MODELING OF THE NONLINEAR BEHAVIOR OF SEMI-RIGID CONNECTIONS IN STEEL FRAMED STRUCTURES AND ITS INFLUENCE ON THREE DIMENSIONAL ANALYSIS OF STRUCTURAL SYSTEMS

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ABSTRACT

MODELING OF THE NONLINEAR BEHAVIOR OF SEMI-RIGID CONNECTIONS IN STEEL FRAMED STRUCTURES AND ITS INFLUENCE ON THREE DIMENSIONAL ANALYSIS OF STRUCTURAL SYSTEMS

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In steel frame structures, introducing the nonlinear force-deformation behavior of frame members with flexible joints will show closer results to the actual behavior. In this thesis, a mixed formulation frame finite element is developed from a nonlinear force-based method that can include localized semi-rigid connection response. The formulation of the element uses the three-fields Hu-Washizu-Barr principle, where displacement shape function approximation is omitted with the use of a force-based approach. The proposed element formulation can accurately capture the spread of plasticity along element length and section depth with a single element for each beam and column member. Introducing flexible connections to frame members does not necessitate additional nodes where the degrees of freedom do not increase. Also, nonlinear geometric effects and a correct shear area definition are applied to the elements with the use of the proposed element. Accuracy and robustness of the proposed element are presented at both member level and structural level for both the two-dimensional and three-dimensional rigid and semi-rigid steel frame structures. Verifications are conducted by considering studies presented in the literature, as well as the results obtained using advanced nonlinear finite element programs.

Keywords: Steel structures, finite element modeling, semi-rigid connections, vibration characteristics, nonlinear analysis

ÖZ

ÇELİK YAPILARDA YARI RİJİT BAĞLANTILARIN DOĞRUSAL OLMAYAN DAVRANIŞININ MODELLENMESİ VE ÜÇ BOYUTLU YAPISAL ÇÖZÜMLEMELERE OLAN ETKİSİNİN ARAŞTIRILMASI

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Çelik çerçeve yapılarında, çerçeve bağlantı elemanlarının esnek bağlantılarla doğrusal olmayan kuvvet deformasyon davranışının uygulanması, yapının gerçek davranışa daha yakın sonuçlar gösterecektir. Bu tezde, yarı-rijit bağlantının etkisini içerebilen, doğrusal olmayan kuvvet bazlı yöntemden türetilmiş karma formülasyonlu çerçeve sonlu elemanı geliştirilmiştir. Elemanın formülasyonunda, kuvvet-bazlı yaklaşımın kullanılmasıyla deplasman şekli fonksiyonu yaklaşımının ihmal edildiği üç boyutlu Hu-Washizu-Barr prensibini kullanılmıştır. Önerilen eleman formülasyonuyla, her kiriş ve kolon elemanı için tek bir elemanla birlikte, plastisitenin eleman uzunluğu ve kesit derinliği boyunca yayılması hassas bir şekilde yakalanabilmektedir. Çerçeve elemanlarına esnek bağlantıların tanıtılmasıyla ilave düğüm noktalarının eklenmesine gerek duyulmayarak elemanın serbestlik derecesi artmamıştır. Ayrıca, önerilen model doğrusal olmayan geometrik etkilerini ve doğru bir kesme alanı tanımını elemana uygulamaktadır. Önerilen elemanın doğruluğu ve çözümsel sağlamlığı hem iki boyutlu hem de üç boyutlu rijit ve yarı-rijit çelik çerçeve yapılarda hem eleman seviyesinde hem de yapısal düzeyde sunulmaktadır. Literatürde sunulan çalışmaların yanı sıra, ileri düzey doğrusal olmayan sonlu elemanlar programları kullanılarak elde edilen sonuçlar da dikkate alınarak doğrulamalar yapılmıştır.

Anahtar Kelimeler: Çelik yapılar, sonlu eleman modeli, yarı-rijit bağlantılar, titreşim özellikleri, doğrusal olmayan analiz

To My Family

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C_j	Curve fitting parameter
E	Elasticity modulus
$\boldsymbol{F_c}$	Concentrated load
I	Moment of inertia
L	Length of frame
L_b	Length of beam
M	Moment
M_0	Initial moment
M_u	Ultimate moment capacity
M_y, M_z	Moments about y and z axes
N	Axial force
R_k	Stiffness ratio
R_{kf}	Strain hardening stiffness of the connection
R_{ki}	Initial stiffness ratio
R_{ks}	Secant stiffness ratio
T	Torsional moment
V_y, V_z	Shear force in y and z directions
a	transformation matrix
a, b	Curve-fitting parameters
a_s	Compatibility matrix of the section
b	Force interpolation matrix
b_f	Width of flange
c	Damping matrix
d	Depth of web
e	Section deformation vector
$\boldsymbol{f}$	Flexibility matrix
$\boldsymbol{f_B}$	Body forces
f_{Con}	Contribution of connection for flexibility matrix
f_{Frame}	Contribution of frame for flexibility matrix
$\boldsymbol{f_S}$	Traction forces
k	Element stiffness matrix
$\mathbf{k_m}$	Material tangent modulus
m	Mass

m, n	Flange parameters
$\mathbf{m_s}$	Mass matrix
n	Shape parameter
nIP	Total number of sections
nSC	total number of semi-rigid connections
$\mathbf{p}$	Complete system nodal forces
p_{app}	Applied load
$\mathbf{q}$	Basic element forces
$\hat{s}$, s	Element section forces
$\mathbf{s_p}$	Particular solution for constant distributed loads
t_f	Thickness of flange
t_w	Thickness of web
u	Displacement along element axis
$\mathbf{u}$	Complete system nodal displacements
$\boldsymbol{u'}$	Virtual displacement
$\dot{\boldsymbol{u}}$	Velocity of complete system
u_x, u_y, u_z	x, y and z-direction displacements
$\ddot{\boldsymbol{u}}$	Acceleration of complete system
$\mathbf{v}$	Basic element displacements
v, w	Transverse displacement
v_{Con}	Contribution of connection for basic element displacements
v_{Frame}	Contribution of frame for basic element displacements
wIP	Weight of integration
w_x	Axial distributed loads
w_y	Transverse distributed loads
y, z	Distance to neutral axis
α	Scaling factor
δ	Variation
Δ_{SC}	Deformations of semi-rigid connections
δ_{SC}^{axial}	Axial deformation of semi-rigid connections
$\delta_{Y,SC}^{shear}$, $\delta_{Z,SC}^{shear}$	Shear deformation of semi-rigid connections
$\boldsymbol{\varepsilon'}$	Virtual strain
ε_a	Axial strain of the section
ε_{xx}	Normal strain
γ	Shear deformation of the section
γ_{xy}, γ_{xz}	Shear strain

φ	Torsional deformation of the section
φ_{SC}	Torsional deformation of semi-rigid connections
κ, κ_y, κ_z	Curvature of the section
κ_s, κ_{sy}, κ_{sz}	Shear correction factor
λ	Joint stiffness coefficient
ν	Poisson's Ratio
θ_r, θ	Section rotation
θ_x, θ_y, θ_z	Section rotation in x, y, z
$\theta_{Y,SC}$, $\theta_{Z,SC}$	Rotation of semi-rigid connections
ρ	Density
$\boldsymbol{\sigma}$	Stress

CHAPTER 1

INTRODUCTION

1.1. General

Steel framed structures are constructed on site from prefabricated members. Steel members are connected to each other either through welding or by the use of bolts. For the purposes of structural analysis, behavior of the connection regions in steel framed structures is mostly idealized as either shear type (pinned) or moment type (rigid) in practice. While this provides a simple analysis approach, assessment of the real response of structures both in practice and research necessitates consideration of the force-deformation characteristics of all parts. In reality, moment connections have some flexibility and shear connections have some rigidity combined in their real behavior with nonlinearity included. Therefore, this behavior exposes another category for connections as partially restrained or also called as semi-rigid for the purpose of design and as well as modeling and analysis. The issue to incorporate especially the nonlinear behavior of semi-rigid connections prompts the need for an accurate modeling of both the monotonic and the hysteretic behavior of the connections and accurate representation of spread of inelasticity along each frame element length. Although significant amount of experiments and analytical modeling of connections have been performed by researchers, implementation and use of an accurate and robust modeling of steel framed structures with semi-rigid connections into structural analysis is still an ongoing research issue.

Researchers have studied the effects of dynamic behavior of steel structures in the past and many decided to take the effect of flexible connection regions on describing the behavior of structures. Laboratory tests demonstrate that the experimental results and numerical simulations better match each other when the linear flexible behavior of connections are introduced in the mathematical models. Another issue according to the researchers is the need for accurate and robust modeling of the nonlinear behavior; thus, inelastic material behavior with nonlinear geometry is introduced to the element formulations. For this purpose, researchers simply added springs and dashpot systems at the ends of the frame members that are modeled as linear elastic members in order to obtain the energy dissipation arising due to connection behavior besides element end plastification during seismic excitations on steel structures. Besides the flexibility at connection regions, it is also realized that including a realistic shear deformation behavior of steel members will allow the capture of drift estimations of buildings more accurately. Thus, for an accurate modeling of steel frame members, the use of Timoshenko beam theory is followed, where there is also a need for the use of a realistic shear correction coefficient for the widely used steel sections in practice.

According to the literature, introducing flexibility to the joints will affect the design of steel frames, where conservative solutions and less reliable design can be converted to economic and safer results by introducing semi-rigid joint behavior on the structure. The use of hybrid systems that contain both semi-rigid and fully rigid connections in an optimized way can provide a better performing structural system during earthquake excitations. There are studies that focus on the development of replaceable energy dissipating beam-column connections in steel structures, as well. Despite these efforts, the main design philosophy in practice is still mostly to protect the connection region from yielding in moment resisting frames and keep the plastification to the beams for the purpose of performance-based design. However, the connection regions of structures built in the past do not all conform to the guidelines that have evolved especially after Northridge Earthquake in 1994. Thus, for the purpose of structural assessment in practice and research, it is crucial to capture all the nonlinear actions that can take place in a steel framed structure under extreme events.

The amount of studies on the emphasis of introducing connection behavior on structural analysis of steel frame structures reveal the importance of the issue. The pursuit for accurate and robust analysis of steel framed structures first of all starts with an accurate frame element formulation that can capture deformation characteristics, spread of inelasticity and implement localized connection response in an accurate and robust manner. These motivation points led to the research study undertaken in this thesis.

1.2. Objective and Scope

Modeling of the nonlinear force-deformation behavior of frame members with connection regions in steel frame structures will present closer results to the actual behavior. For this purpose, a mixed formulation frame finite element that can incorporate localized semi-rigid connection response is developed from nonlinear force/flexibility method in this thesis.

The formulation of the element starts with a variational form that bases on the use of three-fields Hu-Washizu-Barr principle by [1] and [2]. The proposed element incorporates Timoshenko beam theory assumptions with an accurate representation of shear area for widely used steel sections. The element can also model the linear or nonlinear behavior of connection regions through a localized inclusion of the connection region at any point along its length without further specification of nodes and degrees of freedom at element or structural level. Consideration of connection regions is merely additional monitoring points that are just as the same as the monitoring sections that track the spread of inelasticity along element length.

The formulation of the element uses force interpolation functions and does not need the description of displacement field along its length and localized connection regions.

In a displacement-based finite element approach, the presence of localized connections would introduce discontinuous deformations at a connection region on member length and thus this necessitates the introduction of nodes and degrees of freedom to the structural model at an expense in modeling and implementation effort. Furthermore, the displacement-based elements would also require additional effort in correct description of shape functions for varying geometry, stiffness and mass distributions even for the linear elastic case. On the other hand, the proposed element in this thesis can calculate the stiffness and mass distributions over the whole element and connection regions through the use of force-based interpolation functions that are always continuous along a member even when localized connections are present. Furthermore, the element formulation circumvents the need for further placement of nodes and degrees of freedom at both member and structural level.

The accuracy of the element is verified under static and dynamic cases considering linear and nonlinear responses. First, the dynamic characteristics of members with varying geometry and material distribution is assessed with the presence of semi-rigid connections. Vibration frequencies and mode shapes for both the rigid and semi-rigid connection cases are studied at member level and structural system level for validation of the proposed formulation with results available in the literature as well as simulations undertaken in available simple and advanced finite element programs.

The proposed element formulation can also accurately capture spread of plasticity along element length and section depth through the use of single element use per span for each beam and column member even in the presence of nonlinear connection regions. Accuracy and robustness of the proposed element are presented at both member level and structural level for both the planar and three-dimensional rigid and semi-rigid steel frame structures. Verifications are undertaken by considering available studies presented in the literature, as well as the results obtained through the use of advanced nonlinear finite element programs.

In this thesis, there are five chapters. Besides the introduction chapter that is presented up to now, the second chapter covers the literature survey including the frame finite element models with displacement based and force-based approaches, nonlinear analysis of frame elements with semi-rigid connections and vibration characteristics of frame elements again with the semi-rigid connections. In the third chapter, the derivation of two- and three-dimensional forced based frame element with the semi-rigid connection is presented. After this derivation, the validation is conducted through two- and three-dimensional benchmark and generated examples for vibration characteristics and nonlinear analysis of the systems in the fourth chapter. Finally, conclusions are made in the fifth chapter which is the last chapter.

CHAPTER 2

LITERATURE REVIEW

2.1. Finite Element Method

The term finite element was first used by Clough [3]. In different fields of engineering, finite element method (FEM) became the most effective analysis method to be used both in research and practice. FEM has based on approximate solutions through the use of discrete number of independent elements aggregated and mapped to represent the whole continuous structure. The assemblage of these discrete members presents the structure with idealized elements. Researchers give popularity to this tool with its increasing developments.

The physical problems are induced to mathematical problems in engineering analysis and structural design. The physical problem contains a structural system with applied loads. The alteration of the physical problem is achieved with possible assumptions that are directing to differential equations representing the mathematical model. After a physical problem is converted to a mathematical model, the finite element analysis is used to solve the problem. The solution technique of finite element model is a numerical procedure; therefore, the drawbacks of a numerical procedure are inherently present in FEM, as well. The accuracy of the solution should be satisfied otherwise solution has to be refined with different parameters, such as using more elements to represent the continuum (called as mesh refinement in FEM) until enough accuracy is obtained. The finite element solution is unique to its problem and all the assumptions should be formed to the foreseen response of the model. That is why the mathematical model should correctly idealize the properties and behavior of the actual physical problem that will be obtained by numerical analysis.

The finite element method is a numerical method that is as accurate as the accuracy of the representation of the utilized mathematical model, which is derived from a physical situation. However, the exact match between mathematical and physical models is impossible, that is why the possible close solution to the real situation is enough to obtain sensible results. In finite element analysis, different types of procedures exist to deal with the physical model for more accurate results. There are two methods that can be utilized. These are Displacement Based Finite Element Method and Force Based Finite Element Method. These methods will be discussed in the following sections of this chapter.

Force method and displacement method are used in structural analyses. Both methods are widely used and have different outcomes on the solution. Force method is used to determine the element forces through satisfying joint or node compatibilities. On the other hand, the displacement method calculates joint displacements to satisfy the equilibrium equations. Both formulation methods are used in structural analysis, yet, displacement method gained popularity in the last century due to its ease with regards to its implementation for structural model development and analysis, especially with two-dimensional plane stress elements, three dimensional solid elements, and plate and shell finite elements. This led to the dominance of displacement based finite element methods throughout all finite element formulations, from static to dynamic, as well as linear to nonlinear analysis.

Despite the popularity of displacement-based elements, the use of force-based formulation remained in use at least for linear elastic case for frame finite elements, i.e. the flexibility matrix of the element is obtained and then inverted to get the linear elastic element stiffness matrix. Extension of this approach to the nonlinear case was thought to be the difficulty in the adaptation of this approach within a displacement-based finite element solution platform that tries to find the solution of applied loads to the resistance of the structure.

The breakthrough development happened in late 1990s after the formulation of nonlinear force-based frame finite element model by researchers at UC Berkeley and University of Rome La Sapienza [4] The research studies in the last 2 decades further demonstrated the extreme superiority of the force-based approach in frame finite element model development, and the current thesis provides further contribution to this development.

The finite element model is based on the assumptions on the geometry of the system, material properties, boundary conditions, loading, kinematics of the body. The problem is discretized to small elements that are dividing the model into discrete finite elements. Hence, element response is determined by the final state of the variables. The method in essence is a trial procedure of changing the sizes of the elements with possible minimum calculation time, which could be called as robustness of solution, i.e. speed with high accuracy.

In the following sections, different application approaches of finite element methods that are "Displacement Based Finite Element Method" and "Force Based Finite Element Method" will be discussed. After the explanation of finite element methods, the flexible connection, in other words semi-rigid connections, in the structural members will be presented. Semi-rigid connection will be discussed in detail since it is the main scope issue of this thesis. Also, geometric corrections such as shear area correction of steel structural members and secondary effects of the members will be discussed in the proceeding sections.

2.1.1. Displacement Based Finite Element Method

Displacement based finite element analysis is a method of structural analysis where joint displacements are state variables determined by boundary conditions, geometry of the structure, material model and loading. Then, joint displacements are used to calculate internal forces and stresses. Algorithm is straightforward and easy to apply, displacement based finite element analysis are chosen for the analysis of the structures. The principal of virtual work is conducted through the presentation of the displacement based finite element analysis. Virtual work principle denotes that the total internal virtual work done is equated to the total external virtual work done by applying a virtual small displacement to the system.

$$\int_V \boldsymbol{\varepsilon}'^T \boldsymbol{\sigma}\, dV = \int_V \boldsymbol{u}'^T \boldsymbol{f_B}\, dV + \int_V \boldsymbol{u}'^T \boldsymbol{f_S}\, dV + \sum_i \boldsymbol{u}_i'^T\, (\boldsymbol{F_c})_i \qquad (2.1)$$

where f_B denotes body forces, f_S is traction forces, F_c is concentrated loads, ε' is the virtual strains, u' are the virtual displacements and σ are the stresses which are in equilibrium with the external loads.

The nodes and element equilibrium are equated in the displacement-based finite element analysis whether the analysis type is linear or nonlinear, but not the differential equilibrium. The stress-strain relationship, strain-displacement, and displacement limitation are fully met [5]. Displacement-based finite element satisfies equilibrium of displacement conditions. However, fully satisfying the boundary conditions is not possible and internal stresses can be far from correct. Although granting the continuity of the derivatives in differentials, increasing the element mesh or changing the shape functions may lead to better results. Yet, this leads to more calculation time.

2.1.2. Force Based Finite Element Method

Force based finite element method is based on deriving unknown displacements strains and stresses in the variational form of total potential energy. Force based finite element formulation is also called as mixed formulation finite element because formulation is presented with independent displacements and the element forces or stresses.

In the more generalized form, the variational function of the solution has three-fields, namely the displacements, stress and strains, and is also called as Hu-Washizu functional [2]. The functional is written as follows:

$$\Pi_{\mathrm{HW}} = \int_0^L \delta\boldsymbol{e}^T \left(\hat{\boldsymbol{s}}\big(\boldsymbol{e}(x)\big) - \boldsymbol{b}(x,L)\boldsymbol{q} - \boldsymbol{s}_p(x)\right)\, dx \\ -\delta\boldsymbol{q}^T \int_0^L \boldsymbol{b}^T(x,L)\boldsymbol{e}(x)\, dx - \delta\boldsymbol{u}^T\boldsymbol{p}_{app} = 0 \tag{2.2}$$

This functional formulation is the weak form of the compatibility and equilibrium of the system. The force-based approach depends on exact equilibrium solution within the basic system. Equilibrium is exact in between elements and section. Force based finite elements can be implemented to work in displacement-based finite element solutions through static condensation of element stresses and strains.

Force-based finite element model is noticeable where structural problems with the locking effect of the shear on the members. The displacement-based model presents smaller displacements under the locking effect and this tends to wrong behavior under both linear and nonlinear analysis. That is why in this study force based finite element is adapted for nonlinear structural analysis of framed structures that also include semi-rigid connections.

2.2. Semi-Rigid Connections

In practice, beam to column connections is either as shear type (pinned) or moment type (rigid) in the implementation of finite element analysis of steel structures. However, the actual behavior can be summarized as moment connections with some flexibility or shear connections with some rigidity, together with nonlinearity. The American Institute of Steel Construction (AISC) classifies two types of buildings as fully restrained (FR) and partially restrained (PR) in the Load and Resistance Factored Design Specification. In addition, type PR includes two cases which are depending on whether the connection is restrained or not. The case without restraint is called a simple connection. If the connection is restraint, strength, stiffness, and ductility are included in the design of the connection. The European standard Eurocode 3 describes three types of framing as simple, continuous and semi-continuous. These definitions decide that there are three types of connections and the and that the degree of semi-rigid action is largely dependent on the type of structure.

The behavior of the semi rigid connection can be defined by moment rotation (M- θ_r) curves. The rotation in these curves represents the rotation of the section among the neutral axis. The typical moment rotation curves for the beam-column can be obtained from several databases: Lui and Chen [6], Kishi and Chen [7], etc. Also, connections can be modelled by various methods. These are direct implementation through laboratory tests, component models and detailed finite element analysis models.

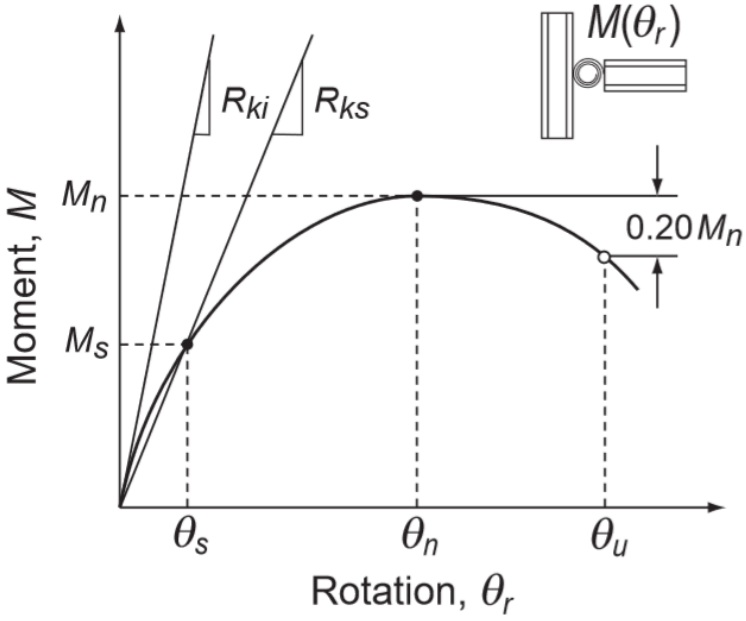

Figure 2.1. Semi-rigid Connection Behavior Representation [7]

Initial stiffness ratio, R_{ki} and secant stiffness ratio, R_{ks} are stiffness parameters for semi-rigid connections in Figure 2.1. These ratios are defined in different approaches and one should choose properly. Since the nonlinear behavior of the connection can occur at low-stress levels, the initial stiffness value of the connection is not enough to define the response of the connection. For this case, the secant stiffness ratio can be used. The rotational stiffness ratio is defined with the formula;

$$R_k = \frac{\lambda EI}{L_b} \tag{2.3}$$

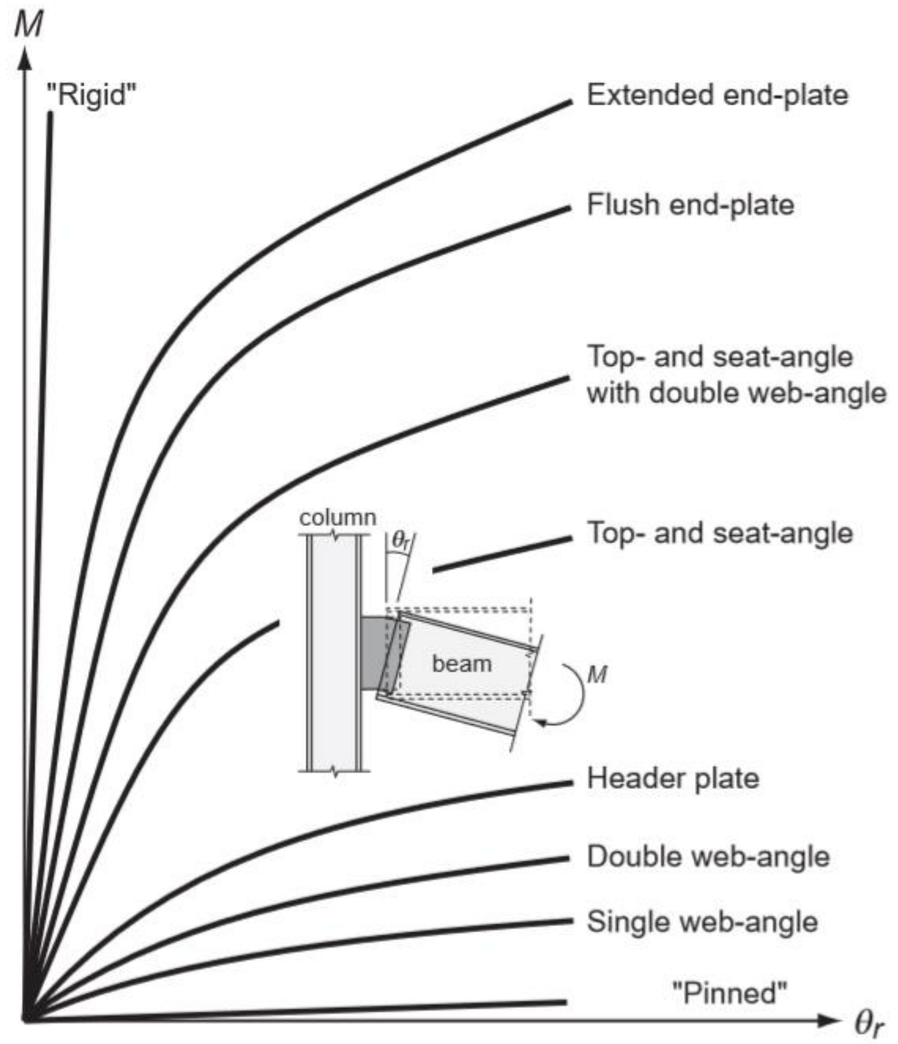

Figure 2.2. Moment Rotation Behavior of Semi-rigid Connection [7]

Rotational stiffness ratio in above equation is determined with the length and inertia of the member, elasticity modulus and coefficient λ. The rotational stiffness of semi-rigid connections is defined to range between values of λ 2 and 20. When λ is equal and less than 2, the connection is considered as a simple connection (shear type) and when λ is equal and more than 20, the connection is considered as a rigid connection (moment type), where the values are provided by AISC Specification [8]. In between, the connection is considered as semi-rigid. In the Figure 2.2, moment rotation curves of several connection types are plotted where they can be considered as semi-rigid since their behavior is in between the rigid and pinned connection behaviors. Other specifications on level of semi-rigidity is presented by, Eurocode [9], Bjorhovde et al. [10], Nethercot et al. [11] and Goto and Miyashita [12].

There are different types of connection formulations in the literature. The one presented in Equation (2.3) is a representation of the linear model. However, the characteristics of the semi-rigid connections are nonlinear as members. That is why, researchers developed several formulations for the semi-rigid connections. In this study, the Power Model and Exponential Model are used. Yet besides these formulations, there are Polynomial Model, Bounding Line Model, Richard-Abbot Model and Hardening Models. The power model and exponential model will be discussed in the following paragraphs.

There are several power models that were developed for the different types of connection. There are two or three parameters in their functions. A two-parameter model [13], [14] has the form of;

$$\theta_r = aM^b \tag{2.4}$$

where a and b are curve-fitting parameters obtained from experiments. Colson and Louveau [15] presented a three-parameter power model as:

$$\theta_r = \frac{|M|}{R_{ki}} \frac{1}{(1 - |M/M_u|^n)} \tag{2.5}$$

where R_{ki} is the initial stiffness, M_u is the ultimate moment capacity of the connection, and n is the shape parameter. Kishi and Chen [16] proposed also a three-parameter power model which is obtained by removing the strain-hardening stiffness of the Richard-Abott model [17]:

$$\theta_r = \frac{M}{R_{ki}\left\{1 - \left(\frac{M}{M_u}\right)^n\right\}} \tag{2.6}$$

where R_{ki}, M_u, and n are the same as above. Kishi and Chen model is derived from Richard and Abott model [17] with removing the strain hardening stiffness. The shape parameter n can be developed with the method of least squares for the variances between the forecast moments and the experimental test data [18], [19]. This power model can implement the second-order nonlinear structural analysis more accurately [20].

Lui and Chen [6] utilized an exponential function in order to curve-fit the experimental data. This model is good at presenting the monotonic nonlinear joint behavior. The exponential model is denoted by a function as in the following form;

$$M = M_0 + \sum_{j=1}^{n} C_j \left[1 - exp^{-\frac{|\theta_r|}{2j\alpha}}\right] + R_{kf}|\theta_r| \tag{2.7}$$

As discussed previously, in addition to the two ideal cases considered in practice (simple/shear/pinned connection case and rigid/moment connection case), the third case is the semi-rigid case. The joints assumed as rigid or pinned in the analysis should be implemented to meet rigid joint and nominally pinned joint classifications according to the design codes, respectively. The semi-rigid frame model should be dealt carefully. Thus, connections can be implemented as springs with moment-rotational relationships that can vary from linear elastic to non-linear type, allowing the degree of ductility of the connections. The design of a linear-elastic model of structural analysis requires linear-elastic modeling of connections. An elastic perfectly plastic behavior, a bilinear joint model is necessary for analysis. Hence, complexity of the model is determined by the connection behavior. Different analysis results cannot be covered in where nonlinear joint modeling exists. The analysis of member and connection forces, displacements and frame stability are determined with semi-rigid connections when the valid connection response is included. Detailing the problem depends on the problem itself. It is important to use sophisticated approaches where necessary. Further detailing may lead to loss of time. That is why separate analyses are made for the design of semi-rigid frames for their serviceability design and ultimate limit designs.

Linear springs with initial stiffness are enough at a service load level for the analysis under serviceability when the joint is far below that of the strength. However, under factored loads for design case, the response of the connection is important, and the analysis should be consisted to ensure the characteristics of the real behavior. Since the response would be nonlinear when the forces reach to the strength of the connections. Therefore, material and geometry nonlinearities as well as stability checks should be taken into account.

2.2.1. Nonlinear Analysis with Semi-rigid Connections

In order to denote the actual behavior of steel structures many researchers tend to take the effect of the semi-rigid connections on the behaviors of the structures. [21] and [22] conducted laboratory studies on the semi-rigidly jointed steel frames convoyed with numerical analyses. Studies showed a perfect match with the laboratory test results and the numerical studies when introducing the semi-rigid joints. Connections in steel structures are described in two categories to specify design and analysis phases. The behavior of steel connections is deliberated as simple (shear) or moment type (fixed). Meanwhile the expected behavior of a connection should show a relative rotation together with moment transfer. This is called semi-rigid connection, that is the actual response of the connection.

The framing of a structural system is the geometric arrangement of the beams, columns, braces and shear walls. The joints of these members are referred to as the connections of the elements. However, it is crucial to make enough strength of members and the necessary connections for the demand. This framing system is set to carry design loads as well as the serviceability states of the structure. Design loads of the structure are gravitational and lateral loads. Hence, a structure could be a combination of braced members and unbraced members, which are determined and designed with sophisticated approaches. At this point, a designer should decide whether the framing is rigid or flexible using the connections of the members.

The lateral stiffness of a rigid frame is mainly due to the rigid joints connected to the bending of the frame elements connections. The joints should have enough strength, stiffness and small deformation. Deformations should be small to not affect the distribution of internal forces and moments on frames. A rigid frame may withstand additional loads without bracing if there is additional support for stability. The frame has to resist the design forces on its own, including gravity and lateral forces. Hence, it has sufficient lateral stability against lateral vibration when exposed to horizontal wind and earthquake loads. Rigid frame systems perform better in the case of cyclic loads or earthquakes, even if rigid connections result in a less economical structure.

A simple frame attributed to a structural system where the beams and columns are flexibly connected, and the system cannot withstand significant lateral loads. The stability is satisfied by attaching the simple frame to bracing systems. Brace systems support lateral loads, and both frame and brace systems support gravity loads. The lateral load response is small in many cases of braced systems so that the second-order effects of the frame design can be neglected. Figure 2.2 shows the representative sketches of simple and rigid connection frames.

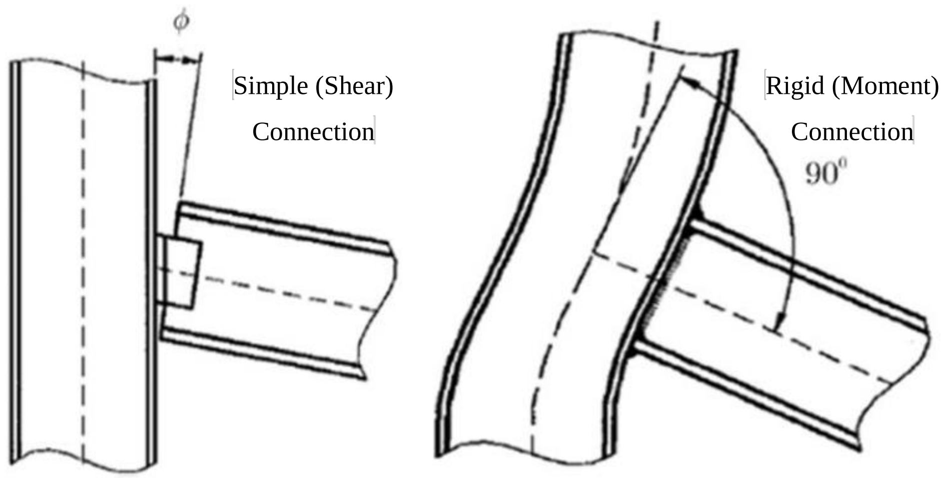

Figure 2.3. Representation of Simple (Left) and Rigid (Right) Connections [23]

Pinned connection frames are manufactured and assembled easier and for steel structures, it is better to join the members without connecting the flanges. The bolted joints are normally preferred instead of welded joints which require careful application and more qualified workers. If the system is prepared with simple connections, the sizing of beams and columns is a simple process. Reducing the horizontal drift with simple connections and brace systems is more convenient than using rigid frame systems with rigid connections.

2.2.2. Frame Elements of Semi-rigid Connections

Assessment of the seismic loads represented on structures is affected by their vibration characteristics. Therefore, implementing accurate finite element models becomes important for vibration study and dynamic analysis of steel framed structures. At this point, the presence of connections in steel structures further increases the importance of the employed numerical models. In order to simplify the analysis, shear deformations can be neglected in slender members, for example, beams and columns that are the framing elements in steel structures; nevertheless, the accumulation of error due to this simplification in a 10 story steel building with 6 m bays and a 40 story steel building with 3 m bays yields to 10% and 30% underestimation of roof drifts, respectively [24]. Therefore, neglecting shear deformations and rotary inertia effects in the mass matrix may further create errors due to the usage of lumped mass matrix and single element discretization along the member. It is also vital to deliberate the actual behavior of the connections that connect the structural members.

Researchers have broadly studied the influences of dynamic behavior of steel framed structures in the last two decades. Many researchers suggested considering the effect of the semi-rigid connections in the analysis and design stages [25], [26]. According to the studies [27], introducing semi-rigidity to pinned joints will affect the design, hence pinned connection design tends to conservative solutions and less consistent design. Razavi and Abolmaali [28] revealed that semi-rigid frames showed better results than fully rigid ones under the study of flexibility of the connections in high-rise steel buildings.

Tests on flexible connections in steel framed structures are conducted by Chui and Chan [21] and Nader and Astaneh-Asl [22] which is conducted with numerical analyses and showed the reputation of considering semi-rigid connection in structural models. Hence, consideration of semi-rigidity at connection region is important, it is also vital to consider possible inelastic behavior and nonlinear geometric effects on frame members in conducting dynamic analysis [29]–[31]. The studies present significant amount of research on the study on the dynamic behavior of steel framed structures which is carried with semi-rigid connections through the use of finite element method [31]–[37]. Present design codes also attempt to deliver the effect of semi-rigid connection response for steel framed structures under dynamic loads, and studies try to assess the code suggestions. Sophianapoulos [38] related Eurocode approach with closed form solutions in one of similar studies, where the outcomes from both methods presented that responses show good match with each other in the fundamental modes of vibration; but, in the higher modes, the difference of the semi-rigid connections became distinct and results varied from each other. Studies on the stability of structures which are proposed by Shakourzadeh et al. [39] and Minghini et al. [40] presented substantial change on critical loading on members, stability and strength responses because of the presence of semi-rigid connection. Both studies taken the displacement-based finite element formulations. The comparisons were done for small structures with small amount of connections in these studies. Numbers of degrees of freedom were limited. However, increasing number of members and connections in large structures would need to increase matrix sizes and solution times with displacement-based element analyses in many cases of practice. Hence increasing element number in order to catch nonlinear material response would present accuracy problems. However, force-based elements have been confirmed to perform vigorously [41]–[43].

Galvo et al. [34], da Silva et al. [33], Al-Aasam and Mandal [37] have established FEM formulations to inspect the dynamic behavior of steel frames with semi-rigid connections. These studies expose that proper modeling of the connections have important part on the dynamic behavior of steel framed structures. Therefore, it is necessary to mention that structural design codes propose the effect of structures under dynamic response. Present design codes provide the influence of structures under dynamic response.

Finite element models should distinguish the modeling of the mass and stiffness matrices for both beam and column members in order to get transverse shear deformations and rotary inertia along a member’s length accurately. The partial fixity is presented by the existence of connections. Partial fixity at connection areas of steel structures significantly affects the vibration characteristics of steel framed structures. Also, it is also important to consider possible inelastic behavior and nonlinear geometric effects on frame members [29]–[31] in carrying out the dynamic analysis with using of semi-rigidity at the connection region. The studies cover a noteworthy number of researches on the examination of the dynamic behavior of steel framed structures with semi-rigid connections with the use of the finite element method [31]–

[37]. Present design codes also try to deliver the effect of semi-rigid response for steel structures under dynamic actions. Research studies also attempt to assess the code proposals.

Studies of [43] showed the effects of semi-rigid connections under cycling loading. These studies conducted with experimental results for two-dimensional steel semi-rigid framed structures. The dynamic behavior of steel structures depends on the description of semi-rigid joints on the structure. The study of [44] presented the requirement of defining semi-rigid joints for exact modelling of vibration characteristics on structures. Again in dynamic point of view for semi-rigid connections, three-dimensional studies of [31] revealed the consequence of flexible joints on steel framed structures under cyclic loading.

The study by Özel, Saritas and Tasbahji [45] showed the need of presenting connection in the structure for correct modeling of vibration characteristics. Base-plate connections also require similar tendency on connection behavior. Abdollahzadeh and Ghobadi [46], showed the assessment of column base or base-plate with experimental, analytical and FEM model under static loading that was presented by [47]. The basic mathematical formulations for column base-plates were conducted by Stamatapoulos and Ermopoulos [47] to exhibit the flexibility on joint behavior of column base-plates under dynamic loading.

The strength along the members of structure controls the performance of the building. On the other hand, deformations control the serviceability of structures. Hence, deformations in different directions have consequence on each other due to the continuum phenomenon of the bodies. Shear deformations tend to determine the lateral flexibility of steel frames. The study by Charney et al. [48] presents the influences of the shear deformations on the members. The use of the shear effects on the members are presented with the definition of effective shear area. In this thesis, the effective shear area model is adopted from the study of Charney et al. [48]. The studies on the shear properties of the sections on linear basis [49] and nonlinear basis [50], [51] display the importance of this study among the behavior of systems.

In order to achieve a precise dynamic analysis of steel framed structures, vibration characteristics of steel beams, braces and columns with semi-rigid connections should be studied. For a better representation of the semi-rigidity, the exact behavior of the members should be conducted.

The brace frame members are another element of the steel framed structures. In order to mention a complete system, it is important to focus carefully on the behavior of these members. The behavior of brace end connections for steel framed structures is conducted in [52], [53]. Most of the cases for the brace end connections are under the axial load deformation [54]–[56] and only the flexibility of axial deformation should be taken into account.

Nonlinear three-dimensional frames with semi-rigid connection studies are carried out by researchers. The study of Nguyen and Kim [57] presents three-dimensional semi-rigid steel frames accounting for the second-order effects with the use of stability functions which are generated from the solution of beam–columns under axial force and bending moments and the semi-rigid beam-to-column connection is generated by a 3D nonlinear multi-spring element. Chiorean studied on large deflection distributed plasticity analysis of three-dimensional semi-rigid frames [58]. In study [58], Maxwell-Mohr rule and second-order force-based functions are used to generate second-order inelastic flexibility-based element and semi-rigid connections are introduced with zero-length elements. Another study is done by Thai and Kim [59]. Thai and Kim proposes a distributed plasticity analysis of semi-rigid steel frames with geometric and material nonlinearities [59]. P-delta effects, residual stresses and inelastic behavior of materials are considered using a fibre model and the semi-rigid connections at beam ends are generated with a zero-length connection element.

Studies on semi-rigid steel framed structures for trying to reveal better predictions shows the necessity for accurate modeling approaches. Therefore, a frame finite element with semi-rigid connections is generated in this study for static analysis and vibration assessment of steel framed structures. The element formulation depends on the three-field Hu-Washizu-Barr principle with the implementation of force-based interpolation functions. The model lets precise determination of vibration frequencies of members with semi-rigid connections by using single element without the need for displacement field. Hence, this approach limits the calculation time and error in the analysis of steel framed structures. Also, an accurate shear correction factor for I-sections is considered to obtain closer match with exact solutions. Available finite element programs (SAP2000, ANSYS and OpenSees) and benchmark examples are utilized for the verification of the proposed model.

As an important benchmark, the proposed model with semi-rigid connections is compared with advanced research oriented finite element program [60], which also incorporates force based element, but where semi-rigid connections can only be included through the introduction of extra springs with new nodes, degrees of freedom and constraint conditions to be specified. Thus, the approach in OpenSees requires an increase in modeling effort and sizes of matrices to be stored and inverted in the solution of nonlinear equations. The proposed model and OpenSees's model elements are compared in the validation studies. Another novelty in current model is the force-based mass matrix use as proposed by [61], while OpenSees models use lumped mass matrix approximation.

CHAPTER 3

FRAME ELEMENT FORMULATION

3.1. Frame Element Formulation for 2D Case

In this section, the formulation of two-dimensional frame element is presented. The cases of static and dynamic variational formulations are undertaken through the use of three-fields variational formulation and force-based approach. Spread of inelasticity through the element is presented by fiber discretization of the section.

3.1.1. Basics of Element Formulation for 2D Case

A two-dimensional formulation is conducted through the element formulation for a cantilever beam with three free degrees of freedom at its free end, which is then transformed to the complete system that has six degrees of freedom. The kinematics of deformation through the continuum of beam is defined with Timoshenko beam theory, which allows section rotations to be independent from the derivative of the beam deflection. As a result, the difference between the section rotation and slope of beam's axis allow for the inclusion of shear deformations along the length of the beam, which is by the way set to zero in the Euler Bernoulli beam theory. The section displacements on a material point which deforms in xy-plane can be calculated by deliberating Timoshenko beam theory as follows;

$$\begin{Bmatrix} u_x(x,y) \\ u_y(x,y) \end{Bmatrix} = \begin{Bmatrix} u(x) - y\theta(x) \\ v(x) \end{Bmatrix} \tag{3.1}$$

where u_x and u_y are the *x* and *y*-direction displacements which are the displacement functions in x and y-directions at any point on the cross-section. u(*x*) is the displacement of any point along the beam's axis, i.e. x-axis. v(*x*) is the transverse deflection along on any point at (*x*,*0*) in *y* direction. Rotation of the beam around z-axis is θ(*x*).

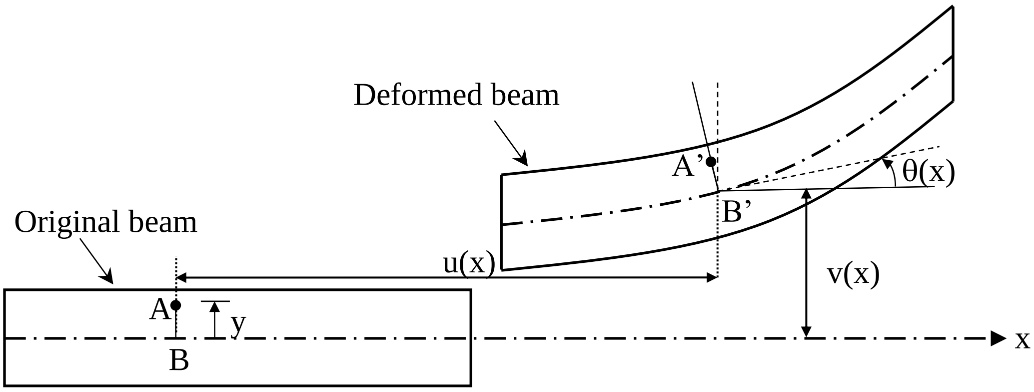

Figure 3.1. Deformed Beam Sketch

The strain, ε contains the normal strain, ε_{xx} along the member and shear strain, γ_{xy}, where these parameters are computed as follows;

$$\varepsilon_{xx} = u'(x) - y\theta'(x) = \varepsilon_a(x) - y\kappa(x) \tag{3.2a}$$

$$\gamma_{xy} = -\theta(x) + v'(x) \tag{3.2b}$$

$$\boldsymbol{e}(x) = [\varepsilon_a(x) \quad \kappa(x) \quad \gamma(x)]^T \tag{3.2c}$$

$$\varepsilon = \begin{Bmatrix} u'(x) - y\theta'(x) \\ -\theta(x) + v'(x) \end{Bmatrix} = \begin{Bmatrix} \varepsilon_a(x) - y\kappa(x) \\ \gamma(x) \end{Bmatrix} = a_s(y, z)\ e(x) \tag{3.2d}$$

where **e**(*x*) is the section deformation vector. $\varepsilon_a(x)$ is the axial strain in the normal direction of the section, $\gamma(x)$ the shear deformation (sliding) of the section and $\kappa(x)$

the curvature of the section about z-axis. $\boldsymbol{a}_s(y,z)$ is the compatibility matrix of the section and is calculated as follows;

$$\mathbf{a}_s(y,z) = \begin{bmatrix} 1 & -y & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.3}$$

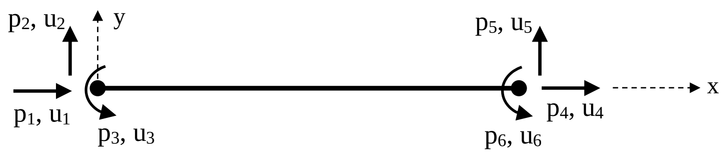

Figure 3.2. Two-dimensional Complete System

Element formulation is conducted as a frame element with two end nodes in xy-plane (Figure 3.2). The element in complete system has 3 degrees of freedom per node and 6 degrees of freedom in total, where this is transformed to a basic system by removing the 3 rigid body modes of displacement through a transformation that gives the 3 deformation modes left for the basic system. The transformed element is based on the cantilever basic system that is shown in Figure 3.3. The transformation matrix, **a** interacts the whole system nodal forces **p** and nodal displacements **u** to the basic equation elements **q** and element deformations **v** along the length L of the beam through the following equation;

$$\boldsymbol{v} = \boldsymbol{a}\boldsymbol{u} \tag{3.4a}$$

$$\mathbf{p} = \mathbf{a}^T\boldsymbol{q} \tag{3.4b}$$

$$\mathbf{a} = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & -L & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \tag{3.4c}$$

Figure 3.3. Cantilever Basic System Forces and Deformations

Element section forces **s**(x) are composed of axial force $N(x)$ along beam's x-axis, shear force in perpendicular direction $V(x)$ along y direction, and moment $M(x)$ around z-axis. **s**(x) can also be defined with basic element forces **q** at free nodes by employing the force interpolation matrix **b**(x,L) and $\mathbf{s}_p(x)$ which is the particular solution for constant distributed loads in the axial and transverse directions, namely w_x and w_y, respectively.

$$\boldsymbol{s}(x) = [N(x) \quad M(x) \quad V(x)]^T \tag{3.5a}$$

$$\boldsymbol{s}(x) = \boldsymbol{b}(x, L)\boldsymbol{q} + \boldsymbol{s}_p(x) \tag{3.5b}$$

$$\boldsymbol{b}(x, L) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (L-x) & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{3.5c}$$

$$\mathbf{s}_p(x) = \begin{bmatrix} (L-x) & 0 \\ 0 & (L-x)^2/2 \\ 0 & (L-x) \end{bmatrix} \begin{Bmatrix} w_x \\ w_y \end{Bmatrix} \tag{3.5d}$$

3.1.2. Variational Method and Finite Element Formulation with Vibration for 2D Case

Independent element nodal displacements **u**, element basic forces **q**, and section deformations **e** create the variational form of the element by using three-fields Hu-Washizu functional and applied as part of the beam finite element formulation [62]. Derivation of the dynamic case is reached through applying the inertial forces $\boldsymbol{m\ddot{u}}$ acting at nodes by taking into account D'Alembert's principle to become the following variational form of the element.

$$
\begin{aligned}
\delta \Pi_{\mathrm{HW}} = & \int_0^L \delta \boldsymbol{e}^T \left(\hat{\boldsymbol{s}}\big(\boldsymbol{e}(x)\big) - \boldsymbol{b}(x,L)\boldsymbol{q} - \boldsymbol{s}_p(x) \right) \, dx \\
& -\delta \boldsymbol{q}^T \int_0^L \boldsymbol{b}^T(x,L)\boldsymbol{e}(x) \, dx + \delta \boldsymbol{q}^T \boldsymbol{a}\boldsymbol{u} \\
& +\delta \boldsymbol{u}^T \boldsymbol{a}^T \boldsymbol{q} + \delta \boldsymbol{u}^T \boldsymbol{m}\ddot{\boldsymbol{u}} - \delta \boldsymbol{u}^T \boldsymbol{p}_{app} = 0
\end{aligned}
\tag{3.6}
$$

The above equation is also found by considering the general Hu-Washizu variational form with dynamic case by Barr [1]. Equation (3.6) ought to embrace for arbitrary $\delta\boldsymbol{u}$, $\delta\boldsymbol{q}$ and $\delta\boldsymbol{e}$. Therefore, the following three equations must be fulfilled in order for the Hu-Washizu-Barr variational to be zero.

$$\boldsymbol{m}\ddot{\boldsymbol{u}} + \boldsymbol{p} \equiv \boldsymbol{p}_{app} \tag{3.7a}$$

$$\mathbf{p} = \mathbf{a}^T \boldsymbol{q} \tag{3.7b}$$

$$\boldsymbol{v} \equiv \int_0^L \boldsymbol{b}^T(x,L)\boldsymbol{e}(x)dx \tag{3.8a}$$

$$\mathbf{v} = \mathbf{a}\boldsymbol{u} \tag{3.8b}$$

$$\hat{\boldsymbol{s}}(\boldsymbol{e}(x)) \equiv \boldsymbol{b}(x, L)\boldsymbol{q} + \boldsymbol{s}_p(x) \tag{3.9}$$

The equation of motion that is Equation (3.7a and 3.7b) holds for linear or nonlinear material responses, hence this equation can be generated for each element to attain the structure's equation of motion. A numerical time integration scheme can be conducted to obtain a solution as given in [63]. The effect of viscous damping can be attained by implementing $\boldsymbol{c}\dot{\boldsymbol{u}}$ to the left-hand side of the equation. Where **c** is the damping matrix. It is also possible to get resisting forces **p** not only in terms of displacements, **u** but also as a function of velocities, $\dot{\boldsymbol{u}}$ using a material model that takes into account time-dependent properties, such as visco-elastic or visco-plastic material models, but this approach is not undertaken in the analysis of civil engineering structures. Any energy dissipation which could not be modeled with hysteretic nonlinear models could be modeled through the use of viscous damping matrix **c** in structural analysis. For linear section response, section deformations can be obtained as $\boldsymbol{e} = \boldsymbol{k}_s^{-1}\ \hat{\boldsymbol{s}}$ to get the section deformations from section forces with the usage of section stiffness matrix $\boldsymbol{k}_s$. The change of section deformations **e** to Equation (3.8a) thus gives:

$$\boldsymbol{v} = \boldsymbol{f}\ \boldsymbol{q}; \tag{3.10a}$$

$$\mathbf{f} = \int_0^L \boldsymbol{b}^T(x, L)\boldsymbol{f}_s(x)\ \boldsymbol{b}(x, L)dx \tag{3.10b}$$

In the equations (3.10a and 3.10b), **f** is the flexibility matrix of the element. The section flexibility matrix is $\mathbf{f}_s$ which is calculated by taking inversion of the section stiffness matrix $\mathbf{k}_s$. Additional substitution of above equation presents;

$$\boldsymbol{m}\ddot{\boldsymbol{u}} + \boldsymbol{k}\boldsymbol{u} = \boldsymbol{p}_{app} \tag{3.11a}$$

$$\mathbf{k} = \mathbf{a}^T \boldsymbol{f}^{-1} \boldsymbol{a} \tag{3.11b}$$

where $\mathbf{k}$ is the 6×6 element stiffness matrix in the whole structure.

3.1.3. Presence of Semi-rigid Connections in Element Formulation for 2D Case

Incidence of semi-rigid connections is adapted with the discretized version of continuous integrals above for the calculation of basic element deformations;

$$\boldsymbol{v} = \boldsymbol{v}_{\text{Frame}} + \boldsymbol{v}_{\text{Con}} \tag{3.12a}$$

$$\boldsymbol{v}_{\text{Frame}} = \sum_{i=1}^{nIP} \boldsymbol{b}^T(x_i)\boldsymbol{e}_i wIP_i \tag{3.12b}$$

$$\mathbf{v}_{\text{Con}} = \sum_{i=1}^{nSC} \boldsymbol{b}^T(x_i)\boldsymbol{\Delta}_{SC,i} \tag{3.12c}$$

where *nIP* is the total number of sections used for the capture of the nonlinear response of the element and *nSC* is the whole number of semi-rigid connections current along the element; *wIP* is the weight of integration equivalent to corresponding integration location for a discretized section, and finally $\boldsymbol{\Delta}_{SC} = [\delta_{SC}^{axial} \quad \theta_{SC} \quad \delta_{SC}^{shear}]^T$ is the vector of deformations of semi-rigid connection with axial deformation δ_{SC}^{axial}, rotation θ_{SC} and shear δ_{SC}^{shear}. Introducing a semi-rigid connection along member in Figure 3.1 does not modify the force vector under small deformations, hence above Equations (3.1) to (3.6) are not affected by this operation. Element flexibility matrix is similarly generated as follows:

$$\boldsymbol{f} = \boldsymbol{f}_{\text{Frame}} + \boldsymbol{f}_{\text{Con}} \tag{3.13a}$$

$$\boldsymbol{f}_{\text{Frame}} = \sum_{i=1}^{nIP} \boldsymbol{b}^T(x_i)\boldsymbol{f}_{s,i}\boldsymbol{b}(x_i)wIP_i \tag{3.13b}$$

$$\mathbf{f}_{\text{Con}} = \sum_{i=1}^{nSC} \boldsymbol{b}^T(x_i)\boldsymbol{f}_{SC,i}\boldsymbol{b}(x_i) \tag{3.13c}$$

Equations (3.8) and (3.9) are linked to the element state determination, which means these equations can be conducted independent of Equation (3.7), and then the solution can be reduced into Equation (3.7). The equations of motion are accumulated for all elements. This process was shown for the linear elastic case above. Generally, state determination of the element involves an iterative solution in the case of nonlinear behavior, where Equations (3.7) to (3.9) are required to be solved. This solution needs the generation of element flexibility matrix **f** under nonlinear response, where taking derivative of element deformations **v** wherein Equation (3.8) according to the element forces **q** which finalizes into the same flexibility integration expression given in Equation (3.10); however, the section stiffness will be nonlinear this time. Finally, specifics of the solution for the dynamic case at element level is analogous to the nonlinear static case offered in [43].

3.1.4. Section Response for 2D Case

Section response can be attained by the basic assumptions presented in Timoshenko beam theory before. The plane sections before deformation remain plane after deformation along the length of the beam by the usage of section compatibility matrix $\mathbf{a}_s$ given in Equation (3.2). At this point, a correction term will be introduced to this matrix, called as the shear correction factor κ_s as follows;

$$\boldsymbol{a}_s = \boldsymbol{a}_s(y) = \begin{bmatrix} 1 & -y & 0 \\ 0 & 0 & \sqrt{\kappa_s} \end{bmatrix} \tag{3.14}$$

Shear correction factor κ_s can be obtained as the inverse of the κ suggested by Cowper [64] for I-section:

$$\begin{aligned} \kappa = & \frac{1}{10(1+\nu)(1+3\text{m})^2}[(12+72\text{m}+150\text{m}^2+90\text{m}^3) \\ & +\nu(11+66\text{m}+135\text{m}^2+90\text{m}^3) \\ & +30\text{n}^2(\text{m}+\text{m}^2)+5\nu\text{n}^2(8\text{m}+9\text{m}^2)] \end{aligned} \tag{3.15a}$$

$$\text{m}=\frac{2b_f t_f}{d\ t_w} \quad \text{and} \quad \text{n}=\frac{b_f}{d} \tag{3.15b}$$

Derivation of the above equation for the rectangular shapes vanishes the terms with flange parameters. Then, setting Poisson's ratio to zero generates the form factor for the rectangular section as 6/5. In the study presented by Charney et al. [48], major axis shear deformations were examined for I-sections in the AISC section database, and the subsequent easy form factor was anticipated.

$$\kappa=0.85+2.32\frac{b_f t_f}{d\ t_w} \tag{3.16}$$

In this study, both of the form factors presented in Equations (3.15) and (3.16) have been utilized in executing the validation studies.

By utilizing the section compatibility matrix $\mathbf{a}_s$, section forces can be retrieved by integration of the stresses. This satisfies the material constitutive relations $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\boldsymbol{\varepsilon})$, and the derivative of section forces **s** with the section deformations results wherein the section stiffness matrix $\mathbf{k}_s$.

$$\boldsymbol{s} = \int_A \boldsymbol{a}_s{}^T \boldsymbol{\sigma} \; dA \tag{3.17a}$$

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} \\ \sigma_{xy} \end{pmatrix} \tag{3.17b}$$

$$\mathbf{k}_s = \int_A \boldsymbol{a}_s{}^T \boldsymbol{k}_m \boldsymbol{a}_s \; dA \tag{3.17c}$$

where the material tangent modulus $\mathbf{k}_m$ is gained by the stress-strain relation by the use of $\mathbf{k}_m = \partial\boldsymbol{\sigma}(\boldsymbol{\varepsilon})/\partial\boldsymbol{\varepsilon}$. Hence, inverse of the section stiffness matrix, which is the section flexibility matrix is utilized in generation of element flexibility matrix in Equation (3.13).

Behavior of the generated local connection response can be nonlinear generally. The comprehensive evidence is given in [43] according to the types of mathematical models that can be utilized in generation of the flexibility influence of the connection in Equation (3.13).

3.1.5. Force Based Consistent Mass Matrix for 2D Case

The mass matrix used for the proposed element with semi-rigid connections is attained by the use of a force-based approach presented by Soydas and Saritas [61]. This approach was first applied by Molins et al. [65]. This method indirectly deliberates the displacement field along the element length which is calculated in a simple manner using unit dummy load method. The generation of the consistent mass matrix is done by the determination of the section mass matrix. The mass is considered as a distributed load by the length of the beam in the basic cantilever system. The section mass matrix is generated by the following equation:

$$\boldsymbol{m}_s(x) = \int_A \boldsymbol{a}_s^T \rho(x,y) \ \boldsymbol{a}_s \ dA \tag{3.18}$$

In calculation of the section mass matrix in above equation, shear correction factor, should not be applied in conducting section compatibility matrix $\mathbf{a}_s$. Mass matrix of the force-based element can be presented in the complete system which is used in Equation (3.7a), which is in 6×6 form as:

$$\boldsymbol{m} = \begin{bmatrix} \boldsymbol{m}_{00} & \boldsymbol{m}_{0L} \\ \boldsymbol{m}_{L0} & \boldsymbol{m}_{LL} \end{bmatrix} \tag{3.19}$$

The components of element mass matrix are generated in 3×3 sub-matrices

$$\boldsymbol{m}_{LL} = \boldsymbol{f}^{-1} \int_0^L \boldsymbol{b}^T(x,L)\boldsymbol{k}_s^{-1}(x) \left(\int_x^L \boldsymbol{b}^T(x,\xi)\boldsymbol{m}_s(\xi)\boldsymbol{f}_p(\xi)\boldsymbol{f}^{-1} d\xi \right) dx \tag{3.20a}$$

$$\boldsymbol{m}_{L0} = \boldsymbol{f}^{-1} \int_0^L \boldsymbol{b}^T(x,L)\boldsymbol{k}_s^{-1}(x) \left(\int_x^L \boldsymbol{b}^T(x,\xi)\boldsymbol{m}_s(\xi)(\boldsymbol{b}^T(0,\xi) - \boldsymbol{f}_p(\xi)\boldsymbol{f}^{-1}\boldsymbol{b}^T(0,L)) d\xi \right) dx \tag{3.20b}$$

$$\boldsymbol{m}_{0L} = \boldsymbol{m}_{L0} = -\boldsymbol{b}(0, L)\boldsymbol{m}_{LL} + \int_0^L \boldsymbol{b}(0, x)\boldsymbol{m}_s(x)\boldsymbol{f}_p(x)\boldsymbol{f}^{-1} dx \qquad (3.20c)$$

$$\boldsymbol{m}_{00} = -\boldsymbol{b}(0, L)\boldsymbol{m}_{L0} + \int_0^L \boldsymbol{b}(0, x)\boldsymbol{m}_s(x)\left(\boldsymbol{b}^T(0, x) - \boldsymbol{f}_p(x)\boldsymbol{f}^{-1}\boldsymbol{b}^T(0, L)\right) dx \qquad (3.20d)$$

The element flexibility matrix **f** is taken as given in Equation (3.10). The partial flexibility matrix $\mathbf{f}_p$ is calculated as follows:

$$\boldsymbol{f}_p(x) = \int_0^x \boldsymbol{b}^T(\xi, x)\boldsymbol{k}_s^{-1}(x)\boldsymbol{b}(\xi, x) d\xi \qquad (3.21)$$

The integral terms in Equations (3.20) and (3.21) are presented to the existence of semi-rigid connections without the necessity for more discretization of the element.

3.2. Frame Element Formulation for 3D Case

In this section, the formulation of a three-dimensional frame element is presented. The cases of static variational formulations are conducted with a force-based frame element. Spread of inelasticity through the element is presented by fiber discretization of the section similar to the previous two-dimensional case.

3.2.1. Kinematic Relations for 3D Case

The three-dimensional formulation is conducted through the element formulation of a cantilever beam again. The continuum is satisfied with reduced degrees of freedom to a cantilever beam. Timoshenko beam theory allows the beam rotation independent of beam's deflection and this creates shear deformation of the beam which is not included in the Euler Bernoulli beam theory. Displacements on a material point on the section of a beam that deforms in xyz-space can be obtained by calculating Timoshenko beam theory as follows;

$$\begin{Bmatrix} u_x(x,y,z) \\ u_y(x,y,z) \\ u_z(x,y,z) \end{Bmatrix} = \begin{Bmatrix} u(x) - y\theta_z(x) + z\theta_y(x) \\ v(x) - z\theta_x(x) \\ w(x) + y\theta_x(x) \end{Bmatrix} \tag{3.22}$$

where u_x(x,y,z), u_y(x,y,z) and u_z(x,y,z) the x, y and z-direction displacements which are the displacement functions in x, y and z directions at any point in the cross-section. $u(x)$ is the displacement of any point along the x-axis. $v(x)$ and w(x) are the transverse deflections along on any point at $(x,0)$ in *y and z*-direction. Rotation of the beam around x-axis is $\theta_x(x)$, around y-axis is $\theta_y(x)$ and around z-axis is $\theta_z(x)$. The strain, ε contains the normal strain, ε_{xx} along the member and shear strain, γ_{xy}, where these parameters are computed as follows;

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xy} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} u'(x) - y\theta_z'(x) + z\theta_y'(x) \\ -\theta_z(x) + v'(x) - z\theta_x'(x) \\ \theta_y(x) + w'(x) + y\theta_x'(x) \end{Bmatrix} \tag{3.23a}$$

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_a(x) - y\kappa_z(x) + z\kappa_y(x) \\ \gamma_y(x) - z\varphi(x) \\ \gamma_z(x) + y\varphi(x) \end{Bmatrix} \tag{3.23b}$$

$$\boldsymbol{\varepsilon} = \boldsymbol{a}_s(y,z)\ \boldsymbol{e}(x) \tag{3.23c}$$

where $\mathbf{e}(x)$ is the section deformation vector.

$$\boldsymbol{e}(x) = [\varepsilon_a(x) \quad \kappa_z(x) \quad \kappa_y(x) \quad \gamma_y(x) \quad \gamma_z(x) \quad \varphi(x)]^T \tag{3.24}$$

$\varepsilon_a(x)$ is the axial strain along the beam's x-axis, $\varphi(x)$ is torsional rate of twist along beam's x-axis, $\gamma_y(x)$ and $\gamma_z(x)$ are the shear deformations of the sections along y and z-axis, respectively, and $\kappa_y(x)$ and $\kappa_z(x)$ are curvatures of the sections about y and z axes. Section deformations are generated from section displacements from the comparison of the terms of Equation (3.23). $\boldsymbol{a}_s(y,z)$ is the compatibility matrix of the section and is given as follows;

$$\boldsymbol{a}_s(y,z) = \begin{bmatrix} 1 & -y & z & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -z \\ 0 & 0 & 0 & 0 & 1 & y \end{bmatrix} \tag{3.25}$$

3.2.2. Basic System without Rigid Body Modes and Force Interpolation Functions for 3D Case

Element formulation is conducted as a frame element with two end nodes in xyz. Thus, the complete structure system is reduced to a basic system which is derived to interpret the member elements state determination of the structure. The complete system in Figure 3.4 is reduced from 12 displacement modes that contain 6 rigid body modes to only 6 deformation modes through the use of a basic system. The produced element is based on the cantilever basic system where the left-hand side of the system only permits rotations in y and z-direction, and the right-hand side permits the only displacement in x-direction and rotations in x, y and z-direction that is shown in Figure 3.5. The transformation matrix, a interacts the whole system nodal forces p and nodal displacements u to the basic equation elements q and element deformations v along the length L of the beam through following equation;

$$\boldsymbol{v} = \boldsymbol{a}\boldsymbol{u} \tag{3.26}$$

$$\boldsymbol{p} = \boldsymbol{a}^T\boldsymbol{q}; \quad \text{and} \quad \mathbf{a} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & -L & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & L & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.27}$$

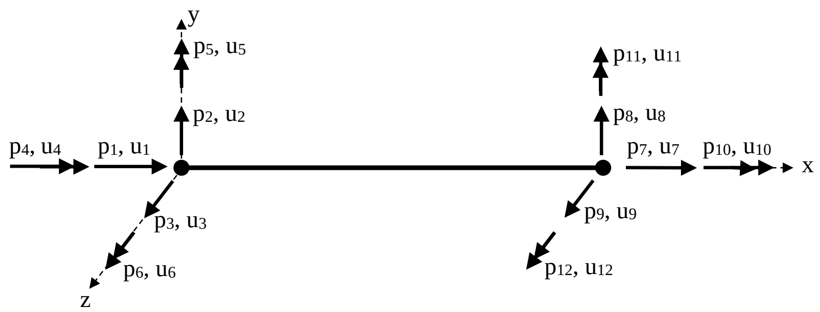

Figure 3.4. Three-dimensional Complete System

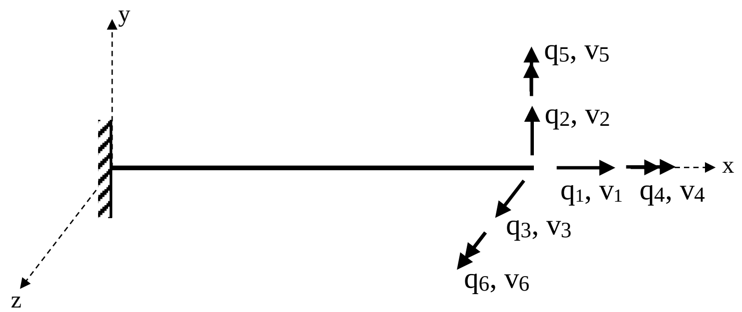

Figure 3.5. Basic System Forces and Deformations

Basic element forces at free end, $\boldsymbol{q}$ are given in Figure 3.5 and presented in Equation (3.27). These forces are internal section forces, $\boldsymbol{s}(x)$ by using the force interpolation matrix $\boldsymbol{b}(x, L)$ for the cantilever beam as follows;

$$\boldsymbol{s}(x) = [N(x) \quad M_z(x) \quad M_y(x) \quad V_y(x) \quad V_z(x) \quad T(x)]^T = \boldsymbol{b}(x, L)\boldsymbol{q} \tag{3.28a}$$

$$\boldsymbol{b}(x,L)=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & (L-x) & 0 & 0 & 0 & 1\\ 0 & 0 & (x-L) & 0 & 1 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\end{bmatrix} \tag{3.28b}$$

By using Equation (3.28), it can get exact equilibrium between the forces at free end of the element and forces at any section that is *x* units away from the fixed end. Axial force $N(x)$, shear force in *y and z* directions $V_y(x)$, $V_z(x)$ and moments about *x*, *y* and *z*-axis $T(x)$, $M_y(x)$, $M_z(x)$ are section forces.

3.2.3. Variational Base and Finite Element Formulation for 3D Case

Independent element nodal displacements **u**, element basic forces **q**, and section deformations **e** create the variational form of the element by using three-fields Hu-Washizu functional and applied as part of beam finite elements by [41] and [62].

$$\delta\Pi_{\mathrm{HW}}=\int_0^L \delta\boldsymbol{e}^T\left(\hat{\boldsymbol{s}}\big(\boldsymbol{e}(x)\big)-\boldsymbol{b}(x,L)\boldsymbol{q}-\boldsymbol{s}_p(x)\right)\,dx-\delta\boldsymbol{q}^T\int_0^L \boldsymbol{b}^T(x,L)\boldsymbol{e}(x)\,dx-\delta\boldsymbol{u}^T\boldsymbol{p}_{app}=0 \tag{3.29}$$

Above equation can also be retrieved by considering the general Hu-Washizu variational form. Equation (3.29) should hold for arbitrary $\delta\boldsymbol{u}$, $\delta\boldsymbol{q}$ and $\delta\boldsymbol{e}$, thus the following three equations should be satisfied in order for the Hu-Washizu variational to be zero.

$$\boldsymbol{v}\equiv\int_0^L \boldsymbol{b}^T(x,L)\boldsymbol{e}(x)dx \tag{3.30a}$$

$$\mathbf{v} = \boldsymbol{a}\boldsymbol{u} \tag{3.30b}$$

$$\hat{\boldsymbol{s}}(\boldsymbol{e}(x)) \equiv \boldsymbol{b}(x, L)\boldsymbol{q} \tag{3.31}$$

Section deformations can be obtained as $\boldsymbol{e} = \boldsymbol{k}_s^{-1}\ \hat{\boldsymbol{s}}$ to get the section deformations from section forces with the usage of section stiffness matrix $\boldsymbol{k}_s$ for linear elastic material response. The change of section deformations **e** to Equation (3.30a) gives:

$$\mathbf{a}\boldsymbol{u} = \boldsymbol{v} = \boldsymbol{f}\ \boldsymbol{q}; \quad \text{where} \quad \mathbf{f} = \int_0^L \boldsymbol{b}^T(x, L)\boldsymbol{f}_s(x)\ \boldsymbol{b}(x, L)dx \tag{3.32}$$

In the equation above, **f** is the flexibility matrix of the element. The section flexibility matrix is $\mathbf{f}_s$ which is calculated by taking inversion of the section stiffness matrix $\mathbf{k}_{s.}$ Additional substitution of above equation presents;

$$\boldsymbol{k}\boldsymbol{u} = \boldsymbol{p}_{app}; \quad \text{where} \quad \mathbf{k} = \mathbf{a}^T\boldsymbol{f}^{-1}\boldsymbol{a} \tag{3.33}$$

where **k** is the 12×12 element stiffness matrix in the whole system.

3.2.4. Presence of Semi-rigid Connections in Element Formulation for 3D Case

At this point in the element formulation, presence of semi-rigid connections will be adapted through the following discretized version of above the calculation of basic element deformations:

$$\boldsymbol{v} = \boldsymbol{v}_{\text{Frame}} + \boldsymbol{v}_{\text{Con}}; \tag{3.34a}$$

$$\mathbf{v}_{\text{Frame}} = \int_L \boldsymbol{b}^T(x)\boldsymbol{e}(x)dx; \tag{3.34b}$$

$$\boldsymbol{v}_{\text{Con}} = \sum_{i=1}^{nSC} \boldsymbol{b}^T(x_i)\boldsymbol{\Delta}_{SC,i} \tag{3.34c}$$

$$\boldsymbol{\Delta}_{SC} = \begin{bmatrix} \delta_{SC}^{axial} & \theta_{Z,SC} & \theta_{Y,SC} & \delta_{Y,SC}^{shear} & \delta_{Z,SC}^{shear} & \varphi_{SC} \end{bmatrix}^T \tag{3.34d}$$

The first integral can be numerically calculated by using a quadrature rule to get inelastic behavior along the member *nIP* is the total number of sections used for the design of the nonlinear response of the element and $\boldsymbol{\Delta}_{SC}$ is the vector of deformations of semi-rigid connection with axial deformation δ_{SC}^{axial}, rotation $\theta_{Y,SC}$, $\theta_{Z,SC}$ and shear $\delta_{Y,SC}^{shear}$, $\delta_{Z,SC}^{shear}$ and torsion φ_{SC}.

Introducing a semi-rigid connection along the member in Figure 3.5 does not modify the force vector under small deformations, hence the above Equations (3.22) to (3.29) are not affected by this operation. Element flexibility matrix is similarly generated as follows:

$$\boldsymbol{f} = \boldsymbol{f}_{\text{Frame}} + \boldsymbol{f}_{\text{Con}}; \tag{3.35a}$$

$$\mathbf{f}_{\text{Frame}} = \int_L \boldsymbol{b}^T(x)\boldsymbol{f}_s(x)\boldsymbol{b}(x)dx; \tag{3.35b}$$

$$\mathbf{f}_{\text{Con}} = \sum_{i=1}^{nSC} \boldsymbol{b}^T(x_i)\boldsymbol{f}_{SC,i}\boldsymbol{b}(x_i) \tag{3.35c}$$

Equations (3.30) and (3.31) are linked to the element state determination. Generally, state deterioration of the element involves an iterative solution in the case of nonlinear behavior, where Equations (3.30) to (3.31) are required to be solved. This solution needs the generation of element flexibility matrix **f** under nonlinear response, where taking derivative of element deformations **v** wherein Equation (3.30) according to the element forces **q** which finalizes into the same flexibility integration expression given in Equation (3.32); however, the section stiffness will be nonlinear this time.

3.2.5. Section Response for 3D Case

Section response can be attained by the basic assumption. The plane sections before deformation keep on plane after deformation among the length of the beam by the usage of section compatibility matrix $\mathbf{a}_s$ given in Equation (3.23). The section compatibility matrix has the shear correction factor κ_s as follows;

$$\boldsymbol{a}_s = \boldsymbol{a}_s(y) = \begin{bmatrix} 1 & -y & z & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\kappa_{sy}} & 0 & -z \\ 0 & 0 & 0 & 0 & \sqrt{\kappa_{sz}} & y \end{bmatrix} \tag{3.36}$$

Shear correction factor κ_s can be obtained as the inverse of the form factor suggested by [48] for I-section about the major bending axis:

$$\kappa_s = 1/\kappa \tag{3.37a}$$

$$\kappa = 0.85 + 2.32\,\frac{b_f t_f}{d\ t_w} \tag{3.37b}$$

By utilizing the section compatibility matrix $\mathbf{a}_s$, section forces can be obtained by integration of the stresses. This satisfies the material constitutive relations $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\boldsymbol{\varepsilon})$, and the derivative of section forces **s** with the section deformations results wherein the section stiffness matrix $\mathbf{k}_s$

$$\boldsymbol{s} = \int_A \boldsymbol{a}_s{}^T \boldsymbol{\sigma}\ dA\,; \quad \text{where} \quad \boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} \\ \sigma_{xy} \\ \sigma_{xz} \end{pmatrix} \tag{3.38}$$

$$\boldsymbol{k}_s = \frac{\partial \boldsymbol{s}}{\partial \boldsymbol{e}} = \int_A \boldsymbol{a}_s{}^T \frac{\partial \boldsymbol{\sigma}(\boldsymbol{\varepsilon})}{\partial \boldsymbol{e}}\ dA = \int_A \boldsymbol{a}_s{}^T \boldsymbol{k}_m \boldsymbol{a}_s\ dA \tag{3.39}$$

where the material tangent modulus k_m is gained by the stress-strain relation by the use of $\mathbf{k}_m = \partial\boldsymbol{\sigma}(\boldsymbol{\varepsilon})/\partial\boldsymbol{\varepsilon}$. Numerical evaluation of the integrals in (3.38) and (3.39) can use Gauss-quadrature, the midpoint or the trapezoidal rule. While Gauss-quadrature gives improved results for smooth strain distributions and stress-strain relations, and the midpoint rule is desirable for strain distributions and stress-strain relations with discontinuous slopes. Hence, the section stiffness matrix inverse, which is the section flexibility matrix is utilized in generation of element flexibility matrix in Equation (3.35).

CHAPTER 4

NUMERICAL VERIFICATIONS

Verification studies for the proposed frame element with localized semi-rigid connections for steel structures are conducted in this chapter. For this purpose, both two dimensional (2D) and three dimensional (3D) examples under various loading and boundary conditions are considered in order to assess the accuracy of the frame element proposed in this thesis. Comparisons are conducted with respect to available examples and solutions in the literature, as well as example problems generated in this thesis. When necessary, available finite element programs used in practice and research are considered in order to provide comparison of the similarities and differences between various solution platforms, as well. First, vibration studies are considered in order to provide the accuracy of proposed element in capturing stiffness and mass distributions under linear elastic response. Then, nonlinear behavior of frame elements and structures with the presence of semi-rigid connections is studied with the use of proposed element. For both cases of validation, member level and structure level examples are presented.

4.1. Verification of Vibration Characteristics

4.1.1. Cantilever Beam Example

The first validation is done for the proposed approach with different commercially available programs. A cantilever I beam that is rigidly fixed at one end is modeled in ANSYS as shown in Figures 4.1 where a finite element analysis can be conducted for a detailed analysis. This model in ANSYS is chosen as the control (benchmark) model for the proposed frame element in the absence of connections, i.e. representing the rigid connection case. The closed form solution for a cantilever beam with solid circular section is compared in [66] and the results showed very good match which can be understood as the proposed frame finite element formulation presents a good accuracy when compared with the closed form solutions. The cantilever beam geometry is selected as IPE270 section with varying length to depth (L/d) ratios (cross-section properties of this section are later shown in Figure 4.2). The radius fillet of IPE270 section is not applied to the model. For analysis, L/d aspect ratio of the beam is varied as 10, 5 and 2 for the study of long to short beam cases. Elasticity modulus, Poisson's ratio and density of the beam are taken as 200 GPa, 0.3 and 7832 kg/m^3, respectively. ANSYS finite element model is created with the use of 3D Solid187 element discretized with fine mesh in order to get numerically converged exact solution with the use of that program.

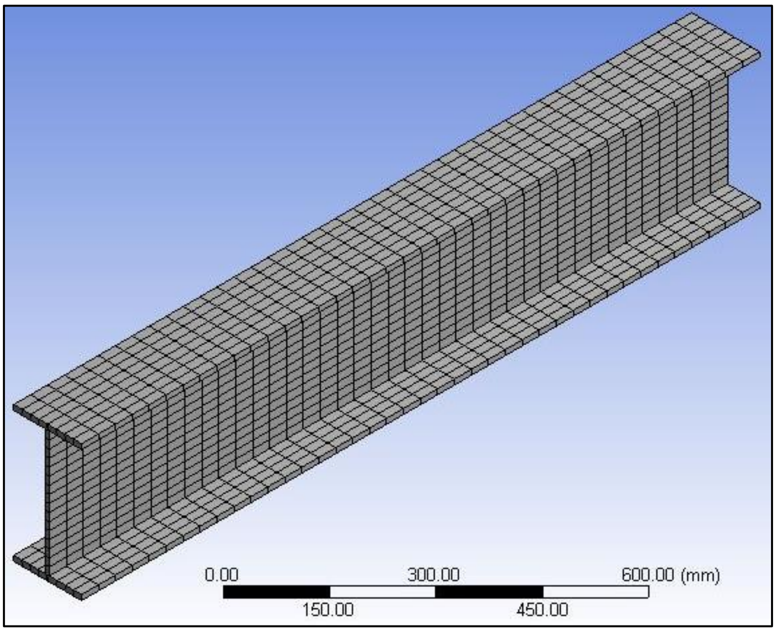

Figure 4.1. Representative 3D I Beam Generated in ANSYS

Fundamental vibration frequencies obtained from ANSYS simulations are presented in Table 4.1.

Table 4. 1 *ANSYS Results for Cantilever I-Beam with Rigid Connection*

L/d	1st Bending (Hz)	2nd Bending (Hz)	1st Axial (Hz)
10	42.154	227.13	468.35
5	155.66	649.25	937.13
2	671.28	NA	2358.7

The results obtained from ANSYS and with the use of proposed element will also be compared with the popularly used commercial structural analysis program SAP2000. The model in SAP2000 is prepared by the use of linear elastic frame elements, as well.

In Table 4.2, results obtained with the use of proposed frame element and SAP2000 solutions are presented. The analyses with the use of proposed element and SAP2000 are carried out with the use of 4 and 32 elements

The accuracy of SAP2000 elements is greatly influenced by the use of lumped mass matrix, while the proposed element uses distributed mass matrix calculated from force-based approach. In order to capture 2nd bending mode accurately, at least 4 elements should be used with proposed approach, and 32 elements solution gives perfect match with ANSYS. Accuracy of the proposed approach is observed to be due to the use of force-based stiffness and mass matrix calculations, as well as due to the use of an accurate shear correction coefficient proposed by [48].

With respect to the short beam case, the kinematics of deformation observed in ANSYS model was not exactly replicated with the use of Timoshenko beam theory assumptions, the error in vibration mode calculations are increased to about 2-3% in

1st bending mode with the use of 32 elements with proposed approach, and this is accepted as a reasonable error.

Table 4. 2 *SAP2000 and Proposed Model Results for Cantilever I-Beam with Rigid Connection*

L/d	Mode Type	SAP 2000				Proposed Model			
		N_{el} =4	Err. %	N_{el} =32	Err. %	N_{el} = 4	Err. %	N_{el} = 32	Err. %
10	1st Bending (Hz)	41.31	2.0	42.42	0.6	42.10	0.1	42.08	0.2
	2nd Bending (Hz)	213.86	5.8	232.29	2.3	229.21	0.9	226.95	0.1
	1st Axial (Hz)	464.90	0.7	467.95	0.1	470.91	0.5	467.95	0.1
5	1st Bending (Hz)	154.23	0.9	157.83	1.4	155.51	0.1	155.32	0.2
	2nd Bending (Hz)	643.50	0.9	692.52	6.7	671.87	3.5	659.15	1.5
	1st Axial (Hz)	930.23	0.7	935.45	0.2	941.83	0.5	935.90	0.1
2	1st Bending (Hz)	692.52	3.2	702.25	4.6	686.56	2.3	684.10	1.9
	2nd Bending (Hz)	2136.75	NA	2277.90	NA	2080.12	NA	2030.62	NA
	1st Axial (Hz)	2325.58	1.4	2341.92	0.7	2354.57	0.2	2339.74	0.8

After studying the cantilever beam with IPE270 section, different steel sections as shown in Figure 4.2 are going to be investigated. The considered sections will provide different vibration characteristics to the cantilever beam as a result of the variation of their depth to width as well as flange and web thicknesses.

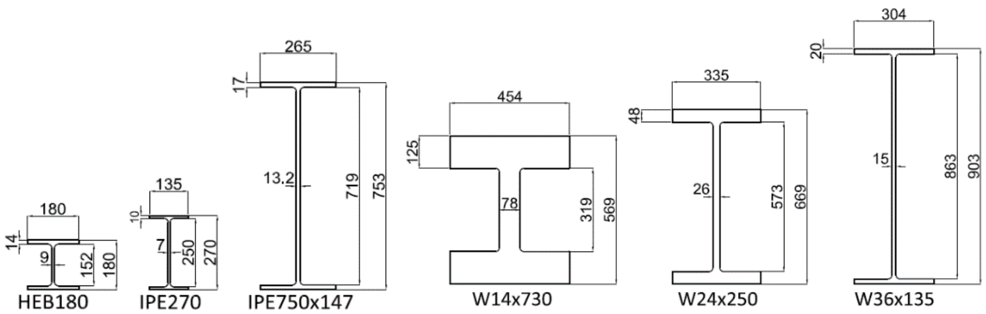

Figure 4.2. Sections used in Validation Analysis (dimensions in mm)

The considered beam sections are divided into two groups. The first group of I-sections consists of European sections: HEB180, IPE270 and IPE750×147 (Figure 4.2). The second group of I-sections considered in this study was also considered in the literature [37], and these are W36×135, W24×250 and W14×730 (Figure 4.2). The accuracy of the mass and stiffness matrices resulting from the proposed element formulation are verified for these I-sections. Length to depth (L/d) ratio of the beam is chosen again as 10, 5 or 2 in order to study both the long beam and short beam cases. Elasticity modulus, Poisson's ratio and density are taken as 200 GPa, 0.3 and 7832 kg/m^3, respectively.

The proposed model is first tested with the 3D model created in ANSYS environment. ANSYS model is accepted as the governing or control model for this comparison study. It is important to take into consideration some modelling approximations that have a crucial effect on the finite element analysis, as the element type, meshing elements, boundary conditions. In order to eliminate the influence of vibrations due to local flange and web distortions that could take place in some of the sections considered in Figure 4.2, the web of I-beam in ANSYS model is stiffened with plates in Figure 4.3 that have negligible mass and stiffness, and as a result one to one comparison with the proposed frame element became possible. It is important to recall that the kinematics of Timoshenko beam theory considered in the proposed frame element formulation does not allow any flange and web distortion to take place. For this purpose, the geometry of the wide flange beams in ANSYS is stiffened with 1 mm thick stiffeners, with 1×10^{-10} kg/m^3 density. These are supplemented along the length of the elements to restraint the flanges and decrease their distortion behavior; hence, assuming such a flange restraining method converges to a realistic behavior of wide-flange beams in structures. After employing the geometry, 3D solid mesh is produced using the SOLID187 element in ANSYS as shown in below figure, where this element provides 10 nodes and quadratic displacement behavior to the elements with improved strain formulation.

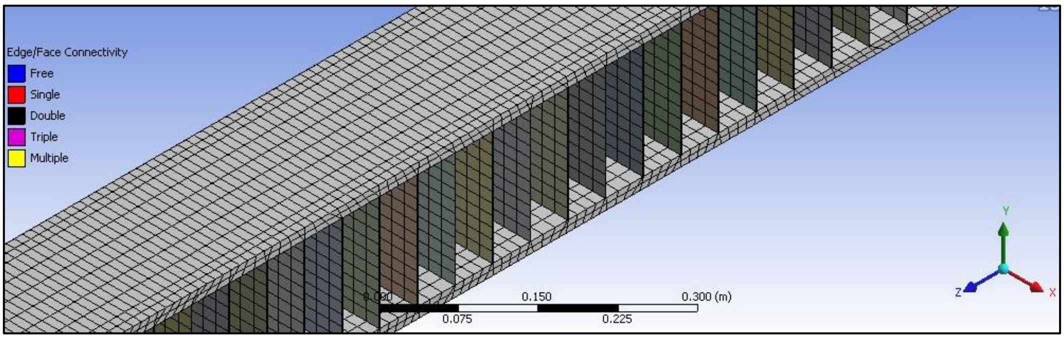

Figure 4.3. Representative 3D I Beam Generated in ANSYS with Stiffeners

The ratios of the natural frequencies obtained from proposed model to ANSYS model are presented in Figures 4.4 and 4.5. The ANSYS FEM model is taken as a control model for the steel sections presented in Figure 4.2. Therefore, this ratio will denote the error between the benchmark (control) model and the proposed model. The results of the cantilever beams are denoted for its first bending, second bending and axial modes of beams for different length to depth ratios. For the two groups of cross-sections, compact and non-compact sections present significantly smaller error, but, for the deeper sections, approximately 10% error is retrieved for the short beam aspect ratios.

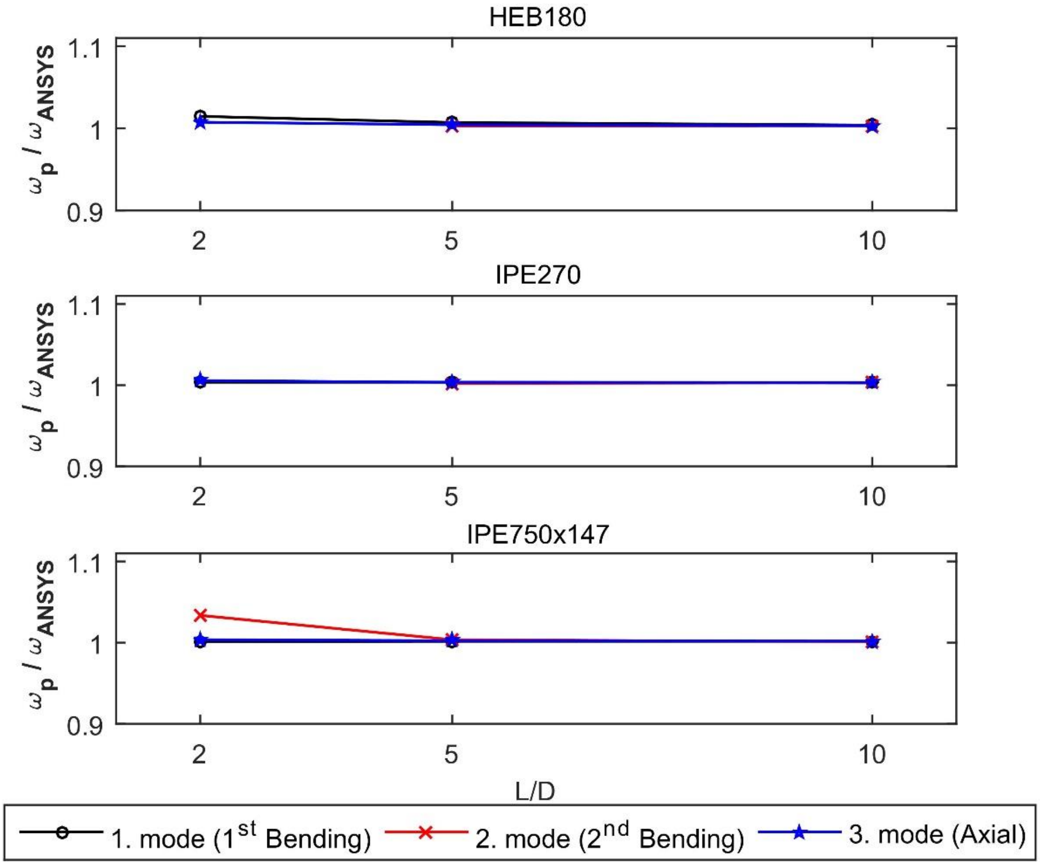

Figure 4.4. Proposed Model over ANSYS Natural Frequencies for European Sections

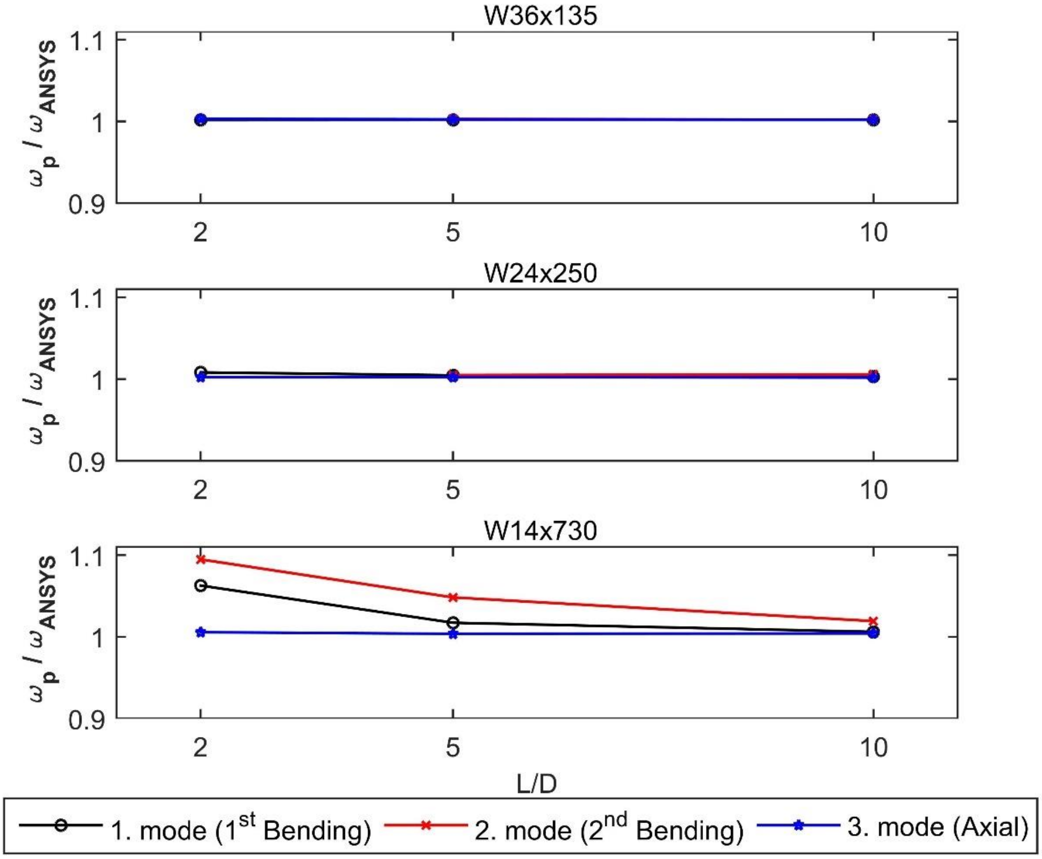

Figure 4.5. Proposed Model over ANSYS Natural Frequencies for American Sections

From the comparisons with ANSYS results for the rigid connection case, it is observed that the proposed frame element model overall provides better match than SAP2000 to the results obtained in ANSYS. In the next stage, the case of semi-rigid connections will be studied through the use of proposed frame element and SAP2000 program as a means for comparison of the differences between the two solution approaches in capturing the vibration characteristics of flexibly connected members. For this purpose, IPE270 section is considered for the analysis of the cantilever beam with flexible connection at its base. SAP2000 program is chosen for the simplicity of defining flexible connections with acceptable accuracy which is mentioned earlier in cantilever beam examples with rigid connections.

The base of the cantilever beam is introduced with a flexible connection with flexibility provided with λEI/L formula. E, I and L are the elasticity modulus, the moment of inertia of cross section and length of the beam, respectively. The connection stiffness ratio for the semi-rigid case is varied as $\lambda = 2$, 11 and 20, where λ is the ratio of connection stiffness to flexural rigidity EI/L of the beam, where $\lambda = 2$ represents a behavior that is closer to pinned connection, and $\lambda = 20$ represents a behavior that is approach rigid connection.

Analyses results obtained with the use of SAP2000 solutions are presented in Table 4.3 with the use of 4 and 32 elements, and the results obtained with the proposed frame element analysis are presented with the use of 1 and 4 elements in Table 4.4. It is evident that the introduction of semi-rigid connection in the proposed frame element highly changes its accuracy with the use of 1 element solution for it, and more element discretization is now needed even to capture 1^{st} bending mode accurately. Introduction of localized connection results in a discontinuity in element's response through single element discretization for the capture of vibration modes. The results of the proposed element analysis with 4 elements is observed to be very close to the results of SAP2000 solution with 32 elements, which clearly demonstrates the superiority of the proposed element.

Table 4. 3 *SAP2000 Results for Cantilever I-Beam with Semi-Rigid Connection*

	L/d	10		
N_{el}	Mode Type	λ=2	λ=11	λ=20
4	B1(Hz)	24.29	35.68	37.92
	B2(Hz)	178.86	197.90	203.54
	N(Hz)	444.44	459.14	464.25
32	B1(Hz)	24.75	36.52	38.86
	B2(Hz)	192.75	214.04	220.46
	N(Hz)	467.95	467.95	467.95
	L/d	5		
N_{el}	Mode Type	λ=2	λ=11	λ=20
4	B1(Hz)	94.66	135.34	142.94
	B2(Hz)	596.66	624.61	631.71
	N(Hz)	930.23	930.23	930.23
32	B1(Hz)	96.39	138.22	146.11
	B2(Hz)	640.21	671.14	678.89
	N(Hz)	935.45	935.45	935.45
	L/d	2		
N_{el}	Mode Type	λ=2	λ=11	λ=20
4	B1(Hz)	508.13	644.33	664.89
	B2(Hz)	2136.75	2136.75	2136.75
	N(Hz)	2325.58	2325.58	2325.58
32	B1(Hz)	515.20	653.17	673.86
	B2(Hz)	2277.90	2277.90	2277.90
	N(Hz)	2341.92	2341.92	2341.92
B1: 1^{st} Bending, B2: 2^{nd} Bending, N: 1^{st} Axial				

Table 4. 4 *Proposed Model Results for Cantilever I-Beam with Semi-Rigid Connection*

	L/d	10		
N_{el}	Mode Type	λ=2	λ=11	λ=20
1	B1(Hz)	41.43	42.05	42.19
	B2(Hz)	352.54	373.32	378.90
	N(Hz)	515.94	515.94	515.94
4	B1(Hz)	25.10	36.54	38.76
	B2(Hz)	199.00	215.61	220.42
	N(Hz)	470.91	470.91	470.91
	L/d	5		
N_{el}	Mode Type	λ=2	λ=11	λ=20
1	B1(Hz)	153.81	156.51	157.09
	B2(Hz)	1031.87	1031.87	1031.87
	N(Hz)	1149.86	1182.72	1190.30
4	B1(Hz)	97.03	137.22	144.62
	B2(Hz)	637.96	658.35	663.43
	N(Hz)	941.83	941.83	941.83
	L/d	2		
N_{el}	Mode Type	λ=2	λ=11	λ=20
1	B1(Hz)	699.33	712.83	715.15
	B2(Hz)	2579.68	2579.68	2579.68
	N(Hz)	3137.40	3123.86	3121.53
4	B1(Hz)	505.54	639.38	659.39
	B2(Hz)	2062.07	2074.72	2076.94
	N(Hz)	2354.57	2354.57	2354.57
B1: 1st Bending, B2: 2nd Bending, N: 1st Axial				

In the following graphs, proposed model and SAP2000 model results are presented with the increased number of mesh sizes for the rigid and semi-rigid connection cases with flexibility ratio varied as 2, 11 and 20. The relative error of SAP2000 results with respect to the proposed model results are plotted for aspect ratio L/d=2 in Figure 4.6, L/d=5 in Figure 4.7, and L/d=10 in Figure 4.8. The element numbers are chosen as 2, 4, 8, 16 and 32 in both analyses.

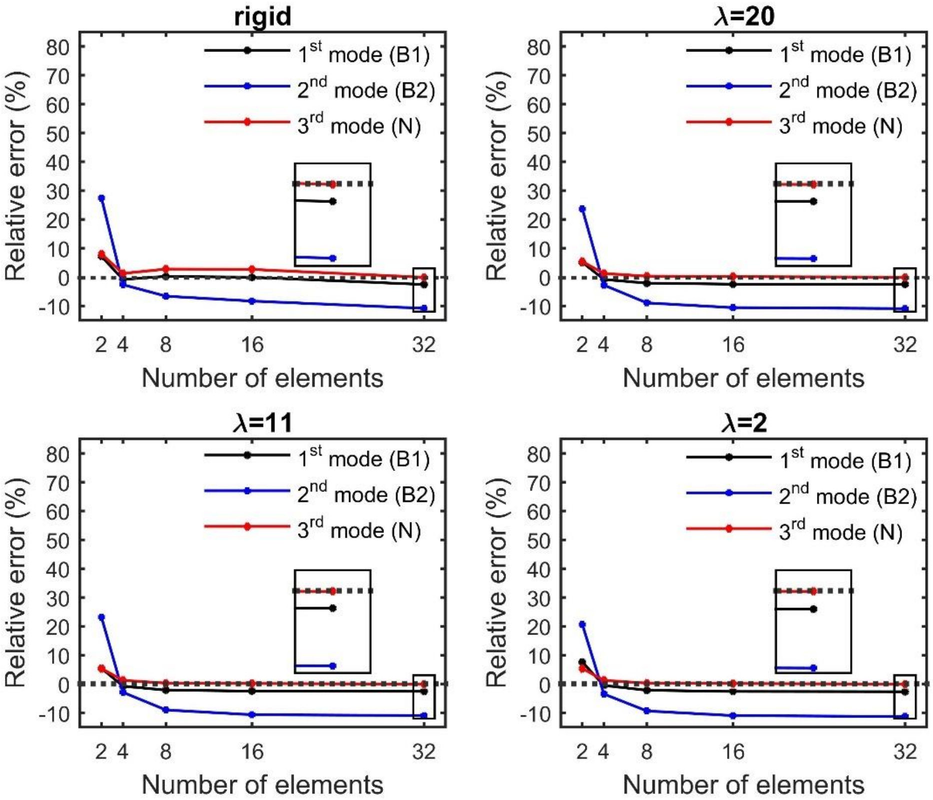

Figure 4.6. Relative Error (%) vs. Number of Elements for Rigid and Semi-rigid Connections, L/d=2

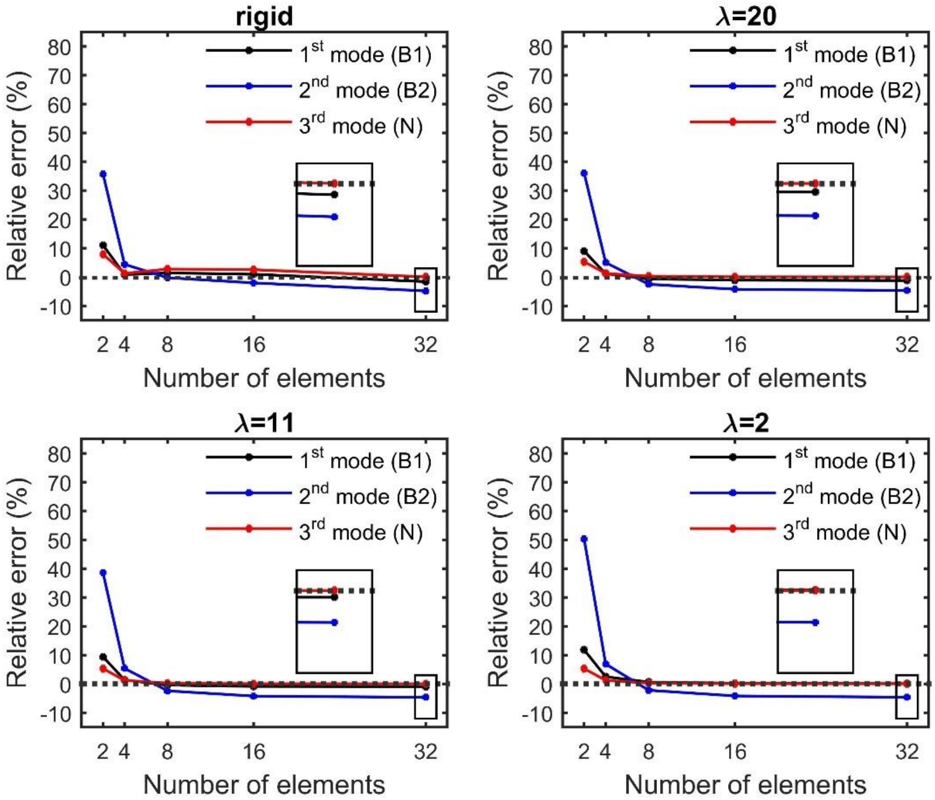

Figure 4.7. Relative Error (%) vs. Number of Elements for Rigid and Semi-rigid Connections, L/d=5

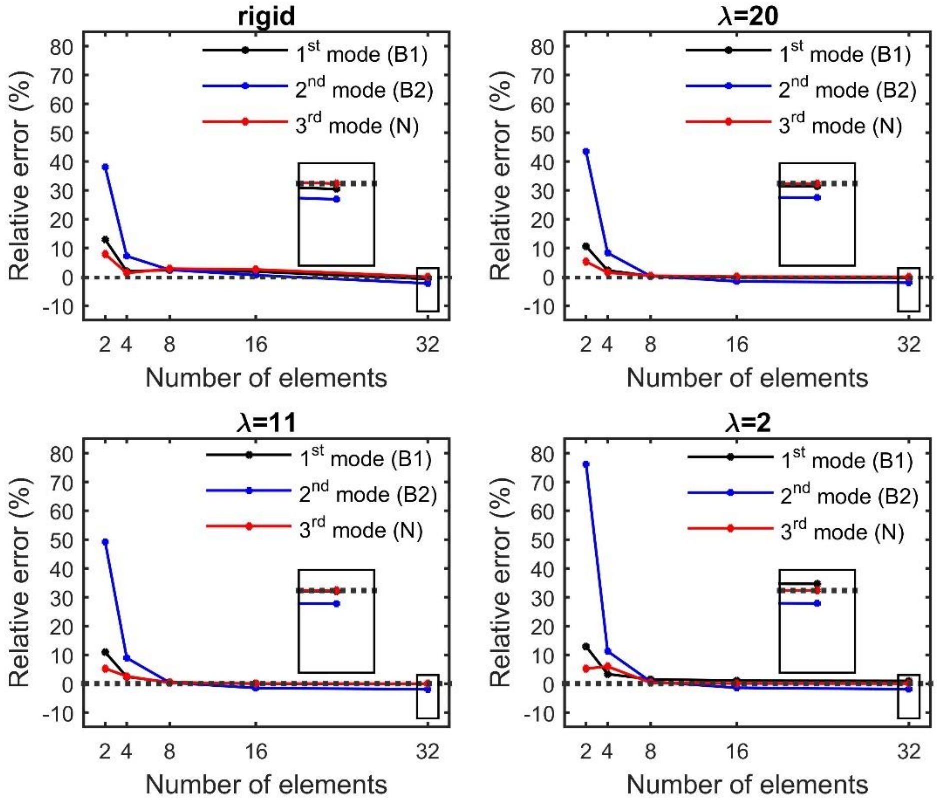

Figure 4.8. Relative Error (%) vs. Number of Elements for Rigid and Semi-rigid Connections, L/d=10

It is observed that the first bending and axial vibration frequencies obtained in both models are very close to each other for fine mesh cases, but SAP2000 results demonstrated higher error compared to the proposed model for the 2nd bending mode for the short beam case L/d=2 and intermediate beam case L/d=5. However, the plots clearly demonstrate that as the length of the beam increases accuracy of SAP2000 for 2nd bending mode also improves. It is suspected that the shear area considered in SAP2000 by default is the reason for the error, while the proposed model uses a correct shear are for this section. Overall, it is concluded that SAP2000 requires at least 4 to 8 elements to provide a reasonable estimation of even the 1st bending mode, and users of SAP2000 should consider finer mesh discretization for dynamic analysis of small structural systems that have few structural members.

4.1.2. Two-Dimensional Portal Frame Example

The second verification example is a portal frame taken from the study by [37]. The semi-rigid connections are presented at both ends of the beam in this structure. British UB254x146x37 section is placed for the beam and UC203x203x60 sections are placed for the columns. Modulus of elasticity is taken as 200GPa. The length of the beam is 2.9 m and the height of the columns is 3.0 m (see Figure 4.9).

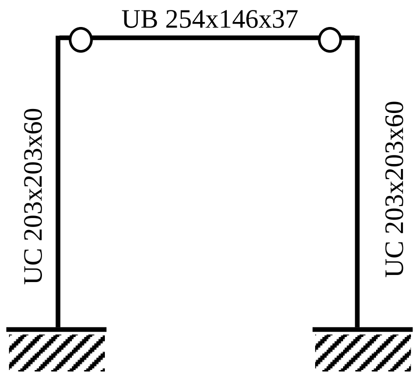

Figure 4.9. Sketch of the Portal Frame [37]

Above portal frame is analysed with proposed model as well as by the use of SAP2000 program for joint stiffness ratio λ ranging from pinned to rigid case, i.e. joint stiffness is equal to λ times EI/L of beam. The joint stiffness is defined by partial fixity at the ends of the frame element in SAP2000 model which introduces additional nodes and degrees of freedom to the system. The element numbers for the SAP2000 model are varied as 1, 2, 4, 8 and 32 per member, and the proposed model uses only 1 element per member, where the results are presented in Figure 4.10. It is evident that 1 and 2 element solutions in SAP2000 gives highly inaccurate solutions for all joint stiffness ratios, and at least 4 elements per member are needed in SAP2000 in order to get the accuracy reached by the use of single element with proposed frame element model. By the way, fundamental vibration mode shapes obtained from the proposed model are presented in Figure 4.11 for representative purposes.

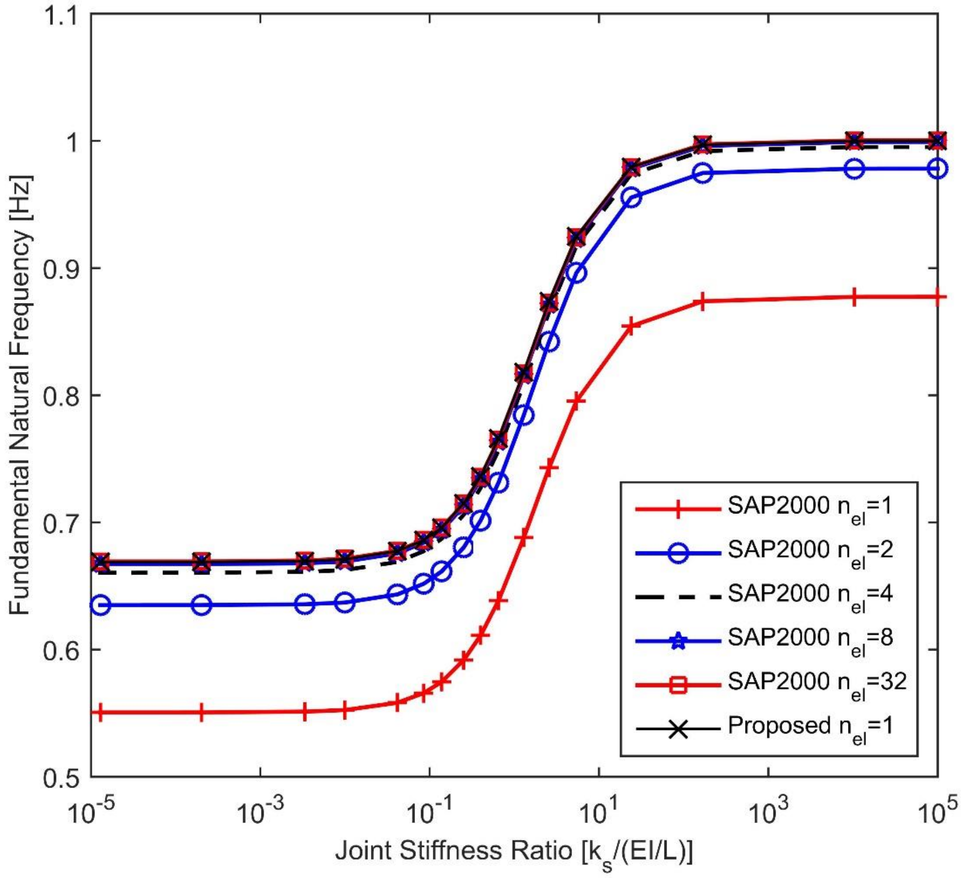

Figure 4.10. Fundamental Natural Frequency vs. Joint Stiffness Ratio for Various Sizes of SAP2000 Model Mesh

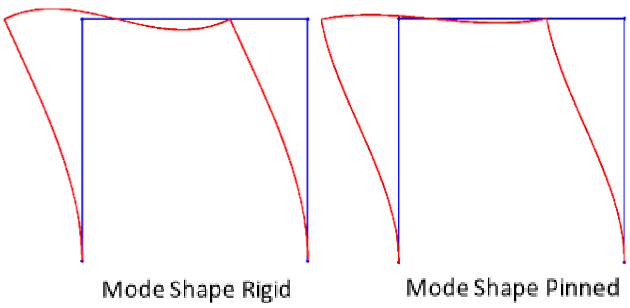

Figure 4.11. Mode Shapes of the Portal Frame for Rigid and Pinned Connections

4.1.3. Three Bays Six Stories Planar Frame Example with Beam to Column Connections

This example is a 3 bays and 6 stories steel frame composed of European sections HEB260 for columns and IPE300 for beams which is proposed in the study by [37]. HEB260 section has 260 mm height and width, 7.5 mm flange and 10 mm web thicknesses; IPE300 section has 300 mm height, 150 mm width, and 10.7 mm flange and 7.1 mm web thicknesses. The beams are 6.0 m in length and the columns are 3.75 m in height.

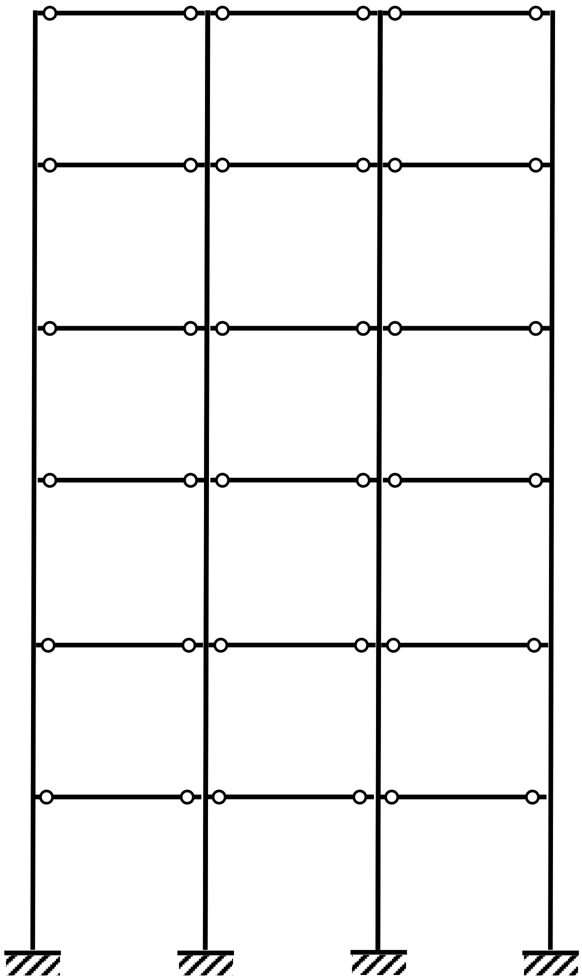

Figure 4.12. 3 Bays 6 Stories Frame with Beam to Column Semi-Rigid Connections

The proposed model results are compared with the results of SAP2000 analysis and the results provided in the study by [37] that provides an approximate closed-form solution as well as analyses undertaken in ABAQUS. The ABAQUS solutions were undertaken by the use of beam finite elements and the connection regions were modeled through the use of spring elements with the introduction extra nodes and degrees of freedom in [37]. Furthermore, SAP2000 also requires the introduction of extra nodes in order to introduce spring elements to represent connection region. On the other hand, the proposed element does not require such a modeling effort, and furthermore the matrix sizes are not increased due to such requirement.

In this example, semi-rigid connections are present at both ends of each beam, where connection stiffness is varied from pinned to rigid cases. Results from different analysis cases are provided in Figure 4.13 for the fundamental vibration frequency. Comparison of the results show that all three numerical simulations (proposed, SAP2000 and ABAQUS) clearly provide very close match, where the proposed model provides exact solution through the use of 1 element per member, while SAP2000 requires at least 4 elements per member. Furthermore, approximate closed-form solution by [37] overestimates the fundamental frequencies especially for the rigid connection case. The impact of using 1 element per member in capturing the vibration characteristics of the given large structural systems clearly demonstrate reduced effort in modeling and analysis through smaller mass and stiffness matrix assemblies, where this will especially provide much more robustness in time history analysis. The proposed model uses in total 42 elements, while 204 elements are used in SAP2000 (details of ABAQUS analysis are not present in [37]).

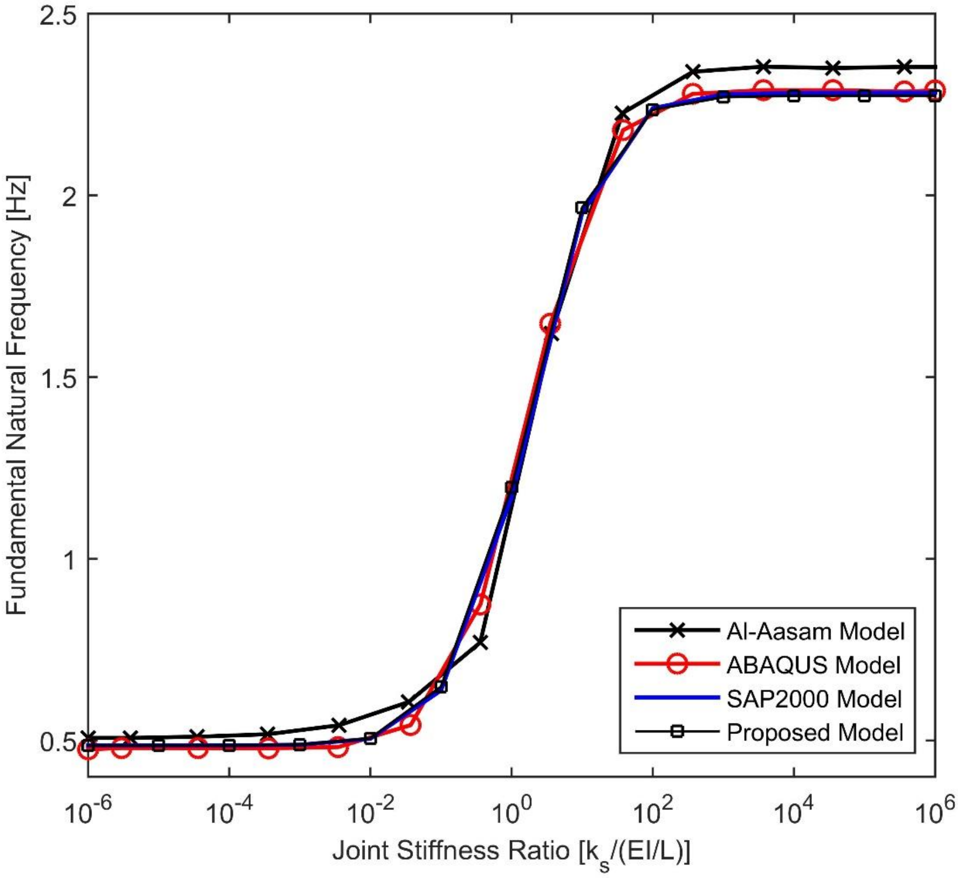

Figure 4.13. Fundamental Natural Frequency vs. Joint Stiffness Ratio for 3 Bays 6 Stories Steel Framed Structure

For mid-rise to high-rise structures, it is known that the higher modes of vibration play a significant role on the dynamic forces acting on the structures. Therefore, representation of higher modes of vibration in numerical analysis is critical in correctly estimating the real performance of structures. higher structures, considering high order vibration modes becomes vital. For this purpose, 2nd and 3rd vibration modes of the given structure in Figure 4.13 are studied, where [37] also provided ABAQUS solutions in their study. Following the same discretization considered for proposed element with 1 element per span and for SAP2000 with 4 elements per span, the results of three simulations are now presented in Figures 4.15 and 4.16 for 2nd and 3rd fundamental vibration frequencies for the structures, respectively.

The results clearly demonstrate perfect match between the proposed model and SAP2000 model, while ABAQUS solution slightly overestimates the frequencies for the 2nd mode. For the 3rd mode, ABAQUS solutions give more than 5% error compared to proposed model, where we suspect that this could be due to discretization mistake in [37], and this clearly demonstrate the numerical implications of using less accurate elements in modeling the behavior of structural systems.

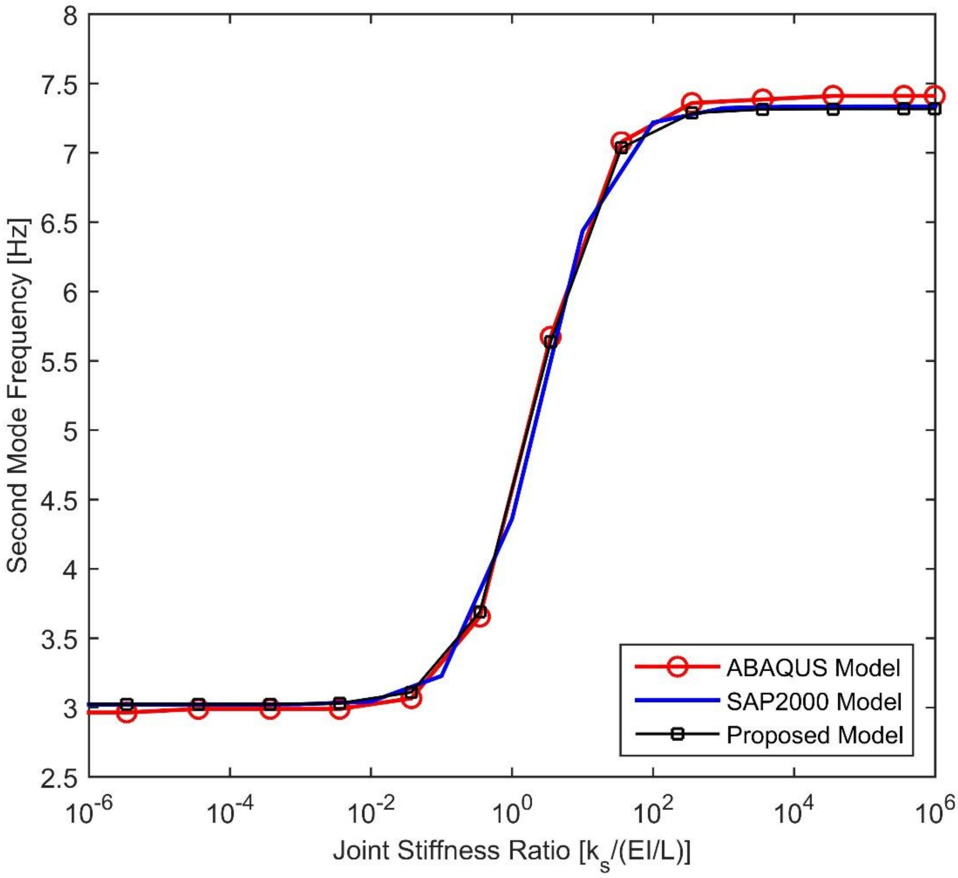

Figure 4.14. Second Mode Frequency vs. Joint Stiffness Ratio for 3 Bays 6 Stories Steel Framed Structure

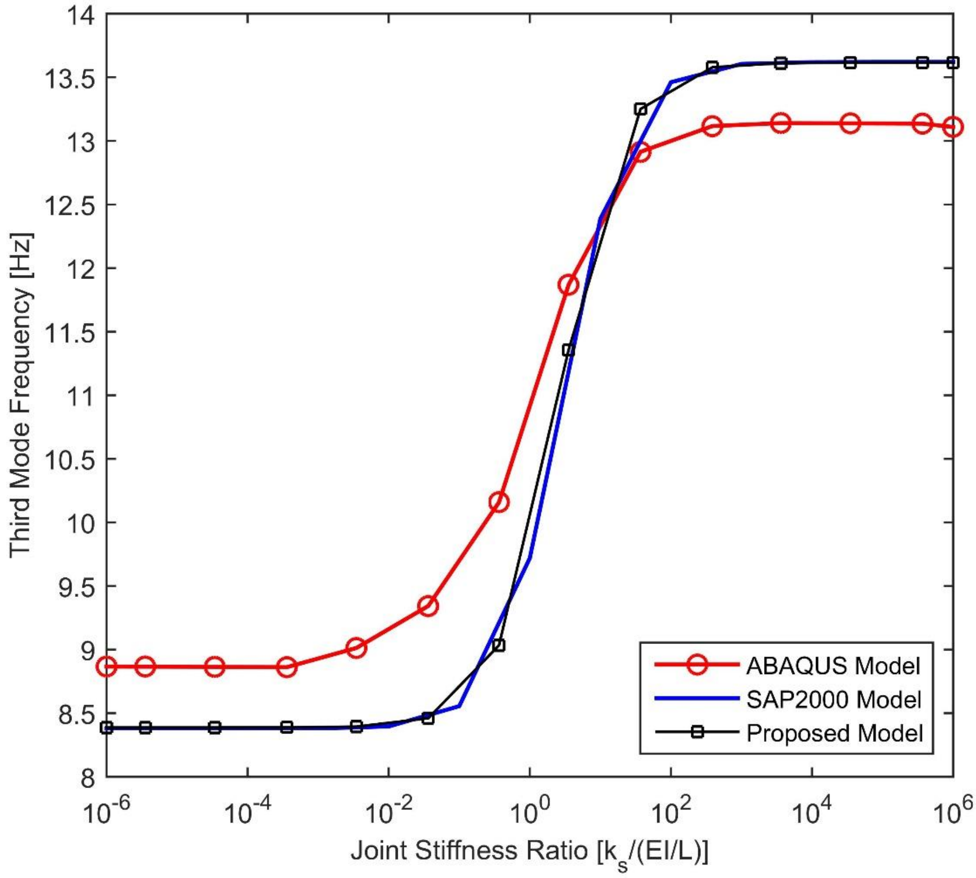

Figure 4.15. Third Mode Frequency vs. Joint Stiffness Ratio for 3 Bays 6 Stories Steel Framed Structure

For the purposes of representation, vibration shapes for 1st, 2nd and 3rd modes acquired with the proposed model are presented in Figure 4.16 for the cases ideally approaching rigid and pinned connections in the simulations.

The verification analysis on this structure evidently displays the necessity to give distinct attention for the use of structural analysis programs for the calculation of vibration characteristics of structures and the evaluation of seismic loads represented on these structures. Incorrect definition of shear effects, neglecting rotary inertias by using a lumped mass matrix or not knowing the accuracy of the used structural analysis program in general could result in significant mistakes in the analysis and design stages of structural systems.

Finally, it is important to mention that the presence of superimposed story masses in buildings should be taken into account in analysis, as well. with the proposed frame element. It is known that the occurrence of story superimposed dead loads in the analysis of buildings decreases the necessity for the use of the consistent mass matrix in capturing the fundamental modes of vibration. However, there is still the necessity to capture vibration characteristics of many structural systems with high precision. This can only be done by representing shear stiffness and considering rotary inertia effects through the use of accurate frame element models. The proposed frame element model with semi-rigid connections is demonstrated to provide better results and vigorous solutions for steel structures compared to available programs and techniques in the literature.

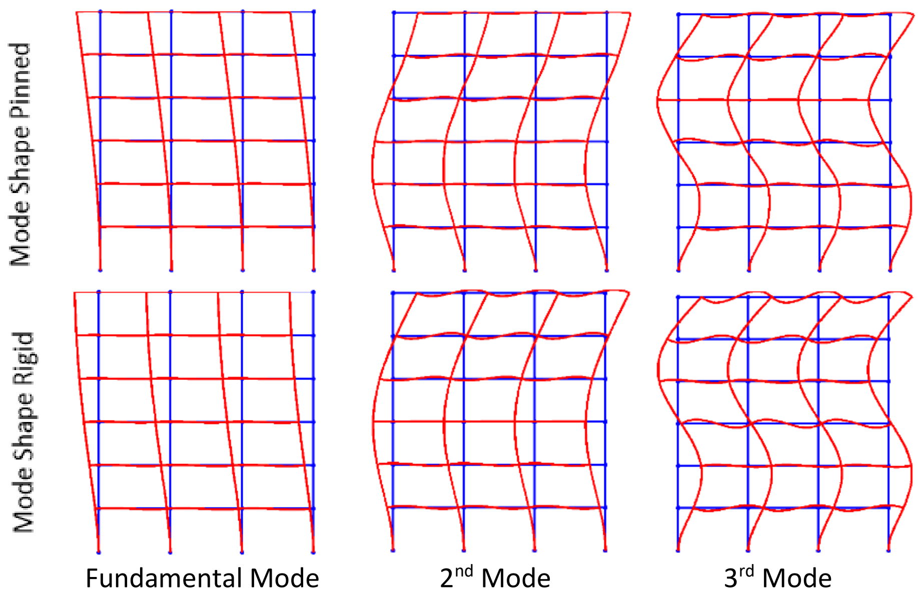

Figure 4.16. Mode Shapes of the 3 Bays 6 Stories Structure

4.1.4. Three Bays Six Stories Frame Example with Column Base Connection

The previous example, the 3 bay and 6 stories steel frame, is introduced with column baseplate connection as well as beam-column connection which is presented in Figure 4.17. Again, the beams are 6.0 m in length and the columns are 3.75 m in height. The columns and beams of the structure contain HEB260 beams and IPE300 columns. Semi-rigid connections are assigned at both ends of each beam, and column bases.

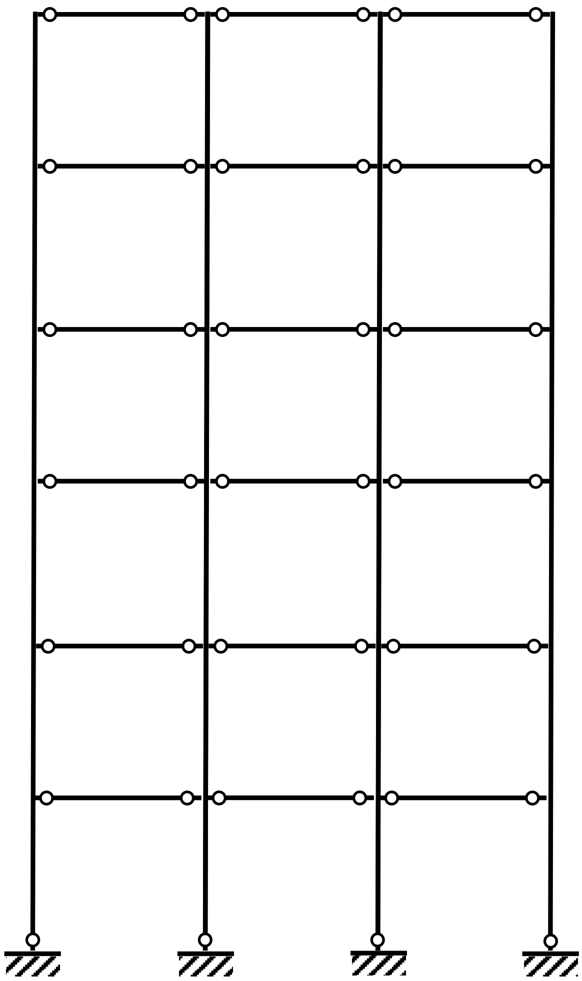

Figure 4.17. 3 Bay 6 Story Frame with Column Base Semi-Rigid Connections

The presence of the column base connection affects the vibration characteristic of structure. In the Figure 4.18, the change of fundamental vibration frequency is plotted against varying beam to column and column base joint stiffness ratios. The outcome of the semi-rigid connection at both beam-column joints and column bases show an important variation on the dynamic behavior of the steel structure that should be correctly represented carefully. The bold blue region on the 3D graph shows both beam-column and column base connections tend to behave as pinned connection and the bold red region at the top of the plot denotes connection behaviors that are close to fixed connection. In between these results are denoted as semi-rigid behavior of connection on both beams ends and column bases. For representative purposes, vibration shapes obtained from the proposed model are plotted in Figure 4.19 for rigid beam to column connections with rigid and pinned column base connections.

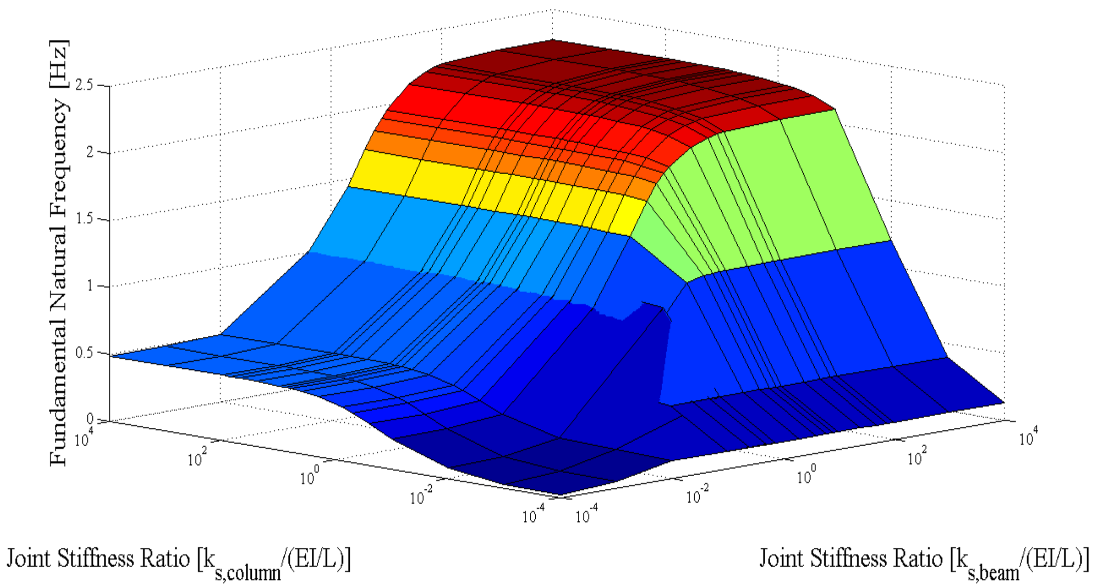

Figure 4.18. Fundamental Natural Frequency vs. Joint Stiffness Ratio of Beam-column and Column Base Connection Interaction for 3 Bays 6 Stories Steel Framed Structure

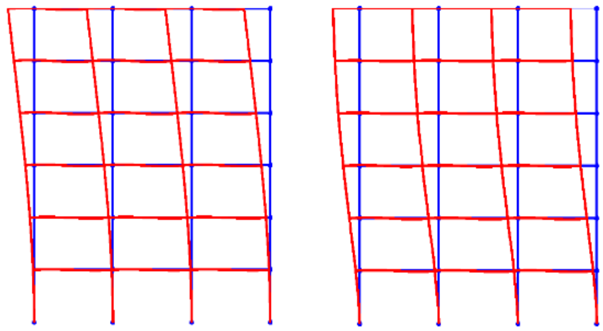

Figure 4.19. Mode Shapes of the 3 Bays 6 Stories Structure with Column Base Flexibility only

4.1.5. Two-dimensional Portal Frame Example with Brace Member

This example presents the importance of gusset plate axial flexibility on the vibration characteristic on a structure. The portal frame with brace that is presented in Figure 4.20 is considered for this purpose, and braces are introduced as shown in Figure 4.20. The height of the portal frame is 3 m and the span is 2.9 m with UC203X203X60 columns, UB254X146X37 beam and RHS70*5 brace. This example is utilized for the verification of proposed element's ability to present axial stiffness in an inclined component's response as brace members. Semi-rigid connection does not necessitate to introduce new nodes and new degrees of freedom to the structure even for the case of axial deformation effects that will represent the gusset plate for the current example.

Gusset plates are considered at the ends of the brace and the flexibility of these gusset plates are designated with a ratio, λ that is gained by axial rigidity of the brace member divided by the length, which is EA/L. The range of the λ value is plotted in Figure 4.20 with corresponding normalized fundamental natural frequency of the braced portal frame. This study is also analyzed with SAP2000 and compared with proposed model which presents a very good match between proposed model and SAP2000 model in Figure 4.20.

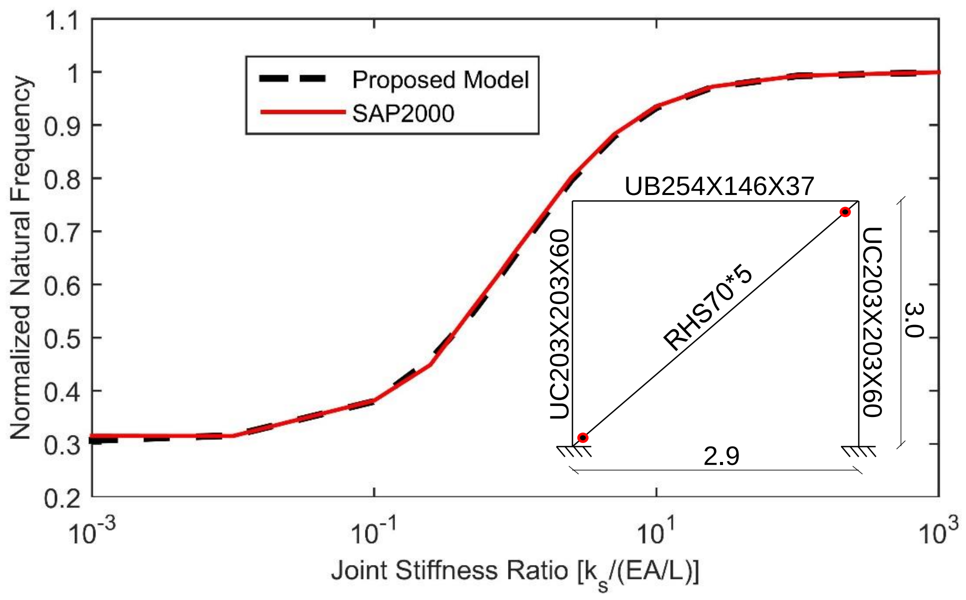

Figure 4.20. Normalized Natural Frequency vs. Joint Stiffness Ratio for Braced Portal Frame

4.2. Verification of Nonlinear Analysis

4.2.1. Three-dimensional Portal Frame Example

A portal frame model in three dimensions as shown in Figure 4.21 is generated with OpenSees software and by using the proposed frame element model. OpenSees software is a good tool to generate three dimensional examples with its frame finite element formulation. Model height is taken as 3 m and bays in each direction are taken as 6 m. Each node on the elements has six degrees of freedom. All beam and column elements have a linear geometry transformation. That means second order P-Delta effects are ignored. For all columns HEB180 steel section is considered, and for all beams IPE240 steel section is considered. The structure is first considered to be all connected rigidly to each other and to the ground, where this case is named as rigid case. Alternative case is created by considering beam to column connections to be semi-rigid while column bases are fixed to the ground rigidly, and this case is named as the semi-rigid case. Elasticity modulus, Poisson's ratio, yield stress and density are taken as 210 GPa, 0.3, 275 MPa and 7832 kg/m3, respectively. In this model, 5 monitoring sections are used for beams and columns with Gauss integration. Cross-sections of the beams are divided into 5 layers on flanges and 10 layers on web and the thickness of the flanges and web are divided into 2 layers and cross-sections of the columns are divided into 10 layers on flanges and web and the thickness of the flanges and web are divided into 2 layers.

For semi-rigid case, connection stiffness ratio is chosen as λ=2, 11 and 20, where λ is the ratio of connection stiffness over flexural rigidity EI/L of the connecting beam. Bilinear material of steel with a strain-hardening ratio of 1×10^{-6} is utilized to define both beams and columns and they are generated with nonlinear force beam-column element in OpenSees. A lateral displacement of 200 mm is imposed such that the columns bend about their strong axes.

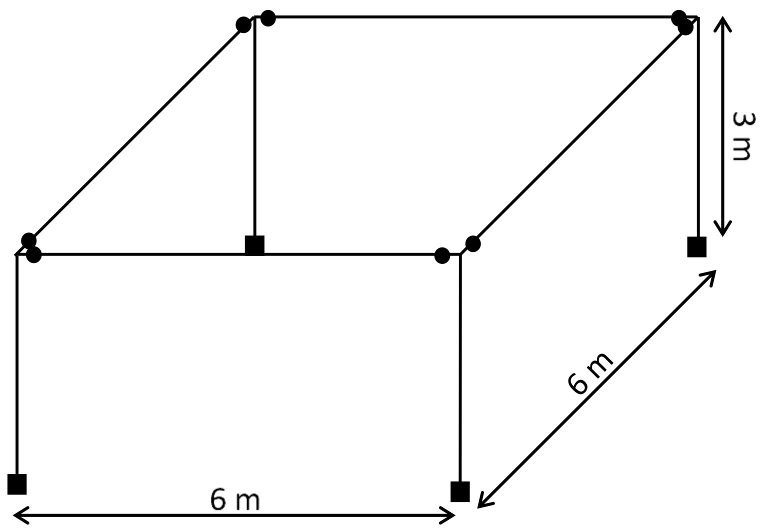

Figure 4.21. 3D Portal Frame Model

In order to model a semi-rigid beam-column connection in the OpenSees model, the node which joins beam to the column is duplicated and a zero-length rotational spring is added between the additional node and joint, and furthermore constraint conditions are imposed for the rest of the degrees of freedom between the two nodes. While such a modeling effort is necessary in OpenSees, the proposed frame element model does not require the introduction of extra nodes and degrees of freedom and the specification of constraint conditions in order to introduce the localized connection response to the structural system.

In order to verify the linear elastic and nonlinear behavior of proposed frame element model with OpenSees, connection response is taken as first linear elastic and the results are compared in Figure 4.22. In the second case, connection responses are considered as bilinear with hardening slope of 1×10^{-6} of initial elastic slope. The yield moment of the connections are taken as $M_{y,c}=\beta M_{p,beam}$ with $\beta=0.5$, where $M_{p,beam}$ is the plastic moment capacity of the beam calculated from the multiplication of yield stress σ_y of steel with the plastic modulus Z for IPE240 section, and the load-displacement plots are presented in Figure 4.23 for both the proposed model simulation and OpenSees. For all simulations, the results show perfect match for all cases that are considered. It is demonstrated that without the need to define extra nodes, degrees of freedom and the use of zero-length rotational springs, the proposed model is able to perfectly capture the nonlinear spread of plasticity along element length, section depth, and furthermore also capture the presence of nonlinearity localizing at the connection region. In Figure 4.23, the lateral load carrying capacity of the structure is reduced due to the presence of nonlinearity at connection regions as demonstrated by [67]. As a result, the beam ends cannot reach to their capacity but remain elastic, while the connections yield as its capacity is chosen less than beam's plastic capacity, and thus limit the lateral capacity of the structure.

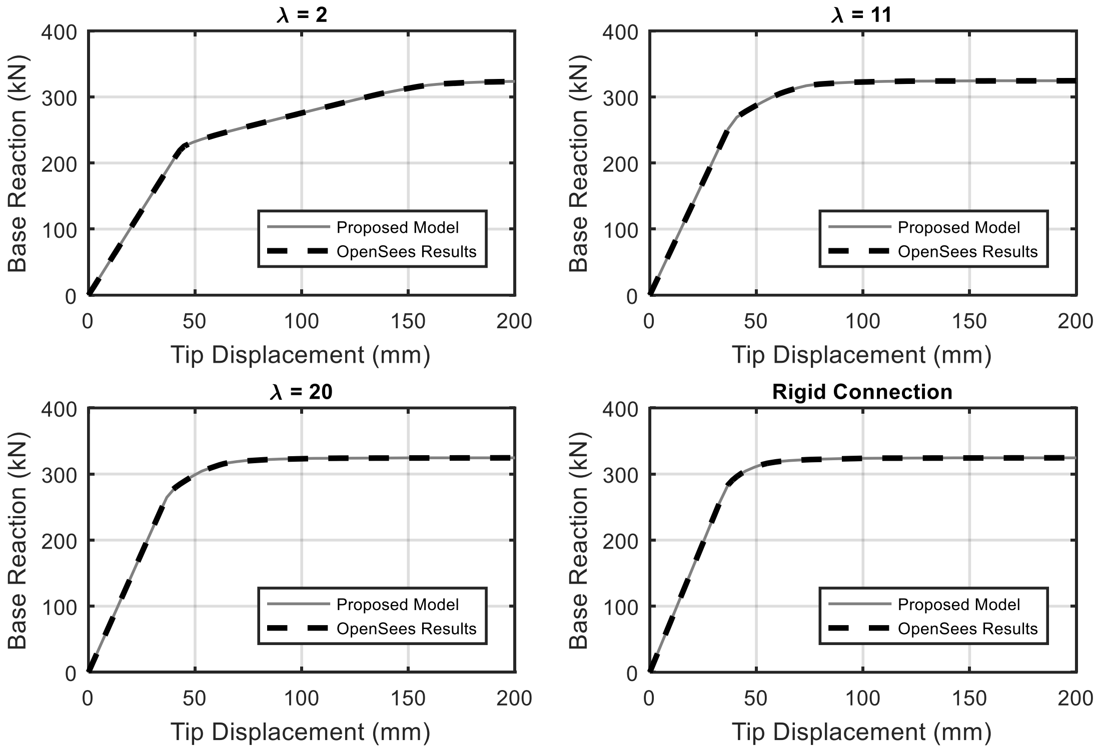

Figure 4.22. Response of the Structure under Bilinear Material Behavior for Beams and Columns and Linear Elastic Behavior for Connections

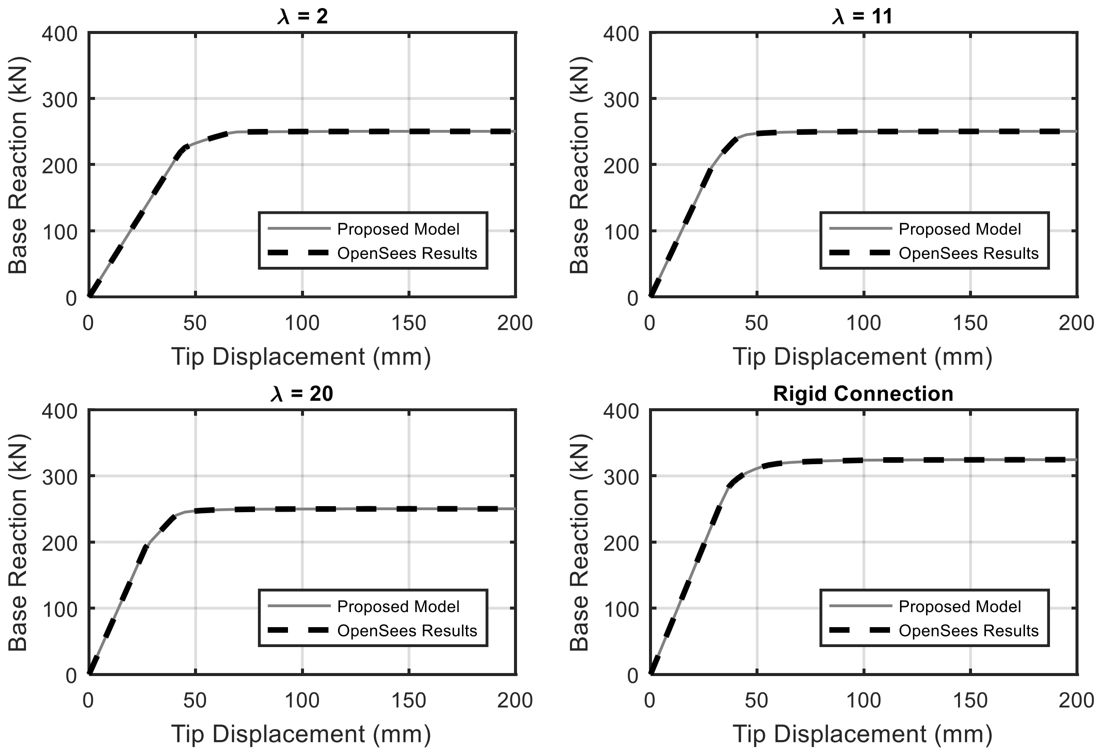

Figure 4.23. Response of the Structure under Bilinear Material Behavior for Beams and Columns and Bilinear Behavior for Connections

The effects of the linear elastic flexibility of connections on the nonlinear response of structure are presented in Figure 4.24 more clearly, where this figure contains the cases shown in Figure 4.22. It is evident that even the linear elastic flexibility of the connection greatly affects the energy dissipation characteristics of steel structural systems, let alone the nonlinearity that can occur in the connection regions.

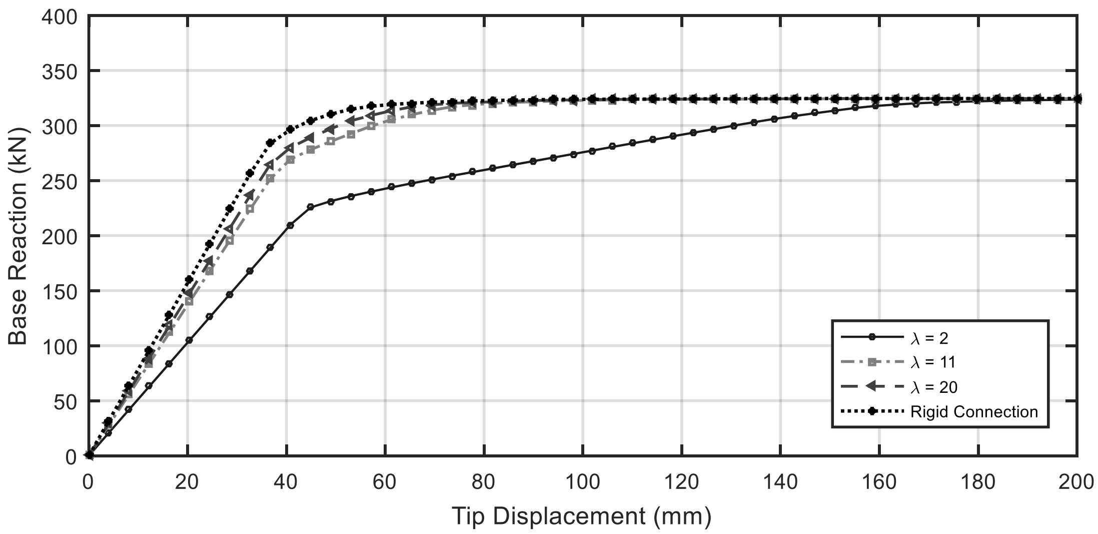

Figure 4.24. Proposed Model Response with Linear Elastic Connection Responses and Rigid Connection Response

This time, lateral displacement of 200 mm on left and 100 mm on right of the floor are applied in the strong direction of the column tips to create torsion on the floor of the portal frame. The purpose is to visualize a 3D deformation on the structure. The bilinear material behavior and reduced bilinear connection behavior is selected for this comparison study. In Figure 4.25, the rotation of the floor versus torque generated at the base of the portal frame with different λ values of the beam end connections are presented. The comparison with proposed model and OpenSees model showed perfect match.

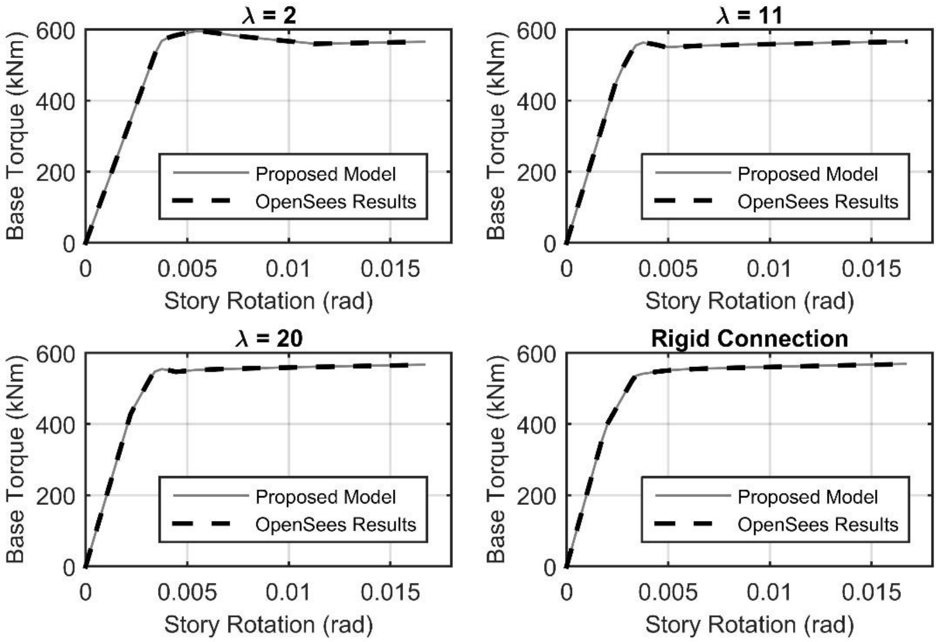

Figure 4.25. Torsional Response of the Structure under Bilinear Material Behavior for Beams and Columns and Bilinear Behavior for Connections

After the validation of the basic accuracy of the proposed model, examples present in the literature are now considered in the next for verification of the accuracy of the proposed model in capturing the responses of three-dimensional complex structural systems with or without the presence of semi-rigid connections.

4.2.2. Three-dimensional 20 Story Structure with Plan Irregularity and Rigid Connections

This 20 stories structure has all rigid connections and a plan irregularity as shown in Figure 4.26, and it was first analyzed in the study of [68]. Due to the complexity of the structure, this is considered as one of the benchmark examples to pursue in assessing the nonlinear behavior of developed structural analysis programs in research. In this thesis, the analysis of this building is pursued, and the results provided recently in the literature by Liu et al. [69] and Chiorean [58] are given, as well.

This building has the following section properties as shown in plan view in Figure 4.26 and perspective view in Figure 4.27: beams are assigned W12x26, W21x57 and W16x36 sections and columns are assigned W8x31, W10x60, W12x87, W12x106, W14x132, W14x145, W14x159 and W14x176 sections. The geometric properties of these sections are presented in Table 4.5. There are two loading cases on the system. Lateral wind load is defined as area load of 0.96 kN/m^2. The wind load is applied in the global Y-direction as point load on the nodes and uniform floor load of 4.8 kN/m^2 is applied, as well. These loads are distributed to the nodes with magnitude of their own tributary areas.

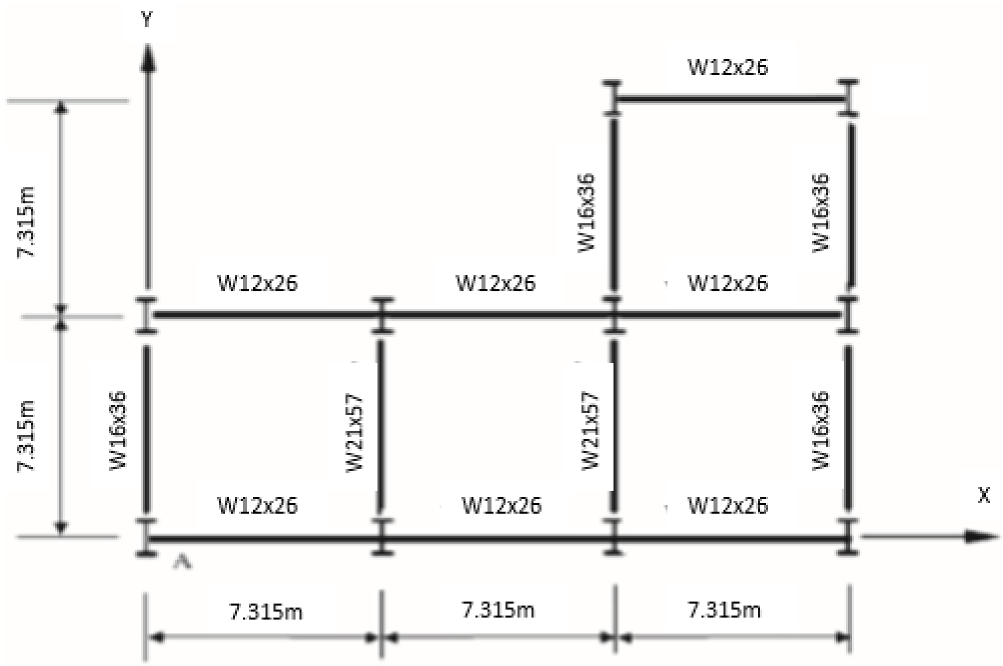

Figure 4.26. 20 Story Structure Plan View [58]

Elasticity modulus, Poisson's ratio and yield strength is taken as 200 GPa, 0.3 and 344.8 MPa, respectively. In the proposed frame element model analysis, 5 monitoring sections are used for beams and columns with Gauss-Lobatto integration. Cross-sections of the beams are divided into 6 layers on flanges and 9 layers on web along their width directions, and the thickness of the flanges and web are divided into 2 layers each; cross-sections of the columns are divided into 12 layers on flanges and 9 layers on web along their width directions, and the thickness of the flanges and web are divided into 2 layers each. In this analysis, geometric transformation of the beams is assumed as linear and rigid-end zone offsets are ignored, and P-delta transformation is considered for the columns. In the present study, torsional constant is calculated from the Galambos formula [70] which is $J = \left(2b_f t_f + d'' t_w\right)/3$ where b_f is width of the flange, t_f is the thickness of the flange, d'' is net web height and t_w is web thickness. However, further definition of torsional effects with warping could be necessary for I beam sections, yet, for steel structures, this can be neglected with plausible amount of error. Consideration of torsional rigidity in the study by Liu et al. [69] and Chiorean [58] are not discussed and could be polar moment inertia of the sections.

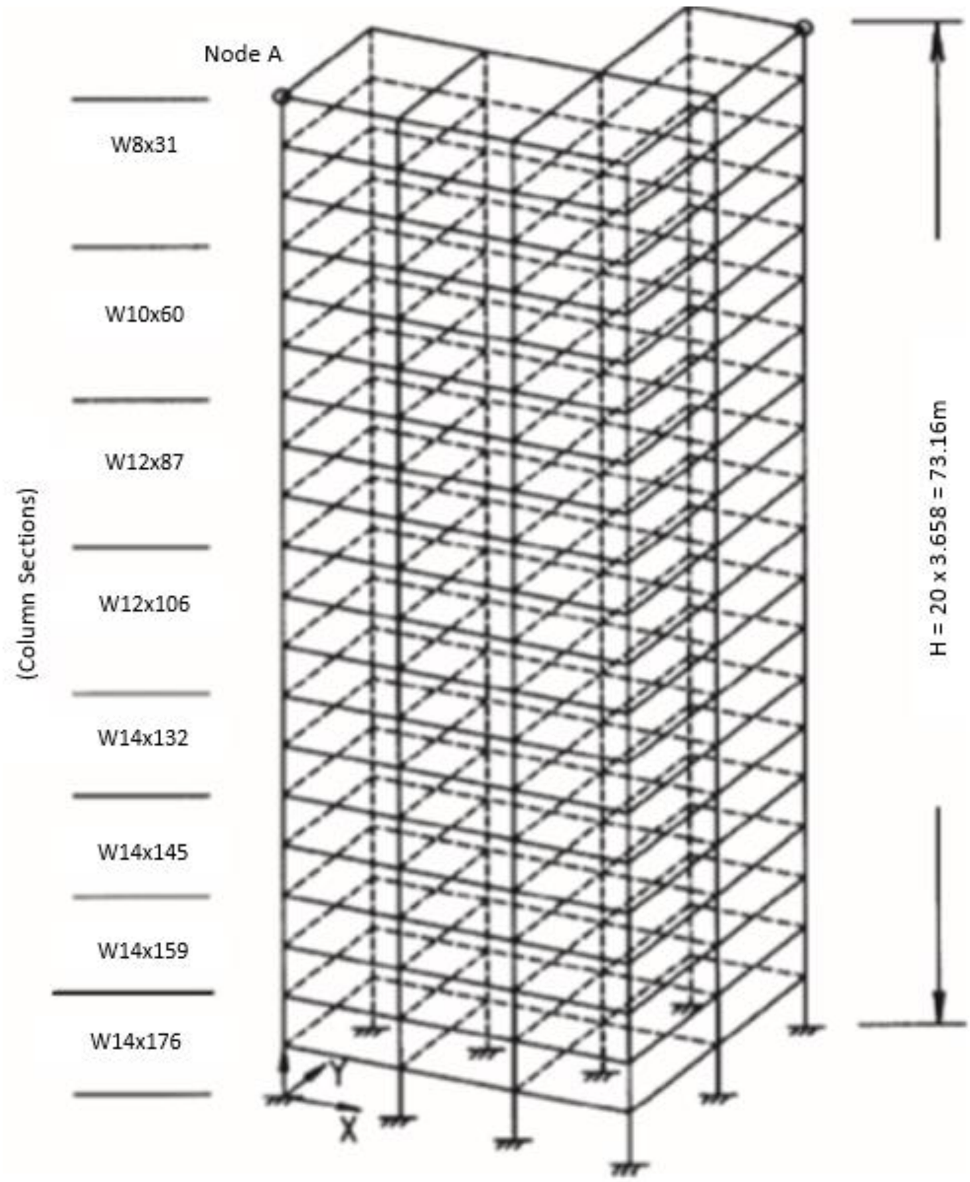

Figure 4.27. 20 Story Structure 3D Perspective View [58]

Table 4. 5 *Section Properties for 20 Story Frame*

	Depth (h) mm	Width (w) mm	Web Thickness (t_w) mm	Flange Thickness (t_f) mm	Area mm^2	I_x mm^4	I_y mm^4
W12x26	310.388	164.846	5.842	9.652	4967.732	84911210.82	7200803.66
W16x36	402.844	177.419	7.493	10.922	6838.696	186471678.7	10197669.9
W21x57	535.94	166.624	10.287	16.51	10774.17	486990768	12736681.6
W8x31	203.2	203.073	7.239	11.049	5870.956	45785456.82	15442185.9
W10x60	259.588	256.032	10.668	17.272	11354.82	141934916.1	48282845.4
W12x87	318.262	307.975	13.081	20.574	16516.1	308011254.9	100311774
W12x 106	327.406	310.388	15.494	25.146	20128.99	388343920.1	125285659
W14x132	372.364	374.015	16.383	26.162	25032.21	636834081.2	228094821
W14x145	375.4	393.7	17.3	27.7	27548	711756000	281789000
W14x159	380.5	395.4	18.9	30.2	30129	790840000	311341000
W14x176	386.6	397.5	21.1	33.3	33419	890735000	348802000

This example is analyzed with proposed frame elements through incremental application of the lateral load and recording of the displacement of node A in global Y-direction shown in Figure 4.27 in terms of base shear divided by total applied load which is referred as base shear ratio. The results obtained with proposed frame element are compared with the results presented by Liu et al. [69] and Chiorean [58] in Figure 4.38. The model by Liu et. al [69] presents a refined plastic hinge method and Chiorean [58] uses second-order force-based element. In the study by Chiorean [58], the cross-section properties of I-sections are more accurately modeled in order to include the fillet regions, and the residual stresses were also considered, while in proposed model these fillet regions are neglected and residual stresses are not considered in the simulations. Modeling and formulation approaches are suspected to be the cause of slight variation in the results. As stated above, whether or not rigid-end zone offsets are present, or the definition of torsional rigidity could also cause slight variations in the response. Overall, the nonlinear load-displacement behavior of this complex irregular structure with rigid connections is satisfactorily followed with respect to benchmark solutions presented in the literature, and the accuracy of the proposed frame element formulation for rigid connection case confirmed. Also it is valuable to mention that the P-delta transformation on columns causes the base shear ratio versus story drift curves flatten where can be seen in Figure 4.28.

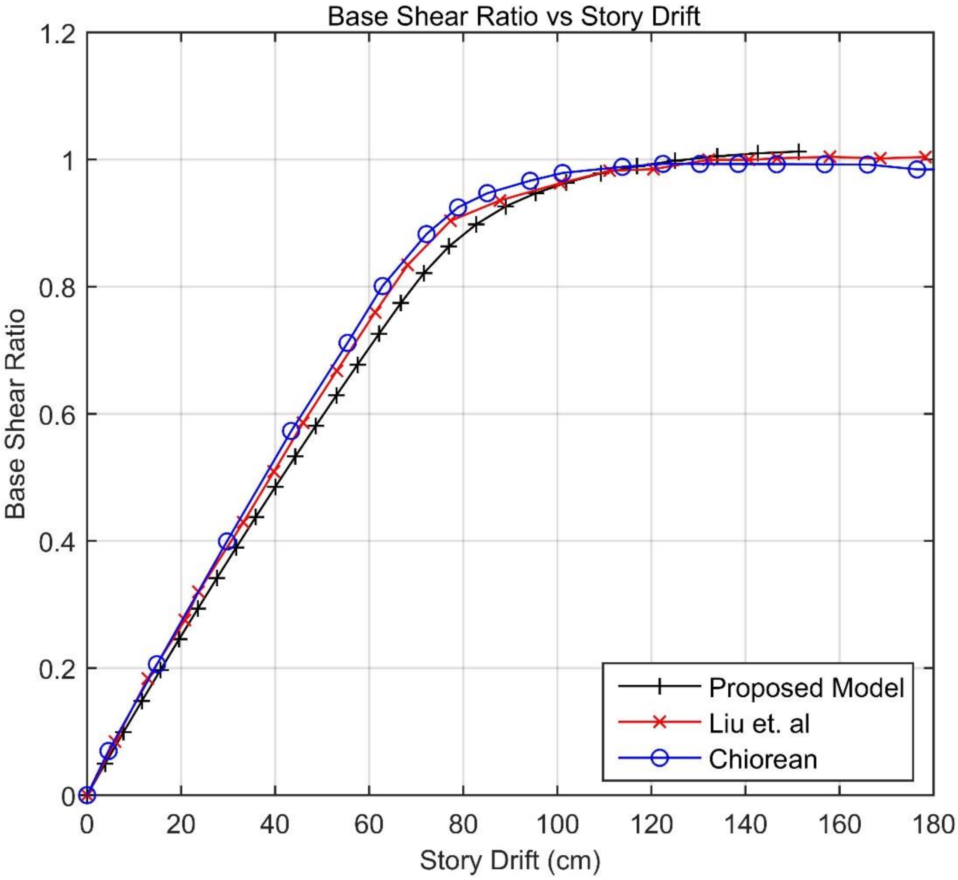

Figure 4.28. 20 Story Structure Base Shear Ratio versus Roof Drift

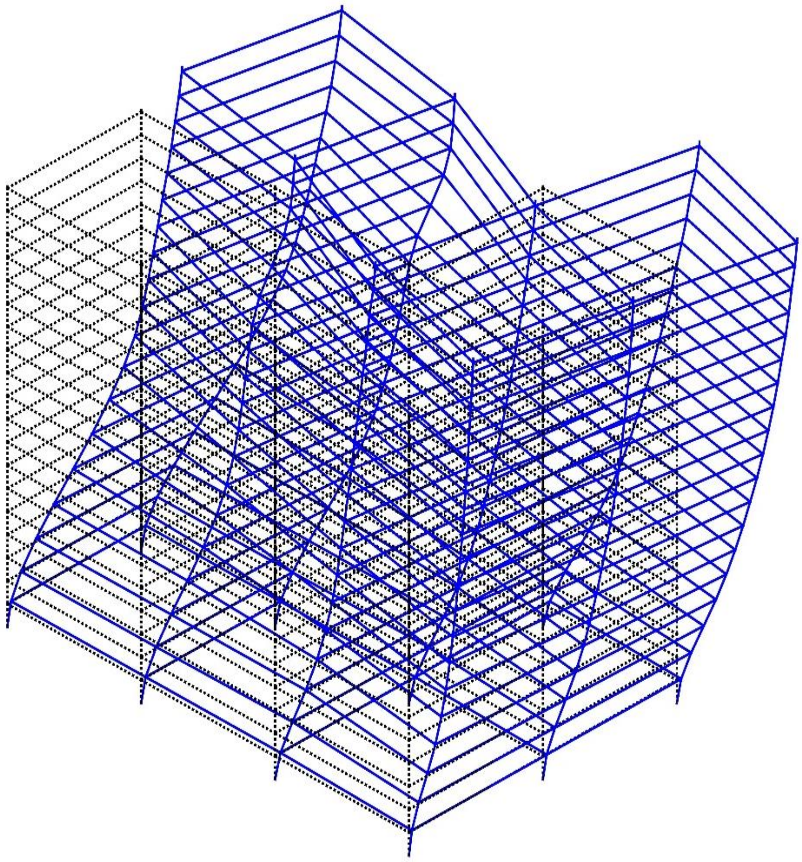

Figure 4.29. 20 Story Structure Deformed Shape

4.2.3. Three-dimensional 6 Story Structure with Elevation Irregularity and Semi-rigid Beam to Column Connections

In this example, 6 story structure first analyzed by [68] with rigid connections is considered. The structure has elevation irregularity as shown in Figure 4.30, and height of the floors is 3.658 m and the length of the beams in each direction on the floor is 7.315 m. Total height of the structure is 21.948 m.

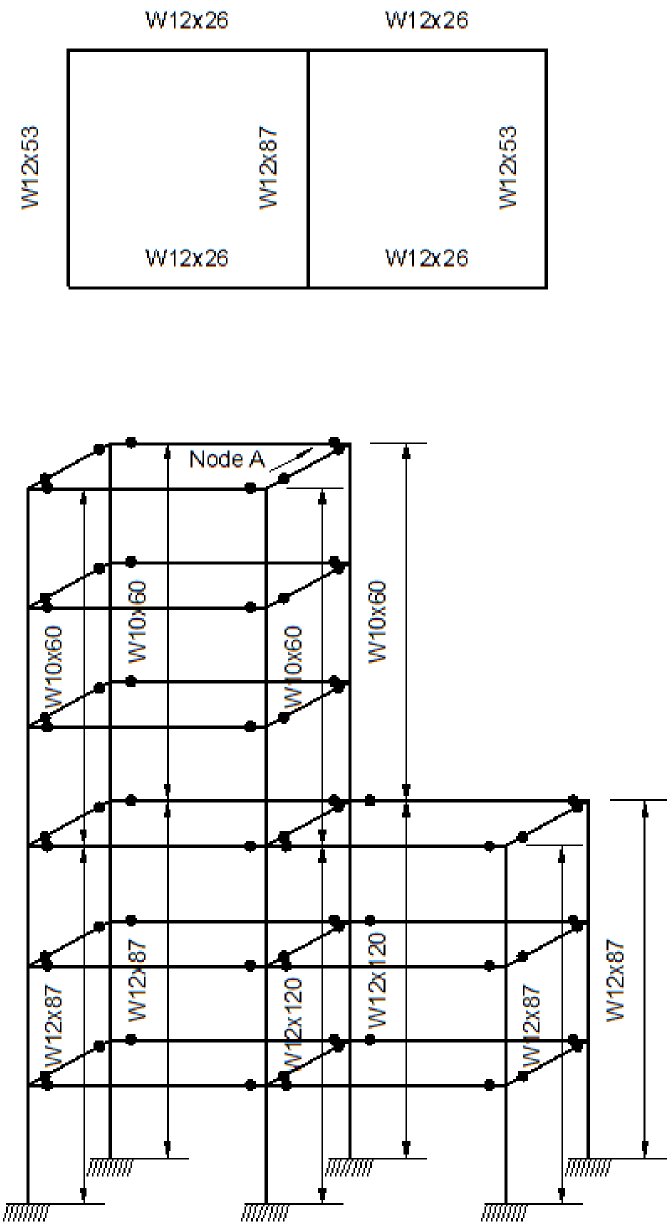

Figure 4.30. 6 Story Structure with Beam to Column Semi-rigid Connections 3D Representation

This example was later on analyzed with semi-rigid connections and currently stands as the only complex 3D steel structure with semi-rigid connections with benchmark solutions. [68] used a plastic hinge approach for the analysis of this frame, where rigid connection behavior was assumed. Nguyen and Kim [57], Thai and Kim [59] and Chiorean [58] used frame element models with second-order force based approach, where the inclusion of semi-rigid connections was implemented through extra spring elements and addition of nodes and degrees of freedom to the structural model. Further details on the formulation and modeling assumptions of these studies are presented in literature review chapter of this thesis.

The original example stated that the material properties of the frame members would represent A36 steel, where elasticity modulus and shear modulus are set as 206850 MPa and 79293 MPa in the literature, accordingly. Yield stress is 250 MPa for all the members and Poisson's ratio is 0.3. Behavior of the steel material is modeled with bilinear with hardening slope of 1×10^{-6} of initial elastic modulus. In the proposed model in this thesis, each member is modeled with single proposed frame element, and elements have 5 monitoring sections with location and weights assigned from Gauss-Lobatto integration rule. Cross-sections of the beams are divided into 6 layers on flanges and 9 layers on web along width directions, and the thickness of the flanges and web are divided into 2 layers each; cross-sections of the columns are divided into 12 layers on flanges and 9 layers on web along width direction, and the thickness of the flanges and web are divided into 2 layers. Geometric transformation of the beams is assumed as linear and P-delta transformation is considered for the columns, and rigid end zone regions are ignored. The connections of the beam to column elements is considered as bolted top and seat angle connections in the literature, where the power model by Kishi and Chen [7] is adopted for the definition of the semi-rigid connections. Strong and weak axis of the frames and the connections are considered in this example, and the properties of the connections for power model are documented in Table 4.6. As in the previous example, the torsional rigidity is adopted from the Galambos Formula.

Table 4. 6 *Semi-Rigid Connection Properties for both Strong and Weak Axis*

Beam Sections	Bending Axis	M_u (kNm)	R_{ki} (kNm/rad)	n
W12x87	Major	300	160503.2	1.57
	Minor	300	52267.75	1.57
W12x53	Major	300	92185.09	1.57
	Minor	300	20776.82	1.57
W12x26	Major	200	44247.8	0.86
	Minor	200	3752.54	0.86

The loading on the structure consists of a lateral load obtained from wind loads and uniform floor loads. The wind load is applied in global Y direction as point load of 53.376 kN at every beam to column connecting node and uniform floor load of 9.6 kN/m^2 is applied. These loads are distributed to the nodes with magnitude of their own tributary areas.

The results of lateral load in terms of base shear ratio and lateral displacement of node A in Y direction are plotted for the proposed frame element model analysis and compared with the study by [57] in Figure 4-31. The study [57] uses three-dimensional semi-rigid steel frames, where the spread of plasticity is taken by uniaxial stress-strain relation from cross section of the elements. The second order P-delta effects are taken into account. Semi-rigid beam column connection is generated by a 3D multi spring element which accounts for rotational response in both major and minor axis bending, and thus requires increase in the number of nodes and degrees of freedom to the structural model.

Evident from the comparison in Figure 4.31, the proposed model response provides good match with the literature and can actually follow the peak load response of roof displacement even further than the result presented by [57]. Also again, the P-delta effect on columns causes the base shear ratio versus story drift curves flatten which is presented in Figure 4.31 and 4.32.

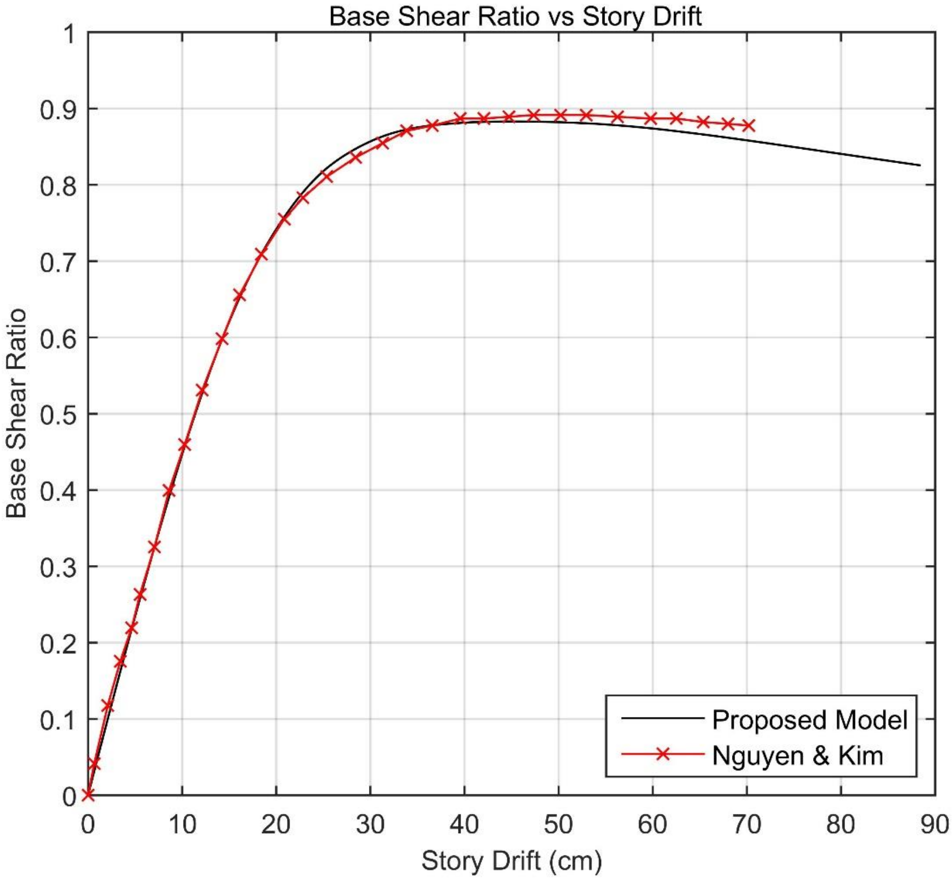

Figure 4.31. The Comparison of Base Shear Ratio versus Story Drift with Proposed Model and Nguyen and Kim Study [57]

Response of the proposed model is also compared with the results provided by Chiorean [58] and Thai and Kim [59]. Both of these studies also implement force-based shape functions but need the introduction of extra nodes and degrees of freedom to the structural model to consider semi-rigid connection response as spring elements. Further details on these models are presented in literature review chapter. Comparison of the plots clearly indicate satisfactory match between the solutions, and the differences in the results are thought to be due to the modeling variations that were listed and discussed in the previous example with regards to the definition of rigid end zones, torsional rigidity, and description of fillet regions of I-sections in especially Chiorean [58].

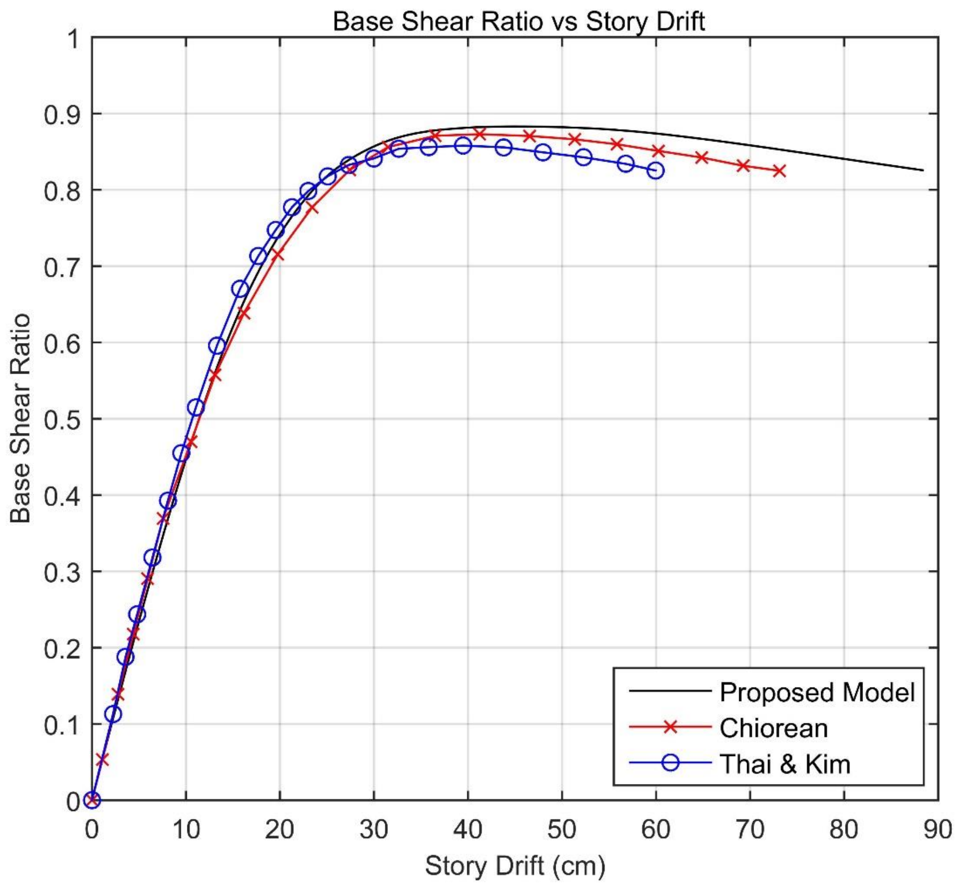

Figure 4.32. The Comparison of Base Shear Ratio versus Story Drift for Chiorean [58] and Thai and Kim [59] and Proposed Model

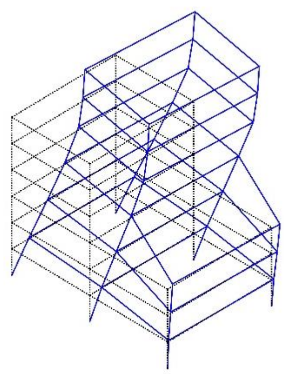

Figure 4.33. 6 Story Structure Deformed Shape

CHAPTER 5

CONCLUSION

5.1. Summary

In this thesis, a force-based formulation frame finite element is derived from three-fields Hu-Washizu-Barr functional. In the formulation presented in this thesis, consistent mass matrix is implemented in a consistent manner with the functional and obtained through force-based approach without any need for the derivation of displacement field along element length. Proposed approach allows the determination of vibration characteristics of structural members with varying geometry and material distribution. Furthermore, the proposed model is also able to present localized connection responses at any section on the element without additional requirement for the specification of different displacement shape functions for every situation. Hence, proposed model does not need additional discretization in the form of introduction of nodes and degrees of freedom to the element or structural model.

The proposed model exploits fiber discretization of sections that monitor the spread of inelasticity along element length and section depth. Introducing semi-rigid connections either linear or nonlinear behavior along the member does not require any increase in number of degrees of freedom even in the case of nonlinear behavior. The element is generated with the choice of taking nonlinear geometric effects. The proposed element is developed with correct element behavior and flexibility on the connections in order to capture the biased analysis of steel structures.

Numerical examples are conducted to validate the results taken from the proposed finite element models with regards to other available structural and finite element analysis programs. The evolution of this study initiates from member level examples in 2D with a cantilever beam example, which is followed by planar framed structures. Through the use of member level study, vibration characteristics obtained through the proposed frame element is verified. The validity of the proposed force-based frame element formulation with localized connections is first evaluated for the rigid connection case with ANSYS through the use of 3D solid finite element models, and then with frame elements available in SAP2000 programs. After this basic validation study, the proposed frame element's accuracy is compared with available results in the literature for steel framed structures. With regards to the nonlinear validation of the proposed element, both 2D and 3D examples generated in this thesis and that are available in the literature are considered. For 3D examples, semi-rigid connection behavior is taken to be present in the strong and weak axis bending directions using linear and nonlinear models for the connection.

5.2. Conclusions

Discussion of the results from the research work undertaken in this thesis provide the following conclusions:

- Consistent mass matrix of the element is determined from force method. Thus, the proposed approach permits determination of vibration frequencies of members with differing geometry and material distribution in an accurate and robust manner and allows modeling of the localized connection response at any section on the element without additional application of different displacement shape functions. Furthermore, the element response does not need additional discretization because of the presence of discontinuous deformations that take place at localized connections. Also, the proposed model considers the transverse shear deformations through the use of Timoshenko beam theory and implements an accurate shear correction factor for widely used steel sections in practice.

- Member level validation studies on the estimation of vibration characteristics demonstrate that the proposed element can determine the first bending and axial modes in addition to the second bending mode for I-beams for rigid connection case accurately when compared with ANSYS finite element simulations undertaken through the use of 3D solid finite elements. Comparisons are undertaken for long beam to short beam cases, and proposed element provides highly accurate results for intermediate to long beams when compared with ANSYS solutions, while its accuracy drops down to give 2-3% error for the fundamental vibration modes for the short beam case.

- Member level comparisons for the proposed element is also pursued through comparison with SAP2000 structural analysis program. Comparison of the results of proposed element for both rigid and semi-rigid connection cases clearly demonstrate the superiority of proposed approach in capturing vibration characteristics of both rigidly and flexibly connected steel members, where SAP2000 needs to use higher number of elements to acquire the same level of accuracy with respect to proposed element.

- Validation studies for structural systems is pursued through a comparison of 2D portal frame analysis with SAP2000 for both rigid and semi-rigid connection cases. With system level examples, the superiority of proposed approach demonstrates itself through the use of fewer number of nodes. Through the use of 4 proposed frame elements per span without any extra need for the presence of semi-rigid connections, the results of higher vibration modes match to the results obtained in SAP2000 with the use of 32 elements per span.

- Last validation study on vibration characteristics considered 3 bays 6 story 2D frame structure with semi-rigid connections. The analysis with proposed frame element through the use of single element per span provided perfect match with the benchmark results. Further studies on this example extended comparisons of this structure with SAP2000 outputs acquired for higher modes of vibrations and shapes.

- The proposed frame element is also demonstrated to model the presence of flexible regions at column bases and the ends of braces in gusset regions. Validation studies are undertaken through the use of SAP2000 program, and the accuracy of proposed element in introducing axially flexible regions besides deformation prescribed in terms of moment-rotation is demonstrated with high accuracy. The proposed element can incorporate any type of connection behavior without further specification of nodes and degrees of freedom to the element or structural model.

- Validation of the nonlinear behavior of proposed element is pursued first with respect to OpenSees structural analysis program through 3D portal frame example, which is analyzed for both rigid and nonlinear semi-rigid connection cases. Both the proposed element and the elements used in OpenSees base on force-based approach and provide identical results. OpenSees lacks force-based mass matrix and the inclusion of connections in its force-based elements. Presence of localized connections in OpenSees require the introduction of extra nodes and degrees of freedom and the description of constraint conditions, which are not needed in the proposed frame element. The advantages of proposed element formulation clearly give a clear edge to it for the solution of large structural systems at both structural modeling and analysis levels.

- Validation of the nonlinear behavior of proposed frame element with connections is also done with benchmark 3D examples. A 20-story structure with rigid connections is analyzed with proposed element with nonlinear material and geometry. For the torsional behavior in 3D, warping effects are neglected in proposed element, and torsional response for I-sections is specified through the approach proposed by Galambos. The results obtained through proposed model showed good match with benchmark solutions in capturing spread of plasticity in the presence of P-delta effect on the columns.

- As a second 3D structural system example, an irregular structure with 2 Bays and 6 stories is considered. This structure is studied in the literature and it has results presented in the presence of semi-rigid connection for both bending axis at the ends of beams connecting to the columns. Nonlinear connection response is modeled through the use of power model proposed by [7] in both the proposed element and the analysis undertaken in the example that considers extra springs through the inclusion of nodes and degrees of freedom to the structural model at the expense of increased matrix sizes and computation time. Comparison of the results for the nonlinear behavior of this complex structure demonstrates good match with the benchmark results, and clearly demonstrates the accuracy and robustness of the proposed element for performance assessment of steel framed structures with the presence of localized connections.

5.3. Future Study

Description of more complex deformation behaviors of steel beam and column members can be investigated, especially with respect to the capture of warping effects due to torsional behavior, lateral torsional global buckling of members, flange and web local buckling of widely used steel sections and eigenvalue buckling analysis.

Development of column base connection models that can incorporate axial force and bending moment interaction at connection region can be pursued. Development of macro models to describe the complex nonlinearities that arise from nonlinear material, geometry and contact nonlinearities that occur at connection regions in steel structures can be investigated. There are such complex macro models proposed in literature, but these are not efficiently used in conjunction with framed structural analysis. Such macro models can be implemented to work along with the proposed frame element model towards nonlinear dynamic analysis of steel framed structures.

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PUBLICATIONS

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HOBBIES

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