MATHEMATICAL ONTOLOGY: THE QUESTION OF THE MATHEMATICAL SOURCE OF OBJECTIVITY

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ABSTRACT

MATHEMATICAL ONTOLOGY
THE QUESTION OF THE MATHEMATICAL SOURCE OF OBJECTIVITY

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This thesis traces the source of mathematical objectivity, as an approach to justify mathematical properties to be real, through how our mind and language were evolved. In the mirror of the indispensability argument, and the unreasonable effectiveness of mathematics, it will be argued that the reason why the world and the mind exhibit ontologically similar structures (and properties) is because they have the same ontological origin.

Accordingly, it will be shown that (1) why/how that "the world and the mind have the same ontological origin" explains "the world and the mind exhibit ontologically similar structures (and properties)" And (2) to bring out the self-evidence of the sameness of the ontological origin.

Keywords: Indispensability Argument, the Unreasonable Effectiveness of Mathematics, Mind, Realism, the Unreasonable Effectiveness of the language

ÖZ

Matematiksel Ontoloji
Matematiksel Objektivitenin Kaynağı Sorusu

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Bu tez, zihnimiz ve dilimizin nasıl evrimleştiğinden hareketle, matematiksel niteliklerin gerçekliğini gerekçelendirmeye yönelik bir yaklaşım olarak, matematiksel objektivitenin kaynağını araştırmaktadır. Kaçınılmazlık argümanının ve matematiğin akıl dışı etkililiğinin ışığında, dünya ve zihnin neden ontolojik olarak aynı yapılarda (ve niteliklerde) olduğunu, onların ontolojik olarak aynı kökenden geldikleri şeklinde bir argüman ile savunacağız. Bu bağlamda, göstermek istediklerim (1) "zihin ve dünya aynı ontolojik kökene sahiptir" ifadesinin nasıl ve neden "zihin ve dünya ontolojik olarak benzer yapı ve niteliklere sahiptir" ifadesini açıklaması, ve (2) ontolojik köken bakımından aynılığın kendini kanıtlar nitelikte olduğudur.

Anahtar Sözcükler: Vazgeçilmezlik argümanı, Matematiğin akıl dışı etkililiği, Zihin, Realizm, Dilin akıl dışı etkililiği

To all my Teachers

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CHAPTER 1

INTRODUCTION

There have been many debates about the nature of mathematical entities and their properties among realists and anti-realists, with all their branches, for centuries. One of the critical issues that has been the subject matter of debates for both realists and anti-realists is the relation of mathematical properties to our mind. For example, the claim that mathematical objects exist independent of our mind is known to be the main feature of the robust type of realism[1]. On the other hand, we have the opposite idea in the camp of anti-realists, which rejects any abstract or non-abstract corresponding entities concerning our mathematical statements.

Throughout my thesis, I will attempt to defend a type of non-robust realism[2] which claims that although mathematical properties exist (and I should be able to clarify what I mean by "existence"), they are mind-dependent. In line with this investigation, I will try to shed light on the question of how mathematical properties (as phenomena that appears through some linguistic processes) could be generated from our mind.

Here, although I claim that mathematical properties are mind-dependent, this dependence does not necessitate that the scope of "the mathematical" is all and only in mind; since I acknowledge the existence of mathematical entities in some dimension(s)

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[1] Also known as Platonistic realism.

[2] To refer to a position that recognizes our mathematical properties to be mind-dependent in their existence (in some literatures called anti-platonistic realism, see Mark Balaguer, "Realism and Anti-Realism in Mathematics." In Philosophy of Mathematics (Handbook of the Philosophy of Science), ed. Andrew Irvine, (Amsterdam: North Holland, 2009), p.36

of the external world (in addition to the structure of the mind).[3] Thus, from this perspective, a realist position holds because the structure of the mind has been formed through the interaction with the world[4]

I claim that this communication (between our conscious mind and the world, to extract/invent mathematical structures) has become possible mainly because of **a)** the experience that we have had with the world while evolving; where, through this evolved capacity of the mind (i.e., the capability of the linguistic abstraction) the mathematical forms (intuitively derived) became a rational tool to enable the mind to reflect on itself (on what evolution has put in its formation.)[5]

And **b)** because mathematical structures can be considered to indicate that the effect of interacting with the world (to shape our constitutive faculty), in line with the evolution of language, provided us with clues that enabled us to combine the language with what our evolved brain would confirm to be intuitive.

The result of such a construction turned to be amazing in terms of consistency and bringing up new theorem which we analytically did not know. We started to use this new dimension of existence to apply in science and make successful predictions. It was "unreasonably effective" and has become the primary tool for us to quantify over

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[3] This must be the major difference between my view (MP'ism) and the traditional psychologism. In the coming two chapters, I will try to explain how MP'ism encompasses different features of both actualist and possibilist versions of psychologism. [I tried to find a good expression for what I have in mind, but any label seemed misleading to me (I tried super-psychologism, conscious-materialism, naturalistic monism, naturalized Platonism (a phrase which has been used by McDowell, but I was not sure if this term would be compatible with what I have in mind) and etc. So, just for now, I will call my view as MP'ism (to stand for materialistic platonism).]

[4] As I will later elaborate more, the mathematical (and linguistic, in general) connection between mind and world has become possible because of a sort of correspondence that exists between the world and the type of being that our mind has become to be, or better to put, has been shaped to be).

[5] One may call it the constitutive feature of the mind, but what I have in mind is something more. And although, as I will discuss language was formed to effectively reflect our experiences with the world, the process of mathematical abstraction engages more than our linguistic capability since, mathematical type of abstraction is based on something we usually refer to as mathematical intuition; you say, intuition of (I would rather call it 'the capability of abstracting the notion of) "oneness", "addition", etc.

our theoretical objects. All these indicate that something real must be going on in the way that we do mathematical abstraction.

In the first chapter[6] of my thesis, I will very briefly go over some ideas among realists, which are mostly related to my non-robust position. Naturally, I have to be selective to be able to direct and concentrate the discussion to the core problematic of my research. So, in this chapter, I will introduce the two main versions of realism (namely, Platonism and Anti-Platonistic realism). I also go over the two branches of psychologism (both to be forms of the anti-Platonistic interpretations of realism) and structuralism. Then, I will attempt to provide a ground for in what sense psychologism and structuralism could be defended in the light of what I call MP'ism[7]

Before I make my points about this possible notion of the mathematical reality, in chapter 2 and 3 I will try to provide the reader with a vivid picture of two objections against platonistic realism (i.e., <u>the epistemological objection</u>, and the <u>non-uniqueness objection</u>). And then, two arguments in support of realism in general (i.e., the "<u>unreasonable effectiveness of mathematics</u>" (UEM) raised by Eugen Weigner, and <u>the indispensability argument</u> (IA), or Quine-Putnam's indispensability argument in mathematics.) I will later try to use these arguments to both clarify and substantiate my view on mathematical ontology. By commenting on these two critiques, I will also attempt to clear the path for presenting the idea of MP'ism.

The fourth chapter is allocated to the main argumentation of this thesis, where I will provide the reader with one way of defining what can be counted as "real" in terms of mathematical structures. In the fifth chapter, I will go over the non-robust *concept-platonism* position argued by Daniel Isacsson (1998). I chose concept-platonism as one appealing response to the objections mentioned above, and this choice was, first, because I found major similarities this view has with some features that I developed as

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[6] This chapter is mostly based on Balaguer's essay "Realism and Anti-Realism in Mathematics", 2009.

[7] Although this view has some core ingredients of psychologism and structuralism in it, what I mostly wanted to be the general argument of MP'ism is a sort of monism.

of MP'ism. Secondly, and most importantly, because of the differences that Isacsson and I have in viewing mathematical emergence. I realized that analyzing these similarities and differences can be immensely helpful to enable me to trace some relevant and significant historical issues that non-robust realists have had in defending their positions.

Finally, I will finish my thesis by suggesting a form of Platonism where mathematical apprehension (and structures) to be seen as shadows for what (and how) the world is.

1.1 Realism vs. Anti-Realism

Generally, the view which is known as realism in mathematics implies that our mathematical theories are true descriptions of a part of some entities, either abstract of non-abstract. On the other hand, we have the anti-realism in mathematics which asserts that mathematical realism is false (in a way, that there is nothing that our mathematical theories are about.) Consequently, according to this view, our mathematical theories are not a true description of any part of the physical or abstract world.

Among mathematical realists, we can mention Platonists who believe in the abstract and non-spatio-temporal mathematical objects. The other group is the anti-platonist realists who think that our mathematical theories are about some spatio-temporal objects. But even among the anti-platonist realists, we can distinguish between advocates of psychologism and physicalism. Psycholigism is the view that our mathematical theories are true descriptions of our mental objects; whereas, physicalism says that our mathematical theories are true descriptions of some physical and non-mental objects.

The Platonistic realism is a, generally, well-known view in the philosophy of mathematics. But let us talk more about anti-platonistic realism (especially, psychologism). Whereas as physicalism indicates the view that mathematics is all about physical objects and treat mathematics as if it is an empirical science (although a general kind of it). So, for example, John Stuart Mill [1843, book II, chapters 5 and 6], as a prominent advocate of physicalism, says that mathematics is about physical things in the sense that when we add something to the same kind, we have two of them. So, this, does not indicate the existence of abstract objects, and very briefly speaking, there is no such thing as abstract objects.

1.2 Psychologism

On the other hand, we have psychologism that claims that mathematical objects are mental objects (and more specifically, are ideas in our mind). Therefore, for example, when we say that 8 is an even number, we are talking about one particular mental object (to be number 8). We can refer to two different types of psychologism, namely actual psychologism and possibilist psychologism.

Actual psycholgism is a view that believes that our mathematical statements correspond to our actual mental objects. Therefore, '8 is an even number' is about an actual mental object 8, which has the property of being even. Whereas, *possibilist psychologism* is a view that says that the mathematical statements are about our mental object, which is possible to be constructed by the mind. For example, the statement "There is a prime number between $10^7$and (10^7+ 2)" says that it's possible to construct such a number, even if no one has ever constructed one.[8,9]

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[8] Mark Balaguer (2009) thinks that possibilist psychologism is not a genuinely psychologistic view at all, because it doesn't involve the adoption of a psychologistic ontology for mathematics. He points out that possibilist psychologism would collapses into either a platonistic view (i.e., a view that takes mathematics to be about abstract objects) or an anti-realist view (i.e., a view that takes mathematics not to be about anything - i.e., a view like deductivism, formalism, or fictionalism that takes mathematics not to have an ontology).

[9] Later, after defining what I understand of mathematical objects (or properties) to be about, I will show that one can, in manner, hold a combination of actualist-possibilist psychologism without falling into any of anti-realist or pure (traditional)-Platonistic views.

1.3 Some Critiques against Psychologism

Psychologism, in a way, is a diluted version of realism since from their ontological perspective on mathematics, they claim that the mathematical objects used in our mathematical theories do not have any independent existence.

Therefore, from one point of view, psychologism has similarities with fictionalism in the sense that both of them claim that mathematical objects are made by us (as compared to objects that have their independent existence). The difference, however,

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To give a general clue about the standpoint of this thesis, I am going to discuss that mathematics is about a kind of construction, as a potentiality that has been dependent on time, environment (including the technological) and the stage of (mental-) mathematical progression; and not as *objects* which has absolute and separate existence from our being. Instead, mathematical existences are possibilities (of some structures of the mind which heavily intertwined with how our mental-linguistic capability was evolved) manifested through the usage of our mathematical language; such possibilities (you read such mental properties) that for their emergence and abstract constructions, linguistic tools (and probably in the case of applied mathematics, pragmatic thinking) is necessary.
I will also argue that indispensability argument (and some other facts about mathematics, like UEM) are indicators to provide us a picture for why our mathematical structures match so well with our confirmed scientific theories. Furthermore, I will elaborate on the fact that the origin of the mind and the non-mental objects have been the same, the indispensability argument (IA) and unreasonable effectiveness of mathematics (UEM) can be taken to indicate this sameness.
So, I see it very natural to think of mathematics as a part of the mind's projection/production (irrespective whether it is used and merged in our scientific theories or not). I also see it intuitive to think that we have closer access to the logical-mathematical possibilities of our mind when it is compared to having access to the properties of the empirical world (and that is why relatively mathematics is such a strong language while it talks about forms and models), and when we try to make sense of the world using our sense perceptions through using experimental tools and tests. However, as far as it is related to this part of the discussion (possibilist psychologism), to ask whether a set of mental properties (resulted from the mind's self-reflections) exist or not is a tricky question which will clarify in the coming chapters.
Now, before I continue talking about the hybrid version of psychologism, I want to claim that (for now, only to present my ontological position) mathematical structures can be considered as an essential property of the mind that has become possible to be revealed through the mind's self-reflection and language, using intuitive axioms, and intuitive logics for such construction. I also assume that the structure of the external world and our mind have some commonality. It may sound very vague to say so. However, if we have evolved from the same type of materials, then would it be surprising to say that brain (and its mental and conscious phenomena) can have some affinity, relation, and communication with the external world?
Well, let us take the external world to be mentally (if we can attribute such a thing to objects) static and passive without having any power or control to make changes in their structure. So, if we take our own mind as active (progressive to understand itself and the world), then all these linguistic, mathematical, scientific and mental reflections are there to understand and change the mind through this understanding. Thus, as a part of this reflection and becoming, I see that the mathematical production (established on intuitive axioms/ logical for the mind) is, in a way, revealing a dimension of the mind's and nature's structure. I will later try to provide more proofs for this claim.

is that in fictionalism the claim is that our mathematical theories are based on fictional stories (and therefore, there is no truth behind them), whereas in psychologism the claim is that these theories are true for the actors of these stories and do exist in our mind. However, in Balaguer's words "*this is a rather empty sort of truth, and so psychologism does not take mathematics to be actual in a very deep way*" (p.82).

However, the problem is that psychologism, when it comes to explaining the indispensability and application of mathematics in empirical sciences, face the same sort of questions that fictionalism has to deal with to defend its position. For, for any commonsensical observer, it will become a question of how concepts and objects in our mind turn out to be so necessary and applicable for natural sciences.

The difference, however, between psychologism and fictionalism is that although fictionalism admits that, for example, one can have the idea of number 8 (in a mathematical theory) and therefore '8 is an even number' means that my idea about number 8 is an idea about an even number. Now, for psychologists, unlike fictionalists, claims that our mathematical theories are about "ideas in our head." So, in the above example '8 is an even number' and my idea about number 8 to be an even number are exactly (by ontological reference) the same. Nonetheless, as mentioned, similar to fictionalism, psychologism needs to explain Quine-Putnam's indispensability argument. (Colyvan, p.652)

Mark Balaguer points out four objections against and problems of psychologism, which any account that contains psycholigistic assumption must be able to explain. He states:

> First of all, psychologism seems incapable of accounting for any talk about the class of all real numbers, since human beings could never construct them all[10]. Second, psychologism seems to entail that assertions about very large numbers (in particular, numbers that no one has ever thought about) are all untrue; for if none of us has ever

[10] Although I will ,partly, explain how constructing any mathematical property might become possible in ch.4 & 5: yet, I do not have any definite position towards the claim that there exist "*the class of all real numbers*" since the notion of infinity would be involved, and my theory would be silent either to account or reject it.

> constructed some very large number, then any proposition about that number will, according to psychologism, be vacuous[11]. Third, psychologism seems incapable of accounting for mathematical error: if George claims that 4 is prime, we cannot argue with him, because he is presumably saying that his 4 is prime, and for all we know, this could very well be true[12]. And finally**,** psychologism turns mathematics into a branch of psychology, and it makes mathematical truths contingent upon psychological truths, so that, for instance, if we all died, '2 +2 = 4' would suddenly become untrue. (There is as Frege says, "Weird and wonderful ... are the results of taking seriously the suggestion that number is an idea[13].) (p.82)

1.4 Mathematical Platonism

Mathematical Platonism implies that there are such abstract (non-spatiotemporal) mathematical entities that exist independent of us (and our mental activities), Frege and Gödel are among modern figures who embraced this kind of view. Nevertheless, it must become clear what we mean when we talk about *abstract* objects. So, I hope that throughout this thesis, I can also provide an approach that (as a step to clarify the platonistic belief of objectivity of mathematical properties) furnish the way for a kind of holistic view of our mathematical practices.

There are different types of platonism proposed by different philosophers (some examples are Frege and Gödel, and some parts of Quine). In his own version of platonism, Mark Balaguet presents a view which he calls “full-blooded platonism”, where he put forward the claim that” all the mathematical objects that (logically possibly) could exist actually do exist, i.e., that there actually exist mathematical

[11] For MP’ism, as far as the natural numbers are concerned, since the structure is there, the structure of *any* (regardless of how large it is) number is there (for the same question regarding real number go to footnote 48.)

[12] Not the case with MP’ism since the development of the human mind is not one individual’s mind but the “human mind” in perceiving the abstract through our common intuition.

[13] MP’ism explains that the existence will never become inexistence, mathematics is a reflection of mind (to rise from existence) on itself, and although contingent to time and environment of the time to bring up some possibilities to be perceived, it is relatively true.

objects of all logically possible kinds." However, this is a critical point which requires clarification about what "logically possible" is. [14]

So, imagine an alien with a different type of existence. They are mainly, difference in the way their mind function, living in a different planet with, relatively, different type of time experience (as general relativity indicates) who can logically formulize some mathematics form their existential experience (or even some abstract notions rising from their type of consciousness and being) in a manner that is incomprehensible to us. Do these logical (at least, for the alien) properties exist?

Again, if I refer to my approach for an answer, we can say that since this aliens are a product of this world, the reflection of the alien's consciousness (consciousness based on an evolved enough brain-like processor to do mathematical abstraction[15]) would enable them to establish a connection with some parts of their intuition. The intuition to derive logical principles from their (mental and bodily) existence is what may make them able to perceive and construct structures (to match with their intuition) after going through (any possible) language.

So, if these structures show compatibility with their empirical theories to explain the world (or consistency within their other rational constructions), to be called something like mathematics, it just mean that the language was capable of, at least partly, reflecting one dimension of their mind-on-mind / mind-world mapping/matching (and hence, shows that their construction was based on true mathematical assumptions) and

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[14] If we test the above claim about being logically possible with the kind of image I will depict for mathematical ontology (in chapter 4&5), we realize that since the concept of "logically" in the *logically possible*, to be understood rationally is problematic. Since we do not exactly know what those possibilities are, we do not know what could be the structures that our metaphysical condition of the time may justify us to count as logical, and we do not know the breakthrough innovations that science may bring up for us to assume some mathematical structures to be logical. So, I find it difficult to account for the acceptance of these possibilities (in the platonic sense and as pure and abstract entities) to exist.
At this point, I do not want to accept or reject this conclusion (since it does seem to draw upon a clear image of what logically possible means). However, I want to ask a question that may help us clarify the path to how we may approach the issue.

[15] I am assuming that evolution is a universal phenomenon.

thus enables them to establish a linguistic communication with the existence (the nature).

1.5 Structuralism

One of the popular positions in the philosophy of mathematics is the view that our mathematical theories are not about specific abstract objects (to explain them), rather they are like containers that can be filled with any collection of objects that fits that structure. So, according to this view, the ontological status of particular mathematical objects (like number 8) is undermined to *exist*. Instead, what matters in this perspective is the pattern that exists to connect those mathematical objects. [16]

So, for example, in Balaguer's expression,

> What structuralists maintain is that arithmetic is concerned not with some particular one of these ω -sequences, but rather, with the structure or pattern that they all have in common. Thus, according to structuralists, there is no object that is the number 3; there is only the fourth position in the natural-number pattern. (p.42)[17]

1.6 Mathematical Anti-Realism

Although, generally speaking, I am not supporting any specific anti-realistic view, I see some of the defenses (and critiques) put forward by fictionalism so valuable to contribute in suggesting a defendable position concerning mathematical ontology,

[16] Some famous names who endorse structuralism are Benacerraf (1965), Hellman (1989) [both to advocate anti-platonistic structuralism], Resnik (1981) and Shapiro (1989) [both to endorse platonistic structuralism]. (Balaguer,p.42)

[17] The thesis that I will try to defend is partly congruent with platonic structuralism. I will come back to structuralism in chapter 5, when I take a closer look at Daniel Isacsson's concept-platonism.

more specifically when it comes to presenting my own view. Before anything, though, let us go over some basic definitions and categories of anti-realism.

Anti-realism is the view that mathematics does not have an ontology, i.e., that our mathematical theories do not provide us with true descriptions of some part of the abstract of physical world[18].

There are different versions of anti-realism. But the common problem among them is to provide a satisfactory account for applicability and indispensability of mathematics. In other words, to justify IA and UEM.

One version of anti-realism which I must point at is factionalism.[19]According to Balaguer,

> [for] fictionalists, mathematical sentences and theories are fictions; they are comparable to sentences like 'Santa Claus lives at the North Pole.' This sentence is not true, because 'Santa Claus' is a vacuous term, that is, it fails to refer. Likewise, '3 is prime' is not true, because '3' is a vacuous term - because just as there is no such person as Santa Claus, so there is no such thing as the number 3.[…]. Thus, the real difference between sentences like '2 + 1 = 3' and sentences like '2 + 1 = 4' is that the former are part of our story of mathematics, whereas the latter are not. (p.47)

There are un-negligible problems about fictionalism stance. The major critiques could be "how one story is recognized by everyone," why (in Daniel Isacsson's[20] terminology) mathematics is *invariant with respect to change of objects* (unlike fictional objects like a unicorn), how is it that mathematics is indispensable and

[18] In this sense, I cannot relate myself to anti-realists. Since, as it will be explained, language, in its interaction with the world, is getting its formation (existence) from our sensations. I obviously think that our mathematical language (at least the one with strong intuitive axioms) has roots in how the mind was structured. So, since I assume that the structure of the mind (in its source) has commonality with the structure of the external world, I understand that the mathematical self-reflections of the mind on itself benefit from a level of existence.

[19] First introduced by Hartry Field [1980; 1989]

[20] Daniel Isacsson, "Mathematical Intuition and Objectivity." In Mathematics and Mind (Logic and Computation in Philosophy, ed. Alexander George, (Oxford University Press, 1994), p.118-140

applicable in science, why is this specific mathematical story is aesthetically pleasing?
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[21] Although because of UEM, IA, and the other reasons, I do not embrace fictionalism in its totality, I find the fictionalist notion of the “alternative stories” interesting to contribute to my own view. So, if, in line with my narration, I push the alternative form of a mathematical story to be dependent on our biology and mental structure, we can imagine that, on the ground of different environment, different body (different physical evolution) and/or mind’s type, a different sort of symbols (matched and in relation to that conscious entity) and intuitive axioms could be formed.
Here, I want to refer to Stanisław Lem’s (a Polish philosopher who is mostly known for his science fictions) position towards understanding an alien language (which I found very useful to talk about to what extent the development of a language can affect, or not affect, the structure of mathematics.)
So, imagine we receive a message from stars. Is it possible for us to decode it into our own language and understand it? Lem provides us with an answer which might not very much promising. His argument, briefly, is that we cannot make sense of that message mainly because of two impassable barriers, i.e., linguistic and intelligence gaps.
What he means by the linguistic gap is that we do not share any of the reference points we rely on for language. So, for example, for us the sentence “grandmother dead, funeral Wednesday,” can be translated in all languages. But this translation is possible because we share the same cultural and biological reference. However, what if, for example, those aliens reproduce like amoeba? Then the concept of “grandmother” would automatically vanish from their culture. The same applies to the notion of death (for example, what if instead of decomposing, they are divided at the end of their lifespan). So, even if they (or we) are able to translate their message into our language (because of having different type of biology and being), there is no guarantee that we can make a good sense even of one word (that if they construct such a thing to be called a “word”!)
So, even if they have such a thing we call grammar, it would be far from the construction of our mental possibility to understand and discover them. Plus, and a non-cultural language may be very limiting and require some common scientific and technological codes; “This would mean the communication we're receiving isn't just the message itself, such as a message in binary code.” Astronomy, 2016)
But Lem seems to be optimistic about mathematics as a universal language (although he is not an optimist about it as a mean that would facilitate the communication between aliens and us. Since, in using the language of pure mathematics, each proposition may refer to many many different things.)
Lem says “*with mathematics, one can say nothing about the world — it is called 'pure' for the very reason that it has been purified of all material dross, and its absolute purity is its immortality. But precisely therein lies its arbitrariness, for it can beget any sort of world, as long as that world is consistent.”*
What he simply means is that without using language (I understand natural language) we cannot use mathematics as a mean for communication. “With mathematics one may signal only that one Is, that one Exists,” [You can read the whole article from William Herkewitz, "The cynics' case for why we may never understand extraterrestrial communication," *Astronomy*.]

However, as I have been trying to show, I doubt mathematics to be pure from material dross, our biology, and natural language.
[Please note that I am not rejecting the idea that some aliens may have similar type of grammar that we have. Indeed it is notable to contemplate about claims like the ones of linguists, Bridget Samuels and Jeffrey Punske, when they say:

> The whole universe is subject to the same laws of physics. For example, there are not that many ways a signal can be transmitted, particularly over large distances," or, "[W]e can

CHAPTER 2

TWO CRITIQUES OF PLATONISM

There are two major critiques which were raised against Platonism, and although, in my approach, I am not defending (at least the classical type of) Platonism that claims mathematical objects have the characteristic of existing mind-independently, I find it very useful to elucidate these two well-known critiques in the light of what I later call MP'ism.

In this chapter, instead of trying to derive any definite ontological conclusion from them, it suffice only to introduce the two arguments and give some brief comments on

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> expect that extraterrestrial languages ... have a vocabulary consisting of building blocks of meaning that can be combined to create more complex meanings." (See Erick Mack, "Alien Languages might not be that Different from Ours. Cnet)

But I find this view somehow flawed. The problem is that first, we do not master the science (laws) of physics to know how an alien consciousness would interact with its environment (we do not know much about different environment as well) in order to have an awareness about linguistic images of the world around it, second, we do not know, evolutionary speaking, what will happen to the structure of our mind after some million years, third, and consequently, we do not know about that alien's biology.]
I will try to show that mathematical logic to be extracted from the same language which has developed parallel to the development of the language (and language on the basis of how our intuition of time, number and sequences, etc. were shaped by the experience), so, in many cases our mathematical production must be created with reference (and by) the structure of the language. So, what Lem said about the differences in language, probably have *some critical effects* on the way we construct our mathematics.
Nevertheless, it appears to me that mathematics consists of some unique structures (to bring up the claim of objectivity). So, if, as might be the case, the mathematical dimension of the world is, potentially, reflectable by any developed enough consciousness in the universe, it is possible that aliens' mind can find a way to interpret a good part of mathematics, formalized by us, to deduce a model where the same structures understandable for them. [the concept of forcing in set theory extends our understanding of logical entailment]

the way that they are formulated. Nevertheless, in the coming chapters, I will take them precisely as useful tools while I will try to articulate and substantiate my stance on mathematical ontology. Before we discuss any ontological implication of such objections, though, a clear description of the arguments are necessary.[22]

2.1 The Epistemological Objection Against Platonism

Mark Balaguer (2009) formulizes the Epistemological Argument against Platonism as follows:

(1) Human beings exist entirely within space-time.

(2) If there exist any abstract mathematical objects, then they exist outside of space-time.

Therefore, it seems very plausible that

(3) If there exist any abstract mathematical objects, then human beings could not attain knowledge of them.

(4) If mathematical Platonism is correct, then human beings could not attain mathematical knowledge.

(5) Human beings have mathematical knowledge.

Therefore,

[22] I must note that although both epistemological and non-uniqueness objections were, initially, raised by Benecerraf (1965) to emphasis the problem of how human mind and mathematical objects could causally interact (for us to have access to those abstract entities), I use the versions which were developed by Mark Balaguer.

(6) Mathematical Platonism is not correct.[23]

2.2 The Ontological Non-Uniqueness Objection

The non-uniqueness problem (or what Charles Parsons calls "multiple reduction" problem[24]) points to the Platonist perspective that (as was discussed in the previous chapter) sees our mathematical theories to describe unique collections of abstract objects, which turns to be problematic. So (borrowing Balaguer's formulation of this objection), the idea comes as follows[25]:

(1) If there are any sequences of abstract objects that correspond with the axioms of Peano Arithmetic (PA), then there are infinitely many such sequences.

(2) There is nothing "metaphysically special" about any of these sequences that makes it stand out from the others as the sequence of natural numbers.

Therefore,

(3) There is no unique sequence (inductive set) of abstract objects that is the natural numbers (many sets can model the same structure)

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[23]Although, at this point, I do not intend to discuss the validity of these propositions (and the conclusion) one by one, as a general comment, I think that (1) is an obscure and problematic assumption; since we really do not know what is it *to be* in the space-time and what time and space exactly are (I will briefly talk about the notion of time in chapter 5.) The same applies to (2). As one purpose of this thesis, I will try to show why the problem is more complicated than how some platonists believe about *abstraction* and *existence*. So, one of my objectives in articulating these concepts will be to cast doubt on the idea that mathematical properties (I prefer "properties" or "structure" over "objects") have their existence independent of us.

[24] Parsons, *Mathematical Thought and its Objects*, p.48 [from "Cognition, Content and the A Priori" by Robert Hanna, p.387]

[25] I will later elaborate on Daniel Isacsson's proposal (which he calls "concept platonism") and there we will have a closer look at the non-uniqueness objection.

But,

(4) Platonism entails that there is a unique sequence of abstract objects that is the natural numbers.

(5) Platonism is false.

So, in Robert Hanna's words (2015),

> This problem flows from the fact that many different models satisfy the abstract structure of any logical system rich enough to express Peano Arithmetic, hence the second-order logic of Peano Arithmetic underdetermines the identity conditions of the natural numbers.[26]

It requires another extensive research to enter into the details of how, at the beginning of our mathematical adventure, the apprehension of natural numbers was possibly constituted[27] within the construction of our intuition. Nevertheless, it seems to me that the capacity of abstract counting (for example, to understand what it would mean to talk about n+1, n/2, 2*n …), as mental structures that are laid in our very basic intuitions about the world (and this mental ability) started to be formed and developed from when the conscious mind (of any evolving entity) started to interact with the world (to be discussed in details in chapter 4.)

However, it appears as an objective fact that even before we talk about twoness, there were entities of the similar type to be called "this and another one of this." So, it seems that evolution has provided us with an efficient[28] linguistic tool to become able of

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[26] Robert Hanna. Cognition, Content, and the A-Priori: A Study in the Philosophy of Mind and Knowledge.

[27] I do not and am not referring to the standard set theoretic model (unless one says that the standard model mirrors and captures exactly the same notion of numbers as we have in mind). Since even before Frege and Dedekind and Zermelo and others we were able to count relying on our basic intuition (and regardless of the concern of whatever the set theoretic construction of a number might be.) Rather, here, I want more to focus on the metaphysical ground (and process) that made the act of counting possible.

[28] Indeed, by efficient I do not mean "perfect", rather, language development has turned to become an "unreasonably effective" tool (as we will see, in a way, is different from unreasonable effectiveness of mathematics) through the way that we explain and constitute (put together) the world in our mind.

counting as we do now (including the apprehension of the notions like what is like to be “more” or “less” and the ability to put them into linguistic forms.)

So, the reality of these mathematical structures (in the other words, the reason why I would call them “real”) comes from the fact that it was through nature (to include all that exists) that our mental capacity was shaped (on the basis of our mental reality[29]) in a way that we can construct our hypotheses (through the clues that we get from outside about the world, using our linguistic-constitutive faculty) by reference to our intuition (which, as I said, was shaped by and through nature).

So, all that we discover about mathematics is a mirror on how things (in our mind, in the form of properties that we call mathematical) could be in terms of relations and structures (regardless of what that thing is.)[30]

Nevertheless, oneness or twoness (or any of the natural numbers) can be attributed to anything (even when those physical things are not present), and these numbers are used in our scientific theories and appears to us to be the real and objective language of nature, which brings us to discuss the unreasonable effectiveness of mathematics, and the indispensability argument.

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[29] What a mental reality is a grand question that should be discussed in relation to what consciousness might be. Here, nevertheless, by mental reality I simply refer to the fact that there is something (say, a matter) that, in Descartes words, result in the phenomena to say that “I am” and that (regardless of how this being is) we are aware of this existence.

[30] It is as if nature develops a machine (from itself), and then the machine, based on the ingredients (and structures) given to him by nature, on a ground of a conscious mind, starts the process of abstraction using its own creative construction (on the ground of the same nature from where he emerged.

CHAPTER 3

INDISPENSIBILITY ARGUMENT AND THE UNREASONABLE EFFECTIVENESS OF MATHEMATHICS

It is very strange that mathematicians are led by their sense of mathematical beauty to develop formal structures that physicists only later find useful, even where the mathematician had no such goal in mind. [...] Physicists generally find the ability of mathematicians to anticipate the mathematics needed in the theories of physics quite uncanny. It is as if Neil Armstrong in 1969 when he first set foot on the surface of the moon had found in the lunar dust the footsteps of Jules Verne.

[Weinberg, 1993, p. 125][31]

3.1 The Indispensability Argument

Indispensability argument (IA) was explained by Shapiro (2005) as follows:

> Quine and others, such as Putnam [1971], propose a hypothetical-deductive epistemology for mathematics. Their argument begins with the observation that virtually all of science is formulated in mathematical terms. Thus, mathematics is ‘‘confirmed" to the extent that science is. Because mathematics is indispensable for science, and science is well confirmed and (approximately) true, mathematics is well confirmed and true as well. This is sometimes called the indispensability argument.[32] (p.14)

[31] Mark Colyvan, "Mathematics and the World." In *Philosophy of Mathematics (Handbook of the Philosophy of Science)*, ed. Andrew Irvine, (Amsterdam: North Holland, 2009), p.689

[32] Stewart Shapiro. The Oxford Handbook of Philosophy of Mathematics and Logic.

So, from the Quine-Putnam argument one may want to, legitimately, conclude that fictionalists cannot account for these indispensable applications of mathematics to empirical science.

Mark Balaguer (p.85) nicely comments on indispensability argument[33] to say that:

> The central idea behind this view is that because abstract objects are causally inert, and because our empirical theories don't assign any causal role to them, it follows that the truth of empirical science depends upon two sets of facts that are entirely independent of one another One of these sets of facts is purely platonistic and mathematical, and the other is purely physical (or more precisely, purely nominalistic).

As a response to this argument, a fictionalist may want to argue that the applicability of mathematics in empirical science (in a dispensable manner) does not necessarily refute fictionalism.

In his analysis of the IA, Mark Balaguer invites us to take the sentence (A): The physical system S is forty degrees Celsius, as an example. Here we have S to refer to physical and 40 to refer to an abstract object, and there is no causal relationship between these two objects (how abstract object can have a causal relationship with the physical world). So, "it is not saying that the number 40 is *responsible* in some way for the fact that S has the temperature it has."(p.85)

But then how can scientific facts be true based on two distinguished types of facts (that are held or do not hold independent of one another), namely, purely nominalistic facts and a set of purely abstract (Platonistic) facts? He concludes that there is no such things as platonistic (purely abstract) objects[34], or mathematics appears in empirical

[33] Which opens the way for one to contemplate on how (on the ground of our constitutive faculty) we are able to merge and relate mathematical properties (as from what I understand, the empty containers) to empirical explanations and theories"

[34] For example, for a fictionalist to claim that "*the picture that empirical science paints of the physical world could still be essentially accurate, even if there are no such things as mathematical objects*"(p.86). Furthermore, Balaguer argues that it would be an acceptable argument to say that mathematics appears in empirical science as a descriptive aid; this I did not find quite satisfactory since mathematics not only does provides us with an easy way of talking about the empirical world, it is also a part of seeing and quantifying the nature around us (it is in the heart of scientific thinking when it

science as a descriptive aid; that is, it provides us with an easy way of saying what we want to say about the physical world.

3.2 The unreasonable effectiveness of mathematics

"Unreasonable effectiveness of mathematics" is an expression used by Eugene Wigner (Noble prize winner in physics) in 1959 (later published in an article in 1960). Wigner thought that the application of mathematics (as a product independent of our empirical consideration) is extraordinary and miraculous "*The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve*."(P.14)[35]

Wigner's idea was that since we do not have a clear understanding for why mathematics is unreasonably (unexpectedly) applicable to our empirical science, we cannot (philosophically speaking, as I understand) comment on our scientific theories if the theory (intertwined with mathematical forms) is uniquely suitable to explain our phenomenon or not. (Sarukkai, 2005)

Throughout his analysis, Wigner points to a mysterious fact (which he calls to have "no rational explanation") in how mathematics is useful in natural sciences (Sarukkai).

In this inquiry to investigate the unreasonable effectiveness of mathematics, I will be mostly focused on the use and emergence of mathematical language that, apparently, turned to be unreasonably effective. A considerable portion of my analysis in this part can be counted as a reaction to Sundar Sarukkai's essay (*Revisiting the 'unreasonable*

comes to numbers and quantities). Thus, I see it very improbable that the process for our empirical knowledge construction, cognitively speaking, is independent from how our mathematical production capability was developed.

[35] Eugene Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," *Communications on Pure and Applied Mathematics*, VOL. 13, NO. 1, February 1960: p.14

effectiveness' of mathematics, 2005) who concludes that it is not only mathematics (as a language) but even our (natural) language is also unreasonably effective

> Mathematics cannot be applied to the world but only to some descriptions of the world. This description occurs through the medium of language and models, thus leading us to consider the role of mathematics as language. The use of a language like English to describe the world is itself 'unreasonably effective' and the puzzle with mathematics is just one reflection of this larger mystery of the relation between language and the world.

That being said, apart from this kind of language-oriented approach, one may go to a detailed investigation based on how scientific methods are constructed, the aesthetic nature of mathematics or whole complete research on the notion of "applicability" when we talk about the mathematical application in natural science.[36]

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[36] Mark Steiner is the one who provides an extensive analysis for the notion of applicability of math in science, and although it would worth to go over his suggestion in details, the main goal of this thesis (add to it the space restriction) does not allow me to properly elaborate on Steiner's Ideas. However, as the reader may realize, at the end of my thesis, we will see that some of his concerns, as far as I can see, are relatively answered. Here, to give a general image of what Steiner says, I will quote Mark Colyvan (2009) describing Steiner's (and also partly his own) idea on unreasonable effectiveness:

> Steiner claims that it is important to distinguish the different senses of 'applicability' because some of the associated puzzles [Steiner believes that UEM consists of a number of puzzles] are easily solved while others are not. For example, Steiner argues that the problem of the (semantic) applicability of mathematical theorems was explained adequately by Frege [1995]. There is, according to Steiner, however, a problem which Frege did not address. This is <u>the problem of explaining the appropriateness of mathematical **c**oncepts for the description of the physical world.</u>
>
> Of particular interest here are cases where the mathematics seems to be playing a crucial role in making predictions. Moreover, Steiner has argued for his own version of Wigner's thesis. According to Steiner, the puzzle is not simply the extraordinary appropriateness of mathematics for the formulation of physical theories, but concerns the role mathematics plays in the very discovery of those theories. In particular, this requires an explanation that is in keeping with the methodology of mathematics _ a methodology that does not seem to be guided at every turn by the needs of physics.
>
> The problem is epistemic: why is mathematics, which is developed primarily with aesthetic considerations in mind, so crucial in both the discovery and the statement of our best physical theories? Put this way the problem may seem like one aspect of a more general problem in the philosophy of science _ the problem of justifying the appeal to aesthetic considerations such as simplicity, elegance, and so on. This is not the case though. Scientists and philosophers of science invoke aesthetic considerations to help decide between two theories that are empirically equivalent. Aesthetics play a much more puzzling role in the Wigner/

It is not a new idea that people think of nature to be written in the language of mathematics (a statement which is attributed to Galileo). Sarukkai says that this idea

> has been echoed for centuries after by figures such as Newton, Einstein, and Feynman. Wigner too joins this chorus and begins by correctly noting that only some mathematical concepts are used in the formulation of laws of nature and these concepts are not chosen arbitrarily.(p.3)

What I can understand in this regard is that (the so-called, external) world exists parallel to our mind's type of existence, and mathematical emergence and manifestation occurs (and is about) on the ground of the interaction and the type of communication between the mind and the world. Mathematics came to existence from the same source, and to answer why "some mathematical concepts are used in the formulation of laws of nature," and why they are not arbitrary, I would say it is like tuning a musical instrument.

We (scientists) try to tune (or find) a mathematical language with respect (from) our natural language (the natural language through which we use to describe the world). Yet, I see mathematics to be systematically (and logically) deducted structure of the same natural language (as far as one can define a logical, sequential and causal justification for that system, and as far as this system could justify itself as "a possibility" of how the world and the things, no matter how much abstract, could be).

I will finish this chapter by an imaginary conversation between Sarukkai and me. Sarukkai says:

> One of the elements contributing to the mystery of mathematics lies in the physicist stumbling upon a mathematical concept that best describes a phenomenon only to find that the mathematician has already <u>developed that</u>

Steiner problem. Here aesthetic considerations are largely responsible for the development of mathematical theories. These, in turn, […] play crucial role in the discovery of our best scientific theories. In particular, novel empirical phenomena are discovered via mathematical analogy. In short, aesthetic considerations are not just being invoked to decide between empirically equivalent theories; they seem to be an integral part of the process of scientific discovery. (Colyvan. P.690 & 691)

> concept independently.[37] As examples, Wigner cites complex numbers and functions, the appropriateness of which is especially manifested in the formulation of the complex Hilbert space which is so essential to quantum mechanics. The surprising (to the common sense) and necessary role of complex numbers and functions along with the idea of analytic functions is one example of the 'miracle' of mathematization. (P.3)

Omid: Mathematical capability, as a part of the evolution (I'd rather call it progression) of the mind is laid in our intuition, and the structure of language (I will explain how language was developed through the mind and the world interaction) is progressive (see image 3). Indeed, mathematics is not separable from the structure of intuition and language and since, in Isacsson's words (1998, p.119), "*Thought is the only basis on which the truths of mathematics is obtainable….Thought is the only medium by which the facts which are reflected in its truths impinge upon us."* But, for me, the question is, do we think "mathematically" when we try to explain the world? Is there such a thing as the logic of natural language?

Sarukkai: "The important argument here is that mathematical concepts are not accidentally useful but are necessary in the sense that they are the 'correct language' of nature." (P.3)

Omid: I think that it is not very accurate to say "the correct language." Language has been formed and evolved on the ground of the conscious mind's capacity, so, as I have argued (and will argue), the fountainhead of both the logic of mathematical language and scientific language is in the mind, such a mind that has the capability of intuitive understanding of the mathematical abstraction (I will say more about the concept of intuition). But, what do you mean by "necessary"?

> Sarukkai: Wigner offers three examples to illustrate this necessary relation. The first is that of Newton's law. Not only was this law based on 'scanty observations', it also contained the physically non-intuitive idea of the second derivative and yet exhibited an extremely high sense of accuracy. The second

[37] Isacsson (1994) whose ideas will be discussed in chapter 5 also comments on mathematical independence "[…] *mathematics, in its pure form, unique among the sciences in that its notions make no reference to the physical world*." (p.119)

> example is the matrix formulation of quantum mechanics. The miracle in this case, according to Wigner, lay in the fact that one could apply these matrix methods even in cases where Heisenberg's initial rules did not apply, as illustrated in the calculation of the lowest energy level of helium. The third example is that of quantum electrodynamics, particularly the theory of Lamb shift, a theory which again showed extremely high accuracy with experiment. From this, Wigner concludes that mathematical concepts, 'chosen for their manipulability', are not only appropriate but are also accurate formulation of the laws of nature.
>
> For him, these laws, together with the laws of invariance are the foundation of the mathematical method in sciences. Finally, he considers the uniqueness of theories in physics and asks whether mathematics alone can help adjudicate which theories are essentially right. The problem here is that some theories which are known to be false also give 'amazingly accurate results'. The examples he gives of these 'false' theories are Bohr's early model of the atom, Ptolemy's epicycles and the free-electron theory. (P.4)

Omid: So, one basic question here should be whether we can use mathematics as a criterion to know if a scientific theory is true. It is very difficult to answer this question, right? Since apart from the notion of trueness (which in addition to the empirical confirmation, needs to appear reasonable and comprehensible on the logical background of our mind), as I can infer from your (Sarukkai's) anomaly examples, the primacy should be given to the scientific explanation to see how successfully it can fulfill the basic expectation that we have about occurrence of a phenomenon (like causality, generality and the images we have in mind from the previously established scientific theories). [38]

However, as I previously discussed, to combine mathematics with our scientific theories is like tuning the mind's logico-mathematical vibration with a part of a more general type of language. This tuning is needed because the expression of existence (in

[38] Nevertheless, I think it could be a great research topic to investigate and see whether there is a significant difference between the theories which have benefited from intuitive and pure mathematics and those which have the kind of mathematics that were initially created for pragmatic purposes. Also to search which types of theories are more explanatory; those which primarily started from mathematics (as a possibility of how the abstract things, the universe [Existence] for example, is) or those which fundamentally started from the empirical tests and then tried to find a proper mathematical tool for its structure.

the mind, or the realm of possibilities) becomes distorted, broken in parts and filtered through the application of language (it is so because of the type of consciousness and sense perception that we have.)

So, a careful observation (and mathematical literacy) is needed for us to find out how some of our deductive and inductive (mathematical) conclusions match with what we have come up through our sense perception as systematic, causal, and scientific explanations. The major tool is to examine (and build) our scientific understanding with respect to our abstract language of mathematics (which, because of its, sort of, consistency, we have come to believe in mathematics to have some abstract truth in its existence.)[39]

I understand that for this tuning (also when we construct our scientific theories) we use the already produced mathematics (in its pure form) for most of the times. However, we know that a mathematical structure can be created for practical purposes as well (invention/derivation). [40]

So, I assume that the source of our mathematical capacity is our experience (while the qualitative properties of our) consciousness was exposed to the external world through our senses.) Moreover, the source of our scientific explanations are the empirical tests (and in a way our experience). But, how is it that this evolutionary capacity (the use and development of mathematical language) turned to meet (and reconcile) with our scientific theories? I will come back to this important question in the coming pages.

[39] More to be discussed on the notion of truth and existence.

[40] This would be another question to search if the criterion of consistency is enough for a newly practical-based mathematical invention to be called true in the same manner that the pure mathematical theorems are.

CHAPTER 4

MATHEMATICAL ONTOLOGY

To start the inquiry about how mathematical production becomes possible, I want to ask three questions: “in what way(s) mathematics is the production of mind?”, “What happens when a mathematician discovers (or, as one might say, invents) a new system, relation, proportionality, etc.?” Furthermore, assuming that it is the human mind that puts mathematical entities and their relations together, “to what extent can we defend mathematical properties to be real?”

So, before we engage in any of these discussions about the ontological status of mathematical objects, I should be assuming that mathematical entities are mind-dependent in *their formation and manifestation* (throughout this chapter, I will elaborate more on mind dependency of mathematical emergence).

As far as it is related to the question of “how are we able to understand (and exercise) mathematics,” I suggest that mathematics, as human understands it, is a partial-structural dimension of the reality, being formulated by the mind's capacity. This capacity (in mind) manifests its different levels of "being" depending on the metaphysical means that the condition of the external (mental and material) reality provides for the mind; for the mind to reflect on itself (on the way that it is constituted to perceive and apprehend the world) and create/discover a proper linguistic (intuitively logical within its construction) forms to project the mathematical properties on the mind and exteriorize it.

Indispensability argument (IA), as explained in chapter 2, provides us with a reliable method of reasoning to include that, at least, some mathematical structures have some sort of reality in their manifestation (in their type of being). From this perspective, (and in line with the controversial argument on the independence of the existence of the abstract mathematical objects), mathematics can be seen as a property of the external reality in the sense that, in the mathematical level of abstraction, there really exist such harmonies, proportionalities, relations, etc. *somewhere* in a dimension of the world; where these properties can be shown in the forms of mathematical language (to conclude that this correspondent and relatively confirmed-truth is a part of the nature of the mathematical construction).

Yet, as I suggested, mathematical structures are conceived and arise from the individual's deliberate (intentional, intuitional and linguistic) relation between some specific (pre-given) properties of the mind and the tools that the external reality provides for such a construction; which we can call *the capacity of mathematical thinking.*

4.1 The Capacity of Mathematical Thinking

Although, in a general picture, our encounter with the chain of the pre-established mathematical forms (created/discovered by people in the past), the environment of the time and even the instrumental necessities for the new scientific theories are counted to be considered as elements of experience (therefore, from this perspective, mathematics cannot be seen to be purely *a priori*) yet, mathematical emergences, at least in their pure mathematics sense, are not empirical in the same way that scientific theories are.

Mathematical knowledge and the way it receives confirmation for their validity are different with respect to their ontological status compared to those other elements that figure into scientific theories which are already conceived as existing in space and time;

while the existence of mathematical objects in space and time is something to be questioned.

The capacity for relating the basic concepts and axioms, and the formation of such a deductive, linguistic and complex construction arise from the fact that our intuition (or generally, common sense) confirms the linguistic form of that axiom (with regard to the network that they are supposed to form. For example, axioms with regards to the network that concepts like oneness, twoness, accumulation/addition, absence/subtraction, dot, line, etc. would create). However, since the mathematical process is a linguistic one, as I will discuss, it is “broken” and time-based.

I tend to take it as obvious that mathematics has something to do with the nature of the mind-world interaction[41] (which requires our consciousness, sense perception, the way we experience time and space, language, and etc.) This interaction exists in our current type of being (human-world communicative form), and it is so because this is how we have evolved to be. I also take it as one of my basic assumptions that our mathematical intuitions are what they are as a result of having evolved this way.[42]

We are coming from the same nature, so the human mind is not to be conceived on an ontological plane that is distinct from the rest of nature in a way that creates an

[41] Here by the world I mean the physical objects, their essential properties and sensible manifestations. Yet it should be kept in mind that they are not completely mind-independent; in the sense that they are dependent on our mental-mathematical constructions and projection via the technology at a given time, which is also dependent on the dynamics of our social structures. In other words, it should be kept in mind that social structures and technology play an important role in the formation of our logical-mathematical structures.

[42] It would worth (and also would be interesting and relevant) if I quote Richard Joyce when he says

> There is…evidence that the distinct genealogy of [mathematical] beliefs can be pushed right back into evolutionary history. Would the fact that we have such a genealogical explanation of…‘1 + 1 = 2’ serve to demonstrate that we are unjustified in holding it? Surely not, for we have no grasp of how this belief might have enhanced reproductive fitness independent of assuming its truth.(R.Joyce.p.182)

So, as the reader will see, I will try to give a metaphysical explanation for such an evolutionary process, and at the same time hold the idea that mathematics has some relative truth (in other words, to be *about)* how the reality of the world is.

irreducible dualism. But human being has got the capacity to develop language to mirror his mind, the intuitive concept producing (as it is our subject of concern here, mathematical properties) on the world through language, and to become able to explain it.

So it should also be noted that, despite reliance of this thesis on the concept of “intuition,” the position which I defend is not precisely as, what Daniel Isacsson calls, concept platonism[43] either since I see the phenomena of the mathematical perception as essentially broken and discrete in its emergence.

I am saying it is essentially broken since while the division and individualization emanated from *the one* (we were part of nature before that), the mind and the world have separated and became two in their interaction, they have become two in their vibration, reflection, and reality as it is perceived through our individualized bodily being (see image 2).

So, the communication (of the conscious mind to relate itself to the world through the perceptory sensors and rational tools) through mathematical language is partial[44] (not exactly reflecting the reality of the mind/world) and frozen in the form of language, as a means to interact with the mind’s intuition (the part of the mind which has been evolved to, and continues to, establish an *effective* relation with the world through linguistic abstraction).[45] It is in this process of interaction that we reflect on our mind's

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[43] Isacsson elaborates on concept platonism as follows: “a [mathematical] structure is given by concepts, [so] it favors an account of mathematical reality in terms of the reality of mathematical concepts” (p.125). Somewhere else he says “philosophy of mathematics must take account of the fact that thought is the only medium by which the mathematical facts impinge upon us. The locus of our contact with concepts is the process of thinking about, or with, them.” Isacsson calls his understanding, which is based upon the *objective reality of mathematical concepts*, “concept Platonism”. (p.126) more to be discussed in chapter 5.

[44] Since language is developed within the boundaries and limitation of our physical (environmental and thus sensorial) evolution and there is a gap between what our (scientific or mathematics) language may count as (approximately) true, and what is really there (in the Existence).

[45] In a sense, this is Hegelian. Since for Hegel, mathematical concepts are dead (“inert and lifeless”) in that they are the result of abstractions (See Brenda Larvor, In. “Lakatos’ Mathematical Hegelianism." *Academia*,). This, for Hegel, is not to deny that they are good and useful, but he thought that we go

property (mind's type of existence) where language (as a natural product of such a mind) becomes an objective foundation for our abstract and mathematical contemplations.

Moreover, as I mentioned, time and the environment of the time play role in the mathematical type of emergence, something that may give the implication that mathematics is progressive; (normatively speaking, if it keeps roots in the essential parts of the principles of rationality (intuitive and logical consistency, for instance), and not merely in our formal use of language, or practical necessities)[46]

4.2 The Main Argument: An Old Solution in a New Dress

So, Mathematics is a property of mind just because those mathematical entities, axioms, and objects were discovered/ synthesized/produced by the mind (and, then, discussed and agreed upon by the mathematical community).

It is this phenomenon that, from a bunch of simple mathematical definitions, axioms, and relations (defined in intuitive-linguistic forms), we came to have such, scientifically, effective and complex systems of mathematics. This is how, by reference to IA and the UEM, one might be convinced that there are, in a way, mathematical properties in the structure of reality; and consequently, as it is the argument of this thesis, in our mind.

What appears to be is that mathematical relations and abstractions are produced by us, both through sensations and rationalization. In other words, in the beginning, I suppose, mathematics was very empirical (since it had not found the proper language,

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wrong if we start taking this dead abstraction as the ultimate truth and forget that it's merely an abstraction and the dialectic of the mind doing the abstraction is where "the truth" is.

[46] That could be another discussion to ask whether math produced for pragmatic reasons are less real. However, it appears that for naturalists it would not be a problem, whereas for logicists, even if it serves pragmatism, it should not be there, and it could be serious problem.

and the environment, for its development) where, as I showed in Image **2**, they were pre-linguistic concepts to be intuitively felt. Then, it started to have some vague connotation of the nominal (concept-identification) in mind (for example, number one as is used for "one apple" or "one son"), and were used for purely practical purposes.[47]

[47] Although there are notable differences between how I depict the mathematical dimension of our mind to develop compared the kind of picture that Isacsson sketches, I found this part of his explanation, congruent with my view when he talks about how different notions of mathematical conceptions arise from the experience. He states:

> If, for example, our attention is directed to collection of discrete (physical) objects and processes of putting them in an order, abstraction may lead to the mathematical notion of "first element" and "successor of". We may then reach the notion of natural number and arithmetic operations on natural numbers. If, by contrast, our attention is focused on the movement in space of a physical object, we may be led to the notion of continuum of points, and so to real numbers in mathematics (p.126)

I will discuss the similarities and differences between my view and those of Isacsson's in the coming chapters. However, in this context, Isacsson also points out that "*most mathematical concepts arise not from experience in the external world, but from mathematical experience itself (P.126)*". So for him, some notions like exponentiation "*seems already to be a purely mathematical extension from existing mathematical notions*". Nevertheless, how I view this "mathematical extension" is not purely separate from our time of the mind, environment and experiences. Throughout evolution (and in general the universal chains of changes) the Existence pushes the time-dependent mind to see new realms of possibilities, thus any mathematical emergence must be understood in the light of these possibilities of becoming (to have the chance to be reflected by the mind).

Yes, it is possible that the life of the mathematician and his/her individual experiences have something to do with the kind of mathematics s/he produces. Yet since mathematics, by its nature, seeks objectivity in its creation, the mathematician's mind, to a great extent, must be under a more general notion of the mind (to keep the connection between the language which he uses and those, apparently, objective concepts).

Moreover, I think it is very legitimate to ask for an explanation of how "mathematical extension" resulted into the discovery (emergence) of concepts like π, e, or golden ratio φ; whether these real numbers (deduced from our mathematical system) are based on (founded upon) the intuitive concepts constructed by mathematical facts (in Isacsson's examples, discrete physical objects or the movement of a physical object), would these real numbers have the same type of reality (having roots in the intuition abstracted from the experience we had in the world) as the natural numbers? Or, we have to separate them as somethings to do with the intuitive concept of a line?

Is the concept of natural number consistent with the way we use real numbers? Are there two types of realities; since I can think of "a" and "b" to be the unit "1" in a^2+b^2=c^2 but how do I want to project √2 as a number if I can assign no unit or proportionality to it other than referring to a not-very-clear geometrical derivation? (So, for instance, we know that we can have two apples, but is there, in reality, a way for us to have √2 apple?) Or do we need to rethink and carefully scrutinize the possible epistemological gaps in how we assign units (natural numbers) to lines? Is not it because of these possible gaps that we have paradoxes like the one of Zeno's? Is not it a crucial difference that the unit in our mind (number one) has some differences with the one we assign to a line? Are numbers and geometrical (imaginable spatial shapes) represent two different realm of existence?

Obviously, to answer questions of these sort would require a more extensive research than my master's thesis, since it is related to how we started to apply natural numbers in real world, how our

It was at the latter time that we started to use our capacity to generalize notions like "adding" or "categorizing" in a more abstract manner. But this was the starting point.[48]

And the abstraction started; it was because of its fascinating deductive characteristic that Pythagoreans and probably in a more complex manner, Plato) came to believe that there is a realm of mathematics that is real (and even more real than this world) (Moslehian, p.36[49]) Nevertheless, this chain of deductive (and synthetic, in Kant's expression) discoveries were not that perfect for providing mathematicians always with satisfactory and intuitive answers. We had examples like the discovery of irrational numbers as earthquakes in the mathematical worldview which required us to revise our assumptions about mathematical reality in different senses (the separation of arithmetic and geometry for Plato's disciples was one early attempt to make such reconciliation).

So, mathematicians, along with the mathematical dilemmas in the history of mathematics, had to find ways to cope with these discrepancies. Dilemmas that, sometimes, resulted from our basic axioms. So, we had to search in the mind in attempts to find what could affirm the intuition to invent, adopt or add some other definitions and systems to our already made mathematics (like the case of the non-Euclidian geometry.)

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brain and language was developed, and how different phenomena started to represent themselves on our consciousness (by consciousness, I mostly mean what Ned Block calls access consciousness, 1995) to shape our mental being and its mathematical capacity.

[48] It, probably, requires more explanation to show how mathematics started to grow and manifest itself in human's mind after a period of practical usage. But, I am inclined to conclude that, somehow, and after a specific stage, this practice appeared to accompany with the joy of certainty (which usually philosophers seek to reach and had made the Pythagoreans joyful, since it was a divine capability, for them, to explain and make predictions).
I assume that was the most appealing part of mathematical idealism for the early mathematicians. But to give an explanation in line with the metaphysical stand that I am taking, I can say that it was the joy, or maybe the illusion, of the mind seeing its property in the form of absoluteness that exist in mathematics (in Platonic terminology, the joy of remembering a flavor of what the soul had seen in the realm of Form).

[49] Moslehian, Mohammad Sal, *Philosophy of Mathematics.* Vajegan-e Kherad Publication, 2005 (in Farsi)

However, despite all these *breaks* in the processes of mathematics, it would *not* be much reasonable to say that mathematics is a matter of mere invention as well. I say “breaks” because the mind is limited in mirroring and understanding its formation in its totality. So, it seems clear that the defined (mostly based on our intuition’s) assumptions are not only related to how we perceive time and space, but also language-dependent (both in its intuitive formation and also the way we express them), and thus fragmentary in reflecting[50] the reality (the unity of the mind and the world in its consistency). In other words, and again in the Kantian sense, there is always this gap between noumena and phenomena (what a deliberate thought can apprehend, and say, about mathematical object’s state of being in its symbolic and linguistic form).

In image**1**, I have tried to show how when the mind tries to mirror itself (its *naturally* formed structures) on the plane of some mathematical axioms and definitions it ends up providing us with some possible models of how things might objectively be (what could be the structures of the things, in their abstract forms.)

[50] It is legitimate to ask how mathematics is the reflection of the mind on itself, and at the same time as a rational extraction (through abstraction) of the natural language. As I see it (and I think I have enough reasons for such an assumption) mathematical potentiality is the one that has been formed parallel to the evolution of the brain and in line with the development of the natural language. It has been this relation between the mind and the world outside mind that has created the capacity to construct notions like numbers, the space, dots, lines and so many other abstract objects within the construction of our brain (for our consciousness to have access to these natural concepts).
So, numbers, dots and lines are concepts (or concept- images), yet “how to put these concepts into language (how to define them) and how these mathematical systems turned to be true” in some interpretations (as the fact of consistency and trueness of the theorems resulting from them) goes back to “how the pre-linguistic structure of the mind (see image 1) imposed (projected) itself on the language.”
I previously explained how, through an evolutionary process, the mathematical capacity of the mind has been realized, and how the abstract arrangements of the things (or a dimension, of their existence) reveal themselves to the mind (in other words, how it has become communicable to the mind through mathematics because/through of the commonality they have both in the source, but also because the human consciousness, that is a ground for the creation of such a mechanism, through mind, is based on our physical existence (constructed from the same materials outside of us). So, inevitably, this (deliberate) reflection is essentially compatible and can be tuned (to be extended to the world outside of the mind.) So, the fact that our mathematical capacity was given to us through evolution, does not mean that mathematical structures are merely our mind invention.

Moreover, it is enough to look at mathematics' successful functions in the complex scientific theories to admit that mathematics is a manifestation of *some streaks of reality* and not a mere invention (the idea that connects us to the indispensability argument.)

In the process of reflection on mathematical objects and the abstractions over such entities to be applied on empirical theories, in a fashion, we scan the external reality (which are being represented to us through empirical explanations) with the available cognitive tools, to match a proper (strong enough) mathematical language with what we think to be a justification for a physical phenomenon.[51]

As it is the case for many of our encompassing scientific theories, the success of the applied mathematical languages has been extraordinarily explanatory. And although, for many cases, those scientific theories may become accompanied with anomalies, using Thomas Kuhn's terminology, (and I am inclined to take this fact as another reason to confirm the partial, one- dimensionality and discreteness of the mathematical production in the process of mind-mind, and mind-world reflection), the level of explanations, applications, and predictions are so high that we cannot think of mathematical structures to be considered as merely a mental game having no connection with outside reality. They say something directly about the world, and thus, are streaks of reality[52].

To give a summary of what I have said so far, a systematic arrangement of some of my assumption and conclusions might be useful.

(a), (b) and (c) are my assumptions (I take them as given), whereas,

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[51] We know that we have a theory like the Newtonian mechanics (or calculus as conventional tool) which was invented alongside with his calculus. But even in this case, *Principia* was significantly rooted in the works done by the past mathematicians. Furthermore, Leibniz developed almost the same system in the same period, which provides us another reason to argue about the dependency of mathematical manifestations on history.

[52] Thus, unreasonable effectiveness of mathematics can be counted as branch (and bold expression) of what I have been articulating so far about mathematics as a product of mind and its relative truth.

(d) and (e) are my conclusions:

(a) The world exists.

(b) Our minds exist.

(c) The way the mind is structured is the result of evolution.

(d) Strict ontological dualism is to be rejected, in the sense that the ontological origin of the mind and the world are the same.

(e) As a consequence of (d) the world and the mind exhibit ontologically similar structures and properties.

Like so, in line with the main argument of this thesis, I argue that the reason why the world and the mind exhibit ontologically similar structures (and properties) is because they have the same ontological origin.

I will argue that they are structurally similar, and show that this seems to call for an explanation in that it seems like a miracle (to use Eugene Wigner's expression when articulated the unreasonable effectiveness of mathematics).

So, I want to advocate that they have the same ontological origin is the explanation. Accordingly, I want to show that (1) why/how that "the world and the mind have the same ontological origin" explains "the world and the mind exhibit ontologically similar structures (and properties)" And (2) to bring out the self-evidence of the sameness of the ontological origin).

So, the argument in a short and systematic manner would be as follows:

(f) Through the use of language, thought, and observation, we can develop scientific theories that explain the world and make reliable predictions.

(g) The development of scientific theories is inextricably interwoven with mathematical structures.

(h) The understanding, use, and development of mathematical structures are relatively independent of empirical considerations (to be derived from UEM).

(i) If our best scientific theories are confirmed to be relatively true, then the mathematical formations used within such theories must be true as well (otherwise, in Putnam's words, we commit "intellectual dishonesty") [Putnam, 1971/1979, p. 347][53]

(j) From (h) and (i), if the act of our abstract thinking can be seen to be confirmed to indicate how the external world is, then there must be similarities (commonality) between how our thought is (in the relatively independent operations of mathematical thought to reveals itself through the application of mathematical theories and scientific thought).

(k) The immediate bases of this commonality are that our thoughts come from our conscious bodily existence.

(l) From (j) and (k): The basis of the correspondence between the pure and abstract mathematical thought and the world must be in our conscious existence.

(m) Our mental being emerges from our physical being

(n) From m: Our mathematical abstraction is *about* how we are.

(o) From (f), (g), (h), (i), (j), (k), (l), (m), and (n): our mathematical thoughts are about how the world is.

(p) From (a), (b), and (o): Our abstract mathematical thought has, in an expression, incomplete (see (**s**)) access to the quiddity of existence.

(q) Evolution connects the conscious mind to the quiddity of the things only through our developing sensory apparatus.

(r) Our sensory organs are limited in reflecting the reality as it is (through the way that it has formed our language and perceptions of the world).

(s) Mathematical constructions are partial, one-dimensional images of the Existence.

[53] M. Colyvan, p.651.

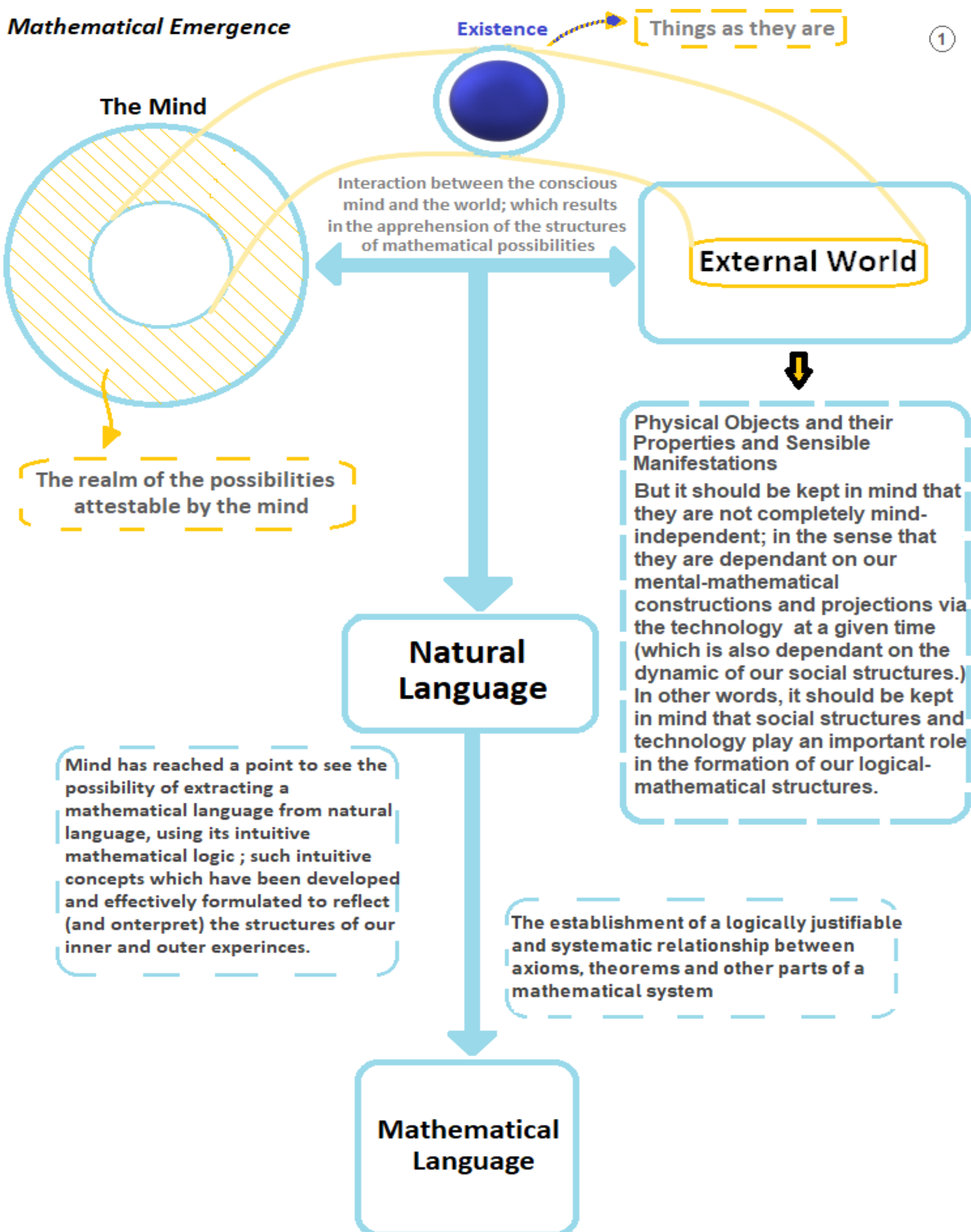

Figure 1: Mathematical Emergence

4.3 Mind and the Creation of the Mathematical Language

It is incredible to investigate the problem of how it was possible for the mind to generate such magnificent apparatus (mathematical systems) out of a limited number of axioms. It seems that, as I showed, there has been great mind-corresponding truth about the reality of some of our basic axioms and assumptions that the (natural) abstraction of the mind has captured them through its reflection. It seems that it is only through this synthetic construction that the reality makes sense of itself to us.

Therefore, it seems that the mind, in its rational (concept-producing) and linguistic manifestation, has some properties that match with those of reality (since the mental abstraction over some innovative notions turned to be extraordinarily successful, both to explain, and also to interact with the world through a channel, a possibility, or a dimension that mathematics opens to us). Hence, although our mathematical products do not reflect the absolute picture of Existence (the noumena[54]), mathematics says something *directly* about Existence in general (since it belongs to and is rooted in it, as illustrated in (l) and (P).)

As I previously tried to demonstrate, I am making the claim that mathematics is not a totally *a- priori* phenomenon since a) it is the conditions of time and the environment that pave the way for some specific mental structures to emerge and to be formulized; b) in addition to the fact that language is directly dependent on the pre-linguistic concepts and our intuitive apprehensions of the world, in many cases, mathematical innovative-discoveries requires us to borrow hints from outside, either because of some practical necessities, pervious models, by the consideration of the available linguistic forms, or other factors that bounds the rationality of the mind to the time and to the rest of the world.

Well, although I have tried to demonstrate why mathematics is more than innovation, I should better say some more words for why mathematics can be counted as innovation too. Mathematics is innovation because to exteriorize itself and appear, it has to be

[54] Keeping in mind Kant's critique of pure reason and the limits of understanding.

constructed through the innovative mind (mind's rational and technical methods, such a mind that is, at a specific point of the time, is hugely under the effect of its environment), through linguistic forms.

So, briefly, mathematics, in a manner, can be counted as discovery because the synthetic consistent production of its structure reveals a property (or a shadow, in a Platonic terminology) of the reality. Then, we pick an-open-for-interpretation axiom or notion, and the creative discovery/invention starts again.

I also find it very useful to approach the reality of mathematical properties by investigating how new inventions take place in the field (and how the mind adjusts itself facing critical discrepancies, and the edges of those broken mirrors (as opposed to being a consistent and complete and perfectly deductive system) or, how "the creative mind" comes up with new forms and possibilities.

If I am going to answer this question considering the theoretical ground I provided so far. So, the answer to those questions could be similar to this:

Mind and reality (mind-world) have a common set of properties which enables them to communicate (enables the conscious mind to communicate with the world through the perception sensors), and for those mathematicians who are equipped with the formal language (and are concentrated and creative enough), through "mind-on-Mind, and mind-world[55]" reflections, and the application of proper linguistic forms, some of these properties are discoverable, and derivable.

Therefore, mathematical systems present only some (limited) dimension of the reality (in its metaphysical sense) that have the features (and the condition) of being reflected on the (time- dependent)[56] innovative mirror of mind (again, I am saying innovative

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[55] The "mind-world" reflection may be understood in the sense of creating the (infrastructural) capacity for mathematical progression, since it is the mind-world before mind-mind interaction which introduces new stimulation that opens up possibilities for the mind to adjust and develop itself.

[56] In a more relaxed expression, I would describe it as something like the basic sensorial property of the brain that emerges in particular (historical and environmental) conditions, provided that the

because language is discrete in reflecting phenomena, and we have to be selective in choosing a statement, or the system of the language in general).[57]

Thus, what I can understand about mathematicians is that they are mostly the laborer of their time, they consider the already built structure, get inspiration from their scientific theories and technological tools, and can see only a limited horizon of mathematical possibilities (being brought to us by nature) for their judgment and contribution. So, this endeavor is, in a way, progressive; and the mathematical dimension of the mind, in a Hegelian picture, progressively "becomes" in its reflections.

person is capable of concentrating and finding a proper language for that case (for that imagination, puzzle, or possible modality of a certain condition; either mathematical or physical). However, since the basic intuitive axioms are widely accepted, the construction of a relatively unanimous language turned out to be easier. The reason for this unanimity, or everyone's confirmation of the objectivity of mathematical statements, must be sought in how our mind was structured (and developed) to make mathematical abstractions (even) in perception (in space and time, as the dimension we perceive in and through, our abstractions, though not *a priori* as Kant says).

[57] Which would depend on the metaphysical condition of the time on how to find, for example, one mathematical logic justifiable and reliable (which itself depends on the previous works of mathematicians, and probably the stage of our scientific level, the dynamic of our social order and etc. (I say metaphysical because I want to draw attention to the fact that these social and historical contingencies open up ontological possibilities for realization).

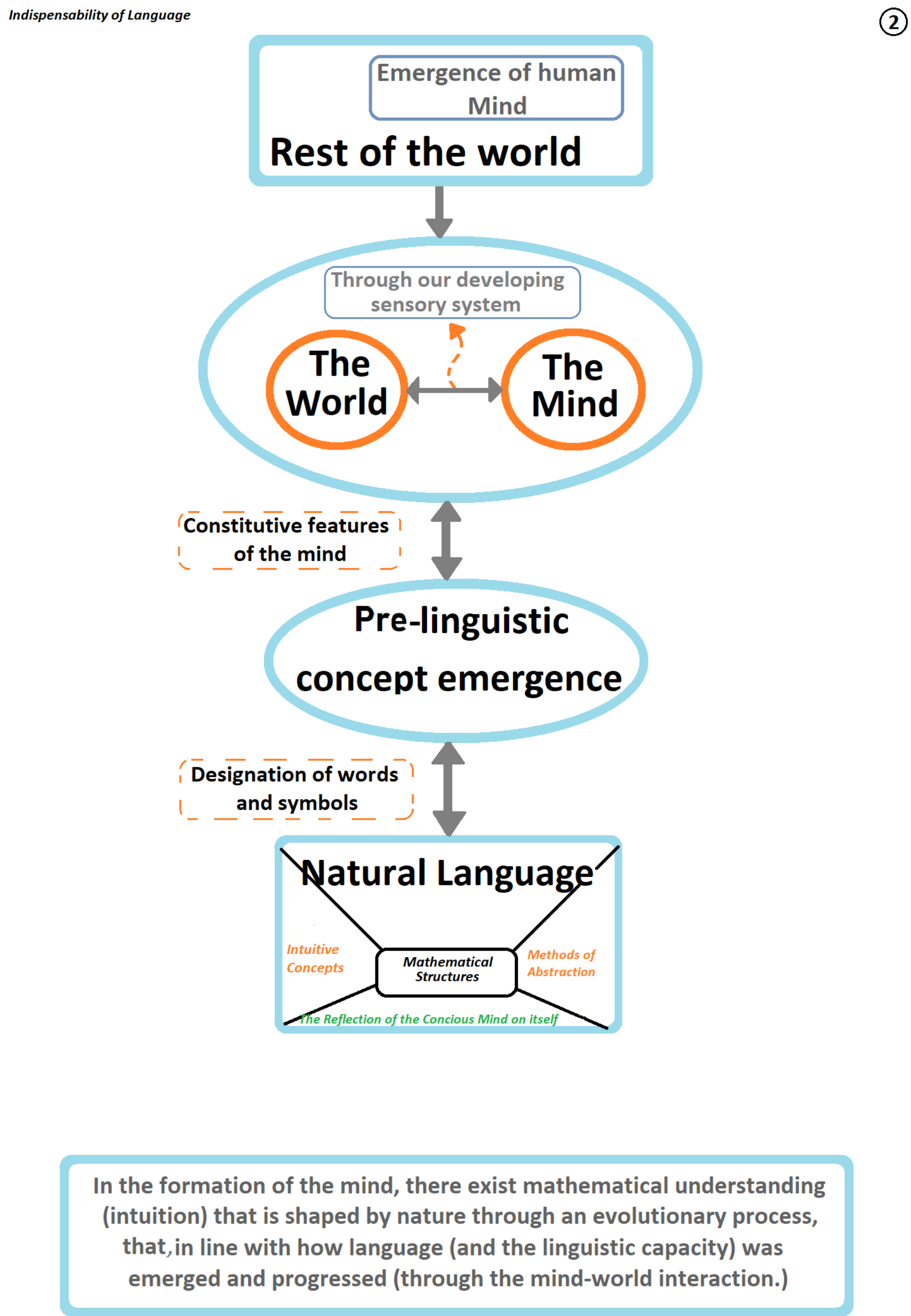

Figure 2: The Indispensability of Language

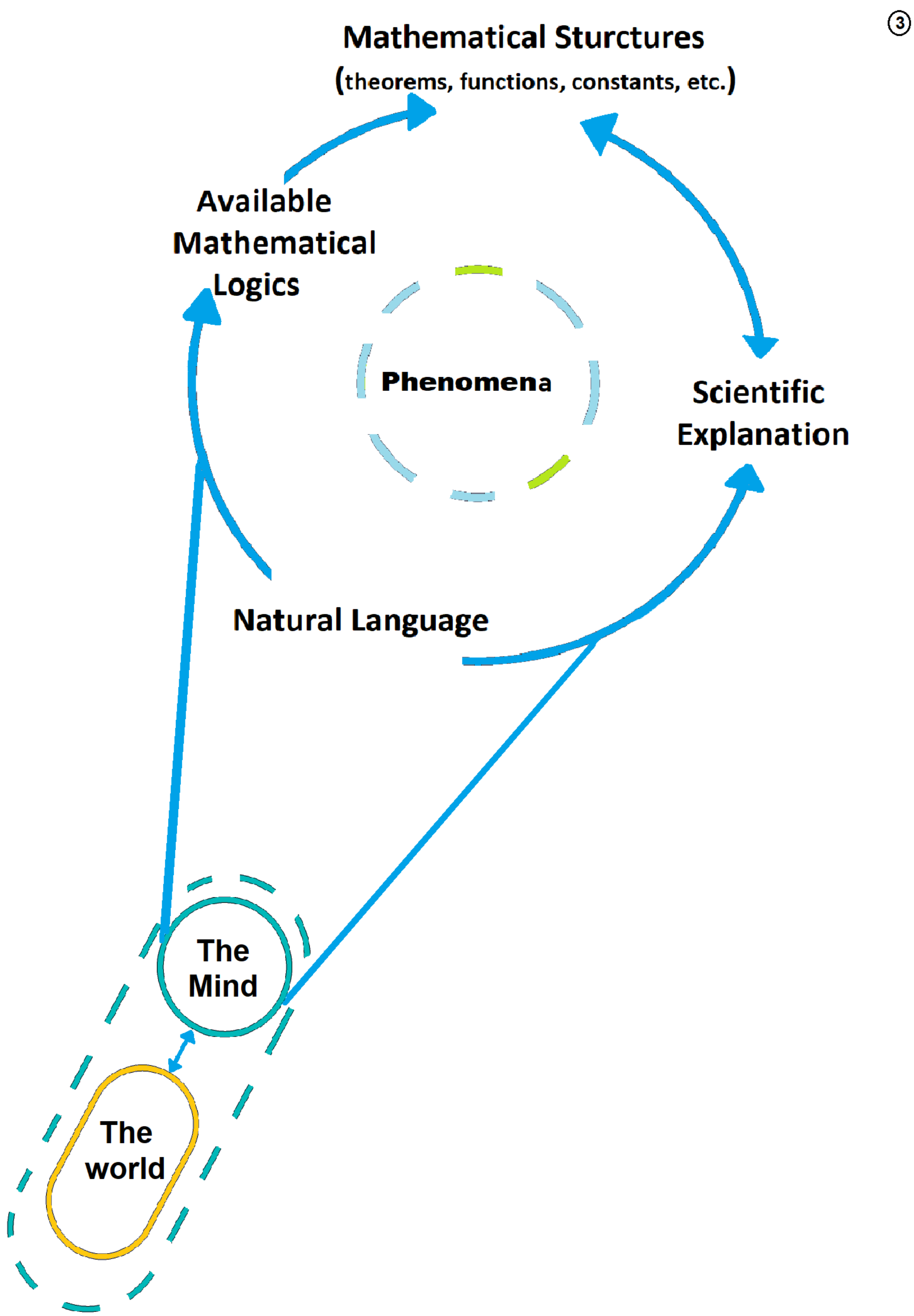

Figure 3: The Relation of the Language to the World

CHAPTER 5

CONCEPT PLATONISM VS. MATERIALISTIC PLATONISM

5.1 Concept Platonism

In his article, “Mathematical Intuition and Objectivity,” Daniel Isacsson (1994) introduces a view which he calls “concept Platonism.” In this chapter, I am going to talk about this view for two primary reasons. As I already mentioned in the Introduction, I found some considerable similarities this view has some features of what I developed (and called Materialistic Platonism). Moreover, I realized that analyzing the similarities and differences between Isacsson and my views can be immensely helpful to enable us to trace some relevant and significant historical issues non-robust realists have had in defending their positions.[58]

To develop his argument, Isacsson starts by talking about “mathematical truth” and the objective reality of mathematics. He provides us with two quotations. One from *Principia Mathematica* (1910, 1927), by Russell and Whitehead, and the other one from Hilbert’s famous essay “On the Infinite” (1926). The first quote is as follows: “*In proportion as the imagination works easily in any region of thought, symbolism (except for the express purpose of analysis) becomes only necessary as a convenient shorthand writing to register results obtained without its help”(P.119)*

[58] In the same manner, this comparison will help me to show (or, at least, try to show that) why my narration might provide a better ground for our understanding of mathematical ontology. So, although I have discussed views of different camps in chapter1, in this chapter I will be more focused on the view that, chiefly, takes mathematical properties to be mind-dependent.

And the second quote, by Hilbert, is:

> …as a condition for the use of the logical inferences and performance of logical operations, something must already be given to our faculty of representation [in der vorstellung], certain extralogical concrete objects that are intuitively [anschualich] present as immediate experience prior to all thought. If logical inferences is to be reliable it must be possible to survey these objects completely in all their parts, and the fact that they occur, that they differ from one another, and that they follow each other or are concatenated, is immediately given intuitively together with the objects, as something that neither can be reduced to anything else nor requires reduction. This is the basic philosophical position that I consider requisite for mathematics and, in general, for all scientific thinking, understanding and communication. And, in mathematics, in particular, what we can consider is the concrete signs themselves, whose shape, according to the conception we have adopted, is immediately clear and recognizable. (P.120)

But the question must be this: do we use symbols, as Russell and Whitehead had pointed, only to *register* the results and, as I understand from the quote, has no effect over the results?", meaning that the mathematical realm of the mind (mathematical structures) is objective to the way we put them together (more specifically, the way we use our logical and mathematical language)? Or, are these *objects* (if we take them as linguistic tools), as Hilbert puts it, are being presented to our intuition *prior to all thoughts*? How does, (in the picture that I have been trying to depict in the previous chapter) mathematical language play a role to project a mathematical reality?[59]

It seems that, in one interpretation, Isacsson is more on the side of Russell and Whitehead rather than Hilbert, so he presents his view for such an advocacy[60].

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[59] Clearly one (depending on the way s/he wants to interpret these two quotes) may not find any considerable contradiction between these two quotes. Yet I am taking the opportunity to follow Isacsson strategy to spell out my main argument and to clarify some involved concepts.

[60] He points out that "*It seems to me that Hilbert is either wrong here or means something actually compatible with the view that thought is the only medium by which the facts of mathematics impinge upon us.*"(p.120) somewhere else, Isacsson states"

> […] physical displays and strings of written symbols are not themselves mathematics, but only devices that serve as aids to thoughts. They no more mean that mathematics consists of physical configurations than notating a symphony or writing down a poem or a novel shows these creations of mind to be distribution of ink on paper".(p.119)

Nevertheless, let us go back to the two quotes I mentioned above. As for the first part of Hilbert's quote:

> […] as a condition for the use of the logical inferences and performance of logical operations, something must already be given to our faculty of representation [in der vorstellung], certain extralogical concrete objects that are intuitively [anschualich] present as immediate experience prior to all thought."

It appears that we can agree with Hilbert that, at least, some mathematical forms (indeed, I am not referring to any symbol here, but rather structures) are there. And by "there," I mean in the formation of the mind, shaped by nature through an evolutionary process (to enable him to have effective interaction with the world), and in line with how language and linguistic capacity emerged and progressed through the "conscious-mind/world" interaction.

Well, although I can understand that there are similarities between how we write down a poem or the note of a symphony and doing mathematics, but I do not see the analogy to be perfectly working; since writing a poem or a symphony (or even understanding them) are, sort of, subjective and partial in terms of their truth value (in terms of the correspondence to the real feeling of the poet.)
But on the other hand, if I take this analogy seriously, it probably can provide me with a tool to reaffirm my claim about the nature of language, in general. So, imagine you write down a poem. To what extent does this piece of poem reflects your feeling? Is it perfectly the same? Seems not. Assuming that you master the language, it just reflect your feeling on the mirror of some discrete words and sentences by inviting your subjective feeling to create its subjective images. Now, with all the differences that poem (that is seeking to reflect the howness of your feeling) might have with mathematics, let us ask the same question about mathematics.
To what degree our mathematical language represents the objective reality (in my view, the Existence)? This time, although not as radical as the discrepancy that exist between a feeling and a poem, still (as I have been arguing) our mathematical language is not perfectly capable to justify, capture and embrace the totality of the synthetic results of our intuitive concepts and axioms (look at the problem of infinity, natural numbers vs real numbers, incompleteness theorem, etc.) [I include incompleteness as an example because, in Isacsson's words, "*Gödel's incompleteness theorem shows that for basic arithmetic, and every extension of it, there can be no uniform procedure by which every statement is established or refuted*" (p.131).
In my words, the totality of the synthetic result is not fully captured by our progressive mathematics since for this completeness we would need all the axioms. But all the axioms are not perceivable by us now. They probably need infinite duration of time, and the exposition to different types of realities that the metaphysical condition of the time must bring up in front of the mind's eye.]

As for what Isacsson quotes from Russell and Whitehead (which gives me an impression of structuralism[61]) and under the light of the view of MP'ism, I more tend to disagree rather than agreeing with this statement, although my position is not absolute and definite. Remember the quote:"*In proportion as the imagination works easily in any region of thought, symbolism (except for the express purpose of analysis) becomes only necessary as a convenient shorthand writing to register results obtained without its help*"

So, here, if by a symbol, we mean the appearance of that symbol, we all know that they are changeable (and therefore, Russell and Whitehead are right). But if, by symbolism, and the use of symbolism, they mean it does not affect how one definition is *formulated* and how definitions are connected (which probably is the interpretation they meant to be), then this would push us to enter the grey zone of *how* possibly some intuitive mathematical facts (and properties) came to represent themselves to the mind without having any symbolic form.[62]

Moreover, regardless of how much precisely and skillfully the language is chosen, mathematical emergences act only as a broken mirror of the reality. Since, basically, our sensory apparatus, as the ground, the complier and the gatherer of the inputs for

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[61] Recall the definition of structuralism given by Balaguer, "According to this view, our mathematical theories are not descriptions of particular systems of abstract objects; they are descriptions of abstract structures, where a structure is something like a pattern, or an "objectless template" - i.e., a system of positions that can be "filled" by any system of objects that exhibit the given structure."

[62] Nevertheless, although I do not necessarily oppose such an interpretation from the above quote, it seems to me that mathematical definitions are language dependent; where the language is intertwined with the mathematical dimension of the mind and was developed parallel to the development of our mathematical capability (So, for example for the number 5 to be, and be referred to, as a property of a collection of things requires us to have the conception of fiveness, and then intentionally use, or think about, them; even if no collection is present to be called five. It appears to me that we need language for this kind of abstraction).

the conscious mind to evolve, has *not* been perfect in reflecting other existences (since these inputs were filtered to be proportionate to and dependent on our physic in order to be experienced), in spite of the fascinating progression in our mathematical construction, the reality behind our mathematical abstraction (evolved to be understood and constituted in line with our conscious apprehension of time and space and the physical world), in its totality (if there is any complete system, in the form of language, to be called "mathematical"), remains in the *core of our mind,* in the *Existence.*

Following the question of whether our mathematical intuition indicates the existence of concrete mathematical objects, Isacsson gives his reason why he rejects Platonistic account of mathematics, which he calls object Platonism[63] (in contrast with what he later will call concept platonism).

First, he reminds us of the same classical questions I also raised in the previous chapters about the nature of mathematical objects, and how causality works for us to have access to those entities (the objection which is, famously, attributed to Paul Benacerraf to challenge any view that claims to give an account for our mathematical knowledge). But, then, he argues that the main reason why he rejects object Platonism is not because of these critiques. Rather, he constitutes his main argument upon what I quoted from Mark Balaguer (in chapter 2) as the non-uniqueness objection. Isacsson says:

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[63] As an explanation for object Platonism, he writes:

> It is not difficult to feel that our experience of mathematics is an experience of objects, those objects that mathematics is about, such as natural numbers, rationals, real and complex numbers, functions, sets, geometrical figures, metric spaces, topologies, differentiable manifolds, and so on. The language in which we express our mathematical thinking has the same grammatical categories of substantival reference as does our talk of the physical world that is singular terms and the apparatus by which we speak of everything or something in a given domain. These consideration may lead us to the view that mathematics is about particular mathematical objects, and that the objectivity of a mathematical statement is explained by the existence of those mathematical objects that the statement is about. Let us designate such a view "object Platonism. (p.121)

> The compelling and immediate reason for rejecting the idea that mathematics is about a particular objects is that for any mathematical theory the domain of objects which that theory is taken to be about can always be replaced by a domain consisting of different objects, so long as the second domain has a structure isomorphic to that the first. (P.123)

So, in his view, the structure which must be held by isomorphism depends on the notions we use in that theory where "in all cases, mathematics is inherently to do with structure." (p.125)

I found Isacsson's detailed analysis of mathematical structures very appealing. In its basic formation, his thought and assumption (in the primacy of structure over objects) are very similar to the way I portrayed the extraction of mathematical language from natural language using our mathematical intuition (image**3**).[64] He explains more on the notion of isomorphism and divides isomorphism in mathematics into two, one trivial and the other, he calls, nontrivial.

So, for example, according to him, mapping the domain of natural numbers to even numbers (which is also its proper subset) is a trivial isomorphism[65]. However, the methods of constructing real numbers by Dedekind's cuts of rationals and classes of Cauchy sequences of rationals under the equivalence relation of equiconvergence is a nontrivial isorphisim.[66] So, in his view, no particular individual (we name through

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[64] However, it appears to me, that Isacsson puts less weight on how come that we turned to have this capacity in his analysis; and thus, is not providing us with such a ground metaphysical theory in his essay, nor he claims to be doing so,

> In holding that philosophy of mathematics must respect our sense of the objective reality of mathematics, I am not thereby offering a philosophy of mathematics, nor prejudging a philosophical issue. This demand says nothing in itself as to the nature of any such objective reality. That is for philosophical inquiry to elucidate. (p.118)

[65] As an example, I may say that it is imaginable that, for example, an alien take a pair of things to indicate oneness for them.

[66] Isacsson also notes: "*Of course I do not mean that there is no mathematical difference between Dedekind's cuts and equivalence classes of Cauchy sequences, which clearly is. But they have equal claim to be the real numbers*."(p.36)

using a symbol for it) can make a difference to mathematical structures, rather it is the relation between objects that matters for mathematicians.

As for the claim that sets are exceptions and that they do deal with a collection of particular entities (rather than structure)[67], Isacsson thinks that sets actually "exemplify, rather than refute, the general claim that the entities of mathematics are not particular things" chiefly because "the two-place relation of set membership can be variously interpreted."

[Similar to {{Ø}} and {Ø, {Ø}} where both to be used as ways to construct number two.]

But, before entering into Isacsson's main argument, let us recall the reason why he rejected Platonism (or, as he calls, object Platonism). The key concept in Isacsson's position is the belief of the "<u>invariance of mathematical truth with respect to isomorphism</u>" and this was taken by him as the main reason for rejecting Platonism of any kind (either the Fregean platonism where he believed in any number as a self-subsistent object, or numbers as the Gödelean independent objects of intuition)

For Frege, in line with his anti-psychologist stance, numbers are not objects of intuition. By stating "always to separate sharply the psychological from the logical, the subjective from the objective", he insists on *context principle* to search for a number's identity (since he believed that we cannot have any picture or intuition of it before realizing its objectivity). So, for Frege, our problem becomes this: To define the sense of a proposition in which a number would occur.

In a way, both Gödel and Frege recognize the independent existence of mathematical objects. For Gödel, it was the existence of mathematical objects (objects of intuition)

[67] A claim which is based on the definition of the empty set (that set which has no elements) through which, using the standard model, one might be able to construct numbers (based on the definition of the empty set to act as ontological atoms for such construction).

that explains the objectivity of mathematics, and for Frege, it is the objectivity of mathematics to imply the existence of mathematical objects.

“Frege’s enterprise was then to identify objects as the reference of number words in statements of arithmetic” (Isacsson, p.129). Isacsson says “what is important is just the point that the structural invariance of mathematics tells us that any account of the objects of mathematics that claims to consider to reveal which unique thing each number is must be mistaken.”

5.2 Concept Platonism vs. Materialist Platonism

As we have explained, for Isacsson, the fact that mathematics is invariant with respect to isomorphism is alone enough to discard Platonism (or, as he calls, object platonism). That is so because for him, “a structure is given by concepts, [so] it favors an account of mathematical reality in terms of the reality of mathematical concepts” (p.125).[68]

Moreover, for a number of times, Isacsson insists “that any philosophy of mathematics must take account of the fact that thought is the only medium by which the mathematical facts impinge upon us.[69]The locus of our contact with concepts is the process of thinking about, or with, them.”

Isacsson calls his understanding, which is based upon the objective reality of mathematical concepts, “concept platonism.” He states, **“**the genesis of our

[68] Which, although similar but is not exactly compatible with what I have been trying to demonstrate. That, because of the gap I recognize to exist between the reality in itself and the intuition that was formed in us in perceiving mathematical forms (and generally, our descriptive world) when we use our abstraction power over concepts (or empirical facts.)

[69] Obviously, I am not very comfortable using the verb “impingement” since I think we are taking and extracting the proportionality and the structures by hints and application of the constitutive, rational and (intuitively) logical part of our mind (which has been shaped to *effectively* reflect the possible quiddity of Existence to our conscious mind.)

mathematical concepts reflects *both constitutive features of mind* and *elements abstracted from experience in the world around us*." [70]

Isacsson also thinks that abstraction from the experiences that we have of the external world means that there is some contingency in the development of mathematical concepts, since differences in experience and interest may give rise to differences in the choice of concepts with which to do mathematics. (p.126) I found his description of contingency in the development of mathematical concepts vague and not very much satisfactory (see footnote 47.)

Although I am not certain whether by experience Isacsson means one individual experience or humanity in general, I guess he means the former. In the mirror of my view, nature, around and in us, paves the way for the emergence of new types of mathematical structures (through the innovative-discovery process I tried to illustrate in the previous chapter[71]). And evolution, as one mechanistic tool for this progression (since we do not what could be other undiscovered factors in the universe and within

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[70] This part of his argument, which must also be considered as the corner stone of his ideas, has the biggest similarity with the view that I have been presenting. Nonetheless, as I will try to show, the conclusions (and the subsidiary definitions and assumptions that he develops later) seems not very clearly explanatory to me.

[71] If we imagine to have different levels of existence (let us, for now, just take it as an assumption), then it is probably acceptable if we say that our mind and the world are connected on the plane (the fluid/level) of the same sort of existence (which basically is the reason why they are interactable). So, when I say that the mind becomes capable of formulizing some structures of the reality in mathematical language, it is as if nature is continuously nurturing a part of our mind (mind with small letter, to indicate "the realm of possibilities" in the image**1)**, to become able to shed more light on Existence (or higher grounded MIND).
But It would be far and beyond this thesis to give a theory of mind and time here. Nevertheless, what I am trying to verbalize is that natural tools (environment, evolution, time, and etc.) induce and give rise to some sort of ideas in our mind. It is these (by its nature, holistic) ideas that pushes mathematics forward; where through some innovative linguistic tools, the mind takes these ideas to reveal new sort of structures from the source of existence.
But if one is going to talk about concept platonism while taking the mind solely in its evolutionary sense, s/he should also reply to the question whether these structures have been there in pre-homo sapiens human species' s mind too, or the pre-planetary life or even before that. If yes, where (since a mind is needed to apprehend such concepts and form such structures). As I said, one can think of another MIND that brings the potentials (for human beings) to their sight. Such a mind that is in the source of existence, or maybe *is* the source of existence.

the mechanism of time and space that affects our social and technological progression, in micro or macro scales) transmits those activated capabilities (you say through our genes and changes that have occurred in the environment), of these contingencies, generation by generation in a very complex probabilistic manner. So, considering the evolution of the language (parallel to other reasons that I provided as clues for mathematical progression) it cannot be one individual's experience (as might be Isacsson's point).[72]

I should note that my position is not in contradiction with such a claim that says even before we exist, mathematical structures were there in the world,[73] you say, in the mind of any developed enough creature (although not necessarily the same as we practice mathematics in human language form). As far as that creature's mind is a part of nature's progression (which naturally, must be) mathematical dimension of the world should manifest itself through that creature's mind using language or any sort of analytical apparatuses that that aliens might possess.[74]

However, the manifestation of mathematical structures (as a logical and intuitive construction) for us, human beings, is through language and the kind of relationship it has with our mind.

What I see is that the world is *becoming*, expanding through synthesizes after synthesizes, and the human's rational mind, on the ground of a universal mind, is expanding while finding itself in more and more complex situations. And evolution, in its biological sense, is only one scientifically visible (sensible) dimension of this expansion.

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[72] As partly explained in footnote 47.

[73] Uri D. Leibowitz and Neil Sinclair. *"Introduction", Explanation in Ethics and Mathematics, p.16*.

[74] It is so because in my view, and partly Isacsson's, the constitutive part of our mind and the development of concepts (and language) are dependent on our senses. So, if an alien has some less (or more) sense preceptors, it would directly affect the way their constitutive mind and the way their mind is shaped to form (to be called *intuitive* for them) concepts.

In a point of this becoming, we were able to deduce (through abstraction) the/a systematic dimension of our constitutive mind using the innovative tools of the language; while language itself seems to be indispensable to make the abstraction over some intuitive concepts possible (and basically has been intertwined with such mental and conceptual apprehension), the result of such abstractions appears to say something about the reality of the (core of the) mind, and of the world.

Therefore, I see what Isacsson refers to as the "abstraction from experience of the external world" (to give rise of different concepts) to be meaningful only when there is an evolving human mind (or as I elaborated, other conscious non-human minds, even with different types of analytical tools). But to have fair comments on Isacsson's account on mathematical reality, let us take a more in-depth look at his ideas.

As I previously mentioned, it is not very clear, from his article, how we can have access to what he calls mathematical concepts. Yet, there is a part where he gets close to the clarification of what this source is when he says:

What are the constitutive features of the mind that enables us to think of mathematical concepts? Thought is the capacity to consider the absent object. Iteration of this capacity brings us to the properties of structure abstracted from the particular, that is, to mathematics. The nature of thought as the capacity to consider the absent object shows itself in the intentional nature of thinking. (p.126)[75]

It seems that his statements have more than a grain of truths in them. I interpret his statement (which obviously is not necessary the one he means) to mean that when the conscious mind repeatedly faces with objects of the same property, it starts to categorize them or (along with these experiences) the constitutive part of the mind is formed to reflect the external world's structures in its mirror. Yet, Isacsson appears to

[75] Isacsson makes it clear that when he uses the term structure, he does not mean the Tarskian notion of structure (which is a particular set-theoretic object, itself composed of objects) (p.127) since "*the concepts that determine a structure in this sense are given by their extension*." Whereas, for Isacsson, mathematical concepts are primary and concepts in the sense required are not given in extension.

be vague in pointing to “mathematical concepts” and their nature. Plus that the notion of “capacity” and how it brings us to the properties of a structure from some particulars leaves us with a question to be speculated about, *how*? [76]

Unlike the howness question, Isacsson makes it well explained for what he means by mathematical concepts. Mathematical concepts for him involve inherently type of understanding, and by a mathematical structure, he refers to a body of thought whose concepts are mathematical, in the sense that what can be expressed in terms of these concepts is” *invariant with respect to change of objects* “and “*we see here the sense in which pure mathematics* is *abstract thought*.”

As a general depiction, the image that Isacsson provide is understandable, but in addition to my previous questions (of how from particular we come to properties of the structure, and how come that we have this capacity, and how much these concepts are real) another concern may be raised as well. Considering that thought and language are intertwined, Isacssons explanation requires a bit more inspection to see what exactly he (or, we) mean by pure mathematics and abstract thought.

One interesting part of his argument is when he tries to illuminate his metaphysical stance of these concepts. To distinguish himself from platonism, he states that mathematical concepts are not concepts of objects.

> The paradoxical sounding truth of the matter is that mathematics is about objects, but at the same time there are no mathematical objects. The resolution of whatever air of paradox there may be to this formulation resides not in any program of ontological reduction but in reflection upon the nature of mathematical thought.(P.127)

However, as I have shown, mathematical thoughts are directly related to the ontological roots (physical world) of how those thoughts are becoming *possible*. Moreover, through the picture that I have presented, one can see that this seeming paradox of being about objects where there are no mathematical objects is organically and automatically resolved. Since these concepts, which Isacsson calls platonic, are

[76] I provided my narration of how this capacity was formed in the previous chapter.

not pure to be called platonic concepts[77]; but are the reflection of the mind (to detect some structures when they come to the horizon of our rational possibilities) on the source of its existence (materials).

That being said, Isacsson makes sure to provide us with what he means to be when he talks about mathematical reality; "*the reality of mathematics is to be understood in terms of the reality of its concepts by which a structure is characterized*."(S.129) So, for him, "*the reality of a structure lies in the reality of the concepts that characterize it*."(S.129)

I previously elaborated about one possible ontological explanation for mathematical emergence (image 1 & 2) here, although I do not reject Isacsson's idea about some intuitive concepts forming our mathematical structures, I think that he did not really clarify the metaphysical source of being/not-being about objects' paradox.

[77] As the reader might have realized, there are ingredients of Platonism in what I have drawn as MP'ism. So, apart from the points I mentioned about language, the role of invention in our mathematical construction and its impurity with respect to Existence, if I use the Platonic analogy, mathematical structures to the Forms in the intelligible realm is like shadows to originals in the visible realm.

CHAPTER 6

CONCLUSION

So, I hope that I have been successful in clarifying my view in explaining the emergence and use of mathematical concepts. I argued that it was through the evolution of language and the constitutive and rational faculty of our mind (to make the act of abstraction possible) that such concepts (and consequently, structures) were efficiently formed and emerged from the conscious mind. Moreover, I have tried to offer a defendable narration for how we became able to construct (and project) some imperfect mathematical dimension of reality using both language and intuition (with the claim that these two were essentially evolved together).[78]

Furthermore, in this view that I have been defending, mathematics is *about* objects (in themselves). So, if we agree that the phenomenon of our conscious mind resides and

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[78]In addition to what I have discussed, one may reasonably take my perspective as a compromise between intuitionism and formalism, where I take both intuition and language to have roots in how we are evolved to be (they are interwoven in the structure of our brain). That, because I take language to be more than a mere tool for our intuition to express itself (although the game-like and innovative part of a mathematical constructions would have more to do with linguistic and formal manipulation of a system).
Otherwise, it appears to me, language and intuition could not successfully go along with each other. However, I think that we can take it as an obvious assumption that our *evolved* intuitions of things (including mathematical abstract properties) are imperfect in the sense that they may misguide us (take the Aristotelian assumption that objects fall proportionate to their masses as an example). The reason would be because our intuition, in a relax way of interpretation, has tried to reconcile the different phenomena around us (generally speaking, it seeks to do so without deep mathematical and metaphysical analysis) to avoid paradoxes (at least in their apparent sense.)
Nevertheless, on the other hand, language also has its own kind of limitations in encompassing what we see, or feel, as a part of intuition (plus that there might be no deep intuition behind our mathematical formulation when we do formalistic mathematics.)

relies on materials that have been evolutionarily constituted, and if we consider Sarukkai's argument (with some modification that I made) of language as being efficiently explanatory, plus my arguments about mathematics as the property of our brain to be reflected on the broken (imperfect) mirror of our mind, it appears that we get close to the Platonic understanding of mathematics as shadows of the Forms. .However, I understand that for some fundamental and detailed explanation, more comprehensive research on the relation between the emergence of consciousness and mathematics is necessary.

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APPENDECIES

A. TURKISH SUMMARY/ TÜRKÇE ÖZET

Matematiksel antitelerin doğasına ve niteliklerine dair tüm alt kategorileri ile birlikte realistler ve anti-realistler olmak üzere iki kamp arasında yüzyıllardır süregelen pek çok tartışma vardır. Hem realistlerin hem de anti-realistlerin tartışmalara konu edindiği kritik meselelerden biri de matematiksel niteliklerin zihnimizle olan ilişkisidir. Örneğin, matematiksel nesnelerin zihnimizden bağımsız bir şekilde var oldukları iddiası, katı realizmin temel özelliklerinden biri olarak bilinmektedir[79]. Diğer yandan, bunun karşıtı olan iddiayı, yani matematiksel ifadelere karşılık gelen gerek soyut gerekse soyut olmayan tüm antiteleri reddeden iddiayı anti-realistlerin kampında görmekteyiz.

Ben ise, tezim boyunca matematiksel niteliklerin var olmalarına rağmen bunların zihne bağlı olduklarını öne süren ("var olma" ifadesinde ne anladığımı da belirterek) katı olmayan realizmin bir türünü savunma teşebbüsünde bulunmaktayım.[80] Bu soruşturma ile paralel bir şekilde, (bazı linguistik süreçler aracılığıyla bizlere görünen fenomenler olmaları bakımından) matematiksel niteliklerin zihnimiz tarafından nasıl türetilebildiği konusuna da ışık tutmaya çalışacağım.

[79] Platonistik realizm olarak da bilinmektedir.

[80] Matematiksel niteliklerin var olmaları bakımından zihne bağlı olduğunu savunan bir görüş için (bu bazı kaynaklarda anti-paltonistik realizm olarak adlandırılır), Mark Balauger'in şu metnine bakınız: "Realism and Anti-Realism in Mathematics." In Philosophy of Mathematics (Handbook of the Philosophy of Science), ed. Andrew Irvine, (Amsterdam: North Holland, 2009), s.36

Burada her ne kadar matematiksel niteliklerin zihne bağlı olduğunu iddia etsem de, bu bağlılık "matematiksel olan" ifadesinin kaplamını zihinde var olanlara indirgememektedir, çünkü (zihnin yapısına ek olarak) dış dünyanın bazı boyut(lar) bakımından matematiksel antitelerin varlığını içerdiğini belirtmekteyim.[81] Dolayısıyla, bu perspektiften bakıldığında, realist bir pozisyon tutarlılık arz etmektedir, zira zihnin yapısı dış dünya ile girilen etkileşim sonucunda şekillenmektedir.[82]

(Bilinç sahibi zihnimiz ile dünya arasında matematiksel yapıların icat/keşif edilmesine dayalı) bu iletişimin mümkün olduğunu iki gerekçe ile açıklayacak olursak **a)** evrim sürecinde dünyaya dair tecrübelerimiz, zihnin (linguistik soyutlama gibi) evrimleşmiş kapasiteleri aracılığıyla (sezgisel olarak edinilmiş) matematiksel biçimler zihnin kendi kendisi üzerine düşünebilmesini mümkün kılan araçlar olarak (evrimin kendi oluşumuna yaptığı bir katkı olarak) evrimleşmesidir.[83] Ve **b)** Matematiksel yapılar (bizi biz yapan fakültenin şekillenmesi bakımından) dünya ile karşılıklı ilişkimizin etkilerinin, dilin evrimleşmesi ile birlikte, bizleri dil ve evrimleşmiş beynimizin sezgisel olarak nitelediği şeyleri bir araya getirebilmesine imkan tanıyan bir işaret olarak yorumlanabileceğidir.

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[81] Bu, benim görüşüm (MP'izm) ve geleneksel psikolojizm arasındaki en temel fark olmalı. İlk iki bölümde, MP'izmin psikolojizmin hem aktüelist hem de posibilist versiyonlarının farklı özelliklerini kapsadığını anlatmaktayım. [Her ne kadar kendimi ifade etmek için uygun bir ifade aradıysam da bulduğum tüm ifadeler görüşlerimi yanlış ifade ediyormuş gibi geldi. Süper-psikolojizm, bilinçli-materyalizm, natüralistik monizm, doğallaştırılmış Platonizm (sonuncusunu McDowell kullansa da bana uygun olup olmadığı konusunda kararsız kaldım) gibi kavramlar düşündüklerimden bir kaç tanesi. Dolayısıyla, şimdilik kendi görüşümü Materyalistik Platonizm'in kısaltması olacak şekilde MP'izm olarak adlandıracağım.]

[82] Bu çalışmada, zihin ve dünya arasındaki matematiksel (ve genel anlamıyla linguistik) bağ mükün hale gelmektedir, çünkü dünya ve evrilmekte olan, ya da evrim tarafından şekillendirilmekte olan zihnimiz arasında bir bağ vardır).

[83] Bir kimse buna zihnin kurucu özelliği adını verebilir, fakat aklımdaki fikir bundan daha fazlası. Dilin dünya ile tecrübemiz üzerine etkili bir şekilde düşünmek için şekillendiği konusunu ileride tartışıyor olsam da, diğer yandan matematiksel soyutlama edimi linguistik kapasitemizden çok daha fazla şeyle birlikte hareket eder. Zira matematiksel türden soyutlama genel olarak matematiksel sezgi dediğimiz şey üzerine kuruludur. Dolayısıyla, bu kimsenin "birlik" *sezgisi*, "toplama" *sezgisi* vs dediği şeye ben "... nosyonunu soyutlayabilme yeteneği" adını vermekteyim.

Bu türden bir inşanın sonuçları, analitik olarak bilemeyeceğimiz yeni teoremlerin geliştirebilmemiz ve bunların tutarlılığı konularına gelindiğinde oldukça şaşırtıcı bir hal almaktadır. Bizler bu yeni varlık boyutunu bilime de uygulayarak başarılı tahminlerde bulunabilmekteyiz. Aslında bu, "akıldışı etken" adını verdiğimiz bir etken olup, teoretik nesneleri kaplamın içerisinde sayısal olarak türetebilmemize yardımcı olan temel araçtır. Tüm bunlar, matematiksel soyutlamalar yaparken zihnimizde gerçekten bir şeylerin vuku bulduğunu göstermektedir.

Tezimin ilk bölümünde, oldukça öz bir şekilde realistlerin benim katı-olmayan pozisyonumla ilgili olan bazı fikirlerini ortaya koymaktayım. Doğal olarak, çalışmamın merkezi problemine yoğunlaşmak ve lafı dolandırmamak için seçici davranmaktayım. Dolayısıyla, bu bölümde realizmin iki türünü (yani, Platonizm ve Anti-Platonizmi) ele almaktayım. Buna ek olarak, psikolojizmin (her ikisi de anti-Platonistik olan) iki alt dalına ve yapısalcılığa da değinmekteyim. Akabinde, benim MP'izm[84] adını verdiğim duruşu temel alarak psikolojizm ve yapısalcılığın nasıl savunulabileceğini göstermeye çalışmaktayım.

Matematiksel gerçekliğin bu mümkün nosyonu hakkındaki iddialarımı dile getirmeden önce, bölüm 2'de ve 3'te okuyuculara platonistik realizme yöneltilmiş iki itirazın (yani, epistemolojik itiraz ve biricik-olmama itirazının) renkli bir resmini sunmaya çalışacağım. Akabinde, genel anlamıyla realizmi destekleyen iki argümanı (yani Eugen Weigner tarafından öne sürülen "matematiğin akıldışı etkenliği" (UEM) argümanı ve matematikte Quine-Putnam ikilisinin vazgeçilmezlik argümanı olarak bilinen (IA) argümanını) açıklayacağım. Sonrasında ise, bu argümanları benim matematiksel ontoloji konusundaki görüşlerimi temellendirmek ve açıklamak amacıyla kullanacağım. Bu iki eleştiri hakkında yorum yaparak MP'izm fikrini açıklamanın ön hazırlığını da yapmış olacağım. 4. bölüm, bu tezin ana argümantasyonunu konu edinmekte olup, burada okuyucuya matematiksel yapılar bakımından "gerçek" sözcüğünün bir tanımını sunacağım. Beşinci bölümde ise Daniel Isacsson (1998)

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[84] Her ne kadar bu görüş psikolojizmin ve yapısalcılığın bazı temel öğelerini içerse de, benim MP'izmin genel argümanı olarak düşündüğüm şey bir tür monizmdir.

tarafından öne sürülen katı-olmayan *kavram-platonizmi* pozisyonunu gözden geçireceğim. Kavram-platonizmini yukarıda bahsettiğim eleştirilere etkili bir cevap olması bakımından tercih etmekteyim, zira bunun ilk gerekçesi bu görüş ve MP'izme dair geliştirdiğim bazı görüşler arasında yakın benzerlik olmasıdır. İkinci ve daha önemli gerekçe ise Isacsson ve benim matematiksel oluşlar hakkındaki görüş farklılığımızdır. Bu farklılıkların ve benzerliklerin analiz edilmesi, katı-olmayan realistlerin kendi pozisyonlarını savunurken akılda tuttukları bazı yakından ilişkili ve belirgin tarihi mülahazaların izini sürmemde oldukça yardımcı olacağını fark etmiş bulunmaktayım. İddiam ise, matematiğin sadece (bizi vazgeçilmezlik argümanıyla bağlantılandıran) bir icattan ibaret olmadığını görebilmek için matematiğin *gerçekliğin bazı izlerinin* bir tür manifestosu olduğunu ortaya koyan bazı karmaşık bilimsel teorilerde matematiğin yerine getirdiği başarılı işleve göz atmanın yeterli olduğudur. Matematiksel nesneler ve bu nesnelere dair yaptığımız bazı soyutlamaların ampirik teorilerde kullanarak, bir bakıma, (bazı ampirik açıklamalar ile önümüze serilen) dışsal gerçekliği tarayabilir ve elimizdeki bilişsel araçların yardımıyla, gayet yerinde ve (yeterince güçlü) bir matematiksel bir dili, bir fiziksel fenomenin gerekçesi olarak düşündüğümüz şeyle ilişkilendirebiliriz.[85]

Bize yol gösteren pek çok bilimsel teori içinde geçerli bir durum olması itibarıyla, uygulamaları matematiksel dillerin başarısı da bizler için sıra dışı bir açıklayıcılık arz etmektedir. Her ne kadar pek çok örnekte olduğu üzere bilimsel teorilere eşlik eden kuraldışı durumlar olsa da (ve ben bu durumları zihin-zihin ve zihin-dünya etkileşiminin kısmiliğini, tek-boyutluluğunu ve soyutluğunu açıklamak için kullanma eğilimindeyim), Thomas Kuhn'un terminolojsini kullanacak olursak, açıklamaların, uygulamaların ve tahminlerin seviyeleri o kadar yüksek ki matematiğin dışsal gerçeklikten tamamen kopuk zihinsel bir oyun olduğunu düşünmek artık mümkün

[85] Newtoncu mekanik (ya da konvansiyonel bir araç olarak *calculus*) gibi bir teorinin yine kendi *calculus* aracı ile birlikte ortaya konulduğunu bilmekteyiz. Bu durumda bile, *Principia* belirgin bir şekilde daha önceki matematikçilerin çalışmalarına dayanmaktaydı. Dahası, Leibniz de aynı dönemde aşağı yukarı aynı sistemi geliştirmişti. Tüm bunların bize matematiksel manifestoların tarihe bağımlılığına işaret ettiğini düşünmekteyiz.

olmamaktadır. Bu durumlar, dorudan dünya hakkında ve dolayısıyla gerçekliğin izlerine dair bir şeyler söylemektedir[86].

Buraya kadar söylediklerimi özetlememin, bazı varsayımlarım ve sonuçlarımı sistematik bir şekilde yeniden ifade etmemin yaralı olacağını düşünüyorum:

(a), (b) ve (c) benim varsayımlarım olup, bunların verili olduğunu düşünmekteyim. (d) ve (e) ise benim çıkardığım sonuçları ifade etmektedir:

(a) Dünya vardır.
(b) Zihinlerimiz vardır.
(c) Zihnin halihazırdaki yapısı evrimin bir sonucudur.
(d) Katı ontolojik düalizm reddedilmeli ve bu anlamda dünya ve zihnin ontolojik kaynağı bir ve aynıdır.
(e) "d" şıkkının bir sonucu olarak dünya ve zihin ontolojik olarak benzer ve nitelik ve yapılardadırlar.
Aynı şekilde, tezimin ana argümanı ile uyumlu olacak şekilde, dünya ve zihnin ontolojik olarak benzer yapı ve nitelikte olmasını onların ontolojik olarak aynı kökenden gelmeleri ile açıklamaktayım.

Bunların yapısal olarak benzer oldukları iddiası, adeta mucize gibi bir açıklama yapma zorunluluğunu da beraberinde getirmekte olup, mucize, Eugene Wigner'in matematiğin akıldışı etkenliğini dair kullandığı ifadedir. Dolayısıyla, niyetim bunların açıklanmasının aynı ontolojik köken üzerinden yapılabileceğini savunmaktır. Bu konuyla ilintili olarak, göstermek istediklerim arasında (1) "zihin ve dünya aynı ontolojik kökene sahiptir" ifadesinin nasıl ve neden "zihin ve dünya ontolojik olarak benzer yapı ve niteliklere sahiptir" ifadesini açıklaması, ve (2) ontolojik köken bakımından aynılığın kendini

[86] Dolayısıyla, matematiğin akıldışı etkililiği , benim zihnin ve onun göreceli doğruluğu arsında bir bağ olması bakımından matematiğe dair söylediklerimin bir dalı (ya da kabaca ifade edilmiş bir biçimi) olarak düşünülebilir.

kanıtlar nitelikte olduğudur. Dolayısıyla, sistematik bir dille ifade ettiğimizde argüman şu şekilde karşımıza çıkmaktadır:

(f) Dil, düşünce ve gözlem kullanarak dünyayı açıklayan ve güvenilir tahminler ortaya koymamızı sağlayan bilimsel teoriler geliştirebiliriz.

(g) Bilimsel teorilerin gelişimi matematiksel yapılarla ayrılamaz bir örüntü içindedir.

(h) Matematiksel yapıların anlaşılması, kullanımı ve geliştirilmesi amirik mülahazalardan göceli bir şekilde bağımsızdır (yani UEM'den çıkarımlanabilirler).

(i) Eğer en iyi bilimsel teorilerin göreceli şekilde doğru olduklarını söyleyecek olursak, o halde bu teorilerde kullanılan matematiksel biçimlemelerin de doğru olduğunu söylememiz gerekir (öteki türlü, Putnam'ın deyişiyle, "entellektüel sahtekatlık" yapmış oluruz.) [Putnam, 1971/1979, s. 347][87]

(j) (h) ve (i) şıklarında hareketle, eğer soyut düşünme edimlerimiz dış dünyanın nasıl olduğunu göstermekte ise, o halde aralarında düşüncemizin de (kendilerini matematiksel teoriler ve bilimse düşüncede açık eden ve matematiksel düşüncenin göreceli olarak bağımsız olan operasyonların) nasıl olduğunu gösteren benzerlikler (ortaklıklar) da var olmalıdır.

(k) Bu ortaklığın doğrudan temeli niteliğindeki şeyler, bilinçli bedensel varlığımızdan kaynaklanan düşüncelerimizdir.

(l) "j" ve "k" şıklarında hareketle: Soyut ve saf matematiksel düşünce ve dünya arasındaki karşılıklılığın temeli bizim bilinçli varlığımızda yatmaktadır.

(m) Zihinsel varlığımızın kaynağı bedensel varlığımızdır.

(n) "m" şıkkından hareketle: Matematiksel soyutlamamız bizim nasıl olduğumuz *hakkındadır*.

(o) "f", "g", "h", "i", "j", "k", "l", "m", ve "n" şıklarından hareketle: bizim matematiksel düşüncelerimiz dünyanın nasıl olduğu hakkındadır.

(p) "a", "b", ve "o" şıklarından hareketle: bizim soyut matematiksel düşüncemiz, var olmanın neliğine yönelik sınırlı bir erişime sahiptir. (Bkz: **s**)

[87] M. Colyvan, s. 651.

(q) Evrim, bilinçli beden ve şeylerin neliğini ancak duyusal araçlarımızın gelişmesi ile birbirine bağlayabilir.
(r) Duyusal organlarımız gerçekliği olduğu gibi (yani gerçekliğin dilimizi ve dünyaya dair algılarımızı nasıl şekillendirdiğini) yansıtma konusunda sınırlıdır.
(s) Matematiksel inşaalar, Varlığa dair kısmi ve tek-boyutlu resimlerdir.

Göstermeye çalıştığım üzere, matematiğin tamamen a priori bir fenomen olmadığı iddiasında bulunmaktayım, çünkü (a) bazı özel zihinsel yapıların ortaya çıkması ve şekillenmesi için zaman ve çevre durumu öncelikli bir öneme sahiptir; (b) dilimizin pre-linguistik kavramlar ve dünyaya dair sezgisel kavrayışımıza doğrudan bağımlı olmasının yanı sıra, pek çok durumda matematiksel-sezgisel keşifler, ya bazı pratik gereklilikler, önceki modeller veya elimizdeki verili linguistik biçimler nedeniyle, yahut zihnin akılsallığını zaman ve dünyanın geri kalanına bağlayan başka etmenler nedeniyle bizim dış dünyadan ip uçları almamızı gerektirmektedir.

Yine açıkladığımız üzere, Isacsson açısından Platonizmi (ya da onun nesne platonizmi adını verdiği görüşü) saf dışı bırakmak için matematiğin izomorfizm açısından çeşitlilik arz etmemesi bile tek başına yeterlidir. Onun böyle düşünmesinin nedeni, "kavramlar bize yapıyı verir, [dolayısıyla bir yapı] matematiksel kavramların gerçekliği bakımından matematiksel gerçeklik zemini üzerinde yer alır"(s.125).[88] Isacsson, matematiksel kavramların objektif gerçekliğine dayandırdığı bu görüşü "kavram platonizmi" olarak adlandırır. Onun iddiasına göre "matematiksel kavramlarımızın çıkış noktası iki şeyi yansıtır: *zihnin kurucu özellikleri* ve *çevremizdeki dünyada edindiğimiz tecrübelerden soyutlayarak elde ettiğimiz öğeler.*" [89]

[88] Bu her ne kadar benim söylediğime benzese de ortaya koymaya çalıştığım görüşle tutarlılık arz etmemektedir. Bunun nedeni ise, kendinde şey olarak gerçeklik ve kavramlar yahut ampirik olgular üzerinde soyutlama kabiliyetimizi kullandığımızda algıladığımız matematiksel formlara dair bizde oluşan sezgiler (daha genel bir ifade ile betimsel dünyamız) arasında bir boşluk olduğunu düşünmemdir.
[89] Onun argümanının fikirlerine kilit taşı teşkil eden bu kısmı benim fikirlerimle en çok benzeştiği yerdir. Fakat, ileride izah etmeye çalışacağım üzere onun vardığı sonuçlar (ve yan tanımların yanı sıra daha sonra geliştirdiği varsayımlar) bana pek tatmin edici bir açıklama olarak görünmemektedir.

Isacsson'un matematiksel yapılara dair önerdiği bu analizi oldukça cazip bulmaktayım. En temel biçimiyle, onun (yapıların nesnelere önceliği konusundaki) düşünce ve varsayımları, matematiksel sezgiyi kullanarak matematiksel dilin doğal dilden damıtılmasına dair ortaya koyduğum resme oldukça benzemektedir (Bkz: Şekil 3).[90]

Son olarak, tezimi matematiksel kavrayışın (ve yapıların) dünyanın ne (ve nasıl) olduğunun birer gölgesi olarak anlaşıldığı bir tür Platonizm önerisi ile bitirmekteyim.

Benim bakış açımda dünya *olmakta*dır, sentezlerle giderek genişlemekte, ve evrensel bir zihin temelinde insanoğlunun rasyonel zihni de genişlemekte ve kendini daha karmaşık durumlar içinde bulmaktadır. Ve biyolojik anlamıyla evrim, bu genişlemenin sadece görülebilen (duyumlanabilen) bilimsel bir boyutudur.

Bu olma sürecinin bir noktasında, (soyutlama aracılığı ile) bizler kurucu zihnimizin sistematik (bir) boyutunu dilimizin yenilikçi araçlarını da kullanarak çıkarsayabilir hale geldik; nitekim dilin kendisi bazı mümkün sezgisel kavramlardan soyutlamalar yapabilmemiz için vazgeçilmez olup (zira bu zihinsel ve kavramsal kavrayış ile temelden örüntülüdür), bu soyutlamaların sonucu da zihnin ve dünyanın gerçekliği (ve özü) hakkında bir şeyler ifade etmektedir.

Bundan ötürü, Isacsson'un "dış dünyanın deneyiminden yapılan soyutlama" şeklinde ifade ettiği şeyi (farklı kavramların türetilmesi için) , ancak sadece evrilen bir insan zihninin varlığı koşulunda (ya da yukarıda belirttiğim üzere diğer bilinçli insan-dışı akılların, hatta çeşitli analitik araçların varlığı koşulunda) anlamlı bulmaktayım. Ama

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[90] Isacsson nasıl olup da bu fakülteye sahip olduğumuz konusuna, yaptığı analizde çok az önem vermektedir, dolayısıyla, metninde ne bu konuda temel teşkil edecek bir metafizik de sunmakta ne de kendi sözlerinden açık olduğu üzere, böyle bir niyet taşımaktadır:

> *Matematik felsefesinin bizde var olan matematiğin objektif gerçekliği hissine saygı duyması gerektiği fikrini benimsemekle ne bir matematik felsefesi önermekte, ne de felsefi bir yargıda bulunmaktayım. Bu talepsöz konusu objektif gerçekliğin doğasına dair herhangi bir şey söylememektedir. Bu konu, felsefi soruşturmanın sonucunda aydınlatılabilecek bir husustur*.(s.118)

Isacsson'un matematiksel gerçeklik konusundaki görüşlerini haksızlık yapmadan değerlendirebilmek için daha yakından ele almamız gerekmekte.

Gösterdiğim üzere, matematiksel düşünceler, onların nasıl *mümkün* olmakta olduklarına dair ontolojik köklere (yani fiziksel dünyaya) doğrudan bağlantılıdır. Dahası, sunduğum resme bakarak, ortada matematiksel nesneler yokken *nesne hakkında olmak* şeklindeki görünüşteki paradoksun da otomatik ve doğal bir yoldan çözüme kavuştuğunu herkes görebilir. Isacsson'un platonik olarak adlandırdığı bu kavramlar tamamen saf platonik kavramlar olmayıp[91]; (akılsal imkanlarımızın ufkuna giren yapıları tespit edebilmek için) zihnimizin kendi kökenleri (maddi varlıklar) üzerine düşüncesinden ibarettir.

Dolayısıyla, matematiksel kavramların nasıl ortaya çıktıkları ve nasıl kullanıldıklarını da açıklamış oldum. Bunu ise, dilin ve (soyutlama edimini mümkün kılmak amacıyla) zihnimizin kurucu rasyonel fakültesinin evrimi aracılığıyla bu tür kavramların (ve nihayetinde yapıların) etkili bir şekilde bilinçli zihin ile türetilip biçimlendiği iddiası ile dile getirdim. Dahası, hem dili hem de sezgiyi kullanarak (bunların da temelde birlikte evrimleştiğini iddia ederek), gerçekliğin tamamlanmamış matematiksel bazı boyutlarını da nasıl kurabildiğimizi (ve öngörebildiğimizi) açıklayan, savunulabilir bir anlatı ortaya koydum.

Bunlara ek olarak, dile getirdiğim görüş matematiğin (kendinde şeyler olarak) nesnelere *dair* olduğunu savunmaktadır.

Dolayısıyla, eğer bir fenomen olarak bilinçli zihnimizin evrimsel olarak kurulmuş materyallerle birlikte kurulduğu ve bu materyallere bağlı olduğunu kabul edecek olursak, aynı zamanda Sarukkai'nin (benim yaptığım küçük eklemelerle birlikte) dilin etkin bir açıklama aracı oluduğu görüşüne ek olarak, matematiğin beynimizin bir

[91] Benim MP'izm adını verdiğim görüş Platonizmin öğelerini içermektedir. Dolayısıyla, dil hakkında ifade ettiğim hususlar bir kenara, eğer Platonik analojiyi kullanacak olursak, matematiksel inşada keşfin rolü ve Var olmak bakımından saf olmamasını değerlendirdiğimizde, matematiksel yapıların düşünülebilen alandaki Formlara olan durumu, gölgelerin görülebilen alandaki asıllara olan durumu gibidir.

niteliği olup zihnimizin kırık (ve eksik) aynasında yansıtılması gerektiği şeklindeki argümanımı göz önünde bulundurursak, görünen o ki matematiği Formların gölgeleri olarak gören Platonik matematik anlayışına yaklaşmış oluruz. Lakin daha temelden ve ayrıntılı bir açıklama için, bilincin ortaya çıkışı ve matematik arasında daha kapsamlı bir çalışmanın yürütülmesi gerektiği kanaatindeyim.

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