CONDITION BASED MAINTENANCE POLICIES
FOR A CRITICAL UNIT

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ABSTRACT

CONDITION BASED MAINTENANCE POLICIES FOR A CRITICAL UNIT

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This thesis analyses a maintenance optimization problem of a critical unit used in military systems. It is aimed to model the stochastic deterioration behavior of these units and develop some maintenance policies. For this purpose, periodic inspection times which maximize expected time to failure are proposed to be able to provide preventive maintenance before failures.

In the scope of this work, resistance measurements from the field are gathered for ten months. First, functions representing the increase in resistance are obtained. Then they are transformed into the deterioration condition of the unit. Then optimal maintenance policies depending on these conditions are found for various objectives using Markov models.

Keywords: Condition Based Maintenance, Markovian Deterioration, Markov Decision Process

ÖZ

KRİTİK BİR BİRİM İÇİN DURUMA BAĞLI BAKIM POLİTİKALARI

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Yüksek Lisans, Endüstri Mühendisliği Bölümü
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Bu tez, askeri sistemlerde kullanılan kritik bir birimin bakım optimizasyon problemini analiz etmektedir. Bu çalışmada bu birimlerin rassal bozulma davranışlarını modellemek ve bazı bakım politikaları geliştirmek amaçlanmıştır. Arızadan önce önleyici bakım faaliyetlerinin uygulanabilmesini sağlamak adına, arızaya kadar süreyi en çoklayan periyodik gözlemler önerilmiştir.

Bu çalışma kapsamında, on ay boyunca sahadan direnç ölçümleri alınmıştır. Önce, dirençteki artışı temsil eden fonksiyonlar elde edilmiştir. Sonra bu fonksiyonlar ünitenin “bozulma durumu”na dönüştürülmüştür. Daha sonra eskime sürecinin Markov modelleri kullanarak ve değişik amaçlar altında duruma bağlı en iyi bakım politikaları bulunmuştur.

Anahtar kelimeler: Duruma Bağlı Bakım, Markov Bozulma, Markov Karar Süreci

To my lovely daughter Zeynep Duru…

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CHAPTER 1

INTRODUCTION AND LITERATURE SURVEY

1.1. Introduction

Maintenance optimization is a significant discipline to preserve functionality of systems or to apply timely and cost effective solutions in case of failures. Lack of enough effort for this field causes cost of detecting design problems after fielding to consume very high portion of budgets and correspondingly dramatic decrease in customer satisfaction. This fact and some other related factors like rising support costs, necessities to reduce logistic footprint and financial risks make organizations to think more about their designs.

There are some general policies in maintenance optimization to determine maintenance tasks and intervals. Generally, Preventive Maintenance (PM) and Condition Based Maintenance (CBM) methodologies are used to eliminate undesirable effects of failures. PM is a scheduled maintenance task, while CBM involves monitoring the condition of the system and deciding to apply a maintenance task according to its condition. Gillespie (2015) defines Condition Based Maintenance as a way of determining an on-condition activity for the equipment that ages according to planned operational profile. In other words, it means monitoring a deteriorating equipment and taking suitable actions at decision points. Shin and Jun (2015) sort benefits of adopting CBM methodology as foreseeing forthcoming failures, effectively managing maintenance policies and avoiding risks that may cause costly problems.

Nowadays, we see various technologies becoming parts of design solutions to allow condition monitoring effectively. It may even be possible to remotely monitor instant conditions of systems that operate very far away. In this study, we evaluate the preventive maintenance suggestion of a critical unit from Condition Based Maintenance point of view due to high failure frequency.

Problem studied in this thesis is related with Condition Based Maintenance optimization of a radar. A critical unit of a radar, called slip ring, is inspected and a maintenance policy for these units is tried to be established. Slip rings provide wireless transmission of information for rotational movements and deteriorates with daily usage. Since it is a critical unit of radars frequent failure is not acceptable not only for system owners but also for the company. Therefore, it is aimed in this study to be able to offer some improvements by assessment of natural deterioration of slip rings and optimizing maintenance policies.

In the literature, Markovian behaviors of these aging equipment are studied intensively. Stochastic modeling of deterioration problems is established for various infrastructures and systems. Generally, estimation of transition probabilities and determination of inspection or replacement intervals form the main constituents of these problems studied in the literature. Some additional aspects like life cycle cost considerations, maintenance policies under available actions and different objectives and constraints are incorporated into these problems. Within the scope of this thesis, we try to estimate transition probabilities and determine optimum maintenance policies with different objective function considerations.

Following part briefly describes the context of chapters:

In Chapter 1, purpose of the study is outlined along with the introduction part and short technical information is given about the unit under inspection. It also discusses some literature review related to study conducted. Main results of some studies are given.

Chapter 2 explains the problem that is subject to this work in detail and describes some linear and nonlinear models which are used to investigate details of the problem.

Chapter 3 explains the analysis methods, computational results and interpretations. It also contains parametric analysis because of the probabilistic nature of the problem.

Chapter 4 is the final chapter of this thesis and states conclusions and advises some future works that can be part of further researches.

1.2. Literature Survey

There exist many studies about stochastic deterioration modeling of systems in the literature. In this field, Condition Based Maintenance models are quite common and can be classified according to many factors and some steps similar to our work generally take part in these studies.

Condition Based Optimization models can be studied for both discrete state and continuous state deterioration problems. This classification made by Alaswad and Xiang (2017) is further detailed in one level and classified for single and multi-unit systems. In this thesis, a discrete state deterioration problem for a single unit is considered. Park et al. (2011) develop an equipment modeling which is a continuous time Markov Chain for three types of units. Due to ease of computations, they convert it to discrete time Markov chain by uniformization. Kallen and Noortwijk (2005) apply a continuous time Markov process for bridges in Netherlands to determine an optimized periodic inspection time. They discuss the benefit of using continuous time Markov process instead of discrete time Markov process which neglects aging property. Main concern of this study is modeling the uncertain time between state transitions with a probability distribution with increasing failure rate.

Alaswad and Xiang (2017) go through literature and summarize Condition Based Maintenance optimization studies for modeling stochastic deterioration problems. They review these models based on various factors like inspection frequency, inspection quality, optimization criteria etc. Some problems also differ in method of estimating transition probabilities.

Models can differ in inspection frequency. Time between consecutive inspections effect maintenance policies. Observed systems can be monitored continuously, periodically or non-periodically. Ye et al. (2015) propose a dynamic policy that generates optimal inspect/replace decisions and inspection interval for the next observation. In that regard, we want to propose optimal and non-periodic inspection intervals which have to be adjusted in accordance with the observed conditions.

CBM optimization models can be solved under various optimization criteria such as cost minimization, downtime minimization and multi objective. We intend to find optimal maintenance policies for different objective function considerations including cost minimization and downtime minimization. Ferreira et al. (2009) propose a decision model which simultaneously satisfy decision maker's two different objective to determine optimal inspection intervals. They use delay time analysis for that purpose. Masoumi (2014) uses cost minimization to obtain optimal policies. Jin (2016) benefits from stochastic approaches to conduct a material selection process. He uses life cycle cost estimates as decision criteria. Three types of sewer pipes are used as alternatives and three life cycle stages and relevant costs are assessed to make an evaluation for best alternative. Kurt and Kharoufeh (2010) work on a system to satisfy maintenance optimization with a number of repair constraint which sets a limit for this system to be repaired before replacement. They try to minimize total cost of operation and maintenance activities. They also work on cumulative number of repairs effect on the system's condition. As the number of repair increases, system gets more prone to failures.

Aging factor can also be incorporated into these studies. Commonly as good as new and as bad as old assumptions are used for analyzing data. Intermediate assumptions are also studied in the literature. Do et al. (2015) suggest a hybrid maintenance policy that includes both imperfect and perfect maintenance activities. At the first part of their study they evaluate the advantages and disadvantages of imperfect repairs. Chan and Asgarpoor (2006) try to find maintenance policies by assuming that repair of random failures do not bring the component's state to an as good as new condition while repairs due to deterioration make the component as good as new. For our case, we assume that maintenance activities bring observed units to as good as new condition.

Different decision variables can take place such as inspection intervals, deterioration limits, maintenance policies etc. Chan and Asgarpoor (2006) use Markov Decision Process to find optimum maintenance policies and calculate mean time to preventive maintenance. Their model consist of both random failures and failures due to aging. Butt et al. (1994) use dynamic programming and try to eliminate maintenance and repair options. Park et al. (2011) apply a dynamic programming model for a modified semi Markov chain to find optimal inspection intervals. They develop a semi Markov chain which adopts arbitrarily distributed times between events and propose optimal inspection times. Wirahadikusumah et al. (1998) also uses nonlinear regression approach and dynamic programming to assess condition of sewer systems and develop maintenance policies. Kallen and Noortwijk (2005) propose optimal inspection intervals in accordance with expected costs. Masoumi (2014) aim to find optimal maintenance policies for bridges in Turkey and Jiang et al. (1988) want to obtain transition probabilities. In our case we use actions to apply at decision points and expected first passage times as decision variables.

Some differences in the data used for transition matrix estimation are observed. Jin (2016) states main challenge of his study as lack of historical data which is essential for a condition deterioration curve and with the help of relevant associations' data empirical curves that range for life cycle of pipes are used. In contrast to that, in order to reflect the real situation for a system that operates in a dynamic environment collection of real records are considered as important. Jiang et al. (1988) calculate transition probabilities with the help of a historical data. From those records, number of total transitions for system level or subcomponent level is calculated from one state to another and used for estimation of transition probabilities.

We see that condition based monitoring is very common for the infrastructure elements since maintaining the existing structure is more cost effective than building new ones. Masoumi (2014) studies different failure mechanisms and corresponding bridge elements and considers maintenance optimization for every single element.

Estimated transition probabilities and determined actions are used in Markov Decision Process to obtain optimal policies. At the end of the study he creates a bridge management system to give management capability to authorized bodies in Turkey. Masoumi and Akgul (2012) also study part of this bridge management system before. Masoumi (2014) suggests a prioritization approach to determine criticality of the maintenance actions and satisfy the financial constraints.

We also reviewed papers which consists different transition matrix estimation methods. In this content, when it gets difficult to obtain sufficient data to use in Markov processes, some empirical methods are used to create curves that represents natural deterioration behaviors. It is better to evaluate transition probabilities with real data but challenges may lie in to obtain or use real data to evaluate transition probabilities. Following studies include instructions about transition probabilities generation methods.

Jiang et al. (1988) proposes a Markov chain model to predict bridge deteriorations. They use both percentage prediction model and nonlinear regression model for estimation of transition probabilities. Since, Markov Chain requires time homogeneity of transition probabilities, nonlinear regression approach is also used to satisfy the time homogeneity property.

Butt et al. (1994) propose a methodology to create an efficient pavement management system. They apply a dynamic programming approach that uses Markovian transition probabilities estimated by nonlinear regression approach as input to and try to eliminate some maintenance and repair options within constraints. Recommended output is then evaluated to stay within the budget limits and prioritize the options with the help of cost/benefit analysis.

Masoumi (2014) and Jin (2016) also use nonlinear regression approach to estimate transition probabilities. Generally, curves that represents deterioration process and provides condition states are used in this approach. For this part, we find a curve which fits to directly observable information of deterioration behavior and then convert this information into the states.

Madanat et al. (1995) also work on transition probabilities generation methods, Poisson regression model and ordered probit model, in two different studies and proposes them as viable options with case studies.

First study of Madanat et al. (May 1995) is about application of Poisson regression model to generate transition probabilities of an infrastructure. During the development of a discrete incremental deterioration model, it is assumed that number of state transitions over an inspection period is the dependent variable of Poisson regression model. Within this framework Poisson probability mass function is used to specify that dependent variable. Stated shortcoming of nonlinear regression models, affecting variables, also integrated into the problem. A deterministic exponential equation is used to model these variables in order to satisfy non-negativity of dependent variable and Maximum Likelihood Estimation technique used to estimate unknown parameters. Once the parameters are estimated, transition probabilities are calculated.

In another study, Madanat et al. (June 1995) develop a model for transition probabilities to be estimated by using Ordered Probit Model which is used in social sciences to incorporate hidden variables or characteristics into models. Main assumption is a hidden continuous random variable that exists behind the unobservable part of the process. This establishment is helpful to overcome latency drawback of nonlinear regression approach since the deterioration itself is not directly measurable. Both Poisson regression approach and ordered probit model are assessed to be useful for modeling high number of state options because number of drops in condition states will not be meaningful for low number of states.

Baik et al. (2006) also use Ordered Probit Model to estimate transition probabilities for a wastewater management system. They discuss the suitability of this method against the nonlinear optimization approach and explain the need of data collection for explanatory variables.

Since the deterioration process is probabilistic in nature, some of the reviewed papers consist of consistency evaluations. Jin (2016) suggests an uncertainty evaluation approach to avoid wrong estimates of deterioration and performs simulations to choose most cost-effective option at the end of the study. Jiang et al. (1998) conduct chi-squared goodness of fit tests to check whether real values are close to predicted values or not.

In this thesis, steps that are similar to given literature above and required to be able to fit a Markovian model for a natural deterioration process are followed. Different than the previous studies different optimization models are used and optimal policies are compared.

CHAPTER 2

PROBLEM DEFINITION AND MODELING

2.1. Problem Definition

In this thesis, Condition Based Maintenance (CBM) method is our concern to analyze a real life problem of a defense company. Operational life of military systems requires high readiness values. Especially very high availability requirements are projected into contracts to ensure that operational duties are achieved with high success rates. For that reason, some parts of the systems may require more care.

We explain the problem under consideration in detail in this section. The units observed for the study belong to air defense radars of a defense company. Radar is a detection system that determines the distance and location of objects with the help of radio waves. As the definition implies, the word “Radar” stands for “Radio Detection and Ranging” and radars provide air and surface surveillance and target identification within various ranges. Therefore, they hold an important place for national security.

Complexity of radars may increase in parallel to design requirements. They constitute many subsystems, assemblies and equipment. Among these, we observe a critical unit of an air defense radar. The units that are observed in radars are slip rings and their functionality is essential for continuous operation. Generally, they are used in radar systems, periscopes, vehicles etc. In the present case, a slip ring serves as a bridge between the two important subsystems of the radars: antennas and processing sections; they take the responsibility of communication of these major subsystems.

Antenna subsystem has the 360° scanning capability and signals gathered by the antennas flow down to be processed with the help of these slip rings. In other words, slip rings provide contactless transmission between antenna and processing section of the radar. They provide contactless transmission for various voltages and even for high rotational speeds. Pictures of a slip ring and its pins can be seen in Appendix A.

Rotation of slip rings means the action of turning about its center in accordance with the movement of the antenna. According to data gathered from the field, operating hours of these slip rings are in terms of number of rotations. Depending on the rotating speed of the antenna, slip rings make 30 rotations per minute and according to this usage profile numbers of rotations are transformed into operating hours of radars.

Each radar under consideration has one slip ring whose deterioration has to be monitored. In every slip ring, there are yellow rings called pins and fixed brushes whose brush wires enter between the canals of yellow rings. This electrical activity creates a natural resistance at the pins of slip rings and resistance of these pins effects radar functionality. Environmental conditions and time of use increase the resistance of the pins, as a result it results in the loss of communication which is a very critical problem.

Only resistance of a single pin reaching a critical level causes a communication error. Therefore, life of a slip ring is equivalent to life of a pin. So, we can think that the pins of a slip ring work in series configuration and reliability of the slip rings is the reliability of the least reliable pin.

Every slip ring consists of 7 pins[1] and an unused slip ring embodies pins with 0.5 Ω (ohm) resistances. While these slip rings are in operation, pin resistances randomly increase due to operating conditions. But rapid increase in pin resistances is not an expected situation. However, for our case very early failures are in picture and this is a critical problem for the mission profiles that have very high strategic importance.

[1] Slip rings contain more than 90 pins but in the scope of this study 7 pins that cause failures are evaluated.

We observe 9 failures in 10 months. This is a major reason to look for a solution for this problem. Table 1 shows the all failures occurred during the observation period.

Table 1 Failure Records of Slip Rings

Slip Ring	Rotations	Corresponding Operating Hour	Failure Notice
9	1,486,800	826	1st Failure
10	2,107,800	1,171	1st Failure
11	2,165,400	1,203	1st Failure
11	1,348,200	749	2nd Failure
13	1,274,400	708	1st Failure
13	2,557,800	1,421	2nd Failure
19	1,303,202	724	1st Failure
19	1,146,600	637	2nd Failure
20	2,884,860	1,603	1st Failure

An unused slip ring embodies pins with 0.5 ohm resistances. As the operational life begins, pin resistance starts increasing. These slip rings are planned to be operational with a requirement of 30 rotations per minute which corresponds to approximately 8333 hours of operation without maintenance. According to preventive maintenance proposal of the producer, a slip ring would need a lubrication of pins at every 15 million rotations. This implies 4 or 5 years between consecutive lubrication periods. However, compelling operational life or design problems may prevent them to operate as planned. Generally, this problem causes dramatic increase of costs for the utilization stages of radars because of long service life demand of users.

As the operation time increases, equipment wear-out becomes inevitable and if required precautions are not taken, customer satisfaction and critical military operations suffer from this. It is aimed to prevent failures of the radar due to fact that long downtimes affect availability. It is interesting to note that original equipment manufacturer of this unit recommends a preventive maintenance period which equals to approximately fifteen times of observed average failure time.

Preventive maintenance of slip rings involves lubrication of pins and brushes with a special type of grease to smooth rotation, control the heat and prevent increased resistances of pins. It is not only applied at predefined periods but also at the time of slip rings failures. Since the failure of this unit is very undesirable, a subcontractor field technician is assigned to make periodic resistance measurements of the pins. Thus, pin resistances are measured once a month with a multimeter or an ohm meter.

The producers of slip rings offer a service life that requires low maintenance needs. Radars comprising such units have to be useful very long times without any intervention. In parallel with this assumption, it is proposed to apply a preventive maintenance for every 15 million rotations. However, currently the monitoring frequency is increased to collect data, 4 radars and 8 slip rings are observed and resistance measurements are collected. Normally 4 slip rings are operational for 4 radars and other slip rings are used as spares. Radars under consideration operate in South East of Turkey; exact locations are not given due to confidentiality. In here, we want to say radars operate at the climatically same environment.

Importance of early intervention before failures is another reason to study this problem. A failure takes 3 or 4 days to retain the radars back in operation since we intervene after failures and cause some waiting times. However, if a sound condition monitoring is implemented, it will only take few hours to keep radars in operation with an as good as new state. After maintenance, slip rings are assumed to be as good as a new unit, because offered preventive maintenance returns resistance values to new unit conditions. As the number of preventive maintenances increases, the assumption of returning to an unused slip ring’s condition may be unrealistic because of the aging and preventive maintenance intervals may decrease. But this issue is not included in the scope of this study.

Since support strategies of these radars do not constitute an effective Condition Based Maintenance policy, the units could be intervened after the failures. During the observation period it took 3-4 days to recover from failures. Therefore, we aim to reduce downtimes of radars.

In this thesis, we

i. modeled the stochastic deterioration behavior of the slip rings,
ii. fitted a Markov model,
iii. developed some control policies to optimize the long run costs or availability defined in several measures.

2.2. Condition Definition

Throughout the thesis study, every month resistance measurements are collected from 4 radars and estimated values like transition probabilities, policies are updated. Before the final analysis period, there exists a 10-month data collection period that can be thought as preliminary work or preparation period of the thesis.

Radars are placed at remote points and difficult to reach frequently. On the other hand, slip rings do not provide information about their conditions by remote monitoring; only a physical check can show the amount of actual deterioration. For that reason, we could only collect limited data about the conditions and failure times of these slip rings.

From the expert opinion and from the resistance data, we observed that the resistance of a pin is non-decreasing, mostly increasing in time (rotation). Our aim is to fit a Markov model to this deterioration behavior of the pins. Since we have few data points to evaluate deterioration process for each radar separately, we aim to use data that comes from separate radars as if it comes from a single source. For this purpose, we want to check if the slip rings are similar. Correspondingly we want to check if the pins are similar. With the limited number of observations on the resistance measurements of the pins, it is not possible to perform any test homogeneity of the lifetime distributions of the pins. Rather, we simply check the most appropriate distribution for each pin resistance and check how close these distributions are.

Due to operation restrictions and costs we could collect pin resistances once a month for the 10-month observation period. In order to use the collected data in estimation of transition probabilities, we use a life data analysis software, Weibull ++, to check lifetime distributions of seven pins separately. Since the operation conditions, mission profiles and external factors like temperature, vibration etc. are assumed to be common for radars, we intended to see lifetime distributions and mean life calculations of each pin and using them we decide their similarity in terms of deterioration behavior.

In order to use data from all radars together in estimation of transition probabilities and check the appropriateness of our assumption, we use a life data analysis software which is named Weibull++.

Table 2 Weibull ++ Results

Pin	Proposed Lifetime Distribution	Parameters	Mean Life (Hours)
1-2	Lognormal	Mean: 7.2372 Std: 0.4687	1552
4-5	Lognormal	Mean: 7.2105 Std: 0.3798	1455
7-8	Weibull – 2 Parameter	Beta: 5.0094 Eta: 1563.1167	1435
	Lognormal	Mean: 7.2804 Std: 0.3353	1536
10-11	Lognormal	Mean: 7.1339 Std: 0.3630	1339
13-14	Lognormal	Mean: 7.2105 Std: 0.3798	1455
16-17	Lognormal	Mean: 7.1436 Std: 0.4339	1391
19-20	Lognormal	Mean: 7.1436 Std: 0.4339	1391

Table 2 shows the lifetime distributions of different pins. We assume that distributions suggested by Weibull++ are similar enough and that these pins are identical. The slip ring resistances that are the maxima of 7 pin resistances will also be identical.

Next, we model the "deterioration" behavior of slip rings and then estimate the corresponding Markovian transition probabilities. To model the "deterioration" process, we prefer to use the data monitoring the maximum of the pin resistances which represents the slip ring deterioration rather than modeling individual pin resistances. This approach allows us to put data on top of each other and provides more data points.

Figure 1 shows all measurements for a slip ring used in the radars under no maintenance. It consists of inspection hours and measured resistances that correspond to condition states according to our condition state classification. Last inspection shows failure of this unit. Depending on the amount of data collected this analysis could be conducted for individual slip rings as well.

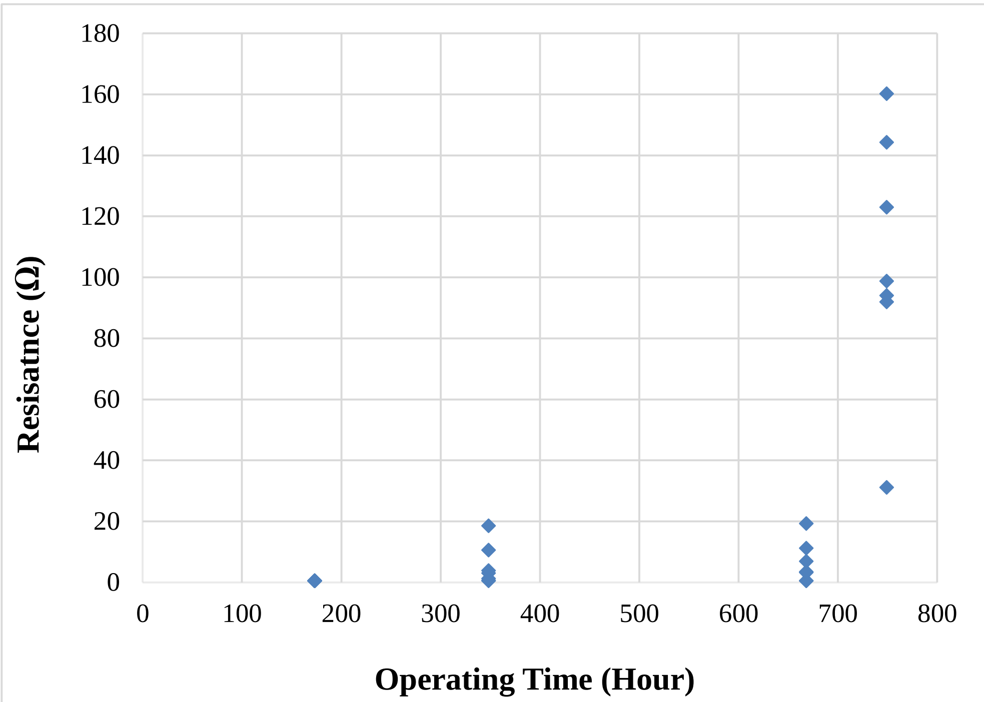

Figure 1 Resistances for an Individual Slip Ring (SN: 11)

The slip ring resistance is found to be between 30 Ω and up to 200 Ω when it has failed. That's why slip ring resistance levels of 30 and above is defined as failure thresholds. As the base case, 3 (condition) states based on the resistance of a slip ring are defined as given in Table 3. We also use a 4 state classification given in Table 4 to evaluate effect of increasing the number of states. State classifications are made intuitively.

Table 3 Condition State Classification for 3-State Case

State	Slip Ring Resistance (Ω)
1	0-5
2	5-25
3	>25

Table 4 Condition State Classification for 4-State Case

State	Slip Ring Resistance (Ω)
1	0-3
2	3-16
3	16-25
4	>25

At that point we define condition states according to resistance levels of slip rings. They are classified according to highest pin resistance measured. We define state spaces as

- S = {As good as new (1), Up (2), Down (3)} for the 3 state case,
- S = {As good as new (1), Up (2), Degraded (3), Down (4)} for the 4 state case.

In here, we can state once more that slip ring resistance is actually the maximum of resistances of the 7 pins in that slip ring.

2.3. Modeling

In order to analyze and optimize maintenance policies of sling rings Markov Processes are used. A base case and some other variances in the name of parametric analysis are evaluated.

Let stochastic variable X(t) represent the condition of a slip ring at time t and we have discrete state space S = {As good as new (1), Up (2), Down (3)} as defined before. Since our maintenance action brings radars back to as good as new state and future condition states are assumed to depend on only present states for identical pins, we model deterioration of pins as a Markov Chain. In other words, pin resistances increase arbitrarily and they do not depend on history. Figure 2 describes the order of methodology used in this thesis.

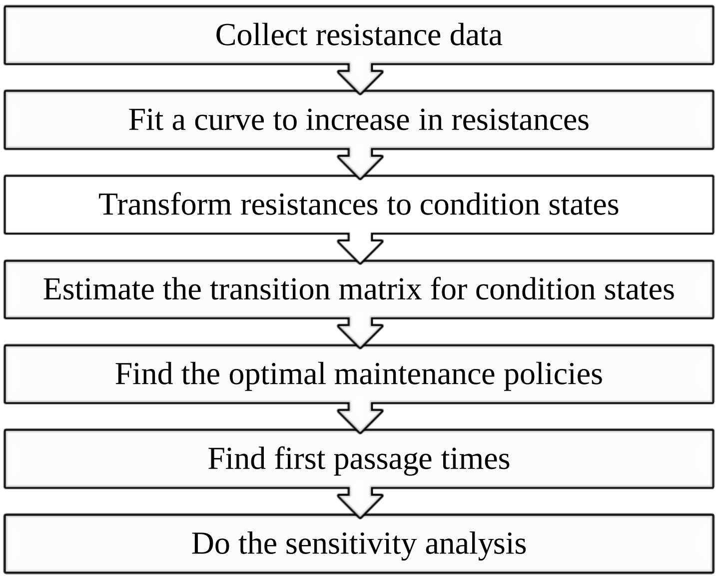

Figure 2 Methodology

2.3.1. Curve Fitting Model

The observed slip ring resistances can be seen in Figure 3. Points above threshold value (30 ohm) which is shown as a red line on the graph correspond to 9 slip ring failures. This figure is the graphical representation of complete data.

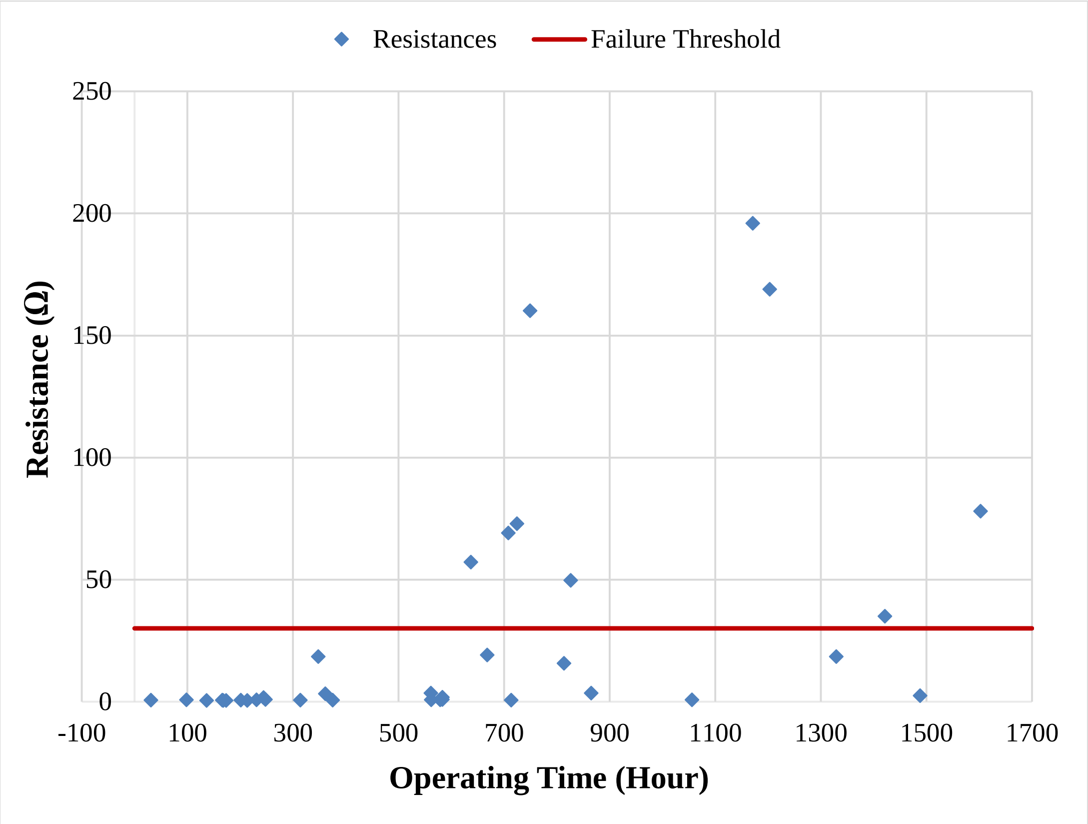

Figure 3 Observed Behavior of Slip Ring Resistances

According to state classifications given in Table 3 and Table 4 resistance values are converted to the condition states. We fit curves to both observed resistances and the corresponding states with respect to time t. We define R(t) as the resistance fit and S(t) as the state fit of a slip ring at time t.

We need these functions to estimate the transition matrix of the Markovian model that represents the state behavior if no interference is applied. It should represent the natural deterioration behavior of the slip rings. We expect a non-decreasing resistance fit with respect to time. Various functions like exponential, polynomial, power etc. are tried for those fits.

Goodness of these fits is measured using R^2, the coefficient of determination, and Root Mean Square Error. Since this is a single variable fit, these measures are sufficient. Besides, we have very few data points that is discouraging to conduct a detailed statistical analysis.

Table 5 Measurement Data

Slip Ring Serial	Pin	Rotation	Operating Hour	Resistance	Cond. State
17	10-11	55,800	31	0.7	1
15	4-5	176,400	98	0.8	1
11	19-20	626,400	348	18.5	2
19	1-2	649,800	361	3.3	1
19	7-8	1,146,600	637	57.2	3
11	19-20	1,202,400	668	19.2	2
11	13-14	1,348,200	749	160.2	3
10	13-14	1,463,400	813	15.7	2
9	16-17	1,486,800	826	49.7	3
13-2	7-8	1,557,000	865	3.5	1
10	19-20	2,107,800	1,171	196	3
20	13-14	2,392,200	1,329	18.5	2
13	7-8	2,557,800	1,421	35	3
17	19-20	2,678,400	1,488	2.5	1
20	4-5	2,884,860	1,603	78	3

Problem is analyzed with 3 condition states as the base case. Maximum values of resistance measurements and corresponding condition states with observation times are partially given in Table 5. Operating hour column is created from a simple conversion of number of rotations as explained before.

2.3.1.1. Fit to Observations

According to data, we use models given in Table 6 to find the best fits. We evaluate two options for curve fitting. Since we use single variable regression models, we try to check suitability of the model with R-square and Root Mean Square Error Measures which shows the difference between actual and estimated data.

Table 6 shows models providing highest measures for resistance fit alternative. As can be seen in Table 6, R^2 values are low. The best fits in that sense, Gaussian with 2 terms, give an undesirable behavior; the fitted curves do not have monotone behavior as we expect. The best fits can be seen in Figure 4 and Figure 5.

Table 6 Types of Models and Goodness of Fit Results 3-State

Model	**Data**	**R-Square**	**Root Mean Square Error**
Gaussian with 2 terms	Condition State versus Operating Time S(t)	0.4111	0.7220
Sum of Sine with 2 terms		0.4065	0.7248
Fourier with 2 terms		0.4031	0.7269
Polynomial 4th Degree		0.4016	0.7160
Power		0.3550	0.7098
Exponential		0.3161	0.7309
Gaussian with 2 terms	Resistance versus Operating Time R(t)	0.8174	23.4137
Sum of Sine with 2 terms		0.4331	41.2532
Fourier with 2 terms		0.3508	44,1439
Polynomial 4th Degree		0.3222	44.3747
Power		0.2314	45.1201
Exponential		0.1753	46.7374

Root Mean Square Error (RMSE) measures how much error there is between two data sets.

Figure 4 shows the Gaussian model for state fit which seems applicable in contrast to other evaluated models.

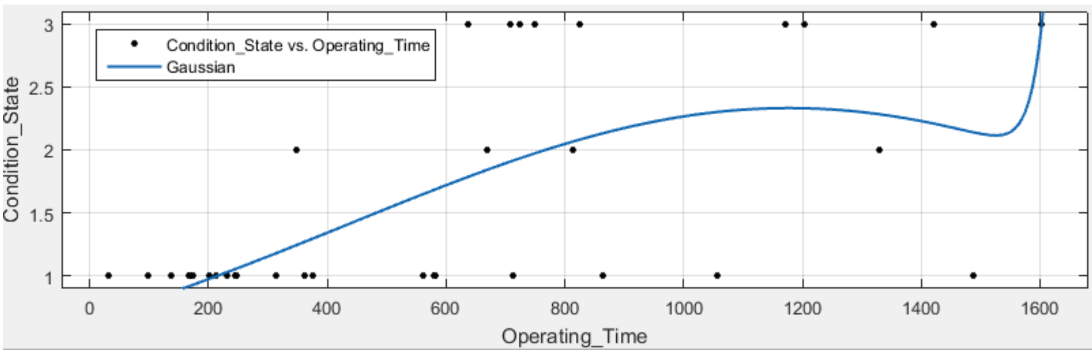

Figure 4 Gaussian Model for State Fit

Figure 5 shows the Gaussian model for resistance fit which seems applicable in contrast to other evaluated models.

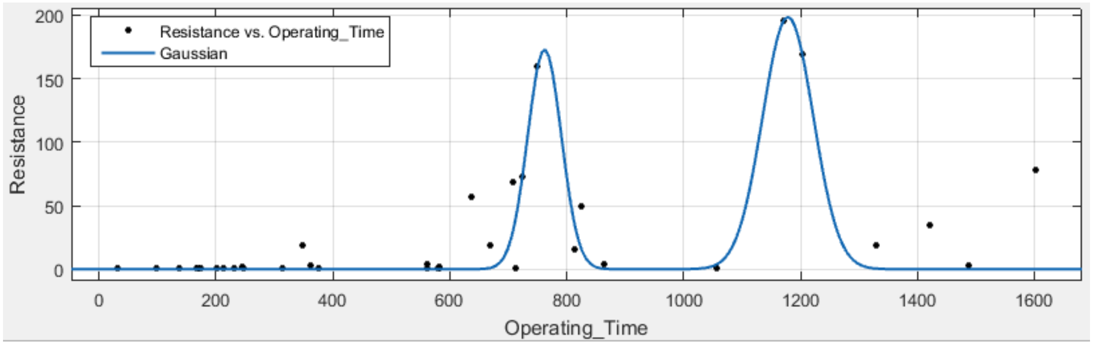

Figure 5 Gaussian Model for Resistance Fit

We observe similar results for the 4 state case of the problem. Table 7 shows models providing highest measures.

Table 7 Types of Models and Goodness of Fit Results 4-State

Model	**Data**	**R-Square**	**Root Mean Square Error**
Gaussian with 2 terms	Condition State versus Operating Time S(t)	0.4959	0.9955
Sum of Sine with 2 terms		0.4128	1.0743
Fourier with 2 terms		0.4139	1.0733
Polynomial 4^{th} Degree		0.4139	1.0559
Power		0.3789	1.0380
Exponential		0.3131	1.0915
Gaussian with 2 terms	Resistance versus Operating Time R(t)	0.8174	23.4137
Sum of Sine with 2 terms		0.4331	41.2532
Fourier with 2 terms		0.3508	44.1439
Polynomial 4^{th} Degree		0.3222	44.3747
Power		0.2314	45.1201
Exponential		0.1753	46.7374

As a solution we try to smooth the data by taking averages over the slip rings. In this case, we only fit curves for average resistances for the following reason: converting resistances to condition states causes some information loss about the slip rings. When we use averages as observations, number of observations decreases to 8 which does not seem to be sufficient for a limited response. Instead, corresponding condition states are determined from the fitted resistances at every time point.

2.3.1.2. Fit to Averages

In this section, we find a function that will represent the behavior of slip rings in other words, deterioration of resistances. Based on the mean life observation of slip rings, we bring together the measurements of each slip ring and try curve fitting. Since first failure times of these units are ranging from 637 to 1603 hours, the data cause our curve to have some fluctuations which may cause misrepresentation. This non-monotone behavior does not represent increasing resistance/deterioration of a slip ring well.

Therefore, we look at this part with averages of 200 hours of resistance, as well. This approach restricts us to study with less data points but provides higher R-square measures. Table 10 shows average resistances.

Table 8 200 Hours Resistance Averages

Time Interval (Hour)	Average Resistance	Condition State (3 State)	Condition State (4 State)
0-200	0.62	1	1
200-400	3.10	1	2
400-600	1.58	1	1
600-800	63.22	3	4
800-1000	22.97	2	3
1000-1200	98.40	3	4
1200-1400	93.75	3	4
1400-1600	38.50	3	4

According to data in Table 8, we check goodness of fits for the same models used in fit to observations section. Table 9 shows the fitting results for resistance fit that is both applicable for 3 and 4 state cases. As stated before we do not apply curve fitting for operating time versus state data.

Table 9 Types of Models and Goodness of Fit Results 3-State

Model	Data	R-square
Power	Resistance versus Operating Time	0.7822
Polynomial 4th Degree		0.7710
Gaussian with 1 term		0.7281
Exponential		0.7061
Fourier		N/A
Sum of Sine		N/A

In consideration of above assessments, we pick Power model for average resistances and continue to analysis process explained in Figure 2. Figure 6 shows the Power model for both 3 state and 4 state resistance fits.

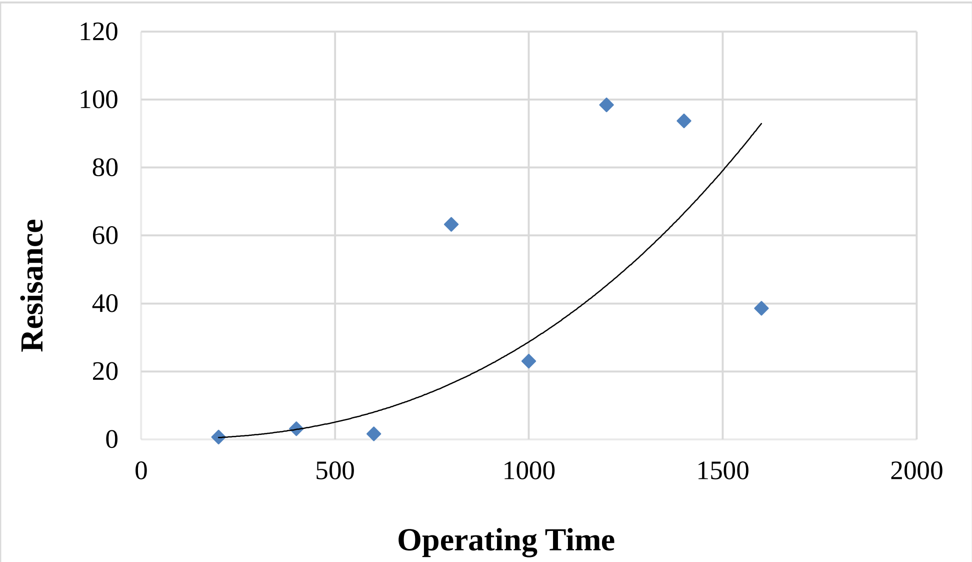

Figure 6 Power Fit – R(t)

From the fitting, coefficients for the Power function are obtained as

$$R(t) = 9x10^{-7}t^{2.5016} \quad (2.1)$$

According to Power fitting function Table 10 shows the time increments and corresponding condition states. Then, we move to next step, "estimate the transition matrix" with these results. All models that are used in this section are given in Appendix.

Table 10 Condition States for Time Increments

Operating Time	$\mathbf{R(t) = 9x10^{-7}t^{2.5016}}$	**Condition State (3 State)**	**Condition State (4 State)**
200	0.51	1	1
400	2.91	1	1
600	8.02	1	1
800	16.47	2	3
1000	28.78	3	4
1200	45.41	3	4
1400	66.77	3	4
1600	93.25	3	4

2.3.2. Markov Chain Model

Let X(t) be the condition of the slip ring at time t, X(t) in S = {1, 2, 3}. Here we use the fact that the resistances are non-decreasing over time and assume that slip ring condition state may increase more than one state. Under this setting, the transition matrix is an upper triangular matrix as in (2.2) if not interfered. So, for a 3 state chain, there are 3 transition probabilities to estimate. Under do nothing action p_{33} equals one.

$$P = \begin{bmatrix} p_{11} & p_{12} & 1 - p_{11} - p_{12} \\ 0 & p_{22} & 1 - p_{22} \\ 0 & 0 & p_{33} \end{bmatrix} \tag{2.2}$$

The estimation of these probabilities is performed in two steps:

i. Fit a curve to the deterioration of the resistance of a slip ring over time (described in the previous section)
ii. Estimate the transition probabilities that produce closest expected states to that curve

Along with the fitted function, the Markov Chain model forms the objective function of a nonlinear regression approach to estimate the transition probabilities. The initial probability distribution over states $I(0)$ and transition matrix $P_{ij}(t)$ are the main constituents of this model. Estimating the probability distribution over states by multiplying the initial state vector and power of transition matrix is known as a Markovian property used in the estimation process.

Let I(t) be the initial probability distribution at time t and let P(t)=[$p_{ij}(t)$] be the t step transition matrix of the process. Then we have

$$I(t) = I(0) \text{ x } P^t \tag{2.3}$$

and the expected state at time t under P is

$$E(X(t), P) = \sum_{i \in S} \sum_{j \in S} j \, P\{ X(t) = j \mid X(0) = i \} P\{X(0) = i\}$$

$$= \sum_{i \in S} \sum_{j \in S} j \, p_{ij} \, P\{X(0) = i\}$$

$$= I(t) \text{x } R = I(0) \text{ x } P^t \text{ x } R \tag{2.4}$$

where R is the transpose of state vector [1, 2, 3].

We estimate the transition probabilities in (2.2) so that total “distance” between the state obtained from the fitted curve and expected value of the state from Markov chain model is minimized. Minimum time increment that the inspections are performed in data is 200 hours and the last inspection is at 1600. Hence, we need P^t, t=1, 2, …, 8 to construct (2.4). Maximum number of matrix power is simply the division of last inspection time by 200 hours of time increment.

The model to give the unknown transition probabilities (decision variables of the model) p_{ij} is

$$\text{Minimize } \sum_{t=1}^{8} |Y(t) - E[X(t), P)]| \qquad (2.5)$$

Subject to

$$0 \leq p_{ij} \leq 1 \qquad (2.6)$$

where

$Y(t)$ is the condition state at time t obtained from fitted function,

$E[X(t), P)]$ is the expected value of the condition state as in time t given in (2.4).

2.3.3. Optimization with Markov Decision Process

Markov Decision Process (MDP) is a tool to select the appropriate action at value of the observed state of a Markovian process so that an objective is achieved in the long run. Let X(t) be the state of the process/slip ring at time t taking values from *the state space* S = {As good as new (1), Up (2), Down (3)}. Suppose we observe the state of the process at every period t (at every 200 hours). We can either i) "*do nothing*" or ii) "*maintain*" depending on the state of the process. We call A = { do nothing (0), maintain (1) } *the action set*. If do nothing action is applied then X(t) evolves according to the transition matrix P(do nothing) = [$p_{ij}(0)$] estimated from (2-5)-(2.6). If maintain action is applied then the states improve according to a different transition matrix P(maintain) = [p_{ij} (1)]. We assume various forms for P(maintain) in computations.

The following linear programming formulation is used to obtain the optimal steady state probabilities, and equivalently, the optimal policy with respect to a linear objective function:

$$\text{Minimize } \sum_{i \in S} \sum_{a \in A}^{A} c_{ia} \pi_{ia} \quad (2.7)$$

Subject to

$$\sum_{a \in A} \pi_{ja} - \sum_{i \in S} \sum_{a \in A} \pi_{ia} p_{ij}(a) = 0 \quad j \in S \quad (2.8)$$

$$\sum_{i \in S} \sum_{a \in A} \pi_{ia} = 1 \quad (2.9)$$

$$\pi_{ia} \geq 0, \; i \in S, \quad a \in A \quad (2.10)$$

where the decision variables

$$\pi_{ia} = \lim_{t \to \infty} P\,\{X(t) = i, \; A(t) = a\}$$

and the parameters

c_{ia} is the expected cost of taking action a in state i,

$p_{ij}(a)$ is the transition probability from state i to state j under action a,

are used.

It is well known that, due to the coefficient matrix in (2.8)-(2-9) there is a deterministic optimal policy obtained by

$$\lim_{t \to \infty} P\,\{A(t) = a |\, X(t) = i\} = \frac{\pi_{ia}}{\sum_{a \in A} \pi_{ia}}$$

$$= \begin{cases} 1 & \text{if } \pi_{ia} > 0 \\ 0 & \text{otherwise} \end{cases}$$

A policy gives which action to use at each state. Optimal policies naturally differ with respect to the objective functions. We used several optimization criteria in this study:

- Minimize long run average cost: $\sum_{i \in S} \sum_{a \in A} c_{ia} \pi_{ia}$
- Maximize / minimize long run probability of up or down states: $\sum_{a \in A} \pi_{ia}$
- Maximize first passage time from up states to a down state

The cost quantities c_{ia} in (2.7) are difficult to measure because maintenance is really cheap except the travel costs. Failure on the other hand is disastrous and long downtimes may cause catastrophic results.

In MDP methodology, usually objective functions that are linear functions of the steady state probabilities are employed as extensions of above linear program. In the present study, we also used the maximization of the expected first passage time to failure state. For a given transition matrix $P=[p_{ij}]$, expected first passage times μ_{ij} time from state i to state j can be computed by solving the following system of linear equations:

$$\mu_{ij} = 1 + \sum_{\substack{k\neq j \\ k\in S}} p_{ik}\, \mu_{kj} \qquad i, j \in S \tag{2.11}$$

However, when the policy is unknown, expected first passage times are not linear functions of the unknown steady state probabilities of the linear program. We can rewrite equation (2.11) in terms of $\pi_{ia} = \lim_{t\to\infty} P\,\{X(t) = i,\ A(t) = a\}$ as follows:

$$\mu_{iF} = 1 + \left[\sum_{\substack{j\neq F \\ j\in S}} \sum_{a\in A} \lim_{t\to\infty} P\{A(t) = a \,|\, X(t) = i\}\, p_{ij}(a)\right] \mu_{jF}$$

$$\mu_{iF} = 1 + \left[\sum_{\substack{j\neq F \\ j\in S}} \sum_{a\in A} \frac{\pi_{ia}}{\sum \pi_{ia}}\, p_{ij}(a)\right] \mu_{jF}$$

Hence, the following nonlinear model gives the steady state probabilities that maximize first passage times to failure state from any state i:

$$\text{Maximize } \mu_{iF} \tag{2.12}$$

Subject to

$$\mu_{iF} = 1 + \left[\sum_{\substack{j \neq F \\ j \in S}} \sum_{a \in A} \frac{\pi_{ia}}{\sum \pi_{ia}} p_{ij}(a)\right] \mu_{jF} \tag{2.13}$$

(2.8), (2.9), and (2.10).

The optimal solution of this MDP formulation is expected to produce randomized policies, i.e., π_{ia} is positive for more than one action. These policies are difficult to apply in general.

In order to produce deterministic optimal policies, we introduce the following constraints:

$$\prod_{a \in A} \pi_{ia} = 0 \qquad \forall i \tag{2.14}$$

CHAPTER 3

COMPUTATIONAL RESULTS

The maintenance of the slip rings is quite straight forward requiring simple lubrication. In fact, such an activity renews any pin to a state that is as good as new. When visited, all the pins are lubricated leaving a slip ring that is also as good as new. The difficulty of monitoring and maintaining is in their remoteness. However, we need the probability distribution after one transition. We can use various assumptions for that. We assumed that all states evolve as state 1 evolves under do nothing action in one period.

We first give the results for base case with S = {As good as new (1), Up (2), Down (3)}. We then conduct parametric analysis for two different deterioration assumptions after a "Maintain" action is performed. For this case, we keep the first row of transition matrix under do nothing action as they estimated.

We use the following two transition matrices under maintain (1) action as two separate cases:

$$P(1) = \begin{bmatrix} p_{11}(0) & p_{12}(0) & p_{13}(0) \\ \alpha & \beta & 1-\alpha-\beta \\ \alpha & \beta & 1-\alpha-\beta \end{bmatrix} \tag{3.1}$$

$$P(1) = \begin{bmatrix} p_{11}(0) & p_{12}(0) & p_{13}(0) \\ \alpha & 1-\alpha & 0 \\ \beta & 1-\beta & 0 \end{bmatrix} \tag{3.2}$$

We also give the results for 4 state case with S = {As good as new (1), Up (2), Degraded (3), Down (4)} and conduct parametric analysis, as well. At this time, we assume same parameters for all state transitions.

$$P(2) = \begin{bmatrix} \alpha & \beta & 1-\alpha-\beta & 0 \\ \alpha & \beta & 1-\alpha-\beta & 0 \\ \alpha & \beta & 1-\alpha-\beta & 0 \\ \alpha & \beta & 1-\alpha-\beta & 0 \end{bmatrix} \tag{3.3}$$

3.1. Base Case: 3 Condition States

Matrix structure under "Do Nothing" action consists of 3 states and probabilities for the upper triangular part which implies it is not probable to observe an improvement on the measured resistances and radar will stay in state 3 unless an intervention is made or an action is taken.

$$P(0) = \begin{bmatrix} p_{11} & p_{12} & 1-p_{11}-p_{12} \\ 0 & p_{22} & 1-p_{22} \\ 0 & 0 & 1 \end{bmatrix}$$

Open form of objective function that belongs to nonlinear minimization model can be written as $Y(t) - E(X(t), P) = Y(t) - I(0).P(t).R$ and running of this optimization model gives us transition probabilities.

As described in the previous chapter

- I(0) is initial state vector and in this thesis it is always assumed as I(0) = [1 0 0].
- P (t) is the t^{th} power of the transition matrix. In this problem, we took the powers of transition matrix from 1 to 8 and R is the transpose of the state vector.

Close and open forms of nonlinear optimization model are given below:

$$\text{Min} \sum_{t=1}^{8} |Y(t) - E[(X(t), P)]|$$

$$0 \leq p_{ij} \leq 1$$

$\text{Min}\ |Y(1) - E[(X(1), P)]| + |Y(2) - E[X(2), P)]| + |Y(3) - E[X(3), P)]| + |Y(4) - E[X(4), P)]| + |Y(5) - E[X(5), P)]| + |Y(6) - E[X(6), P)]| + |Y(7) - E[X(7), P)]| + |Y(8) - E[X(8), P)]|$

$$0 \leq p_{11} \leq 1$$

$$0 \leq p_{12} \leq 1$$

$$0 \leq p_{22} \leq 1$$

This is equivalent to

$\text{Minimize}\ \mid 1 - (3 - 2x(1) - x(2))) + (1 - (-2x(1)^2 - x(1)x(2) - x(2)x(3) + 3)) + (1 - (-2x(1)^3 - x(2)x(3)^2 - x(1)^2x(2) - x(1)x(2)x(3) + 3)) + (2 - (-2x(1)^4 - x(2)x(1)^3 - x(2)x(1)^2x(3) - x(2)x(1)x(3)^2 - x(2)x(3)^3 + 3)) + (3 - (-2x(1)^5 - x(2)x(1)^4 - x(2)x(1)^3x(3) - x(2)x(1)^2x(3)^2 - x(2)x(1)x(3)^3 - x(2)x(3)^4 + 3)) + (3 - (-2x(1)^6 - x(2)x(1)^5 - x(2)x(1)^4x(3) - x(2)x(1)^3x(3)^2 - x(2)x(1)^2x(3)^3 - x(2)x(1)x(3)^4 - x(2)x(3)^5 + 3)) + (3 - (-2x(1)^7 - x(2)x(1)^6 - x(2)x(1)^5x(3) - x(2)x(1)^4x(3)^2 - x(2)x(1)^3x(3)^3x(3)^3 - x(2)x(1)^2x(3)^4 - x(2)x(1)x(3)^5 - x(2)x(3)^6 + 3)) + (3 - (-2x(1)^8 - x(2)x(1)^7 - x(2)x(1)^6x(3) - x(2)x(1)^5x(3)^2 - x(2)x(1)^4x(3)^3 - x(2)x(1)^3x(3)^4 - x(2)x(1)^2x(3)^5 - x(2)x(1)x(3)^6 + x(2)x(3)^7 + 3)) \mid$

$$x(1) + x(2) < 1$$

$$0 < x(3) < 1$$

In this formulation X(1), X(2) and X(3) correspond to p_{11}, p_{12} and p_{22} respectively. In order to solve this model a Matlab code [2] is used. Matlab code scans all possible regions according to possible (0, 1) range for unknown variables and perform lots of iterations to search for minimum objective function value. At first 10 initial points are generated and transition probabilities and corresponding matrices are obtained. Then more points are generated and same results are found to search an optimal point.

Determined matrix structure under do nothing action consists of 3 unknown probabilities and they are generated as p_{11}=0.8136, p_{12}=0.1038 and p_{22}=0.5730 with the minimum objective function value of $1.11e^{-10}$ which yields a transition probability matrix:

$$\mathrm{P}(0) = \begin{bmatrix} 0.8136 & 0.1038 & 0.0826 \\ 0 & 0.5730 & 0.4270 \\ 0 & 0 & 1 \end{bmatrix}$$

Under maintain action it is assumed that rows of transition probability matrix is equal to each other and radar wears out with the probabilities of first row of the transition matrix under do nothing action.

$$\mathrm{P}(1) = \begin{bmatrix} 0.8136 & 0.1038 & 0.0826 \\ 0.8136 & 0.1038 & 0.0826 \\ 0.8136 & 0.1038 & 0.0826 \end{bmatrix}$$

Estimated probabilities yield a condition state for every time increment. Open form of objective function given in (2.5) is used to check the performance of prediction. x, y and z in the model corresponds to unknown probabilities respectively P_{11}, P_{12} and P_{22} and Table 11 shows related points generated from fitting function and Markov Chain model.

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[2] All Matlab codes used for solving linear and nonlinear optimization models are given in APPENDIX section of the thesis.

Table 11 3-State Results

t	R(t)	S(t)	E(X(t),P)
200	0.51	1	1.26900
400	2.91	1	1.58309
600	8.02	1	1.84650
800	16.47	2	2.06145
1000	28.78	3	2.23639
1200	45.41	3	2.37873
1400	66.77	3	2.49453
1600	93.25	3	2.58875

We use Table 11 to see how close the estimated points are. Generated points and comparison can be shown in a graph, as well. Figure 7 shows the differences between Y(t) and E[X(t),P)] along with the observed states which are transformed from average resistances.

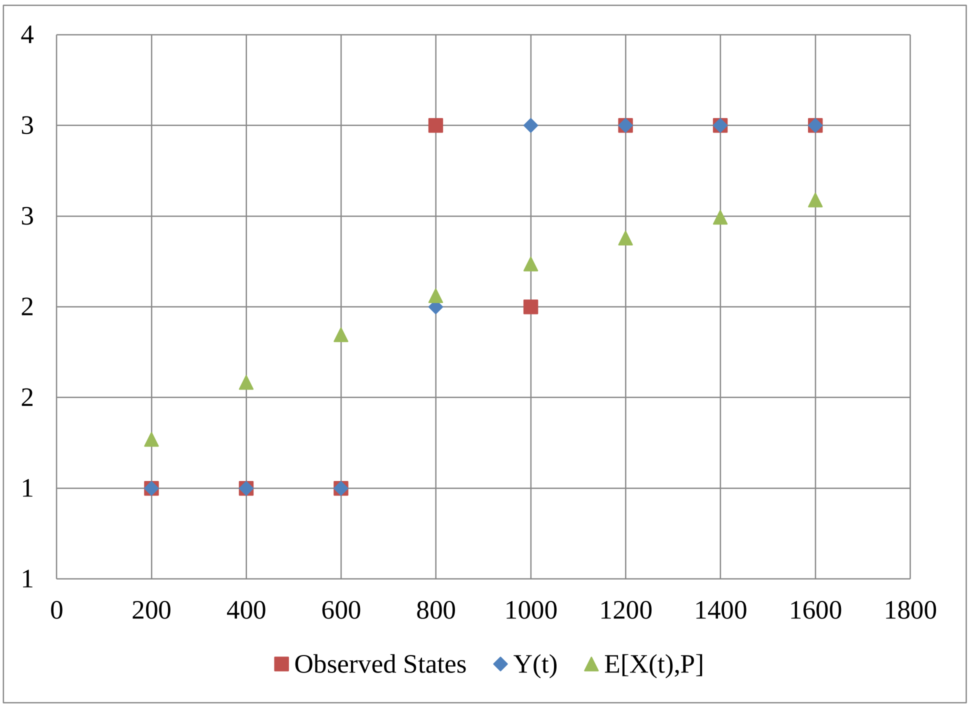

Figure 7 State Estimations

Second step of the optimization problem is Markov Decision Process. Model given in objective function (2.7) and constraints (2.8), (2.9) and (2.10) is used to obtain steady state probabilities and policies. This is our first model for the base case.

Objective function is determined as minimizing operation costs which are comprised of replacement cost of 1000 and failure cost of 10000. But these cost values are used for evaluation purposes. They do not reflect real values. Three states and 2 possible actions create a cost matrix of

$$C = \begin{bmatrix} 0 & 1000 \\ 0 & 1000 \\ 10000 & 1000 \end{bmatrix}$$

Open form of optimization problem is written as:

Min $1000\,\Pi_{11}+1000\Pi_{21}+10000\Pi_{30}+1000\Pi_{31}$

Subject to

$\Pi_{10}+\Pi_{11}+\Pi_{20}+\Pi_{21}+\Pi_{30}+\Pi_{31}=1$

$(1-P_{11}(0))\,\Pi_{10}+(1-P_{11}(1))\,\Pi_{11}-\Pi_{20}P_{21}(0)-\Pi_{21}P_{21}(1)-\Pi_{30}P_{31}(0)-\Pi_{31}P_{31}(1)=0$

$-P_{12}(0)\,\Pi_{10}-P_{12}(1)\,\Pi_{11}+(1-P_{22}(0))\Pi_{20}+(1-P_{22}(1))\,\Pi_{21}-\Pi_{30}P_{32}(0)-\Pi_{31}P_{32}(1)=0$

$-P_{13}(0)\,\Pi_{10}-P_{13}(1)\,\Pi_{11}-P_{23}(0)\,\Pi_{20}-P_{23}(1)\,\Pi_{21}+(1-P_{33}(0))\,\Pi_{30}+(1-P_{33}(1))\,\Pi_{31}=0$

$\Pi_{ia} \geq 0$ i=1, 2, 3 and a=0, 1

Optimal steady state probabilities

$$\Pi_{ia} = \begin{bmatrix} 0.6545 & 0 \\ 0.1956 & 0 \\ 0 & 0.1499 \end{bmatrix}$$

determine optimal maintenance policy as do nothing in states 1 and 2 and maintain in state 3.

Next, expected first passage times are calculated based on the optimal policy. Transition probability under the above optimal policy is

$$P = \begin{bmatrix} 0.8136 & 0.1038 & 0.0826 \\ 0 & 0.5730 & 0.4270 \\ 0.8136 & 0.1038 & 0.0826 \end{bmatrix}$$

First passage times to failure state 3 is calculated from the state 1 and state 2 with the following equations:

$$\mu_{1F} = 1 + P_{11}\mu_{1F} + P_{12}\mu_{2F} \tag{3.4}$$

$$\mu_{2F} = 1 + P_{21}\mu_{1F} + P_{22}\mu_{2F} \tag{3.5}$$

- Expected first passage time to failure state from state 1: 6.67 transition periods
- Expected first passage time to failure state from state 2: 2.34 transition periods

Results mean that failure time from state 1 under policy obtained with above MDP model is approximately is 1330 hours and failure time from state 2 is 460 hours in accordance with the transition of 200 hours.

In this regard, our policy is very similar to company's current approach against unexpected failures of slip rings. We intervene the units upon failures and expect a failure time from state 1 to be approximately 1330 hours. Then we can suggest checking these units with state 1 before 1300 hours of operation and in the same manner units with state 2 before 400 hours of operation. We think that this intervals suit the recent situation well. But we are aware of the need for sufficient data in order to make more accurate suggestions.

We next solve the second model which maximizes first passage time to failure state from state 1.

In here, μ_{iF} is the expected first passage time to failure state from starting state i. After incorporating transition probabilities under do nothing and maintenance actions the model becomes

Maximize μ_{1F}

Subject to

$$\pi_{10} + \pi_{11} + \pi_{20} + \pi_{21} + \pi_{30} + \pi_{31} = 1$$

$$\pi_{10}\left(1 - P_{11}(0)\right) + \pi_{11}\left(1 - P_{11}(1)\right) - \pi_{20}P_{21}(0) - \pi_{21}P_{21}(1) - \pi_{30}P_{31}(0) - \pi_{31}P_{31}(1) = 0$$

$$-\pi_{10}P_{12}(0) - \pi_{11}P_{12}(1) + \pi_{20}\left(1 - P_{22}(0)\right) + \pi_{21}(1 - P_{22}(1)) - \pi_{30}P_{32}(0) - \pi_{31}P_{32}(1) = 0$$

$$-\pi_{10}P_{13}(0) - \pi_{11}P_{13}(1) - \pi_{20}P_{23}(0) - \pi_{21}P_{23}(1) + \pi_{30}\left(1 - P_{33}(0)\right) + \pi_{31}(1 - P_{33}(1)) = 0$$

$$\pi_{ia} \geq 0 \qquad \forall\, a, i$$

$$\mu_{1F} = 1 + \left[\frac{\pi_{10}}{\pi_{10}+\pi_{11}} P_{11}(0) + \frac{\pi_{11}}{\pi_{10}+\pi_{11}} P_{11}(1)\right] \mu_{1F} + \left[\frac{\pi_{10}}{\pi_{10}+\pi_{11}} P_{12}(0) + \frac{\pi_{11}}{\pi_{10}+\pi_{11}} P_{12}(1)\right] \mu_{2F} \qquad (3.6)$$

$$\mu_{2F} = 1 + \left[\frac{\pi_{20}}{\pi_{20}+\pi_{21}} P_{21}(0) + \frac{\pi_{21}}{\pi_{20}+\pi_{21}} P_{21}(1)\right] \mu_{1F} + \left[\frac{\pi_{20}}{\pi_{20}+\pi_{21}} P_{22}(0) + \frac{\pi_{21}}{\pi_{20}+\pi_{21}} P_{22}(1)\right] \mu_{2F} \qquad (3.7)$$

$$\mu_{iF} \geq 0 \qquad \text{for } i = 1 \text{ and } 2$$

The nonlinear equations (3.6) and (3.7) of the model can be re-written as

$$0.1864\mu_{1F} - 0.1038\mu_{2F} = 1$$

$$0.4270\pi_{20}\mu_{2F} + 0.8962\pi_{21}\mu_{2F} - 0.8136\pi_{21}\mu_{1F} - \pi_{20} - \pi_{21} = 0$$

The optimal solution is

$\pi_{10} = 0.3387$

$\pi_{11} = 0.3387$

$\pi_{21} = 0.086$

$\pi_{30} = 0.1673$

$\pi_{31} = 0.0688$

$\mu_{1F} = 12.1065$

$\mu_{2F} = 12.1065$

which is highly randomized. Randomized policies are very difficult to implement. In order to avoid this, we add the following constraints to obtain deterministic optimal policies:

$\pi_{10}\pi_{11} = 0$

$\pi_{20}\pi_{21} = 0$

$\pi_{30}\pi_{31} = 0$

Then the optimal solution is

$\pi_{10} = 0.8136$

$\pi_{21} = 0.1038$

$\pi_{31} = 0.0826$

$\mu_{1F} = 12.1065$

$\mu_{2F} = 12.1065$

which uses do nothing action in state 1 and maintain in states 2 and 3. If we adopt this policy, we can suggest higher inspection frequency with higher costs.

Our third model given below finds maintenance policies with the objective of minimizing the steady state probability of failure state.

Min $\Pi_{30} + \Pi_{31}$

Subject to

$\Pi_{10} + \Pi_{11} + \Pi_{20} + \Pi_{21} + \Pi_{30} + \Pi_{31} = 1$

$(1-P_{11}(0))\,\Pi_{10} + (1-P_{11}(1))\,\Pi_{11} - \Pi_{20}P_{21}(0) - \Pi_{21}P_{21}(1) - \Pi_{30}P_{31}(0) - \Pi_{31}P_{31}(1) = 0$

$-P_{12}(0)\,\Pi_{10} - P_{12}(1)\,\Pi_{11} + (1-P_{22}(0))\,\Pi_{20} + (1-P_{22}(1))\,\Pi_{21} - \Pi_{30}P_{32}(0) - \Pi_{31}P_{32}(1) = 0$

$-P_{13}(0)\,\Pi_{10} - P_{13}(1)\,\Pi_{11} - P_{23}(0)\,\Pi_{20} - P_{23}(1)\,\Pi_{21} + (1-P_{33}(0))\,\Pi_{30} + (1-P_{33}(1))\,\Pi_{31} = 0$

$\Pi_{ia} \geq 0$ i=1, 2, 3 and a=0, 1

Steady state probabilities are

$\pi_{10} = 0.8136$

$\pi_{21} = 0.1038$

$\pi_{31} = 0.0826$

which uses do nothing action in state 1 and maintain in states 2 and 3.

Table 12 Comparison of Optimization Modes for the Base Case

	$\text{Min} \sum_{i \in S} \sum_{a \in A}^{A} c_{ia} \pi_{ia}$	Max μ_{13}	Min $\Pi_{30} + \Pi_{31}$
Cost	149.95	186.4	186.4
$\Pi_{30} + \Pi_{31}$	0.1499	0.0826	0.0826
μ_{13}	6.67 transitions	12.11 transitions	12.11 transitions
Policy	1: DN 2: DN 3: Maintain	1: DN 2: Maintain 3: Maintain	1: DN 2: Maintain 3: Maintain

Table 12 summarizes the results of optimization models. We obtain different policies for different objectives. It is clear from the table that if it is desirable to have longer expected first passage times to failure state, higher costs have to be accepted.

Since the transition probability matrix under maintain action is an assumption, a parametric analysis is conducted to examine sensitivity of the problem.

3.1.1. Case 1

Two different matrix structures are developed for parametric analysis. Main purpose of this section is to see how suitable maintenance policies change in accordance with the transition probabilities. Case 1 is analyzed with the following transition matrix:

$$P(1) = \begin{bmatrix} 0.8136 & 0.1038 & 0.0826 \\ \alpha & \beta & 1-\alpha-\beta \\ \alpha & \beta & 1-\alpha-\beta \end{bmatrix}$$

Matlab code to obtain steady state probabilities for base case is slightly modified for case 1 and α and β are increased with 0.1 increments to see differences in maintenance policies. Table 13 shows some of the solutions.

Let Policy A be maintain in 3 only (states 1 and 2 are not observed),

Policy B be do nothing in 2, maintain in 3 (state 1 is not observed),

Policy C be do nothing in 1 and 2, maintain in 3.

Table 13 Case 1 Results

α	**β**	**$\pi 10$**	**$\pi 11$**	**$\pi 20$**	**$\pi 21$**	**$\pi 30$**	**$\pi 31$**	**Policy**
0	0						1.0000	A
0	0.1			0.1898			0.8102	B
0	0.2			0.3190			0.6810	B
0	0.4			0.4837			0.5163	B
0	0.6			0.5842			0.4158	B
0	0.7			0.6211			0.3789	B
0	0.8			0.6520			0.3480	B
0	0.9			0.6782			0.3218	B
0	1			0.7008			0.2992	B
0.1	0	0.3218		0.0782			0.5999	C
0.1	0.1	0.2822		0.1918			0.5260	C
0.1	0.2	0.2512		0.2804			0.4683	C
0.1	0.3	0.2264		0.3516			0.4220	C
0.1	0.4	0.2060		0.4099			0.3841	C
0.1	0.5	0.1890		0.4586			0.3524	C

Results are illustrated in Figure 8. We assumed increasing deterioration probabilities represented by $\alpha > \beta > 1 - \alpha - \beta$.

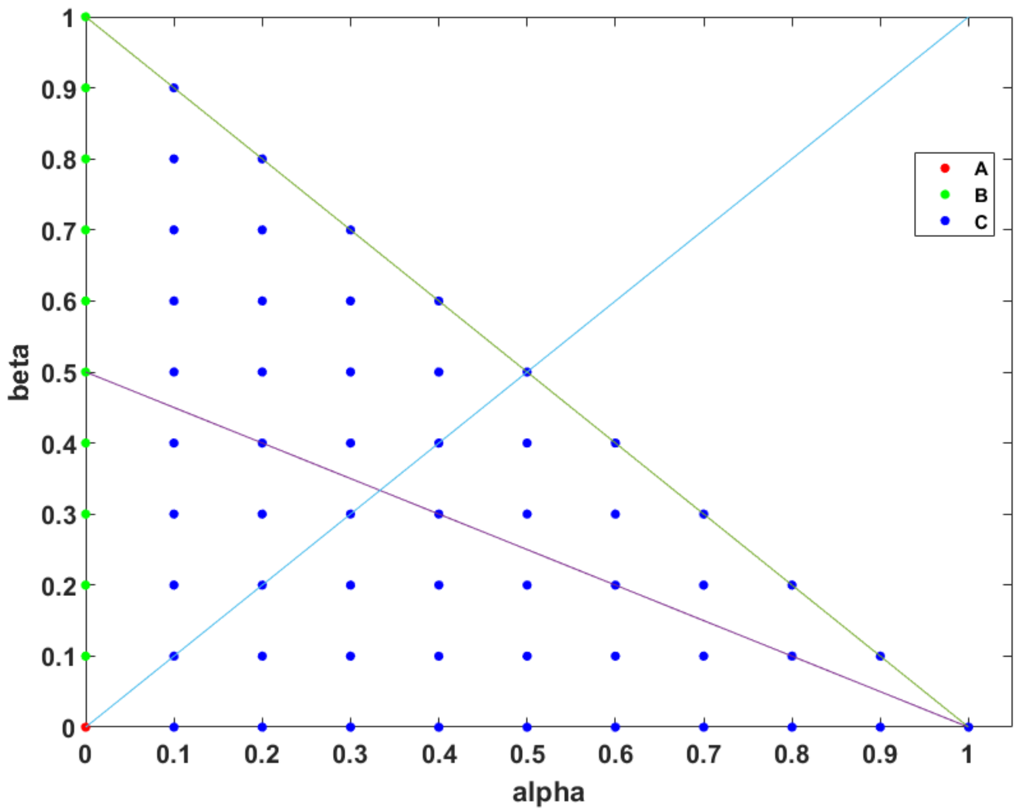

Figure 8 Changes in Optimal Policy wrt α and β for Case 1

Figure 8 shows that for every value of α except zero we obtain policy C which we do nothing for states 1 and 2 and perform a maintenance for state 3. If α =0 meaning it is not possible to completely renew the unit after maintenance, the optimal policy eliminates state 1. Optimal policy does not change although our assumption is not valid for some regions of the graph.

3.1.2. Case 2

Another case in the name of parametric analysis, Case 2, is analyzed with the following matrix structure.

$$P(1) = \begin{bmatrix} 0.8136 & 0.1038 & 0.0826 \\ \alpha & 1-\alpha & 0 \\ \beta & 1-\beta & 0 \end{bmatrix}$$

In this case it is assumed that if we perform a replacement while in state 2, we will not see state 3 at the next inspection. Same rule applies for state 3, too. We use Matlab to obtain steady state probabilities. Table 14 shows some of the solutions. Policy B and C are the same policies found in case 1. Different from the case 1, we find policy D.

Let Policy B be do nothing in state 2, maintain in 1 only (states 1 is not observed),

Policy C be do nothing in 1 and 2, maintain in 3,

Policy D be do nothing in 1, maintain in 2 and 3.

Table 14 Case 2 Results

α	**β**	**π10**	**π11**	**π20**	**π21**	**π30**	**π31**	**Policy**
0	0			0.7008			0.2992	B
0	0.1	0.1421		0.5929			0.2649	C
0	1	0.6995		0.1701			0.1304	C
0.1	0			0.7008			0.2992	B
0.1	0.1	0.1421		0.5929			0.2649	C
0.1	0.8	0.6308		0.2222			0.1470	C
0.1	0.9	0.6672		0.1946			0.1382	C
0.2	0			0.7008			0.2992	B
0.2	0.1	0.1421		0.5929			0.2649	C
0.3	0.7	0.5895		0.2536			0.1570	C
0.4	0			0.7008			0.2992	B
0.4	0.1	0.1421		0.5929			0.2649	C
0.5	0.4	0.4230		0.3798			0.1971	C
0.5	0.5	0.4872		0.3311			0.1816	C
0.6	0	0.7177			0.2230		0.0593	D
0.6	0.1	0.1421		0.5929			0.2649	C
0.6	0.3	0.3469		0.4376			0.2155	C
0.6	0.4	0.4230		0.3798			0.1971	C
0.7	0	0.7414			0.1974		0.0612	D
0.8	0.1	0.7661			0.1706		0.0633	D
1	0	0.7880			0.1469		0.0651	D

Figure 9 shows the results. Here, we assume both α and β to be greater than 0.5 in order to satisfy $p_{21}>p_{22}$ and $p_{31}>p_{32}$. For this case three different maintenance policies are in picture, as well. Our expectation for α and β shows that policy C is optimal. For this policy we do nothing for states 1 and 2 and maintain for state 3. If β is less than 0.5, p_{32} will be greater than p_{31} and this may imply policy D for the values of α greater than 0.6. Policy D makes us to do nothing for state 1 and perform a maintenance for states 2 and 3 since we have more probability for seeing the slip rings in state 2 after a maintenance for slip rings at state 3. This means being more precautious than the policy C.

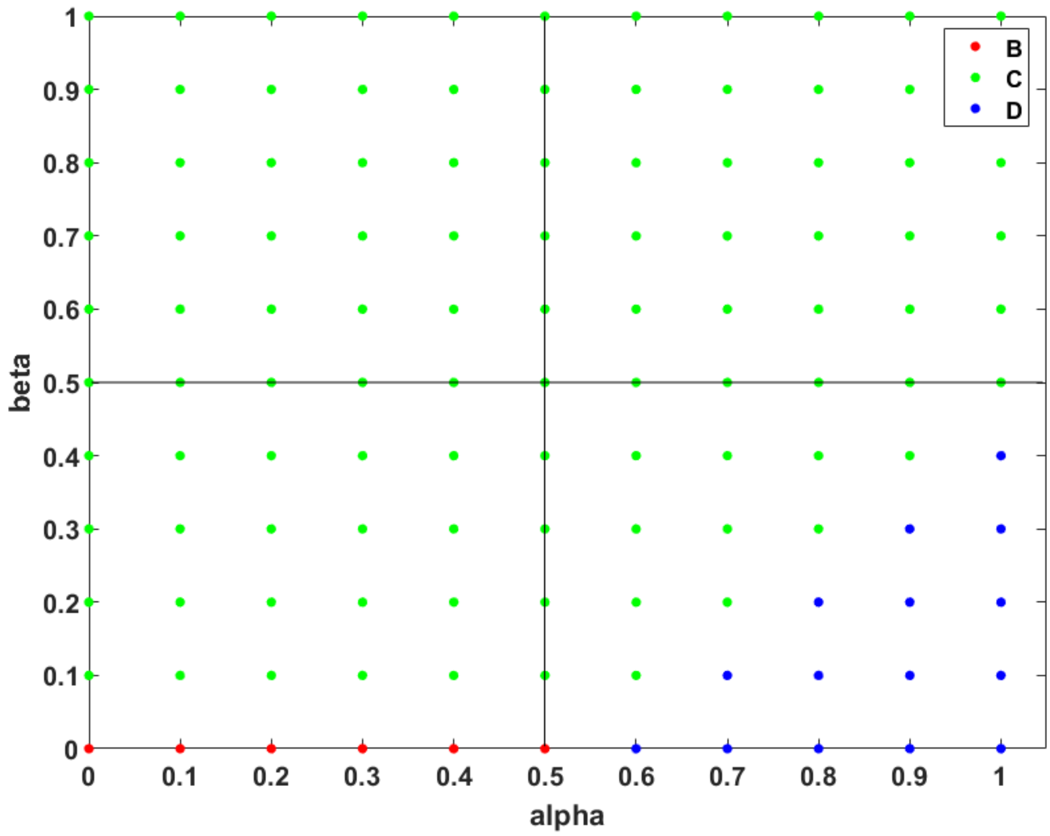

Figure 9 Changes in Optimal Policy wrt α and β for Case 2

3.2. 4-State Case

In this case, one more condition state is added to condition state classification and same data given in Table 5 is used again. Table 15 partially shows the state classification according to measured resistances.

Matrix structure for this case is given below and does not constitute a major difference from the case with 3 states. Matrix structure under do nothing action:

$$P(0) = \begin{bmatrix} P_{11} & P_{12} & P_{13} & 1 - P_{11} - P_{12} - P_{13} \\ 0 & P_{22} & P_{23} & 1 - P_{22} - P_{23} \\ 0 & 0 & P_{33} & 1 - P_{33} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Condition states are classified in parallel with the new case.

Table 15 Measurement Data

Slip Ring Serial	Pin	Rotation	Operating Hour	Cond. State	Resistance
17	10-11	55,800	31	1	0.7
15	4-5	176,400	98	1	0.8
11	19-20	626,400	348	3	18.5
19	1-2	649,800	361	2	3.3
19	7-8	1,146,600	637	4	57.2
11	19-20	1,202,400	668	3	19.2
11	13-14	1,348,200	749	4	160.2
10	13-14	1,463,400	813	2	15.7
9	16-17	1,486,800	826	4	49.7
13-2	7-8	1,557,000	865	2	3.5
10	19-20	2,107,800	1,171	4	196
20	13-14	2,392,200	1,329	3	18.5
13	7-8	2,557,800	1,421	4	35
17	19-20	2,678,400	1,488	1	2.5
20	4-5	2,884,860	1,603	4	78

Next, we use Markov Chain model for determining transition probabilities. As stated in the previous cases a nonlinear optimization model is used to estimate unknown transition probabilities. Close form of nonlinear optimization model is given below:

$$\text{Min} \sum_{t=1}^{8} |Y(t) - E[(X(t), P)]|$$

$$0 \leq p_{ij} \leq 1$$

In this formulation X(1), X(2), X(3), X(4), X(5) and X(6) corresponds to p_{11}, p_{12}, p_{13}, p_{22}, p_{23} and p_{33} respectively. With initial starting point generation assumption, our code tries to scan all possible regions according to possible (0, 1) range for unknown variables and perform lots of iterations to search for minimum objective function value for 4 state problem structure.

Transition matrix under do nothing action is obtained as

$$P(0) = \begin{bmatrix} 0.6721 & 0.2187 & 0.0568 & 0.0524 \\ 0 & 0.6036 & 0.1964 & 0.2000 \\ 0 & 0 & 0.5446 & 0.4554 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Under maintain action it is assumed that rows of transition probability matrix is equal to the first row of do nothing transition matrix.

$$P(1) = \begin{bmatrix} 0.6721 & 0.2187 & 0.0568 & 0.0524 \\ 0.6721 & 0.2187 & 0.0568 & 0.0524 \\ 0.6721 & 0.2187 & 0.0568 & 0.0524 \\ 0.6721 & 0.2187 & 0.0568 & 0.0524 \end{bmatrix}$$

Same rule is applied as in the previous cases and transition probabilities and corresponding matrix are obtained with the aim of finding an optimum policy.

Estimated probabilities yield a condition state for every time increment. Open form of objective function given in (2.5) is used to check the performance of prediction. Table 16 shows related points generated from fitting function and Markov Chain model.

Table 16 4-State Results

t	R(t)	S(t)	E(X(t),P)
200	0.51	1	1.48950
400	2.91	1	1.79900
600	8.02	1	2.41330
800	16.47	3	2.78570
1000	28.78	4	3.08820
1200	45.41	4	3.32600
1400	66.77	4	3.50810
1600	93.25	4	3.58880

In order to check the performance of the estimates generated points can again be shown in a graph. Figure 10 shows the differences between Y(t) and E[X(t),P)] along with the observed states which are transformed from average resistances.

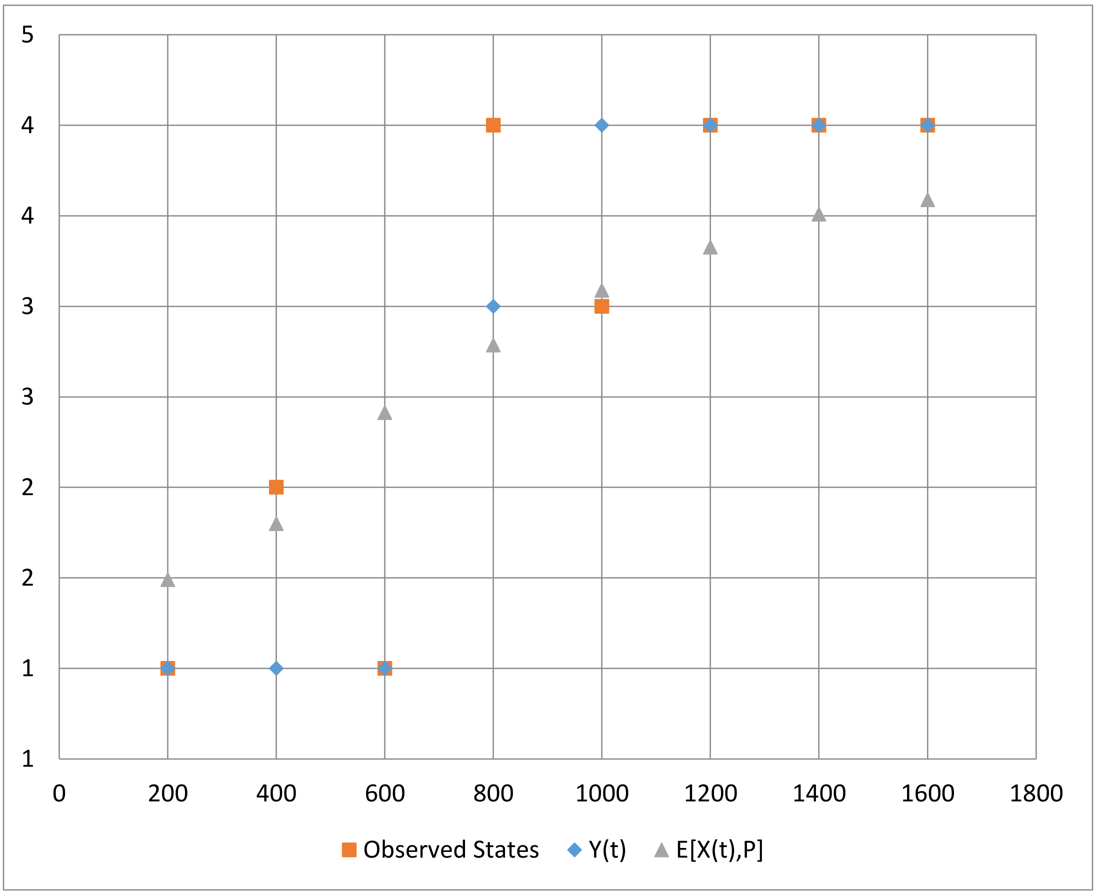

Figure 10 Estimated versus observed states

First model of the optimization models is then applied with the same objective function and constraints structure given before. As stated before objective function is to minimize operation costs which are comprised of replacement cost of 1000 and failure cost of 10000 and 4 states and 2 possible actions create a cost matrix below.

$$C = \begin{bmatrix} 0 & 1000 \\ 0 & 1000 \\ 0 & 1000 \\ 10000 & 1000 \end{bmatrix}$$

Open form of the problem is written as:

Min $1000\,\Pi_{11}+1000\Pi_{21}+0\Pi_{30}+1000\Pi_{31}+10000\Pi_{40}+1000\Pi_{41}$

Subject to

$\Pi_{10}+\Pi_{11}+\Pi_{20}+\Pi_{21}+\Pi_{30}+\Pi_{31}+\Pi_{40}+\Pi_{41}=1$

$(1-P_{11}(0))\,\Pi_{10}+(1-P_{11}(1))\,\Pi_{11}-\Pi_{20}P_{21}(0)-\Pi_{21}P_{21}(1)-\Pi_{30}P_{31}(0)-\Pi_{31}P_{31}(1)-\Pi_{40}P_{41}(0)-\Pi_{40}P_{41}(1)=0$

$-P_{12}(0)\,\Pi_{10}-P_{12}(1)\,\Pi_{11}+(1-P_{22}(0))\Pi_{20}+(1-P_{22}(1))\,\Pi_{21}-\Pi_{30}P_{32}(0)-\Pi_{31}P_{32}(1)-\Pi_{40}P_{42}(0)-\Pi_{40}P_{42}(1)=0$

$-P_{13}(0)\,\Pi_{10}-P_{13}(1)\,\Pi_{11}-P_{23}(0)\,\Pi_{20}-P_{23}(1)\,\Pi_{21}+(1-P_{33}(0))\,\Pi_{30}+(1-P_{33}(1))\,\Pi_{31}-\Pi_{40}P_{43}(0)-\Pi_{40}P_{43}(1)=0$

$-P_{14}(0)\,\Pi_{10}-P_{14}(1)\,\Pi_{11}-P_{24}(0)\Pi_{20}-P_{24}(1)\,\Pi_{21}-\Pi_{30}P_{34}(0)-\Pi_{31}P_{34}(1)+\Pi_{40}\,(1-P_{43}(0))+\Pi_{41}\,(1-P_{43}(1))=0$

$\Pi_{ia} \geq 0$ i=1, 2, 3, 4 and a= 0, 1

We find steady state probabilities as

$$\Pi_{ia} = \begin{bmatrix} 0.3511 \\ 0 \\ 0.2882 \\ 0 \\ 0.1894 \\ 0 \\ 0 \\ 0.1713 \end{bmatrix}$$

This policy can be stated as acting upon failures as in the base case.

Next, expected first passage times are calculated based on the optimal policy. Transition probability matrix is generated from do nothing or maintenance actions decisions obtained with MDP.

$$P = \begin{bmatrix} 0.6721 & 0.2187 & 0.0568 & 0.0524 \\ 0 & 0.6036 & 0.1964 & 0.2000 \\ 0 & 0 & 0.5446 & 0.4554 \\ 0.6721 & 0.2187 & 0.0568 & 0.0524 \end{bmatrix}$$

$$\mu_{1F} = 1 + P_{11}\mu_{1F} + P_{12}\mu_{2F} + P_{13}\mu_{3F} \tag{3.6}$$

$$\mu_{2F} = 1 + P_{21}\mu_{1F} + P_{22}\mu_{2F} + P_{23}\mu_{3F} \tag{3.7}$$

$$\mu_{3F} = 1 + P_{31}\mu_{1F} + P_{32}\mu_{2F} + P_{33}\mu_{3F} \tag{3.8}$$

We solve the above equations and the results are given below:

- Expected first passage time to failure state from state 1: 5.84 transition periods
- Expected first passage time to failure state from state 2: 3.61 transition periods
- Expected first passage time to failure state from state 3: 2.20 transition periods

In accordance with the transition period of 200 hours, we convert estimated first passage times to operating hours:

μ_{1F} = 5.84 transitions ≅ 1100 hours

μ_{2F} = 3.61 transitions ≅ 700 hours

μ_{3F} = 2.20 transitions ≅ 400 hours

We can again adjust inspection times of slip rings, if we adopt this policy. In here, we can share an important observation. Every constituent for this model except the number of states is kept the same with the base case, same policy is obtained but expected first passage times estimations get closer to real situation.

We next solve our second model which maximizes first passage time to failure state from state 1 for the 4 state case.

Max μ_{14}

St

$\Pi_{10} + \Pi_{11} + \Pi_{20} + \Pi_{21} + \Pi_{30} + \Pi_{31} + \Pi_{40} + \Pi_{41} = 1$

$0.3279\ \Pi_{10} + 0.3279\ \Pi_{11} - 0.6721\Pi_{21} - 0.6721\Pi_{31} - 0.6721\Pi_{41} = 0$

$-0{,}2187\Pi_{10} - 0.2187\Pi_{11} + 0.3964\Pi_{20} + 0.7813\Pi_{21} - 0.2187\Pi_{31} - 0.2187\Pi_{41} = 0$

$-0{,}05687\Pi_{10} - 0.0568\Pi_{11} - 0.1964\Pi_{20} - 0.0568\Pi_{21} + 0.4554\Pi_{30} + 0.9432\Pi_{31} - 0.0568\Pi_{41} = 0$

$-0{,}0524\Pi_{10} - 0.0524\Pi_{11} - 0.2\Pi_{20} - 0.0524\Pi_{21} - 0.4554\Pi_{30} - 0.0524\Pi_{31} + 0.9476\Pi_{41} = 0$

$0.3279\mu_{1F} - 0.2187\mu_{2F} - 0.0568\mu_{3F} = 1$

$0.3964\mu_{2F}\ \Pi_{20} + 0.7813\mu_{2F}\ \Pi_{21} - 0.6721\mu_{1F}\ \Pi_{21} - 0.1964\mu_{3F}\ \Pi_{20} - 0.0568\mu_{3F}\ \Pi_{21} - \Pi_{20} - \Pi_{21} = 0$

$0.4554\mu_{3F}\ \Pi_{30} + 0.9432\mu_{3F}\ \Pi_{31} - 0.6721\mu_{1F}\ \Pi_{31} - 0.2187\mu_{2F}\ \Pi_{31} - \Pi_{30} - \Pi_{31} = 0$

$\Pi_{10}\ \Pi_{11} = 0$

$\Pi_{20}\ \Pi_{21} = 0$

$\Pi_{30}\ \Pi_{31} = 0$

$\Pi_{40}\ \Pi_{41} = 0$

$\pi_{ia} \geq 0, \quad i \in S, \quad a \in A$

$\mu_{iF} \geq 0, \quad i = 1, 2, 3$

Following steady state probabilities which mean a different policy are obtained:

$\pi_{10} = 0.4331$

$\pi_{20} = 0.3556$

$\pi_{31} = 0.1064$

$\pi_{41} = 0.1049$

$\mu_{1F} = 7.42$

$\mu_{2F} = 5.17$

$\mu_{3F} = 5.34$

Optimal policy for this model means doing nothing for state 1 and state 2 and maintaining for states 3 and 4. This policy has following transition matrix:

$$P = \begin{bmatrix} 0.6721 & 0.2187 & 0.0568 & 0.0524 \\ 0 & 0.6036 & 0.1964 & 0.2000 \\ 0.6721 & 0.2187 & 0.0568 & 0.0524 \\ 0.6721 & 0.2187 & 0.0568 & 0.0524 \end{bmatrix}$$

If we solve the equations (3.4) and (3.5) we obtain following first passage times as in the model for deterministic policy:

- μ_{1F} = 7.42 transitions ≅ 1400 hours
- μ_{2F} = 5.17 transitions ≅ 1000 hours
- μ_{3F} = 5.34 transitions ≅ 1000 hours

If we adopt this policy, we can suggest higher inspection times but this policy causes higher costs.

Finally, our third model to minimize the probabilities of being in state 4 under do nothing and maintain actions gives the following results:

Min $\Pi_{40} + \Pi_{41}$

Subject to

(2.8), (2.9) and (2.10)

Steady state probabilities:

$\pi_{10} = 0.6721$

$\pi_{21} = 0.2187$

$\pi_{31} = 0.0568$

$\pi_{41} = 0.0524$

This model gives a different policy from both first and second optimization models. We do nothing for state 1 and maintain for the rest.

This policy yields following transition matrix and expected first passage times:

$$P = \begin{bmatrix} 0.6721 & 0.2187 & 0.0568 & 0.0524 \\ 0.6721 & 0.2187 & 0.0568 & 0.0524 \\ 0.6721 & 0.2187 & 0.0568 & 0.0524 \\ 0.6721 & 0.2187 & 0.0568 & 0.0524 \end{bmatrix}$$

Expected first passage times:

- μ_{1F} = 19.08 transitions ≅ 3800 hours
- μ_{2F} = 19.08 transitions ≅ 3800 hours
- μ_{3F} = 19.08 transitions ≅ 3800 hours

Table 17 summarizes the results of the optimization models solved for 4 state case:

Table 17 Comparison of Optimization Modes for the 4 State Case

	$\text{Min} \sum_{i \in S} \sum_{a \in A}^{A} c_{ia} \pi_{ia}$	Max μ_{14}	Min $\Pi_{40} + \Pi_{41}$
Cost	171.28	211.3	327.9
$\Pi_{40} + \Pi_{41}$	0.1713	0.1049	0.0524
μ_{14}	5.84	7.42	19.08
Policy	1: DN 2: DN 3: DN 4: Maintain	1: DN 2: DN 3: Maintain 4: Maintain	1: DN 2: Maintain 3: Maintain 4: Maintain

3.2.1. Parametric Analysis – 4-State

Parametric analysis to see how proposed policies change is performed for the 4 state case. Following matrix structure is assumed suitable for this case and Matlab code is modified.

$$P(1) = \begin{bmatrix} \alpha & \beta & 1-\alpha-\beta & 0 \\ \alpha & \beta & 1-\alpha-\beta & 0 \\ \alpha & \beta & 1-\alpha-\beta & 0 \\ \alpha & \beta & 1-\alpha-\beta & 0 \end{bmatrix}$$

Different combinations imply different policies. Table 18 shows the results partially.

Let Policy X be maintain in 3 only (states 1, 2 and 3 are not observed),

Policy Y be maintain in 2 and 3 (states 1 and 4 are not observed),

Policy Z be maintain in 2 only (states 1, 3 and 4 are not observed),

Policy W be maintain in 1 and 3 (states 2 and 4 are not observed),

Policy V be maintain in 1 and 2 (states 3 and 4 are not observed),

Policy U be maintain in 1, 2 and 3 (state 4 is not observed),

Policy T be maintain in 1 only (states 2, 3 and 4 are not observed).

Table 18 Parametric Analysis Results

α	**β**	**π10**	**π11**	**π20**	**π21**	**π30**	**π31**	**π40**	**π41**	**Policy**
0	0						1.00			X
0	0.1				0.10		0.90			Y
0	0.2				0.20		0.80			Y
0	0.3				0.30		0.70			Y
0	0.4				0.40		0.60			Y
0	0.5				0.50		0.50			Y
0	0.6				0.60		0.40			Y
0	0.7				0.70		0.30			Y
0	0.8				0.80		0.20			Y
0	0.9				0.90		0.10			Y
0	1				1.00					Z
0.1	0		0.1				0.90			W
0.1	0.1		0.1		0.10		0.80			W
0.1	0.2		0.1		0.20		0.70			W
0.4	0.5		0,4		0,5		0,1			U
0.4	0.6		0,4		0,6					V
0.5	0		0,5				0,5			W
0.5	0.2		0,5		0,2		0,3			U
1	0		1							T

Lines on Figure 11 show the restrictions $\alpha > \beta > 1 - \alpha - \beta$ we assumed. For the 4 state case, we observe more policies in contrast to base case. Among these policy U which means performing a maintenance for state 1, 2 and 3 is optimal. If we do not have a chance to see slip rings in state 3 or 4 after a maintenance performed while they are in any of the states, then policy V will be optimal. Optimal policy remains the same although our assumption is not valid for some regions of the graph as in the case 1 of base case.

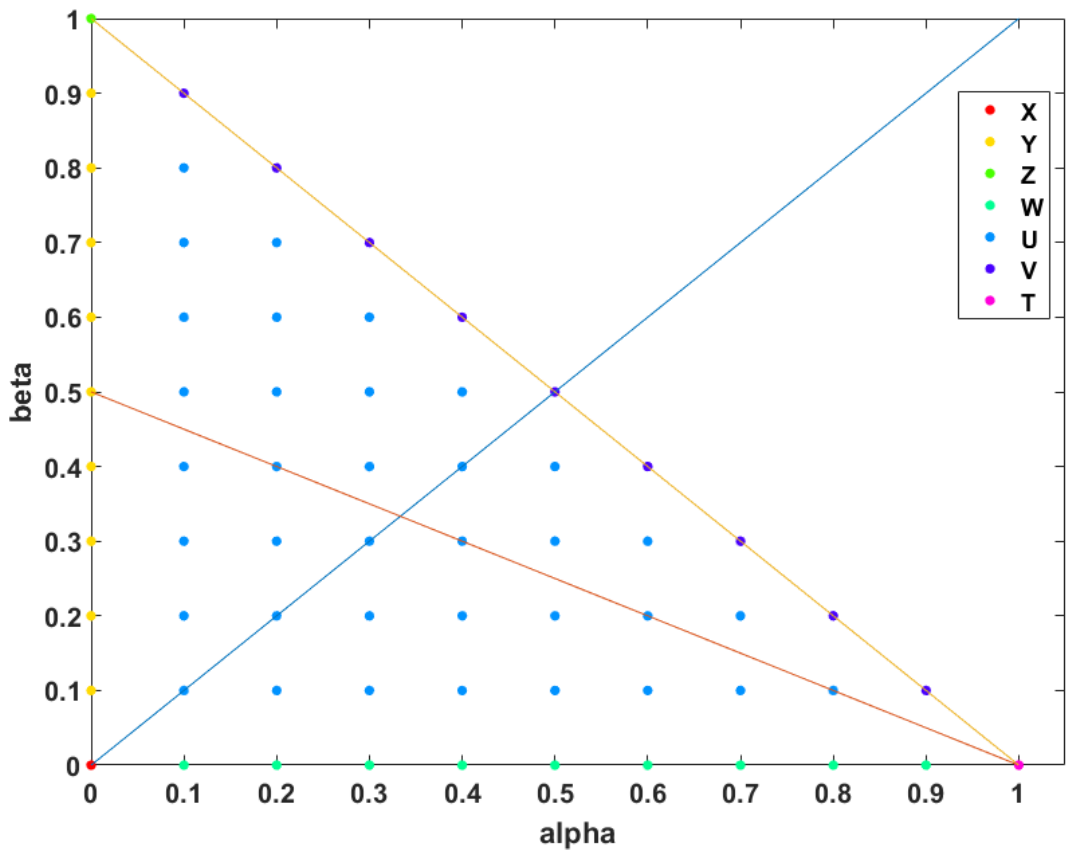

Figure 11 Changes in Optimal Policy wrt α and β with 4 state

CHAPTER 4

CONCLUSION

4.1. Conclusion

In this thesis, we evaluate maintenance history of a fielded unit which stochastically deteriorates by daily usage and causes a critical failure. Failure records are used and lifetime distributions of each pins located inside the observed units are checked to be able to observe an identical behavior. We then look for a curve that represents the expected behavior of deterioration for all units together. Two different aspects: fit to observations and fit to averages are implemented to find the best curve. We make use of this curve and solve a nonlinear optimization model to estimate transition probabilities of deterioration process.

In order to prevent undetected failures of units, we benefit from MDP and propose maintenance policies under different optimization criteria: cost minimization, long run probability minimization of down states and expected first passage time maximization. We look for optimal policies consisting of two possible actions: “Do nothing” and “Maintain”. We assume different transition matrices under maintain actions. Under each setting or policy, we calculate first passage times and determine inspection periods accordingly. We evaluate this real-life problem under two state classifications: base case and 4 state case. We observe that if we act upon failures and keep every factor same for each classification, the results get better with increased number of states for the same policies. Detailed state classification increases the precision.

We conduct a parametric analysis to see how policies can change depending upon the transition probabilities under maintain action.

As a result, we propose to monitor resistance/conditions of the unit closely. Collecting sufficient data, defining models with larger number of states, a condition based maintenance will improve the performance of the units significantly.

4.2. Future work

It would be beneficial if a more detailed state classification were possible although a larger number of observations would be needed. Larger number of states would decrease too early or too late maintenance.

Environmental measurements like temperature, vibration etc. can also be recorded and used in resistance fits. Then a proper regression analysis could be performed rather than a simple curve fitting.

The objectives can be considered simultaneously incorporating multi objective decision making tools. The trade of between them could be evaluated involving the real decision makers of the units.

In this thesis, units are assumed to be as good as new after repair. However, this may not be a realistic assumption for some systems. Maintenance types and restoration percentages can be added to the problem studied.

It is difficult to provide realistic cost measures for this problem. Maintenance action is cheap itself but being in a down state can result in disastrous outcomes since it is a matter of life. Therefore, cost parameterization can be adopted, as well.

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APPENDIX A

PICTURES OF SLIP RING

1. A slip ring

Figure 12 Slip Ring Unit of Radar

2. Pins of a slip ring

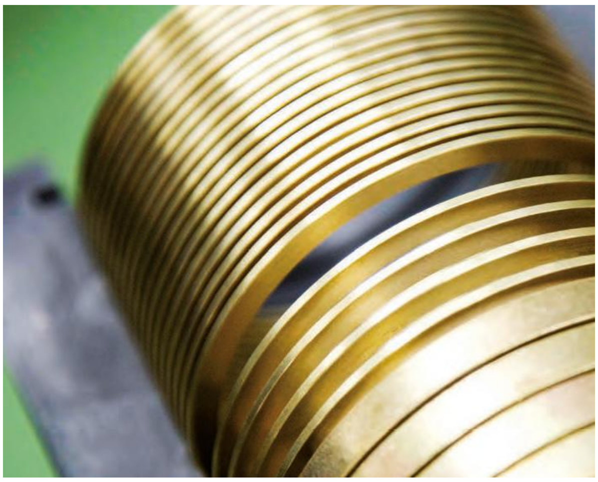

Figure 13 Pins of Slip Ring

APPENDIX B

MATLAB CODES

1. Matlab code to estimate transition probabilities / base case

```
clc;
clear all;
tic;
y=1000;
options = optimoptions('fmincon','Algorithm','interior-point');
objective=@(x) abs((1-(3-2*x(1)-x(2)))+(1-(-2*x(1)^2-x(1)*x(2)-x(2)*x(3)+3))+(1-
(-2*x(1)^3-x(2)*x(3)^2- x(1)^2*x(2)-x(1)*x(2)*x(3)+3))+(2-(-2*x(1)^4-
x(2)*x(1)^3-x(2)*x(1)^2*x(3)-x(2)*x(1)*x(3)^2-x(2)*x(3)^3+3))+(3- (-2*x(1)^5-
x(2)*x(1)^4-x(2)*x(1)^3*x(3)-x(2)*x(1)^2*x(3)^2-x(2)*x(1)*x(3)^3-
x(2)*x(3)^4+3))+(3-(-2*x(1)^6-x(2)*x(1)^5-x(2)*x(1)^4*x(3)-x(2)*x(1)^3*x(3)^2-
x(2)*x(1)^2*x(3)^3-x(2)*x(1)*x(3)^4-x(2)*x(3)^5+3))+(3-(-2*x(1)^7-x(2)*x(1)^6-
x(2)*x(1)^5*x(3)-x(2)*x(1)^4*x(3)^2-x(2)*x(1)^3*x(3)^3-x(2)*x(1)^2*x(3)^4-
x(2)*x(1)*x(3)^5-x(2)*x(3)^6+3))+(3- (-2*x(1)^8-x(2)*x(1)^7-x(2)*x(1)^6*x(3)-
x(2)*x(1)^5*x(3)^2-x(2)*x(1)^4*x(3)^3-x(2)*x(1)^3*x(3)^4-x(2)*x(1)^2*x(3)^5-
x(2)*x(1)*x(3)^6+x(2)*x(3)^7+3)));t=1;
k=[1,1,1];
n=[1,1,1];
for d1=0:0.25:0.75;
  for d2=0:0.25:0.75;
      for d3=0:0.25:0.75;
for j=2:2:2;
   for i=1:j;
      t1=d1+0.25*rand(1);
      t2=d2+0.25*rand(1);
          if (t1+t2>1)
             break
          end
      t3=d3+0.25*rand(1);
      x0=[t1,t2,t3];
      disp(['Initial guess: ' num2str(x0)]);
      disp(['Initial objective: ' num2str(objective(x0))]);
      A= [1,1,0];
      b= (1);
      Aeq=[ ];
```

```
        beq=[ ];
        nonlcon=[ ];
        lb=[0 0 0];
        ub=[1 1 1];
[x,fval,ef,output,lambda,hessian]=fmincon(objective,x0,A,b,Aeq,beq,lb,ub,nonlcon,
options);
        disp(['Final probabilites: ' num2str(x)]);

          if objective(x)<y;
             y=objective(x);
             for l=1:3;
                k(t,l)=x(1,l);
             end
             for l=1:3;
                n(t,l)=x0(1,l);
             end
          end
        disp(['Final objective: ' num2str(objective(x))]);
        m=[x(1) x(2) 1-x(1)-x(2);0 x(3) 1-x(3); 0 0 1];
        disp(m);
        z(t)=y;
     end
     t=t+1;
     break
     disp(['Min objective: ' num2str(z(t))]);
end
        end
    end
end
disp(['Min of min objectives: ' num2str(y)]);
toc;
```

2. Matlab code to estimate steady state probabilities / base case

```
clc;
clear all;
tic;
objectivecoefficients=[0;1000;0;1000;10000;1000];
 Aeq= [1 1 1 1 1 1; 0.1864 0.1864 0 -0.8136 0 -0.8136; -0.1038 -0.1038 0.4270
0.8962 0 -0.1038; -0.0826 -0.0826 -0.4270 -0.0826 0 0.9174];
beq=[1;0;0;0];
lb=zeros(6,1);
options = optimoptions('linprog','Algorithm','simplex');
[x,fval,exitflag,output,lambda] =
linprog(objectivecoefficients,[],[],Aeq,beq,lb,[],[],options);
disp(x);
```

```
disp(['objective: ' num2str(fval)]);
toc;
```

3. Matlab code to estimate first passage times / base case

```
syms x;
syms y;
m=[0.8136 0.1038 0.0826;0 0.5730 0.4270;0.8136 0.1038 0.0826];
x=1+m(1,1)*x+m(1,2)*y;
y=1+m(2,1)*x+m(2,2)*y;
[solx,soly] = solve(x==1+m(1,1)*x+m(1,2)*y,y==1+m(2,1)*x+m(2,2)*y)
```

4. Matlab code to estimate steady state probabilities / Parametric case 1

```
clc;
clear all;
tic;
objectivecoefficients=[0;1000;0;1000;10000;1000];
t=1;
for a=0:0.1:1
    for b=0:0.1:(1-a)
        Aeq= [1 1 1 1 1 1; 0.1864 0.1864 0 -a 0 -a; -0.1038 -0.1038 0.4270 (1-(1-a)) 0 -
(1-b); -0.0826 -0.0826 -0.4270 0 0 1];
        beq=[1;0;0;0];
        lb=zeros(6,1);
        options = optimoptions('linprog','Algorithm','simplex');
        [x,fval,exitflag,output,lambda] =
linprog(objectivecoefficients,[],[],Aeq,beq,lb,[],[],options);
        disp(x);
        disp(['objective: ' num2str(fval)]);
         y=transpose(x);
            for l=1:6;
                z(t,l)=y(1,l);
            end
          t=t+1;
     end
 end
toc;
```

5. Matlab code to estimate steady state probabilities / Parametric case 2

```
clc;
clear all;
tic;
objectivecoefficients=[0;1000;0;1000;10000;1000];
t=1;
for a=0:0.1:1
   for b=0:0.1:(1-a)
       Aeq= [1 1 1 1 1 1; 0.1864 0.1864 0 -a 0 -b; -0.1038 -0.1038 0.4270 (1-(1-a)) 0 -
(1-b); -0.0826 -0.0826 -0.4270 0 0 1];
       beq=[1;0;0;0];
       lb=zeros(6,1);
       options = optimoptions('linprog','Algorithm','simplex');
       [x,fval,exitflag,output,lambda] =
linprog(objectivecoefficients,[],[],Aeq,beq,lb,[],[],options);
       disp(x);
       disp(['objective: ' num2str(fval)]);
        y=transpose(x);
          for l=1:6;
              z(t,l)=y(1,l);
          end
         t=t+1;
     end
 end
toc;
```

6. Matlab code to estimate transition probabilities / 4 state case:

```
clc;
clear all;
tic;
y=1000;
options = optimoptions('fmincon','Algorithm','interior-point');
objective = @(x) abs ((1-(4-2*x(2)-x(3)-3*x(1))) + (2-(4-x(2)-x(6)*x(3)-
2*x(1)*x(2)-x(1)*x(3)-3*x(1)^2-2*x(4)*x(2))) + (1-(4-x(6)^2*x(3)-2*x(1)^2*x(2)-
x(1)^2*x(3)-3*x(1)^3-x(4)*x(5)*x(2)-x(5)*x(6)*x(2)-2*x(4)*x(1)*x(2)-
x(5)*x(1)*x(2)-x(6)*x(1)*x(3)-2*x(4)^2*x(2))) + (4-(4-x(6)^3*x(3)-2*x(1)^3*x(2)-
x(1)^3*x(3)-3*x(1)^4-x(4)^2*x(5)*x(2)-x(5)*x(6)^2*x(2)-2*x(4)*x(1)^2*x(2)-
2*x(4)^2*x(1)*x(2)-x(5)*x(1)^2*x(2)-x(6)*x(1)^2*x(3)-x(6)^2*x(1)*x(3)-
x(4)*x(5)*x(6)*x(2)-x(4)*x(5)*x(1)*x(2)-x(5)*x(6)*x(1)*x(2)-2*x(4)^3*x(2))) +
(3-(4-x(6)^4*x(3)-2*x(1)^4*x(2)-x(1)^4*x(3)-3*x(1)^5-x(4)^3*x(5)*x(2)-
x(5)*x(6)^3*x(2)-2*x(4)*x(1)^3*x(2)-2*x(4)^3*x(1)*x(2)-x(5)*x(1)^3*x(2)-
x(6)*x(1)^3*x(3)-x(6)^3*x(1)*x(3)-2*x(4)^2*x(1)^2*x(2)- x(6)^2*x(1)^2*x(3)-
x(4)*x(5)*x(6)^2*x(2)-x(4)^2*x(5)*x(6)*x(2)-x(4)*x(5)*x(1)^2*x(2)-
x(4)^2*x(5)*x(1)*x(2)-x(5)*x(6)*x(1)^2*x(2)-x(5)*x(6)^2*x(1)*x(2)-
x(4)*x(5)*x(6)*x(1)*x(2)-2*x(4)^4*x(2))) + (4-(4-x(6)^5*x(3)-2*x(1)^5*x(2)-
```

```
x(1)^5*x(3)-3*x(1)^6-x(4)^4*x(5)*x(2)-x(5)*x(6)^4*x(2)-2*x(4)*x(1)^4*x(2)-
2*x(4)^4*x(1)*x(2)-x(5)*x(1)^4*x(2)-x(6)*x(1)^4*x(3)-x(6)^4*x(1)*x(3)-
2*x(4)^2*x(1)^3*x(2)-2*x(4)^3*x(1)^2*x(2)-x(6)^2*x(1)^3*x(3)-
x(6)^3*x(1)^2*x(3)-x(4)*x(5)*x(6)^3*x(2)-x(4)^3*x(5)*x(6)*x(2)-
x(4)*x(5)*x(1)^3*x(2)-x(4)^3*x(5)*x(1)*x(2)-x(5)*x(6)*x(1)^3*x(2)-
x(5)*x(6)^3*x(1)*x(2)-x(4)^2*x(5)*x(6)^2*x(2)-x(4)^2*x(5)*x(1)^2*x(2)-
x(5)*x(6)^2*x(1)^2*x(2)-x(4)*x(5)*x(6)*x(1)^2*x(2)-x(4)*x(5)*x(6)^2*x(1)*x(2)-
x(4)^2*x(5)*x(6)*x(1)*x(2)-2*x(4)^5*x(2))) + (4-(4-x(6)^6*x(3)-2*x(1)^6*x(2)-
x(1)^6*x(3)-3*x(1)^7-x(4)^5*x(5)*x(2)-x(5)*x(6)^5*x(2)-2*x(4)*x(1)^5*x(2)-
2*x(4)^5*x(1)*x(2)-x(5)*x(1)^5*x(2)-x(6)*x(1)^5*x(3)-x(6)^5*x(1)*x(3)-
2*x(4)^2*x(1)^4*x(2)-2*x(4)^3*x(1)^3*x(2)-2*x(4)^4*x(1)^2*x(2)-
x(6)^2*x(1)^4*x(3)-x(6)^3*x(1)^3*x(3)-x(6)^4*x(1)^2*x(3)-
x(4)*x(5)*x(6)^4*x(2)-x(4)^4*x(5)*x(6)*x(2)-x(4)*x(5)*x(1)^4*x(2)-
x(4)^4*x(5)*x(1)*x(2)-x(5)*x(6)*x(1)^4*x(2)-x(5)*x(6)^4*x(1)*x(2)-
x(4)^2*x(5)*x(6)^3*x(2)-x(4)^3*x(5)*x(6)^2*x(2)-x(4)^2*x(5)*x(1)^3*x(2)-
x(4)^3*x(5)*x(1)^2*x(2)-x(5)*x(6)^2*x(1)^3*x(2)-x(5)*x(6)^3*x(1)^2*x(2)-
x(4)*x(5)*x(6)^2*x(1)^2*x(2)-x(4)^2*x(5)*x(6)*x(1)^2*x(2)-
x(4)^2*x(5)*x(6)^2*x(1)*x(2)-x(4)*x(5)*x(6)*x(1)^3*x(2)-
x(4)*x(5)*x(6)^3*x(1)*x(2)-x(4)^3*x(5)*x(6)*x(1)*x(2)-2*x(4)^6*x(2))) + (4-(4-
x(6)^7*x(3)-2*x(1)^7*x(1)-x(1)^7*x(3)-3*x(1)^8-x(4)^6*x(5)*x(2)-
x(5)*x(6)^6*x(2)-2*x(4)*x(1)^6*x(2)-2*x(4)^6*x(1)*x(2)-x(5)*x(1)^6*x(2)-
x(6)*x(1)^6*x(3)-x(6)^6*x(1)*x(3)-2*x(4)^2*x(1)^5*x(2)-2*x(4)^3*x(1)^4*x(2)-
2*x(4)^4*x(1)^3*x(2)-2*x(4)^5*x(1)^2*x(2)-x(6)^2*x(1)^5*x(3)-
x(6)^3*x(1)^4*x(3)-x(6)^4*x(1)^3*x(3)-x(6)^5*x(1)^2*x(3)-
x(4)*x(5)*x(6)^5*x(2)-x(4)^5*x(5)*x(6)*x(2)-x(4)*x(5)*x(1)^5*x(2)-
x(4)^5*x(5)*x(1)*x(2)-x(5)*x(6)*x(1)^5*x(2)-x(5)*x(6)^5*x(1)*x(2)-
x(4)^2*x(5)*x(6)^4*x(2)-x(4)^3*x(5)*x(6)^3*x(2)-x(4)^4*x(5)*x(6)^2*x(2)-
x(4)^2*x(5)*x(1)^4*x(2)-x(4)^3*x(5)*x(1)^3*x(2)-x(4)^4*x(5)*x(1)^2*x(2)-
x(5)*x(6)^2*x(1)^4*x(2)-x(5)*x(6)^3*x(1)^3*x(2)-x(5)*x(6)^4*x(1)^2*x(2)-
x(4)*x(5)*x(6)^2*x(1)^3*x(2)-x(4)*x(5)*x(6)^3*x(1)^2*x(2)-
x(4)^2*x(5)*x(6)*x(1)^3*x(2)-x(4)^2*x(5)*x(6)^3*x(1)*x(2)-
x(4)^3*x(5)*x(6)*x(1)^2*x(2)-x(4)^3*x(5)*x(6)^2*x(1)*x(2)-
x(4)^2*x(5)*x(6)^2*x(1)^2*x(2)-x(4)*x(5)*x(6)*x(1)^4*x(2)-
x(4)*x(5)*x(6)^4*x(1)*x(2)-x(4)^4*x(5)*x(6)*x(1)*x(2)-2*x(4)^7*x(2))));
t=1;
k=[1,1,1,1,1,1];
n=[1,1,1,1,1,1];
for d1=0:0.25:0.75;
  for d2=0:0.25:0.75;
      for d3=0:0.25:0.75;
          for d4=0:0.25:0.75;
              for d5=0:0.25:0.75;
                  for d6=0:0.25:0.75;
for j=1:1:2;
   for i=1:j;
      t1=d1+0.25*rand(1);
      t2=d2+0.25*rand(1);
```

```
        t3=d3+0.25*rand(1);
        t4=d4+0.25*rand(1);
        t5=d5+0.25*rand(1);
        t6=d6+0.25*rand(1);
            if (t1+t2+t3>1)
                break
            end
            if (t4+t5>1)
                break
            end
        x0=[t1,t2,t3,t4,t5,t6];
        disp(['Initial guess: ' num2str(x0)]);
        disp(['Initial objective: ' num2str(objective(x0))]);
        A=[1,1,1,0,0,0;0,0,0,1,1,0];
        b=[1;1];
        Aeq=[];
        beq=[];
        nonlcon=[];
        lb=[0 0 0 0 0 0];
        ub=[1 1 1 1 1 1];
[x,fval,ef,output,lambda,hessian]=fmincon(objective,x0,A,b,Aeq,beq,lb,ub,nonlcon,
options);
        disp(['Final probabilites: ' num2str(x)]);
           if objective(x)<y;
               y=objective(x);
               for l=1:6;
                   k(t,l)=x(1,l);
               end
               for l=1:6;
                   n(t,l)=x0(1,l);
               end
           end
        disp(['Final objective: ' num2str(objective(x))]);
        m=[x(1) x(2) x(3) 1-x(1)-x(2)-x(3);0 x(4) x(5) 1-x(4)-x(5); 0 0 x(6) 1-x(6);0 0 0
1];
        disp(m);
        z(t)=y;
    end
    t=t+1;
    break
    disp(['Min objective: ' num2str(z(t))]);
end
                    end
                end
            end
        end
   end
```

```
end
disp(['Min of min objectives: ' num2str(y)]);
toc;
```

7. Matlab code to estimate steady state probabilities / 4 state case

```
clc;
clear all;
tic;
objectivecoefficients=[0;1000;0;1000;1000;1000;10000;1000];
Aeq= [1 1 1 1 1 1 1 1; 0.3279 0.3279 0 -0.6721 0 -0.6721 0 -0.6721; -0.2187 -
0.2187 0.3964 0.7813 0 -0.2187 0 -0.2187; -0.0568 -0.0568 -0.1964 -0.0568 0.4554
0.9432 0 -0.0568;-0.0524 -0.0524 -0.2 -0.0524 -0.4554 -0.0524 0 0.9476];
beq=[1;0;0;0;0];
lb=zeros(8,1);
options = optimoptions('linprog','Algorithm','simplex');
[x,fval,exitflag,output,lambda] =
linprog(objectivecoefficients,[],[],Aeq,beq,lb,[],[],options);
disp(x);
disp(['objective: ' num2str(fval)]);
toc;
```

8. Matlab code to estimate first passage times / 4 state case

```
clc;
clear all;
tic;
syms x;
syms y;
syms z;
m=[0.6721   0.2187  0.0568  0.0524; 0   0.6036  0.1964  0.2;0.6721  0.2187  0.0568
0.0524;0.6721   0.2187  0.0568  0.0524];
x=1+m(1,1)*x+m(1,2)*y+m(1,3)*z;
y=1+m(2,1)*x+m(2,2)*y+m(2,3)*z;
z=1+m(3,1)*x+m(3,2)*y+m(3,3)*z;
[solx,soly,solz] =
solve(x==1+m(1,1)*x+m(1,2)*y+m(1,3)*z,y==1+m(2,1)*x+m(2,2)*y+m(2,3)*z,z=
=1+m(3,1)*x+m(3,2)*y+m(3,3)*z);
```

9. Matlab code to estimate steady state probabilities / 4 state parametric

```
clc;
clear all;
tic;
objectivecoefficients=[0;1000;0;1000;1000;1000;10000;1000];
t=1;
for a=0:0.1:1
    for b=0:0.1:(1-a)
        Aeq= [1 1 1 1 1 1 1 1; 0.3279 1-a 0 -a 0 -a 0 -a; -0.2187 -b 0.3964 (1-b) 0 -b 0
        -b; -0.0568 -(1-a-b) -0.1964 -(1-a-b) 0.4554 (1-(1-a-b)) 0 -(1-a-b);-0.0524 0 -
        0.2 0 -0.4554 0 0 0];
        beq=[1;0;0;0;0];
        lb=zeros(8,1);
        options = optimoptions('linprog','Algorithm','simplex');
        [x,fval,exitflag,output,lambda] =
        linprog(objectivecoefficients,[],[],Aeq,beq,lb,[],[],options);
        disp(x);
        disp(['objective: ' num2str(fval)]);
        y=transpose(x);
         for l=1:8;
             z(t,l)=y(1,l);
         end
        t=t+1;
    end
 end
toc;
```

10. Matlab code to estimate steady state probabilities to max μ_{1F} / 3 state

```
clc;
clear all;
tic;
fun=@(x)-x(7);
x0=[0 0 0 0 0 0 1 0];
lb=zeros(8,1);
ub=[];
disp(['Initial objective: ' num2str(fun(x0))]);
A=[];
b=[];
Aeq=[1 1 1 1 1 1 0 0; 0.1864 0.1864 0 -0.8136 0 -0.8136 0 0; -0.1038 -0.1038 0.4270
0.8962 0 -0.1038 0 0; -0.0826 -0.0826 -0.4270 -0.0826 0 0.9174 0 0;0 0 0 0 0 0 0.1864
-0.1038];
beq=[1;0;0;0;1];
nonlincon=@nlcon_2;
options = optimoptions('fmincon','Algorithm','interior-point');
```

```
[x,fval,exitflag,output,lambda] =
fmincon(fun,x0,A,b,Aeq,beq,lb,ub,nonlincon,options);
disp(x);
disp(['Final objective: ' num2str(fun(x))]);
toc;

function [c,ceq]=nlcon_2(x)
c=[];
ceq(1) = 0.4270*x(3)*x(8)+0.8962*x(4)*x(8)-0.8136*x(4)*x(7)-x(3)-x(4);
ceq(2) = x(1)*x(2);
ceq(3) = x(3)*x(4);
ceq(4) = x(5)*x(6);
```

APPENDIX C

MEASUREMENT DATA

Field data used in analysis process is given in Table 19.

Table 19 Measurement Data

No	Slip Ring Serial	Pin	Rotation	Operating Hour	Cond. State (3 State)	Cond. State (4 State)	Resistance
11	17	10-11	55,800	31	1	1	0.7
21	15	4-5	176,400	98	1	1	0.8
20	10	1-2	244,800	136	1	1	0.5
16	17	1-2	298,800	166	1	1	0.7
33	19	4-5	302,400	168	1	1	0.5
8	11	1-2	311,400	173	1	1	0.5
14	13-2	7-8	361,800	201	1	1	0.7
37	9	10-11	383,400	213	1	1	0.5
22	15	1-2	415,800	231	1	1	0.8
1	13	16-17	439,200	244	1	1	1.7
2	9	13-14	446,400	248	1	1	0.9
17	17	4-5	565,200	314	1	1	0.7
12	11	19-20	626,400	348	2	3	18.5
3	19	1-2	649,800	361	1	2	3.3
15	13-2	10-11	675,000	375	1	1	0.7
9	19	10-11	1,009,800	561	1	2	3.5
29	10	7-8	1,011,600	562	1	1	0.8
27	15	10-11	1,042,200	579	1	1	0.8
6	9	10-11	1,049,400	583	1	1	1.9
23	13-2	19-20	1,049,400	583	1	1	0.9
35	19	7-8	1,146,600	637	3	4	57.2
18	11	19-20	1,202,400	668	2	3	19.2
7	13	4-5	1,274,400	708	3	4	69.1
25	17	13-14	1,283,400	713	1	1	0.7

Table 19 Continued

13	19	19-20	1,303,200	724	3	4	72.9
19	11	13-14	1,348,200	749	3	4	160.2
30	10	13-14	1,463,400	813	2	2	15.7
10	9	16-17	1,486,800	826	3	4	49.7
24	13-2	7-8	1,557,000	865	1	2	3.5
28	15	10-11	1,900,800	1,056	1	1	0.8
31	10	19-20	2,107,800	1,171	3	4	196
36	11	4-5	2,165,400	1,203	3	4	169
4	20	13-14	2,392,200	1,329	2	3	18.5
32	13	7-8	2,557,800	1,421	3	4	35
26	17	19-20	2,678,400	1,488	1	1	2.5
5	20	4-5	2,884,860	1,603	3	4	78

APPENDIX D

CURVE FITTING MODELS

1. Polynomial Models

A Polynomial model is defined with the following equation:

$$y = \sum_{i=1}^{n+1} p_i x^{n+1-i}$$

where

n+1 is the order of the polynomial and n is the degree of the polynomial

1. Exponential Models

An Exponential model is defined with the following equation:

$$y = ae^{bx}$$

where

a and b are the coefficients.

2. Fourier

Fourier series are defined with the following equation:

$$y = a_0 + \sum_{i=1}^{n} a_i \cos(iwx) + b_i \sin(iwx)$$

where

a_0 is constant

w is the fundamental frequency of the signal

n is the number of terms in series

3. Gaussian

Gaussian models are described with the following equation:

$$y = \sum_{i=1}^{n} a_i e^{\left[-\left(\frac{x-b_i}{c_i}\right)^2\right]}$$

where

a is the amplitude

b is the centroid

c is related peak width

n is the number of peaks

4. Power

Power series are defined as:

$$y = ax^b$$

where

a and b are the coefficients.

5. Sum of Sine

These models are used for fitting periodic functions:

$$y = \sum_{i=1}^{n} \sin(b_i x + c_i)$$

a is the amplitude

b is the frequency

c is the phase constant for each wave

n is the number of terms

APPENDIX E

PARAMETRIC ANALYSIS

1. 3-State - Case 1

Table 20 Case 1 Results

α	β	π_{10}	π_{11}	π_{20}	π_{21}	π_{30}	π_{31}	Policy
0	0						1.0000	A
0	0.1			0.1898			0.8102	B
0	0.2			0.3190			0.6810	B
0	0.3			0.4127			0.5873	B
0	0.4			0.4837			0.5163	B
0	0.5			0.5394			0.4606	B
0	0.6			0.5842			0.4158	B
0	0.7			0.6211			0.3789	B
0	0.8			0.6520			0.3480	B
0	0.9			0.6782			0.3218	B
0	1			0.7008			0.2992	B
0.1	0	0.3218		0.0782			0.5999	C
0.1	0.1	0.2822		0.1918			0.5260	C
0.1	0.2	0.2512		0.2804			0.4683	C
0.1	0.3	0.2264		0.3516			0.4220	C
0.1	0.4	0.2060		0.4099			0.3841	C
0.1	0.5	0.1890		0.4586			0.3524	C
0.1	0.6	0.1746		0.4999			0.3255	C
0.1	0.7	0.1623		0.5353			0.3025	C
0.1	0.8	0.1515		0.5660			0.2825	C
0.1	0.9	0.1421		0.5929			0.2649	C
0.2	0	0.4598		0.1118			0.4285	C
0.2	0.1	0.4178		0.1928			0.3894	C
0.2	0.2	0.3829		0.2602			0.3569	C
0.2	0.3	0.3534		0.3173			0.3293	C
0.2	0.4	0.3281		0.3662			0.3058	C

Table 20 Continued

0.2	0.5	0.3061		0.4085			0.2853	C
0.2	0.6	0.2870		0.4456			0.2675	C
0.2	0.7	0.2701		0.4783			0.2517	C
0.2	0.8	0.2550		0.5073			0.2377	C
0.3	0	0.5364		0.1304			0.3333	C
0.3	0.1	0.4975		0.1933			0.3091	C
0.3	0.2	0.4639		0.2478			0.2883	C
0.3	0.3	0.4346		0.2954			0.2700	C
0.3	0.4	0.4088		0.3373			0.2540	C
0.3	0.5	0.3858		0.3745			0.2397	C
0.3	0.6	0.3653		0.4077			0.2270	C
0.3	0.7	0.3469		0.4376			0.2155	C
0.4	0	0.5851		0.1422			0.2727	C
0.4	0.1	0.5500		0.1937			0.2563	C
0.4	0.2	0.5188		0.2394			0.2418	C
0.4	0.3	0.4910		0.2801			0.2288	C
0.4	0.4	0.4661		0.3167			0.2172	C
0.4	0.5	0.4435		0.3498			0.2067	C
0.4	0.6	0.4230		0.3798			0.1971	C
0.5	0	0.6189		0.1504			0.2307	C
0.5	0.1	0.5871		0.1940			0.2189	C
0.5	0.2	0.5585		0.2333			0.2082	C
0.5	0.3	0.5325		0.2689			0.1985	C
0.5	0.4	0.5089		0.3014			0.1897	C
0.5	0.5	0.4872		0.3311			0.1816	C
0.6	0	0.6436		0.1565			0.1999	C
0.6	0.1	0.6148		0.1942			0.1910	C
0.6	0.2	0.5885		0.2287			0.1828	C
0.6	0.3	0.5643		0.2604			0.1753	C
0.6	0.4	0.5421		0.2895			0.1684	C
0.7	0	0.6625		0.1611			0.1764	C
0.7	0.1	0.6362		0.1943			0.1694	C
0.7	0.2	0.6120		0.2251			0.1630	C
0.7	0.3	0.5895		0.2536			0.1570	C
0.8	0	0.6775		0.1647			0.1578	C
0.8	0.1	0.6533		0.1945			0.1522	C
0.8	0.2	0.6308		0.2222			0.1470	C
0.9	0	0.6896		0.1676			0.1428	C
0.9	0.1	0.6672		0.1946			0.1382	C
1	0	0.6995		0.1701			0.1304	C

2. 3-State - Case 2

Table 21 Case 2 Results

α	β	π_{10}	π_{11}	π_{20}	π_{21}	π_{30}	π_{31}	Policy
0	0			0.7008			0.2992	B
0	0.1	0.1421		0.5929			0.2649	C
0	0.2	0.2550		0.5073			0.2377	C
0	0.3	0.3469		0.4376			0.2155	C
0	0.4	0.4230		0.3798			0.1971	C
0	0.5	0.4872		0.3311			0.1816	C
0	0.6	0.5421		0.2895			0.1684	C
0	0.7	0.5895		0.2536			0.1570	C
0	0.8	0.6308		0.2222			0.1470	C
0	0.9	0.6672		0.1946			0.1382	C
0	1	0.6995		0.1701			0.1304	C
0.1	0			0.7008			0.2992	B
0.1	0.1	0.1421		0.5929			0.2649	C
0.1	0.2	0.2550		0.5073			0.2377	C
0.1	0.3	0.3469		0.4376			0.2155	C
0.1	0.4	0.4230		0.3798			0.1971	C
0.1	0.5	0.4872		0.3311			0.1816	C
0.1	0.6	0.5421		0.2895			0.1684	C
0.1	0.7	0.5895		0.2536			0.1570	C
0.1	0.8	0.6308		0.2222			0.1470	C
0.1	0.9	0.6672		0.1946			0.1382	C
0.1	1	0.6995		0.1701			0.1304	C
0.2	0			0.7008			0.2992	B
0.2	0.1	0.1421		0.5929			0.2649	C
0.2	0.2	0.2550		0.5073			0.2377	C
0.2	0.3	0.3469		0.4376			0.2155	C
0.2	0.4	0.4230		0.3798			0.1971	C
0.2	0.5	0.4872		0.3311			0.1816	C
0.2	0.6	0.5421		0.2895			0.1684	C
0.2	0.7	0.5895		0.2536			0.1570	C

Table 21 Continued

0.2	0.8	0.6308		0.2222			0.1470	C
0.2	0.9	0.6672		0.1946			0.1382	C
0.2	1	0.6995		0.1701			0.1304	C
0.3	0			0.7008			0.2992	B
0.3	0.1	0.1421		0.5929			0.2649	C
0.3	0.2	0.2550		0.5073			0.2377	C
0.3	0.3	0.3469		0.4376			0.2155	C
0.3	0.4	0.4230		0.3798			0.1971	C
0.3	0.5	0.4872		0.3311			0.1816	C
0.3	0.6	0.5421		0.2895			0.1684	C
0.3	0.7	0.5895		0.2536			0.1570	C
0.3	0.8	0.6308		0.2222			0.1470	C
0.3	0.9	0.6672		0.1946			0.1382	C
0.3	1	0.6995		0.1701			0.1304	C
0.4	0			0.7008			0.2992	B
0.4	0.1	0.1421		0.5929			0.2649	C
0.4	0.2	0.2550		0.5073			0.2377	C
0.4	0.3	0.3469		0.4376			0.2155	C
0.4	0.4	0.4230		0.3798			0.1971	C
0.4	0.5	0.4872		0.3311			0.1816	C
0.4	0.6	0.5421		0.2895			0.1684	C
0.4	0.7	0.5895		0.2536			0.1570	C
0.4	0.8	0.6308		0.2222			0.1470	C
0.4	0.9	0.6672		0.1946			0.1382	C
0.4	1	0.6995		0.1701			0.1304	C
0.5	0			0.7008			0.2992	B
0.5	0.1	0.1421		0.5929			0.2649	C
0.5	0.2	0.2550		0.5073			0.2377	C
0.5	0.3	0.3469		0.4376			0.2155	C
0.5	0.4	0.4230		0.3798			0.1971	C
0.5	0.5	0.4872		0.3311			0.1816	C
0.5	0.6	0.5421		0.2895			0.1684	C

Table 21 Continued

0.5	0.7	0.5895		0.2536			0.1570	C
0.5	0.8	0.6308		0.2222			0.1470	C
0.5	0.9	0.6672		0.1946			0.1382	C
0.5	1	0.6995		0.1701			0.1304	C
0.6	0	0.7177			0.2230		0.0593	D
0.6	0.1	0.1421		0.5929			0.2649	C
0.6	0.2	0.2550		0.5073			0.2377	C
0.6	0.3	0.3469		0.4376			0.2155	C
0.6	0.4	0.4230		0.3798			0.1971	C
0.6	0.5	0.4872		0.3311			0.1816	C
0.6	0.6	0.5421		0.2895			0.1684	C
0.6	0.7	0.5895		0.2536			0.1570	C
0.6	0.8	0.6308		0.2222			0.1470	C
0.6	0.9	0.6672		0.1946			0.1382	C
0.6	1	0.6995		0.1701			0.1304	C
0.7	0	0.7414			0.1974		0.0612	D
0.7	0.1	0.7479			0.1903		0.0618	D
0.7	0.2	0.2550		0.5073			0.2377	C
0.7	0.3	0.3469		0.4376			0.2155	C
0.7	0.4	0.4230		0.3798			0.1971	C
0.7	0.5	0.4872		0.3311			0.1816	C
0.7	0.6	0.5421		0.2895			0.1684	C
0.7	0.7	0.5895		0.2536			0.1570	C
0.7	0.8	0.6308		0.2222			0.1470	C
0.7	0.9	0.6672		0.1946			0.1382	C
0.7	1	0.6995		0.1701			0.1304	C
0.8	0	0.7601			0.1771		0.0628	D
0.8	0.1	0.7661			0.1706		0.0633	D
0.8	0.2	0.7722			0.1640		0.0638	D
0.8	0.3	0.3469		0.4376			0.2155	C
0.8	0.4	0.4230		0.3798			0.1971	C
0.8	0.5	0.4872		0.3311			0.1816	C

Table 21 Continued

0.8	0.6	0.5421		0.2895			0.1684	C
0.8	0.7	0.5895		0.2536			0.1570	C
0.8	0.8	0.6308		0.2222			0.1470	C
0.8	0.9	0.6672		0.1946			0.1382	C
0.8	1	0.6995		0.1701			0.1304	C
0.9	0	0.7754			0.1606		0.0640	D
0.9	0.1	0.7809			0.1546		0.0645	D
0.9	0.2	0.7866			0.1485		0.0650	D
0.9	0.3	0.7923			0.1423		0.0654	D
0.9	0.4	0.4230		0.3798			0.1971	C
0.9	0.5	0.4872		0.3311			0.1816	C
0.9	0.6	0.5421		0.2895			0.1684	C
0.9	0.7	0.5895		0.2536			0.1570	C
0.9	0.8	0.6308		0.2222			0.1470	C
0.9	0.9	0.6672		0.1946			0.1382	C
0.9	1	0.6995		0.1701			0.1304	C
1	0	0.7880			0.1469		0.0651	D
1	0.1	0.7932			0.1413		0.0655	D
1	0.2	0.7984			0.1356		0.0659	D
1	0.3	0.8037			0.1299		0.0664	D
1	0.4	0.8091			0.1241		0.0668	D
1	0.5	0.4872		0.3311			0.1816	C
1	0.6	0.5421		0.2895			0.1684	C
1	0.7	0.5895		0.2536			0.1570	C
1	0.8	0.6308		0.2222			0.1470	C
1	0.9	0.6672		0.1946			0.1382	C
1	1	0.6995		0.1701			0.1304	C

3. 4-State

Table 22 Parametric Results – 4-State

α	β	π_{10}	π_{11}	π_{20}	π_{21}	π_{30}	π_{31}	π_{40}	π_{41}	Policy
0	0						1			X
0	0.1				0.1		0.9			Y
0	0.2				0.2		0.8			Y
0	0.3				0.3		0.7			Y
0	0.4				0.4		0.6			Y
0	0.5				0.5		0.5			Y
0	0.6				0.6		0.4			Y
0	0.7				0.7		0.3			Y
0	0.8				0.8		0.2			Y
0	0.9				0.9		0.1			Y
0	1				1					Z
0.1	0		0.1				0.9			W
0.1	0.1		0.1		0.1		0.8			U
0.1	0.2		0.1		0.2		0.7			U
0.1	0.3		0.1		0.3		0.6			U
0.1	0.4		0.1		0.4		0.5			U
0.1	0.5		0.1		0.5		0.4			U
0.1	0.6		0.1		0.6		0.3			U
0.1	0.7		0.1		0.7		0.2			U
0.1	0.8		0.1		0.8		0.1			U
0.1	0.9		0.1		0.9					V
0.2	0		0.2				0.8			W
0.2	0.1		0.2		0.1		0.7			U
0.2	0.2		0.2		0.2		0.6			U
0.2	0.3		0.2		0.3		0.5			U
0.2	0.4		0.2		0.4		0.4			U
0.2	0.5		0.2		0.5		0.3			U
0.2	0.6		0.2		0.6		0.2			U
0.2	0.7		0.2		0.7		0.1			U
0.2	0.8		0.2		0.8					V
0.3	0		0.3				0.7			W
0.3	0.1		0.3		0.1		0.6			U
0.3	0.2		0.3		0.2		0.5			U
0.3	0.3		0.3		0.3		0.4			U
0.3	0.4		0.3		0.4		0.3			U
0.3	0.5		0.3		0.5		0.2			U
0.3	0.6		0.3		0.6		0.1			U

Table 22 Continued

0.3	0.7		0.3		0.7					V
0.4	0		0.4				0.6			W
0.4	0.1		0.4		0.1		0.5			U
0.4	0.2		0.4		0.2		0.4			U
0.4	0.3		0.4		0.3		0.3			U
0.4	0.4		0.4		0.4		0.2			U
0.4	0.5		0.4		0.5		0.1			U
0.4	0.6		0.4		0.6					V
0.5	0		0.5				0.5			W
0.5	0.1		0.5		0.1		0.4			U
0.5	0.2		0.5		0.2		0.3			U
0.5	0.3		0.5		0.3		0.2			U
0.5	0.4		0.5		0.4		0.1			U
0.5	0.5		0.5		0.5					V
0.6	0		0.6				0.4			W
0.6	0.1		0.6		0.1		0.3			U
0.6	0.2		0.6		0.2		0.2			U
0.6	0.3		0.6		0.3		0.1			U
0.6	0.4		0.6		0.4					V
0.7	0		0.7				0.3			W
0.7	0.1		0.7		0.1		0.2			U
0.7	0.2		0.7		0.2		0.1			U
0.7	0.3		0.7		0.3					V
0.8	0		0.8				0.2			W
0.8	0.1		0.8		0.1		0.1			U
0.8	0.2		0.8		0.2					V
0.9	0		0.9				0.1			W
0.9	0.1		0.9		0.1					V
1	0		1							T