STRUCTURAL COUPLING OF TWO NONLINEAR STRUCTURES

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ABSTRACT

STRUCTURAL COUPLING OF TWO NONLINEAR STRUCTURES

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In mechanical design, modelling and analysis of a complex structure can be simplified with dividing the structure into substructures; therefore, any change in the structure can be addressed easily which is referred as "structural coupling".

Utilization of proper coupling techniques, it is possible to understand the behavior of the whole structure by considering the behavior of its substructures. For linear structures, coupling is a common technique; however, in most of the engineering structures, nonlinearities are also encountered; therefore, it is required to extend linear coupling methods to nonlinear systems. Although, studies on nonlinear coupling are available in literature, existing methods are limited to coupling of structures where one substructure is linear and the other is nonlinear or two linear substructures coupled with a nonlinear element.

In this thesis, a structural coupling method is proposed to couple two-nonlinear substructures. Similar to linear coupling methods, the proposed method considers the compatibility of internal forces at the connection degrees of freedom in addition to displacements. The proposed method is simulated with two different conditions which are coupling of identified substructures and coupling of identified substructure with neural network trained substructure. Since, the substructures are nonlinear, the resulting system of nonlinear differential equations are converted into

a set of nonlinear algebraic equations by using Describing Function Method, which are solved by using Newton's method with arclength continuation.

Keywords: Structural Coupling, Nonlinear Structural Coupling, Vibration of Nonlinear Structures

ÖZ

DOĞRUSAL OLMAYAN İKİ SİSTEMİN YAPISAL BİRLEŞİMİ

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Mekanik tasarımda, karmaşık bir yapının modellenmesi ve analizi, bu yapıyı kendisini oluşturan altyapılara bölmekle daha kolay gerçekleştirilir. Bu sayede tüm sistem üzerinde meydana gelecek olan herhangi bir değişiklik altyapıların tekrar birleştirilmesiyle gözlemlenebilir. Bu olguya yapısal birleşme denilir.

Uygun birleştirme yöntemleriyle, tüm sistemin davranışını onu oluşturan alt sistemlerin davranışını anlayarak çözmek mümkündür. Doğrusal yapılar için birleşme çok uygulanan bir yöntem iken doğrusal olmayan yapılar için bu yöntemin nasıl uygulanacağına dair çok örnek bulunmamaktadır. Günümüzde çoğu mühendislik yapılarında doğrusal olmayan sistemlerin de görüldüğünü düşünürsek, doğrusal birleştirme yöntemlerini, doğrusal olmayan yapılara taşımak gerekli hale gelmiştir. Literatürde doğrusal olmayan sistemleri içinde barındıran birleştirme yöntemleri bulunsa da, bu yöntemler, doğrusal yapıyı doğrusal olmayan yapı ile birleştirme ya da iki doğrusal yapıyı doğrusal olmayan elemanlarla birleştirme ile sınırlıdır.

Bu tezde, iki doğrusal olmayan yapıyı birbirine birleştirme konusunda çalışılacaktır. Doğrusal olmayan yapıların birleşiminde doğrusal olan sistemlerin birleşimine benzer bir şekilde, iki yapının birleştiği sırada koruması gereken kuvvet ve yer değiştirme değerleri kullanılacaktır. Önerilen metodu doğrularken iki farklı

yöntem izlenecektir. Bu yöntemleri; özellikleri bilinen sistemlerin birbirleriyle birleşimi ve özellikleri bilinen sistemle sinirsel ağ ile tanımlanan sistemin birbirleriyle birleşimi olarak özetlemek mümkündür. Yapılar doğrusal olmadığı için çözülmesi gereken denklemler cebirsel doğrusal olmayan denklem setine dönüştürülecek ve bu denklem setleri sürdürülür yöntemlerle çözülecektir.

Anahtar Kelimeler: Yapısal Birleşim, Doğrusal Olmayan Yapısal Birleşim, Doğrusal Olmayan Sistemlerin Titreşimi

To My Beloved Love
Mısra Ayşe ADSIZ

To My Parents
Şükran TEPE, Abdullah TEPE

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LIST OF SYMBOLS

$\{x\}$: Generalized displacement vector

$\{f\}$: Internal forcing vector

$[Z]$: Impedance matrix

$\left[D_{coupling}\right]$: Dynamic stiffness matrix of coupling elements

$\{F\}$: External forcing vector

$[M]$: Mass matrix

$[C]$: Viscous damping matrix

$[H]$: Structural damping matrix

$[K]$: Stiffness matrix

$\{f_N\}$: Nonlinear forcing vector

n : Nonlinear restoring force

y : Relative displacement

$\{X\}$: Complex amplitude vector of the displacement

ν : Describing function

$\{N\}$: Complex amplitude vector of internal forces

m : Number of harmonics

$[\Delta]$: Nonlinearity matrix

k_c : Coefficient of cubic stiffness

k : Contact stiffness between rubbing surfaces

N : Constant normal force

μ : Dry friction coefficient

$[J]$: Jacobian matrix

$\{q\}$: Vector of unknown in arc-length continuation method

s : Arc-Length parameter

$\{x\}_0$: Improved initial guess of displacement vector

ω_0 : Improved initial guess of frequency

$\left[D_{coupling}^{NL} \right]$: Nonlinear dynamic stiffness matrix

$\{w\}$: Weight vector

b : Bias term

o : Output of the neuron

N_R : Number of elements in the input vector

MSE : Mean squared error

NN : Neural network transfer function

CHAPTER 1

INTRODUCTION

1.1   Introduction to the Problem

In the design of mechanical systems, engineers should test and analyze each
prototype created in order to provide a qualified and optimized design which has a
wide range of requirements. Over the last 40 years, engineering structures are
analyzed with the finite element method which is proven as reliable tool. In order
finalize the design, whole structure has to be analyzed several times; therefore, an
alternative approach is required in order to decrease the number of analyses and tests.
In former years, this requirement is disposed with proper structural coupling
techniques, and utilizing structural coupling, modeling and analysis of a complex
structure can be simplified by dividing the structure into substructures and applying
the required changes only on one or some of the substructures, where each
substructure can be analyzed individually. In this way, complexity of whole structure
may be avoided.

Although, coupling is a common technique for linear structures, in most of the
engineering structures, nonlinearities are also encountered with the increasing
demand of high precision mechanical components; therefore, it is required to extend
linear coupling methods to nonlinear systems.

1.2   Literature Survey

Substructure analysis of linear systems is a well-known subject dated back to 1960s
by the works of Bishop and Johnson [1] on Receptance method and Hurty [2] on

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Component Mode Synthesis which was a simplified version of the method developed by Craig and Bampton [3]. Many different substructure and coupling methods for linear structures are developed by Rubin [4], Przemieniecki [5], Urgueira [6], Ewins [7], Klosterman and Lemon [8] and Ren and Beards [9]. All of these methods are developed for linear systems and the methodology is based on the compatibility of internal forces at the connection degrees of freedom in addition to the compatibility of the displacements. However, extention of linear coupling methods to non-linear systems is essential; since many structures, which are considered as linear, are nonlinear in reality.

Analysis of nonlinear systems is much more complicated compared to linear systems [10] due to their response dependent behavior. In this thesis, Describing Function Method (DFM) is used for the solution of nonlinear systems which was introduced by Krylov and Bogolyubov [11] in order to analyze nonlinear control problems based on an earlier work of Van der Pol [12]. Later, Taylor [13] replaced each nonlinear element with a quasilinear descriptor to define this approach.

Solution of multi degree of freedom nonlinear system with symmetrical nonlinearities is introduced by Budak and Özgüven [14, 15], which utilizes a special algebra. Later, Tanrıkulu [16] and Tanrıkulu et. Al.[17] extended this formulation for any type of nonlinearity by replacing this special algebra with describing functions. Other studies, which may be shown as an example of vibration analysis of nonlinear structures, are made by Siller [18] and Abat [19].

Although, several studies on structural coupling of linear systems and modelling systems with nonlinearities are available in literature, the numbers of studies that consider nonlinear structural coupling are limited. Existing studies on nonlinear structural coupling are focused on coupling of structures where one substructure is linear and the other one is nonlinear or coupling of two linear substructures with a nonlinear coupling element.

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Watanabe and Sato [10] suggested "Nonlinear Building Block" approach, for coupling of linear substructures with nonlinear coupling elements.

Cömert and Özgüven [20] proposed a method for coupling of linear substructures with nonlinear connecting elements by using DFM, in which FRFs of the linear substructures are used. Kalaycıoğlu [21] suggested a modification/coupling technique that couple two linear structures with nonlinear elements.

Murakami and Sato [22] suggested a method to predict the response of linear structures with non-linear joints using a describing function approach. Later, Wyckaert et. Al. [23] is extended the approach with cubic stiffness and friction damping type non-linearity.

Ferreira and Ewins [24] introduced a new "Nonlinear Receptance Coupling Approach" and Ferreira [25] extended the approach with "Multi-Harmonic Nonlinear Receptance Coupling Approach". Both approaches are capable of coupling a linear structure with a nonlinear structure with different types of joints.

Chong and Imregün [26] managed coupling of nonlinear systems with linear systems with an iterative algorithm.

1.3   Objective

It has been thought that the main motivation behind the coupling procedure is computational efficiency because of fact that if the system is subdivided into two equal subsystems the solution time may be expected to be reduced by a factor of 4. [26]. However, avoiding the complexity of a whole system with dividing it into substructures is outweighs the computational efficiency by far [26]. With proper coupling techniques, each subsystem should be solved with preserving numerical

accuracy [26]. Most complex structures are consisted of assembled substructures which are designed by different engineering groups, at different times and in different locations [25]. So, proceeding designs and modifications as independently as possible is desired. Nevertheless, computational time may be decreased with proper methods such as domain decomposition method which is used parallel processing to solve each substructure.

From an engineering viewpoint, there is a need to use proper coupling methods if complexity of the system is needed to be avoided. Also, extend linear coupling methods to nonlinear systems is essential with the increasing demand for high precision, so if more accurate results with less time and cost is wanted, nonlinear structural coupling is required.

In this thesis, an approach is going to be developed to dynamic reanalysis of nonlinear substructures. Different from the existing methods in literature, proposed method is going to be capable of coupling of two nonlinear substructures. Proposed method should solve the coupled system even any DOFs have nonlinearities. Moreover, with the proposed coupling method, in addition to linear coupling elements, nonlinear coupling elements can as well be used. Beside avoidance of complexity, proposed method should also be studied it is time efficient or not even if it is not the main objective.

1.4   Scope of the Thesis

The outline of the thesis is given as follows:

In Chapter 2; firstly, brief information about linear structural coupling is given. After that, theory behind the nonlinear modelling is explained. DFM method, which is also used in this thesis, is introduced in detail and describing function of cubic stiffness and dry friction are given. Lastly, several numerical methods are introduced to solve

nonlinear system equations. Newton's method and Newton's method with arc-length continuation are described in detail.

Chapter 3 reviews the proposed method which is used in coupling of two nonlinear structures. Firstly, brief introduction is made about the problem. Then, a new nonlinear structural coupling method is proposed to couple two nonlinear substructures. First part of this chapter is about couple two nonlinear substructures which are identified already. So, spatial model of substructures are used. After that, compliance of this method is shown such that identification of any substructure is not essential to proceeding the operation. This achievement is provided with the help of neural networks which are developed via MATLAB®.

In Chapter 4; verification of the proposed method will be demonstrated in two groups of case studies which are also dividing into two. Main section of case studies is about coupled DOF, while subsections are departed usage of artificial neural networks or not. In first part of case studies, two substructures are coupled each other with one coupled DOF. In second part, two substructures are also coupled each other such that one of the substructures is trained with neural network already. Solution of trained data is used coupling procedure. In third and fourth part of case studies, same procedure is used for multiple coupled DOF.

In Chapter 5, brief summary will be given about work done with discussions. The conclusion of thesis is given in this chapter. Finally, contributions to nonlinear structural coupling are summarized.

CHAPTER 2

THEORY

2.1   Introduction

In this chapter, the theory of nonlinear structural coupling method, which underlies the basis of this thesis, is proposed. In section 2.2, theory of structural coupling is introduced. In section 2.3, modeling of nonlinear structures is presented using describing function method. Later, in the same section the types of nonlinearities investigated in this study are explained in detail. In section 2.4, solution of nonlinear equation of motion is introduced.

2.2   Structural Coupling of Linear Substructures

Consider two substructures A and B, shown in Figure 2.1, where internal DOFs are represented by subscripts, i_A and i_B respectively and the connection DOFs are represented by subscripts c_A and c_B, respectively.
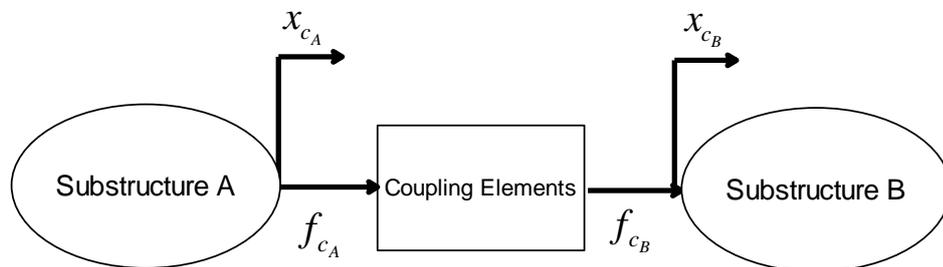
Figure 2.1 Schematic view of structural coupling

The corresponding equilibrium of each substructure can be written as

$$\left\{ \begin{matrix} \{f_{i_A}\} \\ \{f_{c_A}\} \end{matrix} \right\} = \begin{bmatrix} [Z_{i_A i_A}] & [Z_{i_A c_A}] \\ [Z_{c_A i_A}] & [Z_{c_A c_A}] \end{bmatrix} \left\{ \begin{matrix} \{x_{i_A}\} \\ \{x_{c_A}\} \end{matrix} \right\}, \tag{2.1}$$

$$\left\{ \begin{matrix} \{f_{i_B}\} \\ \{f_{c_B}\} \end{matrix} \right\} = \begin{bmatrix} [Z_{i_B i_B}] & [Z_{i_B c_B}] \\ [Z_{c_B i_B}] & [Z_{c_B c_B}] \end{bmatrix} \left\{ \begin{matrix} \{x_{i_B}\} \\ \{x_{c_B}\} \end{matrix} \right\}, \tag{2.2}$$

where $\{x_{i_A}\}$ and $\{x_{i_B}\}$ are generalized displacement vectors for internal DOFs, $\{x_{c_A}\}$ and $\{x_{c_B}\}$ are generalized displacement vectors for coupled DOFs of substructures A and B, respectively. $\{f_{i_A}\}$ and $\{f_{i_B}\}$ are internal forcing vectors for internal DOFs, $\{f_{c_A}\}$ and $\{f_{c_B}\}$ are coupled forcing vectors for internal DOFs of substructures A and B, respectively. Lastly, $[Z_A]$ and $[Z_B]$ are the impedance matrices of substructures A and B. Equilibrium of the forces between the connection DOFs can be written as

$$\{f_c\} = \{f_{c_A}\} + \{f_{c_B}\}, \tag{2.3}$$

where $\{f_c\}$ is the external force acting on the connection DOFs. Considering the compatibility of displacements of the substructures the following relation can be written

$$[D_{coupling}]\left(\{x_{c_A}\} - \{x_{c_B}\}\right) = \{f_{c_A}\}, \tag{2.4}$$

where $[D_{coupling}]$ is dynamic stiffness matrix of coupling elements. Substituting compatibility and equilibrium equations, i.e. Eqs. (2.3) and (2.4), into Eqs. (2.1) and (2.2), the overall impedance of the assembled system can be written as

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$$[Z] = \begin{bmatrix} \begin{bmatrix} Z_{i_A i_A} \end{bmatrix} & [0] & \begin{bmatrix} Z_{i_A c_A} \end{bmatrix} \\ [0] & \begin{bmatrix} Z_{i_B i_B} \end{bmatrix} & \begin{bmatrix} Z_{i_B c_B} \end{bmatrix} \\ \begin{bmatrix} Z_{c_A i_A} \end{bmatrix} & \begin{bmatrix} Z_{c_B i_B} \end{bmatrix} & \begin{bmatrix} Z_{c_A c_A} \end{bmatrix} + \begin{bmatrix} Z_{c_B c_B} \end{bmatrix} \end{bmatrix}. \tag{2.5}$$

2.3 Modelling of Nonlinear Structures

2.3.1 Describing Function Method

In nonlinear structure modelling, if the system may exhibit periodic oscillations, describing function method is frequently used. The describing function method linearizes the nonlinearity by defining the transfer function as the relation of the fundamental components of the input and the output to the nonlinearity.

The equation of motion of nonlinear MDOF system excited with harmonic external forcing $\{f(t)\}$, can be written as

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + i[H]\{x(t)\} + [K]\{x(t)\} + \{f_N(t)\} = \{f(t)\}, \tag{2.6}$$

where $[M]$, $[C]$, $[H]$ and $[K]$ are the mass, viscous damping, structural damping and stiffness matrices of the linear system. $\{x(t)\}$ is the generalized displacement vector and $\{f_N(t)\}$ is the nonlinear forcing vector. The kth element of vector $\{f_N(t)\}$ can be expressed as a series of the form,

$$\{f_N(t)\}_k = \sum_{j=1}^{N} n_{kj}, \tag{2.7}$$

where n_{kj} denotes the nonlinear restoring force acting between the coordinates k and j for $k \neq j$ and between ground and the coordinate k for $k = j$ and N is the number of elements of vector $\{f_N(t)\}$. Note that,

$$n_{kj} = n_{jk} .$$ (2.8)

The nonlinear restoring force n_{kj} is a function of relative displacement y_{kj} and its derivatives.

$$n_{kj} = n_{kj}(y_{kj}, \dot{y}_{kj}, \ddot{y}_{kj}, \ldots) ,$$ (2.9)

where,

$$\begin{aligned} y_{kj} &= x_k - x_j \quad \text{for } k \neq j \\ y_{kj} &= x_k \qquad \text{for } k = j \end{aligned} .$$ (2.10)

If the external forcing $\{f(t)\}$ is periodic, then it can be expressed as,

$$\{f(t)\} = \{F\}_0 + \text{Im}\left[\sum_{m=1}^{\infty} \{F\}_m \cdot e^{i \cdot m \cdot \psi} \right] ,$$ (2.11)

where $\{F\}_m$ is the amplitude vector of the mth harmonic. Then the response of the system, $\{x(t)\}$ can as well be assumed periodic which can be expressed as follows,

$$\{x(t)\} = \{X\}_0 + \text{Im}\left[\sum_{m=1}^{\infty} \{X\}_m \cdot e^{im\psi} \right] ,$$ (2.12)

where $\{X\}_m$ is the complex amplitude vector of the mth harmonic of the displacement. The intercoordinate displacement responses between arbitrary two coordinates k and j, y_{kj}, can be written as:

$$\{y_{kj}\} = \text{Im}\left[\sum_{m=0}^{\infty}\{Y_{kj}\}_m \cdot e^{i \cdot m \cdot \psi}\right], \qquad (2.13)$$

where,

$$\{Y_{kj}\}_m = \{X\}_{m_k} - \{X\}_{m_j} \quad \text{for } k \neq j$$
$$\{Y_{kj}\}_m = \{X\}_{m_k} \qquad\qquad \text{for } k = j \qquad (2.14)$$

Accordingly, a complex and periodic nonlinear function, n_{kj} can also be represented in the form of Fourier series as

$$\{n_{kj}\} = \sum_{m=0}^{\infty}\{n_{kj}\}_m \cdot e^{i \cdot m \cdot \psi} = \{N_{kj}\}_0 + \text{Im}\left[\sum_{m=1}^{\infty}\{N_{kj}\}_m \cdot e^{i \cdot m \cdot \psi}\right], \qquad (2.15)$$

where,

$$\{N_{kj}\}_m = \frac{i}{\pi}\int_0^{2\pi} n_{kj}(y_{kj}, \dot{y}_{kj}, \ddot{y}_{kj}, \ldots) \cdot e^{-i \cdot m \cdot \psi} \cdot d\psi \quad \text{for } m = 1, 2, 3, \ldots$$
$$\{N_{kj}\}_m = \frac{i}{2\pi}\int_0^{2\pi} n_{kj}(y_{kj}, \dot{y}_{kj}, \ddot{y}_{kj}, \ldots) \cdot d\psi \qquad\qquad \text{for } m = 0 \qquad (2.16)$$

The describing function, v_{kj} of order m corresponding to $\{n_{kj}\}$ can be defined as

$$v_{kj_m} = \frac{\{N_{kj}\}_m}{\{Y_{kj}\}_m}. \tag{2.17}$$

Inserting Eq. (2.16) into Eq. (2.17), the describing function, v_{kj} can be written as,

$$v_{kj_m} = \frac{i}{\pi \cdot Y_{kj_m}} \int_0^{2\pi} n_{kj}(y_{kj}, \dot{y}_{kj}, \ddot{y}_{kj}, \ldots) \cdot e^{-i \cdot m \cdot \psi} \cdot d\psi \quad \text{for } m = 1, 2, 3, \ldots$$

$$v_{kj_m} = \frac{i}{2\pi \cdot Y_{kj_m}} \int_0^{2\pi} n_{kj}(y_{kj}, \dot{y}_{kj}, \ddot{y}_{kj}, \ldots) \cdot d\psi \qquad \text{for } m = 0 \tag{2.18}$$

The internal nonlinear forces, n_{kj} can be represented in terms of describing functions as,

$$\{n_{kj}\} = v_{kj_0} \cdot Y_{kj_0} + \text{Im}\left[\sum_{m=1}^{\infty} v_{kj_m} \cdot Y_{kj_m} \cdot e^{i \cdot m \cdot \psi}\right]. \tag{2.19}$$

Eq. (2.7) can also be written as,

$$\{f_N(t)\} = \sum_{m=0}^{\infty} \{N_m\} \cdot e^{i \cdot m \cdot \psi}, \tag{2.20}$$

where, $\{N_m\}$ is the complex amplitude vector of internal forces for the mth harmonic. Combining Eqs. (2.19) and (2.20), $\{N_m\}$ can be written as,

$$\{N_m\}_k = \sum_{j=1}^{N} v_{kj_m} \cdot Y_{kj_m}. \tag{2.21}$$

Inserting the expressions used for the periodic excitation, periodic response and nonlinear forces in complex form into Eq. (2.6), the following equation set is obtained.

$$\left[-(m\cdot\omega)^2\cdot[M]+i\cdot m\cdot\omega\cdot[C]+i\cdot[H]+[K]\right]\cdot\{X\}_m+\{N\}_m=\{F\}_m, \quad (2.22)$$

where, $\{N_m\}$ can be rewritten using describing functions,

$$\{N\}_m=[\Delta]_m\{X\}_m, \quad (2.23)$$

where, $[\Delta]$ referred as "nonlinearity matrix", is a function of the unknown displacement amplitude vector, $\{X\}$. The elements of nonlinearity matrix are defined as

$$\begin{aligned}[\Delta]_{kj}&=\sum_{j=1}^{n}\nu_{kj} \quad \text{for } k=j\\ [\Delta]_{kj}&=-\nu_{kj} \quad \text{for } k\neq j\end{aligned} \quad (2.24)$$

Substituting Eq. (2.23) into Eq. (2.22), equation of motion can be obtained as

$$\left[-(m\cdot\omega)^2\cdot[M]+i\cdot m\cdot\omega\cdot[C]+i\cdot[H]+[K]+[\Delta]\right]\cdot\{X\}_m=\{F\}_m. \quad (2.25)$$

In this thesis, describing function method is employed for the determination of the nonlinear algebraic equations by considering only the first harmonic term. Therefore, Eq. (2.25) can be reformed as

$$\left[-\omega^2\cdot[M]+i\cdot\omega\cdot[C]+i\cdot[H]+[K]+[\Delta]\right]\cdot\{X\}=\{F\}. \quad (2.26)$$

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2.3.2   Types of Nonlinearities Considered

In this thesis, cubic stiffness and hysteretic dry friction are used as nonlinear elements in the substructures. In this section, quasi-linearization of nonlinearity types is shown.

2.3.2.1   Cubic Stiffness

Cubic stiffness is the most common nonlinearity type used in structural dynamics. Nonlinear force in the case of cubic stiffness can be written as

$$n(x) = k_c \cdot x^3, \tag{2.27}$$

where, k_c is the coefficient of cubic stiffness nonlinearity. k_c can be either positive or negative. If $k_c > 0$, cubic stiffness shows a hardening behavior, in other words, level of excitation increases the restoring force introduced is greater than a linear spring. On the other hand, if $k_c < 0$, cubic stiffness shows a softening behavior, in other words, level of excitation increases the restoring force introduced is lower than a linear spring. Assuming for a single harmonic input,

$$x = X \sin(\psi), \tag{2.28}$$

where X is the amplitude of the harmonic input x and ψ is the replacement term of the ωt . According to Eq. (2.18), describing function of this nonlinear force can be written as

$$v = \frac{i}{\pi \cdot X} \int_0^{2\pi} k_c [X \sin(\psi)]^3 \cdot e^{-i \cdot \psi} \cdot d\psi .$$
(2.29)

Describing function of cubic stiffness can be written in a simple form as

$$v = \frac{3}{4} \cdot k_c \cdot X^2 .$$
(2.30)
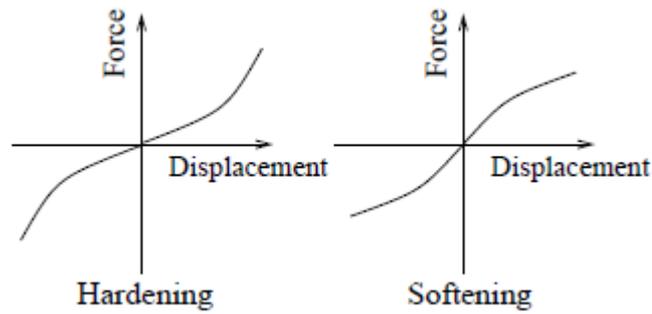
Figure 2.2 Characteristic of hardening and softening cubic stiffness elements [27]

Because of the characteristic of cubic stiffness nonlinearity, system response is bending around resonant frequency towards forward for hardening systems and towards backward for softening systems.

2.3.2.2 Hysteretic Dry Friction

There exists several friction models in the literature and in this thesis; a one-dimensional Coulomb friction model with constant normal load is used. One-dimensional dry friction element and the corresponding hysteresis curve for a single harmonic input are given in Figure 2.3 [28].
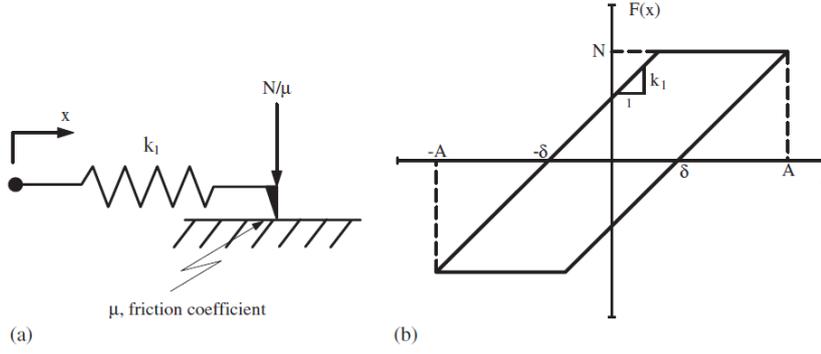
Figure 2.3 (a) Schematic drawing, (b) corresponding hysteresis curve for dry friction nonlinearity [28]

Nonlinear force in the case of hysteretic dry friction nonlinearity can be written as

$$n(x) = -\mu N + k \cdot (x + \delta) \quad \text{for} \quad \frac{\pi}{2} \leq \psi \leq \psi_1$$
$$n(x) = -\mu N \qquad\qquad \text{for} \quad \psi_1 \leq \psi \leq \frac{3\pi}{2} \tag{2.31}$$

where, k is the contact stiffness between rubbing surfaces, N is the constant normal force, μ is the dry friction coefficient and ψ_1

$$\psi_1 = \pi - a\sin\left(\frac{k \cdot X - 2 \cdot \mu \cdot N}{k \cdot X}\right). \tag{2.32}$$

Describing function of this nonlinear force can be written as

$$v = \frac{2 \cdot i}{\pi \cdot X} \cdot \int_{\pi/2}^{\psi_1} [-\mu N + k \cdot (x + \delta)] \cdot e^{-i \cdot \psi} \cdot d\psi \int_{\psi_1}^{3\pi/2} -\mu N \cdot e^{-i \cdot \psi} \cdot d\psi . \tag{2.33}$$

Describing function of hysteretic dry friction can be written in a simple form as

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$$
v_{re} = \begin{cases} \left[\frac{1}{\pi}\left(k - \frac{2 \cdot \mu \cdot N}{X} \right)\sqrt{1 - \left(\frac{k \cdot X - 2 \cdot \mu \cdot N}{k \cdot X} \right)^2} + \frac{k \cdot \psi_1}{\pi} - \frac{k}{2} \right] & \text{for } |k \cdot X| > \mu N \\ k & \text{for } |k \cdot X| \le \mu N \end{cases}
\tag{2.34}
$$

$$
v_{im} = \begin{cases} -\left[\frac{4 \cdot \mu \cdot N \left(\mu \cdot N - k \cdot X \right)}{\pi \cdot k \cdot X^2} \right] & \text{for } |k \cdot X| > \mu N \\ 0 & \text{for } |k \cdot X| \le \mu N \end{cases}
\tag{2.35}
$$

2.4   Solution of Nonlinear Algebraic Equations

General numerical methods, used in solving nonlinear algebraic system of equations, are introduced and discussed in this section. Solution of the nonlinear equation set may give more than one result for a single frequency where jump phenomena occurs [29] so, using a path following method should be used to investigate all possible solutions. In this thesis, for this purpose, Newton's method with arc-length continuation is used.

Nonlinear algebraic equation set which is to be solved can be expresses as follows

$$
\{ R((x), \omega) \} = \{ 0 \} .
\tag{2.36}
$$

Further, expanding the nonlinear residual vector, $\{ R((x), \omega) \}$, for Eq. (2.26), following equation may be written as

$$
([D(\omega)] + [\Delta]) \cdot \{ X \} - \{ F \} = 0 .
\tag{2.37}
$$

2.4.1   Newton's Method

Newton's method is one of the popular root-finding numerical solution techniques based on the first order Taylor series expansion. Using Newton's method, solution of a set of nonlinear algebraic equations can be obtained iteratively as follows [30, 31, 32]

$$\{x\}_{new} = \{x\}_{old} + \{\Delta x\},$$

(2.38)

$$\{x\}^{i+1} = \{x\}^i - \left[J((x)^i, \omega)\right]^{-1} \cdot \{R((x)^i, \omega)\},$$

(2.39)

where i is the iteration number, $\{x\}^i$ is the solution vector at the ith iteration and $\left[J(x, \omega)\right]$ is the Jacobian matrix which can be written as

$$\left[J(x, \omega)\right] = \frac{\partial \{R(x, \omega)\}}{\partial \{x\}}.$$

(2.40)

2.4.2 Arc-Length Continuation Method

Because of the nature of the some nonlinear systems, frequency response may curve turns back. So, whole solution may not be obtained properly with Newton's method due to increasing the frequency results in a jump up or down. Also, path following using Newton's method may encounter two main problems. First, Jacobian of the residual vector is close to singular at the turning points, second a good initial guess assumption is required around turning points. In the arc-length continuation method, a new parameter, s, is added to the nonlinear equation set which makes the Jacobian matrix non-singular at the turning points. Moreover, the arc-length parameter is the path following parameter instead of frequency. Arc-length parameter s is defined as the radius of a hypothetical sphere on which the next solution point is to be obtained [30, 31].

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For this new system, frequency, ω , is an unknown in the nonlinear equation set with the addition of the arc-length parameter. So, the vector of unknowns may be written as

$$\{q\} = \begin{Bmatrix} \{x\} \\ \omega \end{Bmatrix}. \tag{2.41}$$

The required additional equation is the equation of the hypothetical sphere which has a radius s and centered at the previous solution point.

$$(\{x\}^i - \{x\}^{i-1})^2 + (\omega^i - \omega^{i-1})^2 = s^2, \tag{2.42}$$

where $\{x\}^i$ is the response of the nonlinear system at the ith frequency point, ω_i. So, the iterative formula of the Newton's method can be written as [32, 33]

$$\{q\}^{i+1} = \{q\}^i - \left[\bar{J}\left(\{x\}^i, \omega^i\right) \right]^{-1} \cdot \left\{ \bar{R}\left(\{x\}^i, \omega^i\right) \right\}, \tag{2.43}$$

where $\bar{R}(\{x\}^i, \omega^i)$ is the new nonlinear algebraic equation set and $\bar{J}(\{x\}^i, \omega^i)$ is the new Jacobian matrix

$$\left[\bar{J}\left(\{x\}^i, \omega^i\right) \right] = \begin{bmatrix} \left[J\left(\{x\}^i, \omega^i\right) \right] & \dfrac{\partial R\left(\{x\}^i, \omega^i\right)}{\partial \omega} \\ \dfrac{\partial h\left(\{x\}^i, \omega^i\right)}{\partial \{x\}} & \dfrac{\partial h\left(\{x\}^i, \omega^i\right)}{\partial \omega} \end{bmatrix}, \tag{2.44}$$

$$h\left(\{x\}^i, \omega^i\right) = \left(\{\Delta q\}^i\right)^T \{\Delta q\}^i - s^2 = 0, \tag{2.45}$$

$$\{\Delta q\}^i = \begin{Bmatrix} \{\Delta x\}^i \\ \Delta \omega^i \end{Bmatrix} = \begin{Bmatrix} \{\Delta x\}^i - \{\Delta x\}^{i-1} \\ \Delta \omega^i - \Delta \omega^{i-1} \end{Bmatrix}, \tag{2.46}$$

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$$\left\{ \bar{R}\left(\{x\}^{i},\omega^{i}\right)\right\} = \left\{ \begin{array}{c} \left\{ R\left(\{x\}^{i},\omega^{i}\right)\right\} \\ h\left(\{x\}^{i},\omega^{i}\right) \end{array}\right\}. \tag{2.47}$$

Using tangent predictor to determine the initial guess of the next step should give better results such that it increases the rate of convergence and decreases the computational time. Initial guess for $\{x\}$ at the next solution point can be written according to tangent predictor as [31, 34].

$$\{x\}_0^i = \{x\}_0^{i-1} - \left[J\left(\{x\}^{i-1},\omega^{i-1}\right)\right]^{-1} \cdot \left(\frac{\partial R\left(\{x\}^{i-1},\omega^{i-1}\right)}{\partial \omega^{i-1}}\right), \tag{2.48}$$

where $\{x\}_0^i$ is the improved initial guess for the next i^{th} iteration, $\{x\}_0^{i-1}$ is the solution at the previous solution point and

$$\left[J\left(\{x\}^{i-1},\omega^{i-1}\right)\right] = \frac{\partial R\left(\{x\}^{i-1},\omega^{i-1}\right)}{\partial \{x\}^{i-1}}, \tag{2.49}$$

Initial guess for frequency, ω, can be written as [35]

$$\omega_0^i = \omega_0^{i-1} \pm \frac{s}{\sqrt{\left(\left[J\left(\{x\}^{i-1},\omega^{i-1}\right)\right]^{-1} \cdot \frac{\partial R\left(\{x\}^{i-1},\omega^{i-1}\right)}{\partial \omega^{i-1}}\right)^2 + 1}} \tag{2.50}$$

There are two solutions available for ω_0^i. Therefore, correct sign is needed to be chosen in order to follow the path. Choosing correct sign according to the sign of determinant of Jacobian matrix works quite well for most of the cases [35].

CHAPTER 3

A NEW NONLINEAR STRUCTURAL COUPLING METHOD

3.1   Introduction

In this chapter, a new nonlinear structural coupling method is developed to dynamic reanalysis of nonlinear substructures. Different from the existing methods in literature, proposed method is capable of coupling of two nonlinear substructures. Moreover, with the proposed coupling method, in addition to linear coupling elements, nonlinear coupling elements can as well be used. The proposed method considers the compatibility of internal forces at the connection degrees of freedom in addition to displacements. Since, the substructures are nonlinear, the resulting system of nonlinear differential equations are converted into a set of nonlinear algebraic equations by using describing function method, which are solved by using Newton's method with arc-length continuation.

3.2   Structural Coupling of Nonlinear Substructures

The equation of motion of the nonlinear substructures A and B excited with a harmonic external forcing $\{f(t)\}$, can be written as

$$
\begin{aligned}
[M_A]\{\ddot{x}_A(t)\}+[C_A]\{\dot{x}_A(t)\}+i[H_A]\{x_A(t)\}+[K_A]\{x_A(t)\}+\{f_{N_A}(t)\}=\{f_A(t)\} \\
[M_B]\{\ddot{x}_B(t)\}+[C_B]\{\dot{x}_B(t)\}+i[H_B]\{x_B(t)\}+[K_B]\{x_B(t)\}+\{f_{N_B}(t)\}=\{f_B(t)\}
\end{aligned}
, (3.1)
$$

where $[M]$, $[C]$, $[H]$ and $[K]$ are the mass, viscous damping, structural damping and stiffness matrices of the linear system and $\{f_N(t)\}$ is the nonlinear forcing vector. Here subscripts A and B indicate the coupled substructures. Generalized displacement vectors $\{x_A(t)\}$ and $\{x_B(t)\}$ can be written as

$$\{x_A(t)\} = \begin{Bmatrix} \{X_{i_A}\} \\ \{X_{c_A}\} \end{Bmatrix}, \quad \{x_B(t)\} = \begin{Bmatrix} \{X_{i_B}\} \\ \{X_{c_B}\} \end{Bmatrix}, \tag{3.2}$$

where $\{X_{i_A}\}$ and $\{X_{i_B}\}$ are generalized displacement vectors acting on internal DOFs and $\{X_{c_A}\}$ and $\{X_{c_B}\}$ are generalized displacement vectors acting on the coupled DOFs. External forcing vectors $\{f_A(t)\}$ and $\{f_B(t)\}$ can be written as

$$\{f_A(t)\} = \begin{Bmatrix} \{F_{i_A}\} \\ \{F_{c_A}\} + \{f_{c_A}\} \end{Bmatrix}, \quad \{f_B(t)\} = \begin{Bmatrix} \{F_{i_B}\} \\ \{F_{c_B}\} + \{f_{c_B}\} \end{Bmatrix}, \tag{3.3}$$

where $\{F_{i_A}\}$ and $\{F_{i_B}\}$ are external forcing vectors acting on internal DOFs and $\{F_{c_A}\}$ and $\{F_{c_B}\}$ are external force vectors acting on the coupled DOFs. If the external forcing, $\{f(t)\}$ is periodic, response of the system, $\{x(t)\}$, can as well be assumed periodic, which can be expressed as follows

$$\{f(t)\} = \{F\}_0 + \mathrm{Im}\left[\sum_{m=1}^{\infty} \{F\}_m \cdot e^{im\psi} \right], \tag{3.4}$$

$$\{x(t)\} = \{X\}_0 + \mathrm{Im}\left[\sum_{m=1}^{\infty} \{X\}_m \cdot e^{im\psi} \right]. \tag{3.5}$$

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Utilizing Describing Function Method (DFM) and substituting Eqs. (3.2) and (3.3) into Eq. (3.1) as the following result is obtained

$$
\begin{aligned}
\left(-\omega^2[M_A]+i\cdot\omega[C_A]+i[H_A]+[K_A]+[\Delta_A]\right)\begin{Bmatrix}\{X_{i_A}\}\\\{X_{c_A}\}\end{Bmatrix} &= \begin{Bmatrix}\{F_{i_A}\}\\\{F_{c_A}\}+\{f_{c_A}\}\end{Bmatrix}\\
\left(-\omega^2[M_B]+i\cdot\omega[C_B]+i[H_B]+[K_B]+[\Delta_B]\right)\begin{Bmatrix}\{X_{i_B}\}\\\{X_{c_B}\}\end{Bmatrix} &= \begin{Bmatrix}\{F_{i_B}\}\\\{F_{c_B}\}+\{f_{c_B}\}\end{Bmatrix}
\end{aligned}
\tag{3.6}
$$

where internal forcing vector $\{f_{c_A}\}$ can be written as

$$
\{f_{c_A}\}=\left[D_{coupling}^{NL}\right]\left(\{x_{c_A}\}-\{x_{c_B}\}\right) ,
\tag{3.7}
$$

where, $\left[D_{coupling}^{NL}\right]$ is the nonlinear dynamic stiffness matrix of the connection elements;

$$
\left[D_{coupling}^{NL}\right]=\left(-\omega^2[M_c]+i\cdot\omega[C_c]+i[H_c]+[K_c]+[\Delta_c]\right).
\tag{3.8}
$$

Substituting Eqs. (2.3) and (3.7), into Eq. (3.6), equation of motion can be obtained as

$$
\begin{aligned}
\left(-\omega^2[M_A]+i\cdot\omega[C_A]+i[H_A]+[K_A]+[\Delta_A]\right)\begin{Bmatrix}\{X_{i_A}\}\\\{X_{c_A}\}\end{Bmatrix} &= \begin{Bmatrix}\{F_{i_A}\}\\\{F_{c_A}\}+\left[D_{coupling}^{NL}\right]\left(\{X_{c_A}\}-\{X_{c_B}\}\right)\end{Bmatrix}\\
\left(-\omega^2[M_B]+i\cdot\omega[C_B]+i[H_B]+[K_B]+[\Delta_B]\right)\begin{Bmatrix}\{X_{i_B}\}\\\{X_{c_B}\}\end{Bmatrix} &= \begin{Bmatrix}\{F_{i_B}\}\\\{F_{c_B}\}+\{f_c\}-\left[D_{coupling}^{NL}\right]\left(\{X_{c_A}\}-\{X_{c_B}\}\right)\end{Bmatrix}
\end{aligned}
\tag{3.9}
$$

where,

$$
\{f_c\}=\{f_{c_A}\}+\{f_{c_B}\}.
\tag{3.10}
$$

Eq. (3.9) can be solved by a nonlinear equation solver and in this thesis; Newton's method with arc-length continuation, which is explained in detail at section 2.4, is used. Algorithm of the proposed nonlinear coupling method is given in Figure 3.1.
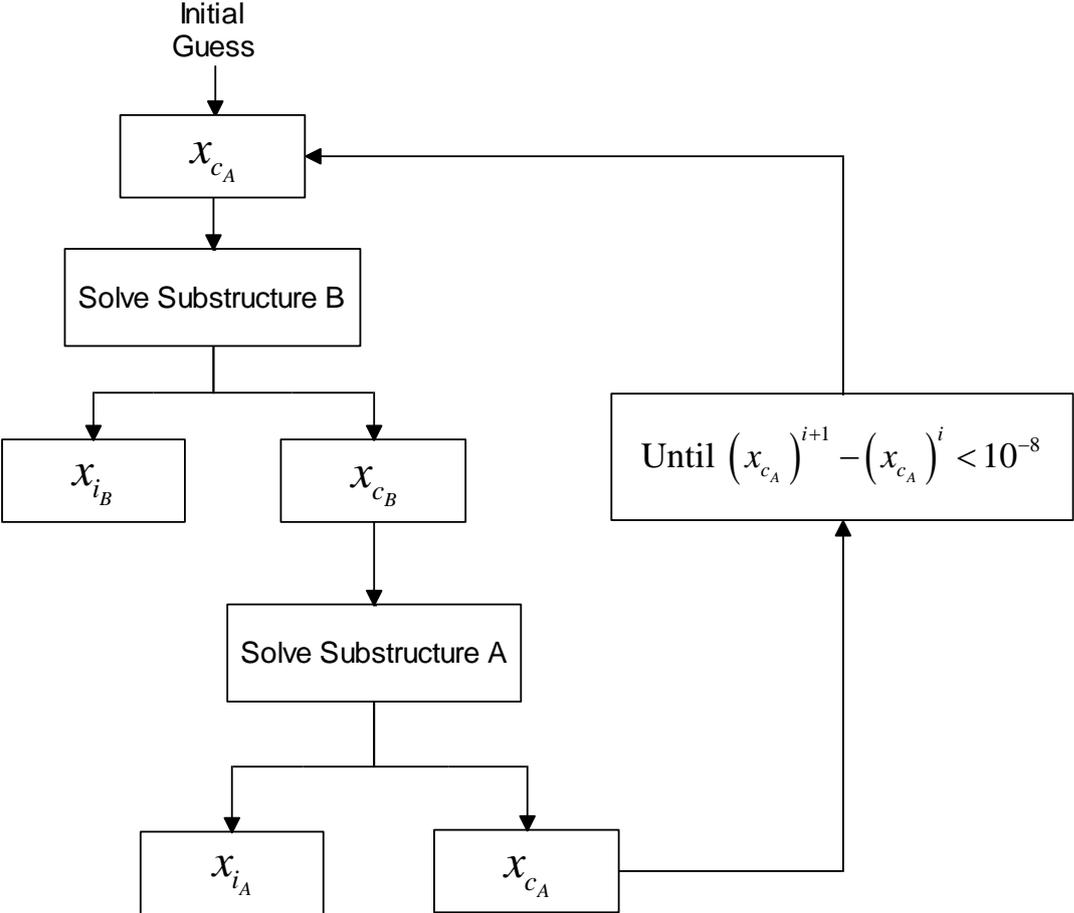
Figure 3.1 Nonlinear Structural Coupling Algorithm

3.3 Nonlinear Structural Coupling Using Artificial Neural Networks

Considering learning abilities of human brain; it is more complex compared with computers. Complexity of human brain is simulated with artificial neural networks which can be used with multi-input and multi-output systems for various applications. Neural networks can simulate very complex, highly nonlinear systems

consisting of artificial neurons that are interconnected. Fundamental element of a neural network is called as neuron. Mathematical model of a simple neuron is

$$o = f\left(\sum_{i=1}^{N_R}[w_i \cdot y_i + b]\right) = f\left(\{w\}^T\{y\} + b\right) = f(n),\qquad(3.11)$$

where y is the input vector, $\{w\}^T$ is the weight vector, b is the bias term, o is the output of the neuron, n is the net input, N_R is the number of elements in the input vector and f is the transfer function used.
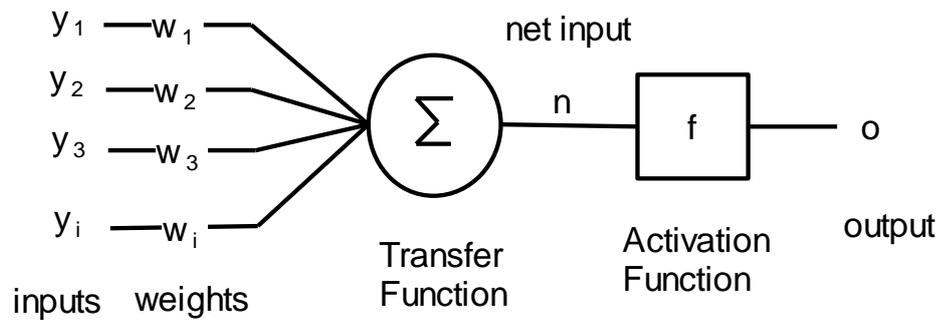
Figure 3.2 Simple neuron model

Neural networks consist of an input layer, an output layer and hidden layers between these two. These layers are made of neurons. Additional layers and multi neurons can be employed in neural network structures to increase capability of the network. In this thesis, a neural network, which has two layers and twenty neurons, is used.
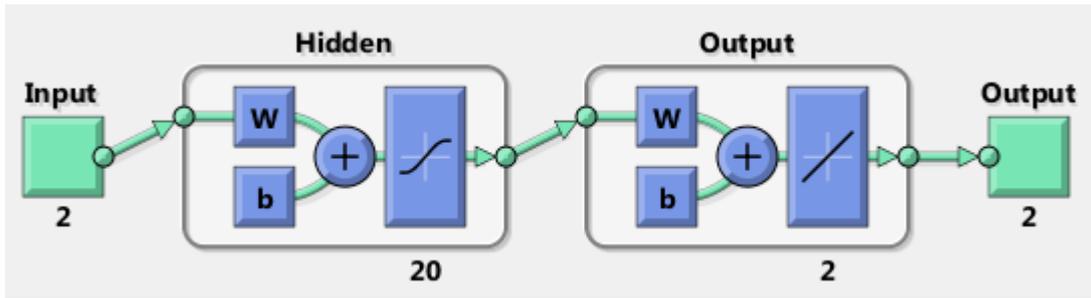
Figure 3.3 Classification of neural network [36]

Additional layer can be employed in neural network structures to increase capability of the network. It can be observed that output of each layer becomes the input of the next layer.

Minimizing the error between target and output vectors is the main reason of neural network training. Performance of a network is quantified through mean squared error (MSE) between the network output vector, $\{o\}$, and target vector, $\{t\}$ as:

$$MSE = \frac{1}{N}\int_{i=1}^{N} e_i^2 = \frac{1}{N}\int_{i=1}^{N}\left(o_i - t_i\right)^2 \tag{3.12}$$

Minimizing MSE via tuning the elements of weight matrices and bias vectors is the main idea behind neural network theory. The tuning process is called as training, where weight matrices and bias vectors are updated according to "the training algorithm".

In this study, MATLAB Neural Network Toolbox is used for operating the proposed method. One of the substructures is trained via Neural Network Toolbox and so, a transfer function, which transforms internal forcing and frequency into displacements, is created. Schematic view of such operation is shown in Figure 3.4.
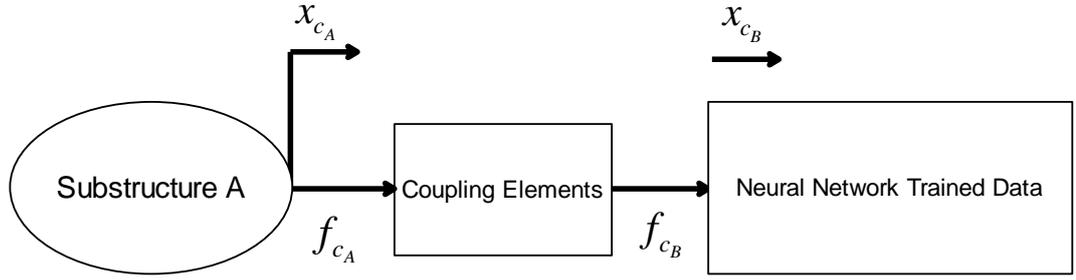
Figure 3.4 Schematic view of structural coupling via neural networks

Then, Eq. (3.9) can be written as:

$$\left(-\omega^2\left[M_A\right]+i\cdot\omega\left[C_A\right]+i\left[H_A\right]+\left[K_A\right]+\left[\Delta_A\right]\right)\begin{Bmatrix}\{X_{i_A}\}\\\{X_{c_A}\}\end{Bmatrix}=\begin{Bmatrix}\{F_{i_A}\}\\\{F_{c_A}\}+\left[D_{\text{coupling}}^{NL}\right]\left(\{X_{c_A}\}-\{X_{c_B}\}\right)\end{Bmatrix}$$

$$\{X_{c_B}\}=\{NN(\omega,f_{c_B})\}$$

,

(3.13)

where $\{NN\}$ is the neural network transfer function which is created by training substructure B via Neural Network Toolbox. Before starting to the training process, collected data is required to be divided into three subsets as: training, validation and test data sets. Weight matrices and bias vectors are updated based on the training data set. All the data subsets should represent the entire data set. Otherwise there might be large discrepancies between targets and outputs. Optimal division of training data set is one of the main concerns in neural network training. In this study, via several trials, 70% of samples are allocated as training data set, 15% of samples are allocated as the validation data set and the remaining 15% is used as the test data set.

Training data set is created from frequency, ω, and internal forcing vector, $\{f\}$. However, there might be more than one response near a resonance frequency because of the nonlinearity present in the system, so, in these frequency points, neural

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network does not expected to operate with only frequency and internal forcing vector inputs. In this study, to overcome this problem, a dummy variable is introduced to the system as a third input, so that, it is started with zero and increased if sign of $\Delta \omega$ is changed. Classification of such neural network is shown in Figure 3.5.
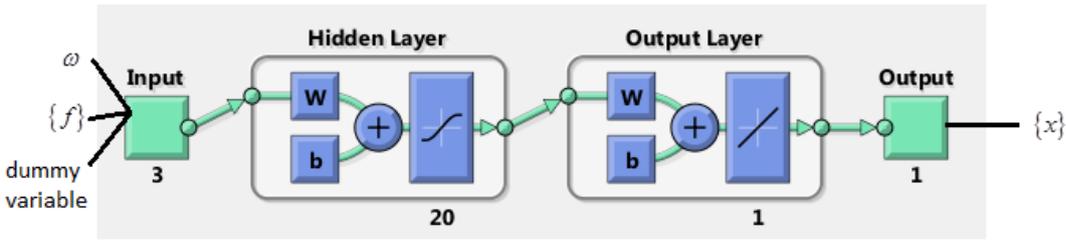
Figure 3.5 Classification of neural network with dummy variable [36]

After a proper training is done, performance of the neural network should be checked via MSE graph and error histograms. If performance of neural network is sufficient, Eq. (3.12) can be solved by a nonlinear equation solver and in this thesis; Newton's method, which is explained in detail at section 2.4, is used.

Initial
Guess

f_{c_A}

Equilibrium
Condition

f_{c_B}

Until $\left(\left\{ X_{c_A} \right\} - \left\{ X_{c_A}^{NN} \right\} \right) < 10^{-8}$

Solve Substructure A

Neural Network
Solution

X_{c_B}

X_{i_A}

X_{c_A}

Compatibility
Condition

X_{c_A}
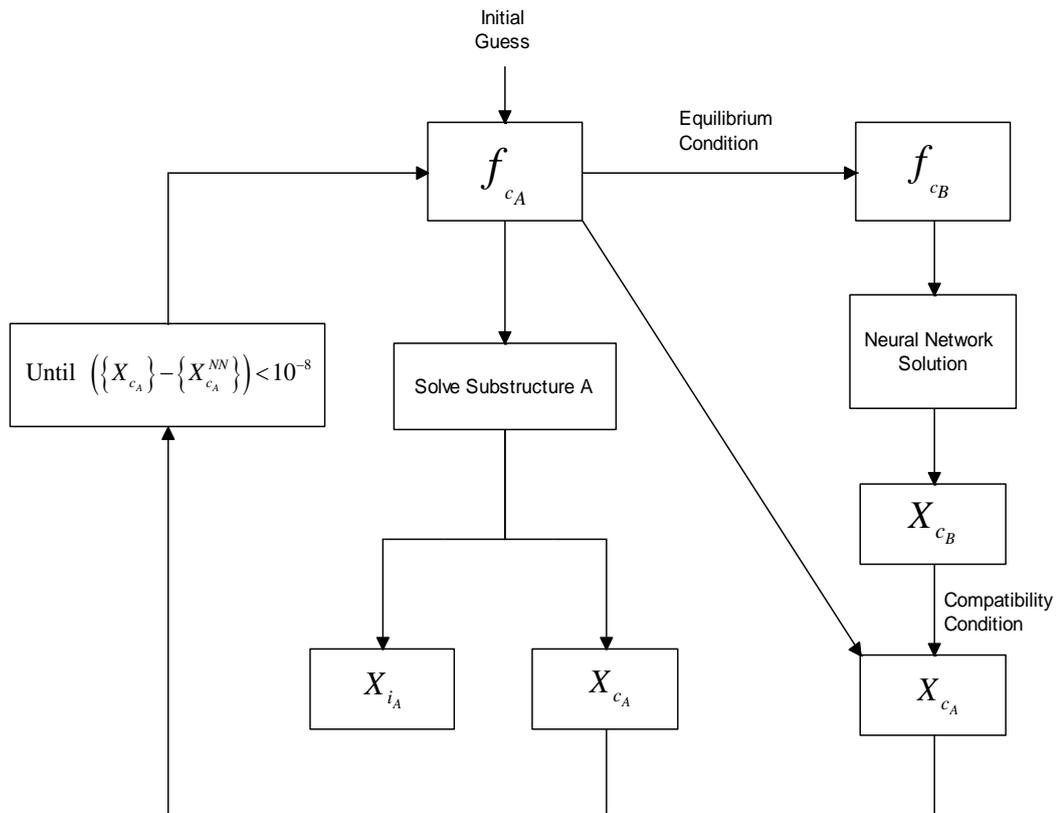
Figure 3.6 Nonlinear Structural Coupling Algorithm using Artificial Neural
Networks

CHAPTER 4

**CASE STUDIES OF STRUCTURAL COUPLING METHOD TO
NONLINEAR SYSTEMS**

In this chapter, the proposed coupling method is demonstrated on different models. In the first one, substructures are coupled from a single DOF whereas in the second one, they are coupled from multiple DOFs. In both models, two main works are done such that, proposed coupling method is used for coupling of one identified and one neural network trained substructures, and two identified substructures. The results obtained by proposed method will be compared with those obtained via solving the coupled system entirely.

4.1 Nonlinear Structural Coupling from a Single DOF

4.1.1 Nonlinear Structural Coupling with Two Identified Substructures

In the first example, application of the proposed approach is presented on a simple 8-DOF system shown in Figure 4.1. Parameters of substructures *A* and *B* are given Table 4.1 and coupling elements at Table 4.2. The nonlinear elements used in the first case study, case study 1, are defined in Table 4.3.
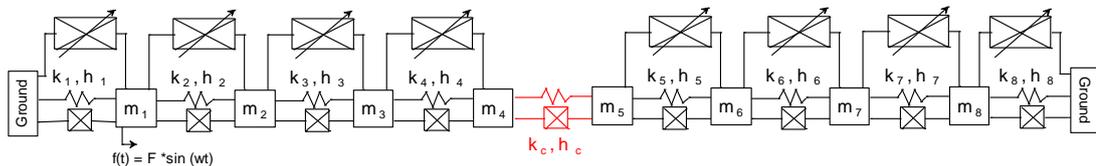
Figure 4.1 Schematic view of 8-DOF coupled system

Table 4.1 Parameters of substructures A and B

Substructure A			
m_1 [kg]	m_2 [kg]	m_3 [kg]	m_4 [kg]
1	0,75	2	1
k_1 [N/m]	k_2 [N/m]	k_3 [N/m]	k_4 [N/m]
5000	2000	4000	6000
h_1 [N/m]	h_2 [N/m]	h_3 [N/m]	h_4 [N/m]
50	20	40	60

Substructure B			
m_5 [kg]	m_6 [kg]	m_7 [kg]	m_8 [kg]
0,75	1	1	2
k_5 [N/m]	k_6 [N/m]	k_7 [N/m]	k_8 [N/m]
3000	2000	5000	3000
h_5 [N/m]	h_6 [N/m]	h_7 [N/m]	h_8 [N/m]
30	20	50	30

Table 4.2 Parameters of coupling elements

k_c [N/m]	h_c [N/m]
4000	40

Table 4.3 Nonlinear elements of substructures A and B in case study 1

Nonlinear Connection DOFs	Nonlinearity Type	Nonlinearity Coefficients
1-Ground	Cubic Stiffness [N/m^3]	-1x10^5
1-2	Cubic Stiffness [N/m^3]	-2x10^5
2-3	Cubic Stiffness [N/m^3]	-10x10^5
3-4	Cubic Stiffness [N/m^3]	-1x10^5
5-6	Cubic Stiffness [N/m^3]	-1x10^5
6-7	Cubic Stiffness [N/m^3]	-3x10^5
7-8	Cubic Stiffness [N/m^3]	-1x10^5
8-Ground	Cubic Stiffness [N/m^3]	-5x10^5

Normalized responses of the 1st, 3rd, and the 8th DOFs obtained from the proposed nonlinear coupling method and by solving the entire system directly are given in Figure 4.2, Figure 4.3, and Figure 4.4. The response of the system is obtained for three different excitation amplitudes, 8N, 12N and 16N in order to observe the effect of cubic stiffness nonlinearity.
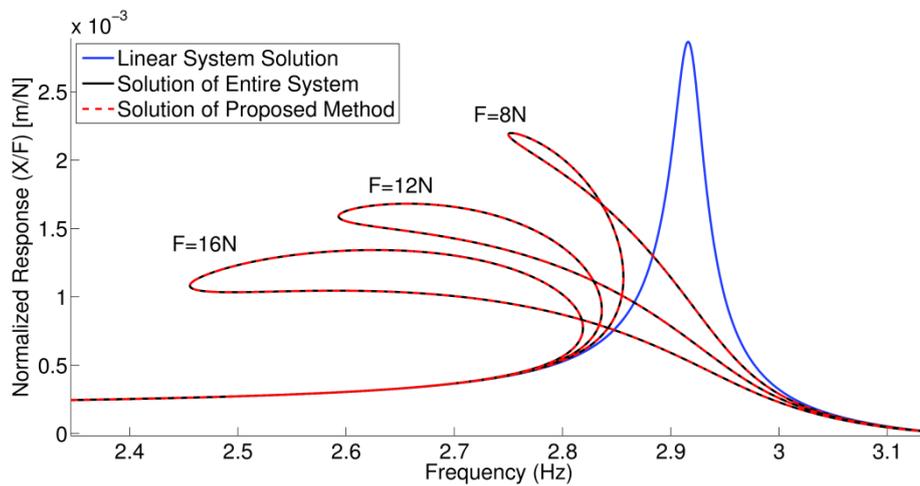
Figure 4.2 Normalized response of the 1st DOF in case study 1
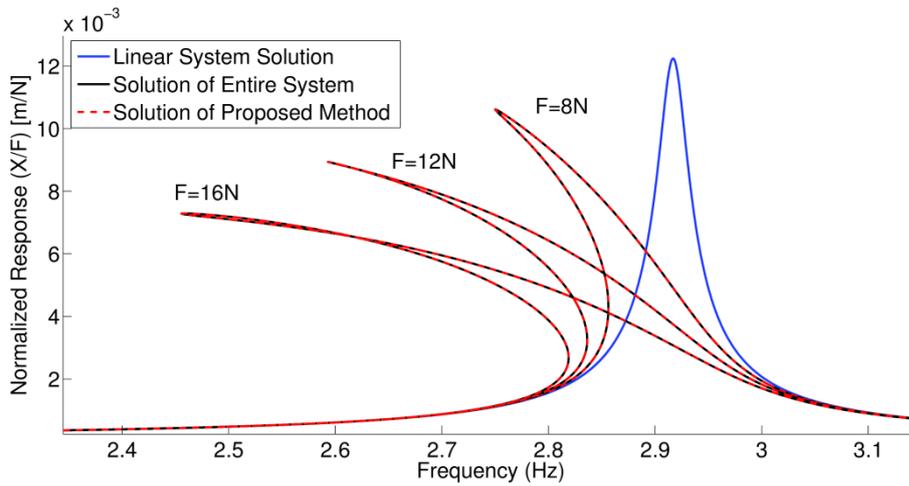
Figure 4.3 Normalized response of the 3rd DOF in case study 1
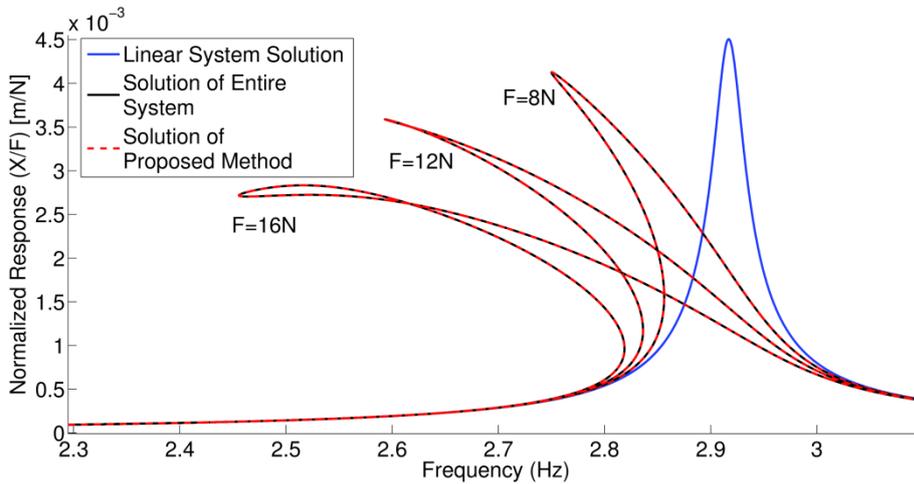
Figure 4.4 Normalized response of the 8th DOF in case study 1

It can be seen from the Figure 4.2, Figure 4.3, and Figure 4.4 that, natural frequency is shifted due to cubic stiffness nonlinearity. Furthermore, more importantly the proposed method is in exact agreement with the ones obtained from entire system solution, even in unstable regions where the path turns back or intersects itself.

Phase angle of imaginary and reel part of the 1th DOF in case study 1 is shown in Figure 4.5.

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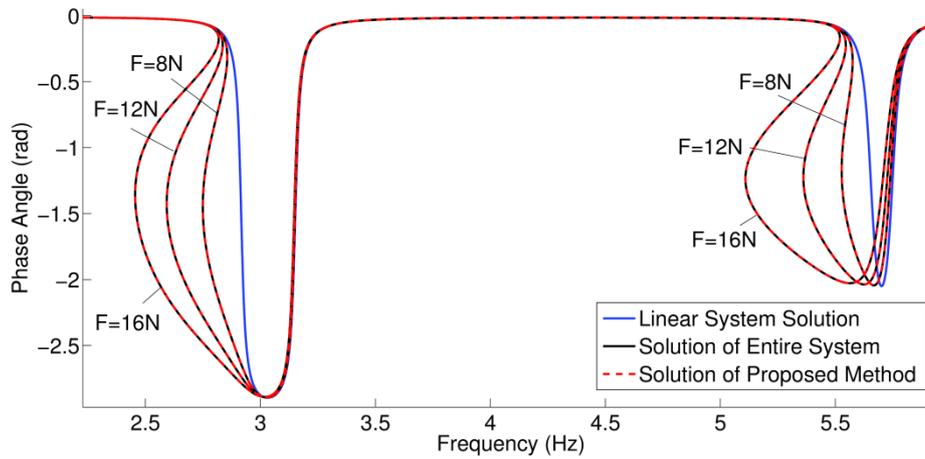
Figure 4.5 Phase angle of the 1th DOF in case study 1

In the second case study, case study 2, 4-DOF system is obtained from the coupling of a two 2-DOF systems as shown in Figure 4.6. Parameters of substructures A and B, and coupling elements are given in Table 4.4 and Table 4.5, respectively. The nonlinear elements present in the system are defined in Table 4.6.
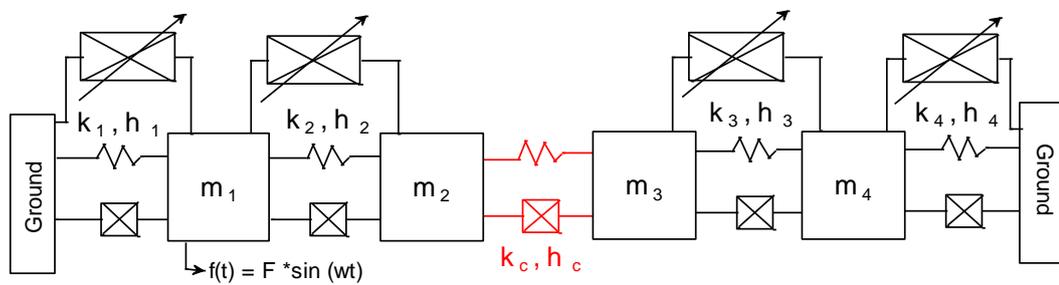
Figure 4.6 Schematic view of 4-DOF coupled system

Table 4.4 Parameters of substructures A and B

Substructure A		Substructure B	
m_1 [kg]	m_2 [kg]	m_3 [kg]	m_4 [kg]
1	0,75	0,75	1
k_1 [N/m]	k_2 [N/m]	k_3 [N/m]	k_4 [N/m]
5000	4000	3000	5000
h_1 [N/m]	h_2 [N/m]	h_3 [N/m]	h_4 [N/m]
50	40	30	50

Table 4.5 Parameters of coupling elements

k_c [N/m]	h_c [N/m]
5000	50

Table 4.6 Nonlinear elements of substructures A and B in case study 2

Nonlinear Connection DOFs	Nonlinearity Type	Nonlinearity Coefficients
1-Ground	Dry Friction [N]	μN
1-2	Dry Friction [N]	μN
3-4	Dry Friction [N]	μN
4-Ground	Dry Friction [N]	μN

Normalized responses of the 1st and 3rd DOFs obtained from the proposed nonlinear coupling method and by solving the entire system directly are given in Figure 4.7, and Figure 4.8. Responses of the coupled structure are given for an external forcing of $F = 30$N and for different slip loads. Perfect agreement between the results obtained from the proposed nonlinear coupling method and the entire system solution is observed which verifies the developed coupling method.
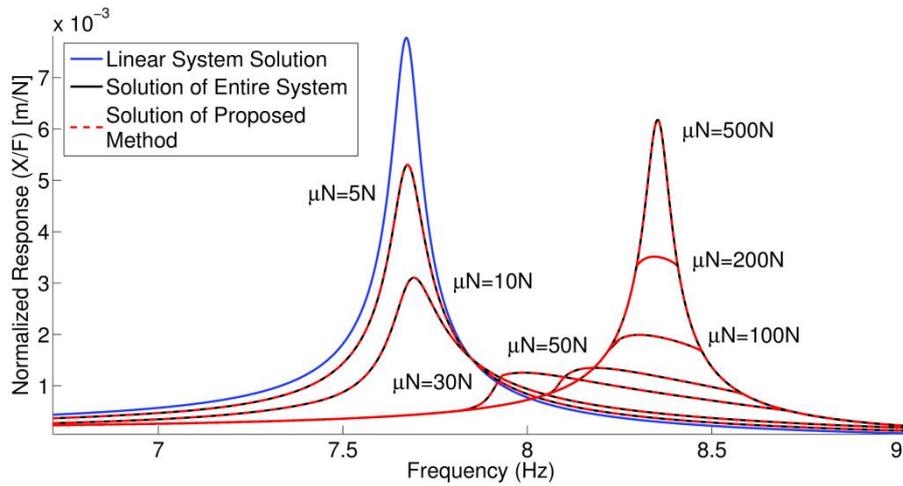
Figure 4.7 Normalized response of the 1st DOF in case study 2
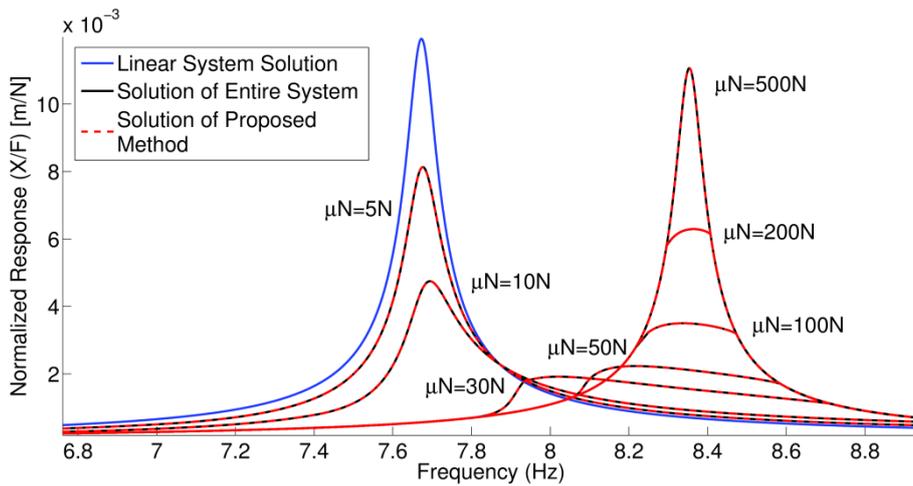
Figure 4.8 Normalized response of the 3rd DOF in case study 2

In the third case study, case study 3, 8-DOF system is obtained from the coupling of a 6-DOF, 2-DOF systems as shown in Figure 4.9. Parameters of substructures A and B, and coupling elements are given in Table 4.7 and Table 4.8, respectively. The nonlinear elements present in the system are defined in Table 4.9.
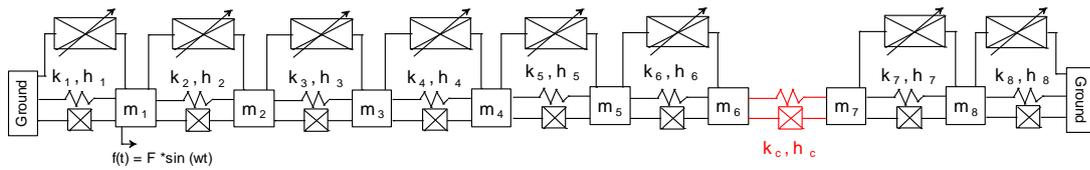
Figure 4.9 Schematic view of 8-DOF coupled sytem

Table 4.7 Parameters of substructures *A* and *B*

Substructure *A*					
m_1 [kg]	m_2 [kg]	m_3 [kg]	m_4 [kg]	m_5 [kg]	m_6 [kg]
0,75	2	1	1	2	0,75
k_1 [N/m]	k_2 [N/m]	k_3 [N/m]	k_4 [N/m]	k_5 [N/m]	k_6 [N/m]
3000	5000	4000	6000	6000	5000
h_1 [N/m]	h_2 [N/m]	h_3 [N/m]	h_4 [N/m]	h_5 [N/m]	h_6 [N/m]
60	100	80	120	120	100

Substructure *B*	
m_7 [kg]	m_8 [kg]
2	2
k_7 [N/m]	k_8 [N/m]
3000	5000
h_7 [N/m]	h_8 [N/m]
60	100

Table 4.8 Parameters of coupling elements

k_c [N/m]	h_c [N/m]
2000	40

Table 4.9 Nonlinear elements of substructures A and B in case study 3

Nonlinear Connection DOFs	Nonlinearity Type	Nonlinearity Coefficients
1-Ground	Cubic Stiffness [N/m^3]	1×10^5
1-2	Cubic Stiffness [N/m^3]	2×10^5
2-3	Cubic Stiffness [N/m^3]	1×10^5
3-4	Dry Friction [N]	$\mu N,\ k = 2000$ N/m
5-6	Cubic Stiffness [N/m^3]	1×10^5
6-7	Cubic Stiffness [N/m^3]	3×10^5
7-8	Cubic Stiffness [N/m^3]	1×10^5
8-Ground	Dry Friction [N]	$\mu N,\ k = 2000$ N/m

Corresponding response plots of the coupled structure obtained from the proposed nonlinear coupling method and by solving the entire system directly are compared in Figure 4.10 and Figure 4.11 for the 1st DOF. In Figure 4.10, the response of the coupled system is obtained for 12N, 24N and 36N excitation force amplitudes, while the slip load of dry friction nonlinearities are kept constant as $\mu N = 100$N. In Figure 4.11, responses of the coupled structure are given for an external forcing of $F = 12$N and for different slip loads. Perfect agreement between the results obtained from the proposed nonlinear coupling method and the entire system solution is observed which verifies the developed coupling method.
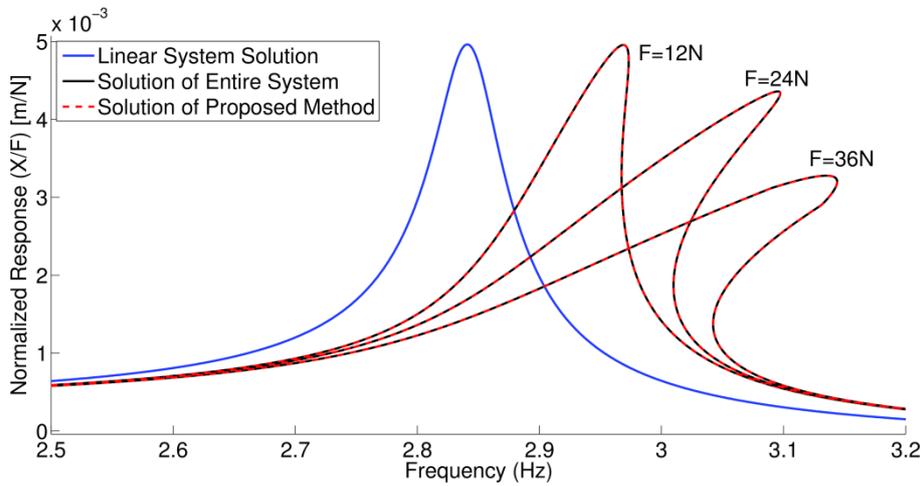
Figure 4.10 Normalized response of the 1ˢᵗ DOF in case study 3
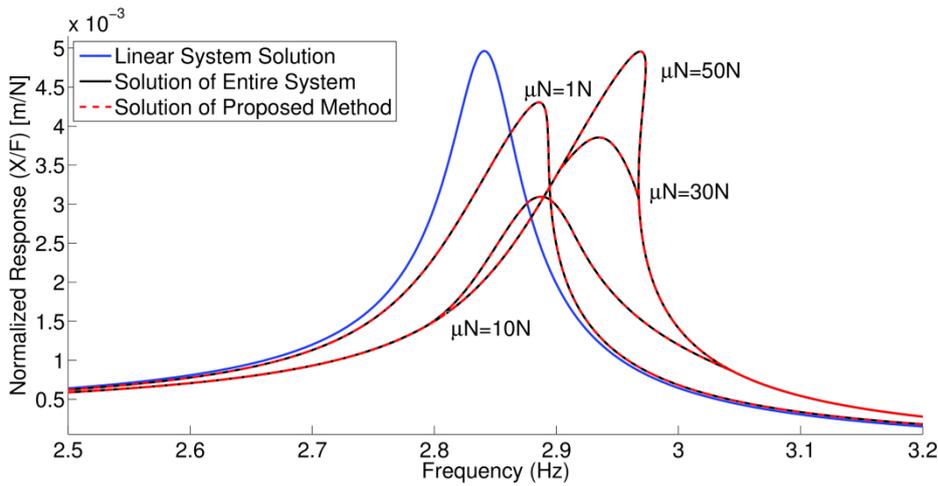
Figure 4.11 Normalized response of the 1ˢᵗ DOF in case study 3

4.1.2 Nonlinear Structural Coupling using Neural Networks

In the fourth case study, case study 4, 4-DOF system is obtained from the coupling of a two 2-DOF systems as shown in Figure 4.12. As it can be seen from Figure 4.12 substructure *A* is chosen as identified substructure, while substructure *B* is chosen as neural network trained substructure. Although substructure *B* is added to

coupling equations as a black box, schematic view of it can be seen from figure 4.13. Parameters of substructures A and B, and coupling elements are given in Table 4.10 and Table 4.11, respectively. It should be noted that coupling elements are not linear and there exists friction damping nonlinearity in the coupling elements. The nonlinear elements present in the system are defined in Table 4.12.
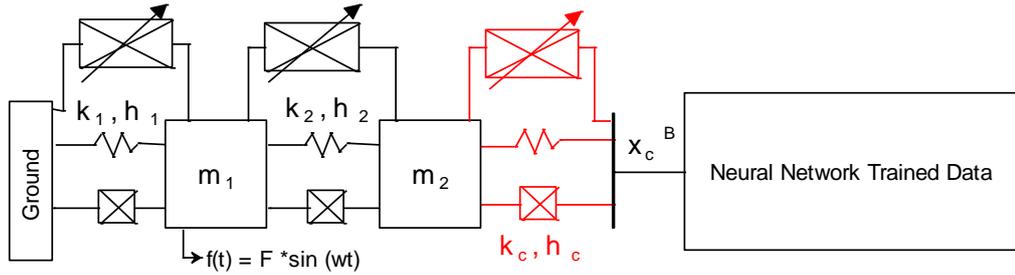
Figure 4.12 Schematic view of 4-DOF coupled system
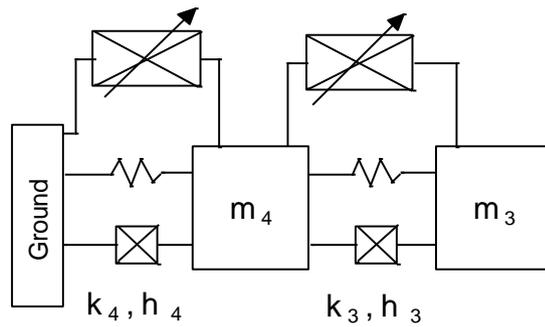
Figure 4.13 Schematic view of neural network trained substructure

Table 4.10 Parameters of substructures A and B

Substructure A		Substructure B	
m_1 [kg]	m_2 [kg]	m_3 [kg]	m_4 [kg]
1	2	0,75	0,75
k_1 [N/m]	k_2 [N/m]	k_3 [N/m]	k_4 [N/m]
5000	4000	3000	5000
h_1 [N/m]	h_2 [N/m]	h_3 [N/m]	h_4 [N/m]
50	40	30	50

Table 4.11 Parameters of coupling elements

k_c [N/m]	h_c [N/m]	Δ_c
5000	50	μN_c, $k = 2000$ N/m

Table 4.12 Nonlinear elements of substructures A and B in case study 4

Nonlinear Connection DOFs	Nonlinearity Type	Nonlinearity Coefficients
1-Ground	Dry Friction [N]	μN, $k = 2000$ N/m
1-2	Dry Friction [N]	μN, $k = 2000$ N/m
3-4	Dry Friction [N]	μN, $k = 2000$ N/m
4-Ground	Dry Friction [N]	μN, $k = 2000$ N/m

Normalized responses of the 1st, and the 2nd DOFs obtained from the proposed nonlinear coupling method with using neural network and by solving the entire system directly are given in Figure 4.14, and Figure 4.15. While, lin-lin scale is used in "(a)", log-lin scale is used "(b)". Responses of the coupled structure are given for an external forcing of $F = 25$N. In first part of case study 4, slip load between coupled elements, μN_c, and slip loads between internal elements, μN, are same. Perfect agreement between the results obtained from the proposed nonlinear coupling method with using neural network and the entire system solution is observed which verifies the developed coupling method.
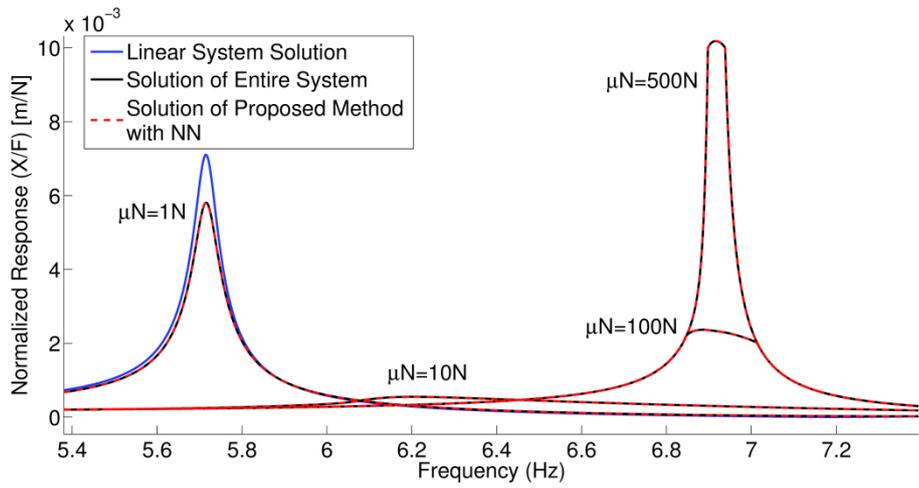
Figure 4.14 (a) Normalized response of the 1st DOF in case study 4
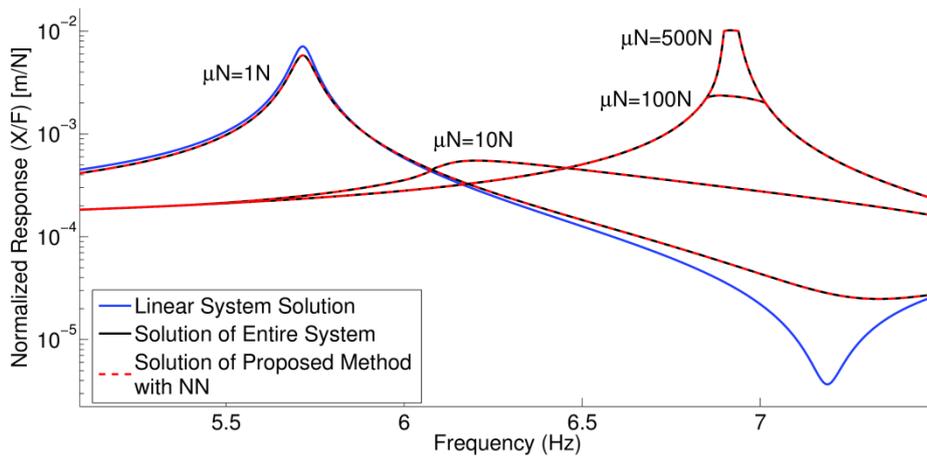
Figure 4.14 (b) Normalized response of the 1st DOF in case study 4
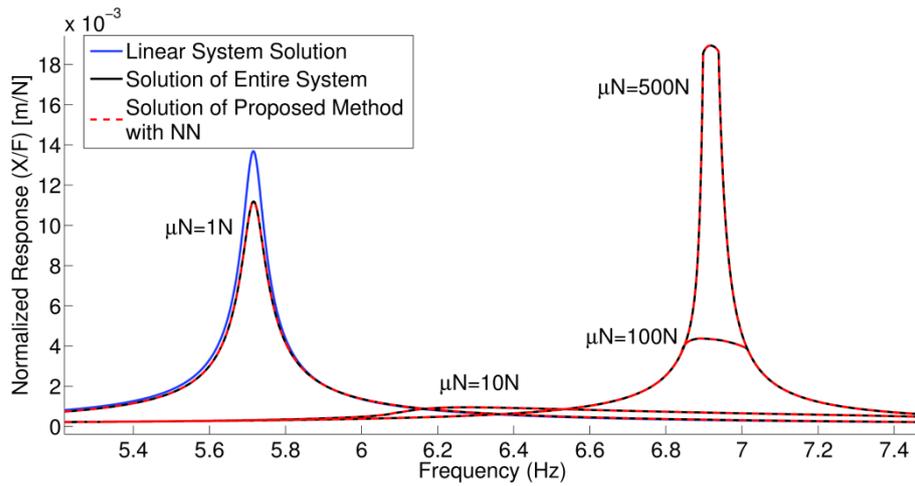
Figure 4.15 (a) Normalized response of the 2nd DOF in case study 4
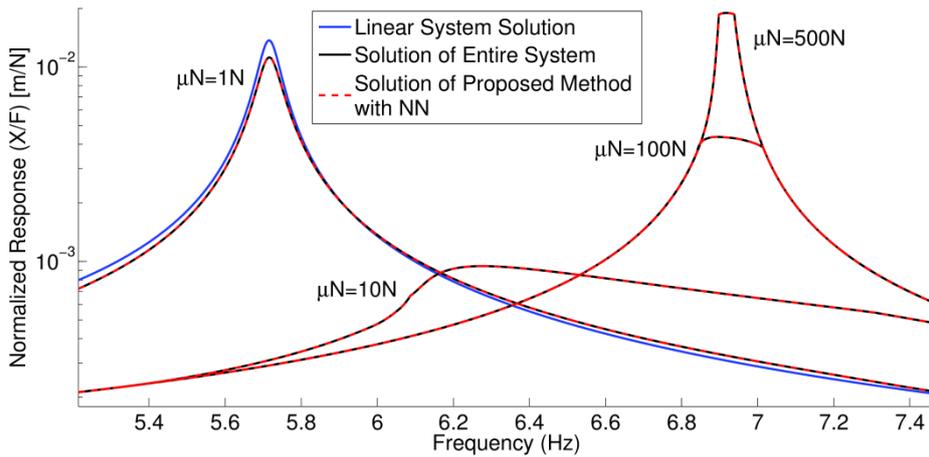
Figure 4.15 (b) Normalized response of the 2nd DOF in case study 4

In second part of case study 4, slip loads between internal elements are chosen as $\mu N = 1\text{N}$ while slip load of the coupling element, μN_c is varied. Normalized responses of the 1st DOFs obtained from the proposed nonlinear coupling method with using neural network and by solving the entire system directly are given in Figure 4.16. Responses of the coupled structure are given for an external forcing of $F = 25\text{N}$. Perfect agreement between the results obtained from the proposed nonlinear coupling method with using neural network and the entire system solution is observed which verifies the developed coupling method.

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Phase angle of imaginary and reel part of the 1$^{\text{th}}$ DOF in case study 4 is shown in Figure 4.17.
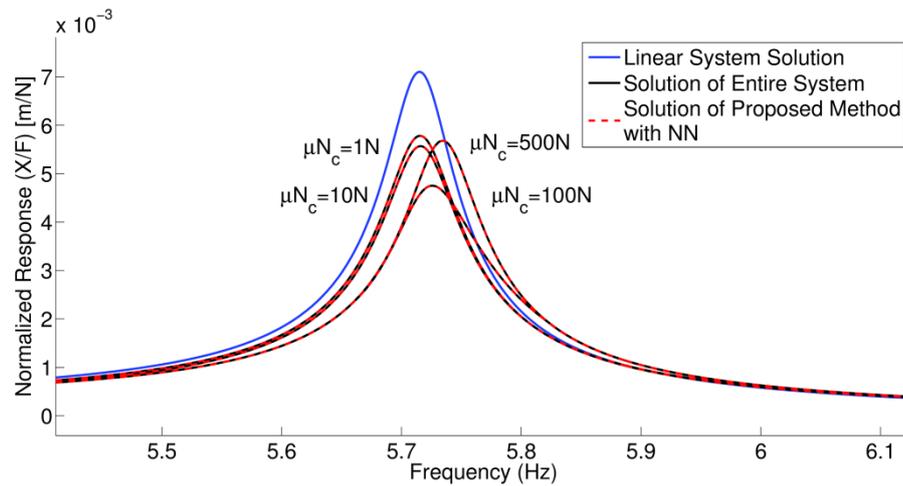
Figure 4.16 Normalized response of the 1$^{\text{st}}$ DOF in case study 4
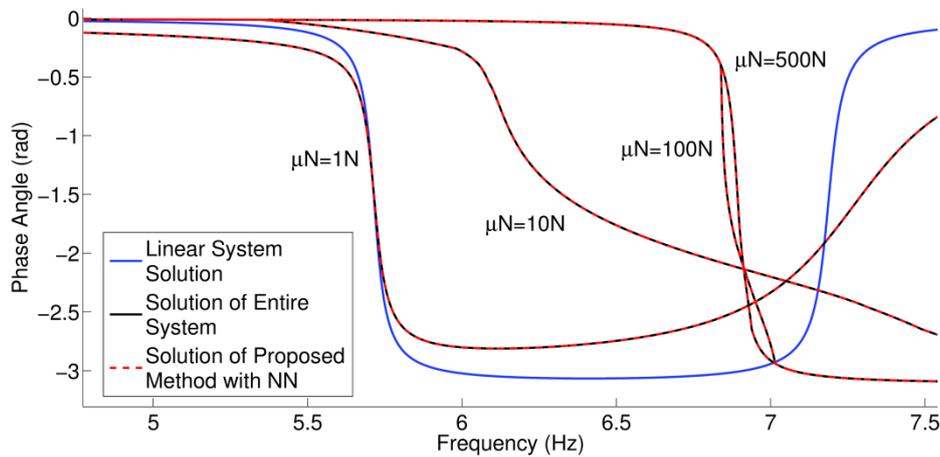
Figure 4.17 Phase angle of the 1$^{\text{th}}$ DOF in case study 4

In the fifth case study, case study 5, 6-DOF system is obtained from the coupling of a 2-DOF, 4-DOF systems. Different from case study 4, in case study 5, a 4-DOF system is used as substructure B, shown in Figure 4.18. Parameters of substructures

A and *B* , and coupling elements are given in Table 4.13 and Table 4.14, respectively. The nonlinear elements present in the system are defined in Table 4.15.
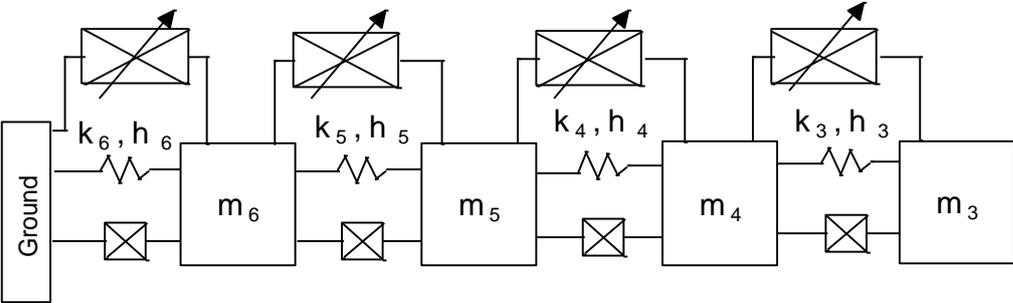
Figure 4.18 Schematic view of neural network trained substructure

Table 4.13 Parameters of substructures *A* and *B*

Substructure *A*	
m_1 [kg]	m_2 [kg]
0,8	0,5
k_1 [N/m]	k_2 [N/m]
7000	4000
h_1 [N/m]	h_2 [N/m]
70	40

Substructure *B*			
m_3 [kg]	m_4 [kg]	m_5 [kg]	m_6 [kg]
1	0,8	0,75	1
k_3 [N/m]	k_4 [N/m]	k_5 [N/m]	k_6 [N/m]
3000	4000	5000	3000
h_3 [N/m]	h_4 [N/m]	h_5 [N/m]	h_6 [N/m]
30	40	50	30

Table 4.14 Parameters of coupling elements

k_c [N/m]	h_c [N/m]
4000	40

Table 4.15 Nonlinear elements of substructures A and B in case study 5

Nonlinear Connection DOFs	Nonlinearity Type	Nonlinearity Coefficients
1-Ground	Dry Friction [N]	μN, $k = 1500$ N/m
1-2	Dry Friction [N]	μN, $k = 1500$ N/m
3-4	Dry Friction [N]	μN, $k = 1500$ N/m
4-5	Dry Friction [N]	μN, $k = 1500$ N/m
5-6	Dry Friction [N]	μN, $k = 1500$ N/m
6-Ground	Dry Friction [N]	μN, $k = 1500$ N/m

Normalized responses of the 1st, and the 2nd DOFs obtained from the proposed nonlinear coupling method with using neural network and by solving the entire system directly are given in Figure 4.19, and Figure 4.20. While, lin-lin scale is used in "(a)", log-lin scale is used "(b)". Responses of the coupled structure are given for an external forcing of $F = 25$N and for different slip loads.
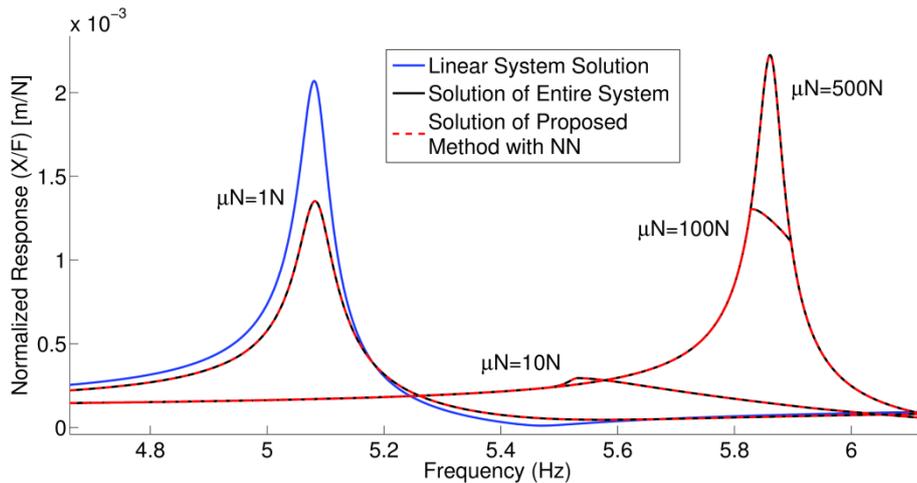
Figure 4.19 (a) Normalized response of the 1st DOF in case study 5
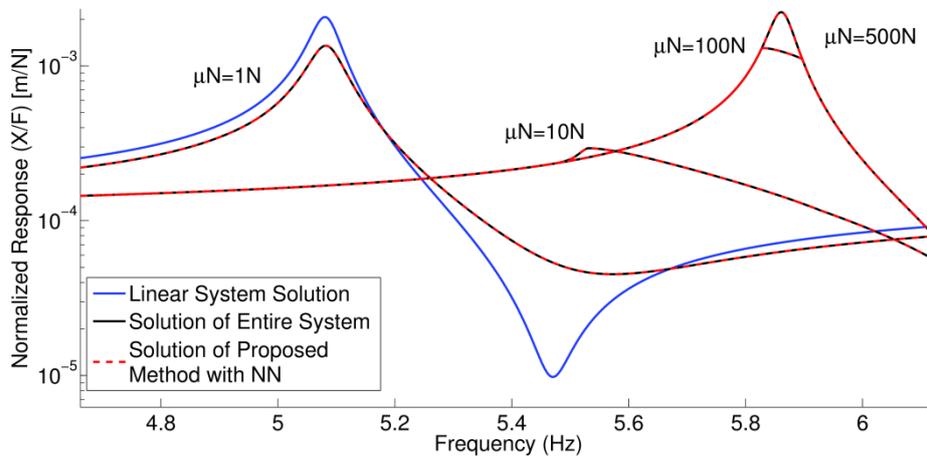
Figure 4.19 (b) Normalized response of the 1st DOF in case study 5
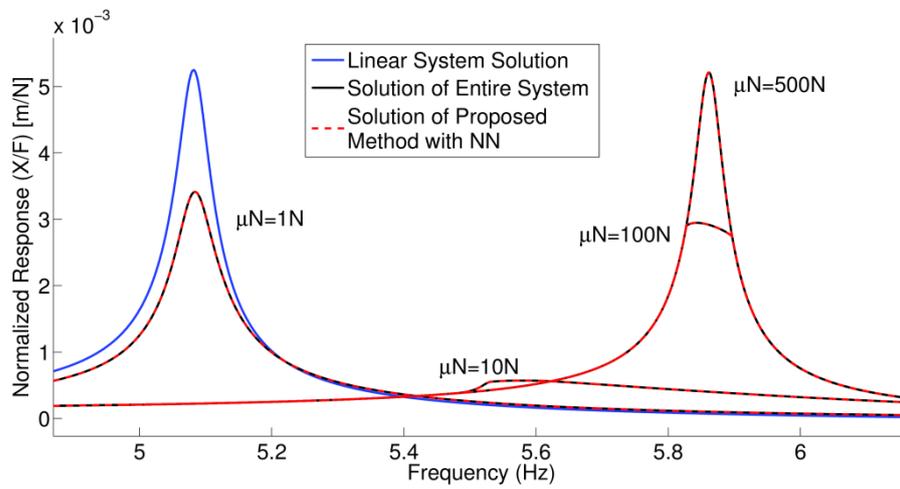
Figure 4.20 (a) Normalized response of the 2nd DOF in case study 5
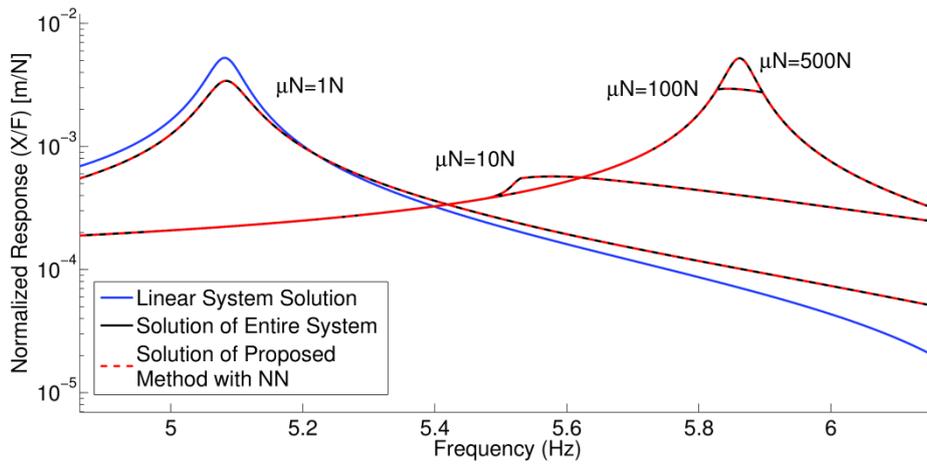
Figure 4.20 (b) Normalized response of the 2nd DOF in case study 5

The proposed method with using neural network is in exact agreement with the ones obtained from entire system solution, even in unstable regions where the path turns back or intersects itself.

In the sixth case study, case study 6, 8-DOF system is obtained from the coupling of a 2-DOF, 6-DOF systems. Different from case study 4 and 5, in case study 6, a 6-DOF system is used as substructure B, shown in Figure 4.21. Parameters of substructures A and B, and coupling elements are given in Table 4.16 and Table 4.17, respectively. The nonlinear elements present in the system are defined in Table 4.18.
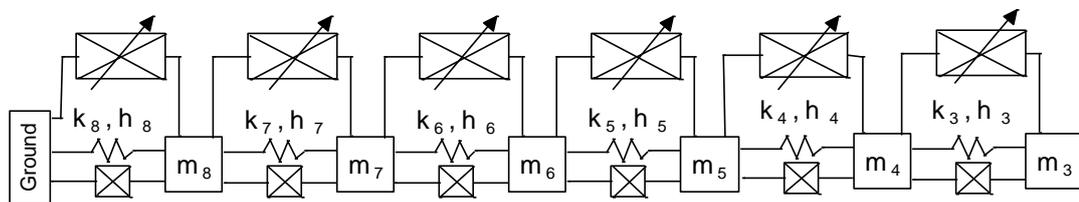
Figure 4.21 Schematic view of neural network trained substructure

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Table 4.16 Parameters of substructures A and B

Substructure A	
m_1 [kg]	m_2 [kg]
1	1,5
k_1 [N/m]	k_2 [N/m]
7000	4000
h_1 [N/m]	h_2 [N/m]
70	40

Substructure B					
m_3 [kg]	m_4 [kg]	m_5 [kg]	m_6 [kg]	m_7 [kg]	m_8 [kg]
0,6	1	0,8	1	0,75	1,2
k_3 [N/m]	k_4 [N/m]	k_5 [N/m]	k_6 [N/m]	k_7 [N/m]	k_8 [N/m]
3000	4000	5000	3000	4000	5000
h_3 [N/m]	h_4 [N/m]	h_5 [N/m]	h_6 [N/m]	h_7 [N/m]	h_8 [N/m]
30	40	50	30	40	50

Table 4.17 Parameters of coupling elements

k_c [N/m]	h_c [N/m]
4000	40

Table 4.18 Nonlinear elements of substructures A and B in case study 6

Nonlinear Connection DOFs	Nonlinearity Type	Nonlinearity Coefficients
1-Ground	Dry Friction [N]	μN, $k = 2500$ N/m
1-2	Dry Friction [N]	μN, $k = 2500$ N/m
3-4	Dry Friction [N]	μN, $k = 2500$ N/m
4-5	Dry Friction [N]	μN, $k = 2500$ N/m
5-6	Dry Friction [N]	μN, $k = 2500$ N/m
6-7	Dry Friction [N]	μN, $k = 2500$ N/m
7-8	Dry Friction [N]	μN, $k = 2500$ N/m
8-Ground	Dry Friction [N]	μN, $k = 2500$ N/m

Normalized responses of the 1st, and the 2nd DOFs obtained from the proposed nonlinear coupling method with using neural network and by solving the entire system directly are given in Figure 4.22, and Figure 4.23. While, lin-lin scale is used in "(a)", log-lin scale is used "(b)". Responses of the coupled structure are given for an external forcing of $F = 25$N and for different slip loads. Perfect agreement between the results obtained from the proposed nonlinear coupling method with using neural network and the entire system solution is observed which verifies the developed coupling method.
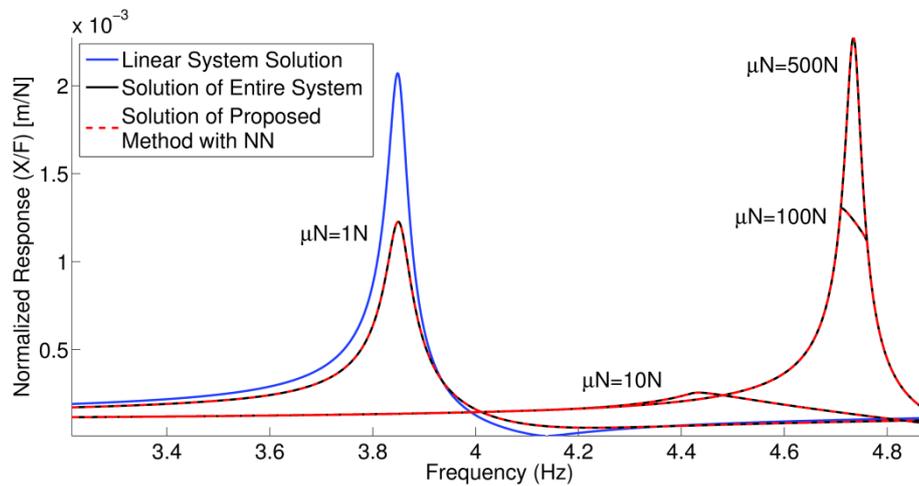
Figure 4.22 (a) Normalized response of the 1st DOF in case study 6
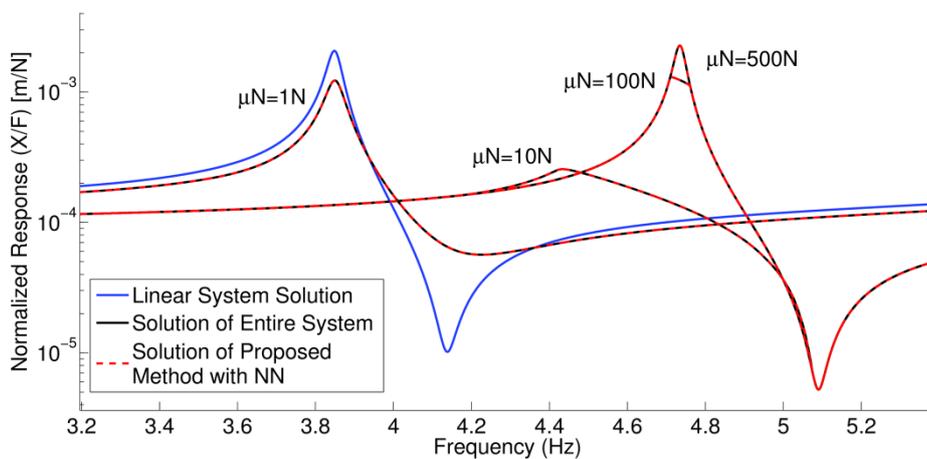
Figure 4.22 (b) Normalized response of the 1st DOF in case study 6
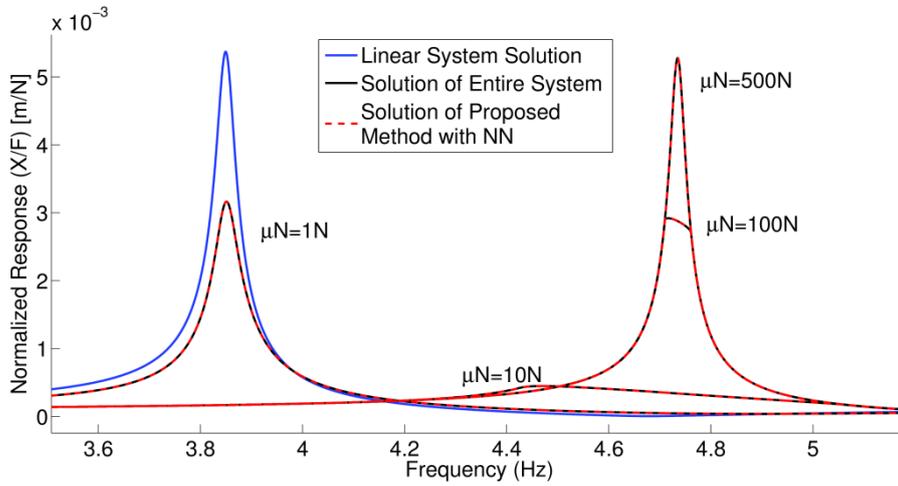
Figure 4.23 (a) Normalized response of the 2nd DOF in case study 6
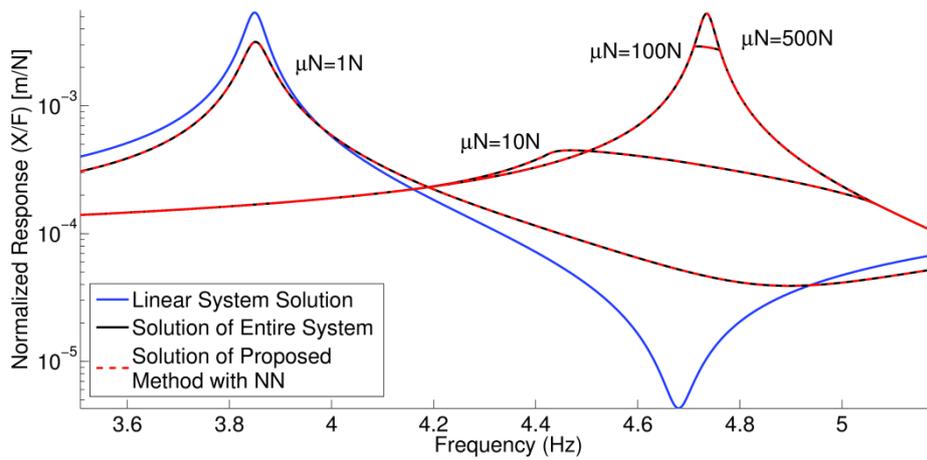
Figure 4.23 (b) Normalized response of the 2nd DOF in case study 6

4.1.3   Comparison of Computational Time Required

In this section, developed nonlinear structural coupling approach is compared with entire system solution. For this comparison two main parts are considered. In the first part, time requirement for solution is compared for coupling of two identified substructure, while in the second part, it is done for one identified and neural network trained substructure. In the first part, time requirement is compared for

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different forces and different substructure sizes. In the second part, neural network trained substructure size, so, size of the coupled system is different.

The calculations are done on a computer having a processor Intel Core i7-Q720 CPU @ 1.60 GHz with 4,00 GB of RAM.

In the first part, computational time requirement is compared for a simple 8-DOF system shown in Figure 4.1. The response of the system is obtained for three different excitation amplitudes, 8N, 12N and 16N in order to observe the effect of external forcing in computational time. From the results in Table 4.19, it can be observed that time requirement in proposed method is more than entire system solution for all external forcings. However, usage of parallel processing to solve each substructure may increase efficiency of proposed method. Although, it is a reasonable assumption, it is not achieved via MATLAB parallel processing tool. It is thought that it should be managed with more proper programs which are directed to this topic.

Table 4.19 Comparison of calculation times for different external forcing values

External Forcing [N]	Proposed Method [s]	Entire System Solution [s]
8	21.44	15.20
12	30.8	22.05
16	39.52	30.40

Although, application of proposed method in two identified system is not observed as time efficient, it can be said that it is important to show validation of proposed method. Furthermore, benefit of the proposed method in computational time is going to be shown in the following part.

In the second part, computation time is compared between entire system solution and proposed method with using neural networks. In all of case studies, substructure A is

a 2-DOF system, while neural network trained substructure B is varying between a 2-DOF, 4-DOF, 6-DOF, Therefore, comparison which can be seen from Table 4.20 is managed for different size of coupled system. For an effective comparison external forcing is chosen same in all of case studies as $F = 25\text{N}$.

Table 4.20 Comparison of calculation times for different coupled system sizes

# of DOF of Coupled System	Proposed Method with using Neural Networks [s]	Entire System Solution [s]
4-DOF	658,6	22,18
6-DOF	667,4	41,08
8-DOF	673,6	73,98

It can be observed from Table 4.20 that time requirement for entire system solution is increased with increasing of number of DOF of coupled system while computational time of the proposed method remains nearly the same. It is a reasonable result, since neural network trained substructure is not solved again for coupling procedure; therefore, time requirement of the proposed method remains nearly constant. In Figure 4.24, computation time requirement of the entire system solution and the proposed method with neural networks is shown. It can be concluded that the proposed method is more efficient than solving the entire system for large systems.
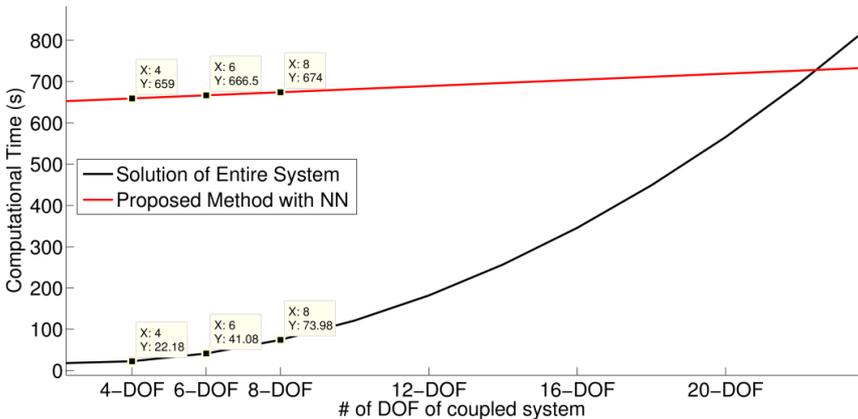
Figure 4.24 Comparison of computational time between proposed method with neural networks and entire system solution

4.2 Nonlinear Structural Coupling from Multiple DOFs

4.2.1 Nonlinear Structural Coupling with Two Identified Substructures

In the seventh example, application of the proposed approach is presented on a simple 4-DOF system shown in Figure 4.25. Different from section 4.1 structural coupling is done from multiple DOFs. Parameters of substructures A and B are given Table 4.21 and coupling elements at Table 4.22. The nonlinear elements used in the seventh case study, case study 7, are defined in Table 4.23.
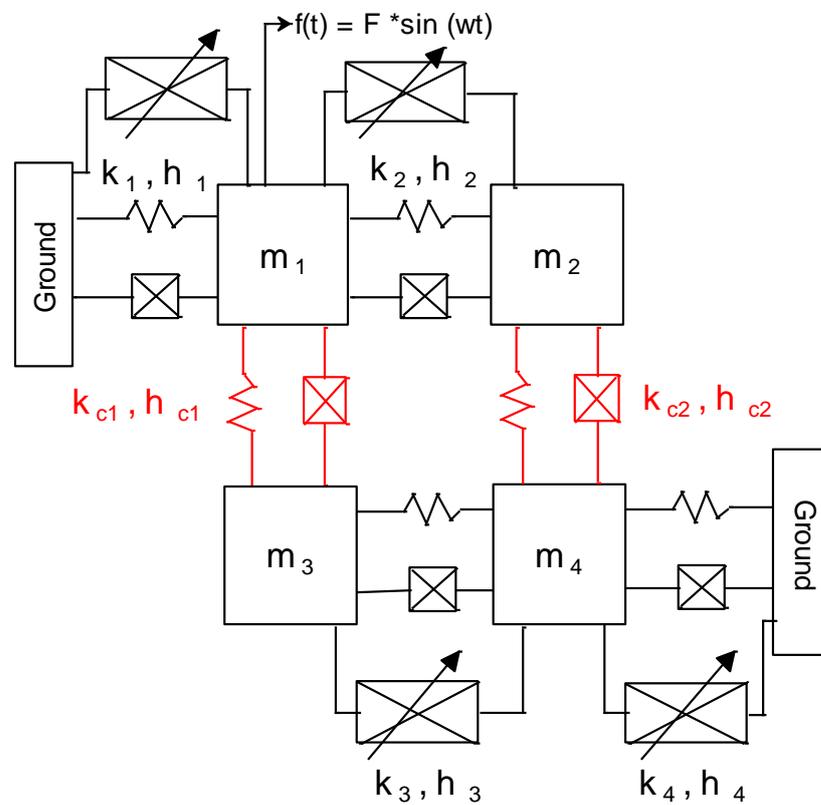
Figure 4.25 Schematic view of 4-DOF coupled system

Table 4.21 Parameters of substructures A and B

Substructure A		Substructure B	
m_1 [kg]	m_2 [kg]	m_3 [kg]	m_4 [kg]
1	0,75	0,75	1
k_1 [N/m]	k_2 [N/m]	k_3 [N/m]	k_4 [N/m]
3000	4000	3000	5000
h_1 [N/m]	h_2 [N/m]	h_3 [N/m]	h_4 [N/m]
30	40	30	50

Table 4.22 Parameters of coupling elements

k_{c1} [N/m]	h_{c1} [N/m]	k_{c2} [N/m]	h_{c2} [N/m]
5000	50	4000	40

Table 4.23 Nonlinear elements of substructures A and B in case study 7

Nonlinear Connection DOFs	Nonlinearity Type	Nonlinearity Coefficients
1-Ground	Cubic Stiffness [N/m^3]	-0.5x10^5
1-2	Cubic Stiffness [N/m^3]	-1x10^5
3-4	Cubic Stiffness [N/m^3]	-2x10^5
4-Ground	Cubic Stiffness [N/m^3]	-1x10^5

Normalized response of the 1st DOF obtained from the proposed nonlinear coupling method and by solving the entire system directly is given in Figure 4.26. The response of the system is obtained for three different excitation amplitudes, 5N, 8N and 10N. The proposed method is in exact agreement with the ones obtained from entire system solution, even in unstable regions where the path turns back or intersects itself.
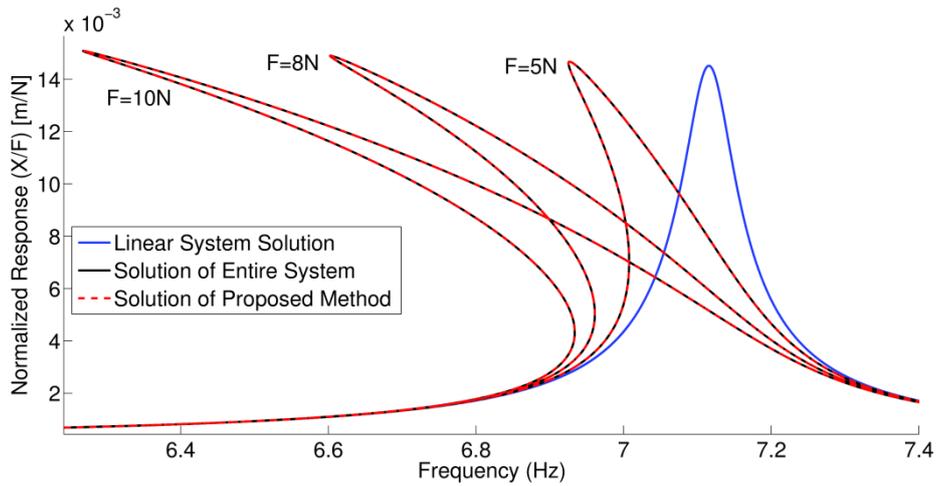
Figure 4.26 Normalized response of the 1st DOF in case study 7

In the eigthth case study, application of the proposed approach is obtained from the coupling of a 4-DOF, 2-DOF systems as shown in Figure 4.27. Different from section 4.1 structural coupling is done from multiple DOFs. Parameters of substructures *A* and *B* are given Table 4.24 and coupling elements at Table 4.25. The nonlinear elements used in the eighth case study are defined in Table 4.26.
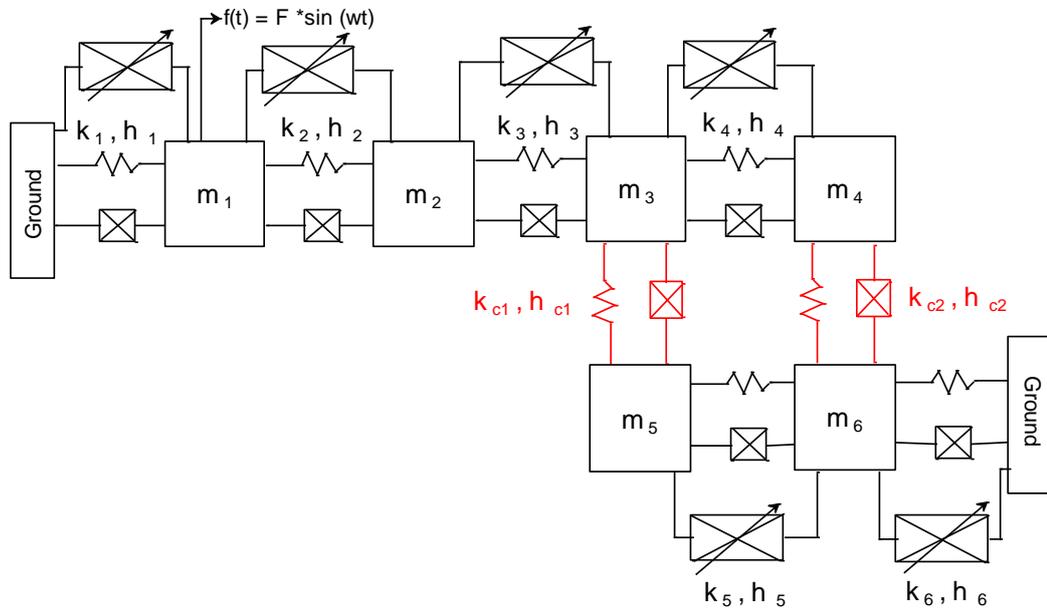
Figure 4.27 Schematic view of 4-DOF coupled system

Table 4.24 Parameters of substructures A and B

Substructure A				Substructure B	
m_1 [kg]	m_2 [kg]	m_3 [kg]	m_4 [kg]	m_5 [kg]	m_6 [kg]
1	0,75	2	1	0,75	1
k_1 [N/m]	k_2 [N/m]	k_3 [N/m]	k_4 [N/m]	k_5 [N/m]	k_6 [N/m]
5000	4000	3000	4000	3000	5000
h_1 [N/m]	h_2 [N/m]	h_3 [N/m]	h_4 [N/m]	h_5 [N/m]	h_6 [N/m]
25	20	15	20	15	25

Table 4.25 Parameters of coupling elements

k_{c1} [N/m]	h_{c1} [N/m]	k_{c2} [N/m]	h_{c2} [N/m]
5000	25	3000	15

Table 4.26 Nonlinear elements of substructures A and B in case study 8

Nonlinear Connection Coordinates	Nonlinearity Type	Nonlinearity Coefficients
1-Ground	Cubic Stiffness [N/m³]	1×10^5
1-2	Cubic Stiffness [N/m³]	2×10^5
2-3	Cubic Stiffness [N/m³]	1×10^5
3-4	Cubic Stiffness [N/m³]	2×10^5
5-6	Cubic Stiffness [N/m³]	1×10^5
6-Ground	Cubic Stiffness [N/m³]	2×10^5

Normalized response of the 1st DOF obtained from the proposed nonlinear coupling method and by solving the entire system directly is given in Figure 4.28. The response of the system is obtained for three different excitation amplitudes, 6N, 9N and 12N. It is observed that the results obtained from the proposed nonlinear coupling method and the entire system solution are in perfect agreement, for this case as well.

Phase angle of imaginary and reel part of the 1th DOF in case study 10 is shown in Figure 4.29.
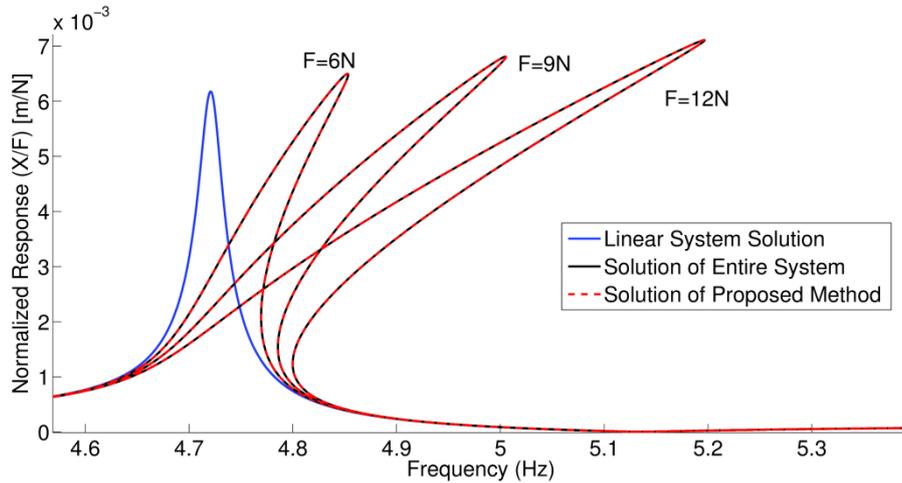
Figure 4.28 Normalized response of the 1st DOF in case study 8
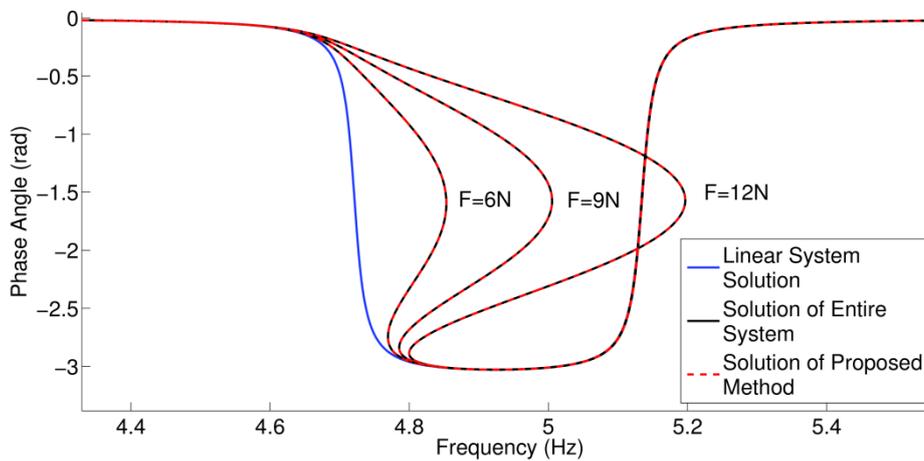
Figure 4.29 Phase angle of the 1th DOF in case study 8

4.2.2 Nonlinear Structural Coupling with using Neural Networks

In the ninth case study, application of the proposed approach is obtained from the coupling of two 2-DOF systems as shown in Figure 4.30. As it can be seen from

Figure 4.30 substructure *A* is chosen as identified substructure, while substructure *B* is chosen as neural network trained substructure. Schematic view of substructure *B* is shown at Figure 4.31. Parameters of substructures *A* and *B* are given Table 4.27 and coupling elements at Table 4.28. The nonlinear elements used in the ninth case study are defined in Table 4.29.
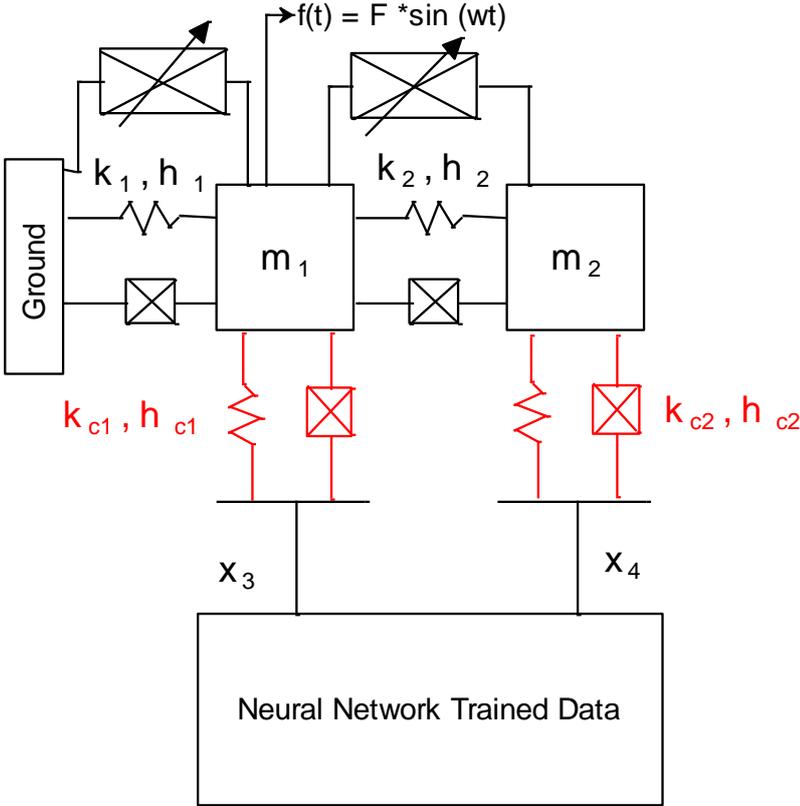
Figure 4.30 Schematic view of coupled system
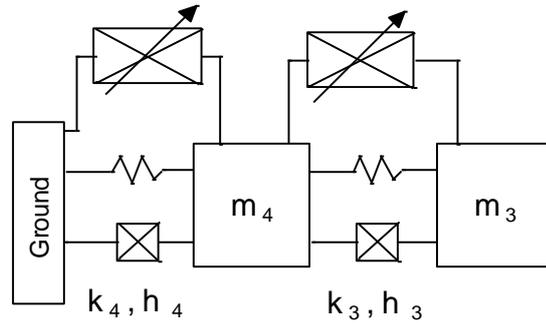
Figure 4.31 Schematic view of neural network trained substructure

Table 4.27 Parameters of substructures A and B

Substructure A		Substructure B	
m_1 [kg]	m_2 [kg]	m_3 [kg]	m_4 [kg]
1,2	2	0,7	0,75
k_1 [N/m]	k_2 [N/m]	k_3 [N/m]	k_4 [N/m]
4000	4000	3000	7000
h_1 [N/m]	h_2 [N/m]	h_3 [N/m]	h_4 [N/m]
40	40	30	70

Table 4.28 Parameters of coupling elements

k_{c1} [N/m]	h_{c1} [N/m]	k_{c2} [N/m]	h_{c2} [N/m]
300	3	200	2

Table 4.29 Nonlinear elements of substructures A and B in case study 9

Nonlinear Connection DOFs	Nonlinearity Type	Nonlinearity Coefficients
1-Ground	Dry Friction [N]	μN, $k = 1500$ N/m
1-2	Dry Friction [N]	μN, $k = 1500$ N/m
3-4	Dry Friction [N]	μN, $k = 1500$ N/m
4-Ground	Dry Friction [N]	μN, $k = 1500$ N/m

Normalized response of the 1st DOF obtained from the proposed nonlinear coupling method with using neural network and by solving the entire system directly is given in Figure 4.32. While, lin-lin scale is used in "(a)", log-lin scale is used "(b)". Responses of the coupled structure are given for an external forcing of $F = 25$N and for different slip loads. It is observed that the results obtained from the proposed nonlinear coupling method with using neural network and the entire system solution are in perfect agreement.
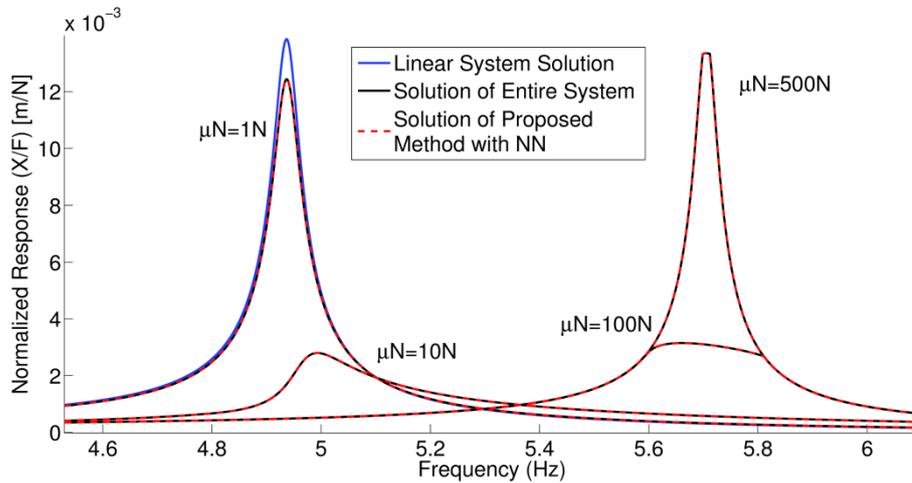
Figure 4.32 (a) Normalized response of the 1st DOF in case study 9
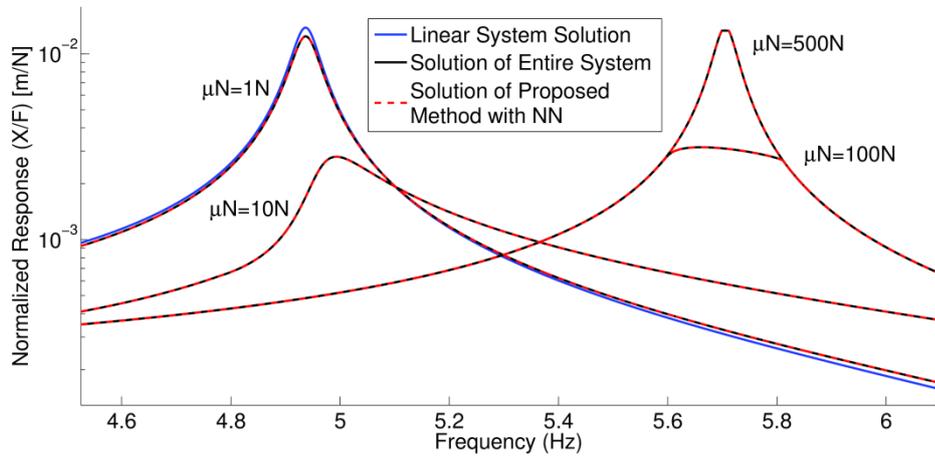
Figure 4.32 (b) Normalized response of the 1st DOF in case study 9

CHAPTER 5

DISCUSSION AND CONCLUSION

In this study, a new structural coupling method is introduced which is capable of coupling of two nonlinear substructures, where the connection elements can be nonlinear as well.

A nonlinear solution method utilizing describing function method with a single harmonic is used to obtain solution of substructures, which are employed in the solution of the coupled system and training of the substructures with using neural networks. Cubic stiffness and hysteretic dry friction are used as nonlinear elements in the substructures, so brief information is given about these nonlinearities. Numerical solution techniques are introduced to solve nonlinear algebraic equations obtained by using describing function method.

Compatibility and equilibrium equations, which are derived from existing linear coupling methodology, are added to nonlinear equations of motions in order to model coupled system. Model of coupled system consists of two ways, that coupling of two identified substructures or coupling of one identified and one neural network trained substructures. The resulting nonlinear equations of motion of the coupled system are solved by using Newton's method with arc-length continuation.

Applications of the proposed nonlinear coupling approach are demonstrated by numerical case studies. Two main parts, which has two subsections, are considered in the case studies. In the first one two substructures are coupled from a single DOF; whereas, in the second example, two substructures are coupled from two DOFs. In the subsections difference between two identified substructure coupling and

identified and neural network trained substructure coupling is studied. Normalized responses of the selected DOFs obtained from the proposed nonlinear coupling method and by solving the entire system directly are compared in order to verify the proposed method for different nonlinear systems. The results obtained from the proposed method and the ones obtained by directly solving the entire system agree perfectly with each other, which verifies the developed nonlinear coupling method. Also time requirement in selected case studies is studied. Time requirement to solve coupled system with neural networks is seen more efficient for large systems.

Proposed method is capable of solving the coupled system with nonlinearities at any DOFs. This is especially important, since location of the nonlinearities is not important to solve the coupled system. It should be noted that in large systems time efficiency can be seen clearly.

In this study, coupling of nonlinear systems is achieved successfully using the proposed method. However, there are some aspects needed to be studied. First of all, training data generation is one of the most crucial parts of the second part of the method. There may be practical limits to generate training data as the number of possible nonlinear systems increases. Moreover, proposed method is seen less time efficient in small systems. The method may be improved such that instead of neural network toolbox, same work which is done by neural network toolbox may be done with a user defined code.

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APPENDIX A

CONFERENCE PAPER

STRUCTURAL COUPLING OF TWO-NONLINEAR STRUCTURES

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ABSTRACT

In mechanical design, modeling and analysis of a complex structure can be simplified with dividing the structure into substructures; therefore, any change in the structure can be addressed easily which is referred as "structural coupling". Utilization of proper coupling techniques, it is possible to understand the behavior of the whole structure by considering the behavior of its substructures. For linear structures, coupling is a common technique; however, in most of the engineering structures, nonlinearities are also encountered; therefore, it is required to extend linear coupling methods to nonlinear systems. Although, there exists studies on nonlinear coupling, existing methods are limited to coupling of structures where one substructure is linear and the other is nonlinear or two linear substructures coupled with a nonlinear element. In this paper, a structural coupling method is proposed to couple two-nonlinear substructures. Similar to linear coupling methods, the proposed method considers the compatibility of internal forces at the connection degrees of freedom in addition to displacements. Since, the substructures are nonlinear, the resulting system of nonlinear differential equations are converted into a set of nonlinear algebraic equations by using Describing Function Method, which are solved by using Newton's method with arc-length continuation.

Keywords: Structural Coupling, Nonlinear Structural Coupling, Vibration of Nonlinear Structures

1. INTRODUCTION

In the design of mechanical systems, engineers should test and analyze each prototype created in order to provide a qualified and optimized design which has a wide range of requirements. Over the last 40 years, engineering structures are analyzed by the finite element method which is proven to be a reliable tool. However, in order finalize the design, whole structure has to be analyzed several times; therefore, an alternative approach is required in order to decrease the number of analyses and tests. Utilizing structural coupling, modeling and analysis of a complex structure can be simplified by dividing the structure into substructures and applying the required changes only on one or some of the substructures, where each substructure can be analyzed individually.

Substructure analysis of linear systems is a well-known subject dated back to 1960s by the works of Bishop and Johnson [1] on Receptance method and Hurty [2] on Component Mode

Synthesis which was a simplified version of the method developed by Craig and Bampton [3]. Many different substructure and coupling methods for linear structures are developed by Rubin [4], Przemieniecki [5], Urgueira [6], Ewins [7], Klosterman and Lemon [8] and Ren and Beards [9]. All of these methods are developed for linear systems and the methodology is based on the compatibility of internal forces at the connection degrees of freedom in addition to the compatibility of the displacements. However, there is a need to extend linear coupling methods to non-linear systems; since many structures, which are considered as linear, are nonlinear in reality.

Analysis of nonlinear systems is much more complicated compared to linear systems [10] due to their response dependent behavior. In this paper, Describing Function Method (DFM) is used for the solution of nonlinear systems which was introduced by Krylov and Bogolyubov [11] in order to analyze nonlinear control problems based on an earlier work of Van der Pol [12]. Later, Taylor [13] replaced each nonlinear element with a quasilinear descriptor to define this approach.

Solution of multi degree of freedom nonlinear system with symmetrical nonlinearities is introduced by Budak and Özgüven [14, 15], which utilizes a special algebra. Later, Tanrıkulu [16] and Tanrıkulu et. Al.[17] extended this formulation for any type of nonlinearity by replacing this special algebra with describing functions.

Although, several studies on structural coupling of linear systems and modelling systems with nonlinearities are available in literature, the numbers of studies that consider nonlinear structural coupling are limited. Existing studies on nonlinear structural coupling are focused on coupling of structures where one substructure is linear and the other one is nonlinear or coupling of two linear substructures with a nonlinear coupling element. Watanabe and Sato [10] suggested "Nonlinear Building Block" approach, for coupling of linear substructures with nonlinear coupling elements. Cömert and Özgüven [18] developed a method for coupling of linear substructures with nonlinear connecting elements by using DFM, in which FRFs of the linear substructures are used. Ferreira and Ewins [19] proposed a new Nonlinear Receptance Coupling Approach and Ferraira [20] extended the approach with Multi-Harmonic Nonlinear Receptance Coupling Approach. Both approaches are capable of coupling a linear structure with a nonlinear structure with different types of joints. Chong and Imregün [21] suggested an iterative algorithm for the coupling of nonlinear structures with linear ones.

In this paper, an approach is developed to dynamic reanalysis of nonlinear substructures. Different from the existing methods in literature, proposed method is capable of coupling of two nonlinear substructures. Moreover, with the proposed coupling method, in addition to linear coupling elements, nonlinear coupling elements can as well be used. The proposed method considers the compatibility of internal forces at the connection degrees of freedom in addition to the displacements, and uses both of these equations to couple nonlinear substructures. Since, the substructures are nonlinear, the resulting system of nonlinear differential equations are converted into a set of nonlinear algebraic equations by using Describing Function Method, which are solved by using Newton's method with arc-length continuation.

2. THEORY

2.1. Structural Coupling of Linear Substructures

Consider two substructures A and B, shown in Fig. 1, where internal DOFs are represented by subscripts, i_A and i_B respectively and the connection DOFs are represented by subscripts c_A and c_B, respectively.

Fig. 1 Schematic view of structural coupling

The corresponding equilibrium of each substructure can be written as

$$
\left\{ \begin{array}{c} \{f_{i_A}\} \\ \{f_{c_A}\} \end{array} \right\} = \left[\begin{array}{cc} \left[Z_{i_A i_A}\right] & \left[Z_{i_A c_A}\right] \\ \left[Z_{c_A i_A}\right] & \left[Z_{c_A c_A}\right] \end{array} \right] \left\{ \begin{array}{c} \{x_{i_A}\} \\ \{x_{c_A}\} \end{array} \right\} , \tag{1}
$$

$$
\left\{ \begin{array}{c} \{f_{i_B}\} \\ \{f_{c_B}\} \end{array} \right\} = \left[\begin{array}{cc} \left[Z_{i_B i_B}\right] & \left[Z_{i_B c_B}\right] \\ \left[Z_{c_B i_B}\right] & \left[Z_{c_B c_B}\right] \end{array} \right] \left\{ \begin{array}{c} \{x_{i_B}\} \\ \{x_{c_B}\} \end{array} \right\} , \tag{2}
$$

where $\{x_{i_A}\}$ and $\{x_{i_B}\}$ are generalized displacement vectors for internal DOFs, $\{x_{c_A}\}$ and $\{x_{c_B}\}$ are generalized displacement vectors for coupled DOFs of substructures A and B, respectively. $\{f_{i_A}\}$ and $\{f_{i_B}\}$ are internal forcing vectors for internal DOFs, $\{f_{c_A}\}$ and $\{f_{c_B}\}$ are coupled forcing vectors for internal DOFs of substructures A and B, respectively. Lastly, $\left[Z_A\right]$ and $\left[Z_B\right]$ are the impedance matrices of substructures A and B. Equilibrium of the forces between the connection DOFs can be written as

$$
\{f_c\} = \{f_{c_A}\} + \{f_{c_B}\} , \tag{3}
$$

where $\{f_c\}$ is the external force acting on the connection DOFs. Considering the compatibility of displacements of the substructures the following relation can be written

$$
\left[D_{coupling}\right]\left(\{x_{c_A}\} - \{x_{c_B}\}\right) = \{f_{c_A}\} , \tag{4}
$$

where $\left[D_{coupling}\right]$ is dynamic stiffness matrix of coupling elements. Substituting compatibility and equilibrium equations, i.e. Eqs. (3) and (4), into Eqs. (1) and (2), the overall impedance of the assembled system can be written as

$$
[Z] = \left[\begin{array}{ccc} \left[Z_{i_A i_A}\right] & [0] & \left[Z_{i_A c_A}\right] \\ [0] & \left[Z_{i_B i_B}\right] & \left[Z_{i_B c_B}\right] \\ \left[Z_{c_A i_A}\right] & \left[Z_{c_B i_B}\right] & \left[Z_{c_A c_A}\right] + \left[Z_{c_B c_B}\right] \end{array} \right] . \tag{5}
$$

2.2. Structural Coupling of Two Nonlinear Substructures

The equation of motion of the nonlinear substructures A and B excited with a harmonic external forcing $\{f(t)\}$, can be written as

$$
\begin{aligned}
[M_A]\{\ddot{x}_A(t)\}+[C_A]\{\dot{x}_A(t)\}+i[H_A]\{x_A(t)\}+[K_A]\{x_A(t)\}+\{f_{N_A}(t)\}=\{f_A(t)\} \\
[M_B]\{\ddot{x}_B(t)\}+[C_B]\{\dot{x}_B(t)\}+i[H_B]\{x_B(t)\}+[K_B]\{x_A(t)\}+\{f_{N_B}(t)\}=\{f_B(t)\}
\end{aligned}
\tag{6}
$$

where $[M]$, $[C]$, $[H]$ and $[K]$ are the mass, viscous damping, structural damping and stiffness matrices of the linear system and $\{f_N(t)\}$ is the nonlinear forcing vector. Generalized displacement vectors $\{x_A(t)\}$ and $\{x_B(t)\}$ can be written as

$$
\{x_A(t)\}=\left\{\begin{matrix}\{X_{i_A}\}\\\{X_{c_A}\}\end{matrix}\right\}, \quad \{x_B(t)\}=\left\{\begin{matrix}\{X_{i_B}\}\\\{X_{c_B}\}\end{matrix}\right\},
\tag{7}
$$

and external forcing vectors $\{f_A(t)\}$ and $\{f_B(t)\}$ can be written as

$$
\{f_A(t)\}=\left\{\begin{matrix}\{F_{i_A}\}\\\{F_{c_A}\}+\{f_{c_A}\}\end{matrix}\right\}, \quad \{f_B(t)\}=\left\{\begin{matrix}\{F_{i_B}\}\\\{F_{c_B}\}+\{f_{c_B}\}\end{matrix}\right\},
\tag{8}
$$

where $\{F_{i_A}\}$ and $\{F_{i_B}\}$ are external forcing vectors acting on internal DOFs and $\{F_{c_A}\}$ and $\{F_{c_B}\}$ are external force vectors acting on the coupled DOFs. If the external forcing, $\{f(t)\}$ is periodic, response of the system, $\{x(t)\}$, can as well be assumed periodic, which can be expressed as follows

$$
\{f(t)\}=\{F\}_0+\mathrm{Im}\left[\sum_{m=1}^{\infty}\{F\}_m\cdot e^{im\psi}\right],
\tag{9}
$$

$$
\{x(t)\}=\{X\}_0+\mathrm{Im}\left[\sum_{m=1}^{\infty}\{X\}_m\cdot e^{im\psi}\right].
\tag{10}
$$

Utilizing Describing Function Method (DFM) [14, 22] and substituting Eqs. (7) and (8) into Eq. (6) as the following result is obtained

$$
\begin{aligned}
\left(-\omega^2[M]+i\cdot\omega[C]+i[H]+[K]+[\Delta]\right)\left\{\begin{matrix}\{X_{i_A}\}\\\{X_{c_A}\}\end{matrix}\right\}=\left\{\begin{matrix}\{F_{i_A}\}\\\{F_{c_A}\}+\{f_{c_A}\}\end{matrix}\right\} \\
\left(-\omega^2[M]+i\cdot\omega[C]+i[H]+[K]+[\Delta]\right)\left\{\begin{matrix}\{X_{i_B}\}\\\{X_{c_B}\}\end{matrix}\right\}=\left\{\begin{matrix}\{F_{i_B}\}\\\{F_{c_B}\}+\{f_{c_B}\}\end{matrix}\right\}
\end{aligned}
\tag{11}
$$

where, $[\Delta]$ is the "nonlinearity matrix", which is function of the displacement vector. The elements of nonlinearity matrix are defined as

$$\begin{aligned}
\left[\Delta\right]_{kj} &= \sum_{j=1}^{n} v_{kj} \qquad k = j \\
\left[\Delta\right]_{kj} &= -v_{kj} \qquad k \neq j
\end{aligned} \tag{12}$$

where v_{kj} is describing function of the nonlinearity between the kth and the jth degrees of freedom, which is a quantity complex in general. For $k = j$ nonlinearity is between the kth degrees of freedom and the ground. Details of DFM can be found in [14, 22].

Internal forcing vector $\{f_{c_A}\}$ can be written as

$$\{f_{c_A}\} = \left[D_{coupling}^{NL}\right]\left(\{X_{c_A}\} - \{X_{c_B}\}\right) = \left(-\omega^2[M] + i \cdot \omega[C] + i[H] + [K] + [\Delta]\right)\left(\{X_{c_A}\} - \{X_{c_B}\}\right), \tag{13}$$

where, $\left[D_{coupling}^{NL}\right]$ is the nonlinear dynamic stiffness matrix of the connection elements. Substituting Eqs. (3) and (13), into Eq. (11), equation of motion can be obtained as

$$\begin{aligned}
\left(-\omega^2[M] + i \cdot \omega[C] + i[H] + [K] + [\Delta]\right)\begin{Bmatrix}\{X_{i_A}\}\\\{X_{c_A}\}\end{Bmatrix} &= \begin{Bmatrix}\{F_{i_A}\}\\\{F_{c_A}\} + \left[D_{coupling}^{NL}\right]\left(\{X_{c_A}\} - \{X_{c_B}\}\right)\end{Bmatrix} \\
\left(-\omega^2[M] + i \cdot \omega[C] + i[H] + [K] + [\Delta]\right)\begin{Bmatrix}\{X_{i_B}\}\\\{X_{c_B}\}\end{Bmatrix} &= \begin{Bmatrix}\{F_{i_B}\}\\\{F_{c_B}\} + \{f_c\} - \left[D_{coupling}^{NL}\right]\left(\{X_{c_A}\} - \{X_{c_B}\}\right)\end{Bmatrix}
\end{aligned}. \tag{14}$$

Eq. (14) can be solved by a nonlinear equation solver and in this paper; Newton's method with arc-length continuation [23] is used. Algorithm of the proposed nonlinear coupling method is given in Fig. 2.
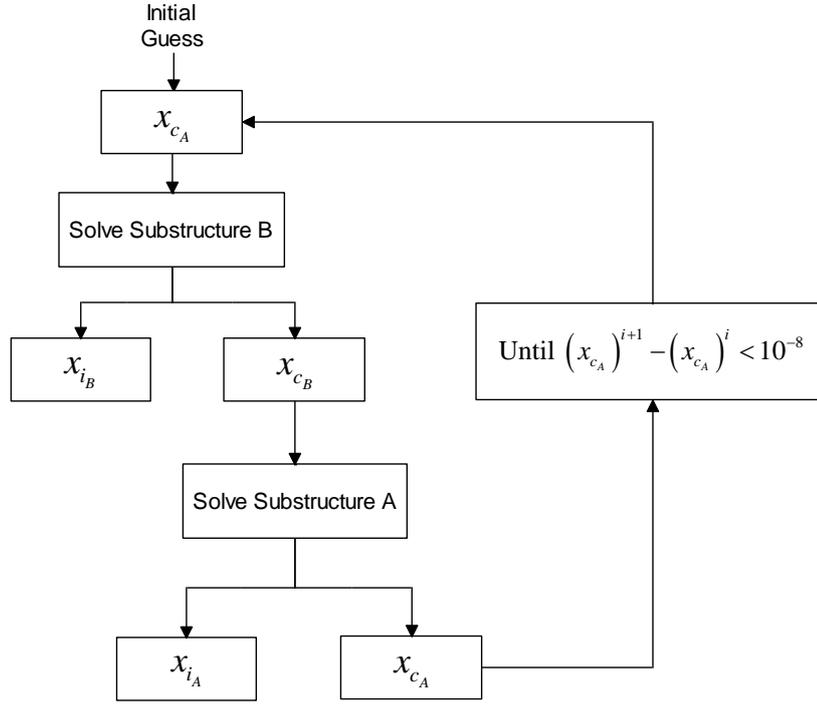
Fig. 2 Algorithm of the proposed method

2.3. Describing Functions of the Nonlinear Elements Used

In this paper, cubic stiffness and hysteretic dry friction are used as nonlinear elements in the substructures. The nonlinear forcing in a cubic stiffness element can be given as

$$F_N = k_c \cdot x^3 \,, \tag{15}$$

where k_c is the coefficient of the cubic stiffness nonlinearity. Describing function of the cubic stiffness nonlinearity is given as

$$v = \frac{3}{4} \cdot k_c \cdot X^2 \,, \tag{16}$$

where X is the amplitude of the relative displacemetn between the two ends of the cubic stiffness element.

There exists several friction models in the literature and in this paper, a one-dimensional Coulomb friction model with constant normal load is used. One-dimensional dry friction element and the corresponding hysteresis curve for a single harmonic input are given in Fig. 3.
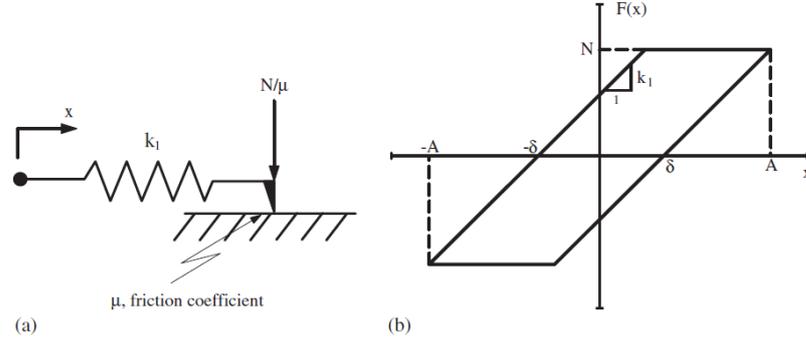
Fig. 3 (a) Schematic drawing, (b) corresponding hysteresis curve for dry friction nonlinearity [22] Describing function of the hysteresis curve given in Fig. 3(a) can be written as [22, 24]

$$
v = \begin{cases} \left[\dfrac{1}{\pi}\left(k - \dfrac{2 \cdot \mu \cdot N}{X}\right)\sqrt{1 - \left(\dfrac{k \cdot X - 2 \cdot \mu \cdot N}{k \cdot X}\right)^2} + \dfrac{k \cdot \psi_1}{\pi} - \dfrac{k}{2}\right] - i\left[\dfrac{4 \cdot \mu \cdot N(\mu \cdot N - k \cdot X)}{\pi \cdot k \cdot X^2}\right] & \text{for } |k \cdot X| > \mu N \\ k & \text{for } |k \cdot X| \le \mu N \end{cases}
$$

$$\tag{17}$$

where, k is the contact stiffness between rubbing surfaces, N is the constant normal force, μ is the dry friction coefficient and ψ_1

$$
\psi_1 = \pi - a\sin\left(\frac{k \cdot X - 2 \cdot \mu \cdot N}{k \cdot X}\right). \tag{18}
$$

3. CASE STUDIES

In this section, the proposed coupling method is demonstrated on different model. In the first one, substructures are coupled from a single DOF whereas in the second one, they are coupled from two DOFs.

3.1. Example 1: Coupling from a Single DOF

In the first example, application of the proposed approach is presented on a simple 8-DOF system shown in Fig. 4. Parameters of substructures A and B are given Table 1 and coupling elements at Table 2. The nonlinear elements used in the first case study, case study 1, are defined in Table 3.
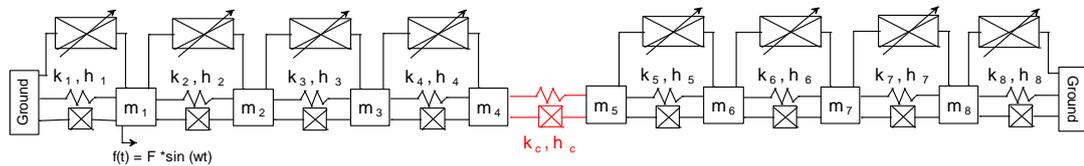
Fig. 4 Schematic view of 8-DOF coupled system

Table 1 Parameters of substructures A and B

Substructure A				Substructure B			
m_1 [kg]	m_2 [kg]	m_3 [kg]	m_4 [kg]	m_5 [kg]	m_6 [kg]	m_7 [kg]	m_8 [kg]
1	0,75	2	1	0,75	1	1	2
k_1 [N/m]	k_2 [N/m]	k_3 [N/m]	k_4 [N/m]	k_5 [N/m]	k_6 [N/m]	k_7 [N/m]	k_8 [N/m]
5000	2000	4000	6000	3000	2000	5000	3000
h_1 [N/m]	h_2 [N/m]	h_3 [N/m]	h_4 [N/m]	h_5 [N/m]	h_6 [N/m]	h_7 [N/m]	h_8 [N/m]
50	20	40	60	30	20	50	30

Table 2 Parameters of coupling elements

k_c [N/m]	h_c [N/m]
4000	40

Table 3 Nonlinear elements of substructures A and B in case study 1

Nonlinear Connection DOFs	Nonlinearity Type	Nonlinearity Coefficients
1-Ground	Cubic Stiffness [N/m^3]	-1x10^5
1-2	Cubic Stiffness [N/m^3]	-2x10^5
2-3	Cubic Stiffness [N/m^3]	-10x10^5
3-4	Cubic Stiffness [N/m^3]	-1x10^5
5-6	Cubic Stiffness [N/m^3]	-1x10^5
6-7	Cubic Stiffness [N/m^3]	-3x10^5
7-8	Cubic Stiffness [N/m^3]	-1x10^5
8-Ground	Cubic Stiffness [N/m^3]	-5x10^5

Normalized response of the 1st and the 8th DOFs obtained from the proposed nonlinear coupling method and by solving the entire system directly are given in Fig. 5 and Fig. 6. The response of the system is obtained for three different excitation amplitudes, 8N, 12N and 16N in order to observe the effect of cubic stiffness nonlinearity.
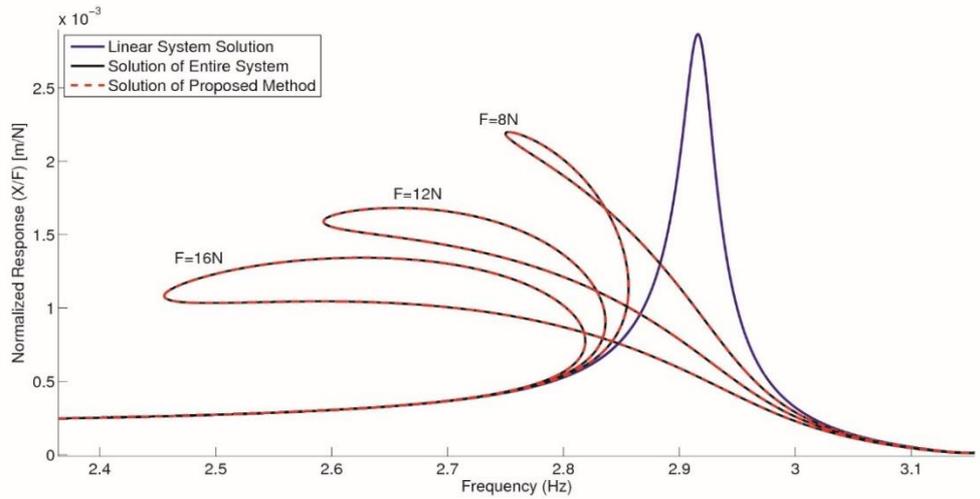
Fig. 5 Normalized response of the 1st DOF in case study 1
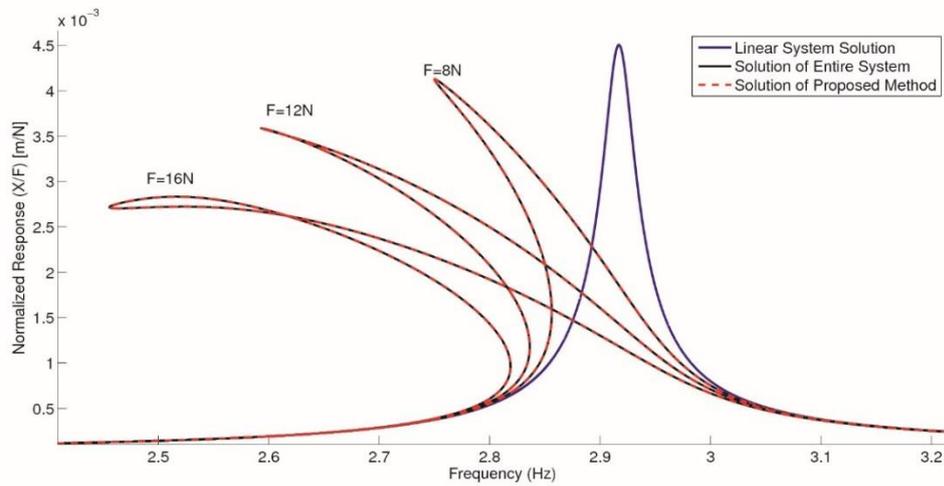
Fig. 6 Normalized response of the 8th DOF in case study 1

It can be seen from the Fig. 5 and Fig. 6 that, natural frequency is shifted due to cubic stiffness nonlinearity. Furthermore, more importantly the proposed method is in exact agreement with the ones obtained from entire system solution, even in unstable regions where the path turns back or intersects itself.

In the second case study, case study 2. 8-DOF system is obtained from the coupling of a 6-DOF, 2-DOF systems as shown in Fig. 7. Parameters of substructures *A* and *B* , and coupling elements are given in Table 4 and Table 5, respectively. The nonlinear elements present in the system are defined in Table 6.
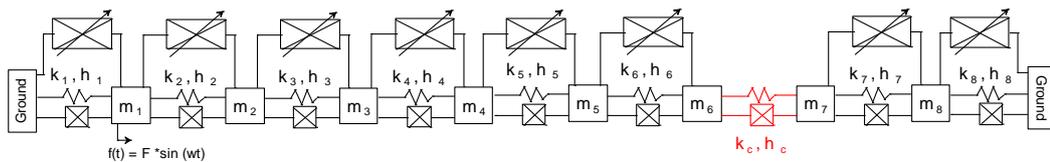
Fig. 7 Schematic view of 8-DOF coupled system

Table 4 Parameters of substructures A and B

		Substructure A				Substructure B	
m_1 [kg]	m_2 [kg]	m_3 [kg]	m_4 [kg]	m_5 [kg]	m_6 [kg]	m_7 [kg]	m_8 [kg]
0,75	2	1	1	2	0,75	2	2
k_1 [N/m]	k_2 [N/m]	k_3 [N/m]	k_4 [N/m]	k_5 [N/m]	k_6 [N/m]	k_7 [N/m]	k_8 [N/m]
3000	5000	4000	6000	6000	5000	3000	5000
h_1 [N/m]	h_2 [N/m]	h_3 [N/m]	h_4 [N/m]	h_5 [N/m]	h_6 [N/m]	h_7 [N/m]	h_8 [N/m]
60	100	80	120	120	100	60	100

Table 5 Parameters of coupling elements

k_c [N/m]	h_c [N/m]
2000	40

Table 6 Nonlinear elements of substructures A and B in case study 2

Nonlinear Connection Coordinates	Nonlinearity Type	Nonlinearity Coefficients
1-Ground	Cubic Stiffness [N/m^3]	1×10^5
1-2	Cubic Stiffness [N/m^3]	2×10^5
2-3	Cubic Stiffness [N/m^3]	1×10^5
3-4	Dry Friction [N]	μN
4-5	Cubic Stiffness [N/m^3]	1×10^5
5-6	Cubic Stiffness [N/m^3]	3×10^5
7-8	Cubic Stiffness [N/m^3]	1×10^5
8-Ground	Dry Friction [N]	μN

Corresponding response plots are plotted in Fig.8 and Fig.9 for the 1st DOF. Normalized responses of the coupled structure obtained from the proposed nonlinear coupling method and by solving the entire system directly are compared in Fig. 8 and Fig. 9. In Fig. 8, the response of the coupled system is obtained for 12N, 24N and 36N excitation force amplitudes, while the slip load of dry friction nonlinearities are kept constant as $\mu N = 100$N. In Fig.9, responses of the coupled structure are given for an external forcing of $F = 12$N and for different slip loads. Perfect agreement between the results obtained from the proposed nonlinear coupling method and the entire system solution is observed which verifies the developed coupling method.
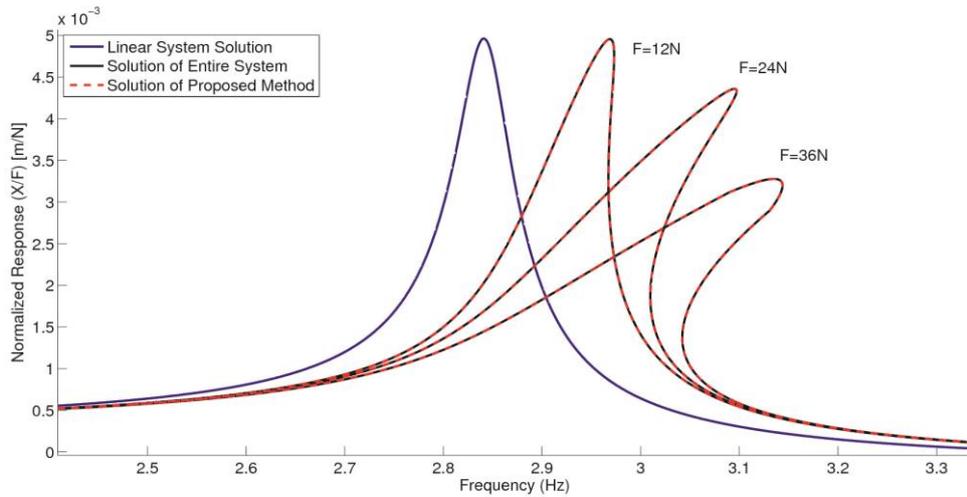
Fig. 8 Normalized response of the 1th DOF in case study 2
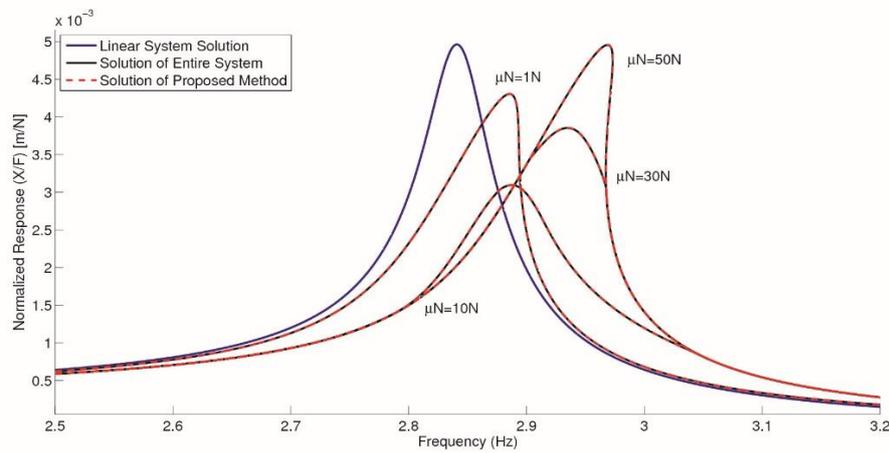
Fig. 9 Normalized response of the 1th DOF in case study 2

3.2. Example 2: Coupling from Multiple DOFs

In this section, a 6-DOF system is used as a case study as shown in Fig. 10. Parameters of substructures *A* and *B* , and coupling elements are given in Table 7 and Table 8. The nonlinear elements used in the third case study, case study 3, are defined in Table 9.
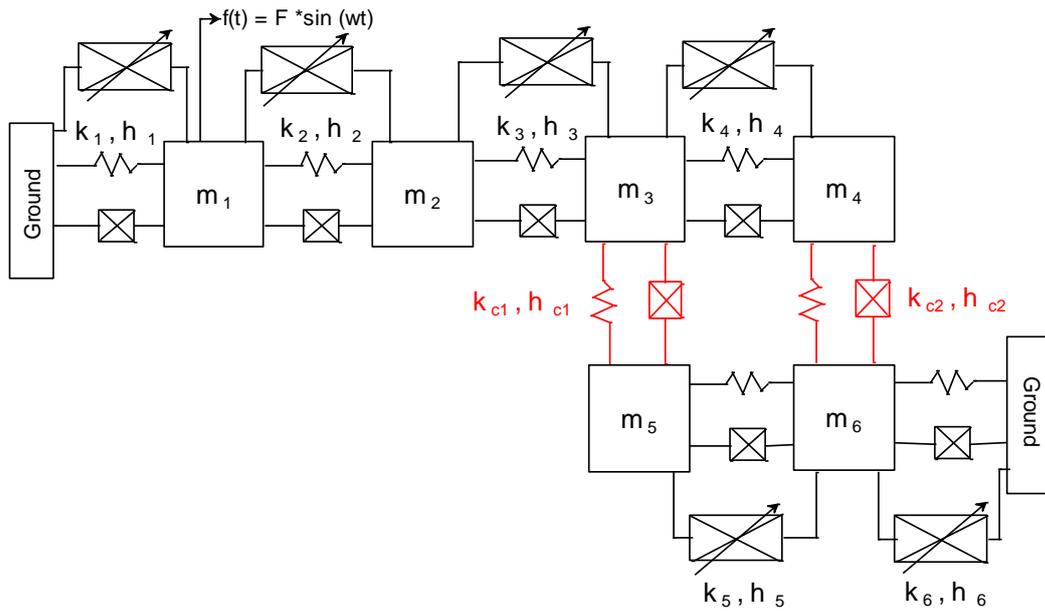
Fig. 10 Schematic view of 6-DOF coupled system

Table 7 Parameters of substructures *A* and *B*

Substructure *A*					Substructure *B*	
m_1 [kg]	m_2 [kg]	m_3 [kg]	m_4 [kg]		m_5 [kg]	m_6 [kg]
1	0,75	2	1		0,75	1
k_1 [N/m]	k_2 [N/m]	k_3 [N/m]	k_4 [N/m]		k_5 [N/m]	k_6 [N/m]
5000	4000	3000	4000		3000	5000
h_1 [N/m]	h_2 [N/m]	h_3 [N/m]	h_4 [N/m]		h_5 [N/m]	h_6 [N/m]
25	20	15	20		15	25

Table 8 Parameters of coupling elements

k_{c_1} [N/m]	h_{c_1} [N/m]	k_{c_2} [N/m]	h_{c_2} [N/m]
5000	25	3000	15

Table 9 Nonlinear elements of substructures A and B in case study 3

Nonlinear Connection Coordinates	Nonlinearity Type	Nonlinearity Coefficients
1-Ground	Cubic Stiffness [N/m^3]	1x10^5
1-2	Cubic Stiffness [N/m^3]	2x10^5
2-3	Cubic Stiffness [N/m^3]	1x10^5
3-4	Cubic Stiffness [N/m^3]	2x10^5
5-6	Cubic Stiffness [N/m^3]	1x10^5
6-Ground	Cubic Stiffness [N/m^3]	2x10^5

Normalized response of the 1st DOF obtained from the proposed nonlinear coupling method and by solving the entire system directly is given in Fig. 11. The response of the system is obtained for three different excitation amplitudes, 6N, 9N and 12N. It is observed that the results obtained from the proposed nonlinear coupling method and the entire system solution are in perfect agreement, for this case as well.
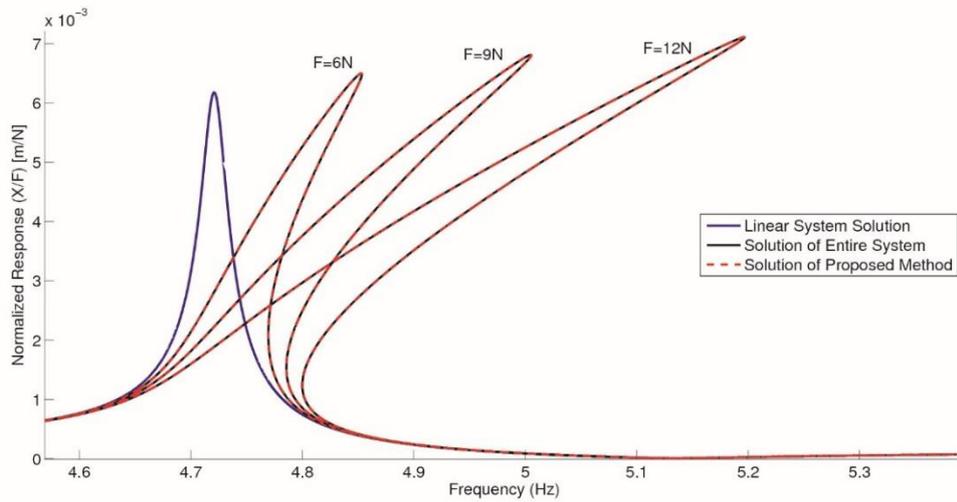
Fig. 11 Normalized response of the 1th DOF in case study 3

4. DISCUSSION AND CONCLUSION

In this paper, a new structural coupling method is introduced which is capable of coupling of two nonlinear substructures, where the connection elements can be nonlinear as well. Compatibility and equilibrium equations, which are derived from existing linear coupling methodology, are added to nonlinear equations of motions in order to model coupled system. The resulting nonlinear equations of motion of the coupled system are solved by using Newton's method with arc-length continuation. Cubic stiffness and hysteretic dry friction are used as nonlinear elements in the substructures. Applications of the proposed nonlinear coupling approach are demonstrated by numerical case studies. Two examples are considered in the case studies. In the first one two substructures are coupled from a single DOF; whereas, in the second example, two substructures are coupled from two DOFs. Normalized responses of the selected DOFs obtained from the proposed nonlinear coupling method and by solving the entire system directly are compared in order to verify the proposed method

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for different nonlinear systems. The results obtained from the proposed method and the ones obtained by directly solving the entire system agree perfectly with each other, which verifies the developed nonlinear coupling method.

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