MIDDLE SCHOOL MATHEMATICS TEACHERS' MATHEMATICAL PRACTICES IN TEACHING SLOPE, LINEAR EQUATIONS, AND GRAPHS IN TECHNOLOGY ENHANCED CLASSROOM ENVIRONMENT

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Prof. Dr. Meliha ALTUNIŞIK
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Doctor of Philosophy.

Prof. Dr. Özgül YILMAZ TÜZÜN
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Doctor of Philosophy.

Prof. Dr. Mine IŞIKSAL BOSTAN
Supervisor

Examining Committee Members

Prof. Dr. Erdinç ÇAKIROĞLU
(METU, MSE)
Prof. Dr. Ayhan Kürşat ERBAŞ
(METU, MSE)
Prof. Dr. Mine IŞIKSAL BOSTAN
(METU, MSE)
Assoc. Prof. Dr. Elif YETKİN ÖZDEMİR
(Hacettepe Unv., MSE)
Assist. Prof. Dr. Reyhan TEKİN SİTRAVA
(Kırıkkale Unv., MSE)

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Name, Last name: Seçil YEMEN-KARPUZCU

Signature:

ABSTRACT

MIDDLE SCHOOL MATHEMATICS TEACHERS’ MATHEMATICAL PRACTICES IN TEACHING SLOPE, LINEAR EQUATIONS, AND GRAPHS IN TECHNOLOGY ENHANCED CLASSROOM ENVIRONMENT

Yemen Karpuzcu, Seçil

Ph. D., Department of Elementary Mathematics Education

Supervisor: Prof. Dr. Mine IŞIKSAL BOSTAN

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The aim of this study was to examine an in-service middle school mathematics teacher’s mathematical practices in teaching slope, linear equations and graphs in a technology enhanced classroom environment while enacting an instructional sequence that was designed based on the national curriculum. The technology utilized in the study was GeoGebra materials within computers that were developed by the teacher and the researcher to be used for the instructional sequence. The data was collected during the spring semester of 2012-2013 while she was teaching to eight graders in her mathematics classrooms. While the primary data source was classroom teaching sessions of the teacher, other data sources (i.e. individual pre-interview and post-interview, planning and analyzing instruction sessions, classroom activity sheets, and field notes) were considered for a coherent examination of the teacher’s mathematical practices. The data was analyzed by using qualitative methods. The findings of the study were presented within the scope of the teacher's mathematical practices such as bridging, trimming and decompressing in two contexts, when the teacher used GeoGebra materials and when she made mathematical explanations while not using

GeoGebra materials. The findings indicated that the teacher utilized various sets of mathematical practices that were composed of several actions. While bridging practices were the most seen mathematical practices, they also fostered and advanced trimming and decompressing practices. All these practices were strongly interrelated in teaching using GGB materials when compared to the interrelations in teaching without using GGB materials in a technology-enhanced classroom environment.

Keywords: Middle school mathematics teachers, mathematical practices, teaching slope, technology-enhanced classroom environment, design experiment

ÖZ

ORTAOKUL MATEMATİK ÖĞRETMENLERİNİN TEKNOLOJİ DESTEKLİ SINIF ORTAMINDA EĞİM, DOĞRU DENKLEMLERİ VE GRAFİKLERİ ÖĞRETİMİ SIRASINDAKİ MATEMATİKSEL UYGULAMALARI

Yemen Karpuzcu, Seçil

Doktora, İlköğretim Matematik Eğitimi Bölümü

Tez Yöneticisi: Prof. Dr. Mine IŞIKSAL BOSTAN

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Bu çalışmanın amacı, bir ortaokul matematik öğretmeninin, öğretim programına dayalı olarak tasarlanan bir öğretim dizisini sınıfa koyması sürecinde, teknoloji destekli bir sınıf ortamında eğim, doğru denklemleri ve grafikleri öğretimi sırasındaki matematiksel uygulamalarını incelemektir. Bu çalışmada kullanılan teknoloji, öğretim dizisisi için bilgisayarlarla kullanılmak üzere öğretmen ve araştırmacılar tarafından geliştirilen GeoGebra materyalleridir. Veriler 2012-2013 bahar döneminde öğretmen kendi matematik sınıflarında sekizinci sınıflara öğretim yaparken toplanmıştır. Birincil veri kaynağı öğretmenin sınıf oturumlarının video kayıtları iken, öğretmenin matematiksel uygulamalarının tutarlı bir değerlendirmesi için diğer veri kaynakları (bireysel ön-görüşme ve son-görüşme kayıtları, planlama ve analiz oturumları kayıtları, sınıf etkinlikleri kâğıtları ve alan notları) dikkate alınmıştır. Veriler nitel araştırma yöntemleri kullanılarak analiz edilmiştir. Çalışmanın bulguları, öğretmenin bağlama (bridging), kırpma (trimming) ve açma (decompressing) gibi matematiksel uygulamaları kapsamında, öğretmen GeoGebra materyalleri kullandığı sırada ve öğretmen GeoGebra materyalleri kullanmazken matematiksel açıklamalar yaptığı sırada olmak üzere iki kapsamda sunulmuştur. Bulgular göstermiştir ki, öğretmen birçok eylemden oluşan matematiksel uygulamaların türlü gruplarını kullanmıştır. Bağlama uygulamaları en sık görülen matematiksel uygulamayken, bu uygulamalar aynı zamanda kırpma ve açma uygulamalarını desteklemiştir ve geliştirmiştir. Teknoloji destekli bir sınıf ortamında bağlama, kırpma, ve açma uygulamaları arasındaki ilişkiler karşılaştırıldığında bu ilişkilerin, GeoGebra materyalleri kullanmadan yapılan öğretime kıyasla, GeoGebra kullanılarak yapılan öğretimde daha güçlü olduğu görülmüştür.

Anahtar kelimeler: Ortaokul matematik öğretmenleri, matematiksel uygulamalar, eğim öğretimi, teknoloji destekli sınıf ortamı, tasarım deneyi

To my family

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LIST OF ABBREVIATIONS

GGB	GeoGebra
CCSSI	Common Core State Standards Initiative
CCSM	Common core standards for mathematics
MoNE	Ministry of National Education
NCTM	National Council of Teachers of Mathematics
SMK	Subject Matter Knowledge
PCK	Pedagogical content knowledge
KAT	Knowledge of Algebra for Teaching
HLT	Hypothetical Learning Trajectory

CHAPTER 1

INTRODUCTION

When it is looked from the global perspective of algebra in curricula, in many countries content of algebra programs is shaped in parallel with the social context of the class that teaching occurs (Kendal & Stacey, 2004). That is, if the classes are not layered in terms of the students’ ability, the country has a comprehensive system. If not, it has a streamed system (Kendal & Stacey, 2004). In Turkey, while algebra is taught, students are in a comprehensive system up until grade 10. Even though there are two types of middle schools and many different types of high schools, the subjects in the mathematics program is same for all schools except vocational high schools. Therefore, children who have different abilities and interests learn mathematics together in the middle school grades (grades 5 to 8). This system is parallel with most of the algebra programs in other country’s comprehensive school systems (Kendal & Stacey, 2004) and Turkey’s middle school mathematics curriculum does not give earlier emphasis on the use of symbolic algebra and functions. Another issue that has structured algebra programs is how algebra is adopted into curriculum, as a separate course or as a learning area in an integrated mathematics approach (Kendal & Stacey, 2004). For example, in Turkey’s curriculum, the integrated mathematics approach provides students to take algebra topics as units under the mathematics course, which is an opportunity to make connections across other topics.

According to what algebra is conceived of, every country’s curriculum has different generalities, different uses of technology, and different approaches to functions. From this point of view, in Turkey, algebra is conceived of as a way of expressing generality and pattern in grades 5 and 6, a study of symbol manipulation and solving equations, and table and graphs of the equations in grades 7 and 8 (Ministry of National Education [MoNE], 2009, 2013a), and a study of functions and their transformations in grades 9-12 (MoNE, 2013b). Therefore, school algebra has a vital degree that is more than just general number properties and relations. MacGregor (2004) emphasized algebraic competence, and emphasized the significance of it in compulsory years of schooling. Additionally, the teaching of algebra should mean to provide students opportunities, to develop abstract ideas and satisfy them from using the symbol system of algebra to support logical thinking (MacGregor, 2004). Beyond solving linear equations, linearity and representations of linear relationships (i.e. tables, graphs, and equations) are important aspects for algebraic competence (National Council of Teachers of Mathematics [NCTM], 2000).

In consideration of curricula all over the world, the concept of slope is taught within the context of linear equations and their graphs in school algebra from elementary mathematics classrooms to secondary mathematics classrooms. In the U.S. context, NCTM (2000) explained the goals in grade bands (e.g. grades 6-8) in Principles and Standards for School Mathematics and Common Core State Standards Initiative [CCSSI] (2010) clarified and specified the standards for each grade in common core standards for mathematics [CCSM]. In CCSM, the slope concept is taught in grades 7-8. In detail, students start to learn “unit rate” (CCSSI, 2010, p. 46) as constant of proportionality in graphing proportional relationships in grade 7. In addition, students learn slope in representing, analyzing and solving problems on linear equations and the system of linear equations in grade 8. They learn slope with multiple representations and various conceptions in curriculum (Nagle & Moore-Russo, 2014). In Turkey’s national middle school mathematics curriculum for grades 5-8 (MoNE, 2013), students learn constant of proportionality without mentioning unit rate in graphs and tables of proportional relationships and linear relationships in tables, graphs, and linear equations in grade 7. Additionally, in grade 8, students learn slope with using models and relating linear equations, graphs, and tables. As a revised form of the elementary mathematics curriculum in 2009 (MoNE, 2009), MoNE (2013a) have the same objectives about slope with emphasis on the relating between representations of linear equations, graphs and tables in grade 8.

There are various reasons that make the teaching of slope prominent and foundation in school algebra. It is also important to conceptualize and use the concept of slope in a real life context and in a mathematical context with a wide-variety of experiences in real-life contexts and in mathematical contexts in mathematics learning and teaching. There are various conceptualizations about slope that require us to consider slope from different points of views. This situation also requires us to understand the links between the different conceptualizations of slope. Furthermore, the concept of slope connected with proportional reasoning (Lesh, Post, & Behr, 1988; Lobato & Siebert, 2002), quantitative reasoning (Lobato, Ellis, & Munoz, 2003; Lobato & Bowers, 2000), and covariational reasoning (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002; Johnson, 2012). Therefore, the concept of slope is one of the cornerstones in algebra learning and teaching in middle school mathematics before high school mathematics.

Research regarding slope learning mentioned students' and teachers' conceptions and difficulties about slope (Cheng, 2010, 2015; Cheng, Star, & Chapin, 2013; Cho & Nagle, 2017; Coe, 2007; Dündar, 2015; Hoffman, 2015; Lobato & Siebert, 2002; Nagle & Moore-Russo, 2013a, 2013b; Stump, 1997, 2001a, 2001b; Teuscher & Reys, 2010; Zaslavsky, Sela, & Leron, 2002). For example, there were studies that were conducted with high school students to reveal their conceptions of slope as a measure (Stump, 2001b) and to reveal their understanding of slope in relation with steepness and rate of change (Teuscher & Reys, 2010), with college students to reveal their difficulties with slope considering procedural and conceptual aspects (Cho & Nagle, 2017). There were also found some studies that had a focus on middle school students and revealed the relations between their conceptualizations of slope and reasoning proportionally (Cheng, 2010, 2015; Cheng, Star, & Chapin, 2013). In addition, there are few in-depth studies to reveal middle school students' understanding of slope. For example, Lobato and Siebert (2002) investigated a student's -who finished eighth-grade- understanding of slope and quantitative reasoning in different situations during a teaching experiment for a mixed group of students from 8^{th} grade to 10^{th} grade.

On the other hand, there are the limited availability of research studies about slope teaching that were conducted with teachers. It was seen that some studies were conducted with preservice teachers and some were conducted with both preservice and in-service teachers. The studies that were conducted with preservice teachers revealed their knowledge of slope through their definitions, concept images and conceptualizations of slope for high school level (Stump, 1997, 2001a) and middle school level (Dündar, 2015). The studies conducted with both in-service and preservice teachers gave information about their (without giving the level) approaches of slope of linear function in coordinate system in relation with scale and angle (Zaslavsky, Sela, & Leron, 2002). In addition, Nagle and Moore-Russo (2013b) presented preservice and in-service teachers' (without giving the level) conceptions of slope and their planned instructional materials for slope teaching within the scope of a graduate course. Furthermore, there were less studies with a focus on in-service teachers. For examples, a research indicated high school teachers' conceptions about rate of change and slope (Coe, 2007) and the other one was indicated middle school teachers' understanding of slope (Hoffman, 2015). .However, while these studies give insight about teachers' mathematical understanding of slope, they do not inform us about middle school mathematics teachers' use of these understandings in teaching practice, especially in classroom teaching. Therefore, it is difficult to understand teachers' mathematical practices that they use of their own mathematical understanding in slope teaching.

It is an obvious premise that teachers are the indispensable and core part of the development of mathematics education in a classroom teaching based on a curriculum (Remillard, 2005; Sherin & Drake, 2004). Research on mathematics curriculum reforms and teachers shows that teachers' knowledge about mathematics content, teaching practices, their personal theories about learning and teaching mathematics have a great influence on their values and implementation of programs (Manouchehri & Goodman, 1998). We accepted that there are prominent frameworks of mathematics teaching, which are models of mathematical knowledge for teaching by Ball, Thames, and Phelps (2008) and Hill, Ball and Schilling (2008) and knowledge quartet by Rowland, Turner, Thwaites, and Huckstep (2009). However, although these frameworks provide tremendous contributions, how teachers' knowledge is used and developed in classroom mathematics teaching practice for specific learning areas, is still an open issue when it comes to combining theory and practice (Doerr, 2004). Additionally, for the development of teachers, their personal model of mathematics teaching is seen as an important aspect of mathematics teaching in addition to the knowledge components (Simon, 1997). This model of teaching "guides and constraints instructional decision making and defines the teacher's role in relation to students' learning" (Simon, 1997, p.81). In this regard, learning-teaching trajectories are suggested as a means to provide conjunctions between the general objectives in curricula and teaching, while at the same time presenting "a conceptual framework for didactical decision making" (van den Heuvel-Panhuizen & Wijers, 2005, p.305). However, it is difficult to understand teachers' mathematics teaching when theory and practice are not connected.

In specifically, investigating teachers' mathematical practices in slope teaching in school algebra requires to have a distinctive spectacles. In this regard, knowledge of algebra for teaching (KAT) framework (McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012) that specifies knowledge and practices for algebra teaching is seen meaningful to use. The mathematical practices dimension in the framework involves three main categories as bridging, trimming and decompressing. The bridging category that corresponds to the practices of making mathematical connections can provide an understanding about how teachers make connections across slope related concepts, topics, representations, and domains. The trimming category that corresponds to the practices of removing the mathematical complexity can provide an understanding about how teachers transfer or eliminate the complexities in slope teaching. The decompressing category that corresponds to the practices of making explicating the mathematical complexity can provide an understanding about how teachers highlighted or magnified the complexities in slope teaching. However, even though this framework primarily provides opportunities for assessing teachers in terms of their mathematical content knowledge in algebra, how all these mathematical practices emerge is an open issue during teaching an algebra topic in real classroom teaching practice.

The results in the literature indicated that mathematics teachers had a tendency to put themselves into an information provider's position in teaching algebra (Boaler, 2003) and they followed the procedures from the textbook and gave routine exercises and examples (McKnight, Travers, Crosswhite, & Swafford, 1985). However, while teaching algebra, there are many different conceptions of algebra to how it is related with the meaning of variables. These are generalized arithmetic, the set of procedures used for solving problems, the study of relationship among quantities, and the study of structures (Usiskin, 1988). In other words, to what extent we teach algebra is related to the ways of using of variables. When considering a new approach to teach algebra, the pattern-based approach becomes prominent because it involves generality, understanding of functional relationships and their algebraic description, building on the functional relationship to how to formulate and solve equations, algebraic letters as pattern generalizers, and letter-as-variable in equations (Stacey & MacGregor, 2001). In addition, as a result of students' and teachers' rudimentary understanding of algebra, the pattern based approach to algebra is adopted to most of the curriculum (CCSSI, 2010; MoNE 2009, 2013a; NCTM, 2000). Therefore, middle school mathematics teachers try to follow this approach in Turkey. However, what kind of slope teaching is going on in a middle school mathematics teacher's classes is not known while the teacher used this approach of the curriculum.

Furthermore, the use technology (dynamic software, calculators, spreadsheets, etc.) is emphasized for algebra and other learning areas in the middle school mathematics curriculum (MoNE, 2013a). Meanwhile, the challenge of developing new teaching practices for a new approach or with a new tool or technology cannot be underestimated for mathematics teachers (Doerr, Ärlebäck, & O'Neil, 2013; Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010; Drijvers, Tacoma, Besamusca, Doorman, & Boon, 2013; Monaghan, 2004; Ruthven, Deaney, & Hennesy, 2009). Therefore, understanding mathematics teachers' ways of demonstrating teaching practices with or without using instructional tools in a technology enhanced classroom environment and the teachers' skills and knowledge in these practices become important for supporting teacher education or development (Lagrange & Özdemir-Erdoğan, 2009; Drijvers et al., 2013). However, how teachers' mathematical practices emerge while using technologies in teaching specific topics within the curricula is not fully disclosed.

1.1 Purpose of the Study

Progress in research on teaching algebra notifies the significance of understanding of slope (Lobato & Thanheiser, 2002; Moore-Russo, Conner, & Rugg 2011) and understanding slope in linear equations and graphs (Greenes, Chang, & Ben-Chaim, 2007). The review of literature indicated that students and teachers have difficulties about understanding various conceptualizations of slope (Coe, 2007;

Hoffman, 2015; Stump, 1997, 2001a). These results are an indication of the puzzling nature of slope teaching. In addition, the need for studies with teachers about slope teaching to help students learn slope and related concepts in a connected way (Cheng, Star, & Chapin, 2013).

The recent changes in school algebra learning in Turkish middle school mathematics curriculum involves making a connection between slope of line graphs and equations of line graphs using tables, graphs, and symbolic representations in grade eight. However, these change does not guarantee to relate the understanding of slope within the conceptualizations of linearity, proportionality, ratio and rate in classrooms. On the other hand, it is suggested that mathematics teachers can develop students' understanding of the concept of slope by supporting their understanding of ratio and proportion, and the concepts of ratio and proportion will be advanced with the multiplication in mathematics classes (Cramer, Post, & Currier, 1993). In addition, it is suggested to grasp the logic of slope formula in different situations (Cheng, 2010; Crawford & Scott, 2000; Lobato & Thanheiser, 2002). Although one of the big ideas is slope in teaching middle school mathematics curricula, we do not know the middle school mathematics teachers' mathematical practices in slope teaching during implementing an instructional sequence. In connection with both curricula and the students' learning issues, middle school mathematics teachers also need for an instructional sequence to improve their students' mathematical understanding of slope, linear equations and graphs in grade eight.

Effective mathematics teachers contextualize the big ideas in the mathematics curriculum for the purpose of organizing the instruction (Sowder, 2007). In this regard, the teacher's role is indispensable in curricular decision-making and it is an important process in teaching with these questions: "What should be taught and how should it be taught? Who should decide? Are teachers supposed to decide what to teach? If so, how should they decide?" (Ball & Feiman-Nemser, 1988, p. 405). In addition, when considered the results of the studies (Ball & Bass, 2003; Ball et al., 2008; Hill, Schilling, & Ball, 2004) and Doerr's (2004) perspective, it is obvious that there is a need of understanding mathematics teaching to be fused with the theory and practice into a cohesive whole. That is, "how mathematics teacher knowledge is enacted and the relationship with classroom practice remains poorly understood" (Hodgen, 2011,

p. 39). In a sense, teachers' knowledge is considered as situated and social and the teachers' communities in and out of the classroom become more of an issue while understanding their teaching (Hodgen, 2011). However, mathematics educators need to have an understanding about teachers' mathematical work of teaching in practice to provide teacher education or development in addition to the having the theoretical approaches. Although this is a complex issue, one way to solve this is for the researchers and teacher educators to provide an opportunity to investigate teachers' practices in real classroom environments (Wood, 1995). From this point of view, the primary goal of this study is to analyze the middle school mathematics teachers' mathematical practices in classroom teaching for slope, linear equations and graphs while enacting an instructional sequence based on the national curriculum.

To emerge mathematics teachers' mathematical practices in the context of teaching slope, linear equations, and graphs, it necessitates a specific algebra lens on what teachers do. In this regard, research on knowledge of algebra teaching (Ferrini-Mundy, McCrory, Floden, Burrill, & Sandow, 2005; McCrory et al., 2012) can enlighten this process. The mathematical practices dimension of this framework can be used and developed for investigating particular mathematical practices of mathematics teachers as their didactical performance in a classroom. In this regard, interpreting mathematical practices of the middle school mathematics teachers in slope teaching from the perspective of algebra teaching can provide in depth understanding about mathematical practices in the work of mathematics teaching.

Above all, mathematics teachers and mathematics teacher educators should consider the research on teaching slope, linear equations, and graphs in a technology enhanced classroom environment since using technology in teaching is emphasized by both researchers (Bozkurt & Ruthven, 2017; Oncu, Delialioglu, & Brown, 2008) and curricula (CCSSM, 2010; MoNE, 2013). For example, studies conducted with multi-representational software for learning of the function concept with its representations gave results that these technologies positively influenced the students' conceptual understanding of the functions and relations between the representations (Schwarz & Hershkowitz, 1999). Moreover, the revised middle school mathematics curriculum (MoNE, 2013a) attached a particular importance to the use of technology in teaching with an emphasis on the representations of concepts and interrelations, between these representations, and exploring these interrelations by the students. In relation to algebra teaching, various technological tools and environments (e.g. Dynamic software) consider the pattern based approach with its emphasis on multiple representations and dynamic properties. Dynamic mathematics software is seen as the indispensable part of mathematics teaching in 21st century. GeoGebra is one of them that provides dynamic mathematical representations and integrates graphics view, algebra view, spreadsheet and various tools within a dynamic environment, which is used in this study.

On the other hand, in parallel with these developments and educational revisions, it is the fact that what mathematics teachers do to teach in their classrooms while teaching algebra topics is a mystery. However, although there are research studies that integrate new curricula, approaches, and technology into algebra teaching in the classroom (Bozkurt & Ruthven, 2017; Drijvers et al., 2010; Hallagan, 2004; Kendal & Stacey, 2001), there is still a need to study the practice of slope teaching within technological environments (Kieran, 2007). Especially in Turkey, there has been a few rare theories and practices for teacher development and instructional practices that were based on in-service middle school mathematics teachers and their students in real settings. For these reasons, the focus of the current study is determined as investigating middle school mathematics teachers' mathematical practices in teaching slope, linear equations, and graphs in a technology enhanced classroom environment for eight graders while enacting an instructional sequence that was designed based on the national curriculum. However, the teachers can demonstrate mathematical practices while using instructional tools, and while explaining mathematical ideas without using instructional tools in the moments of teaching in a technology enhanced classroom environment. Therefore, this current research study has contextualized middle school mathematics teachers' mathematical practices in teaching slope, linear equations, and graphs both using GeoGebra materials and not using GeoGebra materials in a technology enhanced classroom environment in grade eight.

Despite the important developments of research and curricula, changing of a teacher's practices in real mathematics classrooms is not that simple as it was suggested. That is, developing mathematical teaching practices in a technology enhanced classroom environment can be challenging for the mathematics teachers. In professional development, design experiments has been utilized as a means to emerge and enhance teaching practices of mathematics teachers, e.g. multi-tiered design experiments (Lesh, Kelly, & Yoon, 2008; Zawojewski, Chamberlin, Hjalmarson, & Lewis, 2008). Therefore, to provide a source for teachers and educators in this subject, a design experiment to support teachers' mathematical work in teaching is seen as a suitable method to explore the teachers' mathematics teaching practices (Kelly, 2013). In sum, this study aimed to investigate a middle school mathematics teacher's set of mathematical practices in teaching slope, linear equations and graphs in a technology enhanced classroom environment while enacting a curriculum-based instructional sequence for eight graders throughout a design experiment.

1.2 Research Questions

This study investigated a middle school mathematics teacher's mathematical practices in teaching slope, linear equations and graphs in a technology enhanced classroom environment while enacting an instructional sequence with GeoGebra materials as instructional tools that was designed for eight graders based on the national curriculum. Considering the purpose of the study, the research questions that drive the study are in the following.

1. What mathematical practices of middle school mathematics teachers emerge while using GeoGebra materials in a technology enhanced classroom environment in enacting the instructional sequence?

    1.1 What is the nature of mathematical practices of bridging, trimming, and decompressing that middle school mathematics teachers demonstrated while using GeoGebra materials in a technology enhanced classroom environment in enacting the instructional sequence?

    1.2. How do these mathematical practices interrelate while middle school mathematics teachers are using GeoGebra materials in a technology enhanced classroom environment in enacting the instructional sequence?

2. What mathematical practices of middle school mathematics teachers emerge while explaining mathematical ideas without using GeoGebra materials in enacting the instructional sequence?

2.1 What is the nature of mathematical practices of bridging, trimming, and decompressing that middle school mathematics teachers demonstrated while explaining mathematical ideas without using GeoGebra materials in enacting the instructional sequence?

2.2 How do these mathematical practices interrelate while middle school mathematics teachers are explaining mathematical ideas without using GeoGebra materials in enacting the instructional sequence?

1.3 Significance of the Study

This dissertation study can provide important points with different aspects. In the first aspect, it provides a picture of a teacher's mathematical practices and the thinking behind these practices. When considering the emphasis of research about mathematics teaching practice (Kilpatrick, Swafford, & Findell, 2001) in classroom environments, design experiments are suggested for understanding teachers' practices and interpretive systems (Lesh et al., 2008; Zawojewski et al., 2008). "The conceptual systems that teachers use for their educational practice can be called interpretive systems for teaching and learning mathematics" (Zawojewski et al., 2008, p. 225). While these systems are complex and not easy to define accurately, they are mostly mathematical. From this point of view, data that comes from teachers' practices in actual classroom teaching and what they do to plan and revise the lessons can have a rich potential to reach the ways of teachers' thinking in mathematical practices. With considering Cobb, Stephan, McClain, and Gravemeijer's (2001) approach which relates the social and the psychological perspectives in design experiments, this study clarifies a teacher's mathematical practices in both terms of the act of individual process, that she reorganizes her mathematical interpretations, and the act of participation in the mathematical practices with a community of students. By the way, as the design researchers suggested (Gravemeijer, Bowers, & Stephan, 2003, Stephan & Akyüz, 2012), these practices were presented with the descriptions of instructional products (activities, learning sequence, tools and technology) with the emphasis on students' learning. That is, the focus of this research study, while most of other studies that were made in this area have focused on students' mathematical practices and students' learning in classroom environment or tutor-learner interaction (Cobb et al., 2001; Moschkovich, 2004), is on teachers' mathematical practices in classroom teaching of slope, linear equations and graphs. Therefore, using the design experiment approach which was primarily decided by teachers' concern on teaching for student's understanding as a setting and designing principles for sessions with teachers in this setting to investigate teachers' mathematical practices, this study could provide both pragmatic and theoretical utility for educational improvement (Cobb & Jackson, 2015). In this sense, since this study considered the teachers' mathematical practices within the consideration of their mathematical knowledge upon the KAT framework as an interpretive framework, it can provide a practical understanding of teachers' content knowledge in classroom teaching.

Another significant aspect of this study is that an instructional sequence was developed to provide a teaching approach for students'' understanding of slope, linear equations and graphs is emerged in the context of middle school mathematics curriculum (MoNE 2009, 2013a) and the conceptual aspects on ratio, rate of change, linearity and graphs are already in the literature. Considering these ideas, the instructional sequence was structured through a transition from ratio in the physical situations to rate in functional situations. Therefore, the trajectory in this instruction has been able to present teacher's and the students' ways of understandings and difficulties in this transition. In a sense, this instruction can be given as an offer to teachers and teacher educators for understanding this teaching approach of slope in classroom and in the curriculum context. While curriculum developers hold the same opinion with researchers that understanding slope is indispensable for handling and solving problems in real life, these ways of understanding slope rarely exist in middle school mathematics classroom. Moreover, there are very few empirical studies reporting teachers' ways of teaching about slope in the middle school classroom setting (Lobato et al., 2003).

In addition, the teacher's instructional sequence and the classroom teaching is emerged in a technology enhanced classroom environment in this study. In a technology enhanced classroom environment, the mathematics teaching can involve the moments of teaching in which tools (i.e. concrete materials and concrete objects) and technology (i.e. Dynamic software) are used and not used. Therefore, mathematics teachers' teaching practices emerge when the tools and technology are used and the tools and technology are not used. In line with this, this study examines teachers' mathematical practices when GeoGebra materials are used and when GeoGebra materials are not used. The reasons why GeoGebra software was chosen to investigate were that teachers easily accessed this open software and they were familiar with using them and the software was appropriate for slope teaching as mentioned above. Furthermore, other middle school mathematics teachers can also create and use similar materials considering the instructional sequence that was developed in this study. Therefore, this study also clarifies the how GeoGebra materials become advantageous or disadvantageous for middle school mathematics teachers' mathematical practices in a classroom teaching. In addition, teachers' each mathematical practice can provide different support for students' learning. In sum, the use of GeoGebra materials in a technology-enhanced classroom environment can provide an additional perspective for teaching slope to be shared with teachers and teacher educators.

What's more significant is that, frameworks that guided this current study assembled the complex parts of teaching in technology enhanced classroom environment. A conceptual framework of Knowledge of Algebra for Teaching (KAT) (McCrory et al., 2012) composed the basis for framing teachers' mathematical practices. The in-depth data of a teacher's mathematical practices showed how the key three practices in KAT framework were combined while a teacher taught in a technology enhanced classroom environment. Furthermore, conceptions of slope (Stump, 1997, 1999, Moore-Russo et al., 2011), covariational reasoning framework (Carlson et al., 2002) and theory of quantitative reasoning (Thompson 1993, 1994b) enlightened the teachers' mathematical thinking under these practices. The conceptions of slope, covariational reasoning framework, and quantitative reasoning supplied a wide-angle lens in interpreting the teachers' practices in her actions and perspectives. This integration may contribute to connecting the frameworks and to develop connections and transitions with the conceptions of slope.

1.4 Definitions of Terms

Middle school mathematics teacher

A middle school mathematics teacher is an in-service teacher that teaches mathematics from grade 5 to 8 in the age group of 10 to 13 in public middle schools (lower secondary education). A middle school mathematics teacher has a bachelor degree from Elementary Mathematics Teacher Education Program in Education Faculties.

Slope

The concept of slope is considered in the context of middle school mathematics curriculum in this study. In the middle grades, the concept of slope is contextualized in physical situations with straight objects and linear situations with lines and line graphs.

In physical situations, slope of an object is a quantity as a result of rate of change of vertical distance relative to horizontal distance of that object/feature (Lobato & Thanheiser, 2002). Slope of an object as a physical situation can be computed by amount of change in vertical distance per unit change in horizontal distance.

In linear situations, slope of a line is a quantity as a result of "rate of change in one quantity relative to the change of another quantity, where the two quantities covary" (Lobato & Thanheiser, 2002, p. 163). This definition is constructed on the assumption that "the behavior of the graphs of a function is invariant" (Zaslavsky et al. 2002, p.138). The constant slope of a line is a result of a linear situation that can be computed through any interval on the line graph. Slope of a line as a linear functional situation can be computed by amount of quantity in y-variable per one unit of quantity in x-variable by operating division. For a linear equation, $= mx + n \; or \; ax + by + c = 0$, slope is m or $-\frac{\boldsymbol{a}}{\boldsymbol{b}}$ as parametric coefficient (Stump, 1999).

Mathematical practices

The mathematical practices are described for the middle school mathematics teachers in classroom mathematics teaching in this study. In a sense, a teacher's mathematical practices involve actions with specific goals and perspectives on particular mathematical ideas that are centralized through the interactions with students.

This study focuses on the mathematical practices in specific to algebra teaching considering KAT framework (McCrory et al., 2012). KAT framework transform the use of mathematics knowledge in teaching to mathematical practices as the moves or actions of teachers (McCrory et al., 2012; Wasserman, 2015). Considering Cobb et al.'s (2001) notion of classroom mathematics practice and McCrory et al.'s (2012) notion of mathematical practice for teaching, a mathematics teacher's mathematical practices in classroom teaching can be defined as ways of actings that have emerged sharing mathematics to reason, argue, and symbolize mathematical ideas in the moments of algebra teaching. Therefore, a teacher's mathematical practices evolve with his/her acts of reasoning on the use of mathematical content knowledge and are structured through the shared mathematical procedures with the students in a classroom community.

As part of KAT framework, mathematical practices are categorized as bridging, trimming, and decompressing. Bridging practices are described as ways of making connections across topics, assignments, representations, and domains (McCrory et al., 2012). In this study, bridging refers to middle school mathematics teachers' mathematical practices of connecting topics, representations, and domains in teaching slope, linear equations, and graphs in eighth-grade mathematics.

Trimming practices are described as ways of removing complexity of mathematical ideas while holding mathematical integrity (McCrory et al., 2012). In this study, trimming refers to middle school mathematics teachers' mathematical practices of removing complexity of slope, linear equations, and graphs while holding the mathematical integrity in both eighth-grade mathematics and advanced mathematics.

Decompressing is described as making explicit complexity of mathematical ideas in ways that make them comprehensible (McCrory et al., 2012). In this study, decompressing refers to middle school mathematics teachers' mathematical practice of unpacking the complexity of slope, linear equations, and graphs in ways that make them comprehensible to eight-grade students.

Technology enhanced classroom environment

Technology-enhanced learning environments are defined in general as "technology-based learning and instruction systems through which students acquire skills or knowledge, usually with the help of teachers or facilitators, learning support tools, and technological resources" (Wang & Hannafin, 2005, p. 5). In this study, when considered a classroom environment as a learning environment, a technology enhanced classroom environment is a classroom setting that involves the technology-enhanced learning and instruction through which students perform mathematical practices with the guidance of teacher, learning support tools, and technological resources. In the classroom environment, teacher and students can use personal computer(s) and computer-based tools. The teacher's computer monitor is projected on the board or on the projector screen. The teacher and the students use dynamic mathematics software (i.e. GeoGebra) for teaching and learning on their computers. In addition, teachers can use additional instructional tools in such an environment in connection with the technology (i.e. concrete materials, objects, etc.). In such an environment, teaching and learning practices are composed of all the moments in which those instructional tools are used and not used.

GeoGebra: GeoGebra (GGB) is a dynamic mathematics software. GGB integrates geometry, algebra, spreadsheet in a connected way and within a dynamic environment. GGB has 2D and 3D graphics views, algebra view, spreadsheet and various tools (Edwards & Jones, 2006). GGB is an open source software that everyone access freely. In this study, GGB is used through pre-prepared GGB materials and activity sheets for these materials in the classroom environment.

1.5 Overview of the Study

The dissertation is organized into five chapters. In chapter 1, there is an introduction into the research area with providing the purpose of the study, research questions and the significance of the study. In addition, definitions of terms is given.

In chapter 2, there is a review of the literature to provide an understanding for investigating teachers' knowledge and practices in teaching and for guiding the design principles. This chapter involves sections on mathematics teachers' knowledge, mathematics teachers' mathematical practices in teaching, knowledge and practices in teaching of algebra, and slope. The slope section is composed of three subsections that involve ideas on slope and conceptions of slope, understandings, misconceptions and difficulties about the slope, and teaching of the slope, linear equations and graphs.

In chapter 3, there is method of the study in line with the research problem. This chapter involves ten sections. These sections are (1) restatement of the purpose and research questions, (2) research design, (3) the context of the study, (4) research process, (5) data analysis, (6) trustworthiness of the study, (7) researcher role, (8) ethical issues, and (9) limitations of the study.

Chapter 4 describes the findings of the study. This chapter involves three sections. In the first section, the teacher's instructional sequence is given. In the second chapter the set of mathematical practices in using GGB materials are given. In the third section, the set of mathematical practices in explaining mathematical ideas without using GGB materials are given.

Chapter 5 gives discussion, conclusion, and implications in six sections. These sections are the teacher' mathematical practices in teaching the slope of an object, the teacher's mathematical practices in teaching slope of a line, the teacher's mathematical practices in teaching the solution of a system of equations and the slope relation, interrelations among the mathematical practices in the moments of teaching, suggestions on.

CHAPTER 2

LITERATURE REVIEW

The aim of this study was to investigate a middle school mathematics teacher's set of mathematical practices in teaching slope, linear equations and graphs for grade eight in technology enhanced classroom environment while enacting an instructional sequence that was designed based on the national curriculum. The review of literature provided an understanding for investigating teachers' knowledge and practices in teaching slope. Therefore, this chapter is organized under four sections: mathematics teachers' knowledge, mathematics teachers' mathematical practices in teaching, knowledge and practices in teaching of algebra, and slope.

2.1 Mathematics Teachers' Knowledge

What should be required of teachers to teach in the lessons and how they accomplish this process was started by Shulman (1986). Shulman provides a theoretical framework in categories for understanding teachers' knowledge. These are categorized under the content knowledge as "subject matter knowledge, pedagogical content knowledge and curricular knowledge" (Shulman, 1986). Pedagogical content knowledge (PCK) differs from subject matter knowledge because it includes knowledge for teaching and PCK includes "the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations- in a word, the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p. 9). Curricular knowledge is another category which is separated into three categories of content knowledge of Shulman. In Shulman's conception, curricular knowledge includes

> the knowledge of alternative curriculum materials for a given subject or topic within a grade,… ability to relate the content of a given course or lesson to topics or issues being simultaneously in other classes [lateral curriculum knowledge], … being familiar with the topics and issues that have been and will be taught in the

same subject area during the preceding and later years in school, and materials that embody them (p. 10).

In Shulman's and his colleagues' later writings the characterization of PCK was expanded and elaborated, especially with the work of case studies of teachers working in different subject areas (Graeber & Tirosh, 2008). In the meantime, Grossman (1990) suggested four central components of PCK: knowledge of students' understanding, the teacher's knowledge of curriculum, strategies, and purposes for teaching. Thus, Grossman (1990) expanded PCK to include aspects of Shulman's category of curricular knowledge. We cannot see the components of PCK as independent from the Grossman and Shulman theories. Correspondingly, knowledge about the purposes for teaching, is an overarching conception of teaching mathematics, and thus serves as a conceptual idea map in decision making, in enactment of teaching, a base for deciding instructional strategies and evaluation of teaching objectives, textbooks, and curricular materials (Borko & Putnam, 1995).

In the last two decades, the description of knowledge for teachers has moved beyond just teacher content knowledge and for this reason, the domain of PCK has changed along with the more recent version of PCK as *mathematics knowledge for teaching* (MKT) that is regarded now as acceptable and the most up-to-date source of mathematical knowledge (Ball & Bass, 2000, 2003). The MKT is considered a specific mathematical knowledge basis for teaching and is different from the knowledge of other professions as research, mathematics or engineering (Ball & Bass, 2000). Knowing teaching is not only applying prior understanding but also being able in all and challenging situations (Lampert & Ball, 1999).

Hill et al. (2008) show forms of MKT with a domain map as a model. Mathematical teaching knowledge is composed of two components which are subject matter knowledge with three sub-domains of "common content knowledge (CCK), specialized content knowledge (SCK) and horizon content knowledge (HCK)", and the other component is pedagogical content knowledge with three sub-domains containing "knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of content and curriculum (KCC)" (Hill et al., 2008, p. 377). The relation between subject matter knowledge and pedagogical content knowledge can be seen in Figure 1.

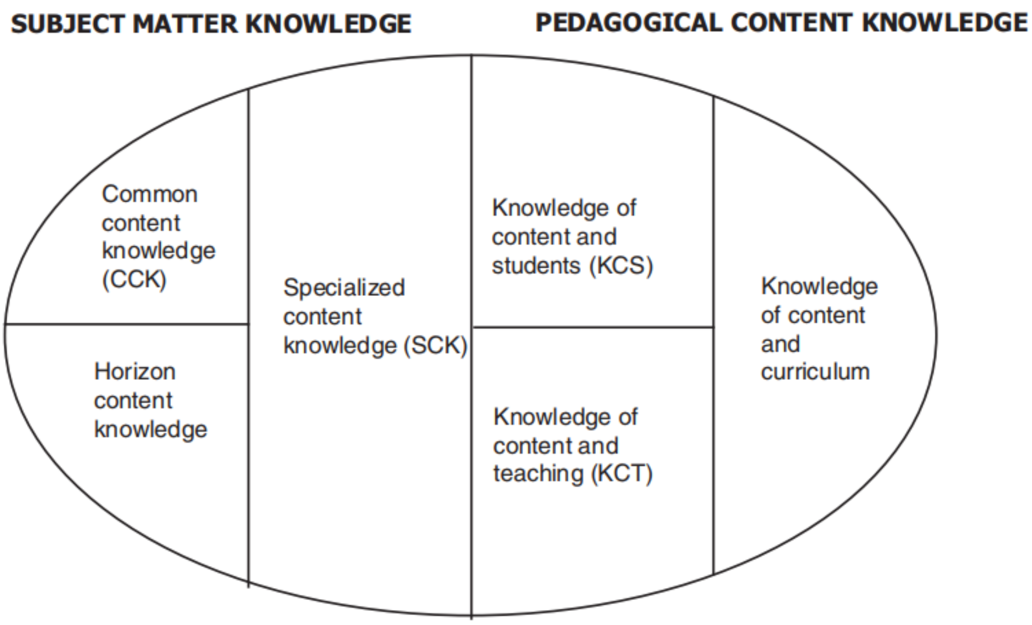

Figure 1. Domains of mathematical knowledge for teaching.

Reprinted from "Content knowledge for teaching: What makes it special?," by D. L. Ball, M. H. Thames, and G. Phelps, 2008, *Journal of Teacher Education*, *59*(5), p. 403. Copyright [2008] by Sage Publications.

In the components of subject matter knowledge (SMK), while CCK is mathematical knowledge that is common or base knowledge for other professions, SCK is a unique knowledge in the work of teaching for particular tasks (Hill et al, 2008). For example, teachers with CCK can solve linear equations, recognize wrong answers in solving procedures, and use algebraic notation correctly. In addition, teachers with SCK can understand different interpretations of mathematical ideas (e.g. equations), explain mathematical ideas (e.g. why the sign of a number/unknown changes when it goes to the other side of the equation), and use mathematical representations (e.g. connect multiple representations of linear equations). One important feature of knowing mathematics *for teaching* is that "mathematical knowledge needs to be *unpacked*" (Ball & Bass, 2003, p. 11). That is, it requires not to compress the mathematical content but to decompress one's own mathematical knowledge uncovering its elements in composition of a mathematical idea. This feature indicates the importance of SCK. HCK, which is not centered in SMK, is of knowledge of the mathematical horizon related with mathematical topics in the curriculum (Hill et al., 2008). This can be considered as the understanding of the school mathematics and advanced mathematics in a coherent manner. Thus, these categories can be seen as the distinction between "knowing how to do mathematics" and "knowing mathematics in ways that enables its use in teaching practices" are key to knowledge of mathematics for teaching (Ball, 2000). This indicates the difference between being a mathematics teacher and being a mathematician.

In the pedagogical content knowledge (PCK) component, knowledge of mathematics teaching, KCS is defined as the knowledge of students' mathematical knowledge which involves knowing what tasks students will find confusing, interesting, easy or hard. In KCS, it is also necessary to know about mathematics and how to remedy their student's errors. These tasks "requires an interaction between specific mathematical understanding and familiarity with students and their mathematical thinking" (Ball et al. 2008, p. 401). In addition, KCT is knowledge of instructional design in mathematics (Hill, Sleep, Lewis, & Ball, 2007). For example, teachers with KCT are able to decide which example is the best to solve first, when a multi-faceted example or question is given, and how to use appropriate representation for the instruction. Lastly, KCC, which is not a main focus, entails familiarity of curriculum materials for teaching and being aware of the curriculum guidelines.

In similar vein, based on Shulman's (1986) conceptualization of SMK and PCK, Rowland et al. (2009) developed a framework for primary mathematics teachers' content knowledge in classroom teaching namely Knowledge Quartet. They explained the teachers' performance using four units/dimensions of Knowledge Quartet focusing on mathematical aspects of mathematics lessons. That is, this framework as a study of teaching, involves units of foundation, transformation, connection and contingency with sub-categories under each unit. Foundation includes teachers' background and beliefs about mathematics, and mathematics pedagogy. Transformation involves teachers' decisions and actings on transforming a mathematical idea to present and to make students access their mathematical ideas. Connection refers to the sequence of episode(s) and lesson(s) in a coherent way and connectivity of concepts, ideas, topics, strategies. Contingency refers to teachers being engaged in a repartee (response) appropriately with students' unexpected actions, such as comments, questions, and answers. Rowland et al. (2009) asserted that these headings in the framework can be used as a tool for understanding teachers' mathematical content knowledge in teaching.

Teachers' mathematical knowledge in teaching can be categorized under different dimensions. There may be other components of mathematics teachers' mathematical knowledge which are not mentioned in the aforementioned frameworks. Therefore, the process of aggregating and combining continues since more studies are needed to formalize the theory of teachers' knowledge base for different learning areas in mathematics. On the other hand, while examining teachers' activity in teaching, one of the indispensable construct is teacher practice (Ponte & Chapman, 2006). Teacher practice has a wide scope from classroom to other professional settings. While theories on teacher knowledge enlighten the development of mathematic teaching, investigating practices in teaching is needed to realm of mathematics teaching. In the current study, we focused on mathematical practices and in the act of teaching itself.

2.2 Mathematics Teachers' Mathematical Practices in Teaching

In classroom mathematics teaching, classroom practice can be defined as a complement of features of discourse, norms, and building relationship (Franke, Kazemi, & Battey, 2007). Specifically, routines of practice are seen as the core idea of the classroom practice that should be considered for each feature within different mathematical and social contexts (Franke et al., 2007). Therefore, the evolvement of set of practices in any feature of classroom practice may be vary depending on the perspectives of constituent of the educational context (i.e. student learning, teaching, social and cultural context, etc.). Since the focus of this current study on teachers, understanding the classroom practice from the perspective of teachers comes to the forefront. Teaching is a complex issue that requires various qualifications both in planning practice, classroom practice and reflecting practice. In general, teaching is described through the process of decision making that develops in the interactions between teachers' knowledge, goals, and beliefs (Schoenfeld, 1998, 2000). In addition, similar to my perspective, in a classroom teaching, a teacher's practices and students' practice emerge in interactive ways through the discourse and the mathematical activities (Wood, 1995).

Teaching practices and mathematics teaching practices (mathematical practices) are also described through the teachers' actions and moves in classroom teaching (Lampert, 2004; Stein, Engle, Hughes, & Smith, 2008; Wasserman, 2015). These actions and moves are considered as the translations of teachers' content knowledge (Wasserman, 2015). In this regard, this study considered the actions as being mathematical in nature and that is a kind of mathematical movement depending mostly on teachers' mathematical knowledge. Therefore, this current study focused on teachers' mathematical practices considering how teachers participate in the practice of classroom community and how teachers establish mathematical practices as they apply their mathematical knowledge in the moments of teaching.

The notion of mathematical practices is discussed and developed through different perspectives in the literature (Cobb, Wood, & Yackel, 1993; Lampert, 1986; Lave & Wenger, 1991; Moschkovich, 2013; Schoenfeld, 1992). When considered these studies, it is seen that in some way mathematical practice of a mathematician and mathematical practice in classroom compared. In addition, the connections between mathematical practices and mathematical activity is questioned. These studies explained students' mathematical practices in the mathematical activities of classroom teaching in a communal.

Mathematical practices were explained both in classroom environment and in an individual tutoring. Cobb et al. (2001) introduced the notion of "classroom mathematical practice" (p. 117) from a social perspective for students' learning. They explained classroom mathematical practices as acting of "taken-as-shared ways of reasoning, arguing, and symbolizing established while discussing particular mathematical ideas" (Cobb et al., 2001, p. 126). They explained students' mathematical practices on taken-as-shared purpose, their ways of reasoning and forms of mathematical argumentation. Similarly, Moschkovich (2004) utilized Vygotskian perspective to explain a learner's practices with a tutor on the mathematical practices for functions. Therefore, when considered what mathematical practices are and what they involve, Moschkovich (2004) summarized this notion:

> Mathematical practices are social and cultural, because they arise from communities and mark membership in communities. They are cognitive, because they involve thinking and semiotic, because they involve signs, tools, and meanings. Mathematical practices involve values, points of view, and implicit knowledge. (p.55)

For example, in Moschkovich’s (2004) study there were two mathematical practices for functions that were “treat functions as objects and connecting lines to their equations” (p. 56). By the way, she explained the mathematical practices of a student on the aspects of “actions and goals (e.g. connecting equations and lines) *and* perspectives and shifts between perspectives” (p. 57). For example, the student used comparing positive and negative slopes as a goal to explore the signs of in the equation of $y = mx + b$ while the student and tutor exploring the parameters.

Teachers’ classroom teaching practices is also described with the notions of instrumental orchestration and instrumental genesis. Instrumental genesis explains the complexity of behaviors in using tools and technology (object of artefact as an instrument) in a classroom. (Artigue, 2002). Orchestration refers to teachers' techniques of enhancing their students to be able to use instruments in individual or communal mathematical situations in the classroom. After Trouche (2004) introduced the notion of instrumental orchestration as a metaphor for the teaching in a learning environment (e.g. a classroom) to arrange students’ instrumental genesis, Drijvers and Trouche (2008), Drijvers et al., (2010), and Drijvers (2012) described the elements of instrumental orchestration into three components. These components are “didactical configuration, exploitation mode and didactical performance” (Drijvers et al., 2010, p.215). In general, the notions of didactical configuration, which is a configuration of a teaching setting and its artefacts, and exploitation mode, which is the way the teacher decide to execute a didactical configuration, were used to explain teaching practices in an observed learning environment. Besides, the component of the didactical performance refers to the teachers’ decisions that are made during the actual teaching performance in the chosen didactic configuration and exploitation mode. Based on these components various orchestration types emerged (Drijvers et al, 2010; Drijvers, et al., 2013). For example, while using a dynamic mathematics tool, the *technical-demo* orchestration involves enabling teachers to “project the computer screen” as a didactical configuration and “demonstrating technique in a new situation or task” (e.g. drawing a graph) (Drijvers et al., 2010). When considered specifically mathematical aspects of teaching, teachers’ mathematical practices can be explained through the notion of didactical performance. However, studies did not expand on this dimension (Drijvers et al., 2010; Drijvers et al., 2013; Tabach, 2011). For example, Drijvers et al.

(2013) explained 8^{th} grade mathematics teachers' teaching practices through the orchestrations in general without focusing on the didactical performance. Besides, they combined the instrumental orchestration and TPACK model to identify the type of skills and knowledge needed to put the teachers' orchestration in a computerized learning environment. This study showed that a way of investigating teaching practices and knowledge can be thought in an integrated way.

In the subsequent studies, the examples are given of their findings on teachers' mathematical practices. These studies were conducted from different perspectives. Sacristán, Sandoval, and Gil's (2007) study combined practices and reflections. Bozkurt and Ruthven's (2017) study combined classroom practice and craft knowledge. Li's (2011) study combined mathematical knowledge for teaching and pedagogical routines for mathematical practices.

Sacristán et al. (2007) conducted a research study with in-service mathematics teachers from primary, middle and college ages with their reflections about changes in their practice that incorporated digital technologies (i.e. spreadsheets, Dynamic geometry, Logo and CAS) in the classrooms. They showed that teachers usually used technology to show algorithmic memorizations and textbooks or sheets digitally but not to make explorations with mathematical activities. Teachers did not have a particular aim to use technology through a plan and they use superficially just for giving an activity to students. Additionally, it was seen that teachers who did not trust their own mathematical knowledge, like conceptual notions, felt incompetent to use digital technology. Therefore, we could not gather rich mathematical practices from this study.

Li (2011) investigated a mathematics teacher's mathematical practices in solving quadratic equations in grade 9 for three lessons. The emerged mathematical practices were categorized under the themes of representations (i.e. symbolic, verbal, rhythmic, and metaphorical), exemplifications, derivation, justification, connections, applications, and comparisons. While there was a sequence between some practices, the teacher gave examples after the representations and the teacher guided students to make justification after derivation of solving and transforming the equations, she made connections on equations throughout the lessons. In addition, the teacher made comparisons after have given four methods of solving quadratic equations.

Bozkurt and Ruthven (2017) conducted a case study with an expert secondary mathematics teacher that used GeoGebra (GGB) in teaching transformations and circle theorems. They investigated the teacher's (i.e. Chris) classroom practice and craft knowledge (Ruthven, 2009) in his teaching. They found that Chris used GGB to make students explore and evaluate mathematical ideas both given by the teacher and developed by the students, themselves within multiple approaches. He guided students to "investigate and prove conjectures that lead to different circle theorems" (p. 16) and assisted students to lead themselves to act and develop their higher-order reasoning. While doing those, Chris used the technique of spot-and-show that involves showing students' work to make discussion as a whole class. He also devoted his teaching structure to enable students to work independently by predicting and testing with GGB. According to the teacher as a reflection, what makes it distinctive to use GGB in teaching is that it provides an additional interactive way for students to connect to math as well as between the student and teacher interaction to grow and develop.

As a consequence, a teacher's mathematical practices in classroom teaching are limited to teaching a particular mathematical topic. In this regard, there is a need to understand teachers' mathematical practices in specific mathematical domains. Therefore, we will dig into literature on teachers' knowledge and practices in specific regards to teaching of algebra for the topics specifically of slope, linear equations, and graphs.

2.3 Knowledge and Practices in Teaching of Algebra

While investigating teachers' knowledge and practices in teaching of algebra, a good discrimination of dilemmas about the teachers' knowledge and practice were discussed by Doerr (2004). There were two dilemmas that are related to my research. One is that the difficulty in communication of the teachers' knowledge. The word knowledge as a noun seems to be static. However, teachers' knowing as a verb is more preferable. This form of articulation makes it understood that "the nature of what it is that teachers need to know is dynamic, fluid, situational, reflected in action, and situated in specific cultures and in specific social settings within those cultures"

(Doerr, 2004, p. 270). Therefore, ‘teachers’ knowledge’ is written in a dynamic meaning of ‘teachers’ knowing’ if the language limitations exist. In this context, it was mentioned that teachers’ knowing has a focus on teachers’ learning and teachers’ reasoning in context (Doerr, 2004). For that reason, how teachers reason about that a particular instance and how teachers learn from such instances are engaging issues for the researchers studying teaching. That is, characterizations of teachers’ development of thinking over time and across settings is an important need for teachers of algebra (Doerr, 2004). Another dilemma is the difficulty of situating claims to be made within the larger body of research on teacher development in articulating the nature and development of teachers’ awareness for teaching of algebra. Therefore, which approach is internalized by teachers for teaching algebra is important to criticize teachers’ mathematical content knowledge. In the following I tried to review the perspectives both in knowledge and practices of teachers in specific to algebra teaching.

To organize knowledge for teaching algebra for measuring teachers’ competence, Artigue and her colleagues proposed a three dimensional framework including epistemological, cognitive, and didactic/pedagogical dimensions, which are interrelated dimensions of knowing (Artigue, Assude, Grugeon, & Lenfant, 2001). This framework is useful for researchers to “organize the descriptions of the knowledge for teaching algebra and to suggest research perspectives for investigating that knowledge” (Doerr, 2004, p. 284). The epistemological dimension involves knowing as follows, “the content of algebra, the structure of algebra, the role and place of algebra within mathematics, the nature of valuable algebra tasks for learners, and the connections between algebra and other areas of mathematics and to physical phenomena” (Artigue et al., 2001). This dimension specifies the subject matter knowledge for the teaching of algebra. Cognitive dimension, separately from the epistemological dimension, describes the importance of knowing the subject matter in a way that can lead to effective teaching of that subject matter (Doerr, 2004). The cognitive dimension includes knowing: “the development of students’ algebraic thinking, interpretations of algebraic concepts and notation, misconceptions and difficulties in algebra (often referred to as epistemological and cognitive obstacles), different approaches taken by learners, ways to motivate learners, and theories of learning". The didactic/pedagogical dimension includes knowing: the curriculum (including the dynamic interrelations between the mathematical content, the specific teaching goals, the teaching methods or strategies, and the assessment practices), the resources (textbooks, technology, manipulatives, and other curriculum materials), different instructional representations, varied practices and approaches taken by other teachers, the connections across the grade levels, and the nature and development of effective classroom discourse. Whilst knowledge of content, teaching and knowledge of curriculum are sub-dimensions of pedagogical content knowledge in Hill et al.'s (2008) model, Artigue et al. (2001) emphasizes that curricular knowledge includes the knowledge of teaching methods and the relationship between those teaching methods and the mathematical content of algebra.

What is crucial here is that teachers' have a shared knowledge, i.e. "teachers' knowing the practices and approaches taken by other teachers" (Doerr, 2004, p. 282). Teachers build professional knowledge beyond the particular knowledge of one teacher (Doerr, 2004). At one point, while having an education in mathematics and being able to do mathematics is seen as socially and culturally situated activity (Schoenfeld, 1998, 2000), teaching mathematics cannot be thought as an individual activity. Most researchers see teachers knowledge as craft knowledge (Brown & McIntyre, 1993; Ruthven, 2009) that provides tools for identifying and analyzing types of features of work of teaching in a classroom (Drijvers et al, 2010; Ruthven, 2009). Therefore, teaching mathematics as an activity requires an "expertise rather than knowledge" (Ruthven, 2014, p. 390) and mathematics teaching practices are the consequences of this type of expertise in the classroom (Hodgen, 2011). In this sense, we focused on the expertise dimension of teaching in specific to the algebra area, this urged me to adopt a framework on the use of teachers' content knowledge as teachers' mathematical practices in algebra teaching.

There is also one more framework for algebra teaching, namely knowledge of algebra for teaching (KAT) (McCrory et al., 2012) that involves dimensions of both practices and knowledge. Thus, this current study adopted this framework to understand teachers' practices in classroom teaching (McCrory et al., 2012, p.585). Ferrini-Mundy et al. (2005) carried out a project called "Knowledge of Algebra for Teaching" as a counterpart of Ball and colleagues' framework. They seek to outline a conceptual framework for the knowledge for teaching algebra when considering the importance of algebra teaching for middle schools and secondary schools (Figure 2). When in development, this framework Ferrini-Mundy et al. (2005) involved three dimensions, namely *overarching categories* as mathematical practices, which are decompressing, trimming, and bridging, *tasks of teaching*, where teachers apply their mathematical knowledge, and *categories of knowledge*, which are involved in teaching algebra. These categories, tasks, and practices were not written in any specific meaningful order. Since they were focused on assessment of teacher's knowledge, they did not consider the specific tasks of teaching in further stages of the framework. Therefore, in the following of stage of framework, McCrory et al. (2012) re-approximated the categories in two dimensions, explained in the following pages. One is mathematical content knowledge that involves categories of "school knowledge, advanced knowledge, and teaching knowledge" (McCrory et al., 2012, p. 595). Another one is the teaching practices that involves categories of "trimming, bridging, and decompressing" (McCrory et al., 2012, p. 595).

In the dimension of mathematical content knowledge of the KAT framework, school knowledge for primary and secondary algebra involves the content of algebra area that is taught in curriculum and textbooks. Advanced knowledge involves the content of algebra at the university level that is broader and deeper than primary and secondary school algebra. The mathematics-for-teaching knowledge or briefly teaching knowledge involves the content of mathematics classes for teachers which is useful for teaching and the content has already been taught in their teaching practice. Beyond the purely mathematical knowledge, teaching knowledge involves many ways of thinking about mathematics that are based on the mathematical ideas in teachers' applications and ways of interpreting students' mathematical language (McCrory et al., 2012). According to McCrory et al., this category of teaching practice knowledge overlaps with the specialized content knowledge category in Ball et al.'s (2008) framework.

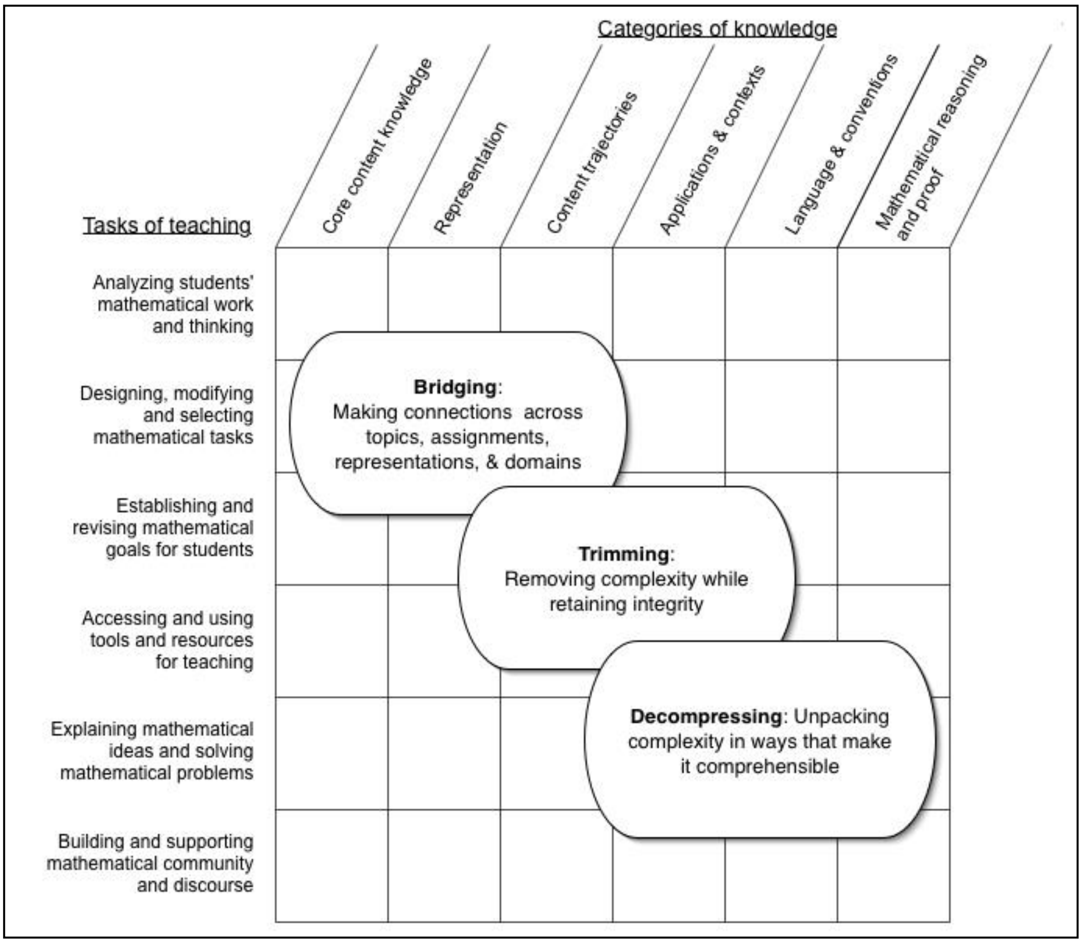

Figure 2. A conceptual framework characterizing knowledge for teaching school algebra.

Retrieved from http://www.educ.msu.edu/kat/Figure1.doc, by J. Ferrini Mundy, R. Floden, R. McCrory, "Knowledge for Algebra Teaching Framework."

In the dimension of teaching practices of the KAT framework, McCrory et al. (2012) categorized teachers' use of mathematical knowledge in the tasks of algebra teaching. These "key practices of algebra teaching" are bridging, trimming, and decompressing (McCrory et al., 2012, p. 608). In the KAT framework, bridging means to connect and link "mathematics across topics, courses, concepts, and goals, linking one area of school mathematics to another" (McCrory et. al., 2012, p. 606). In other words, bridging includes different kinds of junctures between students' understanding and teachers' goals. Some of the teachers' goals are for their students to connect ideas of school algebra, and those of abstract algebra, and one area of school mathematics and another area of it. Besides, bridging involves making connections between representations (e.g. graphs and verbal expressions), subjects such as linking mathematics to science, geography or other contents, and making connections between primary and secondary school mathematics and advanced mathematics. For example, a teacher makes a connection between a table and an expression, or makes a connection across the meanings of mathematical language by changing the instructional terms.

In the KAT framework, as another practice in the category of mathematical practices, trimming practice entails that teacher makes mathematics accessible to all students by removing the complexity while preserving the continuity of mathematical integrity (McCrory et. al., 2012). According to McCrory et al (2012), this notion corresponds to the terms of "intellectually honest" teaching (Bruner, 1960), shortcutting as a component of horizontal mathematisation (Treffers & Goffre, 1985), and "recontextualization and repersonalization" of teaching as a component of didactical transposition (Brousseau, 2002, p. 23). Trimming means to transform mathematical ideas from a more advanced or rigorous form to more understandable form but at the same time preserving the essence as well as considering students' backgrounds and knowledge. The question of what explanation is appropriate, accurate or complete in this learning context needs to be thought about. Also it needs to be taken into consideration of how a teacher trims a mathematical idea for her mathematics instruction. Therefore, trimming can involve arranging the instruction by omitting or adding details in a clear and simple form of a mathematical idea. Trimming requires having an understanding of a key content in a topic with the underlying mathematical ideas. For instance, use of examples to illustrate a particular procedure in both a simple and an honest way corresponds to trimming.

As the last category of mathematical practices, when described briefly, decompressing includes the unpacking of teachers' own knowledge in the practice of teaching and in the design of lessons and tasks. Decompressing practice entails a teacher unpacking the underlying complex meaning in the concepts, words, mathematical instructions and procedures to make them comprehensible. Ball and Bass (2000) and Cohen (2004) explained this situation within the category of unpacking that parallels decompressing (McCrory et al, 2012). Unlike to compressing or constructing, decompressing involves unpacking computational algorithms used in arithmetic operations, unpacking algorithms for solving equations and systems of equations, for simplifying expressions, for moving among representations, unpacking the meaning of identities, equations, helping students grasp the logic of the procedures, attaching fundamental meaning of symbols and algorithms, presenting a mathematical difficulty in a way that addresses student thinking (McCrory et. al., 2012). For example, a mathematics teacher could decompress what a word means in a given mathematical context.

The KAT framework is particularly prepared for measuring middle and high school mathematics teachers' knowledge and effectiveness in teaching. In the literature, Reckase, McCrory, Floden, Ferrini-Mundy, and Senk (2015) used the framework to develop a test for investigating mathematical knowledge of pre-service and in-service teachers. On the other hand, it is used by Shimuzu (2013) to examine mathematics teachers' work to make students understand mathematical ideas in algebra. While making investigations on seven middle and high school mathematics teachers, she observed a few lessons in different topics in algebra.

In this sense, the current study adopted the KAT framework for understanding teachers' mathematical practices in teaching slope, linear, equations, and graphs in eighth grade technology enhanced classroom environment. The concept of the 'slope' is one of the most prominent terms in primary and secondary school algebra content. What makes it important to learn and teach slope, linear equations and graphs in middle school algebra is that it provides a basis for students while acquiring proportional reasoning, linearity, quantitative reasoning, and covariational reasoning. Meanwhile, it appears throughout the middle and high school mathematical curriculum for primary and sometimes secondary goals (MoNE, 2009, 2013a, 2013b). For that reason, there is a need of a section for the concept of slope under the linear equations and graphs context. Reviewing of the literature, which will be given in subsequent section, on what the conceptions of slope are, why those conceptions emerges, how students and teachers misunderstand and understand this concept and how these notions are taught and guided. This is the current research in both understanding of the teacher's teaching, analyzing their mathematical practices, and interpreting their findings.

2.4 Slope

This section is composed of three subsections that involve ideas on: (1) the concept of slope and conceptions of slope, the importance of slope with related concepts, (2) understandings, misconceptions and difficulties about the slope, and (3) teaching of the slope, linear equations and graphs. To examine teachers' mathematical practices in teaching the slope we need the mathematical underpinnings of the slope teaching, which requires epistemological basis as mentioned in section 2.4.1, conceptual basis as mentioned in section 2.4.2, and practical basis in teaching as mentioned in section 2.4.3.

2.4.1 Slope and Conceptions of the Slope

While describing the conceptions of the slope concept, basic terms that are used are quantity, extensive, intensive, rate and ratio. Thompson (1994b) explained quantity as a conceptual entity and schematic with four characteristics. These are "it is composed of an object, a quality of the object, an appropriate unit or dimension, and a process by which to assign a numerical value to the quality" (Thompson, 1994b, p. 184). For example, a train is an object, when a train moves it has a motion. The unit of speed of the train can be kilometers per hour. The motion of the train provides a measurement for its speed and its measurement gives it a numerical value. As another example, a tree is an object and has a height. The appropriate unit of height of the tree can be in meters or in inches. When it is measured it has a numerical value. There are two types of quantities (i.e. extensive and intensive) that are classified according to measurability (Schwartz, 1988). An extensive quantity can be counted or measured directly, like instances, distance, temperature, height, weight, etc. Intensive quantity can be measured indirectly as a ratio that is derived with extensive quantities, like speed, price, or the sweetness of a mixture. In addition, intensive quantities are categorized as an entity and as parts that show the quantity in a certain rate with a ready word (e.g. price-the cost per unit, speed-miles per hour) and comprise a ratio without a ready word (e.g., amount of sugar to amount of mixture, women's earnings to men's earnings), respectively (Lobato & Thanheiser, 1999).

The concepts of ratio and rate are described and used in a changing form in various sources. In this study, I considered the following distinctive characteristics of these terms and with Thompson (1994b)'s viewpoint. As a perspective used in previous studies, ratio and rate both means a comparison between quantities, however, these quantities differ in nature. That is, quantities of one are like nature (same attribute) and the others are unlike nature (different attributes) (Vergnaud, 1983). While a ratio is a comparison between quantities in same attributes, rate is a comparison of quantities in different attributes (Vergnaud, 1983). As triggering definition of the current perspective, while a ratio is a binary relation which involves ordered pairs of quantities (relationship between them) (Lesh et al., 1988), a rate is a single quantity that shows a relationship between two quantities which one is a unit quantity (Lesh et al., 1988). In other words, rate can be considered as an extended ratio (Lamon, 2006). That is, while a ratio is for an exact situation, rate is for a range of situations that involves the relation between two quantities in the same way (Lamon, 2006). From the viewpoint of Thompson (1994a, 1994b), he made distinctions between these concepts through a person's mental operations while conceiving rate and ratio situations. That is, for Thompson (1994b), how one comprehends the quantitative operations in a situation influences the ways of conceiving the situation. Then, how one comprehends a situation influences his/her classification of the situation. Therefore, Thompson (1994b) defined ratio as the "result of comparing two quantities multiplicatively" regardless of the units of the quantities (p.190) and rate as "a reflectively abstracted constant ratio" (p. 192). That is, it depends on the way of people's conceiving multiplicative comparison. While ratio is static, independent and applied just to a situation, rate is applied to generally outside of the originally conceived bounded situation. Rate can be defined as a result of comparison of two quantities multiplicatively that vary in a relation (Thompson, 1994b).

Lamon's (2006) example for rate context can show this situation. The question is "the soccer team has a ratio of 3 wins to 2 losses so far in the season. At this rate, what will be their record at the end of the 15-game season?" (Lamon, 2006, p. 193). The ratio is 3 wins to 2 losses that applies just to this situation. However, the statement of winning and losing at this rate allows it to extend it to 15-games total that makes the question if there is 3 wins per 5 games, how many wins will be per 15 games. Besides, this rate helps us for finding the number of wins at the end of 10 and 15 games not for 6, 7, 8, 9, 11, 12, and 13 games. From this point of view, the concept of slope can be explained using both ratio and rate concepts in different situations.

As one of the conceptions, slope is conceptualized in the context of physical objects and features. In measuring the steepness of a physical object in a physical situation, Lobato and Thanheiser (2002) conceptualized slope of the object as "the rate of change of the vertical distance relative to horizontal distance" (p.163). The slope of the object is calculated using an interval and the horizontal and vertical components for this interval as the ratio of vertical distance to the horizontal distance. For instance, a ramp is a physical object. The slope of a ramp is calculated as a ratio of vertical distance to horizontal distance for an interval of the object. This interval can be the whole of the ramp or any part of the ramp. What makes this choice of interval optional is that the ramp has a constant slope because of the ramp linearity. Therefore, the slope is also expressed as a measure of steepness or slantiness of the object (Lobato & Thanheiser, 2002, Stump, 2001a).

In the Cartesian plane, there are various forms of conceptions for slope. Walter and Gerson (2007) defined it as "a slope between two points is often conceptualized as the ratio of the change in y-coordinates to the change in the associated x-coordinates" (p. 204). Van de Walle, Karp, and Bay-Williams (2013) described the slope under the part-to-part type of a ratio as "In algebra, the slope of a line is a ratio of rise for each unit of horizontal distance (called run)." (p. 358). On the other hand, as another example in the Cartesian plane, the slope of a line is conceptualized as a rate of change in one quantity relative to the change of another quantity, where the two quantities covary (Lobato & Thanheiser, 2002). This conception is meaningful when the axes represent measured quantities that are involved in the relationship in an aspect of situation or object. For instance, as a linear functional situation in a distance-time graph, slope of the line shows the velocity as a measure of the object's motion. In addition, slope is calculated with using a pair of points on the line as the ratio of the change in-y coordinates to the change in the associated x-coordinates. This calculation needs reasoning on constant rate of change for a line. While a straight line is a graphical representation of the linear equation $ax + by + c = 0$ or $y = mx + b$, the equation is an algebraic representation of the straight line. Therefore, the slope is calculated algebraically through the algebraic expressions $\frac{\Delta y}{\Delta x}=\frac{y2-y1}{x2-x1}$ on the line or parametric coefficient of the equation (m=$-\frac{a}{b}$).

Although Stump (1999) explained slope of a line is a measure of its steepness, Lobato and Thanheiser (2002) questioned on the necessity of measuring the steepness of a line. One of the reasons of this questioning is that a line is not a physical object. Therefore, it does not seem meaningful to calculate the steepness of a line. A second reason is that, while steepness of two lines in different coordinate systems looks different, their calculations of slopes can be equal since the ratios of axes (scaling) in the coordinate systems can be different for each coordinate system. Therefore, the look for steepness can mislead the interpretation of slope that is why the line is considered as "an infinite collection of ordered pairs of values of covarying quantities" (Lobato & Thanheiser, 2002, p.164). Thus, the slope of a line can be interpreted through the rate of change of the covarying quantities.

As a trigonometric conception, the slope of a line is also described as an angle between the line and the x-axis (Stump, 2009). This angle, which is called the angle of inclination of the line, gives the slope. In addition, the slope of a physical object is also considered as a tangent of the inclination angle. The trigonometric ratio of the tangent angle is also used in the similarity of triangles. As a calculus conception, the slope is considered with a limit and derivative concepts. In this consideration, the importance of understanding a slope is as an average rate of change and instantaneous rate of change emerges with various forms of graphs for functional situations (Stump, 2009; Kertil, 2014).

The importance of the slope concept

The concept of slope connects algebra and geometry with using symbolic and graphical representations since the concept of slope is related with various concepts. These concepts are ratio, rate of change, covariation, lines, graphs, similarity, division and fractions. Primarily, slope is connected to the ratio concept since proportional thinking is at the core of the slope concept. Proportional situations involve a multiplicative relationship rather than an additive relationship and ratio as a number expresses multiplicative relationship (part-part and part-whole).

Secondarily, slope is connected to the rate that represents an infinite set of ratios (Lobato & Ellis, 2010). Specifically, slope is connected to rate of change that is vital in algebra. While connecting the concepts of slope and rate, covariation is the focus of this relation. Specifically, “Covariation means that two quantities vary together” (Van de Walle et al., 2013, p. 362). That is, covarying quantities involve simultaneous and interdependent changes (Johnson, 2012). The connection between slope and covariation is seen within the ratio of two quantities (measures) in the same setting. When considered, covariational thinking involves “coordinating two varying quantities while attending to the ways in which they change in relation to each other” (Carlson et al., 2002, p. 355), it is seen that the level of thinking of rate designates a student’s covariational reasoning abilities. Students’ covariational reasoning abilities are categorized developmentally by specifying the level of the students’ behaviors (Carlson et al., 2002). This categorization is valid for all the ways that graphs can change (i.e. linear and non-linear changes). Therefore, interpreting a linear graph requires “the coordination of the relative magnitudes of change in the x and y variables” (Carlson, et al., 2002) in the context of a graph.

Slope is connected to linear situations that can be either proportional or not proportional. Proportional situations are linear situations that are represented by the lines that go through the origin. For example, for the equation of the line $y = mx$ or $ax + by = 0$, the slope is m $(-\frac{a}{b})$ that represents one of the equivalent ratios that falls along the straight line (Lamon, 2006). In proportional situations it is called as constant of proportionality. The slope is constant which shows the invariant relationship between quantities that vary together. On the other hand, the lines that represent linear situations as non-proportional also have a constant slope. For example, the slope is m $(-\frac{a}{b})$ for the equation of the line $y = mx + c$ or $ax + by + c = 0$. Therefore, the slope is related with the lines and graphs. In other words, “A graph is the picture of the rate of change of one variable in terms of the other.” (Van de Walle et al., 2013, p. 278). However, in linear graphs, the slope is limited with the constant rate. Besides, in nonlinear graphs, the slope is interpreted through a varying rate of change with instantaneous rate of change. In sum, from this point of view, we can see that the slope has a multidirectional relation with those concepts above rather than a one-way relationship.

In geometry, slope of line is also connected with the similarity of geometric shapes. These connections can be presented by the similarity ratio. Van de Walle et al. (2013) explained this connection by exemplifying proportional rectangles on the coordinate plane with the common corner at the origin. When a line is drew by joining the corner of the rectangles, the slope of this line gives the ratio of the sides of rectangles.

When considered the advanced mathematics, the slope concept is connected to derivative, rate of change of functions, and gradient notions. The connections between these concepts were also investigated on calculus students' understanding (Byerley, Hatfield, & Thompson, 2012). These connections will become indispensable to interpret non-linear graph of a function with understanding its instantaneous rate of change (Carlson, et al., 2002). Although these studies are out of scope for this study, findings on students' understanding can guide to interpret teachers' subject matter knowledge of slope and linear equations. In addition, the literature on misconceptions and difficulties about slope and the underlying reasons of these issues give an insight to emerge the critical mathematical ideas (concepts, skills, procedures, generalizations) that is needed and for bridging, trimming, and decompressing practices of teachers.

2.4.2 Understandings and Difficulties about the Slope

In research, the slope related studies interconnected with three mathematics education areas, that are functions and graphs, proportional and algebraic thinking (Lobato & Thanheiser, 1999), and covariational reasoning (Carlson et al., 2002).

The literature indicated that students, preservice and in-service teachers conceptualized slope from various aspects. Therefore, the research on conceptualizations of slope developed through the studies on students' conceptions, (Stump, 2001b) based on preservice and in-service secondary mathematics teachers' conceptions (Stump, 1997, 1999, 2001a), secondary mathematics education graduate students (Moore-Russo, et al., 2011), middle school mathematics education undergraduate students from first year to fourth year (Dündar, 2015); and on documents of mathematics standards (Stanton & Moore-Russo, 2012). Among these studies, only Moore-Russo, et al. (2011) investigated conceptions of slope in three dimensions. These conceptions of the slope in two dimensions and three dimensions are categorized under eleven titles as explained in the following Table 1.

Table 1. Conceptualizations of slope

Conceptualizations	**Slope**
Geometric ratio	Rise over run
	Vertical displacement (distance, change) over horizontal displacement (distance, change)
Algebraic ratio	Change in y over x
	Representation of ratio with algebraic expressions, $\frac{y_2-y_1}{x_2-x_1}$
Physical property	Property of line often described using expressions like “steepness” (“slant”, “pitch”, etc.); “how high up” or “it goes up”
Functional property	Constant rate of change between variables
Parametric coefficient	Coefficient $\boldsymbol{m}$ in equation $\boldsymbol{y = mx + b}$
Trigonometric conception	Tangent of a line’s angle of inclination
	Direction component of a vector
Calculus conception	Limit
	Derivative
	Tangent line to a curve at a point
Real-world situation	Static, physical situation (e.g., wheelchair ramp)
	Dynamic, functional situation (e.g., distance vs. time)
Determining property	Property that determines parallel, perpendicular lines
	Property with which a line can be determined, if you are also given a point
Behavior indicator	Real number with sign which indicates increasing, decreasing, horizontal trends of line
	Real number with magnitude which indicates amount of increase/decrease of line
	Real number that, if positive or negative, indicates line must intersect the x-axis
Linear constant	Property that is unaffected by translation
	Constant property unique to “straight” figures
	Constant property independent of representation

Note: This table is adapted from Moore-Russo et al. (2011, p.9).

Similar to Walter and Gerson’ s (2007) study that was conducted with in-service elementary teachers, preservice and in-service secondary mathematics teachers’ common slope conceptualization was a geometric ratio (Stump, 1999). Similarly, in Dündar’s study (2015), preservice middle school mathematics teachers from the first to fourth year usually used geometric conception of slope in Turkey. Such understanding of slope as a fraction mediates instrumental understanding (Skemp, 1976). Therefore, this understanding can distract the understanding of slope as rate and slope can change.

In Stump's (1999) study, physical property of slope as steepness was usually mentioned by in-service teachers. This could be related with in-service teachers' knowledge of students that students have previous conceptions of slant and steep about slope before learning slope of a line (Herscovics, 1992). In addition, Dündar's (2015) study physical property of the slope was usually mentioned by first-year preservice middle school mathematics teachers.

Algebraic ratio and functional property conceptualizations were more frequent in preservice teachers' conceptions than that of in-service mathematics teachers (Stump, 1999). However, in Dündar's (2015) study, preservice middle mathematics teachers rarely conceptualized the slope algebraically and functionally. On the other hand, Finnegan (2009), as a high school teacher, emphasized the understanding of rate and change in nature of slope concept. While teacher's conceptions of rise-over-run is dominant, providing teachers a support on representing slope with additive structures for recursive relation in linear functions could develop their understanding of slope as constant rate of change (Walter & Gerson, 2007). Stump (1999) also investigated teachers' knowledge of rate of increase for an increasing line graph of a linear equation. While there are teachers who could give a correct answer for a rate of an interval's connections with slope and rate, there are teachers who could not make that relation between slope and rate (Stump, 1999). Similary, in Kertil, Erbaş and Çetinkaya's (2017) study, preservice middle school mathematics showed difficulties in thinking rate of change. Specifically, the preservice teachers had difficulties on interpreting the slope in a functional situation context about rate of change. Furthermore, in a case study related to preservice middle school mathematics teacher's covariational reasoning, it was found that developing an understanding about instantaneous rate of change in a non-linear graph of a dynamic event strongly related with the understanding of slope of a graph at a point (Yemen-Karpuzcu, Ulusoy, & Işıksal-Bostan, 2017). The reason of those difficulties were related to both the nature of rate of change and the use of ratio and rate concepts in Turkish curriculums (Kertil et al., 2017; Yemen-Karpuzcu et al., 2017).

In addition, preservice secondary mathematics teachers who described what a slope is with the word of angle and/or steepness, lack the understanding of connection between ratio, rate and slope (Stump, 2001b). In Stump's (1999) study with preservice and in-service secondary mathematics teachers, slope has been rarely conceptualized as a parametric coefficient, trigonometric and calculus (derivative) representations (Stump, 1999). Teachers also had difficulties understanding slope in the equation (i.e. $l - l_0 = al_0\ (t - t_0)$. (Stump, 1999). On the other hand, preservice middle school teachers in the fourth year usually used the trigonometric conception of slope (Dündar, 2015).

In the context of a price situation (unit price for unit time), preservice and in-service teachers calculated the slope and interpreted slope as a process (like $\frac{change\ in\ price}{change\ in\ time}$) or as an object (like cost for minute of a phone call) (Stump, 1999). However, teachers had difficulties understanding slope in the context of a speed situation (position-time graph) (Stump, 1999).

As Thompson (2015) found teachers' meaning of a slope is related with the numerical value and triangle. In their research, they asked teachers what would be the numerical value of slope for the equation of $y = mx + b$ when the y-axes is scaled up by a factor of 2. Most of the teachers did not realize the slope of the line remains unchanged. They concluded that when teachers considered slope as "a property of a triangle" (p. 443), they reduced the slope value to half or doubled the slope value (Thompson, 2015). Comparatively, when teachers consider slope symbolically, they usually would think of a slope as unchanged (Thompson, 2015).

Students also had difficulties and misgeneralizations about slopes and equations of lines (Bell & Janvier 1981; Lobato, 2006). They had difficulties about the ratio-as-measure conception of a slope (Lobato & Thanheiser, 1999; Lobato & Thanheiser, 2002; Swafford & Langrall, 2000). It can be thought that the case of speed or the case of price (i.e. ratio as an entity) is easier for students than the case of sweet density for a mixture (i.e. ratio as parts) and vice versa. However, students had different difficulties in both types of intensive quantities (i.e. entity and parts). Students' sources of difficulties were given in detail under three titles. First one is that students had difficulties on relating relevant attributes in a situation involving entity quantity that is measured indirectly with derived nature. For instance, in a speed situation high school students focused on the length of the object while also measuring the speed of the object. The second issue is that the students and teachers had a difficulty of determining the quantities affecting the slope in physical situations (e.g. ramp) (Lobato & Thanheiser, 1999; Stump, 2001b). For instance, in a physical situation (e.g. a ramp situation), students did not provide an understanding of slope as a ratio that represent measuring of steepness and an understanding of the number value of slope as a ratio (Stump, 2001b). The expression of the "ratio of rise to run" was made by a very small number of students (Stump, 2001b, p. 84). Specifically, they could not specify the length of the object is not related with the steepness (i.e. slope) of the object. In addition, they had a misunderstanding of "longer makes it steeper" for the length of the base of the object (Lobato & Thanheiser, 1999, p. 295). Students could think that when there is an increase of one quantity, the other also increases. Furthermore, they treated height as steepness (known as slope as height confusion). Students also had a difficulty on understanding the changing situation of ramp as a dynamic instead of static in nature.

The last problem for high school students is that their real life experience/familiarity on quantities (e.g. speed) did not enable them to form ratios (Lobato & Thanheiser, 1999). They used them as a label (i.e. miles per hour) and did not respond the distance that a person could go in a given time for a given speed or time that it took to go for a distance. That is, they were aware of quantities but not be able to form the ratio. Therefore, they cannot understand the covarying quantities in speed as a rate. That is, in the motion context, students have difficulties on understanding speed as a rate (Thompson & Thompson, 1996). For instance, the students moved a distance through a motion but not consider the amount of time simultaneously (Thompson & Thompson, 1996). Similarly, Stump (2001b) mentioned that high school students did not utilize slope as a measure of rate of change of the line with the variables. For instance, in a functional situation (e.g. slope of the line in a bicycle situation-number of pedal revolution graph), many students did not express any ratio or rate in the graph (Stump, 2001b). These students' responses (see Stump (2001b) showed that they had coordination (Level1) and direction (level 2) reasoning without making any connection with slope as Carlson et al. (2002) mentioned. In another functional situation (e.g. the profit-the number of tickets sold graph), many students did not determine the cost of the ticket (Stump, 2001b) that means they could not read the points in the graph, not compute the slope and not explain the slope in the situation (Stump, 2001b). On the other hand, a high school student can imagine slope in quantitative graphs with a tool (e.g. a computer based-tool) by reflecting on the productions of the tool (i.e. graphs) (Nemirovsky & Noble, 1997). In sum, visual representations of linear functions (Arcavi, 2003) and experiencing learning tools (e.g. computer-based tool) that are used to see slope in graphs had an important role in understanding slope in mathematical visualization (Nemirovsky & Noble, 1997). Therefore, it could be inferred that characteristics of the tools and actions in the lived places (examples: a graphical space in a software, experiencing with a physical object) are significant in the understanding of the slope and equation of line as well as the mathematical visualization.

In a cross-cultural study on linearity, line graphs and slope, Greenes et al. (2007) found high school students have various difficulties about those notions. As parallel to the findings of Schoenfeld, Smith and Arcavi (1993), those students could not make a connection among the Cartesian coordinate system, points of a line graph, and an equation of the line. That is, they could not make the procedure of satisfying an equation of a line using coordinates of points of this line and using the coordinates of points of this line in the table (Greens et al., 2007). In detail, students had many difficulties in identifying the slope of lines on the third quadrant that they considered the slope as a negative even the line as an increasing graph. Moreover, students did not connect between the direction of a line and sign of slope the line. Additionally, while making computations on slope, students made judgments from visual aspects rather than finding the exact coordinates. Similarly, they did not consider the scaling in the graph that is scaled as 30:1. Furthermore, students had confusions on linearity and proportionality. That is, when a line graph is proportional students made a correct calculation of slope using a point on the graph. However, when the line graph is linear but not proportional, students made an incorrect calculation just only using coordinates of a point. Therefore, making suggestions the ways of coping with those difficulties requires having understanding of the algebra teaching and the curriculum.

2.4.3 Teaching of the Slope, Linear Equations and Graphs

In mathematic classrooms in Turkey, the slope concept is taught in different grades step by step through the national curriculum. In the middle school grades, linearity and linear relationship is the focus in algebra (MoNE, 2013a). Linear equations and straight lines as their graphical representations (graphs) connected to situations representing linearity. In the eighth grade, the slope concept is taught under the linear equations and graphs topics. First, students learn slope in physical situations and then in functional situations through multiple representations. Therefore, in the eighth grade, slope is a constant for linear equations and graphs. In Turkish math classrooms, the slope algorithm is connected with the phrase of ‘vertical over horizontal’. However, this phrase does not discriminate the slope of a physical object and a functional situation. Additionally, in high school and in college mathematics, students will learn slope as in a complex equations and nonlinear graphs. In order to develop mastery learning in the future, students should grasp the idea of slope as a rate. Thus, students can interpret slope as average rate of change and instantaneous rate of change (Carlson et al., 2002).

In the U.S. context, the slope concept and its various conceptualizations has an importance in the standards for both upper elementary and secondary mathematics classrooms (CCSSI, 2010). Similar to CCSSI (2010), NCTM (2006) generated detailed conceptualizations of slope in the eighth grade. Eventually, in the U.S. math classrooms, slope algorithms were connected with the rise-over-run phrase while students are still taught the slope of a line in a Cartesian plane (Walter & Gerson, 2007).

Research showed that teachers’ knowledge of mathematics for teaching of slope has some flaws in understanding slope conceptually (Simon & Blume, 1994; Stump, 1999). For example, preservice elementary teachers’ ways of understanding slope as ratio-as-measure differs in steepness and velocity situations (Simon & Blume, 1994). While using real world situations in teaching slope, teachers, especially in-service teachers, were prone to use physical situations rather than functional situations (Stump, 1999). Few preservice teachers and fewer in-service teachers used both physical and functional situations in teaching (Stump, 1999). Researchers showed that preservice secondary teachers had difficulties to integrate both physical and functional situations of slope in lesson plans even when they had taken method courses about functional representations of slope (Stump, 2001a). In addition, while teaching in a classroom, even mathematics teachers utilized the physical and/or functional situations they hadn't mention the meaning of a slope in these situations (Stump2001a). Therefore, although they made connections among the representations of a slope, they did not have a rich perception for rate of change (Stump, 2001a). Similar with students lack of connection among the rate of change, average rate of change, proportionality, and slope (Lobato 2006; Lobato & Thanheiser, 2002), teachers also do not associate these ideas in related contexts. Coe's (2007) study on meanings of rate of change showed that secondary school teachers had weak connections between slope and constant rate of change and did not reach any connection among constant rate of change, average rate of change, instantaneous rate of change, proportionality, and slope. For example, even an experienced secondary teacher lingered with the connection of speed to fastness.

In-service and preservice teachers rarely considered and connected function concepts and functional relationship, however, they usually considered concepts of coordinate system, points, graphs, variables, equations, and graphing linear equations (both geometry and algebra) while getting their students ready to understand slope (Stump, 1999).

Research showed that while teachers explained their students' difficulties of slope, they mostly focused on the process of slope even for different conceptions. Secondary mathematics teachers mentioned that their students had difficulty in applying the formula of slope as an algebraic ratio with considering the order of points on the line and slope as ratio of y over x (Stump, 1999). In addition, even teachers mentioned that students' had deficiencies about meaning of slope in manner of speaking, they could not also give rich definitions of a slope (Stump, 1999).

CHAPTER 3

METHOD

This chapter is composed of ten sections. These sections are restatement of the purpose and research questions, research design, the context of the study, research process, data analysis, trustworthiness of the study, researcher role, ethical issues, and limitations of the study.

3.1 Restatement of the Purpose and Research Questions

The purpose of this study was to investigate a middle school mathematics teacher's set of mathematical practices in teaching slope, linear equations and graphs in a technology enhanced classroom environment while enacting a curriculum-based instructional sequence for eight graders. In line with this purpose, the current study was conducted throughout a multitiered design experiment in terms of teacher-tier (Lesh et al., 2008; Zawojewski et al., 2008), while a teacher was a participant of design experiment process that involved designing, enacting and analyzing the instruction based on the curriculum in a technology enhanced classroom environment. In this technology enhanced classroom environment, the teacher used the GGB software, which is a dynamic mathematics software technology, to prepare GGB materials for the classroom activities. While there were two middle school mathematics teachers that attended designing an instructional sequence in different schools in the whole research process, the second teacher that attended the research in the main study was the focus of the study. Therefore, considering the purpose of the study, the following research questions were answered upon the instructional sequence that the teacher enacted.

1. What mathematical practices of middle school mathematics teachers emerge while using GeoGebra materials in a technology enhanced classroom environment in enacting the instructional sequence?

1.1 What is the nature of mathematical practices of bridging, trimming, and decompressing that middle school mathematics teachers demonstrated while using GeoGebra materials in a technology enhanced classroom environment in enacting the instructional sequence?

1.2. How do these mathematical practices interrelate while middle school mathematics teachers are using GeoGebra materials in a technology enhanced classroom environment in enacting the instructional sequence?

2. What mathematical practices of middle school mathematics teachers emerge while explaining mathematical ideas without using GeoGebra materials in enacting the instructional sequence?

2.1 What is the nature of mathematical practices of bridging, trimming, and decompressing that middle school mathematics teachers demonstrated while explaining mathematical ideas without using GeoGebra materials in enacting the instructional sequence?

2.2 How do these mathematical practices interrelate while middle school mathematics teachers are explaining mathematical ideas without using GeoGebra materials in enacting the instructional sequence?

3.2 Research Design

The purpose of this study was to bring together both practical problems and support for theoretical issues. To this end, this study designed an intervention for mathematics teachers and their students in the real mathematics classroom. To do such an intervention, this study required a cyclic approach of design, evaluation, and revision (van den Akker, Gravemeijer, McKenney, & Nieveen, 2006). In the design process, the aim was to understand mathematical practices of mathematics teachers while improving the interventions for the teachers and their students in real classrooms. The researcher can improve interventions for developing a supportive sequence in teachers' learning while having them improve interventions for developing an instructional sequence in their students' learning. In this process, the design should be based upon educational principles, and the testing of the design should make contributions to the frameworks on mathematics teaching and learning.

Therefore, these needs overlap with the general characteristics of a design research (design experiments) in the educational context: “interventionist, iterative, process oriented, utility oriented, and theory oriented” (van den Akker et al., 2006, p. 5).

The concern of design research or design experiments from learning perspective is twofold: intending a progress in particular forms of learning and providing design environments for this process of learning (Cobb & Gravemeijer, 2008). This idea of design experiments emerged as opposed to the idea of controlling variables in the environment (Cobb, 2003). Irrespective of where it takes place, laboratory environment or classroom environment, most of the design experiments center on the process of learning (Prediger, Gravemeijer, & Confrey, 2015). In addition to this learning perspective, design experiments are suggested for investigating the use of technology in educational settings (Reeves, 2006) and developing curricular materials for scale of classrooms (McKenney, Nieeven, & van den Akker, 2006). Furthermore, Cobb, Zhao, and Dean (2009) and Zawojewski et al. (2008) proposed the idea of using design experiments for understanding teachers’ learning and development. Therefore, design research methods can be used to understand teachers’ knowledge base and expertise within the complexity of classroom teaching (Zawojewski et al., 2008). In other words, design research methods can provide opportunities for investigating the mathematical practices of in-service middle school mathematics teachers in actual classroom situation. Therefore, we decided to conduct a design experiment within an individual in-service teacher’s classroom and school setting to characterize her mathematical practices in classroom teaching while enacting an instructional sequence that is designed based on the curriculum. In this regard, multi-tiered design experiment (Lesh & Kelly, 2000; Lesh et al., 2008), which is a type of design research methodology and which involves understanding the development of interactively connected subjects (e.g. students, teachers, and researchers), is appropriate to embrace the complexity of teaching in classroom environment (Zawojewski et al., 2008). While Lesh and Kelly (2000) illustrated a three-tiered design experiment with tiers of students, teachers, and researchers, Zawojewski et al. (2008) added a facilitator tier as a constituent in the design process. Therefore, multitiered models involve design cycles, the constituent interactions, and the models developed by tiers. In these studies, the tier of students develop mathematical ideas and artifacts; the tier of teachers develop tools for teaching while examining students' understanding of mathematical ideas; the tier of researchers produce interpretations on teacher's and students' experiences and understanding (Lesh & Kelly, 2000); and the tier of facilitators develop sessions for teachers (Zawojewski et al., 2008). In line with the purpose of the current study, the teacher-tier aspect of the multitiered design experiment was focused on (Zawojewski et al., 2008).

In this study, it is assumed that adopting methods of design experiment could develop the process of investigating teachers' mathematical practices in classroom teaching based on the suggestions of usefulness of design research methodology situated in instructional environments (van den Akker et al., 2006). In this regard, the researcher organized the sessions for and with a teacher from the perspective of classroom design experiment (Cobb & Gravemeijer, 2008) with a focus on teachers' mathematical practices. The research agenda with a teacher was shaped by the idea of providing a real environment for investigating a teacher's mathematical practices in a design experiment process in which the teacher's concern was to develop a particular approach to classroom teaching for teaching slope, linear equations, and graphs in a technology enhanced classroom environment using the GGB software in computers. Therefore, this primary intent of the teacher's research that could be used in her classroom and adapted by other teachers discriminates the teacher's work from action research (Stephan & Cobb, 2013).

Design experiments are conducted through phases and cycles. Even the names of the phases can be different in various studies (Cobb & Gravemeijer, 2008; McKenney, 2001; Plomp, 2007; Reeves, 2000, 2006), the phases are generally named as preliminary phase, prototyping (development) phase, and assessment (Plomp, 2007). In parallel with these studies, Wademan (2005) developed a generic design research model and Plomp (2007) illustrated that model in a general form to be applied in various studies (See Figure 3). This current design experiment followed that model (Plomp, 2007) includes the preliminary and the prototyping phases (i.e. prototype study and main study). The development of each phase with the transitions between the phases was described in the following sections.

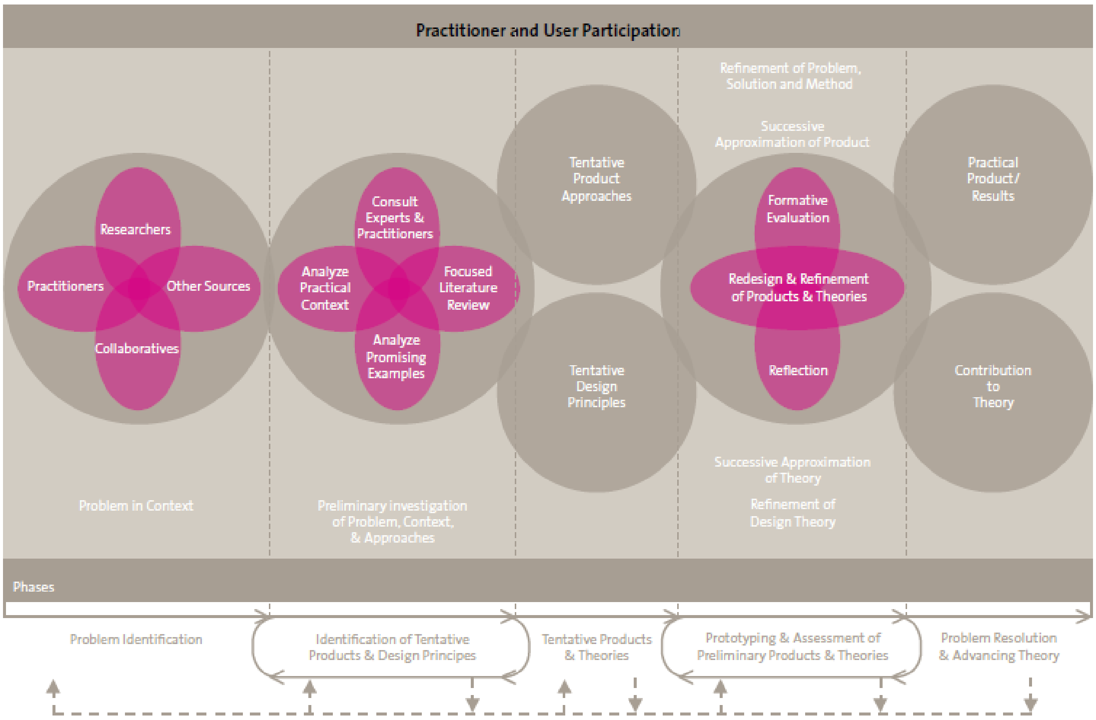

Figure 3. Plomp's illustration of Wademan's (2005) Generic Design Research model.

Reprinted from *Educational design research: An introduction* (p. 16), by T. Plomp, 2007, Enschede: SLO.

The scheme of design experiment process of the study presents the whole process of this study (See Figure 4). The scheme displays that the study has systematically progressed and the flow of the progress is shown with the arrows. The researcher constructed this scheme combining the phases of the design experiment and the related aspects within these phases of the design experiment. In the beginning, the research problem was identified with the participation of researchers and teachers in the context. Then, in the preliminary phase, need analysis and review of literature were conducted; experts and teachers were consulted, and practical context was analyzed through the participation of researchers and teachers. Based on this phase, tentative design principles for sessions with the teacher and data collection tools were developed.

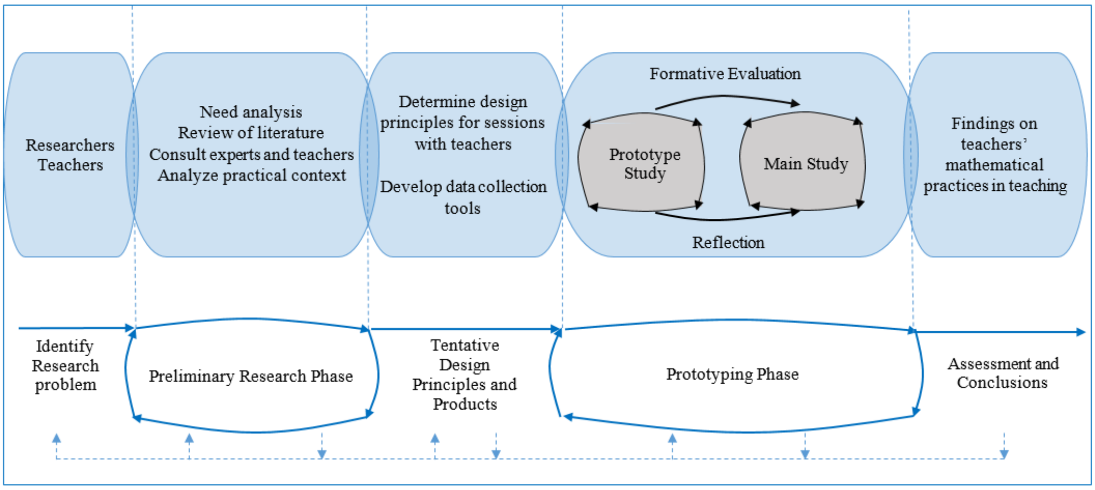

Figure 4. Design experiment process of the study.

Adapted from *Educational design research: An introduction* (p. 16), by T. Plomp, 2007, Enschede: SLO.

In the following, the prototyping phase included prototype study and main study. Each phase in the prototyping phase was conducted with different middle school mathematics teachers in different public schools. During the prototype study, the researcher evaluated and revised the study through formative evaluation, and reflection that was made to plan a research agenda with the participant of the main study. Considering the aforementioned issues, the main study was conducted with the participant teacher that was the focus of this dissertation. The main study involved four stages that were the first stage with pre interview with the teacher and observation of regular teaching, the second stage as meeting with the teacher about use of GGB materials in classroom teaching, the third stage as iterative cycles of planning-enacting-analyzing instruction in accordance with the objectives in curriculum, and the last stage with post interview with the teacher.

Afterwards, as the last stage of the whole process of this study, the researcher made an analysis for describing and explaining the data on mathematical practices of the teacher in teaching. The instructional sequence and the findings on the teacher's mathematical practices were presented. Then, the researcher drew conclusions from the findings.

3.3 The Context of the Study

This section presents the details about the context of the study under two sub-sections: Turkish Middle Schools and Mathematics Education and Middle School Mathematics Teachers Zehra and Oya

3.3.1 Turkish Middle Schools and Mathematics Education

Turkey has a strong centralized structure of compulsory primary and secondary education with a centralized educational system for the schools of Ministry of National Education (MoNE) (Çakıroğlu & Çakıroğlu, 2003). Before the change of the National Education Law in March 2012, 8-year compulsory primary education schools involved elementary education for 5 years (grade 1 to 5) and middle school education for 3 years (grade 6 to 8). After the change of the National Education Law in April 2012 to be applied with effect from 2012-2013 school year, there are now 3 stages (4+4+4) before higher education-two stages in primary education and one stage for upper secondary education. With this change (4+4+4), primary education system involves elementary education schools for 4 years (grade 1 to 4) in the age group of 6 to 9 and middle school education schools (lower secondary education) for 4 years (grade 5 to 8) in the age group of 10 to 13. Students who graduate from primary school (grade 4) start middle school (grade 5). While there was one type of Middle School under primary education until 2013, there are now two types of public middle schools which are general middle schools and imam-hatip middle schools after this change. In addition to public schools, there are private schools for all stages. This study was conducted within two public middle schools (one for the prototype study, the other one for the main study) in the second semester of 2012-2013 school year. The middle schools involved grades from 6 to 8 when the data was collected.

The middle school mathematics education curriculum and standards are determined by the Board of Education in Ministry of National Education. The middle schools used the MoNE (2009) mathematics curriculum when the study was conducted. The curriculum for grades 6 to 8 was prepared in 2005, was implemented gradually grade by grade, and was developed until 2009 with the decision (numbered 187 and dated 30/06/2005) of the Board of Education. The middle school mathematics curriculum (MoNE, 2009) involves 5 learning areas which are Numbers, Geometry, Measurement, Probability and Statistics, and Algebra. The standards in the sub-learning areas of algebra in middle school mathematics (MoNE, 2009) are given through grades 6-8 in the table in Appendix A. Middle school mathematics teachers use the mathematics textbook written in line with the middle school mathematics education curriculum (MoNE, 2009) and distributed free of charge to students and teachers. Teachers also use a teacher guide book involving yearly plan and guides for activities parallel with the mathematics textbook. During the data collection period, the eight-grade mathematics textbook (MoNE, 2012) was distributed to all middle school students in Turkey by the Ministry of the National Education. Besides, teachers can use additional sources with the textbook.

Instructional Unit

In the study, the researcher determined the sub-learning area of equations to investigate a teacher's mathematical practices in a particular topic of algebra. Specifically, the focused mathematical ideas were slope, linear equations and graphs topics that were given under a unit in the offered textbook and the yearly plan for eight-graders. In MoNE's (2009) curriculum, the related objectives of this unit are:

- Explain slope of a straight line with models
- Relate slope of a straight line and equation of a straight line.
- Solve linear equation systems using graphs (p. 290)

After the change in the education system (4+4+4), middle school mathematics curriculum and standards were revised and renewed for grades 5 to 8 (decision numbered 8 and dated 01/02/2013) to be used for grade 5 in 2013-2014 school year, and for grades 6, 7 and 8 gradually after the 2013-2014 school year. While conducting this study, the principles and objectives for slope, equations, and graphs topics in the revised curriculum (MoNE, 2013a) were also considered since the participant teachers were a part of that change team. In MoNE's curriculum (2013a), the related objectives in this topic are:

- Explain slope of a straight line with models.
- Relate linear equations, graphs and related tables of linear equations with slope.
- Relate solutions of system of linear equations and graphs of lines of those linear equations (p. 37)

In the revised curriculum, within these objectives, the importance of learning these mathematical ideas in real-life situations and with appropriate technologies has also been emphasized. In this study, the research team (the author, the supervisor and the teachers) considered the revised curriculum.

Middle School Mathematics Teachers

In Turkey, middle school mathematics teachers have a bachelor degree from Elementary Mathematics Teacher Education Program in Education Faculties. The program (four-year) involves courses on Mathematics, Mathematics Teaching Methods, and General Education for teaching from grade 4 to 8. After graduation, teacher candidates have to take the Public Personnel Selection Examination (PPSE) and should get the required score to be recruited to the public middle schools by the MoNE. The MoNE appoints a certain number of middle school mathematics teachers within a PPSE score interval according to the annual teacher shortage in public schools. When the participant teachers passed the PPSE, the exam that was held once a year at the end of the school year was composed of two parts. One part involved 'General Knowledge' test on history, geography, basic civics, general cultural subjects and 'General Ability' test on verbal ability and mathematical ability. The other part involved 'Educational Sciences' test on developmental and learning psychology, evaluation and assessment, methods of teaching, and guidance and counseling. In addition to those two parts, Elementary Mathematics Teaching Content Knowledge Test has been given in PPSE since 2013. The test involves subject matter knowledge part on calculus, algebra, geometry, applied mathematics, and pedagogical content knowledge. However, the participant teachers took this exam when it was composed of General Knowledge, General Ability and Educational Sciences parts.

3.3.2 Middle School Mathematics Teachers: Zehra and Oya

The participant teachers Oya and Zehra were in-service middle school mathematics teachers in public middle schools. The researcher met them through the contacts of her supervisor. There were three reasons for selecting them as participants. First, they use technology in their classes and their students could use dynamic software in math classes. Second, they had research experience on mathematics education and were eager to teach through the research process. Last, they were experienced teachers with a rich background. Therefore, they seemed to have a potential for producing data for the study. They participated voluntarily in the study. The details about them were given in the following sections.

3.3.2.1 Teacher Zehra

Teacher Zehra was the participant of the prototype study. Zehra graduated from elementary mathematics education program in a public university in Ankara in 2006. She took 'Calculus, Linear Algebra, Analytic Geometry, Geometry, Discrete Mathematics, Modern Algebra, Statistics and Probability' courses, Methods of Teaching Mathematics (1 and 2) courses, and general education courses. In addition, she took technology courses to teach mathematics using Microsoft Office programs. She was a successful undergraduate with a graduation grade of 3.03 (over 4.00). Zehra had a master degree in Elementary Mathematics Education in 2013. She took courses on GGB in graduate education. Her master thesis was about the effects of teaching linear equations with dynamic mathematics software on seventh grade students' achievement. She used GGB software in teaching for her study.

Zehra has been a teacher in public schools for 6 years (at the data collection time). She has been teaching to 6^{th}, 7^{th}, and 8^{th} grades. She has been working in this school for two and a half years. Zehra is eager to use technology in her classroom and to conduct projects about the use of technology. For example, she was involved in an e-twinning project about the use of technology in teaching. She also used other technologies (e.g. PowerPoint, Word) for showing and lecturing rather than using technology as a means to reinvent mathematical ideas. Besides, she was one of the members of the commission selected for writing middle school textbooks.

Zehra's school

The school is a public middle school with middle socioeconomic status children in Ankara. It is in a middle socio-economic level district and 9 km away from the city center. The number of teachers was 30 and there were two mathematics teachers. The size of classrooms was roughly 20 students in the school. Each middle grade (6^{th}, 7^{th}, and 8^{th}) had three classes in the school.

Zehra's classroom

Zehra's observed classroom was a regular classroom reserved for an eight-grade class in the school. This classroom was bright with windows across a wall and with a large space between the desks. This classroom had a computer, a projector and a white board. The white board was also used as a projection screen. There were students' posters and other works on the classroom wall. In classroom arrangement, students pairs were seated on their desks in three straight columns facing the board (see Figure *5*). This seating was the regular seating of the classroom. Students usually worked in pairs or individually.

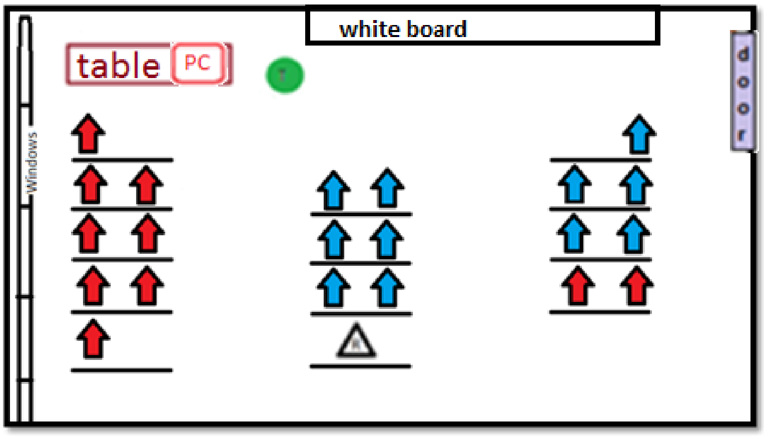

Figure 5. An example of a classroom seating arrangement of teacher Zehra.

Note. T: teacher (green circle); R: researcher (white triangle); Red arrows: girls, Blue arrows: boys; PC: personal computer.

In classroom teaching, Zehra uses the mathematics textbook offered by the MoNE. She prepares activity sheets or worksheets as supplementary materials so that student could understand a specific topic or the practice questions. She uses technology in her classes. For example, she uses PowerPoint presentations, Excel activities and GGB activities. In addition, she uses the computer laboratory for teaching mathematics as well as the classroom. In other words, she uses technology in various ways for different purposes in teaching. During the design experiments, the teacher used the computer and a Bluetooth mouse that students could use to access the personal computer on the teacher's table while they were sitting on their desks. When it was necessary, she allowed students to use the mouse to use GGB materials that were prepared for the experiment. Teacher gave activity sheets to students that were connected to the GGB materials.

While conducting this study, the teacher taught two eighth grade classrooms. In this study, only one eight-grade classroom was observed in line with the teacher's request. The classroom was an eighth grade with 21 students (10 girls, 11 boys). The eight graders in this school knew how to use GGB in class as they studied with it before. The teacher described eighth graders as successful students but they were anxious about the national exam. Therefore, she expressed that the students tended to learn procedures rather than being interested in the meaning of the concepts as they tend to solve questions in the national exam.

3.3.2.2 Teacher Oya

Teacher Oya was the participant of the main study. Oya graduated from elementary mathematics education program in a public university in Ankara in 2005. She took "Calculus 1/2/3/4, Abstract Mathematics, Linear Algebra, Analytic Geometry, Geometry, Statistics and Probability 1/2" mathematics courses, "Teaching Geometry, Material Development, Instruction Methods and Techniques" mathematics education courses and general education courses. She also took teaching mathematics with technology and tools courses such as "Computer Assisted Mathematics Education, Specific Teaching Methods". She learned using MS office programs, WinGeom, and the Geometers' Sketchpad software. She was eager to learn and was a successful undergraduate with a high graduation grade above 3.5 (over 4.00). Oya had a master degree in mathematics education from another public university in Ankara. Her master thesis was about investigating the teaching of translation in geometric transformations in a computer assisted environment for middle grades. She used inquiry approach and WinGeom, which is dynamic geometry software, to investigate students' understanding in teaching experiments. She knows GGB and its properties in general but she has never used it in classroom teaching until this research study.

Oya has been a teacher in public middle schools for 8 years (at the data collection time). She has been teaching to 6^{th}, 7^{th} and 8^{th} grades. The current school is the second school that she worked. She has been working in this school for 3 years. Oya is eager to develop and use different instruction strategies with her rich background. For example, she uses drama techniques since she attended creative drama education courses in Creative Drama Association in Turkey. In addition, she had geometry and origami projects with her students and other mathematics related projects with Comenius partnerships. Besides, she was one of the members of the commission selected for the revision of the curriculum for middle school mathematics. She worked with university members and teachers to revise the curriculum. Therefore, she knew the standards of revised middle school mathematics curriculum (MoNE 2013). In addition, thanks to this commission work, she had experience in analyzing various mathematics textbooks from different countries.

Oya's school

The school is a public middle school with low and middle socio economic status children in Ankara. The school is in a low socio economic district, and it is 10 km away from the city center. The number of teachers was roughly 35 and the number of classrooms was 20 in the school. There were 10 middle grade (5-8) classrooms in the school. The size of the classrooms was roughly 20-30 students. Different classrooms existed for each course and students changed classrooms according to the courses in their daily schedule. In the school, there were two mathematics classrooms for mathematics courses. Since there were two mathematics teachers in the school, each mathematics teacher had her own mathematics classroom.

Oya's mathematics classroom

Oya had a mathematics classroom in the current school. Her classroom was bright with windows across a wall and there was a large space between the desks. This classroom had a white board, a computer, a projector, and a material storage with various concrete materials (See Figure *6*). In the Figure, it is seen that there was a projection screen for projector on the white board. At that moment, she was teaching geometric solids using dynamic software. She could use both the white board and the projection screen by lifting up the screen. There were students' posters and other works on pin boards and classroom wall.

Figure 6. A view of the teacher's classroom.

Note. The artistic effect was added to the photograph of the classroom for the confidentiality of the participants.

She arranged the class seating plan considering group work in collaboration (see Figure *7*). She has been using this seating arrangement since she started teaching. The students were seated as groups, with around 6 students in each group. The seats of groups change every week since she wanted the students to sit on different seats at different parts of the classroom during the year. The groups were heterogeneous involving students with different levels of achievement. While some responsibilities of the students in a group changed from class to class, all students in a group shared the same responsibilities like participating in producing a solution to a task, sharing and discussing solutions with each other when working individually on a task in the activity sheet (or notebook), and writing the results of their group on their own activity sheet. Other responsibilities of the students in a group were changed in rotation. These responsibilities were the 'controller' that controls the group members to participate in a solution within a consensus and the 'leader' that informs the group about sharing the task. In addition, the teacher encourages students who have more advanced understanding on a task to share their ideas, solutions, questions, or suggestions in order to stimulate other students' mathematical activity. She allows students to study and discuss within a group and between groups.

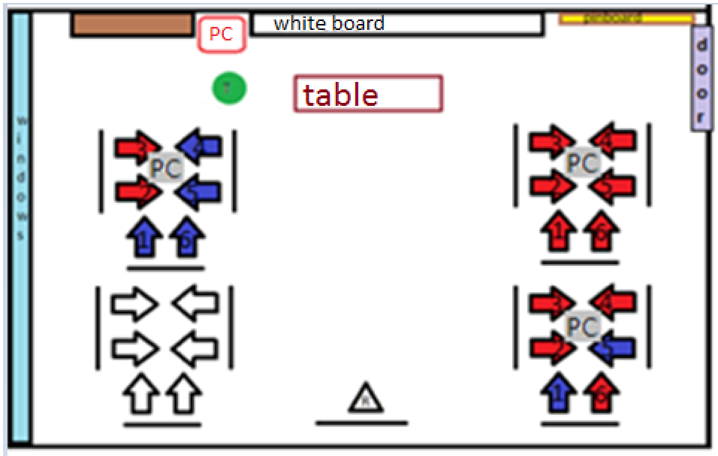

Figure 7. An example of the classroom seating arrangement of teacher Oya.

Note. T: teacher (green circle); R: researcher (white triangle); Red arrows: girls, Blue arrows: boys; PC: personal computer.

In classroom teaching, she uses the mathematics textbook offered by the MoNE. She prepares activity sheets or worksheets as supplementary materials so that students can understand specific topics or practice questions. She uses concrete materials and technology in her classes. For example, she uses office programs (i.e. power point, excel) and dynamic software (i.e. WinGeom for teaching geometry). She uses computer technology in the classroom throughout a lesson/an activity or a part of a lesson/ an activity. She uses technology in a topic, in an activity or in solving problems in a semester for different purposes in teaching. For example, she used dynamic software to teach volume of a geometric solid. In addition, she uses concrete materials (e.g. geometric solids) in the classroom throughout a lesson/an activity or a part of a lesson/an activity. Therefore, the teacher designed such a classroom environment that students could also manipulate materials, use dynamic software, and discuss mathematical ideas.

While the aforementioned features were observed in the teacher's regular classroom teaching and were also acquired during interviews in design experiment with the teacher, some additional features emerged during the design experiment. In design experiment, there were three laptop computers which are given to the groups of students. When it was necessary, she guided students to use the laptops (one laptop for a group of students) that involved GGB materials prepared for the experiment. In the meantime, she gave activity sheets that were connected to the GGB materials. In addition, she was able to give her Bluetooth mouse to students to use the screen as a computer. Besides, she could project an activity sheet on the board when students worked on their own activity sheets. Moreover, she used battens as concrete objects that were fixed length wooden strips (i.e. battens).

While conducting this study, Oya taught two eight-grade classrooms, which were 8A and 8B. In 8A, the number of students was 20, 12 of whom were girls and 8 of whom were boys, but two students did not attend regularly during the experiment. In 8B, the number of students was 23, 12 of whom were girls and 11 of whom were boys, but three students did not attend regularly during the experiment. Both classrooms had one student who had learning disabilities.

3.4 Research Process

While designing the intervention of the current study, it was aimed to provide a way to understand a complex issue in the educational context from the design research perspective (Reeves, 2006; van den Akker, 1999). In the study, this complex issue is mathematical practices of teachers in teaching to improve students' understanding of slope, linear equations and graphs in a technology enhanced classroom environment. In this regard, it was proposed that when teachers are a part of improving a teaching-learning environment, they could develop such practices that involve mathematical actions to improve students' learning during the instruction (Lesh & Kelly, 2000; Lesh et al., 2008; Zawojewski et al., 2008). Therefore, the function of the intervention is primarily to gain an insight into the forms of middle school mathematics teachers' mathematical practices in teaching in such a process that they enact an instructional sequence for supporting students' conceptual understanding about slope within the linear equations and graphs unit in a technology enhanced classroom environment based on Turkish national curriculum standards.

In this process, the researcher used the stages of classroom design experiment as a tool for improving mathematical work of teachers. As the researchers, the author and the supervisor designed sessions for teachers to understand their knowledge, goals, and beliefs about slope teaching in middle school mathematics and to emerge their mathematical practices in slope teaching. As the team, the researchers (the author and the supervisor) and a teacher designed instructional activities with tools and technology in a sequence in which the teacher had a decision-making priority and was responsible for teaching in her classrooms. Therefore, the researchers primarily aimed at investigating mathematics teaching in practice rather than testing a theory. What should be the students' contributions, what tools and technology should be used by students, what should be the teacher's contributions, how the teacher should use tools and technology for the sake of students' learning were the essential issues in this process which involves cycle(s) of design, test and analysis of instructional sequences (Cobb, 2003). Meanwhile, the use of design experiment in this study could perform a basis for designing the learning environment in technology-enhanced classroom teaching based on a national curriculum.

As mentioned in section of research design above, this study includes the preliminary phase and the prototyping phases (i.e. prototype study and main study). The development of each phase with the transitions between the phases was described in the following sections. In the preliminary phase, some of the data collection tools for teachers were obtained, and the design characteristics of instruction were decided. In the prototyping phase, the prototype investigation study was conducted through micro-cycles and modifications during those cycles. After that, the main study was conducted through micro-cycles. While transiting between the phases, criteria for high qualitative interventions based on formative evaluation with its methods (Nieeven, 2007) were considered to improve the intervention.

3.4.1 Preliminary Phase of the Study

The preliminary research phase involves "needs and context analysis, review of literature, development of a conceptual or theoretical framework for the study" (Plomp, 2007, p.15). Based on the need and context analysis and review of literature, this study focused on a design for investigating teachers' mathematical practices in teaching slope, linear equations and graphs in a technology enhanced classroom environment. This specific algebra topic led the researcher to focus on a framework on algebra teaching as the interpretative framework for understanding teachers' mathematical work. In this regard, the KAT framework (Ferrini-Mundy et al., 2005;

McCrory et al., 2012) provided guidance for the researchers both for implementation and analysis.

While developing the preliminary phase of the study, the theory of teaching in context (Schoenfeld, 1998) guided the researchers for gathering data from the teachers. This theory oriented the researchers to consider the teachers' beliefs and goals as elements of teaching in their context while investigating teachers' mathematical practices. In this regard, the researchers prepared and developed a pre-interview protocol. In addition, a teacher information protocol was prepared to gain an insight into the background and current situation of the teacher. These protocols and their development process were explained in detail under the main study section.

While developing sessions with teachers and designing instructional sequence for prototyping phase of the research (i.e. prototype study and main study), realistic mathematics education (RME) heuristics guided the teachers and the researchers (i.e. the team) in supporting students' learning (Gravemeijer et al., 2003). One of this heuristics is to provide an "experientially real" activity as a starting point (Gravemeijer et al., 2003) as mentioned in the mathematics curricula (MoNE, 2013a). Therefore, the researcher should guide the teacher to start the classroom activities within a realistic context. It is important to note that the participant teachers in this study tended to contextualize the mathematical tasks in realistic situations. As for examples in the prototyping phase of the study, in the batten activity in the main study, students needed to measure the slope of battens in different situations. In the *Mobile Operators* activity in the main study, students needed to compare and contrast the line graphs of the mobile operators to make a decision on choosing the more affordable operator.

The other important heuristic is guided reinvention which is based on Freudenthal's (1991) idea of mathematics as a human activity for developing a conjectured learning sequence for all the students in a classroom. Therefore, instructional activities in the learning sequence provide an environment to make students reorganize and construct increasingly sophisticated mathematical understanding based on the justifiable initial activity (Cobb, 2010). In this regard, the instructional sequence is guided by the notion of hypothetical learning trajectory (HLT) (Gravemeijer et al., 2003; Stephan & Akyüz, 2012). While the notion of HLT that was introduced by Simon (1995) illuminates a teacher's consideration for the path of students' engagement in learning and thinking in a process, this notion of HLT is developed under the RME and design experiment studies. In the current study, the researcher considered the notion of HLT as a means of guiding the teacher in planning and analyzing sessions. In planning sessions with a teacher, the notion of HLT should be used for planning the instructional sequence with the dimensions of learning goals, tools, activities, and classroom discourse. In the prototyping phase of this study, the researcher introduced and used these dimensions as a guide for the teachers while the teacher planned and revised the instructional sequence during the sessions. In analyzing sessions with a teacher, the researcher focused on to guide the teacher primarily to analyze and reflect on how classroom discourse developed and secondarily to reflect on how tools and activities supported these dimensions.

Another important heuristic is the tools that will provide support to students for developing mathematical practices and constructing in the reinvention process. That is, the tools should help students develop mathematical practices from models of their informal mathematical activity to models for formal mathematical reasoning in the instructional sequence (Gravemeijer et al., 2003; de Beer, Gravemeijer, & van Eick, 2017). This transition is considered as students' way of constructing mathematical relations and activities with the model. Therefore, the aim is to reify the process of mathematical activity and reasoning with the model rather than to make a model (Gravemeijer, 1999). In this regard, various tools can be used such as physical objects, pictures, graphs, symbols, and notations in the instructional sequence. For example, tools could be pacing, unifix cubes, footstrips (created by students), and a measurement strip in teaching measurement (Stephan, Cobb, & Gravemeijer, 2003), net worth statements, symbols of '+' and '-', and empty number line in teaching integers (Stephan & Akyüz, 2012), and a computer simulation, 'visual representations of the filling process' in teaching instantaneous speed (de Beer et al., 2017). In this study, concrete objects were decided to be used as physical tools and the GGB materials in computers were used as technological tools.

Based on the aforementioned principles, it was decided to make the planning sessions with the teachers enriched with examples of HLTs for middle grades and with examples of GGB materials to give an insight into instructional tools. In addition, in these sessions, the teacher should concentrate on instructional sequence enriched with the instructional tools based on the dynamic mathematics software (i.e. GGB). For each instructional goal, an activity sheet and a GGB material should be prepared. In this planning instruction process, the researcher should provide teachers with the dimensions of HLT to guide the process after the teachers made the conceptual analysis of the slope within linear equations and graphs context for eight grade mathematics. On the other hand, the researcher do not direct teachers about what to do in the classroom teaching and how to teach the topic. However, the researcher can help the teachers to construct GGB materials in line with their request and demand. In addition, it was decided to observe the classroom teaching without interrupting the instruction. Finally, there should be meetings for the purpose of analyzing sessions to assess and revise daily teaching after the instruction was enacted. It was decided to make the analyzing sessions with the teachers enriched with video-records of the classroom sessions, activity sheets of the students, observation notes and questions of the observer (i.e. the researcher) about students' understanding and teacher's mathematical actions. During this process, the team should analyze students' understanding of slope and designed or revised the activities and materials the next-day.

After determining the tentative design principles and products of the intervention and developing the data collection tools, the researcher moved on with the prototyping phase which involves two phases, namely the prototype study and the main study. While moving from the preliminary phase to the prototype phase, the quality of the interventions was considered with the relevancy and consistency of the study (Nieveen, 2007). In this regard, the design and products of the intervention were based on the "scientific knowledge" (Nieveen, 2007, p. 94) analysis that focused on the literature review, expert opinions, and the practical solutions of the products developed for similar problems. In addition, the consistency of the intervention of students was interpreted as follows. First, the development of the intervention followed the components of HLT learning goals, tools, activities, and classroom discourse based on the national curriculum. Second, the development of intervention considered with whom the students would learn (i.e. individual, pair, group), the characteristics of where they would learn (i.e. classroom, seating plan), and the time when they would learn (i.e. time for each activity, time for instructional goals). Furthermore, the development of intervention of teachers followed the engagement of teachers in cyclic process of classroom-based design experiment to develop an instructional sequence.

3.4.2 Prototype Study

The prototype study was conducted with teacher Zehra. The steps of the research process with Zehra were given in Table *2*.

Table 2. Research process with Teacher Zehra

Research Process	Duration	Explanations
-Teacher Information protocol -Pre-Interview on teacher's goals and beliefs for mathematics teaching	60 min	Audio record was done.
Planning Instruction	90 min	Classroom norms and activities considered
Planning Instruction	20 min	Organization on activity sheets and materials
Classroom Observation	90 min	Enactment of instruction
Planning Instruction and Analyzing Instruction	20 min	-Evaluated and revised enacted instruction -Audio record of the classroom teaching was used as a tool -Based on analyzing interview organization on activity sheets and materials
Classroom Observation	90 min	Enactment of instruction
Analyzing Instruction	90 min	-Evaluated and revised enacted instruction -Audio record of the classroom teaching was used as a tool
Planning Instruction and Analyzing Instruction	20 min	Based on analyzing interview organization on activity sheets and materials
Classroom Observation	40 min	Enactment of instruction
Analyzing Instruction	20 min	-Evaluated and revised enacted instruction -Audio record of the classroom teaching was used as a tool
Classroom Observation	40 min	Enactment of Instruction
Analyzing Instruction and Post-interview	60 min	-Evaluated and revised enacted instruction -Audio record of the classroom teaching was used as a tool -Analyzing the whole process
Total	*660 min*	

Before the plan-enact-analyze instruction iterations, teacher information protocol and pre-interview on teacher's goals and beliefs for mathematics teaching were conducted. The information protocol gave appropriate background information about the teacher and the pre-interview protocol worked as expected and the teacher answered all the questions clearly. The interview was recorded by an audio-recorder.

The planning instruction sessions were carried out as semi-structured interviews that were conducted while the teacher was planning the instructional sequence. In the beginning, the researcher gave the dimensions of HLT with two examples: one was about patterns and was prepared by professors (e.g. Michael Stephan) in a design research session (by Dr. Paul Drijvers and Dr. Michiel Doorman) in Freudenthal Institute, Utrecht University in 2011, and the other one was about measurement (Gravemeijer et al., 2003). These interviews involved questions as to how Zehra taught this topic before, how she would use GGB for teaching slope and how she would construct GGB materials and the teacher's self-expressions about the lesson. In addition, the researcher gave examples of pre-prepared GGB materials (by the researcher and from https://www.geogebra.org/materials/) to provide a basis for the discussion and the development for the intervention. During the planning sessions, the instructional goals were determined; GGB materials and activity sheets were designed and developed; and classroom activity structure and students' roles were determined. All sessions were recorded via a voice recorder.

While enacting the instructional sequence in the classroom, the teacher's implementation of instruction was recorded with voice recorders and the researcher made observations and took detailed field-notes. There were two voice recorders, one on the teacher desk in front of the white board and one on the researcher desk at the back of the classroom. The researcher could not use video camera since the teacher did not want to be recorded by a video camera. In addition, full motion recording of screen activity of computers was used in the classroom that records the motion of teachers' computer screen activity and the voice. The researcher used the observation protocol that fit the purpose of the study and decided which type of questions she would ask after the lesson during this process. In this process, the researcher reached conclusions on what questions to create and ask during the analysis sessions to understand a teacher's actual practices. Therefore, in the sessions of analysis of the instruction, stimulated recall interviews were conducted to evaluate the instruction and investigate teacher's mathematical practices. The teacher made reports about her teaching during the analysis sessions after listening to the selected parts of audio records of the lesson and thinking aloud regarding her thoughts in the lesson episodes. The selected parts mainly involved using GGB. Besides, the researcher used her observation notes and questions to learn about the teacher's use of mathematical knowledge in the lesson.

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While making transition from the prototype study to the main study, the researcher made modifications based on the criteria for high qualitative interventions (Nieveen, 2007) (See Table *3*).

Table 3. Transition between the studies

	Prototype Study	Modifications
Practicality	*Expected:* Audio records can be used to understand teachers' mathematical practices in classroom while analyzing the instruction. *Actual:* Audio records provide limited resources for understanding the mathematical practices	Video records. Changes in questions to ask the teacher after the enactment.
	Expected: It is useful to make students use activity sheets of GGB materials for conceptual understanding of slope when the teacher uses the GGB materials on the computer. *Actual:* Students benefited from GGB materials only as the teacher used them. This provides limited discussion on conceptual understanding of slope with the activity sheets of GGB materials.	Students can use GGBmaterials on teacher's computer with Bluetooth mouse. Students can use GGB materials on a different computer than the teacher.
	Expected: Teacher uses GGB materials to conceptualize the meaning of slope. *Actual:* Teacher tended to use the materials to show the ways of computing slope in different situations.	The team should have a meeting about the properties and use of GGB for teaching slope.

In practicality, the usability of the intervention in real classroom and teacher setting was evaluated. After evaluating the enactment of instruction in classroom and the analysis of instruction in classroom, it was concluded that the video records of the instruction are certainly needed. The reasons were to make the questions precise and meaningful for the teacher by providing an effective stimulus for reflecting on classroom teaching in analyzing instruction interviews, to make data analysis by watching and writing the whole process, and to understand teachers' mathematical practices both in enactment and in the analysis of instruction. In addition, even though the teacher knew how to use the dynamic software, the researcher decided to have a meeting about the properties and use of GGB on teaching with the teacher.

3.4.3 Main Study

In this section, the research process of the main study which was constructed after the reorganizations based on the prototype phase of the study is given (see Figure *8*). The preparation stage was composed of two steps. In the former step, the researcher conducted an interview as pre-interview on the teacher's goals for and beliefs in teaching mathematics and gathered additional information about the teacher with Teacher Information Protocol. In the latter step, the researcher observed and recorded the participant teacher's regular classroom teaching with a video-camera and a voice recorder to gather more information about the teacher's mathematics teaching perspective in terms of the classroom structure and norms. The second stage involved meeting with the participant teacher on using GGB in classroom teaching since the teacher had not use the GGB software in classroom before. These two stages were informative for the researcher in that the researcher had the opportunity to learn more about the teacher and her perspectives on mathematics teaching with GGB materials in classroom environment. In addition, this process gave time to the teacher and the researcher to know each other for conducting the study.

Upon setting the research agenda with the main participant, the third stage was initiated which included a macro-cycle with three micro-cycles of design experiment in accordance with the objectives in curriculum. These cycles of plan, enact, and analyze (evaluate/revise) on daily basis are named as “design minicycles” (Cobb & Gravemeijer, 2008, p. 76). These cycles were progressed iteratively cycle by cycle. In this regard, in the main study, it was focused on teaching experiment in three micro-cycles as a macro-cycle (the whole instructional sequence) on teaching slope, equations, and graphs. While designing the instruction, the teacher planned the instructional sequence on slope concept and slope of linear equations and graphs based on the envisioned learning trajectory of her students. The researcher, as the facilitator of enacting the design experiment methods in this process and the helper in executing teacher’s needs in designing and revising the GGB materials, revealed the details of instructional activities in the instructional sequence in the teacher’s planning and made suggestions on the sequence and technology use considering the literature and the prototype study. In sum, the researchers’ main focus was to investigate the mathematical practices of the teacher during the classroom teaching experiment.

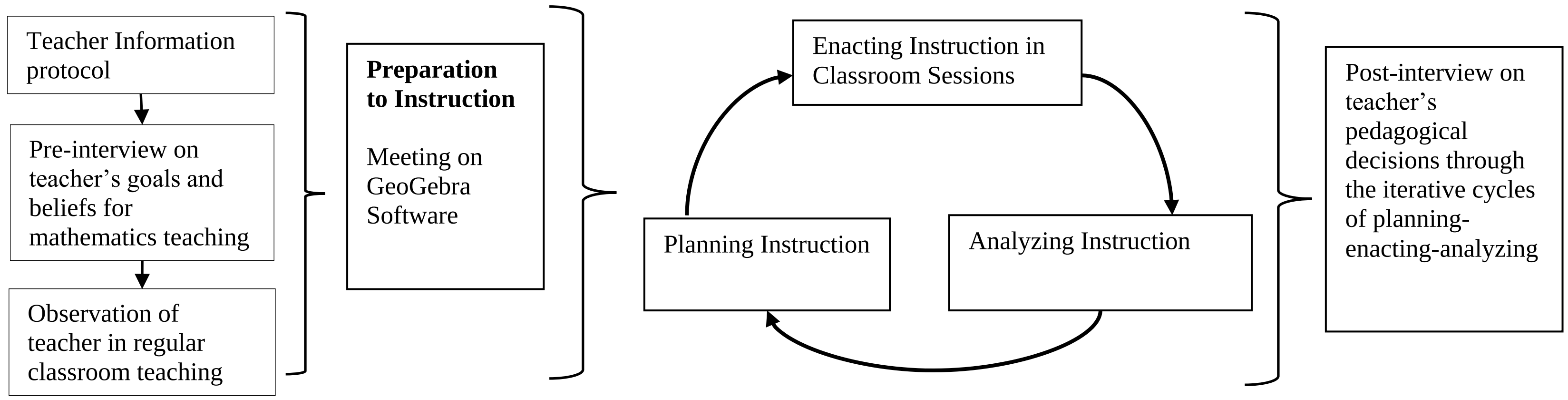

Figure 8. The diagram of process of the main study that aims to develop a teacher's set of mathematical practices in classroom teaching sessions

In the enactment of the instructional sequence, the teacher implemented the lessons in two eighth grade classes, which was different from the prototype phase. After the first classroom teaching was implemented, as suggested by Cobb and Gravemeijer (2008), the team considered the current conjectures about students' learning to use them in constructing the next instructional activities to be used after a day or two. Therefore, after the lesson implementations, instruction was analyzed with the teacher to evaluate and revise the instructional sequence. Considering the previous lessons, the teacher revised her plan when she needed a change. Therefore, while the beginning sessions with interviews on lesson plans was named as "planning sessions", the interview sessions made after the implementation of the lesson to evaluate the enacted lesson and to revise further lessons were named as "analyzing sessions" as mentioned in the prototype phase. In addition, based on the teacher's envisioned classroom activity sheets, the team planned and constructed GGB materials to be used with those activity sheets in classroom teaching. When necessary, the teacher changed these materials or activity sheets considering the consistency between an activity sheet and a material. After the first cycle, the analysis of the instruction was integrated with the planning for cycle 2 and cycle 3. After the second cycle, the analysis of the instruction was integrated with the planning for cycle 3. In the last stage of the main study, the researcher conducted an interview as a post-interview about the teachers' pedagogical decisions through the whole process as iterative cycles of planning-enacting-analyzing. Ongoing analysis was made during this process.

The detailed research schedule was given under the table regarding the research agenda with teacher Oya (See Table *4*). The sources of data collection were explained under the subsections: teacher information protocol, teacher pre-interview, meeting with the teacher on GGB, planning the instruction sessions, classroom teaching, and analyzing the instruction sessions.

Table 4. Research agenda with Teacher Oya

Data Sources	Int. time	Obsv. time	Description of the Sources
Teacher Pre-interview	90 min		Interview and teacher information protocol
Meeting on GGB	100 min		Use of GGB materials in classroom
Classroom observation		180 min	Prism (8A; 8B)
Classroom observation		90 min	Prism & Pyramid (8B)
Planning instruction	60 min		Conceptual analysis of the slope, preparing instructional sequence using HLT dimensions, planning Battens, Fire Truck and Tent activities
Classroom observation		90 min	Prism & Pyramid (8A)
Planning instruction	60 min		Planning the activities of Designing building, Leaking container, Slopes and equations of lines, and Mobile operators
Planning instruction	120 min		Organization of all the activities, preparing HLT, classroom activity structure
Classroom Sessions		180 min	Batten, Fire Truck and Tent activities (8A; 8B)
Analyzing instruction	90 min		
Classroom Sessions		90 min	(8B) Building Design, Leaking Container, slopes and equations of lines, Mobile operators activities
Analyzing instruction	90 min		
Classroom Sessions		90 min	(8A) Building Design, Leaking Container, slopes and equations of lines, Mobile operators activities
Analyzing instruction	60 min		
Planning instruction	60 min		Planning Stores and Equations systems activities
Classroom sessions		90 min	(8B) Mobile operators, Equation systems, Stores M and N
Classroom sessions		90 min	(8A) Mobile operators, Equation systems, Stores M and N
Analyzing instruction	80 min		
Teacher Post-interview	40 min		
Total	840 min	900 min	1710 min

3.4.3.1 Teacher Information Protocol

The teacher information protocol was structured to get accurate data about the participant teachers' past and present. It includes questions on undergraduate and graduate education (questions 1-9), teaching experiences in the past (questions 10-12) and teaching experiences in the current school (questions 12-20) (see Appendix B). In other words, the protocol was applied to understand the teacher's background and experience before the design process within the context of the study.

3.4.3.2 Teacher Pre-Interview

The teacher pre-interview was conducted out of the classroom environment by the researcher and was recorded by an audio-recorder. The teacher pre-interview was conducted with a semi-structured pre-interview protocol and provided an insight for the researcher into the teacher's goals and beliefs about the nature of teaching and learning mathematics before the design process. The teacher pre-interview protocol was developed considering the theory of teaching-in-context (Schoenfeld, 1998) and the studies on beliefs about mathematics teaching (Haser, 2006; Çetinkaya 2006). Schoenfeld (1998) emphasized that the goals and beliefs of the teachers regarding their teaching practice within the actual classroom context were the indispensable part of teaching. In this regard, while examining the actions of teachers, it is necessary to gather information about teachers' professed goals and beliefs to interpret teachers' actual behaviors with the indicated goals that they possessed (Schoenfeld, 1998). Therefore, the interview protocol involves questions on goals (A), beliefs about the nature of mathematics (B.1), beliefs about learning mathematics (B.2), beliefs about teaching mathematics (B.3), beliefs about particular students and classes of students (B.4), and other questions (B.5) (see Appendix C).

In the protocol, the questions on goals and beliefs were arranged from general thoughts about learning and teaching mathematics to specific thoughts about learning and teaching slope, linear equations, and graphs. For example, in the questions regarding goals (part A), while asking about short term goals, the third question is about slope and equations objective. The questions on goals in part A were structured through the guidelines and examples of Schoenfeld (1998). The belief questions in part B were structured through the guidelines of Schoenfeld (1998) and Haser's (2006) study. The pilot study of the pre-interview was done with two middle school mathematics teachers. After the first piloting, metaphor questions on learning and teaching mathematics (Çetinkaya, 2006) were added into the process. During the second piloting, it was thought that these metaphor questions could help teachers to express their thoughts clearly. In the first version of the study, there were also questions on teachers' knowledge of algebra and algebra teaching. However, the opinions of two professors about these questions were that they could shape or manipulate teachers' knowledge and teaching, which is considered as the researcher effect, and they could be asked or encountered in the planning sessions with the teachers. Therefore, those questions were removed from this phase to ask during the planning sessions phase to gain an insight into the teacher's behaviors in consideration with the subject that will be taught and into the teacher's mathematical knowledge for teaching (i.e. knowledge of slope computation algorithm).

3.4.3.3 Meeting with the Teacher regarding GeoGebra

After the researcher conducted a pre-interview on the teacher's goals and beliefs about teaching mathematics with the teacher, they met to talk about the use of GGB software in mathematics teaching. Before the meeting, the teacher examined some GGB materials that the researcher gave her. Since the teacher did not use GGB or any other dynamic software in algebra teaching before, the researcher explained the properties and tools of GGB to the teacher in the meetings. They discussed the usefulness and the properties of GGB materials that the researcher constructed before and got from geogebra.org. Then, they discussed which tools and properties of GGB should be used for developing classroom activities. Then, the teacher decided to develop a GGB material for each activity with an activity sheet. Besides, during the meeting, they also talked about how GGB could be used in teaching slope, linear equations, and graphs on different GGB materials. After the meeting, the teacher re-examined some GGB activities for algebra teaching that the researcher gave her. While the meeting was recorded by an audio-recorder, it was not used in data analysis.

3.4.3.4 Planning Instruction

While planning instruction sessions were done, the researcher conducted planning interviews. These interviews were conducted in a meeting setting which involved a planning process designed by the teacher and planning questions asked by the researcher. The purpose of making planning interviews was to accompany the teacher in the process of planning the instructional sequence and instructional activities for classroom teaching. These interviews were used to gain an insight into the teacher's planned mathematical practices in classroom teaching and the classroom activity structure. There were four planning interviews. Three of them were made before the instructional sequence, while one was made during the instructional sequence. All the sessions were recorded by an audio-recorder.

In the planning phase of the instructional sequence, the team (the teacher and the researchers) considered the instructional goals based on both the national curriculum in use (MoNE, 2009) and the revised version of the curriculum (MoNE, 2013a) even though it was not in use in eight grades at the time of design experiment. Moreover, while clarifying the goals, they questioned the goals in the curricula. The teacher realized that while multiple representations of slope were emphasized in the curriculum, what is meant by model of slope was unclear. In this regard, they concluded that the problem was about the conceptions of slope with its computational algorithm. Therefore, they conjectured an instructional sequence considering slope as ratio-as-measure. This approach led the transformation from the objective of "Explain slope of a straight line with models" to *"Explain slope of a physical object/feature"* with instructional goals of *"Notice and describe slope as a measurement for steepness of a physical object", "Relate vertical and horizontal distances to the slope of a physical object", "Compute slope for a given physical object", "Evaluate the effects of horizontal and vertical distances on computation of slope a physical object as a ratio"* without mentioning direction and sign issues and not explaining them as a model of slope of line. In addition, considering the previous year's objectives and mathematical activities and to provide a connectedness between the ideas of linear equations, graphs, and slope, and a real-life context for slope of line, the idea of slope of line was explained through the functional situations in realistic contexts without mentioning the concept of function to students. Therefore, the objective of "Relate slope of a straight line and equation of a straight line" evolved as *"Relate slope of a straight line and equation of a straight line for functional situations"* with instructional goals of *"Compute slope of straight line relating table, equation, and graph of the* *functional situations"* and "*Interpret magnitude and sign of slope of line for functional situations/graphs*". At last, they concluded that linear equations systems could not be thought independently from the slopes of these equations. Then, the objective of "Solve linear equation systems using graphics" evolved as *"Solve linear equations systems relating graphs and slopes of lines and solutions of system of equations"*.

In the planning phase of the instructional sequence, while organizing the instructional goals, the teacher relied on students' previous knowledge and experiences on learning ratio, rate, linear equations and graphs since she had taught them for 2 years. She also relied on previous year's eight-graders' understanding on slope, linear equations, and graphs. The researcher relied on students' understandings in the prototype phase which was conducted in a different teacher's mathematics classroom with students who had similar profiles and backgrounds. Based on these experiences, it was concluded that most of the students did not develop an understanding of slope with different conceptions and just calculated the slope of a line without interpreting the computational algorithm of slope. In addition, they did not see slope as an indirect measure that is computed by a dynamic ratio. As a theoretical context, the researcher situated the slope, linear equations and graphs topics in classroom experiments with the guidance of the quantitative reasoning framework (Thompson, 1993, 1994b), conceptions of slope (Stump, 1999, 2001b, Moore-Russo et al., 2011), and understanding of ratio-as-measure for slope (Lobato & Thanheiser, 2002). In addition, the team's perspective on learning and teaching is parallel to RME heuristics (Gravemeijer, et al., 2003) that were explained in section of Preliminary Phase of the Study and Greeno and Engeström's (2014) situativity of learning that was based on the idea of "everything that people can do is both social and individual" (Greeno, 1997, p.9). Further detail on instructional sequence was given under the section titled teacher's instructional sequence with the envisioned and actual considerations in Chapter four.

In beginning of the aforementioned process, the researcher guided the teacher to make a conceptual analysis about slope, linear equations, and graphs and provided the teacher with the HLT dimensions to guide her in composing the instructional sequence. Based on the dimensions of HLT and mostly on the teacher's knowledge of students' weak understanding of slope, linear equations, and graphs, various means to support the classroom activity structure were identified by the team after making a conceptual analysis of slope, linear equations, and graphs. As two important means, activity sheets and materials (GGB materials and other materials) were developed and revised considering the purpose of the activity, mathematical acts and students' possible mathematical understandings. As another mean, the nature of classroom discourse was clarified with what students should do individually and in group, which mathematical questions should be asked by students and the teacher, which mathematical comments and ideas might arise in classroom discussion while using technology and while not using technology, why students might have difficulty in an activity or with a GGB material, and what the teacher could do for misunderstandings. In this process, the teacher emphasized supporting students' learning by using concrete objects and multiple representations considering switching among multiple representations as means of supporting and organizing conceptions of slope, computation of slope, and computation algorithm of slope in students' reasoning.

As an example, when preparing the first experiment, the focus was the starting activity. It was a challenge to provide a support to make students reason on computational algorithm of slope in physical situations since they tend to just operate on numbers. In addition, it was a challenge to experience slope in a physical situation in class and to bring together various positions of physical object/feature to analyze slope on them. However, the teacher conjectured that students need to experience the process of concreting slope for the purpose of answering how to measure the slope of an object rather than calculate it. Furthermore, the teacher conjectured that the students could do that both by using concrete object of batten to create slope situations by hand work and by using GGB material that computerized the various slope situations dynamically to develop reasoning on the slope measurement. In this regard, how and when to give the computational algorithm of slope was a significant decision. While making this decision, the teacher decided that students should analyze the relevant and irrelevant factors affecting the slope of the battens. For example, one of the tasks of the first activity was to compare the slopes of different positions of the battens on equal length using measurements of horizontal and vertical lengths of those battens without knowing how to compute the slope of a batten.

During the planning interviews, the researcher and the teacher also made explanations and posed questions about the means of support. The examples of explanations and questions that the researcher and the teacher made and posed during these meetings were given below (see Table 5 and Table 6).

Table 5. Examples of the researcher's questions or explanations in planning meetings

Descriptions	The Researcher's Questions/Explanations
Discussing batten activity	• Were you making an introduction to the slope topic in the same way in previous years? • How do you make an introduction to the topic and how do you continue?
Discussing slope of graphs	• What do you think you should remind students at this point?
Question for activity sequence	• Which one would you like to start with? I think you said the physical situation.
Discussing Leaking container activity	• Actually, we are examining the rate of change, not the rate of the values here. We are interested in the rate of change. We did not focus on the concept of rate of change before, so we have a deficiency here. What ratio is actually the ratio of $y2 - y1$ to $x2 - x1$?
Discussing the activity of equations systems	• Do you think we can relate equation systems to slope? • We expressed this relationship in a graph. How can we express it algebraically? • Will we consider ratio again?

Table 6. Examples of the teacher's questions or explanations in planning meetings

Descriptions	The Teacher's Questions/Explanations
Discussing Batten activity	• For example, students measure the lengths of the battens and we enter these values in a GGB material • Earlier on, I ask questions like "Which one would make you tired?", "What happens if we do this?" • Let's adapt this to GGB, but we should use concrete objects as well. It is difficult to imagine some things although they (the students) see them in GGB.
Discussing activities for slope of graphs of linear equations	• I think students should first see the linear relationship through a situation. This is a topic in 7th grade. The opening price for the taximeter is 2 Turkish liras and 1.5 Turkish liras are paid per kilometer. If we draw a table, turn this table into a graph, and ask students to make a line graph considering the relationship, it is certain that the graph will be linear.
Explanation for GGB material	• Let's connect this variable in the leaking container situation to the slider tool in GGB (for the Leaking container activity)
Discussing the Leaking Container activity	• Here, while one increases, the other one decreases. I did not know this. I learned it from you the other day. That the minus in the slope comes from there… • Proportional relationship and increasing relationship or decreasing relationship still confuse my mind. We say proportional, but these two are different as expressions. We should not say so. We can say: While one (variable) decreases, what happens to the other (variable)? If the student realizes this, if the student realizes that while one decreases the other increases, we can generalize that the slope is negative. We should not form a relationship with proportion • …The difference between is 1 to 2 as ratio. If the rates of change are equal, can we say they are directly proportional? • Shall we ask about the relationship between the changes? They will surely say that they are the same. And that is the slope. Then, let's consider the rate of change between any two points. You know we always take the sequential points. We can ask such questions.

3.4.3.5 Enactment of Instruction in Classroom Sessions

For enacting instruction in the classroom teaching sessions, the team made decisions on data collection procedures and enacted the phases of the design and analysis. For data collection, it was decided to observe the teacher and take field notes, to record classroom sessions of two 8^{th} grade classes with a video-camera and a voice-recorder, to record the computer screens of groups of students, and to get copies of all the students' written work. There were 450 minutes of video records and audio records for each 8^{th} grade class. Totally, there were 900 minutes video records and audio records of the classroom teachings. All the video records of the classroom were transcribed verbatim.

During the data collection process, the researcher entered the classroom with the teacher. Before starting lessons, the researcher organized personal computers (laptops) for the groups of the students, voice recorder and video camera. Then, she sat down near the video camera at the back of the classroom (See Figure 9). The researcher used a voice-recorder and a video camera to record the teacher, computer-teacher and student-teacher interactions. The voice recorder was placed on the teacher's desk in front of the board. The video recorder was placed at the back of the classroom. The researcher followed the teacher and controlled the video camera on a tripod at the back of the classroom. For example, when the teacher had a discussion with a specific group of students, the researcher moved the camera toward the teacher and this group of students. The T-shape video camera icons in Figure *9* represent the movements of the camera. The green circles in Figure *9* represent the teacher's movements in classroom sessions.

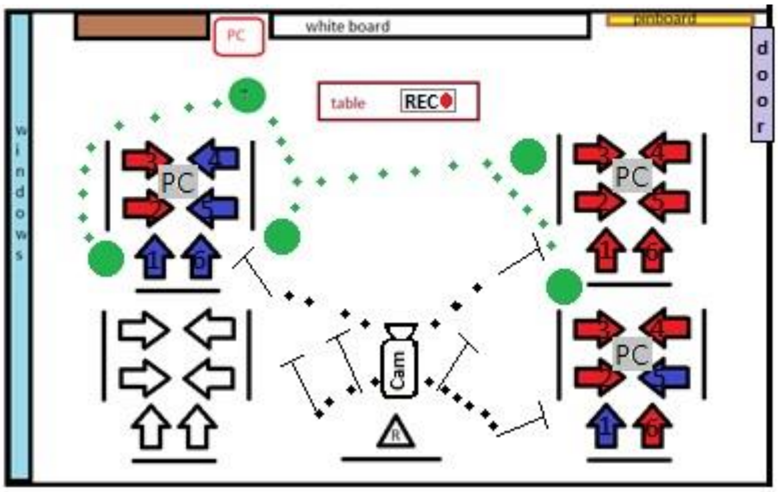

Figure 9. The positions of the video camera and audio recorder in the classroom.

Note. Cam: Video camera (T shape icons represent the movements of the camera), REC: voice recorder, T: teacher (green circles represent the movements of the teacher), Red arrows: girls, blue arrows: boys, PC: personal computer, R: researcher (triangle represents the researcher).

In computer screen recording, after students opened their laptops, the screen recorder program (Youcam) captured the desktop of the laptop in full screen and recorded the students' voices as a voice recorder. The program saved the document as a video file. In getting the students' written work, the copies of students' activity sheets were obtained by scanning. The researcher also collected student's self-assessment sheets in which the teacher asked students what they learned during this teaching experiment. Therefore, the team had evidence on students' understanding of the lesson to evaluate and revise the instruction after the lesson.

As mentioned above, the researcher observed the teacher while she was teaching in two 8^{th} grade classes. The researcher participated the classes as a non-participant observer. For disclosing the researcher's role to the students, the teacher introduced her to the students as a researcher to observe her teaching and her interactions with them and not to evaluate their achievement. The teacher told students that they could behave just as in regular class hours. Since the researcher observed and video recorded the teacher within the classroom before the actual study, the students seemed to be not anxious about the researcher's presence in the classroom and not distracted by the video camera and the researcher's presence.

During the observations, the researcher purposefully took field notes on the teacher's mathematical practices while watching and listening (see Appendix D). The notes were descriptions, thoughts about the teacher's practices which involved the teacher's actions, acts, behaviors, routines, exceptional questions, and explanations in teaching, and questions about teachers' practices to be asked during the interviews conducted to analyze the instruction. In detail, the researcher wrote questions on how/why she decided to ask such a question or to make such a statement, and how/why she used GGB at that time. The researcher noted whether things did not happen when they were expected or planned to happen. Besides, the researcher noted questions about why she had changed her planned practices in teaching if she did not execute the planned activities in the classroom. Some examples of these questions were "What did you do mathematically while asking this question to students?", "Why did you ask this question to the students while using GGB?", "Why did you use trace tool at that moment?", and "Why didn't you use GGB while asking/explaining this mathematical idea?" With those notes and questions, the researcher tried to get both the nature of the teacher's mathematical practices and the teacher's rationale for acting in that way in the lessons. While the primary goal of data collection was to investigate the teacher's mathematical practices in classroom teaching, interpreting students' activity and learning in classroom after the enactment provided additional sources for triangulating the characterizations of the teacher's mathematical practices.

Observing the teacher in the classroom provided the researcher a key understanding of what the teacher did to create a classroom environment that supports her students' learning and teaching. In addition, the researcher came to understand how the teacher approached the students in different ways that varied through their understanding. Furthermore, considering the interviews conducted with the teacher for planning, the researcher better understood how she made connections between her students' understanding and the curriculum mathematics, and how she used her knowledge of teaching mathematics in classroom.

3.4.3.6 Analyzing Instruction

After the classroom teachings, the teacher and the researcher met to evaluate the lesson and students' learning and to make revisions for the further instructional activities. There were four analyzing instruction sessions with interviews (Appendix E). While collecting data during the analysis sessions, the researcher used the Interpersonal Process Recall Interview method using video-recordings of the classroom teaching. In this process, the teacher watched the episodes of video recording of the lesson. The teacher reflected and commented on the practices in her classroom teaching experience. In addition, the researcher posed questions on teaching practices in the classroom sessions considering the notes that were taken during classroom teaching and comments that the teacher made while watching the video recording. Specifically, these practices were the teacher's decisions for supporting students' mathematical understanding. The examples of the questions were "What did you think or expect while asking this question to the whole class?", "What did you think while students were making computations using the GGB material?", "Why did you use slider tool at that moment?", and "How do students understand the slope in this process?"

While interpreting student activity and learning during the instruction, the team considered students' both individual and group mathematical practices. In this process, the notion of classroom mathematical practices in the interpretive framework (Cobb & Yackel, 1996) guided the interpretations of the team. What students learned/did not learn, how they participated/did not participate in mathematical activities, how they used tools (concrete objects and GGB materials) in mathematical activities and the relations were identified in the social context of the classroom based on the interpretive framework. For example, in position of battens activity, the team considered whether students constructed the mathematical practice of determining which distances affect the slope of the batten. The teacher reflected on how they made these discriminations while watching this episode in the video-record. She concluded that the activity of comparing several measurements of distances (horizontal and vertical lengths) made by themselves and other students and evaluating their exact values on the GGB material provided a shift for students to get the idea factors of slope. In addition, the teacher concluded that while students considered the length of the batten as a factor of slope of batten, they saw that they could get different slope positions with the equal lengths of battens. These interpretations for the instructional sequence were made considering both the activities of individual students and groups of students with the teacher in the social structure of the classroom (Cobb & Yackel, 1996). The data obtained from these meetings provided a triangulation for the teacher's mathematical practices in classroom teaching.

3.4.3.7 Teacher Post-Interview

After the mini cycles were finished, the researcher and the team met to analyze the whole instructional sequence retrospectively. In this process, the researcher used a semi-structured interview protocol (Appendix F) to have the teacher reflect on her pedagogical decisions. This process provided information to the researcher about the teachers' analysis of the instructional sequence and her mathematical practices in classroom teaching. This process also gave feedback to the researcher about the design of the instructional sequence in which learning is supported or constrained (Stephan & Cobb, 2003).

In summary, the sources of data collection and the intended purpose for each data source were presented in Table *7* below. After the minicycles of plan, teach, and analysis and teacher post-interview, a retrospective analysis (Cobb & Gravemeijer, 2008) for the whole process of experimenting was conducted to evaluate the teacher's mathematical practices for developing students' learning and the means (activities, tools, classroom structure, and classroom discourse) that were used to support this learning. The details were given in the data analysis section.

Table 7. Data sources and the purpose of collecting them

Data Source	The purpose
Teacher Information Protocol	To describe the context and the teacher of the study
Teacher Pre-interview	To describe teacher's goals and beliefs that complete the model of mathematical practices of the teacher
Planning sessions	To investigate planned mathematical practices for teaching
Classroom Teachings, Field Notes	To investigate mathematical practices of the teacher
Analyzing sessions	To triangulate mathematical practices in teaching by reflecting on them
Teacher Post-interview	To investigate the teacher's analysis of the design process

3.5 Data Analysis

This study used grounded theory methods to analyze data (Strauss & Corbin, 1998). Grounded theory methods is a way of a discovery for developing theories and concepts based on systematic data collection, abstraction of actions (interactions, relations), and making interrelations of categories (Strauss & Corbin, 1998). Comparative analysis method of grounded theory is used to collect accurate evidence, to provide empirical generalizations, to specify unit of analysis, to generate a theory or to verify the existing or emergent theories (Glaser & Strauss, 2006). In this regard, the constant comparative method was used in this study (Glaser & Strauss, 2006).

While this study had a huge data set with rich data sources, the data that was focused was a teacher's mathematical practices in classroom teaching sessions in the main study which involves three phases that were developed based on the objectives in the national curriculum. That is, the focus of these phases in the study was the mathematical practices in the enactment of instruction in classroom teaching. In Table 8, it was showed that the data were analyzed under these phases in accordance with the research questions. In classroom teaching, the teacher's mathematical practices were analyzed in twofold.. In the first fold, they were analyzed while the teacher used GGB materials and concrete objects. Although the research problem was structured with the aim of investigating mathematical practices during the use of technological tools (i.e. GGB materials) and did not considered any other instructional tools, the teacher used concrete objects as instructional tools in classroom teaching. Therefore, the mathematical practices that were performed when the teacher used concrete objects did not ignored in data analysis and the mathematical practices that were demonstrated when the teacher used GGB materials and concrete objects were presented under the same section in the findings with the intention of giving a coherent understanding of the teacher's mathematical practices during using instructional tools. In the second fold, the teachers' mathematical practices were analyzed while the teacher explained mathematical ideas without using GGB materials and concrete objects. While analyzing the data, this study mainly used the knowledge for algebra teaching framework as an interpretive framework for the teacher's mathematical practices. In the following section, the framework and the analysis of mathematical practices in classroom teaching were described.

Table 8. Categories of mathematical practices in tasks of teaching

	Phase 1		**Phase 2**		**Phase3**	
Tasks of teaching / **Practices**	**UT**	**EMI**	**UT**	**EMI**	**UT**	**EMI**
Bridging **Trimming** **Decompressing**	RQ 1	RQ2	RQ1	RQ2	RQ1	RQ2

Note: UT: Use tools (i.e. GGB materials and concrete objects); EMI: Explain mathematical ideas

3.5.1 Use of Knowledge of Algebra for Teaching (KAT) framework

The KAT framework (Ferrini-Mundy et al., 2005; McCrory et al., 2012) provided guidance for analyzing teachers' mathematical practices in the current study. The aim of this framework is to describe how teachers use their knowledge in their activities as mathematical practices for developing an assessment of teacher's knowledge for teaching algebra. The framework describes teacher's mathematical practices based on their hypothesized mathematical content knowledge (advanced, school and teaching knowledge). These practices are categorized as trimming, bridging, and decompressing. Indeed, these practices are teachers' concrete ways of leveraging their content knowledge in their teaching (Stockton & Wasserman, 2017). In other words, these practices are ways of professional work particularly mathematics teaching (Wasserman, 2015). Therefore, this study explored how these forms of mathematical practices emerged in technology enhanced classroom setting.

The term of teaching practice or mathematical practice is not thoroughly described or defined in the KAT framework. Besides McCrory et al. (2012), Wasserman (2015) explained these mathematical practices as moves in classroom teaching that describe actions and responses involving mathematics in nature and mathematical consideration. In this regard, the researcher constructed the analysis on the following definition that is parallel with the perspective of the KAT framework. While making this definition, the researcher benefited from Cobb, Stephan, McClain, and Gravemeijer (2001) and Moschkovich's (2004) studies that explained mathematical practice from students' and learner's perspective and Drijver et al 's

(2010) study on the notion of instrumental orchestration that explains teacher behavior. Therefore, a mathematics teacher's mathematical practices in classroom teaching can be defined as ways of acting that have emerged while sharing mathematics to reason, argue, and symbolize mathematical ideas in a didactical performance. A mathematics teacher's didactical performance that is described with the metaphor of instrumental orchestration involves "ad hoc decisions" of the teacher on actual actions of her teaching in intentionally preferred ways of mathematics teaching in an arranged classroom setting (Drijvers et al., 2010, p. 215). That is, the work of teaching mathematics requires moves that involve mathematical actions. In this regard, these actions and responses can be thought both in taken-as-shared ways (Cobb et al., 2001) and unidirectional ways since a teacher can also act in a mathematical way even though students cannot discuss her understanding of the meaning of ideas. In other words, a teacher can construct or initiate mathematical practices even though her students cannot act with her in a didactical performance. However, when students and teacher share an act while discussing a mathematical idea, this can provide a base for a teacher's didactical performance.

In McCrory et al.'s (2012) KAT framework, bridging practices are described as ways of making connections across topics, assignments, representations, and domains. That is, bridging means "providing students with the big picture of mathematics, making explicit connections across topics, keeping a range of ideas in play in the classroom, and presenting mathematics as a coherent, connected endeavor" (McCrory et al., 2012, p. 608). In this regard, a teacher manifesting these actions of bridging can have an understanding of mathematics as a connected and coherent realm.

As another practice in KAT framework (McCrory et al., 2012), trimming practices are described as ways of removing the complexity of mathematical ideas while holding mathematical integrity. That is, trimming means "retaining the integrity of the mathematical ideas, with special care not to embed ideas that are correct for the current context but lead to problems in more advanced mathematical work" (McCrory et al., 2012, p. 606). In this regard, for a teacher, actions of trimming can require to have an understanding of advanced mathematics, school mathematics, and the curriculum in use.

As another category of the mathematical practice in KAT framework (McCrory et al., 2012), decompressing is described as making the complexity of mathematical ideas explicit in ways that make them comprehensible. Similar to Ball and Bass's (2000) unpacking, this category involves actions that "intentionally highlight and describe some local complexity within a topic" (Stockton & Wasserman, 2017). That is, decompressing means revealing or unpacking the complexity of mathematics for students (McCrory et al., 2012).

To clarify these practices, the related terms and the studies that referred to similar practices were given in Table *9*

Table 9. Teaching practices and corresponding terms used to explain these practices

Practices	**Explained in a similar vein**
Bridging	Connections between fractions in primary grades and algebraic skills and concepts in upper grades (Wu, 2001) Algebrafing arithmetic in primary grades to formalize algebraic skills in upper grades (Van Dooren, Verschaffel, & Onghena, 2002) "recontextualization and repersonalization" of the knowledge in teaching as a part of didactical transposition (Brousseau, 2002, p. 23)
Trimming	"trimming away the reality" as a step of mathematization process (OECD, 2003, p. 37) Teaching in an "intellectually honest" way (Bruner, 1960, p. 33) Mathematical integrity (Ball & Bass, 2000)
Decompressing	Decompression as "deconstructing owns mathematical knowledge into less polished and final form" (Ball & Bass, 2000, p. 98) Unpacking (Cohen, 2004)

Note: This table was prepared through the citations from McCrory et al. (2012) and Ferrini-Mundy et al. (2005).

3.5.2 Analysis of Mathematical Practices in Classroom Teaching

While collecting data, the researcher started to analyze data which came from the classroom observation write-ups that were converted from field notes and video records. After the interviews and visiting the regular classroom setting, the researcher reflected on the contact using the Contact Summary Form. This form was prepared by using an example in Miles and Huberman (1994). An example of the contact summary form was given in appendices (see Appendix G). The researcher used this form to outline how to use the KAT framework in analysis of teaching and to consider the teacher's classroom structure and norms before analyzing video recordings of the classroom teaching. Therefore, the ongoing analyses of field notes that came from the observation of lesson implementation in classroom teaching, as guided by the KAT framework, attempted to outline the characteristics of the categories of mathematical practices of the teacher in classroom teaching that are bridging, trimming, and decompressing.

Before the analysis of classroom teaching, video recordings of the classroom teaching sessions were transcribed in addition to audio recordings of the classroom teaching sessions and video recordings of student groups' computers. In data analysis, the researcher started with the microanalysis of the transcripts through line-by-line analysis. This analysis was used to generate initial categories and to suggest relationships among categories with open coding and axial coding. While doing this analysis, the teachers' acts, events and activities were conceptualized and classified when the actions of the teacher occurred in a pattern during the classroom teaching. Moreover, theoretical comparisons were made to identify the derived properties and dimensions of the concepts from the incidents in the literature. In this process, the conceptual framework (McCrory et al., 2012) was appropriate to use as an analytic tool to analyze data.

While using KAT framework in the process of data analysis, the researcher used bridging, trimming, and decompressing practices as the main themes of the coding process. However, the categories as the practices under the themes and the sub-categories as the teacher's actions under the practices were not determined. The researcher started to code the actions considering the bridging, trimming and decompressing themes. In this process the researcher did open coding, the researcher named the sub-categories considering the memos, the literature, and in vivo codes that were words of the teacher herself. Then, the concepts were grouped into the categories with related dimensions and properties. In axial coding, the researcher linked the related categories and subcategories considering structural conditions, the process and the actions. As a result, under the aforementioned themes, the categories as the practices under the themes were used to combine sub-categories as the sub-practices. The sub-categories under the practices were used to combine the codes as the teacher's actions. For example, under the decompressing theme, there were a category of decompressing practice with sub-practices involving the actions. In addition, the researcher defined a category of preliminary decompressing. The researcher used this term to describe a practice in which a teacher provides an action to unpack the complexity of a mathematical idea but continued to describe the idea with another decompressing practice or did not fully describe the idea in a comprehensible way. Besides, the researcher gave this practice as a category under the decompressing theme in the findings of the study rather than as another theme of mathematical practice. Therefore, preliminary decompressing can be considered as an introductory level of decompressing. After that, a larger scheme and frame were formed for the study by integrating and refining categories relationally as selective coding. In this scheme, the researcher approximated these practices while the teacher using the concrete objects and GGB materials and not using the concrete objects and GGB materials. Furthermore, during the analysis, questioning and comparison techniques were used as analytic tools. For example, the researcher looked for opposite examples to bring out the significant properties of a category. In this regard, the researcher gave the opposite examples in the findings.

While the primary framework that is used for the data analysis is KAT framework, the researcher utilized different frameworks conceptually. More specifically, the researcher benefited from the covariational reasoning framework (Carlson et al., 2002) to clarify the terms of coordinating, direction, variables, rate of change in the practices. In addition, Thompson's (1994b) quantitative reasoning framework enlightened the decomposition of quantity, ratio, rate and slope interrelations in the practices. Furthermore, since teachers used concrete objects and GGB materials, the researcher needed to understand the teachers' mathematical work considering these tools. In this sense, the researcher benefited from Drijvers et al.'s (2010) instrumental orchestration framework considering their way of explaining and categorization of instrumentation types based on the elements of didactical configuration and exploitation mode. The researcher decided to adopt their way of explaining a teacher's instrumentation type from a global perspective to categorization of the teacher's mathematical practices as ways of didactical performance. In this regard, when the researcher described the actions she gave detail on how she performed the mathematical idea. For example, in the theme of trimming practices, the researcher give the actions in a detailed way that whether the teacher directly used a tool of the GGB material or encouraged the students to use the tool or trace the students' understanding while teacher/students using the tools.

In this process, the transcripts of the planning and analyzing sessions were used for triangulation of the enacted actions. For example, while naming a code as a practice, the researcher considered the teachers' planned classroom actions during the planning sessions and the teacher's reflections during the analyzing sessions. As another example, while analyzing the teacher's mathematical practices in class, the researcher used the teacher's reflections on her classroom mathematical practices during the analyzing sessions to gain understanding about the teacher's mathematical intention in class. In addition, the researcher watched the videos of classroom teachings and audios of planning and analyzing sessions when she needed. In a similar vein, students' documents were used to support the intended mathematical practices with their written responses. Therefore, the researcher investigated whether there was sufficient repetition of the observed actions in the data sources to support the observed actions as a practice. In this process, the transcripts of the two-hour implementation of classroom teaching were coded by two co-coders in a full consensus. Furthermore, the participant teacher checked the analysis of the transcripts of the two-hour implementation of classroom teaching. While using the constant comparative method, the researcher also investigated the regularity of the actions throughout the three phases and two eight-grade classes. To provide coherence, integrity, and fluency in reading findings, the practices were explained through one eight-grade class in Chapter four.

3.6 Trustworthiness of the Study

In this study, Bakker and van Eerde's (2012, 2015) organization of trustworthiness which is based upon the guidelines of Miles and Huberman (1994) was taken into consideration under the dimensions of internal validity, external validity, internal reliability, and external reliability. Various techniques that were used for providing the trustworthiness of this study were explained below under the validity and reliability issues.

Internal validity as credibility in qualitative research denotes the quality of data collections and arguments (Bakker & van Eerde, 2012). In this study, several techniques were used for internal validity. Context-rich and thick descriptions were provided in the method and findings sections. Both the configuration and the temporal issues of the elements of the context, e.g., appearance of the classroom with physical and social properties, time schedule of the teacher's teaching were considered. In the analysis, conjectures of practices tested in a specific episode of classroom teaching were tested from a specific episode to another episode. During this testing stage in the analysis, counterexamples of the conjectures of practices made in episodes were searched for. In addition, those practices were tested by other data sources for data triangulation. For example, field notes, interviews with the teacher, and students' written work were used. In addition, the episodes were analyzed with multiple theoretical instruments of analysis for theoretical triangulation. Therefore, triangulation of theory was satisfied by the model of Hill et al. (2008) and the framework of McCrory et al. (2012). Moreover, the concepts in the findings were related systematically with each other, i.e. relating categories and subcategories at the level of their properties and dimensions.

To have valid interpretation of the findings, the researcher tried to overcome the biases about the teacher and the school environment. That is, the researcher began to engage the class before the research and tried to identify the classroom and to be familiar with the classroom to prevent the researcher effect in the settings. For example, she used the video camera to record the lesson implementation of the teacher in the classroom before the actual research topic. Thus, it was tried to clarify the researcher bias from the outset of the study for readers and to understand the researcher's position in the classroom. A second coder for the data was not used only for reliability but also for handling the overgeneralization and researcher bias.

External validity or transferability refers to the generalizability of the results (Bakker & van Eerde, 2012). The way the results were presented may help other researchers to accustom themselves to their own local contexts as an audit trail. This means that the researcher framed issues as instances of something more general (Gravemeijer & Cobb, 2001). Then, a setting where the findings of this study could be followed and tested in further research was suggested. Besides, the characteristics of the participants and the context were described to allow for comparisons with other samples. The limiting effects of sample selection and the setting were mentioned to examine the threats to generalizability.

Internal reliability as dependability indicates reliability within a research project; that is, independency of data collection and analysis from the researcher (Bakker & van Eerde, 2012). To improve internal reliability, the researcher used the following methods. The researcher collected data with audio-recorder and video-recorder. In analysis, systematic coding was done through open coding, axial coding and selective coding through the process. Moreover, reliability was ensured by coding of the data by second coders (investigator triangulation). The transcripts of the two-hour implementation of classroom teaching were coded by two co-coders. In addition, the transcripts of one hour planning interview and one hour analyzing interview were coded by one of the co-coders. The co-coders were informed about the teacher's classroom structure, norms, classroom activities, concrete objects, GGB materials, and KAT framework. The co-coders used and looked into the related activity sheets and video-records when they needed to understand the setting. After the research questions and the purpose of the study were described to them, the researcher explained and gave them a list of the codes with related examples and explanations. The coders and the researcher had an agreement on the codes of the transcripts with full consensus. The co-coders were doctoral students in mathematics education and had a teaching experience.

In addition, the researcher ensured reliability and validity of the data analysis by member-checking that was done with the participant teacher for the transcripts of the two-hour implementation of classroom teaching. The document of the open coding of the transcripts and the researcher's commentary on lesson implementation were given to the teacher, and the teacher checked the coding of her implementation. The teacher agreed on the coding and commentaries about her lesson implementation.

Last but not least, the researcher ensured the reliability by peer examination within three ways. The supervisor were in involved in the research design and research process with the teachers. There were weekly meetings with the supervisor during the research process. In addition, the dissertation monitoring committee and the supervisor, who are professors in the field, helped the researcher to cope with the problems and irresolution experienced in the setting and while analyzing data. Then, a mathematics teacher, who is a doctoral student, helped the researcher to discuss and interpret the data from a teacher perspective independently from the theory. This helped the researcher to interpret the data from a teacher perspective.

External reliability refers to *replicability,* which means “the conclusions of the study should depend on the subjects and conditions in the research” rather than on the researcher (Bakker & van Eerde, 2012, p. 445). Therefore, the current study was documented in such a way that the method and findings of the study were described explicitly and in detail to be followed. The researcher particularly explained the teacher and the context to help understand the conditions in detail. Multiple data sources were used in a composition (data triangulation) to draw results and conclusions about the study. A criterion for virtual replicability is *trackability* (Gravemeijer & Cobb, 2001), which means that others can follow the researcher’s learning process and construction of the study (Bakker & van Eerde, 2012). Considering trackability, the researcher explained the research process and the analysis procedures as they actually occurred for those who want to track the same or a similar research path in the future.

3.7 Researcher Role

In this study, I had a dual role. From one aspect, I was a non-participant researcher; from another aspect I was a participant researcher. Before the study, I revealed the purpose of the study to the participants with a full disclosure. I explained what my role was, my possible interventions, and how teachers can engage in the setting during the study.

In the planning process, my intention in the meetings was to support the teacher about what she wanted to do while creating a classroom teaching experiment through a HLT. After I explained the dimensions of HLT for a classroom teaching experiment, I considered the teacher as a decision-maker of the whole process. I was primarily there for technical assistance and secondarily for discussing and suggesting ideas for the activities that teacher wanted to develop and use through the HLT. At that moment, I was a technical supporter for preparing the GGB materials through the teacher’s plans and constructions. Therefore, in the whole process, the teacher decided what activity she wanted to do and when to use which activity. For example, in the first activity (i.e. positions of batten), teacher Oya wanted to integrate both concrete objects and technology in teaching, and wanted to design a GGB material and an activity sheet in parallel with the use of concrete object. I constructed the objects in the GGB material for the activity in accordance with the teacher's wishes and the teacher intervened as she wanted or changed her decision. As another example, while introducing slope of line, teacher Oya wanted to develop an activity (i.e. building design activity) based on the activity that she used for her students to teach linear equations when they were in grade 7. That is, she extended one of the activity that she used in previous years and adapted it to teach the slope of line with GGB. Although there were planned interview questions, the questions were structured more or less at that time through the meetings. In such an interaction with the teacher, I had a deep understanding about the teacher's knowledge of using dynamic software for mathematics teaching and how this knowledge guides the planning of a lesson and relates mathematical practices for teaching.

During the enactment process, I deployed myself as a non-participant observer. While the teacher was teaching the lesson, I did not interrupt the teacher's actions. I opened the students' laptops, and audio and video recorders. I controlled the video camera and took field notes. Before the study, the teacher introduced me to the students as a researcher that investigates her teaching. I had interactions with the students during the breaks. I answered their questions about the current study in such a way that they would not think they are being assessed. In addition, I asked what they are thinking about learning algebra topics with GGB. Most of the students explained that they like learning with GGB since they can see various possibilities at the same time and they can make mathematical interpretations while using GGB.

After the enactment, during the analysis process, my intention in the meetings was to provide classroom video records, students' works, and field notes to the teacher and to pose her questions to reveal her enacted mathematical actions during the instructional sequence in classroom teaching. While asking those questions, I did not criticize the teacher and her teaching. In addition, the teacher made reflections on her teaching (i.e. the how of mathematical usability of the technology in her teaching and the why of her mathematical actions in her teaching) and on students' learning (i.e. how they conceptualize the mathematical ideas and how they used concrete objects and technology in this process).

I believe that I built relations based on trust with the teachers while gathering data. During the process, I was at the school 3 days a week. Besides observing them in the classroom, I conducted interviews before and after the enactments and I spent time with them and with other teachers in the teachers' room and at lunch. In line with the purpose of the study, I was a witness to the whole process of planning-enacting-evaluating and revising of the lessons. In addition, I took notes of the informal conversations between us that were related to the study. This intensive process enabled me to be a witness of a day of mathematics teacher inside and outside the classroom in her school environment.

As a researcher, I believe that understanding school culture is important because teachers are in community in their school and they are interdependent parts of the school culture with school principal and vice-principal. Having this point of view, I visited the teachers' room with the mathematics teacher during breaks with other teachers' appraisal. After introducing myself as a mathematics education researcher working with the participant teacher in her classes, I informally observed the interactions among teachers and principals. During these moments, I saw that the participant teacher Oya was one of the passionate teachers in the school. She had active interaction with an English teacher and a Science teacher. They spoke about lessons and students and discussed the related topics in their field. However, she did not have an active interaction with the other mathematics teacher (There were 2 mathematics teachers in the school). She thought that the other mathematics teacher was "reluctant to learn new things in teaching such as GGB". While she offered her to share and show the GGB activities in the current study, the other mathematics teacher did not show any interest. On the other hand, she had a good relation with the principals. The vice-principal helped and supported the teacher and the students whenever any technological problem arose. The vice-principal and the teacher spoke on the applications of their projects and the vice-principal asked whether they need anything else. Besides, in her first year in this school, she convinced the principals and the teachers about changing the labels of the classrooms from grades (8A, 8B, 7A, etc.) to the subjects (mathematics, science, etc.). I noticed that she was ready to do anything that was necessary for her teaching, her students, and the school, and she was eager to learn and produce new things.

3.8 Ethical Issues

The research proposal of the study including the research agenda and all the other necessary documents were submitted for the approval of Human Research Committee in Applied Ethics Research Center in METU. After the approval of the committee, the documents were submitted to National Education Directorate of Ankara in Ankara Governorship and the required permissions were obtained to study in public middle schools (see Appendix H for ethics forms). The data collection procedures started after the required approvals were obtained. In line with this process, the stakeholders did not get any physical damage. The participants were not forced to participate in any moment of the study that they voluntarily participated. For keeping the privacy and the confidentially of the participants, the names of the participants were not used throughout the dissertation since pseudonyms was used. In the photographs, the faces of the subjects were not displayed. The data gathered during the research was not shared with someone and nobody could get access to the data. Besides, there was not any risk for the participants to attend such a study. They were informed clearly and explicitly about the purpose of the study and the implications of the study for the future. The research was conducted in a naturalistic classroom and school environment where neither teachers nor students encountered any destructive effect.

CHAPTER 4

FINDINGS

The purpose of this study was to examine middle school mathematics teachers' mathematical practices in a technology enhanced classroom environment, while enacting an instructional sequence that was designed about slope, linear equations, and graphs subjects for eight-graders. In line with this purpose, the current study was conducted throughout a multi-tiered design experiment in terms of teacher-tier that involved cycles of planning, enacting and analyzing the instructional sequence for classroom teaching in a technology enhanced classroom environment. In this technology enhanced classroom environment, the middle school mathematics teachers had the flexibility to decide whether or not to use GeoGebra (GGB), which is a dynamic mathematics software technology. In this regard, both situations of the teachers' mathematical practices in classroom teaching were taken into consideration: when GGB materials (i.e. GGB materials that were prepared with GGB software to be used within computers) were used and when GGB materials were not used in a technology-enhanced classroom environment. This was a two-phase study with prototype study and main study, the participants of which were two teachers in different public schools. In the research process, the first teacher was the part of the prototype study and the second teacher was part of the main study.

The findings of the study reported in this chapter are based on the findings obtained from the teacher in the main study, who implemented instruction in two eighth grade classes. The data obtained from both classes were analyzed. The teacher's mathematical practices are thoroughly explained in this chapter to ensure coherence, integrity, and fluency in reading the findings. This chapter is organized in three sections. First, the teacher's instructional sequence is summarized to provide a more meaningful reading about the mathematical practices of the teacher although the main purpose of the study is to reveal and elaborate on the mathematical practices. Thus, the teacher's instructional sequence that was enacted in this process turned out to be another product of this study. Second, the teacher's mathematical practices with the use of tools (i.e. GGB materials and concrete objects) are presented. In this section, the teacher's mathematical practices while using concrete objects are also presented with the practices while using GGB materials since they are also considered as tools for the instruction. Third, the teachers' mathematical practices in explaining mathematical ideas without using the tools (i.e. GGB materials and concrete objects) are presented.

4.1 The Teacher's Instructional Sequence

The instructional sequence for the subject of slope was divided into three phases. Each phase is comprised of a micro-cycle that involves the cycle of planning-enacting-analyzing. Each phase corresponds to the curriculum objectives in the algebra learning domain for Grade 8 (MoNE, 2009, 2013a) and the units on slope, linear equations, and graphs in the mathematic textbook (MoNE, 2012). However, the objectives in the national curriculum were revised by means of the conceptual analysis and the phases in this process (see Table *10*).

Table 10. Phases of the Instructional Sequence

	Phase 1	**Phase 2**	**Phase 3**
Objectives	Explain the slope of a physical object/feature	Relate the slope of a straight line and the equation of a line for functional situations	Solve linear equation systems relating graphs, slopes of lines, and solutions of equation systems
Activities	-Positions of Battens -Fire Truck -Tent	-Building Design -Leaking Container -Slopes and Equations of Lines	-Mobile Operators -Equation Systems -Stores

In phase 1, the principles were conceptualizing the term slope in physical situations, understanding slope as a measure of steepness in physical situations, and understanding the computational algorithm of slope as a ratio of vertical length to horizontal length in physical situations. Then, in phase 2, the principles were conceptualizing the slope of a line for a linear relationship in functional situations, understanding the slope of a line as an algebraic ratio and rate of change in functional situations, understanding computational algorithm of the slope of a line as a ratio of change in vertical dimension (variable) to change in horizontal dimension (variable). Last, in phase 3, the principles were conceptualizing system of linear equations in functional situations, understanding the solution of the system of linear equations in functional situations and in a coordinate system, and understanding the relations among a solution set, position of lines, slopes, and equations in a system of equations. Therefore, the envisioned learning trajectory in the instructional sequence was designed to support students' mathematical understanding of slope in physical situations and in linear functional situations with linear equations and graphs as follows:

1. Noticing and describing slope as a measurement for steepness of a physical object.

Determining the influential quantities: The students compare and contrast a physical object in different positions that affect the slope of the physical object.

Measuring the influential quantities: The students measure the lengths of the quantities of a physical object that affect the slope of the object.

Organizing the influential quantities and slopes: The students discuss the slope of a physical object as a ratio of vertical length to horizontal length. Here, students compare various slopes, horizontal and vertical length values of an object on GGB material by using the dynamic text of computation. While doing so, the students start to develop an understanding of the relationships between horizontal length, vertical length, and slope.

2. Structuring the computation of slope of a given physical object. The students explain the computation of a slope as a ratio of vertical length to horizontal length. The students compute the slopes of a physical object in different positions that they create.
3. Evaluating the effects of horizontal and vertical lengths on the computation of the slope of a physical object as a ratio.

The students reason about slope. They develop a computational algorithm of the slope of a physical object that might be supported by referring to the changes in the quantities of the object on the GGB material. While doing so, in one activity the horizontal length remains constant (does not vary), whereas the vertical length and slope vary, and in another activity, while the vertical length remains constant, the horizontal length and slope vary.

4. Using the ratio as a measure of steepness in a computation text as a means of scaffolding and communicating the computation algorithm of a slope within various situations. Here, the students compare various slope, horizontal length and vertical length values of an object on GGB by using the object (line segment), a slider and dynamic text tools.
5. Noticing and describing the slope as an algebraic ratio for linear equations and their graphs.

Determining the effect of quantities on the slope of a line graph representing a linear relationship. The students determine the dimensions of vertical change and horizontal change in the line graph, which affect the slope of the line graph representing a linear relationship in a functional situation.

Determining the constant value of the slope for a line graph. The students determine the slope of a line with a constant value, which is independent of the points on the line for a functional situation.

6. Structuring the computation of slope of a line graph for a given linear relationship in a functional situation and on the coordinate plane. The students explain the computation of the slope of a line as a ratio of vertical change (in y-variable) to horizontal change (in x-variable) in a functional situation and on the coordinate plane. The students compute the slopes of lines in different positions.
7. Evaluating the effects of the attributes in the y-variable (vertical change) and in the x-variable (horizontal change) on the computation of the slope of a line graph in a functional situation.

The students reason about magnitude and the sign of the slope. They develop an understanding of the computational algorithm of the slope of a line, which might be supported by referring to the changes between two points on a line graph for a functional situation on the GGB material. While doing so, in one activity, the functional situation involve a linear relationship with a positive slope, and in another activity, the functional situation involve a linear relationship with a negative slope.

The computation algorithm of a slope in a functional situation is given within a dynamic text, in which the students can change the values (slope, horizontal change, vertical change) in the computation by dragging the points of the line graph of the functional situation.

8. Using rate of change for a linear equation to generalize the slope of a line representing a linear relationship for a functional situation. The students develop the computation of a slope as the rate of change in the y-variable in terms of the x-variable in a linear functional situation. The students coordinated the amount of changes in the variables while computing the slope of the line for a functional situation. The students use rate of change as a measure of speed, price, or any other indirect measure for a linear functional situation.
9. Evaluating the relation between the slope of a line and the equation of a line in a functional situation and in the coordinate system. The students develop an understanding of the relationships among slope, linear equation, and line graph by using drawings, tables, graphs, symbolic expressions, and verbal expressions in a functional situation and the coordinate system.
10. Noticing and describing the solution set for a system of linear equations algebraically and geometrically in the context of functional situations and the coordinate system.

Noticing one solution: The students determine the intersection point of the lines in the system of equations, which has one solution in the context of functional situation and in the coordinate system.

Determining the coordinates of the intersection point in the context of functional situation and in the coordinate system: The students contextualize the coordinates of an intersection point as the solution set of a system of equations, which has one solution in the functional situation.

Noticing no solution: The students determine no intersection point of the lines in the system of equations, which has no solution in the coordinate system. The students determine that the system of equations of parallel lines has no solution in the coordinate system.

11. Structuring the solution set for a system of equations algebraically and geometrically. The students explain the solution set of an equation system of in the coordinate system geometrically. The students make algebraic computations to solve an equation system.

12. Evaluating the relationship among a solution set of an equation system for two lines, the positions of the lines, and the slopes of the lines.

The students reason about the solution set. They develop an understanding of the relationships among the number of solutions and the positions and slopes of the lines in the equation system.

To explain the instructional sequence for classroom teaching in more detail, the activities in each phase are explained as follows:

In the first phase, three activities accompanied with activity sheets were designed for the students to explain the slope of a physical object. The first activity sheet involved the *Positions of Battens* activity (see Appendix I). The second activity sheet involved the *Fire Truck* and *Tent* activities (Appendices J and K). In these activities, the teacher used the terms *vertical length* and *horizontal length* for the vertical and horizontal dimensions of an inclined object. Therefore, to provide coherence in reading this section on the instructional sequence, it is important to note that the term *length* is used for vertical length, horizontal length, and the length of the object.

The aim underlying the *Positions of Battens* activity was to enable the students to relate the slope of a physical object to vertical and horizontal lengths of the object (i.e. batten as concrete object). The reason why this was the first activity to be administered to the students was to have the student structure the quantities (i.e. horizontal and vertical lengths of the batten) that had an impact on the slopes of the battens of equal length and to isolate the slope from the other quantity (i.e. length of the batten) that did not affect the slopes of the battens. In other words, the aim was to have the students structure how the slopes of the objects changed when the vertical and horizontal lengths of the object changed and when the lengths of the objects were equal. In this activity, the students were given four battens to lean against the wall as they wished to create a ramp. Three of the battens were given to the three student groups and one of the battens was given to a student with learning differences to do the activity with the teacher's help. The activity sheet involved four tasks, which were comparing the positions of the battens, measuring the vertical and horizontal lengths of the battens to compute the slopes of the battens, justifying the slope of the batten with the biggest slope value, and justifying the slope of the batten with the smallest slope value (Appendix I)

In the *Positions of Battens* activity, the students determined and measured the horizontal and vertical lengths of their battens with the meters they had and then they compared the battens in different positions. Therefore, the students could structure that the slopes of an object could be different in different positions, and it was independent of its length. In this process, the students made connections between their concrete objects (i.e. horizontal length and vertical length of the battens) and visual representation of the batten on the GGB material of the activity. The GGB material was used for allowing students to measure the real horizontal length and vertical length more easily by representing these lengths with line segments in the GGB graphics view (see Figure 10). For example, in this situation (i.e. leaning the batten against the wall), the students could structure the vertical length as the height of the batten and the horizontal length as the distance between a point at which the batten touches the wall and the other point that is aligned with the point that touches the ground.

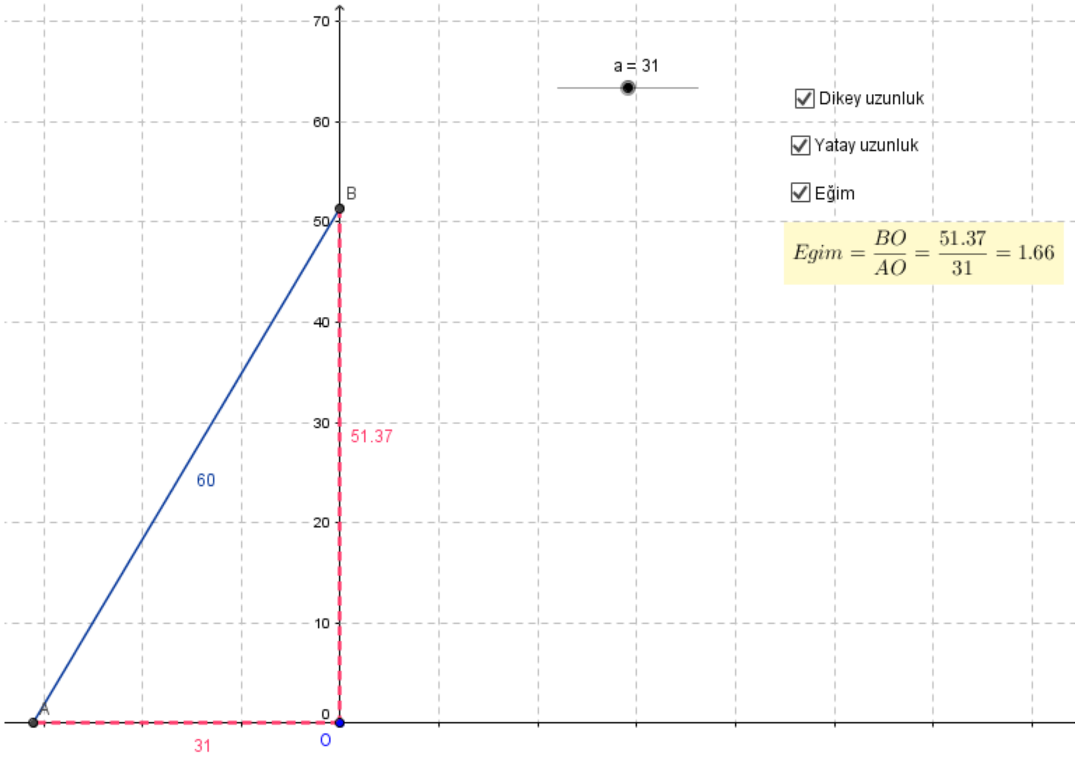

Figure 10. Positions of Battens activity: the GGB graphics view

In Figure 10, the GGB graphics view of the GGB material involves a constant length line segment (AB is 60 cm), slider-a, which is set with an interval between 1 and 60 and with the increment of 1, vertical length, horizontal length, and slope checkboxes, vertical line segment that represents varying vertical length, horizontal line segment that represents varying horizontal length. In the given GGB graphics view, vertical length, horizontal length, and slope checkboxes are clicked. When the vertical length and horizontal length checkboxes are clicked, the vertical (BO) and horizontal (AO) line segments appear together with their length values. When the slope checkboxes are clicked, the dynamic text of slope computation is shown by using the dynamic vertical length and horizontal length (horizontal length is connected to slider-a and position of the line segment (i.e. batten)). In this GGB material, the coordinate system was not used intentionally since representing a physical situation visually does not require a coordinate system.

Until that point, the students could measure the quantities affecting the slope of the batten, but could not measure the slope of the batten since they did not know the computation algorithm of the slope. Then, the students discussed how the slope changed when the quantities of their battens changed. In addition, the students could visually compare various slope representations (without given the mathematical values) that correspond to different positions of the batten with different horizontal and vertical length values on GGB as the teacher positioned the line segment (i.e. batten) with a slider tool (slider-a, Figure 10). It was anticipated that the discourse would focus on the students' reasoning about the computational algorithm of the slope that provided an indirect measurement of the slope as a ratio. In this computation, the students could compare the slope, horizontal length and vertical length and could structure the slope as a ratio of vertical length to horizontal length. After the students made computations of the slope as a ratio of vertical length to horizontal length, they again discussed this computational algorithm while comparing various slopes in the visual representations with slope values (i.e. dynamic text of the slope computation on GGB) that corresponded to different positions of the batten with different horizontal and vertical length values on GGB as the teacher positioned line segment with a slider tool. Then, all groups of students checked the computations of the slopes of the battens and the horizontal and vertical lengths of the battens as their horizontal length values were provided by the GGB slider that was connected to the horizontal length of the batten (remember, the length of the batten was constant). The aftermost discourse involved students' reasoning on how the slope of the batten increased or decreased considering the changes in the horizontal and vertical lengths on the GGB material using the trace of the batten, the slider tool, the dynamic text of the slope computation. The GGB material was designed to provide the students with the opportunity to derive meaning out of the computational algorithm of a slope in a physical situation. An important feature of this GGB material was that the change in the trace of the position of the batten, the change in the slope of the batten using the dynamic text of the computational algorithm, the change in the horizontal and vertical lengths, and the constant length of the batten were easily compared and related to each other.

After the *Positions of Battens* activity, the lesson was continued with the *Fire Truck* activity. The aim of the *Fire Truck* activity was to enable students to evaluate the effect of the vertical length of an object on the slope of the object when the horizontal length of the object was constant. The reason why this activity was administered was to have students reason on how the slope of an object (i.e. the fire truck ladder) varied with a quantity that affected the slope (i.e. vertical length of the ladder) when the other quantity (i.e. horizontal length) was constant after the students structured the affecting quantities in the computation algorithm of the slope of an object in the *Positions of Battens* activity. In other words, the aim was to enable the students to reason on the computational algorithm of the slope of an object in the form of $slope = \frac{vertical\ length}{horizontal\ length}$ when the horizontal length was constant. In the context of this activity, there was a building fire and a fire truck ladder accessing the burning floors of the building. The activity sheet included three tasks, which were computing the slope of the ladder in different positions when the horizontal length was constant, explaining the smallest slope in the situation, interpreting the effect of the height of the building as vertical distance on slope (Appendix J).

In the *Fire Truck* activity, the students computed the slope of the fire truck ladder in different positions after they created the situations on GGB using the slider tool (slider-f) that changed the vertical lengths of the ladder (See Figure 11). Then, they checked their computations after clicking the slope checkbox that showed the computation of slope within a dynamic text (See Figure 11). After these computations, the students discussed how the slope changed when the vertical length of the ladder changed and the horizontal length was constant. In this process, the students compared and connected various visual representations of the slope (i.e. line segments on GGB represented different positions of the ladder), the slope values and the computation algorithm of the slope (i.e. dynamic text) with varying vertical lengths and constant horizontal length using a slider tool. That is, the teacher wanted the students to comprehend the logic of how slope varies considering the change in vertical length when the horizontal length is constant.

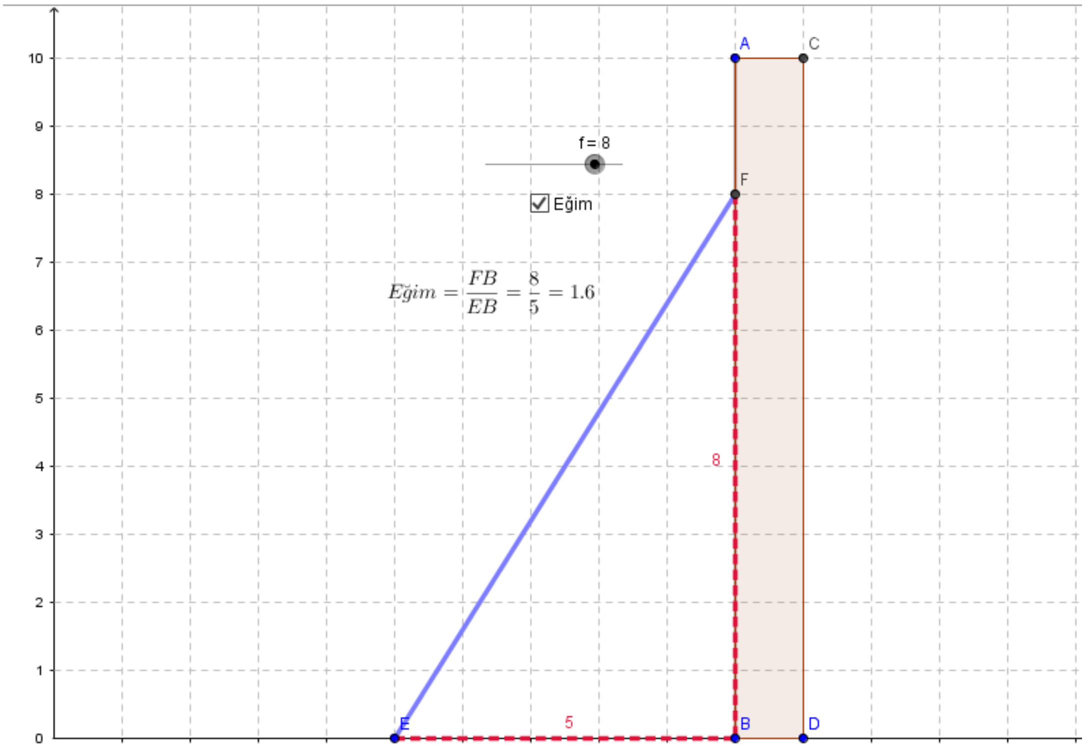

Figure 11. Fire Truck activity: the GGB graphics view.

In Figure 11, the GGB graphics view of the GGB material involves a rectangle that represents a building fire, a line segment that represents a fire truck ladder, two line segments that represent a varying vertical length and a constant horizontal length (EF represents 5 units), slider-f, which is set with an interval between 0 and 10 and with the increment of 1, and a slope checkbox. In addition, slider-f changed the vertical distance of the ladder, and thus, the position of the ladder and the length of the ladder. Students could see how the ladder looked when the floor (point F) was moved using the slider. In the given GGB graphics view, the slope checkbox is clicked. When the slope checkboxes are clicked, the dynamic text of the slope computation is shown with a constant value of horizontal length and a dynamic value of vertical length, and that is connected to slider-f and to the position of the line segment (i.e. fire truck ladder). In this GGB material, the coordinate system was not used intentionally since it was a physical situation.

Until that point, the students could reason on the computational algorithm of the slope as they discussed how the slope of the fire truck ladder increased or decreased with the vertical length when the horizontal length was constant on the GGB material by using the ladder object, the slider tool, and the dynamic text of slope computation.

In addition, they discussed the value and computation of the slope when the vertical length was zero in the fire truck ladder situation on the GGB material. The GGB material was designed to provide students with opportunities for developing reasoning on the computational algorithm of the slope in a physical situation when one quantity is constant and another quantity varies. An important feature of this material was that the change in the position of the ladder, the change in the slope of the ladder within a dynamic text of computational algorithm, the change in the vertical length and the constant horizontal length of the ladder are easily compared and related to each other.

The first phase ended with the *Tent* activity. The goal of the *Tent* activity was to evaluate the effect of the horizontal length of an object on the slope of the object when the vertical length of the object was constant. The reason why students were administered this activity after the *Fire Truck* activity was to enable the students to reason on how the slope of an object (i.e. the tent rope) varied with a quantity that affected the slope (i.e. horizontal length of the rope) when the other quantity (i.e. vertical length) was constant after they structured the direct variation between slope and vertical length. In other words, the aim was to enable students to reason on the computational algorithm of the slope of an object in the form $slope = \frac{vertical\ length}{horizontal\ length}$ when the vertical length was constant. In this activity, there was a tent and a rope from the tent that was tied to the ground. The activity sheet involved three tasks, which were computing the slope of the tent rope in different positions, explaining the biggest slope in the situation, interpreting the effect of the horizontal length of the rope on the slope (Appendix K).

In the *Tent* activity, the students individually computed the slope of the rope in different positions after they created the situations on GGB using the slider tool that changed the horizontal length of the rope (see Figure 12). Then, they checked their computations within their groups after they clicked the slope checkbox that showed the computation of slope in the dynamic text (see Figure 12). In addition, they discussed the maximum horizontal length value in this situation. After these computations and discussions, the students discussed how the slope changed when the horizontal length of the rope changed. In this process, the students compared and connected various visual representations of the slope (i.e. line segments on GGB), slope values and the computational algorithm of the slope (i.e. dynamic text on GGB) with varying horizontal length and constant vertical length that correspond to different positions of the rope as they positioned the rope (i.e. line segment on GGB). That is, the teacher wanted the students to comprehend the logic of how slope varies when considered the change in the horizontal length and constant vertical length.

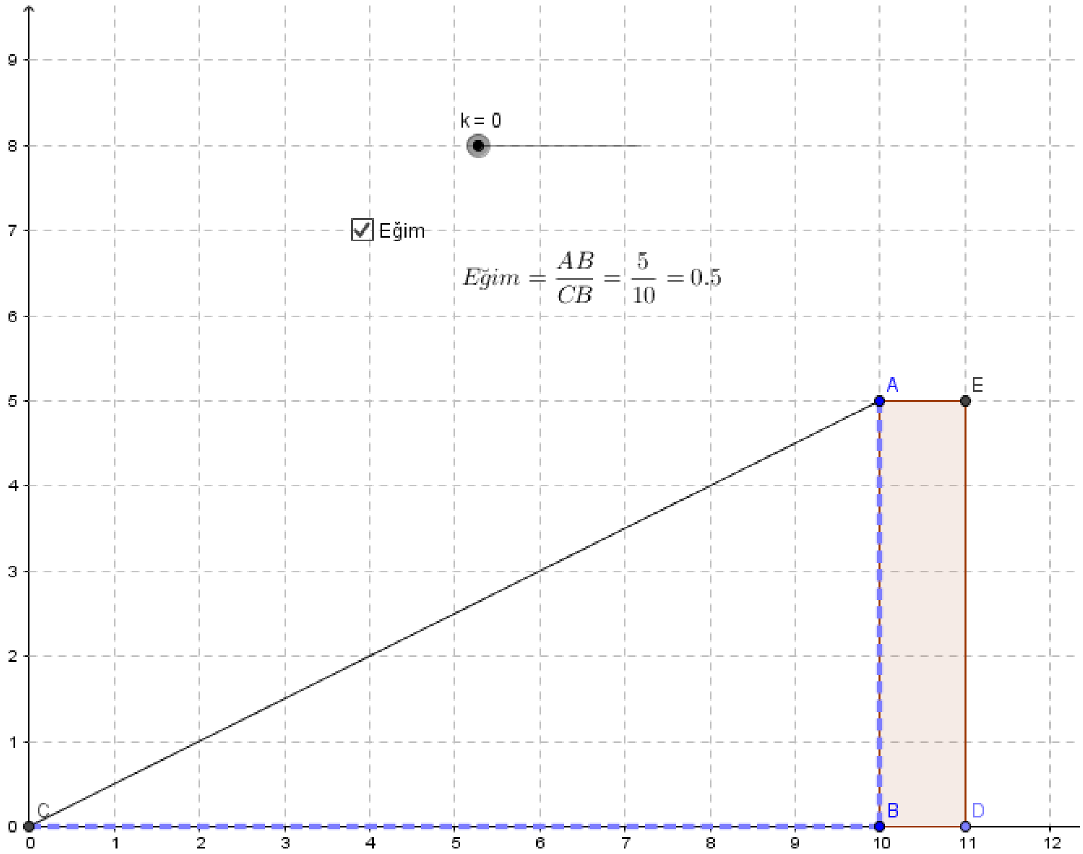

Figure 12. Tent activity: the GGB graphics view.

In Figure 12, the GGB graphics view of the GGB material involves a rectangle that represents a tent (rectangle *BDEA*), a line segment that represents a rope with varying length, two line segments that represent the constant vertical length (*AB* represents 5 unit), horizontal lengths that vary, slider-k set with an interval between 0 and 10 and with the increment of 1, and a checkbox for the slope to show the dynamic text of the computational algorithm of the slope. In addition, slider-k changed the horizontal length of the rope, and thus, the position of the rope and the length of the rope. In the given GGB graphics view, the slope checkbox was clicked. When the slope checkbox was clicked, the dynamic text of the slope computation was shown with a constant value of the vertical length and the dynamic value of the horizontal length, which is connected to slider-k and the position of the line segment (i.e. rope). On this GGB material, the positive direction of the coordinate axes were used to guide the students to measure the horizontal and vertical lengths, which do not represent the slope in coordinate system.

Until that point, the students could reason on the computational algorithm of the slope as they discussed how the slope of the tent rope increased or decreased with the horizontal length when the vertical length was constant on GGB material using the rope object, the slider tool, and the dynamic text of the computational algorithm of the slope. In addition, they discussed the slope value and the slope computation when the horizontal distance was zero in the tent rope situation on the GGB material. The GGB material was designed to provide students with the opportunities for developing reasoning on the computational algorithm of the slope in the physical situation when a quantity was constant and another quantity varied. An important feature of this material was that the change in the position of the rope, the change in the slope of the rope using the dynamic text of the computational algorithm, the change in the horizontal length, and the constant length of the vertical length of the rope (i.e. height of the tent) were easily compared and related to each other. The students could see how the rope looked when the point at which the rope was tied to the ground (point C) was moved using the slider. The students could see how the rope looked when the distance to the point at which the tent was tied to the ground (point C) from the tent (point B) was moved using the slider (i.e. horizontal length). In addition, they could see the change in the slope formula when the horizontal length (distance from the tent) varied. The students could see how the rope looked when the point at which the tent was tied to the ground (point C) was moved using the slider.

The second phase started with the concept of slope based on linear equations and line graphs in the coordinate system and in functional situations. In the second phase, there were three activity sheets prepared based on the following objective: 'Relate slope of a straight line graph and equation of the line graph for functional situations'. The activity sheets involved the *Building Design* activity (Appendix L), the Leaking Container activity (Appendix M) and *Slopes and Equations of Lines* activity (Appendix N). While preparing the activities of *Building Design* and *Leaking Container* to enable students' comprehension of the slope of a line, the teacher intentionally created the tasks for functional situations on quadrant 1 of the coordinate plane. The teacher wanted to students to focus on the slope of the line graph representing a linear relationship for a functional situation and to relate the changes in the variables in the functional situation on the first quadrant of the coordinate system without showing the other quadrants, which might be the reasons underlying students' confusions regarding the sign of a slope.

In the *Building Design activity*, the primary goal was to enable the students to relate the slope of a line graph to linear relationship, change in horizontal variable, change in vertical variable, and rate of change on the graph. The reasons why this was the first activity to be administered to the students were to provide background information about tables, equations, and graphs for linear relationships, to structure the quantities (number of floors and number of windows) that affect the slope computation of the line graph as the rate of change in one variable with respect to another variable with multiple representations, and to interpret a positive slope situation before a negative slope situation. In this activity sheet, there was a building design situation in which the number of windows of the basement (2 windows) and the number windows for each floor (3 windows) were given. The activity sheet involved four tasks: (1) writing the linear equation that shows the linear relation between total number of floors and the total number of windows after filling in the table for the given situation, (2) drawing the line graph that shows the linear relation between the total number of floors and the total number of windows, (3) finding the number of windows for a five-story building using the line graph and the linear equation, and (4) computing the slope of the line using any two points on the line graph after determining them (Appendix L).

In the *Building Design* activity, the students identified the variables in the situation, denoting x for the number of floors and y for the number of windows. After filling in the table for the situation, some of the students experienced difficulties in drawing the line graph that represented the linear relationship in the situation. The teacher started to use the GGB material after giving students time to work on the activity sheet. In the beginning of the activity, students did not see the graph, the point and the guidelines for slope computation. The teacher drew the points using the trace of point H when it was moved by using slider-h (see Figure 13). The students saw the points as ordered pairs in a table when they moved slider-h (it was shown as $h = 0$ in the first position, see Figure 13) for the starting point. Then, the teacher drew a straight line that was on those points using the drawing tool on the GGB graphics view. She also explained the linear equation for the situation after the students worked on it. Then, the teacher and the students discussed the slope of a line by comparing and relating it to the slope of an object. After the students developed an understanding of how to determine an interval on a line between two points and to compute the slope on this interval, the teacher allowed the students to make computations in their own way. Then, the teacher clicked the horizontal change and vertical change checkboxes to discuss and show the amount of change in the number of windows considering the change in the number of floors by using the guidelines of the points (i.e. red and blue dash-line arrows. See Figure 13). Therefore, the students could see how the changes in the vertical variable occurs with respect to uniform changes in the horizontal variable using the guidelines of the points (i.e. red and blue dash-line arrows). In addition, when using the two-different-point checkbox, they could see any two points with their guidelines on the line. After clicking the two-different-point checkbox, the slope-checkbox was used to show the slope value as the rate of change of the vertical change relative to the horizontal change with the formula $\frac{vertical\ change}{horizontal\ change} = \frac{y_2 - y_1}{x_2 - x_1}$ by using two dynamic points that emerged with the two-point checkbox and that could move on the line.

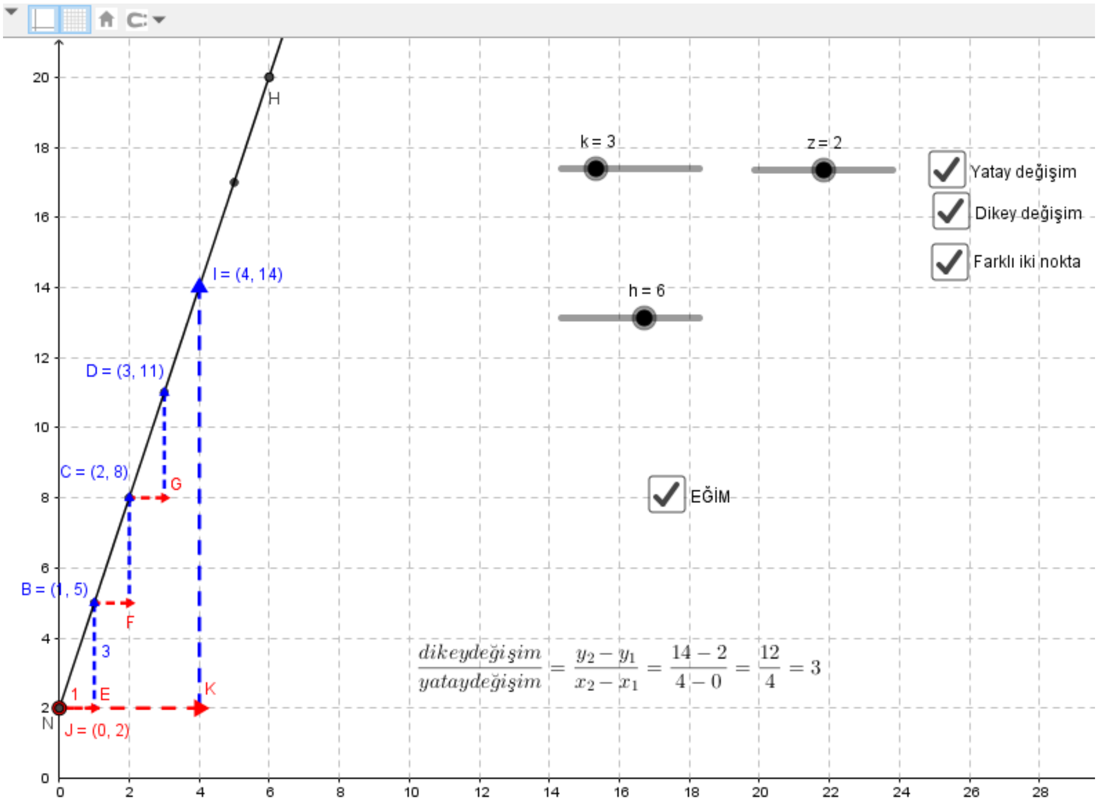

Figure 13. Building design activity: the GGB graphics view.

In Figure 13, the graphics view of the GGB material involved slider-k, slider-z, slider-h, vertical change, horizontal change, two-different-points, and slope checkboxes. In the given GGB graphics view, slider-k was 3, and slider-z was 2, which signified the equation $y = 2x + 3$ for the situation. In the given GGB graphics view, all the checkboxes were clicked. The checkboxes of vertical change and horizontal change showed the change in the vertical variable with respect to uniform increments (1 unit) in the horizontal variable with the dotted line segments between the points (0,2), (1,4), (2,6), and (3,8). The checkbox of two-different points showed two dynamic points (I and J) with their coordinate values on the line, which was also connected to the dynamic text of slope computation. In this GGB material, quadrant 1 of the coordinate system was used intentionally since it was sufficient to represent the functional situation and not to establish an unintended relationship between the slope of the line and the position of the line on the quadrants in the coordinate system.

After the *Building Design* activity, the second phase was continued with the *Leaking Container* activity. The reason why this was the second activity was to enable students to structure and evaluate the slope of a line representing a linear relationship in a functional situation with a decreasing line after an understanding of a functional situation with an increasing line was developed the in *Building Design* activity. The aim of the *Leaking Container* activity was to enable students to structure the slope of a line graph and evaluate the computation algorithm of the slope with a negative sign in a functional situation. In other words, the goal of this activity was to construct the slope of the line graph that represented the situation by understanding horizontal change, vertical change, and rate of change on a decreasing line graph. In this activity, there was a linear functional situation of leaking water from a container with a hole in the base. In the situation, from the 10-liter container, one liter of water leaked per one minute continuously. Specifically, the teacher prepared the tasks on quadrant 1 of the coordinate plane to relate line graph with a real-life situation and purposefully composed a line with a negative slope. In this activity, students used both the activity sheet and the related GGB material. The activity sheet involved four tasks: (1) writing a linear equation using the table for the given situation (i.e. the relation between the amount of water in the can and time), (2) drawing the line graph that showed the linear relation mentioned above, (3) finding the amount of water at the 4^{th} minute using the linear equation and the line graph above, (4) finding the slope of the line using any two points on the line graph (see Appendix M).

In the beginning of the activity, the students worked on their activity sheets and they worked on the visual representation of the situation, the table for the situation, and the equation of the situation. While the students were drawing the line graph for the situation, they started to use the GGB material. The students saw a dynamic figure that represented the leaking container situation, which involved varying amounts of water with respect to time, in the GGB material (see Figure *14*) and connected to point Z. As they moved point Z, which was one of the points on the graph, and moved through the line by moving slider-b, the students determined the points of the graph and compared them with their drawings in their activity sheets. They also traced on point Z and drew the entire graph for the situation. Then, the teacher guided the students through the work on reading the graph in the situation and to compute the slope of the line graph. The teacher allowed the students to compute the slope of the line by hand as an algebraic ratio using any two different points and use the checkboxes of horizontal and vertical changes between different points for comparing different intervals (see Figure 15).

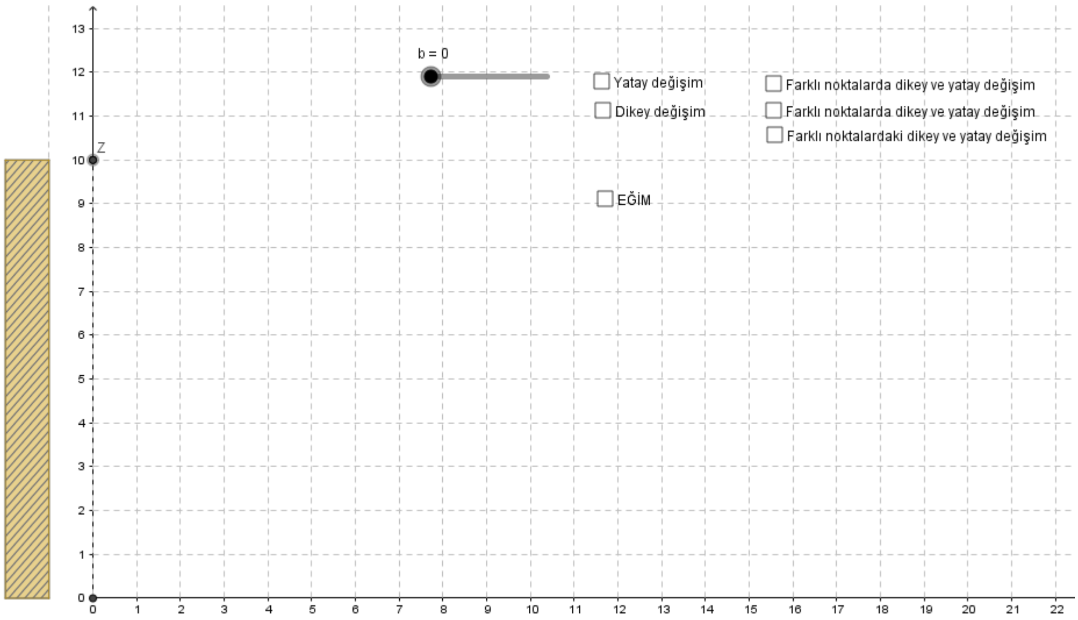

Figure 14. Leaking Container activity: the GGB graphics view-1.

In Figure 14, in the GGB graphics view, the shaded rectangle represents the leaking container full of 10 liters of water.

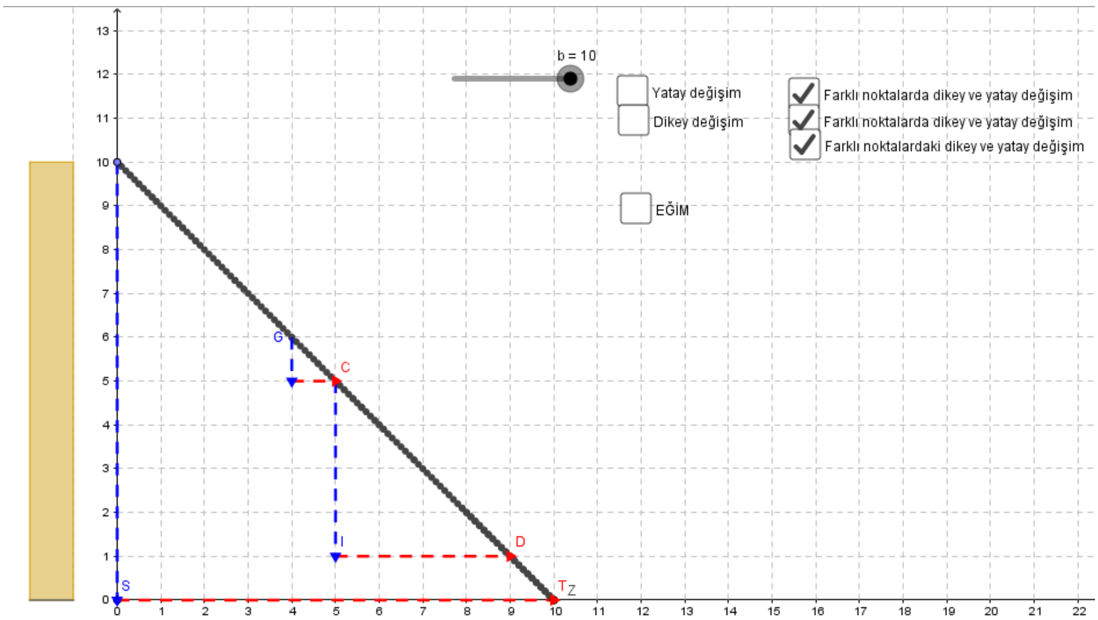

Figure 15. Leaking Container activity: the GGB graphics view-2.

When the students experienced difficulties in the computation of the slope of the line graph for the container situation, the teacher allowed the students to use the slope checkbox and the horizontal and vertical change checkboxes. Therefore, when they checked the horizontal and vertical change checkboxes, they could see how y-values changed when x-values changed by unit, or different units with the guidelines of the points (i.e. red and blue dash-line arrows. See Figure *16*). Then, they could interpret the computation of the slope in the line graph using the slope checkbox that showed the slope of the line as the rate of vertical change to horizontal change by with the formula of $\frac{y_2-y_1}{x_2-x_1}$ by using the coordinates of the two points on the line that emerged with the horizontal and vertical change checkboxes.

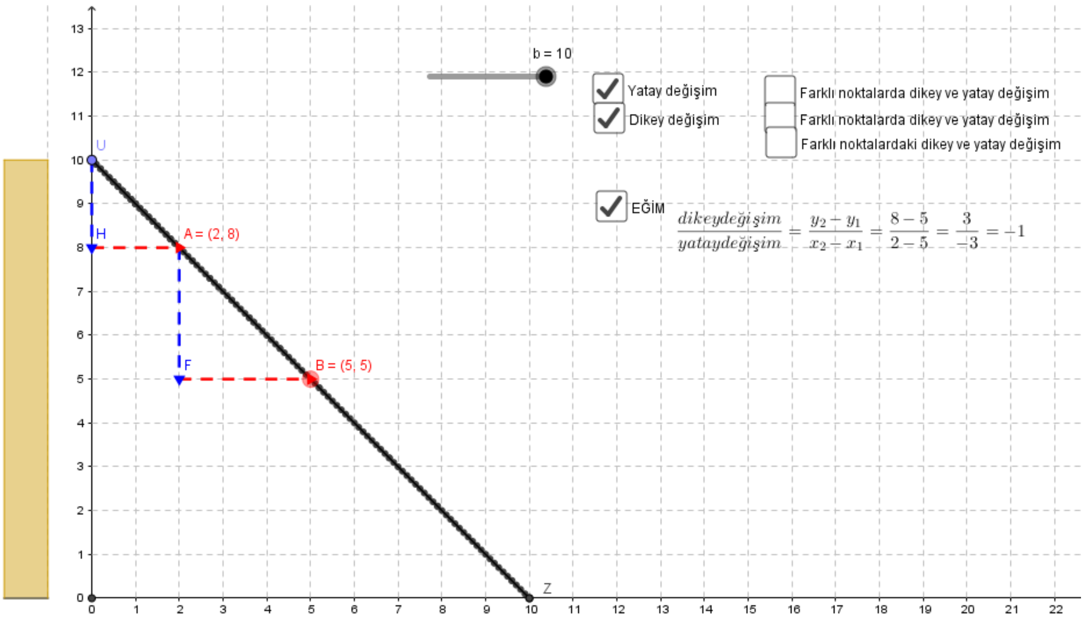

Figure 16. Leaking Container activity: the GGB graphics view-3.

In the second phase, the last activity was the *Slopes and Equations of Lines* activity (See Appendix N). By the time the students started doing this activity, they had developed an understanding of computation and had evaluated the effects of the y-variable and x-variable attributes on the computation of the slope of a line graph for a linear functional situation. As a further step, the goal of this activity was to relate the slope of a line and the equation of a line by using various equations, lines, and slope values. To this end, the first task involved the equations in the activities above (i.e. Building Design and Leaking Container). The students compared the differences and similarities in these equations and their slopes by writing them in a table in the activity sheet. The second task involved writing various linear equations and their slopes, constructing them using GGB material, and relating a linear equation and the slope of a line.

In the activity, the GGB material involved a full coordinate plane, two sliders that controlled the m and n values of the $y = mx + n$ equation corresponding to a line, points-on-the-line checkbox, which showed two points and horizontal and vertical changes between these points with guideline line segments, a slope checkbox that showed the dynamic text of slope computation connected to the coordinates of the two points (see Figure 17).

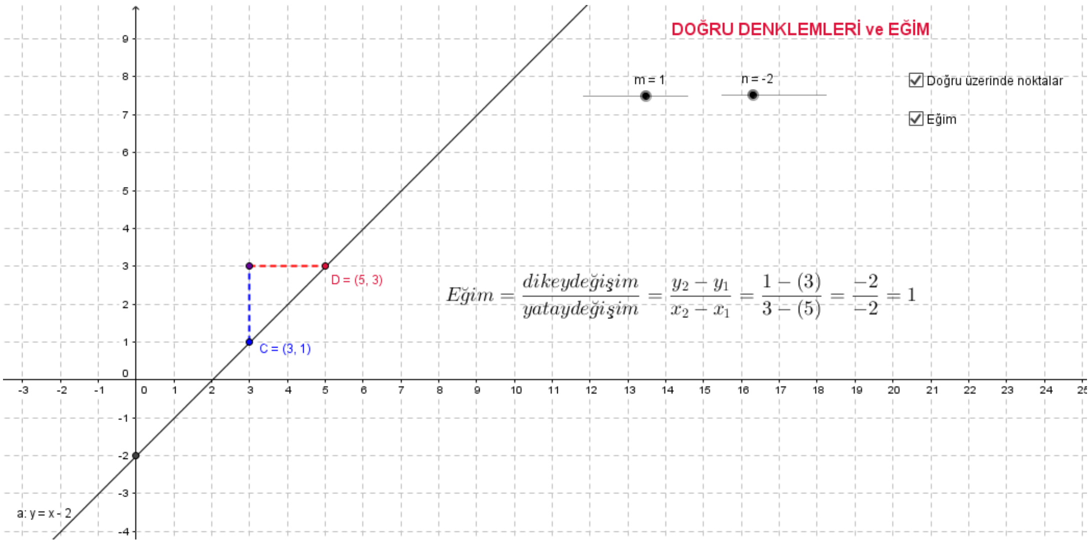

Figure 17. Slopes and Equations of Lines: the GGB graphics view.

In Figure 17, the GGB graphics view was shown when slider-m was 1, slider-n was -2, and the line equation was $y = x + 2$. In the GGB graphics view, when the point-on-the-line and slope checkboxes were clicked, the dynamic points of D and C and the dynamic text of slope computation were shown, respectively.

The third phase was based on the concept of linear equation system both in a functional situation and in a coordinate system, understanding the solution set of the system of equations algebraically and geometrically, and understanding the relations among position of lines, slopes, a solution set, and equations in the system of equations. In the third phase, three activity sheets were prepared. The first one was the *Mobile Operators* activity (see Appendix O). The second one was the *Equation*

Systems activity (see Appendix P) and the third one was the *Stores* activity (see Appendix R).

The aim of the *Mobile Operators* activity was to enable the students to structure a solution set for a system of equations in functional situations both graphically and algebraically after noticing the intersection point of lines as one solution for the system of equations and determining the coordinates of intersection point as the solution set in the context of functional situations. The reason why this was the first activity was that the students could build an awareness of the coordinates of the intersection point as the single solution for an equation system in the easy-to-follow functional situations with increasing line graphs with positive slopes. In other words, as a beginning activity for linear equation systems, the teacher purposefully chose two equations for increasing straight lines that would be more comprehensible for her students. In this activity, the real-life functional situations involved two mobile operators− T-cell and Z-cell− and a person who had to make a decision about which company to choose depending on the talk time (see Appendix O). The tasks in the activity sheet involved filling in tables, writing the linear equations for the situations, drawing the graphs for the situations on the coordinate system, comparing the charges for 4-, 5- and 7-minute talks after drawing the price-talk-minute graphs for each company in order to decide when and which operator they would use.

In the activity, the students could reason that mobile operators charged the same amount (10 TL) at 5 minutes, and this was the solution of the system of linear equations, which was an ordered pair of (5,10) and the intersection points of the lines for the equations. In addition, the students could reason that the slope of the equations showed the rate of mobile operators. During this process, the students compared the values of dependent variables in two equations considering the intersection point of the lines of these equations, which gave the solution set for the system. Corresponding to this activity, there was a GGB material as in the figure below (see Figure *18*). The GGB material involved two sliders (sliders t and z) that controlled the points T and Z. When one student moved slider-t after tracing on point T, point T moved depending on the equation for the T-cell price. Similarly, when one student moved slider-z after tracing on point Z, point Z moved depending on the equation for Z-cell price. Students could draw the line by using any two points for each equation or clicking the equation of the line in algebra view.

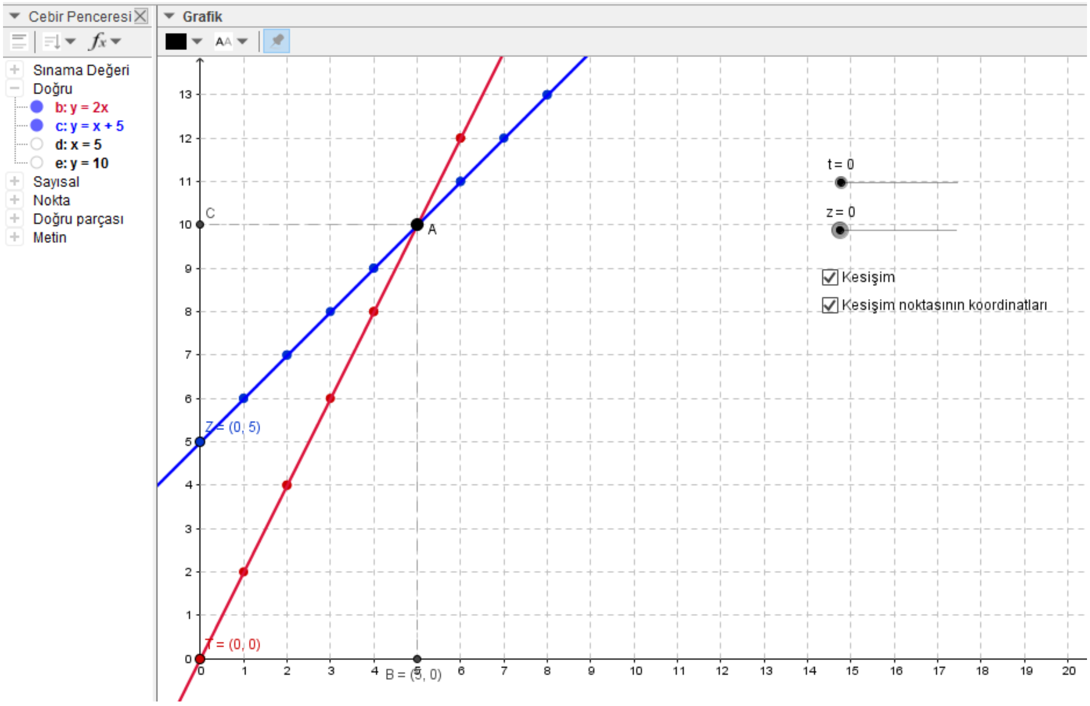

Figure 18. Mobile Operators activity: GGB algebra view and GGB graphics view.

In the GGB graphics view in Figure 18, there was slider-t, which showed the points for the situation of T-cell, and slider-z, which showed the points for the situation of Z-cell. The lines corresponding to the situations were drawn through these points. There were also checkboxes for intersection point and coordinates of intersection point, which showed the location of the intersection of point-A and the coordinates of the intersection point, respectively. Point T and Point Z were connected to slider-t and slider-z and produced the points on the graphs, respectively. In the GGB graphics view, the graphs were drawn after point T and point Z were traced on by moving slider-t and slider-z. In the GGB algebra view, there were symbolic expressions of linear equations in the system.

After the *Mobile Operators* activity, the second activity sheet− the *Equation Systems*− was administered to the students. The primary goal of this activity was to enable students to evaluate the relationship among the solution set of a system of equations for two lines, the positions of the lines, and the slopes of the lines. A secondary goal was to enable students to develop an understanding of slope as rate of change and reason about magnitude and the sign of the slope while computing or verbalizing the slopes of the lines. In the activity sheet, the students wrote various systems of equations, the positions of the lines for the system, the solution set, the slopes of the lines/equations in a table and drew the lines on the coordinate system in the activity sheet (see Appendix P).

There were two GGB materials for this activity. The first one, Equation Sysytems-1, involved systems of equations for two parallel lines (See Figure *19*). The second one involved systems of equations for two intersecting lines (See Figure *19*). The students used each GGB material with the activity sheet. They wrote the equations of the lines and computed their slopes. Then, they found the solution set and compared the positions of the lines. The teacher wanted the students to reason on the solution set of linear equation systems by taking into consideration the slope of the lines and the positions of the lines.

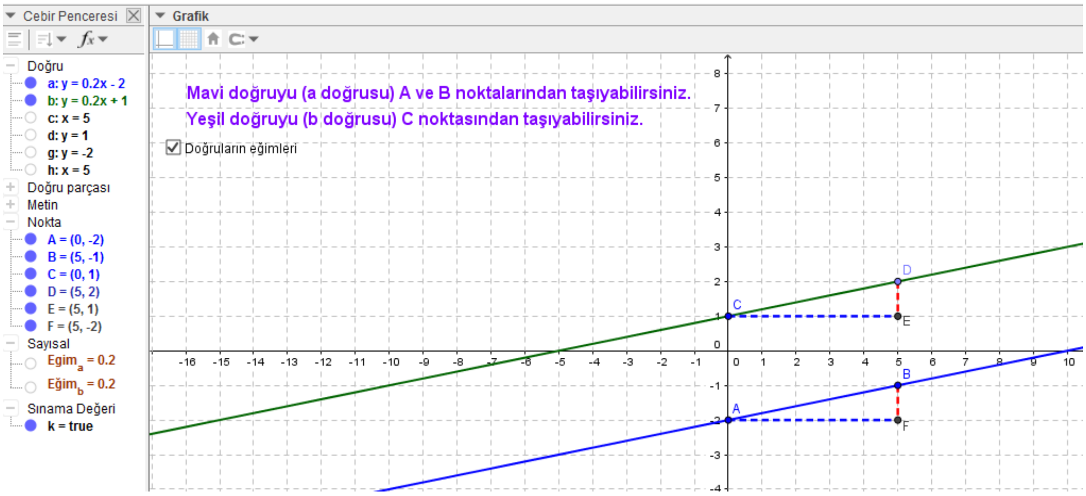

Figure 19. Equation Systems-1 GGB material for Equation Systems activity: GGB algebra view and GGB graphics view.

In the GGB graphics view of Equation Systems-1 GGB material, points A, B, and C were dynamic in that they were moved to create a new system of equations with parallel lines. When the checkbox of the slopes of the equations was clicked, the guiding dotted line segments (*CE*, *DE*, *AF*, and *BF*) for the vertical and horizontal changes were shown for each line. In algebra view, the teacher showed the equations of lines *a* and *b*, the coordinates of the points, and the slopes of the lines.

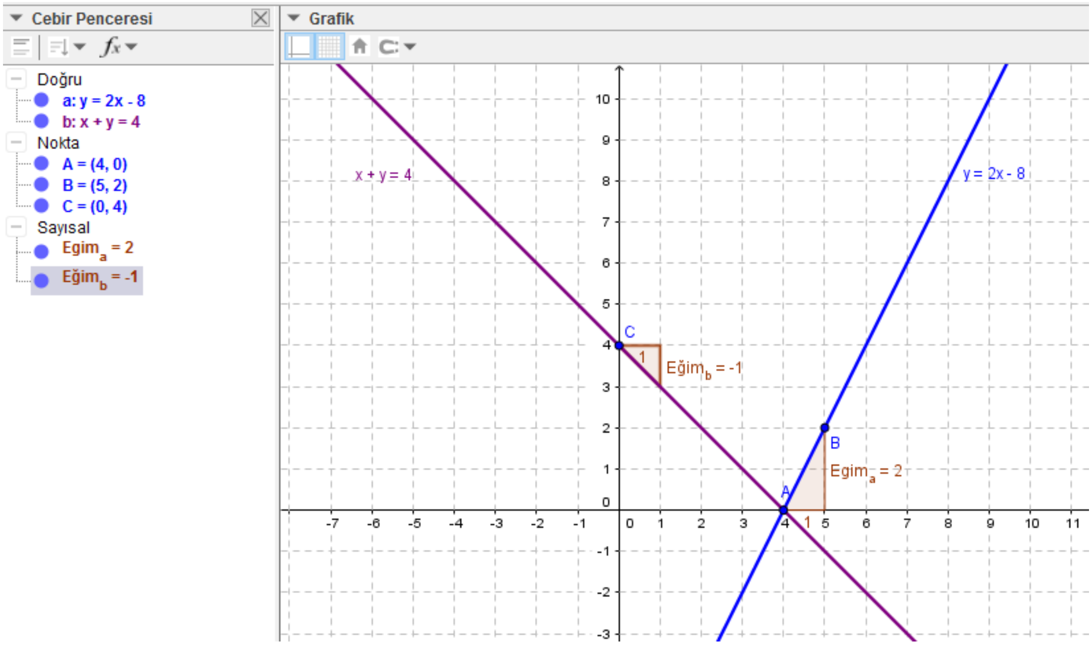

Figure 20. Equation Systems-2 GGB material for Equation Systems activity: GGB algebra view and GGB graphics view.

In the GGB graphics view of Equation Systems-2 GGB material in Figure 20, points A, B, and C were dynamic in that they were moved to create a new system of equations with intersecting lines. In the GGB algebra view, the teacher showed the equations of lines *a* and *b*, the coordinates of the points, and the slopes of the lines. When the number values of the slopes of lines a and b ($Eğim_a$ and $Eğim_b$) were clicked in the GGB algebra view, the values of the slopes were shown with the guiding line segments for the vertical change in terms of unit change in the horizontal dimension were shown for each line.

The third activity in this phase was the *Stores* activity. The aim of the *Stores* activity was to enable students to structure a solution set for a system of equations in functional situations both graphically and algebraically after noticing the intersection point of lines as one solution for the system of equations and determining the coordinates of the intersection point as the solution set in the context of functional situations. The reason why this was the last activity was that the students could build an awareness of the coordinates of an intersection point as the only one solution for a system of equation that was involved in a functional situation with a decreasing line graph with a negative slope and a functional situation with an increasing line graph with a positive slope. In other words, as the last activity for linear equation systems, the teacher purposefully chose two equations with opposite sign slopes to provide a comparison between the line with a positive slope and the line with a negative slope in a system of equations as a further step for the students (Even though these lines were perpendicular to each other, the teacher did not have a purpose to relate the slopes of the perpendicular lines). In the stores activity, there were two stores which were called M and N and the number of products sold each day were presented in tables (see Appendix R). The tasks in the activity sheet involved writing the linear equations for the situations, drawing the line graphs for the situations on the coordinate system, relating the slopes of the lines for the situations, finding the solution set of the equations for the system, relating the slopes of the lines to the equations for the situations, and evaluating the solution set for the situations.

While working on the *Stores* activity sheet, the students used the GGB material also. In the beginning, they worked on writing the equation for each store and the teacher guided them in constructing the linear equation by discussing the explicit relationship. Some students could also draw the line graphs of the equations that represents the linear relationships before using GGB for each equation but some could not. Then, the students opened the GGB material with the GGB graphics view to draw the points on the coordinate system in the GGB graphics view. Expecting that condition, the teacher had the students turn on the trace command for the starting points (points B and C in the GGB graphics view) of the table in the activity sheet and had the students move the sliders to create the points of the graphs for each store (see Figure *21*). In this way, the students could see the ordered pairs in the table and draw the corresponding line graphs. The students also drew the graphs on their activity sheets. Assuming that there was a line on the graph, the students drew the line for a situation using the equation of the line in the GGB algebra view (see Figure *22*). Then, they closed the GGB algebra view. In this way, they were able to see the system of equations graphically. In addition, the teacher allowed the students to compute the slopes of the lines and conceptualize the slope in terms of the degree of increase/decrease in stores in consideration of the increase in number of days. After the students clicked the slope checkboxes and equation checkboxes in the GGB graphics view, they discussed the relationship among the solution set, the intersection point, and equations and the relationship among the slopes of the lines and equations of the lines. They also evaluated the intersection point in the system of equations for functional situations. The students found the slope values both on the lines and the equations and explained what the positive and negative slope meant in this activity context.

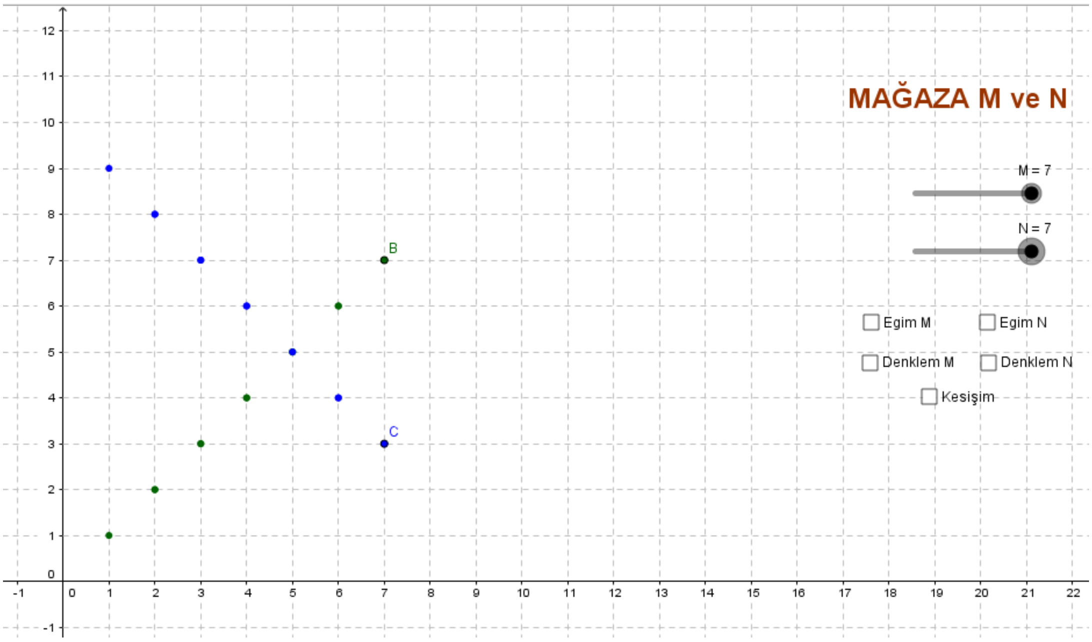

Figure 21. Stores activity: the GGB graphics view-1.

In the GGB graphics of the GGB material in Figure 21, point B is connected to slider-M, and point C is connected to slider-N. In the GGB graphics view, the sliders moved from 1 to 7, and then points B and C were traced on to show the points for the situations.

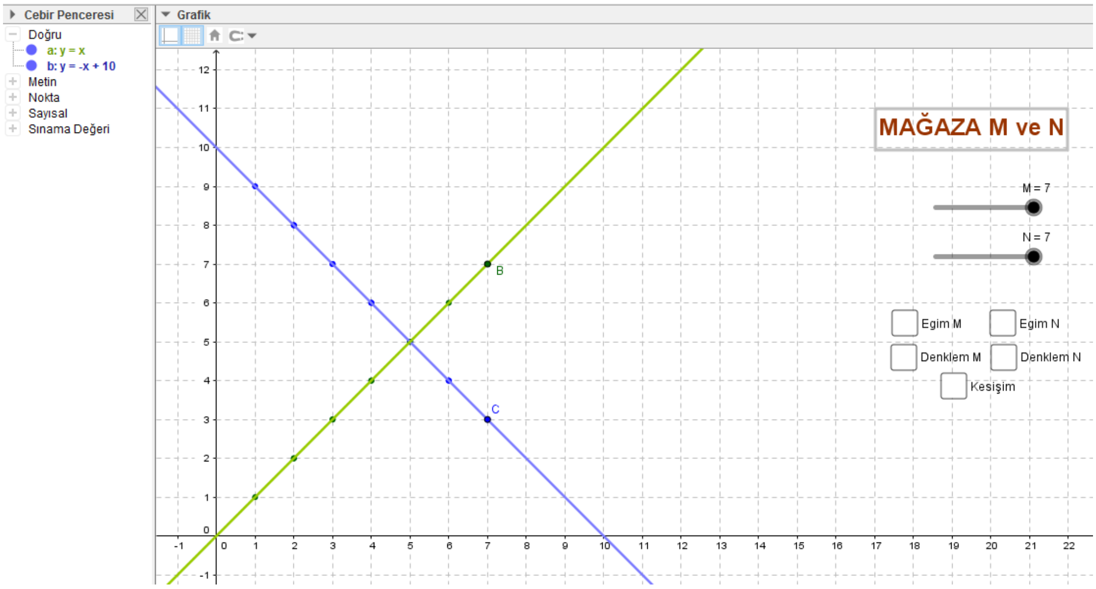

Figure 22. Stores activity: GGB algebra view and GGB graphics view.

In the GGB material in Figure *23*, the equations were clicked in the GGB algebra view, and the lines for the equations emerged in the GGB graphics view.

Figure 23. Stores activity: the GGB graphics view-2.

In the GGB material in Figure 23, the equations were clicked in the GGB algebra view, and the lines for the equations emerged in the GGB graphics view. Slope M, slope N, equation M, equation N, and the intersection checkboxes were clicked. The slope M and slope N checkboxes showed the slope values with the guiding line segments for the change in vertical length in terms of unit change in horizontal length were shown for each line. The equation M and equation N checkboxes showed the equations in the slope-intercept form. The intersection checkbox showed the position of the intersection point of the lines and the coordinate values of the point as an ordered pair.

After the students completed the tasks of the activity of *Stores*, as a conclusion of the instructional sequence, the teacher initiated a discussion on constructing slope as rate of change or unit rate in functional situations considering the activities in phase 3 and phase 2. During this process the teacher did not use any GGB material but she referred to the previous GGB materials. In addition, they had a discussion on relating the slope as rate of change to slope of an equation for a linear functional situation considering the functional situations in the phases and other real-life functional situations. Therefore, the teacher ended the instructional sequence in that way.

In the following sections of this chapter, the teacher's mathematical practices were analyzed under two sections based on tasks of teaching during the aforementioned phases (see Figure *24*). While the first section (4.2) involved mathematical practices during the time when the teacher used GGB materials and concrete objects in classroom teaching. The second section (4.3) involved mathematical practices during the time when the teacher explained mathematical ideas without using GGB materials and concrete objects in classroom teaching. For each teaching task, the mathematical practices were analyzed under the categories of: (1) bridging practices, (2) trimming practices, and (3) decompressing practices. Each category was described throughout the phases of the design, which were teaching the slope of an object in Phase 1, teaching the slope of a line in Phase 2, and teaching the solution of the system of equations and slope relation in Phase 3. In this regard, the mathematical practices in classroom teaching are explained under the subsections of bridging, trimming and decompressing practices in using GGB materials and concrete objects and in explaining mathematical ideas without using GGB materials and concrete objects. Therefore, this sequence of sections did not indicate the flow of instruction in classroom teaching nor did it indicate a priority for any mathematical practice.

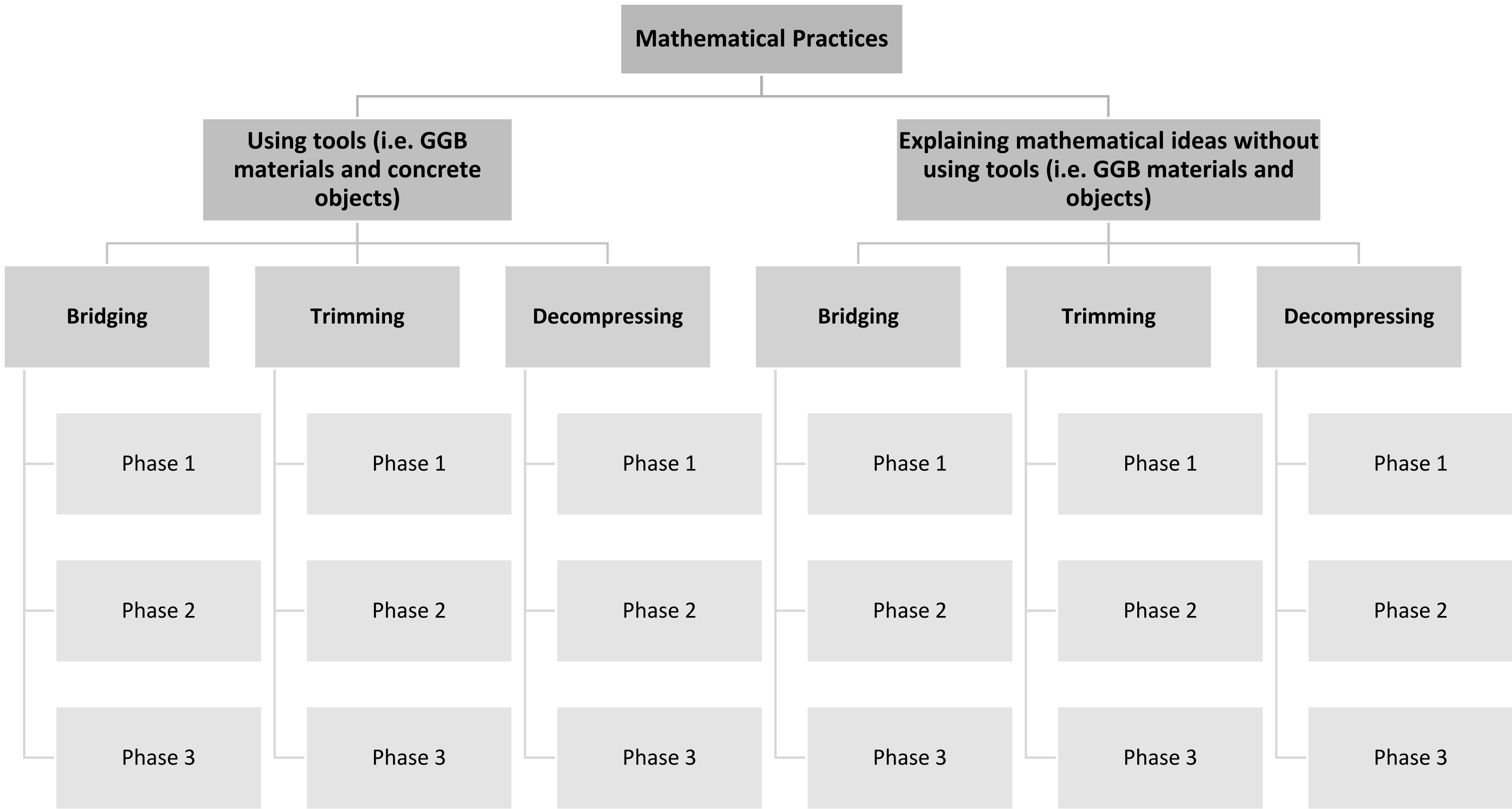

Figure 24. The organization of the findings.

4.2 Mathematical Practices in using GGB materials and Concrete objects

This section involved the teacher's mathematical practices while using GGB materials and concrete objects as instructional tools in teaching in a technology enhanced classroom environment. The data that were obtained from the classroom teaching sessions were present considering bridging, trimming, and decompressing categories. The practices were reported under three sub-sections for Phase 1, 2 and 3: (1) Bridging practices in using GGB materials and concrete objects, (2) Trimming practices in using GGB materials and concrete objects, and (3) Decompressing Practices in using GGB materials and concrete objects.

4.2.1 Bridging Practices in using GGB materials and Concrete objects

In this section, the teacher's bridging practices were reported within each phase when she used GGB materials and concrete objects as the instructional tools in classroom teaching. From the perspective of McCrory et al.'s (2012) KAT framework, bridging practices in teaching algebra are described as mathematical practices of making connections among topics, representations, and domains in teaching slope, linear equations, and graphs in eighth-grade mathematics. Therefore, the practices were categorized by identifying the teacher's actions in classroom teaching in which the teacher provides mathematical practices to provide connectivity and coherence of mathematics across mathematical ideas, representations, topics, and other domains while using concrete objects and GGB materials. Here, GGB materials and concrete objects are also seen as a form of representing mathematics.

Each phase (phase 1, phase 2, and phase 3) involved themes of bridging practices and the actions under each theme within a table. The flow of actions in the practices are explained under the classroom activities for each phase. At the end of this section, bridging practices are summarized considering all the phases.

4.2.1.1 Bridging practices in teaching the slope of an object for Phase 1

The bridging practices in teaching the slope of an object with GGB materials and concrete objects are explained in this subsection. In the first phase, there were three activities (*Positions of Battens*, *Fire Truck*, *and Tent*) with activity sheets and GGB materials. The teacher used situations entitled as batten as a ramp, fire truck ladder and rope of a tent as physical situations. In addition, she used battens as concrete objects for *Positions of Battens* activity and GGB materials for the activities to represent the situations dynamically with a line segment in the GGB graphics view.

While teaching the slope in physical situations, the teacher connected the concrete object, the GGB materials with GGB graphics view and GGB tools, mathematical situations, concepts, and processes, conceptualizations of mathematical concepts, and students' activity sheets in various combinations. The concrete object dimension of the codes involved the batten. In the GGB dimension of codes, the teacher used the line segment (as batten, ladder, rope, horizontal length, and vertical length, dynamic or static), the trace of line segment, the slider tool, the slope checkbox tool, static and dynamic text of computation of the slope. The mathematical dimension of codes involved (1) physical situations as mathematical situations, (2) line segments, horizontal and vertical lengths of the line segments, factors of slope, and slope as mathematical concepts, and (3) computation of slope and slope formula as a geometric ratio as mathematical processes. The conceptualizations of concept(s) codes involved representations of factors of slope and representations of undefined slope. The activity sheet dimension involved visual representations in the students' activity sheet. In consequence of these dimensions, bridging practices were gathered under these six themes:

Practice 1: Connecting the concrete object and the mathematical situation/concept;

Practice 2: Connecting the concrete object, the mathematical situation/concept, and the GGB material;

Practice 3: Connecting the mathematical situation/concept and the GGB material;

Practice 4: Connecting the mathematical situation/concept/process and the GGB material;

Practice 5: Connecting the mathematical situation/concept/process, the GGB material, and the activity sheet;

Practice 6: Connecting conceptualizations of concept(s) and the GGB material.

The practices and the actions under each practice with sample descriptions are given in Table *11*. The detailed descriptions of these practices have been presented in the sequential flow of activities (Positions of Battens, Fire Truck, and Tent) during phase 1. In the table, the practices of the teacher are enumerated to follow each practice easily in the findings, and this enumeration does not indicate the instructional sequence of the teacher. In other words, the teacher did not enact these practices in the classroom in same order as in the table.

Table 11. Bridging practices of Phase 1 while using GGB materials and concrete objects

Practice	**Action**	**Sample description**
Practice 1: Connect concrete object and mathematical situation/concept		
Connect concrete object, physical situation, and slope (latent)	Connect positions of battens and slopes in different physical situations	Connect different positions of battens and slopes in different situations without mentioning the term "slope" (P)
	Connect positions of battens and slopes within the road analogy (walking on roads)	Connect different positions of battens and walking on the inclined roads/straight roads without mentioning the term "slope" (P)
Connect concrete object, physical situation, and slope	Connect positions of battens and factors of slope	Connect different positons of battens, base and horizontal length (P)
		Connect different positions of battens, height and vertical length (P)
Connect concrete object, physical situation, angle, and slope (latent)	Connect positions of battens, angle, slopes in different situations	Connect different positions of battens, the angle between batten and base, and slopes of battens in different situations without mentioning the term "slope" (P)
Practice 2: Connect concrete object, mathematical situation/concept, and the GGB material		
Connect concrete object, physical situation, line segment, GGB graphics view	Connect positions of battens, line segment, GGB graphics view	Connect different positions of battens and line segment as a batten (model of the physical situation) in GGB graphics view (P)
		Connect horizontal/vertical lengths of batten and horizontal/vertical lengths of line segment (model of the physical situation) in GGB graphics view (P)

Table 11 (continued)

Practice 3: Connect mathematical situation/concept and GGB material		
Connect physical situation, line segment, GGB graphics view, GGB tools	Connect positions of the object (i.e. batten, ladder, rope), line segment, slider in GGB graphics view	Connect positions of battens, line segment and slider for horizontal length in GGB graphics view (P)
		Connect change in positions of the ladder for different floors of the burning building, line segment and slider of vertical length in GGB graphics view (F)
		Connect the change in the positions of rope, line segment and slider for horizontal length in GGB graphics view (T)
	Connect horizontal length, distance (real life experience), line segment, slider in GGB graphics view	Use slider and line segment to connect distance between the fire truck and the building *and* horizontal length of the ladder, which has a constant value in different slope situations in GGB graphics view (F)
		Use slider and line segment to connect real-life experience and limited value of horizontal length in GGB graphics view for the Tent activity (T)
Connect physical situation, line segment, slope, GGB graphics view, GGB tools	Connect slopes of battens, line segments, slider in GGB graphics view	Use slider for horizontal length to connect different slope positions and line segments as visual representation of the slopes in GGB graphics view (P)
	Connect distance and horizontal length of rope, slope, slider, line segment, GGB graphics view	Use slider and line segment of rope to connect distance and horizontal length in GGB graphics view (T)
		Use slider and line segment of rope to connect slope and horizontal length in GGB graphics view (T)
	Connect zero slope, real life experience and position of line segment, slider in GGB graphics view (F)	Make connection between zero slope, real life experience and position of line segment, and slider in GGB graphics view for the fire truck ladder situation (F)
Practice 4: Connect mathematical situation/concept/process, and GGB material		
Connect slope computation, line segment, GGB graphics view, GGB tools	Connect dynamic slope computation text, line segment, checkbox, and slider in GGB graphics view	Connect checkboxes, slider for horizontal length, positions of line segments of different slopes, line segments of horizontal and vertical lengths, dynamic slope computation text for battens in GGB graphics view (P)

Table 11 (continued)

	Connect dynamic slope computation text, line segment, trace, checkbox, slider in GGB graphics view	Connect trace of line segment, checkboxes, slider for horizontal length, each line segment, each line segment's slope value, horizontal length and vertical length values within the dynamic slope computation text in GGB graphics view (P)
	Connect slope computation, line segment, checkbox, and dynamic slope computation text in GGB graphic view	Connect slope computation by hand and line segment of the ladder, slope checkbox, dynamic slope computation text for ladders in GGB graphics view (F)
Practice 5: Connect mathematical situation/concept, GGB material, and activity sheet		
Connect physical situation, line segment, GGB graphics view, GGB activity sheet	Connect physical situation, line segment, GGB graphics view and activity sheet	Connect the fire truck ladder situation and line segment as visual representation in GGB graphics view and activity sheet (F)
Practice 6: Connect conceptualizations of concept(s) and GGB material		
Connect representations of factors of slope, line segment, GGB graphics view	Connect verbal, symbolic, and visual representations of length, line segment in GGB graphics view	Connect base and horizontal length terms and horizontal length line segment with OA symbol in GGB graphics view (P)
		Connect height and vertical length terms and vertical length line segment with OB symbol in GGB graphics view (P)
Connect representations of undefined slope, factors of slope, GGB graphics view, GGB tools	Connect undefined slope, representations of undefined slope, horizontal length, slider, dynamic slope computation text in GGB graphics view	Connect undefined slope, symbolic and visual representations of slope, horizontal length, slider for horizontal length, dynamic text of slope computation in GGB graphics view (T)

Note: The abbreviations in the sample description column of the table indicate the activity in which an action emerged. P: Positions of Battens activity, F: Fire Truck activity, T: Tent activity.

The teacher started with *Positions of Battens* activity, which involved the positions of an object (i.e. battens with constant length) as a physical situation. As can be seen in Table 11. *Bridging practices of Phase 1 while using GGB materials and concrete objects*, in Positions of Battens activity, the teacher's bridging practices were combined under the practices of connecting the concrete object and the mathematical situation/concept (Practice 1), connecting the concrete object, mathematical situation/concept, and the GGB material (Practice 2), connecting the mathematical situation/concept and the GGB material (Practice 3), connecting the mathematical situation/concept/process and the GGB material (Practice 4), and connecting conceptualizations of concept(s) and the GGB material (Practice 6).

As practices for connecting the concrete object and the mathematical situation/concept (Practice 1), the teacher made connections among concrete object, physical situation, and slope (latent), made connections among concrete object, physical situation, angle, and slope (latent), and made connections among concrete object, physical situation, and slope in the Positions of Battens activity. In the beginning of the lesson, the teacher gave battens as concrete objects (i.e. each batten was 60 cm) to each group of students. The teacher gave the battens to the groups of students to lean the battens against the classroom wall. Each group of students took a batten and placed the batten in different positions. As an evidence of practice 1, she posed questions to connect the different positions of the battens and slopes in different situations without expressing the term ‘slope’; for example, *“Are the positions of the battens the same? Can we change them?”*. After the students noticed that the angle between the batten and the base of the batten was different for all battens, the teacher posed questions to establish a connection among the positions of the battens, the angle between the batten and the base, and slopes in different situations without expressing the term “slope”; for example, *“What happens to the positions of battens when the angles [of the battens] are different?”.* In addition, after the students again expressed that the angles were different when the positions of the battens were different, the teacher asked, “*What is affected by the change of the angle in these battens? What is affected by the change in the positions of the battens?”*. She also used the road analogy (i.e. what makes walking on a road harder/easier) and related walking on the inclined roads or straight roads to different positions of the battens by posing such questions as *“What if these battens were roads? Which one would be easy to walk on? This one or the other one?”*. The teacher’s purpose of asking these types of questions without explicitly mentioning the concept of slope was to evoke the students’ sense of slope and to have the students notice the connection among slope, other mathematical concepts, and real-life within the context of the physical situations. Thus, the above dialogues emerged during the practice of establishing connections among concrete object, physical situation, and slope latently and among concrete object, physical situation, angle, and slope latently. In this process, the students articulated the term of slope and bigger/smaller slope themselves.

The teacher made aforementioned connections as a preparation to the practice of connecting concrete object, physical situation, and slope in Practice 1. In detail, by posing questions to the students, the teacher had the students think about the connection between the positions of the battens and the related factors of slope that should be considered in measuring the slope of a physical object. As evidence of this practice, for example, she asked, "*When you think about your positions of battens, what could affect the battens' slopes?"*. Another example of a question posed upon a student's response, which was *"The height of the batten can affect the slope of the batten"*, was as follows: *"Where is the height here (The teacher shows the batten)?"*. Therefore, the teacher connected the related factors (base/horizontal length and height/vertical length) of the slope and the positions of the battens.

As a further step in the *Positions of Battens* activity, the teacher used the GGB material of the activity, which was a model of the situation of the positions of the battens. First, as evidence connecting the concrete object, the mathematical situation/concept, and the GGB material (Practice 2), the teacher, by posing questions and making explanations, made connections between the positions of battens as concrete objects and a dynamic line segment as the batten in the GGB graphics view. In addition, she connected horizontal and vertical lengths of the battens and horizontal and vertical lengths of the dynamic line segment as the batten in the GGB graphics view by posing such questions as follows: "*Let's think of this as the wall (The teacher shows the vertical ray in the GGB graphic view) and this as the ground (The teacher shows the base line in the GGB graphic view) that the batten touches. At which point is the batten material touching the wall in GGB?".* That is, she asked the length of the line segment that represents a 60 cm-long batten in the GGB graphics view and asked the contact points of the batten concrete object, which were given as points A and B in the GGB graphics view.

As practices of connecting the mathematical situation/concept and the GGB material (Practice 3), the teacher connected physical situation, line segment, GGB tools, and GGB graphics view, and connected physical situation, slope, line segment, GGB tools, and GGB graphic view. As an action, by posing question, she connected the slider of horizontal length (slider-a), the dynamic line segment, and change in the positions of line segments as battens in the GGB graphics view. In addition, by posing question, she connected the slider of horizontal length (slider-a), slopes in different positions of battens, and the dynamic line segment as visual representations of the slopes. At this point, the teacher used the slider tool in connection with the line segment in the GGB graphics view while posing the questions. She had the intention of using the line segment to create a model of a batten at the beginning of the students' learning process and then a model for reasoning about the slope as steepness at end of their learning experience.

Then, as a practice of connecting conceptualizations of concept(s) and the GGB material (Practice 6), she connected representations of factors of slope and line segment in the GGB graphics view. As evidence of this practice, the teacher posed questions to make connections between the terms height and vertical length (verbal representations) and vertical length line segment with *BO (*visual and symbolic representation*)* in the GGB graphics view. As further evidence, the teacher again posed questions to build connections between the terms of base and horizontal length (verbal representations) and horizontal length line segment with *AO* (visual and symbolic representations) in the GGB graphics view. Therefore, the teacher made connections among horizontal length, vertical length, and slope value in the GGB graphics view for further connections among division, ratio, and computation of slope.

After the students measured the horizontal and vertical lengths of their battens using tape measures (meters) and computed the slopes of their battens as a geometric ratio by hand, the teacher made connections among the mathematical situation/concept/process and the GGB material (Practice 4). That is, the teacher connected slope computation, line segment, GGB graphics view, and GGB tools. As an evidence of this practice, by posing questions, she connected slope checkbox, horizontal length and vertical length checkboxes, the slider of horizontal length, the dynamic slope computation text, and the line segment as visual representations of the slopes of different battens in the GGB graphics view. That is, after the teacher received a response of a horizontal length value of a batten from a group of students, she moved the slider to this horizontal length value, which showed the position of the line segment for this value, and clicked the horizontal length and vertical length checkboxes that showed their line segments and values, and the slope checkbox that showed the dynamic computation text of the slope of the batten that corresponds to this situation. The teacher repeated this for all battens and the students checked their computations.

To sum up the activity, she again made connections among the mathematical situation/concept/process and the GGB material (Practice 4). That is, she connected the slider for horizontal length, the slope checkbox, horizontal length and vertical length checkboxes, the dynamic text of slope computation, and the command of trace on for line segment in the GGB graphics view. The following dialogue emerged during this practice (T: the teacher, S: a student, SS: students):

(T) …Okay then, I want you to see how the slope value changes when I change the [position of] the batten. I clicked trace on command in GGB. Look at both the batten [line segment] and the slope. The value of the slope computation. What happens to the slope? [The teacher moved the slider to the lower part, which increases the horizontal length of the batten, and eventually moves the batten] *(see Figure 25 for the computer screen view)*
(S) Decreases!
(T) It decreases, doesn't it? Look it is 1.17 and then 1.12. What happens gradually?
(SS) Lessens. Becomes smaller.
(T) The slope decreases more and more. What happens when it moves to the other part?
(SS) Increases!

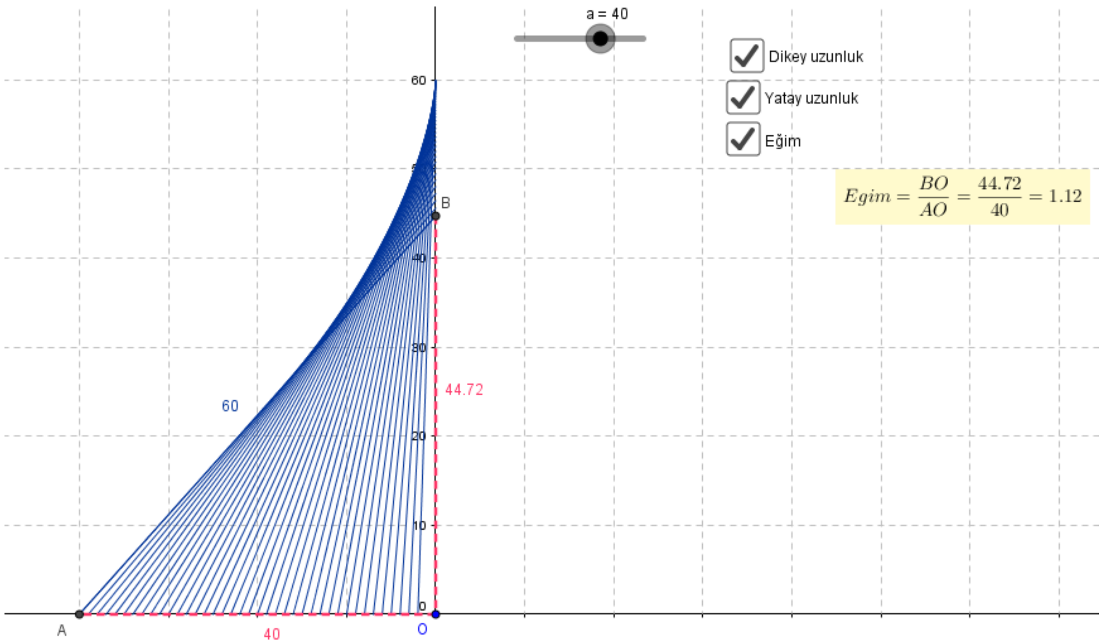

Figure 25. The teacher's screen view of the graphics view in the GGB material of the Positions of Battens activity.

In the GGB graphics view in Figure 25, the vertical length and horizontal length checkboxes, which showed BO and AO, were clicked, the slope checkbox, which dynamically showed the computation of the slope of line segment *AB*, was clicked, the trace of line segment *AB* was turned on, and slider-a, which changed the horizontal length of the batten, was moved from 1 cm to 40 cm. The blue line segments showed the trace of the line segment *AB.*

That is, after turning on the trace of line segment (i.e. batten) and clicking on the checkboxes, the teacher used the slider that moved the line segment and left the traces of line segment to connect each line segment as a visual representation of the slopes and each line segment's slope value (as a ratio and decimal value), horizontal length and vertical length values within the dynamic slope computation text (i.e. slope formula and numerical values) and in the GGB graphics view. As can be seen in Figure *25*, the teacher made connections among slider-a (horizontal length value (*AO*) of the line segment *AB*), line segment *AB*, slope checkbox and dynamic text of slope computation, which showed the slope formula and numerical values of vertical length, horizontal length, and slope as

$slope = \frac{BO}{AO} = \frac{dynamic\ value\ of\ vertical\ length}{dynamic\ value\ of\ horizontal\ length} = dynamic\ decimal\ value$ in the GGB graphics view.

After the *Positions of Battens* activity, the teacher continued with *Fire Truck* activity, which again involved positions of an object (i.e. a fire truck ladder that can be of different lengths and that has a fixed horizontal length) as a physical situation. In the *Fire Truck* activity, the teacher's bridging practices were categorized as practices connecting the mathematical situation/concept and the GGB material (Practice 3), connecting the mathematical situation/concept/process and the GGB material (Practice 4), and among the mathematical situation/concept/process, the GGB material, and the activity sheet (practice5).

In the *Fire Truck* activity, the students used the GGB material within their groups and the activity sheet individually. As a practice for connecting mathematical situation/concept, GGB material, and activity sheet (Practice 5), the teacher started by making connections among positions of the fire truck ladder, the line segment as visual representation of the ladder situation in the GGB graphics view, the verbal description and the visual representation of the ladder situation in the activity sheet by posing questions and making explanations. As an evidence of this practice, she asked, "*But the fire truck ladder doesn't have to be on 10th floor. On which floor could it be?... Look at the structure on GGB. For instance, on which floor does the fire truck ladder seem to be?".*

Then, as practice of connecting mathematical situation/concept and GGB material (Practice 3), the teacher made connections among physical situation, line segment, GGB graphics view, and GGB tools. As evidence for this practice, she connected the slider of the vertical length (slider-f), the dynamic line segment as the visual representation of the ladder positions, and the change in the positions of the line segment as the ladder for different floors of the building in the GGB graphics view. For example, she said, *"Here you see the slider [slider-f]. Click the slider and move it with the direction keys on the keyboard. You can change the floor number [the horizontal length value] with it? For example, in the question [in the activity sheet], which floor is asked?".*

After the students made the aforementioned connections, they computed the slope of the given situation in the activity sheet. Subsequently, as a practice of connecting mathematical situation/concept/process, and GGB material (practice 4), the teacher connected slope computation, line segment, GGB graphics view, and GGB tools. As an evidence, after the students computed the slope for a situation by hand and moved the slider for a situation (e.g. fire truck ladder leans to the 3rd floor in the building, which corresponds to 3 units), the teacher made connections between the students' computations of the slopes by hand and the positions of the ladders of different slopes (i.e. line segments) as the students clicked the slope checkbox that showed the dynamic slope computation text (see Figure 26). For instance, she asked, *"Okay [the students make the computation], do you see the slope checkbox on the screen? Click on it. What came up on the screen?"*.

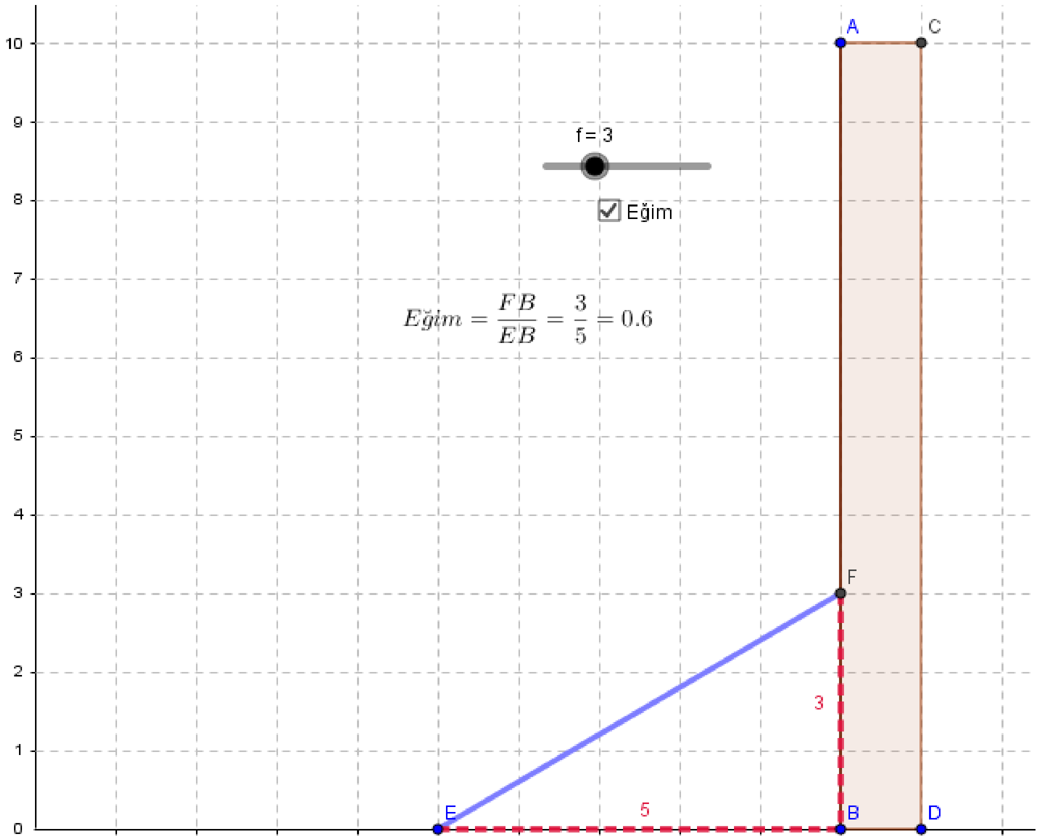

Figure 26. The students' screen view of the GGB material in the Fire Truck activity.

In students' GGB graphics view in Figure 26, the slope checkbox that dynamically showed the computation of the slope of line segment *EF* was clicked after slider-f, which changed the vertical length of the ladder, was moved to 3 units. Rectangle *BDCA* represented the building and line segment *EF* represented the fire truck ladder.

In addition, as practices of connecting mathematical situation/concept and GGB material (Practice 3), the teacher connected physical situation, line segment, and GGB graphics view again and connected physical situation, slope, line segment, and GGB graphics view. At first, she made a connection between the horizontal length of the ladder (i.e. fixed length line segment has constant value of 5 units) in different positions *and* the distance between the fire truck and the building (*EB*) in the GGB graphics view by posing questions. As an evidence, she asked, *"Why does horizontal distance remain constant? How close is the fire truck to the building?"*. Secondly, by posing questions, she made a connection between zero slope, real life experience (i.e. the position to lean the ladder) in the situation, the position of the line segment in the situation in the GGB graphics view. As an evidence, she asked, *"Which floor is it the simplest to lean the ladder on? What happens there?".* In this way, the students discussed the zero slope in the situation and the GGB graphics view.

In the third phase, the teacher ended the idea of slope in physical situations with the *Tent* activity, which again involved positions of an object (i.e. rope of a tent that can be of different lengths and that has a fixed vertical length). In the *Tent* activity, the teacher's bridging practices were categorized as practices for connecting the mathematical situation/concept and the GGB material (Practice 3) and connecting conceptualizations of concept(s) and the GGB material (Practice 6).

In the *Tent* activity, the students used the GGB material within their groups and the activity sheet individually. As a practice for connecting mathematical situation/concept and GGB material (Practice 3), the teacher started by making a connection among the physical situation (positions of the rope), the line segment, the GGB graphics view and the GGB tools. That is, after an introduction about the physical situation in the activity, the teacher made a connection among the change in the positions of the rope (i.e. line segment) in different distances, the line segment as visual representation of the rope situation in the GGB graphics view, and slider for horizontal length in the GGB graphics view by posing questions. As evidence, the teacher asked such questions as follows: *"Here, when you change slider-k [value], how many meters* *away is the rope?", "For example, to tie it one unit away, where will you move k?", "You tie the tent's rope to point C. How many units away are you from the tent?".* Then, in another practice, the teacher made connections among physical situation, line segment, slope, GGB graphics view, and GGB tools. That is, while the students used the slider to move the line segment (i.e. rope), the teacher posed questions successively to connect distance and horizontal length and to connect slope and horizontal length in the GGB graphics view. For example, as evidence of connecting distance and horizontal length, the teacher asked, *"What changes? Explain its reason. What goes up when the point becomes distant? What happens? What does the slope depend on?", "Okay. What changes as it [rope] get farther away?".* In addition, while the teacher was using the slider to summarize the activity, she again posed questions to connect the slope and the horizontal length of the line segment (i.e. rope) for various examples of rope situations. For instance, she said, *"Let's look at it in this way and see how the slope changes" (The horizontal distance value increases as the teacher moves the slider.), "What happens as we do this"? (The teacher moves the slider and the value decreases).* Moreover, as another practice 3 of connecting physical situation, line segment, GGB graphics view, and GGB tools, the teacher connected horizontal length, distance (real life experience), line segment, slider in GGB graphics view. More specifically, when one student asked such a question about the horizontal length of the rope as *"Could the place you're showing with the rope be infinite?",* she responded by establishing a connection between real-life experience and the limited value of horizontal length of the rope (i.e. line segment) in the GGB graphics view. To be specific, she moved the rope to the farthest point by using the slider and said, *"I can lengthen this horizontal length but my rope here can only extend to this point in GGB. In this case, it can extend a maximum of 10 units that is to point zero".*

In the *Tent* activity, as practice for connecting conceptualizations of concept(s) and GGB material (Practice 6), the teacher connected representations of undefined slope, factors of slope, GGB graphics view, and GGB tools. In detail, the teacher connected undefined the slope, the symbolic and visual representations of the slope, and the horizontal length, the slider and the dynamic text of the slope computation in the GGB graphics view. The following dialogue emerged at the beginning of this practice (T: the teacher, S: a student, SS: students).

(T) What is the slope value here? *(The teacher moves the slider to make the horizontal length of the rope zero)*

(S) Zero.

(SS) Infinite.

(T) The GGB shows *(The teacher mentions the dynamic text of the slope computation)* as if it [slope] is infinite, but what is the horizontal length here? *(see Figure 27 for the computer screen view)*

In addition, after the students made computations for the undefined slope, she continued with the explanation on the GGB graphics view:

> The reason is that the result of the computation is undefined. However, GGB makes a different computation, and it indicates it as being infinite. It perceives the result as being tremendous and extreme. However, the slope cannot be computed. Thus, while the GGB software used '∞' as a symbolic representation of the undefined slope in the dynamic text of slope computation in the GGB graphics view, the teacher continued to connect the symbolic representation of the slope and the undefined slope by making the explanation above.

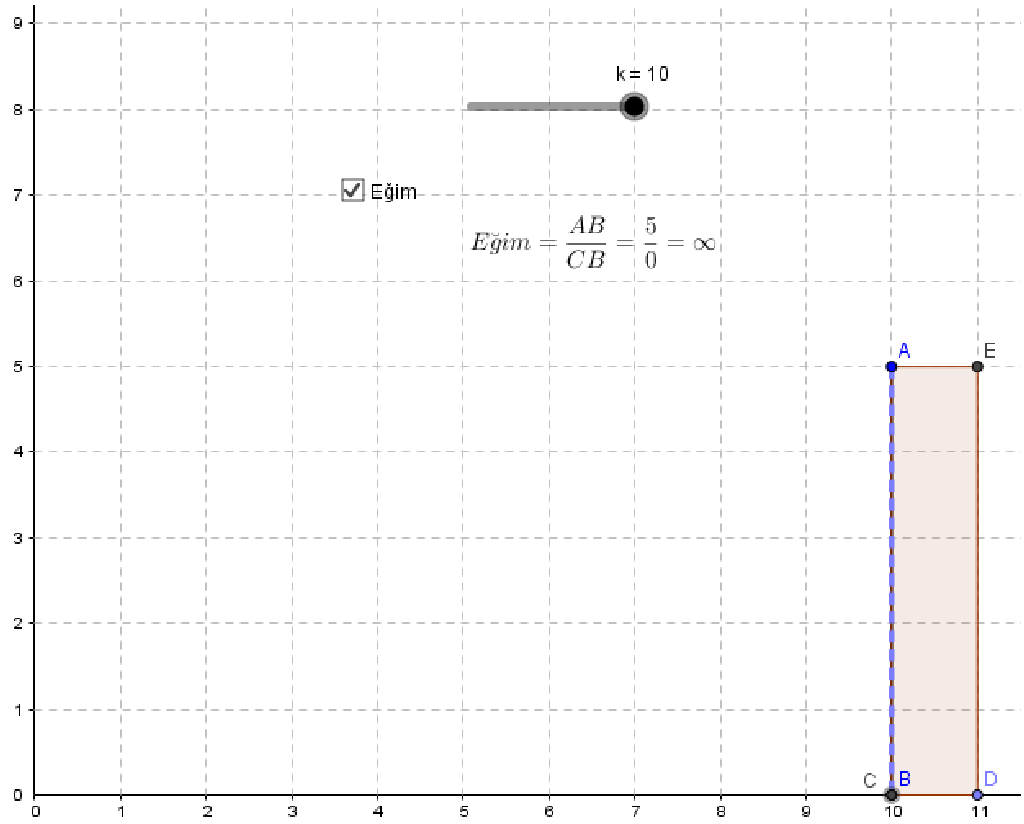

Figure 27. The students' screen view of the GGB material in the Tent activity.

In the GGB graphics view in Figure *27*, the slope checkbox, which showed the dynamic text of the computation of slope of line segment (*AC*), was clicked and slider-k, which changed the horizontal length of the rope, was moved to make horizontal length 0 (zero). The rope of the tent (line segment *AC*) and the side (line segment *AB*) of the tent (rectangle *BDEA)* coincide in this position of the rope.

4.2.1.2 Bridging practices in teaching the slope of a line for Phase 2

The bridging practices in teaching the slope of a line with GGB materials and concrete objects are explained in this subsection. In the second phase, there were three activities accompanied with activity sheets and GGB materials. In the activity sheet of *Building Design,* the teacher used pictures of buildings, a table for the number of floors and the corresponding number of windows, and a grid for the graph of window-floor relation. In the activity sheet of *Leaking Container,* she used pictures of containers, a table for the amount of water and time, and a grid for the graph of the amount of water-time relation. While starting to teach the slope of a line graph, the teacher used these activities to get the students involved in functional situations in real life. In the activity sheet of *Slopes and Equation of Lines,* she used tables for relating the linear equations of line graphs and the slopes of lines, and a coordinate plane grid. In addition, she used GGB activity materials to represent these situations dynamically in GGB graphics and GGB algebra views.

While teaching the slope of a line, the teacher connected concrete object, GGB views, GGB tools, and mathematical situations, concepts, processes, conceptualizations of mathematical concepts, and students' activity sheets in various combinations. The concrete object codes involved a batten and pen. In the GGB dimension of the codes, the teacher used points, graphs, the slope of a line, the drawing line tool, the points checkboxes for intervals, the slope checkbox, the slope computation text, slider tools, the trace of points, and the dynamic figure of the functional situation in the GGB graphic view, and equations and forms of equations in the GGB algebra view. The mathematical dimension of codes involves (1) functional situations as mathematical situations, (2) points, points of line graph, line graph, linear relation, equation, standard form of equation, slope-intercept form of equation, similarity, change, segments for interval, slope, y-intercept, and constant term as mathematical concepts, and (3) computation of slope as mathematical process. In addition, she connected GGB material and other dynamic software. The activity sheet dimension of codes involved tables, students' points of graphs, and students' line graphs. In consequence of these dimensions, bridging practices consisted of six themes.

Practice 1. Connecting the mathematical situation/concept, the GGB material, and the activity sheet;

Practice 2. Connecting the mathematical situation/concept and the GGB material;

Practice 3. Connecting the mathematical situation/concept/process and the GGB material;

Practice 4. Connecting the concrete object, the mathematical process, and the GGB material;

Practice 5. Connecting conceptualizations of concept(s) and the GGB material;

Practice 6. Connecting different software.

The practices and actions under each practice, together with sample descriptions, are outlined in Table 12. The detailed descriptions of these practices are given within the sequential flow of the activities during phase 2. In the table, the practices of the teacher are enumerated to follow each practice easily in the findings, and this enumeration does not indicate the instructional sequence of the teacher. In other words, the teacher did not enact these practices in the classroom in same order as in the table.

Table 12. Bridging Practices of Phase 2 while using GGB materials and concrete objects

Practice	**Action**	**Sample Description**
Practice 1: Connect mathematical situation/concept, GGB material, and activity sheet		
Connect functional situation, GGB tools, and activity sheet	Connect functional situation, slider, and descriptions in activity sheet	Connect constant value in functional situation of *Building Design*, slider-z tool produces the constant value of equation, and description about number of windows in the basement in activity sheet (B)
		Connect change in functional situation of *Building Design*, slider-k tool produces the coefficient of x-variable of equation, and description of number of windows in the basement in activity sheet (B)
Connect graph, GGB graphics view, and students' activity sheets	Connect students' drawings of graph in activity sheets and graph in GGB graphics view (graphical representations)	Connect students' points in activity sheet and points of the graphs in GGB graphics view to have students check their answers by posing a question (H)
		Connect points of graph in students' activity sheet and the graph in GGB graphics view by making explanations (H)
Connect equation, line graph, slope, GGB graphics and algebra views, and activity sheet	Connect equation, line, slope, GGB graphics view, GGB algebra view, and table in activity sheet	Connect equation of a line in GGB algebra view and slope of a line in GGB graphics view and table of equations and slopes in activity sheet for a situation (S)
		Connect equation of line in GGB algebra view and slope of line in GGB graphics view and table of equations and slopes in activity sheet for various situations (S)
Practice 2: Connect mathematical situation/concept and GGB material		
Connect points, functional situation, GGB graphic view, and GGB tools	Connect points of line graph for functional situation, GGB graphics view, slider tool, trace tool	Connect point-h-slider, trace of point H in GGB graphics view, points of line graph for functional situation of Building Design (B)
Connect line graph, and linear relation, GGB graphics view, and GGB tools	Connect line graph and linear relation for functional situation, GGB graphics view, line tool	Connect drawing line tool, line graph in GGB graphics view and linear relation for functional situation by making explanations (B)
Connect concepts of coordinate system, GGB graphics view	Connect point, equation, line graph, GGB graphics view	Connect coordinates of point, equation and line graph in the coordinate system in GGB graphics view (solution of equation) (B)

Table 12 (continued)

Connect functional situation, points, dynamic figure, GGB tools, GGB graphics view	Connect items of graph, dynamic figure of situation, slider tool	Connect points of the graph of situation, time-slider, change of amount of water in dynamic figure by making explanations and posing questions (H)
	Connect items of graph, graph, dynamic figure of situation, slider tool, trace tool	Connect time-slider, trace of point, points of graph, graph, change of amount of water in dynamic figure in GGB graphics view by making explanations (H)
Connect equation and line, GGB algebra view, GGB graphics view	Connect equation and line in algebra and graphics views of GGB	Connect equation of line in GGB algebra view and line in GGB graphics view by making explanations (S)
		Connect constant value in functional situation and in equation, y-intercept point value of line in GGB algebra view and GGB graphics view by posing questions (S)
Connect equation, line, slope, GGB algebra view, GGB graphics view, and GGB tools	Connect slope, equation and/or line, slider in GGB algebra and graphics views	Connect equation, slope value of equation/line and m-slider as symbolic notation of slope (m) by posing questions (S)
		Connect line, slope value of equation/line, m-slider in GGB graphics view and GGB algebra view by posing questions and making explanations (S)
Connect, equation, line, y-intercept, GGB algebra view, GGB graphics view, and GGB tools	Connect y-intercept, equation and/or line, slider in GGB algebra and graphics views	Connect n-slider as n-symbolic notation, constant term in equation, and y-intercept of line by posing questions and making explanations (S)
Practice 3: Connect mathematical situation/concept/ process and GGB material		
Connect similarity, slope computation, GGB graphics view, GGB tools	Connect similarity, computation of slope, checkboxes of changes, GGB graphic view	Connect similarity in similar triangles, computation of slope of line, and checkboxes of horizontal and vertical changes for different intervals in GGB graphic view by posing questions (B)
Connect interval between two points, slope computation, GGB tools, GGB graphic view	Connect segments for intervals, slope value, checkbox tools, GGB graphics view	Connect points checkboxes for intervals, slope checkbox for slope value in GGB graphics view by making explanation (H)
	Connect segments for an interval, slope algorithm, checkbox tools, GGB graphics view	Connect horizontal vertical changes checkboxes for an interval, slope checkbox for text of slope computation algorithm in GGB graphics view by making explanations (H)
	Connect moving point, different intervals, slope algorithm, checkbox tools, GGB graphic view	Have students connect moving points on a graph, different intervals on a graph, slope checkbox for text of slope computation algorithm in GGB graphic view by making explanations (H)

Table 12 (continued)

Practice 4: Connect concrete object, mathematical process, and GGB material		
Connect concrete object, objects in GGB graphics view, slope computation	Connect a physical object (i.e. pen), line segment in GGB graphics view, computation of slope of line	Connect computation of slopes of an object (i.e. pen) and a line for the functional situation of Building Design in GGB graphics view by making explanations and posing questions (B)
Connect concrete object, similarity, slope computation, objects in GGB graphics view	Connect a physical object (i.e. batten), similarity, computation of slope of line	Connect similarity in similar triangles (previous knowledge) and computation of slope of a line for the functional situation of Building Design using an object (i.e. batten) by posing questions and making explanations (B)
Practice 5: Connect conceptualizations of concept(s) and GGB material		
Connect behavior of slope and symbolic notation in algebraic expression, GGB graphics view	Connect behavior of slope and algebraic expression of slope computation, GGB graphics view	Have students connect sign as behavior of slope and operations in algebraic expression of slope computation text in GGB graphics view by posing questions (H)
		Connect sign of slope and ratio in slope computation text in GGB graphics view by posing questions (H)
Connect change in dimensions and operations in slope computation, GGB graphics view	Connect change in vertical and subtraction in slope computation, GGB graphics view	Connect vertical change and subtraction in algebraic expression of slope computation text in GGB graphics view by posing questions (H)
Connect forms of equations, GGB algebra view, GGB graphics view	Connect standard and slope-y-intercept forms of equation in GGB algebra view, lines in GGB graphics view	Connect standard and slope-y-intercept forms of equation by converting in GGB algebra view, and their lines in GGB graphics view by posing questions (S)
Practice 6: Connect different software		
Connect GGB and WinGeom	Connect using of line tool in GGB and WinGeom	Connect drawing with line tool, GGB and WinGeom software by making explanations (B)

Note: The abbreviations in the sample description column of the table indicate the activity in which an action emerged. B: Building Design activity, H: Leaking Container activity, S: Slopes and Equations of Lines activity.

As can be seen in Table 12, in the *Building Design* activity, the teacher's bridging practices were categorized as practices for connecting the mathematical situation/concept, GGB material, and the activity sheet (practice 1), connecting the mathematical situation/concept and the GGB material (practice 2), connecting the mathematical/situation/concept/process and GGB material (practice 3), connecting the concrete object, the mathematical process, and GGB material (practice 4), and connecting different software (practice 6).

In the beginning of the *Building Design* activity, the students filled in the table, wrote the equation and drew the graph for the functional situation that relates to the numbers of windows and floors in their activity sheets. As practice for connecting the mathematical situation/concept, the GGB material, and the activity sheet (practice 1), the teacher first connected the functional situation, the GGB tools, and the activity sheet. In detail, she connected the sliders (slider-k and slider-z produced the equation $y = z + kx$ in the background of the GGB material) and the number of windows on each floor with the number of windows in the basement of the building, which are the coefficient of a term and the constant value of the equation for the functional situation, relatively. As an evidence, she asked, *"The number of windows on a floor is K (The teacher shows it on slider k). What is the number of windows on each floor?"*. She then explained, *"It [the number of windows] increases 3 at a time. Then, in this case? I moved slider-k to 3, okay?"*. In addition, as practice for connecting the mathematical situation/concept and GGB material (Practice 2), the teacher connected the points, the functional situation, the GGB graphics view, and the GGB tools. In detail, she connected slider-h (slider-h produced the points of the graph), the trace of point H, which was moved with slider-h in the GGB graphics view, the points of the line graph for the functional situation in the GGB graphics view by making explanations and posing questions. Therefore, the students could structure the points of the graph on the coordinate plane in the GGB graphics view. After showing the points on the coordinate plane, the teacher connected the line graph, the linear relation, the GGB graphics view, and the GGB tools. While doing this, she used the line tool while connecting the linear relation and the line graph. As an evidence, she said, *"What should we do with these points? We can draw the line by connecting the points. Didn't you do that too?"*. After this process, the teacher connected different software (Practice 6). In detail, she made explanations to make a connection between the use of the GGB and the WinGeom software in terms of drawing lines with the line tool (685). She said, *"You are drawing in the same way as you did in WinGeom before, right?"* Since the teacher and her students had used WinGeom in class, the teacher intended to make such connections for the students' mathematical constructions in GGB. She thought that her students could quickly comprehend how to use GGB by using the GGB materials.

Then, as practices for connecting concrete object, mathematical process, and GGB material, the teacher connected the concrete object, the line segment, the slope computation, and the GGB graphics view, and connected the concrete object, similarity, slope computation, and the GGB graphics view. As a starting point for enabling the students to understand the slope of a line, the teacher established a connection between the computation of the slope of an object (i.e. pen as a concrete object) she was holding and the computation of the slope of the graph for the functional situation in the *Building Design* activity sheet by posing questions. To illustrate, she asked successively, *"Does this have a slope (The teacher shows the line graph in the GGB graphics view)?"*, *"But this is leading to infinity. How can there be a slope? How should we calculate it? This pen has a slope, doesn't it?"*, and *"What do you do when you are calculating the slope of this pen? How will you calculate the slope of a line?"*. Then, she connected similarity in similar triangles and the computation of the slope of the line in the GGB graphics view by showing the intervals between different points on a batten by posing questions. As an evidence, for example, she asked, *"Would the slope value vary if you calculate the slope between this point and that point (The teacher shows the points of an interval on the batten), and the slope between here and here? (The teacher shows the start and end points of the batten)"*. A couple of minutes later, as practice for connecting the mathematical situation/concept/process and the GGB material (practice 3), she again connected the concept of similarity in similar triangles and the computation of the slope of line in the GGB graphics view using checkboxes, which showed horizontal and vertical changes for different intervals on the line, by posing questions. As an evidence, she asked, *"We took lots of triangles. Are there any differences among them in terms of computing the slope? (The teacher clicks on the two checkboxes that show the arrows for change in the y-variable with unit change in the x-variable)"* (See Figure *28* for the teacher's computer screen view at that moment).

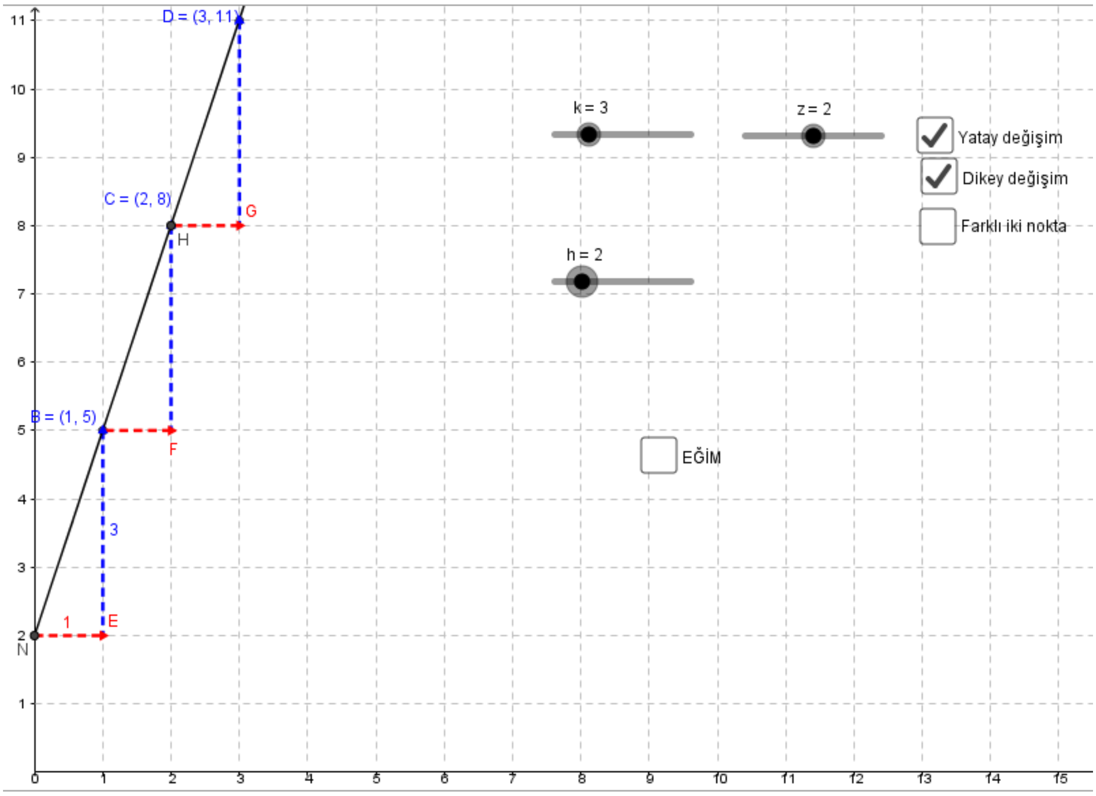

Figure 28. The teacher's screen view of the GGB material in the Building Design activity.

In the GGB graphics view in Figure *28*, the checkboxes (vertical change and horizontal change) were clicked after the sliders were moved and the line graph was drawn. The clicked checkboxes showed how the number of windows increased when the number floors increased by 1 with the dashed arrows.

As can be seen in Table 12, in the *Leaking Container* activity, the teacher's bridging practices were categorized as practices connecting the mathematical situation/concept, GGB material, and the activity sheet (practice 1), connecting the mathematical situation/concept and GGB material (practice 2), connecting the mathematical situation/concept/process and GGB material (practice 3), connecting conceptualizations and GGB material (practice 5).

In the *Leaking Container* activity, the students used the GGB material within their groups. The students used GGB material after they filled in the table and wrote the equation for the situation in their activity sheets. Since most of the students could not draw the graph for the functional situation that related the amount of water in the container to the time in the activity sheet, the teacher provided the GGB material at that moment. As practice for connecting the mathematical situation/concept, the GGB material, and the activity sheet (practice 1), the teacher established a connection between students' drawings of points in their activity sheets and points of the graph in the GGB graphics view by having students check their drawings. As evidence, she asked, *"…You can follow on GGB. For instance, what happens at the first minute?"*. Therefore, the students showed the points of the graph for the situation when they moved the slider (slider-b), which was connected to a point (point-Z was defined through the equation for the situation) in the GGB graphics view. In addition, as practice for connecting the mathematical situation/concept and GGB material (practice 2), the teacher established connections among the functional situation, points, the dynamic figure, GGB tools, and the GGB graphics view. That is, to enable students to draw the points of the graph for the situation, the teacher connected the points of the graph for the situation, slider-b for time variable, and the change in the amount of water in the dynamic figure of the situation in the GGB graphics view, by making explanations and posing questions. As evidence, when the students looked at the GGB material, she said, *"Now are you moving the slider at the same time? Click on that slider. Now at the same time look here. (The teacher points to the coordinate system) and here (The teacher points to the dynamic figure)"*. Immediately afterwards, by making explanations, she established a connection between the points on the graph and the graph for the functional situation, changed the amount of water in the dynamic figure of the situation, the time-slider, and the trace of point-Z in the GGB graphics view. As evidence, for example, she said, "*Click on your slider. Move it continuously (The graph is seen in the GGB graphics view). Look at the water (The teacher refers to the dynamic figure of amount of water in the container). What happens to the amount of water? Do you see the change of water on the left side?*" (See Figure 29 for the computer screen view of the students at that moment).

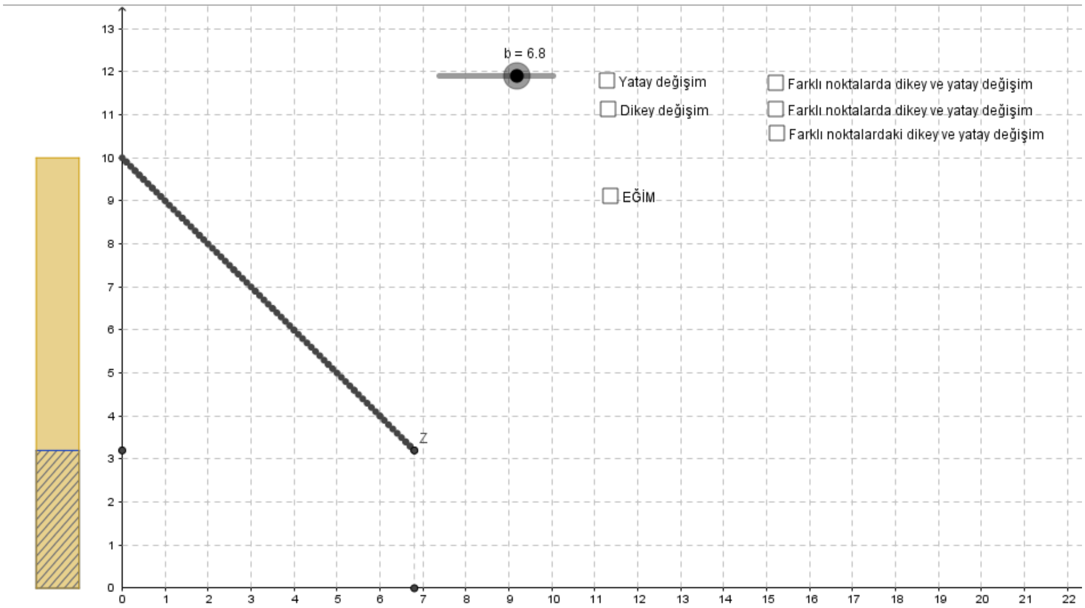

Figure 29. The students' screen view-1 of the GGB material for the Leaking Container activity.

In Figure *29*, the trace of point-Z was on and slider-b was moved to 6.8. Slider-b was connected to point-Z and the dynamic figure. The dynamic figure on the left side showed the amount of water in the container in relation to time.

Moreover, as practice for connecting the mathematical situation/concept, GGB material, and the activity sheet (practice 1), the teacher again established a connection between students' drawings of points of graph in their activity sheets and the graph in the GGB graphics view by making explanations. For example, the following dialogue emerged in the practice.

(T) What is this graph for? *(The teacher shows the points of the graph in Figure 29).* And look at your graph. What do you need to do right now?

(S) We need to place the intervals between [the points] *(The students draw the points for the integer values of the time variable in their activity sheet but do not draw a line).*

(T) Yes. That is, you should draw a line. Because it is a continuous event. There is water between 2 minutes and 1 minute. So, what do you do? That is, we need to bring together the set of points between them.

After the teacher ensured that the students understood the equation and the graph for the functional situation in the *Leaking Container* activity and enabled the students to overcome their difficulties in points, axes and graphs on the coordinate system, the students computed the slope of the line graph by using any two points. When the students experienced difficulty in computing, the teacher provided the GGB material to the students again. In this process, as practice for connecting the mathematical situation/concept/process and GGB material (practice 3), the teacher make connections among interval between two points, slope computation, GGB tools, and GGB graphics view. That is, she established connections between points checkboxes for different intervals between two points, and the slope checkbox for slope value in the dynamic slope computation text in GGB Graphics view, by making explanations. As an evidence, she said, *"If we use the computer, it could perhaps give a clue for us. For example, click on the boxes [checkbox] on the right side. Click on the two-different-points checkboxes"*. Immediately afterwards, she connected the horizontal change and the vertical change checkboxes for the horizontal change and vertical change segments of an interval between two points and the slope checkbox for the dynamic text of the slope computation algorithm in the GGB graphics view, by making explanations. She also connected moving a point of the interval, different intervals on the graph, which were composed by moving the point on the interval and the slope checkbox for the dynamic text of slope computation in the GGB graphics view, by making explanations. As evidence, for example, she said, *"Now do it like this. Remember you opened up [clicked the checkboxes] horizontal and vertical change segments. So point-A emerged there, right? Click on point A and drag it. See how the slope changes"* (See Figure 30 for the computer screen view of the students at that moment).

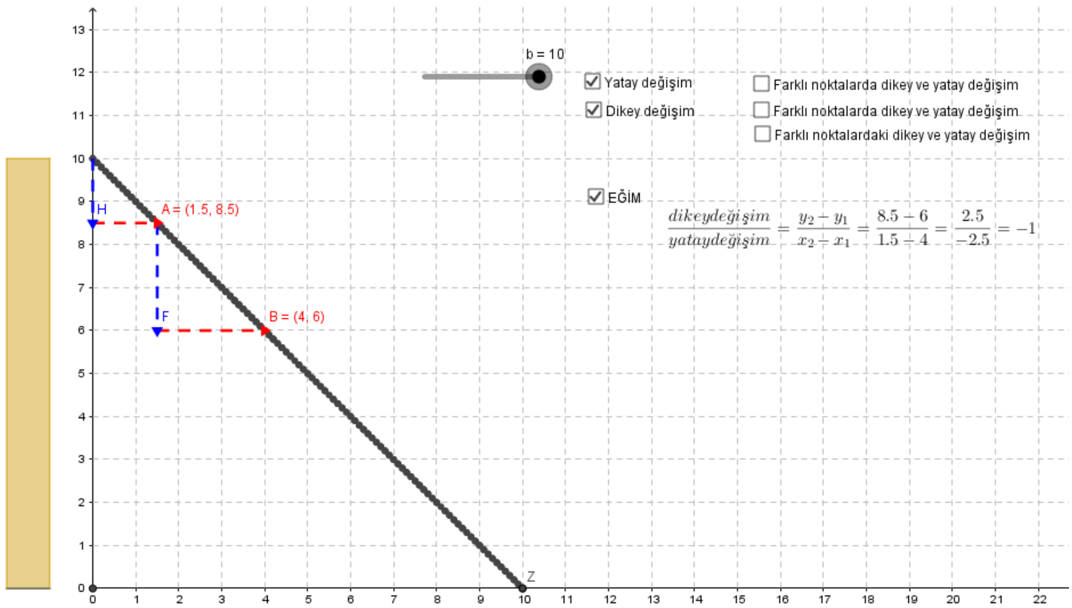

Figure 30. The students' Screen view-2 of the GGB material for the Leaking Container activity.

In the GGB graphics view in Figure 30, the horizontal change, vertical change and slope checkboxes were clicked. The slope checkbox showed the dynamic slope computation text, which was connected to the dynamic points of A and B, which were shown with the horizontal change and vertical change checkboxes.

Then, as practices for connecting conceptualizations of slope and GGB graphics view (practice 5), the teacher established connections among the behavior of a slope, the symbolic notation of slope computation, and the GGB graphics view and among the change in the dimensions, the operations in slope computation, and the GGB graphics view. As practice, the teacher established connections between the behavior of slope and the algebraic expression of slope computation in the GGB graphics view. More specifically, she established a connection between the sign of the slope as behavior of slope and operations in algebraic expression of dynamic text of slope computation in GGB graphics view by posing question. As evidence, for example, she asked, *"When it decreases, what result does this operation yield? In subtraction (in slope computation)?"*. Thus, in this example, the teacher related the negative sign of the slope to the behavior of a decrease and the result of the operation of subtraction. In addition, she related the sign of the slope to the ratio in the dynamic text of slope computation in GGB graphic view by posing a question. She asked, *"So* *what happened to [the sign of] the ratio? Negative…"*. When the students made computations, as another practice for making connections among conceptualizations of change, she related vertical change to subtraction in the algebraic expression of the dynamic slope computation text in the GGB graphics view. For example, she asked, *"So how many units is the vertical length of the triangle (The teacher refers to the interval between two points) we formed? Does it have any relation to the value we found? "*. Thus, the teacher wanted to have the students structure the relation among the sign of the slope, the operations in the computation of the slope, and the ratio in the slope computation of the slope.

In the *Slopes and Equations of Lines activity,* the teacher's bridging practices were categorized as practices for connecting the mathematical situation/concept, the GGB material, and the activity sheet (practice 1), connecting the mathematical situation/concept and GGB material (practice 2), and connecting conceptualizations and the GGB material (practice 5) (see Table 12).

In the *Slopes and Equations of Lines* activity, the students used the activity sheet and the GGB material in coordination. As practices for making connections between the mathematical situation/concept and the GGB material (practice 2), the teacher established a connection among equation, line, GGB algebra view, and GGB graphics view and among equation, line, slope, GGB algebra view, GGB graphics view, and GGB tools. That is, after the students got engaged in the activity sheet, the teacher started to use the GGB material to connect the equation of the line in the GGB algebra view and the corresponding line in the GGB graphics view by making explanations. Then, she connected an equation in the GGB algebra view and the slope value of the equation/line and m-slider as a symbolic notation of slope (m) in the GGB graphics view by posing questions. As an evidence, for example, the following dialogue emerged during these practices.

(T) … Everything, every line, point we have structured here *(The teacher shows the GGB Graphics view)* has a mathematical display here *(she shows the GGB algebra view)*. It has a figurative image here (the GGB graphics view) … When we click on that line *(she clicks on an equation of a line in the GGB algebra view)* look at whose color changes *(she moves the mouse on the equation in the GGB algebra view and the line is highlighted in the GGB graphics view)*? What is this? It is the equation of that line *(she moves on the line in the GGB* *graphics view and the line sparkled in the GGB graphics view)*. That is, it is the line of y equals x minus two *(she shows the GGB algebra view)*. So what is the slope of this line?

(SS) One.

(T) It is one. So which of these sliders was one? (*she mentions the sliders on the GGB graphics view*)

(SS) M.

(T) So what is m?

In addition, she connected the line, slope value of the equation/line and m-slider in the GGB graphics view and the equation in the GGB algebra view by posing questions and making explanations. For example, she said, *"We changed the slope (The teacher mentions moving slider-m for slope). When the slope changed, did the appearance of the line change too? Those with a different slope appear differently, don't they? So what happened to the equation? "*.

As practice for connecting mathematical situation/concept, GGB material, and activity sheet (practice 1) in the *Slopes and Equations of Lines* activity, the teacher established connections among equation, line, slope concepts, GGB material and the students' activity sheets. In detail, by posing questions and making explanations, she made connections among the equation of a line in the GGB algebra view, the slope of a line graph in the GGB graphic view, and the table of the equations and the slopes in activity sheet for a situation and for various situations. That is, while the teacher asked questions, such as *"Did the image of the line change when you changed the value of the slope? (The students moved slider-m at that moment). What happened to the equation?"*, the students created various slopes for different equations and lines in the GGB material, and then brought them together on the table in the activity sheet to build connections among them.

Then, as practice for connecting mathematical situation/concept and GGB material (practice 2), she again connected equation, line, GGB algebra view, and GGB graphics view. In detail, when the teacher focused on the other items of the equations, she connected the constant value in the functional situation and the equation in GGB algebra view and y-intercept point value of the line in GGB graphics view by posing questions. As another for connecting mathematical situation/concept and GGB material (practice 2), she connected equation, line, y-intercept, GGB algebra view, GGB graphics view, and GGB tools. That is, she connected constant term in equation in GGB algebra view, n-slider as n-symbolic notation and y-intercept value of line in GGB graphics view by posing questions and making explanations.

As a conclusion for this activity, the teacher connected forms of equations, GGB algebra view, and GGB graphics view as practice for connecting conceptualizations of concept(s) and GGB material (practice 5). In detail, she connected the standard form of an equation and slope-intercept form of an equation, and the line in GGB graphics view by converting the equations to each other in GGB algebra view by making explanations and posing questions. For example, *"Are both the same? (She right clicks on the equation and changes the form of the equation)"*.

4.2.1.3 Bridging practices in teaching the solution of a system of equations and the slope relation for Phase 3

The bridging practices in teaching the solutions of systems of equations and the slope relation with GGB materials are explained in this subsection. In the third phase, the teacher and the students used three activity sheets: *Mobile Operators*, *Equation Systems*, and *Stores*. They used two GGB materials for the *Equation Systems* activity, one GGB material for the *Mobile Operators* activity, and one GGB material for the *Stores* activity. In the *Mobile Operators* activity, two equations represented two functional situations that had positive slope values. In the *Stores* activity, two equations represented two functional situations in which one had a positive slope and the other had a negative slope value. In the *Equation Systems* activity, various systems of equations were created on the GGB material to analyze the systems of equations.

While using GGB materials, the teacher connected GGB views, GGB tools, and mathematical situations, concepts, and processes, conceptualizations of mathematical concepts, and students' activity sheets in various combinations. In the dimension of GGB material codes, the teacher used graphs and points, slope value, slope checkboxes, equation checkboxes, slider tools, and trace of points in GGB graphics view and slope value, equations and forms of equations in the GGB algebra view. The dimension of mathematical codes involves (1) functional situations as mathematical situations, (2) equations of lines, line graphs, solution(s) of equations, point(s), ordered pair(s), coordinates of point, coordinates of intersection point, intersection point, solution set of system of equations, position of lines, slope, slope of line, slope of equation, slope-intercept form of equation, sign of slope, and behavior of lines as mathematical concepts, and (3) ways of finding a solution set of a system of equations, and computation of a slope as mathematical processes. The activity sheet dimension of codes involved a table, students' points of graphs, and students' line graphs. In consequence of these dimensions, bridging practices consisted of the following themes:

Practice 1. Connecting the mathematical situation/concept and the GGB material;

Practice 2. Connecting the mathematical situation/process and the GGB material;

Practice 3. Connecting conceptualizations and the GGB material;

Practice 4. Connecting the mathematical concept, the GGB material, and the activity sheet;

Practice 5. Connecting the mathematical situation/concept, GGB material, and activity sheet.

The practices and the actions under each practice with sample descriptions are presented in Table *13*. The detailed descriptions of these practices in Table *13* are given in the sequential flow of the activities during phase 3. In the table, the practices of the teacher are enumerated to follow each practice easily in the findings, and this enumeration does not indicate the instructional sequence of the teacher. In other words, the teacher did not enact these practices in the classroom in same order as in the table.

Table 13. Bridging Practices of Phase 3 while using GGB materials

Practice	**Action**	**Sample Description**
Practice 1: Connect mathematical situation/concept and GGB material		
Connect equations, line graphs, GGB algebra view, and GGB graphics view, and/or GB tools	Connect equations of lines in GGB algebra view, line graphs in GGB graphics view	Connect equations of lines in GGB algebra view and line graphs in GGB graphics view for the functional situations of Mobile Operators by posing questions (M)
		Connect equations of lines in GGB algebra view and line graphs in GGB graphics view for various situations in Equation Systems activity (E)

Table 13 (continued)

	Connect equations of lines in GGB algebra view and points and line graphs in GGB graphics view	Connect equations of lines in GGB algebra view, points of graphs and line graphs for functional situation of store-M/N in GGB graphics view (S)
	Connect equations, line graphs, checkbox in GGB graphics view	Connect line graphs and equation checkbox that showed the equations of the line graphs in GGB graphics view (S)
Connect equations, line graphs, solutions, GGB tools, and GGB graphics view	Connect points of lines, solutions of equations, coordinates of the points functional situations, GGB graphics view, slider tool	Connect points of line graphs, solutions of equations of lines, coordinates as ordered pairs in functional situations of Mobile Operators using time-sliders for situations and graphs in GGB graphics view by making explanations and posing questions (M)
	Connect points, solutions of equations, GGB graphics view, slider tool, trace tool	Connect points of line graphs, solutions of equations of lines for functional situations of Stores M/N using time-sliders for situations and graphs, trace of points in GGB graphics view (S)
	Connect point, solution, GGB graphics view	Connect a point in GGB graphics view and a solution of equation for functional situation of Store-M/N by posing questions (S)
Connect slope, line graph, GGB graphics view, and/or GGB tool	Connect slopes, line graphs, checkbox, GGB graphics view	Connect slopes of line graphs and slope checkboxes in GGB graphics view (S)
	Connect sign of slope and behavior of line graph, GGB graphics view	Connect positive and negative sign of slopes and increasing and decreasing behavior of line graphs in GGB graphics view (S)
Practice 2: Connect mathematical situation/process and GGB material		
Connect process of solving system of equations GGB graphics view	Connect ways of finding solution set of system of equations in GGB graphics view	Connect algebraic ways and geometric ways of finding solution set of linear equation systems for functional situations of Mobile Operators in GGB graphics view by responding student's answer and making explanations (M)
		Connect algebraic ways and geometric ways of finding solution set of system of equations using GGB graphics view (E)
Connect computation of slope, GGB graphic view, and/or GGB algebra view	Connect slope value in GGB algebra view and computation of negative slope of a line	Connect student's use of slope value in GGB algebra view and student's computation of slope of a decreasing line by posing a question (E)

Table 13 (continued)

	Connect computation of slope and vertical change/horizontal change in GGB graphics view	Connect conception of vertical change/horizontal change in GGB graphics view and computation of slope of line by making explanation and posing question (E)
	Connect computation of negative slope, forms of equations, GGB algebra view	Connect slope of equation and slope-intercept form of equation using GGB algebra view (E)
	Connect computation of slope of line, triangle region in GGB graphics view	Connect computation of slope of line and triangle that is bounded by the line and the axes in GGB graphics view (E)
Practice 3: Connect conceptualizations of concept(s) and GGB material		
Connect representations of intersection point, GGB graphics view	Connect symbolic and verbal representations of intersection point in GGB graphics view	Connect coordinates and verbal description of intersection point for the functional situations of Mobile Operators in GGB graphics view by posing questions (M)
Connect conceptualizations of solution set, GGB graphics view	Connect intersection point, solution set of equations, GGB graphics view	Connect intersection point of lines and solution set of system of equations for functional situations of Mobile Operators in GGB graphics view by posing questions and making explanations (M)
		Connect coordinates of intersection point and solution set of system of equations for functional situations of Stores in GGB graphics view by posing questions and making explanations (S)
		Connect coordinates of intersection point and solution set of system of equations for lines in GGB graphics view (E)
Connect conceptualization of linearity, GGB graphics view	Connect functional situation, visual representation and verbal description of linearity, GGB graphics view	Connect functional situation of store-M, verbal description of linear relation, points of the line graph in GGB graphics view (S)
Practice 4: Connect mathematical concept, GGB material, activity sheet		
Connect slopes of equations, lines, solution set, GGB material of another activity, and table in the activity sheet	Connect GGB graphics view of another activity, slopes of lines of equations in a system of equations, solution set of system of equations, and table in the activity sheet	Connect slopes of lines/equations and solution set of system of equations in GGB graphics view of Mobile Operators activity and table in the activity sheet of the Equation Systems activity by posing questions and making explanations (E)

Table 13 (continued)

Connect equations, position of lines, GGB algebra view, GGB graphics view, and table in activity sheet	Connect equations and position of lines in GGB algebra view and GGB graphics view, table in activity sheet	Connect equations of lines in GG algebra view and position of lines in GGB graphics view, and table in activity sheet of Equation Systems for various systems (E)
Connect equations, position of lines, slopes of lines, solution set, GGB algebra view, GGB graphics view, and/or GGB tools, and table in activity sheet	Connect equations, position of lines, slopes of lines in GGB algebra view and graphics view, slope checkbox, table in activity sheet	Connect equations of lines, position of lines, slope of lines using slope checkbox in GGB algebra view and graphics view, and table in activity sheet Equation Systems for various systems (E)
	Connect equations, position of lines, slopes of lines, solution set, GGB algebra view and graphics view, table in activity sheet	Connect equations of lines, positions of lines, slopes of lines, and solution set of system of equations using GGB graphics view and algebra view and table in activity sheet by making explanations (486) (E)
Practice 5: Connect mathematical situation/concept, GGB material, and activity sheet		
Connect graphs of functional situations, GGB graphics view, and students' activity sheet	Connect points of graph in activity sheet, GGB graphics view, points	Connect student's points of graph in activity sheet and GGB graphics view to draw points of graph for functional situation of store-M (595) (S)
	Connect line graph in activity sheet, GGB graphics view, line	Connect student's line graph in activity sheet and line graph in GGB graphics view to check the line graphs of Store-M and N (603, 605, 618) (S)

Note: The abbreviations in the sample description column of the table indicate the activity in which an action emerged. M: Mobile Operators activity, E: Equation Systems activity, S: Stores activity.

In the *Mobile Operators* activity, the teacher's bridging practices were categorized as practices for making connections between the mathematical situation/concept and the GGB material (practice 1), between the mathematical situation/process and the GGB material (practice 2), and between conceptualizations of concept(s) and the GGB material (practice 3) (see Table 13).

After the students worked on the tables, equations, and graphs in their activity sheets for Mobile Operators, the teacher connected equations, line graphs, GGB algebra view and GGB graphics view as practice for connecting mathematical situation/concept and GGB material (practice 1). In detail, the teacher enabled students to use both GGB algebra view and GGB graphics view to connect equations of lines in GGB algebra view and the corresponding lines in GGB graphics view for functional situations (i.e. T-cell and Z-cell mobile operators' time versus price relations), by posing questions. As evidence, for example, the following dialogue among a group of students and the teacher emerged during the practice.

(T) *(when the students looked at to the GGB material of the activity)* Look at your first line *(the teacher refers to the equation in the GGB algebra view)* and clicks it. Did you draw this relation *(The teacher shows the line in the GGB graphics view)*?

(SS) Yes, we did *(the students clicked the equation* $y = 2x$ *in the GGB algebra view and the line is seen on the GGB graphics view).*

(T) Click the second one. Before this, is your equation correct? That is, is it *y* equals to *2x*?

(SS) Yes.

(T) What is your second equation?

(S) Y equals to x plus 5 *(the students clicked the equation* $y = x + 5$ *in the GGB algebra view and the line is seen on GGB graphics view)*

Subsequently, as another practice for connecting mathematical situation/concept and GGB material (practice 1), the teacher connected equations, line graphs, solutions, GGB tools, and GGB graphics view. That is, the teacher had students use slider tools (slider-t and slider-z for the time variable in the situation) that elicit the points of the graphs for the equations in GGB graphics view to connect points of the graphs, solutions of equations, and ordered pairs in the functional situations by making explanations and posing questions. As evidence, the teacher said,

> For example, in your activity sheet, which minute is asked for (She asks for the seventh minute)? Can you move the sliders to the seventh-minute? (She means two sliders that move the points for the two situations). Move the sliders to seven. We can see which one [price] is higher. How much do you pay for each?

In further connections, as practice for connecting conceptualizations of concept(s) and GGB material (practice 3), the teacher connected representations of intersection point and the GGB material. The teacher had students use the GGB graphics view (graphical representation of intersection point) to connect coordinates of intersection point (symbolic representation) and verbal description of intersection point (verbal representation) for the functional situations, by posing questions. During this activity, as practice for connecting mathematical situation/process and GGB material (practice 2), the teacher connected the process of solving system of equations and GGB graphics view. In detail, she connected algebraic and geometric ways of finding solution set of linear equations system for the functional situations using GGB graphics view, by responding to the students' answers and making explanations. For example, after a student found the intersection point of the lines by solving the system of equations algebraically and showed her solution to the teacher, the teacher said, *"You used the equations. It's okay. Can we use the graphs? (The teacher refers to the GGB graphics view)"*.

Then, another practice for connecting conceptualizations of concept(s) and GGB material (practice 3), the teacher connected conceptualizations of a solution set and the GGB graphics view. Specifically, she enabled students to connect the intersection point of the lines and the value of solution set of the system of equations for the functional situations of Mobile Operators in GGB graphics view. For example, the following dialogue emerged to show that the intersection point represented the solution set to a system of equation after the students solved the system of an equation by hand.

(T) You know, there is an intersection point here *(the teacher refers to the point on the GGB graphics view)*. So, what is this intersection point considering the equations of the two lines?

(S) The solution set.

(T) It is the solution set of the system of equations, isn't it? Do you see it? What do the linear equations have? Two variables, x and y, and various x and y values. If the two lines for the two linear equations intersect, there is one solution set.

Thus, for this activity, the teacher explained that the value of the solution set of a system of equations for intersecting lines is an ordered pair, which were the coordinates of the interception point.

In the activity of *Equation Systems*, the teacher's bridging practices were categorized as practices for making connections between mathematical situation/concept and GGB material (practice 1), between mathematical situation/process and GGB material (Practice2), connect conceptualizations of concept(s) and GGB material (practice 3), and among mathematical concept, GGB material, and the activity sheet (Practice 4) (see Table 13).

In this activity, the teacher prepared two GGB materials that involved systems of equations for parallel lines (the Equation Systems-1 material) and systems of equations for intersecting lines (the Equation Systems-2 material). In the activity, the students used the GGB materials within their groups and the activity sheet individually. The activity sheet involved tables for analyzing systems of equations with their equations, slopes, a solution set, relative positions of lines, and a coordinate system for drawing lines of equations in the systems of equations (see Appendix H).

As practice for connecting the mathematical concept, the GGB material, and the activity sheet (practice 4), the teacher connected slopes of equations, lines, solution set, GGB material of another activity, and table in the activity sheet. As a starting point, the teacher used the GGB material of Mobile Operators to connect the slopes of the lines/equations and the solution set of the system of equations in the GGB graphics view of the previous activity and the table in activity sheet of Equation Systems by posing questions and making explanations. As evidence, she said, *"One line represents the equation of y equals to 2x and the other one represents y equals to x plus 5. The slope of the first one is 2 and the slope of the second one is 1. What is the solution set for this system?"* Thus, the students started to bring together the value of the solution set of a system of equations, the slopes of the lines/equations both in the GGB material and the table of the activity sheet.

Then, as practice for connecting the mathematical situation/concept and GGB material (practice 1), the teacher used the Equation Systems GGB materials to connect equations of lines in the GGB algebra view and the corresponding line graphs for various situations in the GGB graphics view. That is, the students bring together various systems of equations in the GGB algebra view and their graphical representations in the GGB graphics view by moving the dynamic points that were on the lines (e.g. system of equations for parallel lines, see Figure *31*).

In addition, as another practice for connecting the mathematical concept, the GGB material, and the activity sheet (practice 4), the teacher connected equations, position of lines, GGB algebra view, GGB graphics view, and the table in the activity sheet. For example, while the students were using the Equation Systems-1 GGB material for parallel lines, the teacher enabled the students to use both the GGB material and the activity sheet simultaneously to connect the equations of the lines in the GGB algebra view and position of lines in the GGB graphics view, and the table in the activity sheet for various systems by making explanations. Following this practice, the teacher connected the equations of the lines in the GGB algebra view, the position of the lines, and the slopes of the lines using the checkbox in the GGB graphics view, and the table in the activity sheet for various systems of equations by posing questions and making explanations. For example, when the practice above was enacted, the screen view of the Equation Systems-1 GGB material in Figure 31 appeared.

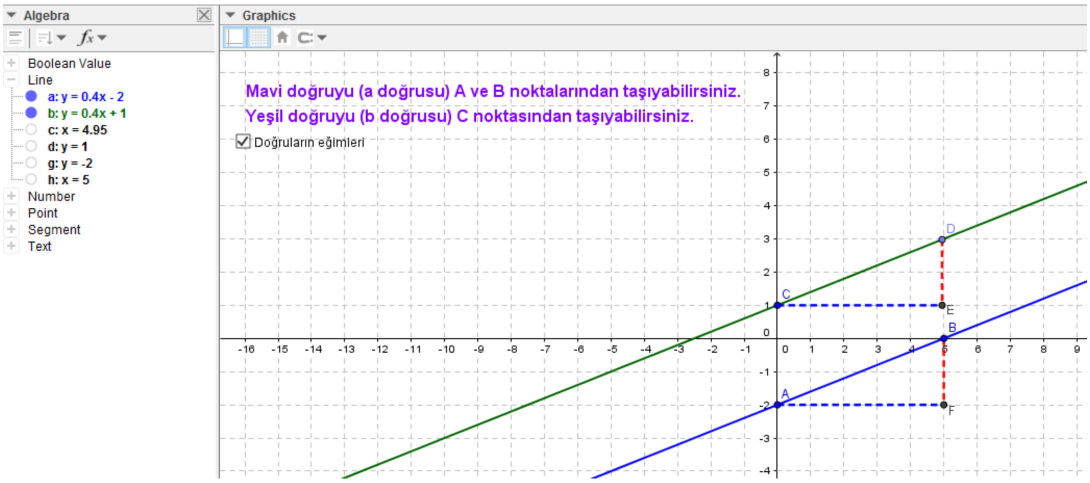

Figure 31. Screen view of the Equation Systems-1 GGB material.

In Figure *31* for the Equation Systems-1 GGB material, the algebra view of the GGB material shows the following equations:$y = 0.4x - 2$ and $y = 0.4x + 1$. The graphic view of the GGB material showed the lines of the equations, the slope checkbox that was clicked and the guiding segments for slope computation. The dynamic lines could be moved by using dynamic points A, B, and C.

While the students were working on the Equation Systems-2 GGB material for intersecting lines, the teacher connected conceptualizations of solution set and the GGB material (practice 3). That is, she connected coordinates of an intersection point as an ordered pair and the solution set of the system of equations as an ordered pair algebraically for the intersecting lines in the Equation Systems-2 GGB material, by posing questions and making explanations. As evidence, the following dialogue emerged during the practice after a student mentioned that she could not understand the screen view of the GGB material.

(T) So, we can say that the intersection point of these two lines represents their solution set [two equations of the lines]. What is the coordinates of these points *(the teacher shows the point on the GGB graphics view)*? What is the value of x?
(S) Four.
(T) What is y?
(S) Zero.

(T) You can see points four to zero *(the teacher shows point (4, 0) in the GGB graphics view)*. Can you solve the system of equations that you created?

During the aforementioned practices were enacted in this activity, the teacher also connected the mathematical situation/process and GGB material (practice 2). She made connections among the process of finding a solution set, computation of slope, and conceptions of slope computations in the activity of Equation Systems. More specifically, she made connections among algebraic ways and geometric ways of finding solution set of system of equations using GGB graphics view, by posing questions. In addition, while the teacher guided students in computing the slopes of the equations of lines, she connected the students' use of slope value in GGB algebra view for the decreasing line graph and the students' computation of slope using the slope-intercept form of the equation of the line. For example, she said, *"You saw the slope from the algebra view. Ok, let's find the slopes now. We'll compare it with GGB. What should we do in the equation?"*.

In a system of equations in the Equation Systems-2 GGB material, she again connected conceptualizations of solution set and GGB material (practice 3). That is, she connected the coordinates of the intersection point and the solution set of the system of equations for the lines in the GGB graphics view. In detail, after the students solved a system of equations algebraically, the teacher posed questions to have students connect x-and-y values in the ordered pair as the solution of the equations and the coordinates of the points of the intersecting lines for the equations in the system. Then, she connected the computation of a slope and GGB material as practice 2. In detail, she connected the computation of the slope of the line and the triangle region that was bounded by the line and the axes in the GGB graphics view by posing questions. When the teacher referred this triangle region, she actually referred to computation of slope as an algebraic ratio using two points on the line. Then, while computing the slope of the lines, the teacher connected the computation of slope and GGB material (in practice 2). That is, she connected vertical change/horizontal change in GGB graphics view and computation of slope of a line in the GGB graphics view by making explanation and posing question. In this the way, as practice for connecting computation of slope and GGB graphics view (practice 2), she converted from the standard form of equation to the slope-intercept form of equation in the GGB algebra view to connect the computation of slope line for negative slope and slope-intercept form of equation of the line by making explanations.

While finishing the activity, the teacher again connected mathematical concept, GGB material, and the activity sheet (practice 4). That is, she connected equations of lines, positions of lines, slopes of lines, and solution set of system of equations using the GGB graphics view and the GGB algebra view and the table that was filled in for various systems of equations in the activity sheet, by making explanations. For example, the following dialogue emerged during the practice.

(T) Let's make a generalization. For example, look at the systems [of equations] and the lines that you created. You can say something about parallel lines. Sema (a student) made some explanations. Look at your equations and positions of lines. Compare these situations. For example, what happens to the slope or solution set? Yes, İrem? *(A student was asked to speak.)*

(S) If the lines intersect, there is a solution set. If they are parallel, it is an empty set.

(T) Yes. This is one of the important results.

(S) The slopes of parallel lines are equal.

(S) The slopes of parallel lines are equal but the slopes of intersecting lines are different.

In the activity of *Stores*, the teacher's bridging practices were categorized as making connections between the mathematical situation/concept and GGB material (practice 1), between conceptualizations of concept(s) and GGB material (practice 3), and among mathematical concept, GGB material, and activity sheet (practice 4) (see Table 13).

As practice for relating the mathematical situation/concept to GGB material (practice 1), the teacher established a connection among equations, lines, solutions, GGB tools, and GGB graphics view. In detail, the teacher enabled the students to use the GGB material to focus on the GGB graphics view in order to connect a point with its coordinates and the solution of an equation for the functional situations of Store M and Store N by posing questions. For example, the teacher said, *"Let's open the GGB file. You know how to find points. Can you mark the points? For example, what is the first point? Point one to one [(1, 1)]. Let's find this point in the Cartesian graph. To which store's graph does the point belong?"* and "*Which store makes one sale per* *day?".* Immediately afterwards, as another practice for relating the mathematical situation/concept to GGB material (practice 1), she guided the students to connect time-sliders and the trace of the points and the points of the line graphs for Store M and Store N by making explanations. As evidence, for example, she said, *"Now, let's right click on this point and trace on. Right click on the point.".* Then, after the students moved the slider, she continued by asking the following questions: *"How many sales does it make on the 3rd day?*" and *"How many sales does it make on the 4th day?".* Besides, as practice for connecting conceptualizations of linearity and the GGB graphics view (practice 3), she used points of the graph in GGB graphics view to connect linear relation and functional situation of store-M, by posing questions. For example, while showing the GGB graphics view, she asked, *"What do these points show? Is it a linear relationship?".* Thus, she connected the visual representation of the linear relationship and the verbal description of the linear relationship on the GGB graphics view.

Then, she continued with practices for connecting mathematical situation/concept and GGB material (practice 1). That is, she connected equations, line graphs, the GGB algebra view, and the GGB graphics view. In detail, she had students use the equation of the line in the GGB algebra view to connect the points of the line graph with the line graph for the functional situation of store-M in the GGB graphics view by making explanations. For example, after the students drew the points for a situation, she said, "*...You can do the same thing when you click on the line in the GGB algebra view. Open the algebra view. Open the first line here (the teacher mentioned the equation of the line in the GGB algebra view)".*

After making connections for the line graphs of the situations, the teacher posed questions to connect the sign of the slope and the behavior of the line graphs in the GGB graphics view as practice for connecting mathematical situation/concept and GGB material (practice 1). For example, when some of the students gave the slope values of the situations, she asked, *"Let's say the slope of Store M. How do we compute it? First, can we say whether it is positive or negative? How (do variables) increase or decrease?"* and *"Can you understand the sign of the slope, whether it is negative or positive?".* Thus, the students considered the direction of the change (increasing straight line or decreasing straight line) in the y-variable (number of sold products) while considering the changes in the x-variable (number of days) to determine the sign of slope. In addition, she made explanations to connect the slope of line graphs with the slope checkboxes in the GGB graphics view and to connect the equations of graphs with the equation checkboxes in the GGB graphics view. Thus, the students related their results to the algebraic and geometric views of the GGB material. After these connections, as practice 3, the teacher connected conceptualizations of the solution set and the GGB material. More specifically, by posing questions and making explanations, she connected the coordinates of the intersection point and the solution set of the system of equations as ordered pair for functional situations of Stores in GGB graphics view. As evidence, the teacher asked, *"What do we call this intersection point? (The teacher mentioned the intersection point of the lines in the GGB graphics view.) What does this point tell about the [system of] equations?"*. After the students said that it was the solution set, she continued, *"So, if we solve these equations, what will the solution set of the system be? The x value is 5 and the y value is 5"*.

In the meantime, with the aforementioned bridging practices, the teacher made connections among graphs of functional situations, GGB graphics view and students' activity sheet as practice for connecting the mathematical situation/concept, GGB material and the activity sheet (practice 5). More specifically, she connected a student's points of graph in the activity sheet and the GGB graphics view to draw the points of the graph for the functional situation of Store-M and connected a student's line graph in the activity sheet and the line graph in the GGB graphics view to check the line graphs for the functional situations of Store-M and Store-N. As evidence, for example, while the students drew the line graph for the functional situation of Store-M, she said, *"You can draw a line with those points. They are not connected to each other on your sheet. ..."* and *"Did you do it that way? (The teacher refers to the graph in the GGB graphics view)"* . Similar bridging practices emerged while the students drew the line graph for the functional situation of Store-N.

Summary of bridging practices in using GGB materials and concrete objects:

Table 14. Bridging practices while using GGB materials and concrete objects

Practice	Number of phase
Connecting concrete object and mathematical situation/concept	1
Connecting concrete object, mathematical situation/concept, and GGB material	1
Connecting mathematical situation/concept/process, GGB material, and activity sheet	1
Connecting mathematical situation/concept/process, GGB material	1, 2
Connecting mathematical situation/concept and GGB material	1, 2, 3
Connecting conceptualizations of concept(s) and GGB material	1, 2, 3
Connecting mathematical situation/concept, GGB material, and activity sheet	2, 3
Connecting concrete object, mathematical process, and GGB material	2
Connecting different software	2
Connecting mathematical situation/process, and GGB material	3

4.2.2 Trimming Practices in using GGB materials and Concrete objects

In this section, the teacher's trimming practices are reported within each phase when she used the concrete objects and the GGB materials of the activities in classroom teaching. From the perspective of McCrory et al.'s (2012) KAT framework, trimming practices in teaching algebra are described as mathematical practices of retaining the integrity of slope, linear equations, and graphs in eighth-grade mathematics and eliminating the complexity of slope, linear equations, and graphs in a correct way both in school and advanced mathematics. Accordingly, the practices were categorized by identifying the teacher's actions in classroom teaching in which the teacher provides mathematical practices to refine those mathematical ideas within the accuracy of mathematical context while using concrete objects and technology.

Each phase (phase 1, phase 2, and phase 3) involved themes of trimming practices and the actions under each theme are presented in a table. The flow of actions in the practices are explained under the classroom activities for each phase. In addition, when a given trimming practice is related with a bridging practice, which was explained in the previous section, the relation in the flow of the actions was also reported. It was assumed that two mathematical practices are related when they occurred concomitantly or when one occurred as a background for the other one at the moments of teaching. It was meant by the background practice that it emerged through a practice by giving the priority to the other practice upon the supposed intention of the teacher. At the end of this section, trimming practices are summarized considering all the phases.

4.2.2.1 Trimming practices in teaching the slope of an object for Phase 1

The trimming practices in teaching the slope of an object with GGB materials and concrete objects are explained in this subsection. In the first phase, the teacher used three activities: Positions of Battens, Fire Truck, and Tent. In addition, she used a concrete object (i.e. batten) and GGB materials for the activities. In this phase, the categories of trimming practices were grouped, by the researcher, under four themes:

Practice 1. Guiding to trim the use of concrete objects;

Practice 2. Trimming mathematical situation by using GGB material;

Practice 3. Trimming computation by using GGB material;

Practice 4. Addressing mathematical difficulties within conditions of GGB material.

In the category of guiding students to trim the use of concrete object, the teacher acted as a guide for students in using the concrete object (i.e. batten), in making measurements on the batten, in computing the slope of the batten, and in comparing the slope values with the batten. In the category of trimming mathematical situations by using GGB material, the teacher trimmed the physical situations by guiding students through the content of the activities and the analysis of critical issues for a situation. In the category of trimming computation by using GGB material, the teacher guided the students in computing the slope, showing/checking lengths, showing/checking changes in a slope, showing/checking the computation of a slope, and showing the conditions that verify the computation. In the category of addressing mathematical difficulty by using GGB material, the teacher traced students' misunderstandings/difficulties within the conditions of GGB material and responded to the students' mathematical questions in using GGB material. While using the GGB materials of the activities, she used the GGB graphics view, the slider and the checkbox tools, length objects (dynamic and static), and text tools (dynamic and static) in various combinations during the practices.

The practices and the actions under each practice accompanied with sample descriptions are given in Table 15. The detailed descriptions of these practices are given in the flow of the activities (Positions of Battens, Fire Truck, and Tent) during phase 1. The enumeration of these practices in the table is not indicative of the order in which the practices were implemented. That is, a practice for trimming mathematical situation by using GGB material and a practice for trimming computation by using GGB material could emerge among the practices for trimming the use of concrete object. In addition, two different practices under the category of trimming computation by using GGB material interwove that the one practice does not have to finish to start another practice.

Table 15. Trimming practices of Phase 1 while using GGB materials and concrete objects

Practice	**Action**	**Sample description**
Practice 1: Guide to trim the use of concrete object		
Guide for use of concrete object	Explain how to use batten	Have students understand the use of a batten for leaning it against the wall by making explanations (P)
Guide for making measurements on concrete object	Explain how to make measurements on a batten	Have students do correct measurements of the horizontal and vertical lengths of the batten with tape measure by posing question and making explanation (P)
	Analyze students' failures/difficulties about measuring	Analyze students' measuring failures/difficulties for horizontal and vertical lengths of the battens with tape measure by posing a question (P)
	Consider and response mathematical questions about measuring productively	Consider and respond to students' mathematical questions about measuring productively to prevent student difficulties (P)
Guide for computing slope on concrete object	Encourage to compute slope of batten	Encourage students to compute the slopes of battens as geometric ratio (P)
Guide for comparison of slopes with concrete object	Give hints to compare slopes with concrete object	Give hints to help students find the answer of her question/difficulty about comparing slopes in GGB graphics view with concrete object by posing questions (T)

Table 15 (continued)

Practice 2: Trim mathematical situation by using GGB material		
Guide for content of the activity using GGB material	Explain how to show/change positions of battens using slider tool	Have students understand slider of horizontal length to show/change the positions of battens by making explanations (P)
	Trace students' understanding about content of the activity in GGB graphics view	Have students understand Fire Truck activity GGB graphics view by posing questions (F)
		Have students understand Tent activity GGB graphics view by posing questions and making explanations (T)
		Have students understand the horizontal length as horizontal distance on the horizontal line of the coordinate plane in GGB graphics view by making explanations (T)
Analyze critical issues in a situation using GGB material	Trace students' understanding about critical variables on GGB graphics view	Have students show the constant value as horizontal length (i.e. Distance between fire truck and building) in GGB graphics view by posing questions (F)
	Trace students' understanding about the critical points of horizontal and vertical lengths in GGB graphics view	Have students show the critical points of horizontal and vertical lengths in the fire truck ladder situation in GGB graphics view by posing questions (F)
	Trace students' understanding about the critical points of horizontal lengths in GGB graphics view	Have students show the critical points of horizontal lengths in the Tent activity situation in GG graphics view by posing questions (T)
Practice 3: Trim computation by using GGB material		
show and/or check lengths using GGB material	Show values of horizontal/vertical lengths of line segment using checkboxes and slider	Use slider and checked horizontal and vertical lengths checkboxes to show the values of batten (i.e. line segment) in GGB graphics view (P)
		Use slider to show the values of horizontal and vertical lengths of ladder in GGB graphics view (F)
show and/or check change of slope using GGB material	Encourage students to use GGB slider to see the change of slope	Encourage students to use slider of vertical length that moves the height of the floor in burning to show the change of the slope of fire truck ladder (i.e. line segment in GGB graphics view) (F)
	Encourage students to use GGB slider to see the change of horizontal length	Encourage students to move the horizontal length slider tool that shows the value of the point where the rope is tied on line to be seen the change of horizontal length of the rope (T)
show and/or check computation of slope using GGB material	Show and check slope computation using checkboxes and slider	Use slope checkbox with slider and horizontal and vertical lengths checkboxes to show and check the correctness of the computation of slope of batten (P)

Table 15 (continued)

	Encourage students to check slope computation using slider and checkbox	Encourage students to use slider and slope checkbox to show and check the correctness of their computation of slope of ladder (F)
		Encourage students to use slope checkbox with checking the horizontal length slider to show and check the correctness of computation of slope (T)
	Show and check slope computation using checkbox	Use slider and slope checkbox to trace correctness of students' mathematical responses by posing question (F)
Show the conditions that verify the computation using GGB material	Make understand conditions of zero slope using slider in GGB graphics view	Show affecting factor of zero slope (i.e. Vertical length is zero) using slider in GGB graphics view (F)
	Make understand conditions of slope using slider in GGB graphics view	Explain the variables that are either constant or varying in the situation and the quantities of these variables in GGB graphics view (F)
	Make understand conditions of undefined slope in GGB graphics view	Show the affecting factor of computation of undefined slope (i.e. Horizontal length is zero) in GGB graphics view by posing questions (T)
Practice 4: Address mathematical difficulties within conditions of GGB material		
Trace students' understanding/misunderstanding/difficulties within the conditions of GGB material	Have students understand the limited conditions of the situation in GGB graphics view	Make explanations to show the limited value of the horizontal length with slider in GGB graphics view (T)
Respond students' mathematical questions using GGB material	Respond students' mathematical questions in GGB graphics view	Respond to students' mathematical question about horizontal length in GGB graphics view productively by posing questions and/or making explanations (T)

Note: The abbreviations in the sample description column of the table show the activity in which an action emerged. P: Positions of Battens activity, F: Fire Truck activity, T: Tent activity.

In the *Positions of Battens* activity, the teacher's trimming practices were combined under the practices of guiding to trim the use of concrete object (practice 1), trimming mathematical situation by using GGB material (practice 2), trimming computation by using GGB material (practice 3) as can be seen in Table 15.

As practice for guiding to trim the use of concrete object (practice 1), the teacher guided the students in the use of concrete object. That is, by making explanations, the teacher explained to the students how to use a batten for leaning it against the wall. As evidence, she said, *"Let's take our battens (each group of students takes a batten). Lean it against the wall in such a way that the batten touches the floor"* and *"Kids, It [the batten] does not have to stand straight. You can do as you wish. It* *can stand in this way or that way (the teacher shows one student's batten position)"*. Thus, the teacher guided the students to create a physical situation for a slope as a mathematical situation using the battens without elaborating on the slope of a line. In the meantime, as practice for trimming mathematical situation by using GGB material (practice 2), the teacher guided through the content of the activity by using GGB material. While using the GGB material for trimming the mathematical situation, the teacher explained how to show and change positions of battens (i.e. line segments in the GGB graphics view) using the slider that changed the horizontal length of a line segment. As evidence, for example, she said,

> Look, here is such a feature [in GGB]. When I change point A *(The teacher moves the slider of the horizontal length)*, the position of the batten changes. Some of your battens stand as such, don't it? *(The teacher pointed to the line segment in the GGB graphics view on the screen).*

Therefore, while the teacher's priority was to connect the slider for the horizontal length (slider-a), the dynamic line segment, and the change in positions of line segments (i.e. battens) in the GGB graphics view as bridging (Bridging practice 3: connect the mathematical situation/concept and the GGB material in Table *11*), she concomitantly maintained the accuracy of the mathematical situation by using the GGB material.

In response to students' questions, as practice for guiding to trim the use of concrete object (practice 1), the teacher continued to guide the students in making measurements on concrete object. That is, by posing questions and making explanations, she continued to enable students to do correct measurements on the horizontal and vertical lengths of the battens using a tape measure. As evidence, for example, while the students worked in groups and made measurements, the teacher said:

> Yes, we will talk about how those lengths [horizontal and vertical lengths of a batten] affect the slope. Find them. Mark the point at which the batten touches the wall without removing the batten *(see Figure Figure 32 for the students' work)*. Then, mark the point at which the batten touches the ground. You'll see more clearly.

Figure 32. A group of students measuring horizontal and vertical lengths of a batten using a tape measure.

Then, while the students were measuring the horizontal and vertical lengths of their battens using a tape measure, the teacher analyzed the difficulties students experienced in measuring horizontal and vertical lengths of the battens, by posing questions. In addition, she took into consideration the students' mathematical questions about measuring the horizontal and vertical lengths of their battens and responded to them productively to prevent the students from experiencing difficulties in their practices of measurement. Therefore, these actions provided a basis for the mathematical accuracy of measurement of a slope as a physical situation.

After the students' hands-on measurements and the teacher's synthesis of the components of slope computation using GGB material, which is explained in the next section, the students continued to use concrete object as in practice 1 to guide students in computing slope on concrete object. That is, the teacher encouraged the students to compute the slopes of their battens as a geometric ratio (i.e. vertical length over horizontal length) by making explanations. For example, she said, *"So, what is the slope? It is the ratio of vertical length over horizontal length. Let's find the slope of your own batten…"*. Thus, the teacher retained the slope computation on the batten material as a physical situation.

After the students' hands-on computations of slopes, as practice for trimming computation by using GGB material (practice 3), the teacher showed and checked the lengths by using GGB material. That is, the teacher used both the slider and the length checkboxes that showed values of horizontal and vertical lengths of battens (i.e. line segment) in the GGB graphics view for different positions by posing questions. As evidence, for example, she said,

> Let's look at the screen *(The teacher refers to the GGB material)*. You see slider-a, which we can change continuously. What if we click on this horizontal length and this vertical length *(The teacher clicks on the checkboxes of the lengths of the line segment).* It is slider-a *(The teacher moves slider-a, which changes the horizontal length directly and the vertical length and slope indirectly).* What changed?

Thus, the students and the teacher followed the change in the vertical and horizontal lengths of the battens on the GGB material dynamically. In other words, the teacher provided clear dynamic views for the vertical and horizontal lengths of the batten in different positions on the GGB material dynamically.

Then, as practice for trimming computation by using GGB material (Practice 3), the teacher showed and checked the computation of the slope by using GGB material. More specifically, by posing questions, she used horizontal length and vertical length checkboxes and the slope checkbox together with the slider, to show and check the accuracy of the computation of the slope of the batten. For example, she said,

> Yes. You take the vertical length of the batten as 57 cm *(The teacher checks the value of the vertical length on the computation after moved slider to make students' vertical length of the batten)*. Let's look. Is the value of your slope like this? *(The teacher shows the value of the slope on the slope computation text in the GGB graphics view).*

That is, she moved the slider to create the students' battens and then showed and checked these values on the GGB material. Thus, she both constrained the slope computation as ratio of vertical length to horizontal length and provided various slope computations with different values.

In the *Fire Truck* activity, the teacher's trimming practices were combined under the practices of trimming mathematical situation by using GGB material (practice 2) and trimming computation by using GGB material (practice 3) (see Table 15).

As the first practice for trimming mathematical situation by using GGB material (practice 2), the teacher guided the students through the content of the activity using the GGB material of the activity. More specifically, the teacher traced students' understanding about the content of the activity in the GGB graphics view. To illustrate, the teacher enabled the students to understand the GGB material of the activity that represented a fire truck leaning its ladder against a burning building by posing a question. Then, the teacher showed an example for trimming the computation by using GGB material (practice 3). That is, as practice for showing and checking the change of the slope by using GGB material, the teacher made explanation to encourage the students to use the vertical length slider tool that moves the height of the burning floor to show the change of the slope in the fire truck ladder (i.e. line segment in the GGB graphics view). In addition, while using the GGB material of this activity for this practice, the teacher made connections among the physical situation, the line segment, the GGB graphics view, and the GGB tools (as bridging practice 3 in phase 1, see Table 11) in the background.

After the students' hands-on computations, as another practice for trimming computation using GGB material (practice 3), the teacher showed and checked the computation of the slope by using GGB material. More specifically, the teacher encouraged the students to show and check the accuracy of their slope computations of the ladders as ratio by using the slider that moved the ladder and the slope checkbox, which showed the dynamic text of the slope computation text in the GGB graphics view. The following dialogue showed a section which emerged during the practice:

(T) You found [the slope as] 0.6. Check if it is correct? You clicked the slope *(The students clicks the slope checkbox that show the slope computation text in the GGB graphics view)*. Do it for the second ladder?

(S) Yes, we did *(The students moved the slider from 3 to 7 in the GGB graphics view).*

(S) Then, the second one will be have a steeper slope *(The students clicks the checkbox in the GGB graphics view after they computed the slope for the 7th floor. See* Figure *33).*

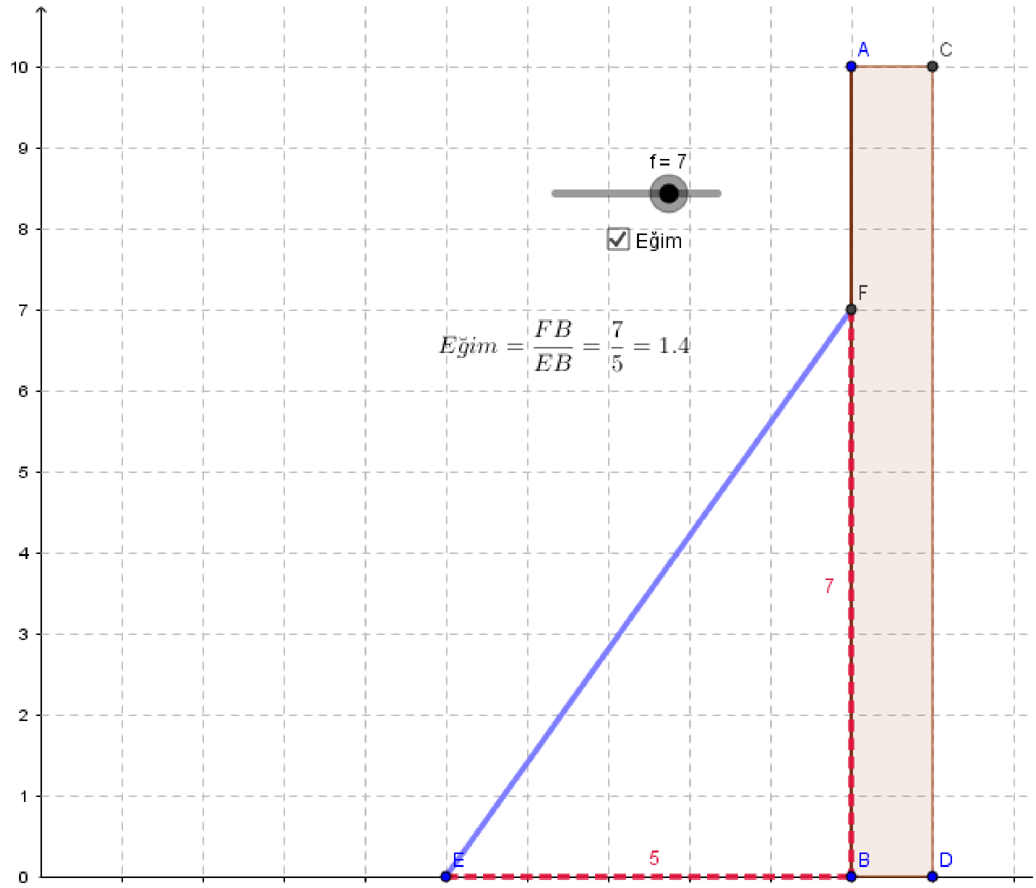

Figure 33. The students' screen view-2 of the GGB material of Fire Truck activity.

In Figure 33, the slope checkbox that dynamically showed the computation of the slope of line segment *EF* was clicked after slider-f, which changed the vertical length of the ladder, was moved to the 7th unit. The rectangle *BDCA* represented the building and the line segment *EF* represented the fire truck ladder. Thus, while using the GGB material of this activity for this practice, the teacher connected slope computation, line segment, the GGB graphics view, and the GGB tools as practice for connecting the mathematical situation/concept/process, and the GGB material (as bridging practice 4 in phase 1) in the background.

After making computations for the given vertical distance values in the Fire Truck activity sheet, as a practice for trimming mathematical situation by using GGB material (practice 2), the teacher examined the critical issues in the physical situation in terms of slope. Specifically, she traced students' understanding of critical variables for a slope on the GGB graphics view. In the action, she posed questions to get the students to show the constant value as horizontal length (i.e. distance between the fire truck and the building) in the GGB graphics view. For example, she asked, *"Well. In this case, which (quantity) is constant?".* Then, she traced students' understanding of critical points as regards horizontal and vertical lengths in the GGB graphics view. In the action, she posed questions to encourage the students to show the critical points of horizontal and vertical lengths in the fire truck ladder situation in the GGB graphics view. Thus, as evidence, she specifically asked, *"What's at point E?"*.

For the practice of trimming computation by using GGB material (practice 3), she showed and checked the lengths by using GGB material. That is, she also used the slider to show the values of the horizontal and vertical lengths of the ladder in different positions in the GGB graphics view by making explanations (348). Then, the teacher showed and checked the computation of the slope by using GGB material. That is, she also used the slider and the slope checkbox tool to trace the accuracy of students' mathematical responses regarding slope computation for different heights, by posing questions. In addition, the teacher showed the conditions that verified the computation by using GGB material. More specifically, she enabled the students to understand the conditions of zero slope in the situation. Thus, she posed questions to show the factors affecting zero slope (i.e. vertical length is zero) for the fire truck ladder (i.e. line segment in the GGB graphics view). For example, she asked, *"So is there a slope there? (The teacher used the slider to make the vertical length zero)".* To complete the activity, she had the students understand the conditions of the slope using a slider in the GGB graphics view. More specifically, she made explanations about the constant and varying variables in the situation and the values of these variables in the GGB graphics view. Thus, the teacher set the limit for the interpretation about the effect of the vertical length on the slope when the horizontal is invariant.

In the *Tent* activity, the teacher's practices were combined under the categories of guiding to trim the use of concrete object (practice 1), trimming the mathematical situation by using GGB material (practice 2), trimming computation by using GGB material (practice 3), and addressing mathematical difficulties within conditions of GGB material (practice 4) (see Table *15*).

In the activity, as practice for trimming the mathematical situation by using GGB material (practice 2), the teacher guided students through the content of the activity by using GGB material. That is, by posing questions and making explanations, the teacher helped the students understand the GGB material of the activity that represents a tent, which is tied with a rope to the ground with different distance. While using the GGB material for this practice, the teacher connected physical situation, line segment, GGB graphics view, and GGB tools as practice for connecting mathematical situation/concept and GGB material (bridging practice 3 in phase 1 in Table 11) in the background. That is, when the teacher started to guide the students to make them understand the GGB activity for the mathematical activity, she continued this trimming practice with the bridging practice concomitantly. Then, as another sample of practice 2, as indicated in Table 15, she made explanations to have students understand the horizontal length as horizontal distance on the horizontal line of the coordinate plane. That is, she guided students to use the coordinate grid to determine the lengths in the physical situation not to determine the points on the coordinate system. However, since the objects were placed in the coordinate plane in the GGB graphics view, she wanted the students to ignore this coordinate background.

As practice for trimming computation by using GGB material (practice 3), the practice of showing/checking change by using GGB material emerged between the aforementioned two practices. That is, the teacher encouraged the students to use the slider tool, which moves the point where the rope is tied on the ground, to see the change of the horizontal length of the rope (i.e. distance between points B and C in the GGB graphics view), by posing questions. For example, she said, *"For that reason (The teacher refers to a student's response that the rope was 2 meters away from the tent for a position), where will you move the slider?"* (See Figure 34 for the students' screen view of the GGB material for the position of the rope).

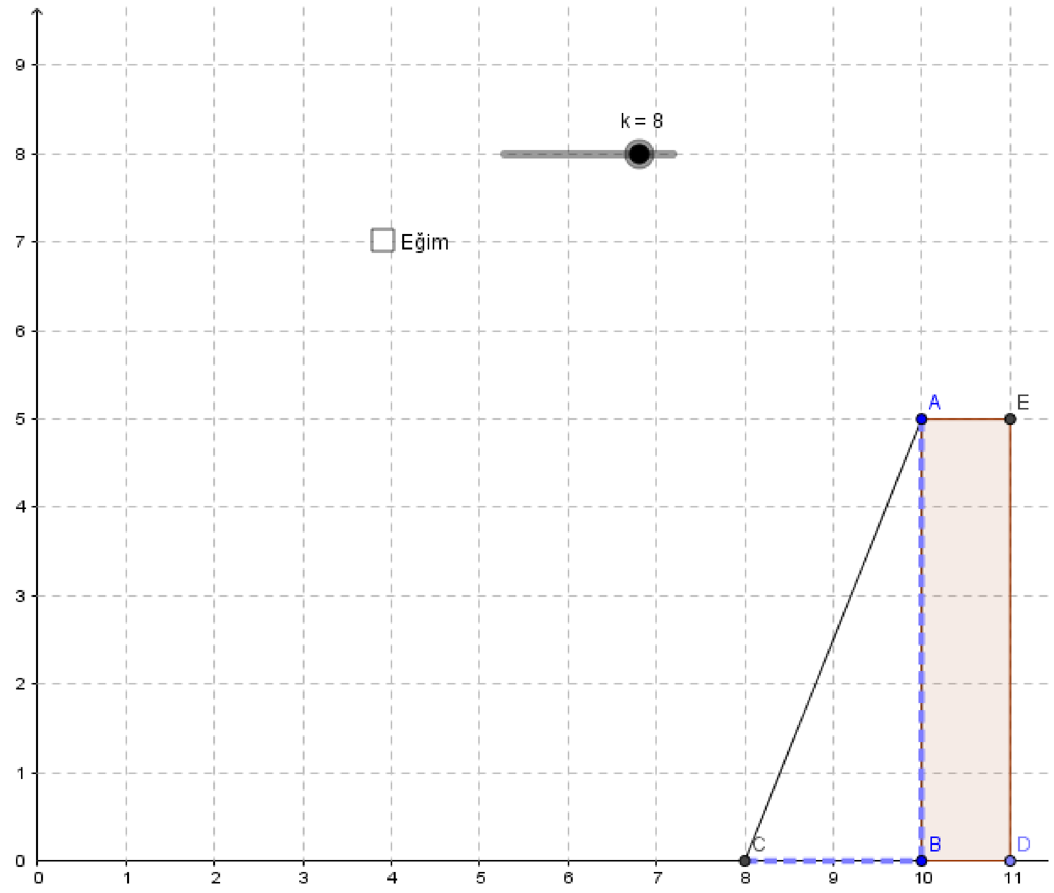

Figure 34. The students' screen view-2 of the GGB material for the Tent activity for a position of the rope.

After the students computed the slopes of the ropes for different distances, the teacher had the students use the horizontal length slider by clicking on the slope checkbox, which showed the text of the slope computation to show and check the accuracy of the computations of slopes. That is, as practice for trimming computation by using GGB material (practice 3), the teacher showed and checked the slope computation in the GGB material. As evidence, for example, while a group of students worked on the activity sheet and GGB material, she asked, *"Did you compute it? Are you looking at the slope (The teacher refers to the slope checkbox in the GGB graphics view). Did you check it?"*. Thus, she could provide students with different ways of computation (hands-on computation, GGB computation, dynamic view for the computation) to maintain a clear understanding of slope computation for a physical situation.

As practice for trimming mathematical situation by using GGB material (practice 2), critical issues in a situation were examined by using GGB material through the practice of checking the computations of the slopes of the tent ropes. That is, the teacher traced the students' understanding of the critical points in the horizontal lengths in the GGB graphics view. More specifically, she posed questions to have the students show the critical points in the horizontal lengths in the physical situation in the GGB graphics view.

As practice for addressing the students' mathematical difficulties while using GGB material (practice 4), the teacher traced students' difficulties within the limited conditions of the situation in the GGB graphics view. Therefore, she made explanations to show the limited conditions of the situation which has an interval of horizontal length with slider in GGB graphics view. For example, she said,

> … I can move away as much as I want from the tent in real. But the structure we established on the computer as such, until there [that point]. Otherwise, if we wanted, we could tie the rope much farther away [depending on the rope length in real].

While using GGB material for this practice, the teacher also connected physical situation, factors of slope, and the GGB graphics view as bridging practice (bridging practice 3: connecting the mathematical situation/concept and the GGB material in Table 11). In the way, while the students were using GGB material of the activity, the teacher responded to the students' mathematical questions productively by posing questions and/or making explanations.

As another practice for trimming computation by using GGB material (practice 3), the teacher showed the conditions that verified the computation using GGB material. That is, the teacher wanted the students to understand the conditions of the undefined slope for the situation in the GGB graphics view. Therefore, she posed the question to show the factor affecting the computation of the undefined slope (i.e. horizontal length is zero) for rope of the tent in the GGB graphics view (i.e. line segment in the GGB graphics view). For example, the following dialogue emerged during this practice:

(T) What is the slope value here? *(The teacher moves the slider to make the horizontal length of the rope zero)*

(S) Zero.

(SS) Infinite.
(T) The GGB shows it as if it [slope] is infinite, but what is the horizontal length here? *(see Figure 27 for the computer screen view)*
(S) Zero.
(T) Zero, isn't it? What computation do you do when the horizontal length is zero?

While using the GGB material for this practice, the teacher concomitantly connected the representations of the undefined slope, the factors of the slope, the GGB graphics view, the GGB tools as practice for connecting conceptualizations of concept(s) and GGB material (Bridging Practice 6 in Table 11). Therefore, even though she did not prepare a section on undefined slope in the activity sheet, she explained the undefined slope on the division operation when the divisor was zero and on visual representation in the GGB graphics view.

4.2.2.2 Trimming practices in teaching the slope of a line for Phase 2

The trimming practices in teaching the slope of a line with GGB materials are explained in this subsection. In the second phase, the teacher used GGB materials of the three activities, which were Building Design, Leaking Container, and Slopes and Equations of Lines, under different themes of trimming practices. In this phase, the practice themes were grouped, by the researcher, under five headings:

Practice 1. Trimming mathematical situation by using GGB material;

Practice 2. Trimming computation by using GGB material;

Practice 3. Trimming interrelations in equations and lines by using GGB material;

Practice 4. Doing the mathematical entailments of GGB;

Practice 5. Addressing mathematical difficulties within conditions of GGB material.

In the category of trimming mathematical situation by using the GGB material, the teacher guided the students through the content of the activities by using GGB materials and guided them to move between the GGB algebra view and the GGB graphics view. In the category of trimming computation by using GGB material, the teacher showed/checked the points, the change in slope, the computation of slope, and the drawing. In the category of trimming interrelations in equations and lines using the GGB material, the teacher showed a relation, an independency, and different forms of equations. In the category of doing mathematical entailments of GGB, the teacher showed specific mathematical representations for GGB. In addressing mathematical difficulties within conditions of GGB material, the teacher responded to the students' mathematical difficulties in using GGB material. While using the GGB materials of the activities, the teacher used the GGB graphics view, the slider tools, the trace of points, the dynamic text tools, the checkbox tools, the GGB algebra view, and the convertor of forms of equations in the GGB algebra view.

The practices and the actions under each practice accompanied with sample descriptions are presented in Table *16*. The detailed descriptions of these practices are given in the flow of the activities (Building Design, Leaking Container, and Slopes and Equations of Lines) during phase 2. The enumeration of these practices in the table is not indicative of the order in which the practices were implemented.

Table 16. Trimming practices of Phase 2 while using GGB materials and concrete object

Practice	**Action**	**Sample Description**
Practice 1: Trim mathematical situation by using GGB material		
Guide for content of the activity using GGB material	Trace students' understanding about items of the equation in the activity using slider tool	Have students understand slider-z to show the constant value of the number of windows in basement in the equation by posing questions (11) (B)
		Have students understand the slider-k to show the coefficient value of the number of windows in each floor in the equation by posing questions (B)
	Trace students' understanding about items of line of the equation in the activity using slider	Have students understand slider tool to show each point by posing question (B)
		Have students understand slider that show each point by posing questions and making explanations (H)
	Trace students' understanding about items of line of the equation in the activity using slider and trace	Have students understand slider and trace tools to show all points for the activity by posing questions (B)
		Have students understand slider and trace tools that show points of the graph by posing questions and making explanations (H)
	Trace students' understanding of drawing of line in the activity using line tool	Have students understand line tool to draw a line through the points by making explanations (B)

Table 16 (continued)

	Trace students' understanding of coordinate system using GGB graphics view	Use GGB graphics view to assess students' previous knowledge about quadrants (B)
		Use GGB graphics view to assess students' previous knowledge about points as ordered pairs by posing questions (B)
		Use GGB graphics view to assess students' previous knowledge of equations on coordinate system (B)
	Explain how to use GGB views and tools with activity sheet	Explain how to use GGB algebra view and graphics view with sliders and slope computation text (S)
		Explain the purpose of the use of GGB material within the activity (S)
Guide students to move between GGB algebra view and GGB graphics view	Trace students' understanding about using slope computation text tool in GGB graphics view and GGB algebra view for equation of a line, and sliders for slope and y-intercept of a line (constant value of an equation) in GGB graphics view in the activity context	Have students show slope of line using slope computation text for a line in GGB graphics view by posing questions (S)
		Show linear equation as algebraic expression of line using GGB algebra view by making explanation and posing questions (S)
		Have students show m-slider as slope of equation/line in GGB graphics view by posing question, responding student's answer, and making explanations (S)
		Have students show n-slider as constant value of equation in GGB algebra view and y-intercept of line in GGB graphics view by posing questions (S)
Practice 2: Trim computation by using GGB material		
Show and check points by using GGB material	Enable students to understand points on computing slope using points and tools in GGB graphics view	Have students compute the slope of line using points in GGB graphics view by posing questions (B)
		Have students compute the slope of line using points, ray tool and checkbox tool by posing questions (B)
		Show the coordinates of the points on the line in GGB graphics view by making explanations (B)
Show and check change by using GGB material	Show vertical/horizontal changes by using checkbox	Use vertical/horizontal change checkboxes to show the change in y over x (804) [calculate as algebraic ratio] (B)
	Encourage students to use the GGB slider to see the points and change in the y-value	Encourage students to move slider tool to show and check the points of the graph (H)
		Encourage students to move slider tool to show and check the points of the graph and the change in the water (H)
Show and check computation by using GGB material	Constitute the computation algorithm of slope of a line by using the coordinates of the points on the intervals in GGB graphics view	Compute the slope of the line on an interval using a pair of points (an interval) in the GGB graphics view by posing questions [calculate as algebraic ratio] (B)
		Have students use checkbox tool to show any two points for an interval to compute the slope of graph (H)

Table 16 (continued)

		Compute the slope of the line on an interval using a pair of points (an interval) in the GGB graphics view by posing questions (H)
	Show the computation algorithm of slope of a line by using slope checkbox and dynamic computation text	Show the computation algorithm of slope by using slope checkbox to show the computation with a combination of static and dynamic text as algebraic ratio (B)
		Have students use slope checkbox tool to show and check computation of slope with a combination of static and dynamic text as algebraic ratio (H)
	Summarize the steps of computation of slope of a line as an algebraic ratio using checkboxes and dynamic text	Compute slope of the line on an interval by using a pair of points, vertical change and horizontal change checkboxes, which shows the vertical and horizontal dashed arrows as the change of y-variable with change in x-variable, and slope checkbox, which shows dynamic computation text, in GGB graphics view by posing questions (H)
	Explain difference of the slope computation algorithm between mathematical situations by using checkboxes and dynamic text	Explain students to perceive the difference of the slope computation algorithm between physical situations and functional situations using checkboxes of slope, vertical change, and horizontal change (376) (H)
Show and check drawing with GGB material	Show the drawing of the graph using GGB graphics view	Encourage students to draw the graph in their sheets using GGB graphics view (H)
Practice 3: Trim interrelations in equations and lines with using GGB material		
Show a relation by using slider tools, GGB graphics and algebra views	Encourage students to investigate relation between slope, line, and equation using slider in GGB graphics view and GGB algebra view	Show students how equation, line and slope relates each other for various lines by using slider-m, GGB graphics view and algebra view by posing questions (S)
	Encourage students to investigate relations between y-intercept, line and equation by using the slider in GGB graphics view and GGB algebra view	Show students how equation in GGB algebra view, y-intercept value of line in GGB graphics view relates to each other by using slider-n by posing questions and making explanations (S)

Table 16 continued

Show independency by using slider tools, GGB graphics and algebra views	Encourage students to investigate the independence of slope of equation from y-intercept by using slider in GGB graphics view and GGB algebra view	Show students how slope of equation in GGB algebra view is independent of the y-intercept by using slider-n in GGB graphics view and GGB algebra view by posing questions and making explanations (S)
Show forms of equations for slope computation in GGB algebra view	Show converting forms of equations by using the property of GGB algebra view	Show standard form of equation using convertor in GGB algebra view by making explanations and posing questions (S)
		Show slope-intercept form of equation as normal way of equation for slope computation in GGB algebra view by making explanations and posing questions (S)
Practice 4: Do Mathematical entailments of GGB		
Show mathematical representation specific to GGB	Show differences in the decimal point in the GGB context and the Turkish mathematics context	Show representations of decimal point on GGB by responding to a student's comment (S)
		Show how GGB uses the dot to represent the decimal point rather than comma as is the case in Turkey (S)
Practice 5: Address mathematical difficulties within conditions of using GGB material		
Respond to students' mathematical difficulties using GGB material	Respond to students' wrong answers on GGB graphics view and using slider	Respond to students' wrong answer(s) about the coordinate system on GGB graphics view and with slider by making explanations and posing questions (42-44) (B)

Note: The abbreviations in the sample description column of the table indicate the activity in which an action emerged. B: Building Design activity, H: Leaking Container activity, S: Slopes and Equations of Lines activity.

In the *Building Design* activity, the teacher's trimming practices were combined under the themes of trimming a mathematical situation by using GGB material (practice 1), trimming computation by using GGB material (practice 2), and addressing students' mathematical difficulties within conditions of GGB material (practice 5) (see Table 16).

As a practice for trimming a mathematical situation by using GGB material (practice 1), the teacher guided students through the content of the activity by using GGB material. While using the GGB material, the teacher traced students' understanding of the items of the equation for the functional situation by using the slider. That is, she enabled the students to understand slider-z to show the constant value of the equation as the number of windows in the basement in the situation by posing questions. For instance, she asked, *"Here, (The teacher shows slider-z in the GGB graphics view) z means the number of windows in the basement. How many windows are there in the basement in our question?"*. She also enabled the students to understand slider-k to show the coefficient value of the equation as the number of windows on each floor in the situation by posing questions. Besides, while guiding the students through the content of the activity by using GGB material, she traced students' understanding of the items in a line of the equation for the functional situation by using the slider. That is, she enabled the students to understand slider-h in order to show each point for the graph of the equation in the situation by posing questions. In addition, she used both the slider tool and the trace tool for tracing students' understanding of the components of the line. That is, she enabled the students to understand the slider and the trace tools to show all the points in the graph of the equation in the situation by posing questions. While using GGB material for those practices, the teacher connected the mathematical situation/concept, the GGB material, and the activity sheet (bridging practice 1 in phase 2 in Table 12) in the background as explained in the bridging section for phase 2. The following dialog serves as evidence for those practices:

(T) I click on point H here *(in the GGB graphics view)*. It has a property. When I trace on it, it shows all the points *(she traces on point H).* Now, I have traced on it. How many floors does the building have? You see, there are 2 windows. Then I continue. What did I do? *(The teacher moved slider-h to 1.)*

(SS) One-storey building.

(T) How many windows does a one-storey building have? *(The teacher uses slider-h, which moves point H from (0,2) to (1,5))*

(S) Five.

(T) Did you do the same thing?

(SS) Yes.

(T) Good. Let's move on. Two-storey building? *(The teacher uses slider-h, which moves point H.)*

(S) Eight.

(T) Three-storey building? This goes on and on. I will not continue now. That's enough for now. Is there anyone who does not understand?

After that conversation, the teacher continued to guide students through the content of the activity by using GGB material (practice 1). She traced students' understanding of drawing a line for the functional situation using the line tool in the GGB graphics view. More specifically, the teacher enabled the students to understand the line tool to draw a line through the points of the situation that were shown with the trace tool by making explanations. She said,

> Ok then, what will I do with these points *(The teacher points to the trace of points)*? I will connect them and create a line. Didn't you do that too? *(The teacher draws the line on the points with the line tool in the GGB graphics view).*

In continuation of this activity, the teacher traced students' understanding of the coordinate system using the GGB graphics view. That is, she used the GGB graphics view to assess students' previous understandings of quadrants, points as ordered pairs, and equations on the coordinate system by posing questions.

In the *Building Design* activity, the teacher trimmed the computation by using the GGB material (practice 2). These trimming practices involve showing/checking points, change, and computation by using GGB material. More specifically, she enabled the students to understand the points of a line in computing slope by using points and tools in the GGB graphics view. In addition, by posing questions, she had the students compute the slope of the line by using the points of the line in the GGB graphics view and by using the points, ray tool, and a checkbox tool in the GGB graphics view. She also showed the coordinates of the points on the line in GGB graphics view, by making explanations. That is, she said, *"I will show the coordinates of point-I here (The teacher right clicks on the point and shows the coordinates). OK? Now, what are the coordinates of point-I?"*. That is, she enable the students to gain an understanding of points like a prerequisite to make the computation of the slope of a line. As practice for trimming the computation for showing/checking change by using GGB material (practice 2), she showed the vertical and horizontal changes by using checkboxes in the GGB graphics view. That is, she used vertical change and horizontal change checkboxes to show horizontal and vertical dashed arrows as the change in y-variable with unit change in the x-variable by making explanations. She said, *"We took lots of triangles. Is there any difference among them in computing the slope? (The teacher clicks on the two checkboxes that show arrows for changes in the y-variable with unit change in the x-variable. See* Figure 28)". When using GGB material for this practice, the teacher connected similarity in similar triangles and computation of the slope of a line in the GGB graphics view as bridging practice (bridging practice 3 in phase 2 in Table 12) concomitantly.

In the following, as practice for showing/checking computation with using GGB material (practice 2), the teacher constituted the computation algorithm of the slope of a line using the coordinates of the points on the intervals in the GGB graphics view. More specifically, she computed the slope of the line as an algebraic ratio on an interval using a pair of points in the GGB graphics view by posing questions. As an evidence, for example, the following dialogue emerged during the practice.

(T) It doesn't matter [whether to take any interval]. One of the points is this point B. What are the coordinates of point B? (The teacher refers to the GGB graphics view)

(SS) One to five *(The students read the point (1, 5) in the GGB graphics view).*

(T) Let's look at your GGB screens *(The students look at their computer screens).* You all *(the groups of students)* make computations with different points. In this case, look at point B and point I. Is it okay? Does it matter?

(S) It is okay.

(S) Nope. It doesn't matter.

(T) Well. When you move from point-I to point-B, how does the floor number change?

(S) It increases.

(T) What is the value of the point? How does it change?

(S) It was 2 and now it's 5 *(The student's response was wrong).*

(T) This is for the number of windows. We are talking about the number of floors now.

(S) Hmm. It was 0 and now it's 1 *(The student's response was correct).*

(T) There was 0 window. Then, there is 1 window *(The teacher explains the change in the number of windows as the y-variable in the GGB graphics view).* Isn't there? We can do the similar thing again. The coordinates of point-I was 0 to 2. Point-B?

(SS) 1 to 5.

(T) Yes. How many units does it change *(The teacher refers to the change in the x-variable in the GGB graphics view)* ?

(S) It increased by 1 unit.

(T) Yes. 1 unit. Well. How many units does the ordinate or y-axis change while moving from point-I to point-B? That is, how many units did the number of windows change?

(S) It increases.

(T) Yes. Both of them increase. And since both of them increase, what did you do while finding the difference [between the points]? Both [changes] have the same sign. That is, here, you subtract 0 from 1. Then what did you do? You subtract 2 from 5. And what did you find? (The teacher explains the computation of $\frac{5-2}{1-0} = 3$)

(SS) Three.

In addition, she showed and had the students check their computation algorithm of the slope as an algebraic ratio using the slope checkbox, which shows the computation with a combination of static and dynamic text in the GGB graphics view:

$$\frac{vertical\ change}{horizontal\ change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{dynamic\ subtraction}{dynamic\ subtraction}$$

$$= \frac{dynamic\ value\ of\ the\ difference}{dynamic\ value\ of\ the\ difference} = dynamic\ decimal\ value$$

by making explanations. For example, showing the dynamic text, she said, *"We should show the relation between two things. Let's look how GGB computes the slope?"*. Besides, as practice 5, she considered the mathematical difficulties students experienced in using GGB material. That is, she responded to students' wrong answers about the coordinate system using the slider in the GGB graphics view by making explanations and posing questions. For example, when a student mentioned only the ordinate value (8) when referring to the coordinates of the point (2, 8), she said, *"Kids, 8 would not be a point in the coordinate system. You should say the point's coordinates. A number and 8?"* Then she used the slider that showed the point in the GGB graphics view after a student said that it was 2 and 8.

In the *Leaking Container* activity, the teacher's trimming practices were combined under the categories of trimming mathematical situation by using GGB material (practice 1) and trimming computation by using GGB material (practice 2), as can be seen in Table 16.

As practice for trimming the mathematical situation by using GGB material (practice 1), the teacher guided students through the content of the activity using GGB material by tracing students' understanding of the components of the line of the equation for the situation. Specifically, she enabled the students to understand slider-b, which shows each point for the graph of the equation in the situation by posing questions and making explanations. For instance, she said, "*At the same time, move the slider (The students move the slider). What happens?"*. In addition, she enabled the students to understand both the slider and the trace tools that showed the points of the graph by posing questions and making explanations. For instance, she said, *"Trace on point Z. Then, move your slider again. So, you have changed the time. Examine what has happened there."*. While using the GGB material for this practice, the teacher established an association among the functional situation, the points, the dynamic figure, the GGB tools, and the GGB graphics view as a bridging practice (practice 2: connecting mathematical situation/concept and GGB material in phase 2, see Table 12) in the background.

As practices for trimming computation using GGB material (practice 2) in *Leaking Container* activity, the teacher showed/checked the change, the computation, and the drawing by using GGB material. After the students understood the content of the activity in the GGB material, the teacher showed and checked the change by using the GGB material. That is, she encouraged the students to move the slider tool to show and check their own points for the change in the graph of the given situation. In addition, while connecting the functional situation, the points, the dynamic figure, the GGB tools, and the GGB graphics view (bridging practice 2 in phase 2, see Table 12) in this process, she encouraged them to move the slider tool to show and check the change in the amount of water. After all, as practice for showing and checking the drawing by using the GGB material (in practice 2), she encouraged the students to draw the graph of the situation on their activity sheets using the GGB graphics view by posing questions and making explanations. As evidence, the teacher asked such questions to the groups of students as:

> What is this graph for? *(The teacher shows the points of the amount of water-time graph in Figure 29)*. And look at your graph. What do you need to do right now? ... Did you draw your line? I see that some of your graphs are not drawn? There are only points. There is a line graph, isn't there?

At this point, after the teacher analyzed the students' drawings on their activity sheets, she tried to maintain mathematical coherence in the graphical representations of the functional situation in the students' drawings by referring to the graphics that the students composed in the GGB graphics view of the GGB material. Besides, while the teacher was using GGB material for this practice, the teacher concomitantly connected the mathematical situation/concept, the GGB material, and the activity sheet (bridging practice 1 in phase 2 in Table 12) as explained in the bridging practices section.

As practice for trimming computation by using GGB material (practice 2) in the *Leaking Container* activity, the teacher provided an environment for computing the slope of the graph by using GGB material. When she observed that some of the students had difficulties in computing the slope, she enabled the tools in the GGB material to show and check their computations by using GGB material. That is, she had the students use different-points-checkboxes to show any interval between any two points with horizontal and vertical dashed arrows to compute the slope of the graph. She also had the students use the slope-checkbox to show and check the computation of the slope as an algebraic ratio, combined with a static and dynamic text in the GGB graphics view:

$$\frac{vertical\ change}{horizontal\ change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{dynamic\ subtraction}{dynamic\ subtraction}$$

$$= \frac{dynamic\ value\ of\ the\ difference}{dynamic\ value\ of\ the\ difference} = dynamic\ decimal\ value$$

Thus, the students clicked the slope checkbox, and the text of the slope appeared in the GGB graphics view. While using GGB material for this practice, the teacher concomitantly connected mathematical/situation/process and GGB material (bridging practice 3 in phase 2 in Table 12) as explained in the section of bridging practices.

After the students used GGB material together with its tools, the teacher also used the GGB material to summarize the steps in computing the slope of a line as an algebraic ratio in the activity as trimming practice for showing and checking computation by using the GGB material. That is, she computed the slope of the line on an interval by using a pair of points, vertical change and horizontal change checkboxes, which show the vertical and horizontal dashed arrows as the change in y-variable with the change in x-variable, and the slope checkbox, which shows the dynamic computation text in the GGB graphics view by posing questions. In other words, she maintained the vertical and horizontal changes on the dashed arrows while integrating the direction of the change and the slope of the line. Then, she explained the distinction between the slope computation algorithm in physical situations and functional situations using the checkboxes. In brief, she emphasized the sign of the slope of a line and the directions in changes, that is, whether they increased or decreased in the computations. The following dialogue shows the practice that emerged.

(T) Now, if we consider point A and point B. Let's look at the screen on the board *(The teacher clicks on the checkboxes in the GGB graphics* *view)*. For instance, if we want to compute the slope between A and B, *(She drags point A on the line in the GGB graphics view)*. You said it doesn't matter which point it is but let's look. When you are computing the slope here, what unit of vertical distance would you find *(She points to the vertical change with dashed arrow in the GGB graphics view)*? *(see Figure 35)*

(SS) Two.

(T) Okay then, what will the unit of its vertical length be? (She points to the horizontal change dashed arrow in the GGB graphics view)?

(SS) Two.

(T) [The slope appears to be] two over two, that is one. However, it is a line. While computing the slope of a line, the sign of the amount of change is important. Isn't it? The slope of a line can be positive or negative. Let's think about why the slope is negative in that situation. There is a decrease from 8 to 6. And, from 2 to 4 – *(The teacher shows the points A(2,8) and B(4,6) in the GGB graphics view)*

(S) Increases.

(T) There is an increase. Therefore, the change in one is positive, and the change in the other one is–

(S) Negative.

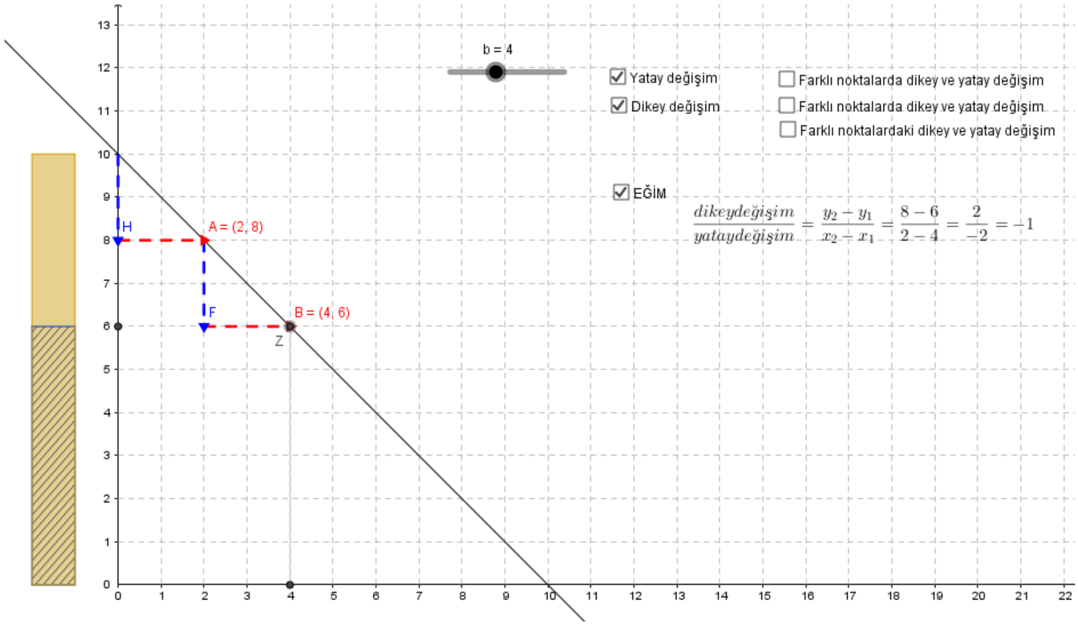

Figure 35. Teacher's screen of the Leaking Container activity on the board.

In Figure 35, the teacher computed the slope of the line by dragging point A to (2, 8) and then clicking on slope checkbox, which showed the computation accompanied with a dynamic text as an algebraic ratio.

In the *Slopes and Equation of Lines* activity, the teacher's trimming practices were collected under the categories of trimming mathematical situation by using GGB material (practice 1), trimming interrelations in equations and lines by using GGB material (practice 3), and doing mathematical entailments of GGB (practice 4).

As practices for trimming the mathematical situation by using GGB material (practice 1), the teacher guided the students through the content of the activity using GGB material and in moving between the GGB algebra view and the GGB graphics view. In the beginning, for guiding through the content of the activity, she explained to the students how to use the GGB algebra view, the GGB graphics view, and the tools (the sliders and the dynamic slope computation text) by using an activity sheet in line with the purpose of the activity by making explanations. The purpose was to trim the idea that the slope of line could be interpreted through algebraic and geometrical methods. In addition, she guided the students to move between the GGB algebra view and the GGB graphics view while tracking their understanding of using a dynamic slope computation text for a line in the GGB graphics view and using the GGB algebra view for the equation of a line in the activity context. That is, she showed a linear equation as an algebraic expression of line (in the GGB graphics view) by using the GGB algebra view by making explanations and posing questions. Then, she had the students show the slope of a line for the equation in the GGB algebra view using the slope computation text for a line in the GGB graphics view by posing questions. As evidence, the following excerpts of explanations made by the teacher to the whole class can be given an example for these practices.

Excerpt	Practices
Now everybody look at the board because in a while you will do it on the computer. Look carefully. And make sure that it's not the same people who are using the computer all the time. Change places. I'm telling you what to look at on GGB. The section of GGB here is the algebra checkbox. Everything, every line, point we have structured here *(The teacher shows the GGB Graphics view)* has a mathematical display here *(she shows the GGB algebra view)*. It has a figurative image here (the GGB graphics view). You learned these as well *(She shows the sliders)*. The slider of a continuously changing value. Everybody understood this. Here there is a formula checkbox *(The teacher shows the slope checkbox)*. There are many line sections on your activity sheet too. You can create these lines on GGB.	Guiding students through the content of the activity using GGB material

Now, I want this. When we click on that line *(she clicks on an equation of a line in the GGB algebra view)* look at whose color changes *(she moves the mouse on the equation in the GGB algebra view and the line is highlighted in the GGB graphics view)*? What is this? It is the equation of that line *(she moves on the line in the GGB graphics view and the line sparkled in the GGB graphics view)*. That is, it is the line of y equals x minus two *(the teacher shows the GGB algebra view)*. So what is the slope of this line?	Guiding students to move between the GGB algebra view and the GGB graphics view

Then, she had students show slider-m as the slope of the equation/line in the GGB graphics view by posing questions, responding to students' answers, and making explanations. For example, while explaining to a group of students that they need to change the slope of the lines in the GGB graphics view, she said, *"Kids, the letter m is used for showing the slope (The teacher refers to the slider-m in the GGB graphics view)"*. In a similar vein, she also had students show slider-n as a constant value of an equation in the GGB algebra view and the y-intercept of a line in the GGB graphics view by posing questions. While using GGB material for these practices, the teacher established a connection among not only the equation, the line, the GGB algebra view, and the GGB graphics view but also the equation, the line, the slope, the GGB algebra view, the GGB graphics view, and GGB tools (bridging practice 2: connecting the mathematical situation/concept and the GGB material in phase 2, see Table *12*) in the background.

In the *Slopes and Equations of Lines* activity, the teacher trimmed interrelations in equations and lines by using GGB material (practice 3). In the beginning, she showed a relation by using slider tools, the GGB graphics view and the GGB algebra view. Specifically, she encouraged students to show how an equation, a line and slope relates to each other for various lines using slider-m, the GGB graphics view and the GGB algebra view by posing questions. For example, she asked, *"Did the image of the line change when you change the slope value? (The students move slider-m at that moment) What happened to the equation?"*. While using GGB material for this practice, the teacher established a connection among the concepts of equation, line, and slope, the GGB material and the students' activity sheets (bridging practice 1: connecting the mathematical situation/concept, the GGB material, and the activity sheet in phase 2, see Table *12*) in the background.

In addition, she also encouraged students to show how an equation in the GGB algebra view and the y-intercept value of the line in the GGB graphics view relate to each other using slider-n by posing questions and making explanations. For example, she said, *"Let's change the n value [slider-n]. Look, what changes in the equation when the n value changes? What will be the equation if n is minus one and a half?"*. While using GGB material for this practice, the teacher connected the constant term in the equation in the GGB algebra view, slider-n as an n-symbolic notation and the y-intercept value of the line in the GGB graphics view (Bridging practice 2: connecting the mathematical situation/concept and the GGB material in phase 2, see Table *12*) in the background. Then, she showed an independency by using the slider tools and the GGB views. That is, she encouraged the students to show how the slope of an equation in the GGB algebra view is independent of the y-intercept by using slider-n in the GGB graphics view and the GGB algebra view by posing questions and making explanations. For example, to develop students' understanding of this relationship on various situations, she asked, *"What happens to the slope? Change n [slider-n]? You should do this for ten [lines of] equations".* In this way, the students created various (almost 10) equations and lines in GGB algebra view and GGB graphics view using slider-m and slider-n.

Subsequently, as another practice for trimming interrelations in equations and lines by using GGB material (practice 3) In the *Slopes and Equations of Lines* activity, the teacher showed how to convert forms of linear equations for slope computation using a property of the GGB algebra view. That is, she showed the standard form of an equation using the convertor in the GGB algebra view by making explanations and posing questions. For example, she said,

> Look at the lines. I right clicked the lines. There is another way to represent the lines *(The teacher right clicks on the equation of the line and converts the equation to the standard form in the GGB algebra view).* Look at the algebra view. What do you see?

While using the GGB material for this practice, the teacher connected forms of equations, the GGB algebra view, and the GGB graphics view (bridging practice 5: connecting conceptualizations of concept(s) and the GGB material in Phase 2, see Table *12*) in the background. She also showed the regular slope-intercept form of an equation as the usual way of representing equations for a slope computation in the

GGB algebra view by making explanations and posing questions. The reason why the teacher used the slope-intercept form of the equation was to enable the student to relate the equation to the slope concept and the graphical representation in this form.

Subsequent to a student's comment on decimal representation, as practice for doing mathematical entailments of GGB (Practice 4), the teacher showed the mathematical representation that was specific to GGB. As evidence for this practice, while using the GGB material, the teacher made an explanation about decimals as in the following dialogue. The dialogue started with a student's awareness of GGB being used as a different mathematical representation for a decimal in the equation that the representation she used.

(S) Teacher there is times 1 there *(The student refers to the decimal point in the equation in the GGB algebra view).*
(T) That's not times 1. That's one and a half. Kids, GGB does not use a comma while writing a decimal point in decimals. GGB uses a dot in decimals *(The teacher shows the GGB algebra view).*
(S) But it places a comma between the coordinates.
(T) A comma? Very good, you've paid attention to that. So that this and that point does not get confused. Did you understand? If it placed a comma for this *(decimal representation)* and that *(coordinate values)* too, what would happen?
(S) It would become confusing.

Thus, the teacher showed the difference between the decimal point representation in the GGB context and that in the Turkish mathematics context. More specifically, she showed the representation of a decimal point in GGB by responding to a student's comment and showed that GGB uses a dot to indicate the decimal point rather than a comma, as is the case in Turkey, by responding to the student's comment. Based on the previous comments, the teacher also explained that some countries use a dot and some countries use a comma as the decimal mark in decimal separation as a mathematical convention.

4.2.2.3 Trimming practices in teaching the solution of a system of equations and the slope relation for Phase 3

The trimming practices in teaching the solutions of systems of equations and the slope relation with GGB materials are explained in this subsection. In the third phase, the teacher used three activity sheets: Mobile Operators, Equation Systems, and Stores. In addition, she used two GGB materials for the Equation Systems activity, one GGB material for the Mobile Operators activity, and one GGB material for the Stores activity. In this phase, the categories of trimming practices were grouped under four headings:

Practice 1. Trimming mathematical situation by using GGB material;

Practice 2. Trimming graphical representation by using GGB material;

Practice 3. Trimming computation by using GGB material;

Practice 4. Addressing mathematical difficulties within conditions of GGB material.

In the mathematical situation category, the teacher showed/checked the equations by using the GGB material, showed how to satisfy an equation by using the GGB material, and created different mathematical situations by using the GGB graphics view. In the graphical representation category, the teacher showed/checked how to draw a graph by using GGB material, showed/checked the intersection point by using GGB material, showed/checked the points in the situations by using GGB material, guided the students through content of the activity by using GGB material, showed/checked the solution set by using the GGB graphics view, and limited the use of the GGB material in drawing a graph. In the slope computation category, the teacher showed/checked the slope by using the GGB graphics view, limited the use of the GGB material in relation to slope, and compared the slopes of parallel lines/equations by using GGB material. In the mathematical difficulties category, the teacher used GGB to prevent students' from forming misconceptions regarding the slopes of parallel lines. While using the GGB materials of the activities, the teacher used the GGB graphics view, slider tools, checkbox tools, trace of points, the dragging property, static texts, the GGB algebra view, expressions for lines and numbers in the GGB algebra view in various combinations during the practices.

The practices and actions under each practice accompanied with sample descriptions are presented in Table 17. The detailed descriptions of these practices are given in the sequential flow of the activities (Mobile Operators, Equation Systems, and Stores) during phase 3. The enumeration of these practices in the table is not indicative of the order in which the practices were implemented.

Table 17. Trimming practices of Phase 3 while using GGB materials

Practice	Action	Sample Description
Practice 1. Trim mathematical situation using GGB material		
Show and/or check equations using GGB material	Use equation in GGB algebra view for equation of line to check students' response	Have students use equation in GGB algebra view for constructing equation to check their equation in activity sheet by posing questions and making explanations (M)
	Use equation checkbox for equation of line in GGB graphics view	Have students use equation checkboxes that show equations of lines (text) in GGB graphics view to check students' answers (S)
Show satisfying equation using GGB material	Use values of a point in GGB graphics view to show satisfying an equation	Encourage students to calculate the y-value of intersection point on equation using the x-value of an intersection point in GGB graphics view to show the point satisfies equation (M)
Create different mathematical situations using GGB graphics view	Use dragging function of point to create different lines for systems of equations using GGB graphics view	Have students use the dragging function of point on a line in GGB graphics view to create different system of equations (E)
		Drag point on line in GGB graphics view to create lines of different system of equations (E)
Guide for content of the activity using GGB material	Trace students' understanding of parallel lines using GGB graphics view.	Have students understand that parallel lines do not intersect by using GGB graphics view by posing a question (E)
	Explain when to use the checkbox in GGB graphics view	Encourage students to use slopes of lines checkbox that show horizontal and vertical changes with dashes between two points in GGB graphics view to have students compute slopes of parallel lines by making explanations (E)
	Trace students' understanding of crossing lines using GGB graphics view	Use GGB graphics view to show the solution set of crossing lines by making explanations (E)
Practice 2. Trim graphical representation using GGB material		
Show and/or check drawing of graph using GGB material	Use equations in GGB algebra view for drawing line graphs to check students' responses	Have students use equation of line in GGB algebra view for drawing line graph in GGB graphics view to check their graph in activity sheet by making explanations and posing questions (M)
	Use a start point in GGB graphics view for drawing line graphs	Use a start point in GGB graphics view to have students draw the graph of store-M in (S)
	Use slider and trace tools for drawing line graphs	Have students understand each point of the graph compose functional situation of store-M/N using slider and trace tools by posing questions (S)

Table 17 (continued)

	Use equation in GGB algebra view for drawing line graphs	Have students draw the line for functional situation of store-M/N using equation of line in GGB algebra view (S)
Show and/or check intersection point by using GGB material	Use equations in GGB algebra view for drawing intersection point of graphs	Have students show intersection point of the line graphs of equations in GGB graphics view with using equations of lines in GGB algebra view to check their graphs in activity sheets by posing question (M)
	Use checkbox for drawing intersection point of graphs	Use intersection point checkbox to show intersection point and its values (text) in GGB graphics view by posing question (S)
		Have students use intersection point checkbox in GGB graphics view that shows coordinates of the point to check their graphs by making explanations (M)
	Use values of intersection point in GGB graphics view	Use values of coordinates for intersection point in GGB graphics view by posing questions and making explanations (M)
Show and/or check points in situations using GGB material	Use slider for the situations to show points in GGB graphics view	Have students use sliders to show the points of the graphs at a given time in GGB graphics view by making explanations (M)
	Use sliders to show values of points for the situations in GGB graphics view	Encourage students to use sliders to show different time values for the situations in GGB graphics view by making explanations (M)
Show and/or check solution set by using GGB graphics view	Show that intersection point of lines is a solution set of system of equations by using GGB graphics view	Have students show solution set of equations system is coordinates of intersection point of lines of equations in GGB graphics view by posing question (E)
		Use lines in GGB graphics view to show intersection point as a solution set of system of equations by posing questions and making explanations (E)
	Check algebraic solution and graphical solution of system of equation by using GGB graphics view	Have students use intersection point in GGB graphics view to check algebraic solution for solution set of system of equations (E)
Limit the use of GGB material for drawing	Close the algebra view for drawing the graph in GGB graphics view	Do not use algebra view when drawing points of graph for functional situation of store-M (S)
Practice 3. Trim computation with using GGB material		
Show and/or check slope by using GGB material	Compute slope of line in GGB graphics view	Have students compute slopes of lines in GGB graphics view by posing questions (E)

Table 17 (continued)

		Have students compute slopes of parallel lines that have equal slopes using GGB graphics view by responding to student's comment (E)
		Encourage students to compute slope of line by using points on axes in GGB graphics view (E)
		Encourage students to compute slope of line as algebraic ratio using points on axes in GGB graphics view by responding to a student's answer (E)
	Show slope value with checkbox in GGB graphics view	Have students use slope checkboxes for the lines that show the slope values to check students' answers (S)
Limit the use of GGB material for slope computation	Limit the checkbox tool for slope computation	Postpone using slope checkbox to compute slopes of lines by hand (E)
	Limit the GGB algebra view for slope computation	Do not use GGB algebra view when computing slopes of lines by hand (S)
Compare slopes of parallel lines/ equations by using GGB material	Use GGB graphics/algebra view to compare slopes of lines/equations	Have students compare slopes of parallel lines in GGB graphics view by posing question and responding student's comments (E)
		Have students compare slopes of equations of parallel lines in GGB algebra view by posing question (E)
Practice 4. Address mathematical difficulties within conditions of GGB material		
Use GGB to prevent students' misconceptions of slopes of parallel lines	Use dragging function to prevent students' misconceptions regarding the equality of slopes of parallel lines	Drag a point on one of parallel lines in GGB graphics view to prevent students' misconceptions of points in alignment showing equal slope of parallel lines by responding to students' comments (E)

Note: The abbreviations in the sample description column of the table indicate the activity in which an action emerged. M: Mobile Operators activity, E: Equation Systems activity, S: Stores activity.

In the *Mobile Operators* activity, the teacher's trimming practices were collected under the practices of trimming the mathematical situation by using GGB material (practice 1) and trimming the graphical representation by using GGB material (practice 2) (see Table 17). More specifically, the teacher trimmed the mathematical situation of time versus price by using the GGB material and the graphical representation for the situations by using GGB material. In addition, she trimmed the mathematical situation by showing/checking equations and showing how to satisfy the equation by using GGB material. Furthermore, she trimmed the graphical representation by showing/checking the drawing of a graph, the intersection point, and points in situations by using GGB material.

In the beginning, as practice for trimming graphical representation by using GGB material (practice 2), the teacher showed and checked the drawing of a graph by using GGB material. That is, by making explanations and posing questions, the teacher had the students draw a line graph in the GGB graphics view by using the equation of a line in the GGB algebra view so that they could check their graphs in the activity sheet. For example, while the students were working on the GGB material of the *Mobile Operator* activity, she said, *"Look at your first line (the teacher refers to the equation in the GGB algebra view) and clicks it. Did you draw this relation (The teacher shows the line in the GGB graphics view)?".* As can be seen, while using GGB material for this trimming practice, the teacher concomitantly connected the equations, the line graphs, the GGB algebra view and the GGB graphics view as practice for connecting the mathematical situation/concept and the GGB material (Bridging practice 1 in phase 3 in Table *13*). In addition, the teacher showed and checked the intersection point by using the GGB material. That is, she also had students show the intersection point of the line graphs of the equations in the GGB graphics view by using the equations of the lines in the GGB algebra view so that they could check their graphs in the activity sheets by posing questions. As evidence, she asked, *"Will these two lines intersect at a point? I want you to find this intersection point in GGB. Those two lines will intersect, won't they? I want you to find the intersection point…".* While using the GGB material for this trimming practice, the teacher also connected the graphical, symbolic and verbal representations of the intersection point by using the GGB graphics view as practice for connecting conceptualizations of concept(s) and GGB material (Bridging practice 3 in phase 3 in Table *13*). Thus, the teacher localized the solution of the system of equations to this situation that involved intersecting lines while using the GGB material.

Meanwhile, as practice for trimming a mathematical situation by using GGB material (practice 1), by posing questions and making explanations, the teacher got the students to use (click) an equation in the GGB algebra view to construct an equation to check their equations in the activity sheets. For example, she said, *"Now can you* *look at the algebraic image? Is the first equation y equals 2x? (She shows the equation in the GGB algebra view by dragging the mouse cursor on it.)"*. Thus, the teacher ensured the mathematical accuracy for the functional situation.

Then, the teacher continued to trim the graphical representation using the GGB material (practice 3) by showing/checking the points in the situations and the intersection point using the GGB material. That is, by making explanations, she had the students use sliders to show the points of the graphs at a given time in the GGB graphics view. As evidence, for instance, while the teacher guided a group of students, she said,

> For example, in your activity sheet, which minute is asked *(She asks about the price of seventh minute)*? Can you move the sliders to the seventh-minute? *(She meant two sliders that moved the points for the two situations)*. Move the sliders to seven. We can see which one [price] is higher. How much do you pay for each?

As can be seen, while the teacher used the GGB material for this trimming practice, the teacher concomitantly connected equations, line graphs, solutions, GGB tools, and GGB graphics view as practice for connecting the mathematical situation/concept and GGB material (bridging practice 1 in phase 3, see Table *13*). In addition, she encouraged the students to use slider-t and slider-z to show the prices at different time values for the situations in the GGB graphics view by making explanations. As evidence, while the teacher guided another group of students, she said, *"You can give such an example. Use the slider to find how much money will be paid at any time"*. As practice for showing and checking intersection point using GGB material, the teacher had the students use the checkbox of the intersection point in the GGB graphics view, which showed the coordinates of the point (i.e. position of the point on the coordinate plane) to check their graphs by making explanations. In the meantime, she also used the label of the coordinate values for the intersection point in the GGB graphics view by posing questions and making explanations as a preparation for interpreting the intersection point for the equations. While using the GGB material for this trimming practice, the teacher also connected representations of the intersection point and the GGB graphics view as practice for connecting conceptualizations of concept(s) and GGB material (Bridging practice 3 in phase 3, see Table 13). Thus, the teacher ensured the accuracy of the representations of the linear equations system when the lines were intersecting and representing functional situations by using the GGB material.

Lastly, as practice for trimming the mathematical situation by using GGB material (practice 1) in the *Mobile Operators* activity, the teacher showed how to satisfy an equation using GGB material. That is, she encouraged students to calculate the y-value of an intersection point on the equations by using the x-value of the intersection point in the GGB graphics view to show that the point satisfies the two equations for the Mobile Operators situations by posing questions.

In the *Equation Systems* activity, the teacher's practices were collected under the categories of trimming a mathematical situation by using GGB material (practice 1), trimming a graphical representation by using GGB material (practice 2), trimming computation by using GGB material (practice 3), and addressing students' mathematical difficulties within conditions of GGB material (practice 4) (see Table 17).

In the beginning of the Equation Systems activity, as practice for trimming a mathematical situation involving two lines for a system of equations by using GGB materials (practice 1), the teacher guided the students through the content of the activity by using GGB material. First, by posing questions, the teacher traced students' understanding of parallel lines by using the GGB graphics view. That is, she enabled the students to understand the equation of parallel lines and the fact that parallel lines do not intersect by using the GGB graphics view. For example, she said, *"There should be two lines in the equation system in your GGB file. Do these lines intersect? Do blue and green lines intersect?".* While using GGB material for this practice, the teacher connected equations of lines in the GGB algebra view and the line graphs in the GGB graphics view for various situations in the activity (Bridging practice 1: connect the mathematical situation/concept and the GGB material in phase 3, see Table 13) in the background. Then, as another practice for guiding students through the content of the activity by using GGB material (in practice 1), she explained when to use the checkbox in the GGB graphics view. That is, by making explanations, she encouraged the students to use the slope checkbox to show horizontal and vertical changes between two points with dashed segments for computing slopes of lines in the GGB graphics view. As evidence, the following dialogue emerged during the practice.

(T) Let's click on that *(The teacher encouraged the students to click on the slopes of lines checkbox in the Equation Systems-1 GGB material, see Figure 31. Screen view of the Equation Systems-1 GGB material).* It may help you to do it [the computation of slopes of lines] easily.
(S) Vertical over horizontal.
(T) Yes. Let's do that ratio.
(S) Vertical change over horizontal change.
(T) Yes. It is vertical change over horizontal change. What is it?

In addition, as another practice for guiding students through the content of the activity by using GGB material (in practice 1), she used the GGB material for intersecting lines to enable the students to understand crossing lines as a system of equations in the GGB graphics view by posing questions. To this end, she provided the students with a guide for a mathematical understanding of the system of equations with two GGB materials, which involved parallel lines (the Equation Systems-1 GGB material) and intersecting lines (the Equation Systems-2 GGB material) with various situations for parallel lines and intersecting lines. That is, she limited the system of equations to two linear equations which had no solution set or one solution set while using the GGB materials.

As another practice for trimming a mathematical situation by using GGB material (practice 1) in the Equation Systems activity, the teacher created different mathematical situations by using the GGB graphics view. That is, she had the students drag the line in the GGB graphics view to create different systems of equations by making explanations. To illustrate, a dialogue such as the following emerged during the practice:

(T) Can you give the mouse? *(The teacher takes the Bluetooth mouse from group A).* Let's change the equations of the lines. Ayşe, take the mouse *(The teacher gives the mouse to group C to use the computer).* Let's change the line in the screen. Click on point A and move it.
(S) Okay *(She moves point A on the x-axis and changes the position of the line in the GGB graphics view).*
(T) You can drag the point up. Can't you?
(S) Okay. *(She dragged point A up to quadrant 1 in the GGB graphics view. See Figure 36. The GGB view with lines of a system of an equation after a student dragged point A to create a different system)*

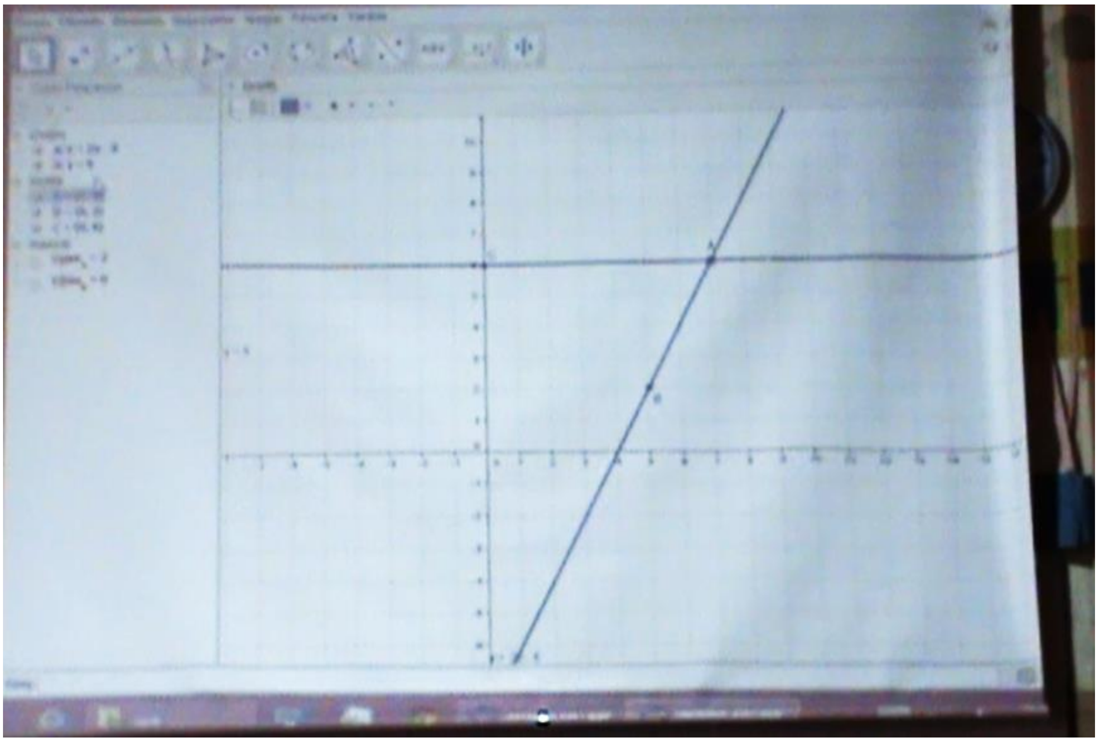

Figure 36. The GGB view with lines of a system of an equation after a student dragged point A to create a different system.

As practice for trimming a graphical representation by using GGB material (practice 2) in the Equation Systems activity, the teacher showed and checked a solution set by using the GGB graphics view. That is, the teacher showed that the intersection point of lines is a solution set of a system of equations by using the GGB graphics view of the Equation Systems-2 GGB material. More specifically, she used the lines in the GGB graphics view to show the intersection point of lines as a solution set of the system of equations by posing questions and making explanations. In addition, she had the students show that the solution set of a system is the coordinates of the intersection point of the lines of equations in the GGB graphics view by posing questions. For example, the following dialogue illustrates the teacher's questions:

(T) What are the coordinates of the intersection point of the lines?
(SS) 7 and 6.
(T) What are the equations of the lines?
(S) *Y* equals to *2x* minus 8. And *y* equals to 6.
(T) If you want to find the solution set of the equations of these lines, which point will you find?
(S) Point A.

Then, as practice for showing and checking the solution set by using the GGB graphics view, the teacher checked the algebraic solution and the graphical solution of the system of equations by using GGB graphics view. That is, she had the students check the algebraic solution for the solution set of a system of equations using the intersection point of lines in the GGB graphics view by posing questions. For example, after the students algebraically solved the system of equations by using a pen that they created in the GGB material and placed the value of the intersection point in the GGB graphics view into the equations, she asked, *"Is the solution set point seven and six (7,6)? (The teacher points to the GGB graphics view)"* to check the algebraic solution and the graphical solution for the system of equation by looking at whether the coordinates of the point satisfies the equations.

As practices for trimming computation by using GGB material (practice 3), the teacher used the GGB materials for showing/checking the computation of slopes of lines, limiting the use of GGB material for slope computation, and comparing slopes of parallel lines/equations. The teacher had the students compute slopes of lines in the GGB graphics view by posing questions. When she saw that students had found the slope on an equation, she made explanations and posed questions to encourage them to compute the slope of a line by using points on axes in the GGB graphics view by making explanation and posing question. While using GGB material for this practice, the teacher also connected slope computation of a line and GGB graphics view (Bridging practice 2: connecting the mathematical situation/process and the GGB material in phase 3 in Table 13). In response to a student's answer for an algebraic ratio of a slope, she encouraged them to compute slopes of lines as an algebraic ratio by using points on axes in the GGB graphics view. While trimming the computation, she limited the use of GGB material for slope computation. That is, she postponed using the slope checkbox after the students computed the slopes by hand.

In the equations systems activity, another theme for the trimming practices was addressing the mathematical difficulties experienced by the students by using GGB material (practice 4). In response to a student's comment about the equality of the slopes of parallel lines, the teacher dragged a point on one of the parallel lines in the GGB graphics view to prevent the student's misconception that two points in alignment on parallel lines show that the slopes of these two parallel lines are equal.

After that consideration, the teacher continued to trim slope computation by comparing slopes of parallel lines by using GGB material. More specifically, by posing questions and responding to students' comments, she had the students compare slopes of parallel lines in a system of equations in the GGB graphics view and compare slopes of equations of parallel lines in the GGB algebra view. For example, she said, *"Your equations are there [in the GGB algebra view]. Write their slopes and fill in your table. Did you write four systems of equations? (The teacher checked the students' activity sheets). Let's make a generalization through these four systems"*.

In the *Stores* activity, the teacher's practices were collected under the categories of trimming mathematical situation by using GGB material (practice 1), trimming graphical representation by using GGB material (practice 2), and trimming computation by using GGB material (practice 3) (see Table 17).

In the beginning, the teacher trimmed the graphical representation of the situations by using GGB material. As practice for showing/checking the drawing of graphs, the teacher used a starting point in the GGB graphics view to enable students to draw the graphs of store-M and store-N. Then, by posing questions, she ensured that the students understood that each point on the graph composes the functional situations of store-M and store-N using slider tools and trace of a point. While using the GGB material for this practice, the teacher connected equations, line graphs, solutions, the GGB graphics view, and the GGB tools (Bridging practice 1: connecting mathematical situation/concepts and GGB material in Phase 3, see Table 17) in the background. After those drawings of points for the graphs, she had students draw the lines for functional situations of store-M and store-N using equations of lines in GGB algebra view by making explanation. For example, she said that "*…You can do the same thing when you click on the line in the GGB algebra view. Open the algebra view. Open the first line here (The teacher mentioned the equation of the line in the GGB algebra view)".* While using GGB material for this practice, the teacher connected equations, line graphs, the GGB algebra view, and the GGB graphics view (bridging practice 1: connecting mathematical situation/concept and GGB material) in the background. On the other hand, she limited the use of GGB material for drawing graphs. She particularly wanted students not to use the GGB algebra view when drawing the points on the graph for the functional situation of store-M by saying, *"Now, close the algebra view. You can use the slider that moves point M here."*. Thus, before or during the use of GGB material for this practice, the teacher guided the students to connect trace of the points, solutions of equations, the points on the line graphs of Store M and Store N, time-sliders in GGB graphics view (bridging practice 1: connecting a mathematical situation/concept and GGB material in phase 3, see Table 13).

In the following activity on Stores, as practices for trimming slope computation (practice 3), the teacher limited the use of GGB material for slope computation and showed/checked slope by using GGB material. That is, at first, the teacher limited the use of the GGB algebra view for slope computation when computing slopes of lines by hand. Then, she had the students use the slope checkboxes for the line graphs of the stores, which showed the slope values on their graphs, to check students' answers by making explanations (see Figure 37. The view of GGB material for the Stores activity when the students clicked the slope checkboxes for store-M and store-N line graphs.). That is, students showed the slope values with a preplaced slope interval accompanied with a slope value, which is connected to the slope tool in the GGB graphics view. Thus, while using GGB material for this practice, the teacher also connected slope, line graphs, the GGB graphics view, and GGB tools (bridging practice 1: connecting the mathematical situation/concept and GGB material in phase 3, see Table 13).

While proceeding with the practice of trimming graphical representation by showing/checking the intersection point, the teacher used the intersection point checkbox to show the intersection point of the graphs and its coordinate values with a text in the GGB graphics view by posing questions. In addition, she trimmed the equation of a mathematical situation by showing/checking equations by using GGB material. That is, by making explanations, she had the students use equation checkboxes for the situations of stores, which showed the equations of the lines with a text in the GGB graphics view in order to check students' answers. For example, she said, "*Let's look at these lines (She clicks on the equation checkboxes). This line is y equals to x and this line is y equals to x plus 10. What is this Point A here? (She clicks on the intersection checkbox)"*.

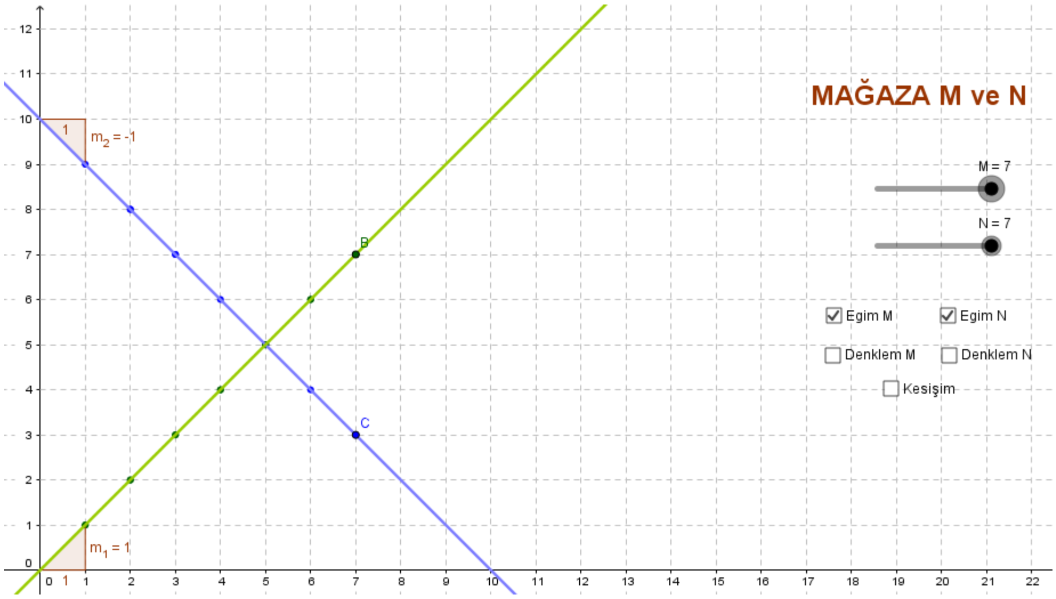

Figure 37. The view of GGB material for the Stores activity when the students clicked the slope checkboxes for store-M and store-N line graphs.

Summary of trimming practices in using GGB materials and concrete objects

Table 18. Trimming practices while using GGB materials and concrete objects

Practice	Number of phase
Guiding to trim the use of concrete object	1
Trimming a mathematical situation by using GGB material	1, 2, 3
Trimming computation by using GGB material	1, 2, 3
Trimming interrelations in equations and lines by using GGB material	2
Trimming graphical representation by using GGB material	3
Doing mathematical entailments of GGB	2
Addressing mathematical difficulties within conditions of GGB material	1, 2, 3

4.2.3 Decompressing Practices in using GGB materials and Concrete objects

In the previous two sections, the teacher's mathematical practices were explained in terms of two dimensions (bridging and trimming) in the KAT framework as the teacher used concrete objects and GGB materials in the classroom teaching sessions. In this section, the teacher's decompressing practices specific to teaching slopes, lines, and equations in Grade 8 are explained in terms of the decompressing dimension of the KAT framework. Therefore, this section involves the teacher's decompressing practices within each phase when she used concrete objects and GGB materials during the activities in classroom teaching. From the perspective of McCrory et al.'s (2012) KAT framework, decompressing practices in teaching algebra are described as mathematical practices of highlighting or deconstructing the complexity of mathematical ideas to make them comprehensible. Therefore, the decompressing practices were categorized by identifying the teacher's actions in classroom teaching in which the teacher provides mathematical practices to make explicit those mathematical ideas for a comprehensible mathematical context.

Each phase (phase 1, phase 2, and phase 3) involved the themes of decompressing practices and the actions under each theme in a table. The flow of actions in the practices were explained under the classroom activities for each phase. In addition, when a given decompressing practice was related with a bridging practice and/or a trimming practice that were explained in the previous sections, it was also reported those relations in the flow of the actions. It was assumed that two or three mathematical practices are related when they occurred concomitantly or when one occurred as a background for the other one at the moments of teaching. It was meant by the background practice that it emerged through a practice by giving the priority to the other practice upon the supposed intention of the teacher. In the end of this section, decompressing practices were summarized considering all phases.

4.2.3.1 Decompressing practices in teaching the slope of an object for Phase 1

The decompressing practices in teaching the slope of an object with GGB materials and concrete objects are explained in this subsection. In the first phase, the teacher used three activities accompanied with their activity sheets: Positions of Battens, Fire Truck, and Tent. In addition, she used a concrete object (i.e. batten in this phase) and GGB materials of the three activities. While using GGB materials of the activities, the teacher used the GGB graphics view, line segments (dynamic and static), trace of line segments, slider tools, checkbox tools, and text tools (dynamic and static texts). In the activities of the first phase, the teacher decompressed the computation algorithm of a slope, the measure of which is a ratio. The process of this phase involved understanding the slope in physical situations, the components of the slope, which were factors of the slope of an object, and the computation of a slope by comparing, exploring and interpreting. More specifically, the categories of the decompressing practices were grouped under four headings:

Practice 1. Preliminary comparisons

Practice 2. Interpretation of the components of the slope of an object by using concrete object

Practice 3. Interpretation of the components of the slope of an object by using GGB material

Practice 4. Appreciation of the use of Dynamic software

The practices and the actions under each practice accompanied with sample descriptions are given in Table 19. The practice for preliminary comparisons are named with 'preliminary' term since they were started to highlight the complexity of slope by comparisons but did not involve a completed interpretation or examination. The detailed descriptions of these practices are given in the flow of the activities (Positions of Battens, Fire Truck, and Tent) during phase 1. The enumeration of these practices in the table is not indicative of the order in which the practices were implemented.

Table 19. Decompressing practices of Phase 1 while using GGB materials and concrete object

Practice	**Action**	**Sample description**
Practice 1: Preliminary comparisons		
Making comparisons using concrete object	Compare positions of battens by using object	Have students compare the positions of battens while using their concrete objects (i.e. Batten, 60 cm) (P)
	Compare positions of battens, and concepts (slope & angle) by using material	Create connections across positions of battens, angle, and slope by posing questions while using concrete objects (P)
	Compare positions of battens by using material and analogy	Have students assume their battens as roads and they were walking on these inclined roads to compare the positions of battens by posing questions (P)

Table 19 (continued)

Practice 2: Interpret components of slope of an object by using concrete object		
Interpreting factors of slope by using concrete object	Explore the non-influential factors of slope of battens	Get students to explore the factors that do not affect the slope of battens by posing questions and making explanations (P)
	Explore the affecting factors of slope of battens	Get students to explore the factors that affect the slope of battens by posing questions (P)
		Disclose the influential factors of slope as quantities of horizontal length and vertical length for the battens (P)
		Encourage students to interpret the effects of vertical and horizontal lengths on slope while using their battens by making explanations (P)
Practice 3: Interpret components of slope of an object by using GGB material		
Interpret factors of slope by using GGB material	Explore the influential/non-influential factors of slope of batten using GGB graphics view	Get students to explore the factors that affect and do not affect the slope of batten (i.e. Line segment represents the physical object of batten) in GGB graphics view by posing questions (P)
	Explore the factors affecting/not affecting slope of batten using slider	Use horizontal length slider tool to get students to explore the influential factors of the slope of a line segments in GGB graphics view by posing questions (P)
Interpret logic in computation of slope by using GGB material	Interpret slope computation algorithm using slider and/or checkbox with dynamic text of slope computation	Related horizontal length, vertical length, and slope values to have student explore the computation algorithm using slider in GGB graphics view (P)
		Have students interpret the change in computation of slope while using vertical length slider tool after clicked the slope checkbox with dynamic slope computation text in GGB graphics view. (F)
		Interpret the change of slope when the horizontal length is constant using slider in GGB graphics view (F)
		Have students interpret the change in computation of slope while using horizontal length slider tool after clicking slope checkbox with dynamic slope computation text in GGB graphics view (T)

Table 19 (continued)

	Interpret slope computation algorithm using slider, trace, and checkbox with dynamic text	Use slider tool by tracing on the line batten (i.e. Line segment) and clicked the slope checkbox with dynamic slope computation text to enable the students to interpret the change of the slope of batten (P)
		Use slider tool by tracing on the line segment (i.e. Batten) and clicking the slope checkbox, which showed dynamic slope computation text, to compare the slope values considering the horizontal and vertical lengths as affecting factors of a slope (P)
	Interpret the change in slope using slider	Get students to compare different values of a slope in different vertical lengths while using vertical length slider tool by posing questions (F)
		Have students compare different values of slope while using horizontal length slider tool by posing questions (T)
Interpret special cases of slope by using GGB material	Interpret undefined slope in GGB graphics view	Interpret the meaning of undefined slope while using the horizontal length slider tool in GGB graphic view (T)
Practice 4: Appreciate the use of Dynamic software		
Appreciate the use of GGB	Appreciate the use of GGB to interpret mathematical ideas	Help students to understand the usefulness and importance of using GGB in mathematics to understand the algebra in a generalized and compressed form (F)

Note: The abbreviations in the sample description column of the table show the activity in which an action emerged. P: Positions of Battens activity, F: Fire Truck activity, T: Tent activity.

In the *Positions of Battens* activity, the teacher's decompressing practices were collected under the themes of preliminary comparisons (practice 1), interpretation of the components of the slope of an object using concrete object (practice 2), interpretation of the components of the slope of an object using GGB material (practice 3) (see Table 19).

In the activity, as preliminary comparisons (practice 1) for decompressing, the teacher wanted the students to make comparisons using concrete object. More specifically, she had the students compare the positions of the battens by posing questions while using the concrete object (i.e. batten, 60 cm). As evidence, she asked, *"Are the positions of the battens the same? Can we change them?"*. Here, the teacher's action took place concomitantly with the bridging practice of connecting concrete object and the mathematical situation/concept (practice 1 in phase 1 in Table 11). Thus, the students could begin to examine the reason underlying the differences between battens in different positions without expressing the concept of slope. After the students connected the concept of angle to positions of battens, the teacher had the students compare positions of battens and concepts (slope and angle) while using concrete object. That is, she created connections across positions of battens, angle, and slope by posing questions. As evidence, for example, while each group of students positioned the battens in their hands, the teacher asked, *"What influences having different angles for the battens? What influences the differences in the positions of the battens?"*. In addition, the teacher had the students compare positions of battens using material and analogy. That is, she wanted the students to assume that their battens were roads and they were walking on these inclined roads to compare the positions of battens again by posing questions. This decompressing practice also took place concomitantly with the bridging practice that connected concrete object, physical situation, and slope latently (Bridging practice 1: connecting concrete object and the mathematical situation/concept in phase 1, see Table 11).

After the aforementioned preliminary comparisons in the *Positions of Battens* activity, the theme that emerged within decompressing practice was interpreting the components of the slope of an object by using concrete object or GGB material. This theme involved the practice in which the teacher asked the students to interpret the factors of a slope by using concrete object. More specifically, the teacher explicitly posed questions to get the students to explore the factors that affected the slope of battens. For example, she asked, *"When you think about the positions of your battens, what could affect their slopes?"*. Meanwhile, she posed questions and made explanations to get the students to explore the factors that did not affect the slope of the battens. For example, while appreciating a student's response, she said, *"Yes. The part of the batten that is perpendicular to the ground affects the slope of the batten. You are right. Is there any other part that affects the slope? Does the length of your batten affect the slope?"*. Immediately afterwards, she disclosed the affecting factors of slope as quantities of horizontal length and vertical length for the battens by making explanations. As evidence, she said *"Yes. We can name this as horizontal length (The teacher shows the distance on a batten) and this as vertical length (The teacher shows the distance on a batten). And, the slope is related with these two notions."*. While using concrete object for decompressing the related factors of slope in this practice, the teacher concomitantly connected the positions of battens and factors of a slope (i.e. first, base and horizontal length; second, height and vertical length) as a bridging practice (Practice 1: connecting concrete object and the mathematical situation/concept in phase 1, see Table 11). In these actions, the teacher's intention was to make an introduction to developing a mathematical understanding of slope and its relations to the factors of slopes. After students made connections and explored what affects the slope of a batten, she continued to encourage the students to reason on the reason underlying this relation. That is, she encouraged the students to interpret the effects of vertical and horizontal lengths on slope while using their battens, by making explanations. To illustrate, she said:

> Yes, horizontal and vertical lengths have an impact [on the slope]. We will talk about them and how they affect the slope together. First, mark the point on the ground that your batten touches. Then, you can make an interpretation easily.

In the meantime, as practice for interpreting the components of the slope of an object using GGB material (practice 2), the teacher wanted the students to interpret factors related to slope by using GGB material. That is, the teacher got the students to explore the factors that affected and did not affect the slope of the batten (i.e. line segment object in the GGB graphics view represents the physical object of batten) in the GGB graphics view. As evidence, the following dialogue emerged during the practice.

(T) We talked about steep roads or sloping roads. Similar to battens, we can change the slope of this one [line segment as the object of a batten in the GGB graphics view]. Yes. What affects the slope of this *(The teacher points to the line segment in the GGB graphics view)*. Does the length of this *(The teacher points to the line segment in the GGB graphics view)* have any effect?
(SS) No.
(T) Which part? What do we call it in GGB?
(S) Its height.
(T) Where is the height? Let's see. It is point O *(The teacher points to point-O in the GGB graphics view)*. So, does the length of OB affect it?
(SS) Yes!!
(T) Is there any other one?
(S) OA
(T) Yes. [The length of] OA has an effect on the slope.

Therefore, while using the GGB material for this practice, the teacher connected representations of factors of slope and line segment in the GGB graphics view (bridging practice 6: connecting conceptualizations of concepts and the GGB material in phase 1, see Table 11) in the background. Then, as another practice of interpreting the factors of a slope by using GGB material (in practice 2), the teacher used the slider of the horizontal length of the batten to get the students to explore the affecting factors of the slope of the battens in the GGB graphics view by posing questions. As evidence, a part of the discussion with a group of students is given below:

(T) What makes the slope steeper here? *(The teacher moves the slider and then the batten object is moved in the GGB graphics view)*
(S) Its height.
(T) Yes. Look at the change in the horizontal at the same time *(The teacher pointed the batten in GGB graphics view and moved the slider)*. What kind of a change in the horizontal distance makes the slope increase?
(S) Small.
(S) Decrease.

The teacher made this interpretation to develop an understanding of how a change in horizontal distance and vertical distance for an object affect the slope of an object.

After the aforementioned interpretations of the factors of the slope of an object (i.e. batten), the teacher interpreted the logic behind slope computation by using GGB material as another practice for interpreting the components of the slope of an object by using GGB material (practice 3). That is, related relate vertical length and horizontal length values and slope of the batten visually (without the slope computation or value) to have students explore the computation algorithm in GGB graphics view. More specifically, she enabled the students to comprehend the logic behind slope computation algorithm by using the slider in the GGB graphics view by posing questions and making explanations as in the following dialogue:

(T) How can we relate a slope to both of them, I mean the vertical and horizontal lengths? What computation is possible?
(S) Division.
(T) Dividing what by what?
(S) [Dividing] The vertical by the horizontal length.
(T) So, what is slope? It is the ratio of vertical length over horizontal length. Let's find the slope of your own batten! I want you to write it [slope] using decimals. Please, do not write just the fraction.

Thus, the teacher structured the algorithm for the slope computation of an object with the ratio of vertical distance to horizontal distance by considering the increases and decreases in vertical distances and horizontal distances for an object with different slopes. While using concrete object for this practice, the teacher also encouraged the students in computing slope (as geometric ratio) on the concrete object (trimming practice 1: guide to trim the use of concrete object in phase 1, see Table 15) concomitantly. The teacher's thought was based on the assumption that a student could construct a slope as a ratio by directly relating vertical distance and slope and by inversely relating horizontal distance and slope. However, the teacher did not mention this relationship between a slope and a ratio in the classroom sessions since she wanted the students to develop an understanding of slope computation by means of the other two activities (*Tent* and *Fire Truck*) by making the horizontal or vertical distances constant (In the *Fire Truck* activity, the horizontal distance is constant. In the *Tent* activity, the vertical distance is constant).

When the teacher interpreted the logic underlying slope computation by using GGB material as practice for interpreting components of the slope of an object by using GGB material (practice 3), the teacher wanted the students to interpret the change in slope computation by using the slider tool, trace of line segment, and the checkbox tool. More specifically, she used the slider tool (the slider of the horizontal length of batten) by tracing on the batten (i.e. line segment) and clicking on the slope checkbox that showed the computation within a dynamic text to enable the students to interpret the change in the slope values of the batten. Then, she used the slider tool by tracing on the batten (i.e. line segment) and clicking on the slope checkbox, which showed the computation within a dynamic text, in order to compare the slope values of the batten considering the horizontal and vertical lengths as influential factors on the slope. As evidence, the following dialogue related to the change in slope emerged during the practice:

(T) …Ok then, I want you to see how the slope value changes when I change the [position of] batten. Even I clicked trace on command in GGB. Look at both the batten [line segment] and the slope. The value of the computation of the slope. What happens to the slope? *(The teacher moves the slider to the lower part, which increases the horizontal length of the batten, and eventually that moves the batten) (See Figure 25 for the computer screen view).*

(S)	Decrease!
(T)	It decreases, doesn't it? Look it is 1.17 and then 1.12. What happens gradually?
(SS)	Lessens. Becomes smaller.
(T)	The slope decreases more and more. What happens when it moves to the other part? *(The teacher moves the slider.)*
(SS)	Increases!
(T)	So, can you answer the next question in your activity sheets?
…	*The students discuss it in their groups.*
(T)	Good. Is there anything else? What happens while the slope increases and the height [vertical length] increases?
(S)	The horizontal length decreases.
(S)	Our batten has the longest vertical length and the least horizontal length with the biggest slope.
(T)	Yes. That's why it happens. You can think of numerator and denominator values. Did you do that?

While the teacher used the GGB material for this decompressing practice, she connected the slider of the horizontal length, the slope checkbox, the horizontal length and vertical length checkboxes, the dynamic text of slope computation, and the trace on the command for line segment in the GGB graphics view (bridging practice 4: connecting the mathematical situation/concept/process and the GGB material in phase 1, see Table 11) at the same time. Therefore, while structuring the understanding of the slope of an object, the teacher enacted some decompressing practices in unison with bridging practices.

In the *Fire Truck* activity, the teacher's decompressing practices were collected under the following themes: interpretation of the components of the slope of an object by using GGB material (practice 3) and appreciation of the use of the Dynamic software (practice 4) (see Table 19).

In the *Fire Truck* activity, as practice for interpreting the components of the slope of an object using GGB material (practice 3), the teacher wanted the students to interpret logic in slope computation by using the GGB material. Therefore, she sought to interpret the change in slope by using GGB material. That is, she got the students to compare different slope values (i.e. the slope of the fire truck ladder) in different vertical lengths while the students were using the slider of the vertical length in the GGB graphics view by posing questions. For example, after the teacher encouraged the students to show and check the slope computations of the ladders using the slider and the slope checkbox (trimming practice 3: trimming computation using GGB material in phase 1, see Table 15), the following dialogue emerged during the decompressing practice.

(T) Which ladder is more inclined? *(The teacher refers to the three positions of the ladders in the GGB graphics view)* Look at the numbers. 0.6 or 1.4?

(S) The ladder on the 9th floor. *(The students compare the slope values in the GGB graphics view for the 3rd floor, 7th floor, and 9th floor, which were asked in the activity sheet. The slope values were 0.6, 1.4, and 1.8, respectively)*

(T) Fine. Which floor would have the most [inclined] ladder in this apartment if there were a fire?

(S) 10th floor.

…

(T) On the 10th floor, is the slope the biggest one?

(S) Yes.

(T) Why? Write it down [on the activity sheet].

(S) The slope is 2 *(The students compare the slope values in the GGB graphics view).*

(T) I want you to write and explain why the slope is the biggest on the 10th floor.

(S) We see it as 2.

(T) You know it is 2. Fine, why?

(S) Because the height is the longest.

The students wrote their reasoning on their activity sheets and the teacher observed them. (See a student's (Ekin) activity sheet in Figure *38.* A student's (Ekin) activity sheet for the Fire Truck activity.*)*

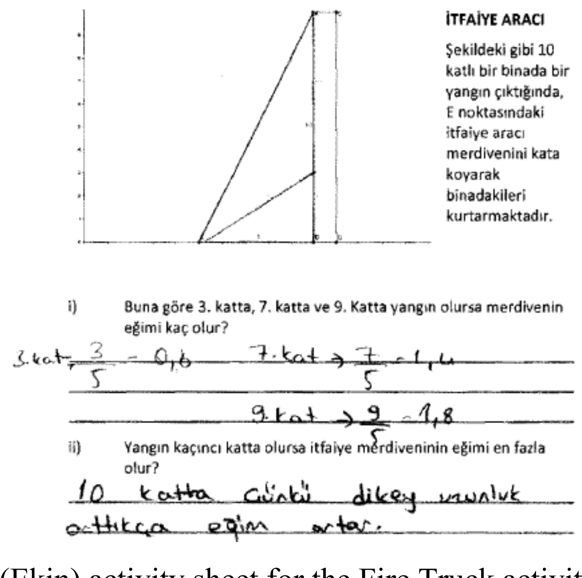

Figure 38. A student's (Ekin) activity sheet for the Fire Truck activity.

As seen in Figure 38, while explaining the biggest slope in the situation, the student named Ekin wrote, *"10th floor. When the vertical length increases, the slope increases,"* like many other students who wrote in a similar way. In addition, most of them also added to this sentence *"when the horizontal length is constant"* in parenthesis after the following practice.

As a concluding practice for interpreting components of the slope of an object by using GGB material (practice 3), the teacher wanted to interpret the logic behind the computation of a slope by using GGB material. More specifically, she interpreted the change in the slope when the horizontal length was constant by using the slider in the GGB graphics view, by posing questions and making explanations. For example, she said, *"We have two variables. One is the vertical and the other is the horizontal length (The teacher shows the GGB graphics view). In this question, which one is constant?"*. The students responded by saying the horizontal was constant. The teacher added, *"The horizontal distance is constant. Thus, we can see how the vertical distance affects the slope. Could we see clearly the effect of one variable on the slope when the other one is constant?"*. While using the GGB material for this decompressing practice, the teacher also enabled the students to understand the conditions of the slope that verified the computation (i.e. variables that are either constant or varying in the situation) by using the GGB graphics view (trimming practice 2: trimming computation by using the GGB material in phase 1, see Table 15) concomitantly. In addition, by posing questions and making explanations, she had the students interpret the change in computation of a slope while using the vertical length slider tool after clicking on the slope checkbox, which showed the computation with dynamic text of

$$slope = \frac{FB}{EB} = \frac{dynamic\ number\ value}{5} = dynamic\ decimal\ value$$

As evidence, for example, she said, *"I'm showing the slope checkbox at the same time (The teacher clicks on the checkbox). If we increase the vertical length of the ladder (The teacher moves the slider), how does the slope change?"*. Thus, the students could reason that slope values are directly related with vertical length values when the horizontal length is constant.

At the end of the activity, the teacher appreciated the use of the dynamic software in mathematics (practice 4). Specifically, she helped the students to understand the usefulness and importance of using GGB in mathematics to understand algebra in a generalized and compressed form by making explanations. For example, after a student made a comment that it was important to interpret the factors of a slope, the teacher responded by saying, *"You see that you could make interpretations since you saw different situations at the same time on GGB. It is very useful when making interpretations about the slope"*.

In the *Tent* activity, the teacher's decompressing practices were collected under the practice of interpreting the components of the slope of an object by using GGB material (Practice 3) (see Table 19).

As practice for interpreting the components of a slope of an object by using GGB material (Practice 3), the teacher wanted the students to interpret the logic behind slope computation by using GGB material. That is, she had the students interpret the change in the slope by using the slider in the GGB graphics view. More specifically, she got the students to compare different values of slope while the students were using the slider tool by posing questions. For example, the following dialogue within a group of students emerged during the practice:

(T) At which point is the slope at a minimum? To which point do you tie the rope?
(S) 10 meters away from the tent *(The students use the slider in the GGB graphics view).*
(T) Why?
(SS) Hmm [silence].
(T) Why does the rope at this point have the least slope?
(S) The vertical is the highest when the height [vertical distance] is constant.

In addition, the teacher had the students interpret the slope computation algorithm by using the slider and the checkbox within a dynamic text in the GGB graphics view. That is, she had the students interpret the change in slope computation while using the slider tool for horizontal length after clicking on the slope checkbox, which showed the dynamic text of the slope computation in the GGB graphics view. The teacher showed the actions of this practice before and after the practice described above. As evidence of this practice, she started with such questions as *"What changes?*

Explain the reason. What goes up when the point becomes distant? What happens? What does the slope depend on?". Then, she continued with the question, *"Look at this. How does the slope change?"*, while the students interpreted the graphics view of the situation, which is connected to the slope computation text as follows:

$$slope = \frac{AB}{CB} = \frac{5}{dynamic\ number\ value} = dynamic\ decimal\ value$$

As can be seen in the text, the value of the slope was given as a ratio and a decimal dynamically, which they changed according to the dynamic number value of the horizontal length of the rope connected to a slider having an interval between 0 and 10. In addition, when the students used the GGB material for this decompressing practice, the teacher concomitantly connected distance and horizontal length and connected slope and horizontal length in the GGB graphics view as practice for connecting the physical situation, line segment, slope, GGB graphics view, GGB tools (Bridging practice 3: connecting the mathematical situation/concept and GGB material in Table 11).

At the end of the activity, as practice for interpreting special cases of the slope by using GGB material, the teacher interpreted undefined slope in the GGB graphics view. That is, she interpreted the meaning of undefined slope while using the horizontal length slider tool in the GGB graphics view by making explanation. Before using GGB material for this decompressing practice, the teacher concomitantly connected representations of undefined slope, factors of slope, GGB graphics view, GGB tools as practice of connecting conceptualizations of concept(s) and GGB material (Bridging practice 6 in Table 11) and showed the conditions (of undefined slope) that verified the situation (i.e. horizontal length is zero in slope computation) by using GGB material as practice for trimming computation by using GGB material (Trimming practice 3 in Table 15). Then, as evidence for this decompressing practice in relation to the meaning of an undefined slope, the teacher said,

> It is usually said that the slope of a perpendicular object cannot be calculated. The reason is that the result of the computation is undefined. However, GGB makes a different computation, and it indicates it as being infinite. It perceives the result as being tremendous and extreme. However, the slope cannot be computed.

And she continued, *"Although GGB says as such, the slope is undefined"*. Thus, it was seen that this decompressing practice was ensued after the bridging and trimming practices for undefined slope.

4.2.3.2 Decompressing practices in teaching the slope of a line for Phase 2

After the phase on the slope of an object as a physical situation, the second phase was carried out with the purpose of developing students' understanding of the slope of a line as a functional situation and the slope computation of a line. The decompressing practices in teaching the slope of a line with GGB materials and concrete objects are explained in this subsection. The teacher used the GGB materials of the three activities: Building Design, Leaking Container, and Slopes and Equations of Lines. While using the GGB materials of the activities, the teacher used the GGB graphics view, sliders, checkboxes, texts (static and dynamic), lines (dynamic), points (dynamic), trace of points, and the GGB algebra view. In the activities of the second phase, the teacher decompressed the computation algorithm of the slope of a line, which is an algebraic ratio. The process of this phase involved understanding the slope in a functional situation, the components of slope of a line, which were computation of a slope, the sign of the slope, and the relation between slope and term in a line equation by comparing, interpreting, and examining. Therefore, in this phase, the categories of practices were grouped under two headings:

Practice 1: Preliminary comparisons

Practice 2: Interpretation of the components of the slope of a line by using GGB material

The practices and the actions under each practice accompanied with sample descriptions are presented in Table 20. The detailed descriptions of these practices in Table 20 are given in the flow of the activities during phase 2.

Table 20. Decompressing practices of Phase 2 while using GGB materials and concrete objects

Practice	Action	Sample description
Practice 1: Preliminary comparisons		
Have comparisons for computations of slopes by using concrete object and GGB material	Compare computations of slope of an object and slope of a line graph in GGB graphics view	Have students compare computation of slope for an object on concrete object and a line graph in GGB graphics view by posing questions (B)
		Have students compare the vertical dimension for an object on concrete object and a line graph in response to student's comment in GGB graphics view by posing question (B)
	Compare computations of slopes of an object for different intervals on concrete object	Have students compare computations of slopes for different two pairs of points (intervals) on the object by posing questions (B)
	Compare computations of slope of a line graph for different intervals in GGB graphics view	Have students compare the computations of slope of a line graph on different intervals in GGB graphics view by posing questions (B)
Practice 2: Interpret components of slope of line by using GGB material		
Interpret the logic in the computation of slope of line as algebraic ratio in GGB material	Interpret variables in computation of slope of line using points of line in GGB graphics view	Have students interpret the change in x as change in number of floors (x variable) by using points of line in GGB graphics view by posing questions (B)
		Have students interpret the change in y as change in number of windows (y variable) by using points of line in GGB graphics view by posing questions (B)
	Interpret computation of slope of line by using checkboxes, computation text, and dynamic points	Have students interpret symbolic representation of computation of slope of line by using dynamics points of the line, the slope checkbox in GGB graphics view (B)
		Have students use vertical and horizontal changes checkboxes that show dynamic points of the line connected to slope computation and slope checkbox that shows computation algorithm text to interpret the computation of slope of line in GGB graphics view (H)
		Have students interpret computation of slope by using dragging property of dynamic points connected to slope computation and slope checkbox by posing questions (H)
	Interpret constant slope of line in computation by using dragging property of dynamic points and slope checkbox, and computation text	Have students interpret constant slope of the line graph in computation by using dragging property of dynamic points, slope checkbox, and computation text by making explanations (H)

Table 20 (continued)

Interpret logic of the sign of the slope as behavior of the line in GGB material	Obviate pictorialization of positive slope in GGB graphics view	Obviate pictorialization of positive slope through quadrants of the coordinate plane in GGB graphics view by responding to students' incorrect answers and posing questions (B)
	Interpret the relation between positive slope and change in horizontal and vertical variables in GGB	Interpret the relation between positive slope and change in line that both changes in y and change in x increase in GGB graphics view by posing questions and making explanations (B)
		Interpret the relation between positive slope and change in line that both change in y and change in x decrease in GGB graphics view by posing questions (B)
	Interpret negative sign in computation of slope of line by using slope checkbox with computation text, and GGB graphics view	Have students interpret the negative sign within their computation of slope of line graph in GGB graphics view (H)
		Interpret negative sign of slope within the computation of slope of line graph in GGB graphics view (H)
	Interpret negative sign in computation of slope of line by using slope checkbox, computation text, changes checkboxes, dynamic points and GGB graphics view	Have students use vertical and horizontal change checkboxes, which show dynamic points and intervals connected to slope computation and slope checkbox with computation algorithm text, to interpret negative sign of the slope in GGB graphics view by posing questions (H)
		Have students use dragging property of dynamic points connected to slope computation and slope checkbox with computation algorithm text to interpret negative sign of the slope in GGB graphics view by posing questions (H)
Interpret slope-term relation within an equation of line in GGB material	Examine slope of an equation and terms in an equation in GGB graphics view and in GGB algebra view	Have students examine slope-term relation in equation using GGB graphics view and GGB algebra view (S)
	Interpret slope of an equation and terms in an equation by using slider in GGB algebra view, and Graphics view	Have students interpret the relation between an equation and its slope for various lines by using slider-m, GGB graphics view and algebra view by posing questions and responding to students' answers (S)
		Have students reveal no relation between constant term in equation and slope of equation by using slider-n, GGB graphics view and algebra view by posing questions (S)

Note: The abbreviations in the sample description column of the table show the activity in which an action emerged. B: Building Design activity, H: Leaking Container activity, S: Slopes and Equations of Lines activity.

In the Building Design activity, the teacher's decompressing practices were collected under the practices of preliminary comparisons (practice 1) and interpretation of the components of the slope of a line by using GGB material (practice 2) (see Table 20).

In the *Building Design* activity, as preliminary comparisons for decompressing, the teacher made comparisons for computations of slopes using concrete object and GGB material. These comparisons involved comparing computations of the slope of an object and the slope of a line graph in the GGB graphics view, comparing computations of slopes of an object for different intervals on concrete object, and comparing computations of slopes of a line graph for different intervals in the GGB graphic view. More specifically, as one of the comparisons, the teacher had the students compare the computation of a slope for a physical object on concrete object and the computation of a slope for a line in the GGB graphics view by posing questions. For example, she said:

(T) This is a line and it is infinite [she meant unlimited] *(The teacher shows the GGB graphics view)*. How do we compute the slope? Let's look at this pen *(She shows the inclined pen in her hand).* Does it have a slope?
(S) Yes.
(T) How did we compute it before? We took points *(She points to the object)* and computed this *(She points to the horizontal distance)* and this *(She points the vertical distance).* Now, how do you compute the slope of a line?

It was seen that while the teacher used concrete object and GGB material for this decompressing practice, she connected the physical object, the line segment in the GGB graphics view, and the computation of the slope of a line as practice for connecting concrete object, mathematical process, and GGB material (bridging practice 4 in phase 2 in Table 12) in the background. Then, she had the students compare the vertical dimensions of slope computations for an object (i.e. batten) and for a line by responding to a student's comment in the GGB graphics view with a question. As evidence, in response to a student's comment that they could draw a vertical length, she said, *"The line is infinite [She meant unlimited]. There is no starting point or end point. There are no limits for both sides of the line. How do we determine the horizontal or vertical length?"*. These questions enabled the students to reason on a taking limited part of the line to compute the slope of the line. Thus, the teacher prepared the students for the conceptualization of the slope of a line algebraically.

As another practice for preliminary comparisons in the Building Design activity, the teacher had the students compare the computations of the slopes of the object (i.e. batten) for different two pairs of points (intervals) on the object, by posing questions and making explanations. For example, she said, *"There are similar triangles. Do you see? Is the ratio of vertical over horizontal in this triangle equal to the ratio of vertical over horizontal in that triangle?" (The teacher positions the batten and creates different intervals for slope computation*, see Figure 39).

Figure 39. The teacher showing how to compute the slope of an object for different intervals.

While the teacher used concrete object (i.e. batten) and GGB material (building design GGB material) for this decompressing practice of the slope without given the formula, the teacher connected concrete object, similarity, slope computation, objects in the GGB graphics view as practice for connecting concrete object, mathematical process, and GGB material (Bridging practice 4 in phase 2 in Table 12) in the background.

As the last comparison, the teacher have the students compare slope computations of a line on different intervals in the GGB graphics view by posing questions. As evidence, for example, she asked, the following questions:

- "So, here is such a line graph *(The teacher shows the GGB graphics view)*. It is linear. Would there be any problem if we computed the slope of a line between any two points on the line?"
- "Is the slope always the same [for the line]?", and
- "Does it have to be the same as the points you set and the points your friends set or the points I set?"

When all these practices are considered as a whole, it was seen that the teacher compared different ways of computing slope within different intervals for an object and for a line. Thus, the teacher laid the foundation for the transition from a constant slope of a linear object to a constant slope of a line. On the other hand, she did not prepared a question or a task for this transition in the activity sheet and she developed this transition in the moments of teaching.

Following the comparisons and students' computations of slope by hand, the teacher initiated a discussion on the sign of the slope and the computation of the slope of the line in the Building Design activity. As practice for interpreting the components of the slope of a line using GGB material (practice 2), the teacher interpreted the logic underlying the sign of the slope as a behavior of the line in the GGB graphics view of the activity material. More specifically, she obviated students' pictorialization of a positive slope through the quadrants of the coordinate plane in the GGB graphics view, by responding to students' incorrect answers and by posing questions. The following dialogue emerged during the practice.

(S) In the coordinate plane, this line is in here, positives *(The student shows quadrant-1 of the coordinate system in the GGB graphics view)*, so its slope is positive.

(T) If you say that the reason is a coordinate system, it will be incorrect. Why? Let's continue on the line. It can be in negatives *(The teacher shows the quadrant-3 in the GGB graphics view)*. Then, will the slope below *(The teacher shows quadrant 3)* and the slope above *(The teacher shows the quadrant-1 in the GGB graphics view)* be different for a line? Remember, we said that the slope is the same [constant] for each point on the line.

Thus, when the students picturized the position of the line (i.e. overgeneralized the position of a line in quadrant-1 to be a positive slope), the teacher converted this misunderstanding of the student to an interpretation for the sign of the slope.

In addition, as practice for interpreting the logic behind the sign of a slope as a behavior of the line in the GGB graphics view of the activity material, the teacher interpreted the relation between positive slope and change in vertical variable and horizontal variable in the GGB graphics view. That is, she interpreted the relation between a positive slope and a change in the line as an increase in changes in both the y-variable and the x-variable in the GGB graphics view by posing questions. While using the GGB material, the teacher constructed this decompressing practice with the trimming practice of showing/checking computation (i.e. constitute the computation algorithm of slope of a line by using the coordinates of the points on the intervals) by using GGB material (trimming practice 2: trimming computation by using GGB material in phase 2 in Table 16). Then, she also interpreted the relation between a positive slope and a change in the line as decreases in both the y-variable and the x-variable in the GGB graphics view by posing questions. The following dialogue illustrates the practice.

(T) Now, this was what we did while going from B to C. What happens while going from C to B? *(The teacher shows the GGB graphics view)*
(S) It reduces.
(T) It reduces. Do they both reduce at the same time?
(SS) Yes.
(T) If they reduced at the same time, I mean if they both changed by increasing or decreasing, would the slope be positive or negative?

In the meantime, the teacher continued decompressing by interpreting the logic underlying the computation of the slope of the line as an algebraic ratio in the GGB material of the Building Design activity. That is, she had the students interpret the change in x as a change in the number of floors (x variable, a dimension of computation) by using the points of the line in the GGB graphics view by posing questions. She said, *"Now, what did your number of floors turn out to be while going from point I to point B?"* and *"Yes. How much unit does the coordinates between these points change?"*. Then, she had the students interpret the change in y as change in the number of windows (the y variable, a dimension of the computation) by using the points of the line in the GGB graphics view by posing questions. While using the GGB material for this decompressing practice, the teacher trimmed with the practice of showing/checking computation (i.e. constitute the computation algorithm of slope of a line by using the coordinates of the points on the intervals) by using the GGB material (trimming practice 2: trimming computation by using the GGB material in phase 2 in Table 16).

In addition, as a conclusion for the *Building Design* activity, she had the students interpret the symbolic representation of the computation of the slope of the line as an algebraic ratio by using the slope checkbox that shows the algorithm using dynamic points of the line graph and dynamic text of slope computation algorithm by posing questions. As an evidence, for example, she asked, "*Let's think! What does ' y_2 minus y_1 ' mean here? (She points to the static part $(y_2 - y_1)$ of the computation* *text of the slope) What is y_2?"*and *"Is 'y_2 minus y_1' valid for all the points on the line? What is?"*.

In the *Leaking Container* activity, the teacher's decompressing practices were collected under the practice of interpreting the components of the slope of the line by using the GGB material (practice 2) (see Table 20). This theme involves the practices of interpreting the logic underlying the computation of the slope of the line as an algebraic ratio in the GGB material.

As practice for interpreting the logic underlying the computation of the slope of the line as an algebraic ratio in the GGB material, the teacher interpreted the computation of the slope of the line by using checkboxes, the computation text, and dynamic points. That is, the teacher had the students use vertical and horizontal change checkboxes that showed the dynamic points connected to the slope checkbox that showed the slope computation algorithm within a static and dynamic text as

$$\frac{vertical\ change}{horizontal\ change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{dynamic\ subtraction}{dynamic\ subtraction}$$
$$= \frac{dynamic\ value\ of\ the\ difference}{dynamic\ value\ of\ the\ difference} = dynamic\ decimal\ value$$

in order to interpret the slope computation in the GGB graphics view by posing questions. As evidence for the practice, for example, she said, *"Think about where you made a mistake? By the way, the text does not mention the coordinates, does it? Click on those horizontal and vertical checkboxes. Now click on the slope checkbox. How did it compute the slope? Which value did it subtract from which value? Why is there a minus 1 there?"*. In, a group of students' screen view as a response to these questions posed by the teacher is portrayed. Then, while using the GGB material for this decompressing practice, the teacher connected the horizontal and vertical change checkboxes for the horizontal and the vertical change segments of an interval between two points and the slope checkbox for the dynamic text of the slope computation algorithm in the GGB graphics view (Bridging practice 3: connecting the mathematical/situation/concept/process and the GGB material in phase 2 in Table 16) in the background. In addition, the teacher had the students interpret the computation of a negative slope using the dragging of dynamic points (points A and B, Figure 40) and the slope checkbox by posing questions. As an evidence, for example, the following dialogue emerged during the practice.

(T) Drag the points. (*The teacher mentions points A and B in the GGB graphics view, see Figure 40*)
(S) It doesn't change.
(S) No change.
(T) The slope doesn't change. What happens to the numbers in the subtraction? What are these numbers in the subtraction?

While the teacher continued to use GGB material for this decompressing practice mentioned above, the teacher also connected moving a point on the interval, different intervals on the graph, which were composed by moving the point on the interval, and the slope checkbox for a dynamic text of slope computation in the GGB graphics view (Bridging practice 3: connecting the mathematical/situation/concept/process and the GGB material in phase 2 in Table 16) in the background.

In the meantime, the teacher had the students interpret the constant slope of the line graph in the computation by dragging the dynamic points, the slope checkbox, and the computation text by making explanations. That is, when the students used GGB material, she said, *"You have all taken different points [for the computation of the slope]. Your set of points are different from the set of points that your friend has taken. However, what is the slope of the line for all the points on the line?"*.

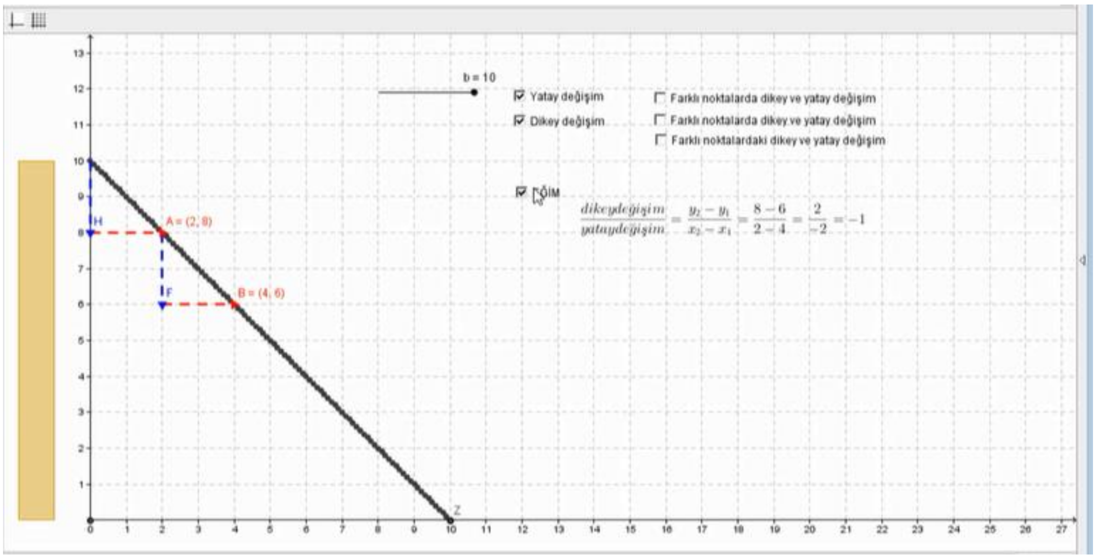

Figure 40. Students' screen view while interpreting the slope computation algorithm in the GGB graphics view of Leaking Container activity.

[The rectangle figure on the left represented the leaking container that was empty at that moment (t=10), the graph show the amount of water- time relationship in the situation.]

As practice for interpreting the logic of the sign of the slope in the GGB material, the teacher had the students interpret the negative sign within their computation of the slope of the line graph by using slope the checkbox with the computation text in the GGB graphics view by posing questions in response to a student's answer about the slope. Then, the teacher had the students use vertical and horizontal change checkboxes that showed the dynamic points and intervals connected to the slope computation and the slope checkbox with the computation algorithm text in order to interpret the negative sign of the slope in the GGB graphics view by posing questions. In addition, the teacher had the students use the dragging tool of the dynamic points connected to slope computation and the slope checkbox with a computation algorithm text to interpret the negative sign of a slope in the GGB graphics view by making explanations and posing questions. For example, she said, *"Let's talk about this. Why is the slope negative here? While the slope is positive in the building design activity, the slope is negative in this activity. Why?"*. Thus, as the teacher revealed the meaning of the slope of a line in a functional situation, the students gave explanations for the reason underlying a negative slope as they coordinated the amount of the changes in the y-variable with changes in x-variable. For example, a student said, *"As the time increases, the amount of water in the container decreases"*. While using the GGB material for these decompressing practices, the teacher connected the behavior of slope and the algebraic expression of slope computation in the GGB graphics view concomitantly with decompressing practices (bridging practice 5: connecting conceptualizations of the slope concept and the GGB graphics view in phase 2 in Table 12) in the background.

In the end, while summarizing the *Leaking Container* activity, the teacher interpreted the negative sign of the slope within the slope computation of the line graph as the ratio of amount of changes in the variables in the GGB graphics view, by making explanations. For example, she said,

(T) [The slope appears to be] two over two, that is one. However, it is a line. While computing the slope of a line, the sign in the amount of change is important. Isn't it? The slope of a line can be positive or negative. Let's think about why the slope is negative in that situation. There is a decrease from 8 to 6. And, from 2 to 4 there is– *(the teacher shows the points A(2,8) and B(4,6) in the GGB graphics view)*

(S) An increase.

(T) There is an increase. Therefore, one of the changes is positive, and the other change is-

(S) Negative.

(T) So, what is the sign of the ratio [slope]? Negative.

Thus, the teacher disclosed the sign of the slope of a line with digging the amount of changes in the variables (amount of water and amount of time) using any to points on the line.

In the *Slopes and Equations of Lines activity*, the teacher's decompressing practices were collected under the practice of interpreting the components of the slope of a line by using GGB material (practice 2) (see Table 20).

In the *Slopes and Equations of Lines* activity, the teacher's decompressing practices for interpreting the components of the slope of the line using GGB material involved practices on the relation between slope and term. As practice of interpreting the slope-term relation within a line equation in the GGB material, the teacher explored and interpreted the slope-term relation in equations by using the GGB graphics view and the GGB algebra view. That is, she had the students interpret the relation between the slope value of a line and the equation of a line by using the GGB graphics view and the GGB algebra view by posing questions. More specifically, after the teacher explained the students how to use the GGB algebra view, the GGB graphics view, and the tools with the activity sheet (Trimming practice 1: trimming a mathematical situation by using the GGB material in phase 2, see Table 16 and the students created various slopes, lines and equations in the GGB material, she asked, *"... Can we find the relation between the slope and the equation?"*.

Then, the teacher interpreted the slope of an equation and the terms in the equation by using slider in the GGB graphics view and the GGB algebra view. That is, she had the students interpret the relation between an equation and its slope for various lines by using slider-m, the GGB graphics view and the algebra view by posing questions and responding to a student's answer. As evidence, she said, *"We are going* *to continuously change slider-m. Do you think we will be able to figure out the relation between the equation and the slope?"* and *"I mean examine the 10 lines and their equations in such a way that you can find what the slope is related with.".* While the using GGB material for this practice, the teacher connected an equation in the GGB algebra view, the slope value of the equation/line and slider-m as a symbolic notation of slope (m) in the GGB graphics view (Bridging practice 2: connecting the mathematical situation/concept and the GGB material in phase 2, see Table 12) in the background.

In addition to the slope-equation relation, the teacher had the students reveal that there is no relation between a constant term in an equation and the slope of an equation by using slider-n, the GGB graphics view and the algebra view by posing questions. As evidence, for example, she asked, *"When we changed slider-n, did the slope change (The students moved slider-n in the GGB graphics view)?"*. Then, the students looked at both the equations and their terms in the GGB algebra view and the lines and the slopes of the lines in the GGB graphics view while using slider-n. Thus, the teacher supported students' reasoning on the relation between a slope and an equation (coefficient of term x) and noticing no relation between the slope and the constant term for the slope-intercept form of the equation.

4.2.3.3 Decompressing practices in teaching the solution of a system of equations and the slope relation for Phase 3

After the phase on developing an understanding of the slope of a line as a functional situation and the computation of the slope of a line, the third phase took place with the purpose of enabling students to understand the solutions of systems of equations and the slopes in the systems of equations. The decompressing practices in teaching the solutions of systems of equations and the slope relation with GGB materials are explained in this subsection. The teacher used three activity sheets: Mobile Operators, Equation Systems, and Stores. In addition, she used two GGB materials for the Equations Systems activity, one GGB material for the Mobile Operators activity, and one GGB material for the Stores activity. In the activities of the third phase, the teacher decompressed the representations of the system of equations for the two lines and the ways of representing solutions of a system of equations. The process of this phase involved understanding the equation and the line in a functional situation, intersection point on functional situations, and the components of a solution set for the system of equations by comparing and interpreting. In addition, the teacher used the slope concept as a way to reveal the solutions of a system of equations by interpreting. In this phase, the categories of decompressing practices were grouped under three headings:

Practice 1: Preliminary comparisons

Practice 2: Interpretation of the components of the solutions of a system of equations by using GGB material

Practice 3: Interpretation of the components of the slope of a line by using GGB material

While using GGB materials of the activities, the teacher used points, sliders, and the GGB graphics view. The practices and the actions under each practice accompanied with sample descriptions are given in Table 21. The detailed descriptions of these practices in Table 21are given in the flow of the activities during phase 2.

Table 21. Decompressing practices of Phase 3 while using GGB materials

Practice	**Action**	**Sample description**
Practice 1: Preliminary comparisons		
Making comparisons of functional situations by using GGB material	Compare the situations by using points with the sliders	Have students compare two situations at a given time using points with the sliders in GGB graphics view by posing questions (M)
		Have students compare two situations at different intervals of time by using points with the sliders in GGB graphics view by responding to students' comments and posing questions (M)

Table 21 (continued)

Practice 2: Interpret components of solution of system of equations by using GGB material		
Interpret intersection point on functional situations by using GGB material	Interpret the intersection point for the situations in GGB graphics view	Have students interpret the intersection point of the graphs for the situations in GGB graphics view as it provides the same price for both of the situations by posing questions and approving students' responses (M)
	Interpret the intersection point for the equations in GGB graphics view	Have students interpret intersection point for each equation as its' coordinates satisfy each equation of line by posing questions and making explanations (M)
	Interpret intersection point as solution set in GGB graphics view	Have students interpret intersection point of the lines as solution set for the equations of lines using point as ordered pair in the equation (M)
Interpret components of a solution set for system of equations using GGB material	Interpret positions, slopes, solution set and system of equations by using GGB graphics view and table	Have students generalize the relation between positions of lines, slopes of lines, solution set, and system of equations of lines by using GGB graphics view and table in activity sheet by posing a question (E)
		Have students' interpret the components of intersecting lines for a system of equation in GGB graphics view (S)
	Interpret positions, solution set and system of equations by using GGB graphics view and table	Have students generalize the relation between positions of lines, solution set, and system of equations of lines by using GGB graphics view and table in activity sheet by posing questions (E)
Practice 3: Interpret components of slope of line by using GGB material		
Interpret computation of slopes for system of equations by using GGB material	Interpret computation of slope with changes between two points in GGB graphics view	Interpret computation of slope of line through the changes in horizontal and vertical between two points in GGB graphics view by responding to students' answers and posing questions (E)
	Interpret slope and solution set in system of equations by using GGB graphics view	Have students interpret the slopes of lines with solution set in different positions of lines (intersect/parallel) using GGB graphics view and table in activity sheet (E)

Note: The abbreviations in the sample description column of the table show the activity in which an action emerged. M: Mobile Operators activity, E: Equations Systems activity, S: Stores activity.

In the *Mobile Operators* activity, the teacher's decompressing practices were categorized as preliminary comparisons (practice 1), interpretation of the components of the solutions of a system of equations by using GGB material (practice 2), and interpretation of the components of the slope of a line by using GGB material (practice

3) (see Table 21). The category of the preliminary comparisons involved practices for making comparisons of functional situations by using GGB material.

As practice for preliminary comparisons, the teacher compared functional situations using points with the sliders. That is, she had students compare two situations at a given time by using points with the sliders in the GGB graphics view by posing questions. As evidence, for examples, she said, *"…Can you move the sliders to the seventh-minute? (She meant two sliders that moved the points for two situations). Move the sliders to seven. We can see which one [price] is higher. How much do you pay for each?"* and *"Which mobile operator is more appropriate?"*. As it was seen, while using GGB material for preliminary comparisons of decompressing, the teacher connected line graphs, equations, solutions, the GGB graphics view, and GGB tools (bridging practice 1 in phase 3 in Table 13) and showed/checked points in situations and at an intersection point by using GGB material (trimming practice 3 in phase 3 in Table 17).

In addition, she had the students compare two situations at different intervals of time by using the points through the sliders in the GGB graphics view by responding to students' comments and by posing questions. For example, she said, *"Let's make a generalization together. If you were going to talk less than 5 minutes, which [operator] would you choose?"* and *"Why? Because until that part, an operator (t-cell) looks more decisive. What if it is a person talking more than 5 minutes?"*

The category of interpreting the components of the solutions of a system of equations by using GGB material (practice 2) involved practices of interpreting the intersection point on functional situations by using GGB material. As practices of interpreting the intersection point on functional situations by using GGB material, the teacher interpreted the intersection point for the situations in the GGB graphics view, the intersection point for the equations in the GGB graphics view, and the intersection point as a solution set in the GGB graphics view. More specifically, the teacher had the students interpret the intersection point of the graphs for the situations in the GGB graphics view as it provides same prices for both of the situations by posing questions and approving students' responses. For examples, she asked, *"What do we understand regarding this point? What do we understand from 5 and 10 [the point (5,10)]?"and*

"… What does it mean to intersect". While using the GGB material for this decompressing practice, the teacher sometimes showed and checked the intersection point by using the GGB material (trimming practice 2: trimming graphical representation by using the GGB material in phase 3 in Table 21). Then, the teacher had the students interpret the intersection point for each equation as its coordinates satisfy each line equation by posing questions and making explanations. For example, she said, *"What does point 5 comma 10 [(5,10)] mean in the first equation? For the first equation, what kind of a relationship is there between the equation and the point? Is the point on the y equals to two x [$y = 2x$] equation or not?"* and *"Is the point 5 comma 10 [5,10] a point that satisfies both of them?"*. In addition, the teacher had the students interpret the intersection point of the lines as a solution set for the line equations by using points as an ordered pair in the equation in the GGB graphics view. For example, she said, *"Now, what is the relation of the intersection point on the GGB screen to the equations of those two lines"* after the students solved the solutions of the two equations in the system by hand. While using the GGB material for this decompressing practice, the teacher concomitantly connected the intersection point, the solution set of equations, the GGB graphics view as practice for connecting conceptualizations of a solution set, the GGB graphics view (Bridging practice 3: Connecting conceptualizations of the concept(s) and the GGB material in phase 3 in Table 13).

In the *Equations Systems* activity, the teacher's decompressing practices were categorized as interpreting the components of the slope of a line by using GGB material and interpreting the components of the solutions of a system of equations by using GGB material. There were two practices under the category of interpreting the components of a slope by using GGB material. First, the teacher interpreted the computation of the slope of a line as the changes in the horizontal and vertical axes between two points in the GGB graphics view, by responding to a student's answer and posing questions. Second, the teacher had the students interpret the slopes of lines with a solution set of a system of equations for different positions of the lines (intersecting/parallel) using the GGB graphics view and the table in the activity sheet, by posing questions.

There were two practices under the category of interpreting the components of a solutions of a system of equations by using the GGB material of the Equations Systems activity. First, the teacher had the students generalize the relation among position of lines, slopes of lines, solution set, and a system of line equations by using the GGB graphics view and the table in the activity sheet by posing questions. She said:

> Now let's generalize by taking a look at these 4 lines. Like by looking at these lines, saying this happens in parallel lines… A while ago Sema initiated an interpretation. Look at the line equations you wrote. Look at the situation of the lines. What happens as a result of what? Analyze the four situations. What happens to the slope? What happens to the solution set?

In other words, the teacher wanted the students to conclude that two equations for intersecting lines have one solution set and two equations for parallel lines have no solution set. In addition, the slopes of parallel lines are equal and the slopes of intersecting lines are different. Therefore, she wanted the students to deduce the positions of lines in a system of equations from the slopes of equations in the system. Second, the teacher had the students generalize the relation among position of lines, solution set, and system of line equations by using the GGB graphics view and the table in the activity sheet by posing questions. The following dialogue showed the practices mentioned above.

(T) Well, if I give you two equations. If say that they do not have a solution set. What would you say?
(SS) Parallel.
(T) If there were one element in the solution set, what would you say?
(S) Then they will intersect.
(T) Then, what if the solution set consists of real numbers? What would happen in that case? What were real numbers?
(S) Real numbers.
(T) You know if every point satisfied both lines, then what would those two lines be?
(S) Intersecting.
(S) They were be parallel.
(T) But imagine, it doesn't satisfy only one point, but all points. How will those two lines be?
(S) Then, they will be on top of each other.
(T) How?
(S) One on top of the other.

In the Stores activity, the teacher's decompressing practice was categorized as interpreting the components of the solutions of a system of equations by using the

GGB material. Specifically, the teacher had the students interpret the components of the intersecting lines for an equation system in the GGB graphics view by posing questions. For example, she asked, *"What did we know about intersecting lines?* and *"There is a solution set. What else did we know?"*. That is, the students could deduce that intersecting lines have different slopes and one solution set.

Summary of decompressing practices while using GGB materials and concrete objects

Table 22. Decompressing practices while using GGB materials and concrete objects

Practice	**Number of phase**
Preliminary comparisons	1, 2, 3
Interpretation of the components of the slope of an object by using concrete object	1
Interpretation of the components of the slope of an object by using GGB material.	1
Interpretation of the components of the slope of a line by using GGB material	2, 3
Appreciation of the use of a Dynamic software	1
Interpretation of the components of the solutions of a system of equations by using GGB material	3

4.3 Mathematical Practices in Explaining Mathematical Ideas without using GGB materials and Concrete objects

In this study, the teacher implemented the instructional sequence about on the topics of slopes, linear equations, and graphs in grade 8 in a technology enhanced classroom environment. However, she also made mathematical explanations without using technology (i.e. GGB materials) and concrete objects during the classroom sessions. In this regard, this section dwells on the teacher's mathematical practices based on exposing students to mathematical ideas and providing them with opportunities to engage in reasoning on these mathematical ideas without GGB materials and concrete objects in a technology enhanced classroom environment. The practices are reported under three sub-sections: (1) Bridging practices for Phase 1, 2 and 3, (2) Trimming practices for Phase 1, 2, and 3, and (3) Decompressing Practices for Phase 1, 2, and 3.

4.3.1 Bridging Practices in Explaining Mathematical Ideas without using GGB materials and Concrete objects

In this section, the teacher's bridging practices are reported within each phase during which she explained the mathematical ideas without using GGB materials and concrete objects in classroom teaching. As previously mentioned, from the perspective of McCrory et al.'s (2012) KAT framework, bridging practices in teaching algebra are described as mathematical practices of connecting topics, representations, and domains in teaching slope, linear equations, and graphs in eighth-grade mathematics. Accordingly, the teacher's practices were categorized by identifying the teacher's actions in classroom teaching in which the teacher implemented mathematical practices to provide connections and coherence in mathematics across mathematical ideas, representations, topics, and other domains while explaining mathematical ideas without using GGB materials and concrete objects.

Thus, the themes of bridging practices that were grouped by the researcher and the phases that involved those practices are as follows:

Table 23. Summary of Bridging Practices while explaining mathematical ideas without using GGB materials and concrete objects

Practice	Number of phase
Practice 1: Connecting concept(s) and representations	1, 2, 3
Practice 2: Connecting concepts	1, 2, 3
Practice 3: Connecting activities	1,2, 3
Practice 4: Connecting topics	2, 3

The emerging themes of bridging practices and the actions under each theme are presented for each phase in Table 24. Thus, there were four themes of bridging practices and all of them emerged in all the phases, except for the theme of connecting topics in phase 1. The flow of actions in the practices are explained under the classroom activities for each phase.

Table 24. Bridging Practices while explaining mathematical ideas without using GGB materials and concrete objects

Practice	**Action**	**Sample description**
Practice 1: Connect concept(s) and representations		
Connect concept and real world representations	Connect slope and physical situations in real life	Connect slope and road situations in real life (1P)
		Connect slope and rope of tent (1R)
	Connect negative slope and real life situations	Connect negative slope and real life situations (2B)
	Connect system of equations and functional situations in real life	Connect graphs of the system of equations and functional situations in real life (3M)
Connect concept and table/ graphical representations	Connect linear relationship, table, and graphical representation	Connect linear relationship, table, and line graph (2B) (3M)
	Connect linear relationship, and graphical representation	Connect linear relationship and line graph (2B) (2H)
	Connect linear relationship and table representation	Connect linear relationship and table (3S)
Connect concept and symbolic representation	Connect linear relationship and symbolic representation	Connect linear relationship and linear equation symbolically (2B)
Connect concept and multiple representations	Connect linear relationship, verbal explanation, table, picture, symbolic representation	Connect linear relationship, verbal explanation, table, drawings of steps, linear equation symbolically (2H)
Practice 2: Connect concepts		
Connect conceptualizations of a concept	Connect conceptualizations of distance	Connect horizontal distance and distance for rope of a tent (1R)
	Connect conceptualizations of slope	Connect change in dimensions of the slope of a line and slope as algebraic ratio of difference in y-values to difference in x-values (2B)
		Connect the subtraction operation in algebraic ratio (slope) and vertical change (distance) between two points (in slope) (2H)
	Connect conceptualizations of ordered pair on the graph	Connect coordinates of a point as ordered pair on the graph, meanings of a point as an ordered pair in the functional situation (3M) (3M: 748)
	Connect conceptualizations of a solution set of a system of equations	Connect ordered pair as one solution of a system of equations algebraically and coordinates of an intersection point graphically

Table 24 (continued)

Connect slope and other concepts	Connect slope, ratio, division and decimal	Connect slope, ratio, division, and decimal, converting between them in the context of physical situation (1P) (1F)
	Connect slope and line graph	Connect slope of an object and slope of a line considering students' previous knowledge (1F)
		Connect slope and line graph in the context of functional situation (2B)
	Connect computation of the slope of a line and triangle	Connect computation of the slope of a line between two points and a triangle (2B)
	Connect computation of the slope of a line, triangle, horizontal and vertical change	Connect computation of slope between two points a and triangle, horizontal and vertical change (2B)
	Connect slope of line and ratio	Connect slope of line and unit of ratio (2B)
	Connect slope, direction, and ratio,	Connect sign of slope, direction, and ratio of vertical over horizontal (2B)
	Connect slope, horizontal length and change in horizontal	Connect horizontal length in slope of an object and change in horizontal in slope of a line (2B)
	Connect slope, coefficient term of x-variable, rational number	Connect slope of a line, coefficient term of x-variable, rational number in linear equations (2S)
	Connect slope and ways of finding slope on the equation and on the two points of the line	Connect slope and ways of finding slope on the equation and on the two points of the line of equation (3E)
	Connect slope, unit rate (price), functional situation/equation	Connect slope, unit rate (price), functional situation/equation (e.g. stores, mobile operators, price of detergent, taxi fare, price of tomato) (3S)
	Connect slope, rate of change, functional situation/equation	Connect slope, rate of change, functional situation/equation (e.g. taxi fare, price of tomato, the speed of train, fuel consumption rate, density formula, velocity) (3S)
Connect linear equation (linear function) and other concepts	Connect recursive relationship, rate of change, and linear equation	Connect numerical relationship and numeric value of rate of change in a linear function (2B)
	Connect explicit relationship and linear equation	Connect explicit relationship and linear equation (2B) (3S)
	Connect linear equation and line graph	Connect solutions of linear equation as ordered pair of numbers, points of line graph (2B)
	Connect forms of equations and explicit relationship	Connect forms of equations and explicit relationship (3S) (3S: 556, 565-569-573)

Table 24 (continued)

Connect slope and linear equation	Connect slopes and linear equations for different functional situations	Connect slopes and linear equations for different functional situations in a table (2S)
Connect solution set and other concepts	Connect a solution set of the system of equations and intersection point	Connect coordinates of the intersection point of two lines and values of a solution set of the system of equations with one solution (3M) (3E)
	Connect a solution set of the system of equations and positions of lines of equations	Connect solution a set of the system of equations and positions of parallel lines (3E)
		Connect solution a set of the system of equations and positions of intersecting lines (3E)
Practice 3: Connect activities		
Connect activities in a phase	Connect activities for slope of an object in phase 1	Connect the Fire Truck and Tent activities for computation of the slope of an object (1R)
		Connect the Fire Truck, Tent, Positions of Battens activities for the computation of the slope of an object (1R)
	Connect activities for slope of a line in phase 2	Connect Building Design, a hole in the container, and Slopes and Equations of Lines activities (2S)
	Connect activities for slopes of lines in phase 3	Connect Mobile Operators and Equation Systems activities (3E) (3E: 137)
Connect activities in different phases	Connect building design activity in phase 2 and phase 1 activities for slope	Connect building design activity and phase 1 activities for slope (2B)
Practice 4: Connect topics		
Connect topics in math	Connect topics in 7th grade and 8th grade mathematics	Connect linear relations and slope topics (2B)
		Connect patterns and linear equations (3S)
	Connect 8th math and high school math	Connect functions, analytic geometry, and slope, linear equations and graphs (3S)
Connect topics in math and science	Connect topics in science and math	Connect delta x, displacement, and change (2B)
		Connect density, speed, and slope (3S)

Note: The abbreviations in the sample description column of the table indicate the activity in which an action emerged. Phase 1 activities; 1P: Positions of Battens activity, 1F: Fire truck activity, 1T: Tent activity. Phase 2 activities; 2B: Building Design activity, 2H: Leaking Container activity, 2S: Slopes and Equations of Lines activity, Phase 3 activities: 3M: Mobile Operators activity, 3E: Equations Systems activity, 3S: Stores activity.

4.3.1.1 Bridging practices in teaching the slope in physical situations for Phase 1

In phase 1, the teacher started with Positions of Battens activity, and then continued with the Fire Truck activity and the Tent activity. These activities were based on the slopes of objects in a physical situation, which were a batten object in the Positions of Batten activity, the fire truck ladder in the Fire Truck activity, and a rope of a tent in the Tent activity. In Phase 1, the teacher's bridging practices were combined under the practices of connecting concept(s) and representations (practice 1), connecting concepts (practice 2), and connecting activities (practice 3), as can be seen in Table 24. In these practices, the concepts were slope, distance, ratio, division, and decimal. The representations were physical situations in the real world.

In the *Positions of Battens* activity, the teacher's bridging practices were combined under the themes of connecting concept and representations (practice 1) and connecting concepts (practice 2) (see Table 24).

As practice for connecting concept and representations (in practice 1 in Table 24), the teacher connected slope and physical situations in real life. More specifically, the teacher connected slope as steepness and road situations in real life. For example, this practice was introduced with the teacher's question, *"Do we use the term 'slope' in roads?"* and the following dialogue took place.

> (T) For instance, my house is down the school. Some of you live down the school, don't you? For example, what kind of a road do those people use on their way to school?
> (S) An inclined road.
> (T) Then, what kind of a road do those living on this side use? *(The teacher points to the houses aligned with the school.)*
> (S) Straight road.
> (T) Straighter road? Little slope or no slope. What should we call it mathematically? We will talk about this.
> (S) It is easy [to walk] to come to school but it is hard to go back from school.
> (T) Yes, it is sometimes easy. But what do we call these types of roads?
> (S) Inclined.
> (T) Yes. An inclined road or a road with a slope.

Thus, she made explanations that the word 'slope' is used in road situations in daily life and that the terms 'inclined' are used together with the term 'slope' term. She posed questions about the slopes of the roads that the students walked on while they were coming to school. That is, the teacher connected the concept of slope and road situations as the students' real-life experiences.

As practice for connecting slope and other concepts (in practice 2 in Table 24), the teacher connected slope, ratio, division, and decimal concepts, and converting between them in the context of physical situation the context of physical situation (i.e. positions of battens). As evidence, for example, after the students interpreted the slope of a batten as a ratio and the teacher approved the slope as the ratio of vertical length to horizontal length, she said, *"I want you to write it [slope] with decimals. Please, don't just write the fraction"*. As another example, while a group of students experiencing difficulties in dividing prime numbers (as the vertical length value), she said, *"It (slope) doesn't have to be a whole number. Find the decimal number. We can make divisions with prime numbers. That's why we use decimals."*

In the *Fire Truck* activity, the teacher's bridging practices were combined under the theme of connecting concepts (practice 2), as can be seen in Table 24.

As practice for connecting slope and other concepts (in practice 2 in Table 24), the teacher connected slope, ratio, division, and decimal, converting between them in the context of physical situation (i.e. positions of fire truck ladder). As evidence, for example, while the students were making computations of slope after they wrote the ratio of 3 over 4, the teacher asked, "*Did you divide it?"*. As another practice for connecting slope and other concepts (in practice 2 in Table *24*), the teacher connected slope and line graph, considering students' previous knowledge and guided them to think of slope in different situations. As evidence, for example, the following dialogue emerged during the practice:

(T) Do only the objects we see in daily, such as roads, roofs, sleds, have slopes? I wonder if the other things we learned in math have slopes too. What are they that we showed like this? *(The teacher draws a line in the air with her hand-gesture)*
(S) Triangles.
(T) The triangles and hypotenuses can have slopes. What did we learn in the previous year? There was a geometrical figure that showed linearity *(The teacher draws a line in the air hand- gesture).*
(S) Line.
(T) Can we speak about the slope of a line?

In the *Tent* Activity, the teacher's bridging practices were combined under the themes of connecting concept(s) and representations (practice 1), connecting concepts (practice 2), and connecting activities (practice 3), as can be seen in Table 24.

As practice for connecting concept and representations (in practice 1 in Table 24), the teacher connected slope and physical situations in real life. More specifically, the teacher connected slope and the rope of a tent in real life. As evidence, for example, this practice emerged while the teacher was talking about making a camp and using tents with the students, and the teacher posed such questions as *"Have you ever set up a tent? Why is it [slope] related to horizontal [distance]"*, and *"Isn't the place where you will fix the end of the rope in the ground depend on your free will?"*. Thus, the teacher connected the concept of slope and rope of tent situations as the students' real-life experiences.

As practice for connecting conceptualizations of concept(s) (in practice 2 in Table 24), the teacher connected conceptualizations of distance. More specifically, she connected distance and horizontal distance in the tent situation. As evidence, she asked, *"What is the distance from the tent? Horizontal distance or vertical distance?"* and continued, *"Yes. It is horizontal distance."*. Therefore she connected the terms that the students used before and the terms that were used for the slope of batten.

As practice for connecting activities in a phase (in practice 3 in Table 24), the teacher connected the Fire Truck and the Tent activities for the slope computation of an object and connected the Fire Truck, the Tent, the Positions of Battens activities for the slope computation of an object. As evidence, for example, she said, *"What did we learn from these two activities and how do we make a generalization? Let's summarize them: Fire Truck and Tent. What did we learn about slope from them?"*. As another example, after the students made conclusions about horizontal length and vertical length, she said,

(T) When we consider the Positions of Battens activity, both of them [horizontal length and vertical length] can change also.

(S) Yes

(T) Therefore, when we talk about them [slope, horizontal length, and vertical length], it is incorrect to say the longer the horizontal, the less slope. We should consider that the horizontal is longer, but what about

> the vertical? Is that right? However, if one of them is constant, it is easy to talk about the others. Is it clear?

4.3.1.2 Bridging practices in teaching the slope of a line for Phase 2

In Phase 2, there were activities on the slope of a line in functional situations and in the coordinate system, which were the *Building Design* activity, which entailed a line graph representing the linear relation between the number of windows in a building and the number of floors of a building, the *Slopes and Equations of Lines* activity, entailing relating linear equations, lines, and slopes, and the *Leaking Container* activity, including a line graph representing the linear relation between the amount of water (volume) in a container and time (minutes). In Phase 2, the teacher's bridging practices were combined under the practices of connecting concept(s) and representations (practice 1), connecting concepts (practice 2), connecting activities (practice 3), and connecting topics (practice 4), as can be seen in Table 24. In these practices, the concepts were negative slope, linear relationship, and linear equation.

In the *Building Design* activity, the teacher's bridging practices were combined under the themes of connecting concept(s) and representations (practice 1), connecting concepts (practice 2), connecting activities (practice 3), and connecting topics (practice 4) (see Table 24).

As practice for connecting topics in math (in practice 4 in Table 24), the teacher connected topics in 7th grade and 8th grade mathematics. More specifically, the teacher connected linear relations and slope topics by using the building design task that was also used for linear relationship in grade 7. She also said, *"Do you remember this activity. In the previous year, we did a similar one. How many windows are there when the apartment has 1 floor?".*

As practice for connecting concepts and table/graphical representations (in practice 1 in Table 24), the teacher connected linear relationship, table, and graphical representation, and connected linear relationship and graphical representation. As evidence, for example, while the students were drawing the graph for the situation using the number values in the table, she asked, *"How many windows [are there] in a 1-floor apartment? Where do you mark?".* As practice for connecting concept and symbolic representation (in practice 1 in Table 24), the teacher connected linear relationship and symbolic representation of the linear equation. As evidence, she asked a student, *"To find the number of windows for a 5-floor apartment, what do you do? How do you use the equation? What are the meanings of x and y in the equation?"*.

As practice for connecting linear equation and other concepts (in practice 2 in Table 24), the teacher connected recursive relationship, rate of change, and linear equation, connected explicit relationship and linear equation, and connected linear equation and line graph. To explain the action in more detail, the teacher connected numerical relationship (i.e. recursive relationship) in the situation and the numerical value of rate of change in a linear function. As evidence, while explaining the linear equation, she posed successively the questions of *"How does it increase?"* and *"Yes. It increases three by three. But this does not give us the number of windows. You have a constant value [constant number of windows for the basement floor] ..."*. She also connected explicit relationship and linear equation by posing questions. For instance, she said, *"...It is not enough to multiply by 3 [number of floors]. You add the constant value [to this]. So, what does y equal to?"*.

As practice for connecting activities in different phases (in practice 3 in Table 24), the teacher connected the building design activity in phase 2 and activities in phase 1 for slope. More specifically, she brought together the notions of the slope in physical situations and the slope of a line before explaining the computation of the slope of a line. For example, after she mentioned that they learned the slope concept in the previous lesson activities, she posed the following question: *"Does it [the line graph for the situation) have a slope?"*.

As practice for connecting slope and other concepts (in practice 2 in Table 24), the teacher connected slope and line graph, connected computation of slope of line and triangle, connected computation of slope of line, triangle, horizontal change and vertical change, connected slope of a line and ratio, connected slope, direction, and ratio, and connected slope, horizontal length and change in horizontal dimension. To give the details of an action, the teacher connected the sign of the slope, the direction of the line, and the ratio of vertical over horizontal change. As evidence, after the students computed the slope of the line between two points for the situation as 3, she asked, *"So, what is the ratio of the changes in the vertical and changes in the* *horizontal?"*. After she received the response "positive" from the students, she continued by saying, *"Yes, positive. What is the ratio of negative to negative?"*. In other words, she connected the sign of the slope of the line to the ratio of the sign of the changes in the horizontal and vertical changes, since she computed the slope of the line for the situation considering how both y-values change when x increases and y-value changes when x decreases.

As practice for connecting conceptualizations of concept(s) (in practice 2 in Table 24), the teacher connected the conceptualizations of slope as change in dimensions of the slope of a line and as algebraic ratio of the difference in y-values to difference in x-values. As evidence, after the students' comments about the slope of a line as 'ratio of vertical to horizontal' and 'divide y_2 minus y_1 by x_2 minus x_1', she asked, *"What does this formula mean? Combine your comments and think again"* (76), and continued by saying, "*We found the horizontal and vertical [changes] between these two points. So, can you think about this again with considering what said, 'y_2 minus y_1'?"*

As practice for connecting topics in math (in practice 4 in Table 24), the teacher connected the topics in science and math. More specifically, she connected delta x, displacement, and change. As evidence, while ending the activity, she posed such questions about change as *"What is change here? Did you learn the displacement concept in science?"* However, the students did not give productive responses about displacement and the teacher did not continue to focus on this notion.

In *Leaking Container* activity, the teacher's bridging practices were combined under the themes of connecting concept and representations (practice 1), and connecting concepts (practice 2) (See Table 24).

As practice for connecting concept and multiple representations (in practice 1 in Table 24), the teacher connected linear relationship, verbal explanation, table, picture, and symbolic representations. More specifically, she connected linear relationship, verbal explanation, table, drawings of steps, and linear equation in the context of Leaking Container activity. As evidence, while the students were working on the activity sheet that involved drawings of the initial steps, the following dialogue emerged during the practice:

(T) Ok, you filled in the table. Now try to form the equation. What happens? What did you do to find *y* for any *x*th minute?
(S) Is It 10 minus *x?*
(S) Aha, we should subtract from the [amount of water in the] container.
(S) Is it minus 1?
(S) We should subtract time from the [amount of water in the] container.
(T) What is the amount of water in the container?
(S) 10.
(T) Yes 10.
(S) 10 minus *x*.
(T) What equals to 10 minus *x*?
(S) *y*
(T) And, what is *y*?
(S) Amount of water [in the container]

As practice for connecting concept and graphical representation (in practice 1 in Table 24), the teacher connected linear relationship and line graph while reading the graph in the context of the Leaking Container activity. As evidence, she posed the following questions: *"Let's look at the graph at first. How do you understand how much water there is at 4th minute?"* and *"Where is the 4th minute in the graph?"*.

As practice for connecting conceptualizations of a concept (in practice 2 in Table 24), the teacher connected conceptualizations of slope. More specifically, she connected the subtraction operations in the slope as algebraic ratio and change in the vertical dimension of the slope of a line between two points. As evidence, the following dialogue emerged during the practice:

(T) I will ask another question. You determined two points [while finding the slope of the line]. What is this [vertical] distance [between the points]? Don't you count it?
(S) This part increases.
(S) We subtracted.
(S) When we make this subtraction, we find this part.
(T) Yes. When you make the subtraction, you find [the vertical] distance.

In the *Slopes and Equations of Lines* activity, the teacher's bridging practices were combined under the themes of connecting concepts (practice 2) and connecting activities (practice 3) (see Table 24).

As practice for connecting activities in a phase (in practice 3 in Table 24), the teacher connected the activities for the slope of a line in phase 2. More specifically, the teacher connected the Building Design, the Leaking Container, and the Slopes and Equations of Lines activities. That is, while introducing the current activity, the teacher wanted the students to look at the building design and leaking container situations, their equations and slopes. Then, she wanted them to write the equations [$y = 2x + 3; y = 10 - x$] and the slopes [$2, -1$]in their tables in the activity sheets of Slopes and Equations of Lines.

As practice for connecting slope and other concepts (in practice 2 in Table 24), the teacher connected slope, the coefficient term of x variable, and rational number. More specifically, the teacher connected the slope of a line and the coefficient term of the x-variable in linear equations, and rational number. As evidence, while the students were finding the slope value in the linear equation ($y = \frac{x}{4} - 9$), the teacher said, *"You found 1 over 4. You know this [slope] is a rational number"*.

4.3.1.3 Bridging practices in teaching the solution of a system of equations and the slope relation for Phase 3

In Phase 3, there were activities on the system of equations in functional situations and in the coordinate system. They were the *Mobile Operators* activity based on the solution of two linear equations for price (TL)-time (minutes) linear relations of two mobile operators, the *Equations Systems* activity based on relating linear equations in a system, slopes, solution set, positions of lines, and drawing lines in a system, and the *Stores* activity based on the solution of two linear equations of amount of product- time (day) linear relations of two Stores and the relation between slopes, solution, and graphs. In addition, at the end of this phase the teacher revised the slope of a line graph in functional situations. In phase 3, the teacher's bridging practices were combined under the practices of connecting concept(s) and representations (practice 1), connecting concepts (practice 2), connecting activities (practice 3), and connecting topics (practice 4) (see Table 24).

In the *Mobile Operators* activity, the teacher's bridging practices were combined under the practices of connecting concept(s) and representations (practice 1) and connecting concepts (practice 2) (see Table 24).

As practice for connecting a solution set and other concepts (in practice 2 in Table 24), the teacher connected the solution set of system of equations for mobile operators and the intersection point. More specifically, the teacher connected the coordinates of the intersection point of two lines and the values of a solution set of system of equations with one solution. After the students solved the system algebraically, the following dialogue emerged during the practice:

(T) You found x and y values. You know what a solution set is. Do you remember?
(S) Yes x is 5 *(The x-value of the solution set of the system of equation is 5).*
(T) What is y?
(SS) 10.
(T) You have already had an intersection point? What is that?
(S) 5.
(T) Is that so?

As practice for connecting conceptualizations of a concept (in practice 2 in Table 24), the teacher connected conceptualizations of an ordered pair on the graphs of the mobile operators. More specifically, the teacher connected coordinates of a point as an ordered pair on the graph, and the meanings of a point as an ordered pair in the functional situation. As evidence, while the students were working on the tasks in the activity sheet and explaining their results, the teacher asked, *"So, you understood how to read the graphs. The meanings of the points on the graphs. For example, in a pair of points [ordered pair], one is price and the other one is time. Do you see that?"*.

As practice for connecting concept and real world representation (in practice 1 in Table 24), the teacher connected the system of equations and functional situations in real life. More specifically, the teacher connected graphs of a system of equations and functional situation (i.e. mobile operators' time versus price relations) in real life. For example, while ending the activity, she said:

> You know there are many mobile operators. We cannot say one of them is economic all the time. It is related with the length of time that we talk on the mobile. So, you should know how long you talk [on the phone] and then make a computation to decide on one of them.

In the *Equations Systems* activity, the teacher's bridging practices were combined under the practices of connecting concepts (practice 2) and connecting activities (practice 3) (see Table 24).

As practice for connecting activities in a phase (in practice 3 in Table 24), the teacher connected the activities for slopes of lines in phase 3. More specifically, she connected the Mobile Operators and Equations Systems activities. That is, while introducing the Equations Systems activity, the teacher wanted the students to look at the Mobile Operators activity to write down equations, positions of lines, slopes of lines and a solution set into their tables in the activity sheet of Equations Systems. While the students did not compute the slopes of the lines in the Mobile Operators activity, they made connected that activity to slope with this activity in the background.

As practice for connecting slope and other concepts (in practice 2 in Table 24), the teacher connected slope and ways of finding slope on the equation and on the two points of the line. More specifically, after the students wrote down an equation in their activity sheets, the teacher wanted them to find the slope by using both the equation and the graph. That is, she said, *"Let's write your equations in your table [in the activity sheet]. Then find their slopes. Find it using any two points on the line. Then, find it using the equation."*.

As practice for connecting a solution set and other concepts (in practice 2 in Table 24), the teacher connected a solution set of a system of equations with one solution and an intersection point. More specifically, the teacher connected the coordinates of the intersection point of two lines and the values of the solution set of system of equations with one solution. After the student solved the system algebraically, she asked, *"You solved it [algebraically]. Is it the same with the graphics?"*. In addition, the teacher connected the solution set of the system of equations and the positons of the lines of equations. More specifically, the teacher connected the solution set of the system of equations and positions of parallel lines and connected the solution set of the system of equations and positions of intersecting lines. As evidence, while students were solving the system of equation with no algebraic solution and found no solution of an empty set, she said *"Is it [having no solution, no common point] the same for the parallel lines?"*.

In the *Stores* Activity, the teacher's bridging practices were combined under the practices of connecting concepts and representations (practice 1), connecting concepts (practice 2), and connecting topics (practice 4) (see Table 24).

As practice for connecting concepts and table representation (in practice 1 in Table 24), the teacher connected linear relationship and the table for the situations of stores M and N. As evidence, while the students were working on the table in the activity sheet at the beginning of the activity, she posed such questions as *"Can you* *look at the relationship between the two [variables in the table]. 1 and 9, 2 and 8. What do you see in each?".*

As practice for making connections among concepts (practice 2 in Table 24), the teacher connected linear equation and other concepts. More specifically, the teacher connected explicit relationship and linear equation. That is, she explained the equation $y = 10 - x$ as the relationship in the situation after the students found the relationship between the x and y values in the situation. In addition, as practice for connecting topics in math (in practice 4), the teacher connected patterns and linear equations for the linear equation mentioned above.

As practice for connecting slope and other concepts (in practice 2 in Table 24), the teacher connected slope, unit rate (price), functional situation/equation and connected slope, rate of change, and functional situation/equation. More specifically, after the students finished the task in the Stores activity, the teacher posed questions to connect slope, unit rate, and functional situation/equation. For example, while talking about the Mobile Operators situation, she asked, *"... How much do you pay per unit time for the mobile operator T-cell? What does unit time mean"*. As another example, while talking about taxi fares, she asked, *"What is the unit price for this taxi?".* Then, she gave other examples on the price of tomatoes per kilogram, price of detergent per kilogram. In addition, she continued to connect slope, rate of change, and functional situation/equation. For example, she mentioned the situation of the taxi fare again, and said,

> In that situation ($y = 2x + 2.5$), slope describes how much the price increases considering a one-kilometer increase in distance. That is, when I go 1 kilometer, how much does the price increase? We look at the rate of increase between them. Don't we? When we say unit, we look at increments one by one. Even if we do not need to compute the ratio. You know vertical over horizontal. We do not even do this computation if we write the rate of change because one of them [vertical, denominator] is 1. It increases by one. Remember, we say if I go one more kilometer, I will pay 2 liras.

Then she continued to connect slope and rate of change with functional situation or equations for speed as kilometers per hour, density as mass per volume, and fuel consumption rate as liters per kilometer.

At the time the aforementioned practice took place, the teacher also connected topics in math and connected topics in math and science as practice for connecting topics (practice 4 in Table 24). That is, in terms of connecting topics in math, she connected the topics of functions, analytics geometry, slope, linear equations, and graphs. In addition, in terms of connecting topics in science and math, she connected the concepts of density, speed, and slope.

4.3.2 Trimming Practices in Explaining Mathematical Ideas without using GGB materials and Concrete objects

In this section, the teacher's trimming practices are reported within each phase during which she explained the mathematical ideas without using GGB materials and concrete objects in classroom teaching. As previously mentioned, from the perspective of McCrory et al.'s (2012) KAT framework, trimming practices in teaching algebra in eighth-grade mathematics are described as mathematical practices of retaining the integrity of slope, linear equations, and graphs and eliminating the complexity of slope, linear equations, and graphs accurately both in school and advanced mathematics. Therefore, the practices were categorized by identifying the teacher's actions in classroom teaching during which the teacher provides the students with mathematical practices to refine those mathematical ideas within the accuracy of the mathematical context while explaining mathematical ideas without using GGB materials and concrete objects.

In this section, the teacher's trimming practices are reported within each phase during which the teacher explained mathematical ideas without using GGB materials and concrete objects in classroom teaching. Thus, the themes of trimming practices that were grouped by the researcher and the phases that involved those practices are as follows:

Table 25. Summary of trimming practices while explaining mathematical ideas without using GGB materials and concrete objects

Practices	Number of phase
Practice 1: Explaining a mathematical value	1, 2, 3
Practice 2: Explaining the solution method/procedure	1, 2, 3
Practice 3: Determining the relevance of a concept	1, 2
Practice 4: Explaining mathematical relations	1, 2, 3
Practice 5: Explaining the context of an activity	1, 2, 3
Practice 6: Using appropriate mathematical terms	1, 2, 3
Practice 7: Revealing students' mathematical difficulties	3

The emerging themes of trimming practices and the actions under each theme are presented for each phase in Table 26 by the researcher. Thus, there were seven themes of trimming practices and five of them emerged in all the phases. The flow of actions in the practices are explained under the classroom activities for each phase. In addition, when a given trimming practice is related with a bridging practice explained in the previous section, those relations have also been included in the flow of the actions.

Table 26. Trimming Practices while explaining mathematical ideas without using GGB materials and concrete objects

Practice	Action	Sample description
Practice 1: Explain mathematical value		
Explain decimal as a mathematical value of slope	Explain converting ratio into decimal by means of division	Provide explanations for converting decimals into slope computation by using students' errors/difficulties (1P)
Explain the sign of the slope of a line	Explain slope of line in functional situations	Explain slope of line in functional situations by using negative and positive terms (2B)
Explain slope value within different symbolic representations	Explain equivalence of slope value within different symbolic representations of algebraic expressions	Explain equivalence of slope value within different symbolic representations of algebraic expressions as (2S)
Explain solution set value of system of equations	Explain solution set value of system of equations having one solution	Explain the solution set value of a system of equations with a point as ordered pair (3E)
Explain slope values of lines/equations in a system of equations	Question to compare slopes of parallel lines	Question to compare slope values of parallel lines in a system of equations (3E)
Practice 2: Explain solution method/procedure		
Explain computation of slope	Explain the computation of slope as geometric ratio for physical situations	Explain the computation of slope as geometric ratio for physical situations (1F)

Table 26 (continued)

	Explain computation of slope of a line graph as algebraic ratio	Explain computation of slope of a line graph on different intervals using two different points for the functional situation (2B) (2H)
		Explain/Encourage students to compute the slope of a line graph as the ratio of vertical change to horizontal change (3E) (3S)
	Explain computation of the slope of a line graph as direction of change for functional situations	Explain computation of slope of a line graph as direction of change in the vertical variable with change in the horizontal variable for functional situations (2B)
Explain solutions of equations/graphs of equations for linear relations	Explain solutions of equations to calculate the value of the y-variable when a value of x-variable is given	Explain how to calculate the value of a y-variable when a value of x-variable is given for a linear equation (2B)
	Explain points of graphs as solutions of corresponding equations to read the value of the y-variable when a value of x-variable is given	Explain how to read the value of y-variable when a value of x-variable is given for a line graph (2B) (2H)
	Explain points of graphs as solutions of corresponding equations to read the value of an x-variable when a value of y-variable is given	Explain points of graphs as solutions of corresponding equations to read the value of an x-variable when a value of y-variable is given (2H)
Explain converting forms of linear equations	Explain how to convert the standard form to the slope-intercept form of linear equation algebraically	Explain/question how to convert the standard form of an equation to the slope-intercept form of an equation algebraically on the board (2S)
		Explain the division of an algebraic expression by a number in converting forms of equations in converting forms of equations (2S)
	Explain the slope of a linear equation by converting forms of linear equations	Explain slope of a linear equation in slope-intercept form (2S) (3E)
Explain solutions of a system of equations	Explain the solutions of a system of equations algebraically	Make calculation to show the coordinates of an intersection point as a solution for a system of equations by satisfying the equations (3M)
		Explain algebraic ways of solving the solution of a system of equations (3M)
		Explain/Encourage solving a system of equations by means of algebraic substitution (3M) (3E)
		Encourage students to solve a system of equations algebraically (3E)

Table 26 (continued)

Practice 3: Determine the relevance of a concept		
Determine the relevance of a concept in different mathematical situations	Determine the relevance of slope in different mathematical situations than physical situations	Determine the relevance of slope for geometrical situations by providing examples (1F)
	Constrain the relevance of the unit of slope in different functional situations (opposite example)	Overlimit the slope of graphs for linear functional situations as having no unit (2B)
Practice 4: Explain mathematical relations		
Localize factors of slope	Explain the slope-vertical length relation	Explain the slope-vertical length relation in the physical situation of fire truck ladder (1F)
	Explain the slope-vertical length-horizontal length relation	Explain the slope-vertical length-horizontal length relation in physical situations (1F) (1R)
Localize relationship for the linear equation	Localize recursive relationship for the linear equation in the table	Localize recursive relationship for the linear equation in the table/drawings (2B)
	Localize the explicit relationship between the coefficient of x and the y-values for the equation	Localize verbally the explicit relationship between the coefficient of x and y-values for the linear equation (2B) (2H) (3M) (3S)
	Localize symbolic representation of the explicit relationship (linear equation)	Localize symbolic representation of the explicit relationship as the linear equation (2B) (3M) (3S)
Localize slope of line in the linear equation	Localize slope of line as the coefficient of x-variable of the linear equation in the slope-intercept form	Localize slope of line as the coefficient of x-variable in the linear equation in the slope-intercept form of the equation (2S) (3E)
		Localize slope values of equations as coefficients of the x-variable in a system of equations with one solution (3S)
	Explain slope-linear equation relation by excluding the constant value for functional situation	Localize slope-equation relation by excluding the constant value in the equation for a functional situation (3S)
	Compare slopes of lines for functional situations	Compare slopes of lines for functional situations (3S)
Provide examples of equations and slopes for a slope-equation relation	Provide distinguishable examples of equations/slopes in random sequence for a slope-equation relation	Provide slopes and equations (2S)
Provide examples of equations for relating forms of equations	Provide distinguishable examples of equations in random sequence for converting equations	Provide equations to show how to convert standard forms of equations to slope-intercept forms of equations algebraically on the board (2S)
Provide examples of systems of equations for relating solution sets and positions of lines	Provide distinguishable examples of systems of equations in random sequence for relating a solution set and positions of lines	Provide examples of systems of equations in random sequence for relating a solution set algebraically and positions of lines (3E)

Table (26 continued)

Practice 5: Explain activity context		
Explain activity procedure by concerning the construction of mathematical concepts or processes	Explain activity procedure regarding slope construction	Explain activity procedure by considering slope construction by posing questions, or giving real life examples (1P) (1R)
	Explain activity procedure by means of filling in the table	Explain activity procedure by means of filling in the table for linear relation (2B) (2H) (3M)
		Explain activity procedure by means of filling in the table with equations and slope values for slope-equation relations (2S)
		Explain activity procedure by means of filling in the table with system of equations, slopes of equations, solution set of equations, positions of lines of equations (3E)
	Explain activity procedure for positions of lines	Explain activity procedure for determining positions of lines in a system of equations (3E)
Explain context of activity for the graphing	Explain context of activity for labels of axes in the graphing	Explain context of activity for labels of axes in the graphing of the situation (2B) (2H) (3M) (3S)
Explain how to construct the graph	Explain how to construct the graph of a linear equation	Explain how to construct the graph of a linear equation by determining the ordered pairs of points (2B) (3M)
	Trace students to construct the points of the graph of a linear equation	Trace students to construct the graph of a linear equation by determining the ordered pairs (2B) (3M)
	Trace students to draw the graph	Trace students to draw the graph by connecting points (2B) (3M)
Explain how to construct the equation	Explain how to construct the linear equation	Explain how to construct the linear equation by writing an equation, which is different from writing an algebraic expression (2H) (3M)
Practice 6: Use appropriate mathematical terms		
Use appropriate mathematical terms	Use appropriate terms for the slope of an object	Use appropriate mathematical terms for factors that affect the slope of an object (1R)
	Use appropriate terms for points of a line graph	Use appropriate terms for coordinates of points of a line graph for an equation (2S)
	Use appropriate terms for coincident lines	Use appropriate terms for coincident lines for a system of equations (3E)
Use inappropriate mathematical terms (opposite practice)	Use inappropriate mathematical terms in line graphs	Use infinite term to express the line is unlimited (2B)
Use inappropriate mathematical representations for graphs (opposite practice)	Use an inappropriate graphical representation for a functional situation	Draw a continuous graph (i.e. ray) for a discrete situation (2B)

Table 26 (continued)

Practice 7: Reveal students' mathematical difficulties		
Reveal students' difficulties	Reveal students' difficulties in solving a system of equations	Reveal students' difficulties in solving a system of equations having one solution graphically (3E)
	Reveal students' difficulties in computing the slope of physical objects	Reveal students' difficulties in computing the slope of physical objects as the length of the physical object (3S)

Note: The abbreviations in the sample description column of the table indicate the activity in which an action emerged. Phase 1 activities; 1P: Positions of Battens activity, 1F: Fire Truck activity, 1T: Tent activity. Phase 2 activities; 2B: Building Design activity, 2H: Leaking Container activity, 2S: Slopes and Equations of Lines activity, Phase 3 activities: 3M: Mobile Operators activity, 3E: Equations Systems activity, 3S: Stores activity.

4.3.2.1 Trimming practices in teaching the slope in physical situations for Phase 1

In Phase 1, the teacher's trimming practices were combined under the practices of explaining a mathematical value (practice 1), explaining the solution method/procedure (practice 2), determining relevance of a concept (practice 3), explaining mathematical relations (practice 4), explaining the context of an activity (practice 5), and using appropriate mathematical terms (practice 6) (see Table 26).

In the *Positions of Battens* activity, the teacher's trimming practices were combined under the themes of explaining a mathematical value (practice 1) and explaining the context of an activity context (practice 5) (see in Table 26).

As practice for explaining the context of an activity (practice 5 in Table 26), the teacher explained the activity procedure considering the construction of a slope. More specifically, while the students started to work on the activity sheet, the teacher wanted them to fill in the horizontal length and vertical length cells for a batten but did not want them to make the computation of the slope. After each group wrote their batten's horizontal and vertical lengths, all the groups shared their horizontal length and vertical length values for further comparisons.

As practice for explaining mathematical value (practice 1 in Table 26), the teacher explained the decimal as a mathematical value of a slope. More specifically, the teacher made explanations about converting decimals in the computation of a slope based on students' errors/difficulties. As evidence, when some of the students had difficulties in computing the slope as a decimal using prime numbers, she said:

> When you have a decimal, you tend to think that you cannot do it. We all use decimals in real life everywhere. I mean don't be afraid of these numbers. We can make divisions with prime numbers and we can write the slope down as a decimal [value] …

In the *Fire Truck* activity, the teacher's trimming practices were combined under the themes of explaining the solution method/procedure (practice 2), determining the relevance of a concept (practice 3), and explaining mathematical relations (practice 4) (seen Table 26).

As practice for explaining the solution method/procedure (practice 2 in Table 26), the teacher explained slope computation as a geometric ratio for the physical situation. More specifically, the teacher explained the computation of a slope as vertical over horizontal length. For example, after the students created the situation in which there was a ladder with a horizontal length of 3 units and a vertical length of 5 units and computed the slope in their activity sheets, the teacher explained this computation to the class again and said, *"3 over 5. We divided 3 by 5. We can do it. Write the slope [of the ladder] for 3rd floor"*. To give another example, for another positon of the ladder, the teacher asked, *"For this situation, how do you construct the ratio?"*

As practice for explaining mathematical relations (practice 4), the teacher localized the factors of slope in physical situations. More specifically, the teacher explained the slope-vertical length relation in the physical situation of the fire truck ladder in different positions and explained the slope-vertical length-horizontal length relation in the physical situation of batten. As evidence, while completing the Fire Truck activity, she said,

> Therefore, we cannot always say that the longer the vertical [distance], the more [steeper] the slope. We should look at the horizontal [length]. But, if the horizontal is constant [for various slope situations], it is easy to say. If we know both of them (horizontal and vertical lengths), how will the slope change as the vertical increases and the horizontal decreases?

As practice for determining the relevance of a concept in different mathematical situations (in practice 3 in Table 26), the teacher determined the relevance of a slope for geometrical situations with providing examples. That is, she guided the students to approve the slope in different mathematical situations by giving examples of triangles and lines. For example, after a student's comment about slopes in triangles, she said that triangles and hypotenuses could have slopes. To give another example, she said that a line as a geometrical figure could have a slope. While providing explanations for this trimming practice, the teacher also connected slope and other concepts (bridging practice 2: connecting concepts in Table 26) in the background.

In the *Tent* activity, the teacher's trimming practices are combined under the themes of explaining mathematical relations (practice 4), explaining the context of an activity (practice 5), and using appropriate mathematical terms (practice 6) (see Table 26).

As practice for explaining the context of an activity (practice 5), the teacher explained the activity procedure considering the construction of a slope. More specifically, while the students were beginning to work on the activity sheet, the teacher wanted them to understand the context of the activity, which involved a constant tent and a rope with a changing length, a changing horizontal length and a constant vertical length using real life experiences. While providing explanations for this trimming practice, the teacher also connected slope and a real life representation, namely the rope of a tent situation (bridging practice 1: connecting concept and representations in Table 24).

As practice for using appropriate mathematical terms for the slope of an object (in practice 6), the teacher used appropriate mathematical terms for the factors that affected the slope of the rope. More specifically, after the students drew conclusions that as the distance from the tent increases, the slope increases for the positions of the rope and as the vertical length increases, the slope increases for the positions of the ladder, the teacher said, *"Yes. We saw how horizontal and vertical lengths affect the slope of an object".*

As practice for explaining mathematical relations (practice 4), the teacher localized the factors of the slope of an object. More specifically, she made explanations about the slope-vertical length-horizontal length relation in physical situations. As evidence, she said, *"Therefore, we cannot always say that the longer the horizontal [distance] is, the less slope. We should look at the vertical [length]. But, if one of them is constant, it is easy to talk about the others."* While explaining in this trimming practice, the teacher connected the Fire Truck, Tent, and Positions of Battens activities for computing the slope of the object (bridging practice 3: connecting activities in Table 24) concomitantly.

4.3.2.2 Trimming practices in teaching the slope of a line for Phase 2

In Phase 2, there were activities on the slope of a line in functional situations and in the coordinate system, which were the *Building Design* activity, entailing the linear relation based on a line graph of the number of windows in a building and the number of floors of a building, the *Slopes and Equations of Lines* activity, based on relating linear equations, lines, and slopes, and *Leaking Container* activity, based on a line graph representing the linear relation between the amount of water in a container and time (minutes). In Phase 2, the teacher's trimming practices were combined under the practices of explaining a mathematical value (practice 1), explaining the solution method/procedure (practice 2), determining the relevance of a concept (practice 3), explaining mathematical relations (practice 4), and explaining the context of an activity (practice 5) (see Table 26).

In the *Building Design* activity, the teacher's trimming practices were combined under the themes of mathematical value (practice 1), explaining the solution method/procedure (practice 2), explaining mathematical relations (practice 4), and explaining the context of the activity (practice 5) (see Table 26). In this activity, there were also given the examples of non-trimming practices named as opposite examples. These examples for the trimming practices displayed the teacher's actions that were in opposition to the given trimming practice. These opposite examples were combined under the trimming practices for determining the relevance of a concept (practice 3) and using appropriate mathematical terms (practice 6) to show the opposition (see Table 26).

As practice for explaining the context of an activity (practice 5), the teacher explained the activity procedure, considering the construction of the mathematical concept or process. More specifically, the teacher explained the activity procedure, which required filling in the table for the linear relation in the Building Design situation. While explaining this trimming practice, the teacher connected the topics in 7^{th} grade mathematics (linear relation) and 8^{th} grade mathematics (slope, linear equation, and graph) in the background. Since the teacher prepared this activity to connect 7th grade and 8th grade algebra topics, she also mentioned this to the students and referred to the mathematical ideas that the students learned in the previous year.

As practice for explaining mathematical relations (practice 4), the teacher localized the relationship for the linear equation for the *Building Design* situation. More specifically, the teacher localized the recursive relationship for the linear equation in the table/drawings. Then, she localized the explicit relationship between the coefficient of x and y-values (the number of windows) for the linear equation verbally. In addition, she localized the symbolic representation of the explicit relationship as the linear equation. As evidence, for example, while the students were working on the table to find the relationship among the variables in the situation in their activity sheets, she guided the students by making such explanations as *"While thinking about this [relation], think about the number of windows for the building with x floors. Look at it step by step. The relationship from one [step] to another [step]."*. She then approved students' responses, saying "*Yes it increases by three. We multiply it [x] by 3. But is it enough?"* While providing explanations for this trimming practice about recursive relationship, the teacher also connected linear equation, recursive relationship, and rate of change (Bridging practice 2: connecting concepts in Table 24). As another evidence for explicit relationship, while the students working on writing the relationship between the number of windows and the number of floors in the Building Design situation using the table, the following dialogue emerged during the practice:

(S) It is 3 times x plus 2 *(A group of students make comments on the equation).*
(S) It increases by three.
(T) Yes. It increases by three. But you cannot reach the result [y-values] when there is 3 times [of the x] because there is a need for a constant [the number of windows in the basement floor].
(S) Plus 2.
(T) Yes, plus 2. So you have an equation. What does y equal to? Then, continue to the graph.
(The teacher goes to the desk of another group of students.)
(S) It increases by three.
(T) Yes. If there is three times the number of floors. That is, three times x. What is the result? For example, what would be the result for the first floor?
(S) 3
(T) But there are actually 5 [windows].

(S) *(There are murmurs)*
(T) It is 3 times x plus what [do we need more]?
(S) Plus 2.

Furthermore, while providing explanations for the trimming practice about the symbolic representations of explicit relationships, the teacher connected explicit relationship and linear equation (bridging practice 2: connecting concepts in Table 24) concomitantly.

As another practice for explaining the context of an activity (practice 5), the teacher explained how to construct the graph for the *Building Design* situation. More specifically, the teacher explained to the students how to construct the graph of a linear equation by determining the ordered pairs of points. While providing explanations for this trimming practice, the teacher connected linear relationship, table and graph representations (bridging practice 1: connecting concept and representations in Table 24). In the meantime, as another trimming practice, the teacher traced the students' graphing while determining the ordered pairs in constructing the points of the graph for the linear equation and traced the students' graphing while connecting the set of points in drawing the graph.

As an opposite example for the trimming practice of using appropriate mathematical terms (practice 6), the teacher gave inappropriate terms for the line graph. That is, she described the line graph as being infinite in order to express the meaning of unlimited. For example, she said, *"These lines are infinite. How do we determine the vertical [distance] of this line?"* In other words, she used the infinite term to express a line having no limit and end points rather than express the infinite set of points on a line. In addition, as another opposite example for the trimming practice for using appropriate mathematical terms (practice 6), the teacher gave an inappropriate graphical representation for a functional situation. That is, she draw a continuous graph (i.e. drew a ray) for a discrete situation. On the other hand, she used this graphing to express the slope of a line graph and she sometimes made explanations, such as thinking of this graph as a line graph. Therefore, it was seen that the teacher used this representation because she believed that this method made it easy to understand the slope of a line graph in a functional situation.

As a practice for explaining the solution method/procedure (practice 2), the teacher explained solutions of equations/graphs of equations for linear relation. More specifically, the teacher explained the solutions of equations to calculate the value of the y-variable when a value of x variable was given for the linear equation of *Building Design* situation. That is, the teacher explained to the students how to calculate the value of the y-variable when a value of x-variable is given for a linear equation. As evidence, while the students were working on finding the number of windows for a 5-floor building by using the equation for the situation, the teacher said, *"5 replaces x in the equation. What do you do?"* and posed the following question: *"How do you find the number of windows [for a 5-floor building] in the equation?"*. Then, some students said that since x was the number of floors in the equation, they substituted x with 5, and some said they multiplied 3 by 5 and then added 2, the teacher appreciated their responses and guided them to solve the equation.

As another practice of explained solutions of equations/graphs of equations for linear relations (in practice 2), the teacher explained points of graphs as solutions of corresponding equations to read the value of y-variable when a value of x-variable is given. That is, the teacher explained students to read the value of y-variable when a value of x-variable is given for the line graph of the *Building Design* situation. As evidence, after the students showed the axes as the number of windows and the number of floors in the coordinate system, the teacher said, *"Can you find the number of windows for a 5-floor building? If you go there (The teacher shows 5 on the x-axes) and [then] go [to the line] (The teacher moves her finger from 5 on the x-axes to point (5, 17) on the line)"*. Then, she posed the following question: *"How do you find [the number of windows] for a 2-floor building [on the line]?"*. As another evidence, the following dialogue emerged during the practice:

(T) Assume that you do not know the equation. You just have the graph where this is [x-axes] number of floors and that is [y-axes] number of windows. Does anyone see the number of windows for a building? For example, does anyone say that there are 2 windows for a 0-floor building? *(The teacher shows the point (0, 2) on the coordinate system).*

(S) Yes.

(T) Okay. Does she know the number of windows for a 2-floor building?

(S) Eight.

(T) Yes. There are 8 floors. Okay. Tell me what the coordinates of this point are? *(The teacher points to (1, 5) with her finger on the graph).*
(SS) 5
(S) 1
(S) 1 to 5.
(T) Okay. If this is 1 to 5 [point (1, 5)], could we see that there are 5 windows for a 1-floor building?
(S) Yes
(T) So, the context does not have to be given verbally or in a table. We can understand it on the graph by reading the graph.

As practice for explaining the solution method/procedure (practice 2), the teacher explained the slope computation of a line graph as algebraic ratio. More specifically, the teacher explained the slope of a line graph on different intervals using two different points for the functional situation. That is, after the students stated that the slope could be computed with two points and the teacher supported this idea by showing any two points could be used to compute the slope of a line, she said:

> Okay. Then, your groups identify two points on the line. If you determine the points that intersect with the axes, it will be easy to compute. Each of your groups used different pairs of points, but one group of students used the same pair of points.

Thus, by making similar explanations to the aforementioned excerpt, the teacher guided each group of students to determine and use any two points on a line for computing the slope of the line for the *Building Design* situation. While doing this, the teacher did not guide the groups to use the same pair of points to gain further understanding of slope as constant for a line.

As an opposite example for the trimming practice of determining the relevance of a concept (practice 3), the teacher did not provide the relevance of the unit of a slope in different functional situations. More specifically, she overlimited the slope of graphs for linear functional situations as having no unit. As evidence, after a student asked whether the slope had a unit, she said *"Think about it. What is the unit of the numerator? Unit [number of windows].What is the unit of the denominator? Unit [number of floors]. Cancel the units. So is there a unit for the slope [of the line]?"*. Then, she continued by saying, *"Why? Because both of them (numerator and denominator) are the same quantity. The same measure. That's why there is no unit [of slope]"*. Although this comment, that the slope has no unit, can be accepted for the Building Design situation when considered the rate concept in Turkish curriculum, it is not acceptable for all linear functional situations. In addition, her explanation entailed the reason why there was no unit in this situation, and the students could follow such a process to determine whether a slope of a line graph for a functional situation had a unit (of measure) and if so, what this unit was. Therefore, when the teacher made this explanation by removing the slope in functional situations as speed (distance-time relation), density (mass-volume relation) or consumption rate (liter-time relation), which had units (e.g. km/h), the teacher did not secure the mathematical accuracy of the slope of a line for a linear functional situation.

As a practice for explaining mathematical value (practice 1), the teacher explained the sign of the slope of a line in the functional situations. More specifically, she explained the slope of a line in functional situations with positive and negative terms. That is, after the students computed the slope of the line graph for the Building Design situation, she emphasized by saying, *"We find that the slope is 3. But there is one more important thing. Yes, the slope is 3. But, we see that the slope [of a line graph] can be positive or negative in such [functional] situations. Have you ever seen that?"*, and asked that *"Have you ever computed a negative slope?"*.

As another practice for slope computation (in practice 2), the teacher explained the slope of a line graph as the direction of change for functional situations. More specifically, she explained the slope computation of a line graph with the direction of change in the vertical variable and the direction of change in the horizontal variable for functional situations. That is, she explained the slope of the line for the Building Design situation by computing the slope using two points by considering the change in the y-values when both x-values increase (positive direction) and x-values decrease (negative direction). Therefore, for each computation, the sign of the slope was positive with the ratios of positive to positive and negative to negative. While providing explanations for this trimming practice, the teacher connected the sign of a slope, the direction of a line, and the ratio of vertical over horizontal (bridging practice 2: connecting slope and other concepts in Table 24) concomitantly.

In *Leaking Container* activity, the teacher's trimming practices were combined under the practices of explaining the solution method/procedure (practice 2), explaining the mathematical relations (practice 4) and explaining the context of the activity (practice 5) (see Table 26).

As practice for explaining the context of an activity (practice 5), the teacher explained the procedure of the activity regarding filling in a table, explained the context of the activity for graphing, and explained how to construct the equation. More specifically, the teacher explained the activity procedure regarding filling in the table for the linear relation between the amount of water in the container and time relation. As evidence, she asked questions successively and got correct responses from the students as can be seen in the following dialogue:

(T) I want you to read the question and fill in the table. Please do it with your group. Let's start with the 0^{th} minute. How much water exists in the 0^{th} minute?

(S) 10

(T) Then, how much water remains at 1st minute?

(SS) 9

(T) Then, I want you to work by using this table. Therefore, you should fill in the table first.

As practice for explaining mathematical relations (practice 4), the teacher localized the relationship for the linear equation. More specifically, the teacher verbally localized the explicit relationship between the coefficient of x and y-values for the linear equation. As evidence, after the teacher explained the activity procedure regarding filling in the table for the linear relation between the amount of water in the container and time, the following dialogue emerged during the practice while they were also working on the table and the drawings of the situation:

(T) Okay, you filled in the table. Then try to calculate it. What happens? What did you do to find y for any xth minute?

(S) Is It 10 minus x?

(S) Aha! We should be subtracted from the [amount of] container.

(S) Is it minus 1?

(S) We should subtract time from the [amount of] container.

(T) What is the amount of water in the container in the beginning?

(S) 10.

(T) Yes 10 liters.

(S) 10 minus x.

(T) What equals to 10 minus x?

(S) *y*.

(T) And, what is *y*?

(S) Amount of water [in the container].

Thus, the teacher guided the students in writing the linear equation by using the explicit relationship as a connection between the values of minute to the amount of water at that minute for the container. While providing explanations for this trimming practice, the teacher also connected linear relationship, verbal, table, picture, and symbolic representations (bridging practice 1: connecting concept and multiple representations in Table 24).

As another trimming practice for explaining the context of the activity (practice 5), the teacher explained the context of the activity for labels of axes in the graphing of the functional situation. Furthermore, as practice 5, the teacher explained how to construct the linear equation by writing an equation, which is different from writing an algebraic expression.

As practice for explaining the solution method/procedure (practice 2), the teacher explained solutions of equations/graphs of equations for linear relations. More specifically, the teacher explained to the students how to read the value of y-variable when an x-variable value was given for a line graph for the container situation. Since the teacher wanted the students to read the graph in their activity sheet, she wanted them to find the amount of water at the fourth (4^{th}) minute in their graphs. Then she asked, *"How do you understand directly the amount of water at 4^{th} minute while using the graph?"*. She also asked similar questions to the other groups. While making explanations for this trimming practice, the teacher connected linear relationship and line graph (bridging practice 1: connecting concept and representations) concomitantly. In addition, as practice for explaining solutions of equations/graphs of equations for linear equations, the teacher made explanations to the students about points of graphs as solutions of corresponding equations in order to read the value of the x-variable when the value of y-variable is given.

As another practice for explaining the solution method/procedure (practice 2), the teacher explained the computation of a slope of a line graph. More specifically, the teacher explained the computation of the slope of a line graph on different intervals using two different points for the functional situation. As evidence, for example, while the students were working on the slope of the line graph for the amount of water in the container-time relations, the following dialogue with the groups of students emerged during the practice.

(T) Yes. The next question [is about slope]. I am asking what we need to find the slope. What did we do in the Building Design activity?
(S) Horizontal and vertical
(T) Bu we do not see them.
(S) Two points
(T) Which? Let's determine two points here [line graph]. Please determine the point with your friends in your group. Talk to each other.
(SS) It is 3. We can take this *(The students talk to each other in their own group).*
(T) Have you determined your points? *(The teacher talks to another group of students that are different from the students above)*
(SS) Yes.
(T) Okay. Let's find the slope. Where are your points? To compute the slope between these points, find the horizontal and vertical [changes].

In the *Slopes and Equations of Lines* activity, the teacher's trimming practices were combined under the practices of explaining mathematical value (practice 1), explaining the solution method/procedure (practice 2), explaining mathematical relations (practice 4), explaining the context of an activity (practice 5), and using appropriate mathematical terms (practice 6) (see Table 26).

As practice for explaining the context of an activity (practice 5), the teacher explained the activity procedure regarding filling in the table. More specifically, the teacher explained to students the activity procedure about filling in the table with equations and slope values for the slope-equation relations. For example, she said, *"Now, fill in in your tables with the equations and slopes in the Building Design and the Leaking Container activities."* While explaining this trimming practice, the teacher connected activities in phase 2 (bridging practice 3: connecting activities in Table 24).

As practice for explaining mathematical relations (practice 4), the teacher localized the slope of a linear equation. More specifically, the teacher localized the slope of a line as the coefficient of x-variable in the linear equation in the slope-intercept form. For example, after the students wrote the equations and slope values of the Building Design and the Leaking Container activities in the table in their activity sheets and one of the students asked whether the slope was related with the signs of the equations, the following dialogue emerged during the practice:

(T) It could be. Look! Your friend made a comment. It's a good start.

(S) In the equation of y equals to three x plus two $y = 3x + 2$, the slope is 3 and positive.
(T) Yes. Can you repeat?
(S) In the equation of y equals to three x plus two $y = 3x + 2$, the slope is 3 and positive. In the equation of y equals to ten minus x $y = 10 - x$, the slope is -1 and negative.
(T) So, you think that it could be related with this negative sign.

It was seen that the teacher had not explicitly given the answer or the relation, but she guided the students by posing questions, appreciating their responses and revoicing their comments because she wanted the students to examine various equations and then to draw a conclusion.

As practice for explaining mathematical relations (practice 4), the teacher provided the students with examples of equations and slopes for a slope-equation relation. More specifically, the teacher provided the students with distinguishable examples of equations/slopes in random sequence for such slope-equation relations as $y = 8x, m = 1/2,\ y = 6x - 4, y = \frac{x}{2} + 2, y = \frac{x}{4} - 9$.

It was seen that these examples of equations and slopes were distinguishable as they involved a different slope value or a constant value in the equation. For example, she asked, *"Can you establish an equation that has a slope of 1 over 2?"*. As another example, she asked, *"Can you say the equation for the slope is y equals to 6x minus 4?"*. In the meantime, as practice for explaining mathematical value (practice1), the teacher explained the slope value within different symbolic representations. More specifically, the teacher explained the equivalence of a slope value within different symbolic representations of an algebraic expression such as $\frac{1}{2}x = \frac{x}{2}$ and $\frac{x}{4} = x/4$. As evidence, after the students interpreted the relationship between the slope and an equation, the teacher asked the students various equations to find their slopes. For example, after the students expressed the equation that had a slope of 1 over 2 as y equals to 1 over 2 x minus 5 ($y = \frac{1}{2}x + 5$), she asked, *"It can also be written differently. What can it be?"* Upon receiving a reply, she appreciated the student's response by saying, *"Yes. Y equals to x over 2 ($y = \frac{x}{2} + 5$). I mean, don't be confused when you see that."*

As practice for using appropriate mathematical terms (practice 6), the teacher used appropriate terms for points of a line graph. More specifically, the teacher used appropriate terms for coordinates of points of a line graph for an equation. As evidence, while the students explained the constant value in the equation with an inappropriate term as the coordinate of y, the teacher said, *"But which y? You mean the point on the line that intercepts with the y-axis. That is, the y-intersect of the line. Kids, listen! There are various x and y values on a line. Aren't there?"* She continued by saying:

> There are various points. And these points have x and y values. The equation is a general formula of these points. For example, in the equation $y = x + 1$, it means that if you add 1 to all x values in the line, you will get all the y values of the line. In addition, there are points that intersect with the axes. The y-intercept value is the constant value in the equation. You all say something. You know this but you do not express it in this way. You should express it correctly. Some of you said 'value of y'. However, there are various y values [on the line]. Which y? ... The y value [of the line] that intersects with the y-axis.

As practice for explaining the solution method/procedure (practice 2), the teacher explained how to converting forms of linear equations. That is, the teacher explained how to convert a standard form to a slope intercept form of linear equation algebraically. More specifically, the teacher explained or questioned how to convert a standard form of an equation to a slope-intercept form of an equation or vice versa algebraically on the board. In addition, she explained the division of an algebraic expression by a number while converting forms of equations. Furthermore, she explained the slope of a linear equation by converting forms of linear equations. As evidence, for example, the teacher converted a slope-intercept form of the equation $y = x + 3$ to a standard form of the equation $-x + y = 3$ as follows:

(T) How do we do it? We eliminate this [x]. How do we eliminate [x]?
(S) Minus x
(T) Adding minus x to both sides of the equation ($-x + y = -x + x + 3$). Here minus x plus y, here these ($-x + x + 3$) eliminate each other and what remains is 3 ($-x + y = 3$).

Subsequently, as practice for explaining mathematical relations (practice 4), the teacher provided the students with examples of system of equations for relating forms of equations. More specifically, the teacher provided distinguishable examples of equations in a random sequence for converting equations. That is, she showed equations ($2y + x = 6$ and $y = \frac{-x}{2} + 3$) to show how to convert a standard form of equation to a slope-intercept form of equation algebraically on the board.

4.3.2.3 Trimming practices in teaching the solution of a system of equations and the slope relation for Phase 3

In Phase 3, there were activities on the system of equations in functional situations and in the coordinate system. They were *Mobile Operators,* based on the solution of two linear equations for price (TL)-time (minutes) linear relations of two mobile operators, the *Equations Systems,* based on relating linear equations in a system, slopes, solution set, position of lines, and drawing lines in a system, and the *Stores* activity, entailing the solution of two linear equations based on the amount of product-time (day) linear relations of two Stores and the relation between slopes, solution, and graphs. In addition, at the end of this phase the teacher reconsidered the slope of a line graph in functional situations. In Phase 3, the teacher's trimming practices were combined under the practices of explaining mathematical value (practice 1), explaining the solution method/procedure (practice 2), explaining mathematical relations (practice 4), explaining the context of an activity (practice 5), using appropriate mathematical terms (practice 6), revealing students' mathematical difficulties (practice 7) (see Table 26).

In the *Mobile Operators* activity, the teacher's trimming practices were combined under the practices of explaining the solution method/procedure (practice 2), explaining mathematical relations (practice 4), and explaining the context of the activity (practice 5) (see Table 26).

As practice for explaining the context of the activity (practice 5), the teacher explained the activity procedure regarding filling in the table for the linear relations in the T-cell and Z-cell mobile operators situations, similar to the Building Design and the Leaking Container activities. Therefore, the students read the verbal expression of the situations and filled in the tables with x–values for the time (minute) and y-values for the price (TL) for each mobile operator. As another practice for explaining context of the activity (practice 5), the teacher explained how to construct the equation. More specifically, she explained how to construct the linear equation by writing an equation, which is different from writing an expression. That is, after the students used the expressions for the situations ($x + 5$ and $2x$), she emphasized that they were not equations that showed the relation and corrected them as $y = x + 5$ and $y = 2x$.

As practice for explaining mathematical relations (practice 4), the teacher localized the relationship for the linear equation. More specifically, the teacher verbally localized the explicit relationship between the coefficient of x and the y-values (i.e. amount of price (TL) for the linear equation and localized the symbolic representation of the explicit relationship as the linear equation. As evidence, the following dialogue emerged during the practice:

(T) In the T-cell situation, what did you do to find the y values by using the time values (x- values)? Add two or multiply by 2?
(S) Multiply by 2.
(T) Okay. So you will multiply x by 2 [to find x].

As another evidence, the following dialogue emerged during the practice:

(T) [In the T-cell situation], do we pay for the initial subscription?
(S) No.
(T) There is no subscription fee. But how much do we pay for each minute?
(S) 2
(S) 2 by 2.
(T) Yes. In this case, I pay twice as much as the length of time I talk. Therefore, what is the equation?
(S) Two x.
(SS) Y equals to two x.

As another practice for explaining the context of the activity (practice 5), the teacher explained the activity context for the graphing. More specifically, she explained the activity context for the labels of the axes in the graphing of the situation. That is, she mentioned that the x-axis showed the time as minutes and the y-axis showed the price as liras. Furthermore, in practice 5, the teacher explained how to construct the graph. More specifically, the teacher explained to the students how to construct the graph of a linear equation by determining the ordered pairs of points. In the meantime, she traced students' graphing while determining the ordered pairs to construct the graph of a linear equation and connecting set of points to draw the graph, similar to the Building Design activity. While providing explanations for this practice, the teacher connected linear relationship, table and line graph representations (bridging practice 1: connecting concept and representations in Table 24) concomitantly.

As practice for explaining the solution method/procedure (practice 2), the teacher explained the solutions of a system of equations. More specifically, she explained the solutions of a system of equations algebraically. That is, she made calculations to show that the coordinates of an intersection point is a solution for a system of equations by satisfying the equations. In addition, she provided the students with explanations about algebraic ways of solving the solution of a system of equations, and explained or encouraged students to algebraically solve a system of equations by substitution. As evidence, for example, after the students graphically solved the system of equations and the teacher explained that there were also algebraic ways of solving this system, the following dialogue emerged:

(T)	Can you apply substitution? For example, what can you write for y? Can you write $2x$ in the equation ($y = x + 5$)? Let's see your solutions.
(SS)	*(Students talk to each other about the solution.)*
(T)	You have two equations ($y = 2x$ and $y = x + 5$). Let's solve [the system] using the substitution method.
(S)	Both of them?
(T)	You have two equations [in the system]. They are two-variable equations.
(S)	*(Students were working on their sheets.)*
(T)	Did you put ($2x$) into the equation ($y = x + 5$)? (The teacher checked students' sheets)
(SS)	Yes!
(T)	So, you find x and y values.

In the *Equations Systems* activity, the teacher's trimming practices were combined under the practices of explaining mathematical value (practice 1), explaining the solution method/procedure (practice 2), explaining mathematical relations (practice 4), explaining the context of an activity (practice 5), using appropriate mathematical terms (practice 6), and revealing students' mathematical difficulties (practice 7) (see Table 26).

As a practice for explaining the context of the activity (practice 5), the teacher explained the activity procedure regarding filling in the table. More specifically, the teacher provided the students with explanations about the activity procedure of filling in a table with the system of equations, the slope of equations, the solution set of equations, and the positions of the lines of equations. For example, while the students were working on the activity sheet, she said, *“You should write the slope values of these equations. You know you can find the slope in two ways. Slope of the line or slope of the equation.”*. In addition, as practice for explaining the context of the activity (practice 5), the teacher explained the activity procedure for positions of lines. More specifically, the teacher provided the students with explanations about the activity procedure to determine the positions of the lines in a system of equations. To illustrate, while the students were working on the system of equation in the Mobile Operators activity, she emphasized the positions by posing the following question: *“Are they parallel [lines] or intersecting [lines]?*”.

While explaining the context for the activity, as practice for explaining the solution method/procedure (practice 2), the teacher explained the computation of a slope. More specifically, the teacher explained the computation of the slope of a line graph as algebraic ratio. That is, the teacher explained or encouraged the students to compute a slope of a line graph as the ratio of vertical change to horizontal change, similar to the Building Design and the Leaking Container activities. In this process, the students were working on the equations in the Mobile Operators activity. Therefore, while providing the students with explanations in this trimming practice, the teacher connected activities on slopes of lines in phase 3 (bridging phase 3: connecting activities in Table 24). In addition, as practice for explaining mathematical relations (practice 4), the teacher localized the slope of a line in a linear equation. More specifically, the teacher localized the slope of a line as the coefficient of the x-variable in a linear equation in the slope-intercept form. While providing explanations for this trimming practice, the teacher connected slope and ways of finding slope in an equation and by using two points of a line (bridging practice 2: connecting concepts in Table 24) in the background.

Subsequently, as practice for explaining mathematical value (practice 1), the teacher explained the solution set value of a system of equations. That is, the teacher provided the students with explanations about the solution set value of a system of equations that has one solution with a point as ordered pair. In the meantime, as practice for explaining the solution method/procedure (practice 2), the teacher explained the solutions of a system of equations algebraically. More specifically, the teacher encouraged the students to solve a system of equations algebraically by using any method and explained or encouraged students to solve a system of equations by means of the substitution method algebraically.

While solving the system of equations for parallel lines, as practice for explaining mathematical relations (practice 4), the teacher provided examples of a system of equations for relating solutions sets and positions of lines. More specifically, the teacher provided examples of a system of equations in random sequence for relating a solution set algebraically and positions of lines. As evidence, the teacher gave the following system of equations that had no solution and parallel lines:

$y = 0.4x + 1$, $y = 0.4x - 2$ and $= x + 1$, $y = x + 2$.

As practice for explaining the solution method/procedure (practice 2), the teacher explained how to convert forms of linear equations. More specifically, the teacher explained the slope of a linear equation by converting forms of linear equations (i.e. in slope-intercept form). That is, after some of the students used GGB material to find the slope in the standard form of the equation and did not relate the slope and equation, she wanted the students to convert the standard form of the equation to the slope-intercept form to find the slope by using the equation by hand.

In addition, as practice for revealing students' mathematical difficulties (practice 7), the teacher revealed students' difficulties regarding solving a system of equations that graphically had one solution. That is, she revealed that some of the students did not develop an understanding of the solution set related to the coordinate values of the intersection point of the lines and considered the point as merely a figure. Then, she specifically asked, *"Did you understand how to find a solution set [algebraically] or the graphical way?"* In this regard, she explained that the intersection point of the two lines was the common point of the lines and its coordinates were common x and y values of the equations.

As another practice for explaining mathematical value (practice 1), the teacher explained slope values of lines/equations in a system of equations. That is, the teacher asked the students to compare slope values of parallel lines in a system of equations.

As practice for using appropriate mathematical terms (practice 6), the teacher used appropriate mathematical terms for coincident lines. More specifically, the teacher used appropriate terms for coincident lines in a system of equations. That is, while the teacher organized this activity on the intersecting lines and parallel lines, she also mentioned the coincident lines and the system of equations that has an infinite number of solutions. However, she did not give further examples on this type of a system of equations.

In the *Stores* activity, the teacher's trimming practices were combined under the practices of explaining the solution method/procedure (practice 2), explaining the mathematical relations (practice 4), explaining the context of the activity (practice 5), and revealing students' mathematical difficulties (practice 7) (see Table 26).

As practice for explaining mathematical relations (practice 4), the teacher localized the relationship for the linear equation. More specifically, the teacher verbally localized the explicit relationship between the coefficient of x and y-values for the equation and localized the symbolic representation of the explicit relationship as the linear equation. In this regard, the following dialogue emerged during the practice.

(T) Can you look at the relationship between the two [variables in the table]. 1 and 9, 2 and 8. What do you see for each?
(S) Inverse
(T) All the time?
(S) One of them is directly proportional, the other one is indirectly proportional *(The student interrupts the teacher's speech)*
(S) *(Students talk to each other.)*
(T) Yes. Talk about the relation between 1 and 9. *(The teacher responds to one of the students in the group.)*
(S) One increases, the other one decreases.
(T) Did you find y, when you added x and y? *(The teacher responds to the student's answer of* $x + 9 = y$*)*. If you add 1 and 9, is the sum 9? Look at the relationship between these two *(The teacher shows the columns of x and y in the table)* What do 2 and 8 make? What do 1 and 9 make? And 3 and 7?
(SS) *(Students talk to each other. The teacher watches and listens to them.)*
(T) Consider 2 and 8, 3 and 7. *(The teacher talks to a student)* So, who cannot find the equation of the second store [store-N]?
(S) Increases.
(T) Yes, but that is not the question. Did you find an equation? Y equals to what?
(S) *(Some of the students raise their hands.)*

(T) Okay. Look at again. You are looking at the relationship in the value table. Do you see the similarity of the relationship between 2 and 8 and the relationship between 1 and 9?
(S) It decreases by 1.
(T) Do not look the below [the relationship between y values]. Look at the relationship between x and y. What is the relationship between 1 and 9? What is the relationship between 2 and 8? For example, if I know 1, how do I find 9?
(S) Plus 8.
(T) Is it correct for the second one [2 and 8]?
(S) No.
(S) X plus y [equals to] 10.
(T) Yes. Is it correct for all?
(S) Yes. There is also y equals to 10 minus x.

Thus, the teacher localized the linear equation for store-N by posing questions and by using the number values in the table. Therefore, while explaining for this trimming practice, the teacher also connected linear relationship and table representation (Bridging practice 1: connecting concept and representations) concomitantly. On the other hand, the students could write the linear equation for the other store situation (store-M). It was seen that the students could easily write linear equations with a positive slope. In addition, in another dialogue for localizing the symbolic representation of the explicit relationship as the linear equation ($y = 10 - x$), the teacher connected the topics of patterns and linear equations in math (bridging practice 4: connecting topics in Table 24). That is, after the students equated y to 10 minus x, she said, *"It may be difficult to see this relation at a single glance. But this is related with your understanding of and practices in patterns and relations in previous years."*

As a practice for explaining the context of the activity (practice 5), the teacher explained the context of the activity for labels of axes in the graphing of the Stores situation, similar to the Mobile Operators activity in phase 3 and the Building Design and the Leaking Container activities in phase 2. This showed that students had difficulties regarding this issue, which involved transferring the table to the coordinate plane. Being aware of this issue, the teacher tried to compensate for the gap in this transition.

As practice for explaining the solution method/procedure (practice 2), the teacher explained the computation of slopes. More specifically, the teacher explained and encouraged students to compute the slope of a line graph as the ratio of vertical change to horizontal change by using two different points. As evidence, after the students responded to the slope values of the equations for the Stores situations, she asked, *"Why? Did you determine the horizontal and vertical [changes] with the two points?"*

As practice for explaining mathematical relations (practice 4), the teacher localized the slope of a line in a linear equation. More specifically, the teacher localized slope values of equations as the coefficient of an x-variable in a system of equations with one solution. As evidence, the teacher specifically asked the following questions: *"Can you explain the relationship between the slope of the lines and equations? Could you see the slope on the equation?"*, and *"How do you arrive at this understanding?", and "Is this related with the coefficient of x?"*

After the tasks of the Stores activity was completed, as practice for revealing students' mathematical difficulties (practice 7), the teacher revealed students' difficulties in computing the slope of physical objects. While the teacher was talking about a question regarding the slope of a physical object in the exam, one of the students said, *"Some of us made an incorrect computation. Since the object is a special triangle 3-4-5, they thought 5 was the slope."* In response to this comment, the teacher asked questions to reveal the students' misunderstandings and to make them think over it *"Why? What did they think about the slope?"* Then, the students admitted their misunderstanding that the slope was the length of the object. In response to this comment, the teacher again asked, *"Is the slope a length?"* The students answered by saying, *"Slope is a ratio. Ratio of vertical to horizontal"*. Then, the practice in the following paragraph emerged in the session.

As practice for explaining mathematical relations (practice 4), the teacher again localized the relationship for the linear equation. More specifically, the teacher localized the symbolic representation of the explicit relationship as the linear equation with the examples of the taxi fare equation, the tomato price equation, and the speed equation. While the teacher provided explanations in this trimming practice, she also connected slope, rate of change, and functional situation/equation (bridging practice: connecting concepts in Table 26). As evidence, while the students working on a train's distance-time relation, the following episode emerged during the practice:

(T) Okay. The train goes 300 km per hour. What is the relation between the distance and the time relationship?
(S) Y equals to 300 x.
(T) Okay. What is the distance per unit time?
(S) 300.
(T) What is the slope?
(S) 300.
(T) What is the coefficient of x?
(S) 300.

In addition, as another practice for localizing the slope of line in a linear equation (in practice 4), she explained the slope-linear equation relation by excluding the constant value for a functional situation. More specifically, while the students were discussing the equation of $y = 2x + 2.5$ for a taxi fare situation, the teacher said, *"The slope is not related with this constant value. That is it not related with* 2.5*. Is it?"*

4.3.3 Decompressing Practices in Explaining Mathematical Ideas without using GGB materials and Concrete objects

In the previous two sub-sections, the teacher's mathematical practices were explained on two dimensions (bridging and trimming) of the KAT framework as the teacher explained mathematical ideas without using GGB materials and concrete objects in the classroom teaching sessions. In this section, the teacher's decompressing practices specific to teaching slopes, lines, and equations in Grade 8 are explained based on the decompressing dimension of the KAT framework. Therefore, this section involves the teacher's decompressing practices implemented within each phase when she explained mathematical ideas without using concrete objects and GGB materials of the activities in classroom teaching. From the perspective of McCrory et al.'s (2012) KAT framework, decompressing practices in teaching algebra are described as mathematical practices of highlighting or deconstructing the complexity of mathematical ideas to make them comprehensible. Therefore, the decompressing practices were categorized by identifying the teacher's actions in classroom teaching in which the teacher provides mathematical practices to make explicit those mathematical ideas for a comprehensible mathematical context. However, some of those practices were also characterized as preliminary decompressing practices that involve preliminary interpretations in which the teacher started to highlight the complexity of slope by interpretations but did not finished a completed interpretation or examination.

Therefore, the themes of decompressing practices, which were grouped by the researcher, and the phases that involved those practices are as follows:

Table 27. Summary of Decompressing practices while explaning mathematical ideas without using GGB materials and concrete objects

Practices	**Number of phase**
Practice 1: Interpreting the slope of a line	2, 3
Practice 2: Interpreting the system of equations	3

However, decompressing practices were not observed during the explanations of mathematical ideas without using GGB materials and concrete objects in phase 1. Thus, there were two themes of decompressing practices. While phase 3 involved both practices, phase 1 involved one practice and phase 2 involved no decompressing practices. The emerging themes of decompressing practices and the actions under each theme are presented for each phase in Table 28. The flow of actions in the practices are explained under the classroom activities for each phase. In addition, when a given decompressing practice was related with a bridging practice and/or a trimming practice, which were explained in the previous sections, those relations were also presented in the flow of the actions.

Table 28. Decompressing practices while explaning mathematical ideas without using GGB materials and concrete objects

Practice	**Action**	**Sample description**
Practice 1: Interpret slope of a line (preliminary)		
Interpret constant slope of the line	Interpret constant slope of the line graph for the linear equation	Interpret constant slope of the line graph for the linear equation comparing different intervals (2B)
	Deconstruct slope of the line on the direction of changes	Deconstruct slope of the line on the coordination of the direction of changes in variables (2H)
Interpret the sign of the slope of a line	Interpret the sign of the slope of a line graph for a linear equation/functional situation	Interpret the sign of the slope of a line graph for a linear equation in functional situation (2B)
		Deconstruct the sign of the slope of a line on the coordination of the direction of changes in variables (2B)

Table 28 (continued)

		Examine negative slopes for real life situations (2B)
		Interpret negative slopes of a line graph for afunctional situation (2H)
Interpret change in the slope of the line	Interpret change in slope dimensions	Interpret change in vertical in slope of a line (2B) (2H)
Interpret slope as (constant) rate of change	Interpret slope as unit rate	Interpret slope as unit rate for stores in functional situations (3S)
		Question the unit rate for stores and mobile operators as slope (3S)
		Question the unit rate as slope for price functions (3S)
	Interpret positive slope as rate of change in functional situations	Explain slope in the taxi situation as rate of change, which involves coordinating amount of change of price with respect to a uniform increment of the km (3S)
		Question slope as rate of change in amount of mass per unit volume (3S)
	Interpret negative slope as rate of change in functional situations	Question and explain negative slope as rate of change in the gasoline consumption situation (decrease of gasoline per unit km) (3S)
Practice 2: Interpret system of equations (preliminary)		
Interpret system of equations	Interpret system of equations that has one solution	Interpret system of equations that has one solution for two functional situations in reference to an intersection point (3M) (3S)
	Interpret solution set for a system of equations that has no solution	Interpret solution set for a system of equations that has no solution (3E)
	Interpret slopes and positions of lines for system of equations	Interpret slope of lines and positions of lines for a system of equations that has one solution or no solution (3E)
	Interpret the relation between slope, positions of lines, and solutions for system of equations	Question the relation between slopes of lines, positions of lines, and solutions for a system of equations (3E) (3S)

Note: The abbreviations in the sample description column of the table indicate the activity in which an action emerged. Phase 1 activities; 1P: Positions of Battens activity, 1F: Fire Truck activity, 1T: Tent activity. Phase 2 activities; 2B: Building Design activity, 2H: Leaking Container activity, 2S: Slopes and Equations of Lines activity, Phase 3 activities: 3M: Mobile Operators activity, 3E: Equations Systems activity, 3S: Stores activity.

4.3.3.1 Decompressing practices in teaching the slope of an object for Phase 1

In Phase 1, there were activities on the slope of an object in a physical situation, which were a batten object in the Positions of batten activity, a fire track ladder object in the Fire Truck activity, and a rope of the tent object in the Tent activity. In phase 1, no decompressing practices emerged while the teacher was explaining mathematical ideas without using GGB materials and concrete objects. Therefore, the conception of slope as a geometrical ratio and as a physical situation was elaborated on while the teacher used GGB materials and concrete objects.

4.3.3.2 Decompressing practices in teaching the slope of a line for Phase 2

In Phase 2, there were activities on slope of a line in functional situations and in the coordinate system, which were the *Building Design* activity, based on a line graph representing the linear relation between the number of windows in a building and the number of floors of a building, the *Slopes and Equations of Lines* activity, based on relating linear equations, lines, and slopes, and *Leaking Container* activity, based on a line graph representing the linear relation between the amount of water in container and -time (minutes). However, no decompressing practices emerged while the teacher was explaining mathematical ideas without using GGB materials and concrete objects in the *Slopes and Equations of Lines* activity. In Phase 2, the teacher's decompressing practices were combined under the practice of interpreting the slope of a line (practice 1) (see Table 28). In these practices, the teacher enabled the students to develop an understanding of the meaning of constant slope, sign of slope and change in slope. The details of the practices in the activities are explained below.

In the *Building Design* activity, the teacher's decompressing practices were combined under the practice of interpreting the slope of a line, as can be seen in Table 28. In this practice, the concept of slope was developed on the line graph for the linear equation in the situation by comparing different intervals on the line and on the sign of slope of the line graph.

As practice for interpreting the slope of a line (practice 1), the teacher interpreted the constant slope of the line. More specifically. The teacher interpreted the constant slope of the line graph for the linear equation by comparing different intervals. As evidence, while the groups of students were working on the computation of the slope of the line graph by using two points on the graph and each group chose different pairs of points, the teacher saw that each group of students correctly computed the slope in their activity sheets and the following dialogue emerged during the practice:

(S) We found 3.
(T) This group found that the slope was 3. You? *(The teacher asks another group)*
(SS) 3.
(T) You? *(The teacher asks the other groups)*
(SS) 3.
(T) And you are all correct. Do you have the same points while computing the slope?

Thus, by posing questions, the teacher enabled the students to develop an understanding that the slope of a line is constant and whatever the points or the intervals are chosen it is not changed. These questions of the teacher could be thought as an introductory or preliminary level of decompressing. In the further stage of the activity, the teacher developed this idea in detail by giving exact points and intervals in the graphics view on the GGB material of the activity.

As practice for interpreting the slope of a line, the teacher interpreted the sign of the slope of the line. More specifically, the teacher interpreted the sign of the slope of the line graph for the linear equation in the functional situation. As evidence, after the students computed the slope of the line graph for the Building Design situation and the teacher trimmed the positive value of the slope of the line in the situation (trimming practice 1: explaining mathematical value in phase 2 in Table 26), the following dialogue emerged during the practice:

(T) What could be related with the positive slope? What could the negativity of the slope be related to?
(S1) Its direction.
(S2) Its position.
(T) Do you mean that it depends on just its position? Is it posited like this or that (She points to the board-gesture)
(S3) The direction of the slope.
(S4) The horizontal and vertical [line].
(S5) The angle
(T) All lines have vertical and horizontal [lengths].
(S4) The angle
(S5) But the lengths…
(S6) If it is on this part of the coordinate plane.

Thus, the teacher enabled the students to reason on the meaning of the sign of the slope. This questioning could be thought of as an introductory or preliminary level of decompressing. In a further process, the teacher developed this idea in detail by using the GGB material of the activity.

After the aforementioned practices, the teacher continued to interpret the sign of the slope of the line, as practices for interpreting the slope of a line (practice 1). More specifically, she deconstructed the sign of the slope of the line on the coordination of direction of changes in variables. As evidence, after the students worked on the logic of the sign of the slope on different intervals by using the GGB material, the following dialogue emerged during the practice:

> (T) But, [despite using different intervals], you all found the slope as 3. Then we'll look at it in this way. You will look at the x and y axes. That is, look at the relationship between the floors and windows in this situation. While one [variable] increases, the other one [variable]-
> (S) Increases.
> (T) Increases. Or while one [variable] decreases, the other one-
> (S) Decreases.
> (T) Decreases. Therefore, the ratio of these changes… That is, what is the ratio of vertical change to horizontal change?
> (S) Positive.
> (T) Yes positive. What is the ratio of negative [change] to negative [change]?
> (S) Positive
> (T) What is the ratio of positive [change] to positive [change]?
> (S) It's also positive.
> (T) You got it. So, while commenting on whether the slope is positive or negative, we do not say, 'if it is here, it will be positive' or 'if it is here, it will be negative'. We look at the variables. We look at whether one variable increases when the change in the other variable increases. Then we construct the ratio. We decide about the sign by considering the changes [in the variables].

Thus, while helping the students to interpret the complexity of the sign of the slope of a line on the direction of the changes, the teacher trimmed the computation of slope (trimming practice 2: in Table 26) and connected the sign of a slope, the direction of a line, and the ratio of vertical over horizontal (bridging practice 2: connecting slope and other concepts in Table 24).

In addition, as another practice for interpreting the sign of the slope of the line (in practice 1), the teacher helped the students to examine the negative slope in the real life situations. For example, she asked, *"Could you give an example in a daily life situation? Could you mention a situation in which there is a negative slope? One [variable] increases, while the other one [variable] decreases."*. Then, the students gave examples of physical situations like road and staircase, she again said, *"How do we use the staircase? We should look at the relationship by considering two things* *[quantities, variables]"*. However, the students did not produce examples of negative slope situations and the teacher did not explicitly explain them. The teacher tried to enable students' to reason on negative slope in linear functional situations. Therefore, this questioning could be considered as an introductory or preliminary level of decompressing. In further activities, the teacher developed this idea in detail by using the GGB materials. In addition, while providing explanations in this decompressing practice, the teacher connected negative slope and real life situations (bridging practice 1: connecting concept and representations in Table 24) concomitantly. That is, she conveyed the idea that a negative slope could be seen in real life situations.

At the end of the Building Design activity, the teacher interpreted the change in the slope of the line as a practice for interpreting the slope of a line (practice 1). More specifically, she wanted the students to interpret the vertical change in the slope of a line. That is, after the teacher showed the formula of the slope as the ratio of change in vertical to change in horizontal, she again said, *"We use a new term. Vertical change. What is that? What have we been saying so far?"* and *"We have used the vertical length before. But, what is vertical change? Think about it. In the previous activities [in phase 1] we called it vertical length, but now we call it a vertical change. Why?"* Therefore, the teacher wanted the students to develop an understanding of the meaning of change in slope and computation of slope in functional situations.

In the *Slopes and Equations of Lines* activity, no decompressing practices emerged while the teacher was explaining mathematical ideas without using GGB materials and concrete objects.

In *Leaking Container* activity, the teacher's decompressing practices were combined under the practice of interpreting the slope of a line (practice 1), as can be seen in Table 28. In this practice, the concept of slope was developed on the slope of a line graph by coordinating the direction of changes in variables in the situation, on sign of slope of the line graph, and on the change in slope dimensions in the computation.

As practice for interpreting the slope of a line (practice 1), the teacher interpreted the constant slope of the line. More specifically, the teacher deconstructed the slope of the line on the coordination of the direction of changes in variables within the amount of water in a leaking container and time (minutes) relation. As evidence, while most of the groups were working on reading the line graph for the situation and the teacher did not provide any guidance on slope, one of the groups computed the slope of the line for the situation by using the algebraic formula of the slope, and the following dialogue emerged during the practice:

(T) Do you know the meaning of this formula *(The teacher asks the group)*? Or have you made this computation since your friend said the formula? What does it mean y2 minus y1?
(S) Hmmm. Height.
(S) In y, we find the height. In x we find the vertical [change].
(T) Think about your computation again. There is an error in the subtraction. Let’s think about the relation between the amount of water in the container and time. While one of them increases, what happens to the other?
(S) The amount of water decreases as the time increases.

Thus, the teacher enabled the students to develop an understanding of the meaning of the slope of the line graph in computation by conceptualizing the coordination between the variables in the graph. However, she developed this meaning in the forthcoming steps of the activity while using GGB materials.

As the practice for interpreting the slope of a line (practice 1), the teacher interpreted the sign of slope of the line. More specifically, the teacher interpreted the negative slope of the line graph for the functional situation. As evidence, while a group of students found the slope value of the line graph, the following dialogue emerged during the practice:

(S) Teacher, we found the slope as minus 1.
(T) Minus 1? When did you put this minus sign in the computation?
(S) When 5 is subtracted from 7. It is minus 2. When it is divided by [2], the result is minus 1.
(T) Why?
(S) It is subtraction.
(S) We make this subtraction. And there is inverse proportion.
(T) Please think about it together with your group friends.

Thus, the teacher tried to develop students’ understanding of the meaning of the negative sign of the slope in computing the slope of a line. Even though a student had incorrectly related negative slope with inverse proportion, the teacher did not correct her and gave the students opportunities to reason on this idea in the forthcoming steps of the activity while using the GGB material.

As another practice for interpreting the slope of line (practice 1), the teacher interpreted change in the slope of the line. More specifically, the teacher interpreted vertical change in the slope of a line as an algebraic ratio. While providing explanations during this decompressing practice, the teacher also connected the subtraction operation in the algebraic ratio and the vertical change between two points during the practice of connecting conceptualizations of the slope (bridging practice 2: connecting concepts in Table 24).

4.3.3.3 Decompressing practices in teaching the solution of a system of equations and the slope relation for Phase 3

In Phase 3, there were activities on the system of equations in functional situations and in the coordinate system. They were the *Mobile Operators* activity, based on the solution of two linear equations for price (TL)-time (minutes) linear relations of two Mobile Operators, the *Equations Systems* activity, based on relating linear equations in a system, slopes, solution set, position of lines, and drawing lines in a system, and the *Stores* activity, based on the solution of two linear equations reflecting the amount of product-time (day) linear relations of two Stores and the relation between slopes, solution, and graphs. In addition, at the end of this phase, the teacher reconsidered the slope of a line graph in functional situations. In Phase 3, the teacher's decompressing practices were combined under the practices of interpreting the slope of a line (practice 1) and interpreting the system of equations (practice 2) (see Table 28). In these practices, the teacher enabled the students to develop an understanding of the meaning of slope as rate of change and system of equations with solutions, positions of lines, and slopes.

In the *Mobile Operators* activity, the teacher's decompressing practice were combined under the practice of interpreting the system of equations (practice 2).

As practice for interpreting system of equations (practice 2), the teacher interpreted the system of equations that had one solution. More specifically, the teacher interpreted the system of equations that had one solution for two functional situations in reference to intersection. For example, after the students identified the intersection point of the lines in the system of equations and worked on the GGB material of the activity, the following dialogue emerged during the practice:

(T) Then, we can make a generalization together. If you talk less than 5 minutes, which operator will you use?
(S) T-cell
(T) Why? Because it is more economical at this point. If someone talks more than 5 minutes, which operator will he use?
(S) Z-cell
(T) If you talk exactly 5 minutes, which operator will you use?
(S) It doesn't matter.
(T) You know there are many mobile operators. We cannot say one of them is economical all the time. It is related with the length of time that we talk on the mobile. So, you should know how long you talk and then make a computation to decide on one of them.

While providing explanations in this decompressing practice, the teacher also connected system of equations and functional situations in real life (bridging practice 1: connecting concepts and representations).

In the *Equations Systems* activity, the teacher's decompressing practices were combined under the practice of interpreting the system of equations (practice 2) (see Table 28).

As practice for interpreting a system of equations (practice 2), the teacher interpreted the solution set for a system of equations. More specifically, the teacher interpreted the solution set for a system of equations with no solutions. As evidence, after the students created a system that involved parallel lines and the equations of $y = 0.4x - 2$ and $y = 0.4x - 1$ and solved the system of equations algebraically, the following dialogue emerged during the practice.

(T) Yes, you equated the equations ($0.4x - 2 = 0.4x - 1$). Can this equation be solved? Or can you find the x value after you solve it?
(S) No we can't.
(T) Why not?
(S) The X terms are eliminated. There is nor a y term.

As another evidence, the following dialogue emerged for the equations of $y = x + 2$ and $y = x - 1$, which the teacher wrote on the board.

(S) Plus 2 equals minus 1(The students read: $+2 = -1$).
(T) The equation is x plus 2 equals x minus 1. Isn't it? What happens when the x terms are eliminated?
(S) Plus 2 equals minus 1.
(T) Could it be possible?
(S) No.
(T) Is such an equality possible at any time?

(SS) No.

(T) Since it isn't, we say that the solution set is an empty set.

Thus, the teacher equalizes the solution set in $x + 2 = x - 1$ to the solution set in $+2 = -1$, which could guide students to understand the meaning of the solution of a system of equations that has no solution for parallel lines.

As another practice for interpreting the system of equations (practice 2), the teacher interpreted the slopes and positions of the lines for the system of equations. More specifically, the teacher questioned the relation between slope values and positions of lines for a system of equations. As evidence, after the teacher asked the students to compare slope values of parallel lines in a system of equations (trimming practice 1: explaining mathematical value in Table 26) and the students compared the slopes of the equations of parallel lines, the following dialogue emerged the practice.

(T) You have those equations. And their slopes. Therefore, you wrote four systems of equations and saw their graphs. Based on these, let's make a generalization. Your friend Sema said something about that. Do you agree with her?

(S) I agree with her. *(The student said that the slopes of parallel lines are equal)*

(T) We talked about the positions of two lines on the plane. What did we get?

(S) Intersecting lines and parallel lines.

As another practice for interpreting the system of equations (in practice 2), the teacher questioned the relation between slope of lines, position of lines, and solution set for system of equations. As evidence, while completing the activity, the practice was emerged in the following episode that was occurred after the previous episode.

(T) Let's make a generalization. Look at the lines for the systems. Sema *(a student)* said something about that. What did you learn in this course? Look at the equations of lines. Look at the positions of the lines. What happens when anything does happen? Compare these four situations. What happened to the slope, solution set?

(S) If the lines intersect, there is an exact [point] as a solution set. If the lines are parallel, the solution set is empty.

(T) Yes. This is one of the important results.

(S) The slopes of the parallel lines are equal.

(S) The slopes of the parallel lines are equal, but the slopes of the intersecting lines are different.

(T) The slopes of the intersecting lines are different. Yes. It is the other important result. Therefore, can you explain the positions of the two lines if we know that their slopes are equal?

(S) Parallel.
(T) Okay. Consider two equations [in a system] that have no solution.
(S) Parallel lines
(T) If there is one solution in the solution set, then what?
(S) They intersect.
(T) What if the solution set consists of real numbers? That is, all points satisfy two lines of the equations. What is the positions of these two lines?
(S) They intersect.
(S) Parallel.
(T) Think about it; the common point is not just one point. All points satisfy the lines. What are the positions of these two lines?
(S) They are on top of each other.
(T) How?

This dialogue continued with the explanation of coincident lines as practice for using appropriate mathematical terms for coincident lines (trimming practice 6: using appropriate mathematical terms). However, the teacher did not consider that the lines could be parallel or coincident if the slopes of the two lines were equal since she had prepared the activity on parallel lines and intersecting lines. Therefore, the teacher separated various systems of equations from each other based on the differences in their graphical and algebraic representations, which required showing the changes in slopes, relative positions, and solution sets. This process seems to be metaphorically similar to distillation in chemistry.

In the *Stores* activity, the teacher's decompressing practices were combined under the practices of interpreting the slope of a line (practice 1) and interpreting the system of equations (practice 2) (see Table 28).

As practice for interpreting the system of equations (practice 2), the teacher interpreted the relation between slope, positions of lines, and solutions for the system of equations. More specifically, the teacher questioned the relation between slopes of lines, positions of lines, and solutions for a system of equation in the Stores activity. As evidence, after the students wrote the equations and drew the line graphs for stores M and N, the following dialogue emerged during the practice:

(T) … Does everybody draw graphs? Good. What happened to these two lines? Parallel?
(S) No.
(S) They intersect.
(T) What do we know about intersecting lines?

(S) They are actually perpendicular.
(S) There is one solution.
(T) Yes. What else?
(S) Their slopes are different *(They do not compute the slopes but they can see the difference.).*
(T) Yes. Their slopes are different. Good.
(S) Teacher, they intersect perpendicularly.
(T) Oh, yes! We could look at whether these results are related with their slopes. Let's look at the questions *(in the activity sheet)* now.

Thus, again the teacher gave a system of equations that had one solution and one line with a positive slope and a line with a negative slope.

As practice for interpreting the system of equations (practice 2), the teacher interpreted the system of equations having one solution. That is, the teacher interpreted the system of equations that had one solution for two functional situations in reference to an intersection point. As evidence, when the teacher talked with each group of students, the following dialogue emerged during the practice:

(T) How are the equations for the two stores (Store M and Store N) related?
(S) The number of sold products are equal on the 5^{th} day.
(T) Yes. The number of sold products are equal. Therefore, this 5 comma 5 [point (5,5)] is the solution of these equations.
(T) When do both of the stores sell an equal number of products? What does it mean for the equations? *(The teacher talks with another group of students.)*
(S) On 5th day.
(T) What do they sell?
(S) 5 products.
(T) 5 products on 5^{th} day. What does this mean?
(S) Intersection and solution set.
(T) Yes, this is the intersection point of the lines and the solution set of the system of equations.

After the tasks of the Stores activity was completed, the teacher decompressed the slope considering functional situations in the previous activities and new examples. As a practice for interpreting the slope of a line (practice 1), the teacher interpreted the slope as unit rate. More specifically, the teacher questioned the unit rate based on the number of sold products per day relation in the stores situation and the price-minute relation as the slope in Mobile Operators situation. In addition, the teacher questioned unit rate as slope for other price functions. As evidence, while discussing the Stores situations, the following dialogue emerged during the practice:

(T) Let's talk about another aspect of stores. What is the amount of daily sales for a store? What is the number of sold products per day?
(S) Ten.
(T) No. You said the sum of the products. I asked for the [amount of sold products] per unit change.
(S) It increases considering the change per day.
(S) One of them increases by 1; one of them decreases by 1.
(S) They are the same as the slopes.
(T) So, is this?
(S) Yes the slope is -1.
(T) For example, let's think about the mobile operators again. Open your sheets. Let's look at the T-cell mobile operator. How much money do you pay per unit time to T-cell? What is unit time?
(S) Every passing minute.
(T) Let's consider the detergent; it costs 10 TL for 5kg. How much does 1 kg [of detergent] cost?
(S) Two liras.
(T) What is that? This is the unit price of the detergent. While buying a detergent, you look at this.
(S) Yes.
(T) So, for these mobile operators, what are the unit prices for each one?
(S) Two liras.
(T) What is the slope of this operator [T-cell]?
(S) Two liras.
(T) Look at Z-cell [the other operator in the situation]. In unit time, per minute.
(S) One.
(T) What is the slope of this operator (Z-cell)?
(S) One.
(T) So, could we call the slope by another term?
(SS) *(There is silence.)*
(T) You say the slope is related to vertical over horizontal. Do we have to speak about horizontal or vertical [dimensions] for slope?
(S) We can say amount of increase in price per unit time.
(T) Yes. It depends on the quantity that changes.
(S) The amount of change per unit.

Thus, the teacher gave the meaning of the slope as a unit rate, which was an introduction to the constant rate of change. While providing explanations in this decompressing practice, the teacher connected slope, unit rate (price), and functional situation/equation (bridging practice2: connecting concepts in Table 24).

As another practice for interpreting the slope of a line (practice 1), the teacher interpreted the slope as a (constant) rate of change in a functional situation. More specifically, the teacher explained the positive slope in the taxi situation with rate of change that involved coordinating the amount of price with respect to a uniform increment of the kilometer. In another example, the teacher questioned the slope as rate of change in the amount of mass per unit volume. As evidence, while talking about a taxi fare, the following excerpt emerged during the practice:

> In that situation ($y = 2x + 2.5x$), slope describes how much the price increases when an increase of one kilometer in distance is considered. That is, when I go 1 kilometer, how much does the price increase? We look at the rate of increase between them. Don't we? When we say unit, we look at increments one by one. We do not actually need to compute the ratio. You know, the computation involves vertical over horizontal. We do not even do this computation if we know the rate of change because the one [vertical, denominator] is 1. It increases by one. Remember, we say if I go one more kilometer, I will pay 2 liras.

In addition, in practice 1, the teacher interpreted the negative slope as rate of change in a functional situation. More specifically, the teacher questioned and explained the negative slope in the gasoline consumption situation as rate of change, which is the decrease in amount of gasoline per unit km. As evidence, after the construction of a situation of gasoline conception and its equation as practice for localizing symbolic representation of the explicit relationship (linear equation) (trimming practice: 2 explain mathematical relations in Table 26), the following dialogue emerged during the practice:

(T) In this equation ($y = 40 - 0.5x$), how much decrease is there per one kilometer?
(S) Zero point 5.
(T) Yes. Five tenths. What is the coefficient of x here?
(S) 0 point 5.
(T) Here, there is a decrease. Since there is a decrease, what is the slope?
(S) Minus 0 point 5.
(T) So, what is the coefficient of x?
(S) Minus 0 point 5.
(T) Yes the equation is y equals to 40 minus five-tenths of x.
(S) Minus five tenths.
(T) You got it? So, slope is not just a concept specific to the lines. That is, while finding the slope, you do not have to compute the slope of a line for a situation. For example, you can find the change per unit time.

Thus, the teacher expressed the meaning of slope as rate of change for linear functional situations. Even if she did not emphasize the constant rate of change, it was assumed that the teacher had presented these linear situations to represent the constant rate of change. While providing explanations in this decompressing practice, the teacher also connected slope, rate of change and functional situation/equation (bridging practice 2: connecting concepts in Table 24).

CHAPTER 5

CONCLUSIONS AND DISCUSSION

In the findings of the current study, an in-service middle school mathematics teacher's mathematical practices in teaching slopes, linear equations, and graphs while enacting an instructional sequence for teaching in a technology enhanced classroom environment were presented in two parts to enlighten the actions in the practices: during the time when the teacher used GGB materials and concrete objects in classroom sessions and during the time when the teacher explained mathematical ideas without using GGB materials and concrete objects in classroom sessions. The curriculum-based instructional sequence was developed by the teacher through the guidance of the researcher to help students understand slope through a design experiment. The main objectives in the instructional sequence were (1) to explain the slope of a physical object, (2) to relate the slope of a straight line graph and the equation of a line graph for functional situations, and (3) to solve linear equation systems relating graphs, slopes of lines, and solutions of equations systems. Therefore, the goal of the current study was to thoroughly portray the set of mathematical practices of the teacher in a technology enhanced classroom environment related to these objectives from a design experiment perspective. While a detailed analysis was given in the findings section, this conclusions and discussion section involves the reflections on the meaning and understanding of the findings, implications and limitations of the study and suggestions for further research. Consequently, this chapter consists of four parts. In the first part, the teacher's mathematical practices are characterized for teaching the slope of a physical object, the slope of a line in functional situations, and system of equations related with slope. In the second part, the interrelations among the bridging, trimming, and decompressing practices are discussed. In the third part, the suggestions on teaching of slope in middle school mathematics context are drawn. In the fourth part, implications and suggestions for future research are drawn. Finally, the limitations of the study are drawn.

5.1 Mathematical Practices in Teaching Slope of an Object

While designing and enacting an instructional sequence on slope, linear equations and graph unit, the teacher separated the slope models for physical situations and for linear functional situations and the coordinate system. In this regard, the first phase was formed with the aim of developing an abstraction of slope of an object in physical situations. While teaching slope of an object as the first phase of the instructional sequence, a wide range of mathematical practices emerged using GGB materials and concrete objects, whereas less mathematical practices emerged without using GGB materials and concrete objects. That is, in this phase, the practices resulting from the use of GGB materials and concrete objects were so dominant that decompressing practices did not emerge at all while the teacher was explaining mathematical ideas without using GGB materials and concrete objects (see Table 29). Thus, in general, while teaching slope of an object teachers should have the understanding of what is the meaning of the mathematical idea of slope as a concept of steepness and as a process of measurement and computation in physical situations, and how this meaning of slope is in connection with measurement, ratio, rate, and division. The mathematical practices under the categories of bridging, trimming, and decompressing that emerged in this phase were discussed in the following sub-sections.

Table 29. The teacher's mathematical practices in teaching slope of an object

	Practices		
	Bridging	Trimming	Decompressing
Mathematical practices in using GGB materials and concrete objects	• Connect concrete object and mathematical situation/concept [P] • Connect concrete object, mathematical situation/concept, and GGB material [P] • Connect mathematical situation/concept and GGB material [P, F, T] • Connect mathematical/situation/concept/process and GGB material [P, F] • Connect conceptualization(s) of concepts and GGB material [P, T] • Connect mathematical situation/concept, GGB material, and activity sheet [F]	• Guide to trim the use of concrete object [P, T] • Trim mathematical situation using GG material [P, F, T] • Trim computation using GGB material [P, F, T] • Address mathematical difficulties within conditions of GG material [T]	• Preliminary comparisons [P] • Interpret the components of slope of an object using concrete object [P] • Interpret the components of slope of an object using GGB material [P, F, T] • Appreciate the use of Dynamic software [F]
Mathematical practices while explaining mathematical ideas without using GGB materials and concrete objects	• Connect concept and representations [P, T] • Connect concepts [P, F, T] • Connect activities [R]	• Explain mathematical value [P] • Explain activity context[P] • Explain solution method/procedure [F] • Determine the relevance of a concept [F] • Explain mathematical relations[F, R] • Give appropriate mathematical terms [R]	-

Note: The abbreviations in the table show the activity in which a practice emerged. P: Positions of Battens activity, F: Fire Truck activity, T: Tent activity.

5.1.1 Bridging practices in teaching the slope of an object

While teaching the slope of an object as the first phase of the instruction by using GGB materials and concrete objects, the bridging practices of the teacher included connecting concrete object and mathematical situation/concept; connecting concrete object, mathematical situation/concept, and GGB material; connecting mathematical situation/concept and GGB material; connecting mathematical situation/concept/process and GGB material; connecting mathematical situation/concept, GGB material, and activity sheet; and connecting conceptualizations of concept(s) and GGB material (see Table *29*). In these practices, the teacher usually posed questions to have students connect the mathematical ideas and concrete objects and GGB materials.

The practice of connecting concrete objects (i.e. battens) and mathematical situation/concept included practices such as connecting concrete object, physical situation, and slope latently, connecting concrete object, physical situation, angle, and slope latently, and connecting concrete object, physical situation and slope. More specifically, while making connections to slope latently, the teacher made connections among the positions of battens, slopes of battens without using the term slope, walking on the roads, and the angle between the batten and the base of the batten (ground). Such a way of practicing a mathematical idea could help teachers to develop a conversation about the mathematical concept and to have students notice the mathematical concept (i.e. slope in this instruction) in daily life situations. In addition, while making connections among concrete object, physical situation and slope, the teacher provided actions for connecting the quantities of batten related to slope of the batten. For example, in the Positions of Battens activity of this phase, the teacher made connections among the horizontal length and vertical length of the batten and the slopes of the batten in different positions. In sum, the teacher used batten as a concrete object to model a physical situation (e.g. ramp) not a symbolic process for slope computation. Therefore, with these bridging practices, the teacher provided a mathematical environment for the students to have make sense of the activities (Thompson, 1992). This was also related with the teacher's understanding that the students should have first experience with the slope concretely and they should express their understanding using these concrete objects. Thus, these bridging practices can be seen as the translation of teacher knowledge of content and students (Hill et al., 2008). In sum, the teachers should have a mathematical understanding of what kind of materials provide an environment students to incorporate them in expressing their mathematical ideas about slope of an object in a meaningful way.

While making a transition from the use of concrete object to the use of GGB material, the teacher connected concrete object, mathematical situation/concept and GGB material. In this practice, the teacher provided actions for connecting positions of battens, line segment figure, and GGB graphics view. For example, in the Positions of Battens activity, the teacher made connections between the positioning of the battens in students' hands and figures of line segments as length of the batten, horizontal length of the batten, and vertical length of the batten in GGB graphics view. Therefore, in this bridging practice, the teacher used this way of connecting concrete object to visual representation in GGB before developing students' abstraction of slope. Thus, teachers should have the mathematical understanding of translating the concrete examples of a mathematical ideas into the visual representations of that mathematical idea.

As the prevailing bridging practice in this phase, the practice of connecting mathematical situation/concept and GGB material consisted of two practices: connecting physical situation, line segment, GGB graphics view, and GGB tools *and* connecting physical situation, line segment, slope, GGB graphics view, and GGB tools. In these practices, the teacher used the line segment as the visual representation of the object and the related quantities of the object (i.e. horizontal and vertical length of the object) in the physical situation. The practice of connecting physical situation, line segment, GGB graphics view, and GGB tools included actions such as connecting positions of the object (i.e. batten, ladder, or rope), line segment as the object in the physical situation and slider of horizontal length/vertical length of the object in GGB graphics view, and connecting horizontal length in different slope situations, distance (real life experience), and line segment in GGB graphics view. The first action in the practice was seen in the beginning of all activities in this phase since the teacher wanted to have students recognize the changing positions of the object in connection with the dynamic line segment as the object and the slider of horizontal/vertical length of that line segment that controlled the line segment in GGB graphics views of the activities. That is, the teacher used the slider tool and objects in the GGB graphics view within this bridging practice to introduce the mathematical situation representationally in GGB graphics view. In this sense, the actions involving transitions between the concrete object and the GGB material requires the understanding of connecting the concrete with the visual. Thus, teachers should have the understanding of the relationship between the concrete and visual and the way of giving this relationship considering the students' experiences. In addition, teachers should have a mathematical understanding of how to give visual representations not also for translating the selected concrete examples but also for generating all possible situations with the concrete object using the dynamic properties of the software. In this regard, while using dynamic software to represent such situations, teachers' knowledge of content and technology come to the forefront. Therefore, while the KAT framework (McCrory et al., 2012) and other studies (Wasserman, 2015; Stockton & Wasserman, 2017) did not mentioned a knowledge dimension related with the technology, it is considered that teachers knowledge of teaching mathematics with technology should be considered as a source for fostering the bridging practices in classroom teaching.

The second action in the aforementioned practice was seen in the Fire Truck and Tent activities since the teacher connected horizontal length of the object (i.e. ladder and rope) as the distance between the objects in the ground (i.e. distance between fire truck ladder and building and distance between the tied point of the rope to the ground and the tent) using slider in GGB graphics view. For example, in the Tent activity, the teacher used slider to connect the distance between the tent and the end of the rope in the ground as a real life experience and limited the value of horizontal length of the rope in GGB graphics view. In the practice, the teacher completely represented the objects using dynamic line segments and/or sliders in the context of the physical situations. Thus, when considered the bridging practices above, teachers should have and use the mathematical language accurately while using GGB materials. This understanding of teachers are seen as the foundation of knowledge of logical structures while making definitions and applying an algorithm (Stockton & Wasserman, 2017).

In addition to the aforementioned practice, the practice of connecting physical situation, line segment, slope, GGB graphics view, and GGB tools included actions such as connecting the slopes of the objects (i.e. battens), line segment as the visual representation of the slopes of the objects, and the slider of horizontal length in GGB graphics view; connecting zero slope, real life experience, position of the line segment (i.e. ladder), and slider in GGB graphics view; and connecting distance, horizontal length of the object (i.e. rope), slope of the object, line segment and slider in GGB graphics view. In this practice, the teacher completely represented the slope of the objects visually using dynamic line segments and/or sliders in the context of the physical situations. Furthermore, the practice of connecting mathematical situation/concept, GGB material, and activity sheet in the Fire Truck activity served similar purposes as the previous practice given above. The action in this practice included connecting the visual representation both in GGB graphics view and the activity sheet while students drew the positions of the fire truck ladder when the horizontal length is constant.

As one of the important behaviors in the representational and abstract form of mathematics teaching, the practice of connecting conceptualizations of concept(s) and GGB material consisted of two practices in two different activities, namely connecting representations of slope, line segment, GGB graphics view and connecting representations of undefined slope, factors of slope, GGB graphics view, and GGB tools. The former practice in the Positions of Battens activity included the actions for connecting verbal, symbolic, and visual representations of length and line segment dynamically in GGB graphics view. The latter practice in the Tent activity included the actions for connecting verbal, symbolic, visual representations of undefined slope, horizontal length of the object, slider of horizontal length, dynamic text of slope computation in GGB graphics view.

As the eventual bridging practice at the end of the activities of Positions of Battens and Fire Truck, the practice of connecting mathematical situation/concept/process and GGB material included the practice of connecting slope computation, line segment GGB tools, and GGB graphic view. This practice included actions such as connecting slope computation, line segments (as the objects and horizontal and vertical length of the objects), checkbox (for slope and/or horizontal/vertical length), dynamic text of slope computation (as the ratio of vertical length to horizontal length), and slider of horizontal/vertical length in GGB graphics view *and* connecting slope computation, line segments (as the objects and horizontal and vertical length of the objects), trace of the line segment (i.e. the object), checkbox (for slope and horizontal/vertical length), dynamic text of slope computation (as the ratio of vertical length to horizontal length), and slider of horizontal length in GGB graphics view. In this kind of bridging practices, teachers dynamically connected visual representation of slope with the visual representations of related quantities and symbolic representation of slope computation as ratio of vertical length to horizontal length in a physical situation in GGB materials. Therefore, teachers should have an understanding on the bridge between the visual and symbolic and the way of fostering this bridge with use of the dynamic software materials.

In this phase, the teacher also provided actions for bridging practices without using the GGB materials and concrete objects. As the most frequently seen practice in this phase, the practice of connecting concepts consisted of practices like connecting conceptualizations of concepts and connecting slope and other concepts. In these practices, the teacher connected slope, ratio, division, and decimal in the context of physical situation; connected slope of an object and slope of a line considering the students' previous knowledge; and connected conceptualizations of distance (e.g. horizontal distance and distance between rope of tent and the tent). In connecting the concepts of slope, the teacher had students connect the given concepts while computing the slopes of the objects in the activities of Positions of Battens and Fire Truck. Furthermore, in connecting slope and line graph, the teacher had students to consider idea of slope of a line crossing the limits of slope as a physical property in different situations. It is important to note that the teacher did not connect the computation of slope of an object to computation of slope of a line.

As another bridging practice, the practice of connecting concept and representations consisted of the practice of connecting concept and real world representation with the action of connecting slope and physical situations in real life (e.g. road situations in the Positions of Battens activity, rope of tent situations in the Tent activity). As the last bridging practice in this phase, the practice of connecting activities consisted of the practice of connecting the activities that included the actions of connecting activities or slope of object. It was seen that the teacher connected the activities for the computation of slope of an object to the last activity (i.e. rope of tent). Therefore, it was seen that teachers should give the activities in a connected way so that students can connect the computations of slope of an object for different physical situations. Moreover, teachers should develop this kind of bridging practices to have students meaningful connections for a concept or a computational algorithm since each activity has a contribution to the computation of the slope of an object from different aspects.

5.1.2 Trimming practices in teaching the slope of an object

While teaching the slope of an object with GGB materials and concrete objects, trimming practices of the teacher included guiding to trim the use of concrete objects, trimming mathematical situation using GGB material, trimming computation using GGB material, and considering mathematical difficulties within the conditions of GGB material (see Table 29). While teaching slope of an object, the teacher considered how much of the complete explanation is needed to compute an object's slope. In this regard, the teacher started to remove the complexity of slope of an object using concrete objects to have students create various situations with them and measure the slope of these concrete objects (i.e. battens in the Positions of Battens activity). Since the teacher knew that the students had an incorrect understanding that the slope is the length of the object/feature or hypotenuse of a triangle, she removed some aspects of the materials (i.e. using similar materials with equal lengths). Thus, she provided the actions in the practice of guiding for making measurements on the concrete object to have students pay attention to the related aspects before giving the slope formula. In addition, she provided similar actions in guiding for the content of the activity using GGB material to have students visualize these aspects in GGB graphics view of the GGB material. After the students justified the slope of an object using concrete object and GGB material, the teacher guided students to compute the slope on concrete objects by encouraging them to compute the slopes of battens as the ratio of vertical length to horizontal length.

Following this phase, the teacher wanted to remove the complexity of slope that has both changing vertical lengths and changing horizontal lengths in physical situations. In this regard, she continued to remove the complexity of slope using GGB materials to have students create various situations that have a constant horizontal length or constant vertical length. Since the teacher knew that students had difficulties in understanding the relations between horizontal length-slope and vertical length-slope, she removed some aspects of the objects in GGB material (i.e. changing lengths of a fire truck ladder with a constant horizontal length and changing length of the rope of a tent with a constant vertical length). Thus, she started to provide the actions in the practices of guiding for the content of the activity using GGB material and analyzing critical issues in a situation using GGB material to have students visually pay attention to variant and invariant related aspects of the slope in a physical situation with GGB materials. For example, the teacher traced the students' understanding of the critical points of the horizontal and vertical lengths in GGB graphics view in the Fire Truck activity. This process was also useful for the students that they determine and express the critical attributes for a slope of an object. Thus, trimming practices can be considered by teachers to help students to access the complexity of determining related attributes of the slope of an object by make sensing of the local idea in the classroom (Wasserman, 2015).

In the meantime, the teacher trimmed the computation of slope using GGB material that included the practices of showing and/or checking lengths using GGB material, showing and/or checking change of slope using GGB material, showing and/or checking computation of slope using GGB material, and showing the conditions that verify the computation using GGB material. In these practices, the teacher provided the actions for mathematical values and relations in the computation of slope of an object using the GGB tools (i.e. slider of vertical length or horizontal length, checkbox of slope and lengths, and dynamic text of slope computation) and the GGB graphics view that were the values of horizontal and vertical lengths, the change of slope and lengths, computation of slope of an object, and conditions of zero slope and undefined slope. In these actions, the teachers' pedagogical approaches were different such that she sometimes wanted students to use a GGB tool and material to check their responses and sometimes wanted to use GGB material to show the conditions of zero slope and undefined slope. While the teacher was paying attention to undefined slope and zero slope, she considered the mathematical integrity of slope outside of the physical situations in which it would be more abstract to the students to compute undefined slope and zero slope for a line graph. Specific to the last activity (i.e. Tent activity), the teacher removed students' mathematical difficulties within the limited conditions of GGB material with the practices of tracing students' difficulties within the conditions of GGB material and responding to students' mathematical questions within the conditions of GGB material. For example, since the horizontal length value of the rope in the rope of tent situation was demonstrated within an interval using a slider in GGB graphics view and since the students could not comprehend the limited value for the horizontal length of the rope in the GGB graphics view, the teacher indicated that this was the result of the GGB material and one can, in reality, use a rope with the length of his/her own choice. That is, even if the GGB graphics view represented various rope situations to show the changing value of the slope when the vertical length was constant and the horizontal length changed, the teacher needed to remove the complexity of varying horizontal length in GGB graphics view.

In the overall process for teaching slope of an object, the teacher provided the actions for trimming without using the GGB materials and concrete objects. One of these actions was explaining how to convert ratio into decimal using division in the practice of explaining mathematical value. That is, the teacher needed to make explanations on decimal converting while the students had difficulties in converting a rational number to decimal in writing slope value. In addition, since the slope would be taught in the coordinate system and in functional situations, the teacher wanted students to develop an awareness about slope in different situations. Therefore, she provided the actions in the practice of determining the relevance of a concept (i.e. slope) in different situations than physical situations. However, even though she wanted to have students think about slope in graphical situations and functional situations, she accepted and revoiced a student's comment about slopes of geometrical figures (i.e. triangles). Furthermore, the teacher localized the factors of slope of an object (i.e. vertical length-slope relation) and gave appropriate terms for slope of an object to convey the meaning of slope in simplicity. When considered from this point view of the teacher, it can be said that she trimmed the content of the slope of an object in narrower contiguity of slope in physical situations. Wasserman (2015) referred to this kind of trimming practices as 'concealing' since he parsed out trimming as concealing and abridging in terms of level of complexity (i.e. local or non-local content). In this regard, this parsing is thought to be practicable when the teacher removed the local complexities of the slope of an object. Thus, in this phase, the teacher removed the local complexities of measurement, lengths, ratio, decimal, and she used these practices for giving the slope of an object. On the other hand, the teacher's paying attention to the situations of the undefined and zero slope and changing of slope when the horizontal length/vertical length is constant both served to the meaning of the slope of an object and the slope of a line. Therefore, the teacher's trimming practices in removing the complexity of slope in physical situations considering the slope of a line can be considered both out of the narrow context of physical situations in the narrow context of middle school mathematics. Thus, from this point of view, it is thought that partitioning the trimming practices by determining the context as local or non-local or a level setting may be an exhausting issue for a teacher and teacher education. Instead of leveling which context is local or non-local, teachers and teacher educators should consider how far to remove or avoid the complexity of a mathematical idea. In either case, the teacher need to have an understanding of the content knowledge situated in the mathematical domain beyond the curricula. This form of knowledge is characterized as horizon content knowledge (HCK) (Ball et al, 2008, Ball & Bass, 2009). Jakobsen, Thames, Ribeiro, and Delaney (2012) also argues that HCK provide teachers "providing teachers with a sense for how the content being taught is situated in and connected to the broader disciplinary territory" (p. 4642). Thus, trimming practices can be oriented through HCK in which requires both familiarity with the advanced mathematics and sensitivity to the mathematical content (Jakobsen et al., 2012).

5.1.3 Decompressing practices in teaching the slope of an object

While teaching slope of an object in physical situations by using GGB materials and concrete objects, decompressing practices of the teacher included preliminary comparisons, interpreting the components of slope of an object using concrete object, interpreting the components of slope of an object using GGB material, and appreciating the use of Dynamic software (see Table 29). Preliminary comparisons while using concrete objects had an initiator role as an introductory level of decompressing. During these comparisons, the teacher posed questions to have students compare positions of battens relating to the slopes of battens without expressing the slope term or the computation of slope of an object. Therefore, preliminary comparisons were considered as the preparation for interpreting the components of slope of an object using concrete object. In this practice, the teacher helped students notice the idea of slope in different positions of battens as a physical situation. Furthermore, interpreting the components of the slope of an object using concrete object or GGB material played a central role in decompressing. The practice of interpreting the components of the slope of an object using concrete object included actions such as exploring the factors that affect and do not affect the slope of an object using battens. In these practices, the teacher's purpose was to develop the understanding of slope concept as a ratio in the physical situation. Therefore, she highlighted the slope with the idea of measuring the slope of an object. In this regard, she emphasized the quantities that affect the slope of the battens to get students determine the factors (e.g. horizontal and vertical lengths of the batten) that affect the slope (e.g. the slope a batten) without giving the slope formula.

The practice of interpreting the components of the slope of an object using GGB material included practices such as interpreting factors of slope using the GGB material and interpreting logic in slope computation using GGB material. More specifically, while interpreting the factors of slope using the GGB material, the teacher provided actions of exploring the factors that affect and do not affect the slope of an object using slider and GGB graphics view that are similar to the actions in the practice of interpreting the components of the slope of an object using concrete object. Although concrete objects enabled the teacher to show the related attributes of an object in concrete terms, the students could not make correct measurements of these attributes. On the other hand, the use of GGB material provided the teacher and the students the exact values of horizontal and vertical lengths of the batten for various batten situations. Thus, while the concrete object helped students to see the related/unrelated attributes of an object related to its slope in reality without computing the slope value, most of the students could visually see the exact values of related/unrelated attributes of the slope of an object visually without computing the slope value by means of dynamic view of the GGB material. In addition, while interpreting logic in slope computation using GGB material, the teacher provided actions for interpreting slope computation algorithm using slider, and or trace, and/or checkbox with dynamic text of slope computation. For example, in the Positions of Battens activity, the teacher started to structure the computation algorithm of slope of an object by relating vertical length and horizontal length values and slope of the object visually in the GGB material. While doing this, she provided students varying vertical length and horizontal length values and slope situations for a constant length object in the GGB graphics view. In this way, the students could evaluate the changing values of horizontal length and vertical length in accordance with the slope of the object visually in the GGB graphics view. In this process, some of the students could deduce that their object had the least slope since it had the least vertical length and the most horizontal length. Then, she gave the meaning of slope as the ratio of vertical length to horizontal length for an object.

As another example, in the Fire truck and the Tent activities, the teacher continued to structure the computation algorithm of slope of an object by relating vertical length and horizontal length values and slopes of the objects both visually and symbolically (i.e. dynamic text of slope computation) in the GGB material. In these actions, the teacher highlighted the complexity of relations between horizontal length, vertical length, and slope while making horizontal length or vertical length invariant and varying others for forming a general understanding of slope of an object. In this process, it was seen that while the teacher and/or the students were using the GGB material, it was important to ask the questions of: What changes in the situation? How does slope change? What does slope depend on? Furthermore, while interpreting logic in slope computation using GGB material, the teacher provided actions for interpreting the change of slope using slider. For example, in the Fire Truck activity, the teacher had students evaluate the biggest slope as they compared the different vertical lengths and slopes in that local environment. In these actions, the teacher highlighted the relation between the biggest slope value and the related factor (i.e. vertical length) when the horizontal length was constant. In this process, it was seen that while the teacher and/or the students were using the GGB material it was important to ask the questions of: Which has the biggest slope? Why is the slope the biggest in that situation (i.e. 10^{th} floor in the Fire Truck activity context)? Thus, with decompressing practices of positions of battens, fire truck and tent activities, it was seen that the teacher had the understanding of proportional reasoning underlying the slope computation for an object (i.e. the slope and the vertical length directly proportional when the horizontal length is constant, slope and horizontal length indirectly proportional when the vertical length is constant). Current studies also emphasized this understanding of slope as steepness within the relationship between proportional reasoning (Cheng, 2015; Cheng, Star, & Chapin, 2013; Lobato & Thanheiser, 2002).

Additionally, while interpreting logic in slope computation in special cases using the GGB material, the teacher provided the actions for interpreting undefined slope using slider and dynamic text of slope computation. In this action, by using slider and dynamic text of slope computation in GGB graphics view, the teacher interpreted the meaning of undefined slope within the slope computation through the explanations of the perpendicular object and undefined result of division of a number by zero. While there are some studies about slope as steepness, childrens' and teacher's understanding about undefined slope is not examined up to time. Thus, it was seen that this way of teaching slope should require the knowledge of division and division by zero (Ball, 1990).

In the overall process for teaching slope of an object, the teacher did not provide the actions for decompressing without using the GGB materials and concrete objects. She unpacked the direct relationship between the vertical length of an object and the slope of an object in the computation of slope of an object using concrete objects and the GGB material. In a similar vein, the teacher unpacked the indirect relationship between the horizontal length of an object and the slope of an object in the computation of slope of an object. That is, she constructed the meaning of slope using multiplicative relationship rather than telling the formula using the dynamic properties of the GGB materials. Thus, all things considered, it was concluded that the teacher developed a deep understanding of why the slope of an object is conceptualized as a ratio and why this ratio is the ratio of vertical length to horizontal length. In addition, she could use this knowledge both to highlight the complexity of slope with the conception of the ratio as an indirect measure (e.g. inclination or steepness). She also used the concrete objects and the GGB materials for the sake of decompressing the slope of an object. In sum, when considered the decompressing practices of the teacher in this phase, the teachers should have a mathematical understanding of why the slope is ratio of vertical length to horizontal length and why it is not horizontal length over vertical length, how this conception of slope can be transferred into the understanding of slope of a line.

5.2 Mathematical Practices in Teaching the Slope of a Line

While designing and enacting the instructional sequence on slope of a line, the teacher developed the activities considering primarily the linear relationship writing multiple representations (i.e. verbal description, picture, table, equation, graph, and real life context) and secondarily the slope of a line and linear equation relation. In this regard, the second phase was formed with the aim of developing an abstraction of the slope of a line in linear functional situations and in the coordinate system. Formerly, the teacher intentionally used a context of linear situation to represent the slope of a line graph that was located in the first quadrant of the coordinate system. Later on, the teacher intentionally did not use a context to relate the slope of a line and the linear equation of a line in the coordinate system. In this phase, fewer mathematical practices emerged when the teacher explained mathematical ideas without using GGB materials and concrete objects compared to the mathematical practices which included using GGB materials in a technology enhanced classroom environment (see Table 30). Thus, in general, while teaching slope of an object, teachers should have the understanding of what is the meaning of the mathematical idea of slope of line as a rate of change between variables and as a ratio of vertical change to horizontal change (Moore-Russo et al. 2011) and as a process of measurement and computation in linear functional situations. In addition, they should have an understanding of how this meanings of slope is in connection with measurement, ratio, and rate of change, covariation *and* differs from slope of an object considering the behavior of a line. The mathematical practices under the categories of bridging, trimming, and decompressing that emerged in this phase were discussed in the following sub-sections.

Table 30. The teacher's mathematical practices in teaching slope of a line

	Practices		
	Bridging	Trimming	Decompressing
Mathematical practices in using GGB materials and concrete objects	• Connect mathematical situation/concept, GG material, and activity sheet [B, H, S] • Connect mathematical situation/concept and GG material [B, H, S] • Connect mathematical situation/concept/ process and GGB material [B, H] • Connect concrete object, mathematical process, and GG material [B] • Connect different software [B] • Connect conceptualizations of concept(s) and GGB material [H, S]	• Trim mathematical situation using GG material [B, H, S] • Trim computation using GG material [B, H] • Trim interrelations in equations and lines using GG material [S] • Do mathematical entailments of GG [S] • Address mathematical difficulties within conditions of GG material [B]	• Preliminary comparisons [B] • Interpret the components of slope of line using GG material [B, H, S]
Mathematical practices while explaining mathematical ideas without using GGB materials and concrete objects	• Connect concept and representations [B, H] • Connect concepts [B, H, S] • Connect activities [B, S] • Connect topics [B]	• Explain mathematical value [B, S] • Explain solution method/ procedure [B, H, S] • Determine the relevance of a concept [B] • Explain mathematical relations [B, H, S] • Explain activity context [B, H, S] • Give appropriate mathematical terms [S]	• Interpret the slope of a line [B, H, S]

Note: The abbreviations in the table show the activity in which a practice emerged. B: Building Design activity, H: Leaking Container activity, S: Slopes and Equations of Lines activity

5.2.1 Bridging practices in teaching the slope of a line

While teaching the slope of a line by using concrete objects and GGB materials, bridging practices of the teacher included: (1) connecting mathematical situation/concept, GG material, and activity sheet, (2) connecting mathematical situation/concept and GG material, (3) connecting mathematical situation/concept/process and GGB material, (4) connecting concrete object, mathematical process, and GG material, and (5) connecting conceptualizations of concept(s) and GGB material (See Table 30).

The practice of connecting mathematical situation/concept, GGB material, and activity sheet consisted of practices such as connecting functional situation, GGB tools, and activity sheet in the Building Design activity, connecting graph of function situation, GGB graphics view, and students' activity sheets in the Leaking Container activity, and connecting equation, line graph, slope, GGB graphics view, GGB algebra view, and activity sheet in the Slopes and Equations of Lines activity. The teacher's intentions in the these practices were to help the students recognize the mathematical situation both in the GGB graphics view and in their activity sheets (i.e. in the Building Design activity), to have them compare and check their drawings in their activity sheets with the GGB graphics view (i.e. in the Leaking Container activity), and to have students combine slopes, equations, and lines in various situations. Thus, it was seen that this bridging practice served different purposes in different activities in this phase about slope of a line. It was also related with the purpose of the activities in the phases. Therefore, teachers should use bridging practices for different purposes considering the purpose of the activity, the students' responses, and the stage of teaching (recognize/combine/construct).

The practice of connecting concrete object, mathematical process and GGB material consisted of two actions: connecting concrete object, slope computation, objects in GGB graphics view and connecting concrete object, similarity, slope computation, objects in GGB graphics view in the Building Design activity. These actions enabled the teacher to guide the students to have them recognize both the relation and differences between the computations. Therefore, she rebuilt the slope concept considering the similar triangles but enlarging the slope of a line to the intervals between the points of the line. In addition, the practice of connecting mathematical situation/concept/process and GGB material consisted of two practices about the slope computation: connecting similarity, slope computation, GGB tools and GGB graphics view and connecting interval between two points, slope computation, GGB tools and GGB graphics view. While the teacher was having the students construct the computation of slope of a line graph, she started by connecting similar triangles (equivalence of the ratios) to the ratio of vertical change to horizontal change for an increasing straight line graph in the Building Design activity and continued by connecting slope to the interval between two points for a decreasing straight line graph in the Leaking Container activity that helped the teacher to give the slope of a line with the vertical change and horizontal change terms beyond vertical length/distance and horizontal length/distance. Thus, the teacher connected mathematical ideas of similarity, computation of slope of a line between two points on an interval of the line, and ratio of vertical change to horizontal change in linear situations. That is she used similarity while transiting from computation of slope of an object to computation of slope of a line. This connection can also be related the teacher's understanding of the ratio that is dominant in conceptualizing slope. In sum, teachers can use such a geometric approach to adapt it to the algebraic approach of slope computation.

As the prevailing bridging practice in this phase, the practice of connecting mathematical situation/concept and GGB material consisted of three practices: connecting functional situation, line graph, GGB tools, and GGB graphics view; connecting point, line graph, and GGB graphics view; and connecting line/slope of a line, equation/terms in an equation, GGB algebra view, and GGB graphics view. The practice of connecting functional situation, line graph, GGB tools, and GGB graphics view included actions like connecting points of line graph for functional situation, slider tool, and trace tool in GGB graphics view; connecting line graph and linear relationship for functional situation, line tool in GGB graphics view; and connecting points of line graph, dynamic figure of the functional situation, and slider tool in GGB graphics view. In these actions, the teacher used or encouraged students to use the dynamic GGB tools in GGB graphics view of the GGB materials while guiding them to draw the line graph for a given linear functional situation. Therefore, before constructing the slope of a line, teachers should give students time to construct the graph of the functional situation using or letting the students use the dynamic features of GGB. Instead of finding any two points and connecting them to draw a line, the teacher showed the successive points of the line graph in a coordinating the variables in a linear situation. This way of constructing the graphs could also serve as a way to reason about the graph in terms of coordination between the variables and teachers attempted to use this reasoning in constructing slope of a line as rate of change in further practices. In other words, teachers should have an understanding that the line graph for a linear situation is the representation of covarying quantities and should use this understanding before computing slope of a line graph of a linear situation.

In addition, the practice of connecting line/slope of a line, equation/terms in an equation, GGB algebra view, and GGB graphics view included actions such as connecting equation and line in GGB algebra view and GGB graphics views; connecting equation, slope of a line/an equation, slider in GGB graphics view and GGB algebra view; and connecting line, equation, terms in equation, GGB graphics view and GGB algebra view. This practice of the teacher that emerged in the Slopes and Equations of Lines activity was refinished in different actions. While the connection was between a line and an equation in the first action above, there were connections among coefficient of x and constant value in an equation and lines for various equations/lines in the second and third actions. Therefore, the teacher specifically gave these formal connections in the Slopes and Equations of Lines activity after the students developed an understanding of the equation and graph of functional situation and the slope of a line graph of a linear situation in the previous activities.

The practice of connecting conceptualizations of concept(s) and GGB material consisted of connecting conceptions of slopes and GGB graphics view and connecting forms of equations, GGB graphics view, and GGB algebra view. In the former practice, for the Leaking Container activity, the teacher provided actions for making connections between the sign of slope as the behavior of line graph and algebraic expression of slope computation text in GGB graphics view and between the vertical change as a dimension of magnitude of slope and algebraic expression of slope computation text in GGB graphics view. Therefore, in this bridging practice, the teacher used the slope conception of the behavior indicator of line graph (Moore-Russo et al., 2011). Moreover, she used this conception in connection with the conception of slope as an algebraic ratio. Thus, teachers should gather these conceptions of slope in their bridging practices for further understanding about the sign of slope and the magnitude of slope. In the latter practice, for the Slopes and Equations of Lines activity, the teacher provided actions for making connections between standard and slope-intercept forms of equations in GGB algebra view and lines of equations in GGB graphics view. In that bridging practice, the teacher used GGB as a convertor and showed on the screen that two lines of equations that seem different actually show the same graph. Thus, teachers can use GGB materials for both connector of conceptions of slopes and a convertor of form of equations in bridging practices.

5.2.2 Trimming practices in teaching the slope of a line

In the second phase, even though the practice while using GGB materials drived this phase, the teacher also provided actions for bridging practices without using concrete objects and the GGB materials. It was seen that the teacher usually provided actions of these bridging practices (See Table 30). while starting an activity or a task in an activity. For example, at the beginning of the activities Building Design and the Leaking Container, the teacher provided actions of connecting linear relationship and representations. On the other hand, the teacher provided actions of connecting topics while concluding an activity in connection with science. In addition, she provided actions of connecting conceptualizations of slope while students were making computations of slopes of the line graphs in functional situations.

While teaching the slope of a line by using concrete objects and GGB materials in the second phase, the prominent trimming practices of the teacher included trimming mathematical situation using GGB material, trimming computation using GGB material, and trimming interrelations in equations and lines using GGB material. The practice of trimming mathematical situation using GGB material consisted of the practices of guiding for content of the activity using GGB material, and guiding students to move between GGB algebra view and GGB graphics view. In the actions of these practices, the teacher removed the complexity of equations of linear situations and line graphs of the equations using the GGB tools and GGB graphics view. Since the teacher knew that some students had difficulties in graphing and constructing equations, she constructed the activity considering these issues and considered these local complexities in the moments of teaching. In addition, the teacher helped the students to make transitions between algebra view and graphics view of the GGB easily to convey the slope computation, linear equation, slope and y-intercept of a line in a coherent manner. In addition, the practice of trimming computation using GGB material consisted of the practices of showing and/or checking points using GGB material, change (i.e. horizontal change and vertical change) using GGB material, showing and/or checking slope computation (i.e. computing the slope of a line on an interval) using GGB material (i.e. dynamic text of slope computation), and showing and/or checking drawing with GGB material. Furthermore, the practice of trimming interrelations in equations and lines using GGB material consisted of the practices of showing a relation (i.e. slope-line-equation) using GGB tools, GGB graphics view and GGB algebra view, showing an independency (i.e. slope of a line of an equation is independent from the value of y-intercept of the line) using GGB tools, GGB graphics view and GGB algebra view, and showing forms of equations for slope computation in GGB algebra view. In these practices, the teacher considered giving the slope of a line in this middle school context and used the language of vertical change, horizontal change, and ratio. Even though she showed the relations between the quantities in a graph, she did not use the term of rate of change. However, she had the idea of giving the slope of a line as rate of change in the planning of the instruction.

In the second phase, the teacher also provided actions for trimming practices without using concrete objects and the GGB materials. As the most seen practice in this phase, the practice of explaining the solution method/procedure consisted of explaining the computation of slope of a line graph both as an algebraic ratio and as the direction of change, explaining solutions of equations/ graphs of equations, and explaining how to convert forms of linear equations. In the actions of these practices, the noteworthy one was explaining the slope as a direction of change of a line graph in vertical variable with change in horizontal variable since this was a lower level behavior in covariational reasoning (Carlson et al., 2002). However, the teacher could not take this behavior (i.e. the behavior of coordinating the direction of change in variables) to the further behaviors of covariational reasoning. The other actions in these practices usually emerged with the teacher's consideration of clarifying the meaning of solution of an equation and algebraic process of converting the forms of equations. As the other frequent practice in this phase, the practice of explaining mathematical relations consisted of the practices of localizing the relationship for the linear equation and localizing the slope of line in a linear equation, providing examples of equations and slopes for slope-equation relation and for relating forms of equations.

On the other hand, in some of the actions, the teacher trimmed too much which could produce incorrect explanations even in the elementary context and these actions were named as opposite examples of trimming by the researcher. For example, in the direction of a student's question in the Building Design activity, the teacher could not determine the relevance of the unit of slope considering different contexts of linear functional situations. Based on her knowledge of the curriculum about the ratio and rate (relatively, 'birimli oran' for comparison of two similar quantities and 'birimsiz oran' for comparison of two quantities with different units in Turkish curriculum (MoNE, 2009, 2013a), she explained the unit of slope is nonexistent in that activity. Therefore, as she removed the unit of slope of a line, she could not retain the mathematical integrity of the slope even for the contexts of linear functional situations in middle school context. Here, the teacher had the knowledge of curriculum about the description of ratio and rate in the Turkish curriculum; however, having this knowledge may not helped the teacher to explain the unit of slope since the rate is not conceptualized within the covariational aspect in the curriculum. On the other hand, at that moment, she should have considered the other linear functional situations (i.e. the situations that the slope has a unit such as slope as a measure of speed in the linear graph of distance-time relation) since she was going to enact the activity of leaking container that involved the linear graph of amount of water (volume)-time relationship in which the slope gave the measure of speed of leaking. In this regard, it is concluded that both knowledge of advanced mathematics and curriculum are important in promoting the development of trimming practices in the moments of teaching.

5.2.3 Decompressing practices in teaching the slope of a line

While teaching slope of a line by using GGB materials and concrete objects, decompressing practices of the teacher included preliminary comparisons, and interpreting the components of slope of a line using GGB material (see Table 30).

Preliminary comparisons while using concrete objects and the GGB material had an initiator role as an introductory level of decompressing the meaning of slope of a line. During these comparisons the teacher posed questions to have students compare the slope of an object on a concrete object and the slope of a line in GGB graphics view without giving the formula of slope of a line. Therefore, preliminary comparisons were considered as the preparation for transiting from slope of an object to slope of a line. In this practice, the teacher had students compare the computations of slope of an object and slope of a line graph (i.e. the graph of number of windows-number of floors relation in the Building Design activity) in GGB graphics view, compare computations of slopes of an object for different intervals on the concrete object so that this understanding of invariance of slope of an object (i.e. linear object) in different intervals can be transferred to computing the slope of a line between any two points, and also compare the computations of slope of a line graph on different intervals in GGB graphics view. For example, she posed questions about the essence of the slope of an object by comparing and contrasting the differences between an object (a limited length, limited horizontal and vertical dimensions) and a line (unlimited length, unlimited horizontal and vertical dimensions) and comparing the height and lengths of different intervals on the object that had the same slope.

The practices of interpreting the components of slope of a line using GGB material included practices such as interpreting logic in computation of slope of a line in GGB material, interpreting logic of the sign of the slope as behavior of a line in GGB material, and interpreting slope-equation relation in GGB material. More specifically, while interpreting logic in computation of slope of a line in GGB material, the teacher provided the actions for interpreting variables in computation of the slope of a line using points of a line in GGB graphics view, interpreting computation of slope of a line using dynamic points of a line, checkboxes (i.e. slope, vertical change, horizontal change) and dynamic text of slope computation in GGB graphics view, and interpreting constant slope of a line in computation using dragging dynamic points, slope checkbox, and dynamic text of slope computation in GGB graphics view. These actions showed how the teacher unpacked the meaning of slope of a line algebraically as the ratio of vertical change to horizontal change by using the GGB material. For example, in the Building Design and the Leaking Container activities, the teacher had student interpret the computation algorithm of the slope of a line with the following text in the GGB graphics view:

$$\frac{vertical\ change}{horizontal\ change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{dynamic\ subtraction}{dynamic\ subtraction}$$

$$= \frac{dynamic\ value\ of\ the\ difference}{dynamic\ value\ of\ the\ difference} = dynamic\ decimal\ value$$

In addition, although the GGB material of the Leaking Container activity provided opportunities for the teacher to interpret the situation while coordinating the constant rate of change of the dependent variable (volume of the water) with uniform changes in the dependent variable (time) that could be used to interpret slope of a line as constant rate of change, she did not highlight the rate of change in this process. These actions showed the importance of having upper covariational reasoning level in teaching slope for the practicing teachers (Carlson et al., 2002). Therefore, middle school mathematics teachers should have an understanding about the rate of change in covarying quantities for linear functional situations and should use this understanding in classroom sessions to unpack the meaning of slope of a line for narrow understanding about the constant rate of change and for broader understandings about average rate of change of a curve for secant lines and instantaneous rate of change of a curve for slope of a line tangent, and for understanding derivative further away (Kertil, 2014).

In addition, while interpreting logic of the sign of the slope as behavior of a line in GGB material, the teacher provided actions for obviating pictorialization of positive slope in GGB graphics view, interpreting the relation between positive slope and change in horizontal and vertical variables, and interpreting the negative sign in computation of slope of a line using GGB tools in GGB graphics view. These actions showed that the teacher highlighted the sign of the slope of a line with the direction of the changes in the variables in linear functional situations. Here, the teacher coordinated how the direction of the change in the dependent variables varied when the independent variable increased or decreased. In other words, the teacher interpreted the slope of a line as the points on the graph move from both left to right and right to left. However, the students mainly used the way of interpreting the sign of the slope of a line with coordinating the direction of the line when the independent variable increases. Therefore, the term ‘direction’ was used for the direction of the line (e.g. increasing straight line or decreasing straight line) not only when the independent variable increases but also the independent variable decreases. Thus, when the teacher was highlighting the meaning of the sign of slope of a line, she also coordinated the direction of the variables. In this regard, it was concluded that teachers could use their reasoning on coordinating the direction of the variables to unpack the sign of the slope. While this reasoning was seen as a lower level reasoning (i.e. level 2 direction) in the covariational reasoning framework (Carlson et al., 2012), it should be used in teaching slopes of graphs of linear equations.

Furthermore, while interpreting slope-term relation within an equation of a line in GGB material, the teacher provided the actions for exploring slope of an equation and terms in an equation in GGB graphics view and GGB algebra view, and interpreting slope of an equation and terms in an equation using slider in GGB graphics view and GGB algebra view. Therefore, the teacher had a conceptualization of parametric coefficient for the slope (Stump, 2001b). In addition, she highlighted the reason behind this conception by creating various lines, equations, and slopes using the GGB material. In this regard, the teacher effectively transit from GGB algebra view to GGB graphics view. More specifically, while examining the slope of a line and the slope of an equation, she had students interpret both the algebraic representation (slope-intercept form of the equation [$y = mx + n$] in GGB algebra view), graphical representation (line of the equation in GGB graphics view) and dynamic tools that check the line and the equation (slider-m and slider-n in GGB graphics view). Therefore, she helped students to dig the interrelations between the slope and the terms in the equation. It was concluded that since the teacher had a strong conceptualization of slope as parametric coefficient, she could use these conceptualization for unpacking the meaning of the slope in the equation with the GGB material.

On the other hand, when the teacher interpreted the slope of a line without using concrete objects and GGB materials, she provided the practices of interpreting constant slope of line, interpreting the sign of slope of a line, and interpreting change in slope of a line. However, these practices were mainly considered as the preliminary decompressing practices. Even though the actions of these practices were started to highlight the slope of a line for giving the concept in a comprehensible way, the teacher usually did not complete these actions while making explanations without using concrete objects or GGB materials and usually continued with the decompressing practices while using concrete objects and GGB materials.

5.3. Mathematical Practices in Teaching the Solution of a System of Equations and the Slope Relation

While designing and enacting the instructional sequence on the solution of a system of two linear equations and the relation between the slopes of the equations and the solution of the system, the teacher developed activities considering primarily the solution value of a system of equations for two intersecting line graphs with positive slopes within the representations of table, equation, graph, and real life context; secondarily the interrelations between equation, slope, solution set, the relative position of two lines; and thirdly the solution value of a system of equations for two intersecting line graphs with positive and negative slopes within the representations of table, equation, graph, and real life context. In this regard, the third phase was formed with the aim of developing an understanding of the solution set of system of equations graphically in a context and in relation with the slopes of the lines of the equations. During the moments of teaching, various mathematical practices emerged in using GGB materials and in explaining mathematical ideas without using GGB materials in a technology enhanced classroom environment (see Table 31). The mathematical practices under the categories of bridging, trimming, and decompressing that emerged in this phase were discussed in the following sub-sections.

Table 31. The teacher's mathematical practices in teaching the solution of a system of equations and the slope relation

	Practices		
	Bridging	Trimming	Decompressing
Mathematical practices in using GGB materials	• Connect mathematical situation/concept and GG material [M, E, S] • Connect mathematical situation/process and GG material [M, E] • Connect conceptualizations of concept(s) and GG material [M, E, S] • Connect mathematical concept, GG material, activity sheet [E, S] • Connect mathematical situation/concept, GG material, and activity sheet [S]	• Trim mathematical situation using GG material [M, E, S] • Trim graphical representation using GG material [M, E, S] • Trim computation using GG material [E, S] • Address mathematical difficulties within conditions of GG material [E]	• Preliminary comparisons [M] • Interpret the components of solutions of system of equations using GG material [M, E, S] • Interpret the components of slope of line using GG material [E]
Mathematical practices while explaining mathematical ideas without using GGB materials	• Connect concept and representations [M, S] • Connect concepts [M, E, S] • Connect activities [E] • Connect topics [S]	• Explain solution method/procedure [M, E] • Explain mathematical relations [M, E, S] • Explain activity context [M, E, S] • Explain mathematical value [E] • Give appropriate mathematical terms [E] • Reveal students' mathematical difficulties/ misunderstandings [E, S]	• Interpret the system of equations [M, E, S] • Interpret the slope of a line [S]

Note: The abbreviations in the table show the activity in which a practice emerged. M: Mobile Operators activity, E: Equations Systems activity, S: Stores activity.

5.3.1 Bridging practices in teaching the solution of a system of equations and the slope relation

While teaching the solution of system of equations and slope relation by using GGB materials, bridging practices of the teacher included connecting mathematical situation/concept and GGB material; connecting mathematical situation/process and GGB material; connecting conceptualizations of concepts(s) and GGB material; and connecting mathematical concept, GGB material and activity sheet (see Table 31).

As the prevailing bridging practice in this phase, the practice of connecting mathematical situation/concept and GGB material consisted of three practices: (1) connecting equation, line graph, GGB algebra view, GGB graphics view, and/or GGB tools, (2) connecting equation, line graph, solutions, GGB tools, and GGB graphics view, and (3) connecting slope, line graph, GGB graphics view and/or GGB tools. In the practice of connecting equation, line graph, GGB algebra view, GGB graphics view and/or GGB tools, there was the action of connecting equation of a line in GGB algebra view and line graph in GGB graphics view for two situations in the Mobile Operators activity and for various situations in the Equations Systems activity. As the other actions of this practice, the teacher provided two actions, namely, connecting equation of a line graph in GGB algebra view and points of the line graph in GGB algebra view and connecting equation, line graph, checkboxes (i.e. showing equations within a static text) in GGB graphics view in the Stores activity. In this practice, the teacher posed questions and made explanations to guide students about making connections among the algebraic and graphical representations of linear relationship as a primary understanding of slope of a line/a linear equation. Additionally, in the practice of connecting equation, line graph, solutions, GGB tools, and GGB graphics view, the teacher provided three actions, which are connecting points of a line graph, solutions of an equation of the line graph, coordinates of the points, and slider tool (i.e. sliders of time variable for two situations) in GGB graphics view in the Mobile Operators activity; connecting a point in a line graph and a solution of an equation for a functional situation of the number of sold products-time relationship in GGB graphics view; and connecting the points of a line graph, solutions of an equation of the line graph for a functional situation of the number of sold products-time relationship, the slider tool (i.e. time sliders), and the trace tool (i.e. trace of a point for a line) in GGB graphics in the Stores activity. Therefore, in this practice, the teacher posed questions and made explanations to guide students about making connections about algebraic, graphical and verbal representations of solutions of an equations in a context as a primary understanding of solution set of system of equations. Furthermore, in the practice of connecting slope, line graph, GGB graphics view and/or GGB tools, the teacher provided two actions, namely, connecting sign of the slope and behavior of the line graph in GGB graphics view and connecting the slopes of line graphs and checkboxes (i.e. showing the slope values of the line graphs with slope tool) in GGB graphics view. Therefore, in this practice, the teacher posed questions and made explanations to guide students about making connections between slopes and line graphs on the behavior of line graphs (i.e. increasing or decreasing) in which one of the graphs is an increasing line with a positive slope and the other is a decreasing line with a negative slope.

The practice of connecting mathematical situation/process and GGB material consisted of practices of (1) connecting process of solving system of equations and GGB graphics view, and (2) connecting computation of slope, GGB graphics view or GGB algebra view. In the practice of connecting computation of slope and GGB graphics view or GGB algebra view, the teacher provided actions for making connections between slope value in GGB algebra view and computation of slope of a decreasing line, slope of an equation and slope-intercept form of the equation in GGB algebra view, computation of slope of a line and the triangle region that is bounded by axes in GGB graphics view, *and* computation of slope of a line and the conception of vertical change/horizontal change in GGB graphics view. That is, she made explanations and posed questions to guide students to make computation of the slope of the line using the algebraic ratio of the vertical change to the horizontal change by expanding the conception of vertical/horizontal using the GGB graphics view in the Equations Systems activity.

As one of the important behaviors in the representational and abstract form of algebra teaching, the practice of connecting conceptualizations of concept(s) and GGB material consisted of three practices: (1) connecting representations of intersection point and GGB graphic view, (2) connecting conceptualizations of solution set and

GGB graphics view, and (3) connecting conceptualizations of linearity and GGB graphics view. In the practice of connecting representations of intersection point and GGB graphic view, the teacher provided the action for connecting symbolic representation with coordinates and verbal description of intersection point for the functional situations in the Mobile Operators activity. In addition, in the practice of connecting conceptualizations of solution set and GGB graphics view, the teacher provided the action for connecting the intersection point of lines (coordinates and position) and the solution set of system of equations. Therefore, the teacher posed questions and made explanations to guide students about making connections between the intersection point and solutions set for the system of equations that has one solution. Furthermore, in the practice of connecting conceptualizations of linearity and GG graphics view, the teacher provided the action for connecting visual representation and verbal description of linearity, functional situation, and GGB graphics view. That is, the teacher emphasized the linearity and the graphical representation with the points of the line graph in the Stores activity. As a result, it was seen that even though the purpose of this phase was not to develop the conception of linearity, the teacher mentioned this understanding while guiding the students to draw the line graph for a functional situation.

The practice of connecting mathematical concept, GGB material, and activity sheet consisted of three practices in the Equations Systems activity: (1) connecting GGB material of another activity, slopes of equations/lines, solution set, and activity sheet, (2) connecting equations, positions of lines, GGB algebra view, GGB graphics view, and activity sheet, and (3) connecting equations, position of lines, slopes of lines, solution set, GGB algebra view, GGB graphics view and/or GGB tools, and activity sheet. The teacher provided the actions of these practices in a sequence while using GGB algebra view, GGB graphics view, the slope checkbox that showed the slope of the lines in GGB graphics view, and the table in the activity sheet of Equations Systems that combined equations in a system of equations, positions of lines of the equations, slopes of lines, solution set of a system of equations for various situations. Thus, it was seen that the teacher used this activity mainly to create various systems of equations with intersecting lines or parallel lines and to connect the aforementioned concepts. Even though the previous activity involved a system of equation, she emphasized the connection among the slopes of the lines and the solution set of the system of equations in this activity to have the students make their own connections considering various situations after understanding a system of equations in the context of the previous activity (i.e. the Mobile Operators activity).

As it was seen in the previous phase, the practice of connecting mathematical situation/concept, GGB material, and activity sheet consisted of the practice of connecting the graph of functional situation, GGB graphics view, and students' activity sheets. In this practice, the teacher provided two actions. First, she made connections between the points in students' activity sheets and the points in the GGB graphics view for the graph of functional situations in the Stores activity. Second, she made connections between the line graph in students' activity sheet and the line graph in the GGB graphic view for the functional situations in the Stores activity. The teacher may feel the necessity of performing these actions to have students check their own graphing process from drawing points to line graph.

In the third phase, the teacher also provided bridging practices without GGB materials. As opposed to the previous phases, the teacher made stronger connections among slope, rate, and functional situation/equation in this phase. Moreover, she provided these practice at the end of the phase to give various examples of functional situations and/or their equations. Furthermore, the teacher made connections between these concepts and high school mathematics and science not to give the meaning of advanced concepts but to encourage students to have a high opinion of mathematics they learned. Thus, teachers can use this kind of advanced mathematics knowledge in accordance with school mathematics knowledge to provide a coherent understanding of mathematics. This can lead students to develop a mathematical understanding in a connected way.

5.3.2 Trimming practices in teaching the solution of a system of equations and the slope relation

While teaching the solution of system of equations and slope relation, the teacher considered how much of the complete explanation is needed to understand the solution of system of equations and the relation between the solution set of the system of equations and the slopes of the equations. While teaching the slope of a line by using

GGB materials, the prominent trimming practices of the teacher included trimming mathematical situation using GGB material, trimming graphical representation using GGB material, and trimming computation using GGB material (see Table 31). The practice of trimming mathematical situation using GGB material consisted of the practice of showing and/or checking equations using GGB material, showing satisfying equation using GGB material, creating different mathematical situations using GGB graphics view, and guiding for the content of the activity using GGB material. In the actions of these practices, the teacher had students use GGB material (e.g. equation in GGB algebra view) to check their constructions of equations for the system of equations in a functional context and the coordinate system. Then, the teacher had students use the GGB material to create different systems for parallel lines and intersecting lines in the coordinate system. In this regard, the teacher removed the mathematical complexities of the equations and the positions of lines of the equations relative to each other in these actions.

In addition, the practice of trimming graphical representation using GGB material consisted of the practice of showing and/or checking the drawing of graph using GGB material, showing and /or checking intersection point using GGB material, showing and/or checking points in situations using GGB material, showing and/or checking solution set using GGB graphics view, and limiting the use of GGB material for drawing. In the actions of these practices the teacher paid attention to the students' understanding of graphical context of the system of equations. Since the teacher knew that the students had difficulties in graphical representation of a system of equations, she paid attention students convey the conceptual meaning of intersection point geometrically as the coordinates of the point in the coordinate system and as the solution set in the system of equations. In this process, the teacher mainly used the GGB material to help students check their graphs. In addition, she had the moments that constrain the GGB algebra view to have students focus on the GGB graphics view, e.g. the Stores activity.

Furthermore, the practice of trimming computation using GGB material consisted of the practices of showing and/or checking the slope of a line using GGB material, limiting the use of GGB material for slope computation, and comparing the slopes of parallel lines/equations using GGB material. In the actions of these practices, the teacher paid attention to have students easily recognize the slopes of the lines in the systems of equation using the GGB material and she continued to have students compute the slope of a line as the algebraic ratio. Therefore, the teacher's intention was to provide the integrity of the slope-system of equations relations in the middle school context. However, while the teacher removed the complicated issues about the equations and lines given in a related sequence with the use of GGB material (e.g. dynamic parallel lines and intersecting lines in equations were created using the GGB materials), she did not use a GGB material for a system of equations for the coincident lines.

On the other hand, when the teacher taught considering hiding the complexities about the solution of system of equations and the relation between the solutions set of the system of equations and the slopes of the equations, there were practices of explaining mathematical value (i.e. the solution set value of system of equations and the slope values of lines/equations in a system of equations, explaining solution method/procedure (i.e. computation of slope of a line and solving solutions of system of equations), explaining mathematical relations (i.e. localizing the relationship for the linear equation, localizing the slope of a line in a linear equation, providing examples of systems of equations to relate solution sets and positions of lines), and explaining the mathematical activity (i.e. explaining activity procedure considering construction of the table, context for graphing, constructing the graph, the equation), using appropriate mathematical terms (i.e. the terms of coincident lines), and revealing students' difficulties about the system of equations and slope. It was seen that even the teacher used the GGB materials for removing the complexity of a mathematical idea, she could need to make mathematical explanations without using the GGB material. For example, both for this phase and for the previous phase, the most prominent action in these practices was explaining the labels of the axes in the graphs of functional situations. There was usually a student in the class that needed to be explained the labels of the axes within the activity context. Therefore, teachers should clarify the dependent and independent variables in the graph of the functional situation for understanding both the slopes of the line graphs and the solution set of the system of equations in the graphical representation. As another example, the teacher provided the action of explaining solutions of system of equations in the activities by making the calculation to show the coordinates of the intersection point as a solution and explaining and/or encouraging the students to use algebraic ways of solving system of equations. The teacher needed to simply explain the solution process for a system of equations algebraically through verbal descriptions and on the board. Therefore, even the teachers prepared the activities within the GGB materials, they should clarify the solution process of the system of equations algebraically to have a clear understanding on the solution set of system of equations in the coordinate system.

5.3.3 Decompressing practices in teaching the solution of a system of equations and the slope relation

While teaching solutions of system of equations and the slope relation by using GGB materials, decompressing practices of the teacher included preliminary comparisons, interpreting components of the solution of a system of equations using GGB material, and interpreting components of slope of line using GGB material (see Table 31). Preliminary comparisons while using the GGB material had an introductory level of decompressing the meaning of solution of a system of equations. During these comparisons, the teacher posed questions to have students compare the solutions of the functional situations on the lines of the equations in the GGB graphics view. Therefore, the preliminary comparisons were considered as a transition from interpreting a point as or an interval for the solution of an equation within the context of the functional situation to interpreting the solution set of a system of equation.

In addition, the practices of interpreting the components of solutions of system of equations using GGB material included practices such as, interpreting intersection point on functional situations using GGB material, and interpreting the components of solution set for system of equations using GGB material. In these practices, the prominent action about slope was interpreting positions, slopes, solution set, and system of equation. The teacher highlighted this complex relationship to have students generalize the relationship between the positions of lines, slopes of lines, solution set, and system of equations using GGB graphics view and the table in the students' activity sheet. Furthermore, the practices of interpreting the components of slope of line using GGB material included the practice as interpreting computation of slope for system of equations. In this practice, the teacher provided actions for interpreting the computation of slope with changes between two points in GGB graphics view, and interpreting the slope and solution set in a system of equations using GGB graphics view. Therefore, the teacher had conceptualization of slope as a determining feature for the system of equations and she used this conception to make students understand the solution of system of equations with parallel and intersecting lines graphically by using GGB material. On the other hand, most of the teachers did not have these conceptions (Stump, 1999, 2001a) and some of the teachers did not use this conception in teaching even they had it (Nagle & Moore-Russo, 2013b). In this regard, it was a worthwhile practice that a teacher highlighted the slope relation with the system of equations even though the curriculum (i.e. MoNE 2009, 2013a) did not mention this property. All the more amazing, the recently revised middle school mathematics curriculum (MoNE, 2017) removed all the objectives about the system of equations that neither algebraic nor geometric ways of solutions of system of equations be seen in the algebra standards.

Interestingly, but not surprisingly, while the teacher was teaching about relating the slope and the system of equations, she interpreted the slope of a line as rate of change without using the GGB materials. Even though the teacher did not interpret slope as rate of change in the previous classes with or without using GGB materials, she posed questions and make explanations to have students interpret the slope as unit rate and rate of change and the negative slope and the positive slope with the rate of change conception. This might be the result of the teacher's development of mathematical understanding of slope in this process since she watched and reflected on the previous classroom sessions before that teaching. In addition, this might be the result of the teacher's development of knowledge of teaching slope as rate of change. Furthermore and most probably, this might be the result of the fact that the teacher could not integrate her mathematical understanding of rate of change in teaching with GGB material. That is, even though she could use the GGB materials of the system of equations to highlight the slope of a line as rate of change in the functional situations, she preferred to interpret this meaning of slope at the end of the lesson without using technology. In this regard, whether system of equations is taken part in the curriculum or not, teachers should have the aforementioned mathematical understanding about system of equations and slope relation for having a rich conceptualization of slope in the moments of teaching.

Consequently, it was portrayed the mathematical practices of a middle school mathematics teacher in teaching the slope of an object, and the slope of a line, and the solution of a system of equations and the slope relation in terms of both using GGB materials and not explaining mathematical ideas without using GGB materials during the classroom sessions.

5.4 Interrelations among the Mathematical Practices in the Moments of Teaching

While responding to the question of how the teacher interrelates the mathematical practices both the teacher used concrete objects and GGB materials and not used concrete objects and GGB materials during enacting the instructional sequence in a technology enhanced classroom environment, it was seen that there were various conjectures among these practices.

While most of the bridging practices emerged with the trimming practices and decompressing practices, bridging practices also exist primarily for connecting mathematical ideas, concrete objects and/or GGB materials. In this regard, it was concluded that a teacher needs bridging practices primarily for showing the mathematical connections among the mathematical ideas and tools, and secondarily for fostering or advancing trimming and/or decompressing practices. In this process, the use of concrete objects (i.e. battens) provided a ground for conversation about slope both for teacher-student interactions and student-student interactions in bridging, trimming, and decompressing practices. Parallel to Thompson's (1994b) suggestions, the nature of this conversation was the ways of thinking about battens and on the meanings of various actions with battens. In addition, the use of GGB materials are also provided a ground for accessing the accuracy of the mathematical ideas (i.e. concepts and processes) and unpacking the complexity of the mathematical ideas beyond providing a ground for conversation.

Furthermore, decompressing practices were also emerged with tagged behind the trimming practices. However, there were also moments that the decompressing practices were followed just the bridging practices. In this regard, it can be concluded that the interrelations among these practices varies depending on the mathematical idea, on the students' content knowledge in the context, the teacher's understanding of this mathematical content and her knowledge of students' understanding in this content. For example, while interpreting the logic in slope computation algorithm using GGB material as a decompressing practice, the teacher also needed to show the accurate computation using GGB as a trimming practice. This was also related with planned aim of the activity and the properties of the tools in the activity. In this sense, the teacher should have a conscious that what is she doing with the mathematical content for what in the moments of teaching. Thus, these practices may be seen as the core practices in mathematics teaching. In this regard, even though we investigated these practices in classroom teaching practice, it should have been noteworthy to examine the existence of these practices and the interrelations among them in planning practice.

5.5 Suggestions on the Teaching of Slope in the Middle School Mathematics Context

Considering the findings regarding a middle school mathematics teacher's mathematical practices in teaching slope to eight graders and the students' understanding in this research process, suggestions were presented about what kind of mathematical practices teachers should have in an instruction focusing on learning-teaching slope in grade eight in the context of Turkish middle school mathematics curriculum. Considering the findings from a middle school mathematics teachers' mathematical practices in teaching slope to eight graders and the students' practices and understanding in this process in a technology-enhanced classroom environment, we present suggestions about what kind of mathematical practices teachers should have in an instruction focusing on learning-teaching slope in grade eight in the context of Turkish middle school mathematics curriculum. While providing suggestions considering the national curriculum, the researcher presents her ideas on the dimensions of learning goals, tools, activities and mathematical practices in the classroom discourse (Gravemeijer et al., 2003). In the national curriculum (MoNE 2013a, 2017), the computation of slope is contextualized both in physical situations and linear functional situations. However, in the curriculum, physical situations are introduced as if they were the models of the slope of a line in real life situations which may constraint slope of a line to steepness and geometric ratio. Similar constraints regarding conceptions of slope of a line was also mentioned in various studies (Dündar, 2015; Nagle & Moore-Russo, 2013b; Stump, 2001a). While some of them mentioned the relation between the teachers' conceptions of slope and their ways of instruction about slope (Nagle & Moore-Russo, 2013b), they were not make a discrimination of conception of slope of an object and slope of a line. Therefore, the discrimination between slope of an object and the slope of a line it should be precisely made while mathematics teachers and curriculum developers consider the connections between them. In this regard, different from the middle school mathematics national curriculum (MoNE 2009, 2013a, 2017), the researcher suggested not to giving slope of an object under the objective about the slope of a line for better teaching of slope and understanding of slope. Therefore, mathematical understanding of slope should be developed by calculating and justifying the slope both in physical situations and linear functional situations in middle school mathematics.

On the one hand, the slope is conceptualized and computed for an object/a feature (i.e. straight) in the context of physical situations. Therefore, teachers should consider this understanding of slope as one of the main learning goals. This meaning of slope should be given before the slope of a line since it is more concrete for students to make connections between their real life experience and their previous mathematical understandings. The slope of an object can be thought in two ways as the ratio of vertical distance to horizontal distance and as the rate of change of vertical distance to horizontal distance. Therefore, teachers should also have a deep understanding of ratio, rate, and proportional relationship to give slope of an object conceptually within the procedural understanding. In this regard, as mentioned in the trimming, decompressing, and bridging practices of the teacher in the findings section, teachers should be aware of what the consequences may be when they remove the complexity of rate, highlight the complexity of rate, and make connections between slope, ratio and rate for the slope of an object. It must be clear that the slope of an object can be explained both as a ratio and a rate and the conception a teacher will use depends entirely on the teacher's reasoning and on students' level of understanding about ratio and rate.

For the aforementioned process, teachers may start by conceptualizing slope as a ratio to give the relationships between the slope, vertical distance and horizontal distance of an object (Lobato & Thanheiser, 2002) and continue by conceptualizing slope as a rate of change to give the relationship between the vertical distance of the object and horizontal distance of the object that covary together (Thompson, 1994b). In addition, teachers may use concrete objects that have equal lengths to have students create different slopes with the same objects in a context of physical situation (e.g. the Positions of Battens activity). This process can be started with the activities of learning the terms of horizontal, vertical, and slope, and continued by determining the related and unrelated quantities of the slope of an object without giving the slope formula and by comparing the slope situations in which the slope is more/less. In this process, teachers can use a GGB material that represents the concrete object and give the slope value for a given situation.

Later on, teachers may use concrete objects that have different lengths to have students create the same slopes with the different objects in a context of physical situation. This process can be started with the activity of comparing the vertical distances and horizontal distances of the objects and determining the slopes of the objects and can be continued with the activity of constructing the ratio of vertical distance to horizontal distance. While constructing this ratio, teachers should mathematize the physical situation to interpret the slope as a multiplicative comparison of vertical distance to horizontal distance. In addition, teachers should represent these situations that are created with the concrete objects on the GGB materials dynamically. In detail, teachers should use GGB materials both for dynamic visualization of the situations with the dynamic objects in GGB graphics view and for algebraic representation of proportional relationship between the horizontal distance and vertical distance when the slope is constant for the set of concrete objects with the dynamic text of slope computation.

Furthermore, teachers may use additional activities and GGB materials that involve visualizations of slope positions in a physical situation that has invariant vertical distance/horizontal distance and varying slope and horizontal distance/vertical distance (e.g. the Fire Truck activity and the Tent activity). This process can be started with the activity of relating slope and vertical distance and continued by evaluating slopes when the horizontal distance is constant both visually using dynamic objects in GGB graphics view and algebraically using the dynamic text of slope computation in

GGB material. A similar process can be repeated for the slope and horizontal distance relation when the vertical distance is constant. While evaluating this slope computation, teachers should mathematize all the situations, that is, constant length of object with varying slope, horizontal distance and vertical distance values, constant slope with varying horizontal distance and vertical distance values, constant horizontal distance with varying slope and vertical distance values, and constant vertical distance with varying slope and horizontal distance values, respectively.

On the other hand, the slope concept appears under the subject of linear equations and graphs in middle school algebra without relating the functions. Therefore, teachers may think that they do not need to have an understanding about functions and the rate of change concept. However, teachers should have deep mathematical understanding about rate of change, linear relationship, covariational relationship, and functions to teach slope of a line conceptually within the procedural understanding. Here, it was a critical moment for the teacher to transfer from 'rate of change of vertical distance to horizontal distance' to 'rate of change of vertical change (i.e. dependent variable) to horizontal change (i.e. independent variable). In this regard, the teacher should be aware of what the consequences may be when they remove the complexity of rate of change, highlight the complexity of rate of change, and make connections between slope, rate of change and linear relationship for the slope of a line/line graph.

In addition, since the use of table and graphical representations are the important visual tools, it was vital to integrate this representations to the algebraic process of slope of a line. It must be made clear that this meaning of slope is different from the slope of an object, so the slope of a line should not be computed on the slope of an object or not be exemplified on an object. While giving slope of a line, one of the critical conceptualizations that the teacher need is understanding the sign of the slope of line. Teachers also should be aware of whether the graph is discrete and continuous that they used within the linear situation.

5.6 Implications and Suggestions for Future Research

In the light of the findings and conclusions, the researcher introduced some implications and suggestions for future research. In this regard, mathematics education researchers and teacher educators, and curriculum developers may benefit from this process and develop the research area of professional development of teachers and mathematics teaching.

The first implication is about using frameworks in analyzing teachers' knowledge of algebra teaching. In this study, the researcher used a teacher's process of developing an instructional sequence to understand the teacher's set of mathematical practices while enacting the instruction in her classrooms. The use of KAT framework (McCrory et al., 2012) in this process was appropriate to understand how the teacher performed mathematical practices and how she used her mathematical knowledge through this process in algebra context. However, it was seen that while analyzing the teacher's mathematical practices specifically in slope teaching, the researchers should have additional frameworks about covariational reasoning (Carlson et al., 2002) and quantitative reasoning (Thompson, 1994b) in addition to the conceptualizations of slope (Nagle & Moore-Russo, 2013a; Stump, 2001a, 2001b) to characterize the mathematical practices of teachers. In this regard, it is suggested that the researchers should combine related frameworks that can be used for a deep understanding of teachers' mathematical knowledge and practices.

The second implication is about the development of the objectives in the middle school mathematics curriculum. It was seen that while teaching the system of equations in eighth grade, the teacher could use the slope as a determining feature of the solution of system of equations. That is, having this conception of slope can provide a way of interpreting the solution of systems of equations in grade eight. In this regard, the curriculum developers should work on this subject. While NCTM (2006) recommended this property for parallel and perpendicular lines, it may be useful for students to investigate this relation within the systems of equations for coincident, parallel, and intersecting lines for two reasons. First, students can continue to develop the understanding of the slope of a line by evaluating and comparing the slopes of lines in system of equations. Second, students can develop a deep understanding of the solution set of system of equations. In this regard, teachers and teacher educators should develop this understanding of slope in their classrooms and in their mathematical knowledge for teaching.

The another implication for the curriculum is about the related objectives of the slope understanding. These objectives are so tangled in the middle school curriculum that teachers need to clarify what the slope of the object is, what the slope of the line is, and in which order and how these notions should be given. This study has shown that discriminating the objectives about slope for physical situations and for linear situations can be applicable in classroom teaching. Such a discrimination may be useful for teachers for two reasons. First, teachers can give slope in contextual situations beyond giving formula of the slope. Second, teachers can give the meanings of the slope of a line beyond steepness.

For researchers that have interest in developing or renovating the conceptual frameworks, two dimensions can be pointed out. The first dimension is about algebraic thinking for studying mathematics teaching and learning. While Carlson et al. (2002) described levels of covariational reasoning for non-linear graphs in dynamic events, this framework may be revised in further studies considering the linear function graphs and non-linear function graphs for using in teaching of slope through covariational reasoning from middle school to university. When the teacher's understanding of coordinating quantities for linear functional situations is considered, there might be a mezzo level between the direction level (level 2 in the covariational reasoning framework) and the average rate of change level (level 4 in the covariational reasoning framework) in addition to quantitative coordination (level 3 in the covariational reasoning framework). At this level, there are behaviors and mental actions for understanding constant rate of change for a line graph in a dynamic linear functional situation. Thus, students are expected to coordinate the constant rate of change of the function with uniform increments in the independent variables. In this regard, this type of reasoning should be used in preparing tasks and in teaching slope in eight grade to make an introduction to conceptualizing the rate of change in high school and university mathematics.

The second dimension is about knowledge of teaching mathematics, and we want to indicate a way of developing a framework for teachers' subject-specific (i.e. algebra) mathematical understanding and teachers' subject-specific instructional performances for and in classroom teaching. When considered the algebra teaching, the KAT framework provides ways of understanding about teachers' use of mathematical knowledge in algebra teaching in their mathematical moves, but the tools that practicing teachers have used in mathematical tasks are needed to be specified in these mathematical moves. These tools can be concrete objects and a dynamics software technology. At this point, the *instrumental orchestration framework* (Drijvers et al., 2010, 2013) provide a way to investigate teachers behaviors, but the didactical performance dimension of this framework can be reinterpreted with the teachers mathematical moves. In this regard, the KAT framework can be reframed upon the didactical performance dimension of instrumental orchestration (Drijvers et al., 2010). This study has shown that it can be possible to interpret teachers' mathematical practices considering their usage of tools. Therefore, in further research, integrating frameworks of instrumental orchestration and mathematical practices may provide additional insights on mathematics education research.

For researchers who want to modify and reconsider this study, the researcher wants to draw attention to two issues considering the various ways of repeating this study. The primary issue is that students' being gifted or being at average level in primary and secondary schools and teachers' being more/less competent in using dynamic software may have an influence on the findings. We believe that average level or upper level students may have fewer difficulties about linear equations, and graphs even though they follow parallel experiences on understanding conceptualizations and procedural aspects of slope. In addition, we believe that technologically competent teachers may create and use GGB in different ways but have a corresponding mathematical practices in the moments of teaching. Therefore, in further research, studying with advanced level students and/or technologically competent teachers may provide additional insights on the mathematical practices of the teachers.

The second issue is about examining teachers' mathematical practices. To examine teachers' ways of collaborations among the mathematical practices, reflections, and understandings during the construction of mathematical practices and to examine how they will make contributions to teachers' use of mathematical knowledge in teaching are noteworthy areas for future studies about professional development of teachers and teacher education. In addition, teachers' mathematical practices can be investigated in different communities. We believe that teachers may have various mathematical practices in different ways and in different communities (e.g. communities of teachers in a graduate course, communities of teachers in a school setting). Therefore, in future research, studying with a community of teachers in a school setting may provide additional insights on the mathematical practices of the teachers from a social perspective.

5.7 The Limitations of the Study

There are some limitations of this study. As one of the limitations, this study involved part of a multitiered design experiment based on classroom based design experiments that focused on the teacher-tier. While a study conducted with a full multitiered design experiment requires multiple constituents (researchers, teachers, facilitators, students) in a longitudinal manner, the conclusions from this study present how each tier (constituent) think and how constituent interactions improve these ways of thinking (Lesh et al., 2008). The reasons behind choosing teacher tier were twofold. First one is time constraints. The students had limited time in the second semester because of the national exam, and teachers had limited time for teaching a topic since they prepared students for the national exam. In this regard, I could not focus on the student tier. Second, it was the constraints of human resources. I was the only researcher in the setting, which restricted my mobility in different settings. These constraints directed me to focus on teachers and the aforementioned research questions.

As another limitation, although the participant teachers were experienced teachers, I could reach limited number of teachers during the participants' selection process. The reason for this selection was that it was not easy to reach a teacher that provides the conditions of using technology in her/his classroom, volunteering to be recorded in classroom teaching, and volunteering to be involved in a design experiment. While the research process with teachers was deep, the duration of this process was limited with one eight-grade class for one teacher (Zehra) and two eight-grade classes for the other teacher (Oya) who used dynamic software technology in their classrooms, allowed to be recorded in classroom teaching, and were disposed to be involved in a design experiment. Therefore, the findings of this study were limited to these teachers, their classes, and their setting. In addition, the selected participant teachers did not continue to the research for years because of different reasons (pregnancy and change of schools). Moreover, the data of the study was limited to teaching of slope, linear equations, and graphs unit in eight grade in one semester.

The additional issue is about the development of the mathematical practices of teachers. Although this study presents that developing an instructional sequence for a classroom teaching can serve as a professional process that mathematical practices of teachers can develop, investigating the specific developments of the didactical performance is out of the scope of this study. Therefore, I do not intend to answer the question of how the mathematical practices of teachers experiencing the development of instructional sequence through a design experiment can promote the mathematical practices of teachers. This study has shown that analyzing the enacted classroom session while transiting from a class to another class is useful for developing mathematical practices (e.g. interpreting the slope as rate of change), and this process of developing an instructional sequence with a design experiment perspective provides teachers with opportunities to reflect on mathematical understandings of their students and their own. However, we did not examine what the characteristics of these reflections and understandings are and how these reflections and understandings are collaborate with the mathematical practices of teachers. This study takes a step by characterizing the mathematical practices of a middle school mathematics teacher while enacting an instructional sequence and can provide a basis for the further characterizations mentioned above.

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APPENDICES

A. Algebra Standards in Middle School Mathematics

Ortaokul matematik öğretim programında cebir kazanımları (MoNE, 2009)

		Sınıf 6	Sınıf 7	Sınıf 8
Alt öğrenme alanları	Örüntüler ve ilişkiler	1. Sayı örüntülerini modelleyerek bu örüntülerdeki ilişkiyi harflerle ifade eder. 2. Doğal sayıların kendisiyle tekrarlı çarpımını üslü nicelik olarak ifade eder ve üslü niceliklerin değerini belirler.	1. Tam sayıların kendileri ile tekrarlı çarpımını üslü nicelik olarak ifade eder. 2. Sayı örüntülerini modelleyerek bu örüntülerdeki ilişkiyi harflerle ifade eder.	1. Özel sayı örüntülerinde sayılar arasındaki ilişkileri açıklar.
	Cebirsel ifadeler	1. Belirli durumlara uygun cebirsel ifadeyi yazar.	1. Cebirsel ifadelerle toplama ve çıkarma işlemleri yapar. 2. İki cebirsel ifadeyi çarpar.	1. Özdeşlik ile denklem arasındaki farkı açıklar. 2. Özdeşlikleri modellerle açıklar. 3. Cebirsel ifadeleri çarpanlarına ayırır. 4. Rasyonel cebirsel ifadeler ile işlem yapar ve ifadeleri sadeleştirir.
	Eşitlik ve denklem	1. Eşitliğin korunumunu modelle gösterir ve açıklar. 2. Denklemi açıklar, problemlere uygun denklemleri kurar. 3. Birinci dereceden bir bilinmeyenli denklemleri çözer.		
	Denklemler		1. Birinci dereceden bir bilinmeyenli denklemleri çözer. 2. Denklemi problem çözmede kullanır. 3. Doğrusal denklemleri açıklar. 4. İki boyutlu Kartezyen koordinat sistemini açıklar ve kullanır. 5. Doğrusal denklemlerin grafiğini çizer.	1. Doğrunun eğimini modelleri ile açıklar. 2. Doğrunun eğimi ile denklemi arasındaki ilişkiyi belirler. 3. Bir bilinmeyenli rasyonel denklemleri çözer. 4. Doğrusal denklem sistemlerini cebirsel yöntemlerle çözer. 5. Doğrusal denklem sistemlerini grafikleri kullanarak çözer.

Tablo (devam)

	Eşitsizlikler			1. Eşitlik ve eşitsizlik arasındaki ilişkiyi açıklar ve eşitsizlik içeren problemlere uygun matematik cümleleri yazar. 2. Birinci dereceden bir bilinmeyenli eşitsizliklerin çözüm kümesini belirler ve sayı doğrusunda gösterir. 3. İki bilinmeyenli doğrusal eşitsizliklerin grafiğini çizer.

B. Teacher Information Protocol

Öğretmeni Tanıma Bilgileri Protokolü

Bu form bir matematik öğretmeninin, eğitim sürecini, geçmiş öğretmenlik sürecini ve çalışmakta olduğu okuldaki öğretmenlik sürecini kapsayacak şekilde tanılayıcı 21 soru içermektedir. Bu sorular matematik öğretmeninin altyapısını anlamak için sorulmaktadır.

1) Lisansta hangi bölümden/üniversiteden mezun oldunuz? :
   Lisansa başlama tarihi (ay/yıl):
   Bitirme tarihi (ay/yıl):
   Mezuniyet ortalamanız:
2) Kendinizi bir lisans öğrencisiyken başarılı/başarısız, istekli/isteksiz, meraklı/meraksız olarak nasıl değerlendirirsiniz?

3) Üniversitede matematik alanında hangi dersleri aldınız?

4) Üniversitede matematik öğretimine yönelik hangi dersleri aldınız? Bu derslerde, hangi kaynakları kullandınız?

5) Üniversitede teknoloji kullanıma yönelik hangi dersleri aldınız? Bu derslerde, hangi teknolojileri öğrendiniz?
   -teknoloji donanımları (örn: projeksiyon cihazı, bilgisayar):

   -yazılımlar & programlar (örn: ofice-word, excel, powerpoint-, html kullanımı (yada frontpage), logo, derive, dinamik yazılımlar, vb.):

6) Yüksek lisansta hangi bölümden/üniversiteden mezun oldunuz? :
   Yüksek lisansa başlama tarihi (ay/yıl):
   Bitirme tarihi (ay/yıl):
7) Yüksek lisans öğretimde teknoloji kullanımına yönelik hangi dersleri aldınız?

8) Yüksek tezinizin adı/konusu nedir?

9) Yüksek lisans tezinizde hangi programı/yazılımı kullandınız? Bu programı daha önceden kullanıysanız, nasıl kullandınız?

10) Devlete öğretmenlik yapmadan önce öğretmenlik deneyiminiz oldu mu? Olduysa, nerede?

11) Devlette öğretmenlik yapmaya ne zaman başladınız?

12) Derslerinizde, hangi teknolojileri kullanıyorsunuz? Yada daha önceden kullandınız? (Powerpoint, Word, Excel, Dinamik yazılımlar, vd.)

13) Derslerinizde teknolojiyi hangi sıklıkla kullanıyorsunuz. (Aşağıdakilerden bir veya birden fazlasına evet diyebilirsiniz)

Bir ders süresinin tamamında:
Bir ders süresi içinde bazı aralıklarda:
Bir hafta içerisinde konuya girişte:
Bir hafta içinde soru çözerken:
Bir ay içerisinde bazı konularda:
Bir ay içerisinde bazı sorularda:
Bir ay içerisinde bazı etkinliklerde:
Bir dönem içerisinde bazı konularda:
Bir dönem içerisinde bazı sorularda:
Bir dönem içerisinde bazı etkinliklerde:

14) Okulunuzda bilgisayar laboratuarı var mıdır? Varsa, ders işlemek için kullanıyor musunuz?

15) Okulunuzda bir matematik sınıfı var mıdır? Bu sınıf size ait midir?

16) Okulunuzda hangi sosyo-ekonomik düzeydeki çocuklar vardır?

17) Okulunuzda kaç yıldır öğretmenlik yapıyorsunuz?

18) Okulununuzda hangi sınıf seviylerinde ders anlattınız?

19) Okulunuzda bu dönem hangi sınıf seviyelerinde ders anlatıyorsunuz?

20) Bu dönem okuttuğunuz sekizinci sınıfların dersine geçen senelerde girdiniz mi? Girdiyseniz, hangi seviyede derslerine girdiniz?

21) Burada sormadığım, ama sizinle ilgili bahsetmek istediğiniz bir şey varsa açıklarsanız sevinirim.

C. Teacher Pre-Interview Protocol

Öğretmen Öngörüşme Protokolü

Bu görüşme matematik öğretmenlerinin öğretim hedefleri ve inanışları ile ilgili bilgi edinmek için hazırlanmıştır. Bu görüşmede, bir matematik öğretmenin genel matematik öğretimi hedeflerine ve inanışlarına yönelik sorularla birlikte eğim, doğru denklemleri ve grafikleri konusuna özgü öğretim hedeflerine ve inanışlarına yönelik soruları da içermektedir. Aşağıdaki soruların cevapları hakkında neler düşündüğünüzü açıklayınız. Burada önemli olan sizin düşündüklerinizdir, bu sorularda doğru veya yanlış bir cevap aranmamaktadır.

A. Hedeflere yönelik görüşme soruları [Goals related questions]

1. Matematik öğretimine yönelik öğrenciler için uzun-vadede hedefleriniz neler? [Bir sene sonra, bir dönem sonra, vb.]

   Örneğin: "Öğrencilerin .... olmalarını isterim." Cümlesini tamamlayınız.

2. Matematik öğretimine yönelik öğrenciler için orta-vadede hedefleriniz neler? [bir kazanımın öğretimi süresi; 2-3 hafta]

   Örneğin: "Öğrencilerin denklemler alt öğrenme alanında .... Anlamalarını isterim/beklerim." Cümlesini tamamlayınız.

3. Matematik öğretimine yönelik öğrenciler için kısa-vadede hedefleriniz neler? (bir ders süresi)

   Örneğin: "Doğrunun eğimi ile denklemi arasındaki ilişkiyi belirler kazanımına yönelik bir derste öğrencilerin .... Anlamlarını/yapmalarını isterim/beklerim." Cümlesini tamamlayınız.

4. Bu … Hedefin /hedeflerin dersi planlarken vereceğin kararlarını/eylemlerini ne kadar etkiler? Bu hedeflerini nasıl göz önüne alıyorsun?
5. Bu … hedefin/hedeflerin ders sırasındaki kararlarını/eylemlerini ne kadar etkiler? Bu hedeflerini nasıl göz önüne alıyorsun?
6. Ders sırasında beklenmedik ya da planlamadığınız durumlar gerçekleştiğinde bu durum hedeflerinizi nasıl etkiler?
7. Bu beklenmedik durumları derse girmeden önce bunu öngörebiliyor musun?

8. Ders sonrasında hedeflerinizde değişikler olursa bunlar nelerdir? Bu değişimi etkileyenler nelerdir?

B. İnanışlara yönelik görüşme soruları [Beliefs related questions]

Aşağıdaki soruların cevapları hakkında neler düşündüğünüzü açıklayınız. Burada önemli olan sizin düşündüklerinizdir, bu sorularda doğru veya yanlış bir cevap aranmamaktadır.

B.1 Matematiğin doğasına yönelik inanışlar [Beliefs about nature of mathematics]

Aşağıdaki sorular matematiğin doğasına yönelik inanışlarınızla ilgili olacaktır.

1. Matematik ne için vardır? Matematiğin amacı nedir?
2. Matematik ne açıdan önemlidir?
3. Matematiği bilmek ne demektir? Matematiksel bilgi nelerden oluşur?
4. Sizin düşüncenize göre matematiksel kavram neye dayanır? Matematiksel kavram nedir?

   Örneğin, denklem ve eğim kavramını matematiksel olarak ifade etmedeki sebep nedir?
5. Bir matematiksel kavramı soran bir soru örneği verebilir misiniz?

   Örneğin, öğrencilerde o kavramı sorgulamaya dair nasıl sorular yönlendirirsin?
6. Sizce 'Matematiksel işlem (procedure)' nedir? Sizce 'Matematiksel işlem' neye dayanır? *Örneğin, bir matematiksel işlem soran bir soru örneği verebilir misiniz?*

B.2 Matematik öğrenme sürecinin doğasına yönelik inanışlar [Beliefs about nature of learning mathematics]

Aşağıdaki sorular matematik öğrenme sürecine yönelik inanışlarınızla ilgili olacaktır.

7. Matematiği öğrenmek ne demektir?

   Örneğin, Bir şeyi bilmek demek öğrenmek demek midir?

   Örneğin:

Matematik öğrenmek	ev inşa etmek tarife göre yemek yapmak kilden bir heykel yapmak	gibidir.

ağaçtan meyve toplamak

yap-boz yapmak

film seyretmek

seri üretim yapan bir fabrikanın montaj hattında çalışmak

deney yapmak

____

Yukarıda verilen analojilerden matematik öğrenmeyi en iyi şekilde temsil edeni veya kendi analojinizi belirtiniz, seçiminizi açıklayınız.

Yukarıda verilen analojilerden matematik öğrenmeyi pek temsil etmeyeni seçiniz, seçiminizi açıklayınız.

8. Matematik dersinde öğrencinin rolü nedir? Bu roldeki/rollerdeki öğrencilerin sınıf içerisindeki görevleri nelerdir?
9. Matematik öğrenirken öğrencinin yapması gerekenler nelerdir?
   Örneğin: öğrenci eğim öğrenirken önemli olan şeyler nelerdir?
10. Matematik dersi sonunda bir öğrenci neyi öğrenmelidir?
11. Matematiği anlamak nedir? Matematiği anlamanın önemi nedir?
   Örneğin: Bir formülü bilmekle anlamak arasında nasıl fark vardır?
   Örneğin: Denklemler ve eğim üzerinden konuşabiliriz.
   Örneğin: Formüller ne kadar önemlidir? Açıklamalar ne kadar önemlidir? Nitel akıl yürütme ne kadar önemlidir? İspat ne kadar önemlidir?
12. Bir matematik problemi çözerken, öğrencilerin yapması/öğrenmesi gerekenler nelerdir?
13. Bir öğrencinin matematiksel bir problemi anladığını nasıl bilirsiniz?
14. Öğrencilerin matematik öğrenmesini nelerin etkilediğini düşünüyorsunuz?
   Örneğin; içsel olarak bireysel deneyimleri, dışsal olarak çevre vb…

B.3 Matematik öğretmeye ve çeşitli öğretim türlerine yönelik inanışları [Beliefs about teaching mathematics and various roles of various kinds of instruction]

Aşağıdaki sorular matematiği öğretmeye yönelik inanışlarınızla ilgili olacaktır.

15. Neden matematik öğretiyoruz? Matematik öğretmek neden gereklidir?

16. Matematik öğretmek hakkında ne hissediyorsunuz? Matematiği öğretmek size ne hissettiriyor?

17. Matematik öğretmeni olarak sizin rolünüz nedir? Bu rolü nasıl açıklarsınız?

Örneğin:

	Turist Rehberi	Doktor	
	Antrenör	Misyoner	
	Kumandan	Jokey	
	İnşaat Ustası	Komedyen	
Matematik öğretmeni	Haber spikeri	Gardiyan	gibidir.
	Bahçıvan	Heykeltıraş	
	Orkestra Şefi	Aşçıbaşı	
	Mühendis	__________	

Yukarıda verilen analojilerden bir matematik öğretmenini en iyi şekilde temsil edeni seçiniz veya en sondaki boşluğa kendi analojinizi yazınız ve seçiminizi açıklayınız.

Yukarıda verilen analojilerden bir matematik öğretmenini pek temsil etmeyeni seçiniz veya en sondaki boşluğa kendi analojinizi yazınız ve seçiminizi açıklayınız.

18. Bir matematik öğretmeninin görevleri nelerdir? En önemli görevi nedir?

Ders öncesinde/planlarken …

Ders sırasında …

Ders sonrasında …

19. Matematik öğretmek için bir öğretmenin sahip olması gereken özellikler nelerdir? İhtiyaç duyduğu şeyler nedir?

20. Bir matematik öğretmenin bilmesi gereken bilgiler nelerdir? Matematik öğretmeninin sahip olması gereken bilgileri sıralaya bilir misin/kategorilere ayırabilir misin?

21. Sizce, çeşitli öğretim yöntemlerinin matematik öğretmedeki rolleri nelerdir?

Ayrıca, bu yöntem ve tekniklerin ne kadar etkili olduğuna inanıyorsunuz?

22. Çeşitli araçların matematik öğretmedeki rolleri nelerdir?

Örneğin, bilgisayar teknolojisinin ve yazılımların matematik öğretmedeki rolleri nelerdir?

Ayrıca, Kullandığınız araçların ne kadar etkili olduğuna inanıyorsunuz?

23. Öğrencilerin öğrenme düzeyleri sizin öğretim yönteminizi ne kadar etkiliyor?
24. Öğrencilerin öğrenme şekilleri sizin öğretim yönteminizi ne kadar etkiliyor?
25. Matematik öğretmedeki güçlü yönleriniz neler?

Öğretmen olmadan önce bu güçlü yönlerinizin farkında mıydınız?

26. Matematik öğretmede zayıf yönleriniz neler?

Öğretmen olmadan önce bu zayıf yönlerinizin farkında mıydınız?

27. Matematik öğretmekten keyif alıyor musunuz? Nelerden ne kadar keyif alıyorsunuz/almıyorsunuz?

Sizce keyif almak matematiği öğretmede önemli midir? Ne açıdan önemlidir?

B.4 Öğrencilerine ve sınıflarına yönelik inanışlar [Beliefs about particular students and classes of students]

28. Sınıf içi disiplin ne kadar önemlidir? Sınıf içi disiplin sizce nedir? İyi bir sınıf sessiz bir sınıf mıdır?
29. Grup çalışması veya bireysel çalışmayı seçmende nelerin etkili olduğunu düşünüyorsunuz?
30. Ders anlattığın sınıf(lar) hakkında neler düşünüyorsun? (özellikle sekizinci sınıflar)

    a) Bu sınıftaki öğrenciler dersteki konuları ne kadar anlıyorlar?

    b) Bu sınıftaki öğrenciler dersteki konuları ne kadar seviyorlar?

    c) Bu öğrenciler öğrenmeye istekliler mi?

31. Hangi sınıflarda matematik öğretmek isterdin?

B.5 Diğer: Matematik öğretmede kendilerini nasıl hissediyorlar? [Others]

32. Matematik alanında kendinizi nasıl buluyorsunuz?

Örneğin .... açıdan iyi olduğuma/kötü olduğuma inanıyorum.

33. Matematiği öğretme alanında kendinizi nasıl buluyorsunuz?
Örneğin …. açıdan iyi olduğuma/kötü olduğuma inanıyorum.

34. Sizin eklemek istediğiniz yada keşke sunu sorsaydın dediğiniz bir şey var mı? Varsa, açıklayınız.

D. Observation Protocol for Lesson Implementation

			Name of the Teacher:	
			Class:	
			Date:	
Subject:				
Start time:		End time:		Note:

Speaker [teacher or Student(s)]	Statement	Actions/ Ideas	Use of tool (concrete material/ technology)	Researcher's comment	Researcher's question

E. Teacher's Analysis Sessions Interview Questions: An example

Analiz Oturumunda gerçekleşen Yarı Yapılandırılmış Görüşme Soruları: Bir örnek

Aşağıdaki görüşme soruları ders gözlemi sırasında şekillenmiştir. Ancak görüşme sırasında öğretmenin yorumları doğrultusunda ilerlemiştir. Bu sorular bir görüşmenin bir kısmına aittir.

Video izlerken sorulacak sorular:

1. Bir öğrenci(x) …. hakkında soru sordu. Neden?
2. Öğrenci (ler) … hakkında açıklama yaptı. Neden?
3. Öğrencilerin böyle cevaplar verdiğinde/açıklamalar yaptığında ne düşünüyorsun?
    a. Öğrencilerin neden öyle düşünüyor olabilir?
4. Öğrenciler … konuya/kavrama takıldılar. Neden?
5. Öğrencilerin … arasındaki ilişkiyi anladıklarını düşüyormusun? Sence bu anlamaları nasıl gelişti?
6. Hangi anda somut nesneden GGB materyaline geçmeye karar verdin?
    a. GGB materyalini vermeden önce neyden emin olmak istedin?
7. Öğrencilerin cevaplarını aldıktan sonra GGB geçmenin sebebi neydi?
8. Öğrenciler GGB araçlarını kullanırken sence ne anladılar/öğrendiler?
    a. Öğrencilerin … arasındaki ilişkiyi geliştirmelerinde, GGB nasıl faydalı oldu yada olmadı?
9. Özellikle …. ya vurgu yapıyorsun. Neden?
10. Senin bu etkinliklerle ve materyallerle öğretim hakkında ilgili söylemek istediklerin neler?

F. Teacher Post-Interview Protocol

Öğretmen Songörüşme Protokolü

Bu görüşmede matematik öğretmenin geliştirdiği öğretim dizisi ve bu gelişim süreci hakkında neler düşündüğüne yönelik olarak hazırlanmıştır. Ayrıca, bu süreçte kullanılan dinamik yazılımın (GeoGebra) öğretim sırasındaki rolü hakkında sorular içermektedir.

1. Uyguladığın öğretim dizisi hakkında neler hissediyorsun?
2. Tüm bu öğretim dizisi den öğrencilerin kazandıkları hakkında neler düşünüyorsun?
3. GeoGebra kullanımının bu öğretim dizisine yardım ettiğini düşünüyor musun? Neden?
    a. Öğrencilerin matematiksel uygulamaları nasıl ilerledi?
    b. Öğrencilerin bu matematiksel uygulamaları ilerletirken, sınıf ortamı nasıl gelişti?
    c. Sınıf ortamı bu matematiksel uygulamaların ilerlemesinde nasıl katkı sağladı?
4. Öğrencilerin GeoGebra kullanımına verdikleri tepkileri hakkında neler düşünüyorsun?
5. Bu öğretim dizisinde neler değiştirirdin?
6. GeoGebra senin öğretim biçimini değiştirdi mi? Nasıl?

Yukarıdaki sorulardan sonra, takip eden olası sorular:

a. ….. dan bahsettin, bunu benim için daha detaylı anlatır mısın?
b. …. dan bahsettin, bununla ilgili belirli bir örnek anlatır mısın?

G. Contact Summary Form

Bağlantı Özet Formu

Bağlantı Özet Formu #1 (Contact Summary Form #1)

Bağlantı türü: Ders gözlemi — Katılımcı: Oya

Yer: 8A/matematik sınıfı — Bugünün Tarihi: 06.05.2013

1) Bu bağlantı sırasında dikkatini çeken temel konular veya temalar neydi?

Öğretmen öğrecileri ile dersi sorgulamalar ve küçük soruların birleşimi ile işliyor.

Öğretmen –öğrenci etkileşimi var. Öğretmen öğrencileri derse katıyor ve cevaplarına değer veriyor.

Öğrenci-öğrenci etkileşimi var. Öğrenciler grup olarak fikir üretiyor.

Öğretmen derste materyal ve teknoloji kullanıyor.

Öğretmen öğrencilerin material kullanmasına fırsat veriyor.

Öğretmenin öğretim programı bilgisi ile matematik bilgisi etkileşim içinde.

2) Bu görüşme için aklında olan sorular üzerinden aldığın (ya da alamadığın) bilgileri özetle.

Soru	Bilgi (Information)
Sınıfın yapısı	-Öğrenciler 5li veya 6lı gruplar halinde oturuyor -Öğretmen sınıfta dolaşıyor. -Öğretmen grupları teker teker geziyor -Öğretmenin grupların yanına oturarak o gruba ayrı anlattığı da oluyor
Sınıfın değerleri	-Öğrencileri cevaplarını söylemeleri için cesaretlendiriyor -Öğrencileri birbilerinin cevaplarını dinlemeleri, birbirlerinden öğrenmeleri için görevler vererek cesaretlendiriyor -Öğrencilere bireysel veya grup olarak öğrenme sorumluluğu veriyor.

	-Öğrenciler birbirlerinin sözünü, yorumunu veya sorusunu dinliyor.
Ört. Mat. Uygulamaları	-Öğretmen geometrik cisimler arasında ilişkiler kuruyor. -Öğretmen hacim hesapları arasında ilişkiler kuruyor. -Öğretmen hacim formülleri arasındaki ilişkiyi somut materyal ile gösteriyor.
Öğrt Mat. Açıklamaları	-Öğretmen hacim hesabınının açıklamasını yapıyor. -Öğrenci cevaplarını açıklamalar yaparken kullanıyor. -Öğretmen öğrencilerin verdiği cevapları matematiksel olarak uygun şekilde ediyor.
Öğrt. Mat. Öğretimi. Bilgisi	-Öğretmen öğrencilerin nerelerde hatalar yapabileceğini biliyor (öğrencilerin hatalarından öğretimi sürdüyor). -Öğretmen farklı gösterimleri –dinamik yazılım, şeffaf geometrik cisimler takımı, sembolik, sözel-bir araya getiriyor.

3) Bu görüşmede göze çarpan, ilginç, fikir veren veya önemli diyebileceğin her hangi bir şey dikkatini çeken şeyleri, varsa, açıkla.

Öğretmen öğrencilerin sorularına soruyla cevap veriyor.

Öğretmen materyal kullanırken sınıfta karışıklık olmuyor ve öğrenciler materyali öğretimsel amacına uygun olarak kullanıyor.

4) Bu öğretmenle bir sonraki bağlantını düşündüğünde yeni hedef soruların neler?

Öğretmen materyali veya dinamik yazılımı dersin hangi anında kullanacağına nasıl karar veriyor?

Öğretmen kavramlar ve formüller arasındaki ilişkileri kurarken bunun arkasındaki bilgi öğretim programı bilgisi mi üst düzey matematik bilgisi mi?

Öğretmenin fiziksel araç, materyal kullanırkenki matematiksel eylemleri neler?

Öğretmenin teknoloji kullanırkenki matematiksel eylemleri neler?

H. Ethics forms

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

DUMLUPINAR BULVARI 06800
ÇANKAYA ANKARA/TURKEY
T: +90 312 210 22 91
F: +90 312 210 79 59
ueam@metu.edu.tr
www.ueam.metu.edu.tr

Sayı: 28620816/30 -264

19 Mart 2013

Gönderilen: Doç. Dr. Mine Işıksal Bostan
İlköğretim Bölümü

Gönderen : Prof. Dr. Canan Özgen
IAK Başkanı

İlgi : Etik Onayı

Danışmanlığını yapmış olduğunuz İlköğretim Bölümü Doktora öğrencisi Seçil Yemen Karpuzcu'nun "Ortaokul Matematik Öğretmenlerinin Teknoloji Destekli Sınıf Ortamında Cebir Öğretimine Dair Yansımalarının ve Pedagojik İçerik Bilgilerinin İncelenmesi" isimli araştırması "İnsan Araştırmaları Komitesi" tarafından uygun görülerek gerekli onay verilmiştir.

Bilgilerinize saygılarımla sunarım.

Etik Komite Onayı

Uygundur

19/03/2013

Prof.Dr. Canan ÖZGEN
Uygulamalı Etik Araştırma Merkezi
(UEAM) Başkanı
ODTÜ 06531 ANKARA

ÖĞRENCİ İŞLERİ
DAİRESİ BAŞKANLIĞI
Ev. Arş. Md. Sa... :

T.C.
ANKARA VALİLİĞİ
Milli Eğitim Müdürlüğü

Sayı : 14588481/605.99/601950 15/04/2013
Konu: Araştırma izni

ORTA DOĞU TEKNİK ÜNİVERSİTESİNE
(Öğrenci İşleri Daire Başkanlığı)

İlgi: a) MEB Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 2012/13 nolu Genelgesi.
b) 08/04/2013 tarih ve 3899 sayılı yazınız.

Üniversiteniz Eğitim Fakültesi Araş. Gör. Seçil YEMEN KARPUZCU' nun **"Ortaokul matematik öğretmenlerinin teknoloji destekli sınıf ortamında cebir öğretimine dair yansımalarının ve pedagojik içerik bilgilerinin incelenmesi"** konulu tezi kapsamında çalışma yapma talebi Müdürlüğümüzce uygun görülmüş ve araştırmanın yapılacağı İlçe Milli Eğitim Müdürlüğüne bilgi verilmiştir.

Uygulama örneklerinin araştırmacı tarafından uygulama yapılacak sayıda çoğaltılması ve çalışmanın bitiminde iki örneğinin (cd ortamında) Müdürlüğümüz Strateji Geliştirme Bölümüne gönderilmesini arz ederim.

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Müdür a.
Şube Müdürü

Güvenli Elektronik İmzalı
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16/04/2013

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e-posta: istatistik06@meb.gov.tr

Ayrıntılı bilgi için: Emine KONUK
Tel: (0 312) 221 02 17/135

I. Positions of Battens Activity Sheet

TAHTALARIN DURUŞU

Elimizde 60 cm uzunluğunda tahta parçaları vardır. Bu tahta parçaları duvara yaslanmak isteniyor.

i) Buna göre bu tahtalar duvara yaslandığında duruşları her zaman aynı mıdır?

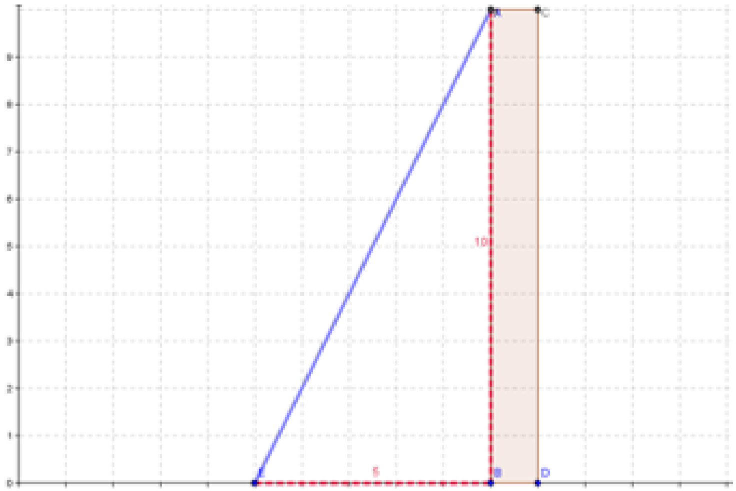
ii) Tahtalar duvara yaslandığında, her bir tahta için aşağıdaki tabloyu doldurunuz.

	Tahta 1	Tahta 2	Tahta 3	Tahta 4
Dikey uzunluk				
Yatay uzunluk				
Eğim				

iii) Bu tahtalar içinde en büyük eğimi olan hangisidir? Sebebi nedir?

iv) Bu tahtalar içinde en küçük eğimi olan hangisidir? Sebebi nedir?

J. Fire Truck Activity Sheet



İTFAİYE ARACI

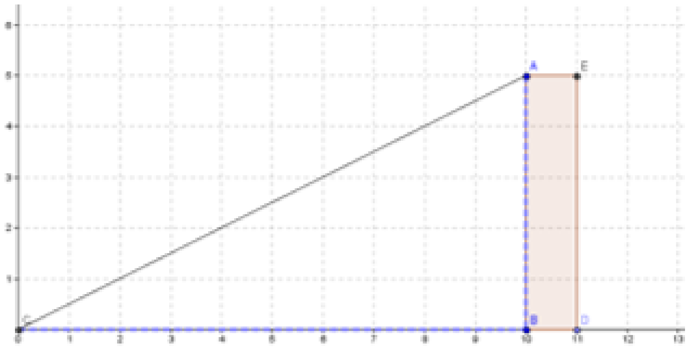
Şekildeki gibi 10 katlı bir binada bir yangın çıktığında, E noktasındaki itfaiye aracı merdivenini kata koyarak binadakileri kurtarmaktadır.

i) Buna göre 3. katta, 7. katta ve 9. Katta yangın olursa merdivenin eğimi kaç olur?

ii) Yangın kaçıncı katta olursa itfaiye merdiveninin eğimi en fazla olur?

iii) ***Katlar yükseldikçe*** eğim nasıl değişiyor?

K. Tent Activity Sheet



ÇADIR

Şekildeki gibi 5 metre yüksekliği olan bir çadır tepe sol köşesinden zemine bir ip ile bağlanıyor.

i) Buna göre ip çadırdan 2 metre, 4 metre ve 5 metre uzağa bağlanırsa ipin eğimi kaç olur?

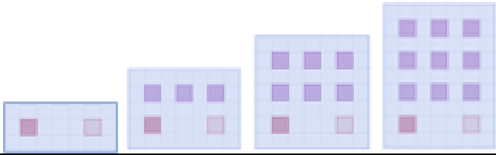
ii) İp zeminde hangi noktaya bağlanırsa ipin eğimi en az olur?

iii) Buna göre ***zemine bağlanan noktanın çadırdan uzaklaşmasıyla*** eğim nasıl değişiyor?

L. Building Design Activity Sheet

BİNA TASARIMI

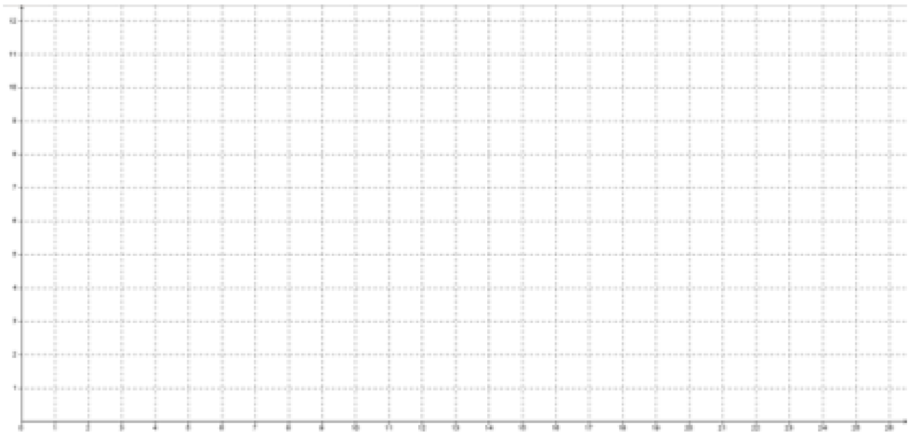
Bir mimarlık firması bina tasarımı yapmaktadır. Bu tasarımda **zemin katta 2** pencere vardır. Zemin üstünde ise **her katta** 3 pencere vardır. Bu tasarıma göre, 1 katlı bir binada toplamda 5 pencere, iki katlı bir binada toplamda 8 pencere, 3 katlı bir binada toplam 11 pencere vardır.



Toplam Kat sayısı							x
Toplam Pencere Sayısı							y=……………

a) Toplam kat sayısı-Toplam pencere sayısı arasındaki ilişkiyi gösteren doğru denklemini yazalım.

b) Binadaki *toplam kat sayısı-toplam pencerece sayısı* arasındaki ilişkiyi gösteren grafiği çizelim.



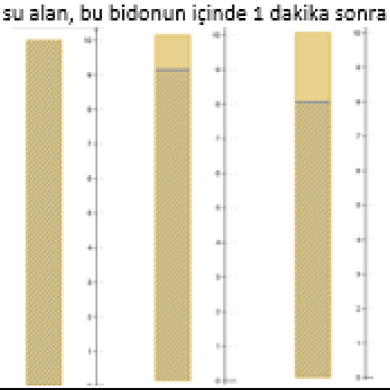
c) Bu tasarımda 5 katlı bir binanın toplamda kaç penceresi olduğunu hesaplarken ***doğru denkleminden*** ve ***grafikten*** nasıl yararlanılır?

d) Bu doğru grafiği üzerinde, herhangi iki nokta belirleyelim ve doğrunun eğimini hesaplayalım.

M. Leaking Container Activity Sheet

DELİK BİDON

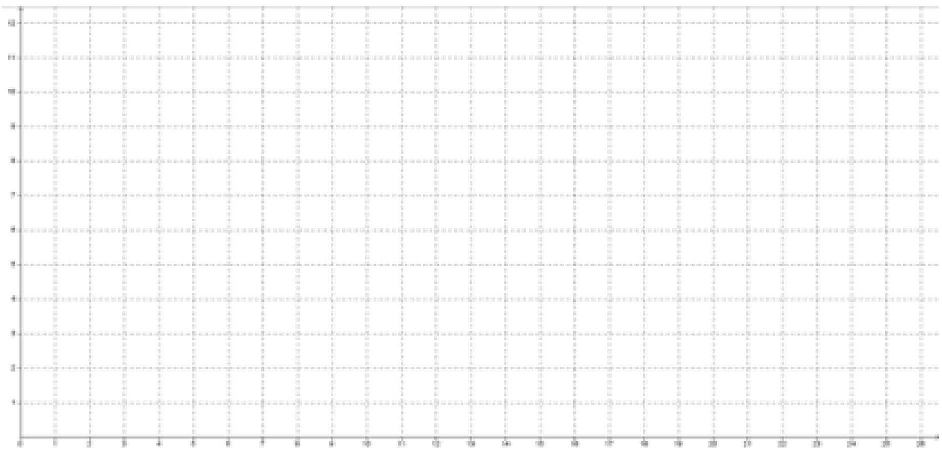
10 litrelik bir bidon dibinde bir delik olduğu için süreli olarak dakikada 1 litre su sızdırmaktadır. Buna göre başlangıçta 10 litre su alan, bu bidonun içinde 1 dakika sonra 9litre, 2 dakika sonra 8 litre su kalmaktadır.



Zaman (dakika)						x
Bidonun içindeki su miktarı (litre)						y=................

a) Bidonda kalan Suyun hacmi (litre) ile zaman (dakika) arasındaki ilişkiyi gösteren doğru denklemini yazalım.

b) Bidonda kalan suyun hacmi (litre) ile zaman (litre) arasındaki ilişkiyi gösteren grafiği çizelim.



c) Bu bidonda 4 dakika sonra toplamda kaç litre su olduğunu hesaplarken ***doğru denkleminden*** ve ***grafikten*** nasıl faydalanılır?

d) Bu doğru grafiği üzerinde doğrunun eğimini hesaplayalım.

N. Slopes and Equations of Lines Activity Sheet

Adı-Soyadı:

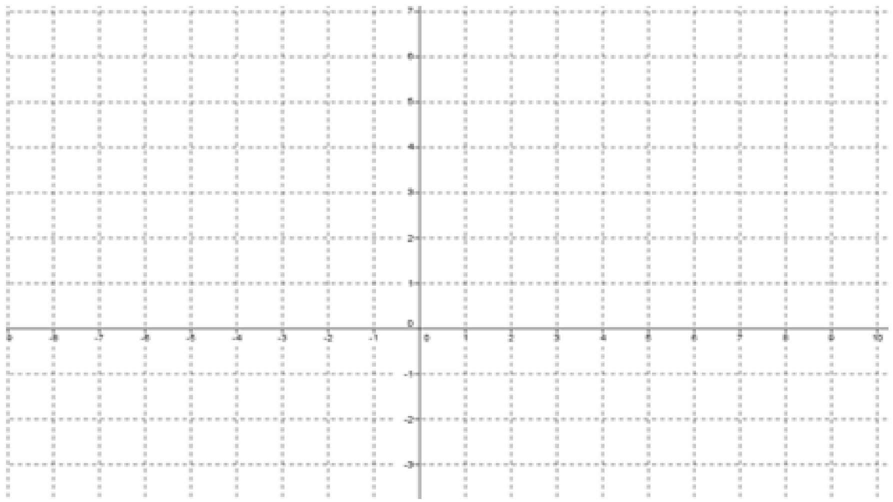
Doğru Denklemleri ve Eğim

1) Apartman etkinliğinde bulduğumuz doğru denklemi ve o doğrunun eğimini, bidon etkinliğinde bulduğumuz doğru denklemini ve o doğrunun eğimini aşağıdaki tabloya yazalım. Bu doğruların grafikleri ve eğimleri arasındaki fark nedir?

	Doğru Denklemi	Doğrunun eğimi
1		
2		

2) Aşağıdaki tabloya farklı doğru denklemleri ve bu doğruların eğimlerini yazalım. Bu doğruların denklemleri ve eğimleri arasında nasıl bir ilişki vardır?

	Doğru Denklemi	Doğrunun eğimi
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		



3) x+2y=6 doğru denkleminin eğimi kaçtır?

O. Mobile Operators Activity Sheet

Telefon Operatörleri

İki telefon operatörü **Tcell** ve **Zcell**'in aylık yurtdışı konuşma paket ücretini aşağıda verilmiştir.

Tcell

Paket giriş	**0 TL**
Dakika ücreti	**2 TL**

Zcell

Paket giriş	**5 TL**
Dakika ücreti	**1 TL**

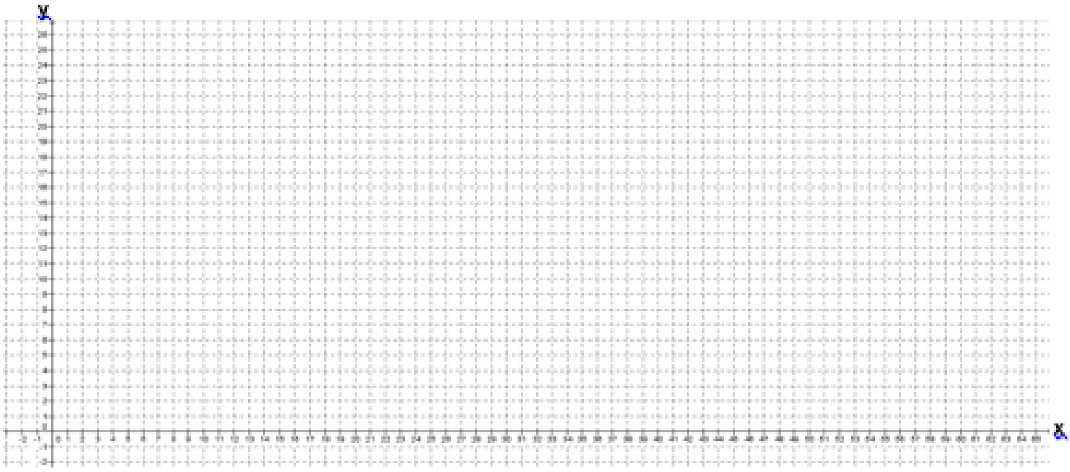
Tcell tablosu

Zaman (dak.)	Fatura Ücreti (TL)	(x,y)
0		
1		
2		

Zcell tablosu

Zaman (dak.)	Fatura Ücreti (TL)	(x,y)
0		
1		
2		

Buna göre aşağıdaki soruları bu operatörlerin ücret-dakika grafiğini çizerek cevaplayınız.



a) Aylık 4 dakika konuşmayı planlayan bir kişi hangi operatörü tercih etmelidir? Neden?

b) Aylık 5 dakika konuşmayı planlayan bir kişi hangi operatörü tercih etmelidir? Neden?

c) Aylık 7 dakika konuşmayı planlayan bir kişi hangi operatörü tercih etmelidir? Neden?

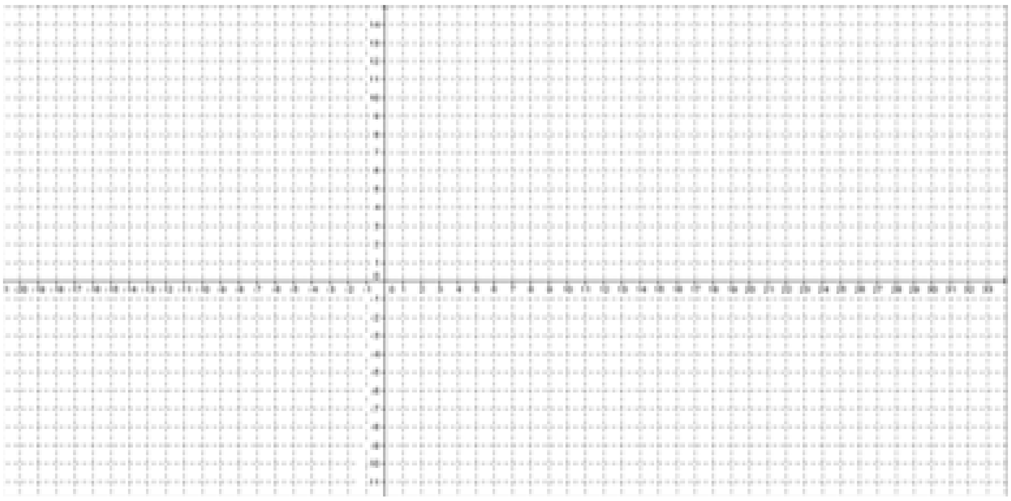
d) Buna göre hangi paketin ne zaman daha hesaplı olduğunu açıklayınız.

P. Equation Systems Activity Sheet

Denklem Sistemleri

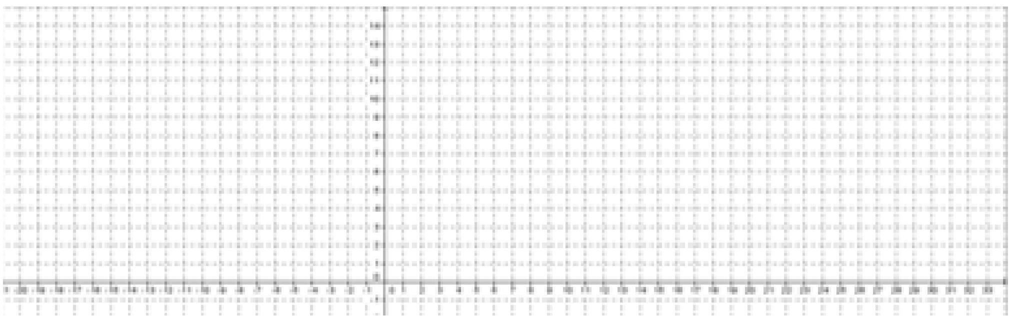
Denklem Sistemi 1:

	Doğru Denklemi	Eğim	Çözüm Kümesi	Doğruların birbirlerine göre durumları
1				
2				



Denklem Sistemi 2:

	Doğru Denklemi	Eğim	Çözüm Kümesi	Doğruların birbirlerine göre durumları
1				
2				



R. Stores Activity Sheet

MAĞAZA M ve N

Mağaza M		
Gün (x)	Satılan ürün Sayısı (y)	İlişki
1	1	
2	2	
3	3	
4	4	
5	5	
6	6	
7	7	y=...

Mağaza N		
Gün (x)	Satılan ürün Sayısı (y)	İlişki
1	9	
2	8	
3	7	
4	6	
5	5	
6	4	
7	3	y=...

İki mağazanın Mayıs ayının ilk haftasında, hangi gün kaç ürün sattığının verileri aşağıdaki gibidir.

Buna göre bu mağazanın satılan *ürün sayısı-gün* satış grafiğini çizelim.

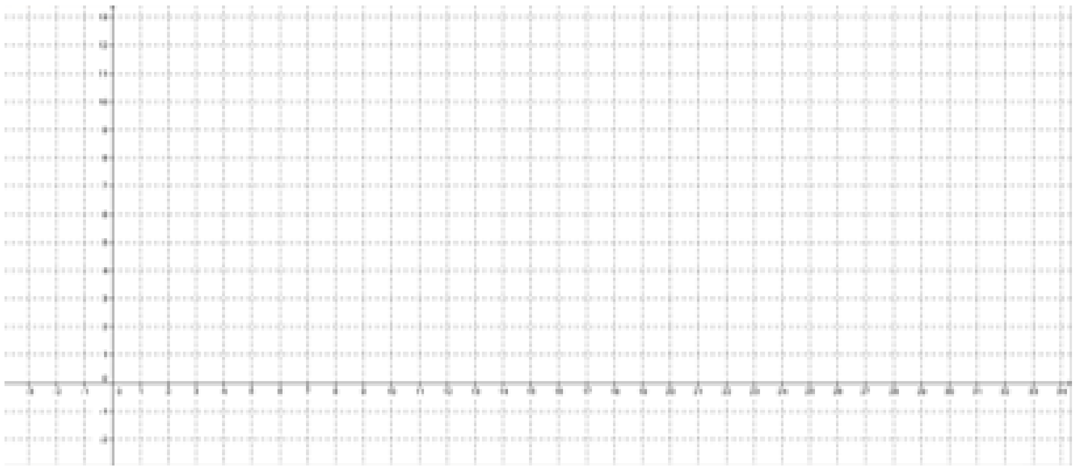
a) Bu grafiklerden bir doğru geçtiğini varsayalım. Bu doğruların denklemlerini ve grafiklerini inceleyelim.

b) Her iki mağazanın satış grafiğinin eğimleri arasında nasıl bir ilişki vardır?

c) Her iki mağazanın denklemleri arasında nasıl bir ilişki vardır?

d) Bu iki doğrunun eğimleri arasındaki ilişkiyi denklemleri üzerinden açıklayınız.

e) Her iki mağaza hangi gün eşit sayıda ürün satmıştır? Bu değer her iki doğru denklemi için ne ifade eder?



S. Turkish Summary/Türkçe Özet

ORTAOKUL MATEMATİK ÖĞRETMENLERİNİN TEKNOLOJİ DESTEKLİ SINIF ORTAMINDA EĞİM, DOĞRU DENKLEMLERİ VE GRAFİKLERİ ÖĞRETİMİ SIRASINDAKİ MATEMATİKSEL UYGULAMALARI

1. Giriş

Öğretim programlarındaki cebir küresel perspektifinden bakıldığında, birçok ülkede cebir programlarının içeriği öğretimin gerçekleştiği sınıfın sosyal bağlamı ile paralel olarak şekillenir (Kendal & Stacey, 2004). Yani, sınıflar öğrencilerin becerileri açısından katmanlı ise ülke akışlı bir sisteme sahipken, sınıflar katmanlı değilse ülke genel bir sisteme sahiptir (Kendal & Stacey, 2004). Türkiye'de, cebir öğretilirken, öğrenciler onuncu sınıfa kadar genel bir sistemin içinde yer alır. İki türde ortaokul ve birçok farklı türde liseler olmasına rağmen, matematik programındaki konular meslek liseleri haricinde tüm okullar için aynıdır. Dolayısıyla, farklı becerilere ve ilgi alanlarına sahip çocuklar ortaokul düzeyinde (5-8. Sınıflar) birlikte matematik öğrenirler. Bu sistem genel okul sistemine sahip diğer ülkelerdeki birçok cebir programına paraleldir. Bu bağlamda, Türkiye'de genel öğrencilere hitap eden ortaokul matematik öğretim programı sembolik cebir kullanımı ve fonksiyonlar üzerine durmaz. Cebir programlarını şekillendirmiş diğer bir konu ise cebirin bir öğretim programında, ayrı bir cebir dersi olarak mı veya entegre bir yaklaşımda bir öğrenme alanı olarak mı yer aldığıdır. Örneğin, Türkiye'deki öğretim programında, entegre matematik yaklaşımı öğrencilere cebir konularını üniteler olarak matematik dersi altında görmelerini sağlamaktadır, ki bu diğer konular arasında bağlantılar yapmak için bir fırsattır.

Cebirin nasıl algılandığına bağlı olarak, her ülkenin öğretim programı farklı genellemelere, farklı teknoloji kullanımlarına ve fonksiyonlara farklı yaklaşımlara sahiptir. Bu açıdan bakıldığında, Türkiye'de cebir 5. ve 6. sınıflarda genelleme ve örüntü ifade etme çalışması olarak, 7. ve 8. sınıflarda sembolik yönlendirme ve denklemleri çözme ve denklemlerin tablo ve grafikleri çalışması olarak (MoNE, 2009, 2013a), ve 9. sınıftan 12. sınıfa kadar ise fonksiyonlar ve onların dönüşümleri çalışması olarak tasarlanmıştır. Dolayısıyla, okul cebiri, genel sayı özellikleri ve ilişkilerinden ötedir ve bu nedenle önem taşımaktadır. Dahası, cebirsel yetkinliğin ve zorunlu okul yıllarında cebirsel yetkinliğin kazanılmasının önemi araştırmacılar tarafından vurgulanmaktadır (MacGregor, 2004). Bu bağlamda, doğrusal denklemleri çözmenin ötesinde, doğrusallık ve doğrusal ilişkilerin gösterimleri (yani, tablolar, grafikler ve denklemler) cebirsel yetkinlik için önemli unsurlardır (NCTM, 2000).

Dünyadaki öğretim programları göz önüne alındığında, eğim kavramı doğru denklemleri ve grafikleri bağlamında ortaokul matematik sınıflarından lise matematik sınıflarına kadar okul cebirinde öğretilmektedir. Amerika Birleşik Devletlerinde Temel Devlet Standartları Girişimi'nde (CCSSI, 2010), eğim kavramı 7 ile 8. sınıflarda öğretilmektedir. Detaylı olarak belirtmek gerekirse, öğrenciler yedinci sınıfta birim oranı orantısal ilişkilerin grafiklerini çizerken öğrenmeye başlar (CCSSI, 2010). Buna ek olarak, öğrenciler eğimi sekizinci sınıfta doğrusal denklemler ve doğrusal denklem sistemleri üzerine problem çözerek, analiz yaparak ve gösterimleyerek öğrenirler. Diğer deyişle, öğrenciler öğretim programında eğimi çoklu gösterimlerle ve çeşitli kavrayışlarla öğrenirler (Nagle & Moore-Russo, 2014). Türkiye'nin ulusal ortaokul matematik öğretim programında (MoNE, 2013), öğrenciler yedinci sınıfta, birim orandan bahsedilmeden, orantısal ilişkilerin tablolarında ve grafiklerinde orantı sabitini öğrenirler. Ayrıca, doğrusal ilişkileri tablolar, grafikler ve doğrusal denklemlerle öğrenirler. Öğrenciler sekizinci sınıfta ise eğimi modeller kullanarak ve doğrusal denklemler, grafikler ve tablolarla ilişkilendirerek öğrenirler. 2009 ilköğretim matematik öğretim programının yeniden düzenlenmiş bir hali olarak 2013 öğretim programı (MoNE, 2013a) sekizinci sınıf düzeyinde, doğrusal denklemler, grafikler ve tabloların gösterimleri arasındaki ilişkiler üzerine daha çok vurgu yaparak, eğim kavramına yönelik aynı kazanımları sunar.

Okul cebirinde eğim öğretmeyi öne çıkaran ve temel gösteren pek çok çalışma vardır. Ayrıca, matematik öğrenme ve öğretmede, eğim kavramının gerçek yaşam bağlamında ve matematiksel bağlamda çok çeşitli deneyimlerle kavramsallaştırılması ve kullanılması önemlidir. Ancak, eğimi farklı açılarından düşünmemizi gerektiren çeşitli kavramsallaştırmalar vardır. Bu durum ise bu çeşitli eğim kavramsallaştırmalarını anlamamızı gerektirmektedir. Dahası eğim kavramı orantısal muhakeme, niceliksel muhakeme ve kovaryasyonel muhakeme ile bağlantılıdır (Lesh, Post, & Behr, 1988; Lobato & Siebert, 2002; Lobato, Ellis, & Munoz, 2003; Lobato & Bowers, 2000; Carlson, Jacobs, Coe, Larsen, & Hsu, 2002; Johnson, 2012). Bu sebeple, eğim kavramı, lise matematiğinden önce ortaokul matematiğinde, cebir öğrenmede ve öğretmede temel taşlardan biridir.

Eğim öğrenme üzerine yapılan çalışmalar öğrencilerin ve öğretmenlerin eğim hakkındaki kavrayışlarından ve zorluklarından bahsetmiştir (Cheng, 2010, 2015; Cheng, Star, & Chapin, 2013; Cho & Nagle, 2017; Coe, 2007; Dündar, 2015; Hoffman, 2015; Lobato & Siebert, 2002; Nagle & Moore-Russo, 2013; Stump, Stump, 1997, 2001a, 2001b; Teuscher & Reys, 2010; Zaslavsky, Sela, & Leron, 2002). Örneğin, bir ölçüm olarak eğim kavrayışlarını ortaya çıkarmak için ve diklik ve değişim oranı ile ilişkili olarak eğim anlamalarını ortaya çıkarmak için lise öğrencileri ile (Stump, 2001b; Teuscher & Reys, 2010), kavramsal ve işlemsel yönleri düşünülerek eğim hakkındaki zorlukları ortaya çıkarmak için üniversite öğrencileri ile çalışmalar yürütülmüştür (Cho & Nagle, 2017). Ayrıca, ortaokul öğrencilerine odaklanan ve onların orantısal muhakemeleri ve eğim kavrayışları arasındaki ilişkiyi ortaya çıkaran çalışmalar görülmüştür (Cheng, 2010, 2015; Cheng, Star, & Chapin, 2013). Ek olarak, ortaokul öğrencilerinin eğim anlamalarını derinlemesine inceleyen az sayıda çalışma vardır. Örneğin, Lobato ve Siebert (2002) sekizinci sınıfı bitirmiş bir öğrencinin farklı durumlarda eğimi anlamasını ve niceliksel muhakemesini bir öğretim deneyi boyunca incelemiştir.

Diğer yandan, eğim üzerine öğretmenlerle yürütülmüş az sayıda çalışma vardır. Bazı çalışmaların sadece öğretmen adayları ile bazılarınınsa hem öğretmen adayı hem öğretmenlerle yürütüldüğü görülmüştür. Lise düzeyinde ve ortaokul düzeyinde öğretmen adayları ile yürütülen çalışmalar, öğretmen adaylarının eğim bilgilerini, tanımlar, kavram imajları ve kavrayışlar yolu ile ortaya koymuştur (Dündar, 2015; Stump, 1997, 2001a). Hem öğretmenlerle hem de öğretmen adayları (düzey belirtilmemiş) ile yapılan çalışmalar, onların koordinat sisteminde bir lineer fonksiyonun eğimine olan yaklaşımlarını ölçek ve açı kavramları ile ilişkili olarak açıklamaktadır (Zaslavsky, Sela, & Leron, 2002). Ek olarak, Nagle ve Moore-Russo (2013) öğretmenlerin ve öğretmen adaylarının eğim kavrayışlarını ve onların eğim öğretmede planladıkları öğretimsel materyalleri bir lisansüstü ders kapsamında sunmuştur. Ancak, sadece öğretmenlere odaklanarak yapılmış az sayıda eğim çalışması bulunmuştur. Örnek olarak, bir çalışma lise matematik öğretmenlerinin değişim oranı ve eğim kavrayışlarını belirtirken, diğer bir çalışma ortaokul matematik öğretmenlerinin eğim anlamalarını belirtmiştir (Coe, 2007; Hoffman, 2015). Ancak, bu çalışmalar öğretmenlerin eğim anlayışlarına ilişkin içerik sunarken, ortaokul matematik öğretmenlerinin öğretme pratiği sırasında bu anlayışlarının kullanımı hakkında bilgi vermezler. Dolayısıyla, eğim öğretiminde öğretmenlerin kendi matematiksel anlamalarını kullandıkları matematiksel uygulamaları anlamak zordur.

Öğretmenlerin, öğretim programına dayalı bir sınıf öğretiminde matematik eğitiminin gelişiminde vazgeçilmez ve temel bir parça olduğu apaçık ortada bir sayıltıdır (Remillard, 2005; Sherin & Drake, 2004). Öğretmenler ve öğretim programı reformları üzerine yapılan çalışmalar göstermektedir ki öğretmenlerin matematik alan bilgileri, öğretim uygulamaları ve onların öğrenme ve öğretme üzerine kişisel teorileri programların uygulanması ve onların değerleri üzerine önemli bir etkiye sahiptir (Manouchehri & Goodman, 1998). Matematik eğitiminde, Ball, Thames ve Phelps'in (2008) öğretim için matematiksel bilgi ile Rowland, Turner, Thwaites, ve Huckstep'in (2009) bilgi dörtlüsü modelleri gibi öne çıkan çerçeveler vardır. Ancak, bu çerçeveler muazzam katkılar sağlarken, iş teori ve uygulamayı birleştirmeye geldiğinde öğretmenlerin bilgilerini sınıf içinde matematik öğretiminde nasıl kullandığı ve geliştiği hala ucu açık bir durumdur (Doerr, 2004). İlaveten, öğretmenlerin gelişimi için, bilgi bileşenlerine ek olarak öğretmenlerin kişisel matematik öğretme modeli, matematik öğretmenin önemli yönlerinden biri olarak görülmektedir (Simon, 1997). Bu öğretim modeli "öğretimsel karar vermeye hem rehberlik eder hem de onu kısıtlar ve öğretmenin rolünü öğrencilerinin öğrenmeleri ile ilişkili olarak tanımlar" (Simon, 1997, s. 81). Bu açıdan, öğrenme-öğretme yörüngeleri, "didaktiksel karar verme için bir kavramsal çerçeve sunarken" (van den Heuvel-Panhuizen & Wijers, 2005, s. 305), öğretme ve öğretim programındaki kazanımlar arasında kavuşum sağlamak için de bir yol olarak önerilmektedir. Ancak, teori ve uygulama bağlanmadığında öğretmenlerin matematik öğretmesini anlamak zordur.

Özellikle, okul cebirinde eğim öğretmede öğretmenlerin matematiksel uygulamalarını incelemek ayırt edici bir çerçeveye sahip olmayı gerektirmektedir. Bu bakımdan, cebir öğretme için uygulamaları ve bilgileri belirten *Öğretme için Cebir Bilgisi* çerçevesini [KAT framework] kullanmak anlamlı görülmektedir. Bu çerçevedeki matematiksel uygulamalar boyutu üç ana kategori olan bağlama (bridging), kırpma (trimming) ve açma (decompressing) uygulamalarını içermektedir. Matematiksel bağlantılar yapma uygulamalarına karşılık gelen bağlama uygulaması, öğretmenlerin eğimle ilişkili kavramlar, konular, gösterimler ve alanlar arasında nasıl bağlantılar yaptığına ilişkin bir anlama sağlayabilir. Matematiksel karmaşıklığı ortadan kaldırma uygulamalarına karşılık gelen kırpma uygulaması, öğretmenlerin eğim öğretmedeki karmaşıklıkları nasıl anlaşılır hale dönüştürdüğüne ve nasıl giderdiğine ilişkin bir anlama sağlayabilir. Matematiksel karmaşıklığı belirginleştirme uygulamalarına karşılık gelen açma uygulaması, öğretmenlerin eğim öğretmedeki karmaşıklıkları nasıl vurguladığına ilişkin bir anlama sağlayabilir. Ancak, bu çerçeve öğretmenlerin cebirdeki matematik alan bilgisi açısından onları değerlendirmede fırsatlar sunarken, tüm bu matematiksel uygulamaların gerçek bir sınıf ortamında bir cebir konusu öğretimi boyunca nasıl ortaya çıktığının ucu açıktır.

Alan yazındaki sonuçlar göstermektedir ki öğretmenler cebir öğretmede kendilerini bilgi sağlayıcı olarak görme eğilimindedirler (Boaler, 2003) ve öğretmenler ders kitaplarındaki izlekleri takip etmekte ve rutin alıştırmalar ve örnekler vermektedir (McKnight, Travers, Crosswhite, & Swafford, 1985). Cebir öğretirken yeni yaklaşımlar düşünüldüğünde, örüntü tabanlı yaklaşım öne çıkmaktadır. Çünkü bu yaklaşım, denklemleri formüle etmeyi ve çözmede fonksiyonel ilişki, örüntü genelleyicileri olarak cebirsel ifadeler ve denklemlerdeki değişkenler üzerine kurarak, genellemeyi ve fonksiyonel ilişkileri anlamayı içermektedir (Stacey & MacGregor, 2001). Öğrencilerin ve öğretmenlerin başlangıç cebir anlamalarının bir sonucu olarak, cebire örüntü tabanlı yaklaşım birçok öğretim programı tarafından benimsenmektedir (CCSI, 2010; MoNE 2009, 2013a; NCTM, 2000). Dolayısıyla, Türkiye'de ortaokul matematik öğretmenleri bu yaklaşımı takip etmeye çalışmaktadır. Ancak, bir ortaokul matematik öğretmeni öğretim programındaki bu yaklaşımı kullanırken öğretmenin sınıfında ne tür bir eğim öğretimi gerçekleştirdiği bilinmemektedir.

Dahası, ortaokul matematik öğretim programında cebri öğrenme alanı ve diğer öğrenme alanlarında teknoloji (dinamik yazılımlar, hesap makineleri, hesap çizelgeleri, vd.) kullanımı da önerilmektedir (MoNE, 2013a). Aynı zamanda, matematik öğretmenleri için yeni bir yaklaşım için veya yeni bir araç ve teknoloji ile öğretim uygulamaları geliştirmenin zorluğu küçümsenmemelidir (Doerr, Ärlebäck, & O'Neil, 2013; Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010; Drijvers, Tacoma, Besamusca, Doorman, & Boon, 2013; Ruthven, Deaney, & Hennesy, 2009; Monaghan, 2004). Dolayısıyla, öğretmen eğitimi ve gelişimi için, matematik öğretmenlerinin teknoloji destekli bir sınıf ortamında öğretimsel araçlar kullanarak veya kullanmayarak öğretim uygulamalarını gösterme yollarını ve bu uygulamalardaki becerileri ve bilgilerini desteklemek önemli bir hal almaktadır (Lagrange & Ozdemir-Erdogan, 2009; Drijvers et al., 2013). Ancak, öğretim programına dayalı olarak teknoloji kullanarak belli bir konuyu öğretmede öğretmenlerin matematiksel uygulamaları nasıl ortaya çıkıyor tam olarak ortaya konmamıştır.

Araştırmalardaki tüm bu boşluklar göz önüne alınarak, bu çalışmada birincil amaç ortaokul matematik öğretmenleri bir öğretim programına dayalı olarak eğim, doğru denklemleri ve grafikleri için tasarlanan bir öğretim dizisini ortaya koyarken onların sınıftaki matematiksel uygulamalarını analiz etmektir. Bu doğrultuda, cebir öğretme perspektifinden matematiksel uygulamaları yorumlamak ise matematik öğretme işinde matematiksel uygulamalar hakkında derin bir anlama sağlayabilir. Ayrıca eğitim reformları ve güncel araştırmalar göz önünde bulundurularak çalışmanın odağı, ortaokul matematik öğretmenleri öğretim programına dayalı olarak tasarlanan bir öğretim programını ortaya koyarken, öğretmenlerin sekizinci sınıflar için teknoloji destekli bir sınıf ortamında eğim, doğru denklemleri ve grafiklerini öğretmedeki matematiksel uygulamalarını incelemek olarak belirlenmiştir. Bu çalışmada eğitim teknolojisi olarak Bozkurt ve Ruthven (2017) ve Drijvers vd.'nin (2010) çalışmalarına benzer şekilde, dinamik bir yazılım olan GeoGebra (GGB) kullanılarak geliştirilen GGB materyallerini kullanma kararı alınmıştır. Ancak, teknoloji destekli bir sınıf ortamında öğretim anlarında, öğretmenler bu GGB materyallerini kullanırken de ve bu GGB materyallerini kullanmadan matematiksel açıklamalar yaparken de matematiksel uygulamalar gösterebilir.

Dolayısıyla, bu çalışma eğim, doğru denklemleri ve grafikleri öğretiminde ortaokul matematik öğretmenlerinin matematiksel uygulamalarını sekizinci sınıfta teknoloji destekli bir ortamda hem 'GGB materyalleri kullanma' ve hem de 'GGB materyalleri kullanmadan matematiksel açıklamalar yapma' kapsamlarında ele almıştır.

Bu amaç doğrultusunda, aşağıdaki araştırma sorularına cevap aranmıştır.

1. Ortaokul matematik öğretmenlerinin bir öğretim dizisini ortaya koyması sırasında teknoloji destekli bir sınıf ortamında GGB materyalleri kullanırken ortaya çıkan matematiksel uygulamaları nelerdir?

1.1. Ortaokul matematik öğretmenlerinin bir öğretim dizisini ortaya koyması sırasında teknoloji destekli bir sınıf ortamında GGB materyalleri kullanırken gösterdiği bağlama, kırpma ve açma matematiksel uygulamalarının doğası nedir?

1.2 Ortaokul matematik öğretmenlerinin bir öğretim dizisini ortaya koyması sırasında teknoloji destekli bir sınıf ortamında GGB materyalleri kullanırken öğretmenler bu matematiksel uygulamaları birbiriyle nasıl ilişkilendirmektedir?

2. Ortaokul matematik öğretmenlerinin bir öğretim dizisini ortaya koyması sırasında GGB materyalleri kullanmadan matematiksel fikirleri açıklarken ortaya çıkan matematiksel uygulamaları nelerdir?

2.1 Ortaokul matematik öğretmenlerinin bir öğretim dizisini ortaya koyması sırasında GGB materyalleri kullanmadan matematiksel fikirleri açıklarken gösterdiği bağlama, kırpma ve açma matematiksel uygulamalarının doğası nedir?

2.2 Ortaokul matematik öğretmenlerinin bir öğretim dizisini ortaya koyması sırasında GGB materyalleri kullanmadan matematiksel fikirleri açıklarken öğretmenler bu matematiksel uygulamaları birbiriyle nasıl ilişkilendirmektedir?

2. Yöntem

Bu bölümde araştırmanın yöntemi, çalışmanın bağlamı, araştırma evreleri ile veri toplama ve veri analizi bölümlerinden oluşmaktadır.

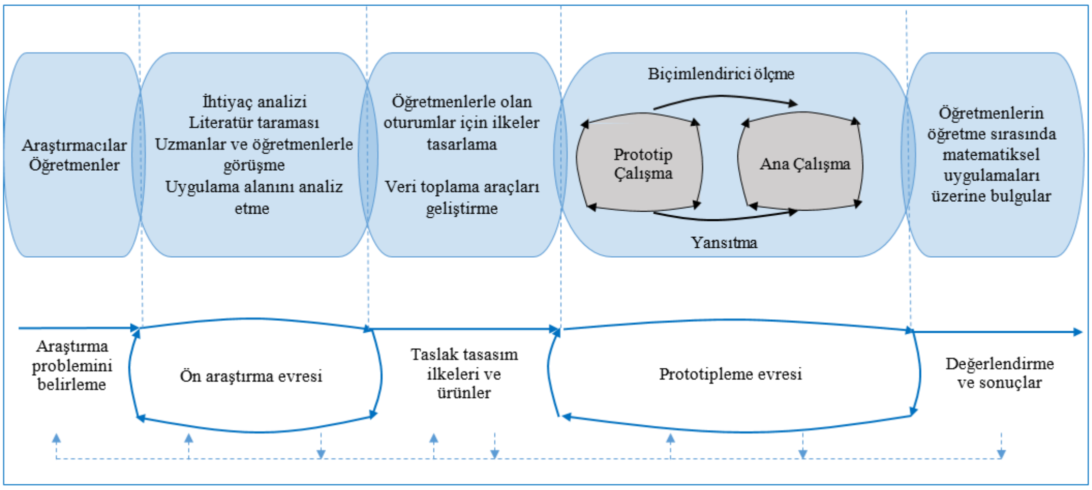
2.1 Araştırma Yöntemi

Bu çalışmada, öğretmenlerin sınıf içinde matematik öğretmedeki matematiksel uygulamalarını ortaya çıkarmak ve geliştirmek amacıyla bir müdahale tasarlanmıştır. Bu müdahaleyi gerçekleştirmek içinse, bu çalışmada tasarla, değerlendir ve yeniden gözden geçir döngüsel yaklaşımına ihtiyaç duyulmaktadır (van den Akker, Gravemeijer, McKenney, & Nieveen, 2006). Bu tasarım sürecinde amaç, gerçek sınıf ortamlarında öğretmenler ve öğrenciler için müdahaleler geliştirirken, matematik öğretmenlerinin matematiksel uygulamalarını anlamaktır. Araştırmacı, öğrencilerinin öğrenmelerinde bir öğretim dizisi oluşturmak için öğretmenlerin müdahaleler geliştirmesini sağlarken, öğretmenlerin öğrenmelerinde destekleyici bir dizi geliştirmek için kendisi de müdahaleler geliştirebilir. Bu süreçte, tasarım eğitimsel ilkelere dayalı olmalı ve tasarımın test edilmesi matematik öğretme ve öğrenme üzerine çerçevelere katkılar sağlamalıdır. Dolayısıyla, bu ihtiyaçlar eğitimsel bağlam içinde bir tasarım araştırmasının (tasarım deneyleri) genel karakteristikleri ile örtüşmektedir: "müdahaleci, yinelemeli, süreç odaklı, işe yararlık odaklı, ve kuram odaklı" (van den Akker et al., 2006, p. 5).

Tasarım deneyleri farklı amaçlarla kullanılmakla birlikte, Cobb, Zhao, ve Dean (2009) ile Zawojewski vd. (2008) tasarım deneylerini öğretmenlerin öğrenmeleri ve gelişimlerini anlamak için kullanma fikrini önermiştir. Dolayısıyla, tasarım araştırma yöntemleri sınıf içi öğretimdeki karmaşıklıkla birlikte öğretmenlerin bilgi ve deneyimlerini anlamak için kullanılabilir (Zawojewski vd., 2008). Bu bağlamda, tasarım araştırma yöntemi tiplerinden biri olan ve interaktif olarak bağlantılı öznelerin (öğrenciler, öğretmenler, araştırmacılar) gelişimini anlamayı içeren, çok katlı tasarım deneyi sınıf ortamındaki karmaşıklığı sarmada uygundur (Lesh & Kelly, 2000, Zawojewski vd., 2008). Bu çalışmada öğretmen-katı açısından çok katlı tasarım deneyine odaklanılmıştır.

Bu doğrultuda, araştırmacı, sınıf tasarım deneyi perspektifinden, öğretmenle ve öğretmen için oturumlar düzenlemiştir. Bir öğretmenle araştırma takvimi, öğretmenin ilgisinin 'bilgisayarlarda GGB materyalleri kullanıldığı teknoloji destekli bir sınıf ortamında eğim öğretimi için belli bir yaklaşım geliştirmek olduğu' bir tasarım deneyi sürecinde, bir öğretmenin matematiksel uygulamalarını incelemek için gerçek bir ortam sağlama fikri ile şekillenmiştir. Dolayısıyla, öğretmenin bu birincil niyeti öğretmenin işini eylem araştırmasından ayırmaktadır (Stephan & Cobb, 2013).

Tasarım deneyleri evreler ve döngüler yoluyla yürütülmektedir. Bu evrelerin isimleri çeşitli çalışmalarda faklı görülebilmesine rağmen (Cobb & Gravemeijer, 2008; McKenney, 2001; Plomp, 2007; Reeves, 2000, 2006), genel olarak ön araştırma evresi, prototipleme evresi ve değerlendirme olarak isimlendirilir (Plomp, 2007). Bu çalışmanın tasarım deneyi sürecinin şeması bu çalışmanın tüm sürecini sunmaktadır (Bakınız Şekil 1). Bu şema, bu çalışmanın sistematik olarak geliştiğini göstermektedir ve bu sürecin akışını oklarla belirtmektedir. Araştırmacı bu şemayı, tasarım deneyi araştırmasının evrelerini ve tasarım deneyinin bu evreleri ile ilişkili yönlerini bir araya getirerek kurmuştur. Başlangıçta, araştırmacıların ve öğretmenlerin katılımı ile araştırma problemi belirlenmiş ve araştırma soruları süreç içinde şekillenmiştir. Daha sonra, ön araştırma evresinde, ihtiyaç analizi, alan yazın taraması yürütülmüş, uzmanlarla ve öğretmenlerle görüşülmüş, uygulama bağlamı araştırmacılar ve öğretmenlerin katılımı ile analiz edilmiştir. Bu aşamaya dayalı olarak, taslak tasarım ilkeleri ve ürünler (yani, veri toplama araçları, öğretmenle gerçekleşecek oturum içerikleri tasarımı) geliştirilmiştir. Devamında, prototipleme evresi prototip çalışma ve ana çalışma evrelerini içermiştir. Prototipleme evresindeki her bir evre farklı devlet okullarında farklı öğretmenlerle yürütülmüştür. Prototip çalışma boyunca araştırmacı çalışmayı biçimlendirici ölçme ve yansıtma yoluyla ölçmüş ve yeniden gözden geçirmiştir. Buna göre ana çalışmadaki katılımcı ile bir araştırma takvimi gelişmiştir. Yukarıdaki hususlar göz önüne alınarak, bu tezin odağı olan katılımcı öğretmenle ana çalışma yürütülmüştür. Ana çalışma dört aşamadan oluşmuştur: öğretmenle ön görüşme, sınıf içinde öğretim sırasında GGB kullanımı hakkında öğretmenle bir buluşma, öğretim programındaki kazanımlar doğrultusunda yinelemeli öğretimi planlama-ortaya koyma-analiz etme döngüsü ve öğretmenle son görüşme. Devamında, araştırmacı öğretmenin öğretme sırasındaki matematiksel uygulamalarına ilişkin verileri açıklamak için analiz yapmıştır. Son olarak, öğretim dizisi ve öğretmenin matematiksel uygulamalarına ilişkin bulgular sunulmuştur.



Şekil 1. Bu çalışmanın tasarım deneyi süreci

2.2 Çalışmanın Bağlamı

Bu çalışma Türkiye'de devlet ortaokullarındaki matematik eğitimi bağlamında gerçekleşmiştir. Bu bağlamda, bu çalışma iki devlet ilköğretim okulunun orta kısmında 2012-2013 öğretim yılının ikinci döneminde yürütülmüştür. Veri toplama sürecinde, bu orta kısım 6-8. Sınıfları kapsamıştır. Ancak şunu hatırlatmak faydalı olacaktır: Nisan 2012'de Milli Eğitim Kanunun değişmesi (yani, 4+4+4 değişikliği) ile ortaokul kısmı ilköğretim okullarından ayrı bir aşama olarak ortaokullar altında ve 5. Sınıftan 8. Sınıfa olmak üzere 4 yıllık olmuştur. Ayrıca, 2013'e kadar tek tip bir ortaokul kısmı varken, şimdi iki tip devlet ortaokulu (genel ortaokullar ve imam hatip ortaokulları) bulunmaktadır.

Bu çalışma yürütüldüğü sırada, ortaokullar 2009 matematik öğretim programını (MoNE, 2009), öğretmen ise bu programa göre hazırlanmış sekizinci sınıf ders kitabını (MoNE, 2012) kullanmaktaydı. Bu dönemde yer alan cebir kazanımları eklerde verilmiştir (Appendix A).

Bu çalışmada, araştırmacı cebirin belirli bir konusunda bir öğretmenin matematiksel uygulamalarını incelemek için denklemler alt öğrenme alanını belirlemiştir. Özellikle, sekizinci sınıflar için önerilen matematik kitabında ve yıllık planda yer alan eğim, doğru denklemleri ve grafikleri konusu odaklanan matematiksel fikirler olmuştur. MoNE'nin (2009) öğretim programında, bu ünite ile ilgili kazanımlar şu şekildedir:

- Doğrunun eğimini modelleri ile açıklar
- Doğrunun eğimi ile denklemi arasındaki ilişkiyi belirler
- Doğrusal denklem sistemlerini grafikleri kullanarak çözer (MoNE, 2009, s. 290).

Eğitim sistemindeki değişikliğin ardından, 5. sınıftan 8. sınıfa ortaokul matematik öğretim programı ve kazanımlar 2013-2014 öğretim yılında 5. sınıflardan itibaren uygulanmak üzere yeniden düzenlenmiştir. Bu çalışma yürütülürken, katılımcı öğretmenler bu değişim ekibinin bir parçası olduğundan, yeniden düzenlenmiş bu öğretim programındaki (MoNE, 2013a) eğim, doğru denklemleri ve grafikleri üzerine olan kazanımlar da dikkate alınmıştır. MoNE'nin yeniden düzenlenmiş öğretim programında, bu konu ile ilgili kazanımlar şu şekildedir:

- Doğrunun eğimini modellerle açıklar
- Doğrusal denklemleri, grafiklerini ve ilgili tabloları eğimle ilişkilendirir
- Doğrusal denklem sistemlerinin çözümleri ile bu denklemlere karşılık gelen doğruların grafikleri arasında ilişki kurar (MoNE, 2013a, s.37).

Yeniden düzenlenmiş bu öğretim programında, bu kazanımlarla birlikte, bu matematiksel fikirlerin gerçek yaşam durumlarında ve uygun öğretim teknolojileri ile öğrenmenin ve öğretmenin önemi de vurgulanmıştır. Bu çalışmada, araştırma ekibi (tez yazarı, tez yöneticisi ve öğretmenler) yeniden düzenlenmiş öğretim programını dikkate almıştır.

2.2.1 Ortaokul Matematik Öğretmenleri

Katılımcı öğretmenler Zehra ve Oya devlet okullarında çalışan iki ortaokul matematik öğretmenidir. Bu öğretmenler, sınıflarında teknolojiyi kullanan, araştırma deneyimi olan ve zengin deneyimleri olan öğretmenlerdir. Bu sebeple bu çalışma için veriler üretebilecek potansiyele sahip oldukları düşünülmüştür. Ayrıca, gönüllü olarak bu çalışmaya katılmışlardır. Zehra öğretmen prototip çalışmanın katılımcısı iken Oya öğretmen ana çalışmanın katılımcısıdır. Aşağıda Oya öğretmen hakkında detaylar verilmiştir.

2.2.1.1 Oya Öğretmen

Oya öğretmen ilköğretim matematik öğretmenliği programından başarı ile mezun olmuş ve matematik eğitimi alanında yüksek lisans derecesini almıştır. Eğitim ve meslek hayatı boyunca çeşitli teknolojilerin matematik öğretirken kullanılması üzerine dersler almış ve uygulamalar yapmıştır. Ancak, GGB programını genel olarak bilmesine rağmen, bu çalışmaya kadar GGB'yi sınıf öğretiminde kullanmamıştır. Oya öğretmen devlet okullarında 8 yıldır öğretmenlik yapmaktadır (veri toplama sürecinde). Şu an çalıştığı okul onun ikinci okuludur ve 3 yıldır burada görev yapmaktadır. Oya zengin altyapısı ile farklı öğretim stratejileri kullanmaya ve geliştirmeye isteklidir. Ulusal ve uluslararası çeşitli projelerde yer almış ve öğretim programının yeniden düzenlenmesi için toplanan bir komisyonun üyesi olmuştur.

Oya'nın okulu Ankara'da düşük ve orta sosyoekonomik düzey aile çocuklarını içeren bir devlet okuludur. Oya'nın yönlendirmesi ile okulda her ders için ayrı derslikler oluşturulmuştur. Okuldaki iki matematik öğretmeni için iki ayrı matematik dersliği vardır.

Oya'nın matematik dersliğinde bir beyaz tahta, bir bilgisayar, bir projeksiyon cihazı ve perdesi, ile çeşitli somut materyallerin olduğu bir materyal dolabı vardır. Ayrıca, öğrenci posterleri ve çalışmaları duvarlarda ve panolarda yer almaktadır. Oya, sınıf oturma planını işbirliği içinde grup çalışmasını gözeterek hazırlamıştır. Öğrenciler genelde 6şarlı gruplar halinde otururken, grupların oturma yeri her hafta değişir ve öğrenciler sınıfın farklı alanlarında otururlar. Gruplar farklı başarı seviyelerindeki öğrencilerle heterojendir.

Oya sınıf öğretiminde, Milli Eğitim tarafından önerilen ders kitabını kullanır. Ayrıca, destekleyici materyaller olarak etkinlik kâğıtları veya çalışma kâğıtları hazırlar. Sınıflarında somut materyaller ve teknoloji kullanır (örneğin, ofis programları, dinamik yazılımlar). Bilgisayar teknolojilerini bir konuda, bir etkinlikte veya bir problem çözümünde farklı amaçlarla kullanır. Ayrıca somut materyalleri ve nesneleri de bir ders veya bir etkinlik sırasında kullanır. Dolayısıyla, öğretmen öğrencilerinin matematiksel fikirleri tartıştığı, dinamik yazılımlar ve materyaller kullandığı bir sınıf ortamı tasarlamıştır.

Yukarda bahsedilen özellikler öğretmenin sıradan sınıf öğretiminde gözlenirken ve öğretmenle yapılan mülakatlar sırasında görülürken, bu tasarım deneyi sırasında ek özellikler ortaya çıkmıştır. Bu tasarım deneyinde öğrenci gruplarına üç adet dizüstü bilgisayar verilmiştir. Öğretmen bu deney için hazırlanmış GGB materyallerini içeren dizüstü bilgisayarları kullanmaları için yönlendirmiştir. Aynı zamanda, GGB materyalleri ile ilişkili olan etkinlik kâğıtlarını da öğrencilere vermiştir. Ayrıca, öğretmen somut nesneler olarak eşit uzunlukta tahta parçaları kullanmıştır. Bu çalışma yürütülürken ise, Oya öğretmen iki sekizinci sınıf şubesinde öğretim yapmıştır. Bu şubelerde öğrenci sayısı 20 (12 kız, 8 erkek) ve 23tür (12 kız, 11 erkek).

2.3 Araştırma Evreleri ve Veri Toplama

Yukarda bahsedilen tasarım deneyi süreci doğrultusunda, bu araştırmada ön araştırma evresi ve prototipleme evresini içermektedir. Ön araştırma evresinde, araştırmacı öğretmenlerin matematiksel uygulamalarını incelemenin ve geliştirmenin ihtiyacını ortaya koymuş, bu doğrultuda *Öğretme için Cebir Bilgisi* çerçevesinin yorumlayıcı bir çerçeve olarak uygun olduğu belirlenmiştir.

Bu çalışmada veriler öğretmenlerden, 2012-2013 bahar döneminde, öğretmenler kendi matematik sınıflarında sekizinci sınıflara öğretim yaparken toplanmıştır. Birincil veri kaynağı öğretmenin sınıf oturumlarının video ve ses kayıtları iken, öğretmenin matematiksel uygulamalarının tutarlı bir değerlendirmesi için diğer veri kaynakları (bireysel ön-görüşme ve son-görüşme kayıtları, planlama ve analiz oturumları kayıtları, sınıf etkinlikleri kâğıtları ve alan notları) dikkate alınmıştır. Aşağıda bu adımlara kısaca açıklanmıştır.

2.3.1 Prototip Çalışma

Prototip çalışma bir devlet okulunda bir matematik öğretmeni ile yürütülmüş. Buradan elde edilen deneyimler ve ölçümler ana çalışmanın yürütülmesine ve öğretmenlerin matematiksel uygulamalarının incelenmesine yön vermiştir.

2.3.2 Ana Çalışma

Bu bölümde, prototip çalışmaya dayalı olarak yeniden yapılan düzenlemelerin ardından oluşturulan ana çalışma araştırma süreci verilmektedir (Bakınız Şekil 2).

Hazırlık aşaması iki adımdan oluşmaktadır. İlk adımda araştırmacı öğretmenin matematik öğretimi için hedefleri ve matematik öğretimindeki inanışları üzerine bir görüşme yapmıştır ve öğretmen hakkında ek bilgileri Öğretmeni Tanıma Bilgileri Protokolü ile toplamıştır. Sonraki adımda, öğretmenin matematik öğretme perspektifi hakkında sınıf yapısı ve değerleri açısından daha fazla bilgi edinmek için öğretmenin sıradan sınıf öğretimi gözlenmiş ve kamera ve ses kayıt cihazı ile kaydedilmiştir. İkinci aşama, öğretmen GGB yazılımı daha önceden sınıfta kullanmadığı için, sınıf içinde öğretimde GGB kullanımı üzerine bir buluşmayı içermiştir. Üçüncü aşama üç mikro-döngüden oluşan bir makro-döngüyü içermiştir. Tasarım mini döngüleri olarak da adlandırılan bu planla-ortaya koy-analiz et döngüleri günlük olarak ve yinelemeli olarak gerçekleşmiştir. Bu bağlamda ana çalışmada bu ana döngüye odaklanılmıştır. Öğretim tasarlanırken, öğretmen, öğretim dizisini, öğrencileri için öngörülen öğrenme yörüngesine dayanarak, eğim kavramı ve doğrusal denklemlerin ve grafiklerinin eğimi üzerine planlamıştır. Araştırmacı ise, bu süreçte tasarım deneyi yöntemlerini ortaya koymada bir yöneten ve GGB materyallerini tasarlamada ve düzenlemede öğretmenin ihtiyaçlarını yerine getirirken bir yardımcı olarak, öğretim etkinliklerinin detaylarını ortaya koymuştur. Sonuç olarak, araştırmacının asıl odağı sınıf öğretimi deneyi boyunca öğretmenin sınıf içindeki matematiksel uygulamalarını incelemektir. Öğretmenle yürütülen bu araştırma takvimi tez içinde verilmiştir (Bakınız Tablo 3.2). Veri toplama kaynakları ise aşağıda kısaca verilmiştir.

Öğretmeni Tanıma Bilgileri Protokolü, katılımcı öğretmenin geçmişi ve bugünü ile ilgili kesin bilgi edinmek üzerine yapılandırılmıştır. Bu protokol, öğretmenin lisans eğitimi, geçmişte ve şu an çalıştığı okuldaki öğretmenlik deneyimi üzerine sorular içermektedir (Appendix B).

Öğretmenle ön görüşme, araştırmacı tarafından sınıf ortamı dışında yürütülmüş ve ses kayıt cihazı ile kaydedilmiştir. Bu görüşme protokolü hedefler, matematiğin doğası hakkında inanışlar, matematik öğrenme hakkında inanışlar, matematik öğretme hakkında inanışlar, belirli öğrenciler ve öğrenci sınıfları hakkında inanışlar ve diğer sorulardan oluşmaktadır (Appendix C). Bu protokol Schoenfeld (1998), Haser (2006) ve Çetinkaya'nın (2006) çalışmaları doğrultusunda hazırlanmıştır.

GGB hakkında öğretmenle görüşme, matematik öğretmede GGB kullanımını konuşmak üzere yapılmıştır. Bu görüşmede araştırmacı öğretmede daha önceden hazırlanmış GGB materyallerini ve bu materyaller üzerinde GGB araçlarını göstermiştir. Ayrıca, sınıf etkinliklerinde GGB'nın nasıl kullanılacağı üzerine tartışılmıştır.

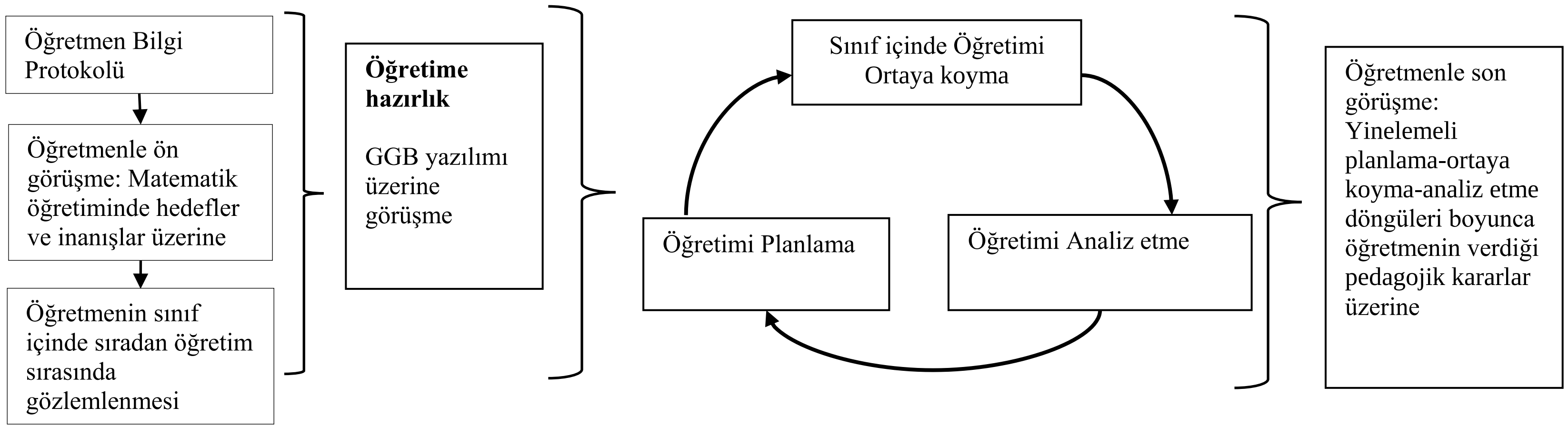
Öğretimi planlama oturumları yapılırken planlama mülakatları yapılmıştır. Bunlar bir buluşma ortamında gerçekleşmiştir. Planlama oturumlarında araştırmacı öğretmenin planladığı matematiksel uygulamaları anlamayı amaçlamıştır. Öğretim dizisinin planlanması evresinde, bir takım olarak araştırmacı ve öğretmen, öğretim programındaki kazanımlara dayalı olarak öğretim hedeflerini dikkate almıştır. Bu süreç sırasında öğretim hedefleri aşağıdaki gibi şekillenmiştir:

- Fiziksel bir nesnenin eğimini hesaplar.
- Fonksiyonel durumlarda bir doğrunun eğimi ile bir doğru denkleminin eğimi arasında ilişki kurar.
- Grafikleri, doğruların eğimlerini ve denklem sistemlerinin çözümlerini ilişkilendirerek doğrusal denklem sistemlerini çözer.

Öğretimin sınıf içinde ortaya konması sürecinde, sınıf oturumları video ve ses kayıt cihazları ile kaydedilmiş, araştırmacı alan notları almış, öğrencilerin çalışmalarının birer kopyası alınmış ve bilgisayarların ekran görüntüsünü kaydetmiştir. İki adet 8. Sınıf şubesinin 450şer dakikalık video kayıtları toplamıştır. Araştırmacı öğretim sırasında sınıfın en arkasında oturmuş ve kamerayı kontrol etmiştir. Araştırmacı katılımcı olmayan bir gözlemci rolüyle sınıflara katılmıştır. Gözlemler sırasında öğretmenin matematiksel uygulamaları üzerine alan notları almıştır (Bakınız Appendix D).

Öğretimi analiz etme oturumlarında öğretmen ve araştırmacı öğrenci öğrenmelerini değerlendirmek ve sonraki öğretim etkinliklerinde yeniden düzenlemeler yapmak üzere bir araya gelmiştir. Bu oturumlarda veri toplarken, ders kayıtlarının videolarını kullanarak öğretmen kişiler arası süreci hatırlama görüşme yöntemini kullanmıştır. Örneğin şu şekilde sorular sorulmuştur: "Bu soruyu tüm sınıfa sorarken ne düşünmüştün veya ne beklemiştin?", "Öğrenciler GGB materyallerini kullanarak hesaplamalar yaptığında ne düşündün?” “O anda neden sürgü aracını kullandın?”, ve “bu süreçte (ders sırasındaki bir zaman aralığına işaret ederek) öğrenciler eğimi nasıl öğrendi?”

Öğretmenle yapılan son görüşmede, araştırmacı ve öğretmen tüm öğretim dizisini geriye dönük olarak analiz etmek için bir araya gelmiştir. Ayrıca, bu görüşme araştırmacı tarafından sınıf ortamı dışında yürütülmüş ve ses kayıt cihazı ile kaydedilmiştir. Araştırmacı bu görüşmede, öğretmenin süreç içinde verdiği pedagojik kararlara yönelik yarı yapılandırılmış bir görüşme protokolü kullanmıştır (Bakınız Appendix F). Bu görüşme süreci, araştırmacıya hem öğretmenin öğretim dizisini ve matematiksel uygulamalarını nasıl analiz ettiğine ilişkin bir geri bildirim hem de öğretim dizisinin tasarımına ilişkin bir geribildirim vermiştir.



Şekil 2. Sınıf içinde öğretmede bir öğretmenin matematiksel uygulamalarının bütününü geliştirmeyi amaçlayan ana çalışmanın sürecinin şeması

2.3 Veri Analizi

Bu çalışmanın veri analizinde gömülü teori yöntemleri kullanılmıştır (Strauss & Corbin, 1998). Gömülü teorinin karşılaştırmalı analiz yöntemi kesin kanıtlar toplamak, ampirik genellemeler sağlamak, analiz birimini belirlemek, ve teori üretmek veya var olan veya gelişmekte olan teorileri doğrulamak için kullanılmaktadır (Glaser & Strauss, 2006). Bu bağlamda, bu çalışmada karşılaştırmalı analiz yöntemi kullanılmıştır.

Bu çalışma çeşitli veri kaynakları ile çok büyük bir veri grubuna sahipken, odaklanılan veri grubu, öğretim programındaki kazanımlara dayalı olarak gelişip üç mikro-evreden oluşan ana çalışma ve buradaki öğretmenin sınıf içindeki öğretimidir. Diğer bir deyişle, bu çalışmanın veri analizinde odak, öğretmenin sınıf içinde öğretimi ortaya koyma sırasındaki matematiksel uygulamalardır. Öğretmenin matematiksel uygulamaları iki koldan analiz edilmiştir. Birinci kolda, bu uygulamalar öğretmen GGB materyalleri ve somut nesneler kullanırken analiz edilmiştir. Araştırma problemi teknolojik araçların kullanımı sırasındaki matematiksel uygulamaları incelemek amacıyla şekillenmesine ve diğer öğretim araçlarını göz önüne almamasına rağmen, katılımcı öğretmen somut nesneleri öğretim araçları olarak kullanmıştır. Dolayısıyla, veri analizinde öğretmen somut nesneler kullandığında gösterilen matematiksel uygulamalar göz ardı edilmemiş ve öğretmen GGB materyalleri ve somut nesneleri kullandığında gösterilen matematiksel uygulamalar bulgularda aynı bölüm altında bütüncül bir anlayışla sunulmuştur. İkinci kolda, öğretmenin matematiksel uygulamaları, öğretmen GGB materyalleri ve somut nesneleri kullanmadan matematiksel fikirleri açıklarken analiz edilmiştir. Verileri analiz ederken, *Öğretme için Cebir Bilgisi* çerçevesi kullanılmıştır.

3. Bulgular

Çalışmanın bulguları, öğretmenin bağlama (bridging), kırpma (trimming) ve açma (decompressing) gibi matematiksel uygulamaları kapsamında, öğretmen GGB materyalleri ve somut nesneler kullandığı sırada ve GGB materyalleri ve somut nesneler kullanmazken matematiksel açıklamalar yaptığı sırada olmak üzere iki kapsamda sunulmuştur.

Bu araştırma sırasında geliştirilen öğretim dizisi üç evreden oluşmaktadır. Birinci evrede, “Fiziksel bir nesnenin eğimini hesaplar” kazanımı doğrultusunda üç etkinlik geliştirilmiştir. İlk olarak *Tahtaların Duruşu* etkinliğinde sabit uzunlukta bir nesnenin farklı pozisyonlardaki eğimi yorumlanmış ve burada eğim hesabı değerlendirilmiştir. İkinci olarak, *İtfaiye aracı* etkinliğinde, yatay uzunluğun sabit olduğu bir durum içinde değişen uzunluktaki bir yangın merdiveninin eğim hesaplamaları yapılmış ve yorumlanmıştır. Üçüncü olarak, *Çadır* etkinliğinde dikey uzunluğun sabit olduğu bir durum içinde değişen uzunluktaki bir çadır ipinin eğim hesaplamaları yapılmış ve yorumlanmıştır. Her bir etkinlikle ilişkili olarak bir GGB materyali hazırlanmıştır. Bu GGB materyalleri, zeminde bir ızgara görünümü üzerinde koordinat sistemi ile ilişkilendirmeden, dinamik nesneler (örneğin, tahta parçası nesnesi için doğru parçası), dinamik metinler (örneğin, eğim hesabı için dinamik metin) ve statik şekiller (örneğin, itfaiye etkinliğinde bina gösterimi için sabit bir dikdörtgen) içermiştir.

İkinci evrede, “fonksiyonel durumlarda bir doğrunun eğimi ile bir doğru denkleminin eğimi arasında ilişki kurar” kazanımı doğrultusunda üç etkinlik geliştirilmiştir. Burada fonksiyonel doğrusal durumlar ile sınırlı kalınmıştır.

Üçüncü evrede, “Grafikleri, doğruların eğimlerini ve denklem sistemlerinin çözümlerini ilişkilendirerek doğrusal denklem sistemlerini çözer” kazanımı doğrultusunda üç etkinlik geliştirilmiştir.

3.1 GGB materyalleri ve somut nesneler kullanma sırasında Matematiksel Uygulamalar

Bu bölüm, teknoloji destekli bir sınıf ortamında GGB materyalleri ve somut nesnelerle öğretim sırasında, öğretmenin matematiksel uygulamalarını içermektedir. Bu matematiksel uygulamalar öğretim dizisinin her bir evresi bağlama uygulamaları, kırpma uygulamaları ve açma uygulamaları başlıkları altında sunulmuştur.

3.1.1 Bağlama uygulamaları

Öğretmenin bağlama uygulamaları aşağıdaki uygulamalar altında toplanmıştır. Bu uygulamalar farklı boyutların çeşitli kombinasyonlarından oluşmaktadır.

Tablo 1. GGB materyalleri ve somut nesneleri kullanmada bağlama uygulamaları

Uygulama	**Evre sayısı**
Somut nesne ve matematiksel durumu/kavramı bağlama	1
Somut nesne, matematiksel durum/kavram, ve GGB materyali bağlama	1
Matematiksel durum/kavram/süreç, GGB materyali, etkinlik kağıdı bağlama	1
Matematiksel durum/kavram ve GGB materyali bağlama	1, 2
Matematiksel durum/kavram ve GGB materyali bağlama	1, 2, 3
Kavram(lar)ın kavrayışlarını ve GGB materyali bağlama	1, 2, 3
Matematiksel durum/kavram, GGB materyali, ve etkinlik kağıdını bağlama	2, 3
Somut nesne, matematiksel süreç ve GGB materyali bağlama	2
Farklı dinamik yazılımları bağlama	2
Matematiksel durum/süreç ve GGB materyali bağlama	3

Bu uygulamalara bakıldığında, matematiksel durum/kavram ve GGB materyali bağlama uygulamasına ve kavramların kavrayışlarını ve GGB materyali bağlama uygulamasına tüm evrelerde rastlandığı görülmüştür. Dolayısıyla, bu uygulama altında gelişen eylemler tüm öğretim dizisini yönlendirmiştir. Somut nesne ile ilişkili bağlama uygulamaları ise temelde birinci evrede görülmüştür.

Birinci evre için yukarıdaki uygulamalar çeşitli boyutlarla ortaya çıkmıştır. Somut nesne boyutu tahta parçasını içermiştir. GGB boyutu, GGB grafik görünümünde, doğru parçalarını (yani, tahta, merdiven, ip, yatay uzunluk, dikey uzunluk- dinamik veya statik-temsili için), doğru parçasının iz bırakmasını, sürgü aracını, eğim işaret kutusunu, statik ve dinamik eğim hesap metnini içermiştir. Matematiksel boyutu şunları içermiştir: 1) matematiksel durum olarak fiziksel durumları (yani, tahtanın duruşu, yangın merdiveninin duruşu, çadır ipinin duruşu), 2) matematiksel kavram olarak doğru parçaları, doğru parçalarının dikey ve yatay uzunlukları, eğimin faktörleri ve eğimi, 3) matematiksel süreç olarak eğim hesaplama ve geometrik oran olarak eğim formülü. Kavrayışlar boyutu eğimin faktörlerinin gösterimleri ve tanımsız eğimin gösterimlerini içermiştir.

İkinci evre için yukardaki uygulamalar çeşitli boyutlarda ortaya çıkmıştır. Somut nesne boyutu tahta parçasını ve bir kalemi içermiştir. GGB boyutu şunları içermiştir: 1) GGB grafik görünümünde noktaları, grafikleri, bir doğrunun eğimini, doğru çizme aracını, noktalar için işaret kutularını, eğim işaret kutusunu, dinamik eğim hesaplama metnini ve 2) GGB cebir görünümünde denklemleri ve farklı denklem formlarını (standart form ve eğim-kesişim formu). Ayrıca öğretmen GGB yazılımı ile daha önceden kullandığı bir dinamik yazılımı matematiksel özellikleri açısından bağlamıştır. Matematiksel boyutu şunları içermiştir: 1) matematiksel durum olarak fonksiyonel durumları, 2) noktaları, doğru grafiğinin noktalarını, doğru grafiklerini, doğrusal ilişkiyi, denklemi, standart denklem formunu, eğim-kesişim denklem formunu, benzerliği, değişimi, aralıklar için parçaları, eğimi, y-kesişim, ve sabit terimi, 3) matematiksel süreç olarak eğim hesabını. Etkinlik kâğıdı boyutu ise tabloları, öğrencilerin nokta grafiklerini ve doğru grafiklerini içermiştir.

Üçünü evre için de yukardaki uygulamalar çeşitli boyutlarda ortaya çıkmıştır. GGB boyutu şunları içermiştir: 1) GGB grafik görünümünde grafikleri, noktaları, eğim işaret kutusu, eğim değerini, denklem işaret kutularını, sürgü aracını, ve noktaların izi bırakmalarını ve 2) GGB cebir görünümünde eğim değerini, denklemleri ve farklı denklem formlarını. Matematiksel boyutu şunları içermiştir: 1) matematiksel durum olarak fonksiyonel durumları, 2) matematiksel kavram olarak doğruların denklemlerini, doğru grafiklerini, denklemlerin çözümleri, noktalar, sıralı ikililer, nokta koordinatları, kesişim noktasının koordinatları, eğim, bir doğrunun eğimi, bir denklemin eğimi, eğim-kesişim denklem formu, doğruların davranışları, 3) matematiksel süreç olarak bir denklem sisteminin çözüm kümesini bulmayı ve eğim hesaplamayı. Etkinlik kâğıdı boyutu ise tabloyu, öğrencilerin nokta grafiklerini ve doğru grafiklerini içermiştir.

3.1.2 Kırpma uygulamaları

Öğretmenin kırpma uygulamaları aşağıdaki uygulamalar altında toplanmıştır. Bu uygulamalar öğretmenin konu ile ilgili karmaşıklığı ortadan kaldırmak veya konuyu sadeleştirmek için çeşitli uygulamalardan oluşmaktadır.

Tablo 2. GGB materyalleri ve somut nesneleri kullanmada kırpma uygulamaları

Uygulama	**Evre sayısı**
Somut nesne kullanımı kırpmak için yönlendirme	1
GGB materyali kullanarak matematiksel durumu kırpma	1, 2, 3
GGB materyali kullanarak hesaplamayı kırpma	1, 2, 3
GGB materyali kullanarak denklemler ve doğrular arasındaki ilişkiyi kırpma	2
GGB materyali kullanarak grafiksel gösterimleri kırpma	3
GGB'nın matematiksel gerekliliklerini yapma	2
GGB materyali koşullarında matematiksel zorlukları irdeleme	1, 2, 3

Bu kırpma uygulamalarından GGB materyali kullanarak matematiksel durumu kırpma, GGB materyali kullanarak hesaplamayı kırpma ve GGB materyali koşullarında matematiksel zorlukları irdeleme uygulamaları tüm evrelerde görülmüştür. Somut nesneye ilişkin olarak ise nesne kullanımına ilişkin bir kırpma dışında bir matematiksel uygulama görülmemiştir.

Ayrıca bu kırpma uygulamalarının bağlama uygulamaları ile ilişkili olarak verildiği görülmüştür. Örneğin, *Tahtaların Duruşu* etkinliğinde öğretmenin önceliği bir 'Matematiksel durum/kavram ve GGB materyali bağlama' uygulaması iken matematiksel doğruluğu sağlamak adına aynı zamanda 'GGB materyali kullanarak matematiksel durumu kırpma' uygulaması da göstermiştir.

3.1.3 Açma uygulamaları

Öğretmenin açma uygulamaları aşağıdaki uygulamalar altında toplanmıştır. Bu uygulamalar öğretmenin konu ile ilgili karmaşıklığı ortaya koyduğu çeşitli uygulamalardan oluşmaktadır.

Bu açma uygulamalarında 'ön karşılaştırmalar' uygulaması her evrede görülmüştür. Bu uygulamaların ön' açma' olarak isimlendirilmesinin nedeni, öğretmenin buradaki eylemleri açma uygulamalarına hazırlayıcı unsurlar içermekle birlikte matematiksel karmaşıklığı tam olarak ortaya koymamaktadır. Ayrıca GGB materyali kullanarak bir doğrunun eğim bileşenlerini yorumla uygulaması ikinci ve üçüncü evrelerde görülmüştür. Burada öğretmenin bir nesnenin eğim bileşenleri ile bir doğrunun eğim bileşenlerini ayrı ele aldığı görülmektedir.

Tablo 3. GGB materyalleri ve somut nesneleri kullanmada kırpma uygulamaları

Uygulama	Evre sayısı
Ön Karşılaştırmalar	1, 2, 3
Somut nesne kullanarak bir nesnenin eğiminin bileşenlerini yorumlama	1
GGB materyali kullanarak bir nesnenin eğim bileşenlerini yorumlama	1
GGB materyali kullanarak bir doğrunun eğim bileşenlerini yorumlama	2, 3
Dinamik yazılım kullanımına değer verme	1
GGB materyali kullanarak denklem sistemlerinin çözümlerinin bileşenlerini yorumlama	3

Bu uygulamanın diğer uygulamalarla olan ilişkilerine bakıldığında, açma uygulamalarının kırpma uygulamaları ile ilişkili olarak verildiği görülmüştür. Ayrıca bağlam uygulamaları ile de ilişki olarak verildiği görülmüştür. Ek olarak, bu üç uygulamanın birbirini izlediği veya ilişkilendirildiği de görülmüştür.

3.2 GGB materyalleri ve somut nesneler kullanmadan matematiksel fikirler açıklama sırasında Matematiksel Uygulamalar

Bu bölüm, GGB materyalleri ve somut nesneler kullanmadan matematiksel fikirler açıkladığı sırada öğretmenin matematiksel uygulamalarını içermektedir. Bu matematiksel uygulamalar öğretim dizisinin her bir evresine ilişkin olarak bağlama uygulamaları, kırpma uygulamaları ve açma uygulamaları başlıkları altında sunulmuştur.

3.2.1 Bağlama uygulamaları

Öğretmenin GGB materyalleri ve somut nesneler kullanmadan matematiksel fikirler açıklama sırasındaki bağlama uygulamaları aşağıdaki uygulamalar altında toplanmıştır. Bu uygulamalar farklı boyutların çeşitli kombinasyonlarından oluşmaktadır.

Tablo 4. GGB materyalleri ve somut nesneleri kullanmadan matematiksel fikirler açıklarken bağlama uygulamaları

Uygulama	**Evre Sayısı**
Kavramları ve gösterimleri bağlama	1, 2, 3
Kavramları bağlama	1, 2, 3
Etkinlikleri bağlama	1,2, 3
Konuları bağlama	2, 3

3.2.2 Kırpma Uygulamaları

Öğretmenin GGB materyalleri ve somut nesneler kullanmadan matematiksel fikirler açıklama sırasındaki kırpma uygulamaları aşağıdaki uygulamalar altında toplanmıştır. Bu uygulamalar öğretmenin konu ile ilgili karmaşıklığı ortadan kaldırmak veya konuyu sadeleştirmek için çeşitli uygulamalardan oluşmaktadır.

Tablo 5. GGB materyalleri ve somut nesneleri kullanmadan matematiksel fikirler açıklarken kırpma uygulamaları

Uygulama	**Evre Sayısı**
Matematiksel bir değeri açıklama	1, 2, 3
Bir çözüm yöntemini/işlemi açıklama	1, 2, 3
Bir kavramın uygunluğunu belirleme	1, 2
Matematiksel ilişkileri açıklama	1, 2, 3
Etkinliğin bağlamını açıklama	1, 2, 3
Uygun matematiksel terimleri kullanma	1, 2, 3
Öğrencilerin matematiksel zorluklarını ortaya çıkarma	3

Çok sık olmasa da bu uygulamaların bağlama uygulamaları ile ilişkilendirildiği görülmüştür.

3.2.3 Açma Uygulamaları

Öğretmenin GGB materyalleri kullanmadan matematiksel fikirler açıkladığı sıradaki açma uygulamaları aşağıda verilmiştir. Ancak bu uygulamalar ‘ön açma’ uygulamaları olarak ortaya çıkmıştır. Yani bu uygulamalar matematiksel karmaşıklığı tam olarak ortaya koymaktan ziyade hazırlayıcı açma unsurları içermektedir.

Tablo 6. GGB materyalleri ve somut nesneleri kullanmadan matematiksel fikirler açıklarken kırpma uygulamaları

Uygulama	**Evre sayısı**
Doğrunun eğimini yorumlama	2, 3
Denklem sistemlerini yorumlama	3

Bu uygulamalar birinci evrede hiç görülmemiştir. Diğer evrelerde ise görülmüştür ancak çok yoğun değildir.

4. Sonuçlar

Bu çalışmada alan yazında belirtildiği gibi, öğretim programına dayalı olarak tasarlanan bir öğretim dizisini ortaya koyması sürecinde, eğim, doğru grafikleri ve denklemleri konusunu teknoloji destekli bir sınıf ortamında öğretmede bir matematik öğretmenin matematiksel uygulamaları açıklanmıştır. Bu açıklamalar bir öğretmenin çeşitli bağlama, açma ve kırpma uygulamaları ve bu uygulamalar altında çeşitli eylemleri gösterdiği görülmüştür.

Bu süreçte, bağlama uygulamalarının en sık kullanılan matematiksel uygulamalar olduğu görülmüştür. Ayrıca bu uygulamaların kırpma ve açma uygulamalarını desteklediği veya geliştirdiği düşünülmektedir. Bunun yanı sıra, kırpma ve açma uygulamalarının da birbirini geliştirdiği düşünülmektedir. Teknoloji destekli bir sınıf ortamında bağlama, kırpma ve açma uygulamaları arasındaki ilişkiler karşılaştırıldığında bu ilişkilerin, GeoGebra materyalleri kullanmadan yapılan öğretime kıyasla, GeoGebra kullanılarak yapılan öğretimde daha güçlü olduğu görülmüştür.

T. Curriculum Vitae

Personal Information

Surname, Name: Yemen-Karpuzcu, Seçil

Nationality: Turkish

Data and Place of Birth: 23.08.1985, İzmir

Marital status: Married

Email: secilyemen@gmail.com

Education

Degree	Institution	Year
Doctor of Philosopy	Elementary Mathematics Education, Middle East Technical University, Ankara, Turkey	2017
Master of Science	Elementary Mathematics Education, Dokuz Eylül University, İzmir, Turkey	2009
Bachelor of Science	Elementary Mathematics Teacher Education, Kocaeli University, Kocaeli, Turkey	2007

Work Experience

Years	Place	Enrollment
December 2010-November 2016	Department of Elementary Education, Middle East Technical University, Ankara, Turkey	Research Assistant

Publications

Journal Papers

Yemen-Karpuzcu, S., Ulusoy, F., & Işıksal-Bostan, M. (2017). Prospective middle school mathematics teachers' covariational reasoning for interpreting dynamic events during peer interactions. *International Journal of Science and Mathematics Education, 15* (1), 89-108. doi 10.1007/s10763-015-9668-8

International Conference Papers and Presentations

Kandil, S., **Yemen-Karpuzcu, S.** & Işıksal-Bostan M. (2017). Prospective middle school mathematics teachers' solution strategies regarding multiplication with decimals. *Paper presented at European Conference on Educational Research (ECER)*, 2017 Aug 21-25, Copenhagen, Denmark.

Işıksal-Bostan, M., & Yemen-Karpuzcu, S. (2017). The role of definitions on classification of solids including (non)prototype examples: The case of cylinder and prism. 10th Congress of European Research in Mathematics Education (1-5 February, 2017), Dublin *(in press)*

Işıksal-Bostan, M., & **Yemen-Karpuzcu, S.** (2016). Prospective middle school mathematics teachers' knowledge on cylinder and prism: Generating Definitions and Relationship. *Paper presented at the 13th International Congress on Mathematical Education (ICMI),* 2016 July 24-31. Hamburg, Germany.

Yemen-Karpuzcu, S., & Işıksal-Bostan, M. (2014). A case study of in-service mathematics teacher's use of knowledge in teaching algebra with dynamics software. In Oesterle, S., Nicol, C., Liljedahl, P., & Allan, D. (Eds.). (2014). *Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education* (Vol. 6), p. 269.

Yemen-Karpuzcu, S., Ayan R., & Işıksal-Bostan, M. (2014). Prospective middle school mathematics teachers' computational estimation strategies for addition. In Şahin, İ., Kıray, A., Alan, S. (Eds.), *International Conference on Education in Mathematics, Science and Technology Proceeding Book,* p. 1284. Konya: ICEMST.

Yemen-Karpuzcu, S., Ulusoy, F., & Işıksal-Bostan, M. (2013). A case study on pre-service mathematics teachers' covariational reasoning in peer learning. In A. M. Lindmeier & A. Heinze (Eds.), *Proceedings of the 37th Conference of the*

International Group for the Psychology of Mathematics Education, Vol5, p. 200. Kiel, Germany: PME.

Yemen-Karpuzcu, S., & Aygün, B. (2012). 6th grade students' conceptions of reflective symmetry in a dynamic geometry environment. *Paper presented at the European Conference on Educational Research 2012.* Cadiz, Spain.

Öner, T., Özen, D., **Yemen, S.**, & Keşan,C. (2008). The effect of technology assisted algebra instruction to success on force and motion unit in science and technology. In B. Çavaş (Ed.), *Proceedings of the XIII. IOSTE Symposium on the Use of Science and Technology for Peace and Sustainable Development*, Vol 2, pp. 725-729. Ankara, Turkey: Palme Publications & Bookshops Ltd. Co.

Özen, D., **Yemen, S.**, Öner, T., & Keşan, C. (2008). The effect of technology assisted transformation geometry instruction on light and voice unit in science and technology. In B. Çavaş (Ed.), *Proceedings of the XIII. IOSTE Symposium on the Use of Science and Technology for Peace and Sustainable Development*, Vol 1, pp. 125-129. Ankara, Turkey: Palme Publications & Bookshops Ltd. Co.

Yemen, S., Öner, T., Özen, D., & Keşan, C. (2008). The opinions of students: The effects of technology assisted mathematics instruction on success in science and technology. In B. Çavaş (Ed.), *Proceedings of the XIII. IOSTE Symposium on the Use of Science and Technology for Peace and Sustainable Development*, Vol 3, pp. 1364-1368. Ankara, Turkey: Palme Publications & Bookshops Ltd. Co.

National Book Chapter

Yemen-Karpuzcu, S., & Işıksal, M. (2013). Geometrik cisimler: silindir, prizma, koni, piramit ve kürenin matematiksel anlamı. In İ. Ö. Zembat, M. F. Özmantar, E. Bingölbali, H. Şandır, A. Delice. *Tanımları ve Tarihsel Gelişimleriyle Matematiksel Kavramlar*, pp. 273-290. Ankara: Pegem Akademi.

National Conference Papers and Presentations

Kandil, S., **Yemen-Karpuzcu, S.**, & Işıksal-Bostan, M. (2016). Ortaokul matematik öğretmen adaylarının ondalık gösterimlerle çarpma işlemine dair pedagojik alan bilgileri. *12. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi*, 28-30 Eylül, pp.162. Trabzon.

Yemen-Karpuzcu, S., Kandil, S., & Işıksal-Bostan, M. (2016). Dikdörtgenler prizmasının hacmi: prizma tasarımı etkinliği. *Matematik Öğretiminde Örnek Uygulamalar Konfreransı-1 (30-31 Ocak 2016*). Ankara. (elektronik kitap hazılanıyor)

Yemen-Karpuzcu, S., & Işıksal-Bostan (2015). GeoGebra ile Cebir Öğretiminde Bir Ortaokul Matematik Öğretmeninin Matematiksel Uygulamaları ve Matematik Öğretimi Bilgisi. *Türk Bilgisayar ve Matematik Eğitimi Sempozyumu 2, 16-18 Mayıs.* Adıyaman.

Yemen-Karpuzcu, S., Ayan, R., S., & Işıksal-Bostan, M. (2014). İlköğretim matematik öğretmen adaylarının kesirlerde çarpma işlemine dair pedagojik alan bilgileri. Fettahoğlu, P., Keleş, O, Çilingir, E., Turhan B., Yeşilpınar M., Karaduman, B., Demircioğlu T., Işık, D. (Eds), *Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi Bildiri Özetleri Kitapçığı,* pp. 1230-1231. Adana: UFBMEK.

Aygün, B., & **Yemen-Karpuzcu, S.** (2014). İlköğretim matematik öğretmen adaylarının simetri eksenine ilişkin hatalarının ve zorluklarının incelenmesi. Fettahoğlu, P., Keleş, O, Çilingir, E., Turhan B., Yeşilpınar M., Karaduman, B., Demircioğlu T., Işık, D. (Eds), *Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi Bildiri Özetleri Kitapçığı, pp. 500-501.* Adana: UFBMEK.

Ayan, R., **Yemen-Karpuzcu, S.**, & Işıksal-Bostan, M. (2014). Matematik öğretmen adaylarının alan bilgileri: sabit çevre değişen alan, sabit yüzey alanı değişen hacim. Fettahoğlu, P., Keleş, O, Çilingir, E., Turhan B., Yeşilpınar M., Karaduman, B., Demircioğlu T., Işık, D. (Eds), *Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi Bildiri Özetleri Kitapçığı,* pp. 1070-1071. Adana: UFBMEK.

Yemen-Karpuzcu, S., Işıksal-Bostan, M., & Ayan, R. (2013). İlkögretim matematik öğretmen adaylarının tam sayılarda çıkarma işleminde ögrenci hatalarını gidermeye yönelik ögretimsel açıklamaları. In Ö. Akın, & A. Tercan (Eds.), *12. Matematik Sempozyumu: Toplumda Matematik Sempozyum Özetleri Kitabı*, pp 123-126. Ankara.

Aygün, B., & **Yemen-Karpuzcu, S.** (2013). İlköğretim matematik öğretmen adaylarının sınıf düzeylerine göre yansıma simetrisi düşünme düzeylerinin ve hatalarının incelenmesi. In Ö. Akın, & A. Tercan (Eds.), *12. Matematik*

Sempozyumu: Toplumda Matematik Sempozyum Özetleri Kitabı, pp 118-122. Ankara.

Yemen-Karpuzcu, S., Ulusoy, F., Işıksal, & M. (2012). İlköğretim matematik öğretmen adaylarının geometrik cisimler ile yükseklik hacim grafiklerini yorumlamaları üzerine bir çalışma. In S. Mandacı-Şahin, O. Çetin, M. N. Aydede, E. Çalışkan, M. Özel, N. Özal, ... T. Daşöz (Eds.), *X. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi Bildiri Özetleri Kitabı*, p. 342. Ankara: Pegem Akademi.

Keşan, C., Özen, D., **Yemen, S.**, Öner, T., & Mesut, M. (2008). Temel matematik dersinde sınıf öğretmenliği bölümü öğrencilerinde karşılaşılan kavram yanılgıları. In A. Akdemir, D. Köksal, B. Güven (Eds.), *VII. Ulusal Sınıf Öğretmenliği Eğitimi Sempozyumu Bildiriler Kitabı*, pp. 351-356. Ankara: Nobel Yayın.

Yılmaz, S., Keşan, C., Turgut, M., **Yemen, S.**, Öner, T., & Özen, D. (2008). İlköğretim matematik öğretmen adaylarının öğrenme stillerinin ve geometrik düzeylerinin incelenmesi. *VIII. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi Özetler Kitabı*, p. 263. Bolu: Özkan Matbaacılık.

Keşan, C., Öner, T., **Yemen, S.**, & Özen, D. (2008). Sınıf öğretmenliği öğrencilerinin geometrik düşünme düzeyleri ve geometriye yönelik tutumları üzerine bir inceleme. *VII. Matematik Sempozyumu (13-15 Kasım, 2008) Özet Kitapçığı,* p. 44. İzmir: Bassaray Matbaası.

National Poster Presentations

Yemen, S., & Keşan, C. (2008). Altıncı sınıf geometri öğrenme alanında kenarlarına ve açılarına göre üçgenler konusu için hazırlanmış etkinlik çalışması. *VII. Matematik Sempozyumu (13-15 Kasım, 2008) Özet Kitapçığı,* p. 93, Bassaray Matbaası: İzmir. *Poster*

Projects

Işıksal-Bostan, M., **Yemen-Karpuzcu, S.** (01/2013-12/2013). Ortaokul matematik öğretmenlerinin teknoloji destekli sınıf ortamında cebir öğretimine dair yansımalarının ve pedagojik içerik bilgilerinin incelenmesi. Proje no: BAP-07-03-2013-023.

Scholarships and Grants

-Scholarship of the Scientific and Technological Research Council of Turkey (TUBITAK 2210) for PhD Degree, Turkey (2010-2015)

-Scholarship of the Scientific and Technological Research Council of Turkey (TUBITAK 2210) for Master Degree, Turkey (2007-2009)

-TUBITAK 2224 Yurt Dışı Bilimsel Etkinliklere Katılım Desteği (2012)

-Financial Support for 37th Conference of the International Group for the Psychology of Mathematics Education, (2013)

Others

-1. Kademe Yardımcı Satranç Antrenör Yetiştirme Kursu. (6 Haziran 2007). Ali Nihat Yazıcı ve Murat Kul tarafından, Kurs no: 41-647, Türkiye Satranç Federasyonu, Kocaeli. [Assistant Coach of Chess Training. June 2007]

Social Activites

1997-1999 Member of Aliağa Alp Oğuz High School Folklore Team

1999-2003 Member of İzmir Aliağa Municipality Folklore Team

2003-2004 Member of Kocaeli University Folklore Team

2010-2011 Member of METU Life-Saving and First Aid Society

Hobbies

Dancing, Singing, Swimming, and Cooking

U. Tez Fotokopisi İzin Formu

ENSTİTÜ

Fen Bilimleri Enstitüsü	☐
Sosyal Bilimler Enstitüsü	☒
Uygulamalı Matematik Enstitüsü	☐
Enformatik Enstitüsü	☐
Deniz Bilimleri Enstitüsü	☐

YAZARIN

Soyadı : YEMEN KARPUZCU
Adı : SEÇİL
Bölümü : İlköğretim

TEZİN ADI (İngilizce) : MIDDLE SCHOOL MATHEMATICS TEACHERS' MATHEMATICAL PRACTICES IN TEACHING SLOPE, LINEAR EQUATIONS, AND GRAPHS IN TECHNOLOGY ENHANCED CLASSROOM ENVIRONMENT

TEZİN TÜRÜ : Yüksek Lisans ☐ Doktora ☒

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
3. Tezimden bir bir (1) yıl süreyle fotokopi alınamaz. ☒

TEZİN KÜTÜPHANEYE TESLİM TARİHİ: