INVESTIGATING MIDDLE SCHOOL PRESERVICE MATHEMATICS TEACHERS' CONCEPTIONS OF ALGEBRA AND KNOWLEDGE OF TASK PURPOSES AND STUDENT THINKING

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ABSTRACT

INVESTIGATING MIDDLE SCHOOL PRESERVICE MATHEMATICS TEACHERS' CONCEPTIONS OF ALGEBRA AND KNOWLEDGE OF TASK PURPOSES AND STUDENT THINKING

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Starting with the beginning of the 21^{th} century, teaching algebra in the early grades has gained more attention. Since teachers are one of the crucial factors in teaching early algebra, this study aimed to understand middle school pre-service mathematics teachers' (PSMTs') awareness about the underlying algebraic structure of given tasks, their conceptions of algebra, expectations about possible student solutions, and the changes after attending the algebra weeks in the Methods of Teaching Mathematics Course. With this aim, a qualitative study was conducted with third year middle school pre-service mathematics teachers who were enrolled to the Methods of Teaching Mathematics Courses in the Elementary Mathematics Education program at a public university in Ankara, Turkey. The data were collected throughout hour-long, semi-structured, task-based individual interviews. The pre-interviews were conducted with eight participants before the two weeks focus on algebra chapter in the tenth week of the first semester and the post-interviews were conducted with seven of these participants after the algebra weeks in the fourth week of the second semester. The findings of the study indicated that the PSMTs were successful in their awareness of task purposes and knowledge of student’s possible solutions except anticipating student misconceptions regarding the equal sign in the pre-interviews. In the post-interviews, PSMTs were more successful at this. While, in the pre-interviews, PSMTs’ categorization of student solutions seemed narrow focusing on symbol manipulation than on relational thinking, this situation changed in the post-interviews. However, PSMTs were not found to hold consistent conceptions of algebra during the interviews.

Keywords: Early Algebra, Middle School Pre-service Mathematics Teachers, Algebra Conceptions, Knowledge of Student Thinking and Task Purposes

ÖZ

ORTAOKUL MATEMATİK ÖĞRETMEN ADAYLARININ CEBİR HAKKINDAKİ ALGILARININ VE SORU AMACI VE ÖĞRENCİ ÇÖZÜMLERİ HAKKINDAKI BİLGİLERİNİN İNCELENMESİ

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21. yüzyılın başlarından bu yana erken yaşlardaki cebir öğretimi önem kazanmıştır. Öğretmenler cebir öğretiminin en önemli unsurlarından biri oldukları için bu çalışma, ortaokul matematik öğretmen adaylarının soruların amaçları hakkındaki bilgilerinin, cebir algılarının, ve öğrenci çözümleri hakkındaki bilgilerinin ve Özel Öğretim Yöntemleri dersindeki cebir haftalarından sonraki değişimlerinin incelenmesini amaçlamıştır. Bu amaca dayanarak, Ankara ilinde Türkiye'de bulunan bir devlet üniversitesindeki Özel Öğretim Yöntemleri dersini alan üçüncü sınıf ortaokul matematik öğretmen adayları ile nitel bir çalışma yapılmıştır. Veriler, yaklaşık bir saat süren, yarı yapılandırılmış ve soru odaklı görüşmeler yoluyla toplanmıştır. Ön görüşmeler, birinci dönemin onuncu haftasında iki haftalık cebir konularından önce sekiz katılımcı ile, son görüşmeler ise ikinci dönemin dördüncü haftasında cebir konularından sonra aynı katılımcılardan yedisi ile yapılmıştır. Çalışmanın sonuçları, ön görüşmelerde

öğretmen adaylarının verilen soruların amaçlarını ve öğrencilerin muhtemel cevaplarını tahmin etmede, eşittir işareti ile ilgili olan kavram yanılgısı dışında, başarılı olduklarını göstermiştir. Son görüşmelerde öğretmen adayları bunda daha başarılı olmuşlardır. Ön görüşmelerde, öğretmen adaylarının öğrenci çözümlerini sınıflandırmaları ilişkisel düşünmeden çok sembol ve işleme dayanırken, bu durum son görüşmelerde değişmiştir. Fakat öğretmen adaylarının tutarlı bir cebir algısına sahip olmadıkları bulunmuştur.

Anahtar Kelimeler: Erken Cebir, Ortaokul Matematik Öğretmen Adayları, Cebir Algısı, Öğrencilerinin Düşünme Biçimlerine ve Soruların Amaçlarına Yönelik Bilgileri

For those who share their wisdom and love generously with others.

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Elementary Mathematics Education: EME

Knowledge of Content and Students: KCS

Knowledge of Content and Teaching: KCT

Methods of Teaching Mathematics: MoTM

Ministry of National Education: MoNE

Pedagogical Content Knowledge: PCK

Pre-Service Mathematics Teacher: PSMT

CHAPTER 1

INTRODUCTION

Romberg and Kaput (1999) stated that the 21st century demands people who have a deeper mathematical understanding. However, Kaput (1999) indicated algebra as a gatekeeper to higher mathematics. Kaput (2008) argued that the school algebra worldwide is mostly based on symbol manipulation. He also claimed that what algebra is depends on how we approach it.

Several researchers (e.g., Blanton and Kaput, 2011; Carpenter, Franke, & Levi, 2003; Ryan & Williams, 2007) advocated that algebraic thinking should be developed in cooperation with arithmetic thinking starting from the early grades. The researchers stated that the focus on the symbol manipulation and the separation of arithmetic and algebra seems to prevent students from building sophisticated mathematical understanding (e.g., Cai & Knuth, 2011; Carpenter et al., 2003). Kaput (1999) argued that, in the school, algebra has been usually taught following some procedures to simplify algebraic expressions, solve equations without making a connection with real life and mathematical ideas. In the school, we need an education which expands our view of algebra with deeper and meaningful mathematical and practical connections (Kaput, 2008). As Blanton and Kaput (2005) stated teachers are the key point to develop algebraic thinking in the classrooms.

Teachers should give importance to mathematical processes and relational thinking to broaden students' algebraic understandings. Teachers' "algebrafication" strategies could be summarized in three main facets according to Blanton and Kaput (2005, p. 71), which are instructional materials, finding and supporting students' algebraic thinking, and, creating a classroom culture and teaching practices that promote algebraic thinking. Additionally, many studies (e.g., Blanton & Kaput, 2004; Blanton et al., 2015; Carpenter & Levi, 2000) showed that when students were led to focus on relations, discuss mathematical ideas, and were challenged through questioning, they were found to be able to make generalizations and generate relational thinking. That is why the "algebraization" (Cai & Knuth, 2011, p. viii) skills of the teachers are the key point of fostering students' algebraic thinking.

Since the teachers are crucial in eliciting and triggering students' algebraic thinking, having insight about pre-service mathematics teachers' conceptions of algebra and their knowledge of content and student in relation to algebra could give an opportunity to make inferences about what they will give importance and what they will focus on in their future lessons in terms of algebra. There are quite a few studies conducted with PSMTs in this area so far, and they mostly focused on equivalence and equations, and variable areas (e.g., Didiş Kabar & Amaç, 2018; Gökkurt, Şahin, & Soylu 2016; Stephens, 2006; Tanisli & Kose, 2013). However, these studies did not focus on how the algebra weeks in the Methods of Teaching Mathematics Courses in the teacher education programs might have an influence on PSMTs' conceptions of algebra and their pedagogical content knowledge in relation to algebra.

This study focused on this gap and attempted to draw a general frame about middle school PSMTs' awareness about the underlying algebraic structure of a given task, their conceptions of algebra, and anticipation of students' possible solutions, and lastly, the changes, if any, after attending the algebra weeks in the Methods of Teaching Mathematics course in their third-year in the teacher education program. These weeks focused on teaching algebra following the course book similar to focusing on teaching other content areas like geometry. Therefore, the algebra weeks were not designed as an intervention, but rather they were part of ongoing MoTM courses.

1.1 Motivation for the Study

During my teaching experience in 4^{th} and 5^{th} grades for two years, I had an opportunity to observe students' misconceptions, their various types of reasoning and solution strategies, and their development when provided with appropriate instruction. Additionally, I realized that my students were ready to generate various ideas when provided with the best practices that were relevant to them, so I realized the importance of teachers' role in the "algebraization" process. Therefore, it was my desire to understand future teachers' awareness of the task purposes, their conceptions of algebra and the possible student solutions in this study.

1.2 Research Questions

This study was conducted with middle school PSMTs who were in their third year in the Elementary Mathematics Education program and enrolled to the Methods of Teaching Mathematics courses in a public university in Ankara, Turkey during the fall and spring terms in the 2017-2018 academic year. The study focused on answering the following research questions:

1. To what extent are middle school pre-service mathematics teachers aware of the underlying algebraic structure of a given task?
2. What are middle school pre-service mathematics teachers' conceptions of algebra?
3. What are middle school pre-service mathematics teachers' awareness about possible student solutions provided to the tasks?
4. How do middle school pre-service mathematics teachers' conceptions of algebra, awareness of task purposes and possible student solutions provided to the tasks change after they attend a Methods of Teaching Mathematics Course?

1.3 Significance of the Study

There are some studies in Turkey which were conducted with pre-service mathematics teachers to understand to what extent they could identify the students' errors and which strategies they use to handle these errors (e.g., Dede & Peker, 2007; Didiş Kabar & Amaç, 2018; Gökkurt et al., 2016; Tanisli & Kose, 2013). In the international literature, there are some studies (e.g., Asquith, Stephens, Knuth, & Alibali, 2007; Stephens, 2006, 2008) which focused on pre-service teachers' understandings of core algebraic concepts and conceptions of algebra. Also, these studies were found to focus on one or two big ideas of algebra such as equivalence and equations, variables among the five big ideas which are equivalence and equations, generalized arithmetic, functional thinking, variable and quantitative reasoning (Blanton, Levi, Crites, & Dougherty, 2011).

This study aimed to focus on the three big ideas which are equivalence and equations, functional thinking, and variable to draw a general frame about PSMTs' awareness about the underlying algebraic structure of a given task, their conceptions of algebra, and anticipation of students' possible solutions, and lastly, the changes, if any, after attending the algebra weeks in the Methods of Teaching Mathematics course in their third-year in the four-year teacher education program. This study might be important about what we might need to know in terms of mathematical knowledge for teaching focusing on algebra. The study might also provide suggestions about how to design "Teaching Algebra" course in the new teacher education programs.

1.4 Definition of Important Terms

Algebraic Reasoning: In this study, algebraic reasoning is defined as "the route [that] involves generalizing and expressing that generality using increasingly formal languages, where the generalizing begins in arithmetic, in modeling situations, in geometry, and in virtually all the mathematics that can or should appear in the elementary grades." (Kaput, 1999, para. 4)

Conception: "A general notion or mental structure encompassing beliefs, meanings, concepts, proportions, rules, mental images, and preferences" (Philipp, 2007, p. 259).

Early Algebra: It is defined as algebra in the early grades which "to encompass algebraic reasoning and algebra-related instruction among young learners—from approximately 6 to 12 years of age" (Carraher & Schliemann, 2007, p. 670).

Middle School Pre-service Mathematics Teachers: The college students who were in their third year in a four-year Elementary Mathematics Education

(EME) program at a public university in Ankara, Turkey. The graduates of the program are certified to teach mathematics between 5th and 8th grades (middle school).

CHAPTER 2

LITERATURE REVIEW

This study aimed to identify the middle school pre-service mathematics teachers' perceptions about the underlying algebraic structure of a given task, their conceptions of algebra, their awareness about possible solutions of students, and the possible changes in all these three categories before and after the "algebraic thinking" chapter. The relevant literature was divided into three sections: in the first part, theoretical frameworks will be described. In the second part, elementary and middle student thinking and misconceptions regarding equivalence and equations, functional thinking, and variable will be summarized. Then, studies related to teacher knowledge of students' algebraic thinking will be presented. Finally, algebra in the national curriculum will be summarized.

2.1 Theoretical Frameworks

In this study, two theoretical frameworks were used. The first framework, Kaput's framework for algebraic reasoning (2008), was used to clarify PSMTs' conceptions of algebra. The second framework, Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008) was used to understand to what extent PSMTs' knowledge of content and students, specifically, their awareness of the underlying algebraic structure of a given task, and possible correct and incorrect student solutions. In this section, firstly Kaput's framework for algebraic reasoning will be summarized. In the following section, Mathematical Knowledge for Teaching (MKT) framework will be reviewed.

2.1.1 Kaput's Framework for Algebraic Reasoning

According to Kaput (2008), algebraic reasoning comprises five complementary strands as forms of reasoning (Figure 2.1). As reported by Kaput, the first two forms of the reasoning (Core Aspects A & B) are the core aspects, which diffuses into the three forms of reasoning (Strands 1, 2, & 3).

The Two Core Aspects
A. Algebra as systematically symbolizing generalizations of regularities and constraints. B. Algebra as syntactically guided reasoning and actions on generalizations expressed in conventional symbol systems.
Core Aspects A & B Are Embodied in Three Strands
1. Algebra as the study of structure and systems abstracted from computations and relations, including those arising in arithmetic (algebra as generalized arithmetic) and in qualitative reasoning. 2. Algebra as the study of functions, relations, and joint variation. 3. Algebra as the application of cluster of modeling languages both inside and outside of mathematics.

Figure 2.1 Core Aspects and Strands in Kaput's Framework of Algebraic Reasoning. Reprinted from *Algebra in the early grades* (p. 11), by J. J. Kaput, 2008, Mahwah, NJ: Lawrence Erlbaum/Taylor & Francis Group.

According to Kaput (2008), Core Aspect B which focuses on manipulation of formalism should be advanced after Aspect A which focuses on regularities, relations and making generalizations in order to build deep and meaningful understanding. Indeed, the relational understanding should be developed first, then the rule-based actions on symbols should be focused on. As Kaput stated (2008), Strands 1, 2 and 3 are embodied by Core Aspects A and B. Among these three strands, Strand 1 could be explained as a syntactic form of transition of arithmetical structure to algebra by making generalizations (i.e., generalized arithmetic). In this process, the focus is on making arithmetic expressions according to its form, not the value that we get when it is computed e.g. generalized arithmetic and quantitative reasoning. The following strand, Strand 2, is about functions. The strand focuses on representing regularities and systematic variations with the base of generalization e.g. functional thinking. Strand 2 comprises the important part of the school algebra, and it depends upon syntactic view of algebra, e.g. writing a function rule by using symbolization. The last strand, Strand 3, is based on three types of modeling. The first category of modeling is the number or quantity specific modeling in which the syntactic notion represents the unknown, not the variable, in an equation. The second category of modeling includes Core Aspect A. In this category, generalization, which is the form of expressions of a function is modeled. The third category of modeling refers to modeling generalization to make the relation to be grasped by comparing it with other situations.

Kaput and Blanton (2008) indicated that generalization and symbolization are essential parts of algebraic thinking. Actually, these generalization and symbolization concepts refer to Kaput's Core Aspects A and B. Since Kaput (2008) hypothesized that these two main aspects are embodied in the three strands, Core Aspects A and B will be used in this study to understand PSMTs' conceptions of algebra.

2.1.2 Mathematical Knowledge for Teaching Framework

Shulman (1986) defined the content knowledge as "the amount and organization of knowledge per se in the mind of the teacher" (p. 6). Furthermore, Shulman claimed that without pedagogical knowledge, merely content knowledge is not practical. Therefore, Shulman suggested evaluating content knowledge by dividing it into three main domains which are subject matter content knowledge, curricular knowledge and pedagogical content knowledge (PCK). Since PCK is "subject matter knowledge *for teaching*" (p. 7), Shulman stated that the teachers have to be aware of the opportunity of various representations and students' current conceptions and misconceptions. Also, the teachers should be able to present the topic by taking into consideration the grade level of the students.

Ball, Thames, and Phelps (2008) detailed Shulman's (1986) categorization, and they worked on a framework, Mathematical Knowledge for Teaching (MKT). In their framework, MKT consists of two main parts as subject matter knowledge (SMK) and PCK. They also defined three domains under SMK and PCK (see Figure 2.2). When the SMK part is examined, common content knowledge (CCK), horizon content knowledge (HCK), and specialized content knowledge (SCK) would be seen under it.

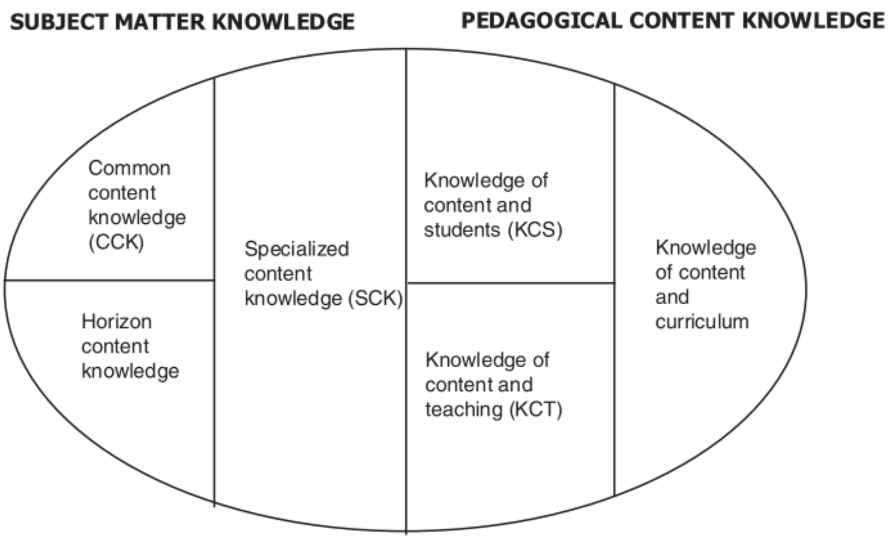


Figure 2.2 Domains of Mathematical Knowledge for Teaching. Reprinted from "Content knowledge for teaching: What makes it special?" by D. L. Ball, M. H. Thames, & G. Phelps, 2008, *Journal of Teacher Education*, *59*(5), p. 403.

Under the PCK, they also described three domains; knowledge of content and student (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). KCS was defined as the knowledge about mathematics and students' thinking. It includes being aware of students' common misconceptions and what students find difficult. KCT was defined as a combination of knowledge about teaching and mathematics. Regarding KCT, teachers should be able to prepare and choose tasks to make connections with other contents and build deeper mathematical understanding. Therefore, teachers should have the adequate mathematical knowledge to make appropriate task design and implementation. As the last domain of PCK, knowledge of content and curriculum could be defined as the knowledge of the followed curriculum, objectives at the related grade level, preparing tasks according to corresponding objectives and level of the students. Also, teachers should know what students learn in the previous years and what they will learn in the following years related to the teaching area. "Methods of Teaching Mathematics I" (MoTM I) and "Methods of Teaching Mathematics II" (MoTM II) courses mostly focus on the PCK, and this study aimed to identify the middle school pre-service mathematics teachers' perceptions about the underlying algebraic structure of a given task, their conceptions of algebra, their awareness about possible solutions of students which are related to KCS and the knowledge of content and curriculum. That is why this study focused on the PCK part of the MKT framework in particular.

2.2 Elementary and Middle School Students' Algebraic Thinking and Misconceptions

The teachers should be aware of students' possible solutions and their different ways of thinking and misconceptions to help them. This section will summarize international and national studies about students' algebraic thinking. In the first part, students' misconceptions and difficulties will be reviewed. Students' algebraic thinking will be attempted to summarize in the following part.

Elementary and Middle School Students' Difficulties and Misconceptions. The equal sign is defined as "the relation between two equal quantities" (Carpenter, Franke, and Levi, 2003, p. 9) or "a symbol manipulation that represents a relation of equivalence" by Blanton et al. (2011, p. 25). In the elementary grades, many students focus on the equal sign as "performing a computation", "the answer", or "the total" (Blanton et al., 2011; McNeil & Alibali, 2005; Yaman, Toluk, & Olkun, 2003). A study conducted by Falkner, Levi, and Carpenter (1999) showed that even middle school students have difficulty in interpreting the equal sign as a relation between two quantities. In their study, the question "8 + 4 = ... + 5" was asked to students from grades 1-2, 3-4 and 5-6 and only 5% of the grades 1-2, 9% of the grades 3-4, and 2% of the grade 5-6 gave the correct answer as 7. The rest gave a response as 12 or 17. In Turkey, Kızıltoprak and Köse (2017) had similar findings. For example, 10 6th grade students were asked the question "3 + 8 =...+ 5" and four students responded by adding 3 and 8, while three students responded the question by adding 3, 8, and 5.

Apart from the difficulty in understanding equivalence, many studies show that students have various difficulties in interpreting the variable (Asquith, Stephens, Knuth, & Alibali, 2007; Dede, Yalın, & Argün, 2002). Blanton et al. (2011) defined five meaning of variables; "(1) symbols in generalized pattern, (2) fixed but unknown numbers, (3) quantities that vary, (4) parameters, and (5) abstract placeholders in an algebraic process" (p. 38). Since the concept of variable might have more than one meaning, students might have several difficulties with the variable. "Letter ignored" is one of the typical student thinking in the variable question (Küchemann, 1978, p. 25). In particular, the result of a study (Dede, Yalın, & Argün, 2002) which was conducted with 8^{th} grade students in Turkey showed that students mainly gave a response by ignoring the letter in the variable question. In the study, 60% of the students gave incorrect answer to algebraic expression question "$2 + 5x = ?$" and one group of the students' gave "7" as a response by ignoring x in the algebraic expression. The other group of the students' responses was related to the "acceptance of lack of closure" (Collis, 1975 as cited in Küchemann, 1978), and they pretended there was a number (for example, a "0") on the other side of the equal sign, then they tried to solve the equation.

As Ryan and Williams (2007) stated, another typical error in the variables is "substitution" (p. 108). They defined this error as assigning a specific value to the unknown for instance, $a = 1$, $b = 2$, or $c = 3$. Also, MacGregor and Stacey (1997) mentioned in their study that students coded a is equal to 1 or b is equal to 2 because of the alphabetical order or that they might have a tendency to put 1 instead of a letter. Students' another confusion about the variables were found to stem from the use of x in arithmetic as a multiplication sign e.g., Ryan and Williams (2007) exemplified it as "$5x$ may be read as '5 times'" (p. 108).

The aforementioned common misconceptions were also observed in the study conducted by Soylu (2008) in Turkey. Additionally, the researcher indicated another limitation of students' understandings about variables. The researcher conducted the study with the purpose of defining Turkish students' interpretation of a variable. In the scope of the study, 50 7th grade students were asked to respond to eight open-ended questions about variables. The result of the study showed that, similar to Ryan and Williams (2007) and MacGregor and Stacey (1997), students put a numerical value instead of a variable. For example, in the question $3(n + 5)$, 17 students found a numerical result by putting a random number instead of *n*, e.g. $n = 10$, in all the questions, participants assigned a value instead of the unknown. Furthermore, students were found to ignore a variable similar to the "letter ignored" defined by Küchemann (1978, p. 25). For instance, in the question "$5x + 4 = ?$" the students gave $9x$ or 9 as a result. In addition to assigning a number instead of an unknown or ignoring a variable, the study presented another limitation of students, which is the students' preference to use *x* in their solutions instead of the given symbolization such as, *h*, *m*, *n*, *y* in the given tasks.

Apart from the equivalence and equations, and variable, students were also found to have some difficulties in functional thinking. As it is stated by Blanton et al. (2011), functions have an important role in developing algebraic understanding. Since the functions express the relation between quantities, they support meaningful understanding of symbolic notation. According to Blanton and Kaput (2004), building a meaningful functional thinking, patterns are used as a transition, but just focusing on recursive patterns might prevent students from developing sophisticated functional thinking. In the study conducted by Isler et al. (2015), in the pre-tests, the majority of the 3rd, 4th, and 5th graders were found to focus on recursive relationships than covariational relationship or functional relationship in words and variables when asked to describe the patterns that they saw.

To sum up, as defined by many researchers (e.g., Asquith et al., 2007; Küchemann,1978; MacGregor and Stacey, 1997; Ryan and Williams, 2007) students have difficulties and misconceptions around fundamental algebraic concepts. As presented in the studies (e.g., Dede et al., 2002; Soylu, 2008) students, even in middle school, were mostly observed to have these misconceptions.

Elementary and Middle School Students' Algebraic Thinking. In this part, the studies which show students' ability to perform algebraic thinking will be reviewed under functional thinking, and equivalence and equations. Since the variable infuses into these two big ideas, it will be summarized under these categories. Firstly, the studies regarding students' functional thinking, and secondly the studies regarding equivalence and equations will be summarized.

Studies conducted regarding students' functional thinking. The first two study (Blanton & Kaput, 2004; Isler et al., 2015) will present how the students from different grade levels can develop functional thinking after instructional interventions. The following two studies (Ng, 2018; Tanışlı, 2011) will be summarized to present students' ways of functional thinking.

Blanton and Kaput (2004) conducted a study to investigate how student develop functional thinking. The data were collected from a 6-year-project which was about teacher development in terms of increasing teachers' classroom practices about algebraic reasoning. Data were gathered from Pre-K – 5th grade students' responses to a task aiming to assess how children build functional relationships, and also interviews were conducted with teachers. The task was asking "If there was one dog, how many eyes would there be? What if there were two dogs? Three dogs? 100 dogs?" and "How many eyes and tails are there for one dog? Two dogs? Three dogs? 100 dogs?" (p. 136). The results showed that the pre-kindergarten students drew a t-chart with the help of the teacher, also they found the far function values by counting without making a prediction. In kindergarten, students recorded the data by drawing a dot for each eye and drawing a notch for each tail, or they drew a t-chart and focused on a pattern. In the 1st grade, students drew the t-chart without the help of the teacher, and they noticed the recursive pattern. For example, they realized that the number of the eyes increases by 2 and the number of the eyes and tiles increases by 3. In the 2nd grade, students were able to identify the multiplicative relationship that the number of eyes is the two times the number of the dogs. Also, they predicted far function values by using this relationship. In the 3rd grade, in addition to drawing a chart, realizing recursive pattern and multiplicative relationship, they also described the relation by writing “n *x* 2” or “2 *x* n” (p. 138). In the 4th and 5th grades, students could perform the aforementioned ones, the only difference was that they could realize the pattern and write the function rule by using fewer data. This study showed how students developed functional thinking in each grade level, to what extent they were able to realize the patterns and relations, how they used representations, and when they were able to use symbols to represent the relationships.

A study conducted by Isler et al. (2015) focused on how a year-long teaching experiment developed students’ functional thinking. The study was conducted on two classes each from the 3rd, 4th, and the 5th grade. Before the teaching experiment started, a pretest was conducted to assess students’ prior knowledge. During the teaching experiment, students worked on some problems in their small study groups. These problems aimed to enable the students to work on different kinds of functional relationships including recursive, covariational and correspondence relationships with the help of a teacher facilitating group discussions by asking triggering questions. After the small group discussions, students were asked to share their ideas with the whole class. According to the pre-test, although students had difficulty in identifying covariational thinking and writing functional relationship in symbols and words, in the post-test, students from all grade levels made significant progress.

Tanışlı (2011) working with four 5th graders, conducted task-based interviews aiming to understand students’ use of functional thinking. The interview consisted of 16 questions about linear function tasks, and they all were shown to students on the function table. Since the 5th graders in Turkey are not exposed to using letters as a variable in the curriculum, the researcher represented the dependent and independent variables by circles and triangles. The results of the study were examined in two main contexts: realizing a pattern and determining their ways of functional thinking. The researcher observed that students focused on finding a recursive pattern primarily by focusing on the change either in the dependent variable or in the independent one in the tables. When the results about functional thinking were examined, students identified the correspondence relationship by using additive and multiplicative relationship. Although the 5th graders were not able to use a letter as an unknown, they explained the correspondence relationship by using semi-symbolic rules. For example, "If we subtract four out of the numbers of triangle, we can find the difference [the second instrumental pattern], then we add up the difference with the numbers of triangle ... we find this [the first instrumental pattern] ... then we add up the difference [the first instrumental pattern] with the numbers of triangle we find the numbers of square" (p. 221). As the result of the study indicated, the 5th graders were successful in realizing correspondence relationships and making a generalization. Moreover, the study also demonstrated that the students were able think more than one way to make a generalization, so the teachers should be aware of students' alternative thinking ways to support their functional thinking.

Lastly, a study conducted by Ng (2018) aimed to understand how students make a generalization in the function tasks. The participants were 10 students from 1st to 6th grades. The interview was prepared in two different levels: one level of the interview for a lower primary grade which included from 1st to 3rd graders, and the other level for an upper primary grade level which included from 4th to 6th graders. The interview was based on function-machine tasks which focused on input number, output number, finding a rule which make input number to output number, and writing a general rule. The interview task was designed in the increasing structural complexity, that is, it started with a single operation and went up to writing a functional rule by using a letter. The researcher assumed that students should be made to think to see the relationship between the input and output and write a general rule on the basis of the various tasks. In the lower primary grades, since the students were not able to use letters, when they realized the relationships between numbers, they wrote the function rule by using a semi-symbolic rule. All in all, although these students did not receive an intervention, the students at each grade level were found capable of noticing a relationship and making a generalization when a task with increasing structural complexity was provided.

Studies conducted regarding students' thinking on equivalence and equations. As mentioned in the students' difficulties and misconceptions part, students could interpret the equal sign as "performing a computation", "the answer", or "the total" (Blanton et al., 2011; McNeil & Alibali, 2005; Yaman et al., 2003). In order to handle this misconception, Carpenter et al. (2003) suggested that students should be challenged with this misconception by using open-number and true-false sentences. The following studies will summarize to what extent the students can build relational understanding of the equivalence and equations by using open-number and true-false sentences.

Carpenter and Levi (2000) conducted a study in order to understand how students in the early grades develop a sense of equality as one of the subdomains of the algebraic thinking. They planned eight lessons to be conducted in a month with an experienced teacher. Their participants were eight students from 1^{st} and 2^{nd} grade. At first, the students were asked true-false questions with the addition of the two numbers and a single answer after the equal sign. After similar examples, when the teacher showed them another true-false question e.g. "4 + 3 = 5 + 2" (p. 7), the students claimed that it is not possible to write such a number sentence., then they conducted a discussion about the meaning of the equal sign. During the other lessons, the teacher focused on the open number sentences, firstly with one variable, secondly with two variables, and then with repeated variables e.g. "$\square + \square + \square - \square = 10$" (p. 10). In the last lessons, the teacher also focused on making a generalization, and students were asked to find numbers to make the sentence true and make a relation between numbers e.g. "$\square + \square = \triangle$ " (p. 10). This study showed that 1^{st} and 2^{nd} graders were mostly successful at the end of the intervention at realizing a relation and making a generalization with the help of the open-number and true-false sentences.

A study conducted by Stephens et al. (2013) with 104 3^{rd} grade, 108 4^{th} grade and 78 5^{th} grade students aimed to assess students' prior knowledge before they receive any specific algebraic instructional intervention. Their prior knowledge was assessed by using an hour-long written assessment which focused on equivalence and equations. At first, students were asked the meaning of the equal sign in the number sentence “3 + 4 = 7” (p. 176). Only six out of 290 students provided a relational meaning of the equal sign which means both sides are the same, and the majority of the students focused on operational thinking which means interpreting the equal sign as a total. Stephens et al. (2013) used two different codes for relational thinking; relational-structural and relational-computational thinking. Relational-structural code was used when the students focused on the underlying structure of a task. For example, in an open number sentence “7 + 3 = ... + 4” (p. 176), if the students stated that “6 should be placed in the blank in 7 + 3 = ... + 4 because 4 is one more than 3, so the number in the blank must be one less than 7” (p. 176) it was coded as relational-structural. The answers which focused on computation to find the unknown number were coded as relational-computational. For example, in the previous number sentence, if the students stated that “6 should be placed in the blank in 7 + 3 = ... + 4 because the sum of each side would be 10” (p. 176), it was coded as relational-computational. In the previous open number sentence, if the students interpreted the equal sign as a total and said that the unknown number should be 10, these responses were coded as operational. As a result of the study, students’ understanding of the equal sign mostly depended on the operational meaning. The researchers suggested that by using open number sentences and true/false questions, students ’understandings should be challenged and they should be helped to focus on relational thinking.

A study conducted by Blanton et al. (2015) aimed to understand the effect of the intervention on the third grades students’ algebraic thinking. The study conducted with 106 third graders and 39 of them received an intervention. The interventions were planned during the academic year totally consisting of 19 one-hour long lessons. These lessons were designed in order to develop students’ algebraic concepts and practices. Each lesson started with a group task around the big ideas of the algebra e.g. equivalence and equations. Pre- and post-written assessments were conducted at the beginning and the end of the intervention. The students were asked two questions about equivalence and equations that consisted of open-number sentences (7 + 3 = ... + 4, p. 51) or true-false number sentence (57 + 22 = 58 + 21, true or false, p. 51). The responses of the students were coded as structural, computational or operational. In the pre-assessment, the students in the intervention and non-intervention groups mostly had operational understanding in both open-number and true-false sentences questions, while none of them used structural strategy. When the post-assessment results were examined, it was seen that, in the intervention group, 61% of the students used computational (e.g., "7 + 3 = 10 and 6 + 4 = 10," p. 51) and 16% of the students used structural strategy (e.g., "if you take one away from the 7 and add it to the 3 you have 6 left," p. 51) in the task, "7 + 3 = ... + 4" (p. 51). However, almost all students in the non-intervention group continued to have the operational strategy. When the pre- and post-assessment results were examined, it was seen that operational understanding of the equal sign did not change in the group who did not receive the intervention, while the students in the intervention group developed relational thinking. This study shows how appropriate instruction might support students' algebraic thinking.

An experimental study aiming to understand students' relational thinking development in 5th grade was conducted by Kızıltoprak and Köse (2017) with six students in Turkey. A clinical interview which focused on equivalence and equations were conducted firstly before the teaching process to understand to what extent the students can think relationally. Then the teaching process which based on interaction between students themselves and teacher-student interaction was designed. Totally eight sessions were conducted, and these sessions focused on building a relational understanding of the equal sign. Lastly, post-clinical interviews were conducted, and almost all of the students including who interpreted the equal sign as "the total" and who were not aware of the relational understanding of the equal sign in the pre- interviews, were found to be successful in the post- interviews.

Although the elementary and middle school students were found to have various difficulties and misconception, the aforementioned national and international studies (e.g., Blanton & Kaput, 2004; Carpenter & Levi, 2000; Isler et al., 2015; Kızıltoprak & Köse, 2017; Ng, 2018; Tanışlı, 2011) showed that the students' algebraic thinking can be developed, starting in the early grades, when the students are presented lessons and tasks that lead them to questioning and thinking about the relationships. Also, when teachers ask triggering questions, use multiple representations and create a learning environment based on reasoning and discussions, the results seem to be successful.

2.3 Teachers' Pedagogical Content Knowledge About Elementary and Middle School Students' Algebraic Thinking and Misconceptions

This part addresses national and international studies which are about Pedagogical Content Knowledge (Ball et al., 2008). Students have various thinking ways and the teachers should be aware of their ways of thinking (Ball et al., 2008, Lannin, Barker, & Townsend, 2006; Yetkin, 2003). In addition, teachers' ability to be aware of students' difficulties and misconceptions makes a valuable contribution to the meaningful learning process (Yetkin, 2003). That is why teachers' awareness of the ways that the students think and that the misconceptions they possibly hold are important components of fostering algebraic thinking (Blanton & Kaput, 2003). According to Thompson (1992), there is a strong relation between teachers' conceptions of mathematics and their instructional practices, also their conceptions of teaching and their conceptions of students' mathematical knowledge. Also, Thompson (1992) stated that the studies conducted with preservice teachers showed that their conceptions are not easy to change because they assimilate the new ideas instead of internalizing them by accommodating. To change teachers' conceptions permanently, the researchers claimed that teachers should be more familiar with students' thinking (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). In the following parts, studies which are about the teacher pedagogical content knowledge about elementary and middle school students' thinking and misconceptions regarding the equivalence and equations and functions will be summarized.

A study which was conducted by Stephens (2006) aimed to understand PSMTs' awareness of possible student misconceptions and underlying algebraic opportunities of the given tasks about equivalence and relational thinking. The researcher studied with 30 elementary PSMTs who were at the third-semester in a five-semester program. The participants were enrolled in their first course which was related to teaching mathematics, and the study was conducted at the beginning of the course term to assess PSMTs' readiness. Semi-structured interviews which consisted of five tasks about equivalence and relational thinking was conducted. According to the findings of the study, the majority of the PSMTs were found to have awareness about the purpose of the tasks addressing relational thinking mathematical equivalence. Although the PSMTs recognized the underlying relational structures of the task, some of them additionally specified the aim of the task as symbol manipulation or performing computational procedures. For the purpose of having insights about the PSMTs' knowledge of students' thinking, the PSMTs were asked possible student solutions provided to the given tasks. The findings suggested that although in some tasks, participants anticipated the relational thinking solutions, they tended to pose computational strategies more frequently. In some tasks, participants were presented with student work including relational thinking strategy (relational structural strategy as categorized in Stephens et al., 2013). The findings indicated that the PSMTs were successful in summarizing strategies that were employed by the students. Finally, the last research question aimed to describe PSMTs' knowledge of students' misconceptions. Although operational thinking of the equal sign is one of the most common misconception of students as discussed earlier, only six PSMTs out of 30 anticipated this misconception. Afterwards, a student solution including the operational understanding of the equal sign was presented to the participants, and 26 participants could recognize students' lack of understanding in the meaning of the equal sign, while other participants based their explanations on students' lack of attention. However, when another task with student solution ("False, because if you minus nine it will not still equal 31" regarding the task 16 + 15 = 31 is true, 16 + 15 – 9 = 31 – 9 true or false?, p. 270) including the operational understanding of the equal sign was presented to the PSMTs, they were found not successful in clarifying the student's misconception. Only seven out of 30 PSMTs based their explanation on the meaning of the equal sign, while 17 out of 30 PSMTs referred to the student "didn't see" or "didn't notice" minus nine (p. 269). The researcher stated that since the second task and regarding student solution were not as straight forward as the first one, PSMTs might have had difficulty to identify the student's misconception.

A similar study that aimed to understand pre-service middle school mathematics teachers' knowledge about students' conceptions of equality and equation, and variable was conducted by Tanisli and Kose (2013). The fourth-year PSMTs were chosen from two different state universities in Turkey. They chose fourth year students since the researchers required them having to complete the Mathematics Teaching I and II courses which PSMTs focused on pedagogical content knowledge. Sixty participants from one university and 70 participants from the other university were chosen to participate in the study. A questionnaire that included three open-ended questions to investigate participants' knowledge about the thinking process of students, ability to ask questions to identify students' errors and anticipating students' possible false answers took place. In the study, for instance, one of the questions was "Ayse is 4 cm. taller than Seda. If Seda is *n* cm. tall, how tall is Ayse?" (p. 5), and an example of a presented student solution to PSMTs was "Aral: Ayse's height is 4n" (p. 7). When the PSMTs were asked to how to handle this difficulty, their responses were found not at the expected level. Example of their suggestions were "The expression 4 cm. taller requires adding, not multiplying in mathematics" or "Does the question state that Ayse is four times taller than Seda, or 4 cm. taller than Seda? If Seda's height is *n*, and Ayse is 4 cm. taller than Seda, aren't we required to add 4 to Seda's height" (p. 8). As it was seen, the PSMTs asked instructional questions which included guiding students too much instead of having students to realize their mistake.

Likewise, in a study conducted by Asquith et al. (2007), the researchers focused on the teachers' knowledge of student understanding regarding the equal sign and variables. In the scope of the research, 20 middle school teachers were asked possible student strategies for five tasks. In the variable task, teachers successfully anticipated students' use of these symbols such as the variable which means that "the symbol can stand for any number" and the unknown which means that "the symbol can stand for one specific number only" (p. 257). In the task related to the equal sign, although most of the participants were aware of the students' misinterpretation of the equal sign as an operational symbol, the teachers did not anticipate students can give these answers, and they anticipated possible student responses including relational thinking. Teachers stated that students' exposure to the equal sign since kindergarten might have had an influence on their operational thinking. For example, one of the teachers said "because they've used the equal sign a lot since kindergarten" (p. 268). The teachers were found successful in anticipating possible student solution regarding the variable and the equal sign. However, although they were aware of the student misconception related to the equal sign, teachers thought that students do not hold such a misconception since they have been exposed to the equal sign for years.

Several studies in Turkey addressed PSMTs' knowledge of student thinking in relation to algebra in the recent years. Gökkurt et al. (2016) aimed to understand the middle school pre-service mathematics teachers' abilities to realize students' misconceptions about variables. As an initial step of the study, eight open-ended questions were asked to 72 7^{th} grade students. Based on the data, the researchers chose six questions with the most common student misconceptions. In the following stage of the study, 63 pre-service middle school mathematics teachers, who were fourth year students in a state university in Turkey, were presented six questions with the incorrect student responses, and they were expected to realize the students' misconceptions. As a result of the study, it was found that the PSMTs were not quite successful in identify students' misconceptions and where these misconceptions stem from. For example, one of the PSMTs stated "S/he misunderstood the question, s/he should have read the question slowly" (p. 22). The result of the study suggested that the PSMTs' PCK is not at the sufficient and expected level to recognize and overcome these misconceptions.

Likewise, Didiş Kabar and Amaç (2018) worked with 44 pre-service middle school mathematics teachers who were in their third-year in a four-year program and enrolled in the Methods of Teaching Mathematics I course in a state university in Turkey. The study aimed to understand middle school PSMTs' anticipation of the students' difficulties and misconceptions and their PCK to handle with these difficulties and errors regarding the variable. As a first stage of the study, the researchers collected data from 49 7th graders by applying Küchemann's (1978) variable test to identify students' common difficulties and misconceptions. Then, the researchers developed an interview protocol that consisted of six task-based open-ended questions to evaluate middle school PSMTs' knowledge of students' misconceptions and abilities to handle these misconceptions. The findings from the study suggested that the PSMTs performed inadequate performance to anticipate students' errors and misconceptions and to explain where they stemmed from. The PSMTs explained students' errors mostly by using general explanations without detecting students' specific misconceptions. Moreover, PSMTs' instructional strategies to handle these misconceptions varied from task to task. For example, in some questions, they preferred they preferred giving direct information by making a description and showing the mistake to students, in some questions they preferred having students realize their errors. The researchers interpreted the inconsistencies in instructional strategies as a result of insufficient PCK.

Similarly, a study conducted by Dede and Peker (2007) aimed to understand PSMTs anticipation of students' errors and their instructional abilities to overcome these difficulties. In this context, the researchers conducted a study that comprised two stages, similar to the design of Gökkurt et al. (2016). In the first stage of the study, 99 middle school students including 7th and 8th graders were applied a test. The data collection tool included 10 open-ended questions about variables. After the researchers analyzed the most common students' errors and misconceptions, the same test was applied to the PSMTs to understand their anticipation of students' errors and misconceptions and their instructional solutions to handle these difficulties. Sixty-five secondary PSMTs and 55 middle school PSMTs who were in their fourth year in a four-year teacher education program were chosen as participants from a state university in Turkey. The result of the study suggested that the PSMTs could anticipate generally one type of student error and there were also some participants who could not anticipate any possible student solutions or misconceptions. Additionally, most of the PSMTs could not make suggestions to overcome these misconceptions and difficulties. In particular, the PSMTs' instructional suggestions that were offered were not based on deepening students' algebraic thinking, but they were mostly about teacher-centered explanations. For example, in the question "$k + 7 = 10$" (p. 41), for the possible wrong student solution, PSMTs suggested similar instructional strategies to overcome it e.g. "It should be explained that the sign will change when numbers are passed to other side of the equality" (p. 44).

As seen in the summarized studies, PSMTs' anticipations of students' misconceptions and instructional strategies to overcome their difficulties were not found at the expected level to help students.

2.4 Algebra in the National Grades 1-8 Mathematics Curriculum

In this part, analysis of the national curriculum developed by the Ministry of National Education (MoNE, 2018) will be presented according to the algebra objectives that were included in the different grade levels. When the national curriculum was examined, it could be seen that the learning area for algebra is specified in the middle school, in the 6^{th} grade, for the first time. Although algebra was not specifically mentioned in the curriculum before Grade 6, there are some objectives which are about the big ideas of algebra: equivalence and equations, generalized arithmetic, functional thinking, variable, and quantitative reasoning (Blanton et al., 2011). In this regard, the related objectives in the Grades 1-8 National Curriculum provided by the Ministry of National Education (MoNE, 2018) will be summarized, respectively.

The objectives addressing algebra in Grades 1-4 were shown in Table 2.1.

Table 2. 1

Learning objectives addressing algebra in Grades 1-4

Grades	Numbering in the Curriculum	Objectives
1st Grade	M.1.1.2.2.	Students perform addition with the numbers which the sums up to 20 (20 included). a) The sign of the addition (+) and the equal sign (=) are introduced and their meanings are emphasized.
	M.1.1.2.3.	Students notice that the sum does not change when the order of the addends change.
	M.1.2.3.1.	Students find the rule of a pattern consisting of objects, a geometric object or figure, and completes the pattern by identifying the missing objects in the pattern.
	M.1.2.3.2.	Students form a pattern that has three items at most by geometric objects or figures.
2nd Grade	M.2.1.1.6.	Students identify number patterns that has a constant difference, find the rule of the pattern and complete the pattern by determining the missing item.
	M.2.1.3.5.	Students realize the meaning of the equal sign as an "equality" between the mathematical expressions.
	M.2.1.4.2.	Students multiply natural numbers. c) Students are made to notice that changing the order of the multipliers would not change the product.
3rd Grade	M.3.1.1.7.	Students expand and generate the number patterns that has a constant difference.
	M.3.1.2.2.	Students realize that, adding two numbers in different order does not change the result.
4th Grade	M.4.1.4.2.	Students show that changing the order of the multipliers in multiplication with three natural numbers does not change the result.
	M.4.1.5.7.	Students identify the value that is not given in one of the two equal mathematical expressions and explain that the equality holds. For instance, 8 + __ = 15 – 3 12 : 4 = __ + 1 6 × __ = 48 – 12
	M.4.1.5.8.	Students explain the operations that must be performed to make two mathematical expressions that are not equal. For instance, students focus on what to do to make the equality hold in $8 + 5 \neq 12 - 3$.

In the middle school grades, algebra as a learning area officially takes place in the Grades 6-8 (see Table 2.2).

Table 2. 2

Learning objectives addressing algebra in Grades 5-8

Grades	Numbering in the Curriculum	Objectives
5th Grade	M.5.1.1.3.[1]	Students construct the required steps when given the rule of the pattern for number and shape patterns.
6th Grade	M.6.2.1.1.	Students write an algebraic expression for the given verbal situation and write a verbal situation for the given algebraic expression.
	M.6.2.1.2.	Students compute the value of the algebraic expression for different natural number values that the variable can take.
	M.6.2.1.3.	Students explain the meaning of simple algebraic expressions.
	M.7.1.1.2.[2]	Students use the properties of addition as a strategy for fluent operations. a) For example, in the addition of 5 + 7 + (-5) = ?, the commutative, associative, inverse element, and identity element (additive identity) properties are shown and the operation is done like: 5 + 7 + (-5) = 5 + ((-5)+7) = (5+(-5)) + 7 = 0 + 7 b) The commutative, associative, inverse element, and identity element (additive identity) properties of the addition are worked on.
	M.7.2.1.1.	Students perform addition and subtraction with algebraic expressions.
	M.7.2.1.2.	Students multiply an algebraic expression by a natural number.
	M.7.2.1.3.	Students express the rule of the number patterns using letters and finds the asked term of the pattern when the rule was expressed by letters.
	M.7.2.2.1.	Students understand the principle of the preservation

[1] Although there is no algebra domain in 5th grade the curriculum, the objective M.5.1.1.3., it was found related to the big idea of functional thinking under algebra.

[2] Although the objective M.7.1.1.2. was not categorized under the algebra domain in the curriculum, it was found related to the big idea of generalized arithmetic under algebra.

Table 2.2 (continued)

7th Grade		of an equivalence. a) In order to keep the equations in balance like in 7 + 2 = __ +3, students find what to put in the place of __ . b) The scales and balance models are shown in order to show the preservation of equivalence in the case of addition and subtraction. c) The preservation of equivalence is worked on in the case of addition or subtraction of the same number from the both sides of the equation and in the case of multiplication or division by the same number.
	M.7.2.2.2.	Students identify linear equations with one unknown and construct a linear equation with one unknown corresponding to the given real-life situations.
	M.7.2.2.3.	Students solve linear equations with one unknown.
	M.7.2.2.4.	Students solve the problems that require constructing linear equations with one unknown.
	M.8.2.1.1.	Students understand simple algebraic expressions and write them in different forms.
	M.8.2.1.2.	Students multiply algebraic expressions.
8th Grade	M.8.2.1.3.	Students explain the algebraic identities with models.
	M.8.2.1.4.	Students factorize the algebraic expressions.
	M.8.2.2.1.	Students solve the linear equations with one unknown.
	M.8.2.2.2.	Students identify the coordinate system with its characteristics and shows the coordinates.
	M.8.2.2.3.	Students express how one of the variables change in relation to the other using a table and an equation when there is a linear relationship between the variables.
	M.8.2.2.4.	Students draw the graph of linear equations.
	M.8.2.2.5.	Students formulate equations, tables and graphs for real life situations involving linear relationships and interpret them.
	M.8.2.2.6.	Students explain the slope of the line with models and associate the linear equations and graphs with the slope.
	M.8.2.3.1.	Students write relevant mathematical sentences for daily life situations that involve linear inequalities with one unknown.
	M.8.2.3.2.	Students show the linear inequalities with one unknown on the number line.
	M.8.2.3.3.	Students solve the linear inequalities with one unknown.

2.5 Summary of the Literature Review

In the related literature, firstly, theoretical frameworks were reviewed and explained in terms of their use in this study. At first, Kaput's (2008) algebraic reasoning framework which includes Core Aspect A that focuses on regularities, relations and making generalizations and Core Aspect B that focuses on manipulation of formalisms was reviewed. Secondly, Mathematical Knowledge of Teaching (MKT) framework (Ball et al., 2008) was reviewed. These frameworks were reviewed to help explain PSMTs' perceptions about the underlying algebraic structure of a given task, their conceptions of algebra, and their awareness about possible solutions of students. Next, the studies which addressed elementary and middle school students' algebraic thinking and misconceptions were summarized.

Then, the studies about teachers' pedagogical content knowledge that focused on algebraic concepts were reviewed to gather information. As a result of the studies about elementary and middle school students' algebraic thinking and misconceptions, many students were found to have the "the answer", or "the total" (Blanton et al., 2011; McNeil & Alibali, 2005; Yaman et al., 2003) understanding regarding to the equal sign. Regarding the variable they had various difficulties; "letter ignored" (Küchemann, 1978, p. 25), "acceptance of lack of closure" (Collis, 1975 as cited in Küchemann, 1978), "substitution" (Ryan and Williams, 2007; MacGregor & Stacey, 1997), use of *x* in arithmetic as a multiplication sign (Ryan & Williams, 2007).

Even though, the students have had various difficulties and misconceptions regarding basic algebraic concepts, they were also found to have capabilities of performing algebraic thinking as early as pre-kindergarten. As Carpenter et al., (2003) suggested, when students were triggered in appropriate ways, they can build equivalence understanding of the equal sign. Additionally, the study conducted by Kızıltoprak and Köse (2017) also indicated that when a relational thinking-based lesson and classroom environment was built, students can have opportunities to develop functional thinking. Regarding the functional thinking, the intervention study conducted by Isler et al. (2014) and the studies without intervention conducted by Ng (2018) and Tanışlı (2011) showed that students from different grade levels can demonstrate abilities of functional thinking and making a generalization when they were provided with well-structured tasks. Additionally, even though the students in the early grades were not able to use variables, they were found to represent generalizations by using semi-symbolic representations.

The studies conducted with pre-service teachers to understand their awareness of the students' possible solutions and misconceptions showed that PSMTs have difficulties in predicting the underlying reasons of students' misconceptions (e.g., Dede & Peker, 2007; Didiş Kabar & Amaç, 2018; Gökkurt et al., 2016; Stephens, 2006; Tanisli & Kose, 2013). Several researchers (e.g., Didiş Kabar & Amaç, 2018; Gökkurt et al., 2016; Tanisli & Kose, 2013) indicated that it could stem from the teacher education programs as they might not provide enough experiences to help PSMTs develop their PCK in algebra.

Lastly, the national curriculum (MoNE, 2018) objectives were reviewed to see algebraic topics addressed and their respective grade levels. Although the algebra learning area officially takes place starting in the 6th grade, there were many objectives addressed in the early grades which were found related to big ideas of algebra that include patterns, the order of the operations, the meaning of the equal sign and equalities.

CHAPTER 3

METHODOLOGY

Throughout this chapter, the information about details of the research design and the components will be provided. This chapter will be divided into the following parts: restatement of the research questions, design of the study, participants, data collection methods, instrument, data analysis procedures, trustworthiness of the study, assumptions of the study, limitations of study, and ethics.

3.1 Restatement of the Research Questions

The research questions of the study are designed as follows:

To what extent are middle school pre-service mathematics teachers aware of the underlying algebraic structure of a given task?

What are middle school pre-service mathematics teachers' conceptions of algebra?

What are middle school pre-service mathematics teachers' awareness about possible students' solutions provided to the tasks?

How do middle school pre-service mathematics teachers' conceptions of algebra, awareness of task purposes and possible student solutions provided to the tasks change after they attend a Methods of Teaching Mathematics Course?

3.2 Design of the Study

Research questions drive the methodology as a qualitative research since "Qualitative research uses a naturalistic approach that seeks to understand phenomena in context-specific settings" (Golafshani, 2003, p. 600). The purpose of this study is to understand the middle school pre-service mathematics teachers'

(PSMTs) conceptions of algebra. Qualitative case study research methodology was employed to investigate the research questions. Creswell (2007) defines it as:

> Case study research is a qualitative approach in which the investigator explores a bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving multiple source of information (e.g., observations, interviews, audiovisual material, and documents and reports), and reports a case description and case-based themes (p.73).

The instrumental case study fits the nature of the current study. Stake (2005) characterized an instrumental case study as "mainly to provide insight into an issue or to redraw a generalization. The case is of secondary interest, it plays a supportive role, and it facilitates our understanding of something else" (p. 437).

The study was conducted in the 2017-2018 Fall and Spring semesters with PSMTs who were the undergraduate students of the Elementary Mathematics Education (EME) program. EME students were observed in the "Methods of Teaching Mathematics I" (MoTM I) and "Methods of Teaching Mathematics II" (MoTM II) courses during two terms. The data for this study came from the individual pre-interviews and post-interviews which were carried out with some of the course participants.

The focus of the following parts is to give detailed information about the department and the classroom environment.

3.2.1 Department Context

Elementary Mathematics Education (EME) program is one of the five programs under the Department of Mathematics and Science Education at a public university in Ankara. English is the medium of instruction at the university. To be qualified as the graduate of EME program, students should complete an eight-semester teacher education program. The program offers 36 must (e.g., physics, history, language, and majorly mathematics, educational sciences, elementary mathematics education) and six elective courses (See Table 3.1). The content courses are offered by the respective departments (e.g., Mathematics, Statistics, Physics, History, Modern languages, Turkish language, Computer Education and

Instructional Technology), the Educational Sciences courses such as the Introduction to Education are offered by the Educational Sciences department, and lastly, and elementary (mathematics) education courses are offered by the Mathematics and Science Education department. The graduates of this program are certified to teach mathematics in middle schools, Grades 5 to 8.

Table 3.1

Undergraduate curriculum for Elementary Mathematics Education (EME) program

Semesters	Course Name
First Semester	Fundamentals of Mathematics
	Analytic Geometry
	Calculus I
	Introduction to Education
Second Semester	Discrete Mathematics
	Basic Algebraic Structures
	Calculus II
Third Semester	Introduction to Differential Equations
	Introduction to Probability & Statistics I
	Instructional Principles and Methods
	Educational Psychology
Fourth Semester	Elementary Geometry
	Introduction to Probability & Statistics II
	Measurement and Assessment
Fifth Semester	Basic Linear Algebra
	Methods of Teaching Mathematics I
	Elective I
	Elective II
Sixth Semester	Community Service
	Instructional Technology and Material Development
	Methods of Teaching Mathematics II
	Classroom Management
	Restricted Elective I
Seventh Semester	Research Methods
	School Experience
	Nature of Mathematical Knowledge for Teaching
	Restricted Elective II
	Elective III
Eighth Semester	Practice Teaching in Elementary Education

Table 3.1 (continued)

	Turkish Educational System and School Management
	Guidance
	Elective IV

3.2.2 Course Context

As seen in the Table 3.1, EME students are required to enroll in the "Methods of Teaching Mathematics I" (MoTM I) course during their fifth semester and "Methods of Teaching Mathematics II" (MoTM II) course in their sixth semester in their teacher education program. Also, MoTM I was the prerequisite course for MoTM II. These courses were offered four class hours in a week and focus on both theory and practice following mainly the book "Elementary and Middle School Mathematics: Teaching Developmentally by Van de Walle, Karp, and Bay-Williams" (2013). The courses require one micro-teaching per chapter starting with the mathematics content area, numbers. At the end of these courses, PSMTs are expected to be able to:

> Construct connections among mathematical ideas in elementary mathematics curriculum, analyze students' misconceptions related to the school mathematics, use representations to organize, record, and communicate mathematical ideas, design and implement plans and activities, design and employ tools for effective teaching of school mathematics, participate in productive classroom discourse, be confident in teaching mathematics (Academic Catalog, 2018).

The main learning areas of these courses are numbers, algebra, geometry, measurement, probability, and data analysis, which are the learning areas in the Turkish national mathematics curriculum (MoNE, 2018). Although in almost all these chapters, algebra connections are mentioned, there is a special chapter ("algebraic thinking") just focusing on algebra in the textbook. Almost two and a half weeks is allocated to this chapter throughout the second term.

3.2.3 Classroom Context

There were 25 students in the course MoTM I, where 6 of them were male and 18 of them were female. In MoTM II course, there were 26 students, 6 of whom were male and 19 of whom were female. Apart from the instructor, there was also a teaching assistant to supervise peer discussions and group work, to help prepare class materials, and to help the instructor and the students in the class when additional support was needed.

The MoTM I and II sections were one of two sections that were offered for this course each semester. The instructor launched each lesson by asking “What did we talk about in the last lesson?”, trying to activate PSMTs’ previous knowledge and terminology. Sometimes, the instructor showed interesting short videos or prepared a warm-up game to start a lesson. PSMTs were required to read the chapter before coming to each class, the instructor asked what big ideas[3] and the new terminology they arrived while they were reading the chapter.

During the lesson, the instructor gave importance to the learning environment so she built the lessons on small groups, pair and whole-class discussions. In each lesson, think-pair-share time was a considerable part of the lesson. Experiencing many different possible strategies seemed to help PSMTs to develop different points of view to use multiple representations and solutions and connect mathematical ideas. Before starting a task, the instructor always wanted to be sure all students fully understood what was asked in a task, if the instructor realized that someone hesitated, she tried to make his/her conception clear. During the lesson, the classroom routines were mostly the same, if there was something unclear for students, the instructor facilitated discussion around making sense of one another’s ideas. When PSMTs met with new terminology, the instructor encouraged students to construct a definition for new terms. In such a situation, the instructor used a holistic approach to come to a conclusion, and she behaved like each member in the class had something unique to add to the learning process. As described above, since the instructor gave importance to sharing multiple experiences, methods, and strategies, she appreciated the students when they shared the points that they agreed on and those they disagreed on. The instructor mostly closed the lesson with a summary discussion which were based

[3] Big ideas are defined at the beginning of each chapter in the book and were expected to be arrived at by the PSMTs.

on a whole-class discussion by asking the big ideas of the chapter and new mathematical terminology of the current chapter.

The courses included several homework assignments that include reflection of the main points of the chapter. In addition, there were individual and group projects. For instance, each group of two or three was supposed to prepare activities and implement in the class related to the content of the week. MoTM I course also included one project, that is making an interview with a student and MoTM II course included two projects, making an interview and conducting a campus math trail (see Appendix A for the syllabi). Besides the assignments, there were one paper and pencil midterm and final in both courses.

3.2.3.1 Algebraic Thinking Chapter

The textbook (Van de Walle, Karp, and Bay-Williams, 2013) suggested that PSMTs should internalize and make their future instruction around the following big ideas at the end of the algebraic thinking chapter:

1. Algebra is a useful tool for generalizing arithmetic and representing patterns and regularities in our world.
2. Symbolism, especially involving equality and variables, must be well understood conceptually for students to be successful in mathematics, particularly algebra.
3. Methods we use to compute and the structures in our number system can and should be generalized. For example, the generalization that $a + b = b + a$ tells us that $83 + 27 = 27 + 83$ without computing the sums on each side of the equal sign.
4. Patterns, both repeating and growing, can be recognized, extended, and generalized.
5. Functions in K-8 mathematics describe in concrete ways the notion that for every input, there is a unique output.
6. Understanding of functions is strengthened when they are explored across representations, as each representation provides a different view of the same relationship. (p. 258)

Generalization, patterns and functions were covered throughout the algebraic thinking chapter as part of an ongoing teaching by following the activities in the book to make the PSMTs be able to reach the objectives of the chapter. Additionally, the instructor showed the PSMTs a TED video that explained where the symbol *x* came from and the other one was an interview with an elementary student and the teacher were challenging her to help build relational thinking.

3.3 Participants

Since the current study is a case study, the PSMTs were observed in their natural settings in the MoTM I and MoTM II courses. Purposeful sampling methodology (Creswell, 2012) was employed for the current study. Hence the study was qualitative research, the PSMTs who were willing to talk and would likely to give more information in semi-structured interviews were chosen as participants. Eight PSMTs were chosen to get detailed insights about the PSMTs' perceptions about the underlying algebraic structure of a given task, their conceptions of algebra, and their awareness about possible student solutions. To be able to see the possible changes in all these three categories before and after the "algebraic thinking" chapter, pre- and post-interviews were conducted. The participants were enrolled in MoTM I and MoTM II courses respectively in their fifth and sixth semesters in the teacher education program. Among eight participants, two PSMTs were male and the remaining were female. The gender of the chosen participants was in approximate proportion with the number of the male and female PSMTs enrolled in the courses. When the MoTM I course was complete, the MoTM II course was taken as a new course, and one[4] of the participants dropped out of the study due to her participation in an overseas ERASMUS Program. Therefore, the post-interviews were conducted with seven PSMTs as participants.

3.3.1 Researcher's Role

At the beginning of the Fall 2017 term, the researcher started to attend the MoTM I course with the permission of the instructor and introduced herself to the class and described her study. The researcher is a two-year experienced middle school mathematics teacher. During the Fall 2017 and Spring 2018 terms, the researcher attended all the methods of teaching mathematics classes to observe the

[4] PSMT 3 went to abroad for a semester to attend an ERASMUS program.

class and took notes to describe the learning environment including participants' perceptions and responses. The researcher also attended group work and pair discussions trying not to affect PSMTs reasoning and change the flow of the lesson.

Although the researcher made observations and took notes during the classes, two cameras were used to record the classes in the algebra chapter "Algebraic Thinking: Generalizations, Patterns and Functions" (Van de Walle, Karp, & Bay-Williams, 2013) not to miss any important parts. The researcher wanted to gain insights about the PSMTs' natural attitude, so to make them get used to presence of the cameras, video recording started two weeks before the chapter. This was not aimed to use as data in the study.

3.4 Data Collection Methods

Data were collected via pre and post interviews with the aim of getting detailed information about PSMTs' conceptions of algebra and observation was made, necessary documents (in class papers) were collected and videos were recorded to get supplementary information. Data collection procedure started when the approvals were obtained from the University Human Subjects Ethics Committee (See Appendix B). After the written consent forms from the students were collected, the video recording stage of the study started. Approval of the students was also taken to be able to use their class materials. This is also explained to the PSMTs as one of the requirements of the study.

3.5 Instrument

With respect to data collection, detailed information about PSMTs' conceptions of algebra were collected through semi-structured individual interviews. The participants' responses helped the researcher to ask follow-up questions and to investigate the PSMTs' conceptions further. The interviews were recorded and notes were taken as well.

In this study, pre and post interviews were conducted to examine the research questions of the study. To assess the development of PSMTs' perceptions about the underlying algebraic structure of a given task, their conceptions of algebra, their awareness about possible student solutions, and the changes in all these three categories after attending Methods of Teaching Mathematics Courses, the same interview protocol was applied before and after the algebra chapter, specifically in the tenth week of the MoTM I, the pre-interviews were conducted, and in the fourth week of the MoTM II, in the second semester, post-interviews were completed. The questions in the interview were taken from different related resources, and they were adapted to make them suitable for the purpose of this study where necessary. Even though the medium of language in the university was English, the interviews were conducted in Turkish to have the participants feel comfortable to talk in their native languages. Most of the questions used in the interview were translated from English into Turkish. When the instrument was prepared, content validation was checked by a mathematics education researcher who was interested in algebra and teacher education. Content validation includes an evaluation whether the instrument assesses what it is supposed to assess, clarity of language and directions, and appropriateness of language (Fraenkel et al., 2012).

The interview protocol consisted of three parts. Part I included questions about the demographic information. Part II started with the question "How would you describe what algebra is to someone who has never heard it before?" which was taken directly from Stephens (2004). Part II included four tasks and for each task, the PSMTs were first asked the purposes, then they were asked whether they addressed algebra or not with their reasons, and lastly, they were asked what responses students might provide to these tasks. This continued in the same order for each task. The tasks focused on the three fundamental ideas (Blanton et al., 2011) of early algebra which are equivalence and equations, functional thinking, and variable (see Figure 3.2). Task 1 and Task 2 which focused on relational thinking and corresponding student solutions were adopted from a doctoral thesis (Stephens, 2004) which focused on PSMTs' conceptions of algebra. Task 3 and corresponding student solutions were adopted from a study which aimed to understand students' algebraic thinking (Blanton et al., 2015). Task 3, specifically, focused on functional thinking that is writing the rule of an equation.

Lastly, Task 4 and one of the student solutions were adopted from a study that focused on students' misconceptions about algebra conducted by Dede and Peker (2007). The other student solution (Seçil's solution) was developed by the researcher. Task 4 focused on the ability of collecting like terms.

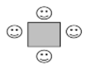
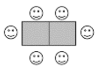
Tasks	Big Ideas Addressed
Task 1 What number goes in the □ ? 37 + 54 = □ + 55	Equivalence and Equations Variable
Task 2 The solution to the equation $2n + 15 = 31$ is $n = 8$. What is the solution to the equation? $2n + 15 – 9 = 31 – 9$	Equivalence and Equations Variable
Task 3 Nehir is having her friends over for a birthday party. She wants to make sure she has a seat for everyone. She has square tables. She can seat 4 people at one square table in this way:  If she joins another square table to the first one, she can seat 6 people:  If Nehir has 100 tables, how many people can she seat?	Functional Thinking Variable
Task 4 Write the simplest form of $5 + 4x + 2x$.	Equivalence and Equations Variable

Figure 3. 1 Tasks and the big ideas addressed by the tasks

The last part of the interview protocol, Part III, consisted of two student solutions provided to each of these four tasks. The participants were asked whether the solution of students is algebraic or not and why. Regarding each task, student solutions were chosen carefully to reveal teachers' conceptions of algebra (see Figure 3.3). In Task 1, two different student solutions who used relational-computational and relational-structural were presented. Regarding Task 2, again, two different solutions, preservation of equivalence and solving an equation were presented. In the following task, Task 3, a solution based on writing an equation and a solution based on continuing a recursive pattern using a table were chosen. In the last task, Task 4, two student solutions respectively which included collecting like terms by using representation and symbol were presented.

Tasks	Students' Solutions	Codes
Task 1	Burak's solution 36 goes in the box because 37 plus 54 is 91, so I had to figure out what plus would be 91. 36 plus 55 is 91, so it is 36.	Relational-Computational Thinking
	Nur's solution 36 goes in the box. 55 is one more than 54, so the number in the box has to be one less than 37, so it is 36.	Relational-Structural Thinking
Task 2	Kerem's solution $2n + 15 - 9 = 31 - 9$ $2n + 6 = 22$ $-6 \quad -6$ $\frac{2n}{2} = \frac{16}{2}$ $n = 8$	Solving Equation
	Defne's solution It is the same, $n = 8$ because you are subtracting the same thing from both sides.	The Preservation of Equivalence

Figure 3. 2 Students' solutions for the tasks and corresponding codes

<table>
<tr><td rowspan="2">Task 3</td><td>Kemal’s solution
The people column goes up by 2s. So, if I extend the table as below, that would be 202 people that can be seated at 100 tables.
<table><tr><th>Number of tables</th><th>Number of people</th></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>6</td></tr><tr><td>.</td><td>.</td></tr><tr><td>.</td><td>.</td></tr><tr><td>100</td><td>202</td></tr></table>*Kemal fills out the table.</td><td>Recursive pattern by using a table</td></tr>
<tr><td>Dilay’s solution
The number of people is 2 more than 2 times the number of tables. So, the rule is $2n + 2 = m$ where
n = number of tables and
m = number of people.
At 100 tables,
$2 \times 100 + 2 = 202$ people can be seated.</td><td>Constructing an Equation</td></tr>
<tr><td rowspan="2">Task 4</td><td>Seçil’s solution
Let’s have x that much
I have 4 groups of this,

Then, I add 2 groups of this;

Now I have 6 groups of this, also I add 5;
 + 5
So, I have $6x + 5$.</td><td>Collecting Like Terms by Using a Representation</td></tr>
<tr><td>Gizem’s solution
I have 4 groups of x. Then I add 2 groups of x. Now, I have 6 groups of x, so it is $6x$. Then I add 5, $6x + 5$.</td><td>Collecting Like Terms by Using Symbolizations</td></tr>
</table>

Figure 3. 2 (continued)

After the interview protocol was prepared, a pilot study was conducted by using the finalized interview protocol with a PSMT, who was not part of the actual study, enrolled in the MoTM I course in December 2017. Following the pilot study, a clarification upon a solution provided by a student Task 4 was needed. After all the revisions were made, the interview protocol (see Appendix

C) was finalized. The interviews took approximately an hour and were recorded to be transcribed later.

Table 3. 2

Implementation time of the pilot study and interviews

	Term/Course	Interview Time
Pilot Study	2017 Fall	December 2017
Pre-Interviews	2017 Fall/MoTM I	December 2017, the tenth week
Post-Interviews	2018 Spring/MoTM II	March 2017, the fourth week

3.6 Data Analysis

As the first step of data analysis the recorded eight pre-interviews and seven post-interviews were transcribed. As Merriam (2009) suggests, the interviews were transcribed by the researcher, since it helped the researcher to make sense of the data by gaining insight. The researcher used tables to be able to organize the transcriptions in a meaningful and manageable way. Manuel coding and analysis were preferred instead of software since it was a small-scale study. The researcher preferred to analyze and mark (color coding) the qualitative data by hand as Creswell (2012) suggested.

In the analysis of the data, initial coding was used as a first cycle coding method which is defined as an “open-ended approach to coding the data with some recommended general guidelines” (Saldaña, 2009, p. 81). The first cycle coding is open to codes and categories which are driven from the data. The codes that come from the literature including Kaput (2008), Stephens (2006), and Stephens et al. (2013) were used as preexisting codes (see Figure 3.3). After the first cycle of the coding, in the second cycle, focused coding was used. According to Charmaz (2006) focused coding is employed after the initial coding since it “requires decisions about which initial codes make the most analytic sense to categorize your data incisively and completely” (p. 57).

Interrater agreement was obtained by randomly selecting a 20% of the data and coding it independently by a second coder who was a mathematics educator researcher with a doctoral degree focusing on qualitative studies and algebra in her research to assess the reliability of coding. In the cases where the agreement between two coders was lower than 80%, the codes were discussed and revisions were reflected to the analysis until 80% agreement between the two coders was reached.

3.7 Trustworthiness of the Study

Qualitative study is different from the quantitative one. In qualitative studies, a research study starts by broad research questions to learn more from participants via exploration to understand underlying phenomena in a particular situation. Unlike the quantitative study which seeks an answer how often and why something occurs and what is the tendency, the qualitative study aims to explore and describe the big picture in detail by using a holistic approach (Creswell, 2007; Fraenkel, Wallen, & Hyun, 2012). As Golafshani (2003) stated, the terms *validity* and *reliability* in quantitative study are not enough to define qualitative study because of its different nature. The terms suggested by Lincoln and Guba (1985) are credibility instead of internal validity, transferability instead of external validity, and consistency or dependability instead of reliability. On that account credibility, transferability and consistency or dependability were employed to assess the trustworthiness of the study and are explained in the following sections.

3.7.1 Credibility and Transferability

As Merriam (2009) defined the credibility, it “deals with the question of how research findings match reality” (p. 213). Creswell (2007) suggested eight validation strategies and he recommended that a qualitative study should have at least two of these strategies. The strategies that Creswell mentioned are persistent observation, triangulation, peer review, negative case analysis, clarifying researcher’s bias, member checking, thick description, and external audits. In this study, three of them which are triangulation, thick description, and clarifying researcher’s bias were employed to increase the credibility of the study.

Triangulation method was employed to increase the credibility of the study. Among Denzin's (1978) four types of triangulation methods which are the use of multiple methods, multiple sources of data, multiple investigators, or multiple theories, the multiple source of data was employed. The PSMTs conceptions of algebra were investigated through an open-ended question in the first, in the second part, they were shown tasks and were asked whether they addressed algebra or not, and lastly, in the last part, they were shown various student solutions and were again asked whether they used algebra or not. This helped the researcher to get at the PSMTs conceptions of algebra in multiple ways.

In order to provide thick description, the context of the department, the classroom and the course information, participants, and the data categorization and analysis procedures were tried to be explained in detail in the respective parts. The researcher tried to be clearer in her writing giving rich description of the findings and used direct quotations from PSMTs' responses as much as possible.

As for the possible biases of the researcher, it is possible to say that, she was a novice researcher in this field. The data were collected by the researcher during the courses of the MoTM I and II. The participation in the study depended on participants' willingness. Before conducting the study and during the interviews it was reminded that PSMTs will not be judged because of their answers, the aim is just to understand their reasoning in conceptualizing algebra. During the interview, the researcher was careful about not confirming and guiding participants' answers.

Merriam (2009) defined transferability "the extent to which the findings of one study can be applied to other situations" (p. 223). The present study aimed not to make a generalization, in fact, it aimed to understand the PSMTs' perceptions about the underlying algebraic structure of a given task, their conceptions of algebra, their awareness about possible student solutions, and the changes in all these three categories after attending Methods of Teaching Mathematics Courses by conducting in-depth interviews. On the other hand, some degree of generalization could be possible in similar contexts given the thick description.

According to Creswell (2007), to ensure the transferability of a study, thick description is needed to transfer the findings accurately to readers. Also, Merriam (2009) advocated that with the help of rich and detailed description, the readers could be able to decide in what degree their context is conformed with the context of the current study and to what extent they can transfer the findings. In the present study, transferability was aimed to be increased by detail description of the design and context of the study, participants, interviews, and data analysis process which were expressed in the respective parts. On that account, the readers can have the opportunity to see to what extent they could generalize the findings to a similar context.

3.7.2 Consistency or Dependability

Reliability is defined as a principle that "refers to the consistency of these inferences over time, location, and circumstances" (Fraenkel et al., 2012, p.460). As Lincoln and Guba (1985) suggested in Merriam (2009) preferred to use *dependability* and *consistency.* She advocated that if the findings are parallel to the data and it makes sense to the reader, the study is dependable and consistent. That is to say the researcher should be explicit in her explanation about the process of conducting research, findings and interpretation of data. With the help of this clear explanation, the readers could be able to understand why the preferred methodology was applied, how the data were interpreted and ideas were developed. Hence, the readers could have the opportunity to conclude whether they come to same results with the researcher or not (Flick, 2007). The strategies that could be used to make a study consistent and dependable are triangulation, peer examination, investigator's position, and the audit trail (Merriam, 2009). In this study, the triangulation method and investigators' position were employed and explained in the credibility and transferability section.

3.8 Assumptions of the Study

There were two assumptions of the study. First, it was assumed that participants gave sincere information in interviews. Also, it was assumed that the interview protocol assessed what it was supposed to assess.

3.9 Limitations of the Study

There are some limitations in this study such as the number of the participants, the researcher's role, interview protocol and interviewing process. The details of the limitations and how the researcher tried to minimize these limitations were aimed to be explained in this section.

The study was conducted with seven participants in a public university in Ankara, Turkey. Since the number of the participants were limited, it can be one of the limitations of the study. Nonetheless the aim of the study was to understand PSMTs' conceptions of algebra, their perceptions about the underlying algebraic structure of a given task, their awareness about possible student solutions, and the changes in all these three categories after attending Methods of Teaching Mathematics Courses through task-based interviews not to make a generalization, so the limited number of the participants may not be a big handicap in the current study.

Another limitation is the fact that the researcher had to use an instrument that she prepared in order to get the intended understandings about the PSMTs' conceptions The study is limited with the questions asked in the interview protocol. Different tasks or questions might have provided different findings.

Lastly, the pre-interview might have had an effect on PSMTs' awareness about the subject of algebra. Similarly, since the same interview protocol was employed as data collection tool in pre- and post-interviews, the PSMTs could have remembered the questions. To deal with these limitations the pre- and post-interviews were applied at intervals of two months. Also, the questions in the interviews were open-ended and rather than the responses the PSMTs gave, the reasoning behind their responses was paid attention.

3.10 Ethics

Confidentiality and anonymity are important in terms of ethical reliability (Flick, 2007). The collected data and participants' name and personal information are kept confidential. Participants were coded assigning a number, so the second coder did not have any personal information about the participants. Moreover, the transcriptions were done by the researcher. This also helped to keep confidentiality standards.

Participants gave consent before they were recorded in the interviews, and they were informed that they had a chance to stop recording or they could leave from the research whenever they wanted to. During the interviewing process, the researcher was careful about not to judge, hurt or make the participants embarrassed with the interpretation of the responses.

CHAPTER 4

FINDINGS

In this chapter, research findings are presented under three main sections. In the first section, pre-interview results and in the second section, post-interview results, and in the third, and the last section, changes between pre-interview and post-interviews are reported. Findings about PSMTs' awareness of the underlying algebraic structure of a given task, PSMTs' conceptions of algebra and PSMTs' awareness of students' possible misconceptions are presented respectively in each section.

4.1 Findings of the Pre-Interviews

4.1.1 Middle School Pre-service Mathematics Teachers' Awareness of the Underlying Algebraic Structure of a Given Task

This section provides the findings about to what extent middle school pre-service mathematics teachers can perceive the underlying algebraic structure of a given task. To have insights about participants' awareness of the underlying algebraic structure of given tasks, the question "Why would a teacher might pose this question?" was asked for each task.

Regarding Task 1 (see Figure 4.1), seven out of eight PSMTs stated that a teacher could ask this question to have the students build relational structural thinking. For example, PSMT 1 stated "The aim is most likely to have [students] understand the question as a whole and realize the relation between 54 and 55 without adding up and to make [them] write 36 in place of the empty box." One participant, PSMT 8, stated that the teacher could ask this question to have students build relational computational thinking. PSMT 8 reported "To me, it is the sum of these two numbers, relevant to the equal sign, and the sum of the other two numbers, what to write in the gap, and what the space (empty box) in this sense means."

What number goes in the □?
37 + 54 = □ + 55

Figure 4. 1 Task 1

Regarding Task 2 (see Figure 4.2), responses of all the PSMTs indicated that they recognized the preservation of equivalence opportunity of the task. For example, PSMT 5 said:

> So [the teacher] may have asked to make them [the students] realize that the balance is not lost and to make them realize that the value of n is still the same. [The teacher] subtracts 9 from both sides to show that the equation has not changed.

The solution to the equation $2n + 15 = 31$ is $n = 8$.
What is the solution to the equation?
$2n + 15 – 9 = 31 – 9$?

Figure 4. 2 Task 2

Upon examining Task 3 (see Figure 4.3), PSMTs' responses were found to have focused on noticing a pattern, constructing an equation, and constructing an equation or a correspondence relationship through noticing a pattern. Two out of eight PSMTs stated that the teacher could ask this question to make students find a pattern. For example, PSMT 7 said "A teacher asks this question to have students find the rule for the pattern." Two out of eight participants stated that the aim is to have students construct an equation. For instance, PSMT 3 reported:

> I think the teacher here actually wants them [the students] to discover something [...] The teacher wants them to build equations, similar to the formulas there [...] I would ask this question to have them [the students] find $2n + 2$.

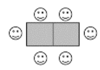
The remaining four participants stated that the teacher could ask this question to have students construct an equation or a correspondence relationship through noticing a pattern. For example, PSMT 5 stated:

> Joining one, two, three tables, seating respectively 4, 6, …people, could help [the students] see something like a pattern and then [the teacher] would want [the students] to set up an equation and then place 100 in that equation and then find the result.

In addition to building an equation through noticing a pattern, two PSMTs also mentioned making generalization as a potential aim of the teacher.

Nehir is having his friends over for a birthday party. She wants to make sure he has a seat for everyone. She has square tables.

She can seat 4 people at one square table in this way:
If he joins another square table to the first one, she can seat 6 people:



If Nehir has 100 tables, how many people can she seat?

Figure 4. 3 Task 3

In Task 4 (see Figure 4.4.), seven out of eight participants stated the aim of the task as collecting like terms together. For instance, PSMT 3 said "Probably, [a teacher] asks to have students understand adding *x*'s with *x*'s." One participant, PSMT 7 did not directly mention collecting like terms, but she had a close explanation to others. PSMT 7 stated "It is obvious that the question asked to state what this unknown expression means is that $4x$ and $2x$ do not specify different things, they are multiples of the same unknown." Among the participants, PSMT 4 also stated taking out a common factor as a teacher's aim:

> [A teacher] could assess taking out the common factor and placing it in front of the parenthesis. [...] Because, [a student] realizes *x* is something different. What could be added up with *2x*, *4x* could be added up, because there is *x* also. I think that taking out the common factor $(5 + x\ (4 + 2))$ is also assessable here.

Write the given expression in the simplest form $5 + 4x + 2x$.

Figure 4. 4 Task 4

4.1.2 Middle School Pre-service Mathematics Teachers' Conceptions of Algebra

4.1.2.1 PSTMs' responses to "How would you describe what algebra is to someone who has never heard of it before?"

In this part, participants' responses to the open-ended question "How would you describe what algebra is to someone who has never heard of it before?" were examined. Four PSMTs based their algebra definitions on the presence of an unknown or an equation. For instance, PSMT 7 stated "I try to explain the equations, what the equation is, what it is like." Another example for this category was that "Equations which include x's, y's come to my mind directly" (PSMT 5). Three out of eight PSMTs' definitions of algebra were related with the operations. For instance, PSMT 6 stated "Mathematical questions that basically include four operations." Lastly, one participant, PSMT 8, mentioned use of modeling. PSMT 8 reported "As far as algebra is concerned, algebra tiles come to my mind. [...] It is modeling in my mind."

Further information about PSMTs' conceptions of algebra was gathered by the task-based questions which will be presented next.

4.1.2.2 How Did Middle School Pre-service Mathematics Teachers Classify the Tasks and the Related Students' Solutions?

In this section PSMTs' algebra categorization of interview tasks and the related students' solutions are examined. The question "Would you consider this to be an algebra problem?" was asked in each task to understand PSMTs' conceptions of algebra. Moreover, for each task, two different student solutions were presented and they were asked whether the students used algebra or not in their solutions.

Participants' categorization of Task 1. The PSMTs' categorizations of Task 1 as algebra or not are provided in Table 4.1.

Table 4. 1

Participants' categorization of Task 1

Task	Algebra	Non-algebra
What number goes in the □? 37 + 54 = □ + 55	6	2

Six out of eight participants evaluated Task 1 as algebraic. Among these six participants, three PSMTs made their decisions based on the presence or absence of an unknown or an equation. For example, PSMT 1 stated "There is an unknown, there is an equality and [the student] is asked to find out the [value of an] unknown by developing a method." Similarly, PSMT 5 stated "Because of equality, it is like an equation. If we see an unknown such as *x*, *y* in the place of the empty box, it seems to me as an algebra [question]." Apart from these three participants, two PSMTs categorized the task as algebra by referring to the relational computational thinking. For instance, PSMT 4 said "Because […] it is a task that we perform operations abstractly on numbers." Lastly, one participant, PSMT 6, categorized the task as an algebra problem focusing on the relational structural thinking. PSMT 6 stated "It (the question) does not only aim to assess students' performing four operations [but], it also [have students] figure out the relationship [between numbers], and this is higher level [than performing four operations]."

On the other hand, two out of eight participants, who evaluated Task 1 as non-algebraic, also based their reasoning on the presence or absence of an unknown or an equation and relational structural thinking, respectively. PSMT 8 focused on the presence or absence of an unknown or an equation. PSMT 8 said "I think it is not an algebra question. [...] I mean, if I had seen an *x* here, I would have called it an algebra question." PSMT 3, who categorized this problem as not an algebra task focused on relational structural thinking and assessed it as number sense stating “I say that it is not an algebraic expression, because in this question we assess number sense […] In the algebraic expressions as a teacher, I don’t ask such a question.”

Participants’ classification of students’ work on Task 1. Two student solutions (see Figure 4.5), were asked to the PSMTs to see whether they categorize the responses as algebra or not.

Burak’s solution	Nur's solution
36 goes in the box because 37 plus 54 is 91, so I had to figure out what plus would be 91. 36 plus 55 is 91, so it is 36.	36 goes in the box. 55 is one more than 54, so the number in the box has to be one less than 37, so it is 36.

Figure 4. 5 Students’ solutions for Task 1

Table 4.2 provides information about PSMTs’ categorization of students’ solutions for Task 1.

Table 4. 2

Participants’ categorization of students’ solutions for Task 1

Burak’s solution		Nur's solution	
Algebraic	Non-Algebraic	Algebraic	Non-Algebraic
5	3	6	2

As seen in Table 4.2, five out of eight participants evaluated Burak’s solution as an algebraic solution. Among these participants, four PSMTs focused on Burak’s use of a computationally based strategy. For instance, PSMT 2 stated “Burak worked one by one, he did not use one-up one-down. Burak’s solution is an algebraic solution, because he did the operation step by step and reached the result.” The remaining participant, PSMT 3, had a different justification focusing on the presence of an unknown:

> PSMT 3: In Burak's thing (solution), I noticed that there is an expression saying that “what plus 55 would be 91”, in fact it

> confused my mind. In algebraic expressions, let's say $x + 5 = 10$, what plus, I mean what is the x? Since we think like that, what x would be to get the thing [10] by adding 5, the solution is algebraic I think.
> R: What would it be if Burak said 37 + 54 is equal to 91, 91 out of 55, I got 36.
> PSMT 3: Then I would think it would not be an algebraic solution. It seems to me we put x in the place of the empty box by saying "What would be."

Three out of eight participants evaluated Burak's solution as non-algebraic offering similar reasoning to the participants who categorized Burak's solution as algebraic—the use of a computationally based strategy, and additionally two of them mentioned the absence of the relational structural thinking. For example, PSMT 1 reported:

> I think this is not an algebraic solution. What is required here, I mean in the algebraic expressions, is realizing the one-up one-down relation between the two numbers. I think the aim of the question is not assessing adding skills, but [the aim is] understanding the relation between the two numbers.

Upon assessing Nur's solution, six out of eight participants categorized it as algebraic, and two of them as not algebraic. Five participants among the six participants, who evaluated Nur's solution as algebraic, justified their answers by emphasizing Nur' s relational structural thinking. For example, PSMT 5 stated "Because she focuses on equality and equivalent equations. I mean she thinks this way, this increases one, this will decrease one to reach equality." The sixth participant, PSMT 8, based his justification on the presence of an unknown saying "There is an unknown, so to find it, we perform algebraic operations."

Two out of eight participants, on the other hand, evaluated Nur's solution as non-algebraic. PSMT 2 emphasized Nur's relational structural thinking strategy as using logic saying "Nur uses logic directly, [she thinks that] if it increases 1, it has to decrease 1. But she did not have an algebraic solution." The other participant, PSMT 3, justified her own response by associating Nur's solution with number sense. PSMT 3 stated "Nur's solution is not an algebraic one. By saying if 55 is 1 more than 54, [the empty box] has to be 1 less than 37, she uses number sense as I said before (in the task categorization).”

Participants’ categorization of Task 2. Seven out of eight PSMTs categorized Task 2 as an algebra question (See Table 4.3). One participant, PSMT 7, specified that the categorization could be algebraic or non-algebraic according to the aim of the task, and PSMT 7 changed her mind when she started to evaluate the students’ solutions for Task 2 categorizing it also as an algebra task. Overall, all participants evaluated Task 2 as an algebra task.

Table 4. 3

Participants’ categorization of Task 2

Task	Algebra	Non-algebra
The solution to the equation $2n + 15 = 31$ is $n = 8$, What is the solution to the equation? $2n + 15 - 9 = 31 - 9$	8	0

Half of the participants directly cited the presence of an unknown or an equation as their justifications. For instance, PSMT 3 reported “As for algebra, as I said before, expressions that include *x* comes to my mind, like there should be an unknown.” Three participants specified the preservation of equivalence as their justifications in their categorization. For instance, PSMT 4 reported “Because, it is a relation between numbers, indeed. Actually, it examines the operational property (subtraction property of equality). In equations, subtracting the same number from both sides of the equation seems to me like an operational property.” Among these three participants, PSMT 7, on the other hand, at first stated that Task 2 could be categorized as algebraic or non-algebraic based on the aim of the task. PSMT 7 said:

> I think it is an algebra question, because I think [a student] will solve the new system after making subtraction. But when I think of the teacher's purpose, then this question is not an algebra question. If [a teacher] wants to have [a student] understand that subtracting

> the same number from both sides does not change anything (equality), when the aim is not to solve a new equation (second equation in the question), it does not look like an algebra question. [So, it is] both algebra and non-algebra.

When PSMT 7 started to categorize students' solution for Task 2, she clarified her justification and categorized Task 2 as an algebra question. PSMT 7 reported:

> Now my previous ideas have begun to change, I think this question is already an algebra [question], because understanding the logic of the equation that it maintains the equality is also algebraic, right? This requires an algebraic operation, too, I subtract 9 from both sides [of the equation] and nothing has changed.

Lastly, one participant, PSMT 2, justified her categorization of algebra by focusing on solving equations, "We have a numerical expression, and we expect children to solve equations. That's why it's an algebraic question."

Participants' classification of students' work on Task 2. PSMTs were also asked to categorize the provided students' solutions for Task 2 (see Figure 4.6).

Kerem's solution	Defne's solution
$2n + 15 - 9 = 31 - 9$ $2n + 6 = 22$ $-6 \quad -6$ $\frac{2n}{2} = \frac{16}{2}$ $n = 8$	It is the same, $n = 8$ because you are subtracting the same thing from both sides.

Figure 4. 6 Students' solutions for Task 2

The majority of the participants categorized both solutions as algebraic (see Table 4.4), but Kerem's solution was rated as more algebraic than Defne's solution.

Table 4. 4

Participants' categorization of students' solutions for Task 2

Kerem's solution		Defne's solution	
Algebraic	Non-Algebraic	Algebraic	Non-Algebraic
7	1	5	3

Seven out of eight participants, who evaluated Kerem's solution as algebraic, mentioned solving an equation in their justifications. For example, PSMT 4 said "Kerem is trying to establish the relationship between the numbers. He said $2n + 6 = 22$, to isolate *n*, I should subtract 6 from both sides. Then he divides *n* [by 2] and finds the [value of] *n*." Among these participants, one of them, PSMT 1, additionally stated that Kerem could understand that two equations in the question were the same. PSMT 1 stated:

> Even if he solved it in different ways, or by following the necessary ways, [at the end] he noticed that [the value of *n*] did not change, because the aim is to make [the students] realize that they are the same.

The only participant, PSMT 6, who categorized Kerem's solution as not algebraic, based his justification also on solving an equation. PSMT 6 stated:

> Kerem also solves equation and gets the right result, but he uses just mathematics (arithmetic). [...] In my opinion, algebra requires logic, but Kerem [solved it] procedurally, and he got the right answer. That is why it is not algebraic.

The five participants, who categorized Defne's solution as algebraic, based their justifications on Defne's understanding of the preservation of an equivalence. For instance, PSMT 1 said "Let's think of a pair of scales. I subtract 9 from both sides. Does my balance change? No, why should it change, it did not change. I think, this is an algebraic approach." Among these five participants, one of them believed that Defne used algebra for the same reason, but still she felt uncomfortable since she did not perform a particular procedure. PSMT 5 stated:

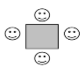
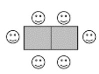
> This is algebraic, I think. Nine was added to both sides, nothing is changed so it is eight. But it seems that she solved it by reasoning. [...] It seems to me that it is not algebra, when I do not see the process. [...] It is coded in my mind if there are variables with *x, y*, it seems more algebraic.

Three out of eight participants categorized Defne's solution as a non-algebraic solution basing their justification on Defne's use of the preservation of an equivalence. For instance, PSMT 2 reported "Similar to Nur's solution, you [the question] give the result, and Defne knows that it does not change, but its solution is not an algebraic solution either. She just realizes that it is the same." Additionally, one participant, PSMT 3, focused on the lack of operations in Defne's response stating "If we subtract the same number from both sides, the results would stay the same; therefore, she did not perform an algebraic operation."

Participants' categorization of Task 3. All of the participants categorized Task 3 (see Table 4.5) as algebraic. The participants' categorizations depended on different justifications.

Table 4. 5

Participants' categorization of Task 3

Task	Algebra	Non-algebra
Nehir is having his friends over for a birthday party. She wants to make sure he has a seat for everyone. She has square tables. She can seat 4 people at one square table in this way:  If he joins another square table to the first one, she can seat 6 people:  If Nehir has 100 tables, how many people can she seat? Would you consider this to be an algebra problem?	8	0

When the participants' responses for Task 3 were examined, their responses were grouped under two categories which are constructing an equation or a correspondence relationship and the presence of an unknown. Seven out of eight participants justified their categorizations by referring to constructing an equation or a correspondence relationship. For example, PSMT 7 stated "While finding out the pattern rule, it is a necessity to look at the relationship between the numbers. Looking at the relationship between numbers is something that requires an equation." Among these seven participants, PSMT 6 also mentioned making a generalization. PSMT 6 said "S/he will make a generalization which includes *n*. If you put 100 tables side by side, s/he will have to make a generalization." The last participant, PSMT 3, who also evaluated Task 3 as an algebra question mentioned that the task included an unknown: "The reason why it is an algebraic question is that we try to find an unknown."

Participants' classification of students' work on Task 3. PSMTs were asked to categorize two students' solutions and explain their justifications (see Figure 4.7).

<table>
<tr><th>Kemal's solution</th><th>Dilay's solution</th></tr>
<tr><td>The people column goes up by 2s. So, if I extend the table as below, that would be 202 people that can be seated at 100 tables.
<table>
<tr><th>Number of tables</th><th>Number of people</th></tr>
<tr><td>1</td><td>4</td></tr>
<tr><td>2</td><td>6</td></tr>
<tr><td>3</td><td>8</td></tr>
<tr><td>4</td><td>10</td></tr>
<tr><td>5</td><td>12</td></tr>
<tr><td>6</td><td>14</td></tr>
<tr><td>7</td><td>16</td></tr>
<tr><td>8</td><td>18</td></tr>
<tr><td>9</td><td>20</td></tr>
<tr><td>10</td><td>22</td></tr>
<tr><td>.</td><td>.</td></tr>
</table></td>
<td>The number of people is 2 more than 2 times the number of tables. So, the rule is $2n + 2 = m$ where n = number of tables and m = number of people.

At 100 tables,
$2 \times 100 + 2 = 202$ people can be seated.</td></tr>
</table>

.	.		
99	200		
100	202	*Kemal fills out the table.	

Figure 4. 7 Students’ solutions for Task 3

Seven out of eight participants evaluated Kemal’s solution as non-algebraic, and all the participants categorized Dilay’s solution as algebraic (see Table 4.6).

Table 4. 6

Participants’ categorization of students’ solutions for Task 3

Kemal’s solution		Dilay's solution	
Algebraic	Non-Algebraic	Algebraic	Non-Algebraic
1	7	8	0

One out of eight participants evaluated Kemal’s solution as an algebraic solution because of Kemal’s awareness of a pattern. PSMT 4 stated:

> He has a sense of logic. Actually, when a table is added, [the number of] people is also increasing by 2. But he could not construct an equation, that is why he wrote all of them (he filled out the table).

Seven out of eight PSMTs who evaluated Kemal’s solution as non-algebraic, similarly referred to his use of a pattern in their justifications. For example, PSMT 5 stated:

> In fact, he also realizes the pattern, the increase by 2, but instead of constructing the equation, he used a table and completed the table without thinking. I think he has not realized something algebraic here.

In addition to focusing on his awareness of a pattern as their categorizations for non-algebra, two participants also mentioned the absence of an unknown. For instance, PSMT 1 stated “It (the solution) does not make an algebraic sense, because Kemal did not mention unknowns.”

All participants based their justifications of Dilay's solution as algebraic on constructing an equation. For instance, PSMT 2 stated:

> She formed an equation, then she specified that *n* is the number of the table, *m* is the number of people. We already know that *n* refers to a 100 table, and she finds the number of the people from the equation.

Participants' categorization of Task 4. During the categorization of Task 4, participants had some difficulty since the question did not ask for the value of *x*. Frequencies of participants' responses as algebraic or non-algebraic are shown in Table 4.7.

Table 4. 7

Participants' categorization of Task 4

Task	Algebra	Non-algebra
Write the given expression in the simplest form $5 + 4x + 2x$	5	3

Five participants who classified the task as algebraic emphasized the presence of an unknown and collecting like terms together. Four out of five participants based their reasoning on the presence of an unknown. For example, PSMT 7 reported "Absolutely, it is an algebra question. Because there is an unknown, and we perform an operation with the unknown." One of these five participants focused on collecting like terms together in her justification of algebra. PSMT 4 stated:

> We are taking linear algebra [course] now. What are we doing in linear algebra? It is also like relationship between numbers. If it [the question] assesses the ability of using a parenthesis for the common factor, could it be the thing [algebra]? I could not decide whether this property is under algebra. There is a common multiplier parenthesis in the $5 + 4x + 2x$ by using x (5 + x (4 + 2)), could it be labeled operational property? [...] It is like collecting like terms together.

Three out of eight participants evaluated Task 4 as non-algebraic. These participants based their reasoning on the lack of an equation or an equivalence. For example, PSMT 1 said “No, it is not [an algebraic question]. There is not an equation, there is not an equality.” Among these three participants, PSMT 6 evaluated the task based on the lack of an equation or an equivalence, even though she noticed collecting like terms. PSMT 6 said:

> In the algebra questions, we get a result by performing four operations. [...] For example, would it be an algebra question if *x* was given a value? [In this situation] again we get a result as a numerical solution, we perform operations and get a result. But in this question, we leave it in the simplest form, we do not get a result. How does it lead [the students] to make sense here? It leads to make [students] understand that the same kind of data can be added up, and that the other must stay out of it. But it is not algebraic in that matter because it does not use operations much.

Participants’ classification of students’ work on Task 4. Two student responses (see Figure 4.8) were shown for Task 4. Six out of eight participants evaluated Seçil’s solution as algebraic while the other two PSMTs evaluated it as a non-algebraic solution, and all participants categorized Gizem’s solution as an algebraic solution.

Seçil’s solution	Gizem’s solution
Let’s have *x* that much  I have 4 groups of this,  Then, I add 2 groups of this;  Now I have 6 groups of this, also I add 5;  + 5 So, I have 6*x* + 5.	I have 4 groups of *x*. Then I add 2 groups of *x*. Now, I have 6 groups of *x*, so it is 6*x*. Then I add 5, 6*x* + 5.

Figure 4. 8 Students’ solutions for Task 4

Table 4.8 provides information about PSMTs’ categorization of students’ solutions for Task 4.

Table 4. 8

Participants' categorization of students' solutions for Task 4

Seçil's solution		Gizem's solution	
Algebraic	Non-Algebraic	Algebraic	Non-Algebraic
6	2	8	0

As seen in Table 4.8, six PSMTs evaluated Seçil's solution as algebraic. Four of these participants focused on collecting like terms by using a representation. For example, PSMT 4 stated:

> Actually, Seçil used modelling, she used bar instead of *x*. [...] She puts a bar in place of *x*, actually she makes it concrete a little more. Seçil said that I have 4 bars, if I add 2 bars I will have 6 bars. There is also 5, so it is 6 groups of *x* plus 5.I think it is an algebraic solution, because she is also trying to build a relationship.

The other two participants had different justifications. One participant, PSMT 7, based her reasoning on the presence of an unknown. PSMT 7 stated "It is an algebraic [solution], she used *x*." The last participant, PSMT 8, focused on Seçil's use of modelling. PSMT 8 said "It is an algebraic solution, because she used the modeling method that I mentioned before (when asked "How would you describe what algebra is to someone who has never heard of it before?")."

On the other hand, two PSMTs, who evaluated Seçil's solution as non-algebraic, emphasized Seçil's use of a representation. For instance, PSMT 1 reported:

> I do not expect it to be an algebraic solution, because I do not think it is an algebraic question as I said. [...] To me, it is not meaningful to represent *x* in this way, *x* could be equal to zero, so I think it is not logical to make it concrete.

All participants categorized Gizem's solution as an algebraic solution, and six of them referred to Gizem's collecting like terms by using symbolizations. For instance, PSMT 2 stated:

> She [Gizem] tried to solve it more numerically [and] she shows 5 + 6*x* directly. There is no *x* beside the 5, so she left it alone. She said

> something like collecting the *x*'s together. Gizem symbolically summarized.

The other two participants, emphasized the "abstractness" of Gizem's solution in their categorizations of algebra. For instance, PSMT 7 stated "Gizem's solution is an algebraic solution because I evaluate the equation [*sic.*] as an abstract thing."

4.1.3 Middle School Pre-service Mathematics Teachers' Awareness of Students' Possible Solutions

In this study, middle school pre-service mathematics teachers' awareness of students' possible solutions was attempted to investigate. In each task, participants were asked possible solutions (correct and incorrect) that students could provide in response to the tasks. In this part, the number of the total responses could be more than the number of the participants since each participant were asked to provide as many solutions as they could. The findings will be presented task by task. Responses that were provided less than by two participants and that were not particularly interesting were categorized under the other category.

PSMTs' views about possible student solutions regarding Task 1 were presented in Table 4.9. In Task 1, seven participants mentioned relational-computational as a possible student strategy. For instance, PSMT 4 reported "Firstly, he or she could add 34 and 54, then he or she thinks to subtract 55 from it [the total]." Also, four participants provided relational-structural strategy as a possible student response. For example, PSMT 7 said "Probably, he or she will think to increase one on this side. He or she will use that method, this one increased by one, so this one will decrease by one." Although the participants were good at anticipating possible correct solutions, only two participants anticipated students' possible misconception of the equal sign regarding that the answer comes right after it. For example, PSMT 8 stated "They might add 37 and 54 and write the result directly." As a common possible incorrect student solution, five participants emphasized a mathematical equivalence mistake. For instance, PSMT 5 reported "Maybe s/he could not think it will decrease, and s/he thinks like if 54 increases by one, 37 also increases by one and the result would be 38." Lastly, four participants emphasized an algebraic manipulation mistake. For example, PSMT 4 stated "They may make a mistake in the addition of 37 and 54 or in subtraction of 55."

Table 4. 9

Participants' responses regarding possible student solutions for Task 1

Possible Student Solutions	Frequency of the Given Responses by Participants
Relational - computational strategy	7
Relational - structural strategy	4
Operational thinking	2
Mathematical equivalence mistake	5
Algebraic manipulation mistake	4

Participants' responses to possible student solutions regarding Task 2 were summarized in Table 4.10. In Task 2, seven out of eight participants emphasized that students could ignore the first equation, and they could solve the second equation. For example, PSMT 8 stated "By subtracting 9 from 15 and 9 from 31, and by following certain procedures he or she gets the result." Moreover, six participants anticipated realizing the preservation of an equivalence as a possible student solution. For instance, PSMT 6 said "We expected students to say that we have subtracted the same thing from both sides, the equation stayed the same, and nothing has changed." Besides, five participants mentioned an algebraic manipulation mistake. For instance, PSMT 2 said "They may make a mistake in addition or subtraction." Finally, one participant, PSMT 5 anticipated two other solutions that could not be categorized under the aforementioned categories. PSMT 5 stated "Maybe, s/he puts 8 [in the place of *x*] in the equation. [...] and finds 22." She also stated "Well, I don't know if s/he thinks this way, but since *n* is equal to 8, maybe s/he subtracts 8 from 9."

Table 4. 10

Participants' responses regarding possible student solutions for Task 2

Possible Student Solutions	Frequency of the Given Responses by Participants
Solving an equation	7
Preservation of an equivalence	6
Algebraic manipulation mistake	5
Other	2

PSMTs' responses to possible student solutions for Task 3 were presented in Table 4.11. In Task 3, six participants emphasized that students could write a correct function rule to predict far function values. For instance, PSMT 3 stated:

> If he or she represents the [number of] table by *n*, there are *n* people sitting on one side, and there are *n* people sitting on the other (opposite) side, and there are two people on the sides of the table. So, he or she can find $2n + 2$.

Besides, five participants expected that students would write an incorrect function rule to predict far function values. PSMT 8 said "the answer may be that four people can sit on each table, and then 100×4, 400 people can sit." Three participants emphasized that students could identify a recursive pattern and use it to predict near data. For instance, PSMT 1 said "I think as a first step they (the students) represent the 3rd step. Then, they may count until 100 [tables] by thinking 4, 6, 8... it increases by 2." Furthermore, three participants expected that students would use geometric visualization to find the number of people for 100 tables to solve the task. For example, PSMT 3 said "If there a 100 table, 100 people will seat at the upper side [of the rectangle], 100 people will seat at the lower part [of the rectangle] and two people will seat on the sides." As another example of a visualization strategy, PSMT 6 stated "There is one [table] at both edges. Three people [are sitting] at the first table and are people [are sitting] at the last table. At the tables [between the first and the last] there are 98 tables ..." Lastly, three participants gave answers which were not categorized under the aforementioned categories. For example, PSMT 1 said "Without doing any calculations, s/he can say 200 or 300. Alternatively, s/he can say if it asks that much, the result will definitely be 100."

Table 4. 11

Participants' responses regarding possible student solutions for Task 3

Possible Student Solutions	Frequency of the Given Responses by Participants
Using a correct function rule to predict far function values	6
Using an incorrect function rule to predict far function values	5
Identifying a recursive pattern and use it to predict near data	3
Using geometric visualization to find the number of people for 100 tables	3
Other	3

Lastly, participants' responses about possible student solutions regarding Task 4 were presented in Table 4.12. In Task 4, six participants stated the students who understand the like terms can collect them as $5 + 6x$. For instance, PSMT 7 said "The right result can be reached. If s/he thinks the same type of terms could be added, if s/he got the idea, s/he will get the correct result." Also, five PSMTs expected that the students would ignore like terms, which means that in the expression "$5 + 4x + 2x$," they would ignore x's, and, for example add 5, 4, and 2. For instance, PSMT 1 stated "By ignoring x, he or she directly sees 5, 4, and 2." Likewise, PSMT 6 said:

> If the child is not aware that the numbers that have the same coefficient are added, and the others should not be added, he or she could give $11x$ as a result. I mean, he or she thinks like there is an x beside 5.

Four participants stated that the students could interpret x as a multiplication sign. For example, PSMT 6 reported "He or she does not think x's like multiplication, does s/he?" Moreover, two participants noted that students can assign a value for x. For example, PSMT 1 stated "By assigning a value for x, like giving 1 as the value of x, they can think that the result would be 11." Lastly, two participants gave responses which were coded under the "other" category. For example, PSMT 4 reported "This may look like the simplest form to them, so they may not be able to do anything."

Table 4. 12

Participants' responses regarding possible student solutions for Task 4

Possible Student Solutions	Frequency of the Given Responses by Participants
Collecting like terms	6
Ignoring like terms	5
Interpreting *x* as a multiplication sign	4
Assigning a value for the unknown	2
Other	2

4.2 Findings of the Post-Interviews

4.2.1 Middle School Pre-service Mathematics Teachers' Awareness of the Underlying Algebraic Structure of a Given Task

The same interview protocol was implemented after the algebra weeks in the methods course. Because of one drop out from the study[5], the results will be provided out of seven PSTMs. This part will present findings about to what extent PSMTs could perceive the underlying algebraic structures of given tasks. The question "Why would a teacher pose this question?" was asked to participants in each task and will be presented respectively.

When Task 1 was shown to PSMTs, all participants focused on relational thinking in their responses. In particular, five out of seven participants emphasized the relational-structural thinking as the purpose of the task. For example, PSMT 1 stated "1 plus 54 is 55, there is 1 more in there (at the right side of the equation), there must be one less [to keep balance], so it is 36." Two out of seven participants focused on the relational-computational thinking and they

[5] PSMT 3 went to abroad for a semester to attend an ERASMUS program.

emphasized the meaning of the mathematical equivalence and the equal sign. For instance, PSMT 8 reported:

> In order to understand what the equal sign means and how it makes sense to students. As we discussed in the class, [students] immediately write the answer after the equal sign. However, students can not realize that equal sign is actually used to show that both sides are equal to each other. [A teacher asks the question] to understand whether they know that the results of both sides are equal or whether there is a problem with this issue [meaning of the equal sign].

When the purpose of Task 2 was asked to participants, all PSMTS participants stated the purpose of the task as the preservation of equivalence. For example, PSMT 6 stated:

> The aim is to provide the understanding of the concept of equality on both sides. If I remove the same thing from both sides of the scales, does the equilibrium or the equality change? For example, the same amount is added to both sides. Will this affect the balance? I think this question is asked to have students to make sense of the equality and not to make them perform operations.

Regarding Task 3, the responses of the participants revealed categories including noticing a pattern and constructing an equation or a correspondence relationship through noticing a pattern. Six out of seven participants stated constructing an equation or a correspondence relationship through noticing a pattern as the main aim of the question, additionally these participants mentioned generalization. For instance, PSMT 4 reported:

> The teacher asks the question to have [students] reach a generalization by realizing the pattern. Yes, to make them (students) understand the relations between them. For example, the student is going to draw the 3^{rd} step, and he will think about how many people there are. There are four (in the first step), there are six (in the second step), and in the 3^{rd} step, there will be 8 [people]. Then, how many people are going to be in the next step? Here, [students] are going to realize a pattern and make a generalization about the n^{th} term.

One participant, PSMT 7, only described noticing a pattern as the purpose of the task without referring to making a generalization or constructing an equation. PSMT 7 stated:

> Because [the teacher] may want to have children build relations between the successive steps. Because there is always a rule between the steps and between the previous step and the next steps. Actually, what is the rule? Increasing by 2. [The teacher] asks this question to make students understand the rule, the sequence.

When the aim of Task 4 was asked to the participants, all participants emphasized that the aim would be collecting like terms. For example, PSMT 7 stated:

> In fact, he (the teacher) wants to [make students understand] that the *x's* are the same unknown numbers and the same values. If the student interprets these two ($4x$ and $2x$) in a different way, he or she cannot put them together and write $6x$.

Among these seven participants, one participant, PSMT 4, additionally stated that the teacher could ask this question to assess students' ability to take out the common factor and placing it in front of the parenthesis as well. PSMT 4 reported "Or he (the teacher) may assess the [students' ability of] performing the ability of taking out the common factor."

The analysis of the participants' conceptions of algebra in the post-interviews will be presented next.

4.2.2 Middle School Pre-service Mathematics Teachers' Conceptions of Algebra

In this part, PSMTs' conceptions of algebra were examined under two main sections. Firstly, the responses to the question "How would you describe what algebra is to someone who has never heard of it before?" was examined. Secondly, the PSMTs' algebra categorization of interview tasks and student solutions were examined task by task.

4.2.2.1 How would you describe what algebra is to someone who has never heard of it before?

In this part, participants' responses to the question "How would you describe what algebra is to someone who has never heard of it before?" were examined. Five out of seven participants based their justifications on the presence of an unknown or an equation. For example, PSMT 2 stated "equations where there is more than one unknown, and variables are used to obtain the result." Among these participants, PSMT 5 also emphasized operations saying "There is unknown and also operational things."

Two out of seven participants based their reasoning of algebra on making a generalization. For example, PSMT 8 reported:

> That is what I call algebra, the generalization of some terms, series, or some rules. A certain order based on a specific generalization, a certain rule. [The question] such as after finding the 1st, 2nd, 3rd step and generalizing it to a certain thing can be called algebra.

Further information about PSMTs' conceptions of algebra in the post-interviews was gathered by task-based questions which will be presented next.

4.2.2.2 How Did Middle School Pre-service Mathematics Teachers Classify the Tasks and the Related Students' Solutions

In this part, the participants' algebra conceptions will be investigated through their evaluation of tasks and student solutions as algebraic or not. To have insights about their algebra conceptions in each task, the question "Would you consider this to be an algebra problem?" was asked to the participants. Also, two different student solutions were displayed for each task in order to have detailed information about PSMTs' algebra conceptions.

Participants' categorization of Task 1. As it is in Table 4.13, six out of seven participants evaluated the task as an algebra question, and one participant evaluated it as a non-algebra question.

Table 4. 13

Participants' categorization of Task 1

Task	Algebra	Non-algebra
What number goes in the □ ? 37 + 54 = □ + 55	6	1

Among those evaluating the task as an algebra question, the participants' responses were divided into three categories. One of the categories was the presence or absence of an unknown or an equation, while the others were related to the relational thinking which are relational-computational and relational structural. Two participants emphasized the presence or absence of an unknown or an equation in their justifications. For example, PSMT 2 stated "Because there is an unknown. [A student] has to perform a certain operation to find the unknown, and [the student] finds the answer at the end of performing operations." Four participants justified their algebra categorizations focusing on the relational-thinking. Three of them focused on relational-structural thinking. For instance, PSMT 7 said:

> At first, I thought it is just related with addition and subtraction. But then, I realized that it is the relation between the numbers again when we consider the fact that it increases 1 (from 54 to 55) and in this case, it should also decrease by 1 (from 37 to 36). [...] It seems to me as algebra.

One participant, PSMT 5, based her reasoning on the relational-computational thinking. PSMT 5 stated "There is also operations. [...] I mean, there is an equality, and s/he builds a relationship."

PSMT 8, who categorized Task 1 as non-algebraic, initially evaluated the question as algebraic because of the relational structure of the task. PSMT 8 stated, "Because there is an unknown, and when I examine the operation, I think rather than the unknown, there is [a relation] 1 more 1 less." However, after examining Task 2, PSMT 8 changed his mind and decided that the Task 1 is non-algebraic. He focused on generalization stating "Previous question (Task 1) is not supposed to be algebra, either. We can write *n* instead of the empty box, there is also an unknown. If the unknown is here, instead of generalizing, we try to find it. So, it could go under the category of the equations."

Participants' classification of students' work on Task 1. In this section, participants' conceptions of algebra were investigated through having them reflect on student responses and categorize as algebra or not (see Table 4.14). Three out of seven participants evaluated Burak's strategy as algebraic, while four of them evaluated it as non- algebraic, and all of the participants evaluated Nur's strategy as algebraic. To understand PSMTs' post-interview algebra conceptions, their justifications were analyzed.

Table 4. 14

Participants' categorization of students' solutions for Task 1

Burak's solution		Nur's solution	
Algebraic	Non-Algebraic	Algebraic	Non-Algebraic
3	4	7	0

Three participants, who evaluated Burak's solution as algebraic, emphasized Burak's computationally based strategy. For instance, PSMT 4 stated "I think it is an algebraic solution. All in all, he tries to find the unknown. He thinks that he can find the unknown by adding this (37) and this (54), and subtracting 55 from this (total)."

Four out of seven participants, who evaluated Burak's solution as non-algebraic, also based their reasoning on Burak's use of a computationally based strategy, additionally three of them mentioned the absence of the relational structural thinking. For example, PSMT 7 reported:

> The only reason why I evaluated this question as algebra question is that [the question] is based on the meaning of balance provided by the equal sign and the relation between numbers. Here the child did not use this meaning. [...] I mean, he just performed operations. It is a solution just consisting of addition-subtraction.

All participants evaluated that Nur's solution as algebraic. Six out of seven participants evaluated her solution as algebraic because of the use of relational structural thinking. For example, PSMT 5 said "She realized the relationship here, she says that if this increases by one, the other one will decrease by one."

Although the remaining one participant, PSMT 4, classified the solution as algebraic, she focused on finding the unknown in Nur's solution. PSMT 4 reported:

> I think it is also an algebraic solution since she tries to find the unknown. […] There is not much difference between them (Burak's and Nur's solutions), but of course her thinking is more practical. But this practical thinking is related with her success in number sense.

Participants' categorization of Task 2. The frequency of the participants' categorization of Task 2 was shown in Table 4.15. Four out of seven participants evaluated the task as an algebraic task while three participants evaluated it as non-algebraic. Their justifications were categorized under the preservation of equivalence, solving an equation, and presence of an unknown or an equation.

Table 4. 15

Participants' categorization of Task 2

Task	Algebra	Non-algebra
The solution to the equation $2n + 15 = 31$ is $n = 8$, What is the solution to the equation? $2n + 15 – 9 = 31 – 9$	4	3

Two out of the four participants who evaluated the task as algebraic focused on the preservation of equivalence. For example, PSMT 6 reported "Will the students get the result by performing operations or will the students realize it (the result) as 8 again without performing the operations? Since he or she [the teacher] tries to understand that, it is an algebra question." Another two participants, who evaluated Task 2 as algebraic, justified their reasoning by referring to the presence of an unknown or an equation. For example, PSMT 5 stated, "There is again an equation and equality, that is why [it is an algebra task]."

When two participants' responses who evaluated the task as non-algebraic were examined, it was seen that these participants emphasized that there was no need to solve an equation. For example, PSMT 2 explained as follows:

> It is not [an algebraic task]. There is already a result, actually, the child knows the result is 8. There is an unknown, but in the (second equation) below nothing is changed, so he or she directly knows that *n* is equal to 8, and it is same with the one (the first equation) above. The important thing here is realizing that equivalence will not change the result. [...] There is an unknown, but there is also the result of the unknown above, so the student does not need to perform any operations.

The last participant, PSMT 8, who categorized the task as non-algebra, reported:

> There is an unknown. It is like an equation, but I do not know whether the equation is algebra. [The task] is mostly about finding the unknown, instead of making a generalization. That is why it may not be algebraic.

Participants' classification of students' work on Task 2. In this section, participants' conceptions of algebra were examined according to their reflections about student strategies provided for Task 2. Four out of seven participants evaluated Kerem's solution as algebraic while three of them evaluated it as non-algebraic (see Table 4.16). The participants, who evaluated Kerem's solution as algebraic, based their reasoning on solving an equation. For example, PSMT 4 stated "I think it is an algebraic solution. Because he isolates the *n* at one side [of the equation]. Without it (performing operation), it does not seem to be an algebraic task."

Table 4. 16

Participants' categorization of students' solutions for Task 2

Kerem's solution		Defne's solution	
Algebraic	Non-Algebraic	Algebraic	Non-Algebraic
4	3	5	2

Three out of seven participants, who evaluated Kerem's solution as non-algebraic, based their reasoning also on solving equations. They emphasized that Kerem's solution does not focus on relational thinking but just on solving an equation. For instance, PSMT 1 stated:

> After subtracting the same number from both sides, he performed the operations step by step. He subtracted 9 from 15, then he tried to eliminate that 6. He did not consider subtracting the same thing [from both sides] as important. The only thing he cared about it was to isolate the *n* on one side and to isolate the numbers on the other side.

When participants' reflections on Defne's solution were examined, two categories which are the preservation of an equivalence and the lack of performing operations were revealed. Five out of seven participants, who evaluated Defne's solution as algebraic, emphasized Defne's awareness of the preservation of an equivalence. For example, PSMT 5 reported "I think Defne's response was also algebraic. She also makes use of the equivalence. She used the equivalence rather than solving the question step-by-step. She thought adding to both sides would not change anything."

Among these five participants, one of them also mentioned generalization in his explanation. PSMT 8 stated:

> Because, as I said before adding and subtracting the same number means a generalization at a certain level. For example, if 10 is added to one side, while 8 is added to the other side, [the student] may see the number at this side (the side which 8 was added) will be 2 more. If [the student] sees that the same things (quantity) were subtracted from both sides, he or she can understand what happens to the relationship when different numbers are added or subtracted.

The two participants, who evaluated Defne’s solution as non-algebraic, based their reasoning on the lack of performing operations. For example, PSMT 2, “She got the result without performing any operations, just by using the information which was given above (in the first equation). But Kerem went through a step-by-step solution.”

Participants’ categorization of Task 3. As seen in Table 4.17, all participants categorized the functional thinking task as algebraic. Five out of seven participants focused on constructing an equation or a correspondence relationship through noticing a pattern. Among the five participants, PSMTs 4 and 8 additionally emphasized making a generalization. PSMT 8 stated:

> I think it is clearly an algebra task. As I said before, there is a certain order. When one table is added, a certain number of people is increasing. [...] Again there is a certain relationship, there is a certain pattern between them. He or she needs to find a general rule to explain it in by algebraic terms so that he or she can find any step. I mean there is a generalization, that is why it is an algebra task.

The two out of seven PSMTs justified their reasoning based on constructing an equation or a correspondence relationship without referring to a pattern, and they additionally mentioned making a generalization. For example, PSMT 5 said “The only difference from the previous ones (tasks) is constructing the equation on his or her own and his or her ability to make a generalization.”

Table 4. 17

Participants’ categorization of Task 3

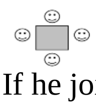
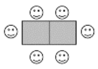
Task	Algebra	Non-algebra
Nehir is having his friends over for a birthday party. She wants to make sure he has a seat for everyone. She has square tables. She can seat 4 people at one square table in this way:  If he joins another square table to the first one, she can seat 6 people:	7	0

Table 4.17 (continued)



If Nehir has 100 tables, how many people can she seat?

Participants' classification of students' work on Task 3. Six out of seven participants categorized Kemal's solution as non-algebraic (see Table 4.18). Three of them justified their reasoning based on Kemal's only being aware of a pattern. Among these three participants, PSMT 1 also mentioned about the absence of a generalization in Kemal's solution. For example, PSMT 1 stated "He noticed the pattern, he noticed the system and realized the arithmetic increase [...] he could not make a generalization. He could not represent the number of the table by a letter [...] he did basic counting."

Table 4. 18

Participants' categorization of students' solutions for Task 3

Kemal's solution		Dilay's solution	
Algebraic	Non-Algebraic	Algebraic	Non-Algebraic
1	6	7	0

Two out of six participants, who evaluated Kemal's solution as non-algebraic, based their reasoning only on the absence of a generalization. For instance, PSMT 4 said "Kemal could not make a generalization. [...] It (his solution) is not a higher order thinking." The remaining participant, PSMT 2, who also evaluated Kemal's solution as non-algebraic, focused on the lack of performing operations stating:

> There are four [people] at one table, and then six [people] at the second one. It is like guess and check method. I never think that this is an algebraic solution. Instead of performing an operation and finding the value, the unknown *n*, he drew until 100 [tables].

The only participant, PSMT 5, who evaluated Kemal's solution as algebraic said:

> Well, even though there is not an unknown, there is not an equation, he continued arithmetically as a result, in some way he counted by 2. Actually, I think these arithmetic things go under the category of algebra. There is again counting and numbers, that is why it is algebraic.

When participants' reflections on Dilay's solution were evaluated, all participants were found to categorize it as an algebraic solution and based their justifications on constructing an equation. For instance, PSMT 2 clarified:

> Dilay represented the certain number of table by an unknown, and she represented the certain number of people by an unknown. She constructed an equation by using these unknowns. When we said "If the number of the table is 100, what is the number of the people?", she found the value of the unknown by using the equation. Therefore, this is an algebraic solution.

Among the participants, three of them also emphasized making a generalization. For instance, PSMT 6 reported "Dilay comprehended the concept of generalization. She has defined n and m properly... I mean if it is a 1000 table, she will find it (the number of the people). That is why it is an algebraic solution."

Participants' categorization of Task 4. The frequency of participants' responses for Task 4 in their algebra categorization was shown in Table 4.18.

Table 4. 19

Participants' categorization of Task 4

Task	Algebra	Non-algebra
Write the given expression in the simplest form $5 + 4x + 2x$	5	2

Two out of five participants, who evaluated Task 4 as an algebra task, based their reasoning on the presence of an unknown. PSMT 1 said "It is definitely an algebra question. I am not kidding; I evaluate it as an algebra question when I see an unknown. It (algebra) is the word of the unknown after all." The other participant, PSMT 7, also made her decision on the categorization of algebra by referring to the presence of an unknown, but the participant hesitated because of the lack of an equation or an equivalence in the problem:

> PSMT 7: Normally I would not evaluate it as an algebra question, but in this case, I evaluate it as an algebra question because of the unknown. [...] We studied variables in the algebra chapter, and dependent and independent variables. Since we mentioned them, I think it (the task) goes under the category of algebra.
> R: What would be the reason for this task not to be seen as an algebra question?
> PSMT 7: Because it does not lead to a result or an operation..., because I am looking for the equal sign.

One PSMT stated her justification of algebra referring to collecting like terms. PSMT 6 said "The students should definitely know what *x* is. *x*'s can be added and combined together. In the sake of teaching this, it is an algebra question." One participant, PSMT 5, made her decision by looking for the presence of performing operations. PSMT 5 reported "There are numbers, values and an operation. [...] There is again an operation, like 4*x*, 5, there is a thing [...], I mean there is an operation." Lastly, PSMT 8 justified his reasoning by focusing on generalization as he did in previous evaluations. PSMT 8 said:

> I think it might be an algebra question. For example, $6x + 5$ could be a rule of a pattern or something that is generalized. I mean, there is not something like [$6x + 5$] is equal to 10. If there were [e.g., $6x + 5 = 10$], it would not be an algebraic task, but now I think it is.

Two of the participants, who evaluated the task as non-algebra, referred to in their explanations. For example, PSMT 2 clarified, "I cannot get a certain result, I do not know what *x* is related to. [...] For example, if I said $5 + 6x = 11$, then *x* would be equal to 1, this time it would be an algebra question." The other participant, PSMT 4, had a similar reasoning with PSMT 2 on finding the value of *x*, but she made her final decision by referring to taking out the common factor. PSMT 4 stated "Is finding *x* an algebra, or the presence of *x* algebra? [...] I think this (task) is not an algebra task since it assesses the ability of taking out the common factor."

Participants' classification of students' work on Task 4. Regarding participants' evaluation of students' work on Task 4 (see Table 4.19), five out of seven participants categorized Seçil’s solution as algebra, while two participants evaluated it as a non-algebraic solution. The five PSMTs, who evaluated Seçil’s solution as algebraic, referred to Seçil’s awareness of collecting like terms by using a representation. For example, PSMT 1 stated “She describes the amount of quantity that she does not know by visualization. There is two of it and four of it. She knows that 4*x* means four of that quantity.”

Table 4. 20

Participants’ categorization of students’ solutions for Task 4

Seçil’s solution		Gizem's solution	
Algebraic	Non-Algebraic	Algebraic	Non-Algebraic
5	2	6	1

Two participants evaluated Seçil’s solution as non-algebraic. PSMT 4 focused on the lack of an equation:

> To me, the task should ask for the value of *x*. That is why I evaluated that task as non-algebraic, and that is why the solution of the task is not algebraic. In there, they add *x*’s, they add the terms with the same thing (coefficient). It seems to me it is not related with algebra.

The last participant, PSMT 6, based her decision of non-algebra on the concreteness of Seçil’s solution. PSMT 6 stated “But that's a bit more in a concrete level, she could not reach the abstract level. [...]. If she stays at this level, it will not be an algebraic solution.”

Six out of seven participants, who categorized Gizem’s solution as algebraic, emphasized Gizem’s awareness of collecting like terms by using symbolizations. For example, PSMT 7 said “She represented by *x*, by a letter. Then, she added two more *x*, and she got six groups of *x*, so she said 6*x* + 5.” Among these participants, PSMT 6 additionally mentioned the “abstractness” of Gizem’s solution as she also mentioned the “concreteness” of Seçil’s solution. The remaining participant, PSMT 4, who evaluated Gizem’s solution as non-algebraic, based her justification on the same idea—the lack of an equation in the task—which is presented in the reasoning about Seçil's solution above.

4.2.3 Middle School Pre-service Mathematics Teachers' Awareness of Students' Possible Solutions

In this part, middle school pre-service mathematics teachers' awareness of students' possible solutions in the post-interviews will be presented. In each task, participants were asked possible solutions (correct and incorrect) that students could provide. The findings were presented for each task separately. Note that the number of the total responses could be more than the number of the participants since each participant were asked to provide as many solutions as they could. Also, responses that were provided less than by two participants and that were not particularly interesting were categorized under the Other category.

In response to Task 1, six participants stated that students could demonstrate relational-structural thinking. For example, PSMT 6 stated "He or she could see that this one (55) is 1 more than this one (54). Then, to make both sides equal and unchanged, this should be one less (37), and it is 36." Also, four participants emphasized relational-computational thinking. For instance, PSMT 1 said "They may think like that 37 plus 54 is equal to 91, and then 91 minus 55." In addition to these possible student solutions, participants also emphasized misconceptions that students could hold. Six participants emphasized the misconceptions regarding the meaning of the equal sign, and they focused on students' interpretation of the equal sign as an operational symbol. For instance, PSMT 7 reported "The children mostly see the equal sign as a sign which leads to the result. Unfortunately, by ignoring 55, they may write the total." In addition, six participants mentioned a mathematical equivalence mistake. For example, PSMT 6 stated "A student can think like that, 54 plus 1 is 55, so 37 should increase 1, and the s/he can write 38." Lastly, two participants emphasized that students can perform an algebraic manipulation mistake. For instance, PSMT 1 stated "They may find something like 26 or 46 when they subtract 54 from 91."

Table 4. 21

Participants' responses regarding possible student solutions for Task 1

Possible Student Solutions	Frequency of the Given Responses by Participants
Relational- structural strategy	6
Relational - computational strategy	4
Operational thinking	6
Mathematical equivalence mistake	6
Algebraic manipulation mistake	2

Participants' anticipated student solutions for Task 2 were summarized in Table 4.21. All participants stated the preservation of an equivalence as a possible student solution. For example, PSMT 5 said "- 9 has been added to both sides, so the equality has not changed. He or she could say *n* is equal to 8 by saying it is the same with the one above (the first equation)." In addition, four participants emphasized students' solving an equation in their responses. For instance, PSMT 2 reported:

> He or she may perform operations step by step, like $2n + 6 = 31 - 9$ and continues. He or she finds the same solution ($n = 8$) again, but he or she cannot realize the meaning of the equality.

Moreover, six participants mentioned that students can do an algebraic manipulation mistake when they are solving the equation. Lastly, two participants mentioned some solutions which were not related to the aforementioned categories. For example, PSMT 7 reported "A student can do 2 times 8 is equal to 16, 16 minus 9 [will give the result]."

Table 4. 22

Participants' responses regarding possible student solutions for Task 2

Possible Student Solutions	Frequency of the Given Responses by Participants
Preservation of an equivalence	7
Solving an equation	4
Algebraic manipulation mistake	6
Other	2

In Task 3 responses (see Table 4.22), participants indicated using a correct function rule to predict far function value, using geometric visualization to find the number of the people for 100 tables, and identifying a recursive pattern and using it to predict near data as possible student solutions. Particularly, six participants referred to using a correct function rule to predict far function values as a possible student strategy. For instance, PSMT 8 said "He or she may get the right answer by thinking like how many people can seat in the 1^{st} step, how many people can seat in the 2^{nd} step, and how many people can seat in the 3^{rd} step. Then he or she may find a certain rule." Additionally, four participants emphasized writing an incorrect function rule to predict far function values as another solution strategy, and three participants mentioned the geometric visualization to find the number of people for 100 tables. For example, PSMT 6 stated "There is a 100-table looking face to face. The student will count as 2, 4, 6, ..., 200, and there are two [people] on the sides (at the first and the last table)." Furthermore, one participant, PSMT 7, emphasized that the students may solve the question by identifying a recursive pattern and use it to predict near data. PSMT 7 reported "He or she could find the result by thinking it increases by 2." Lastly, two participants provided strategies that are not categorized under the aforementioned categories. For instance, PSMT 1 stated "Maybe a student makes a mistake, if it asks for 100 table, the result could be 100."

Table 4. 23

Participants' responses regarding possible student solutions for Task 3

Possible Student Solutions	Frequency of the Given Responses by Participants
Using a correct function rule to predict far function values	6
Using an incorrect function rule to predict far function values	4
Using geometric visualization to find the number of people for 100 tables	3
Identifying a recursive pattern and use it to predict near data	1
Other	2

Lastly, in Task 4, participants listed collecting like terms, ignoring like terms, interpreting *x* as a multiplication sign, and assigning a value for the unknown as possible student solutions (see Table 4.23). Five participants emphasized collecting like terms as a possible student solution. For instance, PSMT 2 said “Since he or she knows the difference between the number and the *x*’s, it is 5 + 6*x*.” Moreover, all participants stated ignoring like terms as a possible misconception. For example, PSMT 5 reported “By ignoring the *x*’s, he or she directly add 5, 4, 2 and find 11.” Also, three participants reported students’ possible interpretation of the letter *x* as a multiplication sign. For instance, PSMT 8 said “I do not know whether they interpret *x* as a multiplication sign; they could.” Lastly, one participant mentioned assigning a value for the unknown. PSMT 1 said “By giving a value to *x* by themselves, for example [*x* is equal to] 1, then [by adding] 4 and 6, they may find 11.” Finally, one participant mentioned solution that are not categorized in the aforementioned categories. PSMT 4 reported “For example, adding 5 and 4 equals to 9, then s/he may add *x* and 2*x*, like 9 + 3*x*.”

Table 4. 24

Participants’ responses regarding possible student solutions for Task 4

Possible Student Solutions	Frequency of the Given Responses by Participants
Collecting like terms	5
Ignoring like terms	7
Interpreting *x* as a multiplication sign	3
Assigning a value for the unknown	1
Other	1

4.3 Changes Between Pre- and Post-Interview Findings

In this section, the changes between pre- and post-interview findings will be outlined in terms of PSMTs’ awareness of the algebraic purpose of a given task, PSMTs’ conceptions of algebra and PSMTs’ awareness of students’ possible solutions, respectively.

4.3.1 Changes in Middle School Pre-service Mathematics Teachers' Awareness of the Underlying Algebraic Structure of a Given Task

Upon Task 1, all participants in pre- and post-interviews stated that a teacher could ask the question to have the students build relational thinking (see Table 4.25). In the pre-interviews, seven out of eight PSMTs emphasized the aim of the task as building relational structural thinking while one participant focused on relational computational thinking. When the post-interview results were examined, it was seen that five out of seven PSMTs specified the aim as building relational structural thinking while the remaining two participants mentioned relational computational thinking.

Table 4.25

Participants' responses regarding underlying algebraic structure of Task 1

What number goes in the ☐? $37 + 54 = \square + 55$			
Pre-interviews		Post-interviews	
Relational Structural Thinking	Relational Computational Thinking	Relational Structural Thinking	Relational Computational Thinking
7	1	5	2

As seen in Table 4.26, upon Task 2, all participants identified the purpose of the question as using the preservation of equivalence in both the pre- and post-interviews.

Table 4.26

Participants' responses regarding underlying algebraic structure of Task 2

The solution to the equation $2n + 15 = 31$ is $n = 8$. What is the solution to the equation? $2n + 15 – 9 = 31 – 9$?

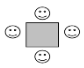
Table 4.26 (continued)

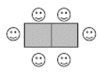
Pre-interviews	Post-interviews
Preservation of the Equivalence	Preservation of the Equivalence
8	7

Upon Task 3, categories that are noticing a pattern, constructing an equation, and constructing an equation or a correspondence relationship through noticing a pattern were revealed in the pre-interviews (see Table 4.27). In the post-interviews, none of the participants provided a purpose related to only constructing an equation category, while two participants in the pre-interview referred to constructing an equation. In the pre-interviews, four PSMTs gave responses which were coded as constructing an equation or a correspondence relationship through noticing a pattern, while this number increased to six in the post-interviews. Besides that, all these six participants also mentioned making a generalization in the post-interviews, while two participants mentioned it in the pre-interviews.

Table 4.27

Participants' responses regarding underlying algebraic structure of Task 3

Nehir is having his friends over for a birthday party. She wants to make sure he has a seat for everyone. She has square tables. She can seat 4 people at one square table in this way: 

If he joins another square table to the first one, she can seat 6 people: 
If Nehir has 100 tables, how many people can she seat?

Pre-interviews			Post-interviews	
Noticing a Pattern	Constructing an Equation	Constructing an Equation or a Correspondence Relationship Through Noticing a Pattern	Noticing a Pattern	Constructing an Equation or a Correspondence relationship Through Noticing a Pattern
2	2	4	1	6

Upon Task 4, all participants identified the purpose of the task as collecting like terms in both the pre- and post-interviews (see Table 4.28). Among these participants, one participant, PSMT 4, additionally mentioned taking out a common factor as a teacher's aim in both interviews.

Table 4.28

Participants' responses regarding underlying algebraic structure of Task 4

Write the given expression in the simplest form $5 + 4x + 2x$.	
Pre-interviews	Post-interviews
Collecting Like Terms	Collecting Like Terms
8	7

4.3.2 Changes in Middle School Pre-service Mathematics Teachers' Conceptions of Algebra

4.3.2.1 Changes to PSTMs' responses to "How would you describe what algebra is to someone who has never heard of it before?"

Examining the participant's responses to "How would you describe what algebra is to someone who has never heard of it before?" derived the categories of the presence of an unknown or an equation and performing operations in the pre-interviews. In the post-interviews, a category "making a generalization" was revealed instead of performing operations (see Table 4.29). In the pre-interviews three participants emphasized performing operations, and in the post-interviews two participants emphasized making a generalization. Also, the responses of four PSMTs in the pre-interviews and five PSMTs in the post-interviews were related to the presence of an unknown or an equation in their algebra definitions.

Table 4.29

Participants' responses regarding the question

How would you describe what algebra is to someone who has never heard of it before?				
Pre-interviews			Post-interviews	
The Presence of an Unknown or an Equation	Performing Operation	Modeling	The Presence of an Unknown or an Equation	Making a Generalization
4	3	1	5	2

4.3.2.2 How Did Middle School Pre-service Mathematics Teachers Classification of the Tasks and the Related Students' Solutions Change?

Upon the participants' categorization of Task 1, six out of eight participants evaluated the task as algebraic in the pre-interviews (see Table 4.30). Among these six participants, three PSMTs made their decisions based on the presence or absence of an unknown or an equation, two PSMTs made their decisions based on relational-structural, and, lastly, one participant based her justification on relational-computational. In the post-interviews, six out of seven participants evaluated the task as an algebra question. Among these six participants, two of them emphasized the presence or absence of an unknown or an equation, three of them emphasized relational structural, and one of them emphasized relational computational thinking in their justifications.

Table 4.30

Participants' categorization of Task 1 in the pre- and post-interviews

Pre-interviews		Post-interviews	
Algebraic	Non-Algebraic	Algebraic	Non-Algebraic

Table 4.30 (continued)

The Presence or Absence of an Unknown or an Equation	Relational Thinking	The Presence or Absence of an Unknown or an Equation	Relational Thinking	The Presence or Absence of an Unknown or an Equation	Relational Thinking	Other
3	3	1	1	2	4	1

Upon the participants' classification of students' work on Task 1, five out of eight participants evaluated Burak's solution as an algebraic solution in the pre-interviews, and four of them focused on Burak's use of a computationally based strategy while one of them focused on the presence of an unknown (see Table 4.31). In the post-interviews, three out of seven participants evaluated Burak's strategy as algebraic, and they based their justifications on the presence of a computationally based strategy. In the pre-interviews, three out of eight participants evaluated Burak's solution as non-algebraic by referring to his use of a computationally based strategy, and additionally two of them mentioned the absence of relational structural thinking in Burak's solution. Similarly, in the post-interviews, four participants provided the same justification to explain their non-algebraic evaluation of Burak's solution, and additionally three of them mentioned the absence of relational structural thinking in Burak's solution. When the participants' categorizations of Nur's solution between pre- and post-interviews were compared, it was seen that in the pre-interviews, six out of eight participants evaluated Nur's solution as algebraic while in the post-interviews, all participants evaluated it as algebraic. Indeed, in the pre-interviews, five participants who categorized it as algebra based their justifications on the relational structural thinking. Likewise, in the post-interviews, six out of seven participants justified their reasoning by referring to her use of relational structural thinking.

Table 4.31

Participants' categorization of Burak's solution for Task 1 in the pre- and post-interviews

Burak's solution 36 goes in the box because 37 plus 54 is 91, so I had to figure out what plus would be 91. 36 plus 55 is 91, so it is 36				
Pre-interview			Post-interview	
Algebraic		Non-algebraic	Algebraic	Non-algebraic
Computationally Based Strategy	Presence of an Unknown	Computationally Based Strategy	Computationally Based Strategy	Computationally Based Strategy
4	1	3	3	4

Table 4.32 shows PSMTs' categorization of Nur's solution for Task 1 in the pre- and post- interviews.

Table 4.32

Participants' categorization of Nur's solution for Task 1 in the pre- and post-interviews

Nur's solution 36 goes in the box. 55 is one more than 54, so the number in the box has to be one less than 37, so it is 36.					
Pre-interview			Post-interview		
Algebraic		Non-algebraic	Algebraic		Non-algebraic
Presence of an Unknown	Relational Thinking	Relational Thinking Strategy as Using Logic	Finding the Unknown	Relational Thinking	-
1	5	2	1	6	0

Regarding Task 2, in the pre-interviews, all participants evaluated Task 2 as an algebra task, while in the post-interviews, four out of seven participants evaluated it as algebraic. In the pre-interviews, half of the participants based their justifications of algebra on the presence of an unknown or an equation while this number was found to decrease to two in the post-interviews (see Table 4.33). Also, the number of the PSMTs who specified the preservation of an equivalence as their algebra justifications in the pre- and post-interviews were three and two, respectively. While one participant in the pre-interviews justified her response as categorization of algebra by referring to solving an equation, two PSMTs in the post-interviews referred to the same justification, solving an equation, in their classifications of the task as non-algebra. Additionally, among the three participants, who categorized Task 2 as non-algebraic, two of them focused on the preservation of an equivalence, and they emphasized there is no need to solve an equation.

Table 4.33

Participants' categorization of Task 2 in the pre- and post-interviews

Pre-interviews				Post-interviews			
Algebraic			Non-Algebraic	Algebraic		Non-Algebraic	
The Presence of an Unknown or an Equation	Preservation of an Equivalence	Solving Equation	-	The Presence of an Unknown or an Equation	Preservation of an Equivalence	Solving Equation	Other
4	3	1	0	2	2	2	1

Upon the participants' classification of students' work on Task 2 (see Table 4.34), seven out of eight participants, who evaluated Kerem's solution as algebraic, mentioned solving an equation in their justifications in the pre-interviews. In the post-interviews, four out of seven participants categorized it as algebraic based their reasoning also on solving equations. They emphasized that Kerem's solution does not focus on relational thinking but just on solving an equation.

In the examination of Defne's solution in the pre-interviews, five out of eight participants who categorized Defne's solution as algebraic referred to Defne's understanding of the preservation of equivalence. Likewise, in the post-interviews five out of seven participants evaluated Defne's solution as algebraic because of the same justification. When the categorizations of the participants' who evaluated Defne's solution as non-algebraic were examined in the pre-interviews, it was seen that these three participants justified their reasoning based on Defne's use of the preservation of an equivalence, and one PSMT additionally focused on the lack of operations in the response. In the post-interviews, the two participants who categorized Defne's answer as non-algebraic based their reasoning on the lack of performing operations.

Table 4.34

Participants' categorization of Kerem's solution for Task 2 in the pre- and post-interviews

Kerem's Solution

$$2n + 15 - 9 = 31 - 9$$

$$2n + 6 = 22$$

$$\underline{\quad -6 \quad -6 \quad}$$

$$\frac{2n}{2} = \frac{16}{2}$$

$$n = 8$$

Pre-interview		Post-interview	
Algebraic	Non-algebraic	Algebraic	Non-algebraic
Solving an Equation	Solving an Equation	Solving an Equation	Solving an Equation
7	1	4	3

Table 4.35 shows PSMTs' categorization of Defne's solution for Task 2 in pre- and post- interviews.

Table 4.35

Participants' categorization of Defne's solution for Task 2 in the pre- and post-interviews

Defne's solution It is the same, *n* = 8 because you are subtracting the same thing from both sides.			
Pre-interview		Post-interview	
Algebraic	Non-algebraic	Algebraic	Non-algebraic
Preservation of an Equivalence	Preservation of an Equivalence	Preservation of an Equivalence	Lack of Performing Operations
5	3	5	2

Upon the participants' categorization of Task 3, all participants evaluated the task as algebraic in both the pre- and post-interviews (see Table 4.36). In the pre-interviews, PSMTs' responses were grouped under two categories which are constructing an equation or a correspondence relationship and the presence of an unknown. Seven out of eight participants justified their categorizations by referring to constructing an equation or a correspondence relationship, while one participant based her reasoning on the presence of an unknown. In the post-interviews, none of the participants mentioned the presence of an unknown. In fact, the responses of the five participants in the post-interviews revealed a new category "constructing an equation or a correspondence relationship through noticing a pattern. The remaining two PSMTs indicated constructing an equation or a correspondence relationship without referring to a pattern. Furthermore, in the post-interviews, four PSMTs additionally mentioned making a generalization in their algebra justifications for the task, while in the pre-interviews only one PSMT mentioned it.

Table 4.36

Participants' categorization of Task 3 in the pre- and post-interviews

Pre-interviews			Post-interviews		
Algebraic		Non-Algebraic	Algebraic		Non-Algebraic
The Presence of an Unknown	Constructing an Equation or an Correspondence Relationship	-	Constructing an Equation or an Correspondence Relationship Through Noticing a Pattern	Constructing an Equation or an Correspondence Relationship	-
1	7	0	5	2	0

Upon the participants' classification of students' work on Task 3, one participant from the pre-interviews and one participant from the post-interviews evaluated Kemal's solution as algebraic by referring to his awareness of a pattern (see Table 4.37). In the pre-interviews, seven out of eight participants evaluated Kemal's solution as non-algebraic by again referring to his using a pattern in their justifications. In the post-interviews, six out of seven participants evaluated Kerem's solution as non-algebraic. Among these six participants, three of them referred to Kemal's use of a pattern, and one of them additionally emphasized the absence of a generalization in Kemal's solution. Also, two other participants, who evaluated Kemal's solution as non-algebraic in the post-interviews, based their reasoning on the absence of a generalization only. In the examination of Dilay's solution, all participants in both the pre- and post-interviews categorized it as an algebraic solution. When participants' responses were analyzed, it was seen that all participants based their justifications on Dilay's constructing an equation in her response in both the pre- and post-interviews. The difference was that, in the post-interviews, three participants also emphasized making a generalization in her solution.

Table 4.37

Participants' categorization of Kemal's solution for Task 3 in the pre- and post-interviews

Kemal's solution

The people column goes up by 2s.

So, if I extend the table as below,

that would be 202 people that can

be seated at 100 tables.

*Kemal fills out the table

Number Tables	Number of People
1	4
2	6
3	8
4	10
.	.
.	.

Pre-interview		Post-interview			
Algebraic	Non-algebraic	Algebraic	Non-algebraic		
Awareness of a Pattern	Only Being Aware of a Pattern	Other	Only Being Aware of a Pattern	Absence of an Operation	Absence of a Generalization
1	7	1	3	1	2

Table 4.38 shows PSMTs' categorization of Dilay's solution for Task 3 in the pre- and post- interviews.

Table 4.38

Participants' categorization of Dilay's solution for Task 3 in the pre- and post-interviews

Dilay's Solution The number of people is 2 more than 2 times the number of tables. So, the rule is $2n + 2 = m$ where n = number of tables and m = number of people. At 100 tables, $2 \times 100 + 2 = 202$ people can be seated			
Pre-interview		Post-interview	
Algebraic	Non-algebraic	Algebraic	Non-algebraic
Constructing an Equation	-	Constructing an Equation	-
8	0	7	0

Upon the participants' categorization of Task 4, five out of eight participants who classified the task as algebraic emphasized the presence of an unknown and collecting like terms in the pre-interviews (see Table 4. 39). In fact, four participants based their reasoning on the presence of an unknown, and one participant focused on collecting like terms in their justifications of algebra. In the post-interviews, similarly, five out of seven participants classified the task as algebraic. Indeed, two out of five participants based their reasoning on the presence of an unknown while one out of five participants based her reasoning on collecting like terms. Unlike the categories in the pre-interviews, one participant made her decision by looking for the presence of performing operations and the other one emphasized the making a generalization in the post-interviews. Three out of eight participants, on the other hand, evaluated Task 4 as non-algebraic in the pre-interviews basing their reasoning on the lack of an equation or an equivalence. Similarly, two out of seven participants, who evaluated Task 4 non-algebraic in the post-interviews, referred to the same justification.

Table 4.39

Participants' categorization of Task 4 in the pre- and post-interviews

Pre-interviews			Post-interviews			
Algebraic		Non-Algebraic	Algebraic			Non-Algebraic
The Presence of an Unknown	Collecting Like Terms	The Lack of an Equation and an Equivalence	The Presence of an Unknown	Collecting Like Terms	Other	The Lack of an Equation and an Equivalence
4	1	3	2	1	2	2

Upon the participants' classification of students' work on Task 4, six out of eight participants evaluated Seçil's solution as algebraic in the pre-interviews (see Table 4.40). Four of these participants focused on the use of collecting like terms by using a representation. In the post-interviews, five out of seven participants evaluated Seçil's solution as an algebraic solution by referring to her use of collecting like terms by using a representation. Two participants, who categorized Seçil's solution as non-algebraic in the pre-interviews, emphasized Seçil's use of a representation. Also, in the post-interviews, two participants evaluated Seçil's solution as non-algebraic by referring to the lack of an equation and the concreteness of Seçil's solution, respectively. In the examination of Gizem's solution, in the pre-interviews, all participants categorized Gizem's solution as an algebraic solution, and six of them referred to her collecting like terms by using symbolizations. The other two participants' responses emphasized the "abstractness" of Gizem's solution. In the post-interviews, six out of seven participants evaluated Gizem's solution as algebraic and emphasized Gizem's collecting like terms by using symbolizations One of these participants additionally mentioned the "abstractness" of Gizem's solution. The remaining participant, who evaluated Gizem’s solution as non-algebraic, based her justification on the lack of an equation in the task.

Table 4.40

Participants’ categorization of Seçil’s solution for Task 4 in the pre- and post-interviews

Seçil’s Solution

Let’s have *x* that much 

I have 4 groups of this,

Then, I add 2 groups of this;

Now I have 6 groups of this, also I add 5;

So, I have 6*x* + 5.

Pre-interview			Post-interview		
Algebraic		Non-algebraic	Algebraic	Non-algebraic	
Collecting Like terms by Using a Representation	Other	Use of a Representation	Collecting Like terms by Using a Representation	The Lack of an Equation	Other
4	2	2	4	2	1

Table 4.41 shows PSMTs’ categorization of Gizem’s solution for Task 4 in the pre- and post- interviews.

Table 4.41

Participants’ categorization of Gizem’s solution for Task 4

Gizem’s Solution

I have 4 groups of *x*. Then I add 2 groups of *x*. Now, I have 6 groups of *x*, so it is 6*x*. Then I add 5, 6*x* + 5.

Pre-interview		Post-interview	
Algebraic	Non-algebraic	Algebraic	Non-algebraic

Table 4.41 (continued)

Collecting Like Terms by Symbolizations	Other	-	Collecting Like Terms by Symbolizations	The Lack of an Equation
6	2	0	6	1

4.3.3 Changes in Middle School Pre-service Mathematics Teachers' Awareness of Students' Possible Solutions

Firstly, the PSMTs' responses to students' possible solutions for Task 1 were examined (see Table 4.42). In the pre-interviews, four out of eight PSMTs and in the post-interviews, six out of seven PSMTs emphasized relational-structural strategy. In addition, seven out of eight participants in pre-interviews and four out of seven participants anticipated relational-computational strategy. Although only two out of eight participants could realize the students' possible misconception regarding the operational understanding of the equal sign in the pre-interviews, six out of seven participants reported that in the post-interviews. Lastly, in the pre-interviews, five out of eight PSMTs reported on a mathematical equivalence mistake, while in the post-interviews, six PSMTs provided it.

Table 4.42

Participants' responses regarding possible student solutions for Task 1

Task 1	Pre-interview	Post-interview
Relational - Computational Strategy	7	4
Relational - Structural Strategy	4	6
Operational Thinking	2	6
Mathematical Equivalence Mistake	5	6
Algebraic Manipulation Mistake	4	2

Secondly, when the participants' responses to possible student responses for Task 2 were examined (see Table 4.43), it was seen that all participants in the pre-interviews and four out of seven participants in the post-interviews indicated anticipated student solutions about the preservation of an equivalence. Additionally, six out of seven participants in the pre-interviews and four out of seven participants in the post-interviews anticipated students solving the second equation.

Table 4.43

Participants' responses regarding possible student solutions for Task 2

Task 2	Pre-interview	Post-interview
Preservation of an Equivalence	6	7
Solving an Equation	7	4
Algebraic Manipulation Mistake	5	6
Other	2	2

Regarding Task 3, six participants referred to using a correct function rule to predict far function values as a possible student strategy in both the pre- and post-interviews (see Table 4.44). Furthermore, three participants in both the pre- and post-interviews expected that students would use a geometric visualization to find the number of people for 100 tables to solve the task. Moreover, in the pre-interviews, three PSMTs anticipated identifying a recursive pattern and using it to predict near data as a possible student solution, this number decreased to one in the post-interviews. Also, in the pre-interviews, five PSMTs emphasized that students could write an incorrect function rule such as multiplying the number of tables by 4 to find the number of people to predict far function values, and this number decreased to four in the post-interviews.

Table 4.44

Participants' responses regarding possible student solutions for Task 3

Task 3	Pre-interview	Post-interview
Using a Correct Function Rule to Predict Far Function Values	6	6
Using an Incorrect Function Rule to Predict Far Function Values	5	4
Identifying a Recursive Pattern and Use it to Predict Near Data	3	1
Using Geometric Visualization to Find the Number of People for 100 Tables	3	3
Other	3	2

Lastly, regarding Task 4, Lastly, six out of eight participants in the pre-interviews, and five out of seven participants in the post-interviews focused on collecting like terms as a possible student solution (see Table 4.45). Also, in the pre-interview five out of seven participants mentioned ignoring like terms, while all participants mentioned it in the post-interviews. Half of the participants in the pre-interviews and three out of seven participants in the post-interviews mentioned that students could interpret *x* as a multiplication sign. Moreover, two out of eight participants in the pre-interviews and one out of seven participants in the post-interviews emphasized assigning a value for the unknown as a possible student solution.

Table 4.45

Participants' responses regarding possible student solutions for Task 4

Task 4	Pre-interview	Post-interview
Collecting Like Terms	6	5

Table 4.45 (continued)

Ignoring Like Terms	5	7
Interpreting *x* as a Multiplication Sign	4	3
Assigning a Value for the Unknown	2	1
Other	2	1

CHAPTER 5

DISCUSSION AND IMPLICATIONS

5.1 Middle School Pre-service Mathematics Teachers’ Awareness of the Underlying Algebraic Structure of a Given Task

When the pre-service teachers' awareness of the underlying algebraic structure of a given task was examined on a task-based basis, it was seen that all the participants stated that the task purpose in Task 1 as relational thinking (seven relational-structural and one relational-computational strategy in the pre-interviews, five relational-structural and two relational-computational strategy in the post-interviews), in Task 2 as the preservation of equivalence, and in Task 4 as collecting like terms in both pre- and post-interviews. When the participants’ responses regarding the purpose of Task 3 were examined, it was observed that the number of the participants who stated constructing an equation or a correspondence relationship through noticing a pattern as a task purpose increased from four to six, from pre- to post-interviews. Additionally, these six participants, in the post-interviews, mentioned making a generalization in their justifications, while only two participants had emphasized it in the pre-interviews. Also, the number of the PSMTs who emphasized noticing a pattern as a task purpose decreased from two to zero, from pre- to the post-interviews. As it can be seen in the presented results above, PSMTs’ awareness of the underlying algebraic structure of a given task was already high especially in Tasks 1, 2, and 4 in the pre-interviews. That is why there might be no remarkable change. The PSMTs were found successful in noticing the underlying algebraic structure of a given task, this might be due to their experiences in a prior course, Measurement and Assessment, which they took in their fourth semester and focused on writing mathematical tasks provided the objectives. This might have affected their awareness about task purposes in a positive way.

In a study conducted by Stephens (2006) which aimed to identify pre-service elementary teachers' awareness of task purposes about relational thinking and equivalence (Tasks 1 and 2), similar results were also observed. PSMTs were found successful in noticing the underlying purposes of the tasks that addressed relational thinking and equivalence.

In Task 3, when pre- and post-interview findings were compared. It was seen that in Task 3, the PSMTs' focus on generalization increased in the post-interviews with respect to their awareness of the underlying algebraic structure of a given task. This might have stemmed from the focus on generalization in the instruction and the textbook.

5.2 Middle School Pre-service Mathematics Teachers' Conceptions of Algebra

5.2.1 PSTMs' responses to "How would you describe what algebra is to someone who has never heard of it before?"

In the pre-interviews, the participants' responses to "How would you describe what algebra is to someone who has never heard of it before?" were analyzed, and these results revealed the categories of the presence of an unknown or an equation and performing operations. However, responses to the same question in the post-interviews revealed "making a generalization" category instead of performing operations. Nevertheless, when the distribution of the participants' responses into these categories was examined, it was seen that the majority of the participants based their algebra conceptions on the presence of an unknown or an equation. Indeed, four participants in the pre-interviews and five participants in the post-interviews referred to presence of an unknown or an equation in their algebra definitions. The PSMTs algebra conceptions seems too narrow given that Kaput (2008) described the Core Aspect B "Algebra as syntactically guided reasoning and actions on generalizations expressed in conventional symbol systems" as mainly focusing on "rule-based actions on symbols" (p. 11). On the other hand, when the other categories were examined, it was seen that two participants in the post-interviews referred to making a generalization, while none of them mentioned it in the pre-interviews. Making a generalization is aligned with Kaput's (2008) Core Aspect A "Algebra as systematically symbolizing generalizations of regularities and constraints" (p. 11). Another encouraging finding was that although three participants in the pre-interviews associated algebra with operations, none of the participants mentioned it in the post-interviews. To sum up, while the increase in their focus on generalization seems to be encouraging, the fact that the majority of PSTMs' conceptions' being related to the presence of an unknown or an equation could be discouraging.

5.2.2 How Did Middle School Pre-service Mathematics Teachers Classify the Tasks and the Related Students' Solutions?

In the pre- and post-interviews, PSMTs' algebra conceptions were also examined through their categorizations of the interview tasks and the related students' solutions. The question "Would you consider this to be an algebra problem?" was asked in each task. Moreover, two different student solutions per task were presented, and they were asked whether the students used algebra or not in their solutions.

When the responses regarding Task 1 categorizations without focusing on algebra and non-algebra were examined, it was observed that four participants based their justification on relational–structural thinking in both interviews. Regarding Task 1, although in the pre- and post- interviews, all participants had clarified the purpose of the task as building relational-structural thinking or relational-computational thinking by referring to the meaning of equivalence, in the task categorization as algebra or not, only three PSMTs in the pre-interview and four PSMTs in the post-interviews used these for their justifications for task categorization as algebra. The rest of the participants' algebra or non-algebra justifications (four in the pre- and two in the post-interviews) who categorized the task by focusing on the presence or absence of an unknown or an equation depended on manipulations of symbols.

The PSMTs justifications for Task 1 student solutions varied. Regarding Burak's solution, half of the participants in the pre-interviews and three out of seven participants in the post-interviews made their algebra categorization by focusing on the presence of a computationally based strategy. On the other hand, it was also seen that the majority of the PSMTs (five PSMTs in pre and six PSTMs in the post-interviews) categorized Nur's solution as algebraic because of her using relational-structural thinking. Looking at both the task and student solution justifications for Task 1, one could say that the PSMTs did not hold consistent conceptions of algebra.

Regarding Task 2, in the pre- and post-interviews, all participants had clarified the purpose of the task as the preservation of equivalence. When their categorizations of Task 2 were examined, it was seen that all participants evaluated Task 2 as algebraic. However, when the participants' justifications were examined, it was seen that only three PSMTs in the pre- and two PSMTs in the post-interviews referred to the preservation of equivalence in their algebra categorizations. On the contrary, five PSMTs in the pre- and four PSMTs in the post-interviews based their algebra justifications on the presence of an unknown or an equation or solving an equation. Therefore, we could see that both in the pre- and post- interviews, the number of PSMTs who focused on the surface features such as the existence of a variable or manipulation of formalism was high. These findings were parallel with the findings of Stephens (2004) in the same tasks, Tasks 1 and 2. Stephens also found that the majority of the PSMTs in her study were found to focus on manipulation of formalism in their justification although they were aware of the task purposes. These findings might give us opportunity to make inferences about PSMTs future classes in terms of focusing on algebra, and one could interpret that the PSMTs may not give importance to build such relational thinking in their classrooms because of not seeing them as algebra.

Regarding the student solutions provided to Task 2, almost all of the PSMTs in the pre-interviews and more than half of the PSMTs in the post-interviews referred to solving an equation in their justifications for Kerem's solution as algebra. Regarding Defne's solution, five PSMTs both in the pre- and post-interviews categorized it as algebra referring to her use of preservation of equivalence. This was again similar to the findings of about Task 1 and Nur's solution in that while the majority of the PSMTs justified Defne's solution as algebra based on her use of the preservation of equivalence, this was not the case in their task justifications for their categorizations of Task 2.

In Task 3, PSMTs made their decisions without any hesitation. This was one of the tasks which the PSMTs stated purposes and algebra justifications for their categorizations were mostly parallel. All participants categorized the task as algebraic in the pre- and post-interviews, and in the pre-interviews, the majority of the PSMTs (five out of eight) based their justifications on constructing an equation or a correspondence relationship, in the post-interviews the majority of them (five out of seven) focused on the relationships within and between variables, and the category of constructing an equation or a correspondence relationship through noticing a pattern was revealed. Also, there was an increase in their focus on generalization from pre to post. In fact, while two participants mentioned making a generalization in the pre-interviews, four participants mentioned it in the post-interviews.

Regarding student solutions provided to Task 3, it was noticed that while the majority of the PSMTs justified Kemal's solution as non-algebra focusing on his awareness of a pattern only in the pre-interviews, two PSMTs in the post-interviews justified their decisions based on his not being able to make a generalization. Regarding Dilay's solution in Task 3, while all PSMTs categorized it as algebra, they also had the same justifications in both pre- and post-interviews that is her construction of an equation. Also, in the pre-interviews, while none of the PSMT mentioned her ability to generalize, three PSMTs mentioned it in the post-interviews. As it is seen, the PSMTs' awareness about making a generalization was noticeably higher in the post-interviews than in the pre-interviews both in task and student solutions justifications for their categories. Actually, the textbook that was mainly followed during the methods of teaching mathematics course focused on making a generalization and asking students “Is it always true?” or “Does it always work?” in several chapters focusing on numbers and algebra. This might have helped PSMTs develop awareness in their conceptions of algebra around making a generalization.

In Task 4, the PSMTs had the most difficulty to make a decision and spent the most time to give a response in the pre- and post-interviews. To remind, all PSMTs stated the task purpose as collecting like terms. Both in the pre- and post-interviews, five PSMTs categorized this task as algebra; however, both in the pre- and post- interviews, only one participant focused on collecting like terms in their justifications as algebra. On the contrary, half of the PSMTs in the pre- and two PSMTs in the post-interviews made their justifications on the presence of an unknown in their categorizations of algebra. Although it is encouraging that the number of PSMTs who focused on surface features decreased in the post-interviews, the ratio (two out of seven) was high. Interestingly, three PSMTs and two PSMTs focused on the lack of an equation or an equivalence in the pre- and post-interviews, respectively, in their categorization of the task as non-algebra. This might have stemmed from PSMTs’ misconceptions around “the lack of closure” issue that is not being able to accept expressions as they are (Kieran, 1981, p. 319). Similarly, as presented in the study conducted by Tanisli and Kose (2013), the pre-service teachers had some misconceptions about the concept of variable. Indeed, one of the misconceptions in their study, that was exemplified by participant responses such as “the symbol *n* does not mean anything since the expression $4n+7$ is not equal to anything” or “it does not represent anything unless there is an equality (p. 15) was parallel to the misconception that few PSMTs held in this study.

When the student solutions provided to Task 4 were examined, it was seen that both Gizem’s and Seçil’s solutions in the pre- and post-interviews were categorized as algebra by the majority of the PSMTs (four, in both the pre- and post-interviews for Seçil and six, in both the pre- and post-interviews for Gizem) referring to their use of collecting like terms. This was in contrast to their task categorization since only one participant focused on collecting like terms as algebra. This inconsistency in their reference points could show that the PSMTs did not have consistent algebra conceptions.

Additionally, three participants during the pre-interviews and five participants during the post-interviews requested to change their past categorizations of algebra and non-algebra, while they were examining the other tasks, or the related student solutions provided to the tasks later. Therefore, there was evidence that the PSMTs conceptions of algebra were unstable during both the pre- and post-interviews.

Besides, in the pre- and post-interviews, when the PSMTs' justifications for their algebra categorizations were examined, it was observed that some of the participants based their reasoning on their views from the linear algebra course, MATH 260, Basic Linear Algebra, which most of them were taking during their fifth semester in the eight-semester teacher education program. The content of the course MATH 260 includes matrix algebra, linear system of equations, and determinants. PSMTs were sometimes found to make their justifications based on the course experiences. For instance, PSMT 1 stated "Linear Algebra (Basic Linear Algebra course) comes to my mind, I think this is an algebra task. Because in algebraic expressions or in a system, we use equations" in the pre-interview while examining Task 2. Similarly, PSMT 6 reported "I think it is an algebraic solution ... I think absolutely it is, I am thinking about the courses that I took, it (course) is abstract" in the post-interview while examining Gizem's solution to Task 4. The reason of why PSMTs might have been influenced more by the mathematics content courses that focused on algebra than the Methods of Teaching Mathematics course could stem from the time that they spent in these. Although the students took the linear algebra course during a semester, they spent only two weeks on the algebra chapter in the Methods of Teaching Mathematics course.

5.3 Middle School Pre-service Mathematics Teachers' Awareness of Students' Possible Solutions

The participants were mostly successful at anticipating students' possible solutions; however, this was not always true about student misconceptions.

Regarding Task 1, four PSMTs in the pre-interviews and six participants in the post-interviews anticipated relational structural thinking strategy as a possible student solution. As another possible student solution, the relational computational thinking strategy was anticipated by seven PSMTs in the pre-interviews and four PSMTs in the post interviews. When these findings were examined, it could be seen that in the post-interviews, PSMTs focused on the structure more than the computation as a possible student solution. Participants were also asked to anticipate incorrect student solutions. The misconception regarding the operational thinking of the equal sign was anticipated by only two PSMTs in pre-interviews, while this was expected by six PSMTs in the post-interviews. Stephens (2006) and Isler and Knuth (2013) also found out that the pre-service teachers were not much familiar with the misconception regarding the operational thinking of the equal sign. In this study, the two PSMTs, who anticipated the operational thinking of the equal sign, indicated that the instructor mentioned this misconception in the previous year in Instructional Principles and Methods course, which they were asked to design lesson plans considering student expected solutions in various learning areas, and the PSMTs also referred to their own one-to-one teaching experiences. These two PSMTs asked a similar question to the student they tutored last year, and they shared that their students answered the question by making this operational mistake. For instance, PSMT 8 said "I experienced it with my student. When the instructor mentioned (operational thinking of the equal sign) last year, I was surprised a lot. She mentioned that the students directly write the result by adding these two (addends) after the equal sign. I asked it to my 4^{th} grade student, and he added these (showed 37 and 54 in Task 1) two and wrote the result. Without caring about this one (showing 55 in Task 1)." This finding could show that the teacher education programs should include courses which offers PSMTs opportunities to experience a variety of correct and incorrect student strategies including misconceptions to broaden their algebraic knowledge of content and students as also stated by several researchers (e.g., Didiş Kabar & Amaç, 2018; Gökkurt, Şahin, & Soylu, 2016; Tanisli & Kose, 2013; Tirosh, 2000). In addition, when the post-interview findings were examined, it was seen that six out of seven participants anticipated the operational thinking of the equal sign as a possible student solution. The remarkable increase in their awareness about this misconception could stem from the instructor's and textbook's emphasis on the issue during the method courses.

Another important point was that while majority of the participants were found to realize the algebraic structure of the tasks when asked the teacher's aims as discussed in the first part, some participants did not anticipate these answers as a possible student solution. For instance, although in Task 1, seven out of eight participants stated the aim of the task as building a relational-structural thinking, only four participants anticipated that the students could respond in this way in the pre-interviews. As Stephens (2008) stated, this could stem from PSMTs' narrow algebra conceptions, that is why even though they had identified the algebraic structure of a task, they might have not evaluated it as a solution method.

Regarding Task 2, PSMTs' anticipation of the preservation of equivalence were found quite successful (six PSMTs in pre-interviews and seven PSMTs in post-interviews). Similar to Task 1, even though all participants in both pre- and post-interviews clarified the purpose of the task to have students realize the preservation of the equivalence, some of the PSMTs (two in the pre-interviews and one in the post-interviews) did not anticipate this as a possible student solution. As another possible student solution, seven participants in the pre-interviews and four participants in the post interviews anticipated solving an equation. In this matter, when the findings across Tasks 1 and 2 were examined, it could be seen that the PSMTs' anticipations depended more on a computational thinking than structural thinking in the pre-interviews. Stephens (2006) working with the pre-service teachers in Tasks 1 and 2 at the beginning of a methods course also found that the PSMTs focused more on computational thinking than structural thinking. However, this situation changed in the post-interviews.

Indeed, regarding Task 1, six participants anticipated relational structural thinking while four PSMTs anticipated relational computational thinking. Regarding Task 2 four participants anticipated solving an equation, while seven PSMTs anticipated the preservation of equivalence in the post- interviews. The differences between pre- and post-interviews could have stemmed from the focus on relational thinking in the textbook and instruction.

Regarding Task 3, six PSMTs anticipated using a correct function rule to predict far function values, and also three PSMTs anticipated using geometric visualization to find the number of people for 100 tables in both the pre- and post-interviews. As an incorrect solution, five PSMTs in the pre-interviews and four PSMTs in the post-interviews anticipated using an incorrect function rule to predict far function values. As it can be seen, there was almost no variation in the PSMTs' responses regarding the anticipation of student solutions for Task 3 as this task might have been a familiar "algebra" task to them since it involved variables and equation.

Regarding Task 4, six participants in the pre-interviews and five participants in the post-interviews anticipated collecting like terms. As an incorrect solution, five PSMTs in the pre-interviews and seven PSMTs in the post-interviews anticipated ignoring like terms. Also, several PSMTs anticipated interpreting x as a multiplication sign (four in the pre-interviews and three in the post-interviews) or assigning a value for the unknown (two in the pre-interviews and one in the post-interviews), the numbers did not change much between pre- and post-interviews.

As it seen, participants were found mostly aware of the possible student solutions. The PSMTs' awareness about possible student solutions could stem from the course, Instructional Principles and Methods, that they took in their third semester. In this course the PSMTs were asked to design lesson plans considering expected student solutions in various learning areas.

5.4 Implications

In this part, the implications of the present study will be presented. As it is indicated above, the two weeks focus in the Methods of Teaching Mathematics (MoTM) course on algebraic thinking might have not been enough to broaden the PSMTs' conceptions of algebra. Even though the PSMTs were found successful on addressing the aims of the task and anticipating student solutions mostly, their conceptions were majorly related to the traditional symbol manipulation aspect of algebra. In order to help the PSMTs broaden and transform their conceptions of algebra, in relation to the knowledge of content and students specific to algebra, the algebraic thinking should be handled as a course instead of a chapter in the teacher education programs. As also suggested by Gökkurt, Şahin, and Soylu (2016), teacher education programs do not offer courses which might enable pre-service teachers to have enough experiences to develop their PCK. As an alternative solution, Tanışlı and Köse (2013) suggested that teacher educations programs could offer more elective courses which aim to broaden pre-service teachers' PCK of algebra especially knowledge of content and students to be more familiar with students' misconceptions.

Indeed, in this study, during the pre- and post-interviews, some participants were found to change their task categorizations when they were shown student solutions provided to the tasks. Therefore, designing the algebra chapter by focusing on a task-based lesson addressing the big ideas of algebra that are equivalence and equations, generalized arithmetic, functional thinking, variable, and quantitative reasoning (Blanton et al., 2011) which might also include different student solutions and discussion of them in terms of the "algebraic" nature might help PSMTs broaden their conceptions of algebra. As mentioned by Thompson (1992), teacher conceptions are resistant to change and as argued by Thompson (1992) and Carpenter et al. (1989), facing with tasks in their teacher education program is a helpful way to broaden conceptions of teachers and preservice teachers. Designing the algebra chapter by focusing on a task-based lesson addressing the big ideas of algebra may also broaden the PSMTs' thinking about students' possible correct and incorrect solutions and how to approach the students to help develop their algebraic thinking. As also suggested by Didiş-Kabar and Amaç (2018), teacher education programs should supply both theoretical and practical education which might provide an opportunity to PSMTs to experience different student solutions and thinking ways. Likewise, Gökkurt et al. (2016) suggested that the Methods of Teaching Mathematics courses should be redesigned to help pre-service teachers have more experiences about students' misconceptions and instructional ways to overcome these misconceptions. When all suggestions by the national and international researchers taken into consideration, it could be recommended that the new course "Teaching Algebra" which is required for preservice teachers in their sixth term by the Council of Higher Education (2018) should include a variety of algebra tasks and corresponding students solutions. The teacher educators should give importance to develop pre-service teachers' algebra conceptions besides developing their knowledge of student thinking including their common difficulties and misconceptions. The instructor of the course can make use of the tasks and the student solutions that were used in the present study in their courses.

Some recommendations for future studies can be made in light of this study. In order to examine the PSMTs' conceptions of algebra, in addition to exposing them to various tasks and student solutions, they could be asked to design a lesson plan that focuses on algebra both in elementary and in middle school. In that way, it could be seen what kind of tasks the PSMTs include in the lesson plans, what student solutions they anticipate, what questions they ask, which can provide evidence about their algebra conceptions. Additionally, in the interviews of this study, the participants were asked to categorize student solutions as algebraic or not, but in the post-interviews, for instance, more student solutions were categorized as "algebraic." Therefore, additionally the PSMTs can be asked "Which student solution seems more algebraic to you and why?" to understand their conceptions deeper.

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APPENDICES

APPENDIX A: SYLLABI OF MoTM I AND MoTM II COURSES

Methods of Teaching Mathematics I Section 2

Course Description:
This course focuses on the basic concepts of school mathematics and how they are taught. More specifically MoTM I includes a study of techniques, materials, strategies, and current research used in the teaching of mathematical concepts to elementary and middle grade students. Students will study contemporary approaches in teaching mathematics and recent curriculum changes. They will develop an awareness for the professional resources, materials, technology, and information available for teachers; prepare unit and lesson plans with related assessment procedures on a variety of topics.

Course Objectives:

- Understand the basic concepts related to school mathematics
- Understand the basic concepts and recognize connections among mathematical ideas in elementary mathematics curriculum
- Prepare and present plans for mathematics instruction that utilize different teaching methods.
- Use a variety of resources for mathematics teachers (e.g., websites, publications)
- Understand the misconceptions related to school mathematics
- Recognize connections among mathematical ideas and other disciplines
- Use representations to organize, record, and communicate mathematical ideas
- Apply a variety of appropriate strategies to solve problems
- Analyze mathematical thinking of other classmates
- Be self-confident in teaching mathematics
- Have positive attitude toward teaching mathematics.
- Be motivated to teach mathematics

Course Community:
My intent and expectation is to fully include all students in this course. Please let me know if you need any accommodations to allow you to fully participate. We are committed to creating a dynamic, diverse and welcoming learning environment for all students and has a non-discrimination policy that reflects this philosophy. Disrespectful behaviors or comments addressed towards any group or individual are unacceptable in this class.

Course Principles:

We are a community of learners. The process of learning requires curiosity courage, determination, honesty, humility, and humor. I expect us to support and encourage each other in our learning.

Ideas, not individuals, are open to critique. We all have opinions and ideas, some of which we hold or believe in strongly. As we are all here to learn from each other, we must all contribute to the establishment and maintenance of a safe, social environment that allows us as participants to engage critically with ideas but avoids attacking or disparaging individuals.

Questions represent an opportunity to learn. Sometimes students hesitate to ask questions because they fear they may "sound dumb" or go against what is thought to be the opinion of the majority. Questions, however, can be an indication of one's engagement with the subject matter. Do not self-censor; your questions may lead to an improved understanding for the whole class.

Participants assume responsibility for their own learning and success. This is another way of a somewhat trite (but true) expression, "you get out of this what you put into it." If there is any way I can be helpful to your learning, please email, call, or visit me. I am committed to your becoming an excellent mathematics teacher and will do whatever I can help you reach that goal.

Required Textbook:

Van De Walle, J. A., Karp, K. S., & Bay-Williams J. M. (2013). *Elementary and middle school mathematics: Teaching developmentally* (8th ed.). Boston, MA: Pearson Education, Inc.

Additional Resources:

Books

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Talim ve Terbiye Kurulu Başkanlığı (2013). Ortaokul matematik dersi öğretim programı, 5-8. sınıflar. Retrieved from http://ttkb.meb.gov.tr/www/guncellenen-ogretim-programlari/icerik/151.

Talim ve Terbiye Kurulu Başkanlığı (2015). İlkokul matematik dersi öğretim programı, 1-4. sınıflar. Retrieved from http://ttkb.meb.gov.tr/www/ogretim-programlari/icerik/72.

Talim ve Terbiye Kurulu Başkanlığı (2017). Matematik dersi öğretim programı (İlkokul ve ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. Sınıflar). Retrieved from http://mufredat.meb.gov.tr/Dosyalar/2017717175055350-02MATEMATIK%201-8.pdf

Journals

Teaching Children Mathematics (TCM), Mathematics Teaching in the Middle School (MTMS)

There will be some readings that are assigned from the resources above and beyond. These readings will be provided to you in PDF or in paper form.

Course Requirements:

Attendance and Class Participation – 10%

Your participation in our class activities and discussions is extremely important, not only for your own learning but also for the learning of others. You are expected to be in class on time and participate in every class. If it is absolutely necessary for you to miss a class, please request permission from me on email, in advance, giving your reasons. The first absence results in a 1-point deduction; two absences result in an additional 2-point deduction (a total of 3 points deducted). Missing four sessions will result in a drop from the class.

If you do miss a class meeting:

(1) Talk in detail with at least one classmate about what we did during class.

Preferably talk with two classmates, so you get more than one perspective.
(2) Check Moodle for all new postings, emails, etc.
(3) If you are absent during a class meeting where a HW check is occurring, send your work electronically via email. For full credit, send it by the beginning of the class meeting. You are responsible for any and all information that occurred during your absence.

Reading Reflections and Homework Assignments – 15%
In this assignment, you are required to read assigned chapters and articles and come to class prepared to discuss/reflect and write the main points of the reading(s) and/or submit the homework assignment for the week. During the semester you will have 5 homework assignments.

In-class Activities/Presentations – 10%
During the last six weeks, you will be asked to prepare activities related to the topic and implement them during the class hour. You will be asked to work in groups.

Quizzes – 10%
There will be several unannounced quizzes during the semester. These will be related to the readings, homework assignments and/or class discussions.

Midterm Exam – 15%
You will have a midterm exam that addresses the book chapters, class discussions and presentations. The midterm exam will be held in the week of November 6th.

Project – 15%
In this assignment, you will be asked to interview a student to see what the student knew in order to solve a task and what was learned as a result of doing the task. The task might help uncover any misconceptions the student might have, which we will discuss during the semester. You will develop and submit the task and the questions you will ask during the interview in advance for feedback, incorporate the feedback from one of the peers and/or the instructor, conduct the interview and write up a summary of the interview and your interpretation of student thinking. Depending on the permissions, you can audio- or videotape the interview. Further details about this assignment and the evaluation will be provided in the class.

Final Exam — 25%
There will be a final examination that assesses the knowledge of the topics studies in the course. The date of the final exam will be announced.

Evaluation Criteria:

Course requirement	Due date	% of final grade

Attendance and Class Participation	Weekly	10%
Reading Reflections and Homework	Weekly	15%
Quizzes	Unannounced	10%
In-class Activities/Presentations	For the last six weeks	10%
Project	January 5th (last day of classes)	15%
Midterm Exam	In the week of November 6th	15%
Final Exam	To be announced	25%

NOTE: Class schedules, policies, and assignments are subject to change as the instructors deem appropriate

Email and Moodle

We post assignments, documents shown in class, URLs, some readings, questions about the readings, and other important information regularly to Moodle

(1) You are expected to check Moodle and email regularly.
(2) To contact us, please send an email to the instructors (see emails on the first page).

Course Policies

Tardiness
Students are expected to arrive promptly and come prepared for class by having completed the readings and assignments due that day. On-time arrival to each class session is required. We have a short time together, and we will need to use all of it to accomplish the goals in the course. Tardiness not only is detrimental to the person who is late (who will miss important information and/or activities); it is disruptive to others. However, I know that occasionally life intervenes. Please inform me if you know you have an unavoidable conflict and will be late to class.

Late Work
Each day an assignment is turned in late, students will lose 10% of the possible points. The 10% late work penalty is applied starting immediately after the specified due date and time. Please make sure you save your work frequently and keep backup copies of your files. Computer accidents, while very unfortunate, are not an acceptable excuse to avoid penalties for late work.

Lost Assignments
You should always keep a copy of every computer file or paper you turn in until your work is graded and you have received your course grade.

Cell phones, newspapers, etc.
Please turn cell phones off during class. Please do not send text messages during class. If I have to ask you twice not to text, you will accrue an absence. If you have an unusual circumstance, please inform me. Also, please do not bring newspapers or other outside reading materials to class—we have plenty to do together to keep us busy!

Academic Ethics:
All assignments you hand in should be the result of your effort only. Academic dishonesty, including any form of cheating and plagiarism will not be tolerated and will result in failure of the course and/or formal disciplinary proceedings usually resulting in suspension or dismissal. Cheating includes but is not limited to such acts as; offering or receiving unpermitted assistance in the exams, using any type of unauthorized written material during the exams, handing in any part or all of someone else's work as your own, copying from the Internet. Plagiarism is a specific form of cheating. It means using someone else's work without giving credit. Plagiarism is a literary theft. Therefore, you have to acknowledge the sources you use in your assignments.
You have to adapt the texts/activities you use AND provide the appropriate citations and references.

NOTE: I expect every student to read the assigned readings prior to class hour. The assigned readings are given below. Additional papers will be assigned according to the topics.

Tentative Schedule:

Week	Date	Topic	Readings/Assignments Due
1	Oct 3	Introduction to the course	Syllabus, overview of the class materials
	Oct 5	Teaching Mathematics in the 21st Century	Van De Walle Chapter 1

2	Oct 10	Exploring What It Means to Know and Do Mathematics	Van de Walle Chapter 2
	Oct 12		
3	Oct 17	Teaching Through Problem Solving	Van de Walle Chapter 3
	Oct 19		
4	Oct 24	Planning in the Problem-Based Classroom	Van de Walle Chapter 4
	Oct 26		
5	Oct 31	Building Assessment into Instruction	Van de Walle Chapter 5
	Nov 2		
6	Nov 7	Teaching Mathematics Equitably to All Children	Van de Walle Chapter 6

	Nov 9	Using Technological Tools to Teach Mathematics	Van de Walle Chapter 7
The midterm exam will be held in the week of November 6th. The date and time will be announced.			
7	Nov 14	Overview of Elementary Turkish Mathematics Curriculum, Grades 5-8	Grades 1-4 & 5-8 Turkish Elementary Mathematics Curriculum (See websites at the end of the syllabus)
	Nov 16		
8	Nov 21	Developing Early Number Concepts and Number Sense	Van de Walle Chapter 8
	Nov 23		
9	Nov 28	Developing Meanings for the Operations	Van de Walle Chapter 9
	Nov 30		
10	Dec 5	Helping Students Master the Basic Facts	Van de Walle Chapter 10

	Dec 7	Developing Whole-Number and Place-Value Concepts	Van de Walle Chapter 11
11	Dec 12	Developing Strategies for Addition and Subtraction Computation	Van de Walle Chapter 12
	Dec 14		
12	Dec 19		
	Dec 21		
13	Dec 26	Developing Strategies for Multiplication and Division Computation	Van de Walle Chapter 13 PROJECT IS DUE
	Dec 28		
14	Jan 2		
	Jan 4		

The date and time for the final exam will be announced.

NOTE: Class schedules, policies, and assignments are subject to change as the instructors deem appropriate.

Methods of Teaching Mathematics II Section 2

Course Description:
Mathematics problems and mathematical problem solving. Importance of mathematical problem solving, categorization of mathematics problems, purposes and processes of problem solving. Teaching how to solve word problems and ill-structured mathematics problems. Teaching whole numbers, operations with whole numbers, fractions, ratio and proportion, data analysis, and geometry in elementary school. Problem-based learning. Lesson planning, presentation and evaluation.

Course Objectives:
Students completing this course will have a critical understanding of teaching and learning processes in Numbers/Algebra/Geometry/Measurement/Probability and Data Analysis learning areas.

- Construct the concepts and connections among mathematical ideas in related mathematics learning areas effectively.
- Analyze students' misconceptions related to mathematics learning areas.
- Use representations to organize, record, and communicate mathematical ideas.
- Design and implement plans and activities for mathematics instruction with different teaching strategies specific to mathematics including problem solving approaches.
- Design and employ materials and resources for effective teaching of school mathematics.
- Participating in productive classroom discourse including teaching activities and mathematical ideas.
- Express interest, self-confidence, and motivation in teaching mathematics.

Course Community:
My intent and expectation is to fully include all students in this course. Please let me know if you need any accommodations to allow you to fully participate. We are committed to creating a dynamic, diverse and welcoming learning environment for all students and has a non-discrimination policy that reflects this philosophy. Disrespectful behaviors or comments addressed towards any group or individual are unacceptable in this class.

Course Principles:

We are a community of learners. The process of learning requires curiosity courage, determination, honesty, humility, and humor. I expect us to support and encourage each other in our learning.

Ideas, not individuals, are open to critique. We all have opinions and ideas, some of which we hold or believe in strongly. As we are all here to learn from each other, we must all contribute to the establishment and maintenance of a safe, social environment that allows us as participants to engage critically with ideas but avoids attacking or disparaging individuals.

Questions represent an opportunity to learn. Sometimes students hesitate to ask questions because they fear they may "sound dumb" or go against what is thought to be the opinion of the majority. Questions, however, can be an indication of one's engagement with the subject matter. Do not self-censor; your questions may lead to an improved understanding for the whole class.

Participants assume responsibility for their own learning and success. This is another way of a somewhat trite (but true) expression, “you get out of this what you put into it.” If there is any way I can be helpful to your learning, please email, call, or visit me. I am committed to your becoming an excellent mathematics teacher and will do whatever I can help you reach that goal.

Required Textbook:
Van De Walle, J. A., Karp, K. S., & Bay-Williams J. M. (2013). *Elementary and middle school mathematics: Teaching developmentally* (8th ed.). Boston, MA: Pearson Education, Inc.

Additional Resources:
Books

Atatürk, Gazi Mustafa Kemal (2015). *Geometri*. Ankara: Türk Dil Kurumu Yayınları.

Lannin, J. K., Ellis, A. B., & Elliott, R. (2011). *Developing essential understanding of mathematical reasoning for teaching mathematics in prekindergarten-grade 8*. Reston, VA: NCTM.

Lobato, J., Ellis, A., Charles, R., & Zbiek, R. M. (2010). *Developing essential understanding of ratios, proportions, and proportional reasoning for teaching mathematics in grades 6-8*. Reston, VA: NCTM.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Hillsdale, NJ: Erlbaum.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: NCTM.

Talim ve Terbiye Kurulu Başkanlığı (2017). Matematik dersi öğretim programı (İlkokul ve ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. Sınıflar). Retrieved from http://mufredat.meb.gov.tr/Dosyalar/2017717175055350-02MATEMATIK%201-8.pdf

Journals

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There will be some readings that are assigned from the resources above and beyond. These readings will be provided to you in PDF or in paper form.

Course Requirements:

Attendance and Class Participation – 10%

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If you do miss a class meeting:

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(2) Check Moodle for all new postings, emails, etc.

(3) If you are absent during a class meeting where a HW check is occurring, send your work electronically via email. For full credit, send it by the beginning of the class meeting.

You are responsible for any and all information that occurred during your absence.

Reading Reflections and Homework Assignments – 15%

In this assignment, you are required to read assigned chapters and articles and come to class prepared to discuss/reflect and write the main points of the reading(s) and/or submit the homework assignment for the week. Please do not use Turkish characters in your file names and name them as Surname_RQ1 as an example for the first reading question.

In-class Activities – 15%

During the last six weeks, you will be asked to prepare activities related to the topic and implement them during the class hour. You will be asked to work in groups.

Quizzes – 5%

There will be several unannounced quizzes during the semester. These will be related to the readings, homework assignments and/or class discussions.

Midterm Exam – 15%

You will have a midterm exam that addresses the book chapters, class discussions and presentations. The midterm exam will be held in the week of March 27th. Further information will be provided.

Project 1 – 10%

For this assignment, you will be asked to relate the mathematics we talk in the class to your campus environment and potential future school environments. Further details about the project will be provided in the class.

Project 2 – 10%

In this assignment, you will be asked to interview a student or two to see what the student knew in order to solve a task and what was learned as a result of doing the task. The task might help uncover any misconceptions the student might have, which we will discuss during the semester. You will develop and submit the task and the questions you will ask during the interview in advance for feedback, incorporate the feedback from one of the peers and/or the instructor, conduct the interview and write up a summary of the interview and your interpretation of student thinking. Depending on the permissions, you can audio- or videotape the interview.

Final Exam — 20%

There will be a final examination that assesses the knowledge of the topics studies in the course. The date of the final exam will be announced.

Evaluation Criteria:

Course requirement	Due date	% of final grade
Attendance and Participation	Weekly	10%
Reading Reflections and Homework	Weekly	15%
Quizzes	Unannounced	5%
In-class Activities	Weekly	15%
Project 1	March 15th	10%
Project 2	May 10th	10%

Midterm Exam	In the week of March 27^{th}	15%
Final Exam	To be announced	20%

NOTE: Class schedules, policies, and assignments are subject to change as the instructors deem appropriate

Email and Moodle

We post assignments, documents shown in class, URLs, some readings, questions about the readings, and other important information regularly to Moodle.

(3) You are expected to check Moodle and email regularly.
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use AND provide the appropriate citations and references. Please check the Academic Integrity Guide for Students on the Moodle.

<u>NOTE: I expect every student to read the assigned readings prior to class hour and be ready for discussion.</u>

Week	Date	Topic	Readings/Assignments Due
1	Feb 13	Introduction to the course	Syllabus, your expectations, my expectations
	Feb 15	Algebraic Thinking: Generalization, Patterns, and Functions	Van de Walle Chapter 14
2	Feb 20	Algebraic Thinking: Generalization, Patterns, and Functions	Van de Walle Chapter 14
	Feb 22		
3	Feb 27	Developing Fraction Concepts	Van de Walle Chapter 15
	March 1		
4	March 6	Developing Strategies for Fraction Computation	Van de Walle Chapter 16
	March 8		
5	March 13	Developing Concepts of Decimals and Percents	Van de Walle Chapter 17
	March 15		*Project 1 is due March 15th*
6	March 20	Proportional Reasoning	Van de Walle Chapter 18
	March 22		

The midterm exam will be held in the week of March 27th. The date and time will be announced.			
7	March 27	Developing Measurement Concepts	Van de Walle Chapter 19
	March 29		
8	April 3		
	April 5		
9	April 10	Geometric Thinking and Geometric Concepts	Van de Walle Chapter 20
	April 12		
10	April 17		
	April 19		
11	April 24	Developing Concepts of Data Analysis	Van de Walle Chapter 21
	April 26		
12	May 1 (no class)	Exploring Concepts of Probability	Van de Walle Chapter 22
	May 3		
13	May 8	Developing Concepts of Exponents, Integer, and Real Numbers	Van de Walle Chapter 23 *Project 2 is due May 10th*
	May 10		
14	May 15		
	May 17		

The date and time for the final exam will be announced.

NOTE: Class schedules, policies, and assignments are subject to change as the instructors deem appropriate.

APPENDIX B: APPROVAL OF THE UNIVERSITY HUMAN SUBJECTS ETHICS COMMITTEE

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

DUMLUPINAR BULVARI 06800
ÇANKAYA ANKARA/TURKEY
T: +90 312 210 22 91
F: +90 312 210 79 59
Sayı: 28620816 / 746
www.ueam.metu.edu.tr

07 KASIM 2017

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Yrd.Doç.Dr. Işıl İŞLER ;

Danışmanlığını yaptığınız yüksek lisans öğrencisi Burcu ALAPALA'nın **"İlköğretim Matematik Öğretmen Adaylarının Erken Cebir Hakkındaki Görüşlerinin İncelenmesi"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek gerekli onay **2017-EGT-163** protokol numarası ile **06.11.2017 – 31.12.2018** tarihleri arasında geçerli olmak üzere verilmiştir.

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Ş. Halil TURAN
Başkan V

Prof. Dr. Ayhan SOL
Üye

Prof. Dr. Ayhan Gürbüz DEMİR
Üye

BULUNAMADI
Doç. Dr. Yaşar KONDAKÇI
Üye

Doç. Dr. Zana ÇITAK
Üye

Yrd. Doç. Dr. Pınar KAYGAN
Üye

Yrd. Doç. Dr. Emre SELÇUK
Üye

APPENDIX C: INTERVIEW PROTOCOL

BÖLÜM I

DEMOGRAFİK BİLGİ FORMU

Cinsiyet: Kadın ☐ Erkek ☐

Yaş:

Özel Öğretim Yöntemleri I dersini daha önce aldınız mı?

Matematik eğitiminden aldığınız seçmeli dersler nelerdir?

Öğretmenlik ile ilgili bir tecrübeniz var mı? Varsa bahseder misiniz?

BÖLÜM II

1. Cebirin ne olduğunu daha önce hiç duymamış birine nasıl tanımlarsınız?

2. Aşağıdaki soruların bir cebir sorusu olup olmadığına karar veriniz.
   a) Boş kutu yerine hangi sayı gelmelidir?
   37 + 54 = ☐ + 55

   - Sizce bir öğretmen öğrencilerine böyle bir soruyu neden sorar?
   - Cebir sorusu: ☐
   - Cebir sorusu değil ☐
   - Bu sonuca nasıl ulaştınız/Bu kararı nasıl verdiniz?
   - Öğrencilerinizden hangi doğru veya yanlış cevapları vermelerini beklerdiniz? Neden?

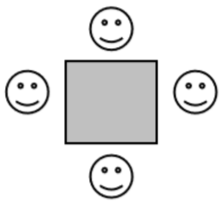
b)

> 2*n* + 15 = 31 denkleminin çözümünde *n*=8'dir.
>
> 2*n* + 15 – 9 = 31 – 9 denkleminin çözümü nedir?

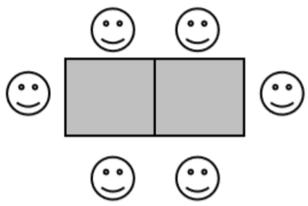
- Sizce bir öğretmen öğrencilerine böyle bir soruyu neden sorar?
- Cebir sorusu:
- Cebir sorusu değil
- Bu sonuca nasıl ulaştınız/Bu kararı nasıl verdiniz?
- Öğrencilerinizden hangi doğru veya yanlış cevapları vermelerini beklerdiniz? Neden?

c) Nehir doğumgünü partisine arkadaşlarını davet ediyor. Kare şeklindeki masaların etrafında her arkadaşı için oturacak bir yerin olduğundan emin olmak istiyor.

Dört kişiyi bir masanın etrafına şekildeki gibi oturtabiliyor.



Eğer bu masaya bir masa daha eklerse 6 kişi oturtabiliyor.



Eğer Nehir 100 masayı yan yana koyarsa kaç arkadaşını oturtabilir?

- Sizce bir öğretmen öğrencilerine böyle bir soruyu neden sorar?
- Cebir sorusu:
- Cebir sorusu değil:
- Bu sonuca nasıl ulaştınız/Bu kararı nasıl verdiniz?
- Öğrencilerinizden hangi doğru veya yanlış cevapları vermelerini beklerdiniz? Neden?

d) 5+4x+2x ifadesini en sade şekilde yazınız.

- Sizce bir öğretmen öğrencilerine böyle bir soruyu neden sorar?
- Cebir sorusu: ☐
- Cebir sorusu değil: ☐
- Bu sonuca nasıl ulaştınız/Bu kararı nasıl verdiniz?
- Öğrencilerinizden hangi doğru veya yanlış cevapları vermelerini beklerdiniz? Neden?

BÖLÜM III

3. Aşağıdaki öğrenci çözümlerinin cebirsel olup olmadığına karar veriniz.

a) Boş kutu yerine hangi sayı gelmelidir?

37 + 54 = ☐ + 55

Burak'ın çözümü:

Boş kutu yerine 36 gelmelidir, çünkü 37 artı 54, 91'e eşittir. Bu sebeple benim ne ile 55'i toplarsam 91 olacağını bulmam gerekir. 36 ve 55'in toplamı 91'dir, bu sebeple cevap 36'dır.

Nur'un çözümü:

Boş kutu yerine 36 gelmelidir. 55, 54'den bir fazladır, bu sebeple boş kutudaki sayı 37'den bir eksik olmalıdır.Bu sebeple cevap 36'dır.

Burak'ın çözümü:

- Cebirsel bir çözümdür ☐
- Cebirsel bir çözüm değildir: ☐
- Bu sonuca nasıl ulaştınız/ Bu kararı nasıl verdiniz?

Nur'un çözümü:

- Cebirsel bir çözümdür: ☐
- Cebirsel bir çözüm değildir: ☐
- Bu sonuca nasıl ulaştınız/Bu kararı nasıl verdiniz?

b)

$2n + 15 = 31$ denkleminin çözümünde $n=8$'dir.

$2n + 15 - 9 = 31 - 9$ denkleminin çözümü nedir?

Kerem'in çözümü:

$$2n + 15 - 9 = 31 - 9$$

$$2n + 6 = 22$$

$$-6 \quad -6$$

$$\frac{2n}{2} = \frac{16}{2}$$

$$n = 8$$

Defne'nin çözümü:

n=8 olarak aynı şekilde kalır, çünkü iki taraftan da aynı sayıyı çıkarıyoruz.

Kerem'in çözümü:

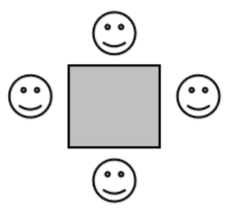
- Cebirsel bir çözümdür: ☐
- Cebirsel bir çözüm değildir: ☐
- Bu sonuca nasıl ulaştınız/ Bu kararı nasıl verdiniz?

Defne'nin çözümü:

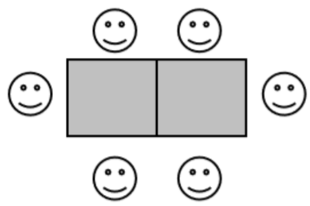
- Cebirsel bir çözümdür: ☐
- Cebirsel bir çözüm değildir: ☐
- Bu sonuca nasıl ulaştınız/Bu kararı

c) Nehir doğumgünü partisine arkadaşlarını davet ediyor. Kare şeklindeki masaların etrafında her arkadaşı için oturacak bir yerin olduğundan emin olmak istiyor.

Dört kişiyi bir masanın etrafına eklerse 6 kişi oturtabiliyor.



Eğer bu masaya bir masa daha şekildeki gibi oturtabiliyor.



Eğer Nehir 100 masayı yan yana koyarsa kaç arkadaşını oturtabilir?

<table>
<tr>
<td>Kemal’in çözümü:

Kişi sayısının bulunduğu sütun ikişer ikişer artarak gidiyor. Eğer masa sayısını 100’e kadar artırırsam oturacak kişi sayısı 202 olur.
<table>
<tr><td>Masa Sayısı</td><td>Kişi Sayısı</td></tr>
<tr><td>1</td><td>4</td></tr>
<tr><td>2</td><td>6</td></tr>
<tr><td>3</td><td>8</td></tr>
<tr><td>4</td><td>10</td></tr>
<tr><td>5</td><td>12</td></tr>
<tr><td>6</td><td>14</td></tr>
<tr><td>7</td><td>16</td></tr>
<tr><td>8</td><td>18</td></tr>
<tr><td>9</td><td>20</td></tr>
<tr><td>10</td><td>22</td></tr>
<tr><td>.</td><td>.</td></tr>
<tr><td>.</td><td>.</td></tr>
<tr><td>.</td><td>.</td></tr>
<tr><td>99</td><td>200</td></tr>
<tr><td>100</td><td>202</td></tr>
</table>

*Kemal tüm tabloyu dolduruyor.</td>
<td>Dilay’ın çözümü:

Kişi sayısı masa sayısının iki katıdan iki fazladır. Kural:
$2n + 2 = m$
n = masa sayısı
m = kişi sayısı

100 masa olduğunda;
$2 \times 100 + 2 = 202$ kişi oturabilir.</td>
</tr>
</table>

Kemal’in çözümü:

- Cebirsel bir çözümdür: ☐
- Cebirsel bir çözüm değildir: ☐
- Bu sonuca nasıl ulaştınız/Bu kararı nasıl verdiniz?

Dilay’ın çözümü:

- Cebirsel bir çözümdür: ☐
- Cebirsel bir çözüm değildir: ☐

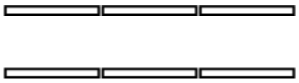
- Bu sonuca nasıl ulaştınız/Bu kararı nasıl verdiniz?

d) 5+4x+2x ifadesini en sade şekilde yazınız.

Seçil'in çözümü:

Bu x kadar olsun *,*
Elimde bundan 4 tane var,

Daha sonra bunlardan 2 tane daha ekliyorum;



Şimdi elimde bunlardan 6 tane var, bir de 5 ekliyorum;

 *+ 5*

Yani elimde 6x+5 oldu.

Gizem'in çözümü:

Elimde 4 tane x var. Daha sonra 2 tane daha x ekliyorum. Şimdi elimde 6 tane x var, yani 6x. Bir de 5 ekliyorum, 6x+5.

Seçil'in çözümü:

- Cebirsel bir çözümdür: ☐
- Cebirsel bir çözüm değildir: ☐
- Bu sonuca nasıl ulaştınız/ Bu kararı nasıl verdiniz?

Gizem'nin çözümü:

- Cebirsel bir çözümdür: ☐
- Cebirsel bir çözüm değildir: ☐
- Bu sonuca nasıl ulastınız/Bu kararı nasıl

APPENDIX D: TURKISH SUMMARY \ TÜRKÇE ÖZET

ORTAOKUL MATEMATİK ÖĞRETMEN ADAYLARININ CEBİR HAKKINDAKİ ALGILARININ VE SORU AMACI VE ÖĞRENCİ ÇÖZÜMLERİ HAKKINDAKI BİLGİLERİNİN İNCELENMESİ

GİRİŞ

Romberg ve Kaput (1999), 21. yüzyılın daha derin bir matematik anlayışına sahip insanlara ihtiyaç duyduğunu belirtmiştir. Kaput (1999) ise cebiri yüksek matematiğe geçişte bir kapı olarak tanımlamıştır. Aynı zamanda Kaput (2008) dünya çapında okullarda öğretilen cebirin sembol manipülasyonuna dayandığını ve cebirin ne olduğunun ona nasıl yaklaştığımıza bağlı olduğunu ileri sürmüştür.

Birçok araştırmacı (örn., Blanton & Kaput, 2011; Carpenter, Franke, & Levi, 2003; Ryan & Williams, 2007) cebirsel düşünmenin erken yaşlardan başlayarak, aritmetik düşünce ile birlikte geliştirilmesi gerektiğini savunmuştur. Araştırmacılar (Cai & Knuth, 2011; Carpenter vd., 2003), sembol manipülasyonuna odaklanmanın ve aritmetik ile cebirin birbirinden ayrılmasının, öğrencilerin sofistike matematik anlayışları geliştirmelerini önlediğini belirtmişlerdir. Kaput (1999) cebirin, okullarda gerçek yaşam ve matematiksel fikirlerle bir bağlantı kurmadan cebirsel ifadeleri sadeleştirmek, denklemleri çözmek için bazı prosedürleri takip etmek olarak öğretildiğini iddia etmiştir. Okullarda cebir algımızı daha derin ve anlamlı matematiksel ve uygulamalı bağlantılar ile geliştiren bir öğretime ihtiyacımız vardır (Kaput, 2008). Blanton ve

Kaput (2005) öğretmenlerin derslerde cebirsel düşünceyi geliştirmek için kilit nokta olduklarını belirtmişlerdir.

Öğretmenler, öğrencilerin cebir algılarını geliştirmek için matematiksel süreçlere ve ilişkisel düşünceye önem vermelidir. Öğretmenlerin "cebirselleştirme" ("algebrafication") stratejileri, Blanton ve Kaput'a (2005, s. 71) göre üç ana yönüyle özetlenebilir: öğretim materyalleri, öğrencilerin cebirsel düşünüşünü keşfetme ve destekleme ve son olarak, cebirsel düşünmeyi teşvik eden bir sınıf kültürü ve öğretim uygulamaları oluşturma. Ek olarak, birçok çalışma (örn., Blanton & Kaput, 2004; Blanton vd., 2015; Carpenter & Levi, 2000), öğrencilerin ilişkilere odaklanmaya yönlendirildiklerinde ve matematiksel fikirleri tartıştıklarında genellemeler yapabildikleri ve ilişkisel düşünebildiklerini ortaya çıkarmıştır. Bu nedenle, öğretmenlerin "cebirselleştirme" ("algebraization") (Cai & Knuth, 2011, s. viii) becerileri öğrencilerin cebirsel düşüncesini teşvik etmenin kilit noktasıdır.

Matematik öğretmen adaylarının cebir algıları ve cebir ile ilgili olarak sahip oldukları pedagojik alan ve öğrenci bilgilerine odaklanmak, öğretmen adaylarının gelecek yıllarındaki derslerinde cebir anlamında neye önem verecekleri ve neye odaklanacakları konusunda çıkarımda bulunma fırsatı verir. Bu alanda bugüne kadar öğretmen adayları ile yapılan çok az çalışma vardır ve bunlar çoğunlukla denklik ve denklemlere ve değişkenlere odaklanmıştır (örn. Didiş Kabar & Amaç, 2018; Gökkurt, Şahin & Soylu 2016; Stephens, 2006; Tanisli & Kose, 2013). Ancak bu çalışmalar, Özel Öğretim Yöntemleri dersinin bir parçası olan cebir konusunun öğretmen adaylarının cebir algıları ve cebir alanındaki pedagojik alan bilgileri üzerinde nasıl bir etkiye sahip olacağına odaklanmamıştır.

Bu çalışma, alanyazındaki bu boşluğa odaklanmakta ve ortaokul öğretmen adaylarının verilen bir sorunun barındırdığı cebirsel amaç hakkındaki farkındalıkları, cebir kavramları ve öğrencilerin olası çözümlerine yönelik beklentileri ve son olarak da İlköğretim Matematik Öğretmenliği programının üçüncü yılında yer alan Özel Öğretim Yöntemleri dersindeki cebir konusundan sonra bu üç alandaki değişimlerine ilişkin genel bir çerçeve çizmeye çalışmıştır.

Bu ders kapsamında cebire odaklanılan yaklaşık iki hafta ders kitabını takip ederek diğer konuların (örn., sayılar, geometri) öğretimi bölümlerdeki gibi işlenmiştir. Dolayısıyla, cebire odaklanılan haftalar deneysel bir çalışma olarak tasarlanmamış, devam etmekte olan Özel Öğretim Yöntemleri dersinin bir parçası olarak verilmiştir.

Araştırma Soruları

Bu çalışma, İlköğretim Matematik Öğretmenliği programının üçüncü yılında olan ve 2017-2018 akademik yılı Sonbahar ve İlkbahar dönemlerinde Ankara'da bir devlet üniversitesinde Özel Öğretim Yöntemleri derslerine kayıtlı olan ortaokul matematik öğretmen adayları ile yürütülmüştür. Bu çalışma, aşağıdaki araştırma sorularını cevaplamaya odaklanmıştır:

1. Ortaokul matematik öğretmen adaylarının verilen bir sorunun cebirsel amacı hakkındaki farkındalıkları nasıldır?
2. Ortaokul matematik öğretmen adaylarının cebir algıları nelerdir?
3. Ortaokul matematik öğretmen adaylarının olası öğrenci çözümleri hakkındaki farkındalıkları nelerdir?
4. Ortaokul matematik öğretmen adaylarının verilen bir sorunun cebirsel amacı hakkındaki farkındalıkları, cebir algıları ve olası öğrenci çözümleri hakkındaki farkındalıkları Özel Öğretim Yöntemleri dersine katıldıktan sonra nasıl değişir?

ALANYAZIN TARAMASI

Alanyazın taraması üç ana başlık altında incelenmiştir. İlk olarak kuramsal çerçeve, ikinci olarak ilkokul ve ortaokul öğrencilerinin eşitlik ve denklemler, fonksiyonel düşünme ve değişkenler üzerine düşünme ve kavram yanılgıları, üçüncü olarak ise öğretmenlerin öğrencilerin cebirsel düşünmeleri ve kavram yanılgıları üzerine bilgilerine odaklanan çalışmalar özetlenmiştir.

Kuramsal Çerçeve

Bu çalışmada iki farklı kuramsal çerçeve kullanılmıştır. İlk olarak, öğretmen adaylarının cebir algılarını incelemek amacıyla Kaput'un (2008) Cebirsel Akıl Yürütme kuramsal çerçevesi kullanılmıştır. İkinci olarak, öğretmen adaylarının pedagojik alan ve öğrenci bilgilerinin, verilen bir sorunun amacını fark etme hakkındaki farkındalıkları ve olası öğrenci çözümleri üzerine farkındalıklarını değerlendirmek için ise Matematik Öğretmek İçin Gerekli Bilgi (Ball, Thames, & Phelps, 2008) kuramsal çerçevesi kullanılmıştır.

İlkokul ve Ortaokul Öğrencilerinin Cebirsel Düşünme ve Kavram Yanılgıları

Öğretmenler öğrencilerine yardımcı olabilmek için onların farklı düşünme biçimlerinin, olası çözüm yöntemlerinin ve kavram yanılgılarının farkında olmalıdırlar. Bu bölümde öğrencilerin cebirsel düşünmelerini konu alan ulusal ve uluslararası çalışmalar özetlenmiştir.

İlkokul ve Ortaokul Öğrencilerinin Zorlukları ve Kavram Yanılgıları. Öğrencilerde sıklıkla karşılaşılan hatalardan birisi öğrencilerin eşit işaretini "cevap", "toplam" olarak yorumlamalarıdır (Blanton vd., 2011; McNeil & Alibali, 2005; Yaman, Toluk, & Olkun, 2003).

Öğrenciler değişken kavramında da birçok kavram yanılgısına sahiplerdir. Örneğin, "işlem yaparken değişkenleri (harfleri) dikkate almama" (Soylu, 2008, s. 1) ("Letter Ignored," Küchemann, 1978, s. 25) bunlardan birisidir. Diğer kavram yanılgılarından birisi ise öğrencilerin eşit işareti bulundurmayan cebirsel ifadeleri eksik kabul etmesidir ("acceptance of lack of closure," Küchemann, 1978, s. 25). Bu yanılgıda öğrenciler eşit işaretinin diğer tarafında bir sayı var gibi davranmaktadırlar (Örn., eşit işaretinin diğer tarafında "0" olduğunu düşünerek eşitlik çözmek). Diğer bir yanılgı ise öğrencilerin "harflerin alfabetik sıralamada olduğu gibi sayısal konum belirttikleri" (Akkaya & Durmuş, 2006, s. 3) ("substitution" Ryan & Williams, 2007, s. 108) yanılgısına düşmesidir. Bu yanılgıda öğrenciler verilen bir bilinmeyen harf yerine spesifik bir rakam koyma eğilimi göstermektedirler. Örneğin, alfabedeki sıralamadan dolayı *a*'ya 1, *b*'ye 2 değerini verme gibi. Bir başka yanılgı ise öğrencilerin bilinmeyen olarak kullanılan *x* işaretini çarpma işareti olarak yorumlamalarıdır. Örneğin, "5*x,* 5 kere olarak okunabilir" (Ryan & Williams, 2007, s. 108). Soylu (2008), tarafından yapılan bir çalışmada ise belirtilen bu kavram yanılgıları dışında öğrencilerin "değişkenleri belli harflerle sınırlandırma" yaptıkları görülmüştür (s. 1). Örneğin, sorularda *h, m, y* gibi sembolizasyonlar kullanılmasına rağmen öğrencilerin çözümlerde bu sembolleri kullanmak yerine *x* kullandıkları görülmüştür.

Fonksiyonlar, öğrencilerin cebirsel bir anlayış geliştirebilmeleri için önemli bir adımdır (Blanton vd., 2011). Blanton ve Kaput'a (2004) göre örüntüler anlamlı bir fonksiyonel düşünmeye geçiş olarak kullanılabilir, fakat sadece yinelemeli örüntülere odaklanmak öğrencilerin derin fonksiyonel düşünmelerini önleyebilir. 3., 4. ve 5. sınıflarla yapılan bir çalışmada (Isler vd., 2015), öğrencilerin birlikte değişimden ve değişkenler arasındaki ilişkiden çok yinelemeli örüntüye odaklandıkları görülmüştür.

Özetlemek gerekirse, ilkokul ve ortaokul öğrencilerinin temel cebirsel kavramlar üzerinde çeşitli zorluklar ve kavram yanılgılarına sahip oldukları görülmüştür.

İlkokul ve Ortaokul Öğrencilerinin Cebirsel Düşünmeleri. Eşitlik ve denklemler ve fonksiyonel düşünme değişken kavramını içinde barındırdığı için bu alandaki çalışmalar fonksiyonel düşünme ve eşitlik ve denklemler başlıkları altında incelenmiştir.

Bu alanda yapılan çalışmalar incelendiğinde (Blanton & Kaput, 2004; Isler vd., 2015; Ng, 2018; Tanışlı, 2011) öğrencileri ilişkisel düşünmeye (relational thinking) yönlendirecek dersler içeren deneysel çalışmalar yapıldığında ya da onları ilişkisel düşünmeye yönlendirecek iyi tasarlanmış aktiviteler sunulduğunda öğrencilerin, ilkokul seviyelerinden itibaren, cebirsel düşünmede başarılı oldukları gözlenmiştir.

Eşitlik ve denklemler konusunda yapılan deneysel çalışmalar incelendiğinde (Blanton vd., 2015; Carpenter & Levi, 2000; Kızıltoprak & Köse, 2017), öğrencilerin ön değerlendirmelerde çeşitli zorluklara ve kavram yanılgılarına sahip olmalarına rağmen cebirsel düşünmeleri doğru-yanlış ve boşluk soruları ile desteklendiği ve öğrencileri ilişkisel düşünmeye k yönlendirdiği görülmüştür.

Öğretmenlerin Öğrencilerin Cebirsel Düşünmeleri ve Kavram Yanılgıları Üzerine Bilgileri

Öğrenciler çeşitli düşünme şekillerine sahiptirler ve öğretmenler bu farklı düşünme şekillerinin farkında olmalıdırlar (Ball vd., 2008, Lannin, Barker, & Townsend, 2006; Yetkin, 2003). Öğretmenlerin, öğrencilerin zorluklarının ve kavram yanılgılarının farkında olmaları, anlamlı bir öğrenme sürecine olumlu katkılar yapar (Yetkin, 2003).

Öğretmen adayları ile yapılan çeşitli çalışmalar (örn., Dede & Peker, 2007; Didiş Kabar & Amaç, 2018; Gökkurt vd., 2016; Stephens, 2006; Tanisli & Kose, 2013) göstermiştir ki, öğretmen adayları öğrencilerin kavram yanılgılarını tanımlamakta ve olası çözüm yollarını öngörmekte zorluklar yaşamaktadırlar.

İlkokul ve Ortaokul Matematik Dersi Öğretim Programı'nda Cebir

Matematik dersi öğretim programı incelendiğinde (MEB, 2018) cebir öğrenme alanı ile ilk defa 6. sınıf seviyesinde karşılaşılmaktadır. Fakat 1.sınıftan başlayarak 5. sınıf dahil olan kazanımlar incelendiğinde öğretim programında cebir olarak adlandırılmasa da cebir ile ilişkili birçok kazanım olduğu görülmektedir. 1. ve 4. sınıf arası cebir ile ilişkili kazanımları görmek için Tablo 2.1'e, 5. ve 8. sınıf seviyeleri arası cebir ile ilişkili kazanımları görmek için Tablo 2.2'ye bakınız.

YÖNTEM

Bu çalışmada nitel araştırma yöntemlerinden durum çalışması (case study) (Creswell, 2007) kullanılmıştır. Araştırmanın odak noktası durumun kendisi değildir, durum sadece genel bir çerçeve çizip öngörü oluşturabilmek amacıyla bir araç olarak kullanıldığı için bu araştırma araçsal durum çalışması (instrumental qualitative study) (Stake, 2005) olarak düşünülebilir. Öğretmen adaylarının verilen bir sorunun cebirsel amacı hakkındaki farkındalıkları, cebir algıları, olası öğrenci çözümleri hakkındaki farkındalıkları ve Özel Öğretim Yöntemleri dersindeki cebirsel düşünme bölümünden sonra bu üç alandaki değişimlerini öğrenmek için ön ve son görüşmeler yapılmıştır.

Bölümün İçeriği

İlköğretim Matematik Öğretmenliği programı araştırmanın yapıldığı Türkiye'deki bir devlet üniversitesinin Matematik ve Fen Bilimleri Eğitimi bölümünün altındaki beş programdan biridir. Öğretmen adaylarının bu programdan mezun olabilmesi için sekiz dönemlik öğretmen eğitimi programını tamamlamaları gerekmektedir. Program boyunca öğretmen adaylarının alması gereken dersleri Tablo 3.1'de bulabilirsiniz. Bu programdan mezun olan öğretmen adayları ortaokul 5 ve 8. sınıflar arasında görev yapmaktadırlar.

Dersin İçeriği

İlköğretim Matematik Öğretmenliği programına kayıtlı olan öğrenciler, Tablo 3.1' de görüldüğü gibi beşinci dönemlerinde "Özel Öğretim Yöntemleri I" ve altıncı dönemlerinde "Özel Öğretim Yöntemleri II" dersini almışladır. Bu dersler haftada dört saat olmak üzere verilmiş ve "Özel Öğretim Yöntemleri I" dersi "Özel Öğretim Yöntemleri II" dersinin ön koşulu olarak tanımlanmıştır. Bu dersler hem teori hem pratiğe odaklanmış ve ana kaynak olarak "İlkokul ve Ortaokul Matematiği: Gelişimsel Yaklaşımla Öğretim" (Van de Walle, Karp, & Bay-Williams, 2013) kitabı takip edilmiştir. Bu dersin ana öğrenme alanlarını sayılar, cebir, geometri, ölçme, olasılık ve veri analizi oluşturmuştur. Bu öğrenme alanlarının birçoğunun içeriğinde cebirsel bağlantılardan bahsedilmesine rağmen, kitapta özel olarak "cebirsel düşünme" olarak ayrılmış bir bölüm bulunmaktadır. Bu bölüme yaklaşık olarak iki buçuk hafta ayrılmıştır.

Sınıf Ortamının İçeriği

"Özel Öğretim Yöntemleri I" dersine 6'sı erkek, 18'i kadın olmak üzere 25 öğrenci, "Özel Öğretim Yöntemleri II" dersine ise 6'sı erkek 19'u kadın olmak üzere 26 öğrenci kayıt olmuştur. Derslerde grup tartışmalarına ve çalışmalarına destek olmak, ders materyallerini hazırlamaya yardımcı olmak ve ihtiyaç halinde ders hocasına ve öğrencilere yardım edebilmek için bir araştırma görevlisi de çoğunlukla derslere katılmıştır.

Dersin hocası öğrenme ortamına önem verdiği için derslerini grup çalışmalarına, ikili ve sınıf tartışmalarına dayalı bir şekilde işlemiştir. Her dersin başında bir önceki bilgileri hatırlatmak amacıyla sınıf paylaşımı yapılmıştır. Ayrıca yeni konuya geçmeden önce, konu ilgili kısa bir video ya da ısınma etkinliği ile derse giriş yapılmıştır ve ders sonlarında genellikle sınıfça yapılan bir özet ile dersler sonlandırılmıştır.

Derslere gelmeden önce öğrencilerin ilgili bölümü okuması ve bununla ilgili belirli haftalarda ödevler yapmaları beklenmiştir. Aynı zamanda öğrenciler bazı bireysel ve grup ödevlerinden sorumlu olmuşlardır. Örneğin iki ya da üç kişiden oluşan her grup, hazırladıkları aktiviteleri sınıf ortamında uygularlar. Derslerin detayları için izlencelere Ek A'da bakınız.

Cebirsel Düşünme Bölümü

Bu çalışma deneysel bir çalışma olmadığı için, bu haftalarda dersin hocası diğer haftalarda olduğu gibi (örn. sayıların öğretimi) dersin akışını genel olarak kitaptaki etkinliklere ve sorulara odaklanarak devam ettirmiştir.

Katılımcılar

Katılımcılar İlköğretim Matematik Öğretmenliği programında üçüncü sınıf öğrencisi olup, "Özel Öğretim Yöntemleri I" dersine kayıtlı olan öğretmen adayları arasından seçilmiştir. Katılımcıları seçmek için amaçlı örnekleme yöntemi kullanılmıştır. Yapılan çalışma nitel bir çalışma olduğu için öğretmen adaylarından konuşkan olan ve yarı yapılandırılmış görüşmede daha çok bilgi vermesi muhtemel olan adaylar seçilmiştir. Bu amaca dayalı olarak sınıftaki kadın-erkek sayısı dağılımı ile orantılı olacak şekilde iki erkek ve altı kadın katılımcı olmak üzere toplamda sekiz katılımcı seçilmiştir. "Özel Öğretim Yöntemleri I" dersi tamamlandıktan sonra bir katılımcı ERASMUS programına katılmak amacıyla yurt dışına gittiği için son görüşmeler kalan yedi katılımcı ile tamamlanmıştır.

Veri Toplama Aracı

Çalışmada kullanılan yarı yapılandırılmış veri toplama aracı (bakınız Ek C), alan yazındaki ilgili kaynaklardan yararlanarak oluşturulmuştur. Veri toplama aracının ilk bölümü demografik bilgiler içeren sorulardan (cinsiyet, yaş gibi) oluşur. İkinci bölüm ise, "Cebirin ne olduğunu daha önce hiç duymamış birine nasıl tanımlarsınız?" (Stephens, 2004) sorusu ile başlar. Bu bölüm dört matematik sorusunun altında, bu sorunun amacını soran, sorunun cebirsel bir soru olup olmadığını soran ve bu sorulara cevap olarak verilen olası öğrenci çözümlerini soran alt soruları içermektedir. Bu matematik soruları, eşitlik ve denklemler, fonksiyonel düşünme ve değişkenler olmak üzere üç ana fikre odaklanmıştır (Şekil 3.2'ye bakınız). Eşitlik ve denkleme odaklanmış olan birinci ve ikinci matematik soruları öğretmen adaylarının cebir algılarını ölçen bir doktora tezinden (Stephens, 2004), üçüncü matematik sorusu ise fonksiyonel düşünmeye odaklanmış olup öğrencilerin cebirsel düşünmelerini ölçen bir çalışmadan (Blanton vd., 2015) ve son soru ise değişkenlere odaklanmış olup öğrencilerin cebir hakkındaki kavram yanılgıları üzerine yapılan bir çalışmadan (Dede & Peker, 2007) alınmıştır. Veri toplama aracının üçüncü ve son bölümünde ise, ikinci bölümde verilen dört matematik sorusunun her birine karşılık verilen iki farklı öğrenci çözümü yer almıştır. Bu bölümde öğretmen adaylarından verilen öğrenci çözümlerinin cebirsel olup olmadığına karar vermeleri istenilmiştir. Her bir öğrenci çözümü öğretmen adaylarının cebir algılarını daha detaylı anlayabilmek için matematik sorularının seçildiği alan yazındaki çalışmalardan seçilmiştir (bakınız Şekil 3.3). Birinci soruya karşılık verilen öğrenci çözümlerinden birisi ilişkisel-yapısal (relational-structural) bir çözüme dayanırken diğeri ilişkisel-hesaplamaya (relational-computational) dayanmaktadır. İkinci soruda ise bir öğrenci çözümü eşitliğin korunumuna dayanırken, diğeri denklem çözmeye dayanmaktadır. Birinci ve ikinci soruya karşılık verilen öğrenci çözümleri birinci ve ikinci sorunun alındığı doktora tezinden (Stephens, 2004) herhangi bir değişiklik yapılmadan alınıp Türkçeye çevrilmiştir. Üçüncü soruda verilen bir öğrenci çözümü denklem yazmayı, diğeri yinelemeli örüntüyü (recursive pattern) kullanarak tablo oluşturmayı içermektedir. Üçüncü soruya karşılık verilen öğrenci çözümleri de soru ile aynı kaynaktan, Blanton vd. (2015)'ten doğrudan alınıp Türkçeye çevrilmiştir. Son sorudaki iki farklı öğrenci çözümleri ise sırasıyla benzer terimleri gösterim ve sembolle toplamayı içermektedir. Bir öğrenci çözümü dördüncü soru aynı ile kaynaktan, Dede ve Peker (2007)'den alırken, diğer öğrenci çözümü (Seçil'in çözümü) araştırmacı tarafından geliştirilmiştir.

Veri Analizi

Veri analizinin ilk aşaması olarak ön görüşme ve son görüşme kayıtlarının deşifreleri yapılmıştır. Veri analizine başlarken ilk aşama olarak ilk kodlama ("initial coding") (Saldaña, 2009, s. 81) kullanılmıştır. İlk kodlamada alan yazından gelen kodlar (örn., denklem çözme, eşitliğin korunumu vb.) ve veriden çıkan kodlar kullanılmıştır. Veri analizinin ikinci aşamasında ise odak kodlaması ("focused coding") (Charmaz, 2006, p.57) kullanılmıştır.

Verilerin analizi sırasında, kodlama güvenilirliği sağlamak için verilerin %20'si (ön ve son görüşmelerde ikişer görüşme) rastgele seçilerek araştırmalarında nitel araştırma yöntemlerine ve cebire odaklanmış olan doktoralı bir matematik eğitimcisi tarafından bağımsız olarak kodlanmıştır. Kodlayıcılar arasındaki güvenirlik %80'e ulaşana kadar kodlama devam etmiş, ardından karşılıklı kodlar tartışılıp uzlaşmaya varılmıştır. Ortaya çıkan değişiklikler tüm analize yansıtılmıştır.

BULGULAR

Bulgular, araştırma sorularına paralel olacak şekilde üç farklı bölümde incelenmiştir: ortaokul matematik öğretmen adaylarının soru amacı hakkındaki bilgileri, ortaokul matematik öğretmen adaylarının cebir algıları, ortaokul matematik öğretmen adaylarının olası öğrenci çözümleri hakkındaki bilgileri ve ortaokul matematik öğretmen adaylarının verilen bir sorunun cebirsel amacı hakkındaki farkındalıkları, cebir algıları ve olası öğrenci çözümleri hakkındaki farkındalıklarının Özel Öğretim Yöntemleri dersindeki cebirsel düşünme bölümünden sonraki değişimi.

Bulgulara göre, öğretmen adayları verilen dört matematik probleminin de amacını tahmin etmede hem ön görüşmede hem de son görüşmede başarılı bulunmuşladır.

Öğretmen adaylarının cebir algılarını anlayabilmek için öncelikle "Cebirin ne olduğunu daha önce hiç duymamış birine nasıl tanımlarsınız?" (Stephens, 2004) sorusu sorulmuştur. Bu soruya verilen cevaplar ön görüşmede bilinmeyenin veya denklemin varlığı ve işlem yapma kategorilerini ortaya çıkarırken, son görüşmede verilen cevaplar bilinmeyenin veya denklemin varlığı ve genelleme yapma kategorilerini ortaya çıkarmıştır. Daha sonraki bölümde öğretmen adaylarının cebir algıları verilen dört matematik sorusu ve bunlara karşılık verilen öğrenci cevapları bazında incelenerek detaylandırılmaya çalışılmıştır.

Eşitlik ve denklemler ve değişkenlere odaklanan birinci soruya verilen ön ve son görüşmedeki cevaplar incelendiğinde (bakınız Tablo 4.1 ve Tablo 4.13) öğretmen adaylarının çoğunun bu soruyu cebirsel olarak değerlendirdiği görülmüştür. Öğretmen adaylarının birinci soruyu cebirsel ya da değil olarak değerlendirmelerine bakmaksızın değerlendirme sebeplerine bakıldığında ön görüşmede katılımcıların yarısının bir bilinmeyen veya denklemin varlığı veya yokluğuna odaklanırken diğer yarısının da ilişkisel düşünmeye odaklandığı görülmüştür (bakınız Tablo 4.30). Son görüşmelerde ise dört katılımcı ilişkisel düşünmeye odaklanırken , iki katılımcı bilinmeyen veya denklemin varlığı veya yokluğuna odaklanmıştır ve kalan bir katılımcı ise sorunun genelleme yapmaya değil, bilinmeyeni bulmaya odaklandığını belirtmiştir (bakınız Tablo 4.30). Bu soruya karşılık gelen öğrenci çözümleri hakkındaki değerlendirmelere bakıldığında ise ilişkisel-hesaplama içeren öğrenci çözümünü (Burak) değerlendiririrken ön görüşmede, son görüşmede üç katılımcı (bakınız Tablo 4.31) çözümün hesaplama içermesine odaklanarak bu çözümü cebirsel olarak değerlendirmişlerdir. İlişkisel-yapısal çözüme dayanan öğrenci çözümü (Nur) hakkındaki verilen cevaplar incelendiğinde ön görüşmelerde beş öğrencinin Nur'un ilişkisel-yapısal çözümüne odaklanarak bu çözümü cebirsel olarak kategorize ettiği, iki katılımcının ise Nur'un çözümünü mantıksal bir çözüm olarak değerlendirerek cebirsel olmayan bir çözüm olarak değerlendirdiği bulunmuştur. Bir katılımcı ise ön görüşmede bilinmeyenin varlığına odaklanarak çözümü cebirsel olarak sınıflandırmıştır (bakınız Tablo 4.31). Son görüşmelerde ise tüm katılımcılar Nur'un çözümünü cebirsel olarak değerlendirmiştir. Katılımcılardan 6'sı gerekçelerinde ilişkisel-yapısal çözüme odaklanırken, bir kişi bilinmeyeni bulmaya odaklanmıştır (bakınız Tablo 4.31).

Eşitlik ve denklem ve bilinmeyenlere odaklanan ikinci soru incelendiğinde, ön görüşmelerde tüm katılımcıların bu soruyu cebirsel olarak değerlendirdiği görülmüştür. Katılımcıların bu sınıflandırmadaki gerekçeleri incelendiğinde, üç kişinin eşitliğin korunumuna, dört kişinin bilinmeyenin veya denklemin varlığına ve bir kişinin de denklem çözümüne vurgu yaptığı görülmüştür (bakınız Tablo 4.33). Son görüşmelere bakıldığında ise bu soruyu dört kişi cebirsel olarak sınıflandırırken, üç kişinin cebirsel olmayan bir soru olarak sınıflandırdığı görülmektedir. Katılımcıların sınıflandırmaları incelendiğinde, son görüşmelerde cebirsel olarak sınıflandırma yapan dört kişiden ikisi eşitliğin korunumundan bahsederken diğer iki kişi bilinmeyenin ya da denklemin varlığından bahsetmiştir. İkinci soruyu cebirsel olarak değerlendirmeyen üç kişiden ikisinin cevabı ise denklem çözümüne odaklanırken kalan bir kişinin cevabı diğer kategorinde kodlanmıştır (bakınız Tablo 4.33). Bu soruya karşılık gelen öğrenci çözümleri için yapılan sınıflandırmalar incelendiğinde denklem çözümüne dayanan öğrenci (Kerem) çözümünü, ön görüşmelerde yedi katılımcı denklem çözümü olmasını öne sürerek cebirsel olarak sınıflandırırken bir katılımcı yine aynı sebeple cebirsel olmayan bir çözüm olarak sınıflandırmıştır (bakınız Tablo 4.34). Son görüşmeler incelendiğinde Kerem'in çözümünü cebirsel olarak değerlendiren dört, cebirsel değil olarak değerlendirenüç kişi de Kerem'in denklem çözdüğünden bahsetmiştir (bakınız Tablo 4.34). Eşitliğin korunumunu içeren öğrenci (Defne) çözümününe yönelik cevaplar incelendiğinde, ön görüşmede beş katılımcı Defne'nin çözümünü eşitliğin korunumunun kullanılmasını belirterek cebirsel olarak sınıflandırırken, üç katılımcı aynı sebeple cebirsel olmayan bir çözüm olarak sınıflandırmıştır. Son görüşmelere bakıldığında da beş katılımcı Defne'nin çözümünü eşitliğin korunumunu kullanmasını vurgulayarak cebirsel olarak sınıflandırırken, iki katılımcı Defne'nin işlem yapmamış olduğunu belirterek çözümünü cebirsel olmayan bir çözüm olarak sınıflandırmıştır.

Fonksiyonel düşünme ve bilinmeyenlere odaklanan üçüncü soruda ise hem ön görüşmede hem de son görüşmede tüm katılımcılar soruyu cebirsel olarak değerlendirmiştir. Ön görüşmede bir katılımcı sınıflandırma sebebi olarak bilinmeyenin varlığını belirtirken diğer yedi katılımcı gerekçelerinde denklem kurma ya da sayılar arasında ilişki kurmaya odaklanmıştır (bakınız Tablo 4.36). Son görüşme sınıflandırılmaları incelendiğinde beş kişinin yinelemeli örüntüyü fark ederek denklem kurma veya ilişki kurmayı belirttiği, iki kişinin ise yalnızca denklem kurma ya da ilişki kurmaya odaklandığı görülmüştür (bakınız Tablo 4.36). Bu soruya karşılık gelen öğrenci cevapları incelendiğinde ön görüşmelerde yinelemeli örüntüyü kullanarak tablo oluşturmayı içeren öğrenci (Kemal) çözümünü bir kişi cebirsel, kalan yedi kişi ise cebirsel olmayan bir çözüm olarak sınıflandırmıştır ve gerekçe olarak her iki grup da Kemal'in örüntüyü fark etmiş olması olarak belirtmiştir. Son görüşmelerde ise bir katılımcı Kemal'in örüntüyü fark ettiğini ama o an için denklemi yazamadığını belirterek bu çözümü cebirsel olarak sınıflandırırken, diğer altı katılımcı cebirsel olmayan bir çözüm olarak sınıflandırma yapmışlardır. Bu altı katılımcıdan üçü Kemal'in sadece yinelemeli örüntüyü fark ettiğinden, iki kişi genelleme yapmadığından ve bir kişi ise işlem yapmadığından bahsetmiştir (bakınız Tablo 4.37). Denklem yazmayı gerektiren diğer öğrenci (Dilay) çözümünü ise hem ön görüşmede hem de son görüşme de katılımcıların hepsi denklem kurmuş olmasını belirterek cebirsel olarak değerlendirmişlerdir (bakınız Tablo 4.37).

Eşitlik ve denklem ve de değişken içeren dördüncü soru incelendiğinde ise ön görüşmelerde sekiz öğrenciden beşi bu soruyu cebirsel olarak sınıflandırırken, üçü cebirsel olmayan bir soru olarak sınıflandırmıştır. Cebirsel olarak sınıflandırma yapan katılımcılardan dördü gerekçe olarak bilinmeyenin varlığından bahsederken, kalan bir katılımcı benzer terimleri bir araya toplamaktan bahsetmiştir. Cebirsel olmayan bir soru olarak sınıflandıran üç katılımcı ise soruda eşitlik veya denklemin olmamasından bahsetmişlerdir. Son görüşme bulguları incelendiğinde ise beş katılımcının soruyu cebirsel, iki katılımcının ise cebirsel değil olarak sınıflandırdığı görülmüştür. Cebirsel bir soru olarak sınıflandıran katılımcılardan ikisi bilinmeyenin varlığını, bir kişi benzer terimlerin bir araya toplanmasını, bir kişi işlem yapmanın varlığını ve son bir kişi de genelleme yapmayı öne sürmüştür. Cebirsel olarak sınıflandırmayan iki kişi ise denklem ve eşitliğin eksikliğinden bahsetmiştir (bakınız Tablo 4.39). Bu soru ile ilgili öğrenci çözümleri incelendiğinde ön görüşmelerde benzer terimleri gösterim kullanarak toplayan öğrenci (Seçil) çözümünü sekiz öğrenciden altısı cebirsel olarak sınıflandırırken, ikisi cebirsel değil diye sınıflandırmıştır. Altı kişiden dördü gerekçe olarak benzer terimleri gösterimle toplamayı ifade ederken, bir kişi bilinmeyenin varlığını, kalan bir kişi de Seçil'in modelleme kullanmış olmasını ifade etmiştir (bakınız Tablo 4.40). Son görüşmeler incelendiğinde ise yedi kişiden dördünün Seçil'in çözümünü cebirsel olarak sınıflandırırken, üçünün cebirsel değil olarak sınıflandırdığı görülmüştür. Cebirsel olarak sınıflandıran dört katılımcı da benzer terimleri gösterimle toplamayı vurgulamıştır. Cebirsel olmayan bir çözüm olarak değerlendiren üç katılımcıdan ikisi denklemin eksikliğinden, bir kişi ise Seçil'in çözümünün somutluğundan bahsetmiştir (bakınız Tablo 4.40). Benzer terimleri semboller kullanarak toplama yapan diğer öğrenci (Gizem) çözümüne verilen cevaplar incelendiğinde, ön görüşmede tüm katılımcıların bu çözümü cebirsel olarak değerlendirdiği bulunmuştur. Bu katılımcılardan altısı Gizem'in benzer terimleri toplarken sembol kullanmış olmasından bahsederken, diğer iki katılımcı bu çözümün soyutluğuna vurgu yapmıştır (bakınız Tablo 4.41). Son görüşme sonuçları incelendiğinde, Gizem'in çözümünü yedi kişiden altısının cebirsel, bir kişinin ise cebirsel değil olarak sınıflandırdığı görülmüştür. Bu çözümü cebirsel olarak değerlendiren altı kişi benzer terimleri sembol kullanarak toplamadan bahsederken, cebirsel olmayan bir çözüm olarak değerlendiren bir kişi ise denklemin eksikliğine vurgu yapmıştır.

Son olarak öğretmen adaylarının olası öğrenci çözümleri hakkındaki farkındalıkları incelendiğinde öğretmen adaylarının verilen dört matematik sorusundaki olası öğrenci çözümlerini ön ve son görüşmelerde tahmin etmedeki farkındalıklarının genel olarak yüksek olduğu gözlenmiştir. Fakat eşitlik ve denklemlere odaklanan ve eşit işaretinin anlamına vurgu yapan birinci soruda, eşit işaretinin "toplam" olarak algılanmasına yönelik kavram yanılgısı ön görüşmelerde yalnızca iki öğretmen adayı tarafından belirtilirken, bu sayı son görüşmelerde altıya yükselmiştir (bakınız Tablo 4.42).

TARTIŞMA VE ÖNERİLER

Çalışma bulgularında görüldüğü gibi öğretmen adayları verilen soruların amacını tahmin etmede hem ön hem son görüşmelerde başarılı bulunmuşlardır. Öğretmen adaylarının bu farkındalığında üçüncü dönemde alınan Öğretim Yöntem ve Teknikleri ve dördüncü dönemlerinde alınan Ölçme ve Değerlendirme derslerinin etkisi olabileceği düşünülmektedir. Bu derslerde verilen kazanımlara göre matematik sorusu geliştirme, ders planı oluşturma çalışmaları yapılmıştır.

Verilen dört matematik sorusu bazında ve ilgili öğrenci çözümleri bazında öğretmen adaylarının cebir algıları incelendiğinde, öğretmen adaylarının tutarlı bir cebir algısı sergilemedikleri görülmüştür. Ayrıca ön görüşmelerde ve son görüşmelerde ise bazı katılımcılar yaptıkları soru sınıflandırmalarını diğer soruları ya da öğrenci çözümlerini görünce değiştirme talebinde bulunmuşlardır. Bu da öğrencilerin tutarlı bir cebir algısına sahip olmadıklarının bir göstergesidir. Çalışma bulguları göstermektedir ki Özel Öğretim Yöntemleri dersinde odaklanılan iki haftalık cebir öğretimi öğretmen adaylarının cebir algılarını geliştirmek için yeterli bir süre değildir. Öğretmen adaylarının cebir algıları ön ve son görüşmelerde genel olarak geleneksel sembol manipülasyonu ile ilişkili bulunmuştur. Bu da Kaput'un cebirsel düşünme kurumsal çerçevesinden genellemelerin sembol sistemleri ile ifade edilmesi görüşü (Core Aspect B) ile ilişkili bulunmuştur. Aynı zamanda öğretmen adaylarının verilen soru ve çözümleri cebirsel veya değil olarak sınıflandırma yaparken beşinci dönemlerinde aldıkları lineer cebir dersine vurgu yaptıkları görülmüştür. Öğretmen adaylarının karar verme sürecinde Matematik Bölümü'nden aldıkları bu dersten, Özel Öğretim Yöntemleri dersinde işledikleri cebir bölümünden daha çok etkisinde kalmalarının sebebi bu derslerde harcadıkları zaman olabilir. Örneğin, öğretmen adayları sadece alan bilgisi içeren lineer cebir dersini bir dönem boyunca alırken, pedagojik alan bilgisine odaklanan Özel Öğretim Yöntemleri dersinde cebirsel düşünme bölümünü sadece iki haftada işlemektedirler.

Öğretmen adayları, olası öğrenci çözümlerini tahmin etmede genel olarak başarılı bulunmuşlardır. Bunda da yine öğrencilerin üçüncü dönemlerinde aldıkları Öğretim İlke ve Yöntemleri dersinin etkili olabileceği düşünülmektedir. Bu ders kapsamında öğretmen adaylarının olası öğrenci çözümlerini düşünerek ders planları oluşturmaları beklenmektedir. Öğretmen adayları genel olarak olası öğrenci çözümlerini tahmin etmede başarılı bulunmuş olsalar da ilk matematik sorusunda öğrenciler arasında yaygın olarak görülen kavram yanılgısını yani eşit işaretini "toplam" olarak yorumlamayı ön görüşmelerde iki öğretmen adayı belirtmiştir. Bu öğretmen adaylarının bu sorudaki açıklamaları incelendiğinde üçüncü dönemde aldıkları Öğretim İlke ve Yöntemleri dersinde dersi veren hocanın bu konudan bahsettiğini belirttikleri, bu kavram yanılgısını ilginç buldukları için kendi özel ders öğrencilerine aynı soruyu sorup bu kavram yanılgısını gözlemlediklerini belirttikleri görülmüştür. Bu durumdan anlaşılabilir ki öğretmen adaylarının bilgilerinin kalıcı olabilmesi ve içselleştirebilmeleri için onlara doğru ve yanlış çeşitli öğrenci çözümlerini görme imkânı sunularak pedagojik alan bilgilerini derinleştirme imkânı sunulmalıdır, birçok araştırmacı bu konuda benzer önerilerde bulunmuştur (örn., Didiş Kabar & Amaç, 2018; Gökkurt vd., 2016; Tanisli & Kose, 2013).

Çıkarımlar

Öğretmen adaylarının cebir algılarını derinleştirebilmek, cebire yönelik pedagojik alan ve öğrenci bilgilerinin geliştirilebilmesi için cebirsel düşünmenin Özel Öğretim Yöntemleri dersinde sadece bir bölüm olarak işlenmesi yerine tek başına bir ders olarak öğretmen eğitimi programına koyulması daha yararlı olabilir. Bu çalışma aynı zamanda yenilenen İlköğretim Matematik Öğretmenliği programının altıncı dönemine koyulan "Cebir Öğretimi" (YÖK, 2018) dersinin içeriğine yönelik öneriler sunmaktadır.

Bu alanda yapılacak olan gelecek çalışmalara bir öneri olarak, öğretmen adaylarının cebir algılarını daha kapsamlı anlayabilmek ve tanımlayabilmek için öğretmen adaylarından cebirsel kazanımları ele alan ders planları oluşturmaları istenebilir. Ayrıca bu ders planlarını dördüncü sınıftaki uygulama okullarında uygulamaları istenebilir ve bu dersler araştırmacılar tarafından gözlenebilir.

APPENDIX E: TEZ FOTOKOPISI İZİN FORMU

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