INVESTIGATION INTO PROSPECTIVE MIDDLE SCHOOL MATHEMATICS TEACHERS' NOTICING OF STUDENTS' ALGEBRAIC THINKING WITHIN THE CONTEXT OF PATTERN GENERALIZATION

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ABSTRACT

INVESTIGATION INTO PROSPECTIVE MIDDLE SCHOOL MATHEMATICS TEACHERS' NOTICING OF STUDENTS' ALGEBRAIC THINKING WITHIN THE CONTEXT OF PATTERN GENERALIZATION

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The purpose of this study was to investigate prospective middle school mathematics teachers' noticing skills of students' algebraic thinking within the context of pattern generalization. In order to obtain in- depth exploration and understanding of issue, the qualitative research method, in particular, the case study design was used. Thirty-two prospective teachers who were studying at one of the public universities located in Ankara were selected via purposive sampling as participants. Data was collected in the fall semester of the 2018-2019 academic year through questionnaire and semi-structure interviews. In the data collection process, the questionnaire was applied to all the participants, and then semi-structured interviews were conducted with eight of them. The data was analyzed using the constant comparative method based on an existing theoretical framework for professional noticing of children's mathematical thinking identified by Jacobs, Lamb and Philipp (2010).

The findings of this study demonstrated that a vast majority of the prospective teachers could attend to students' solutions regarding pattern generalization with robust evidence and emerging evidence. However, it was revealed that prospective teachers had difficulty in interpreting students' algebraic thinking based on their solutions. Also, they had more difficulty in interpreting algebraic thinking of students with incorrect solutions than correct solutions. The findings about prospective teachers' deciding how to respond on the basis of students' algebraic thinking demonstrated that they could support the algebraic thinking of students with incorrect solutions asking follow-up questions. However, they could not extend the existing algebraic thinking of students who solved the problem correctly. They only provided responses by asking a drill or providing a general response.

Keywords: Mathematics Education, Teacher Noticing, Pattern Generalization, Prospective Middle School Mathematics Teachers

ÖZ

ORTAOKUL MATEMATİK ÖĞRETMEN ADAYLARININ ÖĞRENCİLERİN CEBİRSEL DÜŞÜNCELERİNİ FARK ETME BECERİLERİNİN ÖRÜNTÜ GENELLEME BAĞLAMINDA İNCELENMESİ

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Bu çalışmanın amacı, ortaokul matematik öğretmen adaylarının, öğrencilerin cebirsel düşüncelerini fark etme becerilerinin örüntü genelleme bağlamında araştırılmasıdır. Konunun derinlemesine araştırılması ve anlaşılması için temel olarak nitel araştırma yöntemine, daha özelde ise durum çalışması yöntemine başvurulmuştur. Araştırmaya katılmaları için amaçlı örneklem yöntemiyle Ankara'daki bir devlet üniversitesinde okuyan 32 öğretmen adayı seçilmiştir. Veriler, 2018-2019 eğitim-öğretim yılının güz döneminde anket ve yarı-yapılandırılmış görüşmeler aracılığıyla toplanmıştır. Veri toplama sürecinde, tüm öğretmen adaylarına anket uygulanmış, sonrasında bunlardan 8'i ile yarı-yapılandırılmış görüşme yapılmıştır. Çalışmanın amacı doğrultusunda, veriler Jacobs, Lamb ve Philipp (2010)'in kuramsal çerçevesi dikkate alınarak sürekli karşılaştırmalı analiz yöntemiyle analiz edilmiştir.

Bu çalışmanın bulguları, öğretmen adaylarının büyük çoğunluğunun, öğrencilerin örüntü genelleme ile ilgili problem çözümlerini güçlü kanıtlarla açıklayabildiklerini göstermiştir. Fakat öte yandan öğretmen adaylarının öğrencilerin çözümlerinden yola çıkarak, onların cebirsel düşüncelerini analiz edip yorumlamada zorluk çektikleri sonucuna ulaşılmıştır. Dahası, öğretmen adaylarının öğrencilerin yanlış çözümlerini yorumlarken, doğru çözümlere kıyasla daha çok zorlandıkları tespit edilmiştir. Katılımcıların öğrencilere cevap verirken aldıkları kararlarla ilgili bulgulara bakıldığında ise yanlış çözüme sahip öğrencilerin cebirsel düşüncelerini sorularla destekleyebildikleri görülmüştür. Ancak, katılımcıların soruyu doğru çözen öğrencilerin mevcut cebirsel düşüncelerini geliştirememekle birlikte, onlara sadece alıştırma niteliğinde bazı sorular sormak ya da bir takım genel cevaplar vermekle yetindikleri gözlenmiştir.

Anahtar Kelimeler: Matematik Eğitimi, Öğretmenlerin Fark Etme Becerileri, Örüntü Genelleme, Ortaokul Matematik Öğretmen Adayları

To My Family

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ABBREVIATIONS

KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
METU	Middle East Technical University
NCTM	National Council of Teachers of Mathematics
MoNE	Ministry of National Education
PST	Prospective Teacher

CHAPTER 1

INTRODUCTION

Noticing emerged from Latin words which refer to *being known* and *known* and it is defined as something which is done all the time within different contexts (Mason, 2011). The primary characteristic of noticing is that it is an intentional act, not a coincidence (Mason, 2011). For many years, although researchers conducted studies to explore how people recognize or notice their environment, for the last quarter of the century, they have focused on noticing in specific professions (Goodwin, 1994; Mason, 2002; Stewen & Hall, 1998). Teachers' professional vision, as a specific profession, is prerequisite for effective teaching practice (Grossman et al., 2009) and it corresponds to situation-specific skill that combines knowledge and application (Goodwin, 1994; Blömeke, Gustafsson, & Shavelson, 2015). Therefore, when professional vision is adapted to teaching, noticing means seeing and understanding how teachers give meaning to complex classroom environment where everything occurs simultaneously and respnding to everything is not possible (Star & Strickland, 2008; Sherin & Star, 2011; van Es & Sherin, 2002; Sherin, Russ, & Colestock, 2011; Jacobs, Lamb, & Philipp, 2010; Jacobs, Lamb, Philipp, & Schappelle, 2011). Noticing in professional perspective involves the identification and interpretation of noteworthy situations of classroom (Stürmer & Seidel, 2015; van Es & Sherin, 2002). Thus, noticing is a significant part of mathematics education.

Teacher noticing skills is an essential part of mathematics teaching since mathematics classroom is a highly complex environment to be aware of, and teachers must learn to distinguish remarkable actions to pay attention to and handle the complex events that occur in classroom (Star & Strickland, 2008; Sherin & Star, 2011; van Es & Sherin, 2002; Sherin, Russ, & Colestock, 2011; Jacobs, Lamb, & Philipp, 2010). For this reason, teacher noticing has been considered to be an important skill of mathematics teachers and an essential tool for enriching mathematical teaching (Goodwin, 1994 & Mason, 2002). Furthermore, teacher noticing is defined as an active process rather than a static category of knowledge (Sherin, Jacobs, & Philipp, 2011; Mason, 2011). van Es and Sherin listed the dimensions of noticing as;

> (a) identifying what is important and noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about classroom interactions (van Es & Sherin, 2002; p.573).

Ball, Lubienski and Mewborn (2001) proposed "sizing up students' ideas and responding" (p. 453) and stated that being able to use particular knowledge about children's understanding is an effective tool to handle complex classroom situations (Jacobs et al., 2010). In relation to this idea, Jacobs, Lamb and Philipp (2010) focused on how and the extent to which teachers notice students' mathematical thinking rather than what teachers notice. For this reason, the identification of what students practice, making sense of students' ideas, and making in-the-moment decisions about how to respond on the basis of students' mathematical understanding refer to a specialized type of noticing, which is called *professional teacher noticing of children's' mathematical understanding* (Jacobs et al., 2010; van Es, 2011; van Es & Sherin, 2008; Star & Strickland, 2008).

Professional noticing includes three components which are: "(1) attending to children's strategies (2) interpreting children's understanding and (3) deciding how to respond on the basis of children's understandings" (p.169). The first component of professional noticing is about the extent to which teachers attend to the essential details of children's mathematical ideas and approaches (Jacobs et al., 2010). The more details teachers capture about the mathematical essence in students' solutions, the more they make sense of their understanding (Carpenter, Fennama, Franke, Levi, & Empson, 1999).

The second component (interpreting children's understanding) is related to the extent to which teachers make sense of students' mathematical understanding based on the strategies they use (Jacobs et al., 2010). This component also focuses on whether teachers' reasoning is consistent with the mathematical essence of children's specific strategies (Jacobs et al., 2010).

Finally, in the third component of professional noticing (deciding how to respond on the basis of children's understanding) is about teachers' decision when s/he responds on the basis of children's understanding and their reasoning while deciding how to respond (Jacobs et al., 2010). Also, this component emphasizes that there is no best response.

In brief, professional teacher noticing of children's mathematical understanding have two important foci. Firstly, professional teacher noticing focuses on each child's mathematical thinking individually, rather than classroom environment, teacher's pedagogy, and the mathematical understanding of all the students in the classroom. Secondly, professional noticing of children's mathematical understanding is interested in teacher's in-the-moment decisions to respond on the basis of children's understanding in mathematical concepts (Jacobs et al., 2010; LaRochelle, 2018). As this study aims to investigate prospective teachers' noticing skills on the basis of children's understanding, the framework, *Professional Teacher Noticing of Children's' Mathematical Understanding*, was chosen as the theoretical framework of the study. To be more specific, it was aimed to investigate prospective teachers' noticing skills of students' algebraic thinking with the subset of pattern generalization.

One of the mathematical concepts that teachers need to attend to and interpret for effective mathematical teaching is algebra, which is considered as a gatekeeper in mathematics teaching and learning since it serves as a foundation for many advanced mathematical concepts (Knuth, Alibali, McNeil, Weingberg, & Stephens, 2005). According to Kaput (1999), algebra means recognizing the relationship between variables, making a generalization of this relationship, and writing a formula with algebraic expressions based on this generalization. In algebraic activities, algebra is used as a tool through practices of algebraic thinking (Kieran, 1996). Algebraic thinking is defined as the capability of thinking about unknown quantities as known quantities (Swafford & Langrall, 2000), the ability of using different representations to interpret quantitative situations in a relational way (Kieran, 1996), and thinking about functions, how they work and the impact of system's structures on calculations (Driscoll, 1999). According to NCTM (2000) Principles and Standards, middle school students should be able to recognize patterns, relationships, and functions; represent structures with algebraic symbols; use mathematical models to express quantitative relations; and analyze the changes in various contexts. For these reasons, in order to develop algebraic thinking, recognizing relationships, generalizing beyond specific examples, and investigating patterns are critical in the middle school curriculum (Magiera, Van den Kieboom, & Moyer 2013; Van de Walle, Karp, & Bay-Williams, 2013).

In order to teach mathematics effectively, teachers should identify the noteworthy details of students' answers and focus on students' understanding rather than whether their answers are correct or not (van Es & Sherin, 2008; Jacobs et al., 2010). Furthermore, teachers should act based on students' mathematical understanding about a specific topic. In other words, teachers should notice students' mathematical understanding and decide how to respond on the basis of their understanding (Jacobs et al., 2010) as professional teacher noticing is an essential skill for effective teaching. However, many research studies conducted in various contexts about prospective teachers' or teachers' noticing of students' mathematical thinking showed that prospective teachers' and teachers' noticing skills are low (Amador, Carter, & Hudson 2016; Güner & Akyüz, 2017; Kılıç, 2018; Tunç-Pekkan & Kılıç; 2015). It means that although prospective teachers and teachers have pedagogical content knowledge thanks to teacher education programs, they could not notice students' mathematical thinking; they have difficulty in understanding how students solve a problem, which misconceptions students have, and how students make sense of the subject. Since prospective teachers and teachers cannot interpret students' mathematical understanding, they have difficulty in deciding how to respond. In other words, prospective teachers or teachers cannot overcome students' misconceptions about a topic and they cannot carry students' correct understanding forward through responses. Moreover, although there are numerous studies regarding teachers' and prospective teachers' noticing skills of students' mathematical understanding through the lens of various contexts (Jacobs et al., 2010 Osmanoğlu, Işıksal & Koç, 2012; Star & Strickland, 2008; van Es & Sherin, 2002), there are few studies in the literature on teachers' and prospective teachers' professional noticing skills of students' algebraic thinking (Callejo & Zapatera, 2017; Walkoe, 2013). In other words, there are limited studies on how prospective teachers attend to students' strategies, interpret students' algebraic thinking, and decide how to respond on the basis of students' algebraic thinking. For these reasons, in addition to investigating prospective teachers' and teachers' skills of noticing students' mathematical understanding within various contexts, investigation of whether prospective teachers' skills of noticing students' algebraic thinking is necessary. Thus, the aim of this study is to investigate prospective middle school mathematics teachers' noticing skills of students' algebraic thinking within the context of pattern generalization. To be more specific, it is aimed to determine the extent to which prospective middle school mathematics teachers attend to students' solutions and interpret students' algebraic thinking, and determine the nature of prospective middle school mathematics teachers' decisions to respond on the basis of students' algebraic thinking within the context of pattern generalization.

1.1. The Purpose of the Study and Research Questions

The purpose of this study was to examine the extent to which prospective teachers attend to students' solutions, interpret students' algebraic thinking and the nature of the decisions that prospective teachers make to respond on the basis of students' algebraic thinking. In parallel with this aim, the following research questions were addressed:

1. How do prospective middle school mathematics teachers notice students' algebraic thinking within the context of pattern generalization?

    1.1. To what extent do prospective middle school mathematics teachers attend to students' solutions within the context of pattern generalization?

    1.2. To what extent do prospective middle school mathematics teachers interpret students' algebraic thinking within the context of pattern generalization based on students' solutions?

    1.3. What is the nature of the decisions that prospective middle school mathematics teachers make to respond on the basis of students' algebraic thinking within the context of pattern generalization?

1.2. Significance of the Study

"Teaching is one of the most common and also one of the most complicated human activities" (Ball & Forzani, 2010, p.40). In order to teach effectively, teachers have to be expert in recognizing remarkable situations in classroom environment and overcome the complex situations that occur immediately (Mason, 2011; van Es, 2011; Jacobs, Lamb, & Philipp, 2010). Furthermore, it is important for teachers to focus on individual student's mathematical understanding in classroom (Jacobs et al., 2010). More specifically, teachers have to identify the noteworthy aspects of students' strategies, make sense of students' understanding based on their strategies, and make a connection between students' understanding and possible teaching and learning methods/ strategies in the teaching environment (Star & Strickland, 2008; Sherin & Star, 2011; van Es & Sherin, 2002; Sherin, Russ, & Colestock, 2011; Jacobs, Lamb,

& Philipp, 2010). Also, teachers are expected to make students encounter real life problems, and guide students to do group work, express their solution strategies and thinking, and share them (MoNE, 2018). In order to do this, teachers should identify the noteworthy events that occur in mathematics classroom, evaluate students' understanding during the lesson and make in-the-moment decisions to develop instruction based on their inferences (MoNE, 2018). In other words, teachers need to have adequate noticing skills to attend to students' solutions, interpret students' understanding and support/extend their understanding. Therefore, teachers' noticing skill is a critical competency for teachers and prospective teachers. From this point of view, the findings of this study could yield an overall view of the degree of prospective teachers' attending to students' solutions, the degree of their interpreting students' algebraic thinking and the nature of their decisions to respond on the basis of students' algebraic thinking. In the light of the findings of the current study, crucial information and implications might be given to teacher educators and program developers in terms of prospective teachers' noticing skills.

In addition, since prospective teachers are future teachers, examining prospective teachers' noticing skills of students' algebraic thinking is important. This study gives opportunity to prospective teachers to be familiar with the real students' written work. Hence, they analyze and reason students' written work regarding pattern generalization and they foster their decision making skills in the process of this study. Therefore, teachers' noticing of students' algebraic thinking with the subset of pattern generalization might be raised through this study. In addition, they could have the experience of attending to students' solutions, interpreting their algebraic thinking, and deciding how to respond. These experiences will be significant when they become teachers in the future. Thus, the present study provides a learning environment for prospective teachers and it is also essential to help future teachers get ready for the teaching environment. The present study is also significant for middle school mathematic teachers since according to the results of this study, they could criticize themselves about what they should attend to in students' solutions, how they should interpret students' algebraic thinking, and how to respond on the basis of students' algebraic thinking. Hence, in the next process, they could predict students' possible misconceptions or strategies regarding pattern generalization and based on these predictions, they could make lesson plans, implement them, and evaluate students' level of achievement more effectively.

Several researchers agreed that algebra is critical for developing the understanding of high school mathematics, and thus, students' learning fundamental concepts of algebra is a significant issue (Rakes, Valentine, McGatha, & Ronau, 2010). Introduction of algebra topics to students starts in middle school (grade 6-grade 8) and continues throughout high school (grade 9-grade 12) according to the curriculum developed by the Ministry of National Education in Turkey (MoNE, 2018). Although algebra concept has an extensive coverage in Turkish mathematics curriculum and Turkish middle school students' success in this topic is low (Dede & Argün, 2003, Yıldız, Çiftçi, Şengil-Akar, & Sezer; 2015), there are limited studies about teachers' noticing skills of algebraic thinking in both international and national contexts in the literature (Callejo & Zapatera, 2017; Walkoe, 2013). Thus, it is believed that this study will contribute to the literature through the investigation of teachers' noticing skills and by filling the gap related to professional teachers' noticing skills of students' algebraic thinking.

Furthermore, in order to investigate prospective teachers' skills of noticing students' thinking, alternative student solutions are needed. Questionnaire that is prepared to examine their noticing skills should include both correct and incorrect solutions because the correct and incorrect solutions have different properties. For example, students who solved the problem correctly might use different solution ways and strategies. For this reason, teachers should pay attention to students' solution to understand their strategies and interpret their understanding. Moreover, incorrect solutions might involve different students' conceptual and procedural mistakes or misconceptions. For this reason, teachers should understand how students solved the problem and what their difficulties and misconceptions to be able to attend to their strategies and interpret their understanding. Moreover, Jacobs et al. (2010) stated that both the understanding of students' correct solutions has to be extended and the understanding of students' incorrect solutions has to be supported with follow-up questions. Therefore, prospective teachers' or teachers' noticing may vary depending on whether they notice students' correct solutions or incorrect solutions. Hence, in this study, in order to explore prospective teachers' skills of noticing as a whole, questions which is related to both correct and incorrect student solutions were asked to prospective teachers. In this way, prospective teachers' skills of attending, interpreting, and deciding how to respond on the basis of both students' correct and incorrect solutions could be explored, which makes this study significant.

Furthermore, when the literature is reviewed, it can be realized that most of the researchers have collected data through video based learning environment (Osmanoğlu, Işıksal & Koç, 2012; Ulusoy & Çakıroğlu, 2018; van Es & Sherin, 2008) and lesson study (Amador, Carter & Hudson 2016; Güner & Akyüz, 2017) in order to investigate teachers' noticing skills. However, video does not provide prospective teachers to use deeper reasoning skill and they spend too much time discussing students' solutions (LaRochelle, 2018). On the other hand, although written work is an authentic activity for interpreting and responding to students on the basis of their understanding in teaching mathematics (Grosmanet al., 2009; Jacobs & Philipp, 2004), there are limited studies whose data were collected through students' written work (Schack, Fister, Thomas, Eisenhardt, Tassel, &Yoder, 2013; Star & Strickland, 2008). In fact, in this way, how different student solutions affect what prospective teachers notice can be explored. Furthermore, which student solutions are attended and interpreted better and to which student solutions prospective teachers respond more easily or in a more difficult way might be understood. Thus, it would be significant to conduct studies which collect data via students' written works consisting of students' solutions of pattern generalization. Taking all these perspectives into consideration, it was aimed to reveal how prospective middle school mathematics teachers notice students' algebraic thinking as far as the subset of pattern generalization is concerned.

1.3. Definitions of Important Terms

The important terms which are used in this study are given with their meanings below:

Prospective middle school mathematics teachers: Prospective middle school mathematics teachers are students in teacher education programs in their last years. These teachers are educated to teach mathematics to middle school students from 5th grades to 8th grades. They completed most of their courses that included mathematics, pedagogy and education courses.

Noticing: Noticing is the skill through which teachers identify classroom interactions (van Es & Sherin, 2002), and it includes three main components: "(a) identifying what is important in a teaching situation; (b) using what one knows about the context to reason about a situation; and (c) making connections between specific events and broader principles of teaching and learning" (van Es & Sherin, 2002, p.573).

Teacher noticing: Teacher noticing is the expertise in identifying what is important in instructional environment and making sense of one's knowledge of teaching and learning by giving reason to environment (Goodwin, 1994; van Es & Sherin, 2008).

Professional teacher noticing: Professional teacher noticing is teachers' expertise in attending to children's strategies and interpreting children's understanding by making an inference based on the strategies students use and deciding how to respond on the basis of children's understanding (van Es, 2011; Jacobs, Lamb, & Philipp, 2010).

Attending to children's strategy: Attending to children's strategy, which is the first component of professional teacher noticing, refers to highlighting the mathematically noteworthy elements of children's strategies (Jacobs et al., 2010).

Interpreting children's mathematical understanding: Interpreting children's mathematical understanding, which is the second component of professional teacher noticing, refers to making inferences about children's understanding based on their strategies (Jacobs et al., 2010).

Deciding how to respond on the basis of the children's mathematical understanding: Deciding how to respond, which is the third component of professional teacher noticing, refers to the reasoning that teachers use when deciding how to respond (Jacobs et al., 2010).

CHAPTER 2

LITERATURE REVIEW

The purpose of this study was to investigate prospective middle school mathematics teachers' skills of noticing students' algebraic thinking within the context of pattern generalization. In this chapter, the definitions of noticing, the frameworks for teachers' noticing and related studies on teachers' skills of noticing students' mathematical understanding, algebraic thinking and related studies on teachers' noticing skills of students' algebraic thinking are reviewed. At the end of the chapter, a summary of the literature review is provided.

2.1. Noticing

The word *notice* comes from the Latin words *notitia (being known)* and *notus (known)* (Mason, 2011). Noticing is something which is done all the time and in various contexts (Jacobs, Lamb, & Philipp, 2011; Mason, 2011; van Es, 2011), for example, notices were affixed on a noticeboard to attract people' attention. In this way, they will notice them. However, sometimes people do not pay attention to some aspects of situations, so some things might go wrong. Thus, Mason (2011) considered noticing as an intentional act rather than haphazard act. The meaning of noticing from a professional perspective can be thought as a seeing and understanding events (Goodwin, 1994). From this point of view, noticing is a significant part of mathematics education and there are different meanings of teachers' noticing. Mason (2002) defined noticing as "the ability to notice is often perceived to develop over time as it requires extended opportunities to focus on aspects of practice and make connections between teaching and learning" (p.91). Krupa, Huey, Lesseig, Casey and

Monson (2017) focused on notion of awareness that is defined as the ability to direct teachers' attention toward relevant teaching activity by referring to Mason's (2011) construction. Thus, it can be said that since teachers understand the learning environment through noticing, they teach students better. For this reason, the concept of teacher noticing is explained in detail below.

2.1.1. Teacher Noticing

The mathematics classroom is a highly complex environment to be aware of and to respond to everything that is occurring simultaneously. In order to handle complex events that occur in the classroom, teachers must learn to separate remarkable actions to notice. For this reason, teachers need to have noticing skills which are an essential part of mathematics teaching (Star & Strickland, 2008; Sherin & Star, 2011; van Es & Sherin, 2002; Sherin, Russ, & Colestock, 2011; Jacobs, Lamb, & Philipp, 2010). Teacher noticing is related to being able to know what teacher attend and do not attend in class and what the noteworthy aspects of classroom environments are (Star & Strickland, 2008). In other words, the construct of teacher noticing includes teacher's recognition of the mathematical elements of the problems solved by students during instruction (Sanchez–Matamoros, Fernández, & Llinares, 2014). Because of its importance of mathematics teaching, the conceptualization of teacher noticing has been an important phenomenon for researchers, so they define teacher noticing in multitude ways (Sherin, Russ, & Colestock, 2011). Some researchers are interested only in initial filtering of classroom situation and events. For example, Star and Strickland (2008) consider teacher noticing as a process in which teachers primarily see or perceive different aspects of classroom events. Other researchers point out both initial filtering of classroom situation and events, and making sense of what is focused on with the help of existing knowledge (Sherin, 2007; Sherin & van Es, 2009). In other words, teacher noticing is being aware of in what way the students' answers are or are not meaningful for the mathematical learning in addition to focusing on correctness of their answers. Thus, two significant aspects of teacher noticing are emphasized:

attending students' mathematical thinking and analyzing and interpreting students' mathematical reasoning by making reconstruction and inference (Hines & McMahon, 2005; van Es, 2011; Jacobs, Lamb & Philipp, 2010; Holt, Mojica & Confrey, 2013). Detailed information about the conceptual frameworks for teacher noticing are explained in detail below.

2.2. Conceptual Frameworks for Teacher Noticing

"Teaching is one of the most common and also one of the most complicated human activities" (Ball & Forzani, 2010, p.43). Teacher noticing can be considered as a significant component of teaching and it includes many themes, but the widespread theme is how teachers observe and manage complex classroom events (van Es & Sherin, 2008; Jacobs, Lamb& Philipp, 2010). More specifically, the teacher noticing is defined as determining whether students' answers are meaningful rather than determining whether their answers are correct or not (Hines & Mc Mahon, 2005; Holt, Mojica & Confrey, 2013). If teachers have an insight into students' thinking and understanding, they can choose useful and effective teaching strategies. Therefore, in the last two decades, the trend towards researching the construct of teacher noticing has been increasing, so different frameworks have been used in order to conduct these research studies. In the following sections of this chapter, van Es's (2011) and Jacobs et al.'s (2010) frameworks are given and then a conclusion about these frameworks is drawn.

2.2.1. Learning to Notice

In the framework, learning to notice, van Es (2011) investigated teacher noticing based on two general areas; (a) what teachers notice, and (b) how teachers notice. In the first dimension of the framework, what teachers notice, whether teachers professionally observe class as a whole, group of students and particular student or not is focused on. Moreover, this dimension is related to classroom climate, students' behaviors and mathematical thinking and teachers' pedagogical strategies. The second dimension is how teachers analyze what they notice and this dimension includes information about whether teachers can evaluate and interpret what they observe. In other words, teachers' abilities of evaluating noteworthy events in classroom and deciding what is good or bad by making inferences are measured. Furthermore, giving reasons why students have such an idea, understanding the meaning of their particular statements, expressions and gestures, expanding on their analysis, and making connections between students' understanding and teachers' pedagogical knowledge are related to second dimension of this framework.

van Es categorized four levels in teacher noticing skills based on two dimensions of the framework that are what teachers notice and how teachers notice. "These levels are level 1 (baseline), level 2 (mixed), level 3 (focused) and level 4 (extended)" as shown in Figure 2.1 below (van Es, 2011, p. 139).

	Level 1 (Baseline)	Level 2 (Mixed)	Level 3 (Focused)	Level 4 (Extended)
What teachers notice	Attend to whole class environment, behavior, and learning and to teacher pedagogy	Primarily attend to teacher pedagogy Begin to attend to particular student's mathematical thinking and behaviors	Attend to particular student's mathematical thinking	Attend to the relationship between particular students' mathematical thinking and between teaching strategies and student mathematical thinking
How teachers notice	Form general impression of what occurred Provide descriptive and evaluative comments Provide little or no evidence to support analysis	Form general impression and highlight noteworthy events Provide primarily evaluative with some interpretive comments Begin to refer to specific events and interactions as evidence	Highlights noteworthy events Provide interpretive comments Refer to specific events and interactions as evidence Elaborate on events and interactions	Highlights noteworthy events Provide interpretive comments Refer to specific events and interactions as evidence Elaborate on events and interactions Make connections between events and principles of teaching and learning On the basis of interpretations, propose alternative pedagogical solutions

Figure 2.1. Teachers' Noticing Categorization of van Es (2011, p. 139)

As can be seen in the Figure 2.1., at level 1, baseline noticing, teachers' main foci for the first dimension, what teachers notice, are whole classroom environment, classroom management, class's behavior and learning, and teachers' pedagogy. At this stage, teachers generally refer to words of "they", "group of students" and make comments like "The class try to complete a task." However, in the second dimension, how teachers notice, the general impression is emphasized (e.g., "That was a boring lecture" or "That lesson went well"), description of what happened in classroom (e.g., "They were silent in today's lecture") and evaluation of what they had seen (e.g., "The class was not interested in the new topic") and inform little or no evidence to support analysis. As a consequence, teachers' remarks at this stage contain overwhelming descriptions of what occurred in class with evaluative and judgmental comments on pedagogy.

At level 2, mixed noticing, for the first dimension, what teachers notice, teachers point out teacher pedagogy and they make a comment about that (e.g., "They all found the result of multiplication operation by adding numbers repeatedly. Do you teach making a multiplication in this way?) In addition, they refer to issues related to what whole class appeared to understand. To illustrate, teachers at this stage can make a comment like "They seemed to know to distinguish the conceptions of rate and ratio." Also, they began to attend students' mathematical thinking and understanding, but their comments lack evidence and interpretation. In other words, teachers at this level can describe what occurred broadly, but, give no references to particular student comments or do not interpret student's action described. In the second dimension, how teachers notice, teachers focus on the general impression in classroom as it is at level 1. They consider whole class's mathematical thinking and understanding (e.g., "Students could not interpret multiplication as a repeated addition.") In conclusion, teachers with this level of noticing skill can pay attention to noteworthy events in classroom and they begin to evaluate and interpret specific events in the moments they observe. However, they cannot supply detail to support their evaluation and interpretations while they are making analysis.

At level 3, focused noticing, regarding what was noticed, teachers are interested in specific students and their mathematical thinking and understanding. Teachers explain particular student' thinking and reason their strategies at this level clearly while they do not do this at level 2. Thus, teachers' comments on students' solutions predominantly include analytic chunks. To illustrate, teachers may say that "Student did not make estimation, but preferred to use concrete material rather than standard algorithm to solve the problem." In terms of how they noticed, teachers make a deduction through what they had observed and they focus on the noteworthy events and explain why they are important. Another distinct feature is that teachers elaborate on events and interaction. For example, they focus on what the inefficient aspect of lesson is and what should have been done to make a progress. Briefly, at level 3, teachers attend particular students' mathematical understanding, focus on specific events and interaction and draw inferences about these events and interactions rather than focusing only on whole class environment and their thinking.

At level 4, extended noticing, teachers are in the final level in noticing skill. Different from level 3, regarding first dimension, teachers at this stage expand on their analysis in order to investigate the relationship between particular students' mathematical thinking and the teaching strategies. For the dimension of how teachers notice, teachers also can identify a noteworthy student comment and observe their specific actions. Moreover, they give specific reasons why they consider some events or interactions as noteworthy and make connections between events and principles of teaching and learning. Thus, they can explain in detail students' mathematical thinking and understanding using a variety of evaluations and interpretations similar to level 3. Furthermore, at this level, teachers generate alternative teaching approaches or pedagogical suggestions to help students to overcome their difficulties and reach their learning goal by connecting examination about particular student thinking to a specific strategy detected. Even, they comment on some specific topics such as assessment and equity in learning (e.g., "I need to use different assessment techniques to evaluate them better"). In other words, teachers at this level respond to the question of what the factors affecting student learning are and how to improve students' mathematical understanding.

According to van Es' s framework, it can be said that common characteristic of level 1 (baseline noticing) and level 2 (mixed noticing) is that teachers at these two levels make general class observations and they do not focus on students individually. However, at level 3 (focused noticing) and level 4 (extended noticing), teachers point out particular student's strategies and their mathematical understanding. Apart from van Es (2011), Jacobs and his colleagues (2010) constructed a framework to focus on a specialized type of teacher noticing, that is professional noticing of children's mathematical understanding.

2.2.2. Professional Teachers' Noticing of Children's Mathematical Understanding

Jacobs, Lamb and Philipp (2010) focused on the fourth level of Learning to Notice framework -extended level- and chose a particular slice of teaching- teachers' in-the-moment decisions to respond to students. Hence, Jacobs et al. (2010) studied teachers' expertise with specialized type of noticing and they called it *teachers' professional noticing of children's mathematical thinking*. To investigate teachers' professional noticing skills, they constructed a framework consisting of three interrelated skills: attending children's strategies, interpreting children's understanding, and deciding how to respond on the basis of children's understanding by focusing on how and to what extent the teachers notice children's mathematical thinking.

The first and primary component of the Jacobs, Lamb and Philipp's (2010) framework is related to the degree of teachers' attention to essential details relevant to students' mathematical ideas. Jacobs et al. (2010) are interested in "the extent to which teachers attend to a particular aspect of instructional situations: the mathematical details in children's strategies" (p.172) in the first component. This component is significant because the more teachers identify students' strategies, the more they gain an insight into their mathematical understanding. According to their framework, they categorized prospective teachers or teachers' attending skills as an evidence of attending to children's strategies and lack of evidence of attending to children's strategies. Responses that provided evidence of attention to children's strategies include the mathematical essence and substantial details of students' strategies. The description of how children counted by using counters, represented fraction with fraction bars, made multiplication by decomposing numbers are examples of the evidence of attention to children's strategies. However, if teachers mention general features of children's approach without giving detailed descriptions about how they solved, their comments are labeled as a lack of evidence to children's strategies. To illustrate, teacher's comments like "Student's solution was not correct" or "She used fraction bars to solve the problem" can be coded as responses that provided lack of evidence of attention to children's strategies Moreover, irrelevant comments and information that was inconsistent with students' work are categorized as a lack of evidence. For example, "Student's writing is not readable" or "she solved the problem correctly, but she should have preferred a more practical way" can be given as an example of teachers' lack of evidence of attention to children's strategies.

The second component is interpreting children's understanding. In this component, researchers are interested in how teachers interpret children's mathematical understanding embedded in their strategies. Specifically, in this skill, "the extent to which the teachers' reasoning is consistent with both the details of the specific child's strategies and the research on children's mathematical development" (p.172) as well as providing details of students' solution strategies are important. Participants' answers are classified in three categories which are robust evidence, limited evidence and lack of evidence. Responses demonstrating robust evidence of interpretation of children's understandings include different types of inferences about student's understanding. Firstly, teachers express all the details of children's strategies and also recognize how these details reflect children's understanding. To illustrate, teachers might say "Student's strategy demonstrated that she interpreted multiplication as a repeated addition." Secondly, they identify the strategies that are not used and the points that are not understood by students. Briefly, if teachers' responses include interpretation about making sense of strategy details and this interpretation is relevant to mathematical essence of topic and students' mathematical development, it can be said that teachers demonstrate robust evidence of interpretation of children's understanding. Similar to responses with robust evidence, in responses with limited evidence, teachers describe and interpret students' understanding, but this interpretation is more superficial than in responses with robust evidence. In addition, they are interested in which strategies were used and how they use them. However, the specific connection between children's strategies used and their understanding is limited. For example, teachers can make a comment like "Students could not order decimal numbers from larger to small." As a consequence, when teachers draw an overall conclusion related to students' understanding, their responses can be labeled as responses demonstrating limited evidence of interpretation of student' understanding. In some responses, although teachers make a comment regarding students and their understanding, it lacks the evidence to interpret students' mathematical understanding. Furthermore, responses that include positive evaluation of the teaching (e.g. I was happy that no students made calculation error), suggestion for developing teaching (e.g. Teacher needs to use multiple representation), commentary that is irrelevant to mathematical essence and students' understanding (e.g. All of them made effort to solve the problem in class) are evaluated as a response with lack of evidence of interpretation of students' understanding.

The third component of this framework is deciding how to respond on the basis of students' understanding and this component is the most distinct difference between van Es's (2011) categorization and Jacobs et al.'s (2010) categorization regarding professional noticing skills. According to Jacobs et al. (2010), to be professional in noticing, teachers have to be experts in deciding how to respond and give reasoning of their responses as well as being able to attend students' strategies and interpret their understanding. In-the-moment responses in this component are different from planning or long term responses. Thus, teachers need to analyze what students know about the topic and decide on how to respond immediately. Therefore, attending to children' mathematical strategies and interpreting their mathematical thinking are significant criteria for efficient response. Jacobs et al. (2010) do not think that there is a single best response, but they focus on "the extent to which teachers use what they have learned about the children's understanding from the specific situation and whether their reasoning is consistent with the research on children's mathematical development" (p.173). In responses that include robust evidence on deciding how to respond on the basis of students' understanding, teachers notice what students did, which strategy they used and how they used this strategy in detail. And also, according to this capturing, they customized their suggestions for each student. Moreover, responses include particular teaching methods/ possible strategies for the next stage, the rationale behind these methods/strategies proposal and information about how to use them in order to better teach and learn in class. An example of responses showing robust evidence on deciding how to respond on the basis of students' understanding might be as follows.

> Case 1: Example of robust evidence
>
> *Teacher's Response:* For Gözde, I think I should ask the number of the balls in the 52^{th} step of the pattern instead of the 5^{th} step of then pattern.
>
> *Rationale:* Student calculated step by step in order to find the numbers of balls in 5^{th} figure instead of trying to generalize the pattern. When her teacher asks the number of balls for the far term in the same pattern, she will force herself to generalize a pattern.

In case 1, the teacher helps the student to make a generalization and extend his/her knowledge. The teacher also, explains rationale behind his own reasoning. Similar to responses showing robust evidence on deciding how to respond on the basis of students' understanding, responses that include limited evidence include teachers' explanations about students' strategies and alternative ways. However, they do not focus on specific aspects of strategies observed and these responses lack customization. In other words, alternative strategies are proposed for all students instead of for each student specifically. An example of a response which includes limited evidence on deciding how to respond is as follows.

Case 2: Example of limited evidence

Teacher's Response: I think I can use geoboard to explore the area of triangle.

Rationale: All of these students know how to find the area of parallelogram. I think they can make inference about the area of triangle by using the area of parallelogram.

In case 2, although the teacher responds with the rationale that considered the students' past performance, he assumed that all of the class's mathematical understanding is similar and does not customize her response for a particular student. In some responses, there is no evidence of deciding how to respond on the basis of students' understanding. Moreover, if the teachers state the operation used or students' general understanding in solving the given problem, but they do not focus on their strategies and propose unpractical responses, then they can be under the category of lack of evidence. Furthermore, in some responses in this category, teachers write a similar problem to previously asked problems. A case exemplifying lack of evidence can be found below.

Case 3: Example of lack of evidence

Teacher's Response: I will continue with the same type of question, but I will prefer easier numbers.

In case 3, despite the fact that the teacher has to use students' understanding to respond and expand their understandings, she states she will ask the same question with easier numbers.

Briefly, Jacobs and his colleagues' framework concentrates on attending to children's strategies and interpreting their understandings and using these understandings for in-the -moment decision making. Also, these three components unitedly contribute to teachers' responding.

2.2.3. Conclusion Drawn from Categorization of Teachers' Noticing

When teacher noticing frameworks mentioned above are examined from a general perspective, it can be seen that van Es investigated teacher noticing expertise more in detail considering two aspects: what teacher notices and how teacher notices. van Es examined teacher noticing in four levels. However, this categorization had a limited focus on student's thinking and understanding. First three levels (level 1, level 2 and level 3) are related to only classroom environment and group of people while teachers' noticing skills of particular student's thinking is only investigated at level 4. However, Jacobs and his colleagues aimed to examine teachers' skills of deciding how to respond as well as their skills of attending to students' strategies and interpreting their understanding. Therefore, in the Jacobs et al.'s framework, whether teachers can make a connection between students' understanding and possible teaching and learning methods/ strategies or not is investigated. Furthermore, Jacobs et al.'s framework is constructed to examine teachers' professional noticing on specific aspects of student thinking and gives chance to discuss their noticing with common language. Thus, Jacobs et al.'s framework is more detailed to learn teachers' professional noticing skills Since the aim of this study was to explain prospective teachers' noticing skills of students' algebraic thinking in-depth, Jacobs, Lamb and Philipp's categorization was used. Although, these frameworks were used to guide teachers' noticing skills of students' mathematical understanding within various contexts such as geometry and fraction, in this study Jacobs et al.'s framework is preferred in order to investigate prospective middle school mathematics teachers' skills of noticing students' algebraic thinking within the context of pattern generalization.

2.3. Studies about Teachers' Noticing of Students' Mathematical Understanding

Since teacher's noticing skill enables them to teach resiliently and to adapt instruction to accommodate students' ideas, it is a very critical component of teacher's competency (National Council of Teachers of Mathematics [NCTM], 2000; Ball, Lubienski, & Mewborn, 2001). Thus, teachers must be able to pay selective attention to students' strategies observed, make evaluative and interpretive comments regarding their mathematical thinking, make connections between students' thinking and the principles of teaching and learning and decide how to respond to them on the basis of their understanding (van Es & Sherin, 2008; Jacobs, Lamb, & Philipp, 2010). Therefore, in recent years several studies have been conducted to investigate teachers' skills of noticing students' mathematical understanding in an international context (Amador, Carter, & Hudson 2016; Jacobs et al., 2010; Schack, Fister, Thomas, Eisenhardt, Tassel & Yoder, 2013; Star and Strickland, 2008; van Es & Sherin, 2008) and in a national context (Güner & Akyüz, 2017; Kılıç, 2018; Osmanoğlu, 2010; Osmanoğlu, Işıksal & Koç, 2012; Özdemir-Baki & Işık, 2018; Tunç-Pekkan & Kılıç, 2015; Ulusoy & Çakıroğlu, 2018).

2.3.1. Studies about Teachers' Noticing in an International Context

Some studies in the international context investigated to what extent teachers and prospective teachers notice students' understanding within a specific context (Amador, Carter, & Hudson, 2016; Taylan, 2017). There are also research studies that aimed to investigate whether improving teachers' noticing skill is possible or not and in what way it can be done (Jacobs et al., 2010; Schack et al., 2013; Star and Strickland, 2008; van Es & Sherin, 2008).

To illustrate, Amador, Carter and Hudson (2016) focused on what prospective teachers' notice in mathematics classroom during the lesson study via observation. Also, they aimed to state the type of prospective teachers' focused and extended noticing according to van Es's (2011) framework. The data was collected from 24 prospective teachers that were enrolled in method course and field experience through video recordings. According to the results of this study, Amador, Carter and Hudson (2016) stated that examples of focused and extended noticing skills were rare, and they concluded that prospective mathematics teachers had limited noticing skills of students' mathematical understanding. According the output of this research, they modified van Es's (2011) framework and suggested that "Noticing can be considered from the perspective of detailing student strategies, analysis of evidence, quality suggestion and the connection of suggestion to evidence" (p.381). Also, they believed that this framework gives teacher educators a chance to provide more guidance in order to understand students' mathematical understanding while they are watching and observing lecture. In addition to this study, Taylan (2017) examined the noticing skill of a highly successful third grade mathematics teacher within the context of multiplication and division. In that study, the sample included only one teacher who had experience in teaching third grade students for six years and worked one-on-one with a nationally prominent teacher educator and educational researcher in a professional development program for three years. Video records of classes, students' written works and their notes, videotaped interviews and video clips were used in order to gather data from the participant. Taylan (2017) stated that the teacher identified students' answers and strategies. In addition, she focused more on particular student's understanding than whole class instruction. And then, she highlighted the specific events in classroom in relation to students' thinking and made variety of interpretations about students' understanding. As well as attending to students' strategies and interpreting them, she connected her interpretations of students' understanding with general principles of teaching and learning. Contrary to the studies of Amador, Carter and Hudson (2016), the findings of Taylan's (2017) research showed that the teacher had a high level of noticing skill of students' mathematical understanding and this study proposed details about how a teacher constructs her instruction based on her observation and noticing.

There are also many research studies in the international context that aimed to examine whether improving teachers' noticing skill is possible or not and in what way it can be done (Jacobs et al., 2010; van Es & Sherin, 2008; Star and Strickland, 2008; Schack et al., 2013). Jacobs and his colleagues (2010) conducted a cross sectional study to construct a picture of changing of teachers' perspectives engaged in a sustained professional development program focusing on students' mathematical thinking. Thirty-six prospective mathematics teachers and 95 experienced K-3 teachers participated in this study. In order to gather data, the participants were asked to watch a video clip or analyze the students' written work and then to answer prompts about attending, interpreting and deciding how to respond in writing. The analysis of data showed that teaching experience had a critical role in being expert in attending to students' strategies and interpreting their mathematical understanding, whereas there was no similar evidence for expertise in deciding how to respond on the basis of students' understanding. However, they declared that the professional development program helped to enhance expertise in all components of noticing skill. Finally, Jacobs et al. (2010) stated that the participants who took professional development training for 2 or more than 2 years and made leadership activities can more easily interpret students' thinking and use these interpretations in order to respond.

Similarly, van Es and Sherin (2008) explored the changes in teachers' noticing skills in the video club context. The data were collected through video club meetings and interviews from seven fourth and fifth grade elementary teachers with experience ranging from one to over twenty years. As a result of the study, they found that teachers began to attune different features of the classroom by making discussions with their peers in the video club meeting and their depth of interpretation of events in classroom increased. In addition, teachers were aware of different aspects of classroom environment to analyze classroom environment and students' thinking through video club meetings. As a consequence, van Es and Sherin (2008) declared that teachers' noticing can be developed as they interact with colleagues after watching videos.

Parallel to the results of previous studies (Jacobs et al., 2010; van Es & Sherin, 2008), Star and Strickland (2008) conducted a study to investigate whether prospective teachers' classroom observation and noticing skills (attending to) can be developed using video or not. The data obtained from 28 prospective teachers who were enrolled in a semester-long secondary mathematics methods course at Midwestern University in the United States. In order to explore participants' noticing skills, written instruments which are pre-assessment and post-assessment tasks were used. The ability of observation of classroom environment, classroom management and the ability of being attentive to tasks, mathematical content and communication were researched with these instruments. Star and Strickland (2008) stated that in the beginning of the teaching methods course, prospective teachers were weak in the observation of static features of the classroom environment and attending to the issues of mathematical content. However, they realized that after taking teaching methods course in mathematics education, the participants developed their observation and noticing skills.

Finally, Schack, Fister, Thomas, Eisenhardt, Tassel and Yoder (2013) investigated to what extent teacher educators can enhance the progress of prospective elementary school teachers' noticing skills of students' understanding in early numeracy. The data were obtained by using pre and post assessment tasks from 94 prospective elementary students who participated in researcher-developed five session module that respectively nests the three interconnected components of professional noticing-attending, interpreting and deciding how to respond. After watching the video recordings of diagnostic interviews with students carried out by teacher educators, prospective teachers were asked to respond to three prompts which were related to three components of noticing: attending, interpreting and deciding how to respond by making a connection with students' understanding. The results of the study showed that prospective teachers' ability to attend, interpret and decide how to responds in the context of early numeracy improved after participating in researcher-developed five session module. Finally, Schack et al. (2013) claimed that professional noticing is fundamental competency for teaching and can be developed. Jacobs et al. (2010), van Es and Sherin (2008), Star and Strickland (2008) and Schack et al. (2013) had a common idea that noticing skill can be learnt and could be improved with teacher training programs.

In addition to the studies regarding teachers' noticing skills of students mathematical understanding in the international context, there are also many studies that have been conducted in the national context.

2.3.2. Studies about Teachers' Noticing in National Context

In addition to studies conducted in international context, some studies were conducted in order to explore teachers' and prospective teachers' noticing skills of students' understanding within a specific context (Güner & Akyüz, 2017; Kılıç, 2018; Tunç-Pekkan & Kılıç, 2015), while some studies were conducted to investigate whether improving teachers' noticing skill is possible or not and in what way it can be done in national context (Osmanoğlu, 2010; Osmanoğlu, Işıksal & Koç, 2012; Özdemir-Baki & Işık, 2018; Ulusoy & Çakıroğlu, 2018; Güner & Akyüz, 2017).

For example, Güner and Akyüz (2017) conducted a study in order to explore prospective teachers' noticing skills within the context of addition and subtraction on fraction through the lesson study consisting of planning, teaching and discussion parts. The data were obtained from four participants with lesson plan, video records of lecture observation and interview as a part of lesson study. The results of Güner and Akyüz's (2017) study indicated that prospective teachers' noticing skills of students' understanding of addition and subtraction on fraction is low. In fact, it was seen that participants focused on applying lesson plan and using materials rather than students and their understandings.

Parallel to this research, Kılıç (2018) conducted a study to investigate pre-service teachers' noticing skill and scaffolding practices. In this study, six prospective teachers took part in the research and they were matched with a pair of sixth grade students to observe and scaffold these students' mathematical understanding while they were working. The data was obtained from video records of pre-discussion and in-class implementation, and prospective teachers' written reflection regarding noticing during the 14-week course program. According to the participants' answers, they identified mathematical opportunities and they made coding for attending and deciding how to respond. According to the results of this study, although prospective teachers can mostly recognize students' errors and strategies and explain reasoning of their comments, they usually did not use high level scaffolding practice. Namely, participants could not extend or support students' understanding in order to elicit students' misconceptions and improve their understanding.

Additionally, Tunç-Pekkan and Kılıç (2015) carried out a study to examine prospective teachers' noticing skills of students' understanding of fractions similar to Kılıç's (2018) study. The purpose of Tunç-Pekkan and Kılıç's (2015) study was to investigate to what extent the prospective teachers notice mathematical opportunities in relation to fractions and scaffold students' mathematical thinking during interactions. The data were collected from three prospective teachers having experience in teaching middle and high school mathematics and three pairs of 6^{th} grade students through video records of observation and participants' discussion. This study revealed that prospective teachers' lack of content and pedagogical content knowledge negatively affected catching and stating them effectively. Moreover, the researchers claimed that prospective teachers need to further develop appropriate scaffolding activities. Namely, studies demonstrated that Turkish prospective teachers' noticing skills are low and these skills need to be developed (Güner & Akyüz, 2017; Kılıç, 2018; Tunç-Pekkan & Kılıç, 2015).

There are also some studies conducted in Turkey to investigate whether improving teachers' noticing skill is possible or not and in what way it can be done (Güner & Akyüz, 2017; Osmanoğlu, 2010; Osmanoğlu, Işıksal & Koç, 2012; Özdemir-Baki & Işık, 2018; Ulusoy & Çakıroğlu, 2018). To illustrate, Osmanoğlu (2010) conducted a study to explore the changes in prospective teachers' noticing skills regarding teacher and student roles as they watched video cases from real classroom and discussed these videos online within the context of geometry. Online discussion forum was used and data was obtained from fifteen prospective teachers through the participants' reflection papers on video cases from real classroom, online discussion and interview (at the beginning, middle and the end). The results of this study showed that with online video based discussion, prospective teachers' noticing skills regarding teacher and student roles developed.

Moreover, prospective teachers' noticing skills of student roles underlined in the Elementary Mathematics programs were investigated with video-based methodology by Osmanoğlu, Işıksal and Koç (2012). The data were obtained through reflection papers from and interview with fifteen prospective teachers who studied at one of the public universities. This research study indicated that the use of video cases in teacher education gave opportunity to know the expectations of the elementary mathematics program for prospective teachers. Besides, it was revealed that prospective teachers could attend and interpret several issues regarding students' roles by analyzing real mathematics classrooms.

Ulusoy and Çakıroğlu (2018) conducted a study to investigate prospective teachers' noticing skills of students' mathematical thinking in a video-based learning environment and to analyze students' thinking in schools in which prospective teachers made school experience. Also, they aimed to understand their perception regarding the role of analyzing micro case videos on the noticing of students' mathematical thinking. The data was obtained from written reflection for each micro case video, group discussion, and reflection after group discussion, classroom meetings and project report during 14-week elective course program. According to the results of Ulusoy and Çakıroğlu's (2018) research, although prospective teachers simplistically analyzed students' mathematical thinking in the beginning of video-cases analyses, they proposed more profound analyzing by making an interpretation of data and recommend pedagogical strategies. Moreover, micro case videos functioned as a scaffolding to enhance prospective teachers' knowledge about students' thinking. Therefore, Ulusoy and Çakıroğlu (2018) concluded that prospective teachers' noticing skills can be developed by using micro case videos that enable to establish strong knowledge on students' thinking.

Similarly, in a research study, Güner and Akyüz (2017) investigated prospective elementary teachers' noticing skills of 5th grade students' mathematical thinking in the concepts of perimeter, area and surface area. One female and three male prospective elementary teachers participated in this study. The data was obtained from lesson study for eight weeks in which the concepts of perimeter, area and surface area were taught to 5th grade students. Similar to Ulusoy and Çakıroğlu (2018), Güner and Akyüz (2017) stated that although prospective teachers' noticing skill differs in the different stage of lesson study and it is low, it can be developed with lesson study by sharing their thoughts with each other. According to Güner and Akyüz, one of the reasons for this inadequacy might be their lack of teaching experiences. As a result, Güner and Akyüz (2017) stated that lesson study is one of the effective ways to improve teachers' noticing skills.

Parallel to the study of Güner and Akyüz (2017), Özdemir-Baki and Işık (2018) conducted a study to analyze teachers' noticing skills after conducting lesson study that is a popular professional development program. Six teachers who worked at the secondary schools participated in this research study and four of them took part in lesson study, while two of them did not participate. The data was gathered through video recordings of their lectures, participants' evaluation reports and unstructured interviews. The findings of the study revealed that the participants who performed the lesson study paid attention to students' prior knowledge, students' different solutions ways, and their misconceptions. Moreover, the teachers who participated in the lesson study focused on how they corrected students' mistakes and suggested alternative teaching methods. However, teachers who did not attend to the lesson study generally focused on classroom environment and made evaluations regarding teacher pedagogy rather than students' mathematical thinking. Therefore, Özdemir-Baki and Işık (2018) concluded that lesson study is an effective professional development model to improve teachers' noticing skills. Thus, it can be concluded that teacher noticing skills could be improved through teacher education programs or professional development programs (Osmanoğlu, 2010; Ulusoy & Çakıroğlu, 2018; Güner & Akyüz, 2017).

The research studies mentioned above showed that the number of studies in Turkey that examine teachers' professional noticing skills of students' mathematical thinking has increased in recent years and these studies generally concentrated on two areas: investigation of teachers' noticing skills and exploration of whether teachers' noticing skills can be developed or not. According to the results of these studies, researchers agreed that prospective teachers' and teachers' noticing skills of students' mathematical understanding were inadequate (Kılıç, 2018; Tunç-Pekkan & Kılıç, 2015); however, this skill can be enhanced via teacher education programs or professional development programs (Güner & Akyüz, 2017; Osmanoğlu, 2010; Ulusoy & Çakıroğlu, 2018).

Algebra is the primary concept of mathematics in order to build conceptual and deeper understanding of mathematics (Blanton & Kaput, 2005). Thus, the investigation of prospective teachers' noticing skills of students' algebraic thinking is significant. Therefore, in the current study, it was aimed to examine prospective middle school mathematics teachers' noticing skills regarding students' algebraic thinking within the context of pattern generalization. The next section focuses on algebra as a topic of study, algebraic thinking, algebra in curriculum and functional thinking and pattern generalization.

2.4. Algebra as a Topic of Study

Algebra has been one of the important branches of mathematics (Blanton, Stephens, Knuth, Gardiner, Isler, & Kim, 2015) and is considered to be a gatekeeper in mathematics teaching and learning (Knuth, Alibali, McNeil, Weingberg, & Stephens, 2005). According to Radford & Peirce (2006), algebra cannot be thought only as making estimations or using signs, and Howe (2005) addresses algebra as follows:

> Working with variables, and in particular, arithmetic with variables, so the formation of polynomial and rational expressions. This also includes representing, or "modeling" concrete situations with expressions, and setting up equations. It is also often extended to include extracting roots. (If these processes are iterated, they can produce highly complicated expressions. But school algebra does not go very far down this road.) It also includes manipulating expressions and equations, to simplify, solve and interpret (p, 1).

Kaput (1999) gave a detailed definition of algebra and described the following five aspects of algebra in present day mathematics:

1. "Algebra as generalizing and formalizing patterns and constraints, especially, but not exclusively, algebra as generalized arithmetic reasoning and algebra as generalized quantitative reasoning" (p.4).

2. "Algebra as syntactically guided manipulation of (opaque) formalisms" (p.7).

3. "Algebra as the study of structures abstracted from computations and relations" (p.7).

4. "Algebra as the study of functions, relations and joint variation" (p.8).

5. "Algebra as a cluster of modeling languages and phenomena-controlling languages" (p.8).

Thus, algebra means recognizing the relationship between variables, making a generalization of this relationship, and writing a formula with algebraic expression based on this generalization.

According to Rakes, Valentine, McGatha, and Ronau (2010), algebra is core for developing the understanding of high school mathematics, and so, it is important for students to learn the fundamental concepts of algebra. From this point of view, the standards of the National Council of Teachers of Mathematics (NCTM, 2000) stated that students have to achieve four goals for algebra from kindergarten through grade 12. These goals are as follows: "goal 1-understand patterns, relations, and functions; goal 2-represent and analyze mathematical situations and structures using algebraic symbols; goal 3-use mathematical models to represent and understand quantitative relationships, and goal 4-analyze change in various contexts" (p.296).

2.4.1. Algebraic Thinking

Algebraic thinking which is related to algebra has various meanings. To illustrate, Cuoco, Goldenberg and Mark (1996) defines algebraic thinking as a habit of mind and practical ways of thinking about mathematical content. Driscoll (1999, 2001) defines algebraic thinking as a thinking on quantitative situations, which supports forming relationships between variables. He explained that algebraic thinking includes being able to think about functions, how they work and the impact of system's structures on calculations (Driscoll, 1999). Swafford and Langrall (2000) view algebraic thinking as the capability of thinking about unknown quantities as known. Furthermore, Kieran and Chalouh (1993) interpret algebraic thinking as making sense of symbols and operations of algebra in terms of arithmetic. In addition, Kieran (1996) stated that the ability of using different representations to interpret quantitative situations in a relational way is another meaning of algebraic thinking. Radford specified that algebraic thinking entails encouraging young students to become naturally aware of generalizations in numerical and non-numerical contexts and expressing these generalizations using a variety of semiotic signs (2008). However, Radford (2010) reported that algebraic thinking neither comes into existence coincidentally, nor does it arise as the required consequence of cognitive maturation. Therefore, recognizing relationships, generalizing beyond specific examples, and investigating and analyzing patterns have a critical role in the middle school curriculum in order to develop algebraic thinking (Magiera, Van den Kieboom, & Moyer, 2013).

2.4.2. Algebra in Curriculum

Algebra topics are introduced to students firstly when they are 6^{th} graders and taught throughout middle and high school in Turkey (MoNE, 2018). The basic concepts such as coefficient, variable, algebraic expression and constant term are introduced to students at 6^{th} grade in Turkey. Then, operation with algebraic expressions and generalization of pattern are focused on at 7^{th} grade (MoNE, 2018). And then, algebraic expression and identities, linear equations and inequalities are taught to 8^{th} grade students.

As a consequence, engaging in making a generalization from patterns and using variables or algebraic symbols are significant steps to increase students' algebraic thinking (Van de Walle, Karp, & Bay-Williams, 2013). For this reason, in this study, the aim was to investigate prospective teachers' noticing skills of algebraic thinking with the subset of pattern generalization.

2.4.3. Pattern Generalization

Radford stated that algebraic thinking is based on students' possibilities to catch patterns and build functional relationships to find remote and unspecified terms (2011). Radford also suggested that "the linkage of spatial and numerical structures constitutes an important aspect of the development of algebraic thinking" (2011, p.266) because a relationship is created between the figure and the value corresponding to that figure. From this point of view, Mason, Burton and Stacey assert that "Generalizations are the life-blood of mathematics. Whereas specific results may in themselves be useful, the characteristically mathematical result is the general one" (2010, p.8).

According to Radford (2008), generalization process includes three main points which are (1) recognizing a repeated process, (2) generalizing this repeated process to all terms of sequences, and (3) establishing a rule that enables them to directly determine any term of sequence. Rivera and Becker (2009) added justification to these main points and they said that "some kind of explanation that their algebraic generalization is valid by a visual demonstration that provides insights into why they think their generalization is true" (p. 213-214) According to Rivera (2010), the generalization process consists of the coherence of two interdependent actions: "abductive-inductive action on objects, which involves employing different ways of counting and structuring discrete objects or parts in a pattern in an algebraically useful manner, and symbolic actions, which involve translating in the form of algebraic generalization" (p. 300). Consequently, the generalization process enables students to develop their algebraic thinking because this process includes recognition, justification and reasoning (Radford, 2008; Rivera & Becker, 2009; Rivera, 2010).

Although students have difficulty in expressing generalization in algebraic terms and creating inverse functions with symbols (Jurdak & Mouhayar, 2014), their ability of pattern generalization and algebraic thinking can be developed (English & Warren, 1998). If teachers can understand how students made their algebraic activities, they can encourage their algebraic understanding. Thus, pedagogical conditions need to be created in order to enhance students' algebraic thinking (Radford, 2010), and, teachers' noticing skill of students' algebraic thinking is an essential competency for teachers. From this point of view, in recent years, studies that aimed to investigate prospective teachers' noticing skills of students' understanding with the specific context of algebra have been conducted.

2.5. Studies about Teachers' Noticing of Students' Algebraic Thinking

Researchers conducted a study to explore prospective primary school teachers' noticing skills of students' algebraic thinking in international context (Callejo & Zapatera, 2017; Walkoe, 2013). In a research study, Walkoe (2013) explored prospective mathematics teachers' noticing skills of students' algebraic thinking and investigated whether incorporating an eight-week video club intervention could help enhance teachers' noticing skills of students' algebraic thinking or not. The data was collected from 13 prospective secondary mathematics teachers via video club meetings. Similar to Amador et al.'s (2016) study, this study revealed that in the early video club meetings, the participants were weak in making descriptive and evaluative comments, highlighting noteworthy events, and explaining students' algebraic thinking in-depth. However, Walkoe (2013) explained that as video clubs progressed, participants began to discuss classroom environment better and made more interpretive comments regarding students' thinking. In addition to this study, Callejo and Zapatera (2017) conducted a study to explore teachers' competence in noticing students' mathematical thinking in the specific area of the pattern generalization. The data was gathered via a questionnaire from 38 prospective primary school teachers in the second semester of the program. According to the results of Callejo and Zapatera's (2017) study, although the participants identified the mathematical elements of the problems solved by the students, they were not good at interpreting students' understanding of pattern generalization. They stated that while 16 prospective primary school teachers' answers were labeled as high level of identification, only two of them made high level of interpretation. Also, they added that there was no evidence of interpretation for 15 prospective primary school teachers. Thus, the researchers concluded that the teachers could not identify the mathematical essence of the problem in order to understand students' algebraic thinking within the context of pattern generalization. In conclusion, the results of these studies revealed that prospective primary school teachers' noticing skills of students' algebraic thinking and understanding are limited; however, this skill can be developed with the help of teacher training programs.

2.6. Summary of Literature Review

“Teaching is one of the most common -and also one of the most complicated- human activities” (Ball & Forzani, 2010, p.40). In order to teach effectively, teachers must learn to distinguish the remarkable actions to pay attention to and to handle complex events that occur in the classroom; thus, teachers’ noticing skill is an essential competency (Star & Strickland, 2008; Sherin & Star, 2011; van Es & Sherin, 2002; Sherin, Russ, & Colestock, 2011; Jacobs, Lamb, & Philipp, 2010). In the light of the studies reviewed in this section, different models were developed to explain teachers’ noticing skills (van Es, 2011; Jacobs, Lamb, & Philipp, 2010). Although some frameworks focus on teachers’ noticing of both classroom environment and student thinking, others provide an approach related to only teachers’ noticing of students’ understanding.

Our review of the literature indicated that researchers conducted a study in order to examine teachers and prospective teachers’ noticing skills of students’ understanding in international and national context. The results of some studies showed that teachers and prospective teachers have limited noticing skills (Amador, Carter, & Hudson 2016; Güner & Akyüz, 2017; Kılıç, 2018; Tunç-Pekkan & Kılıç; 2015); however, others showed that teachers can learn to notice students’ understanding and this ability can be enhanced through teacher training programs or professional development programs (Güner & Akyüz, 2017; Jacobs, Lamb & Philipp, 2010; Osmanoğlu, 2010; Ulusoy & Çakıroğlu, 2018; Schack, Fister, Thomas, Eisenhardt, Tassel, & Yoder, 2013; Star & Strickland, 2008; van Es & Sherin, 2008).

Algebra is one of the important areas of mathematics, and algebraic thinking is a highly essential skill for learners of mathematics (Kieran & Chalouh, 1993; Kieran, 1996; Cuoco, Goldenberg, & Mark, 1996). Interpreting symbols, semiotic signs or unknowns as arithmetic and known is related to algebraic thinking (Swafford & Langrall, 2000; Kieran and Chalouh, 1993; Kiearn, 1996; Radford; 2008), and the investigation of non-symbolic form of algebraic thinking is highly critical to encourage students to think algebraically. For this reason, enhancing students' pattern generalization skills is an important step (Radford, 2010; Rivera, 2010). Therefore, investigating prospective middle school mathematics teachers' noticing skills within the context of pattern generalization is important. Moreover, the number of investigations on this topic in international context is quite limited (Callejo & Zapatera, 2017; Walkoe, 2013) and there are no studies on prospective teachers' or teachers' noticing in accessible literature in national context. Examining prospective teachers' noticing skills within the context of pattern generalization is believed to contribute theoretically to the literature by filling the gap regarding prospective teachers' noticing skills within the context of pattern generalization. The current study is also assumed to contribute practically to the literature in terms of presenting different real students' solutions strategies. Thus, the aim of this study is to investigate prospective teachers' noticing skills of students' algebraic thinking within the context of pattern generalization.

CHAPTER 3

METHOD

In this study, prospective middle school mathematics teachers' noticing skills of students' algebraic thinking within the context of pattern generalization were examined. In this chapter, the research questions, design of the study, sampling and participants of the study, the context of the study, data collection procedure, the pilot study, data analysis procedure, trustworthiness, researcher role and bias, ethical consideration and limitations of the study are given respectively.

3.1. Research Questions

The research questions that were addressed in this study are as follows:

1.How do prospective middle school mathematics teachers notice students' algebraic thinking within the context of pattern generalization?

> 1.1. To what extent do prospective middle school mathematics teachers attend to students' solutions within the context of pattern generalization?
>
> 1.2. To what extent do prospective middle school mathematics teachers interpret students' algebraic thinking within the context of pattern generalization based on students' solutions?
>
> 1.3. What is the nature of the decisions that prospective middle school mathematics teachers make to respond on the basis of students' algebraic thinking within the context of pattern generalization?

In order to answer these research questions, the research was designed as described below.

3.2. Research Design

In this study, qualitative research method was used in order to investigate prospective middle school mathematics teachers' noticing abilities of students' algebraic thinking within the context of pattern generalization. Qualitative research has been defined differently by many researchers. To illustrate, Denzin and Lincoln (2005) defined qualitative research as follows:

> Qualitative research is a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into series of representations, including field notes, interviews, conversations, photographs, recordings and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative research study things in their natural settings, attempting to make sense of, or interpret phenomena in terms of the meanings people bring to them (p.3).

Patton (2002) defined qualitative research as trying to understand the particular aspects of situations in their context. According to Frankel and Wallen (2006), when researchers want to obtain a more complete picture of teaching, insight into concerns and learning and know more than just "to what extent" or "how well" something is done, qualitative research is preferred. In this type of research, the quality of relationships, activities, situations or materials are investigated (Frankel & Wallen, 2006). Furthermore, Creswell (2007) stated that qualitative research is conducted in order to explore problems or issues and to gain an in-depth understanding of the issue and interpretation of participants. Since the aim of current study was to investigate prospective middle school mathematics teachers' noticing ability regarding students' algebraic thinking within the context of pattern generalization, it was preferred to use qualitative research design in the current study.

Although there are many different qualitative methodology types, some common features that characterize most qualitative research studies are as follows: (1) The researcher is the key instrument and the main source of data to explore the process; (2) During the process of the study, the concepts are interpreted in their own natural settings; (3) Descriptive explanation of what is investigated is produced via words and pictures; (4) Data that is collected through inductive process which is used to improve concepts and theories, or researchers are primarily interested in how people make sense out of their lives to make sense of the concepts within the theory (Bogdan & Biklen, 1998; Merriam, 2009).

Qualitative methodology studies have distinct characteristics as well as common features. For this reason, researchers proposed different types of qualitative research based on their distinctness. Merriam (1998) mentioned five different types of qualitative research: basic or generic, ethnography, phenomenology, grounded theory, and case study. In the current study, I was a part of the study as the researcher, interested in prospective teachers' noticing skills of students' algebraic thinking within the context of pattern generalization and aimed to have a deep understanding of their thinking about the topic. Therefore, qualitative case study is the most appropriate to use. The characteristics of the case study are explained in the following section.

3.2.1. Case Study

Creswell (2007) defined case study as investigating an issue with the help of one or more cases that are in the specific context. Merriam (1998) reported that an in-depth understanding of the issue is obtained through case study design and stated that process, context, and discovery are more important than the outcomes, specific variables, and discovery in case studies. Furthermore, Creswell (2007) and Merriam (1998) specified that a person, a program, or a group should be defined as a case.

Yin (1994) gave the definition of case study as follows:

> A case study is an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident…Case study inquiry copes with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result relies on multiple sources of evidence, with data needing to converge in a triangulating fashion, and as another result benefits from the prior development of theoretical propositions to guide data collection and analysis (p. 13).

In addition, Yin (2003, 2009) categorized case studies into single-case holistic, multiple-case holistic designs and single-case embedded, multiple-case embedded designs. Whether a study is a single-case or multiple case is related to the number of cases, and whether a study is holistic or embedded is related to the number of unit of analysis. Among these designs, single-case embedded design is a common design in case studies where there is more than one unit of analysis to explain the case. The model of the single-case embedded design is given in Figure 3.1 below.

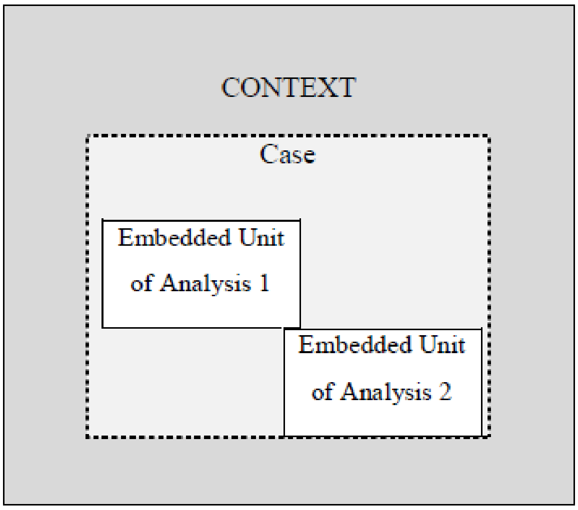


Figure 3.1. Single-case embedded (multiple units of analysis) design (Yin, 2009, p. 46)

The research design of this study was single-case embedded design. The case was thirty-two prospective middle school mathematics teachers, and the prospective teachers' noticing skills of attending to students' solutions, interpreting students' mathematical understanding, and deciding how to respond were the embedded "units of analysis". The participants of the current study are prospective middle school mathematics teachers enrolled in the teacher education program at one of the public universities. In Figure 3.2., the model of the current study with respect to single-case embedded design is given.

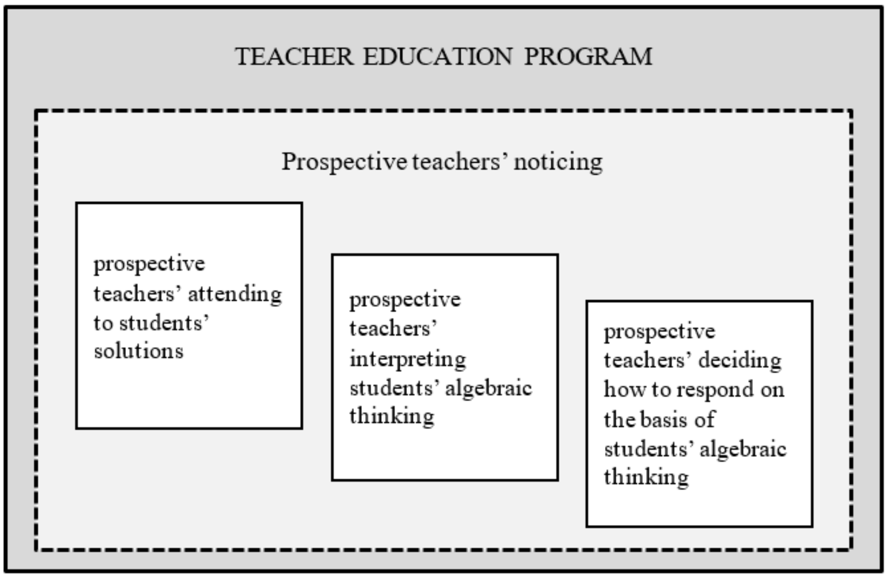


Figure 3.2. Single-case embedded (three units of analysis) design

In sum, in the present study, in an effort to obtain in-depth exploration of prospective middle school mathematics teachers' noticing skills, I wanted to study case-based pedagogy and thus conducted qualitative case study. The sampling and selection process of the participants are explained in the next section.

3.3. Sampling and Selection of the Participants

In qualitative studies, researchers want to elicit crucial information from those who supply them the most and they make connection with participants mostly (Merriam, 1998). Thus, selecting participants has very critical role to achieve the aim of the study. In order to select participants, there are two basic types of sampling methods which are probability sampling and non-probability sampling (Merriam, 2009). The probability sampling method is preferred in order to generalize the results of the study from the sample to the population. Since qualitative research does not aim to make generalizations, the non-probability sampling method was preferred rather than the probability sampling method for sampling (Merriam, 2009). One of the most common forms of non-probability sampling, which is the purposive sampling method, was used in this study. Purposive sampling is based on the assumption that the researcher wants to discover, understand, and gain an insight into the phenomenon and therefore must select a sample from which the most can be learned (Merriam, 2009, p.77). In the light of the definition of purposive sampling given by Merriam (2009), 32 prospective teachers were selected based on three criteria to apply the questionnaire regarding professional noticing skills of students' algebraic thinking.

The first criterion is related to accessibility so that I could easily access the participants. Participants should be close enough to me to make an interview. The second criterion of the sampling procedure was that participants completed the Methods of Teaching Mathematics I-II courses since studying these courses makes prospective teachers more knowledgeable about teaching mathematics. The final criterion is that participants were taking the School Experience course because in this course they have an opportunity to observe classroom environment and gain experiences about noticing classroom environment and students' thinking. For these reasons, in this study, prospective teachers who were easy to reach, who completed Methods of Teaching Mathematics I-II courses, and who were taking the School Experience course were preferred.

To sum up, in the present study, 32 senior prospective middle school mathematics teachers were selected to participate in this study. After applying questionnaire to thirty-two prospective teachers, all of them were asked whether they were voluntary to participate in semi-structured interview. Thus, eight senior prospective teachers agreed to take part in semi-structured interview. The sample of the main study is summarized in Figure 3.1 below.

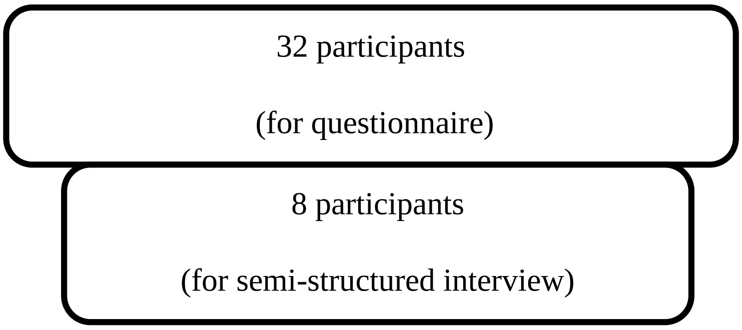


Figure 3.3. Sample of the Study

3.4. Context of the Study

The context of the study was the middle school mathematics teacher education program, which is a four-year undergraduate program at one of the public universities located in Ankara. The program "aims to develop teachers with a sound understanding of how children learn mathematics; with confidence in using technology; with competence in problem-solving; with sensitivity to human rights, democracy, and ethics. The program emphasizes critical thinking, personal reflection, and professional development of prospective mathematics teachers" ("Department of Elementary Mathematics Education", 2018). Students who take this education program become mathematics teachers for 5^{th} to 8^{th} grades. The courses offered in the program are given in Appendix A.

3.5. Data Collection

In qualitative research, data is obtained using multiple techniques such as interview, observation and the examination of documents in order to describe the phenomenon deeply (Frankel & Wallen, 2006; Lodico, Spaulding, & Voegtle, 2006). Interview is the most commonly preferred data collection tool in qualitative studies in order to clearly elicit information from the participants (Merriam, 1998). Observation gives researchers an opportunity to observe participants' behavior as are (Frankel & Wallen, 2006). Another important form of data in qualitative research is "documents produced by key participants in the events being observed" (Slavin, 2007, p. 133) and a document in qualitative research consists of three major sources of data which are personal papers, public records and artifacts (Merriam, 1998). Thus, in this study, documents and interview were used to obtain data.

Prospective teachers' noticing skills of students' algebraic thinking within the context of pattern generalization were investigated through students' solutions; therefore, firstly student solutions regarding pattern generalization were selected through the questionnaire that was applied to students. According to the results of this questionnaire, alternative students' solutions were selected and the questionnaire for prospective teachers was prepared to explore their noticing skills. Interview was also used as a data collection tool in this study.

Table 3.1 presents the time schedule for the data collection process. Then, the data collection tools and data collection procedures are explained in detail in the following sections.

Table 3.1. *Time Schedule for Data Collection*

Date	Events
October 2017 - November 2017	• Development of the instrument (questionnaire for students) • Applying the questionnaire to students • Selecting alternative students' solutions
November 2017 – January 2018	• Development of the instrument (questionnaire for prospective teachers)
January 2018- July 2018	• Pilot study and revision of the data collection tool • Selection of participants for questionnaire in the main study • Development of the instrument (interview)
December 2018- December 2018	• Data collection through the questionnaire for the main study • Pilot study and revision of the data collection tool for interview
December 2018- January 2019	• Selection of the participants for interview in the main study • Data collection through interviews for the main study
January 2019 – March 2019	• Data analysis

3.6. Data Collection Tools

The purpose of this study is to investigate the noticing skills of prospective middle school mathematics teachers regarding students' algebraic thinking within the specific context of pattern generalization. In order to achieve the aim of the study, data was collected via (1) a questionnaire to reveal middle school students' solutions, (2) a questionnaire for prospective middle school mathematic teachers' professional noticing skills, and (3) interviews with prospective teachers following the noticing questionnaire. Detailed information about the data collection tools is given in the following sections.

3.6.1. Questionnaire for Middle School Students

In order for prospective teachers to attend to students' solutions, interpret students' algebraic thinking, and decide how to respond, different student solutions regarding pattern generalization were needed. For this reason, in order to gather alternative student solutions, a questionnaire for middle school students was prepared. Although the curriculum was revised in 2018, the previous curriculum (2013) was implemented when data were collected. Thus, three questions with sub-dimensions were adapted from the literature in accordance with the objective "Students should be able to express the rule of arithmetic sequences by using letters and find the desired term of sequences expressed in letters." (6.2.1.1.) given in the Turkish middle school mathematics curriculum (MoNE, 2013, p.18). Detail information about the questions related to this objective is given below.

Pattern generalization tasks are placed in different categories (Stacey 1989): "questions which can be solved by "step-by-step drawing and counting" (near generalization) and "questions which go beyond reasonable limits of such a step-by-step approach" (p.150), for example, arriving at the number of elements of figure 80 in a series (far generalization). Identifying a model which is the growth pattern of the series is necessary to make a near generalization, while building coordination of two models is important to make a far generalization and write a general rule via functional thinking: the number of elements of the term and the position of each term of series that refers to more complicated relationships (Radford, 2011). Moreover, Warren (2005) revealed that it was important to reverse the process; that is, finding the term with a given number of elements as well as generalizing a pattern from a small position number to large position number. Thus, identifying a term in an inverse functional relationship (finding number of elements of a figure with the number in the figure) is a significant component for pattern generalization.

In the light of these definitions, question 1 examined students' knowledge related to far generalization, while the second question was designed to assess students' knowledge concerning near generalization, far generalization, general rule, and the inverse process. Moreover, Question 3 examined students' knowledge about near generalization, far generalization, and general rule. The questions are as follows:

Question1:

The first four steps are given in the picture below. According to these steps, find the number of squares in the 25th step. While finding the result, please draw a table and write the algebraic expression.

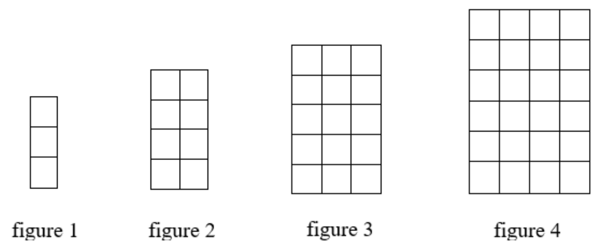


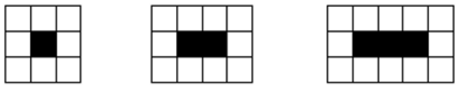
Figure 3.4. Question 1 (Radford, 2000)

In Question 1, whether students can make far generalization or not is explored. To reach a far generalization, students need to form a relationship between the number of squares and the position of the each term of the pattern.

Question 2:

Gardens are framed with a single row of tiles as illustrated below.

(A garden of length 3 requires 12 border tiles.)



length 1 length 2 length 3

a) How many border tiles are required for a garden of length 12?

b) How many border tiles are required for a garden of length n?

c) Show how to find the length of the garden if 152 tiles are used for the garden.

(Find the solutions to questions A, B and C by drawing, creating a table or using numeric or algebraic expressions.)

Figure 3.5. Question 2 (Kriegler, 2008)

In Question 2, sub-question *a* asks students to make a far generalization for the 12^{th} term in the pattern. Then, whether students can write a general rule and inverse the processes or not are investigated in sub-questions b and c, respectively.

Question 3

Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring a group enters that has two more persons than the group that entered on the previous ring,

a. How many guests will have arrived after the 5^{th} ring?
b. How many guests will have arrived after the 100^{th} ring?
c. How many guests will have arrived after the n^{th} ring?

Figure 3.6. Question 3 (Meyer & Sallee, 1983)

In Question 3, students are expected to make a near generalization in sub-question *a* and a far generalization in sub-question *b*. Moreover, the students are asked to write a general rule/formula of the pattern by exploring far generalization for the n^{th} term in sub-question *c*. The Turkish version of these questions are given in Appendix B

The questionnaire which was prepared to obtain alternative student solutions was applied to middle school students. As the objective "Students should be able to express the rule of arithmetic sequences by using letters and find the desired term of sequences expressed in letters." (6.2.1.1.) (MoNE, 2013) is in the 6^{th} grade mathematics curriculum, it was decided to apply the questionnaire to 6^{th} grade students. The accessible 6^{th} grade students were asked whether or not they would like to participate in a study. In addition to being voluntary, it was crucial to be personally convenient to participate in this study in terms of mathematics performance, positive attitude towards mathematics and so on. Based on these criteria, a questionnaire for middle school students were applied to twenty 6^{th} grade students who were enrolled in one of the public middle schools in Ankara to obtain alternative student solutions to put into questionnaire for prospective teachers.

After applying the questionnaire for middle school students, 6^{th} grade students' solutions were analyzed to prepare questionnaire for prospective students. It was realized that the students gave both correct and incorrect answers to the questions. This was an important issue since it is significant to use various solutions reflecting a range of students' understanding to investigate teachers' noticing (Jacobs et al., 2010). Students' correct and incorrect solution have crucial role to examine teachers' responses to extent students' understanding through new problems after the questions were answered correctly and to support the understandings of the students' who answered the problem incorrectly (Jacobs et al., 2010). For these reasons, two correct and three incorrect student solutions which included different important details about pattern generalization were selected to investigate prospective teachers' noticing skills of students' algebraic thinking. The solutions in the questionnaire for prospective teachers were coded as Student A's, B's, C's, D's, and E's solutions. While Student A's, C's and E's solutions were incorrect, Student B's and D's solutions were correct.

More specifically, student A could make near generalization, but s/he was mistaken in far generalization. Student B could make far generalization without using algebra. Student C made a near generalization, but s/he could not write the rule of pattern and make inverse process. Student D could make near and far generalization and write the rule of pattern perfectly. Finally, student E misunderstood the problem, so s/he was mistaken in near generalization, far generalization and writing rule of pattern. All of the students' solutions chosen for questionnaire for prospective students is stated as follows:

Student A's solution:

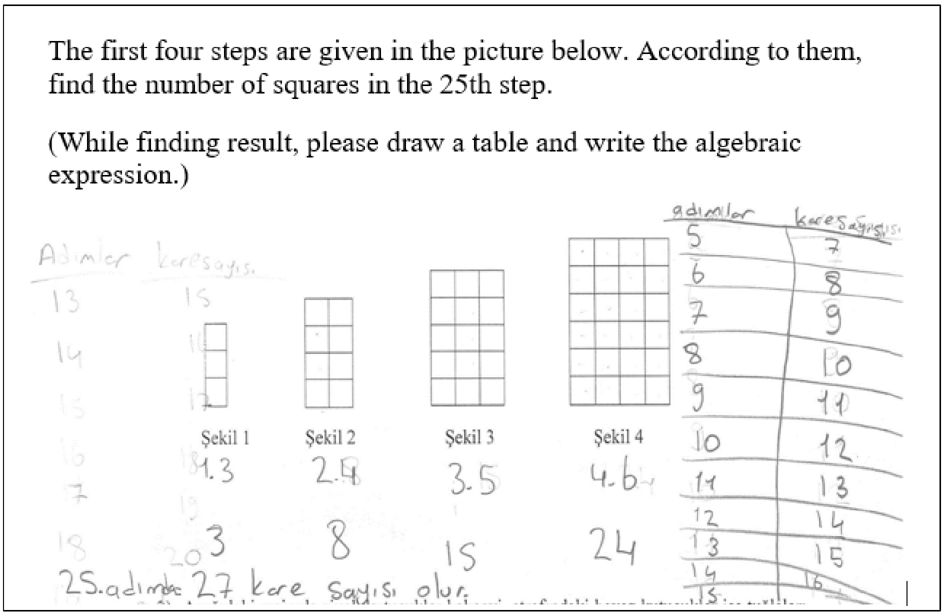
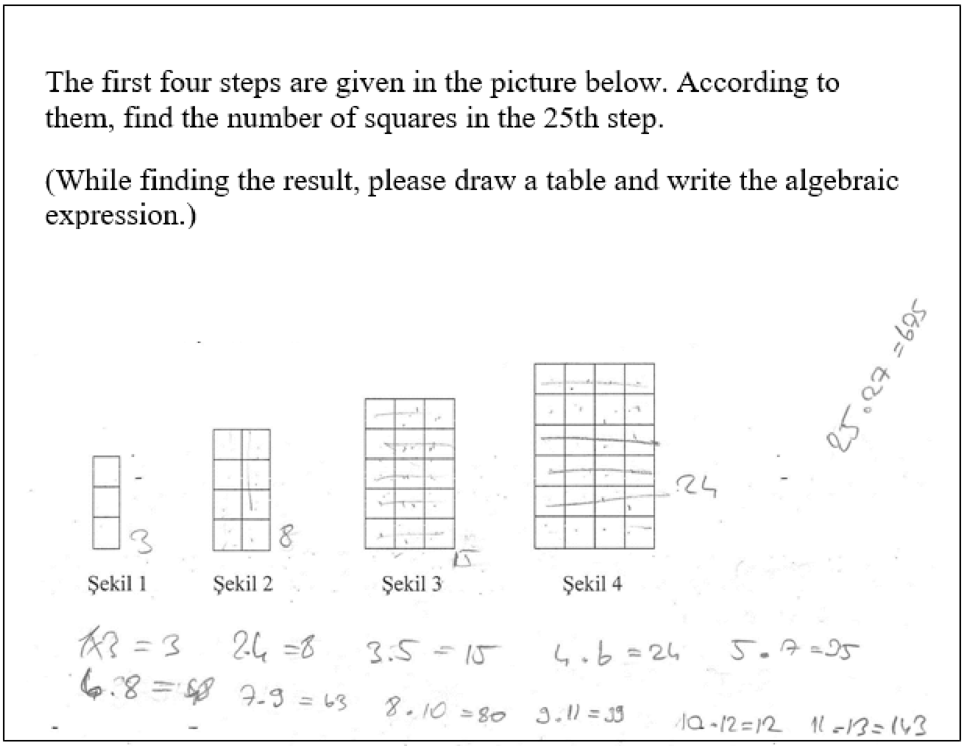


Figure 3.7. Student A's solution

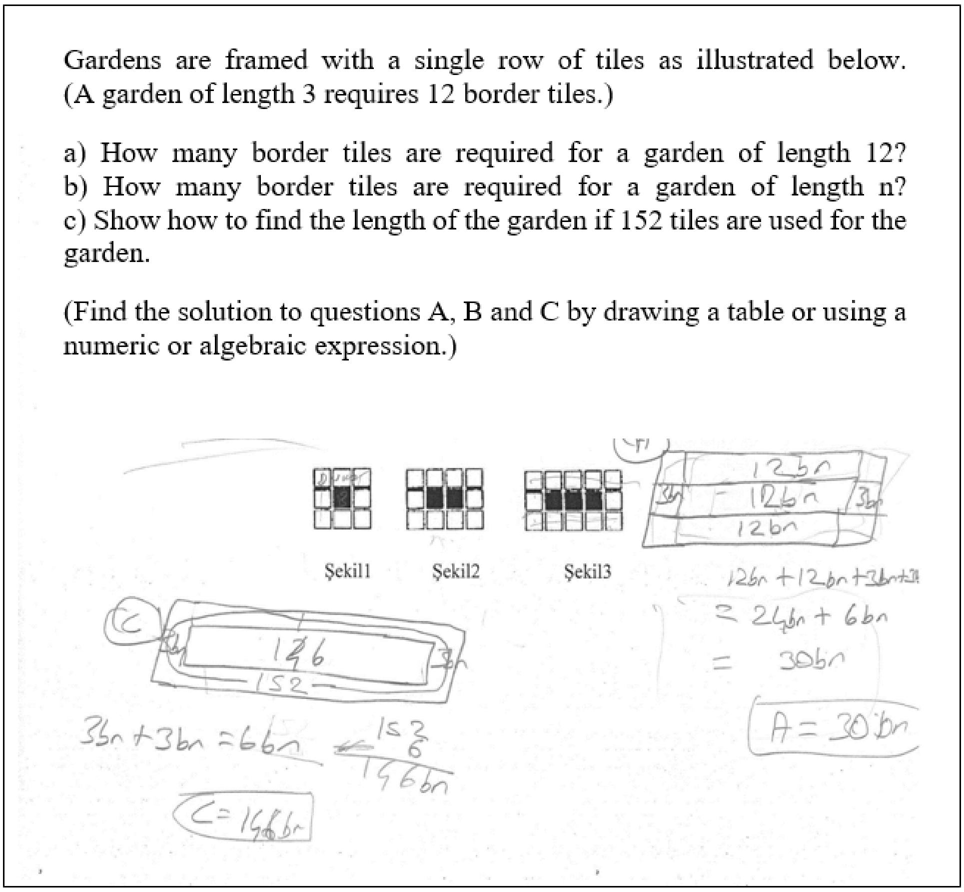
Student B's solution


The first four steps are given in the picture below. According to them, find the number of squares in the 25th step.

(While finding the result, please draw a table and write the algebraic expression.)

Figure 3.8. Student B's solution

Student C’s solution


Gardens are framed with a single row of tiles as illustrated below. (A garden of length 3 requires 12 border tiles.)

a) How many border tiles are required for a garden of length 12?
b) How many border tiles are required for a garden of length n?
c) Show how to find the length of the garden if 152 tiles are used for the garden.

(Find the solution to questions A, B and C by drawing a table or using a numeric or algebraic expression.)

Şekil1 Şekil2 Şekil3

A

12bn

12bn

12bn

3bn 3bn

12bn + 12bn + 3bn + 3bn

= 24bn + 6bn

= 30bn

A = 30bn

C

146

152

3bn 3bn

3bn + 3bn = 6bn

152 − 6 = 146bn

C = 146bn

Figure 3.9. Student C’s solution

Student D's solution

Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring a group enters that has two more persons than the group that entered on the previous ring,

a) How many guests will have arrived after the 5th ring?
b) How many guests will have arrived after the 100th ring?
c) How many guests will have arrived after the nth ring?

a) $\underset{1}{1} \overset{+2}{\frown} \underset{2}{3} \overset{+2}{\frown} \underset{3}{5} \overset{+2}{\frown} \underset{4}{7} \overset{+2}{\frown} \underset{5}{9}$

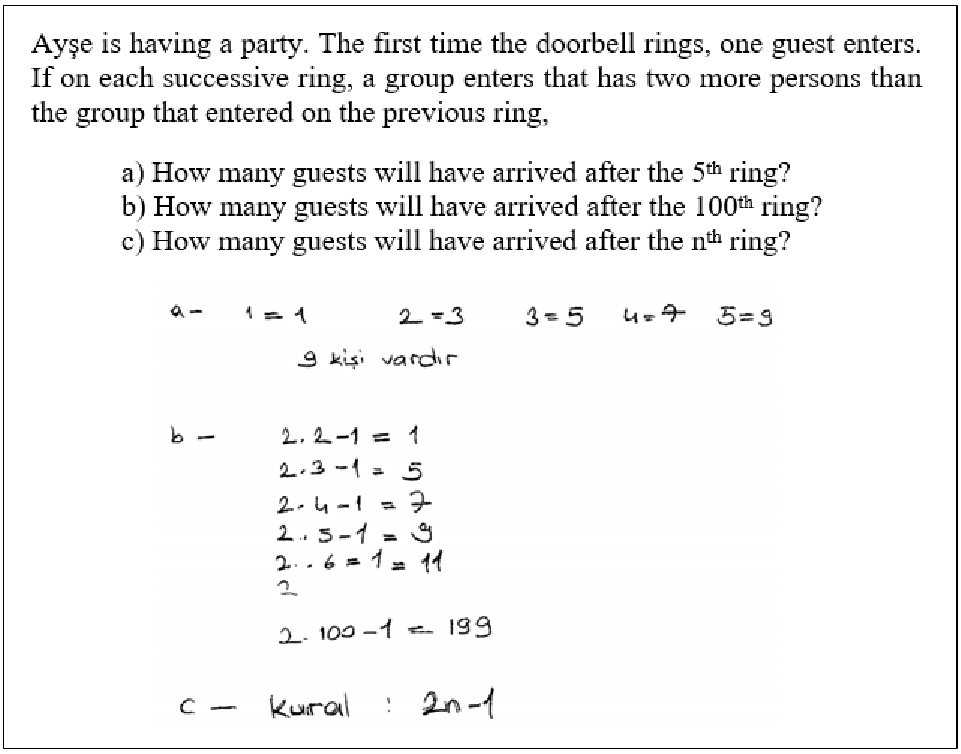
$1 + 3 + 5 + 7 + 9 = 25$

b) 2. kez toplam = 4
3. kez toplam = 9
4 kez toplam = 16
5. kez = 25
6 kez = 36
7 kez = 49
8. kez = 64
9. kez = 81
10. kez = 100
⋮
100 kez = 10000 kişi olur

c) n. kez toplam = $n.n = n^2$

Figure 3.10. Student D's solution

Student E’s solution


Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring, a group enters that has two more persons than the group that entered on the previous ring,

a) How many guests will have arrived after the 5^{th} ring?
b) How many guests will have arrived after the 100^{th} ring?
c) How many guests will have arrived after the n^{th} ring?

a- 1 = 1 2 = 3 3 = 5 4 = 7 5 = 9
9 kişi vardır

b- 2.2-1 = 1
2.3-1 = 5
2.4-1 = 7
2.5-1 = 9
2..6 = 1 = 11
2
2.100-1 = 199

c- Kural : 2n-1

Figure 3.11. Student E’s solution

3.6.2. Questionnaire for Prospective Middle School Mathematics Teachers

In order to explore prospective middle school mathematic teachers’ noticing skills, the noticing questionnaire was developed based on Jacobs, Lamb and Philipp’s (2010) framework using five students’ correct and incorrect solutions that were chosen by researcher. In the questionnaire, there were three open-ended questions with the sub-dimensions related to five different students’ solutions, which helped evaluate the noticing skills of prospective middle school mathematics teachers. The participants were expected to give answers to the open-ended questions in writing. These questions are as follows and also the Turkish version of questions is available in Appendix C.

(1) “Please explain in detail what you think each child did in response to this problem” (Jacobs, Lamb & Philipp, 2010, p.178).

(2) Please explain what you learned about these children’s understanding” (Jacobs, Lamb & Philipp, 2010, p.179).

(3) “Pretend that you are the teacher of these children. What problem or problems might you pose next?” (Jacobs, Lamb & Philipp, 2010, p.179).

These questions were prepared to identify prospective teachers’ noticing skills of students’ algebraic thinking. More specifically, the first question tries to explore to what extent prospective teachers identify students’ solutions and what they understand from their solutions; in the second question how prospective teachers make inferences and interpret students’ thinking is examined; and the third question investigates how prospective teachers respond, namely, how they overcome students’ mistakes/misconceptions or extend their knowledge.

3.6.3. Semi-structured Interview

Interview is one of the most important sources of information in case study research (Yin, 2003), and it is an essential data collection tool to obtain particular information which is not observable (Merriam, 1998). Since researchers cannot observe participant’s feelings, thoughts and intentions, they elicit information regarding participant and enter into the interviewee’s mind by asking questions (Patton, 2002).

According to Merriam (1998), interviews are categorized into three: highly-structured, semi-structured, and unstructured. In highly-structured interviews, the questions and their order are determined before the interview. In semi-structured interviews, before the interview, the questions and issues to be explored are determined; however, the order of these questions can be changed or they can be expanded according to interviewee’s answers. In this type of interview, mostly open-ended questions are preferred to obtain in-depth information about issues (Merriam, 1998). In order to ascertain information about an issue and formulate questions for subsequent interviews, unstructured interviews are preferred. This type of interviews is rarely used to gather data in qualitative research. Thus, in order to clear up and confirm prospective middle school teachers' responses in the questionnaire and draw a holistic picture of prospective teachers' noticing skills, semi-structured interviews were conducted as another data source for this study.

In the interview, researcher interacts with the participants' ideas directly with further questions. The example questions that were asked during the interview were as follows:

PST 3 said about one students' solution that "S/he could form a relationship and make a generalization." At this point, I asked her "How did you come to the conclusion that the student can generalize?" As another example, PST 5 answered the question regarding deciding how to respond on the basis of students' understanding as "I want the student to draw the 5th step of pattern and make comparison between the number of squares in the 5th step and the number of squares in the 25th step". Based on PST 5's answer, I asked the questions "What is your purpose in asking this question?" and "Will you add a new question?" Sample interview questions are given in Appendix D.

3.7. Data Collection Process

The purpose of this study was to explore the prospective teachers' noticing skills of students' algebraic thinking. In order for prospective teachers to attend to students' solutions, interpret, and respond regarding pattern generalization, different students' solutions were needed. In order to select alternative student solutions, data were collected through the questionnaire from twenty 6th grade students who were accessible by researcher. According to the results of the questionnaire for students, researcher selected five student solutions that involve both correct and incorrect solutions to put into the questionnaire for prospective mathematics teachers.

The questionnaire for prospective teachers involved three open-ended questions related to selected students' solutions, and then it was applied to 32 prospective teachers to investigate their noticing skills. Before starting to collect data, information was given to the participants about the study and they were asked whether they were voluntary or not to take part in the study. In order for the participants to complete the questionnaire, sufficient time was given and the researcher promised that the students' responses would be kept confidential and they would be shared only with the advisor and the co-advisor.

In this study, it was necessary to clarify the participants' unclear sentences and confirm their response and draw a holistic picture with the help of another data collection tool. Therefore, semi-structured interviews were conducted with eight prospective teachers. Before conducting an interview, the purpose of the study and the interview were explained to the participants. Participants were videotaped by permission and the researcher promised that nobody else would see responses or listen to their video recording of interviews except for the researcher, advisor and co-advisor. The interviews with one participant lasted approximately 75 minutes. Classrooms were preferred to conduct the interviews so that the participants could feel comfortable.

3.8. Pilot Study

According to Marshall and Rossman (2006), pilot study gives a chance to filter the instruments and rearrange them to increase participants' self-confidence and self-efficacy in conducting the research, and to notice and solve any problems regarding the research before conducting the main study. Thus, a pilot study is needed so that the main study can be conducted effectively.

3.8.1. Pilot Study in Questionnaire for Prospective Teachers

Twelve prospective teachers (4th grade students in the elementary mathematics education program) enrolled at one of the public universities located in Ankara in the

2017-2018 academic year participated in the pilot study. The criteria to choose the participants for the pilot study were similar to the criteria for selecting participants for the main study. The first criterion was accessibility so that I could easily access the participants. I chose the participants who were close enough to me to make the interview. The second criterion of the sampling procedure was that the participants completed the Methods of Teaching Mathematics I-II courses since studying these courses makes prospective teachers more knowledgeable about teaching mathematics. The final criterion was that the participants were taking the School Experience course so that they had an opportunity to observe the classroom environment and had experiences about noticing the classroom environment and students' thinking. For these reasons, in this study, prospective teachers who were easy to reach, who completed the Methods of Teaching Mathematics I-II courses, and who were taking the School Experience course were selected. In the first phase of the pilot study, the questionnaire which is related to teachers' noticing skills was applied to 12 participants in the 2017-2018 academic year. The participants responded to all the questions and wrote their answers in detail in sufficient time.

The instrument was checked through the pilot study and any problems regarding the study were revealed. As a result of the pilot study, it was concluded that the questionnaire can be applied to prospective teachers effectively for the main study.

3.8.2. Pilot Study for the Interview Phase

In the second phase, an interview was conducted with one voluntary participant as the pilot study. This participant was selected among the prospective teachers that completed questionnaire. The pilot interview lasted for nearly 100 minutes. Whether questions can be asked to participants correctly, whether the questions were clear for participants, and whether the researcher can reach her goal with these questions were discussed after conducting the interview. No changes were made in the interview schedule after the pilot study.

3.9. Data Analysis

Data analysis is the process in which order, structure, and meaning to the mass of gained data were obtained (Marshall & Rossman, 2006). According to Bogdan and Biklen (1998), "Analysis involves working with data, organizing them, breaking them into manageable units, synthesizing them, searching for patterns, discovering what is important and what is to be learned, and deciding what you will tell others." (p.157). Merriam (1998) stated when a researcher begins to collect data in qualitative research, the process of analysis of the data starts at the same time.

In order to analyze the data easily, Merriam (1998) has suggested various techniques, which are ethnographic analysis, narrative analysis, phenomenological analysis, the constant comparative method, content analysis, and analytic induction. According to Glaser and Strauss (1967), a researcher can identify a phenomenon, event or set of interest, and generate a theory through the constant comparative method. Since the purpose of this study is to investigate prospective teachers' noticing skills of students' algebraic thinking, the constant comparative method was used in order to analyze the data.

According to Lodico, Spaulding and Voegtle (2006), coding is the process of identifying different segments of the data that describe the related phenomena and labeling these parts using broad category names. Data are identified as major and minor themes while coding the data. According to Lodico et al., "Themes are typically big ideas that combine several codes in a way that allows the researcher to examine the foreshadowed questions guiding the research" (p. 307).

In the current study, the data gathered from the questionnaires and semi-structured interviews were analyzed in order to clarify the in-depth description of prospective middle school mathematics teachers' noticing skills of student' algebraic thinking. Initially, I transcribed the video recordings of the semi-structured interviews with eight prospective teachers and I read the text gained from the questionnaires and all the transcripts of the interviews. The participants' responses were categorized using the codes presented in the framework of Jacobs et al. (2010). After that, according to the similarities and differences of the participants' responses, some changes in the codes were made; new codes were added; some categories were divided into subcategories after discussing with advisor and co-advisor. In order to ensure the inter-rater reliability, a doctoral student at Mathematics Education, who has knowledge and experience in the construct of teacher noticing, coded the data as a co-coder Afterwards, the researcher's coding and the co-coder's coding were compared in order to see commonalities and differences. The interrater reliability was calculated about 93% by using formula suggested by Miles and Huberman (1994). The inconsistencies were discussed one more time, the necessary changes were made and finally consensus was reached. As a consequence, this study included three dimensions for analysis which are *attending to students' solutions*, *interpreting students' algebraic thinking*, and *deciding how to respond on the basis of students' algebraic thinking* by modifying Jacobs et al.'s (2010) professional noticing of children's mathematical thinking framework. Based on this modifications, Jacobs et al.'s framework was extended through this study. The categories and the codes that were added to the framework are explained in detail below.

The first component of professional teacher noticing, which is attending to students' solutions, includes two categories: evidence of attending and lack of evidence of attending (Jacobs et al., 2010). However, in the present study, some responses could not be categorized under evidence of attending or lack of evidence. Thus, in order to represent all prospective teachers' responses, two more categories which are emerging evidence and limited evidence of attending to students' solutions were added considering the common characteristics of responses. As a result, prospective mathematics teachers' noticing skills of *attending to students' solutions* were coded into four categories: *robust evidence*, *emerging evidence*, *limited evidence,* and *lack of evidence of attending to students' solutions*. While the prospective teachers who provide robust evidence correctly identify all the mathematical concepts of the students' solutions and focus on all mathematical details, the participants with responses demonstrating emerging evidence of attention to students' solutions correctly identify their solutions, but they do not capture all the mathematical details. The participants who provide limited evidence of attention to students' solutions make comments reflecting the general features of the solutions and these responses do not provide details about how the problem was solved. Finally, in responses demonstrating lack evidence of attention to students' solutions, the participants incorrectly identify students' solutions. According to the coding of the dimension of *attending to students' solutions* and the mathematical details for each student solution, the characteristic features of each category of the dimension are given in Table 3.2. below.

Table 3.2. *The Features of the Categories in Attending to Students' Solutions Dimension*

Details of solution	**Features of robust evidence of attending to students' solutions**	**Features of emerging evidence of attending to students' solutions***	**Features of limited evidence of attending to students' solutions***	**Features of lack of evidence of attending to students' solutions**
Student A's solution	The prospective teacher states how the student finds the number of squares in the first four steps and makes mistakes in creating the table.	The prospective teacher states how the student finds the number of squares in the first four steps, but does not describe student's mistake in creating the table.	The prospective teacher realizes how the student solves the problem incorrectly and describes student's solution shortly.	The prospective teacher describes student's solution as correct.
Student B's solution	The prospective teacher states how the student finds the number of squares in the first four steps and find the 25^{th} figure.	The prospective teacher states how the student finds the number of squares in the first four steps, but does not describe how student concludes the solution.	The prospective teacher realizes how the student solves the problem correctly and describes the student's solution shortly.	The prospective teacher describes student's solution as wrong.
Student C's solution	The prospective teacher states how the student finds the number of bricks for the garden whose length is 12 units, however; s/he does not find the number of bricks for the garden whose length is n units and the length of the garden in which 152 bricks were used.	The prospective teacher states how the student finds the number of bricks for the garden whose length is 12 units. However, s/he does not describe how the student finds the number of bricks for the garden hose length is n units and the length of the garden in which 152 bricks were used.	The prospective teacher realizes that the student solves sub-question a correctly, but solves sub-question b and c incorrectly and describes student's solution shortly.	The prospective teacher describes student's solution in sub-question a as incorrect or student's solution in sub-questions b and c as correct.

Table 3.2. *The Features of the Categories in Attending to Students' Solutions Dimension (continued)*

Details of solution	**Features of robust evidence of attending to students' solutions**	**Features of emerging evidence of attending to students' solutions***	**Features of limited evidence of attending to students' solutions***	**Features of lack of evidence of attending to students' solutions**
Student D's solution	The prospective teacher describes how the student finds the number of people who are at home in the 5^{th}, 100^{th} and n^{th} rings step by step.	The prospective teacher describes how the student finds the number of people who were at home in 5^{th} ring, but does not describe how the student finds the number of people in the 100^{th} and n^{th} ring.	The prospective teacher realizes that the student solves the problem correctly and describes the student's solution shortly.	The prospective teacher describes the student's solution as wrong.
Student E's solution	The prospective teacher describes how the student incorrectly finds the number of people who were at home in the 5^{th}, 100^{th} and n^{th} rings step by step.	The prospective teacher describes how the student incorrectly finds the number of people who were at home in the 5^{th} ring, but does not describe how the student finds incorrectly the number of people in the 100^{th} and n^{th} rings.	The prospective teacher realizes that the student solves the problem incorrectly and describes the student's solution shortly.	The prospective teacher describes the student's solution as correct.

*Jacobs et al.'s (2010) framework was extended by adding categories to attending to students' solutions dimension.

The second component of professional teacher noticing- interpreting students' algebraic thinking- is coded under three categories which are robust evidence, limited evidence, and lack of evidence (Jacobs et al., 2010). However, in this study, some participants' responses did not correspond to the characteristics of robust evidence or limited evidence and thus, there was a need to add one category, which is named as emerging evidence, between robust and limited evidence. As a consequence, prospective mathematics teachers' skills of *interpreting students' algebraic thinking* were coded into four categories: *robust evidence, emerging evidence, limited evidence, and lack of evidence*. The participants' responses provided robust evidence of interpretation of students' algebraic thinking if the participants made sense of the details of students' solutions. In responses with emerging evidence, prospective teachers made interpretation about students' algebraic thinking but with less detail than the responses including robust evidence. Responses that included comments about only whether students comprehended the topic or not were considered to have limited evidence of interpretation of students' algebraic thinking. Finally, responses that provide wrong evidence of interpretation of students' thinking or irrelevant comments on students' thinking are placed into the lack evidence category. Moreover, some participants focused on attending to students' strategies instead of interpreting students' algebraic thinking. These responses were coded as *no interpretation, just attention*. According to the coding of interpreting students' algebraic thinking and mathematical essences in each student's algebraic thinking, the characteristic features of each category is given in Table 3.3. below.

Table 3.3. *The Features of the Categories in Interpreting Students' Algebraic Thinking Dimension*

Details of solution	Features of robust evidence of interpreting students' algebraic thinking	Features of emerging evidence of interpreting students' algebraic thinking*	Features of limited evidence of interpreting students' algebraic thinking	Features of lack of evidence of interpreting students' algebraic thinking
Student A's solution	The prospective teacher interprets that the student explores the relationship between the number of squares and the number of rows and columns and cannot fill the table according to this relationship.	The prospective teacher either interprets that the student explores the relationship between the number of squares and the number of rows and columns, or interprets that the student makes mistake while filling the table.	The prospective teacher interprets only that the student cannot comprehend the pattern generalization well, but does not refer to the specific points regarding the student's algebraic thinking.	The prospective teacher makes wrong or irrelevant interpretation about the student's algebraic thinking.
Student B's solution	The prospective teacher interprets that the student explores the relationship between the number of squares and the number of rows and columns and makes the generalization of pattern.	The prospective teacher either interprets that the student explores the relationship between the number of squares and the number of rows and columns or interprets that the student makes the generalization of pattern.	The prospective teacher interprets only that the student comprehends the pattern generalization perfectly, but does not refer to the specific points regarding the student's algebraic thinking.	The prospective teacher makes wrong or irrelevant interpretation about student's algebraic thinking.

Table 3.3. *The Features of the Categories in Interpreting of Students' Algebraic Thinking Dimension (continued)*

Details of solution	**Features of robust evidence of interpreting students' algebraic thinking**	**Features of emerging evidence of interpreting students' algebraic thinking***	**Features of limited evidence of interpreting students' algebraic thinking**	**Features of lack of evidence of interpreting students' algebraic thinking**
Student C's solution	The prospective teacher interprets that student explores the relationship between the long length of garden and the number of bricks, but s/he cannot make the generalization of pattern.	The prospective teacher either interprets that the student explores the relationship between the long length of the garden and the number of bricks, or interprets that s/he cannot make generalization of pattern.	The prospective teacher interprets only that the student cannot comprehend the pattern generalization, but does not refer to the specific points regarding the student's algebraic thinking.	The prospective teacher makes wrong or irrelevant interpretation about student's algebraic thinking.
Student D's solution	The prospective teacher interprets that the student explores the relationship between the number of rings and the number of people who were at home and makes the generalization of pattern.	The prospective teacher either interprets that the student explores the relationship between the number of rings and the number of people who were at home or interprets that the student makes generalization of pattern.	The prospective teacher only considers that the student comprehends the pattern generalization perfectly, but does not refer to specific points regarding the student's algebraic thinking.	The prospective teacher makes wrong or irrelevant interpretation about the student's algebraic thinking.

Table 3.3. *The Features of the Categories in Interpreting of Students' Algebraic Thinking Dimension (continued)*

Details of solution	Features of robust evidence of interpreting students' algebraic thinking	Features of emerging evidence of interpreting students' algebraic thinking*	Features of limited evidence of interpreting students' algebraic thinking	Features of lack of evidence of interpreting students' algebraic thinking
Student E's solution	The prospective teacher interprets that the student wrongly explores the relationship between the number of rings and the number of people who were at home and so makes wrong generalization of pattern.	The prospective teacher either interprets that the student wrongly explores the relationship between the number of rings and the number of the people who were at home or interprets that the student makes generalization of pattern wrongly.	The prospective teacher interprets that the student comprehends the pattern generalization, but s/he cannot understand the problem. However, s/he does not refer to the specific points regarding the student's algebraic thinking.	The prospective teacher makes wrong or irrelevant interpretation about the student's algebraic thinking.

* Jacobs et al.'s (2010) framework was extended by adding the category to interpreting students' algebraic thinking dimension.

Furthermore, the third component of professional teacher noticing -deciding how to respond- includes three categories which are robust evidence, limited evidence, and lack of evidence (Jacobs et al., 2010). However, in the current study, prospective teachers responded to students in three different ways. They provided responses by extending/supporting students' algebraic thinking, asking a drill, and providing a general response. Therefore, prospective teachers' skills of deciding how to respond were categorized differently compared to Jacobs et al.'s framework. Hence, the data related to the participants' skill of *deciding how to respond on the basis of their algebraic thinking* were coded under three categories: *extending/supporting students'* *algebraic thinking*, *asking a drill as a response*, and *providing a general response*. When the prospective teachers gave a response in order to extend or support students' existing algebraic thinking, these responses were considered to be in the category of extending/supporting students' algebraic thinking. Responses in which participants asked students to do a drill without extending/supporting their algebraic thinking were coded as asking to do a drill as a response. When the teachers did not consider students' thinking and suggested direct instruction, these responses were coded as providing a general response to students' algebraic thinking.

After ensuring the inter-rater reliability and modifying Jacobs et al.'s (2010) professional noticing of children's mathematical thinking framework with respect to the data of the current study, the characteristic features of each category of three dimensions were clarified. Afterwards, each participant's responses were coded for each dimension based on these features. In order to see the whole picture of prospective teachers' noticing skills of attending, interpreting and responding, the frequency analysis was conducted by calculating the percentage of prospective teachers' responses for each category of three dimensions. The frequency table for each category was given separately in the related part of the findings section. Another important issue for the validity and reliability of the study is trustworthiness which is discussed in the following section.

3.10. Trustworthiness

According to Merriam (2009), whether data is valid and reliable affects the trustworthiness of the research study. For this reason, researchers should consider validity and reliability issues while they are planning the study, analyzing the data, and reasoning the quality of the study regardless of the type of research (Patton, 2002). In quantitative research designs, validity is defined as "referring to the appropriateness, correctness, meaningfulness, and usefulness of the specific inferences researchers make based on the data they collect" (Fraenkel & Wallen, 2006, p. 151) and reliability refers to "the consistency of the scores obtained-how consistent they are for each individual from one administration of an instrument to another and from one set of items to another" (Fraenkel & Wallen, 2006, p. 157). However, these issues are discussed with different terminologies in qualitative research which are credibility, dependability, transferability, and confirmability instead of using the terms internal validity, reliability, external validity, and objectivity (Lincoln & Guba, 1985). Lincoln and Guba (1985) stated that the terms of credibility, dependability, transferability, and confirmability form the trustworthiness of the research design and refer to the quality of the qualitative research.

3.10.1. Credibility

Credibility in qualitative research refers to internal validity in quantitative research which is important criteria to ensure the trustworthiness of the research design (Merriam, 2009; Lincoln & Guba, 1985). According to Merriam (2009), credibility involves the questions of "How congruent are the findings with reality? Do the findings capture what is really there? Are investigators observing or measuring what they think they are measuring?" (p. 213). Although approaching the term of "truth" and "reality" objectively is very difficult for qualitative researchers, there are six strategies suggested to ensure credibility, which are triangulation, member checks, long-term observation, peer-examination or peer debriefing, participatory or collaborative modes of research and the researcher's bias (Merriam, 1998, 2009). In the present study, triangulation, member checks and peer-examination were employed and the researcher's bias was taken into consideration to assure credibility.

One of the strategies used in order to establish credibility is triangulation, which is the "the most well-known strategy to shore up the internal validity of a study (Merriam, 2009). Triangulation is defined by Creswell and Miller (2000) as "a validity procedure where researchers look for convergence among multiple and different sources of information to form themes or categories in a study" (p. 126). According to the literature, there are four types of triangulation which are data triangulation, investigator triangulation, theory triangulation, and methodological triangulation

(Creswell, 2007; Patton, 2002). In the present study, data triangulation, methodological triangulation and investigator triangulation were used to ensure credibility. To make data triangulation, I studied with eight prospective middle school teachers, more than one participant. Furthermore, the data was collected through multiple sources including the questionnaire and semi-structured interviews; thus, methodological triangulation was achieved. In order to increase the credibility of the research study, the investigator triangulation method was applied by analyzing and interpreting the data with more than one researcher. The data was coded by the researcher and a co-coder, and also the coding was reviewed by the advisor and the co-advisor in order to achieve investigator triangulation.

Moreover, member check was used to increase the credibility of the study. According to Merriam (1998), member check enables participants to check the consistency between their responses and researcher’s interpretations. In order to ensure member check, I discussed the participants’ answers during semi-structured interviews in order to confirm their responses. In this way, I checked whether I interpreted the prospective mathematics teachers’ responses correctly or not. Additionally, peer examination was performed to increase the credibility of the study. Peer examination is defined as an examination or review which can be “conducted by a colleague either familiar with the research or one new to the topic" (Merriam, 2009, p.220). In the present study, I studied with one doctoral student in the mathematics education department, who is currently conducting studies related to teachers’ noticing. She took part in the coding and categorization process as a co-coder. I and the co-coder made analysis separately during the process of data analysis and we compared our interpretations, discussed if any inconsistencies occurred, and achieved full-consensus. Additionally, I studied with my advisor and co-advisor regularly. They scanned the data and we discussed the findings of study.

3.10.2. Dependability

Another concern which makes a contribution to the trustworthiness of the research design is dependability which corresponds to reliability in quantitative study. Merriam (1998) defined reliability as "the extent to which research findings can be replicated" (p. 220). However, in the qualitative research, reaching the same result repeatedly is not possible due to the nature of the qualitative research design. For this reason, dependability in qualitative study means finding results that are dependable and consistent with the data (Merriam, 1998). According to the related literature, some strategies such as triangulation and investigator's position are suggested by researchers to assure dependability (Merriam, 1998; Patton, 2002). Triangulation, one of the strategies that was used to increase credibility, establishes dependability of the study at the same time (Merriam, 1998). Data triangulation, investigator triangulation, and methodological triangulation were performed in the present study as discussed above. Another issue to increase the dependability of the research is researcher's position and in order to ensure this, the theory behind the study, participants' selection and the context of the study is need to be explained (Merriam, 1998). Additionally, in order to ensure the dependability of the study, how researcher designs research and how s/he collects, analyzes and interprets the data are need to be discussed clearly (Merriam, 1998). To ensure the dependability of this study, all of these issues were discussed in the methodology part. In addition, triangulation and researcher's position were ensured.

3.10.3. Transferability

Transferability, which is another significant criterion to ensure trustworthiness in qualitative studies, refers to external validity in quantitative research design. The issue of transferability is completely related to whether the findings of the research study can be generalized or not. As making inferences from a small sample and generalizing those to a larger population are not the aims of the qualitative research, transferability is established by giving thick description of the study and conducting the research with sufficient data (Merriam, 1998). In the current study, an in-depth description regarding the study was given by discussing the context of the study, sample selection, data collection tools, data analysis procedures and time schedule in the methodology part. Moreover, in order to reach sufficient data, 32 participants were applied questionnaires, and semi-structured interviews were conducted with eight participants. By collecting the data through different tools, the transferability of the study was increased. Shenton (2004) stated that “it is the responsibility of the investigator to ensure that sufficient contextual information about the fieldwork sites is provided to enable the reader to make such a transfer” (p. 69). Sufficient data and thick description of the study allows researchers to express the findings of the study to other researchers and mathematics educators easily.

3.10.4. Confirmability

The last criterion to ensure the trustworthiness of the qualitative research is confirmability, which corresponds to objectivity in quantitative research. According to Shenton (2004), the findings of a study should be based on participants’ views and experiences and should not be affected by researchers’ characteristics. Shenton (2004) and Lincoln and Guba (1985) proposed strategies to decrease the researcher’s bias so that confirmability can be ensured. One of these strategies to reduce the effects of investigator’s bias is triangulation. Another strategy to reduce the effects of investigator’s bias is elaborative description of the methodology of the research study, and the final strategy is explaining researchers’ roles. Thus, in this study, conformability was ensured through triangulation, the detailed description of the methodology of the study, and the explanation of the researcher role.

3.11. Researcher Role and Bias

In qualitative studies, researcher is an important role for collecting data and analyzing them (Merriam, 1998). Researcher can analyze the data and find the results according to his/her wishes, perspectives and views (Johnson, 1997). Thus, researcher bias is a potential threat to validity since "…qualitative research is open-ended and less structured than quantitative research" (Johnson, 1997, p. 284). In this sense, Merriam said that "Rather than trying to eliminate these biases or subjectivities, it is important to identify them and monitor them as to how they may be shaping the collection and interpretation of data" (2009, p.15). In the rest of this part, the attempts to identify and reduce my biases and my role as a researcher which might have some effects on collection and interpretation of the data were clarified.

Firstly, in order to investigate prospective teachers' noticing of students' algebraic thinking, I selected five students' solutions based on different properties of solutions. The first criterion was selecting both correct and incorrect solutions to investigate prospective students' noticing skills more comprehensively. I believed that, as also emphasized in the study of Chick, Baker, Pham and Cheng (2006), investigating teachers' noticing through students' correct solutions will only provide attending the steps of students' solutions and interpreting students' understanding based on the important issues of the related subject. However, as stated in the previous studies (Crespo, 2002; Ma, 1999), incorrect students' solutions require identifying in which step students made an error and what the reasons for making these errors are. In order to notice students' incorrect solutions, teachers have to attend students' errors/difficulties/misconceptions, to interpret students' understanding which were the reasons for their errors and to decide how they can support students' understanding. For these reasons, as a researcher, I preferred to select incorrect solutions as well as correct solutions which might present more comprehensive findings related to prospective teachers' noticing skills. The second criterion for selecting these solutions was the diversity of the solutions in terms of the critical issues related to pattern generalizations. The important issues of the pattern generalization are near generalization, far generalization, writing a rule of pattern and inverse process. In detail, student A's solution included mistake in far generalization and student C's solution included a mistake in writing the rule of pattern and making an inverse process. Moreover, in student E's solution, student had misunderstanding and mistakes in near generalization, far generalization and writing a rule of pattern. Whereas, student B's and student D's solutions were chosen as correct solutions that were solved differently. Student B could make a generalization correctly, but s/he ignored writing rule of pattern. Student D could make near generalization, far generalization and write the rule of pattern perfectly. Since each student solution involves different mathematical details, student A's, student B's, student C's, student D's and student E's solutions were selected. Hence, my preferences of students' solutions affected questions which were asked to prospective teachers.

Moreover, before the study, I explained the aim of my study and the process of data collection to the prospective teachers. Furthermore, while they were answering the questionnaire, I was flexible about the duration of completion of the questionnaire and I ensured that they completed a task without feeling pressure. Moreover, interview times were arranged according to participants' suitability. At the beginning of the interviews, I had a talk with the participants in order to make them feel comfortable. I emphasized that their deep explanations about their thinking are important for me as the researcher. Also, I explained that there is no correct answer for questions in these interviews.

Briefly, I aimed to reduce researcher bias by giving information about the aim of the research and the data collection process transparently to the prospective teachers, collecting data from voluntary participants, being flexible towards participants during the data gathering process, and checking my understanding of their responses with the help of triangulation.

3.12. Ethical Considerations

In order to carry out the research, permission was received from the Ethics Committee at METU to apply the questionnaire and conduct the semi-structured interviews. The approval of the committee is presented in Appendix E. They declared that this study does not damage prospective middle school mathematics teachers. Additionally, I talked with the prospective teachers and asked whether they were voluntary or not to take part in this study. Then, I noted the names of the prospective teachers who were willing to participate in the present study.

Frankel and Wallen (2006) mentioned three essential concerns regarding ethics in research: avoiding the deception of subjects, protecting participants from harm, and ensuring the confidentiality of the research data. “It is a fundamental responsibility of every researcher to do all in his or her power to ensure that participants in a research study are protected from physical or psychological harm, discomfort, or danger that may arise due to research procedures” (Fraenkel & Wallen, 2006, p.56). For this reason, in this study, I ensured all the participants that there would no damage in the process of research and their rights would be protected. Moreover, once data is collected in a study, researchers should ensure that no one else has access to the data except for the researchers in the study (Fraenkel & Wallen, 2006). From this point of view, I informed them that their personal information, their responses to the questionnaire and their video recordings of semi-structured interviews were confidential and would not be shared with anybody except my advisor and my co-advisor. Additionally, I informed the participants that in order to ensure confidentially, I would give all the participants pseudonyms such as PST1, PST2, PST3 and so on. Finally, I notified them that they could withdraw from the research if they did not want to continue.

3.13. Limitations of the Study

In this study, in order to gather data, participants were selected via purposive sampling and senior prospective teachers who took the Methods of Teaching Mathematics I/II courses and were taking School Experience course in the teacher education program of one public university took part in this study. For this reason, the findings of this study were limited with the responses of participants matching these criteria. Moreover, prospective teachers’ professional noticing skills of students’ understanding were investigated within the scope of pattern generalization. Thus, the objective “Students should be able to express the rule of arithmetic sequences by using letters and find the desired term of sequences expressed in letters.” (6.2.1.1.) (MoNE, 2013, p.18) was the only focus of the study, and the context of the study was also limited. The readers should take these aspects into consideration while assessing the findings of the study.

The second limitation is about the data collection tools of the study. Merriam (1998) stated that “observational data represents a firsthand encounter with the phenomenon of interest rather than a secondhand account of the world obtained in the interview” (p.94). Although in the present study in-depth exploration of the case was ensured through the questionnaire and semi-structured interview, no observations were conducted. If I could have a chance to observe the participants’ behaviors in real classroom environment, then I could have reached a full description of the prospective teachers’ noticing skills of students’ algebraic thinking.

Finally, in this study, prospective teachers’ noticing of students’ algebraic thinking was investigated based on students’ written responses. However, if prospective teachers had an opportunity to watch the video of lecture or observe students’ problem solving process, they might notice students’ algebraic thinking process differently and more efficiently within the context of pattern generalization. For instance, prospective teachers might notice whether or not the students connect their previous knowledge with the knowledge of pattern generalization, whether or not they had difficulty in solving such a problem or how students begin to solve the problem and reach the solution. However, since I preferred to investigate prospective teachers’ noticing skills through students’ written work, the findings of the study are limited in terms of the data collection tool.

CHAPTER 4

FINDINGS

The aim of this study was to examine the prospective middle school mathematics teachers' noticing skills of students' algebraic thinking within the context of pattern generalization. This chapter presents the findings of the research study under three main dimensions which are the dimensions of professional noticing of children's mathematical thinking framework of Jacobs, Lamb and Philipp (2010): attending, interpreting and deciding how to respond. Firstly, the findings related to the first dimension, which is prospective teachers' attending to students' solutions in pattern generalization, is presented. Secondly, the findings related to the second dimension, that is, prospective teachers' interpreting students' algebraic thinking within the context of pattern generalization is summarized. Finally, the findings regarding the third dimension, which is prospective teachers' deciding how to respond on the basis of students' algebraic thinking, is presented. In this chapter, for each of these three dimensions, the coding, properties of students' solutions, the frequency tables and detailed information is presented through the quotations taken from the questionnaires and the interviews, respectively. The Turkish version of the participants' responses of each category in three dimensions are given in Appendix F.

4.1. Prospective Teachers' Attending to Students' Solutions

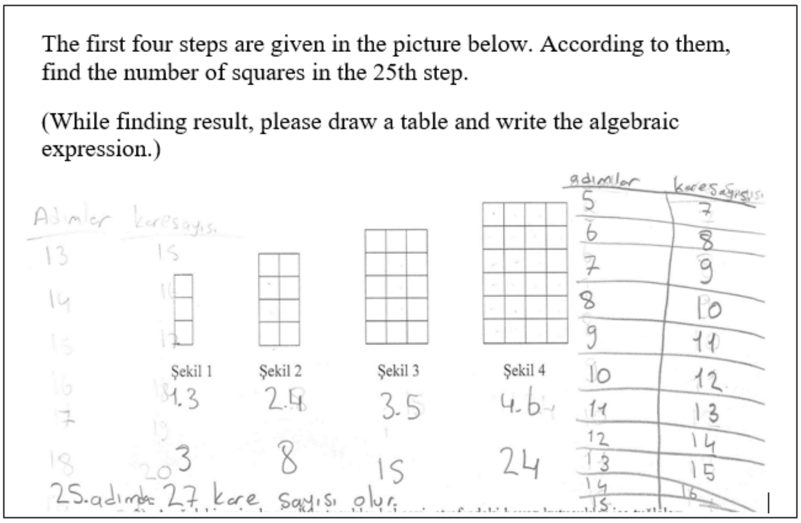
The first dimension of professional teacher noticing -teachers' attending to students' solutions- refers to being aware of their solution ways and the mathematical details in students' solutions. Teachers can window into students' thinking by paying attention to students' solutions. In this study, based on the analysis of the data, the evidence of prospective teachers' attention to students' solutions was classified under four categories: robust evidence of attention to students' solutions, emerging evidence of attention to students' solutions, limited evidence of attention to students' solutions, and lack of evidence of attention students' solutions. Prospective teachers whose responses show robust evidence of attention to students' solutions correctly identify all the mathematical details in students' solutions. In responses demonstrating emerging evidence of attention to students' solutions, prospective teachers correctly identify students' solutions; however, they cannot capture all mathematical details. Responses providing limited evidence of attention to students' solutions include comments which reflect the general features of the solutions, and these responses do not provide details about how the problem was solved. Finally, prospective teachers with responses showing lack evidence of attention to students' solutions incorrectly identify how the problem was solved. The categorization of prospective teachers' attending to students' solutions is summarized in Table 4.1 below.

Table 4.1. *The Coding of Attending to Students' Solutions*

Attending to Students' Solutions	**Coding**
Robust Evidence of Attention to Students' Solutions	Correctly identify students' entire solutions with all the mathematical details.
Emerging Evidence of Attention to Students' Solutions	Correctly identify students' solutions, but not capture all mathematical details.
Limited Evidence of Attention to Students' Solutions	Make comments reflecting the general features of the solutions. Provide no details about how the problem was solved.
Lack of Evidence of Attention to Students' Solutions	Incorrectly identify how the problem was solved.

To determine whether prospective teachers attended to the details of students' strategies or not, firstly the mathematical details of each student's solution were decided. Student A's solution and mathematical details for this solution are:

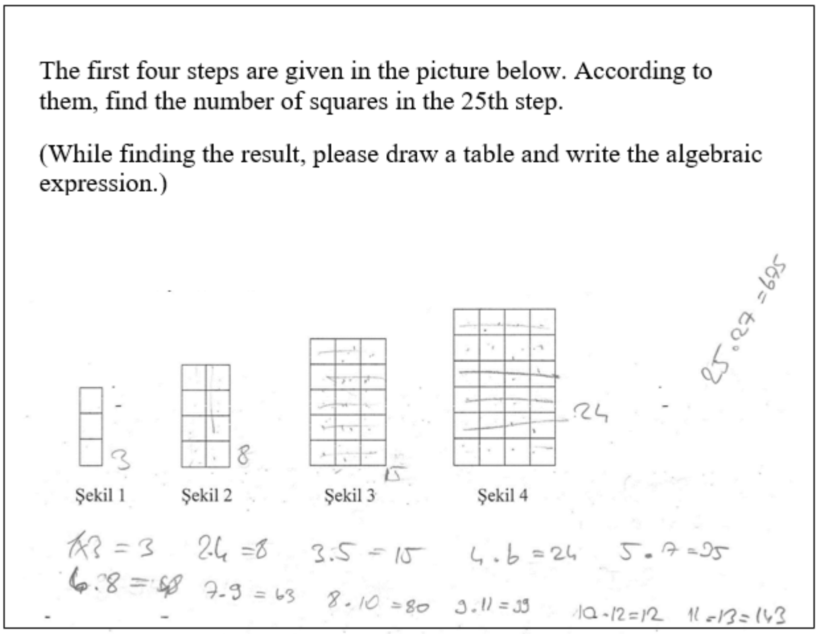
Student A’s solution



Student A’s solution is wrong. As it can be seen, the student firstly finds the number of squares in the first four figures by using the relation between the number of squares and the number of columns and rows. More specifically, s/he finds the number of squares in Figure 1 by multiplying 1 by 3, by multiplying 2 by 4 in Figure 2, by multiplying 3 by 5 in Figure 3, and by multiplying 4 by 6 in Figure 4. Then, s/he fills the table for further steps wrongly and adds 2 to 25, and then reaches an incorrect answer, which is 27 squares.

Student B’s solution and mathematical details for this solution are as follows:

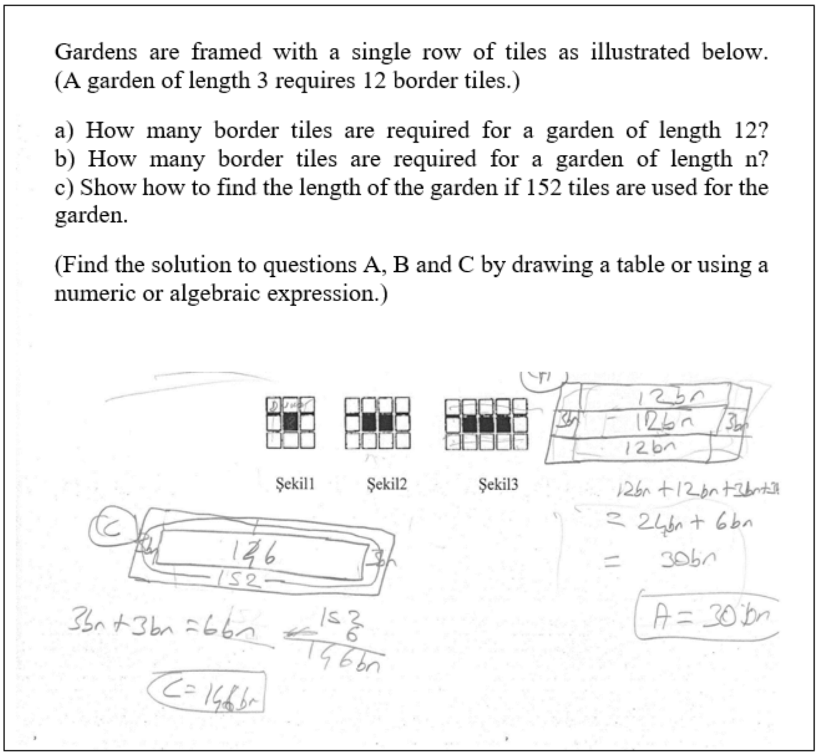
Student B's Solution



Student B solves the problem correctly. At the begining, the student finds the number of squares in the first four figures by using the relation between the number of squares and the number of columns and rows. More specifically, s/he finds the number of squares in Figure 1 by multiplying 1 by 3, in Figure 2 by multiplying 2 by 4, in Figure 3 by multiplying 3 by 5, and in Figure 4 by multiplying 4 by 6. After that, s/he finds the number of total squares for Figure 5 to 11 by writing as in the first four figures and multiplies 25 with 27 and reaches the correct answer, which is 675 squares.

Student C's solution and mathematical details for this solution are as follows:

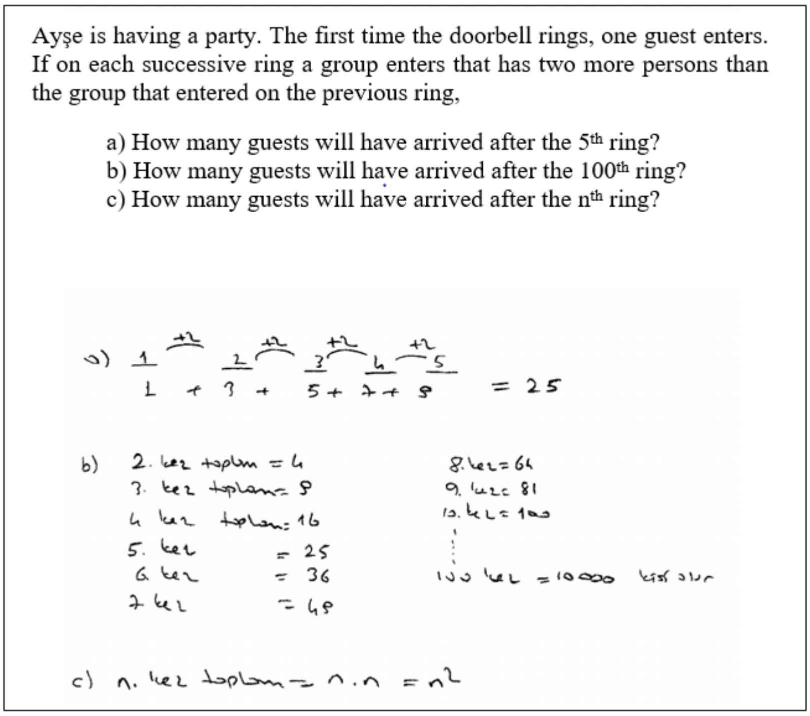
Student C’s solution



Although Student C solves the sub-question a correctly, s/he cannot solve the sub-question b and sub-question c correctly. As it can be seen, the student draws a picture for the 12th step by looking at the figures in the first three steps, and according to the picture, the student puts 12 bricks for each long length of garden and 3 bricks for each short length. Then, s/he makes operations and obtains the correct answer as 30 bricks. However, the student writes only 4n to find the number of bricks for the garden in which the long length is n units, which leads to the incorrect answer. After that, for sub-question c, s/he subtracts 6 from 152, however, does not divide 152 by 2, and thus gets the incorrect answer.

Student D’s solution and mathematical details for this solution are as follows:

Student D's solution



Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring a group enters that has two more persons than the group that entered on the previous ring,

a) How many guests will have arrived after the 5^{th} ring?
b) How many guests will have arrived after the 100^{th} ring?
c) How many guests will have arrived after the n^{th} ring?

a) 1 +2 2 +2 3 +2 4 +2 5
1 + 3 + 5 + 7 + 9 = 25

b) 2. kez toplam = 4
3. kez toplam = 9
4 kez toplam = 16
5. kez = 25
6 kez = 36
7 kez = 49
8. kez = 64
9. kez = 81
10. kez = 100
⋮
100 kez = 10000 kişi olur

c) n. kez toplam = n.n = n^2

Student D's solution is correct. In this solution, the student finds the number of people who are at home in the second, third, fourth, and fifth rings by adding 3 to 1, 5 to 4, 7 to 9, and 9 to 16 respectively, and then obtains the correct answer as 25. After that, s/he makes a list for the first 10 rings and gets the square of 10, and then finds the correct answer for 100 people. Finally, the student multiplies n by n in order to find the number of people who are at home in the n^{th} ring.

Student E's solution and mathematical details for this solution are as follows:

Student E's solution

Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring, a group enters that has two more persons than the group that entered on the previous ring,

a) How many guests will have arrived after the 5th ring?
b) How many guests will have arrived after the 100th ring?
c) How many guests will have arrived after the nth ring?

a- 1 = 1 2 = 3 3 = 5 4 = 7 5 = 9

9 kişi vardır

b – 2.2 − 1 = 1
2.3 − 1 = 5
2.4 − 1 = 7
2.5 − 1 = 9
2..6 = 1 = 11
2
2.100 − 1 = 199

c – Kural : 2n-1

Student E's solution is wrong. As it can be seen, the student writes the number of people who are at home in the second ring as 3, the number of people who are at home in the third ring as 5, the number of people who are at home in the fourth ring as 7, and the number of people who are at home in the fifth ring as 9, and finally reaches a wrong answer. Then, the student makes a list the number of people who are at home for the first 6 rings. S/he multiplies 2 by 100, and then subtracts 1, and then concludes 199 as the number of people who are at home in the 100^{th} ring, but the result is incorrect. Finally, s/he multiplies 2 by n, and then subtracts 1 to find the number of people who are at home in the n^{th} ring and reaches an incorrect answer.

Table 4.2. below demonstrates the percentage of prospective teachers' responses for each category of attending to students' solutions.

Table 4.2. *The Percentage of Prospective Teachers' Responses for Each Category of Attending to Students' Solutions*

Student Solution*	Robust evidence of attending	Emerging evidence of attending	Limited evidence of attending	Lack of evidence of attending
Student A's Solution	53.16%	34.38%	9.37%	3.13%
Student B's Solution	50%	18.75%	21.88%	9.36%
Student C's Solution	37.5%	21.88%	12.50%	28.13%
Student D's Solution	37.5%	31.25%	31.25%	0%
Student E's Solution	25%	21.88%	50%	3.13%

N=32

*The solutions of B and D are correct; A, C and E are incorrect.

As seen in the Table 4.2., about half of the prospective teachers described student A's and student B's solutions by demonstrating robust evidence. Both solutions belong to the same question which included the far generalization concept, but student A's solution is correct whereas student B's solution is incorrect. In other words, these prospective teachers provided robust evidence of attending in the concept of far generalization regardless of the correctness of solutions. On the other hand, almost one third of them provided robust evidence of attending to student C's solution which requires knowing near generalization, writing the rule of pattern and inverse process. Furthermore, in relation to the question including near generalization, far generalization and writing the rule of pattern concepts, one third of prospective teachers demonstrated robust evidence of attending to the correct solution and one fourth of them provided robust evidence of attending to the incorrect solution. On the other hand, less than half of the prospective teachers provided emerging evidence of attending to all solutions. Regarding limited evidence, less than a third of prospective teachers demonstrated limited evidence of attending to each solution except student E's solution. Finally, the percentage of prospective teachers who provided lack of evidence of attending to all solutions is below 30% regardless of the concepts included in each question and regardless of correctness. The detailed information and examples for each category of attending to students' solutions are given below.

4.1.1. Robust Evidence of Attention to Students' Solutions

Responses in which students' entire solutions were identified correctly by giving all mathematical details were labeled as responses presenting robust evidence of attention to students' solutions. The number and percentage of prospective teachers who gave a description that provided robust evidence of attention to student A's B's, C's, D's and E's solutions are given in Table 4.3. below.

Table 4.3. *Number and Percentage of Teachers Providing Robust Evidence of Attending to the Each Solution*

Student Solution*	The number of prospective teachers	Percentage
Student A's Solution	17	53.16%
Student B's Solution	16	50%
Student C's Solution	12	37.5%
Student D's Solution	12	37.5%
Student E's Solution	8	25%

N=32

*The solutions of B and D are correct; A, C and E are incorrect.

Table 4.3 reveals that nearly half of the prospective teachers in the current study described student A's and student B's solutions with robust evidence. Similarly, 37.5% of prospective teachers paid attention to student C's and student D's solutions with robust evidence. Finally, 25% of them provided robust evidence to describe student E' solution. The percentage of prospective teachers who attended to students' solutions by providing robust evidence was high.

The following excerpt is an example of robust evidence of attending to the details of student E's solution that was taken from PST 6's interview transcript.

Student E's solution

Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring, a group enters that has two more persons than the group that entered on the previous ring,

a) How many guests will have arrived after the 5th ring?
b) How many guests will have arrived after the 100th ring?
c) How many guests will have arrived after the nth ring?

a- 1 = 1 2 = 3 3 = 5 4 = 7 5 = 9

9 kişi vardır

b- 2.2-1 = 1
2.3-1 = 5
2.4-1 = 7
2.5-1 = 9
2..6 = 1 = 11
2

2.100-1 = 199

c- Kural : 2n-1

Researcher: *Now, how did the student find the solution such as sub question a?*

PST 6: *The student did not use the equal sign to signify equality. We can think of it as a dash. S/he said when the door was knocked the first time, one person; when the door was knocked the second time, three people; when the door was knocked the third time, five people; when the door was knocked the fourth time, seven people; when the door was knocked the fifth time, nine people came in the house. The student accepted nine as the answer because s/he accepted people* *who come from door each time as the total number of people who came home.*

Researcher: *Okay. What did the student do in sub-question b?*

PST 6: *What did the student do in sub- question b? S/he is a smart child. By using pattern, s/he said that s/he will write data one under the other and try to explore a pattern until the 100th ring. Then, s/he tried to find a pattern that was related to the number of steps. I assume that this is 3, not 1 and I believe s/he could not find the solution due to calculation error. Then, I look…now, hmm 5,7,9…S/he ignored the first step. Do you know how she found the rule?*

Now, s/he tried to form a relationship between the number of steps and the number of people who come home. Then, she said "how do I find three for the second step?"

Researcher: *Okay.*

PST 6: *She said "how do I find five for the third step, how do I find seven for the fourth step?" Then, s/he could find three by multiplying two by two and subtracting 1, and s/he made a calculation error. If we interpret others, s/he applied this- two multiplied by the number of steps minus 1- to all of them. S/he wrote the result of some steps such as fifth, seventh and ninth steps. Then, s/he checked for three steps, so s/he accepted this as a rule. When the doorbell rang a 100 times, s/he accepted that the number of steps was 100 and found the result.*

Researcher: *Okay. What did the student do in c?*

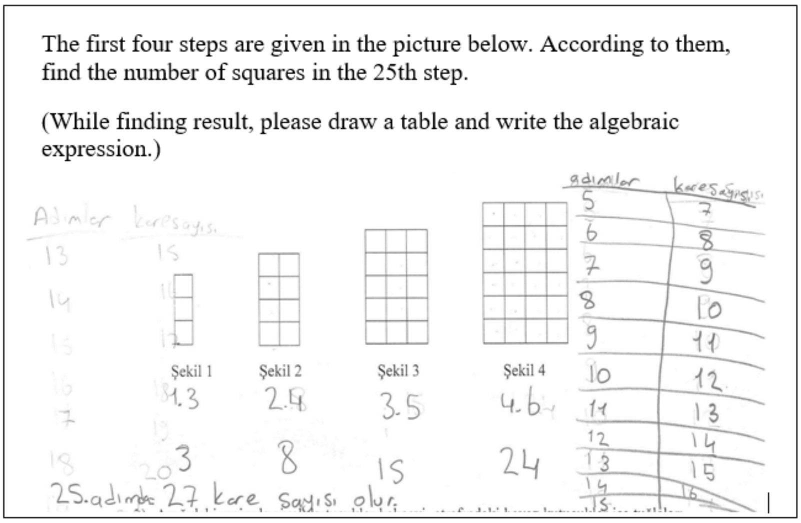
__PST 6:__ S/he accepted that two times n, minus 1 and could write the answer like 2n-1 by saying that n was the number of rings. Smart child expressed that algebraically.

(PST 6, an excerpt from the interview transcript)

In this description, PST 6 *explained in detail how students found* wrongly the number of people at home at the 5^{th}, 100^{th} and n^{th} rings. More specifically, PST 6 explained that the student wrote the number of people who were at home at the second, third, fourth and fifth rings as 3,5,7 and 9, respectively, and the student made a list the number of people for the first 6 rings, and then concluded 199 by multiplying 2 by 100 and subtracting 1. Moreover, s/he explained how the student found the number of people who are at home at the n^{th} ring. Therefore, this response was coded as robust evidence of attention to student's solutions as presented in the Table 3.2.

Another example of response demonstrating robust evidence of attention is PST 2' description of student A's solution is below.

Student A's solution



Wrong. In each figure, student multiplied the number of rows and the number of squares in each row in that figure. When s/he was solving the 5th step, s/he wrote the number of rows in that step instead of writing the total squares in that step. In other words, s/he started the solution with correct reasoning, but when s/he transferred the information to the table, s/he continued it wrongly. S/he continued with the 25th step and s/he said that there are 27 squares in the 25th step since the difference between the number of steps and the number of rows in that step was 2.

(PST 2, an excerpt from the questionnaire)

This response reveals that PST 2 *captured the mathematically important aspects of student A's solution*. Specifically, PST 2 stated that student A found the number of squares in each figure by multiplying the number of columns and the number of rows; however, the student unconsciously made mistakes in transforming knowledge to the table, and thus s/he reached an incorrect answer. As indicated in the Table 3.2., due to these reasons, this response was categorized as the response providing robust evidence of attention to student's solutions.

In addition, PST 5's description of student D's solution was coded as a response that provided robust evidence of attention to students' solutions.

Student D’s solution

Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring a group enters that has two more persons than the group that entered on the previous ring,

a) How many guests will have arrived after the 5^{th} ring?
b) How many guests will have arrived after the 100^{th} ring?
c) How many guests will have arrived after the n^{th} ring?

a) 1 $\overset{+2}{\frown}$ 2 $\overset{+2}{\frown}$ 3 $\overset{+2}{\frown}$ 4 $\overset{+2}{\frown}$ 5

1 + 3 + 5 + 7 + 9 = 25

b) 2. kez toplam = 4
3. kez toplam = 9
4 kez toplam = 16
5. kez = 25
6 kez = 36
7 kez = 49
8. kez = 64
9. kez = 81
10. kez = 100
⋮
100 kez = 10000 kişi olur

c) n. kez toplam = n . n = n^2

The student has done every sub-question correctly. In sub-question a, s/he tried to show the number of doorbell rings and how many people came at each ring with the listing method. S/he found the number of people who came in at that step by adding two to the number of people who came in during the previous step. Then s/he added the numbers that s/he found after the 5th step and reached the correct result. In sub-question b, s/he wrote the results found in each step one under the other, and step by step found how many people came in at each step. And s/he recognized that there was a relation between the number of ringing the doorbell and the people who came in, so s/he saw that in the 100th step, there must be square of 100 people. Thus, s/he reached the correct result. In sub-question c, the student found *the squares of n people who came in when the doorbell rang n times by making a generalization.*

(PST 5, an excerpt from the questionnaire)

In this response, PST 5 *identified the mathematical essence of student D's solution*. PST 5 stated that the student found the number of people who were at home at the 5^{th} ring by adding the number of people who came home at that ring and the number of people who came home at previous rings. Moreover, PST 5 explained how the student found the number of people who were at home at the 100^{th} ring by using the information about the number of people in the first steps. Finally, PST 5 realized that the student multiplied n by n to find the number of people who were at home at the n^{th} ring. Thus, as mentioned in the Table 3.2., this response was coded as robust evidence of attention to student's solutions.

As a consequence, responses demonstrating robust evidence of attention to students' solutions were described in various ways, but the common aspect of these ways is that they captured all the important details about the mathematical essence of the students' solutions.

Apart from robust evidence, the examples of responses demonstrating emerging evidence of attention to students' solutions are presented in the next part.

4.1.2. Emerging Evidence of Attention to Students' Solutions

Responses in which students' solutions were identified correctly, but all the mathematical details were not grasped were labeled as responses presenting emerging evidence of attention to students' solutions. The number and percentage of prospective teachers' responses that provided emerging evidence of attention to student A's B's, C's, D's and E's solutions are presented in Table 4.4. below.

Table 4.4. *Number and Percentage of Teachers Providing Emerging Evidence of Attending to the Each Solution*

Student Solution*	The number of prospective teachers	Percentage
Student A's Solution	11	34.38%
Student B's Solution	6	18.75%
Student C's Solution	7	21.88%
Student D's Solution	10	31.25%
Student E's Solution	7	21.88%

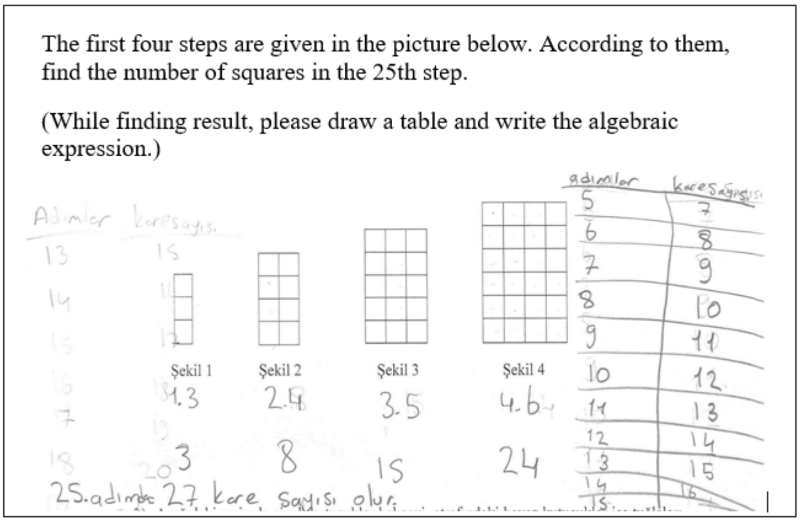
N=32

*The solutions of B and D are correct; A, C and E are incorrect.

As seen in the Table 4.4., the percentage of prospective teachers' responses that provided emerging evidence of attention to all student solutions in the present study is under 50% and average percentage of prospective teachers who attended to students' solutions demonstrating emerging evidence is 25%. The percentage of prospective teachers' attending to student C's and student E's solutions with emerging evidence is the same. Similarly, responses demonstrating emerging evidence of attention to student A's solution and student D's solution had nearly the same percentage. Thus, significant percentage of prospective teachers demonstrated emerging evidence of attending to students' solutions.

PST 9's following description of how student A solved the problem can be given as an example of a response providing emerging evidence to students' solutions.

Student A’s solution



The student realized that the number of rows in each step was two more than the number of steps and s/he concluded his/her solution by stating that there were 27 squares in the 25th step. Student A's solution is wrong because 27 is not the number of squares in the 25th step; actually it is the number of rows.

(PST 9, an excerpt from the questionnaire)

In this response, although PST 9 paid attention to some mathematical essence of student’s solution such as constructing the relation between steps and the number of columns and adding 2 to 25 and reaching 27, s/he did not pay attention to the student’s mistake in constructing the table which led to the incorrect answer. In other words, PST 9 *correctly identified student’s solution, but did not refer the entire solution with all important mathematical details.* Thus, as indicated in the Table 3.2., this response was categorized as a response providing emerging evidence of attention to student’s solution.

Another example of a response including emerging evidence of attention to student E’s solution is as follows:

Student E's solution

Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring, a group enters that has two more persons than the group that entered on the previous ring,

a) How many guests will have arrived after the 5^{th} ring?
b) How many guests will have arrived after the 100^{th} ring?
c) How many guests will have arrived after the n^{th} ring?

a – 1 = 1 2 = 3 3 = 5 4 = 7 5 = 9
9 kişi vardır

b – 2.2 − 1 = 1
2.3 − 1 = 5
2.4 − 1 = 7
2.5 − 1 = 9
2..6 = 1 = 11
2
2.100 − 1 = 199

c – Kural : 2n-1

In sub question a, the student increased the number of people by two in each step. S/he stated the people who came home in the 5^{th} step only, instead of adding people who came home in all steps and responded wrongly by saying 9.

In sub questions b and c, the student made a generalization by continuing this mistake and concluded wrong results.

(PST 9, an excerpt from the questionnaire)

In this response, PST 9 *captured the mathematical essence of student's* solution*, but s/he did not attend to all of them*. PST 9 described how student wrongly concluded the number of people who were at home at the 5^{th} ring. However, s/he stated his/her solutions to sub question b and c in general terms. S/he did not give detailed information about how the student found 199 or 2n-1. So, this response was coded as a response with emerging evidence of attention to student's solutions as presented in the Table 3.2.

Briefly, in this study, responses in which prospective teachers attend to student's solutions without all the mathematical details were considered as responses showing emerging evidence of attention to students' solutions. The findings related to limited evidence of attention to students' solutions are presented below.

4.1.3. Limited Evidence of Attention to Students' Solutions

Responses that included comments reflecting the general features of the solutions and not providing details about how the problem was solved were coded as responses that provide limited evidence of attention to students' solutions. The number and percentage of prospective teachers who gave a description that provided limited evidence of attention to student A's B's, C's, D's and E's solutions are given in Table 4.5 below.

Table 4.5. *Number and Percentage of Teachers Providing Limited Evidence of Attending to the Each Solution*

Student Solution*	The number of prospective teachers	Percentage
Student A's Solution	3	9.37%
Student B's Solution	7	21.88%
Student C's Solution	4	12.50%
Student D's Solution	10	31.25%
Student E's Solution	16	50%

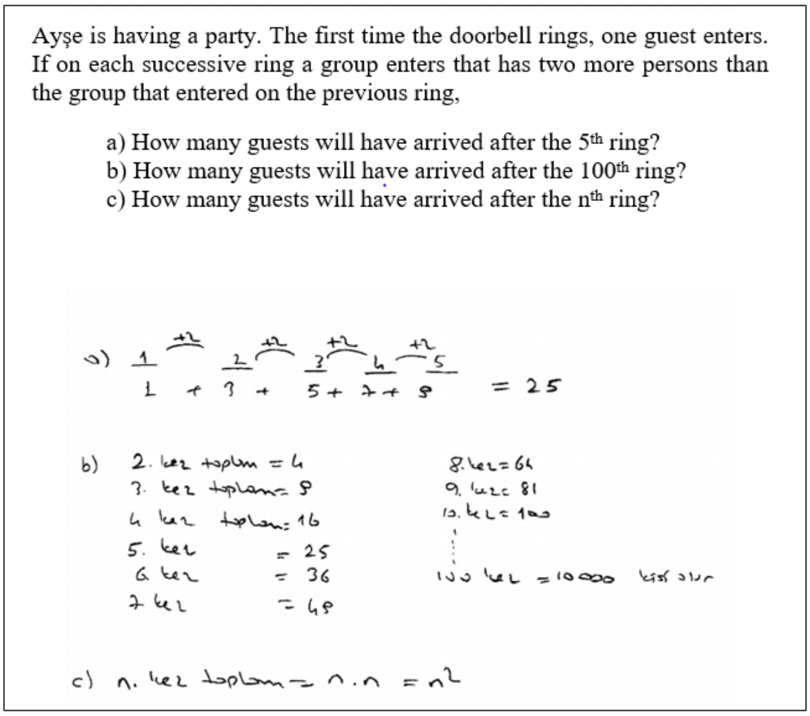
N=32

*The solutions of B and D are correct; A, C and E are incorrect.

According to the Table 4.5., the percentage of prospective teachers' description for student E's solution that provided limited evidence is more than percentage of prospective teachers' descriptions for other students' solutions. The percentage of prospective teachers' description of student A's, B's, C's and D's solutions with limited evidence is less than fifty.

The following prospective teacher's description of student D's solution is an example of a response providing limited evidence.

Student D's solution


Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring a group enters that has two more persons than the group that entered on the previous ring,

a) How many guests will have arrived after the 5^{th} ring?
b) How many guests will have arrived after the 100^{th} ring?
c) How many guests will have arrived after the n^{th} ring?

a) 1 + 3 + 5 + 7 + 9 = 25

b) 2. kez toplam = 4
3. kez toplam = 9
4 kez toplam = 16
5. kez = 25
6 kez = 36
7 kez = 49
8. kez = 64
9. kez = 81
10. kez = 100
100 kez = 10000 kişi olur

c) n. kez toplam = n.n = n^2

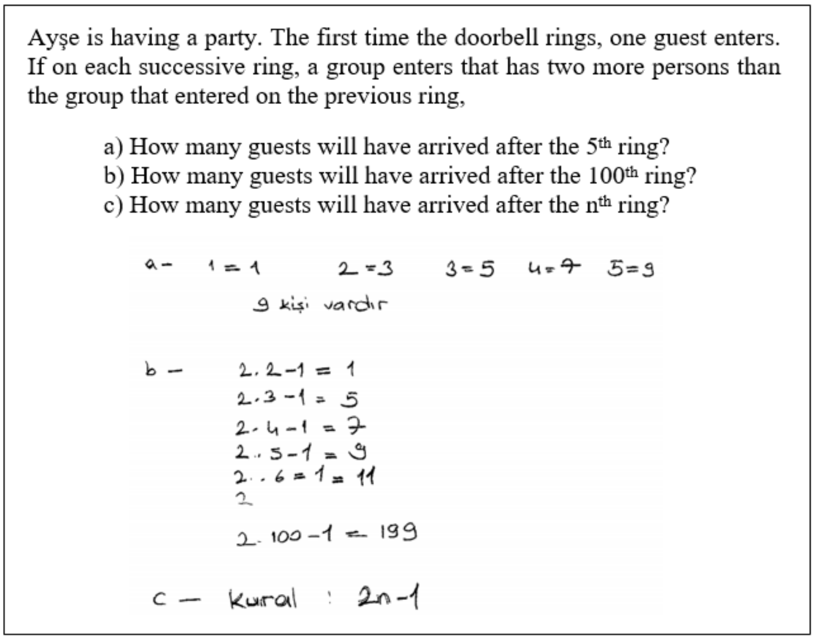
The solution is correct. The student finds a few steps in the beginning and then finds the rules. I think it is a simple and successful approach.

(PST 16, an excerpt from the questionnaire)

In this response, PST 16 *focused on the correctness of the solution and described student's solution in general terms.* While the responses that provided robust evidence or emerging evidence included details of what students did in each step, PST 16's comment regarding this response tended towards general features of that solution and summarized what the student did very shortly. So, as indicated in the Table 3.2., this response was coded as a response with limited evidence of attention to student's solutions.

Another example of a response that was coded as limited evidence of attention to student E's solution is as follows:

Student E's solution



Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring, a group enters that has two more persons than the group that entered on the previous ring,

a) How many guests will have arrived after the 5th ring?
b) How many guests will have arrived after the 100th ring?
c) How many guests will have arrived after the nth ring?

a- 1 = 1 2 = 3 3 = 5 4 = 7 5 = 9
9 kişi vardır

b- 2.2-1 = 1
2.3-1 = 5
2.4-1 = 7
2.5-1 = 9
2..6 = 1 = 11
2
2.100-1 = 199

c- Kural : 2n-1

The student developed a strategy. S/he calculated the number of people who came home in the desired step, but did not calculate the number of all the people at home. The solution is incorrect.

(PST 13, an excerpt from the questionnaire)

In this response, PST 13 stated that the student solved the problem wrongly and only asserted that the student developed a strategy. Moreover, PST 13 gave the general reason why the student's solution was wrong, but *did not provide any details about that solution.* Thus, it was decided that this response provided limited evidence of attention to student's solution due to explanation in the Table 3.2.

Consequently, when responses focused on whether the solution was correct or not and provided general comments regarding the students' solution by omitting the details of the solutions, they were coded as responses demonstrating limited evidence of attention to students' solutions.

In the next part, responses providing lack evidence of attention to students' solutions are exemplified.

4.1.4. Lack of Evidence of Attention to Students' Solutions

The responses in which students' solutions were identified incorrectly were coded as a response providing lack of evidence of attention to students' solutions. The number and percentage of prospective teachers who provided a description with lack of evidence of attention to student A's B's, C's, D's and E's solutions are given in Table 4.6 below.

Table 4.6. *Number and Percentage of Teachers Providing Lack of Evidence of Attending to the Each Solution*

Student Solution*	The number of prospective teachers	Percentage
Student A's Solution	1	3.13%
Student B's Solution	3	9.36%
Student C's Solution	9	28.13%
Student D's Solution	0	0%
Student E's Solution	1	3.13%

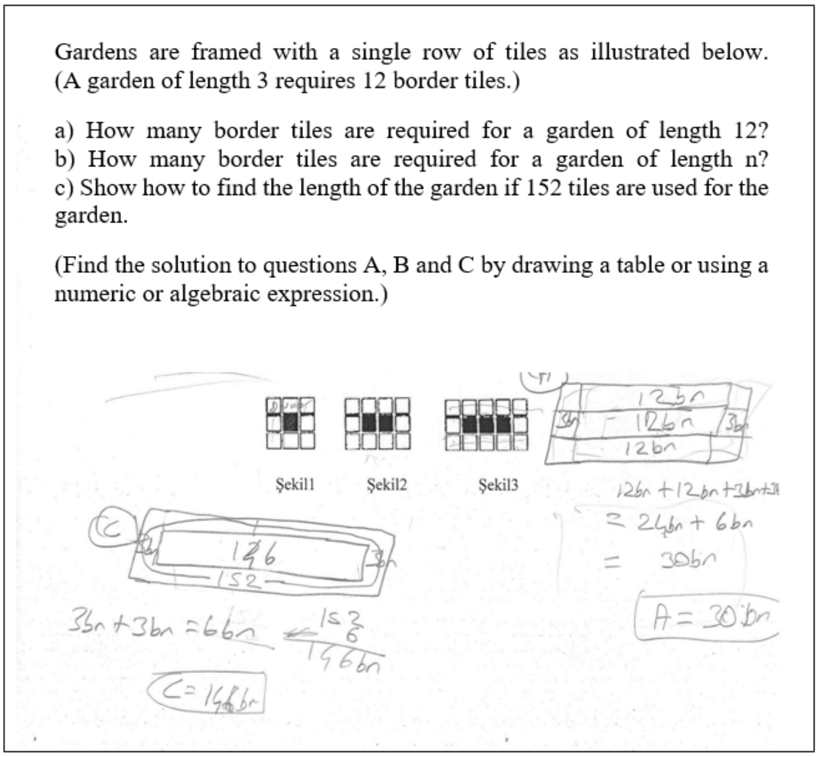
N=32

*The solutions of B and D are correct; A, C and E are incorrect.

As can be observed in the Table 4.6., the percentage of prospective teachers' description with lack of evidence to five students' solutions is very low. To illustrate, only one prospective teacher described student A's and student E' solution with lack of evidence. Moreover, there were three responses that provided lack of evidence of attention to student B's solution. Also, there were nine responses that provided lack of evidence of attention to student C's solution. However, there was no prospective teacher who described student D' solution with lack of evidence.

For example, PST 29's description of student C's solution is an example of a response with lack of evidence.

Student C's solution


Gardens are framed with a single row of tiles as illustrated below. (A garden of length 3 requires 12 border tiles.)

a) How many border tiles are required for a garden of length 12?
b) How many border tiles are required for a garden of length n?
c) Show how to find the length of the garden if 152 tiles are used for the garden.

(Find the solution to questions A, B and C by drawing a table or using a numeric or algebraic expression.)

In the shapes in sub question a, s/he did not see that the short edge and the long edge intersects at the corners and there are common squares. So, the total is supposed to be 39 units, and answer is wrong.

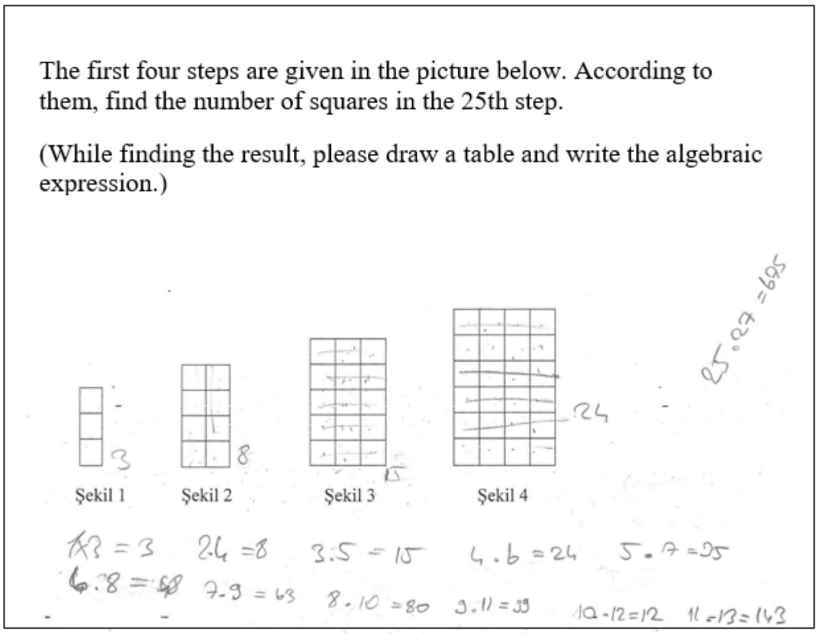
In sub question c, subtracting 6 from 152 is correct; however, it was necessary to divide 146 by 2. The answer is wrong.

(PST 29, an excerpt from the questionnaire)

Although student C solves sub-question a correctly and calculated the number of bricks as 30, PST 29 stated that student solved the problem wrongly; thus, this *prospective teacher incorrectly identified student C's solution.* Therefore, as explained in the Table 3.2., it was decided that this response provided lack of evidence of attention to student's solution

PST 19's description of student B's solution is another example of a response that provided lack of evidence of attention.

Student B's solution



Student's solution is correct. S/he comprehended the logic behind the pattern and expressed it algebraically. S/he applied the pattern of n.(n+2) to the 25th step.

(PST 19, an excerpt from the questionnaire)

In this response, although the student did not express the pattern with an algebraic expression, PST 19 attended to student's solution like that, meaning that PST 19 provided an *incorrect description*. For this reason, according to the Table 3.2., PST 19's description was coded as a response providing lack of evidence of attention.

In conclusion, prospective teachers' responses in which they were mistaken or which incorrectly identified students' solutions were coded as responses that provided lack evidence of attention to students' solutions. In other words, responses that provided lack of evidence did not give any idea about the mathematical essence of the solutions.

The second dimension of professional teacher noticing is interpreting students' algebraic thinking and it is related to whether teachers' reasoning about student's algebraic thinking is consistent with the details of students' solutions. The findings related to interpreting students' algebraic thinking within the context of pattern generalization are given below.

4.2. Prospective Teachers' Interpreting Students' Algebraic Thinking

In the current study, prospective teachers' responses were categorized into four to identify the extent of their interpretation of students' algebraic thinking: robust evidence of interpretation of students' algebraic thinking, emerging evidence of interpretation of students' algebraic thinking, limited evidence of interpretation of students' algebraic thinking, and lack of evidence of interpretation of students' algebraic thinking. Responses providing robust evidence of interpretation of students' algebraic thinking included making sense of the details of the students' solutions. In responses that demonstrated emerging evidence of interpretation, prospective teachers made interpretations about students' algebraic thinking but with less detail than responses including robust evidence. Responses demonstrating limited evidence of interpretation of students' algebraic thinking included comments only about whether students comprehended the topic or not. Responses that provide wrong evidence of interpretation of students' thinking or irrelevant comments to students' thinking were coded as responses with lack of evidence. In addition to these responses, some prospective teachers only paid attention to student's solutions rather than providing interpretation about their algebraic thinking, and some of them did not answer the question in this study.

The summary of categorization of prospective teachers' expertise in interpretation to students' algebraic thinking is presented Table 4.7 below.

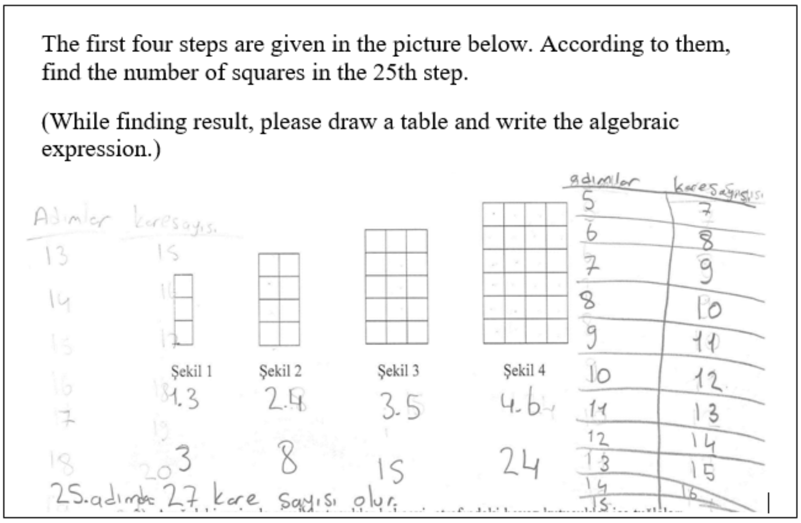
Table 4.7. *The Coding of Interpreting Students' Algebraic Thinking*

Interpreting Students' Algebraic Thinking	**Coding**
Robust Evidence of Interpretation of Students' Algebraic Thinking	Making a sense of details of solutions by providing reasoning.
Emerging Evidence of Interpretation of Students' Algebraic Thinking	Make interpretation about students' algebraic thinking but with less detail than responses included robust evidence.
Limited Evidence of Interpretation of Students' Algebraic Thinking	Make comments about only whether students comprehended the concept or not.
Lack Evidence of Interpretation of Students' Algebraic Thinking	Provide wrong evidence of interpretation of students' algebraic thinking. Make an irrelevant comment to students' algebraic thinking.
No interpretation just attention	
No responses	

To determine whether the prospective teachers interpreted students' algebraic thinking firstly, the mathematically important essences of each student' algebraic thinking were determined.

Student A's solution and mathematically important essence for student A's algebraic thinking are as follows:

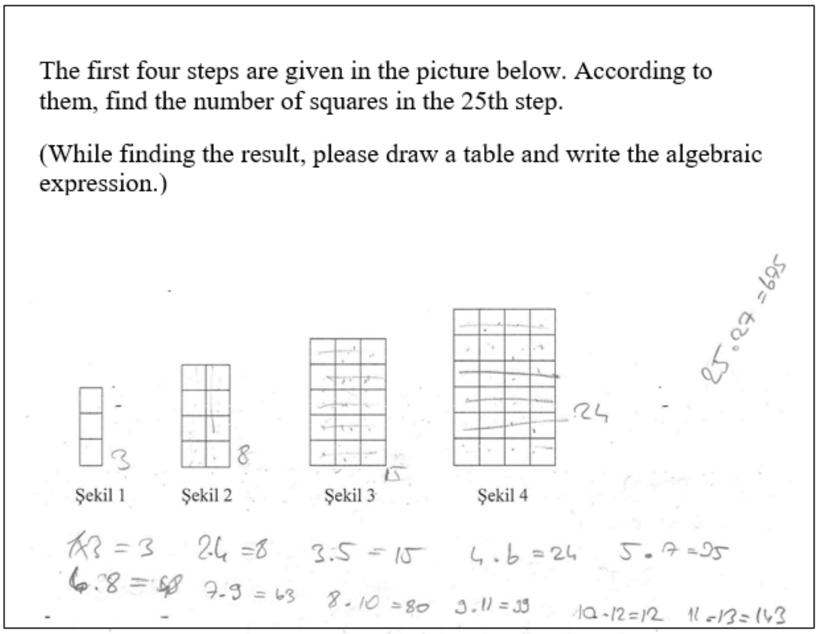
Student A's solution



Student A explores the relation in pattern as the number of squares = (the number of columns) x (the number of rows) or the number of squares = (the short length) x (the long length). However, s/he cannot fill the table according to the relationship that s/he explores in the first four steps in the pattern.

Student B's solution and mathematically important essence for student B's algebraic thinking are as follows.

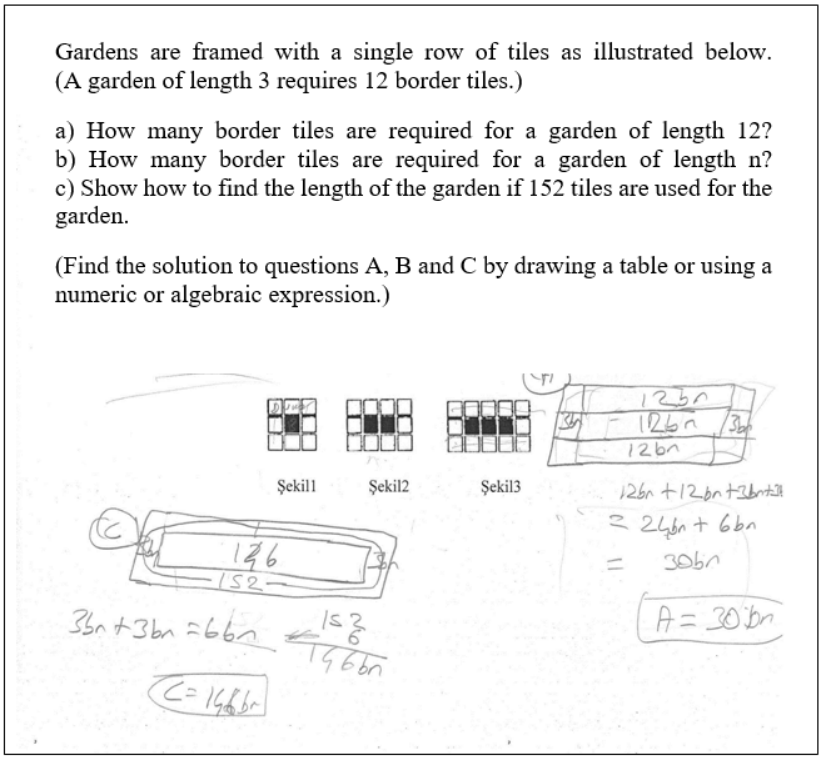
Student B’s Solution



Student B explores the relation in pattern as the number of squares = (the number of columns) x (the number of rows) or the number of squares = (the short length) x (the long length). Then, the student makes a generalization for the 25th term in the pattern.

Student C’s solution and mathematically important essence for student C’s algebraic thinking are as follows:

Student C's solution



Gardens are framed with a single row of tiles as illustrated below. (A garden of length 3 requires 12 border tiles.)

a) How many border tiles are required for a garden of length 12?
b) How many border tiles are required for a garden of length n?
c) Show how to find the length of the garden if 152 tiles are used for the garden.

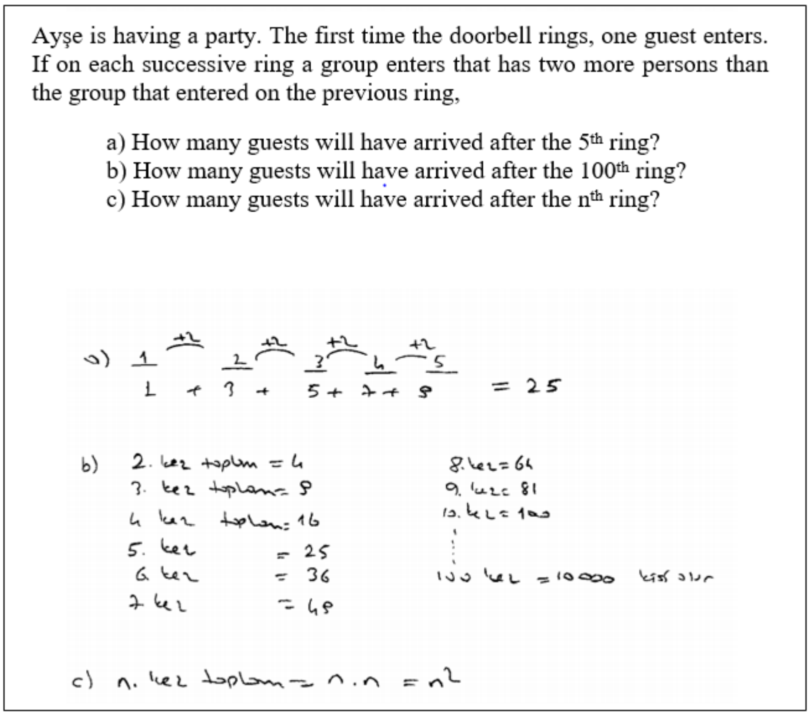
(Find the solution to questions A, B and C by drawing a table or using a numeric or algebraic expression.)

Şekil1 Şekil2 Şekil3

Student C makes a near generalization for the garden in which the long length is 12 units. However, s/he does not write a general rule for the pattern and does not perform an inverse process.

Student D's solution and mathematically important essence for student D's algebraic thinking are as follows:

Student D’s solution



Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring a group enters that has two more persons than the group that entered on the previous ring,

a) How many guests will have arrived after the 5^{th} ring?
b) How many guests will have arrived after the 100^{th} ring?
c) How many guests will have arrived after the n^{th} ring?

a) 1 2 3 4 5
$1 + 3 + 5 + 7 + 9 = 25$

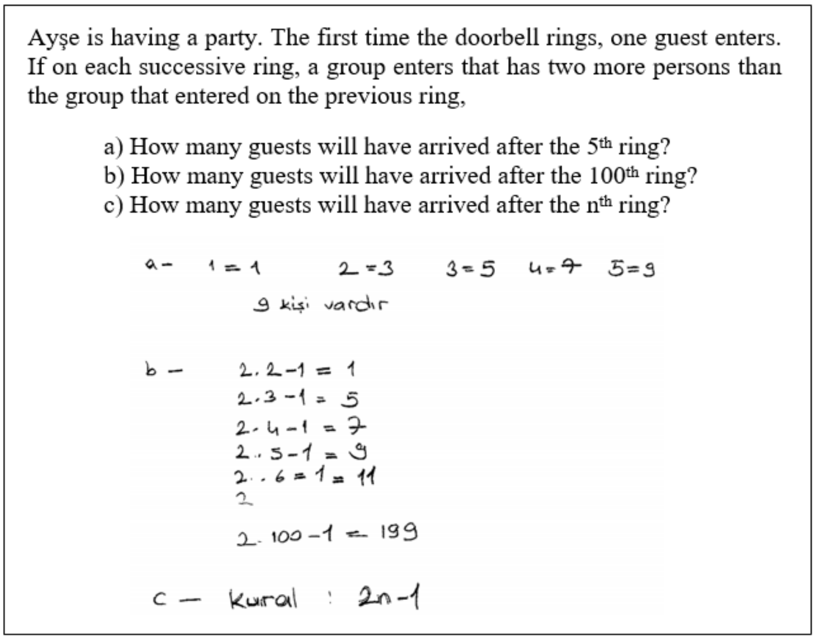
b) 2. kez toplam = 4
3. kez toplam = 9
4 kez toplam = 16
5. kez = 25
6 kez = 36
7 kez = 49
8. kez = 64
9. kez = 81
10. kez = 100
⋮
100 kez = 10000 kişi olur

c) n. kez toplam = $n.n = n^2$

Student D makes a near generalization for the 5^{th} term in the pattern, and then makes a far generalization for the 100^{th} term in the pattern. Moreover, s/he writes a general rule for the pattern.

Student E’s solution and mathematically important essence for student E’s algebraic thinking are as follows:

Student E's solution


Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring, a group enters that has two more persons than the group that entered on the previous ring,

a) How many guests will have arrived after the 5th ring?
b) How many guests will have arrived after the 100th ring?
c) How many guests will have arrived after the nth ring?

a- 1 = 1 2 = 3 3 = 5 4 = 7 5 = 9

9 kişi vardır

b- 2.2-1 = 1
2.3-1 = 5
2.4-1 = 7
2.5-1 = 9
2..6 = 1 = 11
2
2.100-1 = 199

c- Kural : 2n-1

Student E makes a near generalization for the 5th term and makes a far generalization for the 100th term in the pattern, but for a different pattern than the asked pattern. Then, the student writes the general rule, but for a different pattern than the asked pattern.

Table 4.8. shows the percentage of prospective teachers' responses for each category of interpreting students' algebraic thinking.

Table 4.8. *The Percentage of Prospective Teachers' Responses for Each Category of Interpreting*

Student Solution*	Robust evidence of interpreting	Emerging evidence of interpreting	Limited evidence of interpreting	Lack of evidence of interpreting
Student A's Solution	18.75%	18.75%	34.38%	25%
Student B's Solution	31.25%	34.38%	18.75%	9.38%
Student C's Solution	12.50%	31.25%	31.25%	18.75%
Student D's Solution	34.38%	37.5%	28.13%	0%
Student E's Solution	12.50%	40.63%	34.38%	12.5%

N=32

*The solutions of B and D are correct; A, C and E are incorrect.

As can be observed in the Table 4.8., about one fifth of prospective teachers and about one thirds of them interpreted student A's and student B's algebraic thinking by demonstrating robust evidence, respectively. Both solutions belong to the same question which included the far generalization concept, but student A's solution is correct whereas student B's solution is incorrect. In other words, these prospective teachers provided more robust evidence of interpreting correct solution than incorrect solution in the concept of far generalization. On the other hand, one eight of them provided robust evidence of interpreting student C's algebraic thinking which requires knowing near generalization, writing the rule of pattern and inverse process. Furthermore, in relation to the question including near generalization, far generalization and writing the rule of pattern concepts, about one third of prospective teachers demonstrated robust evidence of interpreting student's algebraic thinking with correct solution and one eight of them provided robust evidence of interpreting student's algebraic thinking with incorrect solution. On the other hand, about 30% of the prospective teachers provided emerging evidence of interpreting to all students' thinking. Regarding limited evidence, about one third of prospective teachers demonstrated limited evidence of interpreting algebraic thinking of student with incorrect solutions while less than a third of them provided limited evidence of interpreting algebraic thinking of student with correct solutions. Finally, the percentage of prospective teachers who provided lack of evidence of interpreting all students' algebraic thinking is below 20% except student A's algebraic thinking regardless of the concepts including each question.

4.2.1. Robust Evidence of Interpretation of Students' Algebraic Thinking

Prospective teachers' responses including making sense of the details of students' solutions were coded as response demonstrating robust evidence of interpretation of students' algebraic thinking. The number and percentage of prospective teachers who made interpretations of student A's B's, C's, D's and E's algebraic thinking that included robust evidence are presented in Table 4.9. below.

Table 4.9. *Number and Percentage of Teachers Providing Robust Evidence of Interpreting of Students' Algebraic Thinking*

Student Solution*	The number of prospective teachers	Percentage
Student A's Solution	6	18.75%
Student B's Solution	10	31.25%
Student C's Solution	4	12.50%
Student D's Solution	11	34.38%
Student E's Solution	4	12.50%

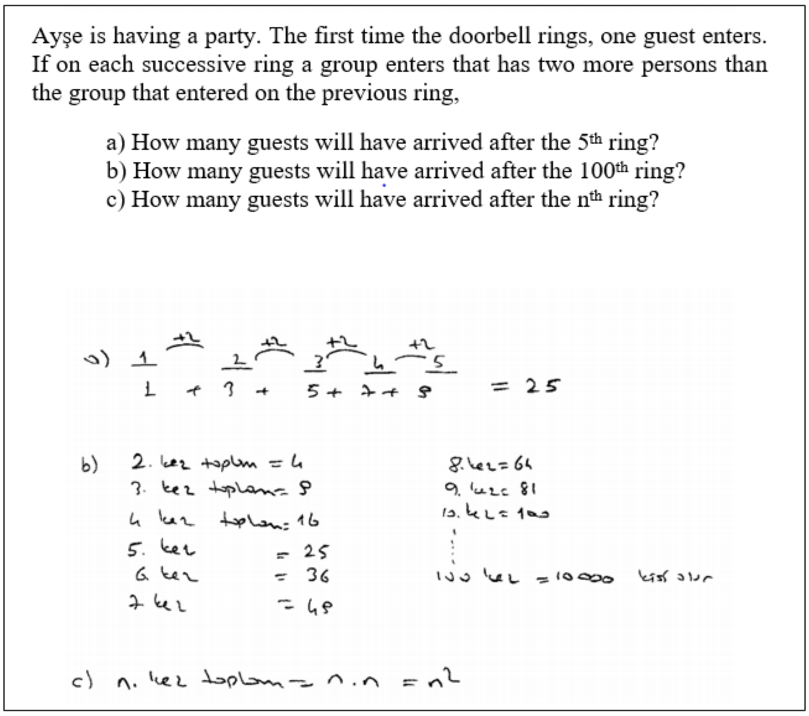
N=32

*The solutions of B and D are correct; A, C and E are incorrect.

The Table 4.9. reveals that the percentage of prospective teachers who took part in the current study and who made interpretations of all students' algebraic thinking by providing robust evidence is under 50%. Moreover, the number of prospective teachers who interpreted student B's and D's algebraic thinking with robust evidence is more than the number of prospective teachers who interpreted other students' algebraic thinking with robust evidence.

An example of robust evidence of interpretation of students' algebraic thinking is as follows.

Student D's solution



Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring a group enters that has two more persons than the group that entered on the previous ring,

a) How many guests will have arrived after the 5th ring?
b) How many guests will have arrived after the 100th ring?
c) How many guests will have arrived after the nth ring?

a) 1 + 3 + 5 + 7 + 9 = 25

b) 2. kez toplam = 4
3. kez toplam = 9
4 kez toplam = 16
5. kez = 25
6. kez = 36
7 kez = 49
8. kez = 64
9. kez = 81
10. kez = 100
100 kez = 10000 kişi olur

c) n. kez toplam = $n.n = n^2$

Researcher: *What can you say about student's understanding based on student's solution in sub-question a?*

PST 24: *I think it is very logical. The student is aware of what s/he did. S/he saw only the relationship between them.*

Researcher: *What is the relationship?*

PST 24: *S/he is aware that it is increasing two by two. Actually, it was given in the question. For example, in the first question, "how many people gathered at home after the doorbell rang 5 times?" Unlike other solutions, s/he was aware of the people who came home at previous rings. She comprehended the concept. In other words, s/he understood the pattern.*

Researcher: *I see. What do you think about student's understanding in sub-question b?*

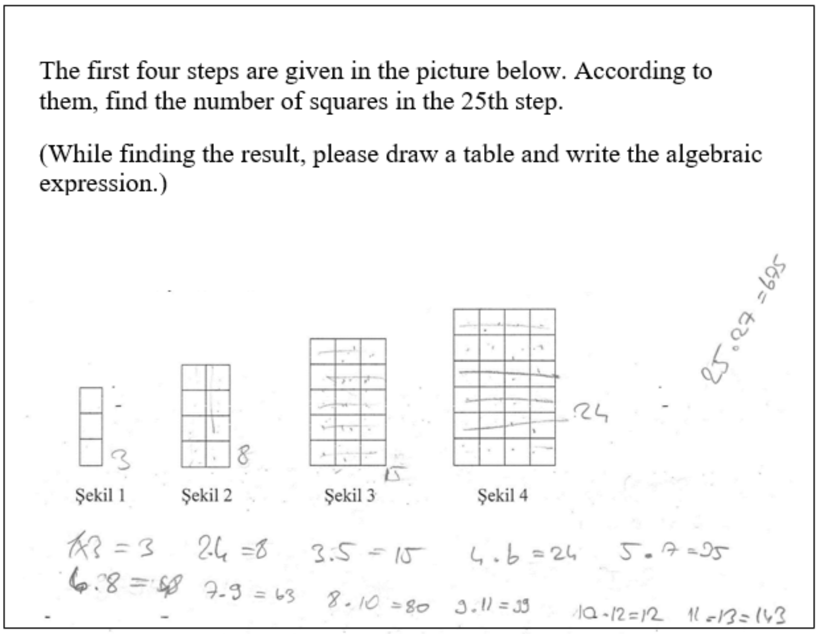
PST 24: *I think student comprehended all the questions. She progressed step by step and found the solution in the fifth step. S/he started from the second time and proceeded to the seventh time. When s/he continued to the tenth step, s/he recognized that the result changed as square. So, when s/he came to the 100th step, s/he could find 10000. S/he made a generalization well and solved the question perfectly since s/he multiplied 100 with 100. S/he found the fifth step and then made a generalization by thinking two, three, four…*

(PST 24, an excerpt from the interview transcript)

In this response, PST 24 *explained that student discovered how to change the number of people who came home at each ring*. Moreover, PST 24 *stated that student could explore the pattern and make a generalization* for the 5th and the 100th term in the pattern. Therefore, it can be said that the prospective teacher made sense of student D's solution with reasoning; thus, this teacher's responses were coded as a robust evidence of interpretation of students' solution due to explanation in the Table 3.3.

PST 7's interpretation of student B's algebraic thinking below is given as an example of the responses that included robust evidence of interpretation to students' solutions.

Student B’s solution


The first four steps are given in the picture below. According to them, find the number of squares in the 25th step.

(While finding the result, please draw a table and write the algebraic expression.)

***Researcher**: What can you say about student’s understanding?*

***PST 7:** Pattern.. one minute.. I looked at the pattern between multiplications. Pattern is actually..*

S/he recognized that the pattern of the number of rows and the number of columns increased one by one in each step. S/he recognized that the difference between the number of rows and columns is two in each step.

***Researcher**: Okay. You said that the student explored the pattern in the questionnaire. How did you make such an inference?*

***PST 7:** The first reason was that the student solved the problem correctly. The second reason was that the student did not write step by step until the 25th step. In other words, after the 11th step, s/he explored the pattern and found it without writing step by step. The* *primary reason for exploring the pattern is to find the result of the far step. Actually, s/he succeeded it here.*

(PST 7, an excerpt from the interview transcript)

In this response, PST 7 made a correct reasoning about the student's algebraic thinking. PST 7 *realized that the student captured the relationship* between the number of rows and columns and the number of squares, and then, the student *explored the pattern correctly*. Therefore, PST 7's interpretation included robust evidence about student's algebraic thinking as indicated in the Table 3.3.

As a consequence, when prospective teachers analyzed students' solutions and made sense of their thinking by providing a reasoning, it means that they interpreted students' algebraic thinking with robust evidence.

Detail information about the responses including emerging evidence of interpretation of students' algebraic thinking is presented below.

4.2.2. Emerging Evidence of Interpretation of Students' Algebraic Thinking

When the responses provided emerging evidence of interpretation of students' algebraic thinking, prospective teachers still interpreted their algebraic thinking, but this interpretation was not in-depth. The number and the percentage of prospective teachers who made interpretations of student A's B's, C's, D's and E's algebraic thinking that included emerging evidence are presented in Table 4.10. below.

Table 4.10. *Number and Percentage of Teachers Providing Emerging Evidence of Interpreting of Students' Algebraic Thinking*

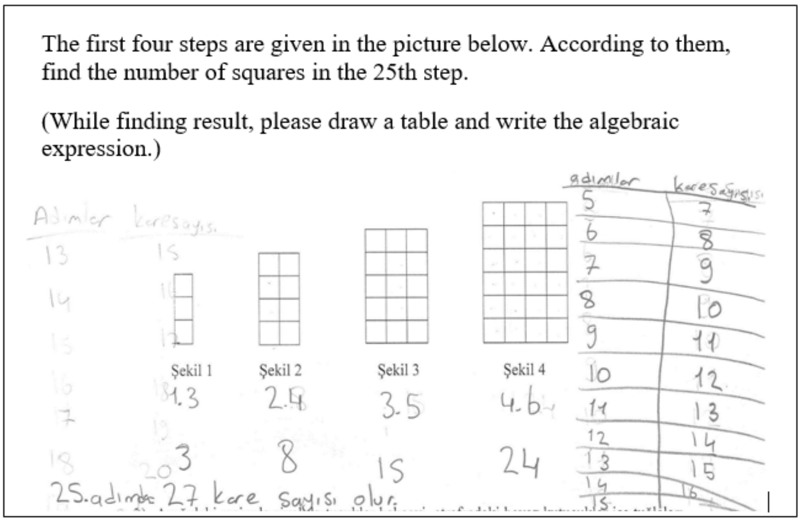
Student Solution*	The number of prospective teachers	Percentage
Student A's Solution	6	18.75%
Student B's Solution	11	34.38%
Student C's Solution	10	31.25%
Student D's Solution	12	37.5%
Student E's Solution	13	40.63%

N=32

*The solutions of B and D are correct; A, C and E are incorrect.

The Table 4.10. shows that while the prospective teachers' interpretation of student E's algebraic thinking has the maximum percentage, their interpretation of student A's algebraic thinking has the minimum percentage. Furthermore, the percentages of their interpretation of student B's, C's and D's algebraic thinking by providing emerging evidence are nearly the same. For example, the interpretation given below provides emerging evidence.

Student A’s solution



He knows that he must multiply the rows and columns to find the number of squares. Also, he correctly forms a relationship between the number of rows in steps. But, he couldn't reach the result. I think there is a lack of attention.

(PST 32 from questionnaire)

PST 32 *noticed that student could recognize the relationship between the number of squares and the number of rows and columns*. However, PST 32 *did not interpret student’s mistake in filling the table.* Briefly, this prospective teacher interpreted student’s thinking, but with less detail; thus, as presented in the Table 3.3., PST 32’s interpretation included emerging evidence.

4.2.3. Limited Evidence of Interpretation of Students’ Algebraic Thinking

Responses that provided comments only about whether students’ comprehension was correct or not were considered as limited evidence of interpretation of students’ algebraic thinking. The number and percentage of prospective teachers who made interpretations about student A’s B’s, C’s, D’s and E’s algebraic thinking demonstrating limited evidence are presented in Table 4.11. below.

Table 4.11. *Number and Percentage of Teachers Providing Limited Evidence of Interpreting of Students’ Algebraic Thinking*

Student Solution*	The number of prospective teachers	Percentage
Student A’s Solution	11	34.38%
Student B’s Solution	6	18.75%
Student C’s Solution	10	31.25%
Student D’s Solution	9	28.13%
Student E’s Solution	11	34.38%

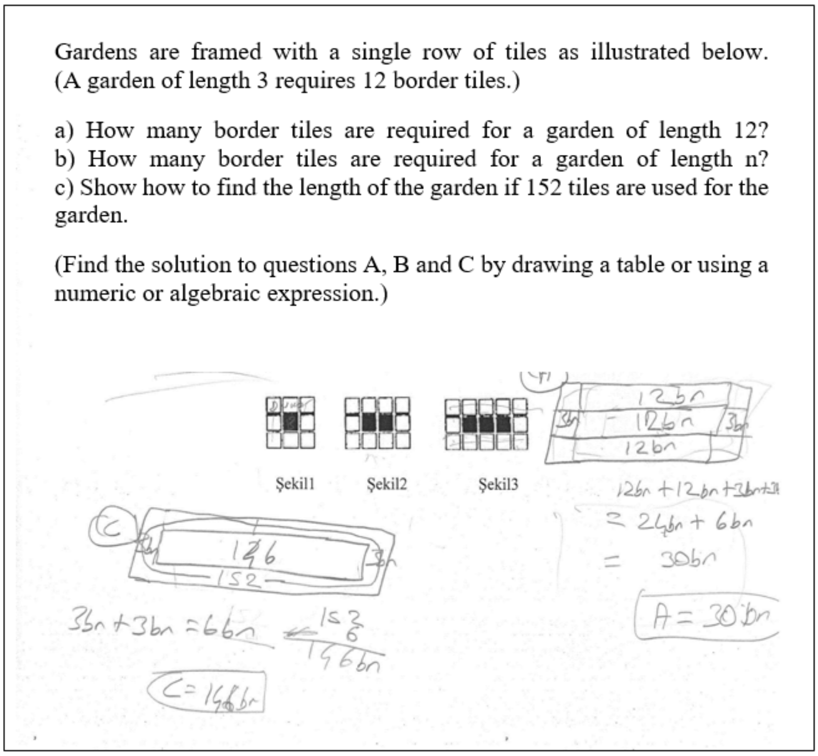
N=32

*The solutions of B and D are correct; A, C and E are incorrect.

As seen in the Table 4.11., the percentages of prospective teachers’ interpretation of student A’ and student E’s algebraic thinking with limited evidence are the same. Also, the percentages of their interpretation of student A’s, C’s D’s and E’s algebraic thinking with limited evidence are close. Furthermore, the prospective teachers’ interpretation of student B’s algebraic thinking by supplying limited evidence has the minimum percentage among the interpretations of five students’ algebraic thinking with limited evidence.

An example of responses demonstrating limited evidence is as follows:

Student C’s solution



I don't think the student could grasp much, except for sub question a. The reasoning in sub question a is good. However, instead of thinking deeply for sub questions b and c, the student adopted a memorization approach.

(PST 16, an excerpt from the questionnaire)

In this example, PST 16 *described the student’s thinking with broad terms*. In other words, the teacher taught that the student comprehended sub-question a, but did not comprehend sub-questions b and c. However, PST 16 *did not refer to specific points regarding student’s thinking* such as exploring the relationship, not making a generalization, or not writing a rule of pattern. Thus, as mentioned in the Table 3.3., PST 16’s interpretation demonstrated limited evidence of interpretation of student’s algebraic thinking.

Briefly, unlike responses demonstrating robust evidence and emerging evidence, when prospective teachers made overgeneralization, their responses were categorized as responses with limited evidence.

4.2.4. Lack of Evidence of Interpretation of Students' Algebraic Thinking

When prospective teachers made a wrong interpretation or made irrelevant comments about students' thinking, these responses were coded as a response providing lack of evidence of interpretation of students' algebraic thinking. The number and percentage of prospective teachers who interpreted student A's B's, C's, D's and E's algebraic thinking with lack of evidence are presented in Table 4.12. below.

Table 4.12. *Number and Percentage of Teachers Providing Lack of Evidence of Interpreting of Students' Algebraic Thinking*

Student Solution*	The number of prospective teachers	Percentage
Student A's Solution	8	25%
Student B's Solution	3	9.38%
Student C's Solution	6	18.75%
Student D's Solution	0	0%
Student E's Solution	4	12.5%

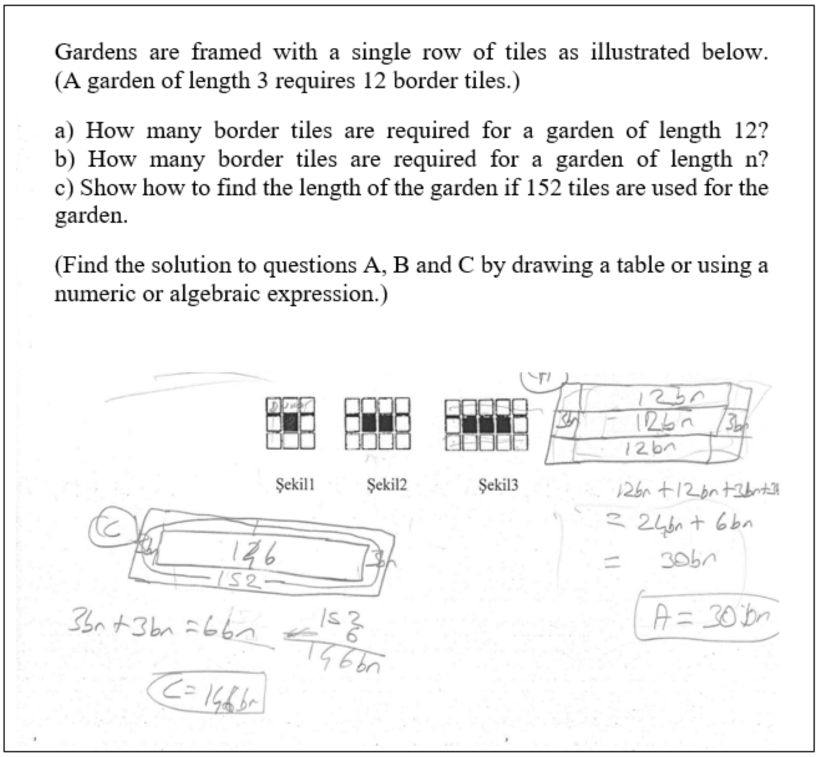
N=32

*The solutions of B and D are correct; A, C and E are incorrect.

As can be observed in the Table 4.12., although the prospective teachers' interpretation of student A's algebraic thinking providing lack of evidence has the maximum percentage (25%), the percentage of prospective teachers' interpretation of student B's, C's and E's algebraic thinking with lack of evidence is low. Even, there is no interpretation of student D's algebraic thinking with lack of evidence.

The following sample interpretations were considered as lack of evidence of interpretation.

Student C's solution


Gardens are framed with a single row of tiles as illustrated below. (A garden of length 3 requires 12 border tiles.)

a) How many border tiles are required for a garden of length 12?
b) How many border tiles are required for a garden of length n?
c) Show how to find the length of the garden if 152 tiles are used for the garden.

(Find the solution to questions A, B and C by drawing a table or using a numeric or algebraic expression.)

The student made a correct drawing in sub-questions a and c, but did not find the correct solution. He does not realize that the result s/he finds in sub question b must be valid for sub question a. Therefore, s/he failed to establish a relationship between them and s/he does not recognize that s/he needs to divide 146 by 2 in sub question c.

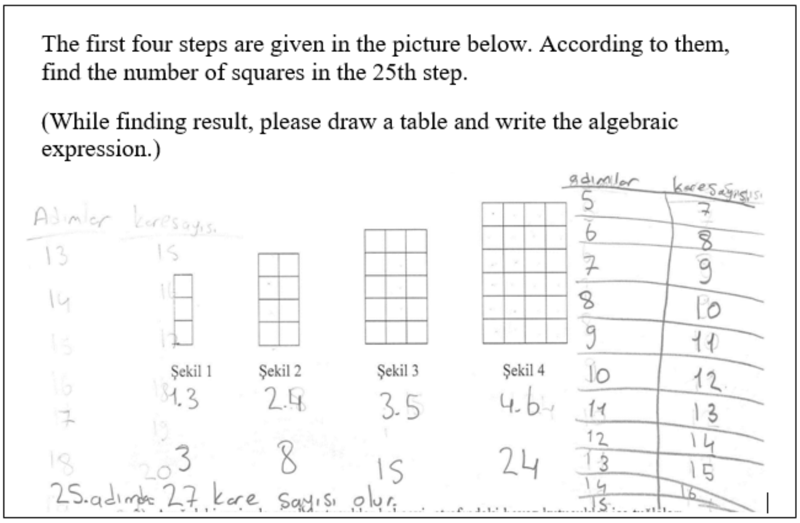
(PST 9, an excerpt from the questionnaire)

In this example, although student C solved sub question a correctly, PST 9 stated that the student solved it wrongly, and thus *interpreted student C's thinking wrongly.*

Therefore, this response has lack of evidence about the interpretation of student's algebraic thinking.

Another example of a response including lack of evidence of interpretation of student thinking is as follows:

Student A's solution



S/he is unable to make sense of the drawing table. S/he made an error when writing the information that was related to the question on the table. S/he used the table as s/he saw from a friend or from previous lessons.

(PST 19, an excerpt from the questionnaire)

This response *did not include any specific comments about student's thinking* such as realizing the relationship, exploring the pattern, or making a generalization. In addition, PST 19 *made irrelevant comments* about student's thinking like "S/he is unable to make sense of the drawing table" and "S/he used the table as s/he saw from a friend or from previous lessons." This response is example of interpretation with lack of evidence due to the Table 3.3.

Consequently, prospective teachers' responses that involved wrong interpretations or making irrelevant comments about students' thinking were coded as providing lack of evidence of interpretation of students' algebraic thinking.

The third dimension of professional teacher noticing is deciding how to respond on the basis of students' understanding. Customizing responding for each student based on their thinking is an important component of teachers' noticing skills. The findings related to this dimension are presented below.

4.3. Prospective Teachers' Deciding How to Respond on the Basis of Students' Algebraic Thinking

In the present study, the prospective teachers' decisions on how to respond on the basis of students' algebraic thinking were grouped under three categories: extending/supporting students' algebraic thinking, asking a drill as a response, and providing a general response. In the first category, which is extending/supporting students' algebraic thinking, prospective teachers gave a response in order to extend or support students' existing algebraic thinking. When the prospective teachers asked students a practice without extending/supporting their algebraic thinking, their responses were coded as asking a drill as a response. Responses in which teachers did not take students' thinking into consideration and suggested direct instruction were coded as providing a general response to students' algebraic thinking. Finally, some prospective teachers did not give response to students' algebraic thinking. The summary of the categorization of prospective teachers' expertise in deciding how to respond on the basis of students' algebraic thinking is given below.

Table 4.13. *The Coding for Deciding How to Respond on the Basis of Students' Algebraic Thinking*

Deciding How to Respond on the Basis of Students' Algebraic Thinking	**Coding**
Extending/supporting Students' Algebraic Thinking	If the student solves the problem correctly, student's existing thinking is extended with a different question. If the student solves the problem wrongly, student's existing thinking is supported by asking a question to make the student recognize his/her mistake.
Asking a Drill as a Response	If the student solves the problem correctly, a similar question is asked in order to reinforce student's knowledge without extending student's algebraic thinking. If the student solves the problem wrongly, a similar question is asked without supporting student's algebraic thinking.
Providing a General Response	Not consider student's thinking. Ask the question that is independent from student's thinking. Suggest direct instruction.
No responses	

The percentage of prospective teachers' responses for each category of deciding how to respond is given in Table 4.14 below.

Table 4.14. *The Percentage of Prospective Teachers' Responses for Each Category of Deciding How to Respond*

Student Solution	Extending/supporting student's algebraic thinking	Asking a drill as a response	Providing a General response
Student A's Solution	68.75%	3.13%	25%
Student B's Solution	21.88%	21.88%	53.13%
Student C's Solution	62.5%	18.75%	12.50%
Student D's Solution	3.13%	34.38%	56.25%
Student E's Solution	62.5%	12.50%	21.88%

N=32

*The solutions of B and D are correct; A, C and E are incorrect.

A seen in the Table 4.14., nearly two thirds of the prospective teachers responded to student A and about one fifth of them responded to student B by extending/supporting student's algebraic thinking. Both solutions belong to the same question which included far generalization concept, but student A's solution is correct whereas student B's solution incorrect. In other words, these prospective teachers responded by extending/supporting thinking of student with incorrect solution more than student with correct solution in the concept of far generalization. On the other hand, more than half of them responded to student C by supporting his/her algebraic thinking which requires knowing near generalization, writing the rule of pattern and inverse process. Furthermore, in relation to the question including far generalization, near generalization and writing the rule of pattern concepts, less than 5% of prospective teachers responded to student with correct solution and more than half of them responded to student with incorrect solution by extending/supporting their thinking. On the other hand, less than half of the prospective teachers asked a drill as a response to all students on the basis of their algebraic thinking. As far as providing a general response is concerned, more than half of the prospective teachers provided general responses to students with correct solutions whereas less than 30% of them provided general responses to students with incorrect solutions.

Example of responses for each category of deciding how to respond on the basis of students' algebraic thinking are given below.

4.3.1. Extending/Supporting Students' Algebraic Thinking

Responses in which prospective teachers guide students who solved the problem correctly by extending their algebraic thinking and responses in which prospective teachers guide students who solved the problem wrongly by supporting their algebraic thinking were coded in this category. The number and percentage of prospective teachers who decided to respond to student A B, C, D and E by extending/supporting students' algebraic thinking are presented in Table 4.15. below.

Table 4.15. *Number and Percentage of Teachers Providing Extending/Supporting Students' Algebraic Thinking Response*

Student Solution	The number of prospective teachers	Percentage
Student A's Solution	22	68.75%
Student B's Solution	7	21.88%
Student C's Solution	20	62.5%
Student D's Solution	1	3.13%
Student E's Solution	20	62.5%

N=32

*The solutions of B and D are correct; A, C and E are incorrect.

According to the Table 4.15., in this study, nearly 69% of prospective teachers decided to respond to student A and 62.50% of prospective teachers decided to respond to student C and student E by extending or supporting the students' algebraic thinking. However, only nearly 3% of prospective teachers decided to respond to student D by extending or supporting the student's algebraic thinking.

In the following example, the prospective teacher's aim of asking the question was to extend students' algebraic thinking.

Student D's solution

Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring a group enters that has two more persons than the group that entered on the previous ring,

a) How many guests will have arrived after the 5^{th} ring?
b) How many guests will have arrived after the 100^{th} ring?
c) How many guests will have arrived after the n^{th} ring?

a) $1 \xrightarrow{+2} 2 \xrightarrow{+2} 3 \xrightarrow{+2} 4 \xrightarrow{+2} 5$

$1 + 3 + 5 + 7 + 9 = 25$

b) 2. kez toplam = 4
3. kez toplam = 9
4 kez toplam = 16
5. kez = 25
6 kez = 36
7 kez = 49

8. kez = 64
9. kez = 81
10. kez = 100
⋮
100 kez = 10000 kişi olur

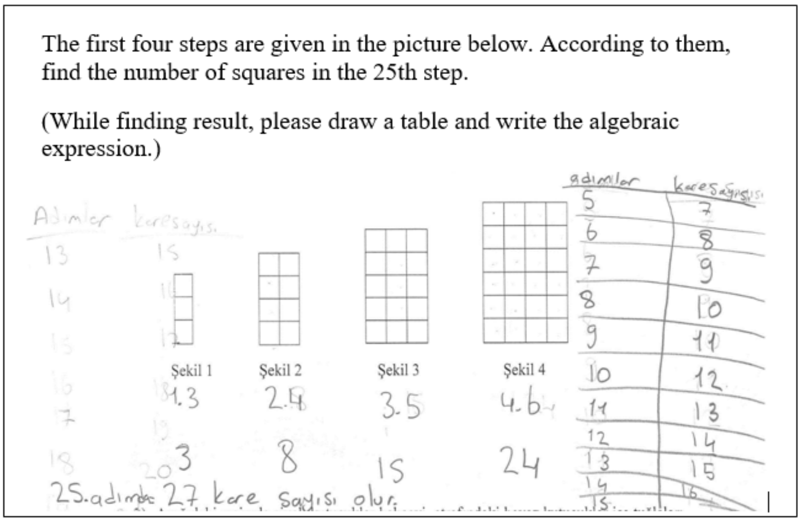
c) n. kez toplam = $n.n = n^2$

In order to respond to student D, PST 3 suggested that *"If there is a total of 144 people at home, how many times the bell has been ringed?"* and explained reasons for asking that question like that "*This student has understood the concept well. Also I asked this question since I wondered what the student will do if the question is asked reversely. Even, the student can connect the question to the concept of square root." (PST 3, an excerpt from the questionnaire)*

PST 3 noticed that student D can form a relationship between the number of rings and the number of people who came home, explored the pattern, and made a generalization. For this reason, s/he *gave the student an opportunity to extend his/her thinking* by asking a question that was related to inverse process. Thus, as presented in the Table 4.13., this response was put into the category of extending/supporting students' algebraic thinking.

Another example of responses supporting students' algebraic thinking is as follows:

Student A's solution



In order to respond to student A, PST 7 suggested that *"(1) You said there were 24 squares in the 4th step, and there were 27 squares in the 25th step. How many shapes were there between figure 4 and figure 25? Do you think it makes sense that the difference between them is 3? (2) Can you draw figure 5? Then can you compare the number you found in figure 5 and figure 24? (3) (I asked the student to make an estimation.) What has changed in the rows and columns after each step? If the number of rows and columns increases by 1, at least how many more squares will there be in figure 5 than figure 4? Can you make an estimation about the number of squares in figure 5? If the number of rows and columns increases by 1, what is the difference in number between the number of squares in figure 5 and the number of steps in figure 4? Can you estimate the number of squares in figure 5?"* Then PST 7 explained reasons for asking these questions like that *"If the student turns back and realizes that the number of steps in the 5th step will have at least 6 squares more than the number of steps in the 4th step, s/he will understand her/his error. My aim in this question is to show adding one row and one column to the figure at each step. After the student understands that, s/he will also understand that the required number of squares for the 24th step is 27 is ridiculous. Moreover, I made the student draw the shape of the 5th*

figure since s/he understands the increase in rows and columns by drawing easily. Furthermore, making an estimation is very important because student can recognize his/her mistake if s/he makes an estimation." (PST 7, an excerpt from the questionnaire)

Although student A could discover the relationship between the number of squares and the number of rows and columns, s/he could not generalize the pattern due to the mistakes in creating the table. Thus, PST 7 tried to make student A recognize his/her mistake through different questions. PST 7 asked the first question so that student can recognize that having 27 squares in the 25th term is not logical while there are 24 squares in the 4th step. In the second question, it was aimed that student A saw that there were 35 squares in the 5th step which is more than 27. After student A realized his/her mistake, PST 7 guided the student to make a generalization through the third question. Therefore, PST 7 *supported his/her algebraic thinking,* and according to the Table 4.13., his/her response was coded in the category of extending/supporting students' algebraic thinking.

In brief, when prospective teachers extend the algebraic thinking of students who solved the problem correctly or support algebraic thinking of students with misconceptions, their responses were coded in the category of extending/supporting students' algebraic thinking.

Different from the responses that extended/supported students' algebraic thinking, some prospective teachers' responses were put into the category of asking a drill as a response to students' algebraic thinking. The details and examples of this category are given below.

4.3.2. Asking a Drill as a Response

In this study, some prospective teachers responded to students by asking a drill as a response. The number and percentage of prospective teachers who decided to respond to student A, B, C, D and E by asking a drill as a response are presented in Table 4.16 below.

Table 4.16. *Number and Percentage of Teachers Asking a Drill as a Response*

Student Solution*	The number of prospective teachers	Percentage
Student A's Solution	1	3.13%
Student B's Solution	7	21.88%
Student C's Solution	6	18.75%
Student D's Solution	11	34.38%
Student E's Solution	4	25%

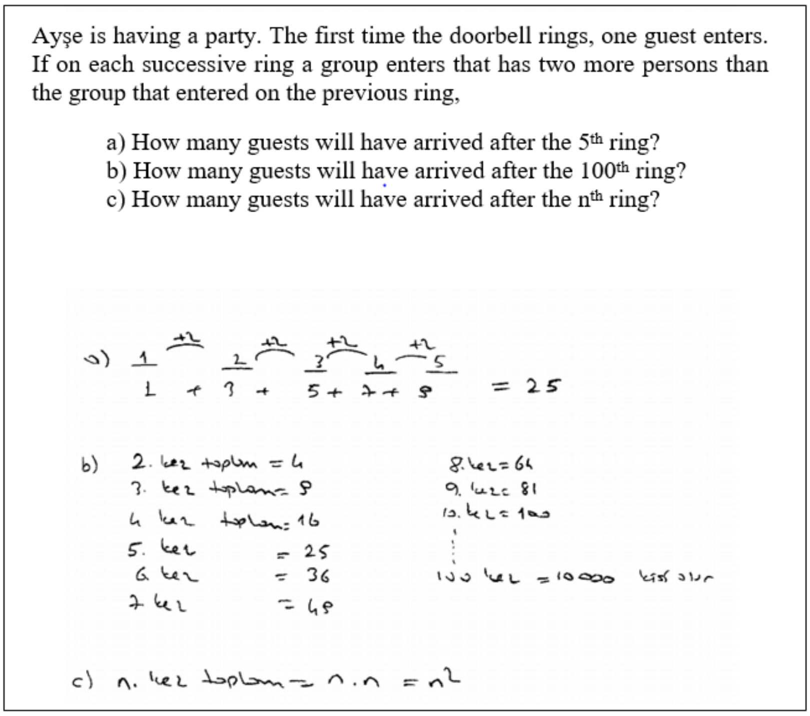
N=32

*The solutions of B and D are correct; A, C and E are incorrect.

The Table 4.16. shows that in the current study, although only 3.13% of prospective teachers responded to student A by asking a drill, the percentage of prospective teachers who responded to other four students by asking a drill was approximately 25%.

The example of a response involving asking a drill is given below.

Student D's solution



Ayşe is having a party. The first time the doorbell rings, one guest enters. If on each successive ring a group enters that has two more persons than the group that entered on the previous ring,

a) How many guests will have arrived after the 5th ring?
b) How many guests will have arrived after the 100th ring?
c) How many guests will have arrived after the nth ring?

a) $\underset{1}{1} \overset{+2}{\frown} + \underset{3}{2} \overset{+2}{\frown} + \underset{5}{3} \overset{+2}{\frown} + \underset{7}{4} \overset{+2}{\frown} + \underset{9}{5} = 25$

b) 2. kez toplam = 4
3. kez toplam = 9
4 kez toplam: 16
5. kez = 25
6 kez = 36
7 kez = 49
8. kez = 64
9. kez = 81
10. kez = 100
⋮
100 kez = 10000 kişi olur

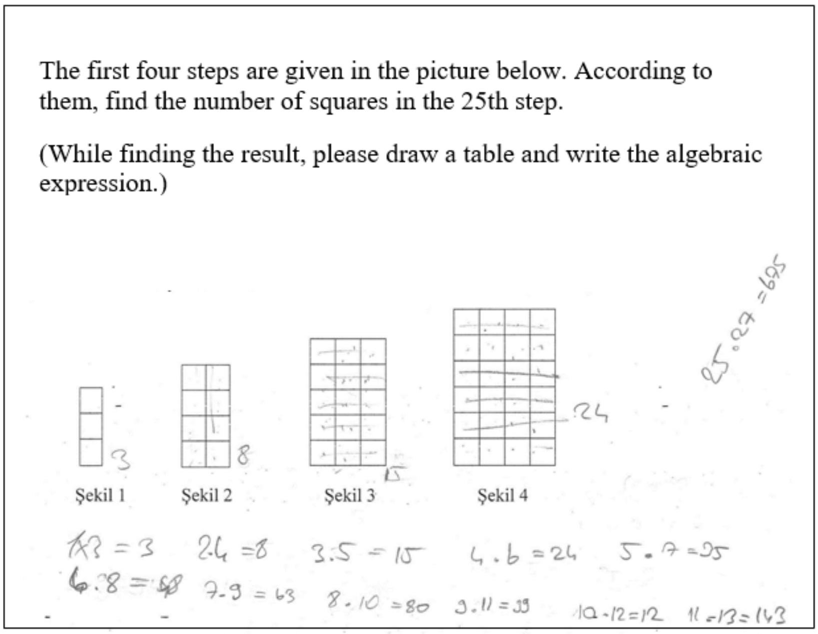
c) n. kez toplam = n.n = n^2

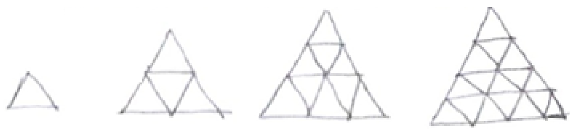
In order to respond to student D, PST 5 suggested that *"I asked the question like at each ring, twice the number of people are coming home than the previous ring*." Then, PST 5 explained reasons for asking that question like that "*I think that the student can understand the concept well. For this reason, I wanted to ask a more complex question since I wanted to check whether the student learned to find a rule and can make a generalization or not. This question will be a more complex question for the student." (PST 5, an excerpt from the questionnaire)*

Although student D explored the pattern, made a generalization, and solved the problem correctly, PST 5 asked a similar problem with the same context and with different operations. For this reason, as indicated in the Table 4.13., this question became a drill for student D and did not give opportunity to extend his/her algebraic thinking.

PST 4’s response is another example of asking a drill as a response to students’ algebraic thinking.

Student B’s solution





In order to respond to student B, PST 4 suggested that *“Find the number of triangles in the 25th step.”* Then PST 4 explained reasons for asking that questions like that *“The student found the number of squares correctly. In order to both reinforce student’s knowledge and evaluate student’ approach to a different type of pattern, this type of question can be asked.” (PST 4, an excerpt from the questionnaire)*

Student B solved the problem correctly by exploring a pattern and making a generalization. However, the prospective teacher *asked a similar problem with just a* *different context* and this question enabled the student to do a practice instead of extending his/her algebraic thinking. Thus, according to the Table 4.13., PST 4's response to student B was coded as asking a drill as a response.

In conclusion, the category of responses that give the opportunity to have a practice, but that do not extend or support students' algebraic thinking were called as asking a drill as a response.

4.3.3. Providing a General Response

Instead of extending/supporting students' algebraic thinking and asking a drill as a response, some prospective teachers' provided general responses to students. In these responses, they did not take students' algebraic thinking into consideration and they gave direct instruction. The number and percentage of prospective teachers who decided to respond to student A, B, C, D and E with a general response are presented in Table 4.17. below.

Table 4.17. *Number and Percentage of Teachers Providing a General Response*

Student Solution	The number of prospective teachers	Percentage
Student A's Solution	8	25%
Student B's Solution	17	53.13%
Student C's Solution	4	12.50%
Student D's Solution	18	56.25%
Student E's Solution	7	21.88%

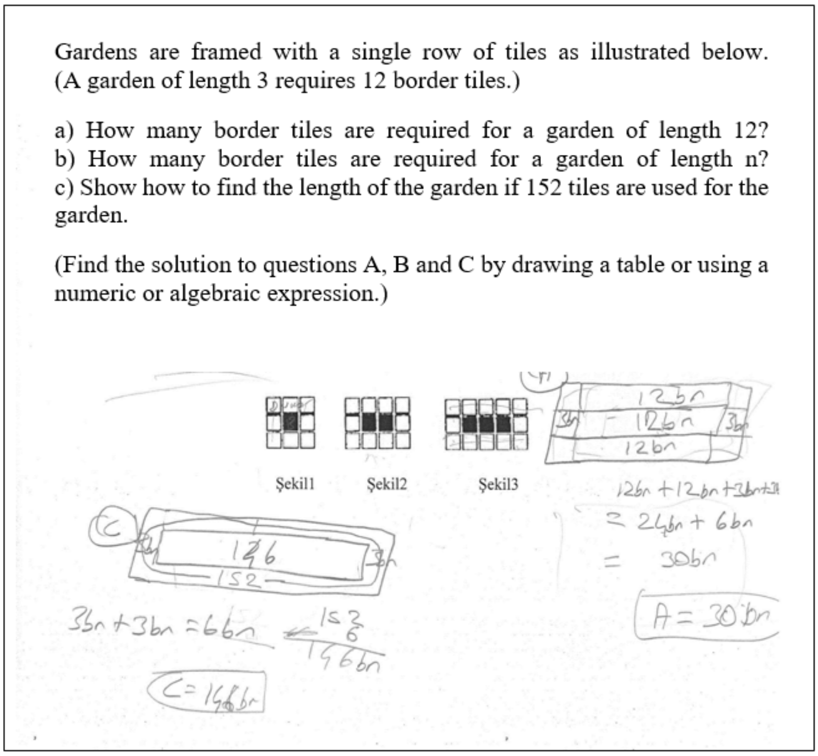
N=32

*The solutions of B and D are correct; A, C and E are incorrect.

According to the Table 4.17., in the present study, nearly half of the prospective teachers provided a general response to student B and student D. However, 25% and nearly 22% of prospective teachers provided general response to student A and student E, respectively. Finally, only 12.5% of prospective teachers gave a general response to student C.

One example of this response is given below.

Student C’s solution

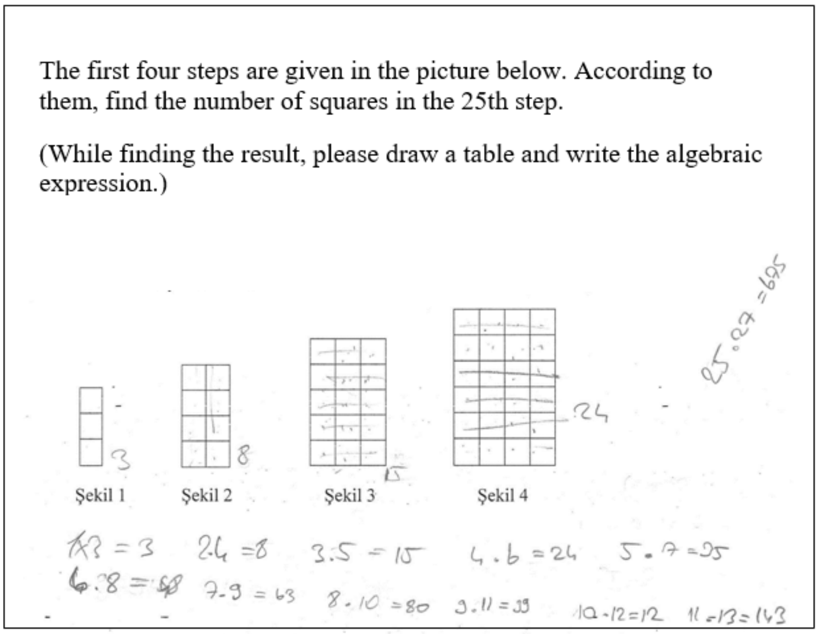


In order to respond to student C, PST 13 suggested that *“I would think that garden is square or a different rectangle”* and explained reason for asking that question like that *“In order to investigate whether the student developed a strategy for this pattern only, I asked the question like that.” (PST 13, an excerpt from the questionnaire)*

PST 13 *asked the question without considering the student's algebraic thinking*. Moreover, this question was independent from student's thinking; thus, as presented in the Table 4.13., the response was placed in the category of providing a general response.

Another example of providing a general response to student B is PST 18's response that is given below.

Student B's solution



In order to respond to student B, PST 18 suggested that *"Even, student B solved the question correctly. I asked a similar problem involving different patterns."* Then, PST 18 explained reason for asking that question like that *"I enabled the student to do some pratice by asking a similar problem involving different patterns." (PST 18, an excerpt from the questionnaire)*

In this response, partipants *did not consider student's algebraic thinking*. Actually, problem was independent from student's thinking since PST 18 only stated which type of question s/he wanted to ask, but did not ask anything. Becuase of these reasons, this response was coded as a category of providing a general response as presented in the Table 4.13.

In the present study, when prospective teachers responded with an irrelavant response,when they could not ask a real question, or only focused on the characteristic of the question that they wanted to ask such as a more complex question, these responses were categorized as providing a general response.

In conclusion, in this study prospective teachers' noticing skills were investigated under three dimensions which are attending to students' solutions, interpreting students' algebraic thinking, and deciding how to repond on the basis of the students' algebraic thinking. While the findings related to attending to students' solutions and interpreting students' algebraic thinking were classified under four categories, the findings related to deciding how to respond on the basis of student's algebraic thinking included three categories.

CHAPTER 5

DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

The aim of this study was to examine prospective middle school mathematics teachers’ noticing skills of students’ algebraic thinking within the context of pattern generalization. In the light of this aim, the findings of this study are discussed with references to previous studies in the literature. In addition, educational implications and recommendations for future research studies are presented in this chapter.

5.1. Discussion

The findings of this study are discussed under three main sections based on the research questions. To be more specific, in the first part, prospective middle school teachers’ attending to students’ solutions regarding pattern generalization is discussed. In the second part, prospective middle school teachers’ interpretation of students’ algebraic thinking within the context of pattern generalization is discussed. In the third part, the nature of prospective middle school teachers’ decisions to respond to students within the context of pattern generalization is discussed. The findings are also compared and contrasted with previous research studies in the literature.

5.1.1. Prospective Teachers’ Attending to Students’ Solutions

The analysis of the data revealed that the majority of the prospective teachers provided robust evidence of attending to students’ solutions. To be more specific, most of the prospective teachers who participated in this study could identify students’ solutions with all the mathematical details. They explained in detail how students solved the problem, which strategies they used, whether their solutions were correct or not, and what their mistakes were. These findings might be considered as consistent with previous researches which reported that prospective teachers and teachers are good at attending to students' solutions (Barnhart & van Es, 2015; Callejo & Zapatera, 2017; Dick, 2013; LaRochelle, 2018; Sánchez-Matamoros, Fernández & Llinares; 2019; Talanquer, Bolger, & Tomanek, 2015). In addition, similar to these previous research studies, Jacobs et al. (2010) revealed that prospective teachers could attend to students' solutions irrespective of whether they are correct or incorrect. The success of prospective teachers in attending to students' solutions might be due to the fact that prospective teachers could focus on important details of students' answers, and they wrote many details of the mathematical elements regarding pattern generalization in students' solutions (Talanquer, Bolger, & Tomanek; 2015). Another reason for prospective teachers' success in attending might be the fact that attending is the easiest component-skill for prospective teachers (LaRochelle, 2018; Sánchez-Matamoros, Fernández & Llinares; 2019). Since understanding how students solved the problem and identifying students' entire solutions step by step were sufficient as far as attending to students' solutions is concerned, many prospective teachers in this study might provide robust evidence of this skill. In addition, since prospective teachers took the Methods of Teaching Mathematics I/II courses, it is not surprising that most of them provided robust evidence of attending to students' solutions in the current study. Prospective teachers mathematically learn each detail and various solution strategies about each topic in learning domains thanks to the Methods of Teaching Mathematics I/II courses. Therefore, taking these courses might have contributed to their skill of attending to students' solutions.

Although the vast majority of the prospective teachers (65%) attended to students' solutions with robust evidence and emerging evidence of attention to students' solutions, the rest could not identify all the mathematical elements of students' solutions. Schoenfeld (2011) stated that knowledge can impact what teachers attend to, and the reason behind this inadequacy might result from lack of prospective teachers' knowledge.

5.1.2. Prospective Teachers' Interpreting Students' Algebraic Thinking

Although a vast majority of the prospective teachers could attend to students' solutions regarding pattern generalization with robust evidence and emerging evidence, the prospective teachers in this study had difficulty in analyzing and interpreting students' algebraic thinking with robust evidence. The findings also revealed that the percentage of prospective teachers who interpreted students' algebraic thinking with limited evidence is significant. This means that many prospective teachers in this study interpreted only whether student could comprehend pattern generalization or not, but did not refer to specific points regarding student's algebraic thinking. These findings are consistent with the findings of previous research studies (Barnhart & van Es, 2015; Sánchez-Matamoros, Fernández & Llinares, 2019; Monson, Krupa, Lesseig, & Casey; 2018; Talanquer, Bolger, & Tomanek, 2015). For example, the findings of Sánchez-Matamoros et al. (2019) reported that although some prospective teachers attended to the mathematical considerations in students' answers, they had difficulties in analyzing students' understanding based on their answers. These findings are also consistent with the study of Sánchez-Matamoros et al. (2019) in which prospective teachers interpreted students' understanding only by saying that correct or incorrect solutions were reported (Sánchez-Matamoros et al., 2019).

The reason for prospective teachers' difficulty in interpreting students' algebraic thinking might be that identifying mathematical essences of students' solution is necessary, but not enough to making sense of students' mathematical understanding (Monson et al., 2018). In other words, in order to make sense of students' mathematical understanding, prospective teachers or teachers should recognize and understand students' strategies and solutions. However, the ability of identifying all the solutions of students with mathematical details does not guarantee that prospective teachers can interpret their understanding. In order to be able to interpret students' understanding, they should make inferences from solution methods, mistakes or misconceptions as well as attending to their solutions.

The another reason for prospective teachers' difficulty in interpreting might be the possibility that the participants were prospective teachers as opposed to in-service teachers. The related studies stated that in-service teachers have the opportunity to see real classroom environment and real students' solutions and they are expected to focus on student ideas and have the skill to interpret their ideas (Barnhart & van Es, 2015). On the other hand, prospective teachers learn most of the knowledge about teaching mathematics theoretically, and they do not have experience to catch students' ideas and analyze them (Barnhart & van Es, 2015). Thus, the participant profile of this study might have affected the findings.

Another reason behind the difficulties prospective teachers experience might be lack of knowledge of content and students (KCS) and knowledge of content of teaching (KCT). KCS is defined as a combination of mathematics knowledge of the teacher and the knowledge of cognitive development of the student. Teachers' foresight of student's possible errors and misconceptions that are expected to be encountered is related to KCS knowledge (Hill, Ball, & Schilling, 2008). Unlike KCS, KCT is a combination of mathematical knowledge and knowledge and skills of the teacher. Being able to prepare a lesson plan for effective and easy teaching of the subject and provide appropriate examples and demonstrations are under KCT (Hill, Ball, & Schilling, 2008). Therefore, the more prospective teachers' knowledge about the content, students, and teaching, the better they could interpret students' mathematical understanding. Hence, prospective teachers' lack of knowledge of content and students (KCS) and knowledge of content of teaching (KCT) might be the reason for prospective teachers' difficulties in interpreting students' mathematical understanding.

In this research study, prospective teachers were asked to notice students' incorrect solutions as well as students' correct solutions. Hence, the final finding was related to prospective teachers' ability of interpreting students' correct and incorrect solutions. The majority of the prospective teachers could provide robust and emerging evidence of interpreting students' correct solutions, whereas many prospective teachers provided emerging and limited evidence of interpreting students' incorrect solutions. The reason for this finding might be that the prospective teachers could easily follow students' correct solutions and make sense of their correct solutions, while they had difficulty in understanding students' incorrect solutions and interpreting what having such mistakes and misconceptions mean for their mathematical understanding. It can be said although most of the prospective teachers could identify students' solutions demonstrating robust evidence or emerging evidence irrespective of their correctness, they analyzed correct solutions better than incorrect solutions. Hence, the fact that attending to students' solutions is easier than interpreting students' algebraic thinking has been confirmed as stated in others studies (Jacobs et al. 2010; Schack et al. 2013; Sa´nchez-Matamoros et al. 2014) based on the skills described by Jacobs et al. (2010).

5.1.3. Prospective Teachers' Deciding How to Respond on the Basis of Students' Algebraic Thinking

The analysis of the data revealed that most of the prospective middle school mathematics teachers responded to students with incorrect answers by supporting their thinking, whereas few prospective teachers provided response by asking a drill or making general comments. Hence, it can be said that most of the prospective teachers could respond to students who had incorrect answers effectively. Similarly, in previous studies it was revealed that prospective teachers could recognize students' mistakes and advance their thinking (Jacobs & Ambrose, 2008; Milewski & Strickland, 2016). The reason for this finding might be that asking students to reflect on how their strategy relates to key mathematical ideas or relationships within the task could help them recognize their mistakes and support student thinking (Jacobs & Ambrose, 2008; Milewski & Strickland, 2016). However, the findings of some research are inconsistent with this finding (Ball, 1993; Crespo, 2002; Lampert, 2001; Milewski & Strickland, 2016; Son & Crespo, 2009). According to these research studies, teachers had difficulty in responding to students (Ball, 1993; Lampert, 2001).

Specifically, it was revealed that they struggled to respond to students who made errors by providing space in order to further students' thinking (Crespo, 2002; Son & Crespo, 2009) and they responded to students with incorrect answers by showing procedures or stating the correct answers (Crespo, 2002; Milewski & Strickland, 2016).

However, the analysis of the data demonstrated that most of the prospective teachers responded to students who had correct answers by asking a drill or providing a general response, while few prospective teachers could extend students with correct solutions. Hence, it was revealed that prospective teachers had difficulty in responding to students who had correct solutions as stated in Taylan's (2018) study. Besides, in some previous research studies, similar findings were reported about responding to students with correct solutions with general responses (Crespo, 2002; Krupa et al., 2017; Milewski & Strickland, 2016). For example, in Krupa et al.'s (2017) study, most of the prospective teachers provided procedural actions or provided general comments as a response. This finding might have resulted from the fact that when students solved the problem correctly, prospective teachers believed that mission was completed. Thus, they did not need to guide students to extend their existing knowledge; on the contrary, they asked a drill or provided general responses. Moreover, teachers might consider that praise is a sufficient response to correct solutions (Crespo, 2002; Milewski & Strickland, 2016) without extending students' mathematical understanding. Briefly, it can be said that prospective teachers respond to students with incorrect solutions more effectively than students with correct solutions because although they easily overcome students' misconceptions, they have difficult time advancing students' algebraic thinking with correct solutions.

As a consequence, it might be stated that in the present study, prospective teachers' skills of deciding how to respond on the basis of students' algebraic thinking is independent from their skills of attending to students' solutions. Moreover, there was no clear relationship between the prospective teachers' interpreting and deciding how to respond on the basis of students' algebraic thinking. However, according to the findings of the study, prospective teachers' ability of responding changed depending on whether students' solutions were correct or incorrect.

Although the findings of the study are largely consistent with the related studies (Barnhart & van Es, 2015; Callejo & Zapatera; 2017; Jacobs & Ambrose, 2018; Milewski & Strickland; 2016; Sánchez-Matamoros, Fernández & Llinares, Taylan, 2018) in terms of the extent to which teachers attend to students' solutions, interpret students' algebraic thinking and decide how to respond, some differences are caused by the nature of the issue focused. The nature of algebra includes "generalizing and formalizing patterns and constraints" (Kaput, 1999, p.4), being able to thinking about unknown quantities as known (Swafford & Langrall, 2000) and making a sense of symbols and operations in terms of arithmetic (Kiearan & Chalouh, 1993), generalizing pattern helps students to transform arithmetic to algebra (Kiearan & Chalouh, 1993). The pattern generalization process consists of four process (Radford, 2008; Rivera & Becker; 2009; Warren, 2005): (1) finding near stage of sequence by drawing and counting (near generalization), (2) noticing the pattern and finding far stage of sequence by making reasoning (far generalization), (3) exploring the general rule of pattern by identifying functional relationship between the stage number and pattern (writing a rule of pattern) and (4) identifying a stage number in inverse functional relationship (inverse process). In detail, in order to generalize the pattern, firstly, students have to find the near stage which is close enough to draw or count (e.g. 5) and they have to find the far stage by using near generalization (e.g. 100). Afterwards, near generalization and far generalization enables them to discover general rule of pattern (Lannin, Barker, Townsend, 2006). Therefore, due to the nature of pattern generalization, students need to make a reasoning in each process of pattern generalization and solve problems related to pattern generalization step by step (Jurdak & El Mouhayar, 2014; Lannin et al., 2006; Radford, 2008). Since pattern generalization has such a nature, selected five student solutions in this study included step by step solutions. Step by step solution might make attending to students' solutions and interpreting students' algebraic thinking easier for prospective teachers in comparison with the other mathematical topics. Moreover, in order to respond to students on the basis of their algebraic thinking within the context of pattern generalization efficiently, prospective teachers might have needed to take consideration into students' algebraic thinking in each process of problem solving such as near generalization, far generalization and writing a rule of pattern. Therefore, deciding how to respond to students who solved problems related to pattern generalization might be hard process due to the nature of pattern generalization.

Finally, when the data was analyzed, it was seen that the categories of attending, interpreting and deciding how to respond based on Jacobs et al.'s framework did not cover all the data in this study. For this reason, Jacobs et al.'s framework was extended through the analysis of the present study.

Consequently, the present study examined prospective middle school mathematics teachers' skills of attending to students' solutions, interpreting their algebraic thinking, and deciding how to respond on the basis of their algebraic thinking. Thus, this study took a deeper look at prospective teachers' skills of noticing students' algebraic thinking within the context of pattern generalization. Therefore, it would be very important to present some implications for educational practices and recommendations for further studies according to the findings of the current study and those of previous studies. In the following sections, the practical and research-based issues are given in line with the findings of this study together with the findings of previous studies.

5.2. Implications for Educational Practices

In this study, middle school prospective mathematics teachers' skills of noticing students' algebraic thinking within the context of pattern generalization were examined. In the light of the findings, this study has several implications for prospective teachers, in-service teachers, curriculum developers, and teacher educators.

In order to conduct effective mathematics teaching, teachers have to identify the noteworthy aspects of students' solutions, make sense of students' understanding from their solutions and build a connection between students' understanding and possible teaching and learning methods/strategies in teaching environment. Therefore, teachers' noticing is a significant competency (van Es & Sherin, 2002; Sherin, Russ, & Colestock, 2011; Jacobs, Lamb, & Philipp, 2010). However, according to the findings of this study, although the majority of the prospective teachers could attend to students' solutions, they had difficulty in interpreting students' algebraic thinking based on their solutions. In the previous section, it was discussed that the reason for this finding might be lack of experience (Barnhart & van Es, 2015). It was also stated that having experience of noticing students' mathematical understanding can be useful in order to enrich prospective teachers' skills of noticing students' mathematical understanding (Jacobs et al., 2010; Barnhart & van Es, 2015). The findings of the present study also revealed that although prospective teachers could support students who solved the problems incorrectly with appropriate questions, they could not extend thinking of the students who provided correct solutions. Therefore, emphasizing how to respond on the basis of different students' thinking is a significant and necessary skill (Jacobs et al., 2010). For this reason, teacher educators might integrate practices with respect to the improvement of noticing skills in the content of courses for the professional development of preservice teachers such as the Methods of Teaching Mathematics I/II courses. To illustrate, teacher educators could involve wrong student solutions that consist of conceptual and procedural errors and correct these solutions in different ways within course. Furthermore, teacher educators can select tasks which provide accurate representation of teachers' or prospective teachers' noticing and can facilitate conversations which involve weighing the affordances and constraints of students' possible responses in the courses of Methods of Teaching Mathematics I/II. Similarly, these kinds of practices and tasks could be integrated into school experience and teaching practice in order to provide prospective teachers with the opportunity to notice real students' works and improve their skills of noticing. In this way, prospective teachers may have the opportunity to practice the skills of attending to students' solutions, interpreting their thinking, and deciding how to respond to students. Moreover, prospective teachers could share their noticing and get feedback from teacher educators and colleagues through interaction in the classroom environment.

In addition, the findings in previous studies also revealed that prospective teachers and teachers can learn to notice students' solutions and this skill can be enhanced through intervention such as teacher training programs or professional development programs (Jacobs, Lamb & Philipp, 2010; Osmanoğlu, 2010; Ulusoy & Çakıroğlu, 2018; Star & Strickland, 2008; van Es & Sherin, 2008). For this reason, teacher educators could provide interventions such as teacher training programs or professional development programs which could be effective in enriching prospective teachers' skills of noticing students' ideas.

Implications for educational practices were touched upon in this section in line with the findings of previous studies and those of the current study. Since some issues emerged from the findings of the present study, a number of recommendations are available in the following section.

5.3. Recommendations for Further Research Studies

Prospective teachers studying in their fourth year at one of the public universities participated in the present study and their skills of noticing students' algebraic thinking were examined. Prospective teachers completed the Methods of Teaching Mathematics I/II courses at the end of the third year and took the Teaching Practice and School experience courses in the fourth year of the Teacher Education Program. The same study might be conducted with prospective teachers who are in their third year in the same context. Hence, the noticing skills of prospective teachers in the third year and fourth year who studied at the same university might be compared. Therefore, the influence of Methods of Teaching Mathematics I/II, Teaching Practice and School experience courses on prospective teachers' skills of attending to students' solutions, interpreting their algebraic thinking, and deciding how to respond on the basis of their algebraic thinking can be explored.

According to the findings of this study, most of the prospective teachers could attend to students' solutions about pattern generalization; however, they had difficulty in interpreting students' algebraic thinking. Moreover, the present study revealed that prospective teachers could not effectively respond to students who had correct solutions. In the previous section, lack of experience was considered as the reason for these inadequacies in prospective teachers' skills of noticing, but how teaching experience affects teachers' noticing was not explored in this study. Therefore, in order to explore the effect of teaching experience on teachers' skills of noticing, this study can be extended through longitudinal research. In order to examine how experience affects prospective teachers' skills of noticing after they become teachers and they gain experience, it is strongly recommended to examine prospective teachers' skills of noticing with the same participants for several years.

In order to conduct this research, thirty-two prospective teachers who were enrolled in their fourth year at one of the public universities, so this study is limited to prospective teachers. Different from prospective teachers, in-service teachers have had the opportunity to see real classroom environment and real students' solutions. Also, they can observe how students learn a subject or what misconceptions or difficulties they have. For these reasons, as a further research, the same study can be conducted with in-service teachers to explore their noticing of students' algebraic thinking. Moreover, in this way, the effect of teaching experience on the teachers' skills of noticing students' algebraic thinking can be explored.

In the present study, the qualitative research method was used in order to explore prospective teachers' skills of noticing students' algebraic thinking within the context of pattern generalization. As a further study, it is strongly recommended that quantitative research methods be used to investigate the noticing skills of prospective mathematics teachers. Hence, a study can be conducted with a sample randomly selected from nationwide universities in such a way that the sample would be representative of all prospective teachers in Turkey. Therefore, findings that are related to the holistic picture of noticing skills of prospective teachers in Turkey can be presented.

Algebraic thinking, which is one of the primary components of algebra, improves through the generalization of pattern (Warren & Cooper, 2006). Hence, students can think about unknown quantities as known and make a transition from arithmetic to algebra. Thus, pattern generalization is a significant topic in order to enhance students' algebraic thinking and mathematical understanding. For this reason, in the present study, prospective teachers' skills of noticing algebraic thinking were assessed within the context of pattern generalization. However, this study was limited in that it investigated prospective teachers' skills of noticing students' mathematical understanding within the context of only one topic. Therefore, prospective teachers' skills of noticing can also be investigated within different contexts, and in this way, a holistic picture of prospective teachers' skills of noticing can be obtained.

In the previous sections, it was discussed that limited skills of attending students' solutions and interpreting their algebraic thinking might result from prospective teachers' lack of knowledge. In addition, although the relationship between professional teacher noticing and mathematical knowledge has been addressed in some research studies (Schoenfeld, 2011; Monson, Krupa, Lesseig, & Casey, 2018), it is not clear how this knowledge affects teachers' attending, interpreting and deciding how to respond on the basis of their solutions. Moreover, this study did not explore the relationship between teachers' knowledge and their skills of noticing. Therefore, this study could be extended with the investigation of the relationship between mathematical knowledge and teachers' noticing students' algebraic thinking and how prospective teachers with this knowledge and without this knowledge attend, interpret, and decide to respond.

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APPENDICES

A. COURSES IN THE ELEMENTARY MATHEMATICS EDUCATION PROGRAM

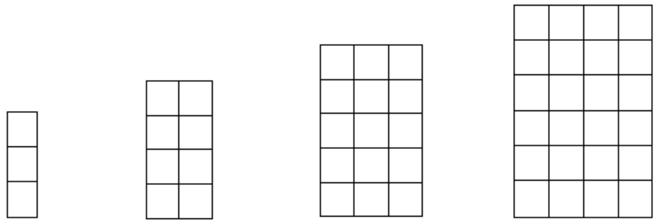
1st YEAR	MATH111	Fundamentals of Mathematics	MATH112	Discrete Mathematics
	MATH115	Analytic Geometry	MATH116	Basic Algebraic Structures
	MATH119	Calculus With Analytic Geometry	MATH120	Calculus for Functions of Several Variables
	EDS200	Introduction to Education	CEIT100	Computer Applications in Education
	ENG101	English For Academic Purposes I	ENG102	English for Academic Purposes II
	IS100	Introduction to Information Technologies and Applications		
2nd YEAR	PHYS181	Basic Physics I	PHYS182	Basic Physics II
	MATH219	Introduction to Differential Equations	MATH201	Elementary Geometry
	STAT201	Introduction to Probability &Stat. I	STAT202	Introduction to Probability & Stat.II
	ELE221	Instructional Principles and Methods	ELE225	Measurement and Assessment
	EDS220	Educational Psychology	ENG211	Academic Oral Presentation Skills
	HIST2201	Principles of Kemal Atatürk I	HIST2202	Principles of Kemal Atatürk II
	HIST2205	History of The Turkish Revolution I	HIST2206	History of The Turkish Revolution II
	MATH260	Basic Linear Algebra	ELE310	Community Service

3rd YEAR	ELE341	Methods of Teaching Mathematics I	ELE329	Instructional Technology And Material Development
	TURK201	Elementary Turkish	ELE342	Methods of Teaching Mathematics
	TURK305	Oral Communication	EDS304	Classroom Management
		Elective	TURK202	Intermediate Turkish
		Elective	TURK306	Written Expression
4th YEAR	ELE301	Research Methods	ELE420	Practice Teaching In Elementary Education
	ELE435	School Experience	EDS416	Turkish Educational System And School Management
	ELE465	Nature of Mathematical Knowledge for Teaching	EDS424	Guidance
		Restricted Elective		Elective
		Elective		

B. TURKISH VERSION OF QUESTIONS IN QUESTIONNAIRE FOR MIDDLE SCHOOL STUDENTS

Soru1:

Aşağıdaki resimde bir örüntünün ilk dört adımı verilmiştir. Buna göre, 25.adımdaki kare sayısını bulunuz. Çözümü yaparken, tablo çiziminden, cebirsel ifade kullanımından yararlanınız.



Şekil 1 Şekil 2 Şekil 3 Şekil 4

Soru 2:

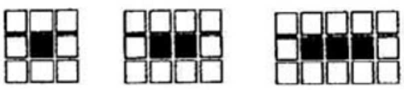
Aşağıdaki resimde siyah kutucuklar bahçeyi, etrafındaki beyaz kutucuklar ise tuğlaları göstermektedir. Her bir bahçenin etrafı bir sıra tuğla ile örülmüştür. (Örneğin, 3.şekildeki uzun kenarı 3 br olan bahçe için 12 tane tuğla kullanılmıştır.)Buna göre;

A) Uzun kenarı 12 br olan bir bahçe için kaç tane tuğla kullanılır?

B) Uzun kenarı n br olan bir bahçe için kaç tane tuğla kullanılır?

C) Toplam 152 tane tuğla kullanılan bahçenin uzunluğunu nasıl bulduğunuzu gösteriniz.

(A, B, C sorularının çözümlerini çizerek, tablo yaparak, sayısal ifade, cebirsel ifade kullanarak yapınız.)



Şekil1 Şekil2 Şekil3

Soru 3:

Ayşe bir parti düzenlemiştir. Kapı ziliniz ilk çalışında partiye bir kişi gelmiştir. Ardışık olarak çalınan her zilde, bir önceki zilin çalınmasıyla gelen kişi sayısının 2 fazlası gelmiştir.

Buna göre,

a. 5.kez kapı zili çalındıktan sonra Ayşe' nin evine toplamda kaç kişi gelmiş olur?

b. 100.kez kapı zili çalındıktan sonra Ayşe' nin evine toplamda kaç kişi gelmiş olur? Sonucu nasıl bulduğunuzu açıklayınız.

c. n.kez kapı zili çalındığında, Ayşe' nin evine toplamda kaç kişi gelmiş olduğunu hesaplamak için nasıl bir formül oluştururdunuz? Açıklayınız.

C. TURKISH VERSION OF QUESTIONS IN QUESTIONNAIRE FOR PROSPECTIVE TEACHERS

1. “Soruyu çözmek için öğrencinin kullandığı çözüm yöntemini detaylı olarak açıklayınız. Sizce öğrencinin çözümü doğru mu? Neden?”

2. Öğrencinin çözümünden onun konuya ilişkin anlamlandırması (kavrayışı) hakkında ne düşünüyorsunuz? Detaylı olarak açıklayınız.

3. Bu öğrencinin öğretmeni olduğunuzu varsayalım. Bu problemin devamında öğrenciye hangi problem ya da problemleri sorardınız? Bu problem ya da problemleri seçmedeki gerekçeniz nedir?” (Jacobs, Lamb & Philipp, 2010, s.178-179).

D. INTERVIEW QUESTIONS

Görüşme Soruları

1. Öğrenci A’ nın soruyu çözerken kullandığı yolları daha detaylı anlatabilir misiniz?
2. Öğrencinin kavrayışı hakkında ………. söylemişsiniz. Neden böyle bir yorumda bulundunuz? Açıklar mısınız?
3. Eklemek istediğiniz bir şey var mı?
4. Öğrenciye ………..problemi sormuşsunuz. Neden böyle bir soru sordunuz?
5. Sorduğunuz …………problemin öğrenciye katkısı ne olacaktır?
6. Öğrenci bu soruyu cevapladıktan sonra, derse nasıl devam etmeyi düşünüyorsun?
7. Farklı sormak istediğiniz ya da değiştirmek istediğiniz bir şey var mı?

E. PERMISSION FROM THE ETHICAL COMMITTE AT METU

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

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11 ARALIK 2018

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Prof.Dr. Mine IŞIKSAL BOSTAN

Danışmanlığını yaptığınız Zeynep ÖZEL'in **"Investigation into Prospective Middle School Mathematics Teachers Noticing Skill regarding Studenst' Algebraic Thinking with the Context of Pattern Generalization"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek gerekli onay **2018-EGT-142** protokol numarası araştırma yapması onaylanmıştır.

Saygılarımla bilgilerinize sunarım.

Prof. Dr. Tülin GENÇÖZ
Başkan

Prof. Dr. Ayhan SOL
Üye

Prof. Dr. Ayhan Gürbüz DEMİR
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Prof.Dr. Yaşar KONDAKÇI (4.)
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Dr. Öğr. Üyesi Ali Emre TURGUT
Üye

Doç. Dr. Emre SELÇUK
Üye

Doç.Dr. Üyesi Pınar KAYGAN
Üye

F. TURKISH VERSION OF PARTICIPANTS' RESPONSES PRESENTED AS AN EXAMPLE OF EACH CATEGORY

Sayfa 93'te verilen örneğin Türkçesi:

Araştırmacı: Şimdi burada nasıl bulmuş bu soruları a şıkkını mesela…

Katılımcı: Şimdi burada bu eşittir i bir kere eşittir için kullanmamış yani. Tire gibi düşünelim onu. Birinci kapı çalışında bir kişi, ikinci kapı çalışında üç kişi demiş, üçüncü kapı çalışında beş kişi dördüncü kapı çalışında yedi kişi beşinci kapı çalışında da dokuz kişi gelir demiş. Dokuzu da cevap olarak kabul etmiş çünkü evine gelen toplamkişi derken kapı önünden giren kişi sayısı olarak almış bunu.

Araştırmacı: Tamam b de ne yapmış?

Katılımcı: b de ne yapmış, işte akıllı çocuk bak burada örüntü kullanmanın ihtiyacını hissederek yüzüncü kapı çalışına kadar bulmak için demiş ki alt alta yazıyım ben bunlar arasında bir örüntü kurmaya çalışayım. Sonra şuradan adım sayısıyla alakalı bir örüntü kurmaya çalışmış. Bir kere şu bir değil üç olduğunu varsayıyorum, işlem hatasından kaynaklı olarak bulamadığını düşünüyorum. Sonra bakıyorum…Şimdi yine arada hımm beş yedi dokuz tane. Aynen birinci adımı atlamış zaten, Tamamen onu bir es geçmiş işlem hatası yaptığı için mi geçmiş. Kuralı nasıl buluyor biliyor musun?

Şimdi şu adım sayılarıyla gelen kişi sayısı arasında bir bağlantı kurmaya çalışmış. Sonra mesela ikincisi için demiş ki ikinci adım için ben buna ne yapsam üçü bulurum.

Araştırmacı: Tamam

Katılımcı: Üçüncü adım için ben buna ne yapsam beşi bulurum, dörtte ben buna ne yapsam yediyi bulurum demiş. Sonra şu iki çarpı iki eksi birden normalde üçü buluyor, işlem hatası yapmış. Diğerlerini yorumlarsak işte bu bulduğu iki çarpı adım sayısı eksi biri hepsine uygulamış yani. Bulduklarına birkaç tane yazmış herhalde bakıyım, beşi yazmış yediyi yazmış dokuzu yazmış. Sonra bakmış üçü içinde tutuyor, o zaman bu bir kuraldır onun için öyle kabul etmiş. Sonrasında da yüzüncü kez kapı çalışında bu sefer adım sayısını yüz olarak kabul edip çünkü yüzüncü kez çalıyor bulmuş cevabı kendince..

Araştırmacı: Tamam c'yi ne yapmış?

Katılımcı: c'de de..Bu dediğini n' e işte kapının çalış sayı diyerek bu bulduğu iki çarpı adım sayısı eksi biri işte iki çarpı n yerine adım sayısını kabul edersek; iki n eksi bir şeklinde yazabilmiş yani. Cebirsel olarak ifade etmiş tatlı çocuk

Sayfa 96'da verilen örneğin Türkçesi:

1a) 1. soruyu çözmek için Öğrenci A'nın kullandığı çözüm yöntemini detaylı olarak açıklayınız. Sizce Öğrenci A'nın çözümü doğru mu? Neden? Yanlış

Her şekilde satır sayısı ve o satırda bulunan kare sayısını çarpmış. 5. adıma geçtiğinde bunu devam etmek yerine adımlarda bulunan kare sayısının yerine o adımda bulunan satır sayısını yazmış. Yani doğru bir mantık ile başlamış fakat tabloya geçtiğinde yanlış devam etmiş. Adım sayısı ve satır sayısı arasında 2 fark olduğu için tabloda 25. adıma kadar devam ettirmiş bu örüntüye ve 25. adımda 27 kare vardır demiş.

Sayfa 97'de verilen örneğin Türkçesi:

4a) 3. soruyu çözmek için Öğrenci D'nin kullandığı çözüm yöntemini detaylı olarak açıklayınız. Sizce Öğrenci D'nin çözümü doğru mu? Neden?

Öğrenci her şıkkı doğru yapmıştır. A şıkkında; zilin çalınma sayısı ve her çalınmada kaçar kişi geldiğini liste yöntemiyle göstermeye çalışmış, Gelen kişi sayısını bir önceki gelen kişi sayısına iki ekleyerek bulmuş ve 5. kezden sonra bulduğu sayıları toplayıp doğru sonuca ulaşmıştır. B şıkkında ise bulduğu değerleri tekrar alt alta dizip her bir zil çaldığında toplamda ne kadar kişi geldiğini adım adım yazmış ve kapının çalınma sayısıyla, gelen kişi arasında tam karelik bir ilişki olduğunu görüp, 100. adımda 100'ün karesini alması gerektiğini görmüştür. Doğru cevaba ulaşmıştır. C şıkkında öğrenci genelleme yaparak n. kez kapı zili çaldığında ve toplam n.n'den n^2 kişinin geldiğini bulmuştur.

Sayfa 100'de verilen örneğin Türkçesi:

1a) 1. soruyu çözmek için Öğrenci A'nın kullandığı çözüm yöntemini detaylı olarak açıklayınız. Sizce Öğrenci A'nın çözümü doğru mu? Neden?

Öğrenci her adımdaki satır sayısının adım sayısından iki fazla olduğunu fark etmiş ve 25. adımda 27 kare sayısı olur deyip çözümünü orada sonlandırmıştır. Öğrenci A'nın çözümü yanlış çünkü 25. adımda bulduğu 27, kare sayısı değil aslında satır sayısıdır.

Sayfa 101'de verilen örneğin Türkçesi:

5a) 3. soruyu çözmek için Öğrenci E'nin kullandığı çözüm yöntemini detaylı olarak açıklayınız. Sizce Öğrenci E'nin çözümü doğru mu? Neden?

a şıkkında her bir adımda 2'şer kişi arttırmış ve 5. adımda hepsini toplamak yerine sadece gelenleri hesaplamış ve 9 kişi diyerek yanlış cevaplamış.
b ve c şıklarında ise bu hatasına devam ederek genelleme yapmış ve hatasına devam edip yanlış sonuçlara ulaşmıştır.

Sayfa 103'te verilen örneğin Türkçesi:

4a) 3. soruyu çözmek için Öğrenci D'nin kullandığı çözüm yöntemini detaylı olarak açıklayınız. Sizce Öğrenci D'nin çözümü doğru mu? Neden?

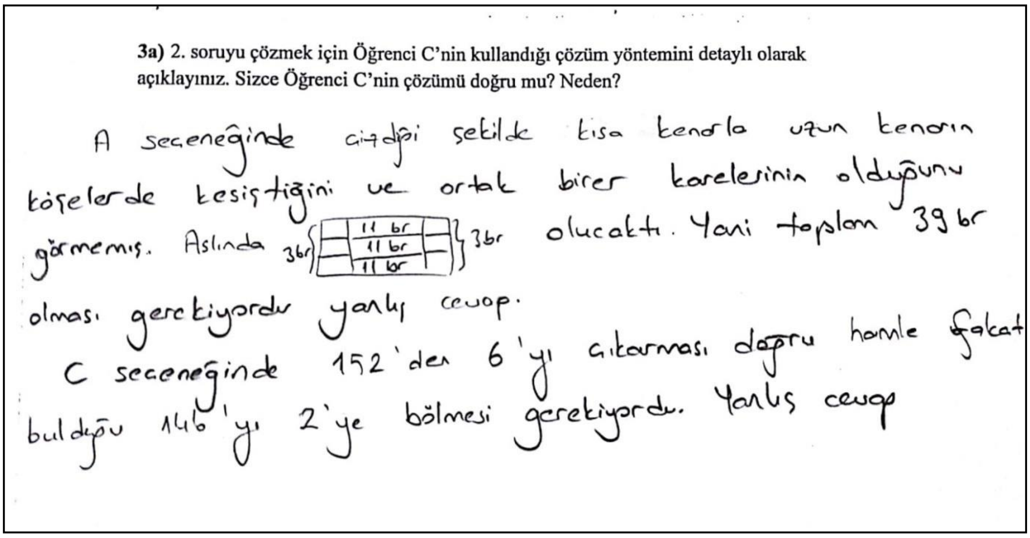
Çözüm doğru Öğrenci başta birkaç adım bularak sonrasında kural buluyor
Bence sade ve başarılı bir yaklaşım.

Sayfa 104'te verilen örneğin Türkçesi:

5a) 3. soruyu çözmek için Öğrenci E'nin kullandığı çözüm yöntemini detaylı olarak açıklayınız. Sizce Öğrenci E'nin çözümü doğru mu? Neden?

-> Öğrenci bir strateji üretmiş fakat istenilen adımda toplam kişi sayısını değil gili gelen kaç kişi olduğunu toplam olarak almıştır. Çözümü yanlıştır.

Sayfa 106'de verilen örneğin Türkçesi:

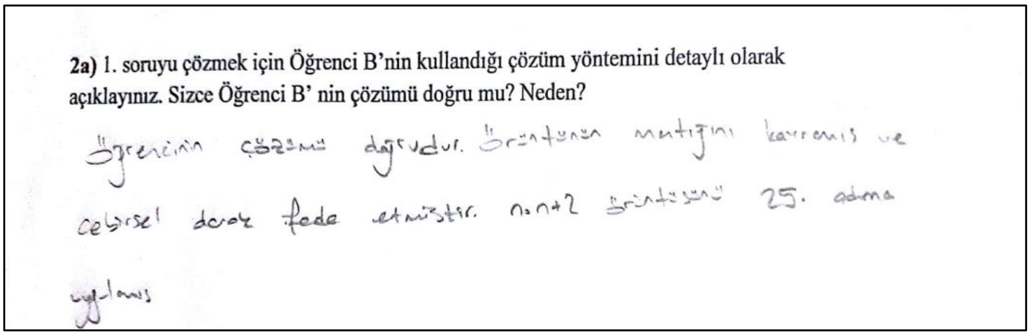


3a) 2. soruyu çözmek için Öğrenci C'nin kullandığı çözüm yöntemini detaylı olarak açıklayınız. Sizce Öğrenci C'nin çözümü doğru mu? Neden?

A seçeneğinde çizdiği şekilde kısa kenarla uzun kenarın köşelerde kesiştiğini ve ortak birer karelerinin olduğunu görmemiş. Aslında 3br 11 br 11 br 11 br 3br olucaktı. Yani toplam 39 br olması gerekiyordu yanlış cevap.

C seçeneğinde 152'den 6'yı çıkarması doğru hamle fakat bulduğu 146'yı 2'ye bölmesi gerekiyordu. Yanlış cevap

Sayfa 108'de verilen örneğin Türkçesi:



2a) 1. soruyu çözmek için Öğrenci B'nin kullandığı çözüm yöntemini detaylı olarak açıklayınız. Sizce Öğrenci B' nin çözümü doğru mu? Neden?

Öğrencinin çözümü doğrudur. Örüntünün mantığını kavramış ve cebirsel olarak ifade etmiştir. n.n+2 örüntüsünü 25. adıma uygulamış

Sayfa 118'de verilen örneğin Türkçesi:

Araştırmacı: Şimdi a şıkındaki çözümüne dair öğrencinin kavrayışı için ne diyebilirsin?

Katılımcı: Ya bence gayet anlamlı. Ne yaptığınında farkında hani. Sadece bu aradaki ilişkiyi görmüş…

Araştırmacı: Ardaki ilişki derken?

Katılımcı: Artı iki artı iki arttığının farkında hani, daha doğrusu o soruda verilmiş zaten de.. Mesela ilk soruda beşinci kapı zili çalındıktan sonra Ayşe'nin evinde toplandı soru. Mesela bir soruda şeydi yaa sadece o basamağı alıyordu mesela bundan önce gelende de ne yaptığının farkında aslında konuyu anlamış, yani kavramış örüntüyü.

Araştırmacı: Anladım. Peki b için ne düşünüyorsun kavrayışı için?

Katılımcı: Bence yaa aslında bütün sorularda bence kavramış. Adım adım ilerlemiş yani beşinci soruda bulmuş. Mesela yirmibeşi bu mesela ikinci kezden başlatmış getirmiş yedinci keze kadar. Onu on kereye kadar yaptığında hep fark etmiş ki karesi karesi şeklinde ilerliyor. O yüzden de yüzüncü keze geldiğinde 10000'i bulabilmiş. Yüzün karesini aldığı için yüzle yüzü çarptığı için bunda da çok güzel bir şekilde genellemiş bence,çok güzel çözmüş yani. Beşinciyi bulmuş, oradan iki üçü dördü hepsini düşünerek genellemeye de ulaşmış.

Sayfa 120'de verilen örneğin Türkçesi:

Araştırmacı: Öğrencinin kavrayışı hakkında ne söyleyebilirsin?

Katılımcı: Buradaki örüntü… Evet bir saniye.. Ben şeylere baktım, çarpımlar arasındaki örüntüyü baktım. Ya örüntü şey arasındaki işte satırın ve sütunun tek tek her seferinde birer birer arttığı örüntüsünü fark etmiş. Birinci olarak, ikinci olarak da her seferinde satır ve sütunlar arasındaki farkın iki olduğunu fark etmiş.

Araştırmacı: Tamam. Testte öğrencinin örüntüyü bulduğunu söylemişsin. Bu çıkarıma nasıl varıyorsun?

Katılımcı: Bu çıkarıma yani birincisi doğru çözdüğü için zaten. İkincisi de belli bir hani en azından yirmi beşinci adıma kadar tek tek yazmamış olmasından. Yani on birinci adımdan sonra bir şeyler dank etmiş kafasına ve hani hiç uğraşmamış o örüntüyü fark etmiş, tek tek yazmadan bulabilmiş. Zaten bizim örüntülerde formül bulmamızın en büyük sebebi bize çok büyük bir adım verildiğinde kolayca bulabilmek ya. Aslında onu başarmış bir şekilde o burada.

Sayfa 123'te verilen örneğin Türkçesi:

1b) Öğrenci A'nın çözümünden onun konuya ilişkin anlamlandırması (kavrayışı) hakkında ne düşünüyorsunuz? Detaylı olarak açıklayınız.

kare sayısını bulmak için satır ve sütunları çarpması gerektiğini biliyor. Ve adımlardaki satır sayıları arasındaki ilişkiyi de doğru bulmuş. Ama sonuca varamamış sadece bulduğu satır sayısında kalmış. Ben dikkat eksikliği olduğunu düşünüyorum.

Sayfa 125'te verilen örneğin Türkçesi:

3b) Öğrenci C'nin çözümünden onun konuya ilişkin anlamlandırması (kavrayışı) hakkında ne düşünüyorsunuz? Detaylı olarak açıklayınız.

A seçeneği dışında çok kavrayabildiğini düşünmüyorum. Sadece A seçeneğinde akıl yürütmesi iyi ancak B ve C seçenekleri için daha derin düşünmek yerine ezbere dayalı bir yaklaşım var.

Sayfa 127’de verilen örneğin Türkçesi:

3b) Öğrenci C’nin çözümünden onun konuya ilişkin anlamlandırması (kavrayışı) hakkında ne düşünüyorsunuz? Detaylı olarak açıklayınız.

Öğrenci A ve C şıklarında doğru bir şekilde çizim yapmış fakat çözümünü doğru yapmamıştır. B şıkkında bulduğu sonucun A'yı sağlaması gerektiğinin farkında değil. Bu yüzden aralarında ilişki kurmada başarısız olmuştur. C şıkkında 146'yı 2'ye bölmesi gerektiğinin farkında değil.

Sayfa 128’de verilen örneğin Türkçesi:

1b) Öğrenci A’nın çözümünden onun konuya ilişkin anlamlandırması (kavrayışı) hakkında ne düşünüyorsunuz? Detaylı olarak açıklayınız.

Tablo çizimini anlamlandıramamıştır. Anladığı soruyu tabloya geçirirken hataya düşüyor. Tablo kullanımı bir arkadaşından veya önceki derslerden gördüğü, hatırladığı kadarını çizmiş.

Sayfa 134’te verilen örneğin Türkçesi:

4c) Öğrenci D’nin öğretmeni olduğunuzu varsayalım. Bu problemin devamında Öğrenci D’ye, hangi problem veya problemleri sorardınız? Bu problem veya problemleri seçmedeki gerekçeniz nedir?

Sorular: Eğer evde toplam 144 kişi varsa, kaç defa kapı çalmıştır?

Sebepleri: Bu öğrenci konuyu net şekilde anlamış, bir de tersten soru sorulursa ne yapacağını merak ettiğim için sordum.
Hatta buradan karekök kavramına bile bağlayabilir.

Sayfa 135’te verilen örneğin Türkçesi:

1c) Öğrenci A’nın öğretmeni olduğunuzu varsayalım. Bu problemin devamında Öğrenci A’ya, hangi problem veya problemleri sorardınız? Bu problem veya problemleri seçmedeki gerekçeniz nedir?

Sorular: ① Şekil 4'teki adım sayısına "24" demişsin. Şekil 25'teki adım sayısına da "27" demişsin. Şekil 4 ve şekil 25 arasında kaç şekil var? Sence aralarındaki farkın 3 olması mantıklı mı?

② Şekil 5'i çizebilir misin? Sonrasında şekil 5'te bulduğun sayıyla, şekil 24 için bulduğun sayıyı karşılaştırır mısın?

[redacted] ③ (Estimation yaptırırdım) Her adım sonrası satır ve sütun sayısında ne gibi değişimler oluyor? Satır ve sütun sayısı 1 artıyorsa, şekil 5'teki kare sayısı, şekil 4'teki adım sayısından en az kaç fazla olur? Şekil 5'teki kare sayısını tahmin edebilir misin?

Sebepleri: Eğer öğrenci geriye dönüp, şekil 5'teki adım sayısının, şekil 4'teki adım sayısından en az 6 kare sayısı daha fazla olacağını farkederse yaptığı hatayı anlar. Bu sorularla hedefim, çocuğun her bir adım sonrası şekle bir satır ve bir sütun daha eklendiğini göstertmek. Bunu anladıktan sonra şekil 24 için gereken kare sayısının 27 olması durumu ona saçma gelecektir. Ayrıca şekil 5'in şeklini de çizdirirdim, çünkü çizerek satır ve sütun sayısının artışını daha iyi anlar. Ayrıca estimation yapması çok önemli çünkü tekrardan yaptığı hatayı farkedecek estimation yaparsa.

Sayfa 138'de verilen örneğin Türkçesi:

4c) Öğrenci D'nin öğretmeni olduğunuzu varsayalım. Bu problemin devamında Öğrenci D'ye, hangi problem veya problemleri sorardınız? Bu problem veya problemleri seçmedeki gerekçeniz nedir?

Sorular: Öğrenciye 2 fazlası yerine bir sonraki zil çalışında ise bir önceki zil çalışında gelen kişilerin sayısının 2 katı kişi gelmektedir' olarak sorarım.

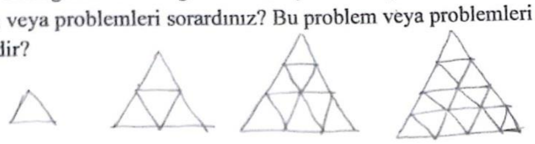
Sebepleri:

Öğrencinin konuyu iyi kavradığını düşünüyorum. Bu yüzden biraz daha zor bir soru sormak istedim. Çünkü kural bulmayı ve genellemeye gitmeyi gerçekten öğrendiğini test etmek istedim. Bu soru öğrenci için biraz daha karmaşık bir soru olacaktır.

Sayfa 139'da verilen örneğin Türkçesi:

2c) Öğrenci B'nin öğretmeni olduğunuzu varsayalım. Bu problemin devamında Öğrenci B'ye, hangi problem veya problemleri sorardınız? Bu problem veya problemleri seçmedeki gerekçeniz nedir?

Sorular:



25. adımdaki üçgen sayısını bulunuz.

Sebepleri: Kare sayısını doğru bulmuş hem bu pekiştirmek hem de farklı türde bir örüntüye yaklaşımını değerlendirmek amacıyla böyle bir soru daha sorulabilir.

Sayfa 141’de verilen örneğin Türkçesi:

3c) **Öğrenci C’nin öğretmeni olduğunuzu varsayalım. Bu problemin devamında Öğrenci C’ye, hangi problem veya problemleri sorardınız? Bu problem veya problemleri seçmedeki gerekçeniz nedir?**

Sorular: Bahçeyi dikdörtgen değil de kare ya da farklı bir dikdörtgen olarak düşünürdüm ve öyle sorardım.

Sebepleri: Öğrencinin sadece o örüntüde kendine bir strateji oluşturmadığını test etmek için böyle bir soru sorardım.

Sayfa 142’de verilen örneğin Türkçesi:

2c) Öğrenci B’nin öğretmeni olduğunuzu varsayalım. Bu problemin devamında Öğrenci B’ye, hangi problem veya problemleri sorardınız? Bu problem veya problemleri seçmedeki gerekçeniz nedir?

Sorular: Öğrenci B zaten soruyu doğru çözmüş. Benzer sorular farklı örüntülerin oluştuğu sorular veririm.

Sebepleri: Farklı örüntülerin olduğu benzer sorular vererek öğrenci B'nin bu konu hakkında pratik yapmasını sağlarım.

G. TURKISH SUMMARY/TÜRKÇE ÖZET

Ortaokul Matematik Öğretmen Adaylarının Öğrencilerin Cebirsel Düşüncelerini Fark Etme Becerilerinin Örüntü Genelleme Bağlamında İncelenmesi

GİRİŞ

Fark etme tesadüfi olmaktan ziyade kasıtlı bir eylemdir (Mason, 2011). Araştırmacılar, son birkaç yüzyıldır, insanların çevrelerini nasıl tanıdıklarını veya fark ettiklerini anlamak için araştırmalar yapmakla beraber, dikkatlerini 20. yüzyılın son çeyreğinde, özellikle bireylerin belirli bir alandaki fark etme becerileri üzerine yoğunlaştırmışlardır (Goodwin, 1994; Mason, 2002; Stewen ve Hall, 1998). Fark etme becerisine getirilen profesyonel bakış açısı öğretime uyarlandığında, fark etme, öğretmenlerin sınıf ortamında eşzamanlı olarak meydana gelen her şeyin farkında olmaları ve bu olayları anlayıp yorumlayabilmeleri anlamına gelmektedir (Jacobs, Lamb ve Philipp, 2010). Ayrıca, öğretme ortamında öğretmenin sahip olduğu fark etme becerisi sabit bilgi yeterliliğinden ziyade aktif bir süreç olarak tanımlanmaktadır (Sherin, Jacobs ve Philipp, 2011; Mason, 2011). Öğretmenlerin fark etme becerileri üzerine çeşitli çalışmalar yapan van Es (2011) ise öğretmenlerin fark etme becerisini incelerken onların ne fark ettikleri ve nasıl fark ettikleri üzerinde durmuştur. Öğretmenlerin fark etme becerileriyle ilgili olarak, Jacobs, Lamb ve Philipp (2010) ise, öğretmenlerin neyi fark ettiklerinden ziyade, onların, öğrencilerin matematiksel düşüncelerini nasıl ve ne ölçüde fark ettiklerine ve öğrencilerin bu düşüncelerini dikkate alarak onlara nasıl cevap verdiklerine odaklanmışlardır. Bu doğrultuda, Jacobs ve arkadaşları (2010) fark etmeyi, (a) öğrencilerin stratejilerine dikkate etme, (b) öğrencilerin matematiksel anlamalarını yorumlama ve (c) öğrencilerin matematiksel anlamalarını göz önünde bulundurarak onlara nasıl cevap vereceklerine karar verme olarak üç boyutta incelemişlerdir. Birinci boyut öğretmenlerin öğrencilerin soruyu nasıl çözdüklerini anlamaları ve onların stratejilerindeki matematiksel detaylara ne kadar dikkat ettikleri ile ilgilidir. İkinci boyutta öğretmenlerin öğrencilerin stratejilerinden yola çıkarak, onların konuyu kavrayışı hakkında çıkarımda bulunabilmeleri ele alınmaktadır. Üçüncü boyutta ise öğretmenlerin öğrencilere ne tür cevaplar verdiklerine ve cevap verirken öğrencilerin konuya dair anlamalarını ne derece dikkate aldıklarına odaklanılır. Ayrıca bu kısımda en doğru cevap diye tanımlanan bir cevap olmamakla birlikte öğretmenin öğrenciye verdiği cevabın gerekçesini sunabiliyor olması önemlidir (Jacobs vd., 2010; van Es, 2011; van Es ve Sherin, 2008; Star ve Strickland, 2007).

Özetle, Jacobs ve arkadaşlarının (2010) tanımladığı fark etme becerisinin iki önemli odaknoktası vardır: İlki sınıf ortamından, öğretmenin pedagojik bilgisinden ve sınıftaki tüm öğrencilerin matematiksel anlayışından ziyade her öğrencinin bireysel olarak matematiksel düşüncesine ve konuyu anlamasına odaklanılmasıdır. İkincisi ise öğretmenlerin, öğrencilerin matematiksel düşüncelerini yorumlayabilmelerinin yanı sıra onlara cevap verme anındaki kararlarının da incelenmesidir. Bu araştırmada öğretmen adaylarının öğrencilerin düşünmelerini fark etme becerilerinin araştırılması amaçlandığı için, Jacobs ve arkadaşlarının (2010) fark etme tanımı temel alınmıştır.

İlgili alanyazını incelendiğinde ise içerisinde toplama-çıkarma, oran-orantı, kesirler ve geometrinin de olduğu birçok konu bağlamında öğretmen ve öğretmen adaylarının fark etme becerileriyle ilgili ulusal (Güner ve Akyüz, 2017; Kılıç, 2006; Osmanoğlu, 2010; Osmanoğlu, Işıksal ve Koç, 2012; Özdemir-Baki ve Işık, 2018; Tunç-Pekkan ve Kılıç, 2016; Ulusoy ve Çakıroğlu, 2018) ve uluslararası (Amador, Carter ve Hudson 2016; Jacobs vd., 2010; Schack, Fister, Thomas, Eisenhardt, Tassel ve Yoder, 2013; Star ve Strickland, 2007; van Es ve Sherin, 2008) birçok çalışmanın yapıldığı gözlenmiştir. Bu çalışmaların sonuçlarına göre, öğretmen ve öğretmen adaylarının fark etme becerilerinin düşük olduğu sonucuna varılmıştır (Amador, Carter ve Hudson 2016; Güner ve Akyüz, 2017; Kılıç, 2006; Tunç-Pekkan ve Kılıç; 2016). Diğer yandan ise, öğretmenlere ve öğretmen adaylarına sağlanan çeşitli destek ve eğitim programlarıyla onların fark etme becerilerinin geliştirilebileceği belirtilmiştir (Güner ve Akyüz, 2017; Jacobs, Lamb ve Philipp, 2010; Osmanoğlu, 2010; Ulusoy ve Çakıroğlu, 2018; Schack, Fister, Thomas, Eisenhardt, Tassel ve Yoder, 2013; Star ve Strickland, 2007; van Es ve Sherin, 2008).

Öğretmenin fark etme becerisinin öğrenci öğrenmesi üzerindeki etkileri göz önünde bulundurulduğunda, öğretmenlerin fark etme becerilerinin matematiğin temel konularından biri olan cebirde de önemli olduğu açıkça görülmektedir (Knuth, Alibali, McNeil, Weingberg ve Stephens, 2005). Kaput (1999) cebiri, veriler arasındaki ilişkiyi bulma, bu ilişkiyi genelleme ve bu ilişkiyi cebirsel ifadeler aracılığıyla formül olarak yazma şeklinde tanımlamıştır. Bu cebirsel işlemler ise bilinmeyeni, bilinen bir değer olarak düşünebilme kabiliyeti olarak tanımlanan cebirsel düşünce vasıtasıyla yapılmaktadır (Swafford ve Langrall, 2000). Cebirsel düşünceyi geliştirmenin en etkili yollarından birinin örüntü genellemesi olduğu vurgulanmaktadır (Van de Walle, Karp ve Bay-Williams, 2013). Tüm bu bilgiler ışığında, örüntülerdeki ilişkiyi fark etmenin bu ilişkiyi genelleyip formül haline getirebilmenin, öğrencinin cebirsel düşüncesini geliştirdiği sonucuna varılabilir (Magiera, Van den Kieboom ve Moyer 2013). Fakat ulaşılabilir alanyazını incelendiğinde, öğretmen ve öğretmen adaylarının fark etme becerilerinin düşük olmasına (Amador, Carter ve Hudson 2016; Güner ve Akyüz, 2017; Kılıç, 2006; Tunç-Pekkan ve Kılıç; 2016) ve cebir konusunun öğrencilerin matematik başarısı için temel teşkil etmesine rağmen (Rakes, Valentine, McGatha ve Ronau, 2010), öğretmen ve öğretmen adaylarının öğrenci düşüncelerini/anlamalarını fark etme becerilerine dair cebir bağlamında yeterli çalışmanın olmadığı gözlenmiştir (Magiera, Van den Kieboom ve Moyer 2013). Bu bağlamda, öğretmen adaylarının öğrencilerin cebirsel düşüncelerini fark etme becerilerinin örüntü genelleme bağlamında incelenmesine ihtiyaç duyulmuştur.

Çalışmanın Amacı ve Araştırma Soruları

Çalışmanın amacı, öğretmen adaylarının öğrencilerin çözüm stratejilerine ne ölçüde dikkat ettiklerini, onların cebirsel düşüncelerini ne ölçüde yorumladıklarını ve öğrencilere cevap verirken aldıkları kararların niteliğini örüntü genelleme bağlamında incelemektir. Bu amaç doğrultusunda aşağıdaki araştırma sorularına cevap aranmıştır.

1. Ortaokul matematik öğretmen adayları, örüntü genelleme bağlamında öğrencilerin cebirsel düşüncelerini nasıl fark ederler?

    1.1. Ortaokul matematik öğretmen adayları, öğrencilerin örüntü genelleme ile ilgili problem çözümlerine ne derece dikkat ederler?

    1.2. Ortaokul matematik öğretmen adayları, öğrencilerin cebirsel düşüncelerini örüntü genelleme bağlamında ne derece yorumlarlar?

    1.3. Ortaokul matematik öğretmen adaylarının, öğrencilerin örüntü genellemesi bağlamındaki cebirsel düşüncelerine cevap vermek için aldıkları kararların nitelikleri nelerdir?

Çalışmanın Önemi

Öğretmenin etkili bir öğretim gerçekleştirebilmesi için, sınıf ortamındaki dikkat çekici durumları kolayca tanımlayabilmesi ve aniden meydana gelen karmaşık durumlarla başa çıkabilme si gerekmektedir (Mason, 2011; van Es ve Sherin; 2011; Jacobs, Lamb ve Philipp, 2010). Ayrıca öğretmenlerin sınıftaki her öğrencinin bireysel matematiksel anlayışına odaklanması önemlidir (Jacobs ve ark. 2010). Bu nedenle, fark etme becerisi, öğretmenler ve öğretmen adayları için önemli bir yeterliliktir. Bu itibarla, bu çalışmanın bulguları, öğretmen adaylarının öğrencilerin stratejilerine dikkat etme becerileri dereceleri, öğrencilerin cebirsel düşüncelerini yorumlama dereceleri ve öğrencilerin cebirsel düşüncelerine cevap verirken aldıkları kararlarının niteliği hakkında genel bir takım fikirler verebileceğinden dolayı önem taşımaktadır. Söz konusu bulguların ışığında, öğretmen adaylarının fark etme becerileri bağlamında, öğretmen eğitimcilerinin ve program geliştiren kişilerin yeni ve önemli bilgiler edinebilmeleri ve yeni bakış açıları kazanabilmeleri mümkün olacaktır.

Öte yandan, geleceğin öğretmenleri olacak öğretmen adaylarının öğrencilerin cebirsel düşüncelerini anlama becerilerinin incelenmesi önemli görülmektedir (Callejo ve Zapatera, 2017; Walkoe, 2013). Ayrıca, bu çalışma, öğretmen adaylarına, öğrencilerin gerçek yazılı çalışmalarına aşina olmaları için fırsat vermektedir. Buna ilaveten öğretmen adayları, öğrencilerin çözümlerine dikkat etme, cebirsel düşüncelerini yorumlama ve cebirsel düşüncelerini dikkate alarak onlara nasıl cevap vereceklerine karar verme konularında tecrübe kazanacaklardır. Bu nedenle, bu çalışma öğretmen adaylarına öğrenme ortamı sağladığı için de ayrıca önem arz etmektedir. Ayrıca, öğretmenler öğrencilerin çözümlerinde nelere dikkat etmeleri gerektiği, öğrencilerin cebirsel düşünceleri nasıl yorumlayıp, onlara nasıl cevap vermeleri gerektiği konularında kendilerini gözden geçirme fırsatı bulacaklardır. Böylece, bir sonraki süreçte öğretmenler, öğrencilerin muhtemel kavram yanılgılarını daha kolay tahmin edebilecek, ders planlarını daha verimli hazırlayıp uygulayabilecek ve öğrencilerin başarı seviyelerini daha etkin bir şekilde değerlendirebileceklerdir.

Cebir, lise matematiği anlayışını geliştirmek için temel teşkil ettiği için, matematiğin temel konularından biridir (Rakes, Valentine, McGatha ve Ronau, 2010). Bu sebeple, Türkiye'de Milli Eğitim Bakanlığı (MEB, 2018) tarafından geliştirilen müfredatta, öğrencilere cebir konularının tanıtılması ortaokulda (6. ve 8. sınıf) başlamakta ve lise boyunca (9. sınıf - 12. sınıf) devam etmektedir. Cebir konusunun matematiğin önemli konularından biri olmasından dolayı Türk öğrencilerinin bu konudaki başarılarını araştıran birçok çalışma mevcuttur (Dede ve Argün, 2003, Yıldız, Çiftçi, Şengil-Akar ve Sezer; 2015). Bu çalışmalar neticesinde, Türk öğrencilerinin cebir konusundaki başarılarının düşük olduğu sonucuna varılmış ve bunun nedenleri de pek çok araştırılmıştır (Dede ve Argün, 2003, Yıldız, Çiftçi, Şengil-Akar ve Sezer; 2015). Öğrencilerin matematiği öğrenmesini etkileyen faktörlerden biri olan öğretmen fark etme becerilerinin araştırılması önemlidir (Jacobs vd., 2010, van Es ve Sherin, 2008).

Fakat hem uluslararası hem de ulusal alanyazınında öğretmen ya da öğretmen adaylarının öğrencilerin cebirsel düşüncelerini fark etmelerine dair sınırlı sayıda çalışma vardır (Callejo ve Zapatera, 2017; Walkoe, 2013). Bu nedenle, bu çalışmanın, öğretmenlerin öğrencilerin cebirsel düşünme becerilerini fark etme becerilerine ilişkin mevcut boşluğu doldurarak, alanyazınına katkı sağlayacağına inanılmaktadır.

Bu çalışma kapsamında, öğretmen adaylarının öğrencilerin cebirsel düşüncelerini fark etme becerileri alternatif öğrenci çözümleri vasıtasıyla incelenmiştir. Öğretmen adaylarının fark etme becerilerini daha kapsamlı incelemek için hem doğru hem de yanlış çözümler kullanılmıştır. Doğru ve yanlış öğrenci çözümlerinde dikkat edilmesi gereken detaylar, öğrencilerin anlamasının nasıl yorumlanacağı ve bu çözümlere sahip öğrencilere nasıl cevap verileceği farklılık arz etmektedir (Jacobs vd., 2010). Bu sebeple çalışma kapsamında araştırmacı tarafından hem doğru hem yanlış soru çözümleri belirlenmiş ve öğretmen adaylarının hem doğru hem de yanlış çözüme sahip öğrenci düşüncelerini fark etme becerileri incelenmiştir. Diğer bir deyişle, öğretmen adaylarının öğrencilerin cebirsel düşüncelerini fark etme becerilerinin öğrencinin soruyu doğru ya da yanlış çözmesine bağlı olarak değişip değişmediğine de odaklanılmıştır. Bu inceleme de çalışmanın önemini artırmıştır.

Ayrıca, alanyazını gözden geçirildiğinde araştırmacıların çoğunun öğretmenlerin fark etme becerilerini araştırmak için video tabanlı öğrenme ortamı (Osmanoğlu, Işıksal ve Koç, 2012; Ulusoy ve Çakıroğlu, 2018; van Es ve Sherin, 2008) ve ders imecesi mesleki gelişim süreci (Amador, Carter ve Hudson 2016; Güner ve Akyüz, 2017) aracılığıyla veri topladığı anlaşılmaktadır. Diğer yandan, yazılı çalışma öğrencilerin matematik öğretimindeki anlayışlarını yorumlamak ve cevaplamak için gerçek bir etkinlik olmasına rağmen (Grosman vd., 2009; Jacobs ve Philipp, 2004), verilerin öğrencilerin yazılı çalışmaları yoluyla toplandığı sınırlı sayıda çalışma bulunmaktadır. (Schack, Fister, Thomas, Eisenhardt, Püskül, ve Yoder, 2013; Star ve Strickland, 2007). Buradan hareketle, öğrencilerin örüntü genelleme ile ilgili problemlerin çözümlerinden oluşan yazılı çalışmalar yoluyla veri toplayan bir çalışma yapılması önemli olacaktır. Tüm bu noktalar göz önünde bulundurulduğunda, ortaokul matematik öğretmen adaylarının örüntü genelleme bağlamında öğrencilerin cebirsel düşüncelerini nasıl fark ettiklerini ortaya koymayı amaçlayan bu çalışmanın ilgili alanyazınına önemli katkılar sağlayacağı düşünülmektedir.

YÖNTEM

Araştırma Yöntemi

Frankel ve Wallen (2006) araştırmacıların konuyla ilgili detaylı bilgi edinmek ve konuya daha bütüncül bir resim ortaya koymak istediklerinde nitel araştırma yöntemini tercih ettiklerini belirtmiştir. Buna ek olarak, Creswell (2007) problem durumunun daha iyi araştırılıp, katılımcıların cevaplarının daha derinlemesine yorumlanabilmesi için nitel araştırma yönteminin kullanıldığını söylemiştir. Bu çalışmada da öğretmen adaylarının fark etme becerileri derinlemesine ortaya koymak ve yorumlamak amaçlandığı için nitel araştırma yöntemi tercih edilmiştir. Nitel araştırma yönteminin bir çeşidi olan durum çalışması ise bir ya da birden fazla durumun olduğu durumlarda belirli bir konu hakkında detaylı araştırma yapmak için kullanılır (Creswell, 2007). Bu çalışmada; konuyu derinlemesine inceleyip araştırma sorularını detaylı cevaplayabilmek için nitel araştırma yöntemi, daha özelde ise durum çalışması (case study) yöntemi kullanılmıştır.

Katılımcılar

Bu çalışmadaki katılımcılar amaçlı örneklem yoluyla bazı kıstaslar göz önünde bulundurularak seçilmiştir. Veri toplama sürecinde araştırmacı katılımcılarla çok vakit geçireceği için ilk olarak katılımcıların araştırmacıya yakın mesafede olması ve kolay ulaşılabilir olması istenmiştir. İkinci olarak, matematik öğretimi konusunda temel bir ders olan Özel Öğretim Yöntemleri I/II derslerini almış olmalarına dikkat edilmiştir. Son olarak ise, katılımcıların Okul Deneyimi dersini alıyor ve bu sayede sınıf ortamını gözlemleme fırsatı yakalıyor olmaları önemsenmiştir. Sonuç olarak bu kıstasları sağlayan ve çalışmaya katılmaya gönüllü olan 2018-2019 eğitim-öğretim yılında Ankara'daki devlet üniversitelerinin birinde İlköğretim Matematik Öğretmenliği programının son yılında öğrenim gören 32 öğretmen adayı katılmıştır. Bu öğretmen adaylarına öncelikle anket uygulanmıştır. Otuz iki öğretmen adayına yarı yapılandırılmış görüşme yapmaya gönüllü olup olmadıkları sorulmuş, katılmayı kabul eden 8 öğretmen adayı ile veri toplama sürecinin ikinci aşaması olarak yarı-yapılandırılmış görüşmeler yapılmıştır.

Veri Toplama Araçları

Bu çalışmada veriler, öğrencilerin cevapladığı açık uçlu sorulardan oluşan anket, öğretmen adaylarına uygulanan anket ve öğretmen adaylarıyla yapılmış yarı yapılandırılmış görüşmeler aracılığıyla toplanmıştır.

Öğrencilere Uygulanan Anket

Öğretmen adaylarının öğrencilerin matematiksel anlamalarını fark etme becerilerini ölçülebilmesi için farklı öğrenci çözümlerine ihtiyaç duyulmuş ve farklı öğrenci çözümleri elde edebilmek amacıyla ulaşılabilinen ve çalışmaya katılmaya gönüllü olan 20 tane 6. sınıf öğrencisine 3 tane örüntü genellemesiyle alakalı problemden oluşan anket uygulanmıştır. Ortaokul matematik öğretim programı 2018'de revize edilmiş olmasına rağmen, verilerin toplandığı sırada önceki öğretim programı (2013) yürürlükte olduğu için, sorular, ortaokul matematik öğretim programında verilen (MEB, 2013) "Sayı örüntülerinin kuralını harfle ifade eder, kuralı harfle ifade edilen örüntünün istenilen terimini bulur." (6.2.1.1) kazanımına uygun olarak hazırlanmıştır. Bu anketten elde edilen verilere göre, 3 tanesi yanlış, 2 tanesi doğru olan beş tane farklı öğrenci çözümü, öğretmen adaylarına uygulanacak olan ölçme aracını hazırlamak için seçilmiştir

Öğretmen Adaylarına Uygulanan Anket

Ankette, ortaokul matematik öğretmen adaylarının, öğrencilerin cebirsel düşünüşlerini fark etme becerilerini değerlendirmek amacıyla her bir öğrenci çözümüyle ilgili üç açık uçlu soru içeren anket hazırlanmış ve bu ankete yazılı cevap vermeleri istenmiştir. Anketteki sorular şunlardır:

(1) "Soruyu çözmek için öğrencinin kullandığı çözüm yöntemini detaylı olarak açıklayınız. Sizce öğrencinin çözümü doğru mu? Neden?

(2) Öğrencinin çözümünden onun konuya ilişkin anlamlandırması (kavrayışı) hakkında ne düşünüyorsunuz? Detaylı olarak açıklayınız.

(3) Bu öğrencinin öğretmeni olduğunuzu varsayalım. Bu problemin devamında öğrenciye hangi problem ya da problemleri sorardınız? Bu problem ya da problemleri seçmedeki gerekçeniz nedir?" (Jacobs, Lamb ve Philipp, 2010, s.178-179).

Yarı Yapılandırılmış Görüşmeler

Yarı yapılandırılmış görüşmeler, bir konu hakkında derinlemesine bilgi almak için tercih edilen nitel veri toplama araçlarından biridir (Merriam, 1998). Çalışmaya katılan öğretmen adaylarının 8'i ile onların ankette verdikleri cevapları netleştirmek, teyit etmek ve daha detaylı cevaplar almak için yarı yapılandırılmış görüşmeler yapılmıştır. Görüşme öncesinde hazırlanan açık uçlu sorular, görüşme sırasında katılımcılara sorulmuştur. Ayrıca, katılımcılardan alınan cevaplara göre başka sorular da eklenmiştir. Bu görüşmeler kamera ile kayıt altına alınmıştır. Yarı yapılandırılmış görüşmelerde sorulan örnek sorular Appendix D' de verişmiştir.

Veri Toplama Süreci

Çalışmanın verileri 2018-2019 eğitim-öğretim yılının güz döneminde toplanmıştır. Öncelikle çalışmaya katılan 32 öğretmen adayına anket uygulanmıştır. Veri toplama sürecinin ikinci kısmı olarak, 32 öğretmenden katılmaya gönüllü olan 8 öğretmen adayı ile yarı-yapılandırılmış görüşme yapılmıştır.

Veri Analizi

Bu çalışmada, anketlerden ve yarı yapılandırılmış görüşmelerden elde edilen veriler, öğretmen adaylarının öğrencilerin cebirsel düşünmelerini fark etme becerilerini derinlemesine açıklamak amacıyla analiz edilmiştir. Başlangıçta, 8 öğretmen adayı ile yapılan görüşmenin video kayıtları yazılı metin haline getirilmiştir. Sonrasında bu metinler ve ankette sorulara verilen cevaplar dikkatli bir şekilde okunmuştur. Katılımcıların cevapları Jacobs ve arkadaşlarının (2010) öğretmen adaylarının öğrencilerin matematiksel düşüncelerini fark etme becerisiyle ilgili olan kuramsal çerçevesi dikkate alınarak kodlanmıştır. Fakat bu kuramsal çerçeve, bu çalışmadaki tüm verileri temsil etmemiştir. Bu nedenle, katılımcıların cevaplarındaki benzerlik ve farklılıklara göre kodlarda bazı değişiklikler yapılmış olup; öğretmen adaylarının tüm cevaplarının kodlanabilmesi için söz konusu çerçeveye yeni kategoriler eklenmiştir. Böylece, mevcut çalışmanın analiziyle, Jacobs ve arkadaşlarının kuramsal çerçevesi genişletilmiştir. Geçerlilik ve güvenirlik çalışması kapsamında matematik eğitimi alanında doktora yapan bir öğrenciden verileri kodlaması istenmiştir. Araştırmacının ve diğer kodlayıcının kodlamaları karşılaştırılıp, Miles ve Huberman' ın (1994) ortaya koyduğu formül kullanılarak kodlayıcılar arası güvenirlik %93 olarak hesaplanmıştır.

Araştırmanın Sınırlılıkları

Bu çalışma İlköğretim Matematik Öğretmenliği programının 4. sınıfında öğrenim gören, Özel Öğretim Yöntemleri I/II derslerini almış ve Okul Deneyimi dersini almakta olan öğretmen adayları ile gerçekleştirilmiştir. Bu yüzden çalışma, belirtilen kıstasları sağlayan öğretmen adaylarının yanıtlarıyla sınırlıdır. Ayrıca, öğretmen adaylarının öğrencilerin cebirsel düşüncelerini fark etme becerileri örüntü genellemesi bağlamında incelendiği için çalışmada sadece "Sayı örüntülerinin kuralını harfle ifade eder, kuralı harfle ifade edilen örüntünün istenilen terimini bulur." (6.2.1.1.) kazanımına odaklanılmıştır (MEB, 2013, p.18). Ayrıca, bu çalışmada anket ve yarı yapılandırılmış görüşmeler yapılarak birden fazla veri toplama yöntemi ile durum derinlemesine araştırılmış olmasına rağmen, sınıf gözlemi yapılmamıştır. Bu da çalışmanın bir diğer sınırlılığıdır.

BULGULAR VE TARTIŞMA

Bu çalışmanın amacı, ortaokul matematik öğretmen adaylarının öğrencilerin cebirsel düşünme becerilerini fark etmelerini örüntü genelleme konusu bağlamında incelemektir. Bu sebeple, öğretmen adaylarını öğrenci çözümlerine dikkat etmelerine, öğrencilerin cebirsel düşüncelerini yorumlamalarına ve öğrenci düşüncelerine bağlı olarak öğrencilere cevap verirken aldıkları kararların niteliklerine dair bulgular elde edilmiştir.

Verilerin analizi, öğretmen adaylarının çoğunun öğrencilerin çözümlerine dikkat ederken güçlü kanıtlar sunduğunu ortaya koymuştur. Bir başka ifadeyle, bu çalışmaya katılan öğretmen adaylarının çoğu, öğrencilerin, hem doğru hem yanlış çözümlerini matematiksel detayların çoğuna vurgu yaparak açıklamıştır. Araştırmaya katılan öğretmen adaylarının, öğrenci çözümlerine iyi derecede dikkat edebilmelerinin sebebi, onların konuya dair matematiksel ayrıntıları iyi biliyor olmaları ve bunlara iyi odaklanabilmeleri olabilir (Talanquer, Bolger ve Tomanek; 2015). Ayrıca, bu bulgu, öğrenci çözümüne dikkat etmenin, öğretmen adayları için fark etme becerisinin en kolay alt becerisi olmasından kaynaklanmış da olabilir (LaRochelle, 2018; Sánchez-Matamoros, Fernández ve Llinares; 2019). Dahası, öğretmen adayları Matematik Öğretme Yöntemleri I / II dersini aldığından dolayı, öğrencilerin çözümlerini açıklamaya yönelik güçlü kanıtlar sunmuş olabilir. Her ne kadar katılımcıların büyük çoğunluğu (%65) öğrencilerin çözümlerine dikkat ederken güçlü kanıtlar sunmuş olsalar da geri kalanları, öğrencilerin çözümlerinin matematiksel unsurlarını eksik tanımlamışlardır. Bu yetersizliğin nedeni ise öğretmen adaylarının bilgi eksikliği olabilir (Schoenfeld, 2011).

Araştırmanın analizinden elde edilen bulgulara göre, öğretmen adayları öğrencilerin cebirsel düşüncelerini analiz etmekte ve yorumlamakta zorlanmışlardır. Ayrıca kayda değer sayıda öğretmen adayı, öğrencilerin anlamasını “kavramıştır” ya da “kavramamıştır” şeklinde yetersiz bir şekilde yorumlamıştır. Bu bulguların nedeni, öğretmen adaylarının henüz öğretmenlik deneyimine sahip olmaması olabilir. Buna ek olarak, öğretmen adaylarının öğrencilerin cebirsel düşüncelerini yorumlarken zorlanmalarının bir diğer nedeni, alan ve öğrenci bilgi (knowledge of content and students) eksikliği ile alan ve öğretim bilgi (knowledge of content and teaching) eksikliği olabilir.

Çalışmaya katılan öğretmen adaylarının öğrencilerin cebirsel düşüncelerini yorumlamalarına dair diğer bir bulgu ise onların doğru öğrenci çözümlerini, yanlış öğrenci çözümlerinden daha iyi analiz edip yorumlayabilmeleridir. Bu bulgunun nedeni, öğretmen adaylarının öğrencilerin doğru çözümlerini daha kolay takip edip anlayabilirken; öğrencilerin yanlış çözümlerini anlamada ve öğrencilerin hatalarını ve kavram yanılgılarını analiz etmekte daha fazla zorluk yaşamaları olabilir.

Öğretmen adaylarının öğrencilere nasıl cevap vereceklerine dair verilerin analizinin neticesinde ortaokul matematik öğretmen adaylarının çoğunun (%60), öğrencilere onların yanlış cevaplarındaki hatalarını fark ettirip, doğru cevaba ulaşmalarını sağlayacak şekilde sorular sordukları sonucuna ulaşılmıştır. Geri kalan öğretmen adaylarının ise yanlış çözüme sahip öğrencilere alıştırma niteliğinde sorularla ya da genel sorularla cevap verdikleri tespit edilmiştir. Bu nedenle, öğretmen adaylarının çoğunun öğrencilerin yanlış cevaplarına etkili bir şekilde cevap verdikleri yorumunda bulunulabilinir. Bu bulgunun nedeni, öğretmen adaylarının, öğrencilerden onların çözümlerindeki unsurlarla ilgili düşünmelerini isteyerek, öğrencilerin yanlışlarını tanımalarına ve doğru düşünceye sahip olmaları konusunda yardımcı olmaları olabilir (Jacobs ve Ambrose, 2008; Milewski ve Strickland, 2016).

Öte yandan çalışmaya katılan öğretmen adaylarının doğru çözüme sahip öğrencilerin cebirsel düşüncelerine katkı sağlayıp, onların bilgilerini artırma hususunda eksik oldukları sonucuna varılmıştır. Çoğu öğretmen adayı, soruyu doğru çözen öğrencilere alıştırma niteliğinde sorular sormakla ya da genel cevaplar vermekle yetinmiştir. Bu nedenle öğretmen adaylarının, Taylan'ın (2018) çalışmasında da belirtildiği gibi, öğrencilerin doğru çözümlerine cevap vermekte zorlandıkları sonucu ortaya çıkmıştır.

Kısaca, mevcut araştırmada, öğretmen adaylarının öğrencilerin yanlış çözümlerine, doğru çözümlerinden daha etkili bir şekilde yanıt verdikleri ve böylece öğretmen adaylarının öğrencilerin kavram yanılgılarını kolayca giderebildikleri fakat soruları doğru çözen öğrencilerin cebirsel düşüncelerini geliştirmekte zorlandıkları gözlemlenmiştir.

Sonuç olarak, bu çalışmada öğretmen adaylarının öğrencilerin cebirsel düşüncelerine nasıl cevap vereceğine karar verme becerilerinin, çözümlerine dikkat etme becerilerinden bağımsız olduğu sonucuna varılmıştır. Ayrıca, öğretmen adaylarının öğrencilerin cebirsel düşünüşlerini yorumlamaları ile onlara nasıl cevap vereceklerine karar vermeleri arasında net bir ilişki tespit edilememiştir. Ancak, bu çalışmanın bulgularına göre, öğrencilerin cevaplarının doğru ya da yanlış olmasına bağlı olarak öğretmen adaylarının cevap verme kabiliyetlerinin değişkenlik arz ettiği görülmüştür.

Çalışmanın bulguları, önceki yapılan çalışmalarla öğretmenlerin öğrencilerin çözümlerine dikkat etmeleri, öğrencilerin cebirsel düşüncelerini yorumlayabilmeleri ve öğrencilere cevap verirken aldıkları kararların nitelği konusunda benzerlik göstermektedir (Barnhart ve van Es, 2015; Callejo ve Zapatera; 2017; Jacobs ve Ambrose, 2018; Milewski ve Strickland; 2016; Sánchez-Matamoros, Fernández ve Llinares, Taylan, 2018). Fakat bu çalışmada öğretmen adaylarının fark etme becerilerinin örüntü genelleme bağlamında incelenmesinden kaynaklı bazı farklı sonuçlar da elde edilmiştir. Cebir, örüntüdeki ilişkinin genellenmesi ve formülle ifade edilmesi olarak tanımlanmıştır (Kaput, 1999). Bu yüzden cebirin doğası gereği örüntü genelleme sayısal ifadelerden sembollere geçişi sağlamaktadır (Kiearan ve Chalouh, 1993). Örüntü genelleme süreci dört önemli aşamayı içermektedir (Radford, 2008; Rivera ve Becker; 2009; Warren, 2005): (1) çizerek ya da sayarak yakındaki terimi bulabilme, (2) örüntüyü keşfetme ve terimler arasındaki ilişkiyi düşünerek uzaktaki bir terimi bulabilme, (3) yaptığı genelleme yardımıyla örüntünün genel kuralımı yazabilme ve (4) herhangi bir terimin sonucu verildiğinde o sonucun hangi terime ait olduğunu bulabilme. Bu süreç birbirine bağlı dört adımı içerdiği için, örüntü genelleme ile ilgili problemler adım adım çözümler içermektedir. Bu doğrultuda, bu çalışmada seçilen örüntü genellemesi ile ilgili çözümler de öğrenciler tarafından adım adım çözülmüştür. Bu sebeple, örüntü genellemenin doğası çalışmaya katılan öğretmen adaylarının öğrenci çözümlerine dikkat etmelerini ve onların cebirsel düşüncelerini yorumlamalarını kolaylaştırmış olabilir. Fakat problemlerin çözümlerinin adım adım yapılmış olması öğretmen adaylarının öğrenci çözümüne dair her adımı değerlendirip öğrencilere cevap verirken karar verme sürecini zorlaştırmış olma ihtimali mevcuttur.

Bu bulgular neticesinde, bu çalışma, öğretmen adaylarının öğrencilerin cebirsel düşüncelerini örüntü genellemesi bağlamında fark etme becerilerinin daha derinlemesine incelenmesine imkân sağlamaktadır. Bu itibarla, mevcut çalışmanın alanyazındaki diğer çalışmalara ek olarak, eğitim uygulamalarına ve konuyla ilgili benzer çalışmalara hareket noktası teşkil edebileceği ve yeni öneriler getireceği açıktır.

Doğurgalar

Öğretmen eğitimcileri, Özel Öğretim Yöntemleri I / II gibi öğretmen adaylarının mesleki gelişimine yönelik derslerin içeriğine, öğretmen adaylarının fark etme becerilerinin geliştirilmesi ile ilgili uygulamalar entegre edebilirler. Derste farklı kavramsal ve işlemsel hatalar içeren örnekler ve farklı stratejilerle çözülmüş öğrenci çözüm örneklerini öğretmen adaylarının analiz edip yorumlamaları istenebilir. Ayrıca öğretmen adayları, sınıf ortamındaki arkadaşlarıyla da etkileşime girerek onlarla düşüncelerini paylaşıp arkadaşlarından da geri bildirim alabilirler.

Benzer şekilde, öğretmen adaylarının fark etme becerilerini geliştirmeye yönelik olarak Okul Deneyimi ve Öğretmenlik Uygulaması derslerine de benzer örnekler ve alıştırmalar entegre edilebilir. Böylece öğretmen adayları gerçek öğrenci çözümleri ve gerçek sınıf ortamıyla karşı karşıya kalarak pedagojik alan bilgilerini güçlendirme ve pratik yapma fırsatı bulacaklardır.

Bunlara ek olarak öğretmen eğitimcileri mesleki gelişim programları kapsamında, öğretmen ve öğretmen adaylarının öğrencilerin anlamalarını fark etme becerilerini geliştirmek için etkinlikler uygulayabilirler.

Öneriler

Aynı içeriğe sahip benzer bir çalışma, aynı üniversitenin 3. ve 4. sınıfında öğrenim gören öğretmen adaylarıyla da yapılabilir. Böylelikle, üçüncü ve dördüncü sınıfta okuyan öğretmen adaylarının fark etme becerileri karşılaştırılabilir ve bu yolla Özel Öğretim Yöntemleri I / II ve Okul Deneyimi derslerinin öğretmen adaylarının fark etme becerisine olan etkisi incelenebilir. Benzer şekilde, aynı çalışma öğretmenlerle de yapılabilir. Böylelikle, öğretmenlik deneyiminin öğretmenlerin öğrencilerin cebirsel düşüncelerini fark etme becerileri üzerindeki etkisi araştırılabilir.

Bu çalışmada öğretmen adaylarının örüntü genelleme bağlamında öğrencilerin cebirsel düşünceleri fark etme becerileri araştırılmıştır. Öğretmen adaylarının söz konusu fark etme becerileri, farklı konu bağlamlarında da araştırılarak öğretmen adaylarının fark etme becerilerine yönelik bütünsel bir tablo elde edilebilir.

Öğretmenlerin fark etme becerileri ve matematik bilgisi arasındaki ilişki bazı araştırmalarda belirtilmiş olsa da (Schoenfeld, 2011; Monson, Krupa, Lesseig, ve Casey, 2018), bu bilgilerin öğretmenlerin öğrenci çözümlerine dikkat etmelerine, yorumlamalarına ve öğrencilere nasıl cevap vereceklerine dair kararlarına etkisi açık değildir. Dolayısıyla benzer bir çalışmayla öğretmenlerin matematik bilgisi ile öğretmenlerin öğrencilerin cebirsel düşüncelerini fark etme becerileri arasındaki ilişkinin ne olduğu da araştırılabilir.