

MULTICELL RESOURCE ALLOCATION FOR MULTICARRIER NETWORKS
WITH MULTIUSER DECODING RECEIVERS

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WITH MULTIUSER DECODING RECEIVERS**

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ABSTRACT

MULTICELL RESOURCE ALLOCATION FOR MULTICARRIER NETWORKS WITH MULTIUSER DECODING RECEIVERS

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In recent years, techniques for exploiting interference on the receiver side to improve the performance of future cellular networks have been of interest. These studies assume multiuser decoding capability of receivers, i.e. the receivers are able to decode the interference signal as well as the intended signal. Additionally, due to the dominance of data in network traffic, user rate demands are becoming crucial for resource allocation of cells in the network.

In this thesis, we propose an efficient transmission scheme for the downlink of an OFDMA multicell multiuser system with multiuser decoding capable receivers. We define a marginal rate maximization problem taking into account minimum rate demands of users and develop practical scheduling (subchannel assignment) and power allocation algorithms using Lagrangian dual decomposition and gradient methods. Through Lagrangian dual decomposition, we derive the optimal power allocation rule using multiuser decoding modes for a given subchannel assignment.

We observe that the method proposed here has low computational complexity and can be easily integrated to next generation networks as well as achieving high performance in practical scenarios.

Keywords: Cellular systems, OFDMA, radio resource management, multiuser decoding, interference cancellation

ÖZ

ÇOK TAŞIYICILI, ÇOK KULLANICILI KOD ÇÖZME YETENEĞİ OLAN ALICILI AĞLAR İÇİN ÇOK HÜCRELİ KAYNAK TAHSİSİ

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Son yıllarda, gelecek nesil hücresel şebekelerin performansının geliştirilmesi için girişimin kullanıldığı teknikler ilgi odağı olmuştur. Anılan çalışmalar çok kullanıcı kod çözme yeteneği olan alıcıların (kendi sinyaliyle birlikte kendisi için gönderilmeyen (girişim) sinyalini de çözebilen alıcılar) varlığını kabul etmektedir. Ayrıca, verinin şebeke trafiğine egemen olmaya başlamasından dolayı, kullanıcıların veri hızı talepleri hücrelerin kaynak dağıtımında önemli bir etmen olmaya başlamıştır.

Bu tezde, çok hücreli, çok kullanıcı ve çok kullanıcı kod çözümü yeteneğine sahip alıcıların yer aldığı bir OFDMA sisteminin aşağı yönlü bağlantısı için verimli bir iletim algoritması önerilmektedir. Kullanıcıların minimum veri hızı talepleri dikkate alınarak marjinal veri hızı maksimizasyon problemi tanımlanmış olup, pratik bir planlama (alt taşıyıcı tahsisi) ve Lagrange ikili ayrışma ve eğim metotları kullanılarak güç dağıtım algoritmaları geliştirilmiştir. Alt taşıyıcı tahsisinin sabit olduğu durumda çok kullanıcı kod çözme modları kullanılarak, Lagrange ikili ayrışma yöntemiyle, optimal güç dağıtım kuralı türetilmiştir.

Bu tezde önerilen yöntemin düşük hesaplama karmaşıklığı olduğu ve pratik senaryolar için yüksek performans sağlamanın yanında gelecek nesil şebekelerine kolaylıkla entegre edilebileceği görülmektedir.

Anahtar Kelimeler: Hücresel sistemler, OFDMA, radyo kaynak yönetimi, çok kullanıcı kod çözümü, girişim giderimi

To my family...

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LIST OF ABBREVIATIONS

3GPP	3 rd Generation Partnership Project
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BS	Base Station
CSI	Channel State Information
CSIT	Channel State Information at Transmitters
EM	Ellipsoid Method
FDMA	Frequency Division Multiple Access
FFR3	Fractional Frequency Reuse with reuse factor of 3.
IC	Interference Channel
INR	Interference-to-Noise Ratio
IWF	Iterative Waterfilling
JD	Joint Decoding
KKT	Karush-Kuhn-Tucker
LTE	Long Term Evolution
LTE-A	Long Term Evolution- Advanced
MAC	Multiple Access Channel
MIMO	Multiple-Input Multiple-Output
MINLP	Mixed Integer Nonlinear Program
MISO	Multiple-Input Single-Output
MUD	Multiuser Decoding
MUST	Multiuser Superposition Transmission
MQAM	M-ary Quadrature Amplitude Modulation
NAICS	Network Assisted Interference Cancellation and Suppression
OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
QoS	Quality of Service
PAPR	Peak to Average Power Ratio
PSM	Projected Subgradient Method

RB	Resource Block
RSFP	Randomized Subchannel Assignment with Fixed Power
Rx	Receiver
SC-OFDMA	Single Carrier- Frequency Division Multiple Access
SD	Single Decoding
SINR	Signal-to-Interference plus Noise Ratio
SNR	Signal-to-Noise Ratio
SIC	Successive Interference Cancellation
TDMA	Time Division Multiple Access
Tx	Transmitter

CHAPTER 1

INTRODUCTION

1.1 Overview

Interference is a performance limiting factor in conventional cellular wireless networks and interference mitigation and dynamic spectrum management have crucial roles in wireless communication networks [1] to improve performance. Interference is mitigated in most systems with one of the following methods: orthogonalizing the communication links in time, frequency and space domains or treating the interference as additive noise by allowing the communication links to share the same degrees of freedom [2]. The first approach directly narrows down the degrees of freedom whether the interference is weak or not. The second approach does not care about the information in the interference signal which can be used in mitigating interference [2].

Along with these methods, finding the best performance for an interference channel (IC) without any a priori assumptions and IC's capacity region have been open problems in the literature for about 30 years [2]. The only case where the capacity region of the IC is characterized is strong interference case found by Carleial in [3] and shown to coincide with the capacity region with the same power constraint and no interference meaning that interference does not degrade capacity when the interference is very strong [4]. In the form of an extension of Carleial's result, the best known strategy for other cases (for 'less' strong interference case) is characterized by Han and Kobayashi in [5] for two user IC. They use the idea of splitting a transmitted signal into a common part which can be decoded by users in adjacent cells and a private part formed only for the intended user [2].

Future generation of mobile wireless communication systems such as the imminent 5G is expected to provide huge data rates. To support high data rates and increase the spectral efficiency, the latest 3GPP releases focus on full frequency reuse to utilize the allocated frequency bands as effectively as possible. On the other hand, implementing full frequency reuse leads to inter cell interference especially for the cell-edge users and performance is degraded severely [6] and receiver side interference management is gaining more traction as well as network side interference management [7].

Receivers that can decode two codewords are being developed for multiple input multiple output (MIMO) receivers in LTE [8]. Interference cancellation is expected to be more important for future cellular systems carrying high volume and bursty data traffic instead of voice [9]. Recently, Network Assisted Interference Cancellation and Suppression (NAICS) which is one of the receiver interference cancellation techniques has been standardized in 3GPP Release 12 [10] and studies on downlink multiuser superposition transmission (MUST) are underway [11]. NAICS is based on detection and decoding of the dominant interferer. In this regard, multiuser receivers are expected to play an important role in downlink and uplink as the number of smaller cells increases hence making interference more dominant and its cancellation more desirable.

Recent 3GPP standards focus on on full frequency reuse to use the allocated frequency bands as effectively as possible and they are based on orthogonal frequency division multiple access (OFDMA) recognized as a promising technique for next-generation wireless communication networks providing high spectral efficiency, reliability and robustness against frequency selective fading [12]–[14].

OFDMA holds an advantage for multiple access communication systems that it enables frequency and time slot allocation very easily hence adaptive interference management techniques can be used [14]. OFDMA is based on orthogonal frequency division multiplexing (OFDM) technique which splits data streams with high rates into lower rate data streams which are transmitted on orthogonal subcarriers at the same time [15]. OFDM has been widely used for today's high speed wireless systems and has become a common signaling scheme for many systems [14]. One of the advantages of OFDM lies in the increase of the symbol duration of low rate data

streams which causes a decrease in the amount of intersymbol interference in time resulting from multipath delay spread [15].

In a single cell multiuser OFDMA scenario, when there is enough spatial separation among the receivers, the channel response of the users can be regarded statistically independent [16]. The multiuser diversity can be exploited as the transmit power for each user and for each subcarrier is adapted according to the channels of each user [15].

In [17], it is stated that the sum capacity of multiuser OFDM systems consisting of a single base station and several users is maximized when each subchannel is assigned to the user with the best signal to noise ratio (SNR) and power allocation is performed by water-filling. A downside of this method is that fairness among users cannot generally be achieved. In [17], to provide each user a pre-defined (required) quality of service, proportional fairness constraints are introduced. Instead of an optimal solution that is computationally complex, a low complexity suboptimal solution having two separate stages are presented. In the subchannel allocation stage, an equal power distribution among subchannels is considered and followed by power allocation in some form of water-filling to maximize the capacity. In [18], a two-step algorithm to maximize energy efficiency (sum rate divided by power used) in a single cell OFDM wireless system considering proportional data rate for users is proposed.

There are a lot of studies for multicell scenarios for single user decoding receivers. On the other hand, for a network with multiple cells and multiple users capable of interference decoding, the resource allocation problem becomes very interesting. In the literature, interference mitigation techniques with multiuser decoding when the transmitters share the same frequency band have been studied. [19] solves a multiuser decoding problem with receivers capable of single decoding (SD), joint decoding (JD) and successive interference cancellation (SIC) in a scenario consisting of 2 transmitters (Tx) and 2 receivers (Rx) with multiuser detection (MUD) capability where each transmitter tries to maximize its own total rate iteratively with an approach similar to iterative water-filling (IWF) [20].

Another work [21] considers a macro-femto network and attempts at optimizing scheduling, power allocation and MUD user pair selection from a system level per-

spective with receivers capable of SD, JD, SIC. [21] implements the scheduling, power allocation and MUD user pair selection steps separately and iteratively to reach a local optimum. [21] considers an OFDMA network and adopts a total rate maximization approach for every time-frequency slot with power allocation. [22] finds an achievable rate region (Pareto boundary) with a multiple-input single-output (MISO) interference channel (IC) setting consisting of 2 Tx and 2 Rx with receivers capable of SD and SIC where [23] provides Nash equilibrium/bargaining solutions for the same setting.

Most interference cancellation studies consider the sum-rate maximization as their objective. However, under such an objective, all radio resources are assigned to users with good channel conditions and users with harsh channel conditions do not receive sufficient (sometimes any) rate from the network [12]. In particular, the problem in [19] does not reflect the multiuser opportunistic interference decoding case and neither subchannel assignment to users nor minimum rate constraints on the users is present in that study.

In the last decade, most of the cellular network traffic has been progressively dominated by data. Different from voice communications, where a satisfactory signal level is sufficient for reliable communications, data communications rely on the received power level as well as the bandwidth assigned to the user for satisfaction of the data on demand for each user. The aforementioned studies considering MUD capable receivers have not taken users' data demands into account. However, in practice, each user in the network may have various data rate demands from the network depending on the active applications on that user's terminal and interests.

We focus on the downlink of an OFDMA multicell multiuser network with single antenna links as shown in Fig. 1.1 where users (receivers) have minimum rate demands. In today's cellular wireless communications networks, data traffic is usually asymmetric and user fairness issue is crucial especially in the downlink, [24], [18]. MUD can be used in the receivers for interference mitigation for each subcarrier. To provide fairness among users in the network, the users in bad channel conditions have to be taken into account. To provide fairness among users in the network, we force a constant ratio (margin) of minimum rate requirements to be satisfied in each of the

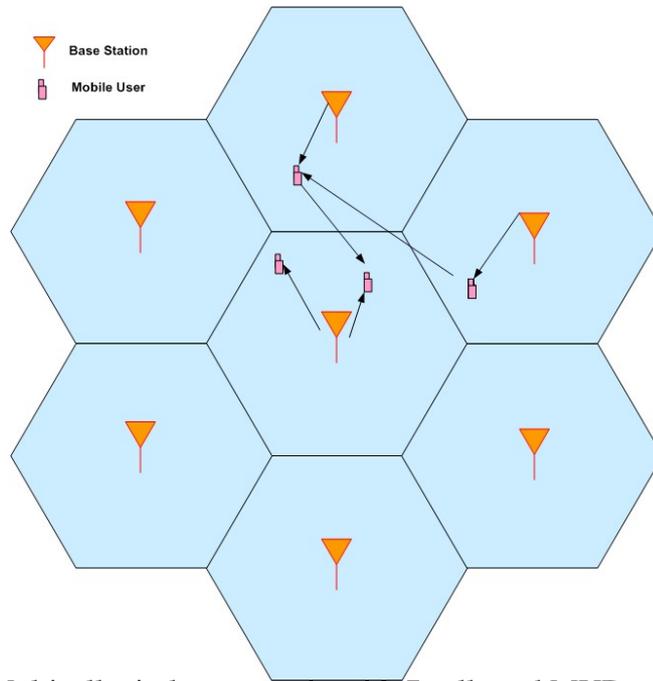


Figure 1.1: Multicell wireless scenario with 7 cells and MUD capable receivers

base stations (BS) and we consider maximization of the user rate margins. User rate margin is defined as the ratio of the rate of a user over its minimum rate requirement.

Feasibility which is a crucial factor for resource allocation problems is guaranteed by defining this rate margin as opposed to many other studies that cannot guarantee it with the constraints under study. Instead, most studies assume the problem is feasible a priori. We extend the single user scenario in [19] to a multiuser scenario with a system level approach defining a rate margin maximization problem that addresses the fairness issue in a network over the users' rate demands.

Alternatively, for the aim of serving to the users in bad channels conditions, a limit can be exercised on the number of subchannels assigned to each user, however this case is not covered in this study.

We consider that the study in [19] can be generalized for multiple macrocells with a system-level approach by designing subchannel assignment and power allocation algorithms with MUD formation steps taking minimum rate constraints into account. The main difference of this study from [19] is using a multicell multiuser scenario with users having minimum rate requirements.

We attempt to propose a distributed approach over the multicell setting in this study.

The BSs have only the knowledge of power transmitted by the other BSs and cross channel responses as well as the rate of the other BSs in each of the subchannels. However, learning the channel strength values for every subchannel is difficult to realize and more studies must be done for efficient channel estimation [25] which is beyond the scope here.

We do not assume any central controller on the network and receivers do not cooperate with each other but BSs share their resource allocation decisions. A solution based on Lagrangian dual decomposition is formulated. Projected subgradient method and ellipsoid methods are proposed to implement the solution and a low complexity algorithm is presented. The algorithm is shown to outperform benchmark methods and a modified multicell algorithm based on [17] for the scenarios under consideration.

In the sequel, we analyze two cases:

- Case 1: The users are considered to have JD and SIC capability so that each user can decode its intended message as well as the message intended for another user in the neighbouring cell where applicable. Hence MUD refers to SIC and JD in this case.
- Case 2: The users are considered not to have JD capability, they can only perform SIC or SD where applicable. Hence MUD refers to SIC in this case.

In the literature, resource allocation is performed exploiting temporal, spectral and/or multiuser diversity. However, in this study, we do not exploit temporal diversity and do not consider ergodic capacity. Resource allocation is performed over frequency and users by considering only the instantaneous rates of the users.

1.2 Contributions

The study in this thesis is a novel study and has some differences from the studies in the literature. There does not exist a resource allocation (subcarrier assignment and power allocation) study trying to satisfy minimum rates of users in a scenario of users with MUD capability. Moreover, the studies trying to satisfy a proportional rate

of users have a single cell scenario and make the subchannel assignment and power allocation in that single cell without considering any interference decoding.

We generalize the study in [19] for multiple macrocells with a system-level approach by proposing novel subchannel assignment and power allocation algorithms and MUD formation steps considering minimum rate constraints. We also characterize the optimal power allocation method when subchannel assignment is given. Since the objective is not solely maximizing the sum capacity, the users in bad conditions are also served with other users according to their minimum rate requirements.

In the subchannel assignment stage, the direct to cross channel ratios are defined different than the studies in the literature. Feasibility is guaranteed by defining a rate margin unlike the studies in the literature which assumes the problems under study are always feasible. A suboptimal low complexity iterative algorithm is proposed and the performance of the algorithm is shown to outperform legacy methods for the scenario in consideration [26]. To the best of our knowledge, this study is the first to attempt to a multicell multiuser OFDMA downlink network with receivers employing MUD subject to minimum rate requirements of each user.

1.3 Outline of the Thesis

The rest of the thesis is organized as follows. In Chapter 2, the background of interference management is introduced and recent studies in the literature and previous algorithms where minimum rate constraints are considered for single cell OFDMA systems and the objective is maximizing the rate subject to power constraints with MUD capable receivers are reviewed. The proposed algorithms are presented in Chapter 3. The performance comparison of the proposed algorithms with legacy methods are presented in Chapter 4 and conclusions are drawn in Chapter 5.

1.4 Notation

In the sequel, $(x)^+ \triangleq \max(0, x)$, small boldface letters are used to denote vectors, capital boldface letters are used to denote matrices, the notation \succeq denotes

component-wise inequality and $\mathcal{E}[\cdot]$ denotes the expectation operation.

CHAPTER 2

INTERFERENCE DECODING AND RESOURCE ALLOCATION STRATEGIES FOR OFDMA NETWORKS

2.1 Interference Decoding

In this section, a brief introduction of the interference decoding for the subchannel and power allocation problem in the downlink of OFDMA multiuser cellular networks is given and Gaussian interference channel and studies about the capacity of this channel are introduced. The system model under consideration is presented first.

2.1.1 System Model

We consider the scenario in Fig. 2.1 for the problem definition.

There are $N = 2$ cells (BSs) and K users are to be served in each BS. In 3GPP Release 12, cancellation of at most one interferer (the dominant interferer) is assumed. In this regard, for $N > 2$ case (this case is considered in Chapter 4), the strongest interfering signal is attempted for decoding and the other interfering signals are regarded as noise. The number of subchannels (resource blocks) is S and each subchannel consists of a fixed number of subcarriers. The modulation scheme is OFDMA and full channel reuse is considered, i.e., channel reuse factor is one. Each subchannel is assumed to be scheduled for only one user for transmission in each BS and the channel conditions of all the subchannels are assumed to be not varying while implementing the resource allocation operations.

Users are interested in maximizing their utilities from the network proportional to their minimum rate demands. Each BS distributes OFDMA subchannels and adjusts

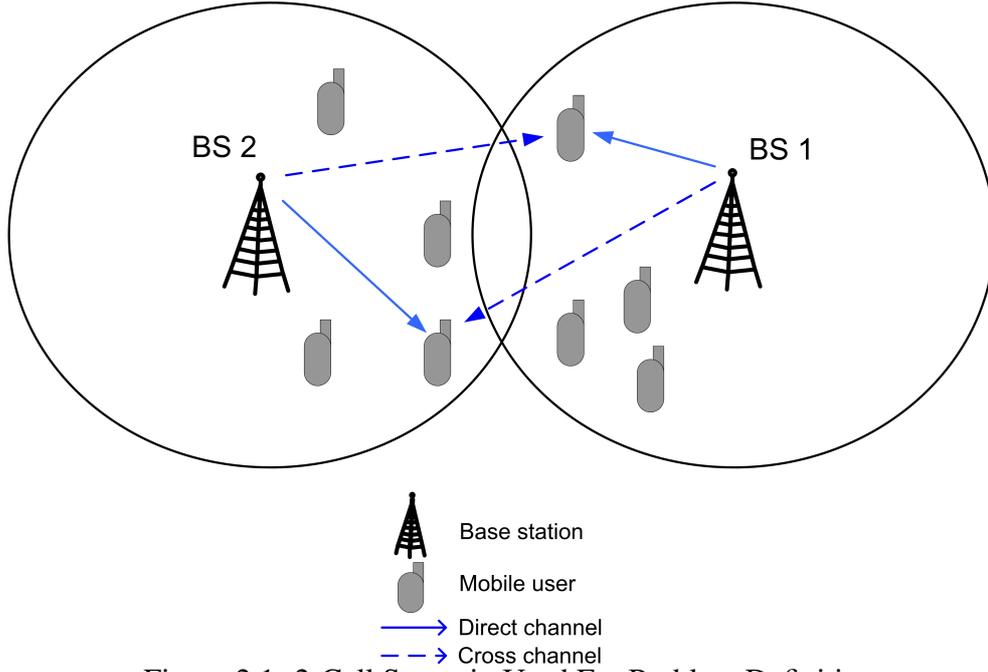


Figure 2.1: 2-Cell Scenario Used For Problem Definition

the power level in each subchannel to maximize the minimum ratio of each user's instantaneous data rate relative to that user's minimum data rate demand from the network. The users and the BSs do not account for the total rate of the network.

There is a limited coordination between BSs such that time and frequency synchronization are assumed to be settled between BSs before transmission. Each BS has full channel knowledge of the users served by itself via channel state information (CSI) feedback. BSs do not know which user is served in the neighbouring BS, however each BS knows the power and the rates of interference signals generated from the neighbouring BS for every subchannel. Each BS transmits independent messages to its users and tries to accommodate only the rates of its users. Each user is assumed to know the (direct) channel to its serving base station and the (cross) channel to the other base station via feedback mechanisms, i.e., pilot channels.

Each BS has a maximum power constraint P_j^{max} for all $j = 1, \dots, N$ and the power used in a subchannel s is p_j^s , hence $\sum_{s=1}^S p_j^s \leq P_j^{max}$ holds for all $j = 1, 2, \dots, N$.

Assume that user l in cell j (denoted as $u_{j,l}$) and user m in cell k (denoted as $u_{k,m}$) are scheduled on a subchannel s . The channel gain from cell k to $u_{j,l}$ on subchannel s is $h_{k,j,l}^s$ (the channel is called direct link if $k = j$; it is called cross link, otherwise)

which is a complex number in general. The received signals $y_{j,l}$ and $y_{k,m}$ at users $u_{j,l}$ and $u_{k,m}$ on subchannel s , respectively are

$$y_{j,l}^s = h_{j,j,l}^s x_j^s + h_{k,j,l}^s x_k^s + z_{j,l}^s \quad (2.1)$$

$$y_{k,m}^s = h_{j,k,m}^s x_j^s + h_{k,k,m}^s x_k^s + z_{k,m}^s \quad (2.2)$$

where x_j^s and x_k^s are transmitted signals from cell j and cell k on subchannel s with zero mean and variances p_j^s and p_k^s , respectively. The terms $z_{j,l}^s$ and $z_{k,m}^s$ are circularly symmetric complex Gaussian noise terms with zero-mean and variance σ^2 at users $u_{j,l}$ and $u_{k,m}$ on subchannel s , respectively. For $N > 2$ case, the noise terms can be considered to include the interference signals generated by the cells other than the cells whose signals can be decoded by the related receivers.

2.1.2 Calculation of Rates of Users and MUD States

Majority of the previous studies assume infinite backlogged traffic streams for all of the users in the network. With this assumption, users' traffic patterns are not taken into account in the resource allocation problem, which simplifies the solution of the problem considerably. On the other hand, in real networks the downlink traffic is not backlogged all the time [12] and such a consideration is left for future studies.

The user $u_{j,l}$ scheduled on s has a rate $r_{j,l}^s$ and a minimum instantaneous rate demand $R_{j,l}^{\min}$ from the BS. Let $c_{j,l}^s$ be 1 when $u_{j,l}$ is scheduled on s and 0 otherwise. Then the total instantaneous rate $R_{j,l}$ of $u_{j,l}$ can be computed as $R_{j,l} = \sum_{s=1}^S c_{j,l}^s r_{j,l}^s$. The nomenclature expressed in [19] which defines SIC, JD and SD rate regions are as follows.

We first assume that the power and rate of $u_{k,m}$ on s is fixed during the transmission of data of user $u_{j,l}$ on s . For case 1, the rate of the user $u_{j,l}$ on s according to the SD, JD and SIC decoding states are (as defined in [19])

$$r_{j,l}^s = \begin{cases} \log_2 \left(1 + \frac{H_{j,j,l}^s p_j^s}{\sigma^2 + H_{k,j,l}^s p_k^s} \right), & \text{if SD} \\ \log_2 \left(1 + \frac{H_{j,j,l}^s p_j^s + H_{k,j,l}^s p_k^s}{\sigma^2} \right) - r_{k,m}^s, & \text{if JD} \\ \log_2 \left(1 + \frac{H_{j,j,l}^s p_j^s}{\sigma^2} \right), & \text{if SIC} \end{cases} \quad (2.3)$$

where $H_{k,j,l}^s \triangleq |h_{k,j,l}^s|^2$, $\forall k, j, l, s$.

The decoding states $MUD_{j,l}^s$ of $u_{j,l}$ depends on the rate of $u_{k,m}$ as

$$MUD_{j,l}^s = \begin{cases} \text{SD, if } r_{k,m}^s > \log_2 \left(1 + \frac{H_{k,j,l}^s p_k^s}{\sigma^2} \right) \\ \text{JD, if } \log_2 \left(1 + \frac{H_{k,j,l}^s p_k^s}{\sigma^2} \right) \geq r_{k,m}^s > \\ \quad \log_2 \left(1 + \frac{H_{k,j,l}^s p_k^s}{\sigma^2 + H_{j,j,l}^s p_j^s} \right) \\ \text{SIC, if } \log_2 \left(1 + \frac{H_{k,j,l}^s p_k^s}{\sigma^2 + H_{j,j,l}^s p_j^s} \right) \geq r_{k,m}^s \geq 0. \end{cases} \quad (2.4)$$

For case 2, the rate of the user $u_{j,l}$ on s according to the SD and SIC decoding states are

$$r_{j,l}^s = \begin{cases} \log_2 \left(1 + \frac{H_{j,j,l}^s p_j^s}{\sigma^2 + H_{k,j,l}^s p_k^s} \right), & \text{if SD} \\ \log_2 \left(1 + \frac{H_{j,j,l}^s p_j^s}{\sigma^2} \right), & \text{if SIC.} \end{cases} \quad (2.5)$$

The decoding states $MUD_{j,l}^s$ of $u_{j,l}$ depends on the rate of $u_{k,m}$ as

$$MUD_{j,l}^s = \begin{cases} \text{SD, if } r_{k,m}^s > \log_2 \left(1 + \frac{H_{k,j,l}^s p_k^s}{\sigma^2} \right) \\ \text{SIC, if } \log_2 \left(1 + \frac{H_{k,j,l}^s p_k^s}{\sigma^2 + H_{j,j,l}^s p_j^s} \right) \geq r_{k,m}^s \geq 0. \end{cases} \quad (2.6)$$

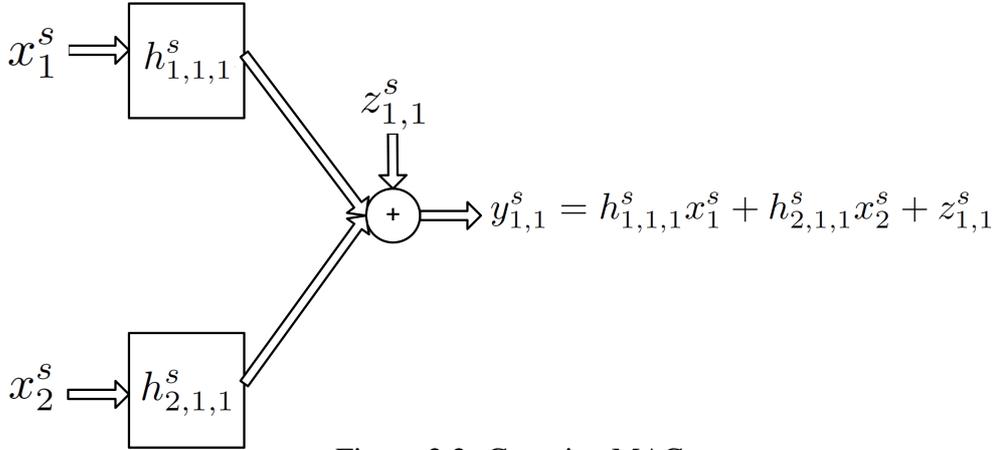


Figure 2.2: Gaussian MAC

Consider the Gaussian Multiple Access Channel (MAC) in Fig. 2.2 illustrated for the user $u_{1,1}$ on subchannel s . The achievable rate regions for the Gaussian MAC are characterized in [27].

SIC refers to the decoding state where each receiver first decodes the interfering message from the other transmitter regarding the intended message as noise corresponding to the rate region for the BS 2: $\log_2 \left(1 + \frac{H_{2,1,1}^s p_2^s}{\sigma^2 + H_{1,1,1}^s p_1^s} \right) \geq r_{2,2}^s \geq 0$ and then decodes

its intended messages afterwards without any interference. In JD mode, the receiver jointly decodes both the intended and the interfering message at the same time, treating the network as a MAC corresponding to the rate region: $\log_2 \left(1 + \frac{H_{2,1,1}^s p_2^s}{\sigma^2}\right) \geq r_{2,2}^s > \log_2 \left(1 + \frac{H_{2,1,1}^s p_2^s}{\sigma^2 + H_{1,1,1}^s p_1^s}\right)$. Each receiver treats the interfering message from the other transmitter as noise in the SD mode corresponding to the rate region of the BS 2: $r_{2,2}^s > \log_2 \left(1 + \frac{H_{2,1,1}^s p_2^s}{\sigma^2}\right)$. The receivers are considered to have capability to decode the signals of at most 2 BSs, hence signals of other BSs are regarded as noise when more than 2 BSs exist. This scenario is considered in simulations in Chapter 4.

The rate regions above and the corresponding rate calculations apply when receivers have MUD capability. Otherwise, we will use only the rate region and rate calculations corresponding to the SD decoding state, where applicable.

Moreover, the above rate calculations are for Gaussian inputs. On the other hand, for finite constellations, the rate is defined

$$r_{j,l}^s \triangleq \log_2 \left(1 + \frac{\gamma_{j,l}^s}{\Gamma}\right) \quad (2.7)$$

where $\frac{\gamma_{j,l}^s}{\Gamma}$ is the effective SINR value depending on the MUD state of subchannel s and Γ is SINR gap value that can be adjusted to reflect the desired BER (bit error rate) and the modulation scheme used. For an uncoded MQAM constellation, SINR gap is $\Gamma = -\frac{\ln(5\text{BER})(M-1)}{1.5}$ for $M \in \{4, 16, 64\}$ [28]–[33].

2.1.3 Gaussian Interference Channel

A general interference channel is shown in Fig. 2.3. The input-output relationship of the two-user Gaussian interference channel is

$$y_1 = g_{11}x_1 + g_{21}x_2 + z_1 \quad (2.8)$$

$$y_2 = g_{12}x_1 + g_{22}x_2 + z_2 \quad (2.9)$$

where $x_1 = x_1^p + x_1^c$ and $x_2 = x_2^p + x_2^c$ are complex variables subject to power constraints P_1 and P_2 ($\mathcal{E}[|x_i|^2] \leq P_i$, $i = 1, 2$) respectively, x_i^p and x_i^c are the private and common message part of the transmitter i and z_1 and z_2 are the additive white

Gaussian noise (AWGN) with power N_0 . Denoting $a_{12} = \frac{|g_{21}|^2}{|g_{11}|^2}$ and $a_{21} = \frac{|g_{12}|^2}{|g_{22}|^2}$ and n_1 and n_2 are AWGN with power 1, the channel is equivalent to the following model from the interference point of view [5]:

$$y_1 = x_1 + \sqrt{a_{12}}x_2 + n_1 \quad (2.10)$$

$$y_2 = \sqrt{a_{12}}x_1 + x_2 + n_2 \quad (2.11)$$

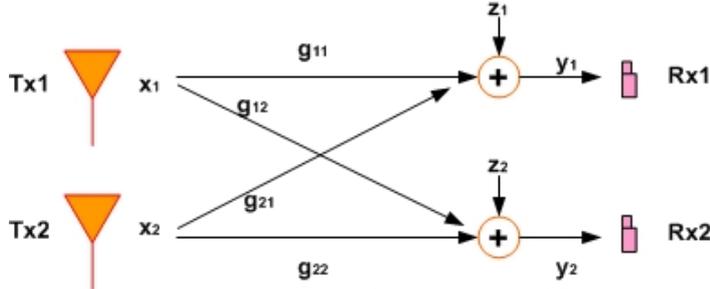


Figure 2.3: Two user Gaussian interference channel

The only case, where the capacity region of the interference channel is characterized, is strong interference case found by Carleial in [3] and is shown to coincide with the capacity region with the same power constraint and no interference.

In the form of an extension of Carleial's result, the best known strategy for other cases (for 'less' strong interference case) is characterized by T. S. Han and K. Kobayashi in [5] for two user interference channel. They use the idea of splitting a transmitted signal into a common part which can be decoded by users in adjacent cells and a private part formed only for the intended user. They use the bright idea that common message decoding can be used to cancel some part of interference. They characterize a new achievable rate region for the two-user interference channel where each transmitter communicates with its respective (intended) receiver through the common channel. In this scenario, the transmit signals are designed to be partially decodable in adjacent users. To provide this, each user's transmit signal is split into a private message part to be decoded only by the intended receiver and a common message part to be decoded by both receivers to mitigate interference. This channel is called the modified interference channel.

Let R_i^p and R_i^c denote private and common message rates of transmitter i , respectively. Han and Kobayashi prove that if the rate quadruple $(R_1^p, R_1^c, R_2^p, R_2^c)$ is achievable for the modified channel, then $(R_1^p + R_1^c, R_2^p + R_2^c)$ is achievable for the interference

channel. This strategy provides the best known achievable rate region for the two-user interference channel and the modification of this strategy provides that of the Gaussian interference channel shown in Fig. 2.3 [2].

The capacity region of the Gaussian interference channel found by Han and Kobayashi under power constraints P_1 and P_2 and $a_{12} \geq 1$ and $a_{21} \geq 1$ contains all rates which is the closure of set of achievable rate pairs (R_1, R_2) satisfying

$$R_1 \leq 0.5 \log_2(1 + P_1), \quad (2.12)$$

$$R_2 \leq 0.5 \log_2(1 + P_2), \quad (2.13)$$

$$R_1 + R_2 \leq \min\{0.5 \log_2(1 + P_1 + a_{12}P_2), 0.5 \log_2(1 + P_2 + a_{21}P_1)\}. \quad (2.14)$$

This result is proven to improve the Carleial's result for Gaussian interference channel [5].

[34] characterizes the capacity region of frequency selective Gaussian interference channels modelling the channel as independent parallel memoryless Gaussian interference channels under strong interference case. The dual problem is shown to be solved and iterative water-filling being a suboptimal scheme is shown to provide close to optimum performance.

In [35], the capacity bounds of vector Gaussian interference channel for multi antenna systems under strong interference case are characterized. In [2], Etkin et. al. show that the capacity region of a two-user interference channel can be achieved within one bit by a simple scheme with setting the private message power at the unintended receiver to be at the background noise level [36]. This result shows that a very simple Han-Kobayashi scheme can achieve rates $(R_1 - 1, R_2 - 1)$ for any (R_1, R_2) in the interference channel capacity region. In some high signal-to-noise ratio (SNR) regimes, these results are shown to be asymptotically optimal. This result shows that common message part can be identified as the part of the out of cell interference signal which is above the background noise level and should be decoded [36].

In [37], Mehanna et. al. study Han Kobayashi rate region more carefully. Unlike [2] which achieves Han-Kobayashi rate region within one bit, [37] obtains a closed form expression by finding the optimal power split ratio between common and private messages assuming no time sharing is allowed for the corresponding maximum Han-

Kobayashi achievable sum rate for two user symmetric Gaussian interference channel ($a_{12} = a_{21}$) with the same input powers ($P_1 = P_2$). Interestingly, it is found that although the channel is symmetric, asymmetric power split ratio for both users can improve the sum rate. Moreover, [37] identifies the regions where simple orthogonal signaling has a better performance than rate splitting.

2.2 Resource Allocation Strategies for OFDMA Networks

In this section, the studies in the literature about resource allocation for OFDMA networks are summarized in two categories.

2.2.1 OFDMA Resource Allocation with Treating Interference as Noise

OFDMA resource allocation without interference decoding has been a research topic in many studies for many years. The main concern of such studies is to distribute the subchannels to the users in order to maximize the overall system throughput. In some studies, the quality of service (QoS) demands of users have been discussed with a point of view different than the one in this study. The common point of the studies falling into this group is that they consider only noise without taking interference into account as a degradation factor to the capacity and having a scenario with single cell. Therefore, multicell subcarrier assignment and power allocation are not considered in the studies below. Comprehensive surveys about OFDMA resource allocation are presented in [31] and [38].

In [39]–[41], resource allocation is studied for single-cell FDMA/ OFDMA systems. [29] considers a single cell multiuser OFDM downlink scenario with adaptive modulation assuming full channel state information at transmitters (CSIT). This paper proposes an algorithm which minimizes the total transmit power with subcarrier allocation and power allocation to users. The study assumes a total rate constraint of the cell (sum throughput) and seeks the minimum power and subcarrier distribution that satisfies that rate.

In [42], dynamic single cell multiuser subchannel allocation in a downlink of an

OFDM system is addressed. As the conventional TDMA (time division multiple access) or FDMA (frequency division multiple access) systems do not care about the users in poor channel conditions, [42] proposes to mitigate this problem. [42] studies maximization of the minimum user rate to provide maximum fairness among users by defining a multiuser convex optimization problem for finding optimal subchannel allocation without considering the rate demands of users. A suboptimal solution to this problem is found with equal power distribution to each subchannel.

[15] and [43] show that by assigning each subchannel to the user with the best subchannel gain and performing power allocation by water-filling algorithm across time and frequency, the ergodic sum capacity is maximized. In neither of these studies, fairness among users is taken into account.

In [25], radio resource allocation (base station and subcarrier assignment along with bit loading) to satisfy QoS demands of users is studied in the downlink of a cellular OFDMA scenario without MUD receivers. A heuristic algorithm is presented to manage radio resources among multiple users according to their QoS demands and maintaining the QoS of already established links in neighbouring cells. The objective of the optimization problem is to minimize the total transmit power. It is shown that the performance of the algorithm is better than classical radio resource management techniques.

[44] exploits time, frequency and multiuser diversity and concludes that utility based cross-layer optimization enhances system performance while achieving proportional fairness by setting the user weights as the reciprocal of the user's average rate [16] in a single cell multiuser scenario.

In [17], it is stated that the sum capacity of multiuser OFDM system (consisting of a base station and several users) is maximized when each subchannel is assigned to the user with the best signal to noise ratio and distribution of power by water-filling after subchannel allocation; but with this method, fairness among the users cannot generally be achieved. In [17], to provide each user a pre-defined (required) quality of service, proportional fairness constraints are introduced. Instead of an optimal solution that is computationally complex, a low complexity suboptimal solution is presented. The proposed suboptimal solution has two separate parts: subchannel allocation and

power allocation. First, subchannel allocation is performed by considering an equal power distribution among subchannels heuristically. Given the subchannel allocation in the previous step, power allocation is performed in a water-filling like technique to maximize the capacity. In [18], a two-step algorithm consisting of subcarrier assignment and power allocation to maximize energy efficiency (sum rate divided by power used) in a single cell OFDM wireless system considering proportional data rates for users is proposed. Proportional rate constraints proposed in [17] and [18] for single cell OFDM networks are special cases for the marginal rate maximization problem formulated in this thesis.

In [45], a subcarrier, rate, and power allocation scheme is proposed for multiuser OFDMA system for a system rate maximization problem under maximum power and minimum rate constraints for each user, considering proportional fairness based on Nash bargaining methods. In [45], interference and interference decoding is not considered.

[46] aims to maximize total rate with joint subcarrier and power allocation scheme (one step approach) in a multiuser OFDM setting while considering proportional fairness amongst users. Joint subcarrier and power allocation algorithm allocates subcarriers to the user with the minimum rate to minimum required rate ratio. Every-time a subcarrier is allocated to a user, the power to be allocated for each subcarrier of that user is calculated by a water-filling algorithm using the total power allocated to that user so far and the rates for all subcarriers of that user are updated with the new power levels calculated with water-filling.

[47] considers the downlink of multiuser OFDM systems and proposes a solution to the problem of maximization of total data rate under QoS constraints on each users' data rates. A suboptimal solution with low complexity for subchannel allocation considering equal power allocation among subchannels is proposed using the Lagrange multipliers method. In [47], interference and interference decoding is not considered and the solution does not care about efficient power allocation.

[16] handles a continuous and discrete ergodic weighted sum rate maximization in a single cell OFDMA scenario without considering interference and an optimal resource allocation algorithm is presented based on dual optimization.

In [48] and [49], low complexity heuristic algorithms are presented for a single cell multiuser OFDMA network considering varying channel conditions and data packets intended for each user.

In [50]–[55], centralized resource allocation for multicell downlink OFDMA networks are studied and suboptimal subcarrier and power allocation algorithms are proposed.

[56] considers an OFDMA system with relays and an optimization problem involving relay selection, subcarrier assignment and power allocation with an objective of maximizing downlink capacity with fairness among users is derived. Since the problem cannot be solved directly, relaxation as a concave optimization problem by time-sharing is performed and dual problem of the relaxed problem is solved using a subgradient method.

In [57], a joint distributed subcarrier, bit and power allocation problem is formulated as a mixed integer nonlinear program (MINLP) and an iterative solution is proposed by decomposing the problem into smaller subproblems.

In [58], a resource allocation problem is set up in multiuser OFDM-based cognitive radio networks consisting of primary and secondary users. The problem is configured to distribute the subchannels to secondary users with minimum power. Finding the optimal solution of the configured problem is stated to be computationally complex, hence a two round method is proposed. In this method, by taking into account subchannel gains and interference, a heuristic subchannel assignment method is proposed as a first round. The subchannels are assigned with maximum SINR rule, first and the remaining subchannels are assigned to provide proportional fairness. In the second round, power is allocated among subchannels assigned to the users in the first round hence maximization of capacity is accomplished.

[59] studies base station coordination for macro multicell and mixed macro-femto multicell OFDMA networks assuming full frequency reuse. A heuristic joint proportionally fair user scheduling, transmit and receive beamforming and power allocation algorithm with an objective of overall network utility maximization considering intracell and intercell interference is proposed. The main idea of the proposed strat-

egy is decoupling scheduling, beamforming and power allocation stages. It is shown that transmitter coordination can significantly improve the overall network throughput compared to a conventional network strategy.

In [60], a joint resource optimization scheme (subcarrier allocation, subcarrier pairing, power allocation and relay selection) is proposed for relay assisted OFDMA networks with multiuser cooperation. The objective is maximizing the total capacity of the system satisfying the QoS requirements of the users. The scenario is a single-cell multiuser scenario. The optimization problem is shown to be a MINLP and an optimization framework to solve such problems is presented [60]. The problem is decomposed into subproblems such as joint relay selection & subcarrier allocation subproblem and joint resource optimization subproblem. A method is developed to modify the former subproblem as a linear assignment problem and the optimal assignment solution is formulated based on the Hungarian method. The solution proposed to the latter subproblem is found through the dual decomposition method.

In [61], sum-rate maximization of a multicell OFDMA downlink network is studied without considering any rate constraints on receivers. Optimal subchannel allocation and optimal power allocation solution is found by a monotonic optimization framework for the multicell scenario.

In [62], in order to maximize frequency reuse factor or spectral efficiency, distributed algorithms for subcarrier allocation are presented for a multicell OFDMA downlink scenario with limited message passing among BSs when intercell interference is present.

In a recent study [63], subchannel assignment and power allocation problems along with user association problem in heterogeneous downlink OFDMA networks with conventional receivers are studied. The objective is maximizing the weighted sum rate and overall problem is divided into two subproblems. Subchannel assignment and user association subproblem for fixed power allocation are solved with optimally using graph theory and power allocation subproblem assuming fixed subchannel assignment is solved using Karush-Kuhn-Tucker (KKT) optimality conditions.

As the above studies suggest, the problem at hand cannot be directly solved by clas-

sical convex optimization techniques but relaxation or dual decomposition are suggested instead to obtain near-optimal solutions. Another common point in the studies in this subsection is absence of considering interference and interference decoding. Moreover, in most of the aforementioned studies in this subsection, the subchannel allocation is done assuming uniform power allocation and the power allocation is subsequently performed by water-filling.

2.2.2 Resource Allocation with MUD Receivers

Available radio resources such as bandwidth and power are scarce but the data usage and demand are rapidly increasing and the interference limits the capacity of the channels. Therefore, trying to realize Han-Kobayashi results for today's multicell wireless networks, practical interference mitigation techniques have attracted attention in order to overcome interference limitations.

Each base station in conventional wireless multicell networks transmits an independent data stream to its users in its own serving area. In this regard, conventional systems do not exploit the interference and treat the interference of outer cells as noise. However, out of cell interference is usually significant and above the noise level. Equipped with multiple antennas, having the ability of forming various beamforming patterns and adjusting transmit powers, modern cellular systems aim at mitigating interference with multiuser detection in the adjacent cells. In this regard, designing decodable interference signals has gained much interest recently.

Mostly, the studies on multi-antenna multicell interference networks deal with the scenario where multiple base stations cooperate with each other and perform beamforming which is a method to distribute the information between transmitters in the downlink to avoid mutual interference as much as possible. As shown in [64]–[67] transmit beamforming at the base station can boost the capacity significantly compared to the uncoordinated scenario.

In [68], a spectrum sharing game is studied where multiple operators share the same frequency band. It is concluded that Nash equilibrium exists when the transmitters transmit simultaneously but the regions heavily depend on the channel realizations,

i.e., some regions can have non-unique Nash equilibrium points. For preventing the non-unique equilibrium points, a sort of priority is proposed between operators.

Aiming to apply information theoretical concepts on practical communication systems, Dahrouj and Yu use the advantage of the idea of common-private message splitting scheme proposed by Han and Kobayashi, for the benefit of cell-edge users in a wireless multicell network in [36]. Dahrouj and Yu consider a multicell downlink system with multiple antenna base stations, single antenna mobile users. In this scenario, multiple users may transmit simultaneously in each cell and downlink beamforming is used to spatially multiplexing users' signals. In this scenario, the users for common message decoding are selected and their downlink beamformers are designed. It is shown that if the beamformers are designed to subtract interference, larger performance gains are possible.

In [36], users have rate constraints and due to these constraints, an objective function of minimizing the total transmit power is considered. It is assumed that a central processor exists and full CSIT is available. Transmit beamformers for private and common messages with fixed user selection (successive decoding strategy is assumed) are optimized. Common-private rate splitting is solved via Semidefinite Programming (SDP) relaxation methodology. Another objective function, maximization of minimum achievable rate across the whole networks is also considered. The problem is posed as minimizing the total transmitter power across the whole network. However, since wireless systems are rate adaptive, another objective function used in [36] is maximizing the minimum rate. In this case, a numerical heuristic greedy discrete optimization is proposed with convex relaxation and the most suitable out of cell users for common message decoding, rate splitting levels and optimal downlink beamforming vectors for common and private messages are determined.

In [36], the optimization problems require to search over all user decoding pairs, common-private rate splitting and possible rate targets. This search can be infeasible for large networks. Instead of this method, so-called interference-to-noise (INR) ratios are used for possible user pairings and the best candidates for common-private message splitting are the users whose INR ratios are the highest. The INRs are assumed to be estimated via pilot signals.

[19] considers a 2 Tx-2 Rx OFDMA system and define interference regions for users as strong interference, moderate interference and weak interference. It is shown that by creating these regions, one can define interference decoding schemes such as SIC, JD or SD for each region in every subchannel and the system can have significant throughput gains over single detection with an iterative power allocation scheme similar to water-filling used in parallel Gaussian interference channels. This study only covers the issue at a link level rather than a system-level [21] and single antenna setting is adopted in the work hence no beamforming is applied.

There are various algorithms in the literature but we elaborate the algorithm in [19] below, since it will be used as a benchmark for comparison with the proposed method(s) in this thesis.

In [19], the system model is defined as follows. There are 2 Tx and 2 respective Rx sharing the same frequency band consisting of S subchannels. The input-output relationship is:

$$y_1^s = h_{11}^s x_1^s + h_{21}^s x_2^s + z_1^s \quad (2.15)$$

$$y_2^s = h_{22}^s x_2^s + h_{12}^s x_1^s + z_2^s \quad (2.16)$$

where x_1^s and x_2^s are transmitted signals from transmitters 1 and 2 and y_1^s and y_2^s are received signals at receivers 1 and 2 on subchannel s and z_1^s and z_2^s are receiver noises (independent circularly symmetric complex Gaussian random variables having zero-mean and unit variance) at receivers 1 and 2, respectively. $h_{i,i}^s$ denotes complex direct link channels from transmitter i to receiver i on subchannel s and $h_{i,j}^s$ denotes complex direct link channels from transmitter i to receiver j .

It is assumed that the receivers are able to estimate the respective direct link channels and the respective cross-link channels from the interferers on every subchannels. Additionally, the transmitters are considered to be able to estimate power and rate of the interference signals and the receivers are considered to be capable of MUD. Since there is a single receiver for each transmitter, subchannel allocation is not a subject of [19]. On the other hand, power allocation is done iteratively until both users' power allocation converges. The power allocation problem for transmitter 1 is denoted as

follows (according to carrier independent coding problem in [19]):

$$\max_{p_1^s \geq 0, \forall s} R_1(p_1^s) \triangleq \frac{1}{S} \sum_{s=1}^S r_1^s(p_1^s) \quad (2.17)$$

$$\text{s.t. } \frac{1}{S} \sum_{s=1}^S (p_1^s) \leq P_1 \quad (2.18)$$

where r_1^s is the rate of transmitter 1 and $R_1(p_1^s)$ is average transmit rate of transmitter 1 with power p_1^s on subchannel s and P_1 is the average transmit power constraint of transmitter 1. The power allocation problem for transmitter 2 can be realized easily from the problem for transmitter 1 over all subchannels by changing the index appropriately.

This problem is proven to be convex hence can be solved via convex optimization techniques [19]. It is solved via the Lagrangian dual decomposition method. The power allocation called iterative spectrum shaping is decomposed into S independent subproblems and solved with a common water level over all subchannels similar to conventional iterative water-filling. The water level is found iteratively by using the bisection method [69] (explained in Appendix B) based on the subgradient of the Lagrangian dual function $\mathcal{L}(p_1^s, \lambda)$ (explained in Appendix A):

$$\mathcal{L}(p_1^s, \lambda) = \frac{1}{S} \sum_{s=1}^S r_1^s(p_1^s) - \lambda \left(\frac{1}{S} \sum_{s=1}^S p_1^s - P_1 \right) \quad (2.19)$$

where λ is non-negative dual variable of the Lagrangian dual function [19]

$$g(\lambda) = \max_{p_1^s \geq 0, \forall s} \mathcal{L}(\{p_1^s\}, \lambda). \quad (2.20)$$

It is proved in [19] that the Lagrangian dual function can be decomposed into S subproblems defined as

$$g_s(\lambda) = \max_{p_1^s \geq 0, \forall s} r_1^s(p_1^s) - \lambda p_1^s, \quad s = 1, \dots, S \quad (2.21)$$

satisfying

$$g(\lambda) = \frac{1}{S} \sum_{s=1}^S g_s(\lambda) + \lambda P_1. \quad (2.22)$$

The power allocated to the receiver on each-subchannel is determined according to the power used by transmitter 2 on that subchannel and the MUD state (SIC, JD or SD) of the receiver 2. Consequently, it is shown that iterative spectrum shaping with MUD receivers performs better than conventional iterative water-filling. In [19], only one user per transmitter is considered to be served.

As another related work, [21] considers a macro-femto network and tries to optimize scheduling, power allocation and MUD user pair selection from a system level perspective with receivers capable of SD, JD, SIC. In [21], authors consider an OFDMA network and adopt a total rate maximization approach for every time-frequency slot with power allocation.

In [8], it is shown that Nash equilibria of a non-cooperative game with a Gaussian interference channel with 2 Tx-2 Rx, where the receivers can decode at most two codewords at a time, exists. It is shown that the equilibrium is achieved where a player reduces its power level for enabling interference cancelling. In [8], the cases SD (referred to as no interference cancellation) and SIC (referred to as interference cancellation) defined in this thesis are studied.

[4] takes the scenario of 2 Tx-2 Rx symmetric Gaussian interference channel with receivers capable of successive interference cancellation. Pareto boundary of the achievable rate region of the channel under investigation is identified.

[22] finds an achievable rate region (Pareto boundary) with a 2 Tx-2 Rx multiple-input single-output (MISO) IC setting with receivers capable of single decoding and successive interference cancellation assuming perfect CSIT. [23] focuses on the achievable rate region with a 2 Tx- 2 Rx MISO IC setting with receivers capable of single decoding and successive interference cancellation and computes Nash equilibrium/bargaining solutions.

The studies stated in this section show that the similar kind of interference decod-

ing problems cannot be analytically solved leading to optimal results but suboptimal heuristic solutions with low complexity and lower computational burden can be found for various scenarios.

CHAPTER 3

PROPOSED ALGORITHMS FOR RESOURCE ALLOCATION IN OFDMA NETWORKS WITH MUD RECEIVERS

We can broadly classify the rate optimization objectives into three:

- Maximize total system throughput- sum of the rates (Max-Sum): the basic method is to assign the subchannels to the users with the maximum marginal rate and this is usually called max-SINR rule.
- Maximize the worst user rate (Max-Min): to provide fairness among users, the maximization of the worst user rate is taken as an objective.
- Maximize the product of rates (Proportional Fairness): the product of the rates is maximized and this method is believed to be the most fair among all optimization rules.

As discussed in Chapter 2, interference cancellation studies mainly take the sum-rate maximization as an objective; however, under such objective, all radio resources are assigned to users with good channel conditions and users with harsh channel conditions do not get enough (sometimes any) rate from the network [12]. In particular, the problem formulation in [19] does not take multiuser opportunistic interference decoding and minimum rate constraints on the users into account.

To provide fairness among users in the network, the users in bad channel conditions have to be taken into account according to some metric. We consider that the study in [19] can be generalized to multiple macrocells with a system-level approach by designing subchannel assignment algorithm and power allocation algorithm with MUD formation steps considering minimum rate constraints to enable the users in bad conditions to be served. We propose an iterative and distributed approach between BSs

with the knowledge of cross channel responses and power transmitted by the other BSs as well as the rate of the other BS in each of the subchannels. We do not assume any central controller on the network. To serve the users in bad channel conditions, we force a constant ratio (margin) of minimum rate requirements to be satisfied in each of the BSs. For this aim, we use max-min fairness which is a fairness measure to balance the resource allocations among users [70], [71]. On the other hand, the user in bad channel conditions limits the overall system performance and max-min fairness has worse overall performance compared to other fairness metrics, when the users have considerably different channel conditions [72].

3.1 Problem Formulation

We define the resource allocation problem as a rate margin maximization problem with parameter α at BS j as follows:

$$\max_{p_j^s, c_{j,l}^s, \forall s} \alpha \quad (3.1)$$

$$\text{s.t. } R_{j,l} = \sum_{s=1}^S c_{j,l}^s r_{j,l}^s \geq \alpha R_{j,l}^{\min}, \quad \forall l = 1, \dots, K \text{ in BS } j \quad (3.2)$$

$$\sum_{s=1}^S p_j^s \leq P_j^{\max} \quad (3.3)$$

$$p_j^s \geq 0, \quad \forall s = 1, \dots, S \quad (3.4)$$

$$\sum_{l=1}^K c_{j,l}^s = 1, \quad \forall s = 1, \dots, S; \quad c_{j,l}^s \in \{0, 1\}. \quad (3.5)$$

The parameter α ensures the feasibility of the problem and it can be determined as the proportional rate margin subject to minimum rate constraints of the users served by the same BS and the value of α depends on the feasibility of the assignment of resources. The variable α satisfies the inequality $\left\{ \frac{R_{j,1}}{R_{j,1}^{\min}}, \dots, \frac{R_{j,K}}{R_{j,K}^{\min}} \right\} \geq \alpha$ hence α is upper-limited with the minimum of $\frac{R_{j,l}}{R_{j,l}^{\min}}, \forall l = 1, \dots, K$. The main aim of this formulation is to keep α greater than or equal to 1 to satisfy the minimum rate constraints, but

this formulation also encompasses the case $\alpha < 1$ when resources are not sufficient to fully support the minimum rate constraints of all the users.

Since $\frac{R_{j,l}}{R_{j,l}^{\min}} \geq \alpha$ is a constraint and α is limited with the minimum of $\frac{R_{j,l}}{R_{j,l}^{\min}}$, we can pose the problem equivalently as follows:

$$\max_{p_j^s, c_{j,l}^s} \min_l \frac{R_{j,l}}{R_{j,l}^{\min}} \quad (3.6)$$

$$\sum_{s=1}^S p_j^s \leq P_j^{\max} \quad (3.7)$$

$$p_j^s \geq 0, \quad \forall s = 1, \dots, S \quad (3.8)$$

$$\sum_{l=1}^K c_{j,l}^s = 1, \quad \forall s = 1, \dots, S; c_{j,l}^s \in \{0, 1\}. \quad (3.9)$$

When the number of users per BS is 1, the problem formulation in this study is equivalent to the problem in [19] because the minimum rate constraint and subchannel assignment are meaningless in that particular case, hence [19] is a special case of this study. Moreover, proportional rate constraints proposed in [17] and [18] for single cell OFDM networks are also special cases for the marginal rate maximization problem formulated in this thesis.

Because of the non-linear constraints, the above optimization problems are non-convex optimization problems and cannot be solved with convex optimization techniques even for single cell. Therefore, finding the optimal solution is rather difficult and computationally complex and it requires searching over all the possible combinations of subchannel assignments, power allocations and MUD pairings. Therefore, suboptimal algorithms are preferred to solve such problems in a reasonable amount of time. From a practical point of view, we have to decompose the main problem into subproblems such as subchannel assignment, power allocation and multiuser detection set composition that can be more easily solved separately as proposed by [21] and [17] for similar kinds of problems.

We decompose the problem to the following subproblems and solve the overall problem in several steps:

- Subchannel assignment,
- Power allocation,
- Multiuser detection pairing

as shown in Fig. 3.1.

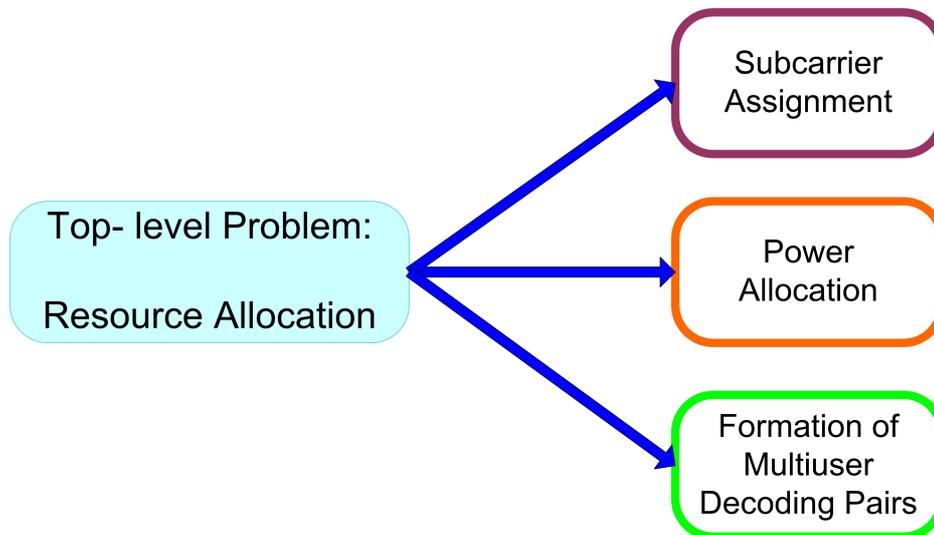


Figure 3.1: Top-level problem and subproblems

Subchannel assignment decisions are made for each subchannel according to the rate demands of users assuming fixed power on all subchannels. Later, in the power allocation step, the powers on all subchannels are optimized. Since the state of MUD for each user on each subchannel depends on the power used on that subchannel, we cannot totally separate the MUD pair formation subproblem from the power allocation subproblem. The MUD pair formation subproblem is integrated with the power allocation subproblem.

The proposed algorithm is a distributed algorithm similar to the ones proposed in [1], [19], [20], [73], [74]. Yet, our study is different from these studies because these studies do not deal with the resource allocation problem for multicell multiuser OFDMA networks with minimum rate requirements and having MUD capable receivers. The updates of subchannel assignments and power levels as well as MUD

pairs for each BS are performed one after another. We assume that all other BSs except BS j have already performed their subchannel assignment and power allocation to their users. Based on the decisions of other BSs about allocated resources, BS j makes the subchannel assignment and determines the decoding states of each user for that subchannel and the corresponding power levels of the subchannels as well as the subchannel rate assigned for that decoding state. The algorithm depicted in Fig. 3.2 can be summarized as follows:

1. The subchannel assignments, power levels and rates for each subchannels in BSs other than BS j are initialized.
2. BS j makes subchannel assignment assuming uniform power distribution over subchannels with the resource allocation knowledge from the other BSs.
3. With this subchannel assignment, it distributes the power over subchannels by determining the MUD states on each of the subchannels with a knowledge of the power levels and resulting rate for the other BSs on each of the subchannels.
4. Power allocation step is performed iteratively until power levels on each subchannel converge or a final number of iterations is reached.

By this methodology, BS j determines the power levels and rates on each of the subchannels. These assignments and allocations are inputs for the other BSs to perform their own subchannel assignment and power allocation over subchannels. The BSs implement the above overall algorithm until convergence is achieved in power levels or a final number of iterations are reached.

The overall algorithm proposed is similar to the IWF method. IWF is a multiuser rate optimization technique which was first suggested in [73] for DSL modems to perform spectral shaping [1], [75]. With this algorithm, each user tries to maximize its own rate by performing single user water-filling iteratively by considering the sum of interference from other users as additive noise. In this algorithm, each user plays a non-cooperative game and the equilibrium is achieved on the convergence point. This algorithm fits well to distributed implementation where a central controller does not exist or global information is not available at all transmitters. In [1], sufficient conditions for the convergence of the algorithm to an equilibrium point in a Gaussian interference channel scenario are presented.

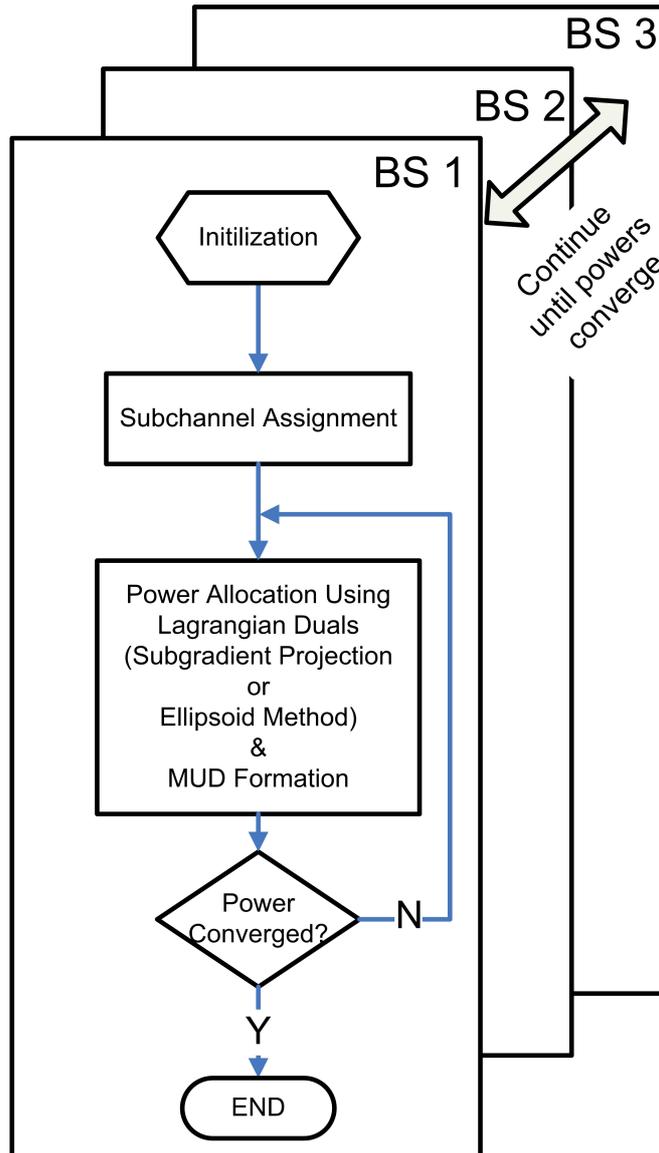


Figure 3.2: Distributed and iterative resource allocation algorithm (Overall Algorithm)

The IWF algorithm is explained for MIMO BC and MAC in [74] and for the single user case in [20]. The IWF algorithm does not try to find the global optimum point [75]. On the other hand, it is proven that at the optimum rate point, each user's power allocation is found by a water-filling solution treating all other users' interference as noise and the rate-sum optimal covariance matrix can be found with an iterative algorithm [20].

The algorithms developed for subchannel assignment and power allocation are elaborated in the following subsections. In some previous studies about single cell OFDMA

resource allocation, it is shown that performance of uniform power allocation to the subchannels is close to optimal water-filling [16], [76]. Hence, the subchannel assignment algorithm starts with uniform power allocation among subchannels. We provide the formulation for Gaussian inputs, however the formulation for MQAM signalling can be derived considering the signal-to-noise ratio gap described in Chapter 2.

3.1.1 Subchannel Assignment

In subchannel assignment step, we do not know the optimal power to be allocated to the subchannels and we assume uniform power distribution $p_j^s = \frac{P_j^{max}}{S}, \forall s \in S$ on the subchannels.

3.1.1.1 MUD-SCA algorithm

In iteration i of this algorithm (for BS j), the user \hat{l} that has the minimum proportional instantaneous rate which is calculated as the ratio of total rate of the user with already assigned subchannels until iteration i to its minimum rate requirement $R_{j,l}^{\min}$ is determined:

$$\hat{l} = \arg \min_{l, \forall l} \left\{ \frac{R_{j,l}}{R_{j,l}^{\min}} \right\}. \quad (3.10)$$

After determining the user, the subchannel \hat{s} that provides the maximum instantaneous rate for that user if assigned to that user is found (assuming uniform power allocation over subchannels) as

$$\hat{s} = \arg \max_{s, \forall s} r_{j,\hat{l}}^s \quad (3.11)$$

where $r_{j,\hat{l}}^s$ is calculated according to (2.3) if Case 1 is applicable and according to (2.5) if Case 2 is applicable, respectively.

In each iteration the rate gained by each user with uniform power is calculated taking MUD decoding states into account and the subchannel is assigned to the user with minimum proportional rate. Then, the assigned subchannel is removed from the available subchannels list. Subchannel assignment continues until all the subchannels are assigned. Subchannel assignment converges to an assignment because in this

step we assume fixed power allocation and the algorithm terminates when there is no remaining unassigned subchannel.

The algorithm for BS j can be summarized in Algorithm 1 (BS k represents the dominant interference source).

Assumptions:

$r_{k,m}^s$ and p_k^s , $\forall s = 1, \dots, S$ ($u_{k,m}^s$ corresponds to the user to which subchannel s is assigned in BS k) and $h_{k,j,l}^s$, $\forall l = 1, \dots, K$ in BS j , are known by BS j

Initialization:

$R_{j,l} \leftarrow 0$, $\forall l = 1, \dots, K$.

while *unassigned subchannel(s) available* **do**

1. Determine the user \hat{l} in BS j with the minimum proportional rate ratio $\frac{R_{j,l}}{R_{j,l}^{\min}}$,
2. Find the subchannel \hat{s} that maximizes $r_{j,\hat{l}}^s$ over all unassigned subchannels,
3. Assign the subchannel \hat{s} to \hat{l} ,
4. Remove \hat{s} from the available subchannel list.
5. Update $R_{j,\hat{l}} \leftarrow R_{j,\hat{l}} + r_{j,\hat{l}}^{\hat{s}}$, accordingly.

end

Algorithm 1: MUD-SCA Algorithm

3.1.1.2 Heuristic SCA algorithm

The idea behind Heuristic SCA algorithm is to assign the subchannels with maximum direct link to cross (interference) link ratio to the user \hat{l} that have minimum proportional instantaneous rate in each iteration as follows:

$$\hat{l} = \arg \min_{l, \forall l} \left\{ \frac{R_{j,l}^i}{R_{j,l}^{\min}} \right\}. \quad (3.12)$$

The assigned subchannel is removed from the available subchannels list and subchannel assignment continues until no subchannel is left unassigned. When there are multiple interferers, the cross link is taken as the cross link of the dominant interference source.

The algorithm for BS j can be summarized in 2 (BS k represents the dominant interference source).

Assumptions:

$r_{k,m}^s$ and p_k^s , $\forall s = 1, \dots, S$ ($u_{k,m}^s$ corresponds to the user to which subchannel s is assigned in BS k) and $h_{k,j,l}^s$, $\forall l = 1, \dots, K$ in BS j , are known by BS j

Initialization:

$R_{j,l} \leftarrow 0$, $\forall l = 1, \dots, K$.

for each user l in BS j do

1. Find the subchannel \hat{s} that maximizes $\frac{H_{j,j,l}^s}{H_{k,j,l}^s}$ ratio over all subchannels,
2. Assign the subchannel \hat{s} to l ,
3. Remove \hat{s} from the available subchannel list,
4. Update $R_{j,\hat{l}} \leftarrow R_{j,\hat{l}} + r_{j,\hat{l}}^{\hat{s}}$, accordingly.

end**while unassigned subchannel(s) available do**

1. Determine the user \hat{l} in BS j with the minimum proportional rate ratio $\frac{R_{j,l}}{R_{j,l}^{\min}}$,
2. Find the subchannel \hat{s} that maximizes $\frac{H_{j,j,\hat{l}}^s}{H_{k,j,\hat{l}}^s}$ ratio over all unassigned subchannels,
3. Assign the subchannel \hat{s} to \hat{l} ,
4. Remove \hat{s} from the available subchannel list.
5. Update $R_{j,\hat{l}} \leftarrow R_{j,\hat{l}} + r_{j,\hat{l}}^{\hat{s}}$, $\forall l = 1, \dots, K$ accordingly.

end**Algorithm 2:** Heuristic SCA Algorithm

3.1.2 Power Allocation

For single user case, it was previously proven that the optimal power allocation strategy is found by water-filling [45] when the subchannel assignment is fixed. Having assigned the subchannels in the previous step (assuming fixed subchannel assignment), under fixed subchannel assignment, we can pose the 'Primary Problem' as follows (for BS j)

$$\begin{aligned} & \max_{\mathbf{p}_j} \alpha \\ & \text{subject to } R_{j,l} \geq \alpha R_{j,l}^{\min}, \forall l = 1, \dots, K \end{aligned}$$

$$\sum_{s=1}^S p_j^s \leq P_j^{\max}$$

$$p_j^s \geq 0, \quad \forall s = 1, \dots, S \quad (3.13)$$

where $\mathbf{p}_j = [p_j^1, \dots, p_j^S]^T$. The primal problem (3.13) is nonconvex due to the objective function hence duality gap (the difference between optimal values of the primal problem (3.13) and the dual problem (3.26)) is not zero. On the other hand, the duality gap can be considered to be negligible for a high number of subcarriers [75], [77], [78] which is also valid for the scenario considered in this thesis since the number of subchannels are assumed to be high. The solution of the primal problem (3.13) requires exhaustive search and it is not practical to solve directly. Therefore, we propose an efficient solution based on the Lagrangian dual method [75].

The Lagrangian of the 'Primal Problem' in (3.13) is

$$\mathcal{L}(\alpha, \mathbf{p}_j, \lambda, \mathbf{q}) = \alpha + \sum_{l=1}^K q_l [R_{j,l} - \alpha R_{j,l}^{\min}] + \lambda \left[P_j^{\max} - \sum_{s=1}^S p_j^s \right] \quad (3.14)$$

where $\mathbf{q} = [q_1, \dots, q_K]^T$ and λ and $\{q_l\}_{l=1}^K$'s are the Lagrangian multipliers for the constraints $\sum_{s=1}^S p_j^s \leq P_j^{\max}$ and $R_{j,l} \geq \alpha R_{j,l}^{\min}, \forall l = 1, \dots, K$, respectively. They are also called dual variables and satisfy the conditions $\lambda \geq 0$ and $\{q_l\}_{l=1}^K \geq 0$. The

Lagrange dual function of (3.13) is defined as

$$\begin{aligned}
g(\lambda, \mathbf{q}) &= \max_{\alpha, \mathbf{p}_j} \mathcal{L}(\alpha, \mathbf{p}_j, \lambda, \mathbf{q}) \\
&= \max_{\alpha, \mathbf{p}_j} \left(\alpha \left[1 - \sum_{l=1}^K q_l R_{j,l}^{\min} \right] \right. \\
&\quad \left. + \sum_{l=1}^K q_l R_{j,l} - \lambda \sum_{s=1}^S p_j^s + \lambda P_j^{\max} \right),
\end{aligned} \tag{3.15}$$

and the Lagrangian dual problem is

$$\begin{aligned}
&\min_{\lambda, \mathbf{q}} g(\lambda, \mathbf{q}) \\
&\text{subject to } \lambda \geq 0, \mathbf{q} \succeq 0.
\end{aligned} \tag{3.16}$$

Applying KKT optimality conditions [69] for the above problem, we have the following expressions to satisfy

$$R_{j,l} \geq \alpha R_{j,l}^{\min}, \forall l = 1, \dots, K, \tag{3.17}$$

$$\sum_{s=1}^S p_j^s \leq P_j^{\max}, \tag{3.18}$$

$$\mathbf{p}_j \succeq 0, \tag{3.19}$$

$$\lambda \geq 0, \tag{3.20}$$

$$\mathbf{q} \succeq 0, \tag{3.21}$$

$$q_l [\alpha R_{j,l}^{\min} - R_{j,l}] = 0, \forall l = 1, \dots, K, \tag{3.22}$$

$$\lambda \left[\sum_{s=1}^S p_j^s - P_j^{\max} \right] = 0, \forall l = 1, \dots, K, \tag{3.23}$$

$$\frac{\partial}{\partial \alpha} \mathcal{L}(\alpha, \mathbf{p}_j, \lambda, \mathbf{q}) = 0 \tag{3.24}$$

$$\frac{\partial}{\partial p_j^s} \mathcal{L}(\alpha, \mathbf{p}_j, \lambda, \mathbf{q}) = 0, \quad \forall s = 1 \dots S. \quad (3.25)$$

From (3.24), $1 - \sum_{l=1}^K q_l R_{j,l}^{\min} = 0$ (Lagrangian is an affine function of α) and α must be positive, it is observed that the dual function $g(\lambda, \mathbf{q})$ is unbounded unless $1 - \mathbf{r}_j^{\min} \mathbf{q} = 0$ where $\mathbf{r}_j^{\min} = [R_{j,1}^{\min}, \dots, R_{j,L}^{\min}]$ [69]. Therefore, the following Lagrangian dual problem is stated as

$$\begin{aligned} & \min_{\lambda, \mathbf{q}} v(\lambda, \mathbf{q}) \\ & \text{subject to } \mathbf{r}_j^{\min} \mathbf{q} = 1, \\ & \mathbf{p}_j \succeq 0, \\ & \lambda \geq 0, \quad \mathbf{q} \succeq 0. \end{aligned} \quad (3.26)$$

where $v(\lambda, \mathbf{q}) \triangleq \max_{p_j^s \geq 0, \forall s} \left(\sum_{l=1}^K q_l R_l - \lambda \sum_{s=1}^S p_j^s + \lambda P_j^{\max} \right)$ denotes the updated Lagrangian dual function. From (3.25),

$$\sum_{s \in \Omega_l} q_l \frac{\partial R_{j,l}}{\partial p_j^s} = \lambda, \quad \forall l = 1, \dots, K \quad (3.27)$$

and therefore, when λ and q_l 's are known, the solution of the Lagrangian dual problem becomes

$$\begin{aligned} & \sum_{s \in \Omega_l^{SIC}} \left(\frac{q_l}{\ln(2)\lambda} - \frac{\sigma^2}{H_{j,j,l}^s} \right)^+ \\ & + \sum_{s \in \Omega_l^{JD}} \left(\frac{q_l}{\ln(2)\lambda} - \frac{\sigma^2 + H_{k,j,l}^s p_k^s}{H_{j,j,l}^s} \right)^+ \\ & + \sum_{s \in \Omega_l^{SD}} \left(\frac{q_l}{\ln(2)\lambda} - \frac{\sigma^2 + H_{k,j,l}^s p_k^s}{H_{j,j,l}^s} \right)^+ = P_l^{tot}, \quad \forall l = 1, \dots, K \text{ in BS } j \end{aligned} \quad (3.28)$$

where Ω_l^{SIC} , Ω_l^{JD} and Ω_l^{SD} are the set of assigned subchannels for SIC, JD and SD decoding ranges for user l , respectively, $\Omega_l = \Omega_l^{SIC} \cup \Omega_l^{JD} \cup \Omega_l^{SD}$ denotes the set of subchannels assigned to user l and $P_l^{tot} = \sum_{s \in \Omega_l} p_j^s$ is the individual total power for each user satisfying $\sum_{l=1}^K P_l^{tot} \leq P_j^{\max}$ as follows from (3.23) since λ is one of the variables determining the water level and $\lambda > 0$. From (3.22), since q_l is another variable determining the water levels and the objective of the primal problem requires that every user must be allocated power, it is apparent that $\min_l q_l > 0$, hence at the

optimum point $[\alpha R_{j,l}^{\min} - R_{j,l}] = 0$ holds. Alternatively, the above problem can also be solved with the decomposition method.

After solving the Lagrangian dual function, the optimum (λ, \mathbf{q}) can be found using the projected subgradient method [79] or ellipsoid method [77], [80] and the following subgradients can be used

$$\mathbf{d}(\lambda) = P_j^{\max} - \sum_{s=1}^S p_j^s, \quad (3.29)$$

$$\mathbf{d}(\mathbf{q}) = \mathbf{r}_j. \quad (3.30)$$

where $\mathbf{r}_j = [R_{j,1}, \dots, R_{j,K}]^T$.

3.1.2.1 Projected Subgradient Method (PSM)

The problem in (3.26) is a minimization problem with an equality constraint. Using the projected subgradient algorithm (explained in Appendix C) [79], we can find the projected subgradient updates for the $(t + 1)^{\text{st}}$ iteration as follows

$$\lambda^{(t+1)} = \left(\lambda^{(t)} - \beta_t \left(P_j^{\max} - \sum_{s=1}^S p_j^s \right) \right)^+ \quad (3.31)$$

$$\mathbf{q}^{(t+1)} = \left(\mathbf{q}^{(t)} - \beta_t \left(\mathbf{I} - \mathbf{r}_j^{\min T} \left(\mathbf{r}_j^{\min} \mathbf{r}_j^{\min T} \right)^{-1} \mathbf{r}_j^{\min} \right) \mathbf{r}_j \right)^+. \quad (3.32)$$

where β_t is the step-size of the updates. The power allocation algorithm for BS j using projected subgradient method can be summarized as follows:

3.1.2.2 Ellipsoid Method (EM)

The details of the ellipsoid method are elaborated in [80] and the ellipsoid algorithm (explained in Appendix D) for equality constrained problems is developed in [81]. The dual variables are then used to obtain the optimum power values on each sub-channel. Power allocation algorithm utilizes EM assuming that the subchannel assignment is fixed. The power allocation algorithm for BS j using EM is summarized in Algorithm 4.

Initialization:

$r_{j,l}^s \leftarrow 0, \forall s = 1, \dots, S \forall l = 1, \dots, K$. Initialize $(\lambda, \mathbf{q}) = (\lambda^{(0)}, \mathbf{q}^{(0)})$

repeat**for each user l in BS j do**

1. Find the power for each subchannel $p_j^s, s \in \Omega_l$ and corresponding MUD state with water level $\frac{q_l}{\ln(2)\lambda}$ from Eq. (3.28).
2. Calculate the rate of each subchannel $r_{j,l}^s$ and update total rate $R_{j,l}$ of each user.

end

Update (λ, \mathbf{q}) by projected subgradient method with update equations (3.31) and (3.32).

until Convergence for optimum $(\lambda^*, \mathbf{q}^*)$ of dual variables is achieved;

Algorithm 3: Optimal Power Allocation (Optimal PA) Algorithm with Projected Subgradient Method (PSM)

Initialization:

$r_{j,l}^s \leftarrow 0, \forall s = 1, \dots, S \forall l = 1, \dots, K$. Initialize ellipsoid $\mathcal{E} = \mathcal{E}^0$ and $(\lambda, \mathbf{q}) = (\lambda^{(0)}, \mathbf{q}^{(0)})$

repeat**for each user l in BS j do**

1. Find the power for each subchannel $p_j^s, s \in \Omega_l$ and corresponding MUD state with water level $\frac{q_l}{\ln(2)\lambda}$ from Eq. (3.28).
2. Calculate the rate of each subchannel $r_{j,l}^s$ and update total rate $R_{j,l}$ of each user.

end

Update (λ, \mathbf{q}) and ellipsoid \mathcal{E} by the ellipsoid method using subgradients in (3.29) and (3.30).

until Convergence for optimum $(\lambda^*, \mathbf{q}^*)$ of dual variables is achieved;

Algorithm 4: Optimal Power Allocation (Optimal PA) Algorithm with Ellipsoid Method (EM)

3.1.2.3 A Heuristic PA Algorithm

In this algorithm, we perform power allocation to each subchannel by the bisection method [69], [82].

Assuming fixed subchannel assignment, the power allocation problem for each BS j can be written as

$$\max_{\mathbf{p}_j} \min_l \frac{R_{j,l}}{R_{j,l}^{\min}} \quad (3.33)$$

$$\sum_{s=1}^S p_j^s \leq P_j^{\max} \quad (3.34)$$

$$p_j^s \geq 0, \quad \forall s = 1, \dots, S. \quad (3.35)$$

We propose an iterative power exchange procedure between user powers for the above problem similar to [83]. The BS performs power exchange. The aim is to provide fairness for users' rates with respect to their minimum rate requirements. Initially the power is distributed uniformly for all users. As the total power for each user $P_{j,l}^{tot}$ is determined, the powers and rates on each subchannel for each user are calculated by the bisection method according to (2.3). The resulting proportional rates are compared, the power of the user that has the most proportional rate is reduced with ΔP and the power of the user that has the least proportional rate is increased with ΔP and the rates and powers on each subchannel are calculated with the updated total powers. This procedure continues until

$$\Delta R = \max_l \left(\frac{R_{j,l}}{R_{j,l}^{\min}} \right) - \min_l \left(\frac{R_{j,l}}{R_{j,l}^{\min}} \right) \quad (3.36)$$

becomes smaller than a pre-defined threshold $\delta > 0$.

As the total power for each user is determined, the rates on each subchannel for each user are calculated as follows. The power allocation procedure for each user l in BS j with total power $P_{j,l}^{tot}$ is stated as

$$\max_{\mathbf{p}_j^y} R_{j,l}(p_j^y) \triangleq \sum_{y=1}^Y r_j^y(p_j^y) \quad (3.37)$$

$$\text{s.t. } \sum_{y=1}^Y p_j^y \leq P_{j,l}^{tot}. \quad (3.38)$$

where $\mathbf{p}_j^Y = [p_j^1, \dots, p_j^Y]^T$ and the subchannels assigned to user l is indexed $1, \dots, Y$.

We will derive the optimal power values by algebra for BS 1 ($j = 1$) considering the dominant interference source is BS 2. A very similar problem with unit variance and single user is proven to be convex in [19] and solved via the Lagrangian dual decomposition method. The Lagrangian dual function $\mathcal{L}(\mathbf{p}_j^Y, \gamma)$ of the above problem is

$$\mathcal{L}(\mathbf{p}_j^Y, \gamma) = \sum_{y=1}^Y r_1^y(p_1^y) - \gamma \left(\sum_{y=1}^Y p_1^y - P_{j,l}^{tot} \right) \quad (3.39)$$

where γ is the non-negative dual variable of the Lagrangian dual function

$$g(\gamma) = \max_{\mathbf{p}_j^Y \geq 0} \mathcal{L}(\mathbf{p}_j^Y, \gamma). \quad (3.40)$$

The dual problem is

$$\min_{\gamma > 0} g(\gamma) \quad (3.41)$$

and this problem can be decomposed into Y independent subproblems that

$$g(\gamma) = \sum_{y=1}^Y g_y(\gamma) + \gamma P_{1,l}^{tot} \quad (3.42)$$

where

$$g_y(\gamma) = \max_{p_1^y \geq 0} r_1^y(p_1^y) - \gamma p_1^y, \forall y = 1, \dots, Y. \quad (3.43)$$

Hence, the dual problem becomes

$$\min_{\gamma > 0} \max_{p_1^y \geq 0} r_1^y(p_1^y) - \gamma p_1^y + \gamma P_{1,l}^{tot}, \quad (3.44)$$

substituting the following rate equation

$$r_1^y(p_1^y) = \begin{cases} \log_2 \left(1 + \frac{H_{1,1,l}^y p_1^y}{\sigma^2 + H_{2,1,l}^y p_2^y} \right), & \text{if SD} \\ \log_2 \left(1 + \frac{H_{1,1,l}^y p_1^y + H_{2,1,l}^y p_2^y}{\sigma^2} \right) - r_2^y, & \text{if JD} \\ \log_2 \left(1 + \frac{H_{1,1,l}^y p_1^y}{\sigma^2} \right), & \text{if SIC} \end{cases} \quad (3.45)$$

in the dual problem, and taking the derivative of the dual problem with respect to p_1^y and equating it to 0, $\frac{\partial g(\gamma)}{\partial p_1^y} = 0$, we obtain two different optimal power levels for

different decoding modes for σ^2 variance noise, as similarly found in [19] for unit variance noise,

$$p_{1f}^y = \max \left(0, \frac{1}{\ln(2)\gamma^*} - \frac{\sigma^2}{H_{1,1,l}^y} \right), \quad (3.46)$$

$$p_{1h}^y = \max \left(0, \frac{1}{\ln(2)\gamma^*} - \frac{\sigma^2 + H_{2,1,l}^y p_2^y}{H_{1,1,l}^y} \right), \quad (3.47)$$

corresponding to the power level when SIC and JD & SD modes are used, respectively. The optimal power can be determined [19] as

$$p_1^{y*} = \begin{cases} p_{1f}^y, & p_{1th}^y > p_{1f}^y \\ p_{1th}^y, & p_{1h}^y < p_{1th}^y < p_{1f}^y \\ p_{1h}^y, & 0 < p_{1th}^y < p_{1h}^y \\ p_{1h}^y, & p_{1th}^y < 0 \end{cases} \quad (3.48)$$

where $p_{1th}^y = \frac{1}{H_{1,1,l}^y} \left(\frac{H_{2,1,l}^y p_2^y}{2^{\gamma^y} - 1} - 1 \right)$ is the threshold power value between JD and SIC decoding modes. The power allocation algorithm for BS 1 can be summarized in Algorithm 5.

3.2 Convergence of the Algorithm

The convergence of the algorithm have two elements. The first one is regarding the convergence of subchannel assignment and power allocation steps in each BS. Subchannel assignment algorithm assumes fixed power allocation with uniform power in each subchannel and this algorithm converges to an assignment because in each iteration one subchannel is guaranteed to be assigned to a user and the algorithm terminates when there is no remaining unassigned subchannel. Power allocation algorithm utilizes PSM or EM assuming the subchannel assignment is fixed. PSM is convergent if an appropriate step size is chosen and EM is convergent if the interval is chosen suitably so as to include the optimal value (the solution) and the objective function is continuous. However, the function under consideration is not continuous and we cannot guarantee that the power allocation algorithm is convergent. On the other hand, in the simulations we see that if the interval is chosen appropriately, the power allocation algorithm in the proposed method, assuming the subchannel assignment is fixed,

Initialization:

$$p_1^y \leftarrow 0, r_{1,l}^y \leftarrow 0, \forall y = 1, \dots, Y, P_{j,l}^{tot} = \frac{P_j^{max}}{K}, \Delta R > \delta > 0, \epsilon > 0.$$

repeat

Given $\gamma_{min} \leq \gamma^* \leq \gamma_{max}$,

for each user l in BS 1 do**while** $\gamma_{max} - \gamma_{min} \geq \epsilon$ **do**

1. $\gamma \leftarrow \frac{\gamma_{max} + \gamma_{min}}{2}$

2. Find the power levels p_1^y with water level $\frac{1}{\ln(2)\gamma}$ for $y = 1, \dots, Y$.

3. Compute p_{1th}^y, p_{1f}^y and p_{1h}^y .

if p_{1th}^y is ∞ , (since $p_2^y = 0, r_2^y = 0$) **then**

| Calculate r_1^y with power $p_1^y = p_{1f}^y$ with SD state with Eq. (2.3),

else if $p_{1th}^y > p_{1f}^y$ **then**

| Calculate r_1^y with power $p_1^y = p_{1f}^y$ with SIC state with Eq. (2.3),

else if $p_{1f}^y > p_{1th}^y > p_{1h}^y$ **then**

| Calculate r_1^y with power $p_1^y = p_{1th}^y$ with SIC state with Eq. (2.3),

else if $p_{1h}^y > p_{1th}^y > 0$ **then**

| Calculate r_1^y with power $p_1^y = p_{1h}^y$ with JD state with Eq. (2.3)

(If case 2 is applicable, calculate with SD state with Eq. (2.5)),

else if $p_{1th}^y < 0$ **then**

| Calculate r_1^y with power $p_1^y = p_{1h}^y$ with SD state with Eq. (2.3),

end**if** $\sum_{y=1}^Y p_1^y < P_{1,l}^{tot}$ **then**

| $\gamma_{max} \leftarrow \gamma$

else

| $\gamma_{min} \leftarrow \gamma$

end**end****end**

1. Compute $\Delta R = \max \left(\frac{R_{j,l}}{R_{j,l}^{min}} \right) - \min \left(\frac{R_{j,l}}{R_{j,l}^{min}} \right)$.
2. $P_{1,w}^{tot} \leftarrow P_{1,w}^{tot} - \Delta P$ and $P_{1,v}^{tot} \leftarrow P_{1,v}^{tot} + \Delta P$
s.t. $w = \arg \max_l \left(\frac{R_{j,l}}{R_{j,l}^{min}} \right)$ and $v = \arg \min_l \left(\frac{R_{j,l}}{R_{j,l}^{min}} \right)$.

until $\Delta R < \delta$;

Algorithm 5: Heuristic PA Algorithm

is convergent. The Heuristic PA algorithm utilizing the bisection method (explained in Appendix B) is always convergent when a relevant ΔP is chosen.

The second one is related to the convergence properties of the overall algorithm. Necessary conditions for convergence of IWF which is a simpler algorithm than the proposed algorithms and does not even consider MUD receivers, have not yet been known completely [19]. As a result, it is very difficult to characterize the convergence properties of the proposed iterative algorithm, however, the algorithm always converged under various scenarios in simulations.

3.3 Complexity Analysis

To find the optimal resource allocation scheme for a BS, for each of the K^S subchannel assignments, corresponding power allocations and rates must be evaluated with the proposed optimal power allocation methods. This exponential complexity is computationally prohibitive, and hence we propose lower complexity algorithms in this study. The complexity of the overall algorithms is the sum of the complexity of subchannel assignment and power allocation algorithms. The complexity of each algorithm is presented in Table 1.

The MUD-SCA subchannel assignment and Heuristic SCA algorithms first find the user that has the least proportional rate and then assign the relevant subchannel to that user hence the complexities of these algorithms are both $\mathcal{O}(KS)$. For the power allocation step, we deal with $K + 1$ dual variables. Therefore, PSM and EM have the complexity of $\mathcal{O}((K + 1)^2) \approx \mathcal{O}(K^2)$, both. The Heuristic PA algorithm requires $\mathcal{O}(K)$ computations and bisection search in the Heuristic PA algorithm has a complexity of $\mathcal{O}(\log(1/\epsilon))$ where ϵ is the accuracy of bisection search, yielding a total complexity of $\mathcal{O}(K \log(1/\epsilon))$. Heuristic SCA and Heuristic PA algorithms facilitate practical implementation while yielding a good performance close to that of MUD-SCA and optimal PA algorithms.

As a result, compared to the brute force approach, proposed algorithms have much lower complexity especially for large K .

Table 3.1: Complexity of Algorithms

SCA Method	Complexity
MUD-SCA	$\mathcal{O}(KS)$
Heuristic SCA	$\mathcal{O}(KS)$
PA Method	Complexity
PA- PSM	$\mathcal{O}(K^2)$
PA- EM	$\mathcal{O}(K^2)$
Heuristic PA	$\mathcal{O}(K \log(1/\epsilon))$

3.4 A Reference Method

To our best knowledge, in the literature there is no study considering resource allocation (subcarrier assignment and power allocation) trying to satisfy minimum rates of users in a scenario of users with MUD capability. To compare the proposed algorithms in the previous section with IWF, we have developed a multicell iterative method using IWF based on the subchannel assignment algorithm in [17]. The scenario in [17] considers single cell scenario with proportional user rate constraints. Interference is not considered in [17] and we extend the study in [17] to multicell scenario with interference. We call this method ‘Multicell-Shen-IWF’. In this algorithm, each BS performs classical waterfilling over the subchannels considering a sum total power over all users in each BS. The receivers are considered to only have SD capability.

The Multicell-Shen-IWF method for BS j can be summarized in Algorithm 6 (BS k represents the dominant interference source).

Assumptions:

$p_k^s, \forall s = 1, \dots, S$ and $h_{k,j,l}^s, \forall l = 1, \dots, K$ in BS j , are known by BS j

Initialization:

$R_{j,l} \leftarrow 0, \forall l = 1, \dots, K.$

for each user l in BS j do

1. Find the subchannel \hat{s} satisfying $H_{j,j,l}^{\hat{s}} \geq H_{j,j,l}^s, \forall s = 1, \dots, S,$
2. Assign the subchannel \hat{s} to $l,$
3. Remove \hat{s} from the available subchannel list,
4. Update $R_{j,\hat{l}} \leftarrow R_{j,l} + r_{j,l}^{\hat{s}}, \forall l = 1, \dots, K$ accordingly.

end**while unassigned subchannel(s) available do**

1. Determine the user \hat{l} in BS j with the minimum proportional rate ratio $\frac{R_{j,l}}{P_{j,l}^{\min}}$ over all users in BS $j,$
2. Find the subchannel \hat{s} satisfying $H_{j,j,\hat{l}}^{\hat{s}} \geq H_{j,j,\hat{l}}^s, \forall s = 1, \dots, S,$
3. Assign the subchannel \hat{s} to $\hat{l},$
4. Remove \hat{s} from the available subchannel list.
5. Update $R_{j,\hat{l}} \leftarrow R_{j,\hat{l}} + r_{j,\hat{l}}^{\hat{s}}$ accordingly.

end

Find the power levels in the assigned subchannels with the water-filling algorithm.

Algorithm 6: Reference Method

CHAPTER 4

NUMERICAL RESULTS

In this chapter, we present simulation results that compare the proposed methods with some benchmark methods in terms of performance and efficiency for various scenarios.

4.1 Simulation Parameters

We simulate and analyze Case 1 and Case 2, both as defined in Chapter 1. We consider the downlink of a LTE OFDMA network with the following parameters in Table 4.1.

Table 4.1: System Parameters

Parameters	Value
Frequency Reuse	1
Number of BSs (N)	3
Cell Radius (r)	2 km
Intercell Distance	$2\sqrt{3}$ km
Max. Transmit Power (P_t^{max})	-10 to 20 dBW
Antenna Gain	15 dBi
Bandwidth	10 MHz
Subcarrier Bandwidth	15 kHz
Resource Block Bandwidth	180 kHz = 12×15 kHz
Number of Resource Blocks	50
Noise Power Density	-174 dBm/Hz
Path Loss (d :distance (km))	$128.1 + 37.6 \log_{10}(d)$
Log Normal Shadowing	10 dB std. dev.

We assume that the coherence bandwidth of the channel covers only one LTE RB of 180 kHz, hence all of the subcarriers in different RB's undergo independent fading and flat fading is assumed for each RB. The path loss is $128.1+37.6 \log_{10}(\text{distance in km})$ for a macrocell at a carrier frequency of 2 GHz assuming that the BS antenna height is located at 15 m above the rooftop [84].

We assume that all the subchannels of 15 kHz bandwidth in the same resource block (RB) of 180 kHz bandwidth (1 RB consists of 12 subchannels) are assigned to the same user and all the subchannels in the same RB have the same channel gains and they behave similarly in terms of multiuser decoding state and used power. Hence, the allocated power to each of the subchannel in the same RB is assumed to be same.

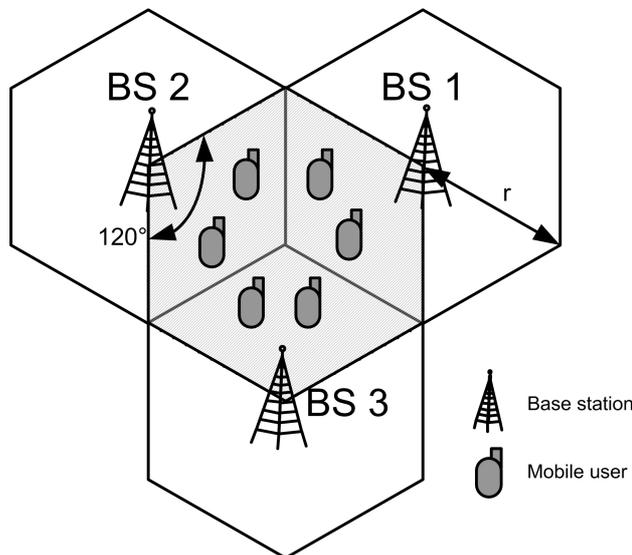


Figure 4.1: Scenario with cell radius of r km.

We consider the scenario in Fig. 4.1. There are $N = 3$ BSs with radius $r = 2$ km and intercell distance of $r\sqrt{3}$ and K users are served in each BS. The scenario is identical to the heterogenous scenario 2a in [10]. We assume that the BSs have sectoral antennas and each BS has 3 sectors and the users are distributed randomly with uniform pdf in the neighbouring sectors (120°) of the cells. Half of the users are called inner users and half of the users are called cell edge users. The inner users and cell edge users are considered to be located in the inner cell area with a radius of $r_{\text{inner}} = \frac{r}{\sqrt{2}}$ km and in the outer cell area between a radius of $r_{\text{inner}} = \frac{r}{\sqrt{2}}$ km and a radius of $r = 2$ km, respectively. A radius of r_{inner} is chosen to be equal to $\frac{r}{\sqrt{2}}$ km so that the area for inner users and area for cell edge users are identical. We assume that in each subchannel the strong interference signal is decoded. We can also extend this scenario to femtocell

networks where the distances between the macrocell center and femtocells are lower than the cell (coverage) radius of the macrocell.

We calculate the rates of users both for Gaussian inputs and MQAM finite constellations. For MQAM constellations we use the rate equation which is defined in Chapter 2 assuming $M = 4$ and desired BER = 10^{-3} .

4.2 Update of the Powers and Dual Variables

For the optimal power allocation, we use PSM and EM methods. Figs 4.2 and 4.3 show the update of dual variables in the Optimal PA algorithm. PSM and EM methods yield the same optimal dual variables, i.e., λ, q_1, \dots, q_K , hence resulting $(\frac{\lambda}{q_i})$'s which determine the water levels for each user are the same for both methods. The complexity of both methods are the same but as observed in the simulations, the PSM method converges faster than the EM method. On the other hand, the Heuristic PA method is the fastest among the power allocation methods.

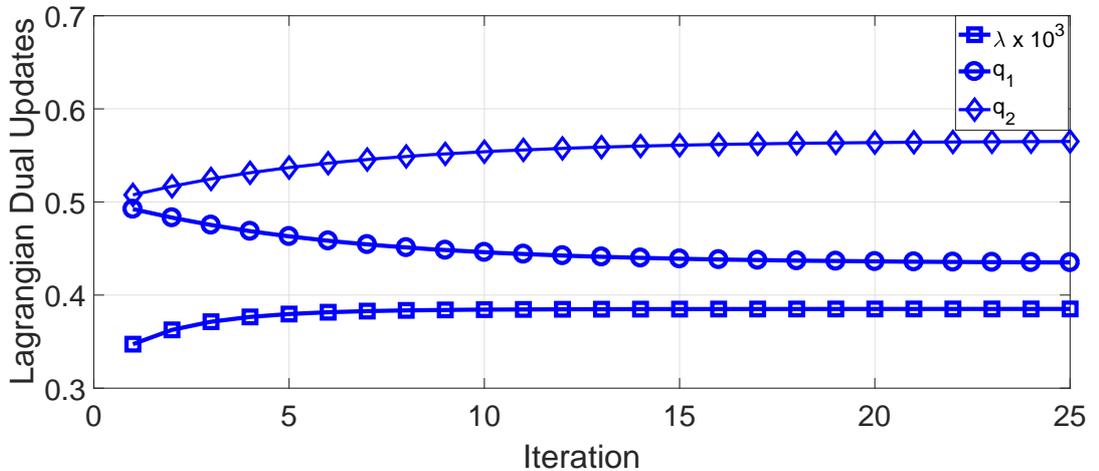


Figure 4.2: Dual variable updates for users ($K = 2$) in BS 1 for 1 channel realization when Optimal PA with PSM (case 1) performed. $R_{1,1}^{\min} = R_{1,2}^{\min}$.

Fig. 4.4 and Fig. 4.5 show the updates of power and rate of the users in the overall algorithm, respectively. As seen in Fig. 4.4, the power levels converge in 5 iterations and in most of the channel realizations the overall algorithm converges in 4-10 iterations. As seen in Fig. 4.4, the power levels of inner users are lower than those of cell

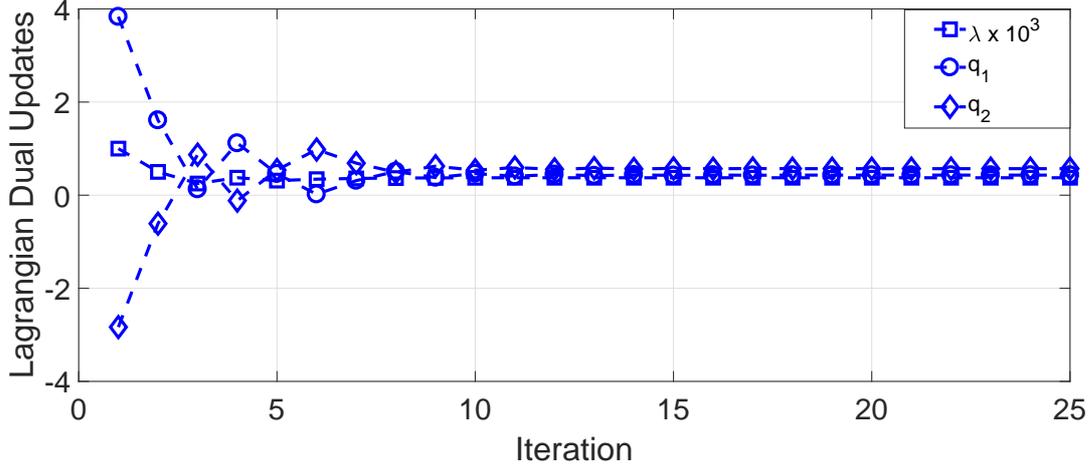


Figure 4.3: Dual variable updates for users ($K = 2$) in BS 1 for 1 channel realization when Optimal PA with EM (case 1) performed. $R_{1,1}^{\min} = R_{1,2}^{\min}$.

edge users to get the same rates, which is an expected outcome. As seen in Fig. 4.5, the rates of users also converge in a few iterations in parallel to the powers.

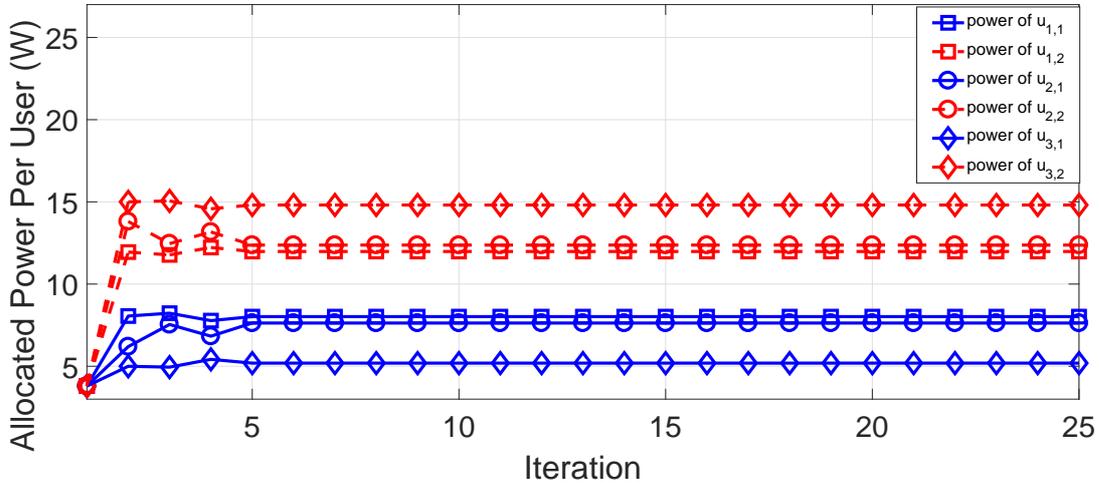


Figure 4.4: Power iterations for users in BS 1, BS 2 and BS 3 for 1 channel realization when MUD+SCA with Optimal PA (case 1) performed. Lines with markers \square , \circ and \diamond are the powers for inner users in BS 1, BS 2 and BS 3, respectively and dashed lines with markers \square , \circ and \diamond are the powers for cell edge users in BS 1, BS 2 and BS 3, respectively. $R_{1,1}^{\min} = R_{1,2}^{\min}$, $R_{2,1}^{\min} = R_{2,2}^{\min}$ and $R_{3,1}^{\min} = R_{3,2}^{\min}$.

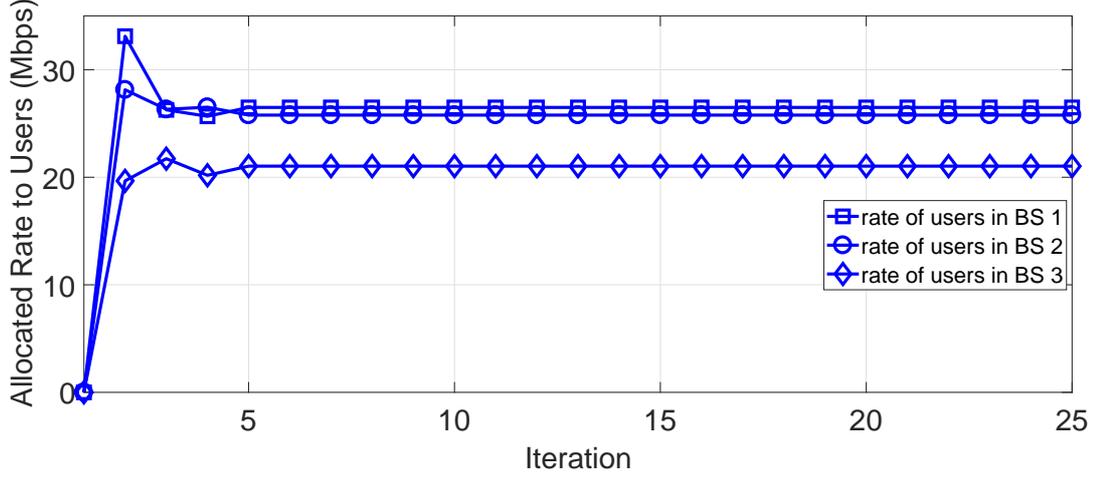


Figure 4.5: Rate iterations for minimum rated users in BS 1, BS 2 and BS 3 for 1 channel realization when MUD+SCA with Optimal PA (case 1) performed. Lines with markers \square , \circ and \diamond are the rates for minimum rated users in BS 1, BS 2 and BS 3, respectively. $R_{1,1}^{\min} = R_{1,2}^{\min}$, $R_{2,1}^{\min} = R_{2,2}^{\min}$ and $R_{3,1}^{\min} = R_{3,2}^{\min}$.

4.3 Comparison of the Proposed Algorithms with Benchmarks

In this section, we first investigate how the proposed algorithm performs in comparison to a benchmark method called Randomized Subchannel Assignment with Fixed Power (RSFP) Method, Multicell-Shen-IWF Method and fractional frequency reuse with reuse factor of 3 (FFR3) with respect to maximum power per BS. RSFP assigns subchannels randomly to users with uniform power so that the number of assigned subchannels is proportional to each user's minimum rate constraint and the users are assumed to have MUD capability. The method, which is originally proposed for single cell systems with proportional rate constraints in [17], is modified in this thesis in Section 3.4 using IWF to be used for multicell systems (receivers without MUD capability) for comparison and it is called Multicell-Shen-IWF Method. FFR3 is fractional frequency reuse with reuse factor of 3 as defined in [85] for macrocells.

P_t^{\max} (maximum transmitter power per BS) varies from -10 dBW to 20 dBW. We consider the scenario in Fig. 4.1. All the results presented here are collected over 500 channel realizations. We consider identical minimum rate requirements for users in each BS, i.e., $R_{1,1}^{\min} = R_{1,2}^{\min}$, $R_{2,1}^{\min} = R_{2,2}^{\min}$ and $R_{3,1}^{\min} = R_{3,2}^{\min}$. As observed in Fig. 4.6, the MUD-SCA with Optimal PA Algorithm (Case 1) and Heuristic SCA

with Heuristic PA (Case 1) Algorithm always perform better than RSFP Method, Shen Method and FFR3 in terms of maximizing the ratio of the rate of the user with minimum rate to that user's minimum rate requirement. Moreover, Heuristic SCA with Heuristic PA Algorithm reaches 98% of the performance of MUD-SCA with Optimal PA Algorithm (Case 1) with lower complexity.

In the low power regime, the performance increase is limited relative to the Shen Method whereas in high power regime, we can observe an increase of more than 25% and 23% compared to Shen Method for the MUD-SCA with Optimal PA Algorithm and Heuristic SCA with Heuristic PA (Case 1), respectively. The proposed algorithms perform 62% - 98% better than RSFP Method and 26% - 138% better than FFR3.

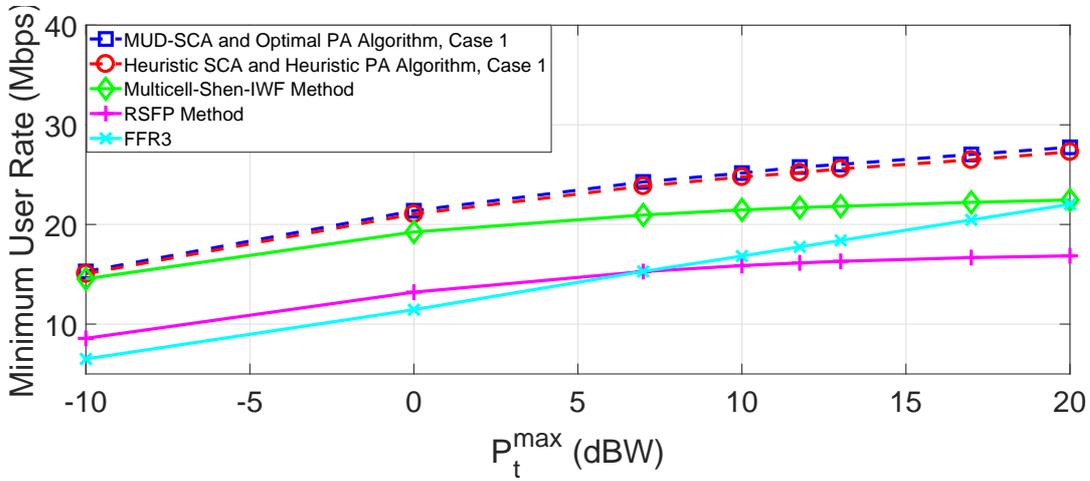


Figure 4.6: Minimum rated user in BS1 vs. P_t^{\max} averaged over 500 channel realizations for comparison of the proposed algorithms (Case 1) (dashed lines with markers \square and \circ) and benchmarks (lines with markers \diamond , $+$ and \times), $R_{1,1}^{\min} = R_{1,2}^{\min}$, $R_{2,1}^{\min} = R_{2,2}^{\min}$ and $R_{3,1}^{\min} = R_{3,2}^{\min}$.

In Fig. 4.7, the MUD-SCA with Optimal PA Algorithm (Case 1) and Shen Method is compared when the number of users per BS change under fixed power of $P_t^{\max} = 20$ W. As the Number of Users changes, the performance of MUD-SCA and Optimal PA algorithm in each BS is always better compared to Shen method.

Next, we inspect how far the performance of the proposed algorithms is to the performance of the case when no inter-cell interference is assumed.

As seen in Fig. 4.8, the performances of MUD-SCA with Optimal PA Algorithm

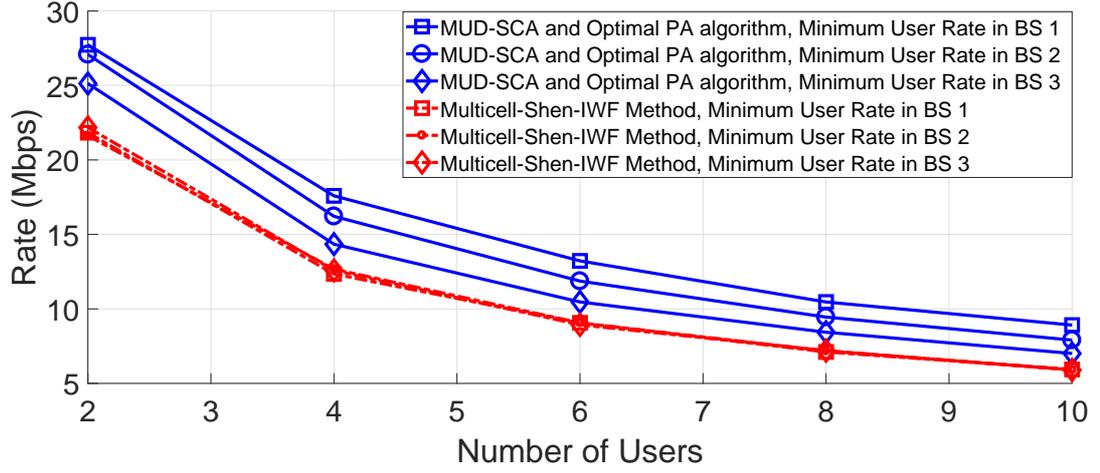


Figure 4.7: Rate of the minimum rated user in BS1, BS2, BS3 vs. Number of Users averaged over 500 channel realizations for comparison of MUD-SCA and Optimal PA algorithm (Case 1) (lines with markers \square , \circ and \diamond) and Multicell-Shen-IWF Method (dashed lines with markers \square , \circ and \diamond). $R_{1,1}^{\min} = R_{1,2}^{\min}$, $R_{2,1}^{\min} = R_{2,2}^{\min}$ and $R_{3,1}^{\min} = R_{3,2}^{\min}$, $P_t^{\max} = 20 W$.

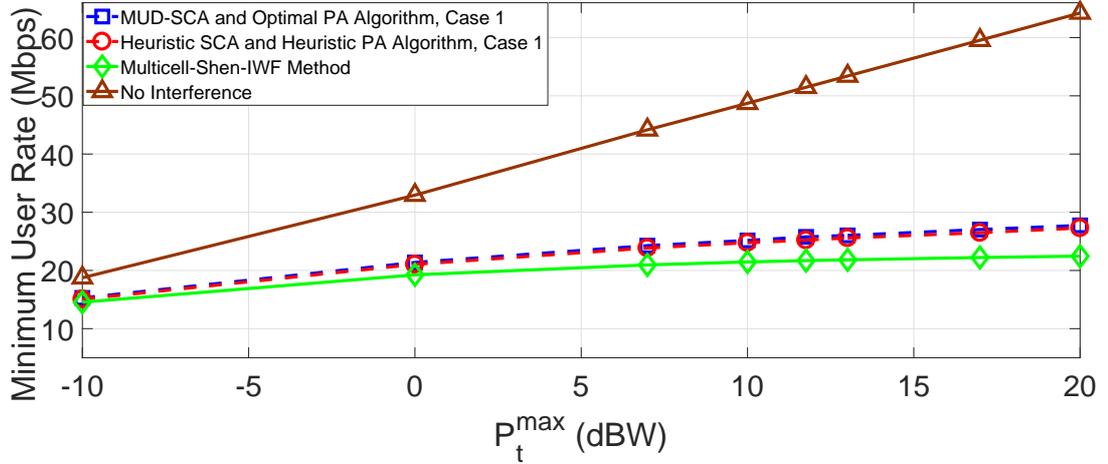


Figure 4.8: Minimum rated user in BS1 vs. P_t^{\max} averaged over 500 channel realizations for comparison of the proposed algorithms (Case 1) (dashed lines with markers \square and \circ), Multicell-Shen-IWF (dashed lines with marker \diamond) and no interference case (dashed lines with marker \triangle), $R_{1,1}^{\min} = R_{1,2}^{\min}$, $R_{2,1}^{\min} = R_{2,2}^{\min}$ and $R_{3,1}^{\min} = R_{3,2}^{\min}$.

(Case 1) and Heuristic SCA with Heuristic PA (Case 1) Algorithm and Multicell-Shen-IWF are worse than that of the (fictional) no interference case, as expected. The rates in no interference case are about 132% better than that of the rates resulted with

the proposed algorithms.

When MQAM constellation inputs are used, the rates of users decrease as expected compared to Gaussian inputs and the advantage of using the proposed algorithms hence using MUD decoding for this constellation is similar to the Gaussian inputs. The MUD-SCA and Optimal PA algorithm always performs better than Multicell-Shen-IWF method especially at high SNR. The ratio of increase in the user rates is about 15%.

An other investigation is on how the asymmetric R_{\min} values for users in the same cell affect the rates of the users. There are $K = 2$ users served per BS. In order to obtain Fig. 4.9 the rate requirements of the inner users in each cell are taken twice of that of the outer users, i.e. $R_{1,2}^{\min} = 2 \times R_{1,1}^{\min}$, $R_{2,2}^{\min} = 2 \times R_{2,1}^{\min}$ and $R_{3,2}^{\min} = 2 \times R_{3,1}^{\min}$. We show the variation of user rates for various simulations in this case in Fig. 4.10 under $P_t^{\max} = 20W$ when we use MUD-SCA with Optimal PA Algorithm (Case 1).

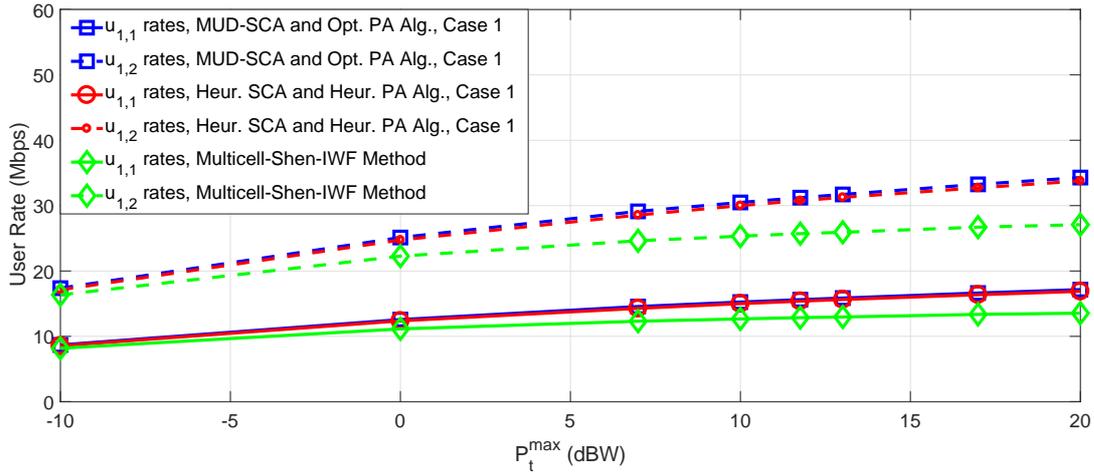


Figure 4.9: Mean of the rates of users in BS1, BS2 and BS3 vs. P_t^{\max} averaged over 500 channel realizations for comparison of the proposed algorithms (Case 1) (dashed lines with markers \square and \circ) and Multicell-Shen-IWF algorithm (line with markers \diamond), $R_{1,2}^{\min} = 2 \times R_{1,1}^{\min}$, $R_{2,2}^{\min} = 2 \times R_{2,1}^{\min}$ and $R_{3,2}^{\min} = 2 \times R_{3,1}^{\min}$.

Mean of the user rates in Fig. 4.10 under $P_t^{\max} = 20W$ for MUD-SCA with Optimal PA Algorithm (Case 1) are shown in Table 4.2.

As seen in Fig. 4.10, the rates of users $u_{1,2}$, $u_{2,2}$ and $u_{3,2}$ are about twice of that of the rates of users $u_{1,1}$, $u_{2,1}$ and $u_{3,1}$, respectively.

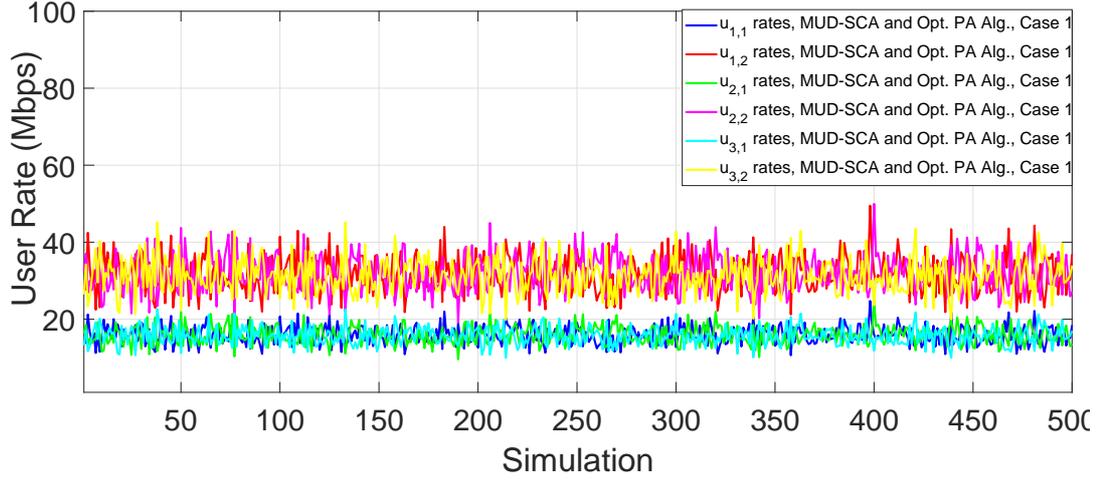


Figure 4.10: Variation of the rates of users in BS1, BS 2 and BS 3 vs. 500 channel realizations with asymmetric R^{\min} when MUD-SCA with Optimal PA Algorithm (Case 1) is used under $P_t^{\max} = 20W$, $R_{1,2}^{\min} = 2 \times R_{1,1}^{\min}$, $R_{2,2}^{\min} = 2 \times R_{2,1}^{\min}$ and $R_{3,2}^{\min} = 2 \times R_{3,1}^{\min}$.

Table 4.2: Mean of the rates of users

User	Instantaneous Rate (Mbps)
$u_{1,1}$	15.84
$u_{1,2}$	31.68
$u_{2,1}$	15.90
$u_{2,2}$	31.81
$u_{3,1}$	15.64
$u_{3,2}$	31.29

In order to obtain Fig. 4.11 the rate requirements of the inner users in each cell are taken twice of that of the outer users, i.e. $R_{1,1}^{\min} = 2 \times R_{1,2}^{\min}$, $R_{2,1}^{\min} = 2 \times R_{2,2}^{\min}$ and $R_{3,1}^{\min} = 2 \times R_{3,2}^{\min}$. We show the variation of user rates for various simulations in this case in Fig. 4.12 under $P_t^{\max} = 20W$ when we use MUD-SCA with Optimal PA Algorithm (Case 1).

Mean of the user rates in Fig. 4.12 under $P_t^{\max} = 20W$ for MUD-SCA with Optimal PA Algorithm (Case 1) are shown in Table 4.3.

As seen in Fig. 4.12, the rates of users $u_{1,1}$, $u_{2,1}$ and $u_{3,1}$ are about twice of that of the rates of users $u_{1,2}$, $u_{2,2}$ and $u_{3,2}$, respectively. This result again shows that by defining R^{\min} requirements for users, we make a fair allocation among the users regardless of

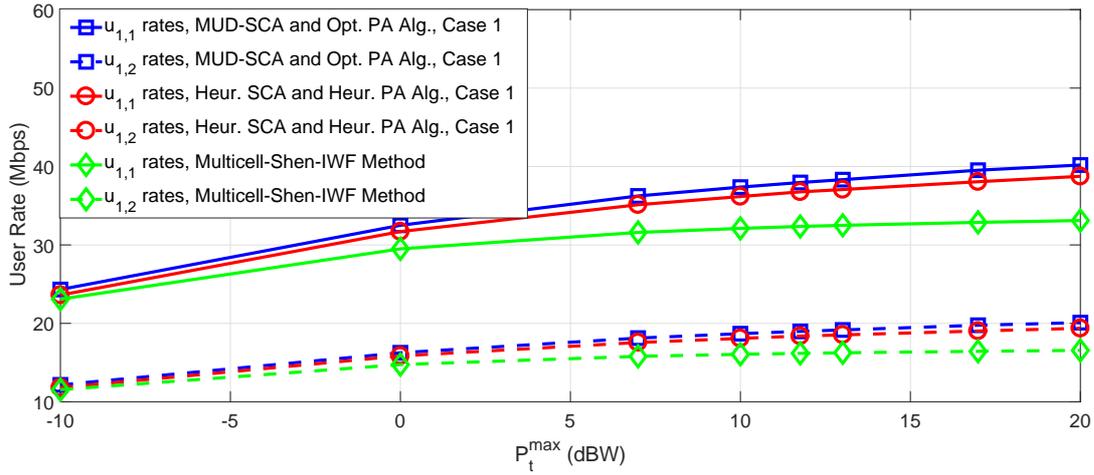


Figure 4.11: Mean of the rates of users in BS1, BS2 and BS3 vs. P_t^{max} averaged over 500 channel realizations for comparison of the proposed algorithms (Case 1) (dashed lines with markers \square and \circ) and Multicell-Shen-IWF algorithm (line with markers \diamond), $R_{1,1}^{\min} = 2 \times R_{1,2}^{\min}$, $R_{2,1}^{\min} = 2 \times R_{2,2}^{\min}$ and $R_{3,1}^{\min} = 2 \times R_{3,2}^{\min}$.

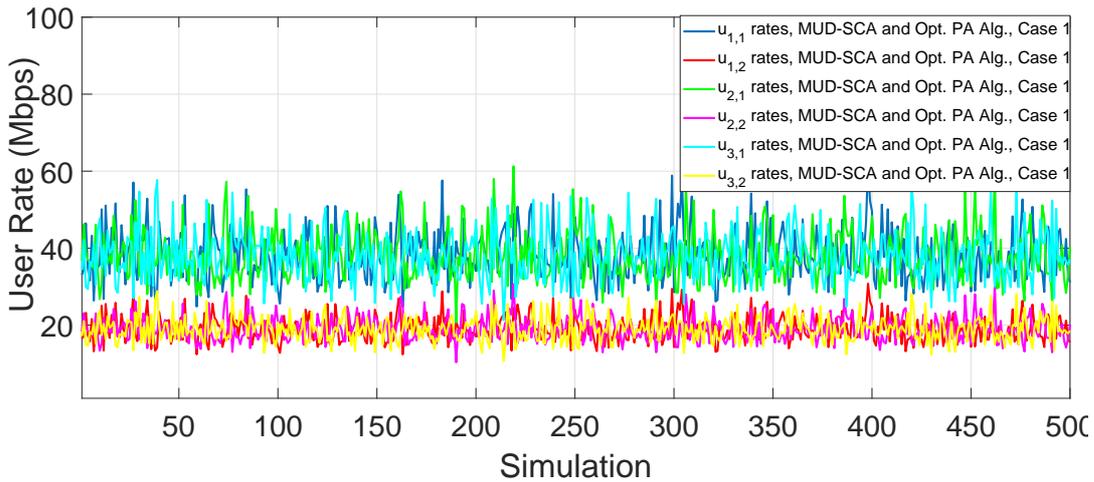


Figure 4.12: Variation of the rates of users in BS1, BS 2 and BS 3 vs. 500 channel realizations with asymmetric R^{\min} when MUD-SCA with Optimal PA Algorithm (Case 1) is used under $P_t^{max} = 20W$, $R_{1,1}^{\min} = 2 * R_{1,2}^{\min}$, $R_{2,1}^{\min} = 2 * R_{2,2}^{\min}$ and $R_{3,1}^{\min} = 2 * R_{3,2}^{\min}$.

their location in the cellular network. As a result, using the proposed algorithms we can control the rates of users by defining R^{\min} values. These values can be defined according to the rate demands of users as well as the resource allocation policy of the cellular network.

As we compare the sum capacity of BSs for asymmetric R^{\min} cases, if we require

Table 4.3: Mean of the rates of users

User	Instantaneous Rate (Mbps)
$u_{1,1}$	38.19
$u_{1,2}$	19.09
$u_{2,1}$	37.99
$u_{2,2}$	18.99
$u_{3,1}$	37.96
$u_{3,2}$	18.98

higher minimum rates for inner users, the sum capacity is higher than that of the other case as expected. When higher minimum rates are required for outer users, more resources would be used for outer users whose channel strengths are weaker and interference link strengths are higher, consequently the sum capacity gets lower.

4.4 A Comparison with Exhaustive Search

To show how the MUD-SCA and Optimal PA Algorithm performs compared to the optimal solution, we find the optimal result with a brute force approach in a 3-cell scenario with $S = 5$ subchannels for $P_t^{max} = 10 W$ and $K = 2$ for each BS. In this network, all subchannel assignment combinations of $2^5 \times 2^5 \times 2^5$ are considered and optimal power allocation scheme is found for each subchannel assignment combination. It is found out that MUD-SCA and Optimal PA algorithm achieves 91% of the optimum in average in the given scenario as seen in Table 4.4. One should note that, this good performance is achieved by a distributed algorithm where each BS conducts its resource allocation with limited information from other BSs.

Table 4.4: Comparison with the optimal results for various channel realizations

Channel Realization	% of Optimal Result
1	100%
2	80%
3	94%
4	85%
5	100%

4.5 Effect of JD on Performance

We also investigate how using JD decoding state affects performance through comparing 'Case 1' and 'Case 2' simulations of the proposed algorithm. As seen in Fig. 4.13, as we compare Case 1 and Case 2 simulations, using JD brings more advantage in the high power regime. The gain in minimum proportional user rates in average is about 15%.

As a result, using JD with SIC brings more advantage to the rates of the users when there is more interference. We can conclude that SIC does not enhance performance for a user without JD in the low power regime since the users cannot always get enough interference power to perform SIC.

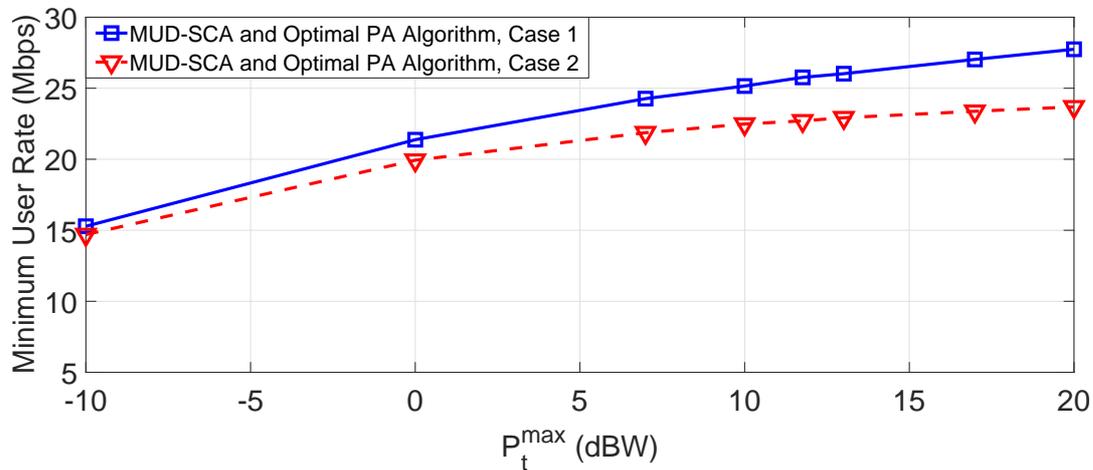


Figure 4.13: Mean of the BS1 user rates vs. P_t^{\max} averaged over 500 channel realizations for comparison of the MUD-SCA+PA Algorithm (Case 1) (line with \square) and MUD-SCA+PA Algorithm (Case 2) (dashed line with ∇), $R_{1,1}^{\min} = R_{1,2}^{\min}$, $R_{2,1}^{\min} = R_{2,2}^{\min}$ and $R_{3,1}^{\min} = R_{3,2}^{\min}$.

4.6 Sensitivity Analysis

In the previous sections, we assume that the channel is perfectly estimated. In this section, we will figure out how the performance of the MUD-SCA and PA algorithm is affected under perfect channel estimation error. We express the estimated channel as the normalized summation of the actual channel matrix and an error matrix whose

entries are normally distributed complex values with mean power σ_E^2 [86], [87]

$$\hat{h}_{k,j,l}^s = \frac{1}{\sqrt{1 + \sigma_E^2}}(h_{k,j,l}^s + e_{k,j,l}^s), \quad (4.1)$$

$$e_{k,j,l}^s \sim \mathcal{N}(0, \sigma_E^2). \quad (4.2)$$

The performance of MUD-SCA and Optimal PA algorithm under perfect channel estimation and imperfect channel estimation is compared in Fig. 4.14. It is observed that the proposed resource allocation scheme is not sensitive to channel estimation errors since errors below $\frac{H}{\sigma_E^2} = 10$ dB is usually not meaningful in real life. Moreover, even under severe channel estimation errors, the performance is comparable to that of Multicell-Shen-IWF Method which is depicted in Fig. 4.6.

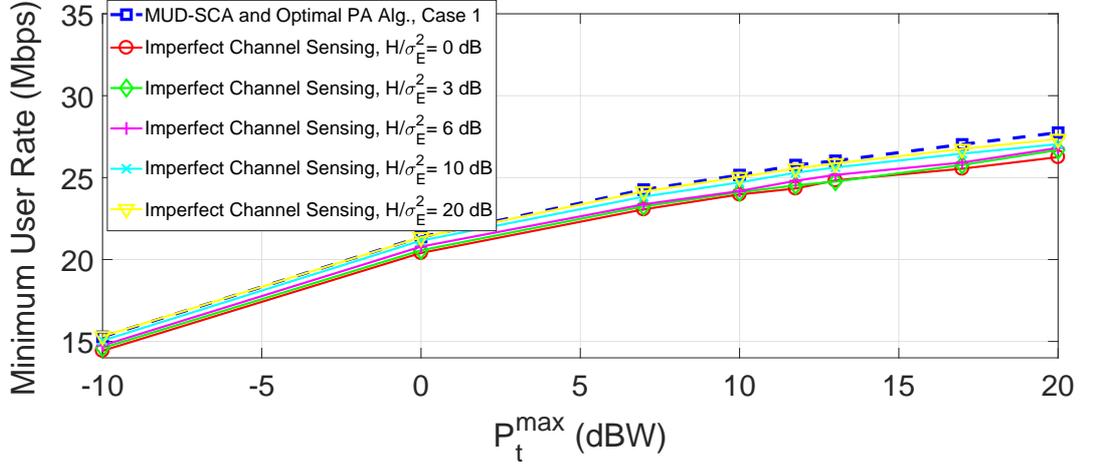


Figure 4.14: Minimum rated user in BS1 vs. P_t^{\max} averaged over 500 channel realizations for comparison of MUD-SCA and Optimal PA Algorithm (Case 1) with perfect channel estimation and imperfect channel estimation (dashed line with marker \square is for perfect channel estimation), $R_{1,1}^{\min} = R_{1,2}^{\min}$, $R_{2,1}^{\min} = R_{2,2}^{\min}$ and $R_{3,1}^{\min} = R_{3,2}^{\min}$ and $H \triangleq |h_{k,j,l}^s|^2, \forall k, j, l, s$.

Next, we figure out how maximum power change in the neighboring BSs affect the rates in a particular BS when MUD-SCA and Optimal PA algorithm is used. As observed in Fig. 4.15, as we decrease the maximum powers of BS 2&3 by 10%, the rates of users in BS1 increase 3.2% and the rates of users in BS 2&3 decrease 1.6% in average. This result shows that the proposed algorithm is not sensitive to the maximum powers of neighboring BSs.

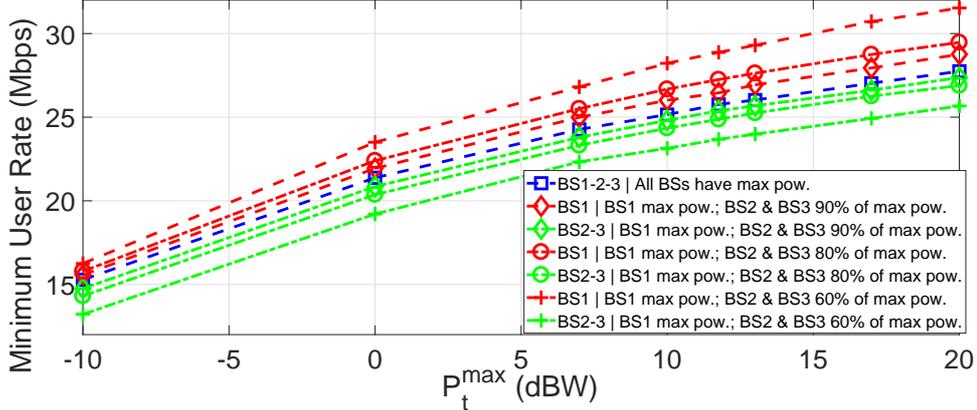


Figure 4.15: Minimum rated user in BS1 vs. P_t^{max} averaged over 500 channel realizations for comparison of MUD-SCA and Optimal PA Algorithm (Case 1) with various maximum powers of neighbouring BSs (dashed line with marker \square is for the case where all BSs have same maximum power), $R_{1,1}^{min} = R_{1,2}^{min}$, $R_{2,1}^{min} = R_{2,2}^{min}$ and $R_{3,1}^{min} = R_{3,2}^{min}$.

Next, we investigate using only SD capable users in neighboring BSs affect performance of MUD-SCA and Optimal PA algorithm. In this particular case, we consider that all the users in BS 1 have MUD capability and the users in BSs 2 and 3 have only SD capability. For this case, the rates of users in BS 1 increase 18% and the rates of users in BSs 2 and 3 decrease 21% in average as observed in Fig. 4.16. This result tells us that the proposed algorithm is sensitive to the MUD capability of users in the connected BS as well as that of users in neighboring BSs.

4.7 Discussions

We achieve near optimal performance with the proposed algorithms. We can obtain rates for every user proportional to their minimum required instantaneous rates. The heuristic algorithms decrease the complexity while not sacrificing much from the performance.

The fundamental difference of this paper with previous studies such as [17], [18] and [78] about resource allocation for multiuser OFDMA downlink systems with minimum rate constraints, is consideration of MUD receivers. By this, interference is exploited and BSs can schedule users on subchannels when there is interference

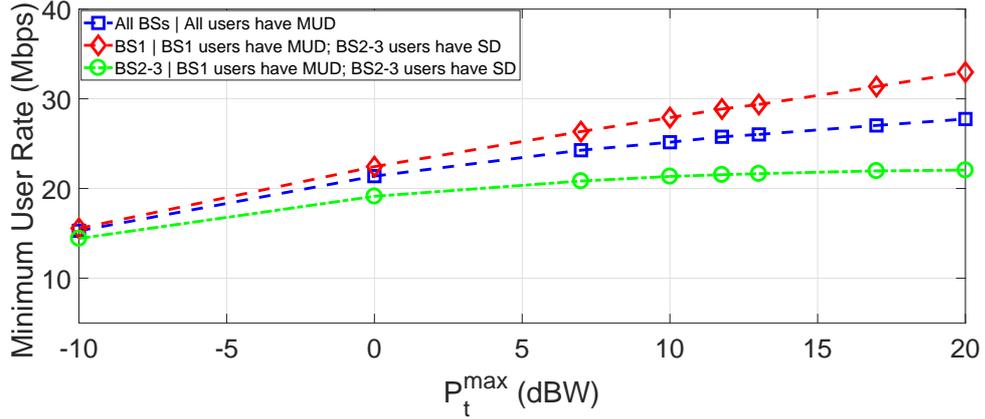


Figure 4.16: Minimum rated user in BS1 vs. P_t^{\max} averaged over 500 channel realizations for comparison of MUD-SCA and Optimal PA Algorithm (Case 1) with various maximum powers of neighbouring BSs (dashed line with marker \square is for the case where all BSs have same maximum power), $R_{1,1}^{\min} = R_{1,2}^{\min}$, $R_{2,1}^{\min} = R_{2,2}^{\min}$ and $R_{3,1}^{\min} = R_{3,2}^{\min}$.

depending on the power on that subchannel and other parameters. MUD decoding regions also constitute a great effect on subchannel assignment and power allocation steps. Additionally, using a rate marginal maximization framework facilitates the implementation and feasibility is always guaranteed as the margin can change depending on the adequacy of resources.

From a practical point of view, there may be a need to decrease the rates when the rates are higher than that of the desired minimum rates. This can be performed trivially by adjusting the used power in every subchannel by applying a lower maximum power constraint or not using some subchannels for the users of the related BS.

CHAPTER 5

CONCLUSION AND FUTURE WORK

In this thesis, we address resource allocation problem in a cellular multicell multiuser OFDMA downlink environment where the users are able to decode interference [26]. We try to exploit interference instead of approaching it as a performance downgrading instrument. We derive a rate margin maximization framework for this network. We separate the problem into subproblems such as subchannel assignment, power allocation and MUD formation steps and present a practical, iterative and distributed algorithm. We propose novel subchannel assignment and optimal power allocation algorithms (for a given subchannel assignment) with PSM and EM methods by using a Lagrangian dual decomposition framework. We observe that MUD-SCA and Optimal PA algorithm performs very close to the optimum in a small network scenario. We also propose lower complexity heuristic subchannel and power allocation algorithms without sacrificing much from performance.

We consider minimum rate requirements different than previous studies in the literature employing MUD receivers. Applying a rate margin constraint for each of the users, we guarantee the feasibility of the problem and provide the proportional distribution of the radio resources among users. We improve [19] which studies 2 Tx-2 Rx OFDMA scenario with MUD receivers by considering a multiuser scenario. Furthermore, we think of a broader context by considering multiuser scenario for each cell. In the subchannel assignment stage, the direct to cross channel ratios are considered different than the previous similar studies. To the best of our knowledge, this study is the first to attempt to a multicell multiuser OFDMA downlink network with receivers employing MUD subject to minimum rate requirements of each user.

MUD-SCA and Optimal PA algorithm improves the rate of the user with minimum

proportional rate requirement compared to legacy methods. By applying minimum rate constraints across users, we provide fairness among users regardless of their locations and distances to the BS through applying minimum rate constraints across users. Minimum rate constraints could also be defined according to the resource allocation policy of the cellular network according to some other criteria like pricing as well as the rate demands of users. Moreover, we observe that applying JD brings an improvement on user rates especially in the high power regime where interference is dominant.

The algorithm developed in this thesis is considered to be used in macrocell OFDMA networks as well as macro-femto OFDMA networks and cognitive radio networks where mitigating interference in full frequency reuse scenarios is one of the major problems. The overall algorithm presented in this study is an iterative algorithm with an assumption of limited information of the users in the neighboring cell. As a future work, this study can be generalized with the addition of a central controller over multiple BSs.

In this thesis, only instantaneous rates of the users are considered. Temporal domain can be exploited for resource allocation problem and ergodic rates of users can be studied in the future. The scenario in this thesis assumes single antenna BSs and users and this study can be trivially generalized for MIMO multicell multiuser scenario.

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APPENDIX A

LAGRANGIAN DUAL FORMULATION

Consider an optimization problem in standard form with variable $\mathbf{x} \in \mathbb{R}^n$:

$$\text{maximize } f_0(\mathbf{x}) \tag{A.1}$$

$$\text{subject to } f_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, m \tag{A.2}$$

$$h_i(\mathbf{x}) = 0, \quad i = 1, \dots, k \tag{A.3}$$

Denote the optimal value of the above problem as \mathbf{x}^* . The Lagrangian of the above problem is defined [69], [75] as :

$$L(\mathbf{x}, \boldsymbol{\lambda}, \mathbf{q}) = f_0(\mathbf{x}) - \sum_{i=1}^m \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^k q_i h_i(\mathbf{x}) \tag{A.4}$$

where λ_i 's and q_i 's are called the Lagrangian multipliers for the constraints $f_i(\mathbf{x})$ and $h_i(\mathbf{x})$, respectively. They are also called dual variables and satisfy the conditions $\lambda_i \geq 0$ and $\mathbf{q} \in \mathbb{R}^k$.

The Lagrangian dual objective is defined as an unconstrained maximization of the Lagrangian function over \mathbf{x} values:

$$g(\boldsymbol{\lambda}, \mathbf{q}) = \max_{\mathbf{x}} L(\mathbf{x}, \boldsymbol{\lambda}, \mathbf{q}) = \max_{\mathbf{x}} \left(f_0(\mathbf{x}) - \sum_{i=1}^m \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^k q_i h_i(\mathbf{x}) \right) \tag{A.5}$$

The dual Lagrangian is ∞ if the Lagrangian is unbounded above.

Consider an arbitrary feasible point, $\tilde{\mathbf{x}}$ for the basic optimization problem. We have

$$- \sum_{i=1}^m \underbrace{\lambda_i}_{\geq 0} \underbrace{f_i(\tilde{\mathbf{x}})}_{\leq 0} + \sum_{i=1}^k q_i \underbrace{h_i(\tilde{\mathbf{x}})}_{=0}, \text{ therefore}$$

$$L(\tilde{\mathbf{x}}, \boldsymbol{\lambda}, \mathbf{x}) = f_0(\tilde{\mathbf{x}}) - \underbrace{\sum_{i=1}^m \lambda_i f_i(\tilde{\mathbf{x}}) + \sum_{i=1}^k q_i h_i(\tilde{\mathbf{x}})}_{\geq 0} \geq f_0(\tilde{\mathbf{x}}). \text{ Then,}$$

$$g(\boldsymbol{\lambda}, \mathbf{q}) = \max_{\mathbf{x}} L(\mathbf{x}, \boldsymbol{\lambda}, \mathbf{q}) \geq L(\tilde{\mathbf{x}}, \boldsymbol{\lambda}, \mathbf{q}) \geq f_0(\tilde{\mathbf{x}}), \forall \tilde{\mathbf{x}}.$$

Therefore, $g(\boldsymbol{\lambda}, \mathbf{q}) \geq \mathbf{x}^*$ if $\boldsymbol{\lambda} \geq 0$.

When $g(\boldsymbol{\lambda}, \mathbf{q}) = \infty$, the dual problem does not give a meaningful upper bound on the optimal value. Therefore, the following dual problem is stated choosing $\boldsymbol{\lambda}$ and \mathbf{q} such that Lagrangian dual function is finite.

$$\text{minimize } g(\boldsymbol{\lambda}) \tag{A.6}$$

$$\text{subject to } \boldsymbol{\lambda} \succeq 0 \tag{A.7}$$

Duality is classified into two categories: weak duality and strong duality. The optimal value, d^* , of the Lagrangian dual problem is the best upper bound on the optimal value, p^* , of the original (primal) optimization problem (A.1). That is stated as by following inequality

$$p^* \leq d^*. \tag{A.8}$$

This equality is valid even if the primal problem is not convex. This situation is called weak duality [69].

The difference $d^* - p^*$ is called optimal duality gap and is a measure for the difference between optimal value of original problem and the optimal value for the Lagrangian dual function. Weak duality is sometimes used to find a upper bound for difficult-to-solve optimization problems.

If the above inequality is satisfied with equality, i.e.,

$$p^* = d^*, \tag{A.9}$$

then the duality gap is 0 and it is stated that 'strong duality' holds (the best upper bound is obtained). Strong duality holds for optimization problems in some certain conditions.

APPENDIX B

BISECTION METHOD

Bisection method for convex optimization is explained in [69]. Suppose we have a convex feasibility "Problem A" at hand. We assume that the interval $[l, u]$ contains the optimal value p^* of A, i.e., $l \leq p^* \leq u$. In each step of the method the lower limit or the upper limit of the interval is updated assuring p^* remains in the interval. This procedure is repeated until the width of the interval is less than some small value, ϵ .

The method is stated as follows:

Given $l \leq p^* \leq u, \epsilon > 0$

repeat

1. $t \leftarrow \frac{l+u}{2}$
2. Solve the convex feasibility Problem A.
3. If the solution is feasible, $u \leftarrow t$ else $l \leftarrow t$.

until $u - l \leq \epsilon$;

APPENDIX C

PROJECTED SUBGRADIENT METHOD

Projected subgradient method is used to solve the following constrained optimization problem

$$\text{minimize } f(\mathbf{x}) \tag{C.1}$$

$$\text{s. t. } \mathbf{x} \in C \tag{C.2}$$

where $x \in \mathbb{R}^n$, $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is a convex function and C is a convex set. The update equation in projected subgradient method is stated as

$$\mathbf{x}^{(t+1)} = \Pi(\mathbf{x}^{(t)} - \beta_t \mathbf{d}^{(t)}) \tag{C.3}$$

where Π is Euclidean projection on C , $\mathbf{d}^{(t)}$ is a subgradient of f at $\mathbf{x}^{(t)}$ and β_t is the step size.

In our case, the constraint is a linear equality constraint

$$\text{minimize } f(\mathbf{x}) \tag{C.4}$$

$$\text{s. t. } \mathbf{A}\mathbf{x} = \mathbf{b}. \tag{C.5}$$

The projection of a vector \mathbf{z} onto the set $\{\mathbf{x} \mid \mathbf{A}\mathbf{x} = \mathbf{b}\}$ is defined as [79]:

$$\Pi(\mathbf{z}) = \mathbf{z} - \mathbf{A}^T (\mathbf{A}\mathbf{A}^T)^{-1} (\mathbf{A}\mathbf{z} - \mathbf{b}) \tag{C.6}$$

$$= \left(\mathbf{I} - \mathbf{A}^T (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{A} \right) \mathbf{z} + \mathbf{A}^T (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{b}. \tag{C.7}$$

The projected subgradient update is ($\mathbf{A}\mathbf{x}^{(t)} = \mathbf{b}$)

$$\mathbf{x}^{(t+1)} = \Pi(\mathbf{x}^{(t)} - \beta_t \mathbf{d}^{(t)}) \tag{C.8}$$

$$= \mathbf{x}^{(t)} - \beta_t \left(\mathbf{I} - \mathbf{A}^T (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{A} \right) \mathbf{d}^{(t)}. \tag{C.9}$$

The projected subgradient method is proven to converge for suitable step sizes.

APPENDIX D

ELLIPSOID METHOD

Ellipsoid method can be considered as a generalization of bisection method for multiple dimensions. For the aim of minimization of a convex function, ellipsoid method creates decreasing volume ellipsoids that are guaranteed to contain the optimal point, using the subgradient of the objective function.

In this thesis, the minimization problem has a linear equality constraint:

$$\text{minimize } f(\mathbf{x}) \tag{D.1}$$

$$\text{s. t. } \mathbf{Ax} = \mathbf{b} \tag{D.2}$$

where $x \in \mathbb{R}^n$, $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is a convex function and A is a full rank matrix. We take an initial ellipsoid $\mathcal{E}^0 = \{(\mathbf{x} - \mathbf{x}^0)^T \mathbf{Q}_0^{-1} (\mathbf{x} - \mathbf{x}^0) \leq 1\}$ that is the smallest ellipsoid containing upper bound \mathbf{U} and lower bound \mathbf{L} which include the optimal value \mathbf{x}^* . \mathbf{x}^0 is the midpoint of \mathbf{U} and \mathbf{L} and \mathbf{Q}_0 is a positive definite and symmetric matrix [80], [81].

Let $\mathbf{d}^{(t)}$ be a subgradient of f at $\mathbf{x}^{(t)}$ and the projection of $\mathbf{d}^{(t)}$ onto the set $\{\mathbf{d}^{(t)} \mid \mathbf{Ad}^{(t)} = \mathbf{b}\}$ is defined as [81]

$$\mathbf{g}^{(t)} = - \frac{\left(\mathbf{Q}_t - \mathbf{Q}_t \mathbf{A}^T (\mathbf{A} \mathbf{Q}_t \mathbf{A}^T)^{-1} \mathbf{A} \mathbf{Q}_t \right) \mathbf{d}^{(t)}}{\sqrt{\mathbf{d}^{(t)} \left(\mathbf{Q}_t - \mathbf{Q}_t \mathbf{A}^T (\mathbf{A} \mathbf{Q}_t \mathbf{A}^T)^{-1} \mathbf{A} \mathbf{Q}_t \right) \mathbf{d}^{(t)}}}. \tag{D.3}$$

The update equations are

$$\mathbf{x}^{(t+1)} = \mathbf{x}^{(t)} + \frac{1}{n+1} \mathbf{g}^{(t)} \tag{D.4}$$

$$\mathbf{Q}_{t+1} = \frac{n^2}{n^2 - 1} \left(\mathbf{Q}_t - \frac{2}{n+1} \mathbf{g}^{(t)} \mathbf{g}^{(t)T} \right). \tag{D.5}$$

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PUBLICATIONS

International Journal Publication

1- Y. K. Yazarel and A. Ö. Yılmaz, “Efficient Scheduling and Power Allocation for Multiuser Decoding Receivers in OFDMA Networks with Minimum Rate Requirements”, to appear in *Physical Communication*, vol. 26, pp. 60–70, Feb. 2018.

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2- Y. K. Yazarel and D. Aktaş, “Downlink beamforming under individual SINR and per antenna power constraints,” in *IEEE Pacific Rim Conference on Commun., Comput., Signal processing (PACRIM)*, pp. 422 – 425, Aug. 2007.

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3- Y. K. Yazarel and D. Aktaş, “Çok Kullanıcılı Çok Antenli Sistemlerde İşbirlikli İletim (Cooperative Transmission for Multiuser MIMO Systems),” in *IEEE 16th Signal Processing, Communication and Applications Conference (SIU)*, Apr. 2008.