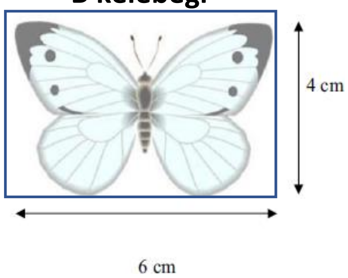
12. Gamze bir işte 12 hafta çalıştığında 600 TL kazanmaktadır. Her hafta aynı miktarda para kazandığına göre, Gamze'nin 200 TL kazanması için kaç hafta çalışması gerekir?

13. Bir baskı makinesi 4 dakikada 18 kitap yazmaktadır. Bu makine 10 dakikada kaç kitap yazabilir?

14. Bir sınıfta öğrenciler her grupta 3 kız öğrenci olmak üzere 5 kişilik gruplar halinde oturmaktır. Sınıfta toplam 25 öğrenci olduğunu göre kız ve erkek öğrencilerin sayısını bulunuz.

15. **A kelebeği** **B kelebeği**





Bilge farklı büyüklüklerde dikdörtgen şeklindeki kağıtlara iki kelebek resmi çizdi. B kelebeğinin kapladığı alan A kelebeğinin kapladığı alanın 2 katıdır.

- Yukarıda verilen ifade için uygun olanı işaretleyiniz:
( ) Doğru ( ) Yanlış
- Neden doğru ya da yanlışı seçtiğinizi boş bırakılan yerlere **işlemlerle açıklayınız**.
- Nedeninize uygun **bir şıkkı** aşağıdakilerden seçiniz..

A. B kelebeğinin kapladığı alan 4 kat daha büyüktür.
B. A kelebeğinin eni B kelebeğinin eninin yarısıdır.
C. B kelebeği A kelebeğinin iki katı kadar uzunluktadır.
D. Bilge A kelebeğinin sadece bir boyutunun ölçüsünü iki katına çıkarmıştır.

16.Sıla, 210 liralık bir MP3 çalar satın almak ister. Annesi, Sıla'nın biriktirdiği her 2 TL için Sıla'ya 5 TL vereceğini söyler. Buna göre Sıla'nın annesi MP3 çalar için Sıla'ya kaç lira verecektir?

17.

A toz deterjanı:

- 5 TL
- 1 kg
- 20 yıkama

B toz deterjanı:

- 6,5 TL
- 1,5 kg
- 30 yıkama

A toz deterjanı daha ekonomiktir.

- Yukarıda verilen ifade için uygun olanı işaretleyiniz:
( ) Doğru ( ) Yanlış
- Neden doğru ya da yanlışı seçtiğinizi boş bırakılan yerlere **işlemlerle açıklayınız**.
- Nedeninize uygun **bir şıkkı** aşağıdakilerden seçiniz.

A. A toz deterjanının fiyatı daha azdır.
B. B toz deterjanının fiyatı azıcık fazladır ama 10 kez daha fazla yıkama yapılabilir.
C. B deterjanında yıkama başına düşen ücret daha azdır.
D. İki toz deterjan da aynı değerdedir.

18. Aynı hızda 6 boyacı bir bahçenin çitlerini 4 günde boyayabilmektedir. Çitlerin 2 günde boyanabilmesi için aynı hızda toplam 10 boyacıya ihtiyaç vardır.
- Yukarıda verilen ifade için uygun olanı işaretleyiniz: **( ) Doğru ( ) Yanlış**

- Neden doğru ya da yanlışı seçtiğinizi boş bırakılan yerlere **işlemlerle açıklayınız**.

- Nedeninize uygun **bir şıkkı** aşağıdakilerden seçiniz.

A. Eğer süre 2 gün azalırsa, boyacı sayısı da 2 azaltılmalıdır. Bu yüzden 4 boyacıya ihtiyaç vardır.
B. Eğer süre 2 gün azalırsa, boyacı sayısı 2 arttırılmalıdır. Bu yüzden 8 boyacıya ihtiyaç vardır.
C. Gün sayısı yarıya düşürüldüğünde daha çok boyacıya ihtiyaç vardır ve iki katına çıkarılmalıdır, bu yüzden 12 boyacıya ihtiyaç vardır.
D. Çitlerin 2 günde boyanabilmesi için boyacı sayısının değişmesine gerek yoktur.

19.

KIRMIZI
2 su bardağı limon suyu
4 su bardağı su

YEŞİL
4 su bardağı limon suyu
6 su bardağı su

Kendi limonatasını yapan Nurgül Hanım kırmızı sürahinin içine 2 su bardağı limon suyu ve 4 su bardağı su koyar. Yeşil sürahinin içine ise 4 su bardağı limon suyu ve 6 su bardağı su koyar. Buna göre hangi sürahideki limonatada daha çok limon tadı vardır? İşlemlerle açıklayınız.

20. Fen bilimleri öğretmeni Aygül, bitkilerle deney yapabilmek için 4 farklı dikdörtgensel saksıya sahiptir. Bu saksıların kenar uzunlukları aşağıdaki şıklarda verilmiştir. Buna göre bu saksılardan hangisinin şekli kareye daha çok benzemektedir? İşlemlerle açıklayınız.

A. 27 cm – 30 cm
B. 17 cm – 20 cm
C. 7 cm – 10 cm
D. 37 cm – 40 cm

APPENDIX B. RUBRIC FOR THE PROPORTIONAL REASONING TEST

1. **Testteki verilmeyen değeri bulma problemlerine ilişkin kullanılan dereceli puanlama anahtarı (İlk 6 problem için)**

0PUAN

- Boş

- Orantısal akıl yürütmenin var olduğuna ilişkin ipucu yok

- Verilerin toplamsal karşılaştırılması var

- Verilerin sayıların ve işlemlerin rastgele kullanımı var

1 PUAN

- Sadece sonuç belirtilmiş

- Orantısal akıl yürütmenin var olduğuna ilişkin ipuçları var (Yanlış değişkenler arasında orantı kurma, görsel verileri kullanarak orantı kurma gibi)

- Orantı çeşidi fark edilmemiş

2 PUAN

- Beklenen değişkenler arasında orantısal akıl yürütme var, ancak sonuca ulaşılamamış
- Beklenen değişkenler arasında orantısal akıl yürütme var, ancak işlem hataları yapılmış

3 PUAN

- Soruyu tam ve doğru çözebilmek için gereken orantısal akıl yürütme var ve sonuca ulaşılmış

2. **Testteki niceliksel karşılaştırma ile ilgili maddelere ilişkin kullanılan dereceli puanlama anahtarı (Son 4 problem)**

0 PUAN

- Boş

- Sadece sonuç belirtilmiş

- Yanlış değişkenler arasında orantı kurulmuş

- Orantısal akıl yürütmenin var olduğuna ilişkin ipucu yok

- Verilerin toplamsal karşılaştırılması var

- Verilerin sayıların ve işlemlerin rastgele kullanımı var

1 PUAN

- Beklenen değişkenler arasında orantısal akıl yürütme becerisini kullanarak ya da kullanmayarak, doğru sonuca ulaşılmış, ancak yanlış yorumlanmış

- Doğru yanıt verilmiş ancak açıklama yetersiz

2 PUAN

- Beklenen değişkenler arasında orantısal akıl yürütme becerisine sahip olunduğu gösterilmiş, doğru sonuca ulaşılmış, ancak yapılan açıklama yetersiz ya da işlem hatası nedeniyle doğru sonuca ulaşılamamış

- Doğru sonuca ulaşmamış olsa da bulunan sonuca göre yapılan doğru yorumlanmış

3 PUAN

- Doğru sonuca ulaşmak için gerekli orantısal akıl yürütme becerisi iyi düzeyde gösterilmiş ve doğru açıklama yapılmış

APPENDIX C. PARENTS' CONSENT FORMS & INFORMATION FORMS

VELİ ONAY FORMU

Sevgili veli,

Bu araştırma, ODTÜ İlköğretim Fen ve Matematik Eğitimi Bölümü Yüksek Lisans öğrencisi Gamze ÖZEN YILMAZ tarafından, Yrd. Doç. Dr. Didem AKYÜZ danışmanlığındaki yüksek lisans tezi kapsamında yürütülmektedir. Bu form sizi araştırma koşulları hakkında bilgilendirmek için hazırlanmıştır.

Bu çalışmanın amacı nedir?

Bu çalışmanın amacı çocuğunuzun da eğitim-öğretim gördüğü okuldaki öğrencilerin (5, 6, 7 ve 8.sınıf) matematik yapmanın temelini oluşturan ve akıl yürütme çeşitlerinden biri olan orantısal akıl yürütme becerileri hakkında bilgi edinmektir.

Çocuğunuzun katılımcı olarak ne yapmasını istiyoruz?

Bu amaç doğrultusunda, çocuğunuzdan 20 sorudan oluşan testi 2 ders saati (80 dakika) süresinde cevaplamasını isteyeceğiz ve cevaplarını yazılı biçiminde toplayacağız. Sizden çocuğunuzun katılımcı olmasıyla ilgili izin istediğimiz gibi, çalışmaya başlamadan çocuğunuzdan da sözlü olarak katılımıyla ilgili rızası mutlaka alınacak.

Çocuğunuzdan alınan bilgiler ne amaçla ve nasıl kullanılacak?

Çocuğunuzdan alacağımız cevaplar sadece araştırmacılar tarafından değerlendirilecektir ve kendi ders öğretmeni tarafından puanlandırılmayacaktır. Elde edilecek bilgiler sadece bilimsel amaçla kullanılacak, çocuğunuzun ya da sizin ismi ve kimlik bilgileriniz, hiçbir şekilde kimseyle paylaşılmayacaktır.

Çocuğunuz ya da siz çalışmayı yarıda kesmek isterseniz ne yapmalısınız?

Katılım sırasında sorulan sorulardan ya da herhangi bir uygulama ile ilgili başka bir nedenden ötürü çocuğunuz kendisini rahatsız hissettiğini belirtirse, ya da kendi belirtmese de araştırmacı çocuğun rahatsız olduğunu öngörürse, çalışmaya sorular tamamlanmadan ve derhal son verilecektir.

Bu çalışmayla ilgili daha fazla bilgi almak isterseniz:

Çalışmaya katılımınızın sonrasında, bu çalışmayla ilgili sorularınız olursa yazılı biçimde cevaplandırılacaktır. Çalışma hakkında daha fazla bilgi almak için İlköğretim Fen ve Matematik Eğitimi Bölümü öğretim elemanlarından Doç. Dr. Didem AKYÜZ ile (e-posta: dakyuz@metu.edu.tr) ya da yüksek lisans öğrencisi Gamze ÖZEN YILMAZ (e-posta: e173307@metu.edu.tr) ile iletişim kurabilirsiniz. Bu çalışmaya katılımınız için şimdiden teşekkür ederiz.

Yukarıdaki bilgileri okudum ve çocuğumun bu çalışmada yer almasını onaylıyorum (Lütfen alttaki iki seçenekten birini işaretleyiniz.

Evet onaylıyorum___ ***Hayır, onaylamıyorum___***

Velinin adı-soyadı: ______________ Bugünün tarihi: _______________

Çocuğun adı soyadı ve doğum tarihi:________________

(Formu doldurup imzaladıktan sonra araştırmacıya ulaştırınız).

Araştırma Sonrası Bilgilendirme Formu

Bu araştırma daha önce de belirtildiği gibi ODTÜ İlköğretim Fen ve Matematik Eğitimi Bölümü Yüksek Lisans öğrencisi Gamze ÖZEN YILMAZ tarafından Yrd. Doç. Dr. Didem AKYÜZ danışmanlığındaki yüksek lisans tezi kapsamında yürütülmektedir. Çalışmanın amacı eğitim-öğretim gördüğünüz okuldaki öğrencilerin (5, 6, 7 ve 8.sınıf) matematik yapmanın temelini oluşturan ve akıl yürütme çeşitlerinden biri olan orantısal akıl yürütme becerileri hakkında bilgi edinmektir.

Orantısal akıl yürütme, ortaokul müfredatının ve cebirin temeli olarak görülen matematiksel akıl yürütme çeşitlerinden biridir. Harita ölçeklerinden yola çıkarak gerçek uzunluğu bulma, kiloyla orantılı olacak şekilde ilaç dozlarını ayarlama, mutfak alışverişi yaparken birim fiyat hesaplama gibi günlük hayatın işlerinde; fizik, kimya, istatistik ve ekonomi gibi diğer disiplinlerde de ihtiyaç duyulan bir akıl yürütmedir. Böylesine önemli bir beceriyi ne yazık ki ortaokul öğrencilerinin, liseden yeni mezun olan çoğu öğrencinin ve hatta yetişkinlerin çoğunun yeterince kazanamadığı görülmektedir. Ortaokul yılları orantısal akıl yürütmenin en kritik dönemi olarak kabul edilmektedir. Orantısal akıl yürütme becerisinin ortaokul döneminde güçlendirilmesi için izlenecek yolların değerlendirilmesi ve geliştirilmesi önemlidir. Bu nedenden dolayı, öğrencilerin orantısal akıl yürütme becerilerinin hangi sınıf kademesinde ne seviyede olduğunu ve bu seviyeler ile akademik başarıları arasında bir ilişki olup olmadığını tespit etmek için bu çalışma yapılmaktadır. Çalışmanın sonuçları öğretmenlerin öğrencilerinin orantısal akıl yürütme becerilerinin gelişmesi için nasıl destek olabilecekleri hakkında fikir verebilecektir.

Bu çalışmadan alınacak ilk verilerin 2018 aralık ayının sonunda elde edilmesi amaçlanmaktadır. Elde edilen bilgiler sadece bilimsel araştırma ve yazılarda kullanılacaktır. Bu araştırmaya katıldığınız için tekrar çok teşekkür ederiz. Araştırmanın sonuçlarını öğrenmek ya da çalışma hakkında daha fazla bilgi almak için İlköğretim Fen ve Matematik Eğitimi Bölümü öğretim elemanlarından Doç. Dr. Didem AKYÜZ ile (e-posta: dakyuz@metu.edu.tr) ya da yüksek lisans öğrencisi Gamze ÖZEN YILMAZ (e-posta: e173307@metu.edu.tr) ile iletişim kurabilirsiniz.

APPENDIX D. PERMISSION FROM METU HUMAN SUBJECTS ETHICS COMMITTEE

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

DUMLUPINAR BULVARI 06800
ÇANKAYA ANKARA/TURKEY
T: +90 312 210 22 91
ueam@metu.edu.tr
www.ueam.metu.edu.tr

Sayı: 28620816 / 577

11 ARALIK 2018

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Dr.Öğretim Üyesi Didem AKYÜZ

Danışmanlığını yaptığınız Gamze Özen YILMAZ'ın **"Ortaokul Öğrencilerinin Orantısal Akıl Yürütme Seviyeleri ve Bu Seviyelerle Akademik Başarıları Arasındaki İlişki"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek gerekli onay **2018-EGT-173** protokol numarası ile araştırma yapması onaylanmıştır.

Saygılarımla bilgilerinize sunarım.

Prof. Dr. Tülin GENÇÖZ
Başkan

Prof. Dr. Ayhan SOL
Üye

Prof. Dr. Ayhan Gürbüz DEMİR
Üye

Prof.Dr. Yaşar KONDAKÇI (4.)
Üye

Dr. Öğr. Üyesi Ali Emre TURGUT
Üye

Doç. Dr. Emre SELÇUK
Üye

Doç.Dr. Üyesi Pınar KAYGAN
Üye

APPENDIX E. PERMISSION OBTAINED FROM MINISTRY OF NATIONAL EDUCATION

T.C.
ANKARA VALİLİĞİ
Milli Eğitim Müdürlüğü

Sayı : 14588481-605.99-E.2016722 29.01.2019
Konu : Araştırma İzni

ORTA DOĞU TEKNİK ÜNİVERSİTESİ REKTÖRLÜĞÜNE
(Öğrenci İşleri Daire Başkanlığı)

İlgi: a) MEB Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 2017/25 nolu Genelgesi.
b) 10/01/2019 Tarihli ve E.75 sayılı yazınız.

Üniversiteniz İlköğretim Anabilim Dalı Yüksek Lisans öğrencisi Gamze ÖZEN YILMAZ'ın **"Ortaokul Öğrencilerinin Orantısal Akıl Yürütme Seviyeleri ve Bu Seviyelerle Akademik Başarıları Arasındaki İlişki"** konulu tez çalışması kapsamında uygulama talebi Müdürlüğümüzce uygun görülmüş ve uygulamanın yapılacağı İlçe Milli Eğitim Müdürlüğüne bilgi verilmiştir.

Görüşme formunun (6 sayfa) araştırmacı tarafından uygulama yapılacak sayıda çoğaltılması ve çalışmanın bitiminde bir örneğinin (cd ortamında) Müdürlüğümüz Strateji Geliştirme Şubesine gönderilmesini rica ederim.

Serkan TOPBAŞ
Vali a.
Milli Eğitim Müdür V.

Güvenli Elektronik İmzalı
Aslı İle Aynıdır.
29../2../2019..

Adres: Alparslan Türkeş cad. Emniyet Mah.4/A
Yenimahalle/ANKARA
Elektronik Ağ: ankara.meb.gov.tr
e-posta: istatistik06@meb.gov.tr

Bilgi için: Ayşe ARDA
Tel: 0 (312) 212 36 00
Faks: 0 (312) 221 02 16

Bu evrak güvenli elektronik imza ile imzalanmıştır. https://evraksorgu.meb.gov.tr adresinden 9dad-d3ee-39ca-a16f-3ae2 kodu ile teyit edilebilir.

APPENDIX F. TURKISH SUMMARY / TÜRKÇE ÖZET

ORTAOKUL ÖĞRENCİLERİNİN ORANTISAL AKIL YÜRÜTME PROBLEMLERİNDEKİ BAŞARILARINI VE KULLANDIKLARI STRATEJİLERİ İNCELEME

Öğretmenlerin matematik eğitimindeki amacı, günümüzün teknolojisine ayak uydurmak, yaşamın sürekli karmaşık olduğu dünyada hayatta kalmayı düşünebilmeyi sağlamak, olaylar arasında ilişkiler kurmak, akıl yürütmeyi kullanmak, tahmin etmek ve sayıları ve işlemleri öğretme ve hesaplama becerileri yerine problem çözme becerileri öğrencilere kazandırmaktır (Umay, 2003). Matematik eğitiminin amacı, sahip olduğu bilgileri aktarabilen, problem çözebilen ve karşılaştıkları durumlara çözüm üretebilen bireyler eğitmektir (MEB, 2013). Akıl yürütme, matematik öğrenmek için önemli becerilerden biridir. Matematiği bilmek ve yapmak oldukça çok önemlidir (NCTM, 1989). Akıl yürütme, özel matematik araçlarını (semboller, tanımlar, ilişkiler vb.) ve düşünme tekniklerini (tümevarım, tümdengelim, karşılaştırma, genelleme vb.) kullanarak yeni bilgi edinme süreci olarak tanımlanabilir (MEB, 2013). Matematiksel olarak akıl yürütebilen bir birey matematiksel kavramlar arasındaki ilişkileri görebilir, geometrik şekilleri ayırt edebilir, orantılı akıl yürütmeyi kullanabilir, üç boyutlu şekiller için uzamsal yeteneği kullanabilir, verilerin farklı gösterimlerini sunabilir ve verileri yorumlayabilir (TIMSS, 2003). Bu yüzden, matematik öğretiminde akıl yürütme becerileri, öğrencilerin kazanması gereken becerilerin başında gelir (İncebacak ve Ersoy, 2016). Akıl yürütme, bir durumu temsil etme veya bir sorunu çözme yöntemine bakılmaksızın, çarpımsal ilişkiye dayandığında orantılıdır. Orantısal akıl yürütme, orantısal akıl yürütme problemlerini çözme, orantılı durumları orantılı olmayanlardan ayırma ve çarpımsal problem durumlarının matematiksel ilişkilerini anlama yeteneğinden oluşur. (Cramer, Post ve Currier, 1993). Bu nedenle, oldukça zor ve karmaşık bir beceridir (Christou ve Papageorgiou, 2002). Her ne kadar çoğu insan içler-dışlar çarpımı stratejisinin kullanımıyla orantısal akıl yürütmeyi tanımlasa da araştırmalar doğru orantısal akıl yürütmenin sadece kesirleri ve rasyonel sayıları anlamayı değil, aynı zamanda oran algısı, göreceli düşünme ve çoklukları bölümlere ayırma, birimlere ayırma ve değiştirme gibi diğer alanlardaki yeterliliği de içerdiğini göstermektedir. (Lamon, 1999). Çoğu öğrenci orantılarla sorun yaşar, çünkü geleneksel öğretim çarpımsal ilişkilerin kavranmasını geliştirmektedir. Öğretmenler, öğrencilerin orantılı akıl yürütme problemlerini çözmeleri için içler-dışlar çarpımı algoritmasını ezberletmektedir. Çoğu öğrenci ise bu anlamsız algoritmayı hiç öğrenemediğinden ya da unuttuğundan çarpımsal akıl yürütme içeren problemleri toplamsal olarak düşünerek çözmeye çalışmaktadır (Vanhille ve Baroody, 2002). Vanhille ve Baroody (2002), öğrenciler bu algoritmayı başarılı bir şekilde uygulamasa bile, öğrencilerin çarpımsal akıl yürütmelerinin gelişmediğini; öğrencilerin orantısal akıl yürütme becerisine sezgisel olarak ulaşması gerektiğini vurgulamaktadır. Diğer taraftan, her ne kadar oran ve orantı kavramları öğrencilere ilk kez 6. ve 7. sınıflarda tanıtılıyor olsa da araştırmalar, oran ve orantı kavramları öğretilmeden de öğrencilerin orantısal akıl yürütme problemlerini çözebildiklerini göstermektedir. Örneğin, Ojose (2015) tarafından yapılan çalışma, tüm sınıf seviyelerindeki öğrencilerin, konunun öğretilmesine ihtiyaç duyulmadan orantı konusunu kavramsal olarak anlayabileceğini göstermiştir. Bulgular çocukların örgün öğretim öncesinde zaten orantılı bir akıl yürütmeye sahip olduğunu göstermiştir. Ayrıca, çoğu çalışma, öğrencilerin sınıf düzeyi arttıkça, orantısal akıl yürütme yeteneğinin de arttığını göstermiştir (Mersin, 2018; Hilton et al., 2016; Toluk Uçar &Bozkuş, 2016; Larson, 2013; Van Dooren et al., 2009). İlgili literatüre bakıldığında, öğrencilerin oransal akıl yürütme becerilerini belirlemek için farklı çözüm stratejileri belirlendiği görülmektedir. Bunlar, birim oran, değişim çarpanı, denk kesirler ve içler-dışlar çarpımı stratejileri (Cramer & Post, 1993), denklik sınıfı stratejisi (Bart, Post, Behr & Lesh, 1994) and arttırma stratejisidir (Ben-Chaim, Fey, Fitzgerald, M., Benedetto & Miller, 1998; Parker, 1999). Aşağıda verilen probleme doğru bir çözüm olacak şekilde, bu stratejilerin her biri kısaca açıklanmıştır:

Bir kitapçıda birbirinin aynısı 4 kitabın değeri 40 TL ise 12 kitabın değerinin kaç TL olduğunu bulunuz.

Birim oran stratejisi ile öncelikle 1 kitabın kaç TL olduğu bulunur. 1 kitabın fiyatı 40 TL : 4 = 10 TL işlemi ile bulunur. Daha sonra 12 x 10 TL = 120 TL işlemi ile 12 kitabın fiyatı bulunur.

Değişim faktörü stratejisi ile 12 kitap, 4 kitabın 3 katı olduğu için 12 kitabın fiyatının, 4 kitabın fiyatının 3 katı olması gerektiği düşünülür. Bu durumda 40 TL x 3 = 120 TL işlemi ile 12 kitabın fiyatı bulunur.

Denk kesirler stratejisi ile oranlar denk kesirler olarak algılanır. Burada amaç verilen kesre denk olan bir kesir bulmaktır.

$\frac{4}{40} = \frac{12}{?}$ $\quad$ $\frac{4x3}{40x3} = \frac{12}{120}$ işlemleri ile 12 kitabın fiyatı bulunur.

Denklik sınıfı stratejisi ile problemde verilen oran ile istenilen oran bulunana kadar oran çiftleri oluşturulur.

$$\frac{4}{40} \equiv \frac{8}{80} \equiv \frac{12}{120}$$

Artırma stratejisi, bir oran içerisinde ilişki kurarak ikinci oranı toplama işlemi ile elde ederek istenilen orana ulaşma yöntemidir.

4 kitap 40 TL

8 kitap 80 TL

12 kitap 120 TL

İçler-dışlar çarpımı stratejisi ile orantı kurulur ve eşitlik çözülür.

4 kitap 40 TL

12 kitap A TL

4xA=12x40 $\quad$ A= $\frac{12x40}{4}$ =120

Literatürdeki çalışmaların çoğu, içler-dışlar çarpımı stratejisini öğrenen öğrencilerin bu stratejiyi orantısal akıl yürütme problemlerini çözmek için yaygın olarak kullandığını göstermektedir. Örneğin, Bal-İncebacak ve Ersoy'un (2016) çalışmasına göre, 7. sınıf öğrencileri farklı orantısal akıl yürütme problemlerini çözmek için çoğunlukla içler-dışlar çarpımı stratejisini kullandılar. Çalışmanın sonucunda, öğrencilerin çoklukları karşılaştırmak için içler-dışlar çarpımı yapmayı tercih ettikleri görülmüştür. Kahraman, Kul ve Aydoğdu-İskenderoğlu (2018) ise 7. ve 8. sınıf öğrencilerinin nicel orantısal akıl yürütme problemlerinde kullandıkları stratejileri öğrenmek için bir çalışma yürütmüştür. Araştırmanın sonucunda 7. sınıf öğrencilerinin en çok birim oran stratejisini, 8. sınıf öğrencilerinin ise içler-dışlar çarpımı stratejisini kullandığı görülmüştür. 7. sınıf öğrencilerinin henüz içler-dışlar çarpımı stratejisini öğrenmedikleri için çoğunlukla birim oran stratejisini kullandıkları belirtilmiştir. Bunlara ek olarak, Artut ve Pelen (2015), 6. sınıf öğrencilerinin orantısal akıl yürütme problemlerini çözmek için kullandıkları stratejileri ve bu stratejilerin problem türü ve problemdeki sayıların yapısına göre değişip değişmediğini araştırmak amacıyla bir çalışma yürütmüştür. Elde edilen sonuçlara göre, 6. sınıf öğrencilerinin çoğunlukla hem eksik değer problemlerinde hem de sayısal karşılaştırma problemlerinde problemlerdeki sayıların yapılarına bakmaksızın değişim stratejisi faktörünü kullandıkları görülmüştür.

Yukarıda bahsedilen çalışmanın doğrultusunda bu çalışmanın amaçları ortaokul öğrencilerinin orantısal akıl yürütme problemlerindeki başarılarını göstermek, bu başarıların problem çeşidine göre nasıl değiştiğini incelemek ve öğrencilerin bu problemleri çözmek için kullandıkları stratejileri incelemektir.

Çalışmanın araştırma soruları ise aşağıdaki gibi belirtilmiştir:

1. Öğrencilerin orantısal akıl yürütme problemleri testindeki akademik başarısı 5. sınıf seviyesinden 8. sınıf seviyesine doğru değişir mi?
    - Bu öğrencilerin akademik başarısı problemlerin çeşidine göre değişir mi?

2. 5. sınıf seviyesinden 8. sınıf seviyesine öğrencilerin orantısal akıl yürütme problemlerinde en sık kullandıkları çözüm stratejileri nelerdir?

Diğer çalışmalardan farklı olarak bu çalışma 5., 6., 7., ve 8. Sınıf öğrencilerin tamamıyla yürütülerek boylamsal bir çalışma özelliği taşımaktadır. Bu yüzden öğrencilerin orantısal akıl yürütme testindeki başarısı, bu başarının problem çeşidine göre nasıl değiştiği ve bu problemlerde öğrenciler tarafından kullanılan stratejilerin neler olduğu 5. sınıf seviyesinden 8. sınıf seviyesine doğru derinlemesine incelenebilmiştir. Çalışmanın örneklemini Ankara'nın Mamak ilçesinde öğrenim görmekte olan 858 ortaokul öğrencisi oluşturmaktadır. Verilerin toplanabilmesi için çoğunluğu literatürden alınmış ve bazıları araştırmacı tarafından hazırlanmış 20 problemlik bir orantısal akıl yürütme testi (PRT) oluşturulmuştur. Testin öğrencilere uygulanabilmesi için 2018-2019 eğitim öğretim yılının güz döneminde hem öğrencilerden hem velilerden hem de okul idaresinden gerekli etik izinler alınmıştır. Çalışmanın ilk araştırma sorusuna cevap bulmak için nicel, ikinci araştırma sorusuna cevap bulmak için nitel metot kullanıldığından çalışmada karma araştırma metodu kullanılmıştır. Nicel metot için uygulama sonrasında öğrencilerin testteki her bir probleme yazdıkları çözümler literatürden alınmış bir dereceli ölçme anahtarına göre 0-3 arasında puanlanarak öğrencilerin testteki akademik başarıları belirlenmiştir. Her bir sınıf seviyesinde öğrencilerin testten aldıkları başarı puanlarının ortalamaları alınarak 5. sınıf seviyesinden 8. sınıf seviyesine doğru öğrencilerin başarılarının nasıl değiştiğine bakılmıştır. Bu amaç doğrultusunda, 5. sınıf öğrencilerinin ortalaması 13,63 (SD=9,27), 6. sınıf öğrencilerinin ortalaması 16,79 (SD=11,40), 7. sınıf öğrencilerinin ortalaması 21.29 (SD=11,32) ve 8. sınıf öğrencilerinin ortalaması 29,20 (SD=13,64), olarak bulunmuştur. 5. sınıf seviyesinden 8. sınıf seviyesine doğru öğrencilerin başarı puanları ortalamasındaki bu artışın istatistiksel olarak anlamlı olup olmadığını tespit edebilmek için ise bağımsız örneklem t-testi yapılmıştır. Testin sonuçlarına göre tüm sınıf seviyelerinin başarı puanları ortalamaları arasındaki fark istatistiksel olarak anlamlı bulunmuştur. Sonuç olarak, sınıf seviyesi arttıkça öğrencilerin orantısal akıl yürütme başarıları da artmaktadır. Çalışmanın bu sonucu literatürdeki çoğu çalışma ile örtüşmektedir (Mersin, 2018; Hilton et al., 2016; Toluk-Uçar & Bozkuş, 2016; Larson, 2013; Van Dooren et al., 2009; Dole et al., 2007). Ayrıca, nicel metot kısmında öğrencilerin başarılarının problem çeşitlerine göre değişip değişmediği de incelenmiştir. Bunun için öğrencilerin her bir problemden aldıkları puanlara göre frekans ve yüzdeleri, eksik değer ve sayısal karşılaştırma problemleri için ayrı ayrı iki tabloda gösterilmiştir. Tablolar incelendiğinde öğrencilerin eksik değer problemlerini çözmekte, sayısal karşılaştırma problemlerine göre daha başarılı olduğu görülmüştür. Çalışmanın bu sonucu literatürdeki araştırmaların sonucuyla da benzerlik göstermiştir (Ben-Chaim et al., 1998; Özgün-Koca & Kayhan-Altay, 2009). Problem çeşidine ek olarak, problemlerdeki değişim faktörünün tam sayı olup olmamasının da öğrencilerin başarısını etkilediği görülmüştür. Öğrencilerin tamamı, değişim faktörünün tam sayı olduğu problemlerde daha başarılı olmuştur. Değişim faktörünün tam sayı olmadığı problemlerde ise öğrencilerin, toplamsal yöntemleri hatalı bir şekilde kullandığı ya da o problemleri çözmeye uğraşmadığı görülmüştür. Bu sonuç literatürdeki çalışmaların sonuçlarıyla benzerlik göstermiştir (Riehl & Steinthorsdottir, 2017; Artut & Pelen, 2015; Dooren, Bock and Verschaffel, 2010; Tjioe & Torre, 2014; Heller et al., 1989).

Çalışmanın nitel kısmı için ise her sınıf seviyesinden en yüksek puanı almış 20 öğrencinin problemlere verdikleri cevaplar tekrar incelenerek her bir problemde kullandıkları çözüm stratejileri belirlenmiştir. Bunun için nitel araştırma yöntemlerimden biri olan içerik analizi kullanılmıştır. İçerik analizi ile öğrencilerin problemleri çözmek için kullanmış oldukları çözüm stratejileri literatürde bahsedilen orantısal akıl yürütme stratejilerine göre kodlanmıştır. Bu doğrultuda, birim oran stratejisi için 1, değişim faktörü stratejisi için 2, denk kesirler stratejisi için 3, denklik sınıfı stratejisi için 4, arttırma stratejisi için 5 ve içler-dışlar çarpımı stratejisi için 6 sayıları kullanılmıştır. Ayrıca, bu çarpımsal stratejileri kullanmayıp problemleri toplama yöntemiyle çözen öğrencilerin çözümleri toplamsal metot olarak adlandırılarak kodlama sırasında 7 sayısı ile ifade edilmiştir. Bunlara ek olarak, toplamsal yöntemleri hatalı bir şekilde kullanan öğrencilerin çözümleri ise toplamsal olmayan metot olarak adlandırılarak kodlama sırasında 8 sayısı ile ifade edilmiştir. Her sınıf seviyesinden yirmişer öğrencinin çözümleri bu kodlamalar ile analiz edildikten sonra SPSS programına girilerek stratejilerin kullanım sıklıkları ve yüzdeleri hesaplanmıştır. Elde edilen sonuçlara göre 5., 6. ve 7. sınıf öğrencilerinin orantısal akıl yürütme problemlerini çözebilmek için en sık kullandığı stratejinin değişim çarpanı stratejisi olduğu görülmüştür. Çalışmanın bu sonucu literatürdeki birkaç çalışmanın sonucuyla benzerlik göstermiştir (Avcu & Avcu, 2010; Avcu & Doğan, 2014). Diğer taraftan, bu sonuç literatürdeki çoğu çalışmanın sonucundan farklıydı, çünkü o çalışmalarda 5., 6. ve 7. sınıf öğrencilerinin sıklıkla kullandıkları strateji birim oran stratejisiydi (Kahraman, Kul & İskenderoğlu, 2018; Küpçü, 2008; Kayhan, 2005; Özgün-Koca & Kayhan-Altay, 2009; Cramer & Post,1993; Christou & Philippou, 2002; Pakmak, 2014). Çalışmada 8. Sınıf öğrencilerinin ise en çok içler-dışlar çarpımı stratejisini kullanarak problemleri çözdüğü tespit edildi. Bu sonuç literatürdeki çoğu çalışmanın sonucu ile örtüştü (Kahraman, Kul & İskenderoğlu, 2018; Küpçü, 2008; Duatepe, Akkuş-Çıkla & Kayhan, 2005; Incebacak & Ersoy, 2016; Cramer & Post, 1993). Aslında içler-dışlar çarpımı stratejisinin sadece 8. sınıf öğrencileri tarafından kullanılması tahmin edilebilir bir sonuçtu, çünkü çalışmanın öğrencilere uygulandığı sırada oran ve orantı konusunun öğretildiği öğrenciler sadece 8. sınıf öğrencileriydi. İçler-dışlar çarpımını kullanan 8. Sınıf öğrencilerinin kağıtlarına bakıldığında eksik değer problemlerini çözebildikleri fakat sayısal karşılaştırma problemlerini çözmekte zorlandıkları ve bu problemleri doğru bir şekilde çözebilen öğrenci sayısının oldukça az olduğu görüldü. Bu durumda, öğrencilerin içler-dışlar çarpımı stratejisini eksik değer problemlerinde ezbere kullandıkları, sayısal karşılaştırma problemlerinde ise orantısal akıl yürütme becerisine sahip olamadıklarından herhangi bir strateji kullanamadıkları düşünülebilir. Öğrenciler, orantısal akıl yürütme problemlerini informal akıl yürütme becerilerini kullanarak çözdükten sonra sayısal karşılaştırma stratejileri geliştirebilir. Bu yüzden, öğrenciler orantısal akıl yürütebilmek için kuralları öğrenmeden önce, kendi informal bilgilerini oluşturabilmeli ve gerekli kavramları geliştirebilmelidir (Uçar & Bozkuş, 2016).

Öğrencilerin kullandıkları stratejileri soru bazında incelemek gerekirse, değişim faktörünün tam sayı olduğu 1. ve 11. problemlerde öğrencilerin birim oran stratejisini daha çok kullandığı görüldü. Diğer taraftan, Christou ve Philippou (2002), problemlerdeki sayılar öğrencilerin birim oranı hesaplayabilmeleri için kolay olmadığında, öğrencilerin en basit yöntemlerinden biri olan arttırma stratejisine yöneldiklerini belirtti. Fakat bu çalışmada, 2. problemdeki gibi birim oranı hesaplamanın kolay olmadığı problemlerde öğrencilerin değişim faktörü stratejisini sıklıkla kullandığı görüldü. Öğrenciler ancak değişim faktörünün tam sayı olmadığı 3. ve 13. problemlerde arttırma stratejisini kullanmaya yöneldiler. 4. ve 14. problemlerde ise 8. sınıf öğrencileri de dahil olmak üzere öğrenciler değişim çarpanı stratejisini sıklıkla kullandılar. 8. sınıf öğrencilerinin bile içler-dışlar çarpımı yerine değişim çarpanı stratejisini sıklıkla kullanmalarında problemin bağlamının parça-parça-bütün olmasının etkili olabileceği düşünüldü. Literatürde en zor bağlamlardan biri olarak kabul edilen benzerlik bağlamına sahip 5. ve 15. problemler, 5. ve 6.sınıf öğrencileri hiçbiri tarafından çözülemezken, 7.sınıf öğrencileri tarafından en çok değişim çarpanı, 8.sınıf öğrencileri tarafından ise içler-dışlar çarpımı kullanılarak çözüldü. Bu bulgular, çözüm stratejilerinin ve öğrencilerin başarı seviyelerinin, problem bağlamından etkilendiği sonucuna ulaşan ilgili literatürdeki çalışma ile benzerlik göstermiştir (De La Cruz, 2013). Orantısal akıl yürütme testindeki (PRT) 7., 9., 17. ve 19. problemlerin sayısal karşılaştırma problemleri olmasına rağmen problemlerde kullanılan çözüm stratejileri farklılık gösterdi. 7. ve 9. problemlerde denk kesirler stratejisi sıklıkla kullanılırken, 17. ve 19. problemlerde değişim çarpanı stratejisi sıklıkla kullanıldı. Bu farklılığın problemlerdeki çoklukların sayısal yapısıyla alakalı olabileceği düşünüldü. Bu durumu örneklemek gerekirse, 7. problemde 20 adet A marka çikolatadan oluşan paketin fiyatı 16 TL, 16 adet B marka çikolatadan oluşan paketin fiyatı ise 12 TL olarak verildi ve öğrencilere hangi çikolata paketinin daha ekonomik olduğu soruldu. Benzer şekilde, 17. problemde ise iki farklı markaya ait çamaşır deterjanının miktarı, kaç yıkama yapabildiği ve fiyatı verilerek öğrencilerden hangi deterjanın daha ekonomik olduğunu bulmaları istendi. Problemde 20 yıkama yapabilen 1 kg A marka deterjanın fiyatı 5 TL ve 30 yıkama yapabilen 1.5 kg B marka deterjanın fiyatı 6.5 TL olarak verilmiştir. Her iki problemde de oranlar arasındaki değişim çarpanının tam sayı olmamasına rağmen 17. problemde değişim çarpanını hesaplayabilmek için daha kolaydı. 17. problemde 1.5 kg, 1 kilogramın 1.5 katı olduğu için 5 TL'nin 1.5 katını hesaplamak öğrenciler için kolaydı. Fakat 7. problemde 20'nin 16'nın kaç katı olduğunu ya da 16'nın 12'nin kaç katı olduğunu hesaplamak öğrenciler için 17.problemdeki gibi kolay değildi. Bu yüzden öğrenciler 17. problemde en çok değişim çarpanı stratejisini kullanırken, 7. problemde denk kesirler stratejisini sıklıkla kullanmayı tercih ettiler. Kısacası, benzer sayısal karşılaştırma problemlerinde farklı stratejilerin kullanılmasının sebebi problemdeki çoklukların sayısal yapısıyla ilişkili olabilir. Bu bulgu Fernandez et al. (2011) tarafından yapılan bir çalışmanın sonucuyla tutarlılık göstermiştir. Ayrıca toplamsal metot kullanımın ve toplamsal metodun hatalı kullanımın en sık görüldüğü problemlerin sayısal karşılaştırma problemleri olduğu görüldü. Bu, öğrencilerin sınıf ortamında sayısal karşılaştırma problemleriyle çok sık karşılaşmadıklarından bu problemlerde nasıl bir çözüm geliştireceklerini bilmemeleri gerçeğiyle açıklanabilir. Tüm bunların yanında, toplamsal metodun hatalı kullanımın 5. sınıf seviyesinde 8. sınıf seviyesine doğru önemli bir şekilde azaldığı görüldü. Bu, öğrencilerin okul uygulamalarıyla deneyim kazanmasının ve yaşın artmasının oran ve oran mantığının geliştirilmesinde belirleyici bir rol oynadığı anlamına gelebilir. Çalışmanın bu bulgusu literatürdeki çoğu çalışma ile uyuşmaktadır (Van Dooren, De Bock & Verschaffel, 2010; Dole, Wright & Clarke, n.d; Dole et Al., 2007; Hilton et Al., 2016; Mersin, 2018; Küpçü & Özdemir, 2011; Doğan & Çetin, 2009). Bu bağlamda, öğretmenlerin, öğrencilerin uygun toplamsal ve çarpımsal akıl yürütme stratejileri geliştirmelerini desteklemek amacıyla orantısal akıl yürütmedeki zorluklarını tanımlamaları yararlı bir ilk adımdır (Van Dooren et al., 2005; Bright et al., 2003).

Bu çalışmanın bulguları, sınıf seviyesi arttıkça öğrencilerin orantısal akıl yürütmedeki akademik başarılarının arttığını göstermiştir. Bu artış, okul uygulamalarının ve yaşın muhtemelen oran ve orantı mantığının geliştirilmesinde belirleyici bir rol oynadığını göstermektedir. Öğrencilerin akademik başarısının sınıf düzeyi ile artmasına rağmen, öğrencilerin ortalama başarı puanları genel olarak düşüktü. Buna ek olarak, öğrencilerin sınırlı sayıda strateji kullandığı, bazı problemleri cevapsız bıraktığı ve bazı problemlerde hatalı stratejiler kullandığı görülmüştür. Bu, öğrencilerin genellikle düşük orantısal akıl yürütme becerisine sahip olduğunun bir göstergesi olabilir. Ortaokul matematik müfredatı birçok önemli kavramı içermesine rağmen, en yaygın olanlardan biri orantıdır. Lise ve kolej seviyesindeki matematiği anlamak için ortaokul yıllarındaki orantıyı anlamak esastır (Johnson, 2010). Bu noktada, öğrencilerin bu ortaokul yıllarında oransal akıl yürütme becerilerini geliştirmeyi amaçlayan durumlarla daha sık tecrübe edilmesi gerekmektedir. Ayrıca, orantısal akıl yürütme yalnızca matematik dersleri için değil, üzerinde durulması gereken bir konu olan diğer birçok disiplin alanı için de gereklidir (Lesh, Post ve Behr, 1989). Bu nedenle, öğretmenlerin öğrencilerin yanlış stratejilerini ve kavramsal ve işlemsel eksikliklerini tespit ederek ders planları hazırlamaları yararlı olabilir. Belli başlı orantısal akıl yürütme problemlerini çözmek yerine, bu çalışmada bahsedilen tüm problem çeşitlerinden öğrencilerin çözmesini sağlamak daha faydalı olabilir. Daha açık olmak gerekirse, öğrencilerin en çok zorlandığı problem çeşitleri olan sayısal karşılaştırma problemleri, değişim faktörünün tam sayı olmadığı problemler ve benzerlik ve karışım bağlamında olan problemler öğrencilere sık sık yöneltilmelidir. Buna ek olarak, öğrencilerin sadece belirli stratejileri kullanmalarını önlemek için, öğretmenlerin sınıfta öğrencilere yönelttiği problemler öğrencileri farklı stratejiler kullanmaya zorlamalıdır. Öğrenciler tarafından çeşitli stratejilerin kullanımında, öğretmenlerin oran ve orantı kavramlarını öğretmede aktif rol almaları gerekmektedir. Öğrencilerin belirli stratejileri kullanmalarının bir diğer sebebi de öğretmenlerin farklı stratejilerin farkında olmamaları olabilir. Bu nedenle, öğretmenlerin bu farklı stratejileri iyice öğrenmesi ve bunları öğrencileriyle paylaşması yararlı olacaktır. Bu noktada, öğretmen adaylarının oransal akıl yürütme becerilerini belirlemeye ve geliştirmeye önem vermek çok daha etkili olabilir. Bu nedenle derslerde daha farklı çözüm stratejileri tartışılmalıdır. Bunlara ek olarak, 8. sınıf öğrencileri zaten oran, orantı ve içler-dışalar çarpımı stratejisini öğrendikleri için çoğunlukla içler-dışlar çarpımı stratejisini kullandı. Ancak, bu stratejiyi kullanan öğrenciler, stratejiyi hatırlayamadıklarında doğru cevabı bulmakta zorluk çektiler. Ne yazık ki, bu yöntem literatürde ezber bir yöntem olarak tanımlanmaktadır ve bu yöntemle problemleri çözen öğrencilerin orantısal akıl yürüttükleri söylenemez. Bu nedenle, içler-dışlar çarpımı stratejisinin öğretilmesi birçok matematik öğretmeni tarafından onaylanmamaktadır (Dole ve Wright, n.d). Bu nedenle, işlemsel ve ezbere stratejilere geçmeden önce kavramsal anlayışı desteklemek önemlidir. Bu şekilde, öğrenciler formülleri hatırlamakta güçlük çektiklerinde problemleri yine de doğru bir şekilde çözme şansına sahip olabilirler.

Daha fazla sayıda öğrenciye katkıda bulunabilmek içim, bu çalışma Türkiye'deki beşinci, altıncı, yedinci ve sekizinci sınıf öğrencilerini temsil edecek şekilde ülke çapında okullardan rastgele seçilen daha büyük bir örneklem ile yapılabilir. Ayrıca, uzunlamasına bir çalışma ile çalışmanın örneklemine ilköğretim ve lise öğrencilerini dahil edilerek, öğrencilerin oransal akıl yürütme becerilerinin gelişimi, ilkokuldan lise yıllarına kadar daha kapsamlı bir şekilde gözlemlenebilir. Öte yandan, araştırmayı daha çok öğrenciye uygulamak yerine daha az sayıda öğrenciye uygulayarak, öğrencilerin orantılı akıl yürütme becerileri görüşmelerle derinlemesine incelenebilir. Böylece, öğrencilerin kullandıkları stratejiler, stratejilerin kötüye kullanımlarının kökenleri veya öğrencilerin bazı problemler türlerinde zorluk yaşamalarının nedeni daha iyi anlaşılabilir. Bu çalışmada öğrencilerin oransal akıl yürütme problemlerini çözmedeki akademik başarıları ve problemleri çözmek için kullandıkları stratejiler üzerinde duruldu. Sonraki araştırmalarda, öğrencilerin orantısal akıl yürütmedeki kavram yanılgılarını önleyebilmek için bu kavram yanılgılarına sahip olma nedenlerine odaklanılabilir. Ayrıca, gelecekteki çalışmalar bu çalışmadaki eksik değer, sayısal karşılaştırma ve ters orantı problemlerini ek olarak nitel karşılaştırma problemleri içeren bir ölçme aracıyla öğrencilerin orantısal akıl yürütme becerilerini inceleyebilir. Ek olarak, öğrencilerin problemleri çözmek için belirli stratejileri sıklıkla kullanmaları, ders kitaplarının veya öğretmenlerin oran ve orantı konusunu nasıl öğrettikleri ile ilgili olabilir. Bu amaçla, ortaokul öğrencileri tarafından kullanılan ders kitaplarında orantı konusunun nasıl öğretildiğini incelemek için bir çalışma yapılabilir. Ayrıca, farklı ortaokullarda çalışan matematik öğretmenlerinden oluşan bir örneklemle, bu öğretmenlerin orantısal akıl yürütme becerilerinin seviyesini ve öğrencilere orantı konusunu nasıl öğrettiklerini anlamak için nitel bir çalışma yapılabilir.

APPENDIX G.TEZ İZİN FORMU / THESIS PERMISSION FORM

<u>**ENSTİTÜ** / INSTITUTE</u>

Fen Bilimleri Enstitüsü / Graduate School of Natural and Applied Sciences	
Sosyal Bilimler Enstitüsü / Graduate School of Social Sciences	X
Uygulamalı Matematik Enstitüsü / Graduate School of Applied Mathematics	
Enformatik Enstitüsü / Graduate School of Informatics	
Deniz Bilimleri Enstitüsü / Graduate School of Marine Sciences	

<u>**YAZARIN** / AUTHOR</u>

Soyadı / Surname : ÖZEN YILMAZ

Adı / Name : Gamze

Bölümü / Department : İlköğretim Fen ve Matematik Eğitimi

TEZİN ADI / TITLE OF THE THESIS (İngilizce / English): INVESTIGATING MIDDLE SCHOOL STUDENTS' ACHIEVEMENT AND STRATEGIES IN PROPORTIONAL REASONING PROBLEMS

TEZİN TÜRÜ / DEGREE: **Yüksek Lisans** / Master [X] **Doktora** / PhD [ ]

1.Tezin tamamı dünya çapında erişime açılacaktır. / Release the entire work immediately for access worldwide. [X]

2.Tez <u>iki yıl</u> süreyle erişime kapalı olacaktır. / Secure the entire work for patent and/or proprietary purposes for a period of <u>two year</u>. * [ ]

3.Tez <u>altı ay</u> süreyle erişime kapalı olacaktır. / Secure the entire work for period of <u>six months</u>. * [ ]

** Enstitü Yönetim Kurulu Kararının basılı kopyası tezle birlikte kütüphaneye teslim edilecektir. A copy of the Decision of the Institute Administrative Committee will be delivered to the library together with the printed thesis.*

Yazarın imzası / Signature .... **Tarih** / Date ....