VOLATILITY INDEXES AND AN IMPLEMENTATION OF THE TURKISH BIST 30 INDEX

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submitted by **CANER KARAKURT** in partial fulfillment of the requirements for the degree of **Master of Science in Financial Mathematics Department, Middle East Technical University** by,

Prof. Dr. Ömür Uğur
Director, Graduate School of **Applied Mathematics** __________

Prof. Dr. Sevtap Kestel
Head of Department, **Financial Mathematics** __________

Prof. Dr. Ömür Uğur
Supervisor, **Scientific Computing, METU** __________

Examining Committee Members:

Assoc. Prof. Dr. Ümit Aksoy
Mathematics, Atılım University __________

Prof. Dr. Ömür Uğur
Scientific Computing, METU __________

Prof. Dr. Sevtap Kestel
Financial Mathematics, METU __________

Date: __________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: CANER KARAKURT

Signature :

ABSTRACT

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Karakurt, Caner
M.S., Department of Financial Mathematics
Supervisor : Prof. Dr. Ömür Uğur

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In 1993, by representing of CBOE Vix, global financial markets met volatility indexes. In 2003, methodology of the CBOE Vix is updated and it took the form which used today. Day after day, volatility indexes have attracted more and more investors and financial institutions, and soon volatility indexes have succeeded in becoming one of the most followed financial indicators. Following these developments, many countries have introduced their implied volatility indexes by using CBOE Vix methodology or its variations.

Although there are different academic studies and opinions about whether volatility indexes are consistent and successful in reflecting the markets volatility expectation or not. CBOE Vix and its derivatives are the most accepted indicators about reflecting investors' volatility expectation.

In this study, firstly development of the CBOE Vix and the methodology of this index will be explained in detail. Then, other important volatility indexes, that are created using this methodology will be mentioned. Lastly, an implementation of this methodology to Turkish BIST 30 index will be made. The interest rates, needed for calculation of this index will be obtained by Svensson method.

Keywords: Implied volatility index, BIST 30 volatility index, XU030 volatility index, Turkish market volatility index, Svensson curve fitting method

ÖZ

DALGALANMA ENDEKSLERİ VE TÜRKİYE BIST 30 DALGALANMA ENDEKSİ UYARLAMASI

Karakurt, Caner
Yüksek Lisans, Finansal Matematik Bölümü
Tez Yöneticisi : Prof. Dr. Ömür Uğur

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1993 yılında Chicago Opsiyon Borsası'nın CBOE Vix endeksini tanıtmasıyla küresel piyasalar dalgalanma endeksi ile tanışmış oldu. 2003 yılında CBOE Vix'in metodolojisi güncellendi ve hali hazırda kullanılmakta olan dalgalanma endeksi oluşmuş oldu. Dalgalanma endeksleri gün geçtikçe daha çok yatırımcının ve finansal kuruluşun ilgisini çekmeye başladı ve çok geçmeden de en çok takip edilen göstergeler arasına girmeyi başardı. Bu gelişmeleri takiben bir çok ülke, genellikle, CBOE Vix metodolojisini veya bu metodolojinin değişik varyasyonlarını kullanarak kendi dalgalanma endekslerini piyasaya sundu.

Dalgalanma endekslerinin piyasanın dalgalanma beklentilerini yansıtma konusunda ne kadar tutarlı ve başarılı olduğu ile alakalı akademik çalışmalar ve değişik görüşler olmasına karşın, yatırımcıların beklenen risk algısını yansıtan en kabul görmüş göstergeler CBOE Vix endeksi ve bu endeksin türevleridir.

Bu çalışmada öncelikle CBOE Vix'in gelişimi ve endeks metodolojisi detaylı olarak anlatılacak, bu endeksten faydalanarak üretilmiş diğer büyük dalgalanma endekslerine değinilecek ve son olarak da CBOE Vix metodolojisi kullanılarak Türkiye BIST 30 dalgalanma endeksi uyarlaması yapılacaktır. Bu dalgalanma endeksinin hesaplanması sırasında gerekli olan faiz oranları Nelson & Siegel modelinin geliştirilmiş hali olan Svensson modeli ile elde edilecektir.

Anahtar Kelimeler: Öngörülen dalgalanma endeksi, BIST 30 dalgalanma endeksi, XU030 dalgalanma endeksi, Türkiye piyasası dalgalanma endeksi, Svensson eğri tamamlama yöntemi

to my beloved sister

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CHAPTER 1

INTRODUCTION

Volatility is a statistical measure of the dispersion of returns for a given security. Volatility is the most important criterion along with the return of a financial instrument. Intrinsically, investors asking for more information about markets volatility expectations to take turn their investments. There are basically two ways to obtain the volatility of an instrument: Historical and implied volatilities. Historical volatility is of course very important while examining the past price movements of an asset and evaluating whether the asset is a safe haven or not, but the real thing is to have an idea about the future volatility of the asset to be invested. Implied volatility is here to find out the fluctuation of a product in the market, based on the price of that asset or considering the price of a contract written on that instrument. Remarkable studies of Christensen and Prabhala (1998) [18] and Fleming (1998) [25], have shown that implied volatility is superior to historical volatility in predicting realized future volatility. After studies in academic circles, implementations on implied volatility indexes have become more intense all over the world. We can think of index options as a priori of volatility index. For this reason, it is needed to start with index of options to talk about studies on volatility indexes. Most striking studies about index of options are works of Gastineau (1977) [27], Galai (1979) [26], Cox and Rubinstein (1985) [20], Brenner and Galai (1993) [13] and Whaley (1993) [48]. When we look at the earlier stages of the volatility indexes, the derivation of a volatility index, and financial instruments based on that index, first appeared in the studies of Menachem Brenner and Dan Galai in 1986 and described in two academic papers [12, 13].

There were effective instruments for hedging versus general changes in depth market directions, yet there were no efficient instruments for hedging against changes in volatility. For this reason Menachem Brenner and Dan Galai suggested the construction of a volatility index which could be based on equity market, bond market and foreign exchange market. In addition to that, with this way volatility options and volatility futures would be produced on this volatility index. Investors who need hedging instruments against volatility benefit from these developments. Menachem and Brennar's volatility index is called Sigma index. They thought that, in calculation of Sigma index values, options and futures could be used. Besides historical data implied values could be integrated into the calculation. In addition, this index would represent the same role as the market index plays for options and futures on the index.

The huge fluctuations in volatility in course of recent years, particularly since the October crash, have highlighted the requirement of instruments for hedging fluctuations in volatility. Menachem Brennar and Dan Galai proposed the formation of futures and options on a volatility index. Financial specialists could set up long or short positions on volatility by exchanging volatility futures and restrict or extend their volatility positions by utilizing volatility options [12].

As stated, there are two different methods to determine the expected volatility. The first one is to use historical volatility, which consists of standard deviation of historical data from a specific time period. The second one is implied volatility. As indicated in the studies of Christensen and Prabhala (1998) [18] and Fleming (1998) [25], implied volatility is superior to historical volatility when forecasting the future volatility of an asset. Implied volatility can be obtained by Black-Scholes (1973) [8] formula or in other ways. Implied volatility first appeared in studies of Dupire (1994) [15], Neuberger (1994) [39] and Carr and Madan (1998) [16]. Britten-Jones and Neuberger (2000) [14]'s study has a crucial role in model-free implied volatility procedure. They studied under the diffusion assumption, and this does not require an assumption of constant volatility. Jiang and Tian (2005) [30] extended the model-free implied volatility of Britten-Jones and Neuberger to asset price processes with jumps. And they showed that model-free implied volatility is superior to Black-Scholes implied volatility regarding information content.

Meanwhile, Chicago Board Options Exchange (CBOE) retained consultant Robert Whailey in 1992 to develop a tradable stock market volatility index based on index option prices. Following that, in 1993, CBOE introduced the CBOE Volatility Index (Vix), which was originally designed to measure the market's expectation of 30-day volatility implied by S&P 100 index (OEX) option prices. The Vix soon became the premier benchmark for U.S. stock market volatility. The Vix is the first and the most known implied volatility index. Vix is followed closely throughout the world, not only in the U.S, it was regarded as the fear index and it is the most important reference for the volatility of the financial world, and regularly featured in the leading financial publications and business news. Implied volatility indexes are assumed to be the most common and the most reliable indicators for equity index performance in the financial world.

In 2003, the CBOE introduced a more detailed methodology, mostly benefited from the study of Demeterfi et al. (1999a) [21]. It is also important to add, as Jiang and Tian (2005) have shown that Demeterfi et al. (1999a)'s methodology is theoretically equivalent to the model-free implied volatility mentioned in Britten-Jones and Neuberger (2000)'s paper. Working with Goldman Sachs, the CBOE developed further computational methodologies, and changed the underlying CBOE S&P 100 index (OEX) to the CBOE S&P 500 index (SPX). While the first version of the Vix, that was introduced in 1993, was using Black-Scholes implied volatility, the revised version of Vix uses model-free approach in index calculation. Hibbert et al. (2008) [29] showed that in their study, new Vix methodology gives better explanations than previous Vix methodology. First version of the Vix was composed of only 8 at the money put and call OEX options volatility, but the revised version of the Vix uses out of the money put and call options over an extensive variety of strikes. In that respect, model-free implied volatility is superior to Black-Scholes implied volatility too, in addition to this with the new implied volatility index measure it is possible to increase the number of volatility derivatives. With this new methodology, Vix was transformed from a theoretical idea into a commonsense standard for trading and hedging volatility.

On the other hand, if we talk about volatility derivatives, Deutsche Börse was the first exchange to list futures contract on implied volatility in 1998. After about 6 years, on March 2004, CBOE introduced the first exchange-traded Vix futures contract on CBOE Futures Exchange (CFE). Two years later in February 2006, CBOE launched Vix options contracts. Since the launch to the today average daily trading volume of the Vix futures and Vix options reached to 800,000 contracts. In 2008, CBOE spearheaded the utilization of the Vix methodology to appraise expected volatility of a number of goods and foreign currencies. Like, The CBOE Crude Oil ETF Volatility index (OVX), CBOE Gold ETF Volatility index (GVZ) and CBOE EuroCurrency ETF Volatility index (EVZ). In 2014, CBOE upgraded the Vix index to incorporate series of SPX Weeklys. The consideration of SPX Weeklys permits the Vix index to be calculated with S&P 500 index option series that most decisively match the 30-day target time span for expected volatility that the Vix index is planned to represent [23]. The successful methodology of Vix has been used to create implied volatility indexes of many countries.

In this thesis, we intend to express volatility indexes and the formulas that are used in index calculations. The aim of this study is to understand all aspects of Vix and other volatility indexes, and in the light of these informations and methods, to design and construct a volatility index for the Turkish BIST 30 index. According to the recent studies there is no volatility index designed for Turkish equity indexes. And also recent financial news stated that, the Central Bank of the Republic of Turkey is going to use Vix as base for decision of interest rate hike. These news point out that Vix and other mostly used volatility indexes are going to have significant impacts on Turkey's economy in the future. Following these developments, Turkish market needs to have a volatility index that can be used as a benchmark for the equity markets status. In Chapter 5, we try to present an implementation of volatility index to the Turkish BIST 30 index. This works biggest aim is to design and construct a model that can be use as a volatility index for Turkish BIST 30 index.

CHAPTER 2

THE FIRST VOLATILITY INDEX : CBOE VIX

In this chapter, CBOE Vix and also history and calculation of this index will be discussed in detail. Vix is the most essential volatility index in the world financial market. It is the first index about volatility and also it is the most used volatility index for years. Not only in the United States of America but also other giants of the financial world take Vix as a reference for volatility. Now, after a short introduction, we can describe CBOE Vix deeply.

2.1 Derivation of the Formula

Firstly, we assume that stock price satisfies:

$$\frac{dS}{S} = (r - q)dt + \sigma dz \tag{2.1}$$

here, r is the risk free interest rate, q is the continuous dividend yield, σ is the volatility, and z is the driftless Brownian motion. By Itô's lemma we have:

$$d\ln(S) = \left(r - q - \frac{\sigma^2}{2}\right)dt + \sigma dz \tag{2.2}$$

By Eq. 2.1 and Eq. 2.2 we get:

$$\frac{dS}{S} - d\ln(S) = \frac{\sigma^2}{2}dt \tag{2.3}$$

Hence, we integrate the last equation from time 0 to time T,

$$\int_0^T \frac{\sigma^2}{2}dt = \int_0^T \frac{dS}{S} - \int_0^T d\ln(S)$$

$$\frac{1}{2} \cdot \sigma^2 T = \int_0^T \frac{dS}{S} - (\ln S_T - \ln S_0) = \int_0^T \frac{dS}{S} - \ln \frac{S_T}{S_0}$$

When we simplify the expression and write V instead of σ^2, we get:

$$V = \frac{2}{T} \int_0^T \frac{dS}{S} - \left(\frac{2}{T}\right) \cdot \ln\left(\frac{S_T}{S_0}\right) \tag{2.4}$$

The expectation $\mathbb{E}\left[\int \frac{dS}{S}\right]$ under risk neutral probability measure, $\mathbb{E}$, is :

$$\mathbb{E}\left[\int_0^T \frac{dS}{S}\right] = \mathbb{E}\left[\int_0^T (r-q)dt\right] + \mathbb{E}\left[\int_0^T \sigma dz\right] = (r-q) \cdot T$$

When we take the expectation $\mathbb{E}[V]$ under risk neutral probability measure $\mathbb{E}$, keeping in mind the above fact, we get:

$$\mathbb{E}[V] = \frac{2}{T} \cdot \mathbb{E}\left[\int_0^T \left(\frac{dS}{S}\right)\right] - \frac{2}{T} \cdot \mathbb{E}\left[\ln\left(\frac{S_T}{S_0}\right)\right] = \frac{2}{T} \cdot (r-q) \cdot T - \frac{2}{T} \cdot \mathbb{E}\left[\ln\left(\frac{S_T}{S_0}\right)\right]$$

$$\mathbb{E}[V] = 2(r-q) - \frac{2}{T} \cdot \mathbb{E}\left[\ln\left(\frac{S_T}{S_0}\right)\right] \tag{2.5}$$

We know that $S_T = S_0 \cdot \exp(X)$, where $X = \left(r - q - \frac{\sigma^2}{2}\right)T + \sigma \cdot z(T)$ is a Gaussian random variable with,

$$\mathbb{E}[X] = \left(r - q - \frac{\sigma^2}{2}\right)T \quad \text{and} \quad Var[X] = \sigma^2 T.$$

Therefore,

$$\mathbb{E}[S_T] = S_0 \cdot \exp\left(\mathbb{E}[X] + \frac{Var[X]}{2}\right) = S_0 \cdot e^{(r-q)T},$$

and hence,

$$\mathbb{E}[V] = \frac{2}{T}\left[\ln\left(\frac{F_0}{S_0}\right) - \mathbb{E}\left[\ln\left(\frac{S_T}{S_0}\right)\right]\right] \quad \text{where} \quad F_0 = \mathbb{E}[S_T] \tag{2.6}$$

On the other hand we need to look at the values of $\int_0^{S_t} \frac{1}{K^2} \cdot \max(K - S_T, 0)dK$ and $\int_{S_t}^{\infty} \frac{1}{K^2} \cdot \max(S_T - K, 0)dK$ in different situations.

We begin with $\int_0^{S_t} \frac{1}{K^2} \cdot \max(K - S_T, 0) dK$ where S_t is any value of S. Here we need to consider 2 cases:

1) If $S_t \leqslant S_T$ then,

$$\int_0^{S_t} \frac{1}{K^2} \cdot \max(K - S_T, 0) dK = 0.$$

2) If $S_t > S_T$ then,

$$\int_0^{S_t} \frac{1}{K^2} \cdot \max(K - S_T, 0) dK = \int_{S_T}^{S_t} \frac{1}{K^2} (K - S_T) dK = \ln\left(\frac{S_t}{S_T}\right) + \frac{S_T}{S_t} - 1.$$

Next, lets look at the values of $\int_{S_t}^{\infty} \frac{1}{K^2} \cdot \max(S_T - K, 0) dK$ where S_t is any value of S. Again, it is needed to be investigated in 2 cases:

1) If $S_t \geqslant S_T$ then,

$$\int_{S_t}^{\infty} \frac{1}{K^2} \cdot \max(S_T - K, 0) dK = 0.$$

2) If $S_t < S_T$ then,

$$\int_{S_t}^{\infty} \frac{1}{K^2} \cdot \max(S_T - K, 0) dK = \int_{S_t}^{S_T} (S_T - K) dK = \ln\left(\frac{S_t}{S_T}\right) + \frac{S_T}{S_t} - 1.$$

Now, combining the two integrals, we have:

$$\int_0^{S_t} \frac{1}{K^2} \max(K - S_T, 0) dK + \int_{S_t}^{\infty} \frac{1}{K^2} \max(S_T - K, 0) dK = \ln\left(\frac{S_t}{S_T}\right) + \frac{S_T}{S_t} - 1.$$

So, for all values of S_t we have:

$$\ln\left(\frac{S_T}{S_t}\right) = \frac{S_T}{S_t} - 1 - \int_0^{S_t} \frac{1}{K^2} \max(K - S_T, 0) dK - \int_{S_t}^{\infty} \frac{1}{K^2} \max(S_T - K, 0) dK. \quad (2.7)$$

We can take the expectation of the last equation under risk-neutral probability measure to obtain:

$$\mathbb{E}\left[\ln\left(\frac{S_T}{S_t}\right)\right] = \mathbb{E}\left[\frac{S_T}{S_t} - 1\right] - \mathbb{E}\left[\int_0^{S_t} \frac{1}{K^2} \max(K - S_T, 0) dK\right] - \mathbb{E}\left[\int_{S_t}^{\infty} \frac{1}{K^2} \max(S_T - K, 0) dK\right].$$

Here, $\mathbb{E}[\max(K - S_T, 0)] = e^{rT} \cdot P(K)$ and $\mathbb{E}[\max(S_T - K, 0)] = e^{rT} \cdot C(K)$. Where $P(K)$ and $C(K)$ are the prices of the put and call options with strike price K, r is the risk-free interest rate and $\mathbb{Q}$ is the risk neutral probability measure respectively. Besides, $R(m)$ denotes the yield to maturity on a treasury bill that will mature in m days:

$$R(m) = \frac{1}{m} \int r(x)dx$$

or equivalently

$$r(m) = R(m) + m \cdot R'(m)$$

Therefore,

$$\mathbb{E}\left[\ln\left(\frac{S_T}{S_t}\right)\right] = \frac{F_0}{S_t} - 1 - \int_0^{S_t} \frac{1}{K^2} e^{RT} P(K)dK - \int_{S_t}^{\infty} \frac{1}{K^2} e^{RT} C(K)dK. \tag{2.8}$$

On the other hand,

$$\mathbb{E}\left[\ln\left(\frac{S_T}{S_0}\right)\right] = \mathbb{E}\left[\ln\left(\frac{S_T}{S_t}\right)\right] + \mathbb{E}\left[\ln\left(\frac{S_t}{S_0}\right)\right] = \ln\left(\frac{S_t}{S_0}\right) + \mathbb{E}\left[\ln\left(\frac{S_T}{S_t}\right)\right]. \tag{2.9}$$

In writing Eq. 2.9 we take advantage of the fact that,

$$\ln\left(\frac{S_T}{S_0}\right) = \ln(S_T) - \ln(S_0) = \ln(S_T) - \ln(S_t) + \ln(S_t) - \ln(S_0) = \ln\left(\frac{S_T}{S_t}\right) + \ln\left(\frac{S_t}{S_0}\right).$$

By combining Eq. 2.6, Eq. 2.8 and Eq. 2.9, we obtain Eq. 2.10, which gives us the expected value of variance from time 0 to T.

$$\begin{aligned}\mathbb{E}[V] = \frac{2}{T} \ln\left(\frac{F_0}{S_t}\right) - \frac{2}{T}\left(\frac{F_0}{S_t} - 1\right) \\ + \frac{2}{T}\left[\int_0^{S_t} \frac{1}{K^2} e^{RT} P(K)dK + \int_{S_t}^{\infty} \frac{1}{K^2} e^{RT} C(K)dK\right]\end{aligned} \tag{2.10}$$

Now, lets assume that, for $1 \leqslant i \leqslant n$, K_i's are the increasing sequence of options' strike prices; besides, lets fix S_t as the first strike price below $F_0 = \mathbb{E}[S_T]$. We define $Q(K_i)$ as follows:

(i) If K_i is smaller than S_t then $Q(K_i)$ is the price of the put option with strike price K_i.

(ii) If K_i is greater than S_t then $Q(K_i)$ is the price of the call option with strike price K_i

(iii) If K_i equals to S_t then $Q(K_i)$ is the average of the prices of the call and put options with strike price K_i.

From this definition we can write:

$$\int_0^{S_t} \frac{1}{K^2} e^{RT} P(K) dK + \int_{S_t}^{\infty} \frac{1}{K^2} e^{RT} C(K) dK = \sum_{i=1}^{n} \frac{\Delta K_i}{{K_i}^2} e^{rT} Q(K_i) \tag{2.11}$$

Where, $\Delta K_i = \dfrac{K_{i+1} - K_{i-1}}{2}$ for $2 \leqslant i \leqslant n-1$, but for upper and lower boundaries of the sequence of the strike prices $\Delta K_1 = K_2 - K_1$ and $\Delta K_n = K_n - K_{n-1}$. When we put the right hand side of Eq. 2.11 into Eq. 2.10 we get:

$$\mathbb{E}[V] = \frac{2}{T} \ln\left(\frac{F_0}{S_t}\right) - \frac{2}{T}\left(\frac{F_0}{S_t} - 1\right) + \frac{2}{T} \sum_{i=1}^{n} \frac{\Delta K_i}{{K_i}^2} e^{rT} Q(K_i) \tag{2.12}$$

It is common to use the Taylor Series approximation of $\ln\left(\dfrac{F_0}{S_t}\right)$ in Eq. 2.12

$\ln\left(\dfrac{F_0}{S_t}\right) = \left(\dfrac{F_0}{S_t} - 1\right) - \dfrac{1}{2}\left(\dfrac{F_0}{S_t} - 1\right)^2 + \dfrac{1}{3}\left(\dfrac{F_0}{S_t} - 1\right)^3 - \ldots$ and

$\ln\left(\dfrac{F_0}{S_t}\right) \approx \left(\dfrac{F_0}{S_t} - 1\right) - \dfrac{1}{2}\left(\dfrac{F_0}{S_t} - 1\right)^2$

Herewith the Eq. 2.7 turns into:

$$\mathbb{E}[V] = -\frac{1}{T}\left(\frac{F_0}{S_t} - 1\right)^2 + \frac{2}{T} \sum_{i=1}^{n} \frac{\Delta K_i}{{K_i}^2} e^{rT} Q(K_i) \tag{2.13}$$

Lastly, we need to change S_t with K_0, the greatest strike price below the forward index level F_0. Besides that, in the place of $\mathbb{E}[V]$ σ^2 can be rewritten. Hereby the formula reached its final approximate form:

$$\sigma^2 = -\frac{1}{T}\left(\frac{F_0}{K_0} - 1\right)^2 + \frac{2}{T} \sum_{i=1}^{n} \frac{\Delta K_i}{{K_i}^2} e^{RT} Q(K_i) \tag{2.14}$$

2.2 Chicago Board Options Exchange Volatility Index

Vix is an index of volatility. It is the first index that is used for measuring volatility expectation of the market. When it was introduced in 1993 there were two objects in mind. It would be:

(1) the main criterion for estimating market's short term volatility.

(2) the base index that is used for creating derivative products on volatility.

The original Vix was based on the S&P 100 index (OEX) option prices, the reason was simple: at that time OEX index options were the most popular index options in the United States. Besides, Vix calculation was based on only at-the-money put and call options, because these options had less bid-ask spreads and these options trading volume was higher. Since its appearance in the world financial markets there have beens two significant changes in the patern of Vix:

(1) The SPX option market's trading volume got ahead and it became the most active index option market in the United States. Besides, S&P 500 option contracts are European-style, that makes them easier to value, but OEX options are American-style and these are harder to price.

(2) Trading processes of market contributors in index option markets changed over time. In the mid 1990s, both index calls and index puts had similar vital parts in speculators' trading lines. Trading volumes were adjusted in 1992, OEX calls and OEX puts had almost the same average daily trading volumes. Over the following years, the index option market dominated by portfolio insurers, who routinely buy out-of-the money and at-the-money index puts for protection purposes. Nearly all 2008 long, the daily average trading volume of SPX puts surpassed SPX calls. The request to buy out-of-the money and at-the-money SPX puts is indeed a key factor in implied volatility measures like Vix.

In September 2003, the Vix calculation was changed to represent both of these significant changes in the index option market structure.

- First, they began to use SPX rather than OEX option prices.
- Second, they started to incorporate out-of-the-money options in the index computation since out-of-the-money put prices, specifically, contain critical data with respect to the requests for portfolio insurance and consequently market volatility. Counting additional option series likewise makes the Vix less delicate to any single option price [49].

2.3 The Vix Calculation

In addition to the changes above in CBOE Vix, CBOE enhanced the Vix index to include series of SPX Weeklys in 2014.

In 2005, 32 years in the wake of presenting the call option, The CBOE began an experimental program with weeklys. They act like monthly options in every way alike, apart from the fact that they exist for eight days. They are presented every Thursday and they terminate eight days after; Fridays (with modifications for occasions). Speculators who have generally delighted in 12 monthly end times –the third Friday of every month– now can appreciate 52 lapses for each year. The consideration of SPX Weeklys permits the Vix index to be calculated with the S&P 500 index option series that most unequivocally match the 30-day target timeframe for expected volatility that the Vix index is proposed to reflect. Using SPX options with more than 23 days and less than 37 days to expiration guarantees that the Vix index will dependably mirror an interpolation of two focuses along the S&P 500 volatility term structure [23].

2.3.1 Generalized Vix Formula

Stock indexes, for example, the S&P 500, are figured using the prices of their stocks. Each index utilizes the choice of component securities and a formula to ascertain index values. The Vix index is an index comprised of options instead of stocks and the price of every option representing the market's view of future volatility. Like classic indexes, Vix has a formula. The generalized formula used in the Vix calculation is:

$$\text{Vix} = 100 \cdot \sqrt{\left[T_1 \cdot \sigma_1^2 \cdot \frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}} + T_2 \cdot \sigma_2^2 \cdot \frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}} \right] \cdot \frac{N_{365}}{N_{30}}} \tag{2.15}$$

where,

$$\sigma_j^2 = \frac{2}{T_j} \cdot \sum_i \frac{\Delta K_i}{K_i^2} \cdot e^{R_j \cdot T_j} \cdot Q(K_i) - \frac{1}{T_j} \cdot \left[\frac{F_j}{K_0} - 1 \right]^2 \tag{2.16}$$

σ_j: Volatility of near term and next term options for $j = 1$, $j = 2$ respectively [1]

T_1 and T_2: Time to expiration of near term and next term options respectively [1]

F_1 and F_2: Desired forward index levels of near term and next term options respectively [1]

K_0: First strike price below the forward index level

K_i: Strike price of the i^{th} out-of-the-money option, a call if $K_i > K_0$; and a put if $K_i < K_0$; both put and call if $K_i = K_0$

ΔK_i: Interval between strike prices - half the difference between the strike on either side of K_i

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2} \text{ [2]}$$

where,

r_1 and r_2 are risk-free interest rates related to near-term and next-term options series respectively

$Q(K_i)$: The mid-point of the bid-ask spread for each option with strike K_i, as mentioned before

N_{T_1}: Number of minutes to settlement of the near term options

N_{T_2}: Number of minutes to settlement of the next term options

N_{30}: Number of minutes in a 30 days (43200)

N_{365}: Number of minutes in a 365-day year (525600)

2.3.2 Components of the Formula

Measuring T: The components of the Vix calculation are near and next term put and call options with over 23 days and under 37 days to lapse. These incorporate SPX options with "standard"3rd Friday expiration dates and "weekly"SPX

[1] Computations of these components are explained by an example in Section 2.5

[2] ΔK for the lowest strike is simply the difference between the lowest strike and the next higher strike. Likewise, ΔK for the highest strike is the difference between the highest strike and the next lower strike

options that terminate every Friday, except the 3rd Friday of every month. Every week, SPX options that will be used in the Vix calculation are evaluated again with regard to new contract maturities.

The Vix calculation measures time to expiration T in date-book days and partitions every day into minutes with a specific end goal to imitate the accuracy that is normally utilized by option traders and volatility merchants. The time to expiration is given by the following expression:

$$T = \frac{M_{\text{Current Day}} + M_{\text{Settlement Day}} + M_{\text{Other Days}}}{\text{Minutes in a year}},$$

where,

- $M_{\text{Current Day}}$: Minutes remaining until midnight of the current day,
- $M_{\text{Settlement Day}}$: Minutes from midnight until 8:30 a.m. for “standard”SPX expirations; or minutes from midnight until 3:00 p.m. for “weekly”SPX expirations,
- $M_{\text{Other Days}}$: Total minutes in the days between current day and expiration day.

Remark 2.1. Calculation of the T discussed above is based on CBOE Vix methodology. Currently used volatility indexes may vary with T's measurement unit. Most of the indexes use CBOE Vix as base and measure T in minutes but some of them, for instance, German VDAX-NEW and Swiss VSMI measures T in seconds. This is because of to create more consistent volatility index. Alternative decisions can be made while measuring T by organisers or developers of the related volatility index.

Determining the risk-free rate: The risk-free rate of interest, r_1 and r_2 are the bond-equivalent yields of the U.S. Treasury-Bill maturing nearest to the expiration dates of pertinent SPX options.

Selecting the options to be used in the Vix calculation: The chosen options are out-of-the money SPX puts revolved around an at-the-money strike value K_0. Just SPX options cited with non-zero bid prices are utilized as a part of the Vix calculation. As volatility rises and falls, the strike value scope of options with non-zero bids have a tendency to grow to contract. Subsequently, the quantity of options used in the Vix calculation may fluctuate from month-to-month, day-to-day, and perhaps even moment-to-moment.

Calculating the forward index level F: For each agreement month, decide forward SPX level F by recognizing the strike price at which the total distinction between the call and put prices is littlest [23].

2.4 Relation Between CBOE Vix and S&P 500 Index

The Vix has been named the ***investor fear gauge*** among financial professionals. High Vix values imply, investors consider remarkable risk that the market will act stingingly, whether downward or upward. The top values of Vix happen when investors expect big moves in both up and down directions are likely. When investors foresee neither noteworthy down directionally risk nor respectable upward potential will the Vix be low. When we look at the historical daily and weekly closing data of S&P 500 index and CBOE Vix between 01.01.2005 and 01.01.2017, it can be implied from the data easily that, correlation of S&P 500 and Vix weekly opening values is -0.50343 and correlation of S&P 500 and Vix daily closing values is -0.50824. These are significant values as a correlation of the two time series. Movements of S&P500 index and Vix in the mentioned time frame can be seen in Table 2.1 and Table 2.2.

Table 2.1: Number of up and down movements of S&P500 Index and CBOE Vix based on daily closing prices between 01.01.2005 and 01.01.2017

	S&P500 Index	*Vix*
UP	1640	1392
DOWN	1379	1612

Table 2.2: Number of up and down movements of S&P500 Index and CBOE Vix based on weekly opening prices between 01.01.2005 and 01.01.2017

	S&P 500 Index	*Vix*
UP	349	304
DOWN	276	320

Table 2.3: Number and percentage of opposite movements of S&P 500 Index and CBOE Vix based on daily closing prices between 01.01.2005 and 01.01.2017.

S&P500 Index UP and Vix DOWN	*Percent of Opposite Movements*
1342	81.83%
S&P500 Index DOWN and Vix UP	*Percent of Opposite Movements*
1100	79.77%

Table 2.4: Number and percentage of opposite movements of S&P 500 Index and CBOE Vix based on weekly opening prices between 01.01.2005 and 01.01.2017.

S&P 500 Index UP and Vix DOWN	*Percent of Opposite Movements*
260	74.50%
S&P 500 Index DOWN and Vix UP	*Percent of Opposite Movements*
216	78.26%

When we observe daily and weekly closing prices in the same data, it is obvious from Table 2.3 and Table 2.4 that CBOE Vix reacts considerably opposite to the S&P 500 index. It can be seen from the Table 2.3, percentage of opposite movements reaches its maximum value of *81.83%* when S&P 500 index moves upward. However the most notable thing is that, percentage of movements when S&P 500 falls and Vix rises is *79.77%*. This explains why the CBOE Vix is named as the investor fear gauge; and it clarifies why financial professionals look closely to the movements of Vix. All the things mentioned here can be seen on, Figure 2.1 and Figure 2.2. Figure 2.1 is based on 3021 daily closing datas in the time period of January 2005 and January 2017, and Figure 2.2 is based on 626 weekly opening datas on the same time period.

Despite all, it can not be said, S&P 500 index and Vix moves one for one with opposite directions. Correlation of S&P 500 and Vix is really strong but it is not enough to say, these indexes move perfectly in the opposite directons. For a reasonable number of instances, the S&P 500 index and Vix move in the same direction. Table 2.5 and Table 2.6 point out these indexes isotropic movements in the same time frame.

Table 2.5: Number and percentage of isotropic movements of S&P 500 (SPX) index and Vix based on daily closing values in the time interval of January 2005 and January 2017

SPX up and Vix up	*SPX up and Vix up / SPX up*	*Percentage of same direction*
292	17.80%	18.58%
SPX down and Vix down	*SPX down and Vix down / SPX down*	
269	19.51%	

Table 2.6: Number and percentage of isotropic movements of S&P 500 (SPX) index and Vix based on weekly opening prices in the time interval of January 2005 and January 2017

SPX up and Vix up	*SPX up and Vix up / SPX up*	*Percentage of same direction*
75	21.49%	21.28%
SPX down and Vix down	*SPX down and Vix down / SPX down*	
58	21.01%	

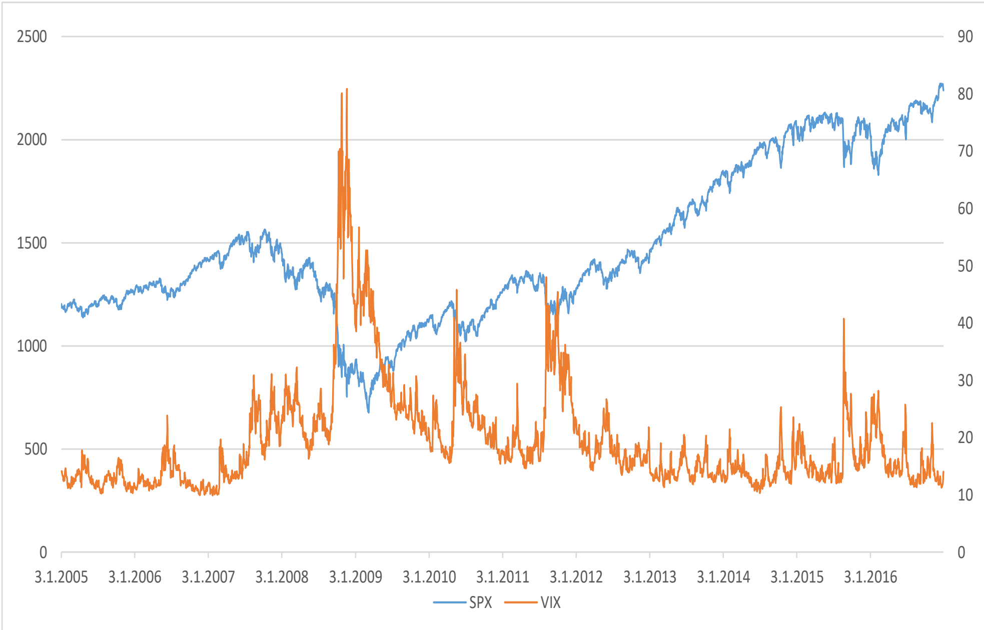


Figure 2.1: Daily closing values of S&P 500 (SPX) and Vix between January 2005 and January 2017

2.5 An Example Of Vix Calculation

The example in this section is taken from the CBOE Volatility Index-VIX White Paper, 2009 [23]. The reason of giving coverage to an example that is taken from White Paper is simple: data we utilize in this example is realistic and it is a possible scenario; and also White Paper is the Chicago Board Options Exchange's official information leaflet.

To begin with, near-term and next-term options need to be determined. In this hypothetical example, let us assume that near-term options are standard SPX options with 25 days and long-term options are SPX weeklys with 32 days to maturity; all calculations will represent prices observed at 9:46 a.m. Chicago time. For the sake of calculation of time to maturity, standard SPX options are assumed to expire the third Friday of the month, and SPX Weeklys are assumed to expire at the close of buying and selling sessions.

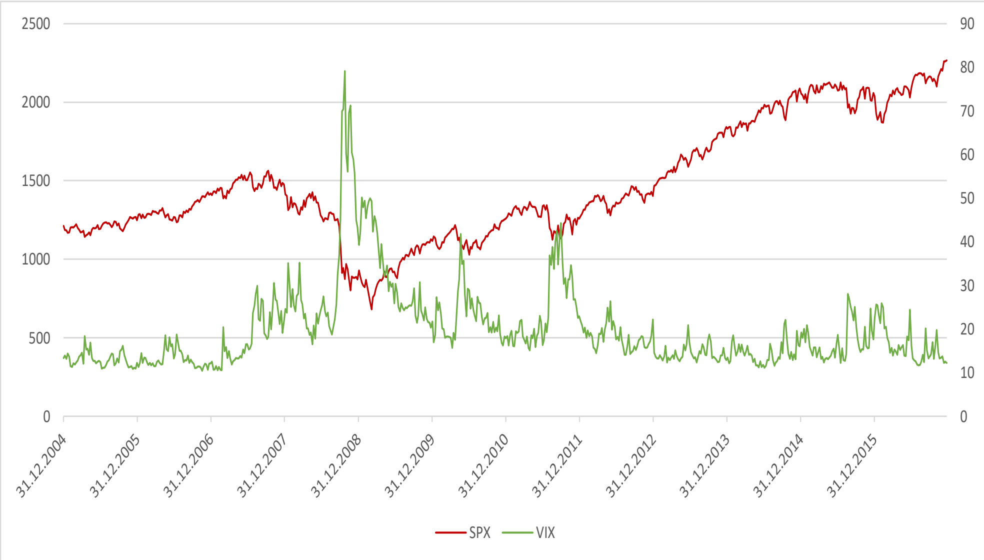


Figure 2.2: Weekly opening prices of S&P 500 (SPX) index and Vix between January 2005 and January 2017

Using 9:46 a.m. as the time of the calculation, T for the near-term and next-term options, respectively T_1 and T_2 are:

$$T_1 = \frac{854 + 510 + 34560}{525600} = 0.0683486,$$

where,

854 : Minutes from 9:46 a.m. to 12:00 p.m.

510 : Minutes from 12:00 p.m. to 8:30 a.m.

34560 : 24 days in minutes

525600 : 1 year in minutes

and

$$T_2 = \frac{854 + 900 + 44640}{525600} = 0.0882686$$

Here,

854 : Minutes from 9:46 a.m. to 12:00 p.m.

900 : Minutes from 12:00 p.m. to 3:00 p.m.

44640 : 31 days in minutes

525600 : 1 year in minutes.

Next, the risk-free interest rates those will be used in the Vix calculation, r_1 and r_2, need to be determined. At this step we will use yields of the U.S. Treasury-Bill expiring closest to the maturity date of relevant SPX options, and $r_1 = 0.0305\%$, $r_2 = 0.0286\%$ on that day.

As the third step of the Vix calculation, we need to find desired forward SPX level F by observing the options trading on the market at the time of calculation. As seen from Table 2.7, difference between the call and put prices is the smallest at the 1965 strike for the near-term options, and at the 1960 strike for the next-term options.

Table 2.7: A part of Near and Next-term call and put options prices listed on the market at 9.46 a.m.

	Near-term Options			*Next-term Options*		
Strike Price	*Call*	*Put*	*Difference*	*Call*	*Put*	*Difference*
.	.	.	.	.	.	.
1940	38.45	15.25	23.20	41.05	18.80	22.25
1945	34.70	16.55	18.15	37.45	20.20	17.25
1950	31.10	18.25	12.85	34.05	21.60	12.45
1955	27.60	19.75	7.85	30.60	23.20	7.40
1960	24.25	21.30	2.95	27.30	24.90	**2.40**
1965	21.05	23.15	**2.10**	24.15	26.90	2.75
1970	18.10	25.05	6.95	21.10	28.95	7.85
1975	15.25	27.30	12.05	18.30	31.05	12.75
1980	12.75	29.75	17.00	15.70	33.50	17.80

And after chosing strike prices for near-term and next-term options, forward index prices, F_1 and F_2 can be calculated, where

F_1 : Forward SPX level obtained from near-term options, and

F_2 : Forward SPX level obtained from next-term options.

F_1 and F_2 are calculated according to:

$$F : Strike\ Price + e^{r \cdot T} \cdot (Call\ Price - Put\ Price) \tag{2.17}$$

$F_1 = 1965 + e^{0.0003 \cdot 0.0683486} \cdot (21.05 - 23.15) = 1962.89996$, and

$F_2 = 1960 + e^{0.00028 \cdot 0.0882686} \cdot (27.30 - 24.90) = 1962.40006$.

Also,

$K_{0,1}$: Strike price for near-term options.

$K_{0,2}$: Strike price for next-term options.

Hereby, strike prices for near-term and next-term calculations, $K_{0,1}$ and $K_{0,2}$ come to hand. $K_{0,1}$ and $K_{0,2}$ are values, instantly below the forward index levels for near-term and next-term estimations, respectively for the second index 1 and 2. At this stage, $K_{0,1}$ and $K_{0,2}$ are both 1960. This value will be used in near-term and next-term Vix calculation as strike price. After specifying strike prices, now, out-of-the-money put and out-of-the-money call options can be determined. Out-of-the-money put options will be selected from options those have strike prices less than $K_{0,1}$ and $K_{0,2}$ for near-term and next-term volatility estimations respectively. Similarly out-of-the-money call options will be chosen from options which have strike prices greater than $K_{0,1}$ and $K_{0,2}$ for near-term and next-term volatility calculations respectively. At the stage of selecting near-term and next-term options, a put option that has a bid price equal to zero will be excluded. And also if two puts with sequential strike prices are found to have zero bid prices, no puts with lower strike than this level are regarded for inclusion. As with the puts, once two successive call options are found to have zero bid prices, no call option with strike prices are marked. The process mentioned can be understood with Table 2.8 and Table 2.9. These operations will be repeated for next term options and lastly both the put and call options with strike price $K_{0,1}$ from among near-term options, once more put and call options with strike price $K_{0,2}$ out of next-term options will be selected.

Table 2.8: Near term out-of-the-money put options

Put Strike	*Bid*	*Ask*	*Include?*
1345	0	0.15	Not Considered (Following two zero bids)
1350	0.05	0.15	Not Considered (Following two zero bids)
1355	0.05	0.35	Not Considered (Following two zero bids)
1360	0	0.35	No
1365	0	0.35	No
1370	0.05	0.35	Yes
1375	0.1	0.15	Yes
1380	0.1	0.2	Yes

Table 2.9: Near term out-of-the-money call options

Call Strike	*Bid*	*Ask*	*Include?*
2095	0.05	0.35	Yes
2100	0.05	0.15	Yes
2120	0	0.15	No
2125	0.05	0.15	Yes
2150	0	0.1	No
2175	0	0.05	No
2200	0	0.05	Not Considered (Following two zero bids)
2225	0.05	0.1	Not Considered (Following two zero bids)
2250	0	0.05	Not Considered (Following two zero bids)

Table 2.10: Near term and next term options with quote prices

Near term Strike	*Option Type*	*Quote Price*		*Next term Strike*	*Option Type*	*Quote Price*
1370	Put	0.2		1275	Put	0.075
1375	Put	0.125		1325	Put	0.15
1380	Put	0.15		1350	Put	0.15
.	.	.		.	.	.
1950	Put	18.25		1950	Put	21.60
1955	Put	19.75		1955	Put	23.20
1960	Put/Call	22.775		1960	Put/Call	26.10
1965	Call	21.05		1965	Call	24.15
1970	Call	18.1		1970	Call	21.10
.	.	.		.	.	.
2095	Call	0.2		2125	Call	0.1
2100	Call	0.1		2150	Call	0.1
2125	Call	0.1		2200	Call	0.08

In Table 2.10, both a put and a call option are selected at strike price $K_{0,1}$ and $K_{0,2}$, while one option, either a put or a call, is utilized at each other strike price. It is also necessary to specify, in Table 2.10, quote prices of call and put options with strike 1960 were founded by averaging quote prices of put and call options with the same strike price, 1960. Now, we are ready to insert our values into Eq. 2.2 in order to calculate near term and next term volatilities. So, we get for near-term and next-term calculations, respectively:

$$\sigma_1{}^2 = \frac{2}{T_1} \cdot \sum_i \frac{\Delta K_i}{K_i^2} \cdot e^{r_1 \cdot T_1} \cdot Q(K_i) - \frac{1}{T_1} \cdot \left[\frac{F_1}{K_{0,1} - 1}\right]^2, \tag{2.18}$$

$$\sigma_2{}^2 = \frac{2}{T_2} \cdot \sum_i \frac{\Delta K_i}{K_i^2} \cdot e^{r_2 \cdot T_2} \cdot Q(K_i) - \frac{1}{T_2} \cdot \left[\frac{F_2}{K_{0,2} - 1}\right]^2. \tag{2.19}$$

In Table 2.11 and Table 2.12, each index option's contribution to the Vix value can be seen.

Table 2.11: Near term options and their contribution to the near term volatility

Near Term Strike	*Option Type*	*Quote Price*	*Contribution by Strike*
1370	Put	0.2	0.0000005328
1375	Put	0.125	0.0000003306
1380	Put	0.15	0.0000003938
.	.	.	.
1950	Put	18.25	0.0000239979
1955	Put	19.75	0.0000258376
1960	Put/Call	22.775	0.0000296432
1965	Call	21.05	0.0000272588
1970	Call	18.1	0.0000233198
.	.	.	.
2095	Call	0.2	0.0000002278
2100	Call	0.1	0.0000003401
2125	Call	0.1	0.0000005536
$\frac{2}{T_1} \cdot \sum_i \frac{\Delta K_i}{K_i^2} \cdot e^{R_1 \cdot T_1} \cdot Q(K_i)$			0.018495

Table 2.12: Next term options and their contribution to the next term volatility

Next Term Strike	*Option Type*	*Quote Price*	*Contribution by Strike*
1275	Put	0.075	0.0000023069
1325	Put	0.15	0.0000032041
1350	Put	0.15	0.0000020577
.	.	.	.
1950	Put	21.6	0.0000284031
1955	Put	23.2	0.0000303512
1960	Put/Call	26.1	0.0000339711
1965	Call	24.15	0.0000312732
1970	Call	21.1	0.0000271851
.	.	.	.
2125	Call	0.1	0.0000005536
2150	Call	0.1	0.0000008113
2200	Call	0.075	0.0000007748
$\frac{2}{T_2} \cdot \sum_i \frac{\Delta K_i}{K_i^2} \cdot e^{R_2 \cdot T_2} \cdot Q(K_i)$			0.018838

The values that we need to know in Eq. 2.2 can be found handily:

$$\frac{1}{T_1} \cdot \left[\frac{F}{K_{0,1}} - 1\right]^2 = 0.00003203$$

$$\frac{1}{T_2} \cdot \left[\frac{F}{K_{0,2}} - 1\right]^2 = 0.00001699$$

Therefore, the volatilities are found to be ${\sigma_1}^2 = 0.01846292$ and ${\sigma_2}^2 = 0.01882101$. As the last step, Vix value will be created via calculating the 30-day weighted average of ${\sigma_1}^2$ and ${\sigma_2}^2$, then taking the square root of this value and multiplying by 100, as in Eq. 2.1. So our Vix value is:

$$\text{Vix} = 100 \cdot \sqrt{\left[0.0683486 \cdot 0.01846292 \cdot \frac{46394 - 43200}{46394 - 35924} + 0.0882686 \cdot 0.01882101 \cdot \frac{43200 - 35924}{46394 - 35924}\right] \cdot \frac{525600}{43200}}$$

Therefore, Vix = 13.69

To see the expected range of S&P 500 index movements we need to infer from this value. To interpret the value, firstly we need to know how to use the Vix value to calculate expected range of the S&P 500 index. Vix is calculated as an annualized value and as it is known volatility is defined as the square root of the variance. Then monthly volatility level can be computed by dividing the Vix value by $\sqrt{12}$, where 12 comes from the number of months in a year.

As continuation of the example above, lets interpret this example's Vix value. To begin with, let us assume that the actual S&P 500 (SPX) index value is 2000 when the Vix calculation is made. In that case,

$$\text{Expected lower bound of SPX} = 2000 - 2000 \cdot \frac{13.69}{\sqrt{12}} \cdot \frac{1}{100} = 1920.9607$$

$$\text{Expected upper bound of SPX} = 2000 + 2000 \cdot \frac{13.69}{\sqrt{12}} \cdot \frac{1}{100} = 2079.0393 \ .$$

Table 2.13: S&P 500 index expected value intervals for different Vix levels

Vix Value	*Expected range of Up and Down Movements of S&P 500 index*
5	$\mp$1.44%
10	$\mp$2.89%
15	$\mp$4.33%
20	$\mp$5.77%
25	$\mp$7.22%
30	$\mp$8.66%
40	$\mp$11.55%
50	$\mp$14.43%
80	$\mp$23.09%

By the Vix, expected levels of S&P 500 index can be computed as it can be seen in Table 2.13 for several values. Apart from that, different Vix levels mean different things in the financial environments but there is not any classification or study with respect to these levels of fear. Just we can say from the historical data, Vix values are swinging from 10 to 20 mostly: see Table 2.14 for the number of occurrences of Vix values historically.

Table 2.14: Frequency of presence of Vix at different levels between January 2005 and January 2017 with daily closing values

Vix level	*Number of occurrence*	*Percentage*
$0 < Vix \leqslant 10$	4	0.13%
$10 < Vix \leqslant 20$	**2024**	**67.02%**
$20 < Vix \leqslant 30$	697	23.08%
$30 < Vix \leqslant 40$	156	5.17%
$40 < Vix \leqslant 50$	83	2.75%
$50 < Vix \leqslant 80$	54	1.79%
$80 < Vix \leqslant 90$	2	0.07%

Exceptionally Vix level have came up to the level of 80s two times in history. Those were November 20, 2008 and October 27, 2008, and also all-time high intra-day Vix value was 89.53 on October 24, 2008. As it can be seen all this values have seen in the time of Sub-prime Mortgage Crisis, which affected the whole global economy deeply.

CHAPTER 3

OTHER IMPORTANT VOLATILITY INDEXES

Globally, the enthusiasm in implied volatility indexes has been developed since the Chicago Board Options Exchange (CBOE) in the USA presented the CBOE Volatility Index (Vix) in 1993. As mentioned in Chapter 2, volatility index leads the market in two different ways: reflecting the anticipation of the market about volatility and being the underlying index for the volatility derivatives. Today, the joint trading activity in Vix options and futures is more than $800,000$ contracts per day and CBOE alone distributes 28 volatility indexes for equity indexes, exchange traded funds (ETFs), interest rates, goods, currencies and so on [23].

Other countries, leading the world economy, have begun presenting volatility indexes for their markets. Several remarkable ones are: VDAX-NEW of Deutsche Börse, VSMI of SMI[1], VCAC of Paris Bourse (also known as Euronext Paris since September 2000), FTSE 100 Volatility Index of NYSE Euronext, VAEX of Euronext Amsterdam and FTSE MIB IVI (Implied Volatility Index) of MIB[2]. Apart from these, VSTOXX of EURO STOXX 50, parenthetically, the Dow Jones Euro Stoxx 50 is an index constituted by the leader and most valuable companies of the 11 countries of the Euro Zone. This index is created to track how Euro zone stocks are sailing. Apart from some multinational companies, those based in the United Kingdom are excluded from this index. The VDAX, VSMI and VSTOXX are developed jointly by Goldman Sachs and Deutsche Börse, but more importantly, all volatility indexes are founded on CBOE Vix-

[1] SMI is the blue chip index that is made up of 20 largest and most liquid stocks of Switzerland market

[2] Milano Italia Borsa

New methodology.

In this chapter, significant points of the above mentioned volatility indexes and relation between these indexes and their adherent equity indexes will be discussed, and at the end of this chapter, interactions between indexes will be mentioned with the help of studies in literature.

3.1 VDAX and VDAX-NEW of Deutsche Börse

In this section, firstly VDAX, the previous version of the VDAX-NEW, will be mentioned and then structure of the VDAX-NEW will be discussed. In this part of the thesis, we mostly benefit from the publication of Aboura and Villa (1999).

VDAX states volatility expectation of the next 30 days for the DAX[3] with percentage value. Because of the DAX's restricted structure, it does not reflect the whole economy. The DAX volatility index has been computed since December 5, 1994. A sub-index is distributed for every DAX-Options' maturity dates and there are 8 sub-indexes with this structure. All indexes are computed interim of 10 seconds. The idea of the volatility sub-indexes has been planned with a specific goal to consider their usage as an underlying for derivative instruments. The VDAX and its sub-indexes are calculated from 8:30 a.m. to 5:00 p.m. Frankfurt time, on every trading day of Deutsche Terminbörse. The VDAX is a linear interpolation of the two sub-indexes which are closest to the remaining life of 45 days. It does not terminate nor it removes the effects of hard fluctuations of volatility, which by and large happen close to expiration. The goal of the VDAX is the development of a volatility index for a moving time interim with a steady span. Four points in time are marked, t_1, t_2, t_3, t_4 and the two lifetimes remaining, $T_j = [t_1, t_2]$ and $T_{j+1} = [t_1, t_4]$ where $t_1 \subset t_2 \subset t_3 \subset t_4$. It is assumed that t_2 be the expiry date of maturity j and t_4 be the expiry date of maturity $j + 1$. t_3 is the conceptual maturity date of the volatility index. In this calculation, $[t_1, t_2]$ interval is assumed as a stationary period and a lifetime

[3] Deutscher Aktienindex or German Stock Index is blue chip stock market index composed of the 30 major German companies trading on the Frankfurt Stock Exchange

remaining of T. The volatility for the T is:

$$\text{VDAX} = \sqrt{\frac{T_{j+1} - T}{T_{j+1} - T_j} \cdot V_j^2 + \frac{T - T_j}{T_{j+1} - T_j} \cdot V_{j+1}^2} \ ,$$

where, V_j and V_{j+1} are volatilities of two sub-indexes, chosen from the eight maturities available for the purpose to encompass fixed lifetime T of the volatility index [1].

The new volatility index of Deutsche Börse with its improved methodology is VDAX-NEW. VDAX-NEW takes into account only at-the-money options. This new index gives access to standard 30 days expected volatility of DAX. VDAX-NEW is traded at the derivatives exchange EUREX[4]. Methodology of the VDAX developed by Deutsche Börse together with Goldman Sachs. The computation strategy of the index is particularly refined so as to permit better replication for derivatives and organized products. In this way the VDAX-NEW sets up volatility as a tradeable and separate asset for speculators. VDAX-NEW have 8 sub indexes for different expiries. These expiration dates are the next 1, 2 and 3 months, the following 1, 2 and 3 quarters. In addition, the coming 2 half year periods are included.

3.2 VSTOXX of Euro STOXX 50

Euro Stoxx 50 index is market capitalization-weighted index of Europe and composed of 50 companies from European Zone's sector leaders. These companies are from 11 Euro-zone countries: Austria, Belgium, Finland, France, Germany, Spain, Ireland, Italy, Luxembourg, Netherlands and Portugal. As noticed Euro Stoxx 50 index does not include any company centered in the United Kingdom. In addition to this, these firms are reappraised annually in September. The VSTOXX has been built up by Goldman Sachs and Deutsche Börse together. VSTOXX uses index options available on the Dow Jones Euro STOXX 50 index. VSTOXX is commonly viewed as Europe's equivalent of the CBOE Vix. That's why, there are more investors in Europe track VSTOXX than in other volatility indexes of Euro-zone. There are 12 main indexes of VSTOXX. These indexes

[4] EUREX exchange is the largest European futures and options market located in Germany

are calculated for next 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 330, 360 days and there are also 8 sub indexes of VSTOXX, counting the next 1, 2, 3, 6, 9, 12, 18, 24 month expirations of Euro STOXX 50 options. Sub-indexes are calculated simultaneously. To get the main VSTOXX indexes, two nearby sub-indexes are interpolated linearly. Options, which take VSTOXX as underlying index, are traded on the EUREX and also, trading volume of these options are highest between EUREX options.

3.3 VSMI OF Swiss Market

Switzerland has great importance with its location in the middle of the European continent, with its economy and industry. Switzerland is known all over the world with its stable economy and banking sector. It is a non-European Union country with very high echelon of welfare. These are sufficient reasons for investor to look at movements of Swiss Market. VSMI, volatility index of SMI, is released to the market on April 20, 2005, and from that day on, VSMI is calculated every trading day, from 8:50 a.m. to 5:30 p.m. in every minutes. Furthermore historical data of this index can be found from January, 1999. Apart from the options, those are used to calculate the index, VSMI is not a different instance of volatility index by its structure and calculation method. The reason is clear, as it was mentioned in the beginning of this chapter, all of the volatility indexes use the CBOE Vix methodology, moreover, VDAX-NEW, VSMI, VSTOXX, were also developed by the same institution. We interpreted the results of Vix calculation before. The same procedure is valid for other volatility indexes. Briefly, to find the expected upper and lower bound of the SMI; firstly, expected amount of change needs to be calculated and this value can be found by multiplying the actual SMI level by $\sqrt{\frac{30}{365}}$ and VSMI value. After the expected change level is founded, the expected upper bound equals to the actual SMI value plus the expected amount of change; likewise, the expected lower bound equals to the actual SMI value minus the expected amount of change.

3.4 VFTSE of United Kingdom

Euronext is a cross-boundary European stock exchange, created on September 22, 2000 with the merger agreement of Paris, Amsterdam and Brussels markets. And these stock exchanges' names are changed with Euronext Paris, Euronext Amsterdam and Euronext Brussels after this integration. In 2002 Lisbon stock exchange (BVLP[5]) participated this exchange group and renamed as Euronext Lisbon. On April 4, 2007, Euronext completed their agreement made in May, 2006 with the NYSE Group, ended in the incorporation of NYSE Euronext. FTSE 100 is the product of this group. NYSE Euronext released the FTSE volatility index in June, 2008. Financial Times Stock Exchange 100 index is composed of the largest 100 companies traded on the London Stock Exchange. These companies have highest market capitalizations among other market participants. According to the data of NYSE Group, these companies are representing approximately 81% of the UK market approximately. VFTSE[6] reflects the volatility expectation of market for the next 30 days. VFTSE uses equity index options those traded in LIFFE[7].

3.5 VCAC of Paris Bourse

MONEP[8] is the subsidiary of Paris Bourse that trades stock and index options. Following the introducing of CBOE Vix, on October 8, 1997 MONEP presented two volatility indexes based on implied volatilities of near at the money CAC 40 index options: VX1 and VX6. While VX1 is the short-term volatility index, VX6 is the next-term volatility index [35]. The CAC 40 index is the main benchmark of NYSE Euronext Paris. CAC 40 consists of 40 blue-chip French equities those are the largest by market capitalization and liquidity. VCAC is the new volatility index for French CAC 40 index. This index is computed since September 3, 2007. The VCAC is calculated according to the CBOE Vix methodology with a few changes. VCAC measures implied volatility in 30-day horizon continuously. In

[5] Bolsa de Valores de Lisboa e Porto
[6] FTSE 100 Volatility Index
[7] London International Financial Futures and Options Exchange
[8] Marché des Options Négociables de Paris

the money options and options that satisfy the inequality

$$\frac{Ask\ Price - Bid\ Price}{(Ask\ Price + Bid\ Price)} > \frac{1}{4}h$$

are excluded from the VCAC calculation.

When we talk about AEX; it is the capitalization-weighted blue chip index of the Euronext Amsterdam, known as Amsterdam Stock Exchange previously. It is one of the major indexes of NYSE Euronext Group. VAEX is the volatility index on AEX index options. Besides, VCAC and VAEX have the same specifications.

3.6 MIB IVI of Borsa Italiana

The FTSE MIB is the benchmark equity index of Borsa Italiana, the National Stock Exchange of Italy. This index was called MIB index until September 2004. Until June 2009 this index was governed by Standard & Poor's, and then, execution of the index passed to the FTSE Group. Since then, this index is called as FTSE MIB. The index consists of the most liquid and capitalized 40 Italian stocks, and holds 80% of the domestic market capitalization according to the FTSE Russell's May 2017 data. FTSE MIB IVI shows volatility expectation of FTSE MIB index, and is calculated by prices of out-of-the-money put and call options of the underlying FTSE MIB index. FTSE MIB IVI represents the market volatility expectation of next 30, 60, 90 and 180 days. FTSE MIB IVI uses the same methodology with VFTSE; evidently, these indexes are administered by the same institution. Unlike the other volatility indexes we mentioned, these indexes are calculated and distributed at the end of trading day. Daily closing prices of the equity indexes and volatility indexes we mentioned above are observed in the time frame of January 2007 and January 2017, parenthetically only CAC 40 and VCAC index daily closing values taken between July 01, 2010 and January 01, 2017 due to the lack of data prior to July 2010. In these time periods the correlations of volatility indexes with their related equity indexes were calculated, it can be seen in Table 3.1. Furthermore, combined plots of these equity and volatility indexes can be seen in Figures 3.1 to 3.5.

Table 3.1: Correlations of volatility and equity indexes

Equity and Volatility Indices	*Correlation*	*Time Period*
FTSE 100 , VFTSE	-0.61492	January 2007 - January 2017
SMI , VSMI	-0.46745	January 2007 - January 2017
DAX , VDAX-NEW	-0.49799	January 2007 - January 2017
Euro Stoxx 50 , VSTOXX	-0.47317	January 2007 - January 2017
CAC 40 , VCAC	-0.47858	July 2010 - January 2017

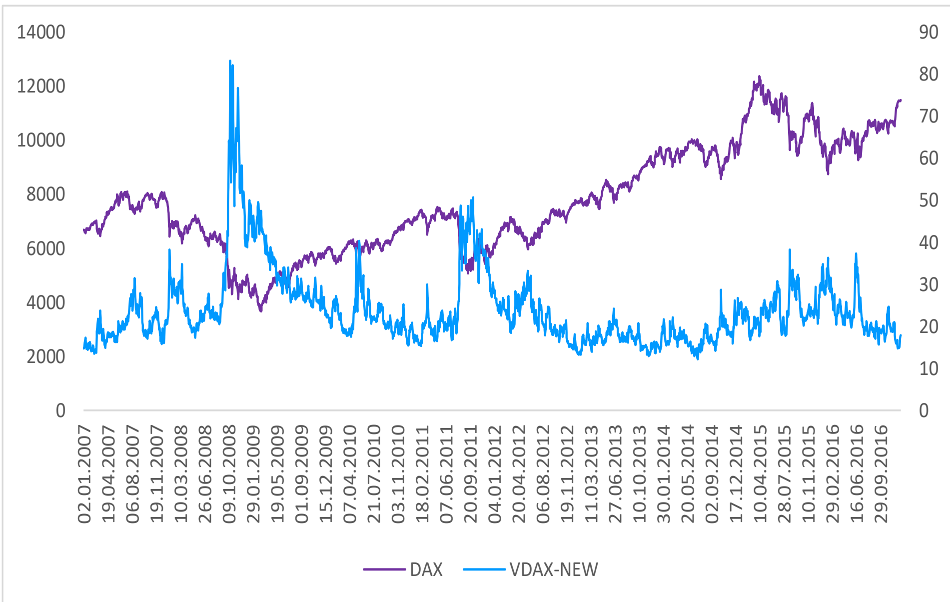


Figure 3.1: German blue chip index versus its new implied volatility index

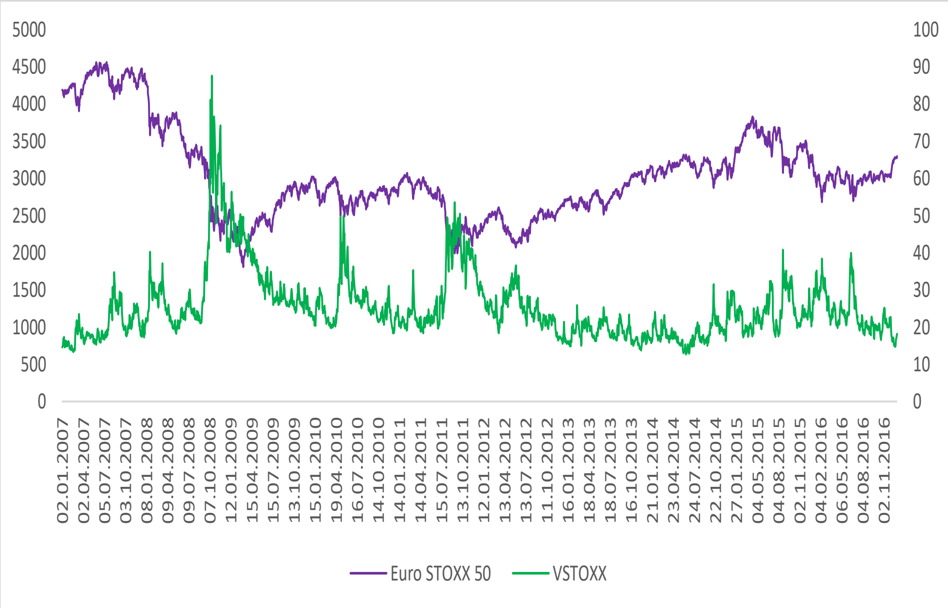


Figure 3.2: Euro STOXX 50 versus VSTOXX

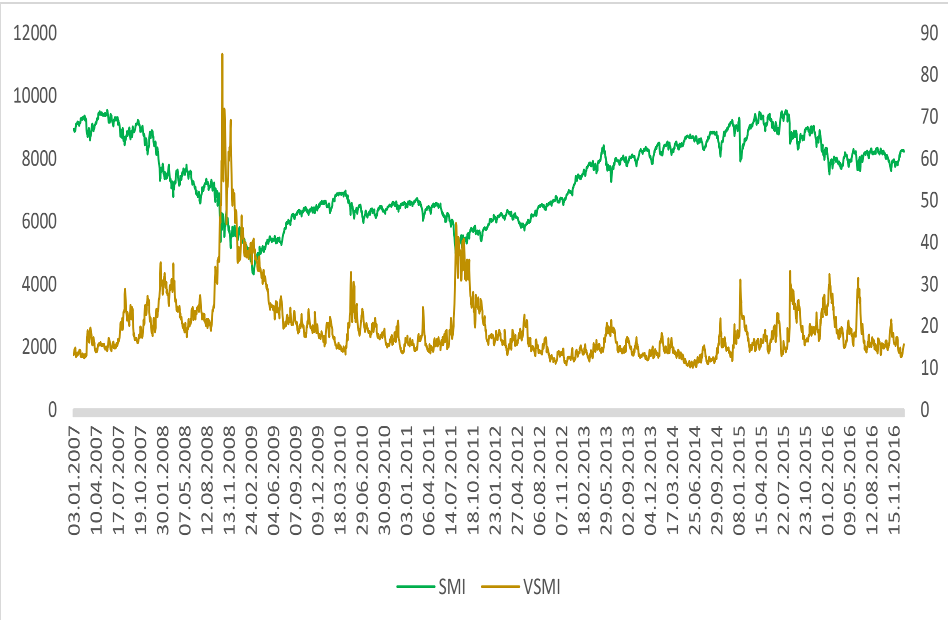


Figure 3.3: Swiss blue-chip index versus its implied volatility index

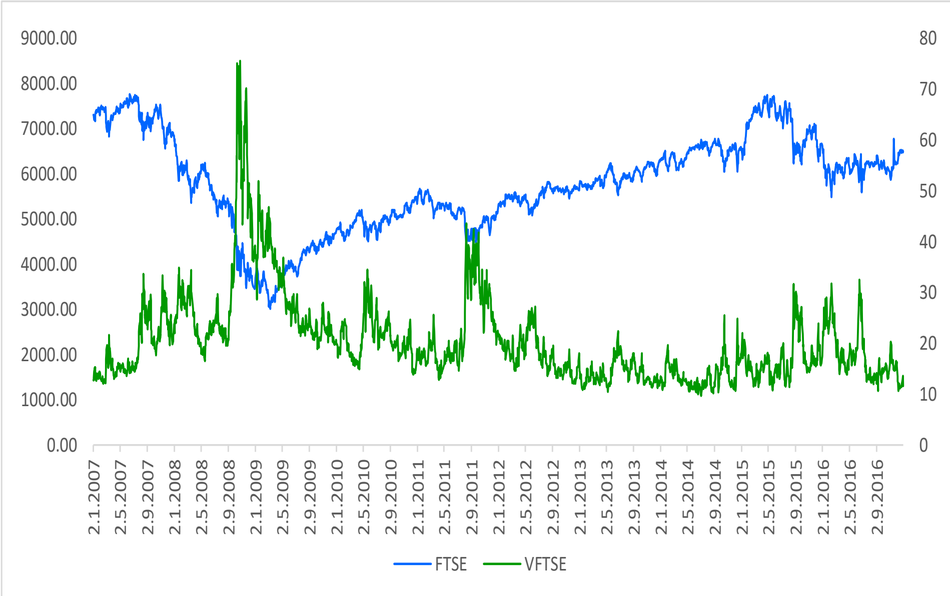


Figure 3.4: FTSE versus VFTSE

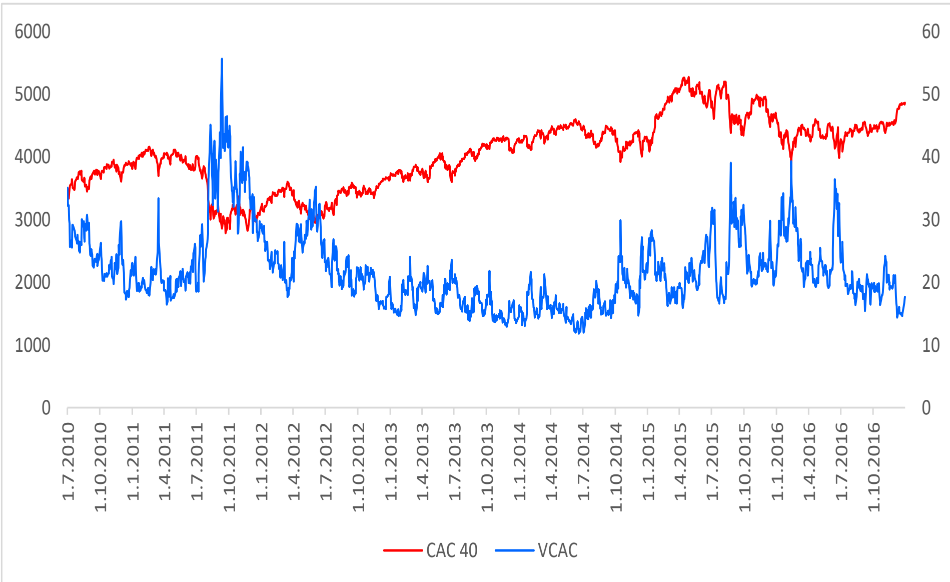


Figure 3.5: French blue-chip index versus its implied volatility index

3.7 Volatility Spillovers Across Markets

Individual investors, institutional investors and researchers follow closely volatility indexes since the CBOE Vix was issued to the market. Not only the volatility expectation of the market, relation between volatility indexes and its underlying equity index, but also how each market affects other markets is a crucial issue in financial decisioning and regulations. For that reason, financial professionals need to know how a financial shock to a market affects another market. However there are a few studies on volatility transmissions across markets. Related studies mostly focus on the answers to these questions:

- Which market leads other markets? In other words, which market is the major supply of the implied information?
- How does a shock to an index affect the other indexes? What is the intense and the direction of such an affect?
- What is the duration of the impact?

In the study of Eun and Shim (1989), time series of daily stock market returns were interpreted via VaR analysis, and the U.S. stock market is found to be, by far, the most influential market in the world. Additionally, against U.S. shocks, all European and Asian-Pacific markets responded strongly with one day delay and most of the responses were completed within two days [22]. In the study of Hamao et al. (1990), it is clearly seen, the existence of price and volatility change had effect on international stock markets. In their study, London, New York and Tokyo markets were examined. Significant spillover effects of U.S. and U.K. stock markets on Japanese market were discovered. At the same time, spillover effects between these two markets were weak [28]. A parallel result was founded in the study of Susmel and Engle (1994). They investigated the relation between New York and London stock markets, and they observed that, there was no strong evidence of volatility spillovers between these markets. In addition, the most significant effects arise around New York stock market's openings [44].

In the paper of Wagner and Szimayer (2004), the behavior of the implied volatility indexes of United States and Germany was investigated: Vix and VDAX particularly. They concentrated on jump risks of implied volatility indexes in the time period of 1992 and 2002. Their findings showed that VDAX had almost 2 times of jump intensity as compared to the Vix. Moreover, jumps were mostly country-specific. They also indicated that foreign exchange market movements might have resulted volatility shocks [47].

Nikkinen and Sahlström (2004) studied the international equity market integration with respect to the uncertainty. This investigation was about the markets of United States, United Kingdom, Germany and Finland. The results revealed a high degree of integration among U.S., U.K. and German markets regarding implied volatility. While U.S. stock market was the leading source of the implied volatility; German market was the leading source for Europe. Uncertainty on U.S. market was spread other markets [40].

Äijö (2008) investigated the term structure linkages between VDAX, VSMI and VSTOXX and he found that, volatility term structures were highly correlated and they were closely linked to each other. Furthermore, it can be seen from this paper, variance of estimation of the implied volatility term structures of SMI and STOXX could be explained as much as 35% and 65% respectively, by the volatility term structure of the DAX, with the variance decomposition analysis [2].

Badshah (2009) studied volatility-return relations and volatility transmissions of Vix, VXN[9], VDAX and VSTOXX. In this study, negative and asymmetric return-volatility relationships were observed between each volatility index and its related equity index in the period of February 2001 and June 2008. Significant spillover effects were founded across volatility indexes. It is obvious from the results, Vix affects VXN, VDAX and VSTOXX significantly. Also a shock to the Vix had an effect on the other three volatility indexes for four to six days. In Europe, VDAX is the major source of the information. In this study, Granger causality, generalized impulse response function and variance decom-

9 CBOE NASDAQ 100 volatility index

position were used to investigate the volatility transmissions. Author stated that, Vix could explain VXN, VDAX and VSTOXX with average forecast error variances of 58%, 31% and 29%, respectively. In addition, VDAX could explain 52% of the forecast error variance of VSTOXX [5]. In the recent paper of Naik and Reddy (2016), they studied the dynamics of linkages between Vix, VDAX, Indian IVIX, South Korean VKOSPI, and Chinese VXFXI. This study showed that Vix was the most influential volatility index, and especially, had a great influence on Indian IVIX. Besides, Indian and Asian volatility indexes had the least interactions among these indexes. Consequently, Vix should have been interpreted as early warning signal for overseas markets [37].

3.8 Comparisons of Volatility Indexes and Realized Volatility

Fleming (1998) studied quality of market volatility forecasts implied by S&P 100 index options. This study reflected that, both the S&P 100 call and put options' implied volatilities were biased estimations. The level of the bias did not appear to be sufficiently large to signal the presence of abnormal trading profits. But also, a linear model using just implied volatility seemed to convey a good estimate of actual volatility. S&P 100 index implied volatility was efficient in these situations; it could be used as a market sentiment index; it might have been an alternative method for asset pricing model; and it might have been useful in estimating stock market returns [25].

In the study of Christensen and Prabhala (1998), they tried to find the answer to the question: Does the implied volatility obtained from S&P 100 index options predict the realized volatility or not? Unlike previous studies on implied volatility and realized volatility comparison, they used lower sampling frequency and non-overlapping data with longer time period. In this way they found that, implied volatility predicted future realized volatility by itself in conjunction with the history of past realized volatility [18].

Shu and Zang (2001) studied the relation of implied and realized volatility of S&P 500 index. They investigated the stability of this relationship with different measurements. They have used four different estimators to compute realized volatility, and two different models to compute the implied volatility. When both the implied volatility and the historical volatility were used to forecast realized volatility, they found out that the implied volatility surpassed the historical volatility and this result was valid for all measurements. On the other hand, all information covered in historical volatility was transmitted on implied volatility, and also historical volatility had no forecasting ability [42].

In Christensen and Hansen (2002), they studied whether implied volatility from OEX options could predict future realized volatility or no. They agreed with Christensen and Prabhala (1998) in that: the implied volatility was an unbiased and efficient forecast of future volatility; besides, it comprised information content of historical volatility. While Christensen and Prabhala (1998) achieved implied volatility from at the money options, they attained implied volatilities from both at the money, and out of the money options. On the other hand, they investigated implied volatilities derived from call and put options separately. Results of this study indicated that implied volatilities obtained from call options were better volatility forecasts than those from implied volatilities of put options [17].

Li and Yang (2008), investigated the relationship between implied and realized volatility of the Australian stock exchange on five years time series from 2001 to 2006. Australian stock exchange (S&P/ASX 200) index options were traded tenuously and in low trading volumes with long maturity cycles. After solving this problem through instrumental variable method, they observed that both put and call implied volatilities were superior to the historical volatility in estimating future realized volatility. And also implied volatilities achieved from call options were a better indicator for future volatility forecasts. It should be also noted that, they focused on the implied volatility from the Black-Scholes model [32].

In their study, Siriopoulos and Fassas (2008) showed that VFTSE contained all information about future volatility those already included in historical returns. In addition, they also showed that VFTSE and FTSE 100 index acted significantly negative and asymmetric [43].

Apart from these, in Figure 3.6 paths of realized volatility of S&P 500 and Vix were depicted. We see a high correlation of 0.91193 in the time period February, 2005–January, 2017.

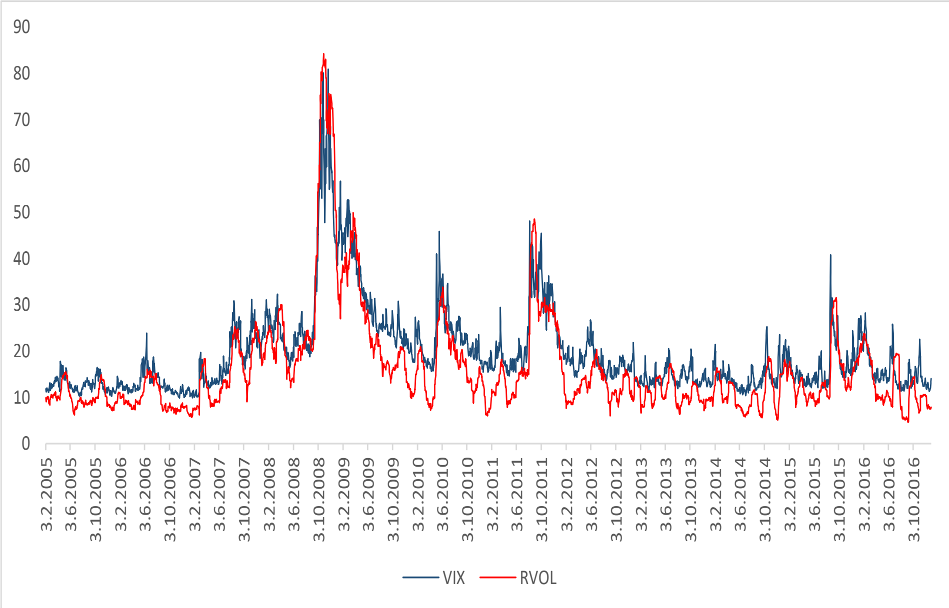


Figure 3.6: Vix and realized volatility of S&P 500 index

CHAPTER 4

YIELD CURVE MODELING

The term structure of interest rates is a characterization of interest rates as a function of maturity, also known as spot rate curve or zero-coupon bond yield curve. In other words, the term structure is a series of discount factors on the same period. On the other hand, forward interest rates are interest rates on loans and investments that begins at a future date, called settlement date, and lasts to a date further into the future, called maturity date. In the absence of forward rates, it needs to be derived from existing interest rates. For example, suppose that the central bank of a country, where the investment is planned to make, has only a certain number of bonds, say 1-year, 2-year, 3-year, 5-year, and 10-year bonds. Assume that the investment is planned for 8 years in this country and it is needed to know the 8-year bonds' yield, but there is no bond issued with maturity of 8 years. In such a scenario, the interest rate for the 8-year bond is needed to be calculated. And this is the process of finding a curve or a function that fits best to the set of observed data.

Reliable information about term structure has great importance for investors, financial professionals, and policy makers. Besides, most of the central banks consider implied forward rates as a monetary policy indicator. In order to provide reliable information about term structure of interest rates, there have been a lot of different techniques to estimate them.

It will be useful to make the following definitions as they will be used in later sections:

Discount function: Price of zero-coupon bond, with a face value of 1 unit of money, as a function of maturity.

Spot yield curve: Spot rates, in other words, zero-coupon bond yields as a function of maturity.

Forward yield curve: Zero-coupon bond forward yields, equivalently, forward rates as a function of maturity.

In general, there are 2 types of curve fitting techniques mostly used to estimate forward rates:

(1) Parametric / Regression methods

(2) Non-parametric methods / Interpolation

(1) *Parametric/Regression Models*: These methods are based on the least squares approach in general. Here, we try to find the smooth curve that best fits the data, but the curve does not need to pass through any data point. Nelson & Siegel method and its expanded version by Svensson are two commonly used parametric methods. The fitting curve can be an exponential, trigonometric or any order polynomial function. For the sake of simplicity, we will mention about linear and polynomial regressions briefly.

(1-a)*Linear regression:* In linear regression, we are trying to best fit a straight line to the set of data (x_1, y_1), (x_2, y_2), $\ldots$, (x_n, y_n). A Linear function is in the form of

$$f(x) = a_0 + a_1 x + e,$$

here a_0 is the intercept, the value of $f(x)$ when $x = 0$, a_1 is the slope and e is the error term. Our goal is to pass the line of this function through the data set with the least possible estimation error. The estimation error is calculated as follows:

$$e_i = y_{i,measured} - y_{i,model} = y_i - (a_0 + a_1 x_i)$$

Our goal is to minimize the error:

$$\underset{a_0,a_1}{\text{minimize}} \sum_{i=1}^{n} e_i^2 = \underset{a_0,a_1}{\text{minimize}}\, E_T(a_0,a_1) = \sum_{i=1}^{n}(y_i - a_0 - a_1 x_i)^2$$

To find a_0 and a_1, we need to take the derivatives of E_T with respect to a_0 and a_1 separately, and then by equating both expressions to zero in order to solve for a_0 and a_1:

$$\frac{\partial E_T}{\partial a_0} = -2\sum_{i=1}^{n}(y_i - a_0 - a_1 x_i) = 0 \tag{4.1}$$

$$\frac{\partial E_T}{\partial a_1} = -2\sum_{i=1}^{n}(y_i - a_0 - a_1 x_i)x_i = 0 \tag{4.2}$$

Below is the graph of the linear regression for the data set consisting of points (1,1), (3,4), (5,6), (9,12), (16,19), (22,23), (35,39), (43,48), (56,62), (78,88), (98,99) and (111,123).

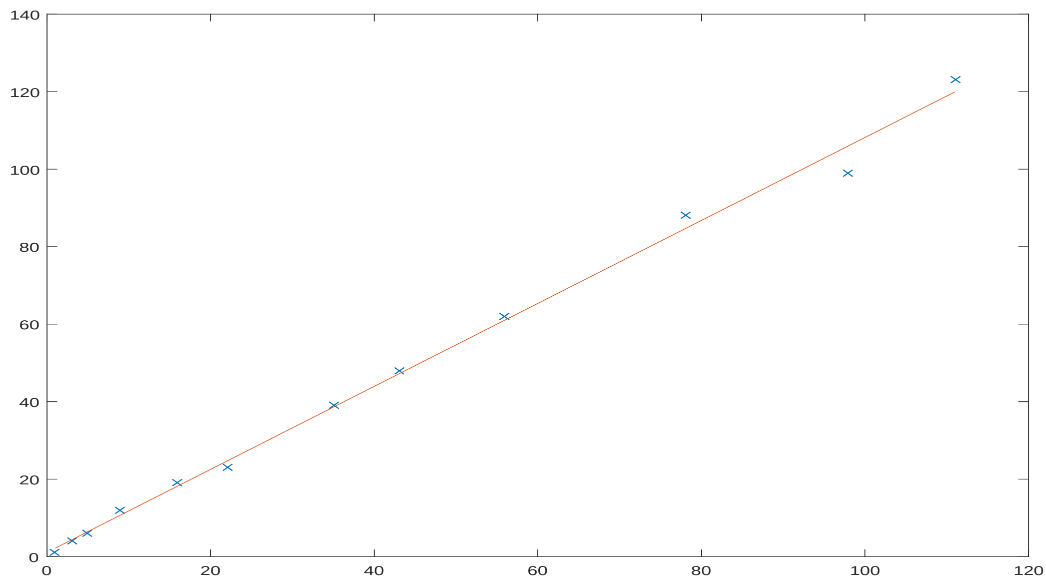


Figure 4.1: An illustration of Linear Regression

(1-b)*Polynomial regression:* Linear regression is the first order polynomial regression, now we will look at n^{th} order polynomial regression in general. An extension of linear regression fitting is n^{th} order polynomial regression, and general form of the n^{th} order polynomial function is:

$$y = f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 + e$$

Degree of the polynomial can be 2, 3 or a higher value. If $n = 2$, polynomial is called quadratic, if $n = 3$ cubic and if $n = 4$ quartic polynomial. Particularly, if $n = 1$ it is coincides with the linear regression. These models reliability increases significantly when they are built on large numbers of data. Polynomial regression models can be represented in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ . \\ . \\ y_m \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 & . & . & . & x_1^n \\ 1 & x_2 & x_2^2 & . & . & . & x_2^n \\ 1 & x_3 & x_3^2 & . & . & . & x_3^n \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 1 & x_n & x_n^2 & . & . & . & x_m^n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ . \\ a_n \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ . \\ . \\ . \\ e_m \end{bmatrix} \tag{4.3}$$

here $m < n$. The matrix on the left side of the equation is $\vec{y}$, the $n \times m$ dimensional matrix is X, the matrix composed of coefficients is $\vec{a}$ and the matrix of the error terms is $\vec{e}$. So, Eq. 4.3 can be rewritten as $\vec{y} = X\vec{a} + \vec{e}$.We proceed, as in the linear regression, to find the coefficients $a_0, a_1, \cdots, a_n$ by minimizing the error term:

$$\underset{\vec{a}=(a_0,\cdots,a_n)}{\text{minimize}}\ E_T(\vec{a}) = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - a_0 - \cdots - a_n x_i^n)^2$$

Thus, we need to solve

$$\frac{\partial E_T}{\partial a_i}, \quad i = 1, 2, \cdots, n.$$

Regression results of different order polynomials can be observed in Figures 4.2 and 4.3.

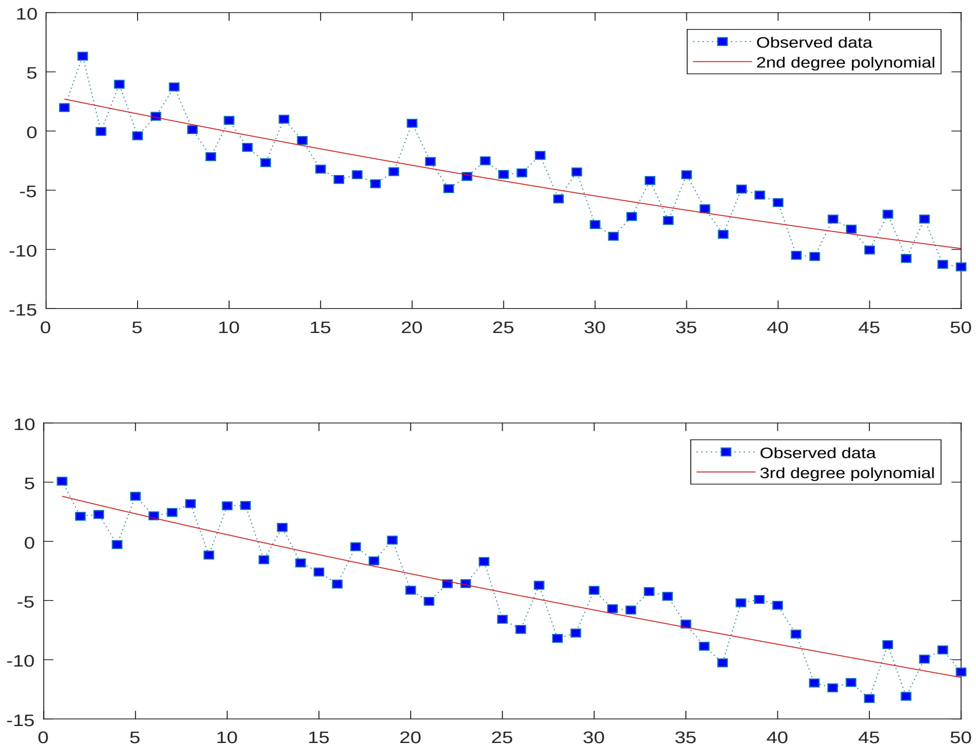


Figure 4.2: Change in fitting ability among 2nd and 3rd degree polynomials.

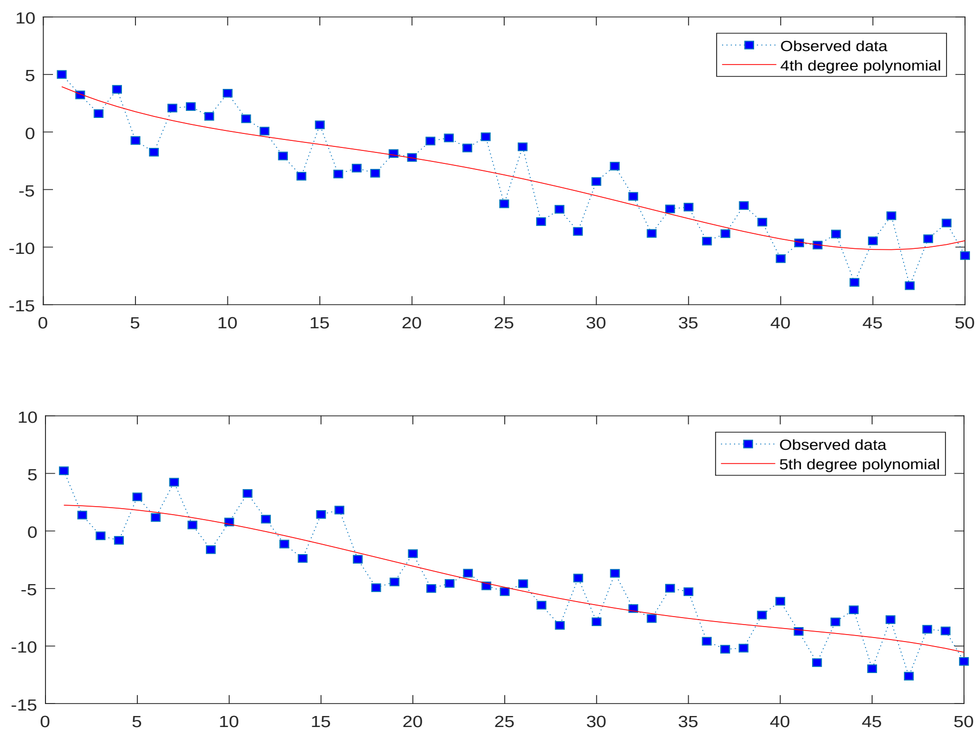


Figure 4.3: Change in fitting ability among 4th and 5th degree polynomials.

(2)*Non-parametric methods / Interpolation*: These types of curve fitting methods are used when a more precise fit is required. Interpolating function passes all the data points, unlike regression methods in sight. We will look at different interpolation functions shortly.

(2-a)*Linear interpolation:* The simplest type of interpolation is linear interpolation. Linear interpolation means that all consecutive data points are linked to each other pair by pair with straight lines. As we have already known, linear interpolating function that crosses points of (x_0, y_0) and (x_1, y_1) is:

$$f(x) = y_0 + \frac{y_1 - y_0}{x_1 - x_0} \cdot (x - x_0).$$

Here we need to find all linear interpolating functions that link every consecutive data points.

For instance, we can see the linear interpolated graph of points (1,30), (2,13), (3,24), (4,50), (5,36), (6,29) can be observed in Figure 4.4. Such kind of interpolation when we have more than two data points is also called piece-wise linear interpolation.

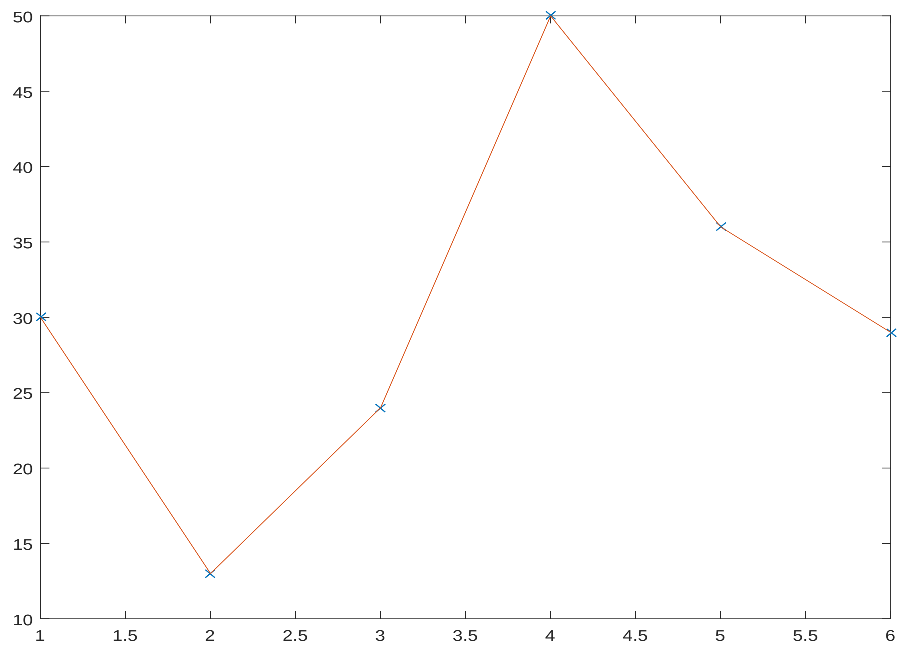


Figure 4.4: An illustration of Linear Interpolation

As it can be seen from the graph, if points are not close to each other, the graph becomes jagged, and may not be a good approximation between the two consecutive data points. Linear interpolation is advantageous in terms of practicality and provides efficient results for most situations.

(2-b) *Quadratic interpolation:* Quadratic interpolating function is created by adding a second order derivative information curvature to linear interpolating function. Here, we assume three different data points (x_0, y_0), (x_1, y_1) and (x_2, y_2) are given. We are going to interpolate these points with a second order polynomial. Our second order interpolating function using Newton's polynomials is of the form:

$$g(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1).$$

Then,

$$g(x_0) = y_0 = a_0, \quad g(x_1) = y_1 = a_0 + a_1(x_1 - x_0) \quad \text{and} \quad a_1 = \frac{y_1 - y_0}{x_1 - x_0}.$$

Similarly,

$$g(x_2) = y_2 = a_0 + a_1(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1)$$

and

$$a_2 = \frac{\left(\dfrac{y_2 - y_1}{x_2 - x_1}\right) - \left(\dfrac{y_1 - y_0}{x_1 - x_0}\right)}{x_2 - x_0}.$$

Sum of the first and the second terms of the $g(x)$ is linear interpolating part of the function, and the third term of the function generates second order curvature. You can see the result of Newton's polynomial interpolation of the same data in linear interpolation example in Figure 4.5.

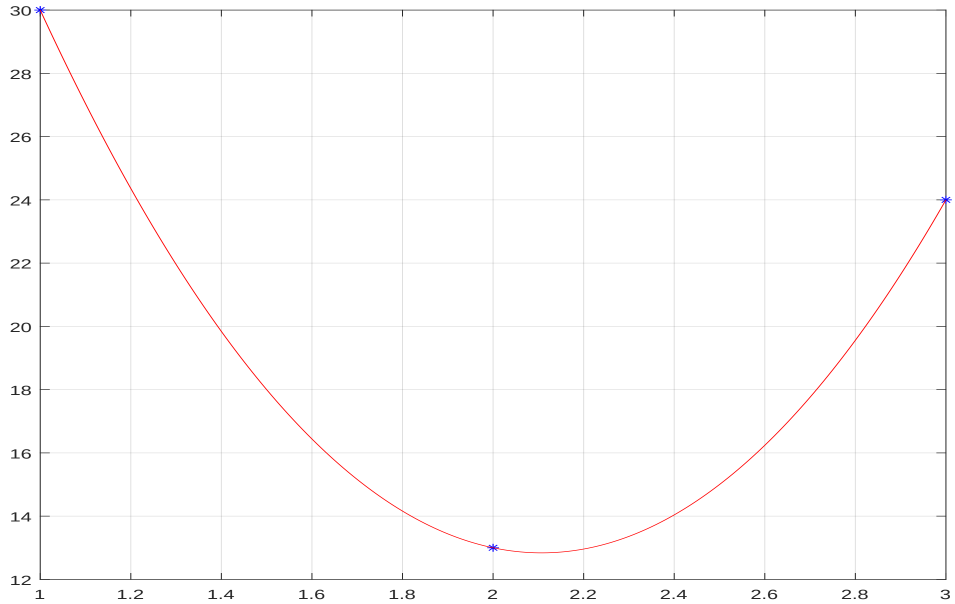


Figure 4.5: Newton's polynomial interpolation

(2-c) *Cubic and Cubic Spline interpolation:* Cubic and cubic spline interpolation techniques are commonly used interpolation methods due to their ability to represent data successfully. In these methods we utilize a third degree polynomial function of the form:

$$h(x) = a_3x^3 + a_2x^2 + a_1x + a_0.$$

The difference between cubic interpolation and cubic spline interpolation is that: In cubic interpolation we need at least four data points to compute the polynomial. By four data points we have three different interpolation intervals, and cubic interpolation is here to connect them with third degree polynomials. On the other hand, cubic spline interpolation can be thought as more aesthetic representation of cubic interpolation. Spline interpolation is achieved by adding the derivatives of the endpoints of each interval to the account. When the same data set is used and interpolated by "Cubic polynomials and Splines"we obtain more satisfactory results, in general, as depicted in Figure 4.6 and Figure 4.7, respectively.

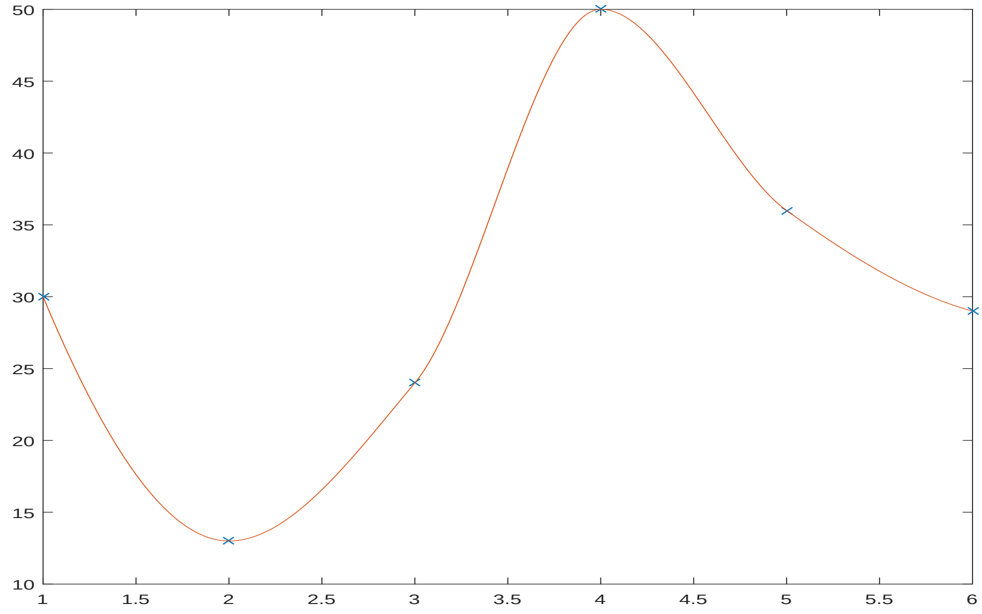


Figure 4.6: Cubic interpolation

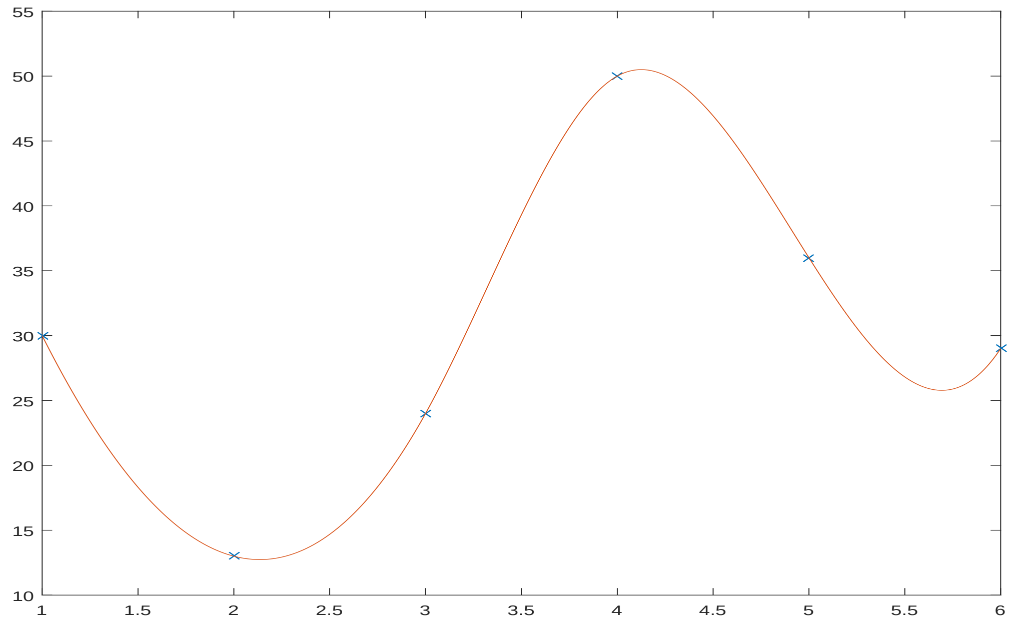


Figure 4.7: Spline interpolation

4.1 Earlier Studies on the Yield Curve Modelling

Development of yield curve fitting techniques started in the seventies with the paper of McCulloch (1971) [34]. Study of Brennan and Schwartz (1979), which was published a few years after McCulloch's, was an important step. They developed a term structure model that assumed the value of a default free bond which might have been written as a function of instantaneous and long term rates [11]. In the study of Cox, Ross, and Ingersoll (1985), they applied a rational asset pricing model to examine the term structure of interest rates, and this model also contained the main factors which were consistent with maximizing behavior and rational expectations [19].

The objective of the study of Langetieg (1980) was to develop a term structure model that was extensive enough to cover a large number of possible macroeconomic relationships. This model required the assumption that the spot rate could have been represented as a linear function of economic factors those followed a joint elastic random walk. The term structure was a simple composition of expected spot rates and the term premium, where the term premium was a deterministic function of the bond's risk vector [31]. As Vasicek (1982) has stated, the spot rate curves derived by the models of Cox et al.(1981), Langetieg (1981), and Vasicek (1977) did not perform well to the observed data on bond yields, moreover existing yield curves exhibited more varied shapes than curves of these models. In addition, the author used exponential spline fitting techniques in term structure estimation.

Vasicek model was one of the earlier stochastic models. This model described the short-term interest rates on the assumption that the instantaneous interest rates were Ornstein-Uhlenbeck process with constant coefficients. This model was a linear model with Gaussian distribution [46]. Estimation of the term structure needed to fit the bond data as much as possible, and a smooth function. These were essential for a good estimation model.

Papers of Mc Culloch (1971) and Vasicek (1982) took an important place among studies on spline methods. McCulloch proposed an approach with a piecewise polynomial spline fitted to price data to the present value function [34]. On the other hand, most attractive term structure estimation studies with parametric models were the works of Cohen et al.(1966), Fisher (1966), Echols and Elliott (1976), Dobson (1978), Chambers et al.(1984) and of course Nelson & Siegel (1987).

In this chapter, we will focus on Nelson & Siegel (1987), and its improved version by Svensson (1994). Nelson & Siegel's study was a milestone in the subject of yield curve modeling. The aim of the study of Nelson & Siegel was to introduce a simple parametric model that was flexible enough to represent monotonic, humped, and S-shaped curves which were generally associated with the yield curves. This model has been accepted by a large group of financial world due to its ease in practice. Many central banks trust and have been using this model for years. Nelson & Siegel tested the procedure on the U.S. Treasury bill yields with four weeks intervals over three year period. The form of the model was motivated by the solution of second order ordinary differential or difference equations. Firstly, we need to give a short description about difference equations; the role of differential operator is similar. Difference equations are sometimes called recurrence relations.

Definition 4.1. Let $w(t)$ be a function of a real (or complex) variable t. The difference operator Δ is defined as:

$$\Delta w(t) = w(t+1) - w(t).$$

The difference operator mentioned above is a first order difference operator, and higher order difference operators or differences can be defined by compounding the difference operator with itself. In this way we may obtain a second order difference, for instance,

$$\begin{aligned}\Delta^2 w(t) &= \Delta(\Delta w(t)) = \Delta(w(t+1) - w(t)) \\ &= \Delta(w(t+1)) - \Delta(w(t)) \\ &= (w(t+2) - w(t+1)) - (w(t+1) - w(t)) \\ &= w(t+2) - 2w(t+1) + w(t).\end{aligned}$$

Similarly, for an n^{th}-order difference, we have,

$$\begin{aligned}\Delta^n w(t) &= w(t+n) - n\cdot w(t+n-1) + \\ &\frac{n\cdot(n-1)}{2!}\cdot w(t+n-2) + \cdots + (-1)^n \cdot w(t) \\ &= \sum_{k=0}^{n} (-1)^n \cdot \binom{n}{k} \cdot w(t+n-k).\end{aligned}$$

Definition 4.2. Let $q(t)$ and $p(t)$ be functions. The first-order linear difference equation is:

$$w(t+1) - q(t)\cdot w(t) = p(t).$$

This equation is called first order difference equation, because it includes the value of w only at t and $t+1$. The n^{th}-order difference equation can be written as,

$$q_n(t)\cdot w(t+n) + q_{n-1}(t)\cdot w(t+n-1) + \cdots + q_0(t)\cdot w(t) = p(t),$$

where $q_0(t), q_1(t), \ldots, q_n(t)$ and $p(t)$ are assumed to be known, and also $q_n(t) \neq 0$ for all t. In addition, if $p(t) \equiv 0$ then equation is said to be homogeneous otherwise, $p(t) \not\equiv 0$, non-homogeneous.

Nelson & Siegel considered the second order differential, or difference, equation and its solution in their well-known model. Here $r(m)$ represents the instantaneous forward rate at maturity m, and modeled as

$$r(m) = \alpha_1 \cdot r(m-1) + \alpha_2 \cdot r(m-2) + \alpha_0. \tag{4.4}$$

If the zeros of $r(m)$ are real and lie outside the unit circle the solution has the form;

$$r(m) = \beta_0 + \beta_1 \cdot \exp\left(\frac{-m}{\tau_1}\right) + \beta_2 \cdot \exp\left(\frac{-m}{\tau_2}\right). \tag{4.5}$$

Here τ_1 and τ_2 are constants representing the time; and β_1 and β_2 are constants to be determined. Different forward rate curves, monotonic, humped or S-shaped can be generated by changing β_1 and β_2. These curves have the asymptote β_0. $r(m) = \beta_0$ as m tends to infinity, and $r(m) = \beta_0 + \beta_1 + \beta_2$ as $m \to 0^+$

The yield to maturity on a bill, called as $R(m)$, is the average of the forward rates:

$$R(m) = \frac{1}{m}\int_0^m r(x)dx.$$

Experiments with the model in Eq. 4.5 showed that the model is over-parameterized, because as the values of τ_1 and τ_2 change, it is possible to get values of the β's that give almost nearly the same fit. Therefore a plainer model could give the same curves; this was achieved by the solution of Eq. 4.4 for the case of two equal real roots:

$$r(m) = \beta_0 + \beta_1 \cdot \exp\left(\frac{-m}{\tau}\right) + \beta_2 \cdot \left[\left(\frac{m}{\tau}\right) \cdot \exp\left(\frac{-m}{\tau}\right)\right]. \quad (4.6)$$

In this representation, β_0 is the contribution of the long term component, β_2 is the medium term components' contribution and β_1 is the short term components' contribution. On the other hand, τ is the time constant which determines the rate of decay of regressors. As it can be deduced that small values of τ mean sudden decays in the regressors, while large values of τ provide slow decays. So, while small numbers of τ's are good at fitting curve at low maturities, big τ's are good at fitting curvature at longer maturities. In Figure 4.8, thin line represents the fitted curve with τ of 20, and thicker one indicates the fitted curve with τ of 100 [38].

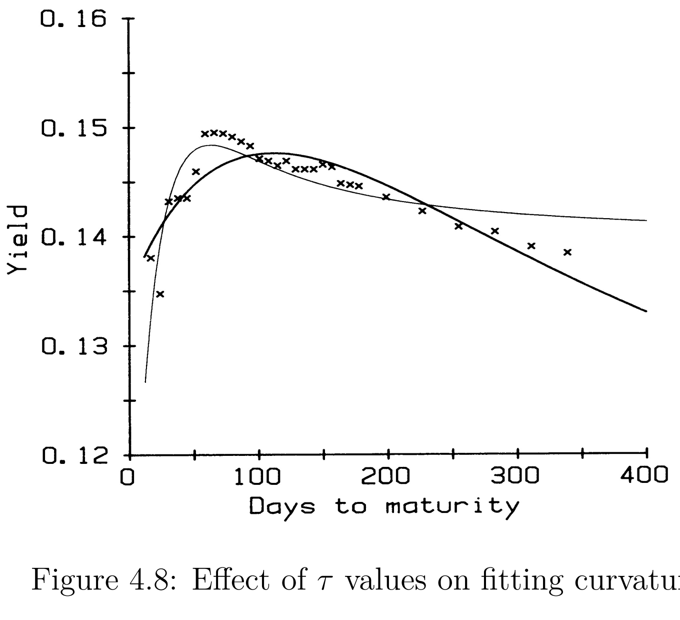


Figure 4.8: Effect of τ values on fitting curvature

The yield to maturity can also be obtain as:

$$R(m) = \frac{1}{m}\int_0^m r(x)dx = \beta_0 + (\beta_1 + \beta_2) \cdot \left[\frac{1 - exp\left(\frac{-m}{\tau}\right)}{\frac{m}{\tau}}\right] - \beta_2 \cdot exp\left(\frac{-m}{\tau}\right). \quad (4.7)$$

Limiting values of $R(m)$ and $r(m)$ can be achieved from Eq. 4.7 and Eq. 4.6, respectively. Intuitively, limiting values of $r(m)$ and $R(m)$ will be the same, because forward rate $R(m)$ is an average of $r(m)$:

$$\lim_{m\to 0} r(m) = \beta_0 + \beta_1, \text{ and } \lim_{m\to\infty} r(m) = \beta_0.$$
$$\lim_{m\to 0} R(m) = \beta_0 + \beta_1, \text{ and } \lim_{m\to\infty} R(m) = \beta_0.$$

In the last section of their study, Nelson & Siegel stated that the parsimonious model presented approximately 96% of the variation in U.S. treasury bills throughout the period of $1981 - 1983$, with a standard deviation of residual errors of 7.25 basis points. On the other hand, when determining coefficients of the curve, firstly an appropriate time constant was selected and then this value was included in the formula to obtain the most suitable coefficients (the β's) with the method of least squares [38].

Svensson (1994) investigated and extended Nelson & Siegel (1987)'s model to a more flexible version. As Svensson (1994) had stated in his study, third term of Eq. 4.6 was generating a hump-shape, or U-shape if β_2 is negative. In order to increase the flexibility of Nelson & Siegel's fit, Svensson (1994) added a new term into the formula, which generated a second hump or U-shape. This new term includes two additional parameters, β_3 and τ_2. Here τ_2 have to be a positive real number. The model reads:

$$r(m) = \beta_0 + \beta_1 \cdot \exp\left(\frac{-m}{\tau}\right) + \beta_2 \cdot \left[\left(\frac{m}{\tau}\right) \cdot \exp\left(\frac{-m}{\tau}\right)\right] + \beta_3 \cdot \left[\left(\frac{m}{\tau_2}\right) \cdot \exp\left(\frac{-m}{\tau_2}\right)\right]. \quad (4.8)$$

The components of the Svensson (1994) model:

- β_0 is a constant and has the same role as in the Nelson & Siegel (1987)'s model.

- $\beta_1 \cdot \exp\left(\frac{-m}{\tau}\right)$ is the exponential term which is monotonically decreasing or increasing depends on β_1. This term also exists in Nelson & Siegel (1987)'s formula Eq. 4.6.

– $\beta_2 \cdot \left[\left(\frac{m}{\tau} \right) \cdot \exp \left(\frac{-m}{\tau} \right) \right]$ term generates hump or U-shape, depending on β_2. If $\beta_2 > 0$, term generates hump shape otherwise, $\beta_2 < 0$, U-shape. This term also exists in Nelson & Siegel (1987) model, too.

– The last and the new term is $\beta_3 \cdot \left[\left(\frac{m}{\tau_2} \right) \cdot \exp \left(\frac{-m}{\tau_2} \right) \right]$. This term is added to Nelson & Siegel (1987) model by Svensson (1994), in order to increase flexibility and fitting performance of the original model. With this fourth component, two parameters were added to the model, β_3 and τ_2. This term generates second hump or U-shape.

Parameters in this model are picked to minimize the sum of squared errors between estimated and observed yields. In most cases, Nelson & Siegel (1987) model gives a reasonable fit, yet at times when the term structure is more intricate the original Nelson & Siegel fit is not flexible enough. At that point the extended version of the model enhances the fit impressively.

The following important information was attached in the appendix part of the Svensson (1994): If the extended Nelson & Siegel model was used it was clear that, perfect multicollinearity resulted in the case of $\tau = \tau_2$. In this case $\beta_2 + \beta_3$ could be determined, but not individually β_2 and β_3. This means that the curve has two hump-shapes on top of each other and the model was over-parameterized. So, it is crucial that appropriate initial values with $\tau \neq \tau_2$ are selected. Besides, if the estimation converged to $\tau = \tau_2$, then appropriate fit is the original Nelson & Siegel fit, but not the extended version of Svensson (1994) [45].

The spot rate in this extended version can be obtained as:

$$R(m) = \frac{1}{m}\int_0^m r(x)dx = \beta_0 + \beta_1 \cdot \left[\frac{1-\exp\left(\frac{-m}{\tau}\right)}{\frac{m}{\tau}}\right] + \beta_2 \cdot \left[\frac{1-\exp\left(\frac{-m}{\tau}\right)}{\frac{m}{\tau}} - \exp\left(\frac{-m}{\tau}\right)\right] + \beta_3 \cdot \left[\frac{1-\exp\left(\frac{-m}{\tau_2}\right)}{\frac{m}{\tau_2}} - \exp\left(\frac{-m}{\tau_2}\right)\right]. \quad (4.9)$$

Table 4.1, from the study of Pereda (2009), which was prepared based on the 2005 data of Bank of International Settlements shows that the original Nelson & Siegel model and extended version by Svensson were met with great interests of most central banks, and also these models are used by many central banks since then [41].

Table 4.1: Term structure models used by central banks (BIS 2005)

Central Bank	*Model*
Belgium	Nelson & Siegel, Svensson
Canada	Svensson
United States	Fisher-Nychka-Zervos
Finland	Nelson and Siegel
France	Nelson & Siegel, Svensson
Germany	Svensson
Italy	Nelson & Siegel
Japan	Fisher-Nychka-Zervos
Norway	Svensson
Spain	Svensson
United Kingdom	Anderson and Sleath (Svensson until 2001)
Sweden	Fisher-Nychka-Zervos (previously Svensson)
Switzerland	Svensson
European Union	Svensson

As is clear in Table 4.1, the model of Fisher, Nychka and Zervos (1995) is also a commonly used spline based estimation method. There is no doubt that, study of Fisher et al. (1995) is the most respected representative of the spline based curve fitting techniques. In the study of Fisher et al. (1995), they claimed that, spline based methods gave better results than parametric models; and they supported their claim with the results they have achieved in their experiments: Nelson & Siegel (1987) model had 2.5 to 3 times larger pricing errors than the spline based methods [24].

"Why other central banks prefer to use parametric models?"Because parametric models give satisfactory results in most situations where it is not necessary to get a very precise estimation; besides, these models are more practical and more applicable. Bliss (1996) made a comparison of five different term structure fitting techniques with series of parametric and non-parametric tests. Those were: Un-smoothed Fama-Bliss method, Smoothed Fama-Bliss method, McCulloch method, Fisher-Nychka-Zervos method and extended Nelson & Siegel method. In this study Bliss had benefited from the most commonly used criteria in this subject for evaluating and comparing methods: Duration-weighted mean of the absolute fitted price errors and hit rate. Based on test results, the Fisher-Nychka-Zervos method was the worst decision among those techniques. Because this model mis-priced short maturities; on the other hand, it was vulnerable to measurement errors of the data. According to the results of this study, un-smoothed Fama-Bliss method was the best performing model; however, the practitioners those wanted to utilize parametric models were advised to use smoothed Fama-Bliss or extended version of Nelson & Siegel model [9].

There are other studies that compare the existing methods. For instance, Muvingi and Kwinjo (2014) compared the original Nelson & Siegel model and the extended version of the model to determine which method was more appropriate to Zimbabwean Bank. T-tests showed that Svensson (1994)'s model was more suitable for this market [36]. In the paper of Aljinović et al. (2012), they investigated which model is more appropriate for the Crotian financial market, Nelson & Siegel (1987) or Svensson (1994). Based on statistical tests they made, Svensson model gave more accurate yield curves on this market than the original model [3]. Marciniak (2006) investigated in detail the most commonly used yield curve estimation methods for the National Bank of Poland. Mostly, comparisons were made among the Svensson model and B-spline model with a variable roughness penalty (VRP). The author mentioned that the low elasticity of the Svensson model at the short end of the yield curve, high degree of instability, non uniqueness of estimations were the weaknesses of this model. Besides, author added that the biggest advantages of Svensson's model was the low complexity and easy computability. This is no longer a big advantage in today's world due to improvements in computer technology. For this reason many institutions changed their choices of yield curve estimation techniques in the direction of piecewise polynomial models with a variable roughness penalty [33].

When the term structure studies related to Turkish market are of interest, we recall the studies of Yoldaş (2002) and Alper et al.(2004). Yoldaş (2002) compared the McCulloch spline method, Nelson & Siegel model and Chambers-Carleton-Waldman exponential polynomial methods on Turkish bond market. Comparisons showed that the Chambers-Carleton-Waldman model was superior to the other two methods in fitting Turkish Treasury bill term structure [50].

Alper et al.(2004), on the other hand, estimated monthly yield curves in Turkish Secondary Government Securities Market with data in the time period of 1992–2003. They utilized both McCulloch spline method and Nelson & Siegel parametric method. Data set used in this study was constructed with monthly volume weighted average price and maturity. In-sample and out-of-sample prediction performances of these two models were compared. Briefly, while Nelson & Siegel model gave better estimation results than McCulloch method at out-of-sample and in the middle region of the curve, McCulloch method was superior to Nelson & Siegel model in in-sample properties [4].

CHAPTER 5

IMPLEMENTATION - BIST 30 VOLATILITY INDEX

Although the history of the İstanbul Stock Exchange[1] dates back to 1866, the basis of the current system was formed with the regulations issued in 1922. With the capital markets law published in July 1981, a new form and content were given to the İstanbul Stock Exchange while the capital markets was being reorganized. At last, on December 26, 1985, İstanbul Stock Exchange opened and on January 3, 1986, trading started officially. Borsa İstanbul A.Ş., mostly known with its abbreviation of BIST was founded on December 30, 2012, and registered on April 3, 2013 as a securities exchange of Turkey. İstanbul stock exchange gathered, under a single roof, all exchanges operating in the Turkish capital markets. This institution was established by bringing together the stock exchange İMKB, futures and options exchange (VOB) and İstanbul Precious Metals and Diamond Market (İAB). The main focus and field of activity of İstanbul stock exchange was stated in their official website as follows: "In accordance with the provisions of the Law and the related legislation, to ensure that capital markets instruments, foreign currencies, precious metals and gems, and other contracts, documents, and assets approved by the Capital Markets Board of Turkey are traded subject to free trade conditions in a facile and secure manner, in a transparent, efficient, competitive, fair and stable environment; to create, establish and develop markets, sub-markets, platforms, systems and other organized market places for the purpose of matching or facilitating the matching of the buy and sell orders for the above mentioned assets and to determine and announce the discovered prices; to manage and/or operate the

[1] It was called as İMKB (İstanbul Menkul Kıymetler Borsası) in Turkey

aforementioned or other exchanges or markets of other exchanges; and to carry out the other activities listed in its Articles of Association". There are three main equity indexes in Turkish stock market; BIST 100 (XU100), BIST 50 (XU050) and BIST 30 (XU030).

- BIST 30 index consists of 30 stocks selected among the stocks of companies traded on BIST Stars[2], BIST Main markets and the stocks of real estate investment and venture capital investment trusts traded on the Collective and Structured Products Market. These 30 stocks have the highest market values and liquidity among BIST 100 stocks.
- BIST 50 index consists of 50 stocks selected among the stocks of companies traded on BIST Stars, BIST Main[3] and the stocks of real estate investment and venture capital investment trusts traded on the Collective and Structured Products Market. BIST 50 index covers BIST 30 index.
- BIST 100 index is the main index of the Borsa İstanbul. Hundred stocks of the BIST 100 index are selected from BIST Stars, BIST Main and Collective and Structured Products Market as BIST 30 and BIST 50 indexes.

In addition, Borsa ṡtanbul operates 324 different stock indices, and 54 of these indices are calculated in real time while the rest of 324 are calculated once at each sessions close. Turkish market is one of the most exciting emerging markets in the world. Turkey's geographical position is one of the main factors that makes this market very important. Turkey's economy is export oriented economy, on the other hand it contains a lot of dynamic companies. Although there are many regional and political risks, interest of foreign investors in Turkish market is not at the least level. For these reasons, Borsa İstanbul is a market with high potential of growth and development. In Figure 5.1 we depict the price movements of major Turkish equity indices, BIST 30, BIST 50 and BIST 100 between January 2007 and January 2017 can be observed.

[2] BIST Stars is the market of companies whose value of traded shares in BIST 100 index and market value is equal or above TRY 100,000,000

[3] BIST Main is the market of companies whose market value is below TRY 100,000,000 according to the actual free float and shares not included in BIST 100 index

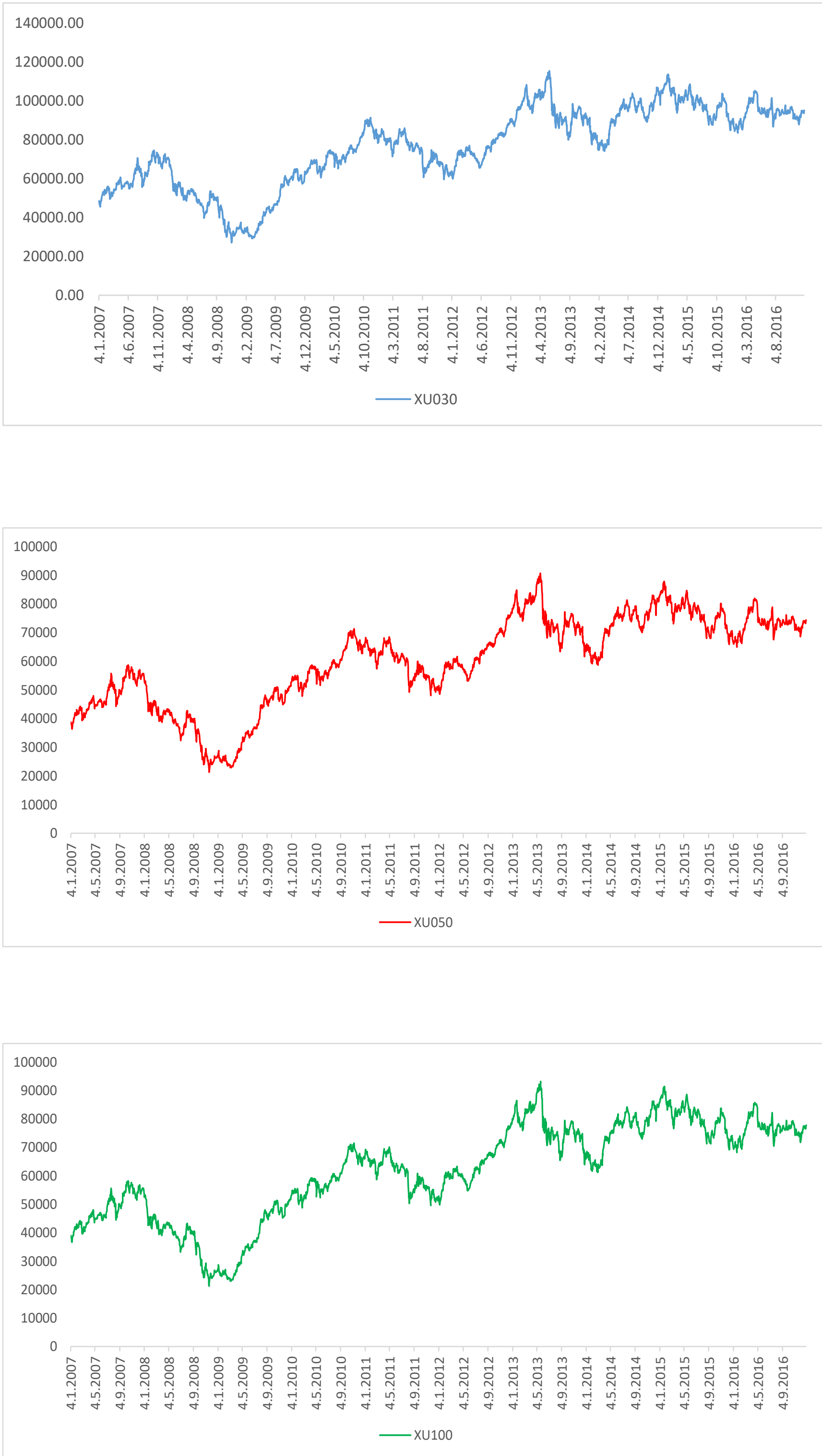


Figure 5.1: Price movements of BIST 30, BIST 50, and BIST 100 indexes from 2007 to 2017, respectively

Turkish market met derivatives products with the opening of IMKB futures and options market on December 21, 2012. Borsa İstanbul A.Ş. VIOP and Turkish Derivatives Exchange (TURKDEX) merged on August 5, 2013. Borsa İstanbul and London Stock Exchange Group (LSEG) have completed a comprehensive agreement covering derivative and index products on January 13, 2015. With this agreement, BIST 30 futures and options are traded on the London Stock Exchange since the second half of 2015 and this agreement gave international exposure to Turkish derivative products.

There are already BIST 30 and mini BIST 30 index options traded in Borsa İstanbul. Mini BIST 30 options are the new ones: Mini BIST option contracts were opened trading on September 19, 2014. Aim of issuing new kind of option series was simple, enabling more retail investors to trade in options market. Major difference between the BIST 30 and Mini BIST 30 options is that, while the contract size of the BIST 30 index options is 100 underlying security, Mini BIST 30 option contract size is only 1 unit of underlying security. Hence, in Mini BIST 30 option contracts, the level of buying and selling price will be more understandable for investors since these prices also show the contract sizes. Before attempting to create BIST 30 volatility index, it is needed to give information about BIST 30 and mini BIST 30 index option contract specifications. Specifications were taken from the official website of Borsa İstanbul in July 2017 [10]:

- Underlying security of these options is $\frac{1}{1000}$ of the BIST 30 equity index value.
- BIST 30 and Mini BIST 30 index options can be either European call or European put options.
- Contract size for the BIST 30 index options is 100 underlying securities, on the other hand this value is 1 for the Mini BIST 30 index options.
- Minimum price movements of the BIST 30 and Mini BIST 30 index options is TRY 0.01 per underlying security
- These options are cash settlement type of options.

– Options are trading from 09:30 to 18:15 continuously according to local time zone.

– Contract months of BIST 30 and Mini BIST 30 index options are February, April, June, August, October and December. Contracts with three different expiration months nearest to the current month are traded on the market at the same time. If December is not one of these three months, a new contract series with a maturity month of December is launched.

– Settlement dates are the first trading day following the maturity date.

– The daily settlement price is the weighted average price of all trades executed within the last 10 minutes of the trading session. If there are less than 10 trades in the last 10 minutes of the session, the weighted average price of the last 10 trades executed during the session will be assigned as the daily settlement price. If there are less than 10 trades in the whole session, the weighted average price of all trades executed during the session will be determined as the daily settlement price.

– Final settlement price of BIST 30 or Mini BIST 30 index call option is calculated by weighting the weighted time average of the index prices of the last 30 minutes of auction and closing value of the index with 80% and 20%, respectively. Then, the difference between the calculated weighted average price (divided by 1000) and strike price is rounded to the nearest price tick and called as the final settlement price of the BIST 30 or Mini BIST 30 index call option. Final Settlement price for BIST 30 or Mini BIST 30 index put option is calculated similarly as final settlement price of call options, the only difference is that, the difference between the strike price and the calculated weighted average price (divided by 1000) is taken, and rounded to the nearest price tick and in this way the final settlement price of the BIST 30 or Mini BIST 30 index put option is achieved.

– Maturity day is the last trading day of the contract month.

– Strike price tick is 2000 index points for BIST 30 index option contracts and 5000 index points for Mini BIST 30 index option contracts

– Maintenance margin for both Mini BIST and BIST options is 75% of the required collateral.

– Strike prices of BIST 30 and Mini BIST 30 options are set between the previous trading day's closing price plus 10% and minus 10% of that value. But also, board may decide to change these limits.

– For each maturity, at least 7 strike prices such that two of them are "in the money", one of them is "at the money" and the other four are "out of the money" shall be opened. And also, new strikes shall be opened automatically during the session according to the price fluctuations of the underlying security.

5.1 An Attempt to Create BIST 30 Volatility Index

Hereby we will try to explain the steps of the attempt to the creation of BIST 30 volatility index in detail and the results of this attempt. In this study, we have used XU030 (BIST 30) index options' daily closing prices, Turkish treasury bill prices, and the other related data on 1-year period of January 2016–January 2017. Data, used in this study, were obtained from the BIST Data Store, an official website of BIST [7].

In the calculation of the experimental volatility index, which was based on the CBOE Vix methodology, it was necessary to make some changes to adapt to our own data. For instance, Vix index is calculated using real-time quotes but this one is calculated using end-of-day data. Nevertheless, a few days we have encountered with exceptional cases those not mentioned in the white paper of Vix. So we had to continue index calculation in line with our own decisions. Changes we made and rare cases we have encountered are:

(1) As we mentioned deeply in the second chapter, in the original Vix formula, time to expiration T, is measured in minutes with three different components: Minutes remaining until midnight of the current day, number of minutes from mid-night until expiration time of options, and lastly total

minutes in the days between the current day and the maturity day. But in our index, we just take into account the remaining days between the current day and expiration day as the variable of T. We used only end-of-day data, so the other components of the time variable did not make sense for our calculation.

(2) In the original CBOE Vix formula, near-term options, which are at least a week from expiration, are taken into calculation. But we could not fulfill this restriction in our volatility index calculation in order to avoid pricing anomalies. Because there were not sufficient number of index options trading on Turkish Derivatives Exchange. If there were a sufficient number of second-nearby index options to calculate volatility index, we had not use index options with maturities lower than 10 days. When we could not find sufficient number of index options, we had to use the same options series till their maturities.

(3) In the original model, if an option has a bid price equals to 0, then that option is excluded from the index calculation. In our index, we excluded options from the calculation which have either bid or ask price equal to 0. The reason behind this limitation was that we could not get robust calculation if we had taken index options with zero ask prices into the calculation. It could be seemed like a contradiction, when we had to go through this limitation while we could not find enough index options, but this change was crucial in terms of consistency and smoothness of the index calculation.

(4) In desired index level F calculations, original formula uses differences of call and put options mid-quote prices, but in our formula we have used absolute differences of mid-quote prices instead of just taking the differences. This change was made to avoid inconsistent cases in the index calculation. As we have stated earlier, strike prices of XU030 index options are listed in 2000 points of intervals; for example, a series of index options have strike prices of $\ldots, 84000, 86000, 88000, 90000, \ldots$. In this case, using the original formula causes bad index option choices. In the U.S. market it does not pose a problem because there are small intervals: 5 points, between strike

prices. But in this study it makes a big difference. To illustrate, in the index calculations of January 5, 2016, if we had calculated according to the original formula, we would get $F = 87999.27327$, and we needed to assign $K = 86000$, besides we had to do index calculations according to this value. This would not reflect market expectations neutrally.

(5) On August 31, 2016 there was only one index option traded, a call option with August 2016 maturity, so on that day the index calculation could not be made.

(6) In the calculations of March 29, 2016 there were two pairs of index options with the same strike price differences for June 2016 maturity. Mid-quote prices were 4.22 and 4.11 for options with strike prices 100000 and 102000 respectively. Here we chose the option with strike price of 100000, due to the fact that if an option's price is higher it means that options strike price reflects a more common expectation of the market participants.

(7) On February 29, 2016 there were just three near-term call options and four near-term put options traded. Besides, among these, only one pair of call and put options had the same strike-price, 98000. So we were able to make near-term volatility calculation based on the pair, and none of the other near-term call options could be included in the calculation. The mentioned case can be seen in Table 5.1.

Table 5.1: An exceptional case encountered in Turkish derivatives market

Near Term Call Options			
Strike Price	*Bid Price*	*Ask Price*	*Mid-Quote Price*
98000	0.21	0.2	0.205
94000	0.01	0.23	0.12
88000	5	5	5
Near Term Put Options			
Strike Price	*Bid Price*	*Ask Price*	*Mid-Quote Price*
102000	10.6	10.6	10.6
98000	2.01	6.05	4.03
92000	0.01	1.99	1
90000	0.01	0.1	0.055

(8) Wide intervals of days to maturities of Turkish treasury bill data reduced the curve fitting ability of the Svensson model. Even more, the model gave negative interest rates in some cases, when days to maturity was below 10 days. When we had faced with such situations, in terms of the consistency of the study we continued to use the Svensson method. If the term structure model change had been made, the same negative results would arose. Besides, these negative values did not affect the index calculation significantly. In Table 5.2 the mentioned case in fitting February 2016 Turkish treasury bill data can be seen.

Table 5.2: Svensson model calculation results for February 2016

Days to Maturity (in years)	*Yield*	*Svensson Model Yield*
0.00274	-	-0.0014621
0.120548	0.01105	0.0105127
0.128767	0.01165	0.0113480
0.131507	0.01227	0.0116265
0.131507	0.01207	0.0116265
0.134247	0.01253	0.0119049
0.153425	0.01444	0.0138540
0.156164	0.01492	0.0141324

As it can be seen in Table 5.2, when we tried to find interest rates for very short-term periods, like a few days, we could not find rational values. The reason behind the mis-pricing in Table 5.2 was the huge gap between the maturity we were looking for, 1-day, and the nearest expiration day, 0.120548 years = 44 days. So, Svensson model could not find the appropriate value for 1-day risk free rate. But, it can be observed from Figure 5.2, Svensson model performs well except for such exceptional cases.

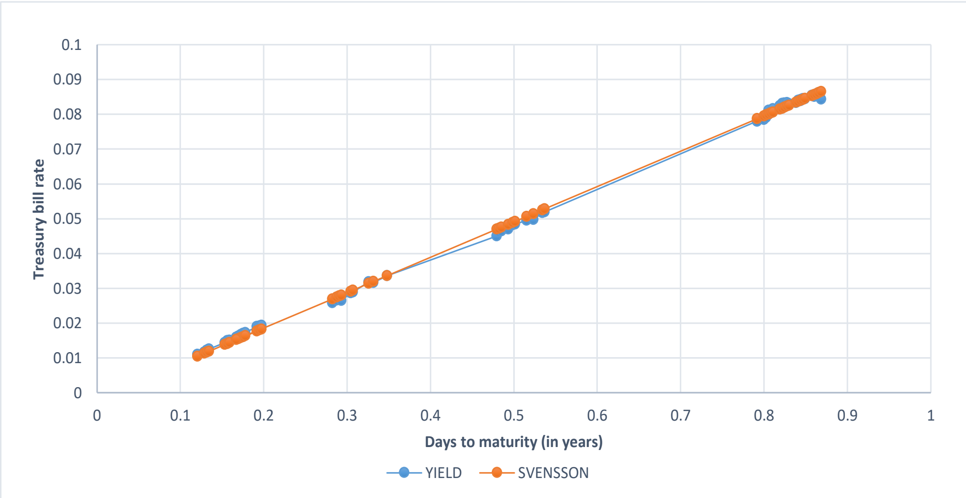


Figure 5.2: Curve fitting with Svensson model for February 2016 Turkish treasury bill data

5.2 Results

In this study we tried to construct an implied volatility index for Turkish market XU030 index using CBOE Vix methodology with constraints and amendments mentioned previously. Results of the study implies that it does not seem possible to create an efficient implied volatility index for the Turkish market with its current form. We will go into details on the reasons that led to this result later in this chapter, but firstly we need to examine the implied volatility index we constructed. We created the index with 2016–2017 period daily closing prices. One can ask that, could this time period be extended to the beginning of 2015? Answer is simple; this was impossible due to the insufficient number of index options traded on the market. While the period we used could not give logical results entirely, the longer periods would be more ineffective. We calculated implied volatilities of each 250 trading days and this index emerged. Index was designed to reflect markets 60 day volatility expectations, and we made a comparison of index results and realized volatility of XU030 index in the same time frame with 60-day intervals. In Figure 5.3 the graph of daily closing prices of XU030 index and daily calculated values of the implied volatility index can be seen. In Figure 5.4 the graph of implied volatility index and realized volatility comparison are shown. Besides, the movements of implied volatility index and XU030 index daily and 60-day returns can be observed in Figure 5.5 and Figure 5.6, respectively.

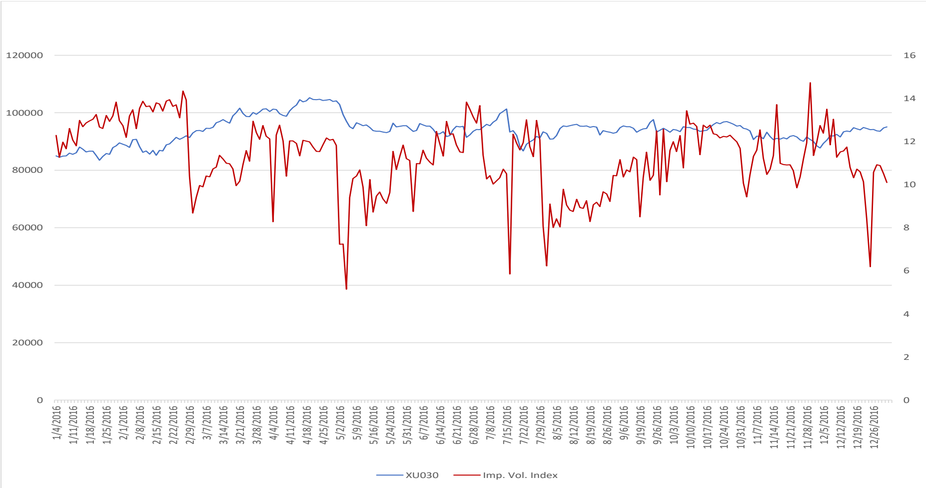


Figure 5.3: XU030 index closing prices vs. implied volatility index

Figure 5.4: XU030 Implied vs. realized volatility

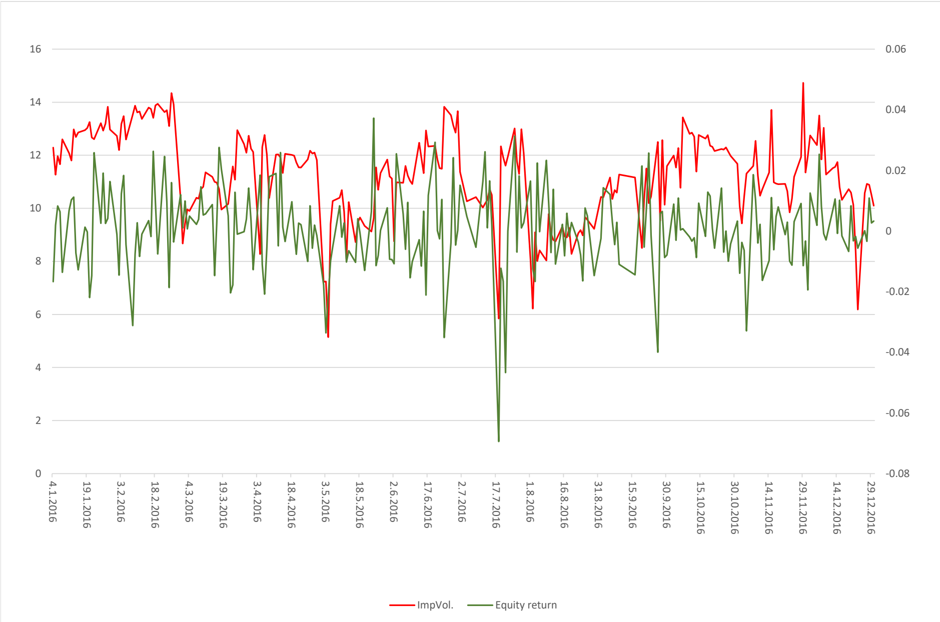


Figure 5.5: XU030 Implied volatility vs XU030 daily return

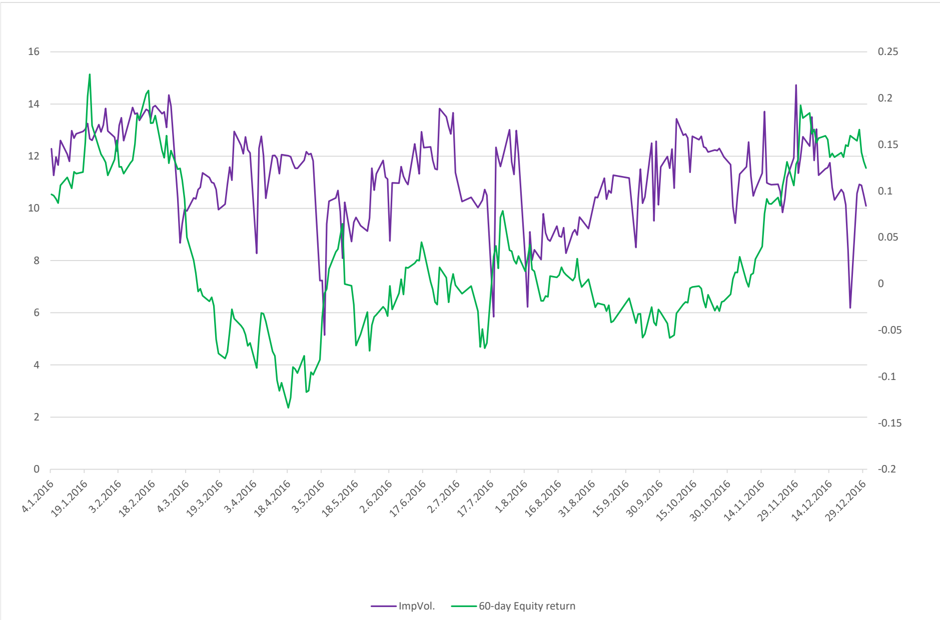


Figure 5.6: XU030 Implied volatility vs XU030 60-day return

Realized volatility and implied volatility follow almost the same path except some times at the line of the implied volatility, unlike the realized volatility. Sharp falls and large jumps were observed in the latter. In this context, we do not say that the implied volatility index failed to reflect market expectations. On the other hand, when we look at the correlation between realized and implied volatilities, -0.016752246, and correlation between equity index and implied volatility index, 0.115551359, we can not say that the implementation was successful. We have already examined relationships between equity indexes and their related implied volatility indexes in Chapter 3, particularly in Table 3.1. When we compare with the correlation values in that table, we see that there is a very weak relationship between the implied volatility index we created and XU030. In addition, we calculated 60-day return of XU030 index using the same time interval with the implied volatility index. We found correlation of these two data set as -0.041190509. It is also important to note that a period of at least 7 years was used in the comparisons of indexes in Table 3.1, but here we were able to calculate for 1-year period. So, if we had the opportunity to work on volatility index for a wider time interval, perhaps different results could be obtained.

5.3 An Additional Investigation

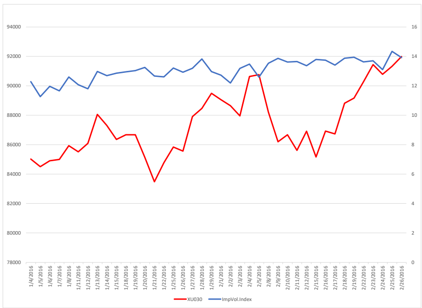
Due to observation of jumps in Figure 5.3, we wanted to investigate the performance of implied volatility index at out of jump times. We considered movements with more than 25% change as a jump and found 15 jumps. Dates and magnitudes of jumps can be seen in Table 5.3. According to jumps, we divided the whole time interval into 8 sub-intervals, which can be seen in Table 5.4. Then, we investigated the relation between XU030 equity index and its implied volatility index in each sub-interval. See Figure 5.7.

Table 5.3: Jumps in implied volatility index

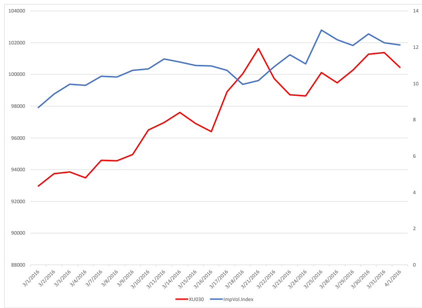
Jump Time	*Change in Imp. Vol Index Value*
February 29. 2016	-25.45 %
April 4, 2016	-31.75%
April 5, 2016	48.71%
May 2, 2016	-38.84%
May 4, 2016	-28.94%
May 5, 2016	82.80%
May 13, 2016	26.47%
June 3, 2016	25.39%
June 18, 2016	-44.33%
June 19, 2016	111.01%
August 1, 2016	-33.17%
August 3, 2016	46.19%
September 28, 2016	32.02%
December 23, 2016	-26.37%
December 26, 2016	70.92%

Table 5.4: Sub-intervals

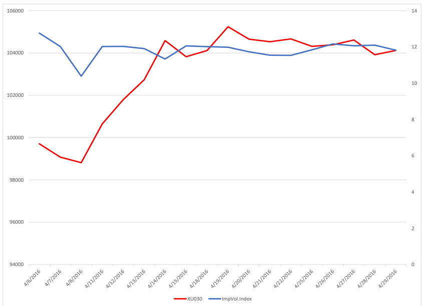
Sub-interval	*Length of interval*
January 4 – February 26	40 trading days
March 1 – April 1	24 trading days
April 6 – April 29	18 trading days
May 16 – June 2	13 trading days
June 6 – July 15	27 trading days
July 20 – July 29	8 trading days
August 4 – September 27	33 trading days
September 29 – December 22	61 trading days



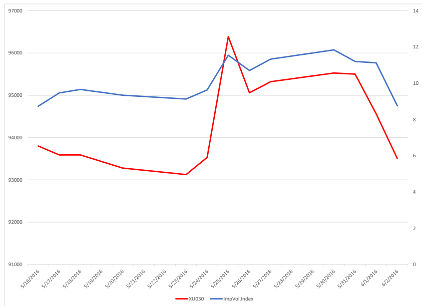
(a) January 4 – February 26



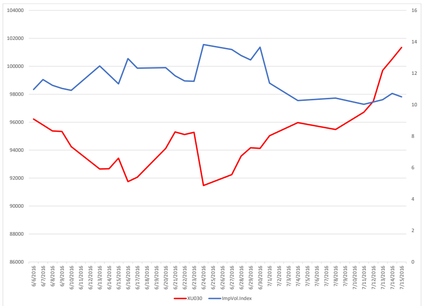
(b) March 1 – April 1



(c) April 6 – April 29

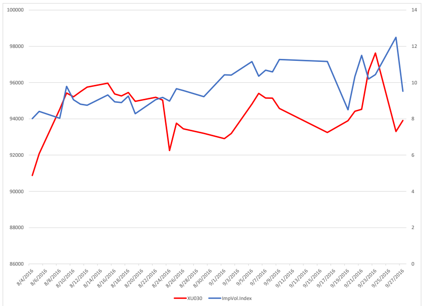


(d) May 16 – June 2

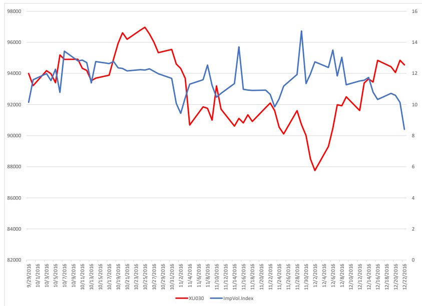


(e) June 6 – July 15

(f) July 20 – July 29



(g) August 4 – September 27



(h) September 29 – December 22

Figure 5.7: Movements of XU030 equity index and its implied volatility index in sub-intervals

Correlation coefficients of XU030 and implied volatility index in these sub-intervals can be seen in Table 5.5. Besides, we excluded all trading days in Table 5.3 from our data set, and re-calculated the correlations of XU030 index and its implied volatility index for 235-day time frame. We achieved -0.286122145 as the correlation coefficient. This value was 0.115551359 before the extraction of jump days. This means, we achieved more successful implied volatility index by excluding trading days with jumps from the calculation period.

Table 5.5: Correlation coefficients in sub-intervals

Sub-interval	*Correlation Coefficient*
January 4 – February 26	0.49812053
March 1 – April 1	0.711026429
April 6 – April 29	0.081671775
May 16 – June 2	0.909046073
June 6 – July 15	-0.73670148
July 20 – July 29	0.151602216
August 4 – September 27	0.101052481
September 29 – December 22	-0.04048215

By observing the values in Table 5.5, we may conclude that implied volatility index was successful in the sub-interval of June 6 – July 15. On July 15, 2016 there was a coup attempt in Turkey; and after this coup attempt XU100 index was opened with 7.08% loss, and XU030 index was opened with 6.71% loss on July 18, 2016. It has been one of the most shocking events in the Turkish stock market.

Also, we reviewed the trading days in which jumps were seen. We noticed that if the desired forward index levels F's were different from the previous trading day's F values then a jump in the implied volatility index value is very likely. Examples of this relation can be seen in Table 5.6. Besides, a change in the number of actively traded options can also be a cause of jumps. We see this case in the jump between index values of December 23 and December 26. Although there was no difference in F values between these two days, there was an increase of 70% in implied volatility index value. This situation can be explained by the lengths of the options series used in the index calculations. While on December 23, 2016, nine near-term and five next-term options were included index calculations. On December 26, volatility index was calculated with ten near-term and ten next-term options. Increase in the number of next-term options affected the index value substantially, and the jump occurred.

Table 5.6: Relation between jumps and change in F values

Trading Day and F Values (Near-term and next-term respectively)	*Previous Trading Day and F Values (Near-term and next-term respectively)*	*Change in Implied Volatility Index Value*
February 29, 2016 98000–92000	February 26, 2016 92000–92000	-25.45%
April 4, 2016 102000–104000	April 1, 2016 100000–102000	-31.75%
April 5, 2016 102000-102000	April 4, 2016 102000–104000	48.71%
May 4, 2016 98000–100000	May 3, 2016 100000–102000	-28.94%
May 5, 2016 98000–98000	May 4, 2016 98000–100000	82.80%
June 3, 2016 96000–98000	June 2, 2016 94000–96000	25.39%
July 18, 2016 98000–98000	July 15, 2016 104000–104000	-44.33%
July 19, 2016 96000–96000	July 18, 2016 98000–98000	111.01%
August 1, 2016 94000–96000	July 29, 2016 94000–94000	-33.17%
August 3, 2016 92000–94000	August 2, 2016 94000–96000	46.19%
September 28, 2016 96000–96000	September 27, 2016 94000–96000	32.02%

5.4 Results of the Implementation

Now we will look at the details of the implementation, and mostly we will focus on the reasons for deficiencies of the index. First of all, options contracts and other financial derivatives are quite new instruments for the Turkish market. The first and the most important reason seems to be that option contracts' not attracting enough demand. For example average daily trading volume of SPX options, underlying security of the Vix, is more than a million since 2016 and more than a hundred thousand since 2002. See Figure 5.8.

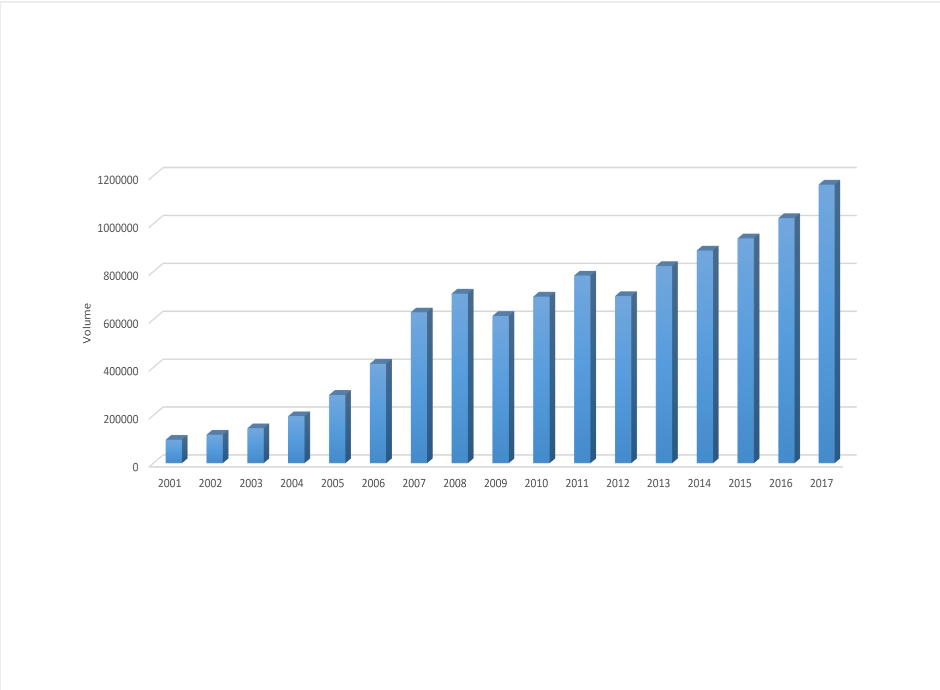


Figure 5.8: S&P 500 index options average daily trading volume since 2001

This figure was created using the data obtained from *www.cboe.com*, official web site of Chicago Board Options Exchange, and it clearly shows the increase in trading volume of SPX index options year by year with only two exceptions, decline after the big Mortgage Crisis and the decline wit the crisis in 2012.

When we talk about Turkish options market, we know that trading volume of the XU030 index options was 763,872,340 TRY in 2014 and 2,234,475,305 TRY in 2015. It means trading volume of the XU030 index options increased 193% from 2014 to 2015. And we had a chance to get access to the index options' trading volume as it can be seen in Table 5.7. These data were obtained from *www.borsaistanbul.com*. The traded volume increased at a significant level between 2014 and 2015, but later, interest on index options contracts decreased between 2016 and 2017. At the same time frame, traded value in US $ seemed to have decreased, but it should not be overlooked that changes in exchange rates between US $ and TL also affected this result.

Table 5.7: BIST index options contracts trading volume and values

Index Options Contracts					
Date	*Open Int.*	*Number of Trades*	*Traded Vol.*	*Traded Value(TL)*	*Traded Value(US $)*
Apr 2013-Dec 2013	6,832	1,142	10,352	94,614,200	47,541,315.41
2014	110,469	3,034	107,344	984,914,140	450,466,861.69
2015	191,522	17,266	290,856	2,836,368,995	1,032,031,788.25
2016	293,451	24,829	315,783	2,969,113,000	987,593,349.17
2017	229,752	20,883	275,030	3,242,741,200	891,538,266.62

In the US, options contracts are actively traded in the market for about 18 years, and the demand is great. Of course we do not compare the US market with the Turkish market. However, high trading volumes of options contracts have great impact on implied volatility indexes' prediction success. As we have mentioned earlier CBOE Vix reflects market expectations nearly perfectly, and it has been used as the fear index not in the US, but also in the global markets. The Turkish market is an emerging market, and it has stridden out in recent years. But as we see, it seems to be too early to establish an implied volatility index for the Turkish market.

We investigated the relation between the implied volatility index we constructed and the XU030 equity index. When we examine the movements of these two indexes together, we clearly understand that there is no relationship we had seen in other implied volatility indexes with their underlying equity indexes. We compared with 1-year data set consisting of 250 daily closing prices that we have used before. In Table 5.8 and Table 5.9 the ratios of parallel and opposite moves of XU030 index and implied volatility index can be observed:

Table 5.8: Number and percentage of isotropic movements of XU030 and its implied volatility index between January 2016 and January 2017

Movement	*Times*	*Percentage*
XU030 Down & Imp. Vol. Index Down	58	23.3%
XU030 Up & Imp. Vol. Index Up	64	25.7%
Total Movements in the Same Directions	122	49%

Table 5.9: Number and percentage of opposite movements of XU030 and its implied volatility index between January 2016 and January 2017

Movement	*Times*	*Percentage*
XU030 Up & Imp. Vol. Index Down	71	28.5%
XU030 Down & Imp. Vol. Index Up	56	22.5%
Total Movements in the Opposite Directions	127	51%

In Chapter 2, we did the same study for S&P 500 index and Vix. Of course there is a big difference in the sense of size of data sets. We examined the movements of S&P 500 index and Vix with a larger data set consisting of 3022 daily closing prices in 12-years time period, from 2005 to 2017. When we look at the results of that study, there is not a big difference in terms of the percentages of isotropic movements, but there is a remarkable difference in the opposite movements. As it can be seen in Table 5.10, percentages of the parallel movements of equity indexes and their implied volatility indexes are not close. When we look at the opposite movements of equity indexes and their implied volatility indexes, we see a big difference. The most important aspect of an implied volatility index is how that index reacts when its related equity index falls or increases. S&P 500 and Vix exhibited exactly what we wanted to see, that is why Vix is called the fear gauge.

One of our biggest expectations was to see rising implied volatility index value while the equity index was falling; however, the results we obtained were far from being satisfactory. Percentage values were very different as compared with Vix, and the differences cannot be explained by the magnitudes of the data sets. See Table 5.10.

Table 5.10: Comparison of movements of XU030 and SPX and their related implied volatility indexes

Parallel Moves	*Percentage*	*Opposite Moves*	Percentage
SPX Up and Vix Up	17.8%	SPX Up and Vix Down	81.83%
XU030 Up and Imp.Vol.Index Up	25,7%	XU030 Up and Imp.Vol.Index Down	28.5%
SPX Down and Vix Down	19.51%	SPX Down and Vix Up	79.77%
XU030 Down and Imp.Vol.Index,Down	23.3%	XU030 Down and Imp.Vol.Index Up	22.5%

While the previous version of Vix included only eight at-the-money options, the updated version incorporates all available options on the market. This development implied that if an option series with a larger scale is used then a more consistent index can be obtained. Therefore we realized one of the reasons for the failure of the index we created. No matter how many options were included in the option series of our index calculations, at-the-money options with near-term maturity were dominant factor in volatility index and other options had almost no effect. Therefore, our implied volatility index reflected the volatility expectations of only these options' holders, and other participants of the market almost had no effect on the value of implied volatility index. And of course this was not a good result when we were trying to measure the volatility expectation of the whole market. Most of the time, our implied volatility index values were moving around the near-term volatility expectations of the market participants. Because with the exception of a few put options contracts, next-term options are barely demanded on the market. This showed that, index options contracts are regarded as short-term hedging instruments in the Turkish market. Implied volatility indexes are obtained by blending near-term and next-term volatility expectations, and when we consider this fact, it is expected that our implied volatility index will be on the side of the near-term volatility expectations of the market.

BIST committee introduced a new type of options contracts to the market, Mini BIST 30 index options, so that small-size investors could take part in the options market. This new financial instrument was presented on September 19, 2014. In our implied volatility index calculations, Mini BIST 30 index options were not added into the calculation. After the failure of implementation, we tried to improve the index to make it more consistent by adding Mini BIST 30 options in the calculation. Nevertheless, this was not possible due to the lack of adequate number of Mini BIST 30 options contracts, with non-zero bid and ask prices. As it can be seen in Figure 5.9, index options contracts correspond just 0.49% of all trading activities made in 2015 in the Turkish derivatives market, which is a very clear indicator of the low demand on index options in VIOP. On the other hand, the percentage of index futures was very high, because Turkish index futures and FX futures were among the top 10 most traded derivative instruments in global markets in 2015.

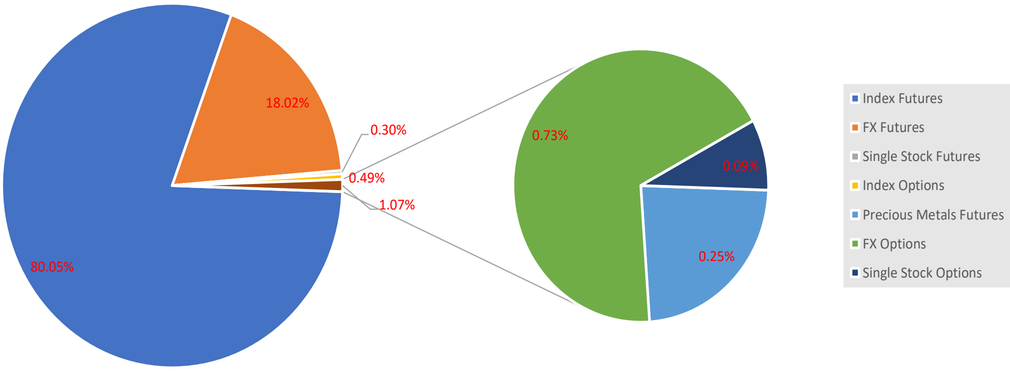


Figure 5.9: The distribution of trading activities in VIOP in 2015

There is another point worth mentioning; distribution of trading volumes of domestic and foreign investors have changed significantly over the last years, and the percentage share of foreign investors has increased obviously. As it can be seen in Figure 5.10, the share of domestic investors declined between 2011 and 2015 except for an increase between 2013 and 2014. From another point of view, interests of foreign investors on the Turkish derivative market products has been increased since 2011.

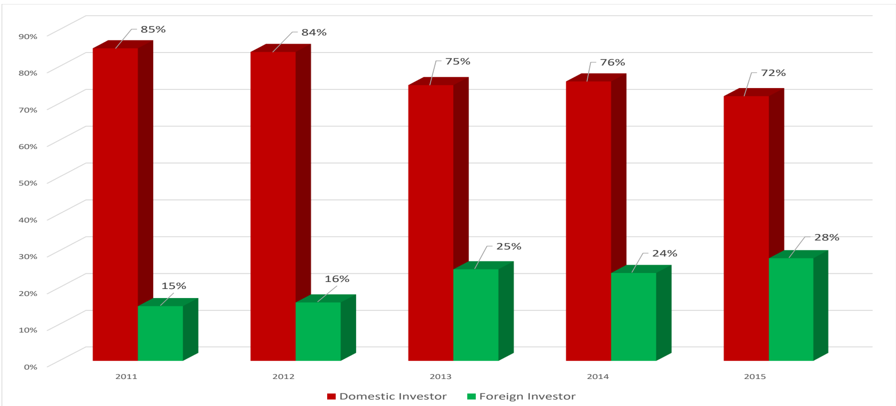


Figure 5.10: Percentages of foreign and domestic investors in VIOP

We tried to give detailed information about the XU030 index options and statistics about it. All the information above shows that investors, both individual and institutional, do not perceive XU030 index options as a financial tool that meets their needs. Investors could not find enough numbers of BIST 30 index options contracts with maturities that fit investors plans, and this was the biggest deficit of BIST 30 index options contracts. Maturities of the index options have to be in a wide range to attract investors.

In Table 5.11, main features of BIST 30 index options can be observed, and also in Table 5.12 and Table 5.13 main features of S&P 500 index options and several important index options can be seen, respectively. In the light of these informations, we see that the Turkish index options have quite limited maturity dates. In addition, XU030 index options series with new maturities are issued to the market less frequently than other index options. Consequently, the Turkish index options have not reached high levels of trading volumes. If an investor wants to take a position in such an European or American index options has lots of alternatives with the same strike prices and different maturities. On the other hand, if an investor wants to buy or sell a Turkish index option, has only three or at most four different maturities with the same strike price. The common problem of most emerging markets, like the Turkish market, is the lack of liquidity and volume. Unless these problems were solved, it is not possible to derive indicators, such as implied volatility indexes, that give consistent and efficient results.

Table 5.11: Specifications of BIST 30 index options

Name of the Instrument	*Underlying Options Chain*	*Contract Size*	*Ticker Symbol*	*Expiration Date*	*Exercise Style*
BIST 30 Index Options	1/1000 of the BIST 30 Index	100 Underlying Assets	XU030X (Bloomberg) 0#XU030*.IS (Reuters)	February, April, June, August, October, December	European
Mini-BIST 30 Index Options	1/1000 of the BIST 30 Index	1 Underlying Asset	XU030XO (Bloomberg) 0#XU030M*.IS (Reuters)	February, April, June, August, October, December	European

Table 5.12: Specifications of S&P 500 index options

Name of the Instrument	*Underlying Options Chain*	*Ticker Symbol*	*Expiration Date*	*Exercise Style*
Traditional				
SPX	SPX	SPX	3rd Friday	European
Non-Traditional				
SPX Friday End-of-Weeks (EOW)	SPX	SPXW	Fridays	European
SPX Wednesday Weeklys	SPX	SPXW	Wednesdays	European
SPX Monday Weeklys	SPX	SPXW	Mondays	European
SPX End-of-Month (EOM)	SPX	SPXW	Last Trading day of Month	European
SPX PM-Settled 3rd Fridays	SPX	SPXW	3rd Fridays	European
Mini-SPX Options (1/10th the Notional Size)				
Mini-SPX Index Options (Weeklys Available)	XSP	XSP	Fridays	European

Table 5.13: Specifications of several European index options

Name of the Instrument	*Trading Unit*	*Ticker Symbol*	*Expiration Months*	*Exercise Style*
Euro STOXX 50 Index Options	€10 multiplied by the value of the index	OESX	The three nearest successive calendar months, the three following quarterly months of the March, June, September and December cycle, the four following semi-annual months of the June and December cyle, and seven following annual months of the December cycle	European
FTSE 100 Index Options	£10 per index value	UKX	Out to 24 months; First 2 non-quarterly months, first 8 quarterly months of March, June, September, December cycle.	European Cash-settled
FTSE 100 Mini-Index Options	\$100 multiplier per 1/10th of the FTSE 100 index	UKXM	Up to 12 near-term month, and also the related exchange may list up to 10 UKXM expiration months that expire 12 to 60 months from date of issuance.	European Cash-settled
CAC40 Index Options	€10 per index value	PXA	Trading is composed of 13 open maturities: Three nearest calendar months in monthly cycle, the following seven quarterly months in March, June, September, December cycle, and the following 3 annual maturities in December cycle	European
BEL20 Index Options	€10 per index value	BEL	1st, 2nd, 3rd, 6th and 9th months according to the March, June, September, December cycle.	European

As the problem is identified, what could be the solution? After analyzing the tables above, we tried to find different solutions to the problem. The most applicable solution is not complex indeed. To increase investors' interest on BIST 30 index options, we need to modify these instruments' specifications. No investor buy or sell a financial instrument that does not suit his or her needs. The biggest problem is that the new index options are not issued to the market before the previous options' expiration.

A suggestion: To solve the problem, as the first step, new index options with maturities of one month, two months and three months need to be issued to the market on each trading day. After continuing this procedure for a while, about two months, it is going to be enough to submit new index options with three months of maturities. The process can be clarified as:

1) On January 1st, index options with expiration dates of the last trading day of January, the last trading day of February, the last trading day of March are issued to the market.

2) On January 2nd, index options with maturity dates of the first trading day of February, March, and April are offered to the market.

3) By applying the same procedure, new index options will be issued to the market every new trading day until the last trading day of January. At the end of the January, we would have approximately sixty index options trading on the market. More importantly, we would have index options with approximately 60 different maturities. This means that, on the last trading day of January, the opportunity to buy or sell an index option that expire every trading day of February, March, and April will be on the market.

4) After the above steps were achieved, on each day issuing new series of index options with three months of maturities is going to be enough.

By applying the procedure above, the VIOP will be enriched in terms of index options, and the problem of finding appropriate index options will be removed.

Another solution is to make calculation of the index when market is stable and there are sufficient number of index options. As it can be seen in Figure 5.4, we have faced with big jumps for several trading days. There were a few reasons behind this. On the last trading day of February, April, June, August, October, and December, index could not give satisfactory results. Because these days were the expiry dates of index options and there were not sufficient number of index options to make an efficient index calculation. Secondly, when F values change, the likelihood of a jump in the index increases at a significant level. Even between two consecutive trading days, F values can be very different, and changes in F values can be a cause of jumps. As we have mentioned in detail, by removing the jump days we achieved more successful index on the entire time frame.

CHAPTER 6

CONCLUSION & OUTLOOK

In this thesis, we studied the evolution of implied volatility indexes, and several illustrations of implied volatility indexes from Europe. However, we mostly focused on CBOE Vix methodology and implementation of the Turkish BIST 30 index. To construct an implied volatility index for the Turkish market, we utilized a modified version of CBOE Vix methodology. Then we examined the forecast ability of this index.

Firstly, we examined the concept of implied volatility index and the first application: CBOE Vix. Then, derivation of the Vix formula and components of the formula were given in detail. For a better understanding of the methodology, the calculation of the Vix index for a single trading day was shown and clarified. Later on, we investigated different illustrations of implied volatility indexes from Europe. All of the indexes we analyzed are using variations of the CBOE Vix methodology and they produced very successful results just like Vix. The success of these type of indexes is measured with the indexes' ability of estimating the underlying indexes' realized volatility. Besides, an implied volatility index needs to indicate the markets volatility expectation at the moments of big stress and major movements to be accepted as an effective indicator. We saw that, all the implied volatility indexes we studied reflected their related equity indexes' future status with a remarkable success. Even, among these indexes, VFTSE gave the most satisfactory results.

Later in Chapter 4, we investigated the yield curve modeling. Studies on yield curve modeling are mainly focused on 2 different methods: Interpolation and regression models. We used Svensson method, a modified version of the Nelson & Siegel model. Svensson model is a parametric method and this model differs from the original Nelson & Siegel model by the addition of the 4^{th} term to the original formula. With this new term, two new parameters, β_3 and τ_2, were added to the Nelson & Siegel model. New term was added to increase flexibility and fitting performance of the original model with its ability to generate second hump or U-shape in yield curves. While β_3 indicates the magnitude and the direction of the second hump, τ_2 shows the location of the hump. Nelson & Siegel model is used in situations where complex curves are not exhibited. Central Bank of Turkish Republic have used Nelson & Siegel model for years, but in this study we wanted to see the fitting performance of the Svensson model in the Turkish treasury bill curve. We concluded that the Svensson model did not make a significant difference in fitting performance according to the original Nelson & Siegel model. We encountered with negative values while we were trying to calculate risk-free rates of short-terms like lengths with less than a week. This values emerged due to the big differences between maturities of the Turkish treasury bills, and the linearity of the Turkish market term structure. In other words, the Svensson model could not provide reasonable values for short-terms when there were large differences between maturities. Nevertheless, Nelson & Siegel model gave almost the same results; hence, it model was not superior to the Svensson model in these scenarios. In addition, a simple linear regression might be sufficient to estimate the Turkish treasury bill data.

Most importantly, at the beginnings of Chapter 5, we described the Turkish market in detail; its structure, process of evolution, important events, and financial products traded within this market, and index options were discussed in detail. All specifications of Borsa Istanbul index options were discussed in this chapter.

Due to the fact that we were not able to use CBOE Vix methodology as it is, due to the conflicts with the Turkish market, we had to made a few modifications in the calculation steps. The changes that were made to get more efficient implied volatility index can be summarized as follows:

(1) Maturities of the index options were not suitable for the 30-day interpolation, therefore we changed the calculation formula so as to reflect volatility estimations of the next 60 days.

(2) We took into calculation the number of days until maturity as the time parameter, and differently from the original Vix model we did not calculate the parameter T in the unit of minutes, because we could do only end-of-day calculations.

(3) While index options with zero bid prices are excluded from the index calculation in original CBOE Vix formula, we excluded both index options with zero bid prices and zero ask prices from our index calculations with a view to increase the stability of the index.

(4) In the original formula, to find desired forward index level differences of call and put options' mid-quote prices are taken, but this computation gave unintended results in our calculations and this step was changed with taking absolute value of differences of call and put options' mid-quote prices.

Along with the changes above, implied volatility index calculations were made for 250 trading days from January 2016 to January 2017, and we constituted an implied volatility index for the XU030 index. An illustration of index calculations can be found in Appendix A. We found the correlation coefficient of XU030 index and implied volatility index as 0.115551359 and correlation coefficient of realized volatility of XU030 and implied volatility index as -0.016752246. These results were far from being satisfactory when we look at the correlation values between FTSE100 and VFTSE, SMI and VSMI, DAX and VDAX-New, Euro STOXX 50 and VSTOXX, CAC40 and VCAC. The main reason of inefficiency was seemed to be having insufficient trading volumes of XU030 index options.

In the light of the results we obtained, we created 8 sub-intervals by removing the jump days from the calculation, and re-calculated the index in these sub-intervals. We achieved more successful index after this process, especially in the 5^{th} sub-interval, June 6 – July 15, index worked nearly perfect. In this sub-interval we obtained correlation coefficient of implied volatility index and XU030 as -0.73670148. This value is even better than correlations of VFTSE and FTSE 100.

Then, we excluded all trading days with jumps from whole dataset and re-calculated the index, and we achieved more successful index than before. By extracting jump days, we achieved correlation coefficient of XU030 and implied volatility index as -0.286122145 for 235 trading days time frame. This situation yields that the index is more successful when it is calculated at certain time intervals.

At the moment, creating an implied volatility index for the BIST 30 index that calculates market's volatility expectation continuously may not be possible. Instead, we can appeal to the index when major events do not take place in the Turkish market. So, instead of calculating the index continuously, it would be more efficient to make index calculation for the intervals at which market is close to stability. This work can be seen as a preliminary study for an implied volatility index that will be generated when the related conditions are matched.

In addition, index warrants issued by İş Investment and Deutsche Bank, are the biggest opponents of XU030 index options contracts in the Turkish market. There are no margin calls and margins on the warrants, besides all the responsibility belongs to the issuer of the warrants. These features make warrants more attractive for individual and institutional investors in the Turkish market. Therefore, instead of BIST 30 index options, BIST 30 index warrants can be used or included in further studies on BIST 30 implied volatility index. On the other hand, in accordance with the studies of Barletta et al.(2017) [6], term structure of the Turkish treasury bills can be investigated in a non-structural method. This method does not need restrictive parameters on the underlying; and hence, it can be used in further studies.

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APPENDIX A

AN EXAMPLE FOR AN INDEX CALCULATION

In this part of the study we tried to give implied volatility calculations of an ordinary day from our sample period to illustrate index calculation steps. We examined implied volatility calculation steps of February 2, 2016.

On February 2, 2016, there were index options with February, April, June and December maturities. We only used options with February and April maturities, because options with June and December maturities did not attract enough demand in the market, and their trading volume was so close to zero. After specifying maturities, as the next step we excluded index options with either zero bid or ask prices. Index options remained after these steps can be seen in Table A.1. Highlighted areas in the table represent excluded options.

Table A.1: Option series included in the index calculation

Near Term Options (Feb.2016)					*Next Term Options (Apr.2016)*				
Strike	*Call Options*		*Put Options*		*Strike*	*Call Options*		Put Options	
	Bid	*Ask*	*Bid*	*Ask*		*Bid*	*Ask*	*Bid*	*Ask*
78000			0.1	0.1	78000			0.84	1.19
80000					80000			1.15	1.56
82000			0.31	0.54	82000			1.25	1.75
84000			0.5	0.75	84000	7	7.64	1.67	2.42
86000	4.08	4.42	0.93	1.17	86000	5.7	6.28	2.22	2.72
88000	2.76	3.12	1.54	1.83	88000	3.98	4.48	2.92	3.42
90000	1.68	1.92	2.47	2.66	90000	2.93	3.43	3.82	4.32
92000	0.81	1.1	3.57	3.87	92000	2.09	2.59	4.92	5.42
94000	0.37	0.61	5.11	5.41	94000	1.45	1.95		
96000	0.15	0.4			96000	0.99	1.24		
98000	0.1	0.25			98000	0.67	0.92		
100000	0.02	0.3	1.01	19.98					
102000									
104000									
106000	0.01	0.01							

The second step is to determine T values for near term and next term. T_1 and T_2 are the time variables of index options with February 2016 and April 2016 maturities respectively. There were 28 days between February 2, 2016 and February 29, 2016, and there were 88 days between February 2, 2016 and April 29, 2016. So,

$$T_1 = \frac{28}{365} = 0.076712329, \text{ and } \quad T_2 = \frac{88}{365} = 0.24109589.$$

Thirdly, after calculating T values, we need to determine risk-free interest rates for near-term and next-term options. As we have already mentioned before, if we do not have the interest rate data for the near term and next term options' expiration day, we need to obtain the data via curve fitting methods. As it can be seen in Table A.2, there were no treasury bill that will expire on February 29, 2016 or April 29, 2016 at the market.

Table A.2: Turkish Treasury bill data for February 2016

Days to Maturity	*Weighted Average Price*	*Yield*
28	-	-
44	98.895	0.01105
47	98.835	0.01165
48	98.773	0.01227
48	98.793	0.01207
49	98.747	0.01253
56	98.556	0.01444
57	98.508	0.01492
58	98.492	0.01508
61	98.406	0.01594
62	98.37	0.0163
63	98.328	0.01672
64	98.297	0.01703
65	98.271	0.01729
70	98.1	0.019

71	98.092	0.01908
72	98.066	0.01934
88	-	-
103	97.416	0.02584
105	97.336	0.02664
106	97.288	0.02712
107	97.346	0.02654
111	97.136	0.02864
112	97.111	0.02889
119	96.812	0.03188
121	96.838	0.03162
127	96.638	0.03362
175	95.501	0.04499
176	95.378	0.04622
177	95.353	0.04647
180	95.3	0.047
182	95.183	0.04817
183	95.157	0.04843
188	95.052	0.04948
191	95.027	0.04973
195	94.832	0.05168
196	94.807	0.05193
289	92.191	0.07809
289	92.199	0.07801
292	92.151	0.07849
292	92.149	0.07851
293	92.052	0.07948
293	92.092	0.07908
294	91.881	0.08119
294	92.011	0.07989
294	91.971	0.08029

295	91.946	0.08054
295	91.924	0.08076
296	91.897	0.08103
296	91.837	0.08163
299	91.755	0.08245
299	91.765	0.08235
300	91.685	0.08315
300	91.725	0.08275
301	91.675	0.08325
301	91.689	0.08311
302	91.672	0.08328
303	91.697	0.08303
306	91.662	0.08338
307	91.597	0.08403
308	91.577	0.08423
308	91.611	0.08389
309	91.557	0.08443
309	91.552	0.08448
310	91.562	0.08438
313	91.461	0.08539
314	91.495	0.08505
314	91.463	0.08537
315	91.41	0.0859
315	91.424	0.08576
316	91.392	0.08608
316	91.377	0.08623
317	91.57	0.0843

We used Excel for calculating approximated values of missing treasury bill rates via Svensson model. To get Svensson model values, the following steps were followed in Excel in order:

(1) Debt instruments data for the month of calculation day, here February 2016 data, was transferred to an Excel sheet. In next steps, just columns of "Days to Maturity"and "Weighted Average Price"are going to be needed. So, importing just these two columns is going to be enough.

(2) We needed to convert days to maturity data to the years to maturity data. So, we divided all members of the column "Days to Maturity"by 365. Then, we needed yields of each treasury-bill. We could get "Yields"column by applying the simple rate of return formula, $\ln\left(\frac{P_t}{P_{t-1}}\right)$, to all elements of "Weighted Average Price"column from the beginning to the end respectively. Meanwhile, do not forget to sort both columns with respect to "Years to Maturity"column from the least to the most and also do not forget to leave the columns we will try to find empty.

(3) In Svensson model, apart from maturity and yield variables, we need to have additional six parameters. We created a column for these parameters and we called these parameters as β_0, β_1, β_2, β_3, τ_1 and τ_2. We assigned 1 to all of these six parameters as the initial value. There was not a specific reason for selecting this initial value, these values will change in next steps. So, 0 or anything else can be assigned.

(4) After those steps, we are ready to create "Svensson Yield"column. At the first row of this column we wrote the Svensson formula (See Eq. A.1), and found the results of this formula for all data respectively.

$$\begin{aligned}\text{Svensson}_t = \beta_0 + \beta_1 \frac{\left(1-\exp\left(\frac{-\text{Maturity}_t}{\tau_1}\right)\right)}{\left(\frac{\text{Maturity}_t}{\tau_1}\right)} + \beta_2 \frac{\left(1-\exp\left(\frac{-\text{Maturity}_t}{\tau_1}\right)\right)}{\left(\frac{\text{Maturity}_t}{\tau_1}\right)} \\ - \beta_2 \exp\left(\frac{-\text{Maturity}_t}{\tau_1}\right) + \beta_3 \frac{\left(1-\exp\left(\frac{-\text{Maturity}_t}{\tau_2}\right)\right)}{\left(\frac{\text{Maturity}_t}{\tau_2}\right)} \\ - \beta_3 \exp\left(\frac{-\text{Maturity}_t}{\tau_2}\right).\end{aligned} \tag{A.1}$$

We created "Svensson Yield"column, and then we needed to create "Residue"column. In this column we calculated the square of the differences of each same rows of "Yield"and "Svensson Yield"columns. "Residue"column can be created easily by using the following short algorithm:

"If $\text{Yield}_t > 0$ then $\text{Residue}_t = (\text{Yield}_t - \text{Svensson Yield}_t)^2$ else do nothing".

Lastly, we created a cell, "Sum of Residues", to conclude our treasury-bill calculation steps. In this cell all of the elements of "Residue"column were summed, and then Excel solver was utilized to calculate minimum value of "Sum of Residues"through changing β_0, β_1, β_2, β_3, τ_1 and τ_2 parameters. After the calculation was done in a meaningful way, we get risk-free rates that were not available in the market. Excel sheets that show the results of these steps can be observed in Figure A.1a, Figure A.1b and Figure A.2.

	A	B	C	D	E	F	G
1	Maturity	Yield	Svensson Yield	Residue	Sum of Residue	beta_0	-3.29259
2	0.076712		0.006057333		5.08589E-05	beta_1	3.290846
3	0.120548	0.01105	0.010512711	2.89E-07		beta_2	-32.8226
4	0.128767	0.01165	0.011348044	9.12E-08		beta_3	156.7399
5	0.131507	0.01227	0.011626486	4.14E-07		tau_1	581.9047
6	0.131507	0.01207	0.011626486	1.97E-07		tau_2	590.6176
7	0.134247	0.01253	0.011904925	3.91E-07			
8	0.153425	0.01444	0.013853952	3.43E-07			
9	0.156164	0.01492	0.014132377	6.2E-07			
10	0.158904	0.01508	0.014410801	4.48E-07			
11	0.167123	0.01594	0.015246062	4.82E-07			
12	0.169863	0.0163	0.015524479	6.01E-07			
13	0.172603	0.01672	0.015802894	8.41E-07			
14	0.175342	0.01703	0.016081307	9E-07			
15	0.178082	0.01729	0.016359719	8.65E-07			
16	0.191781	0.019	0.01775175	1.56E-06			
17	0.194521	0.01908	0.018030151	1.1E-06			
18	0.19726	0.01934	0.018308551	1.06E-06			
19	0.241096		0.022762705				
20	0.282192	0.02584	0.02693807	1.21E-06			
21	0.287671	0.02664	0.027494756	7.31E-07			
22	0.290411	0.02712	0.027773096	4.27E-07			
23	0.293151	0.02654	0.028051435	2.28E-06			
24	0.30411	0.02864	0.029164771	2.75E-07			
25	0.306849	0.02889	0.029443101	3.06E-07			
26	0.326027	0.03188	0.031391362	2.39E-07			
27	0.331507	0.03162	0.031947992	1.08E-07			

(a) Svensson calculations Excel sheet

28	0.347945	0.03362	0.033617841	4.66E-12
29	0.479452	0.04499	0.046974379	3.94E-06
30	0.482192	0.04622	0.047252598	1.07E-06
31	0.484932	0.04647	0.047530815	1.13E-06
32	0.493151	0.047	0.048365455	1.86E-06
33	0.49863	0.04817	0.048921873	5.65E-07
34	0.50137	0.04843	0.04920008	5.93E-07
35	0.515068	0.04948	0.050591086	1.23E-06
36	0.523288	0.04973	0.051425669	2.88E-06
37	0.534247	0.05168	0.052538422	7.37E-07
38	0.536986	0.05193	0.052816606	7.86E-07
39	0.791781	0.07809	0.078680115	3.48E-07
40	0.791781	0.07801	0.078680115	4.49E-07
41	0.8	0.07849	0.079514172	1.05E-06
42	0.8	0.07851	0.079514172	1.01E-06
43	0.80274	0.07948	0.079792187	9.75E-08
44	0.80274	0.07908	0.079792187	5.07E-07
45	0.805479	0.08119	0.080070201	1.25E-06
46	0.805479	0.07989	0.080070201	3.25E-08
47	0.805479	0.08029	0.080070201	4.83E-08
48	0.808219	0.08054	0.080348213	3.68E-08
49	0.808219	0.08076	0.080348213	1.7E-07
50	0.810959	0.08103	0.080626223	1.63E-07
51	0.810959	0.08163	0.080626223	1.01E-06
52	0.819178	0.08245	0.081460244	9.8E-07
53	0.819178	0.08235	0.081460244	7.92E-07
54	0.821918	0.08315	0.081738247	1.99E-06

(b) Svensson calculations Excel sheet cont.

55	0.821918	0.08275	0.081738247	1.02E-06
56	0.824658	0.08325	0.082016248	1.52E-06
57	0.824658	0.08311	0.082016248	1.2E-06
58	0.827397	0.08328	0.082294248	9.72E-07
59	0.830137	0.08303	0.082572246	2.1E-07
60	0.838356	0.08338	0.08340623	6.88E-10
61	0.841096	0.08403	0.083684221	1.2E-07
62	0.843836	0.08423	0.08396221	7.17E-08
63	0.843836	0.08389	0.08396221	5.21E-09
64	0.846575	0.08443	0.084240198	3.6E-08
65	0.846575	0.08448	0.084240198	5.75E-08
66	0.849315	0.08438	0.084518184	1.91E-08
67	0.857534	0.08539	0.085352131	1.43E-09
68	0.860274	0.08505	0.08563011	3.37E-07
69	0.860274	0.08537	0.08563011	6.77E-08
70	0.863014	0.0859	0.085908087	6.54E-11
71	0.863014	0.08576	0.085908087	2.19E-08
72	0.865753	0.08608	0.086186063	1.12E-08
73	0.865753	0.08623	0.086186063	1.93E-09
74	0.868493	0.0843	0.086464037	4.68E-06

Figure A.2: Svensson calculations Excel sheet cont.

As it can be seen in the figures, we got 0.006057 and 0.022763 as the rates of 28-day and 88-day risk-free interest rates, respectively. This estimation was completed with total error term of $5.08589 \cdot 10^{-5}$.

The next step was to find strike prices with the least absolute differences. The comparisons were made between the mid-quote prices of put and call options with same strike prices for near and next-term options. Only examining the put and call options with the same strike prices was enough.

Table A.3: Call and put options with the same strike prices

Near-Term Options				*Next-Term Options*			
Strike Price	*Call Opt. Mid-Quote*	*Put Opt. Mid-Quote*	*Absolute Diff.*	*Strike Price*	*Call Opt. Mid-Quote*	*Put Opt. Mid-Quote*	*Absolute Diff.*
86000	4.25	1.05	3.2	84000	7.32	2.045	5.275
88000	2.94	1.685	1.255	86000	5.99	2.47	3.52
90000	1.8	2.565	0.765	88000	4.23	3.17	1.06
92000	0.955	3.72	2.765	90000	3.18	4.07	0.89
94000	0.49	5.26	4.77	92000	2.34	5.17	2.83
100000	0.16	10.495	10.335				

As it can be seen in Table A.3, index options with strike price 90000 had the least absolute difference of mid-quote prices for both near-term and next-term options. Hence, we have everything to calculate desired forward index level F for near and next-terms:

$$F = \text{Strike Price} + e^{RT} \cdot |\text{Call Price - Put Price}|$$

Here, "Strike Price"is the strike price of the index options with the minimum absolute difference. "Call Price"and "Put Price"represent mid-quote prices of call and put options with strike prices that have minimum mid-quote price differences respectively. Below F_1 and F_2 represent desired forward index levels for near and next-terms respectively. K_1 and K_2 are the highest strike prices below the forward index levels for near and next-terms respectively.

$$F_1 = 90000 + e^{(0.00606 \cdot 0.0767)} \cdot \mid 1.8 - 2.565 \mid = 90000.77 \quad \text{and} \quad K_{0,1} = 90000$$

$$F_2 = 90000 + e^{(0.02276 \cdot 0.2411)} \cdot \mid 3.18 - 4.07 \mid = 90000.89 \quad \text{and} \quad K_{0,2} = 90000$$

Then, we determined the index options included in the calculations according to $K_{0,1}$ and $K_{0,2}$ values we found. For both near-term and next-term implied volatility calculations, same procedure was applied. We took K values as the limiting points, and we chose into calculation the put options with strike prices lower than K and the call options with strike prices higher than K values. In addition to that, we included average of the call and put options with the strike prices equal to the $K_{0,1}$ and $K_{0,2}$ values as a single option. After we listed index options, we calculated contributions of each index options to the volatility index value. We employed Eq. A.2 to find contributions, and the results for February 2, 2016 can be observed in Table A.4.

$$\text{Contribution to the index} = \frac{2}{T} \cdot \sum_i \frac{\Delta K_i}{K_i^2} \cdot e^{R_1 \cdot T_1} \cdot Q(K_i) \tag{A.2}$$

Table A.4: Options' contributions to the index value

Near-Term Options			
Strike Price	*Option Type*	*Mid-Quote Price*	*Contribution*
78000	Put	0.1	6.57768E-08
82000	Put	0.425	1.89707E-07
84000	Put	0.625	1.77237E-07
86000	Put	1.05	2.84069E-07
88000	Put	1.685	4.35378E-07
90000	Put&Call	2.1825	5.39139E-07
92000	Call	0.955	2.25767E-07
94000	Call	0.49	1.10961E-07
96000	Call	0.275	5.97066E-08
98000	Call	0.175	3.64601E-08
100000	Call	0.16	6.40297E-08
106000	Call	0.01	5.34246E-09
Next-Term Options			
Strike Price	*Option Type*	*Mid-Quote Price*	*Contribution*
78000	Put	1.015	3.35498E-07
80000	Put	1.355	4.25768E-07
82000	Put	1.5	4.48618E-07
84000	Put	2.045	5.82838E-07
86000	Put	2.47	6.71604E-07
88000	Put	3.17	8.23204E-07
90000	Put&Call	3.625	8.99987E-07
92000	Call	2.34	5.55973E-07
94000	Call	1.7	3.86907E-07
96000	Call	1.115	2.43302E-07
98000	Call	0.795	1.66467E-07

Total contributions of the near-term options to the near-term implied volatility value was $5.71896E-05$ and total contributions of the next-term options to the next-term implied volatility value was $4.59582E-05$. These numbers were obtained by multiplying the sum of near-term and the sum of next-term contributions with $\frac{1}{T}$.

Next step was to find the value of $\frac{1}{T_j} \cdot \left[\frac{F_j}{K_0} - 1\right]^2$, the second term of Eq. 2.16. This term was equal to $9.42706E-10$, $4.10083E-10$ for near-term and next-term implied volatilities, respectively. Thus, we found σ_1^2 and σ_2^2 as follows:

$$\sigma_1^2 = 5.71887E-05 \quad \text{and} \quad \sigma_2^2 = 4.59578E-05.$$

As the last step of the whole calculation process, we calculated the 60-day weighted average of σ_1^2 and σ_2^2, then we took the square root of that value and multiplied by 100. Finally, we have the implied volatility index value for February 2, 2016 with the calculations of:

$$\text{Volatility Index Value} =$$

$$100 \cdot \sqrt{\left(T_1 \cdot {\sigma_1}^2 \cdot \frac{N_{T_2} - N_{60}}{N_{T_2} - N_{T_1}} + T_2 \cdot {\sigma_2}^2 \cdot \frac{N_{60} - N_{T_1}}{N_{T_2} - N_{T_1}}\right) \cdot \frac{N_{365}}{N_{60}}}$$

$$= 100 \cdot \Bigg[\bigg(0.076712329 \cdot 0.0000571887 \cdot \frac{0.24109589 \cdot 24 \cdot 60 - 60 \cdot 24 \cdot 60}{0.24109589 \cdot 24 \cdot 60 - 0.076712329 \cdot 24 \cdot 60} +$$

$$0.24109589 \cdot 0.0000459578 \cdot \frac{60 \cdot 24 \cdot 60 - 0.076712329 \cdot 24 \cdot 60}{0.24109589 \cdot 24 \cdot 60 - 0.076712329 \cdot 24 \cdot 60}\bigg)$$

$$\cdot \frac{365 \cdot 24 \cdot 60}{60 \cdot 24 \cdot 60}\Bigg]^{\frac{1}{2}} = 12.19396962.$$