OPTIMAL PHASOR MEASUREMENT UNIT (PMU) PLACEMENT FOR THE POWER SYSTEMS WITH EXISTING SCADA MEASUREMENTS

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OPTIMAL PHASOR MEASUREMENT UNIT (PMU) PLACEMENT FOR THE POWER SYSTEMS WITH EXISTING SCADA MEASUREMENTS

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ABSTRACT

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It is extremely important to maintain efficiency, sustainability and reliability of the generation, transmission and distribution of the electrical energy; hence it is mandatory to monitor the system in real time. State estimation has a key role in real time monitoring of a power system. The considered power system has to be observable in order to perform state estimation. Traditionally, power system state estimators employ SCADA measurements. However, as the number of Phasor Measurement Units (PMUs) increase in the system, use of phasor measurements become common as well. There are many Phasor Measurement Unit (PMU) placement methods available in the literature for power system observability. Most of those methods aim to determine the measurement configuration (number of measurement devices and locations of those devices) with minimum cost such that most of the measurements in the resulting design are critical. Since errors associated with those measurements cannot be detected, a measurement configuration with many critical measurements is considered as a bad measurement configuration. Moreover, existing SCADA measurements are not considered by most of the existing methods in the literature. Since utilities have made considerable investments to the SCADA systems in the past, it is not a wise decision to ignore existing SCADA measurements in the system. In this thesis, it is aimed to develop a PMU placement algorithm for robust state estimation considering presence of conventional SCADA measurements. The PMU placement method places optimum number of PMUs to a

known system in computer environment, such that specified measurement redundancy is obtained. The proposed method employs the SCADA measurements already available in the considered system, as well as generation – consumption information provided by the operator. Thus, the cost of the resulting measurement configuration is minimized. The proposed method, is also capable of evaluating PMUs with different current measurement channel number.

Keywords: Phasor Measurement Units (PMU), SCADA, State Estimation, Observability, Optimal Measurement Placement, Binary Integer Linear Programming

ÖZ

SCADA ÖLÇÜMLERİ BULUNDURAN GÜÇ SİSTEMLERİ İÇİN OPTIMUM FAZÖR ÖLÇÜM ÜNİTESİ YERLEŞTİRME

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Elektrik enerjinin üretiminde , iletiminde ve dağıtımında verimliliği, sürekliliği ve güvenilirliği korumak son derece önemlidir. Bu yüzden, sistemi gerçek zamanlı izlemek bir zorunluluktur. Durum kestirimi güç sistemlerinin gerçek zamanlı izlenmesinde çok önemli bir role sahiptir. Durum kestirimi yapabilmek için incelenen güç sistemi gözlemlenebilir olmalı ve böylelikle durum kestiriminin tek bir çözümü olmalıdır. Geleneksel olarak durum kestirimciler SCADA ölçümlerini kullanmaktadır. Buna ragmen, sistemdeki Fazör Ölçüm Üniteleri (FÖÜ) sayısı artmakta ve bu durum fazör ölçümlerinin de durum kestiriminde kullanılmasını yaygınlaştırmaktadır. Literatürde güç sistemlerinde gözlemlenebilirlik için birçok Fazör Ölçüm Ünitesi (FÖÜ) yerleştirme metodu bulunmaktadır. . Bu metotların çoğu ölçüm konfigürasyonunu (ölçüm sayısı ve ölçüm cihazlarının yeri) en düşük maliyete göre yaptıklarından elde edilen sonuçlardaki ölçümler kritik ölçüm olmaktadır. Kritik ölçümlere dayalı hatalar tespit edilemediğinden, kritik ölçümlere dayalı ölçüm tasarımı yetersiz bir tasarım olarak düşünülmektedir. Bu duruma ek olarak sistemde halihazırda bulunan SCADA ölçümleri optimum FÖÜ yerleştirme probleminde dikkate alınmamaktadır. Birçok elektrik sistemi operatörü geçmişte SCADA sistemlerine önemli yatırımlar yaptığından, sistemde bulunan SCADA ölçümlerini ihmal etmek bilgece bir yaklaşım değildir. Bu tezde sistemde halihazırda var olan SCADA ölçümlerini de dikkate alarak gürbüz bir durum kestirimci için Fazör Ölçüm Ünitesi (FÖÜ) yerleştirme algoritması geliştirilmesi hedeflenmektedir.

Geliştirilecek FÖÜ yerleştirme metodu, bilgisayar ortamında modeli bilinen bir sisteme optimum sayıda FÖÜ yerleştirecek ve doğru sonuç verecek bir durum kestirimci için gerekli ölçüm artıklığı elde edilecektir. Önerilecek metot, sistemde bulunan SCADA ölçümlerini operatör tarafından sağlanan üretim ve tüketim bilgilerini de hesaba katarak kullanacaktır. Böylelikle, elde edilen ölçüm tasarımının maliyeti en düşüğe indirgenecektir. Önerilecek yöntem ayrıca değişik akım kanal sayısına sahip Fazör Ölçüm Ünitelerinin sisteme yerleştirilmesi açısından da yetkin bir yöntem olacaktır.

Anahtar Sözcükler: Fazör Ölçüm Ünitesi (FÖÜ), SCADA, Durum Kestirimi, Gözlemlenebilirlik, Optimum Ölçüm Yerleştirme, İkili Tam Sayılı Lineer Programlama

To my family

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PMU	Phasor Measurement Unit
SCADA	Supervisory Control and Data Acquisition
EMS	Energy Management System
WAMS	Wide Area Monitoring System

CHAPTER 1

INTRODUCTION

1.1. Background Information and Literature Review

Maintaining efficiency, sustainability and reliability of the generation, transmission and distribution of the electrical energy is vital for the continuity of the modern life. Therefore, real-time monitoring of the power systems have a critical importance in today's power systems.

State estimates, which are obtained using field measurements, constitute the backbone of Energy Management System (EMS) applications [1]. EMS applications have critical importance on the reliable operation of power systems. State estimators help the system operator by preventing the biased estimates of the power system states. State estimators aim to find the correct bus voltages throughout the system. However, state estimators converge to a unique solution if and only if the system of concern is observable. Moreover, having a robust state estimator is important such that unbiased state estimates will be obtained even in the presence of bad measurements. Although researchers have been working on robust state estimators, it is known that those estimators cannot operate properly without a sufficient measurement redundancy.

In modern power systems there are two types of measurements that contribute to power system observability. They are Phasor Measurement Units (PMUs) and conventional Supervisory Control and Data Acquisition (SCADA) measurements. SCADA measurements are active and reactive power flow, active and reactive power injection, voltage magnitude and current magnitude measurements while PMU measurements are voltage phasor and current phasor measurements. Voltage and current magnitude measurements, which are SCADA measurements, are not taken into account in observability analysis.

Voltage phasor PMU measurements are connected to the corresponding bus via a voltage transformer and current phasor PMU measurements are connected to the corresponding line via a current transformer. Most of the PMUs in the industry have channel limits. Number of the channels of PMUs are the number of lines which a PMU can measure.

Wide Area Monitoring Systems (WAMS), which employ the latest data acquisition technologies, provide fast data acquisition compared to conventional SCADA systems and aim the real-time wide-area grid visibility. PMUs are integral part of WAMS due to their fast refreshing rate, such that PMUs provide measurement updates as frequent as 60 times a second while SCADA updates measurement data in every 2-6 seconds. Moreover, PMU measurements, which are GPS (Global Positioning System) synchronized, are known to be more accurate compared to the conventional power flow measurements. GPS synchronization of PMUs also enables detection of voltage magnitudes and phase angle differences between buses without assigning any slack bus, since the PMUs measure positive sequence voltage and current phasors with respect to the same reference phasor. However, for the systems which are only monitored by SCADA measurements, it is mandatory to assign a slack bus in order to obtain the phase angle differences between busses.

After they had been developed in 1986 by Phadke and Thorp [2] – [3], use of PMUs gradually became popular. Especially after the Northeast Blackout of 2003 (U.S. – Canada), the number of PMUs rapidly increased in the modern power systems. The economic burden of the PMU placement at the power systems has become a critical issue, and hence PMU placement techniques have commonly been examined in the literature.

The optimal PMU placement for system observability has been developed in [4] and [5] using simulated annealing and graph theory. However, in these methods existing conventional SCADA measurements were not considered. Since making the system observable by using only PMU measurements results in high costs, application of these methods is not practical.

Optimal PMU placement method using binary integer programming was developed in [6], which considers the existing conventional measurements. However, the optimization problem is a nonlinear integer programming problem, and hence makes the problem formulation complicated.

Binary integer programming based PMU placement for systems with SCADA measurements was conducted in [7]. However, infinite channel capacity of PMUs were assumed. This approach is not realistic since most of the available PMUs in the industry have a limited channel capacity. Optimal PMU placement was also performed with PMUs with channel limits in [8] and [9]. Those methods do not make use of the existing conventional measurements. Hence, usage of this method will result in an expensive design. PMU placement was also performed for the systems having SCADA measurements in [10] by considering channel limits. However, implementation of this method is complicated for large systems.

Above mentioned studies do not aim the measurement redundancy. Measurement redundancy is needed for the state estimation robustness. In these studies, ost of the placed measurements are critical measurements, which is defined as a measurement whose loss would result in system unobservability, [1]. Hence, loss of a measurement, topology changes, contingencies or bad data acquisition may lead to biased state estimates. In order to prevent the operator from this situation, performing a redundant PMU placement is a wise decision.

For the systems which do not contain SCADA measurements, redundant PMU placement was performed in [11-12]. Obtained measurement configurations guarantee the system observability in cases of single measurement loss and single line contingency. However, those methods do not consider state estimation robustness.

Data injections to the telecommunication systems may highly affect the state estimator. In order to prevent those injections' effects on the state estimators, strategic PMU placement is proposed in [13]. Similarly, communication packet loss may also occur in some situations. In order to make the state estimator converge to a correct solution is assured in [14].

In addition to the single measurement and single line contingency cases for systems free of SCADA measurements, maintaining the system observability in the case of a controlled islanding was accomplished in [15]. Method shown in [16] performs redundant PMU placement against unwanted events by considering a criteria called Optimal Redundancy Criteria (ORC). Following this study, new method which also maintains the system observability in case of a single measurement loss was proposed in [17].

Redundant PMU placement for systems which already have SCADA measurements were also studied in the literature. Redundant PMU placement to obtain system observability even in the presence of a single line contingency was covered in [18]. In addition to the single line contingency, measurement losses were also covered by the algorithms proposed in [19-20]. The paper in [21] propose different approach compared to other studies. In this study, different than topological methods which are employed in other studies, numerical observability analysis was utilized for the cases of single measurement loss and line single contingency. Finally, the study in which measurement losses are considered and criticality analysis for each PMUs was made as given in [22].

Among the PMU placement studies presented in [3-22], none of the studies have considered the robustness of a state estimator. Since EMS applications have a critical role in power system operation, state estimation robustness against bad measurements should be satisfied by the final measurement configuration.

First study which aims to obtain a robust state estimator is given in [23]. In this study branch-type PMUs are used. In the following studies, same problem was formulated in a different form. Methods which are also valid for the PMUs with channel limits, was also proposed in [24-25]. The final study on this subject is [26]. In this study, the PMU placement process is defined as a multi-stage process and a gradual PMU placement algorithm is proposed. All of the studies which aim to have a final measurement configuration for a robust state estimator presented in [23-26], use binary integer programming approach. However, none of the studies have taken the existing SCADA measurements into account. In today's power systems due to the cost constraints this approach is not realistic.

It is possible to obtain unbiased state estimates using robust state estimators without performing a post estimation bad data analysis. . Breakdown point of an estimator can be defined as the smallest amount of contamination (bad measurements) that can cause an estimator to converge to a biased solution [27]. Among the estimators with high breakdown points [28] – [33], the Least Absolute Value (LAV) estimator can be implemented computationally efficient due to the power system's properties, and has the desired properties of a robust estimator [32], [33]. Therefore, this thesis validates the final measurement configurations using the LAV estimator.

1.2. Aim and Scope of the Thesis

This thesis propose a binary integer programming based optimal PMU placement method which guarantees the sufficient measurement redundancy for state estimation robustness.

First two stages of this study, aim to propose observability based PMU placement methods. In fact, obtained methods aim to make the system observable but measurement redundancy is not considered. These two methods improves already existing methods in the literature which are [6] and [23] by incorporating the existing SCADA measurements to the problem formulation such that obtained method will be more realistic to implement in today's power systems.

The proposed method aims to provide a straightforward and efficient formulation of the problem of utilizing the existing conventional measurements in PMU placement compared to the existing methods in the literature. It is convenient to implement the proposed method in binary integer programming solvers. All PMUs in the first stage are assumed to be one-channel (branch type) PMUs while in the second stage multi-channel PMUs are used.

In the proposed methods, firstly the numerical observability analysis [1] is conducted for the system, and the observable islands and the unobservable branches are determined. This procedure does not bring any computational burden, since the numerical observability analysis for the conventional measurements is available in every proper state estimator. Those observable islands are remodeled as boundary buses to reduce the size of the PMU placement problem. Finally a modified version of the previously proposed binary integer programming based PMU placement method is employed to place minimum number of PMUs in order to obtain an observable power system. The modification enables utilization of the boundary injection measurements in PMU placement. The boundary injections are defined as the injection measurements located at either the sending or receiving end of an unobservable branch, which are determined by the numerical observability analysis. Note that, without any extra measurements, those injection measurements do not contribute to the system observability. However, as new measurements are introduced, they may affect the system observability.

In the third stage of the study, previously proposed methods in first two stages are improved and main goal is to provide sufficient measurement redundancy for state estimation robustness. The previously conducted studies in the literature [24-25] are improved in terms of the SCADA measurements. Existing SCADA measurements are treated similarly as in first two stages of this thesis. Redundancy index vector is updated and spanning tree constraint is also added to the optimization problem.

Original values and objectives of this thesis are as listed below:

1. Even though there are PMU placement methods in the literature, most of the methods aim to monitor the system using minimum number of measurements. For this type of measurement configuration, the system can be monitored by the minimum investment cost but the system becomes sensitive to bad measurements Utilizing the proposed method, goal of the algorithm is making it possible to perform EMS applications accurately since the system will not be affected by the existing bad measurements in the existing measurements.
2. In this thesis, existing SCADA measurements in the system taken into account for the first time in the redundant measurement configuration for the robust state estimation. If the method is used by a company, it will be possible to determine number of required PMUs and their correct positions in the power system without modifying the proposed measurement configuration.
3. Most of the applications in the power systems, which depend on the measurement redundancy assume that sufficient redundancy exists. However, this work defines measurement redundancy as an index vector and it becomes possible to realize the measurement configurations numerically.

The organization of this thesis are as below:

Chapter 2 proposes an optimal PMU placement method using branch type PMUs on the basis of system observability.

Chapter 3 proposes an optimal PMU placement method using multi-channel type PMUs on the basis of system observability.

Chapter 4 proposes an optimal PMU placement method on the basis of providing sufficiently redundant measurement configuration for the state estimation robustness.

Chapter 5 analyses the outputs of the work and concludes the thesis.

CHAPTER 2

BINARY INTEGER PROGRAMMING BASED BRANCH PMU PLACEMENT

In this chapter, all PMUs are assumed to have a capability of measuring one branch current phasor along the lines which are incident to the bus, where the PMU voltage phasor measurement is located, in addition to the voltage at that location. Firstly, the conventional observability analysis is conducted for the system, and the observable islands and the unobservable branches are determined according to the results. The buses which are incident to the unobservable branches are labeled and named as boundary buses. After that, observable islands are remodeled as super nodes to reduce the size of the PMU placement problem . Finally, a modified version of a PMU placement method in [23] is employed to find the optimum locations of the PMUs which will be deployed .

Section 2.1 introduces the existing method in the literature. Section 2.2 explains the proposed method. Section 2.3 gives an illustrative example to show the implementation of the method. Section 2.4 shows the simulation results.

This section was presented in 2016 IEEE PES Innovative Smart Grid Technologies (ISGT) Conference Europe in Ljubljana as a part of this thesis , [34].

2.1. Existing Method in the Literature

In this formulation [23], each PMU was assumed to be capable of measuring the voltage phasor on the bus that the PMU is located and the current phasor along the one of the selected lines. Optimal placement of PMUs to the power system with *N* buses and *L* branches can be written as below.

$$\begin{aligned} &\min c^T X \\ &s.t. \quad AX \geq \hat{1} \end{aligned} \qquad (2.1)$$

Where

L number of branches in the system

N number of buses in the system

X binary vector of size L where x_i is the i[th] element of *X*. x_i is either 1 or 0 depending on whether a PMU is placed on that line or not, respectively

c vector of size *L* and its entries, c_i, indicate cost of placement of PMU on branch-i

A binary bus to branch connectivity matrix and it is defined as

$$A_{ij} = \begin{cases} 1, \text{if branch - j is connected to bus - i} \\ 0, \text{otherwise} \end{cases} \qquad (2.2)$$

$\hat{1}$ $\left[1\ 1\ 1 \ldots\ldots\ldots 1\ \ 1\right]^T$

2.2. Proposed Method

The proposed method is based on binary integer programming and is capable of incorporating the existing conventional power measurements. The method utilizes the conventional observability analysis [1] to simplify the problem formulation. The observable islands obtained via the observability analysis are remodeled as super nodes. Once the reduced model is formed, the proposed placement method will be applied.

An observable power system has a unique state estimation solution for a given network topology and observation set. According to topological observability analysis, in an observable power system measured solely by conventional SCADA measurements, once all the measurements are assigned to branches a spanning tree consisting of those branches can be formed. Power flow measurements are assigned directly to the line which they are connected to and power injection measurements are assigned to one of the lines which are incident to bus where the power injection measurement is connected to. Voltage and current magnitude measurements are not considered in observability analysis, [1].

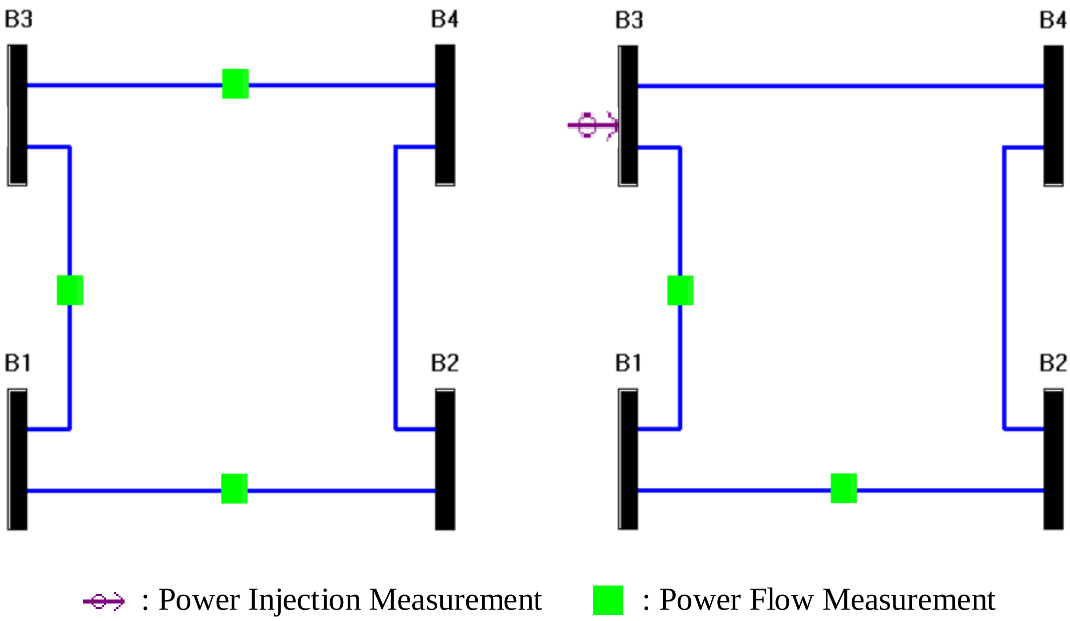


Figure 2-1. Observable Systems Measured by SCADA Measurements

The systems illustrated by Figure 2-1 are both observable. There is a spanning tree in the system which only has power flow measurements. Power flow measurements can be assigned to the line they are connected. The other system with a power injection measurement is also observable since spanning tree can be formed by assigning the power injection measurement on bus-3 to the line between bus-3 and bus-4.

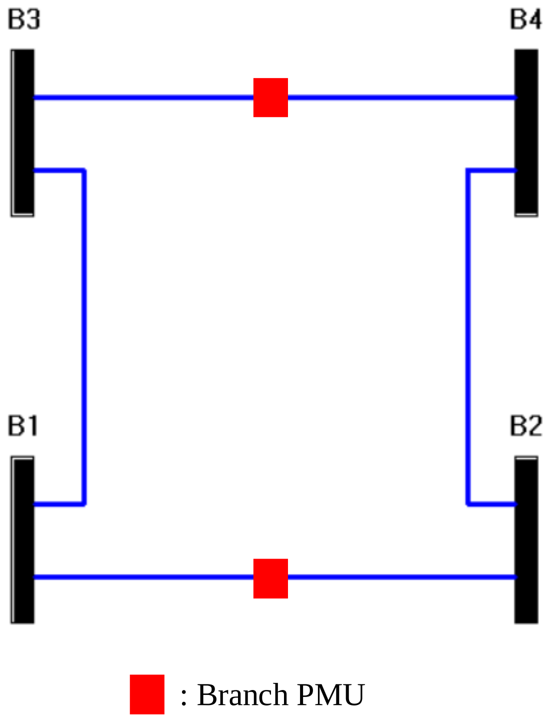


Figure 2-2. Observable Systems Measured by PMU Measurements

For the systems with PMU measurements, formation of a spanning tree is not required for observability since there is a GPS synchronization for the phasor measurements throughout the system. It is possible to determine phase angles of voltages at buses and currents through the lines with respect to the same reference.

The system shown by Figure 2-2, is a system which is observable. There are 2 branch PMUs in the system and each state is measured at least once. Position of the voltage phasor measurement does not make any difference in observability analysis.

In an observable island phase angle differences between buses can be determined. Unobservable branches are defined as the lines which are located between observable islands.

The observable islands found through the topological observability analysis are represented by the boundary buses in observable islands, as shown by Figure 2-3. Note that, all buses except the boundary buses are removed, and the boundary buses in the same observable island are connected by drawing a virtual line. This virtual line indicates that the system states of different buses in the same observable island can be expressed with respect to each other, and it can be assumed that the current flow through this line can also be found.

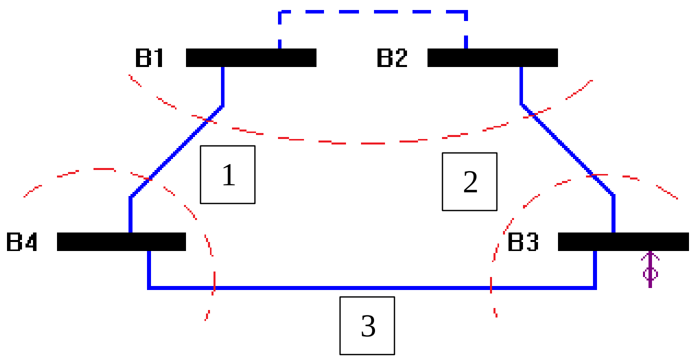
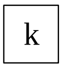


Figure 2-3. Reduced System Model Formed by Boundary Buses



: Branch Number


:Power Injection measurement

Observable islands can be represented as a single super-node instead of representing all of the boundary buses separately as in Figure 2-3. Figure 2-4 shows the reduced model in terms of super-nodes for the same system illustrated in Figure 2-3 .

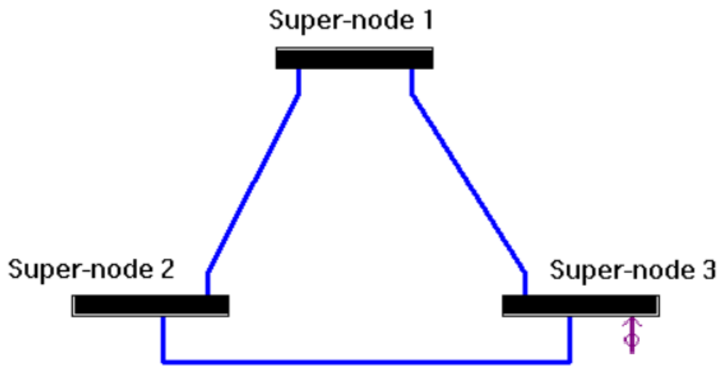


Figure 2-4. Reduced System Model Formed by Super-nodes

Applying conventional observability analysis and forming the reduced model reflects the effect of SCADA measurements to system observability, except the boundary injection measurements. Boundary injection measurements are injection measurements which are incident to unobservable branches that are determined after conducting observability analysis. Those measurements should be kept in the reduced model as illustrated in Figure 2-3. Note that, boundary injection measurements do not affect observability if additional measurements are not added to the measurement set. If PMUs are placed to the system, those measurements may affect the system observability. Formulation in (2.1) should be modified in order to add the boundary injection measurements to optimal PMU placement problem.

As stated in [1] and [35], the numerical observability analysis method is based on the decoupled Jacobian matrix, H_{PP}. For a power system which is measured by SCADA measurements, a decoupled measurement model can be written as:

$$\Delta z_P = H_{PP} \Delta\theta + e_P \tag{2.3}$$

$$\Delta z_Q = H_{QQ} \Delta V + e_Q \tag{2.4}$$

$\Delta\theta$ and ΔV are the changes in the state vector's angle and magnitude rows respectively, while Δz_P and Δz_Q are the changes in the P-Q measurements respectively. In (2.3) and (2.4), e_P and e_Q represent the error in P and Q measurements respectively. H_{PP} and H_{QQ} are the decoupled Jacobian matrices, obtained by neglecting the coupling between V-P and θ-Q variables. For conventional measurements, P and Q measurements are considered in pairs, so only one of the (2.3) and (2.4) is used for observability analysis. P, Q, V and θ represent active power, reactive power, voltage magnitude and voltage phase angle, respectively.

Observability analysis methods do not consider network parameters and operating state of the system. Therefore, by neglecting all line resistances and shunt elements, assuming 1.0 p.u. reactances for all lines and 1.0 p.u. voltages at all buses, real power (P) flow from bus-k to bus-m can be expressed as:

$$P_{km} = sin\left(\ _{k} - \ _{m} \right) \tag{2.5}$$

Applying the first order Taylor approximation around $\theta_{km} = 0$, where θ_{km} is the phase difference between bus-k and bus-m, (2.5) can be approximated to express power injection measurements as given below, where N_k is the set of buses which are incident to bus-k.

$$P_k = \sum_{i \in N_k} \left(\ _{k} - \ _{i} \right) \tag{2.6}$$

As it can be seen in (2.5), power injection measurements carry information about the states of the neighboring buses as well as the bus that the measurement is located. If the states of all the considered buses, except one, are known, the injection measurement will be used to find the remaining state. Having this fact, the proposed method proposes a modification in (2.1).

The proposed modified optimal PMU placement method can be expressed as the following binary integer programming problem.

$$\begin{aligned} &\min c^T X \\ &s.t. \quad B + AX \geq \hat{1} \end{aligned} \tag{2.7}$$

L number of branches in the system

N number of buses in the system

X binary vector of size L and x_i is the i[th] element of X, i[th] entry x_i is 1 or 0 depending on whether the corresponding PMU is placed or not, respectively

c vector of size L and its entries c_i indicates cost of placement of branch PMU on branch-i

A bus to branch connectivity matrix and it is defined as

$$A_{ij} = \begin{cases} 1, \text{if branch - j is connected to bus - i} \\ 0, \text{otherwise} \end{cases} \tag{2.8}$$

B power injection measurement assignment vector of size N. B_i takes the value 1 or 0 depending on whether a power injection measurement assigned to bus-i or not, respectively.

$\hat{1}$ $[1\ 1\ 1 \ldots\ldots\ldots.1\ \ 1]^T$

In an observable system, each state should be observed. Hence a measurement should be assigned to each state. The proposed method assigns the boundary injection to one of the buses that those measurements are related. The solution according to this assignment will be found. Then, other possible assignment combinations will be evaluated. The solution with minimum cost will be accepted as the optimum placement.

If there are more than one boundary injection measurements in the system, the possible assignments of those measurements are represented in a single assignment vector, such that *B* will be an *N*x*1* vector with multiple non-zero entries. Each of those entries will correspond to one of the possible assignment of each boundary injection measurement.

If there isn't any load or a generator connected to a bus in the power system, corresponding bus is called zer injection bus. In order to reduce the number of required PMUs, zero injection buses can be taken into account. Since the net power injections to those buses are known, it can be assumed that there exist injection measurements located at those buses at the beginning of the observability analysis, and the whole process should be followed on the basis of this assumption.

The proposed method also considers the existing PMUs in the system. The corresponding cost of those measurements are assigned as in (6). Therefore, the optimization problem will be forced to place PMUs to those locations where there aren't any PMUs.

Flow chart of the proposed method is illustrated by Figure 2-5.

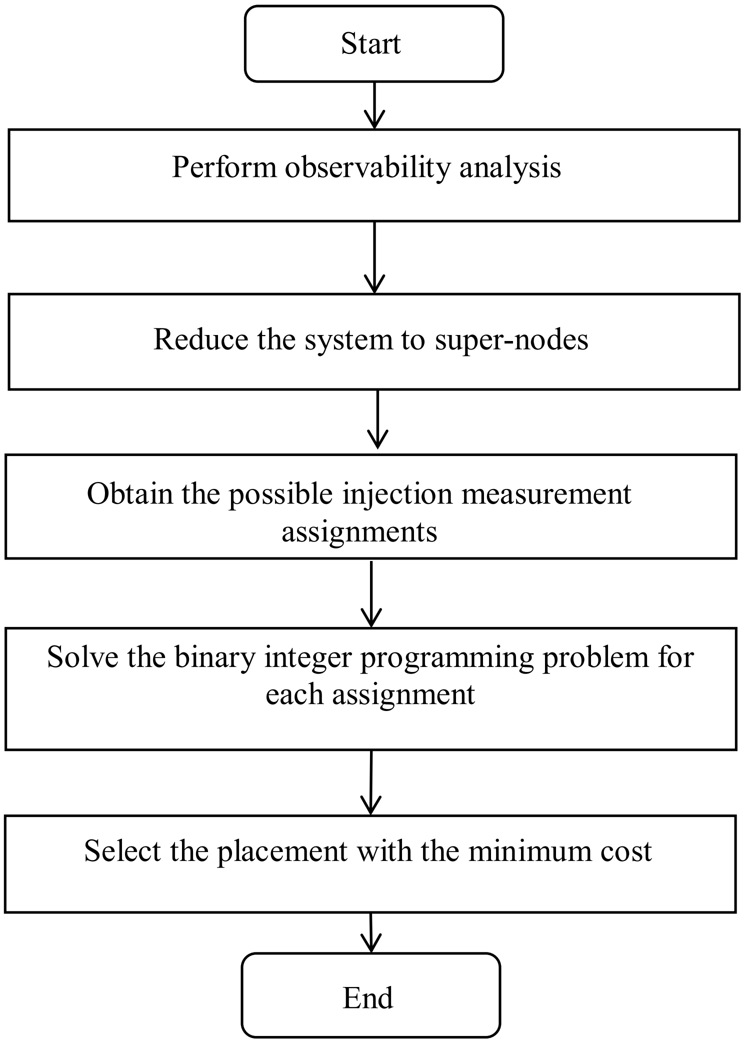


Figure 2-5. Flow Chart of the Proposed Method

2.3. Illustrative Example

Consider the system given in Figure 2-6. The first step of the proposed method is applying the numerical observability analysis.

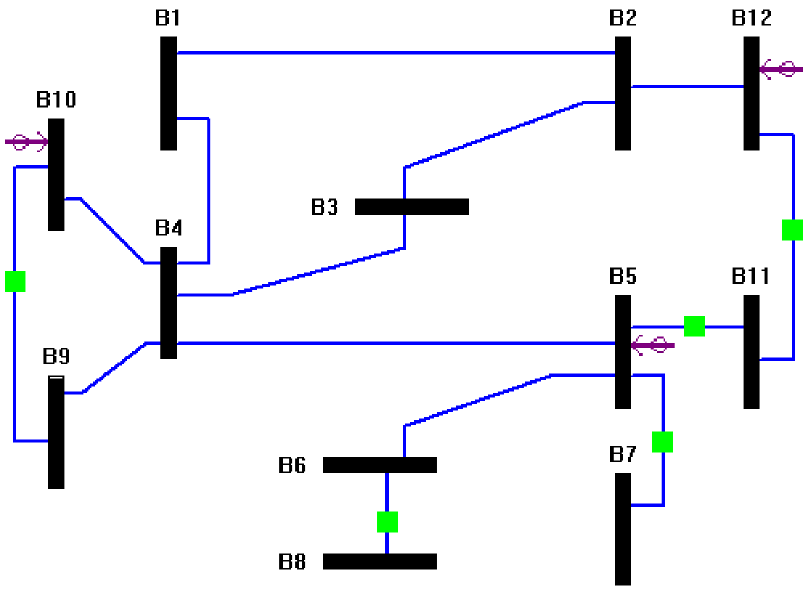


Figure 2-6. 12-bus System Single Line Diagram

: Power InjectionMeasurement : Power Flow Measurement

Once the conventional observability analysis is performed, the observable islands are obtained as shown in Figure 2-7. Unobservable branches are the branches, which are connected between observable islands.

Using the unobservable branch data, a reduced model that is composed of the boundary buses, can be formed as shown in Figure 2-8. Note that, bus-2 and bus-5 are in the same observable island, which means that the phase angle difference between those buses is known. Hence, the states of one of those buses can be represented in terms of the states of the other bus. As indicated in Section 2.2, if branch PMUs are used as, those two buses can be represented as a single super node, as shown in Figure 2-9. However, if multiple current channel PMUs are considered, the reduced model shown in Figure 2-8 should be employed. Each observable island can be represented as a super-node. Unobservable branches are shown by the same branch numbers to explicitly illustrate this step.

The power injection measurement placed on bus-5 (super-node 2) will affect the PMU placement process, and should be taken into account in the problem formulation. This measurement contributes to the computation of the states of the buses 4, 5 and 6 as indicated in Figure 2-8, or to the super-nodes 2, 3 and 5 as shown in Figure 2-9.

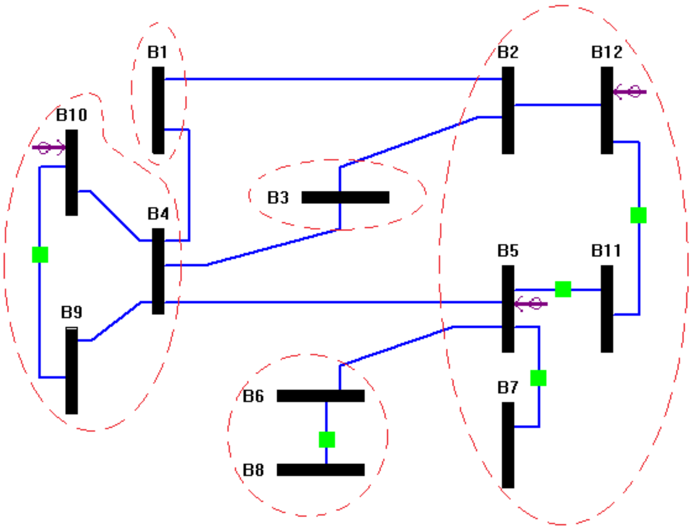


Figure 2-7. Observable Islands for the Measurement Configuration in 12-bus System

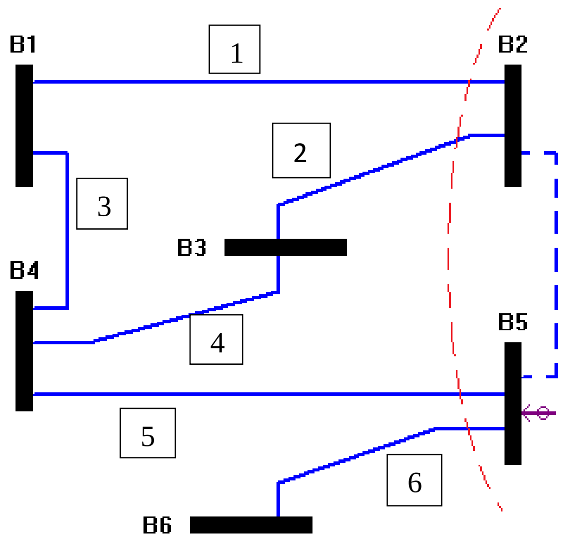


Figure 2-8. Obtained 6-bus System Using Unobservable Branches

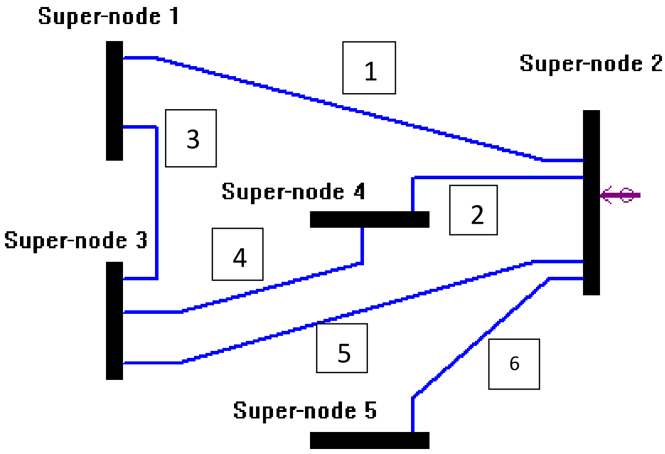


Figure 2-9. Final Reduced System

Considering the reduced model given in Figure 2-9, *A* can be formed as below.

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.9}$$

Since there is not any PMU installed in the system, cost vector can be written as follows.

$$c = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T \tag{2.10}$$

As it can be seen on Fig. 6 there is a boundary injection measurement placed on bus-5, which is located in super-node 2. Once the reduced model given in Figure 2-9 is considered, it may be thought that the boundary injection measurement is related to super-node 1 and 4 as well as 3 and 5. However, since bus-5 is connected to bus-4 and bus-6 which are located in super-node 3 and super-node 5, respectively. Hence, there are 3 possibilities for the power injection assignment vector B, which are given as below.

$$
\begin{aligned}
B_1 &= [0 \quad 1 \quad 0 \quad 0 \quad 0]^T \\
B_2 &= [0 \quad 0 \quad 1 \quad 0 \quad 0]^T \\
B_3 &= [0 \quad 0 \quad 0 \quad 0 \quad 1]^T
\end{aligned}
\tag{2.11}
$$

Finally, three binary integer programming problems will be solved using (2.3). In each solution, one of the B vectors indicated in (2.11) will be used. A matrix in (2.9) remains same for all solutions.

For the case in B_3, the binary vector X will be found as one of the vectors given below.

$$X_1 = [1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]^T \tag{2.12}$$

$$X_2 = [0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0]^T \tag{2.13}$$

The result indicates that optimal locations of PMUs are branch - 1 and branch - 4 or branch - 1 and branch - 2. Since same cost of installations are assumed for each bus, it is possible to obtain more than one optimum solution.

Other power injection measurement assignments indicated by (2.11) lead to the placement of 3 PMUs. Hence, these assignments result in higher cost of installation and they should be neglected.

2.4. Case Studies

Proposed method was applied on IEEE 14 Bus and IEEE 30 Bus systems with measurement configurations shown on the Figure 2-10 and Figure 2-11, respectively. MATLAB's *intlinprog* function was used to solve the binary integer programming problems. In IEEE 14 Bus System, bus 7 is a zero injection bus. In IEEE 30 Bus System, buses 2, 16, 18 and 27 are zero injection buses. The results are shown in Table 2-1 and Table 2-2. Voltage phasor measurements are indicated as sending end bus.

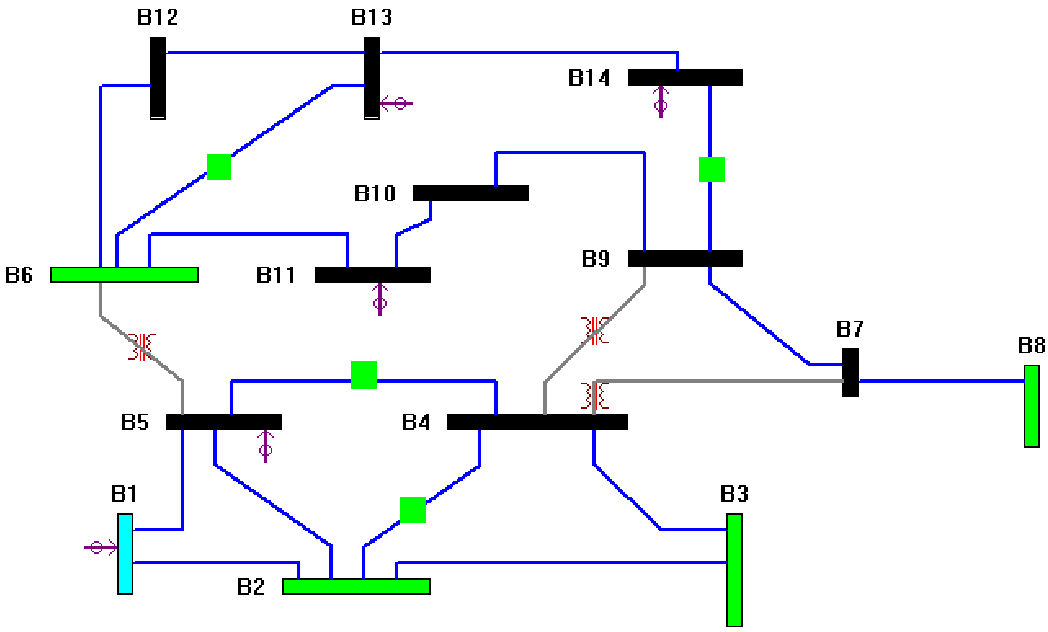


Figure 2-10. IEEE 14-Bus System for the Given Measurement Set

Table 2-1. Optimal Branch PMU Locations for IEEE 14 Bus System

PMU	Sending End	Receiving End
1	2	3
2	7	8
3	10	11

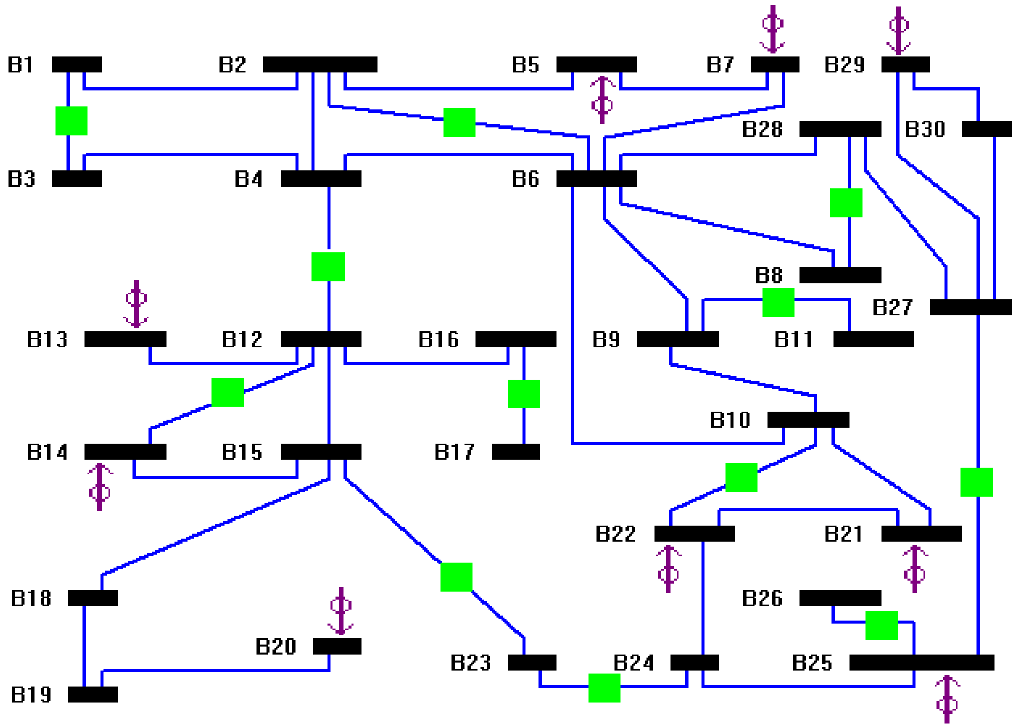


Figure 2-11. IEEE 30 Bus System for the Given Measurement Set

Table 2-2. Optimal Branch PMU Locations for IEEE 30 Bus System

PMU	**Sending End**	**Receiving End**
1	6	9
2	6	10
3	6	28
4	18	19
5	27	29

2.5. Chapter Summary and Conclusions

In this chapter, optimal PMU placement method based on a modified version of the binary integer linear programming formulation was proposed. In the formulation, the installed PMUs and conventional SCADA measurements, i.e. active and reactive power flow and power injection measurements, were used in order to reduce the number of PMUs to be deployed required for system observability. It is obvious that taking the existing SCADA measurements into account decreases the investment cost for system observability. In the derivation, PMUs are assumed to have a capability of measuring one branch current phasor along the lines which are incident to the bus.

The proposed method applies observability analysis to evaluate the system of concern and to reduce the size of the PMU placement problem. Observable islands are reduced to super-nodes in order to simplify the optimization problem. The proposed method considers the effect of boundary power injection measurements, zero injection measurements and existing PMUs in order to decrease the required number of PMUs for observability. The analysis is followed by the proposed modified optimal PMU placement method. The method is applicable to PMUs with single voltage phasor and single current phasor channel measurement capability. The method was illustrated by a tutorial example and validated on IEEE 14-bus and 30-bus test systems.

CHAPTER 3

BINARY INTEGER PROGRAMMING BASED MULTI-CHANNEL PMU PLACEMENT

In this chapter, multi-channel PMUs are used. Similar to the previous chapter, conventional numerical observability analysis method is employed to evaluate the measurement configuration of the system and reduce the size of the PMU placement problem. The conventional numerical observability analysis is followed by a modified version of the optimal PMU placement method based on the binary integer programming in [6]. The proposed method considers already existing PMUs in the system and boundary injection measurements. Those injection measurements do not contribute to the system observability, if there is no additional measurement, and are located at the buses incident to the unobservable branches, which can be determined by the conventional observability analysis. The proposed method is explained using an illustrative example and validated with case studies. This method is the generalized form the method proposed in Chapter 2.

Section 3.1 introduces the existing method in the literature. Section 3.2 describes the proposed method. In Section 3.3, illustrative example is given in order to show the application of the method explicitly . Section 3.4 shows the simulation results.

3.1. Existing Method in the Literature

For an N – bus system with no SCADA measurements and zero injections, the PMU placement problem is solved using the following binary integer programming problem as shown below [6]. Note that, this formulation assumes infinite number of available current channels for each PMU.

$$\begin{aligned} &\min c^T X \\ &s.t. \quad AX \geq \hat{1} \end{aligned} \tag{3.1}$$

N number of buses in the system

X binary vector of size N and x_i is the i[th] element of X, i[th] entry x_i is 1 or 0 depending on whether the corresponding PMU is placed or not, respectively

c vector of size N and its entries c_i indicates cost of installation of a PMU on bus-i

A bus to bus connectivity matrix and it is defined as

$$A_{ik} = \begin{cases} 1, & Bus-i \ and \ Bus-k \ are \ connected \\ 1, & i = k \\ 0, & otherwise \end{cases} \tag{3.2}$$

$\hat{1}$ $[1\ 1\ 1 \ldots\ldots\ldots 1\ \ 1]^T$

3.2. The Proposed Method

WAMS conduct the numerical observability analysis to evaluate the measurement configuration of the system. If the system is found to be unobservable, state estimation cannot be performed. Therefore, any proper state estimation tool runs observability analysis. This work, utilizes the results of topological observability analysis, which reduces the computational load on the PMU placement problem. Moreover, the problem formulation becomes very simplified, without any extra effort.

Once the observable islands and unobservable branches are found, a simplified model of the system in terms of observability can be built. The proposed reduced system is obtained by using the unobservable branches and their sending and receiving end buses, which are called as boundary buses, as well as the injection measurements located at the boundary buses. The proposed method, therefore, will place PMUs to the boundary buses.

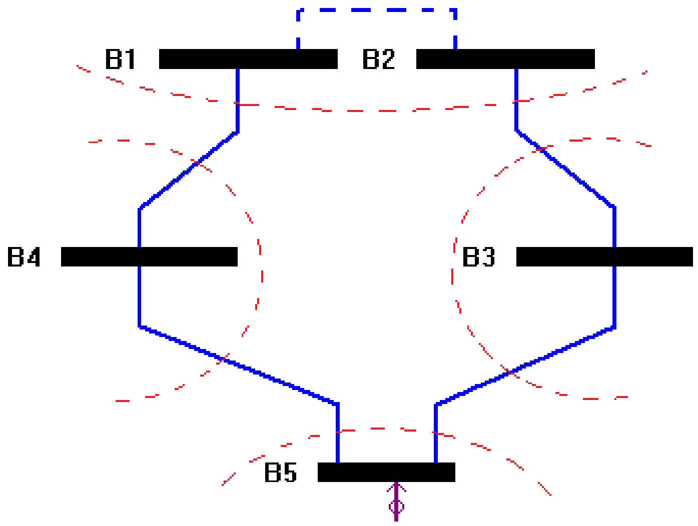


Figure 3-1. Reduced System which is Represented by Boundary Buses

 : Power Injection Measurement

In an observable island, the bus voltage magnitudes and phase angle difference between the bus voltages are known. Consider the reduced system model given in Figure 1. Once the phase angle difference between the buses bus-2 and bus-4 is known, it can be said that the phase angle difference between buses bus-1 and bus-4 is also known, since buses bus-1 and bus-2 belong to the same observable island. Same approach is valid for buses bus-1 and bus-3 as well. Therefore, any unobservable branch which is connected to a boundary bus can be assumed to be connected to other boundary buses in the corresponding observable island. Hence, the proposed method connects the boundary buses which belong to the same observable island via a virtual line as seen in Figure 3-1, which is indicated by a dashed line.

Injection measurements at the boundary buses do not contribute to system observability, despite they provide information about the adjacent buses. If current phasors along the branches that are connected to a bus with an injection measurement are known except one of the branches, this remaining branch current phasor can be calculated using Kirchhoff's Current Law. In the new formulation the conventional PMU placement formulation is updated, utilizing this fact. The proposed optimization problem considers the possible assignments of the boundary injection measurements to the adjacent buses, in order to determine the optimum placement of the PMUs. For Figure 3-1, the boundary injection measurement are assigned to buses bus-3, bus-4 and bus-5. The solution of the optimization problem will employ the placement configuration with the least number of PMUs leading to minimum cost. Zero injection buses can be treated same as buses with injection measurements.

The proposed method can be formulized as follows.

$$\begin{aligned} & min \quad c^T X \\ & s.t. \quad B + AX \geq \hat{1} \end{aligned} \tag{3.3}$$

N number of buses in the system

X binary vector of size N and x_i is the ith element of X, ith entry x_i is 1 or 0 depending on whether the corresponding PMU configuration is selected or not, respectively

c vector of size N and its entries c_i indicates cost of installation of a PMU on bus-i

A bus to bus connectivity matrix and it is defined as

$$A_{ik} = \begin{cases} 1, & Bus-i \text{ and } Bus-k \text{ are connected} \\ 1, & i=k \\ 0, & otherwise \end{cases} \tag{3.2}$$

B power injection measurement assignment vector of size N. B_i takes the value 1 or 0 depending on whether a power injection measurement assigned to bus-i or not, respectively.

$\hat{1}$ $[1\ 1\ 1 \ldots\ldots\ldots 1\ \ 1]^T$

Flow chart of the proposed method is as shown by Figure 3-2. Note that, the solution ensures that the system is observable and each PMU is critical, such that loss of a PMU will make the system lose its observability.

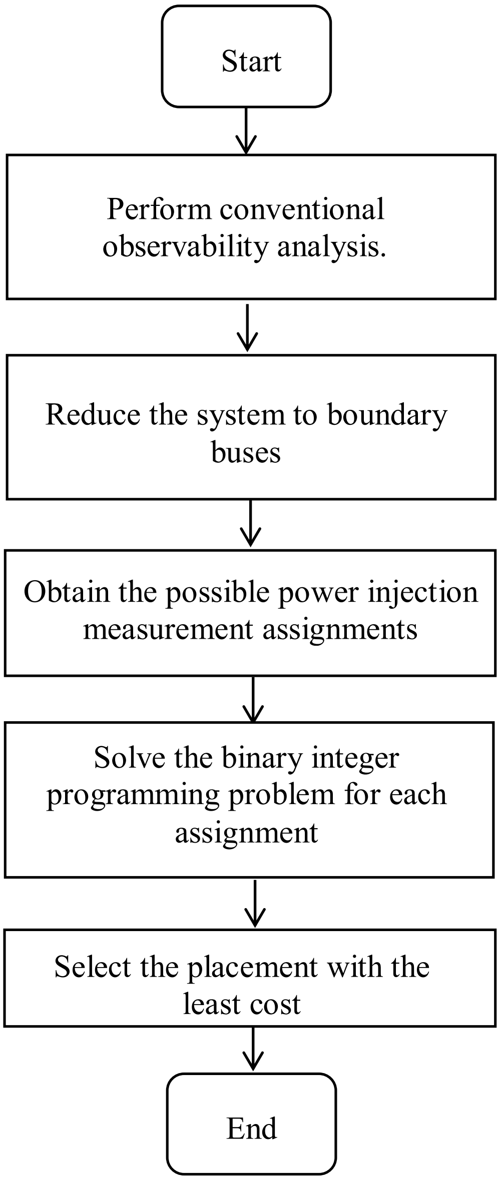


Figure 3-2. Flow Chart of the Proposed Method

Consider the 6 bus system in Figure 3-3. The system is formed by boundary buses. This method is valid for PMUs with infinite channel capacity if *A* matrix is not modified. By modifying the *A* matrix, the method can be made to be applicable for the PMUs with channel limits. Modification of the *A* according to the channel limits will be explained on an example system. Similar approach in [8] and [9] was used for channel limits.

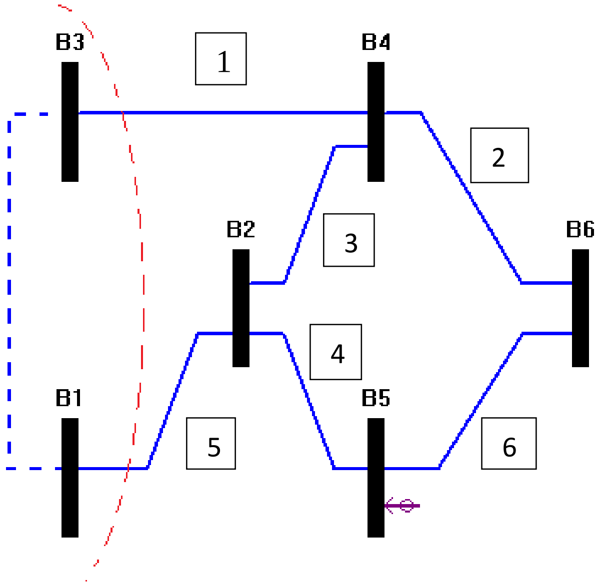


Figure 3-3. 6 Bus System Formed by Boundary Buses

If infinite channel capacity is assumed, *A* matrix will be formed as below.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \quad (3.3)$$

Each entry which is 1 in a column-i, indicates that if a PMU placed on bus-i, voltage magnitudes and angles of the corresponding buses can be determined with respect to each other. For example, if a 3-channel PMU is placed at bus-2, voltage magnitudes and phase angle differences of bus-1, bus-2, bus-3, bus-4 and bus-5 can be determined. In fact, they form an observable island. However, if 2-channel PMUs are used, column corresponding to bus-2 must be modified. Since bus-2 is connected to 3 lines, there are $\binom{3}{2} = 3$ posibilities to connect 2 –channel PMU to bus-4. These PMU current channel configuration posibilities are as follows.

1. Line-1 and line-2
2. Line-1 and line-3
3. Line-2 and line-3

Hence, if 2-channel PMUs are used, there are 3 possible 4th column values for the A matrix. They are given as below.

$$M_1 = [0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1]^T \tag{3.4}$$

$$M_2 = [1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0]^T \tag{3.5}$$

$$M_3 = [1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1]^T \tag{3.6}$$

Note that, since bus-1 and bus-3 are in the same observable island, placing a current phasor on line-1 , will also make a virtual connection between bus-1 and bus-4. Thus, first entries of A_1 and A_2 became 1 , although there is not a physical connection between bus-1 and bus-4.

Similarly, since bus-2 is also incident to 3 lines, there are also 3 possibilities for second column of A . They are as follows.

$$N_1 = [0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0]^T \tag{3.7}$$

$$N_2 = [1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0]^T \tag{3.8}$$

$$N_3 = [1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0]^T \tag{3.9}$$

Other columns of A will remain same since number of lines that other buses are incident to, is less than or equal to 2. Using this information, A matrix can get 3*3=9 values. As it can be seen on Figure 3-3, there is a boundary injection measurement at bus-5 and bus-5 is incident to 2 buses. Hence , there are 2+1=3 possible boundary injection measurement assignments. In the final stage, it can be calculated that there are 9X3= 27 possible optimization problems to be solved by the algorithm.

The final flow chart which corresponds to PMU placement for channel limit PMUs are as in Figure 3-4.

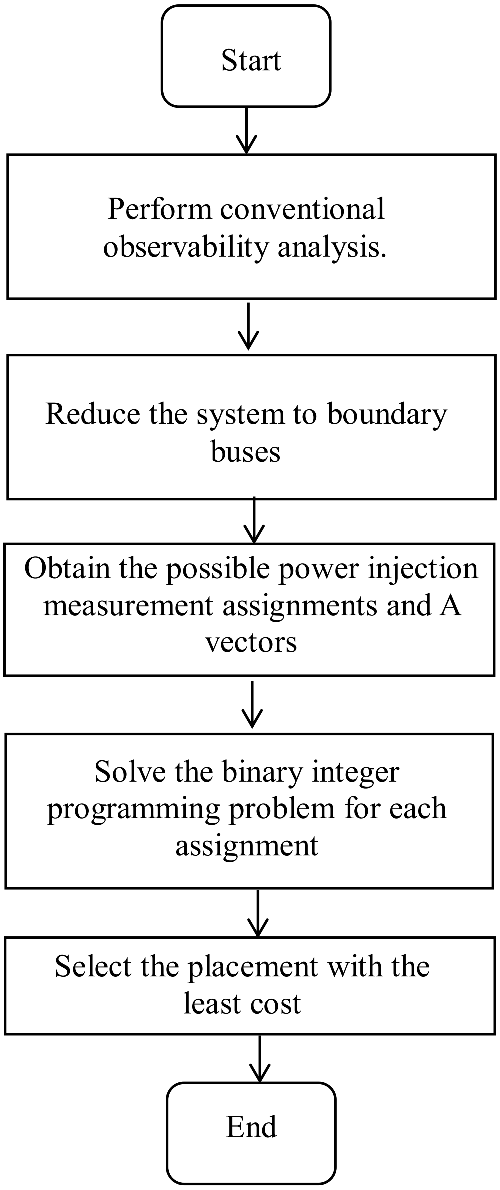


Figure 3-4. Flow Chart for Placement of PMUs with Channel Limits

3.3. Illustrative Example

Consider the 6-bus system shown by the Figure 3-5. The system is the same system which is indicated by Figure 3-3. In this illustrative example, 2-channel and 3-channel PMUs will be placed in order to make the system observable.

Each bus represents the boundary buses that were determined by reducing the observable islands to boundary buses, which are incident to unobservable branches. There is a boundary injection measurement at the bus-5. Note that bus-1 and bus-3 belong to the same observable island.

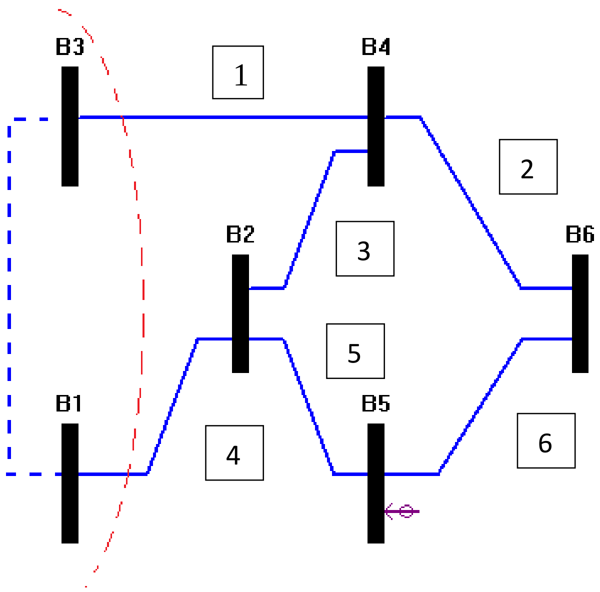


Figure 3-5. 6-bus Tutorial System Formed by Boundary Buses

Noting that buses bus-1 and bus-3 belong to the same observable island, virtual line connecting those two boundary buses was drawn and shown as a dashed line. Any unobservable branch which is connected to bus-1 was assumed to be connected to bus-3. Same approach is also valid for the branch connected to bus-3. For 3-channel PMUs, the binary connectivity matrix, *A*, and the cost vector, *c*, are shown as following.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \quad (3.10)$$

$$c = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]^T \quad (3.11)$$

There are 3 possible boundary injection measurement assignments, B_k, as follows, which are obtained by assigning the injection measurement to the buses the measurement belongs to and to its neighbors. These assignments bring 3 optimization problems to be solved.

$$B_1 = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0]^T \quad (3.12)$$

$$B_2 = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]^T \quad (3.13)$$

$$B_3 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]^T \quad (3.14)$$

Having these vectors, the binary integer programming based PMU placement problem is solved and the following two alternative solutions are obtained.

$$X_1 = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]^T \quad \text{for } B_3 \quad (3.15)$$

$$X_2 = [0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]^T \quad \text{for B}_2 \quad (3.16)$$

This result implies that one 3-channel PMU is proposed to be placed either at bus bus-4 or at bus-6 in order to make the system fully observable. The optimization result regarding to the injection measurement assignment $B_1 = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]^T$, requires to place 2 PMUs to obtain the system observability. Hence this solution will be neglected and other alternatives which result in installation of single PMU will be chosen. Since same cost of installation was assumed for the installation of PMUs to all buses, either one of the possible solution for this case which are X_1 and X_2 can be chosen. If costs of installations are different, placement configuration whose cost is less will be selected by the proposed algorithm.

Similarly, if the same algorithm is applied to the system by using 2-channel PMUs, *A* matrix will also take different values for each different solution. The optimal solutions obtained for the 2-channel PMU case is tabulated in Table 3-1. In this case, there are more than one optimal solutions since cost of placement pf PMUs to different locations are assumed to be same. Every optimal solution which is proposed by the algorithm, results in placement of 2 PMUs.

Table 3-1. Optimal 2-channel PMU Voltage and Current Phasor Measurement Locations

Solution	Voltage Phasor Measurement	Current Phasor Channel	Current Phasor Channel	Voltage Phasor Measurement	Current Phasor Channel	Current Phasor Channel
1	Bus-2	Line-3	Line-4	Bus-4	Line-1	Line-2
2	Bus-2	Line-4	Line-5	Bus-4	Line-1	Line-2
3	Bus-2	Line-4	Line-5	Bus-4	Line-3	Line-2
4	Bus-2	Line-3	Line-4	Bus-6	Line-2	Line-6
5	Bus-4	Line-1	Line-3	Bus-5	Line-5	Line-6
6	Bus-4	Line-1	Line-3	Bus-6	Line-2	Line-6

While making the system observable using 2-channel PMUs, boundary power injection placed on bus-5 doesn't reduce the number of PMUs which will be placed. In order to show the solution steps of the algorithm clearly, A and B matrices are given for the solution 1 and solution 2 as below. Cost vector *c* remains same as in (3-11).

Solution 1

$$A=\begin{bmatrix}1&1&1&1&0&0\\1&1&0&0&1&0\\1&1&1&1&0&0\\0&1&1&1&0&1\\0&0&0&0&1&1\\0&0&0&1&1&1\end{bmatrix} \tag{3.17}$$

$$B_2 = \begin{bmatrix}0 & 0 & 0 & 0 & 1 & 0\end{bmatrix}^T \tag{3.18}$$

Solution 2

$$A=\begin{bmatrix}1&0&1&1&0&0\\1&1&0&0&1&0\\1&0&1&1&0&0\\0&1&1&1&0&1\\0&1&0&0&1&1\\0&0&0&1&1&1\end{bmatrix} \tag{3.19}$$

$$B_3 = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}^T \tag{3.20}$$

3.4. Case Studies

IEEE 14 Bus System shown in Figure 3-6 was analyzed using the numerical observability analysis. Unobservable branches are indicated as red lines on the Figure 3-6. The system was reduced to the system shown in Figure 3-7. In order to make the system fully observable, a 2-channel PMU were used. MATLAB's *intlinprog* function was used to solve the binary integer programming problems.

In order to make the system observable, voltage phasor measurement should be placed on bus-2.

Table 3-2. Optimal 2-channel PMU Voltage and Current Phasor Measurement Location for IEEE 14 Bus System

Voltage Phasor Measurement Location	Current Channel-1	Current Channel-2
Bus-2	Bus-2 to Bus-5	Bus-2 to Bus-4

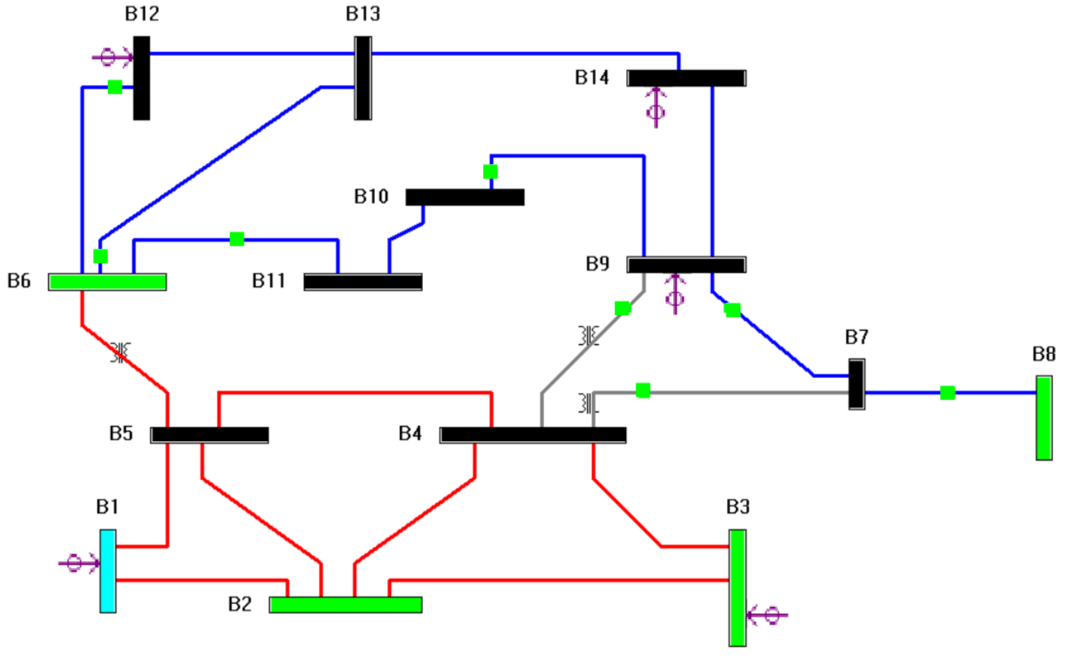


Figure 3-6. IEEE 14 Bus System with Initially Installed SCADA Measurements

 : Power Injection Measurement

 : Power Flow Measurement

IEEE 30 Bus System given in Figure 3-8 with the shown measurement configuration was analyzed using the topological observability analysis. Unobservable branches are indicated as red lines on the Figure 3-8. Bus-4 is a zero injection bus. The system was reduced to the system shown in Figure 3-9.

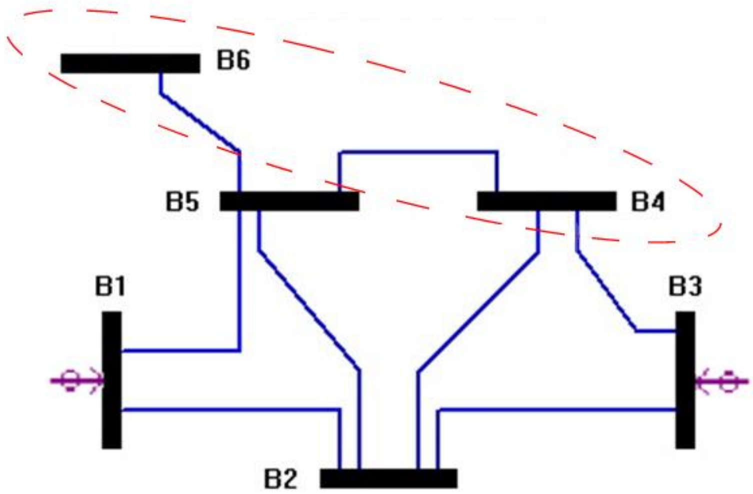


Figure 3-7 - Reduced form of the IEEE 14 Bus System

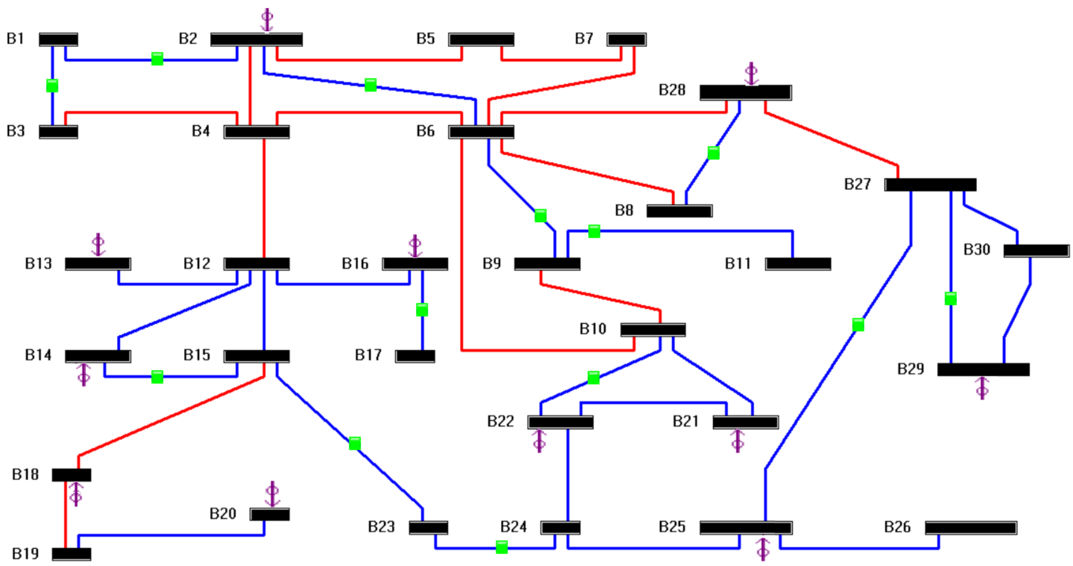


Figure 3-8– IEEE 30 Bus System

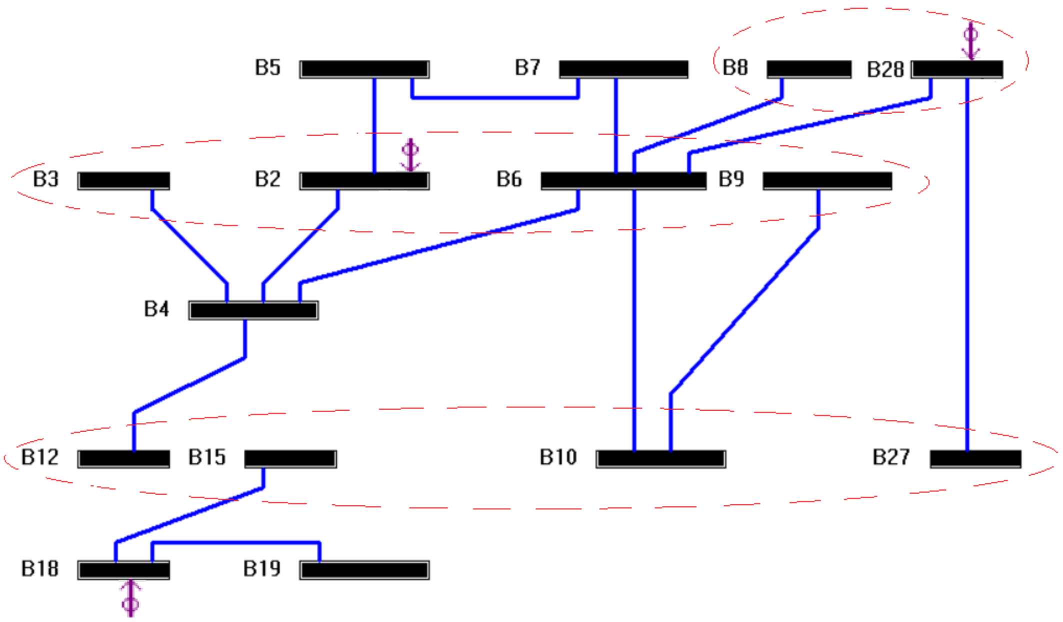


Figure 3-9 - Reduced Form of the IEEE 30 Bus System

Table 3-3. Optimal 2-channel PMU Voltage and Current Phasor Measurement Locations for IEEE 30 Bus System

Voltage Phasor Measurement Location	Current Channel-1	Current Channel-2
Bus-4	Bus-4 to Bus-2	Bus-4 to Bus-12
Bus-6	Bus-6 to Bus-8	Bus-6 to Bus-4
Bus-18	Bus-18 to Bus-19	Bus-18 to Bus-15

3.5. Chapter Summary and Conclusions

In this chapter, the proposed method aims to provide a straightforward and efficient formulation of the problem of utilizing the existing conventional measurements in PMU placement compared to the existing methods in the literature. It is convenient to implement the proposed method in binary integer programming solvers. PMUs are assumed to have a capability of measuring more than one branch current phasors.

In the proposed method, firstly the observability analysis is conducted for the system, and the observable islands and the unobservable branches are determined. This procedure does not bring any computational burden, since the numerical observability analysis for the conventional measurements is available in every proper state estimator. Those observable islands are remodeled as boundary buses to reduce the size of the PMU placement problem. Finally a modified version of the binary integer programming based PMU placement method is employed to place minimum number of PMUs for system observability. The method was illustrated by an illustrated example and validated on IEEE 14-bus and 30-bus test systems.

The modification enables utilization of the boundary injection measurements in PMU placement. The boundary injections are defined as the injection measurements located at either the sending or receiving end of an unobservable branch, which are determined by the numerical observability analysis. Note that, without any extra measurements, those injection measurements do not contribute to the system observability. However, as new measurements are introduced, they may affect the system observability.

CHAPTER 4

PMU PLACEMENT IN THE PRESENCE OF SCADA MEASUREMENTS FOR ROBUST STATE ESTIMATION

The aim the previous chapters was placing minimum number of PMUs for system observability, such that each of those measurements would be critical. Therefore loss of a single PMU would lead to an unobservable system. It is known such a measurement configuration is vulnerable to bad measurements, i.e. even a single bad measurement will bias the state estimation results. This chapter proposes a PMU placement method to improve state estimation robustness against bad measurements. Least Absolute Value (LAV) state estimator will be used as a robust state estimator. The proposed method considers channel limits of PMUs and all available SCADA measurements, and is based on well-developed binary linear programming in [25]. The proposed method is validated in different test systems.

Section 4-1 provides background information on BILP based PMU placement. The proposed method is explained in detail in Section 4-2 . In Section 4.3 an ilustrative example is presented. Simulations and validation of the method are given in Section 4.4.

4.1. Existing Method in the Literature

Binary integer programming based optimal PMU placement problem for robust state estimation can be formulated as follows, if there is no SCADA measurement in the considered n-bus system, [25].

$$\begin{aligned} &\min c^T X \\ &s.t.\ \ AX \geq b \end{aligned} \tag{4.1}$$

where

X binary vector of size N where x_i is the i^{th} element of X. x_i is either 1 or 0 depending on whether the corresponding PMU configuration is deployed or not, respectively

c vector of size N and its entries, c_i, indicate cost of installation of i^{th} PMU in the corresponding configuration.

A PMU configurations matrix of size nxN, for a p-channel PMU located at bus-i, i^{th} entry of the regarding column of A becomes $p+1$, entries corresponding to the receiving end buses become 1. Other entries of the corresponding column are 0.

b index vector of size n, this vector represents the minimum number of measurements that provides information related with the state of the corresponding bus.

The proposed method in the literature is capable of placing PMUs with channel limits. In (4.1), N is the number of possible PMU configurations for all buses and n is the number of buses. Assuming that PMUs are represented as a voltage phasor measurement and p current phasor measurements, a PMU configuration can be defined as a possible assignment of those p current phasor measurements to the incident branches of the considered bus. Considering a PMU placed at bus-i, there are k_i possible ways to assign p-current phasor measurements. If t_i is defined as the number of branches incident to bus-i, k_i can be written as follows:

$$k_i = \binom{t_i}{p} = \frac{t_i!}{p!(t_i - p)!} \tag{4.2}$$

Based on (4.2), total number of possible combinations, *N*, can be found as below.

$$N = \sum_{i=1}^{n} k_i \tag{4.3}$$

In [25] it is stated that index vector entries should be 4 or for depending on the number of the lines which the corresponding bus is incident to. If a bus is incident to a single line, index vector entry becomes 3. Otherwise index vector entry becomes 4. This measurement configuration that enables robust state estimation.

4.2. The Proposed Method

The proposed method aims to incorporate existing SCADA measurements into the problem formulation. Since, power injection and power flow measurements are the measurements that contribute to the power system observability, their existence in the system affects the formulation of the optimal PMU placement problem. Since the existing SCADA measurements provide information related with the states of the system, it can be clearly stated that taking the existing SCADA measurements into account reduces the number of necessary PMUs to make the system fully observable. Therefore, before introducing the proposed method, power system observability in presence of both SCADA and PMU measurements should be discussed.

An observable power system has a unique state estimation solution for a given network topology and observation set. According to topological observability analysis, in an observable power system measured solely by conventional measurements, once all the measurements are assigned to branches a spanning tree consisting of those branches can be formed [1]. However, once PMUs are added to the measurement set, necessity for presence of a spanning tree vanishes thanks to the GPS synchronization of PMUs. Consider the simple system given in Fig. 1. If only the conventional measurements are considered, the system has three observable islands, which are marked as island 1, 2 and 3 in Figure 4-1. However, once the effect of the PMUs on observability added, it will be revealed that the system is observable.

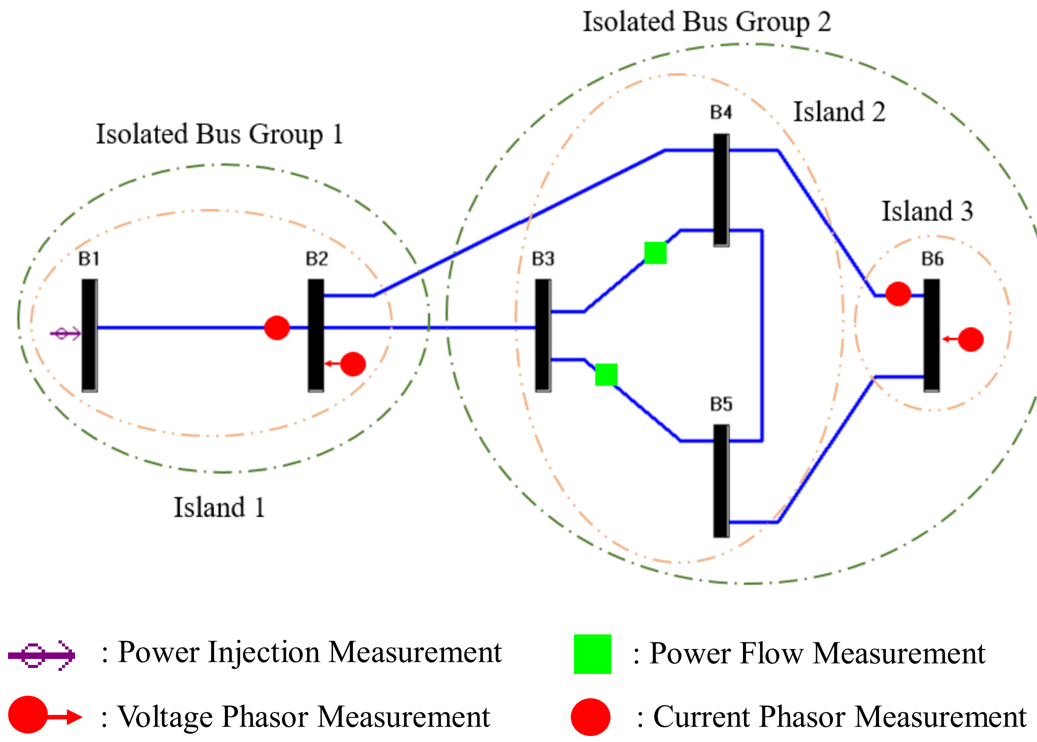


Figure- 4-1. Isolated Bus Groups

Note that, none of the branches between island 1 and island 2 are monitored. Although the system is observable, there is no interaction in terms of information between island 1 and the rest of the system. Those two so-called new islands are called as 'isolated bus groups' [25]. Note that isolated bus groups are not related to observability. Even if a system is observable, it may consist of multiple isolated bus groups due to the measurement configuration, as seen in Fig. 1. Although isolated bus groups do not have a direct relation with observability, they are closely related to the state estimation robustness [24], [25]. In the presence of isolated bus groups, due to information isolation, robust estimators, such as LAV, may fail filtering out bad data. Therefore, in order to obtain state estimation robustness, it is a must to form a spanning tree based on the rules of topological observability analysis. While checking presence of a spanning tree, current phasor measurements are modeled as power flow measurements, and voltage phasor measurements are disregarded. Based on the above discussion, the PMU placement problem for state estimation robustness can be formed as follows.

$$\begin{aligned} &\min c^T X \\ &s.t. \quad F + B + AX \geq d \\ &\qquad a \text{ spanning tree exists} \end{aligned} \tag{4.4}$$

X binary vector of size *N* where x_i is the i^{th} element of *X*. x_i is either 1 or 0 depending on whether the corresponding PMU configuration is deployed or not, respectively

c vector of size *N* and its entries, c_i, indicate cost of installation of PMU ith the corresponding configuration.

A PMU configurations matrix of size *n*x*N*, for a *p*-channel PMU located at bus-i , i^{th} entry of the regarding column of *A* becomes *p+1*, entries corresponding to the receiving end buses become 1. Other entries of the corresponding column are 0.

d index vector of size *n*, this vector represents the minimum number of measurements that provides information related with the state of the corresponding bus.

F Power flow measurement vector of size nx1, its entries indicate how many power flow measurements are incident to the considered bus.

B Power injection measurement assignment vector of size nx1, its entries indicate how many injection measurements are assigned to the considered bus.

In the proposed method, all entries of *b* are assigned as 3 for a breakdown point one. Breakdown point of an estimator can be defined as the smallest amount of contamination (bad measurements) that can cause an estimator to give a biased solution, [27]. Increasing *b* will increase the breakdown point if it is required.

System observability is independent of network parameters and operating state of the system, [35]. Therefore, all line resistances and shunt elements can be neglected, and reactance and voltage values can be assumed to be 1 p.u. Then active power flow from bus-*k* to bus-*m* can be expressed as follows, based on the DC state estimation model.

$$P_{km} = sin(\ _{k} - \ _{m}) \tag{4.5}$$

Applying the first order Taylor expansion around θ_{km}=0, where θ_{km} is the phase difference between buses k and m, (4.5) can be approximated as in (6).

$$P_{km} = (\ _{k} - \ _{m}) \tag{4.6}$$

Similarly, power injection measurements as can be expressed as shown below, where N_k is the set of buses incident to bus-*k*.

$$P_{k} = \sum_{i \in N_k} (\ _{k} - \ _{i}) \tag{4.7}$$

As it can be concluded from (4.6), power flow measurements provide information related with 2 buses and phase angle difference can directly be determined using the available data. However as it can be seen in (4.7), power injection measurements exhibit information related with the states of the adjacent buses in addition to the bus where the measurement is placed. From Kirchhoff's Current Law, if current injection is known at a node together with the current flows in branches incident to that branch except one of the branches, remaining branch current flow can be calculated. Similarly, if the states of all the considered buses, except one, are known, the injection measurement data can be used to find the information regarding the remaining state.

Zero injection buses are also considered same like a bus having power injection measurement.

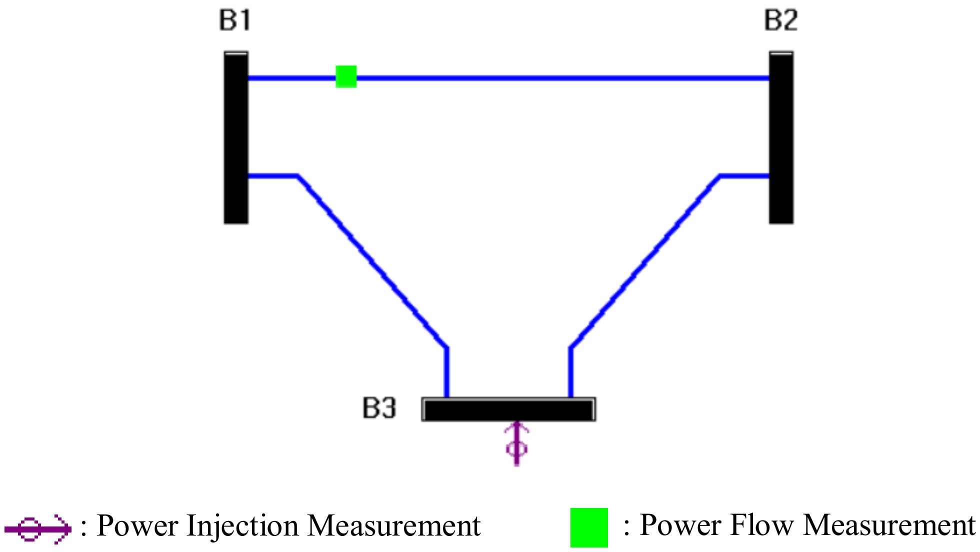


Figure- 4-2. Sample 3-bus System

Considering a system illustrated by Figure 4-2, there is a power flow measurement between bus-2 and bus-3 while there is a power injection measurement which is placed at bus-1. If formulation in (4.3) is used in order to place PMUs to the system, Since there is a power flow measurement between bus-2 and bus-3, second and third rows of *F* became 1. For *B* vector value there are 3 possibilities which are assigning the power injection measurement to bus-1, bus-2 or bus-3. *F* vector and possible *B* vectors are given below.

$$F = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \tag{4.8}$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad B_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{4.9}$$

After obtaining the vectors shown in (4.8) and (4.9), optimization problem in (4.3) need to be solved for different values of *B*. Solution with the least cost will be the optimum solution.

While assigning power injection measurements to bus states, it should be noted that if there are more than one power injection measurement in the system. For each possible assignment, different *B* vectors, which are obtained from different power injection measurements' assignments, should be added to each other in order to obtain the final *D* vector.

The optimization problem shown in (4.3) is complicated in terms of implementation and it may take too much time to solve the problem due to the spanning tree constraint. The optimization problem shown in (4) is complicated in terms of implementation of [36]. Alternatively, the same proposed method can be separated into 2 sub-problems and firstly the one stated below is solved.

$$\begin{aligned} &\min c^T X \\ &s.t. \quad F + B + AX \geq d \end{aligned} \tag{4.10}$$

The formulation given in (4.10) does not consider the spanning tree constraint. Optimization problem formulated by (4.10) is the relaxation of the problem in (4.4). For different power injection measurement assignment vectors, the problem given is solved. Conventional observability analysis [1] assuming current phasor measurements as power flow measurements and neglecting voltage phasor measurements. If the system is found to be observable, a spanning tree exists and solution is optimal. However, if the system is not obervable there will be observable islands. The resulting observable islands found in the analysis are the isolated bus groups. Since there are more than one isolated bus groups, PMUs should be placed on the branches which are located between isolated bus groups in order to ensure that spanning tree is formed.

The system illustrated by Figure 4-2, is a 6-bus system on which the alternative method is applied in order to obtain a measuement design for state estimation robustness. Initially the system does not have any installed measurements. The alternative method is applied to the system and branch PMUs are placed as shown. There are two isolated groups in the proposed design. In order to ensure that spanning tree is formed, a PMU whose current channel is placed on the lines between bus-3 and bus-4 or bus-3 and bus-5 , should be placed.

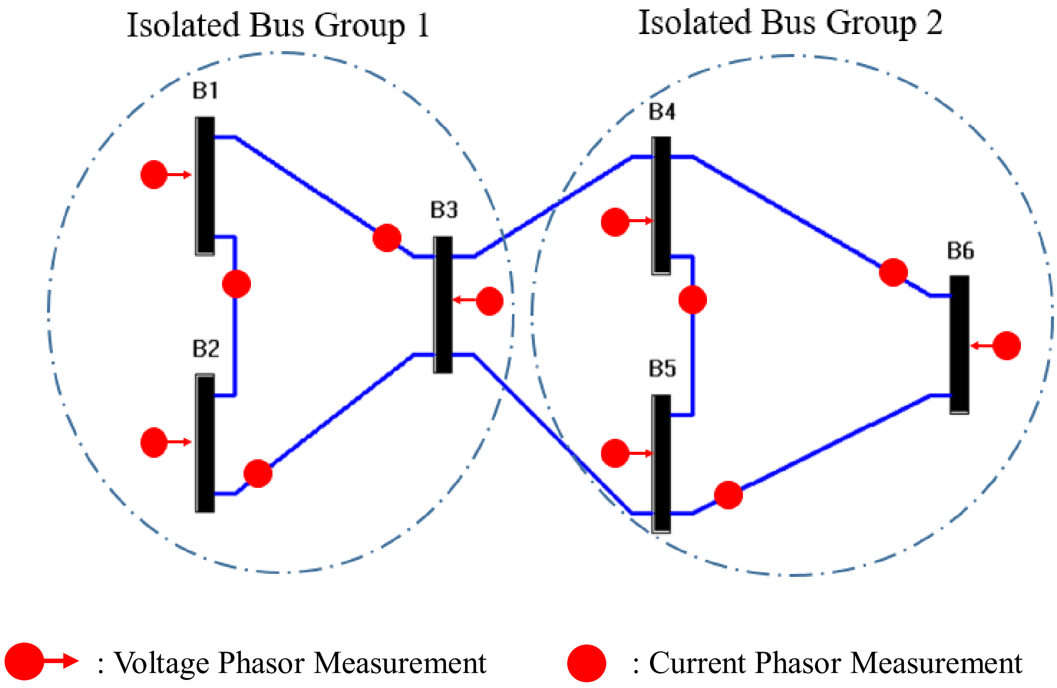


Figure- 4-3. Sample 6-bus System

Although, the alternative approach is easier to implement, it should be noted that in some cases this method may converge to a sub-optimal solution. For example , Figure 4-4 and Figure 4-5 illustrate 2 systems. Initially there is a power flow measurment between bus-1 and bus-2. Alternative approach is used in order to place PMUs to the system. By utilizing the same number of PMUs in Figure 4-4, spanning tree constraint is not satisfied while in Figure 4-5 spanning tree constraint is satisfied. Both designs are obtained by using (4.10) .They are both optimal solutions if the alternative formulation is utilized.

The actual optimal solution to the problem is the design which is depicted by Figure 4-5 since the spanning tree constraint is satisfied. However, for the case in Figure 4-4, by utilizing the proposed alternative approach, a PMU whose current channel is placed between bus-2 and bus-3 is needed in order to obtain a design which guarantees state estimation robustness. Therefore, sub-optimal solution will be obtained.

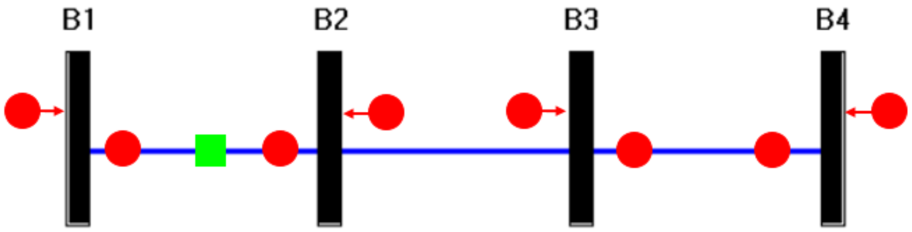


Figure- 4-4. Sample 4-bus System which does not Satisfy Spanning Tree Constraint

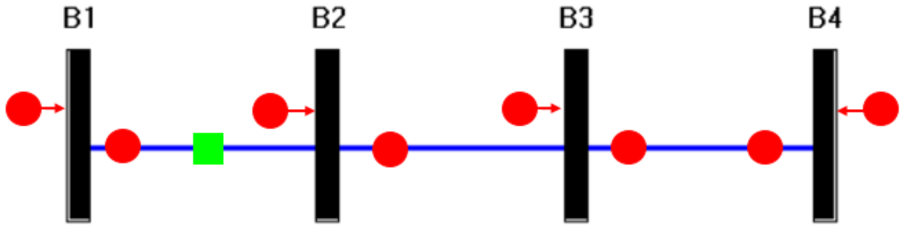


Figure- 4-5. Sample 4-bus System which Satisfies Spanning Tree Constraint

: Voltage Phasor Measurement   : Current Phasor Measurement

: Power Flow Measurement

4.3. Illustrative Example

In order to illustrate the proposed method, the method will be applied on a 6-bus system which is shown by Figure 4-6. The system has SCADA measurements. The objective is to make the system observable and have a robust measurement configuration. In other words, the state estimator estimates will not be affected by the bad data in the system and even if any measurement loss occurs in the system, the system will maintain its observability.

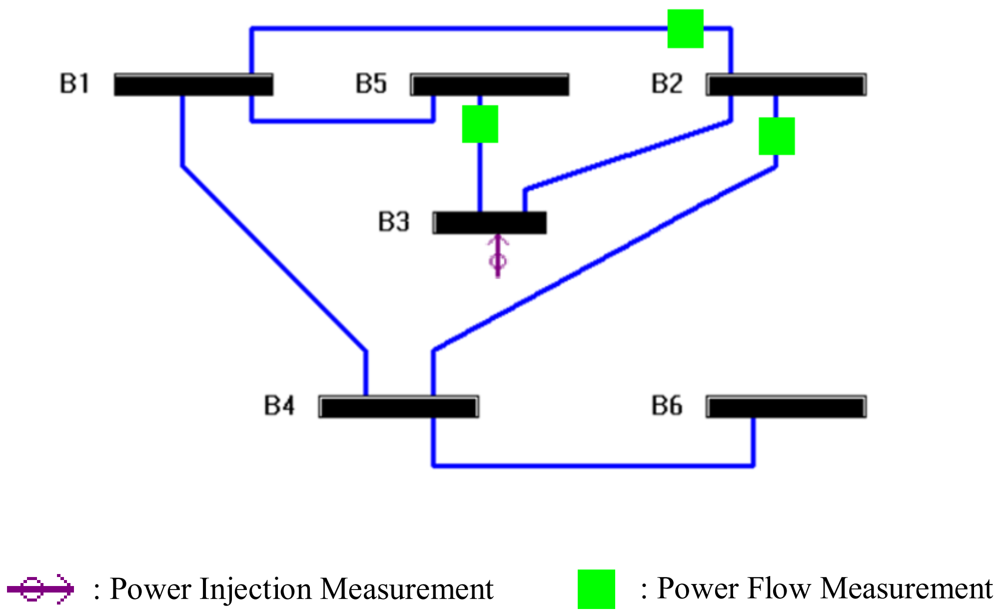


Figure 4-6. 6 Bus Sample System Measured by SCADA Measurements

Branch type PMUs will be used in this example. Branch PMUs have the capability of measuring a voltage phasor on a bus and a current phasor along a single feeder which is connected to the corresponding bus. Using (4.1) and (4.2), total number of ways to assign current phasors along the branches can be calculated as follows.

$$K = \binom{3}{1} + \binom{3}{1} + \binom{2}{1} + \binom{3}{1} + \binom{2}{1} + \binom{1}{1} = 14 \qquad (4.11)$$

After calculating K value, PMU configurations matrix (A) can be written. A is a 6x*14 matrix since* K value is 14 and number of buses in the system is 6.

$$A = \begin{bmatrix} 2 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 \end{bmatrix} \qquad (4.12)$$

Cost vector (c) can be written as in (4.13). Since there isn't any installed PMU in the system, all the entries equal to 1.

$$c = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]^T \tag{4.13}$$

Index vector (b) is written as in (4.14) since it is expected that each bus in the system has to be monitored by at least 3 measurements.

$$d = [3 \quad 3 \quad 3 \quad 3 \quad 3 \quad 3 \quad 3 \quad 3 \quad 3 \quad 3 \quad 3 \quad 3 \quad 3 \quad 3]^T \tag{4.14}$$

While writing power flow assignment vector (F), positions of power flow measurements need to be checked. Since bus-2 is incident to 2 branches which have power flow measurements, its corresponding value will be 2. For the bus- 1, bus-3, bus-4 and bus-5, the corresponding the value will be 1 because of the same reason. Power flow assignment vector F is as follows.

$$F = [1 \quad 2 \quad 1 \quad 1 \quad 1 \quad 0]^T \tag{4.15}$$

Since the power injection measurement is connected to bus-3, the measurement provides information regarding the state of bus-3. Since bus-3 is connected to bus-2 and bus-5, regarding measurement provides information about the state of bus-2 and bus-5 as well. There are 3 possible power injection measurement assignment vectors as below.

$$B_1 = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]^T \tag{4.16}$$

$$B_2 = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]^T \tag{4.17}$$

$$B_3 = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0]^T \tag{4.18}$$

Since there are 3 different possible power assignment vectors, optimization problem in (4.3) will be solved 3 times. Results of the problems are given below.

$$X_1 = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1]^T \tag{4.19}$$

$$X_2 = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1]^T \tag{4.20}$$

$$X_3 = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1]^T \tag{4.21}$$

There are 3 solutions to this problem since the considered study assumes equal cost for all PMU configurations. All of the solutions provide a robust measurement configuration for LAV estimator. If any cost difference occurs in PMU configurations, configuration with the least cost can be selected. Measurement configurations for different B vectors are shown by Figure 4-7, Figure 4-8 and Figure 4-9, respectively.

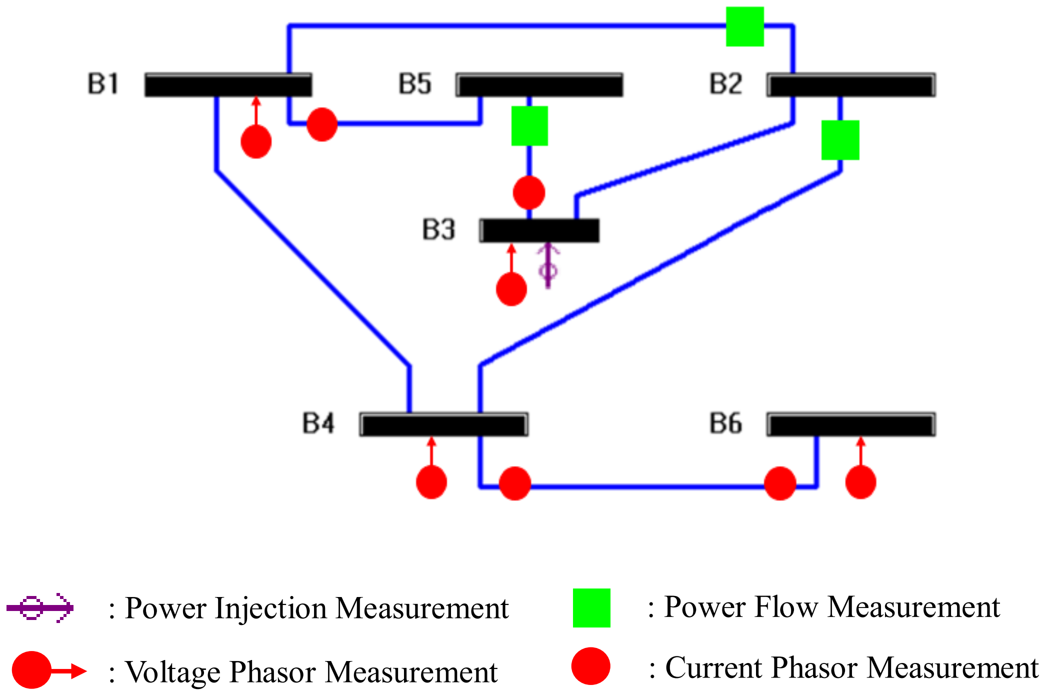


Figure 4-7. Measurement Configuration Based on B_1

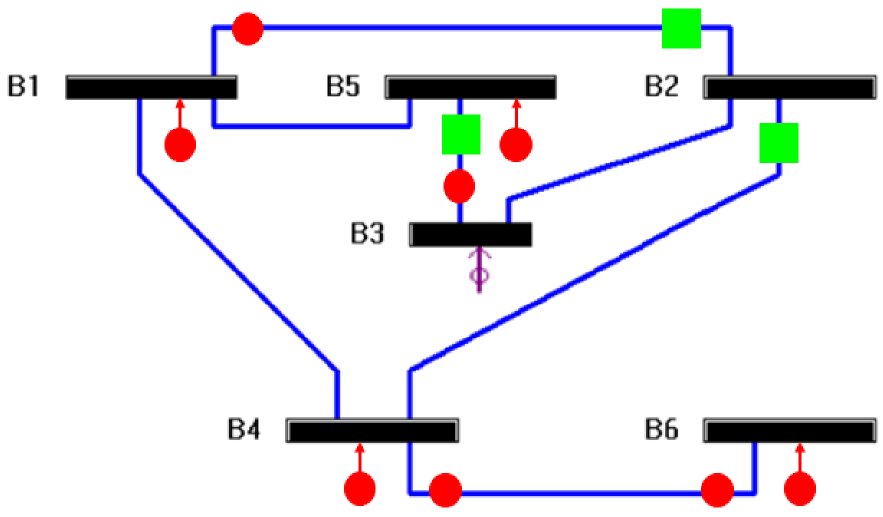


Figure 4-8. Measurement Configuration Based on B_2

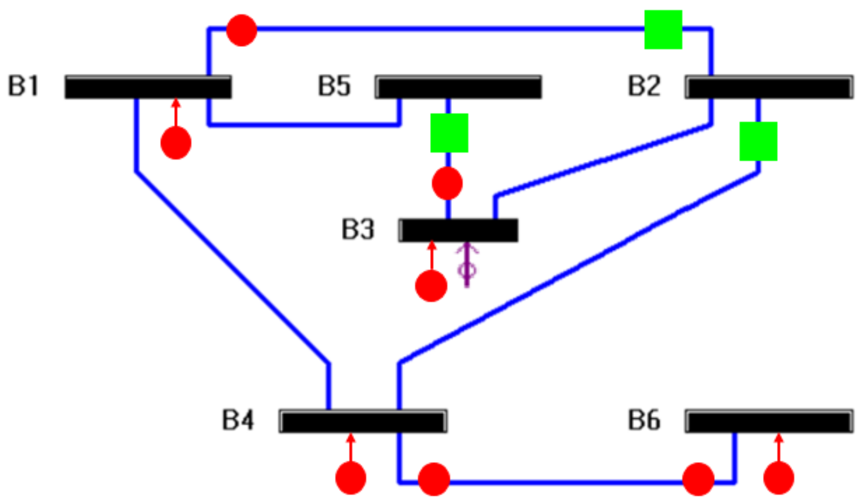


Figure 4-9. Measurement Configuration Based on B_3

4.4. Case Studies

Alternative method was applied to the IEEE 14 Bus Test system. MATLAB's *intlinprog* function was used to solve the binary integer programming problems. The system and the initial measurement configuration is illustrated by Figure 4-10. As it can be realized, there are power injection measurements connected to bus-6 and bus-13. Since bus-6 is connected to 4 buses there are 4+1=5 possible power injection measurement assignment vectors this measurement. Similarly, since bus-13 is connected to 3 buses there are 3+1=4 possible power injection assignment vectors. Therefore, the algorithm will find 4 X 5=20 possible solutions.

Six among those possible 20 solutions require 9 PMUs for state estimation robustness while other solutions require more than 9 PMUs . Those placements are tabulated in Table- 4-1 and Table 4-2. As indicated in these tables, first three solutions satisfy the spanning tree constraint and they can be considered for implementation. To validate the proposed placement method, solution-1 and solution-6 were utilized . LAV state estimation results were compared. Regarding measurement configurations are visualized in Figure 4-11 and Figure 4-12.

A measurement set based on power flow solution was generated for each of the solutions, and Gaussian error was added. Then the state estimation problem was solved using LAV estimator 56 times by assigning one of the measurements as a bad data at each run. Implementation othe LAV state estimator is described in Appendix A. Bad data was generated by multiplying considered measurement with (-1). Estimation bias (EB) and root mean squared error (RMSE) were calculated for each state, and are tabulated in Table 4-3 and Table 4-4. Estimation bias (EB) and root mean squared error (RMSE) values were calculated using the expression in (4.22) and (4.23), respectively

$$EB = \frac{1}{n}\sum_{k=1}^{n} x_k^e - x_k \tag{4.22}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(x_k^e - x_k\right)^2} \tag{4.23}$$

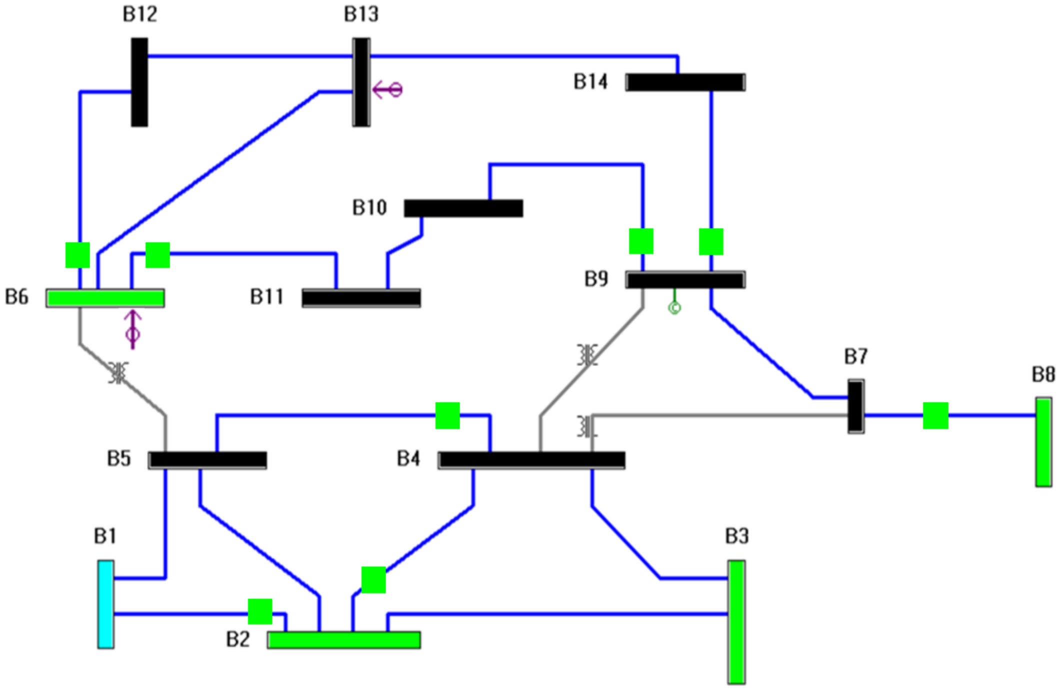


Figure 4-10. IEEE 14 Bus Test System with Initially Installed SCADA Measurements

As it can be realized in Table 4-3 and Table 4-3, state estimates remained unbiased independent of the bad data and its location for solution-1. However, if the spanning tree criterion is not satisfied as in solution-6 , state estimation became vulnerable to gross errors. If states of buses 7 and 8, which constitute an isolated bus group, are examined it can be concluded that their estimated states are biased and isolated bus groups may led to a measurement configurations which is not robust against gross errors.

Table 4-1. Optimal PMU Placement Solutions for the Specified Configuration

(Spanning Tree Exists)

Solution-1		**Solution-2**		**Solution-3**	
Sending End Bus	Receiving End Bus	Sending End Bus	Receiving End Bus	Sending End Bus	Receiving End Bus
Bus-1	Bus-5	Bus-1	Bus-5	Bus-1	Bus-5
Bus-2	Bus-3	Bus-3	Bus-2	Bus-2	Bus-3
Bus-3	Bus-4	Bus-3	Bus-4	Bus-3	Bus-4
Bus-6	Bus-5	Bus-6	Bus-5	Bus-6	Bus-5
Bus-7	Bus-8	Bus-8	Bus-7	Bus-8	Bus-7
Bus-9	Bus-7	Bus-9	Bus-7	Bus-9	Bus-7
Bus-10	Bus-11	Bus-10	Bus-11	Bus-10	Bus-11
Bus-13	Bus-12	Bus-12	Bus-13	Bus-12	Bus-13
Bus-14	Bus-13	Bus-13	Bus-14	Bus-14	Bus-13

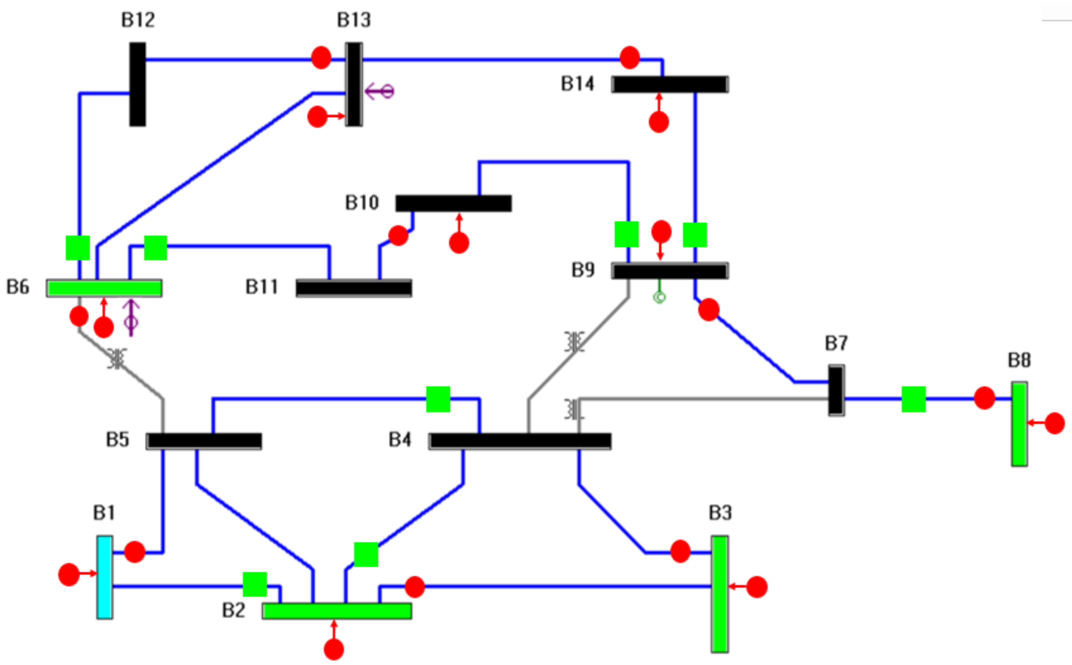


Figure 4-11 . Measurement Configuration in Solution-1

Table 4-2. Optimal PMU Placement Solutions for the Specified Measurement Configuration (Spanning Tree does not Exist)

Solution-4		**Solution-5**		**Solution-6**	
Sending End Bus	Receiving End Bus	Sending End Bus	Receiving End Bus	Sending End Bus	Receiving End Bus
Bus-1	Bus-5	Bus-1	Bus-5	Bus-1	Bus-5
Bus-2	Bus-3	Bus-2	Bus-3	Bus-2	Bus-3
Bus-3	Bus-4	Bus-3	Bus-4	Bus-3	Bus-4
Bus-7	Bus-8	Bus-7	Bus-8	Bus-7	Bus-8
Bus-8	Bus-7	Bus-8	Bus-7	Bus-8	Bus-7
Bus-10	Bus-9	Bus-10	Bus-9	Bus-10	Bus-9
Bus-11	Bus-6	Bus-11	Bus-6	Bus-11	Bus-6
Bus-12	Bus-13	Bus-12	Bus-13	Bus-12	Bus-13
Bus-14	Bus-12	Bus-13	Bus-14	Bus-14	Bus-13

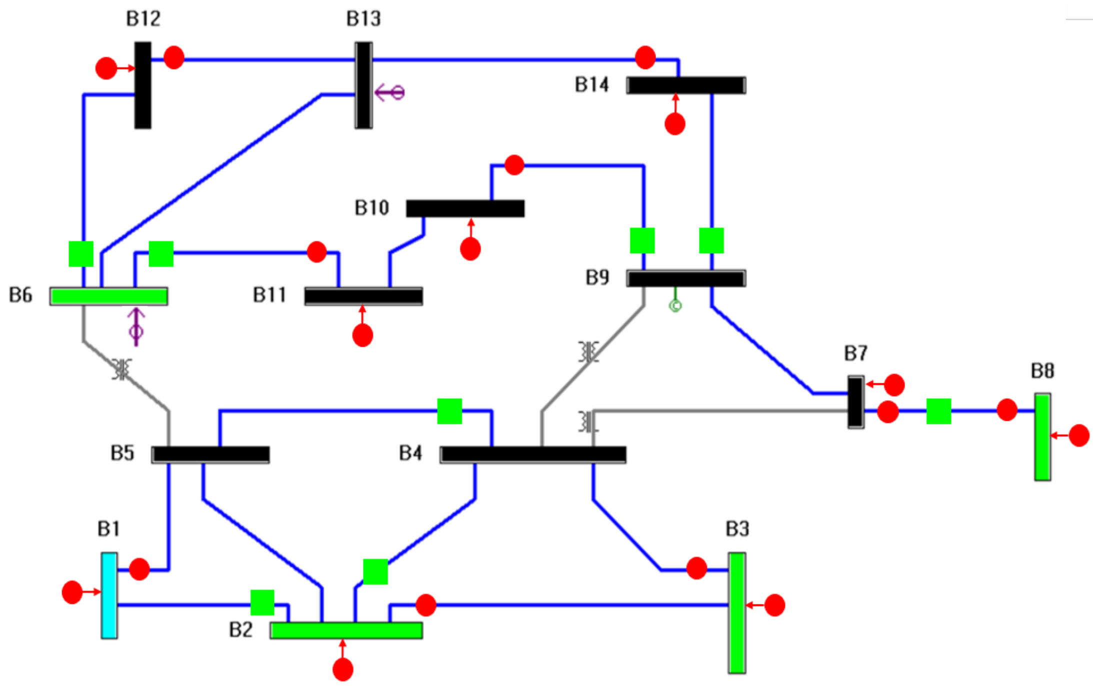


Figure 4-12. Measurement Configuration in Solution-6

Table 4-3. Estimation Bias Values of the Estimated States

	Voltage Magnitude		Phase Angle	
	Solution-1	**Solution-6**	**Solution-1**	**Solution-6**
Bus-1	-0.00044	-0.00298	0	0
Bus-2	-0.00044	-0.00185	0.0071	0.2
Bus-3	-0.00039	-0.00395	0.00452	0.20125
Bus-4	-0.00040	-0.00448	0.03135	0.20647
Bus-5	-0.00014	-0.00442	-0.02178	0.18856
Bus-6	-0.00013	0.02016	0.00150	1.26732
Bus-7	-0.00014	-0.00008	-0.0611	0.09539
Bus-8	-0.00007	-0.00009	-0.06099	0.06126
Bus-9	0.00013	0.00340	-0.00425	0.05023
Bus-10	0.00034	0.00398	0.00154	0.01089
Bus-11	0.00037	0.02051	-0.00895	1.258425
Bus-12	0.00043	0.02945	-0.06605	0.24959
Bus-13	0.0006	0.01619	0.00034	0.89249
Bus-14	0.00074	0.00209	-0.00475	0.16720

Table 4-4. Root Mean Squared Error Values of the Estimated States

	Voltage Magnitude		Phase Angle	
	Solution-1	**Solution-6**	**Solution-1**	**Solution-6**
Bus-1	0.000002	0.000156	0	0
Bus-2	0.000002	0.000103	0.003056	1.7576007
Bus-3	0.000002	0.000111	0.00185	1.89757
Bus-4	0.000002	0.000122	0.009368	1.846418
Bus-5	0.00000007	0.000127	0.095672	2.003124
Bus-6	0.00000008	0.000478	0.050491	2.274058
Bus-7	0.00000009	0.000027	0.1668602	1.2974308
Bus-8	0.0000002	0.000027	0.166886	1.249528
Bus-9	0.00000014	0.000017	0.025805	0.203057
Bus-10	0.00000002	0.000023	0.025363	0.202052
Bus-11	0.000002	0.00049	0.031252	2.250868
Bus-12	0.0000027	0.000924	0.085642	0.422343
Bus-13	0.000004	0.000316	0.037113	1.264121
Bus-14	0.000004	0.00002	0.025272	0.290809

Same approach was also applied on IEEE 33 Bus Distribution test system. System is illustrated by Figure 4-10 with the specified measurement configuration. Proposed solution to this system is tabulated in Table 4-5. LAV State Estimator was also used in order to test the obtained measurement configuration against bad measurements. LAV estimator was run 148 times by assigning one of the measurements as a bad data at each run. Estimation bias and mean squared error values of each state was calculated and tabulated in Table 4-6.

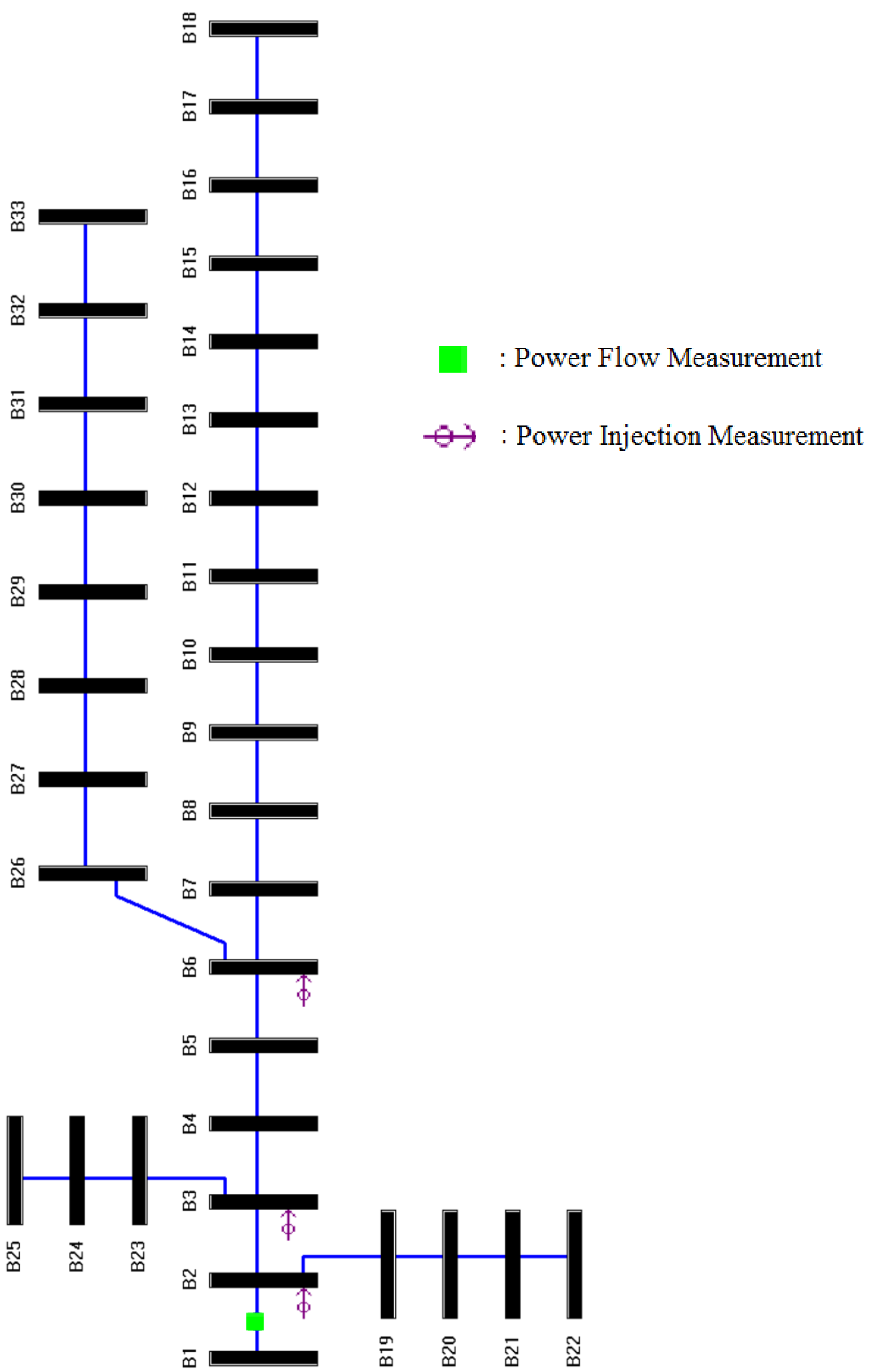


Figure 4-13. IEEE 33 Bus Test System with Initially Installed SCADA Measurements

Table 4-5. Optimal PMU Placement Solution for the Specified Measurement Configuration

Sending End Bus	Receiving End Bus	Sending End Bus	Receiving End Bus
Bus-2	Bus-1	Bus-20	Bus-21
Bus-3	Bus-4	Bus-21	Bus-22
Bus-4	Bus-5	Bus-22	Bus-21
Bus-6	Bus-5	Bus-23	Bus-24
Bus-7	Bus-6	Bus-24	Bus-25
Bus-8	Bus-7	Bus-25	Bus-24
Bus-9	Bus-8	Bus-26	Bus-27
Bus-9	Bus-10	Bus-27	Bus-28
Bus-10	Bus-11	Bus-28	Bus-29
Bus-11	Bus-12	Bus-29	Bus-30
Bus-12	Bus-13	Bus-30	Bus-31
Bus-13	Bus-14	Bus-31	Bus-32
Bus-14	Bus-15	Bus-32	Bus-33
Bus-15	Bus-6	Bus-33	Bus-32
Bus-16	Bus-17	Bus-2	Bus-19
Bus-17	Bus-18	Bus-3	Bus-23
Bus-18	Bus-17	Bus-6	Bus-26
Bus-19	Bus-20		

Table 4-6. Estimation Bias and Root Mean Squared Error Values of the Estimated States

	Voltage Magnitude		Phase Angle	
	EB	**RMSE**	**EB**	**RMSE**
Bus-1	-0.000203	0.00000095	0	0
Bus-2	-0.0001999	0.000000957	-0.0000393	0.0000011
Bus-3	-0.000189	0.00000086	0.002722	0.0000996
Bus-4	-0.000187	0.00000094	-0.001696	0.0002288
Bus-5	-0.0000817	0.000000036	-0.002592	0.0018212
Bus-6	-0.0000832	0.000000038	-0.002826	0.0017249
Bus-7	-0.00008284	0.000000042	-0.003952	0.0017543
Bus-8	-0.00005709	0.000000108	-0.0026188	0.001963
Bus-9	0.0000295	0.000000579	-0.001459	0.002569
Bus-10	0.00011307	0.000001078	-0.0005614	0.0035729
Bus-11	0.0001255	0.00000108	-0.000679	0.0035897
Bus-12	0.00014668	0.0000011	-0.0007835	0.0036219
Bus-13	0.0002353	0.0000017	0.0001352	0.005136
Bus-14	0.000271	0.00000174	0.0012302	0.005452
Bus-15	0.0002928	0.00000178	0.001892	0.005575
Bus-16	0.0003102	0.0000018	0.00231016	0.005634
Bus-17	0.000347	0.0000019	0.00374496	0.0062379
Bus-18	0.000370	0.00000194	0.0022477	0.005953
Bus-19	-0.00018003	0.000000929	-0.00151	0.0000102
Bus-20	-0.000127	0.00000105	-0.000998	0.0005795
Bus-21	-0.0001177	0.00000106	-0.000768	0.0006056
Bus-22	-0.0001096	0.0000001	-0.0000723	0.0006459
Bus-23	-0.0001992	0.000000888	0.002046	0.000153
Bus-24	-0.000098	0.0000015506	0.0032294	0.00189
Bus-25	-0.0000842	0.00000152	0.00264494	0.00191

Table 4-6 (Continued)

Bus-26	-0.0000765	0.00000004	-0.003423	0.00178
Bus-27	-0.0000396	0.000000115	-0.0041462	0.002036
Bus-28	0.000117	0.00000172	-0.005543	0.008249
Bus-29	0.00023495	0.0000026	-0.006737	0.0119411
Bus-30	0.0002854	0.00000275	-0.0082	0.0128656
Bus-31	0.0003486	0.00000303	-0.007059	0.014075
Bus-32	0.000361	0.00000304	-0.006803	0.014152
Bus-33	0.000353	0.000002892	-0.00641	0.014071

In order to detect non-technical loss, in each step of the analysis bad data was given to the measurement at that specific bus. It was found that it is possible to detect non-technical loses if a measurement configuration makes the state estimator robust against bad data.

Proposed methods in this thesis were also applied on IEEE 69 Bus Distribution test system. System is illustrated by Figure 4-14 with the specified measurement configuration.

Effect of existing SCADA measurements on the number of PMUs to be placed, is examined using branch PMUs. Firstly, PMU placement aiming the state estimation robustness was performed by considering the existing SCADA measurements. Secondly, same method was applied by neglecting the existing SCADA measurements. Thirdly, PMU placement for system observability was performed by considering the existing SCADA measurements. Finally, existing SCADA measurements were neglected PMUs were placed aiming the system observability.

Number of PMUs to be placed in each case is tabulated in Table 4-7. Proposed PMU locations are as tabulated in Table 4-8, Table 4-9, Table 4-10 and Table 4-11.

Table 4-7. Numbers of Required PMUs for Various Cases

PMU Placement for State Estimation Robustness		**PMU Placement for System Observability**	
with SCADA Measurements	without SCADA Measurements	with SCADA Measurements	without SCADA Measurements
70	77	30	36

Obtained results indicate that existing SCADA measurements in the system, decreases the number of PMUs to be placed as expected. If there exists higher number of SCADA measurements in the system, effect of the SCADA measurements on the number of required PMUs will be higher.

Since distribution grids have a radial structure, it will be an expensive investment to apply the design in today's technology. Distribution systems also generally don't have conventional power flow and injection measurements. Hence, this will also increase the cost of investment since higher number of PMUs are needed in order to satisfy the required measurement redundancy compared to transmission systems.

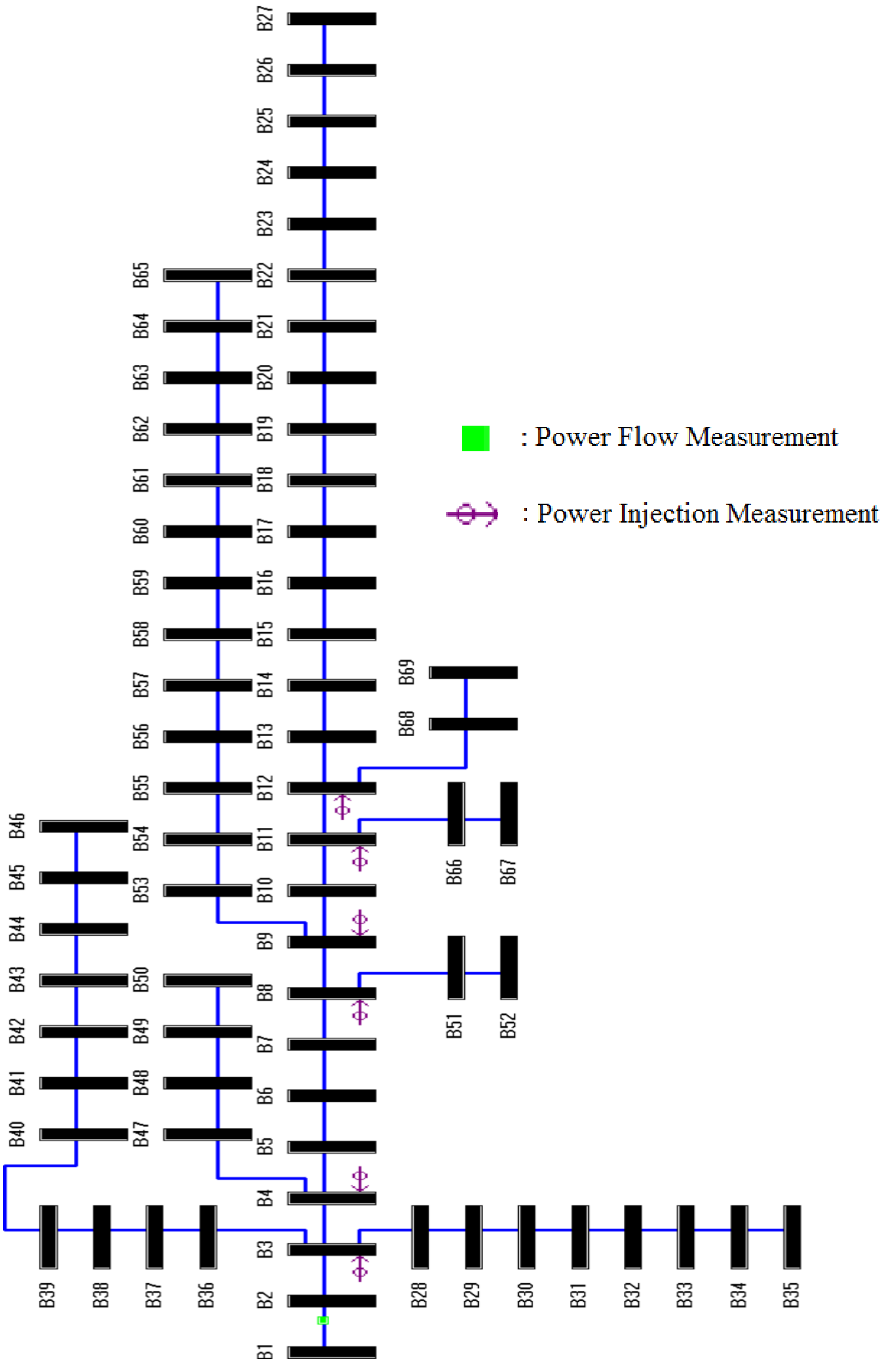


Figure 4-14. IEEE 69 Bus Test System with Initially Installed SCADA Measurements

Table 4-8. IEEE 69 Bus System Optimal PMU Placement Solution for State Estimation Robustness (with SCADA Measurements)

Sending End Bus	**Receiving End Bus**	**Sending End Bus**	**Receiving End Bus**	**Sending End Bus**	**Receiving End Bus**
Bus-1	Bus-2	Bus-26	Bus-27	Bus-48	Bus-49
Bus-2	Bus-3	Bus-27	Bus-26	Bus-49	Bus-50
Bus-4	Bus-5	Bus-3	Bus-28	Bus-50	Bus-49
Bus-5	Bus-6	Bus-28	Bus-29	Bus-51	Bus-52
Bus-6	Bus-7	Bus-29	Bus-30	Bus-52	Bus-53
Bus-7	Bus-8	Bus-30	Bus-31	Bus-53	Bus-54
Bus-8	Bus-9	Bus-31	Bus-32	Bus-54	Bus-55
Bus-9	Bus-10	Bus-32	Bus-33	Bus-55	Bus-56
Bus-10	Bus-11	Bus-33	Bus-34	Bus-56	Bus-57
Bus-11	Bus-12	Bus-34	Bus-35	Bus-57	Bus-58
Bus-12	Bus-13	Bus-35	Bus-34	Bus-58	Bus-59
Bus-13	Bus-14	Bus-3	Bus-36	Bus-59	Bus-60
Bus-14	Bus-15	Bus-36	Bus-37	Bus-60	Bus-61
Bus-15	Bus-6	Bus-37	Bus-38	Bus-61	Bus-62
Bus-16	Bus-17	Bus-38	Bus-39	Bus-62	Bus-63
Bus-17	Bus-18	Bus-39	Bus-40	Bus-63	Bus-64
Bus-18	Bus-19	Bus-40	Bus-41	Bus-64	Bus-65
Bus-19	Bus-20	Bus-41	Bus-42	Bus-65	Bus-64
Bus-20	Bus-21	Bus-42	Bus-43	Bus-66	Bus-67
Bus-21	Bus-22	Bus-43	Bus-44	Bus-67	Bus-66
Bus-22	Bus-23	Bus-44	Bus-45	Bus-68	Bus-69
Bus-23	Bus-24	Bus-45	Bus-46	Bus-69	Bus-68
Bus-24	Bus-25	Bus-46	Bus-45		
Bus-25	Bus-26	Bus-47	Bus-48		

Table 4-9. IEEE 69 Bus System Optimal PMU Placement Solution for State Estimation Robustness (without SCADA Measurements)

Sending End Bus	**Receiving End Bus**	**Sending End Bus**	**Receiving End Bus**	**Sending End Bus**	**Receiving End Bus**
Bus-1	Bus-2	Bus-26	Bus-27	Bus-49	Bus-50
Bus-2	Bus-1	Bus-27	Bus-26	Bus-50	Bus-49
Bus-2	Bus-3	Bus-3	Bus-28	Bus-8	Bus-51
Bus-3	Bus-4	Bus-28	Bus-29	Bus-51	Bus-52
Bus-4	Bus-5	Bus-29	Bus-30	Bus-52	Bus-51
Bus-5	Bus-6	Bus-30	Bus-31	Bus-9	Bus-53
Bus-6	Bus-7	Bus-31	Bus-32	Bus-53	Bus-54
Bus-7	Bus-8	Bus-32	Bus-33	Bus-54	Bus-55
Bus-8	Bus-9	Bus-33	Bus-34	Bus-55	Bus-56
Bus-9	Bus-10	Bus-34	Bus-35	Bus-56	Bus-57
Bus-10	Bus-11	Bus-35	Bus-34	Bus-57	Bus-58
Bus-11	Bus-12	Bus-3	Bus-36	Bus-58	Bus-59
Bus-12	Bus-13	Bus-36	Bus-37	Bus-59	Bus-60
Bus-13	Bus-14	Bus-37	Bus-38	Bus-60	Bus-61
Bus-14	Bus-15	Bus-38	Bus-39	Bus-61	Bus-62
Bus-15	Bus-16	Bus-39	Bus-40	Bus-62	Bus-63
Bus-16	Bus-17	Bus-40	Bus-41	Bus-63	Bus-64
Bus-17	Bus-18	Bus-41	Bus-42	Bus-64	Bus-65
Bus-18	Bus-19	Bus-42	Bus-43	Bus-65	Bus-64
Bus-19	Bus-20	Bus-43	Bus-44	Bus-11	Bus-66
Bus-20	Bus-21	Bus-44	Bus-45	Bus-66	Bus-67
Bus-21	Bus-22	Bus-45	Bus-46	Bus-67	Bus-66
Bus-22	Bus-23	Bus-46	Bus-45	Bus-12	Bus-66
Bus-23	Bus-24	Bus-4	Bus-47	Bus-68	Bus-69
Bus-24	Bus-25	Bus-47	Bus-48	Bus-69	Bus-68
Bus-25	Bus-26	Bus-48	Bus-49		

Table 4-10. IEEE 69 Bus System Optimal PMU Placement Solution for System Observability (with SCADA Measurements)

Sending End Bus	**Receiving End Bus**	**Sending End Bus**	**Receiving End Bus**
Bus-3	Bus-36	Bus-37	Bus-38
Bus-5	Bus-6	Bus-39	Bus-40
Bus-7	Bus-8	Bus-41	Bus-42
Bus-10	Bus-11	Bus-43	Bus-44
Bus-14	Bus-15	Bus-45	Bus-46
Bus-16	Bus-17	Bus-47	Bus-48
Bus-18	Bus-19	Bus-49	Bus-50
Bus-20	Bus-21	Bus-51	Bus-52
Bus-22	Bus-23	Bus-54	Bus-55
Bus-24	Bus-25	Bus-56	Bus-57
Bus-26	Bus-27	Bus-58	Bus-59
Bus-28	Bus-29	Bus-60	Bus-61
Bus-30	Bus-31	Bus-62	Bus-63
Bus-32	Bus-33	Bus-64	Bus-65
Bus-34	Bus-35	Bus-66	Bus-67

Table 4-11. IEEE 69 Bus System Optimal PMU Placement Solution for System Observability (without SCADA Measurements)

Sending End Bus	**Receiving End Bus**	**Sending End Bus**	**Receiving End Bus**
Bus-1	Bus-2	Bus-36	Bus-21
Bus-3	Bus-4	Bus-38	Bus-22
Bus-5	Bus-6	Bus-40	Bus-21
Bus-7	Bus-8	Bus-42	Bus-24
Bus-9	Bus-10	Bus-44	Bus-45
Bus-11	Bus-12	Bus-46	Bus-45
Bus-13	Bus-14	Bus-47	Bus-48
Bus-15	Bus-16	Bus-49	Bus-50
Bus-17	Bus-18	Bus-51	Bus-52
Bus-19	Bus-20	Bus-53	Bus-54
Bus-21	Bus-22	Bus-55	Bus-56
Bus-23	Bus-24	Bus-57	Bus-58
Bus-25	Bus-26	Bus-59	Bus-60
Bus-27	Bus-26	Bus-61	Bus-62
Bus-28	Bus-29	Bus-63	Bus-64
Bus-30	Bus-31	Bus-65	Bus-64
Bus-32	Bus-33	Bus-66	Bus-67
Bus-34	Bus-35	Bus-68	Bus-69

4.5. Chapter Summary and Conclusions

This chapter proposed a binary integer linear programming based PMU placement method, which also considers already existing SCADA measurements. Main goal of the method is to guarantee the state estimation robustness. Branch-PMUs were utilized in the study for simplicity. However, the proposed method is applicable to all channel limited PMUs by modifying the A matrix.

State estimation robustness requires presence of a spanning tree, which is formed based on topological observability analysis, to minimize the required number of PMUs.

Note that, in a system with PMUs, spanning tree is not a constraint for observability. The introduced alternative method neglects the spanning tree constraint while placing PMUs. Formation of a spanning tree is usually encountered at the end of the proposed procedure. However, if a spanning tree cannot be formed, additional PMUs should be deployed. During this step, PMUs will be placed between the observable islands to obtain a spanning tree. Obtained solution may be the sub-optimal solution. One may prefer using this alternative instead of increasing the optimization problem complexity.

The method was explicitly applied using illustrative example and validated on 14-bus 33-bus and 69-bus IEEE test systems. Obtained designs in 14-bus and 33-bus systems were tested using LAV state estimator. 69-bus system was used in order to compare the proposed methods in this thesis and effect of SCADA measurements to the PMU placement process was numerically shown.

CHAPTER 5

CONCLUSION

Considering the fast increase in deployment of Phasor Measurement Units (PMUs) in power systems, this thesis proposes a PMU placement methods based on binary integer linear programming (BILP) based optimal PMU placement methods. These methods consider already existing SCADA measurements together with the already existing PMU measurements. Aim of the proposed methods is satisfying the technical constraints while minimizing the investment cost.

The methods proposed in this study can be classified under two categories which are:

I. Observability of the system is aimed and most of the measurements in the system are critical measurements such that loss of any of the critical measurements will result in an unobservable system. Moreover, errors associated with those measurements cannot be detected. Any bad data existing on that measurements may force the state estimator to give biased estimates.

II. Robustness of the state estimator is aimed and errors of the associated method can be detected. In this measurement configuration, unbiased state estimates will be obtained even in the presence of bad measurements.

Second and third chapters of the thesis focus on the measurement configurations specified in (I). The methods specified in these chapters aim to incorporate existing SCADA measurements to the optimal PMU placement algorithms in the literature which do not consider the existing SCADA measurements in the power system.

In these chapters, proposed methods conduct numerical observability analysis based on the conventional measurements to find unobservable branches. Utilizing the obtained unobservable branches, observable islands are formed. Observable islands are reduced to boundary buses and super-nodes in order to simplify the optimization problem.

Majority of the PMU placement methods in literature assumes no available conventional measurement at the considered system. However, the proposed methods enable the usage of all conventional measurements and hence reduces the cost of a PMU investment for an observable system. Therefore, a system operator can directly use the method to determine the strategic locations of the PMUs to be installed for the proper operation of a robust state estimator.

Note that PMUs with different costs can be employed in the proposed method by modifying the cost vector. Moreover, the zero-injection buses can be modeled as a power injection measurement and hence can be taken into account during the solution of the PMU placement problem.

Forth chapter of the thesis focus on the measurement configuration specified in (II). Main goal of this method is to guarantee the state estimation robustness. State estimation robustness requires presence of a spanning tree, which is formed based on topological observability analysis, to minimize the required number of PMUs. Note that, in a system with PMUs, spanning tree is not a constraint for observability.

The introduced alternative method neglects requirement for a spanning tree while placing PMUs. In general, optimal solution with a spanning tree is encountered at the end of the proposed procedure. However, if a spanning tree cannot be formed by placing PMUs in a configuration that minimum measurement requirement for each bus will be satisfied, additional PMUs should be located. During this step the alternative method will directly place PMUs to obtain a spanning tree, which may not be the optimal solution. One may prefer using this alternative instead of increasing the optimization problem complexity.

Proposed measurement configurations were tested using a LAV State Estimator. For each step, single bad data was given to the measurement set. Obtained estimates were found to be robust against bad data. It is also possible to increase the number of bad measurements for which the state estimator will remain robust by increasing the index number (d). However, this modification will highly increase the investment cost.

Since EMS applications utilize the estimates of a state estimator, robust measurement configuration for a state estimator has a critical importance in modern power system operation. Proposed method guarantees the robustness of the state estimator and during the planning stage, system operators may use the proposed method in order to install PMUs to their systems which have already installed SCADA measurements.

Note that, proposed method is developed for a breakdown point of 1. Required measurement number, which is specified in vector d, should be increased for higher breakdown points.

Branch-PMUs were utilized in the study for simplicity. However, the proposed method is applicable to all channel limited PMUs by modification of the A matrix depending on the channel number of the PMUs.

Computational time of the algorithms were out of the scope of this study, since PMU placement is a one-time process conducted during the planning stage, hence proposed algorithms are independent of the solution time.

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APPENDIX

SIMPLEX BASED ALGORITHM USED FOR LAV STATE ESTIMATION

As it is explained in [1] and [33] , LAV estimation problem can be stated as

$$min \sum_{i=1}^{m} |r_i| \qquad (A.1)$$

$$s.t \quad z_i = h_i(x) + r_i, \quad 1 \leq i \leq m$$

In this formulation z represents the measurement set and z_i becomes i^{th} measurement and $h_i(x)$ is is the measurement function relating state vector x to this measurement. State variables are phase angles and voltage magnitudes.

Assuming an initial solution x^0 for the state and using the first-order approximation of $h_i(x)$ around x^0 , the problem can be expressed as a sucessive set of linear programming problems, each one minimizing the objective function as given below:

$$J(x_k) = \sum_{i=1}^{m} (u_i^k + v_i^k) \qquad (A.2)$$

Where

$u^k - v^k = z - h(x^k) - H(x^k).\Delta x = \Delta z^k - H(x^k).x^k$ represents the measurement residual vector at k^{th} state estimation iteration and H represents the measurement Jacobian.

If superscript k is dropped , by considering the k^{th} iteration problem is formulated as follows.

$$\min \sum_{i=1}^{m} (u_i + v_i)$$

$$s.t \ \ H.\Delta x_u - H.\Delta x_v + u - v = \Delta z \qquad (A.3)$$

$$\Delta x_u , \Delta x_v , u, v \geq 0$$

where $\Delta x = \Delta x_u - \Delta x_v$

This problem can be written in compact form as below.

$$\min c^T.Y$$

$$A.Y = b \qquad (A.4)$$

$$Y \geq 0$$

where

$c^T = [0_n, 0_n, 1_m, 1_m]$

$0_n = [0,.....,0]$, a zero vector size n, all its entries are 0

$1_n = [1,.....,1]$, a vector of size m, all its entries are 1

$b = \Delta z$

$Y^T = [\Delta x_u^T \Delta x_v^T u^T v^T]$

$A = [H \ \ -H \ \ I_m \ \ -I_m]$

I_m , an identity matrix of size m x m

n number of buses

m number of measurements

In the given formulations , the measurement function *h(x)* and *H(x)* matrices have not been defined yet. Their definition will be made below.

For a *n* bus system , the state vector will have *2n*-1 elements. There will be n bus voltages and *n*-1 phase angles. Number of phase angle states being n-1 is due to the assignment of the phase angle of one reference bus arbitrary value , mostly zero. If there is a PMU in a system, global reference time synchronization can be used. The state vector will take the following form if bus-1 is shosen as a reference bus.

$$x^T = [\theta_2\theta_3\theta_4.......V_1V_2V_3.....] \quad \text{(A.5)}$$

Measurement function will use state variables' values in the solution process. The corresponding values of $h_i(x)$ will change in each step.

Considering a π-model of network branches , real and reactive power injections at bus-*i* can be expressed as :

$$P_i = V_i \sum_{j \in N_i} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) \quad \text{(A.6)}$$

$$Q_i = V_i \sum_{j \in N_i} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) \quad \text{(A.7)}$$

Real and reactive power flow from bus-i and bus-j can be written as:

$$P_{ij} = V_i^2 (g_{si} + g_{ij}) - V_i V_j (g_{ij} \cos\theta_{ij} + b_{ij} \sin\theta_{ij}) \quad \text{(A.8)}$$

$$Q_{ij} = -V_i^2 (b_{si} + b_{ij}) - V_i V_j (g_{ij} \sin\theta_{ij} + b_{ij} \cos\theta_{ij}) \quad \text{(A.9)}$$

Line curent flow magnitude from bus-*i* to bus-*j*:

$$I_{ij} = \frac{\sqrt{P_{ij}^2 + Q_{ij}^2}}{V_i} \quad \text{(A.10)}$$

Voltage phasor real and imaginary parts at bus-*i*:

$$Real(\vec{V_i}) = V_i^{m,r} = V_i \cos(\theta_i) \quad \text{(A.11)}$$

$$Imag(\vec{V_i}) = V_i^{m,i} = V_i \sin(\theta_i) \quad \text{(A.12)}$$

Real and imaginary part of the current between bus-*i* and bus-*j*

$$Real(\vec{I_{ij}}) = I_{ij}^{m,r} = g_{ij}(V_i^{m,r} - V_j^{m,r}) - b_{ij}(V_i^{m,i} - V_j^{m,i}) - b_{si} V_i^{m,i} \quad \text{(A.13)}$$

$$Imag(\vec{I_{ij}}) = I_{ij}^{m,i} = g_{ij}(V_i^{m,i} - V_j^{m,i}) - b_{ij}(V_i^{m,r} - V_j^{m,r}) - b_{si} V_i^{m,r} \quad \text{(A.14)}$$

where

$\vec{V}_i$ is the voltage phasor at bus-i

V_i and θ_i are voltage magnitude and phase angle at bus-i

$\theta_{ij} = \theta_i - \theta_j$

$G_{ij} + jB_{ij}$ is the ij^{th} element of the bus admittance matrix

$g_{si} + jb_{si}$ is the admittance of shunt branch directly connected to bus-i

N_i is the set of buses incident to bus-i

The measurement Jacobian, *H*, will have a structure as follows.

$$H = \begin{bmatrix} \frac{\partial P_{inj}}{\partial \theta} & \frac{\partial P_{inj}}{\partial V} \\ \frac{\partial P_{flow}}{\partial \theta} & \frac{\partial P_{flow}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \theta} & \frac{\partial Q_{inj}}{\partial V} \\ \frac{\partial Q_{flow}}{\partial \theta} & \frac{\partial Q_{flow}}{\partial V} \\ \frac{\partial I_{mag}}{\partial \theta} & \frac{\partial I_{mag}}{\partial V} \\ 0 & \frac{\partial V_{mag}}{\partial V} \\ \frac{\partial V_{real}}{\partial \theta} & \frac{\partial V_{real}}{\partial V} \\ \frac{\partial V_{imag}}{\partial \theta} & \frac{\partial V_{imag}}{\partial V} \\ \frac{\partial I_{real}}{\partial \theta} & \frac{\partial I_{real}}{\partial V} \\ \frac{\partial I_{imag}}{\partial \theta} & \frac{\partial I_{imag}}{\partial V} \end{bmatrix} \quad \text{(A.15)}$$