JOINT PRECODER AND DECODER DESIGN IN MULTIUSER DOWNLINK MIMO COMMUNICATIONS FOR COMMON, PRIVATE AND COMMON+PRIVATE INFORMATION TRANSMISSION

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ABSTRACT

JOINT PRECODER AND DECODER DESIGN IN MULTIUSER DOWNLINK MIMO COMMUNICATIONS FOR COMMON, PRIVATE AND COMMON+PRIVATE INFORMATION TRANSMISSION

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The joint precoder/ decoder design in the multiuser downlink multi-input-multi-output (MU-MIMO) communication problem is investigated under three different scenarios. In the first scenario, the same information denoted as common information is delivered to all users, i.e. multicasting scenario. In the second scenario, the base station (BS) transmits independent information bits (private information) to the users, i.e. the conventional communication scenario. Finally, in the third scenario, the BS concurrently delivers both common and private information to the users. The main goal of this thesis is to study precoder/ decoder design for these scenarios. The precoder/ decoder pairs are optimized according to the maximum-minimum fairness (MMF), quality of service (QoS) and minimum total mean square error (TMSE) criteria. In the MMF and QoS problem settings, the optimization problem is non-convex and accomplished by semidefinite relaxation method or another technique known as exact penalty approach. The TMSE minimization is accomplished by sequential design of optimal linear estimator (Wiener filter) at the transmitter and receiver. Simulations with respect to various system parameters such as the number of transmitter and receiver antennas, the number of users and given transmitted power are performed in order to observe the characterization of performance metrics like minimum-mean SNR/SINR, BER or MSE etc. Moreover, the computation times of the

algorithms are also compared in order to evaluate their applicability in practice.

Keywords: multiuser, MIMO communications, downlink, max-min fairness, spectral efficiency, diversity, convex optimization

ÖZ

ÇOK KULLANICILI AŞAĞI BAĞLANTILI MIMO HABERLEŞME TEKNİKLERİNDE ORTAK, ÖZEL VE ORTAK+ÖZEL BİLGİNİN GÖNDERİMİ İÇİN ORTAK ÖN KODLAYICI VE KOD ÇÖZÜCÜ TASARIMI

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Çok kullanıcılı aşağı bağlantılı çok girdili çok çıktılı haberleşme probleminde ortak ön kodlayıcı ve kod çözücü tasarımı üç farklı senaryo altında incelenmiştir. İlk senaryoda yani çoklu gönderim durumunda, ortak bilgi olarak adlandırılan aynı bilgi tüm kullanıcılara gönderilir. Geleneksel haberleşme senaryosu olan ikinci senaryoda baz istasyonu bağımsız bilgi bitlerini (özel bilgi) kullanıcılara gönderir. Son olarak üçüncü senaryoda baz istasyonu aynı anda ortak ve özel bilgileri kullanıcılara gönderir. Bu tezin temel amacı bu senaryolar için ön kodlayıcı ve kod çözücü tasarımı yapmaktır. Ön kodlayıcı ve kod çözücü ikilileri en büyük-en küçük eşitliği (MMF), hizmet kalitesi (QoS) ve en küçük toplam ortalama karesel hata (TMSE) kriterlerine göre optimize edilmiştir. MMF ve QoS problemlerinde, optimizasyon problemi konveks değildir ve bu problem yarı tanımlı rahatlama yöntemi veya tam ceza yöntemi olarak bilinen diğer bir metotla çözülür. TMSE minimizasyonu gönderici ve alıcıda ardışık optimum lineer kestirici (Wiener filtre) yöntemiyle yapılır. Gönderici ve alıcıdaki anten sayıları, kullanıcı sayısı ve göndericinin toplam gücü gibi parametrelere göre minimum ve ortalama sinyal-gürültü veya sinyal-gürültü artı girişim oranı, bit hata oranı ve en küçük karesel hata gibi değişkenlerin karakteristiğini gözlemlemek amacıyla

simülasyonlar yapılmıştır. Ek olarak, algoritmaların hesaplama zamanları gerçek hayattaki uygulanabilirliklerini değerlendirmek amacıyla karşılaştırılmıştır.

Anahtar Kelimeler: çok kullanıcı, çok girişli çok çıkışlı haberleşme, aşağı bağlantı, en büyük en küçük eşitliği, spektral verimlilik, çeşitlilik, konveks optimizasyon

To my dear family and Fehmi...

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LIST OF ABBREVIATIONS

BER	Bit Error Ratio
BS	Base Station
CCI	Co-Channel Interference
CDMA	Code Division Multiple Access
CSI	Channel State Information
EPA	Exact Penalty Approach
FDMA	Frequency Division Multiple Access
KKT	Karush-Kuhn Tucker
MIMO	Multi Input Multi Output
MMF	Max-Min Fairness
MSE	Mean Square Error
MU	Multiuser
PSD	Positive Semidefinite
QCQP	Quadratically Constrained Quadratic Programming
QoS	Quality of Service
SeDuMi	Self Dual Minimization
SDP	Semidefinite Programming Tool
SDR	Semidefinite Relaxation
SINR	Signal-to-Interference-plus-Noise Ratio
SNR	Signal-to-Noise Ratio
SVD	Singular Value Decomposition
TDMA	Time Division Multiple Access
TMSE	Total Mean Square Error
UDD	Uplink-Downlink Duality

CHAPTER 1

INTRODUCTION

In recent years, for wireless communication area, MIMO standing for multi-input multi-output technologies has gained considerable attention as a natural consequence of developments on antenna arrays. Thanks to the MIMO structure, which includes multiple antennas at both transmitter and receiver side, it is possible to transmit many data symbols per channel use [1]. In addition, adding every antenna at transmitter or receiver leads to increase data throughput of the system. As wireless technology improves day by day, there is an increasing demand on allocation of radio spectrum which makes it more valuable and expensive. Therefore, the efficient usage of spectral bandwidth is an important issue leading to place emphasis on MIMO communications. In today's world, many wireless systems benefit from MIMO technology, such as IEEE 802.11n, WiFi, WiMaX, LTE and for future LTE Advanced which are all aimed at high-quality communications [2]-[3].

MIMO communication systems have a lot of advantages in many aspects. The first one is that MIMO structure provides robustness to channel fading. In other words, due to multipath propagation, the copies of the signal reflected from different points may cancel out each other; therefore, this causes to decrease the signal-to-noise ratio (SNR) at the receiver which implies an increase in the number of erroneous bit transmissions. However, by using multiple antennas at the transmitter and receiver, the same signal is transmitted many times over different channel characteristics. By combining the same signal passing from different channels with a proper receiver structure, the probability of signal received cor-

rectly can be increased [4]. This is also known as diversity in the literature. The main reason of the usage of diversity is to provide high reliability. The second advantage is related with MIMO array gain. By transmitting the same signal from many antennas and coherently combining them, the received signal power is amplified by an amount called the array gain [5]. Third and the most important advantage is related with spatial multiplexing [6]. Thanks to multiple antenna structure, one can transmit different data streams simultaneously and also at the same frequency. Contrary to the conventional multiple access techniques such as TDMA or FDMA both of which cause decreasing in spectral efficiency, MIMO provides a prominent advantage at this point, since information bits can be sent at the same time allocating the same bandwidth [7].

There are two types of MIMO communications called as uplink MIMO and downlink MIMO. In downlink MIMO, there is a base station (BS) which transmits the information symbols to the receiver side via a channel. On the other hand, in uplink MIMO communications, the data symbols are transmitted from receiver side to the base station. Both uplink and downlink modes can serve for single user (SU-MIMO) or more than one user (MU-MIMO). In [8]-[10], various single user downlink MIMO techniques based on V-BLAST architecture, SVD techniques etc. are studied. This thesis is mainly concentrated on the solution of multi user downlink MIMO problem.

As well as its significant advantages, there exist some major drawbacks of MIMO communications. One of the most important disadvantage is co-channel interference (CCI) [11]. As explained previously, each user information symbol is transmitted simultaneously by multi antennas and this causes interference problem at the receiver of them. Another disadvantage is related with the channel estimation errors [12]. In practice, the channel can be known with some error. Even if the channel estimation error is small, the precoder-decoder design can be significantly affected due to the interference term. Some robust algorithms can be used in order to decrease performance degradation of MIMO systems due to the channel estimation error [13]- [15].

In order to cope with the interference problem, beamforming strategies, namely

the precoding process at the transmitter side and the decoding process at the receiver side can be applied. In addition to the interference elimination purposes, precoder and decoder blocks can also be used for allocating transmitter power efficiently in order to increase the number of bits which are correctly received. In literature, the optimization of precoder-decoder blocks is analyzed with several cost functions for different MIMO communication scenarios. For the multicasting scenario, where all users receive the same information, the precoder design attracts considerable attention since it ensures to increase the spectral efficiency for highly demanded spectrum regions. Contrary to the traditional approach, namely transmitting the signal isotropically, it is possible to design a precoder aiming to maximize the minimum SNR in the system called the Max-Min Fairness (MMF) problem [16]. However, the MMF problem has non-convex rank-one constraint which might pose difficulties in the optimization. Semi-definite relaxation (SDR) method can be applied in order to solve the problem by using SeDuMi which is a semi-definite programming (SDP) tool [17]. Instead of omitting rank-one constraint directly, [18] tries to solve the MMF problem by using exact penalty approach. This method is superior to SDR, when solution of SeDuMi is not rank-one. In [19]-[20], solution of the MMF problem explained in multicasting scenario is generalized for interfering MIMO broadcast channel. Equivalence of MMF and quality of service (QoS) problems is proven in [19], which is used as the main idea in the solution of the MMF problem for MIMO interfering channel. In [21], the aim is minimizing the total mean square error (MSE) in the system by using transmit precoder and receive decoder filters based on the interference alignment idea. An iterative algorithm is presented in [22] for joint precoder-decoder design to minimize the total MSE. Similarly, in [23], joint precoder-decoder beamforming is achieved but with equally divided per user transmitter power constraint which makes it a suboptimal approach. Precoder design part is based on Schur-Concave optimization problem [24]. Due to the long computation time of SDP tools for the MMF problem, a novel solution method based on uplink-downlink duality principle is presented in [25]. Moreover, in [26], a decomposition method which converts the MU-MIMO channel to many parallel SU-MIMO channels. For this method, precoder and decoder are optimized jointly but there is a tradeoff between the receiver diversity and

the number of transmitter antennas due to null space requirements to eliminate interference completely.

In addition to multicasting and interfering broadcast MIMO channel, there exists some studies on the MIMO broadcasting channel with common and private messages. In general, this subject is analyzed from the information theoretic viewpoint in [27]-[28].

In this thesis, mainly precoder and decoder optimization in the downlink multiuser MIMO communication problem is analyzed for different cost functions under three different scenarios. In the first scenario, optimization is done over three types of cost function, namely maximizing the minimum SNR (MMF), minimizing the total MSE and maximizing the total SNR value in the system. All users receive the same information from the transmitter side, which is known as multicasting in the literature. Therefore, there is no interference in the medium. By using efficient precoding-decoding blocks, according to the CSI, the total transmitter power can be efficiently allocated to increase the spectral efficiency significantly. In the second scenario, each user receives different information called the MIMO interfering broadcasting. In this part, again MMF and minimization of total MSE problems are investigated. Different from the previous one, here there is an interference problem between users, which causes decreasing the signal-to-interference plus noise ratio (SINR). Similarly, by designing precoder and decoder blocks, power allocation and elimination of interference is tried to be achieved as much as possible. Finally, the third scenario is the combination of the previous two, in which the BS transmits one common information and private information of all users together. At the receiver side, each user receives two pieces of information, one of them is the common one and the other is private information. The solution of the MMF and total MSE minimization problems under this scenario are provided. Performance of different precoder-decoder design algorithms for various parameters is analyzed by MATLAB simulations. Furthermore, SeDuMi (Self-Dual Minimization) optimization tool is used for the solution of the MMF problem, details of which are given in the thesis.

Thesis Organization

In this part, a brief organization of this thesis is provided.

In **Chapter 2**, the solution of downlink MU-MIMO communication problem for the multicasting scenario in which a BS transmits a common information to each user is investigated. Precoder-decoder design algorithms for the MMF, minimization of total MSE and maximization of total SNR problems are studied in detail.

In **Chapter 3**, precoder-decoder design for broadcasting downlink MU-MIMO communication problem in which each user receives a private information is described. As in Chapter 2, the MMF and total MSE minimization are analyzed by using different kinds of beamforming algorithms.

In **Chapter 4**, MU-MIMO downlink communication is studied for a combined scenario of Chapter 2 and Chapter 3, in which each user receives one private information and one common information transmitted for all users. Adaptation of precoder-decoder design algorithms existing in the literature and a new design method are analyzed for this scenario.

In **Chapter 5**, MATLAB simulation results for all these three scenarios are provided. Performance comparison of different precoder-decoder design algorithms with respect to different parameters is presented.

In **Chapter 6**, concluding remarks of the thesis are given.

Thesis Contributions:

1) Study of private and common information transmission in MU-MIMO communications.

- **a)** Adaptation of SDR, EPA and Joint-TMSE methods to this scenario.
- **b)** Development of a new method for joint precoder-decoder optimization.

2) Improving SVD-Dec method expressed in [26], in terms of the constraint about the number of transmitter antennas.

3) Presenting a new deterministic method (detA) as an alternative for the randomization part used after semidefinite relaxation part in the MMF problem.

Notation: Throughout this thesis, while boldface lowercase letters represent vectors, boldface uppercase letters represent matrices. $\|\cdot\|_1$, $\|\cdot\|_2$, $\|\cdot\|_F$, $tr(\cdot)$ and $rank(\cdot)$ denotes the l_1 norm, l_2 norm, Frobenius norm, rank and trace respectively. $vec(.)$ implies the transformation of an $M \times N$ matrix into a column vector with $MN \times 1$. $(\cdot)^*$ and $(\cdot)^H$ represent the complex conjugate and Hermitian matrix operators. $(\cdot)_1$ and $(\cdot)_2$ denote the first and second rows of a matrix respectively. By $\boldsymbol{A} \succeq 0$, it is implied that $\boldsymbol{A}$ is a positive semidefinite matrix. Lastly, $\boldsymbol{x} \geq 0$ means that $[\boldsymbol{x}]_l > 0$ for all indices l.

CHAPTER 2

MULTIUSER MIMO DOWNLINK COMMUNICATION PROBLEM WITH COMMON INFORMATION (MULTICAST BEAMFORMING)

In this chapter, a wireless downlink communication problem with a base station (BS) serving a common information to multiple receivers simultaneously is considered. There are several practical applications for this scenario, for example, streaming of a soccer match or a popular video. In such applications, the same information is transmitted to all receivers at the same time within a certain cell area. In this part of the work, the main aim is offering efficient solutions to reach better spectral efficiency values. As stated in [29], traditionally, the common information is transmitted by radiating equal power isotropically to all users. However, by using efficient algorithms, power can be allocated among users in an efficient manner by making beamforming both at the transmitter and receiver side with the knowledge of channel state information (CSI) at transmitter and receiver side. In literature, this method is known as multicasting and it has taken considerable attention in recent years due to its efficient solutions in terms of spectral efficiency to deal with overwhelming traffic demands. Contrary to other MU-MIMO communication problems, there is no interference between users since the same information is transmitted to every user. Here, the key point in precoder and decoder design is enabling the efficient usage of transmission power.

System model for multicasting scenario is defined in Section 2.1. Making efficient power allocation with the help of precoder-decoder blocks under three types of

cost function is the main goal of this chapter. With this regard, firstly the MMF problem is analyzed in Section 2.2. Utilization of SDP tools for the solution of this problem is studied. Next, Section 2.3 is concentrated on precoder-decoder optimization method for the minimization of total MSE problem. Finally, in Section 2.4, total SNR maximization problem is presented as the third type of cost function. Theoretical explanations for optimization strategies are given in detailed manner.

2.1 System Model

In multiuser MIMO downlink scenario, in which a common information is sent to all users, assume that at the transmitter side there is a BS with M antenna elements. At the receiver side, K users are located. Receiver of the ith user has N_i antennas. Throughout this chapter, it is assumed that each user has the same amount of antennas at receiver denoted by N. Each of them receives the common information signal denoted by b which is a zero mean and unity variance uniformly distributed random variable. For the ith user, transmitted signal passes through a flat-fading channel matrix $\boldsymbol{H_i}$ with dimension $N \times M$ whose entries are zero-mean, unity variance complex Gaussian random variables. Channel state information (CSI) is assumed to be known perfectly both at the transmitter and receiver sides. Before the transmission of the signal, $M \times 1$ beamforming precoder vector $\boldsymbol{p}$ is multiplied with information bearing signal. At the receiver of the ith user, $\boldsymbol{n_i}$ denotes the $N \times 1$ vector whose entries are the additive white Gaussian noise with zero mean and variance N_0 and $\boldsymbol{d_i}$ denotes $N \times 1$ decoder vector. System configuration of this scenario is provided in Figure 2.1. As a result, the received signal for the ith user r_i can be expressed in the following form:

$$r_i = \boldsymbol{d_i}^H \boldsymbol{H_i} \boldsymbol{p} b + \boldsymbol{d_i}^H \boldsymbol{n_i} \quad i = 1, 2, \ldots, K. \tag{2.1}$$

In this chapter, mainly solution of three types of optimization problem will be analyzed, namely Max-Min Fairness (MMF) Problem, minimization of total MSE, and maximization of total SNR problem. Design of beamforming vectors in order to increase SNR or decrease MSE in the system could be efficient method

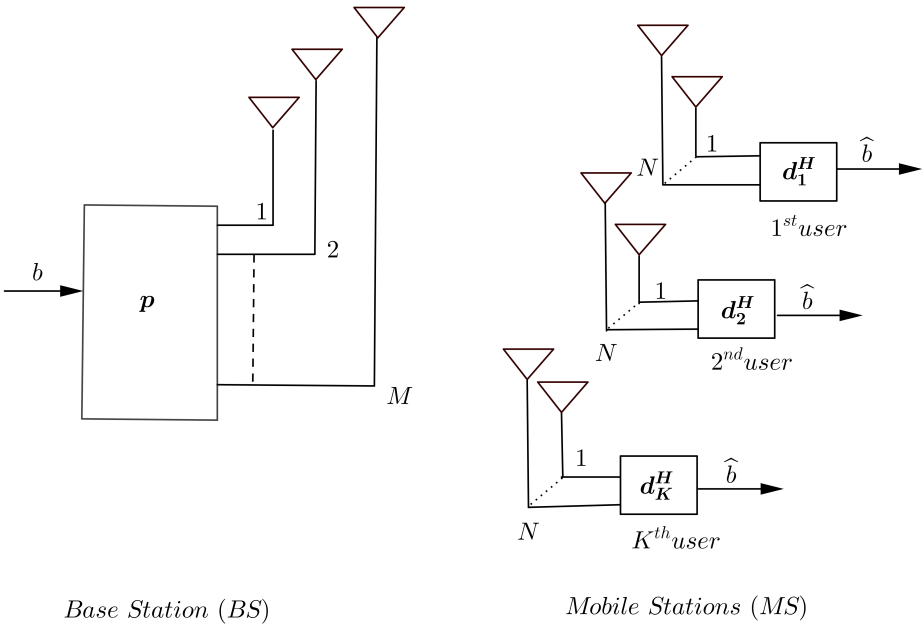

Figure 2.1: System Model for Transmitting Common Information Scenario

to obtain low BER values. The detailed analysis of solution of all these are given in the following parts.

Before starting the analysis of the problems, it should be noted that precoder and decoder design is done jointly in all parts of this thesis work. In addition, in order to simplify the optimization, sequential optimization approach is used. In other words, precoder and decoder vectors are optimized sequentially by keeping one of them fixed and optimizing over the other one.

2.2 Max-Min Fairness (MMF) Beamforming Problem

In the max-min fairness problem, the aim is maximizing the minimum SNR in the system under total power constraint denoted by P_{tot} at the transmitter side. As expressed, since $\mathbb{E}\left\{|b|^2\right\} = 1$, MMF problem could be represented in a mathematical form such that

$$MMF: \max_{\boldsymbol{p}\in\mathbb{C}^M, \boldsymbol{d_1}, \boldsymbol{d_2}, \ldots, \boldsymbol{d_K}\in\mathbb{C}^N} \min_i \left\{ \frac{\left\|\boldsymbol{d_i}^H \boldsymbol{H_i} \boldsymbol{p}\right\|^2}{N_0 \left\|\boldsymbol{d_i}\right\|^2} \right\} \text{ for } i = 1, 2, \ldots, K$$

$$\text{s.t. } \|\boldsymbol{p}\|^2 \leq P_{tot}. \tag{2.2}$$

Since SNR is an increasing function of $\|\boldsymbol{p}\|$, the optimum solution satisfies this inequality constraint with equality. Therefore, the optimization problem turns into

$$MMF: \max_{\boldsymbol{p}\in\mathbb{C}^M, \boldsymbol{d_1}, \boldsymbol{d_2}, \ldots, \boldsymbol{d_K}\in\mathbb{C}^N} \min_i \left\{ \frac{\left\|\boldsymbol{d_i}^H \boldsymbol{H_i} \boldsymbol{p}\right\|^2}{N_0 \left\|\boldsymbol{d_i}\right\|^2} \right\} \text{ for } i = 1, 2, \ldots, K$$

$$\text{s.t. } \|\boldsymbol{p}\|^2 = P_{tot}. \tag{2.3}$$

By using the definition of norm, which is

$$\left\|\boldsymbol{d_i}^H \boldsymbol{H_i} \boldsymbol{p}\right\|^2 = \operatorname{tr}\left(\boldsymbol{d_i}^H \boldsymbol{H_i} \boldsymbol{p}\boldsymbol{p}^H \boldsymbol{H_i}^H \boldsymbol{d_i}\right) = \operatorname{tr}\left(\boldsymbol{p}\boldsymbol{p}^H \boldsymbol{H_i}^H \boldsymbol{d_i} \boldsymbol{d_i}^H \boldsymbol{H_i}\right), \tag{2.4}$$

we define

$$\widetilde{\boldsymbol{R}}_{\boldsymbol{i}} = \frac{\boldsymbol{H_i}^H \boldsymbol{d_i} \boldsymbol{d_i}^H \boldsymbol{H_i}}{N_0 \boldsymbol{d_i}^H \boldsymbol{d_i}} \tag{2.5}$$

where $\boldsymbol{X} = \boldsymbol{p}\boldsymbol{p}^H$, that is $\boldsymbol{X}$ is positive semidefinite and $\operatorname{rank}(\boldsymbol{X}) = 1$. As a result, the MMF problem can be expressed as in the following:

$$\begin{aligned} \max_{\boldsymbol{X}\in\mathbb{C}^{M\times M}, \boldsymbol{d_1}, \boldsymbol{d_2}, \ldots, \boldsymbol{d_K}\in\mathbb{C}^N} \min_i \operatorname{tr}\left(\boldsymbol{X}\widetilde{\boldsymbol{R}}_{\boldsymbol{i}}\right) &\text{ for } i = 1, 2, \ldots, K \\ \text{s.t.} \quad \operatorname{tr}(\boldsymbol{X}) &= P_{tot} \\ \operatorname{rank}(\boldsymbol{X}) &= 1 \\ \boldsymbol{X} &\succeq 0. \end{aligned} \tag{2.6}$$

As can be noted from the expressions above, this problem is a quadratically constrained quadratic problem (QCQP). The cost function is a linear function of $\boldsymbol{X}$; however, it has a non-convex rank-one constraint. This makes the optimization problem NP-hard, the detailed expressions and the proof can be found in [16]. Due to this non-convexity, the optimization problem cannot be solved directly with the help of semidefinite programming solvers such as SeDuMi or CVX, etc. However, by making some modifications, these solvers can still be used. In this thesis work, two methods called "Semidefinite Relaxation (SDR)" and "Exact Penalty Approach (EPA)" are used in order to make the problem solvable by using SDP tools. The decoder design problem is a relatively simpler problem given in the next part.

Decoder Design

For a given precoder vector $\boldsymbol{p}$, the SNR expression for the ith user can be written as

$$\begin{aligned} SNR_i &= \frac{\left\|\boldsymbol{d_i}^H \boldsymbol{H_i} \boldsymbol{p}\right\|^2}{N_0 \left\|\boldsymbol{d_i}\right\|^2} \\ &= \frac{\boldsymbol{d_i}^H \boldsymbol{H_i} \boldsymbol{p} \boldsymbol{p}^H \boldsymbol{H_i}^H \boldsymbol{d_i}}{\boldsymbol{d_i}^H \left(N_0 \boldsymbol{I}\right) \boldsymbol{d_i}}. \end{aligned} \tag{2.7}$$

Let $\boldsymbol{U_1} = \boldsymbol{H_i} \boldsymbol{p} \boldsymbol{p}^H \boldsymbol{H_i}^H$ and $\boldsymbol{U_2} = N_0 \boldsymbol{I}$. Then the SNR_i becomes

$$SNR_i = \frac{\boldsymbol{d_i}^H \boldsymbol{U_1} \boldsymbol{d_i}}{\boldsymbol{d_i}^H \boldsymbol{U_2} \boldsymbol{d_i}}. \tag{2.8}$$

which is in the form of well-known Rayleigh quotient. Therefore, the optimum unit norm decoder vector which maximizes the SNR expression for the ith user is equal to the generalized unit norm principal eigenvector of $\boldsymbol{U_1}$ and $\boldsymbol{U_2}$. There is no need to scale the decoder vector since both the numerator and denominator include this term; therefore, the scaling is arbitrary.

Precoder Design

When decoder vectors for each user is known, under a total transmitter power constraint, precoder design could be achieved with different optimization approaches. Two of them will be given in a detailed manner.

2.2.1 Semidefinite Relaxation (SDR) Method

In fact, the MMF problem has a linear cost function and except for rank-one constraint all of its constraints are convex. Therefore, by omitting rank-one constraint called as "Semidefinite Relaxation", one can solve it with the help of SDP tools [16]. Then, the problem becomes

$$\begin{aligned} \max_{\boldsymbol{X} \in \mathbb{C}^{M \times M}} \min_i \operatorname{tr}\left(\boldsymbol{X} \widetilde{\boldsymbol{R}}_{\boldsymbol{i}}\right) \text{ for } i = 1, 2, \ldots, K \\ \text{s.t. } \operatorname{tr}\left(\boldsymbol{X}\right) = P_{tot} \\ \boldsymbol{X} \succeq 0. \end{aligned} \tag{2.9}$$

Let $\min_i \operatorname{tr}\left(\boldsymbol{X} \widetilde{\boldsymbol{R}}_{\boldsymbol{i}}\right) = u$. Then it could be written that

$$\operatorname{tr}\left(\boldsymbol{X} \widetilde{\boldsymbol{R}}_{\boldsymbol{i}}\right) \geq u \text{ for } i = 1, 2, \ldots, K. \tag{2.10}$$

As a result, the MMF problem can be written as

$$\max_{\boldsymbol{X}\in\mathbb{C}^{M\times M}} u$$

$$\text{s.t} \quad \text{tr}\left(\boldsymbol{X}\widetilde{\boldsymbol{R}}_{\boldsymbol{i}}\right) \geq u \text{ for } i = 1, 2, \ldots, K$$

$$\text{tr}\left(\boldsymbol{X}\right) = P_{tot} \tag{2.11}$$

$$\boldsymbol{X} \succeq 0.$$

Throughout this thesis, SeDuMi is used as SDP tool in MATLAB simulations. SeDuMi which stands for Self-Dual Minimization is a kind of toolbox working on MATLAB, which solves optimization problems with linear, quadratic and semidefinite constraints [17]. In order to use it, it is necessary to formulate the optimization problem as a standard form as given below.

$$\min \boldsymbol{c}^T\boldsymbol{x}$$

$$\text{s.t} \quad \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \text{ where } \boldsymbol{x}_{\boldsymbol{i}} \geq 0 \text{ for } i = 1, 2, \ldots, n, \tag{2.12}$$

where $\boldsymbol{x}$ is a vector of decision variables, $\boldsymbol{A}$ is a matrix, $\boldsymbol{b}$ and $\boldsymbol{c}$ are vectors.

Before solving the optimization problem, for standard form of SeDuMi, adding slack variables denoted by $t_1, t_2, \ldots, t_K$, converting K inequalities into equalities is required. Furthermore, maximization criteria in the cost function should be modified since SeDuMi finds the optimum solution over minimization criteria. At the end, by using the following lemma

$$\text{tr}\left(\boldsymbol{A}^T\boldsymbol{B}\right) = vec\left(\boldsymbol{A}\right)^T vec\left(\boldsymbol{B}\right), \tag{2.13}$$

the problem takes the standard form of

$$\min_{\boldsymbol{X}\in\mathbb{C}^{M\times M}, u, t_i\in\mathbb{R}} -u$$

$$s.t. -u - t_i + vec\left(\widetilde{\boldsymbol{R}}_{\boldsymbol{i}}^T\right)^T vec\left(\boldsymbol{X}\right) = 0,\ t_i \geq 0, \quad \text{for } i = 1, 2, \ldots, K$$

$$vec\left(\boldsymbol{I}\right)^T vec\left(\boldsymbol{X}\right) = P_{tot} \tag{2.14}$$

$$\boldsymbol{X} \succeq 0, u \geq 0.$$

After this point, SeDuMi could be directly used in MATLAB via command **sedumi**$(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c})$ where for transmitting common information scenario $\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}$ are equal to

$$\boldsymbol{A} = \begin{bmatrix} -1 & -1 & 0 & \dots & 0 & vec\left(\widetilde{\boldsymbol{R}}_{\boldsymbol{1}}^T\right)^T \\ -1 & 0 & -1 & \dots & 0 & vec\left(\widetilde{\boldsymbol{R}}_{\boldsymbol{2}}^T\right)^T \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & 0 & 0 & \dots & -1 & vec\left(\widetilde{\boldsymbol{R}}_{\boldsymbol{K}}^T\right)^T \\ 0 & 0 & 0 & \dots & 0 & vec\left(\widetilde{\boldsymbol{I}}\right)^T \end{bmatrix}$$

$$\boldsymbol{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ P_{tot} \end{bmatrix}, \quad \boldsymbol{c} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} u \\ t_1 \\ t_2 \\ \vdots \\ t_K \\ vec\left(\boldsymbol{X}\right) \end{bmatrix}, \tag{2.15}$$

where $\boldsymbol{A}$ is $M^2\left(K+1\right) \times M^2\left(K+1\right)$ matrix, $\boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{x}$ are $M^2\left(K+1\right) \times 1$ vectors.

SeDuMi gives the optimum solution of $\boldsymbol{X}$ denoted by $\boldsymbol{X_{opt}}$. After finding $\boldsymbol{X_{opt}}$, precoder vector $\boldsymbol{p}$ can be found with various methods. The solution of $\boldsymbol{X_{opt}}$ may not be rank-one because of relaxation. If $\boldsymbol{X_{opt}}$ is rank-one, then precoder vector $\boldsymbol{p}$ is directly equal to the scaled version of principal eigenvector of $\boldsymbol{X_{opt}}$. For the case of being $\boldsymbol{X_{opt}}$ not a rank-one matrix, some other techniques should be applied. In this thesis, two types of method called as randA and detA methods are applied on $\boldsymbol{X_{opt}}$ in order to find precoder vector $\boldsymbol{p}$.

Randomization (randA) Method

In convex optimization theory, randomization is a very commonly used technique. In this thesis, a randomization method based on eigenvalue decomposition is studied. As expressed in [16], in order to find the precoder vector $\boldsymbol{p}$, one should make eigenvalue decomposition of $\boldsymbol{X_{opt}} = \boldsymbol{E\Lambda E}^H$ and choose $\boldsymbol{p} = \boldsymbol{E\Lambda}^{1/2}\boldsymbol{v}$ where entries of $\boldsymbol{v}$ are random variables in the form of $e^{j\theta_i}$ for $i = 1, 2, \dots, M$ and $\theta_i's$ are independently and uniformly distributed on the unit circle $[0, 2\pi)$. This randomization method guarantees that $\mathrm{tr}\left(\boldsymbol{X_{opt}}\right) = \mathrm{tr}\left(\boldsymbol{pp}^H\right)$.

Furthermore, it could also be said that $\boldsymbol{X_{opt}} = \mathbb{E}\left\{\boldsymbol{p}\boldsymbol{p}^H\right\}$.

Deterministic Min-Norm Solution (detA) Method

As expressed above, the solution of SeDuMi, i.e. $\boldsymbol{X_{opt}}$, is not required to be rank-one due to relaxation process. Then, one cannot write the equality $\boldsymbol{X_{opt}} = \boldsymbol{p}\boldsymbol{p}^H$. However, at this point, defining an another optimization problem to find precoder vector $\boldsymbol{p}$ from $\boldsymbol{X_{opt}}$ is an efficient approach. In other words, one can solve the below optimization problem written as

$$\min_{p} \left\|\boldsymbol{X_{opt}} - \boldsymbol{p}\boldsymbol{p}^H\right\|^2 \tag{2.16}$$

$$\text{s.t} \quad \text{tr}\left(\boldsymbol{p}\boldsymbol{p}^H\right) = P_{tot}, \tag{2.17}$$

to find the precoder vector $\boldsymbol{p}$.

According to the Karhunen-Loeve Theorem, the solution of Equation 2.16 is the principal eigenvector scaled with $\sqrt{\lambda_{max}}$ where λ_{max} is the principal eigenvalue of $\boldsymbol{X_{opt}}$. In our case, due to the total transmitter power constraint, the principal eigenvector of $\boldsymbol{X_{opt}}$ scaled with a term which satisfies the total precoder power. Furthermore, if $\boldsymbol{X_{opt}}$ is rank-one, then randA method provides a precoder vector $\boldsymbol{p}$ which is the principal eigenvector of $\boldsymbol{X_{opt}}$ scaled with unit norm term. However, it already provides exactly the same SNR value with the one solved by detA method. Therefore, SNR performances of these two methods are exactly the same provided that $\boldsymbol{X_{opt}}$ is rank-one.
The precoder-decoder optimization process continues until the minimum SNR does not change anymore with iterations.

2.2.2 Exact Penalty Approach (EPA)

In the previous part, solution of the MMF problem with non-convex constraints are found by using the SDR method. In the SDR method, it is not guaranteed that the solution found by the SDP tool will be a rank-one. According to the obtained simulation results given in Table 5.2, the solution may not be found rank-one, especially when the number of users is large. Therefore, dropping rank-one constraint may cause to obtain suboptimal solutions.

There is another solution method in literature for the solution of the MMF problem. It is based on "Exact Penalty Approach" which is a common method used in optimization theory expressed in detail in [30]. However, before using this approach, the problem should be converted into an equivalent form. For the statement of equivalent problem, Lemma 2.1 given below is required.

Lemma 2.1: [29]

$$\operatorname{tr}(\boldsymbol{X_1}\boldsymbol{X_2}) \leq \operatorname{tr}(\boldsymbol{X_1})\operatorname{tr}(\boldsymbol{X_2}), \tag{2.18}$$

where $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ are $M \times M$ Hermitian symmetric matrices and $\boldsymbol{X_1} \succeq 0, \boldsymbol{X_2} \succeq 0$. The equality condition is satisfied when both $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ are rank-one matrices and $\boldsymbol{X_2} = c\boldsymbol{X_1}$ for some $c > 0$.

Proof of Lemma 2.1:

First of all, as given in [31], Von-Neumann Inequality states that

$$\operatorname{tr}(\boldsymbol{X_1}\boldsymbol{X_2}) \leq \sum_{i=1}^{M} a_i b_i, \tag{2.19}$$

where a_i and b_i are eigenvalues of $\boldsymbol{X_1} \succeq 0$ and $\boldsymbol{X_2} \succeq 0$ respectively and they are sorted in increasing order $0 \leq a_1 \leq a_2 \leq \cdots \leq a_M$ and $0 \leq b_1 \leq b_2 \leq \cdots \leq b_M$.

From trace properties one can write that

$$\operatorname{tr}(\boldsymbol{X_1}) = \sum_{i=1}^{M} a_i$$

$$\operatorname{tr}(\boldsymbol{X_2}) = \sum_{j=1}^{M} b_j. \tag{2.20}$$

Therefore, the claim can be shown if the following expression can be proven.

$$\operatorname{tr}\{\boldsymbol{X_1}\boldsymbol{X_2}\} \leq \sum_{i=1}^{M} a_i b_i \leq \operatorname{tr}(\boldsymbol{X_1})\operatorname{tr}(\boldsymbol{X_2}).$$

Since,

$$\operatorname{tr}(\boldsymbol{X_1})\operatorname{tr}(\boldsymbol{X_2}) = \sum_{i=1}^{M} a_i \sum_{j=1}^{M} b_j = \left(\sum_{i=1}^{M} a_i b_i\right) + \underbrace{\left(\sum_{i \neq j} a_i b_j\right)}_{\geq 0}. \tag{2.21}$$

In order to satisfy the equality case, it should be noticed that at least one eigenvalue of $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ should be non-zero. If all a_i or b_i values are equal to zero for $i = 1, 2...M$, then $\boldsymbol{X_1}$ or $\boldsymbol{X_2}$ become zero matrices which is the trivial case. Therefore, for the non-trivial case it can be assumed that $a_M > 0$ and $b_M > 0$. Furthermore, for the equality, $\sum_{i \neq j}^{M} a_i b_j$ term should be equal to zero meaning that a_i values for $i = 1, 2, ...M - 1$ are all zero. A similar claim can also be made for b_i values, namely b_i values for $i = 1, 2, ...M - 1$ are all zero. As a result, each of the $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ matrices has only one non-zero eigenvalue which implies that both $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ are rank-one matrices.

Therefore, $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ could be written in eigendecomposition form such that

$$\boldsymbol{X_1} = a_M \boldsymbol{u}\boldsymbol{u}^{\boldsymbol{H}}, \boldsymbol{X_2} = b_M \boldsymbol{v}\boldsymbol{v}^H, \tag{2.22}$$

where $\boldsymbol{u}$ and $\boldsymbol{v}$ are unit norm eigenvectors corresponding to the largest eigenvalues (i.e a_M and b_M) of $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ respectively. As a result of the equality case of Equation 2.18,

$$\operatorname{tr}(\boldsymbol{X_1}\boldsymbol{X_2}) = a_M b_M \operatorname{tr}\left(\boldsymbol{u}\boldsymbol{u}^H \boldsymbol{v}\boldsymbol{v}^H\right) = a_M b_M \rightarrow \operatorname{tr}\left(\boldsymbol{v}^H \boldsymbol{u}\boldsymbol{u}^H \boldsymbol{v}\right) = 1 \rightarrow \left\|\boldsymbol{u}^H \boldsymbol{v}\right\| = 1. \tag{2.23}$$

According to the Cauchy-Schwarz Inequality,

$$\left\|\boldsymbol{u}^H \boldsymbol{v}\right\| \leq \|\boldsymbol{u}\| \|\boldsymbol{v}\|. \tag{2.24}$$

Since $\boldsymbol{u}$ and $\boldsymbol{v}$ are unit norm vectors, $\left\|\boldsymbol{u}^H \boldsymbol{v}\right\| \leq 1$.

From the result of $\left\|\boldsymbol{u}^H \boldsymbol{v}\right\| = 1$, one can say that Cauchy-Schwarz Inequality is satisfied with equality. This is possible when $\boldsymbol{u} = \alpha \boldsymbol{v}$. At the end, $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ can be represented as

$$\boldsymbol{X_1} = a_M \boldsymbol{u}\boldsymbol{u}^H, \boldsymbol{X_2} = \frac{1}{|\alpha|^2} b_M \boldsymbol{u}\boldsymbol{u}^H. \tag{2.25}$$

Therefore, $\boldsymbol{X_1} = c\boldsymbol{X_2}$ where $c = |\alpha|^2 \frac{a_M}{b_M}$. ■

Now, we define an equivalent problem, whose solution is the same with the original MMF problem.

Equivalent Optimization Problem:

$$\max_{\boldsymbol{X_1}\in\mathbb{C}^{M\times M},\boldsymbol{X_2}\in\mathbb{C}^{M\times M}} \alpha_1+\alpha_2$$

$$\text{s.t. } \operatorname{tr}\left(\tilde{\boldsymbol{R}}_{\boldsymbol{i}}\boldsymbol{X_1}\right)\geq\alpha_1 \quad \text{for} \quad i=1,2,...K$$

$$\operatorname{tr}\left(\tilde{\boldsymbol{R}}_{\boldsymbol{i}}\boldsymbol{X_2}\right)\geq\alpha_2 \quad \text{for} \quad i=1,2,...K$$

$$\operatorname{tr}\left(\boldsymbol{X_1}\right)=\operatorname{tr}\left(\boldsymbol{X_2}\right)=P_{tot} \quad \text{for} \quad i=1,2,...K \tag{2.26}$$

$$\operatorname{tr}\left(\boldsymbol{X_1}\boldsymbol{X_2}\right)\geq P_{tot}^2$$

$$\boldsymbol{X_1}\succeq 0, \boldsymbol{X_2}\succeq 0.$$

Claim 2.1: [**29**] Solutions of original MMF problem and equivalent MMF problem are the same.

Proof of Claim 2.1: [**29**] According to Equation 2.18, $\operatorname{tr}\left(\boldsymbol{X_1}\right)=\operatorname{tr}\left(\boldsymbol{X_2}\right)=P_{tot}$ implies that $\operatorname{tr}\left(\boldsymbol{X_1}\boldsymbol{X_2}\right)\leq P_{tot}^2$. When this result is evaluated together with the constraint given in equivalent problem, i.e., $\operatorname{tr}\left(\boldsymbol{X_1}\boldsymbol{X_2}\right)\geq P_{tot}^2$, they imply that $\operatorname{tr}\left(\boldsymbol{X_1}\boldsymbol{X_2}\right)=\operatorname{tr}\left(\boldsymbol{X_1}\right)\left(\boldsymbol{X_2}\right)=P_{tot}^2$. As proved in Lemma 2.1, when equality is satisfied in Equation 2.18, both $\boldsymbol{X_1^*}$ and $\boldsymbol{X_2^*}$ denoting the resulting optimized matrices will be rank-one. As a result, $\boldsymbol{X_1^*}=\boldsymbol{X_2^*}$ due to

$$\operatorname{tr}\left(\boldsymbol{X_1}\right)=\operatorname{tr}\left(\boldsymbol{X_2}\right)=P_{tot} \quad \text{for} \quad i=1,2,...K,$$

constraint in 2.26. Furthermore, when the constraints and cost function of equivalent problem are compared, $\boldsymbol{X_1^*}$ and $\boldsymbol{X_2^*}$ could also solve the original MMF problem independent of each other. In addition to this, optimization problem with respect to either $\boldsymbol{X_1}$ or $\boldsymbol{X_2}$ is the same as the original optimization problem. Therefore, $\boldsymbol{X_{opt}}=\boldsymbol{X_1^*}=\boldsymbol{X_2^*}$. ■

After these procedures, there is still a nonconvex constraint in the equivalent optimization problem, i.e. $\operatorname{tr}\left(\boldsymbol{X_1}\boldsymbol{X_2}\right)\geq P_{tot}^2$. This issue can be dealt with by using "Exact Penalty Approach". Problem turns into convex one by adding non-convex constraint into cost function. This process does not change the solution of original optimization problem as proved in [29]. Note that unlike the SDR method, rank-one constraint is not dropped in this equivalent problem.

Finally, the equivalent optimization problem can be expressed as

$$\begin{aligned}
&\max_{\boldsymbol{X_1}\in\mathbb{C}^{M\times M},\boldsymbol{X_2}\in\mathbb{C}^{M\times M}} \alpha_1+\alpha_2+\mu\left(P_{tot}^2-\operatorname{tr}\left(\boldsymbol{X_1}\boldsymbol{X_2}\right)\right)\\
&\text{s.t. } \operatorname{tr}\left(\tilde{\boldsymbol{R}}_{\boldsymbol{i}}\boldsymbol{X_1}\right)\geq\alpha_1 \quad \text{for} \quad i=1,2,...K\\
&\operatorname{tr}\left(\tilde{\boldsymbol{R}}_{\boldsymbol{i}}\boldsymbol{X_2}\right)\geq\alpha_2 \quad \text{for} \quad i=1,2,...K\\
&\operatorname{tr}\left(\boldsymbol{X_1}\right)=\operatorname{tr}\left(\boldsymbol{X_2}\right)=P_{tot} \quad \text{for} \quad i=1,2,...K\\
&\boldsymbol{X_1}\succeq 0,\boldsymbol{X_2}\succeq 0.
\end{aligned} \tag{2.27}$$

where μ is a penalty parameter.
Since μP_{tot}^2 term is constant, it does not effect the optimization procedure. Therefore, the problem reduces to

$$\begin{aligned}
&\max_{\boldsymbol{X_1}\in\mathbb{C}^{M\times M},\boldsymbol{X_2}\in\mathbb{C}^{M\times M}} \alpha_1+\alpha_2+\mu\left(\operatorname{tr}\left(\boldsymbol{X_1}\boldsymbol{X_2}\right)\right)\\
&\text{s.t. } \operatorname{tr}\left(\tilde{\boldsymbol{R}}_{\boldsymbol{i}}\boldsymbol{X_1}\right)\geq\alpha_1 \quad \text{for} \quad i=1,2,...K\\
&\operatorname{tr}\left(\tilde{\boldsymbol{R}}_{\boldsymbol{i}}\boldsymbol{X_2}\right)\geq\alpha_2 \quad \text{for} \quad i=1,2,...K\\
&\operatorname{tr}\left(\boldsymbol{X_1}\right)=\operatorname{tr}\left(\boldsymbol{X_2}\right)=P_{tot} \quad \text{for} \quad i=1,2,...K\\
&\boldsymbol{X_1}\succeq 0,\boldsymbol{X_2}\succeq 0.
\end{aligned} \tag{2.28}$$

As seen from the problem expression, optimization depends on two variables, namely $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$. Alternating iterative optimization idea can be used to solve it. To be more precise, at the jth iteration, fix $\boldsymbol{X_1}^{j-1}$, solve optimization problem for $\boldsymbol{X_2}^j$ or vice versa. After that, fix the other variable $\boldsymbol{X_2}^j$ and find $\boldsymbol{X_1}^j$. By keeping constant either$\boldsymbol{X_1}$ or $\boldsymbol{X_2}$, note that the equivalent problem is in the standard form which can be solved by using SeDuMi. The $\boldsymbol{A},\boldsymbol{b},\boldsymbol{c}$ for SeDuMi can be easily found like in SDR method. This alternating process continues until both $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ do not change anymore with iterations. See [29] for the proof of convergence of this algorithm.

Steps of Alternating Maximization Algorithm can be summarized as given below [18]:

Alternating Maximization Algorithm

Initialization: Initialize $j = 0$ (j denotes the index of iteration) and keep either $\boldsymbol{X_1}$ or $\boldsymbol{X_2}$ constant. (Assume $\boldsymbol{X_1}$ is fixed.)$\boldsymbol{X_1^{(0)}} = (\sqrt{\frac{P_{tot}}{M}})\boldsymbol{I_{M\times M}}$.

repeat

$j \to j+1$

Step 1) Fix $\boldsymbol{X_1^{(j-1)}}$ and solve optimization problem to find $\boldsymbol{X_2^{(j)}}$.

Step 2) If rank $(\boldsymbol{X_2}) = 1$, terminate the algorithm.

Step 3) Fix $\boldsymbol{X_2^{(j)}}$ and solve optimization problem to find $\boldsymbol{X_1^{(j)}}$.

Step 4) If rank $(\boldsymbol{X_1}) = 1$, terminate the algorithm.

until $\left\|\boldsymbol{X_1^{(j)}} - \boldsymbol{X_1^{(j-1)}}\right\|^2 \leq \epsilon$.

Take the principal eigenvector of the solution matrix (either $\boldsymbol{X_1}$ or $\boldsymbol{X_2}$ depending on termination of algorithm) as the optimum precoder vector $\boldsymbol{p}$ solution.

(In this algorithm, ϵ is taken as 0.001).

In the above part, by using SeDuMi, precoder design for broadcasting MMF problem is solved directly. In addition to the direct solution method, there is also an iterative solution based on the Quality of Service (QoS) problem. Actually for the multicasting scenario, the MMF problem can be solved directly without being required to QoS problem solution due to linearity of SNR constraints and cost function. (However, in the next chapter which focuses on the transmission of different information to each user, the constraints of the MMF problem are not linear due to the interference term and it cannot be solved by standard SDP tools. In order to overcome this difficulty, solution by using the equivalence of the QoS problem and the MMF problem is applied). Introduction for the QoS concept and the proof of equivalence of the MMF-QoS problems is studied in the next section.

2.2.3 Solution of the MMF Problem based on Quality of Service (QoS) Beamforming Problem

In downlink MIMO communication, QoS problem is another problem studied in the literature. Under the same system model explained in the MMF problem set-up, the aim is minimizing precoder power at the transmitter side under the restriction that SNR of each user exceeds a certain SNR threshold. For the ith user, γ_{thr_i} denotes the SNR threshold value for $i = 1, 2, ...K$.

Define

$$\widetilde{\boldsymbol{h_i}}^H = \frac{\boldsymbol{d_i}^H \boldsymbol{H_i}}{\sqrt{N_0 \gamma_{thr_i}}}. \tag{2.29}$$

Then, QoS problem can be expressed mathematically as given in below.

$$QoS: \min_{\boldsymbol{p} \in \mathbb{C}^M} \|\boldsymbol{p}\|^2$$

$$\text{s.t.} \quad \left\| \widetilde{\boldsymbol{h_i}}^H \boldsymbol{p} \right\| \geq 1 \quad \text{for} \quad i = 1, 2, ...K. \tag{2.30}$$

Similar to the MMF problem explained in the previous part, define $\boldsymbol{X} = \boldsymbol{p}\boldsymbol{p}^H$ meaning that $\text{rank}(\boldsymbol{X}) = 1$ and $\widetilde{\boldsymbol{Y_i}} = \widetilde{\boldsymbol{h_i}}\widetilde{\boldsymbol{h_i}}^H$, where $\widetilde{\boldsymbol{Y_i}}$ is positive semidefinite.

The optimization problem can also be expressed as

$$QoS: \min_{\boldsymbol{X} \in \mathbb{C}^{M \times M}} \text{tr}(\boldsymbol{X})$$

$$\text{s.t} \quad \text{tr}\left(\boldsymbol{X}\widetilde{\boldsymbol{Y_i}}\right) \geq 1, \ \text{for } i = 1, 2, ...K$$

$$\boldsymbol{X} \succeq 0 \tag{2.31}$$

$$\text{rank}(\boldsymbol{X}) = 1.$$

Again due to non-convexity of rank-one constraint, problem can not be directly solved by SDP tools, relaxation procedure is required. After omitting the rank-one constraint, it could be written in a standard SDP format with K inequalities such that

$$\min_{\boldsymbol{X} \in \mathbb{C}^{M \times M}, t_i \in \mathbb{R}} vec(\boldsymbol{I})^T vec(\boldsymbol{X}) \tag{2.32}$$

$$s.t. \quad vec\left(\widetilde{\boldsymbol{Y_i}}^T\right)^T vec(\boldsymbol{X}) - t_i = 1, \ t_i \geq 0 \quad \text{for} \quad i = 1, 2, ...K$$

$$\boldsymbol{X} \succeq 0, \tag{2.33}$$

where $t_1, t_2, \ldots, t_K$ are slack variables.

According to above formulations, in standard SeDuMi($\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}$) form, $\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}$ are equal to

$$\boldsymbol{A} = \begin{bmatrix} -1 & 0 & \ldots & 0 & vec\left(\widetilde{\boldsymbol{Y}}_{\boldsymbol{1}}^T\right)^T \\ 0 & -1 & \ldots & 0 & vec\left(\widetilde{\boldsymbol{Y}}_{\boldsymbol{2}}^T\right)^T \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & -1 & vec\left(\widetilde{\boldsymbol{Y}}_{\boldsymbol{K}}^T\right)^T \end{bmatrix}$$

$$\boldsymbol{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \boldsymbol{c} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ vec\left(\boldsymbol{I}\right) \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_K \\ vec\left(\boldsymbol{X}\right) \end{bmatrix}, \tag{2.34}$$

where $\boldsymbol{A}$ is $(M^2 + K) \times (M^2 + K)$ matrix, $\boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{x}$ are $(M^2 + K) \times 1$ vectors.

After using SDP tools, such as SeDuMi, one can find the optimum solution of the QoS problem, i.e. $\boldsymbol{X_{opt}}$ matrix satisfying above constraints. In order to obtain the precoder vector $\boldsymbol{p}$, randomization step is used the same as before. In fact, the QoS problem could be used in order to solve the MMF problem. Before presenting the solution procedure, this claim should be proven. Throughout the proof [16], following notation is used. In all simulations of this thesis work, $\gamma_{thr_i} = \gamma_{thr}$, namely SNR thresholds are the same for all users in the QoS problem.

Notation:

$MMF\left(P_{tot}, \{\widetilde{\boldsymbol{h_i}}\}_{i=1}^K\right) = \gamma_{min}$ means that for the given total transmitted power P_{tot} and channel vectors of all users $\{\widetilde{\boldsymbol{h_i}}\}_{i=1}^K$, the MMF problem returns the minimum SNR in the system denoted by γ_{min}.

$QoS\left(\gamma_{thr}, \{\widetilde{\boldsymbol{h_i}}\}_{i=1}^K\right) = P_{min}$ means that for the given equal SNR thresholds denoted by γ_{thr} and channel vectors of all users $\{\widetilde{\boldsymbol{h_i}}\}_{i=1}^K$, the QoS problem returns the minimum transmitted power denoted by P_{min} required to satisfy SNR thresholds.

Lemma 2.2 [16]: In the QoS problem, if $\gamma_{thr_1} > \gamma_{thr_2}$, then $P_{min_1} > P_{min_2}$ meaning that the minimum total transmitted power is monotonically increasing if SNR threshold γ_{thr} is increased.

Proof of Lemma 2.2 [16]: $\left\|\widetilde{\boldsymbol{h_i}}^H P_{min_1}\right\|^2 \geq \gamma_{thr_1} > \gamma_{thr_2}$.

Define $k = \frac{\gamma_{thr_1}}{\gamma_{thr_2}} > 1$ then, one can write that $\left\|\widetilde{\boldsymbol{h_i}}^H \frac{P_{min_1}}{\sqrt{k}}\right\|^2 \geq \gamma_{thr_2}$. This implies that $P_{min_2} = \frac{P_{min_1}}{\sqrt{k}}$. Since $k \geq 1$, $P_{min_1} > P_{min_2}$ is proven. ■

Claim 2.2 [16]: According to the previously defined notation, for a given $\{\widetilde{\boldsymbol{h_i}}\}_{i=1}^K$ if $P_{tot} = P_{min}$ then $\gamma_{min} = \gamma_{thr}$.

Proof of Claim 2.2 [16]: Let $MMF\left(P_{tot}, \{\tilde{\boldsymbol{h_i}}\}_{i=1}^K\right) = \gamma$. It will be shown that

$$QoS\left(\gamma_{min}, \{\tilde{\boldsymbol{h_i}}\}_{i=1}^K\right) = P_{tot} \Rightarrow MMF\left(P_{tot}, \{\tilde{\boldsymbol{h_i}}\}_{i=1}^K\right) = \gamma_{min}, \tag{2.35}$$

namely $\gamma = \gamma_{min}$.

Firstly, assume that $\gamma < \gamma_{min}$. From $QoS\left(\gamma_{min}, \{\tilde{\boldsymbol{h_i}}\}_{i=1}^K\right) = P_{tot}$, one can say that using total power P_{tot}, SNR values of all users can be made at least γ_{min} for a suitable precoder $\boldsymbol{p}$. Hence $\gamma \geq \gamma_{min}$ which is a contradiction.

Secondly, assume that $\gamma > \gamma_{min}$. Then, from MMF problem with the same total power P_{tot} , $\gamma > \gamma_{min}$ is achieved, which is a contradiction with Lemma 2.2. As a result, it is shown that $\gamma = \gamma_{min}$ meaning that proof of Claim 2.2 is completed. ■

Let $\boldsymbol{p}_{MMF}$ represent the precoder vector designed with MMF problem and $\boldsymbol{p}_{QoS}$ be the precoder vector designed with QoS problem. Another observation on the result of Claim 2 is that the designed precoder vectors $\boldsymbol{p}_{MMF}$ and $\boldsymbol{p}_{QoS}$ are equivalent to each other. It could easily be seen by SNR formula of each user.

Therefore, it is shown that there is an another way to solve MMF problem with the help of QoS problem. To find the solution, first for a given γ value solve the QoS problem and find minimum power P_{min} which satisfies SNR thresholds of all users. After that, scale precoder vector $\boldsymbol{p}$ found by QoS by multiplying with a coefficient which makes the resulting precoder power is equal to P_{tot}.

Precoder and decoder design continues iteratively until the minimum SNR in the system converges. At each iteration, the minimum SNR increases. Furthermore, the convergence of the algorithm is guaranteed since the SNR of every user is upper bounded by the existence of the noise term at the receiver.

2.3 Minimization of Total MSE Problem

For this problem, precoder and decoder vectors are optimized with the cost function of minimizing the total mean-square error (MSE) of system. In other words, under total power constraint at the transmitter side, the total MSE of the system is tried to be minimized. Mathematical expression of the optimization problem is given in the following:

$$\begin{aligned} &\min_{\boldsymbol{p},\boldsymbol{d_1},\boldsymbol{d_2},\ldots,\boldsymbol{d_K}} \sum_{i=1}^{K} MSE_i \\ &s.t.\ \boldsymbol{p}^H\boldsymbol{p} = P_{tot}. \end{aligned} \tag{2.36}$$

MSE of the ith user can be defined as

$$\begin{aligned} MSE_i &= \mathbb{E}\left\{|r_i - b|^2\right\} \quad \text{for} \quad i = 1, 2, \ldots K. \\ MSE_i &= \mathbb{E}\left\{|r_i|^2\right\} + \mathbb{E}\left\{|b|^2\right\} - \mathbb{E}\left\{r_i b^*\right\} - \mathbb{E}\left\{b r_i^*\right\}. \end{aligned} \tag{2.37}$$

Computation of each term in Equation 2.37 is provided below.

1st term:

$$\begin{aligned} \mathbb{E}\left\{|r_i|^2\right\} &= \mathbb{E}\left\{\boldsymbol{d_i}^H\left(\boldsymbol{H_i}\boldsymbol{p}b + \boldsymbol{n_i}\right)\left(b^*\boldsymbol{p}^H\boldsymbol{H_i}^H + \boldsymbol{n_i}^H\right)\boldsymbol{d_i}\right\} \\ &= \boldsymbol{d_i}^H\boldsymbol{H_i}\boldsymbol{p}\underbrace{E\{bb^*\}}_{1}\boldsymbol{p}^H\boldsymbol{H_i}^H\boldsymbol{d_i} + \boldsymbol{d_i}^H\underbrace{\mathbb{E}\{\boldsymbol{n_i}\boldsymbol{n_i}^H\}}_{N_0}\boldsymbol{d_i} \\ &= \boldsymbol{d_i}^H\boldsymbol{H_i}\boldsymbol{p}\boldsymbol{p}^H\boldsymbol{H_i}^H\boldsymbol{d_i} + N_0\boldsymbol{d_i}^H\boldsymbol{d_i}. \end{aligned} \tag{2.38}$$

2nd term:

$$\operatorname{tr}\left(\mathbb{E}\left\{bb^*\right\}\right) = \mathbb{E}\left\{|b|^2\right\} = 1. \tag{2.39}$$

3rd term:

$$\begin{aligned} \mathbb{E}\left\{r_i b^*\right\} &= \mathbb{E}\left\{\left(\boldsymbol{d_i}^H\left(\boldsymbol{H_i}\boldsymbol{p}b + \boldsymbol{n_i}\right)\right)b^*\right\} \\ &= \boldsymbol{d_i}^H\left(\boldsymbol{H_i}\boldsymbol{p}\underbrace{\mathbb{E}\left\{bb^*\right\}}_{1}\right) \\ &= \boldsymbol{d_i}^H\boldsymbol{H_i}\boldsymbol{p}. \end{aligned} \tag{2.40}$$

Since the 4th term is the Hermitian of the 3rd term, it could directly be written as

$$\mathbb{E}\left\{r_i^H b\right\} = \boldsymbol{p}^H \boldsymbol{H}_i^H \boldsymbol{d}_i. \tag{2.41}$$

As a result, for the ith user, MSE_i can be expressed as in Equation 2.42.

$$MSE_i = \boldsymbol{d}_i^H \boldsymbol{H}_i \boldsymbol{p}\boldsymbol{p}^H \boldsymbol{H}_i^H \boldsymbol{d}_i + N_0 \boldsymbol{d}_i^H \boldsymbol{d}_i - \boldsymbol{d}_i^H \boldsymbol{H}_i \boldsymbol{p} - \boldsymbol{p}^H \boldsymbol{H}_i^H \boldsymbol{d}_i + 1. \tag{2.42}$$

Optimization will be done by using the Lagrange multiplier approach. Therefore, the Lagrange objective function can be expressed as [22]

$$\begin{aligned} L\left(\boldsymbol{p}; \boldsymbol{d_1}, \boldsymbol{d_2}, \ldots, \boldsymbol{d_k}; \lambda\right) = \lambda &\left(\boldsymbol{p}^H \boldsymbol{p} - P_{tot}\right) \\ &+ \sum_{i=1}^{K} \left(\boldsymbol{d}_i^H \boldsymbol{H}_i \left(\boldsymbol{p}\boldsymbol{p}^H\right) \boldsymbol{H}_i^H \boldsymbol{d}_i\right) \\ &- \boldsymbol{d}_i^H \boldsymbol{H}_i \boldsymbol{p} - \boldsymbol{p}^H \boldsymbol{H}_i^H \boldsymbol{d}_i \\ &+ N_0 \boldsymbol{d}_i^H \boldsymbol{d}_i + 1, \end{aligned} \tag{2.43}$$

where λ represents the Lagrange multiplier. This problem can be solved by taking the derivative of the Lagrange objective function with respect to the precoder vector $\boldsymbol{p}$ and decoder vectors $\boldsymbol{d_1}, \boldsymbol{d_2}, \ldots, \boldsymbol{d_K}$ and equating the result to zero. Resulting derivative expressions are provided in the following part.

Precoder Design

Take the derivative of Equation 2.43 with respect to $\boldsymbol{p}^*$ and equate the result to zero.

$$\frac{\partial L\left(\boldsymbol{p}, \boldsymbol{d_1}, \boldsymbol{d_2}, \ldots, \boldsymbol{d_k}; \lambda\right)}{\partial \boldsymbol{p}^*} = \sum_{i=1}^{K} \boldsymbol{H}_i^H \boldsymbol{d}_i \boldsymbol{d}_i^H \boldsymbol{H}_i \boldsymbol{p} - \boldsymbol{H}_i^H \boldsymbol{d}_i + \lambda \boldsymbol{p} = 0. \tag{2.44}$$

Decoder Design

Take the derivative of Equation 2.43 with respect to $\boldsymbol{d}_m^*$ and equate the result to zero.

$$\frac{\partial L\left(\boldsymbol{p}; \boldsymbol{d_1}, \boldsymbol{d_2}, \ldots, \boldsymbol{d_k}; \lambda\right)}{\partial \boldsymbol{d}_m^*} = \boldsymbol{H}_m \left(\boldsymbol{p}\boldsymbol{p}^H\right) \boldsymbol{H}_m^H \boldsymbol{d}_m + N_0 \boldsymbol{d}_m - \boldsymbol{H}_m \boldsymbol{p} = 0. \tag{2.45}$$

Therefore, the precoder vector $\boldsymbol{p}$ and the decoder vector $\boldsymbol{d}_m$ become

$$\boldsymbol{p} = \left(\sum_{i=1}^{K} \boldsymbol{H}_i^H \boldsymbol{d}_i \boldsymbol{d}_i^H \boldsymbol{H}_i + \lambda \boldsymbol{I}\right)^{-1} \sum_{i=1}^{K} \boldsymbol{H}_i^H \boldsymbol{d}_i. \tag{2.46}$$

$$\boldsymbol{d_m} = \left(\boldsymbol{H_m}\left(\boldsymbol{p}\boldsymbol{p}^H\right)\boldsymbol{H_m^H} + N_0\boldsymbol{I}\right)^{-1}\boldsymbol{H_m}\boldsymbol{p} \text{ for } m = 1, 2, ...K. \tag{2.47}$$

As seen from formulas of the precoder and decoder, optimum precoder vector $\boldsymbol{p}$ depends on all decoder vectors $\boldsymbol{d_i}$ for $i = 1, 2, ..K$. Likewise, the ith optimum decoder vector $\boldsymbol{d_i}$ depends on the precoder vector $\boldsymbol{p}$. Due to this, finding optimum precoder and decoder via closed form solution is hard. In order to solve this problem, an iterative solution algorithm is used whose details are provided in the following steps.

Joint Precoder/Decoder Design Algorithm for Transmitted Common Information Scenario

Initialization: $t = 0$ (t denotes the index of iteration)
Set all users' decoder vectors $\boldsymbol{d_i^{(0)}}$ for $i = 1, 2, ...K$ to $\boldsymbol{I}_{N\times 1}$ where $\boldsymbol{I}_{m,n} = 1$ for $m = n$ and $\boldsymbol{I}_{m,n} = 0 \quad \text{for} \quad m \neq n$.
repeat
$t \rightarrow t + 1$

Step 1) Calculate the precoder vector $\boldsymbol{p}^{(t)}$ by using the decoder vector of each user $\boldsymbol{d_i^{(t-1)}}$ for $i = 1, 2, ...K$.
Step 2) Calculate $\boldsymbol{d_i^t}$ by using precoder vector calculated in Step 1.

until $\sum_{i=1}^{K} \left\| \boldsymbol{d_i^{(t)}} - \boldsymbol{d_i^{(t-1)}} \right\|^2 \leq \epsilon$ or $t = t_{max}$ where t_{max} is the maximum number of iterations.

Here, ϵ is taken as 0.0005.

After each precoder and decoder calculation step, the total MSE monotonically decreases as expected. On the other hand, it is lower bounded by zero, which means that the algorithm converges. However, due to the general vulnerability of the iterative algorithms, it is not guaranteed that the convergence point is a global optimum solution.

Calculation of Lagrange Multiplier for Multicasting Scenario

The Lagrange multiplier is calculated by using the constraint of the optimization

problem

$$\sum_{i=1}^{K} \boldsymbol{p}^H \boldsymbol{p} = P_{tot}. \tag{2.48}$$

When the precoder vector found in Equation 2.46 is substituted into equality in 2.48, one can obtain

$$\operatorname{tr}\left(\left(\sum_{i=1}^{K} \boldsymbol{H_i}^H \boldsymbol{d_i} \boldsymbol{d_i}^H \boldsymbol{H_i} + \lambda \boldsymbol{I}\right)^{-2} \left(\sum_{i=1}^{K} \boldsymbol{H_i}^H \boldsymbol{d_i}\right) \left(\sum_{i=1}^{K} \boldsymbol{d_i}^H \boldsymbol{H_i}\right)\right) = P_{tot} \tag{2.49}$$

Let $\boldsymbol{A} = \sum_{i=1}^{K} \boldsymbol{H_i}^H \boldsymbol{d_i} \boldsymbol{d_i}^H \boldsymbol{H_i}$ and $\boldsymbol{B} = \sum_{i=1}^{K} \boldsymbol{H_i}^H \boldsymbol{d_i} \sum_{i=1}^{K} \boldsymbol{d_i}^H \boldsymbol{H_i}$. By using trace properties, the Equation 2.48 can also be written as

$$\operatorname{tr}\left(\left(\boldsymbol{A} + \lambda \boldsymbol{I}\right)^{-2} \boldsymbol{B}\right) = P_{tot}. \tag{2.50}$$

Since $\boldsymbol{A}$ is $M \times M$ square matrix, one can apply eigenvalue decomposition to $\boldsymbol{A}$ such that $\boldsymbol{A} = \sum_{i=1}^{K} a_i \boldsymbol{e_i} \boldsymbol{e_i}^H$ where a_i's are eigenvalues of matrix $\boldsymbol{A}$ and $\boldsymbol{e_k}$'s are orthonormal eigenvectors of $\boldsymbol{A}$. Then, Equation 2.50 turns out to be

$$\operatorname{tr}\left(\left(\boldsymbol{A} + \lambda \boldsymbol{I}\right)^{-2} \boldsymbol{B}\right) = \sum_{i=1}^{K} \frac{1}{\left(a_i + \lambda\right)^2} \operatorname{tr}\left(\boldsymbol{B} \boldsymbol{e_i} \boldsymbol{e_i}^H\right) = P_{tot}. \tag{2.51}$$

The $\operatorname{tr}\left(\left(\boldsymbol{A} + \lambda \boldsymbol{I}\right)^{-2} \boldsymbol{B}\right)$ expression given in Equation 2.50 takes values from 0 to ∞ as λ changes from $-\infty$ to ∞. Therefore, one can always find λ values satisfying this equality. By searching within a certain resolution, solution of λ can be found in the intervals $(-\infty, -a_M), (-a_M, -a_{M-1}), \ldots, (a_1, +\infty)$ in which eigenvalues are sorted such that $a_1 \leq a_2 \leq \ldots \leq a_M$. There may be more than one solution satisfying Equation 2.51. In this case, we choose the one which gives the minimum MSE value as a solution of the Lagrange multiplier [22].

In fact, there is no need to search λ values outside the region given by the following inequality in 2.52.

Corollary 2.1: The solution of λ will exist in the interval given below.

$$-\sqrt{\frac{\operatorname{tr}\left(\boldsymbol{B}\right)}{P_{tot}}} - a_{max} \leq \lambda \leq \sqrt{\frac{\operatorname{tr}\left(\boldsymbol{B}\right)}{P_{tot}}} - a_{min}, \tag{2.52}$$

where $a_1, a_2, \ldots, a_K$ are the eigenvalues of matrix $\boldsymbol{A}$ and a_{min} and a_{max} are its minimum and maximum eigenvalues respectively. The proof of sufficiency of this interval is given in the following part.

Proof of Corollary 2.1: According to Von-Neumann Inequality, if $\boldsymbol{A}$ and $\boldsymbol{B}$ are positive semidefinite, the following inequality holds:

$$\operatorname{tr}(\boldsymbol{AB}) \leq \sum_{i=1}^{M} a_i b_i, \tag{2.53}$$

where a_i and b_i are eigenvalues of $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively. As written in Equation 2.50, $P_{tot} = \operatorname{tr}\left[(\boldsymbol{A} + \lambda \boldsymbol{I})^{-2} \boldsymbol{B}\right] \leq \sum_{i=1}^{M} \frac{b_i}{(a_i + \lambda)^2}$.

Then, define $u = \sqrt{\frac{\operatorname{tr}(\boldsymbol{B})}{P_{tot}}}$. Basically, the theorem will be proven by using "'Proof by Contradiction" method.

Step 1: Firstly, prove that $\lambda \leq u - a_{min}$ where a_{min} is the minimum eigenvalue of matrix $\boldsymbol{A}$.

Assume that $\lambda > u - a_{min}$, then $\lambda + a_{min} > u > 0$. Therefore, $\lambda + a_i > u > 0 \ \ \forall\, i$.

Then, $\frac{1}{\lambda + a_i} < \frac{1}{u} \Rightarrow \frac{1}{(\lambda + a_i)^2} < \frac{1}{u^2} = \frac{P_{tot}}{\operatorname{tr}(\boldsymbol{B})}$.

$$P_{tot} \leq \sum_{i=1}^{M} \frac{b_i}{(a_i + \lambda)^2} < \sum_{i=1}^{M} \frac{b_i}{u^2} = \frac{P_{tot}}{\operatorname{tr}(\boldsymbol{B})} \sum_{i=1}^{M} b_i = P_{tot}. \tag{2.54}$$

As a result, $P_{tot} < P_{tot}$ is obtained which is a contradiction. Therefore $\lambda \leq u - a_{min}$ is proven.

Step 2: Secondly, prove that $-u - a_{max} \leq \lambda$ where a_{max} is the maximum eigenvalue of matrix $\boldsymbol{A}$.

Assume that $\lambda < -u - a_{max}$, then $\lambda + a_{max} < -u < 0$. Therefore, $\lambda + a_i < -u < 0 \ \ \forall\, i$.

Then, $\frac{1}{\lambda + a_i} > \frac{1}{-u} \Rightarrow \frac{1}{(\lambda + a_i)^2} < \frac{1}{u^2} = \frac{P_{tot}}{\operatorname{tr}(\boldsymbol{B})}$.

$$P_{tot} \leq \sum_{i=1}^{M} \frac{b_i}{(a_i + \lambda)^2} < \sum_{i=1}^{M} \frac{b_i}{u^2} = \frac{P_{tot}}{\operatorname{tr}(\boldsymbol{B})} \sum_{i=1}^{M} b_i = P_{tot}. \tag{2.55}$$

As a result, $P_{tot} < P_{tot}$ is obtained which is a contradiction. Therefore $\lambda \geq -u - a_{max}$ is proven.

Therefore, it has been shown that

$$-\sqrt{\frac{\operatorname{tr}(\boldsymbol{B})}{P_{tot}}} - a_{max} \leq \lambda \leq \sqrt{\frac{\operatorname{tr}(\boldsymbol{B})}{P_{tot}}} - a_{min}. \blacksquare \tag{2.56}$$

2.4 Total SNR Maximization Problem

In this problem, the aim is maximizing the total SNR in the system with the help of precoder and decoder design. The mathematical statement of the optimization problem can be expressed as in the following form

$$\max_{\boldsymbol{p}\in\mathbb{C}^M} \sum_{i=1}^{K} SNR_i$$

$$s.t. \quad \operatorname{tr}\left(\boldsymbol{p}\boldsymbol{p}^H\right) = P_{tot}. \tag{2.57}$$

Decoder Design

For a given precoder vector, optimum decoders for each user will be designed exactly the same as the one stated in the MMF problem, namely the Rayleigh quotient decoder design.

Precoder Design

For the precoder design, total SNR expression should be written first. It is equal to

$$\sum_{i=1}^{K} \frac{\boldsymbol{d}_i^H \boldsymbol{H}_i \boldsymbol{p}\boldsymbol{p}^H \boldsymbol{H}_i^H \boldsymbol{d}_i}{N_0 \boldsymbol{d}_i^H \boldsymbol{d}_i} = \sum_{i=1}^{K} \operatorname{tr}\left(\boldsymbol{p}^H \widetilde{\boldsymbol{T}}_i \boldsymbol{p}\right), \tag{2.58}$$

where $\widetilde{\boldsymbol{T}}_i = \dfrac{\boldsymbol{H}_i^H \boldsymbol{d}_i \boldsymbol{d}_i^H \boldsymbol{H}_i}{N_0 \boldsymbol{d}_i^H \boldsymbol{d}_i}$.

Note that the total SNR expression is also in the well-known Rayleigh quotient format; therefore, the optimum unit norm precoder vector is equal to the principal eigenvector of $\widetilde{\boldsymbol{T}}_i$. It should then be scaled to satisfy the total transmitter power constraint.

The precoder and decoder are optimized iteratively until the algorithm converges or the maximum number of iterations is reached. The convergence of the algorithm is decided when the total SNR value in consecutive iterations are

sufficiently close. In this optimization problem, it should be noted that maximizing the total SNR may lead to make some users' SNR values very low, which degrades the worst case performance of the system. Therefore, this method may not be suitable if the goal is maximizing the minimum SNR value as in the max-min fairness problem. However, even though methods using SDP tools have better performance in terms of the minimum SNR in the system, the computational complexity of them is very high which leads to difficulties in their implementation in practical applications. On the other hand, the total SNR maximization method is very fast, but comes with some degradation in the minimum SNR performance. The tradeoff between obtaining high minimum SNR and long computation time should be evaluated with the needs and requirements of the related application.

CHAPTER 3

MULTIUSER MIMO DOWNLINK COMMUNICATION PROBLEM WITH PRIVATE INFORMATION (BROADCAST BEAMFORMING)

In this chapter, we examine the downlink MIMO communication problem in which each user receives a different data symbol, namely private information for that user, simultaneously. A base station (BS) of a cell area sends all private information to multiple users at the same time. Therefore, in addition to the desired one, each user receives a combination of all other users' data which causes interference at the receiver. In practice, this scenario can occur when users in a certain cell area are communicating with other users at the same time. Unlike transmitting common information scenario analyzed in Chapter 2, under this communication scenario the interference between users can cause additional problems for the quality of communication. The main goal of this chapter is to analyze the methods of solving the interference problem and provide better communication conditions for users in terms of bit error ratio or capacity etc., by designing precoders and decoders. This communication scenario is well studied in the literature and it is also known as broadcast beamforming.

In this chapter, system model for broadcast beamforming is given in Section 3.1. After that, the optimization of precoder-decoder design for broadcasting scenario is investigated for two different types of cost functions. The first one is the MMF problem. (Section 3.2 focuses on the solution of this problem over methods with or without using SDP tools.) The second one is the minimization of the total MSE, which is studied in Section 3.3.

3.1 System Model

At the transmitter side, consider a BS with M antennas which transmit zero mean unity variance i.i.d. private information signals denoted by $b_1, b_2, \ldots b_k$ to K users located as mobile stations. The ith user has N_i antennas at the receiver side. For simplicity, it is assumed that each user has the same number of antennas at the receiver denoted by N. Unlike the case described in Chapter 2, this time there are several precoder vectors denoted by $\boldsymbol{p_1}, \boldsymbol{p_2}, \ldots, \boldsymbol{p_K}$ to transmit K different private information to K users. Furthermore, to suppress the interference between users, there are K decoders represented by $\boldsymbol{d_1}, \boldsymbol{d_2}, \ldots, \boldsymbol{d_k}$ and each of them is assigned to one user. It is assumed that, channel state information (CSI) is perfectly known both at the transmitter and receiver sides. All users' information is sent simultaneously and firstly pass through an $M \times K$ precoder block denoted by $\boldsymbol{P}$ where $\boldsymbol{P}$ is an augmented matrix whose columns are $\boldsymbol{p_1}, \boldsymbol{p_2}, \ldots, \boldsymbol{p_K}$. For the ith user, corresponding MIMO channel is denoted by $\boldsymbol{H_i}$ with dimension $N \times M$, whose entries are zero mean, unity variance complex Gaussian random variables. At the receiver side, the signal is multiplied by $1 \times N$ decoder block $\boldsymbol{d_i^H}$ in order to extract private information of ith user. Each user's receiver has complex Gaussian $N \times 1$ noise term $\boldsymbol{n_i}$ with zero mean and variance N_0. The system model for interfering broadcast downlink MU-MIMO is given in Figure 3.1. As a result, the received signal for the ith user, r_i can be expressed in a mathematical form such that

$$r_i = \boldsymbol{d_i^H}\left(\boldsymbol{H_i P b} + \boldsymbol{n_i}\right) = \boldsymbol{d_i^H}\left(\boldsymbol{H_i} \sum_{j=1}^{K} \boldsymbol{p_j} b_j + \boldsymbol{n_i}\right) \text{ for } \; i = 1, 2, \ldots, K, \tag{3.1}$$

where $\boldsymbol{P} = [\boldsymbol{p_1}\, \boldsymbol{p_2}\, \ldots \boldsymbol{p_K}]$ and $\boldsymbol{b} = \begin{bmatrix} b_1 & b_2 & \cdots & b_K \end{bmatrix}^T$.

With the selection of precoder and decoder vectors, interference between users is to be decreased in order to increase SINR (signal to interference plus noise ratio) for each user. Note that, differently from Chapter 2, here the decoder design is required in order to suppress interference term. Furthermore, the power allocation between users is to be adjusted according to the CSI.

This chapter is concentrated on the solution of two problems, the max-min fair-

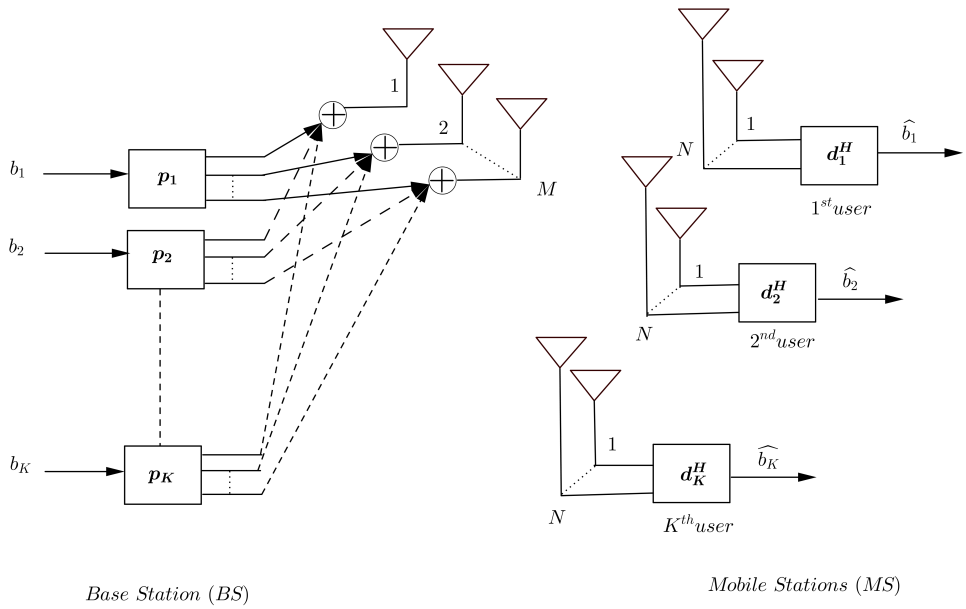

Figure 3.1: System Model for Transmitting Private Information Scenario

ness (MMF) problem and the minimization of total MSE problem. Differently from Chapter 2, here the solution of total SINR maximization is not studied in detailed manner. The reason why that problem is specifically analyzed in Chapter 2 is related with its very low implementation cost. For transmitting private information scenario, there exist some alternative comparatively fast methods.

In this chapter, precoder/decoder design will be achieved jointly and in iterative manner except for one method called SVD-Dec, which is based on decomposition of multiuser MIMO system into parallel single user MIMO systems. Detailed analysis of these methods is given in the following sections.

3.2 Max-Min Fairness Beamforming Problem

In the scenario of transmitting private information to each user, MMF problem can be defined as maximizing the minimum SINR in the system under total power constraint P_{tot}. Accounting also the fact that $\mathbb{E}\left\{|b_i|^2\right\} = 1$, the problem can be written as

$$MMF: \max_{\boldsymbol{p_1},\boldsymbol{p_2},\ldots,\boldsymbol{p_K}\in\mathbb{C}^M,\, \boldsymbol{d_1},\boldsymbol{d_2},\ldots,\boldsymbol{d_K}\in\mathbb{C}^N} \min_{1\leq i\leq K} \left\{ \frac{\left\|\boldsymbol{d_i}^H \boldsymbol{H_i}\boldsymbol{p_i}\right\|^2}{\sum\limits_{j=1,j\neq i}^{K} \left\|\boldsymbol{d_i}^H \boldsymbol{H_i}\boldsymbol{p_j}\right\|^2 + N_0 \left\|\boldsymbol{d_i}\right\|^2} \right\}$$

$$\text{s.t.} \quad \sum_{i=1}^{K} \left\|\boldsymbol{p_i}\right\|^2 \leq P_{tot}. \tag{3.2}$$

Let $\boldsymbol{R_i} = \boldsymbol{H_i}^H \boldsymbol{d_i}\boldsymbol{d_i}^H \boldsymbol{H_i}$ and $\boldsymbol{X_i} = \boldsymbol{p_i}\boldsymbol{p_i}^H$, so the problem can also be expressed as

$$MMF: \max_{\boldsymbol{p_1},\boldsymbol{p_2},\ldots,\boldsymbol{p_K}\in\mathbb{C}^M,\, \boldsymbol{d_1},\boldsymbol{d_2},\ldots,\boldsymbol{d_K}\in\mathbb{C}^N} \min_{1\leq i\leq K} \left\{ \frac{\operatorname{tr}\left(\boldsymbol{R_i}\boldsymbol{X_i}\right)}{\sum_{j=1,j\neq i}^{K} \operatorname{tr}\left(\boldsymbol{R_i}\boldsymbol{X_j}\right) + N_0 \boldsymbol{d_i}^H \boldsymbol{d_i}} \right\}$$

$$\text{s.t.} \quad \sum_{i=1}^{K} \operatorname{tr}\left(\boldsymbol{X_i}\right) \leq P_{tot}. \tag{3.3}$$

Let

$$\min_i \left\{ \frac{\operatorname{tr}\left(\boldsymbol{R_i}\boldsymbol{X_i}\right)}{\sum_{j=1,j\neq i}^{K} \operatorname{tr}\left(\boldsymbol{R_j}\boldsymbol{X_j}\right) + N_0 \boldsymbol{d_i}^H \boldsymbol{d_i}} \right\} = t.$$

Then the problem becomes

$$MMF: \max_{\boldsymbol{p_1},\boldsymbol{p_2},\ldots,\boldsymbol{p_K}\in\mathbb{C}^M,\, \boldsymbol{d_1},\boldsymbol{d_2},\ldots,\boldsymbol{d_K}\in\mathbb{C}^N} t$$

$$\text{s.t.} \quad \left\{ \frac{\operatorname{tr}\left(\boldsymbol{R_i}\boldsymbol{X_i}\right)}{\sum_{j=1,j\neq i}^{K} \operatorname{tr}\left(\boldsymbol{R_i}\boldsymbol{X_j}\right) + N_0 \boldsymbol{d_i}^H \boldsymbol{d_i}} \right\} \geq t \quad \text{for} \quad i = 1, 2 \ldots, K$$

$$\sum_{i=1}^{K} \operatorname{tr}\left(\boldsymbol{X_i}\right) \leq P_{tot}$$

$$\operatorname{rank}\left(\boldsymbol{X_i}\right) = 1 \quad \text{for} \quad i = 1, 2, \ldots, K \tag{3.4}$$

$$\boldsymbol{X_i} \succeq 0 \quad \text{for} \quad i = 1, 2, \ldots, K.$$

Furthermore, unlike the multicasting scenario, this optimization problem cannot be solved as stated, since it has K inequality constraints which are non-linear.

The non-linearity of them is caused by the interference term (t is a variable which depends on $\boldsymbol{X_i}$'s). At this point, the property of equivalence of MMF and QoS problems is used, which is a fact whose proof is provided in Chapter 2. For the transmission of private information scenario, the QoS problem can be expressed as in the following form.

$$
\begin{aligned}
QoS: \quad & \min_{\boldsymbol{p_1},\boldsymbol{p_2},\ldots,\boldsymbol{p_K}\in\mathbb{C}^M,\, \boldsymbol{d_1},\boldsymbol{d_2},\ldots,\boldsymbol{d_K}\in\mathbb{C}^N} \sum_{i=1}^{K} \operatorname{tr}(\boldsymbol{X_i}) \\
& \frac{\operatorname{tr}(\boldsymbol{R_i}\boldsymbol{X_i})}{\sum_{j=1,j\neq i}^{K} \operatorname{tr}(\boldsymbol{R_i}\boldsymbol{X_j}) + N_0 \boldsymbol{d_i}^H \boldsymbol{d_i}} \geq \gamma_i \quad \text{for} \quad i = 1,2,\ldots,K \\
& \operatorname{rank}(\boldsymbol{X_i}) = 1 \quad \text{for} \quad i = 1,2,\ldots,K \\
& \boldsymbol{X_i} \succeq 0 \quad \text{for} \quad i = 1,2,\ldots,K.
\end{aligned}
\tag{3.5}
$$

(Throughout this thesis, without loss of generality all SINR threshold values γ_i's are taken as equal to each other, i.e. $\gamma_i = \gamma \quad \forall i = 1,2,\ldots,K$).

We know that when the minimum power P_{min} found by the QoS problem satisfying SINR threshold values is equal to the MMF total power constraint P_{tot}, then the optimum precoder solution of the QoS problem is identical with the solution of the MMF problem. Therefore, the MMF problem can be solved iteratively, namely by solving a series of QoS problems. To be more precise, for an initial threshold value, we solve the QoS problem and then find the minimum power P_{min} satisfying SINR thresholds. According to the P_{min} value with respect to P_{tot}, we either increase or decrease the SINR thresholds and solve the QoS problem again until the minimum power P_{min} and P_{tot} are nearly equal to each other.

Note that for all optimization methods, the effect of decoder matrices has to be taken into account. Therefore, throughout this chapter except for SVD-Dec method, the optimization is done iteratively in order to simplify its complexity resulting from the non-linearity of the joint precoder/decoder design. In other words, for a given decoder set, the precoder design is optimized and vice versa. This process continues until the overall process converges.

Decoder Design

For a given set of precoder vectors, as in Chapter 2, when the SINR expression of each user is written, it can be noticed that these expressions are decoupled from the viewpoint of decoder vectors. Therefore, each user can design his own decoder by using similar Rayleigh quotient approach expressed in Chapter 2. The derivation of Rayleigh quotient expression for transmitting private information scenario is given in the following part.

Rayleigh Quotient Decoder Design

When the SINR expression for the ith user is written as in Chapter 2, $\boldsymbol{U_1}$ and $\boldsymbol{U_2}$ is obtained as $\boldsymbol{U_1} = \boldsymbol{H_i p_i p_i^H H_i^H}$, $\boldsymbol{U_2} = \boldsymbol{H_i} \left(\sum_{j=1, j \neq i}^{K} \boldsymbol{p_j p_j^H} \right) \boldsymbol{H_i^H}$. Therefore, the optimum unit norm decoder vector should be chosen as the generalized principal eigenvector of $\boldsymbol{U_1}$ and $\boldsymbol{U_2}$. The scaling of decoder vector can be chosen arbitrarily, since its effect will be canceled out due to the existence of the decoder vector both in the numerator and denominator.
In this chapter, throughout the solution of the MMF problem, the decoder design will be optimized by using the Rayleigh quotient approach except for the SVD-Dec algorithm. Therefore, for all these methods, only the precoder optimization is given in detail.

Precoder Design

For a given set of decoder vectors, the precoder design in the MMF problem will be solved by different approaches. The first two methods are identical to the ones in Chapter 2, namely these methods solve the problem with the help of SDP solvers. In addition, there are also other methods of solution without utilizing the SDP tools. All of these approaches are analyzed in detail in the following parts.

3.2.1 Semidefinite Relaxation (SDR) Method

The optimization problem under private information scenario can be solved with the help of QoS problem which also has a non-convex rank-one constraint. As a general approach, in order to solve it by using standard SDP tools, this non-convex constraint should be omitted which is a process called as semidefinite relaxation (SDR). In each iteration, related QoS problem is solved by this tool until the minimum power value found by the solution of the QoS problem is equal to the total transmitter power constraint in the MMF problem [19]. To formulate the problem in the standard SeDuMi form, slack variables denoted by $t_1, t_2, \ldots, t_K$ are introduced. With the introduction of slack variables, for a fixed set of decoder vectors, the QoS problem becomes

$$QoS: \min_{\boldsymbol{p_1}, \boldsymbol{p_2}, \ldots, \boldsymbol{p_K}} \sum_{i=1}^{K} \operatorname{tr}(\boldsymbol{X_i})$$

$$\frac{1}{\gamma} \operatorname{tr}\left(\widetilde{\boldsymbol{R_i}} \boldsymbol{X_i}\right) - \sum_{j=1, j \neq i}^{K} \operatorname{tr}\left(\widetilde{\boldsymbol{R_j}} \boldsymbol{X_j}\right) - t_i = 1, \quad t_i \geq 0 \text{ for } \; i = 1, 2, \ldots, K,$$

$$\boldsymbol{X_i} \succeq 0 \;\; \text{for} \;\; i = 1, 2, \ldots, K, \tag{3.6}$$

where $\widetilde{\boldsymbol{R_i}} = \dfrac{\boldsymbol{R_i}}{N_0 \boldsymbol{d_i}^H \boldsymbol{d_i}}$.

Furthermore, assume that all user's SINR thresholds are equal to each other and denoted by γ. According to the standard form of SDP tool SeDuMi, for the optimization problem under transmitting private information scenario, the inputs $\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}$ are given below.

$$\boldsymbol{A} = \begin{bmatrix} -1 & 0 & \ldots & 0 & \frac{1}{\gamma} vec\left(\widetilde{\boldsymbol{R}}_{\boldsymbol{1}}^T\right)^T & -vec\left(\widetilde{\boldsymbol{R}}_{\boldsymbol{1}}^T\right)^T & \ldots & -vec\left(\widetilde{\boldsymbol{R}}_{\boldsymbol{1}}^T\right)^T \\ 0 & -1 & \ldots & 0 & -vec\left(\widetilde{\boldsymbol{R}}_{\boldsymbol{2}}^T\right)^T & \frac{1}{\gamma} vec\left(\widetilde{\boldsymbol{R}}_{\boldsymbol{2}}^T\right)^T & \ldots & -vec\left(\widetilde{\boldsymbol{R}}_{\boldsymbol{2}}^T\right)^T \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & -1 & -vec\left(\widetilde{\boldsymbol{R}}_{\boldsymbol{K}}^T\right)^T & -vec\left(\widetilde{\boldsymbol{R}}_{\boldsymbol{K}}^T\right)^T & \ldots & \frac{1}{\gamma} vec\left(\widetilde{\boldsymbol{R}}_{\boldsymbol{K}}^T\right)^T \end{bmatrix}$$

$$\boldsymbol{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \boldsymbol{c} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ vec\left(\boldsymbol{I}\right) \\ vec\left(\boldsymbol{I}\right) \\ \vdots \\ vec\left(\boldsymbol{I}\right) \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_K \\ vec\left(\boldsymbol{X_1}\right) \\ vec\left(\boldsymbol{X_2}\right) \\ \vdots \\ vec\left(\boldsymbol{X_K}\right) \end{bmatrix}, \tag{3.7}$$

where $\boldsymbol{A}$ is a $K \times K\left(M^2+1\right)$ matrix, $\boldsymbol{b}$ is a $K \times 1$ vector, $\boldsymbol{c}$ and $\boldsymbol{x}$ are $K\left(M^2+1\right) \times 1$ vectors. SeDuMi gives the solution of $\boldsymbol{x}$ vector, then one can find each $\boldsymbol{X_i}$ matrix using after $(K+1)^{th}$ terms of $\boldsymbol{x}$. After that, in order to obtain the precoder vector $\boldsymbol{p_i}$ from $\boldsymbol{X_i}$, randomization methods (randA) or deterministic method (detA) expressed in Chapter 2 can be used. The summary of steps of the algorithm are provided below.

Iterative MMF Algorithm (based on QoS Problem)

Initialization: Initialize $i = 0$ (i denotes the index of iteration) and γ_{thr} (for example, $\gamma_{thr} = 0\,dB$) $\Delta = \Delta_0 = 10\,dB$.

repeat

$i \rightarrow i+1$

Step 1) Solve QoS problem, find $P_{min}^{(i)}$.

Step 2) If $P_{min}^{(i-1)} < P_{tot} < P_{min}^{(i)}$ or $P_{min}^{(i)} < P_{tot} < P_{min}^{(i-1)}$ make $\Delta \leftarrow \frac{\Delta}{10}$.

Step 3) If $P_{min}^{(i)} < P_{tot}$, increase γ_{thr} by Δ. If $P_{min}^{(i)} > P_{tot}$, decrease γ_{thr} by Δ.

until $\left\| P_{tot} - P_{min}^{(i)} \right\|^2 \leq \epsilon$.

Take the principal eigenvector of $\boldsymbol{X_{opt}}$ found from the Iterative MMF Algorithm and scale it to satisfy the total precoder power constraint to find the optimum precoder vector $\boldsymbol{p}$. (In this algorithm, ϵ is taken as $P_{tot}/1000$).

The joint precoder/decoder optimization continues until the algorithm converges. The convergence criterion of the algorithm is that the minimum SINR is approximately the same with respect to the previous iteration.

3.2.2 Exact Penalty Approach (EPA)

In addition to the SDR method solution, precoder optimization for the MMF problem can also be done by using exact penalty approach. As stated previously, since the solution of the MMF problem for the broadcasting scenario cannot be solved directly by using SDP tools, it is solved with the help of the QoS problem with an iterative approach.

Equivalent Optimization Problem:

Before defining the equivalent optimization problem, following notation will be defined [20]. Let $\boldsymbol{q} = \begin{bmatrix} \boldsymbol{p_1} \\ \boldsymbol{p_2} \\ \vdots \\ \boldsymbol{p_K} \end{bmatrix}$, then the SINR for the ith user can be expressed as in the following form.

$$SINR_i = \frac{\boldsymbol{p_i}^H \widetilde{\boldsymbol{R}_i} \boldsymbol{p_i}}{\sum_{j=1, j\neq i}^{K} \boldsymbol{p_j}^H \widetilde{\boldsymbol{R}_i} \boldsymbol{p_j} + 1} = \boldsymbol{q}^H \boldsymbol{V_i} \boldsymbol{q}, \tag{3.8}$$

where $\boldsymbol{R_i} = \boldsymbol{H_i}^H \boldsymbol{d_i} \boldsymbol{d_i}^H \boldsymbol{H_i}$, $\widetilde{\boldsymbol{R_i}} = \frac{\boldsymbol{R_i}}{N_0 \boldsymbol{d_i}^H \boldsymbol{d_i}}$ and $\boldsymbol{V_i}$ is an $MK \times MK$ block diagonal matrix which is given by

$$\boldsymbol{V_i} = diag\left(\underbrace{-\widetilde{\boldsymbol{R_i}}, -\widetilde{\boldsymbol{R_i}}, \ldots, -\widetilde{\boldsymbol{R_i}}}_{i-1 \text{ terms}}, \frac{1}{\gamma}\widetilde{\boldsymbol{R_i}}, \underbrace{-\widetilde{\boldsymbol{R_i}}, -\widetilde{\boldsymbol{R_i}}, \ldots, -\widetilde{\boldsymbol{R_i}}}_{K-i \text{ terms}} \right). \tag{3.9}$$

Let $\boldsymbol{X} = \boldsymbol{q}\boldsymbol{q}^H$. Assume that for all users' SINR threshold values are equal to each other, i.e. $\gamma_i = \gamma$. Then, the QoS problem becomes

$$\begin{aligned} &\min_{q} \operatorname{tr}(\boldsymbol{X}) \\ \text{s.t} \quad &\operatorname{tr}(\boldsymbol{X}\boldsymbol{V_i}) \geq 1 \\ &\operatorname{rank}(\boldsymbol{X}) = 1 \\ &\boldsymbol{X} \succeq 0. \end{aligned} \tag{3.10}$$

In order to solve the problem by SDP tools, rank-one constraint in the optimization problem can be expressed in a different manner instead of omitting it. Similar to Chapter 2, it can be done by solving an equivalent problem which

gives the same optimization result with the original QoS problem. With the help of equality case of Lemma 2.2 in Chapter 2, (i.e. $(\mathrm{tr}\,(\boldsymbol{X_1})\,\mathrm{tr}\,(\boldsymbol{X_2}) = \mathrm{tr}\,(\boldsymbol{X_1}\boldsymbol{X_2})))$ which implies that both $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ are rank-one, the equivalent problem can be expressed as

$$\begin{aligned}
\min_{\boldsymbol{X_1},\boldsymbol{X_2}} \ & \mathrm{tr}\,(\boldsymbol{X_1}) + \mathrm{tr}\,(\boldsymbol{X_2}) \\
\text{s.t} \quad & \mathrm{tr}\,(\boldsymbol{X_1}\boldsymbol{V_i}) \geq 1 \\
& \mathrm{tr}\,(\boldsymbol{X_2}\boldsymbol{V_i}) \geq 1 \\
& \mathrm{tr}\,(\boldsymbol{X_1})\,\mathrm{tr}\,(\boldsymbol{X_2}) - \mathrm{tr}\,(\boldsymbol{X_1}\boldsymbol{X_2}) = 0 \\
& \boldsymbol{X_1} \succeq 0, \boldsymbol{X_2} \succeq 0.
\end{aligned} \tag{3.11}$$

When above expressions are analyzed, it is noticed that for either $\boldsymbol{X_1}$ or $\boldsymbol{X_2}$ to be constant, the structure of the optimization problem is the same as that of the original QoS problem. Therefore, alternating optimization idea can be used to solve it. In other words, keep $\boldsymbol{X_1}$ constant and optimize $\boldsymbol{X_2}$. Then keep $\boldsymbol{X_2}$ constant and optimize $\boldsymbol{X_1}$. This process continues until the algorithm converges, where each optimization is found by using SDP tools. As proven in Chapter 2, in the optimum case, $\boldsymbol{X^{opt}} = \boldsymbol{X_1^{opt}} = \boldsymbol{X_2^{opt}}$.

On the other hand, the equivalent statement of the fact that both $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ are rank-one given in Equation 3.11 is not a convex constraint. By using the exact penalty approach, it could be added in cost function, then all constraints become convex. After introducing slack variables $t_1, t_2, \ldots, t_K$ and $s_1, s_2, \ldots, s_K$ in order to obtain equalities, the optimization problem becomes

$$\begin{aligned}
\min_{\boldsymbol{X_1},\boldsymbol{X_2}} \ & \mathrm{tr}\,(\boldsymbol{X_1}) + \mathrm{tr}\,(\boldsymbol{X_2}) + \mu\,(\mathrm{tr}\,(\boldsymbol{X_1})\,\mathrm{tr}\,(\boldsymbol{X_2}) - \mathrm{tr}\,(\boldsymbol{X_1}\boldsymbol{X_2})) \\
\text{s.t} \quad & \mathrm{tr}\,(\boldsymbol{X_1}\boldsymbol{V_i}) - t_i = 1 \ \text{ where } \ t_i \geq 0 \\
& \mathrm{tr}\,(\boldsymbol{X_2}\boldsymbol{V_i}) - s_i = 1 \ \text{ where } \ s_i \geq 0 \\
& \boldsymbol{X_1} \succeq 0, \boldsymbol{X_2} \succeq 0,
\end{aligned} \tag{3.12}$$

where μ is a positive scalar.

The problem can be solved by SDP tools with alternating minimization algorithm [20]. As explained in Chapter 2, the problem can be solvable by SeDuMi when $\boldsymbol{X_1}$ or $\boldsymbol{X_2}$ is kept constant. The inputs $\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}$ can be constructed with

similar idea applied in SDR technique. As a result, precoder vectors $\boldsymbol{p_i}$'s found from the solution of QoS problem give the optimum precoder vectors of MMF problem.

Steps of Alternating Minimization Algorithm are provided below.

Alternating Minimization Algorithm

Initialization: $t = 0$ (t denotes the index of iteration)

Start optimization by keeping either $\boldsymbol{X_1}$ or $\boldsymbol{X_2}$ constant. (Assume $\boldsymbol{X_2}$ is fixed). Set $\boldsymbol{X_2^0} = (\sqrt{\frac{P_{tot}}{KM}})\boldsymbol{I}$.

repeat
$t \rightarrow t + 1$

Step 1) Solve the problem for $\boldsymbol{X_1^{(t)}}$.
Step 2) If rank $(\boldsymbol{X_1}) = 1$, terminate the algorithm.
Step 3) Solve the problem for $\boldsymbol{X_2^{(t)}}$.
Step 4) If rank $(\boldsymbol{X_2}) = 1$, terminate the algorithm.

until $\left\|\boldsymbol{X_1^t} - \boldsymbol{X_1^{t-1}}\right\| \leq \epsilon$ or when the maximum number of iteration is reached.

(Here, ϵ is taken 0.01).

Up to this point, two solution methods based on SeDuMi were expressed. In the following part, other techniques not needing an SDP tool are presented.

3.2.3 SINR Balancing Algorithm based on Uplink-Downlink Duality Theory (UDD Algorithm)

For the solution of the MMF problem, there is also a method in literature which is related to the uplink-downlink duality theory. Before discussing the details of this concept, remember that at the beginning of this chapter it is assumed that optimization problems will be studied under downlink communication scenario and downlink SINR expression for the ith user can be written as

$$SINR_{DL}^{i} = \frac{p_i \boldsymbol{v_i}^H \boldsymbol{R_i} \boldsymbol{v_i}}{\sum_{j=1}^{K} p_j \boldsymbol{v_j}^H \boldsymbol{R_i} \boldsymbol{v_j} + N_0 \boldsymbol{d_i}^H \boldsymbol{d_i}}, \tag{3.13}$$

where $\boldsymbol{R_i} = \boldsymbol{H_i}^H \boldsymbol{d_i} \boldsymbol{d_i}^H \boldsymbol{H_i}$ and $\boldsymbol{v_i}$'s for $i = 1, 2, ..., K$ are defined as unit norm precoder vectors. p_i is the ith downlink precoder power; therefore, the ith precoder is equal to $\boldsymbol{p_i} = \sqrt{p_i} \boldsymbol{v_i}$.

SINR expression can also be defined for the uplink system, in which all users send their information to the BS via a channel. As expressed in [32], decoder vectors in the uplink system model can be thought as the precoders in the downlink system. Therefore, with this dual system model, the signal received by BS denoted by y can be expressed as

$$y = \sum_{j=1}^{K} \boldsymbol{v_j}^H \boldsymbol{H_j}^H \boldsymbol{d_j} b_j + n_j, \tag{3.14}$$

where downlink precoder vectors $\boldsymbol{v_j}$'s can be thought as decoder vectors in the uplink system.

According to this model, uplink SINR written for the ith user signal becomes

$$SINR_{UL}^{i} = \frac{q_i \boldsymbol{v_i}^H \boldsymbol{R_i} \boldsymbol{v_i}}{\boldsymbol{v_i}^H \left(\sum_{k=1, k \neq i}^{K} q_k \boldsymbol{R_k} + N_0 \boldsymbol{I} \right) \boldsymbol{v_i}}, \tag{3.15}$$

where q_k denotes the power of b_k in uplink and $\mathbb{E}\left\{ |b_k|^2 \right\} = q_k$.

When uplink and downlink SINR expressions are analyzed, it is noticed that the uplink SINR for the ith user depends on the ith downlink precoder $\boldsymbol{v_i}$ only.

Therefore, each downlink precoder vector can be optimized independently. Furthermore, the optimum solution of the ith precoder vector can easily be found by using the well-known Rayleigh Quotient expression. On the other hand, downlink SINR for the ith user depends on all precoder vectors in the system which makes optimization very difficult. However, thanks to this easy solution of the dual uplink problem, optimum precoder vectors for the downlink scenario will be found by solving its uplink version. Then, by using the uplink-downlink duality property, solution of downlink problem precoders will also be found by solving its uplink dual version, provided that some conditions are satisfied. The details of them are given in subsequent parts of this section.

As explained above, since the precoder vectors in the SINR expression are thought as unit norm vectors, their power allocation should also be done in order to satisfy the power constraint at the transmitter side. In this algorithm, alternating optimization strategy is used between power allocation and precoder design. In other words, for a given power allocation, optimization of precoder vectors is done. After that, the precoder vectors are fixed and corresponding power allocation is calculated. Furthermore, these optimizations are done for a fixed decoder vector set.

Power Allocation in Downlink

For a given unit norm optimum precoder set, denoted by $\widetilde{\boldsymbol{v_1}}, \widetilde{\boldsymbol{v_2}}, \ldots, \widetilde{\boldsymbol{v_K}}$, MMF problem should also be optimized over downlink power vector denoted by $\boldsymbol{x} = [p_1 \;\; p_2 \;\; \cdots \;\; p_K]^T$. Therefore, the problem is reduced to optimizing $\boldsymbol{x}$ such that

$$C_{DL}(\widetilde{\boldsymbol{v_1}}, \widetilde{\boldsymbol{v_2}}, \ldots, \widetilde{\boldsymbol{v_K}}, P_{tot}) = \max_{\boldsymbol{x}} \min_{1 \leq i \leq K} SINR^i_{DL}\left(\widetilde{\boldsymbol{v_1}}, \widetilde{\boldsymbol{v_2}}, \ldots, \widetilde{\boldsymbol{v_K}}, \boldsymbol{x}\right)$$
$$\text{s.t} \quad \|\boldsymbol{x}\|_1 \leq P_{tot}, \tag{3.16}$$

where C_{DL} denotes the optimal minimum SINR.

In order to find the optimum downlink power solution, Lemma 3.1 is given below stated and proved by [25].

Lemma 3.1 [25]: Given the optimum unit norm precoder solution, the optimum downlink power vector $\boldsymbol{x_{opt}}$ should be chosen such that SINR values of all users are equal to each other and total downlink precoder power is equal to P_{tot} [25]. It is also formulated as

$$C_{DL}(\widetilde{\boldsymbol{v_1}}, \widetilde{\boldsymbol{v_2}}, \ldots, \widetilde{\boldsymbol{v_K}}, P_{tot}) = SINR^i_{DL}\left(\widetilde{\boldsymbol{v_1}}, \widetilde{\boldsymbol{v_2}}, \ldots, \widetilde{\boldsymbol{v_K}}, \boldsymbol{x_{opt}}\right) \quad \text{for} \quad i = 1, 2, \ldots, K$$

$$\|\boldsymbol{x_{opt}}\|_1 = P_{tot}. \tag{3.17}$$

Proof of Lemma 3.1 [25]: Suppose that the ith user SINR becomes larger than $SINR_{min}$ in the system. Then, one can always decrease the power of $\boldsymbol{p_i}$ such that $SINR_i$ is larger than $SINR_{min}$. It is easily noticed from downlink SINR expression, $SINR_{min}$ increases, since $\boldsymbol{p_i}$ term is in the denominator of $SINR_{min}$ expression. This means, after applying this procedure, one can obtain larger minimum SINR value, which is a contradiction. In addition, at optimum point $\|\boldsymbol{x_{opt}}\|_1 \leq P_{tot}$ should be satisfied with equality since C_{DL} is a monotonically increasing function of total transmitter power.

At the optimum, when all equal SINR expressions are written one under the other below equation system is obtained.

$$\boldsymbol{x_{opt}} \frac{1}{C_{DL}} = \boldsymbol{Q\Lambda x_{opt}} + \boldsymbol{Q\sigma}, \tag{3.18}$$

$$\text{where} \quad \boldsymbol{Q} = \begin{bmatrix} \frac{1}{\boldsymbol{v_1^H R_1 v_1}} & 0 & 0 & \ldots & 0 \\ 0 & \frac{1}{\boldsymbol{v_2^H R_2 v_2}} & 0 & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \ldots & \frac{1}{\boldsymbol{v_K^H R_K v_K}} \end{bmatrix}$$

$$\boldsymbol{\Lambda} = \begin{bmatrix} 0 & \boldsymbol{v_2^H R_1 v_2} & \boldsymbol{v_3^H R_1 v_3} & \ldots & \boldsymbol{v_K^H R_1 v_K} \\ \boldsymbol{v_1^H R_2 v_1} & 0 & \boldsymbol{v_3^H R_2 v_3} & \ldots & \boldsymbol{v_K R_2 v_K} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{v_1^H R_K v_1} & \boldsymbol{v_2^H R_K v_2} & \ldots & \ldots & 0 \end{bmatrix}, \boldsymbol{\sigma} = \begin{bmatrix} N_0 \boldsymbol{d_1^H d_1} \\ N_0 \boldsymbol{d_2^H d_2} \\ \vdots \\ N_0 \boldsymbol{d_K^H d_K} \end{bmatrix}.$$

Multiply both sides with an $1 \times K$ vector $\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \end{bmatrix}$ denoted by $\mathbf{1}^T$ and obtain

$$\frac{1}{C^i_{DL}} = \frac{1}{P_{tot}} \mathbf{1}^T \boldsymbol{Q\Lambda x_{opt}} + \frac{1}{P_{tot}} \mathbf{1}^T \boldsymbol{Q\sigma}. \tag{3.19}$$

Then, define an extended downlink power vector denoted by $\boldsymbol{x_{ext}} = \begin{bmatrix} \boldsymbol{x_{opt}} \\ 1 \end{bmatrix}$ and extended coupling matrix denoted by $\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{Q\Lambda} & \boldsymbol{Q\sigma} \\ \frac{1}{P_{tot}}\mathbf{1}^T\boldsymbol{Q\Lambda} & \frac{1}{P_{tot}}\mathbf{1}^T\boldsymbol{Q\sigma} \end{bmatrix}$.

When Equation 3.18 and 3.19 are considered together, they form an eigensystem such that

$$\boldsymbol{\Sigma x_{ext}} = \frac{1}{P_{tot}}\boldsymbol{x_{ext}}. \tag{3.20}$$

As expressed in [33], according to Perron-Frobenius Theory, for any nonnegative matrix $\boldsymbol{A}$, there exists a vector $\boldsymbol{u} \geq 0$ such that $\boldsymbol{Au} = \lambda_{max}\boldsymbol{u}$, where λ_{max} is the maximum eigenvalue of $\boldsymbol{A}$. In other words, the maximum eigenvalue and corresponding eigenvector of $\boldsymbol{A}$ are nonnegative. Furthermore, due to the special structure of the $\boldsymbol{\Sigma}$ matrix, there exists no eigenvalue other than the maximum eigenvalue satisfying the eigensystem given in Equation 3.20. As a result, the maximum value of the minimum SINR in the system is equal to $C_{DL} = \frac{1}{\lambda_{max}(\boldsymbol{\Sigma})}$, where $\lambda_{max}(\boldsymbol{\Sigma})$ represents the maximum eigenvalue of $\boldsymbol{\Sigma}$.

The optimum downlink power $\boldsymbol{x_{opt}}$ is equal to the first K components of the eigenvector corresponding to the maximum eigenvalue of $\boldsymbol{\Sigma}$, where the last entry of the eigenvector is scaled to 1.

Power Allocation in Uplink

Similar to the downlink case, the uplink power allocation can also be achieved provided that downlink precoder vectors $\boldsymbol{v_i}$ for $i = 1, 2, \ldots, K$ will be used as decoders at the uplink side along with the same total power constraint. That is, the power optimization problem in the uplink scenario becomes

$$\begin{aligned} C_{UL}(\widetilde{\boldsymbol{v_1}}, \widetilde{\boldsymbol{v_2}}, \ldots, \widetilde{\boldsymbol{v_K}}, P_{tot}) = \max_{y} \quad \min_{1 \leq i \leq K} \quad & SINR_{UL}^{i}(\widetilde{\boldsymbol{v_1}}, \widetilde{\boldsymbol{v_2}}, \ldots, \widetilde{\boldsymbol{v_K}}, \boldsymbol{y_{opt}}) \\ \text{s.t} \quad & \|\boldsymbol{y_{opt}}\|_1 \leq P_{tot}, \end{aligned} \tag{3.21}$$

where $\boldsymbol{y_{opt}}$ is optimum uplink power vector and $\boldsymbol{y_{opt}} = [q_1 \;\; q_2 \;\; \cdots \;\; q_K]^T$.

After applying the same derivation steps as in the downlink scenario, the eigensystem for the uplink can be written as

$$\boldsymbol{\Omega y_{ext}} = \frac{1}{P_{tot}}\boldsymbol{y_{ext}}, \tag{3.22}$$

where $\boldsymbol{y_{ext}} = \begin{bmatrix} \boldsymbol{y_{opt}} \\ 1 \end{bmatrix}$ and last entry of $\boldsymbol{y_{ext}}$ is equal to 1.

Then, in the optimum case for the uplink scenario, maximum value of the minimum uplink SINR is equal to $C_{UL} = \frac{1}{\lambda_{max}(\Omega)}$, where $\boldsymbol{\Omega}$ is defined as the extended uplink coupling matrix

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{Q\Lambda}^T & \boldsymbol{Q\sigma} \\ \frac{1}{P_{tot}}\mathbf{1}^T\boldsymbol{Q\Lambda}^T & \frac{1}{P_{tot}}\mathbf{1}^T\boldsymbol{Q\sigma} \end{bmatrix}. \tag{3.23}$$

As a result, the uplink power vector $\boldsymbol{y_{opt}}$ is equal to the first K elements of the principal eigenvector of $\boldsymbol{\Omega}$, whose last entry is scaled to be equal to 1.

Precoder Design

After completing the power allocation task for a given precoder set, the optimization of precoder vectors can be done. At this point, optimization will be carried out by fixing power values and finding the corresponding optimum unit norm precoder vector set. Thus, optimization problem can be expressed as

$$C_{DL}(P_{tot}) = \max_{\widetilde{\boldsymbol{v_1}},\widetilde{\boldsymbol{v_2}},\ldots,\widetilde{\boldsymbol{v_K}}} \min_{1\leq i\leq K} SINR^i_{DL}\left(\widetilde{\boldsymbol{v_1}},\widetilde{\boldsymbol{v_2}},\ldots,\widetilde{\boldsymbol{v_K}},\boldsymbol{x_{opt}}\right). \tag{3.24}$$

For the solution of the precoder design problem, the famous theorem called as uplink-downlink duality will be used.

Theorem 3.1 (Uplink-Downlink Duality) [32]: The solution of precoder vectors at the downlink side, namely decoder vectors in the uplink side are equal to each other provided that total power constraint in downlink and in uplink are the same. Furthermore, receiver noise values for both uplink and downlink should also be identical for all users in the system. The detailed proof of this theorem is given in [32].

By using the equivalence of the beamforming vector set both for uplink and downlink sides, one can find the optimum downlink precoders by solving the problem at the uplink side which is an easier way, since each user's SINR expression can be optimized independently. In addition, even if the receiver noise levels are not equal for all users, this duality theory can still be applied. By defining $\widetilde{\boldsymbol{R_i}} = \frac{\boldsymbol{R_i}}{N_{0_i}}$ where N_{0_i} represents the noise power for the ith user, effect

of unequal noise terms can be removed by optimizing uplink SINR expressions over $\widetilde{\boldsymbol{R_i}}$ terms.

Then, uplink SINR for user i becomes as in the following:

$$SINR_{UL}^{i} = \frac{q_i \boldsymbol{v_i}^H \widetilde{\boldsymbol{R_i}} \boldsymbol{v_i}}{\boldsymbol{v_i}^H \left(\sum_{k=1,k\neq i}^{K} q_k \widetilde{\boldsymbol{R_k}} + \boldsymbol{I} \right) \boldsymbol{v_i}}. \tag{3.25}$$

Thanks to the independent maximization of all SINR terms, the joint optimization problem is reduced to solve K independent optimization problems. In fact, each optimization problem is in a well-known special form, namely the Rayleigh quotient. Therefore, the ith optimum unit norm precoder vector is equal to

$$\boldsymbol{v_i^*} = \arg\max_{\boldsymbol{v_i}} \frac{\boldsymbol{v_i}^H \widetilde{\boldsymbol{R_i}} \boldsymbol{v_i}}{\boldsymbol{v_i}^H \boldsymbol{Z_i} \boldsymbol{v_i}},$$

$$\text{where} \quad \boldsymbol{Z_i} = \sum_{k=1,k\neq i}^{K} \boldsymbol{q_k} \widetilde{\boldsymbol{R_k}} + \boldsymbol{I} \quad \text{for} \quad i = 1, 2, \ldots, K. \tag{3.26}$$

The solution of the above expression is equal to the generalized principal unit norm eigenvector of $\widetilde{\boldsymbol{R_i}}$ and $\boldsymbol{Z_i}$.

After optimizing precoder vectors, again power allocation task is applied. This alternating optimization idea continues until the algorithm converges. At the convergence point, the difference between current iteration principal generalized eigenvalue of $\widetilde{\boldsymbol{R_i}}$ and $\boldsymbol{Z_i}$ denoted by λ_{max} and that of previous iteration is smaller than a certain threshold value, which means that the C_{DL} term nearly does not change anymore.

The summary of the algorithmic steps is given in following part.

Joint Precoder/Decoder Design Algorithm based on Uplink-Downlink Duality

Initialization: $t = 0$, $l = 0$ (t denotes the index of precoder iteration, l denotes the index of the joint precoder/decoder iteration)
Initialize the decoder of each user $\boldsymbol{d_i^l} = [\ 1 \quad 0 \quad \cdots \quad 0\ ]^T$ for $i = 1, 2, \ldots, K$. Set uplink power vector as all zero vector, $\boldsymbol{y} = [\ 0 \quad 0 \quad \cdots \quad 0\ ]^T$.

repeat

$l \rightarrow l+1$

repeat

$t \rightarrow t+1$

Step 1) For a given uplink power vector $\boldsymbol{y}$, calculate the optimum unit norm precoder vectors $\boldsymbol{v_i}$'s by optimizing each uplink SINR expression independently for $i = 1, 2, \ldots, K$. Namely, take the generalized unit norm principal eigenvector of $\left(\widetilde{\boldsymbol{R_i}}, \boldsymbol{Z_i}\right)$ as the optimum the precoder vector $\boldsymbol{v_i}$.

Step 2) In order to find the uplink power vector, make uplink power allocation by solving the eigensystem given in Equation 3.22. Scale $\boldsymbol{y}_{ext}$ so that its last entry is equal to 1. Then take the first K entries of $\boldsymbol{y}_{ext}$ as the uplink power vector.

until $\lambda_{max}^{(t)} - \lambda_{max}^{(t-1)} \leq \epsilon_1$.

Step 3) When completing the precoder optimization in the uplink part, solution of the optimum unit norm precoders, $\boldsymbol{v_i}$'s for $i = 1, 2, \ldots, K$, for the downlink problem is automatically found by uplink-downlink duality. After that, make downlink power allocation by solving the eigensystem given in Equation 3.20 in order to find the precoder vectors $\boldsymbol{p_i}$ for $i = 1, 2, \ldots, K$.

Step 4) For a given precoder set, decoder optimization is carried out by using Rayleigh Quotient decoder design explained at the beginning of Chapter 3.

until $SINR_{min}^{(l)} - SINR_{min}^{(l-1)} \leq \epsilon_2$ or when the maximum number of iteration is reached.

(Here, ϵ_1 and ϵ_2 values are assumed to be equal to 0.05).

3.2.4 Joint Precoder/Decoder Design Algorithm with Decomposition Approach (SVD-Dec Algorithm)

In the uplink-downlink duality (UDD) method, we noted that due to the interference term in the downlink SINR expression, design of precoders is not a trivial task. Contrary to the uplink case, the ith user downlink SINR expression depends on all precoder vectors which makes the precoder optimization hard. If there were no interference term in the SINR expression, the problem would become much easier to solve. In fact, the main idea of this method is based on this point. In other words, thanks to the interference nulling, it is understood that that the ith user SINR term only depends on $\boldsymbol{p_i}, \boldsymbol{d_i^H}$ and $\boldsymbol{H_i}$, which implies that the multiuser MIMO system can be decomposed into parallel single user MIMO subsystems. With the help of this separation, each user can specify his own beamforming vector; therefore, all precoders and decoders are designed in a fast and parallel manner.

However, different from previous three methods for the MMF problem, in order to decouple the system into subsystems, it should be noticed that each user has an individual power constraint at the transmitter side. In this algorithm, it is assumed that the total transmitter power is divided equally between users, namely $\text{tr}\left(\boldsymbol{p_i}\boldsymbol{p_i^H}\right) \leq P_{tot}/K$. This implies that no power allocation is achieved. In addition, there is another important difference in the design of precoders and decoders. Thanks to the parallel single user systems, precoders and decoders can be independently optimized. In other words, precoders or decoders are not found by keeping one of them constant, both of them are designed at the same time [26].

Precoder Design

In order to make interference term zero, $\boldsymbol{p_i}$'s should satisfy the following equations.

$$\boldsymbol{p_1}, \boldsymbol{p_2}, ...\boldsymbol{p_K} = \underset{\mathrm{tr}\left(\sum_{i=1}^{K} \boldsymbol{p_i}\boldsymbol{p_i}^H\right) \leq P_{tot}}{\arg} \left(\left\{ \begin{array}{c} \boldsymbol{H_1} \sum\limits_{j=1, j\neq 1}^{K} \boldsymbol{p_j} b_j = \boldsymbol{0} \\ \boldsymbol{H_2} \sum\limits_{j=1, j\neq 2}^{K} \boldsymbol{p_j} b_j = \boldsymbol{0} \\ \vdots \\ \boldsymbol{H_K} \sum\limits_{j=1, j\neq K}^{K} \boldsymbol{p_j} b_j = \boldsymbol{0} \end{array} \right\} \right). \tag{3.27}$$

For the ith user, $\boldsymbol{H_i} \sum\limits_{j=1, j\neq i}^{K} \boldsymbol{p_j} b_j = \boldsymbol{0}$ implies that this expression is equal to zero for all b_j with $j \neq i$. Furthermore, b_j's are some scalars which can be arbitrarily chosen; therefore, it can also be said that $\boldsymbol{H_i}\boldsymbol{p_j} = \boldsymbol{0}$ for $i \neq j$. As a result, the solution for the ith user unit norm precoder $\boldsymbol{u_i}$ is equivalent to the solution of the problem which is given in Equation 3.28.

$$\boldsymbol{p_i} = \underset{\mathrm{tr}\left(\sum_{i=1}^{K} \boldsymbol{p_i}\boldsymbol{p_i}^H\right) \leq P_{tot}}{\arg} \left(\left\{ \begin{array}{c} \boldsymbol{H_1}\boldsymbol{p_i} = \boldsymbol{0} \\ \boldsymbol{H_2}\boldsymbol{p_i} = \boldsymbol{0} \\ \vdots \\ \boldsymbol{H_{i-1}}\boldsymbol{p_i} = \boldsymbol{0} \\ \boldsymbol{H_{i+1}}\boldsymbol{p_i} = \boldsymbol{0} \\ \vdots \\ \boldsymbol{H_K}\boldsymbol{p_i} = \boldsymbol{0} \end{array} \right\} \right) \text{ for } i = 1, 2, \ldots, K. \tag{3.28}$$

The above equalities related to interference cancellation implies that $\boldsymbol{p_i}$ will be in the intersection of the null spaces of the $\boldsymbol{H_k}$ for $k = 1, 2, \ldots, K$ and $k \neq i$ which is shown mathematically as

$$\boldsymbol{p_i} \in \bigcap_{k=1, k\neq i}^{K} \ker\left(\boldsymbol{H_k}\right). \tag{3.29}$$

The solution is equivalent to choose precoder vectors from the null space of $\widehat{\boldsymbol{H_i}}$

which is defined as

$$\widehat{\boldsymbol{H_i}} = \begin{bmatrix} \boldsymbol{H_1} \\ \boldsymbol{H_2} \\ \vdots \\ \boldsymbol{H_{i-1}} \\ \boldsymbol{H_{i+1}} \\ \vdots \\ \boldsymbol{H_K} \end{bmatrix}, \tag{3.30}$$

where $\widehat{\boldsymbol{H_i}}$ is a $(K-1)\,N \times M$ matrix where N is the number of receiver antennas for all users.

From the rank-nullity theorem applied on $\widehat{\boldsymbol{H_i}}$, one can say that

$$dim\,(\mathcal{R}) + dim\,(\mathcal{N}) = M, \tag{3.31}$$

where $\mathcal{R}$ and $\mathcal{N}$ denote the range and null spaces of $\widehat{\boldsymbol{H_i}}$, respectively.

Due to the independent identically distributed entries of $\widehat{\boldsymbol{H_i}}$, it can be stated that $\widehat{\boldsymbol{H_i}}$ is a full-rank matrix with probability one. Therefore,

$$\text{rank}(\widehat{\boldsymbol{H_i}}) = \min\{(K-1)\,N, M\}. \tag{3.32}$$

As a result,

$$dim\,(\mathcal{N}) = \max\{0, M-(K-1)\,N\}. \tag{3.33}$$

Furthermore, in order to find a non trivial solution of precoder vector, $dim\,(\mathcal{N})$ should be greater than zero. When these two requirements are evaluated together, it is implied that

$$M > (K-1)\,N, \tag{3.34}$$

which is a constraint on the number of transmitter antennas in the system. Therefore, when the number of receiver antennas increases, the number of transmitter antennas should also be increased for the proper operation of this algorithm, which may be practically infeasible.

Let precoder vector $\boldsymbol{p_i}$ be in the form of $\boldsymbol{p_i} = \boldsymbol{T_i}\boldsymbol{q_i}$. Suppose that the columns of $\boldsymbol{T_i}$ matrix are chosen as an orthonormal basis for $\bigcap_{k=1,k\neq i}^{K} \ker(\boldsymbol{H_k})$. After selection of $\boldsymbol{T_i}$'s, according to the cost function of the system design, $\boldsymbol{q_i}$ component

of the precoder vector $\boldsymbol{p_i}$ can also be specified. As a result, the received signal becomes

$$r_i = \boldsymbol{d_i}^H \boldsymbol{H_i} \boldsymbol{T_i} \boldsymbol{q_i} b_i + \boldsymbol{d_i}^H \boldsymbol{n_i}. \tag{3.35}$$

As can be noted from above formula, the received signal for the ith user only depends on the ith user decoder $\boldsymbol{d_i}$, channel matrix $\boldsymbol{H_i}$ and receiver noise $\boldsymbol{n_i}$. This result leads to decompose multiuser system into smaller subsystems. Therefore, it is possible to divide downlink multiuser MIMO channel into K many parallel independent single user downlink MIMO channels. All techniques which are proper for single user MIMO systems can also be applied for the solution of this multiuser MIMO problem.

With this method, it can be stated that the precoder and decoder design is done at the same time. Firstly, design of $\boldsymbol{T_i}$ (required for precoder vector $\boldsymbol{p_i}$) will be considered. This can be achieved by applying SVD on $\widehat{\boldsymbol{H_i}}$.

$$\widehat{\boldsymbol{H_i}} = \begin{bmatrix} \boldsymbol{H_1} \\ \boldsymbol{H_2} \\ \vdots \\ \boldsymbol{H_{i-1}} \\ \boldsymbol{H_{i+1}} \\ \vdots \\ \boldsymbol{H_K} \end{bmatrix} = \boldsymbol{U_i} \boldsymbol{S_i} \begin{bmatrix} \widetilde{\boldsymbol{T_i}}^H \\ \boldsymbol{T_i}^H \end{bmatrix}, \tag{3.36}$$

where $\widetilde{\boldsymbol{T_i}}$ is $M \times u$ matrix in which $u = M - (K-1)N$. After that, the solution for the term $\boldsymbol{q_i}$ for the precoder design and decoder vectors is given. Thanks to the interference cancellation procedure, SINR terms turn into SNR. Let $\widetilde{\boldsymbol{H_i}} = \boldsymbol{H_i} \boldsymbol{T_i}$. Then SNR term for user i can be written as

$$SNR_i = \frac{\left\| \boldsymbol{d_i}^H \widetilde{\boldsymbol{H_i}} \boldsymbol{q_i} \right\|^2}{N_0 \left\| \boldsymbol{d_i} \right\|^2}. \tag{3.37}$$

Note that, at the beginning of this section, the aim is maximizing the minimum SINR in the system. After converting multiuser MIMO into K parallel single-user MIMO systems, the SNR maximization of each user can be done independently; therefore, it can be thought that the minimum SNR in the system is also tried to be maximized. This may be a suboptimal solution method for

original problem; however, it has advantages in terms of computation time of precoder/decoder optimization.

Decoder Design

As noticed from the SNR_i expression, power of decoder vectors does not change SNR_i term. Therefore, decoder vectors can be considered as a unit norm, i.e. $\|\boldsymbol{d_i}\|^2 = 1$.

Let $\boldsymbol{d_i} = \boldsymbol{x}$ and $\boldsymbol{q_i} = \boldsymbol{y}$ in which $\boldsymbol{x}$ and $\boldsymbol{y}$ are $N \times 1$ and $u \times 1$ vectors, respectively where $u = M - (K-1)N$.

Then we apply SVD method on $\widetilde{\boldsymbol{H_i}}$ such that $\widetilde{\boldsymbol{H_i}} = \boldsymbol{A_i}\boldsymbol{\Sigma_i}\boldsymbol{B_i}$. Let $\boldsymbol{x_0} = \boldsymbol{A_i^H}\boldsymbol{x}$ and $\boldsymbol{y_0} = \boldsymbol{B_i}\boldsymbol{y}$. Therefore, SNR for i^{th} user becomes

$$SNR_i = \frac{\left\|\boldsymbol{x_0^H}\boldsymbol{\Sigma_i}\boldsymbol{y_0}\right\|^2}{N_0}. \tag{3.38}$$

At the beginning of this chapter, it is assumed that power is allocated equally; therefore, $\|\boldsymbol{p_i}\|^2 = \frac{P_{tot}}{K}$. This implies that

$$\|\boldsymbol{p_i}\|^2 = \mathrm{tr}\left(\boldsymbol{T_i}\boldsymbol{q_i}\boldsymbol{q_i^H}\boldsymbol{T_i^H}\right) = \mathrm{tr}\left(\boldsymbol{T_i}\boldsymbol{y}\boldsymbol{y^H}\boldsymbol{T_i^H}\right) = P_{tot}/K \rightarrow \|\boldsymbol{y}\|^2 = \frac{P_{tot}}{K}, \tag{3.39}$$

where $\boldsymbol{T_i}$ is a unitary matrix. Moreover, since $\boldsymbol{A_i}$ and $\boldsymbol{B_i}$ are unitary matrices, $\|\boldsymbol{y_0}\| = \sqrt{\frac{P_{tot}}{K}}$. Furthermore,

$$\boldsymbol{\Sigma_i} = diag\left(e_1, e_2, \ldots, e_K\right),$$

where $e_1, e_2, \ldots, e_K$ are singular values of Σ_i with $e_1 \geq e_2 \geq \cdots \geq e_K$. Define $\boldsymbol{z_0} = \sqrt{\frac{K}{P_{tot}}}\boldsymbol{y_0}$ which is a unit norm vector. When all these definitions are substituted, the resulting SNR maximizing expression becomes

$$\max_{\boldsymbol{x_0},\boldsymbol{z_0}} \left\|\boldsymbol{x_0^H}\boldsymbol{\Sigma_i}\boldsymbol{z_0}\right\|$$

$$\text{s.t} \quad \|\boldsymbol{x_0}\| = 1, \quad \|\boldsymbol{z_0}\| = 1. \tag{3.40}$$

Let denote the entries of $\boldsymbol{x_0}$ and $\boldsymbol{z_0}$ s.t $\boldsymbol{x_0} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$ and $\boldsymbol{z_0} = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_u \end{bmatrix}$. Then,

optimization problem expression becomes

$$\max \left| \sum_{j=1}^{min\{N,u\}} e_j x_j^* z_j \right| . \tag{3.41}$$

Then, by using Cauchy-Schwarz Inequality, it can be written as

$$\left| \sum_{j=1}^{min\{N,u\}} e_j x_j^* z_j \right| \leq \sum_{j=1}^{min\{N,u\}} \left| e_j x_j^* z_j \right| = \sum_{j=1}^{min\{N,u\}} |e_j| \left| x_j^* z_j \right| \leq |e_1| \sum_{j=1}^{min\{N,u\}} \left| x_j^* z_j \right| , \tag{3.42}$$

where e_1 is the maximum singular value. Then,

$$|e_1| \sum_{j=1}^{min\{N,u\}} \left| x_j^* z_j \right| \leq |e_{max}| \underbrace{\sqrt{\left(\sum_{j=1}^{min\{N,u\}} |x_j|^2 \right) \left(\sum_{j=1}^{min\{N,u\}} |z_j|^2 \right)}}_{1} \leq e_1, \tag{3.43}$$

since $\boldsymbol{x_0}$ and $\boldsymbol{z_0}$ are unit-norm vectors. This result states that maximum value of the ith user SNR can be at most the maximum singular value e_1. This bound can be achieved by choosing

$$\boldsymbol{x_0} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{z_0} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} . \tag{3.44}$$

As a result, $\boldsymbol{x} = \boldsymbol{A_i x_0}$ and $\boldsymbol{y} = \sqrt{\frac{P_{tot}}{K}} \boldsymbol{B_i z_0}$. Therefore, the optimal decoder vector $\boldsymbol{d_i} = \boldsymbol{x}$ and precoder defining vector $\boldsymbol{q_i} = \boldsymbol{y}$ is found. In other words, the decoder vector $\boldsymbol{d_i^H}$ is equal to Hermitian of the principal left singular vector of $\widetilde{\boldsymbol{H_i}}$ and $\boldsymbol{q_i}$ is equal to principal right singular vector of $\widetilde{\boldsymbol{H_i}}$ scaled with $\sqrt{\frac{P_{tot}}{K}}$. Finally, $\boldsymbol{q_i}$ vector should be substituted into $\boldsymbol{p_i} = \boldsymbol{T_i q_i}$ expression in order to find precoder vector.

Joint Precoder/Decoder Design Algorithm with Decomposition Approach

Step 1) Calculate orthonormal basis vectors $\boldsymbol{T_i}$ for $i = 1, 2, ...K$ in order to cancel out interference terms.

Step 2) Calculate $\boldsymbol{q_i}$ by taking principal right singular vector of $\widetilde{\boldsymbol{H_i}}$ and scale it by $\sqrt{\frac{P_{tot}}{K}}$ for $i = 1, 2, ...K$.

Step 3) Calculate decoder vectors $\boldsymbol{d_i}$'s by taking Hermitian of principal left singular vectors of $\widetilde{\boldsymbol{H_i}}$ for $i = 1, 2, ...K$.

3.2.5 SVD Based Iterative Joint Precoder/Decoder Design Method (SVD-Iter Algorithm)

In the previous joint precoder/decoder design algorithm based on the decomposition approach, in order to eliminate the interference term, there is a strict constraint on the number of the transmitter antennas. This is a strictly limiting constraint in practice. In order to reduce the impact of this constraint, a different algorithm is proposed which is one of the novelties of this thesis. According to this new method, the constraint on the number of transmitter antennas only depends on the number of users, not on the number of receiver antennas. Similarly, optimization is done jointly but with an iterative manner. Furthermore, different from the previous decomposition algorithm, the power allocation is also optimized. Details of design procedure is given in the following part. Iterative design approach is also used between power allocation and precoder design. Therefore, in the precoder design part, it should be pointed out that precoders are defined as the unit norm vectors. The power allocation will be done after the unit norm precoders are designed.

Precoder Design

Let unit norm precoder vector $\boldsymbol{u_i}$ be in the form of $\boldsymbol{u_i} = \boldsymbol{T_i q_i}$. For a given set of decoder vectors, in order to eliminate interference, $\boldsymbol{u_i}$ should be chosen as an orthonormal basis vector of intersection of null spaces of $\boldsymbol{d_k^H H_k}$ for $k = 1, 2, \ldots, i-1, i+1, \ldots, K$, that is

$$\boldsymbol{u_i} \in \bigcap_{k=1, k\neq i}^{K} \ker\left(\boldsymbol{d_k^H H_k}\right). \tag{3.45}$$

It is equivalent to choose $\boldsymbol{u_i}$ from the null space of $\widetilde{\boldsymbol{H_i}}$ where

$$\widetilde{\boldsymbol{H_i}} = \begin{bmatrix} \boldsymbol{d_1^H H_1} \\ \boldsymbol{d_2^H H_2} \\ \vdots \\ \boldsymbol{d_K^H H_K} \end{bmatrix}. \tag{3.46}$$

In order to have a non-trivial solution for $\boldsymbol{u_i}$, the dimension of null space of $\widetilde{\boldsymbol{H_i}}$ should be a nonzero value. From the rank-nullity theorem for $\widetilde{\boldsymbol{H_i}}$ matrix having

dimensions $(K-1) \times M$, one can say that

$$dim\left(\mathcal{R}\right) + dim\left(\mathcal{N}\right) = M, \tag{3.47}$$

where $\mathcal{R}$ and $\mathcal{N}$ represents the range and null spaces of $\widetilde{\boldsymbol{H_i}}$, respectively.

On the other hand, since $\widetilde{\boldsymbol{H_i}}$ is $(K-1) \times M$ matrix, and due to its independent identically distributed entries, rank $\left(\widetilde{\boldsymbol{H_i}}\right)$ becomes $\min\left\{K-1, M\right\}$ with probability one. Therefore,

$$dim\left(\mathcal{N}\right) = M - (K-1) > 0 \rightarrow M > (K-1)\,, \tag{3.48}$$

for the existence of a non-trivial precoder vector $\boldsymbol{u_i}$.

In the decomposition method, the design of precoder and decoder is achieved together with the SVD. In this algorithm, in order to overcome the number of transmitter antenna constraint, the effect of decoder is taken into account together with the channel matrix by considering an equivalent channel. However, this approach makes precoder/decoder optimization iterative. However, the restriction on the number of transmitter antennas reduces to the number of users, which is practically more advantageous.

As a result, by choosing the columns of $\boldsymbol{T_i}$ component of precoder vector as the orthonormal basis for the null space of $\widetilde{\boldsymbol{H_i}}$, interference term can be canceled out. The remaining $\boldsymbol{q_i}$ term is designed by the optimization criteria of the system. After the interference cancellation procedure, the SINR expression for i^{th} user can be stated as SNR and it can be written in terms of $\boldsymbol{T_i}$ and $\boldsymbol{q_i}$

$$SNR_i = \frac{p_i \boldsymbol{d_i}^H \boldsymbol{H_i} \boldsymbol{T_i} \boldsymbol{q_i} \boldsymbol{q_i}^H \boldsymbol{T_i}^H \boldsymbol{H_i}^H \boldsymbol{d_i}}{N_0 \boldsymbol{d_i}^H \boldsymbol{d_i}}. \tag{3.49}$$

Since $\boldsymbol{u_i}$ is unit norm, $\|\boldsymbol{u_i}\|^2 = 1$ which implies that $\|\boldsymbol{q_i}\|^2 = 1$ and hence SNR_i can also be expressed as

$$SNR_i = \frac{p_i \boldsymbol{q_i}^H \underbrace{\boldsymbol{T_i}^H \boldsymbol{H_i}^H \boldsymbol{d_i} \boldsymbol{d_i}^H \boldsymbol{H_i} \boldsymbol{T_i}}_{\boldsymbol{W_1}} \boldsymbol{q_i}}{\boldsymbol{q_i}^H \underbrace{N_0 \boldsymbol{d_i}^H \boldsymbol{d_i}}_{\boldsymbol{W_2}} \boldsymbol{q_i}}, \tag{3.50}$$

which is a standard Rayleigh quotient format; therefore, $\boldsymbol{q_i}$ should be chosen as a generalized unit norm principal eigenvector of $\boldsymbol{W_1}$ and $\boldsymbol{W_2}$.

Power Allocation

Lemma 3.2: In the optimum case, the total transmitter power should be allocated among the precoders such that all users' SNR are identical.

Proof of Lemma 3.2: Suppose that i^{th} user SNR is larger than minimum SNR in the system. Decrease p_i provided that SNR_i is not below the minimum SNR. Therefore, total precoder power decreases which means with smaller total power one can obtain the same optimum minimum SNR which is a contradiction. ■

Let

$$\boldsymbol{X_i} = \frac{\boldsymbol{d_i}^H \boldsymbol{H_i} \boldsymbol{u_i} \boldsymbol{u_i}^H \boldsymbol{H_i}^H \boldsymbol{d_i}}{N_0 \boldsymbol{d_i}^H \boldsymbol{d_i}}.$$

Equating the SNR of all users $p_i \boldsymbol{X_i} = c$ for $i = 1, 2, \ldots, K$ under the constraint of total power equality $\sum_{i=1}^{K} p_i = P_{tot}$, we obtain the following equation system. Precoder power values p_k's are the solution of this system.

$$\begin{bmatrix} \boldsymbol{X_1} & 0 & \ldots & 0 & -1 \\ 0 & \boldsymbol{X_2} & \ldots & 0 & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \boldsymbol{X_K} & -1 \\ 1 & 1 & \ldots & 1 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_K \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ P_{tot} \end{bmatrix}. \tag{3.51}$$

After finding the optimum precoder vectors for a given set of decoders, the decoder vector optimization is done. This process continues until the algorithm converges. Algorithmic steps are provided in below part.

SVD Based Joint Precoder Decoder Design Algorithm

Initialization: $t = 0$ (t denotes the index of iteration)
Set all decoder vectors $\boldsymbol{d_i}^{(0)}$ for $i = 1, 2, \ldots, K$ to $\boldsymbol{I}_{N\times 1} = [\ 1 \quad 0 \quad 0 \quad \cdots \quad 0\ ]^T$.
repeat
$t \rightarrow t + 1$

Step 1) Calculate unit norm precoder vectors $\boldsymbol{p_i}^{(t)}$ for $i = 1, 2, \ldots, K$ by using decoder vectors of each user $\boldsymbol{d_i}^{(t-1)}$.

Step 2) Make power allocation for precoder vectors.

Step 3) Calculate decoder vectors $\boldsymbol{d_i}^{(t)}$ by using precoder vectors calculated in Step 1.

until $\left|SINR_{min}^{(t)} - SINR_{min}^{(t-1)}\right| \leq \epsilon$ or $t = t_{max}$ where t_{max} is the maximum number of iterations.

Here, ϵ is taken as 0.05 and $SINR_{min}$ denotes the minimum SINR in the system after decoder optimization.

3.3 Minimization of Total Mean Square Error (TMSE) Problem

In this section, the minimization of total mean square error problem (under a total power constraint) will be analyzed for the scenario of private information transmission. As in Chapter 2, the joint optimization of precoders and decoders is considered. Here, two different methods will be analyzed and results will be compared in terms of their total MSE and BER performances.

3.3.1 Joint Precoder-Decoder Design Method (Joint-TMSE Algorithm)

In this algorithm, under a total power constraint P_{tot} at the transmitter side, total MSE is minimized by designing precoders at the transmitter and decoders at the receiver, iteratively. Mathematical representation of the optimization problem is

$$\min_{\boldsymbol{p_1},\boldsymbol{p_2},\ldots,\boldsymbol{p_K},\boldsymbol{d_1},\boldsymbol{d_2},\ldots,\boldsymbol{d_K}} \sum_{i=1}^{K} MSE_i$$
$$s.t. \quad \sum_{i=1}^{K} \boldsymbol{p_i}^H \boldsymbol{p_i} = P_{tot}. \tag{3.52}$$

The MSE for i^{th} user can be expressed as in the following formula:

$$MSE_i = \mathbb{E}\left\{|r_i - b_i|^2\right\} \quad \text{for} \quad i = 1, 2, \ldots, K.$$
$$MSE_i = \mathbb{E}\left\{|r_i|^2\right\} + \mathbb{E}\left\{|b_i|^2\right\} - \mathbb{E}\left\{r_i b_i^*\right\} - \mathbb{E}\left\{r_i^* b_i\right\}. \tag{3.53}$$

Assume that b_i's are zero mean i.i.d., $\boldsymbol{n_i}$'s are zero mean with variance N_0 and b_i's and $\boldsymbol{n_i}$'s are independent of each other for $i = 1, 2, \ldots, K$.

When r_i given in Equation 3.1 is substituted in the MSE formula, the above expression can be written as in the following form.

1st term:

$$\begin{aligned}\mathbb{E}\left\{|r_i|^2\right\} &= \boldsymbol{d_i}^H\left(\boldsymbol{H_i}\sum_{j=1}^{K}\boldsymbol{p_j}b_j+\boldsymbol{n_i}\right)\left(\sum_{l=1}^{K}\boldsymbol{b_l}^*\boldsymbol{p_l}^H\boldsymbol{H_i}^H+\boldsymbol{n_i}^H\right)\boldsymbol{d_i}\\ &= \boldsymbol{d_i}^H\boldsymbol{H_i}\sum_{j=1}^{K}\boldsymbol{p_j}\underbrace{\mathbb{E}\{b_ib_i^*\}}_{1}\boldsymbol{p_j}^H\boldsymbol{H_i}^H\boldsymbol{d_i}+\boldsymbol{d_i}^H\underbrace{E\{\boldsymbol{n_i}\boldsymbol{n_i}^H\}}_{N_0}\boldsymbol{d_i}\\ &= \boldsymbol{d_i}^H\boldsymbol{H_i}\sum_{j=1}^{K}\boldsymbol{p_j}\boldsymbol{p_j}^H\boldsymbol{H_i}^H\boldsymbol{d_i}+N_0\left(\boldsymbol{d_i}^H\boldsymbol{d_i}\right).\end{aligned} \tag{3.54}$$

2nd term:

$$\mathbb{E}\left\{b_ib_i^*\right\}=\mathbb{E}\left\{|b_i|^2\right\}=1. \tag{3.55}$$

3rd term:

$$\begin{aligned}\mathbb{E}\left\{r_ib_i^*\right\} &= \mathbb{E}\left\{\left(\boldsymbol{d_i}^H\left(\boldsymbol{H_i}\sum_{j=1}^{K}\boldsymbol{p_j}b_j+\boldsymbol{n_i}\right)\right)b_i^*\right\}\\ &= \boldsymbol{d_i}^H\left(\boldsymbol{H_i}\sum_{j=1}^{K}\boldsymbol{p_j}\underbrace{\mathbb{E}\left\{b_jb_i^*\right\}}_{\delta[i-j]}\right)\\ &= \boldsymbol{d_i}^H\boldsymbol{H_i}\boldsymbol{p_i}.\end{aligned} \tag{3.56}$$

Since the 4th term is the Hermitian of 3rd term, it could directly be written as

$$\mathbb{E}\left\{r_i^*b_i\right\}=\boldsymbol{p_i}^H\boldsymbol{H_i}^H\boldsymbol{d_i}. \tag{3.57}$$

As a result, for the ith user, the MSE_i can be expressed as in the given formula below.

$$MSE_i=\boldsymbol{d_i}^H\boldsymbol{H_i}\sum_{j=1}^{K}\boldsymbol{p_j}\boldsymbol{p_j}^H\boldsymbol{H_i}^H\boldsymbol{d_i}+N_0\left(\boldsymbol{d_i}^H\boldsymbol{d_i}\right)-\boldsymbol{d_i}^H\boldsymbol{H_i}\boldsymbol{p_i}-\boldsymbol{p_i}^H\boldsymbol{H_i}^H\boldsymbol{d_i}+1. \tag{3.58}$$

In this part, similar to the broadcasting case, optimization will be done by using Lagrange multiplier approach. Therefore, the Lagrange objective function can be expressed as [22]

$$\begin{aligned}L\left(\boldsymbol{p_1},\boldsymbol{p_2},\ldots,\boldsymbol{p_k};\boldsymbol{d_1},\boldsymbol{d_2},\ldots,\boldsymbol{d_k};\lambda\right) &= \lambda\left(\sum_{j=1}^{K}\boldsymbol{p_j}^H\boldsymbol{p_j}-P_{tot}\right)\\ &+\sum_{i=1}^{K}\boldsymbol{d_i}^H\boldsymbol{H_i}\left(\sum_{j=1}^{K}\boldsymbol{p_j}\boldsymbol{p_j}^H\right)\boldsymbol{H_i}^H\boldsymbol{d_i}\\ &-\boldsymbol{d_i}^H\boldsymbol{H_i}\boldsymbol{p_i}-\boldsymbol{p_i}^H\boldsymbol{H_i}^H\boldsymbol{d_i}\\ &+N_0\left(\boldsymbol{d_i}^H\boldsymbol{d_i}\right)+1,\end{aligned} \tag{3.59}$$

where λ represents the Lagrange multiplier. This problem can be solved by taking derivative of Lagrange objective function with respect to each precoder vector $\boldsymbol{p_1}, \boldsymbol{p_2}, \ldots, \boldsymbol{p_K}$ and each decoder vector $\boldsymbol{d_1}, \boldsymbol{d_2}, \ldots, \boldsymbol{d_K}$ and equate the expressions to zero.

Precoder Design

If we take derivative with respect to $\boldsymbol{p_m^*}$ and equate the result to zero, we get

$$\frac{\partial L\left(\boldsymbol{p_1}, \boldsymbol{p_2}, \ldots, \boldsymbol{p_k}; \boldsymbol{d_1}, \boldsymbol{d_2}, \ldots, \boldsymbol{d_k}; \lambda\right)}{\partial \boldsymbol{p_m^*}} = \sum_{i=1}^{K} \boldsymbol{H_i^H d_i d_i^H H_i p_m} - \boldsymbol{H_m^H d_m} + \lambda \boldsymbol{p_m} = 0. \tag{3.60}$$

Decoder Design

If we take derivative with respect to $\boldsymbol{d_m^*}$ and equate the result to zero, we get

$$\begin{aligned}\frac{\partial L\left(\boldsymbol{p_1}, \boldsymbol{p_2}, \ldots, \boldsymbol{p_k}; \boldsymbol{d_1}, \boldsymbol{d_2}, \ldots, \boldsymbol{d_k}; \lambda\right)}{\partial \boldsymbol{d_m^*}} &= \boldsymbol{H_m} \left(\sum_{j=1}^{K} \boldsymbol{p_j p_j^H} \right) \boldsymbol{H_m^H d_m} \\ &\quad + N_0 \boldsymbol{d_m} - \boldsymbol{H_m p_m} = 0.\end{aligned} \tag{3.61}$$

Therefore, the precoder vector $\boldsymbol{p_m}$ and decoder vector $\boldsymbol{d_m}$ becomes

$$\boldsymbol{p_m} = \left(\sum_{i=1}^{K} \boldsymbol{H_i^H d_i d_i^H H_i} + \lambda \boldsymbol{I} \right)^{-1} \boldsymbol{H_m^H d_m} \quad \text{for} \quad m = 1, 2, \ldots, K. \tag{3.62}$$

$$\boldsymbol{d_m} = \left(\boldsymbol{H_m} \left(\sum_{j=1}^{K} \boldsymbol{p_j p_j^H} \right) \boldsymbol{H_m^H} + N_0 \boldsymbol{I} \right)^{-1} \boldsymbol{H_m p_m} \quad \text{for} \quad m = 1, 2, \ldots, K. \tag{3.63}$$

As seen from formulas of precoder and decoder, i^{th} optimum precoder vector $\boldsymbol{p_i}$ depends on optimum decoder vectors $\boldsymbol{d_i}$'s for $i = 1, 2, \ldots, K$. Likewise, the ith optimum decoder vector $\boldsymbol{d_i}$ depends on optimum precoder vectors $\boldsymbol{p_i}$'s for $i = 1, 2, \ldots, K$. Due to this, finding the optimum precoder and the decoder as a closed form solution is computationally hard. In order to solve this problem, an iterative solution algorithm is used whose details are provided below.

Joint Precoder/Decoder Design Algorithm for Transmitting Private Information Scenario

Initialization: $t = 0$ (t denotes the index of iteration)
Set all decoder vectors $\boldsymbol{d_i^{(0)}}$ for $i = 1, 2, \ldots, K$ to $\boldsymbol{I}_{N\times 1} = [\ 1 \quad 0 \quad 0 \quad \cdots \quad 0\ ]^T$.
repeat
$t \rightarrow t+1$

Step 1) Calculate precoder vectors $\boldsymbol{p_i^{(t)}}$ for $i = 1, 2, \ldots, K$ by using decoder vector of each user $\boldsymbol{d_i^{(t-1)}}$.
Step 2) Calculate $\boldsymbol{d_i^{(t)}}$ by using precoder vectors calculated in Step 1.

until $\sum_{i=1}^{K} \left\| \boldsymbol{d_i^{(t)}} - \boldsymbol{d_i^{(t-1)}} \right\|^2 \leq \epsilon$ or $t = t_{max}$ where t_{max} is the maximum number of iterations.

Here, ϵ is taken as 0.0005.

After each precoder and decoder calculation step, the total MSE monotonically decreases as expected. On the other hand, it is lower bounded by zero; therefore, the algorithm is guaranteed to converge. However, it is not guaranteed that the convergence point is the global optimum solution.

Calculation of Lagrange Multiplier for Interfering Broadcasting Scenario

As in Chapter 2, the Lagrange multiplier is calculated by using the constraint

$$\sum_{i=1}^{K} \boldsymbol{p_i^H p_i} = P_{tot}. \tag{3.64}$$

Substituting the precoder formula found in 3.62, we obtain

$$\sum_{i=1}^{K} \mathrm{tr}\left(\left(\sum_{i=1}^{K} \boldsymbol{H_i^H d_i d_i^H H_i} + \lambda \boldsymbol{I} \right)^{-2} \boldsymbol{H_i^H d_i d_i^H H_i} \right) = P_{tot}. \tag{3.65}$$

Let $\boldsymbol{A} = \sum_{i=1}^{K} \boldsymbol{H_i^H d_i d_i^H H_i}$. From the trace properties Equation 3.65 can also be written as

$$\mathrm{tr}\left((\boldsymbol{A} + \lambda \boldsymbol{I})^{-1} \boldsymbol{A} (\boldsymbol{A} + \lambda \boldsymbol{I})^{-1} \right) = P_{tot}. \tag{3.66}$$

By the eigenvalue decomposition of $\boldsymbol{A}$, $\boldsymbol{A} = \sum_{i=1}^{K} a_i \boldsymbol{e_k} \boldsymbol{e_k}^H$ where a_i's are eigenvalues of matrix $\boldsymbol{A}$ and $\boldsymbol{e_k}$'s are orthonormal eigenvectors of $\boldsymbol{A}$, we can write

$$\operatorname{tr}\left(\left(\boldsymbol{A} + \lambda \boldsymbol{I}\right)^{-2} \boldsymbol{A}\right) = \operatorname{tr}\left(\sum_{k=1}^{M} \frac{1}{\left(a_k + \lambda\right)^2} \boldsymbol{e_k} \boldsymbol{e_k}^H \sum_{l=1}^{K} a_l \boldsymbol{e_l} \boldsymbol{e_l}^H\right). \tag{3.67}$$

Since the eigenvectors of $\boldsymbol{A}$ are orthonormal to each other, $\boldsymbol{e_k}^H \boldsymbol{e_l} = 0$ for $k \neq l$ and $\boldsymbol{e_k}^H \boldsymbol{e_l} = 1$ for $k = l$. Then, the Equation 3.67 becomes

$$\sum_{k=1}^{M} \frac{a_k}{\left(a_k + \lambda\right)^2} = P_{tot}. \tag{3.68}$$

As described in Chapter 2, the λ value can be found by using numerical search method in the intervals $\left(-\infty, -a_M\right), \left(-a_M, -a_{M-1}\right), \left(-a_{M-1}, -a_{M-2}\right), \ldots, \left(a_1, +\infty\right)$ in which eigenvalues are sorted such that $a_1 \leq a_2 \leq \cdots \leq a_M$. Furthermore, it is sufficient to search the value of λ in the interval

$$-\sqrt{\frac{\operatorname{tr}\left(\boldsymbol{A}\right)}{P_{tot}}} - a_M \leq \lambda \leq \sqrt{\frac{\operatorname{tr}\left(\boldsymbol{A}\right)}{P_{tot}}} - a_1. \tag{3.69}$$

The proof of this statement is the special case of the one given in "Calculation of Lagrange Multiplier" part in Chapter 2. (In private information scenario, the matrix $\boldsymbol{B}$ is equal to the matrix $\boldsymbol{A}$ with the notation used for this part in Chapter 2.)

3.3.2 Joint Precoder Decoder Design Algorithm Using Matrix Inversion Lemma (Matrix-Inv Algorithm)

In order to find the precoders $\boldsymbol{p_i}$ and decoders $\boldsymbol{d_i}$, the MSE minimization problem is considered. However, the MSE minimization problem is investigated from a different point of view, namely as a Schur-concave optimization problem [24] which is a well-known problem case in the optimization theory. For the solution of this problem, according to the signal model at the beginning of this chapter, the variance for the ith user $\sigma^2_{E_i}$ can be defined in Equation 3.70 as

$$\begin{aligned}\sigma^2_{E_i} &= \mathbb{E}\left\{|r_i - b_i|^2\right\} \\ &= \boldsymbol{d_i}^H \boldsymbol{H_i} \sum_{j=1}^{K} \boldsymbol{p_j}\boldsymbol{p_j}^H \boldsymbol{H_i}^H \boldsymbol{d_i} + N_0 \boldsymbol{d_i}^H \boldsymbol{d_i} - \boldsymbol{d_i}^H \boldsymbol{H_i}\boldsymbol{p_i} - \boldsymbol{p_i}^H \boldsymbol{H_i}^H \boldsymbol{d_i} + 1.\end{aligned} \tag{3.70}$$

For the multiuser case, it is noticed that $\sigma^2_{E_i}$ for the ith user depends on all precoders which leads to a complicated function. To deal with this problem, the ith user decoder and the ith precoder will be optimized by using only the ith user MSE expression. This is a suboptimal approach for total MSE minimization. Furthermore, here the total precoder power P_{tot} at the transmitter side is assumed to be divided equally for all users which may also be suboptimal.

As a result, the optimization problem can be expressed as

$$\begin{aligned}&\min_{\boldsymbol{p_i},\boldsymbol{d_i}} \sigma^2_{E_i} \\ &\text{s.t} \quad \boldsymbol{p_i}^H \boldsymbol{p_i} \leq \frac{P_{tot}}{K} \quad \forall i = 1, 2, \ldots, K.\end{aligned} \tag{3.71}$$

Decoder Design

For this system model, the optimum decoder design in terms of minimization of total MSE is a well-known method in the literature which is also called as the Wiener filter:

$$\boldsymbol{d_i} = \left(\boldsymbol{H_i} \sum_{j=1}^{K} \boldsymbol{p_j}\boldsymbol{p_j}^H \boldsymbol{H_i}^H + N_0 \boldsymbol{I}\right)^{-1} \boldsymbol{H_i}\boldsymbol{p_i}. \tag{3.72}$$

When this expression is multiplied with $\left(\boldsymbol{H_i} \sum_{j=1}^{K} \boldsymbol{p_j}\boldsymbol{p_j}^H \boldsymbol{H_i}^H + N_0 \boldsymbol{I}\right)$ term from

the left side, we obtain that

$$\left(\boldsymbol{H_i}\sum_{j=1}^{K}\boldsymbol{p_j}\boldsymbol{p_j}^H\boldsymbol{H_i}^H + N_0\boldsymbol{I}\right)\boldsymbol{d_i} = \boldsymbol{H_i}\boldsymbol{p_i}. \tag{3.73}$$

Then, multiply both sides with $\boldsymbol{d_i}^H$ term from the left side,

$$\boldsymbol{d_i}^H\boldsymbol{H_i}\sum_{j=1}^{K}\boldsymbol{p_j}\boldsymbol{p_j}^H\boldsymbol{H_i}^H\boldsymbol{d_i} + N_0\boldsymbol{d_i}^H\boldsymbol{d_i} = \boldsymbol{d_i}^H\boldsymbol{H_i}\boldsymbol{p_i}. \tag{3.74}$$

It is noticed that the term above is within the variance formula. When it is substituted,

$$\sigma_{E_i}^2 = 1 - \boldsymbol{p_i}^H\boldsymbol{H_i}^H\boldsymbol{d_i}. \tag{3.75}$$

Again, when $\boldsymbol{d_i}$ expression defined above is substituted into the last equation, it is obtained that

$$\sigma_{E_i}^2 = 1 - \boldsymbol{p_i}^H\boldsymbol{H_i}^H\left(\boldsymbol{H_i}\sum_{j=1}^{K}\boldsymbol{p_j}\boldsymbol{p_j}^H\boldsymbol{H_i}^H + N_0\boldsymbol{I}\right)^{-1}\boldsymbol{H_i}\boldsymbol{p_i}. \tag{3.76}$$

By decomposing $\boldsymbol{H_i}\sum_{j=1}^{K}\boldsymbol{p_j}\boldsymbol{p_j}^H\boldsymbol{H_i}^H + N_0\boldsymbol{I}$ expression into two parts,

$$\boldsymbol{H_i}\sum_{j=1}^{K}\boldsymbol{p_j}\boldsymbol{p_j}^H\boldsymbol{H_i}^H + N_0\boldsymbol{I} = \boldsymbol{H_i}\boldsymbol{p_i}\boldsymbol{p_i}^H\boldsymbol{H_i}^H + \underbrace{\boldsymbol{H_i}\sum_{j=1,j\neq i}^{K}\boldsymbol{p_j}\boldsymbol{p_j}^H\boldsymbol{H_i}^H + N_0\boldsymbol{I}}_{\boldsymbol{R_{(n+I)_i}}}, \tag{3.77}$$

and by using matrix inversion lemma, the following equation can be written.

$$\sigma_{E_i}^2 = 1 - \boldsymbol{p_i}^H\boldsymbol{H_i}^H\left(\boldsymbol{H_i}\boldsymbol{p_i}\boldsymbol{p_i}^H\boldsymbol{H_i}^H + \boldsymbol{R_{(n+I)_i}}\right)^{-1}\boldsymbol{H_i}\boldsymbol{p_i} = \left(\boldsymbol{I} + \boldsymbol{p_i}^H\boldsymbol{R_{H_i}}\boldsymbol{p_i}\right)^{-1}, \tag{3.78}$$

where $\boldsymbol{R_{H_i}} = \boldsymbol{H_i}^H\boldsymbol{R}_{\boldsymbol{(n+I)_i}}^{-1}\boldsymbol{H_i}$.

Precoder Design

After these steps, an iterative algorithm is used for precoder optimization, namely the MSE of each user is optimized independently. In other words, for the iteration l, the ith precoder vector $\boldsymbol{p_i}$ is optimized by using the available precoder vectors $\boldsymbol{p_m}$'s for $m = 1, 2, ..., i-1$ calculated at the lth iteration and also using $\boldsymbol{p_m}$'s for $m = i+1, i+2, ..., K$ calculated at the $(l-1)^{th}$ iteration. For the optimization of $\boldsymbol{p_i}$ precoder, the method of the Schur-concave optimization problem is used.

According to the Schur-concave optimization problem, detailed expression and proof of it is given in [34]

$$\min \sigma_{E_i}^2$$
$$\text{s.t} \quad \boldsymbol{p_i}^H \boldsymbol{p_i} \leq \frac{P_{tot}}{K}. \tag{3.79}$$

The optimum precoder $\boldsymbol{p_i}$ can be found by $\boldsymbol{p_i} = \boldsymbol{V_i}\boldsymbol{\Lambda_i}$, in which $\boldsymbol{V_i}$ is constructed by horizontally concatenating the eigenvectors corresponding to n_i largest eigenvalues of $\boldsymbol{R_{(n+I)_i}}$ matrix where $n_i = \min\{L_i, \text{rank}\{\boldsymbol{H_i}\}\}$. (Here, L_i represents the number of data stream received by the ith user. In the system model of this thesis, it is assumed that each user receives one scalar information meaning that $L_i = 1$). $\boldsymbol{\Lambda_i}$ is an $n_i \times L_i$ matrix which is formed by zero elements except the rightmost main diagonal. Non-zero elements $\lambda_{i,j}$ can be found using the equation

$$\lambda_{i,j} = \left[\sqrt{c}\sqrt{\lambda_{i,j}} - \lambda_{i,j}^{-1}\right]^+, \tag{3.80}$$

where $[x]^+ = x$ for $x \geq 0$ and $[x]^+ = 0$ for $x < 0$. The value of c can be found by using KKT (Karush-Kuhn Tucker) conditions satisfying per user power constraint, namely $\boldsymbol{p_i}^H \boldsymbol{p_i} = \frac{P_{tot}}{K}$.

In [34], the solution of this problem is given for single user case where $\boldsymbol{R_{(n+I)_i}}$ is equal to the noise covariance matrix. However, the same result is valid for any covariance matrix. In multiuser scenario, it is changed with noise plus interference matrix which is also a constant matrix during optimization of any precoder vector even if it depends on decoders. Thanks to the iterative precoder-decoder design, the decoders are fixed when the precoders are optimized. Therefore, all of the results derived for single user case are also valid for multiuser case.

The overall steps of the algorithm can be expressed as in the following form.

Joint Precoder/Decoder Design Algorithm Using Matrix Inversion Lemma

Initialization: $l = 0$ (l denotes the index of iteration)
Initialize all of the precoder vectors $\boldsymbol{p_i^{(l)}}$ as the $M \times 1$ all zero vector.
repeat
$l \rightarrow l + 1$

Set $i = 1$.
repeat
$i \rightarrow i + 1$

Step 1) Optimize i^{th} precoder vectors $\boldsymbol{p_i^{(l)}}$'s for $i = 1, 2, \ldots, K$, by using precoders calculated at the current iteration $\boldsymbol{p_m^{(l)}}$'s for $m = 1, 2, \ldots, i-1$ and the ones calculated at the previous iteration $\boldsymbol{p_m^{(l-1)}}$'s for $m = i+1, i+2, \ldots, K$. In order to do this, calculate $\boldsymbol{R_{(n+I)_i}}$ and apply Schur-Concave optimization. Find $\boldsymbol{V_i}$ and $\boldsymbol{\Lambda_i}$ by substituting c value satisfying KKT condition. Then, find $\boldsymbol{p_i} = \boldsymbol{V_i}\boldsymbol{\Lambda_i}$.

Step 2) Update $\boldsymbol{R_{(n+I)_i}}$ with new precoder vector $\boldsymbol{p_i}$ calculated in Step 1.

until $i = K$
Step 3) By using MMSE formula, calculate optimum decoders $\boldsymbol{d_i^{opt}}$ for $i = 1, 2, \ldots, K$ for the precoder vector set found in Step 1.
Step 4) Calculate the average MSE for the lth iteration ($MSE_{avg}^{(l)}$).

until $|MSE_{avg}^{(l)} - MSE_{avg}^{(l-1)}| \leq \epsilon$ or when the maximum number of iteration is reached.
(Here, ϵ is taken as 0.0005.)

CHAPTER 4

MULTIUSER MIMO DOWNLINK COMMUNICATION PROBLEM WITH COMBINATION OF COMMON AND PRIVATE INFORMATION

In the earlier chapters, multiuser downlink communication techniques are examined for the multicasting and the broadcasting scenarios which are well-known and detailed analyzed cases in the literature. In this chapter, these multiuser downlink communication methods are analyzed under a third scenario which can be thought as the combination of previous two. In the third scenario, each user gets a common information symbol (which is sent to all users) and a private information symbol (which is sent to each user independently). This subject is examined from an information theoretical viewpoint in [27]-[28].

The system model for this scenario is explained in Section 4.1. Similar to the Chapters 2 and 3, it is aimed to solve max-min fairness (MMF) problem and minimization of total MSE problem which are analyzed in Section 4.2 and Section 4.3 respectively. Adaptation of existing algorithms for the third scenario is studied. Furthermore, a new method for the solution of MMF problem in this scenario is also presented.

4.1 System Model

Consider a multiuser MIMO downlink communication system with M transmit antennas at the transmitter side and K users. Each mobile station (MS) has N_i antenna at the receiver for $i = 1, 2, \ldots, K$. Similar to Chapter 2 and 3, it is assumed that at the receiver of each user, an equal number of antennas (denoted by N) is present. b_k for $k = 0, 1, 2, \ldots, K$ represents the data symbol transmitted to all users simultaneously. While b_0 is the common information transmitted to all users, b_k for $k = 1, 2, \ldots, K$ are the private information transmitted to each user. b_k's are zero mean unity variance i.i.d. random variables. The MIMO channel for the ith user is denoted as $\boldsymbol{H_i}$ for $i = 1, 2, \ldots, K$ with dimensions $N \times M$. Assume that the channel is Rayleigh flat fading and channel state information is known both by transmitter and receiver. For each user at the receiver side, a decoder matrix is used to extract the data symbols. For the ith user, it is denoted by $\boldsymbol{D_i}$ which is an $L_i \times N_i$ matrix where L_i is the number of received data stream at the receiver side. In this thesis, for all simulations, $L_i = 2$; since we assume that each user receives one common and one private data symbol per channel use. At the ith user receiver, $\boldsymbol{n_i}$ is the $L_i \times 1$ noise vector whose elements are i.i.d. zero mean complex Gaussian random variables with variance N_0.

In the downlink MIMO communication system, the transmitted data symbols passes through the precoder block denoted by $\boldsymbol{P} = [\boldsymbol{p_0}\ \boldsymbol{p_1}\ \cdots\ \boldsymbol{p_K}]$ which is an $M \times (K+1)$ matrix. Different from Chapters 2 and 3, this time there exist an extra precoder vector $\boldsymbol{p_0}$ is used for common information b_0 sent to all users. After the precoding block, $K+1$ scalar information are transmitted by M transmit antennas via each user MIMO channel $\boldsymbol{H_i}$. Finally, at the receiver side, each user decoder block $\boldsymbol{D_i}$ generates the output vector $\boldsymbol{r_i}$ in the form of $\boldsymbol{r_i} = [\widehat{b_0}\ \widehat{b_i}]^T$ where $\widehat{b_0}$ denotes estimated common data symbol and $\widehat{b_i}$ denotes the ith user estimated private data symbol.

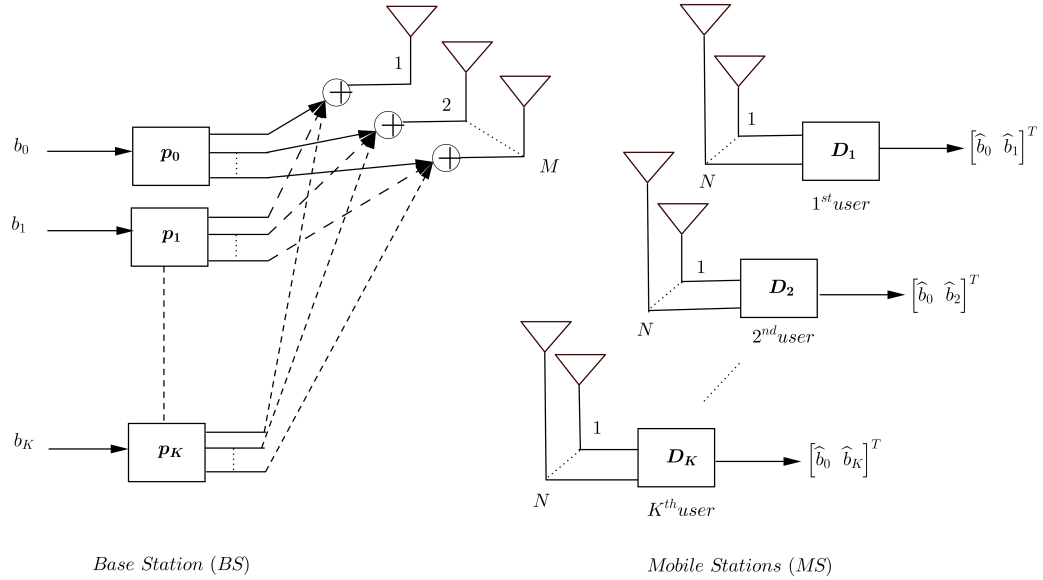

Figure 4.1: System Model for Transmitting Combination of Common-Private Information Scenario

As a result, under this scenario, in the multiuser downlink communication system received signal vector for the ith user can be expressed as

$$\boldsymbol{r_i} = [\widehat{b}_0\ \widehat{b}_i]^T = \boldsymbol{D_i H_i P b} + \boldsymbol{D_i n_i} \quad \text{for} \quad i = 1, 2, \ldots, K, \tag{4.1}$$

where $\boldsymbol{b} = [b_0\ b_1\ \cdots\ b_K]^T$.

According to the system model above, analysis of max-min fairness problem is provided in the next part. Similar to Chapters 2 and 3, the precoder and decoder design is done sequentially and iteratively.

4.2 Max-Min Fairness Problem

Before starting the solution of the MMF problem under common-private combination scenario, the definition of the SINR terms is provided. In this case, for the ith user, there exits two different SINR expressions, namely common and private SINR. According to the received signal formula for the ith user, the received common signal term and the private signal term for the ith user can be respectively expressed as

$$\widehat{b}_0 = \underbrace{(\boldsymbol{D_i})_1 \boldsymbol{H_i p_0} b_0}_{\text{common signal term}} + \underbrace{\sum_{k=1}^{K} (\boldsymbol{D_i})_1 \boldsymbol{H_i p_i} b_i}_{\text{interference term}} + \underbrace{(\boldsymbol{D_i})_1 \boldsymbol{n_i}}_{\text{noise term}},$$

$$\widehat{b}_i = \underbrace{(\boldsymbol{D_i})_2 \boldsymbol{H_i p_i} b_i}_{\text{private signal term}} + \underbrace{\sum_{k=0,k\neq i}^{K} (\boldsymbol{D_i})_2 \boldsymbol{H_i p_i} b_i}_{\text{interference term}} + \underbrace{(\boldsymbol{D_i})_2 \boldsymbol{n_i}}_{\text{noise term}}. \tag{4.2}$$

The reason of defining these new SINR expressions is that both common SINR and private SINR values are optimized individually. (If the summation of common and private signal terms are considered as the signal term of the ith user, the distribution of signal power between common and private signal terms may be unbalanced.) Here, the optimization is over both private and common SINR values. As noted before, since information data symbols have unity power, namely $\mathbb{E}\left\{|b_k|^2\right\} = 1 \quad \text{for} \quad k = 0, 1, \ldots, K$, both SINR expressions for the ith user can be defined as in the following form:

$$SINR^i_{com} = \frac{\left\|(\boldsymbol{D_i})_1 \boldsymbol{H_i p_0}\right\|^2}{\left\|\sum_{k=1}^{K} (\boldsymbol{D_i})_1 \boldsymbol{H_i p_i}\right\|^2 + N_0 \left\|(\boldsymbol{D_i})_1\right\|^2}$$

$$SINR^i_{pri} = \frac{\left\|(\boldsymbol{D_i})_2 \boldsymbol{H_i p_i}\right\|^2}{\left\|\sum_{k=1}^{K} (\boldsymbol{D_i})_2 \boldsymbol{H_i p_i}\right\|^2 + N_0 \left\|(\boldsymbol{D_i})_2\right\|^2}. \tag{4.3}$$

where $SINR^i_{com}$ and $SINR^i_{pri}$ represents common and private SINR terms respectively.

As a result, according to the SINR definitions above, there exists totally $2K$ SINR expressions for K user multiuser MIMO system. Similar to Chapters 2

and 3, here the aim is to maximize the minimum SINR. Adaptation of solution methods given earlier under this common-private combination scenario is provided in the following parts.

Decoder Design

For a given set of precoder vectors, when common and private SINR expressions are examined, it can be noticed that for the ith user, the 1st row of decoder matrices $\boldsymbol{D_i}$ is related with common SINR while the 2nd row is related with private SINR expression. Similar to Chapter 3, both common and private SINR expressions for the ith user is in the form of Rayleigh quotient, namely

$$SINR^i_{com} = \frac{(\boldsymbol{D_i})_1 \underbrace{\boldsymbol{H_i p_0 p_0^H H_i^H}}_{\boldsymbol{Y_1}} (\boldsymbol{D_i})_1^H}{(\boldsymbol{D_i})_1 \underbrace{\left(\boldsymbol{H_i}\left(\sum_{j=1}^{K} \boldsymbol{p_j p_j^H}\right)\boldsymbol{H_i^H} + N_0\boldsymbol{I}\right)}_{\boldsymbol{Y_2}} (\boldsymbol{D_i})_1^H}$$

$$SINR^i_{pri} = \frac{(\boldsymbol{D_i})_2 \underbrace{\boldsymbol{H_i p_i p_i^H H_i^H}}_{\boldsymbol{T_1}} (\boldsymbol{D_i})_2^H}{(\boldsymbol{D_i})_2 \underbrace{\left(\boldsymbol{H_i}\left(\sum_{j=0, j\neq i}^{K} \boldsymbol{p_j p_j^H}\right)\boldsymbol{H_i^H} + N_0\boldsymbol{I}\right)}_{\boldsymbol{T_2}} (\boldsymbol{D_i})_2^H}. \tag{4.4}$$

In addition, each user could design a decoder independently, since the ith user SINR expression only depends on $\boldsymbol{D_i}$.

Therefore, for the ith user, while 1st row of decoder matrix $\boldsymbol{D_i}$ is equal to the Hermitian of generalized principal eigenvector of $(\boldsymbol{Y_1}, \boldsymbol{Y_2})$, the 2nd row of decoder matrix should be chosen as the Hermitian of generalized principal eigenvector of $(\boldsymbol{T_1}, \boldsymbol{T_2})$ in order to maximize the SINR values.

Precoder Design

When decoder matrices for all users are known, the precoder design is achieved with three different approaches.

4.2.1 Semidefinite Relaxation (SDR) Method

Define $\boldsymbol{X_i} = \boldsymbol{p_i}\boldsymbol{p_i}^H$ for $i = 0, 1, 2, \ldots, K$. Then, common and private SINR expressions becomes

$$SINR_{com}^i = \frac{\operatorname{tr}\left(\boldsymbol{Q_i}\boldsymbol{X_0}\right)}{\sum_{k=1}^{K} \operatorname{tr}\left(\boldsymbol{Q_i}\boldsymbol{X_k}\right) + N_0 \operatorname{tr}\left(\left(\boldsymbol{D_i}\right)_1 \left(\boldsymbol{D_i}\right)_1^H\right)}$$

$$SINR_{pri}^i = \frac{\operatorname{tr}\left(\boldsymbol{R_i}\boldsymbol{X_i}\right)}{\sum_{k=0,k\neq i}^{K} \operatorname{tr}\left(\boldsymbol{R_i}\boldsymbol{X_k}\right) + N_0 \operatorname{tr}\left(\left(\boldsymbol{D_i}\right)_2 \left(\boldsymbol{D_i}\right)_2^H\right)}, \tag{4.5}$$

where $\boldsymbol{R_i} = \boldsymbol{H_i}^H \left(\boldsymbol{D_i}\right)_2^H \left(\boldsymbol{D_i}\right)_2 \boldsymbol{H_i}$ and $\boldsymbol{Q_i} = \boldsymbol{H_i}^H \left(\boldsymbol{D_i}\right)_1^H \left(\boldsymbol{D_i}\right)_1 \boldsymbol{H_i}$.

Similar to steps which are explained in detail in Chapter 3, the problem is a non-convex due to the rank-one constraint on $\boldsymbol{X_i}$. In addition, the existence of the interference term leads to a non-linearity of the optimization problem in terms of optimization variables $\boldsymbol{X_i}$ for $i = 0, 1, 2, \ldots, K$. Therefore, MMF problem can not be solved directly. However, by using the idea of the equivalence of the precoder solutions for MMF and QoS problems, it can be solved iteratively. In other words, the MMF problem can be solved by solving QoS problem repeatedly with changing threshold values until reaching the total precoder power constraint of MMF problem. The solution of $\boldsymbol{X_i}$ for $i = 0, 1, 2, \ldots, K$ can be found with the help of an SDP tool. After that, in order to obtain the precoder vectors $\boldsymbol{p_i}$ for $i = 0, 1, 2, \ldots, K$ from $\boldsymbol{X_i}$, relaxation procedures should be applied. In simulations of this part, randA and detA methods are applied for relaxation purposes.

In order to put $2K$ SINR expressions on a standard form $2K$ slack variables, to convert the inequalities to equalities, should be introduced.

Finally, the problem can be expressed as

$$QoS: \min_{\boldsymbol{p_1}, \boldsymbol{p_2}, \ldots, \boldsymbol{p_K}} \sum_{i=1}^{K} \operatorname{tr}\left(\boldsymbol{X_i}\right). \tag{4.6}$$

For private SINR expressions

$$\frac{1}{\gamma} \operatorname{tr}\left(\widetilde{\boldsymbol{R_i}}\boldsymbol{X_i}\right) - \sum_{j=1, j\neq i}^{K} \operatorname{tr}\left(\widetilde{\boldsymbol{R_j}}\boldsymbol{X_j}\right) - s_i = 1, \quad s_i \geq 0 \text{ for } \; i = 1, 2, \ldots, K,$$

For common SINR expressions

$$\frac{1}{\gamma}\operatorname{tr}\left(\widetilde{\boldsymbol{Q}_i}\boldsymbol{X_i}\right)-\sum_{j=1,j\neq i}^{K}\operatorname{tr}\left(\widetilde{\boldsymbol{Q}_j}\boldsymbol{X_j}\right)-t_i=1,\quad t_i\geq 0 \text{ for } \ i=1,2,\ldots,K,$$

$$\boldsymbol{X_i}\succeq 0 \ \text{ for } \ i=1,2,\ldots,K, \tag{4.7}$$

where $\widetilde{\boldsymbol{R_i}}=\frac{\boldsymbol{R_i}}{N_0\left((\boldsymbol{D_i})_1(\boldsymbol{D_i})_1^H\right)}$ and $\widetilde{\boldsymbol{Q_i}}=\frac{\boldsymbol{Q_i}}{N_0\left((\boldsymbol{D_i})_2(\boldsymbol{D_i})_2^H\right)}$.

According to the standard SeDuMi format $\mathbf{sedumi}(\boldsymbol{A},\boldsymbol{b},\boldsymbol{c})$, $\boldsymbol{A},\boldsymbol{b},\boldsymbol{c}$ expressions become

$$\boldsymbol{A}=\begin{bmatrix} -\boldsymbol{A_1} & \boldsymbol{A_2}\end{bmatrix}$$

where $\boldsymbol{A_1}=\boldsymbol{I}_{2K\times 2K}$ and

$$\boldsymbol{A_2}=\begin{bmatrix} -vec\left(\widetilde{\boldsymbol{R_1}}^T\right)^T & \frac{1}{\gamma}vec\left(\widetilde{\boldsymbol{R_1}}^T\right)^T & -vec\left(\widetilde{\boldsymbol{R_1}}^T\right)^T & \ldots & -vec\left(\widetilde{\boldsymbol{R_1}}^T\right)^T \\ -vec\left(\widetilde{\boldsymbol{R_2}}^T\right)^T & -vec\left(\widetilde{\boldsymbol{R_2}}^T\right)^T & \frac{1}{\gamma}vec\left(\widetilde{\boldsymbol{R_2}}^T\right)^T & \ldots & -vec\left(\widetilde{\boldsymbol{R_2}}^T\right)^T \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -vec\left(\widetilde{\boldsymbol{R_K}}^T\right)^T & -vec\left(\widetilde{\boldsymbol{R_K}}^T\right)^T & -vec\left(\widetilde{\boldsymbol{R_K}}^T\right)^T & \ldots & \frac{1}{\gamma}vec\left(\widetilde{\boldsymbol{R_K}}^T\right)^T \\ \frac{1}{\gamma}vec\left(\widetilde{\boldsymbol{Q_1}}^T\right)^T & -vec\left(\widetilde{\boldsymbol{Q_1}}^T\right)^T & -vec\left(\widetilde{\boldsymbol{Q_1}}^T\right)^T & \ldots & -vec\left(\widetilde{\boldsymbol{Q_1}}^T\right)^T \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\gamma}vec\left(\widetilde{\boldsymbol{Q_K}}^T\right)^T & -vec\left(\widetilde{\boldsymbol{Q_K}}^T\right)^T & -vec\left(\widetilde{\boldsymbol{Q_K}}^T\right)^T & \ldots & -vec\left(\widetilde{\boldsymbol{Q_K}}^T\right)^T \end{bmatrix}$$

$$\boldsymbol{b}=\begin{bmatrix}1\\1\\1\\\vdots\\1\end{bmatrix},\boldsymbol{c}=\begin{bmatrix}0\\0\\\vdots\\0\\vec{(\boldsymbol{I})}\\vec{(\boldsymbol{I})}\\\vdots\\vec{(\boldsymbol{I})}\end{bmatrix},\boldsymbol{x_{opt}}=\begin{bmatrix}s_1\\s_2\\\vdots\\s_K\\t_1\\t_2\\\vdots\\t_K\\vec{(\boldsymbol{X_0})}\\vec{(\boldsymbol{X_1})}\\\vdots\\vec{(\boldsymbol{X_K})}\end{bmatrix}, \tag{4.8}$$

where $\boldsymbol{A}$ is an $2K\times(2K+M^2(K+1))$ matrix, $\boldsymbol{x_{opt}}$ and $\boldsymbol{c}$ are $2K+M^2(K+1)\times 1$ vectors and $\boldsymbol{b}$ is a $2K\times 1$ vector.

4.2.2 Exact Penalty Approach (EPA)

The details of EPA method is given in Chapter 3; therefore, in this part, the expressions used in this method is adapted according to the common-private combination scenario.

Define $\boldsymbol{q} = [\boldsymbol{p_0}\ \boldsymbol{p_1}\ \boldsymbol{p_2}\ \cdots\ \boldsymbol{p_K}]^T$. With this definition, the private and common SINR expressions can be expressed as in the following form:

$$SINR_{pri}^{i} = \frac{\boldsymbol{p_i}^H \widetilde{\boldsymbol{R_i}} \boldsymbol{p_i}}{\sum_{j=1, j\neq i}^{K} \boldsymbol{p_j}^H \widetilde{\boldsymbol{R_j}} \boldsymbol{p_j} + 1} = \boldsymbol{q}^H \boldsymbol{V_i} \boldsymbol{q}, \tag{4.9}$$

where $\boldsymbol{V_i}$ is $M\,(K+1) \times M\,(K+1)$ block diagonal matrix with entries such that

$$\boldsymbol{V_i} = diag\left(\underbrace{-\widetilde{\boldsymbol{R_i}}, -\widetilde{\boldsymbol{R_i}}, \ldots, -\widetilde{\boldsymbol{R_i}}}_{i \text{ terms}}, \frac{1}{\gamma}\widetilde{\boldsymbol{R_i}}, \underbrace{-\widetilde{\boldsymbol{R_i}}, \ldots, -\widetilde{\boldsymbol{R_i}}}_{K-i \text{ terms}}\right). \tag{4.10}$$

$$SINR_{com}^{i} = \frac{\boldsymbol{p_0}^H \widetilde{\boldsymbol{Q_i}} \boldsymbol{p_0}}{\sum_{j=1}^{K} \boldsymbol{p_j}^H \widetilde{\boldsymbol{Q_j}} \boldsymbol{p_j} + 1} = \boldsymbol{q}^H \boldsymbol{W_i} \boldsymbol{q} \tag{4.11}$$

where $\boldsymbol{W_i}$ is $M\,(K+1) \times M\,(K+1)$ diagonal matrix with entries such that

$$\boldsymbol{W_i} = diag\left(\frac{1}{\gamma}\widetilde{\boldsymbol{Q_i}}, \underbrace{-\widetilde{\boldsymbol{Q_i}}, \ldots, -\widetilde{\boldsymbol{Q_i}}}_{K \text{ terms}}\right). \tag{4.12}$$

Define $\boldsymbol{X} = \boldsymbol{q}\boldsymbol{q}^H$. After applying the same alternating minimization algorithm, whose details are given in Chapter 3, one can obtain the solution of $\boldsymbol{X_{opt}}$ and by the principal eigenvector of $\boldsymbol{X_{opt}}$, we get $\boldsymbol{q}$. Under this scenario, the inputs of SeDuMi $\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}$ can also be calculated by applying the same procedure expressed in SDR part.

4.2.3 Total SINR Maximization Method (Total SINR Max Algorithm)

Up to this point, the max-min fairness (MMF) problem is solved by using SDR method and EPA. Both of these methods solve the problem with the help of an SDP tool such as SeDuMi. These methods provide very good minimum SINR values; however, their computation time requirements are high in comparison to the other algorithms. A low computational load algorithm is proposed in this section.

Due to the existence of the common information transmitted to each user, maximizing the minimum SINR in the system is a difficult optimization problem. Therefore, instead of maximizing minimum SINR term, maximizing the total SINR cost function is used resulting in a suboptimal but relatively easier problem solution. According to this statement, the optimization problem can be expressed mathematically as

$$\max_{\boldsymbol{p_1},\boldsymbol{p_2},\ldots,\boldsymbol{p_K}\in\mathbb{C}^M,\boldsymbol{D_1},\boldsymbol{D_2},\ldots,\boldsymbol{D_K}\in\mathbb{C}^{L_i\times N}} \sum_{i=1}^{2K} SINR_i$$

$$\text{s.t.} \quad \sum_{i=1}^{K} \operatorname{tr}\left(\boldsymbol{p_i}\boldsymbol{p_i}^H\right) = P_{tot}. \tag{4.13}$$

For this algorithm, again decoder and precoder design is done jointly and iteratively. Firstly, decoder design is proposed.

Decoder Design

For a given precoder set, common and private SINR expressions can be written as follows:

$$SINR_{com}^i = \frac{p_0 \boldsymbol{u_0}^H \boldsymbol{H_i}^H \left(\boldsymbol{D_i}\right)_1^H \left(\boldsymbol{D_i}\right)_1 \boldsymbol{H_i}\boldsymbol{u_0}}{\sum_{k=1}^{K} \boldsymbol{u_k}^H \boldsymbol{H_i}^H \left(\boldsymbol{D_i}\right)_1^H \left(\boldsymbol{D_i}\right)_1 \boldsymbol{H_i}\boldsymbol{u_k} + N_0 \left(\boldsymbol{D_i}\right)_1 \left(\boldsymbol{D_i}\right)_1^H},$$

$$SINR_{pri}^i = \frac{p_i \boldsymbol{u_i}^H \boldsymbol{H_i}^H \left(\boldsymbol{D_i}\right)_2^H \left(\boldsymbol{D_i}\right)_2 \boldsymbol{H_i}\boldsymbol{u_i}}{\sum_{k=1}^{K} \boldsymbol{u_k}^H \boldsymbol{H_i}^H \left(\boldsymbol{D_i}\right)_2^H \left(\boldsymbol{D_i}\right)_2 \boldsymbol{H_i}\boldsymbol{u_k} + N_0 \left(\boldsymbol{D_i}\right)_2 \left(\boldsymbol{D_i}\right)_2^H}, \tag{4.14}$$

where $\boldsymbol{u_0}, \boldsymbol{u_1} \ldots \boldsymbol{u_K}$ are unit norm precoder vectors and $p_0, p_1 \ldots, p_K$ are power value associated with each precoder vector.

By using the trace properties, they can also be expressed as

$$SINR_{com}^{i} = \frac{p_0 \left(\boldsymbol{D_i}\right)_1 \boldsymbol{H_i} \boldsymbol{u_0} \boldsymbol{u_0}^H \boldsymbol{H_i}^H \left(\boldsymbol{D_i}\right)_1^H}{\left(\boldsymbol{D_i}\right)_1 \boldsymbol{H_i} \boldsymbol{H_i}^H \left(\boldsymbol{D_i}\right)_1^H \sum_{k=1}^{K} \boldsymbol{u_k} \boldsymbol{u_k}^H + N_0 \left(\boldsymbol{D_i}\right)_1 \left(\boldsymbol{D_i}\right)_1^H},$$

$$SINR_{pri}^{i} = \frac{p_i \left(\boldsymbol{D_i}\right)_2 \boldsymbol{H_i} \boldsymbol{u_i} \boldsymbol{u_i}^H \boldsymbol{H_i}^H \left(\boldsymbol{D_i}\right)_2^H}{\left(\boldsymbol{D_i}\right)_2 \boldsymbol{H_i} \boldsymbol{H_i}^H \left(\boldsymbol{D_i}\right)_2^H \sum_{k=0,k\neq i}^{K} \boldsymbol{u_k}^H \boldsymbol{u_k} + N_0 \left(\boldsymbol{D_i}\right)_2 \left(\boldsymbol{D_i}\right)_2^H}. \tag{4.15}$$

which is in the Rayleigh quotient format. As understood from SINR expressions, each user can design a decoder independently. Furthermore, the first row of the decoder matrix for the ith user is found from the common SINR while the second row is related to the private SINR of that user. As a result, the first row of the decoder matrix is equal to the generalized principal eigenvector of $\boldsymbol{U_1}$ and $\boldsymbol{U_2}$ where

$$\boldsymbol{U_1} = \boldsymbol{H_i} \boldsymbol{u_0} \boldsymbol{u_0}^H \boldsymbol{H_i}^H \quad \text{and} \quad \boldsymbol{U_2} = \boldsymbol{H_i} \sum_{k=1}^{K} \boldsymbol{u_k} \boldsymbol{u_k}^H \boldsymbol{H_i}^H + N_0 \boldsymbol{I}. \tag{4.16}$$

The second row of the decoder matrix is equal to the generalized principal eigenvector of $\boldsymbol{U_3}$ and $\boldsymbol{U_4}$ where

$$\boldsymbol{U_3} = \boldsymbol{H_i} \boldsymbol{u_i} \boldsymbol{u_i}^H \boldsymbol{H_i}^H \quad \text{and} \quad \boldsymbol{U_4} = \boldsymbol{H_i} \sum_{k=0,k\neq i}^{K} \boldsymbol{u_k} \boldsymbol{u_k}^H \boldsymbol{H_i}^H + N_0 \boldsymbol{I}. \tag{4.17}$$

Precoder Design

At this part, the power allocation of precoder vector is analyzed separately. Therefore, the designed precoder vectors are unit norm. For a given decoder set, when $2K$ SINR expressions are added, due to the interference term in the denominator, it is noticed that the summation includes terms which is the quadratic function of precoder vectors. In order to simplify the problem, we ignore interference terms in the SINR expressions, then the total SINR is equal to

$$\sum_{i=1}^{2K} SINR_i = \boldsymbol{u_0}^H \sum_{i=1}^{K} \frac{\boldsymbol{B_i}}{N_0 \operatorname{tr}\left(\left(\boldsymbol{D_i}\right)_1 \left(\boldsymbol{D_i}\right)_1^H\right)} \boldsymbol{u_0} + \sum_{i=1}^{K} \boldsymbol{u_i}^H \frac{\boldsymbol{A_i}}{N_0 \operatorname{tr}\left(\left(\boldsymbol{D_i}\right)_2 \left(\boldsymbol{D_i}\right)_2^H\right)} \boldsymbol{u_i}, \tag{4.18}$$

where $\boldsymbol{A_i} = \boldsymbol{H_i}^H \left(\boldsymbol{D_i}\right)_2^H \left(\boldsymbol{D_i}\right)_2 \boldsymbol{H_i}$ and $\boldsymbol{B_i} = \boldsymbol{H_i}^H \left(\boldsymbol{D_i}\right)_1^H \left(\boldsymbol{D_i}\right)_1 \boldsymbol{H_i}$.

As understood from summation term, the optimization of common precoder vector $\boldsymbol{u_0}$ is independent from private precoder optimization. Therefore; the optimum unit norm common precoder vector $\boldsymbol{u_0}$ is equal to the principal eigenvector of

$$\sum_{i=1}^{K} \frac{\boldsymbol{B_i}}{N_0 \operatorname{tr}\left((\boldsymbol{D_i})_1 (\boldsymbol{D_i})_1^H \right)}. \tag{4.19}$$

On the other hand, design of optimum private precoder vectors is similar to common precoder vector optimization. In other words, after the elimination of interference terms, $SINR_{pri}^{i}$ only depends on the ith unit norm precoder vector $\boldsymbol{u_i}$; therefore, the optimum unit norm private precoder vectors can be optimized individually. Furthermore, i^{th} private SINR is in the form of Rayleigh quotient. Ignored interference is compensated in the decoder design, since the interference term in SINR expression is taken into account.

Power allocation is another difficulty of this problem. Intuitively, allocation of half of the total precoder power to common precoder $\boldsymbol{p_0}$ may be a feasible approach on the average since $\boldsymbol{p_0}$ is used for every user. Therefore, in this algorithm, unit norm $\boldsymbol{p_0}$ is scaled with a number which makes its power is equal to $P_{tot}/2$. (In terms of private precoders, the problem can be thought as symmetric.) Therefore, by using similar approach proven in uplink-downlink duality part, the optimum power allocation for private precoder vectors is the one at which all private SINR values are equal to each other. By using SINR equality idea, half of the precoder power is divided among private precoders according to the equation system below:

$$\begin{bmatrix} \boldsymbol{X_1} & 0 & \dots & -1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & \boldsymbol{X_k} & -1 \\ 1 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ \vdots \\ p_k \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ P_{tot}/2 \end{bmatrix}, \tag{4.20}$$

where c denotes the equal SINR values. The main steps of the algorithm is given below.

Total SINR Maximization Method for Common-Private Combination Scenario

Initialization: $t = 0$ (t denotes index of iteration) Set for all user's decoder matrices $\boldsymbol{D_i^{(0)}}$ for $i = 1, 2, ...K$ to $\boldsymbol{I}_{L_i \times N}$ where $\boldsymbol{I}_{m,n} = 1$ for $m = n$ and $\boldsymbol{I}_{m,n} = 0$ for $m \neq n$

repeat

$t \rightarrow t + 1$

Step 1) Calculate unit norm common precoder vector $\boldsymbol{p_0}$ and private precoder vectors $\boldsymbol{p_i}$ for $i = 1, 2, \ldots, K$.

Step 2) Make power allocation and multiply each unit norm precoder vector with a scalar according to corresponding allocated power of precoder vector.

Step 3) Calculate decoder matrices $\boldsymbol{D_i}$ for $i = 1, 2, \ldots, K$ by using precoder vectors calculated in Step 2.

until $\left(\sum_{i=1}^{K} SINR_{com}^{i(t)} + SINR_{pri}^{i(t)}\right) - \left(\sum_{i=1}^{K} SINR_{com}^{i(t-1)} + SINR_{pri}^{i(t-1)}\right) < \epsilon.$

(Here ϵ is taken as 0.05.)

4.3 Minimization of Total Mean Square Error (TMSE) Problem

The goal of this part is minimizing the total MSE in the MIMO system by designing precoders and decoders. Design approach is similar to the ones in Chapter 2 and 3, yet due to the new scenario, some differences occur in precoder and decoder formulas. Details of algorithm design is provided below.

4.3.1 Joint Precoder and Decoder Design Method (Joint-TMSE Algorithm)

As in a downlink MIMO communication system with broadcasting and multicasting capabilities, the total MSE can be minimized by designing a linear transmitter and receiver structure in iterative manner, under the total power constraint P_{tot} at the transmitter side.

The optimization problem can be expressed in the following form:

$$\min_{\boldsymbol{p_0},\boldsymbol{p_1},\boldsymbol{p_2},\ldots,\boldsymbol{p_K},\boldsymbol{D_1},\boldsymbol{D_2},\ldots,\boldsymbol{D_K}} \sum_{i=1}^{K} MSE_i$$

$$\text{s.t.} \quad \sum_{i=0}^{K} \boldsymbol{p_i}^H \boldsymbol{p_i} = P_{tot}. \tag{4.21}$$

The MSE for the ith user could be defined as

$$MSE_i = \mathbb{E}\left\{\|\boldsymbol{r_i} - \boldsymbol{b_i}\|^2\right\} \tag{4.22}$$

where $\boldsymbol{b_i} = \begin{bmatrix} b_0 \\ b_i \end{bmatrix}$.

$$\begin{aligned} MSE_i &= \operatorname{tr}\left[\mathbb{E}\left\{(\boldsymbol{r_i} - \boldsymbol{b_i})(\boldsymbol{r_i} - \boldsymbol{b_i})^H\right\}\right] \\ &= \operatorname{tr}\left(\mathbb{E}\left\{\boldsymbol{r_i}\boldsymbol{r_i}^H\right\}\right) + \operatorname{tr}\left(\mathbb{E}\left\{\boldsymbol{b_i}\boldsymbol{b_i}^{\boldsymbol{H}}\right\}\right) - \operatorname{tr}\left(\mathbb{E}\left\{\boldsymbol{r_i}\boldsymbol{b_i}^H\right\}\right) - \operatorname{tr}\left(\mathbb{E}\left\{\boldsymbol{b_i}\boldsymbol{r_i}^H\right\}\right). \end{aligned} \tag{4.23}$$

The received signal could be expressed as another form

$$\boldsymbol{r_i} = \boldsymbol{Q_i}\boldsymbol{b} + \boldsymbol{D_i}\boldsymbol{n_i}, \tag{4.24}$$

where $\boldsymbol{Q_i} = \begin{bmatrix} \boldsymbol{q_{i0}} & \boldsymbol{q_{i1}} & \cdots & \boldsymbol{q_{iK}} \end{bmatrix}$ and $\boldsymbol{q_{ij}} = \boldsymbol{D_i}\boldsymbol{H_i}\boldsymbol{p_j}$ for $i = 1, 2, \ldots, K$ and $j = 0, 1, 2, \ldots, K$.

Therefore, it can be written as

$$\boldsymbol{r_i} = \left(\sum_{j=0}^{K} b_j \boldsymbol{q_{ij}}\right) + \boldsymbol{D_i}\boldsymbol{n_i}. \tag{4.25}$$

Assume that b_j's for $j = 0, 1, \ldots, K$ are zero mean iid, each entry of $\boldsymbol{n_i}$'s are zero mean with variance N_0. b_j's and elements of $\boldsymbol{n_i}$'s are independent of each other for $i = 1, 2, \ldots, K$.

The calculation of each term in the MSE expression is as follows:

1st term:

$$\operatorname{tr}\left(\mathbb{E}\left\{\boldsymbol{r_i}\boldsymbol{r_i}^{\boldsymbol{H}}\right\}\right) = \operatorname{tr}\left(\boldsymbol{D_i}\boldsymbol{H_i}\boldsymbol{p}\boldsymbol{p}^H\boldsymbol{H_i}^H\boldsymbol{D_i}^H\right) + N_0 \operatorname{tr}\left(\boldsymbol{D_i}\boldsymbol{D_i}^H\right). \tag{4.26}$$

2nd term:

Since b_i's are zero mean and unity variance,

$$\operatorname{tr}\left(\mathbb{E}\left\{\boldsymbol{b_i}\boldsymbol{b_i}^H\right\}\right) = \mathbb{E}\left\{|b_0|^2\right\} + \mathbb{E}\left\{|b_i|^2\right\} = 2N_0 = 2. \tag{4.27}$$

3rd term:

$$\begin{aligned}
\operatorname{tr}\left(\mathbb{E}\left\{\boldsymbol{r_i}\boldsymbol{b_i}^H\right\}\right) &= \operatorname{tr}\left(\mathbb{E}\left\{\left(\sum_{j=0}^{K} b_j\boldsymbol{q_{ij}} + \boldsymbol{D_i}\boldsymbol{n_i}\right)\boldsymbol{b_i}^H\right\}\right) \\
&= \operatorname{tr}\left(\mathbb{E}\left\{\left(b_0\boldsymbol{q_{i0}} + b_i\boldsymbol{q_{ii}}\right)\left[b_0^*\, b_i^*\right]\right\}\right) \\
&= \operatorname{tr}\left(\mathbb{E}\left\{\left(b_0\boldsymbol{D_i}\boldsymbol{H_i}\boldsymbol{p_0} + b_i\boldsymbol{D_i}\boldsymbol{H_i}\boldsymbol{p_i}\right)\left[b_0^*\, b_i^*\right]\right\}\right) \\
&= \left(\boldsymbol{D_i}\boldsymbol{H_i}\right)_1\boldsymbol{p_0} + \left(\boldsymbol{D_i}\boldsymbol{H_i}\right)_2\boldsymbol{p_i}.
\end{aligned} \tag{4.28}$$

Since 4th term is the Hermitian of 3rd term, one can write that

$$\operatorname{tr}\left(\mathbb{E}\left\{\boldsymbol{b_i}\boldsymbol{r_i}^H\right\}\right) = \left(\boldsymbol{D_i}\boldsymbol{H_i}\right)_1^*\boldsymbol{p_0}^* + \left(\boldsymbol{D_i}\boldsymbol{H_i}\right)_2^*\boldsymbol{p_i}^*. \tag{4.29}$$

Therefore, the MSE for the ith user can be written as

$$\begin{aligned}
MSE_i = \operatorname{tr}&\left(\boldsymbol{D_i}\boldsymbol{H_i}\left(\sum_{j=0}^{K}\boldsymbol{p_j}\boldsymbol{p_j}^H\right)\boldsymbol{H_i}^H\boldsymbol{D_i}^H\right) - \left(\boldsymbol{D_i}\boldsymbol{H_i}\right)_1\boldsymbol{p_0} - \left(\boldsymbol{D_i}\boldsymbol{H_i}\right)_2\boldsymbol{p_i} \\
&- \left(\boldsymbol{D_i}\boldsymbol{H_i}\right)_1^*\boldsymbol{p_0}^* - \left(\boldsymbol{D_i}\boldsymbol{H_i}\right)_2^*\boldsymbol{p_i}^* + N_0\operatorname{tr}\left(\boldsymbol{D_i}\boldsymbol{D_i}^H\right) + 2.
\end{aligned} \tag{4.30}$$

The optimization problem is solved by the Lagrange multiplier approach. As a result, the Lagrange objective function becomes

$$\begin{aligned}
L\left(\boldsymbol{p_0}, \boldsymbol{p_1}, \boldsymbol{p_2}, \ldots, \boldsymbol{p_k}; \boldsymbol{D_1}, \boldsymbol{D_2}, \ldots, \boldsymbol{D_k}; \lambda\right) = \lambda&\left(\sum_{j=0}^{K}\boldsymbol{p_j}^H\boldsymbol{p_j} - P_{tot}\right) \\
&+ \sum_{i=1}^{K}\operatorname{tr}\left(\boldsymbol{D_i}\boldsymbol{H_i}\left(\sum_{j=0}^{K}\boldsymbol{p_j}\boldsymbol{p_j}^H\right)\boldsymbol{H_i}^H\boldsymbol{D_i}^H\right) \\
&- \left(\boldsymbol{D_i}\boldsymbol{H_i}\right)_1\boldsymbol{p_0} - \left(\boldsymbol{D_i}\boldsymbol{H_i}\right)_2\boldsymbol{p_i} \\
&- \left(\boldsymbol{D_i}\boldsymbol{H_i}\right)_1^*\boldsymbol{p_0}^* - \left(\boldsymbol{D_i}\boldsymbol{H_i}\right)_2^*\boldsymbol{p_i}^* \\
&+ N_0\operatorname{tr}\left(\boldsymbol{D_i}\boldsymbol{D_i}^H\right).
\end{aligned} \tag{4.31}$$

where λ is the Lagrange multiplier. In order to find the optimum precoder vectors and decoder matrices, we can take derivative of the Lagrange objective function with respect to $\boldsymbol{p_0}^*, \boldsymbol{p_1}^*, \boldsymbol{p_2}^*, \ldots, \boldsymbol{p_k}^*, \boldsymbol{D_1}^*, \boldsymbol{D_2}^*, \ldots, \boldsymbol{D_k}^*$ and equate the result to zero.

It should be noted that the derivative expression of $\boldsymbol{p_0}$ is different from that of the private precoder vectors. The results of derivations are given below.

For common precoder vector $\boldsymbol{p_0}$,

$$\frac{\partial L\left(\boldsymbol{p_0}, \boldsymbol{p_1}, \boldsymbol{p_2}, \ldots, \boldsymbol{p_k}; \boldsymbol{D_1}, \boldsymbol{D_2}, \ldots, \boldsymbol{D_k}; \lambda\right)}{\partial \boldsymbol{p_0^*}} = \sum_{i=1}^{K} \boldsymbol{H_i^H D_i^H D_i H_i p_0} - \left(\boldsymbol{D_m H_m}\right)_1^H + \lambda \boldsymbol{p_0} = 0. \tag{4.32}$$

For private precoder vectors $\boldsymbol{p_m}$ where $m = 1, 2, \ldots, K$

$$\frac{\partial L\left(\boldsymbol{p_0}, \boldsymbol{p_1}, \boldsymbol{p_2}, \ldots, \boldsymbol{p_k}; \boldsymbol{D_1}, \boldsymbol{D_2}, \ldots, \boldsymbol{D_k}; \lambda\right)}{\partial \boldsymbol{p_m^*}} = \sum_{i=1}^{K} \boldsymbol{H_i^H D_i^H D_i H_i p_m} - \left(\boldsymbol{D_m H_m}\right)_2^H + \lambda \boldsymbol{p_m} = 0. \tag{4.33}$$

Therefore, the precoder vectors for common and private information are equal to

$$\begin{aligned} \boldsymbol{p_m} &= \left(\sum_{i=1}^{K} \boldsymbol{H_i^H D_i^H D_i H_i} + \lambda \boldsymbol{I}\right)^{-1} \left(\boldsymbol{D_m H_m}\right)_2^H \quad \text{for} \quad m = 1, 2, \ldots, K \\ \boldsymbol{p_0} &= \left(\sum_{i=1}^{K} \boldsymbol{H_i^H D_i^H D_i H_i} + \lambda \boldsymbol{I}\right)^{-1} \sum_{i=1}^{K} \left(\boldsymbol{D_i H_i}\right)_1^H . \end{aligned} \tag{4.34}$$

To simplify the derivation of decoder matrix, the Lagrange objective function can be written in another form

$$\begin{aligned} L\left(\boldsymbol{p_0}, \boldsymbol{p_1}, \boldsymbol{p_2}, \ldots, \boldsymbol{p_k}; \boldsymbol{D_1}, \boldsymbol{D_2}, \ldots, \boldsymbol{D_k}; \lambda\right) = & \sum_{i=1}^{K} \operatorname{tr}\left(\boldsymbol{D_i H_i}\left(\sum_{j=0}^{K} \boldsymbol{p_j p_j^H}\right) \boldsymbol{H_i^H D_i^H}\right) \\ & - \operatorname{tr}\left(\boldsymbol{D_i H_i}\left[\boldsymbol{p_0}\, \boldsymbol{p_i}\right]\right) \\ & - \operatorname{tr}\left(\left[\boldsymbol{p_0}\, \boldsymbol{p_i}\right]^H \boldsymbol{H_i^H D_i^H}\right) \\ & + \operatorname{tr}\left(N_0\left(\boldsymbol{D_i D_i^H}\right)\right). \end{aligned} \tag{4.35}$$

The decoder matrix for the mth user can be found by taking derivative of the Lagrange objective function with respect to $\boldsymbol{D_m^*}$ where $m = 1, 2, ..., K$ and

equate the result to zero,

$$\frac{\partial L\left(\boldsymbol{p_0}, \boldsymbol{p_1}, \boldsymbol{p_2}, \ldots, \boldsymbol{p_k}; \boldsymbol{D_1}, \boldsymbol{D_2}, \ldots, \boldsymbol{D_k}; \lambda\right)}{\partial \boldsymbol{D_m^*}} = \boldsymbol{D_m} \boldsymbol{H_m} \left(\sum_{j=0}^{K} \boldsymbol{p_j} \boldsymbol{p_j^H} \right) \boldsymbol{H_m^H} - \left[\boldsymbol{p_0}\, \boldsymbol{p_m}\right]^H \boldsymbol{H_m^H} + N_0 \boldsymbol{D_m} = 0. \tag{4.36}$$

As a result, the decoder matrix for the mth user is equal to

$$\boldsymbol{D_m} = \left[\boldsymbol{p_0}\, \boldsymbol{p_m}\right]^H \boldsymbol{H_m^H} \left(\boldsymbol{H_m} \left(\sum_{j=0}^{K} \boldsymbol{p_j} \boldsymbol{p_j^H} \right) \boldsymbol{H_m^H} + N_0 \boldsymbol{I} \right)^{-1} \text{ for } \; m = 1, 2, \ldots, K. \tag{4.37}$$

When the formulas of the optimum precoders and decoders are analyzed, it is noticed that the mth private precoder $m = 1, 2, \ldots, K$ is a function of the mth user decoder matrix. Furthermore, common precoder $\boldsymbol{p_0}$ is a function of all user's decoder matrices. This is an expected result, since the common information is transmitted to all users simultaneously. On the other hand, the decoder for the mth user a function of both common precoder $\boldsymbol{p_0}$ and the mth private precoder $\boldsymbol{p_m}$. Due to these reasons, the closed form solutions of both precoders and decoders are difficult to express analytically. However, the problem could be solved numerically. The solution can be summarized as follows.

Joint Precoder/Decoder Design Algorithm for Transmitting Common-Private Information Combination Scenario

Initialization: $t = 0$ (t denotes index of iteration)
Set for all user's decoder matrices $\boldsymbol{D_i^{(0)}}$ for $i = 1, 2, ...K$ to $\boldsymbol{I}_{L_i \times N}$ where $\boldsymbol{I}_{m,n} = 1$ for $m = n$ and $\boldsymbol{I}_{m,n} = 0 \quad \text{for} \quad m \neq n$

repeat

$t \rightarrow t + 1$

Step 1) Calculate common precoder $\boldsymbol{p_0^{(t)}}$ and private precoder vectors $\boldsymbol{p_i^{(t)}}$ where $i = 1, 2, ...K$ by using decoder matrices of each user $\boldsymbol{D_i^{(t-1)}}$.

Step 2) Calculate $\boldsymbol{D_i^t}$ by using common and private precoder vectors calculated in Step 2.

until $\sum_{i=1}^{K} \left\| \boldsymbol{D_i^{(t)}} - \boldsymbol{D_i^{(t-1)}} \right\|_F^2 \leq \epsilon$ or $t = t_{max}$ where t_{max} is the maximum number of iterations.

(Here, ϵ is taken as 0.0005 and $\|\cdot\|_F$ denotes the Frobenius norm.)

With this algorithm, it is obvious that the total MSE has a monotonically decreasing characteristic with the iteration number. Since the MSE is bounded below by zero, the convergence of this algorithm is guaranteed. However, the convergence point might be a local minimum, hence the global optimality is not guaranteed.

Calculation of Lagrange Multiplier for Common-Private Information Transmission Scenario

According to Equation 4.21, we can write

$$\sum_{j=0}^{K} \boldsymbol{p_j}^H \boldsymbol{p_j} = \boldsymbol{p_0}^H \boldsymbol{p_0} + \sum_{j=1}^{K} \boldsymbol{p_j}^H \boldsymbol{p_j} = P_{tot}. \tag{4.38}$$

The common and private precoder vectors are calculated such as

$$\begin{aligned} \boldsymbol{p_m} &= \left(\sum_{i=1}^{K} \boldsymbol{H_i}^H \boldsymbol{D_i}^H \boldsymbol{D_i} \boldsymbol{H_i} + \lambda \boldsymbol{I} \right)^{-1} (\boldsymbol{D_m} \boldsymbol{H_m})_2^H \quad \text{for} \quad m = 1, 2, ...K. \\ \boldsymbol{p_0} &= \left(\sum_{i=1}^{K} \boldsymbol{H_i}^H \boldsymbol{D_i}^H \boldsymbol{D_i} \boldsymbol{H_i} + \lambda \boldsymbol{I} \right)^{-1} \sum_{m=1}^{K} (\boldsymbol{D_m} \boldsymbol{H_m})_1^H . \end{aligned} \tag{4.39}$$

Let $\sum_{i=1}^{K} \boldsymbol{H_i}^H \boldsymbol{D_i}^H \boldsymbol{D_i} \boldsymbol{H_i} = \boldsymbol{A}$ where $\boldsymbol{A}$ is Hermitian. Then, the following equalities can be written:

$$\begin{aligned} \boldsymbol{p_0}^H \boldsymbol{p_0} &= \left[(\boldsymbol{A} + \lambda \boldsymbol{I})^{-1} \sum_{i=1}^{K} (\boldsymbol{D_i} \boldsymbol{H_i})_1^H \right]^H \left[(\boldsymbol{A} + \lambda \boldsymbol{I})^{-1} \sum_{l=1}^{K} (\boldsymbol{D_l} \boldsymbol{H_l})_1^H \right] \\ &= \operatorname{tr} \left\{ (\boldsymbol{A} + \lambda \boldsymbol{I})^{-1} \left[\sum_{l=1}^{K} (\boldsymbol{D_l} \boldsymbol{H_l})_1^H \sum_{i=1}^{K} (\boldsymbol{D_i} \boldsymbol{H_i})_1 \right] (\boldsymbol{A} + \lambda \boldsymbol{I})^{-1} \right\}. \end{aligned} \tag{4.40}$$

$$\begin{aligned} \sum_{j=1}^{K} \boldsymbol{p_j}^H \boldsymbol{p_j} &= \sum_{j=1}^{K} \left[(\boldsymbol{A} + \lambda \boldsymbol{I})^{-1} (\boldsymbol{D_j} \boldsymbol{H_j})_2^H \right]^H \left[(\boldsymbol{A} + \lambda \boldsymbol{I})^{-1} (\boldsymbol{D_j} \boldsymbol{H_j})_2^H \right] \\ &= \operatorname{tr} \left\{ (\boldsymbol{A} + \lambda \boldsymbol{I})^{-1} \left[\sum_{j=1}^{K} (\boldsymbol{D_j} \boldsymbol{H_j})_2^H (\boldsymbol{D_j} \boldsymbol{H_j})_2 \right] (\boldsymbol{A} + \lambda \boldsymbol{I})^{-1} \right\}. \end{aligned} \tag{4.41}$$

Equation in 4.21 can then be expressed as

$$\sum_{j=0}^{K} \boldsymbol{p_j}^H \boldsymbol{p_j} = \operatorname{tr} \left\{ (\boldsymbol{A} + \lambda \boldsymbol{I})^{-1} \boldsymbol{B} (\boldsymbol{A} + \lambda \boldsymbol{I})^{-1} \right\}, \tag{4.42}$$

where $\boldsymbol{B} = \left\{ \sum_{l=1}^{K} (\boldsymbol{D_l H_l})_1^H \sum_{i=1}^{K} (\boldsymbol{D_i H_i})_1 \right\} + \sum_{j=1}^{K} (\boldsymbol{D_j H_j})^H (\boldsymbol{D_j H_j})_2$.
Therefore,

$$P_{tot} = \mathrm{tr}\left((\boldsymbol{A} + \lambda \boldsymbol{I})^{-2} \boldsymbol{B} \right) = \sum_{k=1}^{M} \frac{1}{(a_k + \lambda)^2} \mathrm{tr}\left(\boldsymbol{B} \boldsymbol{e_k} \boldsymbol{e_k}^H \right), \tag{4.43}$$

where $\boldsymbol{e_k}$'s are eigenvectors and a_k's are eigenvalues of $\boldsymbol{A}$.

As expressed in Chapters 2 and 3, the solution of the Lagrange multiplier is in the interval

$$\left[-\sqrt{\frac{\mathrm{tr}(\boldsymbol{B})}{P_{tot}}} - a_{max}, \sqrt{\frac{\mathrm{tr}(\boldsymbol{B})}{P_{tot}}} - a_{min} \right], \tag{4.44}$$

where a_{max} and a_{min} is the maximum and minimum eigenvalues of $\boldsymbol{A}$, respectively.

CHAPTER 5

SIMULATION RESULTS

In this chapter, the simulation results are provided for three downlink MU-MIMO communications scenarios. The performances of different algorithms are compared in terms of minimum SNR/SINR, mean SNR/SINR, average BER and computation time. We use the average BER term to indicate the BER average of all users in the system. The effect of four types of system parameters are examined, namely, the number of transmitter antennas M, the number of receiver antennas at each user N, the number of users served K and the total precoder power P_{tot} at the transmitter side. In addition, according to the specific properties of algorithms, there exist some additional simulation results to show their advantages and disadvantages.

In this study, the effect of channel estimation errors are not examined, the channel is assumed to be perfectly known. Each entry of the channel matrix is taken as an i.i.d. Rayleigh distributed random variable. It is also assumed that the channel has flat characteristic, which stays the same during the transmission of one block data. At each receiver antenna of each user, there is an i.i.d. zero mean, unity variance complex Gaussian noise. Simulation results are averaged over 1000 channels. In order to obtain BER figures, 10000 data symbols are transmitted in one block per channel usage. For all simulations, each user has the same amount of antennas at the receiver. Except for the simulations in which the number of users is swept, the SNR term given in graphics is defined as per user SNR value meaning the ratio of total precoder power to the number of users times the noise power at the receiver of that user. In order to keep the

total precoder power constant in the system where the number of users is swept, the ratio of signal power to noise power at the receiver is used as SNR. Noise power is indicated by N_0.

For the algorithms requiring a semidefinite programming tool usage, the SeDuMi is used as a solver [17].

5.1 Simulation Results for Multicasting Scenario

In this case, a common information is sent to each user and three types of cost function are tried to be optimized by designing precoder and decoder blocks, namely maximizing the minimum SNR (MMF), minimizing the total MSE and maximizing the total SNR under a total precoder power constraint at the transmitter side. The effects of different system parameters are investigated by sweeping one parameter whose effect is desired to be observed and holding the others constant. Finally, some different scenarios are also analyzed to compare results of SDR techniques in the MMF problem, when the solution of SeDuMi for $\boldsymbol{X_{opt}}$ is not found rank-one.

Effect of Total Precoder Power at Transmitter

In this simulation, the aim is to observe the effect of total precoder power P_{tot} for two different cases. In the first case the number of users is a small value while in the second case large number of users are served. For the first case, the system parameters are given below.

The number of transmitter antennas: $M = 6$.

The number of receiver antennas of each user: $N = 1$.

The number of users: $K = 3$.

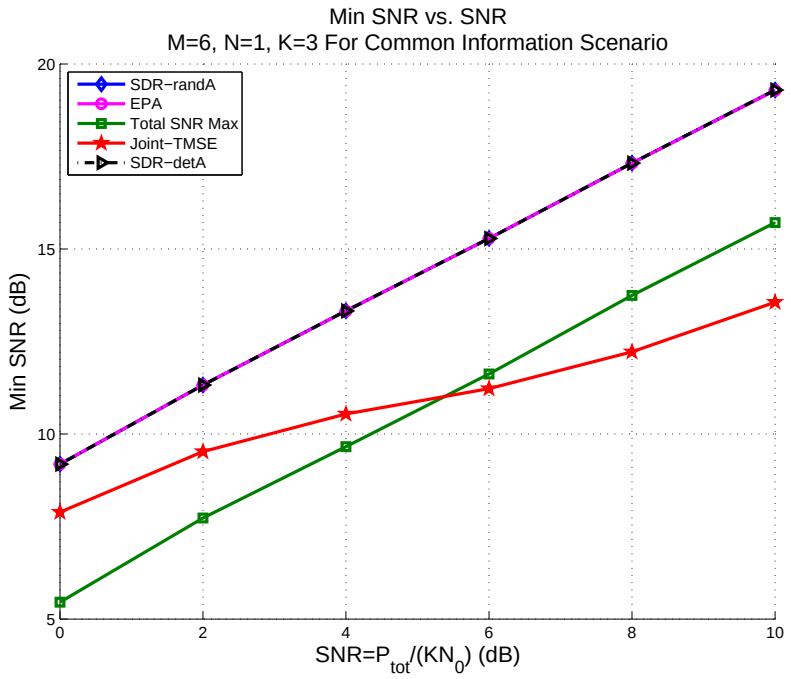

Figure 5.1: Minimum SNR vs. $P_{tot}/(KN_0)$ for Transmitting Common Information Scenario with $M = 6$, $K = 3$, $N = 1$.

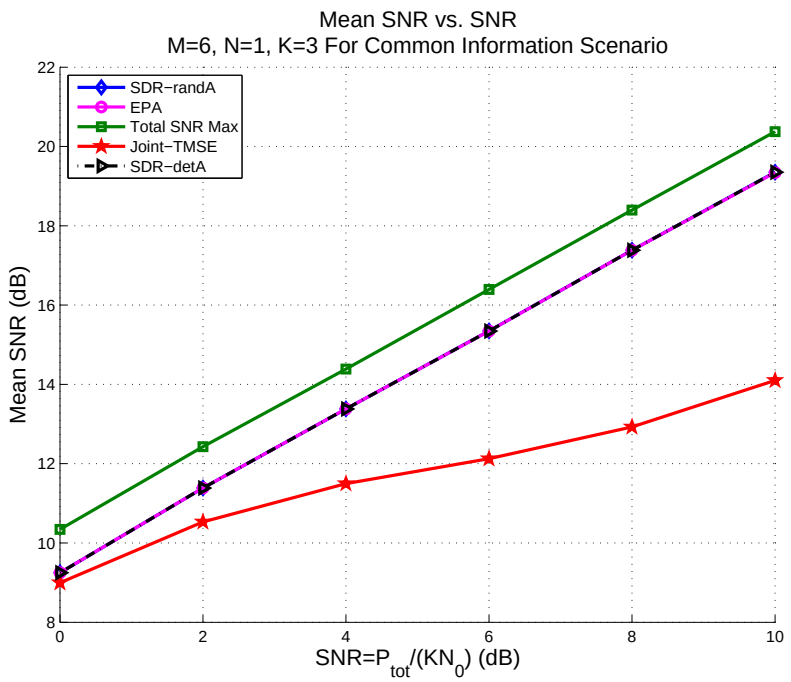

Figure 5.2: Mean SNR vs. $P_{tot}/(KN_0)$ for Transmitting Common Information Scenario with $M = 6$, $K = 3$, $N = 1$.

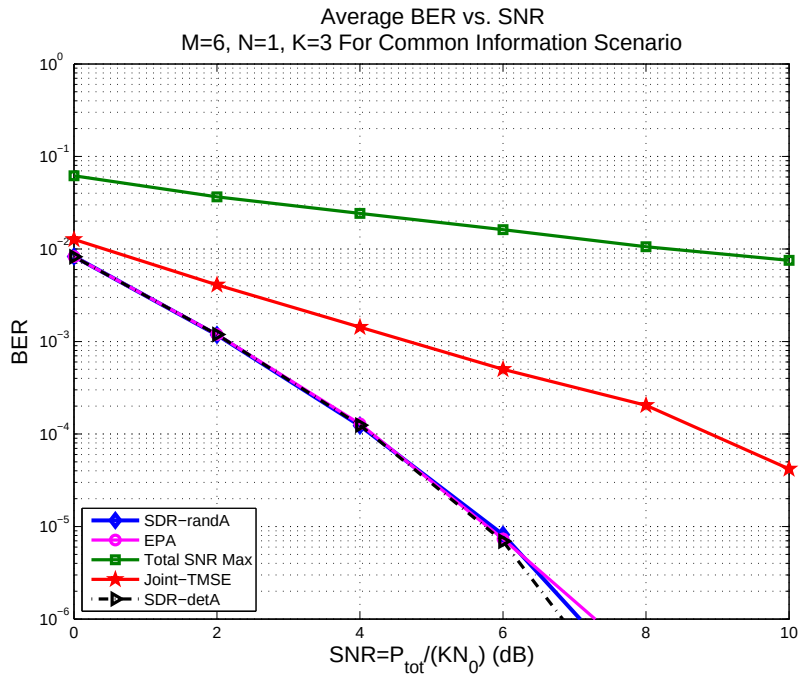

Figure 5.3: Average BER vs. $P_{tot}/(KN_0)$ for Transmitting Common Information Scenario with $M = 6$, $K = 3$, $N = 1$.

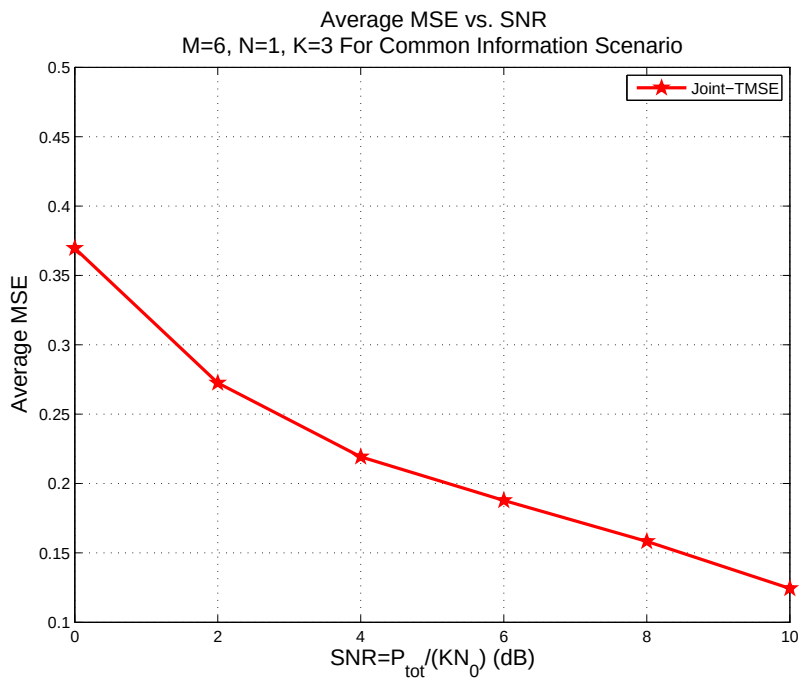

Figure 5.4: Average MSE vs. $P_{tot}/(KN_0)$ for Transmitting Common Information Scenario with $M = 6$, $K = 3$, $N = 1$.

As observed in Figure 5.1, the minimum SNR performance of EPA, SDR-randA and SDR-detA are very similar to each other. Similarity of the results of SDR-randA and SDR-detA implies that according to the notation defined in Chapter 3, MMF part, solution of $\boldsymbol{X_{opt}}$ found from SeDuMi is rank-one. This can also be understood from the percentage of rank-one solutions found in this simulation as presented in Table 5.1. In general, in the case of having small number of users in the system, the solution $\boldsymbol{X_{opt}}$ is mostly rank-one. Therefore, the equivalence of SDR-randA and SDR-detA solutions proved in Chapter 3 is observed at this point. On the other hand, while the Joint-TMSE method is better than the Total SNR Max method at low SNR region, it becomes worse as the SNR increases, since for the Joint-TMSE method, minimum SINR graphic has a saturated characteristic due to the calculation of the Lagrange multiplier. As the SNR increases, finding the Lagrange multiplier for the precoder design of the algorithm requires a higher precision numerical search. For the sake of reasonable simulation time, search precision is chosen as proper value. This might be the reason of saturation at high SNR region. The Total SNR Max algorithm has in general the worst minimum SNR performance, which is indeed reasonable, since the aim of total SNR maximization algorithm is maximizing the mean SNR in the system. As observed in Figure 5.2, the best mean SNR performance belongs to Total SNR Max. In terms of average BER presented in Figure 5.3, similar relation observed in mean SNR is valid for average BER. Lastly, the average total MSE in Figure 5.4 has decreasing characteristic as SNR increases which is also theoretically expected.

Table 5.1: Percentage of rank-one solutions found from SeDuMi in Multicasting Scenario for the Small Number of Users Case, K=3.

M	K	N	SNR (dB)	Percentage of Rank-one Solutions
6	3	1	0	100
6	3	1	2	100
6	3	1	4	100
6	3	1	6	100
6	3	1	8	100
6	3	1	10	100

Now, the simulation results of the second case is given in which the number of users in the system is large and SNR is sweeping. The values of system parameters in this case are provided in the following.

The number of transmitter antennas: $M = 6$.

The number of receiver antennas of each user: $N = 1$.

The number of users: $K = 10$.

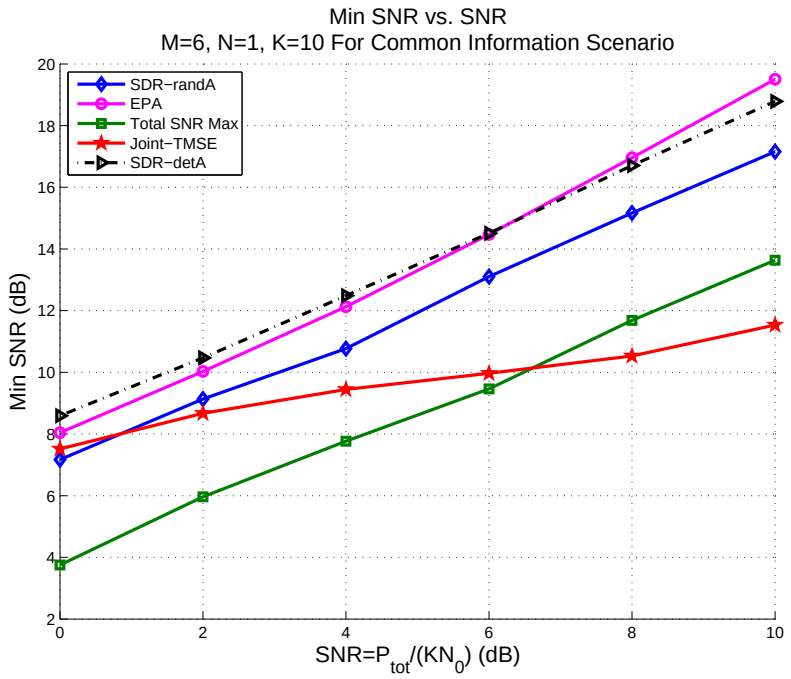

Figure 5.5: Minimum SNR vs. $P_{tot}/(KN_0)$ for Transmitting Common Information Scenario with $M = 6$, $K = 10$, $N = 1$.

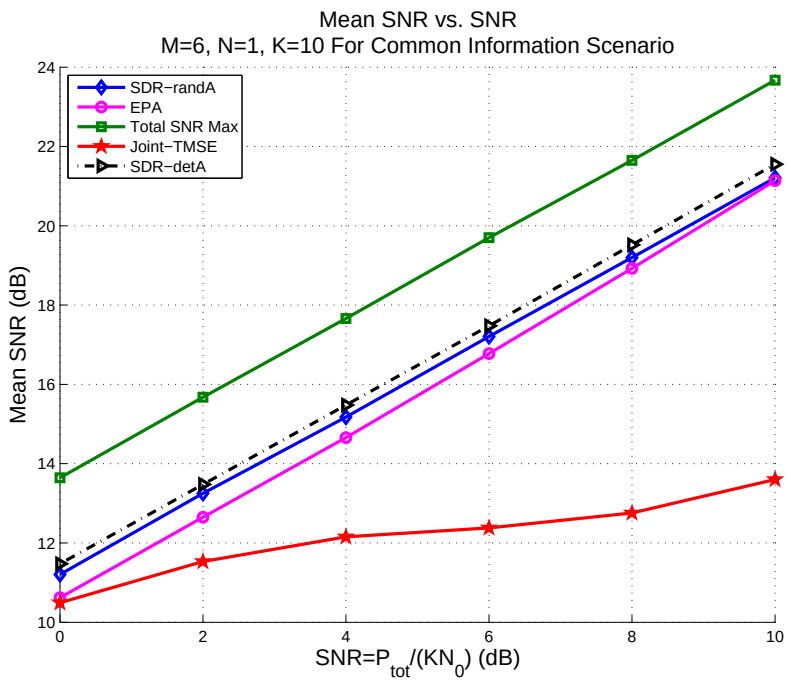

Figure 5.6: Mean SNR vs. $P_{tot}/(KN_0)$ for Transmitting Common Information Scenario with $M = 6$, $K = 10$, $N = 1$.

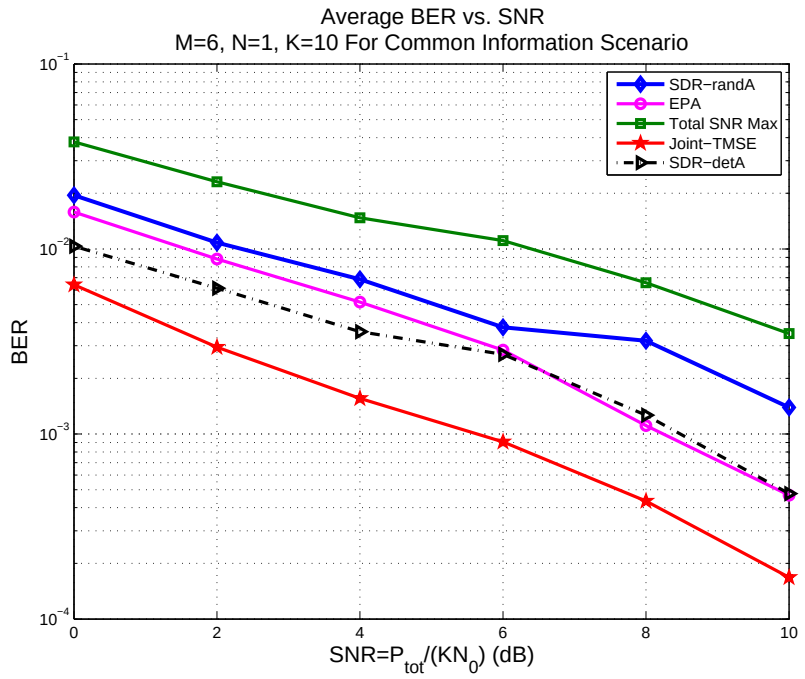

Figure 5.7: Average BER vs. $P_{tot}/(KN_0)$ for Transmitting Common Information Scenario with $M = 6$, $K = 10$, $N = 1$.

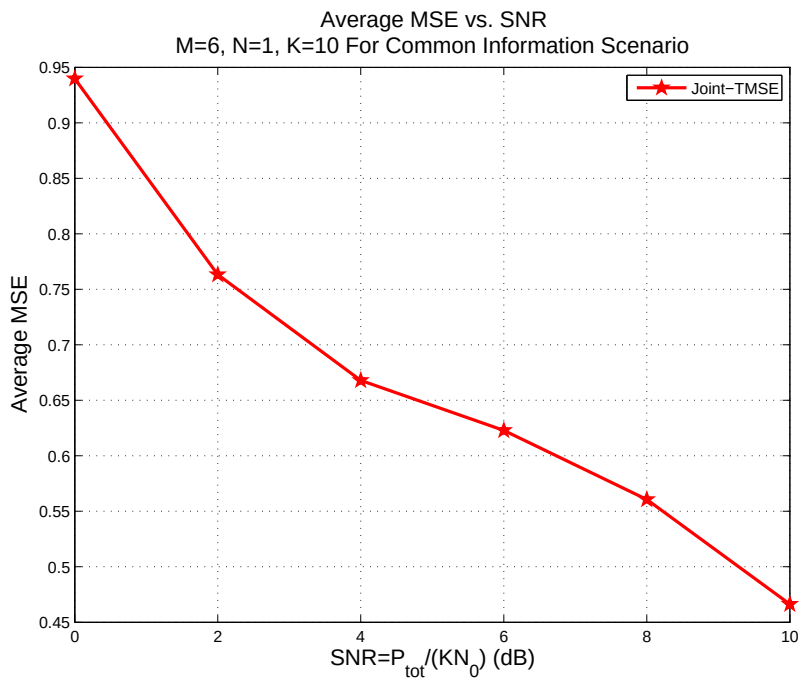

Figure 5.8: Average MSE vs. $P_{tot}/(KN_0)$ for Transmitting Common Information Scenario with $M = 6$, $K = 10$, $N = 1$.

As observed from Figures 5.5-5.6, the minimum and mean SINR is monotonically increasing function of the total precoder power at the transmitter side. In Figure 5.5, it is understood that SDR-detA and EPA methods are the best ones in terms of the minimum SINR. Contrary to the first case expressed, SDR-randA is approximately 1 dB worse than SDR-detA method, which is proposed in this thesis. The reason of the difference between these two methods is due to the fact that solution of SeDuMi for $\boldsymbol{X_{opt}}$ in MMF problem is not rank-one. Percentage of rank-one solutions for this simulation is given in Table 5.2. Having not a rank-one solution is possible in multicasting case especially when the number of users is large. In the case of having not a rank-one solution of $\boldsymbol{X_{opt}}$ it could be said that SDR-detA method performs better than SDR-randA. Moreover, at low SNR region, EPA method is a little bit amount worse than SDR-detA. The reason of this situation is related with long computational time of EPA method as shown in Figure 5.21. It may not be converged until the maximum iteration number of the algorithm specified for feasible computational time is reached. As SNR increases, it has a better performance than SDR-detA method. This is an expected result due to the fact that EPA method always find rank-one solution no matter how system parameters selected provided that the algorithm converges. It directly solves the problem without omitting any constraint as in SDR method. Therefore, in multicasting scenario for the cases when the number of users is large using EPA or SDR-detA methods can be advantageous. For joint TMSE, again due to Lagrange multiplier issues, its performance becomes worse compared to others. For mean SNR performance given in Figure 5.6, Total SNR Max has the best performance due to the same reason given in the first case. In terms of average BER in Figure 5.7, different than first one, joint TMSE is better than the others. This can also be an expected result due to obtaining solutions of SeDuMi not being rank-one. Again Total SNR Max is the worst one. SDR-detA and EPA methods have similar performances both of which are superior than SDR-randA. The average MSE given in Figure 5.8 is a decreasing function of SNR as expected.

Table 5.2: Percentage of rank-one solutions found from SeDuMi in Multicasting Scenario for the Large Number of Users Case, K=10.

M	K	N	SNR (dB)	Percentage of Rank-one Solutions
6	10	1	0	15.2
6	10	1	2	13.5
6	10	1	4	11.2
6	10	1	6	14.7
6	10	1	8	14.7
6	10	1	10	13.7

Effect of The Number of Transmitter Antennas

In this simulation, the aim is to observe the effect of the number of antennas, denoted by M, at the transmitter side. Therefore, other system parameters are fixed whose values are provided below.

SNR: 10 dB.

The number of receiver antennas of each user: $N = 1$.

The number of users: $K = 8$.

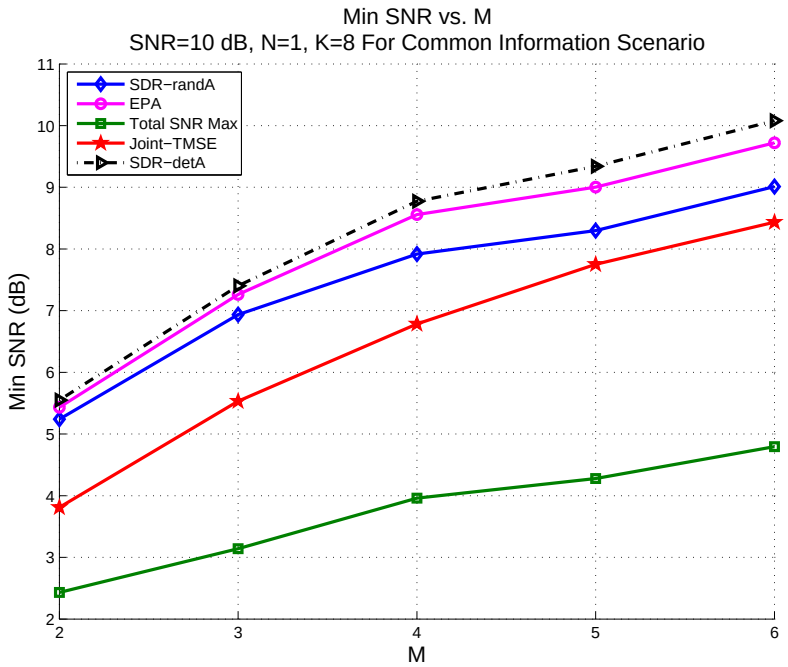

Figure 5.9: Minimum SNR vs. M for Transmitting Common Information Scenario with $SNR = 10$ dB, $K = 8$, $N = 1$.

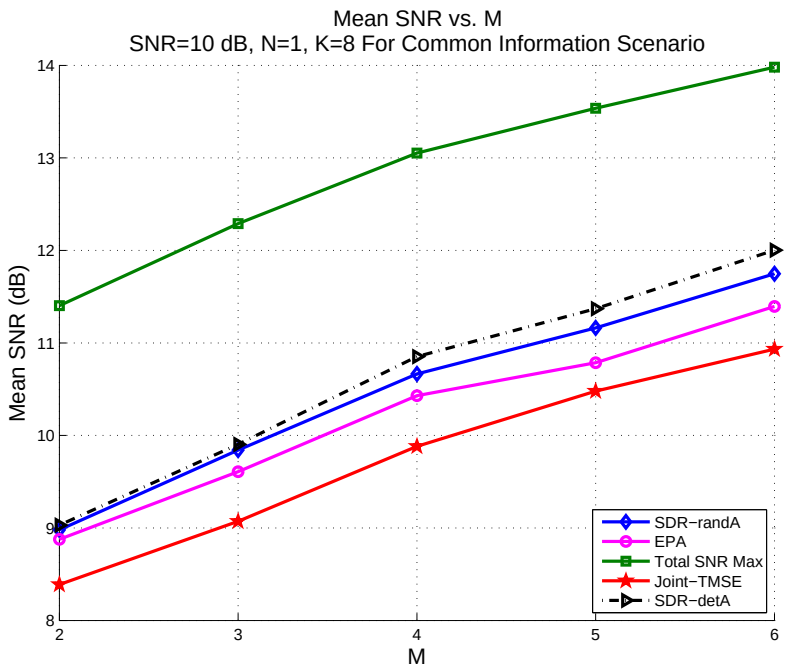

Figure 5.10: Mean SNR vs. M for Transmitting Common Information Scenario with $SNR = 10$ dB, $K = 8$, $N = 1$.

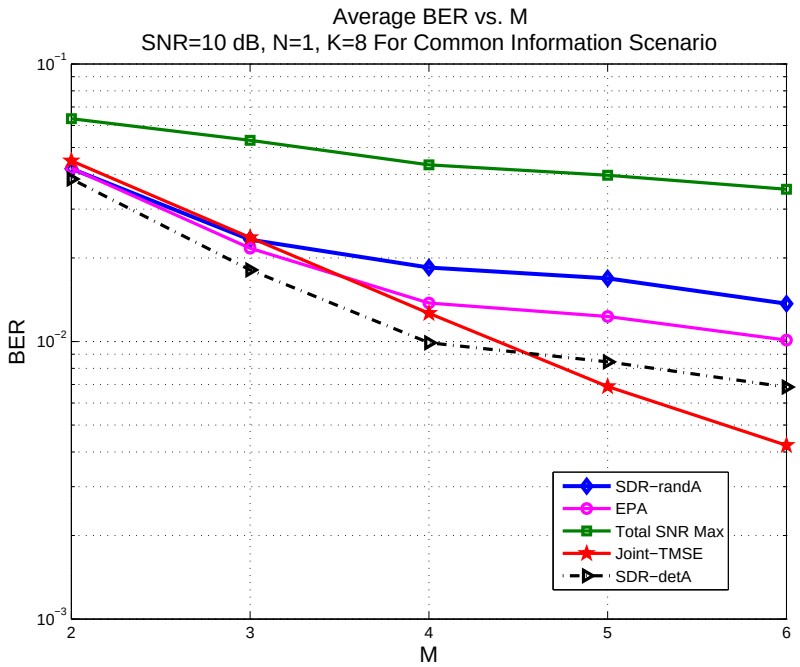

Figure 5.11: Average BER vs. M for Transmitting Common Information Scenario with $SNR = 10$ dB, $K = 8$, $N = 1$.

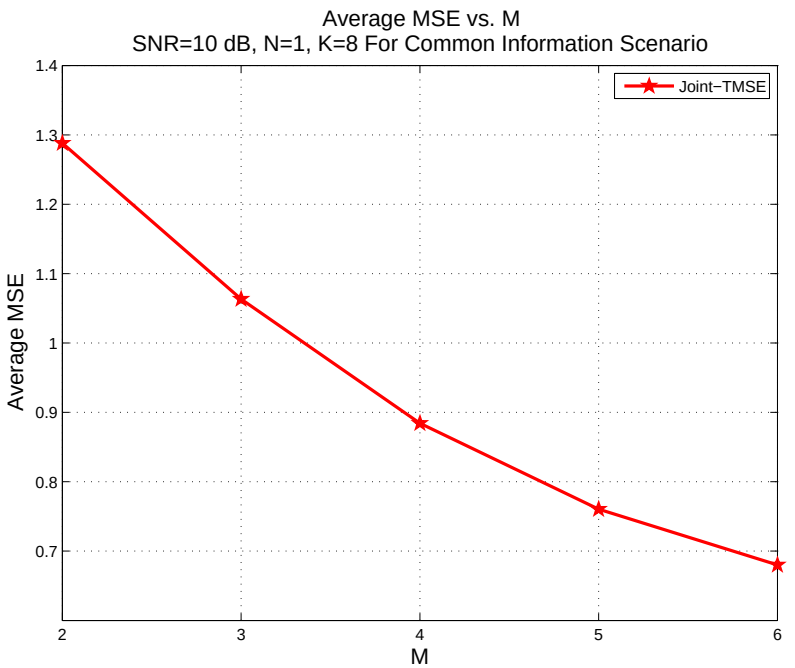

Figure 5.12: Average MSE vs. M for Transmitting Common Information Scenario with $SNR = 10$ dB, $K = 8$, $N = 1$.

When Figures 5.9-5.10 are examined, it is observed that the minimum and mean SNR of the system is increasing function of the number of antennas at the transmitter side due to the transmitter diversity; that is, by using more transmitting antennas, it is possible to sent the same information via more different channel characteristics which increases the possibility of obtaining better signal at the receiver side. The algorithms behave similarly in performance as Figures 5.5-5.6. For the average BER presented in Figure 5.11, as a result of increasing minimum and mean SNR in the system, it decreases as M increases. At high valued number of transmitter antennas, some degradations can be observed in the smoothness of graphics which is due to Monte Carlo simulations that is using a finite number of channels for the feasibility of the simulation time.

Effect of The Number of Users

In this simulation, the aim is to observe the effect of the number of users, denoted by K, in the system. The other system parameters are fixed whose values are provided below.

SNR: 10 dB.

The number of receiver antennas of each user: $N = 2$.

The number of transmitter antennas: $M = 4$.

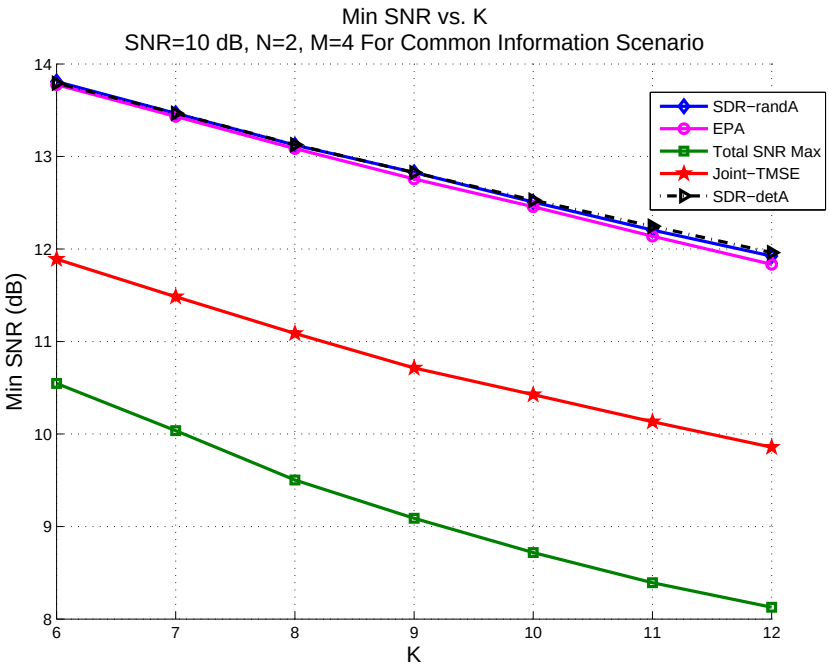

Figure 5.13: Minimum SNR vs. K for Transmitting Common Information Scenario with $SNR = 10$ dB, $M = 4$, $N = 2$.

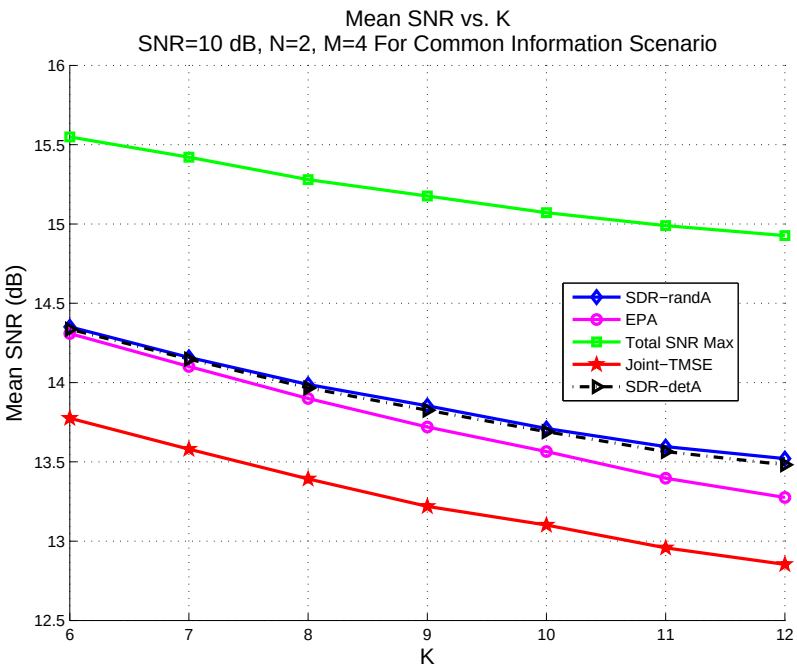

Figure 5.14: Mean SNR vs. K for Transmitting Common Information Scenario with $SNR = 10$ dB, $M = 4$, $N = 2$.

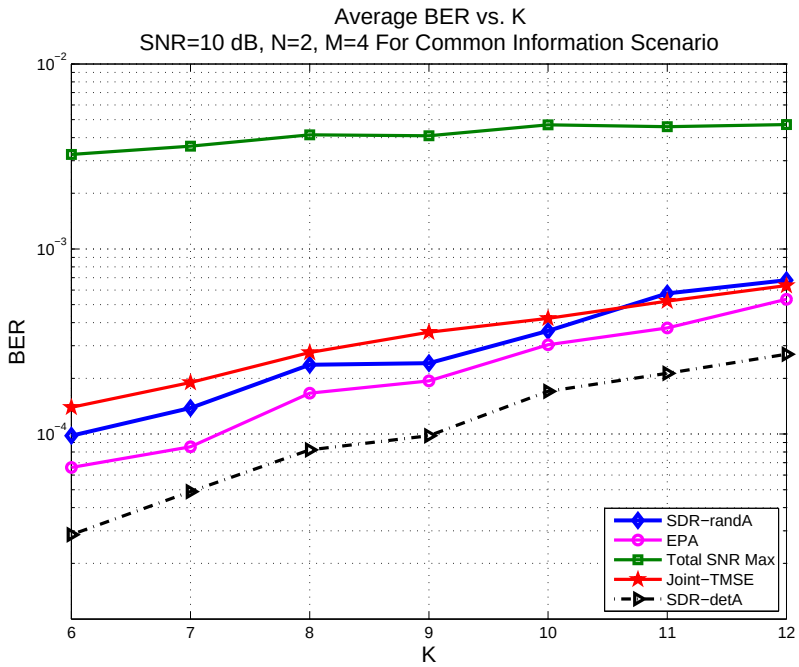

Figure 5.15: Average BER vs. K for Transmitting Common Information Scenario with $SNR = 10$ dB, $M = 4$, $N = 2$.

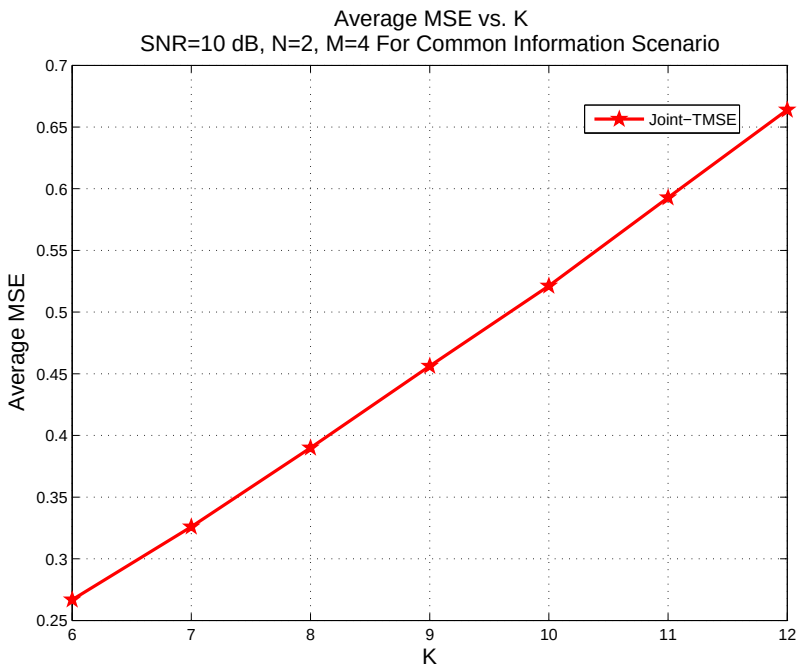

Figure 5.16: Average MSE vs. K for Transmitting Common Information Scenario with $SNR = 10$ dB, $M = 4$, $N = 2$.

As given in Figures 5.13-5.14, the minimum and mean SNR is a decreasing function of the number of users in the system. It should be remembered that in this thesis, throughout the whole simulations in which the number of users is swept, the SNR is defined as the ratio of total transmitter power over N_0. Due to this definition, it should be clear that as the number of users increases, the power per user decreases which directly leads to degradation in obtained minimum and mean received SNR levels. In terms of minimum SNR, SDR-detA, SDR-randA and EPA methods are similar, while the Joint-TMSE and Total SNR Max algorithms are worse than these. For average BER results, given in Figure 5.15, SDR-detA is better than SDR-randA method due to not rank-one solutions found from SeDuMi. According to the simulation results, it is observed that as the number of users in the system increases, the percentage of rank-one solutions found from SeDuMi decreases as shown in Table 5.3. Except for some points which might be due to Monte Carlo fluctuations, in general SDR and EPA methods are better than Joint-TMSE and Total SNR Max since optimizing total MSE or total SNR in the system it can not be guaranteed that all users will be served in a favorable level. Some of the users might have very bad minimum SNR or BER values which effects the performance of overall system drastically. Average MSE of the Joint-TMSE algorithms given in Figure 5.16 also increases as the number of users increases in the system.

Table 5.3: Percentage of rank-one solutions found from SeDuMi in Multicasting Scenario w.r.t. Number of Users.

M	SNR (dB)	N	K	Percentage of Rank-one Solutions
4	10	1	6	98.1
4	10	1	7	95.8
4	10	1	8	93.1
4	10	1	9	90.5
4	10	1	10	87.3
4	10	1	11	82.8
4	10	1	12	78.6

Effect of The Number of Antennas at the Receiver of Each User

In this simulation, the aim is to observe the effect of the number of antennas at the receiver, denoted by N, of each user. Therefore, other system parameters are fixed whose values are provided below.

SNR: 2 dB.

The number of users: $K = 12$.

The number of transmitter antennas: $M = 2$.

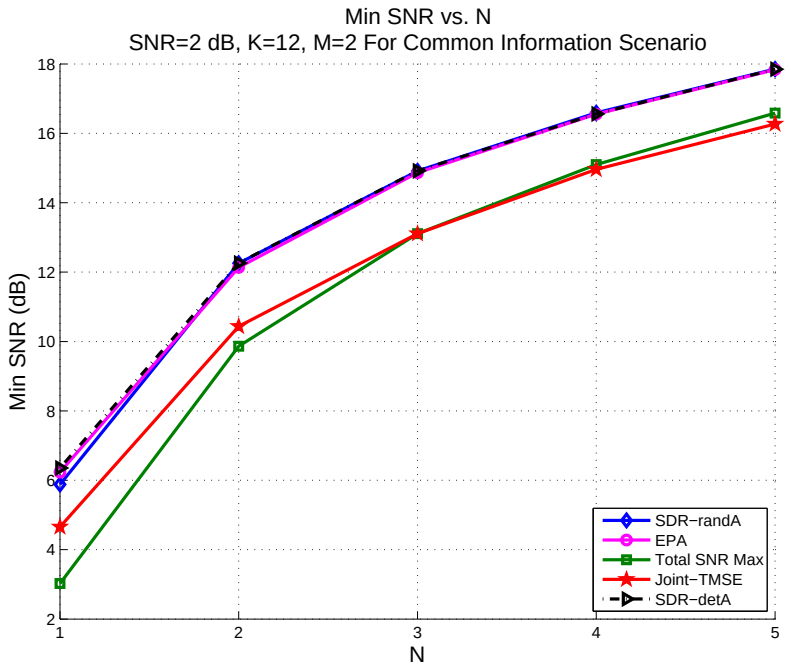

Figure 5.17: Minimum SNR vs. N for Transmitting Common Information Scenario with $SNR = 2$ dB, $M = 2$, $K = 12$.

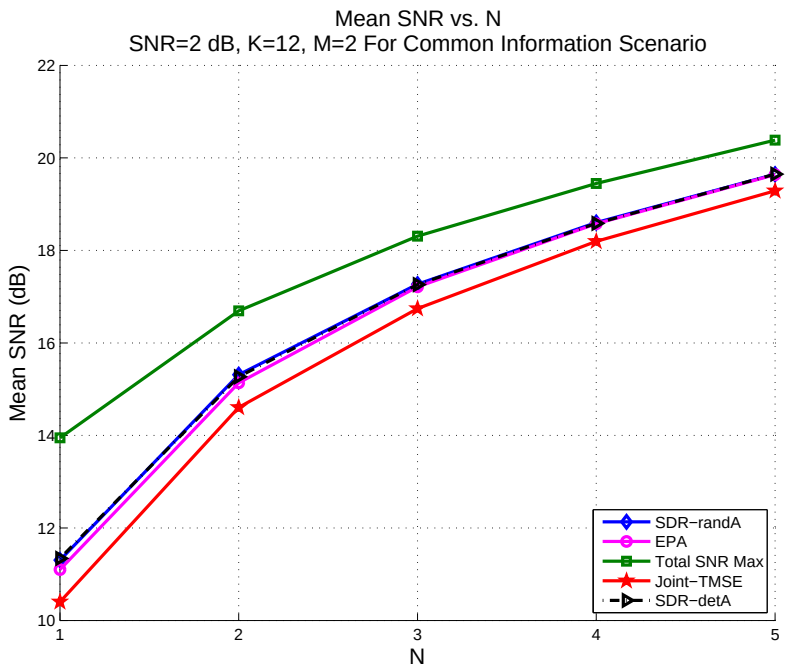

Figure 5.18: Mean SNR vs. N for Transmitting Common Information Scenario with $SNR = 2$ dB, $M = 2$, $K = 12$.

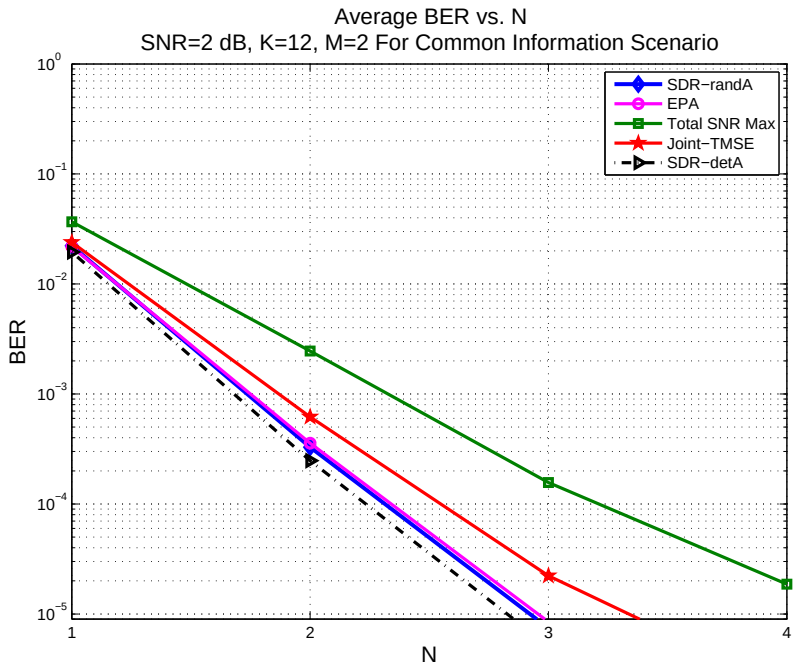

Figure 5.19: Average BER vs. N for Transmitting Common Information Scenario with $SNR = 2$ dB, $M = 2$, $K = 12$.

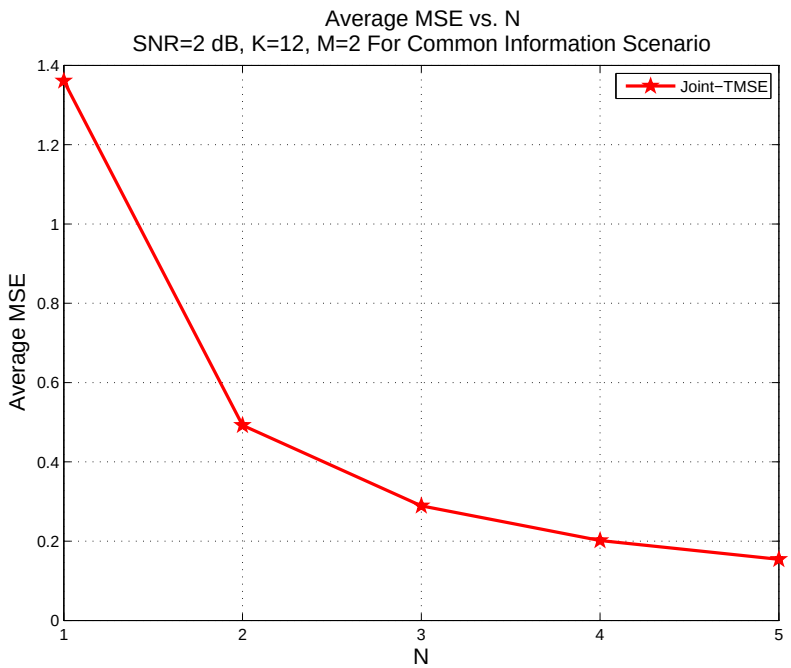

Figure 5.20: Average MSE vs. N for Transmitting Common Information Scenario with $SNR = 2$ dB, $M = 2$, $K = 12$.

Increasing number of receiver antennas for each user affects the system performance significantly as seen from Figures 5.17-5.20. By using multiple antennas, each user have a chance to combine the same signal information passed from different channel characteristics called as the receiver diversity. Contrary to interfering MIMO broadcasting scenario given in Figures 5.34- 5.36, as the number of antennas at the receiver of each user increases, minimum-mean SNR, average BER and average MSE performances of all methods are improved more significantly even in low SNR levels, since there is no interference in the multicasting scenario. Comments given for the cases examined earlier parameters also applies to this case.

Comparison of Computation Time of Algorithms in Multicasting Scenario

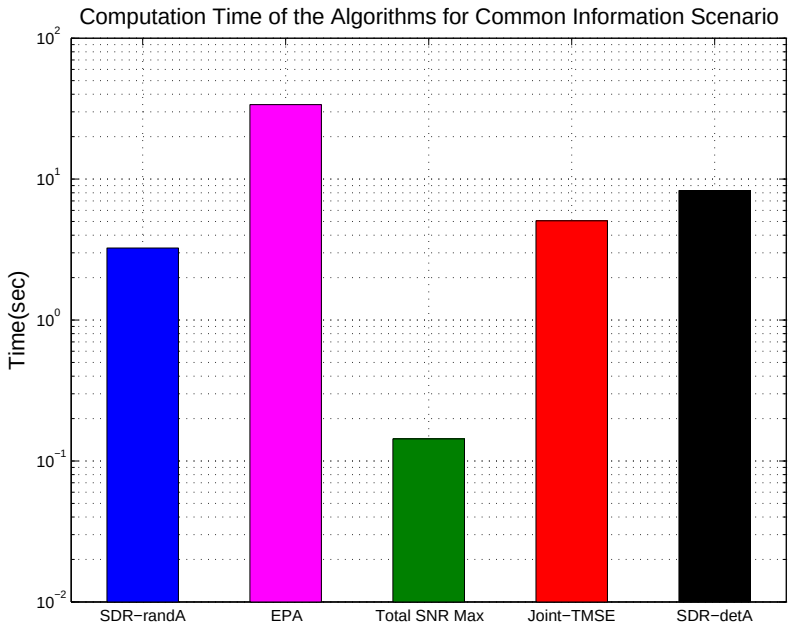

Figure 5.21: Average Computation Time of Algorithms for Transmitting Common Information Scenario with $M = 6$, $N = 1$, $K = 10$ in $0 - 10$ dB SNR range.

In Figure 5.21, the average computational time of all algorithms over the simulation in which the effect of total precoder power is observed. The average shows the average computation time of the algorithms over $0 - 10$ dB SNR range. As observed in Figure 5.21, EPA method has the longest computational

time. It is due to the fact that this method use SeDuMi in many times. On the other hand, performance of the SDR-randA, SDR-detA and Joint-TMSE algorithms are near to each other. In fact, Joint-TMSE method seems simpler to implement; however, finding the Lagrange multiplier task of this algorithm takes significant time due to the numerical search. In fact, this search operation is also done for interfering broadcasting channel, computation time of algorithms of this scenario are given in Figure 5.21. However, for that scenario, the expression for Lagrange multiplier calculation is simplified while in multicasting it can not be simplified. Due to this simplification, Joint-TMSE method is much more computationally efficient than SDR methods in interfering broadcasting scenario while it has similar performance with them in multicasting. Although the SDR methods uses SeDuMi, thanks to the fact that there is no interference in multicasting scenario, the precoder design part of the MMF problem can be directly solved by using SeDuMi at once. Contrary to interfering broadcasting scenario presented in Figure 5.38, the computational time of the SDR methods in this part is not very long. Total SNR Max method is the best one in the sense of computational efficiency. However, its performance in general is far from all the other algorithms in all aspects. The result obtained by SDR-randA and SDR-detA methods have favorable in general and their computational time is also an acceptable level for the multicasting scenario.

In this part, performance of different algorithms are compared under multicasting scenario. Solution of the MMF problem is studied with methods which are all using SDP tools. Minimization of TMSE problem is also analyzed by Joint-TMSE method. It is observed that the methods using the SDP tools have better performance in terms of both minimum SNR and average BER. The EPA method has the longest computational time due to the iterative solution procedure. However, SDR-randA and SDR-detA methods are faster than EPA due to direct solution of MMF problem in multicasting scenario. It is also understood that the average BER performance of SDR-randA, SDR-detA and EPA methods are nearly same for the systems with small number of users while SDR-detA and EPA methods are more advantageous than SDR-randA method where large number of users are served. This is the result of the fact that the percentage

of rank-one solutions found by SDP tools decreases as the number of users in the system increases. As an alternative solution for long computation time of the methods used in the MMF problem, the minimum SNR and the average BER performance of Total SNR Max method is analyzed. This is much faster than the ones using SDP tool; however its min SNR performance is worse as theoretically expected.

5.2 Simulation Results for Interfering Broadcasting Scenario

In the second scenario, the interfering MIMO broadcast channel is analyzed. In other words, each user receives a private information. In this case, due to the interference between channels of different users, instead of SNR terms, SINR expressions will be optimized. Similar to previous case, maximization of minimum SINR and minimization of total MSE problems are simulated. Different from the multicasting scenario in the solution of MMF problem, solution of SeDuMi for $\boldsymbol{X_{opt}}$ is always rank-one as proven in [35]. This result is also shown in Table 5.4.

Effect of Total Precoder Power at the Transmitter

In this simulation, total transmitter power P_{tot} is swept and other system parameters are fixed whose values are provided below:

Number of transmitter antennas: $M = 6$.
Number of receiver antennas at the receiver of each user: $N = 2$.
Number of users: $K = 3$.

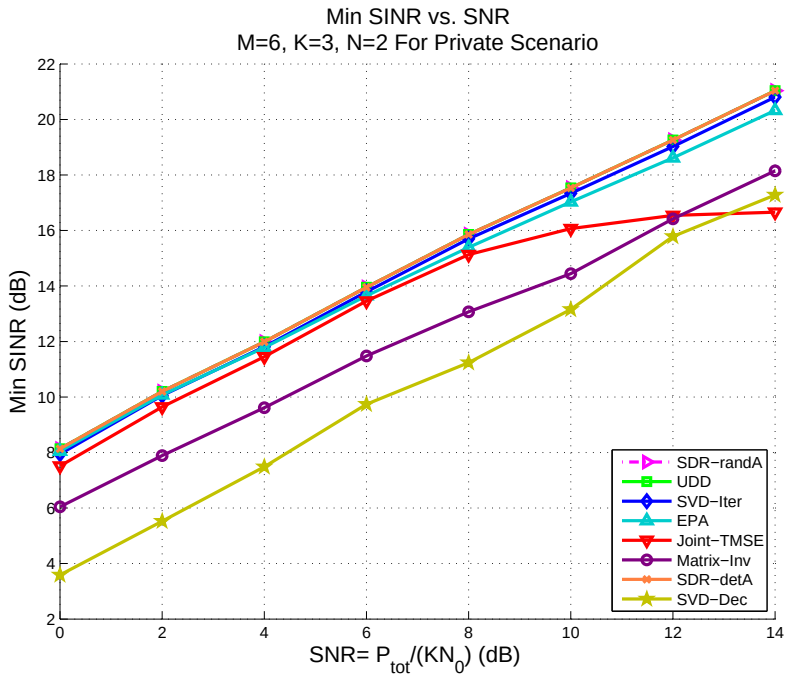

Figure 5.22: Minimum SINR vs. SNR for Transmitting Private Information Scenario with $M = 6$, $K = 3$, $N = 2$.

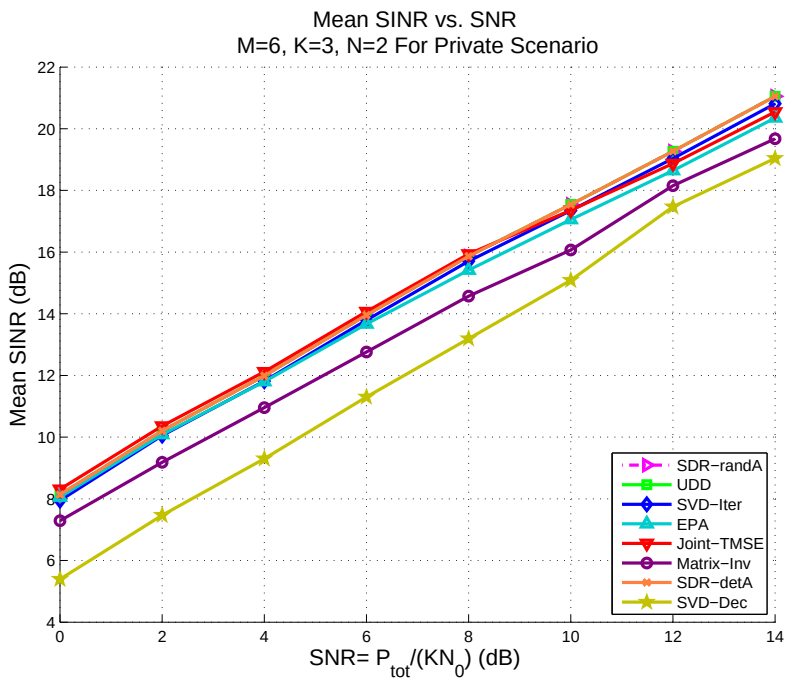

Figure 5.23: Mean SINR vs. SNR for Transmitting Private Information Scenario with $M = 6$, $K = 3$, $N = 2$.

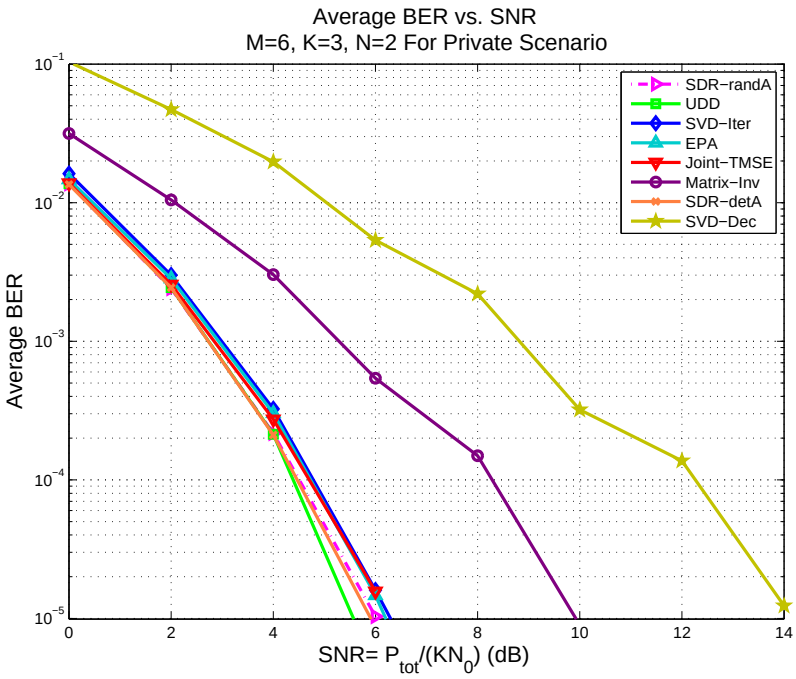

Figure 5.24: Average BER vs. SNR for Transmitting Private Information Scenario with $M = 6$, $K = 3$, $N = 2$.

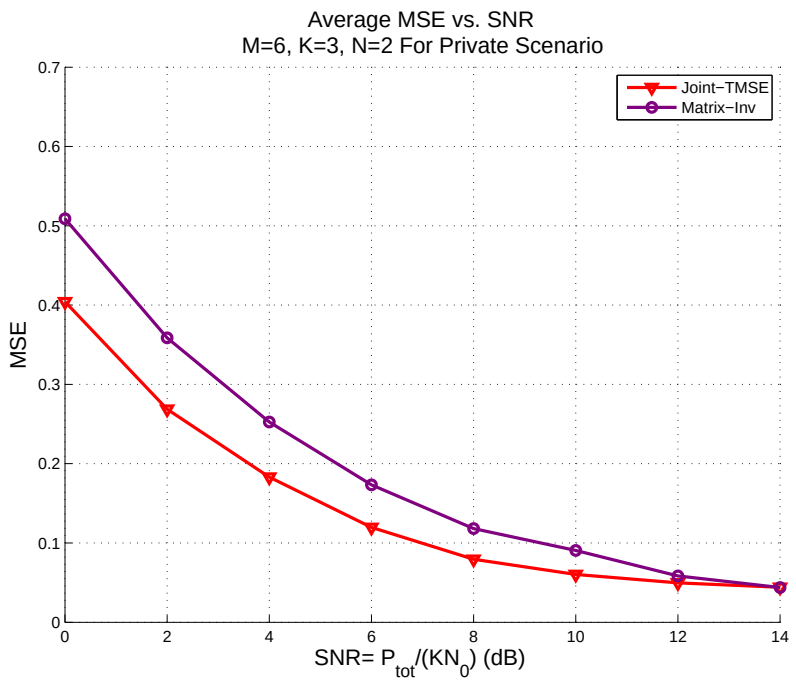

Figure 5.25: Average MSE vs. SNR for Transmitting Private Information Scenario with $M = 6$, $K = 3$, $N = 2$.

In Figure 5.22, the characteristic of minimum SINR with respect to SNR is given. It is noticed that up to 8 dB SNR value, the minimum SINR performances of SDR-randA, SDR-detA, iterated SVD, UDD are nearly the same. The Joint-TMSE method is approximately 0.5 dB inferior in general. This can be an expected result, since the main aim of Joint-TMSE algorithm is minimizing the total MSE in the system. Due to similar reason explained for multicasting MIMO channel simulation results, after 8 dB, joint TMSE method has a saturated behavior which is related with the difficulties in the calculation of the Lagrange multiplier. In terms of mean SINR, given in Figure 5.23, the same algorithms in min SINR case have similar performances, but this time Joint-TMSE method is relatively better. On the other hand, the methods of Matrix-Inv has nearly $2-3$ dB and SVD-Dec method is $4-5$ dB worse performance which is also expected. Remembering that no power allocation is performed in both of these two methods, the transmitter power is equally divided between users. In all other methods, power is dynamically allocated according to the CSI of the users which contributes the performance effectively. Moreover, as seen from Figures 5.22-5.23, minimum SINR and mean SINR monotonically increases with increasing SNR value as theoretically expected.

Similar comparisons can also be given for average BER curve. In Figure 5.24, again SVD-Dec and Matrix-Inv methods are the worst ones. In order to reach the same 10^{-5} BER level, Matrix-Inv method should have 4 dB and SVD-Decomp method should have 8 dB more SNR value with respect to other methods. For the other algorithms, one can say that for the SNR interval of 0-4 dB, their BER values are approximately the same. As SNR increases, it is observed that the UDD, SDR-detA and SDR-randA methods are a little better than the SVD-Iter, Joint-TMSE and EPA. In SVD-Iter method, precoder design will be done by making interference term zero in SINR expression which might be a suboptimal solution in the low SNR region. Moreover, the difference between EPA and SDR methods might be due to Monter Carlo fluctuations.

In Figure 5.25, average MSE performances of Matrix-Inv and Joint-TMSE algorithms are provided. It is observed that as SNR value increases total MSE is monotonically decreases until the algorithms converge. Furthermore, in all

SNR values, the performance of Joint-TMSE algorithm is better than Matrix-Inv method which is an expected result since in Joint-TMSE algorithm power allocation is implemented while in Matrix-Inv it is not.

The percentage of rank-one solutions for this simulation is given in Table 5.4. As can be seen from the results, in private scenario all solutions found from SeDuMi is rank-one. Therefore, performance of SDR-randA and SDR-detA methods are exactly the same as explained in Chapter 3.

Table 5.4: Percentage of rank-one solutions found from SeDuMi in Interfering Broadcasting Scenario.

M	K	N	SNR (dB)	Percentage of Rank-one Solutions
6	3	2	0	100
6	3	2	2	100
6	3	2	4	100
6	3	2	6	100
6	3	2	8	100
6	3	2	10	100
6	3	2	12	100
6	3	2	14	100
6	3	2	16	100

Effect of The Number of Transmitter Antennas

In this simulation, the number of antennas at the transmitter side M is swept and other system parameters are fixed whose values are provided below:

SNR: 5 dB.
Number of receiver antennas at the receiver of each user: $N = 2$.
Number of users: $K = 3$.

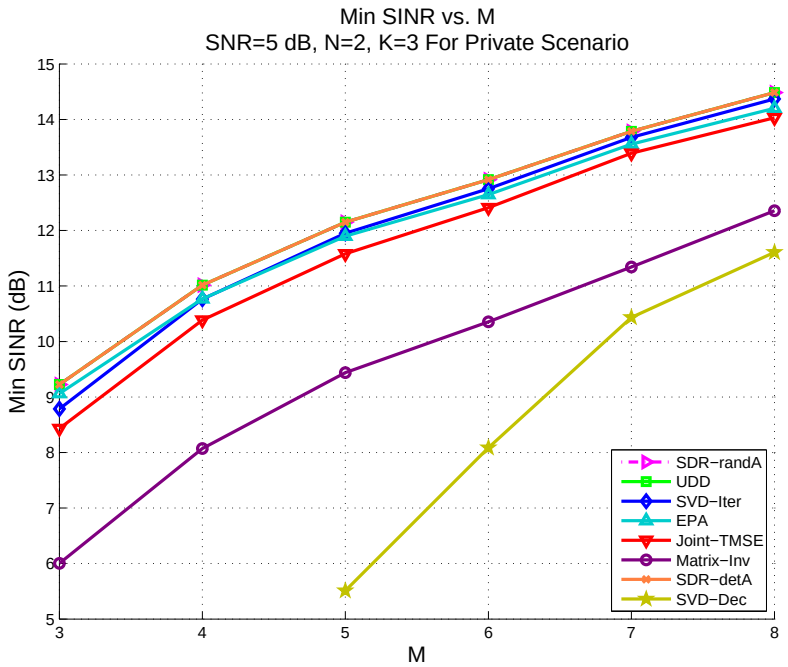

Figure 5.26: Minimum SINR vs. M for Transmitting Private Information Scenario with $SNR = 5$ dB, $K = 3$, $N = 2$.

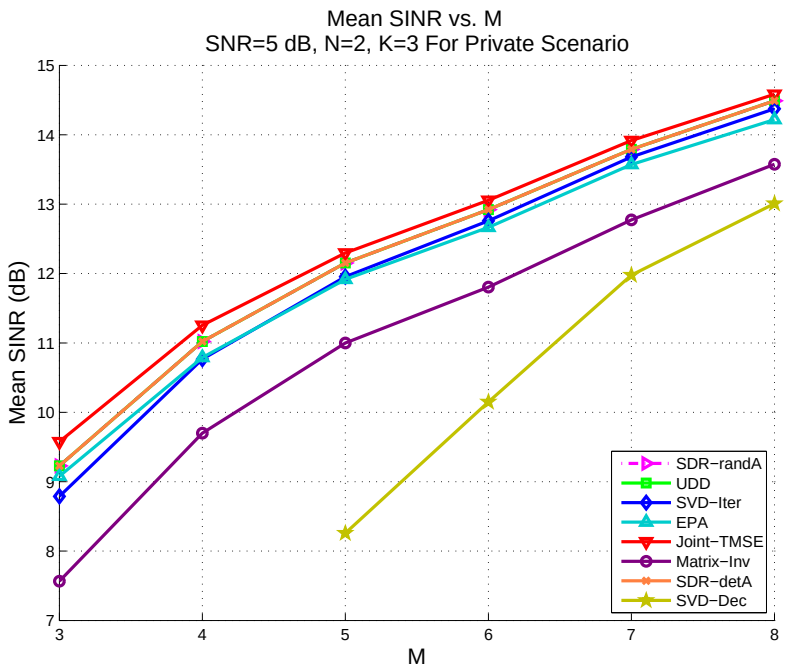

Figure 5.27: Mean SINR vs. M for Transmitting Private Information Scenario with $SNR = 5$ dB, $K = 3$, $N = 2$.

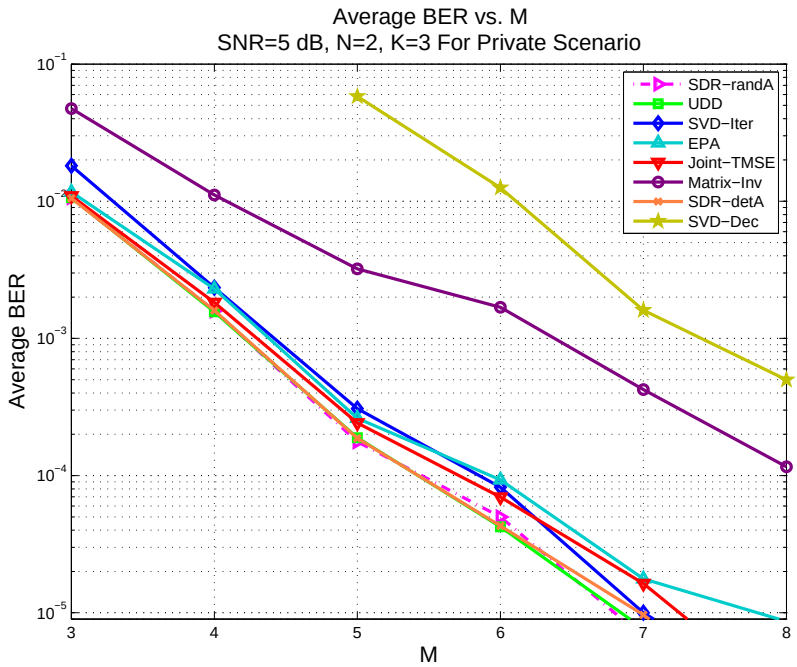

Figure 5.28: Average BER vs. M for Transmitting Private Information Scenario with $SNR = 5$ dB, $K = 3$, $N = 2$.

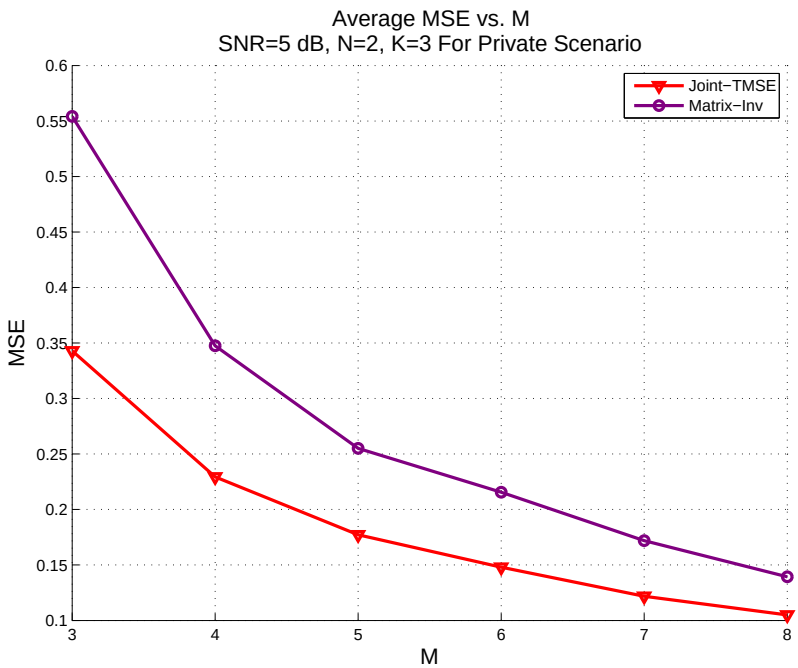

Figure 5.29: Average MSE vs. M for Transmitting Private Information Scenario with $SNR = 5$ dB, $K = 3$, $N = 2$.

As seen from Figures 5.26-5.27, the minimum and mean SINR of all algorithms has a monotonically increasing characteristic as the number of transmitter antennas increases due to the transmitter diversity. In this case, again SDR-detA, SDR-randA and UDD performances are very similar since the solution of SeDuMi is always rank-one. Performances of SVD-Iter and EPA are nearly the same. Moreover, the Joint-TMSE method is approximately 0.5 dB worse than UDD and SDR methods. For mean SINR results given in Figure 5.27, Joint-TMSE is the best one. Except for Matrix-Inv and SVD Dec methods, the other methods have similar mean SINR values. It should be noted that at some transmitter antenna numbers SVD-Dec method has no value since according to simulation parameters, it can not work due to transmitter antenna number constraint ($M > (K-1)N$) expressed in Chapter 3. This result again shows that SVD-Dec algorithm is not practically feasible approach.

The average BER curve given in Figure 5.28 is monotonically decreasing as the number of transmitter antennas increases. Similar relations among performance of the algorithms given in Figure 5.24 is valid for this case. As the number of transmitter antennas increases some degradations are observed in the BER characteristics which is due to finite number of data bits used for simulation purposes. The average MSE also decreases but with a slower rate with respect to the one in which the SNR is swept given in Figure 5.29.

Effect of The Number of Users

In this simulation, the number of users served K is swept and other system parameters are fixed whose values are provided below:

SNR: 5 dB.
Number of receiver antennas at the receiver of each user: $N = 2$.
Number of users: $M = 8$.

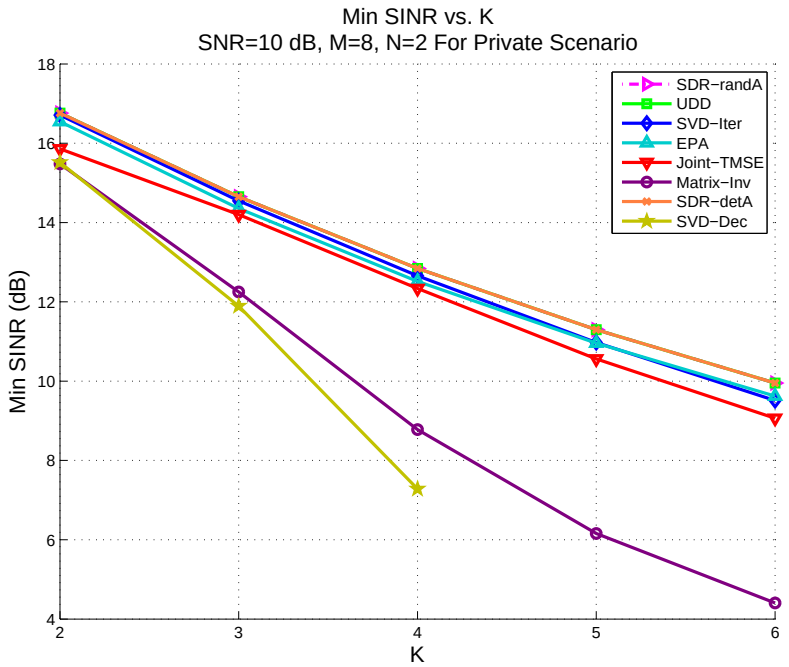

Figure 5.30: Minimum SINR vs. K for Transmitting Private Information Scenario with $SNR = 10$ dB, $M = 8$, $N = 2$.

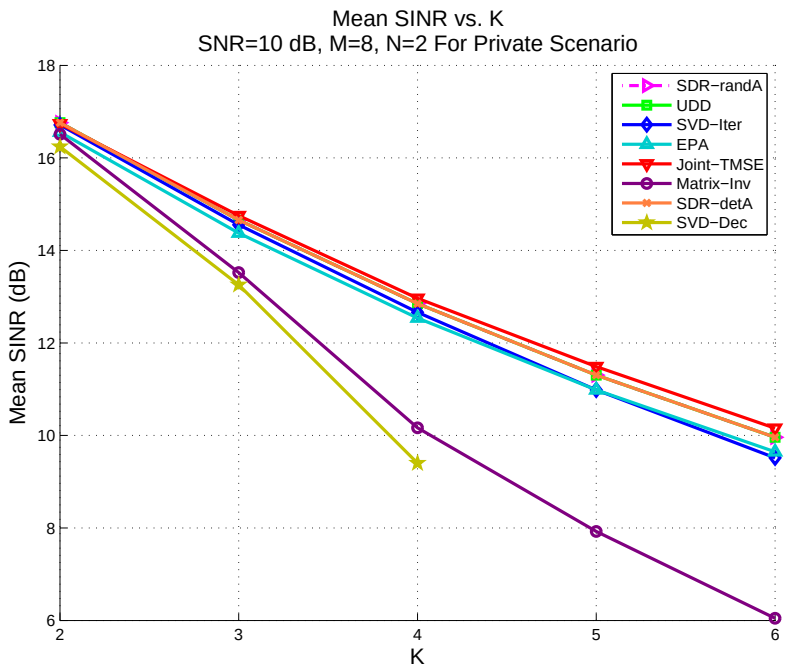

Figure 5.31: Mean SINR vs. K for Transmitting Private Information Scenario with $SNR = 10$ dB, $M = 8$, $N = 2$.

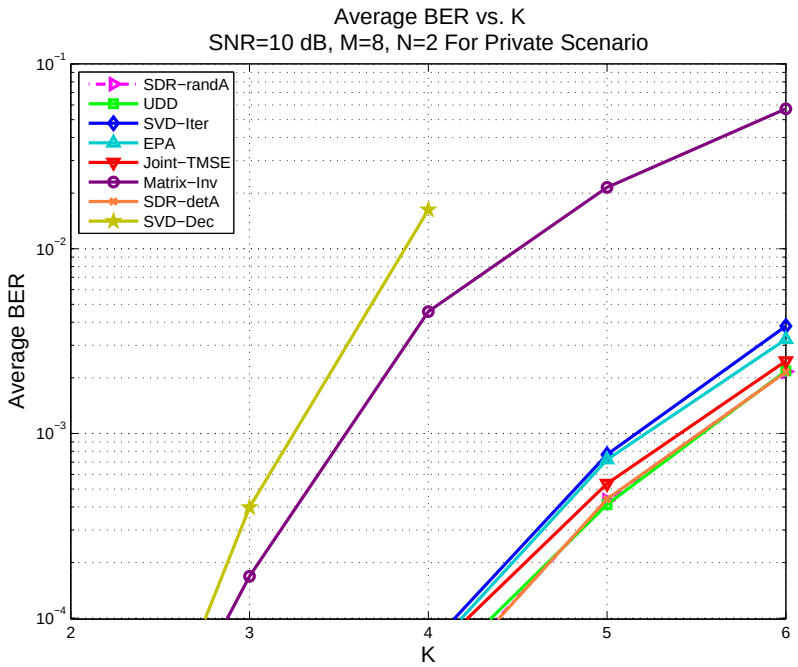

Figure 5.32: Average BER vs. K for Transmitting Private Information Scenario with $SNR = 10$ dB, $M = 8$, $N = 2$.

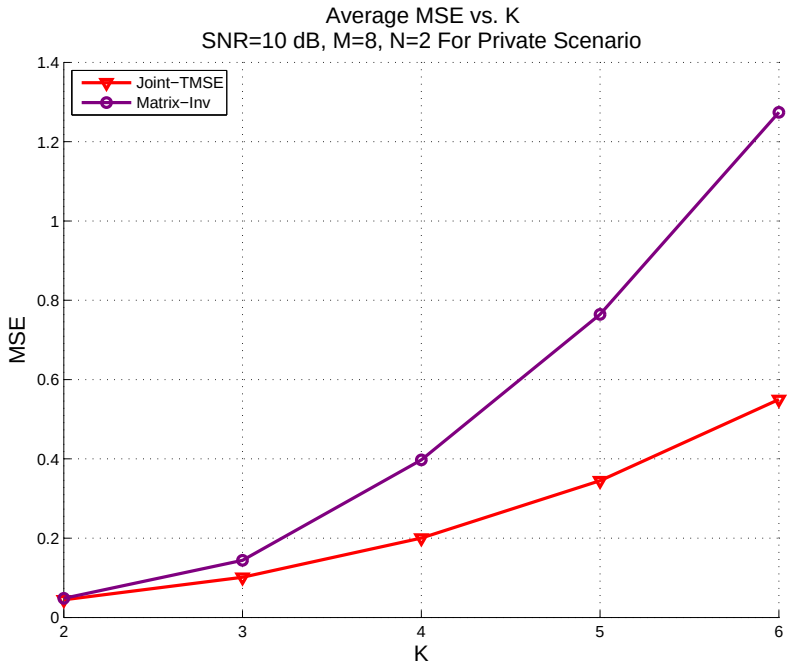

Figure 5.33: Average MSE vs. K for Transmitting Private Information Scenario with $SNR = 10$ dB, $M = 8$, $N = 2$.

As the number of users in the system increases, the minimum SINR and mean SINR of the system decreases in all algorithms due to the increasing the co-channel interference (CCI) between users. Once more, it should be expressed that the SNR parameter used in simulation is defined as the ratio of signal power to noise power N_0 in order to have constant the total precoder power P_{tot} as the number of users increases. Due to same reasons discussed for the effect of sweeping SNR and transmitter antenna number parts, Matrix-Inv and SVD-Dec methods are worse than all the others. The transmitter antenna number constraint on SVD-Dec algorithm leads to the fact that it can not operate when the number of users is greater than 4. Likewise, SDR-randA, SDR-detA, UDD and EPA algorithms shows a similar performance while SVD-Iter is a little worse than others.

As a consequence of increasing interference between users, average BER is getting worse as given Figure 5.32. Finally, as the number of users increases, the average MSE obtained from Matrix-Inv and Joint-TMSE algorithms increases shown in Figure 5.33 fast and the difference between these two becomes larger.

Effect of The Number of Antennas at the Receiver of Each User

In this simulation, the number of antennas at the receiver of each user N is swept and other system parameters are fixed whose values are provided below:

SNR: 5 dB.
Number of transmitter antennas: $M = 7$.
Number of users: $K = 3$.

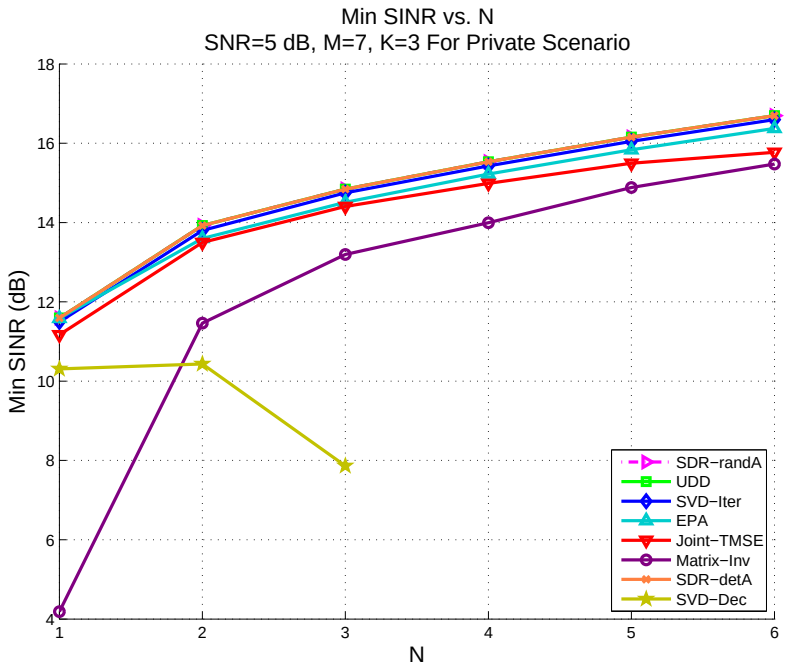

Figure 5.34: Minimum SINR vs. N for Transmitting Private Information Scenario with $SNR = 5$ dB, $M = 7$, $K = 3$.

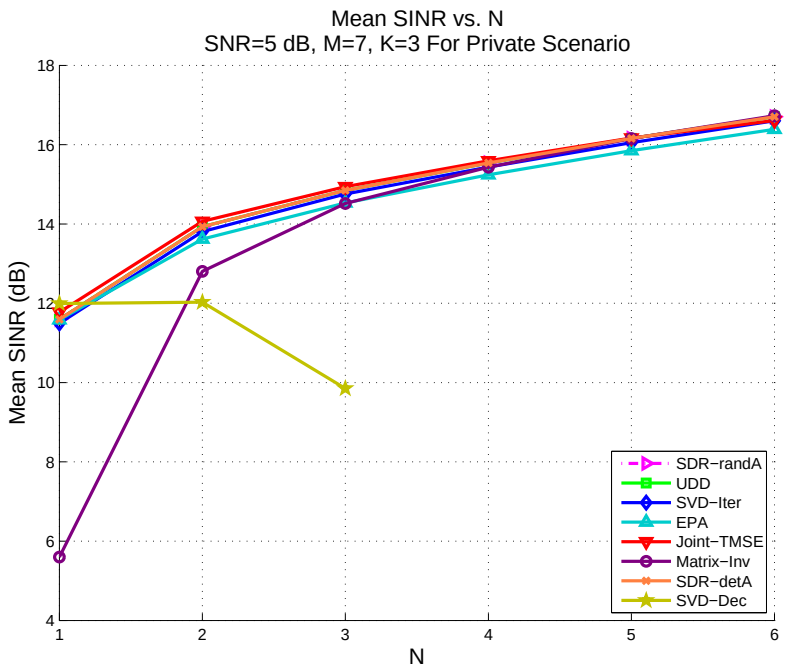

Figure 5.35: Mean SINR vs. N for Transmitting Private Information Scenario with $SNR = 5$ dB, $M = 7$, $K = 3$.

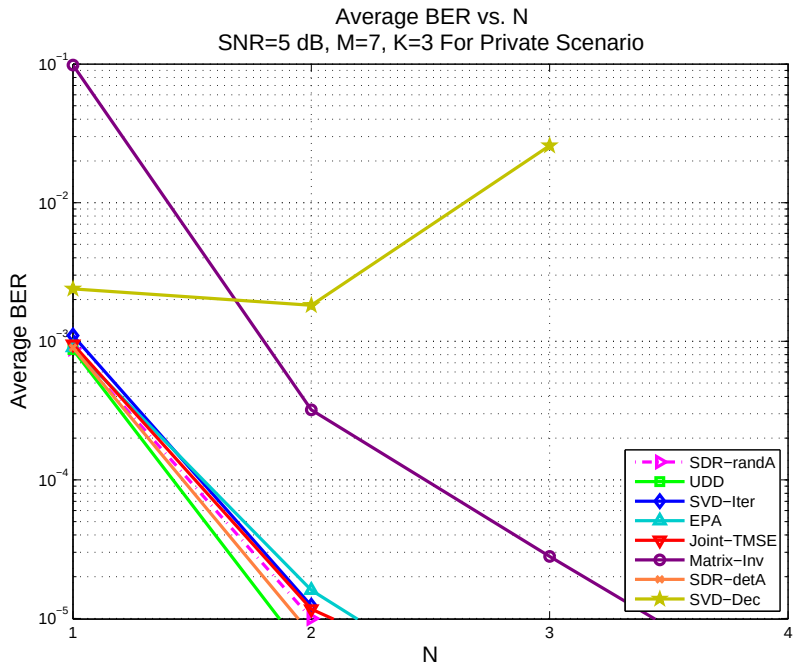

Figure 5.36: Average BER vs. N for Transmitting Private Information Scenario with $SNR = 5$ dB, $M = 7$, $K = 3$.

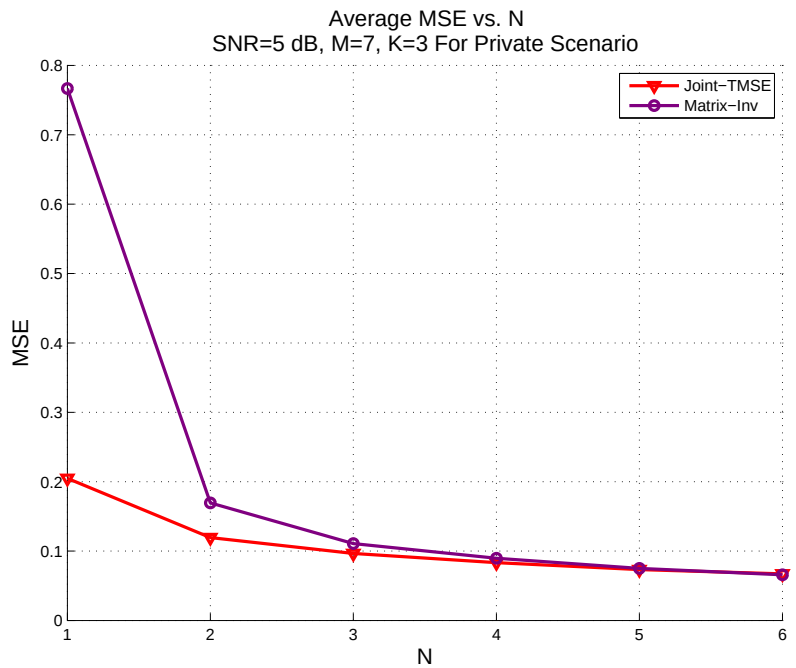

Figure 5.37: Average MSE vs. N for Transmitting Private Information Scenario with $SNR = 5$ dB, $M = 7$, $K = 3$.

As observed in Figures 5.34 and 5.35, as the number of receiver antennas at the receiver of each user increases, thanks to the receiver diversity, minimum SINR and mean SINR in the system increases. More receiver antenna enables combining the signal received from different channels which improves the received signal power. Except for SVD-Dec method the other algorithms have a similar behavior with the case of sweeping of other system parameters. In fact, the behavior of SVD-Dec algorithm is very interesting since it has completely opposite characteristic in minimum SINR, mean SINR and average BER graphs given in Figure 5.34-5.36. The reason of this difference can be explained by some linear algebra. According to the notation defined in In Chapter 3, SVD-Dec algorithm part, it is shown that in order to eliminate interference term the precoder vector $\boldsymbol{p_i}$ is chosen from the subspace of intersection of null spaces of $\widehat{\boldsymbol{H_i}}$ with dimension $M-(K-1)N$. Therefore, as the number of receiver antenna at the receiver of each user increases the dimension of null space of $\widehat{\boldsymbol{H_i}}$ decreases. This implies that precoders are chosen from smaller subspace which might leads to make worse precoder design. It can also be understood from the fact that as the number of constraints on precoder optimization problem increases, it is likely to obtain improper solution. As a result, there is a tradeoff between the operating region of the algorithm and receiver diversity. The number of receiver antennas should be increased to some extent which again shows the infeasibility of this method for practical purposes.

Comparison of Computation Time of Algorithms in Interfering Broadcasting Scenario

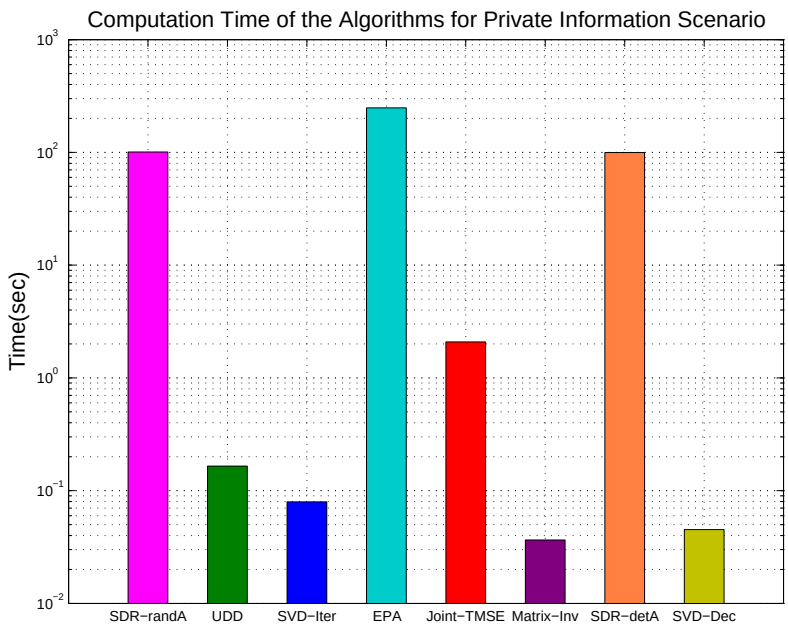

Figure 5.38: Average Computation Time of Algorithms for Transmitting Private Information Scenario with $M = 6$, $K = 3$, $N = 2$ in $0 - 14$ dB SNR range.

The average computational time of the all algorithms are given in Figure 5.38. Here, the average term implies the average of computational times in the $0 - 14$ dB SNR range. As seen from Figure 5.38, the highest computational time belongs to EPA algorithm. It is followed by SDR-randA and SDR-detA methods. It should be noticed that all these three methods uses SeDuMi tool for the solution of optimization problem. Moreover, contrary to multicasting scenario given in Figure 5.21, due to interference term, the MMF problem is solved with the help of QoS problem which leads to the usage of SeDuMi so many times in SDR-randA and SDR-detA methods. Although the performance of these algorithms in terms of minimum SINR, mean SINR and average BER is better in general compared with others as presented in Figure 5.38, their computational time is very high due to complexity of SDP tool. Therefore, they might not be feasible for practical implementations. Moreover, Joint-TMSE method has less computational time and its average BER performance is near to SDR and EPA. On the other hand, remained algorithms namely UDD, SVD-Iter, Matrix-Inv

and SVD-Dec have near computational times; however, among all these UDD performs the best. The performance of proposed SVD-Iter method in terms of average BER and minimum-mean SINR is also very similar to UDD and its computational time is better. Therefore, UDD and SVD-Iter are feasible for real life. Matrix-Inv and SVD-Dec methods seems advantageous due to their computational ease, yet both of them have significantly inferior performance in comparison to the others.

As a result of simulations applied in the private information scenario, it is again observed that the methods using SDP tools are better in general. The solution found from SDP tool for this scenario is always rank-one; therefore, performances of SDR-randA, SDR-detA and EPA methods are nearly the same. However, differently from multicasting scenario, due to the iterative solution procedure of the MMF problem based on the QoS problem, their computation time is much higher than the methods not using SDP tool. For this scenario, there are also some other alternative solution methods such as UDD and SVD-Iter which are computationally fast and very similar minimum SINR and average BER performance with the SDP based methods. In addition, it is easily noticed that the SVD-Dec and Matrix-Inv methods are always worse than the other methods, since both of these do not achieve power allocation task in the precoder design. Furthermore, SVD-Dec method has an infeasible constraint on the minimum number of transmitter antennas and it cannot operate after than a certain value of the receiver antenna number, which implies the tradeoff between the operating region of the algorithm and receiver diversity.

5.3 Simulation Results for Joint Transmission of Private and Common Information

In this scenario, similar optimization criteria (MMF and minimization of total MSE problem) are studied when each user receives one common and one private information at the same time. Simulation results of the adaptation of existing methods in the literature and the proposed one are given in this part.

Effect of Total Precoder Power at Transmitter

In this simulation, the number of antennas at the receiver of each user N is swept and other system parameters are fixed whose values are provided below:

Number of transmitter antennas: $M = 7$.
Number of users: $K = 3$.
Number of antennas at the receiver of each user: $N = 2$.

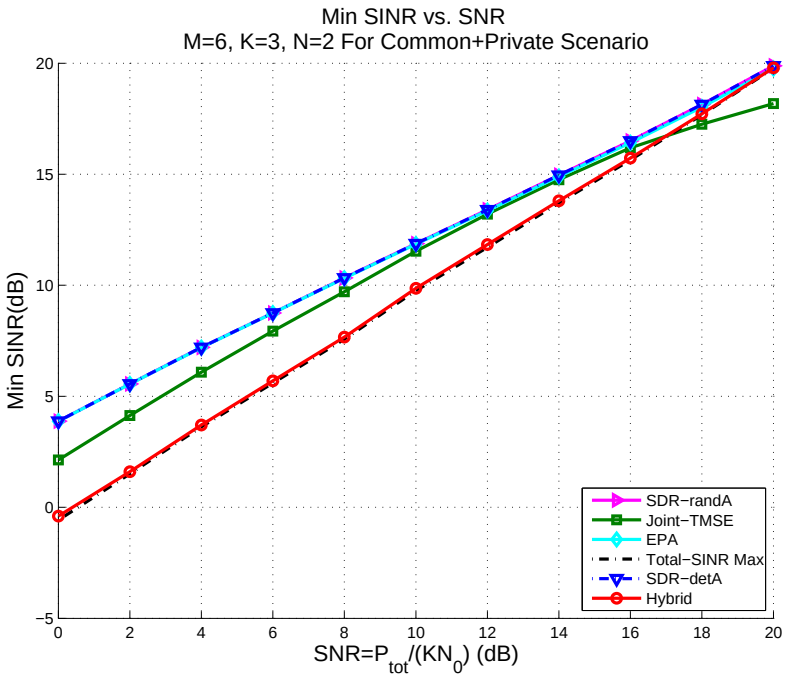

Figure 5.39: Minimum SINR vs. SNR for Transmitting Common and Private Information Scenario with $M = 6$, $K = 3$, $N = 2$.

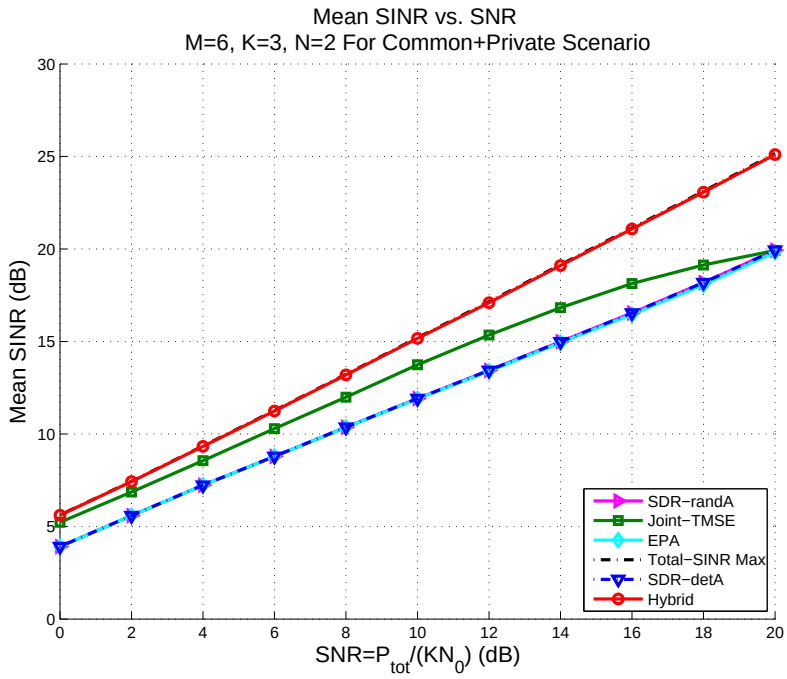

Figure 5.40: Mean SINR vs. SNR for Transmitting Common and Private Information Scenario with $M = 6$, $K = 3$, $N = 2$.

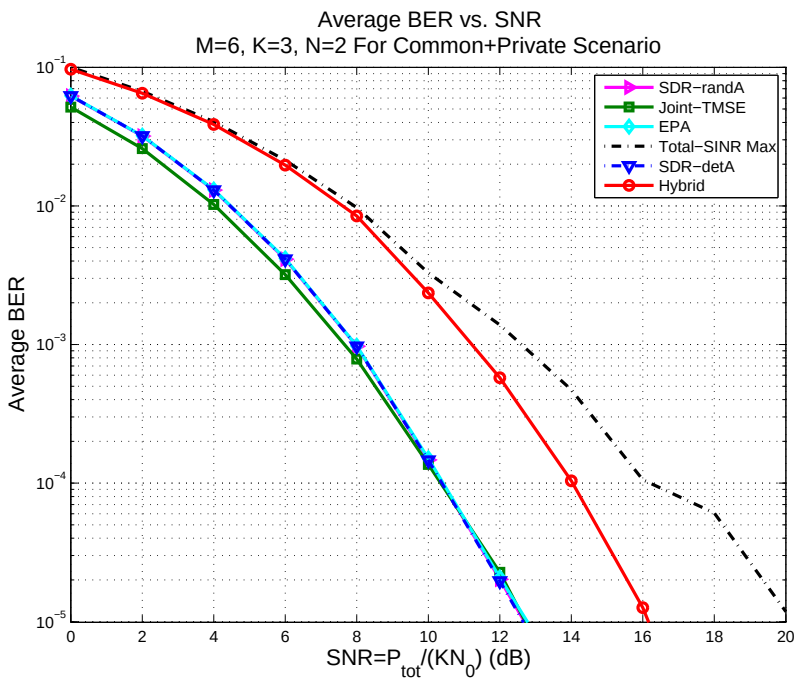

Figure 5.41: Average BER vs. SNR for Transmitting Common and Private Information Scenario with $M = 6$, $K = 3$, $N = 2$.

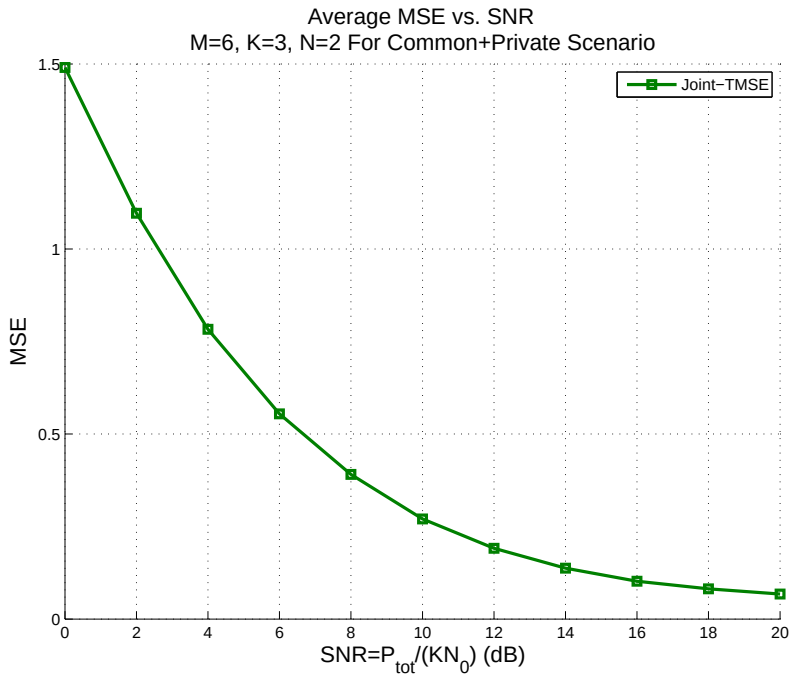

Figure 5.42: Average MSE vs. SNR for Transmitting Common and Private Information Scenario with $M = 6$, $K = 3$, $N = 2$.

As seen from the Figures 5.39-5.40, when the total precoder power is increased, the minimum SINR and the mean SINR in the system monotonically increases, while average BER and average MSE monotonically decreases, as expected. In terms of the minimum SINR given in Figure 5.39, the performances of SDR-randA, SDR-detA and EPA methods are nearly the same and they are better than Joint-TMSE. It is due to the fact that Joint-TMSE aims to optimize TMSE, while the maximization of the minimum SINR is the goal of SDR-randA, SDR-detA and EPA. The curves for Total SINR Max and Hybrid methods are $4-5$ dB below other methods and this difference becomes smaller as SNR value increases. On the other hand, from mean SINR aspect given in Figure 5.40, the best performance belongs to Total-SINR Max and Hybrid methods. At the low SNR region, the performance of Joint-TMSE is very similar to the SDR methods and EPA, but as SNR increases it drops to 5 dB below them. The performances of SDR-randA and SDR-detA are the same which is also proven by the results given in Table 5.5 showing percentage of rank-one solutions found from SeDuMi.

Table 5.5: Percentage of rank-one solutions found from SeDuMi in Combined Scenario.

M	K	N	SNR (dB)	Percentage of Rank-one Solutions
6	3	2	0	100
6	3	2	2	100
6	3	2	4	100
6	3	2	6	100
6	3	2	8	100
6	3	2	10	100
6	3	2	12	100
6	3	2	14	100
6	3	2	16	100
6	3	2	18	100
6	3	2	20	100

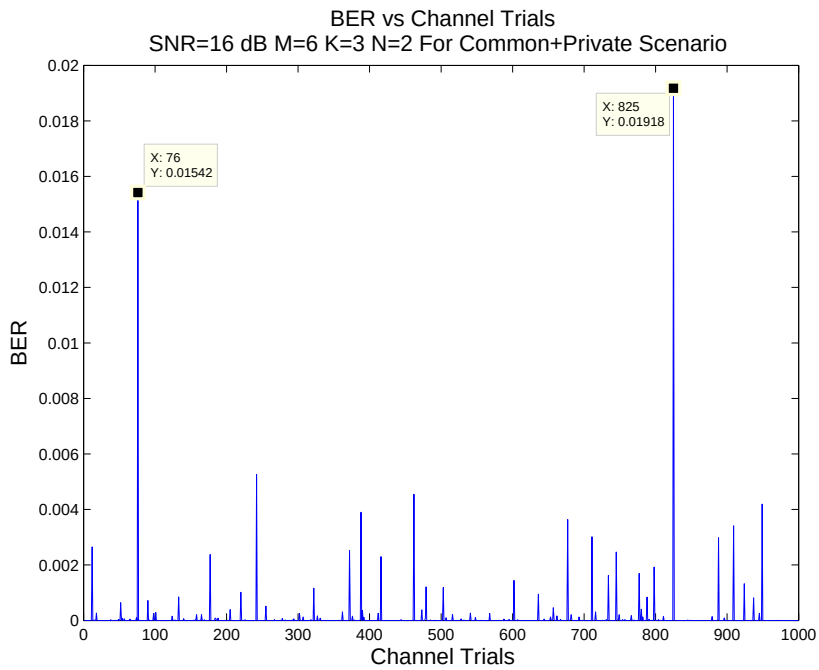

Figure 5.43: BER vs. Channel Trials of Total SINR Max Method for Transmitting Common and Private Information Scenario with $M = 6$, $K = 3$, $N = 2$.

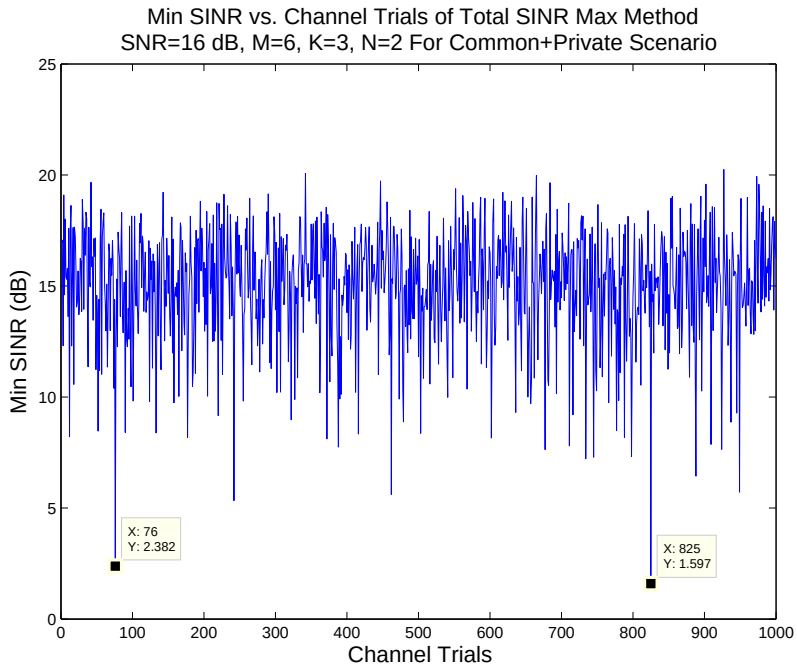

Figure 5.44: BER vs. Channel Trials of Total SINR Max Method for Transmitting Common and Private Information Scenario with $M = 6$, $K = 3$, $N = 2$.

There is a striking result in the average BER graph shown in Figure 5.41. It is noticed that although minimum and mean SINR of proposed Total-SINR Max method approaches to SDR methods as SNR increases, the difference in the average BER characteristics between SDR methods and Total-SINR Max methods is getting larger. Specifically, at 10^{-5} dB BER value is reached approximately with a SNR of 13 dB by SDR methods and EPA while Total SINR Max method reaches the same value at 20 dB. The performance gap between minimum-mean SINR and average BER can be expressed with the help of Figures 5.43-5.44. According to Figure 5.43, in 76th and 825th channel trials the BER obtained via Total-SINR Max method is very large. At these corresponding channels given in Figure 5.44, the minimum SINR value is also small with respect to average. On the average of all channel trials, these bad channels might not affect so much the minimum SINR value obtained at that SNR. However, average BER is degraded drastically. BER is a non-linear function of the minimum SINR in the system. Even if the minimum SINR is a random variable with a small variance, a non-linear function of this random variable, namely BER, is not necessarily a

random variable with a small variance.

As explained in Chapter 4, Total-SINR Max is an intuitive method and for some channels the algorithm may result in bad minimum SINR performance. In order to enhance this algorithm to some extent, a hybrid method is proposed. According to the Hybrid algorithm, for a given channel, when minimum SINR values obtained from Total-SINR Max method is at unfavorable level, the Joint-TMSE algorithm is operated instead of Total-SINR Max method. The unfavorable minimum SINR value should be specified at the beginning. In fact, this is a kind of tuning process. In practice, according to the system parameters which is known by the designer of the downlink MIMO system, minimum SINR values can be measured by many channel trials. After that to eliminate bad results, a minimum SINR threshold can be defined. During this simulation SINR threshold is specified as 7 dB below with respect to given SNR. As a result, thanks to the Hybrid algorithm especially in high SNR region, 10^{-5} BER value is reached at 16 dB SNR which is 4 dB better than Total-SINR Max method.

Moreover, as seen from Figure 5.44, the low performance points of Total-SINR Max algorithm is very rare. The percentage of the number of running Joint-MMSE algorithm throughout Hybrid method operation is given in Table 5.6.

Table 5.6: Percentage of Working of Joint-TMSE Method in Hybrid Algorithm

M	K	N	SNR (dB)	Joint-TMSE Percentage
6	3	2	0	3.4
6	3	2	2	2.3
6	3	2	4	1.9
6	3	2	6	2.2
6	3	2	8	2.4
6	3	2	10	2.5
6	3	2	12	3.5
6	3	2	14	2.7
6	3	2	16	2.8
6	3	2	18	2.1
6	3	2	20	2.5

Finally, as presented in Figure 5.42, the average MSE decreases monotonically as given total power to system increases which is expected.

In this part, the performance of Hybrid algorithm is given for the total precoder power sweep. A similar procedure for specification of minimum SINR threshold can be applied. It is expected that by using Hybrid method, one can also obtain better results for other parameter sweeping scenarios.

Effect of The Number of Transmitter Antennas

In this simulation, the number of antennas at the transmitter M is swept and other system parameters are fixed whose values are provided below:

SNR: 8 dB.
Number of antennas at the receiver of each user: $N = 2$.
Number of users: $K = 3$.

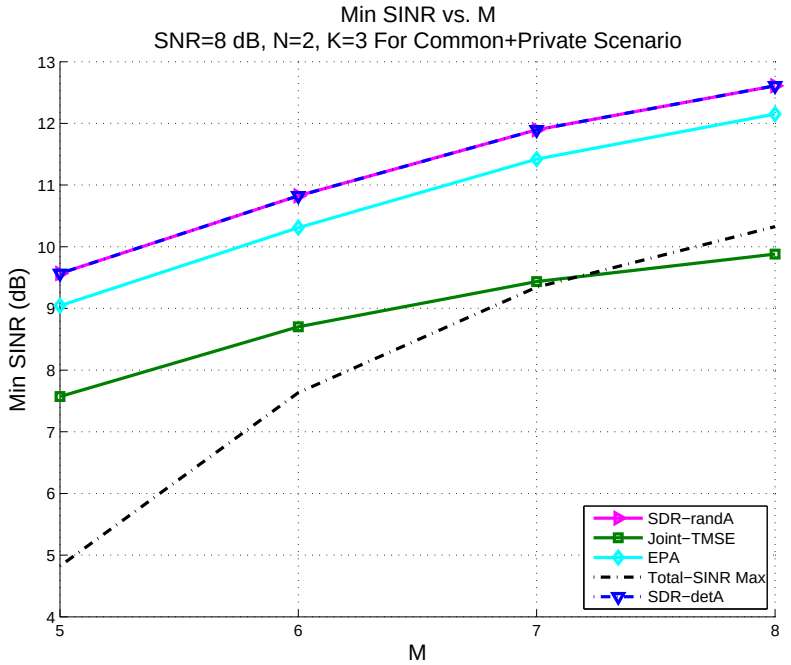

Figure 5.45: Minimum SINR vs. M for Transmitting Common and Private Information Scenario with $SNR = 8$ dB, $K = 3$, $N = 2$.

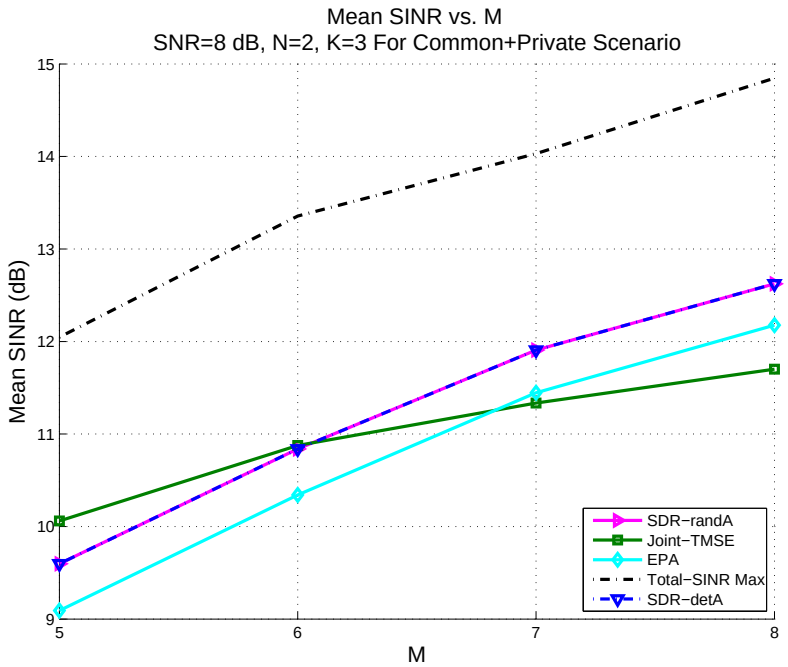

Figure 5.46: Mean SINR vs. M for Transmitting Common and Private Information Scenario with $SNR = 8$ dB, $K = 3$, $N = 2$.

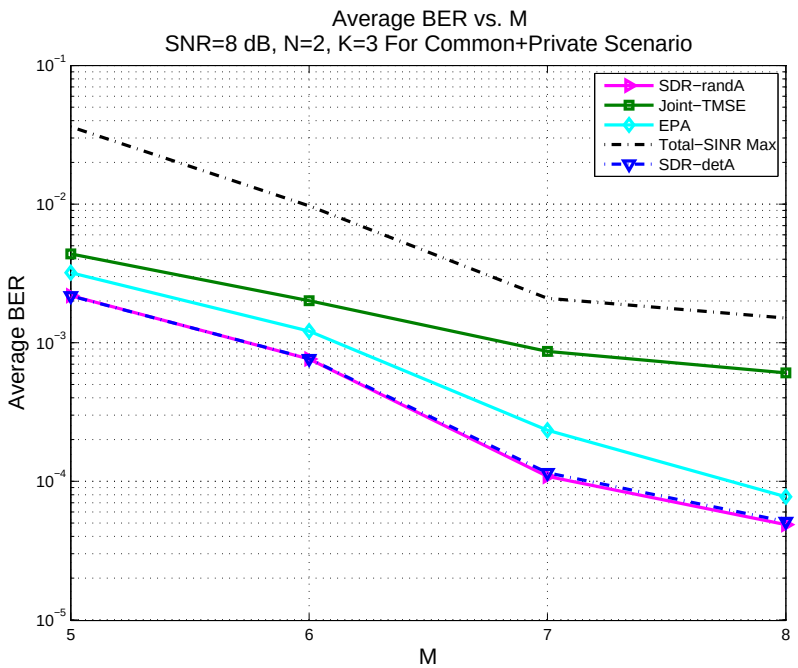

Figure 5.47: Average BER vs. M for Transmitting Common and Private Information Scenario with $SNR = 8$ dB, $K = 3$, $N = 2$.

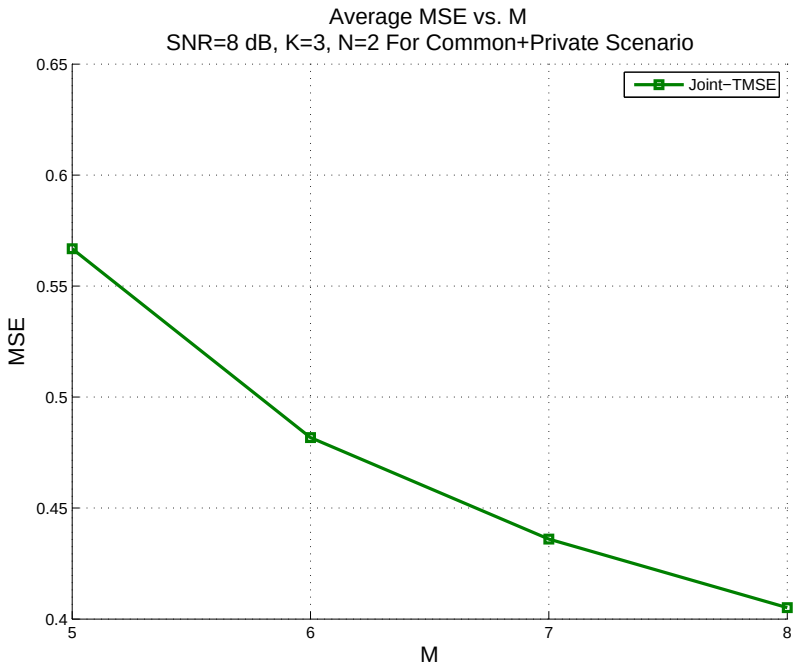

Figure 5.48: Average MSE vs. M for Transmitting Common and Private Information Scenario with $SNR = 8$ dB, $K = 3$, $N = 2$.

In Figures 5.45-5.46, it is observed that as the number of transmitter antennas increases, thanks to the transmitter diversity gain, the minimum and mean SINR value monotonically increases and average BER decreases for all methods. In terms of minimum SINR, given in Figure 5.45, performance of SDR-randA and SDR-detA methods are the best and EPA is nearly 0.5 dB inferior. When the number of transmitter antennas is larger than 7, the Total-SINR Max method becomes better than Joint-TMSE method. For mean SINR, as seen in Figure 5.46, for all transmitter antenna numbers, performance of Total-SINR Max algorithm is much higher than the others since its aim is to optimize total SINR value. Similarity for SDR-randA and SDR-detA methods are also valid for this case due to same rank-one solution of SeDuMi explained in Table 5.5. Moreover, as seen in Figure 5.47, the best average BER characteristic belongs to SDR-randA and SDR-detA methods. Although in terms of minimum SINR, the Total-SINR Max method is better than the Joint-TMSE method for more than 7 antennas at the transmitter, the BER characteristic of joint-MMSE is better than Total-SINR Max method which can be arisen from the same relation be-

tween minimum SINR and BER explained before. Finally, the average MSE in Figure 5.48 has slowly monotonically decreasing characteristic with increasing number transmitter antennas.

Effect of The Number of Users

In this simulation, the number of users K is swept and other system parameters are fixed whose values are provided below:

SNR: 8 dB.
Number of transmitter antennas: $M = 9$.
Number of antennas at the receiver of each user: $N = 2$.

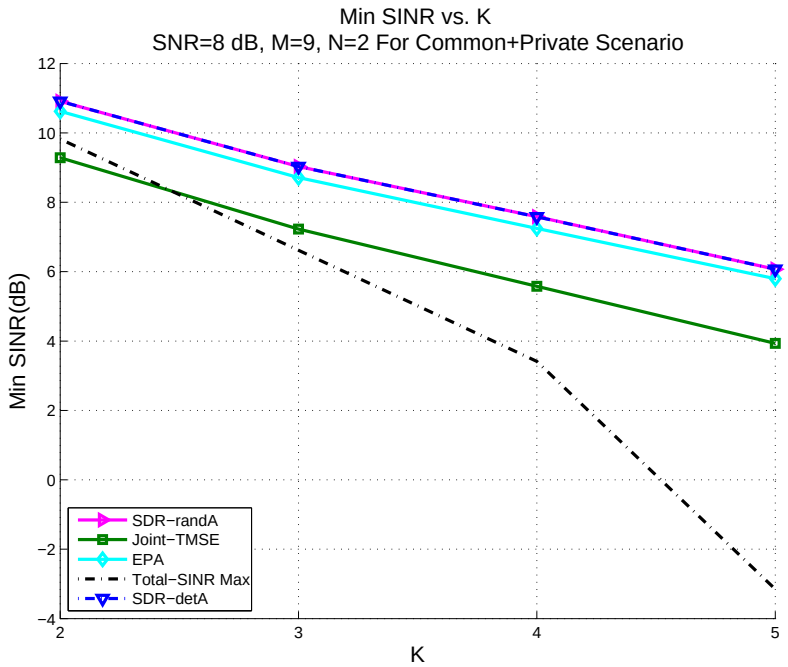

Figure 5.49: Minimum SINR vs. K for Transmitting Common and Private Information Scenario with $SNR = 8$ dB, $M = 9$, $N = 2$.

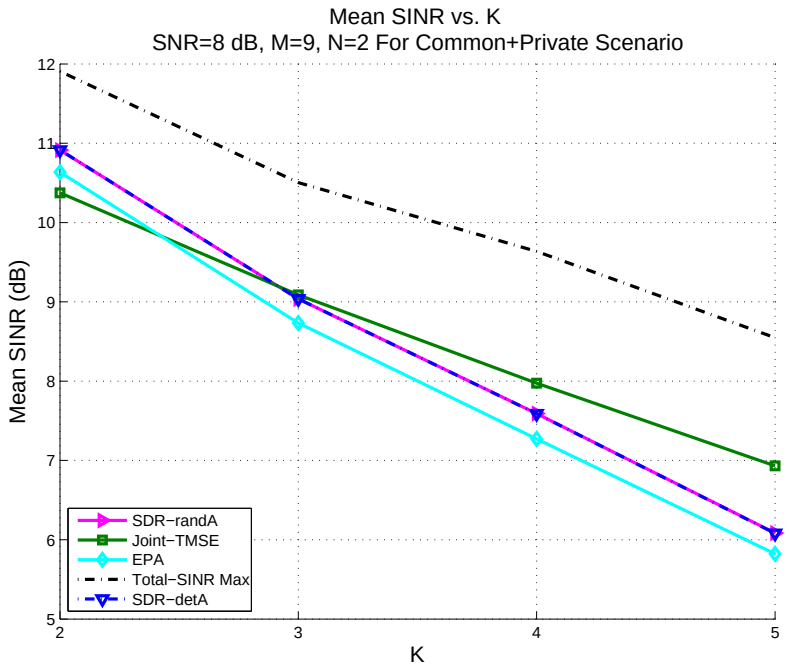

Figure 5.50: Mean SINR vs. K for Transmitting Common and Private Information Scenario with $SNR = 8$ dB, $M = 9$, $N = 2$.

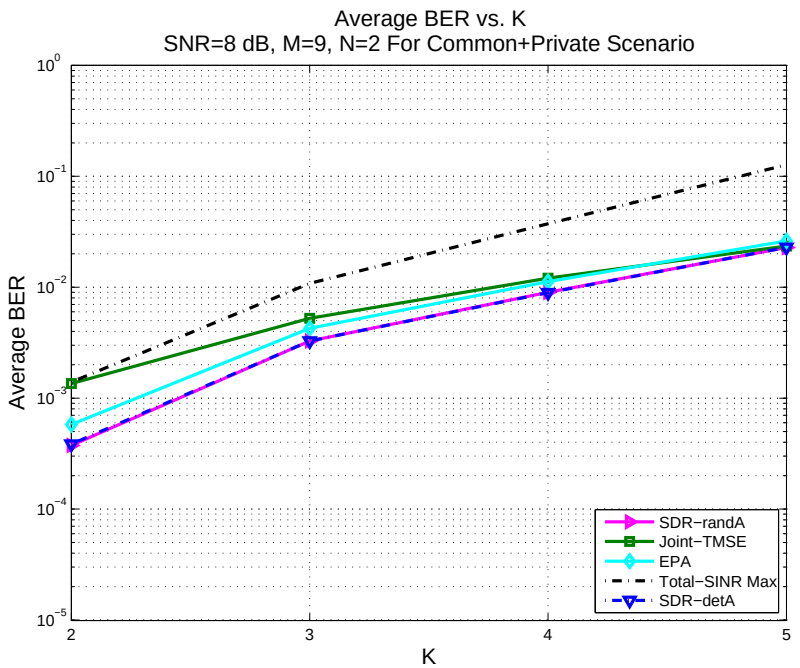

Figure 5.51: Average BER vs. K for Transmitting Common and Private Information Scenario with $SNR = 8$ dB, $M = 9$, $N = 2$.

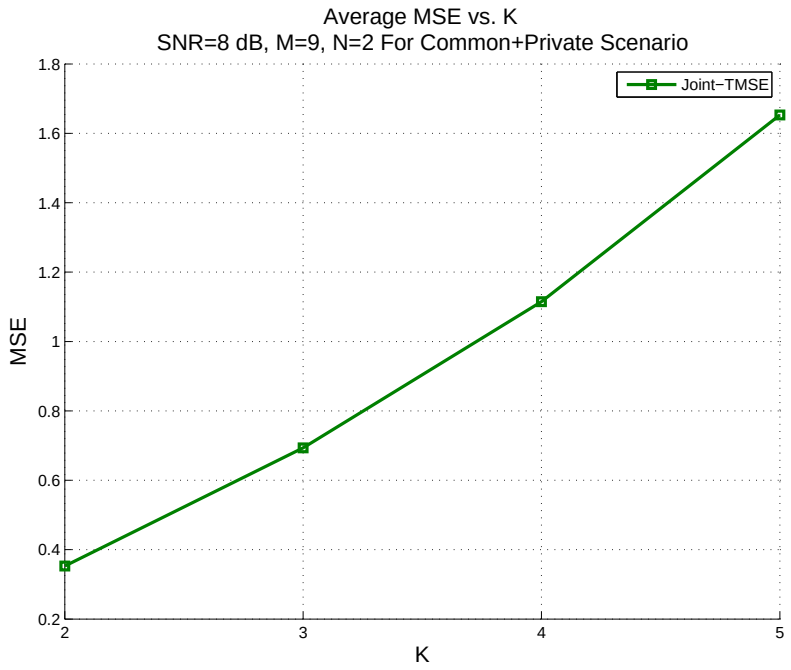

Figure 5.52: Average MSE vs. K for Transmitting Common and Private Information Scenario with $SNR = 8$ dB, $M = 9$, $N = 2$.

As the number of users increases, due to effect of both interference and decreasing power per user, performance of the system degrades in all aspects. Minimum and mean SINR decreases, the average BER and the average MSE increases as shown in Figures 5.49-5.52. Again SDR-randA and SDR-detA have similar performance and they are better than other algorithms.

Effect of The Number of Antennas at the Receiver of Each User

In this simulation, the number of antennas at the receiver of each user N is swept and other system parameters are fixed whose values are provided below:

SNR: 5 dB.
Number of users: $K = 3$.
Number of transmitter antennas: $M = 6$.

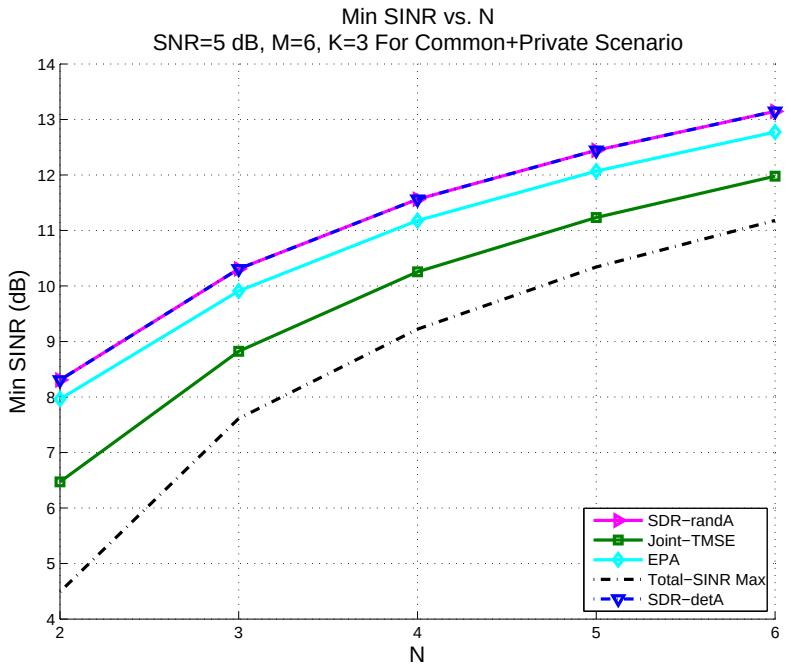

Figure 5.53: Minimum SINR vs. N for Transmitting Common and Private Information Scenario with $SNR = 5$ dB, $K = 3$, $M = 6$.

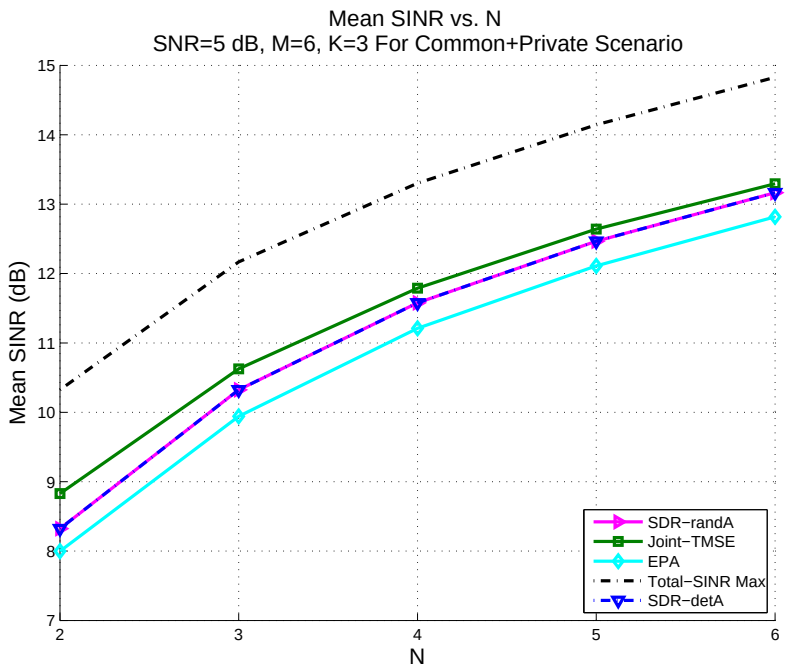

Figure 5.54: Mean SINR vs. N for Transmitting Common and Private Information Scenario with $SNR = 5$ dB, $K = 3$, $M = 6$.

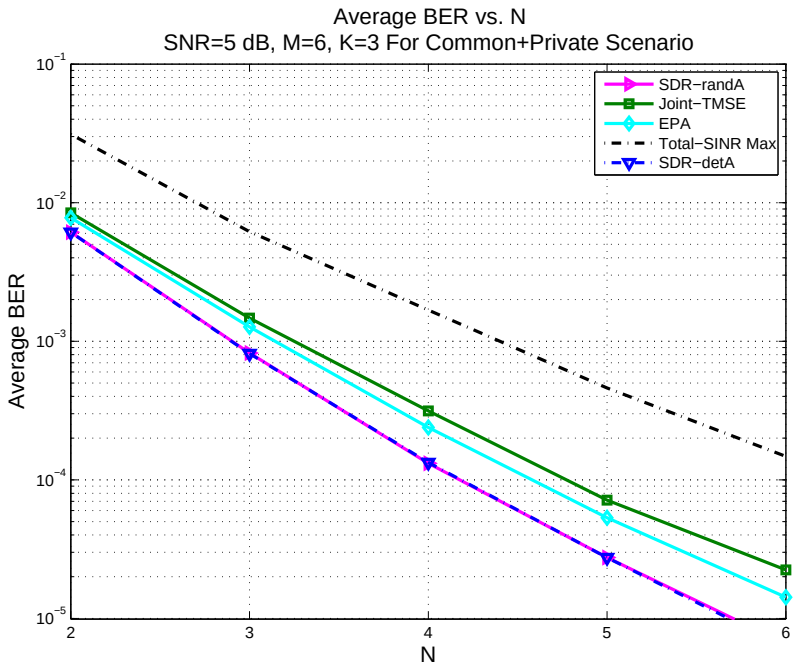

Figure 5.55: Average BER vs. N for Transmitting Common and Private Information Scenario with $SNR = 5$ dB, $K = 3$, $M = 6$.

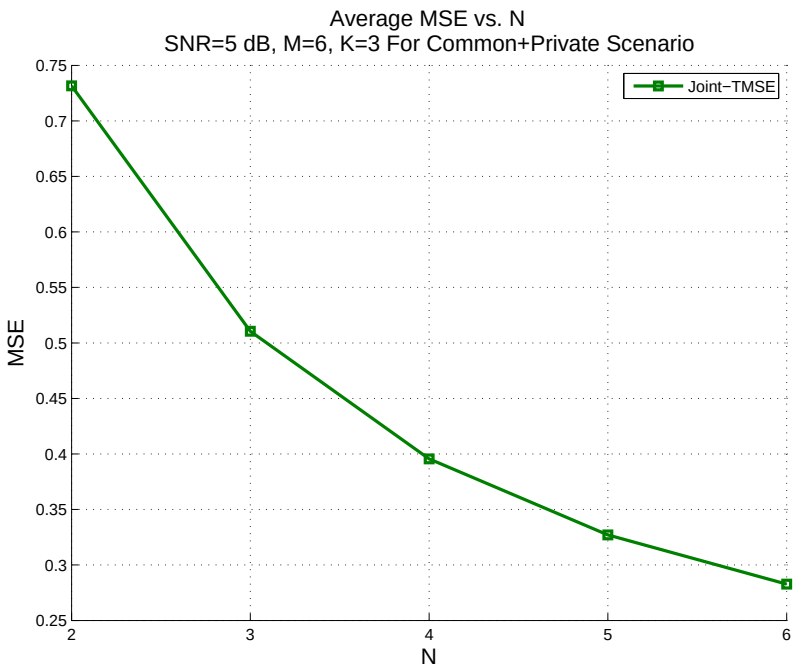

Figure 5.56: Average MSE vs. N for Transmitting Common and Private Information Scenario with $SNR = 5$ dB, $K = 3$, $M = 6$.

As shown in Figures 5.53-5.54, the minimum SINR and the mean SINR monotonically increases as the number of receiver antenna for each user is increased due to the receiver diversity. Similar to the transmitter antenna effect case, again SDR-randA and SDR-detA methods have the best minimum SINR performance. In the mean SINR comparison, Total-SINR Max algorithm is better than the others since its optimization criteria is to maximize the total SINR in the system i.e. maximize the mean SINR. The same conclusions given in minimum SINR graph in Figure 5.53 is also valid for average BER performances of the algorithms as in Figure 5.55 with respect to receiver antenna number. Lastly, the average MSE also monotonically decreases but this time with a fast rate compared to the case in which the number of transmitter antenna is swept.

Comparison of Computation Time of Algorithms in Common-Private Information Transmission Scenario

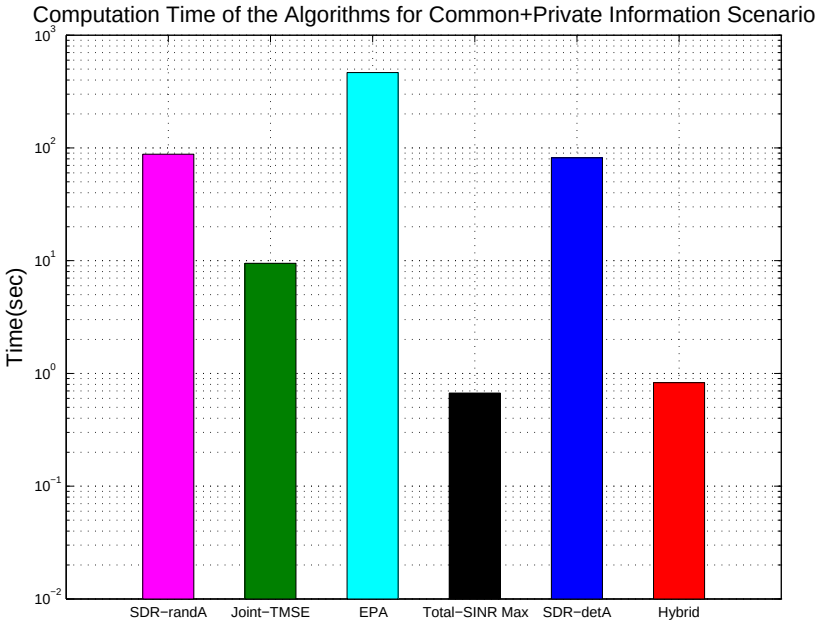

Figure 5.57: Average Computation Time of Algorithms for Transmitting Private Information Scenario with $M = 6$, $K = 3$, $N = 2$ in $0 - 20$ dB SNR range.

For common-private combination scenario, the average computational time of the all algorithms over the SNR range of $0 - 20$ dB is given in Figure 5.57. Similar to the transmitting private information case given in Figure 5.38, the SDP based algorithms, namely SDR-randA, SDR-detA and EPA has the highest com-

putational times. They are followed by Joint-TMSE, Hybrid and Total SINR Max methods respectively. Here, the main result of Table 5.6 can be emphasized once more. In other words, the computational time of Hybrid method is a little amount higher than Total-SINR Max method which denotes the rareness of the unfavorable cases of Total-SINR Max algorithm. However, the Hybrid method provides approximately 4 dB improvement in the sense of SNR for 10^{-5} BER level. With a small amount of additional computational time, which is approximately two times more than the Total-SINR Max method, BER performance of the Hybrid algorithm is getting much better.

In this part of the simulation results, performance of the algorithms which are adapted for common-private combination scenario is examined. Similar to simulation results obtained in the previous two scenario, methods using SDP tools are better in the average BER; however, they are computationally very complex. In order to deal with the complexity issue, a new method not using SDP tool is proposed called the Total SINR Max. The minimum SINR performance of the Total SINR Max is very similar to the SDP based methods; however its average BER is worse due to the bad minimum SINR results. Even if these points are rarely observed, it affects the average BER performance significantly. In order to eliminate the effect of bad results, the Hybrid method is proposed. Thanks to this method SNR is improved 4 dB at 10^{-5} average BER level with only small amount of increasing in computation time. When complexity and average BER issues are evaluated together the Joint-TMSE method can be the most feasible one for this scenario.

CHAPTER 6

CONCLUSIONS

In this thesis, multiuser MIMO (MU-MIMO) downlink communication problem has been investigated under three different scenarios. The optimization of precoder and decoder blocks is the main focus point of this work. Throughout the study, several types of cost functions are optimized by using various methods. In order to observe the practical performance of the algorithms, different kind of simulation scenarios are generated by sweeping some system variables. In general, the effect of four different types of system parameters are analyzed which are the total transmitter power, the number of transmitter antennas, the number of users and the number of antennas at the receiver of each user. Results are evaluated in terms of the minimum-mean SNR/SINR of the system and the average BER for all algorithms; average MSE is also considered for some methods. In all simulations, the design of precoder and decoder blocks is jointly considered. Moreover, for the solution of some optimization problems, a semidefinite programming solver tools are used. In this thesis, SeDuMi is used for this purpose.

In the first scenario, known as multicasting in the literature, MU-MIMO communication problem is studied for the scenario in which all users receive a common information. Here, a BS transmits common information symbol to all users; i.e., there is no interference between users. In this case, the main purpose of the precoder and decoder design is to allocate total transmitter power among users efficiently with the knowledge of CSI to optimize the given design criteria. For the multicasting scenario, three different types of cost functions, namely maxi-

mizing the minimum SNR (MMF problem), minimization of the total MSE and maximizing the total SNR in the system are used as the precoder-decoder optimization criteria. It is known that the solution of the MMF concept is NP-hard and it is quadratically constrained quadratic programming (QCQP) problem with a non-convex rank-one constraint. To overcome this, semidefinite relaxation (SDR) techniques and exact penalty approach (EPA) are used. With this relaxation, the SDR-randA, SDR-detA and EPA methods have generally the best performances in terms of minimum SNR and average BER. EPA algorithm is the one which has the longest computation time. SDR methods and Joint-TMSE have moderate computation times, since at the precoder design stage, in SDR algorithms the problem can be directly solved by using SeDuMi only once. Furthermore, Joint-TMSE takes nearly the same time with SDR methods due to the complexity of the Lagrange multiplier optimization. On the other hand, the Total SNR Max method is the fastest one and its performance is the best in mean SNR as theoretically expected. However, in terms of minimum SNR and average BER, Total SNR Max is far from the others and practical quality of service conditions cannot be guaranteed by using this method. Lastly, in the multicasting scenario, it is deduced that SDR-detA method, which is proposed in this thesis as an alternative solution method, performs better than SDR-randA as the number of users increases. This is due to the fact that with the increasing number of users, the solution found from SeDuMi is not likely to be rank-one.

As second scenario the interfering MIMO broadcasting channel is studied. In this case, each user receives a different message, called as the private information. Different than multicasting, there is CCI between users which is the main concern for the overall system performance. The main aim in designing the precoder and decoder blocks is to make power allocation according to the CSI and to decrease the effect of interference as far as possible. Here, MMF and minimization of total MSE are studied as optimization problems. According to the minimum SINR and average BER performance, UDD, SDR-detA, SDR-randA and EPA methods are better than the others, in general. For this scenario, SDR-detA and SDR-randA performs the same, since for the interfering broadcasting channel, the solution of SeDuMi is always found to be rank-one. On the other hand,

contrary to the transmitting common information scenario, the computational time of SDR-randA, SDR-detA and EPA algorithms are much more higher than others. Due to the interference term, the MMF problem can be solved by using equivalent quality-of-service (QoS) problem iteratively which causes to use SeDuMi multiple times. In this case, Joint-TMSE algorithm requires much less time than the SDR methods and EPA, since Lagrange multiplier expression used in this scenario is much simpler. It should be noted that UDD method performs very well in both minimum SINR and computational time aspects. Here, by using uplink-downlink duality principle, the downlink problem is converted to its uplink version whose solution can be done more easily. Furthermore, the importance of power allocation can be observed via comparison of Joint-TMSE and Matrix-Inv methods. As can be seen from the simulation results, the Joint-TMSE which concerns the power allocation has much better performance than Matrix-Inv method in which the total transmitter power is divided equally between users. Finally, SVD based methods are also studied for this scenario. SVD-Dec method uses decomposition idea in order to divide MU-MIMO system into many parallel single-user MIMO systems. Due to this transformation, all methods for single user MIMO can be applied for MU-MIMO communication problem which simplifies optimization significantly. However, the performance of SVD-Dec method is much worse than the other algorithms, since there is no power allocation in the generation of parallel subsystems. Moreover, this method has a very strict requirement on the minimum number of transmitter antennas used in the system which is practically not feasible in many cases. This condition becomes more restrictive as the number of receiver antennas of each user increases which leads to a tradeoff between the number of transmitter antennas and receiver diversity. In order to overcome this constraint, SVD-Iter method is proposed in this thesis. Thanks to iterative precoder-decoder design of SVD-Iter method, the requirement on the number of transmitter antennas becomes independent from the number of antennas at the receiver of each user. It only depends on the number of users in the system. Furthermore, it is possible to make power allocation which increases the performance of the system significantly. As observed from simulation results, SVD-Iter method has a very close performance to UDD, SDR-randA, SDR-detA and EPA method and requires

less computation time.

In the third and last scenario, the combination of previous two scenarios is analyzed. In this case, both common and private information for each user are transmitted by a BS at the same time. Similarly, the main concern is to design precoders and decoders under this combination scenario. To the best of our knowledge, this subject is studied from the information theoretical viewpoint in the literature. In this thesis, the optimization of beamforming design is studied. Again the solution of MMF and minimization of total MSE problems are considered. SDR methods, EPA and Joint-TMSE algorithms are adapted for the solution of these optimization problems under this combination scenario. It is noticed that SDR-randA and SDR-detA have similar performance as in transmitting private information scenario, since the solution of SeDuMi is found to be rank-one in this case, too. SDR-randA and SDR-detA and EPA methods are better than Joint-TMSE and Total SINR Max algorithms but the required computational time is very long compared to the Joint-TMSE and Total SINR Max methods which makes them practically not useful. The Total SINR-Max method is proposed in this chapter in order to deal with long computation time drawback of SDR methods. This method is intuitive but it is not guaranteed that at every iteration step the optimization criterion improves. Simulation results show that the Total SINR Max method operates much faster than SDR and EPA and also well in terms of minimum SINR in general, except for a few cases in which the obtained minimum SINR is very low. Even if the number of undesired cases from minimum SINR aspect is very low, it affects the average BER performance of Total SINR Max algorithm very drastically while it does not affect the minimum SINR performance on the average. To overcome this, the Hybrid method is proposed which operates the Total SINR Max and Joint-TMSE methods together. By using the Hybrid algorithm, it is noticed that average BER performance is improved especially in high SNR region with a small amount of increase in the computation time of the algorithm with respect to Total SINR Max.

The downlink multiuser MIMO communication methods examined here are advantageous compared with the traditional multiple access techniques such as

TDMA, FDMA or CDMA, since in MU-MIMO communication the information symbols for all users are transmitted at the same time and frequency, which increases the spectral efficiency significantly. Sending all information together causes interference and this can be dealt with using properly designed precoder and decoder blocks.

Throughout this thesis, it is assumed that CSI is known perfectly both at the transmitter and receiver side. However, in practice, the channel is estimated with some error. The performance of the algorithms presented in this thesis can be analyzed under the channel estimation errors and the robust version of them can be investigated, if necessary. Furthermore, the optimum precoder design for the MMF problem for the joint transmission of common-private information case without using SDP tools is not achieved in this thesis. Only the optimum power allocation task is given in the appendix part. This issue can also be examined as a future work. Finally, some other methods can be studied to improve the Total SINR Max method in the common-private combination scenario.

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APPENDIX A

OPTIMUM DOWNLINK POWER ALLOCATION FOR COMBINATION OF COMMON AND PRIVATE INFORMATION SCENARIO

In this part, the optimum power allocation for precoder vectors is given for the solution of MMF problem under common-private combination scenario. The design of precoder block in MMF problem is a tricky issue due to the existence of common information in this case. Therefore, in this thesis only the optimum power allocation task could be achieved which is presented below in detail.

Remember that in a MU-MIMO system with K users, $2K$ SINR expression are defined such that

$$SINR_{com}^{i} = \frac{\left\|(\boldsymbol{D_i})_1 \boldsymbol{H_i p_0}\right\|^2}{\left\|\sum_{k=1}^{K} (\boldsymbol{D_i})_1 \boldsymbol{H_i p_i}\right\|^2 + N_0 \left\|(\boldsymbol{D_i})_1\right\|^2}.$$

$$SINR_{pri}^{i} = \frac{\left\|(\boldsymbol{D_i})_2 \boldsymbol{H_i p_i}\right\|^2}{\left\|\sum_{k=1}^{K} (\boldsymbol{D_i})_2 \boldsymbol{H_i p_i}\right\|^2 + N_0 \left\|(\boldsymbol{D_i})_2\right\|^2}. \tag{A.1}$$

Claim 1: In the optimum case of MMF problem, all the private SINR terms and at least one common SINR term should be equal to the minimum SINR of the system.

Proof of Claim 1: It is obvious that as the total power at the transmitter side is increased, all common and private SINR values increases. For the ith user, $SINR_{pri}^{i}$ monotonically increases with p_i and monotonically decreases with p_k where $k \neq i$. Suppose in the optimum case, i_0^{th} private SINR is greater than the minimum of all common and private SINR terms. Then, decrease p_{i_0} provided

that $SINR_{pri}^{i_0}$ is not smaller than minimum SINR in the system. This leads to a decrease in $SINR_{pri}^{i_0}$ and increase all other SINR expressions in the system meaning that the minimum SINR in the system also increases. As a result, by doing this one can obtain a larger minimum SINR value with less total transmitter power, which is a contradiction. Therefore, all private SINR terms should be equal to minimum SINR in the system.

Now, it is shown that at least one common SINR should be equal to minimum SINR in the system. In the optimum case, suppose that none of the common SINRs is equal to minimum SINR. Then all of the common SINRs are greater than minimum SINR, namely minimum SINR is equal to one of the private SINR. For all users, common SINR's monotonically increase with p_0 and monotonically decreases with p_k where $k = 1, 2, \ldots, K$. Then, decrease p_0 which leads to an increase in all private SINR values and a decrease in all common SINR values in the system. This means that the minimum SINR value also increases after this operation since the minimum SINR is equal to one of the private SINR values. This is a contradiction, since larger minimum SINR value is found even with less total transmitter power; therefore, the proof is completed. ■