DURATION OF MAXIMUM DRAWDOWN IN OIL PRICES

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ABSTRACT

DURATION OF MAXIMUM DRAWDOWN IN OIL PRICES

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Oil price is a vital financial aspect directing the economy in the global world. Changes in the level of oil prices may affect the whole economy; at the same time economic developments may affect the oil prices. For example, while lower oil prices could inhibit financial development and spoil economic and political stability, higher oil prices could cause an increase in inflation and a recession in an economy. Sharp decreases and increases in oil prices have been notable in recent years. These fluctuations in oil prices become one of the key indicators in macroeconomics. Therefore, it is of utmost importance to analyze fluctuations in oil prices.

The purpose of this study is to estimate the duration that is the maximum drawdown of highest possible drop in the oil prices. Maximum drawdown can be defined as the indication of the highest possible market risk in finance. In order to detect the maximum drawdown, super-cycles which are specified as total of an upward and a downward movement are determined in the data set which is the oil prices collected between the year 1948 and 2018. This is done by using filtering methods such as

Hodrick-Prescott and Band Pass filter. Fractional Brownian motion process is used for modeling the oil prices since the increments are observed to be dependent. We have conducted simulation studies for calculating the expected value of the maximum drawdown and expected value of the duration of the maximum drawdown. In this study, comparison of simulation results and observed results from the real life data set is provided in order to make prediction and give discussion on the duration of the maximum drawdown of the oil prices.

Keywords: Oil Prices, Maximum Drawdown, Fractional Brownian Motion, Hodrick Prescott Filter, Band-Pass Filter

ÖZ

PETROL FİYATLARINDA MAKSİMUM DÜŞÜŞ SÜRESİ

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Petrol fiyatı, küresel dünyada ekonomiyi yönlendiren hayati bir finansal unsurdur. Petrol fiyatları seviyesindeki değişiklikler tüm ekonomiyi etkileyebilir; aynı zamanda ekonomik gelişmeler petrol fiyatlarını etkileyebilir. Örneğin, düşük petrol fiyatları finansal gelişmeyi engelleyebilir ve ekonomik ve politik istikrarı bozabilirken, daha yüksek petrol fiyatları enflasyonda artışa ve ekonomide daralmaya neden olabilir. Son yıllarda petrol fiyatlarındaki keskin düşüşler ve artışlar dikkat çekmektedir. Petrol fiyatlarındaki bu dalgalanmalar, makroekonomideki temel göstergelerden biri haline gelmiştir. Bu nedenle, petrol fiyatlarındaki dalgalanmaları analiz etmek son derece önemlidir.

Bu çalışmanın amacı, petrol fiyatlarındaki olası en yüksek düşüşün tamamlanıp tamamlanmadığını görmek ve maksimum kayıp süresini tahmin etmektir. Maksimum kayıp, finansta mümkün olan en yüksek piyasa riskinin göstergesi olarak tanımlanabilir. Maksimum kayıp miktarının tespiti için, veri kümesinde, bir yukarı ve bir aşağı doğru

hareketin toplamı olarak belirtilen süper çevrimler belirlenir. Süper çevrimlerin belirlenmesi, Hodrick-Prescott ve Band Pass gibi filtreleme yöntemleri kullanılarak yapılır. Petrol fiyatlarının modellenmesinde de fiyatlardaki değişimlerin bağımlı davranışları nedeni ile Kesirli Brown Hareketi modeli kullanılmaktadır. Olası maksimum kayıpların boyutunun ve süresinin beklenen değerini kestirebilmek için simülasyon çalışması yapılmıştır. Bu çalışmada, petrol fiyatlarının maksimum kayıp süresinin tahmini ve konunun tartışılabilmesi için, simülasyon sonuçlarınn gerçek veriler ile karşılaştırılması verilmiştir.

Anahtar Kelimeler: Petrol fiyatları, Maksimum Kayıp, Kesirli Brown Hareketi, Hodrick Prescott Filtresi, Band-Pass Filtresi

To my beloved family

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LIST OF ABBREVIATIONS

Bm	Brownian Motion
fBm	Fractional Brownian Motion
GfBm	Geometric Fractional Brownian Motion
HP	Hodrick-Prescott
BP	Band Pass
CF	Christiano Fitzgerald
ACF	Autocorrelation Function
PACF	Partial Autocorrelation Function
KPSS	Kwiatkowski-Phillips-Schmidt-Shin
ADF	Augmented Dickey Fuller

CHAPTER 1

INTRODUCTION

1.1 Introduction

Oil price is a vital financial aspect directing the economy in the global world. Changes in the level of oil prices may affect the whole economy; at the same time economic developments may affect the oil prices. According to the Ferderer's work (1996) on oil price volatility and the macroeconomics [21], deterioration of oil market can have an impact on the macroeconomics because they alter the price levels and cause the oil price volatility increase as well. Because of the fact that volatility of oil prices creates uncertainty, both oil exporting and importing countries' economies cannot be stable and these countries would go through financial crises. According to Yang et. al (2002), the fact that higher prices of oil cause the inflation to escalate and later economic recession in oil consuming countries is resulted from the negative correlation between oil prices and economic activities [55]. For example, lower oil prices could inhibit financial development and spoil economic and political stability.

Sharp decreases and increases in oil prices have been notable in recent years. For instance, Baumeister and Kilian (2016) were interested in the magnitude of the fluctuations in oil prices [3]. They also mentioned that economic decisions tend to be affected by oil price shocks. These fluctuations in oil prices become one of the key indicators in macroeconomics. Therefore, it is of utmost importance to analyze fluctuations in oil prices. In this study, our aim is to estimate the maximum drawdown, i.e. highest possible drop in the oil prices. Leal and Bendes (2003) states that a drawdown is defined as the loss in percentual from the last local maximum to next local minimum in the price of an investment and it is accumulated over non-fixed time intervals and its duration is also a random variable. The reason why the maximum drawdown is important is that it serves as an insurance against adverse movements of the price during a market turmoil [45]. Using duration of maximum drawdown, information about duration of market recovery can be obtained. The data set we used is West Texas Intermediate (Cushing, OK WTI) spot prices.

Data

Despite the fact that there are many different types of petroleum, West Texas Intermediate (WTI), Brent Petroleum and Dubai Fateh petroleum are considered as references for other petroleum prices. WTI, Brent and Fateh petroleum are petroleum products traded on financial markets such as the stock market and forex where non-physical transactions are made.[1]

WTI contracts were first traded on the New York Mercantile Exchange (NYMEX). In 2008, the Chicago Mercantile Exchange purchased NYMEX, and the transactions went through the CME group. WTI Texas is known as light sweet American oil. Also, WTI is the best quality oil in the world. It is ideal for petroleum and diesel fuel production because it is light and slightly sulphurous (sweet). From among crude oil prices, West Texas Intermediate (Cushing, OK WTI) spot price was chosen and data we will use is taken from U.S. Energy Information Administration site (www.eia.gov). Data starts from January 1947 and ends in May 2018. Frequency is monthly. The nominal value of the prices in the economy is measured by the value of the currency at the time, hence the real data is used for showing the price levels corresponding to the Gross Domestic Product. In our analysis, real oil prices are created by using Consumer Price Index (Index 1982-1984=100, Monthly, Seasonally Adjusted) [2]. To convert nominal prices from several different years into real, nominal prices is divided by the GDP of the corresponding year.

[1] Information of oil are taken from the website www.businessinsider.com
[2] CPI data is taken from Federal Reserve Bank of St. Louis (fred.stlouisfed.org)

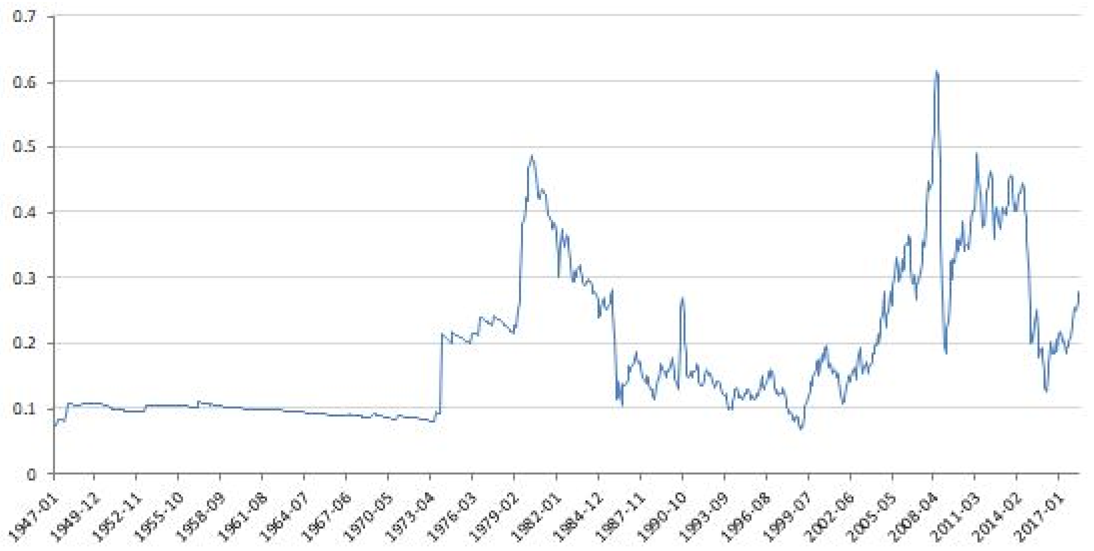


Figure 1.1: Real Spot Crude Oil Price: West Texas Intermediate (WTI)

Kaffel & Abid (2009) and Mostafaei (2013) wanted to obtain the most suitable stochastic process for the crude oil prices in their articles [29], [41]. In these studies, the simulation was conducted on the historical crude oil prices and Geometric Brownian Motion is specified as optimal stochastic model for crude oil prices. For our study, geometric version of Fractional Brownian motion (fBm) process is used for modeling oil prices. fBm is a realistic model, because it is a process that exhibits long-term or short-term dependent behavior and this dependency overlaps with the dependency seen in real life data. This feature in real life coincides with up or down trend of prices. The process $S(t)$ which we use as a notation for oil price at time t is the solution for the following stochastic differential equation driven by fBm:

$$dS(t) = \mu S(t)dt + \sigma S(t)dB^H(t) \tag{1.1}$$

where $S(0) = s$, μ & σ are constants which are greater than 0, and $B^H(t)$ are the constant mean, constant volatility and the fractional Brownian motion respectively. Fractional Brownian motion is identified through its Hurst parameter. Estimation of this parameter takes an important role because fBm which have long-term or short-term dependence and self-similarity properties is characterized by the Hurst parameter. Hurst parameter refers to the “index of dependence” which relates to the autocorrelations of the time series. For instance, if H takes the value in (0.5,1), a time series have long-term dependence.

This master's thesis is organized as follows. In the Second Chapter, fBm will be introduced with its properties and the maximum drawdown and the duration of maximum drawdown will be presented. Maximum drawdown, also known as maximum loss, indicates the highest possible market risk in finance. Therefore, it is especially important both for investors and economists. In the Third Chapter, in order to detect the maximum drawdown for the oil prices super-cycles will be introduced and the very last super-cycle will be identified so that we can focus only on the part of the data we are interested in. Super-cycles are specified as total of an upward and a downward movements. This section describes how super-cycles extract from the series by means of economic filtering methods. The used filtering methods are Hodrick-Prescott and Band Pass filter. These filtering techniques help someone to decompose the trend component from the series and smooth the data. Thanks to these filters, the last super-cycle is identified. The starting point of the last super-cycle gives us beginning of the data which is used for modeling. In Chapter 4, we check whether the oil prices data fulfill the properties of the stochastic differential equation given in Equation 1.1 and appropriateness of this model will be discussed. In order to check the assumptions tests for normality, stationarity and dependency are conducted. The results of each assumptions are given in the same part. Once we confirm, Equation 1.1 could be used as a model for our data set will continue with introducing the methods for estimating the Hurst parameter (H) which are R/S analysis, variance-time Plot and Correlogram. As to verify our simulation codes for generation of fBm, we have conducted simulation for calculating the expected maximum drawdown of standard Brownian Motion. In the last chapter, comparison of simulation results with the real life data set is provided in order to make prediction and give discussion on the duration of maximum drawdown of the oil prices.

CHAPTER 2

MAXIMUM DRAWDOWN AND THE DURATION OF MAXIMUM DRAWDOWN

2.1 Fractional Brownian Motion

Fractional Brownian motion (fBm) process was first studied by Kolmogorov (1940) and Mandelbrot & Van Ness (1968). This process has caught attention of researchers' from different disciplines. For instance some of these disciplines are physics, statistics, biology and economics.

There are different Black-Scholes models established with the different stochastic processes for example the Brownian movement and OrnsteinUhlenbeck process. Although fBm is not a Markov process, it is used for modelling especially financial data by replacing Brownian motion in Black-Scholes formula. Fractional Brownian Motion is more realistic, because fBm is a process that exhibits dependent behavior. fBm is a generalization of Brownian motion (Bm) which captures the dependency seen in real life.

Definition 2.1.1 *Let* $(\Omega, \mathcal{F}, \boldsymbol{P})$ *be a probability space and let* H *be a constant in the interval* $(0,1)$*. fBm,* $B^H(t)$*, with Hurst parameter* H*, is a continuous, Gaussian process with zero mean and has this covariance function (*$t \geq 0$*) [5]:*

$$Cov_H(t) = E[B^H(t)B^H(s)] = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H}).$$

For $H = 1/2$, fBm corresponds to standard Brownian motion [5], where covariance function becomes:

$$E[B^{1/2}(s)B^{1/2}(t)] = \frac{1}{2}(t + s - |t - s|) = min(t, s)$$

Fractional Brownian motion $B^H(t)$ has the following properties:

1. Assume $B^H(0) = 0$ and $E[B^H(t)] = 0$ for all $t \geq 0$.
2. B^H has homogeneous increments, i.e., $B^H(t + s) - B^H(s)$ has the same law of $B^H(t)$ for $s, t \geq 0$.
3. B^H is a Gaussian process and $E[B^H(t)^2] = t^{2H}$, $t \geq 0$, for all $H \in (0, 1)$.
4. B^H has continuous trajectories.

Correlation between two increments

When H parameter takes the value 1/2, the process becomes standard Bm and this process has independent increments. However, the fBm process with $H \neq 1/2$ has dependent increments. From the definition of the fBm, the covariance for the two processes $B^H(t+h) - B^H(t)$ and $B^H(s+t) - B^H(s)$ with $s+h \leq t$ and $t - s = nh$ is

$$\begin{aligned}
&\rho_H(n) = E[((B^H(t+h) - B^H(t))(B^H(s+h) - B^H(s)))] \\
&= E[B^H(t+h)B^H(s+h)] - E[B^H(t+h)B^H(s)] \\
&- E[(B^H(t))B^H(s+h)] + E[B^H(t)B^H(s)] \\
&= \frac{1}{2}[((t+h)^{2H} + (s+h)^{2H} - |(t+h) - (s+h)|^{2H}) \\
&- ((t+h)^{2H} + s^{2H} - |(t+h) - s|^{2H}) \\
&- (t^{2H} + (s+h)^{2H} - |t - (s+h)|^{2H}) \\
&+ (t^{2H} + s^{2H} - |t - s|^{2H})] \\
&= \frac{1}{2}[-2(t-s)^{2H} + (nh - h)^{2H} + (nh + h)^{2H}] \\
&= \frac{1}{2}[-2(nh)^{2H} + ((n-1)h)^{2H} + ((n+1)h)^{2H}].
\end{aligned} \tag{2.1}$$

So we have

$$E[((B^H(t+h)-B^H(t))(B^H(s+h)-B^H(s)))] = \frac{h^{2H}}{2}[(n+1)^{2H}+(n-1)^{2H}-2n^{2H}].$$

In the view of the fact that fBm is a Gaussian process, it has a unique mean and the covariance structure. However, having dependent increments may make somebody to doubt about stationarity of fBm. A stationary process is a stochastic process whose unconditional joint probability distribution does not change when shifted in time. We can show the stationarity property in the following way:

$$\begin{aligned}
&E[((B^H(t+h)-B^H(h))(B^H(s+h)-B^H(h)))] = \\
&E[B^H(t+h)B^H(s+h)] - E[B^H(t+h)B^H(h)] \\
&- E[(B^H(h))B^H(s+h)] + E[B^H(h)^2] \\
&= \frac{1}{2}[((t+h)^{2H} + (s+h)^{2H} - |(t+h)-(s+h)|^{2H}) \\
&- ((t+h)^{2H} + h^{2H} - |(t+h)-h|^{2H}) \qquad (2.2)\\
&- (h^{2H} + (s+h)^{2H} - |h-(s+h)|^{2H}) \\
&+ (h^{2H} + h^{2H} - |h-h|^{2H})] \\
&= \frac{1}{2}[t^{2H} + s^{2H} + |t-s|^{2H}] \\
&= E[B^H(t)B^H(s)].
\end{aligned}$$

Since h does not affect $B^H(t)$. That is, it is obtained that $[B^H(t+h)-B^H(h)]$ equals to $B^H(t)$ in distribution.

Long-range dependence

Definition 2.1.2 *Let $(Y_n)_{n\in N}$ be a stationary time series and the long-range dependent. The covariance of this processes $\rho(n) := Cov(Y_k, Y_{k+n})$ $(Y_n)_{n\in N}$ satisfy:*

$$\lim_{n\to\infty} \frac{\rho(n)}{cn^{-\alpha}} = 1$$

where c is a constant and $\alpha \in (0,1)$. [5]

If n increases in a long-range dependent process, the dependence between Y_{k+n} and Y_k declines gradually as :

$$\sum_{n=1}^{\infty} \rho(n) = \infty.$$

The fact that fBm exhibits long-range dependence property is proved by using the covariance function. The covariance between the increments is:

$$\begin{aligned} \rho_H(n) :&= E[(B^H(t+h) - B^H(t))(B^H(s+h) - B^H(s))] \\ &= \frac{h^{2H}}{2}[(n+1)^{2H} + (n-1)^{2H} - 2n^{2H}]. \end{aligned} \tag{2.3}$$

By using the following expansions;

$$(1+x)^{\alpha} = 1 + \alpha x + \frac{\alpha(\alpha-1)x^2}{2} + ...$$

$$(1-x)^{\alpha} = 1 - \alpha x + \frac{\alpha(\alpha-1)x^2}{2} - ...$$

covariance becomes:

$$\begin{aligned} \rho_H(n) &= 1 + 2H\frac{1}{n} + \frac{2H(2H-1)}{2}\frac{1}{n^2} + 1 - 2H\frac{1}{n} + \frac{2H(2H-1)}{2}\frac{1}{n^2} + ... \\ &\cong \frac{n^H}{2}(2H(2H-1))\frac{1}{n^2} \\ &= H(2H-1)n^{2H-2}. \end{aligned} \tag{2.4}$$

According to the Definition 2.1.2, c equals to $H(2H-1)$ and α equals to $2-2H$. Therefore, for $H > 1/2$ the process has the property of long-range dependence because

$$\lim_{n\to\infty} \frac{\rho_H(n)}{H(2H-1)n^{2H-2}} = 1. \tag{2.5}$$

In conclusion, we get

1. For $H > 1/2$, $\sum_{n=1}^{\infty} \rho_H(n) = \infty$.
2. For $H < 1/2$, $\sum_{n=1}^{\infty} |\rho_H(n)| < \infty$.

We can infer that there is a positive correlation between two increments $B^H(t+h) - B^H(t)$ and $B^H(t+2h) - B^H(t+h)$ for $H > 1/2$. On the contrary, for $H < 1/2$ these two have a negative correlation. Long-range dependence is very useful and significant property when observing the long-term behavior in the financial data. This property can be captured easily in the correlation diagram.

Self-similarity

Definition 2.1.3 *The real-values process $Y_t, t \geq 0$ is self-similar if for all $a > 0$ there exists $b > 0$ such that*

$$(Y_{at})_{t>0} \overset{law}{=} (bY_t)_{t\geq 0}.$$

This means that the finite-dimensional distributions of $Y_{at}, t \geq 0$ are identical to the finite-dimensional distributions of $bY_t, t \geq 0$; i.e., if for every choice to $t_0, ..., t_n$ and any $a \geq 0$,

$$(Y_{at_1}, Y_{at_2}, ..., Y_{at_n}) \overset{d}{=} (bY_{t_1}, bY_{t_2}, ..., bY_{t_n}). \tag{2.6}$$

If $b = a^H$ in Equation 2.6, it can be deduced that the process $Y_t, t \geq 0$ fulfills the self-similarity property [5].

For $H = 1/2$, the self-similarity property of Standard Brownian Motion is recovered as $(B(a^{1/2}t))_{t>0} \overset{law}{=} (a^{1/2}B(t))_{t\geq 0}$.

As a result, if the distribution of a self-similar process is known over the unit interval, this provides us opportunity to find the distribution of the process over the whole time interval, i.e, scaling in time results in scaling in space.

2.2 Duration of the Maximum Drawdown

Maximum drawdown, also known as maximum loss, indicates the highest possible market risk in finance. Therefore, it is especially important for both investors and economists. Financial mathematics provides a motivation for studying maximum drawdown as a function of a stochastic process; see e.g. [45]. To quantify the risk and to measure the performance of a stock, maximum drawdown is used. In addition, it is used in portfolio selection for hedging the risk, [12]. It is also used in pricing of Russian options, [52]. From risk management point of view, the magnitude of risk sometimes is not sufficient enough to build a comprehensive risk evaluation of extreme drawdown risks. For example, it is natural to examine the duration of the maximum drawdown for extreme risks such as natural disasters, financial crises [32]. Caglar and Vardar-Acar (2013) asserts that the maximum possible loss for the logarithm of a price process is almost equal to the maximum drawdown of fBm [8]. In this study, our main goal is to estimate the duration of maximum drawdown of fBm and compare the result to the duration of maximum drawdown in oil prices to check whether the half of the last super-cycle is completed.

Maximum drawdown corresponds to the biggest value of all possible differences between a high and a low value. Eventually, the magnitude of maximum drawdown is a crucial issue that shows us a lot about the current situation of prices in a specific time interval. For that matter, maximum drawdown affects how the investor and government will behave.

Notation: Let $B^H(t)$ denote the fractional Brownian motion. Then, the mathematical definition of Maximum Drawdown before time t is given as

$$\begin{aligned} M_t^H :&= \sup_{0\le u\le v\le t} (B_u^H - B_v^H) \\ &= \sup_{0\le v\le t} ((\sup_{0\le u\le v} B_u^H) - B_v^H) \\ &:= \sup_{0<v<t} D_t^H \end{aligned} \tag{2.7}$$

where D_t^H is called the loss or the drawdown of the process. For M_t^H for some $M > 0$, the corresponding the duration of the maximum drawdown can be defined as;

$$
\begin{aligned}
E(M_t^H) &= E(\sup_{0\le u\le v\le t}(B_u^H - B_v^H)) \\
&= E(\sup_{0\le \frac{u}{t}\le \frac{v}{t}\le 1}(B_u^H - B_v^H)) \\
&= E(\sup_{0\le u'\le v'\le 1}(B_{u't}^H - B_{v't}^H)) \\
&= E(\sup_{0\le u'\le v'\le 1} t^H(B_{u'}^H - B_{v'}^H)) \\
&= t^H E(\sup_{0\le u'\le v'\le 1} t^H(B_{u'}^H - B_{v'}^H)) = t^H E(M_1^H)
\end{aligned}
\tag{2.8}
$$

by the self similarity property of fBm.

Now suppose $M_t^H = m$ for some $m > 0$. Let $H_m^D = \inf(t \geq 0; D_t^H = m)$ and let $\rho = \sup(t \in [0, H_m^D]; D_t^H = 0)$. Then the duration of Maximum Drawdown is defined as:

$$
\begin{aligned}
\tau_m &:= H_m^D - \rho \\
&= \inf(t \geq 0; D_t^H = m) - \sup(t \in [0, H_m^D]; D_t^H = 0).
\end{aligned}
\tag{2.9}
$$

A fractional Brownian motion with Hurst parameter $H = 1/2$ corresponds to a standard Brownian motion. The expected value of the maximum drawdown for the standard Brownian motion is theoretically calculated and the exact value is found as $\sqrt{\pi/2} \cong 1.2533$ (see [17]). For fractional Brownian motion, because of the non-Markovian structure one can only obtain bounds for such calculations [15]. In fact, for fBm theoretical bounds were provided for the expected value of the maximum drawdown in [15].

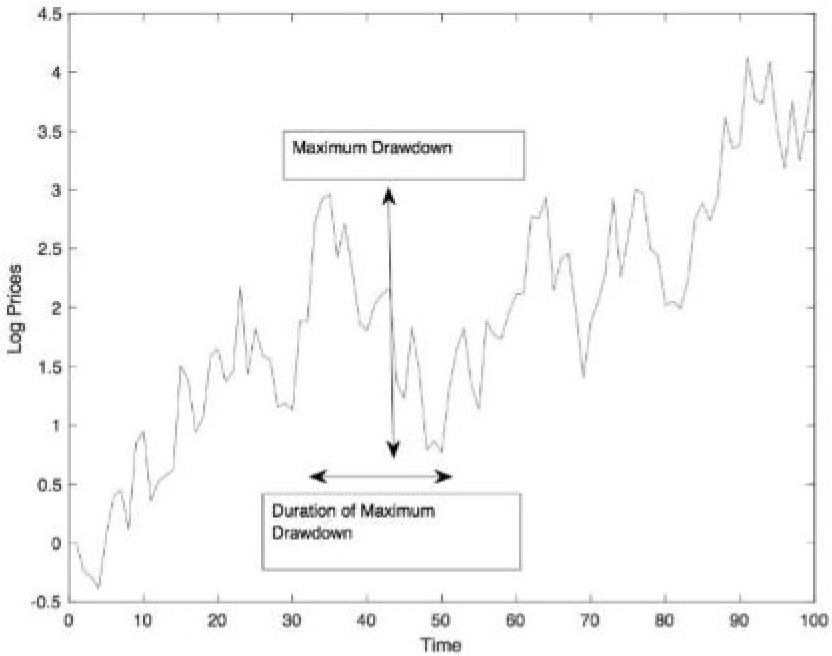


Figure 2.1: Demonstration of Maximum Drawdown and Duration of Maximum Drawdown

Generally, financial data contains lots of descents and ascents in it. The maximum drawdown is one of the fluctuations that should be analyzed in the data set. For our analysis of estimation in order to obtain the maximum drawdown and the duration of it, we need to determine the starting point or level of fluctuations at oil prices. In the next chapter, we focus on distillation the starting point of the last business cycles that has the last and the highest fluctuation in it. This cycle corresponds to the last excursion with maximum height. And the excursion under running supremum with maximum height in fact gives access to the maximum drawdown and its duration.

For analyzing the starting point of the last business cycle filtering techniques are applied to the data set as given in [19] and the last super-cycle has been determined.

CHAPTER 3

SUPER-CYCLES AND FILTERING TECHNIQUES

In this chapter, we present the definition and the types of super-cycles, and filtering methods. Before applying the statistical model and estimating the duration of the maximum drawdown, we need to define the time interval on which we will study from historical data. Examining the super-cycles will help us define this time interval. There has been more than one super-cycle until today. We will be focusing on the last cycle. The time interval of the super-cycles will be determined by using two well-known filtering methods.

3.1 Super-cycles

The fluctuations in the time series constitute cycles. Basically, a cycle is defined as a total of an upward and a downward movement [20]. Especially, price cycles are guide for economists in price series. The duration of maximum drawdown corresponds to the duration of half of a super-cycle. Some techniques could be applied to get information on the duration of market recovery using maximum draw-up which is symmetrically defined as maximum drawdown.

What attracted our interest in this topic is that sharp decreases and increases of oil prices have been notable in recent years. After each sharp increase, tendency to decrease in oil prices cause someone to think of the existence of cyclical pattern. According to Hamilton (2011), when in the comprehensive point of economic activity such pattern is seen that is named as "business cycle" [24]. The business cycle means the fluctuations in the level of economic activities that are formed by expansions and contractions. The long-term growth trend is shown below around these fluctuations:

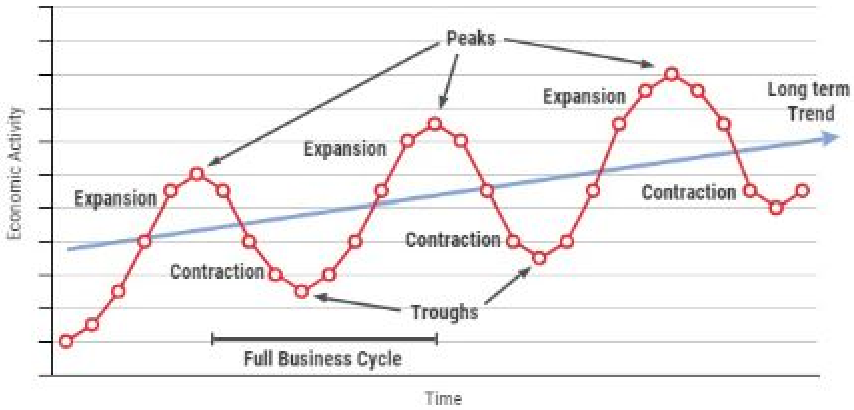


Figure 3.1: Phases of the Business Cycle

Figure 3.1 shows the phases of the business cycles. From one peak to another one, a full business cycle is observed. That is, a full business cycle consists of two peak points and one trough point, besides some contraction and expansion points. A business cycle can take months or years. It is very important to predict the cycles for both economists and investors. It has been a guide to forecast whether economy will shrink or expand.

There are several types of cycles. Palley (2011) states that cycles are categorized as medium and long term cycles. Medium term cycles are also called as the basic cycles and the long term cycles are also called as the super-cycles. He claims that these cycles are "super" in two ways [44]:

1. These cycles are long term cycles which go upward around 10–35 years and are completed in 20–70 years.

2. A large variety of industrial goods such as metals and non-renewable resources are affected by these cycles.

The identification of super-cycles and determination of the current stage of the cycle is important for both policy makers and market players, as it sheds light on price movements in the upcoming period. Recently, studies that analyze the long term cycles in commodity prices have increased. Buyuksahin et. al (2016) states that "Commodity prices tend to go through extended periods of boom and bust, known as super-cycles" [7]. Also, in Erdem and Ünalmış's article (2016), super-cycles in oil prices have been mentioned in detail [19]. In order to identify the period of cycles in our data, some economic filters are used. These filters are used for extracting the cycle by detrending and smoothing [42]. In this study, we will not deal with these filters in detail, we will just apply them to our data set in order to recognize where the last super-cycle starts.

3.2 Filtering Methods and Application to Data

This section describes how super-cycles are extracted from the series by means of economic filtering methods. In the analysis of macroeconomic time series, decomposing the data into trend and cyclical elements is considerable application for segregating short and long term behavior [40]. As mentioned earlier, the filtering techniques help us to decompose the trend component and smooth the data. By means of these features of filtering, we will identify entire super-cycles in the historical oil price data. After the application of filters, the period of super-cycles can be observed graphically. The cycles found from filters will be compared for validation of results. We will be using the Hodrick-Prescott (HP) Filter and Band Pass (BP) Filter which are widely used in identification and extraction of super-cycle components, [19]. According to the comparison of results, the beginning date of the last super-cycle will be identified.

The data has been introduced in the first chapter. Before applying the filtering methods, we have examined the history of the monthly oil price data for both types Brent and WTI. Up to 1970s, crude oil prices have been more stable.

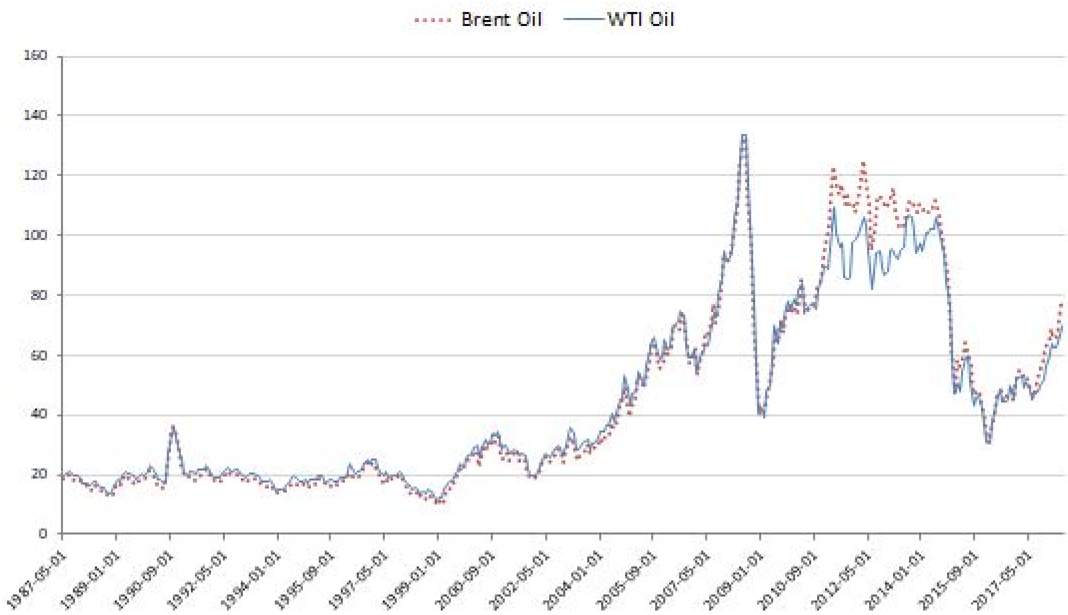


Figure 3.2: Spot Crude Oil Prices

Before the year 2000 there were three major events that affected oil prices in history. There were the 1978 Iranian revolution, the 1980 Iran - Iraq war and the 1990 Iraq invasion of Kuwait, respectively. When it comes to the 2000s, the types of events affecting oil prices have been changed. Throughout the period, there only exists one dramatic event which was in the year 2008. Oil prices experienced a peak value which was 134 dollars per barrel in July 2008. As seen in the Figure 3.2, the price was experienced a sharp decrease to the 40 dollars per barrel level in February 2009 [43]. As Hamilton (2011) says, this oil shock in 2008 resulted from dramatic changes in supply or demand and the speculations, not because of destructive events like war, revolution or invasion [25].

3.2.1 Hodrick-Prescott Filter

The first filtering technique we used in this study is the Hodrick Prescott filter. According to Nilssson and Gyomai (2011), macroeconomic experts widely prefer this filter for detrending and smoothing [42]. Generally, it is thought that the time series consist of a combination of the trend and cycle component. It can be said that the series is complex sinusoidal [42]. The reasons for choosing this filter are the fact that application is more simple and it performs better than other filtering methods. [42]. HP filter is developed in 1981 and named by Hodrick and Prescott [27]. Original form of filter for the trend estimation comes from a result of an optimization problem:

$$Y_t = T_t + c_t$$
$$min_{T_t} \sum_t (Y_t - T_t)^2 + \lambda \sum_t \Big[(T_{t+1} - T_t)(T_t + T_{t-1}) \Big]^2$$

where Y_t is the actual time series, T_t is the trend component, c_t corresponds to the cyclical component and the smoothing parameter is λ [42].

That filter decomposes time series Y_t into a non-stationary time trend (T_t) and a stationary cyclical component (c_t) [27]. That is to say, this filter computes the smoothed series by minimizing the variance of Y around trend and minimizing the curvature shape. The trend T_t is meant to detect the long run growth of Y_t, and is the sum of the squares of its second difference. c_t is taken to reveal a business cycle component, and corresponds to the deviations from trend component of series.

Value of the smoothness parameter λ

λ provides minimization between trend and cyclical component. Before application of filter, the smoothing parameter λ should be determined. As it can be seen in the Table 3.1, λ takes positive values, and generally it equals to 100 for annual data and equals to 14400 for monthly data as given in [46].

Table 3.1: Values of the Smoothness Parameter

frequency	λ
annual	100
quarter	1600
monthly	14400

In this study, we used E-views software to apply the HP filter. Frequency of our data is monthly, but for the practicality annual data is used and so the value of the smoothness parameter λ is taken 100.

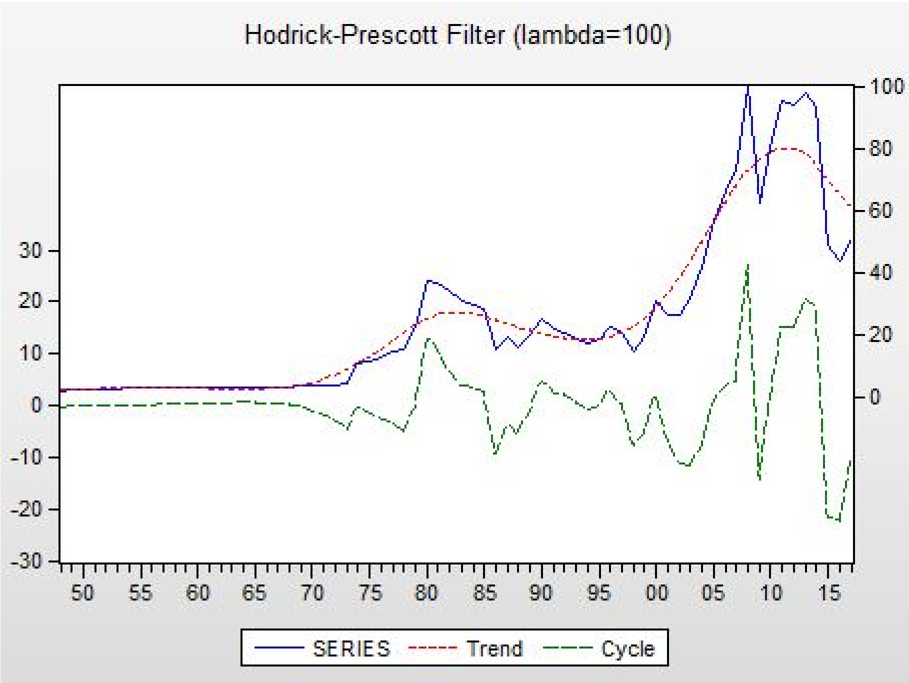


Figure 3.3: HP filter on Real Oil Prices

It is said that the first super-cycle took place between 1861 and 1947, because in 1880s oil has started to be commercialized [19]. The data taken from U.S. Energy Information Administration [1] starts from 1947, therefore the first super-cycle is not included in above figure.

In Figure 3.3, until 1966 oil prices seemed more stable because of price controls and

[1] https://www.eia.gov/

industrialization. Between 1966 and 1996, the second super-cycle can be observed. This period started with the establishment of OPEC and were shaped by the effects of second World War. It can be detected that the new and the last super-cycle has started in the beginning of the 2000s. Also, we can see that oil prices hit their last peak on 2012. [19] say that the beginning of the last super-cycle is around 1996. The reason for evolution of the last super-cycle is the fact that high growth rates in developing countries affect the oil demand positively. As cited in Erdem and Ünalmış (2016), from the research of Pollin and Heintz, there are some opinions arguing that high growth rate causes high liquidity and concentration of capital flows that play an essential roles on increasing oil prices in the expansion period of last super-cycle [19]. Although the financial crisis in 2008 has generated sharp decrease, the rising trend continued until 2014. Based on Erdem and Ünalmış' findings (2016), the previous cycles in oil prices took approximately 25-30 years [19]. Since they indicated that the last super-cycle has started in 1996, it can be said that the expansion period has finished. After 2014, prices has entered a downward trend. Decreases in this period arose from low growth rate of developing countries, especially China [19]. As a result, the time interval can be taken starting from the year 1996.

3.2.2 Band Pass Filter

The second filtering technique we used in this study is the Band Pass (BP) filter, also known as Frequency filter, which is useful in a wide range of economic contexts. This filtering method is developed by [14]. The BP filter extracts the cyclical components of a given times series that lie within a specified range of frequencies or periods. While using that filter, it is needed to specify the lower and upper bounds of periods of the cycles of interest. Christiano and Fitzgerald (2003) states that the idea that different frequency components of the data exists, is supported by the theory of the spectral analysis of time series. They say that this theory does not need to be related to any statistical model of the data and this makes the theory advantageous. It depends on Spectral Representation Theorem in which any time series can be separated into different frequency components. The ideal BP filter can be supplied by using this theory [14].

That is to say, they explain this filtering method derived from spectral representation theorem provides a tool for extracting cyclical components. They stated that the BP filter measures the components of the data at certain frequencies and all other components are annihilated [14]. Similarly, Baxter and King (1999) clarifies that BP filter is a filter passing through components of the time series with periodic fluctuations.

Before the application of that filter, some approximations are needed. In fact, data should be infinite for the application of the ideal band pass filter to the data. In the filtering study of Christiano and Fitzgerald (2003), they built some alternative optimal linear approximations for the data length problem. For the optimal approximation, it is needed to know the true time series representation of the raw data. Because it is impossible to know this representation practically, it must be estimated. Therefore in order to use this approximation, it is assumed that the data are produced by a pure random walk. Then the random walk filter approximation can be used for the data not produced by a random walk.

As mentioned before, in order to perform the ideal BP filter infinite data is needed. Some kind of estimation for finitely data is required for linear approximation. [14]'s suggestion for approximation is mentioned as Christiano Fitzgerald (CF) filter. CF filter is generated from BP filter in order to solve the data length problem and it is not symmetric but time-varying filter [2]. Besides, Nilsson and Gyomai (2011) show that CF filter in the long run converges to the ideal filter [42].

Let X_t denote the data on which BP filter is applied, and Y_t be the raw data. $\widehat{X_t}$ is estimated by X_t of the observed sample Y_t's. CF filter makes $(\widehat{X_t})$ as equal as possible to X_t (object of interest) by diminishing the mean square error criterion in Equation 3.1 into the minimum level. Here, the problem in this minimization arises from the unknown filter weights.

$$\begin{gathered} E[(X_t - \widehat{X_t})^2 | Y] \\ Y = [Y_1, Y_2, ..., Y_T] \end{gathered} \tag{3.1}$$

Christiano and Fitzgerald (2003) applies the filter as follows. By supposing the isolating the component from Y_t with a period of oscillation between p_l and p_u, where $2 \geq p_l < p_u < \infty$[14]:

$$\begin{aligned} \widehat{X_t} &= B_0 Y_t + ... + B_{T-1-t} Y_T + \tilde{B}_{T-1} Y_T \\ &+ B_1 Y_{t-1} + ... + B_{t-2} Y_2 + \tilde{B}_{t-1} Y_1 \end{aligned} \tag{3.2}$$

for $t = 3, 4, ..., T-2$.

$$\begin{aligned} B_j &= \frac{\sin jb - \sin ja}{xj}, j \geq 1 \\ B_0 &= \frac{b-a}{\pi}, a = \frac{2\pi}{p_u}, b = \frac{2\pi}{p_l} \end{aligned} \tag{3.3}$$

and $\tilde{B}_{T-t}$, $\tilde{B}_{t-1}$ are simple linear functions of the B_j's. (see [14] for the details)

As referred earlier, this filter allows someone to extract cyclical component at specified frequencies from time series. While using that filter, it is needed to specify the lower and upper bounds of periods of the cycles of interest. According to study of [56], Band Pass bounds for annual data are chosen as year interval (2,70). They were the first to apply this filter to commodities and oil prices. This results from that equation:

$$\begin{aligned} &Actual \equiv BC(2,8) + IC(8,20) + SC(20,70) + T(70,\infty) \\ &Actual \equiv BP(2,70) + T(70,\infty) \end{aligned} \tag{3.4}$$

where BC is the business cycle, IC is the intermediate cycle, SC is the super-cycle and T is the trend. According to this formula, series are equal to the sum of these components and $BP(2,70)$ corresponds to cyclical component [56]. These bounds of these components come from the year interval of cycles. In this way we separate trend and cycle component from the price series. The calculated trend components are shown in Appendix A.

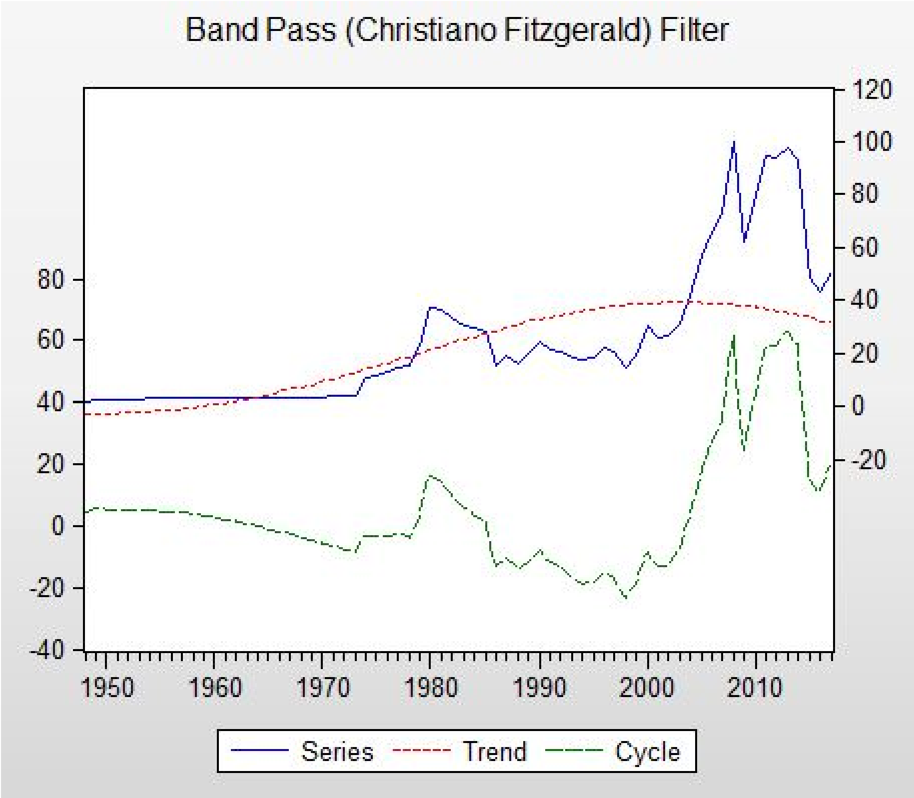


Figure 3.4: BP filter on Real Oil Prices

Likewise in the HP filter, this filter confirms that the last cycle starts around 1996 and reaches its last peak at 2012. Same economic conditions can be seen from the Figure 3.4.

CHAPTER 4

MODELING, ESTIMATION AND SIMULATION

4.1 Modeling using fBm

Typically, the price of a commodity is modeled by Black-Scholes process driven by Brownian motion. Independent increments are needed for this model. However, in real life it is quite difficult to come across with this property. Therefore Black-Scholes model using fBm is more appropriate for capturing the dependence behavior in the data.

4.1.1 Stochastic Model

Geometric fractional Brownian motion (GfBm) is the solution of Black-Scholes model driven by fBm. This model is commonly used for financial data [39]. We implement this model to our data since oil prices exhibits long-range dependence behavior.

Our assumption is that the oil price process $S(t)$ follows the stochastic differential equation:

$$dS(t) = \mu S(t)dt + \sigma S(t)dB^H(t) \tag{4.1}$$

where $S(0) = s > 0$. μ, $\sigma > 0$ and $B^H(t)$ are the drift (constant mean), constant volatility and the fractional Brownian motion, respectively [5]. Explicitly, the solution of the stochastic differential Equation 4.1 is:

$$S(t) = s \cdot exp[\sigma B^H(t) + \mu t - \frac{1}{2}\sigma^2 t^{2H}] \tag{4.2}$$

$$
\begin{aligned}
\int_0^t \frac{dS(t)}{S(t)} &= \int_0^t \mu dt + \int_0^t \sigma dB_t^H \\
&= [\log S(t) - \log S(0)] \\
&= \log \frac{S(t)}{S(0)} = \mu t + \sigma B^H(t) - \frac{1}{2}\sigma^2 t^{2H}
\end{aligned}
\tag{4.3}
$$

or

$$
\begin{aligned}
&= [\log S(t) - \log S(t-1)] \\
&= \log \frac{S(t)}{S(t-1)} = \mu + \sigma B^H(t) - \frac{1}{2}\sigma^2
\end{aligned}
\tag{4.4}
$$

4.2 Assumption Check of fBm for Oil Prices

Referring to the properties of fBm, there are three assumptions to be validated:

1. Stationarity
2. Normality
3. Dependence of increments

Here, the starting year for the data set is chosen as January 1996 because of the fact the last super-cycle starts at this date.

4.2.1 Stationarity

Stationarity is primary property for testing the correlational structure of time series. In a stationary process, the properties of the stochastic process do not change over time [23]. In his article Chen (1991) states that "Time series $(Y_1, Y_2, ...)$ is stationary if the joint distribution of any part of the series of $Y_{t_1}, Y_{t_2}, ..., Y_{t_k}$ have the same distribution with any other part of the series $(Y_{t_{1+\epsilon}}, Y_{t_{2+\epsilon}}, ..., Y_{t_{k+\epsilon}})$, where ϵ can be any integer" [13]. In other words, shifting the time series has no effect on the distribution of series.

For Brownian motion model which is random walk process, the change in the series $(Y_t - Y_{t-1})$ should be random. If this change is not random, then the process is not stationary, and its variance increases with time t. In order to test the stationarity, we are using two popular test: Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test [31] and Augmented Dickey Fuller (ADF) test [16].

KPSS test:

This is one of the most powerful test for testing unit root. The existence of unit root shows non-stationarity [26].

Definition 4.2.1 *Let Y_t be the stochastic process given by ([9]):*

$$\begin{aligned} Y_t &= \alpha_t + \beta t + u_t \\ \alpha_t &= \alpha_{t-1} + \nu_t \end{aligned} \tag{4.5}$$

$t = 1, 2, ..., T$ and where u_t and ν_t are assumed to be two independent stochastic processes. Then, it is implied the variance σ_{ν_t} equals 0. Its hypothesis is:

- *H_0: Y_t is trend (or level) stationary.*
- *H_1: Y_t is a unit root process.*

The KPSS test can be calculated as follows:

- Regress Y_t on a constant and trend and construct the Ordinary Least Squares residuals $e = (e_1, e_2, ..., e_T)'$.
- Obtain the partial sum of the residuals.

$$S_t = \sum_{i=1}^{T} e_i$$

- Find the test statistic

$$KPSS = T^{-2} \sum_{t=1}^{n} \frac{S_t}{\hat{\sigma^2}}$$

where $\hat{\sigma}^2$ is the estimate of the long-run variance of the residuals.

We reject the null hypothesis if KPSS test statistic is large, so this means that the series moving around its mean.

ADF test:

This test is also one of the commonly known unit root tests (see [16]).

The hypothesis of this test is the reverse version of KPSS test.

- H_0: Y_t is a unit root process.
- H_1: Y_t is trend (or level) stationary.

The basic Dickey-Fuller test is augmented in order to comply with higher order ARMA(p,q) models by Said and Dickey. [49].

ADF test equation:

$$Y_t = \Phi Y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta Y_{t-j} + \theta_0 + e_t \tag{4.6}$$

or

$$\Delta Y_t = (\Phi - 1) Y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta Y_{t-j} + \theta_0 + e_t \tag{4.7}$$

which $\delta = (\Phi - 1)$. For this equation, the hypothesis is arranged accordingly:

- H_0: $\Phi = 1$ or $\delta = 0$
- H_1: $|\Phi| < 1$ or $\delta < 1$

The test statistic of the test is

$$DF_T = \frac{\hat{\delta}}{SE(\hat{\delta})}.$$

If the test statistic is less than the critical value ($t_{\Phi=1} < CV$ or $t_{\delta=0} < CV$), the null hypothesis is rejected. By applying the KPSS and ADF tests to monthly oil prices starting from 1996 we observe the following results.

Results:

Table 4.1: Kwiatkowski-Phillips-Schmidt-Shin Test

Null Hypothesis: D(SERIES) is stationary
Exogenous: Constant
Bandwidth: 0 (Newey-West automatic) using Bartlett kernel

		LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic		0.119264
Asymptotic critical values*:	1% level	0.739000
	5% level	0.463000
	10% level	0.347000

*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Table 4.2: Augmented Dickey Fuller Test

Null Hypothesis: D(SERIES) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-13.66587	0.0000
Test critical values:	1% level	-3.454719	
	5% level	-2.872162	
	10% level	-2.572503	

*MacKinnon (1996) one-sided p-values.

As it is seen from the Tables 4.1 and 4.2, the series are stationary. The test statistic of KPSS test is greater than the critical value (0.05), the null hypothesis that series is stationary is not rejected. Also, test statistic of ADF test is less than the critical value (0.05), so the null hypothesis that series has a unit root is rejected.

4.2.2 Normality

In Equation 4.4, the logarithm of the ratio $\frac{S_t}{S_{t-1}}$ corresponds to the differences of logarithms of S_t's and this difference is standardized with its mean and standard deviation in order to apply the normality tests and plots:

$$\log S_t - \log S_{t-1} \sim N(\mu - \frac{1}{2}\sigma^2, \sigma^2)$$

$$\frac{(\log S_t - \log S_{t-1}) - (\mu - \frac{1}{2}\sigma^2)}{\sigma} \sim N(0,1) \tag{4.8}$$

Then, we can easily check normality with tests and plots. According to Marathe and Ryan (2005), histogram can be used for visual check [36]. Also, Jarque Bera normality test is applied as a statistical test. The hypothesis for this test is:

- H_0: The distribution is normal.
- H_1: The distribution is not normal.

P-value of the test will be taken into account, if the observed p-value is greater than the specified level of significance α then the null hypothesis cannot be rejected.

Results:

We can check the normality of our series by using visual and testing tools. According to the results, firstly oil price series fell short of normality expectations. The time series did not distributed normally.

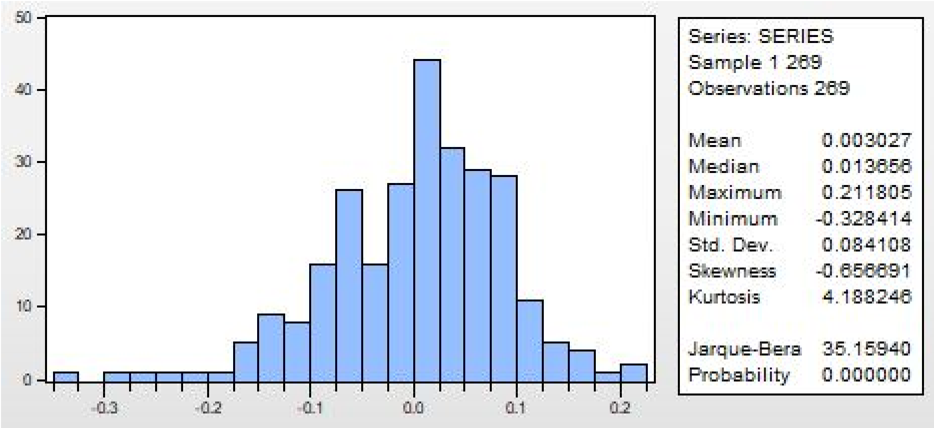


Figure 4.1: Histogram and Normality test

The reason our data has longer tails than normal distribution was the economic shocks especially in year 2008. When the data is analyzed, the time interval, which we are using, contains the 2008 financial crisis. This crisis caused sharp decreases and increases in the oil prices. After reaching the level of 134 dollars per barrel in July 2008, oil prices fell to its lowest level with 40 dollars per barrel in February 2009. To overcome this outlier observations, we applied the Hodrick Prescott filter for the time interval (1996-2018) and the results of filter are replaced by the real time series of the crisis period. After that, normality is tested again.

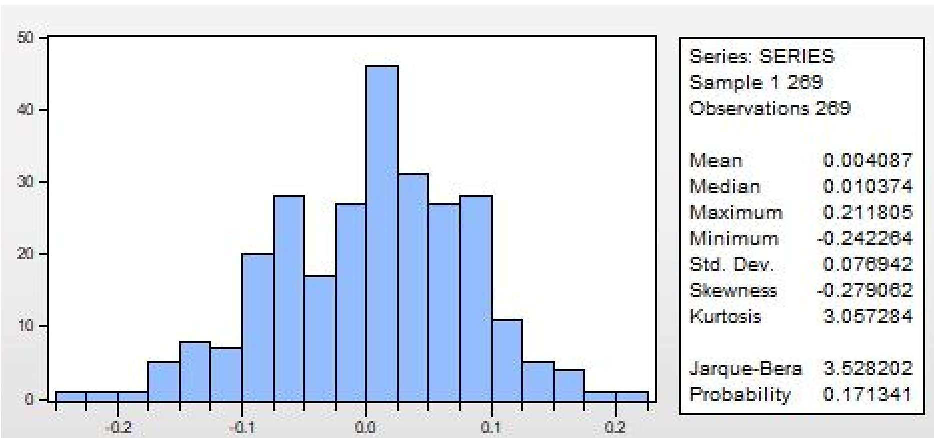


Figure 4.2: Histogram and Normality test-2

In the second tests, with the probability 0.1713 of Jarque Bera test, the null hypothesis that series are normally distributed could not be rejected. Thus, the normality of the data is verified by histogram and Jarque Bera normality test (see Figure 4.2).

4.2.3 Dependency

For checking the assumption of fBm model, it is also necessary to investigate whether the increments are dependent. In order to detect this dependency, the serial correlations diagram using auto-correlation function can be applied.

Definition 4.2.2 *Let X_t be the response at time t with the mean $E[X_t] = \mu_t$ and variance $E[(X_t - \mu_t)^2] = \sigma_t^2$. The auto-correlation function between two responses [10]:*

$$\rho(X_{t_1}, X_{t_2}) = \frac{E[(X_{t_1} - \mu_{t_1})(X_{t_2} - \mu_{t_2})]}{\sigma_{t_1}\sigma_{t_2}}$$

The time series having n observations $X_1, X_2, ..., X_n$, are paired as:

$$(X_1, X_2), (X_2, X_3), (X_3, X_4), ..., (X_{n-1}, X_n)$$

and these pairs will be treated as a bivariate data set. If the correlation between consecutive pairs is computed , they are called as the auto-correlation coefficient or serial correlation coefficient at lag 1 denoted by r_1. The formula is:

$$r_1 = \frac{\sum_{i=1}^{n-1}(X_i - \bar{X})(X_{i+1} - \bar{X})/(n-1)}{\sum_{i=1}^{n}(X_i - \bar{X})^2/n}$$

Likewise, for the lag k the formula is:

$$r_k = \frac{\sum_{i=1}^{n-k}(X_i - \bar{X})(X_{i+k} - \bar{X})/(n-k)}{\sum_{i=1}^{n}(X_i - \bar{X})^2/n}$$

In order to test the serial correlation, correlogram is the best descriptive plot. Correlogram is generated by serial correlations r_k versus the lag k for $k = 0, 1, 2, ..., M$, where $M < n$. If the series have random observations (white noise), it is expected that the serial correlations will be zero. However, for the financial data, it is hard to expect independency of observations from one another. While examining the dependency in correlogram, it is checked whether r_k falls in confidence limits or not. These limits are calculated as follows:

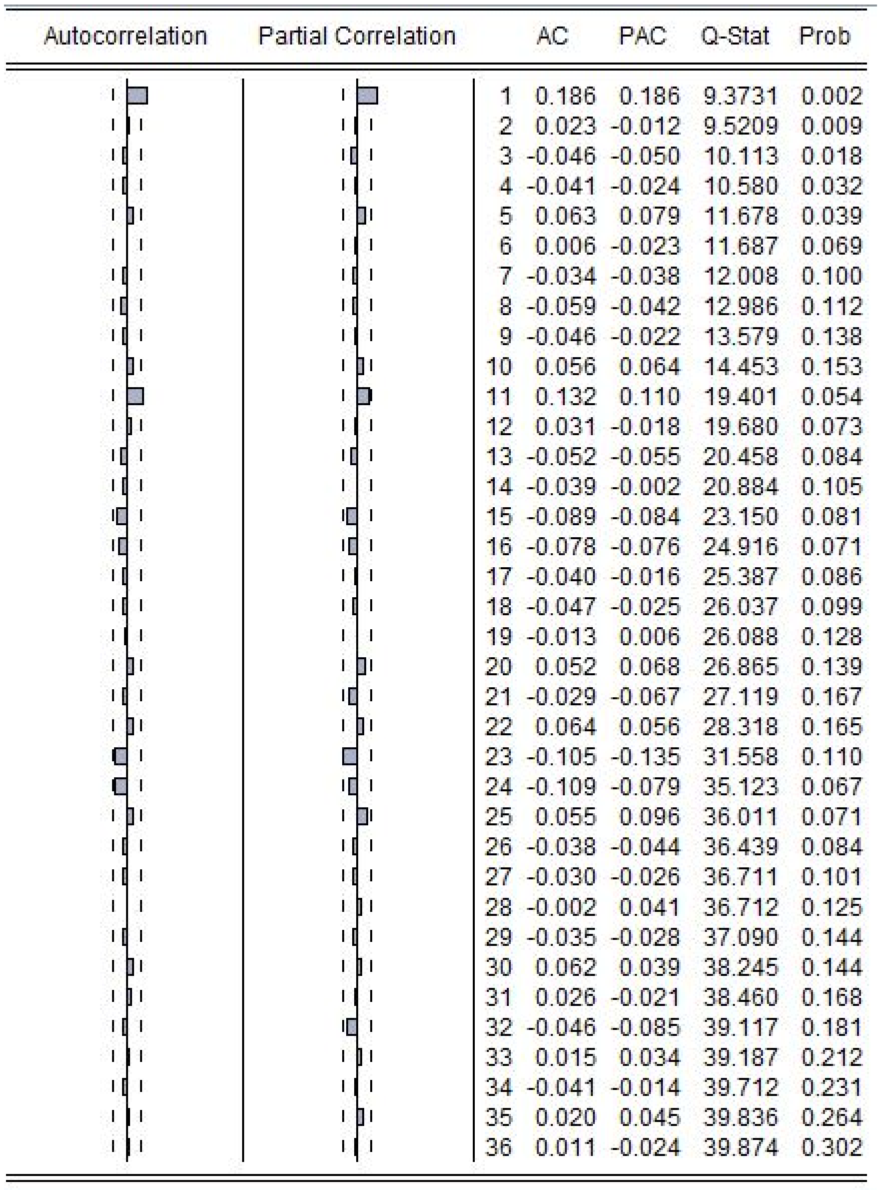
$$-1/(n-1) \pm 1.96/\sqrt{n}$$

If sufficiently enough auto-correlations are not in the standard error bounds, correlation of series is significantly different from zero at the 5 percent significance level.

Results: The hypothesis for the serial correlations is:

- H_0: $\rho_0 = \rho_1 = ... = \rho_k = 0$ (There is no autocorrelation.)
- H_1: At least one is different.

The dotted lines in the serial correlogram shows the confidence limits in the Figure 4.3. Most of the probabilities of the autocorrelations are not significantly different than zero, then we can say that the null hypothesis is rejected. As a result, the increments of the oil prices are dependent and as seen from the Figure 4.3 they exhibit long-range dependence i.e. we expect the Hurst parameter H to be greater than $1/2$.



Lag	AC	PAC	Q-Stat	Prob
1	0.186	0.186	9.3731	0.002
2	0.023	-0.012	9.5209	0.009
3	-0.046	-0.050	10.113	0.018
4	-0.041	-0.024	10.580	0.032
5	0.063	0.079	11.678	0.039
6	0.006	-0.023	11.687	0.069
7	-0.034	-0.038	12.008	0.100
8	-0.059	-0.042	12.986	0.112
9	-0.046	-0.022	13.579	0.138
10	0.056	0.064	14.453	0.153
11	0.132	0.110	19.401	0.054
12	0.031	-0.018	19.680	0.073
13	-0.052	-0.055	20.458	0.084
14	-0.039	-0.002	20.884	0.105
15	-0.089	-0.084	23.150	0.081
16	-0.078	-0.076	24.916	0.071
17	-0.040	-0.016	25.387	0.086
18	-0.047	-0.025	26.037	0.099
19	-0.013	0.006	26.088	0.128
20	0.052	0.068	26.865	0.139
21	-0.029	-0.067	27.119	0.167
22	0.064	0.056	28.318	0.165
23	-0.105	-0.135	31.558	0.110
24	-0.109	-0.079	35.123	0.067
25	0.055	0.096	36.011	0.071
26	-0.038	-0.044	36.439	0.084
27	-0.030	-0.026	36.711	0.101
28	-0.002	0.041	36.712	0.125
29	-0.035	-0.028	37.090	0.144
30	0.062	0.039	38.245	0.144
31	0.026	-0.021	38.460	0.168
32	-0.046	-0.085	39.117	0.181
33	0.015	0.034	39.187	0.212
34	-0.041	-0.014	39.712	0.231
35	0.020	0.045	39.836	0.264
36	0.011	-0.024	39.874	0.302

Figure 4.3: Serial Correlogram

4.3 Estimation of Hurst parameter

In the literature, there are several methods to estimate the parameter Hurst (H), also known as the self-similarity parameter. This is firstly put forward by Mandelbrot & van Ness (1968). This estimation takes an important role in analyzing of series because fBm having long-range dependence and self-similarity is characterized by Hurst parameter. In this section, we introduce three methods for estimation of H, which are R/S analysis, Variance-time Plot and Correlogram.

4.3.1 The R/S Analysis

The R/S analysis is one of the first and most common method for the estimation of H. This method has been existed first in hydrologist Hurst's works on flow at Nile river [28]. Hurst has noticed that after becoming long period of floods,there was a long period of drought. Then, he has studied in order to calculate the flow of river. Mandelbrot and Wallis (1969) has put forward the R/S analysis for the estimation [35].

The statistic for the R/S analysis is calculated step by step as follows [54]:

- Divide a time series of length N into k sub-series of length n
- For each sub-series $j = 1, 2, .., k$
  1. Find the mean E_j and standard deviation S_j
  2. Subtract the sample mean from the data $Y_{i,j}$ in order to centralize: $Z_{i,j} = Y_{i,j} - E_j$ for $j = 1, 2, .., k$
  3. Build a new time series $X_{i,j} = \sum_{t=1}^{i} Z_{t,j}$ for $i = 1, 2, .., n$
- Find the range $R_j = max(X_{1,j}, ..., X_{n,j}) - min(X_{1,j}, ..., X_{n,j})$
- Re-scale the range R_j/S_j

Lastly, the mean value of the re-scaled range for all sub-series of length n is calculated:

$$(R/S) = \frac{1}{k}\sum_{j=1}^{k} R_j/S_j.$$

This ratio gives the re-scaled adjusted range or R/S statistic.
Asymptotically the R/S statistic has this relation:

$$(R/S)_n \sim cn^H.$$

Therefore, in order to calculate the value of H we set a simple linear regression for several values of n

$$\log(R/S)_n = \log c + H \log n.$$

4.3.2 Variance-time Plot

For the estimation of H parameter, the second method is the variance-time plot. This method is based on logarithmic plots and uses the property of long-range dependence. Variance of the sample mean inferred from the Theorem 2.2 in the book of Beran [4] is $Var(\bar{X}) \approx cn^{2H-2}$ where $c > 0$.
For the estimation of H, we follow below steps [57]:

- Divide a time series of length N into m_k sub-series of length k.
- Find the sample means $\bar{Y}_1, \bar{Y}_2, ..., \bar{Y}_{m_k}$ with the integer time lag k lying in $2 \leq k \leq n/2$.
- Find the overall mean by $\bar{Y} = \frac{1}{m_k}\sum_{j=1}^{m_k} \bar{Y}_j(k)$.
- Then, find the sample variance $S^2(k)$ of the sample means $\bar{Y}_j(k), i = 1, 2, ..., m_k$ by

$$S^2(k) = (m_k - 1)^{-1}\sum_{j=1}^{m_k}(\bar{Y}_j(k) - \bar{Y}(k))^2$$

After these calculations, the plot of $\log S^2(k)$ against $\log k$ is drawn and the resulting points are combined. The slope of the plot is used to obtain an estimation of H.

4.3.3 The Correlogram

The last method for the estimation is the correlogram. The correlogram is widely used to generate the plot of serial correlations versus different lag k, ($k = 1, 2, ..., m$ where m is less than the sample size). The sample autocorrelation is

$$\hat{\rho}(k) = \frac{\hat{\gamma}(k)}{\hat{\gamma}(0)}.$$

where $\hat{\gamma}(k)$ and $\hat{\gamma}(0)$ are the serial correlations. While looking at the Autocorrelation function (ACF) plot, two horizantal lines at the levels $\mp 2/\sqrt{n}$ which corresponds to the significance level 0.005 are drawn. Inside the limits there is no correlation, but lines that cross boundaries are considered as correlated. According to Sarker (2007), "if the asymptotic decay of the correlation is hyperbolic, then the points in the plot should be approximately scattered around a straight line with a negative slope of 2H-2 for the long memory processes but for short memory, the points should tend to diverse to minus infinity at an exponential rate."[51]. This method is not widely used and not effective, also its bad sides takes part in article of Mandelbrot and Wallis (1969). H can be estimated via this method by using that equation:

$$\hat{\rho}(k) = \hat{H}(2\hat{H} - 1)k^{2\hat{H}-1}.$$

Application of Hurst parameter Estimation

In this thesis, for estimation of the Hurst parameter, the solution of the model in Equation 5 is used. If we take the differences according to previous value, the distribution of $\log S_t - \log S_{t-1}$ has no H parameter in it. For this reason, the difference $\log S_t - \log S_0$ should be used. The distribution of this difference is:

$$\log S_t - \log S_0 \sim N(\mu t - \frac{1}{2}\sigma^2 t^{2H}, \sigma^2 t^{2H}) \tag{4.9}$$

In R software, “pracma” package is used to estimate the Hurst parameter and “hurstexp” function calculates the estimators with some corrections [6]. After necessary arrangements in the data are conducted, the following results are obtained:

Table 4.3: Results of the Estimation

Simple R/S Hurst estimation:	0.8485069
Corrected R over S Hurst exponent:	1.023286
Empirical Hurst exponent:	1.043707
Corrected empirical Hurst exponent:	1.007416
Theoretical Hurst exponent:	0.5538539

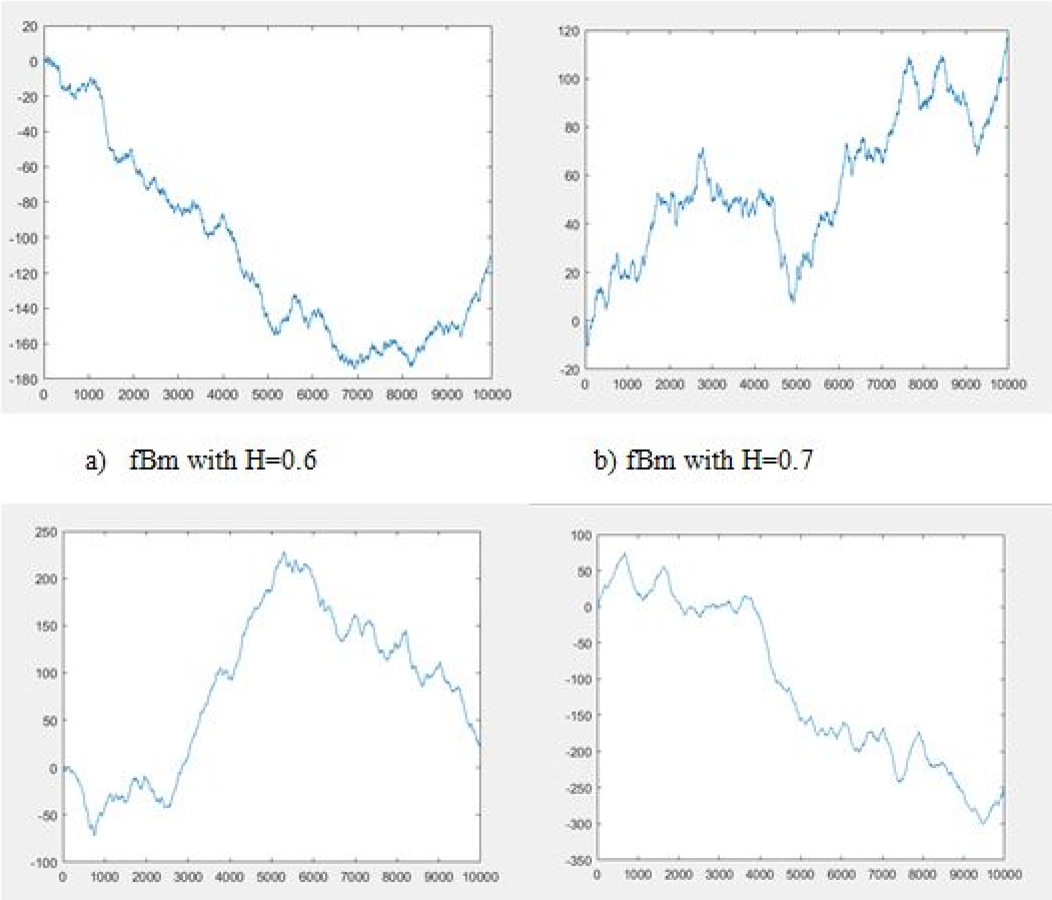
“hurstexp” calculates the Hurst exponent of a time series using R/S analysis, or corrects it with small sample bias with slightly different approaches, see for example Weron [54]. These approaches are a corrected R/S method, an empirical and corrected empirical method, and theoretical Hurst exponent. In this study, we chose the value of the simple R/S Hurst estimation as an H in the Table 4.4.

4.4 Simulation

In order to generate the fBm and find out the duration of the maximum drawdown, simulation study is conducted. We have generated the fBm process through MATLAB code by Abry and Sellan method [1] with $H = 1/2$ up to time 100000 with 100000 simulations, collected maximum drawdown from 100000 paths and compared our results with the theoretical expected value of maximum drawdown to check if we calculated them correctly. We have calculated the maximum loss with the following codes:

```
starttime = cputime;
A=zeros(Length,3);
L = Length;
H = HurstParameter;
N =NumberofSimulations;
data=zeros(N,2);
    for s = 1:N
        fBm= wfbm(H,L);
        maxim(1)= fBm(1);
        loss(1)=0;
            for v=1:L-1;
                if(maxim(v)<fBm(v+1))
                maxim(v+1)=fBm(v+1);
            else
                maxim(v+1)=maxim(v);
            end
        loss(v+1)=maxim(v+1)-fBm(v+1);
    end
A=[fBm' maxim' loss'] ;
```

As an illustration of the results of simulation of fBm please refer to the Figure 4.4.



a) fBm with H=0.6 b) fBm with H=0.7

c) fBm with H=0.8 d) fBm with H=0.9

Figure 4.4: Simulations

From there on we have conducted simulation studies to estimate the maximum drawdown of fractional Brownian motion for $H > 1/2$. We generated a 100000-step fractional Brownian path, then calculated its maximum drawdown and the duration of maximum drawdown. We repeated this algorithm 100000 times in order to calculate the expectation of duration of maximum drawdown. While simulating the process, we used the estimated Hurst parameter from the actual data.

We have used the codes given below to calculate the maximum drawdown of the oil prices:

```
maxloss=0;
maxim(1)=stdln(1);
loss(1)=0;
    for v=1:t-1,
        if(maxim(v)<stdln(v+1));
            maxim(v+1)=stdln(v+1);
        else
            maxim(v+1)=maxim(v);
        end
    loss(v+1)=maxim(v+1)-stdln(v+1);
    end
maxloss=max(loss);
```

Table 4.4: Results of the Simulation Study for Paths of 100000 steps

	Simulation
The Expected Value of Maximum Drawdown	125.33
The Expected Value of the Duration of Maximum Drawdown	4169.3

The expectation of simulation of maximum drawdown and the duration of maximum drawdown are 125.33 and 4169.3, respectively. By using the self-similarity property, results are converted to unit time.

For t=1, the calculation of estimators through simulations with the simple R/S Hurst estimator are that is:

$$HurstEstimate : 0.8485$$

$$
\begin{aligned}
M_1^H = \frac{M_t^H}{t^H} &= \frac{M_{100,000}^{0.8485}}{100,000^{0.8485}} \\
&= \frac{3,980}{100,000^{0.8485}} = 0.2276
\end{aligned}
$$

$$
\begin{aligned}
\tau_{m,1} &= \frac{\tau_{m,t}}{t} \\
&= \frac{\tau_{3,980,100,000}}{100,000} \\
&= \frac{41,658}{100,000} = 0.41658
\end{aligned}
$$

In our data set we have 269 monthly data from the year 1996, so the results of simulation has been converted to the results for paths up to $t = 269$. By the self-similarity property, we convert these values into t=269 by multiplying with $269^{0.8485} = 115.2510$ and 269, respectively which are given in the Table 4.5.

$$
\begin{aligned}
M_{269}^{0.8485} &= M_1^{0.8485} * 269^{0.8485} \\
&= 0.2276 * 115.2510 \\
&= 26.2311 \\
\tau_{26.2311,269} &= \tau_{3,980,1} * 269 \\
&= 0.41658 * 269 \\
&= 112.06
\end{aligned}
$$

Also from the oil prices data set we have calculated the maximum drawdown and corresponding duration for this 269 observations and observed the following results;

$$
M_{269}^{0.8485} = 6.3217 \qquad\qquad \tau_{6.3217} = 75
$$

Table 4.5: Comparison of the Simulation Study with the Actual Data Set for t=269, Hurst Parameter=0.8485

	Simulation	Filtered data
The Expected Value of the Maximum Drawdown	26.2311	6.3217
The Expected Value of the Duration of Maximum Drawdown	112.06	75

For the time $t = 269$, the maximum drawdown and its duration observed from the actual data set are both less than their expected values under fBm model. We conclude the last super-cycle has not completed yet when compared to the expected values of the model despite there is an increasing movement in the prices.

CHAPTER 5

CONCLUSION

In this thesis, our aim was to estimate the duration of the maximum drawdown, i.e. duration of the highest possible drop in the oil prices. The oil prices data were modeled by GfBm. West Texas Intermediate (Cushing, OK WTI) spot price was chosen from among crude oil prices and the data taken from U.S. Energy Information Administration site (www.eia.gov) was used. Monthly data starts from January 1947 and ends in May 2018. The nominal value of the prices in the economy is measured by the value of the currency at the time, hence the real data is used for showing the price level corresponds to the Gross Domestic Product. In order to apply model, real oil prices are created by using Consumer Price Index (Index 1982-1984=100, Monthly, Seasonally Adjusted).

Maximum drawdown, also known as maximum loss, is especially important for both investors and economists. It indicates the highest possible market risk in finance. In order to detect the maximum drawdown in the oil price data set in the light of our aim, super-cycles which are specified as total of an upward and a downward movements were detected. By using economic filtering methods, super-cycles were extracted from the series. The economic filtering methods used were Hodrick Prescott and Band Pass filter. These filtering techniques were applied to the data to decompose the trend component from the series and smooth the data. Then, we identify super-cycles in the historical oil price series. Thanks to these filters, the last super-cycle is identified. The period of the last current super-cycle gives us the time interval of the data which will be used for modeling. HP and BP filter show that the last cycle starts around 1996 and reaches last peak point at 2012.

Before observing the results of maximum drawdown and its duration from the actual data and simulation studies, the assumptions of the fBm were checked for the real life data. First of all, stationarity of increments is looked and the results of KPSS and ADF tests show that series are stationary. Then, histogram and Jarque Bera normality test validated the normality for the differences of logarithm of prices. Finally it has been proven that the series have correlation.

Geometric version of fBm model is used for modeling oil prices. fBm is a realistic model, because fBm is a process that exhibits long-term dependent behavior for $H > 1/2$ and this dependency overlaps with the dependency seen in real life financial data. In the literature, it is well known that Brownian motion gives good results on logarithm of the series and catch the movements of oil prices but not dependency. Therefore Bm can be replaced with fBm. Briefly, as mentioned in the Equations 4.3 and 4.4 the process driven by fBm is:

$$S(t) = S(0)e^{\sigma B_t^H \mu t - \frac{1}{2}\sigma^2 t^{2H}}$$

or

$$\frac{S(t)}{S(t-1)} = e^{\sigma B_1^H \mu - \frac{1}{2}\sigma^2}$$

For our data set oil prices of 269 months, Hurst parameter is estimated from R/S analysis as 0.8485 approximately.

In order to generate the fBm and find out the maximum drawdown of it, simulation was conducted. In these simulations, we have generated 100000 simulations of paths with 100000 steps and calculated the average of 100000 maximum drawdowns and 100000 duration of maximum drawdowns for the purpose of the estimation. In the simulation study, we used the estimated Hurst parameter $H = 0.8485$.

Theoretically the expectation of maximum drawdown is found as 125.33 and the expectation of duration of maximum drawdown is 4169.3. In our data set we have 269 monthly data from the year 1996, so firstly the results of simulation have been converted to the results for paths up to $t = 1$ by the self-similarity property. Then the results for paths up to $t = 269$ have been created. Also from the filtered oil prices data set we have calculated the maximum drawdown and corresponding duration for this 269 observations. Finally the expected maximum drawdown and duration of maximum drawdown from the filtered data equals to 6.3217 and 75, respectively. However simulation results for the maximum drawdown and duration of maximum drawdown for $t = 269$ equals to 26.2311 and 112.06, respectively.

As a result, for the time $t = 269$ the maximum drawdown and its duration observed from the filtered data set are both less than their expected values under fBm model. We conclude the last super-cycle has not completed yet when compared to the expected values of the model despite lately there is an increasing movement in the oil prices.

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APPENDIX A

TABLES OF TIME SERIES

Table A.1: Results of the Filtering Study

Date	Spot Oil Price	HP trend	BP trend
1948	1.841	2.287564267	-0.364722512
1949	2.570	2.379703018	-0.018457508
1950	2.570	2.467376127	0.092534481
1951	2.570	2.54802092	0.432728827
1952	2.570	2.620100964	0.632944593
1953	2.570	2.682299614	0.442771758
1954	2.716	2.732799219	0.027915761
1955	2.820	2.768659129	-0.521042684
1956	2.820	2.786770702	-0.898421071
1957	3.043	2.784538706	-0.550065906
1958	3.058	2.759700202	0.027663133
1959	2.975	2.712576863	0.60825008
1960	2.970	2.64647336	0.949837922
1961	2.970	2.567318595	0.731705942
1962	2.970	2.484276739	0.03491915
1963	2.970	2.410538773	-0.749681619
1964	2.945	2.364152915	-1.168241372
1965	2.920	2.368761991	-0.915532217

Table A.2: Results of the Filtering Study-Cont.

Date	Spot Oil Price	HP trend	BP trend
1966	2.937	2.453817301	-0.057328974
1967	3.027	2.654282523	0.969324979
1968	3.070	3.009953164	1.459159268
1969	3.295	3.564351905	1.266512328
1970	3.351	4.361601893	0.098171352
1971	3.560	5.44313276	-1.260259963
1972	3.560	6.840268117	-2.582644712
1973	3.873	8.565500246	-3.025232061
1974	10.373	10.59851875	3.065256786
1975	11.160	12.87208823	3.045218512
1976	12.645	15.3167181	2.403927282
1977	14.296	17.84579688	0.027230899
1978	14.850	20.34599593	-5.164733556
1979	22.404	22.66848863	-4.03416033
1980	37.375	24.6094884	5.407961821
1981	36.667	25.96256377	1.549796432
1982	33.636	26.64893839	-1.484281517
1983	30.395	26.69688027	-1.864176844
1984	29.276	26.20452806	1.530232989
1985	27.973	25.30700156	4.775184315
1986	15.050	24.17013533	-4.916878964
1987	19.200	22.98642389	0.564459605
1988	15.970	21.85716043	-2.937274009
1989	19.640	20.84577388	-0.267367824
1990	24.530	19.95682158	3.822230757

Table A.3: Results of the Filtering Study-Cont.

Date	Spot Oil Price	HP trend	BP trend
1991	21.540	19.18280311	0.749909294
1992	20.580	18.56194986	0.362371032
1993	18.430	18.15606519	-1.032693778
1994	17.200	18.04713292	-1.829768075
1995	18.430	18.31987627	-0.690183348
1996	22.120	19.0505471	2.575846484
1997	20.610	20.31649852	0.682332587
1998	14.420	22.22577816	-5.658204269
1999	19.340	24.88936867	-0.931556565
2000	30.380	28.34019492	9.11786279
2001	25.980	32.55568809	2.000664023
2002	26.180	37.53367741	-2.881090633
2003	31.080	43.20623523	-5.40274836
2004	41.510	49.39189713	-3.988923521
2005	56.640	55.78793633	1.693845409
2006	66.050	62.01280709	2.3019426
2007	72.340	67.6934843	0.999701462
2008	99.670	72.49731477	21.90387204
2009	61.950	76.13811048	-21.42366096
2010	79.480	78.60141026	-8.798506931
2011	94.880	79.73087184	2.969782991
2012	94.050	79.37893882	1.104549766
2013	97.980	77.54954613	8.238278718
2014	93.170	74.39333926	12.05430655
2015	48.660	70.26526828	-18.44099928
2016	43.290	65.70804984	-5.998734357
2017	50.800	61.04834792	20.27355225