VALUE OF SUPPLIER FLEXIBILITY AND INITIAL SHELF LIFE INFORMATION FOR PERISHABLE ITEMS IN AN EOQ ENVIRONMENT

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ABSTRACT

VALUE OF SUPPLIER FLEXIBILITY AND INITIAL SHELF LIFE INFORMATION FOR PERISHABLE ITEMS IN AN EOQ ENVIRONMENT

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Managing items with shelf life is a challenging task in inventory planning. In this study, we consider an infinite horizon, continuous review inventory model with deterministic stationary demand where the shelf life of the items is uncertain. The initial shelf life of the incoming items from the supplier is a discrete random variable. When the age of the items reaches the initial shelf life, a quality control test for which the outcome is random is applied. According to the result of this test, it is possible to use the items to satisfy demand for an additional time period. Moreover, we also study the supplier flexibilities which are modeled by including a return opportunity at the arrival of the items and the initial shelf life information received before the ordering decision. The aim is to minimize the total expected cost per unit time by using Renewal Theory in order to investigate value of extension test opportunity, value of return opportunity and value of shelf life information. Therefore, several settings are constructed and they are compared by conducting a numerical study.

Keywords: items with shelf life, shelf life extension test, return opportunity, shelf life information

ÖZ

EKONOMİK SİPARİŞ MİKTARI ORTAMINDA RAF ÖMÜRLÜ MALZEMELER İÇİN TEDARİKÇİ ESNEKLİĞİNİN VE BAŞLANGIÇ RAF ÖMRÜ BİLGİSİNİN DEĞERİ

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Raf ömürlü malzemelerin envanter yönetimi zorlu bir problemdir. Bu çalışmada, rassal raf ömrü, rassal olmayan sabit talep, sonsuz ufuk ve sürekli kontrol özelliğine sahip envanter modeli düşünülmüştür. Tedarikçiden gelen malzemelerin ilk raf ömrü rassal bir değişkendir. Malzemeler ilk raf ömrü bitiş tarihine ulaştığında, sonucun rassal olduğu bir kalite kontrol testi uygulanır. Bu testin sonucuna göre, talebi karşılamak için malzemeleri ek bir süre boyunca daha kullanmak mümkün olabilir. Ayrıca, malzemelerin alıcı firmaya ulaştığı anda geri gönderilme fırsatının ve sipariş kararından önce malzemenin raf ömrü bilgisini öğrenme olanağının düşünülmesiyle modellenen tedarikçi esnekliklerini de incelemekteyiz. Amaç, raf ömrü uzatma testi fırsatının değerini, geri gönderme fırsatının değerini ve raf ömrü bilgisinin değerini araştırmak için Yenileme Teorisi'ni kullanarak birim zaman başına toplam maliyeti en aza indirmektir. Bu nedenle, çeşitli kurgular oluşturulmuş ve bunlar sayısal analiz yapılarak karşılaştırılmıştır.

Anahtar Kelimeler: raf ömürlü malzemeler, raf ömrü uzatma testi, geri gönderme olanağı, raf ömrü bilgilendirmesi

To my family

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CHAPTER 1

INTRODUCTION

In many kind of industries, production plants need to procure some perishable items such as adhesives, paints or chemicals to start or perform production (or assembly) of work orders. Such a perishable item has a shelf life which defines the last allowed usage and application time in the production environment. After that time the item becomes useless and must be disposed. Therefore, managing such items in order not to interrupt the production is very important which may result in delay to complete final products; and therefore, unsatisfactory demands at the end. In a classical single item inventory planning problem, the order size and time mainly depend on the review policy (continuous or periodic review), the cost components (fixed ordering, unit purchasing, holding etc.), properties of demand (deterministic or stochastic, stationary or nonstationary), lead time, planning horizon and how stockouts are handled.

In practice, buyers generally become aware of the exact expiration date of the incoming material after receiving. However, depending on the relation between the supplier and the customer, different arrangements might occur. For instance, if the buyer is a reputable buyer for the supplier, it is possible to make an agreement between the buyer and the supplier that the supplier provides the information about the shelf life of its own stock to the buyer before the order is placed. If such an agreement is not preferred, the supplier may allow the buyer to return some of the items as another form of supplier flexibility. In such an agreement, the buyer learns the shelf life after arrival of the batch, and it can return any amount from the batch by getting a certain refund.

Some perishable products may not lose their functionality or characteristics even if their ages reach the expiration date. Therefore, it is possible to use them further rather than dispose. In order to check whether the product is still in good condition, it is possible to conduct some "shelf life extension tests" and examine whether the product still functions as intended. If the extension test is successful, then it is possible to use the product beyond its expiration date. However, if the extension test fails, then that batch is disposed on its expiration date.

Motivated by observations of such applications in defense industry, we would like to investigate the value of expiration date information availability, return opportunity and shelf life extension opportunity for a perishable item with uncertain shelf life. For this purpose, we consider an EOQ setting (deterministic and stationary demand, no initial inventory, lead time is zero, no backorders or lost sales are allowed and unit purchasing cost does not depend on order size) and introduce several settings that involve shelf life information availability, return opportunity cost and shelf life extension test opportunity. For each setting that we consider, we characterize the expected cost per unit time. Through a detailed computational study, we identify and quantify improvements in expected costs provided by supplier relations (information and return) and extension opportunity. Our findings indicate that highest percent improvement is observed when extension opportunity is provided, if there is no shelf life information. However, by investigating all instances, shelf life information opportunity provides generally higher savings than other opportunities. Return opportunity is never more valuable than shelf life information availability, although sometimes it is possible to perform under it as good as under shelf life information opportunity.

The remainder of the thesis is organized as follows: In Chapter 2, we review the studies in the literature which are developed for inventory policies of perishable items. In Chapter 3, detailed problem definition and assumptions are given. The results of the computational study are presented, the outcomes of the relevant settings are compared to each other and they are discussed in Chapter 4. Finally in Chapter 5, we conclude by summarizing our findings and give future research suggestions.

CHAPTER 2

LITERATURE REVIEW

Managing perishable items is a challenging task in inventory planning. There are many studies on inventory policies for perishable and/or deteriorating items. Nahmias [1] provides a review on perishable item inventory theory. This review includes studies for both fixed life perishable items and items subject to continuous exponential decay by considering deterministic or stochastic demands. He reviews the optimal policies, approximately optimal policies, LIFO inventory systems, multiproduct models and multiechelon models for fixed life perishable item inventory problem. Moreover, he provides a review of the studies on inventory management for items with random lifetime which is controlled periodically and inventory model for items with exponential decay. Another review is provided by Karaesmen et al. [2]. They categorize the studies into three main classes. Firstly, they focus on single item, single location systems, then they examine multi-echelon and multi-location systems. Finally, they examine studies considering uncommon features such as multiple products, substitution, multiple-types of customers, pricing etc. They provide subgroups of main topics by considering review policies, lead time types, cost characteristics, information sharing possibilities, and other characteristics.

In this study, we provide a review of studies concentrating on a single item and single location inventory systems; therefore, the studies on multiechelon systems are not considered. The studies in this field can be classified regarding to life time characteristics of perishable products which can be deterministic, deteriorating and random. Although we focus on inventories with random life time in this study, we also examine the studies on other life time characteristics in order to gain an inside. In Section 2.1, studies assuming fixed deterministic life time are reviewed, whereas studies on deterioration and random life are summarized in Section 2.2. Finally, our contribution to existing literature is given in Section 2.3.

2.1 Studies on Fixed Deterministic Life Time

In this section, studies concentrating on fixed deterministic life time are investigated in detail.

Weiss [3] considers an inventory planning problem for a single item with fixed shelf life over an infinite horizon. Demand follows a Poisson process, and lead time is zero. He presents optimal policies for both lost sale and backordering cases by minimizing expected system-wide cost which is composed of fixed ordering cost, purchasing cost per unit, holding cost per unit per unit time, shortage cost per unit per unit time and revenue per unit (as negative cost). He proves that (s, S) policy, where S is order-up-to level and s is the reorder level, with s=0 is optimal for lost sale case, and s $\leq$-1 for backordering case, respectively. Liu and Lian [4] extend the study of Weiss [3] by considering systems where demand is generated by a general renewal process. They study both lost sale and backordering cases with with fixed ordering cost, holding cost per unit per unit time, shortage costs per unit and per unit per unit time, and outdating costs per unit that is incurred for each perished item. They model the inventory level by using Markov renewal equations. They show that optimal policy characterized by Weiss [3] is also valid for a general renewal process demand.

Lian and Liu [5] and Gürler and Özkaya [6] also focus on (s, S) policy in an environment in which backorders are allowed and planning horizon is infinite. Their objective is finding optimal levels of s and S to minimize the average total expected cost per unit time that includes fixed cost per order, holding cost per unit per unit time, shortage penalties per unit and per unit per unit time and replacement cost per unit decayed. Inventory level is formulated as Markov process. In the former study, demand comes in batches with Poisson arrivals, while in the latter study with general distribution function. Lian and Liu [5] give an optimization algorithm for zero lead time, whereas for positive lead time cases heuristics are proposed. Gürler and Özkaya [6] fix a flaw in the study of Lian and Liu [5] and generalize their study for the case where the demand arrivals follow an arbitrary renewal process.

Chiu [7] studies a continuous review (r, Q) inventory model where r is the reorder point and Q is the order quantity. Excess demand is backordered and planning horizon is infinite. Inventory is depleted according to First in First out (FIFO) policy. Lead time is positive and less than the life time of the product and a general demand distribution is studied. The cost is composed of fixed ordering cost, purchasing cost per unit, holding cost per unit per unit time, shortage cost per unit and outdating cost per unit. The author gives an approximate model and compares the minimum cost performance of given model for Poisson demand distribution with cost performances of EOQ and model by Weiss [3] under zero lead time condition. He finds out that proposed approach is a good approximation model but expected shortage per cycle and expected inventory level are underestimated by the proposed model.

Berk and Gürler [8] analyze continuous review (r, Q) policy under continuous review where excess demand is lost, lead time is positive and there is unlimited supply. Demands arrive according to a Poisson process. Authors define the concept of "effective shelf life" which is the remaining shelf life of items and show that the effective shelf-life sequence has the Markov property. The objective is to minimize the total expected costs per unit time which includes fixed cost per order, holding cost per unit per unit time, shortage and outdating costs per unit. They compare exact (r, Q) model with a benchmark policy (Q, r, T) and approximate policy parameters of study of Chiu [7]. Under (Q, r, T) policy, a new order of Q is placed whichever comes first: Either when the inventory level drops to r or when T time units after unpacking of batch have elapsed. They show that the (r, Q) policy is a good approximation for high fixed ordering costs, small shortage or outdating costs and long shelf lives. It is shown that the optimal (r, Q) policy exhibits a maximum difference of 3.5% from the benchmark (Q, r, T) policy, whereas Chiu's model deviates by a maximum of 18%.

Williams and Patuwo [9] study a single period problem for the lost sale case. Lead time and life time are deterministic. Demands in successive periods are assumed to be independent, but not necessarily identically distributed random variables. Inventory is depleted according to FIFO policy. Life time of an item is assumed as two periods. The cost components are purchasing cost per unit, holding cost per unit per unit time, shortage cost per unit and outdating cost per unit. Using Dynamic Programming (DP) the optimal order quantities are derived for cases in which the lead time equals to one period, the lead time equals to two periods (i.e. the life time of items) and the lead time is greater than the life time. The optimal order quantity are given for exponential, triangular and uniform demand distributions. Williams and Patuwo [10] perform a numerical study for different cost parameter values in order to investigate the behavior of optimal order quantities where DP model provided by Williams and Patuwo [9] is used. They find that an increase in the ordering, holding and outdating costs decrease the optimal order quantity, while increase in the shortage cost increases it. They show that the ordering and the shortage costs have a more significant effect than the other cost components. Although it is stated that it is easy to extend the study to a multiperiod finite horizon model, they do not give such an extension.

Minner and Transchel [11] develop a dynamic replenishment policy to meet given service level requirements under periodic review. Lost sale occurs when the demand exceeds the inventory on hand. The planning horizon is infinite. The lead time is deterministic. Demand arrives according to general distribution function. The inventory positions are investigated for both FIFO and Last in First Out (LIFO) depletion cases. They propose a heuristic policy and compare it with base-stock policy (BSP) and constant order policy (COP). Numerical results are given for stationary (Gamma distributed) demand and nonstationary (by assuming some weekly patterns of daily demands) demand and the authors show the superiority of the proposed dynamic policy in terms of inventory and waste levels. They show that a constant-order policy might provide good results when the demand is stationary, shelf life is short, and inventory depleted according to LIFO policy.

Haijema et al. [12] study a heuristic order up to inventory policy for blood platelets under periodic review. Planning horizon is finite which is studied as one week, and lead time is deterministic and taken as one day. There are two types of demand which are called as “any” platelets demand (which follows Poisson distribution) by a patient from general surgery etc., and “young” platelets demand which comes from patients of hematology and oncology. They consider production cost per unit, holding cost per unit per unit time, outdating cost per unit, shortage cost per unit and mismatch (when there is demand for ‘young’ but there are not enough young platelet pools so that older pools have to be used) cost per unit. They assume that a "young" demand is issued in LIFO manner, whereas "any" demands are issued in FIFO manner. They follow a combined Markov DP and simulation approach. They characterize system state with the day of the week, residual shelf life times of inventory on hand and production amount for the next day. For the base test case, DP approach is used for finding order up to levels for each day. However, because of the complexity of the original full-sized problem, they propose local search algorithms which provide near optimal inventory policies.

Parlar et al. [13] examine a system where arrival of the items and demand processes are independent Poisson processes. Planning horizon is as infinity. Lead time is positive and deterministic. Excess demand is lost. The controller does not decide on the size of the replenishment orders and their timings. The aim of that study is to compare extreme issuing policies which are FIFO and LIFO under continuous review regarding to average profits. They derive long run average profit function by considering revenue per unit satisfied demand, holding cost per unit per unit time, shortage cost per unit and purchasing cost per unit. It is shown that if the unit inventory holding cost per unit time is high or unit purchasing cost is low, FIFO outperforms LIFO issuing policy, whereas for low unit inventory holding cost per unit time or high unit purchasing cost levels, LIFO outperforms FIFO.

Muriana [14] provides an Economic Order Quantity (EOQ) model for a system for which the relevant costs are fixed ordering, purchasing cost per unit, holding cost per unit per unit time, shortage and outdating costs per unit. Unlike the traditional EOQ model, demand rate is not fixed and taken as a Normally distributed random variable. Lead time is deterministic and constant. He determines the probability that the products remain in stock beyond the end of their shelf lives. The model provides optimal batch size and cycle time which minimize expected total cost per unit time. A sensitivity analysis by changing the parameters is conducted and the results are discussed in order to support managerial decisions.

The studies reviewed in this chapter include the ones assuming fixed shelf life. The studies which examine items that have uncertain shelf life is explained in details in following Section 2.2.

2.2 Studies on Deterioration and Random Shelf Life

The items that lose their functionality through time, not exactly on the shelf life expiration date are called deteriorating items. Wee [15] defines deteriorating items which are explained as the items that become decayed, damaged, evaporative, expired, becomes invalid, or subject to devaluation etc. through time. Note that as stated by Liu [16], deterioration/decay events and random shelf life are considered as the same concept in papers of Nahmias [1] and Nahmias and Wang [17] when the life time is exponentially distributed. There is a relation between the random life times of the individual items and the proportional decay of the (mean) inventory level. When the life times of individual items are exponentially distributed, the expected inventory level will decrease at a fixed rate equal to the rate of life time distribution. Therefore, the studies which focus on deterioration and random shelf life are investigated in the same classification in this section.

Tripathi and Uniyal [18] study an EOQ model with fixed ordering cost, purchasing cost per unit, holding cost per unit per unit time and shortage cost per unit per unit time. Backordering is allowed and the planning horizon is infinite. Lead time is zero. Demand rate is not fixed, but linearly increasing function of time. This rate is composed of initial constant demand plus demand depending on time. Items deteriorate with a constant rate. Authors construct a mathematical model to find optimal replenishment policy and use differential equations to find optimal ordering quantity, replenishment time and total cost per cycle by considering maximum storage quantity constraint. They perform a numerical study by changing deterioration rate, ordering cost, purchasing cost, shortage cost and holding cost, and investigate the effects of changes in ordering quantity and total cost per cycle time. They show that increase in deterioration rate, purchasing cost, shortage cost and holding cost decrease ordering quantity, whereas increase in ordering cost and initial constant demand increase it. On the other hand, it is stated that increase in only holding cost decreases total cost per cycle. Other cost components result in increase in the cost function.

Dave and Shah [19] study probabilistic EOQ model under periodic review to find optimal order-up to level minimizing expected cost composed of fixed cost per order, purchasing cost per unit and inventory holding cost per unit per unit time. Demand follows a uniform pattern. Planning horizon is finite. Lead time is taken as exactly one cycle length which means that an ordered lot arrives at the time of placing the next order. The items deteriorate with a constant rate according to a continuous exponential decay function. They construct a mathematical model to find optimal order level and the model is solved with respect to different parameter values. The authors state that it is no exact formulation if the model incorporates shortages; therefore, the model they provide excludes the shortages.

Hsu [20] studies an Economic Lot Sizing (ELS) model under periodic review. The excess demand is lost and finite planning horizon is taken into account. Lead time is zero and the demand is deterministic. Deterioration rate and holding cost change regarding to age of the inventory. The objective is the minimize the total cost composed of production cost per unit and holding cost per unit per period. He shows that for age dependent inventory costs (with or without stock deterioration), Zero Inventory Policy does not hold in the optimal solution. Consecutive-Cover-Ordering (CCO) solution where each order is used to satisfy all demands from a number of consecutive indexed periods gives a feasible solution. He shows that minimum objective function value of the best CCO solution is the optimal objective function value of the problem if all production and inventory cost functions are nondecreasing in time and concave. To find the best CCO solution, he presents a DP recursion. The author discusses the special cases where both production and inventory functions are fixed-plus-linear functions which are solvable with reduced complexities.

In a further study, Hsu [21] adds backordering and corresponding cost item, and makes comparisons to Hsu [20] in terms of problem characteristics, cost functions, assumptions and computational complexity. He provides a DP algorithm. Sargut and Isik [22] also study the dynamic ELS model in finite planning horizon and extend the study of Hsu [21] by considering finite production capacity. They show the structural properties of the optimal solution and propose a heuristic procedure which performs satisfactorily when the production periods are taken as given. When production period is taken as 5 and demand is increasing through the time, their heuristic finds the optimal solution in 78.90% of cases and deviates from the optimal solution less than one percent in 88.83% of cases. As demand structure changes and production periods gets longer, their heuristic performs worse.

Chu et al. [23] also work on the same environment in terms of review type, planning horizon, zero lead time and deterministic demand. The excess demand is lost and cost functions to be minimized includes cost of ordering per unit and holding cost per unit per unit time are concave functions. They show that there are instances of the problem where objective function value under Zero Inventory Policy solution may have an arbitrarily large error compared to that of the optimal solution. They analyze the effectiveness of the Consecutive-Cover-Ordering solutions by giving a transformation procedure that transform an optimal solution into a Consecutive-Cover-Ordering solution. It is guaranteed to be no more than 1.52 times of the optimal cost. In addition, if the ordering cost function does not change from period to period, the cost of the best Consecutive-Cover-Ordering policy is no more than 1.5 times of the optimal cost.

Setiawan et al. [24] study a perishable item inventory model with returns. Both deterministic and inventory depended demand structures are examined. Inventory dependent demand is explained as demand at a certain time depends on the available inventory at that time with a certain rate. Backordering is allowed. There is an opportunity that after some period of time (called return time), remaining perishable items may be returned to the supplier at some returning cost. The authors search for the optimal order quantity and the optimal return time. The cost components included in the model are fixed cost per order, purchasing cost per unit, holding cost per unit per unit time, shortage cost per unit, return cost per return and return cost per unit. In a numerical study and sensitivity analysis, they show that as deterioration rate and inventory depended demand factor increase, the optimal return time gets shorter, the optimal order size and the total cost per unit time increase.

Kalpakam and Sapna [25] consider (s, S) inventory policy under continuous review. When the demand exceeds the inventory on hand, it is lost. Infinite planning horizon is studied. Lead time and life time are both exponentially distributed. Demand arrives according to a Poisson Process. The inventory level is modeled as Markov process and the objective is to minimize steady state cost rate. Cost components are fixed ordering, purchasing cost per unit, outdating and shortage costs per unit and holding cost per unit per time. They limit the number of outstanding replenishment orders to, at most, one at any given time. They give the analytic properties of the cost function by allowing only one variable to change, they find the optimal cost, reorder level and order quantity values.

Liu and Yang [26] study a similar model with Kalpakam and Sapna [25], but allow backorders and place no restriction on the number of outstanding orders. They assume that the replenishment orders will be processed one by one in sequence and the processing time (which is a part of lead time) for a replenishment order is exponentially distributed. They model the inventory level as Markov process by considering that an expiration event is not different from a demand arrival for the replenishment decision. They obtain a matrix-geometric solution to the probability distribution of the inventory level in the long-run. The cost components are fixed cost per order, holding cost per unit per time, outdating cost per unit and shortage costs per unit and per unit per unit time. They conduct numerical study by changing cost parameters and replenishment rate which is the inverse of the mean order processing time to investigate optimal reorder size and order-up-to level.

Liu [16] examines (s, S) inventory policy under continuous review for the case where demand arrives according to Poisson process and life times of items are exponentially distributed. Backordering is allowed and the lead time is zero. Since the inventory level over time t can be modeled as a Markov process, he provides appropriate differential equations to describe the behavior of the inventory level and establishes expected cost functions by considering fixed ordering cost, purchasing cost per unit, replenishment cost per unit decayed, shortage costs per unit and per unit per unit time and holding cost per unit per unit time. He shows that optimal reorder point s should be less than equal to -1.

Liu and Shi [27] extend the study of Liu [16] for the case where demand process is a general renewal process. The cost components are same with the former study. The inventory level over time is again modeled as Markov process. They adjust the equations and cost functions accordingly and investigate their properties. They perform numerical analysis with deterministic, Erlang and Hyperexponentially distributed demand. They analyze the impact of change in cost parameters and demand rates on the optimal cost, the optimal order quantity and the optimal number of deteriorated units per cycle.

Kouki et al. [28] examine (T, S) policy in which the inventory level is controlled at equal intervals of time with length T, and a replenishment order is placed to bring the inventory position to the order-up-to-level S every T units of time is considered. They are interested in both lost sales case and backordering case. Planning horizon is infinite. Lead time is positive. Life time of each item is Exponentially distributed and demand arrives according to Poisson distribution. The inventory level over time is modeled as a Markov process. They propose an optimization algorithm for T and S values minimizing expected total cost composed of fixed cost per order, purchasing cost per unit, holding cost per unit per unit time, lost sale/backorder cost per unit and outdating cost per perished unit. They compare the performances of the proposed model with stochastic life time with a benchmark model with no perishability (i.e. infinite life time) and (T, S) policy with fixed life time by conducting numerical study. It is stated that consideration of the randomness of the life time improves the total optimal cost significantly.

Kouki and Jouini [29] focus on periodic review (T, r, Q) policy in which the inventory level is periodically controlled at the beginning of each equal cycle with length T, and if at the observation epoch the inventory level is at or below the reorder point r, a replenishment order of Q units is placed. Excess demand is lost and planning horizon is infinite. Lead time is positive. Demand follows a Poisson Process. The life times are assumed to follow m-Erlang distribution. The objective is to minimize expected average total cost per unit of time equations which includes fixed ordering, holding cost per unit per unit time, purchase cost per unit, lost sales cost per unit and outdate cost per unit of product that perishes in stock. Authors investigate two extreme life time cases which are Exponentially distributed and deterministic life times. For Exponential life time case, first, they provide transition and steady state probabilities for inventory level which is modeled as a continuous time Markov chain and expected operating cost. For deterministic life time, a regenerative cycle is determined according to three possible cases with respect to different relationships among life time, lead time and cycle time. Authors compare two extreme cases of life times by changing cost parameters. For general case analysis, a simulation study is conducted to examine the impact of the life time randomness mixed with the cost parameters on the total cost by changing m from 1 to 10000.

Gürler and Özkaya [30] study (s, S) policy under continuous review where excess demand is backordered over an infinite horizon. Demand arrivals occur in batches. Shelf life of items are random due to imperfect storage conditions. The costs associated with the inventory system are the fixed cost per order, the holding cost per unit per time, outdating cost per unit, the backorder costs per unit and per unit per time. For zero lead time case authors give mathematical models for expected cycle lengths and cycle costs for both discrete and continuous demand batch sizes. They propose a heuristic for positive lead time. In numerical study, the performances of the system under fixed shelf life and random (Gamma, Weibull, Uniform, Triangular and Truncated Gamma) shelf life are compared under unit, geometric and Gamma demand distributions. They show that improving the storage conditions or the production process which results in longer-tailed shelf life distributions may result in significant saving. Moreover, they observe that as coefficient of variation of the shelf life increases, the difference between fixed and random shelf life models also increases considerably and the highest costs are incurred for the Exponentially distributed shelf life.

2.3 Our Contribution to The Existing Literature

We study a single perishable item inventory planning problem in an EOQ environment like Muriana [14], Tripathi and Uniyal [18] and Dave and Shah [19], who also study in EOQ environments under miscellaneous conditions. In our study, the planning horizon is infinite, demand is stationary and deterministic, and the order lead time is assumed to be zero. Therefore, an order is placed whenever the inventory on hand drops to zero. We assume that the expiration date of a new incoming batch is a discrete random variable. There is no deterioration until the shelf life expiration date. With respect to these assumptions, the most relevant studies which give insight and motivate our study are listed in Table 2.1.

Table 2.1: Classification of Reviewed Studies

No	Study	Policy	Review	Excess Demand	Planning Horizon	Lead Time	Shelf life	Demand	Fixed Ordering Cost	Unit Purchasing Cost	Holding Cost	Shortage/ B/O Cost	Outdating Cost
1	Muriana [14]	EOQ	Continuous	Lost	Infinite	Deterministic	Fixed	Random	Yes	Yes	Yes	Yes	Yes
2	Berk and Gürler [8]	(Q, r)	Continuous	Lost	Infinite	Deterministic	Fixed	Random	Yes	No	Yes	Yes	Yes
3	Tripathi and Uniyal [18]	EOQ	Continuous	B/O	Infinite	Zero	Deteriorate (Exp. rand.)	Function of time	Yes	Yes	Yes	Yes	No
4	Dave and Shah [19]	EOQ	Periodic	-	Finite	Deterministic	Deteriorate (Exp. rand.)	Random	Yes	Yes	Yes	No	No
5	Setiawan et al. [24]	Opt.	Periodic	B/O	Infinite	Zero	Deteriorate (Exp. rand.)	Deterministic/ inv. depend.	Yes	Yes	Yes	Yes	No
6	Kalpakam and Sapna [25]	(s, S)	Continuous	Lost	Infinite	Random	Deteriorate (Exp. rand.)	Random	Yes	Yes	Yes	Yes	Yes
7	Liu and Yang [26]	(s, S)	Continuous	B/O	Infinite	Random	Deteriorate (Exp. rand.)	Random	Yes	No	Yes	Yes	Yes
8	Liu [16]	(s, S)	Continuous	B/O	Infinite	Zero	Deteriorate (Exp. rand.)	Random	Yes	Yes	Yes	Yes	Yes
9	Liu and Shi [27]	(s, S)	Continuous	B/O	Infinite	Zero	Deteriorate (Exp. rand.)	Random	Yes	Yes	Yes	Yes	Yes
10	Kouki et al. [28]	(T, S)	Periodic	Lost & B/O	Infinite	Deterministic	Deteriorate (Exp. rand.)	Random	Yes	Yes	Yes	Yes	Yes
11	Kouki and Jouini [29]	(T, r, Q)	Periodic	Lost	Infinite	Deterministic	Random	Random	Yes	Yes	Yes	Yes	Yes
12	Gürler and Özkaya [30]	(s, S)	Continuous	B/O	Infinite	Zero	Random	Random	Yes	No	Yes	Yes	Yes
13	Our study	EOQ	Continuous	-	Infinite	Zero	Random	Deterministic	Yes	Yes	Yes	No	No

Although the studies mentioned in Section 2.2 focus on the random shelf life characteristic, we investigate another random shelf life concept which is "extension test opportunity." An item does not always have to lose its functionality on the shelf life expiration date. In practice, there are some test systems to control the functionality of a perishable item on the shelf life expiration date and may allow them to be used further. In this study, we investigate the effect of shelf life extension test which assess whether the corresponding batch can be used beyond its expiration date or not. The success of the test is a random process; thus, the extension test opportunity concept increases the uncertainty of the length of the replenishment cycle.

Besides, we study the supplier flexibility. The relationship between the buyer and the supplier is examined in terms of shelf life information opportunity before ordering or return opportunity for partial of the batch at arrival (after realizing the expiration date). In the literature, a kind of return policy for perishable items is examined in the study of Setiawan et al. [24]. The authors search for the optimal return time at some returning costs for all unsold items under stochastic demand and deterioration conditions. The supplier replaces them in the next delivery. However, in this study, we investigate a different return opportunity concept. We try to find the optimal return quantity at arrival and the supplier offers a refund for each unit returned which may have different levels.

CHAPTER 3

AN EOQ MODEL WITH RANDOM SHELF LIFE AND SHELF LIFE EXTENSION OPPORTUNITY

The major research questions that we address in this study are:

- How do the random shelf life and shelf life extension opportunity affect the optimal order quantity?
- What is the effect of return policy on costs when an item has a shelf life that can be extended by an additional time period?
- How valuable is it to get shelf life expiration date information before placing an order?
- How valuable is it to get shelf life extension information before placing an order?

In this chapter, in order to gain insights on the questions listed above, we build and analyze several mathematical models. Section 3.1 provides further details on the problem setting upon which we build our mathematical models. The models regarding to sections where they are characterized are given in Table 3.1. To clarify, Section 3.2 introduces the case where there is no shelf life information and no shelf life extension possibility. Section 3.3 includes shelf life extension possibility. In Section 3.4, we introduce a return policy which allows sending items back to the supplier in return for a refund. Section 3.5 focuses on a setting where it is possible to get shelf life expiration date information before placing an order, but shelf life extension is not possible. Shelf life extension opportunity is added to the model in Section 3.6. Finally, the model in Section 3.7 covers not only shelf life expiration date information, but also extension information before ordering.

Table 3.1: Models in related sections

	Return Opportunity	**Extension Opportunity**	**Shelf Life Information**
Section 3.2	No	No	No
Section 3.3	No	Yes	No
Section 3.4	Yes	No	No
Section 3.5	No	No	Partial
Section 3.6	No	Yes	Partial
Section 3.7	No	Yes	Full

3.1 Problem Definition

In order to investigate our major research questions, we consider a single item inventory system where demand is deterministic and stationary, lead time is negligible and planning horizon is infinite. Stockouts are not allowed.

Initial shelf life of an item is defined as the time between its received and expiration date, which we model as a discrete random variable as it might not be possible to know at the time of the order what the expiration date is. For ease of analysis, we assume that the initial shelf life is t_1 with probability α, and it is t_2 with probability $(1 - \alpha)$, where $t_1 < t_2$. The initial shelf life becomes known upon arrival. In this setting, an order of size Q is placed whenever the inventory level drops to zero resulting in a fixed order cost of K and a variable ordering cost of c per unit (See Table 3.2 for notation). The order quantity is determined without perfect information on shelf life in the base setting. Similarly, extension opportunity is ignored in the base setting. On-hand inventory is depleted at a rate of λ until it drops to zero or until the expiration date. An inventory holding cost of h per unit per year is incurred for on-hand inventory. If there is on-hand inventory when expiration date is reached, these units are discarded in the base setting with no additional cost or revenue.

We consider several extensions of the base model in order to analyze effects of the shelf life extension opportunity, a possible return policy, perfect information on initial shelf life and perfect information on shelf life extension information.

Shelf life extension opportunity might arise when the item may be still in a good

Table 3.2: Notation

Parameters & Variables	
K	: fixed ordering cost
c	: ordering cost per unit
h	: holding cost per unit per year
λ	: annual demand rate
t_i	: initial shelf life of items (i=1,2), $t_1 < t_2$
α	: probability that the initial shelf life of the item is t_1
p_i	: probability that shelf life is not extended given that the initial shelf life is t_i
δ	: extension amount of the shelf life in years
Q	: order quantity
$T(Q)$	: time between two successive orders for a given $Q \geq 0$
$P(Q)$	: expected total ordering cost per cycle for a given $Q \geq 0$
$H(Q)$	: expected total holding cost per cycle for a given $Q \geq 0$
$TC(Q)$	: expected total cost per cycle for a given $Q \geq 0$
$AC(Q)$	: expected total cost per year for a given $Q \geq 0$

condition when it reaches the end of its initial shelf life. In order to be able to use that item further, it is possible to run some quality tests on the item, if capable, when the initial shelf life is over. If item passes the tests (which occurs with a certain probability, $(1-p_i)$, when the initial shelf life is t_i, and i=1, 2 in our model), the shelf life of entire batch is extended for a fixed period of time, δ. If the item fails (with probability, p_i), the entire batch is disposed. Notice that the failure/success probability depends on the initial shelf life of the batch, t_i. This extension process determines the final shelf life of the item. It is assumed that there is no cost of extension test.

Another aspect of the problem that we consider is a return agreement between the supplier and the buyer. In this setting, return is possible just after the order is received. The buyer determines the return quantity immediately after the initial shelf life information becomes available and gets a refund per returned item. As another possible supplier-buyer agreement, the supplier might share the initial shelf life information of its own stock just before the buyer places a new order.

We also consider a benchmark setting where the buyer has full information about shelf life and extension possibility. This setting provides lower bound on costs among other ones. Note that if there is perfect (initial or final) information on shelf life, then the buyer is aware of the initial shelf life realization before placing a new order; hence, there is no need to apply a return policy.

Considering different aspects of the setting we presented above, we introduce and analyze the following problem settings:

Setting I: No return, no shelf life information, no extension

Setting II: No return, no shelf life information, possible extension

Setting III: Return is allowed, no shelf life information, possible extension

Setting IV: No return, available information about initial shelf life, no extension

Setting V: No return, available information about initial shelf life, possible extension

Setting VI: No return, full information about shelf life

Besides, another setting called setting III' is also analyzed which is a special case of setting III. The difference is that extension is not possible in setting III' by fixing related parameters of setting III to values which disable extension opportunity. These values are p_1=p_2=1 and δ=0.

Comparisons of the settings listed above will help us assess the value of shelf life extension opportunity, return opportunity and shelf life information. Specifically,

1. I vs. II: Value of shelf life extension possibility when initial shelf life information is not available.

2. IV vs. V: Value of shelf life extension possibility when initial shelf life information is known.

3. I vs. III': Value of return opportunity when extension is not possible.

4. II vs. III: Value of return opportunity when there is extension opportunity.

5. I vs. IV: Value of initial shelf life information when extension is not possible.

6. II vs. V: Value of initial shelf life information when there is extension opportunity.

7. V vs. VI: Value of full shelf life information when initial shelf life is known.

8. II vs. VI: Value of full shelf life information when initial shelf life is unknown.

Note that the inventory level is zero when an order is placed in all of the settings that we consider. The size of the order is the same for every cycle. Furthermore the shelf life of orders and extension realizations (if any) are independent across orders. Hence, if we consider the inventory level at time t, $I(t)$, as a stochastic process, then $I(t)$ regenerates every time that it hits zero. Defining a cycle as time between two consecutive orders represented by $T(Q)$, we can conclude that cycle lengths are independent and identically distributed random variables. $TC(Q)$ can be represented as expected total costs incurred between two consecutive orders which depends on $T(Q)$. Hence, Renewal Reward Theorem can be implemented to find expected cost per unit time (see Theorem 3.16 p. 52 of Ross [31]). By Renewal Reward Theorem, the expected cost per unit time can be calculated as

$$\frac{TC(Q)}{E[T(Q)]}$$

Note that the above argument holds for all settings that we consider.

3.2 Setting I: No Return, No Shelf Life Information, No Extension

In this setting, the buyer does not have shelf life information when placing an order. After the order arrives, although the buyer observes the shelf life of the item, it cannot return any quantity since there is no agreement with the supplier. Moreover, the extension test is not applicable. The optimal order quantity, Q, can be found by solving the following problem optimally:

$$\begin{aligned} \text{minimize} \quad & AC(Q) = \frac{TC(Q)}{E[T(Q)]} \\ \text{subject to} \quad & Q/\lambda \leq t_2 \\ & Q \geq 0 \end{aligned}$$

The objective function is the expected cost per year for orders of size $Q \geq 0$. We only consider instances where $Q/\lambda \leq t_2$, since it is never optimal to place an order that covers a period longer than the maximum shelf life. Depending on the relations among Q/λ, t_1 and t_2, the form of $AC(Q)$ can be represented as

$$AC(Q) = \begin{cases} AC_1(Q) & \text{if} \quad Q/\lambda \leq t_1 \\ AC_2(Q) & \text{if} \quad t_1 \leq Q/\lambda \leq t_2. \end{cases}$$

We characterize $AC_1(Q)$ and $AC_2(Q)$ in sections 3.2.1 and 3.2.2, respectively.

3.2.1 Expected average cost when $Q/\lambda \leq t_1$

If $Q/\lambda \leq t_1$, then the order quantity will definitely be depleted before expiration date. Hence, the total cost over a cycle is independent of the shelf life of the item. In such a condition, the expected cost per unit time is

$$AC_1(Q) = \frac{K\lambda}{Q} + c\lambda + h\frac{Q}{2},$$

which is the cost for a traditional EOQ model. Note that in all other settings when $Q/\lambda \leq t_1$, expected average cost expression is expected average cost under EOQ.

3.2.2 Expected average cost when $t_1 < Q/\lambda \leq t_2$

In every cycle, independent of the shelf life realization, total ordering cost is $P(Q) = K + cQ$. When $t_1 < Q/\lambda \leq t_2$, there are two possible cycle length realizations; t_1 and Q/λ. The cycle length distribution and expected cycle length are as follows

$$P\{T(Q) = t\} = \begin{cases} \alpha & \text{if} \quad t = t_1 \\ (1-\alpha) & \text{if} \quad t = Q/\lambda \end{cases} \tag{3.1}$$

and

$$E[T(Q)] = \alpha t_1 + (1-\alpha)Q/\lambda.$$

If the initial shelf life turns out to be t_1, the cycle ends after t_1 periods due to the shelf life expiration. Then, $(Q - \lambda t_1)$ units of inventory is instantaneously disposed when the order is received at no value and no cost in order to incur less holding cost. Disposal decreases the inventory level to λt_1 (which is shown by Cycle 1 in Figure 3.1). On the other hand, if the initial shelf life is t_2, the cycle is completed exactly after Q/λ periods which means that the entire inventory is depleted by the demand, and there is no disposal (See Cycle 2 in Figure 3.1).

Considering two possible cycle realizations (see Figure 3.1), we get the expected

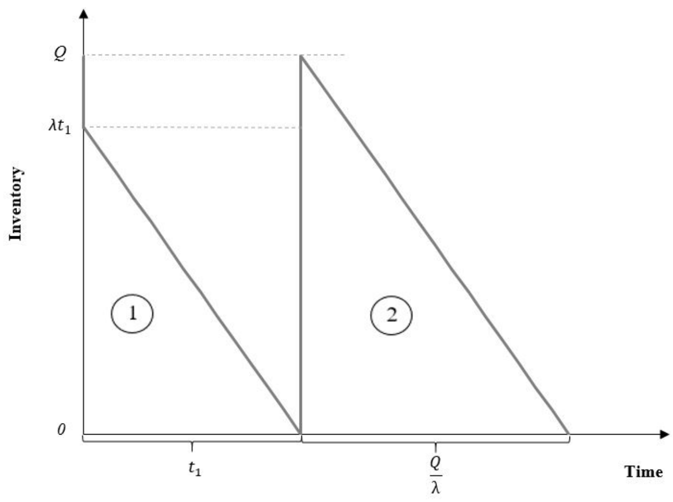

Figure 3.1: Setting I- Possible Cycle Length Realizations when $t_1 < Q/\lambda \leq t_2$

holding cost per cycle as

$$H(Q) = \frac{h}{2}\left(\alpha\lambda t_1^2 + (1-\alpha)\frac{Q^2}{\lambda}\right).$$

Thus, the total expected cost per year for this condition is given as follows:

$$AC_2(Q) = \frac{K + cQ + \frac{h}{2}\left(\alpha\lambda t_1^2 + (1-\alpha)\frac{Q^2}{\lambda}\right)}{\alpha t_1 + (1-\alpha)\frac{Q}{\lambda}}.$$

The optimal order quantity is found by minimizing the total expected cost per year.

3.3 Setting II: No Return, No Shelf Life Information, Possible Extension

In this setting, we include the extension possibility into the base setting that was considered in Section 3.2. When the order is received, the initial shelf life, t_i is observed. Once the initial shelf life is over, the shelf life extension test is conducted. If the batch passes the test, it is allowed to be consumed for an additional δ periods. The probability that the batch passes the test when the initial shelf life is t_i, is $(1-p_i)$. In order to find the optimal order quantity, the following problem must be solved.

$$\begin{aligned} \text{minimize} \quad & AC(Q) = \frac{TC(Q)}{E[T(Q)]} \\ \text{subject to} \quad & Q/\lambda \leq t_2 + \delta \\ & Q \geq 0 \end{aligned}$$

The objective function is the expected cost per year for orders of size $Q \geq 0$. The constraint $Q/\lambda \leq t_2 + \delta$ is introduced because an order that covers a time period longer than the maximum possible shelf life cannot be optimal. Depending on the relations among Q/λ, t_1 and t_2, $AC(Q)$ can be represented as

$$AC(Q) = \begin{cases} AC_1(Q) & \text{if} \quad Q/\lambda \leq t_1 \\ AC_2(Q) & \text{if} \quad t_1 \leq Q/\lambda \leq t_2 \\ AC_3(Q) & \text{if} \quad t_2 \leq Q/\lambda. \end{cases}$$

We characterize $AC_1(Q)$, $AC_2(Q)$ and $AC_3(Q)$, in sections 3.3.1, 3.3.2 and 3.3.3, respectively.

3.3.1 Expected average cost when $Q/\lambda \leq t_1$

If initial shelf life of the item is greater than the value of Q/λ, it means that the inventory is completely depleted by demand before reaching expiration date. Therefore, expected total cost per cycle expression is the same as the cost expression under EOQ as mentioned in 3.2.1, which is

$$AC_1(Q) = \frac{TC(Q)}{E[T(Q)]} = \frac{K\lambda}{Q} + c\lambda + h\frac{Q}{2}.$$

3.3.2 Expected average cost when $t_1 \leq Q/\lambda \leq t_2$

In this case, the average cost function will depend also on the relation between $t_1 + \delta$ and t_2. For this reason we further define $AC_2(Q)$ as

$$AC_2(Q) = \begin{cases} AC_{21}(Q) & \text{if} \quad t_1 \leq Q/\lambda \leq t_1 + \delta \leq t_2 \\ AC_{22}(Q) & \text{if} \quad t_1 + \delta \leq Q/\lambda \leq t_2 \\ AC_{23}(Q) & \text{if} \quad t_1 \leq Q/\lambda \leq t_2 \leq t_1 + \delta. \end{cases}$$

Note that the total ordering cost per cycle is the same, which is $P(Q) = K + cQ$.

We first focus on the case where $t_1 \leq Q/\lambda \leq t_1 + \delta \leq t_2$ and characterize AC_{21}. In such a condition, if the initial shelf life of the item realizes as t_1 and if it is not extended, the cycle is completed exactly after t_1 periods which occurs with probability αp_1. At the end of the cycle, $Q - \lambda t_1$ units are disposed (see Cycle 1 in Figure 3.2). Otherwise, that is if the initial shelf life is t_2 or it is t_1 and the extension is realized, the entire order will be depleted by the demand (see Cycle 2 in Figure 3.2). Hence, when $t_1 \leq Q/\lambda \leq t_1 + \delta \leq t_2$, the cycle length distribution and expected cycle length expression can be given as

$$P\{T(Q) = t\} = \begin{cases} \alpha p_1 & \text{if} \quad t = t_1 \\ 1 - \alpha p_1 & \text{if} \quad t = Q/\lambda \end{cases}$$

and

$$E[T(Q)] = \alpha p_1 t_1 + (1 - \alpha p_1)\frac{Q}{\lambda}.$$

Considering the possible cycle realizations depicted in Figure 3.2, expected holding cost per cycle can be expressed as:

$$H(Q) = \alpha \frac{h}{2} \left((2Q - \lambda t_1) t_1 p_1 + \frac{Q^2}{\lambda}(1 - p_1) \right) + (1 - \alpha)\frac{h}{2}\frac{Q^2}{\lambda}.$$

Therefore, the expected average cost when $t_1 \leq Q/\lambda \leq t_2$ is

$$AC_{21}(Q) = \frac{TC(Q)}{E[T(Q)]} = \frac{K + cQ + \alpha \frac{h}{2}\left((2Q - \lambda t_1) t_1 p_1 + \frac{Q^2}{\lambda}(1 - p_1) \right) + (1 - \alpha)\frac{h}{2}\frac{Q^2}{\lambda}}{\alpha \left(p_1 t_1 + (1 - p_1)\frac{Q}{\lambda} \right) + (1 - \alpha)\frac{Q}{\lambda}}.$$

We next characterize AC_{22} which applies for the case $t_1 + \delta \leq Q/\lambda \leq t_2$. If the initial shelf life is t_1, the cycle ends after at most $t_1 + \delta$ time units due to the shelf life

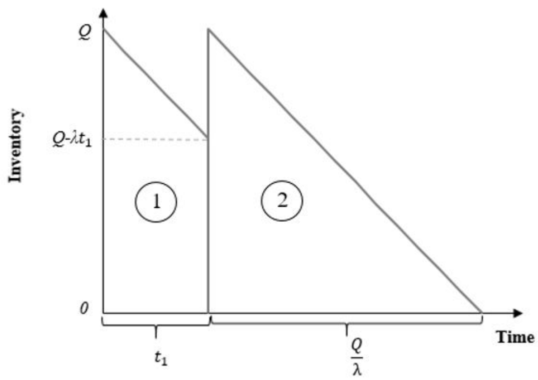

Figure 3.2: Setting II- Possible Cycle Length Realizations when $Q/\lambda \leq t_1 + \delta \leq t_2$

expiration. Hence, $Q - \lambda t_1$ units of inventory is instantaneously disposed when the order is received at no value and no cost in order to incur less holding cost. Disposal decreases the inventory level to $\lambda(t_1 + \delta)$. If extension test fails, then the cycle ends with $\lambda\delta$ units on hand (hence discarded) after t_1 time units with probability αp_1 as in Cycle 1 depicted in Figure 3.3. If the batch passes the extension test, then the cycle ends after $t_1 + \delta$ time units with probability $\alpha(1 - p_1)$ shown by Cycle 2 in Figure 3.3. If the initial shelf life is t_2, the entire inventory is depleted by demand (see Cycle 3 in Figure 3.3). Therefore, the cycle length distribution and expected cycle length are given as

$$P\{T(Q) = t\} = \begin{cases} \alpha p_1 & \text{if} \quad t = t_1 \\ \alpha(1 - p_1) & \text{if} \quad t = t_1 + \delta \\ (1 - \alpha) & \text{if} \quad t = Q/\lambda \end{cases}$$

and

$$E[T(Q)] = \alpha(p_1 t_1 + (1 - p_1)(t_1 + \delta)) + (1 - \alpha)\frac{Q}{\lambda}.$$

As a result, we get expected holding cost per cycle as

$$H(Q) = \alpha\frac{h}{2}\left((\lambda t_1 + 2\lambda\delta)t_1 p_1 + \lambda(t_1 + \delta)^2(1 - p_1)\right) + (1 - \alpha)\frac{h}{2}\frac{Q^2}{\lambda}.$$

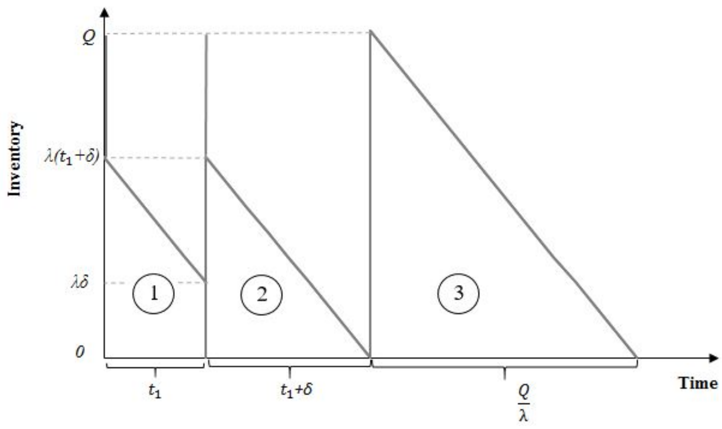

Figure 3.3: Setting II-Possible Cycle Length Realizations: $t_1 + \delta \leq Q/\lambda \leq t_2$

Then, expected average cost can be expressed as;

$$AC_{22}(Q) = \frac{K + cQ + \alpha\frac{h}{2}\left((\lambda t_1 + 2\lambda\delta)t_1 p_1 + \lambda(t_1+\delta)^2(1-p_1)\right)}{\alpha(p_1 t_1 + (1-p_1)(t_1+\delta)) + (1-\alpha)\frac{Q}{\lambda}} + \frac{(1-\alpha)\frac{h}{2}\frac{Q^2}{\lambda}}{\alpha(p_1 t_1 + (1-p_1)(t_1+\delta)) + (1-\alpha)\frac{Q}{\lambda}}.$$

Next, we consider the case where $t_1 \leq Q/\lambda \leq t_2 \leq t_1 + \delta$ and characterize AC_{23}. The cycle length will either be t_1 with probability αp_1 (when the initial shelf life is t_1 and there is no extension), or Q/λ. If the cycle ends after t_1 periods, $Q - \lambda t_1$ units are disposed at the end of the cycle. Otherwise, that is the initial shelf is t_2 or it is t_1 and the extension is realized, the entire order inventory is depleted by demand; and therefore, there is no disposal. Then, corresponding cycle length distribution and the expected cycle length are given by

$$P\{T(Q) = t\} = \begin{cases} \alpha p_1 & \text{if} \quad t = t_1 \\ \alpha(1-p_1) + (1-\alpha) & \text{if} \quad t = Q/\lambda \end{cases}$$

and

$$E[T(Q)] = \alpha\left(p_1 t_1 + (1-p_1)\frac{Q}{\lambda}\right) + (1-\alpha)\frac{Q}{\lambda}.$$

The expected total holding cost can be expressed as

$$H(Q) = \alpha\frac{h}{2}\left((2Q - \lambda t_1)t_1 p_1 + \frac{Q^2}{\lambda}(1-p_1)\right) + (1-\alpha)\frac{h}{2}\frac{Q^2}{\lambda}.$$

Therefore, the expected cost per unit time is

$$AC_{23}(Q) = \frac{K + cQ + \alpha\frac{h}{2}\left((2Q - \lambda t_1)t_1 p_1 + \frac{Q^2}{\lambda}(1-p_1)\right) + (1-\alpha)\frac{h}{2}\frac{Q^2}{\lambda}}{\alpha\left(p_1 t_1 + (1-p_1)\frac{Q}{\lambda}\right) + (1-\alpha)\frac{Q}{\lambda}}.$$

3.3.3 Expected average cost when $t_2 \leq Q/\lambda$

When $t_2 \leq Q/\lambda$, the average cost will depend on the region that Q/λ falls, and the relationship between $t_1 + \delta$ and t_2 as in Section 3.3.2, which can be expressed as

$$AC_3(Q) = \begin{cases} AC_{31}(Q) & \text{if} \quad t_1 + \delta \leq t_2 \leq Q/\lambda \\ AC_{32}(Q) & \text{if} \quad t_2 \leq Q/\lambda \leq t_1 + \delta \\ AC_{33}(Q) & \text{if} \quad t_2 \leq t_1 + \delta \leq Q/\lambda. \end{cases}$$

Total ordering cost over a cycle is $P(Q) = K + cQ$.

We start our analysis with the case where $t_1 + \delta \leq t_2 \leq Q/\lambda$. The cycle length can take four different values with respect to the initial shelf life and extension probabilities. If the initial shelf life is t_1, the cycle ends after at most $t_1 + \delta$ time units due to the shelf life expiration. Then, $Q - \lambda t_1$ units of inventory is instantaneously disposed when the order is received in order to incur less holding cost. Disposal decreases the inventory level to $\lambda(t_1+\delta)$. If the batch fails the test, then the cycle ends with $\lambda\delta$ units on hand after t_1 time units, which occurs with probability αp_1 as shown by Cycle 1 in Figure 3.4. If the batch passes the test, then the cycle ends after $t_1 + \delta$ time units, which occurs with probability $\alpha(1-p_1)$ shown by Cycle 2 in Figure 3.4. If the initial shelf life is t_2 and the extension test fails, then the cycle ends with $Q - \lambda t_2$ units on hand (hence disposed) after t_2 time units, which occurs with probability $(1-\alpha)p_2$ as in Cycle 3 depicted in Figure 3.4. Otherwise, that is the initial shelf life is t_2 and the batch passes the test, the inventory is depleted completely by demand after Q/λ periods, which occurs with probability $(1-\alpha)(1-p_2)$, as in Cycle 4 in Figure 3.4. Therefore, cycle length distribution and expected cycle length expression are given as

$$P\{T(Q) = t\} = \begin{cases} \alpha p_1 & \text{if} \quad t = t_1 \\ \alpha(1-p_1) & \text{if} \quad t = t_1 + \delta \\ (1-\alpha)p_2 & \text{if} \quad t = t_2 \\ (1-\alpha)(1-p_2) & \text{if} \quad t = Q/\lambda \end{cases}$$

and

$$E[T(Q)] = \alpha\left(p_1 t_1 + (1-p_1)(t_1+\delta)\right) + (1-\alpha)\left(p_2 t_2 + (1-p_2)\frac{Q}{\lambda}\right).$$

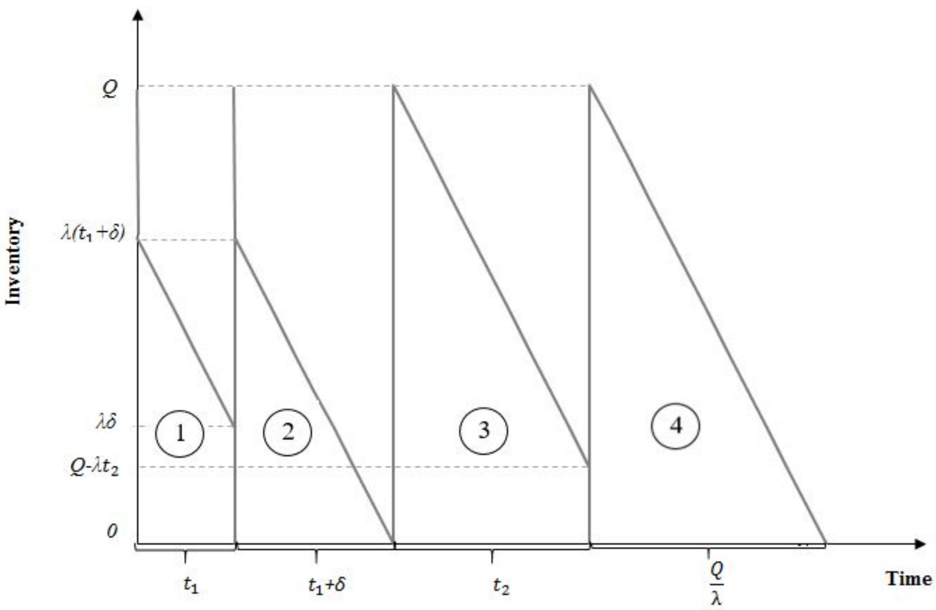

Figure 3.4: Setting II- Possible Cycle Length Realizations when $t_1 + \delta \leq t_2 \leq Q/\lambda$

Then, expected holding cost per cycle is given by

$$\begin{aligned} H(Q) =& \alpha\frac{h}{2}((\lambda t_1 + 2\lambda\delta)t_1 p_1 + \lambda(t_1+\delta)^2(1-p_1)) \\ &+ (1-\alpha)\frac{h}{2}\left((2Q-\lambda t_2)t_2 p_2 + \frac{Q^2}{\lambda}(1-p_2)\right). \end{aligned}$$

Thus, when $t_2 \leq Q/\lambda \leq t_2 + \delta$, the total expected cost per year is;

$$\begin{aligned} AC_{31}(Q) =& \frac{K + cQ + \alpha\frac{h}{2}((\lambda t_1 + 2\lambda\delta)t_1 p_1 + \lambda(t_1+\delta)^2(1-p_1))}{\alpha(p_1 t_1 + (1-p_1)(t_1+\delta)) + (1-\alpha)\left(p_2 t_2 + (1-p_2)\frac{Q}{\lambda}\right)} \\ &+ \frac{(1-\alpha)\frac{h}{2}\left((2Q-\lambda t_2)t_2 p_2 + \frac{Q^2}{\lambda}(1-p_2)\right)}{\alpha(p_1 t_1 + (1-p_1)(t_1+\delta)) + (1-\alpha)\left(p_2 t_2 + (1-p_2)\frac{Q}{\lambda}\right)}. \end{aligned}$$

We next focus on the case $t_2 \leq Q/\lambda \leq t_1 + \delta$. If the initial shelf life is t_1 and it is not extended, then the cycle ends with $Q - \lambda t_1$ units after t_1 periods, which occurs with probability αp_1 (See Cycle 1 in Figure 3.5). Likewise, if initial shelf life is t_2

and it is not extended, then the cycle ends with $Q - \lambda t_2$ units after t_2 periods, which occurs with probability $(1-\alpha)p_2$ (See Cycle 2 in Figure 3.5). If the batch passes the extension test in any initial shelf life realization, the cycle ends after Q/λ periods, which occurs with probability $\alpha(1-p_1)+(1-\alpha)(1-p_2)$, since the demand depletes the inventory entirely before the final expiration date (See Cycle 3 in Figure 3.5). In this condition, the cycle length distribution is as follows

$$P\{T(Q)=t\} = \begin{cases} \alpha p_1 & \text{if} \quad t=t_1 \\ (1-\alpha)p_2 & \text{if} \quad t=t_2 \\ \alpha(1-p_1)+(1-\alpha)(1-p_2) & \text{if} \quad t=Q/\lambda. \end{cases}$$

Therefore, expected cycle length is expression is given by

$$E[T(Q)] = \alpha\left(p_1 t_1 + (1-p_1)\frac{Q}{\lambda}\right) + (1-\alpha)\left(p_2 t_2 + (1-p_2)\frac{Q}{\lambda}\right).$$

Referring to Figure 3.5, the expected holding cost is

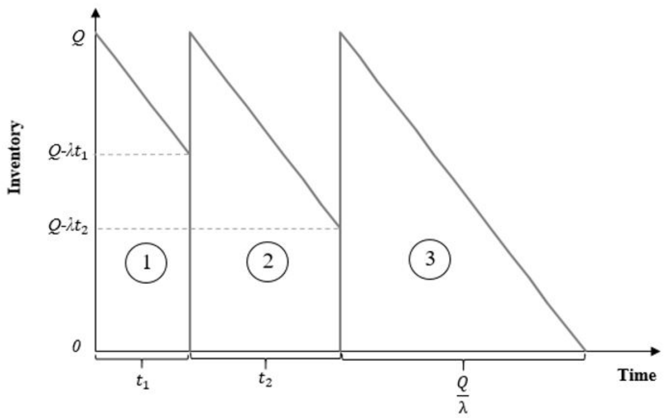

Figure 3.5: Setting II- Possible Cycle Length Realizations when $t_2 \le Q/\lambda \le t_1 + \delta$

$$H(Q) = \alpha\frac{h}{2}\left((2Q-\lambda t_1)t_1 p_1 + \frac{Q^2}{\lambda}(1-p_1)\right) + (1-\alpha)\frac{h}{2}\left((2Q-\lambda t_2)t_2 p_2 + \frac{Q^2}{\lambda}(1-p_2)\right).$$

Hence, expected average cost is given by

$$AC_{32}(Q) = \frac{K + cQ + \alpha\frac{h}{2}\left((2Q-\lambda t_1)t_1 p_1 + \frac{Q^2}{\lambda}(1-p_1)\right)}{\alpha\left(p_1 t_1 + (1-p_1)\frac{Q}{\lambda}\right) + (1-\alpha)\left(p_2 t_2 + (1-p_2)\frac{Q}{\lambda}\right)} + \frac{(1-\alpha)\frac{h}{2}\left((2Q-\lambda t_2)t_2 p_2 + \frac{Q^2}{\lambda}(1-p_2)\right)}{\alpha\left(p_1 t_1 + (1-p_1)\frac{Q}{\lambda}\right) + (1-\alpha)(p_2 t_2 + (1-p_2)\frac{Q}{\lambda})}.$$

Finally, we investigate the condition where $Q/\lambda \geq t_1 + \delta \geq t_2$.

If the initial shelf life is t_1, then $Q - \lambda t_1$ units of inventory is instantaneously disposed when the order is received in order to incur less holding cost. Disposal decreases the inventory level to $\lambda(t_1+\delta)$. If the batch fails the test, then the cycle ends with $\lambda\delta$ units on hand (thus disposed) after t_1 time units, which occurs with probability αp_1. If the batch passes the test, then the cycle ends after $t_1 + \delta$ time units, which occurs with probability $\alpha(1-p_1)$. If initial shelf life of the item is t_2 and it is not extended after the test, the cycle ends with $Q - \lambda t_2$ units after t_2 periods, which occurs with probability $(1-\alpha)p_2$. However, if it passes the test and final shelf life is updated to $t_2+\delta$, the cycle continues until Q/λ periods, which occurs with probability $(1-\alpha)(1-p_2)$, where inventory is depleted completely by demand. Therefore, the cycle length distribution is as follows

$$P\{T(Q) = t\} \quad = \quad \begin{cases} \alpha p_1 & \text{if} \quad t = t_1 \\ \alpha(1-p_1) & \text{for} \quad t = \mathrm{t}_1 + \delta \\ (1-\alpha)p_2 & \text{if} \quad t = t_2 \\ (1-\alpha)(1-p_2) & \text{if} \quad t = Q/\lambda. \end{cases}$$

.

Hence, the expected cycle length is given by

$$E[T(Q)] = \alpha(p_1 t_1 + (1-p_1)(t_1+\delta)) + (1-\alpha)\left(p_2 t_2 + (1-p_2)\frac{Q}{\lambda}\right). \tag{3.2}$$

Then, total expected holding cost is

$$\begin{aligned} H(Q) =& \alpha\frac{h}{2}((\lambda t_1 + 2\lambda\delta)t_1 p_1 + \lambda(t_1+\delta)^2(1-p_1)) \\ &+ (1-\alpha)\frac{h}{2}\left((2Q - \lambda t_2)t_2 p_2 + \frac{Q^2}{\lambda}(1-p_2)\right). \end{aligned} \tag{3.3}$$

Therefore, average cost per unit time is given as

$$AC_{33}(Q) = \frac{K + cQ + H(Q)}{E[T(Q)]}.$$

where $T(Q)$ and $H(Q)$ are given by Equation (3.2) and Equation (3.3), respectively.

3.4 Setting III: Possible refund and return, no shelf life information, possible extension

As in setting II, shelf life information is not available when the order is placed. However, in this case, the buyer has the opportunity to return a portion of the order when the initial shelf life information becomes available. Let R_i be the quantity returned when the initial shelf life of t_i is observed. Based on the agreement between the buyer and the supplier, the supplier offers a refund for each unit returned, $b \leq c$.

In order to find optimal order and return quantities, the following problem must be solved optimally.

$$\begin{aligned} \text{minimize} \quad & AC(Q, R_1, R_2) = \frac{TC(Q, R_1, R_2)}{E[T(Q, R_1, R_2)]} \\ \text{subject to} \quad & (Q - R_1)/\lambda \leq t_1 + \delta \\ & Q/\lambda \leq t_2 + \delta \\ & Q \geq R_1 \\ & Q \geq R_2 \\ & R_1, R_2 \geq 0 \end{aligned}$$

Objective function is the expected total relevant cost per year for given order size $Q \geq 0$ and return quantities $R_1, R_2 \geq 0$. Return quantities should be less than or equal to order size. If the initial shelf life is t_1, the maximum demand during that cycle is $(t_1 + \delta)\lambda$. First constraint ensures that excess quantity is returned. Similarly, $(Q - R_2)/\lambda \leq t_2 + \delta$ constraint can be introduced. However, it is redundant due to the second constraint.

$AC(Q)$ depends on the relative values of Q/λ, t_1 and t_2 for this setting such that

$$AC(Q, R_1, R_2) \quad = \quad \begin{cases} AC_1(Q) & \text{if} \quad Q/\lambda \leq t_1 \\ AC_2(Q, R_1) & \text{if} \quad t_1 \leq Q/\lambda \leq t_2 \\ AC_3(Q, R_1, R_2) & \text{if} \quad t_2 \leq Q/\lambda. \end{cases}$$

AC_1, AC_2 and AC_3 are characterize further in sections 3.4.1, 3.4.2 and 3.4.3, respectively.

3.4.1 Expected average cost when $Q/\lambda \leq t_1$

As mentioned in Section 3.2.1, the entire batch is depleted completely by demand before it expires. Therefore, there is no need to return. Total cost per year is

$$AC_1(Q) = \frac{TC(Q)}{E[T(Q)]} = \frac{K\lambda}{Q} + c\lambda + h\frac{Q}{2}.$$

3.4.2 Expected average cost when $t_1 \leq Q/\lambda \leq t_2$

Since Q units are ordered in every cycle, the total ordering cost is $P(Q) = K + cQ$.

In this case, there are three possible realizations of the cycle length with respect to the initial shelf life and extension probabilities. Cycle 1 in Figure 3.6 corresponds to the cycle where initial shelf life is t_1; (therefore, the buyer returns R_1 units at the beginning of the cycle) and the batch fails the extension test. That cycle ends with $Q - R_1 - \lambda t_1$ units which are disposed instantaneously after t_1 periods, which occurs with probability αp_1. If initial shelf life is t_1 and the extension test succeeds which happens with probability $\alpha(1-p_1)$, $(Q - R_1)$ units will be depleted due to constraint $(Q - R_1)/\lambda \leq t_1 + \delta$ which is illustrated as Cycle 2 in Figure 3.6. When the initial shelf life is realized as t_2, which occurs with probability $(1-\alpha)$, the cycle ends after Q/λ periods and entire ordered quantity is depleted by demand which is shown by Cycle 3 in Figure 3.6. Note that R_2 is trivially zero in this case. The corresponding cycle length distribution and expected cycle length expression are given as follows:

$$P\{T(Q,R_1) = t\} = \begin{cases} \alpha p_1 & \text{if} \quad t = t_1 \\ \alpha(1-p_1) & \text{if} \quad t = (Q-R_1)/\lambda \\ (1-\alpha) & \text{if} \quad t = Q/\lambda \end{cases}$$

and

$$E[T(Q,R_1)] = \alpha\left(p_1 t_1 + (1-p_1)\left(\frac{Q-R_1}{\lambda}\right)\right) + (1-\alpha)\frac{Q}{\lambda}.$$

Hence, we get expected holding cost per cycle as

$$H(Q,R_1) = \frac{h}{2}\left((2(Q-R_1) - \lambda t_1)t_1 p_1 + \frac{(Q-R_1)^2}{\lambda}(1-p_1)\right) + (1-\alpha)\frac{h}{2}\frac{Q^2}{\lambda}.$$

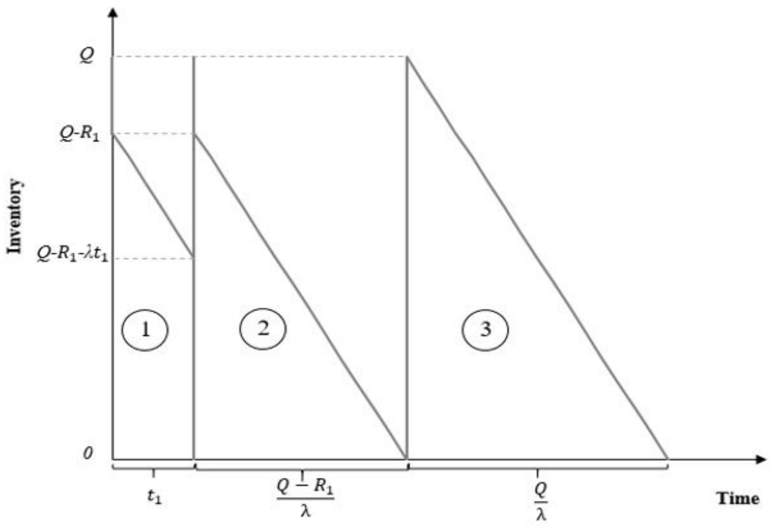

Figure 3.6: Setting III-Possible Cycle Length Realizations when $t_1 \leq Q/\lambda \leq t_2$

Thus, total expected cost per cycle is expressed as below

$$TC(Q, R_1) = K + cQ + H(Q) - b\alpha R.$$

Therefore, expected average cost is given by

$$AC_2(Q, R_1) = \frac{K + cQ - \alpha b R_1 + \alpha \frac{h}{2}\left((2(Q-R_1) - \lambda t_1)t_1 p_1 + \frac{(Q-R_1)^2}{\lambda}(1-p_1)\right)}{\alpha\left(p_1 t_1 + (1-p_1)\frac{Q-R_1}{\lambda}\right) + (1-\alpha)\frac{Q}{\lambda}} + \frac{(1-\alpha)\frac{h}{2}\frac{Q^2}{\lambda}}{\alpha\left(p_1 t_1 + (1-p_1)\frac{Q-R_1}{\lambda}\right) + (1-\alpha)\frac{Q}{\lambda}}.$$

3.4.3 Expected average cost when $t_2 \leq Q/\lambda$

In this condition, there are four possible realizations for the cycle length. Possible cycle length realizations are given in Figure 3.7. If initial shelf life is t_1 and it cannot be extended which occurs with probability αp_1, then the cycle ends with $Q - R_1 - \lambda t_1$ units (see Cycle 1). If initial shelf life is t_1 and shelf life of the batch is extended, which occurs with probability $\alpha(1-p_1)$, the cycle ends after $(Q - R_1)/\lambda$ periods. The entire remaining inventory after return is depleted by demand which is shown by Cycle 2. If initial shelf life is t_2 and it cannot be extended which occurs with probability $(1-\alpha)p_2$, then the cycle ends with $Q - R_2 - \lambda t_2$ units of inventory after

t_2 periods which is shown as Cycle 3. If initial shelf life is t_2 and the extension test is successful, which occurs with probability $(1-\alpha)(1-p_1)$, the cycle ends after $(Q-R_2)/\lambda$ periods. The remaining inventory after return is again depleted entirely by demand (see Cycle 4). The cycle length and the expected cycle length can be expressed as

$$P\{T(Q,R_1,R_2)=t\} = \begin{cases} \alpha p_1 & \text{if} \quad t=t_1 \\ \alpha(1-p_1) & \text{if} \quad t=(Q-R_1)/\lambda \\ (1-\alpha)p_2 & \text{if} \quad t=t_2 \\ (1-\alpha)(1-p_2) & \text{if} \quad t=(Q-R_2)/\lambda \end{cases} \tag{3.4}$$

and

$$\begin{aligned} E[T(Q,R_1,R_2)] =& \alpha\left(p_1t_1+(1-p_1)\frac{(Q-R_1)}{\lambda}\right) \\ &+(1-\alpha)\left(p_2t_2+(1-p_2)\frac{(Q-R_2)}{\lambda}\right). \end{aligned} \tag{3.5}$$

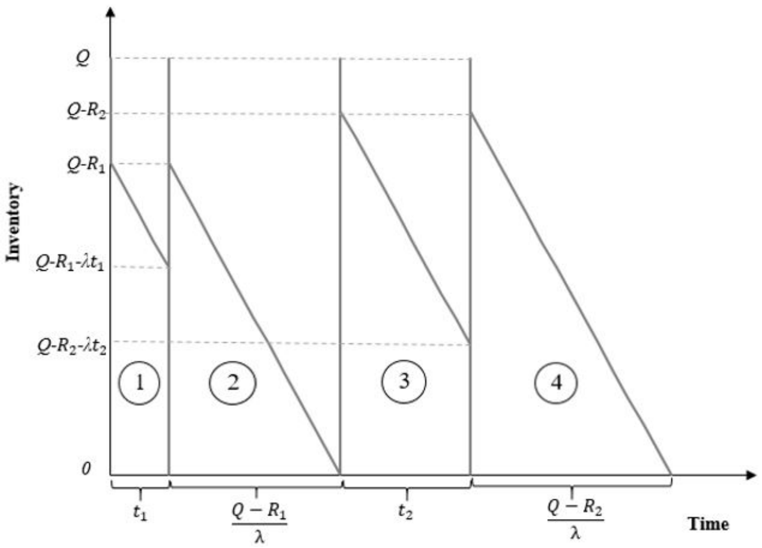

Figure 3.7: Setting III-Possible Cycle Length Realizations when $t_2 \leq Q/\lambda$

Referring to Figure 3.7, expected total holding cost per cycle is given as

$$\begin{aligned} H(Q,R_1,R_2) =& \alpha\frac{h}{2}\left((2(Q-R_1)-\lambda t_1)t_1p_1+\frac{(Q-R_1)^2}{\lambda}(1-p_1)\right) \\ &+(1-\alpha)\frac{h}{2}\left((2(Q-R_2)-\lambda t_2)t_2p_2+\frac{(Q-R_2)^2}{\lambda}(1-p_2)\right). \end{aligned} \tag{3.6}$$

Then, the expected average cost can be expressed as

$$AC_3(Q, R_1, R_2) \quad = \quad \frac{K + cQ - b(\alpha R_1 + (1-\alpha)R_2) + H(Q, R_1, R_2)}{E[T(Q, R_1, R_2)]}.$$

where $E[T(Q, R_1, R_2)]$ and $H(Q, R_1, R_2)$ given by Equation 3.5 and Equation 3.6, respectively.

3.5 Setting IV: No Return, Information about Initial Shelf Life, No Extension

In this section, we consider the setting where the supplier shares initial shelf life information of the batch that will be sent. As a result the buyer has the opportunity to set the order quantity, Q_i, based on the realization of initial shelf life t_i, i=1, 2. The shelf life of the item cannot be extended, since the buyer is not capable to conduct the extension test in this setting. Note that return option is irrelevant here.

Optimal Q_1 and Q_2 quantities can be found by solving the following problem optimally:

$$\begin{aligned} \text{minimize} \quad & AC(Q_1, Q_2) = \frac{TC(Q_1, Q_2)}{E[T(Q_1, Q_2)]} \\ \text{subject to} \quad & Q_1/\lambda \leq t_1 \\ & Q_2/\lambda \leq t_2 \\ & Q_1, Q_2 \geq 0 \end{aligned}$$

Objective function is the expected average cost for given order of sizes $Q_1 \geq 0$ and $Q_2 \geq 0$. The constraints $Q_1/\lambda \leq t_1$ and $Q_2/\lambda \leq t_2$ ensure that the order does not exceed the demand for corresponding cycle. Every time inventory level hits zero, Q_1 units are ordered if the initial shelf life is t_1 with probability α, and Q_2 units are ordered if it is t_2 with probability $(1-\alpha)$. Therefore, expected total ordering cost per cycle is $P(Q_1, Q_2) = K + c(\alpha Q_1 + (1-\alpha)Q_2)$.

The cycle length distribution and expected cycle length expression are as follows

$$P\{T(Q) = t\} \quad = \quad \begin{cases} \alpha & \text{if} \quad t = Q_1/\lambda \\ (1-\alpha) & \text{if} \quad t = Q_2/\lambda \end{cases} \tag{3.7}$$

and

$$E[T(Q_1, Q_2)] = \alpha \frac{Q_1}{\lambda} + (1-\alpha)\frac{Q_2}{\lambda}.$$

In this setting, expected holding cost per unit time is expressed as

$$H(Q_1, Q_2) = \alpha \frac{h}{2}\frac{Q_1^2}{\lambda} + (1-\alpha)\frac{h}{2}\frac{Q_2^2}{\lambda}.$$

Therefore, expected average cost is

$$AC(Q_1, Q_2) = \frac{K + \alpha c Q_1 + (1-\alpha) c Q_2 + \alpha \frac{h}{2}\frac{Q_1^2}{\lambda} + (1-\alpha)\frac{h}{2}\frac{Q_2^2}{\lambda}}{\alpha \frac{Q_1}{\lambda} + (1-\alpha)\frac{Q_2}{\lambda}}.$$

3.6 Setting V: No return, information about initial shelf life, possible extension

In this setting, we include the extension opportunity to the environment described in Section 3.5. In other words, the buyer has the capability and test infrastructure to conduct the shelf life extension tests.

In order to find optimal Q_1 and Q_2, the following problem must be solved

$$\begin{aligned} \text{minimize} \quad & AC(Q_1, Q_2) = \frac{TC(Q_1, Q_2)}{E[T(Q_1, Q_2)]} \\ \text{subject to} \quad & Q_1/\lambda \leq t_1 + \delta \\ & Q_2/\lambda \leq t_2 + \delta \\ & Q_1, Q_2 \geq 0 \end{aligned}$$

As the shelf life can be extended, maximum possible shelf life realizations become $t_1 + \delta$ and $t_2 + \delta$. Based on the relationship between Q_i/λ and t_i values, $AC(Q)$ can be characterized as

$$AC(Q_1, Q_2) = \begin{cases} AC_1(Q_1, Q_2) & \text{if} \quad Q_1/\lambda \leq t_1, Q_2/\lambda \leq t_2 \\ AC_2(Q_1, Q_2) & \text{if} \quad t_1 \leq Q_1/\lambda, Q_2/\lambda \leq t_2 \\ AC_3(Q_1, Q_2) & \text{if} \quad Q_1/\lambda \leq t_1, t_2 \leq Q_2/\lambda \\ AC_4(Q_1, Q_2) & \text{if} \quad t_1 \leq Q_1/\lambda, t_2 \leq Q_2/\lambda \end{cases}$$

where $AC_1(Q_1, Q_2)$, $AC_2(Q_1, Q_2)$, $AC_3(Q_1, Q_2)$ and $AC_4(Q_1, Q_2)$ will be derived in sections 3.6.1, 3.6.2, 3.6.3 and 3.6.4, respectively. For each $AC(Q_1, Q_2)$ function, the total ordering cost per cycle expression is given as $P(Q_1, Q_2) = K + c(\alpha Q_1 + (1-\alpha)Q_2)$.

3.6.1 Expected average cost when $Q_1/\lambda \leq t_1, Q_2/\lambda \leq t_2$

In this case, the inventory is depleted entirely by demand before reaching its shelf life. Therefore, shelf life extension opportunity is irrelevant. There are two possible cycle realizations. If the shelf life of the item is t_1, the cycle ends with no inventory on hand after Q_1/λ periods. Likewise, if the shelf life of the item is t_2, the cycle ends with no inventory on hand after Q_2/λ periods. Hence cycle length distribution and expected cycle length expressions are

$$P\{T(Q_1, Q_2) = t\} = \begin{cases} \alpha & \text{if} \quad t = Q_1/\lambda \\ (1-\alpha) & \text{if} \quad t = Q_2/\lambda \end{cases} \tag{3.8}$$

and

$$E[T(Q_1, Q_2)] = \alpha\frac{Q_1}{\lambda} + (1-\alpha)\frac{Q_2}{\lambda}.$$

Then, the expected holding cost can be expressed as

$$H(Q_1, Q_2) = \alpha\frac{h}{2}\frac{Q_1^2}{\lambda} + (1-\alpha)\frac{h}{2}\frac{Q_2^2}{\lambda}.$$

Therefore, expected total cost per year expression is given by,

$$AC_1(Q_1, Q_2) = \frac{TC(Q_1, Q_2)}{E[T(Q_1, Q_2)]} = \frac{K + \alpha c Q_1 + (1-\alpha)cQ_2 + \alpha\frac{h}{2}\frac{Q_1^2}{\lambda} + (1-\alpha)\frac{h}{2}\frac{Q_2^2}{\lambda}}{\alpha\frac{Q_1}{\lambda} + (1-\alpha)\frac{Q_2}{\lambda}}.$$

3.6.2 Expected average cost when $t_1 \leq Q_1/\lambda, t_2 \geq Q_2/\lambda$

In this case, there are three possible cycle length realizations. If the initial shelf life is t_1 and it is not extended which occurs with probability αp_1, then the cycle ends with $Q_1 - \lambda t_1$ units after t_1 periods (See Cycle 1 in Figure 3.8). If the initial shelf life is t_1 and the batch passes the extension test which occurs with probability $\alpha(1-p_1)$, the entire inventory is depleted by demand and the cycle ends after Q_1/λ periods (See Cycle 2 in Figure 3.8). On the other hand if the initial shelf life is t_2, which occurs with probability $(1-\alpha)$, the cycle ends after Q_2/λ periods, and the demand depletes the inventory entirely before the shelf life is over (See Cycle 3 in Figure 3.8). The cycle length distribution and the expected cycle time are given as

$$P\{T(Q_1, Q_2) = t\} = \begin{cases} \alpha p_1 & \text{if} \quad t = t_1 \\ \alpha(1-p_1) & \text{if} \quad t = Q_1/\lambda \\ (1-\alpha) & \text{if} \quad t = Q_2/\lambda \end{cases} \tag{3.9}$$

and

$$E[T(Q_1,Q_2)] = \alpha\left(p_1t_1 + (1-p_1)\frac{Q_1}{\lambda}\right) + (1-\alpha)\frac{Q_2}{\lambda}.$$

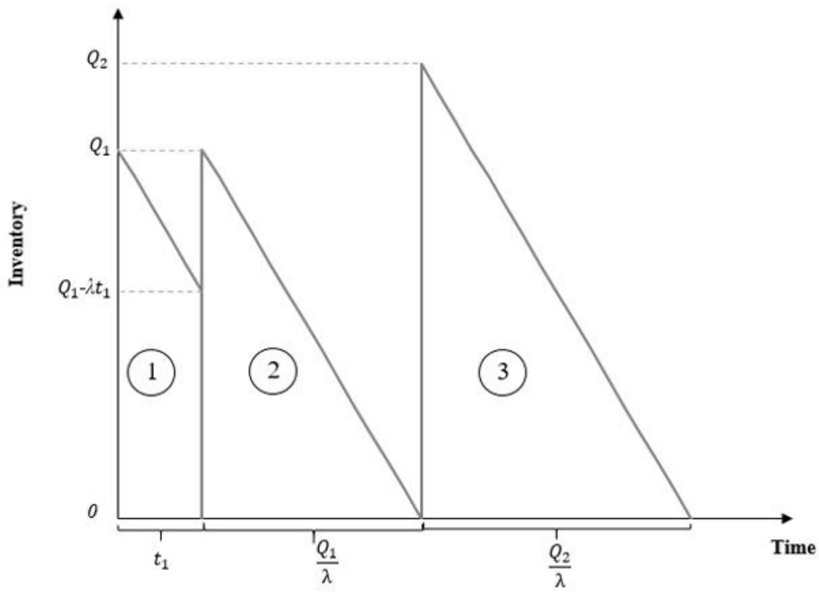

Figure 3.8: Setting V- Possible Cycle Length Realizations when $t_1 \leq Q_1/\lambda, t_2 \geq Q_2/\lambda$

The expected holding cost can be expressed as

$$H(Q_1,Q_2) = \alpha\frac{h}{2}\left((2Q_1 - \lambda t_1)t_1p_1 + \frac{Q_1^2}{\lambda}(1-p_1)\right) + (1-\alpha)\frac{h}{2}\frac{Q_2^2}{\lambda}.$$

Therefore, the expected average cost is given by

$$AC_2(Q_1,Q_2) = \frac{K + \alpha cQ_1 + (1-\alpha)cQ_2 + H(Q_1,Q_2)}{\alpha\left(p_1t_1 + (1-p_1)\frac{Q_1}{\lambda}\right) + (1-\alpha)\frac{Q_2}{\lambda}}.$$

3.6.3 Expected average cost when $Q_1/\lambda \leq t_1, t_2 \leq Q_2/\lambda$

In this case, there are again three possible cycle lengths. If the initial shelf life is t_1, which occurs with probability α, the inventory on-hand is depleted completely by demand before shelf life reaches. The cycle ends after Q_1/λ periods which is shown as Cycle 1 in Figure 3.9. If initial shelf life is t_2, but the extension test fails which occurs with probability $(1-\alpha)p_2$, then the cycle ends with $Q_2 - \lambda t_2$ units (hence discarded) after t_2 periods as Cycle 2 in Figure 3.9. Otherwise the cycle ends after

Q_2/λ periods, which occurs with probability $(1-\alpha)(1-p_2)$, when the inventory is depleted entirely by demand (See Cycle 3 in Figure 3.9) The cycle length distribution and the expected cycle length are given as

$$P\{T(Q_1,Q_2)=t\} \quad = \quad \begin{cases} \alpha & \text{if} \quad t=Q_1/\lambda \\ (1-\alpha)p_2 & \text{if} \quad t=t_2 \\ (1-\alpha)(1-p_2) & \text{if} \quad t=Q_2/\lambda \end{cases}$$

and

$$E[T(Q_1,Q_2)] = \alpha\frac{Q_1}{\lambda} + (1-\alpha)\left(p_2t_2 + (1-p_2)\frac{Q_2}{\lambda}\right). \tag{3.10}$$

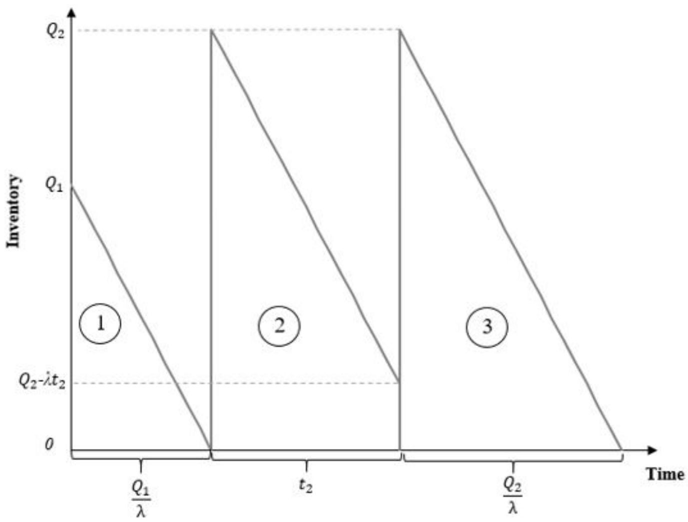

Figure 3.9: Possible Cycle Length Realizations

Referring to Figure 3.9, the expected holding cost can be expressed as

$$H(Q_1,Q_2) = \alpha\frac{h}{2}\frac{Q_1^2}{\lambda} + (1-\alpha)\frac{h}{2}\left((2Q_2-\lambda t_2)t_2p_2 + \frac{Q_2^2}{\lambda}(1-p_2)\right). \tag{3.11}$$

Finally, expected total cost per year for this case can be expressed as

$$AC_3(Q_1,Q_2) = \frac{K + \alpha cQ_1 + (1-\alpha)cQ_2 + H(Q_1,Q_2)}{E[T(Q_1,Q_2)]}.$$

where $E[T(Q_1,Q_2)$ and $H(Q_1,Q_2)$ are given by Equation 3.10 and Equation 3.11, respectively.

3.6.4 Expected average cost when $t_1 \leq Q_1/\lambda, t_2 \leq Q_2/\lambda$

There are four possible cycle realizations in this case. Possible cycles can be seen in Figure 3.10. If the initial shelf life is t_1 but extension test fail which occurs with probability αp_1, then the cycle ends with $Q_1 - \lambda t_1$ units of inventory after t_1 periods as in Cycle 1 depicted in Figure 3.10. If the initial shelf life is t_1 and the batch passes the test which occurs with probability $\alpha(1-p_1)$, then the entire inventory is depleted by demand. The corresponding cycle ends after Q_1/λ periods (see Cycle 2). If the initial shelf life is t_2, but extension test fails which occurs with probability $(1-\alpha)p_2$, then the cycle ends with $Q_2 - \lambda t_2$ units of inventory after t_2 periods as in Cycle 3. Otherwise that is the initial shelf life is t_2 and the batch passes the extension test which occurs with probability $(1-\alpha)(1-p_2)$, the inventory is depleted entirely by demand after Q_2/λ periods which is shown by Cycle 4. The related cycle length distribution and expected cycle length distribution are given as follows

$$P\{T(Q_1,Q_2)=t\} \;=\; \begin{cases} \alpha p_1 & \text{if} \quad t = t_1 \\ \alpha(1-p_1) & \text{if} \quad t = Q_1/\lambda \\ (1-\alpha)p_2 & \text{if} \quad t = t_2 \\ (1-\alpha)(1-p_2) & \text{if} \quad t = Q_2/\lambda \end{cases}$$

and

$$E[T(Q_1,Q_2)] = \alpha\left(p_1t_1 + (1-p_1)\frac{Q_1}{\lambda}\right) + (1-\alpha)\left(p_2t_2 + (1-p_2)\frac{Q_2}{\lambda}\right).$$

Then, expected holding cost can be expressed as follows

$$\begin{aligned} H(Q_1,Q_2) =& \alpha\frac{h}{2}\left((2Q_1 - \lambda t_1)t_1p_1 + \frac{Q_1^2}{\lambda}(1-p_1)\right) \\ &+ (1-\alpha)\frac{h}{2}\left((2Q_2 - \lambda t_2)t_2p_2 + \frac{Q_2^2}{\lambda}(1-p_2)\right). \end{aligned}$$

Finally, expected cost per unit time is given by

$$\begin{aligned} AC_4(Q_1,Q_2) =& \frac{K + \alpha cQ_1 + (1-\alpha)cQ_2 + \alpha\frac{h}{2}\left((2Q_1-\lambda t_1)t_1p_1 + \frac{Q_1^2}{\lambda}(1-p_1)\right)}{\alpha\left(p_1t_1 + (1-p_1)\frac{Q_1}{\lambda}\right) + (1-\alpha)\left(p_2t_2 + (1-p_2)\frac{Q_2}{\lambda}\right)} \\ &+ \frac{(1-\alpha)\frac{h}{2}\left((2Q_2-\lambda t_2)t_2p_2 + \frac{Q_2^2}{\lambda}(1-p_2)\right)}{\alpha\left(p_1t_1 + (1-p_1)\frac{Q_1}{\lambda}\right) + (1-\alpha)\left(p_2t_2 + (1-p_2)\frac{Q_2}{\lambda}\right)}. \end{aligned}$$

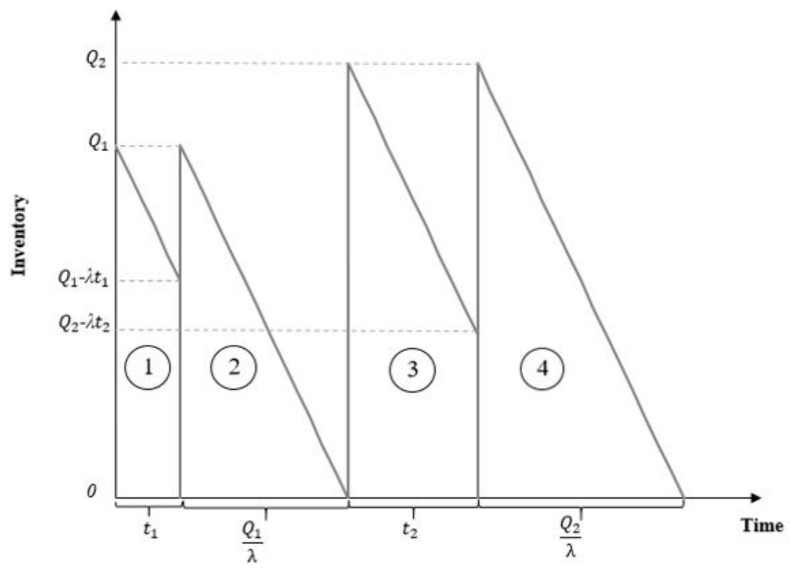

Figure 3.10: Setting V- Possible Cycle Length Realizations when $t_1 \leq Q_1/\lambda, t_2 \leq Q_2/\lambda$

3.7 Setting VI: No return, full information about final shelf life

In this setting, the buyer gets not only the initial shelf life information but also information about whether the shelf life will extend or not before ordering. In other words, the buyer has perfect shelf life information before placing a new order. In this setting, the firm has the opportunity to set the order sizes based on the realization of the shelf life. Let Q_1, Q_2, Q_3 and Q_4 denote the corresponding order sizes for shelf lives t_1, t_2, $t_1 + \delta$ and $t_2 + \delta$, respectively.
In order to find optimal Q_1, Q_2, Q_3, Q_4, the following model must be solved optimally.

$$
\begin{aligned}
\text{minimize} \quad & AC(Q_1, Q_2, Q_3, Q_4) = \frac{TC(Q_1, Q_2, Q_3, Q_4)}{E[T(Q_1, Q_2, Q_3, Q_4)} \\
\text{subject to} \quad & Q_1/\lambda \leq t_1 \\
& Q_2/\lambda \leq t_2 \\
& Q_3/\lambda \leq t_1 + \delta \\
& Q_4/\lambda \leq t_2 + \delta \\
& Q_1, Q_2, Q_3, Q_4 \geq 0
\end{aligned}
$$

The cycle length distribution is given as

$$P\{T(Q)=t\} \;=\; \begin{cases} \alpha p_1 & \text{if} \quad t = Q_1/\lambda \\ (1-\alpha)p_2 & \text{if} \quad t = Q_2/\lambda \\ \alpha(1-p_1) & \text{if} \quad t = Q_3/\lambda \\ (1-\alpha)(1-p_2) & \text{if} \quad t = Q_4/\lambda. \end{cases} \tag{3.12}$$

Thus, expected cycle length expression is given as

$$E[T(Q_1,Q_2,Q_3,Q_4)] = \alpha p_1 \frac{Q_1}{\lambda} + \alpha(1-p_1)\frac{Q_3}{\lambda} + (1-\alpha)p_2\frac{Q_2}{\lambda} + (1-\alpha)(1-p_2)\frac{Q_4}{\lambda}.$$

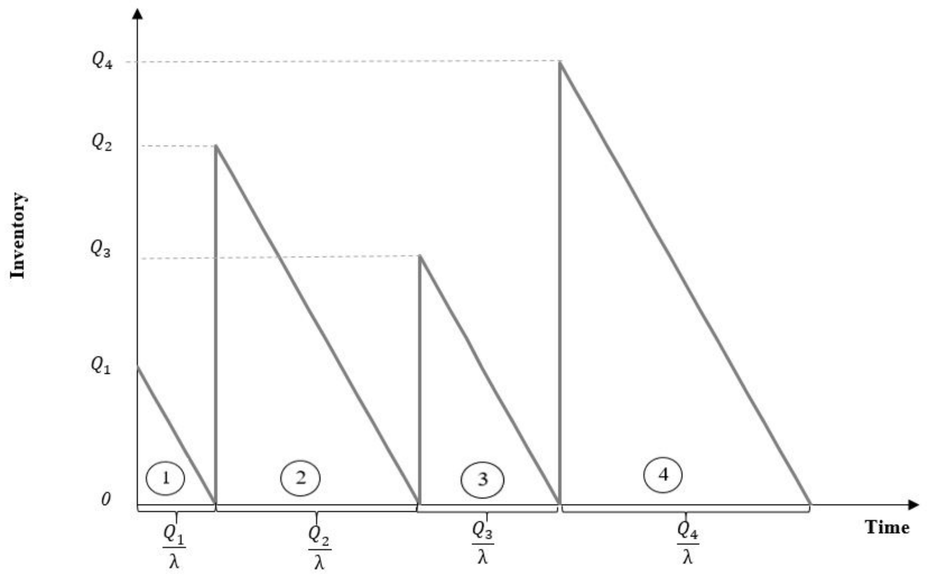

Figure 3.11: Setting VI- Possible Cycle Length Realizations

Referring to Figure 3.11, the expected holding cost per cycle is given by

$$H(Q_1,Q_2,Q_3,Q_4) = \alpha\frac{h}{2}\left(\frac{Q_1^2}{\lambda}p_1 + \frac{Q_3^2}{\lambda}(1-p_1)\right) + (1-\alpha)\frac{h}{2}\left(\frac{Q_2^2}{\lambda}p_2 + \frac{Q_4^2}{\lambda}(1-p_2)\right).$$

The expected ordering cost function can be expressed as

$$P(Q_1,Q_2,Q_3,Q_4) = K + \alpha p_1 c Q_1 + \alpha(1-p_1)cQ_3 + (1-\alpha)p_2 c Q_2 + (1-\alpha)(1-p_2)cQ_4.$$

Finally, expected cost per unit time is given as follows

$$AC(Q_1,Q_2,Q_3,Q_4) = \frac{K + \alpha p_1 c Q_1 + \alpha(1-p_1)cQ_3 + (1-\alpha)p_2 c Q_2 + (1-\alpha)(1-p_2)cQ_4}{\alpha\left(p_1\frac{Q_1}{\lambda} + (1-p_1)\frac{Q_3}{\lambda}\right) + (1-\alpha)\left(p_2\frac{Q_2}{\lambda} + (1-p_2)\frac{Q_4}{\lambda}\right)}$$
$$+ \frac{\alpha\frac{h}{2}\left(\frac{Q_1^2}{\lambda}p_1 + \frac{Q_3^2}{\lambda}(1-p_1)\right) + (1-\alpha)\frac{h}{2}\left(\frac{Q_2^2}{\lambda}p_2 + \frac{Q_4^2}{\lambda}(1-p_2)\right)}{\alpha\left(p_1\frac{Q_1}{\lambda} + (1-p_1)\frac{Q_3}{\lambda}\right) + (1-\alpha)\left(p_2\frac{Q_2}{\lambda} + (1-p_2)\frac{Q_4}{\lambda}\right)}.$$

CHAPTER 4

COMPUTATIONAL STUDY

In this chapter, our aim is to assess the value of shelf life extension possibility, value of return opportunity and value of shelf life information through a detailed computational study. The value of each opportunity above is calculated in terms of the percent improvement in expected costs; that is the percentage decrease in the expected cost as a result of the corresponding opportunity.

Throughout the computational study, we keep some of the parameters unchanged which are listed in Table 4.1 and we perform a full factorial design with the levels of factors investigated are provided in Table 4.2.

Table 4.1: Unchanged Parameter Values Used

λ	c	h	t_2
1000	5	1	1.5

Table 4.2: Levels of Factors in Full Factorial Design

α	δ	(p_1, p_2)	t_1	K
0.2	0.5	(0.25,0.25)	0.5	500
0.5	0.75	(0.5,0.5)	0.75	1000
0.8	1	(0.75,0.75)	1	5000

The rest of the chapter is organized as follows: In Section 4.1, value of extension test opportunity both with and without initial shelf life information is investigated. Section 4.2 focuses on the value of return opportunity under no extension test and possible extension test opportunity. In Section 4.3, value of initial shelf life information under no extension test and possible extension test opportunity is discussed. Findings about value of final shelf life information is provided in Section 4.4. Comparisons of all opportunities are discussed in Section 4.5. And finally, comparisons of return opportunity and initial shelf life information when the extension test is available is examined in Section 4.6.

4.1 Value of Extension Opportunity

Value of extension test opportunity is investigated both with and without shelf life information. Under no shelf life information, settings I and II are compared and findings are discussed in Section 4.1.1. Inferences of value of extension test under initial shelf life information are provided by comparisons of settings IV and V in in Section 4.1.2.
Recall that it is not possible to return any amount item to the supplier after the order is received in these settings.

4.1.1 No Shelf Life Information

In order to evaluate the importance of having shelf life extension possibility under no shelf life information environment, we compare expected costs under the optimal policy for settings I and II which are characterized in Section 3.2 and Section 3.3, respectively. The percent improvement in expected cost can be expressed as:

$$\Delta\% = \frac{AC_I^* - AC_{II}^*}{AC_I^*} \text{x } 100$$

Recall that there is no available shelf life expiration date information of the supplier's stock for both cases.

The results are summarized with respect to the levels of factors in Table 4.3. There are 243 instances in total. Since there are 3 different values for each parameter, there are 81 instances for each subgroup. Below, we list our major findings:

- In 108 out of 243 instances, extension test results in saving. Percent improve-

Table 4.3: Summary of Results for Settings I vs. II

		# of instances with positive $\Delta\%$	**Maximum $\Delta\%$**	**Average $\Delta\%$ (over all instances)**	**Average $\Delta\%$ (over instances with positive $\Delta\%$)**
α	0.2	51	9.18	1.94	3.08
	0.5	33	24.85	3.57	8.78
	0.8	24	31.78	2.97	10.02
δ	0.5	36	23.68	2.35	5.30
	0.75	36	28.61	2.92	6.58
	1	36	31.78	3.21	7.22
p_i	0.25	48	31.78	5.01	8.45
	0.5	36	19.97	2.64	5.95
	0.75	24	9.00	0.84	2.82
t_1	0.5	48	31.78	4.60	7.76
	0.75	33	18.66	2.57	6.32
	1	27	11.08	1.31	3.94
K	500	6	1.83	0.08	1.03
	1000	27	7.33	0.83	2.48
	5000	75	31.78	7.58	8.19

ment in these 108 instances is about 6.36% on average.

- Value of extension test under no shelf life information is the highest when
  - K=5000 (the maximum fixed cost of ordering considered),
  - t_1=0.5 (the shortest shelf life is minimum considered),
  - p_1=p_2=0.25 (the highest extension probability considered),
  - α=0.8 (the shortest shelf life is most likely in our instances) and
  - δ=1 (the extension period is the longest considered).

Under these parameter set, if the initial shelf life is most probably short and there is no extension test possibility, then the cycle length is equal to 0.5 periods with probability 0.8. Therefore, it is optimal to place order quantity which is λt_1=500 for no extension case. On the other hand, if there is an extension possibility, it is reasonable to place a larger order. The cycle length is equal to 1.5 periods ($(t_1 + \delta)$ or t_2) with probability 0.65. Thus, the optimal order quantity is 1500 (placing more than this quantity is risky in terms of disposal).

Since lower order quantity results in more frequent orders, the saving by extension test is highest with these parameters when the fixed ordering cost is the highest. The percent improvement with these parameters is about 31.78%.

- As p_1 and p_2 decrease, the percent improvement increases. This is because of the fact that it is possible to place a larger order which reduces the average cost, as it is still possible to use them beyond the initial shelf life. Therefore, it intuitively makes sense that value of extension test increases with the increase in extension probabilities, i.e. with the decrease in extension test failure probabilities.

- Value of extension test is nondecreasing in δ , at the same levels of other parameters, as expected. It is increasing in 48 out of 108 instances with positive percent improvement. When K=5000, t_1=0.75 and extension probabilities are 0.25 or 0.5, then it is optimal to place the same order size which is λt_2 units for δ=0.75 and δ=1. This quantity covers the entire demand when cycle length is realized as t_1 or t_2, and almost entire demand when it is realized as $t_1 + \delta$. Therefore, taking the risk of ordering a smaller quantity by considering short initial shelf life and taking the risk of disposal by ordering larger are not reasonable.

  When K is 500 or 1000, it is more likely to disregard the extension test, since placing smaller orders does not increase the average cost dramatically.

- As the fixed ordering cost gets higher, the value of extension possibility generally increases. Not only maximum average percent improvement, but also highest average percent improvement occur when K=5000 compared to lower fixed ordering cost values. In 94 out of 108 instances with positive percent improvement, the saving is nondecreasing.

- If α=0.2, and t_1=0.5 when K=5000, then it is optimal to set the order size to λt_2 for both no extension setting and possible extension settings. However, if K=1000, then it is optimal to place an order quantity less than λt_2 under both settings but the optimal order quantities are not equal. Hence, extension possibility improves expected cost for this case when K=1000, whereas it does not make such a significant difference when K=5000.

- If α=0.8, no extension probabilities are high, i.e. equal to 0.75; and t_1 is more than 0.5, the extension is not considered while ordering. Order quantity of λt_1 covers fair enough demand and purchasing and holding more inventory becomes more costly than placing a new order. Therefore; for the parameter set given in Table 4.4, high fixed cost cannot dominate the other cost factors and having extension test gets insignificant.

Table 4.4: Δ%=0 when K=5000

K	t_1	p_1	p_2	α	δ
5000	0.75	0.75	0.75	0.8	0.5
5000	0.75	0.75	0.75	0.8	0.75
5000	0.75	0.75	0.75	0.8	1
5000	1	0.75	0.75	0.8	0.5
5000	1	0.75	0.75	0.8	0.75
5000	1	0.75	0.75	0.8	1

- If fixed cost is K=500, extension test opportunity has almost no value (See Table 4.3). It is expected, since if the fixed cost of ordering is quite low, then giving frequent orders with lowest reasonable quantity almost always gives the optimal results and there is no need to take the risk of disposal. Only when t_1=0.5 and α (probability of having short initial shelf life) is 0.2, ordering more than λt_1 improves the cost function if the extension probabilities are high (i.e. except the instances with p_1=p_2=0.75). However, value of extension test is still insignificant, even if there is a small percent improvement which can be seen in Table 4.5.

- When t_1 value decreases, value of extension test increases. Not only maximum percent improvement, but also highest average percent improvement occur when t_1=0.5 compared to the higher short initial shelf life parameters (See Table 4.3). When the extension test is not applicable, it is optimal to place order quantities as low as possible in order to get rid of disposal risk. Hence, lower initial shelf life leads to lower order quantities. As a result, more frequent orders increase the average cost. On the contrary, if there is an extension test, it is possible to use the item beyond its initial shelf life; therefore, it is optimal to

Table 4.5: Δ%>0 when K=500

K	t_1	p_1	p_2	α	δ	Δ%
500	0.5	0.25	0.25	0.2	0.5	1.825
500	0.5	0.25	0.25	0.2	0.75	1.825
500	0.5	0.25	0.25	0.2	1	1.825
500	0.5	0.5	0.5	0.2	0.5	0.224
500	0.5	0.5	0.5	0.2	0.75	0.224
500	0.5	0.5	0.5	0.2	1	0.224

place larger orders which provides cost advantage. Hence, value of extension test increases as initial shelf life gets shorter.

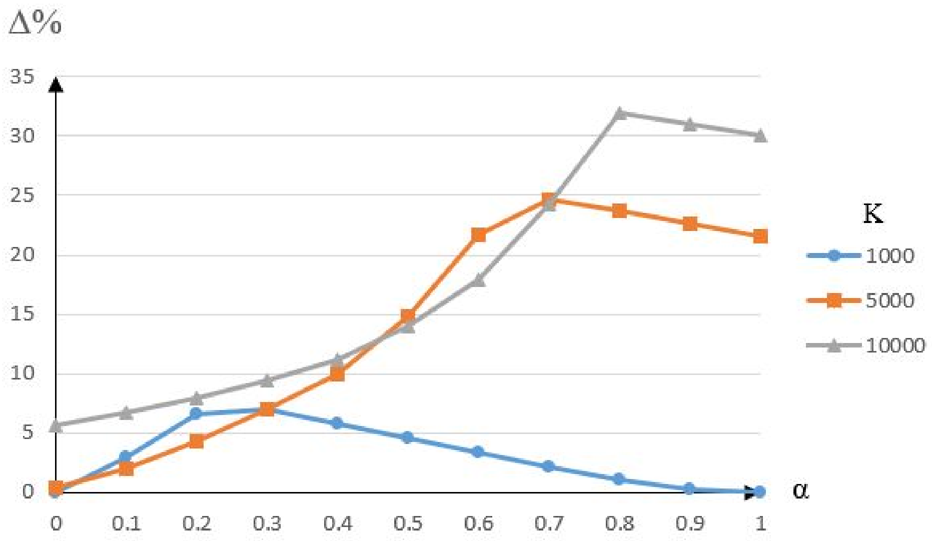

Figure 4.1: Δ% as α increases when K=1000, K=5000 and K=10000 (as p_1=p_2=0.25, t_1=0.5, δ=0.5)

- Maximum percent improvement increases in α, but average percent improvement is the highest when α=0.5. However, since number of instances with positive percent improvement decreases as α increases, average percent improvement over them has a positive relation with α.

  The effects of α on the value of extension opportunity is not straightforward. Hence we conduct further analysis with different parameter sets (see Figure 4.1). Up to a threshold α level, as t_2 is more likely, it is optimal to place larger orders in order to take the advantage of less frequent orders for no extension case. After that α level, as t_1 becomes more likely; it is reasonable to place orders by assuming t_1 will be realized for no extension case. However, there is still a nonnegligible probability of realization of t_2 as initial shelf life. At that threshold α level, extension opportunity provides the highest percent improvement, since if there is an extension opportunity, it is still optimal to place larger orders by considering this opportunity. However, if α value is very close to 1, then it is very likely to get shortest initial shelf life and disadvantage of no extension opportunity decreases. This threshold α level decreases with a decrease in K, since it is reasonable to place frequent orders by assuming t_1 realization which is not very costly and to get rid of disposal risk in no extension setting which can be seen in Figure 4.1.

4.1.2 Initial Shelf Life Information

In order to investigate the value of extension when initial shelf life information is available, we compare the optimal expected costs of Setting IV and V which are given in Section 3.5 and Section 3.6, respectively. The percent improvement in expected cost can be expressed as:

$$\Delta\% = \frac{AC^*_{IV} - AC^*_{V}}{AC^*_{IV}} \text{x } 100$$

The results of our numerical experiments are summarized in Table 4.6. There are again 243 instances in total. Because of 3 different values for each parameter, there are 81 instances for all parameter subgroups.

- In 30 out of 243 instances (compared to 108 instances when information is not available), extension test results in positive percent improvement. Percent improvement in these 30 instances is about 5.44% on the average.
- Just like in no shelf life information environment, value of extension is again the highest when
  - K=5000 (the maximum fixed cost of ordering is considered),
  - t_1=0.5 (the shortest shelf life is minimum considered),

Table 4.6: Summary of Results for Settings IV vs.V

		# of instances with positive $\Delta\%$	**Maximum $\Delta\%$**	**Average $\Delta\%$ (over all instances)**	**Average $\Delta\%$ (over instances with positive $\Delta\%$)**
α	0.2	9	2.73	0.20	1.83
	0.5	9	8.57	0.57	5.10
	0.8	12	18.49	1.25	8.41
δ	0.5	10	13.16	0.56	4.54
	0.75	10	16.37	0.69	5.58
	1	10	18.49	0.77	6.22
p_i	0.25	27	18.49	1.911	5.73
	0.5	3	3.19	0.11	2.84
	0.75	0	0	0	0
t_1	0.5	12	18.49	1.06	7.18
	0.75	9	11.06	0.60	5.43
	1	9	5.94	0.35	3.14
K	500	0	0	0	0
	1000	0	0	0	0
	5000	30	18.49	2.02	5.44

- p_1=p_2=0.25 (the highest extension probability considered),
- α=0.8 (the shortest shelf life is most likely in our instances) and
- δ=1 (the extension period is longest considered).

The logic is the same with no shelf life information environment. The value of percent improvement with these parameters is about 18.49%.

- As in Section 4.1.1, it is observed that value of extension test increases with the decrease in p_1 and p_2 (If extension is less likely, it is not optimal to place orders with larger sizes and take the risk of disposal which results in high average cost). In 27 out of 30 instances with positive percent improvement, we have p_1=p_2=0.25, i.e. the probability that an extension is observed with highest probability in our setting. The remaining 3 instances with positive percent improvement is observed when p_1=p_2=0.5, but the observed percent improvement is lower than the percent improvement by p_1=p_2=0.25 while other parameters are the same ones (see Table 4.7).

Table 4.7: Comparison of $\Delta\%$ under different instances p_1 and p_2 values for $\Delta\%$>0

K	t_1	α	δ		p_1	p_2	$\Delta\%$		p_1	p_2	$\Delta\%$
5000	0.5	0.8	1		0.25	0.25	**18.49**		0.5	0.5	**3.19**
5000	0.5	0.8	0.75		0.25	0.25	**16.37**		0.5	0.5	**2.93**
5000	0.5	0.8	0.5		0.25	0.25	**13.16**		0.5	0.5	**2.39**

- When initial shelf life information is available, the only source of uncertainty is the result of the extension test. The risk of extension test failure is only taken when K=5000 in our parameter set which can be seen in Table 4.6. Note that only 30 out of 81 instances, where K=5000, extension test opportunity provides an improvement. This occurs mostly when p_1=p_2= 0.25, i.e. the probability of extension is higher. The only improvement observed when p_1=p_2=0.5 is when the shortest initial shelf life is minimum and its realization possibility is maximum which is given in Table 4.7.

- When K=500 or 1000, there is no need to take the risk of extension test failure and place larger orders by considering extension opportunity. It is not optimal to conduct an extension test which may result in disposal of unused items. Therefore, it is reasonable to behave as in no extension test environment, even if there is an extension possibility.

- When we examine the instances with positive percent improvement, we observe that value of extension test increases as extension period, δ, increases. As the possible final shelf life gets longer, it is reasonable to place larger orders which reduces the average cost. However, for 213 out of 243 instances, increase in δ provides no savings. This is due to the fact that initial shelf life information already results in sufficient cost reduction and δ becomes ineffective.

- Value of extension with initial shelf life information increases as α increases which is shown in Table 4.6. The batch which arrives with shortest initial shelf life realization, t_1, is affected by extension opportunity more than the batch with t_2. It is more important to use the batch with shortest initial shelf life beyond its shelf life, since the amount (and proportion) of inventory disposed is higher than the batch with longer initial shelf life. It is optimal to order larger quantities which reduces the average cost, if there is an opportunity to use the batch with shortest initial shelf life. Extending t_1 by the amount of δ creates larger effect than extending t_2 by δ. For example, when t_1=0.5 and δ=0.5, the batch can be used beyond twice of its initial shelf life. However, if t_2=1.5, that extension amount increases its shelf life only 1/3 of its initial shelf life. Thus, as probability of observing t_1, which is α, gets higher, the value of extension increases.

- Like in no information settings, value of extension increases with the decrease in t_1. Again not only maximum but also average absolute percent improvement occurs at t_1=0.5 as given in Table 4.6. This is because of the fact that if initial shelf life is long, then order quantity can cover much more demand than short initial shelf life setting. Besides, as the difference between t_1 and t_2 gets smaller as increase t_1 increases, the effect of different initial shelf life possibilities diminish. Therefore, even there is no extension possibility, the system is not affected by the randomness of initial shelf life options for longer initial shelf lives. Hence, extension opportunity is more important for short initial shelf life settings.

4.1.3 The Effects of Shelf Life Information Availability on the Value of Extension Test Opportunity

Extension test opportunity is more valuable when there is no shelf life information. If the initial shelf life information is available, then one of the uncertainties in the environment disappears; and therefore, average costs are smaller. However, if the initial shelf life information is not available, then the extension test opportunity provides higher percent improvement.

Number of instances with positive percent improvement is 108 under no initial shelf life information, whereas there is 30 instances with positive percent improvement under initial shelf life information (See Tables 4.3 and 4.6). Moreover, the maximum percent improvements are 31.78% and 18.49% under no initial shelf life information and under possible initial shelf life information, respectively (See Tables 4.3 and 4.6).

When K=500, there is no positive percent improvement by the extension test, when initial shelf life information is available. On the other hand, if the initial shelf life information is not available, extension test opportunity results in 6 instances with positive percent improvement for K=500 which are insignificant (See Tables 4.3 and 4.6).

If the initial shelf life is known, value of extension test increases with the increase in α, since the extension opportunity gives a chance to use the batch with shortest shelf life which is the most beneficial. However, if the initial shelf life is uncertain, after a threshold α value, placing large orders by hoping success of extension test becomes more risky. Hence, percent improvement starts to decrease.

Effects of other parameters are similar for both cases. With the increase in K, δ, and decrease in p_1, p_2 and t_1, value of extension test increases (See Tables 4.3 and 4.6).

4.2 Value of Return Opportunity

Value of return opportunity is examined under two different settings. Under no extension opportunity, expected costs under optimal policy for settings I and III' (a special case of III) are compared in Section 4.2.1. Under extension opportunity, expected costs under optimal policy for settings II and III are investigated and discussed in Section 4.2.2.
Recall that when return opportunity is available, any quantity can be returned to the supplier after observing the initial shelf life of the batch. The supplier gives a refund for each returned item. Value of return opportunity is investigated for different levels of partial (b=1, 2.5, 4) and full refund (b=c=5).

4.2.1 No Extension Opportunity

In order to evaluate the value of having return opportunity under no shelf life information environment, we compare expected average costs under the optimal policy for settings I (see Section 3.3) and III'. Recall that setting III' is a special case of setting III (see Section 3.4), when p_1=p_2=1 and δ=0. The percent improvement in expected

Table 4.8: Summary of Results for Settings I vs. III'

		# of instances with positive $\Delta\%$	**Maximum $\Delta\%$**	**Average $\Delta\%$ (over all instances)**	**Average $\Delta\%$ (over instances with positive $\Delta\%$)**
b	1	7	3.81	0.44	1.71
	2.5	9	9.52	1.23	3.69
	4	15	15.24	2.70	4.86
	5	24	19.05	4.79	5.38
α	0.2	26	9.14	2.17	3.01
	0.5	18	19.05	3.30	6.61
	0.8	11	17.33	1.40	4.56
t_1	0.5	22	19.05	3.91	6.40
	0.75	19	14.20	2.02	3.83
	1	14	8.10	0.94	2.42
K	500	9	3.56	0.30	1.21
	1000	16	9.14	1.43	3.22
	5000	30	19.05	5.14	6.17

cost can be expressed as:

$$\Delta\% = \frac{AC_I^* - AC_{III'}^*}{AC_I^*} \text{x } 100$$

The results are summarized with respect to different parameters in Table 4.8. There are 108 instances in total. Since b has 4 levels, 27 instances for each b value. Moreover, there are 3 each different values for other parameters, there are 36 instances for each subgroup.

- In 55 out of 108 instances, return policy results in savings. Percent improvement in these 55 instances is about 4.50% on average.
- Value of return opportunity under no extension test setting is highest when
  - b=5 (the full refund considered),
  - K=5000 (the maximum fixed cost of ordering considered),
  - t_1=0.5 (the shortest shelf life is minimum considered) and,
  - α=0.5 (the shortest and longest shelf life is equally likely in our instances).

If K=5000, it is reasonable to place less frequent orders with large sizes. Since the short and long shelf life realizations are equally likely, the uncertainty is the highest. If initial shelf life is realized as t_1 and initial shelf life is quite short, the excess inventory is disposed. Moreover, since the order size is large, purchasing cost becomes also high in the case that t_1 is realized. However, if there is full refund, then it is possible to return the excess inventory which compensate the high purchasing cost. There is no uncertainty of shelf life while deciding the return quantity. Therefore, the return opportunity is most valuable under these parameters and the maximum percent improvement is 19.05%.

- When b=1, which is 20% of unit purchasing cost, percent improvement is quite poor. The maximum percent improvement is 3.81% as can be seen in Table 4.8. When K=500, there is no advantage to have return opportunity. When the fixed cost of ordering is the lowest considered, then it is optimal to place frequent orders with lower sizes. In this case, the incoming batch is depleted by demand entirely, i.e. it is depleted before the shelf life expiration date. Therefore, if the fixed cost of ordering and refund value are the lowest, return opportunity provides no improvement.

- If b= 5, which is full refund, return opportunity is significantly valuable. The maximum percent improvements for our setting is 19.05%. When b=5, giving unnecessarily large orders results in no cost burden. Hence, it is possible to return units as much as wanted without any cost which makes return opportunity very advantageous against no return setting.

- Maximum and average percent improvements are highest when K=5000. In 46 out of 55 instances with positive percent improvement, value of return opportunity increases as K increases while keeping other parameters unvaried. It is optimal to place larger orders when K is high in order to get rid of frequent ordering. If there is a return opportunity, it is possible to return excess amount of the batch after observing the initial shelf life as t_1. On the other hand, if the fixed cost ordering is highest but there is no return policy, it is a necessity to dispose excess amount, if t_1 is observed.

- For instances with positive percent improvement in all b values, as t_1 decreases, the effect of initial shelf life information always increases. There is a risk of disposing large quantities in the case that initial shelf life is realized as t_1, or ordering inadequate amount in the case of t_2 realization. However, when there is a return opportunity, it is possible to get refund for all excess units. Therefore, as the difference between t_1 and t_2 increases, the value of return opportunity also increases.

- As uncertainty of initial shelf life realization increases, value of return opportunity also tends to increase especially under high fixed ordering costs. Hence, maximum and average percent savings are the highest when α=0.5, as can be seen in Table 4.8. This is due to the fact that observing t_1 and t_2 is equally likely when α=0.5; and therefore, the risk of disposal increases. However, if there is a return opportunity, uncertainty of initial shelf life has lower effect.

4.2.2 Possible Extension Opportunity

In order to evaluate the value of having return opportunity under possible shelf life information availability settings, we compare expected average costs for optimal policy for settings II and III which are characterized in Section 3.3 and Section 3.4, respectively. The percent improvement in expected cost can be expressed as:

$$\Delta\% = \frac{AC^*_{II} - AC^*_{III}}{AC^*_{II}} \text{x } 100$$

The results are summarized in Table 4.9. There are 972 instances in total. Since b has 4 levels, we have 243 instances for each b value. Moreover, there are 3 different values for other parameters, and there are 324 instances for each subgroup.

- In 385 out of 972 instances, return policy results in an improvement. Percent improvement in these 385 instances is about 2.47% on average.
- Value of return opportunity under available extension opportunity is highest when
  - b=5 (the full refund considered),
  - K=5000 (the maximum fixed cost of ordering considered),

Table 4.9: Summary of Results for Settings II vs. III

		# of instances with positive $\Delta\%$	**Maximum $\Delta\%$**	**Average $\Delta\%$ (over all instances)**	**Average $\Delta\%$ (over instances with positive $\Delta\%$)**
b	1	18	1.88	0.05	0.71
	2.5	44	4.89	0.27	1.47
	4	108	10.90	0.95	2.15
	5	215	15.35	2.64	2.98
α	0.2	180	7.51	0.93	1.68
	0.5	119	14.89	1.30	3.53
	0.8	86	15.35	0.03	2.64
δ	0.5	145	15.35	1.22	2.72
	0.75	128	14.98	0.93	2.35
	1	112	14.85	0.79	2.28
p_i	0.25	111	10.72	0.51	1.49
	0.5	123	10.00	0.79	2.09
	0.75	151	15.35	1.63	3.50
t_1	0.5	162	15.35	1.79	3.58
	0.75	138	10.73	0.79	1.85
	1	85	6.72	0.36	1.36
K	500	75	3.56	0.26	1.13
	1000	126	8.62	0.92	2.36
	5000	184	15.35	1.75	3.09

- t_1=0.5 (the shortest shelf life is minimum considered),
- α=0.8 (the shortest shelf life is most likely in our instances),
- p_1=p_2=0.75 (the lowest extension probability considered) and
- δ=0.5 (the extension period is shortest considered)

Values of b, K and t_1 when percent improvement is highest are same values under no extension opportunity and the logic is the same. Moreover, when probability of short initial shelf life realization is high, but the extension probabilities and extension period are low, i.e. there is a high disposal risk of high amount of inventory, then being able to return some amount from batch at receiving is more advantageous. Therefore, the return opportunity is most valuable under these parameters and the maximum percent improvement is 15.35%.

- As expected, with an increase in value of b, value of return opportunity under possible extension opportunity increases just like under no extension settings.

- When refund value, b, is equal to 1, there is not a significant advantage of return opportunity. The maximum percent improvement is about 1.88% as can be seen in Table 4.9. The positive, but insignificant percent improvements are observed for b=1, only when K=5000.

- The maximum percent improvement which is observed to be 15.35% occurs when there is full refund. When there is a full refund opportunity, effect of uncertainty of initial shelf life becomes insignificant.

- Under the majority of parameter settings, an increase in K results in an increase in the value of return opportunity. Higher fixed cost of ordering results in larger order, which may result in unnecessarily large inventories especially when short initial shelf life is realized. If there is a refund option, it is possible to take some money back for inventory which is possibly not be used before the initial or final shelf life expiration date. Value of return opportunity is increasing in K in 245 out of 385 instances with positive percent improvement at the same levels of other parameters. Besides, when K=500, it never provides higher percent improvement than other fixed cost of ordering parameters.

- Maximum and average percent improvements are highest when t_1=0.5 as it can be seen in Table 4.9. As t_1 increases, then the need to return because of the short initial shelf life observation decreases. Thus, the value of return policy increases with a decrease of short initial shelf life value, t_1 in the majority of settings.

- Average percent improvements are highest when α=0.5 as it can be seen in Table 4.9. Since the uncertainty of initial shelf life observation is the highest, the risk of disposal or the risk of high fixed order costs due to setting a smaller order quantity increases; and therefore, return opportunity becomes more significant.

- As the probabilities of extension decrease, i.e. p_1 and p_2 increase, value of return opportunity almost always increases. When p_1 and p_2 are low, then the incoming batch can be used beyond its initial shelf life with higher possibility.

Therefore, with an order larger than λt_1, even if the initial shelf life is realized as t_1, that batch can be used beyond t_1, if the extension test is successful. On the contrary, if extension probabilities are low, then the remaining items will be disposed most probably. Therefore, having a return opportunity is generally more beneficial to get rid of taking the risk of disposal when p_1=p_2=0.75.

- Value of return policy is nonincreasing in δ. As δ increases, the period that the batch can be used gets longer. Hence, need of return opportunity because of the initial shelf life uncertainty decreases. However, its effect is not significant even in maximum percent improvements which can be seen in Table 4.9.

4.2.3 The Effects of Extension Test Opportunity on the Value of Return Opportunity

Return opportunity is more valuable when the extension test is not applicable. If there is an extension test opportunity, then there is also an uncertainty in final shelf life. Deciding the return quantity is not trivial due to the possible failure of the extension test which weakens the value of return opportunity. Moreover, the extension test already provides an extra opportunity in expected costs. On the other hand, if there is no extension test, the system becomes deterministic while deciding the return quantity. Return opportunity eliminates the risk of disposal. Therefore, there is no parameter set that return policy under possible extension opportunity provides higher saving than the policy under no extension test.

Number of instances with positive percent improvement is 55 (about 51% of 108 instances) under no extension test opportunity, whereas there is 385 instances with positive percent improvement (about 39% of 972 instances) under extension test opportunity. The maximum percent improvements are 19.05% and 15.35% under no extension test opportunity and under possible extension test opportunity, respectively (See Tables 4.8 and 4.9).

Maximum and average percent improvement increase with the increase in b, K, and decrease in t_1 for return opportunity not only under no extension test opportunity, but also under possible extension test opportunity. Moreover, as α gets closer to 0.5, i.e. the uncertainty of initial shelf life increases, the opportunity of return gets more valuable (See Tables 4.8 and 4.9).

4.3 Value of Initial Shelf Life Information

Value of initial shelf life information is examined under two different environment which are with and without extension opportunity. Under no extension opportunity expected costs under optimal policy for settings I and IV are compared in Section 4.3.1. Under possible extension opportunity settings II and V investigated and discussed in Section 4.3.2.

4.3.1 No Extension Opportunity

To examine the value of initial shelf life information under no extension opportunity, the optimal expected costs of setting I and IV which are given in Section 3.2 and Section 3.5 are compared. The percent improvement in expected cost can be expressed as:

$$\Delta\% = \frac{AC_I^* - AC_{IV}^*}{AC_{IV}^*} \text{x } 100$$

The results are given in Table 4.10. There are 27 instances in total. Since there are 3 different values for each parameter, there are 9 instances for each subgroup.

- In 24 out of 27 instances, initial shelf life information provides in savings. Percent improvement in these 24 instances is about 5.38$\%$ on the average.
- Value of initial shelf life information under no extension test is the highest when
  - K=5000 (the maximum fixed cost of ordering considered),
  - t_1=0.5 (the shortest shelf life is minimum considered) and
  - α=0.5 (the short and long shelf life is equally likely in our instances).

  In this setting, high fixed cost favors large order quantity. However, if the initial shelf life is realized as t_1, the excess inventory is disposed. On the other hand, if there is an initial shelf life information, then the uncertainty disappears.

Table 4.10: Summary of Results for Settings I and IV

		# of instances with positive $\Delta\%$	**Maximum $\Delta\%$**	**Average $\Delta\%$ (over all instances)**	**Average $\Delta\%$ (over instances with positive $\Delta\%$)**
α	0.2	8	9.14	3.89	4.38
	0.5	8	19.05	6.30	7.09
	0.8	8	17.33	4.17	4.69
t_1	0.5	9	19.05	8.24	8.24
	0.75	9	14.20	4.15	4.15
	1	6	8.10	1.97	2.96
K	500	6	3.56	0.98	1.47
	1000	9	9.14	3.69	3.69
	5000	9	19.05	9.70	9.70

Since disposal also results in increased variable cost adding to the highest fixed ordering cost, the initial shelf life information is very valuable. The average percent improvement under these parameters is about 19.05%.

- As t_1 decreases while keeping t_2 constant, value of initial shelf life information increases. There is a high risk that either the number of disposed item may increase, or the batch is depleted by demand completely quite before its shelf life because of the initial shelf life uncertainty if there is no information. Hence, the value of initial shelf life always increases with a decrease in t_1.

- Increase in fixed cost of ordering decreases the value of initial shelf life information up to a threshold α value (see Figure 4.2). If α is low, then it is optimal to place an order with a quantity between λt_1 and λt_2 which is closer to λt_2, when K=1000 and there is no available information. Such an order quantity includes the risk of both disposal due to possible short initial shelf life realization and inventory depletion before expiration date due to long initial shelf life realization which results in unnecessarily frequent orders. As K increases (K=5000), it is optimal to place orders with higher quantities in order to increase the cycle length to save in the total fixed cost of ordering. However, as α is high, i.e. realization of t_1 becomes more likely, then the optimal quantity decreases to λt_1 when K=1000 for no information setting. Since the fixed ordering cost is not very high, giving frequent order is not that costly. However, when K=5000, then the quantity of disposed units increases (because of large order quantity) which results in high total variable cost in addition to high fixed cost in case of t_1 realization for no information setting. Therefore, the value of initial shelf life information is higher for larger K values unless t_2 is very likely.

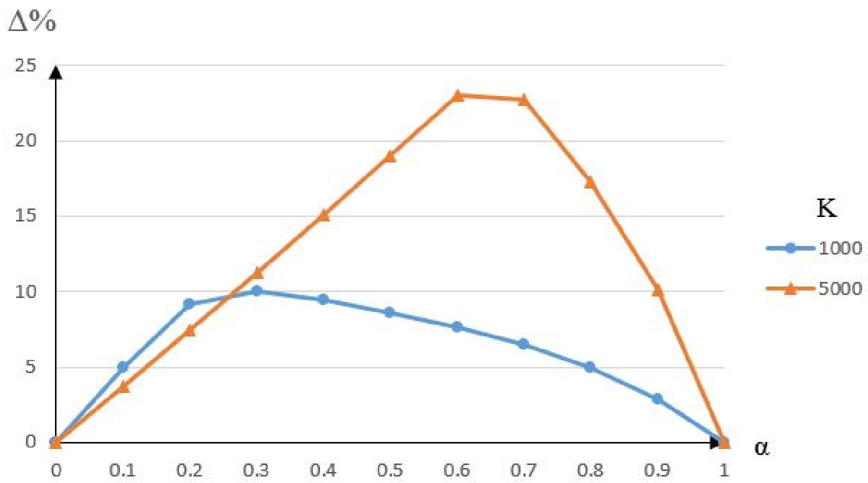

Figure 4.2: $\Delta\%$ for different values of α when K=1000 and K=5000 (t_1=0.5)

- Value of initial shelf life information generally increases until a threshold α, then it starts to decrease. Maximum and average percent improvements are highest when α=0.5 as can be seen in Table 4.10. In order to see the effect of α, further analysis with other α values is conducted. As probability of realizing a certain initial shelf life (α or $(1-\alpha)$) increases, then initial shelf life becomes less uncertain; and therefore, value of information tends to decrease. This threshold α increases with an increase in K. But as expected, if α is equal to 0 or 1, then initial shelf life information becomes useless and there is no improvement regardless of K value (see Figure 4.2).

4.3.2 Extension Opportunity

By comparing the expected cost under optimal solution results under settings II and V, which are explained in Section 3.3 and 3.6, value of initial shelf life information before placing an order is examined. The percent improvement in expected cost can be expressed as:

$$\Delta\% = \frac{AC_{II}^{*} - AC_{V}^{*}}{AC_{II}^{*}} \text{x } 100$$

The results are given in Table 4.11. There are 243 instances investigated in total. Since there are 3 different values for each parameter there are 81 instances for each subgroup.

Table 4.11: Summary of Results for II and V

		# of instances with positive $\Delta\%$	**Maximum $\Delta\%$**	**Average $\Delta\%$ (over all instances)**	**Average $\Delta\%$ (over instances with positive $\Delta\%$)**
α	0.2	72	7.51	2.18	2.46
	0.5	72	14.90	3.36	3.79
	0.8	71	15.35	2.37	2.70
δ	0.5	72	15.35	3.05	3.43
	0.75	72	14.98	2.56	2.88
	1	71	14.85	2.31	2.64
p_i	0.25	72	10.72	1.57	1.76
	0.5	71	10.00	2.31	2.64
	0.75	72	15.35	4.03	4.54
t_1	0.5	80	15.35	4.73	4.79
	0.75	81	10.73	2.18	2.18
	1	54	6.72	1.00	1.50
K	500	54	3.56	0.91	1.36
	1000	81	8.62	2.89	2.89
	5000	80	15.35	4.12	4.17

- In 215 out of 243 instances, initial shelf life information provides percent improvement. The average percent improvement in these 215 instances is about 2.98%.
- Value of initial shelf life information under no extension test is the highest when
  - K=5000 (the maximum fixed cost of ordering considered),
  - t_1=0.5 (the shortest shelf life is minimum considered),
  - p_1=p_2=0.75 (the lowest extension probability considered),

- α=0.8 (the shortest shelf life is most likely in our instances) and
- δ=0.5 (the extension period is shortest considered).

In this setting, the fixed cost of ordering is the highest. Hence, it is optimal to order $\lambda(t_1 + \delta)$=1000 units for no information setting. However, if the initial shelf life is realized as t_1 which occurs with very high probability, and extension test fails, since its success is unlikely, the excess inventory will be disposed. The disposed inventory results in high total variable and holding cost adding to the highest fixed ordering cost. Hence, the initial shelf life information is very valuable. The maximum percent improvement with these parameters is about 15.35%.

- The effect of K depends on the value of α. For high α values, K=5000 generally provides higher percent improvement than other K levels when the other parameters are at the same level. When α=0.8, in 19 out of 27 instances, K=5000 provides higher percent improvement, while in 8 out of 27 instances, K=1000 is the parameter that provides higher percent improvement. On the contrary, for low α values, K=1000 usually generates higher percent improvement than other K values. In 20 out of 27 instances when α=0.2, initial shelf life information under K=1000 is more valuable than other fixed ordering cost settings considered (see Table 4.12). Besides, in order to show the relation between α and K, a further analysis with other parameter setting is conducted (see Figure 4.3).

Table 4.12: Number of instances for different α values at which each K value provides maximum percent improvement

α	K	**# of instances**
0.2	1000	20
	5000	7
0.5	1000	10
	5000	17
0.8	1000	8
	5000	19

- If the fixed cost of ordering is very low, there is some savings especially when

t_1=0.5. However, even the highest percent improvement by K=500 is low and equal to 3.56% as it can be seen in Table 4.11.

- Value of initial shelf life information increases as α increases, and it diminishes after a threshold α value which can be seen in Figure 4.3. Average percent improvement is highest when α=0.5. This is due to the fact that if the probability of realizing a certain initial shelf life increases, i.e. either α or $(1-\alpha)$ is high, then uncertainty of initial shelf life starts to disappear. If the uncertainty of initial shelf life decreases, in the majority of the instances value of information also decreases as expected.

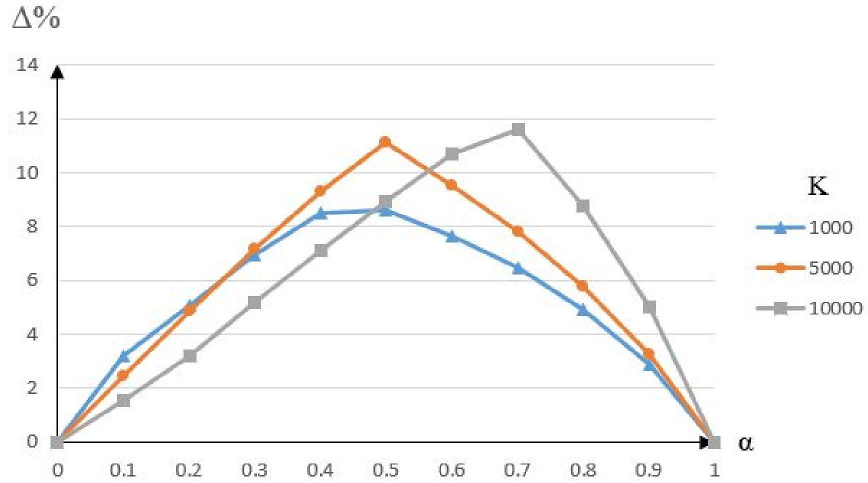

Figure 4.3: $\Delta\%$ for different values of α when K=1000, K=5000 and K=10000 (p_1=p_2=0.5, t_1=0.5, δ=0.5)

- Value of initial shelf life information increases as p_1 and p_2 increase especially for high K values. If the buyer places large orders because of high fixed cost, but t_1 is realized, there is a chance to use that batch beyond its initial shelf life with the extension of shelf life. However, if no extension probabilities are equal to 0.75, then the costumer should discard unused items most probably after t_1 periods. Therefore, as no extension probabilities increase, the value of information increases as well.

- Increase in t_1 decreases the difference between t_1 and t_2. As t_1 and t_2 get closer, value of initial shelf life information almost always becomes less significant as expected.

- Change in δ value creates insignificant effect in terms of initial shelf life information. Maximum percent improvement is 15.35% for δ=0.5, whereas it equals to 14.85% for δ=1 as can be seen in Table 4.11.

4.3.3 The Effects of Extension Possibility on the Value of Initial Shelf Life Information

Initial shelf life information is more significant for no extension test environment. Savings when there is an extension opportunity are always less than or equal to savings when extension is not possible. This is due to the fact that the unique uncertainty is initial shelf life of incoming batch, if there is no extension test; therefore, this information is very critical. On the other hand, extension test already provides an extra opportunity which weakens the value of initial shelf life. Number of instances with positive percent improvement is 24 (about 89% of 27 instances) under no extension test opportunity, whereas there is 215 positive percent improvement instances (about 88% of 243 instances) under extension test opportunity. In other words, in terms of percentage of instances with positive percent improvement over all ones, there is no significant difference between two cases (See Tables 4.10 and 4.11).

The maximum percent improvement is 19.05% under no extension test opportunity, and 15.35% under possible extension test opportunity, respectively. Note that maximum percent improvement values for both no and possible extension opportunities are exactly the same as the percent improvements for both cases due to full refund return policy, respectively (See Tables 4.8, 4.9, 4.10 and 4.11). This is because of the fact that, maximum percent improvement is always provided by full refund for return opportunity. In fact, there is no difference between being informed about the initial shelf life of incoming batch before ordering and returning some units after arrival of the batch and getting full refund in terms of cost functions. Therefore, when there is full refund for return opportunity, then they are actually identical opportunities.

Maximum and average percent improvement increase as K increases and as t_1 decreases for both cases. Besides, the value of initial shelf information increases with the increase in uncertainty of initial shelf life. Therefore, as α gets closer to 0.5, average percent improvement also increase for both under no extension and possible extension cases.

4.4 Value of Final Shelf Life Information

Value of final shelf life information is investigated under two different cases. Firstly, settings II and VI are compared in order to show the importance of final shelf life information against no shelf life information in Section 4.4.1. Then expected costs under the optimal policy for settings V and VI are examined in order to analyze the effect of the final shelf life information comparing to initial shelf life information in Section 4.4.2.

4.4.1 Against No Shelf Life Information

By comparing expected costs under the optimal policy for settings II and VI, value of getting final shelf life information before placing an order rather than no information is studied. Related derivations were given in Section 3.3 and Section 3.7, respectively. The percent improvement in expected cost can be expressed as:

$$\Delta\% = \frac{AC^*_{II} - AC^*_{VI}}{AC^*_{II}} \text{x } 100$$

The results are summarized in Table 4.13. There are 243 instances in total. Since there are 3 different values for each parameter, there are 81 instances for each subgroup.

- In 216 out of 243 instances, final shelf life information provides savings. The percent improvement in these 216 instances is about 6.07% on average.
- Value of final shelf life information against no shelf life information environment is the highest when
  - K=5000 (the maximum fixed cost of ordering considered),
  - t_1=0.5 (the shortest shelf life is minimum considered),
  - p_1=p_2=0.75 (the lowest extension probability considered),

Table 4.13: Summary of Results for II and VI

		# of instances with positive $\Delta\%$	**Maximum $\Delta\%$**	**Average $\Delta\%$ (over all instances)**	**Average $\Delta\%$ (over instances with positive $\Delta\%$)**
α	0.2	72	9.47	4.06	4.56
	0.5	72	18.79	5.97	6.71
	0.8	72	26.24	6.18	6.95
δ	0.5	72	22.12	5.19	5.40
	0.75	72	24.29	5.38	6.05
	1	72	26.24	5.63	6.34
p_i	0.25	72	15.60	4.52	5.09
	0.5	72	18.58	5.73	6.45
	0.75	72	26.24	5.95	6.70
t_1	0.5	81	26.24	8.29	8.29
	0.75	81	16.59	4.84	4.84
	1	54	13.66	3.07	4.61
K	500	54	3.74	1.24	1.86
	1000	81	10.35	3.96	3.96
	5000	81	26.24	10.99	10.99

  - α=0.8 (the shortest shelf life is more likely in our instances) and
  - δ=1 (the extension period is longest considered).

  When K=5000, then it is optimal to place large orders which is λt_2=1500 for no shelf life information case, even if t_1 is more likely. On the other hand, since α=0.8 and p_1=p_2=0.75, there is a high risk of disposal, because of either short initial shelf life realization or extension test failure. Therefore, average cost is very high under no shelf life information with these parameters and final shelf life information is more important in terms of not only initial shelf life information but also extension test result. The highest percent improvement with these parameters is 26.24%.

- Maximum and average percent improvements are the highest when K=5000 as can be seen in Table 4.13. Additionally K=5000 provides higher percent saving then other fixed ordering cost values investigated when the other parameters are at the same level. In no information setting, not only initial shelf life uncertainty, but also unexpected extension test failures should be taken into consideration. Being informed about final shelf life, which includes the extension test result information, creates much more cost saving for larger fixed cost values.

- When K=500, there are 54 instances with positive percent improvement only when t_1 is 0.5 or 0.75. However, these percent improvements are quite low. Even the highest maximum percent improvement is about 3.74% which is shown in Table 4.13.

- As t_1 decreases, value of final shelf life increases. If there is no information, possible disposal quantity increases with an decrease in t_1, when t_1 is realized. For higher values of t_1 this risk decreases; therefore, getting shelf life information starts to lose its advantage and value.

- Value of final shelf life increases with the decrease in probability of extension for K=5000 and t_1=0.5. Because of the high fixed ordering cost, it is reasonable to place large orders. However, when p_1=p_2=0.75, there is a high risk of short initial shelf life realization and/or extension test failure which result in high disposal quantity (since t_1 is very low). Hence, the final shelf life information becomes more valuable for this parameter set combination. However, when the fixed ordering cost is 1000 or 500, change in the probability that extension occurs become less significant. Uncertainty of extension test result which results in frequent order (due to either conservative order quantity, but test success, or large order quantity but test failure) is not that costly.

- Effect of α changes with respect to different K values. Increase in α results in an increase in the value of final shelf life information especially for K=5000 at the same levels of other parameters. If long initial shelf life is realized with high probability, it is reasonable to place large orders especially when K=5000. Thus, final shelf life information is not valuable that much. In 18 out of 27 instances, final shelf life is more valuable under α=0.8 than other α values. In remaining 9 instances under K=5000, α=0.5 provides higher percent improvement than other α values. Note that as K decreases, α value that provides highest percent improvement for that K level, decreases. The relation between α and K is given in Table 4.14.

Table 4.14: Number of instances for different K values at which each α value provides maximum percent improvement

K	α	**# of highest $\Delta\%$ >0**
500	0.2	15
	0.5	3
	0.8	-
1000	0.2	9
	0.5	15
	0.8	3
5000	0.2	-
	0.5	9
	0.8	18

- Change in δ provides low differences. The maximum percent improvement is about 26.24% for δ=1, whereas it equals to 22.12% for δ=0.5 which is significant but low percent improvement which is given in Table 4.13.

4.4.2 Against Initial Shelf Life Information

In both settings V and VI (which were explained in Section 3.6 and in Section 3.7, respectively), it is possible to get initial shelf life information before placing an order. Moreover, in setting VI, the supplier informs the buyer whether the shelf life of the item on his shelf will be extended or not. In this section, we examine the value of final shelf life information in comparison to only initial shelf life information by comparing expected optimal costs of settings V and VI. The percent improvement in expected cost can be expressed as:

$$\Delta\% = \frac{AC_V^* - AC_{VI}^*}{AC_V^*} \text{x } 100$$

The results are summarized in Table 4.15. There are 243 instances in total. Since there are 3 different values for each parameter, there are 81 instances for each subgroup.

- In 216 out of 243 instances, final shelf life information provides savings. Percent improvement in these 216 instances is about 3.23% on average.

Table 4.15: Summary of Results for V and VI

		# of instances with positive $\Delta\%$	**Maximum $\Delta\%$**	**Average $\Delta\%$ (over all instances)**	**Average $\Delta\%$ (over instances with positive $\Delta\%$)**
α	0.2	72	8.05	1.91	2.15
	0.5	72	12.94	2.73	3.07
	0.8	72	18.58	3.97	4.47
δ	0.5	72	11.79	2.26	2.54
	0.75	72	15.60	2.93	3.30
	1	72	18.58	3.44	3.85
p_i	0.25	72	9.56	3.01	3.39
	0.5	72	18.58	3.52	3.96
	0.75	72	13.37	2.09	2.35
t_1	0.5	81	18.58	3.77	3.77
	0.75	81	15.04	2.74	2.74
	1	54	10.86	2.11	3.17
K	500	54	2.19	0.34	0.51
	1000	81	5.98	1.12	1.12
	5000	81	18.58	7.16	7.16

- Value of final shelf life information against initial shelf life information environment is the highest when

  - K=5000 (the maximum fixed cost of ordering considered),
  - t_1=0.5 (the shortest shelf life is minimum considered),
  - p_1=p_2=0.5 (the extension and no extension are equally likely in our instances),
  - α=0.8 (the longest shelf life is more likely in our instances) and,
  - δ=1 (the extension period is longest considered).

  In this parameter set, if α=0.8, then it is optimal to order small sizes in order not to take the risk of disposal. Moreover, the success and failure probabilities of extension test are equally likely. If the information about extension test result is known before ordering, then it would be possible to place larger orders which reduces cost especially when the fixed cost is the highest. The value of percent improvement with these parameters is 18.58%.

- Value of final shelf information is highest when K=5000 in comparison to other fixed ordering cost parameters for all instances. When the fixed ordering cost is highest, the buyer gives orders with large quantities. If the batch fails the extension test, then all remaining units are disposed. Therefore, getting the extension test result information is more crucial when the fixed ordering cost is highest.

- When K=500, even the maximum percent improvement is low which is about 3.74% as can be seen in Table 4.15

- The highest maximum and average percent improvements are observed when p_1=p_2=0.5 which is given in Table 4.15. For these values, failure or success of extension test probabilities are equal; and therefore, the uncertainty of test result is the highest. When p_1=p_2=0.25 or p_1=p_2=0.75, the uncertainty of test result is lower. Therefore, final shelf life information is not valuable that much.

- Increase in α increase the value of final shelf life information against only initial shelf life information especially for the highest fixed cost of ordering. α= 0.8 provides higher saving than other α levels. Therefore, maximum and average percent improvements are highest when α= 0.8. When the initial shelf life is t_1, then it is optimal to place smaller orders than t_2 realization, as expected. If the buyer can be informed about whether its shelf life will be extended, then there is a chance to increase the order size. Thus, when shortest initial shelf life is most likely, getting extension test result information is more valuable.

- Value of final shelf life information increases as t_1 decreases. If the initial shelf life is very short, then the extension opportunity which makes that batch to be used beyond its initial shelf life is very important. However, if the test result is uncertain, then shorter initial shelf life increases the possible disposal quantity in case that the test fails. Therefore, it is reasonable to consider as if the shelf life would be expired after t_1 periods. On the other hand, if the extension test result is known at the beginning of the cycle, then it is possible to place larger orders even when the initial shelf life is t_1.

- Increase in δ also increases the value of final shelf life information when K=5000, since it becomes more risky to miss the chance to order larger sizes due to the lack of extension test result information. For K=1000 and K=500, percent improvement is nondecreasing in δ. Therefore, maximum and average percent improvements are highest when δ=1. On the other hand, its effect is not very significant on the average. Average percent improvement is 3.21% when δ=1; while it is about 2.30% when δ=0.5 which is given in Table 4.15.

4.4.3 The Effects of Final Shelf Life Information

The value of final shelf life information against no information case are always greater than and equal to the value against initial shelf life information. This is an expected result, since no information is a worse case than initial shelf life information. Therefore, getting final shelf life information provides higher percent improvement against the setting under no information.

Number of instances with positive percent improvement are 216 (about 89% of 243 instances) with and without initial shelf life information (See Tables 4.13 and 4.15). Note that for all instances that final shelf life information provides a saving against no shelf life information, it also provides a positive improvement against initial shelf life information.

The maximum percent improvement is 26.24% in comparison to no shelf life information, and 18.58% in comparison to initial shelf life information (See Tables 4.13 and 4.15).

Value of final shelf life information generally increases with an increase in p_1 and p_2, if it is compared to no information. This is due to the fact that, especially when K=5000, placing larger orders is desirable even if t_1 is more likely. However, p_1=p_2=0.75 increases the amount of disposed items for no information setting, especially when t_1 is realized. Hence, final shelf life information against no information gets more significant, as the risk of disposal increases. On the contrary, value of final shelf life information increases, as p_1 and p_2 get closer to 0.5. Since the unique difference between final and initial shelf life information is the extension test result, value of final shelf life information against initial shelf life increases, as the uncertainty about the extension test increases.

Maximum and average percent improvements increase with an increase in K, α, δ, and with a decrease in t_1 for both cases.

4.5 Comparisons of Extension Opportunity, Return Opportunity and Initial Shelf Life Information

The performances of extension opportunity, return opportunity and initial shelf life information are compared to each other in this section. In order to make comparisons, percent cost improvements under corresponding settings over setting I are investigated. The number of problem instances that $\Delta\%$ values of each improvement policy outperforms other policies, is the best (strictly) and worst (strictly) out of the ones considered are given in Table 4.16. Extension opportunity provides higher percent improvement than return policy for 107 out of 243 instances when b=1, for 86 out of 243 instances when b=2.5, for 49 out of 243 instances when b=4 and for 24 out of 243 instances when b=5, respectively. It also provides higher percent improvement than initial shelf life information opportunity for 24 out of 243 instances which are the instances that extension opportunity is strictly better than both policies. On the other hand, return policy provides higher percent improvement than only extension opportunity for 1 out of 243 instances when b=1, for 28 out of 243 instances when b=2.5, for 92 out of 243 instances when b=4 and for 192 out of 243 instances when b=5. Initial shelf life information opportunity provides results better than extension opportunity for 192 out of 243 instances and better than return policy for 216 out of 243 each instances for b=1, b=2.5 and b=4, respectively.

It is observed that return opportunity is not the best option when refund is partial. It mostly provides highest improvements when there is full refund, but it is not the unique best opportunity. Whenever return provides the maximum percent improvement, the initial shelf life information also gives the maximum percent improvement. This is an expected result, since both opportunities are identical. It does not matter that whether the buyer gets information about initial shelf life before ordering and decide the order size accordingly or it has a chance to send back excess units and get full refund. Moreover, when refund is partial, it is possible to perform as well as under the available initial shelf life information for 27 (= 243-216) instances out of

Table 4.16: Number of Instances that $\Delta\%$ Values of Each Improvement Opportunity Outperforms Other Opportunities

		Extension Opportunity	**Return Opportunity**				**Initial SL Info**	**Better than both**	**Worse than both**
			b=1	b=2.5	b=4	b=5			
Extension Opportunity		-	107	86	49	24	24	24	192
Return Policy	b=1	1	-				0	0	107
	b=2.5	28					0	0	86
	b=4	92					0	0	49
	b=5	192					0	0	0
Initial SL Info		192	216	216	216	0	-	192 (except for vs. b=5)	0

243 for each b level.

When the fixed cost is high and extension probabilities are low, return opportunity with full refund and initial shelf life information are much more beneficial than having an extension opportunity. The maximum difference of $\Delta\%$ values for the benefit of return (with full refund) and information opportunities compared to extension opportunity occur under parameters of K=5000, t_1=0.5, p_1=p_2=0.75, α=0.8, δ=0.5 and b=5. Under these parameters, the percent improvement by return opportunity with full refund and initial shelf life information are about 17.33%, whereas the percent improvement by extension test opportunity is only about 2.34%.

Extension test opportunity is more advantageous than other policies when no extension probability is low. When α=0.8, the maximum percent improvements are observed for the benefit of extension test opportunity. Being able to use the batch beyond its shelf life is more important if the probability of having a shorter initial shelf life is high. In such a case, being able to extend the shelf life is more valuable than knowing the initial shelf life. The maximum difference between the benefit of having extension opportunity over the availability of initial shelf life information with respect to $\Delta\%$ values is observed when K=5000, t_1=0.5, p_1=p_2=0.25, α=0.8 and δ=1. Its cost is about 14.45% less.

Initial shelf life information and return opportunity with full refund are the most valuable improvement policies in terms of expected cost in the majority of the instances. In about 89% of instances, they provide any improvement (see Table 4.16). Besides in 11% of instances (which is again the highest proportion of instances investigated for corresponding improvement), they provide a saving which is more than 10% of optimal average cost of Setting I. On the other hand, when b=1, return opportunity provides no saving in 74% of instances. The number of instances in terms of percent deviation (improvement) from expected optimal cost of Setting I is summarized in Table 4.17. When the fixed cost of ordering is low(K=500), and the possible initial shelf life lengths are very close to each other as t_1=1 and t_2=1.5, no policy can provide a percent improvement. Low fixed costs favor frequent orders, and shorter initial shelf life is sufficiently long to cover the demand with corresponding order. Therefore, there is no need to have a remedy against uncertain shelf lives.

Table 4.17: Number of Instances in terms of Percent Improvement

		$\Delta\% = 0$	$0 < \Delta\% \leq 5$	$5 < \Delta\% \leq 10$	$10 < \Delta\%$
Extension Opportunity (I vs. II)		56%	26%	9%	9%
Return Opportunity (I vs. III')	b=1	74%	26%	0%	0%
	b=2.5	67%	26%	7%	0%
	b=4	44%	33%	15%	7%
	b=5	11%	56%	22%	11%
Initial SL Information (I vs. IV)		11%	56%	22%	11%

4.6 Comparisons of Return Opportunity and Initial Shelf Life Information When the Extension Test is Available

The performances of return opportunity and initial shelf life information under available extension test are compared to each other in this section. In order to make comparisons, percent cost percent improvements under these settings over setting II are examined.

Initial shelf life information opportunity provides higher savings than return policy for 216 out of 243 instances and performs as well as return policy for 27 out of 243 instances when b=1, b=2.5 and b=4. In other words, return policy never provides higher percent improvement than initial shelf life information opportunity. As mentioned before, if there is full refund, then there is no risk for initial shelf life uncertainty; therefore, they are same policies in fact. Besides, initial shelf life information and return policy with full refund are again the best options in terms of proportion of instances with positive percent improvement over all instances in comparison to optimal average cost of Setting II. It provides an improvement in about 88% of instances investigated for that policy. The number of instances in terms of percent deviation from optimal average cost of Setting II is given in Table 4.18. (Note that final shelf life information against setting II always generate higher percent improvement than other improvement policies. Therefore, it is not taken into consideration for comparisons regarding to setting II.)

Table 4.18: Number of Instances in terms of Percent Improvement under Extension Test Availability

		$\Delta\% = 0$	$0 < \Delta\% \leq 5$	$5 < \Delta\% \leq 10$	$10 < \Delta\%$
Return Opportunity (II vs. III)	b=1	93%	7%	0%	0%
	b=2.5	82%	18%	0%	0%
	b=4	56%	39%	5%	0%
	b=5	12%	74%	10%	4%
Initial SL Information (II vs. V)		12%	74%	10%	4%

When probabilities of no extension and probability of having a shorter initial shelf life are high, but refund value is low, then the initial shelf life information is significantly more valuable than return opportunity. There is a risk of excess inventory, due to the initial shelf life uncertainty, if there is no information. If the refund is partial and low, then placing large order quantities becomes very costly. Therefore, the maximum difference of $\Delta\%$ values for the benefit of information opportunity compared to return opportunity occur under parameters of b=1 and b=2.5, when K=5000, t_1=0.5, p_1=p_2=0.75, α=0.8 and δ=0.5. The difference between $\Delta\%$ values is about 15.35 %.

CHAPTER 5

CONCLUSION

A huge variety of perishable items are procured in any production environment. Such items have shelf lives, in other words, they become useless after that time; therefore, this characteristic should be taken into consideration while determining when and how much to order. Typically, shelf life constraints increase the inventory related system costs. Hence, we investigate some improvement opportunities for a system that mimics two important aspects of the problem from a buyer in defense industry: (1) Shelf life of the incoming material is not known until the items are placed in stock. (2) A perishable item may be used beyond its shelf life, if it is still in good condition. In order to examine whether it still functions well, a quality control operation which is called "shelf life extension test" is conducted. If the test is successful, then the item gets a new shelf life expiration date. For the environment considered, we concentrate on two different supplier flexibility options: (1) The production buyer can get initial shelf life information before placing a new order rather than learn the expiration date after receiving it. (2) The buyer can get a certain amount of refund by returning some of the items after arrival of a batch, when the shelf life is revealed.

In this study, we specifically focus on the value of extension test, return opportunity and shelf life information. We consider an EOQ setting for items with uncertain shelf lives where demand is deterministic and stationary, lead time is zero, there is no initial inventory, no backorder or lost sale is allowed. We propose some EOQ settings which include shelf life extension test opportunity, shelf life information availability and return opportunity. For each setting that we examine, the objective is to minimize the expected cost per unit time which we characterize.

By conducting a detailed computational analysis under different parameter settings, we compare the performances of improvement policies. Our analyses reveal the following findings:

- Highest percent saving provided is 31.78% which occurs when extension test is available under no shelf life information environment. However, extension opportunity is not the most favorable policy. It is possible to operate better with respect to $\Delta\%$ under it than under return opportunity and initial shelf life information only when no extension probability is low and short initial shelf life is more likely.
- Initial shelf life information is generally more valuable than other improvement policies. Number of instances that provides better savings than other policies is more than half of all instances. Moreover, it never results in lower percent improvement than both opportunities together.
- Return opportunity never gives strictly better percent improvements than other policies together. It provides the maximum percent savings with full refund, whenever the initial shelf life information also does. (Note that initial shelf life information before ordering and getting full refund after arrival of batch result in same settings in fact.)
- When fixed cost is high and no extension probabilities are low, return opportunity with full refund and initial shelf life information are much more favorable than extension test availability.

Our study can be extended by increasing the number of possible initial shelf life realizations and allowing to conduct the extension test more than once. In addition to this, cost of extension test (both equipment and labor) and disposal penalty can be added to the settings as another significant cost factors in practice. Moreover, replenishment lead time can also be taken into consideration. Besides, stochastic demand can be examined instead of deterministic and stationary demand.

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