POLYNOMIAL TRAJECTORY SHAPING GUIDANCE ALGORITHM FOR MULTI-MISSILE SALVO ATTACK

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ABSTRACT

POLYNOMIAL TRAJECTORY SHAPING GUIDANCE ALGORITHM FOR MULTI-MISSILE SALVO ATTACK

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In this thesis, a guidance algorithm is proposed which aims to control impact time via polynomial shaping of the missile trajectory and missile flight path angle. The main motivation of the trajectory design, which is convenient for impact time control, is its application to multiple missile engagement situations.

In the first part of the study, a planar engagement geometry is considered. The trajectory of the missile is defined as a third order polynomial function of downrange. The coefficients of the polynomial function are obtained by solving a set of parametric equations to control impact time and impact angle. The guidance command required for the missile to follow the designed trajectory is defined as the acceleration command applied in the direction that is normal to the missile velocity vector. Nonlinear equations of motion in planar geometry are used to obtain the acceleration command.

By defining the missile flight path angle in terms of polynomial function coefficients, the acceleration command is derived analytically. In the existence of disturbance effects during flight, the use of this acceleration command will cause the missile to deviate from the reference trajectory. For this reason, in order to obtain the guidance commands in feedback form, a virtual target approach is proposed.

After the trajectory design is completed, it has been studied on the application of the proposed guidance method in 3D space. A maneuver-plane approach is used for the application of the trajectory defined in the planar geometry to 3D space. A reference frame is defined so that its x-axis aligns on the missile-target line-of-sight vector direction. Guidance commands are derived in this reference frame and transformed to inertial frame of reference. By solving the navigation equations in this inertial frame, 3D form of the missile trajectory is obtained.

In this study, a salvo attack is defined as the engagement of multiple missiles to a single, valuable target and the simultaneous arrival of the missiles to the target by following trajectories designed according to the scenario requirements. In the second part of the study, the application of the designed guidance method to a salvo attack scenario is emphasized. First, trajectories are designed and then a maneuver plane is defined for each missile that constitutes the salvo attack. The guidance commands are generated on the corresponding maneuver plane.

In the last part of the thesis, example scenarios are constructed to analyze the characteristics and performance of the proposed guidance method. Different salvo attack scenarios for both stationary and maneuvering targets are examined. Simulation results are discussed in terms of important performance parameters.

Keywords: Impact Time Control, Salvo Attack, Trajectory Shaping, Multiple-Missile Attack, Impact Angle Control

ÖZ

SALVO FÜZE ATIŞI İÇİN POLİNOM FONKSİYONLU YÖRÜNGE ŞEKİLLENDİREN GÜDÜM YÖNTEMİ

Bingöl Taştankaya, Neşe Başak

Yüksek Lisans; Havacılık ve Uzay Mühendisliği

Tez Yöneticisi: Prof. Dr. Ozan Tekinalp

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Bu tez çalışmasında, füze yörüngesini ve uçuş yolu açışını bir polinom fonksiyonu olarak şekillendirerek vuruş zamanını kontrol etmeyi amaçlayan bir güdüm yöntemi üzerinde çalışılmıştır. Vuruş zamanının kontrol edilmesini sağlayan yörünge tasarımının temel motivasyonu, yörüngenin çoklu füze angajmanlarındaki uygulamasıdır.

Çalışmanın ilk bölümünde, düzlemsel bir angajman geometrisi ele alınmıştır. Füzenin yörüngesi, menzile bağlı üçüncü dereceden bir polinom fonksiyonu olarak tanımlanmıştır. Polinom fonksiyonun katsayıları, vuruş zamanını ve vuruş açısını kontrol edecek şekilde, parametrik denklem kümesinin çözülmesi ile elde edilmiştir. Füzenin, tasarlanan yörüngeyi takip etmesi için gerekli güdüm komutu, füze hız vektörüne dik doğrultuda uygulanan ivme komutu olarak tanımlanmıştır. İvme komutunun elde edilmesi için, düzlemsel geometrideki doğrusal olmayan hareket denklemleri kullanılmıştır. Füze uçuş yolu açısının polinom yörünge cinsinden tanımlanması ile, ivme komutu analitik olarak türetilmiştir. Uçuş esnasında bu ivme komutunun uygulanması, açık döngü bir kontrol yöntemi olacağından, ortamda bozucu etkiler olması durumunda füzenin yörüngeden sapmasına neden olacaktır. Bu nedenle, güdüm komutlarını geri besleme formunda elde etmek amacıyla bir sanal hedef yaklaşımı kullanılmıştır.

Yörüngenin tasarlanmasının ardından, 3-boyutlu uzaydaki uygulaması üzerine çalışılmıştır. Düzlemsel geometride tanımlanan yörüngenin 3-boyutlu uzaydaki uygulaması için manevra düzlemi yaklaşımı kullanılmıştır. $x-$Ekseni füze-hedef görüş hattı üzerinde kalacak şekilde bir eksen takımı oluşturulmuş ve manevra düzlemi bu eksen takımı üzerinde tanımlanmıştır. Güdüm komutları bu eksen takımında türetilmiş ve ataletsel referans eksen takımında ifade edilmiştir. Navigasyon denklemleri bu eksen takımında çözdürülerek, füzenin 3-boyutlu uzaydaki yörüngesi elde edilmiştir.

Bu çalışmada, salvo atak, bir hedefe birden fazla füze angaje edilmesi ve füzelerin hedefe senaryo gereksinimlerine uygun yörüngeler izleyerek eşzamanlı olarak ulaşması şeklinde tanımlanmıştır. Çalışmanın ikinci bölümünde, tasarlanan güdüm yönteminin bir salvo atak senaryosunda kullanımı üzerinde durulmuştur. Öncelikle, senaryo gereksinimleri gözetilerek yörüngeler tasarlanmıştır. Sonrasında ise, salvo atağı oluşturan her bir füze için bir manevra düzlemi tanımlanmış, güdüm komutları bu düzlem üzerinde oluşturulmuştur.

Tezin son bölümünde, güdüm yönteminin karakteristiği, performansı ve salvo atak senaryolarında uygulamasının incelenmesi amacıyla örnek senaryolar oluşturulmuştur. Simülasyon sonuçları analiz edilmiş ve önemli noktalar üzerinde tartışılmıştır.

Anahtar Kelimeler: Vuruş Zamanı Kontrolü, Salvo Atak, Yörünge Şekillendirme, Çoklu Füze Atışı, Vuruş Açısı Kontrolü

To my family…

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NOMENCLATURE

$\vec{V}_m$: Missile velocity vector

V_m: Constant missile speed

$\vec{V}_t$: Target velocity vector.

V_t: Constant target speed.

$\vec{a}_n$: Missile acceleration command applied in the direction normal to the missile velocity vector.

γ: Fight path angle. Angle between the velocity vector and the inertial reference frame.

γ_d: Desired impact angle

γ_m: Missile flight path angle in planar engagement geometry

γ_{pitch}: Missile flight path angle in the pitch plane of the inertial reference frame

γ_{yaw}: Missile flight path angle in the yaw plane of the inertial reference frame

γ_t: Target flight path angle in planar engagement geometry

γ_0: Missile launch angle in planar engagement geometry

λ: Line of sight angle. Angle between the missile-target line of sight vector and the inertial reference frame

ε: Look angle. Angle between missile velocity vector and missile-target line-of-sight (LOS) vector

PIP: Predicted Intercept Point

$\vec{P}_m$: Missile position vector

$\vec{P}_{mi}$: Position vector of the i^{th} missile

$\vec{P}_t$: Target position vector

R: Instantaneous range between the target and the missile

S_d: Desired trajectory length

t_d: Desired impact time

VT: Virtual Target

$\vec{\omega}_{LOS}$: Missile-target line of sight rate vector

Δt: Constant time delay between the firing times of the missiles that constitute the salvo attack

CHAPTER 1

INTRODUCTION

Rockets have been widely used as weapons and machines of war, for amusement through their colorful and aerial bursts and also for communications or signals since as early as twelfth century [1]. But since there is no control on the intercept point, rockets could not be used for any specific target or a specific area. The requirement of hitting a specific target point resulted in the need of a control on the rockets motion through its flight. This is the difference between a rocket and a missile. A rocket is defined as a projectile weapon that is carrying a warhead and is propelled by an onboard engine [2]. All rockets are fire-and-forget type weapons and have ballistic trajectories, which means they follow the natural laws of motion through their flight. Opposite to that, missiles can be defined as guided rockets which means their trajectory can be controlled to intercept a specific target or area. Intercepting a specific target point requires a sensor feedback through the flight. These sensors are typically inertial sensors, targeting sensors and radio receivers. Type of sensors use define the control method of the missile which is so called the guidance system [3].

Missile systems can be categorized into two which are tactical (homing) missile and strategic (ballistic) missile systems. Strategic missiles generally use inertial sensors and GPS data to follow the required trajectory. Inertial navigation is used to steer the missile to a predetermined set of Earth coordinates. As a result of this, ballistic missiles cannot be used against a moving or maneuvering target. Generally, they are used against long-range, stationary targets such as buildings or military bases.

Hitting a target which has an unpredictable location requires real-time information about the target status which is provided by the homing sensor device. Since the information about the target status updated during the flight, tactical missiles are accurate for maneuvering targets. Tactical missile systems were first originated in Germany during World War II [4]. After the development of the first guided missile ("*Lark"* missile) in 1950s, missile guidance and control gained more importance and different variations of guidance methods started to be studied.

Tactical missile systems are categorized according to the on-board sensors they contain. Three types of homing missiles are defined as: active, semi-active and passive guided missiles. Active missiles are generally fire-and-forget type and contain an on-board radiation source which radiates and reflects from the target. Distinctively, a passive missile does not contain a radiation source, it uses the radiation originated from the target. Semi-active missile is a combination of active and passive types. The radiation source is deployed on ground or carried by another vehicle [5]. The control strategy, so called the guidance method, of the missile primarily depends on the characteristics of the homing system and type of the target which is aimed to be captured.

For all types, homing loop for a tactical missile is shown in Figure 1-1.

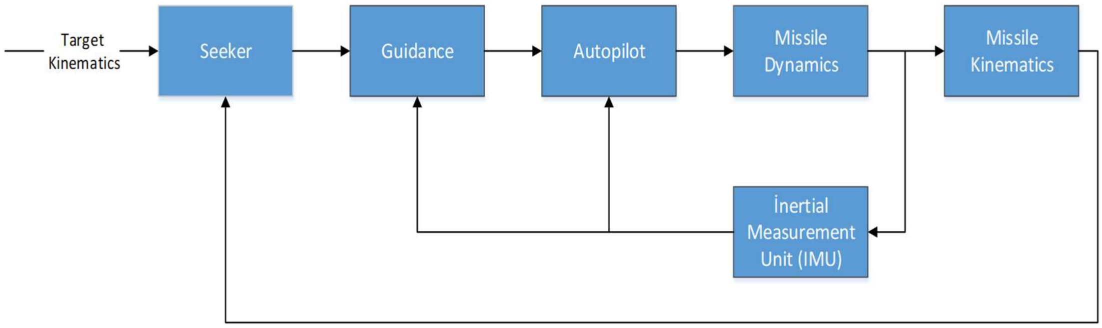

Figure 1-1: Tactical Missile Homing Loop

Seeker is the sensor which perceives the radiation reflected from the target and provides an input for the guidance system. Outputs of the seeker define the missile-target relative kinematics. Most typically used seeker types are IR (infrared), laser and radar seekers. Using the information about the target states which are provided by the missile seeker system, guidance system calculates the necessary commands to steer the target. The system which consists of the guidance, autopilot and missile dynamics generally referred as the Guidance-Navigation and Control (GNC) system.

In literature, guidance is defined as “the process for guiding the path of an object towards a given point, which in general may be moving” [6]. The guided object can be any type of vehicle such as robots, cars, airplanes, UAVs, space crafts or missiles. Missile guidance is a specific and large field of study which includes various applications of control theory. Basically, a guidance law should lead the missile to the target with an acceptable amount of miss distance. Besides this main requirement, some other performance improving and scenario specific requirements can be essential depending on the features of the missile system and the engagement geometry. These sub-requirements can be controlling the impact angle, impact time, applied control effort or any combination of them. An important point to consider while designing a guidance method is the type of the missile system (surface-to-surface, surface-to-air, air-to-surface or air-to-air) and properties and maneuverability capability of the target. For instance, impact angle control is essential especially for anti-tank missile systems to hit the target with a high impact angle and provide a fatal damage. Impact angle control was first studied by Kim et al. [7] by solving the linear quadratic optimal control problem.

In the design process of a guidance method, properties of the target such as minimum and maximum speed, maneuverability, size and other physical characteristics are defined. After that, scenario specific requirements are essential. These requirements include; minimum and maximum range, impact angle, impact velocity, minimum and maximum altitude and other requirements. After the requirements are fixed, guidance method is designed using a control strategy which is appropriate to the situation.

1.1 Literature Survey

With the swiftly developing technology in missile systems and recent advances in missile guidance and control field of study, countermeasure systems technology gained great importance. Modern warfare ships, fighter aircrafts and ground-based military bases have their own self-defense systems against an air attack. For warfare ships, CIWS (close-in weapon systems) is a very common self-defense system which consists of a combination of radars, fire control systems and multiple rapid fire guns [5]. Also engaging a surface-to-air missile to an incoming air threat is another common self-defense technique.

Fighter aircrafts also have countermeasure systems against incoming missile threats. Most common ones are ECM (electronic countermeasure) and flare spread. ECM is very a common countermeasure system of aircrafts against guided missiles to delude any type missiles target detection system. It is composed of various electronical devices especially jammers and can be used against radar, laser or IR guided missiles.

A less complicated countermeasure against missile attack is flare spread. A decoy flare is used against missiles which use IR seeker to track the target. It is a hot-burning metal with a temperature close to the aircraft engine temperature. The aim is to direct the missile seeker to track the flares rather than tracking aircraft engine. Also for IR guided missiles, onboard laser or high-intensity lamp systems coupled with sensors that could detect and track an incoming target missile are used as a countermeasure system. Having detected the missile, the light source would be used to produce a thin, intense beam of infra-red energy that would effectively overload the seeker on the missile, causing it to lose the track of the aircraft. Another countermeasure technique called *Chaff* is used to trick radar guided missiles. In this technique, aircraft sprawls some metallic or plastic particles to jam the tracking radar system. Fighter aircrafts also make aggressive turn maneuvers to get over with the incoming threats.

These countermeasure systems may cause the missile to miss the target. Therefore, in some situations engaging multiple missiles to a single valuable target is essential. If one missile misses the target, the remaining ones can still track and reach the target.

A salvo attack is defined as a many-to-one engagement situation. In a salvo attack scenario, multiple missiles are engaged to a single target with a predefined time delay between their launching times. Missiles can be launched consecutively from the same platform or different platforms. The mission of each missile which constitute salvo attack is to hit the target with an acceptable amount of miss distance at the dictated time. Therefore, the mission of the entire group of missiles can be defined as reaching the Predicted Intercept Point (PIP) simultaneously, which is defined as the estimated location of the target at the instant of impact.

In early studies, main concerns about flight time were minimum time homing, accurate time-to-go estimations and homing techniques without the requirement of time-to-go estimation. The concept of hitting the target at an arbitrary designated flight time was proposed by Lee at all, in their study called *Impact-Time-Control Guidance Law for Anti-Ship Missiles (ITCG)* [5]. In this study, a feedback guidance law to hit the target at the designated impact time is proposed. It is well-known that the conventional PNG (Proportional Navigation Guidance) law is the optimal solution for the minimum control effort problem, when the missile has a constant speed and there is no guidance system lag. This study proposes a method that combines PNG feedback acceleration command with an additional command which controls the impact time. The acceleration command for the proposed guidance law has the form:

$$a_{command} = NV\dot{\lambda} + K_{\varepsilon}\varepsilon_{T} \tag{1.1}$$

The first term is the PNG acceleration command which ensures ZEM (Zero Effort Miss) for a regular scenario. The second term consists of impact time error (difference between the dedicated impact time and estimated impact time) multiplied by a proper gain. This acceleration command is said to be suboptimal since, as the impact time error goes to zero, acceleration command converges to PNG law. Guidance parameters are determined by using a cost function which minimizes applied control effort during flight. The problem is stated in 2-dimensions and the governing equations of the homing problem are linearized to solve the optimal control problem. An important point to concern is that the target is stated to be stationary hence position of the target sets the boundary conditions for the states of the optimal control problem. When the target is moving impact time requirement cannot be satisfied with a diminutive error. ITCG method is also applied for a salvo attack. The designated impact time for the group of missiles is determined as the largest estimated impact time for the case is all missiles were guided with PNG law.

$$t_{go_{desired}} = \max\left\{\hat{t}_{go_i}\right\}, \; i \in \{1, 2, ..., n\} \tag{1.2}$$

This guidance method is convenient for the salvo attack of anti-ship missiles since the target has an ignorable maneuverability capability compared to the missile. For the determination of the designated impact time, all individual missiles should have a common data link to adjust the designated impact time for the calculation of the parameters of the guidance command.
This method sets a precedent for the situation when impact time for salvo attack of group of missiles is achieved using the online communication between missiles. In open literature, different approaches are available for the solution of the simultaneous arrival of a group of missiles. Most commonly, a salvo attack can be achieved in two ways depending whether there is an online communication between the group during the flight or not. Simultaneous arrival to target requires the control of the flight times of each individual missile with a convenient method.

When missiles can communicate to each other during flight, this situation is named as *Cooperative Guidance*. Cooperative guidance is established with different guidance strategies. In the guidance field of study, *Coordination Algorithms* are used to establish the cooperative movement of a group. For information exchange between the individual missiles which form the group, a variable named coordination variable is defined. This variable includes minimal and most essential amount of information and each missile receives this information regarding to other individual missiles during flight. The architecture of the coordination algorithm depends on the communication network of the group. Two types of communication network exist which are; Distributed Communication and Centralized Coordination. In Centralized Coordination, the leading element of the group (so called Coordination Manager) collects all the required data and applies the coordination algorithm. The coordination variable is broadcasted to the group from a single source. Distributed Coordination methods use neighbor-to-neighbor communication in which there are multiple coordination managers and the coordination variable is broadcasted between neighbor elements. These two different strategies are illustrated in Figure 1-2 [8].

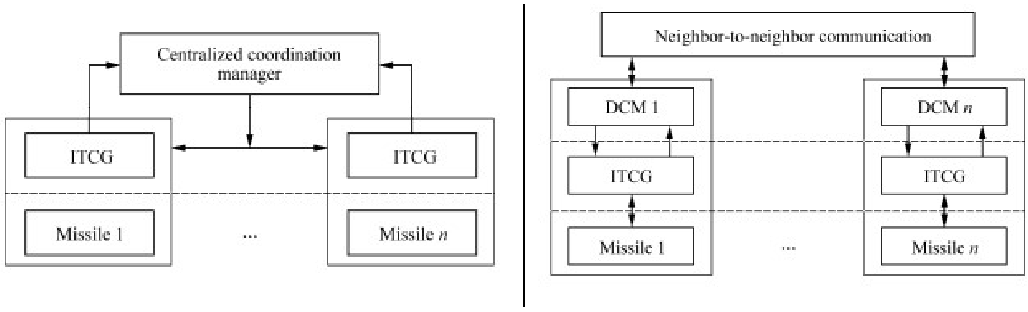

Figure 1-2: Centralized Coordination vs. Distributed Coordination

In a different study which aims to achieve a salvo attack using a Coordination Algorithm, previously proposed ITCG law [5] is used as the guidance law each individual missile uses. Designated impact time is adjusted according to the coordination variable which is estimated time of arrival of the other missiles which form the group [8]. The problem is tackled using both Centralized and Distributed coordination methods. It is concluded that Centralized Coordination strategy shows better performance since all the calculation performed at a single location and all the essential information is broadcasted from a single source which saves considerable amount of time.

In some situations, controlling impact time alone is not sufficient due to the requirements of the engagement geometry. The idea of controlling the orientation of the vehicle velocity vector at the time of interception was first studied by G. Cherry in his study called *A General Explicit Optimizing Guidance Law for Rocket Propelled Spaceflight* [9]. This study reveals the fundamental concepts of the guidance & navigation algorithm of the famous spacecraft *Apollo 11* which was the first spacecraft that landed humans on the moon. This concept, which attempts to control terminal geometry and miss distance at the same time, is entitled as *Explicit Guidance* and there are many studies which deal with various forms of this method [3].

Explicit guidance is parametrized in terms of a design coefficient which determines the degree of curvature of the obtained trajectory. This study is named as *Generalized Vector Explicit Guidance (GENEX)* and aims to achieve zero miss distance and desirable final velocity vector orientation simultaneously [10]. Generalized form of explicit guidance is derived by minimizing a cost function which includes applied control effort multiplied by a constant design parameter.

$$J = \int_0^{T_0} \frac{u^2}{2T^n} dT \tag{1.3}$$

In which $T = t_f - t$ is the time to go to the PIP. Classical minimum control effort optimal control problem is generalized with a user defined design parameter *n*. Hence, choice of different *n* corresponds to a family of cost functions which is in the sense in which explicit guidance is said to be generalized. Implementation of GENEX requires accurate estimation of the PIP and time-to-go to the PIP. The concept of controlling final geometry can be applied to a salvo attack scenario of multiple missiles when the PIP and time-to-go information exist.

An extension of explicit guidance is derived and published as *Hybrid Guidance Law for Angle and Time-of-Arrival Control* [11]. This guidance method is derived using the same time-to-go dependent cost function of GENEX (1.3) but inserts gravity into the system governing equations. Hence it also yields gravity compensation in an optimal way. In this study impact time control is ensured in an indirect way. Arriving the predicted intercept point (PIP) at a desired time is implemented by using time to go as the difference between desired impact time and current flight time. As a consequence, coefficients of the guidance command are calculated using the boundary conditions at the desired impact time.

Application of the explicit guidance to a salvo attack scenario requires an online communication in order to arrange the desired time-to-go values of the missiles. Each missile dictated to hit the target with a different final velocity vector direction to have the attackers approach the target from different directions.

Proportional navigation (PN) has been extensively used for homing missile guidance. It is well known that, in PN acceleration commands are generated proportional to the missile-target line-of-sight rate [12], [13]. Various forms of PN exist according to the forms of commanded acceleration. Biased proportional navigation is commonly used to control impact angle [14].

The control of impact angle is also achieved by shaping the flight trajectory. In the thesis in [15], flight trajectory was defined as a polynomial function and impact angle was controlled by choosing accurate function coefficients.

For a salvo attack scenario, Proportional Navigation Guidance (PNG law is unified with Cooperative Guidance scheme and named as *Cooperative Proportional Navigation* (CPN) [16]. In this methodology, every single missile has a different time varying navigation gain which is adjusted according to the estimated time of arrival data of other missiles.

1.2 Contribution

The contribution of this study can be summarized as follows:

- In literature, polynomial guidance methods have been studied [15]. Although use of polynomial functions as the trajectory function has been used, controlling impact time and impact angle by designing a cubic polynomial reference trajectory is presented as a new guidance concept.

- The proposed polynomial reference trajectory following guidance is used in a 3D salvo attack engagement. This 3D extension is evaluated to be used in practical applications.

- It is considered that this salvo attack strategy increases the hit performance and probability of kill in the existence of target escape maneuvers.

1.3 Thesis Structure

In the first chapter, a brief introduction about missile guidance systems is given. Definition of the multi-missile salvo attack is done and different forms are explained. Then, a literature survey about impact time control guidance laws in literature is presented. Finally, contribution of the thesis is expressed.

The second chapter is named as *Guidance Algorithm Derivation*. In this chapter, a planar engagement geometry between a target and pursuer is investigated first. The parameters defining the engagement scenario are defined. After that, the nonlinear equations motion defining the planar motion of the missile are given. The algorithm of the polynomial trajectory shaping guidance is explained in detail. Acceleration command related to polynomial trajectory shaping guidance is derived by using both analytical approach and virtual target approach.

In the third chapter, 3D extension of the proposed guidance algorithm is given. The application of the method to a 3D geometry is explained in detail.

In chapter 4, the application of the proposed algorithm to a salvo attack scenario is discussed.

In the Simulation Result and Discussions chapter, construction of the missile simulation is explained first. Subsystems of the nonlinear simulation are defined. Then, results of the example scenarios are provided and discussed by means of the guidance performance.

In the final chapter, the thesis is concluded.

CHAPTER 2

GUIDANCE ALGORITHM DERIVATION

As it was mentioned in Chapter 1; a *salvo attack* is defined as a many-to-one engagement situation in which multiple missiles are engaged to a single valuable target. The problem, which is the subject of this study can be defined as the salvo attack of multiple missiles in which an online communication between the individual missiles or the base station does not exist. Therefore, the primary priority task of each individual is to reach the target at a predefined time. Since the missiles do not have any information about the position and velocity of the other individuals, the obtained trajectories should not intercept during flight.

There are various types of guidance methods which ensure impact angle and impact time control separately or simultaneously. These methods do not guarantee that multiple trajectories resulting from the applied guidance command do not intercept during a many-to-one engagement scenario. For instance, impact time control using optimal acceleration command just deals with hitting the target at a desired time by using minimum control effort. Trajectories resulting from this guidance method completely depends on the engagement scenario. It can be concluded that, impact time control alone is not a proper method for a multiple attack. Control of the impact angle in addition to the flight time results in trajectories which approach target from different directions in most cases. But since the trajectory is not directly controlled in this method, interception of the trajectories in some scenarios is unavoidable. To guarantee that the multiple missiles approach the target from different directions and from different paths which never intercept; defining a desired trajectory becomes essential.

The problem of finding a feasible trajectory which meets these requirements is called as the *Trajectory Generation Problem*. A reference trajectory can be any type of algebraic function which satisfies the system constraints. A feasible reference trajectory should satisfy the following requirements:

- Each trajectory should lead the missile to a desired point, i.e. the Predicted Intercept Point (PIP).
- Flight time over each trajectory should be controlled in order to reach the target at the same time. Desired impact time will be given as an input to the guidance system.
- Each single trajectory should approach target from different direction.
- The generated trajectories should not intercept during flight.

In this section, a feasible reference trajectory is defined as a polynomial function and guidance commands are derived to follow this desired trajectory.

2.1 Planar Engagement Geometry and Nonlinear Equations of Motion

In general, any guidance problem should be handled in a three-dimensional geometry. However, if it is assumed that the lateral and longitudinal planes of the missile are decoupled by means of roll control, three-dimensional geometry can be reduced to equivalent two-dimensional planar geometries. In this study, for simplicity, a planar engagement geometry is considered first and the 3-D application will be discussed in the following chapter.

The planar engagement geometry is shown in Figure 2-1. In the figure, subscript m stands for the missile and t stands for the target. Both missile and target are assumed to have constant speeds for the derivation of the guidance algorithm. All angles are defined to be positive in counterclockwise direction.

In the design process of guidance laws, most commonly, effect of gravitational acceleration is neglected in the equations of motion. For the implementation, gravity is considered as a disturbance and effect of gravitational acceleration is compensated from the guidance commands. For this reason, gravitational acceleration is not added to the missile equations of motion.

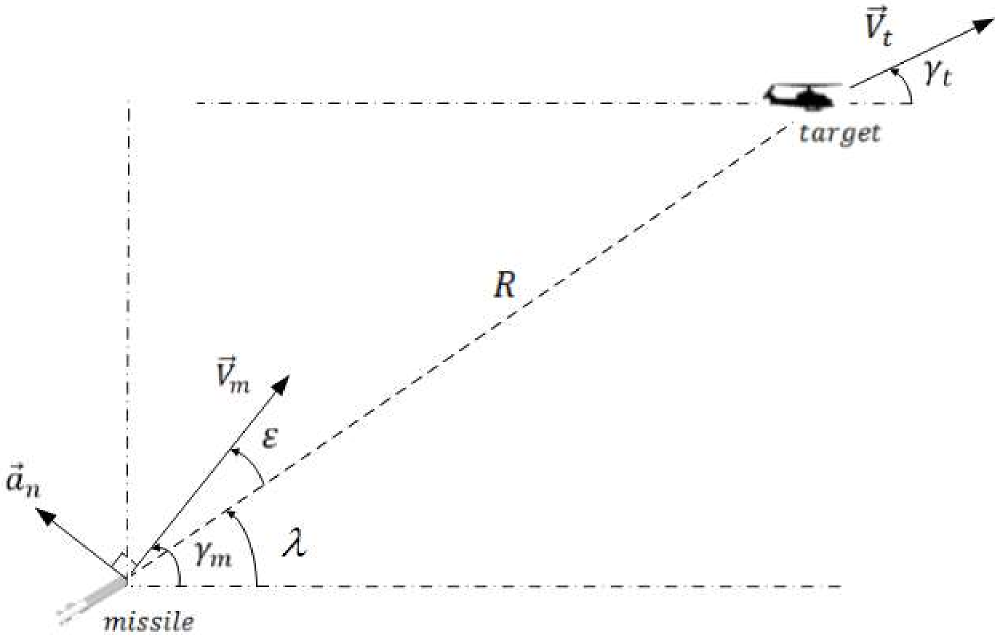

Figure 2-1: Planar Engagement Geometry

The relationship between the crucial angles can be defined as:

$$\varepsilon = \gamma - \lambda \tag{2.1}$$

Equations defining the planar motion of the point mass missile are given in the following equations.

$$\dot{x}(\mathrm{t}) = V_m \cos \gamma(\mathrm{t}) \tag{2.2}$$

$$\dot{z}(\mathrm{t}) = V_m \sin \gamma(\mathrm{t}) \tag{2.3}$$

$$a_n = \dot{\gamma}(\mathrm{t})\,\mathrm{V}_m \tag{2.4}$$

In most endo-atmospheric tactical missiles which are controlled by aerodynamic control surfaces, there is no control authority in the same direction of thrust vector, which is the velocity vector direction. Since the magnitude of the thrust vector is so dominant compared to the force acquired from the fins (aerodynamic control surfaces), the control command is preferred to be applied in the direction normal to the thrust vector. In Eq.(2.4), guidance command, which is the normal acceleration command, steers the missile to the estimated target position by changing the direction of the missile velocity vector while keeping its magnitude constant.

For the derivation of the proposed guidance method, target is assumed to be stationary in the following discussions.

2.2 Derivation of the Guidance Algorithm

Let the initial point of the missile is denoted by point A (x_A, z_A) and the estimated target point is denoted by point B (x_B, z_B). The reference trajectory between points A and B can be defined as any continuous algebraic function of downrange$-x$.

In this study, the reference trajectory is defined as a polynomial function. Let $z(x)$ be an n^{th} order polynomial where n is a nonnegative integer and $a_0, a_1 \dots, a_{n-1}, a_n$ are real numbers.

$$z(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 \tag{2.5}$$

The numbers $a_0, a_1 \dots, a_{n-1}, a_n$ are called the coefficients of $z(x)$ and a_n is called the leading coefficient in the bounded interval $x \in [x_A, x_B]$.

If the trajectory to be followed between the initial pursuer point *A* and *PIP B* is defined to be a polynomial of order n, shape of the trajectory and total flight time can be controlled by accurate selection of the function coefficients. The coefficient selection problem which is referred as the *Reference Trajectory Generation Problem* and the guidance command generation are explained in the following sections.

2.2.1 Calculation of the Total Length of a Given Trajectory

Since the designed reference trajectory is planned to be used in a salvo attack, the total flight time over the trajectory must be controllable. For this reason, total trajectory length should be estimated while the trajectory function coefficients are calculated. When the reference trajectory is defined as a polynomial function as given in Eq.(2.5) the total trajectory length is calculated as follows.

Consider the continuous function $y(x)$ on a closed interval $[x_A, x_B]$. Let this interval is divided into infinitesimally small equal portions with an increment of Δx which is shown in the following figure.

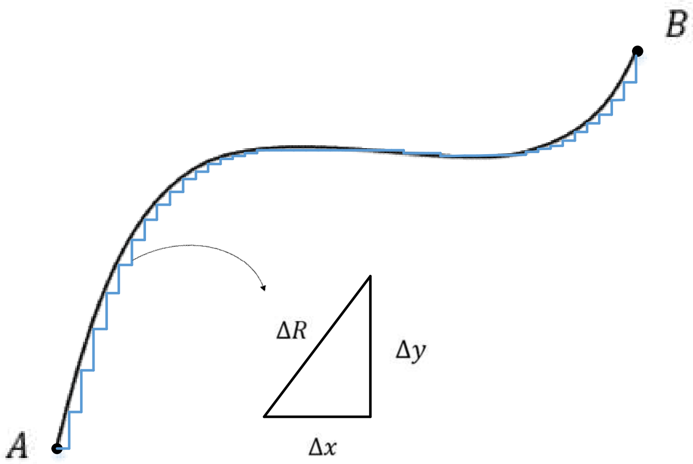

Figure 2-2: Function Portion in a Bounded Interval

If an incremental portion $(\Delta x, \Delta y)$ of the function is taken which is infinitesimally small, the length of this increment becomes:

$$\Delta R = \sqrt{\Delta x^2 + \Delta y^2} \tag{2.6}$$

The function portion is divided to N sub-portions and these sub-portions are summed up to find an approximate value of the trajectory in the defined boundaries. Let S be the total length of the trajectory, then it can be obtained by summing up these incremental lengths from the first to N^{th} sub-portion, i.e.:

$$S \cong \sum_{i=1}^{N} \Delta R \tag{2.7}$$

In order to obtain S accurately, $N \to \infty$ as $\Delta R \to 0$. Consequently, the summation formula converges to a definite integral with $\Delta x = dx$ and $\Delta y = dy$;

$$S = \int \sqrt{dx^2 + dy^2} \tag{2.8}$$

$$S = \int_{x_A}^{x_B} \sqrt{1 + \frac{dy}{dx}^2}\; dx \tag{2.9}$$

In which dy/dx is the derivative of the polynomial function in Eq. (2.5).

$$\frac{dy}{dx} = na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots + a_2 x + a_1 \tag{2.10}$$

Inserting the derivative into the path integral of Eq.(2.9), the following integral equation is obtained.

$$S = \int_{x_A}^{x_B} \sqrt{1 + \left(na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots + a_2 x + a_1\right)^2}\; dx \tag{2.11}$$

Consider the 2^{nd} order quadratic polynomial $f(x)$:

$$f(x) = ax^2 + bx + c \tag{2.12}$$

With the derivative of:

$$\frac{dy}{dx} = 2ax + b \tag{2.13}$$

By inserting this derivative into Eq.(2.11), the integral function which is used to calculate total path length is obtained as:

$$S = \int_{x_A}^{x_B} \sqrt{1+(2ax+b)^2}\ dx \tag{2.14}$$

$$S = \int_{x_A}^{x_B} \sqrt{4a^2x^2+4abx+(b^2+1)}\ dx \tag{2.15}$$

The function $g(x)=\sqrt{4a^2x^2+4abx+(b^2+1)}$ is an integrable function with respect to the variable x . Integral of Eq.(2.15) is evaluated as in Eq.(2.16), in which $\sinh^{-1}(x)$ is the inverse hyperbolic sine function.

$$S = \frac{\sqrt{(2ax+b)^2+1}\,(2ax+b)+\sinh^{-1}(2ax+b)}{4a} + constant \tag{2.16}$$

Total path length between points A and B can be obtained by evaluating the function in Eq. (2.16) at x_A and x_B values.

If the degree of the polynomial (2.12) is increased to 3, as follows,

$$f(x) = ax^3+bx^2+cx+d \tag{2.17}$$

the indefinite integral which defines the total path length becomes:

$$S = \int \sqrt{1+\left(3ax^2+2bx+c\right)^2}\,dx \tag{2.18}$$

This function $g(x)=\sqrt{1+\left(3ax^2+2bx+c\right)^2}$ is a nonintegrable function of x, which means value of S in Eq.(2.18) cannot be obtained analytically. This situation is encountered for the reference trajectory polynomials with a degree of 3 or higher. Consequently, analytical methods cannot be used to evaluate the path length, if the reference trajectory has a degree of 3 or higher. An analytical solution for the integral does not exist for polynomials of degree 3 or higher. Numerical integration methods should be used to evaluate the path integral for a cubic or higher order polynomials.

2.2.2 Numerical Integration for the Calculation of the Trajectory Length

It was stated in the previous section that, it is not possible to calculate the path length of a polynomial trajectory function with a degree of 3 or higher. Therefore, numerical integration methods are used to compute the integral equation of (2.18).

The *Gaussian Quadrature* is a numerical integration method that uses optimal non-uniformly distributed points to achieve the best numerical estimation [17]. It produces accurate results when the function to be integrated is approximated by a polynomial function without singularity in the interval $[-1, 1]$.

$$\int_{-1}^{1} g(x)\mathrm{dx} \approx \sum_{i=1}^{n} \omega_i(x) g(x_i) = \omega_1 g(x_1) + \omega_2 g(x_2) + \ldots + \omega_n g(x_n) \tag{2.19}$$

The function $\omega(x)$ is called the weighting function. When the weighting function $\omega(x) = 1$, the resulting polynomials are named as *Legendre* polynomials and the method is named as *Gauss-Legendre Quadrature.*

For the 3^{rd} order polynomial reference trajectory function, in order to obtain the total trajectory length, the function to be integrated is given below.

$$f(x) = \sqrt{1 + \left(3a_3x^2 + 2a_2x + a_1\right)^2} \tag{2.20}$$

In order to apply the Gauss-Legendre formula, boundaries of the integral should be mapped from $[x_A, x_B]$ into the interval $[-1, 1]$ by using the following coordinate transformation.

$$\begin{gathered} x = \frac{x_B - x_A}{2} t + \frac{x_B + x_A}{2} \\ t = -1 \Rightarrow x = x_A \\ t = 1 \Rightarrow x = x_B \end{gathered} \tag{2.21}$$

Applying this transformation on the function (2.20), the following integral is obtained.

$$\int_{x_A}^{x_B} f(x)\,\mathrm{dx} = \int_{-1}^{1} f\left(\frac{x_B - x_A}{2}t + \frac{x_B + x_A}{2}\right)\left(\frac{x_B - x_A}{2}\right)\mathrm{dt} = \int_{-1}^{1} g(t)\,\mathrm{dt} \qquad (2.22)$$

Equations (2.20) and (2.22) are combined to obtain the new transformed function.

$$g(t) = \left(\frac{x_B - x_A}{2}\right)\sqrt{1 + \left(3a_3\left(\frac{x_B - x_A}{2}t + \frac{x_B + x_A}{2}\right)^2 + 2a_2\left(\frac{x_B - x_A}{2}t + \frac{x_B + x_A}{2}\right) + a_1\right)^2}$$

(2.23)

For the numerical estimation of the path integral of Eq. (2.11), four-point quadrature formula is selected due to its relatively small numerical error. The path integral is estimated by using 4 points as follows:

$$\int_{-1}^{1} g(t)\,\mathrm{dt} \approx \omega_1 g(t_1) + \omega_2 g(t_2) + \omega_3 g(t_3) + \omega_4 g(t_4) \qquad (2.24)$$

The weighting factors ω_i's and the function arguments t_i's used in the 4-point Gauss-Legendre formula are given in the table below.

Table 2-1: 4-Point Gauss-Legendre Formula Coefficients

i	Weighting Factors, ω_i	Function Arguments, t_i
1	0.3478548	−0.861136312
2	0.6521452	−0.339981044
3	0.6521452	0.339981044
4	0.3478548	0.861136312

2.2.3 Reference Trajectory Generation

In this section, a reference trajectory is designed that will reach the missile to the PIP at the desired impact time. This trajectory will be used in a salvo attack of multiple missiles in the following sections.

A polynomial function of order n, has $n+1$ coefficients which are denoted by $a_0, a_1, \ldots, a_n$. So, for a reference trajectory function which is defined by an n^{th} order polynomial, there exist $n+1$ coefficients to be determined. In other words, there exist $n+1$ degrees of freedom. For the problem considered in this study, the function has possessed at least 4 degrees of freedom which are:

- Initial target location should satisfy the polynomial equation
- Location of the PIP should satisfy the polynomial equation
- Flight time over the trajectory should be prescribed.
- Either impact angle or launch angle should be prescribed.

Therefore, to meet these requirements, the reference trajectory function must be at least 3^{rd} order with 4 unknown coefficients to be determined. A cubic polynomial function is proposed to be the reference trajectory function and the function coefficients will be calculated to meet the defined trajectory requirements.

2.2.3.1 Reference Trajectory as a Cubic Polynomial Function

In this part, the reference trajectory between initial missile position and PIP position is defined as a cubic function of x, i.e.

$$z(x) = a_3x^3 + a_2x^2 + a_1x + a_0 \tag{2.25}$$

It is assumed that either the target is stationary, or velocity and position information of the target is provided to the missile guidance algorithm by a radar system. That means, the PIP of missile and target is estimated with high accuracy.

In the calculations, initial position of the missile is denoted by point A and the position of the PIP is denoted by point B. For the simplicity of calculations, the initial position point A is always located at the origin of the coordinate system.

$$(x_A, z_A) = (0,0) \tag{2.26}$$

Since the reference trajectory should lead the missile to the target, both points A and B should lie on that trajectory. In other words, coordinates of points A and B should satisfy trajectory equation.

$$z_A = a_3 x_A^3 + a_2 x_A^2 + a_1 x_A + a_0 \tag{2.27}$$

$$a_0 = 0 \tag{2.28}$$

It is clear that the coefficient a_0 is always equals to zero when the initial target position is located at the origin.

If point B is interested in to Eq.(2.25):

$$z_B = a_3 x_B^3 + a_2 x_B^2 + a_1 x_B \tag{2.29}$$

Relative speed between the missile and target is denoted by V_{rel}:

$$V_{rel} = \left| \overrightarrow{V_m} - \overline{V_t} \right| \tag{2.30}$$

The second important requirement that the reference trajectory should satisfy is that; total flight time over the trajectory should be a user a defined input to the guidance algorithm. If the desired flight time is denoted by t_d, total length of the reference trajectory may be approximated as:

$$S \approx V_{rel}\ t_d \tag{2.31}$$

The function $z(x)$ has 3 parameters a_1, a_2 and a_3 that define the shape of the reference trajectory. Hence 3 independent equations are required to obtain these parameters uniquely. Two of these equations are defined as Eqns. (2.29) and(2.31). A final equation is required to solve the equation set.

In a salvo attack scenario, overlapping of the trajectories of the individual missiles should be avoided. In addition to that, missiles should approach the target from different directions as a countermeasure to the evasive maneuvers of the target and to give fatal damage. The third equation is stated such that either impact angle is controlled or the launch angle is adjusted to arrange the curvature and direction of approach of the reference trajectory.

The equation set to be solved consists of 3 parametric, independent and linear equations. As a summary, the equation set is summarized as follows:

1. PIP satisfies the reference trajectory function.

$$z_B = a_3 x_B^3 + a_2 x_B^2 + a_1 x_B \tag{2.32}$$

2. Flight time is controlled via calculation of total trajectory length.

$$S = V_{rel} t_d \tag{2.33}$$

3. Impact time of launch angle is specified.

$$\left.\frac{dz}{dx}\right|_{x=x_B} = \gamma_f \quad \text{OR} \quad \left.\frac{dz}{dx}\right|_{x=x_A} = \gamma_0 \tag{2.34}$$

By solving this set of equations, parameters a_1, a_2 and a_3 can be obtained uniquely.

2.2.4 Analytical Derivation of the Guidance Command by Polynomial Shaping of the Flight Path Angle

The velocity vector should always be tangential to the trajectory as long as the missile follows the reference trajectory. A planar representation of the reference trajectory tracking is given in the figure below.

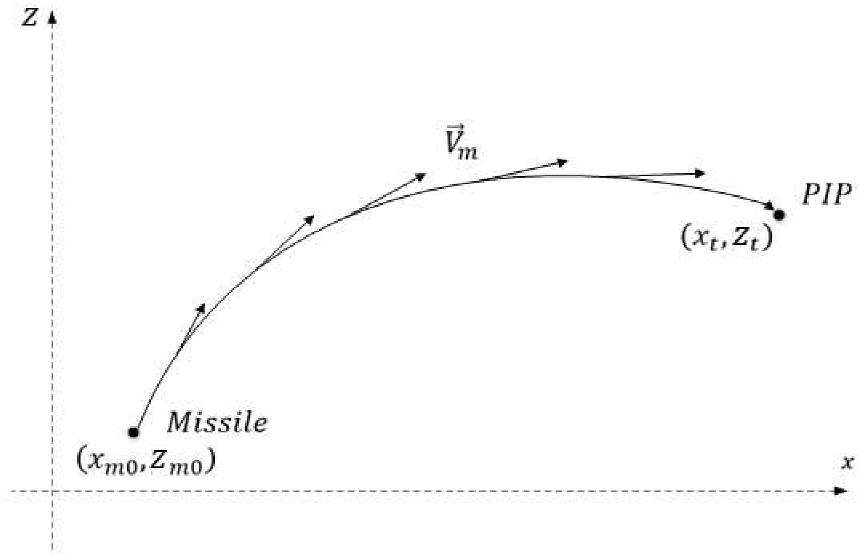

Figure 2-3: Planar Representation of Reference Trajectory Following

Therefore, by definition, the flight path angle can be expressed in terms of the trajectory function.

$$\tan\gamma(x) = \frac{dz}{dx} \tag{2.35}$$

If the small angle assumption is made for γ; for the cubic reference trajectory defined by the function of Eq.(2.25), flight path angle can be defined as a quadratic function of the downrange as:

$$\tan(\gamma) = \frac{dz}{dx} = 3a_3x^2 + 2a_2x + a_1 \tag{2.36}$$

In most guidance laws, in order to obtain a linear set of equations, flight path angle is assumed to be small during flight. However, in the reference trajectory tracking problem considered here, small angle assumption can cause undesirable errors. Hence, it is more convenient to use flight path angle function without making the small angle assumption, i.e.:

$$\gamma(x) = \tan^{-1}\left(3a_3x^2 + 2a_2x + a_1\right) \tag{2.37}$$

The acceleration command normal to the missile velocity vector was stated in the Eq. (2.4). In this equation, guidance command is defined in time domain, since the sensor data about target information is provided also in time domain. Hence, the desired trajectory has the shape of a cubic polynomial of downrange x, it can be also defined as a cubic polynomial of time t. Thus, flight path angle in time domain can be defined as a quadratic function of time, with the polynomial parameters defined in time domain.

The $\gamma(t)$ function can be defined in the closed interval $t \in [0, t_d]$ as:

$$\gamma(t) = kt^2 + mt + n \tag{2.38}$$

If the coefficients of the function (2.38) are obtained at each time instant, the guidance command can be obtained as a function of $\gamma(t)$.

$$a_n = V_m \frac{d\gamma}{dt} \tag{2.39}$$

By taking the time derivative of the quadratic $\gamma(t)$ function, the resulting normal acceleration command which will steer the missile to the predicted intercept point can be obtained. Using chain rule of differentiation, time derivative of flight path angle can be obtained as in the Eq.(2.40).

$$\frac{d\gamma}{dt} = \frac{d\gamma}{dx} \cdot \frac{dx}{dt} \tag{2.40}$$

In which:

$$\frac{d\gamma}{dx} = \frac{2\left(3a_3 x(t) + a_2\right)}{\left(x(t)\left(3a_3 x(t) + 2a_2\right) + a_1\right)^2 + 1} \tag{2.41}$$

Time derivative of $x(t)$ was given in the Eq.(2.2). Combining these and inserting into Eq.(2.40), time derivative of γ is obtained as follows.

$$\frac{d\gamma}{dt} = \frac{2V_m \cos\gamma\left(3a_3 x(t) + a_2\right)}{\left(x(t)\left(3a_3 x(t) + 2a_2\right) + a_1\right)^2 + 1} \tag{2.42}$$

Finally, the normal acceleration command of Eq.(2.4) is obtained as follows.

$$a_n = \frac{2V_m^2 \cos\gamma\left(3a_3 x + a_2\right)}{\left(x\left(3a_3 x + 2a_2\right) + a_1\right)^2 + 1} \tag{2.43}$$

2.2.5 Derivation of the Guidance Command by Virtual Target Approach

The guidance command in Eq. (2.43) is the analytical formula of the required acceleration command that is normal to the missile velocity vector. If this acceleration command is applied during total flight time, missile can reach the estimated intersection point by following the designed reference trajectory. The coefficients of the trajectory function are calculated offline with the provided PIP location and once the coefficients of the flight path angle profile are calculated, no feedback is required during flight to achieve missile-target interception. Hence, it can be said that this acceleration command steers the missile towards a desired trajectory by applying an open loop control. Although it is mathematically correct, it is obvious that this result cannot be used in practical applications due to existence of disturbances.

In order to obtain a guidance command in feedback form, a *virtual target* approach is considered in this section. Let a virtual target follows the reference trajectory defined in Eq. (2.25) with constant speed. The virtual target starts its motion at a point x_{T_0} which is Δx_T away from the origin and stops at the PIP. Figure 2-4 demonstrates the motion of the virtual target.

The virtual target must have the same speed as the missile so that the missile can follow the reference trajectory without ever reaching the virtual target destination. Since the instantaneous velocity and position of the virtual target are known, any type of parallel navigation guidance law can be used to follow this virtual target point.

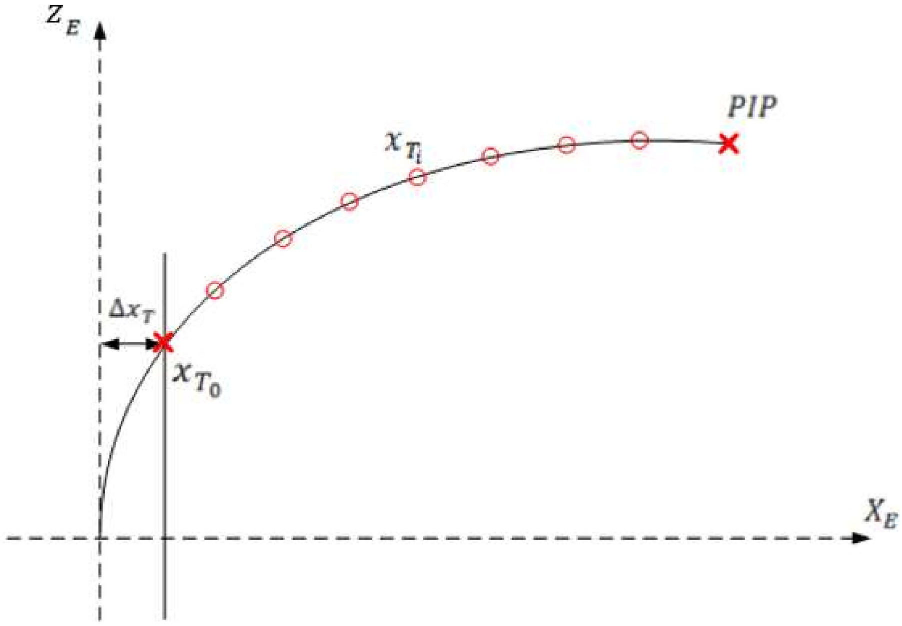

Figure 2-4: Virtual Target Trajectory

Line-of-sight rate between missile and the virtual target can be obtained as follows:

$$\dot{\lambda}_{VT} = \frac{\left(x_{VT} - x_M\right)\left(V_{z_M} - V_{z_{VT}}\right) - \left(z_{VT} - z_M\right)\left(V_{X_M} - V_{X_{VT}}\right)}{R^2} \tag{2.44}$$

In the above formula, R stands for the instantaneous range between missile and virtual target, V_x is the component of the velocity vector in x-direction, V_z is the component of the velocity vector in y-direction and subscript VT stands for the virtual target and M stands for missile.

To generate the guidance commands, proportional navigation is used. The navigation constant N' is usually taken to be greater than or equal to three ($N' \geq 3$).

$$a_n = N' V_M \dot{\lambda}_{VT} \tag{2.45}$$

Increasing N' will cause higher acceleration commands to be generated. Therefore, this will result in a reduction of the deviations due to disturbance effects.

CHAPTER 3

EXTENSION OF POLYNOMIAL TRAJECTORY SHAPING GUIDANCE TO THREE DIMENSIONAL ENGAGEMENTS

The designed polynomial trajectory shaping guidance algorithm has a planar implementation logic. Therefore, to use this algorithm in 3D engagements, it is required to define a maneuver plane and perform the guidance commands to track the desired trajectory in this plane [18]. This maneuver plane on which the desired reference trajectory is defined should be selected such that final velocity vector takes part in this plane. For the 3-D implementation, reference trajectory is defined in $x - z$ plane and the guidance command in y −plane is applied to prevent deviations from the maneuvering plane. Velocity pursuit guidance law is used in yaw plane which aims to make the y −component of the velocity vector zero.

3.1 Reference Frame Definitions

For the 3-D implementation all reference frames are defined as *Inertial Reference Frames* which means they have a constant, rectilinear motion with respect to each other. The origin of the inertial frames can be selected arbitrarily, therefore initial missile location is assigned as the origin of the first maneuver plane for the simplicity of the calculations. The Earth Centered Earth Fixed (ECEF) Frame has its origin fixed to the center of the earth. All other reference frames can be defined relative to the ECEF frame.

Frame-0, $\mathcal{F}_0(0,0,0)$: *Navigation Reference Frame.* This frame is defined as the geodetic East, North, Up Frame. It is named as Navigation Frame since all navigation equations are solved in this frame. Orientation of *Frame*-0 is shown in the following figure.

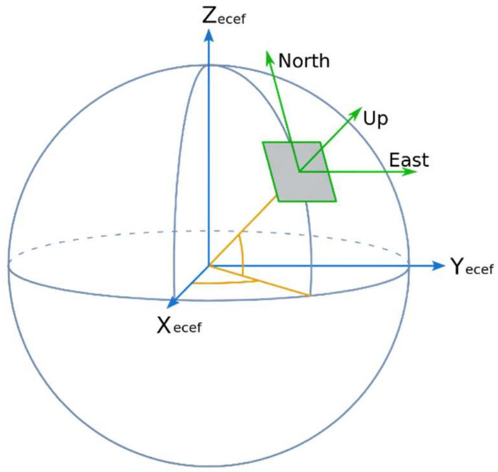

Figure 3-1: Navigation (North-East-Up) Frame Orientation with respect to ECEF

<u>*Frame-1*</u>, $\mathrm{F}_1(0,0,0)$: *Maneuver (Line-of-Sight) Reference Frame.* This reference frame is defined such that the $x-$ axis of F_1 is aligned in the missile-target Line-of-Sight vector direction. The *maneuver plane* is defined in this reference frame, therefore trajectory shaping guidance equations are also to be solved in this reference frame.

Frame-0 is to be rotated by angles ψ_1 and $\theta_1$to obtain *Frame-1* whose $x-$axis aligns along the LOS direction. The rotation angles are defined from the geometry of Figure 3-2:

$$\theta_1 = \tan^{-1}\left(-\frac{z_B}{\sqrt{{x_B}^2 + {y_B}^2}} \right) \tag{3.1}$$

$$\psi_1 = \tan^{-1}\left(\frac{y_B}{x_B} \right) \tag{3.2}$$

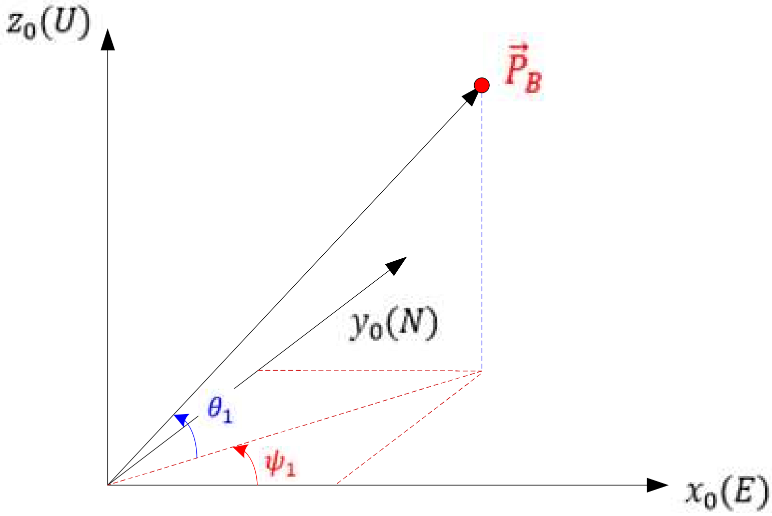

Figure 3-2: $Ƒ_1$ Angle Definitions

Rotation matrices given in the equations (3.3) and (3.4) define the successive rotations around y and z axes.

$$\hat{R}_2(\theta_1) = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 \\ 0 & 1 & 0 \\ -\sin\theta_1 & 0 & \cos\theta_1 \end{bmatrix} \tag{3.3}$$

$$\hat{R}_3(\psi_1) = \begin{bmatrix} \cos\psi_1 & -\sin\psi_1 & 0 \\ \sin\psi_1 & \cos\psi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.4}$$

The $Ƒ_0$ frame is first rotated by an angle ψ_1 around z_0 axis. By this rotation, an interim frame $Ƒ_0{}'$ is obtained. After that, $Ƒ_0{}'$ is rotated by an angle of θ_1 around y_0' axis. Therefore, the transformation (*Direction Cosine Matrix)* matrix between frames $Ƒ_1$ and $Ƒ_0$ can be obtained by using $3-2-1$ rotation.

$$\hat{C}^{(0,1)} = \hat{R}_2(\theta_1).\hat{R}_3(\varphi_1) \tag{3.5}$$

$$\hat{C}^{(0,1)} = \begin{bmatrix} \cos(\psi_1)\cos(\theta_1) & -\sin(\psi_1) & \cos(\psi_1)\sin(\theta_1) \\ \cos(\theta_1)\sin(\psi_1) & \cos(\psi_1) & \sin(\psi_1)\sin(\theta_1) \\ -\sin(\theta_1) & 0 & \cos(\theta_1) \end{bmatrix} \tag{3.6}$$

By definition, the DCM $\hat{C}^{(0,1)}$ is an orthogonal matrix, i.e. the inverse of the DCM is equal to its transpose. Hence, the transformation matrix from $\mathfrak{F}_0$ to $\mathfrak{F}_1$ can be obtained by taking the transpose of the $\hat{C}^{(0,1)}$ matrix.

$$\hat{C}^{(1,0)} = \left(\hat{C}^{(0,1)}\right)^T \tag{3.7}$$

$$\hat{C}^{(1,0)} = \begin{bmatrix} \cos(\psi_1)\cos(\theta_1) & \cos(\theta_1)\sin(\psi_1) & -\sin(\theta_1) \\ -\sin(\psi_1) & \cos(\psi_1) & 0 \\ \cos(\psi_1)\sin(\theta_1) & \sin(\psi_1)\sin(\theta_1) & \cos(\theta_1) \end{bmatrix} \tag{3.8}$$

The $\vec{P}_B$ vector is expressed in the Navigation Frame by using $\hat{C}^{(1,0)}$.

$$\bar{P}_B^{(1)} = \hat{C}^{(1,0)}.\bar{P}_B^{(0)} \tag{3.9}$$

$$\bar{P}_B^{(1)} = \begin{bmatrix} \sqrt{x_B^2 + y_B^2 + z_B^2} \\ 0 \\ 0 \end{bmatrix} \tag{3.10}$$

The location of the PIP in *Frame-1* is shown in the Figure 3-3.

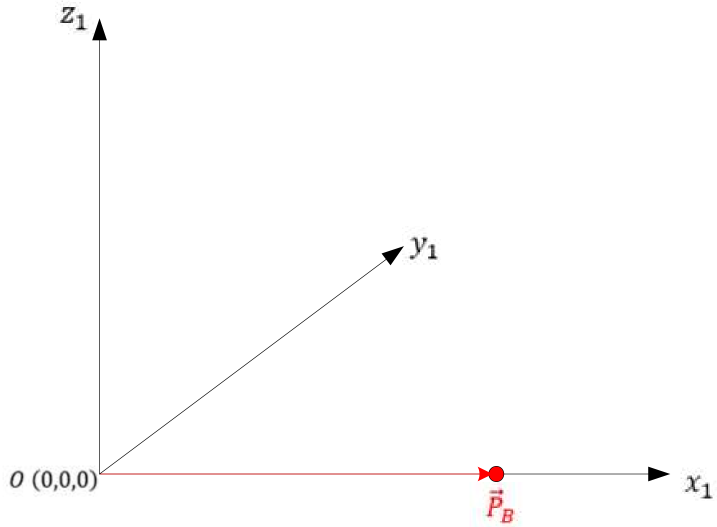

Figure 3-3: Location of the PIP in Frame-1

Reference frames F_0 and F_1 are defined for the implementation of the polynomial trajectory shaping guidance to a 3-D engagement geometry. The following reference frames denoted by $F_i, i = 2, 3, \dots, N$ are used for the implementation of a 3-D *salvo attack* scenario in which *N* denotes the number of missiles that perform the salvo attack. These reference frames are used when all missiles are launched from the same point (origin).

> *Frame-i*, $i = 2, 3, \dots, N$ $F_i(0,0,0)$: *Maneuver Frame* for the i^{th} missile of the salvo attack. This frame is obtained by rotating F_1 around x_1 axis with an angle of ϕ_i.

Rotation Matrix around roll axis is defined as:

$$\hat{R}_1(\phi_i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_i & -\sin\phi_i \\ 0 & \sin\phi_i & \cos\phi_i \end{bmatrix} \tag{3.11}$$

Hence, transformation matrix from F_0 to F_i can obtained by using $\hat{R}_1(\phi_i)$:

$$\hat{C}^{(i,1)} = \left(\hat{R}_1\left(\phi_i\right)\right)^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_i & \sin\phi_i \\ 0 & -\sin\phi_i & \cos\phi_i \end{bmatrix} \tag{3.12}$$

The implementation of this guidance algorithm to a salvo attack scenario will be discussed in the next chapter.

3.2 Three-Dimensional Implementation of Polynomial Trajectory Shaping Guidance Algorithm

The *Maneuver Plane* is defined as the $x_1 - z_1$ plane of the reference frame $\mathcal{F}_1$. Therefore, the desired polynomial trajectory should be generated and tracked in this plane and deviations from maneuver plane should be avoided.

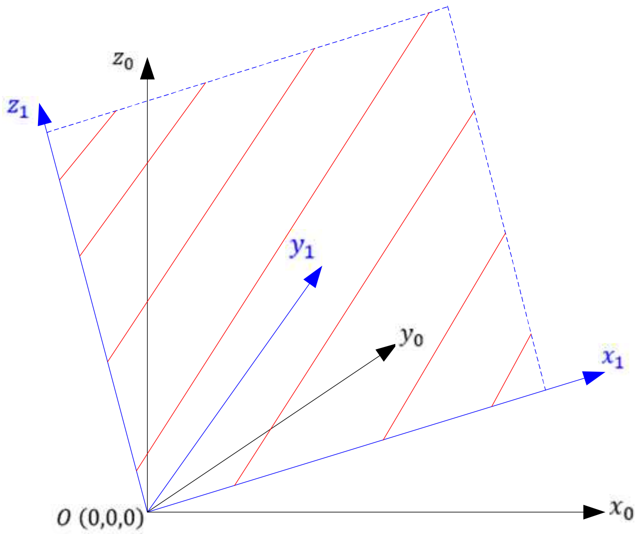

Figure 3-4: Maneuver Plane in $\mathcal{F}_1$

To implement the proposed guidance algorithm in 3-D space, guidance commands in pitch and yaw planes are generated separately.

In yaw plane; *Velocity Pursuit Guidance* is used in which guidance commands are generated proportional to the error that causes deviation from the maneuver plane. Velocity Pursuit acceleration command is given in the following equation, where $\gamma_{ref} = 0$.

$$a_{com} = K\left(\gamma_{yaw} - \gamma_{ref}\right) \tag{3.13}$$

Flight Path Angles in 3-D geometry $(\gamma_{pitch}\ and\ \gamma_{yaw})$ are calculated using the equations given below. Missile velocity, $\vec{V}_m$ resolved in $\mathcal{F}_1$ is stated in Eq.(3.14). By using these velocity components, flight path angles in pitch and yaw planes are calculated in equations (3.15) and (3.16).

$$\overline{V}_m^{(1)} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} \tag{3.14}$$

$$\gamma_{pitch}^{(1)} = \tan^{-1}\left(\frac{V_z}{\sqrt{V_x^2 + V_y^2}}\right) \tag{3.15}$$

$$\gamma_{yaw}^{(1)} = \tan^{-1}\left(\frac{V_y}{V_x}\right) \tag{3.16}$$

The acceleration command in Eq. (2.43) is derived to follow a polynomial reference trajectory by controlling total flight time and impact angle in the considered plane. For the 3-D implementation, this normal acceleration command is modified and redefined in the maneuver-plane.

$$a_n = \frac{2V_m^2 \cos\left(\gamma_{pitch}^{(1)}\right)\left(3a_3\overline{P}_m^{(1)}(1) + a_2\right)}{\left(\overline{P}_m^{(1)}(1)\left(3a_3\overline{P}_m^{(1)}(1) + 2a_2\right) + a_1\right)^2 + 1} \tag{3.17}$$

Where;

$$\overline{P}_m^{(1)} = \hat{C}^{(1,0)} \overline{P}_m^{(0)} \tag{3.18}$$

Finally, total acceleration command in $\digamma_1$ is obtained and given in the equation below:

$$\overline{a}_{com}^{(1)} = \begin{bmatrix} -a_n \sin\left(\gamma_{pitch}^{(1)}\right) \\ K\left(\gamma_{yaw}^{(1)} - \gamma_{ref}\right) \\ a_n \cos\left(\gamma_{pitch}^{(1)}\right) \end{bmatrix} \tag{3.19}$$

Since the navigation equations are solved in the Navigation Frame $\digamma_0$, before integrating, acceleration command of Eq.(3.19) should be moved to $\digamma_0$.

$$\overline{a}_{com}^{(0)} = \hat{C}^{(0,1)} \overline{a}_{com}^{(1)} \tag{3.20}$$

$$\overline{a}_{com}^{(0)} = \begin{bmatrix} -a_n \sin\left(\gamma_{pitch}^{(1)}\right)\left(\cos\psi_1 \cos\theta_1\right) - K\left(\gamma_{yaw}^{(1)} - \gamma_{ref}\right)\sin\psi_1 + a_n \cos\left(\gamma_{pitch}^{(1)}\right)\left(\cos\psi_1 \sin\theta_1\right) \\ -a_n \sin\left(\gamma_{pitch}^{(1)}\right)\left(\cos\theta_1 \sin\psi_1\right) + K\left(\gamma_{yaw}^{(1)} - \gamma_{ref}\right)\cos\psi_1 + a_n \cos\left(\gamma_{pitch}^{(1)}\right)\left(\sin\psi_1 \sin\theta_1\right) \\ a_n \sin\left(\gamma_{pitch}^{(1)}\right)\sin\theta_1 + a_n \cos\left(\gamma_{pitch}^{(1)}\right)\cos\theta_1 \end{bmatrix}$$

(3.21)

This acceleration command is directly used in navigation equations and integrated to obtain velocity and position of the missile. The important point to be considered is the initial condition assignment of the velocity and position integrals. Since at all reference frames, initial missile position is located at the origin, initial value of the position integral is also assigned as zero.

$$\vec{P}_{m0} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \tag{3.22}$$

It was stated in Chapter 2, for a polynomial reference trajectory, initial value of the flight path angle becomes:

$$\gamma_0 = \tan^{-1}\left(a_1\right) \tag{3.23}$$

In 3-D application, this relation corresponds to:

$$\begin{aligned} \gamma_{pitch0} &= \tan^{-1}\left(a_1\right) \\ \gamma_{yaw0} &= 0 \end{aligned} \tag{3.24}$$

It should be noted that, these angles (γ_{pitch0} and γ_{yaw0}) are defined as the missile flight path angles when the velocity vector is resolved in $\mathcal{F}_1$. Hence, initial condition for the velocity vector resolved in $\mathcal{F}_1$ is obtained as follows:

$$\overline{V}_{m0}^{(1)} = V_m \begin{bmatrix} \cos\left(\gamma_{pitch0}\right)\cos\left(\gamma_{yaw0}\right) \\ \cos\left(\gamma_{pitch0}\right)\sin\left(\gamma_{yaw0}\right) \\ \sin\left(\gamma_{pitch0}\right) \end{bmatrix} \tag{3.25}$$

Initial value of the velocity integral is obtained as:

$$\overline{V}_{m0}^{(0)} = \hat{C}^{(0,1)}\ \overline{V}_{m0}^{(1)} \tag{3.26}$$

Finally, by solving the navigation equations, the trajectory of the missile in 3D geometry is obtained.

CHAPTER 4

SALVO ATTACK OF MULTIPLE MISSILES

In this chapter, application of the proposed guidance algorithm to a salvo attack scenario is discussed. The scenario requirements for a salvo attack are defined as:

1. All missiles should arrive to the target point simultaneously.
2. To increase the probability of hit and to give fatal damage to the target, all missiles should approach to the target from different directions.
3. Trajectories of individual missiles should not intercept at any time during flight.

As it was stated before, the *Predicted Intercept Point* (PIP) is defined as the calculated position in space where the target and interceptor coincide. In other words, PIP can be defined as the estimated position of the target at the time of impact. In order to achieve the simultaneous arrival of the all missile group to the PIP, total time of flight of each missile should be controlled. Also, the reference trajectory should end at the PIP with a relatively small miss distance. Since there is no data link (information exchange) between the missiles during flight, the desired flight time, t_d, should be calculated for each missile before launch and this value should be designated to the guidance algorithm.

The second requirement, defined as approaching the target from different directions, is another aim of the salvo attack. This requirement does not have great importance for missile attacks against ground-based targets. However, in a missile attack against an airborne target, the possibility of escape maneuvers is very high when the target has noticed missile attack. In this case approaching the target from different directions will cause narrowing of the area target can escape.

When the missiles are fired from the same location one after the other, the fume from the motor of the front missile will cause the seeker of the next missile to be unable to lock onto the target. Therefore, the trajectories of the missiles should remain outside the field of view of each other's seekers during flight.

Other guidance methods that aim to control impact time, most commonly, do not aim to control the shape of the trajectory directly. The reason for polynomial trajectory shaping guidance algorithm is preferred over other impact time control methods is that; the trajectory to be followed, while controlling impact time, is predetermined and guarantees to meet the requirements of a salvo attack.

Multiple Trajectory Generation Algorithm which is examined in this chapter is a top-level algorithm from the guidance algorithm. It collects the information about the engagement and supplies the inputs listed below to the guidance algorithm.

- Desired flight time
- The location of the PIP
- Desired impact angle
- Launch angle

Logic and the working principle of the algorithm is explained in this chapter.

4.1 Multiple Trajectory Generation Algorithm

Let N be the number of missiles that construct the salvo attack. If the location of the launch point is denoted by $\vec{P}_A$ and the PIP location is denoted by $\vec{P}_B$, initial range between the missile and the collision point is calculated as:

$$R_0 = \left\| \vec{P}_B - \vec{P}_A \right\| \tag{4.1}$$

When the missile velocity is assumed to be constant, initial value of the time-to-go to the target can be estimated simply dividing the initial range to the missile speed.

$$t_{go_{\min}} = \frac{R_0}{V_m} \tag{4.2}$$

This value of $t_{go_{min}}$ can also be referred as the minimum time that the interceptor can achieve to arrive at the PIP. In other words, if the interceptor follows a straight line to arrive at the PIP, total flight time is calculated using this formula.

Multiple missiles can be fired from the same launch point or from different launch points to perform a salvo attack. The example scenario results will be given in the next section in order to examine both situations. It is thought that there is a constant time difference between the firing moments when the missiles are fired from the same location. Such a requirement will not be needed when missiles are fired from different locations. For the situation when the missiles are fired from the same location, it is given that a constant time delay exists between the launching moments. This time delay is denoted by Δt .

In order to define the reference trajectory parameters for each missile, the multi-trajectory generation algorithm follows these steps:

1. Estimated value of the minimum time-to-go is calculated first using Eq.(4.2).
2. The reference trajectory for each missile will be defined as a 3^{rd} order polynomial function. Therefore, lengths of these trajectories must be greater than the linear initial distance R of Eq.(4.1). If the desired impact time of the i^{th} missile is selected larger than the minimum time-to-go value, the corresponding trajectory length becomes larger than the minimum distance of the linear trajectory R_0. First in this step, $t_{go_{min}}$ value is rounded to the largest nearest integer and indicated as desired impact time t_d.
3. Desired impact time of the missile which is fired at the first order should have the largest value to achieve the simultaneous arrival of the whole group. Therefore, the smallest time-to-go value is assigned to N^{th} missile which is fired at the last order.

$$t_d^N = t_d \tag{4.3}$$

4. Concerning the problem definition, i.e. the missiles are fired with a time delay of Δt, the desired impact time of the i^{th} missile is formulated as follows:

$$t_d^i = t_d + (N-i)\Delta t, \quad i = 1,2,...,N \tag{4.4}$$

When the missiles are fired from different locations, t_d value can be assigned as the desired impact time of all group.

5. A 3^{rd} order polynomial reference trajectory function requires one more information to obtain unique function coefficients. This requirement can be satisfied by adding one more constraint to the guidance algorithm. These constraint is defined as:
    a. *Controlling the Impact Angle*

A desired impact angle is assigned to the reference trajectory generation algorithm. The value of this desired impact angle can be determined according to the scenario specifications. In most impact angle control problems, desired impact angle value is determined according to the target specifications. To satisfy this impact angle requirement, a third order constraint equation is added to the system, which is stated in the equation below:

$$\tan(\gamma_d) = 3a_3x_B^2 + 2a_2x_B + a_1 \tag{4.5}$$

This value of γ_d is assigned as the desired impact angle of the missile which is fired at the first order. In planer engagements, desired impact time of the i^{th} missile is determined by using the following relation:

$$\gamma_d^i = (-1)^{i+1}\left[\gamma_d + \Delta\gamma(i-1)\right], \quad i = 1,2,..,N \tag{4.6}$$

The $\Delta\gamma$ term indicates the difference between the desired impact angles assigned to the sequential fired missiles. This parameter also has a value defined by the user. In planar engagements, the larger value of $\Delta\gamma$ results in greater spread between trajectories. The $(-1)^{(i+1)}$ term is used to assign a negative impact angle to one of the sequential fired missiles. Sequential pairs have the closest trajectory lengths to each other. Hence by using this equation, it is ensured that the missiles approach the target from different directions and the intersection of the trajectories is prevented during flight.

In 3D engagements, same desired impact angle value is assigned to whole missile group.

b. Adjusting the Launch Angle

The final equation needed to from the reference trajectory can also be determined by assigning the launch angle. The resulting equation is as follows.

$$\tan(\gamma_0) = 3a_3x_A^2 + 2a_2x_A + a_1 \tag{4.7}$$

The user designation of the launch angle causes the number of sufficient equations to be reached. Therefore, if it is desired to control the impact angle, the launch angle must be determined by the reference trajectory generation algorithm.

4.1.1 Moving Target Engagements

The designed reference trajectory ensures that the missile reaches the estimated collision point (i.e. the PIP) at the desired impact time. When the target is not stationary, a radar system can provide real-time information to the missile guidance system about the position and velocity of the target. But, sampling of the radar data affects the timing of the target information and this timing may cause a delay in guidance commands. As a result, this situation may result in undesirable miss distance.

An onboard seeker system can provide various information according to the type of sensor system and the algorithms of the seeker. But most commonly, onboard seekers do not provide position and velocity of the target directly. Estimation algorithms are required in order to obtain this information. The reference trajectory following guidance algorithm designed in this study, is used as a midcourse guidance algorithm for a missile system that contains both radar and onboard seeker systems.

Moving targets can be divided into two categories as maneuvering and non-maneuvering targets. For non-maneuvering targets the PIP can be estimated by using the following equation.

$$\vec{P}_t = \vec{P}_B + t_d \vec{V}_t \tag{4.8}$$

In the equation, $\vec{P}_B$ denotes initial target position and $\vec{P}_t$ denotes the PIP position. But, it is not possible to estimate the PIP position when the target makes an escape maneuver. For moving targets (for both maneuvering and non-maneuvering) the reference trajectory following is used as a midcourse guidance algorithm.

Terminal guidance is defined as the last phase of the flight in which most commonly an online seeker data is used to generate guidance commands. Guidance commands are switched to terminal guidance phase in the last part of the flight. The well-known Pure Proportional Navigation Guidance (PNG) law is used for the terminal guidance phase. The PNG acceleration command is given below.

$$\overrightarrow{a_n} = N\left(\vec{\omega}_{LOS} \times \vec{V}_m\right) \tag{4.9}$$

In Pure Proportional Navigation Guidance (PPNG), acceleration command is also applied in the direction normal to the missile velocity vector.

a. Maneuvering Target

It is not possible to estimate the PIP when the target makes an escape maneuver at the end of the flight. Therefore, it will not be possible to control the impact time once the target has started to maneuver. From this point on, guidance algorithm must override the need to simultaneously reach the target and focus on the need not to miss the target. For maneuvering target scenarios, guidance commands are switched to terminal phase at the instant target starts the escape maneuver.

a. Non-Maneuvering Target

When the target moves at a constant speed, the PIP location can be estimated using Eq.(4.8). In these scenarios, the flight time should be controllable. To satisfy this requirement, the duration of the terminal guidance phase must be estimated. The time-to-go estimation given below provides an accurate estimation, if PNG is used to as the guidance command.

$$\hat{t}_{go\_PNG} = \frac{\left(1+\frac{(\gamma-\lambda)^2}{10}\right)R}{\left\|\vec{V}_{rel}\right\|} \tag{4.10}$$

During the midcourse guidance phase, this estimation gives the information: if the missile had been guided with the PNG acceleration command how much time left to reach the target point. As a result, the guidance phase is switched to terminal guidance when the $\hat{t}_{go\_PNG}$ value becomes smaller than or equal to a user defined time value $t_{terminal}$. This value may change depending on the desired impact time, but typically is chosen as 2 to 4 seconds.

4.2 Three-Dimensional Implementation of Salvo Attack

It was stated before, the main requirements for a salvo attack are defined as approaching the target from different directions and reaching the PIP simultaneously. To deal with the first requirement in 3-D space; different *Maneuver Planes* are defined for each individual missile that constitute the salvo attack. These maneuver planes are obtained by rotating $\mathcal{F}_1$ around its x −axis at different roll angles, ϕ_i.

$$\Phi = \begin{bmatrix} \phi_2 & \phi_3 & \dots & \phi_N \end{bmatrix} \tag{4.11}$$

The choice of the roll angles is important in determining the scattering of the trajectories. In practical applications, Φ matrix can be selected according to the field of view of missile seeker.

Let *N* missiles constitute a salvo attack. During flight, the i^{th} missile will follow the desired trajectory in the Maneuver Plane defined in $\digamma_i, i = 2, 3, \dots, N$. Therefore, the guidance command for the i^{th} missile becomes:

$$a_n = \frac{2V_m^2 \cos\left(\gamma_{pitch}^{(i)}\right)\left(3a_3 \overline{P}_{mi}^{(i)}(1) + a_2\right)}{\left(\overline{P}_{mi}^{(i)}(1)\left(3a_3 \overline{P}_{mi}^{(i)}(1) + 2a_2\right) + a_1\right)^2 + 1} \tag{4.12}$$

$$\overline{a}_{com}^{(i)} = \begin{bmatrix} -a_n \sin\left(\gamma_{pitch}\right) \\ K\left(\gamma_{yaw} - \gamma_{ref}\right) \\ a_n \cos\left(\gamma_{pitch}\right) \end{bmatrix} \tag{4.13}$$

In these equations, flight path angles γ_{pitc}^{i} and γ_{yaw}^{i} are obtained by resolving velocity vector of the i^{th} missile, $\vec{V}_{mi}$, in the corresponding maneuver plane $\digamma_i$.

Finally, acceleration command is transformed into *Navigation Frame,* $\digamma_0$.

$$\overline{a}_{com}^{(0)} = \hat{C}^{(0,1)}\ \hat{C}^{(1,i)}\ \overline{a}_{com}^{(i)} \tag{4.14}$$

By integrating this acceleration command in $\digamma_0$, velocity and position of i^{th} missile is obtained.

CHAPTER 5

SIMULATION RESULTS AND DISCUSSIONS

In this chapter, behavior and performance characteristics of the proposed guidance algorithm is analyzed through example scenarios. First, characteristics of the polynomial trajectory shaping guidance algorithm is observed with planar one-to-one engagement scenarios. After that, salvo attack of multiple missiles is examined in 3D.

Throughout this chapter, first the main structure of the point-mass missile simulation is explained. A brief information about the subsystems of the simulation is given. After that, simulation results are provided and the performance characteristics of the guidance algorithm are discussed.

5.1 Missile Simulation Structure

The missile is modeled as a point mass moving in a 3D geometry. A simulation model is constructed in MATLAB®/Simulink to investigate the proposed guidance method's behavior by constructing example scenarios. Since a point-mass model is used, only kinematic equations are used and flight dynamics is not considered. Simulation model consists of the following subsystems:

- Target Kinematics
- Missile Kinematics
- Missile-Target Relative Kinematics
- Seeker Model
- Guidance Algorithm
- Termination Conditions

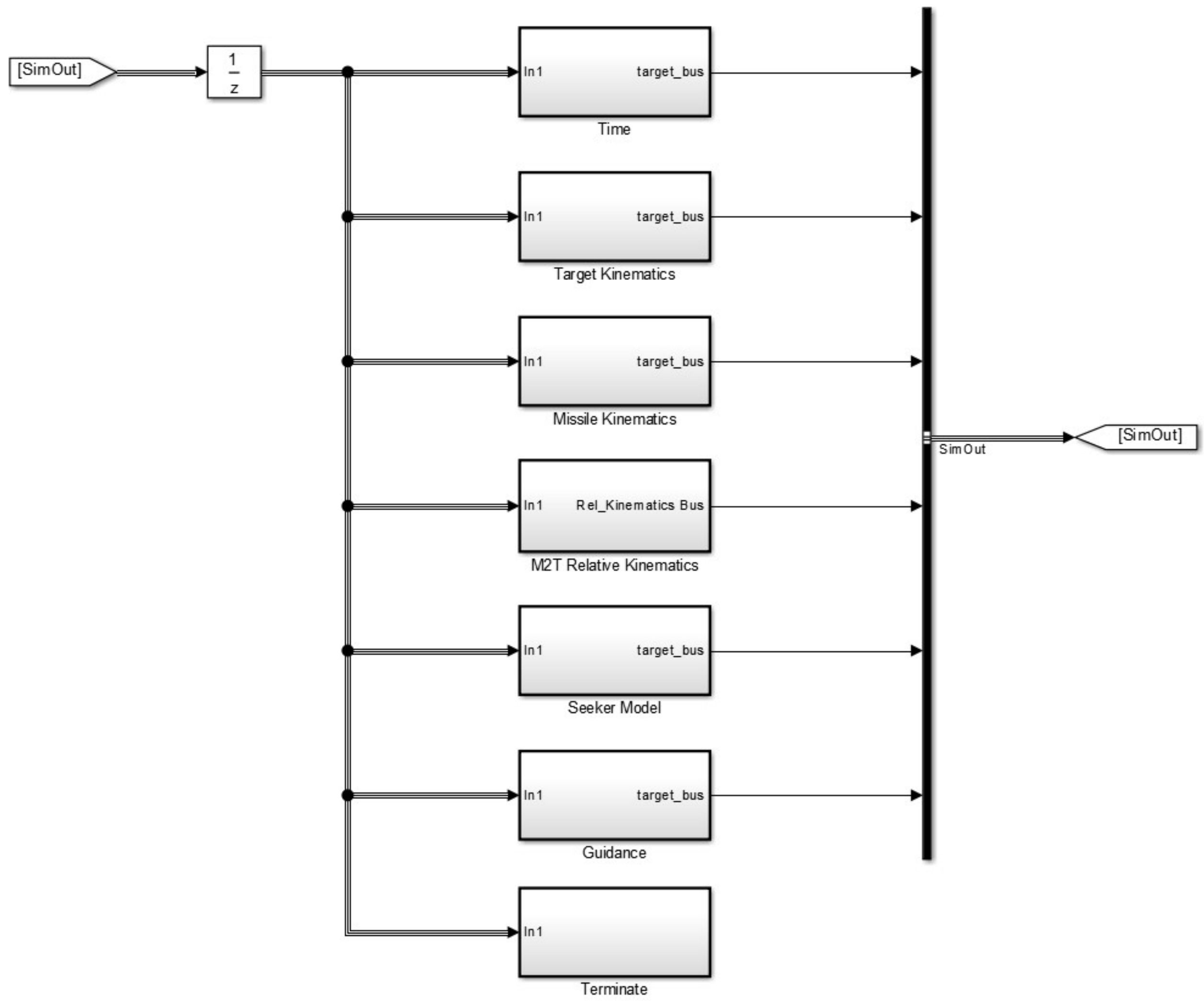

Figure 5-1: Simulation Structure in MATLAB/Simulink Environment

The overview of the missile simulation in Simulink environment is shown in Figure 5-1. The main frame of the model is constructed by bus structure in which outputs of each subsystem are assigned to a single bus and this main bus block is fed back as an input to each of the main subsystems. Unit time delay is added to avoid algebraic loop errors.

Simulation Time subsystem calculates the time of flight and estimates the time-to-go to the target t_{go}.

Missile Kinematics subsystem takes initial values of the missile position, alignment and velocity parameters and integrates them to update missile position and velocity during flight.

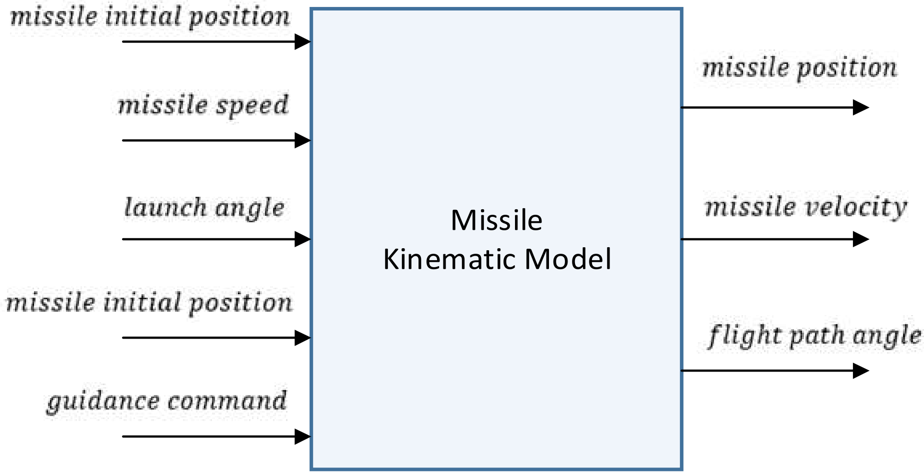

Figure 5-2: Inputs and Outputs of Missile Kinematics Subsystem

Missile-Target Relative Kinematics subsystem calculates the variables regarding to missile-target engagement geometry. Missile-target Line of Sight (LOS) vector, angle and angular velocity are calculated in this subsystem. In addition to that, other parameters related to relative kinematics such as instantaneous missile-target range, relative velocity and relative position are calculated. Inputs and outputs of Missile-Target Relative Kinematics subsystem are shown in Figure 5-3.

Seeker Model subsystem, models the errors and uncertainties resulting from the seeker measurements.

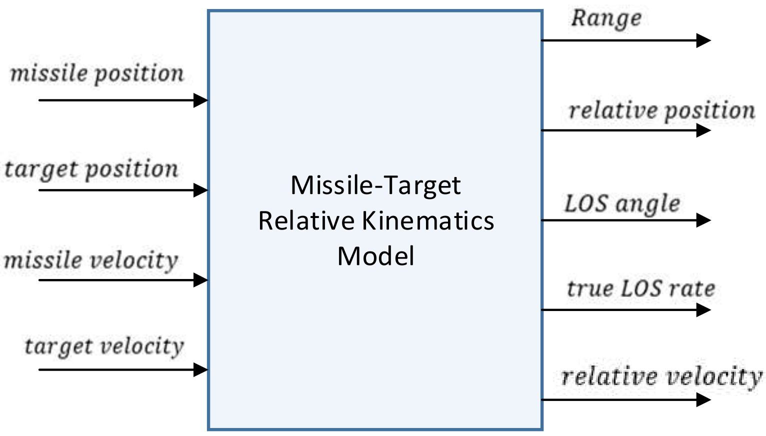

Figure 5-3: Inputs and Outputs of Missile-Target Relative Kinematics Subsystem

Guidance subsystem includes all the guidance algorithms derived in the previous sections. Following block diagram shows inputs and outputs of guidance subsystem.

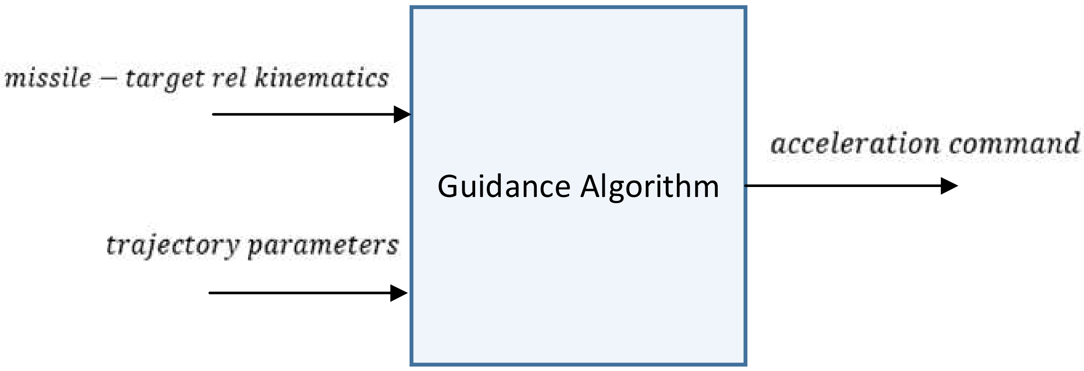

Figure 5-4: Inputs and Outputs of the Guidance Subsystem

Finally, in the *Terminate* subsystem, termination condition of the simulation is checked. Simulation is stopped when:

- Missile-target range is less than $1m$
- Missile misses the target

It is important to note that, for this study it is assumed that the system directly responses the applied acceleration commands. In other words, the *Autopilot* is modelled as a unity transfer function. Since the aim of this study is to focus on the development of the guidance algorithms, autopilot algorithm design is excluded from the scope of this study.

5.1.1 Target Kinematic Model

Target Kinematics subsystem, takes initial conditions of the target position, target velocity and initial alignment and by integrating them calculates position and velocity of the target during flight. Target kinematic model has the same input-output structure as missile kinematic model.

In the target model, target escape maneuver is modelled in addition to the kinematic model. The escape maneuver is modelled as a *g-turn* maneuver in 3-D geometry. The g-turn maneuver is a basic tactical movement performed by a fighter aircraft. Therefore, analyzing the scenarios that the target makes an escape maneuver is important for examining the performance of the guidance algorithm. The constant gravitational acceleration is taken as $g = 9.79 m/s^2$ for the g-turn.

The target maneuver starts at the instant $r_t \leq R$ where the maneuvering distance r_t is an input of the simulation. The maneuver time constant τ_t is set as $0.3s$ and the total maneuver duration is taken as three times of this time constant. The *g-load, n,* of the escape maneuver is changed between simulation runs.

5.2 Simulation Results and Discussions

In this section, simulation results regarding to different engagement scenarios are provided. Characteristics and performance of the proposed algorithm are investigated by discussing example scenarios for both one-to-one and salvo attack engagements. For all simulation runs the following conditions are kept constant.

- The constant missiles speed is taken as $V_m = 300 m/s$
- Initial missile position is always taken as the origin of the inertial reference frame.

5.2.1 Simulation Results for Planar Engagements

Since the polynomial reference trajectory is defined in 2D plane, scenarios are firstly executed for planar engagements in order to discuss the effects of the design parameters on the trajectory. The polynomial reference trajectory is defined in the vertical $x - z$ plane. At first, the coefficients of the reference trajectory function will be determined by adjusting initial flight path angle, γ_0. After that, simulation results will be given for trajectory generation by control of impact angle.

5.2.1.1 Reference Trajectory Generation by Setting the Launch Angle

In this example, launch angle is taken as an input to the guidance algorithm. All scenario parameters which are the *launch angle*, *target location* and *desired impact time* are listed in the table below.

Table 5-1: Parameters of Example Scenario #1

Desired Impact Time, t_d	$19s$
Predicted Intercept Point	$(5000,1500)m$
Launch Angle	27.63^0

Initial range is calculated as $R_0 = 5220m$, so minimum time-to-go is obtained as $\frac{R_0}{V_m} = 18s$. Therefore, the desired impact time is taken as $t_d = 19s$. By using these inputs; the reference trajectory function, which is a cubic function of downrange$-x$ is constructed. Function coefficients are listed in Table 5-2.

Table 5-2: The Cubic Function Coefficients for Example Scenario #1

a_1	0.5236
a_2	1.5144×10^{-4}
a_3	-3.9231×10^{-8}

The resulting trajectory is shown in Figure 5-5.

Figure 5-5: Reference Trajectory Obtained for Scenario #1

Flight path angle, as a function of downrange, is calculated using the equation (2.37) and shown in the Figure 5-6.

When the launch angle is set, impact angle cannot be controlled directly. For this example scenario, by selecting a launch angle of $\gamma_0 = 27.63^o$, impact angle is obtained as $\gamma_f = -42^o$.

The guidance command vs time is shown in theFigure 5-7. The acceleration command is obtained by tracking a virtual target throughout the reference trajectory. For all simulation runs, PNG law with an Effective Navigation Ratio of $N' = 3$ is used to track the virtual target. Positions of the missile and virtual target vs time are provided in the Figure 5-8.

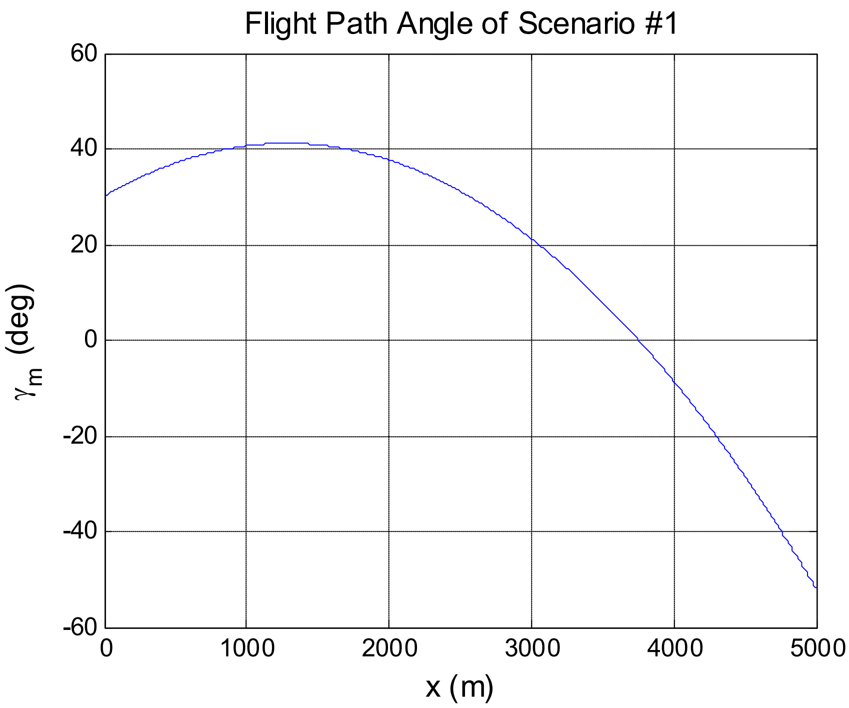

Figure 5-6: Flight Path Angle vs x for Scenario #1

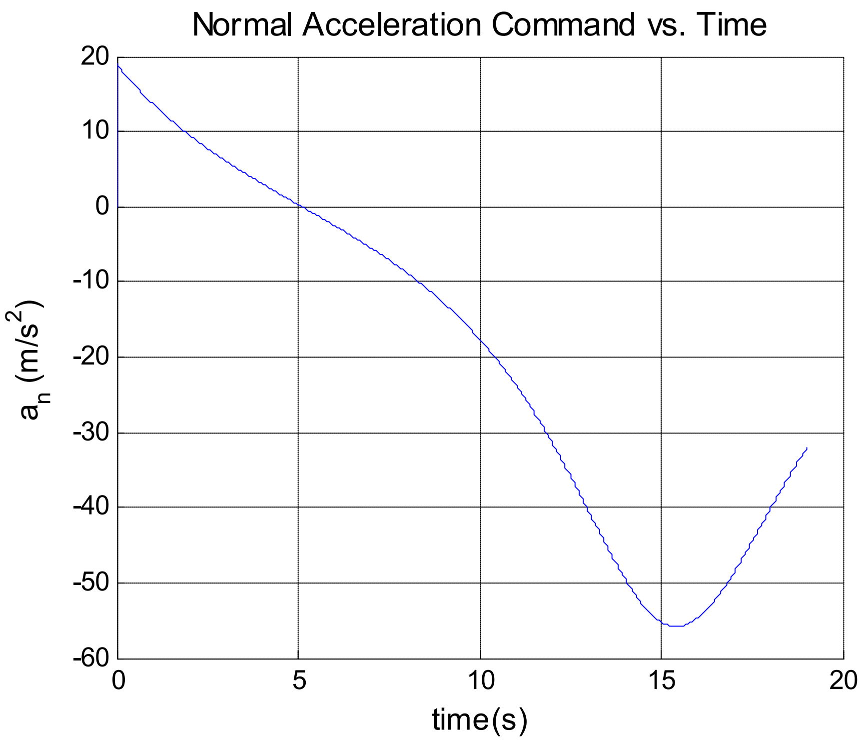

Figure 5-7: Normal Acceleration Command for Scenario #1

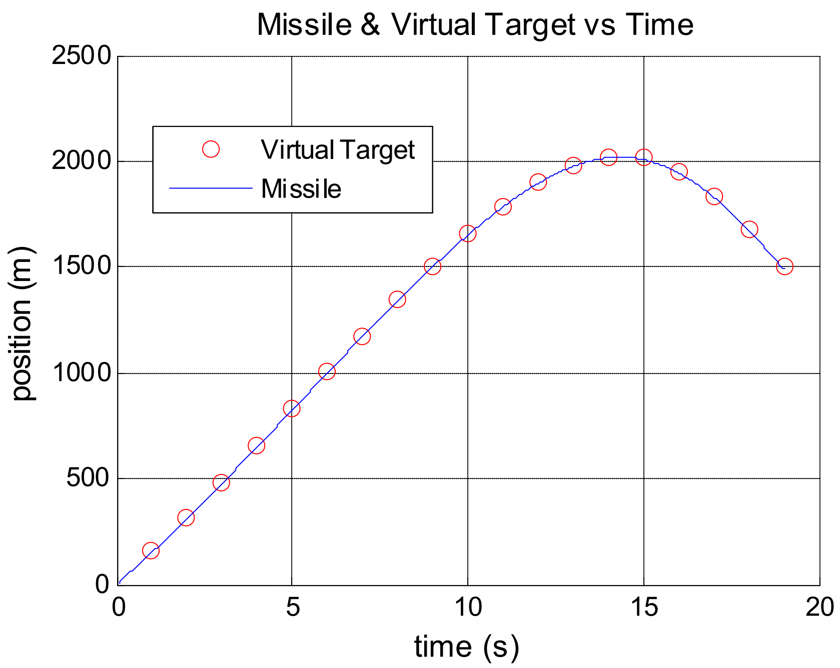

Figure 5-8: Positions of Missile and Virtual Target for Scenario #1

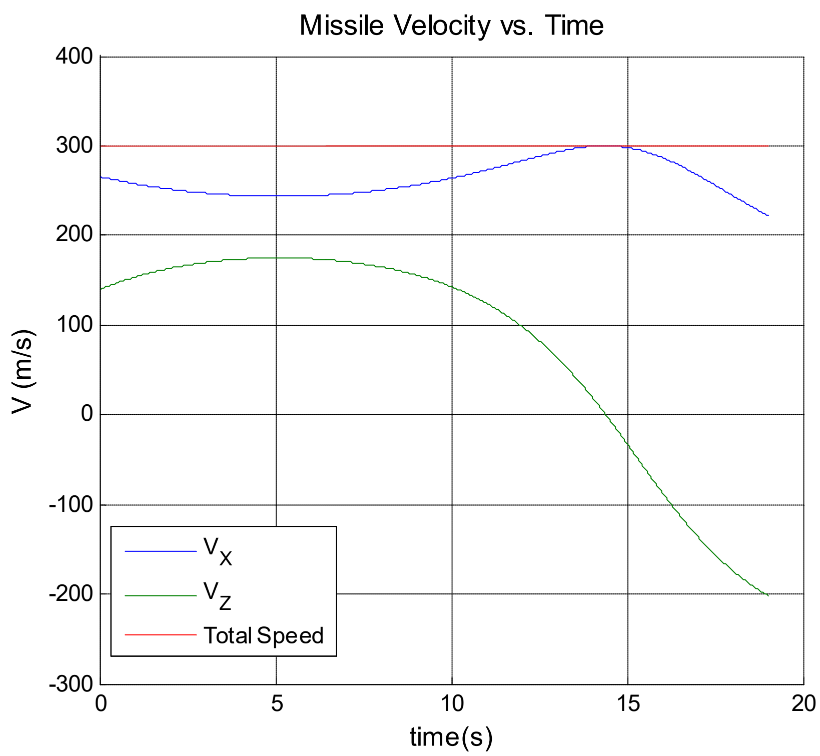

Figure 5-9: Missile Velocity for Scenario #1

The PNG acceleration command to track the VT along the reference trajectory, is applied in the normal direction of the missile velocity vector. It can be seen from the Figure 5-9 that; total missile speed remains constant as the components of the velocity in x and z direction change.

In Figure 5-7, it can be seen that the magnitude of maximum acceleration required to follow the reference trajectory is $55m/s^2$. This max acceleration is required at the maximum altitude of the trajectory. After that point, missile starts the dive maneuver to reach the target. The cubic reference trajectory aims to increase the curvature of the trajectory in order to control the impact time, rather than directly reaching the target. This may lead to a need for more acceleration compared to guidance methods aiming to reach the target directly.

5.2.1.2 Reference Trajectory Generation by Setting the Impact Angle

In the second example scenario, reference trajectory is obtained by controlling the impact time and impact angle at the same time. For the reference trajectory function with 3 coefficients, in order to control impact angle, launch angle should be set free. In other words, launch angle should be determined according to the desired impact angle. Same target location is used to see the difference between the scenarios #1 and #2. Desired impact angle is selected as $\gamma_f = -30^o$. Scenario parameters are listed in the Table 5-3.

Table 5-3: Parameters of Example Scenario #2

Desired Impact Time, t_d	$19s$
Predicted Intercept Point	$(5000,1500)$
Impact Angle, γ_f	-30^o

The obtained reference trajectory is shown in Figure 5-9.

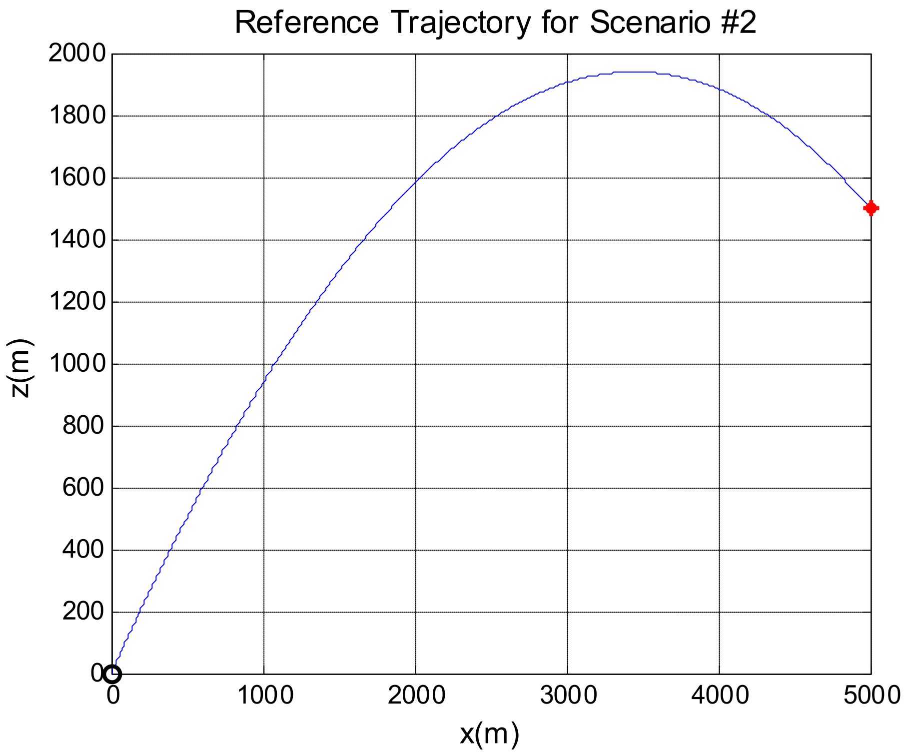

Figure 5-10: Reference Trajectory Obtained for Scenario #2

The associated coefficients of the cubic polynomial reference trajectory are listed in the Table 5-4.

Table 5-4: The Cubic Function Coefficients for Example Scenario #2

a_1	1.0747
a_2	-1.344×10^{-4}
a_3	-4.1073×10^{-9}

Flight path angle vs downrange is presented in Figure 5-11. It can be seen that; flight path angle converges to desired impact angle as missile approaches the target. Another important point to note is that, launch angle must be $\gamma_0 = 47^0$ to ensure a $\gamma_d = -30^0$ impact angle requirement.

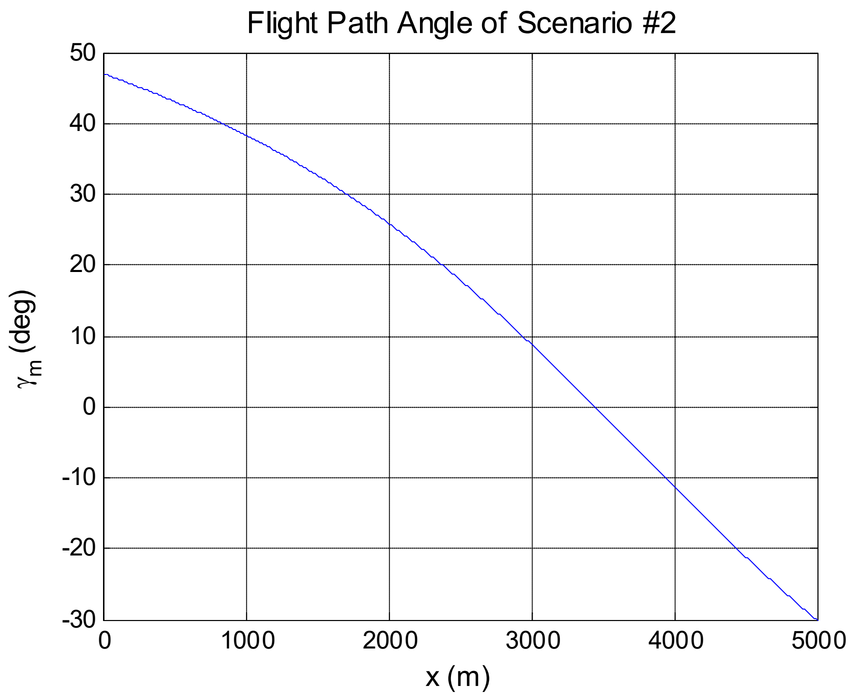

Figure 5-11: Flight Path Angle vs x for Scenario #2

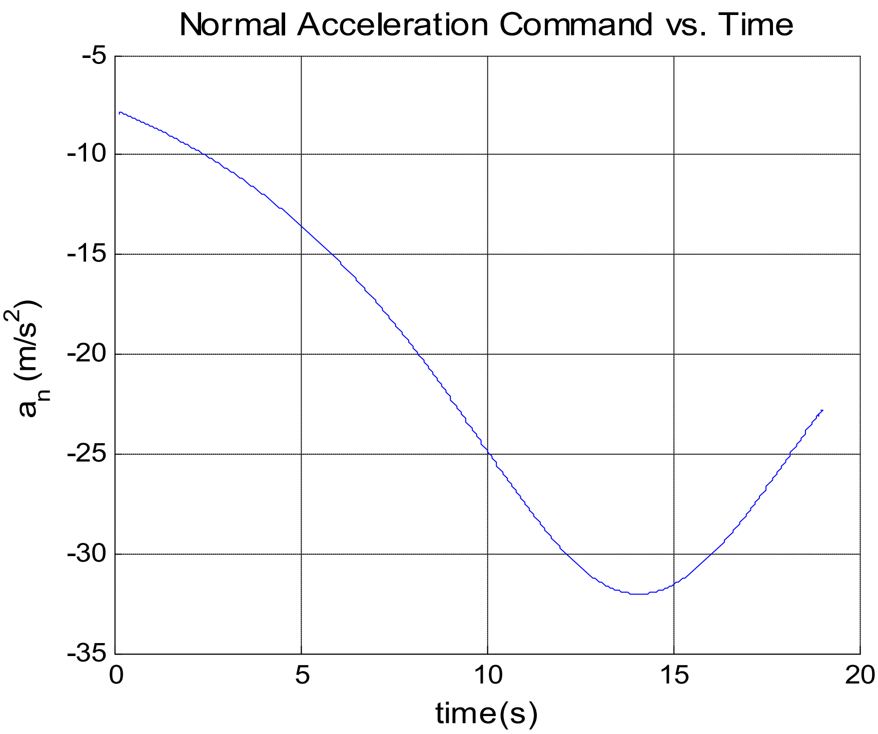

Figure 5-12: Normal Acceleration Command for Scenario #2

The corresponding acceleration command is shown in Figure 5-12. The missile has reached a lower altitude as the impact angle in this scenario is lower in magnitude than the first scenario. Although the trajectory profiles obtained are similar, this situation has led to the need for less acceleration command.

Resulting velocity profile of the missile is shown in Figure 5-13. Again, total missile speed is kept constant, as expected.

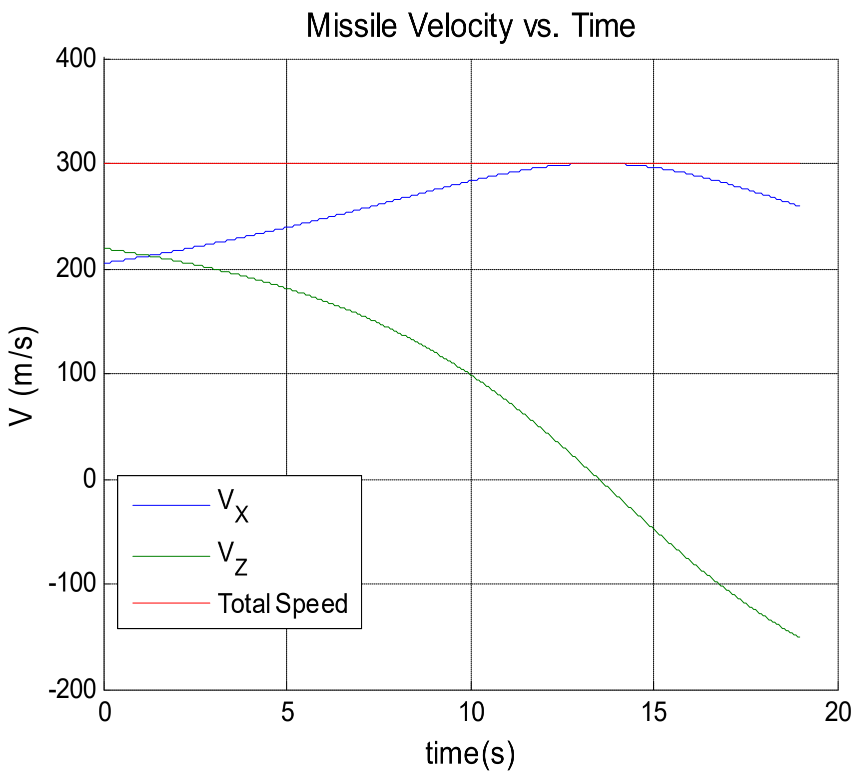

Figure 5-13: Missile Velocity for Scenario #2

In Figure 5-14, the positions (position z-component) of missile and VT are plotted vs time. It can be observed that; missile trajectory overlaps with the VT trajectory during flight.

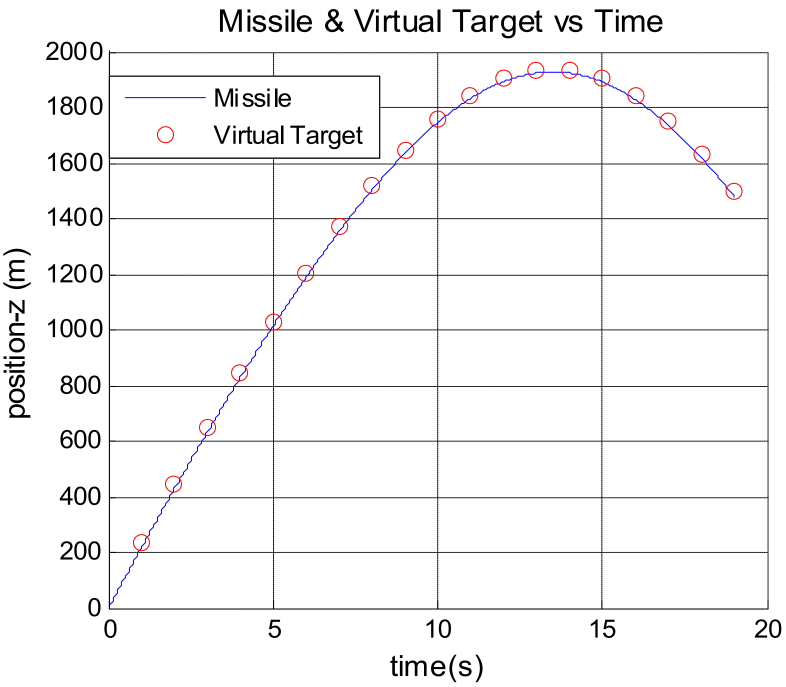

Figure 5-14: Positions of Missile and Virtual Target for Scenario #2

5.2.1.3 Effect of Launch Angle on the Polynomial Reference Trajectory

In order the analyze the effect of launch angle on polynomial reference trajectory, a scenario set is constructed in which launch angle is changed while all other scenario parameters are kept constant. Target location and desired impact time is taken as the same values of example scenarios 1 and 2. Launch angle is changed from -20^0 to 80^0 for the batch simulation run. The input γ_0 matrix is defined as:

$$\gamma_0 = \begin{bmatrix} 20 & 30 & 40 & 50 & 60 & 70 & 80 \end{bmatrix}^T \tag{5.1}$$

The resulting trajectories are shown in Figure 5-15.

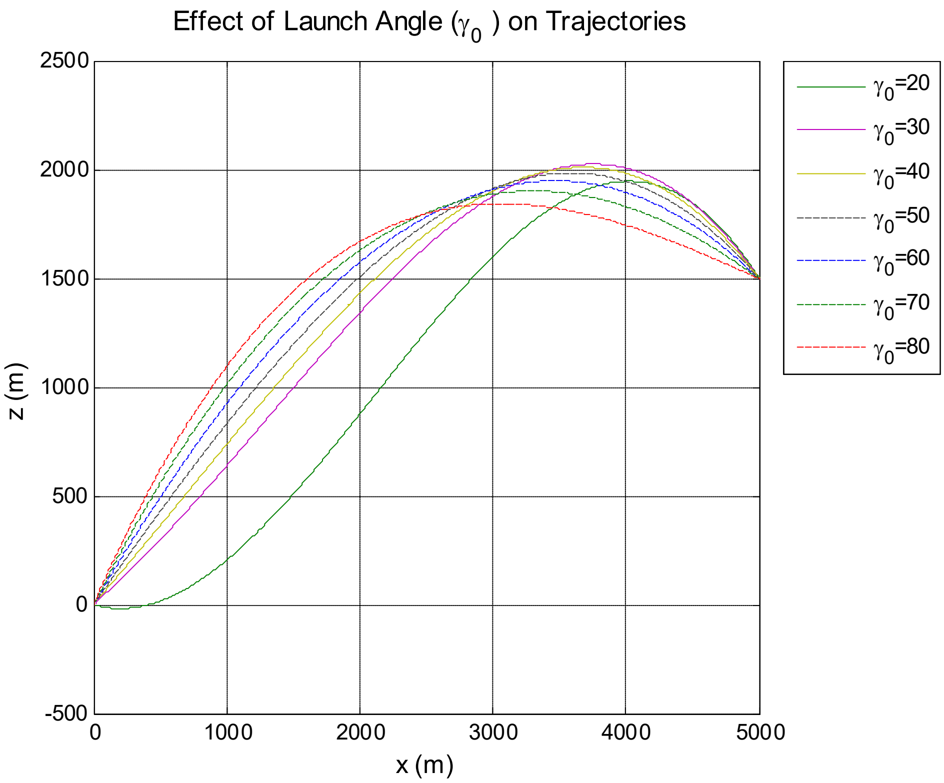

Figure 5-15: Effect of Launch Angle on Polynomial Reference Trajectory

It can be seen from the figure that; different trajectories are obtained for different launch angles, although the target location and desired impact time are the same. However, the obtained trajectories are similar to each other since they all have to satisfy the total trajectory length requirement. In the reference trajectory with a launch angle of 20^o, at the beginning of the flight, the reference trajectory was found to have a lower altitude value than the launch point. Such a reference trajectory is not desired when the launch point is close to the ground level. As a result, attention should be paid to the selection of launch angle, considering the scenario conditions.

5.2.1.4 Effect of Desired Impact Time on the Polynomial Reference Trajectory

In this section, the effect of the desired impact time on the polynomial reference trajectory is investigated. Another scenario set is constructed for different impact time values. Input time matrix is set between $18s$ and $27s$ with an increment of $1s$.

$$t_d = \begin{bmatrix} 17 & 18 & 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 \end{bmatrix}^T \tag{5.2}$$

Same target point is considered and the launch angle is taken as $\gamma_0 = 40^0$ for the batch runs. For the constant missile speed, the desired total trajectory length, S_d, changes between the values 5400 and 8100 with an increment of $300m$. Resulting trajectories are shown in Figure 5-16.

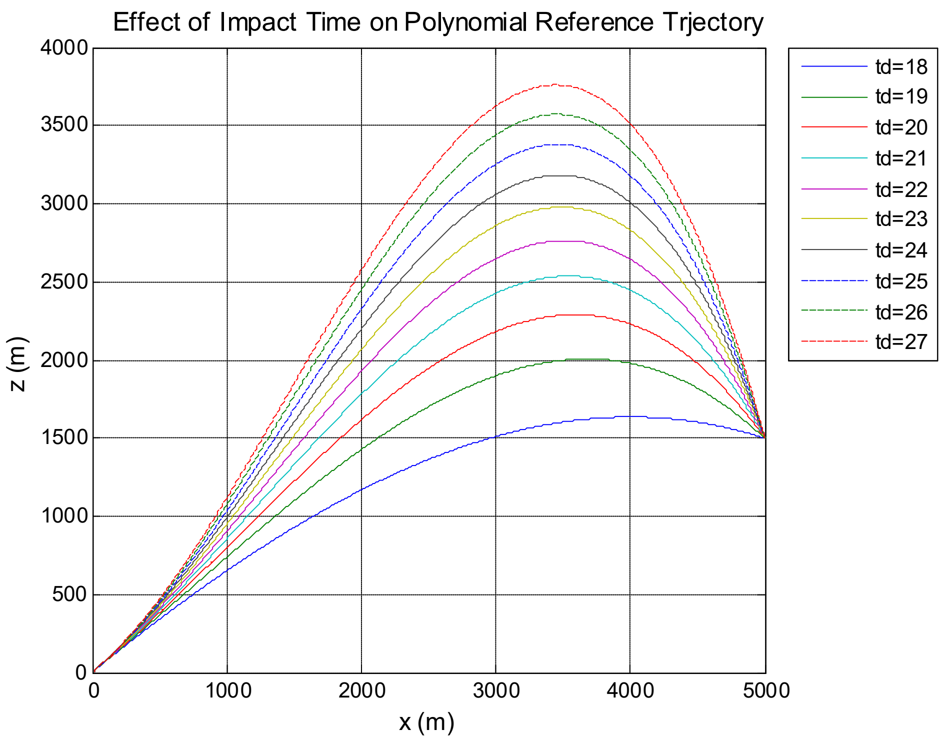

Figure 5-16: Effect of Impact Time on Polynomial Reference Trajectory

It may be observed from the trajectories given in Figure 5-16 that, in order to reach the target point by travelling a larger distance, missile should follow a trajectory with larger curvature. To increase flight time, the missile climbs a higher altitude and approaches the target at a steeper angle. As desired impact time approaches to the minimum time-to-go value, the reference trajectory converges to a linear trajectory.

5.2.1.5 Effect of Desired Impact Angle on the Polynomial Reference Trajectory

Finally, another design parameter, the effect of the impact angle on the polynomial reference trajectory, will be examined. For the analysis, the same target point as scenarios 1 and 2 is used. Desired impact time is changed

$$\gamma_d = -\begin{bmatrix} 10 & 20 & 30 & 40 \end{bmatrix}^T \tag{5.3}$$

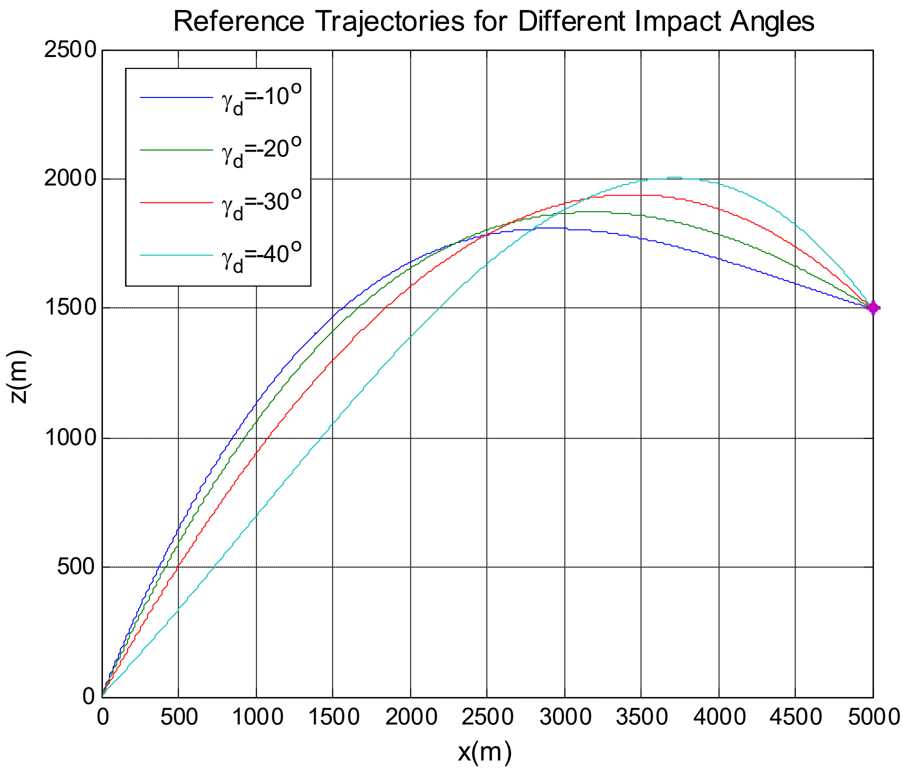

Figure 5-17: Effect of Impact Angle on Polynomial Reference Trajectory

Similar to other known guidance methods in the literature, in order to increase the impact angle, it is generally necessary to climb to a higher altitude and start the diving maneuver later. A similar situation may be seen in Figure 5-17. The reference trajectory assigned the largest impact angle requirement in magnitude, has the highest altitude.

5.2.1.6 Comparison of Analytical Approach and Virtual Target Approach in terms of Disturbance Rejection

It was stated in the previous chapters that; virtual target approach is used to follow the desired reference trajectory to obtain guidance commands in such a feedback form. To see the difference between analytical approach and virtual target approach, a disturbance force is generated and the effects are observed in a simulation run. A disturbance of $10m/s^2$ is added into the acceleration command between $5 \leq t \leq 6$ seconds. Since the analytical approach applies an acceleration command in open loop form, this disturbing effect leads directly to a divergence from the reference trajectory as expected. On the other hand, in virtual target approach, guidance commands are generated in feedback form by using PNG law. As a result of this, the disturbing effects are damped and no deviation occurs from the reference trajectory.

In this case, horizontal plane trajectories are considered and comparison of the applied acceleration commands and the resulting trajectories are shown in the following figures.

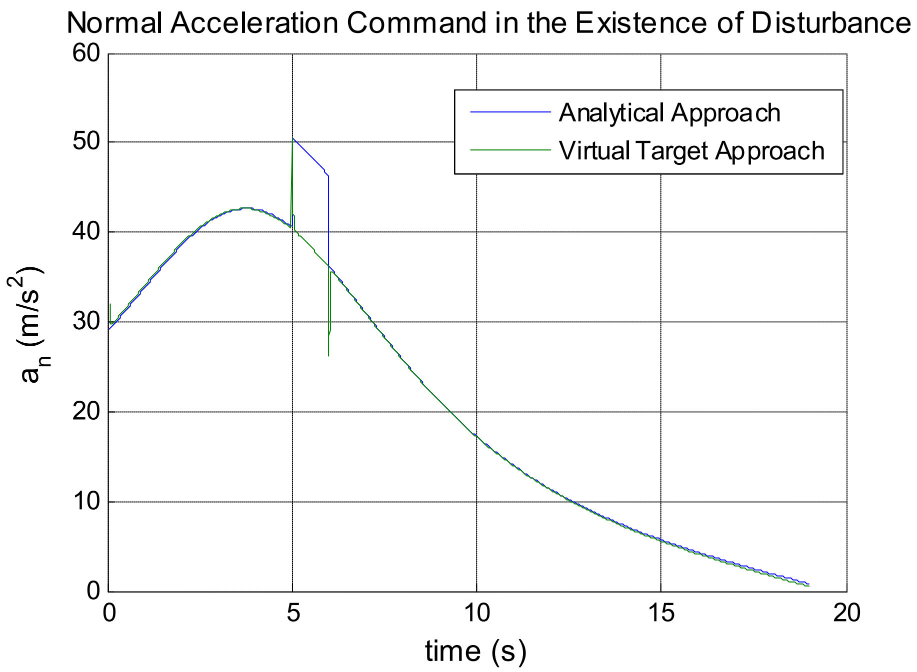

Figure 5-18: Acceleration Command in the Existence of Disturbance

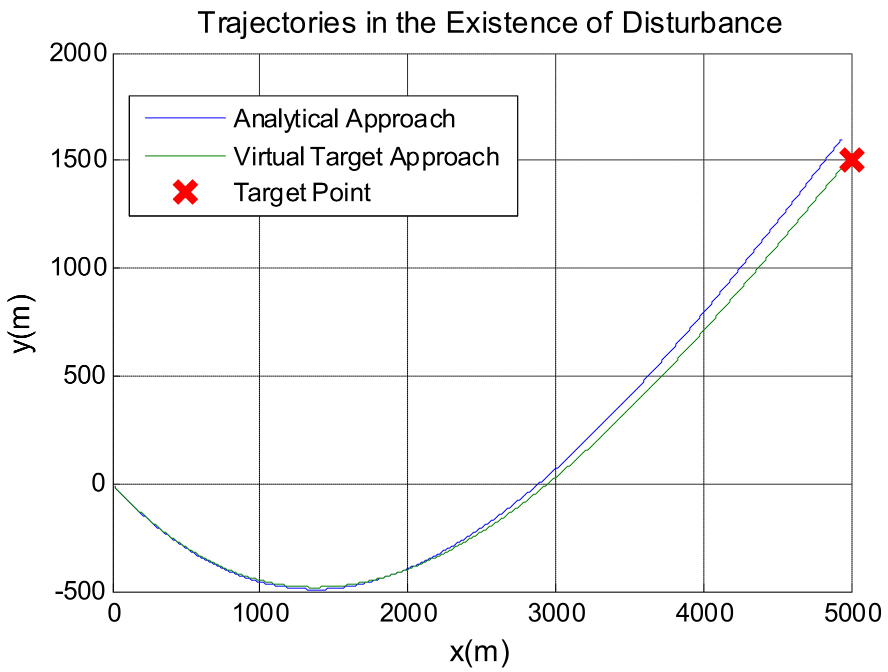

Figure 5-19: Trajectories in the Existence of Disturbance

5.2.2 Simulation Results for Three Dimensional Engagements

In this section, the application of the proposed guidance method to 3D geometry will be discussed through example scenarios. As it was explained in Chapter 3, in order to use the polynomial reference trajectory in 3D, a maneuver plane must be defined and the engagement must be reduced to a planar geometry. 3D trajectories will be generated and analyzed in terms of performance criteria.

5.2.2.1 3D Reference Trajectory Generation by Setting the Impact Angle

The 3D application of the polynomial reference trajectory shaping guidance will be first shown for a stationary target point. The PIP is located at:

$$\overline{P}_B^{(0)} = \begin{bmatrix} 5000 & 500 & 1500 \end{bmatrix}^T \tag{5.4}$$

In order to define the maneuver plane in which guidance commands will be generated, the PIP is expressed in the LOS Frame $\mathfrak{F}_1$:

$$\overline{P}_B^{(1)} = \begin{bmatrix} 5244 & 0 & 0 \end{bmatrix}^T \tag{5.5}$$

Constant missile speed is again taken as $V_m = 300m/s$; therefore, the desired impact time is assigned as $t_d = 19s$. The desired impact angle is set as:

$$\gamma_{pitchF}^{(1)} = -20^{\circ} \tag{5.6}$$

The desired polynomial trajectory obtained in the *Maneuver Plane* is shown in the Figure 5-20. By applying guidance command given in Eq.(3.17), the trajectory is obtained in 3D space and shown in Figure 5-21.

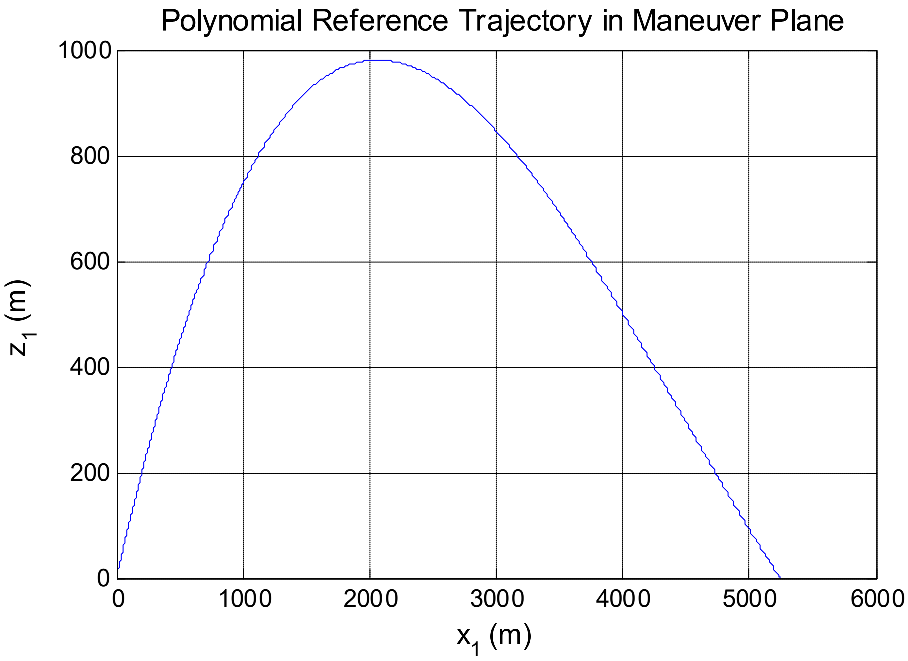

Figure 5-20: Desired Polynomial Reference Trajectory in Frame $\mathcal{F}_1$

The associated coefficients of the cubic polynomial reference trajectory are listed in the Table 5-5.

Table 5-5: The Cubic Function Coefficients for 3D Example Scenario #1

a_1	1.0558
a_2	-3.3327×10^{-4}
a_3	2.5158×10^{-8}

If this trajectory defined in $\mathcal{F}_1$ is expressed in 3D in the Navigation Frame $\mathcal{F}_0$, the trajectory shown in Figure 5-21 is obtained.

The guidance command required to follow this reference trajectory is generated using Eq.(3.21) and shown in Figure 5-22. Since all navigation equations are solved in $\mathcal{F}_0$, the acceleration command must be expressed in this reference frame.

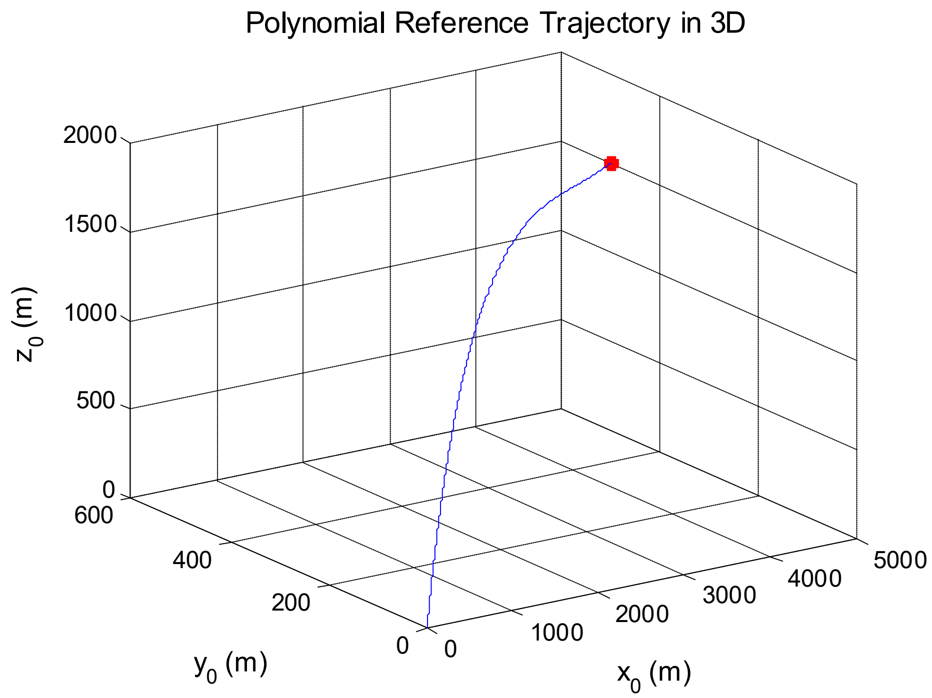

Figure 5-21: Desired Polynomial Reference Trajectory in Frame $\mathcal{F}_0$

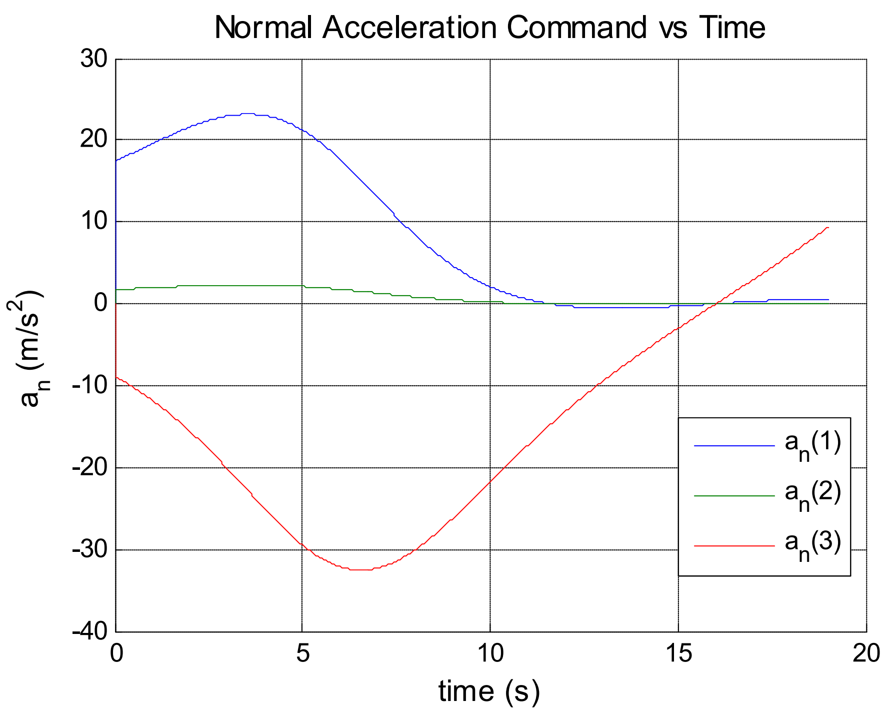

Figure 5-22: Normal Acceleration Command for 3D Scenario #1

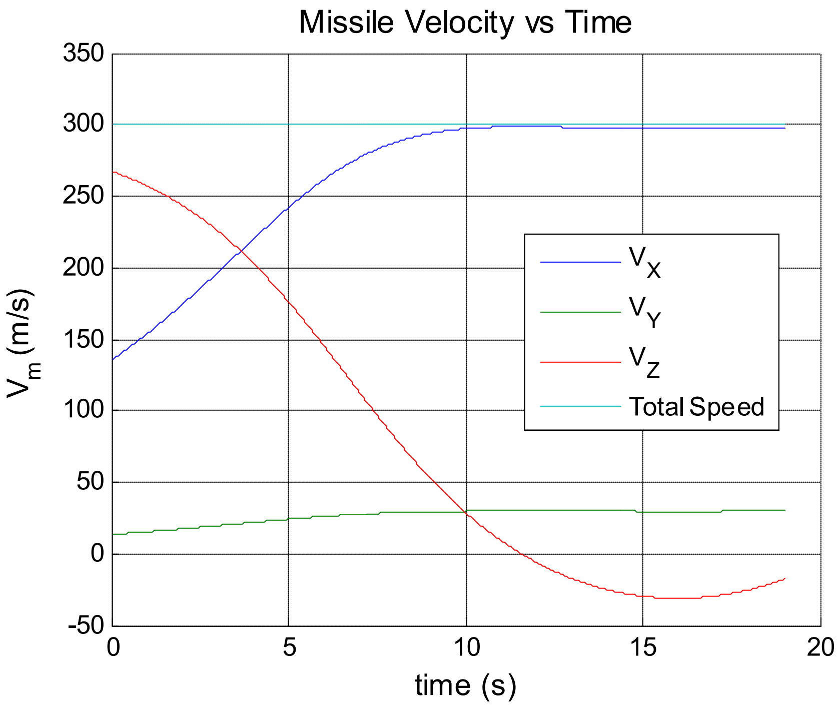

Figure 5-23: Velocity Profile for 3D Scenario #1

5.2.3 Simulation Results for Salvo Attack Scenario

In this section, application of the proposed guidance algorithm to a salvo attack scenario will be analyzed. Salvo attack scenarios will be discussed in 3D engagement and the reference trajectories will be created by impact angle control. The scenarios will be generated for a target that is stationary, moving and maneuvering, respectively and the performance and behavior of the guidance algorithm will be examined under these conditions. The requirements of a salvo attack were described in Chapter 4. The scenarios will be discussed in terms of ensuring these requirements.

5.2.3.1 Salvo Attack Against a Stationary Target

To perform a salvo attack, the missiles can be fired from the same location or from different locations depending on the tactical requirements of the engagement. When the missiles are fired from different locations, the firing moments can be synchronous. However, when they are fired from the same location, there should be a time difference between the firing moments.

In order to investigate the use of the polynomial reference trajectory in a 3D salvo attack, a scenario set is constructed with $N = 5$ missiles. The stationary target location is again set as $\overline{P}_B^{(0)} = \begin{bmatrix} 5000 & 500 & 1500 \end{bmatrix}^T$. Time delay between the firing times of the missiles is set as $\Delta t = 1s$. Desired impact angle is again selected as $\gamma_d = -20^o$ which is equal to the desired impact angle in the maneuver plane, i.e. $\bar{\gamma}_{pitc}^{(1)}$.

The input parameters of the guidance algorithm are listed in Table 5-6.

Table 5-6: Input Parameters of Trajectory Generation Algorithm for Salvo Attack Scenario #1

Missile Number, i	Desired Impact Time, t_d	Desired Trajectory Length, S	Impact Angle, γ_f
1	$22s$	$6600m$	-20^o
2	$21s$	$6600m$	-20^o
3	$20s$	$6000m$	-20^o
4	$19s$	$5700m$	-20^o
5	$18s$	$5400m$	-20^o

The coefficients of the designed trajectory are given in the Table 5-7.

Table 5-7: Coefficients of the Reference Trajectory Function for Salvo Attack Scenario #1

Missile number, i	a_1	a_2	a_3
1	0.4869	-7.1851×10^{-4}	-1.75×10^{-8}
2	1.0429	-6.0427×10^{-4}	-6.76×10^{-8}
3	1.4342	-4.7759×10^{-4}	3.8919×10^{-8}
4	1.7664	-3.2835×10^{-4}	5.0997×10^{-8}
5	0.4869	-1.1628×10^{-4}	6.1890×10^{-8}

A different maneuver plane must be defined for each missile, in order to constitute the trajectories. The reference frame F_i is obtained by rotating F_1 around x_1 axis with an angle of ϕ_i, for $i = 1, \dots, N-1$. Rotation angle matrix, Φ, is a user defined input that determines the scattering of the trajectories. In order to ensure that the entire group reaches the target simultaneously, it is necessary that the N^{th} missile follows the shortest trajectory. For this reason, maneuver plane of the 5^{th} missile is defined in F_1 and the maneuver plane for the i^{th} missile is defined in F_i.

For this scenario, Φ matrix is given in Eq.(5.7).

$$\Phi = \begin{bmatrix} -40^\circ & 40^\circ & -20^\circ & 20^\circ \end{bmatrix}^T \tag{5.7}$$

The obtained trajectories are shown in Figure 5-24.

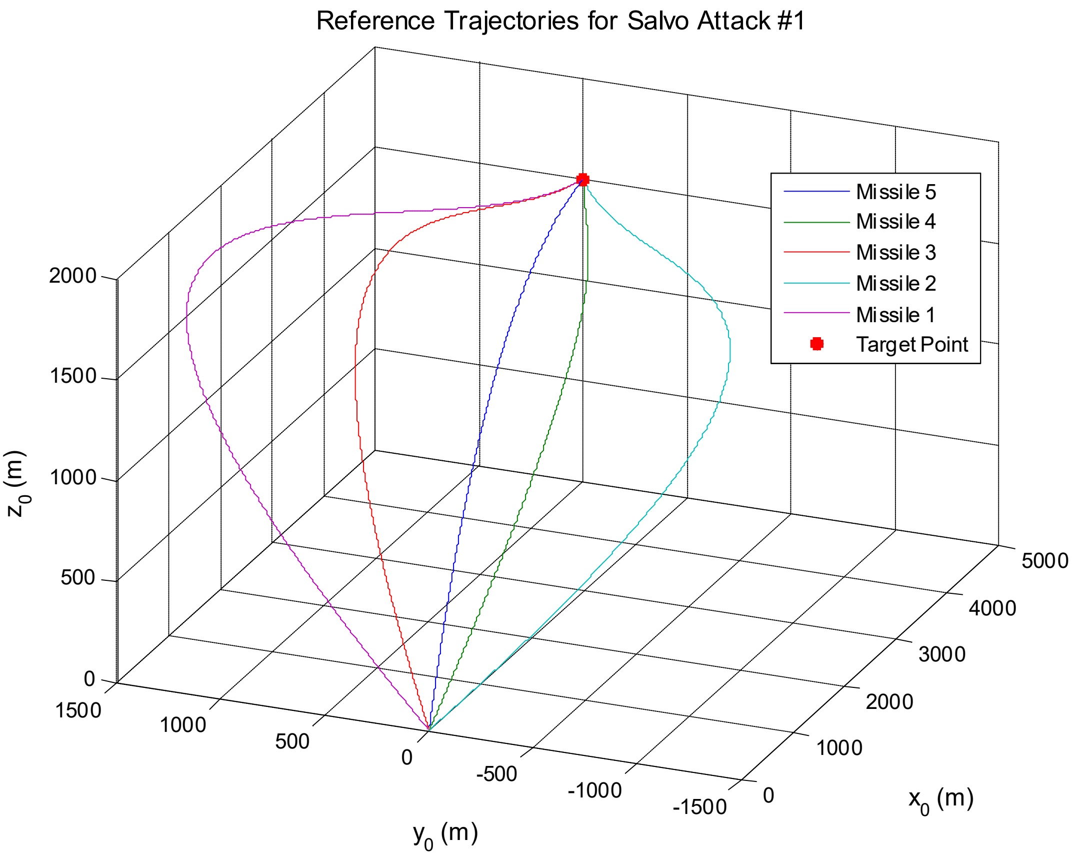

Figure 5-24: Reference Trajectories Obtained for Salvo Attack Scenario #1

Trajectories are also shown in $x - y$ and $x - z$ planes in the Figure 5-25 and Figure 5-26. Especially, by looking at the trajectories in $x - y$ plane, the distances between the trajectories can be clearly understood.

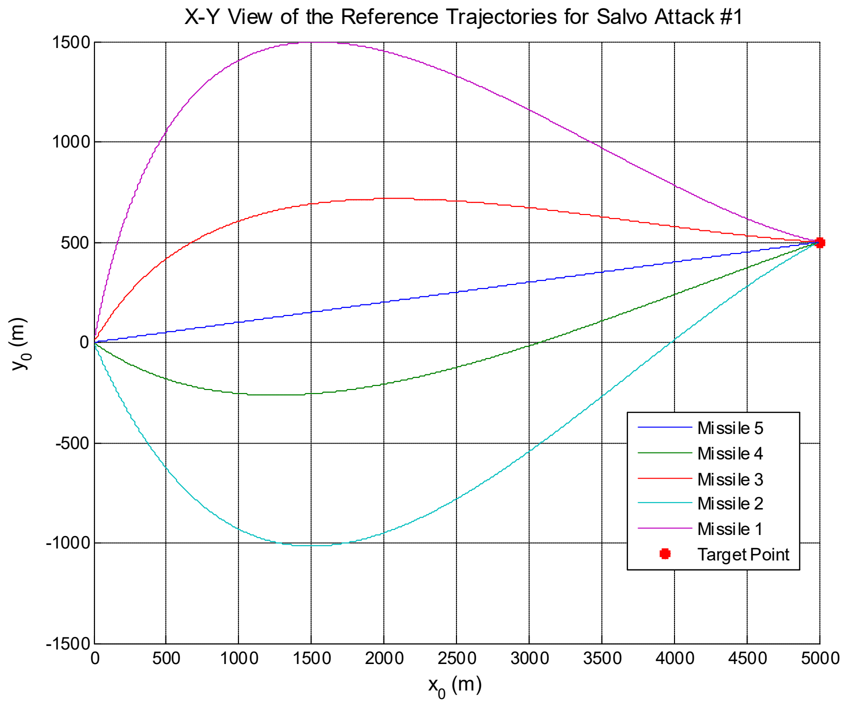

Figure 5-25: X-Y View of the Reference Trajectories for Salvo Attack Scenario #1

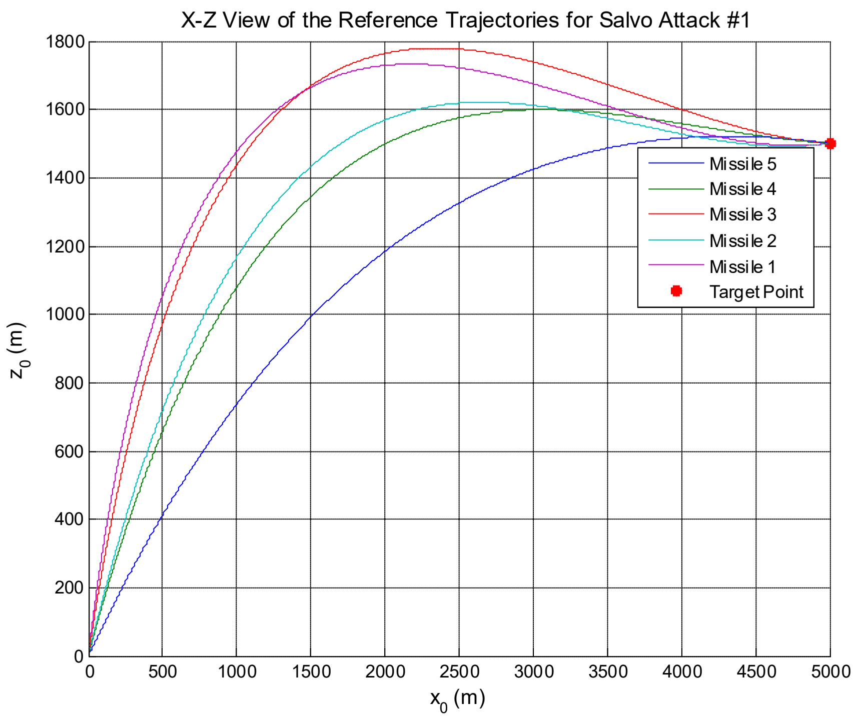

Figure 5-26: X-Z View of the Reference Trajectories for Salvo Attack Scenario #1

5.2.3.2 Salvo Attack Against a Maneuvering Target

In last part of the example scenarios section, salvo attack against a maneuvering target is examined. An incoming target is considered and the target starts to perform an escape maneuver when the missile target range, R becomes smaller than the maneuver distance.

$$R \leq R_{maneuver} \tag{5.8}$$

In terms of better interpretation of the trajectories, despite the same initial location, all missiles use the same reference trajectory function with a desired impact time of $19s$. The guidance commands are switched to terminal guidance phase at the instant missile starts the escape maneuver. Three different maneuvering target scenarios will be discussed. For the scenarios, different escape maneuvers are defined. The target is modelled as an incoming target with a constant velocity of $\vec{V}_t$ and the initial location of the target is denoted as $\vec{P}_{t0}$.

$$\begin{aligned} \vec{V}_t &= V_{tx}\vec{u}_{10} + V_{ty}\vec{u}_{20} + V_{tz}\vec{u}_{30} \\ \vec{P}_{t0} &= P_{tx0}\vec{u}_{10} + P_{ty0}\vec{u}_{20} + P_{tz0}\vec{u}_{30} \end{aligned} \tag{5.9}$$

In Eq.(5.9), $\vec{u}_{10}, \vec{u}_{20}$ and $\vec{u}_{30}$ are the unit vectors of the frame $\mathcal{F}_0$. The components and the initial location of the target are listed in the Table 5-8.

Table 5-8: Target Properties for Salvo Attack Scenarios 2, 3 and 4

$[V_x \;\; V_y \;\; V_z]$	$[-100 \quad 0 \quad 0]\ m/s$
$[P_{tx0} \;\; P_{ty0} \;\; P_{tz0}]$	$[6900\ 500\ 1500]\ m$

Target makes a g-turn maneuver to escape from the incoming missile attack. The $g - load$ of the maneuver is taken as $5g$ with constant g is taken as 9.79 m/s^2.

In the first scenario (Salvo Attack Scenario #2), g-turn maneuver takes part in $x - y$ and the maneuver range is taken as $R_{maneuver} = 2000m$.

For the trajectories, the Φ matrix is chosen as:

$$\Phi = \begin{bmatrix} -60^{o} & 60^{o} & -30^{o} & 30^{o} \end{bmatrix}^{T} \quad (5.10)$$

The resulting trajectories are shown in Figure 5-27.

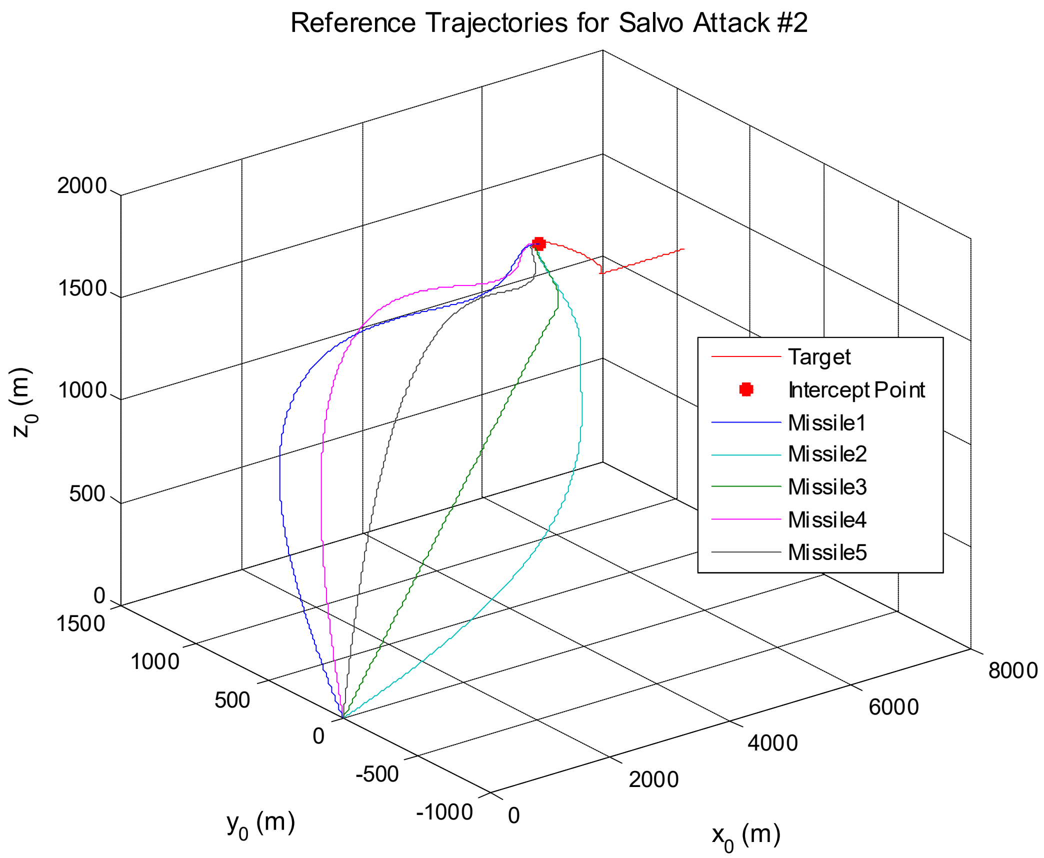

Figure 5-27: Reference Trajectories Obtained for Salvo Attack Scenario #2 Against a Maneuvering Target

The simulation is terminated, when one of the missiles reached the target. For this example scenario, the missile that first reached the target was Missile1 because of its trajectory. Since the escape maneuver takes place in $x - y$ plane, the approach of the missiles from different directions and the interception of Missile1 with the target may be seen more clearly in Figure 5-28. Also, $x - z$ plane trajectories are shown for a clearer understanding of the scenario.

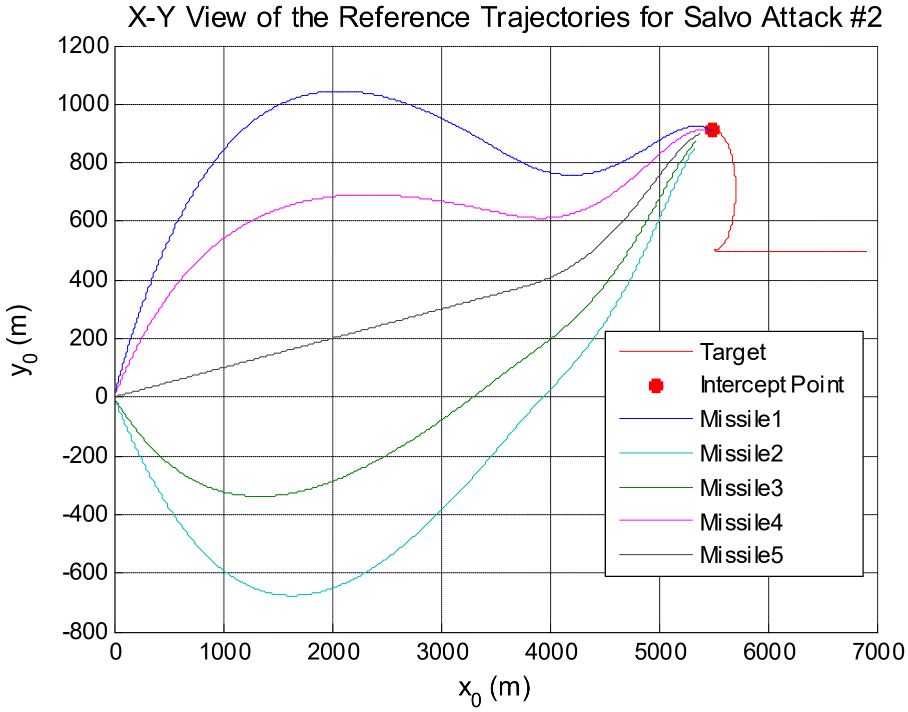

Figure 5-28: X-Y View of the Reference Trajectories for Salvo Attack Scenario #2

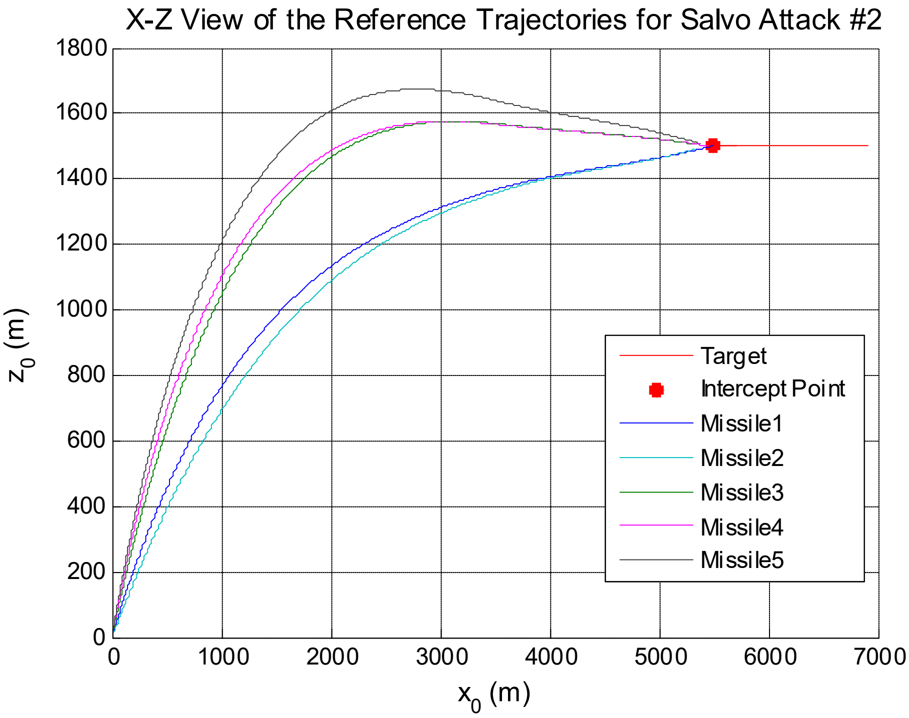

Figure 5-29: X-Z View of the Reference Trajectories for Salvo Attack Scenario #2

In order to analyze different engagement conditions, a scenario is constructed in which target performs the escape maneuver in reverse direction. For this case, a situation is established in which target detects the approaching missiles later. The maneuver distance is taken as $R_{maneuver} = 750m$. The resulting trajectories are shown in Figure 5-30.

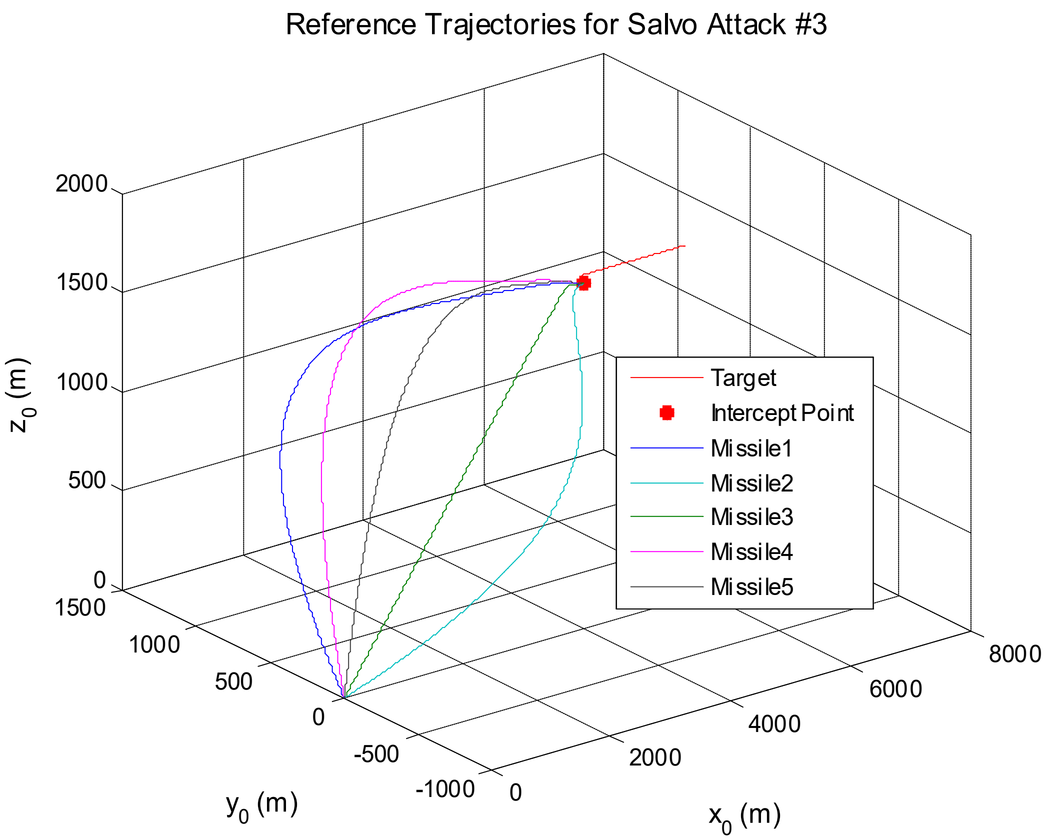

Figure 5-30: Reference Trajectories Obtained for Salvo Attack Scenario #3 Against a Maneuvering Target

In this scenario, Missile2 has become the first missile to reach the target due to orientation of its trajectory. By direction of approach of their trajectories, missiles have narrowed the space for the target escape maneuver. The planer projections of the missile and target trajectories are shown in Figure 5-31 and Figure 5-32.

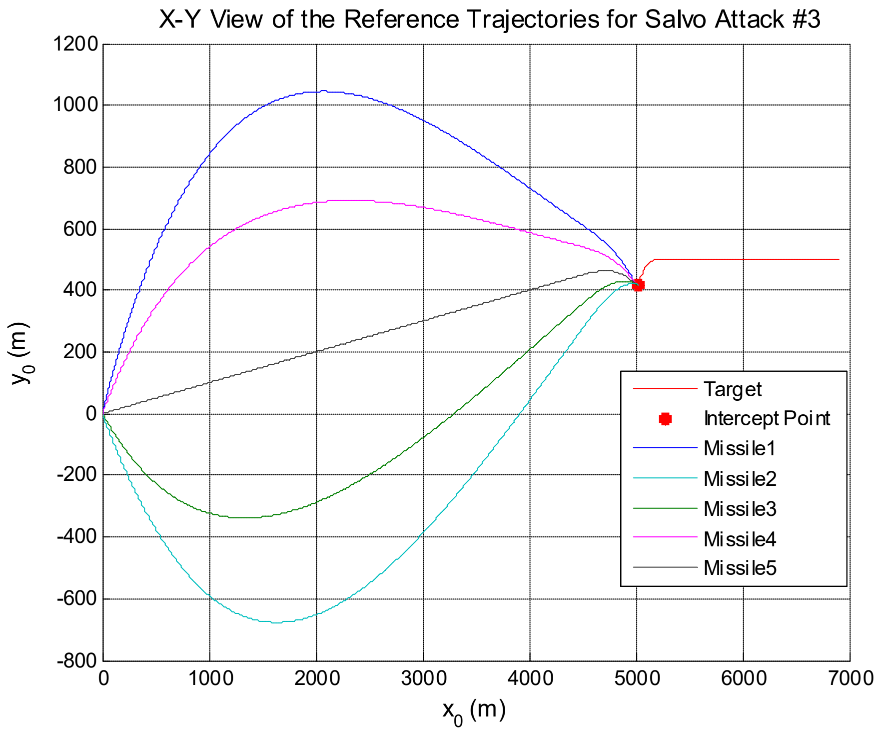

Figure 5-31: X-Y View of the Reference Trajectories for Salvo Attack Scenario #3

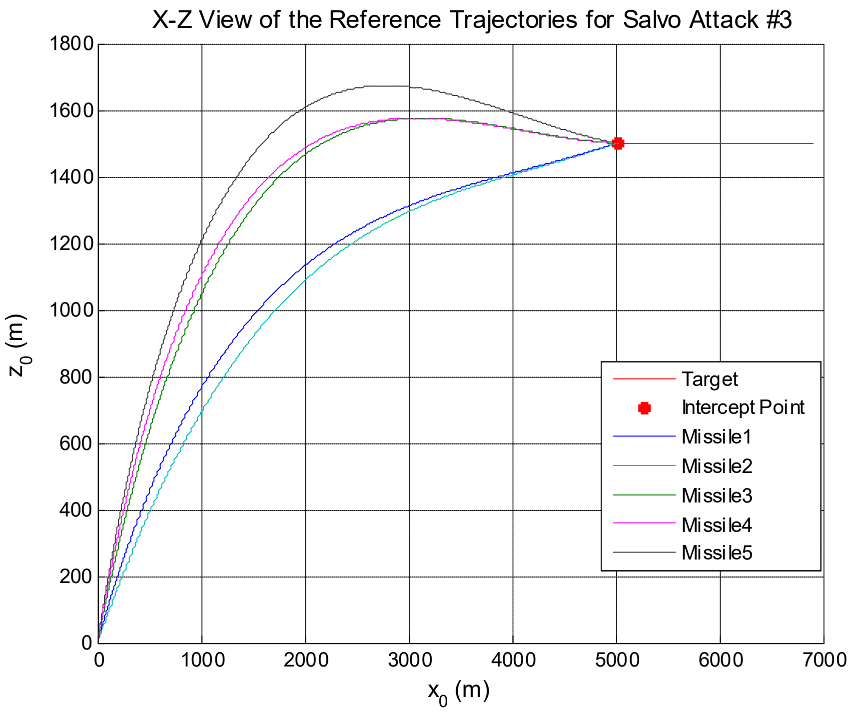

Figure 5-32: X-Z View of the Reference Trajectories for Salvo Attack Scenario #3

Finally, an example scenario in which target performs escape maneuver in both $x - y$ and $x - z$ planes is examined. Maneuver distance is taken as $R_{maneuver} = 1000\ m$. In this scenario, in order to allow the trajectories to spread over a larger area, roll angle matrix is chosen with larger increments.

$$\Phi = \begin{bmatrix} -80^{\circ} & 80^{\circ} & -40^{\circ} & 40^{\circ} \end{bmatrix}^T \tag{5.11}$$

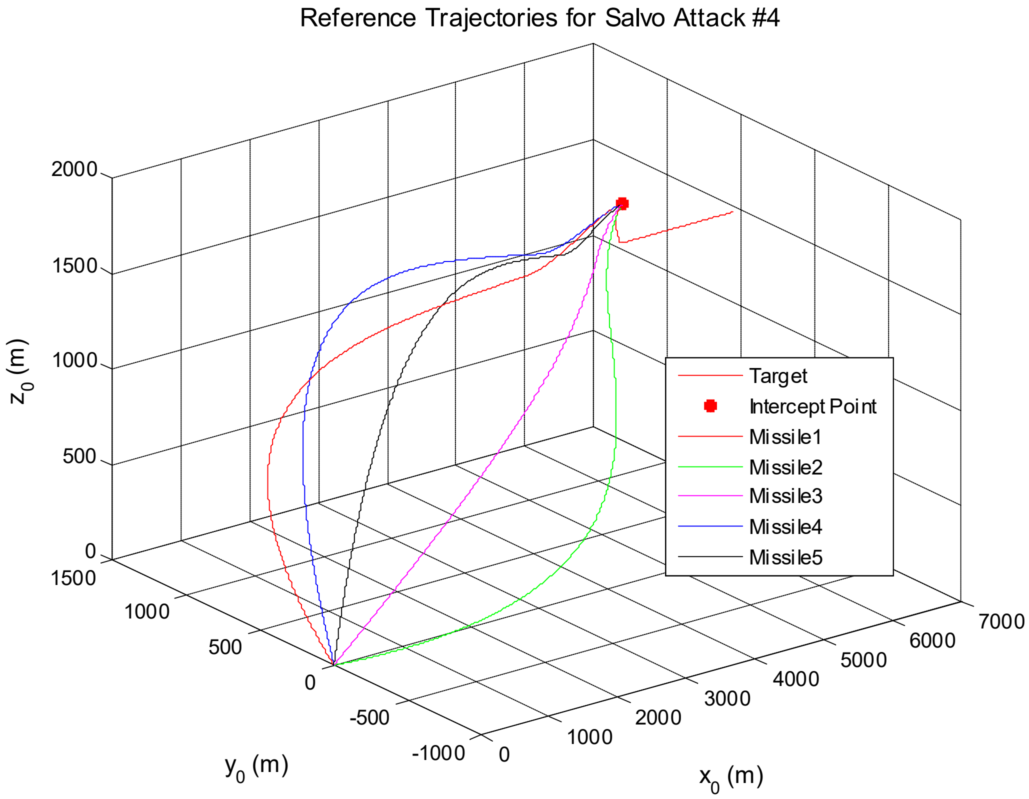

Figure 5-33: Reference Trajectories Obtained for Salvo Attack Scenario #4 Against a Maneuvering Target

For this scenario, Missile3 has become the first missile to reach the target. This situation may be observed more clearly from Figure 5-34 and Figure 5-35. It may be concluded that, approaching the target from a wider area creates an advantage for this type of scenarios.

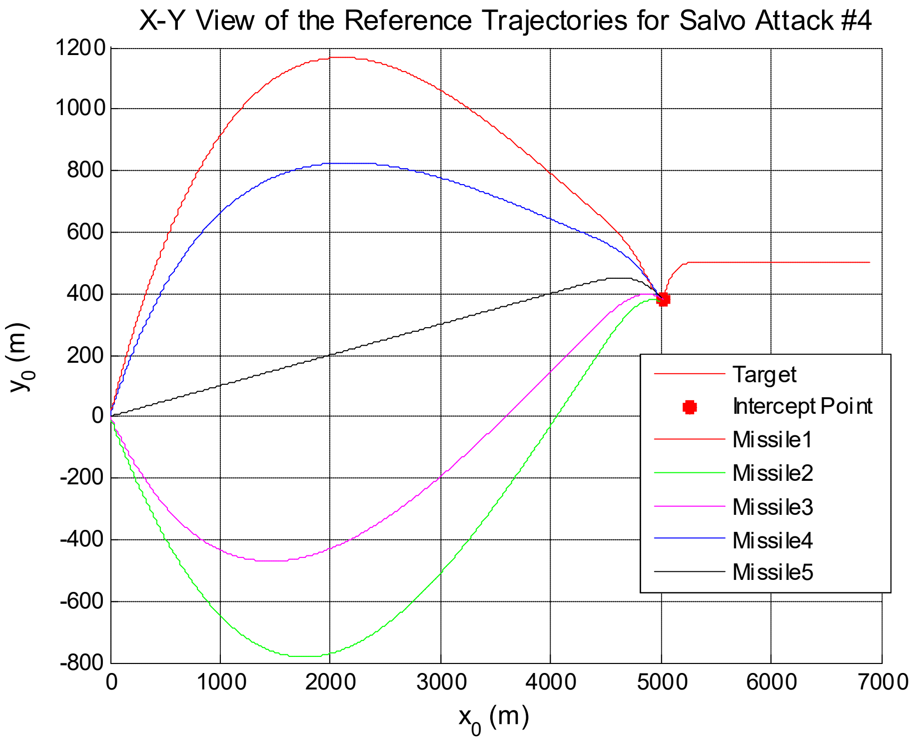

Figure 5-34: X-Y View of the Reference Trajectories for Salvo Attack Scenario #4

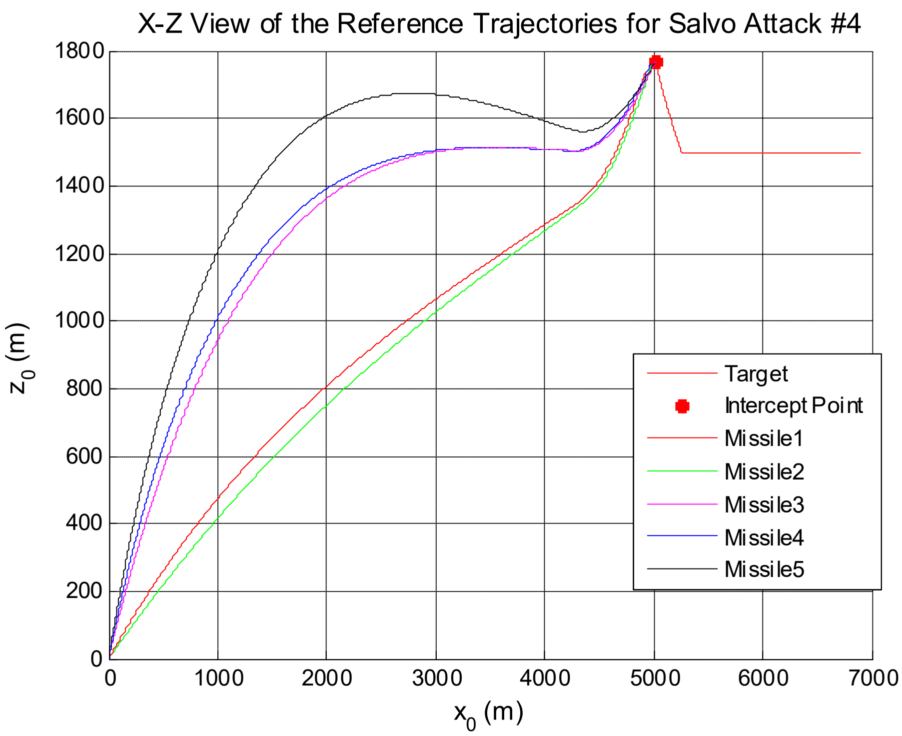

Figure 5-35: X-Z View of the Reference Trajectories for Salvo Attack Scenario #4

For this scenario, missile-target range, R vs time graph for Missile3 is also shown in Figure 5-36.

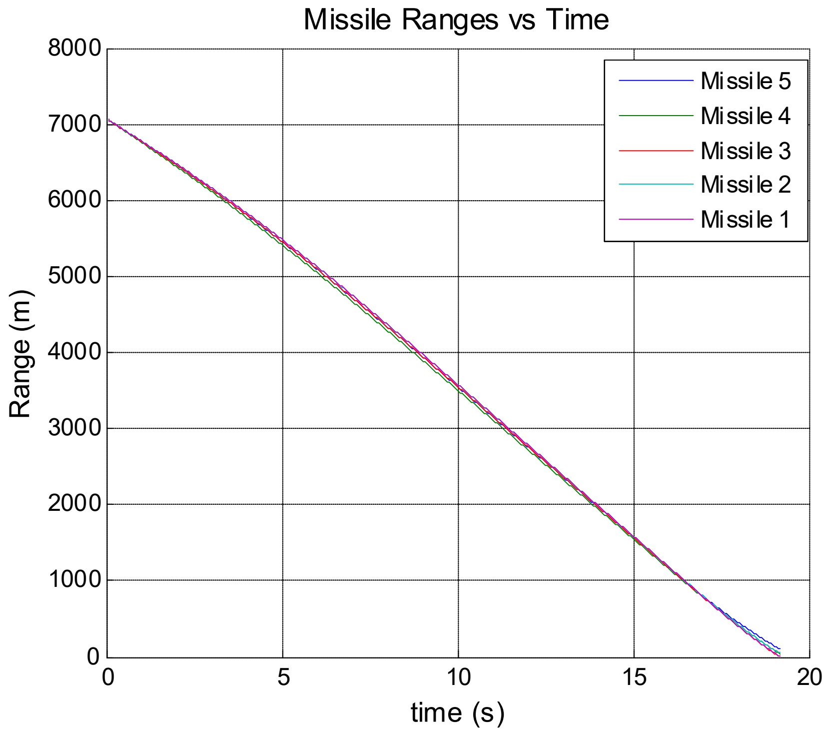

Figure 5-36: Missile Range for Salvo Attack Scenario#4

As it was stated before, impact time control requirement becomes invalid when the target performs an escape maneuver. Despite this, the impact time in the scenario is obtained as $19.2s$ which is very close to the desired impact time value of $19s$.

5.2.3.3 Salvo Attack in the Existence of Acceleration Command Limitation

In practical applications, missiles have a limited acceleration capacity. This limitation can be caused from mechanical and aerodynamical design, control actuation system limits or other sensor limits. Therefore, guidance commands must be limited according to the physical limits of the missile system. For the following simulation runs, an acceleration limit of $a_{\lim} = 100m/s^2$ in magnitude is added to the guidance commands. The results are discussed for a salvo attack of 5 missiles to a maneuvering target. Salvo attack parameters are given in the Table 5-9.

Table 5-9: Scenario Parameters of Salvo Attack #5

Maneuver Distance	$1000m$
Roll Angle Matrix, Φ	$[-80^o \quad 80^o \quad -40^o \quad 40^o]^T$
Maneuver g-load	$8g$
Maneuver Time Constant, τ	$0.3s$
Maneuver Duration	3τ

3D trajectories are shown in Figure 5-37.

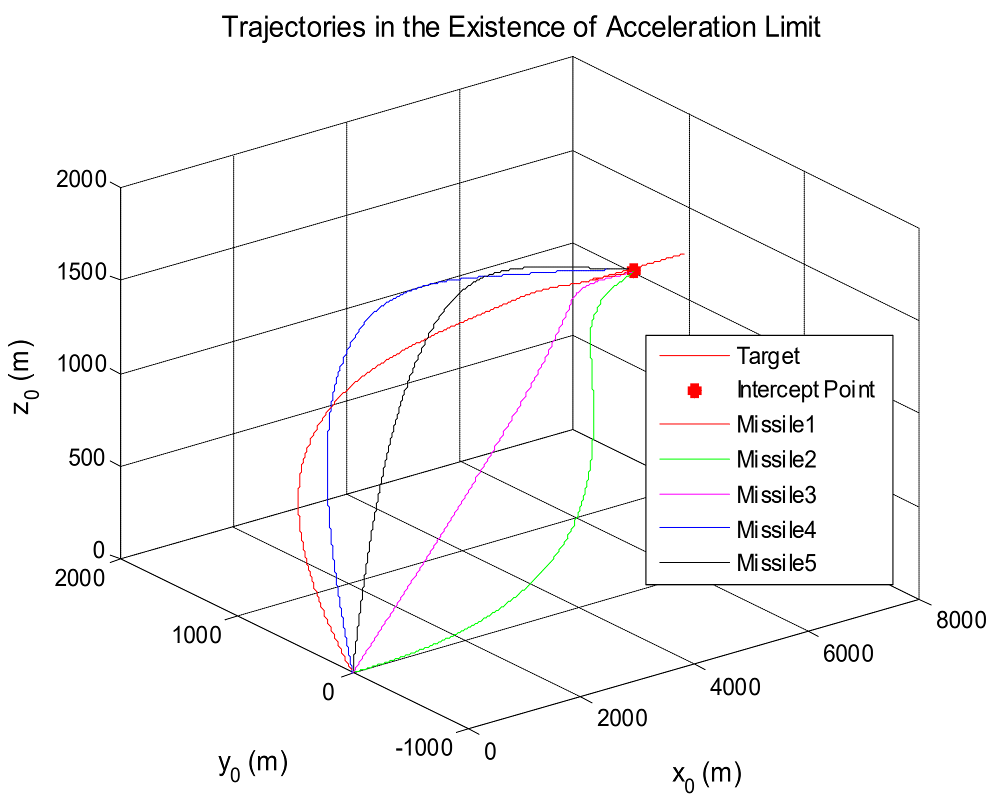

Figure 5-37: Salvo Attack Trajectories in the Existence of Acceleration Limit

The acceleration command for the terminal guidance phase is determined by the position of the missile at the instant target starts the escape maneuver. For this scenario, the acceleration commands of the 5 missiles are shown in Figure 5-38

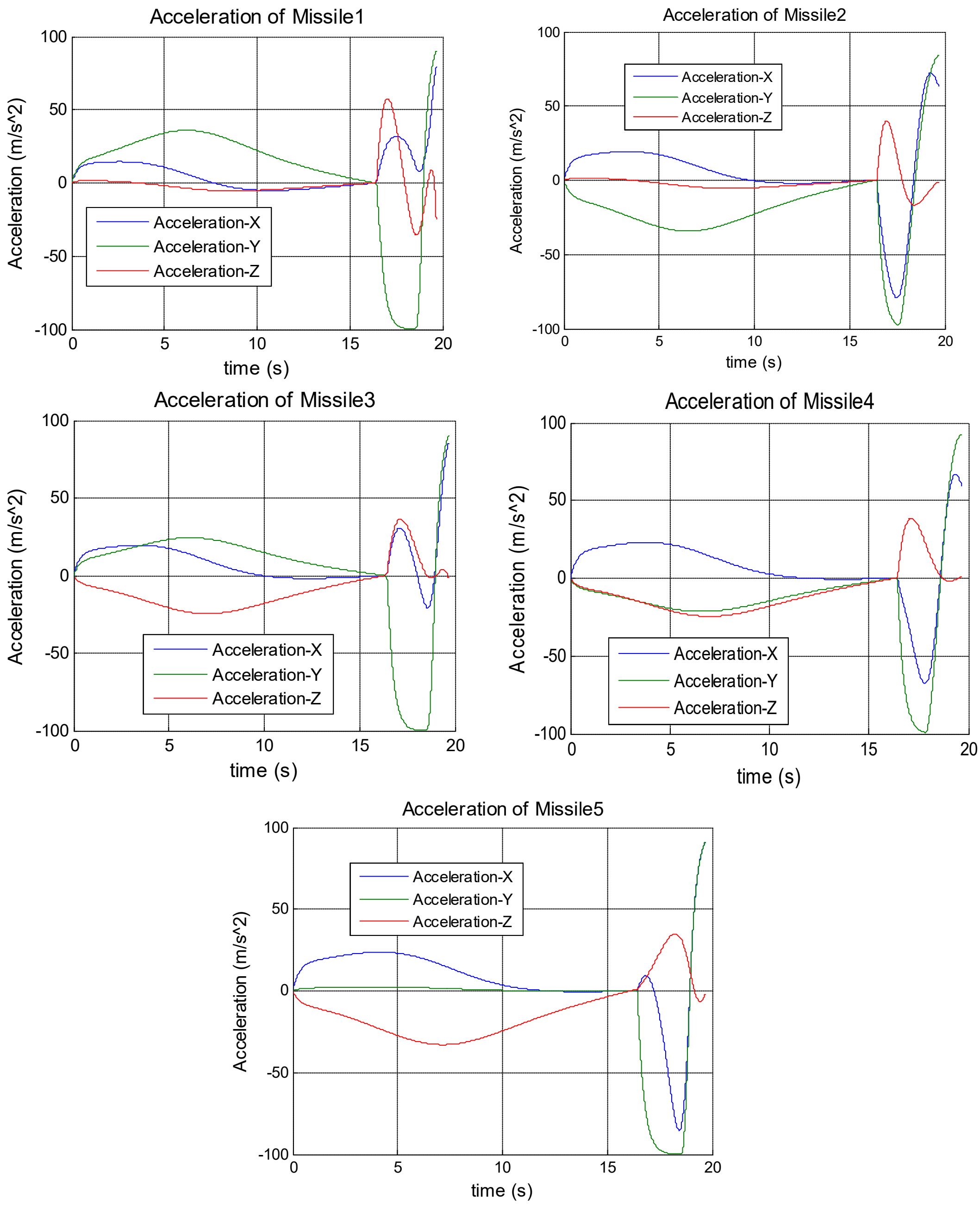

Figure 5-38: Acceleration Command Profiles for Salvo Attack Scenario #4

As it can be seen from the figure, acceleration commands of missiles 1, 3, 4 and 5 are saturated at the terminal guidance phase. With the advantage of its trajectory in midcourse guidance and direction of approach to the target, missile 2 requires less acceleration command to capture the target. This situation has caused missile 2 to hit the target and the others miss. It can be seen more clearly from planar views of the engagement in Figure 5-39 and Figure 5-40.

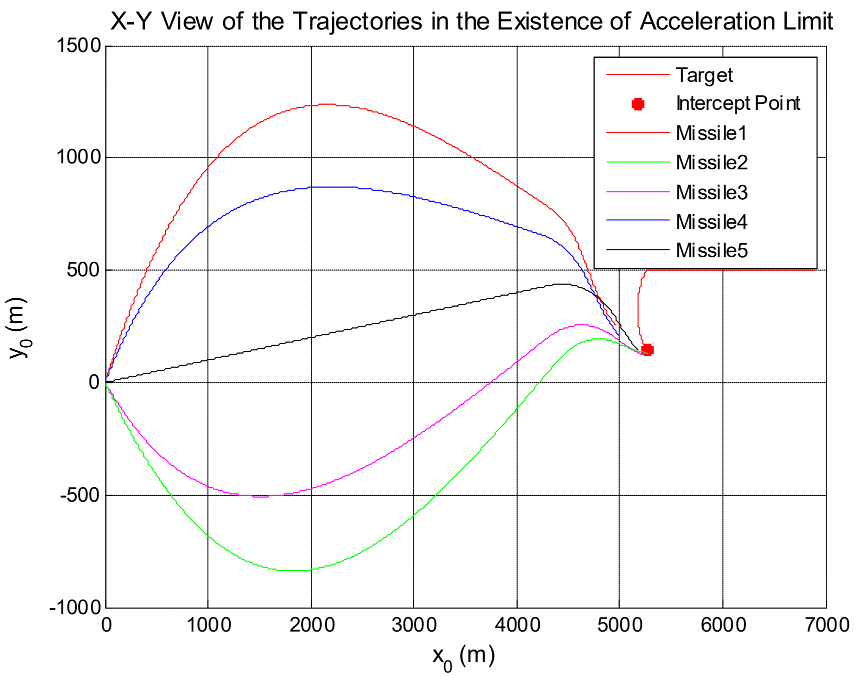

Figure 5-39: X-Y View of the Reference Trajectories for Salvo Attack Scenario #5

In this example scenario, it is observed that, approaching the target from different directions provided the target to be hit. It can be inferred that, the strategy of approaching the target from different directions increases the probability of hit by narrowing the maneuver area of the target.

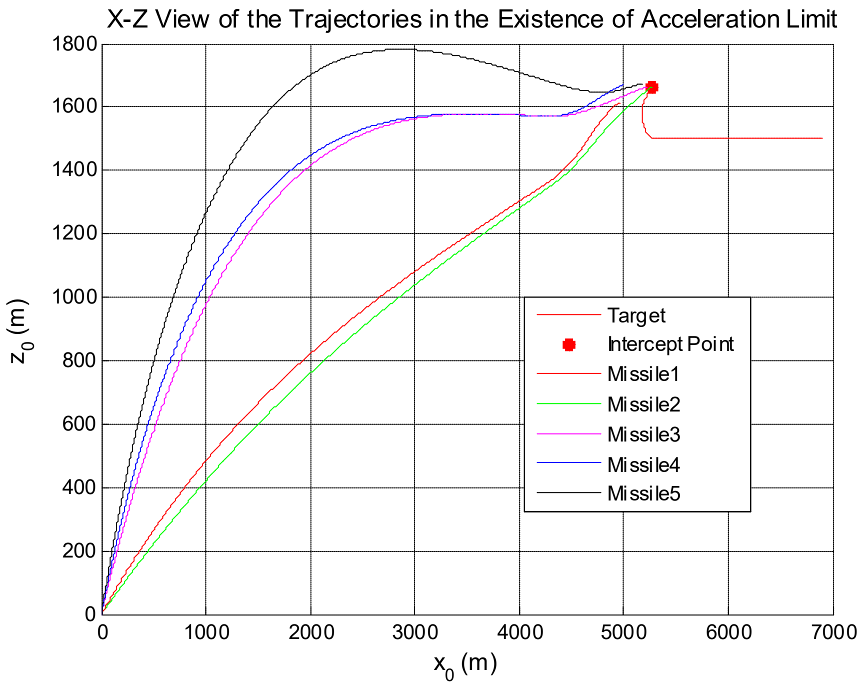

Figure 5-40: X-Z View of the Reference Trajectories for Salvo Attack Scenario #5

CHAPTER 6

CONCLUSION

In this thesis, a guidance algorithm was proposed which aims to control impact time via polynomial shaping of the missile trajectory and missile flight path angle. The main motivation in the trajectory design that provides impact time control is the application of the designed algorithm in a salvo attack scenario.

At first, the desired reference trajectory between the initial missile location and the target point is defined as a 3^{rd} order polynomial function in the 2-D $x - z$ plane. The coefficients of the polynomial function that defines the trajectory are determined to control impact time. In the design process, it has been observed that, order to obtain unique coefficients, one more design parameter should be added to the algorithm. This parameter is defined as the launch angle or impact angle. In some cases, the launch angle may be required to be determined according to the engagement conditions. In these situations, the launch angle must be defined as an input to the guidance algorithm. In cases other than this, impact angle control is a preferred situation in most applications due to its aim of giving maximum damage to the target. For this reason, while designing the reference trajectory, impact angle is determined by user according to the engagement requirements. In order to control impact time over the reference trajectory, total trajectory length is required to be calculated parametrically in terms of polynomial function coefficients. Since the reference trajectory is defined as a 3^{rd} order polynomial function, the integral used to calculate the trajectory length cannot be solved analytically. For this reason, numerical methods have to be used to calculate the trajectory length. Gaussian Quadrature numerical integration method is used since, by using this method, trajectory length is calculated with an error less than 0.5 meters.

After designing the reference trajectory, guidance commands required to follow this trajectory were derived. As in the case with all aerodynamically controlled missiles, guidance commands are derived in the direction normal to the missile velocity vector. Missile equations of motion were used in nonlinear form. Thus, any errors that may arise from linearization are avoided. The normal acceleration command was obtained by analytical formulation. Since the guidance coefficients were determined according to the trajectory function coefficients, applying this acceleration command means that an open loop control method is used. Therefore, in order to obtain the guidance commands in feedback form, virtual target was defined that is moving along the desired reference trajectory. It has been shown that the guidance command derived by the virtual target approach prevents the missile from deviating from the desired trajectory in the existence of disturbance forces.

The reference trajectory was designed for a planar engagement. For the 3D application, a maneuver plane is defined in which the guidance commands are generated. The main reason of the extension of the proposed algorithm to a 3D geometry is the application to a salvo attack scenario. For a salvo attack, a different maneuver plane is defined for each missile. These planes are obtained by rotating the first maneuver plane (LOS frame) around its $x-$axis. By analyzing the example scenarios, it has been concluded that, the use of this strategy in a salvo attack is advantageous since it allows the adjustment of the spread of the trajectories. Hence, it allows to control the direction that each missile approaches the target.

In this study, it was assumed that, during flight, a communication network between the missile group does not exist. For this reason, the parameters of the reference trajectory must be specified before launch. This method can be used in practical applications for targets which are stationary or known to be moving with a constant velocity. However, in situations where the target performs escape maneuvers, a terminal guidance phase is needed in which real-time information about the target is used. The well-known Proportional Navigation Guidance law is used as the terminal guidance algorithm, assuming that the line of sight rate to the target is provided by an onboard seeker. In the simulation analyses, it has been seen that, if the target makes an escape maneuver, the use of the proposed guidance method as a midcourse guidance algorithm narrows the area where the target can escape. In many scenarios, this will lead to a significant increase in probability of hit.

For the last example scenario, the situation in which missile have a limited acceleration capacity was discussed. It had been observed that, for the maneuvering target scenarios, missile may miss the target due the limitation of the guidance commands. In this case, performing a salvo attack and the strategy of approaching target from different directions is seen to increase the hit performance.

Finally, it is concluded that the proposed polynomial trajectory shaping guidance method can be used to control impact time and angle for targets that are stationary or moving with a constant velocity. And also considering all the analyses results, it may be concluded that the proposed method will increase the probability of hit for a salvo attack against a moving target.

Future Works

After this study, following ideas may be able to be studied.

- In this study, the reference trajectory is defined as a third order polynomial function. Additional degrees of freedom can be added to the problem by increasing the degree of the polynomial. Polynomials of degree 4 or higher are aimed to be studied as reference trajectory function candidates.

- In the derivation of the reference trajectory and corresponding guidance commands, missile speed is taken as constant. This is the most common method of guidance algorithm design process. However, in terms of practical applications, it may be studied to adapt the proposed guidance algorithm to variable missile velocity profiles.

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