HIDDEN SECTOR THROUGH DARK HIGGS: A NONMINIMAL EXTENSION

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ABSTRACT

HIDDEN SECTOR THROUGH DARK HIGGS: A NONMINIMAL EXTENSION

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In this thesis, a non-minimal extension of the SM with an additional non-abelian SU(2) symmetry is studied. Hidden sector opens innumerable portals for apprehension of theories beyond the SM, hence there is a wide set of gauge extensions with their own symmetries, providing viable DM candidates. Higgs mechanism and Higgs particle play a crucial role to understand how DM weakly interacts with the SM particles. There may exist unrevealed intensity frontiers in Higgs' decay modes, and the DM can show up through those interaction channels. Although DM has not been observed so far, there are strong evidences and indirect observations for its existence. Imposing these observational constraints on the results of non-minimal extensive model, a consistent region for parameter space of the DM candidates can be expressed via methods of statistical physics. One of the first DM abundance studies has been written by Lee and Weinberg in 1977 by considering evolution of DM after it decouples from the cosmic heat reservoir. But in original work, only annihilation channels of the DM are considered.

However the model can be extended to include coannihilation channels as well, provided that the associated Feynman diagrams are given including dark sector particles. In this study, such extensions will be explored.

Keywords: Non-Abelian Gauge Theory, Hidden Sector, Scalar Portal, Symmetry Breaking, Dark Boson, Dark Matter, Relic Abundance

ÖZ

KARANLIK HİGGS ÜZERİNDEN GİZLİ SEKTÖR: MİNİMAL OLMAYAN BİR UZANTI

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Bu tezde, Standart Model'in abelyen olmayan bir SU(2) simetri uzantısı çalışıldı. Gizli sektör, Standart Model'in ötesindeki teorilerin anlaşılması için sayısız miktarda geçit açmaktadır, bu yüzden de kendi simetrileri dahilinde karanlık madde için aday olan geniş bir ölçü uzantısı var olmaktadır. Higgs mekanizması ve Higgs parçacığı, Karanlık Madde'nin Standart Madde parçacıkları ile nasıl zayıf ölçekte etkileştiğini açıklamak için önemli bir rol oynamaktadır. Karanlık Madde kendini, hâlihazırdaki Higgs parçacığının tamamlanmamış bozunum kanallarından birinde muhtemelen gösterebilir. Her ne kadar Karanlık Madde doğrudan gözlemlenmemiş olsa da, varlığını kanıtlayan kuvvetli kanıtlar ve dolaylı gözlemler mevcuttur. Ve bu gözlemsel sınırlamaları, minimal olmayan genişletilmiş modele uygulayarak, Karanlık Madde parçacıklarının parametreleri hakkında istatistiksel fiziği kullanarak tutarlı bir aralık belirlenebilir. Karanlık Madde kalıntı yoğunluğu hakkında çalışmalardan ilki, Lee ve Weinberg tarafından 1977 yılında, karanlık madde parçacıklarının kozmik ısı kaynağından koptuktan sonraki evrimlerini inceleyerek yazılmıştır. Fakat özgün çalışmada, karanlık maddenin sadece çarpışma kanalları çalışılmıştır.

Lakin modeller, karanlık sektör parçacıklarının ilgili Feynman diagramlarını belirterek, yan çarpışmaları da dahil edecek şekilde genişletilebilir. Bu çalışmada, bu tarz genişletmeler araştırılacaktır.

Anahtar Kelimeler: Abelyen Olmayan Ölçü Teorisi, Gizli Sektör, Skaler Geçit, Simetri Kırılması, Karanlık Bozon, Karanlık Madde, Kalıntı Yoğunluğu

To my family, a unity of dialectical theory and universal action;
for such a relentless struggle against the obstacles in front of my productive force.
Without them, my eternity would remain in darkness.
My true power and worthiness reside in the soil of your labor.

"Güneşin rüzgarına gerilmiş bir badem ağacı gibi...
İçimdeki karanlığı patlatacağım
Ve beynimin en ölümcül yaşlarıyla
ağlaya ağlaya
Yepyeni bir insan
pırıl pırıl bir can bitecek toprağa..."

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Above all else, i would like to deliver my ultimate appreciation to my hero, my mentor and my father. For all the years we have shared together to analyze countless number of universal contradictions and their ultimate effect on the existence of productive obligation. ”Two roads diverged in a wood, and I— ; I took the one less traveled by, And that has made all the difference.”

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LIST OF ABBREVIATIONS

SM	Standard Model
SU(N)	Special Unitary Group with degree of N
GWS	Glashow-Weinberg-Salam
QED	Quantum Electrodynamics
EW	Electroweak Theory
QCD	Quantum Chromodynamics
MEHS	Minimal Extension Hidden Sector
NMEHS	Non-Minimal Extension Hidden Sector
SSB	Spontaneous Symmetry Breaking
NA	Non-Abelian
NADM	Non-Abelian Dark Matter
DM	Dark Matter
DS	Dark Sector
$g_{\mu\nu}$	Mostly Minus Minkowski Metric
EOM	Equations of Motion
VEV	Vacuum Expectation Value
CP	Charge Conjugation, Parity Transformation
PMNS	Pontecorvo–Maki–Nakagawa–Sakata
CKM	Cabibbo–Kobayashi–Maskawa
WMAP	Wilkinson Microwave Anisotropy Probe
CMB	Cosmic Microwave Background
MACHOs	Massive Astrophysical Compact Halo Objects
WIMPs	Weakly Interacting Massive Particles

CDM	Cold Dark Matter
BBN	Big Bang Nucleosynthesis
CM	Center of Mass
LW	Lee-Weinberg
CW	Coleman-Weinberg

CHAPTER 1

INTRODUCTION

For more than a century, dozens of elementary and composite particles have been proposed, modelled and discovered eventually. Countless number of painstaking efforts had been dealt to excavate underlying idea of the fundamental interactions of the universe. Although relativistic quantum field theory had already been formalised, a severe boost came after Gell-Mann's proposal of particle multiplets regardless of the absence of group theory. Countless number of experimental activity have been conducting via colliders day after day, nevertheless theoretical physics has always tried to consolidate experimental studies and also fills the gaps among the fundemental interactions. Studies started with formulation of quantum electrodynamics, which is one of the remarkable outcomes of QFT, inspired further theoretical studies in different aspects of physics. Once the group theory and symmetry properties of particles had been comprehended on behalf of an extended scope, naturally there has been an escalation among physicists to unify symmetry properties and fundamental theories. Einstein's General Theory of Relativity [64] seems to working perfectly on cosmic scales, currently theoretical physis could not propose a consistent mathematical formalism to express gravity in quantum scale. Incompatibility appears due to enormous difference of couplings between gravity and the rest of the fundamental forces and also due to dimensional insufficiency. However, efforts for the quantum gravity scheme have also developed paths for lesser unification models, such as Electroweak Theory and the Grand Unification Theory.

Dynamics of particles are completely determined by the action principle, in which the physics must be invariant with respect to different inertial observers.

Such constraint and dictation for satisfying invariance led to many frameworks which are somehow variant of each other. The SM can still be considered as the best model we have to describe classification of particles and to express the interactions among them. The SM can be mainly considered as gauge field theory, that is combination of group theory and QFT. Hence all basic symmetries of the SM can be considered as gauge symmetries. Cosmic integrity of the most particles are assured by these symmetries. But there are more specifications about the SM particles beside the symmetry properties. One cannot avoid to include their internal properties; which leads to statistical distinction among them. Spin as a pioneer of this 'distinction', plays a crucial role to understand what lies within and beyond.

Furthermore, the SM is not the whole story and far from to be completed. The progress of physics have always left unanswered questions more than it found an explanation to unknown. Hence there remains a set of issues in the SM such as: asymmetry of matter-anti-matter abundance in the universe, origin of the fundamental composition of matter-energy densities like dark matter, dark energy and the SM particle, the Big Bang initial scenario etc.

Consequently, innumerable models have been proposed to extend the SM and try to comprehend what is missing in the theories at current stage. Since we are interested in the Hidden Sector extension models, the SM gauge content will be introduced first via Lagrangian formalism , then we try to extend it with additional symmetry group of 'dark or hidden sector'. Higgs mechanism will play a significant role even in the dark sector. A non-minimal extension of the SM will be introduced via scalar portal. And symmetry breaking mechanism for the extended Lagrangian will reveal one massive scalar boson and three massive vector gauge bosons, namely dark vector bosons as a consequence of additional $SU(2)_d$ gauge symmetry analogous to one in the SM. Moreover these dark gauge bosons will be modelled as dark matter candidates and obviously their internal properties completely depend on the model parameters introduced at the Lagrangian level. Furthermore, absence and presence of additional dark symmetries will support a solid background for DM model to be stable. Then their cosmic abundances left out from Big Bang will be studied via expressions of statistical mechanics.

CHAPTER 2

THE STANDARD MODEL

The Standard Model is a renormalizable gauge field theory constructed with Lie groups $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The SM is classified by bosonic and a fermionic sectors. Bosonic sector is responsible from mediating the three of four interactions in nature. And the fermionic sector is the ingridient of ordinary matter. But the gravity as a fundemental force in the universe cannot be described within the regime of the SM and hence another motivation arises to look for physics beyond it. Firstly the strong interactions of quarks and mediator gluons described by $SU(3)_C$ gauge group. And the second one is the electroweak theory as a unification of weak and electromagnetic interactions of leptons and quarks via by gauge group $SU(2)_L \otimes U(1)_Y$, which was proposed by Glashow-Weinberg-Salam(GWS) model [1, 2, 3]. And the third one is Yukawa interactions mediated by scalar fields ($spin - 0$).

Statistical distinction of fermions and bosons is mainly motivated by their intrinsic quantum numbers. Bosons can occupy the same quantum state and obey Bose-Einstein statistics, whereas fermions cannot occupy in the same state as a result of Pauli exclusion principle, hence they obey Fermi-Dirac statistics. Leptons interact only via electroweak interaction since they have zero color charge. On the other hand, quarks can experience the presence of both electroweak and strong interactions. Fermions interact differently as a result of their helicities, that is handedness, which is the direction of their spin relative to their momenta. Left-handed fermions are $SU(2)$ doublets whereas right-handed fermions are singlets.

Vector gauge bosons in the SM constitue eight massless gluon fields G_μ^a for strong interactions, three massive gauge fields $W_\mu^\pm$, Z^0 for weak interactions and one massless photon field A_μ for electromagnetic interactions.

Underlying idea of the SM Lagrangian formalism and relevant gauge field theory are based on insisting the global phase transformations of each gauge group to hold locally in spacetime. Hence the correction of free-field Lagrangians ends up with a source or current term respectively for each interaction, that ensures the local gauge invariance of the total Lagrangian.

2.1 Quantum Electrodynamics

Quantum electrodynamics(QED) is based on mainly Dirac equation and $U(1)$ local gauge invariance. One of the biggest achievement of QFT led to renormalizable relativistic theory of photons and electrically charged fermions.

To dictate $\mathcal{L}_{free,ferm}$ to hold locally in $U(1)$ phase transformation:

$$\psi' \rightarrow e^{iq\alpha(x)}\psi \tag{2.1}$$

on the fermion field, which is satisfying free field Lagrangian:

$$\mathcal{L}_f = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \tag{2.2}$$

requires $U(1)_{em}$ gauge covariant derivative:

$$D_\mu = \partial_\mu + ieA_\mu \tag{2.3}$$

that fixes the local gauge symmetry of Lagrangian via the kinetic term, whereas the gauge field A_μ satisfies the transformation:

$$A_\mu \rightarrow \ A_\mu + \partial_\mu \lambda(x). \tag{2.4}$$

The final form of the QED Lagrangian is

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{2.5}$$

Local gauge invariance of $U(1)_{em}$ abelian group leads to an interaction term between gauge field A_μ and fermion field ψ is written as[1]

$$J^\mu_{em} = e\bar{\psi}\gamma^\mu Q\psi. \tag{2.6}$$

[1] (For an explicit derivation, see appendix A.8).

Electromagnetic current turns out to be a conserved current in the QED as a consequence of Noether theorem [4]. And the field strength tensor of $U(1)$ group, defines the kinetic term of gauge field

$$F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}. \tag{2.7}$$

Then it becomes apparent why gauge the gauge field A_{μ} has to be massless, since Proca mass term is forbidden by gauge invarience imposed on Lagrangian. And there is no finite region of validity for the local gauge invariance, so the interaction has to extend upto infinity, which assures the mediator to become massless.

In summary, global invariance of fermionic fields is dictated to hold also locally under the given gauge symmetry (2.1). Physical significance of gauge transformations depends on variation from point to point in space. Thus ordinary covariant derivative must be modified to relate the change itself in gauge transformation. On top of that the geometry requires the term A_{μ} to turn into a quantized massless spin-1 field and the corresponding excitation is photon.

Yang-Mills theory [5] had demonstrated that the local gauge symmetries can be extended to larger non-abelian gauge groups. But a various non-trivial problems arise when expressing unified theory of weak interactions and electromagnetism.
$SU(2) \otimes U(1)$ gauge fields must be added into Lagrangian without mass terms to guarantee local gauge invarience of the associated group. The masses of the fermions occurs as another problem since the weak interaction violates parity conservation as a result of coupling variations for different helicity states. From a theoretical point of view, these problems are fixed by the spontaneous symmetry breaking of the related Lagrangian.

2.2 Electroweak Theory

As mentioned earlier, the weak interactions are known to violate parity conservation, and fermion fields are actually combinations of chiral states due to their helicity eigenvalues(± 1)

$$\psi = \psi_L + \psi_R \tag{2.8}$$

with the definiton of the chirality operators:

$$\psi_L = \frac{1-\gamma^5}{2}\psi \quad ; \quad \psi_R = \frac{1+\gamma^5}{2}\psi. \tag{2.9}$$

In the SM, the fermions come as families of left-handed weak-isospin doublets of leptons L_L^i and quarks Q_L^i and also right-handed weak-isospin singlets of leptons $e_R^i(\nu_R^i)$ and quarks $u_R^i(d_R^i)$:

$$L^i = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad ; \quad Q^i = \begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} c \\ s \end{pmatrix}_L , \begin{pmatrix} t \\ b \end{pmatrix}_L \tag{2.10}$$

$$e_R^i = e_R \ , \ \mu_R \ , \ \tau_R \tag{2.11}$$

$$u_R^i = u_R \ , \ c_R \ , \ t_R \quad ; \quad d_R^i = d_R \ , \ s_R \ , \ b_R. \tag{2.12}$$

The left-handed fermions are isospin doublets $(I = \frac{1}{2})$, whereas the right-handed fermions come as isospin singlets $(I = 0)$, and they are invariant obviously under weak isospin rotations. Each doublet family has 3^{rd} component of weak isospin I^3 and I^3 is related to the weak hypercharge (average charge of multiplet $Y = A + S$) in terms of $SU(2)$ generators

$$Q = I^3 + \frac{Y}{2}. \tag{2.13}$$

It is apparent that left-handed Dirac fields transform as doublet under $SU(2)$ while right-handed fields do not. Hypercharge values assure the fermions to possess correct electric charge. Isodoublets differ by unit electric charge. For quarks: $q_u, q_d = \frac{+2e}{3}, \frac{-e}{3}$ and leptons: $q_l, q_\nu = -e, 0$. Quark fields have also color charge and each of them comes in triplet of these 3 colors whereas leptons have zero color charge hence they are singlets.

This property ensures the hypercharge cancellation in each generation hence gauge invariance keeps Lagrangian neutral related to hypercharge [6]. That is,

$$\sum_f Y_f = 0.$$

The electroweak theory obligates the existence of four massless gauge bosons to mediate the unified theory. Two of them are electrically charged and other two are neutral. On the other hand it is well known experimental fact that the weak interaction has short range [7]. So the mediators of this fundemental force are required to be massive.

$SU(2)_L$ non-abelian group symmetry indicates that there should be three vector fields $W_\mu^i (i = 1, 2, 3)$ associated with the generators of given group $T^i = \frac{1}{2}\tau^i$ in terms of generalized Pauli matrices τ^i. Group members are normalized as $\mathrm{Tr}(\tau^i \tau^j) = 2\delta^{ij}$ and are also satisfy the commutation relations $[T^i, T^J] = i\epsilon^{ijk}T^k$. By using chiral doublets and ladder operators of $SU(2)$: $\sigma^{\pm} = \frac{1}{2}(\sigma^1 \pm i\sigma^2)$, weak isospin currents are defined as analogous to J_μ^{em} (2.6)

$$\vec{J}_\mu^{\pm} = \bar{x_L}\gamma_\mu \tau^{\pm} x_L. \tag{2.14}$$

The physical meaning of currents (2.14) is that it reveals the interaction between components of the doublet. Furthermore it is possible to construct 3^{rd} component of weak isospin neutral current:

$$J_\mu^3 = \frac{1}{2}\bar{x_L}\gamma_\mu \tau^3 x_L = \frac{1}{2}\bar{\nu_L}\gamma_\mu \nu_L - \frac{1}{2}\bar{e_L}\gamma_\mu e_L. \tag{2.15}$$

Due to the presence of doublet x_L, triplet form of weak isospin currents can be written. In return, this ends up with the $SU(2)$ symmetry group. However the neutral current in the expression (2.15) is not observed as given form since it only couples to left-handed fermions, meaning it is in the form of $\frac{1}{2}\gamma^\mu(1-\gamma^5)$ vector-axial(V-A) coupling. But experiments has shown that neutral current also couples to right-handed fermions (e.g: neutral weak coupling of Z^0). Hence this observation sparks the sign where the electromagnetic interaction must come into theory. Obviously the electromagnetic interaction couples to both chiral fermion states as a consequence of parity conservation.

As it is seen from equation (2.8), the electromagnetic current:

$$J^{\mu}_{em} = e\bar{\psi}\gamma^{\mu}Q\psi = -\bar{e}\gamma^{\mu}e = -\bar{e_R}\gamma^{\mu}e_R - \bar{e_L}\gamma^{\mu}e_L \tag{2.16}$$

couples only to one member of the chiral doublet. There has to be another neutral current which itself is independent of $SU(2)_L$, meaning, this current must remain as singlet under rotations and electromagnetic current must reveal itself somehow when combined with neutral part of weak isospin current J^i_μ. Then Gell-Mann's relation [8] ($Q = I^3 + \frac{1}{2}Y$) is to be extended for the electroweak mixing phenomena. Weak hypercharge current is defined as

$$\begin{aligned} J^Y_\mu = \bar{\psi}\gamma_\mu\psi = 2J^{em}_\mu - 2J^3_\mu = 2(-\bar{e}\gamma^{\mu}e) - 2(\frac{1}{2}\bar{\nu_L}\gamma_\mu\nu_L - \frac{1}{2}\bar{e_L}\gamma_\mu e_L) \\ = -\mathbf{2}\bar{e_R}\gamma_\mu e_R - \mathbf{1}\bar{e_L}\gamma_\mu e_L - \mathbf{1}\bar{\nu_L}\gamma_\mu\nu_L \end{aligned} \tag{2.17}$$

where the eigenvalues of hypercharge current given in bold numbers. Consequently $SU(2)_L \otimes U(1)_Y$ symmetry group is constructed with chiral states. Various quantum numbers of fermions are given in table (2.1).

Fermion	Q_f	I^3_W	Y_L	Y_R
ν_e , ν_μ , ν_τ	0	$+\frac{1}{2}$	-1	0
e^- , μ^- , τ^-	-1	$-\frac{1}{2}$	-1	-2
u , c , t	$+\frac{2}{3}$	$+\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{4}{3}$
d , s , b	$-\frac{1}{3}$	$-\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{2}{3}$

Table 2.1: Electric charge Q, weak isospin component I^3 and weak hypercharge Y assignments of the fundemental fermions

So far, four currents of $SU(2)_L \otimes U(1)_Y$ symmetry group have been mentioned and the two of them are charged and have the V-A coupling structure in terms of conserved current. And other two remains neutral. One violates the parity conservation whereas the other does not. And the general case of weak isospin currents is

$$\vec{J_\mu} = \frac{1}{2}\bar{x_L}\gamma_\mu\vec{\tau}x_L, \tag{2.18}$$

and the electromagnetic current is summed over particles in doublet

$$J_{\mu}^{em} = \sum_{i=1}^{2} = Q_i(\bar{U_L^i}\gamma_\mu U_L^i + \bar{U_R^i}\gamma_\mu U_R^i). \tag{2.19}$$

In 1967 Weinberg and Salam [2] applied Higgs mechanism to $SU(2)\otimes U(1)$ gauge theory. They claimed that three isospin weak currents $\vec{J_\mu}$ couples to a weak isotriplet of vector boson $\vec{W^\mu}$ with strength g, whereas the weak hypercharge current J_μ^Y couples to and isosinglet B^μ with strength $\frac{g'}{2}$. Thus the interaction term of gauge fields with the weak isospin and weak hypercharge currents appears in the electroweak Lagrangian similiar to expressions (2.5) and (2.6)

$$\mathcal{L}_{EW}^{int} = -i[g\vec{J_\mu}.\vec{W^\mu} + \frac{g'}{2}J_\mu^Y B^\mu]. \tag{2.20}$$

In the light of Yang-Mills theory [5], their strategy was quite similar to $U(1)$ local gauge invariance. But this time insisting on $SU(2)\otimes U(1)$ gauge groups, taken together by gauge transformations on both chiralities

$$x_L \rightarrow x_L^{'} = e^{i\alpha(\vec{x}).\vec{T}+i\beta(x)Y} \quad ; \quad x_R \rightarrow x_R^{'} = e^{i\beta(x)Y}x_R. \tag{2.21}$$

However Lagrangian (2.2) is not invariant under transformation (2.21). Invariance of the Lagrangian is recovered from the gauge covariant derivative of $SU(2)_L\otimes U(1)_Y$:

$$D_\mu^L = \partial_\mu - i\frac{g'}{2}YB_\mu - ig\vec{T}.\vec{W_\mu} \quad ; \quad D_\mu^R = \partial_\mu - i\frac{g'}{2}YB_\mu. \tag{2.22}$$

Altogether, the electroweak part of SM Lagrangian appears in a closed form for both quark and lepton chiralities as

$$\mathcal{L}_{EW} = \sum_{\psi} = \bar{\psi}\gamma^\mu(\partial_\mu - i\frac{g'}{2}YB_\mu - ig\vec{T}.\vec{W_\mu})\psi \tag{2.23}$$

where τ gives non-zero only for left-handed fields, and Y acts on both left and right-handed fields. But the kinetic term of vector fields is not gauge invariant contrary to $U(1)_{em}$, hence it requires a modification for a non-abelian case, since each gauge group defines its own field tensor. It is given for $SU(2)$ group as follows:

$$\vec{W_{\mu\nu}} = \frac{1}{ig}[D_\mu, D_\nu] = \partial_\mu\vec{W_\nu} - \partial_\nu\vec{W_\mu} - g\vec{W_\mu}\times\vec{W_\nu}, \tag{2.24}$$

in which the last term is pure consequence of non-abelian structure of the group members(τ^i). And gauge symmetry of the field components is given by

$$\vec{W}_\mu \rightarrow \vec{W}_\mu - \frac{1}{g}\partial\vec{\alpha} - \vec{\alpha} \times \vec{W}_\mu. \tag{2.25}$$

So the Lagrangian for combined group becomes

$$\mathcal{L}_G = -\frac{1}{4}\vec{W}^{\mu\nu}.\vec{W}_{\mu\nu} - \frac{1}{4}B^{\mu\nu}.B_{\mu\nu}. \tag{2.26}$$

Equations (2.23) and (2.26) give the complete electroweak Lagrangian. Notice that introducing mass terms for both the fermions or the gauge fields, violates the local $SU(2)_L$ gauge invariance (2.37). But such constraint contradicts with experiment since gauge bosons are observed massive. Therefore, interaction of scalar field and gauge fields through broken symmetry plays a paramount virtue for theory to remain renormalizable. The SM Higgs mechanism will be discussed after introducing QCD briefly for completeness and state the full SM Lagrangian at the end.

2.3 Color Gauge Theory and Quantum Chromodynamics

Quantum chromodynamics (QCD) is a non-abelian $SU(3)_C$ gauge field theory that describes the strong interactions. Interaction itself is mediated by eight massless gauge bosons called gluons. Any elementary particle with non-zero color charge experiences the presence of gluons. Since the characteristic that distinguishes quarks from leptons is color, strong interactions and QCD are based on local color gauge symmetry of quark triplets. Unlike QED, QCD vertices could include 3-gluon or 4-gluon couplings since gluons come with bicolor structure and this is purely due to the non-abelian structure of gauge group which will be clearer soon. Hence their vertices are not just based on quark-gluon-antiquark combination. The possibility of color quantum numbers reflects a continous symmetry of the $\mathcal{L}_{strong}$. Underlying idea of group theory of QCD is based on two emprical facts:

- Quarks are color triplets,
- All the known hadrons are color singlets.

Consequently the non-abelian structure of the QCD leads to two important results [9, 10]:

1. Color confinement: QCD coupling constant $g_s = \frac{\alpha_s}{4\pi}$, scales with energy of interaction. At large distances α_s is relatively large, hence the perturbative methods for Feynman rules cannot be applied.

   When a $q\bar{q}$ pair is forced to separate, the colour field mediated by the exchanged gluons increases the coupling constant α_s, in return that leads to creation of a new $q\bar{q}$ pair from the vacuum. Furthermore this phenomena becomes more favorable with increasing energy. Hence this reason explains why quarks are not observed as free state in nature, but the physical states (hadrons) are colorless combinations of quarks bounded together.

2. Asymptotic freedom: The coupling constant decreases at small distances approaching to zero, meaning that quarks can be asymptotically considered as free particles. Thus the small value of α_s allows to use perturbation methods for higher level contributions.

For a color triplet ψ_j, whose dynamics is given by the Lagrangian

$$\mathcal{L}_f = \bar{\psi}_j(i\gamma^\mu\partial_\mu - m)\psi_j. \tag{2.27}$$

The task follows from extension of Yang-Mills theory to $U(3)$ group. And group $U(3)\equiv U(1)\times SU(3)$ transformation: $U \rightarrow e^{i\theta}e^{i\vec{T}.\vec{\alpha}}$; in which the global phase transformation is already invariant. And dictation of local gauge invariance for free Lagrangian similiar to previous cases via $SU(3)_C$ symmetry

$$S = e^{ig_s\vec{\alpha}.\vec{T}} = e^{i\frac{g_s\vec{\alpha}.\vec{\lambda}}{2}} \tag{2.28}$$

where $T^a = \frac{1}{2}\lambda^a$ are the members of $SU(3)$ group and called Gell-mann matrices. Group members have non-abelian structure, hence their commutations and normalization properties are given by $[\lambda_a, \lambda_b] = 2if_{abc}\lambda_c$ and $Tr(\lambda_a\lambda_b) = 2\delta_{ab}$ where the structure constant is an antisymmetric tensor, obeys the following relation

$$f_{abc} = \frac{-i}{4}Tr([\lambda_a, \lambda]\lambda_c).$$

To ensure the gauge invariance (2.28) for the Lagrangian (2.27) with color triplet ψ_j, one must change the ordinary derivative with gauge covariant derivative belonging to gauge group once again

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ig_s\vec{T}.\vec{G}_\mu. \tag{2.29}$$

Unlike the weak interactions, the gauge covariant derivative of $SU(3)$ group is applicable to both chiralities. And the gauge fields satisfy the symmetry:

$$G^a_\mu \to G^a_\mu - \frac{1}{g_s}\partial_\mu \alpha_a - f_{abc}\alpha_b G^c_\mu. \tag{2.30}$$

However all the above expressions do not define the QCD theory completely. To finalize $SU(3)_C$ gauge invariance, the gluon field strength tensor through the kinetic term, must be identified

$$G^a_{\mu\nu} = \frac{1}{ig_s}[D_\mu, D_\nu] = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu \tag{2.31}$$

for all expressions appear in this section $a = 1, ..., 8$. So the QCD Lagrangian appears in $SU(3)_c$ gauge invariant form

$$\mathcal{L}_{QCD} = \bar{\psi}_j (i\gamma^\mu D_\mu - m)\psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a. \tag{2.32}$$

Insisting on local gauge invariance for the free Lagrangian brings a new term defining the interaction between gauge fields and components of color triplet. If Noether theorem and Hamilton principle applied to Lagrangian (2.32) through infinitesimal transformations of $SU(3)_C$, it all ends up with conserved current:

$$\vec{J}^c_\mu = \frac{g_s}{2}\bar{\psi}\gamma_\mu \vec{\lambda}\psi \tag{2.33}$$

that is, the color current plays a source role. But the most distinguishing characteristic comes from the field strength tensor which makes gauge Lagrangian not just purely kinetic, but also allows it to include self-couplings of gluon fields. Extra term appearing in (2.31) brings 3-gluon and 4-gluon interactions from the part of the Lagrangian and these self-couplings are given in fig.(2.1).

$$\mathcal{L} \supset \bar{\psi}\psi + G^2 + g_s\bar{\psi}\psi G + g_s^2 G^4$$

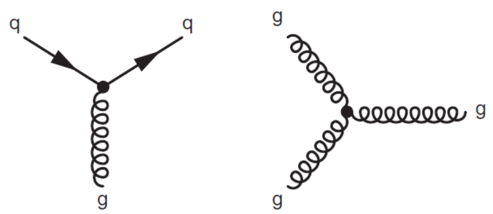


Figure 2.1: The predicted QCD interaction vertices arising from the requirement of SU(3) local gauge invariance.

2.4 The Unified Framework and the SM Lagrangian

It is possible and convenient to add up all symmetries into higher gauge symmetry and hence the Lagrangian of the SM for further convenience can be divided into four parts.

$$\mathcal{L}_{SM} = \mathcal{L}_{kinetic} + \mathcal{L}_{gauge} + \mathcal{L}_{higgs} + \mathcal{L}_{yukawa} \tag{2.34}$$

where the first term includes the kinetic terms of fermions

$$\begin{aligned}\mathcal{L}_{kin} = \bar{Q^i_L}(i\gamma^\mu D_\mu)Q^i_L + \bar{u^i_R}(i\gamma^\mu D_\mu)u^i_R + \bar{d^i_R}(i\gamma^\mu D_\mu)d^i_R \\ + \bar{L^i_L}(i\gamma^\mu D_\mu)L^i_L + \bar{e^i_R}(i\gamma^\mu D_\mu)e^i_R.\end{aligned} \tag{2.35}$$

Larger gauge symmetry group defines now gauge covariant derivative with all gauge field contributions:

$$D_\mu = \partial_\mu - i\frac{g'}{2}YB_\mu - i\frac{g}{2}\tau^i W^i_\mu - i\frac{g_s}{2}\lambda^a G^a_\mu \tag{2.36}$$

and the 3^{rd} term of equation (2.36) is absent when the gauge covariant derivative acts on right-handed fermions. Consequently the 4^{th} term is not present when it acts on leptonic fields since they have zero color charge. At current stage, all the fermions are seen to be massless. Because the chiral states fermions are not in complex conjugated representations. More clearly fermion mass terms:

$$\mathcal{L}_{mass} = -m\bar{\psi}\psi = -m\bar{\psi}(\frac{1-\gamma^5}{2} + \frac{1+\gamma^5}{2})\psi = -m(\bar{\psi_R}\psi_L + \bar{\psi_L}\psi_R), \tag{2.37}$$

mix the left-handed and right-handed chiralities hence they violate the gauge invariance and excluded from the SM Lagrangian. A Majorana mass term $\mathcal{L}_M = -\frac{1}{2}M(\bar{\nu^R_c}\nu_R + \bar{\nu_R}\nu^c_R)$ is not possible since all the fermions carry hypercharge. But the fermion kinetic term possesses 5 global $U(3)$ symmetries for both chiral states of quarks and leptons.

The Yukawa potential [11] expression contains all of the allowed couplings of the Higgs scalar H and the fermion fields, but violates these symmetries.

$$\mathcal{L}_{yuk} = -Y_{ij}\bar{\psi^i_L}\phi\psi^j_R = -Y^u_{ij}\bar{Q^i_L}\epsilon H u^j_R - Y^d_{ij}\bar{Q^i_L}Hd^j_R - Y^e_{ij}\bar{L^i_L}He^j_R + h.c. \tag{2.38}$$

The strength of the interactions between the Higgs and the fermions which is called Yukawa couplings are embeeded into Y^u, Y^d, Y^e, and these are 3×3 complex matrices have again to be added by hand since they cannot be determined theoretically, hence they will be deduced by experiments. And the symmetries of the SM will be broken seperately when the scalar field is situated into vev. The Yukawa interactions are also responsible for fermion masses. The presence of massless mediators: photons and gluons suggest that $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ must be broken down to QCD ($SU(3)_C$), and electromagnetism ($U(1)_{em}$). However the symmetry $SU(3)_C \otimes U(1)$ is unbroken and vector-like, that leads to both chiral states of all fermions transform under the same representation.

Finally the 3^{rd} part of the Lagrangian is the Higgs sector. It contains the Higgs potential

$$V(H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2 \tag{2.39}$$

as well as the kinetic term so that

$$\mathcal{L}_H = (D_\mu H)^\dagger D^\mu H - V(H). \tag{2.40}$$

The mass terms for the gauge bosons will be revealed from the kinetic term after the Higgs field acquires the vacuum expectation value. One can naively consider that whenever 'relevant' gauge covariant derivative acts on Higgs field and subjected to Higgs potential, spontaneous symmetry breaking mechanism ends up with corresponding masses of gauge bosons as a consequence of Goldstone theorem. In the final section of the SM, Higgs mechanism [12, 13, 14] and electroweak symmetry breaking sector will be expressed.

So far, the mass terms of fermions and gauge bosons are excluded from the SM Lagrangian to preserve the gauge invariance. Now, the main objective of Weinberg-Salam model is to generate the massive gauge bosons while preserving the renormalizability of the EW theory. As mentioned in the previous section, the gauge symmetry of the EW interactions belongs to $SU(2)_L \otimes U(1)_Y$ group.

However, breaking of such symmetry and appearance of the massive gauge bosons require at least 3 degrees of freedom and this is the place where the Goldstone theorem [15, 16] comes in. Goldstone bosons give their dofs to the longitudinal polarization component of the massive gauge bosons. Then with a suitable choice of parameters, masses of Z^0 and $W^{\pm}$ bosons can be obtained while keeping the photon massless. After the symmetry breaking process, there will remain one massive scalar corresponding to field excitations in the direction of chosen physical vacuum state.

In 1967, Weinberg [2] considered an invariant $SU(2)$ complex doublet composed of scalar fields:

$$\phi = \begin{pmatrix} \phi^{+} \\ \phi^{o} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) \end{pmatrix} \tag{2.41}$$

where the superscript denotes the electric charge in each component and $\phi_{1,2,3,4}$ are just real scalar fields. Conventionally the lower part of the doublet is needed be electrically neutral, which corresponding to Z^0 and γ with $m_\gamma = 0$. Upper part differs by unit of electric charge to correspond other two massive gauge bosons $W^{\pm}$. Thus the third isospin components of the doublet is written as: $I^3_{\phi_+} = \frac{1}{2}$; $I^3_{\phi_0} = -\frac{1}{2}$. Rewriting the Higgs Lagrangian explicitly (2.40):

$$\mathcal{L}_H = (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \quad , \quad (m^2 < 0, \lambda > 0) \tag{2.42}$$

however this Lagrangian alone is not gauge invariant. It is crucial to keep in mind that the addition of $\mathcal{L}_{gauge}$ is obligatory. And the gauge covariant derivative expression (2.36) reduces to

$$D_\mu = \partial_\mu + i\frac{g'}{2}YB_\mu + i\frac{g}{2}\tau^i W^i_\mu. \tag{2.43}$$

In the regime $(m^2 < 0, \lambda > 0)$ of equation (2.42), rewriting the Higgs potential term by term:

$$V(\phi) = \frac{\mu^2}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) + \frac{\lambda}{4}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)^2.$$

The extremum of the Higgs potential is obtained for the case $(\mu^2 < 0)$

$$\frac{\partial V(\phi)}{\partial \phi_i} = 0 \rightarrow \frac{\partial V(\phi)}{\partial \phi_3} = \mu^2\phi_3 + \lambda\phi_3(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2).$$

Since $\mu^2 < 0$, the extremum of this potential can be chosen by the following relation: $\mu^2 = -\lambda(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)$. But there is an infinite choice for the ground state, so choosing

$$\phi_1 = \phi_2 = \phi_4 = 0 \quad and \quad \phi_3^2 = -\frac{\mu^2}{\lambda} = v^2,$$

breaks the field symmetry appearing in the expression (2.42). Hence the vacuum expectation value occurs as a consequence of the previous choice

$$\langle \phi \rangle_0 = \langle 0 | \phi | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{2.44}$$

Again, this particular v.e.v is chosen as a result of the component ϕ^+, which carries an electric charge, that is absent in the ground state and making it neutral in agreement with the nature. And it will be apparent later that this choice preserves the $U(1)_{em}$ gauge invariance and keeps $m_\gamma = 0$. Consequently the SSB mechanism expresses how the Goldstone bosons transfers their d.o.f's to the polarization vector of the gauge bosons.

Observe that none of the generators individually keeps the vacuum state invariant:

$$\tau^i \phi_0 = (\tau^i) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0 \quad , \quad Y \phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0$$

all broken, however $Q = -\frac{1}{2} + \frac{1}{2}$ with eigenvalue $y_\phi = 1$ keeps the g.s invariant $\phi_0^{'} = e^{i\alpha Q_{\phi_0}} \phi_0 = \phi_0$

$$(\tau^3 + Y)\phi_0 = \left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = 0.$$

So the ground state does not obey the initial $SU(2)_L \otimes U(1)_Y$ symmetry of the Lagrangian but corresponds to sub-symmetry group $U(1)_{em}$, thus the vacuum state remains neutral. This is the underlying concept how a larger symmetry group chooses a smaller gauge symmetry after the symmetry breaking mechanism.

Perturbing the complex scalar doublet around the v.e.v will give rise to the masses of the gauge bosons. In order to describe the excitations from the vacuum, a parametrization of the scalar doublet ϕ around the ϕ_0 has to be identified in terms of the four fields: $\theta_1, \theta_1, \theta_1$ and h and also with the $SU(2)$ generators.

Since physics is invariant under the $SU(2)_L \otimes U(1)_Y$ rotation, vev's of the scalars remains constant in a spacetime rotations. Thus it becomes,

$$\phi(x) = e^{-i\alpha^i(x)\frac{\tau^i}{2}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \to e^{i(\theta^i(x) - \frac{\alpha^i(x)}{2})\tau^i} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} . \quad (2.45)$$

This rotation can also be written with irreducible representation of $SU(2)_L \otimes U(1)_Y$ group [6].

Invariance of the Lagrangian gives us a chance to eliminate the three Goldstone bosons ξ_i. This is called the unitary gauge and it plays a significant role in the Goldstone theorem. Either choosing $B_\mu \to B_\mu^{'} = B_\mu + \frac{1}{gv}(\partial_\mu \xi_i)$ at the Lagrangian level (will be used in next section) or simply choosing $\alpha^i(x) = 2\theta^i(x)$ returns the scalar doublet to the perturbed state around its vev:

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} , \quad (2.46)$$

such that the complex scalar doublet still satisfies vev:

$$\boxed{\langle 0| \phi^\dagger \phi |0\rangle = -\frac{\mu^2}{2\lambda} = v^2 \;\; , \;\; \langle 0| h(x) |0\rangle = 0} , \quad (2.47)$$

whereas the lower component of doublet

$$H(x) = v + h(x) \to \langle 0| H(x) |0\rangle = v$$

The H-field in the vacuum state is constant. And the corresponding particle is Higgs boson, that is the perturbation of the vacuum field with $h(x)$

2.5 The Masses of the SM Gauge Bosons

The mass terms can be identified by writing the Lagrangian such that it respects to the $SU(2)_L \otimes U(1)_Y$ symmetry

$$\begin{aligned}
D_\mu \phi &= (\partial_\mu + i\frac{g'}{2} Y B_\mu + i\frac{g}{2}\tau^i W_\mu^i)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\begin{pmatrix} \partial_\mu + i\frac{g'}{2}B_\mu + i\frac{g}{2}W_\mu^3 & i\frac{g}{2}(W_\mu^1 - iW_\mu^2) \\ i\frac{g}{2}(W_\mu^1 + iW_\mu^2) & \partial_\mu + i\frac{g'}{2}B_\mu - i\frac{g}{2}W_\mu^3 \end{pmatrix}\begin{pmatrix} 0 \\ v + h \end{pmatrix} \\
&= \begin{pmatrix} i\frac{g}{2}W_\mu^+(v+h) \\ \frac{1}{\sqrt{2}}(\partial_\mu h - i(v+h)(\frac{g}{2}W_\mu^3 - \frac{g'}{2}B_\mu) \end{pmatrix} \\
(D_\mu\phi)^\dagger &= \begin{pmatrix} -i\frac{g}{2}W_\mu^-(v+h) & \frac{1}{\sqrt{2}}(\partial_\mu h + i(v+h)(\frac{g}{2}W_\mu^3 - \frac{g'}{2}B_\mu) \end{pmatrix}
\end{aligned}$$

where the eigenvalue of hypercharge for Higgs doublet is chosen as $Y = 1$ and the field ladder operators: $W_\mu^\pm = \frac{1}{\sqrt{2}}[W_\mu^1 \pm iW_\mu^2]$ have already been substituted into the above expression.

Next, performing the contraction of the gauge covariant terms in the Higgs Lagrangian (2.42):

$$(D_\mu\phi)^\dagger(D^\mu\phi) = \frac{g^2}{4}(v+h)^2 W_\mu^- W_+^\mu + \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{8}(v+h)^2(gW_\mu^3 - g' B_\mu)(gW^{3\mu} - g' B^\mu). \tag{2.48}$$

Only the quadratic terms of the gauge fields are required since they correspond to the mass term: $\frac{1}{2}M^2 W_\mu^i W^{\mu i}$

$$\mathcal{L}_{gauge,masses} = \frac{1}{2}(gv)^2 W_\mu^+ W^{\mu-} + \frac{1}{8}v^2 \begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix}\begin{pmatrix} g^2 & -gg' \\ -gg' & (g')^2 \end{pmatrix}\begin{pmatrix} W^{\mu 3} \\ B^\mu \end{pmatrix}, \tag{2.49}$$

and comparing expression (2.49) to the mass term in the Proca Lagrangian, reveals the masses of the gauge bosons $W^\pm$

$$\boxed{M_{W^\pm} = \frac{1}{2}gv}. \tag{2.50}$$

Therefore the masses of the $W^\pm$ bosons are purely determined by the coupling strength of the $SU(2)_L$ gauge interaction and the vev of the Higgs field.

Checking 2^{nd} term in the expression (2.49):

$$\frac{1}{8}v^2 \begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix} \tilde{M} \begin{pmatrix} W^{\mu 3} \\ B^\mu \end{pmatrix} \quad , where \quad \tilde{M} = \begin{pmatrix} g^2 & -gg^{'} \\ -gg^{'} & (g^{'})^2 \end{pmatrix} \tag{2.51}$$

is an off-diagonal mass matrix. But expression (2.51) reveals a physical consequence in which the gauge fields W_μ^3 and B_μ are coupled together. The physical bosonic states propagate as an independent eigenstates of the free particle Hamiltonian. If such basis exists then the related mass matrix should be diagonal and the masses of the neutral gauge bosons correspond to the eigenvalues of the mass matrix.
$\tilde{M}. \rightarrow (g^2 - \lambda)((g^{'})^2 - \lambda) - (gg^{'})^2 = 0 \rightarrow \lambda_1 = 0 \ \ , \ \ \lambda_2 = g^2 + g^{'2}$

If the neutral part of the Lagrangian is rewritten in terms of the eigenvalues of the mass matrix, then it becomes:

$$\mathcal{L}_{neutral,bosons} = \frac{1}{8}v^2[gW_\mu^3 - g^{'}B_\mu]^2 + \mathbf{0}[g^{'}W_\mu^3 + gB_\mu].$$

And now the physical fields A_μ and Z_μ are the eigenvectors of the mass matrix. And the neutral part of the Lagrangian in physical state basis can be written as

$$\mathcal{L}_{neutral,bosons} = \frac{1}{2} \begin{pmatrix} A_\mu & Z_\mu \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{4}v^2(g^2 + g^{'2}) \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}. \tag{2.52}$$

Finally the masses of the neutral gauge bosons can be identified as

$$\boxed{M_A = 0 \ \ , \ \ M_Z = \frac{1}{2}v\sqrt{g^2 + g^{'2}}}. \tag{2.53}$$

And if it is solved for the normalized eigenvectors of the mass matrix, the physical fields appear in the following form:

$$A_\mu = \frac{g^{'}W_\mu^3 + gB_\mu}{\sqrt{g^2 + g^{'2}}} \quad , \quad Z_\mu = \frac{gW_\mu^3 - g^{'}B_\mu}{\sqrt{g^2 + g^{'2}}}. \tag{2.54}$$

The superposition corresponding to the Z^0 boson, which is related to the neutral Goldstone boson of the broken symmetry, has acquired mass through the Higgs mechanism while the photon field has remained massless. And using the Gell-Mann relation [8] on the electroweak currents gives a relation for coupling ratios:

$$\frac{g^{'}}{g} = \frac{g_e \sin\theta_w}{g_e \cos\theta_w} = \tan\theta_w.$$

Due to recent relation, equation (2.54) can be rewritten as follow

$$A_\mu = B_\mu \cos\theta_w + W_\mu^3 \sin\theta_w \quad , \quad Z_\mu = -B_\mu \sin\theta_w + W_\mu^3 \cos\theta_w . \tag{2.55}$$

Hence the mass of the Z^0 boson can be written by using the Weinberg angle:

$$\boxed{M_Z = \frac{1}{2} g v \sec\theta_w = \frac{M_W}{\cos\theta_w}}, \tag{2.56}$$

and the last part of the EW symmetry breaking process is due to Higgs potential. By writing the chosen vacuum in the expression: $\phi^\dagger \phi = \frac{1}{2}(v+h)^2$ of the $V(\phi)$ with a substitution: $v^2 = -\frac{\mu^2}{\lambda}$, rewrites the scalar potential around the vev:

$$V(\phi)_{\phi_0} = \frac{\mu^2 v^2}{2} - \mu^2 h^2 - \frac{\mu^2 h^3}{v} - \frac{1}{4}\frac{\mu^2 h^4}{v^2}.$$

The first term is just constant and therefore has no effect on the equation of motion. And the next term is directly related to the mass term of the Lagrangian. We can deduce the following expression [17]:

$$M_H = \sqrt{2}\mu = \sqrt{2\lambda} v \tag{2.57}$$

Since g and M_W are related to the Fermi constant, one can obtain [18]:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \quad \to v^2 = \frac{1}{\sqrt{2}G_F} \quad \to v \approx 246GeV, \tag{2.58}$$

$$\langle 0 | \phi^\dagger \phi | 0 \rangle = \frac{v^2}{2} \approxeq (174GeV)^2. \tag{2.59}$$

The rest of the terms with h^2 and $W^\pm h$ indicate self-couplings of the h and the gauge bosons $W^\pm$, so the trilinear and quadrilinear couplings can be read off from the Lagrangian and illustrated in fig. (2.2)

$$\mathcal{L}_{SW} = \frac{1}{4} g^2 v^2 W_\mu^- W^{\mu +} + \frac{1}{2} g^2 v W_\mu^- W^{\mu +} h + \frac{1}{4} g^2 W_\mu^- W^{\mu +} hh. \tag{2.60}$$

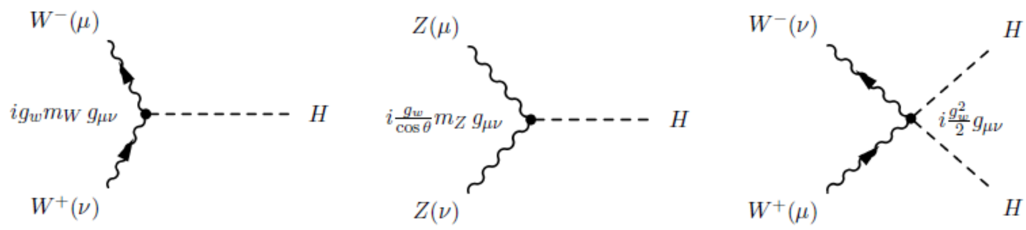


Figure 2.2: Trilinear and quadrilinear couplings of the Higgs field to the gauge bosons

The equation(2.59) sets a scale for the electroweak symmetry breaking process. Such energy density corresponds to a temperature of roughly $10^{15}K$. According to the Big Bang model of the universe, the $SU(2)_L \otimes U(1)_{em} \rightarrow U(1)_{em}$ symmetry breaking had distinguished the electromagnetism and the weak force in a time period $\approx 10^{-11}s$ after the cosmic inflation had begun.

Fermions can also acquire their masses by interacting with the Higgs field. After the symmetry breaking process, the interaction terms between the fermion fields and the Higgs field(arise from the Yukawa potential 2.38), generate the masses of the fermions, and these masses are proportional to the Yukawa couplings: $M_f^i = \frac{1}{\sqrt{2}} g_f^i v$. However neutrinos cannot acquire mass since there is no right-handed neutrino has been observed yet. Furthermore, quarks are triplets under the $SU(3)_C$ color symmetry; similarly the mass matrix must be diagonalized to obtain the mass eigenstates of the physical quarks. But such variation of parameters will lead to various mass eigenstates for the each generation. In return, different generations will be mixed through the weak interactions hence the CP violation occurs.

Finally, by comparing relative couplings of neutral current and charged current of the weak interactions; there has to be a parameter which scales to unity in the SM [19].

$$\rho = \frac{M_W^2}{M_Z^2 (\cos\theta_W)^2} = 1.$$

But to be tested by experimental methods.

As a last word in this section; the SM contains 25 free parameters:

- 2 parameters for the Higgs' vev $\rightarrow \mu$ and λ.
- 12 Yukawa couplings(fermion masses) to the Higgs field(neutrinos are included

since oscillations are observed).

- 8 mixing angles of the PMNS and the CKM matrices [20].
- 3 fundemental couplings g_e, g_w and g_s [21] for the gauge groups $U(1)_Y$,$SU(2)_L$, $SU(3)_c$ respectively.

2.6 Shortcomings of the SM and Theories Beyond the SM

The Standard Model has accomplished many theoretical predictions consistent with the results of particle detectors, hence it is the best elementary particle theory at the current stage. Although a large number of free parameters are available, there are several unanswered questions in which the SM has not found complete solutions yet. First of all, neutrinos are observed as oscillating between the different flavor types [22]. Such phenomenon indicates that they have to be massive. A minimal extension of the SM shows that, by adding right-handed neutrinos, it is possible to express the Majorana mass terms for neutrinos. Second argument follows that the SM is unable to express a rigid theoretical conclusion why there is observed matter-antimatter asymmetry in the universe. This asymmetry is characterized by the baryon-photon ratio [23, 24]:

$$\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma}, \tag{2.61}$$

where the baryon, antibaryon and photon densities are given in the expression (2.61). According to the results of WMAP [25, 26] $\eta \approx 6.19 \times 10^{-10}$.
Thus the baryogenesis theories[27, 28] focuses on explaining why the η is not zero. Then there exists a cosmological observation, indicating that the SM does not include the dark matter, which accounts approximately 83% of the total mass in the universe[29] and known to interact weakly with the SM particles.

2.6.1 Dark Matter

The origin of the dark matter is still unknown and its existence is deduced merely by the gravitational interaction. Inner composition of the dark matter has not been revealed yet. The first proposal of the DM is related to insufficiency of mass through the galaxies [30], since the observed mass could not produce such orbital velocity of the galaxies. In 1933, Fritz Zwicky studied the dynamics of the galaxies through the Coma Cluster[31]. By using the virial theorem on the Newtonian gravity among particles in galaxy and considering nebulae as a uniform spherical shape with radius a R; a simple model ended up with expression: $2\bar{T} = -\bar{U}_g$, where $\bar{U}_g = \frac{-3GM_{tot}^2}{5R}$.

But if that is the case, then the total mass of the observed galaxy should be

$$M_{tot} = \frac{5R\bar{v^2}}{3G}.$$

However, Zwicky observed the brightest nebulae throughout the Coma cluster and came up with an approximation that $M_{tot} \gg \frac{R\bar{v^2}}{5G}$. And the average mass of a galaxy in the Coma Cluster should be approximately $4 \otimes 10^{10} M_{\odot}$ with an average galactic luminosity: $L = 8.5 \times 10^{7} L_{\odot}$. Thus there has to be some non-luminous matter which accounts for the most of the total mass throughout the Coma Cluster. Finally more reliable values of the galaxy masses has been computed by the gravitational lensing method [32], which does not directly depend on the orbital dynamics. And the mass value obtained in this way is in great agreement with the rotation curves [33].

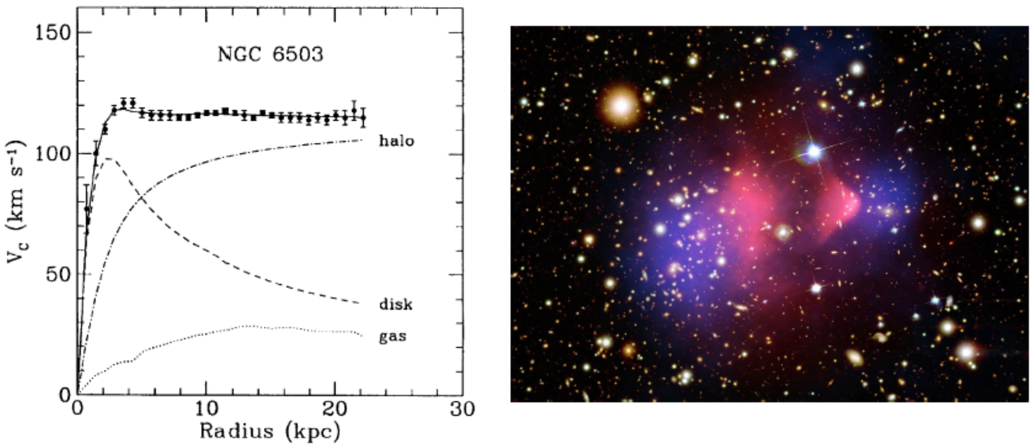


Figure 2.3: Left: The rotation curve of the NGC 6503. The bold line is the contribution of the DM. Right: Observations of the Bullet Cluster. Red cloud indicates a hot gas, observed in the X-ray regime, whereas the blue cloud demonstrates the existence of the DM due to the gravitational lensing

There exist more arguments for the DM, a tangible results of the Cosmic Microwave Background(CMB) [34], which gives information about the evolution of the universe. Nonuniformly distributed CMB propagated through the photon-baryon field until the decoupling stage [35]. Matching out the theory with the peak values of power distribution has concluded conceivable DM density in the universe [36].

There are several dark matter candidates such as baryonic mass as a possible DM candidate, forming dense bodies known as massive astrophysical compact halo objects (MACHOs), and weakly interacting massive particles(WIMPs) [37, 38]. A WIMP can be any particle that interacts with a strength of the order of the weak interaction and a has a mass approximately the weak scale.

Analysis of the structure formation of the DM gives a set of classification for the DM:

- Cold dark matter(CDM), non-relativistic at the decoupling stage [40, 41],
- Warm dark matter(WDM), energies of rest mass at the decoupling. [42],
- Hot dark matter(HDM), relativistic at the decoupling.

Only the CDM model will be considered in the next sections, otherwise relativistic particles(HDM) would cause the universe to be less dense than it is today. And by writing an $SU(2)_d$ symmetric dark sector, the dark gauge boson will be considered as possible DM candidates for a further scope, where the stability of the DM candidates will be given by the custodial symmetry [43].

CHAPTER 3

THE HIDDEN SECTOR

Despite the theoretical physics leaves unanswered questions more than yesterday, physicists try to develop more sophisticated methods to answer questions beyond the SM. And the era of the hidden sector had already begun to comprehend what lies beyond the SM. Each extension methodology brings its own particles, symmetries and interactions, but ”hidden from the everyday observation”, because the experimental insufficiency prevents us from observing the scale of these interactions with the SM particles. But the hidden(or dark) sector had influenced a broad range of study to conceive the nature of the dark matter. There are many possibilities for the DM models in which the DM fields are being designated similiar to the SM fields and mostly the non-/abelian gauge theories are extended to the hidden sector to construct viable DM frameworks.

Regarding a hidden sector, constrained to low energies, includes electrically neutral states and possibly neutral under the SM gauge groups too. Hence a set of interactions regarding the both sectors, can be parametrized in a compact form of the sector operators:

$$\mathcal{L}_{int} = \sum_{k,l,n}^{k+l=n+4} \frac{\mathcal{O}_{HS}^{k}\mathcal{O}_{SM}^{l}}{\Lambda^{n}}$$

where superscripts denote the canonical dimensions of the sector operators and Λ is indicating a cutoff scale $> TeV$

A wide spectrum of the DM studies are being motivated by the gravitational interactions as a consequence of the first observations of Zwicky.

Thus, almost every extension relies on the gauge theory, requires the aid of the Higgs mechanism in the hidden sector or so called "dark Higgs". And obviously there is a 'zoo' for the non-baryonic DM candidates.

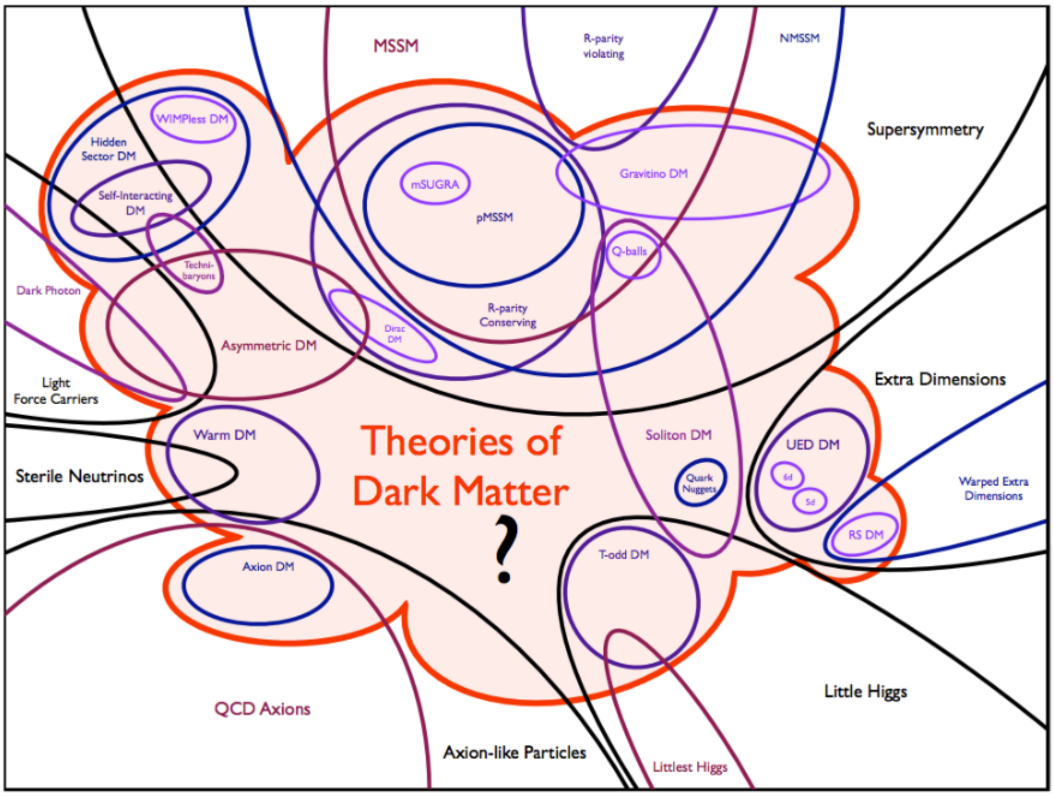


Figure 3.1: Dark Matter related theories beyond the Standard Model

There are also DM candidates at cosmic scales, such as MACHOs, black holes, dwarf type stars. But these are out of scope at this study. However these cosmic structures can include a small fraction of the DM abundance, hence other objects with masses ranging between $10^{-5}eV$ to $10^{4}M_{\odot}$, have been introduced to consolidate the DM relic density. However, it is crucial to keep in mind that, no candidate excludes an other; otherwise it would be trivial to imply that the DM is composed of one substructure alone. The most common candidates are listed in the subsections.

3.1 Neutrino Portal

Neutrinos had been proposed as a viable DM candidates because of their massive presence and weak scale of interactions. But the relic density for the DM cannot be fulfilled with the neutrino masses. Even if neutrinos considered as a part of the DM, there is an upper limit for neutrino abundance: $\Omega_\nu h^2 < 0.07$ [49] is obviously far from total DM abundance. There are more constraints about the neutrino relic density but they are beyond the scope of this study. On the other hand, there exists a proposal about the Sterile neutrinos, interacting only by the gravitational force and considered as the mass generation mechanism for the left-handed neutrinos. One of the logical explanation for the neutrino oscillations can be expressed via the see-saw mechanism. Once a right-handed neutrino is allowed to exist, a possible mass term appears in the Lagrangian. Since right-handed neutrinos do not couple to any gauge field, the Majorana mass term can be expressed for right-handed neutrinos:

$$\mathcal{L}_{majorana} \approx \bar{\nu_R} M_D \nu_L + \bar{\nu_L} M_D \nu_R + \frac{1}{2}\bar{\nu_R} M_M \nu_R^c + \frac{1}{2}\bar{\nu_R^c} M_M \nu_R$$

in terms of the matrix

$$\approx \frac{1}{2}\begin{pmatrix} 0 & M_D \\ M_D & M_M \end{pmatrix}.$$

The mass eigenvalues for the 'physical states': $\frac{1}{4}(M_M \pm \sqrt{M_M^2 + 4M_D^2})$ and with an approximation: $M_D < M_M$, leads to two mass eigenvalues $\frac{M_M}{2}$ and $-\frac{M_D^2}{2M_M}$. One is extremely massive particle and the other is a very light neutrino. And the massive neutrino can be considered as a DM candidate.

3.2 Axion Portal

Axions are pseudo-scalars, which are directly related to the $U(1)$ SSB belongs the Peccei-Quinn model. The related symmetry of that model have been proposed to terminate the strong CP problem of the QCD. However, stellar cooling and supernova processes imposes an approximate limit for the axion masses: $\ll 0.01eV$. Moreover, they are predicted as weakly interacting with the SM particles, hence they could not be in thermal contact with cosmic heat reservoir at the early stage of the universe. Axions are less likely to be considered as a DM candidate than the others, since obtaining their number density is a complex process to end up with associated DM abundance.[45]-[48]

3.3 Vector Portal

It is possible to define a vector portal for the DM through a dark gauge field $Z^{'}_{\mu}$ with additional $U(1)^{'}$ internal dark gauge symmetry. Vector portal Lagrangian also contains an additional scalar field $\Phi^{'}$, which is charged under the additional $U(1)^{'}$ dark symmetry. The dark gauge field $Z^{'}_{\mu}$ obtains mass term through radiative SSB via the CW mechanism. Specific SM fermionic fields can contain $U(1)'$ charge, and the interactions are written in terms of the kinetic mixing between Z'_{μ} and the SM hypercharge field.

A basic approach to the kinetic mixing, in which the SM connected to a dark abelian gauge sector can be expresed as:

$$\mathcal{L}_{int} = \frac{\xi}{2} F^{\mu\nu} Z^{'}_{\mu\nu},$$

where the terms $F^{\mu\nu}$ and $Z^{'}_{\mu\nu}$ are the field strength tensors of the photon and the dark photon respectively, and ξ is the scale of the kinetic mixing between $U(1)$ and $U(1)^{'}$ gauge symmetries. Which is experimentally constrained to a small value ($\xi^2 << 1$).

It should be noted that the generic case for the kinetic mixing, is that the one between the $U(1)_Y$ hypercharge and the $U(1)_{dark}$. But it is sufficient to include only the $U(1)_{QED}$ and the $U(1)_{dark}$ mixing in the regime: $m^2_{\gamma'} \ll m^2_Z$. But for a higher energy scales, $F^{\mu\nu}$ should be replaced with the hypercharge field strength. However, for an energy range of ≈GeV, coupling to the Z is ignored.

Expressing the Higgs portal via the coupling of the new scalar $\Phi^{'}$ to the SM Higgs, the radiative SSB in the dark sector(singlet) triggers the EW SSB process in the Higgs sector. Consequently, the model is forged with an additional Dirac fermion $\psi^{'}$, that is charged under $U(1)'$, which can behave as CDM. Furthermore, the model contains extra observer fermions $\xi^{'}$ and RH neutrinos ν_R, these are just natural consequences to assure anomaly cancellation, nevertheless, they cannot alter the phenomenology.

Once the SSB of the $U(1)^{'}$ sector occurs;

$$\mathcal{L}_{int} \approx \xi Z^{'}_{\mu} J^{\mu}_{em} + \frac{m^2_{Z'}}{v_{Z'}} h^{'} |Z^{'}_{\mu}|^2$$

where the interaction of the dark Higgs $h^{'}$ and $Z^{'}_{\mu}$ bosons has already been included above, and will pose significant signatures for $h^{'}$ boson. Despite this model demonstrates considerable facility, there remains a wide range of sophisticated phenomenological results. Such models with the kinetic mixing have recently been under investigation for the WIMPs DM, since they are charged under the $U(1)^{'}$ too.

There are two domains for the non-trivial DM candidates based on this $U(1)^{'}$ sector:

1. $\xi \leq 10^{-7} - 10^{-6}$ when the sub-GeV gauge bosons may exist with a decay length $c\tau$ several meters. For such case $m_{h'} > m_{Z'}$, permits quick $h^{'}$ decay

2. $\xi \approx 10^{-4} - 10^{-2}$ and $m_{Z'} > m_{h'}$; in such interval, fast decaying vector gauge boson and long-lived $h^{'}$ occurs with a decay width of $\mathcal{O}(\xi^2)$.

Both possibilities are ideal candidates for a fixed target search. For a detailed search see ref. [76]

3.4 Scalar Portal

Scalar portal or more conventionally the Higgs portal, describes a coupling in the form: $H^\dagger H$. Consider a new scalar singlet Φ_{HS}, which belongs to the hidden sector, where its connection to the Higgs portal can be written in a generic form of dimension 3 and 4 operators

$$\mathcal{L}_{scalar,int} = (H^\dagger H)(\lambda\Phi_{DS}^2 + \alpha\Phi_{DS}) = hv(\lambda\Phi_{DS}^2 + \alpha\Phi_{DS}) + ...,$$

One would obviously obtain $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$ after the SSB of the EW symmetry. Furthermore, if $\alpha = 0$, then there would be an additional Z_2 symmetry to ensure the stability of Φ_{DS} scalar. Such constraint suggests a firm assertion for the scalar portal leads to viable DM candidates. Moreover, the hidden sector coupled to the scalar portal may not be totally bosonic, thus one can connect additional fermionic fields X_{HS} to the Φ_{DS} scalar, if the renormalizability of the given model is provided. In such a case, the Lagrangian will simply contain a term of the form $\bar{X}X\mathcal{O}_{SM}^h$.

3.4.1 A Minimal Extension

The minimal extension of the SM with hidden sector can be performed by considering an additional scalar Φ_{DS} in a real singlet form, whereas the Higgs is still a complex doublet. The DM abundance is the underlying motivation for the introduction of such minimal extension. Although a detailed study of the minimal scalar extension has already been studied [50, 51], it will be enlightening to review it in a short discussion.

Starting from the potential of the scalar portal for the minimal extension

$$V(\Phi, \Phi_{DS}) = V(\Phi) - \mu_{DS}^2\Phi_{DS}^2 + \frac{\lambda_{DS}}{4}\Phi_{DS}^4 + \frac{\lambda_{sd}}{2}|\Phi_{DS}|^2|\Phi|^2$$

as mentioned in the section (3.4), Φ_{DS}^3 and $\Phi_{DS}|\Phi|^2$ terms can be added but then, the stability of the DM candidate must be abandoned. Thus the potential expression obeys Z_2 symmetry: $\Phi_{DS} \to -\Phi_{DS}$

The SM scalar doublet obtains a non-zero vev as usual, which breaks the Z_2 symmetry by choosing a particular vacuum.

Minimizing the potential reveals parametric relations through the given minimal extension and the SM. Simply quoting from the identical complex doublet of the SM (2.41), and evaluating the extremum of the potential gives:

$$\frac{\partial V}{\partial \phi_3}\bigg|_0 = \frac{\partial V}{\partial \Phi_{DS}}\bigg|_0 = 0,$$

which leads to the relations:

$$\mu^2 = \frac{1}{4}(\lambda v^2 + \lambda_{sd} v_d^2) \quad , \quad \mu^2_{\Phi_{DS}} = \frac{1}{4}(\lambda_{DS} v_d^2 + \lambda_{sd} v^2).$$

By using a rotational matrix identical to the NMEHS section (3.11), one can diagonalize the mass matrix of the real scalar fields:

$$\tilde{M^2} = \frac{1}{2}\begin{pmatrix} \lambda v^2 & \lambda_{sd} v v_d \\ \lambda_{sd} v v_d & \lambda_{DS} v_d^2 \end{pmatrix}$$

where the mixing angle for the mass eigenstates given by the relation:

$$\tan(2\alpha) = \frac{2\lambda_{sd} v v_d}{\lambda v^2 - \lambda_{DS} v_d^2},$$

with the mass eigenstates:

$$h' = \phi_3 \cos\alpha - \Phi_{DS} \sin\alpha \;\; ; \;\; \Phi'_{DS} = \phi_3 \sin\alpha + \Phi_{DS} \cos\alpha.$$

Indeed the limit $\alpha \to 0$, indicates $v_d = 0$, in which the mass matrix for the neutral fields, becomes diagonal. Furthermore, the Higgs field obviously approaches to the physical Higgs with $h' = \phi_3$ and $m^2_{h'} = \frac{\lambda v^2}{2}$ in which the superposition with dark sector vanishes.

Now, the dark scalar boson is the only candidate for the DM, hence the DM abundance will depend on this mere dark sector particle and the relevant parameters of the minimal extension model. Once the cross sections for the annihilation channels are given:

1. Higgs channel $\Phi'_{DS}\Phi'_{DS} \to h'h'$
2. Vector channel $\Phi'_{DS}\Phi'_{DS} \to W^{\pm}W^{\pm}, Z^0Z^0$
3. fermion channel $\Phi'_{DS}\Phi'_{DS} \to \bar{f}f$,

and relevant Lagrangians for these 4 point interactions:

$$\mathcal{L}_S = \frac{\lambda_{sd}}{4} v h^{'} \Phi^2_{DS} + \frac{\lambda}{4} v h^{'3} + \frac{\lambda_{sd}}{8} h^{'2} \Phi^2$$
$$\mathcal{L}_V = \frac{2M_W^2}{v} g_{\mu\nu} h^{'} W^{\mu +} W^{\nu -} + \frac{M_Z^2}{v} g_{\mu\nu} h Z^{\mu} Z^{\nu}$$
$$\mathcal{L}_F = \frac{M_F^2}{\sqrt{2}v} h^{'} \bar{\psi}_i \psi_i .$$

with the total cross section obtained by[51]: $\sigma_{tot} = \sigma_{h'} + \sigma_{Z^0} + \sigma_{W^{\pm}} + \sum \sigma_f$.

Once the annihilation channels for the $\Phi^{'}_{DS}$ are obtained, an approximate solution for the relic abundance is given by following a short version of the next chapter (3)[1]:

$$\Omega_{DM} h^2 \approx 5.2 \times 10^8 GeV^{-1} \frac{m_{\Phi^{'}_{DS}}}{x_{frozen} \beta(x_{frozen})},$$

where $\beta(x) = \sqrt{\frac{\pi}{45} g_* (\frac{m^{'}_{\Phi_{DS}}}{x})} (8\pi G_N)^{-\frac{1}{2}} \frac{m_{\Phi^{'}_{DS}}}{x^2} \langle \sigma v \rangle (x)$ and the rest of the parameters: x_{frozen}, y_{eq} are identical to the ones in the next chapter(4).

Plotting the relic abundance of the MEHS model relative to the Planck's result [53] versus the mass of the dark scalar is given in fig.(3.2), which is taken from [50, 51]:

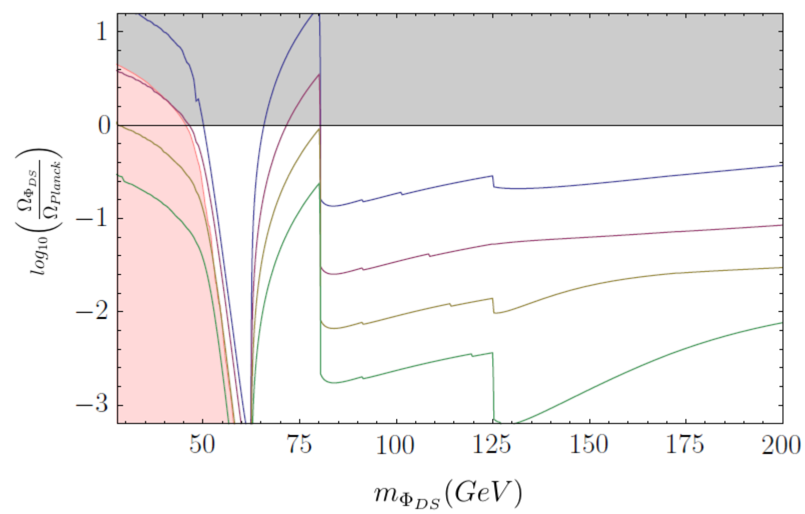


Figure 3.2: Plot for the relative DM abundance versus the mass of the dark scalar. The curves from top to down correspond λ_{sd} values: 0.2, 0.5, 1.0 and 2.0, respectively. Red region excluded by the 2σ limit imposed on $h^{'} \to \Phi^{'}_{DS}\Phi^{'}_{DS}$ decay ratio and gray region excluded by the results of Planck satellite.

[1] It will be sufficient to discuss the results of the minimal extension since a set of derivations for the non-minimal extension will be expressed explicitly in the chapter-4.

3.5 Non-Abelian Dark Higgs Portal

By extending the SM with an additional $SU(2)$ dark gauge symmetry, a possible interaction of the hidden sector vector multiplets and the SM particles can be expressed via the Higgs portal. Since the SM contains $SU(2)$ symmetry, there has to be an extra $SU(2)$ symmetry for the dark sector as well. The kinetic mixing between the SM gauge groups and the non-abelian dark sector $SU(2)_d$ is not allowed as a result of individual gauge invariance, also the scalar potential cannot contain invariant terms of cross sector couples like $\Phi^\dagger_{SM}.\Phi_{DS}$. All the SM particles are considered as singlets under the $SU(2)_d$ gauge group. Then, the only invariant cross sector term is in the form of quadratic scalar interaction: $\lambda_s(\Phi^\dagger_{SM}.\Phi_{SM})(\Phi^\dagger_{DS}.\Phi_{DS})$, that leads the gauge bosons acts as thermal WIMPS. Thus the mixing of both sectors will be proportinal to the scalar coupling.

Furthermore, it is possible to write expressions for the dark sector in analogy with the SM. But the $U(1)$ gauge symmetry is excluded [43] (kinetic mixing $F_{\mu\nu,d}F^{\mu\nu}_Y$ forbidden), so only the $SU(2)$ gauge bosons will be defined. These gauge bosons will be modeled as viable DM fields. The stability of the dark gauge bosons W^a_d is given by the custodial symmetry, which is associated to the gauge symmetry and also to the particle content of the model [44]. And the masses of the dark gauge bosons will appear through the interaction with the dark scalar boson Φ_{DS} which interacts only with the dark sector particles. Similar to the SM, the dark scalar field has a non-zero vev, such that breaks the symmetry when the field is perturbed around this vev. Finally the equivalent Lagrangian reveals the masses of gauge bosons. It will be clearer in the next section that the interaction between the cross sectors is always mediated by the scalars. And this requirement gives an interpretation why the DM and mass content is related through the Higgs portal. Such interaction term $\lambda_{sd}(\Phi^\dagger_{SM}.\Phi_{SM})(\Phi^\dagger_{DS}.\Phi_{DS})$ in the extended Lagrangian, permits intersector matter evolution, hence it will be possible to evaluate the DS and the SM matter densities after the cosmic inflation.

It will be assumed that the DM and SM particles were in thermal equilibrium at early times of the universe. As a consequence of the cosmic inflation, the interaction between them had waned. At some point, the DM has left the heat reservoir and they have evolved independently. Such assumption allows us to determine the DM density.

Analogy of the SM and the custodial symmetry help to write down $G' = SU(2)_d$ gauge covariant derivative to assure the dark sector's invariance as following

$$D_{\mu,d} = \partial_\mu + i\frac{g_d}{2}\tau^a W^a_{\mu,d} = \begin{pmatrix} \partial_\mu + i\frac{g_d}{2}W^3_{\mu,d} & \frac{g_d}{2}(iW^1_{\mu,d} + W^2_{\mu,d}) \\ \frac{g_d}{2}(iW^1_{\mu,d} - W^2_{\mu,d}) & \partial_\mu - i\frac{g_d}{2}W^3_{\mu,d} \end{pmatrix}, \tag{3.1}$$

in which the exclusion of the $U(1)_d$ gauge group mentioned earlier. And the field strength tensor of the $SU(2)_d$ non-abelian gauge group given by:

$$W^a_{\mu\nu,d} = \frac{1}{ig_d}[D_{\mu,d}, D_{\nu,d}] = \partial_\mu W^a_{\nu,d} - \partial_\nu W^a_{\mu,d} - g_d \epsilon^{abc} W^b_{\mu,d} W^c_{\mu,d} \tag{3.2}$$

where the term $W^a_{\mu,d}$ forms dark vector multiplets of the $SU(2)_d$ gauge group and $a = 1, 2, 3$ as usual. The point of interest is based on the scalar potential, in which the conventional Higgs potential constrained by the renormalizability of the model given along with the interaction term of two sectors(dimension-4).

Hence extension of the scalar potential with dark sector becomes

$$\begin{aligned} V(\Phi_{SM}, \Phi_{DS}) = \mu^2_{\Phi_{SM}} \Phi^\dagger_{SM}\Phi_{SM} + \lambda_{SM}(\Phi^\dagger_{SM}\Phi_{SM})^2 \\ + \mu^2_{DS}\Phi^\dagger_{DS}\Phi_{DS} + \lambda_{DS}(\Phi^\dagger_{DS}\Phi_{DS})^2 + \lambda_{sd}(\Phi^\dagger_{SM}\Phi_{SM})(\Phi^\dagger_{DS}\Phi_{DS}) \end{aligned} \tag{3.3}$$

And writing the extended Lagrangian for the two sectors and recall that $\Phi_{SM} \to H$ and $\Phi_{DS} \to \Phi$ for a simpler convention,

$$\begin{aligned} \mathcal{L}_{ext} = \mathcal{L}_{SM,kin} + \mathcal{L}_{SM,gauge} + \mathcal{L}_{yuk} + (D_\mu H)^\dagger (D^\mu H) \\ - \frac{1}{4} W^a_{\mu\nu,d} W^{\mu\nu,a}_d + (D_{\mu,d}\Phi)^\dagger (D^\mu_d \Phi) - V(H, \Phi). \end{aligned} \tag{3.4}$$

Hence the Higgs and the dark scalar doublet Φ expressions are more familiar.

Dark scalar doublet model is defined by three parameters μ^2_Φ,λ_{DS}, and the parameter defining the coupling to the SM: λ_{sd}. Where the first two are the internal property of the dark sector. But λ_{DS} is unconstrained and arbitrary, hence it can be set to a small value for perturbation methods if required for the self interactions in the dark sector. However there are some general constraints on the couplings of the model given, and they ensure the stability of the vacuum and the limitations of the symmetry breaking mechanism.

The presence of the vacuum indicates that the potential must be bounded from below. And the lower limit is assured by the quadratic terms of the potential with the unitary gauge. If the potential $V(H,\Phi)$ is written explicitly with only terms of 4^{th} order of scalars under a slight modification:

$$\begin{aligned} V(H,\Phi) &= \lambda_h(H^\dagger.H)^2 + \lambda_{DS}(\Phi^\dagger.\Phi)^2 - 2\sqrt{\lambda_{DS}\lambda_h}|H|^2|\Phi|^2 \\ &+ 2\sqrt{\lambda_{DS}\lambda_h}|H|^2|\Phi|^2 + \lambda_{sd}|H|^2|\Phi|^2 \end{aligned} \tag{3.5}$$

- $\lambda_h, \lambda_{DS} > 0$
- $\lambda_{sd} > -2\sqrt{\lambda_h\lambda_{DS}}$

for the $V(H,\Phi)$ to remains positive. Conventionally, scalar doublets of the both sectors can be written in terms of 4 real fields as a non-minimal extension to the SM: (2.41)

$$\Phi_{DS} = \begin{pmatrix} \zeta_{DS}^+ \\ \zeta_{DS}^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\zeta_1 + i\zeta_2) \\ \frac{1}{\sqrt{2}}(\zeta_3 + i\zeta_4) \end{pmatrix}. \tag{3.6}$$

The symmetry breaking of the $SU(2)_d$ and the $SU(2)_L \otimes U(1)_Y$ when the potential equation (3.3) is minimized: $\frac{\partial V(H,\Phi)}{\partial \phi_3} = \frac{\partial V(H,\Phi)}{\partial \zeta_3} = 0$. Both the dark scalar and the Higgs boson acquires non-zero vevs:

$$\begin{aligned} v_h^2 &= \frac{\frac{\lambda_{sd}\mu_\Phi^2}{2} - \mu_H^2\lambda_\Phi}{\lambda_h\lambda_\Phi - \frac{\lambda_{DS}^2}{4}}, \\ v_d^2 &= \frac{\frac{\lambda_{sd}\mu_h^2}{2} - \mu_\Phi^2\lambda_h}{\lambda_h\lambda_\Phi - \frac{\lambda_{DS}^2}{4}} \end{aligned} \tag{3.7}$$

and choosing one of them breaks the symmetry of the gauge groups $SU(2)_L \otimes U(1)_Y$ and $SU(2)_d$, then perturbing the Higgs and the dark scalar doublet around their vevs respectively do not change the eom; so the scalar vevs remain invariant. Recalling the Goldstone theorem [16], that justifies to writen down

$$\langle \Phi' \rangle_0 = e^{i\vec{\tau}.\frac{\vec{\xi}}{2v_\Phi}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_d + \eta_d \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_d + \eta_d \end{pmatrix} = \langle \Phi \rangle_0 .$$

Moreover, the unitary gauge gives the freedom to transfer 3 dofs of the Goldstone bosons ξ_d^i into the longitudinal polarization of the gauge bosons by choosing the freedom: $W'_{\mu,d} = W_{\mu,d} + \frac{1}{g_d v_d}(\partial_\mu \xi_d^i)$.

Rewriting the extended Lagrangian (3.4) in terms of the both perturbed scalar fields and substituting the given dark unitary gauge ends up with the following expression:

$$\begin{aligned}\mathcal{L}_{ext} =& \mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu \eta_d^{'})(\partial^\mu \eta_d^{'}) - \frac{1}{4}W^a_{\mu\nu,d}W_d^{\mu\nu,a} + \frac{1}{8}(g_d v_d)^2 W^a_{\mu,d}W_d^{\mu,a} \\ &+ \frac{1}{8}g_d^2 \eta_d^{'2} W^a_{\mu,d}W_d^{\mu,a} + \frac{1}{4}g_d^2 v_d \eta_d^{'} W^a_{\mu,d}W_d^{\mu,a} - \frac{\lambda_{sd}}{2}(\eta_d^{'} + v_d)^2 (H^\dagger H) \\ &- \frac{\mu_\Phi^2}{2}(\eta_d^{'} + v_d)^2 - \frac{\lambda_\Phi}{4}(\eta_d^{'} + v_d)^4\end{aligned} \tag{3.8}$$

In the SM, there is an $SU(2)_L \otimes U(1)_Y$ symmetry since the fermions exist. So the gauge bosons $W^\pm$ and Z^0 are not mass-degenerate and hence they are unstable. But the Lagrangian (3.8) demonstrates an important consequence: there exists a custodial $SO(3)$ symmetry in the dark gauge bosons(W_d^a) component space; in return, this symmetry forbids the $SU(2)_d$ triplets to decay into any $SO(3)$ singlets(SM particles or $\eta_d^{'}$). As it will be clearer soon, these dark gauge bosons will be degenerate in the mass eigenvalues, and their decay modes are absolutely forbidden unless higher-dimensional operators occur in the Lagrangian(3.8). These consequences indicate why the $SU(2)_d$ non-abelian gauge extension offers viable DM candidate since they have survived up to current stage of the cosmic inflation [52].

Furthermore this stability constraint can be seen from the absence of a such term of the form $\partial_\mu W^a_{\mu,d}$, which produces unstable gauge bosons in the theory.

To get the masses of the scalar bosons, rewriting the expression (3.7) with a slight change:

$$\mu_H^2 = -(\frac{v_d^2 \lambda_{sd}}{2} + v_h^2 \lambda_h) \quad , \quad \mu_\Phi^2 = -(\frac{v_h^2 \lambda_{sd}}{2} + v_d^2 \lambda_d). \tag{3.9}$$

However the terms appear in this form(3.9) are not physical mass eigenvalues. If the equation (3.9) is solved for the eigenvalues of the mass matrix:

$$\boxed{\begin{aligned} m_{h'}^2 &= \lambda_h v_h^2 + \lambda_d v_d^2 \mp \sqrt{\lambda_{sd}^2 v_d^2 v_h^2 + \lambda_h^2 v_h^4 - 2\lambda_d \lambda_h v_h^2 v_d^2 + \lambda_d^2 v_d^4} \\ m_{\eta_d^{'}}^2 &= \lambda_h v_h^2 + \lambda_d v_d^2 \pm \sqrt{\lambda_{sd}^2 v_d^2 v_h^2 + \lambda_h^2 v_h^4 - 2\lambda_d \lambda_h v_h^2 v_d^2 + \lambda_d^2 v_d^4}\end{aligned}} \tag{3.10}$$

Depending on the recent experiments, which have not observed the DM particles so far; equation (3.10) gets (+) sign if $\lambda_d v_d^2 > \lambda_h v_h^2$ since the DM particles are considered conventionally heavier than the physical Higgs unless the mixing angleα is large.

But small mixing angle regime is also considered above for the further objectives, thus $(-)$ sign indicates $\lambda_d v_d^2 < \lambda_h v_h^2$. The physical mass eigenstates are those diagonalize the mass matrix $\tilde{M}^2$ and they are given by the linear combination of h and η_d[2]:

$$\begin{pmatrix} h' \\ \eta_d' \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h \\ \eta_d \end{pmatrix} \quad ; \quad \cos\alpha = \left(1 + \frac{\mu_h^2 - 2v_h^2\lambda_h}{\mu_h^2 - 2v_d^2\lambda_d}\right)^{-\frac{1}{2}}. \quad (3.11)$$

The relations among the parameters (4.3) of the NMEHS model can be constructed with a number generator. However the generated values must obey the cosmological constraints deduced by the experimental data of the Planck[53]. Some parameters can be fixed to reduce the degrees of freedom throughout the model given. This non-minimal extension in it's current stage, depends on the parameters $\lambda_h, \lambda_d, \lambda_{sd}, g_d$ and v_h. But $v_h^2 = \frac{1}{\sqrt{2}G_F}$ is already known in the SM and $m_{h'} \approx 125.2GeV$ [17]. Such fixing of the parameters can be used for phenomenological purposes through a modification of the expression (3.10)

$$\lambda_{sd}^2 = \frac{(m_{h'}^2 - 2\lambda_h v_h^2)(m_{h'}^2 - 2\lambda_d v_d^2)}{v_h^2 v_d^2}, \quad (3.12)$$

where the coupling of the scalar mixing has also symmetric characteristic under $m_{h'} \leftrightarrow m_{\eta_d'}$, that is expected from the small mixing angle regime. This is where the WIMPs model will be discussed in the next section. And if α approaches zero, the scalar coupling of the both sector vanishes and familiar results:
$m_{h'}^2 = 2\lambda_h v_h^2$ and $m_{\eta_d'}^2 = 2\lambda_d v_d^2$ occur.

The motivation of the extended model of the SM, is to acquire the masses of the dark gauge bosonsW_d^a, and it can be obtained from the expression (3.8) with a degenerate mass spectrum:

$$\boxed{M_{W_d^a} = \frac{g_d v_d}{2}}. \quad (3.13)$$

This result shows that why g_d is chosen to be an independent parameter, since the dark gauge bosons plays role for the DM fields.

[2] From now on the 3^{rd} components of the doublets will be expressed as $\phi_3 \to h$ and $\zeta_3 \to \eta_d$ for an elegant notation.

Moreover, the linear combination given by the expression (3.11) has an illuminating conclusion: all of the SM particles that can couple to the Higgs field can also interact with the dark scalar field η_d as well. Such conclusion is just one of the arguments why the DM feels the presence of the gravitational interaction. And if required, one can find the physical state based Lagrangian at elsewhere [43].

Unconstrained and arbitrariness condition of the coupling values have an essential importance, these conditions permits to use of the perturbative methods on the higher level Feynman diagrams.

In the last section of this study, statistical methods and the relic abundance for the DM particles will be discussed through the annihilation and the semi-annihilation Feynman diagrams of the gauge bosons by inclusion of the both sectors.

CHAPTER 4

RELIC DENSITY FOR NON-ABELIAN DARK MATTER

Main requirements for the dark matter candidates are listed as follow [39]:

- It must weakly interact with baryons and photon due to cosmological constraints.
- It must be non-baryonic at large amounts since baryonic density $\Omega_b h^2 \approx 0.022$ constituting a small amount throughout entire matter density [36]
- It is probably non-relativistic (CDM), since relativistic DM particles could cause less dense cosmic structures.
- It must have lifetime longer than the age of the universe or its creation-annihilation rates must be equal.

Three of these four requirements are either used or needed as a result of the model extension, that is considered in the previous section. The last requirement can be considered as 'axiom' due to the results of the modern cosmology.

Relic abundance of a particle is defined as remaining or present density that survives after the Big Bang in terms of cosmology literature. Latest results from the Planck Satellite [53] express the DM relic density $\Omega_{DM} h^2 \approx 0.1187$. The DM relic abundance is mainly motivated to study evolution of WIMPs. Conventionally most of the Big Bang models assume that the dark matter and the SM particles were in thermal equilibrium.

Consequently all particle species in the early universe can be given as heat reservoir for the DM and the SM particles, in that case the Boltzmann statistics can be used for the density evolution, otherwise they fall out of thermodynamical equilibrium.

When the interaction strength of two sectors drops below the expansion rate of the universe, equilibrium breaks down and both sectors become decoupled [54]. This is the point where the individual evolutions of the SM and the DM particles had begun.

The relic abundance for the DM is given as a inverse function of the average temperature through the cosmic inflation

$$Y = \frac{n_{DM}}{T^3},$$

where n_{DM} is the number density of the DM candidates, and only way to change the number of particles is through creation or annihilation processes. Once annihilation and creation rates are relatively smaller than the cosmic expansion rate, that is the Hubble constant $H(t)$ [55, 56], then the DM density becomes too small for the annihilations to change the particle number. After the decoupling stage of both sectors, the number of the DM particles will remain unaltered, but the number density n_{DM} will decrease due to the cosmic expansion. In the light of these predictions, cross sections for each of the DM gauge bosons will be discussed throughout the following subsection via statistical mechanics whenever it is needed.

4.1 Boltzmann Statistics and DM Relic Density Calculation

By considering cosmology approach, creation and annihilation of particles in early stage of the universe could occur either in a thermal process or through a phase transformation. But this study based on the former, since WIMPs assumed to be thermal relic. An elementary expression for the annihilation rate per particle is

$$\Gamma = n\sigma v,$$

where n is the number density, v is the thermally averaged velocity of the particles and σ is the cross section of the given interaction.

As the cosmic inflation proceeds, the mean free path of particles increased, hence annihilation rate became less favorable. Furthermore, low average temperature of the universe led to difficulties for the formation of new heavy particles, hence so called 'freeze-out' scenario begun. Fundamental statistical mechanics can be used for the evaluation of the number densities in the non-relativistic regime at equilibrium. In such a case, the number density is

$$n_{eq} = \left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m}{T}}.$$

After the early stage interactions through the entire cosmic expansion [52], the DM was absent in the cosmic heat reservoir, and having a freeze-out energy density. To construct the DM relic abundance; the freeze-out temperature for the DM in which it had left the cosmic heat reservoir, must be expressed. And one of the main assumptions is based on 'mean free path' for the DM particles, hence the further calculations would be simplified. And a conventional way to describe the decoupling stage, is to indicate an initial condition [54]: $\Gamma = H$ where the annihilation rate approaches to the Hubble parameter.

The Boltzmann equation for the number density of the j-particle in the absence of interactions becomes:

$$\frac{dn^j_{eq}}{dt} + 3\frac{\dot{a}}{a}n^j_{eq} = 0. \tag{4.1}$$

That expression ensures that the number of particles in a fixed physical volume, remains constant. But the modification comes from interactions; the number density is decreased in an expanding volume by a factor: $n \propto a^{-3}$

$$a^{-3}\frac{d(n^j a^3)}{dt} = C^j(n_i), \tag{4.2}$$

where the right hand side includes collision terms which depends on the given specific interactions. This is the Boltzmann equation, simply devoted to express time-dependent number density of the given n^j particle in an expanding universe. Focusing on assumptions of CDM mentioned earlier and keeping existence of kinetic energy after the decoupling stage.

And since the interactions of 3 or more particles at initial state is less favorable, strictly focusing on two body interactions:

$$1 + 2 \rightleftarrows 3 + 4,$$

where particle 1 and particle 2 can annihilate producing particles 3 and 4 and vice versa. Following the tracks of the number density n_1, the rate of density change of n_1 given by the difference between the rates for producing and destroying the species. Thus there can be a claim of both terms in the Boltzmann equation

$$a^{-3}\frac{dn^1 a^3}{dt} = -An^1n^2 + Bn^3n^4, \tag{4.3}$$

where the first term in the right hand side stands for the destruction, and the second term for the production of particles with unknown coefficients A and B respectively.

A quick justification comes from the use of both statistical mechanics and collision theory as follow

$$\begin{aligned} a^{-3}\frac{dn_1 a^3}{dt} = \int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} \\ \times (2\pi)^4 \delta^{(4)}(P_1^\mu + P_2^\mu - P_3^\mu - P_4^\mu)|\mathcal{M}|^2 \\ [f_3 f_4 (1 \pm f_1)(1 \pm f_2) - f_1 f_2 (1 \pm f_3)(1 \pm f_4)]. \end{aligned} \tag{4.4}$$

Since the interactions are present, the left hand side of the equation (4.4) indicates that the density multiplied with the scale factor cubed demonstrate the mechanism of the expanding universe, which grows by a^3 with fixed number of particles, hence the number density falls off rapidly by a^{-3}. And the functional terms are probability expressions; these are mere consequence of statistical mechanics of particles. The rate of producing particle-1 is scaled with the occupation functions of particles f_3,f_4 whereas destruction terms are proportional with f_1,f_2. For the term $(1 \pm f)$, plus sign for bosons and minus sign for fermions due to Pauli exclusion principle. And focusing on statistical mechanics of interacting particles in which chemical potential (μ) is present. But assuming thermal equilibrium and sufficiently low temperatures that is smaller than $(E-\mu)$, such a domain allows to ignore quantum statistics namely f_i terms, statistical distribution can be written

$$f(E) \propto e^{\frac{\mu - E}{k_b T}},$$

where k_b is the Boltzmann constant. Furthermore, omitting the occupation factors, the last term of the equation (4.4) can be written approximately

$$[f_3f_4(1 \pm f_1)(1 \pm f_2) - f_1f_2(1 \pm f_3)(1 \pm f_4)] \approx e^{-\frac{(E_1+E_2)}{T}}[e^{\frac{(\mu_3+\mu_4)}{T}} - e^{\frac{(\mu_1+\mu_2)}{T}}],$$

where the energy conservation $E_1 + E_2 = E_3 + E_4$ has already been used. By using the number density of the i^{th} particle via the grand canonical distribution:

$$n^i = g^i e^{\frac{\mu_i}{T}} \int \frac{d^3p}{(2\pi)^3} e^{-\frac{E_i}{T}} = e^{\frac{\mu_i}{T}} n^i_{eq},$$

where the equilibrium number density is simply quoted as

$$n^i_{eq} = g_i \int \frac{d^3p}{(2\pi)^3} e^{-\frac{E_i}{T}}$$

hence the expression can be rewritten as follow

$$[f_3f_4(1 \pm f_1)(1 \pm f_2) - f_1f_2(1 \pm f_3)(1 \pm f_4)] = e^{-\frac{(E_1+E_2)}{T}} \left[\frac{n_3n_4}{n_3^{eq}n_4^{eq}} - \frac{n_1n_2}{n_1^{eq}n_2^{eq}}\right]$$

So, finally introducing the thermally averaged cross section

$$\langle \sigma v \rangle_T = \frac{1}{n_1^{eq}n_2^{eq}} \int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} \times e^{-\frac{(E_1+E_2)}{T}} \times (2\pi)^4 \delta^{(4)}(P_1^\mu + P_2^\mu - P_3^\mu - P_4^\mu)|\mathcal{M}|^2$$

that redefines the Boltzmann equation [57]

$$\boxed{a^{-3}\frac{d(n_1a^3)}{dt} = n_1^{eq}n_2^{eq} \langle v\sigma_{12\to 34}\rangle \left(\frac{n_3n_4}{n_3^{eq}n_4^{eq}} - \frac{n_1n_2}{n_1^{eq}n_2^{eq}}\right)} \tag{4.5}$$

Since the DM is constitution of massive particles and these were assumed to be in the thermal equilibrium with the SM particles through the early stage of universe, thus it will be suitable to check the CDM relic abundance after the decoupling. The Boltzmann equation can be written in a compact form as follow

$$a^{-3}\frac{d(n_1a^3)}{dt} = \langle \sigma v \rangle (n_{1,eq}^2 - n_1^2).$$

Or a more conventional form in terms of the Hubble parameter

$$\boxed{\dot{n}_i + 3Hn_i = -\langle \sigma v \rangle \left[n_1n_2 - \left(\frac{n_1n_2}{n_3n_4}\right)_{eq} n_3n_4\right]} \tag{4.6}$$

At this point, coannihilation parameters are not included yet.

The dark gauge bosons W_d^a of the NMEHS, are the DM candidates, hence the evolution of the dark bosons is to be evaluated once they decouple from the cosmic heat reservoir.The equilibrium densities depend merely on the temperature and the mass terms for the given system. Since the dark gauge bosons are mass degenerate as a consequence of the $U(1)_d$ symmetry absence, n_i^{eq} will be identical for each candidate. So, it is expected to obtain the same annihilation and coannihilation cross sections from the respective Feynman diagrams. All possible annihilation and coannihilation diagrams are given below for the non-abelian dark gauge bosons in figs. (4.1) and (4.2)

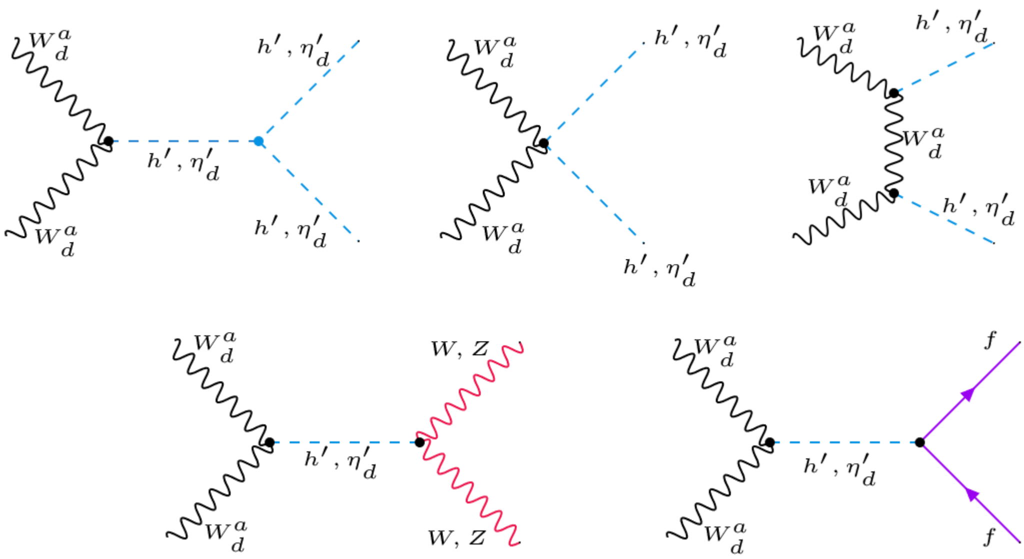


Figure 4.1: Diagrams contributing annihilation cross sections

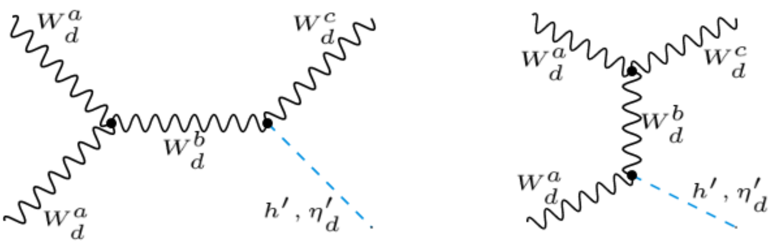


Figure 4.2: Diagrams contributing coannihilation cross sections

It was expressed in equation (3.11), that the physical eigenstates are in the form of linear combination, hence they interact with the SM particles; in return, this interaction leads them to decouple from the cosmic heat reservoir after the DM gauge bosons had left. It is assumed that in the CDM scenario, both densities of the physical scalars remain unaltered after the decoupling, hence they obey a quasistatic equilibrium condition to maintain their equilibrium densities:

$$n_{\eta_d'} = n^{eq}_{\eta_{d,}'} \quad ; \quad n_{h'} = n^{eq}_{h'}.$$

Boltzmann equation is rewritten for all degenerate dark gauge bosons W_d^a ($a = 1, 2, 3$), and this time including the coannihilation cross sections, since they obviously change the DM particle density throughout the model.

The Boltzmann equation for each dark gauge boson is

$$\begin{aligned}
\frac{dn_1}{dt} + 3Hn_1 = &\langle \sigma_{ann} v \rangle \, (n_{eq}^2 - n_1^2) - \langle \sigma_{coann} v \rangle \\
&\times \left[(n_a n_b - \frac{n_c}{n_{eq}} n_{eq}^2) + (n_a n_c - \frac{n_b}{n_{eq}} n_{eq}^2) - (n_b n_c - \frac{n_a}{n_{eq}} n_{eq}^2) \right] \\
\frac{dn_2}{dt} + 3Hn_2 = &\langle \sigma_{ann} v \rangle \, (n_{eq}^2 - n_2^2) - \langle \sigma_{coann} v \rangle \\
&\times \left[(n_b n_c - \frac{n_a}{n_{eq}} n_{eq}^2) + (n_b n_a - \frac{n_c}{n_{eq}} n_{eq}^2) - (n_c n_a - \frac{n_b}{n_{eq}} n_{eq}^2) \right] \\
\frac{dn_3}{dt} + 3Hn_3 = &\langle \sigma_{ann} v \rangle \, (n_{eq}^2 - n_3^2) - \langle \sigma_{coann} v \rangle \\
&\times \left[(n_c n_a - \frac{n_b}{n_{eq}} n_{eq}^2) + (n_c n_b - \frac{n_a}{n_{eq}} n_{eq}^2) - (n_a n_b - \frac{n_c}{n_{eq}} n_{eq}^2) \right]
\end{aligned}$$

Once more, the mass degeneracy of the dark gauge bosons simplifies these equations, time evolution of the densities are symmetric under particle numbers and all of them decouple from the heat reservoir at the same equilibrium density. Thus they simplify

$$\boxed{\dot{n}_a + 3Hn_a = \langle \sigma_{ann} v \rangle \, (n_{eq}^2 - n_a^2) - \langle \sigma_{coann} v \rangle \, n_a (n_a - n_{eq})}. \tag{4.7}$$

The total DM density with degeneracy is just $n_{tot} = 3n$. The expression(4.7) is called as the Lee-Weinberg equation [58], that describes the evolution of WIMPs after they decouple from a heat reservoir. But the solution of this equation is a non-trivial problem and requires some simplifications [59].

Starting from the relation of total entropy density of the universe and the relic abundance parameter $Y = \frac{n}{s}$

$$s_{tot} = 2\pi^2 g_*(T)\frac{T^3}{45}, \tag{4.8}$$

where g_* standas for relativistic degrees of freedom:

$$g_*(T) = \sum_{bosons} g_i(\frac{T_i}{T})^3 + \frac{7}{8}\sum_{fermions} g_i(\frac{T_i}{T})^3 \tag{4.9}$$

Invoking the conservation of entropy per moving volume sa^3=constant, the Lee-Weinberg equation is rewritten as

$$s\dot{Y} = \langle\sigma_{ann}v\rangle\, s^2(Y_{eq}^2 - Y_a^2) - \langle\sigma_{coann}v\rangle\, Y(Y_a - Y_{eq}) \tag{4.10}$$

Furthermore, a new change of parameters introduced as $x = \frac{T}{m_{W_d}}$ recovers modification of the equation (4.7)

$$\frac{dY}{dx} = 2\pi^2 g_*(T)\frac{m_{W_d}^3 x^2}{45H}[\langle\sigma_{ann}v\rangle\,(Y^2 - Y_{eq}^2) + \langle\sigma_{coann}v\rangle\, Y(Y - Y_{eq})] \tag{4.11}$$

For a given average $\langle\sigma v\rangle\,(x)$ the equation (4.11) requires a numerical solution. But there is an approximation for the modified equation. Keeping in mind that the universe was extremely hot at the beginning of the cosmic inflation $x = \frac{T}{m_{W_d}} > 1$ such that fluctuations of the relic density from the equilibrium value was recovered very fast, and that condition allows to use quasistatic approach. As the cosmic inflation goes on, the temperature of the universe decreased. And the DM particles approach to non-relativistic limit $x \ll 1$, in this limit equilibrium relic abundance and its specific value at certain point have been distinguished from each other. Decoupling stage is defined in terms of relative seperation between specific abundance and equilibrium abundance in the order $\delta \approx \mathcal{O}(1)$

$$\delta = \frac{Y_{decouple} - Y_{eq}}{Y_{eq}} \quad ; \quad \frac{dY_{decouple}}{dx} = (1+\delta)(\frac{dY_{eq}}{dx}). \tag{4.12}$$

To find out the temperature of the decoupling stage in freeze-out scenario; adding equation (4.11) into the expression (4.12) with redefinition of the parameters:
$\gamma = x^{-1} = \frac{m_{W_d}}{T}$, and $y_{eq} = e^{\gamma}Y_{eq}$.

Substituting all these definitions into (4.11) and rewriting the following relation:

$$\frac{dY}{dx}\Big|_{x=x_{decouple}} = \left[(1+\delta)\left(\frac{dy_{eq}}{d\gamma} - y_{eq}\right)(-\gamma^2 e^{-\gamma})\right]\Bigg|_{\gamma=\gamma_{decouple}}, \tag{4.13}$$

and using functional iteration of the Mathematica

$$e^{\frac{m_{W_d}}{T}} = \left[\frac{T^4 2\pi^2 g_*(T) y_{eq}^2}{45 m_{W_d} H(\frac{dy_{eq}}{d\gamma} - y_{eq})(1+\delta)}\Big(\langle\sigma_{ann}v\rangle\,\delta(\delta+2) + \langle\sigma_{coann}v\rangle\,\delta(\delta+1)\Big)\right]\Big|_{x=x_{dec}} \tag{4.14}$$

where, the condition $\frac{T}{m_{W_d}}|_{decouple} > 1$ must hold for the Boltzmann distribution in which the equilibrium abundance is given by [60]:

$$Y_{eq} = \frac{n_{eq}}{s_{eq}} = \frac{45 g_n M^2 K_2}{4\pi^4 g_* T^2} \tag{4.15}$$

where $n_{eq} = \frac{m_{W_d}^2 T}{2\pi^2} K_2(x)$ and $K_2(x)$ is defined as the modified Bessel function of the second kind. Using equation (4.14) one gets

$$\boxed{T_{frozen} = m_{W_d}\left[\log\left[\frac{T_{frozen}^4 2\pi^2 g_*(T) y_{eq}^2}{45 m_{W_d} H(\frac{dy_{eq}}{d\gamma} - y_{eq})(1+\delta)}\Big(\langle\sigma_{ann}v\rangle\,\delta(\delta+2) + \langle\sigma_{coann}v\rangle\,\delta(\delta+1)\Big)\right]\right]^{-1}}. \tag{4.16}$$

The equation (4.16) will demonstrate the relation between $T_{frozen} - m_{W_d}$. It is necessary to evaluate annihilation and coannihilation cross sections.

Once $(\frac{m_{W_d}}{T})_{initial}$ value is given logarithmically, relation between the dark gauge bosons and the decoupling temperature can be plotted. And as mentioned before, having the dark gauge bosons degenerate mass will drastically simplify the computation of the diagram amplitudes

4.2 Scattering Amplitudes for DM Gauge Bosons

First of all, the vertex factors must be extracted from each relevant interaction. Writing each interaction term in the Lagrangian (3.8) and changing the derivatives with the momentum operator($i\hbar\partial_\mu \to p_\mu$) and extracting the involved field parameters from the interaction Lagrangians, we obtain the vertex factors for the dark gauge bosons. Then quoting the mixing angle for the physical eigenstates from the equation (3.11)

$$\cos\alpha = \left(1 + \frac{\mu_h^2 - 2v_h^2\lambda_h}{\mu_h^2 - 2v_d^2\lambda_d}\right)^{-\frac{1}{2}}.$$

The vertex factors for VVSS, FFS, VVV, SSS and VVS type interactions are depicted in figs. (4.3) to (4.6)[1]

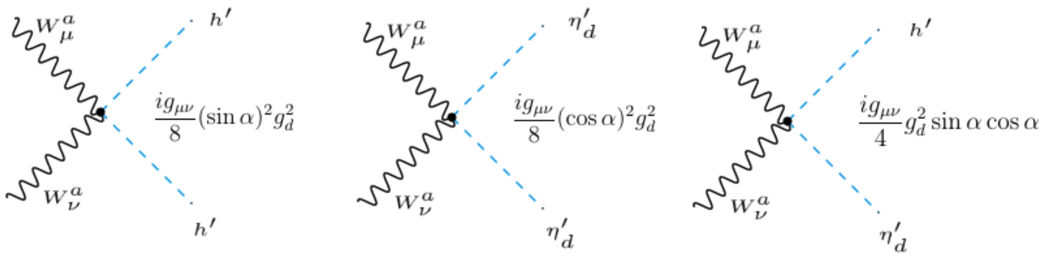


Figure 4.3: The relevant vertex factors for four-point VVSS type of interactions

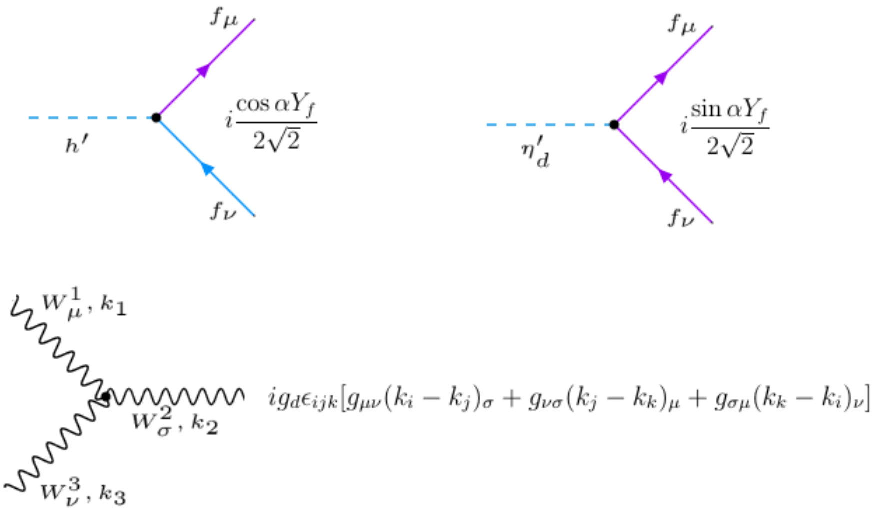
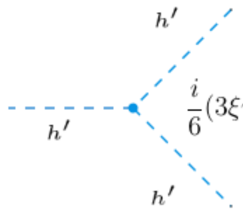
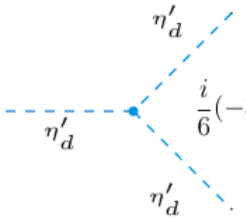


Figure 4.4: The relevant vertex factors for three-point FFS and VVV type of interactions

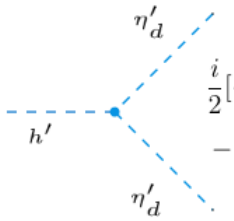
[1] For an elegant notation, redefining $\lambda_{sd} \to \xi$.



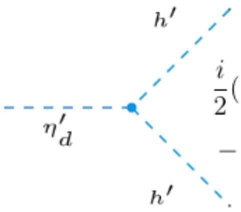
$$\frac{i}{6}(3\xi v_d \sin\alpha\cos^2\alpha - 3\xi v_h \cos\alpha \sin^2\alpha - 6\lambda_h v_h \cos^3\alpha + 6\lambda_d v_d \sin^3\alpha)$$



$$\frac{i}{6}(-3\xi v_d \sin^2\alpha\cos\alpha - 3\xi v_h \cos^2\alpha \sin\alpha - 6\lambda_h v_h \sin^3\alpha - 6\lambda_d v_d \cos^3\alpha)$$



$$\frac{i}{2}[-2v_d\xi\cos^2\alpha\sin\alpha + v_d\xi\sin^3\alpha - \xi v_h\cos^3\alpha + 2\xi\sin^2\alpha\cos\alpha v_h]$$
$$- 6\sin^2\alpha\cos\alpha\lambda_h v_h + 6\cos^2\alpha\sin\alpha v_d\lambda_d$$



$$\frac{i}{2}(-\xi v_d\cos^3\alpha + 2\xi v_d\cos\alpha\sin^2\alpha + 2\xi v_h\cos^2\alpha\sin\alpha - \xi v_h\sin^3\alpha)$$
$$- 6\lambda_h v_h\cos^2\alpha\sin\alpha - 6\lambda_h v_d\cos\alpha\sin^2\alpha$$

Figure 4.5: The relevant vertex factors for three-point SSS type of interactions

And finally coupling SM bosons to dark gauge bosons and dark scalars

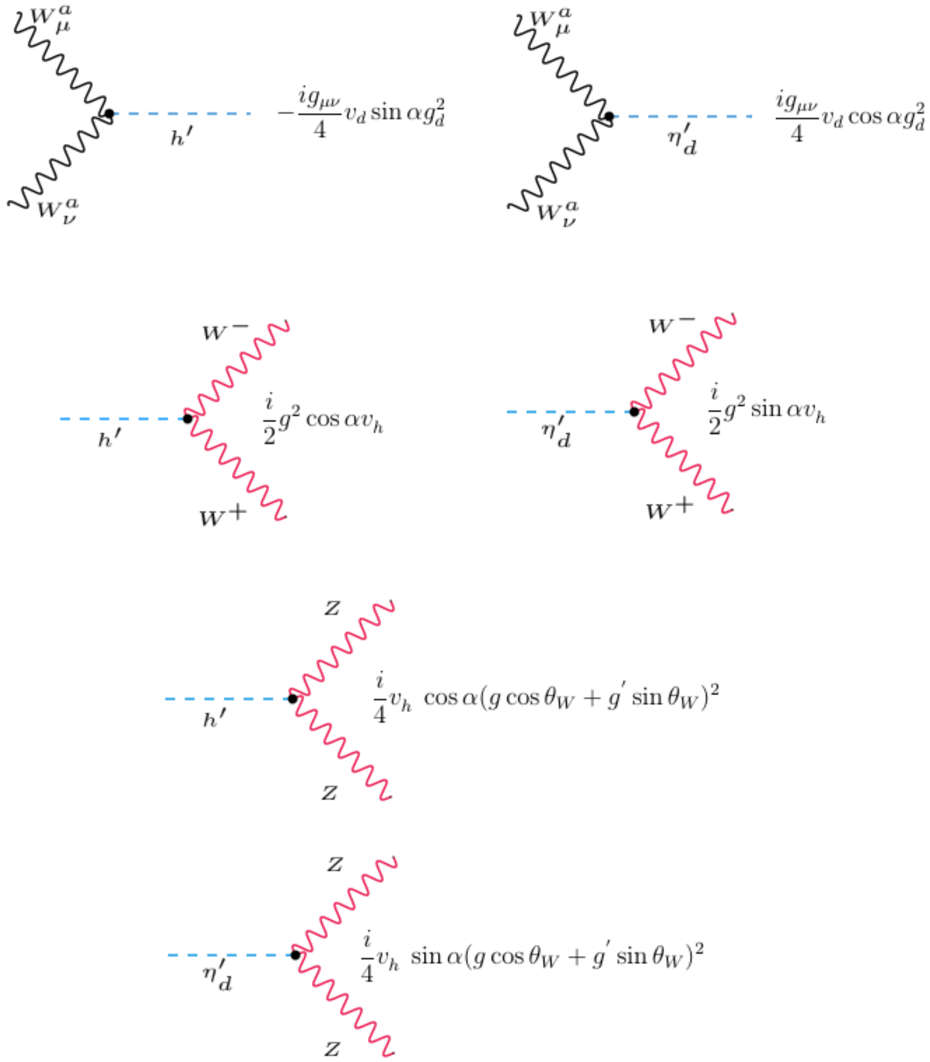


Figure 4.6: The relevant vertex factors for three-point VVS type of interactions

Mandelstam variables and special functions will be introduced whenever required. On the way of calculating the scattering amplitudes, the Higgs boson partial widths are taken from [61]. The analytical matrix element expressions for the following annihilation and coannihilation diagrams shown in figs. (4.1) and (4.2)

$$\mathcal{M}_{W_{d,a}W_{d,b}\to h'h'} = \frac{i}{2}g_{\mu\nu}v_d g_d^2 \frac{1}{(s-m_{h'}^2+im_{h'}\Gamma_{h'})}\Big[3\xi v_d\cos^2\alpha\sin\alpha - 3\xi v_h\sin^2\alpha\cos\alpha$$
$$- 6\lambda_h v_h\cos^3\alpha + 6\lambda_d v_d\sin^3\alpha\Big] - \frac{i}{2}g_{\mu\nu}v_d g_d^2\cos\alpha\frac{1}{(s-m_{\eta_d'}^2)}\times$$
$$\Big[-\xi v_d\cos^3\alpha + 2\xi v_d\sin^2\alpha\cos\alpha + 2\xi v_h\cos^2\alpha\sin\alpha - \xi v_h\sin^3\alpha$$
$$- 6\lambda_h v_h\cos^2\alpha\sin\alpha - 6\lambda_h v_d\sin^2\alpha\cos\alpha\Big] + \frac{i}{2}g_{\mu\nu}g_d^2\sin^2\alpha$$
$$- \frac{i}{4}g_{\mu\sigma}g_{\nu\lambda}v_d^2 g_d^4\sin^2\alpha\frac{1}{(t-m_{W_d}^2)}(\frac{k^\sigma k^\lambda}{m_{W_d}^2} - g^{\sigma\lambda})$$
$$- \frac{i}{4}g_{\mu\sigma}g_{\nu\lambda}v_d^2 g_d^4\sin^2\alpha\frac{1}{(u-m_{W_d}^2)}(\frac{p^\sigma p^\lambda}{m_{W_d}^2} - g^{\sigma\lambda}) \tag{4.17}$$

$$\mathcal{M}_{W_{d,a}W_{d,b}\to \eta_d'\eta_d'} = \frac{i}{2}g_{\mu\nu}v_d g_d^2\cos\alpha\frac{1}{(s-m_{\eta_d'}^2)}[3\xi v_d\sin^2\alpha\cos\alpha + 3\xi v_h\cos^2\alpha\sin\alpha$$
$$+ 6\lambda_h v_h\sin^3\alpha + 6\lambda_d v_d\cos^3\alpha] + \frac{i}{2}g_{\mu\nu}v_d g_d^2\sin\alpha\frac{1}{(s-m_{h'}^2+im_{h'}\Gamma_{h'})}$$
$$\times\Big[-2\xi v_d\cos^2\alpha\sin\alpha + \xi v_d\sin^3\alpha - \xi v_h\cos^3\alpha + 2\xi v_h\sin^2\alpha\cos\alpha$$
$$- 6\lambda_h v_h\sin^2\alpha\cos\alpha + 6\lambda_d v_d\cos^2\alpha\sin\alpha\Big] + \frac{i}{2}g_{\mu\nu}g_d^2\cos^2\alpha$$
$$- \frac{i}{4}g_{\mu\sigma}g_{\nu\lambda}v_d^2 g_d^4\cos^2\alpha\frac{1}{(t-m_{W_d}^2)}(\frac{k^\sigma k^\lambda}{m_W^2} - g^{\sigma\lambda})$$
$$- \frac{i}{4}g_{\mu\sigma}g_{\nu\lambda}v_d^2 g_d^4\cos^2\alpha\frac{1}{(u-m_{W_d}^2)}(\frac{p^\sigma p^\lambda}{m_{W_d}^2} - g^{\sigma\lambda}) \tag{4.18}$$

$$\mathcal{M}_{W_{d,a}W_{d,b}\to W^+W^-} = -\frac{i}{4}g_{\mu\nu}v_d g_d^2\cos\alpha\frac{1}{(s-m_{\eta_d'}^2)}g_{\sigma\lambda}g^2 v_h\sin\alpha$$
$$-\frac{i}{4}g_{\mu\nu}v_d g_d^2\sin\alpha\frac{1}{(s-m_{h'}^2+im_{h'}\Gamma_{h'})}g_{\sigma\lambda}v_h g^2\cos\alpha \tag{4.19}$$

$$\begin{aligned}\mathcal{M}_{W_{d,a}W_{d,b}\to ZZ} =& -\frac{i}{4}g_{\mu\nu}v_d g_d^2\cos\alpha\frac{1}{(s-m^2_{\eta'_d})}g_{\sigma\lambda}v_h\cos\alpha\big(g\cos\theta_W+g^{'}\sin\theta_W\big)^2\\ &-\frac{i}{4}g_{\mu\nu}v_d g_d^2\sin\alpha\frac{1}{(s-m^2_{h'}+im_{h'}\Gamma_{h'})}g_{\sigma\lambda}v_h\cos\alpha\\ &\times\big(g\cos\theta_W+g^{'}\sin\theta_W\big)^2\end{aligned} \tag{4.20}$$

$$\begin{aligned}\mathcal{M}_{W_{d,a}W_{d,b}\to f\bar{f}} =& -\frac{i}{4\sqrt{2}}g_{\mu\nu}v_d g_d^2\cos\alpha\frac{1}{(s-m^2_{\eta'_d})}g_{\sigma\lambda}Y_f\sin\alpha\\ &-\frac{i}{4\sqrt{2}}g_{\mu\nu}v_d g_d^2\sin\alpha\frac{1}{(s-m^2_{h'}+im_{h'}\Gamma_{h'})}g_{\sigma\lambda}Y_f\cos\alpha\end{aligned} \tag{4.21}$$

$$\begin{aligned}\mathcal{M}_{W_{d,a}W_{d,b}\to W_{d,c}h'} =& \frac{i}{2}v_d g_d^3\epsilon_{ijl}\Big[g_{\mu\nu}(k_i-k_j)_\gamma+g_{\nu\gamma}(2k_j+k_i)_\mu+g_{\gamma\mu}(-2k_i-k_j)_\nu\Big]\\ &\times\frac{1}{(k_i+k_j)^2-m^2_{W_d}}\left[\frac{(k_i+k_j)^\gamma(k_i+k_j)^\beta}{m^2_{W_d}}-g^{\gamma\beta}\right]g_{\beta\sigma}\sin\alpha\\ &+\frac{i}{2}v_d g_d^3\epsilon_{ilk}\Big[g_{\mu\gamma}(2k_i-k_k)_\sigma+g_{\gamma\sigma}(2k_k-k_i)_\mu+g_{\sigma\mu}(-k_i-k_k)_\gamma\Big]\\ &\times\frac{1}{(k_i-k_k)^2-m^2_{W_d}}\left[\frac{(k_i-k_k)^\gamma(k_i-k_k)^\beta}{m^2_{W_d}}-g^{\gamma\beta}\right]g_{\beta\nu}\sin\alpha\end{aligned} \tag{4.22}$$

only the mixing angle changes for $W_d-W_d-\eta_d^{'}$ vertex as compared to $W_d-W_d-h^{'}$, thanks to the symmetry of linear combinations of the mass eigenstates, so that

$$\begin{aligned}\mathcal{M}_{W_{d,a}W_{d,b}\to W_{d,c}\eta'_d} =& \frac{i}{2}v_d g_d^3\epsilon_{ijl}\Big[g_{\mu\nu}(k_i-k_j)_\gamma+g_{\nu\gamma}(2k_j+k_i)_\mu+g_{\gamma\mu}(-2k_i-k_j)_\nu\Big]\\ &\times\frac{1}{(k_i+k_j)^2-m^2_{W_d}}\left[\frac{(k_i+k_j)^\gamma(k_i+k_j)^\beta}{m^2_{W_d}}-g^{\gamma\beta}\right]g_{\beta\sigma}\sin\alpha\\ &+\frac{i}{2}v_d g_d^3\epsilon_{ilk}\Big[g_{\mu\gamma}(2k_i-k_k)_\sigma+g_{\gamma\sigma}(2k_k-k_i)_\mu+g_{\sigma\mu}(-k_i-k_k)_\gamma\Big]\\ &\times\frac{1}{(k_i-k_k)^2-m^2_{W_d}}\left[\frac{(k_i-k_k)^\gamma(k_i-k_k)^\beta}{m^2_{W_d}}-g^{\gamma\beta}\right]g_{\beta\nu}\cos\alpha\end{aligned} \tag{4.23}$$

Having all scattering amplitudes for the annihilation and coannihilation channels at hand, cross sections for these interactions can be obtained by using the center of mass frame. Two body scattering amplitude for $1 + 2 \to 3 + 4$ in the CM-frame is given by:

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \left(\frac{1}{8\pi}\right)^2 \frac{S}{(E_1 + E_2)^2} \frac{|\vec{P}_f|}{|\vec{P}_i|} |\mathcal{M}|^2 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \tag{4.24}$$

where $|\vec{P}_f| = \frac{1}{2(E_1+E_2)} \sqrt{\lambda((E_1 + E_2)^2, m_3^2, m_4^2)}$ in terms of triangle function λ. Then total cross section is obtained from integration over entire solid angle

$$\sigma_{CM} = \int_0^{\pi} \int_0^{2\pi} \left(\frac{d\sigma}{d\Omega}\right)_{CM} \sin\psi d\psi d\phi$$

Expressions (4.17) - (4.24) are substituted into the thermal averaged cross section [62, 63] for annihilation and coannihilation channels in terms of modified Bessel functions K_1, K_2 of the first and the second kind respectively:

$$\langle \sigma_{ann,co} v \rangle_T = \int_{4m_{W_d}}^{\infty} \frac{s\sqrt{s - 4m_{W_d}^2} K_1(\frac{\sqrt{s}}{T}) \sigma_{ann,co} v}{16 T m_{W_d}^4 K_2^2(\frac{m_{W_d}}{T})} ds. \tag{4.25}$$

Due to Hambye's model [43, 44], in the mass range $\frac{m_{h'}}{2} < m_{W_d} < m_{h'}$, which is rather less likely; if the physical Higgs is heavier than the dark gauge boson W_d within WIMP regime; the result is just the minus of expression (3.10). Then the annihilation channels of the dark gauge bosons to the dark scalar become Boltzmann suppressed. Hence the annihilation cross sections into the SM particles by Higgs total width $\Gamma_{h'}$ become[61]

$$\langle \sigma v \rangle = \frac{v_d^2 g_d^4 \sin^2\alpha}{8\sqrt{s}} g_{\mu\sigma} g_{\nu\lambda} \frac{\Gamma_{h'}(\sqrt{s})}{(s - m_{h'}^2)^2 + \Gamma_{h'}(m_{h'})} \left(\frac{k_1^\mu k_1^\nu}{m_{W_d}^2} - g^{\mu\nu}\right) \left(\frac{k_2^\sigma k_2^\lambda}{m_{W_d}^2} - g^{\sigma\lambda}\right) \tag{4.26}$$

for k_1, k_2 four-momentum of the incoming dark gauge bosons.

Taking Boltzmann suppresion into account and for small mixing angles in which the Higgs $h^{'}$ approaches the SM Higgs. Hence eliminating the annihilation and coannihilation channels with the dark scalar in the final states are negligible. Let us quote the expression (4.16) for $T_{frozen} - m_{W_d}$ relation:

$$T_{frozen} = m_{W_d}\Bigg[\log\Bigg[\frac{T_{frozen}^4 2\pi^2 g_*(T) y_{eq}^2}{45 m_{W_d} H(\frac{dy_{eq}}{d\gamma} - y_{eq})(1+\delta)} \times \Bigg(\langle\sigma_{ann}v\rangle\,\delta(\delta+2) + \langle\sigma_{coann}v\rangle\,\delta(\delta+1)\Bigg)\Bigg]\Bigg]^{-1}.$$

Using model parameter [44] for average velocity of the CDM in WIMP regime (constrained by theoretical limits) gives $\langle v\rangle_T \approx \sqrt{\frac{3T_f}{m_{W_d}}} \approx \frac{1}{2.8}$ and considering early stage interval for freeze-out parameter $x_{frozen} = \frac{m_{W_d}}{T_f} \approx 25 - 35$, it is ready for further discussion. Once an iterative solution method applied on the expression (4.16) consecutively, relation between m_{W_d} and T_f can be plotted. Choice of the initial value $\gamma_i = \frac{m_{W_d}}{T_f} = 30$ and interval for the mixing angle $\alpha = [0°, 89°]$ ends up with the graph in fig.(4.7)

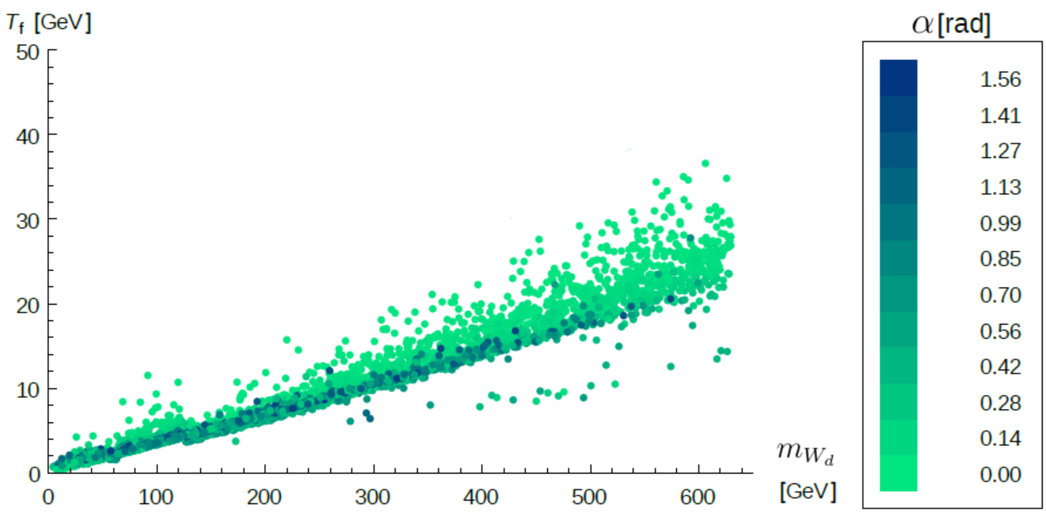


Figure 4.7: Relation between mass of dark gauge boson and decoupling temperature

Since Boltzmann suppresions leads to small angles for mixing, $h^{'}$ is drawn to the SM Higgs limit. Furthermore, for small angles(bright dots), decoupling temperature is relatively higher; that is a pure consequence of cross section at small angles.

4.3 Non-Abelian Dark Matter Relic Abundance

To get a better comprehension about relic density, let us start from the Robertson-Walker metric, which is a solution of the Einstein field equations [64] for isotropic expanding universe given by

$$ds^2 = -dt^2 + a^2(t)\left(r^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2 + \frac{dr^2}{1-Kr^2}\right) = g_{\mu\nu}dx^\mu dx^\nu \quad (4.27)$$

where the constant K is the curvature of space and can take values $K = -1, 0, 1$ depending on the geometry of spacetime: open,flat and closed. Because the cosmic expansion has been observed, the scale factor $a = a(t)$ is time-dependent. Furthermore, evolution of scale factor is given by the Friedmann equation [65], which is an another solution of the Einstein field equations.

$$H^2(t) + \frac{K}{a^2} = \frac{8\pi G_N}{3}\rho_{tot} \quad ; \quad \rho = \frac{\pi^2}{30}g_* T^4 \quad (4.28)$$

in which the total energy density has main components $\rho_{tot} = \rho_{matter} + \rho_{rad} + \rho_{vac}$. It is seen from the Friedmann equation, if $K = 0$ flat universe, then the energy density will be equal to the critical density

$$\rho_c = \frac{3H^2}{8\pi G_N} \quad (4.29)$$

The equation (4.28) can be written in a more conventional form

$$\frac{K}{H^2 a^2} + 1 = \frac{\rho}{\left(\frac{3H^2}{8\pi G_N}\right)} \equiv \Omega \quad (4.30)$$

where $\Omega = \frac{\rho}{\rho_c}$.

In general, energy density parameter of the universe is $\Omega \neq 1$. Although $\Omega_0 = 1$ is predicted by cosmic inflation, $\Omega(t)$ is effected by various number of parameters. Clearly, total energy density has main components $\Omega_0 = \Omega_\Lambda + \Omega_M$ where $\Omega_M = \Omega_{DM} + \Omega_b$. But from the equation(4.30) $\frac{K}{H^2 a^2} = \Omega - 1$, thus the variations from $\Omega_0 = 1$ can be realized as contribution from curvature expansion rate $\Omega_K = -\frac{K}{H_0^2 a_0^2}$. The total parameter can be written in terms of its components $\Omega_{tot} = \Omega_\Lambda + \Omega_M + \Omega_K = 1$

Consistency of observations and the Big Bang nucleosynthesis (BBN) [66] imposes an upper limit to the baryonic density $\Omega_b h^2 \leq 0.019$ [67]. Due to absence of direct measurement of the DM, an estimation of $\Omega_{DM} h^2$ relies on the asymmetry in CMB [35].

Linear relation between dark gauge boson's mass m_{W_d} and T_f utilizes calculation of the non-abelian DM relic density. Taking attention back to the relic density expression for the dark gauge bosons (4.11)

$$\frac{dY}{dx} = 2\pi^2 g_*(T)\frac{m_{W_d}^3 x^2}{45H}[\langle\sigma_{ann}v\rangle\,(Y^2 - Y_{eq}^2) + \langle\sigma_{coann}v\rangle\,Y(Y - Y_{eq})],$$

and considering the stage after decoupling of the DM because of cooling of the universe $Y \gg Y_{eq}$, Y_{eq}^2 term is neglected and the expression (4.11) reduces to

$$\frac{dY}{dx} = 2\pi^2 g_*(T)\frac{m_{W_d}^3 x^2}{45H}\left[Y^2\big(\,\langle\sigma_{ann}v\rangle + \langle\sigma_{coann}v\rangle\,\big) - YY_{eq}\big(\,\langle\sigma_{coann}v\rangle\,\big)\right] \quad (4.31)$$

At this stage, omitting the Y_{eq} term recovers the original solution proposed by [58]. The second term in expression (4.31) has a contribution only within the immediate decoupling stage and it can be neglected if annihilation channels dominate the total cross section $\langle\sigma_{ann}v\rangle \gg \langle\sigma_{coann}v\rangle$

$$\frac{dY}{dx} = 2\pi^2 g_*(T)\frac{m_{W_d}^3 x^2}{45H}Y^2\,\langle\sigma v\rangle\,. \quad (4.32)$$

That is the Lee-Weinberg expression for the relic density with $\langle\sigma v\rangle = \langle\sigma_{ann}v\rangle + \langle\sigma_{coann}v\rangle$. Solution can be obtained numerically [68] as depicted in fig. (4.8)

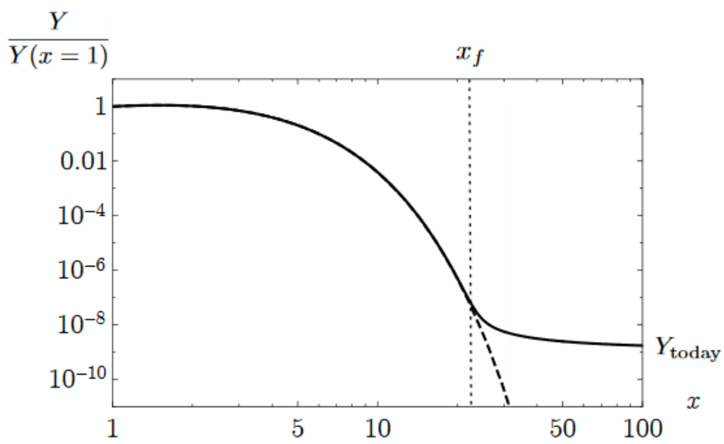


Figure 4.8: The DM relic density as a function of $\frac{m_{DM}}{T}$. Before the decoupling stage ($x < x_f$), density approaches to the equilibrium limit(dashed line). After the decoupling stage, relic density stays almost constant as time evolves(bold line).

However actual solution for the expression (4.31) requires some simplifications. By looking to expression (4.28); $H \propto \sqrt{\rho_{tot}}$ for flat space($K = 0$), and $\rho \propto T^4$, thus $H \propto x^2$. By considering this dependence, a simplification rises; $\frac{x^2}{H}$ remains unaltered and also recall that $\langle \sigma v \rangle_T$ remains constant. Moreover, an approximation for relativistic degrees of freedom as a function of entropy through early stage of the universe is given by [60]:

$$\begin{aligned} g_*(T) \approx g_{*s}(T) = \; & 2 + 6.\theta(T - m_W) + 3.\theta(T - m_Z) + 3.3\theta(T - m_{W_d}) + 3.\frac{7}{4} \\ & + \frac{7}{2}.\sum_{e,\mu,\tau} \theta(T - m_i) + \sum_{scalars} g_i \theta(T - m_i) \\ & + \theta(T - T_{QCD})\left(2.8 + \frac{21}{2}.\sum_{quarks} \theta(T - m_i)\right) \\ & = \theta(T - T_{QCD})\left[2.8 + \frac{21}{2}\left(\theta(T - m_u) + \theta(T - m_d) + \theta(T - m_c)\right.\right. \\ & \left.\left. + \theta(T - m_s) + \theta(T - m_b) + \theta(T - m_t)\right)\right] + 3.\frac{7}{4} + 2 \\ & + \frac{7}{2}\left(\theta(T - m_e) + \theta(T - m_\mu) + \theta(T - m_\tau)\right) + 2.3\theta(T - m_W) \\ & + 3\theta(T - m_Z) + \theta(T - m_{h'}) + \theta(T - m_{\eta_d'}) + 3.3\theta(T - m_{W_d}) \end{aligned} \tag{4.33}$$

where $T_{QCD} \approx 0.25$. The evolution of relativistic degrees of freedom for only the SM particle content is shown in fig.(4.9)

To compute $Y_{present}$; equation (4.31) is to be integrated with the assumptions above.[69, 70]

$$\boxed{Y_{present} \approx \left[2\pi^2 g_*(T) \frac{m_{W_d}^3 x^2}{45H}\left(\langle \sigma_{ann} v \rangle + \langle \sigma_{coann} v \rangle\right)\right]^{-1}_{frozen}} \tag{4.34}$$

Energy density for the dark gauge bosons can be given as

$$\begin{aligned} \rho_{present} = m_{W_d} n_{present} = m_{W_d} & \frac{s_{present}}{s_{frozen}} \frac{H_{frozen}}{(\langle \sigma_{ann} v \rangle_{frozen} + \langle \sigma_{coann} v \rangle_{frozen})} \\ = & \boxed{\frac{g_{*s}(T_{present})}{g_{*s}(T_{frozen})} \frac{\sqrt{8\pi G_N} m_{W_d} T^3_{\gamma,present}}{T_f(\langle \sigma_{ann} v \rangle_{frozen} + \langle \sigma_{coann} v \rangle_{frozen})} \sqrt{\frac{4\pi^3 g_*(T_f)}{45}}} \end{aligned} \tag{4.35}$$

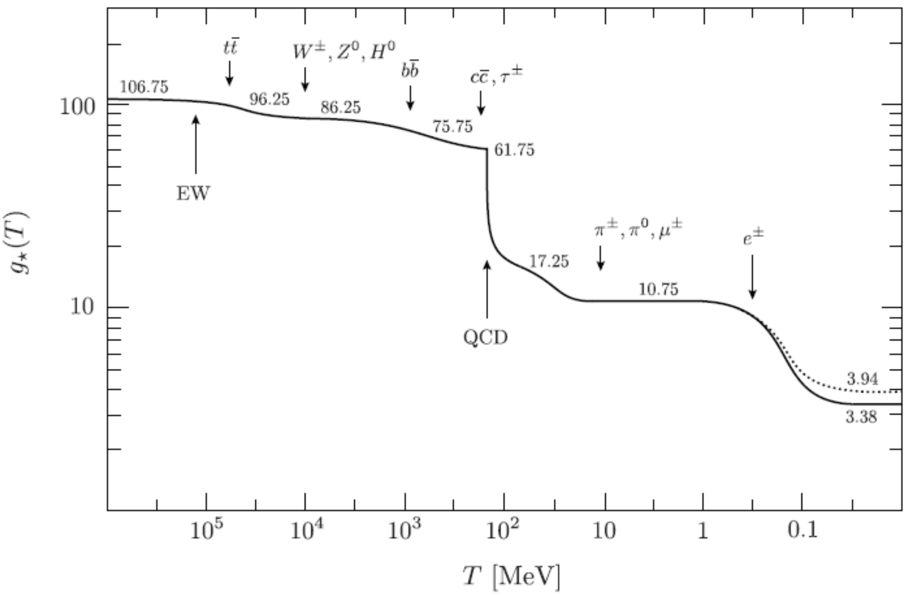


Figure 4.9: The evolution of relativistic dofs $g_*(T)$ in early stage of universe considering the SM particle content [62]. The dotted line indicates the number of effective dof in entropy g_{*s}

And furthermore; given the masses of the dark sector particles, constrained by the limitation of the NADM model, and the respective model parameters (3.4). By substituting the average values $m_{W_d} \approx 325 GeV$ and $m_{\eta'_d} \approx 2950 GeV$. The numerical solutions to expression (4.34) are shown in fig. (4.10)

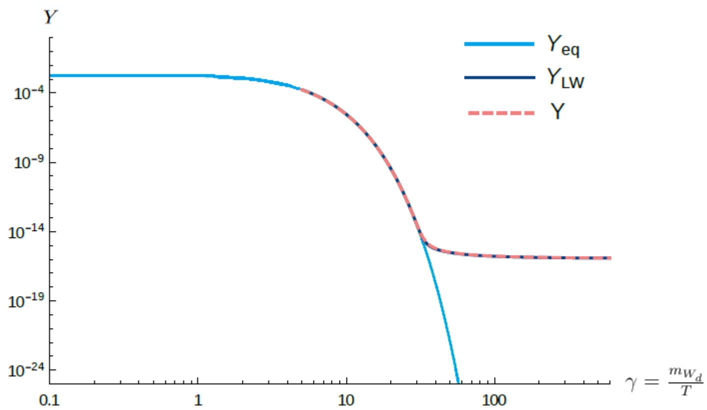


Figure 4.10: A numerical solution for Y. Blue line typically approaches to the equilibrium value. Lee-Weinberg behavior and solution to equation(4.31) without approximation, holds at considerable amount as a result of suppresion.

Finally, relic density for each dark sector particle at the current stage, can be given in terms of critical density[71]

$$\Omega_{W_d^a} h^2 = \frac{\rho_{a,present}}{\rho_c} = \frac{\rho_{a,present}}{\rho_\gamma} \Omega_\gamma h^2 \tag{4.36}$$

Defining the photon energy density and the photon relic abundance at the current cosmic stage as a function of average temperature[2] $\langle T \rangle \approx 2.7K$, and also writing the normalised Hubble parameter explicitly: $h = 0.7$. Using all of them to define the photon relic abundance $\Omega_\gamma^{present}$:

$$\begin{aligned} \varepsilon_\gamma^{present} = \frac{\pi^2}{15\hbar^3 c^3}(k_B T)^4 \rightarrow \quad \Omega_\gamma^{present} = \frac{\rho_\gamma^{present}}{\rho_c} &= \frac{8\pi^3 G_N}{45\hbar^3 c^5 H_{present}^2}(k_b T_{present})^4 \\ &= 2.47 \times 10^{-5} h^{-2}. \end{aligned} \tag{4.37}$$

Adding the equation (4.37) into the equation (4.36), and having the dark gauge bosons mass degenerate in the given model, we simply multiply by factor 3, hence the total relic abundance for the $SU(2)_d$ particles becomes

$$\begin{aligned} (\Omega h^2)_{tot} &= 3\Omega_{W_d^a} h^2 \\ &\approx \boxed{2.58 \times 10^{-10} \left(g_*^{T_f} \left(\frac{m_{W_d}}{T_f} \right) \right)^{-\frac{1}{2}} \frac{1}{(\langle \sigma_{ann} v \rangle_{frozen} + \langle \sigma_{coann} v \rangle_{frozen})}}. \end{aligned} \tag{4.38}$$

In the light of the expression (4.26) and the results of (4.7), the relic abundance completely depends on the number of relativistic dof after the decoupling stage and thermal cross section of the DM candidates. Although the NMEHS model can yield a wide range of DM abundance by considering variation on the model parameters, it is expected to deduce a reasonable abundance by choosing[3] $m_{W_d} \approx 325 GeV$ with $T_f \approx 10.8 GeV$ corresponding to $g_{*s}(T) \approx 100$ with inclusion of the dark sector particles. Hence the final value becomes

$$(\Omega h^2)_{tot} = \frac{2.58 \times 10^{-10}}{10 \times 2.2 \times 10^{-9}} \approx 0.012$$

[2] The present value for the Hubble parameter gives the energy density as $\rho_c = 1.054 \times 10^{-5} h^2 GeV.cm^{-3}$.
[3] Both large and small mixing angle of parameter space can produce the desired DM abundance, however some of the particular regions are excluded by experimental contraints.

Each value for the decoupling condition plotted in fig.(4.7), determines a distinct number for relativistic degrees of freedom. Because each of the SM particles are available in different freeze-out temperature and hence defining possible dofs for the dark gauge bosons to interact. Considering both mixing angle regime, numerical solutions to the equation (4.38) are shown in fig. (4.11)

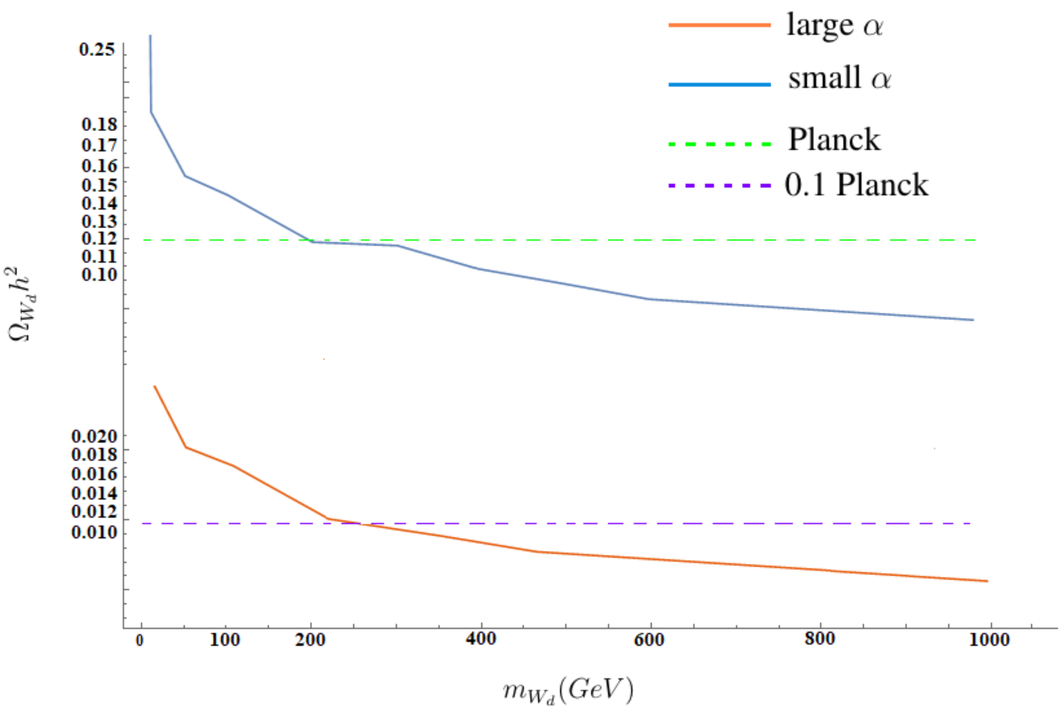


Figure 4.11: The relic abundance given by the $SU(2)_d$ sector gauge bosons, the lower purple dashed line sets a limit as 10% for the $SU(2)_d$ model's contribution to the observed result of Planck satellite and the upper green dashed line stands for the Planck's observation

The upper and lower values of the experimental constraints on the parameters of $SU(2)_d$ gauge group were chosen as a result of logical consistency. Since there is no strict constraint for the DM to be composed of one elementary particle such as W_d, the relic abundance of the non-abelian $SU(2)_d$ DM produced by the given model is set to a lower limit: 10% of the relic abundance observed by Planck satellite, via generating reasonable values for the parameter space. Leaving aside the suppresion of DM co/annihilation channels; logical values of thermal averages are chosen to satisfy the criteria specified above.

In fig. (4.11) we see that for the small angles, $m_{W_d} < 200GeV$ values are discarded since they exceed the relic abundance value observed by the Planck satellite. And for the large mixing angle values, relatively heavier dark gauge bosons $m_{W_d} > 200GeV$ are not so efficient to yield a considerable amount 10% of the Planck's observation. Furthermore, whenever the dark gauge bosons W_d are present, the minimum of the dof will be insufficient to produce relic abundance at the scale of Planck's results, since the relic abundance is inversly proportional to term g_{*s}

The interval for DM relic density due to the given extension can produce higher amounts of density if these parameters are changed depending on the current situation. However a wide range of parameter space is still allowed by the experimental constraints given in fig.(4.12):

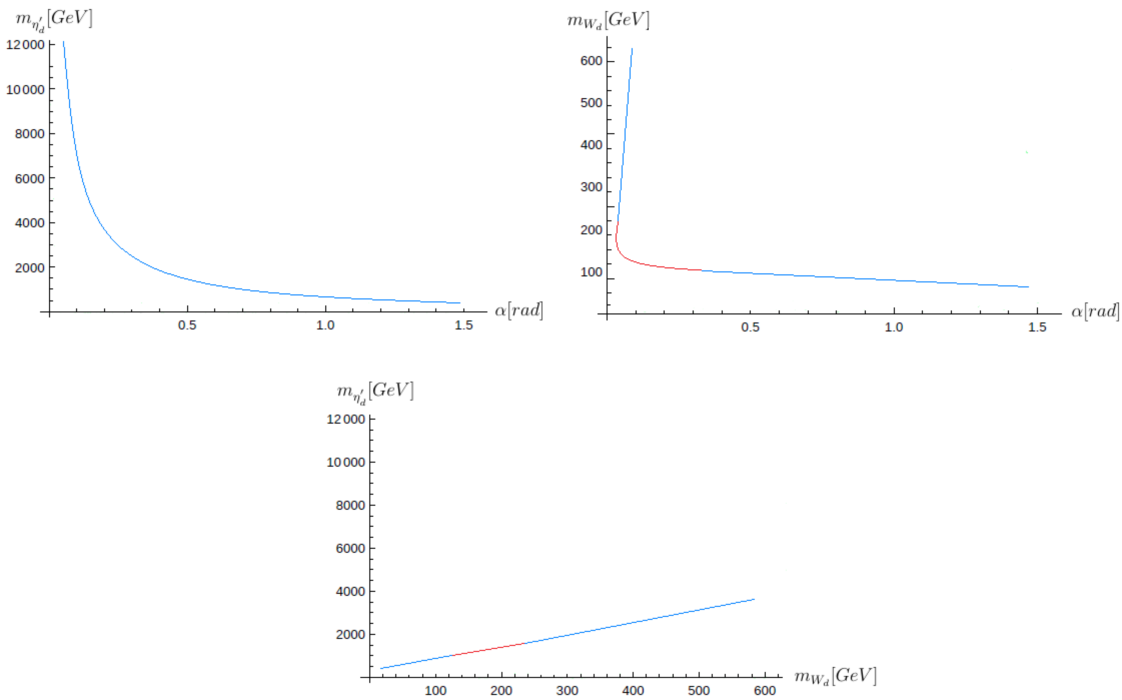


Figure 4.12: The extended plotting for the masses of $SU(2)_d$ dark particles. Blue lines is consistent with the experimental results. Whereas red lines stand for parameter interval, ruled out by observation of Planck satellite.

We can offer a quick justification for the mass interval of the dark gauge bosons W_d along with their co/annihilation cross sections proposed in this study. Assuming the DM is composed of one particle, it is possible to estimate its mass limit for this one particle by comparing the DM relic abundance with the Planck's data. In the regime when the radiation stage is overwhelming, it is still the same strategy to specify the decoupling moment: $H \approx T^2_{decouple}\sqrt{8\pi G_N}$. Thus freeze-out condition gives

$$n_{freeze-out} \approx \frac{T^2_{freeze}\sqrt{8\pi G_N}}{\langle\sigma v\rangle}.$$

If number density for equilibrium case in the non-relativistic regime is substituted as we did in chapter-5(4.1), one gets

$$(m_{DM}T_{freeze})^{\frac{3}{2}}e^{-\frac{m_{DM}}{T_f}} \approx \frac{T^2_{freeze}\sqrt{8\pi G_N}}{\langle\sigma v\rangle}$$

$$\sqrt{x_{freeze}}e^{-x_{freeze}} \approx \frac{\sqrt{8\pi G_N}}{m_{DM}\langle\sigma v\rangle}$$

At this point, assuming energy level at the scale of EW interaction $\sigma \approx G_F^2 m^2_{DM}$ and $m_{DM} \approx 100GeV$, hence $x_{freeze} \approx 20$, the DM relic density can be written with a slight modification

$$\Omega_{DM} = \frac{m_{DM}n_{DM}^{T_{present}}}{\rho_c} = \frac{m_{DM}T^3_{present}}{\rho_c}\frac{n_{present}}{T^3_{present}}$$
$$\approx \frac{T^3_{present}x_{freeze}\sqrt{8\pi G_N}}{\rho_c\langle\sigma v\rangle}$$

where $T_{present} \approx 2.7K$ and substituting other parameters from [72, 73], one finally reaches

$$\Omega_{DM}h^2 \approx \frac{(1.8-4.5)\times 10^{-27}cm^3/s}{\langle\sigma_{ann}v\rangle} \tag{4.39}$$

and considering Planck's result $\Omega_{DM}h^2 \approx 0.118$, and approximate value for average cross section $\langle\sigma v\rangle \approx (2-5)\times 10^{-26}cm^3/s \approx 2\times 10^{-9}GeV^{-2}$ ≈1pb, the experimental relic density can be matched [74].

Cross section at the scale of 1pb corresponds to energy level of weak interactions with the corresponding masses scale around $100-1000GeV$. That falls into the WIMPs regime and it is mainly motivated due to Planck's result and sets a theoretical limit for extended models as if DM is a composed of only one particle type. This can be taken as another motivation for WIMPs to study from the extended hidden sector models.

Although WIMPs' domain consolidating a practical approach for the DM models; there would still be remaining Higgs channels to be checked for in LHC. In return, these decay modes can open up ways for extensive parametrization of the DM models through the hidden sector.

CHAPTER 5

CONCLUSION

For the minimal extension model with singlet scalar, $\Phi^{'}_{DS}$ is the only DM candidate. Moreover, if $\alpha \to 0$, then it gives vanishing vev $v_d = 0$ for the singlet $\Phi^{'}_{DS}$, such condition is necessary if $\Phi^{'}_{DS}$ to be considered as viable DM candidate. However, such vanishing vev for dark scalar ends up with a diagonal mass matrix for neutral fields, hence $h^{'} = \phi_3$ with $m^2_{h^{'}} = \frac{\lambda v^2}{2}$ and $m_{\Phi^{'}_{DS}} = \frac{\lambda_{sd} v^2}{2} - 2\mu^2_{\Phi_{DS}}$ occur. Which is obviously less than $\frac{m_{h^{'}}}{2}$, in return, there can be a Higgs decays channel $h^{'} \to \Phi_{DS}\Phi_{DS}$. But this decay channel is surely 'hidden' from experimental results, since the dark scalar Φ_{DS} does not interact with the SM gauge bosons and fermions. But the inclusion of hidden decay channel affects only total width of Higgs. Thus it can be written as:

$$\frac{\Gamma_{tot}}{\Gamma_{SM,tot}} = 1 + \frac{\Gamma_{hid}}{\Gamma_{SM,tot}}$$

where the unity in the r.h.s surely counts for all the SM Higgs decay channels. And additional hidden decay channel through dark sector puts some limits on the minimal model constraints (e.g. λ_{sd}) once compared to fluctuations of Higgs decay channels observed in LHC. But such limitations in the parameter space of MEHS model are beyond the scope of the study.

If DM abundance for this model is dictated to yield 100% of Planck's observation, mass interval for dark scalar must be in 1-3 TeV scale. But in the NMEHS model, there are two possible DM candidates, dark gauge bosons W^a_d or $\eta^{'}_d$. Without the dark scalar, NMEHS yields less than 10% of the Planck's observation due to LHC data and has problems about the Higgs vacuum stability. Hence to satisfy reasonable value for DM abundance, dark scalars must be contained through the model.

Parameters space of NADM model is completely governed by total relic density due to Planck's observation. And there are two regions for parameter space requiring attention. In the small mixing angle limit, $h^{'}$ tends to the SM higgs, hence two sectors begin to decouple. And the figure (4.12) shows a direct relation between m_{η_d} and m_{W_d}. For a special case $m_{W_d} < \frac{m_{h'}}{2}$, then annihilation through physical higgs will be suppressed kinematically. And such constraint also leads to $m_{\eta_d^{'}} < m_{h'}$, thus α has to get larger values to obtain correct relic density as Planck observed, which can be seen from (4.12); at small angles, low m_{W_d} values are ruled out. Moreover, the interval $m_{W_d} \approx 100 - 200\ GeV$ almost totally is discarded along with parameter space corresponding to this limit, since they give the desired DM abundance. And finally if $m_{W_d} > 200\ GeV$, then dark scalar $\eta_d^{'}$ passes beyond TeV scale and mixing angle tends to small values in which the resolution of two sectors weaken. Although small mixing angle parameter space ends up with higher DM abundance, it sets also an experimental weakness for interactions to be observed.

Changing the parameter space to obtain total DM abundance is theoretically allowed, nevertheless, higher abundance also enhances triviality of the given model. It would be a whimsical proposal to consider DM, corresponding exactly to density given by the $SU(2)_d$ dark gauge bosons, since there are additional firm candidates for DM abundance such as MACHOs,providing a large fraction for relic density.

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APPENDIX A

NOETHER'S THEOREM AND CONSERVED CURRENTS

Continous symmetries and conserved currents of field theories are directly related via Noether's Theorem. If the Lagrangian density of the system has a continous differential symmetry; then there has to be a conserved current for the associated field. And dynamics of the given system is completely described by the Action principle.

Starting from the Lagrangian function, simply $\mathcal{L} = \mathcal{L}(\phi_i, \phi_{i,\mu}, x^\nu)$. The objective is to extract conserved quantitites from $spin - \frac{1}{2}$ Lagrangian, regardless of component number. Furthermore, each $U(N)$ group is also defined as $U(1)\otimes SU(N)$, global phase transformation of each lie group is a crucial tool to obtain those conserved quantities. For a generic dirac field Ψ, consider the following infinitesimal $U(1)$ transformations:

$$\psi \to \psi^{'} = (1 + i\varepsilon)\psi \quad ; \quad \bar{\psi} \to \bar{\psi}' = (1 - i\varepsilon)\bar{\psi} \tag{A.1}$$

where

$$\delta\psi = i\varepsilon\psi \quad ; \quad \delta\bar{\psi} = -i\varepsilon\bar{\psi} \quad ; \quad \delta(\partial_\mu\psi) = i\varepsilon(\partial_\mu\psi) \quad ; \quad \delta(\partial_\mu\bar{\psi}) = -i\varepsilon(\partial_\mu\bar{\psi}) \tag{A.2}$$

The Action principle indicates that $\int \mathcal{L}dt = 0$ or $\delta\mathcal{L} = 0$ where $\mathcal{L} = \mathcal{L}(\psi, \psi_{,\mu}\bar{\psi}, \bar{\psi}_{,\mu})$

$$\begin{aligned} \delta\mathcal{L} &= \frac{\partial\mathcal{L}}{\partial\psi}\delta\psi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\delta(\partial_\mu\psi) + \frac{\partial\mathcal{L}}{\partial\bar{\psi}}\delta\bar{\psi} + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})}\delta(\partial_\mu\bar{\psi}) \\ &= i\varepsilon\frac{\partial\mathcal{L}}{\partial\psi}\psi + i\varepsilon\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}(\partial_\mu\psi) - i\varepsilon\frac{\partial\mathcal{L}}{\partial\bar{\psi}}\bar{\psi} - i\varepsilon\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})}(\partial_\mu\bar{\psi}) = 0 \end{aligned} \tag{A.3}$$

Defining the following term:

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\partial_\mu\psi = \partial_\mu\bigg(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi\bigg) - \bigg(\partial_\mu(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)})\bigg)\psi, \tag{A.4}$$

rewriting (A.3):

$$\delta\mathcal{L} = i\varepsilon\left[\frac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu\Big(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\Big)\right]\psi + i\varepsilon\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi\right)$$
$$- i\varepsilon\left[\frac{\partial\mathcal{L}}{\partial\bar{\psi}} - \partial_\mu\Big(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})}\Big)\right]\bar{\psi} - i\varepsilon\partial_\mu\left(\bar{\psi}\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})}\right). \tag{A.5}$$

It is clearly seen for Lagrange field equations: $\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\phi_{i,\mu}}\right) - \frac{\partial\mathcal{L}}{\partial\phi_i} = 0$; 1^{st} and 4^{th} terms of(A.5) vanish, what remains is:

$$\delta\mathcal{L} = i\varepsilon\partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi - \bar{\psi}\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})}\right] = 0. \tag{A.6}$$

Without loss of generality, for each fermion Lagrangian, kinetic and interaction terms can be written as

$$\mathcal{L}_{EM,EW,QCD} = \sum_{g_a=1}^{3}\bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - g_a\bar{\psi}\gamma^\mu\vec{T}\psi V_\mu^a. \tag{A.7}$$

The contiunity equation is recognized from (A.6) $\partial_\mu J^\mu = 0$, and combined with the Lagrange equations; J_a^μ appears as conserved current in the field:

$$J_\mu^a = g_a\bar{\psi}\gamma_\mu\vec{T}\psi. \tag{A.8}$$

Once the generator of each group acts on the generic fermionic field, we recover the familiar results (2.6), (2.18) and (2.33)

APPENDIX B

GOLDSTONE THEOREM

B.1 Classical Definition

Define a Lagrangian symmetric under the group G_1, and the vacuum state of this system remains invariant under G_2,which is a subgroup of G_1. Expressing $U(G_1)$ as the transformation operator for the given group acting on a field ϕ. Similarly for the subgroup; $U(G_2)$. As long as we stick with constant fields, first order derivative of the given potential vanishes, and the potential remains invariant under $U(G_1)$ for sure.

$$\begin{aligned} V(\phi) &= V(U(G_1)\phi) \\ &= V((1 + i\varepsilon^a T^a)\phi) \end{aligned} \tag{B.1}$$

And the expansion of the potential for infinitesimal transformations:

$$\begin{aligned} V(\phi) =& V(\phi) + \frac{\partial V}{\partial \phi_j} i\varepsilon^a T^a \phi + \mathcal{O}|\varepsilon^2| \\ &\rightarrow \frac{\partial}{\partial \phi_k}\left(\frac{\partial V}{\partial \phi_j} T^a_{jl}\phi_l\right) = 0. \end{aligned} \quad . \tag{B.2}$$

Using the vacuum state expression

$$\frac{\partial}{\partial \phi_k}\left(\frac{\partial V}{\partial \phi_i} T^a_{il}\phi_l\right) = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} T^a_{jl}\phi_l + \frac{\partial V}{\partial \phi_i} T^a_{ik}\bigg|_{\langle \phi_0 \rangle}, \tag{B.3}$$

we obtain the following relation:

$$0 = M_{ki} T^a_{il} \langle \phi_l \rangle_0 + 0. \tag{B.4}$$

No one arguing about invariance of the vacuum under subgroup: $\phi_0 = U(G_2)\phi_0$, however this invariance is not preserved under G_1: $\phi_0 \neq U(G_1)\phi_0 \longrightarrow T^a_{il}\langle \phi_l \rangle_0 \neq 0$

Conclusion: If T^a corresponds to a broken generator, we end up with $T^a \langle\phi_0\rangle \neq 0$, then M_{ik} will have a null eigenvector with null eigenvalues. Which mainly asserts that, there would be massless particles since the eigenvalues of mass matrix corresponds directly to particle states. "Each broken generator brings a massless scalar".

B.2 Quantum Definition

Noether's theorem simply expresses that, every continuous symmetry is associated to its generators T. For the quantum limit this expression is connected to the operators, in which T commutes with the Hamiltonian $[H, T] = 0$. If system has zero vacuum energy $H|0\rangle = 0$ and the vacuum is invariant under the symmetry T then simply we have $e^{iT\theta}|0\rangle = |0\rangle$

Consider an infinitesimal field transformation:

$$\phi_j \rightarrow \phi_j^{'} = \phi_j + \alpha^a(\delta\phi)_j^a, \tag{B.5}$$

where ϕ_j can stand for both fermionic and bosonic fields. It is possible to obtain well known conserved current by using Noether's theorem

$$Q^a = \int J_0^a d^3x,$$

with the conserved current:

$$J_\mu^a = -\frac{\delta\mathcal{L}}{\delta(\partial^\mu\phi_j)}(\delta\phi)_j^a. \tag{B.6}$$

And recalling canonical momentum density:

$$\pi_j = \frac{\delta\mathcal{L}}{\delta(\partial^0\phi_j)}. \tag{B.7}$$

Commutation relations given as

$$[\pi_j(\vec{r_1}), \phi_k(\vec{r_2})] = -i\delta^3(\vec{r_1} - \vec{r_2})\delta_{jk}, \tag{B.8}$$

where the conserved charges obey the algebra:

$$[Q^a, Q^b] = if^{abc}Q^c. \tag{B.9}$$

If we use the above relations; temporal components of currents satisfy the following algebra:

$$[J_0^a(\vec{r_1},t), J_0^b(\vec{r_2},t)] = if^{abc} J_0^c(\vec{r_1},t)\delta^3(\vec{r_1}-\vec{r_2}). \tag{B.10}$$

Now, allow Q be a symmetry charge; say $U(1)$, and the corresponding conserved current J^μ. If one can assert the vacuum state $|0\rangle$ would not be destroyed by this generator, thus it would not stand for actual vacuum state.

$$Q\,|0\rangle \neq 0. \tag{B.11}$$

Invoking current conservation by seperating temporal and spatial parts:

$$\int [\partial_\mu J^\mu, \phi_0] d^3x = \partial_0 \int [J^0, \phi_0] d^3x + \int_S [\vec{J}, \phi_0] d\vec{S} = 0. \tag{B.12}$$

A well known generalization by taking the surface, extending to the infinity where the currents vanish. Hence second term in the r.h.s dissapears, and what remains is

$$\frac{d}{dt}[Q(t), \phi_0] = 0, \tag{B.13}$$

which follows as

$$\langle 0|\,[Q, \phi_0]\,|0\rangle = \xi \neq 0. \tag{B.14}$$

Explicitly ξ can be shown after integrating over $\vec{r_1}$

$$\xi = \sum_n (2\pi)^3 \delta^3(\vec{p}_n) \Big[\langle 0|\, J^0(0)\,|n\rangle \langle n|\, \phi(0)\,|0\rangle\, e^{-iE_n t} - \langle 0|\, \phi(0)\,|n\rangle \langle n|\, J^0(0)\,|0\rangle\, e^{iE_n t} \Big] \neq 0. \tag{B.15}$$

Seen from the above relation that if $E_n \neq 0$, then two distinct eigenvalue parts would not cancel each other, hence expression(B.15) cannot be constant. To provide constancy for the above expression, one must impose that the intermediate states have to be massless; such dictation is assured by the fact $\xi \neq 0$. So there must be a massless state for every broken generator.

$$\langle n|\, \phi_0\,|0\rangle \neq 0 \;\; ; \;\; \langle 0|\, J^0(0)\,|n\rangle \neq 0 \tag{B.16}$$

as a conseqence of Nambu-Goldstone theorem [16].

It is concluded that, spontaneously broken symmetry of the vacuum $Q\,|0\rangle \neq 0$ showing the excitations of the system with a frequency vanishing in the low momentum limit.

Goldstone's theorem foresees massless states in RQM regime, while in the NRQM limit, a typical example comes from solid state that corresponds to collective excitations with zero energy gap.