A GENERALIZED MATHEMATICAL MODEL OF DC CATENARY LINES

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ABSTRACT

A GENERALIZED MATHEMATICAL MODEL OF DC CATENARY LINES

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Most of the subway systems are powered by electricity around the world. Although both AC and DC systems can be employed for energization, DC electrified systems are commonly preferred. The DC catenary line voltage can be calculated trivially for stationary systems at steady state. However, accurate modeling of the catenary voltage during the movement of the subway trains is cumbersome, as it requires solution of differential equation systems.

This thesis proposes a generalized model for catenary voltage variation of DC powered systems. One of the challenges of modeling problem is calculation of the electrical parameters of the system, because of the non-regular shape of the rails, on which the electric current flows. The thesis firstly develops an analytical method for accurate computation of electrical parameters of the system, and validates it using finite elements analysis. After that, by using the electrical parameters the voltage and current variations through the catenary line are investigated analytically for different mode of operations of the subway trains, such as constant speed and accelerating/decelerating operations. Because of the dynamical behavior of the system, both time and position dependent differential equations are defined during the analytical derivations.

The solution of the system based on differential equations is not trivial, and requires significant computational burden, Therefore, this work, proposes a linearized method in order to improve the computational performance of the derived model. The proposed linear method enables solution of the system with improved computational performance, while maintaining a high accuracy. Moreover, the effect of the regenerative braking and the effect of multiple substation and train are investigated. The results of the analytical solution of voltage variation are compared with the real life test results.

Keywords: DC Voltage Variation, DC Catenary Line, Distributed Parameters, DC System Analysis, Regenerative Braking

ÖZ

DC KATENER HATLARININ GENELLEŞTİRİLMİŞ MATEMATİKSEL MODELİ

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Dünyadaki çoğu metro sistemi elektrik ile çalıştırılmaktadır. Her ne kadar AC ve DC sistemlerin ikisi de enerjilendirmede kullanılabilse de, genellikle DC sistemler tercih edilmektedir. (DC katener hattı gerilim tahmini metronun elektriksel güvenliği ve tren kontrolü açısından önemli olduğu için metro operatörleri için önemli bir rol oynamaktadır.) DC katener hattı gerilimi durağan ve denge durumuna ulaşmış sistemler için kolaylıkla çözülebilmektedir. Fakat, metro trenlerinin hareketi sırasında katener hattı geriliminin doğru modellenmesi, diferansiyel denklem sistemlerinin çözümünü gerektirdiği için yavaş olmaktadır.

Bu tez, DC ile enerjilendirilmiş sistemlerin katener gerilim değişimleri için genelleştirilmiş bir model önermektedir. Modelleme probleminin zorluklarından biri, elektrik akımının aktığı rayların düzensiz şekli nedeniyle, sistemin elektrik parametrelerinin hesaplanmasıdır. Tezde, önce sistemin elektrik parametrelerinin doğru hesaplanması için analitik bir yöntem geliştirilmekte ve sonlu elemanlar analizi kullanarak doğrulanmaktadır. Bundan sonar, elektrik parametreleri kullanılarak, sabit hız ve hızlanma/yavaşlama gibi metro treninin farklı çalışma tarzları için katener hattı boyunca gerilim ve akım değişimleri analitik olarak araştırılmaktadır. Sistemin dinamik davranışından dolayı zamana ve konuma bağlı diferansiyel denklemler, analitik çözümlemeler sırasında tanımlanmaktadır.

Sistemin diferansiyel denklemlere dayalı çözümü kolay değildir ve önemli bir hesaplama yükü getirir. Bu sebeple bu çalışma, türetilmiş modelin hesaplama performansını arttırmak için doğrusal bir yöntem önermektedir. Önerilen doğrusal yöntem, yüksek doğruluk oranını korurken, geliştirilmiş hesaplama performansı ile sistemin çözümünü sağlar. Ayrıca, rejeneratif frenleme etkisi ve çok sayıda güç merkezinin ve trenlerin etkileri araştırılmıştır. Analitik çözümün sonucu elde edilen gerilim değişimleri, gerçek hayatta elde edilen test sonuçları ile karşılaştırılmıştır.

Anahtar Kelimeler: DC Gerilim Değişimi, DC Katener Hattı, Dağıtık Parametreler, DC Sistem Analizi, Regeneratif Frenleme

To my family ...

For all their love and support and putting me through the best education possible.

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DC	Direct Current
AC	Alternating Current
MVA	Mega Volt-Amperes
SCMVA	Short Circuit MVA
FEM	Finite Element Method
RPM	Revolutions Per Minute
SiC	Silicon Carbide

CHAPTER 1

INTRODUCTION

1.1 Problem Definition

Transportation technologies are evolving day by day. Previously, people were used to use oil powered engines. However, nowadays electric powered motors are getting popular. When the subway systems specifically examined, it can be seen that today most of the subway systems are powered by electricity. Although both AC and DC systems can be employed for energization, DC electrified systems are commonly preferred.

In DC subway system planning, to foresee the required investments for the railway power supply system, train power system approximators can be used, and these approximators need DC catenary line voltage estimation [1]. According to [2], modeling the DC catenary line voltage is also important for the simulation programs, which are used to find required equipment ratings and to calculate voltage regulation.

In this work it is aimed to develop a generalized analytical solution methodology for the voltage and current variations of moving loads in a DC system. In the specific problem, there exist a DC subway system, in which the voltage across the train is wanted to be simulated. Moreover, the considered system has supercapacitors on the substations, in order to store the energy created in regenerative braking of the trains.

Catenary line, or third rail, can be defined as the line which carries the required power from substation to the subway train. On the other hand, the return path to the electrical circuit is provided through the subway rails, which are also named as return rails in this work. In fact, the conductivity of the actual rails is not as good as the third rail; however, the cheapest solution is using them as the return path [3]. In order to supply DC power there are DC substations, in which there are transformers and twelve-pulse rectifiers.

The subway trains runs over the return rails. They are also connected to the catenary line with a *'shoe'*. The *'shoe'* is a kind of connector between the catenary line and the subway train, like the brushes in DC motors. Generally *'carbon alloy shoes'* are used in the subway systems. Therefore, they may wear rapidly because of the friction between third rail and shoes [4]. Another important part in the subway trains is the gear ratio of the trains. The gear box of the train has a constant ratio and at rated speed of the motors, e.g. is 2000 rpm, the velocity of the train is 17 m/s.

This thesis aims to develop a generalized catenary line model which can simulate the behavior of line voltage and current. To solve this problem, the subway system should be analyzed. The required electrical parameters, namely resistance, inductance and capacitance terms, are strongly related to the geometry and position of the third rail and return rails. Self resistance and inductance terms are the dominant electrical parameters for the system. In [5], during the modeling unit length inductance of the rail, the dependency of the radius is not clearly explained; moreover, the given method in [6] is not applicable for the considered case, which will be explained in *Section 2.1 Series Impedance of the System*. The capacitance formed between the rails and the earth is the last required parameter. In order to calculate this capacitance term, the type of the subway system should be analyzed in terms of stray current control methods. The main idea behind this, stray current phenomena is a result of the system insulation, which also directly affect the system capacitance because of the permittivity of the insulating material.

The topic of this thesis is a sub-topic of a TÜBİTAK project, which is The Project of Developing a New Generation Traction Inverter for Subway and

Light Rail Transit Vehicles, under the TÜBİTAK project number 5150038. This project aims to develop a SiC based traction inverter system and its controller. Another part of the project is, charging the super-capacitors with the generated energy in the regenerative braking operation. The research and development part of this project is the estimation of the catenary line voltage of the subway system, which is the subject of this thesis. The considered system consists of eight subway stops, eight DC substations, subway trains with regenerative braking capability, 28.5 km. railway in total length and super-capacitors and their charging units at substations.

In power system applications power-flow solutions play an important role. With the help of power-flow solutions, the necessary investments and the best operation point for the given power system can be determined easily [7]. However, the most beneficial point of the power-flow solution arises in multi-grid systems. With the help of this solution procedure voltage magnitudes and voltage phase angles of each substation can be predicted easily [8].

In power flow problem, in fact some optimization problems are solved. By using iterative processes, the error between actual value and estimated value is tried to be minimized. However, in multi-grid systems it might become time consuming due to multiple nodes. In order to speed up the process some assumptions can be used, which are beyond the scope of this work.

There are two main types of power flow analysis; AC power flow analysis and DC power flow analysis. In fact, these two power flow analyses have same logic at base level. However, in DC power flow solution, which is an approximation to the classical AC power flow solution, the contribution of the reactive power is neglected, while in AC the effect of this is included.

In classical power flow solutions one of the main assumptions is neglecting the changes in the transmission line parameters, which is a realistic assumption for most cases. However, in this specific case the power carrying line, which is called catenary line, is changing its effective length as the subway train moves. As a result, the electrical parameters vary during the movement of the train. Moreover, the mode of operation of the train changes, such as speeding up or slowing down. Due to the change in operation mode, the required power by the subway train can also change. As a result, current drawn from the catenary line changes with time. Therefore, the variations will create a time dependent characteristic, although the system operates with DC.

Due to the changing impedance of the system, computation of voltage variation across the subway train has a challenging computational burden. In literature there are some existing methods to simulate the subway DC catenary line voltage. Using an iterative load flow analysis for the DC catenary line is one of them. However, during this process, iterations may not converge to a result due to different range of electrical parameters or due to starting point of iterations [9], [10]. For example, in [9] Zollenkopf bifactorization approach is used; however, [2] raises some questions about the convergence of the used approach. Moreover, in [2] it is accepted that, some potential refinements are sacrificed to increase the computational speed. In [11], the current injection and conductance matrix methods are discussed; however, it does not include the time dependent changes in its system model.

In [10] it can be seen that, the effect of the inductance and capacitance are neglected in iterative processes. Besides that, iterative processes may not give a simple solution methodology for the specific problem. Using linear solution method directly [12] also may not be a good way to find the voltage across the train, since it omits the effect of the change of the electrical parameters as the subway train moves. However, in [13] it is shown that well-known transmission line equations can be used in DC rail voltage modeling, and distributed parameter models can be utilized.

As explained above in the considered system there exist super-capacitor storage system to store the energy appear in the regenerative braking. In [14] and [15] the of onboard train storage is investigated. However, they do not mention about the energy storage devices on substation. Moreover, effects of the regenerative braking on the catenary line are discussed in [16]; which states that if the catenary line voltage drops below a predefined level, over-current protection of the train decreases the demand power of the subway train. On the other hand, if the voltage rises above a thresh-hold, then the squeezing control will not give all the energy created in the regenerative braking to the catenary line. In [17], while the substation modeling is explained, also the effect of AC grid to the AC and DC subway systems is explained.

To calculate the subway train voltage accurately, distributed parameter model of the system is used. In normal conditions, distributed parameter solutions and lumped parameter solutions give satisfactory results in short distances. However, change of movement characteristics of the train may result in a different voltage variation characteristic than the expected lumped parameter model. Moreover, the drawn power by the train may vary during different mode of operations, and this may result in instantaneous changes in the catenary line voltage and current. Therefore, it can be said that, the system has both time and position varying characteristics. So that, time and position dependent differential equations are used to develop the generalized solution of the catenary line [18]. However, using both time and position dependency will result in appearing of higher order terms, as a result, the computational burden of the solution increases. In this work a linearized model will be proposed, which will take into account the motion of the train, and the effect of these higher order terms will be eliminated. With the help of linearized model, the computational performance is aimed to be maximized with minimum loss of accuracy. Moreover, it will be shown that the solution of the lumped parameter model will give close results to the solution of distributed parameter model.

1.2 Thesis Outline

In this thesis, to give a thorough approach to all parts, the work is divided into five chapters.

In the Chapter 1, the introduction part, the problem definition is given. In that part, the main motivation behind this work is tried to be explained. Existing solution methods in the literature are also reviewed in this part. Their advantage and disadvantages are explained. Moreover, the innovative side of the proposed method is also explained in that part.

The Chapter 2 is 'Derivation of Electrical Parameters'. The distributed parameter model of the system is going to be used in the proposed solution procedure. To explain the electrical parameter finding methodology, the required information will be given in this chapter. In that part there are two main sections; finding of the series impedance, and the finding of the shunt admittance. The detailed derivation of each parameter will be given.

In Chapter 3, all the proposed solution methodologies will be explained. Firstly, the solution methodology is explained step by step. Time and position dependency of the voltage and current relations are simplified, and solved for the constant speed operation of the train. After that the acceleration of the train, will be investigated. To make the analysis realistic, effect of the multiple substation is going to be included. Furthermore, the effect of the multiple train operation is examined. Finally, the variations during the regenerative braking and the required adaptation will be explained.

The validation of the method is given in Chapter 4. In this chapter, field data and the simulation results are compared. With the help of data provided by the co-operative organization, the proposed method is validated; although, some assumptions have to be made, since some necessary data are not available. Nevertheless, still it can be seen that for the same part of travel, the trends in voltage variation are same in simulation results and measured data.

In the final part of the thesis, which is Chapter 5, the explained work will be summarized and concluded. Moreover, some possible future improvements for the industrial use of the given method are shortly mentioned.

CHAPTER 2

DERIVATION OF ELECTRICAL PARAMETERS

In order to start voltage characteristic modeling of the catenary line, firstly the electrical model of the subway system should be found. To achieve this, firstly the electrical parameters of the rails should be analyzed. The considered system is modeled as a combination of electrical parameters, namely resistance, inductance and the capacitance.

In this part of the thesis, the distributed parameters will be analyzed. Even though both lumped parameter and distributed parameter solutions give satisfactory results, in *Chapter 3 The Proposed System Model* part distributed parameter model will be used as the starting point to the solution procedure. And later it will be shown that lumped parameter assumption, which improves the computational performance, will give satisfactory results, too.

In this Chapter, the resistance and inductance calculations for the railway structure will be given in *Series Impedance of the System* part, and the capacitance calculations of the system will be given in *Shunt Susceptance of the System* part.

2.1 Series Impedance of the System

To examine both the impedance and the susceptance of the system, the physical structure of the subway rails should be known. In Figure 2.1 cross-sectional view of the physical structure can be found:

In this part two main electrical parameters, namely the resistance and the induc-

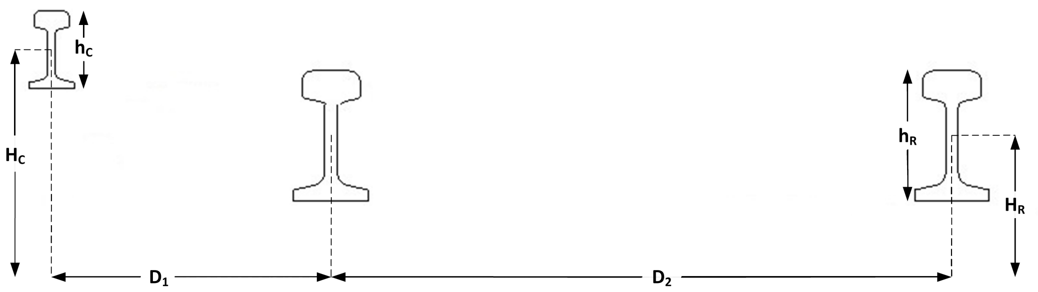

Figure 2.1: Cross-Sectional View of Physical Structure

tance, will be investigated. In [6] and [19], calculation of electrical parameters are explained both in analytical way and experimental way. Firstly, finding the resistance of the system will be given.

According to [8], the well-known resistance formulation for the DC systems can be written as follows:

$$R = \rho \frac{l}{A} \tag{2.1}$$

where ρ, l and A represents the resistivity, length and cross-sectional area of the considered material respectively. Resistivity of the material can be written in terms of conductivity (σ) as follows,

$$\rho = \frac{1}{\sigma} \tag{2.2}$$

According to [20], if the temperature change is limited, the rail conductivity can be linearized as follows:

$$\sigma = \sigma^{'}(1 + \beta T)^{-1} \tag{2.3}$$

In equation 2.3, 'T' represents the temperature difference between the ambient temperature and the reference temperature. While '$\sigma^{'}$' is the conductivity at the ambient temperature, 'σ' represents the conductivity at the reference temperature. Moreover, 'β' is taken as a temperature change coefficient, and it can be taken in between $0.003K^{-1}$ and $0.008K^{-1}$ according to [20]. In [20], temperature range taken in between $-10°$C and $30°$C; however, in the considered case the temperature change will be very small, as the system is an underground system. So that, the variation in conductivity will be much smaller. In [20], the authors assume σ values for the rails as constant; hence, in the considered case taking σ as constant will be a reasonable assumption.

After the constant conductivity (or resistivity) assumption, the resistance of the rails per unit length can be calculated as follows:

$$R_C = \rho_C \frac{1}{A_C} \Omega/m \tag{2.4}$$

$$R_R = \frac{1}{2}(\rho_R \frac{1}{A_R})\Omega/m \tag{2.5}$$

In equation 2.4 and equation 2.5 the subscript C is used for the power carrying catenary line (third rail), and subscript R represents the return rails. In equation 2.5, $(\frac{1}{2})$ term comes from the fact that two parallel return rails are located in the subway system. Moreover, since the resistance per unit length is investigated, in equation 2.4 and equation 2.5 the length of the material is taken as 1 meter (l = 1m).

Another important parameter in the resistance calculations is the contact resistance between the rails and the train. It might play an important role in the total resistance of the system. However, due to the lack of measurements this characteristic cannot be investigated in detail. In order to eliminate the effect of the contact resistance, during the solution procedure a constant contact resistance term can be added in to the calculated resistance value.

Inductances of the rails are the other important electrical parameter for the series impedance of the subway system. Inductance term is strongly depended on locations and geometry of the rails.

In Figure 2.1, the physical configuration of the rails and the corresponding rail parameters can be seen for the considered system in the project. Again, subscript C is used for catenary line, and R used for return rails. While, the distance between two return rails is represented as D_2, the distance between catenary line and the closest return rail can be represented with D_1. Moreover, 'H' represents the distance between the center of the rails and the ground level, and 'h' represents the height of the rail itself.

When the system is electrically analyzed, intuitively inductance term can be seen as unnecessary, due to the DC electrification. However, rapid changes in the power demand of the subway train can increase the importance of the effect of inductance term. Furthermore, even if the current demand remains constant the effect of changing equivalent impedance of the rails caused by the movement of the subway train, gives an additional importance to the consideration of inductance.

It can be seen that the cross-sectional view of the physical structure of the subway system is very similar to the well-known transmission systems. Both of them have three conductors to carry electricity power. In transmission case all lines are balanced; on the other hand, in subway system case, while one rail carrying all the required power to the subway train other two rails just create a return path to the circuit. Therefore, although the rail currents are not balanced, their summation equals to zero. In order to find an analytical solution to the inductance of the physical structure, some assumptions should be made because of the irregular shape of the rails. If the shape of the rails can be assumed as circular, then the well-known inductance relation for transmission lines [7], [8] will be applicable for this case. The formulation for the inductance term in transmission lines is given as follows;

$$L_{avg} = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s} H/m \tag{2.6}$$

In order to be able to use the formulation given in equation 2.6, the conductors should have a circular shape. According to [6] at very low frequencies, rail shapes can be assumed circular, such that the equivalent radius is calculated by using the rail cross-sectional area. Since the considered system has already electrified by DC, the equivalent radius can be calculated as follows based on [6].

$$r_{eq} = \sqrt{\frac{A}{\pi}} \tag{2.7}$$

In equation 2.7, 'A' represents the cross-sectional area of the rail and 'r_{eq}' represents the equivalent radius of the rail for inductance calculations. The equivalent radius is found by utilizing a circle with the same cross-sectional area of the rail (area assumption).

In this study, a new methodology is proposed in order to find the equivalent radius, in which the height of the rail is assumed as the diameter of the equivalent circle. It will be shown that the proposed method will result in a better calculation accuracy compared to equation 2.7. The proposed method can be given as follows;

$$r_{eq} = \frac{h_R}{2} \tag{2.8}$$

In equation 2.8, 'h_R' represents the height of the rail as seen in Figure 2.1. The equivalent radius is found by utilizing a circle with a diameter of 'h_R' (height assumption).

While making this assumption, it was seen that the shape of the rail is very similar to a circle without the side parts. When the finite element method (FEM) analyses are applied to the rail, it was seen that the effects of these side gaps are very low. Especially, at the points far away from the rail the effect of the side gaps can be neglected. The self-inductance FEM result can be seen in Figure 2.2. As a result, in the proposed method, the height of the rail is taken as the diameter of the equivalent circular rail.

In order to compare both approaches, which are given in equation 2.7 and equation 2.8, a basic method should be defined. One of the easiest way to check the accuracy is comparing the FEM analysis of self-inductance of the rail with both approaches.

In FEM analysis a rail is placed in space and the boundary distance 'D' for the

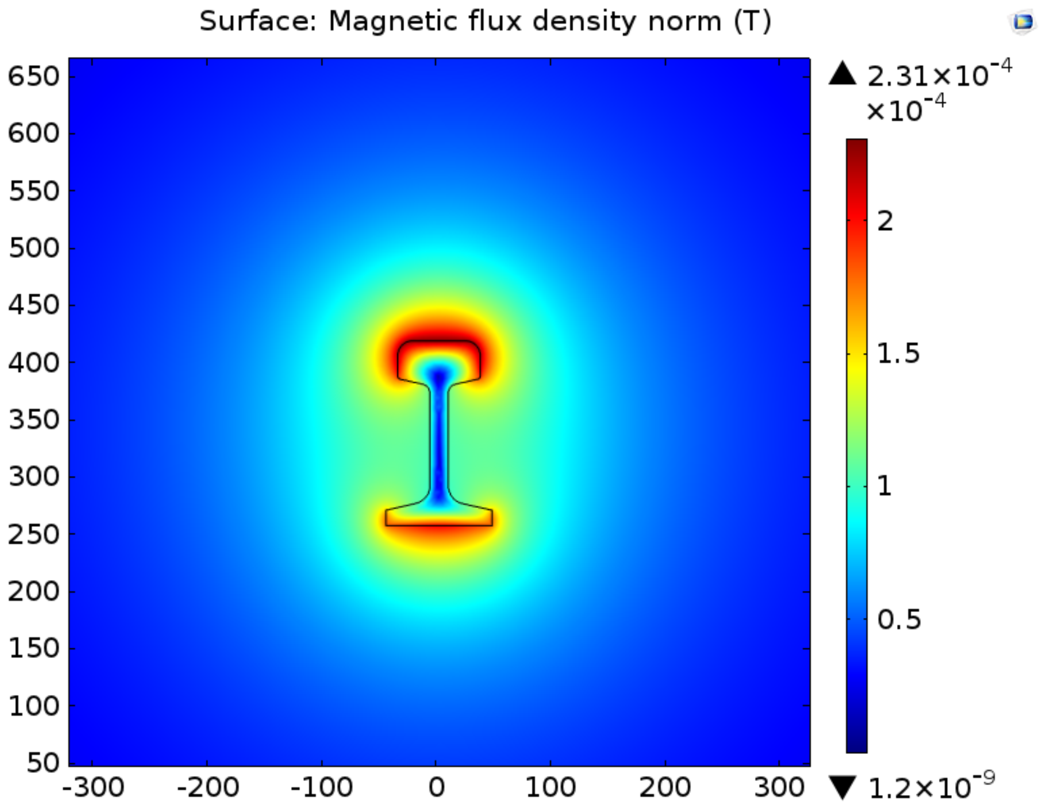

Figure 2.2: Finite Element Method Analysis for Rail ($I_{rail} = 59.796A$)

magnetic field is given by the user. In Figure 2.2, x and y coordinates represents the position of the rail in space. Moreover, the color chart gives the magnetic flux density in 'Tesla'. In order to model the rail, 'Physics-controlled mesh' sequence type is used in 'Extremely fine' element size. During the calculations 10000 A/m^2 external current density is applied to the rail. Hence, the current passing on the rail is observed as 59.796 A. When the adjusted settings are applied and run; the total magnetic energy inside the boundary region can be extracted from the FEM program and the inductance of the material, which gives the self-inductance in this scenario, can be calculated easily with the well-known magnetic energy formulation:

$$W_{magnetic} = \frac{1}{2} L i^2 \tag{2.9}$$

By knowing the magnetic energy, which is represented as $W_{magnetic}$ in equation 2.9, inside the boundary and the current which passes through the rail, represented as ‘i’, the self-inductance of the rail can be found easily as follows;

$$L = \frac{2 \times W_{magnetic}}{i^2} \tag{2.10}$$

Now, in order to compare the approaches, the self-inductance of the rail should be analytically calculated by using both assumptions given in equation 2.7 and equation 2.8. The analytical solution procedure starts from;

$$L = \frac{\lambda}{I} \tag{2.11}$$

where ‘λ’ represents the flux linkage and ‘I’ represents the encircled current.

According to [8], inductance of a conductor due to the internal flux can be found as follows;

$$L_{int} = \frac{1}{2} \times 10^{-7} H/m \tag{2.12}$$

Moreover, the flux linkage between a point external to the conductor and the conductor can be represented as follows;

$$\lambda_{pnt} = 2 \times 10^{-7} I \ln \frac{D}{r_{eq}} H/m \tag{2.13}$$

By using the equation 2.11;

$$L_{pnt} = 2 \times 10^{-7} \ln \frac{D}{r_{eq}} H/m \tag{2.14}$$

Note that, equation 2.12 covers the area from the center of rail to its equivalent radius, which means inner side of the rail (gray area in Figure 2.3). Furthermore, equation 2.14 covers the remaining area, which means the area between the outer side and the external point (yellow area in Figure 2.3). Figure 2.3 shows the covered areas clearly:

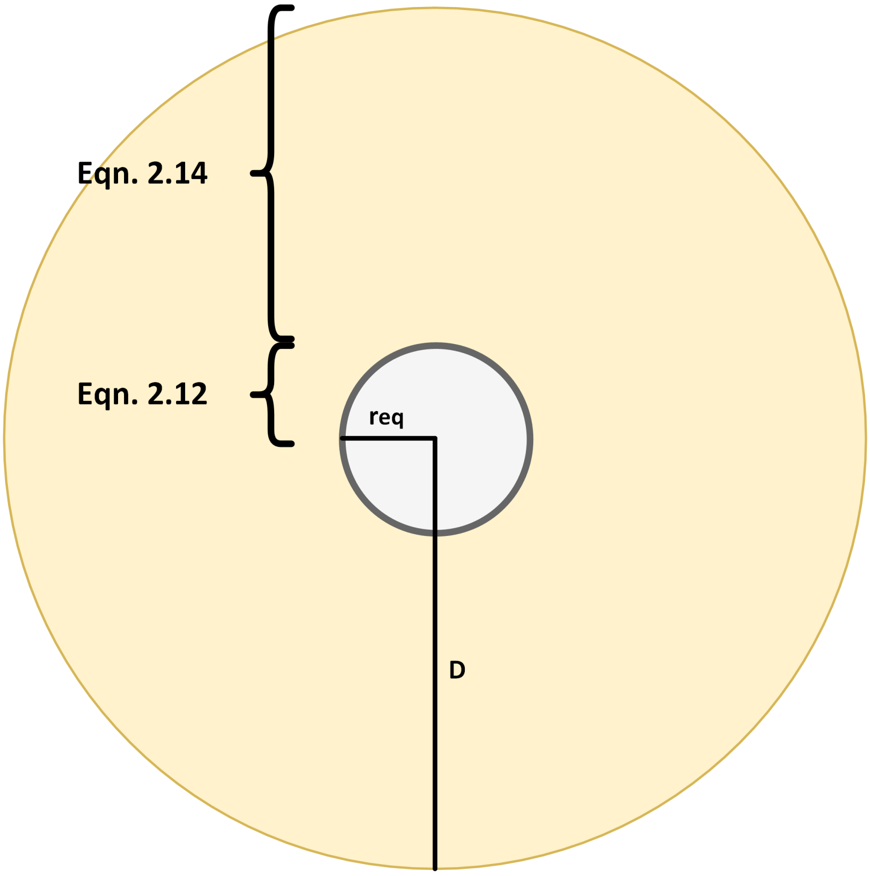

Figure 2.3: Coverage of Equations

When equation 2.12 and equation 2.14 are combined the total inductance will give the total self-inductance in the defined boundary distance ‘D’;

$$L_{self} = (\frac{1}{2} + 2\ln\frac{D}{r_{eq}}) \times 10^{-7} H/m \tag{2.15}$$

When the FEM analysis result is compared with both approaches, which are given in equation 2.7 and equation 2.8, both results give satisfactory results. However, the proposed method gives a much closer result to the FEM analysis result. The results for different boundary distances ‘D’ can be seen in below figure;

In order to understand the accuracy of the proposed method, errors of the approaches with respect to the FEM analysis are given in Figure 2.5;

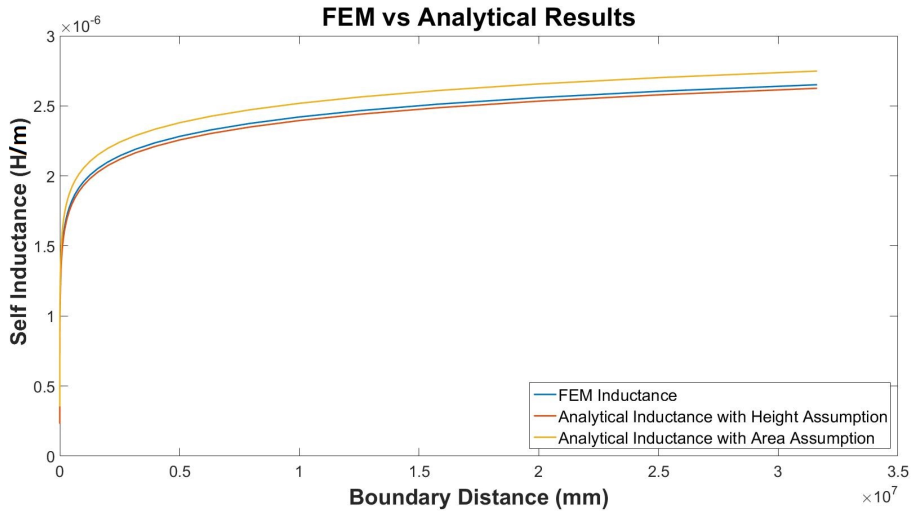

Figure 2.4: Unit Length Self-Inductance Values for Different Boundary Distances

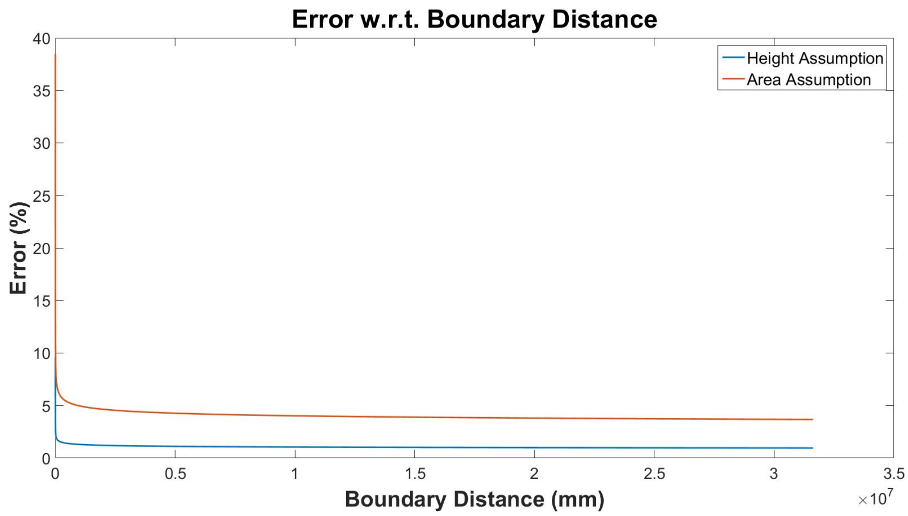

Figure 2.5: Errors of Both Assumptions with respect to FEM Analysis

After proving the accuracy of the proposed assumption, which is given in equation 2.8, the total inductance of the system can be found easily as follows;

$$L_{total} = L_{catenaryline} + L_{returnrails} \tag{2.16}$$

Since in the considered system summation of currents flowing on the catenary line and the return rails is zero, the inductance of the system can be calculated as follows by using the Ampere's Laws [8]:

$$L_{catenaryline} = 2 \times 10^{-7} \times \ln(\frac{\sqrt{D_1 \times (D_1 + D_2)}}{r_C^{'}})H/m \qquad (2.17)$$

$$L_{returnrails} = 2 \times 10^{-7} \times \frac{\ln(\frac{D_1^2}{r_R^{'} \times D_2}) \times \ln(\frac{(D_1+D_2)^2}{r_R^{'} \times D_2})}{\ln(\frac{D_1^2 \times (D_1+D_2)^2}{r_R^{'2} \times D_2^2})}H/m \qquad (2.18)$$

2.2 Shunt Susceptance of the System

The other important parameter for the system is the capacitance between the rails and the earth. In order to calculate the capacitance of the rails, again the well-known transmission line approach in [7], [8] can be used. According to [13], the insulation material used to prevent stray currents, creates a shunt branch. However, in [13], only the conductance of the insulation material is investigated. In this thesis the capacitance between the rails and the earth will be examined. Note that, the permittivity (ϵ) of the insulation material between the rails and the earth affects the capacitance of the system.

The main reason of the insulation material is preventing the stray currents. Stray current is a kind of leakage current that passes through the earth. Normally, current should follow the return rails to create a closed loop. However, due to the current division it may leak to earth, such that the electric current follows the low resistance path. In modern world under the ground there are a lot of metallic pipes. So, one of these pipes may form a good low resistance path for the stray currents. Here the importance of stray current arises. In DC systems, stray currents may lead corrosion both on the rails and on the metallic pipes, which are next to the subway system.

It should be noted that this corrosion is mainly due to the DC system; on the other hand, the effect of the AC systems is very small. Moreover, the main reason of the corrosion is the absence of enough electrons in earth, so metals supply electrons to the earth. Because of that, corrosion is seen only in the places that the current lefts the metallic structure and goes into the earth, due to the lack of free electrons on the earth with respect to the metals [21].

Corrosion causes the aging of the metallic underground pipes earlier. Since they are located under the ground, their maintenance is costly. In order to solve this problem, some stray current control methods are used; because, the prevention of the problem is cheaper than the cost of the consequences. In literature, there are four main stray current control methods namely, the railway earthing systems, stray current collection systems, fully insulated earthed systems and drainage bonding [21]. The considered subway system lies in the fully insulated earthed system category in which the rails are insulated from the sleepers or ballast. However, the return rails are creating a return path and they are connected to the local earth at each substation. The configuration of the subway system can be visualized as given in Figure 2.6.

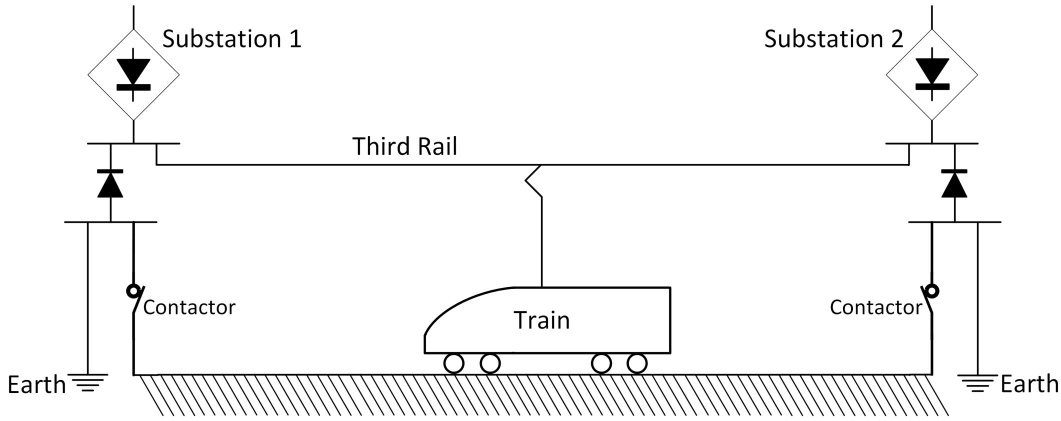

Figure 2.6: The Schematic of the Fully Insulated Earthed System

Since the permittivity of the insulation material is known the capacitance of the rails can be found both analytically and numerically.

According to [7], the voltage drop between two points due to a point charge (q) can be shown as follows:

$$v_{12} = \int_{D_1}^{D_2} E dx = \int_{D_1}^{D_2} \frac{q}{2\pi\epsilon x} dx = \frac{q}{2\pi\epsilon} \ln \frac{D_2}{D_1} \tag{2.19}$$

where D_1 and D_2 represents the distance of the points from the point charge location.

In the considered case, by the use of Figure 2.1. and equation 2.19, the potential difference between the rails and the earth can be written as follows:

$$v_C = \frac{q_C}{2\pi\epsilon} \ln \frac{H_C}{(\frac{h_c}{2})} V \tag{2.20}$$

$$v_{R1} = \frac{q_{R1}}{2\pi\epsilon} \ln \frac{H_R}{(\frac{h_R}{2})} V \tag{2.21}$$

$$v_{R2} = \frac{q_{R2}}{2\pi\epsilon} \ln \frac{H_R}{(\frac{h_R}{2})} V \tag{2.22}$$

In order to find the capacitance between rails and the ground, the relation between q, C and v should be used:

$$C = \frac{q}{v} \tag{2.23}$$

The capacitance values can be found as follows:

$$C_{catenaryline} = \frac{2\pi\epsilon}{\ln \frac{H_C}{(\frac{h_c}{2})}} F/m \tag{2.24}$$

$$C_{R1} = C_{R2} = \frac{2\pi\epsilon}{\ln \frac{H_R}{(\frac{h_R}{2})}} F/m \tag{2.25}$$

It should be noted that, in the subway system, catenary line is used to carry power from the substation to train. And the same current returns from the running rails. So that, the running rails are parallel to each other. Hence the equivalent return rail capacitance can be written as follows:

$$C_{returnrails} = 2 \times \frac{2\pi\epsilon}{\ln \frac{H_R}{(\frac{h_R}{2})}} F/m \tag{2.26}$$

2.3 Validation of the Electrical Parameters

In the previous sections, which are *Section 2.1 Series Impedance of the System* and *Section 2.2 Shunt Susceptance of the System*, the analytical methodology is given for the calculation of required electrical parameters. In this part the required field tests will be explained to validate the analytical results.

According to [19] for the traction systems energized with over-head line, six measurements are required. However, in the considered case, catenary line is placed as a third rail. Therefore, the measurements given in [19] will be modified, such that using only five measurements is sufficient for the considered case.

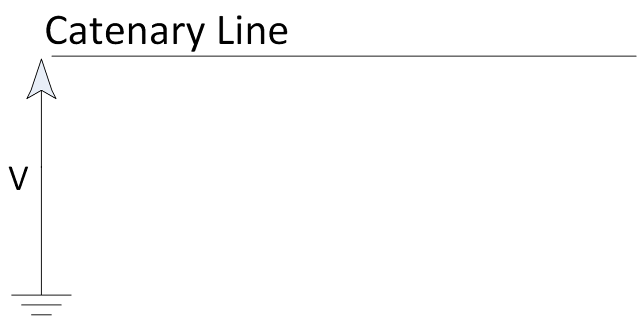

Figure 2.7: Configuration 1 - Open Loop Characteristic of Catenary Line

In the first configuration, which is given in Figure 2.7, a DC voltage will be applied to the one end of the catenary line and the other end of the line will be open circuited. With the help of this configuration, the capacitance between the catenary line and earth will be measured.

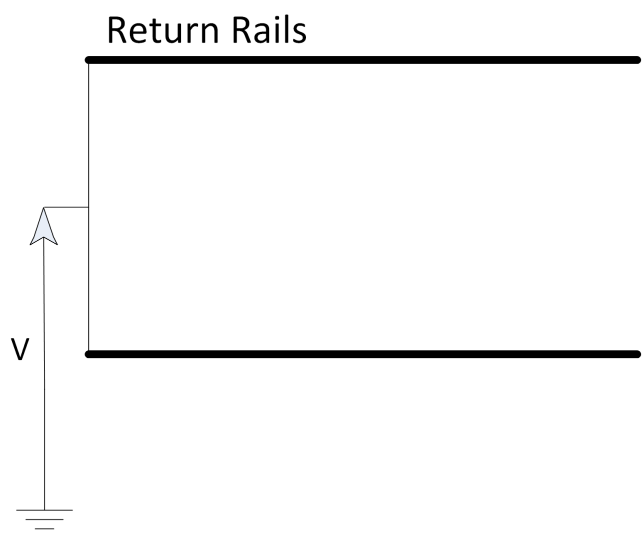

Figure 2.8: Configuration 2 - Open Loop Characteristic of Return Rails

In the second configuration, given in Figure 2.8, one end of the return rails are shorted and a DC voltage will be applied that side. By remaining the other end open circuited, the capacitance between the return rails and earth will be measured.

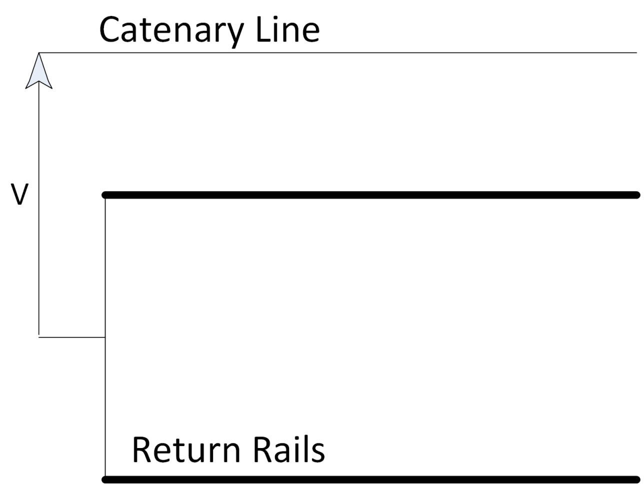

Figure 2.9: Configuration 3 - Open Loop Characteristic of the System

In the third configuration, given in 2.9, one end of the return rails is shorted and a voltage applied between this end of the return rails and the catenary line. The other end of them are remained as open. It is aimed to measure the capacitance between the catenary line and the return rails, with this configuration. In fact, in the determination of the electrical parameters, this capacitance term is not considered. Because, the distance between the rails and the earth is smaller compared to the distance between the rails, and there is an insulation material ,which has relatively high permittivity compared to air, between rails and earth. Therefore, the capacitance between the rails is neglected in *Section 2.2 Shunt Susceptance of the System* when it is compared to the capacitance between rails and earth. This measurement is configured to check whether the given assumptions are applicable or not.

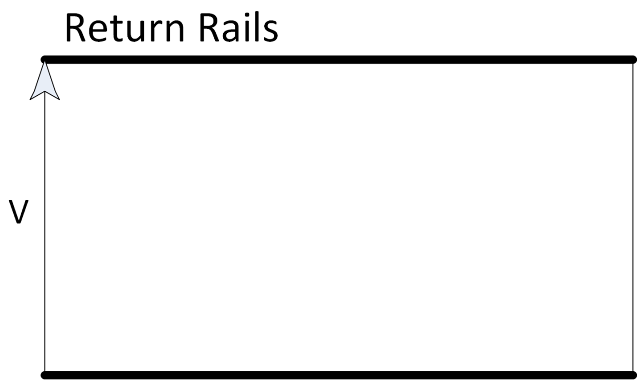

Figure 2.10: Configuration 4 - Short Circuit Loop Characteristic of Return Rails

In the fourth configuration, given in 2.10, a voltage will be applied in between one end of the return rails. The other ends will be short circuited. With the help of this configuration the series impedance of the return rails will be measured.

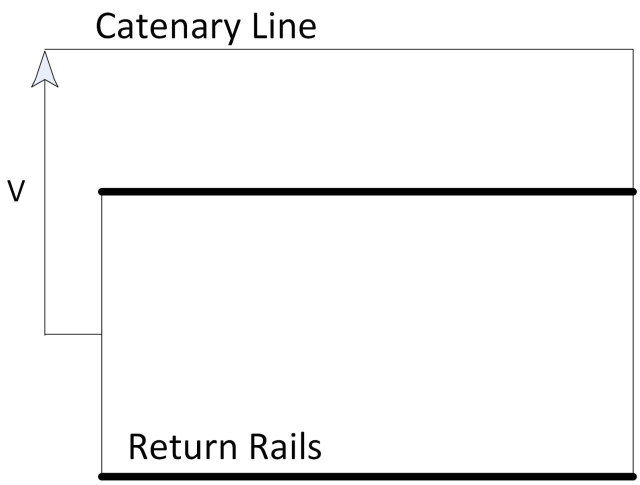

Figure 2.11: Configuration 5 - Short Circuit Loop Characteristic of the System

In the fifth configuration, given in Figure 2.11, return rails will be connected in parallel and a voltage will be applied between return rails and the catenary line. The other end of the catenary line is short circuited to the return rails. With the help of this configuration, it is aimed to measure the series impedance of the system.

In Figure 2.12, an example of short circuit test on the third rail system is shown, [22].

Figure 2.12: An Example of Short Circuit Test of Catenary Line and Return Rail

In this part of the thesis, finding of the electrical parameters of the subway system has explained. Both the series impedance and the shunt susceptance of the rails are investigated. Rather than using the given approaches in the literature, a new approach is proposed in the inductance calculations and verified with the help of FEM modeling. In order to check the correction of the analytical calculations, some tests have been proposed. The configuration of these tests have also been given. However, in this thesis, the verification of the proposed parameter calculation with real data is performed by comparing the model outputs, namely catenary line voltage and currents with field measurements in *Chapter 4*.

CHAPTER 3

THE PROPOSED SYSTEM MODEL

To model the system electrically, first the electrical parameters should be known. In the previous chapter, derivation of the electrical parameters is given in detail. By using the given formulations, capacitance, inductance, and resistance terms can be calculated easily.

In the modeling process, it is important to decide which model will be used. Since the system is a DC fed system, one may use only the resistance values for the modeling [9]-[12]. However, in this study a more generic approach will be used. This approach will add the effect of the movement of the train, in other words it will use the change of equivalent impedance as the train moves. Hence, the use of inductance and capacitance terms is getting important in this approach.

The similarity between transmission lines and subway system was mentioned in previous section. To clarify this, it can be said that in both systems, there are three conductors and the summation of the line currents is equal to zero. Because of that, the well-known methods to find transmission line parameters were applied to the considered subway system in the previous chapter. Similarly, the analytical solutions used in current and voltage relations can be applied to subway system, too.

There exist three types of transmission lines in terms of their lengths, namely short lines, medium-length lines, and long lines, according to [8], [7]. To summarize these approaches, in short lines the shunt admittance of the line is neglected and the series impedance of the line is represented as a lumped parameter, in medium-length lines both shunt admittance and the series impedance of the line represented as lumped parameters. However, in long lines the distributed parameter model, which gives more accurate results compared to other approaches, is used.

In the *Distributed Parameter Model* part, the main principles in long line modeling will be explained and the relevance of this approach to the considered subway system will be discussed. In the *Proposed Modeling* part, the dynamic solution model of current and voltage relations will be given.

3.1 Distributed Parameter Model

Distributed parameter model states that, a line is formed by the series connection of unit length R-L-C circuits. Each R-L-C circuit can be represented as a series R-L impedance and a shunt susceptance, as given in Figure 3.1.

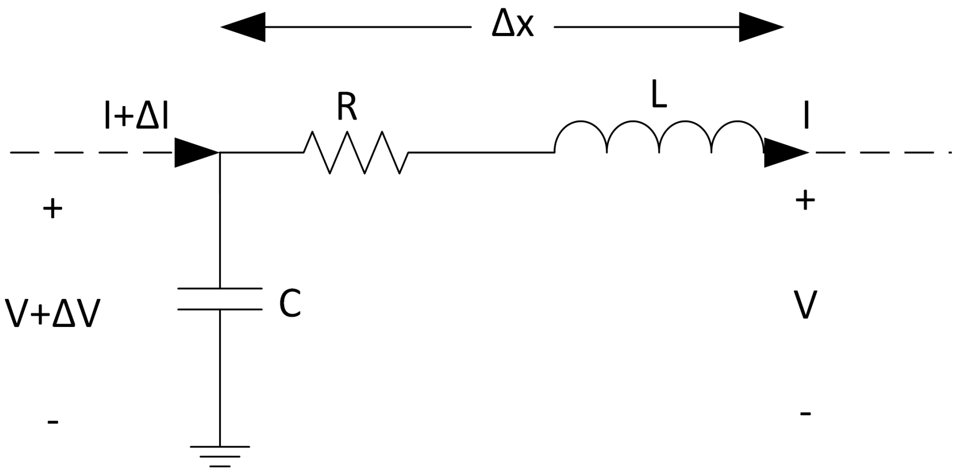

Figure 3.1: Unit Length R-L-C Circuit Used in Distributed Parameter Modeling

In Figure 3.1, R and L represents the series branch components, namely resistance and inductance of the line. C represents the shunt branch component, namely capacitance of the line. While V is representing the potential difference between the corresponding points, I represents the current flowing at the given direction, and Δx shows the unit change in the position. If the left side of the figure is assumed as the sending end and the right side is taken as receiving end side, change in the voltage and the current can be represented as ΔV and ΔI respectively. The voltage and current relations for the transmission lines can be written as follows;

$$V + \Delta V = V + R\Delta x I + L\Delta x \frac{dI}{dt} \tag{3.1}$$

$$I + \Delta I = I + C\Delta x \frac{d(V + \Delta V)}{dt} \tag{3.2}$$

As it can be understood from the equation 3.1 and equation 3.2, distributed parameter modeling gives a solution for the current and voltage relations for a unit length. In order to find the total solution of the line, each unit distance part can be cascaded with the previous part.

Distributed parameter model gives exact solution for the voltage and current relations of the lines, since it considers the unit length circuit models of the line and reaches total length solution. The analytical solution methodology of the distributed parameter model will be given below.

The model can be generalized, by changing the equation 3.1 and equation 3.2, and the following steps can be achieved;

$$V + \Delta V = V + z\Delta x I \tag{3.3}$$

$$I + \Delta I = I + y\Delta x V \tag{3.4}$$

Equation 3.3 and equation 3.4 can be rewritten as follows;

$$\Delta V = z\Delta x I \tag{3.5}$$

$$\Delta I = y\Delta x V \tag{3.6}$$

where, z represents the equivalent series impedance, and y represents the equivalent shunt susceptance.

Since the change in the voltage and the current depends on the change on the position, equation 3.5 and equation 3.5 can be rewritten as follows;

$$\frac{dV}{dx} = zI \tag{3.7}$$

$$\frac{dI}{dx} = yV \tag{3.8}$$

Let's take the derivative of both sides, in equation 3.7 and equation 3.8:

$$\frac{d^2V}{dx^2} = z\frac{dI}{dx} \tag{3.9}$$

$$\frac{d^2I}{dx^2} = y\frac{dV}{dx} \tag{3.10}$$

If the equation 3.8 substituted into equation 3.9, and equation 3.7 substituted into equation 3.10, the following second order differential equations are obtained easily;

$$\frac{d^2V}{dx^2} = zyV \tag{3.11}$$

$$\frac{d^2I}{dx^2} = yzI \tag{3.12}$$

The solution of the second order differential equation of the voltage can be found as follows;

$$V(x) = k_1e^{\sqrt{yz}x} + k_2e^{-\sqrt{yz}x} \tag{3.13}$$

It should be noted that, current equation can be found by substituting the voltage equation, which is found in equation 3.13, into equation 3.7:

$$I(x) = \sqrt{\frac{y}{z}} k_1 e^{\sqrt{yz}x} - \sqrt{\frac{y}{z}} k_2 e^{-\sqrt{yz}x} \quad (3.14)$$

In the solution of the second order differential equations, the constant coefficients k_1 and k_2 are used. These coefficients can be found by the use of boundary conditions. The receiving end side can be taken as the initial point of the line:

$$V(x = 0) = k_1 + k_2 = V_R \quad (3.15)$$

$$I(x = 0) = \sqrt{\frac{y}{z}} (k_1 - k_2) = I_R \quad (3.16)$$

When the equation 3.15 and equation 3.16 are solved for k_1 and k_2, they can be found as:

$$k_1 = \frac{V_R + \sqrt{\frac{z}{y}} I_R}{2} \quad (3.17)$$

$$k_2 = \frac{V_R - \sqrt{\frac{z}{y}} I_R}{2} \quad (3.18)$$

Hence the final solution of voltage and current relations of a line can be given as below;

$$V(x) = \frac{V_R + \sqrt{\frac{z}{y}} I_R}{2} e^{\sqrt{yz}x} + \frac{V_R - \sqrt{\frac{z}{y}} I_R}{2} e^{-\sqrt{yz}x} \quad (3.19)$$

$$I(x) = \frac{\sqrt{\frac{y}{z}} V_R + I_R}{2} e^{\sqrt{yz}x} - \frac{\sqrt{\frac{y}{z}} V_R - I_R}{2} e^{-\sqrt{yz}x} \quad (3.20)$$

As explained above, this approach gives exact solution for all types of lines. In this model basic voltage and current relations are used and these relations are applied for each unit length distances. In the modeling part of the considered system, equivalent rail impedance varies as the train moves. Moreover, the drawn power by the train can change drastically. To take into account these situations, using the distributed parameter model is the best option; since, in this model the exact voltage and current relations between the train and the substations can be extracted depending on the position of the train.

In the following section, the proposed solution to the considered case will be given step by step, starting from the given distributed parameter solution method.

3.2 Proposed Method

In the previous part distributed parameter model was investigated, and its relevance to the considered subway system was discussed. In this part the proposed modeling of the subway system will be given in detail.

In the proposed modeling of the subway system, voltage and current relations will be found by adopting the methods used in distributed parameter modeling. The main motivations behind this choice are the accuracy of this method, and creating a basic start for developing the analytical formulation process. At the end of the proposed method part it will be proved that the use of lumped parameter model is also giving acceptable results.

Firstly, the substations will be analyzed electrically. Secondly, the distributed parameter model of the system will be constructed with the previously found electrical parameters. Finally, the mathematical relation between voltage, current and velocity of the train will be found.

3.2.1 Substation and Subway Train Modeling

Let's focus on the modeling of the substations. In the substations the AC gird voltage is rectified to DC to feed the subway system. For this purpose, 12-pulse rectifiers, where two 6-pulse rectifiers are connected in parallel in such a way that there is a 30° phase difference between them, are used. With this method, a smoother voltage and current waveforms are obtained. Moreover, the harmonics created in 12-pulse rectifier less than the harmonics created in single 6-pulse rectifier [23]. Furthermore, the 12-pulse rectifiers have the simplest winding configuration and highest power density.

In the AC grid, where the 12-pulse rectifiers are connected, some power quality problems can be observed due to the drawn current by the rectifier. The main reason of these power quality problems is the harmonics created at the point of common coupling due to the rectifier currents. To prevent this kind of problems, trap filters are placed in front of the rectifiers. In these filters, current harmonics are trapped inside the filter and the harmonic distortion due to the currents drawn by the rectifier is decreased.

The main purpose of these rectifiers is to supply the system with direct current. However, these rectifiers cannot supply a constant direct current to the system. To solve this problem, some filters are placed at the output of the rectifiers. These filters try to maintain the output voltage of the rectifiers at their maximum value. Although the output current of the rectifiers may fluctuate, these filters balance them with their stored energy.

In the modeling of DC substations in [17], the effect of the diode and equivalent AC source impedance losses are taken into account as $Z_{Thevenin}$. However, in the solution of the power-flow problem, the effect of $Z_{Thevenin}$ is neglected and the source power assumed as equal to the power transferred to DC link [17]. All these specifications of the rectifiers are making them closer to the ideal characteristic of a DC supply. Of course, in reality they do not supply ideal DC voltage and current to the subway system; however, in the modeling there is no problem to show them as an ideal voltage source with an equivalent Thevenin impedance. However, due to the relatively low conductivity of the rails, the Thevenin impedance of the substations can be neglected when they are compared with the impedance of the rails. Therefore, during the modeling of the system, the substations will be modeled as an ideal DC voltage source.

In the considered subway trains there exist three carriages. Only the front one and last one have the motors. In these two carriages there are two bogies in each. Each bogie is connected to two induction motors. Therefore, it can be said that in each subway train there exist eight induction motors. Each induction motor has a rated power of 130 kW.

In order to drive these induction motors, SiC based inverters are used. The modular traction control units have a rated voltage of 750 volts, and they can operate in between 500 volts to 1000 volts. With the flexibility of these SiC based inverter controllers, the induction motors of the subway trains can be driven with a high efficiency. Moreover, these controllers enable the regenerative braking and high efficiency switching.

The most important part for the DC system modeling is the drawn power by the subway trains. Since the power characteristic of the motors may vary depending on the operation scenarios, it is better to model the trains as power sink devices in the following parts.

3.2.2 Construction of the System Model

After finding the substation model, it can be combined with the distributed parameter model of rails. The basic electrical model of the subway system can be represented as Figure 3.2;

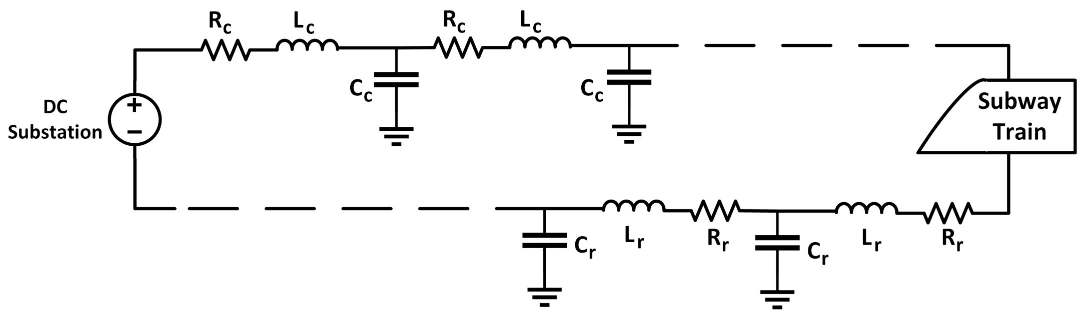

Figure 3.2: Distributed Parameter Model of the System

As explained before, resistance and inductance of the rails are the series components. However, the capacitance term is in between the rails and the earth.

In this modeling a lot of assumptions are made. Firstly, the real railway system is not shown here. Actually, there are lots of substations in the system and subway train moves in between two of them. However, in Figure 3.2 an equivalent model is represented for the sake of simplicity. Another assumption is the number of trains fed by the substations. Actually this number can vary depending on the subway system operators. Thirdly, the subway system was shown here as one-way system; however, in actual system there are two parallel lines and subway trains operates in opposite directions in these lines. Fourth assumption is, the contact resistance between the rails and the shoe is neglected. In fact, this contact might create a large resistance and it might have an important effect. However, due to the lack of field measurements, this parameter could not be modeled in this work. Finally, all the unit length line parameters assumed to be constant; however, depending on the environmental conditions and non-ideality in the production of the rails, there might be some differences in their actual electrical parameters.

3.2.3 Mathematical Modeling the Voltage and Current Relations

In the mathematical modeling of the system, previously explained distributed parameter model analysis methods will be used. However, it should be kept in mind that, in the considered subway system loads are moving and in this study the effect of the movement will be tried to be implemented into the existing distributed parameter solutions.

After constructing the distributed parameter model configuration given in Figure 3.2, a general solution for the catenary line voltage and current can be found for the moving train. To make the problem simpler, return rails are omitted during the derivation. However, the revised problem still has not a simple solution. The desired solution should depend on both time and position of the train. For this purpose, the solution procedure can be started from adding '-V' to both sides of the equation 3.1:

$$\Delta V = R\Delta x I + L\Delta x\frac{dI}{dt} \tag{3.21}$$

Equation 3.21 can also be re-written as follows;

$$\frac{dV}{dx} = RI + L\frac{dI}{dt} \tag{3.22}$$

Similarly, the current relation can be found as;

$$\frac{dI}{dx} = C\frac{dV}{dt} \tag{3.23}$$

By combining equation 3.22 and equation 3.23, two second order equations can be obtained, where one of them depends only the voltage the other one depends only the current;

$$\frac{d^2V}{dx^2} = RC\frac{dV}{dt} + LC\frac{d^2V}{dt^2} \tag{3.24}$$

$$\frac{d^2I}{dx^2} = RC\frac{dI}{dt} + LC\frac{d^2I}{dt^2} \tag{3.25}$$

It should be noted that, both voltage and current waveforms depend on position and time. To solve these equations both time and position dependencies must be covered. Hence, separation of variables technique will be used for that purpose. When the equation 3.24 is solved by using separation of variables (equation 3.25 will have the same solution), firstly the following relation can be written;

$$V(x,t) = X(x)T(t) \tag{3.26}$$

In equation 3.26, voltage equation, which depends on time and position, is written as the multiplication of X, which depends only position, and T, which depends on only time. When this equality is substituted into equation 3.24;

$$\frac{d^2X}{dx^2} = RC\frac{dT}{dt} + LC\frac{d^2T}{dt^2} \tag{3.27}$$

As it can be seen from the equation 3.27, time derivative of X, and position derivative of T have disappeared. When the resulted form, which is equation 3.27, is solved with the separation of variables;

$$X(x) = k_1 e^{\sqrt{-\lambda RLC}x} + k_2 e^{-\sqrt{-\lambda RLC}x} \tag{3.28}$$

$$T(t) = k_3 e^{\frac{t(R+\sqrt{R^2-4\lambda RL^2})}{2L}} + k_4 e^{\frac{t(R-\sqrt{R^2-4\lambda RL^2})}{2L}} \tag{3.29}$$

In order to find the coefficients k_1, k_2, k_3, k_4, and λ, five boundary conditions are required; however, some assumptions still can be made. It should be noted that the position change and the time change can be related to each other. In different motion characteristics of the train, different time & position relations can be found:

- Acceleration of train (assumed to be constant) (independent of time)

  In this mode of operation of the subway train, the acceleration will be assumed as constant. Therefore, a deterministic relation can be written in between the position of the subway train and time, with the help of well known acceleration equations. Hence, the time dependency of the time can be eliminated, and it can be found by using the position of the train.

- Constant speed motion of train (independent of time)

  In this mode of operation, the velocity of the train will be taken as constant. Since the speed is constant, a deterministic relation can be written between the velocity of the train and the position of the train. Therefore, the time dependency of the voltage variation can be eliminated.

- Deceleration of train (assumed to be constant) (independent of time)

  This mode of operation is similar to the *acceleration of train*. Deceleration of the train will be assumed as constant through the mode; therefore, with the help of relation between time and position, the time dependency will be eliminated.

- Initial current demand (inrush) (independent of position)

  This part covers starting of the motors, where huge inrush currents may be drawn. While modeling this mode it should be noted that, the position of the subway train remains constant. There will be a power demand, which varies with time, by the load. Therefore, the solution will be independent of position of the subway train. The system will act as a classical static-load system; however, only the challenge will be the transient solutions for the time varying power demand. In the considered case and new trains, soft starters are used to prevent high inrush currents. So that, the effect of this mode will not be covered in detail.

During the constant speed behavior of the subway train the drawn power could be assumed having a determined function, such as constant power just to overcome friction forces. Moreover, the current will be also a function of drawn power and the voltage. Therefore, the change in the position of the train will directly affect the current amplitude. Formulation of the voltage and current relations will be given in the *Section 3.2.3.1 Constant Speed Movement of Subway Train* part.

Acceleration and deceleration of the train can be treated similarly. The only difference in these two behaviors is the power flow direction, i.e. in acceleration mode train draws the power; however, in deceleration mode the train generates power due to regenerative braking. In deceleration mode of the train generated power could be either dissipated on brake resistors or stored in storage equipment. Based on the operation of the system, an additional component, namely resistor or a capacitor should be located between the proper location and ground to model deceleration.

In this work to reduce the computational burden it is assumed that the voltage variation can be modeled as constant speed movement by assuming that the velocity of the train remains constant for short periods. The main motivation behind this assumption is the fact that electrical time constant is much smaller than the mechanical time constant. The adaptation of the constant speed solution to the acceleration of the train will be explained in the *Section 3.2.3.2 Constant Acceleration Movement of Subway Train* part.

3.2.3.1 Constant Speed Movement of Subway Train

This part is one of the most important parts in the subway operation since it creates a base for other parts of the solution; moreover, it covers the largest part of the travel.

In order to model this part, equation 3.22 will be used again. It should be emphasized that the main motivation in this part is to find a relation between time and position of the train. Therefore, in the equation 3.22 inductance term will be multiply by $\frac{dx}{dx}$;

$$\frac{dV}{dx} = RI + L\frac{dI}{dx}\frac{dx}{dt} \tag{3.30}$$

A similar approach can be applied to the equation 3.23;

$$\frac{dI}{dx} = C\frac{dV}{dx}\frac{dx}{dt} \tag{3.31}$$

The modified equations can be re-written as follows;

$$\frac{dV}{dx} = RI + Lv\frac{dI}{dx} \tag{3.32}$$

$$\frac{dI}{dx} = Cv\frac{dV}{dx} \tag{3.33}$$

In the equations 3.32 and 3.33, v represents the rate of change of the position with respect to time, in other words the velocity of the train in this specific case. As it can be seen from the equations 3.32 and 3.33, with the proposed approach time dependency of the voltage and current relations are disappears. Instead of two independent variables, the relations will depend on one variable, which is position, and one constant, which is velocity of the train. Therefore, there will be no need for complex solution methods, such as separation of variables. Furthermore, with this approach, the number of required boundary conditions will decrease.

To solve the equations 3.32 and 3.33, equation 3.33 will be substituted into equation 3.32;

$$\frac{1}{Cv}\frac{dI}{dx} = RI + Lv\frac{dI}{dx} \tag{3.34}$$

The above equation can be written as follows;

$$\frac{dI}{dx} = \frac{RCv}{1 - LCv^2}I \tag{3.35}$$

The first order ordinary differential equation given in equation 3.35 can be solved as follows;

$$I(x) = k_1 e^{\frac{RCv}{1-LCv^2}x} \tag{3.36}$$

In equation 3.36, the current relation of the catenary line has been found with respect to position. In this equation k_1 is an unknown coefficient. It will be found by using the boundary conditions.

It should be noted that during the unit distance change in the position of the train, the velocity is assumed as constant. The motivation behind this assumption is the high inertia of the train.

If the found current relation given in equation 3.36 is substituted into the equation 3.33, the voltage relation can be found as follows;

$$V(x) = (k_1\frac{1 - LCv^2}{Cv} + Lk_1v)e^{\frac{RCv}{1-LCv^2}x} + k_2 \tag{3.37}$$

Or alternatively, it can be written as;

$$V(x) = \frac{k_1}{Cv}e^{\frac{RCv}{1-LCv^2}x} + k_2 \tag{3.38}$$

In equation 3.38, the voltage relation of the catenary line has been found with respect to position. In this equation k_1 and k_2 are unknown coefficients, where

k_1 is also appears in the current relation of the catenary line.

Since there exist two unknown coefficients, two boundary conditions are required for the solution. In order to find these boundary conditions, the system model should be investigated again. By taking the Figure 3.1 and Figure 3.2 into account, it is assumed that the starting point will be assigned as the position of the train, i.e. position of the train will be taken as x=*0* point. Under this assumption, substation terminals will be taken as the final position, i.e. x=l will be used to indicate the location of substation.

In order to find k_1 and k_2 two boundary conditions are required, as explained above. It should be noted that, for this part only the catenary line current and voltage relations are tried to be found. So that, the effect of the return rails is not included into the boundary conditions; however, in the next step return rail effect will also be considered.

Since the position of the substation is known as x=l, the first boundary condition can be written as;

$$V(x = l) = V_{source} \tag{3.39}$$

It should be noted that, in the *Substation and Subway Train Modeling* part, it is assumed that substations can be shown as ideal DC sources due to their low Thevenin impedances compared to the rail impedances.

It is needed to a second boundary condition. This boundary condition can be extracted from the drawn power by the train. If it is assumed that the effect of the return rails area neglected, then second boundary condition can be written as follows;

$$P = V(x = 0) \times I(x = 0) \tag{3.40}$$

In equation 3.40, the drawn power is represented by P. It should be noted that during this mode of operation train moves with constant speed, in other words its acceleration is equal to zero. Therefore, the applied force as well as the required power is equal to zero. However, there are some friction forces, and these forces makes the drawn power larger than zero.

By using these boundary conditions, unknown coefficients can be found with the following steps;

$$\frac{k_1}{Cv} e^{\frac{RCv}{1-LCv^2} l} + k_2 = V_{source} \tag{3.41}$$

$$\frac{k_1^2}{Cv} + k_1 k_2 = P \tag{3.42}$$

From equation 3.41, k_2 can be found as;

$$k_2 = V_{source} - \frac{k_1}{Cv} e^{\frac{RCv}{1-LCv^2} l} \tag{3.43}$$

By substituting equation 3.43 into equation 3.42;

$$\frac{k_1^2}{Cv} + (V_{source} - \frac{k_1}{Cv} e^{\frac{RCv}{1-LCv^2} l}) k_1 = P \tag{3.44}$$

$$k_1^2 (1 - e^{\frac{RCv}{1-LCv^2} l}) + k_1 V_{source} Cv - PCv = 0 \tag{3.45}$$

It should be noted that, equation 3.45 is in the form of $ax^2+bx+c=0$. For that kind of equations the solution can be found as $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$. As a result, k_1 and k_2 can be found as:

$$k_1 = \frac{-V_{source} Cv \pm \sqrt{(V_{source} Cv)^2 + 4(1 - e^{\frac{RCv}{1-LCv^2} l}) PCv}}{2(1 - e^{\frac{RCv}{1-LCv^2} l})} \tag{3.46}$$

$$k_2 = V_{source} - \frac{-V_{source} Cv \pm \sqrt{(V_{source} Cv)^2 + 4(1 - e^{\frac{RCv}{1-LCv^2} l}) PCv}}{2(1 - e^{\frac{RCv}{1-LCv^2} l}) Cv} e^{\frac{RCv}{1-LCv^2} l} \tag{3.47}$$

As it can be seen from equations 3.46 and 3.47 the results are really complicated, although the return rail contribution is neglected. To find the actual current and voltage relations these coefficients should be substituted into equations 3.36 and 3.38.

If the effect of return rails is added into the solution procedure, one more voltage and current relation will be added to the proposed analytical method. Due to the current and voltage relations of the return rail two more boundary conditions are required. Moreover, the existing boundary conditions should be changed as follows:

$$V_C(x = l) = V_{source} \tag{3.48}$$

$$P = [V_C(x = 0) - V_R(x = 0)] \times I_C(x = 0) \tag{3.49}$$

In the above equations subscript C and R are used to represent catenary line and return rails. It is seen that the drawn power by the subway train is actually the multiplication of drawn current and the train voltage, which is the potential difference between the catenary line and the return rails.

In equations 3.48 and 3.49, previously found boundary conditions are modified for the new case. However, two more boundary conditions are required to be able to solve the system. These boundary conditions can be written as follows:

$$V_R(x = l) = 0 \tag{3.50}$$

$$I_C(x = 0) = I_R(x = 0) \tag{3.51}$$

According to the *Section 2.2 Shunt Susceptance of the System*, the running rails are isolated from the ground with an insulating material. However, they are connected to the earth at each substation location. Therefore, in equation 3.50, the voltage of the return rails at the substation position is taken as zero.

According to Kirchhoff's Current Law, current flow into the train and the current leave the train must be equal. Therefore, in the equation 3.51, it is stated that the current flow from the catenary line to the subway train is same with the current flowing from subway train to the return rails.

With these two new boundary conditions k_1, k_2, k_3 and k_4 can be found, where k_3 and k_4 are the unknown coefficients in the relations of return rails. The solutions of k_1, k_2, k_3 and k_4 are not given in this work, since solving them in computer program is more convenient choice.

After finding these four unknown coefficients, voltage and current relations can be written easily. With the solution of these relations, the modeling of the catenary voltage at the constant speed movement of the subway train has been accomplished.

3.2.3.2 Constant Acceleration Movement of Subway Train

In the previous part constant speed solution has been achieved by solving first order differential equations. Unknown coefficients, which are appeared in these solutions, are obtained with the help of boundary conditions.

In this part voltage and current characteristics of the train will be investigated under the constant acceleration and deceleration mode of operations. In order to model the voltage and current variation behavior, two different techniques can be used. One solution may be the integrating the acceleration with respect to time and finding the velocity of the subway train. In this approach the resulting velocity can be used in the solution, which was found in the previous part. Also boundary conditions should be changed accordingly. However, this approach is computationally expensive, and due to the integration of acceleration any error in this process may lead wrong results easily. In order to decrease the effect of errors, and simplifying the solution acceleration of the subway train can be assumed as constant. But this, may cause loss of accuracy during the non-linear acceleration instants.

In this work another and simpler method will be proposed. In the proposed method, acceleration of the subway train will be discretized. The main motivation behind this approach is the high sampling frequency of the sensors, which is 1kHz. The sampling frequency determines the step-size of the solution, i.e. the higher the sampling frequency, the smaller the sampling-size. Since the sampling-size is small, it can be assumed that the position change of the train is too short between two data samples. This means that, between the two successive time steps it can be assumed that the subway train moves with a constant speed.

So far, it is explained that, between two successive time steps the velocity of the train will be assumed as constant in the proposed method. Hence, the previously found solution methodology can be used in this part, too. However, the difference between the constant speed solution and constant acceleration solution arises in the power demands.

In this part of the solution the required power is related with the speed of the train. When the power demand of this part of the motion is compared with the power demand of the constant speed mode of operation, where the power just overcomes the friction forces, it can be seen that much more power will be drawn when the train accelerates. Moreover, it should be kept in mind that, the given assumptions are acceptable for flat path surfaces. In such a condition the power demand relation can be found as follows;

$$F = m \times a \tag{3.52}$$

$$v = \int a dt \tag{3.53}$$

$$P = F \times v \tag{3.54}$$

As it can be seen in equation 3.54, while the acceleration is constant the required force will also be constant. However, due to the linear increase in velocity, formulated in equation 3.53, power demand of the subway train will also increase with time. In Figure 3.3 the characteristics of power demand of subway train and its speed variation can be seen.

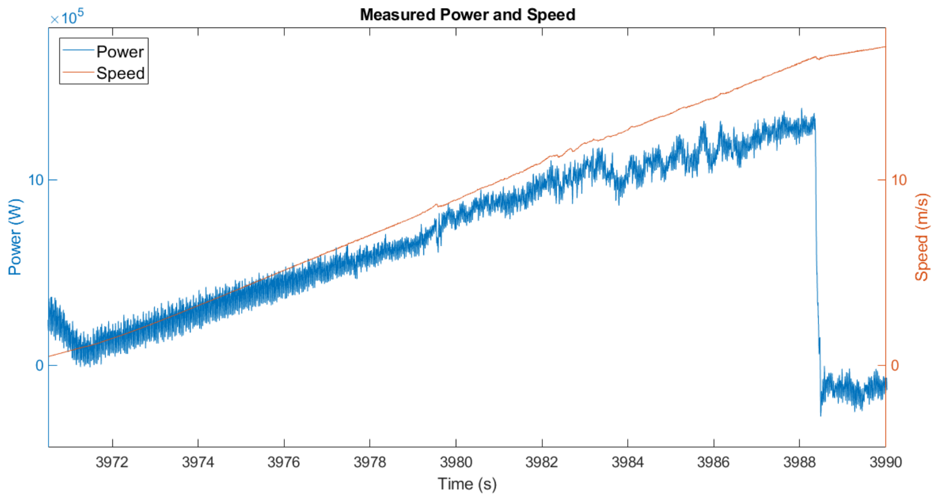

Figure 3.3: Power Demand in Constant Acceleration Mode of Operation

In Figure 3.3, the similarity between the characteristics of the power and the speed data is a validation of the result of equation 3.54. Nevertheless, still it is possible to see some changes in the slope of power, while the acceleration of the train is constant. The slope of the railway is the main reason such a variation in the power demand. According to Figure 3.3, the rate of change of power demand decreases, while the acceleration is constant; therefore, the subway train may go downhill.

In order to apply the proposed method, equations 3.30 and 3.31 will be discretized as follows;

$$\frac{dx}{dt} = \frac{x[k] - x[k-1]}{\Delta t} \tag{3.55}$$

where $x[k]$ is the position of the train at time instant k.

It should be noted that, instead of using constant velocity for the whole solution procedure as given in *Section 3.2.3.1 Constant Speed Movement of Subway Train*, in each solution step a new velocity should be defined and it should be assumed as constant between these two successive time steps, as explained in equation

3.55. In this way varying acceleration situation can also be modeled. Moreover, with this approach it is possible to assume that the power demand of the train remains constant for the considered time interval. Therefore, it can still be used as a boundary condition. This power demand can be calculated based on the equation 3.54.

For the deceleration mode of operation of the subway train, the regeneration mode of the induction motors should be considered. This will be explained in *Section 3.2.6 Regenerative Braking Adaptation.*

So far, in the proposed solution method, to find a mathematical dependency between the speed and the position of the train, the distributed parameter model is used. Therefore, during the simulations, higher order differential equations are solved to reach a solution. However, solving these kind of differential equations is a computational obstacle, and it slows down the computation process. In order to deal with this computational burden, the lumped parameter model of the system can be solved. To find the lumped parameters, distributed parameter values can be multiplied with the distance between the substation and subway train. The configuration of the lumped parameter can be seen in Figure 3.4.

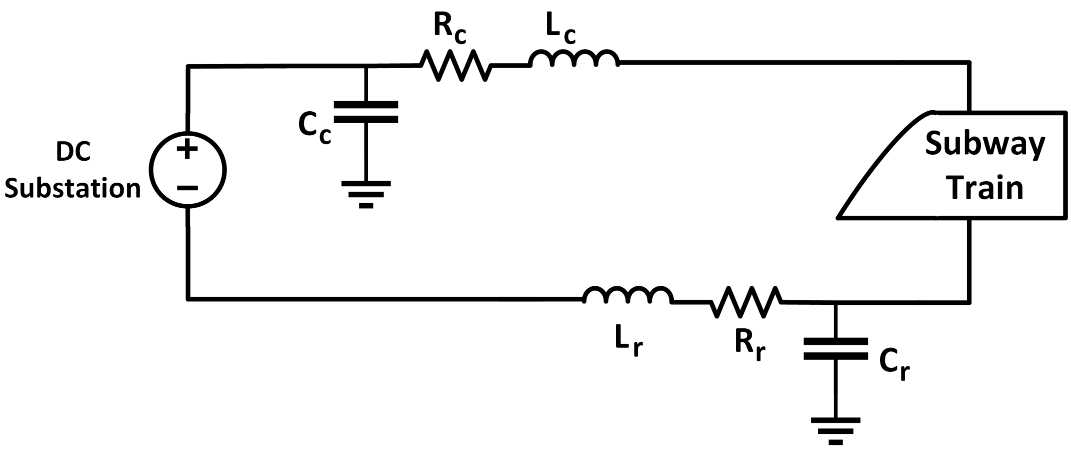

Figure 3.4: Lumped Parameter Model of the System

In Figure 3.4, due to the location of C_C, its effect can be neglected. In fact, the given assumption can be applicable, since the calculated capacitance is small and its effect can be neglected. Then the variation of the catenary line can be written as follows;

$$V_{train} = V_{source} - R_C I_{train} - L_C \frac{dI_{train}}{dt} \tag{3.56}$$

Since V_{train} and I_{train} are the unknowns, another equation is required. Which is the power demand of the train again;

$$P_{train} = V_{train} I_{train} \tag{3.57}$$

In this problem the solution of the V_{train} is not straight-forward, actually, to solve the train voltage some iterations are required. However, solving the initial point with differential equations and linearly solving the other steps will increase the computational performance. Using the equation 3.55 can be used for that purpose. Equation 3.56 can be modified as follows;

$$V_{train} = V_{source} - R_C I_{train} - L_C \frac{I_{train}[k] - I_{train}[k-1]}{\Delta t} \tag{3.58}$$

In 3.58 $I_{train}[k-1]$ is known , due to the solution of previous steps. By using this known parameter, the solution for the train voltage can be written as follows;

$$I_{train} = \frac{LI_{train}[k-1] + V_{source}\Delta t \mp \sqrt{(LI_{train}[k-1] + V_{source}\Delta t)^2 - 4(R\Delta t + L)P\Delta t}}{2(R\Delta t + L)} \tag{3.59}$$

Finally, the catenary line voltage can be found as;

$$V_{train} = \frac{P}{I_{train}} \tag{3.60}$$

The validation of the lumped parameter model can be given in Figure 3.5. In Figure 3.5, while blue line represents the lumped parameter model approach, orange line represents the distributed parameter model approach. As it can be seen both models give close sufficiently close results. The validation of these approaches with the measured data will be explained in *Chapter 4 Validation of the Proposed Method & Simulation Results*.

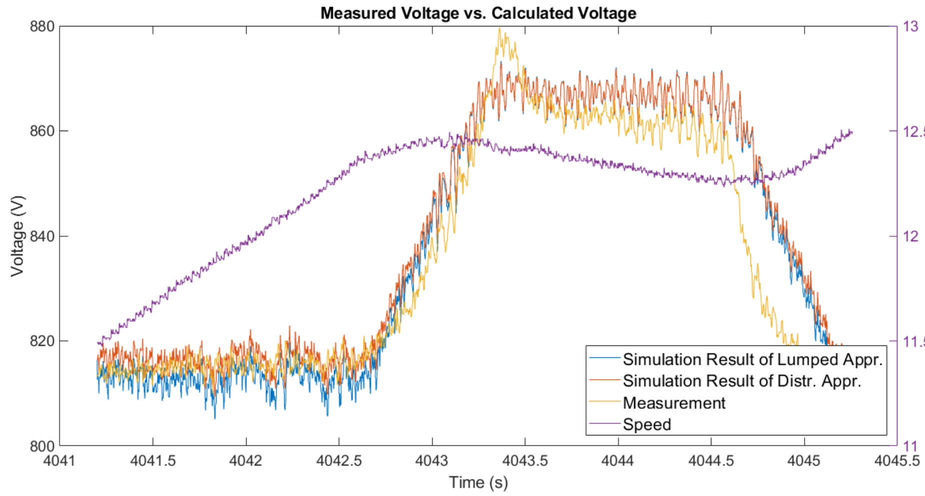

Figure 3.5: Validation of the Lumped Parameter Modeling

3.2.4 Multi Substation Adaptation

In the previous parts, mathematical modeling of the constant speed movement and accelerated movement of the train has been investigated. In these modeling parts, one of the main assumptions is the number of the feeding substations. In the previous parts all the required power was drawn from an equivalent substation; however, in reality there are a lot of substations along the railway.

In the actual subway system, nearly all stations have a DC substation. Therefore, the effect of all these substations should be considered in the model. However, as it was explained before, the Thevenin impedances of the DC substations are nearly negligible compared to the rail impedances.

By using the above assumption, it will be enough to consider two adjacent substations with respect to the location of the subway train. Since the subway train will draw power from only the closest two substations, other parts of the DC system can be omitted for the analytical adaptation of multi-substation case.

In the considered subway system, there are eight DC substations. The distance between these substations can be seen in Table 3.1.

Table 3.1: Distance between DC substations

		Distance (m)
Substation 1	Substation 2	2240
Substation 2	Substation 3	1966
Substation 3	Substation 4	2881
Substation 4	Substation 5	1954
Substation 5	Substation 6	1497
Substation 6	Substation 7	1875
Substation 7	Substation 8	1700

After indicating the distances between adjacent substations, calculation of the effect of each substation can be the next step. To find these effects, a similar way to current division can be used; however, in this case the distances between the subway train and the substations can be used. Since all the parameters are calculated as per unit length, in the proposed solution method, the distance can be used instead of impedance.

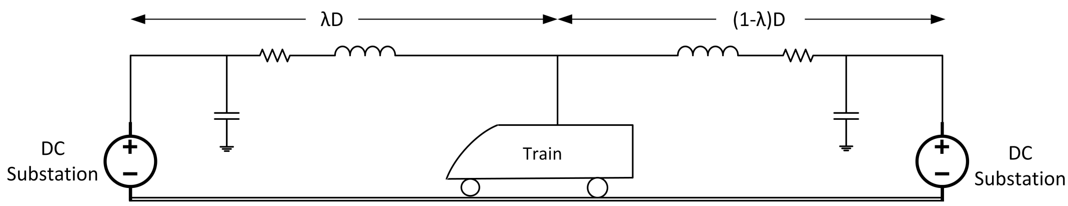

Figure 3.6: Adjacent Substations to the Subway Train

In Figure 3.6, the simplified model of the multi-substation case is represented. It can be seen that the distance between the left-most substation and the right-most substation is shown as D. The coefficient λ represents the ratio of the distances. This coefficient should be in between 0 and 1;

$$0 \leq \lambda \leq 1 \tag{3.61}$$

In the previous parts the voltage and the current relations of the train are developed for different mode of operations of the train, under single substation fed condition. Now, the multi substation case will be adapted into this solution.

Let's assume that the total power drain by the subway train is P. Then the support of the left-most substation can be expressed as;

$$P_{effective,left} = (1 - \lambda) \times P \tag{3.62}$$

The distance between the left-most substation and the subway train can be seen from Figure 3.6 as;

$$l = \lambda \times D \tag{3.63}$$

Now, the voltage and current relations with respect to the position of the train can be found between the left-most substation and the subway train; by the use of equation 3.62 as boundary condition, and equation 3.63 as the total distance parameter. Other boundary conditions will remain as same, since they are independent of the position of the train, or drawn power characteristic.

In order to solve the voltage and current relations between the subway train and the right-most substation, the boundary condition and the total distance parameter can be changed as follows;

$$P_{effective,right} = \lambda \times P \tag{3.64}$$

$$l = (1 - \lambda) \times D \tag{3.65}$$

In this part, firstly the multi substation case simplified to two adjacent substation case, with the help of the assumption which states that the DC substations are modeled as ideal voltage sources, which neglects the variations in AC system. Secondly, the drawn power by the train is distributed to each substation. It is important to note that, the drawn power by each substation is inversely proportional with its distance to the train. After separating the power, finally each substation and corresponding power drawn is solved for single substation model for a time instant (t).

3.2.5 Multi Train Adaptation

In the previous part, the multi substation case problem is separated into two single substation case problems and solved. In this part of the work, the effect of the multiple train will be investigated.

As explained above, in the actual subway system there are multiple substation. Moreover, there are multiple train operating on the subway line. In multi substation adaptation part it was explained that, only two closest substations supply the required power. However, it might be a second train in between the same substations, which might be traveling in the same or opposite direction.

In this work, superposition principle will be used to solve the multiple train problem. In which, each train will be investigated separately for a specific time instant (t), then the resulting power passing through the catenary line will be calculated. With the combination of drawn powers by each train, the voltage and current values can be determined between two substations.

Firstly, the only the effect of two trains will be investigated. After the solution of two trains problem, both multiple train and multiple substation case will be solved, by merging to solutions.

To visualize the multi train case, Figure 3.7 can be studied.

As it can be seen from the Figure 3.7, there are two subway trains fed by a single DC substation. The distance between the substation and the Train-1 is given as l_1, and the distance between Train-1 and Train-2 is given as l_2. Let's assume the power drawn by the trains as P_1 and P_2 respectively.

Basically, the solution procedure for that problem will be same as the previous parts. However, since there exist two different trains and their motion are independent of each other, the voltage variation along the line will be discussed, i.e.

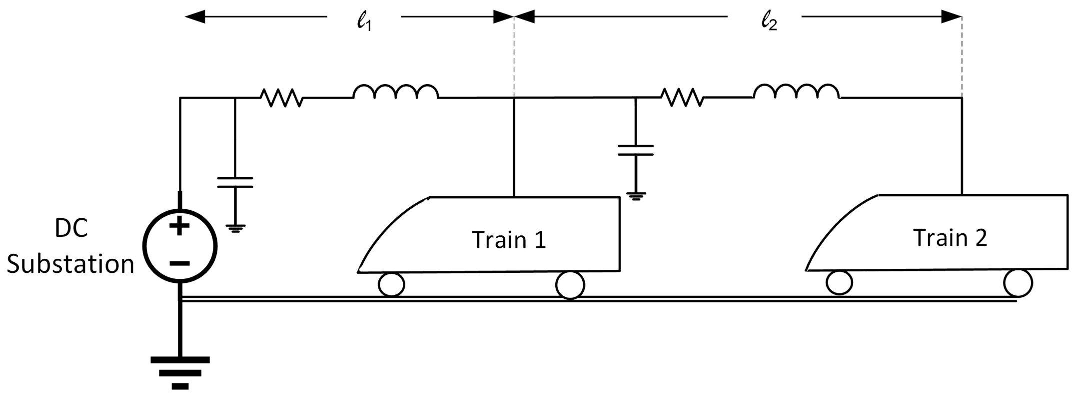

Figure 3.7: Multiple Train Configuration

for a time instant (t) will be analyzed. As explained above, the superposition principle will be utilized. The power transferred on the catenary line in between DC substation and Train-1 is;

$$P_{Substation-Train1} = P_1 + P_2 \tag{3.66}$$

$$P_{Train1-Train2} = P_2 \tag{3.67}$$

By using these two boundary conditions, two separate solutions for the considered trains can be achieved. However, in this case it is hard to find a solution with differential equations; because, the trains can be modeled as dependent current sinks. Therefore, a linearized solution method will be utilized in this section specifically. Which means that, instead of distributed parameter models, lumped parameter model will be used. It should be noted that, both approaches are giving satisfactory results. Furthermore, the shunt admittance and series inductance terms will also be omitted; since the solution is applied for a time instant. As a result, the following equations can be written;

$$V_{Source} - (\frac{P_1}{V_1} + \frac{P_2}{V_2})l_1 R = V_1 \tag{3.68}$$

$$V_1 - \frac{P_2}{V_2} l_2 R = V_2 \tag{3.69}$$

In equation 3.68 and equation 3.69, V_1 and V_2 are the voltages across the Train-1 and Train-2 respectively. R represents the unit length resistance of the line. By solving these two equations voltage across the trains, and current drawn by the trains can be found easily.

Now, multiple substation and multiple train case will be investigated. In Figure 3.8, the configuration of the considered case can be seen.

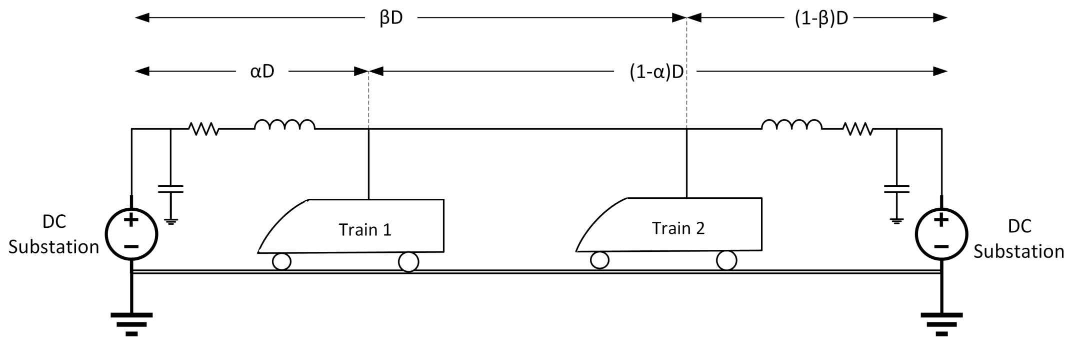

Figure 3.8: Multiple Train and Multiple Substation Configuration

The superposition principle will be used in the solution of this problem. Let's assume that the power demand of Train-1 is P_1 and the power demand of Train-2 is P_2. The distance between two substations is D.

Considering the system in Figure 3.8, $P_{eff,0-\alpha}$, $P_{eff,\alpha-\beta}$ and $P_{eff,1-\beta}$ are power flow from left-most substation to Train-1, power flow from Train-1 to Train-2, and power flow from right-most substation to Train-2. Those flows are defined as follows, assuming P_1 is the demand of Train-1 and P_2 is the demand of Train-2.

$$P_{eff,0-\alpha} = ((1-\alpha) \times P_1) + ((1-\beta) \times P_2) \tag{3.70}$$

$$P_{eff,\alpha-\beta} = -(\alpha \times P_1) + ((1-\beta) \times P_2) \tag{3.71}$$

$$P_{eff,1-\beta} = (\alpha \times P_1) + (\beta \times P_2) \tag{3.72}$$

By the use of above power flows, the solution of the multi-train & multi-substation case can be achieved.

3.2.6 Regenerative Braking Adaptation

So far, the voltage and current relations of the train are obtained for constant speed and accelerated operations. Moreover, the multi substation and multi train adaptations are explained. In this part the regenerative braking modeling will be given in a simple way.

In order to increase the energy efficiency of the motors, regenerative braking is commonly employed in modern electrified traction systems. In the considered subway system, generated power during the regenerative braking is transferred to a super-capacitor.

The location of the super-capacitor is important. If it is located on the train, then there will be no effect of regenerative braking on the catenary line. However, in the considered system, it is located in the DC substation. In such a case, system model should be modified with the detailed model of the super-capacitor, which includes the internal resistance and the actual capacitance. Moreover, there is a DC-to-DC converter between the catenary line and the super capacitor. Like the other power system applications, the super-capacitor and the DC-DC converter will be represented as a power sink.

In order to implement this adaptation to the solution, the following relation will be enough; since, the system model does not change according to the given assumptions. The only change is the direction of the power flow. Normally, the power flows from the DC substation to the train; however, in regenerative braking mode it flows from train to the substation terminals, where the super-capacitor is located.

$$P_{effective} = -|P_{regenerative}| \tag{3.73}$$

For the analytical solution of the system, only the power related boundary condition should be changed according to the equation 3.73. Then, the solution method given in *Section 3.2.3.2 Constant Acceleration Movement of Subway Train* can be used.

CHAPTER 4

VALIDATION OF THE PROPOSED MODEL & SIMULATION RESULTS

In the previous parts, firstly the required preliminary works are done, to investigate the catenary line voltage. And then, the proposed solution methodology is derived step by step. As the specifications of the subway system included to the problem one by one, the required modifications are also applied. Now, in that part, the relevance between the results of the proposed method and the actual system response will be investigated.

Unfortunately, the validation of the proposed method cannot be applied at the field, due to the security protocols of the subway organization. Therefore, during the validation of the solution methodology, some assumptions are made to apply the proposed solution methods. Although, the subway operator does not allow the university for the tests, some measurement data are supplied by the developer organization of the motor controllers. In this part all the validation is done with the help of these data.

In the previous parts there are a lot of analytical solutions to the voltage and current relations of the catenary line. The validation of all these analyses will be given in this part. Step by step the similarities between the analytical solutions characteristic and the real data characteristic will be shown.

In the supplied measurement data, the following parameters are given;

- Drawn power by one induction motor,

- DC voltage across the subway train,
- DC link current flows into the subway train,
- Motor speed in rpm,
- Torque demand,
- Generated torque.

The drawn power by the all induction motors will be used as a boundary condition for the proposed solution method. There is also another option like, determining the power from the torque demand and the velocity of the motors. But in this work it will not be used, since the actual power measurement exists. DC link voltage and current will be used to validate the corresponding simulation results. Motor speed will be used as an input for the proposed solution method, since it is the connection between position and time.

The main drawback of the field measurements is the lack of position data. In the given measurement data, the voltage across the subway train is known, but the location of the subway train is not known. So it is not possible to know the exact position of the subway train. As a result, the total impedance between the train and the substation cannot be find exactly. Therefore, in the simulations an approximate initial distance is selected between train and substation.

Another important handicap in the given measurement data is the effect of other trains on the railway. It is known that, some trains are also working on the same railway during the measurements; however, there is no information or data about the operation of these subway trains.

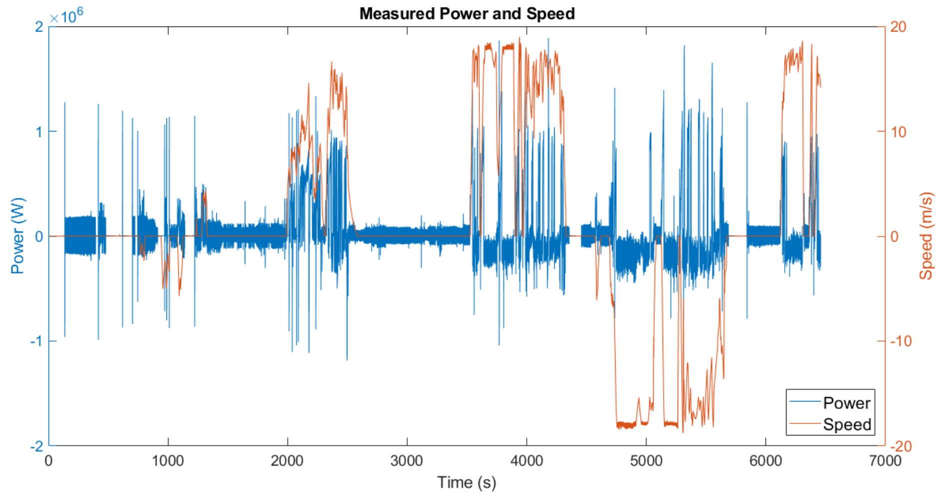

Figure 4.1: Power and Speed Measurements - Field Data

In Figure 4.1 and Figure 4.2 the power drawn by the train is given. The drawn power characteristic of the train is found by the multiplication of the DC link voltage and the DC current flow through the train.

In Figure 4.2, the proportional relation between the required power of the train and the velocity of the train can be seen clearly.

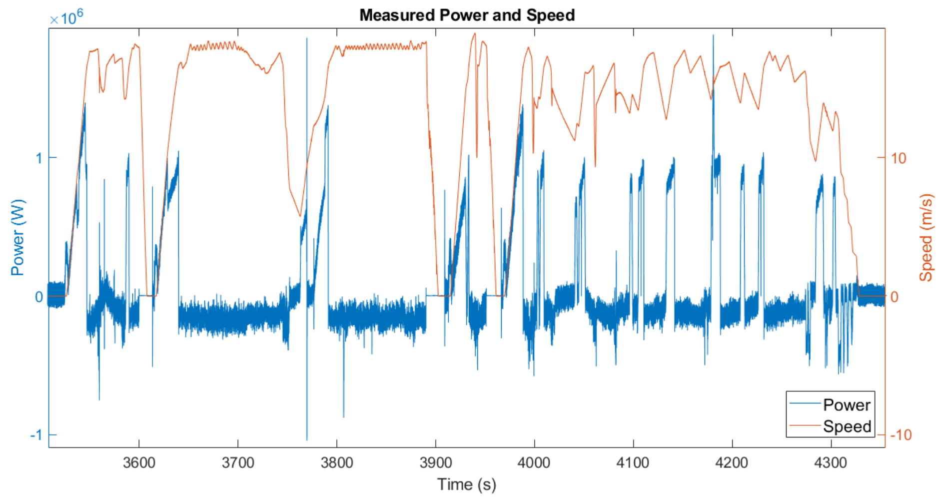

Figure 4.2: Power and Speed Measurements in a Smaller Window

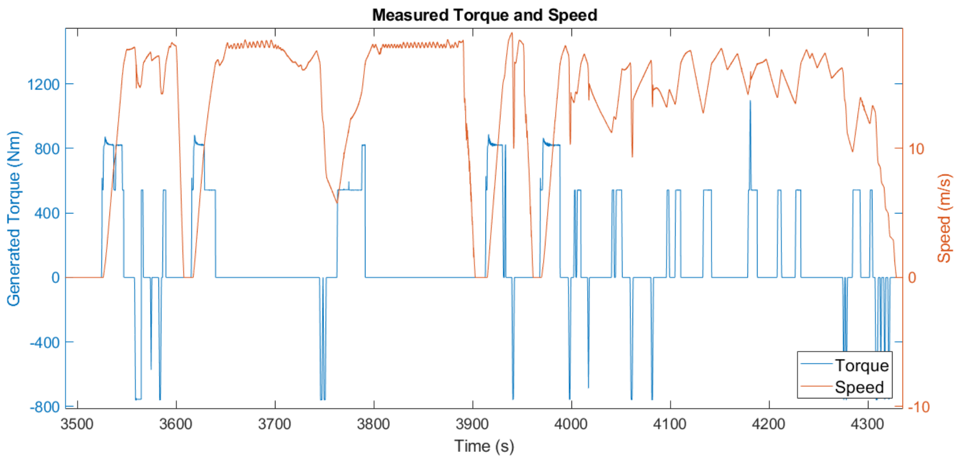

Figure 4.3: Torque and Speed Measurements in a Smaller Window

In Figure 4.3 the speed of the train and the generated torque measurement in the train can be seen. Moreover, in Figure 4.4, the relation between the drawn power by the train and the generated torque is shown.

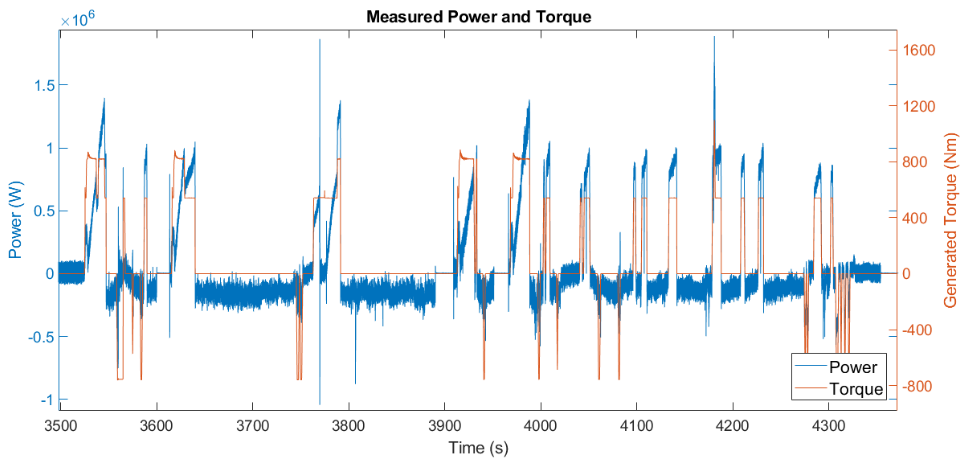

Figure 4.4: Power and Torque Measurements in a Smaller Window

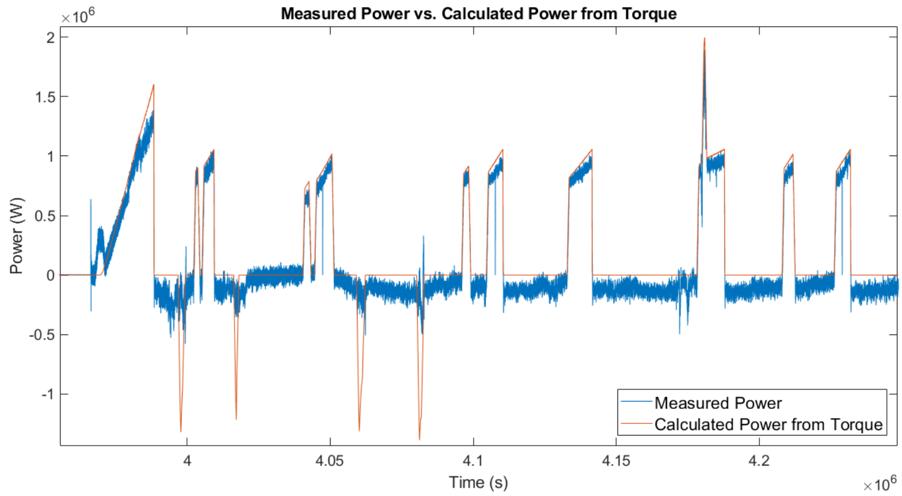

Figure 4.5: Actual Power vs. Calculated Power from Torque Measurements in a Smaller Window

It should be noted that, the power drawn by the train can be found in another way, which is the multiplication of the generated torque and the angular velocity in revolution per second. In Figure 4.5 the similarity between the drawn power characteristic and the calculated power characteristic with the use of torque and speed can be seen.

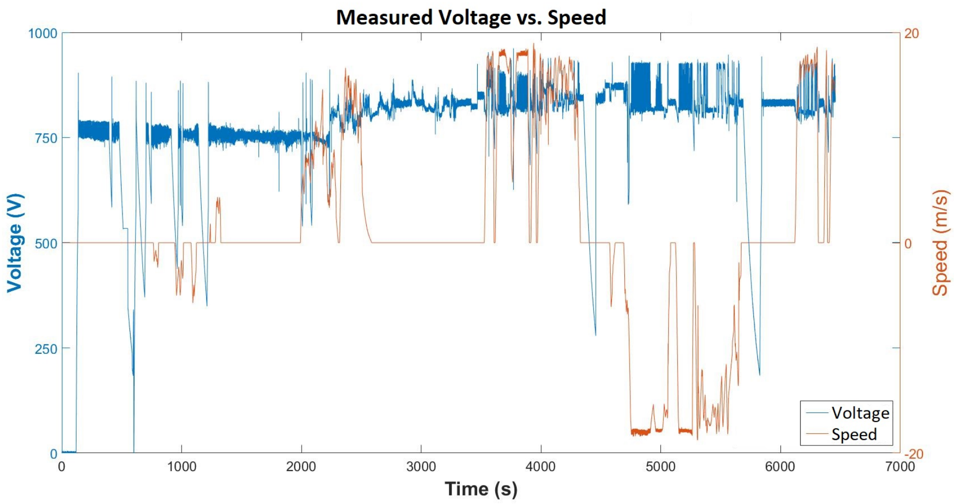

Figure 4.6: Voltage and Speed Measurements - Field Data

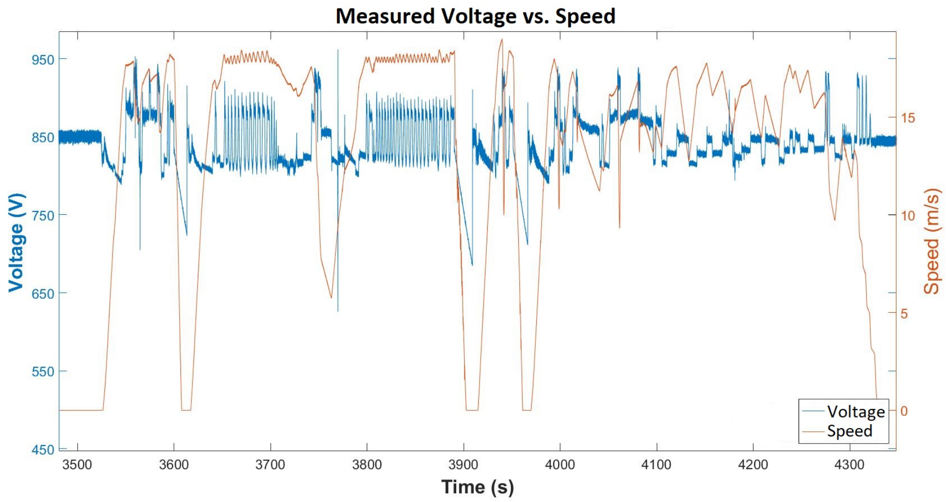

Figure 4.7: Voltage and Speed Measurements in a Smaller Window

In the Figure 4.6 and Figure 4.7, the given measurement data can be seen. While in the Figure 4.6 all the measurement data exist, Figure 4.7 shows a smaller window size of data, i.e. zoomed in version of Figure 4.6. In these figures, While the blue colored lines represent the actual voltage variation across the subway train, the orange colored lines show the actual speed of the train. It should be noted that, as explained above, in the given measurement data the speed of the motors are given in rpm. As explained in *Section 1.1 Problem Definition*, the motor speed can easily converted to velocity of the train.

As it can be seen in Figure 4.6, although the speed of the train is zero, in other words while the subway train stops; some variations can be seen in the voltage at the catenary line in some specific regions. The reason of such a variation is the effect of other trains operating on the line. Therefore, the effect of other trains should be keep in mind while interpreting the results.

If the discretized solution methodology is applied to the system, which is introduced in *Section 3.2.3.2 Constant Acceleration Movement of Subway Train*, the voltage variation in the catenary line can be seen in Figure 4.9 and 4.10. The system model is configured as given in Figure 4.8 in the given simulation results.

In Figure 4.9 and 4.10, orange colored, blue colored, and yellow colored lines

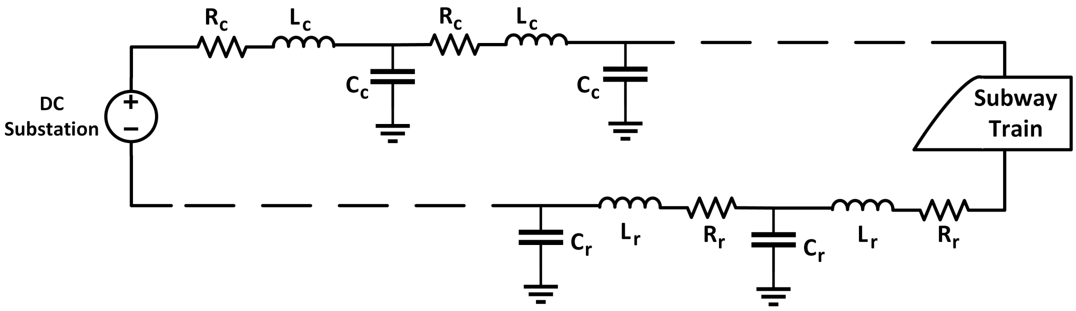

Figure 4.8: Considered Case in the Simulation of Figure 4.9 and 4.10

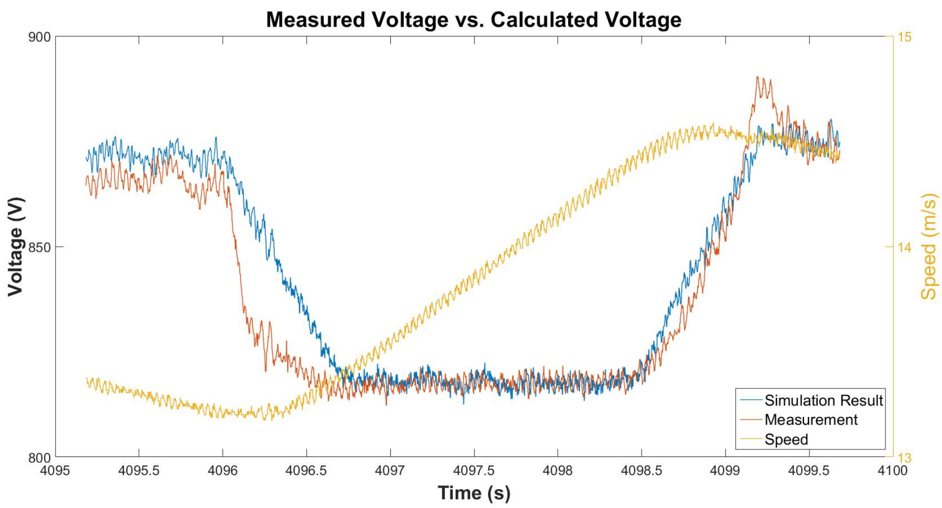

Figure 4.9: Simulation Result and Measurement of the Catenary Line Voltage - Case 1

represent the measured data, the simulation results, and the speed of the train respectively.

In Figure 4.9 and 4.10, it can be seen that as the subway train accelerates, and due to the increase in drawn power, the voltage at the catenary line dramatically decreases. Moreover, when the acceleration of train ends, the voltage shows an increase trend. But the most importantly, the variation characteristics of the catenary line voltage measurements and the simulation results have a lot of similarities. The trends in both measurement and simulation result are consistent. Although the general trends are similar, still there are some major differences in these results.

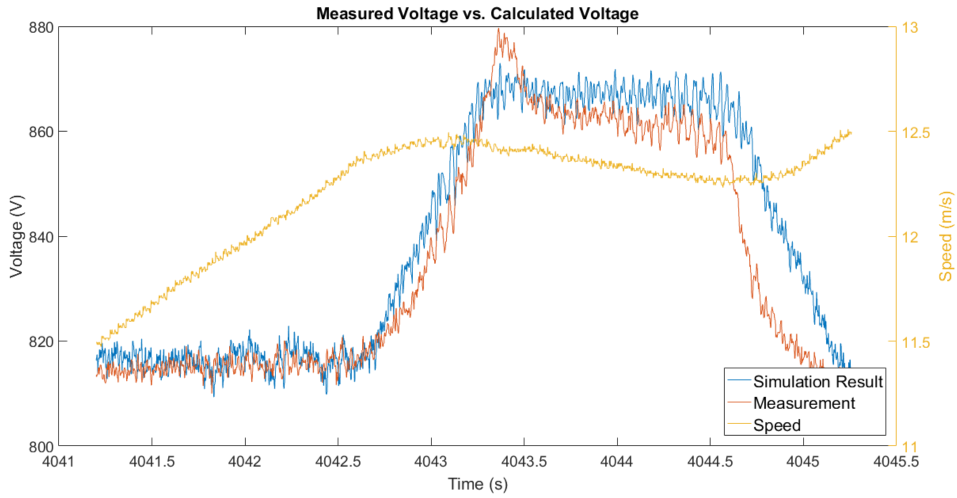

Figure 4.10: Simulation Result and Measurement of the Catenary Line Voltage - Case 2

Firstly, when the acceleration of the train starts, initially the actual catenary line voltage decreases sharply. However, in the simulation results a more linear change can be seen in the catenary line voltage. It should be noted that, this situation is not same for the deceleration of the train. In deceleration mode of the operation, it can be seen that the measured voltage data and simulation results are similar. Therefore, a possible reason of the difference might be the transients. In *Chapter 3 The Proposed System Model* the given solution methodologies are applicable for steady state solutions. The effect of the transient changes in the system are related with the time-constant (τ) of the system model at the considered mode of operation. Since the series impedance of the system has a dominant effect, time constant can be taken as follows;

$$\tau = \frac{L}{R} \tag{4.1}$$

By using the result of equation 4.1, the results of the proposed method can be modified for the transient parts. In order to eliminate this anomaly, a post-process can applied to the results of the proposed method.

In Figure 4.11, the change of the voltage according to the given transient analysis can be seen. For the considered system, time-constant of the catenary line is calculated as; $\tau = \frac{3.294\mu H/m}{24.044\mu\Omega/m} = 0.137s$

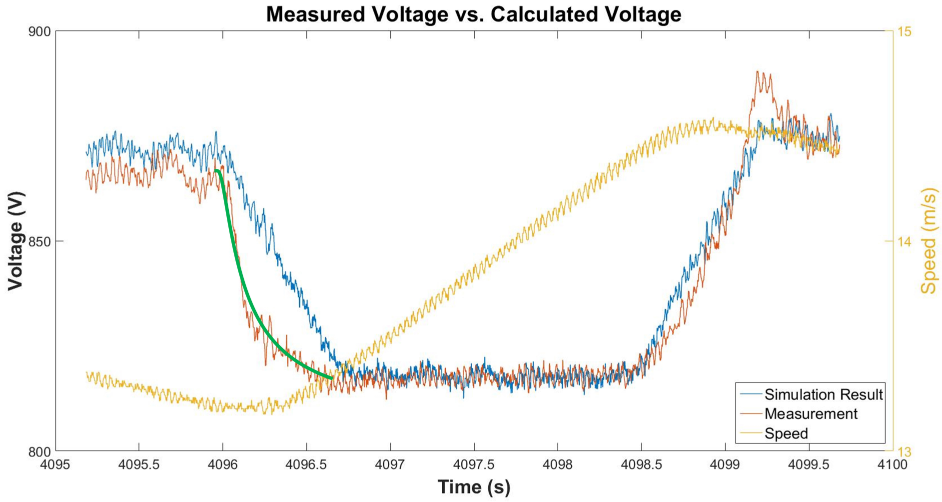

Figure 4.11: Adaptation for the Transients

Secondly, a voltage peak can be seen in the real measurements, at the end of the deceleration of the train. However, such a peak cannot be observed in the simulation results. Moreover, it should be noted that, the catenary line voltage reaches a steady-state value after the peak. The reason might be the uncertainty of the electrical parameters. In this mode of operation system model changes,depending on the connected element, which might be a resistor or a capacitor. However, due to uncertainty of the R_{eq} and L_{eq} of the catenary line modeling of the regenerative braking mode is not possible. However, still some deduction about the system can be made. Normally, when the transient change of the power is modeled with the time-constant of the system, and it is seen that the system acts as a over-damped system. However, in this problem, system acts as a under-damped system. Therefore, the subway train might be carrying a break-resistor to dissipate the regenerative braking power. Since the resistor is in parallel with the catenary line and return rails, the equivalent radius seen by the train decreases. This may result in an under-damped characteristic.

So far, the results of the proposed solution of constant speed and the constant acceleration modes shown. In these results, always the catenary line voltage at the location of the train is investigated, which means the position dependency is not shown. However, it should be noted that, when catenary line voltage relation is examined, dependency of the methodology to both time and position can be seen. Therefore, the solution with respect to position at a specific time instant should also be investigated. For that purpose, the configuration given in Figure 4.12 is analyzed and its results are given in 4.13.

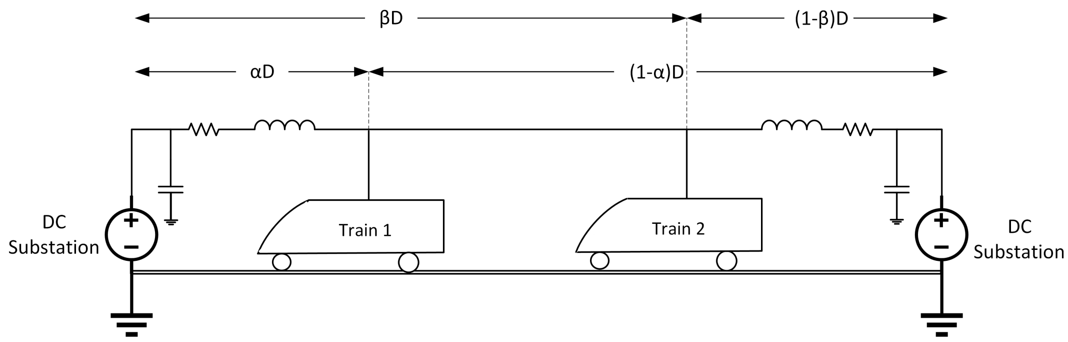

Figure 4.12: Considered Case in the Simulation of Figure 4.13

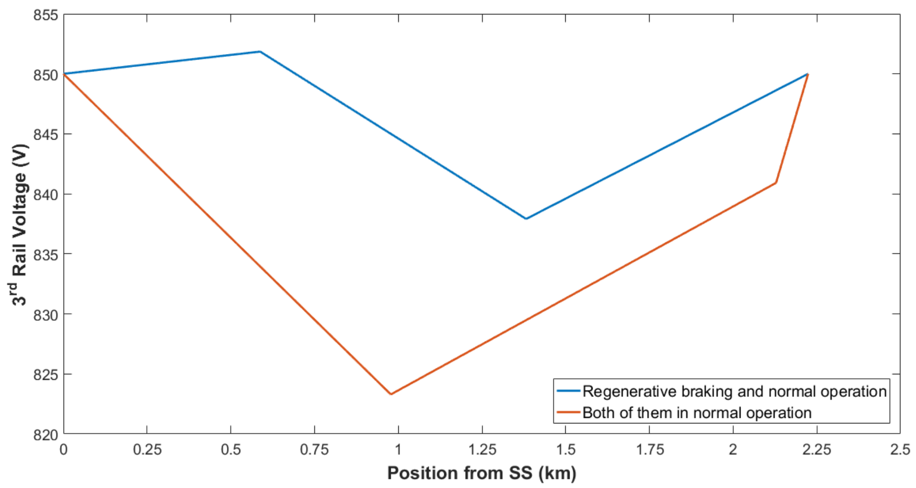

Figure 4.13: Simulations of catenary line voltage between two DC substations under the Operation of Multiple Train

In Figure 4.13, the red line represents the operation of two subway trains under the constant speed or acceleration modes of operation. However, the blue line represents the operation of two trains, while one of them in the deceleration mode of operation the other one is in acceleration mode of operation.

Unfortunately, it is not possible to verify the simulations with the given measured data. In the given data, measurements are taken only on the train; however, in this part the simulation results are showing the voltage level at all points between two DC substations. Therefore, it is impossible to do a verification with these data. Moreover, some assumptions are used to locate the substations in proper locations. To find the locations of the substations for the simulation; firstly the speed data of the subway train is integrated over time to find the position change of the train. Then by using the Table 3.1, the location of the substations are estimated.

In Figure 4.13, it can be seen that the voltage variation along the catenary line is linear. In the adaptation of multiple substation and multiple train operation to solve the problem, it was linearized to simplify the solution. Therefore, it is an expected result to see linear change in Figure 4.13. Moreover, since the system is a DC system and this simulation is done for a time instant, it is normal to see this behavior.

When both trains are drawing power from the catenary line, the voltage variation on the line decreases linearly depending on the position of trains. The voltage levels at the train locations get smaller as the trains move to the middle of the line, due to the high equivalent impedance. The amount of the drawn power is also very important, but the main characteristic is as explained.

When one of the trains is decelerating, the generated power causes a rise in the voltage. Due to the power drawn by the other train, the voltage decreases towards the position of that train.

CHAPTER 5

CONCLUSION

In this paper it is aimed to find an analytic solution for the voltage of subway catenary line. Due to the motion of the train, the electrical parameters are varying, and this makes the calculations harder. In this thesis some assumptions are proposed in order to simplify the calculations. These assumptions decouples time dependence and position dependency of the voltage for various behaviors of the train.

In order to find the parameters an analytic approach is developed and compared to FEM results for validation. It has been seen that proposed method provides better accuracy compared to the approach given in literature, for the considered system. To validate the results of the analytically determined electrical parameters, the required measurement configurations are proposed.

Considering the much faster variation in electrical quantities compared to position of train, the differential relations are solved assuming constant speed at each instant. Only the considered operating conditions, such as the drained power and speed, are changed for different mode of operations. Moreover, it is shown that the lumped parameter model is also giving similar results to the distributed parameter results. In the modeling part of the multiple train and multiple substation case, a linearized approach is used, while investigating the catenary line voltage at a time instant.

In this thesis regenerative braking case is not considered in a detailed way. However, in order to improve the model of this behavior, some modifications can be applied to the existing system model. One change might be the adding the model of the energy storage device to the system under the operation of regenerative braking. Another improvement might be the addition of a detailed substation model. Finally and the most importantly, uncertainty of the electrical parameters should be defeated with the required validation measurements. With these improvements, the problems given in *Chapter 4 Validation of the Proposed Method & Simulation Results*, can be solved.

Another important part for the solution is, the measurement of the contact resistances. This part can play an important role for the voltage characteristic of the system. In this work, this part cannot be done; because, the subway organization did not allow any field measurement to students. Only a limited data is acquired with the help of supportive company.

To conclude, the proposed method makes a connection between the voltage on the train and the speed of train. As it can be seen from the validations, the proposed method gives satisfactory results.

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