A REMANUFACTURING SYSTEM WITH IMPERFECT SORTING: DETERMINISTIC AND PROBABILISTIC MODELS

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ABSTRACT

A REMANUFACTURING SYSTEM WITH IMPERFECT SORTING: DETERMINISTIC AND PROBABILISTIC MODELS

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In this study, we focus on inaccurate sorting process with classification errors in a reverse supply chain comprising a remanufacturer and a collector under deterministic demand in a single time period. The collector acquires used items from end-users and the remanufacturer reproduces them to serve the deterministic demand of remanufactured products. There are two sources of uncertainty: uncertain quality of used items and uncertain quality of sorted items due to imperfect testing. Used items are categorized into two quality states by imperfect inspection: remanufacturable or non-remanufacturable and the actual condition of items is revealed after the remanufacturer's disassembly process. Under this environment, firstly, we construct different settings without incorporation of randomness in the inspection process and compare their optimal solutions to assess the effects of pricing decisions, the change in the agents' roles and sorting location. We observe that the sorting location does not affect the optimal collection quantity under deterministic market demand. However, the optimal solution changes regarding to the change in the agent responsible for sorting under the case where the demand and supply are price sensitive. We also show that the channel leadership does not affect the optimal solution when the transfer price is exogenous. Secondly, we investigate the impact of ignoring randomness due to sorting errors on the optimal solution and profits. The results show that disregarding randomness hurts the collector more than the remanufacturer. Lastly, we conduct an extensive computational study to analyze the effects of problem parameters on this randomness impact.

Keywords: Remanufacturing, Quality Uncertainty, Imperfect Sorting

ÖZ

HATALI SINIFLANDIRMA İÇEREN YENİDEN ÜRETİM SİSTEMİ: DETERMİNİSTİK VE RASSAL MODELLER

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Bu çalışmada, tek bir zaman periyodunda deterministik talep altında bir yeniden imalatçı ve bir toplaycıdan oluşan tersine tedarik zincirinde, sınıflandırma hataları içeren kesin olmayan sınıflandırma prosesine odaklanıyoruz. Toplayıcı, kullanılmış ürünleri son kullanıcılardan satın almakta ve yeniden üretici bu ürünleri yeniden imal edilen ürünlerin deterministik talebini karşılamak için üretmektedir. Belirsizliğin iki kaynağı mevcuttur: kullanılmış ürünlerin belirsiz kalitesi ve hatalı test işlemi sebebiyle sınıflandırılan ürünlerin belirsiz kalitesi. Kullanılmış ürünler hatalı denetimden sonra yeniden üretilebilir ve yeniden üretilmez olmak üzere iki farklı kalite grubuna ayrılmaktadır, ve ürünlerin gerçek kalite bilgisi yeniden üreticinin demontaj işleminden sonra açığa çıkmaktadır. Bu ortamda ilk önce denetim sürecindeki belirsizlik dahil edilmeden, farklı modeller oluşturuluyor ve fiyatlandırma kararlarının, üyelerin rollerindeki ve sınıflandırma yerindeki değişikliğin etkilerini belirlemek için modellerin optimum çözümlerini karşılaştırıyoruz. Deterministik market talebi altında, sınıflandırma yerindeki değişimin optimum toplama miktarını etkilemediğini gözlemliyoruz. Fakat, fiyata duyarlı talep ve arz altında, optimum çözüm sınıflandırma sorumlusundaki değişime bağlı olarak değişmektedir. Ayrıca, transfer fiyatı eksojen olduğunda, kanal liderliğinin optimum çözümü etkilemediğini görüyoruz. İkinci olarak, sınıflandırma hatalarından kaynaklı belirsizliğin göz ardı edilmesinin optimum çözüm ve kar değerleri üzerine etkisini inceliyoruz. Sonuçlar; belirsizliğinin göz ardı edilmesinin, yeniden üreticiye oranla toplayıcıya daha fazla zarar verdiğini göstermektedir. Son olarak, bu etki üzerine problem parametrelerinin etkilerini analiz etmek için kapsamlı bir sayısal çalışma yürütüyoruz.

Anahtar Kelimeler: Yeniden İmal Etme, Kalite Belirsizliği, Hatalı Sınıflandırma

To my family...

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CHAPTER 1

INTRODUCTION

Because of increasing general awareness of the effect of environmental pollution and the better understanding for its negative consequences, and some governmental actions to achieve a sustainable environment, the importance of the recovery processes is increasingly being recognized and managing returns has become a very critical issue for the firms. Remanufacturing is one of the recovery options in order to reduce energy consumption and landfill space, and it also motivates the firms to use the parts that are dismantled from the used products in order to decrease the purchasing cost of the raw materials.

Remanufacturing is a series of activities in which a particular used product is cleaned, disassembled to assess its current condition, inspected for deciding remanufacturable or not, and reassembled to its original conditions and specifications. In many cases, the remanufactured product can be used to satisfy the demand in the primary market as good as the new product (Karvonen et al., 2004).

Remanufacturing activities not only serve both economical and ecological needs but also result in a considerable decrease in the costs related to production compared to manufacturing of the new items. Since remanufacturing processes mostly contribute to the overall profitability of the firms, an increasing number of firms in many industries have been implementing remanufacturing and recycling activities and they try to find the most effective way to maximize their profit under existing governmental regulations and environmental issues. The main sectors involved in remanufacturing operations are aerospace industry, consumer products, information technology products, medical devices, machinery, motor vehicle parts, office furniture, electrical apparatus and retreaded tires (United States International Trade Commission, 2012).

Conditions of used products collected from end-users are not the same regarding to some quality characteristics such as age of the return, degree of physical damage, functional properties etc. For this reason, their operating costs and needed remanufacturing activities are different. Some of them can be used for material recovery or remanufacturing while some of them are not suitable for the remanufacturing operations and they are disposed. Firms engaged in remanufacturing processes prefer remanufacturing items in better quality condition in order to minimize cost. Therefore, getting perfect quality is critical for them. However, perfect inspection requires costly and complete product disassembly and so it also takes a lot of time. Thus, the firms try to find a way in order to inspect items simply and quickly based on the some quality characteristics without using expensive sorting techniques. However, these quick and inexpensive sorting methods are most probably not exactly accurate and the actual condition of items can be known after the disassembly process. In the literature, there are lots of studies that have addressed perfect inspection in the remanufacturing operations, but there is a little interest to fast but inaccurate sorting methods.

In this thesis, we focus on the imperfect sorting process in a remanufacturing system that comprises two independent agents: a collector that is responsible for the collection of used items from end-users, and a remanufacturer who remanufactures used items to satisfy the demand in a single-time period setting. It is assumed that remanufacturing is the single source for the remanufacturer in order to satisfy the demand. Our study is based on the decentralized setting under deterministic demand case studied by Gu and Tagaras (2014). We call this setting as the base model in this thesis, and we discuss its several extensions. We assume that the items are categorized into two quality states imperfectly: remanufacturable and non-remanufacturable. Our main objectives in this study are:

1. to generate different settings in order to analyze the effects of different sorting locations, leadership, pricing decisions for remanufactured and supplied products on the optimal solution and both parties' profits, and also compare the results to the base model according to the optimal solutions,

2. to analyze the effects of disregarding randomness on the optimal solution and

profits.

First of all, we discuss the base model in detail and then reformulate it by giving the sorting responsibility to the remanufacturer. In this setting, the remanufacturer is the single decision maker and he decides on the collection quantity. The observation says that the optimal value of the collection quantity is the same as the base model, but the price ranges in which the remanufacturer is willing to operate are different. Then, we study the case where the remanufacturer takes the responsibility of sorting unlike the base model and there is a price dependent deterministic supply and demand. Hence, the remanufacturer and the collector have the power to set selling price and acquisition price, respectively. Therefore, the remanufacturer can increase the demand of the remanufactured products by offering a lower selling price, and also the collector can affect the supply by changing the acquisition price. Under such settings, we assume that there is exogenous transfer price. It is observed that the change in the roles of agents results to change the sequence of events, but the optimal solution does not change because of the exogenous transfer price. Lastly, we extend the base model to study both price dependent demand and supply, where the collector is responsible for sorting.

In order to assess the effect of disregarding randomness on the results, we reformulate the base model by incorporating randomness in the collector's inspection process. In this setting, we are interested in (i) how much profit is overestimated when randomness is ignored, and (ii) what are the benefits from incorporating randomness, that is, how much profit is lost by disregarding randomness. We also analyze the parameter sensitivity in order to evaluate how much the impact of randomness is dependent on the changes in the value of each parameter. It is observed that the collector is more affected than the remanufacturer from disregarding randomness since for him, there is a source of uncertainty in the sorted item quality due to the sorting errors in addition to uncertainty in the quality of collected used item. For the remanufacturer, on the other hand, the only uncertainty is the quantity of actual remanufacturables sent. Hence, the collector's profit is highly overestimated when randomness is disregarded. Another observation is that when randomness is incorporated, the percentage loss of profit is small for both parties and so disregarding the uncertainty in the collector's inspection process does not hurt the agents significantly for the selected parameter set.

However, the effects of disregarding randomness on the agents change with respect to a change in the parameters.

The rest of the thesis organized as follows: in Chapter 2, we discuss the literature related to closed-loop supply chains and remanufacturing systems focusing on our basic topics. In Chapter 3, we define the problem environment in detail and introduce our basic model assumptions and models. Seven models are constructed under different scenarios and these models are compared to the base model regarding to the optimal solutions, analytically. In Chapter 4, the base model is reformulated by incorporating randomness in the inspection process and a detailed computational study is performed. Lastly, our observations and main results are summarized and the study is concluded in Chapter 5.

CHAPTER 2

LITERATURE REVIEW

Closed-loop supply chain management has attracted attention in both academia and industry. In the literature, there are lots of papers provide a study related to various issues of closed loop supply chain concept. Gungor and Gupta (1999) present a review of literature on environmentally conscious manufacturing and product recovery (ECMPRO). They discuss the concept of ECMPRO which helps to decrease environmental effects of the products by integrating environmental thinking into both design consideration and production system. Topics that are studied in the paper are: the assessment of the life cycle, design for environment of the product, review of the recovery options, disassembly, collection and production planning issues of the used products in recycling and remanufacturing.

Guide and Van Wassenhove (2001) present a framework based on economic value added (EVA) concept that is an approach to assess potential profitability of reuse activities. They mention about the importance of effective acquisition management in EVA concept since it is a key issue to decide whether remanufacturing option is profitable for the firms.

Guide and Van Wassenhove (2002) analyze different product groups that use remanufacturing activities and, present critical and distinguishing properties of their supply chains. They point out that while common reuse activities are used by all remanufacturing firms such as acquisition, inspection and grading, logistics activities, disassembly, disposition, reworking and recycling operations, their supply chains have different key characteristics and management issues in order to succeed in the sector.

Guide et al. (2003) provide an overview of challenging issues in closed loop supply chains. They mention about the main aspects which closed-loop supply chains differ significantly from forward supply chains. The developed studies in the literature are reviewed and the potential future challenges and topics the researchers should focus on in the future are mentioned.

Blackburn et al. (2004) discuss key activities that must be performed in reverse supply chains: product acquisition, reverse logistics, inspection and disposition, remanufacturing and marketing. They mention about two important supply chain structures: cost efficient and responsive supply chain. If the objective is cost minimization then the supply chain is designed as a centralized model. Each returned item is tested and their actual condition are evaluated at central location. On the other hand, if the aim is responsiveness, then the supply chain is designed as decentralized model. Early detection of return condition is made at multiple collection sites before they are sent to the central facility for reuse activities in order to minimize time delays. They present main principles in order to increase efficiency of recovery processes: treat returns as perishable assets, elevate the priority of the return process, make time the essential performance metric, use time value to design the supply chain and use technology to achieve speed at lower cost.

Guide and Van Wassenhove (2008) overview the evolution of the closed loop supply chains from a business perspective. They provide a simple understanding of the evolution of the CLSC by introducing five phases based on conducted studies in the literature: the golden age of remanufacturing as a technical problem, from remanufacturing to valuing the reverse logistics process, coordinating the reverse supply chain, closing the loop, prices and markets. They have discussed the issue from a technical focus on the individual activities to entire reverse chain, to finally considering CLSCs as a interdisciplinary business process.

Akçalı and Çetinkaya (2011) provide a detailed review on quantitative models for inventory and production planning for CLSC systems. They firstly classify the recently studied models in the literature into deterministic and stochastic problems according to the modelling of demand and return processes. They define the critical modelling parameters which affect the complexity of the models and categorize the studies in existing literature according to these parameters.

Çorbacıoğlu and van der Laan (2013) provide a framework in order to analyze recovery systems. The studied framework differs from other constructed schemas in the literature since it has been basically developed from a quality point of view. They have defined the quality as the main source of the variability in the CLSCs and have pointed out the necessity to describe proper definition and all relevant dimensions of the quality in the CLSCs. In order to manage variabilities due to quality issues, the framework has been established. The presented quality framework provides the structural way to deal with possible problems and inefficiencies due to quality uncertainty. The framework offers quality management approach can be used to prevent quality based problems by analyzing potential quality bottlenecks in the early stages.

Kumar and Kumar (2013) provide a brief review on the issues related to CLSCs. They focus on basically recently developed studies for Green Operations (Reverse Logistics), Green Design, Green Manufacturing, Waste Management, and Product Life Cycle Assessment. The paper also indicates main differences between CLSC and traditional form of supply chain and basically mentions about the main aspects which CLSC differs from forward SC.

Ketzenberg et al. (2006) study the value of information (VOI) related to uncertain demand, return quantity and recovery yield rate for remanufacturing firms that serve customer demand from both remanufactured and new manufactured products. When one or more of these uncertainty issues are reduced, then the VOI of related uncertainty is measured as amount of reduction in the total expected cost. Firstly, the case where the value of information is given fully related to one or more sources of uncertainty is considered in a single time period. Demand and supply of returns are independent random variables and both are normally distributed. They compare the VOIs to obtain which type of information is dominant. The results show that there is no significant difference between VOI of three uncertainties and the combined effect of more than one type of information is greater. Therefore, they suggest to use the VOI obtained by reducing more than one source of uncertainties. They study also multi-period model and assume that product return in each period is correlated with realized demand in the previous periods. The value of partial information is also evaluated in the multi period context. Moreover, uniformly distributed demand and return are also studied and the analysis shows that there is robustness in the results regarding to demand and return distributions.

We have constructed alternative settings under imperfect inspection in order to analyze the effects of price dependent demand and/or supply, the change in the agents' roles and sorting location. Therefore, we restrict the literature review part with the studies that are mostly related to the our work. We categorize the studies in our literature review with respect to the following topics:

- Value of sorting information (imperfect and perfect sorting),
- Effects of central/ decentralized sorting (sorting location),
- Pricing of returns and remanufactured products,
- Acquisition management for returns,
- Multiple quality groups.

A detailed comparison of the reviewed papers and our work is given in Table 2.1.

Table 2.1: Reviewed Papers and Related Topics

Studies Reviewed	Effects of central/local sorting	Imperfect Sorting	Sorting	Pricing of Remanuf. Products	Price Depend. Demand	Price Depend. Supply	Acquisition Managem. for Returns	Effects of Decentr. Setting	Multiple Quality Groups
Gu and Tagaras (2014)		+	+				+		
Zikopoulos and Tagaras (2005)	+		+						
Zikopoulos and Tagaras (2007)			+						
Tagaras and Zikopoulos (2008)	+	+	+						
Zikopoulos and Tagaras (2008)	+	+	+						
Wassenhove and Zikopoulos (2010)		+	+						+
Aras et al. (2004)			+						+
Ferguson et al. (2009)			+				+		+
Denizel et al. (2010)			+				+		+
Teunter and Flapper (2011)			+				+		+
Galbreth and Blackburn (2006)			+				+		
Galbreth and Blackburn (2010)			+				+		+
Guide et al. (2003)			+	+	+	+	+		+
Savaskan et al. (2004)				+	+			+	
Atasu et al. (2013)				+	+			+	
Karakayali et al. (2007)			+	+	+	+	+	+	+
Ünal (2009)			+	+	+	+	+	+	+
Ferrer (2003)			+				+		
Bakal and Akcali (2006)			+	+	+	+	+		
Our Work	+	+	+	+	+	+	+	+	

2.1 Value of Sorting Information: Imperfect and Perfect Sorting

The uncertainty about the quality of the used products in reverse supply chains has been addressed by many researchers in recent years. In order to decide whether a used product is suitable for remanufacturing and assign it into the true category, the quality classification of the used product is the important process for the remanufacturers. When the sorting procedure is inaccurate, the actual quality state of a returned item is not revealed until the disassembly process. Since waiting for the disassembly of the used products to evaluate their actual quality state takes a lot of time and also disassembly of every unit is expensive, remanufacturers focus on fast and not fully accurate sorting methods based on some metrics and general quality characteristics.

We analyze and compare our settings to the decentralized model in Gu and Tagaras (2014). This decentralized model is called throughout the thesis as the base model. Gu and Tagaras (2014) study on two alternative settings: decentralized and centralized model. In both settings, there is a single type of used product and two quality states are available: remanufacturable or non-remanufacturable. A two echelon reverse supply chain model with a remanufacturer and collector is considered in a single period setting. The remanufacturer is a Stackelberg leader and decides his optimal order quantity, the collector is a follower and seeks the optimal collection quantity for profit maximization. The sorting activity is performed by the collector and there are two classification errors in the sorting procedure: type I error and type II error. Remanufacturable used products can be misclassified as non-remanufacturable with α probability (Type I error) and non-remanufacturables can be also misclassified as remanufacturable with β probability (Type II error). The important point is that they do not take the uncertainty in the collector's inspection process into account and assume the number of items that are classified as remanufacturable by the inspection process is equal to its expected value. The collector is responsible for the product acquisition from the market and after inspection process, (s)he delivers the remanufacturable used products to the remanufacturer. Since the inspection is not perfectly accurate, some of items tested as remanufacturable may be non-remanufacturable. Until the disassembly process, actual remanufacturable items are not revealed. The remanufacturer dissembles all transported items and reveals their actual conditions. After disassembly, actual remanufacturable items are remanufactured to satisfy the remanufacturer's order quantity and non-remanufacturable items do not move into further process and are disposed. Two settings are analyzed and compared in terms of their optimal solutions under both deterministic and stochastic demand cases.

In the decentralized setting, for the deterministic demand, optimal solution shows that the remanufacturer's optimal order quantity should be equal to deterministic demand and the collector collects items such as the expected number of actual remanufacturables sent is exactly equal to the demand. For stochastic demand case, the remanufacturer's optimal order quantity is a function of the parameters that affect his profit. The collector similarly collects items such as the expected number of actual remanufacturables sent is exactly equal to the remanufacturer's optimal order quantity. In the centralized setting, for the deterministic demand case, the optimal order quantity is again equal to the deterministic demand. However, for the stochastic case, the optimal collection quantity in the centralized model is greater than in the decentralized model. If demand variability increases, the difference between optimal collection quantities in the centralized and decentralized models increases. Therefore, the result shows that higher collection rates are obtained with centralized decision making approach.

Zikopoulos and Tagaras (2008) examine the effects of quick sorting with limited accuracy before disassembly on the expected profit of the entire system in a single period setting. They analyze two configurations: a system without sorting and a system with inaccurate sorting of returned items at the remanufacturing facility. There are two independent agents in the system: the remanufacturing facility and the collection site. The demand of remanufactured products is a continuous random variable. At the collection site, large numbers of returned products are available and the fraction of remanufacturables is distributed continuously with known distribution function. The collection facility collects the items and ordered quantity is transported to the remanufacturing center. The items are classified imperfectly as remanufacturable or non-remanufacturable. After the sorting is performed, the items that are classified as remanufacturable are disassembled and inspected at the remanufacturing location, and their exact conditions are revealed. The decision variables are the transportation quantity and the remanufacturing lot size. Regardless of the existence of sorting, the optimal remanufacturing quantities are the same in both models. However, the optimal procurement quantity changes according to the existence of sorting. In the first step, the optimal remanufacturing quantity is determined in order to maximize expected total profit for a given amount of available remanufacturables at the remanufacturing center. In the second step, the optimal transportation quantity is determined by taking the relationship between the amount of available remanufacturables and the optimal remanufacturing quantity into account. Optimality conditions are derived for both settings. They show when the model with inaccurate sorting is economically attractive. The main result of the study is that the model with imperfect testing before disassembly is preferable when the inspection, disposal and transportation cost are low, the disassembly cost is high and fraction of the remanufacturables at the collection site is low.

Tagaras and Zikopoulos (2008) study a supply chain that includes a remanufacturing facility and N collection sites in an infinite time period setting. The items are grouped imperfectly into two quality states: remanufacturable or non-remanufacturable. There is a single type of used item and they assume that the demand of used products is stochastic with stationary probability distribution. For each collection site, the fraction of remanufacturables is known and the used item quantities are unlimited. The remanufacturing facility continuously reviews its inventory, whenever the amount of used product falls at the reorder level, then the remanufacturer places an order from one or multiple collection sites. Transferred items from collection sites are disassembled by the remanufacturing facility and then their actual conditions are revealed. Three different channel structures are studied: no sorting, sorting at the remanufacturing facility, and sorting at the collection sites. There are two possible sorting errors: misclassification of remanufacturables as non-remanufacturable with proportion α (Type I error) and misclassification of non-remanufacturables as remanufacturable with proportion β (Type II error). For all models, the remanufacturer pays a fixed set up cost in every replenishment cycle and faces the same unit disassembly cost and remanufacturing cost. In all settings, the optimal policy is to procure all used items from the most preferred collection site that has the minimum ratio of cost to yield rate. Regardless of the model selection and existence of sorting, the optimal reorder level and the total procurement quantity are the same, but the most preferable collection site may be different under different channel configurations.

Van Wassenhove and Zikopoulos (2010) study the effects of misclassification of the used products in a two-level supply chain with a remanufacturer and N independent collectors under both deterministic and stochastic demand in a single time period. The collectors supply items from the end-users and inspect them imperfectly based on some quality specifications provided by the remanufacturer, and then group them into M different quality classes. The collection sites can overestimate the quality of an used item, but it is assumed that the condition of items is overestimated at most by one class. The fraction of the available quantity of each quality class at the collection sites is random. The remanufacturer decides which suppliers send the returned products and the supply quantities in each quality class. The returns are transported to the remanufacturer with different acquisition prices and they are sorted by the remanufacturer, and then their actual grades are revealed. It is found that the optimal procurement quantities depend on testing accuracy and the relationship between acquisition costs of the sequential classes. If additional quality classes, which have remanufacturing costs not larger than the costs of existing classes, are introduced to the model, the effect of misclassification errors on the profit reduces since the variance of classification accuracy decreases.

Zikopoulos and Tagaras (2007) analyze the impact of uncertain quality of the used items on the entire system profit and they also discuss the impact of existence of a correlation between uncertain yield rates at the two supply locations on the profits and decision variables. They study a two level supply chain which includes a refurbishing site and two suppliers in a single period context. After supply sources transport used items to the refurbishing facility, all of them are inspected by the remanufacturer. Sorting is error-free and each used item is graded as refurbishable or non-refurbishable. The demand of returned product is stochastic. The problem is examined in two decision stages and solved starting from the second stage. At the second decision making stage, the optimal production lot size is determined by assuming that the exact number of refurbishable items in the procurement quantity from each suppliers are given. After the optimal production quantity is determined, the optimal procurement quantities from the suppliers are determined in order to maximize the overall system profit. They derive optimality conditions for sourcing from only one of the suppliers and from both suppliers. Their analysis shows that if there is any positive correlation between the suppliers' yield rates, this correlation causes to increase the difference between supply quantities. The optimal policy is sourcing from fewer suppliers. Another important result is that if an identical collection site is added to the system with a single sourcing, yield variance between the collection sites decreases, and hence the system profit increases.

2.2 Effects of Sorting Location

Another important issue for the reverse supply chains is the decision of the sorting location. Zikopoulos and Tagaras (2005) consider a reverse supply chain that consists of one collection site and a central remanufacturing facility that faces uncertain demand in a single period context. In the system, the quality of used products is also uncertain. Used products are tested perfectly and they can be classified into two possible quality states: remanufacturable and non-remanufacturable. At the collection site, used item capacity is assumed as unlimited and the fraction of remanufacturables is a continuous random variable. There are two decision variables: transported quantity from the collection site to the remanufacturer and the remanufacturer's production lot size. If the optimal production lot size is smaller than the demand, the penalty cost is incurred per short item. On the contrary, if remanufactured item quantity exceeds the demand, this excess produced amount is disposed at the same cost with non-remanufacturables.

Two alternative models are considered according to the location where inspection activity takes place: sorting at the collection site and sorting at the remanufacturing facility. Sorting costs are different in the alternative settings. When the remanufacturer sorts the used products, there will be two decision variables in the system: quantity of the transported used items from the collection site to the center and the production lot size at the remanufacturing facility. The optimization problem is analyzed in two stages with the goal of the expected profit maximization of the entire system in each problem stage. At the first stage, the optimal production lot size for the remanufacturer is determined under the assumption that the sorting process is completed and remanufacturable item quantity in the transported lot is known. At the second stage, the optimal transported quantity is determined by using optimal production size determined in the first stage. When the sorting activity is performed at the collection site, the information about the quality state of the used items are identified before transportation. Therefore, all transferred used items are remanufacturable and the quantity is equal to the remanufacturer production order size. Thus, there is a single decision variable in the model that is optimal production lot size. The results for two different systems are analyzed analytically and computationally. The numerical study shows that sorting at the collection facility is preferable when the proportion of remanufacturables decreases and higher transportation cost is incurred.

2.3 Pricing of Returns and Remanufactured Products

Another topic addressed in the literature is to analyze the effects of the pricing decisions of the remanufactured products and used items on the optimal solution under both decentralized and centralized settings and to compare the results of the decentralized models to the centralized model regarding to some cost and revenue parameters.

Guide et al. (2003) develop a framework for determining the optimal quality based acquisition prices and the selling price in order to maximize the profit. There are known price dependent demand and supply functions. Their suggested model includes a remanufacturing firm and intermediaries that grade used items perfectly into N different discrete quality classes and transfer them from final users to the remanufacturer in a single time period. It is assumed that all information about the quality and acquisition prices of the returned products is commonly shared.

Karakayali et al. (2007) analyze the effect of channel selection on the pricing decisions and optimal collection quantity in a single time period. The centralized setting, where both collection and processing activities are performed by the remanufacturer, and two different decentralized models, which are the collector driven channel and remanufacturer driven channel, depending on who determines the wholesale price are discussed. In their models, both the demand of remanufactured products and the supply of the used items have a linear price sensitive function. After the used items are collected from the customers, they are perfectly sorted and grouped into m quality classes by the collector in the decentralized channel setting and the remanufacturer in the centralized channel setting. The centralized and decentralized models are compared according to their collection rates. They identify when outsourcing the collection activity can be preferable and under which conditions one of the decentralized models is more profitable than the other. They also discuss how the collection rate can be increased when it is smaller then the optimal value because of the impact of environmental regulations.

Savaskan et al. (2004) illustrate the impact of different channel structures on the used product return rate, the wholesale price and the profit of the system in a single period context. They consider a hybrid system that consists of remanufacturing operations as well as manufacturing operations, and there is no quality difference between manufactured and remanufactured items. The system is modelled as a two-echelon supply chain that includes a remanufacturer and a single retailer. Demand is modelled as a linear price dependent function. It is assumed that manufacturing of a product by using new materials is more costly than remanufacturing a product from returned units. In order to measure the supply chain performance, they define the return rate that is the fraction of the quantity of remanufactured items to quantity of all produced items. This rate shows the customers' incentive to return their used items for remanufacturing process. They study a centralized model and three types of decentralized models. In all different channel structures, the remanufacturer is the Stackelberg leader and tries to find the optimal wholesale price, and the retailer distributes the new produced products to the market. Decentralized models differ in the responsibility for collection activity: manufacturer collecting, retailer collecting and third-party collecting. They analyze these decentralized models and compare them to the centralized setting in terms of the return rates, the wholesale and retail prices and the total system profits. The centralized model has the highest return rate and lowest retail price. Among the decentralized models, the retailer collecting model gives the highest return rate and lowest retail price value. In terms of the total system profit, the retailer collecting model provides the best value in the decentralized structures since the retailer connects to the customers directly and affects the system profit by chancing the selling price.

Atasu et al. (2013) analyze the combined effect of collection and investment cost structures on the remanufacturer's reverse channel choice. A hybrid system is considered and undifferentiated manufactured and remanufactured items are used to satisfy price dependent market demand. A two-echelon supply chain is studied with a manufacturer and a retailer in a single period context. The remanufacturer is the Stackelberg leader and determines the wholesale price. They assume that all returned items are remanufacturable and the customer demand firstly is filled from remanufactured items since remanufacturing cost of a used item is smaller than manufacturing cost of a new item. They define their models based on the channel structure in Savaskan et al. (2004). Three decentralized models are discussed: the channel with the remanufacturer undertaking collection of used items, the channel with the retailer collecting of used items and the channel with the third party managing collection of used items. The main difference from the model in Savaskan et al. (2004) is that they incorporate collection cost component into the total cost function in addition to rate dependent investment cost component. This cost component consists of scale economies and diseconomies in the collection quantity. They compare the models and observe which channel choice is more profitable for the remanufacturer. The results show that both the manufacturer and retailer collecting options dominate the third party-managed collection. When the collection cost displays scale economies or the scale effect in the rate-dependent investment cost structure is high, the model with the retailer-managed collection is optimal although the remanufacturer shares his profit with the retailer. If the collection cost displays diseconomies of scale or the scale effect in the investment cost is not enough strong, then the model with the remanufacturer handling collection activity is optimal.

Ünal (2009) focuses on the model that includes a remanufacturer and a collector in a single time period setting. She aims to show the impact of centralization on the optimal prices and the profit of the system and examine the effects of knowing the quality information of used items before pricing decisions. Centralized and decentralized model structures are analyzed with respect to optimal prices and the system profit. The demand of the remanufactured items is a linear function of the selling price and the supply of the returned items is a deterministic linear function of the acquisition price. The used items are classified into two quality groups: inferior and superior quality class. The fraction of the items with superior quality level is unknown. In the centralized model, the remanufacturer is a single decision maker and collection and inspection activities are performed by the remanufacturer. There are two different centralized model configurations: simultaneous pricing model and sequential pricing model. In the simultaneous pricing model, the remanufacturer sets selling price and acquisition price before the sorting process. Therefore, the pricing decisions do not affect the quality information. On the other hand, in the sequential pricing model, the remanufacturer determines the acquisition price before the sorting operation and sets the selling price after inspection by using quality information. In decentralized models, there is a collector who is the Stackelberg leader and collects the used items from the end users. She inspects returned items and then sends the sorted items to the remanufacturer at a transfer cost that depends on the quality of the used items. She considers two different decentralized settings regarding to the condition of the transfer price: exogenous transfer prices and collector driven transfer prices. The collector driven model has two different configurations in terms of the sequence of the pricing decisions. If the collector determines the transfer prices for two quality classes before the inspection, then the quality does not affect the pricing decisions. However, if the collector sets the transfer prices after the sorting process, the quality knowledge is used to determine optimal transfer prices. In the numerical analysis, the effects of the centralization and postponing pricing decisions on the collection quantity, remanufacturing lot size and the expected profit in both centralized and decentralized models are discussed.

2.4 Acquisition Management for Returns

For the remanufacturing firms, effective acquisition management helps to select an appropriate method in order to maximize the gain from reuse activities. Galbreth and Blackburn (2006) discuss the variability in the quality of the used items and define the quality condition of returns in a continuous scale. They focus on a sorting policy that specifies the rules about whether a returned product is to be remanufactured or scrapped by incorporating the variability in returned product quality. They introduce a model in a single period context comprising a remanufacturer and third party brokers who supply used items to the remanufacturer. It is assumed that whenever used items are needed, there is available stock at the collection site. After the used products are acquired, the remanufacturer sorts all items and groups them into two possible categories: remanufacturable or scrap. There are two important decisions which affect each other and are made jointly: the optimal acquisition quantity of the used items and the remanufacturing quantity. If the acquisition quantity is larger than the remanufacturing quantity, then the remanufacturer is more selective. Only used items with smaller remanufacturing costs are selected for remanufacturing. Therefore, the total remanufacturing cost decreases due to selection of used product with better condition while the acquisition cost increases. On the other hand, if the difference between acquisition quantity and remanufacturing lot size is low, the remanufacturer is less selective. Thus, the optimal sorting policy is driven by the excess amount of the returns relative to the remanufacturing lot size. They formulate and analyze the model with both deterministic and stochastic demand.

Galbreth and Blackburn (2010) discuss the impact of the variation in the quality condition of the returned items on the remanufacturing and acquisition decisions and total cost of the system. A remanufacturing facility can deal with this variation when the collection quantity is greater than the demand and only the used items with higher quality conditions are remanufactured. They model the problem to find the optimal acquisition quantity in order to minimize total expected cost under deterministic demand. They assume that there are continuous quality conditions and the distribution of the acquired lot is uncertain. The remanufacturer firstly decides the acquisition quantity that is generally beyond the demand in order to produce only the used items of high quality. All acquired products are sorted perfectly by the remanufacturer and the used items in the best conditions are remanufactured until the remanufacturing quantity reaches the demand. The excess amount in the collected lot is scrapped at the unit scrap cost. Galbreth and Blackburn (2006) also study with a continuous range of returned item conditions, but they assume that the return condition in the collected lot is distributed with known distribution function. On the other hand, the distribution of used item condition is not known with certainty in this paper. Two simplifications are made: the condition of each item in the acquired lot has uniform distribution and there is a linear remanufacturing cost function. They also analyze the model with two discrete quality classes. The used items are ranked as low cost and high cost and the low cost items in the acquired lot follow binomial distribution. The simple policy is applied: the remanufacturer satisfies the demand from the low cost class first, if there is short amount of demand then that is served from high cost items.

Ferrer (2003) and Bakal and Akcali (2006) assess the benefits of the early detection of yield rate on the pricing decisions. They analyze different model settings in terms of timing of yield realization in order to make pricing decisions.

Ferrer (2003) analyzes the value of early yield information and the impact of late yield information on the optimal procurement and disassembly lot size decisions and the total cost of the system in a single time period. The demand is deterministic and the remanufacturer satisfies the demand from both new parts supplier and remanufacturing facility. The acquired product has a main part that can be recovered and has a stochastic return yield with known probability distribution. The delivery lead time of the order for both the external supplier and the remanufacturing center are assumed to be deterministic. Four alternative scenarios are discussed. Firstly, yield information is delayed and the decisions are made without the realization of the yield rate. The amount of procurement quantity from external supplier and internal remanufacturing quantity and also the number of items to be disassembled are determined. All disassembled parts are inspected in the recovery process. There are two options for the remanufacturer in order to recover the parts: the whole dissembled parts can be processed or the needed parts are only processed to fulfill the demand. Secondly, during the disassembly operation, the items that are recoverable are detected and nonrecoverable parts are disposed. Therefore, yield rate is identified before the remanufacturing process and only remanufacturable parts go into the recovery process. Under this policy, the decision variables are again determined before the realization of the yield. Thirdly, if the external supplier offers short lead times, then whenever the amount of the recovered items are not sufficient to fill the demand, the supplier can place an urgent order. Lastly, if the actual yield rate is known, the remanufacturer uses this yield information to decide the optimal quantities by avoiding penalty and storage costs. These scenarios are analyzed in a numerical experiment and compared in terms of the total expected costs. The results show that when the yield variance is high, the benefit of determining the yield rate before the recovery process is more than having short lead time suppliers. Another observation is that when the shortage cost is high, having short lead time suppliers is more cost effective than others. Lastly, when the holding cost, procurement cost or recovery cost increase, the model with early yield information is economically attractive than others.

Bakal and Akcali (2006) examine the impact of the yield rate on the optimal acquisition and selling prices and the profit of the system. They assume that the demand of the recovered products is a deterministic decreasing linear function of the selling price and the supply of end-of-life vehicles is also deterministic increasing linear function of the acquisition price. A remanufacturer who procures the vehicles from end users, dismantles the main part from the body and inspects the dismantled parts in order to decide they are recoverable or not. Nonrecoverable parts after the inspection process, and the remaining part of the body after the remanufacturable part is dismantled, are salvaged at different salvage values, and recovered parts are sold to satisfy the demand in the secondary market. The proportion of the remanufacturables is defined as recovery yield rate and it depends on the acquisition price. The problem is modelled in order to determine optimal acquisition and selling prices to achieve profit maximization aim.

They introduced different model configurations to show the benefits of early detection of the yield rate on the optimal decisions and the total profit of the firm. Four different models are examined: deterministic pricing model, postponed pricing model, simultaneous pricing model and exogenous pricing model. In the deterministic pricing model, there is a deterministic yield rate and the decisions are made taking into account this yield. In the postponed pricing model, the yield rate is stochastic and the remanufacturer determines the selling price of the recovered parts after inspection process is completed and the yield rate is specified. In third model, both the selling price and acquisition price are decided before the inspection process. In the last model, the acquisition price or the selling price can be exogenous. If the acquisition price is exogenous, the remanufacturer has no effect on the supply quantity of the returned vehicles. If the selling price is exogenous, the demand is known and independent from the remanufacturer's decisions. The major conclusion of their study is that having the yield information before the pricing decisions gives higher profit value. This value increases when the yield rate decreases and the yield variance increases.

2.5 Multiple Quality Groups

In the literature, the value of quality categorization and its impact on the system profitability for the remanufacturing firms are also studied with multiple discrete quality groups by including the quality uncertainty in the model settings.

Ferguson et al. (2009) introduce a tactical production planning model in a finite discrete time horizon context. The quality of returned items is a random variable and the quality grade of each used item is continuously distributed with known probability density function in each period. They assume that the remanufacturer categorizes the returns into three usage options according to their quality conditions: the items for salvaging for material recovery $(0, q_0)$, for salvaging for parts recovery (q_0, q_1), and for remanufacturing $(q_1, 1)$. The remanufacturable items' range is also divided into I quality levels. In each time period, the demand of the used items is stochastic and if the demand is not satisfied from the remanufactured items then the shortage amount is backlogged at a unit penalty cost with λ probability and lost with $1 - \lambda$ probability. The remanufacturer takes into account the remanufacturing cost of the returns and the holding cost of both the returns and remanufactured items, then faces a trade-off for each quality grade between deciding of remanufacturing quantity in each period and holding quantity for returned items and remanufactured items at the end of each individual time period in order to maximize profit. The model is formulated as a stochastic dynamic problem with no capacity constraints. They derive the optimal solution under stochastic demand and returns, then provide a heuristic method to solve the model under the deterministic demand and returns. They also examine the effects of the value of categorization on the profit under the existence of a capacity constraint on the quantity of returns that can be collected from the customers.

Denizel et al. (2010) also study a tactical production planning model in a multi-period time context under a capacity constraint on the amount of the cores which can be remanufactured in each time period. The remanufacturer receives the cores (used items) that are at the end of their lease and inspects them to classify into I quality groups. However, all returned cores may not be inspected, some of them are inspected and remanufactured to satisfy the demand or some amount of returns can be stocked to be sorted in the future period. The graded core may not be used to remanufacture in the current period and they are held in inventory for possible remanufacturing in the future periods. There is a single type of return and deterministic demand. Unmet demand is backlogged at unit shortage cost. The problem is formulated by using stochastic programming. Firstly, the quantity of the returns to be sorted is determined and secondly, for each scenario, the remanufacturing, salvaging and holding quantities are determined for each quality levels. After the formulation of the stochastic problem, they analyze the model numerically and make a sensitivity analysis to show the effects of the parameters on the optimal solution and the profit.

Teunter and Flapper (2011) investigate the impact of uncertainty in the quality of cores under both deterministic and stochastic demand cases in a single time period setting. After the cores are acquired from the collectors, all collected items are inspected and grouped into K quality classes by the remanufacturer with error-free testing. The quality of the cores has a multinomial distribution. The cost minimization problem is formulated and solved in two stages: firstly, the optimal remanufacturing lot size is determined, then the optimal collection quantity is determined. The uncertainty impact on the optimal decisions is analyzed in the numerical study part. They also examine how the optimal solutions and the total cost are affected by ignoring the variability in the quality of the cores.

Aras et al. (2004) study the classification of the returned products according to their quality and highlight the impact of this categorization on the total cost of the system. They examine a hybrid remanufacturing center that remanufactures returned items and also manufactures new items. The remanufactured and manufactured items are assumed as perfect substitutes. The remanufacturing system includes three inventory centers: the serviceable inventory consisting of remanufactured products and the remanufacturable inventories consisting of two different types of returns. Demand follows a Poisson process and it is firstly satisfied from the remanufactured items. Returns arrive according to a poisson process and they are inspected and classified into two quality classes: high and low quality. The remanufacturing time of the returns follows an exponential distribution. After inspection process, some of returns may be disposed because the capacity of the remanufacturable inventories of two quality classes are limited by the disposal levels. The serviceable inventory is reviewed continuously and whenever the inventory level falls at the base level, then the returned items are pulled into the remanufacturing operation. There are two strategies according to priority given to use the remanufacturable inventories: high class first and low class first. They represent the model as a continuous-time Markov Chain and in order to minimize the total cost for both strategies, they determine optimal values for the decision variables: disposal levels for two quality groups and optimal base stock level. For a comparison of the two strategies, they analyze the models under different cost parameter values and conclude that if high class first strategy is selected, the cost savings due to quality categorization is higher. They also analyze the system by using a benchmark model to asses when the quality based classification of the returns is cost effective. In this model, it is assumed that two class returns are stored in a single inventory and there is a single disposal level to be reviewed. They provide a numerical analysis and the result shows that the improvement in the system cost is about ten percent by using quality information of the returns in the model.

We focus on the remanufacturing processes in which sorting process of returns is subject to classification errors and their actual conditions are revealed after the disassembly operation. Although reverse supply chains with error-free inspection have been studied in many papers recently, not much effort has been spent on imperfect sorting process in the closed loop supply chains. Our work relates to previous studies in the literature on the imperfect inspection by Gu and Tagaras (2014), Zikopoulos and Tagaras (2008), Tagaras and Zikopoulos (2008) and Van Wassenhove and Zikopoulos (2010). However, it differs from their studies in an important aspect. They take the uncertainty regarding to return item quality into account but they do not incorporate of the uncertainty in the inspection process and assume that the quantity of remanufacturables and non-remanufacturables determined by the inspection are exactly equal to their expected values. In our study, however, randomness in the sorting process is taken into account in addition to quality uncertainty in used items. Our study can be divided into two parts. First of all, we formulate a number of different settings based on the deterministic demand model in Gu and Tagaras (2014) under inaccurate sorting procedure and analyze the effects of different leader and follower combinations, price dependent demand and supply and sorting location on the optimal solution and the profits. Secondly, we analyze the impact of disregarding randomness in the inspection process on the optimal solution and the profits. We reformulate the decentralized model in Gu and Tagaras (2014) by taking into account of randomness in the collector's inspection process and characterize the optimal solution under imperfect testing. We also discuss how the impact of randomness on the optimal solution and the profits changes with respect to a change in the parameters' values. To our knowledge, this is the first study that formulates the model with incorporation of randomness related to inaccurate sorting process and analyze the effects of ignoring randomness on the results and also seeks the effects of parameters on randomness.

CHAPTER 3

MODEL DESCRIPTION AND ANALYSIS

In this study, we consider a two-echelon supply chain consisting of a remanufacturer and a collector in a single time period. The remanufacturer orders used items from the collector, and after the remanufacturing process, he sells remanufactured products to the customers in order to serve the deterministic demand in the market. The collector acquires used products from the customers and sells them to the remanufacturer. In the literature, there is a limited number of papers that deal with imperfect sorting in reverse supply chains. The paper by Gu and Tagaras (2014) is mostly related to our work since they study a reverse supply chain consisting of a remanufacturer and a collector with deterministic demand and characterize the optimal solution under inaccurate sorting procedure. They analyze both decentralized and centralized settings under deterministic and stochastic demand cases. We only focus on their decentralized setting under deterministic demand case, which is called as the base model. In this chapter, we develop alternative settings to investigate the effects of price sensitive demand and/or supply on the optimal decisions as well as sorting location and channel leadership. We model price sensitive demand as $D(p_r) = a - bp_r$, where $a > 0$ shows the potential market size and $b > 0$ shows the sensitivity of the demand to the selling price. We also study the price dependent supply which is denoted as $S(f) = rf$, where f is the acquisition price and $r > 0$ shows the supply price sensitivity to the acquisition price f. Therefore, the collector can increase the supply by increasing the acquisition price. In order to analyze the effects of the sorting location, we compare the settings where the sorting activity is performed by the collector versus remanufacturer. When the collector sorts, all collected items from end-users are inspected and only the items classified as remanufacturables are transported to the remanufacturer. When the remanufacturer sorts, all collected items are transported by the collector to the remanufacturer and inspected before disassembly. In all models, used items are inspected imperfectly and grouped into two quality states by inspection: remanufacturable and non-remanufacturable. It is noted that imperfect inspection means that actual condition of used items may be estimated incorrectly. That is, remanufacturable used products can be misclassified as non-remanufacturable or non-remanufacturables can also be misclassified as remanufacturable.

The rest of this chapter is organized as follows: Section 3.1 revisits the model that is studied in Gu and Tagaras (2014), that is the base model, and from Section 3.2 to 3.8 different extensions of the base model are discussed and their optimal solutions are compared to the solution of the base model. In Section 3.9, a detailed comparison of the models is performed and main differences from the base model are presented.

3.1 The Base Model

Gu and Tagaras (2014) analyze a recovery system that includes a single remanufacturer and collector in a single time period. Used products have two quality states: remanufacturable or non-remanufacturable. The collector gets Q_0 units of used items from the market at a unit collection cost of c_o, imperfectly sorts them at the unit inspection cost of c_i and transfers Q_C units, which are grouped as remanufacturable, to the remanufacturer at a unit transportation cost of c_t. It is assumed that the fraction of the remanufacturables q within the collected quantity is known and items that are sorted as non-remanufacturable are disposed of at a unit cost of c_{dC} by the collector. The sorting procedure is subject to two classification errors: type I error and type II error with α and β probability, respectively. Remanufacturable used products can be misclassified as non-remanufacturable with probability α and non-remanufacturables can also be misclassified as remanufacturable with probability β. Upon the arrival of the items at the remanufacturer's site, all transported items are disassembled at a unit cost of c_{dis}. After the disassembly process, actual conditions of the used items are revealed and remanufacturable used products are remanufactured at a per unit cost of c_r and sold in order to satisfy the deterministic market demand D and non-remanufacturable items are disposed of at a unit cost of c_{dR} by the remanufacturer. The remanufacturer pays a unit transfer price of w to the collector for each remanufacturable item up to Q_R, which is the remanufacturer's order quantity. If the number of remanufacturable units that is revealed after disassembly is less than Q_R, then the collector has to pay a penalty b_0 per unit short. Otherwise, the remanufacturer does not pay to the collector for the extra remanufacturable units. If the quantity of remanufactured products is less than the demand, then the remanufacturer faces a unit shortage cost of b for unsatisfied demand. In the decentralized setting, the collector and remanufacturer are independent decision makers. The remanufacturer is the first agent to act and decides the optimal order size Q_R. Then, the collector decides optimal collection quantity Q_0 in order to maximize his profit. The sequence of events in the base model is summarized as follows:

1. The remanufacturer orders Q_R units,
2. The collector collects Q_0 units,
3. The collector sorts imperfectly Q_0 units and transports Q_C units to the remanufacturer,
4. The remanufacturer disassembles and sorts actually Q_C units,
5. The remanufacturer remanufactures $min\,\{(1-\alpha)qQ_0, Q_R\}$.

Table 3.1 summarizes the notation used in the base model.

Since the remanufacturer is the Stackelberg leader, the collector's problem is firstly analyzed and the optimal collection quantity(Q_0) is optimized in order to maximize the collector's profit. The collector sorts all collected items imperfectly and sends the remanufacturable items to the remanufacturer. It is assumed that $w > c_o + c_i + c_t$, otherwise collection of used item is not profitable for the collector. We also assume that $w > c_{dC}$, then selling of used item to the remanufacturer is more profitable for the collector than salvaging of the unsold remanufacturable items in the secondary market. Gu and Tagaras (2014) do not take the uncertainty in the collector's inspection process into account and assume that the number of items that are classified as remanufacturable by the inspection process is equal to its expected value, $[(1-\alpha)q+\beta(1-q)]Q_0$ denoted by Q_C. $(1-\alpha)qQ_0$ represents the quantity of actual remanufacturables after the collector's inspection process and $\beta(1-q)Q_0$ represents

Table 3.1: List of Notations for the Base Model

Notation	Description
Q_0	The collected quantity
Q_C	The transported quantity(Items classified as remanufacturables by the collector)
Q_R	Target remanufacturable quantity(Remanufacturer's order quantity)
Q	Actual remanufacturable quantity after disassembly
D	Deterministic market demand
c_o	unit collection cost
c_i	unit inspection cost
c_t	unit transportation cost
c_{dR}	unit disposal cost of remanufacturer after disassembly
c_R	unit salvage value of remanufactured product if $Q_R > D$
c_r	unit remanufacturing cost
w	unit price paid by the remanufacturer to per remanufacturable unit up to Q_R(transfer price)
c_{dC}	unit disposal cost of collector after imperfect sorting
c_{dis}	unit disassembly cost
p_r	unit selling price
b_0	unit penalty below Q_R paid by the collector
b	unit penalty for unmet demand paid by the remanufacturer
q	proportion of remanufacturables in the collected quantity
α	proportion of remanufacturables which are incorrectly classified as non-remanufacturable
β	proportion of non-remanufacturables which are incorrectly classified as remanufacturable

the quantity of non-remanufacturables that are misclassified as remanufacturable. Q_C units are transported from the collector to the remanufacturer and disassembled by the remanufacturer. After the remanufacturer's disassembly and actual sorting processes, the actual quantity of remanufacturables $Q = (1-\alpha)qQ_0$, is revealed. Note that the quantity of actual remanufacturables can either be smaller or larger than the remanufacturer's order quantity. As a result, the collector's expected profit can be expressed as:

$$\Pi_C(Q_0) = \begin{cases} \Pi_C^I(Q_0) & \text{if } Q_0 \leq \frac{Q_R}{(1-\alpha)q} \\ \\ \Pi_C^{II}(Q_0) & \text{if } Q_0 > \frac{Q_R}{(1-\alpha)q} \end{cases}$$

where

$$\begin{aligned} \Pi_C^I(Q_0) &= w(1-\alpha)qQ_0 - c_0Q_0 - c_iQ_0 - c_{dC}[\alpha q + (1-\beta)(1-q)]Q_0 \\ &\quad -c_t[(1-\alpha)q + \beta(1-q)]Q_0 - b_0[Q_R - (1-\alpha)qQ_0] \end{aligned}$$

and

$$\begin{aligned} \Pi_C^{II}(Q_0) &= wQ_R - c_0Q_0 - c_iQ_0 - c_{dC}[\alpha q + (1-\beta)(1-q)]Q_0 \\ &\quad -c_t[(1-\alpha)q + \beta(1-q)]Q_0. \end{aligned}$$

The revenue and cost items that are used in the collector's profit function are given as follows:

- Collection cost of used items: c_0Q_0;
- Collector's inspection cost: c_iQ_0;
- Disposal cost of non-remanufacturables by the collector: $c_{dC}[\alpha q + (1-\beta)(1-q)]Q_0$
- Transportation cost of shipped items: $c_t[(1-\alpha)q + \beta(1-q)]Q_0$
- Penalty cost paid by the collector: $\begin{cases} b_0[Q_R - (1-\alpha)qQ_0] & \text{if } Q_0 \leq \frac{Q_R}{(1-\alpha)q} \\ \\ 0 & \text{if } Q_0 > \frac{Q_R}{(1-\alpha)q} \end{cases}$

- Transfer payment by the remanufacturer: $\begin{cases} w(1-\alpha)qQ_0 & \text{if } Q_0 \leq \frac{Q_R}{(1-\alpha)q} \\ \\ wQ_R & \text{if } Q_0 > \frac{Q_R}{(1-\alpha)q} \end{cases}$

Lemma 1. *$\Pi_C(Q_0)$ is continuous, but not differentiable at $Q_0 = \frac{Q_R}{(1-\alpha)q}$.*

Proof. The proof is provided in Appendix A. ∎

Proposition 1. *Given the remanufacturer's order quantity, Q_R, the optimal collection quantity under deterministic demand is characterized by:*

$$Q_0^* = \begin{cases} \frac{Q_R}{(1-\alpha)q} & \textit{if } (w+b_0)(1-\alpha)q > A_1 \\ \\ 0 & \textit{o/w} \end{cases}$$

where A_1 is given by:

$$A_1 = (c_0 + c_i) + c_{dC}[\alpha q + (1-\beta)(1-q)] + c_t[(1-\alpha)q + \beta(1-q)].$$

Proof. The first derivatives of $\Pi_C^I(Q_0)$ and $\Pi_C^{II}(Q_0)$ with respect to Q_0 are given as:

$$\begin{aligned} \frac{d\Pi_C^I(Q_0)}{dQ_0} &= (w+b_0)(1-\alpha)q - (c_0+c_i) - c_{dC}[\alpha q + (1-\beta)(1-q)] \\ &\quad -c_t[(1-\alpha)q + \beta(1-q)] \end{aligned}$$

and

$$\frac{d\Pi_C^{II}(Q_0)}{dQ_0} = -(c_0+c_i) - c_{dC}[\alpha q + (1-\beta)(1-q)] - c_t[(1-\alpha)q + \beta(1-q)].$$

When $(w+b_0)(1-\alpha)q > A_1$, $\frac{d\Pi_C^I(Q_0)}{dQ_0} > 0$ and $\frac{d\Pi_C^{II}(Q_0)}{dQ_0} < 0$. Hence, $Q_0^* = \frac{Q_R}{(1-\alpha)q}$. Otherwise, $\frac{d\Pi_C^I(Q_0)}{dQ_0} < 0$ and $\frac{d\Pi_C^{II}(Q_0)}{dQ_0} < 0$. Hence, $Q_0^* = 0$. ∎

When $Q_0 < \frac{Q_R}{(1-\alpha)q}$, $A_1 = (c_0+c_i)+c_{dC}[\alpha q+(1-\beta)(1-q)]+c_t[(1-\alpha)q+\beta(1-q)]$ represents the increase in the collector's expected cost and $(w+b_0)(1-\alpha)q$ shows the increase in the collector's expected revenue when the collection quantity increases by one unit. Hence, if $(w+b_0)(1-\alpha)q > A_1$, then collection of additional unit of used item is profitable for the collector and he tries to collect as many units as possible.

The remanufacturer disassembles all transported items at a per unit cost of c_{dis} and observes the actual quantity of remanufacturables Q. If there is a non-remanufacturable item, it is disposed at a unit cost of c_{dR}. Since $Q = (1-\alpha)qQ_0$ and $Q_0^* = \frac{Q_R}{(1-\alpha)q}$ when $A_1 < (w+b_0)(1-\alpha)q$, then $Q = Q_R$. When $A_1 > (w+b_0)(1-\alpha)q$, $Q_0^* = 0$ and so $Q = 0$, then $Q_R = 0$. Hence, $Q = Q_R$.

We now consider the remanufacturer's problem. The remanufacturer produces $\min\{Q, D\}$ units and the unit remanufacturing cost of c_r is incurred. After the remanufacturing process, remanufactured items are used to satisfy the deterministic market demand D and they are sold at a unit selling price p_r. It is assumed that $p_r > c_{dis} + c_r + w$ in order to gain a revenue from remanufacturing. We also assume $p_r + b > c_R$ and $p_r - c_r > c_{dR}$. That is, salvaging a remanufactured product and a remanufacturable used item, respectively is less profitable than selling a remanufactured product. If Q is smaller than D, unsatisfied demand is lost at a unit cost of b. On the other hand, if Q is larger than D, the excess remanufacturables are disposed at unit cost of c_{dR}. It is also assumed that $c_R < (c_{dis} + c_r + w) + (c_{dis} + c_{dR})\frac{\beta(1-q)}{(1-\alpha)q}$, that is, the salvage value for an excess remanufactured product is smaller than the corresponding operational cost. Otherwise, the remanufacturer sets his order size as large as possible regardless to the market demand since he always gains profit from remanufacturing of the used item that exceeds the demand. The remanufacturer determines the optimal order size by taking into consideration Q_0^* and $Q = Q_R$. As a result, the expected profit of the remanufacturer can be expressed as:

$$\Pi_R(Q_R) = \begin{cases} \Pi_R^I(Q_R) & \text{if } Q_R \leq D \\ \\ \Pi_R^{II}(Q_R) & \text{if } Q_R > D \end{cases}$$

where

$$\begin{aligned} \Pi_R^I(Q_R) &= p_r Q_R - c_{dis}[(1-\alpha)q + \beta(1-q)]\frac{Q_R}{(1-\alpha)q} \\ &\quad - c_{dR}\beta(1-q)\frac{Q_R}{(1-\alpha)q} - c_r Q_R - wQ_R - b(D - Q_R) \end{aligned}$$

and

$$\begin{aligned} \Pi_R^{II}(Q_R) &= p_r D + c_R(Q_R - D) - c_{dis}[(1-\alpha)q + \beta(1-q)]\frac{Q_R}{(1-\alpha)q} \\ &\quad - c_{dR}\beta(1-q)\frac{Q_R}{(1-\alpha)q} - c_r Q_R - wQ_R. \end{aligned}$$

The revenue and cost items that are used in the remanufacturer's profit function are:

- Disassembly cost: $c_{dis}[(1-\alpha)q + \beta(1-q)]\frac{Q_R}{(1-\alpha)q}$
- Disposal cost of non-remanufacturables: $c_{dR}\beta(1-q)\frac{Q_R}{(1-\alpha)q}$
- Total remanufacturing cost: $c_r Q_R$
- Total cost paid to the collector for remanufacturables due to the contract: wQ_R
- Revenue from remanufacturing : $p_r \min\{D, Q_R\}$
- Penalty cost for unsatisfied demand: $b \max\{0, D - Q_R\}$
- Salvage revenue from remanufactured items: $c_R \max\{0, Q_R - D\}$

Lemma 2. $\Pi_R(Q_R)$ *is continuous, but not differentiable at* $Q_R = D$.

Proof. The proof is in Appendix B. ■

Proposition 2. *Under deterministic demand, the optimal order quantity for the remanufacturer is characterized by:*

$$Q_R^* = \begin{cases} D & \textit{if } (p_r + b) > A_2 \\ \\ 0 & \textit{o/w} \end{cases}$$

where $A_2 = (c_{dis} + c_r + w) + (c_{dis} + c_{dR})\frac{\beta(1-q)}{(1-\alpha)q}$.

Proof. The first derivatives of $\Pi_R^I(Q_R)$ and $\Pi_R^{II}(Q_R)$ with respect to Q_R are given as:

$$\frac{d\Pi^I R(Q_R)}{dQ_R} = p_r + b - (c_{dis} + c_r + w) - (c_{dis} + c_{dR})\frac{\beta(1-q)}{(1-\alpha)q}$$

$$\frac{d\Pi_R^{II}(Q_R)}{dQ_R} = c_R - (c_{dis} + c_r + w) - (c_{dis} + c_{dR})\frac{\beta(1-q)}{(1-\alpha)q}.$$

Since we assume $c_R < A_2$ for the remanufacturer, $\frac{d\Pi_R^{II}(Q_R)}{dQ_R} < 0$. When $(p_r + b) > A_2$, $\frac{d\Pi_R^I(Q_R)}{dQ_R} > 0$. Hence, $Q_R^* = D$. Otherwise, $\frac{d\Pi_R^I(Q_R)}{dQ_R} < 0$ and $\frac{d\Pi_R^{II}(Q_R)}{dQ_R} < 0$. Hence, $Q_R^* = 0$. ■

In order to order one more additional unit, the remanufacturer compares the marginal revenue to the related marginal cost. When $Q_R \leq D$, $A_2 = (c_{dis} + c_r + w) + (c_{dis} + c_{dR})\frac{\beta(1-q)}{(1-\alpha)q}$ represents the increase in the remanufacturer's cost while $(p_r + b)$ represents the increase in the remanufacturer's revenue when one more additional unit is ordered by the remanufacturer. If $(p_r + b) > A_2$, the resulting increase in the remanufacturer's revenue dominates the related increase in the remanufacturer's cost. Therefore, ordering of one more unit is profitable for the remanufacturer. In this case, $\Pi_R(Q_R)$ is an increasing function of Q_R and $Q_R^* = D$. Otherwise, $\Pi_R(Q_R)$ is a decreasing function of Q_R, hence $Q_R = 0$. When $Q_R > D$, A_2 represents the increase in the remanufacturer's cost whereas c_R shows the increase in his profit when he orders one more unit. Since we assume $c_R < A_2$, then $\Pi_R(Q_R)$ is a decreasing function of Q_R. It means that the related margin loss outweighs the benefit from ordering of one additional used item by the remanufacturer. Therefore, he sets the order quantity Q_R such that $Q_R = 0$.

3.2 Model I-The Base Model with Remanufacturer Managing Inspection

In this model, we consider a different version of the base model. In this setting, the remanufacturer is responsible for sorting the used items, whereas the collector sorts collected items in the base model. We analyze the model under deterministic demand and the problem includes only one decision variable: collection quantity. The shipment size and actual number of remanufacturables are functions of this sole decision variable. Hence, it actually corresponds to a centralized setting. The remanufacturer is the single decision maker and tries to find the optimal collection quantity in order to maximize his profit. The sequence of the events is given below:

1. The remanufacturer determines and orders Q_0 units,
2. The collector collects and transports Q_0 units to the remanufacturer,
3. The remanufacturer imperfectly sorts Q_0 units,
4. The remanufacturer disassembles Q_0 units, then remanufactures $min\{(1-\alpha)qQ_0, D\}$.

The remanufacturer's profit is characterized as follows:

$$\begin{aligned}\Pi_R(Q_0) &= -c_iQ_0 - c_{dR}[\alpha q + (1-\beta)(1-q)]Q_0 - c_{dis}[(1-\alpha)q + \beta(1-q)]Q_0 \\ &\quad -c_{dR}\beta(1-q)Q_0 + (p_r - c_r)\min\{(1-\alpha)qQ_0, D\} \\ &\quad -c_{dR}\max\{0, (1-\alpha)qQ_0 - D\} - w\min\{(1-\alpha)qQ_0, D\}\end{aligned}$$

The revenue and cost items in the remanufacturer's profit function are given below:

- Inspection cost: c_iQ_0;
- Disposal cost of non-remanufacturables after imperfect sorting: $c_{dR}[\alpha q + (1-\beta)(1-q)]Q_0$
- Disassembly cost: $c_{dis}[(1-\alpha)q + \beta(1-q)]Q_0$
- Disposal cost of non-remanufacturables after disassembly: $c_{dR}\beta(1-q)Q_0$
- Disposal cost of excess remanufacturables: $c_{dR}\max\{0, (1-\alpha)qQ_0 - D\}$
- Revenue from remanufacturing : $p_r\min\{(1-\alpha)qQ_0, D\}$
- Remanufacturing cost : $c_r\min\{(1-\alpha)qQ_0, D\}$
- Total cost paid to the collector for remanufacturables: $w\min\{(1-\alpha)qQ_0, D\}$

Lemma 3. *The optimal number of actual remanufacturables cannot exceed demand, e.g.* $(1-\alpha)qQ_0^* \leq D$.

Proof. Consider a collection quantity, Q_0 such that $(1-\alpha)qQ_0^* > D$. Define $\epsilon > 0$ such that $\epsilon = (1-\alpha)qQ_0^* - D$. Then, the remanufacturer's profit at $Q_0 = \frac{\epsilon+D}{(1-\alpha)q}$ can be expressed as:

$$\begin{aligned}\Pi_R\left(\frac{\epsilon+D}{(1-\alpha)q}\right) &= \frac{-(\epsilon+D)}{(1-\alpha)q}\begin{pmatrix} c_i + c_{dR}[\alpha q + (1-\beta)(1-q)] \\ +c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q)\end{pmatrix} \\ &\quad +(p_r - c_r - w)D - c_{dR}\epsilon\end{aligned}$$

When $Q_0 = \frac{D}{(1-\alpha)q}$, the remanufacturer's profit is expressed as follows:

$$\Pi_R\left(\frac{D}{(1-\alpha)q}\right) = \frac{-D}{(1-\alpha)q}\left(\begin{array}{l} c_i + c_{dR}[\alpha q + (1-\beta)(1-q)] \\ +c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q) \end{array}\right)$$

$$+(p_r - c_r - w)D$$

Then,

$$\Pi_R\left(\frac{\epsilon + D}{(1-\alpha)q}\right) - \Pi_R\left(\frac{D}{(1-\alpha)q}\right)$$

$$= \frac{-\epsilon}{(1-\alpha)q}\left(\begin{array}{l} c_i + c_{dR}[\alpha q + (1-\beta)(1-q)] \\ +c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q) \end{array}\right)$$

$$-c_{dR}\epsilon < 0$$

Therefore, for any Q_0 such that $(1-\alpha)qQ_0 > D$, the remanufacturer's profit is smaller than $\Pi_R\left(\frac{D}{(1-\alpha)q}\right)$. Hence, $Q_0 > \frac{D}{(1-\alpha)q}$ can not be optimal.

■

Using Lemma 3, the remanufacturer's maximization problem is rearranged as:

$$\begin{aligned} \max \qquad \Pi_R(Q_0) = & -c_iQ_0 - c_{dR}[\alpha q + (1-\beta)(1-q)]Q_0 \\ & - c_{dis}[(1-\alpha)q + \beta(1-q)]Q_0 - c_{dR}\beta(1-q)Q_0 \\ & + (p_r - c_r - w)(1-\alpha)qQ_0 \end{aligned}$$

$$\text{subject to} \qquad Q_0 \leq \frac{D}{(1-\alpha)q}.$$

Proposition 3. *Under deterministic demand, the optimal collection quantity is characterized by:*

$$Q_0^* = \begin{cases} \frac{D}{(1-\alpha)q} & \textit{if } p_r(1-\alpha)q > A_3 \\ \\ 0 & \textit{o/w} \end{cases}$$

where $A_3 = c_i + c_{dR}[\alpha q + (1-\beta)(1-q)] + c_{dis}[(1-\alpha)q + \beta(1-q)]$
$+ c_{dR}\beta(1-q) + (c_r + w)(1-\alpha)q.$

Proof. The first derivative of $\Pi_R(Q_0)$ with respect to Q_0 is:

$$\begin{aligned}\frac{d\Pi_R(Q_0)}{dQ_0} &= -c_i - c_{dR}[\alpha q + (1-\beta)(1-q)] - c_{dis}[(1-\alpha)q + \beta(1-q)] \\ &\quad -c_{dR}\beta(1-q) + (p_r - c_r - w)(1-\alpha)q\end{aligned}$$

When $p_r(1-\alpha)q > A_3$, $\frac{d\Pi_R(Q_0)}{dQ_0} > 0$. Therefore, $Q_0^*= \frac{D}{(1-\alpha)q}$. Otherwise, $\frac{d\Pi_R(Q_0)}{dQ_0} < 0$ and hence, $Q_0^* = 0$. ■

$p_r(1-\alpha)q$ represents the remanufacturer's marginal revenue associated with ordering one more additional unit of used item. The remanufacturer compares this marginal revenue with the related marginal cost in order to decide whether to order one more item is profitable or not. If $p_r(1-\alpha)q > A_3$, $\Pi_R(Q_0)$ is an increasing function of Q_0. Therefore, ordering of one more additional item is profitable for the remanufacturer. Hence, $Q_0^* = \frac{D}{(1-\alpha)q}$. Otherwise, $\Pi_R(Q_0)$ is a decreasing function of Q_0. Therefore, he sets Q_0 at its lower bound, $Q_0^* = 0$.

Propositions 1 and 3 show that the expressions for the optimal collection quantity are the same in the base model and Model I since the remanufacturer always sets the quantity of exact remanufacturables equal to the deterministic market demand after disassembly. However, the conditions that are used in order to decide whether ordering one more additional unit of used item is profitable are different.

Corollary 1. *The comparison of optimal collection quantity, Q_0^*, in the base model and Model I is given as follows:*

a) If $p_r > (c_{dis} + c_r + w) + (c_{dis} + c_{dR})\frac{\beta(1-q)}{(1-\alpha)q} + \frac{c_i + c_{dR}[\alpha q + (1-\beta)(1-q)]}{(1-\alpha)q}$, then $Q_0^* = \frac{D}{(1-\alpha)q}$ in both Model I and the base model.

b) If $(c_{dis} + c_r + w) + (c_{dis} + c_{dR})\frac{\beta(1-q)}{(1-\alpha)q} - b < p_r \leq \begin{pmatrix} (c_{dis} + c_r + w) \\ +(c_{dis} + c_{dR})\frac{\beta(1-q)}{(1-\alpha)q} \\ +\frac{c_i + c_{dR}[\alpha q + (1-\beta)(1-q)]}{(1-\alpha)q} \end{pmatrix}$, then $Q_0^* = \frac{D}{(1-\alpha)q}$ in the base model and $Q_0^* = 0$ in Model I.

c) If $p_r \leq (c_{dis} + c_r + w) + (c_{dis} + c_{dR})\frac{\beta(1-q)}{(1-\alpha)q} - b$, then $Q_0^* = 0$ in both Model I and the base model.

Proof. The proof directly follows from Proposition 1 and Proposition 3.

■

In order to illustrate the behavior of the optimal collection quantity, we define an example parameter set given in Table 3.2.

Table 3.2: An example of parameter set

D	c_o	c_i	c_t	c_{dC}	c_{dis}	c_{dR}	c_r	w	c_R	b_0	b	q	α	β
25	5	5	5	0	10	5	50	100	60	20	30	0.4	0.2	0.1

For any $p_r \, \epsilon \, (163, 188]$, ordering one more additional unit of used item is profitable for the remanufacturer in the base model, then $Q_R^* = 25$ and $Q_0^* = 78$. However, ordering one more additional unit of used item is not profitable for the remanufacturer in Model I, then $Q_R^* = 0$ and so, $Q_0^* = 0$. For any p_r such that $p_r > 188$, the optimal collection quantity for the base model and the optimal order quantity for Model I are both equal to 78. On the other hand, when $p_r \leq 163$, $Q_0^* = 0$ in both models.

Corollary 1 indicates that there is a price range in which the remanufacturer makes a profit in the base model while he does not gain any profit in Model I. The remanufacturer is only responsible for disassembly and remanufacturing of used items that are collected and sorted by the collector in the base model. On the other hand, the remanufacturer takes responsibility of inspection process in addition to disassembly and remanufacturing activities in Model I. Therefore, his total operation cost increases regarding to additional inspection and disposal costs and hence, he should operate at larger selling price values in order to gain profit from remanufacturing in Model I. From the collector's point of view, he collects the same amount of items when the remanufacturer sorts. However, his total cost changes by $c_i Q_0 + (c_{dC} - c_t)[\alpha q + (1 - \beta)(1 - q)]Q_0$ units.

3.3 Model II-Price Dependent Supply Case under Remanufacturer's Lead

In this section, we extend the base model such that the supply of used items is deterministic and price sensitive, denoted as $S(f) = rf$, where $f > 0$ and the collector determines the acquisition price of f for collection of used items. Therefore, the supply quantity can be changed by changing the acquisition price. Another difference from the base model is that the remanufacturer is responsible for sorting of the used items in this model whereas the collector takes responsibility of the inspection process in the base model. There are two decision variables: collection quantity determined by the remanufacturer and the acquisition price f determined by the collector. We assume that the transfer price is exogenous. The sequence of events in this model is as follows:

1. The remanufacturer orders Q_0 units,
2. The collector determines f and collects $S(f) = rf$,
3. The collector delivers $min\{rf, Q_0\}$ to the remanufacturer at a unit transfer price of w,
4. The remanufacturer sorts $min\{rf, Q_0\}$ and remanufactures $\min\{q \min\{rf, Q_0\}(1-\alpha), D\}$.

Since the remanufacturer is the leader, firstly the collector's problem is analyzed, then the remanufacturer's problem is studied by using the optimal decision of the collector's problem. The collector's profit maximization problem is formulated below in order to find the optimal acquisition price given Q_0:

$$\begin{aligned} \max \quad & \Pi_C(f) = -f(rf) + w \min\{rf, Q_0\} \\ \text{subject to} \quad & f \geq 0. \end{aligned}$$

Lemma 4. *The optimal amount of supply cannot exceed the remanufacturer's order quantity, e.g.* $rf^* \leq Q_0$.

Proof. We will prove this lemma by contradiction. Assume that $rf^* > Q_0$. Define $\epsilon > 0$ such that $\epsilon = rf^* - Q_0$. Then, the collector's profit at $f = \frac{\epsilon+Q_0}{r}$ can be expressed as:

$$\Pi_C\left(\frac{\epsilon + Q_0}{r}\right) = \frac{-(\epsilon + Q_0)^2}{r} + wQ_0$$

For $f = \frac{Q_0}{r}$, the collector's profit is expressed as follows:

$$\Pi_C\left(\frac{Q_0}{r}\right) = \frac{-(Q_0)^2}{r} + wQ_0$$

Then,

$$\Pi_C\left(\frac{\epsilon + Q_0}{r}\right) - \Pi_C\left(\frac{Q_0}{r}\right) = \frac{-(2\epsilon Q_0 + \epsilon^2)}{r} < 0$$

Hence, $f = \frac{\epsilon+Q_0}{r}$ can not be optimal. ■

Lemma 4 is rather intuitive. If the collector sends a larger quantity than the remanufacturer order size, the remanufacturer does not pay anything to the collector for the excess amount. The excess supply only results in a decrease in the collector's profit by their total acquisition cost.

Using Lemma 4, the collector's problem is rearranged as follows:

$$\begin{aligned} \max \quad & \Pi_C(f) = -f(rf) + w(rf) \\ \text{subject to} \quad & rf \leq Q_0 \\ & f \geq 0. \end{aligned}$$

Lemma 5. $\Pi_C(f)$ *is a concave function in* f.

Proof. The first derivative of $\Pi_C(f)$ with respect to f is:

$$\frac{d\Pi_C(f)}{df} = -rf + r(w - f)$$

and the second derivative of $\Pi_C(f)$ with respect to f is:

$$\frac{d^2\Pi_C(f)}{df^2} = -2r < 0.$$

Hence, $\Pi_C(f)$ is concave in f. ■

Proposition 4. *Given the remanufacturer's order quantity Q_0, the optimal acquisition price is given by:*

$$f^* = \begin{cases} \frac{w}{2} & \text{if } r\frac{w}{2} \leq Q_0 \\ \frac{Q_0}{r} & \text{o/w} \end{cases}.$$

Proof. The first derivative of $\Pi_C(f)$ with respect to f is:

$$\frac{d\Pi_C(f)}{df} = -rf + r(w - f)$$

The unconstrained maximizer of $\Pi_C(f)$ is given by $\frac{d\Pi_C(f)}{df} = 0$, which is $\frac{w}{2}$. If $\frac{rw}{2} < Q_0$, it is the optimal solution. Hence, $f^* = \frac{w}{2}$. Otherwise, the constraint is binding and $f^* = \frac{Q_0}{r}$.

■

We now consider the remanufacturer's problem. The profit function is characterized as follows:

$$\begin{aligned} \Pi_R(Q_0) \;=\; & -(c_i + w)\min\{rf^*, Q_0\} - c_{dR}\min\{rf^*, Q_0\}[\alpha q + (1-\beta)(1-q)] \\ & - c_{dis}\min\{rf^*, Q_0\}[(1-\alpha)q + \beta(1-q)] \\ & - c_{dR}\min\{rf^*, Q_0\}\beta(1-q) \\ & + (p_r - c_r)\min\{\min\{rf^*, Q_0\}(1-\alpha)q, D\} \\ & - c_{dR}\max\{0, \min\{rf^*, Q_0\}(1-\alpha)q - D\} \end{aligned}$$

The revenue and cost items that are used in the remanufacturer's profit function are given as follows:

- Inspection cost: $c_i \min\{rf^*, Q_0\}$;

- Transfer cost of shipped items: $w \min\{rf^*, Q_0\}$;
- Disposal cost of non-remanufacturables after inspection:
  $c_{dR} \min\{rf^*, Q_0\} [\alpha q + (1-\beta)(1-q)]$
- Disassembly cost: $c_{dis} \min\{rf^*, Q_0\} [(1-\alpha)q + \beta(1-q)]$
- Disposal cost of non-remanufacturables after disassembly:
  $c_{dR} \min\{rf^*, Q_0\} \beta(1-q)$
- Disposal cost of excess remanufacturables:
  $c_{dR} \max\{0, \min\{rf^*, Q_0\}(1-\alpha)q - D\}$
- Revenue from remanufacturing : $p_r \min\{\min\{rf^*, Q_0\}(1-\alpha)q, D\}$
- Remanufacturing cost : $c_r \min\{\min\{rf^*, Q_0\}(1-\alpha)q, D\}$

Lemma 6. *There exists an optimal solution to the remanufacturer's problem such that* $Q_0^* \leq rf^*$.

Proof. Consider an order quantity Q_0 such that $Q_0 > rf^*$. Define $\epsilon > 0$ such that $\epsilon = Q_0 - rf^*$. Then, the remanufacturer's profit at $Q_0 = rf^* + \epsilon$ can be expressed as:

$$\begin{aligned}
\Pi_R(rf^*+\epsilon) &= -(c_i+w)rf^* - c_{dR}rf^*[\alpha q + (1-\beta)(1-q)] \\
&\quad - c_{dis}rf^*[(1-\alpha)q + \beta(1-q)] - c_{dR}rf^*\beta(1-q) \\
&\quad +(p_r - c_r)\min\{rf^*(1-\alpha)q, D\} \\
&\quad -c_{dR}\max\{0, rf^*(1-\alpha)q - D\}
\end{aligned}$$

When $Q_0 = rf^*$, then the remanufacturer's profit is expressed as follows:

$$\begin{aligned}
\Pi_R(rf^*) &= -(c_i+w)rf^* - c_{dR}rf^*[\alpha q + (1-\beta)(1-q)] \\
&\quad - c_{dis}rf^*[(1-\alpha)q + \beta(1-q)] - c_{dR}rf^*\beta(1-q) \\
&\quad +(p_r - c_r)\min\{rf^*(1-\alpha)q, D\} \\
&\quad -c_{dR}\max\{0, rf^*(1-\alpha)q - D\}
\end{aligned}$$

Then,

$$\Pi_R(rf^*+\epsilon) - \Pi_R(rf^*) = 0.$$

Any solution with $Q_0 > rf^*$, results the same profit for the remanufacturer. ■

Intuitively, for any value of $Q_0 > rf^*$ results the same profit for the remanufacturer since the collector collects and delivers only rf^* units in order to maximize his profit. Therefore, the remanufacturer's optimal order quantity Q_0^* should be less than or equal to rf^*.

The remanufacturer's problem is rearranged by using Lemma 6 as follows:

$$\begin{aligned} \max \qquad \Pi_R(Q_0) = & -(c_i + w)Q_0 - c_{dR}Q_0[\alpha q + (1-\beta)(1-q)] \\ & - c_{dis}Q_0[(1-\alpha)q + \beta(1-q)] - c_{dR}Q_0\beta(1-q) \\ & + (p_r - c_r)\min\{Q_0(1-\alpha)q, D\} \\ & - c_{dR}\max\{0, Q_0(1-\alpha)q - D\} \end{aligned}$$

$$\text{subject to} \qquad Q_0 \leq \frac{rw}{2}.$$

Lemma 7. *In the optimal solution of the remanufacturer's problem, the actual quantity of remanufacturables is always less than or equal to the deterministic market demand, i.e,* $Q_0^*(1-\alpha)q \leq D$.

Proof. Assume that $Q_0^*(1-\alpha)q > D$. Define $\epsilon > 0$ such that $\epsilon = Q_0^*(1-\alpha)q - D$. Then, the remanufacturer's profit at $Q_0 = \frac{\epsilon + D}{(1-\alpha)q}$ can be expressed as:

$$\Pi_R\left(\frac{\epsilon + D}{(1-\alpha)q}\right) = \frac{-(\epsilon + D)}{(1-\alpha)q}\begin{pmatrix} c_i + w + c_{dR}[\alpha q + (1-\beta)(1-q)] \\ +c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q) \end{pmatrix}$$

$$+(p_r - c_r)D + c_{dR}\epsilon$$

Now, consider the remanufacturer's profit at $Q_0 = \frac{D}{(1-\alpha)q}$ is expressed as follows:

$$\Pi_R\left(\frac{D}{(1-\alpha)q}\right) = \frac{-D}{(1-\alpha)q}\begin{pmatrix} c_i + w + c_{dR}[\alpha q + (1-\beta)(1-q)] \\ +c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q) \end{pmatrix}$$

$$+(p_r - c_r)D$$

Then,

$$\Pi_R\left(\frac{\epsilon + D}{(1-\alpha)q}\right) - \Pi_R\left(\frac{D}{(1-\alpha)q}\right)$$

$$= \frac{-\epsilon}{(1-\alpha)q}\left(\begin{array}{l} c_i + w + c_{dR}[\alpha q + (1-\beta)(1-q)] + c_{dis}[(1-\alpha)q + \beta(1-q)] \\ +c_{dR}\beta(1-q) \end{array}\right)$$

$$-c_{dR}\epsilon < 0$$

Therefore, for any $\epsilon > 0$ such that $\epsilon = Q_0(1-\alpha)q - D$, the remanufacturer's profit at $Q_0 = \frac{\epsilon+D}{(1-\alpha)q}$ is smaller than at $Q_0 = \frac{D}{(1-\alpha)q}$. Hence, $Q_0^*(1-\alpha)q \le D$. ■

Intuitively, after the disassembly process, if the number of remanufacturables exceeds the demand, then the remanufacturer only processes D units. $Q - D$ units are not remanufactured and disposed of at a unit cost of c_{dR}. Therefore, the remanufacturer's profit will decrease.

The remanufacturer's problem is rearranged by using Lemma 7 as follows:

$$\begin{aligned} \max \qquad \Pi_R(Q_0) = &-(c_i + w)Q_0 - c_{dR}[\alpha q + (1-\beta)(1-q)]Q_0 \\ &- c_{dis}[(1-\alpha)q + \beta(1-q)]Q_0 - c_{dR}(1-q)\beta Q_0 \\ &+ (p_r - c_r)q(1-\alpha)Q_0 \end{aligned}$$

$$\text{subject to} \qquad Q_0 \le \min\left\{\frac{rw}{2}, \frac{D}{(1-\alpha)q}\right\}.$$

Proposition 5. *The remanufacturer's optimal order quantity under deterministic demand case is characterized by:*

$$Q_0^* = \begin{cases} \min\left\{\frac{rw}{2}, \frac{D}{(1-\alpha)q}\right\} & \text{if } p_r(1-\alpha)q > A_4 \\ \\ 0 & \text{o/w} \end{cases}$$

where A_4 is given by:

$$\begin{aligned} A_4 = &(c_i + w) + c_{dR}[\alpha q + (1-\beta)(1-q)] + c_{dis}[(1-\alpha)q + \beta(1-q)] \\ &+ c_{dR}\beta(1-q) + c_r(1-\alpha)q. \end{aligned}$$

Proof. The first derivative of $\Pi_R(Q_0)$ with respect to Q_0 is:

$$\begin{aligned}\frac{d\Pi_R(Q_0)}{dQ_0} &= -(c_i + w) - c_{dR}[\alpha q + (1-q)] - c_{dis}[(1-\alpha)q + \beta(1-q)] \\ &+(p_r - c_r)(1-\alpha)q\end{aligned}$$

When $p_r(1-\alpha)q > A_4$, $\frac{d\Pi_R(Q_0)}{dQ_0} > 0$, that is, $\Pi_R(Q_0)$ is increasing for $0 \leq Q_0 \leq \min\left\{\frac{rw}{2}, \frac{D}{(1-\alpha)q}\right\}$. Hence, $Q_0^* = \min\left\{\frac{rw}{2}, \frac{D}{(1-\alpha)q}\right\}$. Otherwise, $\frac{d\Pi_R(Q_0)}{dQ_0} < 0$, then $Q_0^* = 0$.

■

Note that $p_r(1-\alpha)q$ present the marginal revenue from remanufacturing of additional one more unit, and $A_4 = (c_i + w) + c_{dR}[\alpha q + (1-\beta)(1-q)] + c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q) + c_r(1-\alpha)q$ represents the related cost margin that is realized by collection and remanufacturing of one more item. If $p_r(1-\alpha)q > A_4$, then $\Pi_R(Q_0)$ is an increasing function of Q_0. Hence, the remanufacturer sets Q_0^* is equal to the its upper bound such that $Q_0^* = \min\left\{\frac{rw}{2}, \frac{D}{(1-\alpha)q}\right\}$. Otherwise, the remanufacturer is better off not to order one more additional unit, hence $Q_0^* = 0$.

The results show that the optimal order quantity in this setting is similar to the optimal collection quantity in the base model. Only difference is that the collector's optimal decision also affects the remanufacturer's optimal order size. In the base model, the remanufacturer sets his order quantity to be equal to the exact number of remanufacturables after disassembly, that is $Q_R = D = Q_0(1-\alpha)q$, and the collector collects $Q_0 = \frac{D}{(1-\alpha)q}$ units of used item to satisfy the remanufacturer's order. In this setting, however, if the amount of supply, rf^*, is smaller than $\frac{D}{(1-\alpha)q}$, then the remanufacturer's order quantity is as much as supply. Otherwise, he sets the order quantity such that the actual quantity of remanufacturables to be exactly equal to the deterministic demand like the base model. Moreover, the collector undertakes the responsibility of sorting in the base model, but the inspection process is performed by the remanufacturer in this model. Because of the deterministic demand, the location of the sorting does not affect the remanufacturer's optimal order size.

3.4 Model III-Price Dependent Supply under Collector's Lead

This model is also generated as Stackelberg game and the remanufacturer is responsible for sorting like the previous model, but in this model Stackelberg leader is the collector and the remanufacturer is the follower. Hence, the difference from the base model is the price dependent supply and the sorting location. Due to the change in the roles of the agents in the game, the sequence of events also changes, which is given as follows:

1. The collector determines f and collects $S(f) = rf$ units,
2. The remanufacturer orders Q_0 units,
3. The collector delivers $min\,\{rf, Q_0\}$ to the remanufacturer,
4. The remanufacturer sorts $min\,\{rf, Q_0\}$ and remanufactures $\min\{q \min\{rf, Q_0\}(1-\alpha), D\}$.

We assume that the transfer price is exogenous. The remanufacturer's problem is analyzed first. Given the acquisition price set by the collector, the remanufacturer aims to maximize his expected profit which is given by:

$$\begin{aligned}
\max\ \Pi_R(Q_0) \quad = \quad & -(c_i + w)\min\{rf, Q_0\} \\
& -c_{dR}\min\{rf, Q_0\}[\alpha q + (1-\beta)(1-q)] \\
& -c_{dis}\min\{rf, Q_0\}[(1-\alpha)q + \beta(1-q)] \\
& -c_{dR}\min\{rf, Q_0\}(1-q)\beta \\
& +(p_r - c_r)\min\{\min\{rf, Q_0\}(1-\alpha)q, D\} \\
& -c_{dR}\max\{0, \min\{rf, Q_0\}(1-\alpha)q - D\}
\end{aligned}$$

The revenue and cost items that are used in the remanufacturer's profit function are explained below:

- Inspection cost: $c_i \min\{rf, Q_0\}$;
- Transfer cost of shipped items: $w \min\{rf, Q_0\}$;

- Disposal cost of non-remanufacturables after inspection:
  $c_{dR} \min\{rf, Q_0\}\,[\alpha q + (1-\beta)(1-q)]$

- Disassembly cost: $c_{dis} \min\{rf, Q_0\}\,[(1-\alpha)q + \beta(1-q)]$

- Disposal cost of non-remanufacturables after disassembly:
  $c_{dR} \min\{rf, Q_0\}\,\beta(1-q)$

- Disposal cost of excess remanufacturables:
  $c_{dR} \max\{0, \min\{rf, Q_0\}(1-\alpha)q - D\}$

- Revenue from remanufacturing : $p_r \min\{\min\{rf, Q_0\}(1-\alpha)q, D\}$

- Remanufacturing cost : $c_r \min\{\min\{rf, Q_0\}(1-\alpha)q, D\}$

Lemma 8. *The remanufacturer's optimal order quantity is less than or equal to the amount of supply, that is $Q_0^* \leq rf$.*

Proof. This proof is similar to the proof of Lemma 6, hence it is omitted. ■

Lemma 9. *In the optimal solution to the remanufacturer's problem, the actual quantity of remanufacturables is always less than or equal to the deterministic market demand such that $Q_0^*(1-\alpha)q \leq D$.*

Proof. This is omitted, since this proof is essentially the same as that of Lemma 7. ■

The remanufacturer's maximization problem is simplified by using Lemma 8 and Lemma 9 as follows:

$$\begin{aligned} \max \qquad \Pi_R(Q_0) = &-(c_i + w)Q_0 - c_{dR}[\alpha q + (1-\beta)(1-q)]Q_0 \\ &- c_{dis}[(1-\alpha)q + \beta(1-q)]Q_0 - c_{dR}\beta(1-q)Q_0 \\ &+ (p_r - c_r)(1-\alpha)qQ_0 \end{aligned}$$

$$\text{subject to} \qquad Q_0 \leq \min\left\{rf, \frac{D}{(1-\alpha)q}\right\}.$$

Proposition 6. *The remanufacturer's optimal order quantity under deterministic demand case is characterized by:*

$$Q_0^* = \begin{cases} \min\left\{rf, \frac{D}{(1-\alpha)q}\right\} & \textit{if } p_r(1-\alpha)q > A_4 \\ \\ 0 & \textit{o/w} \end{cases}$$

where A_4 is defined in Proposition 5 by:

$$\begin{aligned} A_4 = {} & (c_i + w) + c_{dR}[\alpha q + (1-\beta)(1-q)] + c_{dis}[(1-\alpha)q + \beta(1-q)] \\ & + c_{dR}\beta(1-q) + c_r(1-\alpha)q. \end{aligned}$$

Proof. The first derivative of $\Pi_R(Q_0)$ with respect to Q_0 is:

$$\begin{aligned} \frac{d\Pi_R(Q_0)}{dQ_0} \quad = \quad & -(c_i + w) - c_{dR}[\alpha q + (1-\beta)(1-q)] \\ & -c_{dis}[(1-\alpha)q + \beta(1-q)] \\ & -c_{dR}\beta(1-q) + (p_r - c_r)q(1-\alpha) \end{aligned}$$

When $p_r(1-\alpha)q > A_4$, $\frac{d\Pi_R(Q_0)}{dQ_0} > 0$, that is, $\Pi_R(Q_0)$ is increasing for $0 \leq Q_0 \leq \min\left\{rf, \frac{D}{(1-\alpha)q}\right\}$. Hence, $Q_0^* = \min\left\{rf, \frac{D}{(1-\alpha)q}\right\}$. Otherwise, $\frac{d\Pi_R(Q_0)}{dQ_0} < 0$, then $Q_0^* = 0$.

■

Next, we consider the collector's problem which is:

$$\begin{aligned} \max \qquad & \Pi_C(f) = -f(rf) + w\min\{rf, Q_0^*\} \\ \text{subject to} \qquad & f \geq 0. \end{aligned}$$

Lemma 10. *In the optimal solution, the amount of supply cannot exceed remanufacturer's order quantity, such that $rf^* \leq Q_0^*$.*

Proof. This is omitted, since the proof is essentially the same as that of Lemma 4.

■

Using Lemma 10, the collector's constrained problem is rearranged as:

$$\begin{aligned} \text{max} \quad & \Pi_C(f) = -f(rf) + w(rf) \\ \text{subject to} \quad & rf \leq \frac{D}{(1-\alpha)q} \\ & f \geq 0. \end{aligned}$$

Lemma 11. $\Pi_C(f)$ *is a concave function in* f.

Proof. This is omitted, since the proof is essentially the same as that of Lemma 5. ■

Proposition 7. *Given the remanufacturer's optimal order quantity* Q_0^**, the optimal acquisition price is given by:*

$$f^* = \begin{cases} \frac{w}{2} & \textit{if } r\frac{w}{2} \leq \frac{D}{(1-\alpha)q} \\ \frac{D}{(1-\alpha)qr} & \textit{o/w} \end{cases}.$$

Proof. The first derivative of $\Pi_C(f)$ with respect to f is:

$$\frac{d\Pi_C(f)}{df} = -rf + r(w-f)$$

The unconstrained maximizer of $\Pi_C(f)$ is given by $\frac{d\Pi_C(f)}{df} = 0$, which is $\frac{w}{2}$. If $\frac{rw}{2} < \frac{D}{(1-\alpha)q}$, it is the optimal solution. Hence, $f^* = \frac{w}{2}$. Otherwise, the constraint is binding and $f^* = \frac{D}{(1-\alpha)qr}$. ■

The optimal acquisition price and the remanufacturer's order size in the third model are the same as in the second model, since the wholesale price is exogenous. It shows that change in the roles of the supply chain members as leader and follower does not affect the optimal solution for the second and third model. On the other hand, when the supply is price dependent, the optimal order quantity is similar to the optimal collection quantity in the base model because of the deterministic demand. The only difference is that the collector can affect the collection quantity by changing the acquisition price after or before the remanufacturer makes an order. Therefore, the optimal order quantity is not affected by changing the responsibility of the sorting and the model setting.

3.5 Model IV-Price Dependent Supply and Demand under Remanufacturer's Lead

In this model, the demand for remanufactured items is now price sensitive. That is, the remanufacturer can increase the demand by decreasing the selling price p_r of remanufactured products. We assume that in this model the transfer price is exogenous like previous models. The sequence of events in this model is as follows:

1. The remanufacturer determines p_r and so $D(p_r) = a - bp_r$, and orders Q_0 units,
2. The collector determines f and collects $S(f) = rf$,
3. The collector delivers $min\{rf, Q_0\}$ to the remanufacturer at unit transfer price of w,
4. The remanufacturer sorts $min\{rf, Q_0\}$ and remanufactures $\min\{\min\{rf, Q_0\}(1-\alpha)q, D(p_r)\}$.

The collector's problem is exactly the same as the second model. Therefore, the analysis of the second stage is omitted. Recall that the optimal acquisition price for the collector is:

$$f^* = \begin{cases} \frac{w}{2} & \text{if } r\frac{w}{2} \leq Q_0 \\ \\ \frac{Q_0}{r} & \text{o/w} \end{cases}.$$

The remanufacturer optimizes the selling price p_r and the order quantity Q_0 given that the optimal acquisition price f^*. The remanufacturer's profit function is:

$$\begin{aligned}
\Pi_R(Q_0, p_r) = &-(c_i + w)\min\{rf^*, Q_0\} \\
&- c_{dR}\min\{rf^*, Q_0\}[\alpha q + (1-\beta)(1-q)] \\
&- c_{dis}\min\{rf^*, Q_0\}[(1-\alpha)q + \beta(1-q)] \\
&- c_{dR}\min\{rf^*, Q_0\}\beta(1-q) \\
&+ (p_r - c_r)\min\{\min\{rf^*, Q_0\}(1-\alpha)q, D(p_r)\} \\
&- c_{dR}\max\{0, \min\{rf^*, Q_0\}(1-\alpha)q - D(p_r)\}
\end{aligned}$$

The revenue and cost items that are used in the remanufacturer's profit function are:

- Inspection cost: $c_i \min\{rf^*, Q_0\}$;
- Transfer cost of shipped items: $w \min\{rf^*, Q_0\}$;
- Disposal cost of non-remanufacturables after sorting:
  $c_{dR} \min\{rf^*, Q_0\}[\alpha q + (1-\beta)(1-q)]$
- Disassembly cost: $c_{dis} \min\{rf^*, Q_0\}[(1-\alpha)q + \beta(1-q)]$
- Disposal cost of non-remanufacturables after disassembly:
  $c_{dR} \min\{rf^*, Q_0\}\beta(1-q)$
- Disposal cost of excess remanufacturables:
  $c_{dR} \max\{0, q\min\{rf^*, Q_0\}(1-\alpha) - D(p_r)\}$
- Revenue from remanufacturing : $p \min\{q\min\{rf^*, Q_0\}(1-\alpha), D(p_r)\}$
- Remanufacturing cost : $c_r \min\{q\min\{rf^*, Q_0\}(1-\alpha), D(p_r)\}$

Lemma 12. *The remanufacturer's optimal order quantity should be less than or equal to the amount of supply, that is* $Q_0^* \leq rf^*$.

Proof. This proof is the same as the proof of the Lemma 6. ■

The remanufacturer's problem is rearranged by using Lemma 12 as follows:

$$\begin{aligned}
\max \qquad \Pi_R(Q_0, p_r) = & -(c_i + w)Q_0 - c_{dR}[\alpha q + (1-\beta)(1-q)] \\
& - c_{dis}[(1-\alpha)q + \beta(1-q)]Q_0 - c_{dR}\beta(1-q)Q_0 \\
& + (p_r - c_r)\min\{(1-\alpha)qQ_0, D(p_r)\} \\
& - c_{dR}\max\{0, (1-\alpha)qQ_0 - D(p_r)\}
\end{aligned}$$

$$\text{subject to} \qquad Q_0 \leq r\frac{w}{2}.$$

Lemma 13. *In the optimal solution to the remanufacturer's problem, the actual quantity of remanufacturables should be equal to the price dependent deterministic demand such that* $Q = D(p_r)$.

Proof. The proof is provided in Appendix C. ■

Due to Lemma 13, in the optimal solution, we have $Q_0^* = \frac{a-bp_r}{(1-\alpha)q}$ and the remanufacturer's problem is rearranged as follows:

$$\max \qquad \Pi_R(p_r) = \frac{a-bp_r}{(1-\alpha)q}\begin{pmatrix} -c_i - w - c_{dR}[\alpha q + (1-\beta)(1-q)] \\ -c_{dis}[(1-\alpha)q + \beta(1-q)] - c_{dR}\beta(1-q) \end{pmatrix}$$

$$+ (a-bp_r)(p_r - c_r)$$

$$\text{subject to} \qquad \frac{a-bp_r}{(1-\alpha)q} \leq r\frac{w}{2}.$$

Lemma 14. *$\Pi_R(p_r)$ is a concave in p_r.*

Proof. The first derivative of $\Pi_R(p_r)$ with respect to p_r is:

$$\frac{d\Pi_R(p_r)}{dp_r} = \frac{b}{(1-\alpha)q}\begin{pmatrix} c_i + w + c_{dR}[\alpha q + (1-\beta)(1-q)] \\ + c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q) \end{pmatrix}$$

$$+(a - 2bp_r + c_r b)$$

The second derivative of $\Pi_R(p_r)$ with respect to p_r is:

$$\frac{d^2\Pi_R(p_r)}{dp_r^2} = -2b < 0$$

Hence, $\Pi_R(p_r)$ is concave in p_r. ■

Proposition 8. *Let p_r^* denote the optimal selling price for the remanufacturer's problem. Let $p_r^{'}$ denote the solution to $\frac{d\Pi_R(p_r)}{dp_r} = 0$. Then,*

$$p_r^* = \begin{cases} p_r^{'} & \textit{if} \quad \frac{a-bp_r^{'}}{(1-\alpha)q} \leq \frac{rw}{2} \\ \\ \frac{1}{b}\left\{a - \frac{rw}{2}(1-\alpha)q\right\} & \textit{o/w} \end{cases}.$$

Proof. The first derivative of $\Pi_R(p_r)$ with respect to p_r is:

$$\frac{d\Pi_R(p_r)}{dp_r} = \frac{b}{(1-\alpha)q}\left(\begin{array}{l} c_i + w + c_{dR}[\alpha q + (1-\beta)(1-q)] \\ + c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q) \end{array}\right)$$
$$+(a - 2bp_r + c_r b)$$

$p_r^{'}$ is the solution to $\frac{d\Pi_R(p_r)}{dp_r} = 0$. Then,

$$p_r^{'} = \frac{1}{2(1-\alpha)q}\left(\begin{array}{l} (c_i + w) + c_{dR}[\alpha q + (1-\beta)(1-q)] \\ +(c_{dis} + c_{dR})\beta(1-q) \end{array}\right)$$
$$+\frac{1}{2}(\frac{a}{b} + c_r + c_{dis})$$

The unconstrained maximizer of $\Pi_R(p_r)$ is $p_r^{'}$. If $\frac{a-bp_r^{'}}{(1-\alpha)q} < \frac{rw}{2}$, $p_r^{'}$ is feasible and hence, optimal. Otherwise, the constraint is binding and $p_r^{*} = \frac{1}{b}\left\{a - \frac{rw}{2}(1-\alpha)q\right\}$. ■

Intuitively, for $p_r^{'}$ such that $\frac{a-bp_r^{'}}{(1-\alpha)q} < \frac{rw}{2}$, the remanufacturer sets the optimal selling price as $p_r^{'}$ in order to maximize his profit. When $\frac{a-bp_r^{'}}{(1-\alpha)q} \geq \frac{rw}{2}$, the price sensitive demand $a - bp_r^{'}$ is not fully satisfied and $a - bp_r^{'} - (1-\alpha)qr\frac{w}{2}$ units of demand is lost. Hence, the remanufacturer sets the price to make the actual quantity of remanufacturables sent to be equal to price dependent demand.

In this setting, the remanufacturer can affect his order quantity by changing the selling price. Since the demand is price dependent, the remanufacturer always tries to set the price dependent deterministic demand equal to the quantity of exact remanufacturables after the disassembly process. The optimal acquisition price expressions for Model II and Model IV are similar, but the collector's decision is affected by price sensitive demand in this model.

3.6 Model V-Price Dependent Supply and Demand under Collector's Lead

In this model, Stackelberg's leader is the collector and the remanufacturer is the follower unlike the previous model. Regarding to the change in the roles of the agents in the game, the sequence of decision making also changes as follows:

1. The collector sets f and collects $S(f) = rf$ units,
2. The remanufacturer determines p_r and so $D(p_r) = a - bp_r$, and orders Q_0 units,
3. The collector delivers $min\{rf, Q_0\}$ to the remanufacturer at a unit transfer price of w,
4. The remanufacturer sorts $min\{rf, Q_0\}$ and remanufactures $\min\{q \min\{rf, Q_0\}(1-\alpha), D(p_r)\}$.

The remanufacturer optimizes the selling price p_r and the order quantity Q_0 given the acquisition price f. The remanufacturer's expected profit is defined as follows:

$$\begin{aligned}\Pi_R(Q_0, p_r) = &-(c_i + w)\min\{rf, Q_0\} \\ &- c_{dR}\min\{rf, Q_0\}[\alpha q + (1-\beta)(1-q)] \\ &- c_{dis}\min\{rf, Q_0\}[(1-\alpha)q + \beta(1-q)] \\ &- c_{dR}\min\{rf, Q_0\}\beta(1-q) \\ &+ (p_r - c_r)\min\{q\min\{rf, Q_0\}(1-\alpha), D(p_r)\} \\ &- c_{dR}\max\{0, q\min\{rf, Q_0\}(1-\alpha) - D(p_r)\}.\end{aligned}$$

The revenue and cost items that are used in the remanufacturer's profit function are:

- Inspection cost: $c_i \min\{rf, Q_0\}$;
- Transfer cost of shipped items: $w \min\{rf, Q_0\}$;
- Disposal cost of non-remanufacturables after sorting: $c_{dR}\min\{rf, Q_0\}[\alpha q + (1-\beta)(1-q)]$
- Disassembly cost: $c_{dis}\min\{rf, Q_0\}[(1-\alpha)q + \beta(1-q)]$

- Disposal cost of non-remanufacturables after disassembly:
  $c_{dR}\min\{rf, Q_0\}\,\beta(1-q)$

- Disposal cost of excess remanufacturables:
  $c_{dR}\max\{0, \min\{rf, Q_0\}(1-\alpha)q - D(p_r)\}$

- Revenue from remanufacturing : $p_r \min\{\min\{rf, Q_0\}(1-\alpha)q, D(p_r)\}$

- Remanufacturing cost : $c_r \min\{\min\{rf, Q_0\}(1-\alpha)q, D(p_r)\}$

Lemma 15. *In the optimal solution, the remanufacturer's order quantity should be less than or equal to the amount of supply, that is $Q_0^* \leq rf$.*

Proof. The proof is omitted as it is similar to the proof of Lemma 6. ■

Lemma 16. *In the optimal solution to the remanufacturer's problem, the actual quantity of remanufacturables should be equal to the price dependent deterministic demand such that $Q = D(p_r)$.*

Proof. The proof is omitted since it is essentially the same as that of Lemma 13. ■

Due to Lemma 16, in the optimal solution, we have $Q_0^* = \frac{a-bp_r}{(1-\alpha)q}$ and the remanufacturer's problem is rearranged as follows:

$$\max \qquad \Pi_R(p_r) = \frac{a-bp_r}{(1-\alpha)q}\begin{pmatrix} -c_i - w - c_{dR}[\alpha q + (1-\beta)(1-q)] \\ -c_{dis}[(1-\alpha)q + \beta(1-q)] - c_{dR}\beta(1-q) \end{pmatrix}$$

$$+ (a-bp_r)(p_r - c_r)$$

$$\text{subject to} \qquad \frac{a-bp_r}{(1-\alpha)q} \leq rf.$$

Lemma 17. $\Pi_R(p_r)$ *is a concave function in* p_r.

Proof. This is omitted as it is essentially the same as the proof of Lemma 14. ■

Proposition 9. *Let p_r^* denote the optimal selling price for the remanufacturer's problem, and $p_r^{'}$ denote the solution to $\frac{d\Pi_R(p_r)}{dp_r} = 0$. Then,*

$$p_r^* = \begin{cases} p_r^{'} & \text{if } \frac{a-bp_r^{'}}{(1-\alpha)q} \leq rf \\ \\ \frac{1}{b}\left\{a - rf(1-\alpha)q\right\} & \text{o/w} \end{cases}.$$

Proof. The first derivative of $\Pi_R(p_r)$ with respect to p_r is:

$$\frac{d\Pi_R(p_r)}{dp_r} = \frac{b}{(1-\alpha)q}\left(\begin{array}{l} c_i + w + c_{dR}[\alpha q + (1-\beta)(1-q)] \\ + c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q) \end{array}\right)$$
$$+(a - 2bp_r + c_r b)$$

$p_r^{'}$ is the solution to $\frac{d\Pi_R(p_r)}{dp_r} = 0$. Then,

$$p_r^{'} = \frac{1}{2(1-\alpha)q}\left(\begin{array}{l} (c_i + w) + c_{dR}[\alpha q + (1-\beta)(1-q)] \\ +(c_{dis} + c_{dR})\beta(1-q) \end{array}\right)$$
$$+\frac{1}{2}(\frac{a}{b} + c_r + c_{dis})$$

The unconstrained maximizer of $\Pi_R(p_r)$ is $p_r^{'}$. If $\frac{a-bp_r^{'}}{(1-\alpha)q} < rf$, $p_r^{'}$ is feasible. Hence, it is optimal. Otherwise, the constraint is binding and $p_r^* = \frac{1}{b}\left\{a - rf(1-\alpha)q\right\}$.

■

Next, we analyze the collector's problem in order to find the optimal acquisition price. The collector collects rf units of supply at a unit acquisition cost of f and the remanufacturer orders Q_0 units of used items, then the collector sends $min\left\{rf, Q_0\right\}$ units to the remanufacturer. The collector's profit is derived taking into account of the optimal selling price that is determined in the first decision making stage:

$$\Pi_C(f) = -f(rf) + w\min\left\{rf, \frac{a-bp_r^*}{(1-\alpha)q}\right\}.$$

Lemma 18. *In the optimal solution, the amount of supply cannot exceed remanufacturer's order quantity, such that $rf^* \leq \frac{a-bp_r^*}{(1-\alpha)q}$.*

Proof. This is omitted, since the proof is essentially the same as that of Lemma 4. ∎

The collector's problem is rearranged by using Lemma 18 as follows:

$$\max \qquad \Pi_C(f) = -f(rf) + wrf$$

$$\text{subject to} \qquad rf \leq \frac{a - bp_r^*}{(1-\alpha)q}.$$

Lemma 19. $\Pi_C(f)$ *is a concave function in* f.

Proof. This is omitted, since it is the same as the proof of Lemma 5. ∎

Proposition 10. *Given the optimal selling price* p_r^*, *the optimal acquisition price is given by:*

$$f^* = \begin{cases} \frac{w}{2} & \textit{if } r\frac{w}{2} \leq \frac{a-bp_r^{'}}{(1-\alpha)q} \\ \\ \frac{a-bp_r^{'}}{(1-\alpha)qr} & \textit{o/w} \end{cases}.$$

Proof. The first derivative of $\Pi_C(f)$ with respect to f is:

$$\frac{d\Pi_C(f)}{df} = -rf + r(w - f)$$

The unconstrained maximizer of $\Pi_C(f)$ is given by $\frac{d\Pi_C(f)}{df} = 0$, which is $\frac{w}{2}$. If $\frac{rw}{2} < \frac{a-bp_r^{'}}{(1-\alpha)q}$, it is the optimal solution. Hence, $f^* = \frac{w}{2}$. Otherwise, the constraint is binding and $f^* = \frac{a-bp_r^{'}}{(1-\alpha)qr}$. ∎

When $\frac{rw}{2} < \frac{a-bp_r^{'}}{(1-\alpha)q}$, the collector sets the acquisition price to be equal to $\frac{w}{2}$ in order to maximize his profit. Otherwise, $\frac{rw}{2} - \frac{a-bp_r^{'}}{(1-\alpha)q}$ units of supply is not used for remanufacturing and the remanufacturer does not pay anything for this excess amount of supply. Therefore, it only results in an increase in the collector's cost and hence, the collector sets the acquisition price such that $rf^* = \frac{a-bp_r^{'}}{(1-\alpha)q}$.

The optimal acquisition price and selling price are the same in Model IV and Model V, since the wholesale price is exogenous. The results show that the change in the roles of the supply chain members as leader and follower only affects the order of decision making, but it does not affect the optimal value of the collection quantity and the remanufacturer's order quantity. The optimal order quantity in Model IV and Model V are similar to the base model and other three settings, the difference is that the remanufacturer also affects the demand by changing the selling price.

3.7 Model VI-Base Model with Price Dependent Supply

In this model, we consider the base model with price dependent supply in a single period context. The sequence of events and the responsibilities of the agents are the same as the base model. The only difference from the base model is that the acquisition price is also a decision variable. The remanufacturer decides the optimal production lot size Q_R and the collector determines the optimal acquisition price f in order to maximize his profit. The sequence of events in this model is as follows:

1. The remanufacturer orders Q_R units,
2. The collector determines f,
3. The collector sorts imperfectly rf units and transports Q_C units, which is equal to $[(1-\alpha)q + \beta(1-q)](rf)$, to the remanufacturer,
4. The remanufacturer disassembles and sorts actually Q_C units, then remanufactures $min\{(1-\alpha)qrf, Q_R\}$.

Since the remanufacturer is the Stackelber leader, the collector's problem is firstly analyzed and the optimal acquisition price f is determined in order to maximize the collector's profit. The collector's profit is characterized taking into account the relationship between the amount of actual remanufacturables and the remanufacturer's order size as follows:

$$\Pi_C(f) = \begin{cases} \Pi_C^I(f) & \text{if } f \leq \frac{Q_R}{(1-\alpha)qr} \\ \Pi_C^{II}(f) & \text{if } f > \frac{Q_R}{(1-\alpha)qr} \end{cases}$$

where

$$\begin{aligned}\Pi_C^I(f) &= w(1-\alpha)qrf - f(rf) - c_i(rf) - c_{dC}[\alpha q + (1-\beta)(1-q)](rf) \\ &\quad -c_t[(1-\alpha)q + \beta(1-q)](rf) - b_0[Q_R - (1-\alpha)q(rf)]\end{aligned}$$

and

$$\begin{aligned}\Pi_C^{II}(f) &= wQ_R - f(rf) - c_i(rf) - c_{dC}[\alpha q + (1-\beta)(1-q)](rf) \\ &\quad -c_t[(1-\alpha)q + \beta(1-q)](rf).\end{aligned}$$

The revenue and cost items in the collector's profit function are:

- Collection cost of used items: $f(rf)$;
- Inspection cost: $c_i(rf)$;
- Disposal cost of non-remanufacturables:$c_{dC}[\alpha q + (1-\beta)(1-q)](rf)$
- Transportation cost of shipped items: $c_t[(1-\alpha)q + \beta(1-q)](rf)$
- Transfer payment by the remanufacturer: $\begin{cases} w(1-\alpha)qrf & \text{if } f \leq \frac{Q_R}{(1-\alpha)qr} \\ wQ_R & \text{if } f > \frac{Q_R}{(1-\alpha)qr} \end{cases}$
- Penalty cost paid by the collector: $\begin{cases} b_0[Q_R - (1-\alpha)qrf] & \text{if } f \leq \frac{Q_R}{(1-\alpha)qr} \\ 0 & \text{if } f > \frac{Q_R}{(1-\alpha)qr} \end{cases}$

Lemma 20. $\Pi_C(f)$ *is continuous, but not differentiable at* $f = \frac{Q_R}{(1-\alpha)qr}$.

Proof. The proof is provided in Appendix D. ∎

Lemma 21. $\Pi_C^I(f)$ *and* $\Pi_C^{II}(f)$ *are concave functions in* f.

Proof. The first derivatives of $\Pi_C^I(f)$ and $\Pi_C^{II}(f)$ with respect to f are given as:

$$\begin{aligned}\frac{d\Pi_C^I(f)}{df} &= -2rf - c_i r - c_{dC}[\alpha q + (1-\beta)(1-q)]r \\ &\quad -c_t[(1-\alpha)q + \beta(1-q)]r + w(1-\alpha)q\,r + b_0(1-\alpha)q\,r\end{aligned}$$

$$\frac{d\Pi_C^{II}(f)}{df} = -2rf - c_i r - c_{dC}[\alpha q + (1-\beta)(1-q)]r$$
$$-c_t[(1-\alpha)q + \beta(1-q)].$$

The second derivatives of $\Pi_C^I(f)$ and $\Pi_C^{II}(f)$ with respect to f are given as:

$$\frac{d^2\Pi_C^I(f)}{df^2} = \frac{d^2\Pi_C^{II}(f)}{df^2} = -2r < 0$$

The second derivatives of $\Pi_C^I(f)$ and $\Pi_C^{II}(f)$ with respect to f are always less than zero, so it is concluded that they are concave in f.

■

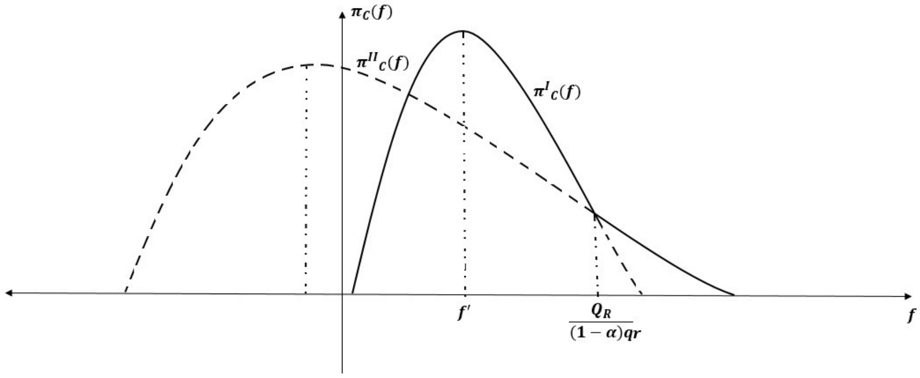
Proposition 11. *Let f^* denote the optimal acquisition fee to the collector's profit function, and f' denote the solution to $\frac{d\Pi_C^I(f)}{df} = 0$. Then, given the remanufacturer's order quantity Q_R, f^* is given by:*

$$f^* = \begin{cases} f' & \text{if } f' \leq \frac{Q_R}{(1-\alpha)qr} \\ \frac{Q_R}{(1-\alpha)qr} & o/w \end{cases}$$

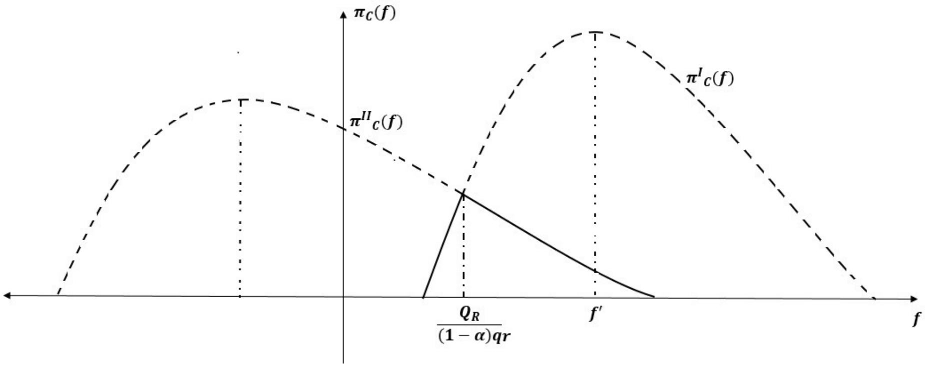
where $f' = \frac{(w+b_0)(1-\alpha)q - c_i - c_{dC}[\alpha q + (1-\beta)(1-q)] - c_t[(1-\alpha)q + \beta(1-q)]}{2}$.

Proof. If $f' \leq \frac{Q_R}{(1-\alpha)qr}$, then $\frac{d\Pi_C^I(f)}{df}|_{f=\frac{Q_R}{(1-\alpha)qr}} < 0$. Hence, $\Pi_C^I(f)$ is decreasing at $\frac{Q_R}{(1-\alpha)qr}$. Since $\frac{d\Pi_C^{II}(f)}{df}|_{f=\frac{Q_R}{(1-\alpha)qr}} \leq 0$, $f^* = f'$. If $f' > \frac{Q_R}{(1-\alpha)qr}$, then $\frac{d\Pi_C^I(f)}{df}|_{f=\frac{Q_R}{(1-\alpha)qr}} > 0$. Hence, $\Pi_C^I(f)$ is increasing at $\frac{Q_R}{(1-\alpha)qr}$. Since $\frac{d\Pi_C^{II}(f)}{df}|_{f=\frac{Q_R}{(1-\alpha)qr}} \leq 0$, $f^* = \frac{Q_R}{(1-\alpha)qr}$. ■

Proposition 11 is depicted in Figure 3.1. In the first graph, the case in which $\Pi_C^I(f)$ and $\Pi_C^{II}(f)$ are decreasing at $\frac{Q_R}{(1-\alpha)qr}$ is depicted. Hence, $f^* = f'$. In the second graph, $\Pi_C^I(f)$ is increasing and $\Pi_C^{II}(f)$ is decreasing at $\frac{Q_R}{(1-\alpha)qr}$. Then, $\Pi_C(f)$ takes maximum value at $\frac{Q_R}{(1-\alpha)qr}$. Hence, $f^* = \frac{Q_R}{(1-\alpha)qr}$.



(a) $f^* = f^{'}$



(b) $f^* = \frac{Q_R}{(1-\alpha)qr}$

Figure 3.1: Collector's Expected Profit for given f^*

The remanufacturer's problem is formulated next. When $f^{'} \leq \frac{Q_R}{(1-\alpha)qr}$, then $f^* = f^{'}$ and $(1-\alpha)qrf^* \leq Q_R$. On the other hand, if $f^{'} > \frac{Q_R}{(1-\alpha)qr}$, then $f^* = \frac{Q_R}{(1-\alpha)qr}$ and $(1-\alpha)qrf^* = Q_R$. Therefore, the actual quantity of remanufacturables that is represented as $Q = (1-\alpha)qrf^*$ is always less than or equal to the remanufacturer's order size Q_R. If the remanufacturer sets Q_R such that $Q_R \leq D$, then his expected profit is expressed by:

$$\begin{aligned}
\Pi_R^I(Q_R) &= -c_{dis}[(1-\alpha)q + \beta(1-q)]rf^* - c_{dR}\beta(1-q)rf^* \\
&\quad -w(1-\alpha)qrf^* + b_0(Q_R - (1-\alpha)qrf^*) + (p_r - c_r)(1-\alpha)qrf^* \\
&\quad -b(D - (1-\alpha)qrf^*)
\end{aligned}$$

Otherwise, if he sets Q_R such that $Q_R > D$, then his expected profit is expressed by:

$$\begin{aligned}
\Pi_R^{II}(Q_R) &= -c_{dis}[(1-\alpha)q + \beta(1-q)]rf^* - c_{dR}\beta(1-q)rf^* \\
&\quad -w(1-\alpha)qrf^* + b_0(Q_R - (1-\alpha)qrf^*) - c_r(1-\alpha)qrf^* \\
&\quad +p_r min\left\{D, (1-\alpha)qrf^*\right\} \\
&\quad -b max\left\{D - (1-\alpha)qrf^*, 0\right\} \\
&\quad +c_R max\left\{(1-\alpha)qrf^* - D, 0\right\}
\end{aligned}$$

$\Pi_R^I(Q_R)$ is rearranged by substituting f^* as follows:

$$\Pi_R^I(Q_R) = \begin{cases}
\begin{array}{l} -c_{dis}[(1-\alpha)q + \beta(1-q)]rf^{'} \\ -c_{dR}\beta(1-q)rf^{'} - w(1-\alpha)qrf^{'} \\ +b_0(Q_R - (1-\alpha)qrf^{'}) \\ +(p_r - c_r)(1-\alpha)qrf^{'} \\ -b(D - (1-\alpha)qrf^{'}) \end{array} & \text{if} \quad f^{'}(1-\alpha)qr \leq Q_R, \\
\\
\begin{array}{l} -c_{dis}[(1-\alpha)q + \beta(1-q)]\frac{Q_R}{(1-\alpha)q} \\ -c_{dR}\beta(1-q)\frac{Q_R}{(1-\alpha)q} - wQ_R \\ +(p_r - c_r)Q_R - b(D - Q_R) \end{array} & o/w
\end{cases}$$

$\Pi_R^{II}(Q_R)$ is rearranged by substituting f^* as follows:

$$\Pi_R^{II}(Q_R) = \begin{cases} -c_{dis}[(1-\alpha)q + \beta(1-q)]rf' \\ -c_{dR}\beta(1-q)rf' - w(1-\alpha)qrf' \\ +b_0(Q_R - (1-\alpha)qrf') \\ -c_r(1-\alpha)qrf' & \text{if } f'(1-\alpha)qr \leq Q_R, \\ +p_r min\left\{D, (1-\alpha)qrf'\right\} \\ -bmax\left\{D - (1-\alpha)qrf', 0\right\} \\ +c_R max\left\{(1-\alpha)qrf' - D, 0\right\} \\ \\ \\ -c_{dis}[(1-\alpha)q + \beta(1-q)]\frac{Q_R}{(1-\alpha)q} \\ -c_{dR}\beta(1-q)\frac{Q_R}{(1-\alpha)q} - wQ_R & o/w \\ -c_r Q_R + p_r D + c_R(Q_R - D) \end{cases}$$

Then, the remanufacturer's profit can be expressed as:

$$\Pi_R(Q_R) = \begin{cases} \Pi_R^I(Q_R) & \text{if } Q_R \leq D \\ \\ \Pi_R^{II}(Q_R) & \text{if } Q_R > D. \end{cases}$$

The revenue and cost items that are used in the remanufacturer's profit function are:

- Disassembly cost: $c_{dis}[(1-\alpha)q + \beta(1-q)]rf^*$
- Disposal cost of non-remanufacturables: $c_{dR}\beta(1-q)rf^*$
- Total payment to the collector: $w(1-\alpha)qrf^*$
- Penalty cost for the collector: $b_0(Q_R - (1-\alpha)qrf^*)$
- Total remanufacturing cost: $c_r(1-\alpha)qrf^*$
- Revenue from remanufacturing: $\begin{cases} p_r(1-\alpha)qrf^* & \text{if } Q_R \leq D \\ p_r min\left\{D, (1-\alpha)qrf^*\right\} & \text{o/w} \end{cases}$

- Penalty cost for unsatisfied demand:
$\begin{cases} b(D-(1-\alpha)qrf^*) & \text{if } Q_R \le D \\ bmax\left\{D-(1-\alpha)qrf^*, 0\right\} & \text{o/w} \end{cases}$
- Salvage revenue from excess remanufactured items:
$\begin{cases} 0 & \text{if } Q_R \le D \\ c_R max\left\{(1-\alpha)qrf^*-D, 0\right\} & \text{o/w} \end{cases}$

Lemma 22. $\Pi_R(Q_R)$ *is continuous, but not differentiable at* $Q_R = D$.

Proof. The proof is provided in Appendix E. ■

From the proof of Lemma 22, when $f^{'}(1-\alpha)qr \le Q_R$, $\frac{d\Pi_R(Q_R)}{dQ_R} = b_0$ and if $b_0 > 0$, $\Pi_R(Q_R)$ is increasing function of Q_R. Thus, the remanufacturer can set Q_R^* as large as possible regardless to the demand. It means that when the actual number of remanufacturables is smaller than Q_R, the remanufacturer always makes a profit from ordering of one more additional unit of item. It does not make much sense since his profit converges to infinity while the collector's profit converges to minus infinity. Therefore, we set b_0, which is paid by the collector to the remanufacturer for the remanufacturer's unsatisfied order, as zero and characterize the optimal solution for the remanufacturer by Proposition 12.

Proposition 12. *The optimal remanufacturer's order quantity is given by:*

$$Q_R^* = \begin{cases} D & \textit{if } p_r + b > A_2 \\ \\ 0 & \textit{o/w} \end{cases}$$

A_2 is defined in Proposition 2 such that $A_2 = (c_{dis} + c_r + w) + (c_{dis} + c_{dR})\frac{\beta(1-q)}{(1-\alpha)q}$.

Proof. When $Q_R < D$, the first derivative of $\Pi_R(Q_R)$ with respect to Q_R is:

$$\frac{d\Pi_R^I(Q_R)}{dQ_R} = \begin{cases} 0 & \text{if } \ f^{'}(1-\alpha)qr \le Q_R \\ \\ -c_{dis} - (c_r + w - p_r - b) \\ -(c_{dis} + c_{dR})\frac{\beta(1-q)}{(1-\alpha)q} & \text{o/w .} \end{cases}$$

When $Q_R > D$, the first derivative of $\Pi_R(Q_R)$ with respect to Q_R is:

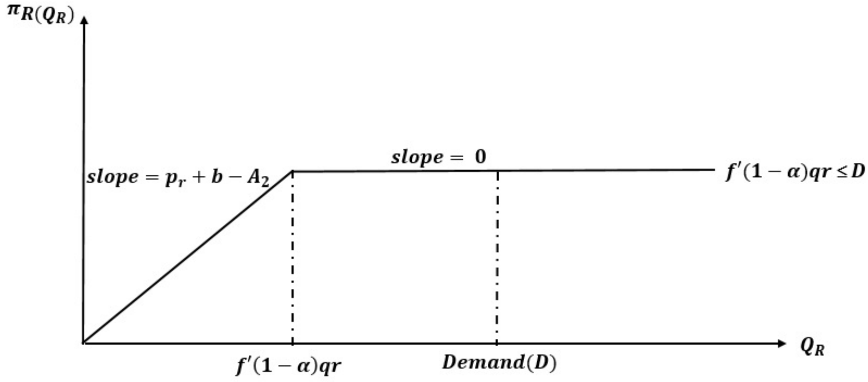
$$\frac{d\Pi_R^{II}(Q_R)}{dQ_R} = \begin{cases} 0 & \text{if } \; f^{'}(1-\alpha)qr \leq Q_R \\ \\ -c_{dis} - (c_r + w - c_R) \\ -(c_{dis} + c_{dR})\frac{\beta(1-q)}{(1-\alpha)q} & \text{o/w} . \end{cases}$$

As we assume $c_R < A_2$ in Section 3.1, we have $\frac{d\Pi_R^{II}(Q_R)}{dQ_R} \leq 0$. If $p_r + b > A_2$ then, $\frac{d\Pi_R^{I}(Q_R)}{dQ_R} \geq 0$. Hence, $Q_R^* = D$. Otherwise, $\frac{d\Pi_R^{I}(Q_R)}{dQ_R} \leq 0$ and $\frac{d\Pi_R^{II}(Q_R)}{dQ_R} \leq 0$. Hence $Q_R^* = 0$.

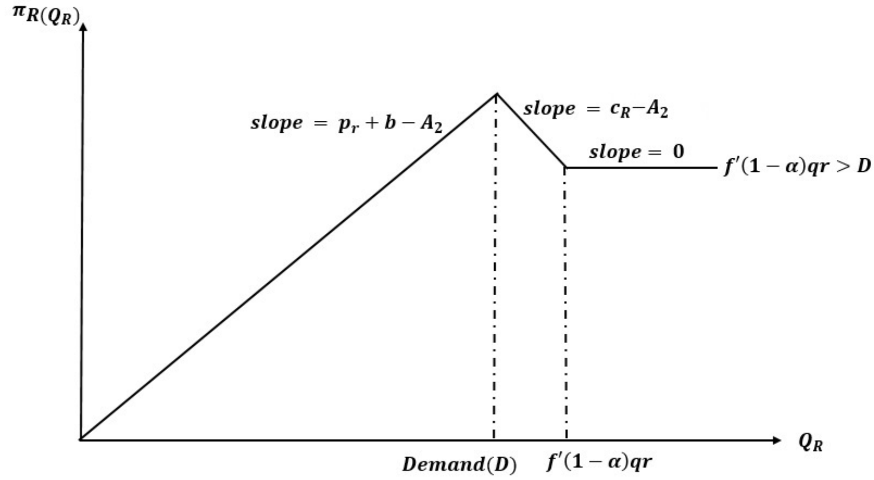
■

The proof is explained in detail in Figure 3.2 and Figure 3.3. The profits are depicted regarding to the relationship between $f^{'}(1-\alpha)qr$ and D. $(p_r + b)$ represents the marginal revenue associated with remanufacturing one more additional unit and $A_2 = (c_{dis}+c_r+w)+(c_{dis}+c_{dR})\frac{\beta(1-q)}{(1-\alpha)q}$ represents the related margin lost. When $p_r+b > A_2$, remanufacturing of one more additional unit is profitable for the remanufacturer.

In part (a) of Figure 3.2, when $Q_R \leq f^{'}(1-\alpha)qr \leq D$, $\Pi_R(Q_R)$ increases with slope p_r+b-A_2 until $Q_R = f^{'}(1-\alpha)qr$. Hence, $\frac{d\Pi_R^{I}(Q_R)}{dQ_R} > 0$. After this point, an increase in Q_R does not affect the remanufacturer's profit. Hence, $\frac{d\Pi_R^{I}(Q_R)}{dQ_R} = 0$ between $f^{'}(1-\alpha)qr < Q_R \leq D$ and $\frac{d\Pi_R^{II}(Q_R)}{dQ_R} = 0$ at $Q_R > D$. In part (b), when $f^{'}(1-\alpha)qr > D$, $\Pi_R(Q_R)$ increases until Q_R reaches the demand with slope $p_r + b - A_2$. Hence, $\frac{d\Pi_R^{I}(Q_R)}{dQ_R} > 0$. After that, between $D < Q_R \leq f^{'}(1-\alpha)qr$, the profit decreases with an increase in Q_R. Hence, $\frac{d\Pi_R^{II}(Q_R)}{dQ_R} < 0$. When $Q_R > f^{'}(1-\alpha)qr$, the remanufacturer's profit does not change and $\frac{d\Pi_R^{II}(Q_R)}{dQ_R} = 0$. From the figure, $\Pi_R(Q_R)$ takes maximum value at $Q_R^* = D$. Hence, when $p_r+b > A_2$ then $Q_R^* = D$. On the other hand, when $p_r + b < A_2$ profits are depicted regarding to the relationship between $f^{'}(1-\alpha)qr$ and D in Figure 3.3. In part (a), when $Q_R \leq f^{'}(1-\alpha)qr \leq D$, $\Pi_R(Q_R)$ decreases until $Q_R = f^{'}(1-\alpha)qr$. Hence, $\frac{d\Pi_R^{I}(Q_R)}{dQ_R} < 0$. After that, the profit does not change, and hence $\frac{d\Pi_R(Q_R)}{dQ_R} = 0$. In part (b), when $f^{'}(1-\alpha)qr > D$, the remanufacturer's profit decreases until $Q_R = f^{'}(1-\alpha)qr$. After this point, the remanufacturer's profit does not change and $\frac{d\Pi_R^{II}(Q_R)}{dQ_R} = 0$. Therefore, $\Pi_R(Q_R)$ takes its maximum value at $Q_R^* = 0$. Hence, when $p_r + b < A_2$ then $Q_R^* = 0$.

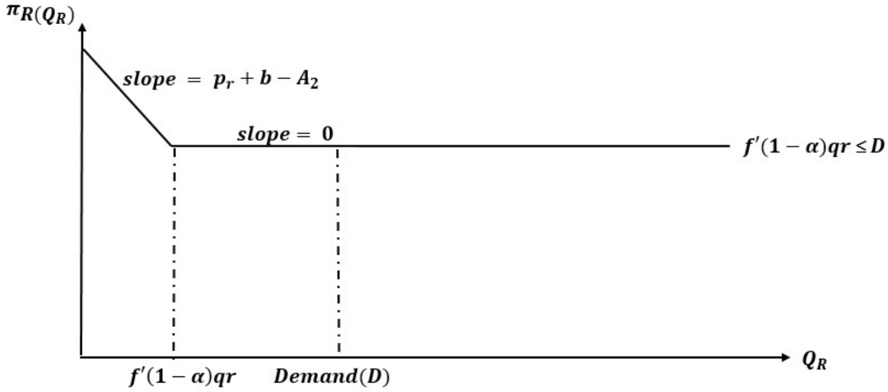


(a) $f^{'}(1-\alpha)qr \leq D$

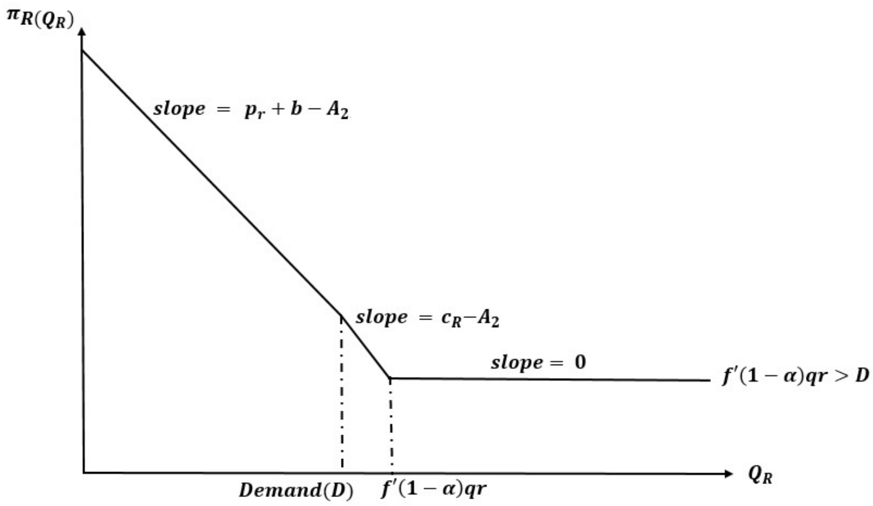


(b) $f^{'}(1-\alpha)qr > D$

Figure 3.2: $\Pi_R(Q_R)$ when $p_r + b > A_2$



(a) $f^{'}(1-\alpha)qr \leq D$



(b) $f^{'}(1-\alpha)qr > D$

Figure 3.3: $\Pi_R(Q_R)$ when $p_r + b < A_2$

The remanufacturer's optimal order size is equal to the deterministic demand. The result is the same as in the base model. The only difference from the base model is that the collector can affect the collection quantity by changing the acquisition price. However, the collector's pricing decision does not affect the remanufacturer's optimal order size because of the deterministic demand.

3.8 Model VII-The Base Model with Price Dependent Supply and Demand

In this model, we consider the base model with price dependent supply and price dependent demand in a single period context. The only difference from the previous model is that the market demand has a deterministic price dependent linear function such that $D(p_r) = a - b_1 p_r$, where $a, b_1 > 0$. Therefore, the remanufacturer can set the demand by changing the selling price. It is noted that we use parameter b_1 to denote the sensitivity of demand to selling price instead of b since b has been already defined as unit penalty cost for unmet demand in the base model.

For this model, we assume that unit penalty cost b_0 paid by the collector to the remanufacturer as zero like Model VI. Otherwise, the remanufacturer can order arbitrarily large quantity of remanufacturables is larger than the price sensitive demand regardless to the actual quantity of remanufacturables sent by the collector. It is not meaningful since the remanufacturer always makes a profit for unsatisfied unit of order while the collector loses profit. When $b_0 = 0$, the remanufacturer sets his order quantity exactly equal to the price sensitive demand. There are two decision variables: the optimal selling price p_r determined by the remanufacturer, and the optimal acquisition price f set by the collector. The sequence of events in this model is as follows:

1. The remanufacturer determines p_r and $D(p_r) = a - b_1 p_r$, then orders $D(p_r)$ units,
2. The collector determines f,
3. The collector sorts imperfectly rf units and transports Q_C units, $[(1-\alpha)q + \beta(1-q)]rf$, to the remanufacturer,

4. The remanufacturer disassembles and sorts actually Q_C units,

5. The remanufacturer remanufactures $min\,\{(1-\alpha)qrf, D(p_r)\}$.

The collector's problem is the same as in Model VI. The only difference is that the order quantity for the remanufacturer is equal to price sensitive demand. Therefore, the analysis of the second stage is omitted. The optimal acquisition price for the collector is:

$$f^* = \begin{cases} f' & \text{if } f' \leq \frac{D(p_r)}{(1-\alpha)qr} \\ \\ \frac{D(p_r)}{(1-\alpha)qr} & \text{o/w} \end{cases}$$

where $f' = \frac{(w+b_0)(1-\alpha)q - c_i - c_{dC}[\alpha q + (1-\beta)(1-q)] - c_t[(1-\alpha)q + \beta(1-q)]}{2}$.

The remanufacturer optimizes the selling price p_r given that the optimal acquisition price f^*. Then, his expected profit is expressed by:

$$\begin{aligned} \Pi_R(p_r) = & -c_{dis}[(1-\alpha)q + \beta(1-q)]rf^* - c_{dR}\beta(1-q)rf^* \\ & - w \min\{(1-\alpha)qrf^*, D(p_r)\} \\ & + (p_r - c_r)\min\{(1-\alpha)qrf^*, D(p_r)\} \\ & - b\max\{D(p_r) - (1-\alpha)qrf^*, 0\} \\ & - c_{dR}max\,\{(1-\alpha)qrf^* - D(p_r), 0\} \end{aligned}$$

The revenue and cost items that are used in the remanufacturer's profit function are:

- Disassembly cost: $c_{dis}[(1-\alpha)q + \beta(1-q)]rf^*$
- Disposal cost of non-remanufacturables: $c_{dR}\beta(1-q)rf^*$
- Total payment to the collector: $w\min\{(1-\alpha)qrf^*, D(p_r)\}$
- Total remanufacturing cost: $c_r\min\{(1-\alpha)qrf^*, D(p_r)\}$
- Revenue from remanufacturing : $p_r\min\{(1-\alpha)qrf^*, D(p_r)\}$
- Penalty cost for unsatisfied demand: $b\max\{D(p_r) - (1-\alpha)qrf^*, 0\}$
- Disposal cost of excess remanufacturables: $c_{dR}max\,\{(1-\alpha)qrf^* - D(p_r), 0\}$

Lemma 23. *In the optimal solution, the price sensitive demand should be equal to the actual quantity of remanufacturables sent by the collector, that is* $a - b_1 p_r^* = (1-\alpha)qrf^*$.

Proof. We will prove this lemma by contradiction. Consider a selling price such that $a - b_1 p_r > (1-\alpha)qrf^*$. Define $\epsilon > 0$ such that $\epsilon = a - b_1 p_r - (1-\alpha)qrf^*$. Then, the remanufacturer's profit at $p_r = \frac{a-(1-\alpha)qrf^*-\epsilon}{b_1}$ can be expressed as:

$$\begin{aligned}\Pi_R\left(\frac{a-(1-\alpha)qrf^*-\epsilon}{b_1}\right) &= -c_{dis}[(1-\alpha)q+\beta(1-q)]rf^* - c_{dR}\beta(1-q)rf^* \\ &\quad -w(1-\alpha)qrf^* + (p_r - c_r)(1-\alpha)qrf^* \\ &\quad -b\epsilon\end{aligned}$$

When $a - b_1 p_r = (1-\alpha)qrf^*$, then the remanufacturer's profit at $p_r = \frac{a-(1-\alpha)qrf^*}{b_1}$ is expressed as follows:

$$\begin{aligned}\Pi_R\left(\frac{a-(1-\alpha)qrf^*}{b_1}\right) &= -c_{dis}[(1-\alpha)q+\beta(1-q)]rf^* - c_{dR}\beta(1-q)rf^* \\ &\quad -w(1-\alpha)qrf^* + (p_r - c_r)(1-\alpha)qrf^*\end{aligned}$$

Then,

$$\Pi_R\left(\frac{a-(1-\alpha)qrf^*-\epsilon}{b_1}\right) - \Pi_R\left(\frac{a-(1-\alpha)qrf^*}{b_1}\right) = -b\epsilon < 0$$

Therefore, for any $\epsilon > 0$, the remanufacturer's profit at $p_r = \frac{a-(1-\alpha)qrf^*-\epsilon}{b_1}$ is smaller than at $p_r = \frac{a-(1-\alpha)qrf^*}{b_1}$. Hence, $a - b_1 p_r = (1-\alpha)qrf^*$ is optimal.

■

If the remanufacturer sets the demand to be larger than the actual quantity of remanufacturables after disassembly, the excess amount of demand is not satisfied from the collector because he collects and sends only rf^* units in order to maximize his profit. Therefore, the remanufacturer's profit decreases by unit shortage cost of b for unsatisfied demand.

The remanufacturer's profit is rearranged by using Lemma 23 as follows:

$$\begin{aligned}\max \qquad \Pi_R(p_r) = &-c_{dis}[(1-\alpha)q + \beta(1-q)]\left(\frac{a-b_1p_r}{(1-\alpha)q}\right) \\ &- c_{dR}\beta(1-q)\left(\frac{a-b_1p_r}{(1-\alpha)q}\right) - w(a-b_1p_r) \\ &+ (p_r - c_r)(a - b_1p_r)\end{aligned}$$

subject to

$$a - b_1p_r = (1-\alpha)qrf^*.$$

Lemma 24. $\Pi_R(p_r)$ *is a concave function in* p_r*.*

Proof. The first derivative of $\Pi_R(p_r)$ with respect to p_r is:

$$\frac{d\Pi_R(p_r)}{dp_r} = b_1\frac{\beta(1-q)}{(1-\alpha)q}(c_{dis} + c_{dR}) - 2b_1p_r + a + b_1(w + c_r + c_{dis})$$

The second derivative of $\Pi_R(p_r)$ with respect to p_r is:

$$\frac{d^2\Pi_R(p_r)}{dp_r^2} = -2b_1 < 0.$$

Hence, $\Pi_R(p_r)$ is concave in p_r. ■

Proposition 13. *Let* p_r^* *denote the optimal selling price for the remanufacturer's problem and* $p_r^{'}$ *denote the solution to* $\frac{d\Pi_R(p_r)}{dp_r} = 0$*. Then,* p_r^* *is:*

$$p_r^* = \begin{cases} p_r^{'} & \textit{if} \quad \frac{D(p_r^{'})}{(1-\alpha)qr} \leq f' \\ \\ \frac{1}{b}\left\{a - (1-\alpha)qrf'\right\} & \textit{o/w} \end{cases}.$$

Proof. The first derivative of $\Pi_R(p_r)$ with respect to p_r is:

$$\frac{d\Pi_R(p_r)}{dp_r} = b_1\frac{\beta(1-q)}{(1-\alpha)q}(c_{dis} + c_{dR}) - 2b_1p_r + a + b_1(w + c_r + c_{dis})$$

Let $p_r^{'}$ denote the solution to $\frac{d\Pi_R(p_r)}{dp_r} = 0$. Then, $p_r^{'}$ is:

$$p_r^{'} = \frac{1}{2}\left(\frac{\beta(1-q)}{(1-\alpha)q}(c_{dis} + c_{dR}) + \frac{a}{b_1} + (w + c_r + c_{dis})\right)$$

The unconstrained maximizer of $\Pi_R(p_r)$ is $p_r^{'}$. If $\frac{D(p_r)}{(1-\alpha)qr} < f'$, $p_r^{'}$ is feasible. Hence, it is optimal. Otherwise, the constraint is binding and $p_r^* = \frac{1}{b_1}\{a - (1-\alpha)qrf'\}$. ■

When the price sensitive demand, $D(p_r^{'})$ is larger than the actual quantity of remanufacturables sent by the collector, the remanufacturer loses $D(p_r^{'}) - (1-\alpha)qrf'$ units of demand. Therefore, he sets his selling price such that $p_r^* = \frac{1}{b_1}\{a - (1-\alpha)qrf'\}$. Hence, the constraint is satisfied. Otherwise, $D(p_r^{'})$ is smaller than or equal to the actual quantity of remanufacturables sent, and hence, the remanufacturer sets the price to be equal to $p_r^{'}$ in order maximize his profit. From the collector's point of view, f' is feasible when $D(p_r^{'})$ is larger than $(1-\alpha)qrf'$. Otherwise, the remanufacturer does not pay for extra $(1-\alpha)qrf' - D(p_r^{'})$ units of remanufacturables. Therefore, the collector sets his acquisition fee to make the actual quantity of remanufacturables sent equal to $D(p_r^{'})$. Hence, $a - b_1 p_r^* = (1-\alpha)qrf^*$ is satisfied.

In this model, the remanufacturer can affect the order quantity by changing the selling price. The remanufacturer tries to set the price dependent deterministic demand equal to the quantity of exact remanufacturables after the disassembly process like model IV and model V. The optimal selling prices in the model IV and model V are similar to the selling price in this model. However, there are some differences in the selling price expressions because of the different sorting locations. In this model the sorting activity is performed by the collector, but the remanufacturer is responsible for sorting in the model IV and model V.

3.9 Detailed Comparison of the Models

In Table 3.3, the models are compared and main results are summarized. Note that the expressions for the selling prices and acquisition fees are given under the table. Based on these results, we present our observations below by comparing the models with each other and the based model.

- Model I differs from the base model regarding to the location of sorting. In the base model, the collector is responsible for sorting and the remanufacturer takes the responsibility of the disassembly and remanufacturing processes. In Model I, on the other hand, the collected items are sorted at the remanufacturer's site in addition to disassembly and remanufacturing operations. Therefore, the remanufacturer is a single decision maker in Model I, and it corresponds to the centralized setting of the base model.

  Given that p_r is large enough, the optimal collection quantity for the base model and Model I are the same. That is, the location of the sorting does not affect the optimal value of the collection quantity under the deterministic market demand, since the remanufacturer always gives an order to make the exact quantity of remanufacturables equal to the deterministic demand. However, it is observed that there is a critical range of p_r such that it is profitable to operate in the base model whereas it is not in Model I. Since the remanufacturer's total cost is higher in Model I than the base model due to additional inspection and disposal costs, he needs to operate at larger selling price values to offset the amount of increase in his total cost. Hence, there is a selling price range in which the remanufacturer operates in the base model whereas he does not operate in Model I.

- In Model II, the sorting activity is performed by the remanufacturer unlike the base model and the supply of used items has a deterministic and price sensitive function such that $S(f) = rf$. The optimal remanufacturer's order quantity of the second model is similar to the optimal collection quantity of the base model and the first model. Only difference is that the collector can manipulate the collection quantity by changing the acquisition price. It is observed that in Model II, the remanufacturer sets his order quantity as $\min\left\{rf, \frac{D}{(1-\alpha)q}\right\}(1-\alpha)q$ whereas in the base model, the remanufacturer adjusts his order quantity as the exact number of remanufacturables after disassembly to be equal to the deterministic demand such that $Q_0^* = \frac{D}{(1-\alpha)q}$. Therefore, the collector's decision affects the remanufacturer's decision in Model II.

- In Model III, the supply is price dependent as in Model II, and hence the collector can affect the collection quantity by changing the acquisition price after the remanufacturer's decision. The optimal acquisition prices for the collector are the same for Model II and Model III since they are considered under the case with exogenous wholesale price. Moreover, the optimal order quantity for the remanufacturer also are the same for both models because of the deterministic market demand. Therefore, in these settings, changing the roles of the supply chain members in the Stackelberg game does not affect the optimal solutions under the case where wholesale price is exogenous and the demand is deterministic.

- In Model IV, the deterministic market demand is also modelled as a function of the selling price of remanufactured items. The remanufacturer sets the price sensitive market demand equal to the quantity of exact remanufacturables after the disassembly process like the base model. The optimal acquisition price expressions for Model II and Model IV are similar, but the remanufacturer optimal pricing decision affects the collector's decision in Model IV because of price sensitive demand. On the other hand, the remanufacturer's optimal order quantity in Model IV is different from Model II. In Model IV, the remanufacturer equates the price dependent demand to the exact number of remanufacturables after disassembly. However, in Model II, the remanufacturer's optimal order quantity is affected from the collector's decision and when the amount of supply, rf, is less than $\frac{D}{(1-\alpha)q}$, the remanufacturer sets his order quantity equal to the supply, rf. As a result, the remanufacturer adjusts the selling price to satisfy all demand and so there is no unsatisfied demand in Model IV. On the other hand, in Model II, there can be unsatisfied demand if the supply amount is less than $\frac{D}{(1-\alpha)q}$.

- Model V differs from Model IV regarding to the change in the agent's roles. In Model V, the collector sets the acquisition fee after the remanufacturer's decision whereas the acquisition price is determined before the selling price in Model IV. The optimal acquisition price for the collector and the selling price for the remanufacturer are the same for Model IV and Model V since the transfer price is exogenous and the demand is deterministic. In Model V, the remanufacturer's order quantity is different from the second and third model. Since the demand is price dependent in Model V, the remanufacturer can adjust the selling price in order to equate the actual quantity of remanufacturables after disassembly to the price sensitive deterministic demand. Thus, all demand is satisfied from the supply like Model IV. In Model II and III, on the other hand, the demand is not fully served when the quantity of supply is less than $\frac{D}{(1-\alpha)q}$.

- The base model is reformulated with the price dependent supply in Model VI. The collector sets the acquisition price so as to maximize his profit. The results show that in the optimal solution, the expected number of remanufacturables sent by the collector, that is $(1-\alpha)qrf$, should be less than or equal to the remanufacturer's order size. Therefore, the collector does not collect excess number of items than his optimal supply quantity to satisfy the remanufacturer's order. For the remanufacturer's point of view, he sets his order quantity to be equal to the deterministic demand like the base model. Therefore, the collector's pricing decision does not affect the remanufacturer's optimal order size because of the deterministic demand. However, the remanufacturer's order quantity, D, is not always satisfied by the collector because the collector can send the actual quantity of remanufacturables less than the deterministic market demand to maximize his profit. Thus, the remanufacturer can face the shortage cost for unsatisfied demand.

- In Model VII, we consider the base model with the price dependent supply and the price dependent demand. The optimal pricing decision for the collector is the same as in the sixth model. The remanufacturer sets the price dependent deterministic demand equal to the quantity of exact remanufacturables after the disassembly process like Model IV and Model V. However, in Model VII, inspection is performed by the collector unlike Model IV and Model V, hence there are some differences between the selling price expressions. The term $\frac{1}{2}\left(\frac{\beta(1-q)}{(1-\alpha)q}(c_{dis}+c_{dR})+\frac{a}{b}+c_r+c_{dis}\right)$ is common in both unconstrained maximizers $(p_r^{'})$ of the remanufacturer's profit. However, $p_r^{'}$ for Model IV and Model V, is larger than $p_r^{'}$ for Model VII, by $\frac{1}{2}\left(\frac{(c_i+w)}{(1-\alpha)q}-w+\frac{c_{dR}[\alpha q+(1-\beta)(1-q)]}{(1-\alpha)q}\right)$ units. In Model IV and Model V, the remanufacturer is responsible for sorting whereas the collector takes the responsibility of sorting in Model VII. Therefore, the remanufacturer's total expected cost in Model IV and Model V is larger than Model VII due to additional inspection and disposal costs. Hence, he sets $p_r^{'}$

in Model IV and Model V to be larger than in Model VII in order to maximize his profit from remanufacturing. From the collector's point of view, his pricing decision is affected from the remanufacturer's optimal selling price decision in Model IV, V and VII. Moreover, the remanufacturer sets the selling price of remanufactured products in order to maximize his profit and there is no unsatisfied demand in Model VII. On the other hand, in Model VI, the deterministic demand D, may not be fully satisfied and the remanufacturer can face some penalty cost for unsatisfied demand.

Table 3.3: Comparison of the Models

Model	Decision Variable(s)	Optimal Collection Quantity	Optimal Order Quantity	Production Lot Size	Sorting Location	Leader/ Follower	Demand	Supply
The Base Model	Q_0(C) Q_R(R)	$\frac{D}{(1-\alpha)q}$	D	D	C	R/C	Constant	Decis. Vari.
Model I	Q_0(R)	$\frac{D}{(1-\alpha)q}$	$\frac{D}{(1-\alpha)q}$	D	R	R/C	Constant	Decis. Vari.
Model II	Q_0(R) f(C)	$rf^*_{(1)}$	$\min\left\{\frac{rw}{2}, \frac{D}{(1-\alpha)q}\right\}$	$\min\left\{\frac{rw}{2}, \frac{D}{(1-\alpha)q}\right\}(1-\alpha)q$	R	R/C	Constant	Price Dep.
Model III	Q_0(R) f(C)	$rf^*_{(1)}$	$\min\left\{\frac{rw}{2}, \frac{D}{(1-\alpha)q}\right\}$	$\min\left\{\frac{rw}{2}, \frac{D}{(1-\alpha)q}\right\}(1-\alpha)q$	R	C/R	Constant	Price Dep.
Model IV	f(C) p_r(R)	$rf^*_{(2)}$	$\frac{a-bp_{r^*(1)}}{(1-\alpha)q}$	$D(p_{r^*(1)})$	R	R/C	Price Dep.	Price Dep.
Model V	f(C) p_r(R)	$rf^*_{(2)}$	$\frac{a-bp_{r^*(1)}}{(1-\alpha)q}$	$D(p_{r^*(1)})$	R	C/R	Price Dep.	Price Dep.
Model VI	f(C) Q_R(R)	$rf^*_{(3)} \leq \frac{D}{(1-\alpha)q}$	D	D	C	R/C	Constant	Price Dep.
Model VII	f(C) p_r(R)	$rf^*_{(4)} = \frac{D(p_{r^*(2)})}{(1-\alpha)q}$	$D(p_{r^*(2)})$	$D(p_{r^*(2)})$	C	R/C	Price Dep.	Price Dep.

R: Remanufacturer, **C**: Collector

$$f^*_{(1)} = \begin{cases} \frac{w}{2} & \text{if } r\frac{w}{2} \leq \frac{D}{(1-\alpha)q} \\ \frac{D}{(1-\alpha)qr} & \text{o/w} \end{cases}.$$

$$f^*_{(2)} = \begin{cases} \frac{w}{2} & \text{if } r\frac{w}{2} \leq \frac{a-bp^*_r(1)}{(1-\alpha)q} \\ \frac{a-bp^*_r(1)}{(1-\alpha)qr} & \text{o/w} \end{cases}.$$

$$f^*_{(3)} = \begin{cases} f^{'} & \text{if } f^{'} \leq \frac{D}{(1-\alpha)qr} \\ \frac{D}{(1-\alpha)qr} & o/w \end{cases}$$

where $f^{'} = \frac{(w+b_0)(1-\alpha)q-c_i-c_{dC}[\alpha q+(1-\beta)(1-q)]-c_t[(1-\alpha)q+\beta(1-q)]}{2}$.

$$f^*_{(4)} = \begin{cases} f^{'} & \text{if } f^{'} \leq \frac{a-b_1p^*_r(2)}{(1-\alpha)qr} \\ \frac{a-b_1p^*_r(2)}{(1-\alpha)qr} & \text{if } o/w \end{cases}$$

where $f^{'} = \frac{(w+b_0)(1-\alpha)q-c_i-c_{dC}[\alpha q+(1-\beta)(1-q)]-c_t[(1-\alpha)q+\beta(1-q)]}{2}$.

$$p^*_{r(1)} = \begin{cases} p^{'}_r & \text{if } \quad \frac{a-bp^{'}_r}{(1-\alpha)q} \leq \frac{rw}{2} \\ \frac{1}{b}\left\{a - \frac{rw}{2}(1-\alpha)q\right\} & \text{o/w} \end{cases}.$$

$$p^{'}_r = \frac{1}{2(1-\alpha)q}\begin{pmatrix} (c_i+w)+c_{dR}[\alpha q+(1-\beta)(1-q)] \\ +(c_{dis}+c_{dR})\beta(1-q) \end{pmatrix} + \frac{1}{2}(\frac{a}{b}+c_r+c_{dis})$$

$$p^*_{r(2)} = \begin{cases} p_r^{'} & \text{if} \quad \frac{D(p_r^{'})}{(1-\alpha)qr} \leq f' \\ \\ \frac{1}{b}\left\{a - (1-\alpha)qrf'\right\} & \text{o/w} \end{cases} .$$

$$p_r^{'} = \frac{1}{2}\left(\frac{\beta(1-q)}{(1-\alpha)q}(c_{dis} + c_{dR}) + \frac{a}{b_1} + (w + c_r + c_{dis})\right)$$

CHAPTER 4

ANALYSIS OF THE BASE MODEL WITH INCORPORATION OF RANDOMNESS: NUMERICAL STUDY AND INSIGHTS

In Chapter 3, we revisited the recovery system analyzed by Gu and Tagaras (2014) that includes a single remanufacturer and collector in a single time period under deterministic demand case, and we introduced several extensions of it. In all of the settings we considered so far, the uncertainty in the collector's inspection process is disregarded and the number of items that are classified as remanufacturable by inspection is assumed to be equal to its expected value. Now, our aim is to analyze the base model with incorporation of randomness and evaluate the effects of disregarding randomness on the optimal solution and profits. For this reason, in this chapter, we firstly reformulate the base model by taking randomness in the sorting process into account. After that, we discuss the effects of randomness on the optimal collection quantity and profits numerically and quantify which of the agent's profit is more sensitive to randomness. Then, a detailed sensitivity analysis on the problem parameters is also conducted to observe how the impact of randomness changes with respect to a change in each parameter and to answer the following questions:

1. When the firm operates ignoring randomness, how does its corresponding expected profit compare to the profit calculated disregarding randomness?

2. When the firm operates taking randomness into account, how does the optimal collection quantity differ from the quantity found by solving the base model without incorporation of randomness, and how do the corresponding expected profits compare?

The rest of this chapter is organized as follows: in Section 4.1, the base model is reformulated by incorporating randomness. In Section 4.2, our research questions are stated explicitly and performance measures to answer these questions are given. In Section 4.3, the results of the base model with incorporation of randomness are analyzed for a selected parameter set and the comparison of the expected and true profits are made numerically. Next, we conduct a sensitivity analysis in order to show the impact of changes in values of individual parameters on the supply chain members' profits and the system profit, and to analyze how the randomness effect changes with respect to a change in each parameter. The optimal values of collection quantities, which are calculated with and without incorporation of randomness into the base model respectively, are compared and then how the related true profits deviate from the expected values is represented.

4.1 Base Model with Incorporation of Randomness

In Section 3.1, the deterministic demand model studied by Gu and Tagaras (2014), which ignores the random nature of the sorting process, was discussed in detail. Now, we incorporate the uncertainty in the inspection process and reformulate the base model in order to analyze the impact of disregarding randomness on the results. In the base model, recall that the remanufacturer is the Stackelberg leader and determines the order quantity Q_R and the collector is the follower and determines the optimal collection quantity Q_0. Note that each collected item is remanufacturable with probability q independent of others. Hence, the total number of remanufacturable items, which we represent by X, has a binomial distribution with parameters Q_0 and q. The collected items are inspected and all items that are sorted as remanufacturable are sent to the remanufacturer by the collector. There is an imperfect sorting procedure that is subject to two classification errors. Each remanufacturable item is classified as non-remanufacturable and discarded before disassembly with probability α. Let X_1 represent the quantity of remanufacturables that is sorted actually as remanufacturable by inspection. Each item is sorted actually as remanufacturable with probability $(1-\alpha)$. Then, for a given value of $X = x$, X_1 has a binomial distribution with parameters x and $1 - \alpha$. Similarly, let X_2 represent the number of non-remanufacturables that are misclassified as remanufacturable in $Q_0 - X$ items ($Q_0 - X$ is the quantity of non-remanufacturables that are collected). Each non-remanufacturable item is wrongly classified as remanufacturable with probability β, hence for a given value of $X = x$, X_2 has a binomial distribution with parameters $Q_0 - x$ and β. As a result, the probability distributions of X, X_1 and X_2 are expressed as follows:

$$P(X = x) = \binom{Q_0}{x} q^x (1-q)^{(Q_0 - x)} \qquad x = 0, 1, 2, ..., Q_0$$

$$P(X_1 = x_1 | X = x) = \binom{x}{x_1} (1-\alpha)^{x_1} \alpha^{(x - x_1)} \qquad x_1 = 0, 1, 2, ..., x$$

$$P(X_2 = x_2 | X = x) = \binom{Q_0 - x}{x_2} \beta^{x_2} (1-\beta)^{(Q_0 - x - x_2)} \qquad x_2 = 0, 1, 2, ..., Q_0 - x$$

The probability of occurrence of a realization $(X = x, X_1 = x_1, X_2 = x_2)$ for $0 \leq x \leq Q_0$; $0 \leq x_1 \leq x$ and $0 \leq x_2 \leq Q_0 - x$ is given as follows:

$$\begin{aligned} & P(X = x, X_1 = x_1, X_2 = x_2) = P(X = x) P(X_1 = x_1, X_2 = x_2 | X = x) \\ = \;& P(X = x) P(X_1 = x_1 | X = x) P(X_2 = x_2 | X = x) \\ = \;& \binom{Q_0}{x} q^x (1-q)^{(Q_0 - x)} \binom{x}{x_1} (1-\alpha)^{x_1} \alpha^{(x - x_1)} \binom{Q_0 - x}{x_2} \beta^{x_2} (1-\beta)^{(Q_0 - x - x_2)} \end{aligned}$$

We start our analysis with the collector's profit function. The collector sorts Q_0 units of used item imperfectly, then delivers x_1 remanufacturable units and x_2 non-remanufacturable units that are wrongly classified as remanufacturable to the remanufacturer. Since the remanufacturer's order quantity is equal to the deterministic demand, the collector's expected profit for $(X = x, X_1 = x_1, X_2 = x_2)$ is expressed by taking the relationship between the quantity of actual remanufacturables after disassembly and the deterministic demand into account as follows:

$$\Pi_C(Q_0|X=x,X_1=x_1,X_2=x_2)=\begin{cases}-c_0Q_0-c_iQ_0 & \\ -c_{dC}(Q_0-x_1-x_2) & \text{if } x_1\le D\\ -c_t(x_1+x_2)+wx_1 & \\ -b_0(D-x_1) & \\ & \\ -c_0Q_0-c_iQ_0 & \\ -c_{dC}(Q_0-x_1-x_2) & \text{if } x_1> D\\ -c_t(x_1+x_2)+wD & \end{cases}$$

The revenue and cost items in the collector's profit function are:

- Collection cost of used items: c_0Q_0;
- Collector's inspection cost: c_iQ_0;
- Collector's disposal cost: $c_{dC}[(x-x_1)+(Q_0-x-x_2)]=c_{dC}(Q_0-x_1-x_2)$
- Transportation cost of shipped items: $c_t(x_1+x_2)$
- Transfer payment by the remanufacturer: $\begin{cases} wx_1 & \text{if } x_1\le D\\ wD & \text{if } x_1>D\end{cases}$
- Penalty cost paid by the collector: $\begin{cases} b_0(D-x1) & \text{if } x_1\le D\\ 0 & \text{if } x_1>D\end{cases}$

Then, the collector's expected profit is expressed as:
$E[\Pi_C(Q_0)]=$

$$\sum_{x=0}^{Q_0}\sum_{x_1=0}^{x}\sum_{x_2=0}^{Q_0-x}\Pi_C(Q_0|X=x,X_1=x_1,X_2=x_2)P(X=x,X_1=x_1,X_2=x_2)$$

Next, we consider the remanufacturer's profit. The remanufacturer receives (x_1+x_2) units and reveals their actual conditions after the disassembly process. x_2 units of non-remanufacturables and excess remanufacturables after disassembly are disposed at a unit cost of c_{dR}. After the remanufacturing process, if there is unmet demand, it is lost at a unit cost of b. The remanufacturer's profit for $(X = x, X_1 = x_1, X_2 = x_2)$ is formulated as:

$$\Pi_R(Q_0|X = x, X_1 = x_1, X_2 = x_2) =$$
$$-c_{dis}(x_1 + x_2) - c_{dR}x_2 + (p_r - c_r - w)\min\{x_1, D\} - c_{dR}\max\{0, x_1 - D\}$$
$$+(b_0 - b)\max\{0, D - x_1\}$$

The revenue and cost items that are used in the remanufacturer's profit function are:

- Disassembly cost: $c_{dis}(x_1 + x_2)$
- Disposal cost of non-remanufacturables: $c_{dR}x_2$
- Disposal cost of excess remanufacturables:$c_{dR}\max\{0, x_1 - D\}$
- Total remanufacturing cost: $c_r \min\{x_1, D\}$
- Total cost paid to the collector for remanufacturables: $w\min\{x_1, D\}$
- Penalty cost for unsatisfied demand paid by the collector: $b_0 \max\{0, D - x_1\}$
- Revenue from remanufacturing : $p_r \min\{x_1, D\}$
- Penalty cost for unsatisfied demand paid by the remanufacturer:
  $b\max\{0, D - x_1\}$

Then, the remanufacturer's expected profit is expressed as:
$E[\Pi_R(X, X_1, X_2)] =$

$$\sum_{x=0}^{Q_0}\sum_{x_1=0}^{x}\sum_{x_2=0}^{Q_0-x}\Pi_R(Q_0|X = x, X_1 = x_1, X_2 = x_2)P(X = x, X_1 = x_1, X_2 = x_2)$$

Now, we can find the optimal collection quantity by complete enumeration and compare the results calculated with and without incorporation of randomness in order to analyze the effects of ignoring randomness for both parties and the system.

4.2 Main Research Questions and Related Performance Measures

In this section, we summarize main research questions that we are interested in. We use some performance measures in order to find answers for these questions and the notation used to express the performance measures throughout the analysis are given below:

- Q_A: Optimal collection quantity that is determined by solving the base model with randomness,
- Q_D: Optimal collection quantity for the base model that is determined by disregarding randomness (Note that this quantity is the same as Q_0^*)
- $\pi_A^C(Q_i)$: The collector's accurately calculated expected profit with incorporation of randomness by using Q_i for $i = A$ and D
- $\pi_A^R(Q_i)$: The remanufacturer's accurately calculated expected profit with incorporation of randomness by using Q_i for $i = A$ and D
- $\pi_A^T(Q_i)$: The expected system profit that is calculated with incorporation of randomness by using Q_i for $i = A$ and D
- $\pi_D^C(Q_D)$: The collector's expected profit that is found without incorporation of randomness by using Q_D
- $\pi_D^R(Q_D)$: The remanufacturer's expected profit that is found without incorporation of randomness by using Q_D
- $\pi_D^T(Q_D)$: The expected value of system profit that is found without incorporation of randomness by using Q_D

The questions that we try to address and corresponding performance measures are given as follows:

1. We aim to investigate the impact of disregarding randomness for both parties and the supply chain. For this purpose, we consider the case where the collection quantity is determined by disregarding randomness, that is Q_D, and we

compare the profits that are calculated with and without incorporating randomness, that are $\pi^i_A(Q_D)$ and $\pi^i_D(Q_D)$. We calculate the error caused by disregarding randomness as $\triangle^i_D = \frac{(\pi^i_D(Q_D)-\pi^i_A(Q_D))}{\pi^i_A(Q_D)}100\%$ for $i = C, R$ and T. This error provides an insight in order to show how much accurate profit is overestimated on the average by using Q_D with disregarding randomness.

2. We will compare the optimal collection quantity that is calculated by incorporating randomness, Q_A to Q_D. We also investigate how true profits $\pi^i_A(Q_A)$ that are calculated by incorporating randomness differ from $\pi^i_A(Q_D)$? For this purpose, we calculate the percentage difference as $\triangle^i_A = \frac{(\pi^i_A(Q_A)-\pi^i_A(Q_D))}{\pi^i_A(Q_A)}100\%$ for $i = C, R$ and T. This difference shows that how much accurate profit is lost on the average by using Q_D instead of Q_A.

3. What are the effects of the problem parameters on the collection quantity and profits? Which parameter(s) is/are more effective on the results? The sensitivity analysis is conducted and the results are discussed to show how collection quantities Q_A and Q_D, respectively and the corrsponding profits are affected by the changes in parameter values.

The first and second questions are related to the effect of randomness on the collection quantity and the profits. The third question's answer will show how the profits and the collection quantity are sensitive to parameters and which parameter is the most effective on the profits. In order to answer these questions, we use the optimal collection quantities and the percentage changes in the profits as performance measures.

4.3 Computational Analysis

In order to compare results for the base model with incorporating randomness to disregarding randomness, a numerical study is performed. We start our analysis with a base parameter set given in Table 4.1 and discuss the results. We then perform a thorough sensitivity analysis.

For the base parameter set, the optimal order quantity is $Q^*_R = D = 25$ and the optimal collection quantity ignoring randomness is equal to $\left[\frac{D}{(1-\alpha)q}\right] = 78$ units. For the

Table 4.1: Base Parameter Set

D	c_o	c_i	c_t	c_{dC}	c_{dis}	c_{dR}	c_R	c_r	w	p_r	b_0	b	q	α	β
25	5	5	5	0	10	0	0	50	100	500	20	0	0.4	0.2	0.1

selected parameter set, expected profits and the related error rates are summarized in Table 4.2. The result shows that the expected profits calculated with disregarding randomness deviate highly from the accurate results. It means that the expected profits that are calculated ignoring randomness do not provide a good approximation about the supply chain members' profits. More specifically, maximum percentage change is observed in the collector's profit. Since the sorting activity is performed by the collector imperfectly and subject to sorting errors, there is uncertainty in the sorted item quality in addition to the uncertainty in the quality of collected used items. This variability results in highly overestimated collector profit value. For the remanufacturer's profit, the uncertainty is only the amount of remanufacturables in the shipped quantity, so deviation from the accurate remanufacturer's profit that is found by taking randomness into account is smaller than the collector's profit.

Table 4.2: Comparison of Profits with and without Incorporation of Randomness

	$\pi_D^i(78)$	$\pi_A^i(78)$	$\pi_A^i(84)$	$\triangle_D^i$	$\triangle_A^i$
Collector(C)	1567.0	1372.9	1390.9	14.1	1.3
Remanufacturer(R)	8453.6	7906.6	8129.7	6.9	2.7
System(T)	10020.6	9279.5	9520.7	8.0	2.5

If the value of any cost item related to the collector's profit in the base parameter set increases while other parameters are kept constant, the effect of randomness on the collector's profit also increases. For example, if c_{dC} is set as 10 instead of 0, then $\pi_D^C(78)$ is 1083.4 and $\pi_A^C(78)$ is 889.3 and $\triangle_D^C$ is 21.8%. If the value of any cost items related to the collector profit decreases, then the effect of randomness on the collector's profit also decreases. For example, if c_o is set as 2 instead of 5, then $\pi_D^C(78)$ is 1801.0 and $\pi_A^C(78)$ is 1606.9 and $\triangle_D^C$ is 12.1%. If the value of any parameter related to the remanufacturer profit changes, its effect on the remanufacturer's profit is small since we set the unit penalty b for unmet demand as zero. For instance, if p_r is set as 300 while others are constant, then $\triangle_D^R$ is 6.7%. If it is set as 200, then $\triangle_D^R$ is 5.5%. For another example, when c_{dis} is increased from 10 to 30, then $\triangle_D^R$ is 7.5%. When the value of b is set different from zero, the percentage change might be high. For example, if b is set as 25 and other parameters are kept constant, then the error is 7.5%. If p_r is set as 200, then $\triangle_D^R$ is 10.6%. As a result, the collector's profit is more affected from randomness and more sensitive due to changes in values of parameters related to his profit since his inspection process includes uncertainty regarding to sorting errors.

Next, our objective is to assess the effects of incorporating randomness on the collection quantity and to investigate how true profits that are calculated by incorporating randomness, $\pi_A^i(Q_A)$, differ from $\pi_A^i(Q_D)$. For the base parameter set, we obtain Q_A as 84 units, and the related accurate profits are also given in Table 4.2. The results show that ignoring randomness does not hurt the supply chain members significantly for the selected base parameter set. If the value of any parameter, which does not affect Q_D, related to the collector's profit changes, then Q_A and the profit differences between $\pi_A^i(78)$ and $\pi_A^i(Q_A)$ also change. For example, if w is increased to 150, then Q_A is 88 units. As for to the percentage losses between profits, $\triangle_A^C$ is 2.4% and $\triangle_A^R$ is 3.7%. As another example, when c_{dC} is increased to 10, then Q_A is also calculated as 78 units and the percentage losses between profits are zero. As a result, the value of Q_A and related accurate profits can change if any parameter, which affects the collector optimal decision, changes. If the value of any parameter affects the remanufacturer's profit, but does not affect Q_A changes, the remanufacturer's profit changes in slight manner while the collector's profit remains constant. For example, if c_{dis} is increased from 10 to 30, then $\pi_A^R(78)$ is 7313.8 and $\pi_A^R(84)$ is 7491.3 and $\triangle_A^R$ decreases from 2.7% to 2.4%. For another example, when p_r is set as 300, then $\triangle_A^R$ is 2.2%. If it is set as 200, then $\triangle_A^R$ is 0.0%. However, when the unit penalty b for unmet demand is set different from zero, the change in any parameter related to the remanufacturer's profit results in higher percentage loss of profit. For example, when b is 45 and others are kept constant, the percentage loss of profit, $\triangle_A^R$, is 3.2%. When b is 45 and p_r is set as 200, $\triangle_A^R$ is 3.8%. If b is set as 80, $\triangle_A^R$ is 3.5% and 7.1% when

p_r is 500 and 200, respectively.

We continue our analysis with a thorough sensitivity analysis in order to show the effects of the system parameters on the optimal collection quantities and the profits. For both base model representation with and without incorporation of randomness, the optimal collection quantities and profits are determined by changing values of each parameter while the other values are kept constant.

We start to assess the impact of inspection accuracy on the collection quantity and profits. The inspection accuracy is related to two classification errors, type I and type II with α and β probabilities, respectively. It is intuitively known that larger error rates have a negative impact on profits, but we try to answer how much individual profits of the supply chain's members change by changing these error probabilities and how the collection quantities of the model with and without incorporation randomness are affected from these changes in error probabilities.

Firstly, the effect of α is analyzed. Note that α is the probability that a remanufacturable used product is misclassified as non-remanufacturable. The optimal collection quantity for the base model, Q_D, is a function of α; $Q_D = Q_0^* = \frac{Q_R}{(1-\alpha)q}$. In our analysis, we increase the value of α from 0 to 0.75 with a stepsize of 0.05 and a subset of results are provided in Table 4.3 for the case where the collection quantities are determined disregarding the uncertainty in the sorting process. Our findings are summarized below:

- For both parties, the errors in profit calculations caused by ignoring randomness, $\triangle_D^C$ and $\triangle_D^R$ increase in α. Since the uncertainty in the quality of the sorted item increases in α and the collector's decision also changes, he highly overestimates profit by disregarding randomness. For the remanufacturer, on the other hand, the uncertainty in the quality of items sent increases, but his optimal decision does not change and so, the increase in the error $\triangle_D^R$ is relatively small comparing to $\triangle_D^C$. As a result, the impact of ignoring randomness on the collector increases significantly in α while it does not hurt the remanufacturer considerably.

- When randomness is disregarded, the collector is willing to operate until α

Table 4.3: Comparison of Profits with and without Incorporation of Randomness when α Increases

α	Q_D	$\pi_D^C(Q_D)$	$\pi_D^R(Q_D)$	$\pi_D^T(Q_D)$	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_D^C$	$\triangle_D^R$	$\triangle_D^T$
0	63	1749.1	8460.2	10209.3	1551.6	7983.0	9534.5	12.7	6.0	7.1
0.1	69	1645.9	8460.2	10106.1	1465.2	7910.3	9375.5	12.3	7.0	7.8
0.2	78	1567.0	8453.6	10020.6	1372.9	7906.6	9279.5	14.1	6.9	8.0
0.4	104	1299.2	8438.0	9737.2	1093.8	7859.9	8953.7	18.8	7.4	8.8
0.5	125	1087.5	8425.0	9512.5	874.2	7838.3	8712.5	24.4	7.5	9.2
0.6	156	763.6	8406.8	9170.4	547.5	7799.3	8346.8	39.5	7.8	9.9
0.7	208	228.0	8375.6	8603.6	6.7	7753.9	7760.7	3285.5	8.0	10.9
0.71	216	156.6	8369.8	8526.5	−71.2	7761.7	7690.5	–	–	–
0.72	223	75.3	8366.4	8441.8	−148.0	7744.5	7596.5	–	–	–
0.73	231	−10.3	8361.9	8351.6	−232.3	7734.2	7501.9	–	–	–

reaches to 0.73 as he assumes positive profits. However, his accurately calculated expected profit starts to take negative values at $\alpha = 0.71$ and he actually incurs a loss after this point and is not willing to operate at all.

- Table 4.4 summarizes the computations for the case where randomness is incorporated. Both Q_A and Q_D increase as α increases in order to compensate for actual remanufacturables wrongly disposed due to imperfect sorting. Obviously, $\pi_A^C(Q_A)$ is larger than $\pi_A^C(Q_D)$ at any value of α since he selects Q_A in order to maximize his profit by incorporating randomness into the model which is not the case for the remanufacturer. When $\alpha > 0.5$, Q_A is smaller than Q_D which hurts the remanufacturer.

Table 4.4: Optimal Collection Quantities and Related Accurate Profits when α Increases

α	Q_A	$\pi_A^C(Q_A)$	$\pi_A^R(Q_A)$	$\pi_A^T(Q_A)$	Q_D	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_A^C$	$\triangle_A^R$	$\triangle_A^T$
0	68	1575.7	8195.4	9771.1	63	1551.6	7983.0	9534.5	1.5	2.6	2.4
0.1	75	1492.6	8160.2	9652.8	69	1465.2	7910.3	9375.5	1.8	3.1	2.9
0.2	84	1390.9	8129.7	9520.7	78	1372.9	7906.6	9279.5	1.3	2.7	2.5
0.4	108	1097.8	7985.3	9083.1	104	1093.8	7859.9	8953.7	0.4	1.6	1.4
0.5	125	874.2	7838.3	8712.5	125	874.2	7838.3	8712.5	0.0	0.0	0.0
0.55	136	729.8	7744.4	8474.3	139	728.3	7825.5	8553.8	0.2	−1.0	−0.9
0.6	149	554.5	7620.3	8174.8	156	547.5	7799.3	8346.8	1.3	−2.3	−2.1
0.7	180	62.1	7117.2	7179.2	208	6.7	7753.9	7760.7	89.1	−8.9	−8.1
0.71	183	−1.7	7079.3	7077.4	216	−71.2	7761.7	7690.5	–	–	–

- When $\alpha < 0.5$, for both the collector and the remanufacturer the percentage loss of accurate profit, $\triangle_A^C$ and $\triangle_A^R$, change slightly. After $\alpha = 0.5$, $\triangle_A^C$ increases, but $\triangle_A^R$ decreases below zero. It means that when $\alpha > 0.5$, the amount of loss for the collector increases, while the remanufacturer makes more profit without incorporation of randomness.

- Both accurately calculated collector profits $\pi_A^C(Q_D)$ and $\pi_A^C(Q_A)$, take a negative value at $\alpha = 0.71$. It means that the collector loses some profit if he operates after this point by using both Q_D and Q_A.

We next discuss the effects of β on the optimal collection quantities and profits. Note that β is the probability that a non-remanufacturable used product is misclassified as remanufacturable. The value of β is changed from 0 to 1 with a stepsize of 0.05 and a subset of results are summarized in Table 4.5 which leads to the following observations:

- Recall that Q_D does not change with respect to a change in β. Since higher β value results an increase in the quantity of non-remanufacturables sorted as remanufacturable, total transportation cost for the collector and total disassembly cost for the remanufacturer increase. Since the unit transportation cost c_t is larger than unit disassembly cost c_{dis} in the base parameter set, any change in β affects the collector more than the remanufacturer. Both expected profits calculated with and without incorporation of randomness, $\pi_D^i(Q_D)$ and $\pi_A^i(Q_D)$, decrease linearly in β.

- $\triangle_D^C$ is larger than $\triangle_D^R$ at any value of β which is reasonable since the collector's profit is more sensitive to randomness due to uncertainty in his inspection process regarding to sorting errors. The impact of a change in β on the profits, $\pi_D^i(Q_D)$ and $\pi_A^i(Q_D)$, and the percentage differences, $\triangle_D^i$, for $i = C, R$ and T, are small for the base parameter set and so, the effect of randomness on both collector's and the remanufacturer's profits are not highly dependent on a change in β.

Table 4.6 summarizes the correct expected profits when β changes with both Q_A and Q_D. Our observations follow:

Table 4.5: Comparison of Profits with and without Incorporation of Randomness when β Increases

β	Q_D	$\pi_D^C(Q_D)$	$\pi_D^R(Q_D)$	$\pi_D^T(Q_D)$	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_D^C$	$\triangle_D^R$	$\triangle_D^T$
0	78	1590.4	8500.4	10090.8	1396.3	7953.4	9349.7	13.9	6.9	7.9
0.1	78	1567.0	8453.6	10020.6	1372.9	7906.6	9279.5	14.1	6.9	8.0
0.2	78	1543.6	8406.8	9950.4	1349.5	7859.8	9209.3	14.4	7.0	8.0
0.4	78	1496.8	8313.2	9810.0	1302.7	7766.2	9068.9	14.9	7.0	8.2
0.6	78	1450.0	8219.6	9669.6	1255.9	7672.6	8928.5	15.5	7.1	8.3
0.8	78	1403.2	8126.0	9529.2	1209.1	7579.0	8788.1	16.1	7.2	8.4
1	78	1356.4	8032.4	9388.8	1162.3	7485.4	8647.7	16.7	7.3	8.6

- When β increases, the quantity of non-remanufacturables wrongly classified as remanufacturable increases. As a result, the collector collects less items in order to offset the increase in the transportation cost. However, Q_A changes in small quantities with an increase in β since any decrease in Q_A also results to a loss of expected payment received from the remanufacturer for the collector. This profit loss is larger than the decrease in the total cost for the base parameter set. Therefore, the collector's accurately calculated profit $\pi_A^C(Q_A)$ decreases as β increases.

Table 4.6: Optimal Collection Quantities and Related Accurate Profits when β Increases

β	Q_A	$\pi_A^C(Q_A)$	$\pi_A^R(Q_A)$	$\pi_A^T(Q_A)$	Q_D	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_A^C$	$\triangle_A^R$	$\triangle_A^T$
0	84	1416.1	8180.1	9596.3	78	1396.3	7953.4	9349.7	1.4	2.8	2.6
0.1	84	1390.9	8129.7	9520.7	78	1372.9	7906.6	9279.5	1.3	2.7	2.5
0.2	84	1365.7	8079.3	9445.0	78	1349.5	7859.8	9209.3	1.2	2.7	2.5
0.4	83	1315.9	7950.6	9266.5	78	1302.7	7766.2	9068.9	1.0	2.3	2.1
0.6	82	1266.3	7821.4	9087.6	78	1255.9	7672.6	8928.5	0.8	1.9	1.8
0.8	82	1217.1	7723.0	8940.0	78	1209.1	7579.0	8788.1	0.7	1.9	1.7
1	81	1168.2	7594.4	8762.6	78	1162.3	7485.4	8647.7	0.5	1.4	1.3

- The collector selects Q_A in order to maximize his profit by incorporating randomness into the model, so $\pi_A^C(Q_A)$ is larger than $\pi_A^C(Q_D)$ for any β. For the remanufacturer, larger collected quantity results an increase in both expected disassembly cost and expected revenue from remanufacturing. However, the increase in the expected profit from remanufacturing is larger than in the total disassembly cost. Therefore, both parties incur a loss by using Q_D instead of Q_A, but the deviation is small between the profits for the selected parameter set and it decreases in β. Hence, ignoring randomness hurts less both parties and the system when β increases.

We next consider the effects of returns quality on the collection quantities and profits. The fraction of remanufacturables in the collected lot, that is q, can be used as a measure of the returns quality. Therefore, larger q values imply that an item is more likely to be remanufacturable. Hence, smaller collection quantity is enough to satisfy the demand. In the model that disregards randomness, recall that the optimal collection quantity, Q_D, is a decreasing function of q; $Q_0^* = Q_D = \frac{D}{(1-\alpha)q}$. The value of q is increased from 0 to 1 with a stepsize of 0.05 and a subset of the results is presented in Table 4.7 when the order quantities, profits and percentage differences are determined ignoring uncertainty in the inspection process. Our findings are listed according to a change in value of q below:

- Q_D decreases in smaller rates in q since the expected number of remanufacturables in the collected lot, qQ_D, closes to the collection quantity Q_D and the expected number of non-remanufacturables closes to zero.

Table 4.7: Comparison of Profits with and without Incorporation of Randomness when q Increases

q	Q_D	$\pi_D^C(Q_D)$	$\pi_D^R(Q_D)$	$\pi_D^T(Q_D)$	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_D^C$	$\triangle_D^R$	$\triangle_D^T$
0.13	240	−134.0	8291.6	8157.6	−357.3	7664.3	7307.1	–	–	–
0.14	223	46.4	8308.5	8354.8	−176.9	7686.5	7509.6	–	–	–
0.15	208	202.0	8323.6	8525.6	−19.3	7701.9	7682.7	–	–	–
0.16	195	338.5	8336.6	8675.1	118.3	7717.7	7836.0	186.2	8.0	10.7
0.2	156	748.0	8375.6	9123.6	531.9	7768.1	8300.0	40.6	7.8	9.9
0.4	78	1567.0	8453.6	10020.6	1372.9	7906.6	9279.5	14.1	6.9	8.0
0.6	52	1840.0	8479.6	10319.6	1670.7	8001.0	9671.7	10.1	6.0	6.7
0.8	39	1976.5	8492.6	10469.1	1836.4	8094.2	9930.7	7.6	4.9	5.4
1	31	2042.0	8502.0	10544.0	1948.9	8180.0	10128.8	4.8	3.9	4.1

- The collector's expected profit, $\pi_D^C(Q_D)$, increases in q. The increase is especially significant at smaller values of q. The remanufacturer's profit, $\pi_D^R(Q_D)$, also increases regarding to the decrease in the disassembly cost, but it is not highly affected from an increase in q. Since the collector always sends the amount of actual remanufacturables to be exactly equal to the demand, the quantity of actual remanufacturables sent does not change by an increase in q.

- The collector starts to operate at threshold value of $q = 0.14$. However, his accurately calculated expected profit $\pi_A^C(Q_D)$ still takes a negative value with an increase in q from 0.14 to 0.16. It means that the collector expects to gain profit after this threshold value of q by disregarding randomness, but actually he incurs a loss and collection of used items is not profitable for him until $q = 0.16$.

- Since the collector's profit is more sensitive due to randomness because of the imperfect sorting, the error caused by disregarding randomness for the collector $\triangle_D^C$ is larger than for the remanufacturer $\triangle_D^R$ at any q. It is observed that the effect of randomness is highly dependent on a change in value of q especially for the collector, and its effect decreases in q.

Table 4.8 shows a subset of the results under the case where randomness is taken into account. Our observations are given below:

- For any value of q, $\pi_A^C(Q_A)$ is larger than $\pi_A^C(Q_D)$ since the collector sets Q_A to maximize his profit. On the other hand, for the remanufacturer, higher collection quantity results an increase in both the expected revenue from remanufacturing and the expected disassembly cost. However, the gain from remanufacturing is higher than the loss due to the increase in the disassembly cost. Therefore, he makes larger profit by using larger collected quantity.

- Until $q = 0.25$, the collector maximizes his profit by using smaller collected quantity with incorporation of randomness than disregarding randomness. The percentage loss of accurate profit, $\triangle_A^C$, is high at smaller values of q regarding to larger difference between Q_D and Q_A and it decreases in q. After $q = 0.25$, $\triangle_A^C$ increases in slight manner. For the remanufacturer, $\triangle_A^R$ is negative up to point of $q = 0.25$. After this point, the remanufacturer makes larger profit by

Table 4.8: Optimal Collection Quantities and the Related Accurate Profits when q Increases

q	Q_A	$\pi_A^C(Q_A)$	$\pi_A^R(Q_A)$	$\pi_A^T(Q_A)$	Q_D	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_A^C$	$\triangle_A^R$	$\triangle_A^T$
0.14	185	-92.9	6844.3	6751.4	223	-176.9	7686.5	7509.6	–	–	–
0.15	179	39.6	7044.1	7083.7	208	-19.3	7701.9	7682.7	148.6	-9.3	-8.5
0.16	173	159.6	7208.8	7368.4	195	118.3	7717.7	7836.0	25.9	-7.1	-6.3
0.2	148	539.6	7562.1	8101.8	156	531.9	7768.1	8300.0	1.4	-2.7	-2.4
0.25	125	864.8	7819.6	8684.3	125	864.8	7819.6	8684.3	0.0	0.0	0.0
0.4	84	1390.9	8129.7	9520.7	78	1372.9	7906.6	9279.5	1.3	2.7	2.5
0.6	58	1709.8	8285.1	9995.0	52	1670.7	8001.0	9671.7	2.3	3.4	3.2
0.8	43	1882.3	8340.2	10222.5	39	1836.4	8094.2	9930.7	2.4	2.9	2.9
1	34	1996.2	8401.7	10397.9	31	1948.9	8180.0	10128.8	2.4	2.6	2.6

using Q_A and $\triangle_A^R$ is positive. It means that the remanufacturer actually gains more profit when randomness is disregarded until $q = 0.25$. However, he can increase his accurate profit by using Q_A instead of Q_D when $q > 0.25$. As a result, the effect of ignoring randomness on the collection quantity is high at smaller values of q and it causes the collector to lose high profit while it does not hurt the remanufacturer until $q = 0.25$.

- The collector starts to make a profit at $q = 0.15$ by using Q_A while he incurs a loss at this point when Q_D is used. Although both profits, $\pi_A^C(Q_A)$ and $\pi_A^C(Q_D)$ are calculated with incorporation of randomness, Q_D is very high compared to Q_A at this point and it results to the collector incur high collection, inspection and transportation cost by using Q_D. His revenue is smaller than the related cost and hence, he actually incurs a loss by using Q_D at $q = 0.15$.

We next consider the effects of changes in the deterministic demand D on the collection quantities and profits. When randomness is disregarded, recall that Q_D is linearly dependent on D such that $Q_D = \frac{D}{(1-\alpha)q}$. D is changed between 0 and 100 with a stepsize of 5 and the computations are summarized in Table 4.9 for some values of D when the collection quantities are determined by disregarding randomness. Observations are provided below:

- Both collector's profits $\pi_D^C(Q_D)$ and $\pi_A^C(Q_D)$ increase with an increase in D. The increase in $\pi_D^C(Q_D)$ is larger than in $\pi_A^C(Q_D)$. When randomness is disregarded, it is assumed that the quantity of actual remanufacturables sent is equal to demand and so, there is no penalty cost. On the other hand, when randomness is taken into account, it can be less than the demand. Therefore, for the collector, the expected value of total revenue received from the remanufacturer decreases and the expected total cost also increases regarding to an increase in the expected penalty cost for unsatisfied demand with incorporation of randomness.

Table 4.9: Comparison of Profits with and without Incorporation of Randomness when D Increases

D	Q_D	$\pi_D^C(Q_D)$	$\pi_D^R(Q_D)$	$\pi_D^T(Q_D)$	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_D^C$	$\triangle_D^R$	$\triangle_D^T$
0	0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	-
10	31	621.5	3382.2	4003.7	503.0	3030.0	3533.0	23.6	11.6	13.3
20	62	1243.0	6764.4	8007.4	1077.3	6256.0	7333.3	15.4	8.1	9.2
25	78	1567.0	8453.6	10020.6	1372.9	7906.6	9279.5	14.1	6.9	8.0
40	125	2512.5	13525.0	16037.5	2263.4	12840.0	15103.5	11.0	5.3	6.2
60	187	3755.5	20289.4	24044.9	3460.0	19424.1	22884.1	8.5	4.5	5.1
80	250	5025.0	27050.0	32075.0	4672.3	26080.2	30752.5	7.5	3.7	4.3
100	312	6268.0	33814.4	40082.4	5883.4	32703.9	38587.3	6.5	3.4	3.9

- The related errors caused by disregarding randomness, $\triangle_D^i$, for $i = C, R$ and T, decrease with an increase in D. The result shows that the effect of randomness on both collector and remanufacturer's profit is higher at smaller values of D, but its effect decreases and it hurts less both parties and system as D increases.

Table 4.10 gives some of the results for the case where the uncertainty in the inspection process is incorporated. Our findings are listed below:

- Q_A takes larger values than Q_D at any value of D since the collector needs to collect more items to handle the effect of randomness and satisfy the demand.
- Both $\pi_A^C(Q_A)$ and $\pi_A^R(Q_A)$ increase in D. It is observed that when Q_D is used instead of Q_A, the profit losses for both parties are small at any values of D for the base parameter set and the amount of loss decreases in D slightly.

Next, we analyze the effects of changes in the transfer price w on the collection quantities and profits. w is changed between 0 and 200 with step size of 10 and a subset of

Table 4.10: Optimal Collection Quantities and the Related Accurate Profits when D Increases

D	Q_A	$\pi_A^C(Q_A)$	$\pi_A^R(Q_A)$	$\pi_A^T(Q_A)$	Q_D	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_A^C$	$\triangle_A^R$	$\triangle_A^T$
0	0	0.0	0.0	0.0	0	0.0	0.0	0.0	-	-	-
10	35	513.4	3174.1	3687.5	31	503.0	3030.0	3533.0	2.0	4.5	4.2
20	68	1095.3	6479.0	7574.2	62	1077.3	6256.0	7333.3	1.6	3.4	3.2
25	84	1390.9	8129.7	9520.7	78	1372.9	7906.6	9279.5	1.3	2.7	2.5
40	132	2286.6	13106.3	15392.9	125	2263.4	12840.0	15103.5	1.0	2.0	1.9
60	197	3493.1	19804.3	23297.5	187	3460.0	19424.1	22884.1	0.9	1.9	1.8
80	261	4707.1	26522.7	31229.8	250	4672.3	26080.2	30752.5	0.7	1.7	1.5
100	325	5926.8	33199.3	39126.1	312	5883.4	32703.9	38587.3	0.7	1.5	1.4

the results are given in Table 4.11 under the case where the collection quantities are calculated by ignoring randomness. We summarize our findings as follows:

- It is obvious that both collector profits $\pi_D^C(Q_D)$ and $\pi_A^C(Q_D)$ increase and both remanufacturer profits $\pi_D^R(Q_D)$ and $\pi_A^R(Q_D)$ decrease while w increases. Since an increase in the remanufacturer's profit is equal to a decrease in the collector's profit for both models with and without incorporation of randomness, the expected total profits $\pi_D^T(Q_D)$ and $\pi_A^T(Q_D)$ remain constant. The increase in $\pi_D^C(Q_D)$ is larger than in $\pi_A^C(Q_D)$ as the actual number of remanufacturables sent may be smaller than the demand when randomness is incorporated and some of demand may not be satisfied. The decrease in $\pi_D^R(Q_D)$ is also larger than in $\pi_A^R(Q_D)$ for the same reason.

Table 4.11: Comparison of Profits with and without Incorporation of Randomness when w Increases

w	Q_D	$\pi_D^C(Q_D)$	$\pi_D^R(Q_D)$	$\pi_D^T(Q_D)$	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_D^C$	$\triangle_D^R$	$\triangle_D^T$
37	78	−8.0	10208.6	10020.6	−261.1	9540.6	9279.5	–	–	–
38	78	17.0	10003.6	10020.6	−74.3	9353.8	9279.5	–	–	–
41	78	92.0	9928.6	10020.6	−4.3	9283.8	9279.5	–	–	–
42	78	117.0	9903.6	10020.6	19.0	9260.5	9279.5	514.8	6.9	8.0
50	78	317.0	9703.6	10020.6	205.8	9073.7	9279.5	54.1	6.9	8.0
80	78	1067.0	8953.6	10020.6	906.0	8373.5	9279.5	17.8	6.9	8.0
100	78	1567.0	8453.6	10020.6	1372.9	7906.6	9279.5	14.1	6.9	8.0
150	78	2817.0	7203.6	10020.6	2540.0	6739.5	9279.5	10.9	6.9	8.0
200	78	4067.0	5953.6	10020.6	3707.1	5572.4	9279.5	9.7	6.8	8.0

- At smaller values of w, the percentage difference between the collector profits, $\triangle_D^C$, is high and it changes significantly with respect to an increase in w. Therefore, the effect of randomness on the collector's profit is highly dependent on a change in the value of w and its effect decreases when w takes larger values. On the other hand, the percentage differences between the remanufacturer profits, $\triangle_D^R$, and total profits, $\triangle_D^T$, do not change. It means that the impact of randomness on the remanufacturer and the system is not affected from a change in w.

- When randomness is disregarded, the collector does not operate until w is equal to 38. At this point, collector starts to make a profit and collects used items from the market. However, his accurate profit, $\pi_A^C(Q_D)$, takes negative values up to $w = 42$. Therefore, if w is between 38 and 42, he expects to make a profit by disregarding randomness, but actually he incurs a loss until the threshold value of $w = 42$.

The accurately calculated expected profits by using both Q_A and Q_D are summarized in Table 4.12. Our observations are given below:

- Intuitively, the collector collects higher Q_A with respect to an increase in w and the remanufacturer prefers a larger collected quantity. When w is smaller than 60, the remanufacturer makes more profit without incorporation of randomness. After that, disregarding randomness causes the remanufacturer to lose some profit. For the collector, at any value of w, $\pi_A^C(Q_A)$ is larger than $\pi_A^C(Q_D)$. Up to point of $w = 60$, for the collector the percentage loss of profit, $\triangle_A^C$, decreases significantly and after this point $\triangle_A^C$ increases slightly. It shows that the change in w highly affects the percentage loss of accurate profit caused by using Q_D instead of Q_A for the collector, but its effect on the remanufacturer and the system is small and ignoring randomness affects the remanufacturer and system positively before $w = 60$.

- When randomness is taken into account, the collector starts to operate at $w = 41$ if Q_A is used, but he loses profit at this point if Q_D is used. However, the profit is only 1.2 by using Q_A, and the profit loss, that is 4.3, is not high by

Table 4.12: Optimal Collection Quantities and the Related Accurate Profits when w Increases

w	Q_A	$\pi_A^C(Q_A)$	$\pi_A^R(Q_A)$	$\pi_A^T(Q_A)$	Q_D	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_A^C$	$\triangle_A^R$	$\triangle_A^T$
40	73	−21.4	8979.8	8958.4	78	−27.7	9307.2	9279.5	–	–	–
41	74	1.2	9030.5	9031.6	78	−4.3	9283.8	9279.5	–	–	–
42	74	23.8	9007.8	9031.6	78	19.0	9260.5	9279.5	20.1	−2.8	−2.7
50	76	206.8	8957.8	9164.6	78	205.8	9073.7	9279.5	0.5	−1.3	−1.3
60	78	439.2	8840.3	9279.5	78	439.2	8840.3	9279.5	0.0	0.0	0.0
80	82	911.8	8544.9	9456.6	78	906.0	8373.5	9279.5	0.6	2.0	1.9
100	84	1390.9	8129.7	9520.7	78	1372.9	7906.6	9279.5	1.3	2.7	2.5
150	88	2603.7	7002.3	9606.0	78	2540.0	6739.5	9279.5	2.4	3.8	3.4
200	90	3827.9	5802.3	9630.2	78	3707.1	5572.4	9279.5	3.2	4.0	3.6

using Q_D at this point. Therefore, he can make a profit by using Q_A instead of Q_D at $w = 41$, but his gain is not high and he may prefer not to operate at this point.

When randomness is disregarded, it is assumed that the collector sends the amount of actual remanufacturables that is exactly equal to the deterministic demand. Therefore, the collector does not incur any shortage cost due to unsatisfied demand. Hence, a change in the unit penalty cost b_0 does not affect the expected results. On the other hand, for the model with incorporation of randomness, the amount of actual remanufacturables sent may be less or more than the demand. Therefore, a change in b_0 affects the optimal collection quantity Q_A and the related accurate profits. In order to observe the effects of changes in b_0 on the results, it is increased from 0 to 100 with a stepsize of 5 and a subset of the results are given in Table 4.13 for the case where the expected profits and percentage differences determined by using Q_D. Our findings with respect to an increase in b_0 are provided below:

- When randomness is disregarded, the collector can operate by using Q_D until $b_0 = 849$. After that, he actually incurs a lose while he expects to gain 1567 units of profit without incorporation of randomness.
- The error due to disregarding randomness for the collector, $\triangle_D^C$ increases while the error for the remanufacturer, $\triangle_D^R$, decreases in b_0. Both collector's and the remanufacturer's profits change by the same amount with an increase in b_0, but the effect of randomness on the collector's profit is changed much strongly than

Table 4.13: Comparison of Profits with and without Incorporation of Randomness when b_0 Increases

b_0	Q_D	$\pi_D^C(Q_D)$	$\pi_D^R(Q_D)$	$\pi_D^T(Q_D)$	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_D^C$	$\triangle_D^R$	$\triangle_D^T$
0	78	1567.0	8453.6	10020.6	1406.0	7873.5	9279.5	11.4	7.4	8.0
5	78	1567.0	8453.6	10020.6	1397.8	7881.7	9279.5	12.1	7.3	8.0
10	78	1567.0	8453.6	10020.6	1389.5	7890.0	9279.5	12.8	7.1	8.0
20	78	1567.0	8453.6	10020.6	1372.9	7906.6	9279.5	14.1	6.9	8.0
35	78	1567.0	8453.6	10020.6	1348.0	7931.5	9279.5	16.2	6.6	8.0
50	78	1567.0	8453.6	10020.6	1323.2	7956.3	9279.5	18.4	6.2	8.0
75	78	1567.0	8453.6	10020.6	1281.7	7997.8	9279.5	22.3	5.7	8.0
90	78	1567.0	8453.6	10020.6	1256.9	8022.6	9279.5	24.7	5.4	8.0
100	78	1567.0	8453.6	10020.6	1240.3	8039.2	9279.5	26.3	5.2	8.0
848	78	1567.0	8453.6	10020.6	0.4	9279.1	9279.5	–	–	–
849	78	1567.0	8453.6	10020.6	−1.2	9280.7	9279.5	–	–	–

its effect on the remanufacturer's profit.

Table 4.14 shows the results for the case where randomness is taken into account. We summarize our observations as follows:

- For the collector, the expected shortage cost for unsatisfied remanufacturer's order increases in b_0. Then, he collects more items in order to diminish the effect of the increase in the expected shortage cost. However, the increase in Q_A is not high since the expected collection, inspection and transportation costs also increase as Q_A increases while the effect of b_0 decreases.

Table 4.14: Optimal Collection Quantities and the Related Accurate Profits when b_0 Increases

b_0	Q_A	$\pi_A^C(Q_A)$	$\pi_A^R(Q_A)$	$\pi_A^T(Q_A)$	Q_D	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_A^C$	$\triangle_A^R$	$\triangle_A^T$
0	82	1411.8	8044.9	9456.6	78	1406.0	7873.5	9279.5	0.4	2.1	1.9
5	82	1406.1	8050.5	9456.6	78	1397.8	7881.7	9279.5	0.6	2.1	1.9
10	83	1400.8	8089.8	9490.6	78	1389.5	7890.0	9279.5	0.8	2.5	2.2
20	84	1390.9	8129.7	9520.7	78	1372.9	7906.6	9279.5	1.3	2.7	2.5
35	85	1378.0	8129.7	9547.1	78	1348.0	7931.5	9279.5	2.2	2.9	2.8
50	86	1367.7	8203.3	9570.0	78	1323.2	7956.3	9279.5	3.2	3.0	3.0
75	88	1350.7	8255.2	9606.0	78	1281.7	7997.8	9279.5	5.1	3.1	3.4
90	89	1342.4	8277.0	9619.4	78	1256.9	8022.6	9279.5	6.4	3.1	3.5
100	89	1337.2	8282.2	9619.4	78	1240.3	8039.2	9279.5	7.3	2.9	3.5

- For both the collector and the remanufacturer, accurate profits calculated by using Q_A are larger than by using Q_D. For the collector, obviously Q_A gives larger profit as he sets the order quantity in order to maximize his profit and the remanufacturer makes more profit with larger collected quantity since higher collection quantity results to higher increase in expected revenue from remanufacturing than the expected disassembly cost. The percentage loss of profit for the collector, $\triangle_A^C$, is high especially at higher values of b_0. On the other hand, for the remanufacturer $\triangle_A^R$ increases slightly. As a result, for smaller values of b_0, disregarding randomness does not hurt both parties and the system considerably, but its effect on the collector increases highly as b_0 increases.

A change in the remanufacturer's unit penalty cost of b for unmet demand does not affect the results of the base model without incorporation of randomness as the collector sets the actual amount of remanufacturables exactly equal to the deterministic demand. On the other hand, when randomness is taken into account, the demand may not be satisfied and a change in the value of b affects the remanufacturer's accurately calculated profits, $\pi_A^R(Q_D)$ and $\pi_A^R(Q_A)$, but does not affect the optimal collection quantity Q_A and the collector's accurate profits, $\pi_A^C(Q_D)$ and $\pi_A^C(Q_A)$. The effects of b on the remanufacturer's profits and the percentage differences are analyzed by increasing its value from 0 to 100 and from 0 to 2000 with a stepsize of 5 and 100, respectively. A subset of the computations is summarized in Table 4.15 and it leads to observed results below:

- It is clear that both the remanufacturer's profits $\pi_A^R(Q_D)$ and $\pi_A^R(Q_A)$ decrease with an increase in b. When $Q_A = 84$ is used instead of $Q_D = 78$, the remanufacturer's expected revenue from remanufacturing and the related disassembly cost and the cost paid to the collector increase while the total expected shortage cost paid to the customers for unsatisfied demand decreases. Since his gain outweighs the related cost increase, his expected profit is larger by using Q_A rather than Q_D.
- The error caused by disregarding randomness, $\triangle_D^R$, increases as b increases and the remanufacturer highly overestimates the expected profit at larger values of b. The percentage loss of profit, $\triangle_A^R$, also increases in b. Unless b takes very

Table 4.15: Comparison of The Remanufacturer's Profits for $Q_D = 78$ and $Q_A = 84$ when b Increases

b	$\pi_D^R(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^R(Q_A)$	$\triangle_D^R$	$\triangle_A^R$
0	8453.6	7906.6	8129.7	6.9	2.7
5	8453.6	7898.3	8125.2	7.0	2.8
10	8453.6	7890.0	8120.6	7.1	2.8
50	8453.6	7823.7	8084.1	8.1	3.2
100	8453.6	7740.8	8038.5	9.2	3.7
200	8453.6	7575.1	7947.3	11.6	4.7
500	8453.6	7077.8	7673.6	19.4	7.8
1000	8453.6	6249.0	7217.4	35.3	13.4
2000	8453.6	4591.5	6305.1	84.1	27.2

high values, the remanufacturer can operate and disregarding of randomness does not hurt him significantly.

Any change in the unit selling price p_r, unit disassembly cost c_{dis}, unit disposal cost of the remanufacturer c_{dR} and unit remanufacturing cost c_r only affect the remanufacturer's profit for both models with and without incorporation of randomness. Since the collection quantities Q_A and Q_D do not change with respect a change in values of these parameters, expected collector profits $\pi_D^C(Q_D)$, $\pi_A^C(Q_D)$ and $\pi_A^C(Q_A)$ remain the same. Therefore, we only analyze the effects of these parameters on the remanufacturer's profit and the percentage differences. First of all, the effects of an increase in value of the selling price are analyzed by increasing its value from 0 to 1000 with a stepsize of 25 and for a subset of the results, Table 4.16 shows the remanufacturer's expected profits and the related percentage differences. Our observations from the results are listed below:

- Both $\pi_D^R(Q_D)$ and actually calculated expected profit $\pi_A^R(Q_D)$ are negative until p_r is equal to 162. The important point is that until $p_r = 170$, the remanufacturer's expected profit calculated without incorporation of randomness, $\pi_D^R(Q_D)$, is smaller than his accurately calculated expected profit, $\pi_A^R(Q_D)$. Hence, actually he makes higher profit than he expects to gain. After this point, the remanufacturer's gain is less than he expects and $\triangle_D^R$ gets higher with an increase in p_r.

Table 4.16: Comparison of The Remanufacturer's Profits for $Q_D = 78$ and $Q_A = 84$ when p_r Increases

p_r	$\pi_D^R(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^R(Q_A)$	$\triangle_D^R$	$\triangle_A^R$
100	−1546.4	−1430.4	−1505.3	–	–
161	−21.4	−6.5	−36.0	–	–
162	3.6	16.9	−11.9	−78.6	–
163	28.6	40.2	12.2	−28.9	−229.9
169	178.6	180.3	156.7	−0.9	−15.0
170	**203.6**	**203.6**	180.8	**0.0**	−12.6
171	228.6	226.9	204.9	0.7	−10.8
200	953.6	**903.9**	**903.9**	5.5	**0.0**
201	978.6	927.2	927.5	5.5	0.0
300	3453.6	3238.1	3312.2	6.7	2.2
500	8453.6	7906.6	8129.7	6.9	2.7
800	15953.6	14909.3	15356.0	7.0	2.9
1000	20953.6	19577.8	20173.6	7.0	3.0

- When randomness is incorporated into the model, the remanufacturer starts to make a profit from remanufacturing at $p_r = 163$, whereas he starts to operate at $p_r = 162$ disregarding randomness. Before $p_r = 200$, $\pi_A^R(Q_D)$ is larger than $\pi_A^R(Q_A)$. It means that ignoring randomness does not hurt the remanufacturer. After $p_r = 200$, the remanufacturer can improve his profit with incorporation of randomness. Therefore, the percentage difference, $\triangle_A^R$, increases below zero until $p_r = 200$ and after that, it increases when p_r increases.

- The important result is that when p_r is between 170 and 200, the remanufacturer makes less profit than he expects without incorporation of randomness, that is $\pi_D^R(Q_D) > \pi_A^R(Q_D)$. On the other hand, his accurate profit $\pi_A^R(Q_D)$ is higher than $\pi_A^R(Q_A)$ in this range of p_r. This implies that although his expected profit $\pi_D^R(Q_D)$ overestimates the accurate profit $\pi_A^R(Q_D)$, but his profit actually improves when randomness ignored.

The effects of a change in the unit remanufacturing cost, c_r, on the remanufacturer's profit are discussed by changing its value from 0 to 500 with a stepsize of 25. For some values of c_r, the computations are summarized in Table 4.17 and the findings with respect to an increase in c_r are given below:

- When randomness is not taken into account, the remanufacturer makes less profit than he expects until $c_r = 380$. When $c_r > 380$, $\pi_A^R(Q_D)$ is larger than $\pi_D^R(Q_D)$ and hence, he gains more profit than he expects.

Table 4.17: Comparison of The Remanufacturer's Profits for $Q_D = 78$ and $Q_A = 84$ when c_r Increases

c_r	$\pi_D^R(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^R(Q_A)$	$\triangle_D^R$	$\triangle_A^R$
0	9703.6	9073.7	9334.1	6.9	2.8
25	9078.6	8490.2	8731.9	6.9	2.8
50	8453.6	7906.6	8129.7	6.9	2.7
100	7203.6	6739.5	6925.4	6.9	2.7
200	4703.6	4405.2	4516.6	6.8	2.5
275	2828.6	2654.6	2710.0	6.6	2.0
349	978.6	927.2	927.5	5.5	0.0
350	953.6	**903.9**	**903.9**	5.5	**0.0**
375	328.6	320.3	301.2	2.6	−6.3
380	**203.6**	**203.6**	180.8	**0.0**	−12.6
381	178.6	180.3	156.7	−0.9	−15.0
387	28.6	40.2	16.2	−28.9	−229.9
388	3.6	12.9	−11.9	−78.6	–
389	−21.4	−6.5	−36.0	–	–
400	−296.4	−263.3	−301.0	–	–

- The remanufacturer operates until $c_r = 388$ when the collector sets the collection quantity with incorporation of randomness. At $c_r = 388$, the remanufacturer actually makes 12.9 units of profit by using $Q_D = 78$ while he loses 11.9 units of profit by using $Q_A = 84$ with incorporation of randomness. Therefore, at this point, the remanufacturer actually could make a profit when the system operates by disregarding randomness.
- When randomness is disregarded, the error, $\triangle_D^R$, decreases when c_r increases until 380. When c_r is equal to 380, both profits are equal and there is no error regarding to randomness. At $c_r > 380$, $\pi_A^R(Q_D)$ is larger than $\pi_D^R(Q_D)$. That is, the remanufacturer makes more profit than he expects. Therefore, the effect

of randomness is high at small values of c_r on the remanufacturer, but his profit is also high.

- When $c_r < 350$, $\pi_A^R(Q_A)$ is larger than $\pi_A^R(Q_D)$. After that, the remanufacturer's accurate profit is higher by using $Q_D = 78$ than $Q_A = 84$ and actually, he can make more profit without incorporation of randomness. Note that when c_r is between 350 and 380, $\pi_D^R(Q_D)$ is larger than $\pi_A^R(Q_D)$ and $\pi_A^R(Q_D)$ is also higher than his accurately calculated profit $\pi_A^R(Q_A)$. This shows that although he overestimates accurate profit by using $Q_D = 78$ without incorporation of randomness, he actually gets more profit than by using $Q_A = 84$ with incorporation of randomness.

Next, the impact of disassembly cost, c_{dis}, on the remanufacturer's profit is analyzed by changing its value from 0 to 300 with a stepsize of 10. In Table 4.18, the remanufacturer's profits, $\pi_D^R(Q_D)$, $\pi_A^R(Q_D)$ and $\pi_A^R(Q_A)$ and the related percentage differences, $\triangle_D^R$ and $\triangle_A^R$, are given for a subset of the results as c_{dis} increases. Our observations are summarized below:

- When randomness is not incorporated, the remanufacturer expects to make a profit from remanufacturing until $c_{dis} = 296$. However, he actually incurs a loss when c_{dis} is between 276 and 296.

- The remanufacturer is not willing to operate after $c_{dis} = 264$ when randomness is incorporated. When c_{dis} is between 264 and 277, he actually makes profit by using $Q_D = 78$, but he loses profit by using $Q_A = 84$ with incorporation of randomness. Therefore, in this range of c_{dis}, disregarding randomness does not hurt the remanufacturer and he can make a profit by using $Q_D = 78$.

- Another observation says that when $c_{dis} < 108$, $\pi_A^R(Q_A)$ is larger than $\pi_A^R(Q_D)$. Hence, he actually makes more profit by using $Q_A = 84$ instead of $Q_D = 78$. When $c_{dis} > 108$, incorporation of randomness into the model results smaller profit for the remanufacturer by using $Q_A = 84$. Therefore, the percentage difference, $\triangle_A^R$, decreases until $c_{dis} = 108$ and after that, it decreases below zero.

Table 4.18: Comparison of The Remanufacturer's Profits for $Q_D = 78$ and $Q_A = 84$ when c_{dis} Increases

c_{dis}	$\pi_D^R(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^R(Q_A)$	$\triangle_D^R$	$\triangle_A^R$
0	8750.0	8203.0	8448.9	6.7	2.9
10	8453.6	7906.6	8129.7	6.9	2.7
50	7268.0	6721.0	6852.9	8.1	1.9
100	5786.0	5239.0	5256.9	10.4	0.3
108	5548.9	**5001.9**	**5001.6**	10.9	**0.0**
150	4304.0	3757.0	3660.9	14.6	-2.6
200	2822.0	2275.0	2064.9	24.0	-10.2
250	1340.0	793.0	468.9	69.0	-69.1
264	925.0	378.04	22.1	144.7	-1614.2
265	895.4	348.4	-9.9	157.0	$-$
276	569.4	22.35	-361.0	2447.5	$-$
277	539.7	-7.3	-392.9	$-$	$-$
295	6.2	-540.8	-967.5	$-$	$-$
296	-23.4	-570.4	-999.4	$-$	$-$

- The error caused by ignoring randomness, $\triangle_D^R$, increases significantly when c_{dis} increases.

For the remanufacturer, lastly the effects of a change in the value of remanufacturer's unit disposal cost, c_{dR}, are analyzed by increasing its value from 0 to 2000 with a stepsize of 50. We summarize the results in Table 4.19 and it leads to following findings:

- When randomness is disregarded, it is assumed that the actual quantity of remanufacturable items is exactly equal to the demand. Therefore, the remanufacturer only disposes of non-remanufacturables that are sorted wrongly as remanufacturable after the disassembly process. On the other hand, the quantity of remanufacturables sent can be larger than the demand with incorporation of randomness into the model, then the remanufacturer disposes of excess number of remanufacturables in addition to the non-remanufacturables. Hence, the decrease in $\pi_A^R(Q_D)$ is larger than $\pi_D^R(Q_D)$ regarding to higher expected disposal cost as c_{dR} increases.
- When randomness is incorporated, the remanufacturer can make higher profit

Table 4.19: Comparison of The Remanufacturer's Profits for $Q_D = 78$ and $Q_A = 84$ when c_{dR} Increases

c_{dR}	$\pi_D^R(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^R(Q_A)$	$\triangle_D^R$	$\triangle_A^R$
0	8453.6	7906.6	8129.7	6.9	2.7
50	8221.6	7591.7	7738.1	8.3	1.9
100	7989.6	7276.8	7346.5	9.8	0.9
145	7780.8	**6993.5**	**6994.1**	11.3	**0.0**
146	7776.2	6987.2	6986.2	11.3	0.0
250	7293.6	6332.2	6171.7	15.2	−2.6
400	6597.6	5387.6	4996.8	22.5	−7.8
750	4973.6	3183.4	2255.5	56.2	−41.1
1000	3813.6	1609.0	297.4	137.0	−441.0
1037	3641.9	1376.0	7.6	164.7	−17965.0
1038	3637.2	1369.7	−0.2	165.5	−
1255	2630.4	3.1	−1699.8	83644.0	−
1256	2625.7	−3.2	−1707.7	−	−
1821	4.2	−3561.3	−6132.9	−	−
1822	−0.5	−3567.6	−6140.8	−	−

with $Q_A = 84$ instead of $Q_D = 78$ until c_{dR} reaches to 145. After that, disregarding randomness results more profit for him by using $Q_D = 78$. Therefore, percentage difference between accurately calculated profits, $\triangle_A^R$, decreases.

- When randomness is incorporated into the model, the remanufacturer operates until $c_{dR} = 1038$. It is important that when c_{dR} changes between 1038 and 1256, the remanufacturer actually makes a profit without incorporation of randomness while he loses profit with incorporation of randomness where $Q_A = 84$ is used. Therefore, for this range of c_{dR}, the remanufacturer could make profit disregarding randomness although the error $\triangle_D^R$ is high.

- When randomness is disregarded, the remanufacturer does not operate when $c_{dR} > 1821$. On the other hand, his accurately calculated profit, $\pi_A^R(Q_D)$, takes negative values after $c_{dR} = 1255$. Therefore, the remanufacturer expects to make a profit while he actually loses profit when c_{dR} is between 1255 and 1822.

- The error by ignoring randomness, $\triangle_D^R$, increases significantly until $c_{dR} = 1256$ where the accurately calculated expected profit, $\pi_A^R(Q_D)$, takes a negative value. After $c_{dR} > 1256$, disregarding randomness causes the remanufacturer to incur a loss and $\triangle_D^R$ decreases. However, it is still very high regarding to the significant effect of a change in c_{dR} on randomness. As a result, the effect of randomness on the remanufacturer is strongly affected from a change in the value of c_{dR}.

Next, we analyze the effects of cost parameters that affect directly collector's profit for both models with and without incorporation of randomness, but affect only remanufacturer's profit that is calculated with incorporation of randomness. These parameters are the unit collection cost c_o, unit inspection cost, c_i, unit disposal cost of the collector, c_{dC}, and unit transportation cost, c_t. When randomness is disregarded, a change in any of these parameters does not affect the collection quantity, Q_D and, the remanufacturer's profits, $\pi_D^R(Q_D)$ and $\pi_A^R(Q_D)$ remain constant. On the other hand, when randomness is taken into account, the optimal collection quantity, Q_A changes and the related remanufacturer's profit, $\pi_A^R(Q_A)$, also changes with an increase in these parameters. The impact of these parameters on both parties' and the system, and the percentage differences are analyzed separately.

Firstly, the effects of an increase in the unit collection cost c_o, and unit inspection cost, c_i, are analyzed together. The collector sorts all collected items before he sends the items sorted as remanufacturable to the remanufacturer. Therefore, he charges both unit collection cost and unit inspection cost for any item collected. We analyse the effects of a change in c_o on the profits and the percentage differences, but the analysis for c_i is omitted since the results for c_o are also valid for c_i. The value of c_o is increased from 0 to 100 with a stepsize of 5 and a subset of the results is given in Table 4.20 which shows the computations for the case where the collection quantities are determined by ignoring randomness. Findings with respect to an increase in c_o are summarized below:

- The collector operates until c_o is equal to 26 by disregarding randomness. However, his accurately calculated expected profit by using $Q_D = 78$ takes negative values after $c_o = 22$. It shows that when the value of c_0 is between 22 and 26, he expects to make a profit by ignoring randomness, but he actually incurs a loss.

Table 4.20: Comparison of Profits with and without Incorporation of Randomness when c_o Increases

c_o	Q_D	$\pi_D^C(Q_D)$	$\pi_D^R(Q_D)$	$\pi_D^T(Q_D)$	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_D^C$	$\triangle_D^R$	$\triangle_D^T$
0	78	1957.0	8453.6	10410.6	1762.9	7906.6	9669.5	11.0	6.9	7.7
5	78	1567.0	8453.6	10020.6	1372.9	7906.6	9279.5	14.1	6.9	8.0
10	78	1177.0	8453.6	9630.6	982.9	7906.6	8889.5	19.7	6.9	8.3
20	78	397.0	8453.6	8850.6	202.9	7906.6	8109.5	95.7	6.9	9.1
22	78	241.0	8453.6	8694.6	46.9	7906.6	7953.5	414.0	6.9	9.3
23	78	163.0	8453.6	8616.6	-31.1	7906.6	7875.5	–	–	–
25	78	7.0	8453.6	8460.6	-187.1	7906.6	7719.5	–	–	–
26	78	-71.0	8453.6	8382.6	-265.1	7906.6	7641.5	–	–	–

- The error caused by ignoring randomness for the collector, $\triangle_D^C$, is highly dependent on a change in c_o. After $c_o > 10$, he highly overestimates the accurate profit without incorporation of randomness. On the other hand, the error for the remanufacturer remains constant since both $\pi_D^R(Q_D)$ and $\pi_A^R(Q_D)$ are not dependent on c_o.

Table 4.21 summarizes the accurate results as c_o increases. Observations are listed below:

- The collected quantity, Q_A, determined with incorporation of randomness decreases in order to reduce the effect of the increase in c_o. Until $c_o = 11$, the percentage loss of profit for the collector, $\triangle_A^C$, decreases, and after that it increases significantly. Especially, when c_0 is larger than 22, the percentage difference, $\triangle_A^C$, is higher than fifty percent. For the remanufacturer and the system, the percentage differences decrease up to $c_o = 11$. After that, they decreases below zero. It means that the remanufacturer and system actually make higher profit without incorporation of randomness when $c_o > 11$.

- The collector's accurate profit, $\pi_A^C(Q_A)$, calculated by using $Q_A = 68$ is positive at $c_o = 23$ whereas $\pi_A^C(Q_D)$, calculated by using $Q_D = 78$ is negative. Therefore, he makes a profit by incorporating randomness, but he actually incurs a loss by disregarding randomness at this point.

Table 4.21: Optimal Collection Quantities and the Related Accurate Profits when c_o Increases

c_o	Q_A	$\pi_A^C(Q_A)$	$\pi_A^R(Q_A)$	$\pi_A^T(Q_A)$	Q_D	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_A^C$	$\triangle_A^R$	$\triangle_A^T$
0	90	1823.8	8256.3	10080.2	78	1762.9	7906.6	9669.5	3.3	4.2	4.1
5	84	1390.9	8129.7	9520.7	78	1372.9	7906.6	9279.5	1.3	2.7	2.5
10	79	983.7	7951.5	8935.2	78	982.9	7906.6	8889.5	0.1	0.6	0.5
11	78	**904.9**	**7906.6**	**8811.5**	78	**904.9**	**7906.6**	**8811.5**	**0.0**	**0.0**	**0.0**
20	71	233.7	7499.9	7733.6	78	202.9	7906.6	8109.5	13.2	−5.4	−4.9
22	69	93.9	7354.9	7448.9	78	46.9	7906.6	7953.5	50.1	−7.5	−6.8
23	68	25.5	7278.1	7303.6	78	−31.1	7906.6	7875.5	−	−	−
24	67	−41.9	7198.4	7156.5	78	−109.1	7906.6	7797.5	−	−	−

We next discuss the effects of a change in the value of unit transportation cost, c_t on the results. The value of c_t is increased from 0 to 100 with a stepsize of 5 and Table 4.22 provides a subset of the results for the case where the order quantities are determined without incorporation of randomness. Our observations follow:

- The collector expects to make a profit until c_t reaches to 58 by disregarding randomness, but he actually incurs a loss when c_t is between 51 and 58. The error caused by ignoring randomness, $\triangle_D^C$, changes significantly in c_t and hence, the randomness impact on the collector very sensitive to a change in the unit transportation cost.

Table 4.22: Comparison of Profits with and without Incorporation of Randomness when c_t Increases

c_t	Q_D	$\pi_D^C(Q_D)$	$\pi_D^R(Q_D)$	$\pi_D^T(Q_D)$	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_D^C$	$\triangle_D^R$	$\triangle_D^T$
0	78	1715.2	8453.6	10168.8	1521.1	7906.6	9427.7	12.8	6.9	7.9
5	78	1567.0	8453.6	10020.6	1372.9	7906.6	9279.5	14.1	6.9	8.0
10	78	1418.8	8453.6	9872.4	1224.7	7906.6	9131.3	15.8	6.9	8.1
20	78	1122.4	8453.6	9576.0	928.3	7906.6	8834.9	20.9	6.9	8.4
35	78	677.8	8453.6	9131.4	483.7	7906.6	8390.3	40.1	6.9	8.8
50	78	233.2	8453.6	8686.8	39.1	7906.6	7945.7	496.6	6.9	9.3
51	78	203.6	8453.6	8657.2	9.5	7906.6	7916.1	2053.8	6.9	9.4
52	78	173.9	8453.6	8627.5	−20.2	7906.6	7886.4	−	−	−
57	78	25.7	8453.6	8479.3	−168.4	7906.6	7738.2	−	−	−
58	78	−3.9	8453.6	8382.6	−198.0	7906.6	7708.6	−	−	−

- Since the collection quantity, Q_D, does not change, the related expected remanufacturer's profits, $\pi_D^R(Q_D)$ and $\pi_A^R(Q_D)$, also do not change. Therefore, the difference between $\pi_D^R(Q_D)$ and $\pi_A^R(Q_D)$ remains constant and the percentage difference, $\triangle_D^R$, is the same at any value of c_t. For the system, both $\pi_D^T(Q_D)$ and $\pi_A^T(Q_D)$ decrease since the related collector's profits decrease and the error, $\triangle_D^T$, increases in slight manner.

Table 4.23 shows accurately calculated profits, $\pi_A^i(Q_D)$ and $\pi_A^i(Q_A)$, when c_t increases with both Q_A and Q_D. We list our findings as follows:

- The collector collects less items in order to diminish the effect of the increase in the transportation cost. Until $c_t = 23$, the collection quantity determined with incorporation randomness Q_A is larger than Q_D, and so $\pi_A^R(Q_A)$, is larger than $\pi_A^R(Q_D)$. Therefore, the remanufacturer makes more profit with incorporation of randomness by using Q_A instead of Q_D. After that, disregarding randomness does not hurt the remanufacturer. For the collector, the percentage loss of profit, $\triangle_A^C$, changes significantly and it is very high at larger values of c_t.

Table 4.23: Optimal Collection Quantities and the Related Accurate Profits when c_t Increases

c_t	Q_A	$\pi_A^C(Q_A)$	$\pi_A^R(Q_A)$	$\pi_A^T(Q_A)$	Q_D	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_A^C$	$\triangle_A^R$	$\triangle_A^T$
0	86	1552.0	8181.3	9733.4	78	1521.1	7906.6	9427.7	2.0	3.4	3.1
5	84	1390.9	8129.7	9520.7	78	1372.9	7906.6	9279.5	1.3	2.7	2.5
10	82	1233.5	8067.4	9300.8	78	1224.7	7906.6	9131.3	0.7	2.0	1.8
20	79	928.4	7951.5	8879.9	78	928.3	7906.6	8834.9	0.0	0.6	0.5
23	78	**839.4**	**7906.6**	**8746.0**	78	**839.4**	**7906.6**	**8746.0**	**0.0**	**0.0**	**0.0**
35	74	494.1	7694.0	8188.0	78	483.7	7906.6	8390.3	2.1	-2.8	-2.5
50	69	87.1	7355.0	7442.0	78	39.1	7906.6	7945.7	55.1	-7.5	-6.8
51	68	60.9	7278.1	7338.9	78	9.5	7906.6	7916.1	84.5	-8.6	-7.9
52	68	35.0	7278.1	7313.1	78	-20.2	7906.6	7886.4	–	–	–
53	68	9.2	7278.1	7287.3	78	-49.8	7906.6	7856.8	–	–	–
54	67	-16.5	7198.4	7182.0	78	-79.5	7906.6	7827.1	–	–	–

- It is also observed that the collector's accurate profit, $\pi_A^C(Q_A)$, is positive until $c_t = 54$, but $\pi_A^C(Q_D)$ is positive until $c_t = 52$. It says that when the collector operates at $c_t = 52$ and $c_t = 53$, he incurs a loss without incorporation of randomness. However, he can make some profit with incorporation of randomness into the model at these values of c_t.

Lastly, the effects of a change in the value of collector's unit disposal cost, c_{dC}, on the expected profits and percentage differences are analyzed. Its value is increased from 0 to 50 with a stepsize of 5. For the selected range of c_{dC}, the computations are provided in Table 4.24 and Table 4.25. We present our observations regarding to an increase in c_{dC} as follows:

- The expected number of items sorted as non-remanufacturable by the collector are equal for both models with and without incorporation of randomness. Therefore, the expected disposal cost for the collector in both models are equal. The collector assumes to gain until $c_{dC} = 33$, but actually he loses some profit when c_{dC} is between 28 and 33 by disregarding randomness. It is observed that $\triangle_D^C$, changes significantly. Hence, the impact of randomness on the collector is highly affected from a change in c_{dC}. Moreover, the percentage difference for the remanufacturer, $\triangle_D^R$, remains the same and the error for the system, $\triangle_D^T$, decreases in slight manner regarding to a decrease in the collector's profit.

Table 4.24: Comparison of Profits with and without Incorporation of Randomness when c_{dC} Increases

c_{dC}	Q_D	$\pi_D^C(Q_D)$	$\pi_D^R(Q_D)$	$\pi_D^T(Q_D)$	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_D^C$	$\triangle_D^R$	$\triangle_D^T$
0	78	1567.0	8453.6	10020.6	1372.9	7906.6	9279.5	14.1	6.9	8.0
5	78	1325.2	8453.6	9778.8	1131.1	7906.6	9037.7	17.2	6.9	8.2
10	78	1083.4	8453.6	9537.0	889.3	7906.6	8795.9	21.8	6.9	8.4
20	78	599.8	8453.6	9053.4	405.7	7906.6	8312.3	47.8	6.9	8.9
28	78	212.9	8453.6	8666.5	18.8	7906.6	7925.4	1031.9	6.9	9.4
29	78	164.6	8453.6	8618.2	−29.6	7906.6	7877.1	–	–	–
30	78	116.2	8453.6	8569.8	−77.9	7906.6	7828.7	–	–	–
32	78	19.5	8453.6	8473.1	−174.6	7906.6	7732.0	–	–	–
33	78	−28.9	8453.6	8424.7	−223.0	7906.6	7683.6	–	–	–

- For the collector the percentage loss of profit, $\triangle_A^C$, is small by using Q_D instead of Q_A at small values of c_{dC}. However, it increases significantly in c_{dC}, and he can gain much more profit when randomness is incorporated into the model at higher values of c_{dC}. For the remanufacturer, when $c_{dC} < 10$, he losses some profit by using Q_D rather than Q_A, but after this point he makes more profit when randomness is disregarded.

Table 4.25: Optimal Collection Quantities and the Related Accurate Profits when c_{dC} Increases

c_{dC}	Q_A	$\pi_A^C(Q_A)$	$\pi_A^R(Q_A)$	$\pi_A^T(Q_A)$	Q_D	$\pi_A^C(Q_D)$	$\pi_A^R(Q_D)$	$\pi_A^T(Q_D)$	$\triangle_A^C$	$\triangle_A^R$	$\triangle_A^T$
0	84	1390.9	8129.7	9520.7	78	1372.9	7906.6	9279.5	1.3	2.7	2.5
5	81	1135.8	8031.8	9167.6	78	1131.1	7906.6	9037.7	0.4	1.6	1.4
10	78	**889.3**	**7906.6**	**8795.9**	78	**889.3**	**7906.6**	**8795.9**	**0.0**	**0.0**	**0.0**
20	73	420.6	7632.6	8053.2	78	405.7	7906.6	8312.3	3.6	−3.6	−3.2
28	69	69.1	7355.0	7424.1	78	18.8	7906.6	7925.4	72.8	−7.5	−6.8
29	68	26.9	7278.1	7304.9	78	−29.5	7906.6	7877.1	–	–	–
30	67	−15.1	7198.4	7183.3	78	−77.9	7906.6	7828.7	–	–	–

- The collector operates until c_{dC} is equal to 30 with incorporation of randomness. On the other hand, he actually loses profit by disregarding randomness after $c_{dC} = 28$. Therefore, when he operates at $c_{dC} = 29$, he can make a profit by using Q_A while he loses some profit by using Q_D.

As a conclusion, we present the behaviors of the collection quantities and expected profits with respect to an increase in each parameter in Table 4.26. The plus sign, '$+$', shows an increase in the related performance measure while the sign '$-$' shows a decrease in the related performance measure. Moreover, if any performance measure is not affected from an increase in the value of the related parameter, the effect is represented with the symbol '$\longleftrightarrow$'.

Table 4.26: Effects of Parameters on the Collection Quantities and Expected Profits

	D	c_o	c_i	c_t	c_{dC}	c_{dis}	c_{dR}	c_r	w	p_r	b_0	b	q	α	β
Q_D	$+$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$-$	$-$	$\longleftrightarrow$
Q_A	$+$	$-$	$-$	$-$	$-$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$+$	$\longleftrightarrow$	$+$	$\longleftrightarrow$	$-$	$-$	$-$
$\pi_D^C(Q_D)$	$+$	$-$	$-$	$-$	$-$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$+$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$+$	$-$	$-$
$\pi_D^R(Q_D)$	$+$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$-$	$-$	$-$	$-$	$+$	$\longleftrightarrow$	$\longleftrightarrow$	$+$	$-$	$-$
$\pi_D^T(Q_D)$	$+$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$\longleftrightarrow$	$+$	$\longleftrightarrow$	$\longleftrightarrow$	$+$	$-$	$-$
$\pi_A^C(Q_D)$	$+$	$-$	$-$	$-$	$-$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$+$	$\longleftrightarrow$	$-$	$\longleftrightarrow$	$+$	$-$	$-$
$\pi_A^R(Q_D)$	$+$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$-$	$-$	$-$	$-$	$+$	$+$	$-$	$+$	$-$	$-$
$\pi_A^T(Q_D)$	$+$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$\longleftrightarrow$	$+$	$\longleftrightarrow$	$-$	$+$	$-$	$-$
$\pi_A^C(Q_A)$	$+$	$-$	$-$	$-$	$-$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$+$	$\longleftrightarrow$	$-$	$\longleftrightarrow$	$+$	$-$	$-$
$\pi_A^R(Q_A)$	$+$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$-$	$+$	$-$	$-$
$\pi_A^T(Q_A)$	$+$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$-$	$+$	$-$	$-$

In Table 4.27, we also summarize how the impact of randomness on both parties' and the system are affected by the changes in each parameter while others are kept constant. Note that we use $\triangle_D$ to measure the error caused by disregarding randomness and $\triangle_A$ to present the percentage loss of accurate profit resulted by using Q_D instead of Q_A. If the amount of change in the errors is smaller than or equal to 5.0%, we assume that the impact of randomness is low. If the amount of change in the errors is between 5.0% and 15.0%, than its impact is assumed as moderate. Otherwise, its impact on the profits is assumed to be high.

Table 4.27: Effects of Parameters on the Randomness

	D	c_o	c_i	c_t	c_{dC}	c_{dis}	c_{dR}	c_r	w	p_r	b_0	b	q	α	β
$\triangle_D^C$	high	high	high	high	high	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	high	$\longleftrightarrow$	moderate	$\longleftrightarrow$	high	high	low
$\triangle_D^R$	moderate	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	high	high	high	low	high	low	high	low	low	low
$\triangle_D^T$	moderate	low	low	low	low	high	high	moderate	$\longleftrightarrow$	moderate	$\longleftrightarrow$	high	moderate	low	low
$\triangle_A^C$	low	high	high	high	high	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	high	$\longleftrightarrow$	moderate	$\longleftrightarrow$	high	high	low
$\triangle_A^R$	low	moderate	moderate	moderate	moderate	high	high	high	moderate	high	low	high	moderate	moderate	low
$\triangle_A^T$	low	moderate	moderate	moderate	moderate	high	high	low	low	low	low	high	moderate	moderate	low

CHAPTER 5

CONCLUSION

In remanufacturing systems, items collected from the end-users have uncertain quality and they need to be inspected in order to determine the appropriated recovery options and to decide whether they are suitable for remanufacturing. This uncertain quality results a difference in related operating costs and remanufacturing operation times, and hence it affects the pricing decisions, optimal collection and remanufacturing quantities and so the agents' and system profits. Therefore, sorting of used items and categorizing them in true categories regarding to some quality based metrics are the key issues to achieve a successful product recovery for remanufacturing firms. However, accurate sorting methods require high technology and complete disassembly of all collected items, and thus they are expensive and time consuming. For this reason, the firms have focused on fast and not accurate sorting techniques without complete disassembly process. These inaccurate methods, however, might result in some classification errors. The quality of items can be either overestimated or underestimated and items may be misclassified in better or lower quality classes. In the literature, perfect sorting processes have been recently addressed by many papers, but there is little attention on the imperfect sorting methods with classification errors.

In this study, we have considered a supply chain that includes two independent agents: a remanufacturer and a collector under deterministic market demand in a single period context. The collector acquires used items from end-users and the remanufacturer is responsible for disassembly and remanufacturing processes. The fraction of remanufacturables in the collected lot is known with certainty and the demand is only served from remanufactured products. It is supposed that items are sorted imperfectly before disassembly and there are two types of item category: remanufacturables which are disassembled and move into further remanufacturing operation and non-remanufacturables which are disposed before remanufacturing process. Because of inaccurate inspection, the items sorted as remanufacturable can be non-remanufacturable, and items sorted as non-remanufacturable can be remanufacturable. These errors occur with α and β probability, respectively. The decentralized setting with deterministic demand under inaccurate inspection by Gu and Tagaras (2014) is highly related to our work. However, there is an important aspect that our study differs from their work. They discuss only the effects of randomness in the collected item quality while randomness in the inspection process is disregarded. On the other hand, we focus on the uncertainty in the inspection process related with the classification errors in addition to the uncertainty in used item quality, and analyze the impact of disregarding randomness in the sorting process on the optimal solution and profits of both parties and the system. We also evaluate how this randomness effect depends on problem parameters. To our knowledge, this is the first study that studies randomness in the inspection process and the effects of ignoring it on the results are analyzed.

Our analysis has two important aspects. First of all, we developed different settings based on the decentralized model under deterministic demand provided in Gu and Tagaras (2014). We call this model as the base model in this study. In the base model the remanufacturer is the leader and the collector is responsible for inspection process. There are two different decision variables which are the collector's collection quantity and the remanufacturer's order size. Firstly, we analyzed the base model by discussing their results in detail. Then, we developed seven different models based on their setting. In the first setting, remanufacturer takes the responsibility of sorting and there is one decision variable: the remanufacturer's order quantity. This is actually centralized version of the base model. In Model II and Model III, price sensitive supply version of the base model was considered where the remanufacturer is responsible for sorting unlike the base model. In these two models, the transfer price is assumed to be exogenous. The impact of central sorting and the change in the roles of the agents on the optimal solution were discussed. In Model IV and V, the price dependent demand of remanufactured products was also incorporated into Model II and Model III. Lastly, both price sensitive demand and the supply were incorporated into the base model in Model VI and Model VII. All models were compared with each other and the base model, and our findings were presented. The observations are summarized below:

- In Model I, the sorting activity is performed by the remanufacturer centrally unlike the base model. When the selling price large enough to operate for both the base model and Model I, the optimal value of the collection quantity does not change due to the deterministic demand. For the remanufacturer, however, the expected marginal cost for ordering one more additional unit in the base model is smaller than Model I. Hence, he can make a profit at smaller values of selling price in the base model while he does not make any profit in Model I.

- When comparing Model II and Model III where the remanufacturer versus the collector is leader, respectively incorporation of the price sensitive supply does not affect the remanufacturer's decision since the demand is constant. The optimal value of acquisition price also are the same for Model II and Model III as they are modelled under exogenous transfer price.

- Model IV and Model V were constructed by incorporating price sensitive demand into Model II and Model III, respectively. The optimal values of acquisition fee and selling price are the same for Model IV and Model V. Moreover, the acquisition price in these models is the same as in Model II and Model III, but the remanufacturer's optimal order quantity is different from Model II and Model III. In Model IV and Model V, the remanufacturer can manipulate the demand by changing the value of selling price and he exactly makes the price dependent demand to be equal to the actual number of remanufacturables sent by the collector to him.

- In Model VI, the base model was reconstructed under price sensitive supply case and we observed that the remanufacturer's decision is the same as the base model because of deterministic demand. Lastly Model VII was modelled as an extension of the sixth model by introducing the price sensitive demand. The remanufacturer sets the price sensitive demand to be equal to the actual quantity of remanufacturables sent by the collector.

In the second part of our study, we reformulated the base model by taking randomness into account and the related optimal collection quantity and profits were calculated. The effects of disregarding randomness in the collector's inspection process for the base model were detected by comparing the expected and accurate results. We also studied on how the impact of disregarding randomness on the collection quantity and the profits depend on changes in the problem parameters. The observations via extensive computational study are presented as follows:

- When randomness is disregarded, the expected profits overestimate accurately calculated profits as expected. The collector faces the uncertainty in the sorted item quality because of the classification errors in the sorting process in addition to the uncertain quality of used items. The remanufacturer, on the other hand, only faces the uncertainty regarding to the quality of items transported by the collector. Hence, ignoring randomness in the collector's inspection process has more impact on the collector than the remanufacturer.
- The error caused by disregarding randomness for the collector depends on a change in the demand, unit collection cost, unit inspection cost, unit transportation cost, unit disposal cost of the collector, transfer price, the fraction of the remanufacturables in the collected lot and the value of α. On the other hand, any change in the unit shortage cost, which the collector charges if the remanufacturer's order is not satisfied, and β do not result in significant deviation for the collector's profit. Moreover, a change in the value of unit disassembly cost, unit disposal cost for the remanufacturer, remanufacturing cost, selling price and the unit penalty cost for unsatisfied demand do not affect both collector's profits calculated with and without incorporation of randomness by using the optimal collection quantity found disregarding randomness.
- For the remanufacturer and the system, the errors result from disregarding randomness are highly affected from a change in values of unit disassembly cost, unit disposal cost of the remanufacturer and the unit penalty cost for an unsatisfied demand.
- We also investigated how much accurate profit is lost by using the optimal collection quantity determined disregarding randomness rather than incorporating

randomness. For the collector, the profit loss gets higher with respect to an increase in the unit collection cost, unit inspection cost, unit transportation cost, unit disposal cost and α. The important point is that the error caused by disregarding randomness significantly changes in demand, unit transfer price and the fraction of the remanufacturables in the collected lot while the percentage loss of accurate profit is not highly affected with respect to an increase in these parameters. This shows that when any of these parameters increases, the collector highly overestimates his accurate profit by ignoring randomness in the sorting process. However, the increase in accurate profit determined ignoring randomness is small when randomness is taken into account.

- For the remanufacturer, the percentage loss between accurately calculated profits changes importantly regarding to an increase in values of unit disassembly cost and unit disposal cost. This observation says that high values of the unit penalty cost for unsatisfied demand result for the remanufacturer highly overestimated profits when randomness is ignored. However, the percentage loss of profit between accurately calculated remanufacturer profits does not change considerably with an increase in the unit penalty cost. Thus, he does not increase his accurate profit so much by using the optimal collection quantity calculated with taking randomness into account.

In this study, we have developed and analyzed models under different scenarios in order to discuss the effects of channel's leadership, local and central sorting, price sensitive demand and supply on the optimal solution with inaccurate inspection. We have also made a detailed analysis to assess the impact of disregarding randomness on the profits and the optimal collection quantity. An interesting extension would be the analysis of these systems under stochastic demand with imperfect inspection and then to solve and compare the settings regarding to the optimal solutions and detect the effect of randomness on the results. Another area for future research may be to work with stochastic fraction of remanufacturable items in the collected lot. For future studies, our work can also be extended with the consideration of more than two quality classes of the used items and also multiple usage options for items after disassembly can be considered such as remanufacturing, selling for part or material recovery, using for recycling processes, repairing etc. In all models, transfer price was assumed to be exogenous and it can be relaxed to be a decision variable, and the cases where the transfer price is set by the collector versus remanufacturer can also be compared. The effects of the uncertainty in the transfer price and the change in the decision maker on the results may lead to interesting extensions of our work. In our study, we searched the collection quantity which maximizes the collector's profit, and analyzed the parameter sensitivity in order to observe the behaviors of this collection quantity and related profits. Our work can be also extended in a case where the collection quantity which maximizes the remanufacturer's profit is determined and the behaviours of the related profits and randomness impact on both parties and the system can be observed with respect to changes in the parameters.

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APPENDIX A

DETAILED PROOF OF LEMMA 1

For $\Pi_C(Q_0)$ to be a continuous function at $Q_0 = \frac{Q_R}{(1-\alpha)q}$,

$$\lim_{Q_0 \to \left(\frac{Q_R}{(1-\alpha)q}\right)^-} \Pi_C(Q_0) = \lim_{Q_0 \to \left(\frac{Q_R}{(1-\alpha)q}\right)^+} \Pi_C(Q_0) = \Pi_C\left(\frac{Q_R}{(1-\alpha)q}\right) \text{ should hold.}$$

$$\begin{aligned}
\lim_{Q_0 \to \left(\frac{Q_R}{(1-\alpha)q}\right)^-} \Pi_C(Q_0) &= \Pi_C^I\left(\frac{Q_R}{(1-\alpha)q}\right) \\
&= wQ_R - \frac{Q_R}{(1-\alpha)q}\begin{pmatrix} c_0 + c_i \\ +c_{dC}[\alpha q + (1-\beta)(1-q)] \\ +c_t[(1-\alpha)q + \beta(1-q)] \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\lim_{Q_0 \to \left(\frac{Q_R}{(1-\alpha)q}\right)^+} \Pi_C(Q_0) &= \lim_{Q_0 \to \left(\frac{Q_R}{(1-\alpha)q}\right)^+} \Pi_C^{II}(Q_0) \\
&= wQ_R - \frac{Q_R}{(1-\alpha)q}\begin{pmatrix} c_0 + c_i \\ +c_{dC}[\alpha q + (1-\beta)(1-q)] \\ +c_t[(1-\alpha)q + \beta(1-q)] \end{pmatrix}
\end{aligned}$$

$$\Pi_C\left(\frac{Q_R}{(1-\alpha)q}\right) = wQ_R - \frac{Q_R}{(1-\alpha)q}\begin{pmatrix} c_0 + c_i + c_{dC}[\alpha q + (1-\beta)(1-q)] \\ +c_t[(1-\alpha)q + \beta(1-q)] \end{pmatrix}$$

$$\lim_{Q_0 \to \left(\frac{Q_R}{(1-\alpha)q}\right)^-} \Pi_C(Q_0) = \lim_{Q_0 \to \left(\frac{Q_R}{(1-\alpha)q}\right)^+} \Pi_C(Q_0) = \Pi_C\left(\frac{Q_R}{(1-\alpha)q}\right) \text{ holds.}$$

Hence, $\Pi_C(Q_0)$ is continuous at $Q_0 = \frac{Q_R}{(1-\alpha)q}$.

For $\Pi_C(Q_0)$ to be differentiable at $Q_0 = \frac{Q_R}{(1-\alpha)q}$, the following left-hand and right-hand limits should exist and should be equal to each other.

$$\lim_{h \to 0^-} \frac{\Pi_C\left(\frac{Q_R}{(1-\alpha)q} + h\right) - \Pi_C\left(\frac{Q_R}{(1-\alpha)q}\right)}{h}$$

$$= \frac{h\begin{pmatrix} (w + b_0 - c_t)(1-\alpha)q + (c_{dC} - c_t)\beta(1-q) - c_{dC}[\alpha q + (1-q)] \\ -(c_0 + c_i) \end{pmatrix}}{h}$$

$$= (w + b_0 - c_t)(1-\alpha)q + (c_{dC} - c_t)\beta(1-q) - c_{dC}[\alpha q + (1-q)]$$
$$-(c_0 + c_i)$$

$$\lim_{h \to 0^+} \frac{\Pi_C\left(\frac{Q_R}{(1-\alpha)q} + h\right) - \Pi_C\left(\frac{Q_R}{(1-\alpha)q}\right)}{h}$$

$$= \frac{h\Big(-c_0 - c_i - c_{dC}[\alpha q + (1-\beta)(1-q)] - c_t[(1-\alpha)q + \beta(1-q)]\Big)}{h}$$

$$= (-c_t)(1-\alpha)q + (c_{dC} - c_t)\beta(1-q) - c_{dC}[\alpha q + (1-q)] - (c_0 + c_i)$$

Left- and right-hand limits exist, but they are not equal unless $w + b_0 = 0$. Hence, $\Pi_C(Q_0)$ is not differentiable at $Q_0 = \frac{Q_R}{(1-\alpha)q}$.

APPENDIX B

DETAILED PROOF OF LEMMA 2

For $\Pi_R(Q_R)$ to be continuous at $Q_R = D$,

$$\lim_{Q_R \to D^-} \Pi_R(Q_R) = \lim_{Q_R \to D^+} \Pi_R(Q_R) = \Pi_R(D) \text{ should hold.}$$

$$\begin{aligned}
\lim_{Q_R \to D^-} \Pi_R(Q_R) &= \Pi_R^I(D) \\
&= (p_r - c_r - w)D - \frac{D}{(1-\alpha)q} \begin{pmatrix} c_{dis}[(1-\alpha)q + \beta(1-q)] \\ +c_{dR}\beta(1-q) \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\lim_{Q_R \to D^+} \Pi_R(Q_R) &= \lim_{Q_R \to D^+} \Pi_R^{II}(D) \\
&= (p_r - c_r - w)D - \frac{D}{(1-\alpha)q} \begin{pmatrix} c_{dis}[(1-\alpha)q + \beta(1-q)] \\ +c_{dR}\beta(1-q) \end{pmatrix}
\end{aligned}$$

$$\Pi_R(D) = (p_r - c_r - w)D - \frac{D}{(1-\alpha)q} \begin{pmatrix} c_{dis}[(1-\alpha)q + \beta(1-q)] \\ +c_{dR}\beta(1-q) \end{pmatrix}$$

$\lim_{Q_R \to D^-} \Pi_R(Q_R) = \lim_{Q_R \to D^+} \Pi_R(Q_R) = \Pi_R(D)$ holds. Hence, $\Pi_R(Q_R)$ is continuous at $Q_R = D$.

To check differentiability of $\Pi_R(Q_R)$ at $Q_R = D$, the following left- and right-hand limits should exist and should be equal to each other.

$$\lim_{h\to 0^-} \frac{\Pi_R(D+h)-\Pi_R(D)}{h}$$

$$= \quad h\frac{\left(p_r - c_r - w - \frac{1}{(1-\alpha)q}\left\{c_{dis}[(1-\alpha)q+\beta(1-q)] + c_{dR}\beta(1-q)\right\} + b\right)}{h}$$

$$= \quad p_r + b - c_r - w - \tfrac{1}{(1-\alpha)q}\left\{c_{dis}[(1-\alpha)q+\beta(1-q)] + c_{dR}\beta(1-q)\right\}$$

$$\lim_{h\to 0^+} \frac{\Pi_R(D+h)-\Pi_R(D)}{h}$$

$$= \quad h\frac{\left(c_R - c_r - w - \frac{1}{(1-\alpha)q}\left\{c_{dis}[(1-\alpha)q+\beta(1-q)] + c_{dR}\beta(1-q)\right\}\right)}{h}$$

$$= \quad c_R - c_r - w - \tfrac{1}{(1-\alpha)q}\left\{c_{dis}[(1-\alpha)q+\beta(1-q)] + c_{dR}\beta(1-q)\right\}$$

Left- and right-hand limits exist, but they are not equal unless $p_r + b - c_R = 0$. Hence, $\Pi_R(Q_R)$ is not differentiable at $Q_R = D$.

APPENDIX C

DETAILED PROOF OF LEMMA 13

The remanufacturer's profit at $(1-\alpha)qQ_0 = a - bp_r$ is:

$$\Pi_R\left(\frac{a-bp_r}{(1-\alpha)q}, p_r\right) = \frac{-(a-bp_r)}{(1-\alpha)q}\begin{pmatrix} c_i + w + c_{dR}[\alpha q + (1-\beta)(1-q)] \\ +c_{dis}[(1-\alpha)q + \beta(1-q)] \\ +c_{dR}\beta(1-q) \end{pmatrix}$$

$$+(p_r - c_r)(a - bp_r)$$

Now, consider any solution with $(1-\alpha)qQ_0 > a - bp_r$. Let $\epsilon = (1-\alpha)qQ_0 - (a - bp_r)$. Then, the remanufacturer's profit at this solution is:

$$\Pi_R\left(\frac{a-bp_r+\epsilon}{(1-\alpha)q}, p_r\right) = \frac{-(a-bp_r+\epsilon)}{(1-\alpha)q}\begin{pmatrix} c_i + w \\ +c_{dR}[\alpha q + (1-\beta)(1-q)] \\ +c_{dis}[(1-\alpha)q + \beta(1-q) \\ +c_{dR}\beta(1-q) \end{pmatrix}$$

$$+(p_r - c_r)(a - bp_r) - c_{dR}\epsilon$$

Then,

$$\Pi_R\left(\frac{a-bp_r+\epsilon}{(1-\alpha)q}, p_r\right) - \Pi_R\left(\frac{a-bp_r}{(1-\alpha)q}, p_r\right)$$

$$= \frac{-\epsilon}{(1-\alpha)q}\begin{pmatrix} c_i + w + c_{dR}[\alpha q + (1-\beta)(1-q)] \\ +c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q) \end{pmatrix} - c_{dR}\epsilon < 0$$

Therefore, for any $\epsilon > 0$ such that $\epsilon = (1-\alpha)qQ_0 - (a - bp_r)$, the remanufacturer's profit $\Pi_R\left(\frac{a-bp_r+\epsilon}{(1-\alpha)q}, p_r\right)$ is smaller than $\Pi_R\left(\frac{a-bp_r}{(1-\alpha)q}, p_r\right)$. Hence, $(1-\alpha)qQ_0 > a-bp_r$ can not be optimal.

Now, consider $(1-\alpha)qQ_0 < a - bp_r$. Let $\epsilon = a - bp_r - (1-\alpha)qQ_0$. Then the remanufacturer's profit can be expressed as:

$$\Pi_R\left(\frac{a-bp_r-\epsilon}{(1-\alpha)q}, p_r\right) = \frac{-(a-bp_r-\epsilon)}{(1-\alpha)q}\begin{pmatrix} c_i + w \\ +c_{dR}[\alpha q + (1-\beta)(1-q)] \\ +c_{dis}[(1-\alpha)q + \beta(1-q)] \\ +c_{dR}\beta(1-q) \end{pmatrix}$$

$$+(p_r - c_r)(a - p_r) - (p_r - c_r)\epsilon$$

Then,

$$\Pi_R\left(\frac{a-bp_r-\epsilon}{(1-\alpha)q}, p_r\right) - \Pi_R\left(\frac{a-bp_r}{(1-\alpha)q}, p_r\right)$$

$$= \frac{\epsilon}{(1-\alpha)q}\begin{pmatrix} c_i + w + c_{dR}[\alpha q + (1-\beta)(1-q)] \\ +c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q) \end{pmatrix}$$

$$-(p_r - c_r)\epsilon < 0$$

Since, $\Pi_R\left(\frac{a-bp_r-\epsilon}{(1-\alpha)q}, p_r\right) - \Pi_R\left(\frac{a-bp_r}{(1-\alpha)q}, p_r\right) < 0$, we deduce that a solution with $(1-\alpha)qQ_0 < a - bp_r$ can not be optimal.

It is concluded that any solution different from $(1-\alpha)qQ_0 = a-bp_r$ results a decrease in the remanufacturer's profit. Hence, $(1-\alpha)qQ_0 = a - bp_r$ is optimal.

APPENDIX D

DETAILED PROOF OF LEMMA 20

For $\Pi_C(Q_0)$ to be continuous at $f = \frac{Q_R}{(1-\alpha)qr}$,

$$\lim_{f \to \left(\frac{Q_R}{(1-\alpha)qr}\right)^-} \Pi_C(f) = \lim_{f \to \left(\frac{Q_R}{(1-\alpha)qr}\right)^+} \Pi_C(f) = \Pi_C\left(\frac{Q_R}{(1-\alpha)qr}\right) \text{ should hold.}$$

$$\begin{aligned}
\lim_{f \to \left(\frac{Q_R}{(1-\alpha)qr}\right)^-} \Pi_C(f) &= \Pi_C^I\left(\frac{Q_R}{(1-\alpha)qr}\right) \\
&= wQ_R - \frac{Q_R}{(1-\alpha)q}\begin{pmatrix} \frac{Q_R}{(1-\alpha)qr} + c_i \\ +c_{dC}[\alpha q + (1-\beta)(1-q)] \\ +c_t[(1-\alpha)q + \beta(1-q)] \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\lim_{f \to \left(\frac{Q_R}{(1-\alpha)qr}\right)^+} \Pi_C(f) &= \lim_{f \to \left(\frac{Q_R}{(1-\alpha)qr}\right)^+} \Pi_C^{II}(f) \\
&= wQ_R - \frac{Q_R}{(1-\alpha)q}\begin{pmatrix} \frac{Q_R}{(1-\alpha)qr} + c_i \\ +c_{dC}[\alpha q + (1-\beta)(1-q)] \\ +c_t[(1-\alpha)q + \beta(1-q)] \end{pmatrix}
\end{aligned}$$

$$\Pi_C\left(\frac{Q_R}{(1-\alpha)qr}\right) = wQ_R - \frac{Q_R}{(1-\alpha)q}\begin{pmatrix} \frac{Q_R}{(1-\alpha)qr} + c_i \\ +c_{dC}[\alpha q + (1-\beta)(1-q)] \\ +c_t[(1-\alpha)q + \beta(1-q)] \end{pmatrix}$$

$$\lim_{f \to \left(\frac{Q_R}{(1-\alpha)qr}\right)^-} \Pi_C(f) = \lim_{f \to \left(\frac{Q_R}{(1-\alpha)qr}\right)^+} \Pi_C(f) = \Pi_C\left(\frac{Q_R}{(1-\alpha)qr}\right). \text{ Hence,}$$

$\Pi_C(Q_0)$ is continuous at $f = \frac{Q_R}{(1-\alpha)qr}$.

For $\Pi_C(f)$ to be differentiable at $f = \frac{Q_R}{(1-\alpha)qr}$, the following left- and right-hand limits should exist and should be equal to each other.

$$\lim_{h\to 0^-} \frac{\Pi_C\left(\frac{Q_R}{(1-\alpha)qr}+h\right)-\Pi_C\left(\frac{Q_R}{(1-\alpha)qr}\right)}{h}$$

$$= \lim_{h\to 0^-} \frac{rh}{h}\left(\begin{array}{l} w(1-\alpha)q-\left(h+\frac{2Q_R}{(1-\alpha)qr}\right)-c_i-c_{dC}[\alpha q+(1-\beta)(1-q)] \\ -c_t[(1-\alpha)q+\beta(1-q)]+b_0(1-\alpha)q \end{array}\right)$$

$$= r\Big((w+b_0-c_t)(1-\alpha)q-(c_t-c_{dC})\beta(1-q)-c_{dC}[\alpha q+(1-q)]-c_i\Big) - \frac{2Q_R}{(1-\alpha)q}$$

$$\lim_{h\to 0^+} \frac{\Pi_C\left(\frac{Q_R}{(1-\alpha)qr}+h\right)-\Pi_C\left(\frac{Q_R}{(1-\alpha)qr}\right)}{h}$$

$$= \lim_{h\to 0^+} \frac{rh}{h}\left(\begin{array}{l} -\left(h+\frac{2Q_R}{(1-\alpha)qr}\right)-c_i-c_{dC}[\alpha q+(1-\beta)(1-q)] \\ -c_t[(1-\alpha)q+\beta(1-q)] \end{array}\right)$$

$$= r\Big(-c_t(1-\alpha)q-(c_t-c_{dC})\beta(1-q)-c_{dC}[\alpha q+(1-q)]-c_i\Big) - \frac{2Q_R}{(1-\alpha)q}$$

Left- and right-hand limits exist, but they are not equal unless $w + b_0 = 0$. Hence, $\Pi_C(Q_0)$ is not differentiable at $Q_0 = \frac{Q_R}{(1-\alpha)q}$.

APPENDIX E

DETAILED PROOF OF LEMMA 22

For $\Pi_R(Q_R)$ to be a continuous function at $Q_R = D$,

$$\lim_{Q_R \to D^-} \Pi_R(Q_R) = \lim_{Q_R \to D^+} \Pi_R(Q_R) = \Pi_R(D) \text{ should hold.}$$

$$\lim_{Q_R \to D^-} \Pi_R(Q_R) = \begin{cases} -c_{dis}[(1-\alpha)q + \beta(1-q)]rf' \\ -c_{dR}\beta(1-q)rf' - w(1-\alpha)qrf' \\ +b_0(D-(1-\alpha)qrf') & \text{if } f'(1-\alpha)qr \leq Q_R, \\ +(p_r - c_r)(1-\alpha)qrf' \\ -b(D-(1-\alpha)qrf') \\ \\ -c_{dis}[(1-\alpha)q + \beta(1-q)]\frac{D}{(1-\alpha)q} \\ -c_{dR}\beta(1-q)\frac{D}{(1-\alpha)q} - wD & o/w \\ +(p_r - c_r)D \end{cases}$$

$$\lim_{Q_R \to D^+} \Pi_R(Q_R) = \begin{cases} -c_{dis}[(1-\alpha)q + \beta(1-q)]rf' \\ -c_{dR}\beta(1-q)rf' - w(1-\alpha)qrf' \\ +b_0(D-(1-\alpha)qrf') & \text{if } f'(1-\alpha)qr \leq Q_R, \\ +(p_r - c_r)(1-\alpha)qrf' \\ -b(D-(1-\alpha)qrf') \\ \\ -c_{dis}[(1-\alpha)q + \beta(1-q)]\frac{D}{(1-\alpha)q} \\ -c_{dR}\beta(1-q)\frac{D}{(1-\alpha)q} - cD & o/w \\ +(p_r - c_r)D \end{cases}$$

$$\Pi_R(D) = \begin{cases} -c_{dis}[(1-\alpha)q+\beta(1-q)]rf' \\ -c_{dR}\beta(1-q)rf' - w(1-\alpha)qrf' \\ +b_0(D-(1-\alpha)qrf') & \text{if } f'(1-\alpha)qr \leq Q_R, \\ +(p_r-c_r)(1-\alpha)qrf' \\ -b(D-(1-\alpha)qrf') \\ \\ -c_{dis}[(1-\alpha)q+\beta(1-q)]\frac{D}{(1-\alpha)q} & o/w \\ -c_{dR}\beta(1-q)\frac{D}{(1-\alpha)q} - wD \\ +(p_r-c_r)D \end{cases}$$

$\lim_{Q_R \to D^-} \Pi_R(Q_R) = \lim_{Q_R \to D^+} \Pi_R(Q_R) = \Pi_R(D)$ holds. Hence, $\Pi_R(Q_R)$ is continuous at $Q_R = D$.

To check differentiability of $\Pi_R(Q_R)$ at $Q_R = D$, the following left- and right-hand limits should exist and should be equal to each other. When $f'(1-\alpha)qr \leq Q_R$, the optimal acquisition price is independent from Q_R. Then the left- and right-hand limits can be expressed as follows:

$$\begin{aligned} \lim_{h \to 0^-} \frac{\Pi_R(D+h) - \Pi_R(D)}{h} &= h\frac{b_0}{h} \\ &= b_0 \end{aligned}$$

$$\begin{aligned} \lim_{h \to 0^+} \frac{\Pi_R(D+h) - \Pi_R(D)}{h} &= h\frac{b_0}{h} \\ &= b_0 \end{aligned}$$

When $f'(1-\alpha)qr > Q_R$, the optimal acquisition price is a function of Q_R. Then left- and right-hand limits can be expressed as follows:

$$\lim_{h \to 0^-} \frac{\Pi_R(D+h) - \Pi_R(D)}{h}$$

$$= h\frac{\left(p_r + b - w - c_r - \frac{1}{(1-\alpha)q}\left\{c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q)\right\}\right)}{h}$$

$$= p_r + b - w - c_r - \frac{1}{(1-\alpha)q}\left\{c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q)\right\}$$

$$\lim_{h \to 0^+} \frac{\Pi_R(D+h) - \Pi_R(D)}{h}$$

$$= h\frac{\left(c_R - w - c_r - \frac{1}{(1-\alpha)q}\left\{c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q)\right\}\right)}{h}$$

$$= c_R - w - c_r - \frac{1}{(1-\alpha)q}\left\{c_{dis}[(1-\alpha)q + \beta(1-q)] + c_{dR}\beta(1-q)\right\}$$

When $f^{'}(1-\alpha)qr \leq Q_R$, the left- and right-hand limits exist and they are equal. When $f^{'}(1-\alpha)qr > Q_R$, the left- and right-hand limits exist but they are not equal unless $p_r + b - c_R = 0$. Hence, $\Pi_R(Q_R)$ is not differentiable at $Q_R = D$.