THE DESIGN OF A HIGH FREQUENCY PULSE WIDTH MODULATION INTEGRATED CIRCUIT WITH EXTERNAL SYNCHRONIZATION CAPABILITY

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ABSTRACT

THE DESIGN OF A HIGH FREQUENCY PULSE WIDTH MODULATION INTEGRATED CIRCUIT WITH EXTERNAL SYNCHRONIZATION CAPABILITY

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The Pulse Width Modulation (PWM) has been playing an active role in circuits and systems for many years. The PWM is an irreplaceable part of the switch-mode power supplies (SMPS) and the class D power amplifiers (PA). The high frequency PWM operation is the key for having more compact SMPSs and class D PAs since it provides size reduction of the passive components, which dominates the size and the cost of these circuits. The digital implementation of the high frequency PWM inherently suffers from the lack of resolution, which leads to a performance degradation. On the other hand, the implementation of the PWM in the analog domain offers infinite resolution by its nature.
This thesis presents the design and implementation of an analog high frequency PWM integrated circuit (IC), which is fabricated in a commercial 0.35 µm CMOS process. The implemented PWM IC employs the natural sampling based PWM generation instead of the uniform sampling method for better spectral response. Taking a coding signal and a digital clock as inputs, the PWM IC creates the PWM signal and its complement, which may be required for some systems (e.g. SMPSs and class D PAs).

The external clock-controlled architecture of the PWM IC brings many features and capabilities. Using this external clock, the PWM IC generates an analog triangle wave so that it provides higher resolution and less quantization error compared to the digital implementations, where step-wise approximation of a triangle wave is used. The selection of the triangle wave as the carrier signal offers lower harmonic distortion compared to the saw-tooth wave. The PWM IC eliminates the frequency error since the frequency of the PWM is directly set by the external clock signal. Besides, the external clock-controlled architecture allows the modulation frequency to be user adjustable, where it can be increased up to 5 MHz, for this implementation. The external clock-controlled architecture of the PWM IC allows multiple chips to be synchronized. This feature enables the use of the presented PWM IC in phased-array systems. The experimental results show that the timing error of the generated triangle wave is 800 ps in a synchronized operation. In addition, the time delay between the modulated AC signals at the output of the synchronized PWM ICs is limited to 4.8 ns when identical AC input signals with the frequency of up to 500 kHz are applied to the multiple PWM ICs. The PWM IC works with a single supply voltage of 5 V while consuming 20 mW of DC power. The full-chip area is 1.8 mm x 1.4 mm including the pad frame. The fabricated IC is protected against electro-static discharge (ESD) with an expected level of Class 2A.

Keywords: PWM, IC, power converter, class D power amplifier
ÖZ

HARİCİ SENKRONLANABİLEN YÜKSEK FREKANSLI DARBE GENİŞLİK MODÜLASYONU TÜMLEŞİK DEVRESİ TASARIMI

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Bu tezde, 0.35 µm CMOS üretim tekniği ile üretilen analog yüksek frekanslı PWM tümleşik devrenin (IC) tasarımı ve uygulamasi anlatılmaktadır. PWM IC, daha iyi bir spectrum tepkisi için PWM oluşum tekniği olarak tek düze örnekleme yöntemi yerine doğal örnekleme metodunu kullanmaktadır. PWM IC, kodlanacak işaret ve sayısal saat işaretini giriş olarak alarak, bazı uygulamalarda (örn: anahtarlamalı güç kaynakları ve D sınıfı güç kuvvetlendircileri) kullanımı gerektirebilir. PWM işaretini ve PWM işaretinin tümleyenini üretir. PWM IC’nin harici bir saat ile kontrol edilebilme mimarisi, ona birçok özellik ve yetenek kazandırmıştır. PWM IC, bu harici saat işaretini kullanarak analog bir üçgen dalga üretir ve PWM IC’nin analog üçgen dalga ile çalışması, ona üçgen dalığın adımsal yaklaşım ile üretildiği sayısal uygulamalar arasında daha yüksek çözünürlük ve daha az nicelime hatası özelliklerini sağlar. PWM IC’nin frekansı, doğrudan harici bir saat işaretini ile belirlenir ve bu sayede frekans hatası ortadan kaldırılır. Harici saat ile kontrol edilebilme mimarisi sayesinde modülasyon frekansı kullanıcı tarafından ayarlanabilir ve bu frekans değeri 5 MHz’e kadar yükseltilebilir. Birden fazla çipin senkronlanması PWM IC’nin harici bir saat ile kontrol edilebilme mumarisiyle gerçekleştirilir. Bu özellik, sunulan PWM IC’nin faz dizili sistemlerde kullanımını sağlar. Ölçüm sonuçlarında, senkron çalışma sırasında elde edilen üçgen dalgalar arasındaki zaman hatasının 800 ps olduğu görülmüştür. Birden fazla PWM IC’ye 500 kHz frekansına kadar eş AC sinyaller uygulandığında, PWM IC’lerin çıkışlarında oluşan modüle edilmiş AC sinyaller arasındaki zaman kayması 4.8 ns olarak ölçülmüştür. PWM IC, 5 V değerindeki tek besleme voltajı ile çalışmakta olup bu voltajda 20 mW güç tüketmektedir. Tüm çip alanı 1.8 mm x 1.4 mm’dir. Üretimi gerçekleştiriliren tümleşik devre, Smif 2A değerinde olması beklenen elektrostatik deşarj (ESD) korumasına sahiptir.

Anahtar Kelimeler: PWM, IC, güç çevircisi, D sınıfı güç kuvvetlendircisi
To Fatma and My Family
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CHAPTER 1

INTRODUCTION

Pulse width modulation (PWM) has been actively used in circuits and systems for many years. Its unique features help it participate in various applications, including motor control, telecommunications, switch-mode power supplies (SMPS), and class D power amplifiers (PA). The PWM is an inevitable part of the SMPSs and the class D PAs among the other major applications. Being a part of these circuits and systems makes the PWM be a part of a huge family of products addressing various markets such as consumer electronics, wearable electronics, automotive, healthcare, industrial, military/defense, and aerospace.

The evolution of a simple PWM chip was first started by Silicon General’s cofounder and power electronics engineer, Bob Mammano, in 1975 [1]. Constant advances in the electronics technology have triggered the evolution of the first PWM integrated circuit (IC) so that the transition from a simple chip to a complete power management IC (PMIC) was achieved.

The significant role of the PWM in wide range of circuits and systems has been motivating many researchers and engineers to develop its theoretical and practical background for many years. Today, the PWM can be implemented in various platforms with different methods. The PWM can be implemented by an analog or a digital application specific integrated circuit (ASIC) or general purpose digital ICs such as a field-programmable gate array (FPGA) or a digital signal processor (DSP). Besides, the PWM can be implemented in discrete circuit level with active and passive electronic components.
1.1. Challenges and Motivation

The trend for a more connected world leads the way for electronic equipment, such as, cell phones, laptops, portable audio devices, and home appliances, getting smarter. Building these complex yet small electronic devices requires high speed and high performance processors working in a relatively compact area. This brings the requirement for more compact and integrated power supply designs. Same situation is also valid for the class D PAs, used for audio applications. They are required to provide high power and quality audio performance in a limited area.

The major area consumption for the SMPSs and the class D PAs is dominated by the discrete passive components such as capacitors, inductors, and transformers employed in the system [2]. Sizes of these components scale inversely proportional to the frequency of operation (PWM). The advances in semiconductor technology introduce new power transistors with higher switching speed and voltage/current capabilities. The increase in switching speed of these transistors is particularly important because it leads to a reduction of the area consumed by the discrete passive components used in the SMPSs and the class D PAs. Besides, as the PWM frequency increases, the required values of the passive components decrease so that their on-chip integration can be satisfied. By this way, higher power densities can be achieved for a smaller area [3], [4].

Many efforts have been put into the implementation of the high frequency PWM in both digital and analog domains. For the digital PWM generation, generic digital ICs such as a FPGA or a DSP, and digital ASICs can be used. The major drawback of the digital implementation is resulted from its discrete nature. It suffers from the limited resolution, which results in reduced accuracy. Besides, the required clock frequency to generate a high frequency/high resolution PWM signal becomes impractical for some applications [5], [6].

In the analog PWM generation, an input signal is compared with a high frequency analog carrier signal (a triangle wave or a saw-tooth wave) by the help of a comparator. In contrast to the digital PWM generation, the analog implementation of the PWM is free of the resolution limitation. The major challenge of implementing a high frequency PWM in an analog platform is the generation of the carrier signal, which
can be a saw-tooth wave or a triangle wave. The carrier signal should be generated with high linearity, which minimizes the distortion so that more precise PWM operation can be obtained. The analog generation of a high-linearity carrier signal requires complex circuits. Another design challenge is the design of the comparator. The comparator makes the decision by comparing the input signal with the carrier signal, where comparison is required to be performed in the range of mV. Thus, the comparator should be designed in such a way that it is immune to noisy environment [7].

1.2. Design Overview

In this work, the design of a high frequency pulse width modulation integrated circuit with external synchronization capability is presented. The PWM IC is fabricated in a commercial 0.35 µm CMOS process.

Since the PWM is a non-linear process, its frequency content is very important, especially for the class D power amplifiers. As will be analyzed later in this dissertation, the sampling method used in the PWM generation has direct correlation to the output frequency content. The PWM IC presented in this work employs the natural sampling based modulation to provide better spectral response.

As mentioned earlier, the high frequency PWM operation is undoubtedly important for the switched-mode power supplies and the class D power amplifiers. For this purpose, the presented PWM IC can generate PWM signals user adjustable up to 5 MHz. The presented IC also generates the complement of the PWM signal enabling its integration into the switch-mode power supplies and the class D power amplifiers, where differential signaling is required.

Moreover, to overcome the resolution limitation of digital implementations, the presented PWM IC performs the PWM generation in an analog manner. As will be detailed later in this report, the presented IC uses a triangle wave as the carrier signal, since it offers lesser harmonic distortion compared to a saw-tooth wave carrier. Thus, the PWM is required to generate a triangle wave with a user adjustable frequency of up to 5 MHz.
The synchronized PWM operation is important for both the SMPSs and the class D PAs used in phased-array applications. Thus, the PWM IC allows multiple chips to be synchronized thanks to its external-clock controlled architecture.

As the ready-to-use analog ready-to-use pulse width modulators in the world market, two products come forward, one is a discrete level solution, and the other one is integrated level solution. The product [8] of Texas Instruments, USA is a discrete pulse width modulation circuit with a maximum PWM frequency of 500 kHz. Besides, this circuit does not have an external synchronization capability. The product [9] of Linear Technology, USA is an integrated pulse width modulator with a maximum PWM frequency of 1 MHz, and without an external synchronization feature.

1.3. Thesis Organization

This thesis includes 6 main chapters and the organization is as follows: Chapter 2 presents the pulse width modulation in terms of its brief description, major applications, types, and spectral analysis. Finally, the selection of the method employed in the PWM IC will be covered.

In Chapter 3, the literature review on the PWM implementations are given. The literature is examined based on the implementation domain as analog and digital. After the brief information for different analog and digital implementation types are covered, this chapter is finalized by their comparison and evaluation.

Chapter 4 provides the design and the implementation steps of the presented PWM IC by analyzing it from the block-diagram level to the circuit level. The design of each sub-block is expressed and the required simulation results are given.

In Chapter 5, experimental results of the fabricated PWM IC is given to validate its functionality.

Chapter 6 includes the conclusion of this thesis and the discussion of the future work.
CHAPTER II

PULSE WIDTH MODULATION

This chapter is dedicated to the overview of the pulse width modulation. Section 2.1 mentions the brief description of the pulse width modulation. In Section 2.2, the major applications of the pulse width modulation are covered. Section 2.3 provides the different types of the pulse width modulation. In Section 2.4, the spectral analysis of different pulse width modulation types is evaluated. Section 2.5 includes the selection of the pulse width modulation type, which is employed in the presented PWM IC.

2.1 Brief Description of Pulse Width Modulation

The Pulse Width Modulation (PWM) is a technique in which a reference signal is coded into a pulse train whose widths correspond to the interpretation of the signal itself [6], [10]. The PWM requires two signals; the original signal, also called the “modulating signal”, which will be coded into a pulse train, and the “carrier signal”, which can be either a triangle wave or a saw-tooth wave. The resulted pulse train is called “modulated signal” which is the PWM signal itself. The PWM signal is generated by comparing the modulating signal with the high frequency carrier signal as depicted in Figure 1.

![Figure 1 Pulse Width Modulation](image-url)
As can be seen in Figure 1, the PWM signal is made of rectangular pulses, which switch between high and low levels. Figure 2 shows a close-up view of a sample PWM signal with a defining function $y(t)$, period $T$, low level value $y_{\text{min}}$, high level value $y_{\text{max}}$, and a duty cycle $D$.

![PWM signal diagram](image)

**Figure 2** A PWM signal with a duty cycle of $D$

The average value of the PWM signal shown in Figure 2 can be expressed as

$$\bar{y} = D \times y_{\text{max}} + (1 - D) \times y_{\text{min}}.$$  \hspace{1cm} (1)

By taking $y_{\text{min}} = 0$, the expression for the average value of the PWM signal becomes $\bar{y} = D \times y_{\text{max}}$. This relation reveals the direct dependence between the average value of the PWM signal and its duty cycle. The ability of controlling the average value of the PWM signal with its duty cycle creates many application areas for the PWM. The following sub-section summarizes some of the major application areas of the PWM.

### 2.2 Major Applications of Pulse Width Modulation

The pulse width modulation has been playing a critical role in many circuits and systems for a long time. Its unique structure makes the PWM participate in various applications. The major applications of the PWM can be listed as follows:

#### 2.2.1. Motor Control

Motor control is one of the major applications of the PWM for many years [11], [12], [5]. Controlling the speed of an electric motor is achieved by controlling the power delivered to it, which is directly proportional to the voltage applied. This control
mechanism is perfectly matched with the idea of the PWM in such a way that the duty cycle of the PWM signal should be decreased to slow down the motor or increased to speed it up. By changing the duty cycle of the PWM signal, its average value is adjusted to control the power delivered to a motor as it is expressed in (1). In PWM-controlled servomotors on the other hand, the servo position is determined by the width of the pulse instead of the duty cycle of the pulse. Controlling a servomotor with respect to the widths of the PWM signal is shown in Figure 3 with an example, where the servomotor used is HS-322HD of Hitec RCD, USA [13]. As can be seen in Figure 3, the specific values of the pulse widths of the PWM signal correspond to the specific rotation angles.

![Figure 3 Controlling HS-322HD servomotor with PWM](image)

2.2.2. Switch-Mode Power Supplies

All electronic circuits and systems need power supplies to function. Power supplies can be categorized into two; linear power supplies and switch-mode power supplies (SMPS). Linear power supplies contain transistors working in the active operation region, causing high voltage drops at high currents. Thus, these types of supplies have large power dissipation resulting in low efficiency [10], [14].

SMPSs use transistors as switches in such a way that they allow current passing through them when they are “ON” and they do not conduct any current when they are “OFF”. For both cases, the power dissipation over the transistors are ideally zero. Therefore, the switch-type operation of the transistors dramatically reduces the power
dissipation of the system resulting in a large improvement in the efficiency. High efficiency, small size and light-weight are the dominant characteristics of SMPSs over linear power supplies helping SMPSs employed in a variety of electronic systems such as personal computers, laptops, and televisions [15].

The PWM is an essential part of most of the SMPS circuits. Kazimierczuk in [10] defines a family of PWM-based circuits consisting of the buck, boost, buck-boost, fly-back, forward, SEpic (single-ended primary input converter), and dual SEpic, which are all single-ended types. Moreover, there are three multiple-switch PWM-based SMPS circuits such as the half-bridge, full-bridge, and push-pull converters. All of these circuits utilize the PWM in their control loop to adjust the output voltage. The detailed analysis of all PWM-based SMPS circuits are beyond the scope of this work, however for the sake of completeness, the PWM operation will briefly be covered for the buck and boost converter circuits.

The circuit topology for the buck converter is shown in Figure 4.

![Figure 4: The circuit topology of the buck converter](image)

The circuit operates under the control of the pulse width modulation as follows: when the PWM signal is high, the transistor $M$ will be “ON” making the diode $D$ reverse-biased. Thus, there will be a current flowing through the inductor $L$ charging the capacitor $C$. When the PWM signal is low, the transistor will be “OFF” and the diode will be forward-biased. Also, the input voltage will be separated from the output since the transistor is “OFF”. Within this time interval, the inductor will behave like a voltage source. In other words, the input voltage will supply the current when the PWM signal is high and the inductor will supply the current when the PWM signal is low. If the value of the current never falls to zero, the operation is called continuous mode and the voltage/current waveforms for this operation is shown in Figure 5.
The relation between the input and the output voltages for the buck converter can be expressed as

\[ V_{OUT} = D \times V_{IN}. \]  \hspace{1cm} (2)

Since the duty cycle of the PWM signal is not more than 1, the output voltage will always be less than the input voltage causing the buck converter is also called as the step-down converter. As it can be seen from (2), the PWM signal directly controls the operation of the buck converter by adjusting its duty cycle.

Another PWM-based DC-DC converter type is the boost converter and its circuit topology is shown in Figure 6. The circuit has the same components with the buck converter however, its design and working principle are different than the buck converter.
When the PWM signal is high, the transistor $M$ will be “ON”, the diode $D$ will be reverse biased, and the current will flow through the inductor $L$ returning back to the input power supply. Meanwhile, the current will be increased resulting in stored energy on the inductor. This stored energy will create a magnetic field in which the positive polarity is at the left side of the inductor. Within this time interval, the current, which will be supplied to the load, is obtained by the discharge of the capacitor $C$. When the PWM signal is low, the transistor will be “OFF” forcing the current drawn from the input supply flow through the inductor and the diode. Within this time interval, the current will be reduced and the magnetic field will be reversed. Thus, the induced voltage on the inductor will act as a voltage source in series with the input voltage charging the capacitor to the output voltage. In other words, when the PWM signal is high, the capacitor will be the source of the current for the load while the current will be drawn from the input voltage supply and it will flow through the inductor and the diode. Similar to the buck converter, the operation is called continuous mode if the value of the current never falls to zero and the voltage/current waveforms for this operation is shown in Figure 7.

The direct dependence of the output voltage on the duty cycle of the PWM signal is expressed as

$$V_{OUT} = \frac{V_{IN}}{1-D}.$$  \hspace{1cm} (3)

The output voltage generated in the boost converter is always greater than the input voltage as can be seen in (3).
2.2.3. Class D Power Amplifier

Class D power amplifiers (PA) have been employed by many electronic devices in which an audio signal amplification is required. There are different types of power amplifiers such as class A, B, and AB classified with respect to the transistor’s quiescent point. The theoretical efficiency values for class A, B, and AB are 25%, 78.5%, and slightly more than 78.5%, respectively [16]. However, battery-powered devices such as cell-phones, portable audio devices, and laptops necessitate low-power circuits in their electronics. For such applications, to achieve higher power efficiency numbers switching mode power amplifiers (i.e. class D) with their theoretical efficiencies of 100% are usually employed. Although this value will be limited due to the switching and the conduction losses in the switching power transistors, class D PAs are still much more efficient than the other classes. The circuit topology for a half-bridge class D power amplifier is shown in Figure 8.
Figure 8 The circuit topology of a half-bridge class D power amplifier

The input signal to be amplified is compared with a high frequency carrier wave so that the PWM signal is generated. The PWM signal is fed to a gate driver, which can be an integrated circuit or a discrete circuit design. The gate driver boosts the current rating of the PWM signal and adjusts its voltage level to be able drive the switching power transistors. After the switching is done, the PWM signal is obtained in such a way that its rails are between the positive and the negative supply voltages. To re-obtain the amplified input signal without harmonics, the amplified PWM signal is filtered with a low-pass filter. With filtering, high frequency components generated due to the carrier wave are removed and we end up with the amplified input signal, which drives the load. The higher efficiency of the class D power arises from the switching architecture of the circuit, for which the PWM operation is perfectly suited.

As mentioned earlier, class D power amplifiers are generally used in cell-phones, laptops and portable audio devices to generate high power audio signal, which drives a speaker. Another usage of class D power amplifiers is in SONAR (SOund Navigation And Ranging) systems. SONAR systems use acoustic waves to detect and classify a target underwater. They are also used for communication and navigation underwater. There are two types of SONAR systems; passive and active [17]. A passive SONAR works based on evaluating the echo coming from the underwater targets. On the other hand, an active SONAR emits acoustic waves using its transmitter and it processes the reflected acoustic signals from the targets. A transmitter of an active SONAR consists of power amplifiers and acoustic transducers. An acoustic transducer is a sensor, which converts an electrical signal into an acoustic signal and it is driven by power amplifiers to generate high acoustic powers [18]. To be able to detect a target from a longer range, the power transmitted by the power amplifiers should be as high as possible [17]. It is
required to satisfy high power values with high efficiency, which makes class D power amplifiers be a good solution for SONAR transmitters.

Until this point, the major applications of the pulse width modulation have been covered. The next part includes the types and the general characteristics of the PWM signal.

### 2.3 Types of Pulse Width Modulation

Pulse width modulation topologies can be categorized into two main classes; Single-Edge Pulse Width Modulation (SEPWM) and Double-Edge Pulse Width Modulation (DEPWM). For the SEPWM pulses, one of the transition edge, either leading (rising) or trailing edge (falling), is fixed while the modulation occurs at the other edge as the input (reference) signal varies. Depending on the modulation edge, the SEPWM is sub-categorized into the Trailing Edge Pulse Width Modulation (TEPWM) and the Leading-Edge Pulse Width Modulation (LEPWM). On the other hand, the modulation occurs at the both edges of pulses in the DEPWM. The main difference between the SEPWM and the DEPWM arises from the type of the carrier wave used in the modulation. The SEPWM uses a saw-tooth carrier while the DEPWM uses a triangle wave carrier.

PWM can further be categorized with respect to the sampling method used in the modulation process. There are two major types in terms of sampling method; Naturally Sampled PWM (NPWM) and Uniformly Sampled PWM (UPWM) [6].

The following subsections provide a short discussion on various PWM classes, along with the associated waveforms.

#### 2.3.1. Single-Edge PWM

As illustrated in Figure 9, for the LEPWM, the trailing edge of the PWM signal is fixed, while the modulation is performed at the leading edge of the PWM pulse. A saw-tooth waveform is used as a carrier to perform the LEPWM.
Figure 9 The leading-edge pulse width modulation (LEPWM)

As shown in Figure 10, for the TEPWM, the leading edge of the PWM signal is fixed, while the modulation is performed at the trailing edge of the PWM pulse. Similar to the LEPWM, the TEPWM employs a saw-tooth waveform as the carrier.

Figure 10 The trailing-edge pulse width modulation (TEPWM)
2.3.2. Double-Edge PWM

In the Double-Edge Pulse Width Modulation (DEPWM), both the leading and trailing edge of a PWM signal is modulated with respect to the input signal variation. The DEPWM differs from the SEPWM types in terms of the carrier type. It uses a triangle wave as a carrier as shown in Figure 11.

![Diagram of Double-Edge Pulse Width Modulation](image)

Figure 11 The double-edge pulse width modulation

2.3.3. Naturally-Sampled PWM

The Naturally Sampled (or Natural Sapling) Pulse Width Modulation (NPWM) is generally considered as the traditional analog implementation of the PWM in the literature [19]. In the NPWM, the switching edges of the PWM pulses occur at the same time with the sampling of the input signal by the carrier signal. The NPWM can use a saw-tooth wave or a triangle wave as a carrier to execute the modulation. This means the NPWM can be used both in the double-edge and in the single-edge modulation forms [20]. An illustration of the NPWM for the single-edge (both leading and trailing edges) version is shown in Figure 12 and for double-edge version can be seen in Figure 13.
Figure 12 The single-edge naturally-sampled PWM and uniformly-sampled PWW
(a) leading-edge (b) trailing-edge

Figure 13 The double-edge naturally-sampled PWM and uniformly-sampled PWM
(symmetrical and asymmetrical)
2.3.4. Uniformly-Sampled PWM

Unlike the NPWM, in the Uniformly Sampled PWM (UPWM), the sampling of the input signal occurs at the top or at the bottom of the carrier instead of at the intersection of the input signal and the carrier signal [21]. The UPWM is also applicable for both double-edge and single-edge types. For the single-edge UPWM, a saw-tooth wave is used as the carrier and the input signal is usually sampled at the beginning of the period. In the case of the double-edge UPWM, one sample or two samples can be used to locate the switching edges of the PWM pulses. If one sample of the input signal at the peak or valley of a triangle wave is taken, the method is called the Symmetrical Uniform Sampling. If the sampling occurs twice at both the peak and the valley of the triangle wave, the method is referred as the Asymmetrical Uniform Sampling [20]. The Double-Edge Symmetrical and Asymmetrical Uniform Sampling modulation types can be seen in Figure 13 and an illustration of the UPWM for the single-edge (both leading and trailing edges) version is shown in Figure 12.

2.4 Spectral Analysis of Pulse Width Modulation

The spectral analysis of the pulse width modulation is important in terms of the selection of the method, which will be employed in the presented PWM IC. The circuits used PWM are generally switched-type such as switch-mode power supplies and Class D power amplifiers. These circuits have many nonlinearity sources such as the passive components and the power transistors in addition to the PWM signal. By its nature, the PWM operation is a nonlinear operation having many harmonic components. Especially, for Class D power amplifiers, reducing the harmonic distortion is very critical since they are mostly used in sound systems in which sound quality means lower harmonic distortion. This chapter will be a guide to select the pulse width modulation type for the presented PWM IC by comparing the spectral response of the different PWM method mentioned in Chapter 2.3.

The spectral analysis of the PWM signals for different types has been studied for many years. Two-dimensional (double) Fourier series (DFS) is the most common technique used in the analysis and there have been many works to analyze the frequency spectrum of different PWM schemes [21], [12], [22], [23]. The analytical complexity of modulated multi-tone signals is much higher than the one of single tone signals.
However, the analysis of single tone modulated signals gives enough information to have a better understanding of the modulation process [22]. Thus, we will only consider the spectral characteristic of the modulation of a single tone input signal without going in too much details of the derivation of the mathematical expressions. Moreover, the spectral comparison of different PWM types will be examined. Considering their generation methods explained in Chapter 2.3, it is inevitable to expect different spectral responses from the NPWM and the UPWM.

The PWM signal has many harmonics in addition to the fundamental component, which contains the input signal. Different harmonics can arise in the frequency spectrum according to the generation method of the modulation. All possible harmonics can be listed as follows:

- Harmonics of the input signal, at frequencies $f = n f_{in}$, for $n = 2, 3, \ldots, +\infty$
- Carrier signal itself and its harmonics, at frequencies $f = m f_{carrier}$, for $m = 1, 2, \ldots, +\infty$
- Intermodulation (IM) components formed by the input and the carrier signals, at frequencies $f = m f_{carrier} + n f_{in}$, for $m = 1, 2, \ldots, +\infty$, $n = \pm 1, \pm 2, \ldots, \pm \infty$

Nielsen and Gong in [22] and [6], respectively, provide the expressions for both the NPWM and the UPWM, including the single-edge and the double-edge modulations. We will focus on the double-edge NPWM (DNPWM) and the double-edge UPWM (DUPWM), which will give enough information to examine their characteristics and make a comparison between them. The equations of the DNPWM and the DUPWM (symmetrical) are expressed as

$$f_{DNPWM}(t) = M \cos(y)$$

$$+ \sum_{m=1}^{\infty} \left(2 J_0 \left(\frac{m\pi M}{2}\right) \sin \left(\frac{m\pi}{2}\right) \cos(mx) \right)$$

$$+ \sum_{m=1}^{\infty} \sum_{n=\pm 1}^{\infty} \left(2 J_n \left(\frac{m\pi M}{2}\right) \sin \left(\frac{(m+n)\pi}{2}\right) \cos(mx + ny) \right)$$

(4)
\[ f_{\text{DUPWM}} = \sum_{n=1}^{\infty} \left( \frac{2J_n(n\pi M/2)q}{n\pi q} \sin \left( (q + 1) \frac{n\pi}{2} \right) \cos(ny) \right) + \sum_{m=1}^{\infty} \left( \frac{2J_0(m\pi M/2)}{m\pi} \sin \left( \frac{m\pi}{2} \right) \cos(mx) \right) + \sum_{m=1}^{\infty} \sum_{n=\pm 1}^{\infty} \left( \frac{2J_n \left( (nq + m) \pi M/2 \right)}{(nq + m)\pi} \sin \left( (m + n(1 + q)) \frac{n\pi}{2} \right) \cos(mx + ny) \right), \tag{5} \]

where, \( M \) is the “modulation index”, which is the ratio between the peak-to-peak amplitudes of the input signal and the carrier signal. \( w_{\text{in}} \) and \( w_{\text{carrier}} \) are the angular frequency of the input and carrier signal, respectively. \( q \) defines the ratio between \( w_{\text{in}} \) and \( w_{\text{carrier}} \). \( J_n \) is the \( n \)th-order Bessel function of the first kind. \( n \) and \( m \) are the indices of the harmonics of the input and carrier signal, respectively. The carrier signal is denoted as \( x = w_{\text{carrier}}t \) while the input signal as \( y = w_{\text{in}}t \). The harmonic components and their amplitudes for DNPWM and DUPWM are tabulated in Table 1 and 2, respectively.

### Table 1 Harmonic Components of the DNPWM

<table>
<thead>
<tr>
<th>Harmonic Component</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>m’th harmonic of carrier</td>
<td>( \frac{2J_0 \left( m\pi M/2 \right) \sin \left( \frac{m\pi}{2} \right)}{m\pi} )</td>
</tr>
<tr>
<td>IM-component of carrier and input</td>
<td>( \frac{2J_n \left( (m + n(1 + q)) \frac{n\pi}{2} \right)}{(nq + m)\pi} \sin \left( \frac{m\pi}{2} \right) \cos(mx + ny) )</td>
</tr>
</tbody>
</table>

### Table 2 Harmonics Components of the DUPWM

<table>
<thead>
<tr>
<th>Harmonic Component</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>n’th harmonic of input</td>
<td>( \frac{2J_n \left( n\pi M/2q \right)}{n\pi q} \sin \left( (q + 1) \frac{n\pi}{2} \right) )</td>
</tr>
<tr>
<td>m’th harmonic of carrier</td>
<td>( \frac{2J_0 \left( m\pi M/2 \right)}{m\pi} \sin \left( \frac{m\pi}{2} \right) )</td>
</tr>
<tr>
<td>IM-component of carrier and input</td>
<td>( \frac{2J_n \left( (nq + m) \pi M/2 \right)}{(nq + m)\pi} \sin \left( (m + n(1 + q)) \frac{n\pi}{2} \right) )</td>
</tr>
</tbody>
</table>
One of the most critical and major outcome of the comparison between the NPWM and the UPWM is that there is no direct harmonics of the input signal in the NPWM contrary to the UPWM [21]. In other words, the UPWM introduces distortions at both the frequency of the input signal itself and its harmonics. This is very important, especially for Class D power amplifiers, because the direct harmonics of the input signal will be involved in the frequency band of interest, which lies between the frequencies of the input signal and the carrier signal. For the NPWM, being free of direct harmonics of the input signal has a serious advantage over the UPWM since the input signal can be re-constructed from the PWM signal by appropriate filtering, while the reconstruction filter will be impractical in the UPWM due to the direct harmonics of the input signal [22], [24].

Vasca and Iannelli in [20] have given a detailed analysis for different NPWM methods. The analyses are performed based on an input signal consisting of a DC component $R_0$ and a single tone sinusoidal component with an amplitude of $R_1$ and a frequency of $w_{in}$ shown as

$$i_n(t) = R_0 + R_1 \cos(w_{in}t).$$

(6)

The NPWM can further be sub-categorized with respect to the voltage rails of the signals forming the modulation procedure; unipolar-modulation and bipolar-modulation [20]. The voltage rails for both modulation types are listed in Table 3. The rest of this part will focus on the analysis of the NPWM for both bipolar and unipolar versions.

<table>
<thead>
<tr>
<th>Unipolar Modulation</th>
<th>Bipolar Modulation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input</strong></td>
<td><strong>Carrier</strong></td>
</tr>
<tr>
<td>Peak</td>
<td>0 to $R_0+R_1$</td>
</tr>
</tbody>
</table>
2.4.1. Unipolar Trailing-Edge Naturally-Sampled PWM

For demonstration purpose, the input signal is chosen as a sine wave with $R_0 = 0.5 \, V$, $R_1 = 0.2 \, V$, $f_{in} = 100 \, kHz$, and the carrier signal is a saw-tooth wave with $f_{carrier} = 1 \, MHz$, which rails between 0 and 1 V ($C_m = 1 \, V$). Similarly, the generated PWM signal switches from 0 V to 1 V ($A = 1 \, V$). The modulation can be seen in Figure 14 and its mathematical expression is given as

\[
f_{NPWM}(t) = \frac{R_0}{C_m} + \frac{M}{2} \cos(y) + \sum_{m=1}^{+\infty} \frac{1}{m\pi} \left( \sin(mx) - J_0(m\pi M) \sin \left( m\pi - 2m\pi \frac{R_0}{C_m} \right) \right) \]

\[
+ \sum_{m=1}^{+\infty} \sum_{n=\pm 1}^{\infty} \frac{J_n(m\pi M)}{m\pi} \sin \left( \frac{n\pi}{2} - mx - ny + 2m\pi \frac{R_0}{C_m} \right). \tag{7}
\]

Figure 14 The unipolar trailing-edge naturally-sampled pulse width modulation
2.4.2. Bipolar Trailing-Edge Naturally-Sampled PWM

For demonstration, the input signal is chosen as a sine wave with $R_0 = 0 \ V$, $R_I = 0.2 \ V$, $f_{in} = 100 \ kHz$ and the carrier signal is a saw-tooth wave with $f_{carrier} = 1 \ MHz$, which swings between $-0.5 \ V$ and $0.5 \ V$ ($C_m = 1 \ V$). Similarly, the generated PWM signal rails from $-0.5 \ V$ to $0.5 \ V$ ($A = 1 \ V$). The modulation can be seen in Figure 15 and its mathematical expression is given as

$$f_{DNPWM}(t) = \frac{M}{2} \cos(y) + \sum_{m=1}^{\infty} \frac{1}{m\pi} (\cos(m\pi) - J_0(m\pi M)) \sin(mx) + \sum_{m=1}^{\infty} \sum_{n=\pm 1}^{\infty} J_n(m\pi M) \frac{m\pi}{m\pi} \sin \left( \frac{n\pi}{2} - mx - ny \right).$$

Figure 15 The bipolar trailing-edge naturally-sampled pulse width modulation
2.4.3. Unipolar Leading-Edge Naturally-Sampled PWM

For demonstration purposes, the input signal is chosen as a sine wave with \( R_0 = 0.5 \) V, \( R_f = 0.2 \) V, \( f_{in} = 100 \) kHz and the carrier signal is a saw-tooth wave with \( f_{carrier} = 1 \) MHz, which rails between 0 and 1 V (\( C_m = 1 \) V). Similarly, the generated PWM signal switches from 0 V to 1 V (\( A = 1 \) V). The modulation can be seen in Figure 16 and its mathematical expression is given as

\[
f_{NPWM}(t) = \frac{R_0}{C_m} + \frac{M}{2} \cos(y) \\
- \sum_{m=1}^{+\infty} \frac{1}{m\pi} \left( \sin(mx) - J_0(m\pi M) \sin \left( mx + 2m\pi \frac{R_0}{C_m} \right) \right) \\
- \sum_{m=1}^{+\infty} \sum_{n=\pm 1} J_n(m\pi M) \sin \left( \frac{n\pi}{2} - mx - ny - 2m\pi \frac{R_0}{C_m} \right).
\]  

(9)

Figure 16 The unipolar leading-edge naturally-sampled pulse width modulation
2.4.4. Bipolar Leading-Edge Naturally-Sampled PWM

For demonstration, the input signal is chosen as a sine wave with \( R_0 = 0 \) V, \( R_I = 0.2 \) V, \( f_{in} = 100 \) kHz and the carrier signal is a saw-tooth wave with \( f_{carrier} = 1 \) MHz, which swings between -0.5 V and 0.5 V (\( C_m = 1 \) V). Similarly, the generated PWM signal rails from -0.5 V to 0.5 V (\( A = 1 \) V). The modulation can be seen in Figure 17 and its mathematical expression is given as

\[
f_{DNPWM}(t) = \frac{M}{2} \cos(y) \\
- \sum_{m=1}^{+\infty} \frac{1}{m\pi} \left( \cos(m\pi) - J_0(m\pi M) \right) \sin(mx) \\
- \sum_{m=1}^{+\infty} \sum_{n=\pm1}^{\infty} \frac{J_n(m\pi M)}{m\pi} \sin \left( \frac{n\pi}{2} - mx - n(y + \pi) \right).
\]  

Figure 17 The bipolar leading-edge naturally-sampled pulse width modulation
2.4.5. Unipolar Double-Edge Naturally-Sampled PWM

For demonstration purposes, the input signal is chosen as a sine wave with \( R_0 = 0.5 \text{ V} \), \( R_I = 0.2 \text{ V} \), \( f_{\text{in}} = 100 \text{ kHz} \) and the carrier signal is a triangle wave with \( f_{\text{carrier}} = 1 \text{ MHz} \), which rails between 0 V and 1 V \( (C_m = 1 \text{ V}) \). Similarly, the generated PWM signal switches from 0 V to 1 V \( (A = 1 \text{ V}) \). The modulation can be seen in Figure 18 and its mathematical expression is given as

\[
f_{NPWM}(t) = \frac{R_0}{C_m} + \frac{M}{2}\cos(y) + \sum_{m=1}^{+\infty} \frac{2}{m\pi} J_0 \left( \frac{m\pi M}{2} \right) \sin \left( m\pi \frac{R_0}{C_m} \right) \cos(mx) + \sum_{m=1}^{+\infty} \sum_{n=\pm 1}^{+\infty} \frac{2}{m\pi} J_n \left( \frac{m\pi M}{2} \right) \sin \left( \left( 2m \frac{R_0}{C_m} + n \right) \frac{\pi}{2} \right) \cos(mx + ny).
\]

Figure 18 The unipolar double-edge naturally-sampled pulse width modulation
2.4.6. Bipolar Double-Edge Naturally-Sampled PWM

For demonstration, the input signal is chosen as a sine wave with $R_0 = 0 \, V$, $R_I = 0.2 \, V$, $f_{in} = 100 \, kHz$ and the carrier signal is a triangle wave with $f_{carrier} = 1 \, MHz$, which swings between -0.5 V and 0.5 V ($C_m = 1 \, V$). Similarly, the generated PWM signal rails from -0.5 V to 0.5 V ($A = 1 \, V$). The modulation can be seen in Figure 19 and its mathematical expression is given as

$$f_{NPWM}(t) = \frac{M}{2} \cos(y)$$

$$+ \sum_{m=1}^{+\infty} \frac{2}{m\pi} f_0 \left(\frac{m\pi M}{2}\right) \sin\left(\frac{m\pi}{2}\right) \cos(mx)$$

$$+ \sum_{m=1}^{+\infty} \sum_{n=\pm1}^{+\infty} \frac{2}{m\pi} f_n \left(\frac{m\pi M}{2}\right) \sin\left((m+n)\frac{\pi}{2}\right) \cos(mx + ny).$$

Figure 19 The bipolar double-edge naturally-sampled pulse width modulation
Considering types of the NPWM, as Vasca and Iannelli stated in [20] that the spectral response of the trailing-edge and the leading-edge NPWM are the same for both bipolar and unipolar modulation versions. Figure 20 shows the comparison between the amplitude spectrum of the bipolar trailing-edge and leading-edge NPWM, which correspond to cases in Chapter 2.4.2 and 2.4.4, respectively.

![Amplitude Spectrum of Bipolar Trailing Edge NPWM](image)

![Amplitude Spectrum of Bipolar Leading Edge NPWM](image)

Figure 20 The amplitude spectrum comparison of the bipolar trailing-edge and leading-edge NPWM

Another critical observation is that the double-edge modulation has far fewer harmonics compared to the single edge modulation (trailing or leading edge), regardless of whether uniform sampling or natural sampling is used [20]. In other words, choosing a triangle wave as a carrier results in a modulation with reduced harmonic distortion compared to a saw-tooth carrier wave [21], [22]. As an example, the amplitude spectrum of the bipolar single-edge NPWM corresponding to the case in Chapter 2.4.2 (trailing-edge) or 2.4.4 (leading-edge) is compared with the amplitude spectrum of the bipolar double-edge NPWM corresponding to the case in Chapter
2.4.6. As it is shown in Figure 21, the double-edge modulation performs better harmonic performance over the single-edge modulation.

![Amplitude Spectrum of Bipolar Single Edge NPWM](image)

![Amplitude Spectrum of Bipolar Double Edge NPWM](image)

Figure 21 The amplitude spectrum comparison between the bipolar single-edge NPWM and the bipolar double-edge NPWM

### 2.5 Selection of Pulse Width Modulation Method

The types and spectral analysis of the different PWM types are covered in Chapter 2.3 and Chapter 2.4, which will be a guide to determine the type of PWM employed in the PWM IC that is presented in this work.

Firstly, it is required to select the sampling type, which will be used in the modulation. As it is covered in Chapter 2.4, the uniform sampling brings distortion at the frequency of the input signal itself and its harmonics besides the other harmonics related to the carrier wave. On the contrary, the natural sampling does not introduce harmonics at the input signal frequency. Because of this reason, the natural sampling is chosen as the sampling method of the presented PWM IC.
Secondly, the double-edge modulation is selected over the single-edge modulation since it performs better harmonics performance compared to the single-edge modulation. This means that a triangle wave will be used as a carrier instead of a sawtooth wave.

Another selection should be made in terms of the voltage polarity of the signals participated in the modulation process. The mathematical expressions for the unipolar double-edge NPWM and the bipolar double-edge NPWM will lead us to make a comparison between them. As it can be seen in (12), the IM harmonics of the input and the carrier signals occurring at the frequency of $mf_{\text{carrier}} + nf_{\text{in}}$ are eliminated if $m + n$ is even due to $\sin[(m+n)\pi/2]$ being equal to zero for the bipolar modulation. In other words, the IM harmonics related to even-order harmonics (where $m$ and $n$ are even) and odd-order harmonics (where $m$ and $n$ are odd) are diminished in the case of the bipolar modulation as it is shown in Figure 21. To obtain the same results from the unipolar modulation, the term

$$\sin\left(2m \frac{R_0}{C_m} + n\right) \frac{\pi}{2},$$

should be equal to zero when $m+n$ is even. The term (13) can only be equal to zero by making the term

$$2m \frac{R_0}{C_m} + n,$$

be an even number. To be able to satisfy this constraint for $m+n$ is even, the term $2R_0/C_m$ should be an odd number.

The bipolar modulation is not preferred since it requires a negative supply voltage as well as a positive supply voltage while the unipolar operation uses a single positive supply voltage. However, the PWM IC should satisfy the term $2R_0/C_m$ be an odd number in its circuit design.

To sum up, the presented PWM IC uses the unipolar naturally sampled double-edge modulation type as the pulse width modulation method.
CHAPTER III

LITERATURE REVIEW ON PULSE WIDTH MODULATION IMPLEMENTATION

This chapter covers the literature background on the implementation of pulse width modulation. The review of the literature is sub-categorized with respect to the implementation domain as digital and analog, which are provided in Section 3.1 and 3.2, respectively. Section 3.3 provides the evaluation of different implementation types.

3.1 Digital Implementation

The PWM can be digitally implemented using an application specific integrated circuit (ASIC) or generic digital integrated circuits such as a field-programmable gate array (FPGA) or a digital signal processor (DSP). FPGAs and DSPs are capable of many digital functions and they are very common in the world-market. Since there are low cost/high performance FPGAs or DSPs available, they are very popular in digital implementation of the PWM. Besides, there are many ASICs for PWM in the literature, where more customized operation is required.

The digital PWM is generally used in the digital control loops of DC-DC switching power converters [25]. The block diagram of a digitally controlled DC-DC switching power converter is shown Figure 22. The digital controller consists of an A/D converter, a compensator and a digital pulse width modulator. Basically, the output voltage of the DC-DC converter is compared with a reference voltage so that an error voltage is generated. The error voltage is converted into a digital signal using an A/D converter. The output of the A/D converter is fed to the compensator generating a
control signal for the digital pulse width modulator. The digital pulse width modulator creates the PWM pulses, which will be fed to DC-DC converter [26].

![Diagram of a DC-DC converter with digital control loop](image)

**Figure 22** The digitally controlled DC-DC converter

The major problem of the digitally controlled DC-DC switching power converters is the limit-cycling (or limit-cycle oscillation), which is caused by the lack of resolution [25], [27], [28]. The limit cycling is a quantization error based problem and it is a critical phenomenon, which should be taken into account, especially for the DC-DC converters requiring strict output voltages. It is inherently obvious that the quantization effects occur both in the digital pulse width modulator and in the A/D converter considering the signal chain in the digital control loop of the DC-DC converter [27].

Theoretically, the control loop should work as follows: when the output voltage gets below its regulation, the control loop should increase the duty cycle of the PWM so that the output voltage will be compensated and vice-versa. On the other hand, both digital pulse width modulator and A/D converter suffer from the quantization error. Let us consider a DC-DC converter with an A/D converter resolution of $n_{A/D}$-bits and a digital pulse width modulator resolution of $n_{PWM}$-bits. The outputs of the A/D converter and the digital pulse width modulator will have voltage quantization steps of $\Delta V_{A/D} = V_{in}/2^{n_{A/D}}$ and $\Delta V_{PWM} = V_{in}/2^{n_{PWM}}$, respectively. If $\Delta V_{PWM}$ is larger than $\Delta V_{A/D}$, the digital pulse width modulator outputs a voltage level, which does not fall into the A/D converter reading of the reference voltage (zero-error bin) as it is shown in Figure 23. In other words, it is not possible to represent the needed duty cycle of the PWM due to the quantization with low resolution. The digital pulse width modulator will try to generate an output voltage of $V_{ref}$ but it will alternate outside the zero-error range.
This behavior results in an unstable output voltage with oscillations due to the limit-cycling. To overcome the limit-cycle oscillation, the bit resolution of the digital pulse width modulator should be increased [27]–[29].

Figure 23 Limit cycling phenomenon in digitally controlled DC-DC converters [27]

In the literature, there are many works and researches for obtaining digital PWM. The detailed reviews of their implementations are beyond the scope of this work. However, the results obtained from different PWM implementations will be discussed by considering the maximum achievable PWM frequency with highest resolution.

3.1.1 Counter-based Digital Pulse Width Modulation

The most common way of the digital PWM implementation is the counter-based digital PWM, which can be employed in both general purpose digital ICs and ASICs. This method can be considered as the digital implementation of conventional PWM [30]. In counter-based method, the carrier signal is generated by using a counter. Thus, the switching edge of the PWM is determined by the counter type. In case of an up-down counter, the modulation is specified as the double-edge modulation. The use of an up-counter depicts the trailing-edge modulation, while the use of a down-counter depicts the leading-edge modulation [31]. As an example, the trailing-edge digital PWM based on counter topology is shown in Figure 24.
Figure 24 Counter-based trailing-edge digital pulse width modulation [31]

Considering the counter-based trailing-edge PWM, the PWM signal is set to the high level at the beginning of the switching cycle. If the duty cycle value $d_{in}$ is lower than the counter, the comparator sets its output to the high level so that the PWM signal switches to the low level from the high level as can be seen in Figure 24. In case of the duty cycle value $d_{in}$ gets higher than the counter within the same switching cycle, the modulator will not change its output and it will wait until the next switching cycle. This situation results in a turn-on delay for counter-based modulation. Similarly, there occurs turn-off delay in case of down-counter. On the other hand, up-down counter-based modulation (double-edge) exhibits both turn-on and turn-off delays but they are smaller compared to that of up-counter based modulation (trailing-edge) and that of down-counter based modulation (leading-edge), respectively [29], [31]–[33].

The major drawback of the counter-based digital PWM is that when high frequency/high resolution digital PWM is desired, the required clock frequency goes to impractical values. The required clock frequency $f_{CLK}$ is directly proportional to the number of bits $n$, and the desired PWM frequency $f_{PWM}$. The relation can be expressed as

$$f_{CLK} = 2^n f_{PWM}. \quad (15)$$

As an example, a clock frequency of around 5.1 GHz is required to generate a sawtooth wave with a 5 MHz frequency and 10-bits resolution. This clock frequency value is impractical for many applications. Even though counter-based approach is
advantageous in terms of its ease in implementation and linearity, it is not suitable for high frequency/high resolution digital PWM applications due to the high clock frequency need [34]. Besides, it exhibits modulation delays.

### 3.1.2 Delay-line based Digital Pulse Width Modulation

One other method is the delay-line based digital PWM as can be seen in Figure 25. It consists of $2^n$ delay cells and $2^n \times 1$ multiplexer for an n-bit digital PWM. The delay of each cell determines the time step and the total time delay selected by the multiplexer generates the PWM signal. The selection of the multiplexer is controlled by the n-bit duty cycle signal $d[n]$. The PWM signal is set to the high level by the clock signal and it is reset when the multiplexer makes its selection.

![Delay-line based digital pulse width modulation](image)

Figure 25 The delay-line based digital pulse width modulation

Delay-line based approach uses the clock frequency as the PWM frequency so that it eliminates the high clock frequency requirement of the counter-based modulation and also minimize the power consumption. The one of the major drawback is the large chip area consumption when high resolution is desired. The case of higher resolution requires more delay cells and larger multiplexer, which results in a larger chip area. In the design, the total delay resulted from the delay cells should be designed in such a way that the maximum delay obtained should be equal to the clock frequency for proper operation. However, the delay of a cell is a function of supply voltage and is vulnerable to process and temperature variations. This can create a degradation in the performance of the operation. Another critical issue is the matching of the high number
of delay cells, where high resolution is required. Device mismatch can create a linearity problem [34]–[37].

The work in [31] presents a digital pulse width modulator integrated circuit, which employs delay-line based architecture. The measurement results reveal that the chip operates at a maximum PWM frequency of 1 MHz with 6-bits resolution. Another delay-line based digital PWM IC is presented in [38]. For this IC, the simulation results offer a digital PWM with a frequency of 1 MHz with 11-bits resolution.

3.1.3 Hybrid Digital Pulse Width Modulation

Hybrid digital PWM is combination of delay-line based and counter-based methods as can be seen in Figure 26. The resolution of the hybrid modulation is arranged in such a way that $n$-bit PWM resolution is obtained using an $n_c$-bit counter and $n_d$-bit delay line, where $n=n_c+n_d$. In PWM generation, the most significant bit (MSB) of the duty cycle is determined by the counter-based part and the least significant bit (LSB) is controlled by the delay-line based part. Moreover, the reverse operation can be applied to determine LSB and MSB of the duty cycle [5], [39].

![Figure 26 Hybrid digital pulse width modulation](image)

Hybrid digital PWM offers higher resolution/high frequency PWM with lower clock frequency as opposed to the counter-based modulation and consumes smaller chip area as opposed to the delay-line based modulation. However, it still falls short in terms of obtaining high frequency/high resolution PWM signal due to its discrete nature, which requires very sharp filters to reduce harmonics and switching glitches [5], [39], [40].
The simulation work in [41] proposes a digital PWM operation with 2 MHz frequency and 4-bits resolution requiring 32 MHz clock frequency. Another simulation work in [42] offers a PWM generation with a frequency of 500 kHz with 11-bits resolution operating at 16 MHz clock frequency. The measurement results of the digital PWM controller IC presented in [39] reveals a digital PWM with 8-bits and 1 MHz frequency running with a clock frequency of 8 MHz. The work in [34] is a FPGA implementation of the hybrid digital PWM and experimental results show a PWM signal with 10-bits resolution and 780 kHz frequency.

Table 4 summarizes the results obtained from several works based on the digital implementations of the PWM.

<table>
<thead>
<tr>
<th>Method</th>
<th>[43]</th>
<th>[38]</th>
<th>[41]</th>
<th>[42]</th>
<th>[39]</th>
<th>[34]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWM frequency</td>
<td>1 MHz</td>
<td>1 MHz</td>
<td>2 MHz</td>
<td>500 kHz</td>
<td>1 MHz</td>
<td>780 kHz</td>
</tr>
<tr>
<td>Resolution</td>
<td>6 bits</td>
<td>11-bits</td>
<td>4 bits</td>
<td>11 bits</td>
<td>8 bits</td>
<td>10 bits</td>
</tr>
<tr>
<td>Clock frequency</td>
<td>1 MHz</td>
<td>1 MHz</td>
<td>32 MHz</td>
<td>16 MHz</td>
<td>8 MHz</td>
<td>25 MHz</td>
</tr>
</tbody>
</table>

### 3.2 Analog Implementation

The implementation of the PWM in analog domain can occur by using an integrated circuit or discrete circuit specified for PWM. The major advantage of the analog implementation compared to the digital one is improved resolution, theoretically infinite, while one of its major design challenge is to generate the carrier signal (either saw-tooth or triangle wave) with good linearity and high frequency [44].

Implementing PWM on discrete level requires many active and passive components. The use of many circuit components results in reduced accuracy due to their tolerances and serious area consumption compared to IC level design. One way of discrete level analog implementation of PWM is based on the circuit realization of the naturally-sampled double-edge PWM. The circuit in [8] is an example of this implementation
and its schematic is shown in Figure 27. It basically involves an error amplifier, a triangle wave generator, and a comparator. The critical issue is to determine the frequency of the triangle wave, which gives the frequency of the PWM. In [8], the frequency of the triangle wave $f_{tri}$ is calculated as

$$f_{tri} = \frac{R_6}{4R_7R_5C_3}.$$  \hfill (16)

The tolerance of capacitor $C_3$ and resistors $R_5$, $R_6$ and $R_7$ are directly responsible for the frequency error of the triangle wave and PWM. The measurement results of the circuit in [8] shows that the circuit generated a triangle wave with a frequency of 477 kHz, where the frequency value is set to 500 kHz in the design. Thus, the circuit dramatically suffers from the frequency error.

Another approach for analog implementation is the phase-locked loop (PLL) based PWM generation. This type of PWM is widely used in power-line and wireless applications, where the PWM is used to drive the power stage of a class-D power
amplifier [7]. The work in [44] is an integrated class-D PA with PLL-based PWM driver and it is presented in Figure 28.

![Figure 28 A Class D power amplifier with PLL-based PWM driver [44]](image)

In PLL-based PWM generation, the phase of a VCO (voltage-controlled oscillator) is obtained by converting the difference between the output signal and the input signal. Then, a phase detector generates the PWM signal as an output by comparing the phase of VCO with that of a reference clock. The major advantage of the PLL-based PWM generation over the triangle or saw-tooth based PWM generation is the replacement of a high-linearity carrier signal with a reference clock, which does not have any requirement on its linearity. This situation gives a way to high frequency PWM operation [7]. On the other hand, the major problem of the PLL-based PWM generation is its harmonic characteristics due to the non-linearity of VCO. As we have discussed in Chapter 2.4, there are many harmonic components in the frequency spectrum of the PWM, which is generated by an input signal with the frequency of $f_{in}$ and a carrier signal with the frequency of $f_c$. The non-linearity of VCO results in further intermodulation harmonics of the spurs related to input and carrier signals. As an example, there are spurs at $f_c+nf_{in}$ and $f_c+(n+2)f_{in}$ in the spectrum of the PWM by its nature. However, the second harmonic distortion of VCO results in down-converting of these spurs into the baseband and appearing as a distortion at $2f_{in}$. Similarly, the third harmonic distortion of VCO can cause that spurs at $f_c+2nf_{in}$ and $2f_c+nf_{in}$ generate third-order intermodulation distortion [44]. In other words, PLL-based approach introduces extra harmonics caused by the non-linear behavior of the VCO in addition to the harmonics related to the input and the carrier signals. These unwanted spurs will be located at baseband. Thus, in order to reduce the harmonic distortion at the baseband, PLL-based PWM ICs are forced to work at higher frequencies, around 100 MHz range even if the input signal frequency is around 10 kHz range. However, the switching loss of a power MOSFET is directly proportional to the switching frequency
(PWM frequency). As the PWM frequency rises to several 10 MHz range, the switching losses of the power MOSFETs increase reducing the efficiency of the system to impractical values.

The work in [45] is an analog implementation of the double-edge uniformly-sampled PWM in IC level. The block diagram of the circuit is shown in Figure 29. The circuit involves a sample/hold circuit to perform uniform sampling, a triangle wave generator and a comparator.

![Sample/Hold Circuit](image1)

![Comparator](image2)

![Triangle Wave Generator](image3)

Figure 29 Analog uniform sampling PWM implementation [45]

For triangle wave generation, the idea of taking integral of a square wave is used, which is implemented with an OPAMP circuit as it is shown in Figure 30. Moreover, the integration is based on charging and discharging of a capacitor with a time constant of $RC$.

![OPAMP integrator circuit](image4)

Figure 30 Ideal OPAMP integrator circuit

The major problem of the integration-based triangle wave generation is the linearity. Also, the slew rate of the OPAMP will limit the operation for high frequency PWM operation.
3.3 Evaluation

The works in the literature about PWM implementation have been covered based on the implementation domain as analog and digital. The design challenge of the digital implementation of PWM is to satisfy high resolution/high frequency PWM operation. However, the digital PWM always suffers from the lack of resolution since it is inherently a discrete solution. Moreover, limited resolution results in reduced accuracy and there occurs additional delays related to sampling and processing times [31], [46]. Considering the analog domain, the discrete level solutions require many active and passive components, which dramatically affect the PWM operation due to their tolerances. Moreover, they consume too much circuit area. PLL-based PWM generation has many unwanted harmonics in their baseband due to the VCO in its circuit topology and they are not preferred especially for audio and SONAR applications. Integration-based PWM implementation is insufficient in terms of generating the linear triangle wave otherwise higher harmonic distortion is obtained.

The next chapter is dedicated to the design, analysis and implementation of the presented high frequency pulse width modulation integrated circuit. The presented PWM IC offers an architecture, that generates the triangle wave with constant current charging/discharging based method so that triangle wave can be obtained with high linearity. It performs naturally-sampled PWM in analog manner eliminating the resolution problem as will be detailed in the following section.
CHAPTER IV

IMPLEMENTATION

In this section, the implementation of the PWM IC is presented. In Chapter 4.1, the design specifications of the PWM IC are determined. Chapter 4.2 covers the design and the circuit analysis of the PWM IC. Its circuit architecture is provided starting with the major blocks down to the sub-blocks. The simulation results of the key sub-blocks are summarized. Chapter 4.3 provides the full-chip layout and the methods that have been followed in the physical design of the PWM IC. In Chapter 4.4, the top-level simulation of the PWM IC is given.

4.1 Design Specifications of the PWM IC

As expressed in Chapter 2.5, the presented PWM IC employs the unipolar double-edge naturally-sampled pulse width modulation. The selection of the modulation type determines some of the specification by itself. The unipolar modulation type requires a single supply operation whereas, the double-edge modulation type requires the generation of a triangular wave as the carrier. The natural sampling indicates that the modulation is required to occur at the same time instant with the sampling of the input signal by the carrier signal. Extra features are considered to make the PWM IC be a candidate for many applications, as will be listed later in this section. The fundamental specifications and features determined for the PWM IC are listed as follows:

4.1.1 High Frequency PWM Operation

The maximum operating frequency of the PWM IC is specified as 5 MHz considering the trend of high frequency operation in the SMPSs and the class D PAs. Another driving point is the SONAR applications, where the high frequency operation (up to 1 MHz) is required [17]. It is obvious that the requirement of generating a PWM signal
up to 5 MHz frequency inherently implies the generation of a triangle wave with 5 MHz frequency. This should also be accomplished by the PWM IC.

4.1.2 Two Output Generation

In the SMPSs and the class D PAs, the PWM signal is fed to a gate driver chip, which drives the switching MOSFETs. Gate drivers being capable of operating at several MHz are available in the market. These products are designed in such a way that they need both a PWM signal and its complement as the inputs to operate [47], [48]. Considering this situation, the PWM IC is required to generate a PWM signal and its complement up to 5 MHz frequency as the outputs.

4.1.3 Adjustable PWM Frequency

The PWM IC is required to generate the PWM signal with adjustable frequency up to 5 MHz. This feature gives an opportunity to the PWM IC for being applicable in various areas having different PWM frequency requirement.

4.1.4 Synchronized Operation

The PWM IC in design should have external synchronization capability. This is required in building phased-array SONAR systems. As it is covered in Chapter 2.2.3, the PWM is used as the driving method in the class D PAs. Many phased-array active SONAR systems use class D PAs in their transmitters. Very similar to the phased-array systems used in the wireless communication and RADAR systems, the acoustic waves generated by the phased array can be electronically steered without moving the transducers. By this way, the beams are targeted to different directions by adjusting the relative phases of transducer elements, without the need of a mechanical movement. In such applications, the accurate controlling of the phases is very important. For example, when the inputs of the PAs are in phase, the time delay between outputs of the PAs should theoretically be zero. If there is a time delay between the outputs of the PAs, there will be a phase shift between them even if no intentional phase shift is given. This situation will degrade the accuracy of the controlling of the phased array. In order to have phase matched outputs from multiples of class D PAs in a phased-array system, the PWM signal driving them should also be in phase. Although it is obvious that there will be some phase-error contribution due
to the fabrication tolerances, generation of PWM signals with the best possible phase matching is important.

Considering two AC sinusoidal signals as in Figure 31, the expression for the phase shift $\Delta \phi$ between them is calculated as

$$\Delta \phi = 360^\circ \times f \times \Delta t,$$

(17)

where $f$ is the frequency of the signals, and $\Delta t$ is the time delay between signals.

Figure 31 Two AC signal with a time delay of $\Delta t$

There is no magic number for the maximum tolerable phase error, since it is a system-dependent parameter, which will change with respect to a number of elements, such as the order of the phased array system. For this application, it was decided to have a maximum of $1^0$ of a phase shift error at a frequency of 500 kHz. In other words, when in-phase AC input signals are applied to several PWM ICs, the coded PWM outputs should have a maximum time-delay skew of at most 5.5 ns.
4.2 Design and Analysis of the PWM IC

By definition, a PWM signal is generated by comparing an input signal with a high frequency carrier signal. The proposed PWM IC follows this basic idea. The block diagram of the presented PWM IC is shown in Figure 32. The PWM IC consists of two main blocks; the comparator, and the triangle wave generator. The triangle wave generator has three sub-blocks; current-reference circuit, the constant source/sink currents generator circuit, and the charge/discharge control circuit. The PWM IC requires an external clock signal and the input coding signal to output the coded PWM signal and its complement. It also requires one off-chip resistor and one off-chip capacitor to operate. The reason for the usage of these two external components will later be discussed.

![Figure 32 The block diagram of the presented PWM IC](image)

4.2.1 Triangle Wave Generator

The triangle wave generator is required to generate a high frequency analog triangle wave with adjustable frequency so that a PWM signal can be obtained with adjustable frequency. For this purpose, the PWM IC employs a method based on the periodic charging/discharging of a capacitor with a constant current. In this method, an external clock signal at the desired operation frequency is used to control the steering mechanism of the constant sink/source currents. This approach can be understood better in the light of the basic equation of the current-voltage relation of a capacitor which is shown as
\[ I = C \frac{\Delta V}{\Delta t}. \quad (18) \]

The parameters participated in (18) have individual meanings for the developed methodology. In (18), \( I \) refers to the constant sink/source current, which will discharge/charge an off-chip capacitor with a capacitance of \( C \). \( \Delta V \) is the peak-to-peak amplitude of the triangle wave, which is denoted as \( C_m \) in Chapter 2.4. \( \Delta t \) corresponds to the half of the PWM (or triangle wave) period which is adjusted by the external clock signal. Equation (18) is obtained as

\[ C_f \frac{f_{PWM}}{2 \Delta V} = \frac{I}{2 \Delta V}, \quad (19) \]

if it is written in terms of the frequency of the PWM signal which is denoted as \( f_{PWM} \).

To generate the triangle wave with user-adjustable frequency, it is required to obtain a relation between the parameters in (19). In case of the values of \( I \) and \( \Delta V \) are held constant, the multiplication of \( C \) and \( f_{PWM} \) is equal to a fixed value. Thus, a simple relation between the external clock frequency and the off-chip capacitance provide the method of generating a triangle wave (or PWM) with adjustable frequency. The values of \( I \) and \( \Delta V \) are chosen to be 1 mA and 1 V, respectively. Thus, the multiplication of \( C \) and \( f_{PWM} \) is set to 0.0005 meaning that if the frequency of the PWM is selected as 5 MHz, the off-chip capacitor should have a capacitance of 100 pF. Some examples of this developed method are tabulated in Table 5. It is obvious that this method requires the employment of an external capacitor. In order to have accurate triangle wave generation, capacitors with low tolerances are used.

**Table 5 The method for adjustment of the PWM (or triangle wave) frequency**

<table>
<thead>
<tr>
<th>Desired PWM(or triangle wave) Frequency</th>
<th>Required External Clock Frequency</th>
<th>Required Off-chip Capacitance</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 MHz</td>
<td>5 MHz</td>
<td>100 pF</td>
</tr>
<tr>
<td>1 MHz</td>
<td>1 MHz</td>
<td>500 pF</td>
</tr>
<tr>
<td>100 kHz</td>
<td>100 kHz</td>
<td>5 nF</td>
</tr>
</tbody>
</table>
The circuit realization of the method for the triangle wave generation with adjustable frequency is shown in Figure 33. The constant source/sink currents $I_{source}$ and $I_{sink}$ are steered by controlling the switches $S_1$ and $S_2$. If the timing of the switches can be arranged with respect to an external clock signal, a triangle wave can be generated with desired frequency by adjusting the frequency of the external clock signal.

Figure 33 The circuit realization of the method for triangle wave generation with adjustable frequency

The charge/discharge control circuit is basically the implementation of the switches $S_1$ and $S_2$ shown in Figure 33. The time management of the charge/discharge control circuit is provided by the external clock signal and its complement. The circuit schematic of the charge/discharge control circuit is shown in Figure 34. It consists of four transmission gates $TG_1$-$TG_4$, and two on-chip resistors $R_1, R_2$.

Figure 34 The circuit schematic of the charge/discharge control circuit
The clock-controlled charging/discharging mechanism works as follows: when the external clock is low, $TG_2$ will be “ON” resulting in charging of the off-chip capacitor, and $TG_3$ will be “OFF” disabling the discharge. When the external clock is high, $TG_2$ will be “OFF” preventing the charging, and $TG_3$ will be “ON” enabling the discharging path of the off-chip capacitor. Besides, his control circuit ensures the peak-to-peak value of the triangle wave to be equal to 1 V as follows: When the external clock is low, $TG_4$ will be “ON” and it will allow the sink current to flow through $R_2$, which will set the voltage of node $H$ to 3 V. This determines the top-peak value of the triangle wave. Similarly, when the external clock is high, $TG_1$ will be “ON”, and the source current will flow through $R_1$ setting the voltage of node $G$ to 2 V. By doing this, a triangle wave with a peak-to-peak amplitude of 1 V is obtained, which is required with respect to the method developed for triangle wave generation with adjustable frequency.

The charge/discharge control circuit will bring another important result. It generates the triangle wave with a peak-to-peak amplitude of 1 V, and a DC offset of 2.5 V. In case of an input signal composed of AC and DC components as in (6), its DC value $R_0$ should be set to 2.5 V for full-range operation. Thus, the value of $R_0$ and $C_m$ shown in (13) are set to 2.5 and 1, respectively, making the term $2D (=R_0/C_m)$ be equal to 5, which is an odd number. Since $2D$ is an odd number, the term $\sin[(2Dm+n)\pi/2]$ will be equal to zero when $m$ and $n$ are both even or odd numbers. As discussed in Chapter 2.4, the PWM IC should satisfy to achieve lower harmonic distortion as in bipolar modulation.

It is critical to design the transistors employed in the transmission gates by considering their switching performance. In case of a MOSFET used as a switch, it will be in the triode (linear) region when the switch is “ON”, and it will be in the cut-off region when the switch is “OFF”. In the triode region, a MOSFET behaves like a resistor, which is called as on-resistance [49]. It can be expressed as

$$R_{on} = \frac{1}{\mu_n C_{ox}(W/L)(V_{GS}-V_{th})},$$

(19)

where $\mu_n$ is the mobility of electrons, $C_{ox}$ is the gate oxide capacitance, $W$ is the width of a MOSFET, $L$ is the effective length of a MOSFET, $V_{GS}$ is the gate-to-source voltage
of a MOSFET, and $V_{th}$ is the threshold voltage of a MOSFET. Besides, $W/L$ is called as the aspect ratio of a MOSFET.

When current passes through a MOSFET in the triode region, a voltage drop across its drain-source terminals occurs, which is equal to the multiplication of the on-resistance and the drain current. This voltage drop can be reduced by lowering the on-resistance value, which can be achieved by increasing the $W/L$ ratio.

The switching performance of a MOSFET is related to its input capacitance. Larger input capacitance limits the switching performance of a MOSFET, and the input capacitance is proportional to $W/L$ ratio. Thus, there is a trade-off between the on-resistance and the switching performance of a MOSFET. The circuit design and optimisation is performed considering this trade-off.

Another sub-block of the triangle wave generator is the constant source/sink currents generator. This circuit is required to generate 1 mA of constant source/sink currents, which will charge/discharge the off-chip capacitor. The circuit uses the basic idea of mirroring a current reference to generate the constant source/sink currents. For copying the current reference, a basic current mirror circuit is designed as is shown in Figure 35.

![Current mirror circuit for generation of the source and sink currents](image-url)
The source current is generated by the current mirror established by the transistors $M_2$, $M_3$, and the sink current is mirrored from the reference current with the help of the transistors $M_4$, $M_5$. Ignoring the channel length modulation, the ratio between the source/sink currents and the reference current are expressed as

$$\frac{I_{\text{sink}}}{I_{\text{reference}}} = \frac{(W/L)_5}{(W/L)_4}$$

(20)

$$\frac{I_{\text{source}}}{I_{\text{reference}}} = \frac{(W/L)_3}{(W/L)_2}.$$  

(21)

It is obvious that this circuitry will suffer from the channel length modulation in practice. This effect will cause an error in the mirroring process [49]. Considering the current mirror formed by $M_4$ and $M_5$, the ratio between the sink current and reference current can be written as

$$\frac{I_{\text{sink}}}{I_{\text{source}}} = \frac{(W/L)_3}{(W/L)_4} \frac{1+\lambda V_{DS5}}{1+\lambda V_{DS4}},$$

(22)

where $\lambda$ is the channel length modulation parameter, $V_{DS4}$ and $V_{DS5}$ are the drain-to-source voltages of $M_4$ and $M_5$, respectively. To minimize the copying error between the reference current and the sink/source currents, it is obvious that the drain-to-source (or source-to-drain) voltages of the transistors participated in the current mirrors should be equal. By this way, the effect of the channel length modulation can be eliminated. For this purpose, the current mirror circuit in Figure 35 is improved to the circuit as shown in Figure 36. The improvement is achieved by three feedback circuits; two of them are for the upper current mirrors, and the other one is for the lower current mirror. The main idea is the equalization the source-to-drain voltages of $M_2$, $M_3$, and the drain-to-source voltage of $M_4$, $M_5$ with the help of the feedback circuits. The feedback circuit for the upper current mirrors includes an OPAMP and a PMOS transistor, while the lower feedback circuit consists of an OPAMP and an NMOS transistor. The feedback circuits create a negative feedback, and the OPAMP will force the non-inverting input voltage to be equal to the inverting input voltage. Thus, the voltages of nodes $B$, $C$, and $D$ will be equal, and similarly the voltages of node $E$ and $F$. By the help of this feedback mechanism, the channel length modulation effect is minimized and the copying error in the current mirrors is reduced.
The design of the OPAMP used in all of these the feedbacks requires some points needed to be considered. An OPAMP in a negative feedback configuration forces the non-inverting input voltage to be equal to the inverting input voltage. The equalization of the inputs is exactly satisfied when an OPAMP has infinite open-loop gain. In reality, an OPAMP has a finite gain, which results in a voltage error between two inputs. [42], [44], [46]. To minimize the error between the inverting and the non-inverting inputs, the open-loop gain should be as high as possible. Besides, the stability of the OPAMP is another parameter that should also be considered for a proper operation.

For the OPAMP used in the feedback circuit, a two-stage OPAMP topology with Miller compensation is chosen since high gain with improved stability can be provided [42], [47]. The circuit schematic of the OPAMP used in the feedback circuit is shown in Figure 37.
As can be seen in Figure 38, the open-loop gain of the designed OPAMP is 88.6 dB and its phase margin (PM) is 61.2°, which points a stable operation. These plots show the stability of the OPAMP in a unity gain configuration. This is however, a conditional stability, for which the stability is only guaranteed for a given set of loading (input and output) loading. To check the stability of all feedback loops, stability analysis (stb) along with the stability probe (iprobe) instance provided by Cadence Spectre was used. To ensure stability over process, temperature and voltage variations, a Monte Carlo simulation of the stability analysis was applied to the OPAMPS $OP_1$, $OP_2$, and $OP_3$ for -40°C, 27°C, and 125°C temperature values. The Monte Carlo simulation has been run for 200 samples by including both process and device mismatches. Since there are nine results for three OPAMPS, it is sufficient to illustrate the worst-case phase margin for each OPAMP.
For $OP_1$, the worst PM is obtained for 125°C. As can be seen in Figure 39, the mean value is 62.85°, where the standard deviation is 0.404°C.

The worst PM of $OP_2$ has a mean value of 62.69° with a deviation of 0.369° for 125°C as illustrated in Figure 40.
The worst phase margin with $57.12^\circ$ mean and $0.370^\circ$ deviation is obtained for $OP_3$, where the temperature is $125^\circ C$, and the result can be seen in Figure 41.
As the results shown in Figures 39, 40, and 41 suggest, the OPAMPs employed in three feedback circuits satisfy a stable operation.

Another sub-block of the triangle wave generator is the current reference circuit, which is required to generate 100 µA of reference current. For successful triangle wave generation, it is mandatory to generate the constant source/sink currents robust against process, voltage, and temperature (PVT) variations, which makes the use of a reference current be necessary. The current reference circuit should be designed in such a way that it can be integrated to the rest of the triangle wave generator circuit as it is shown in Figure 42.

For the current reference, the circuit topology shown in Figure 43 is used. This is basically a circuit that converts a voltage to a current. This topology is adaptable for the rest of the triangle wave generator since it sources a current that can be integrated into the upper current mirror. The resistor shown in the schematic is an external high accuracy resistor. The system uses an external resistor to generate a very accurate reference current. It is quite easy to create stable reference voltages in integrated circuit.
implementations using bandgap references as will be detailed later in this section. However, since the generation of current depends on resistors, which have very large variations in integrated circuit implementations (typically +/- 10%), it is impossible to get stable current generation without trimming. On the other hand, with the usage the circuit defined in Figure 43, stable current generation can be achieved, thanks to the availability of accurate external thin-film resistors. The $I_{\text{ref}}$ value for this implementation was chosen as $100 \, \mu\text{A}$. The value of the resistor $R$ can easily be calculated since the top node of the resistor is forced to the value of the $V_{\text{ref}}$ voltage by the feedback.

To be able to generate a reference current using this circuit, a band-gap reference circuit is required to provide the $V_{\text{ref}}$ voltage. In operation, an OPAMP forces its non-inverting input voltage to be equal to the inverting input voltage so that the reference current can be generated through a resistor and a PMOS transistor.

![Diagram of the current reference circuit](image)

Figure 43 The current reference circuit

With the integration of the current reference, the schematic of the triangle wave generator is shown in Figure 44.
The value of the $V_{\text{ref}}$ voltage should be calculated back from the circuit schematic above, while ensuring all transistors in taking part in the mirroring safely stays in the saturation regime all the time. In operation, the voltage of node $G$ is set to 3 V during the charging of the off-chip capacitor by the control circuit. Considering the upper part of the circuit in Figure 44, the voltages of the nodes $B$, $C$, and $D$ will be equal to the output of the band-gap reference. The output voltage of the band-gap reference should be arranged in such a way that enough saturation voltage headroom is provided for the transistor $M_8$ as shown in Figure 44. For this purpose, the output of the band-gap reference is set to 3.5 V, which means that the value of the resistor is automatically determined as 35 KΩ to satisfy the reference current of 100 µA.

Figure 45 shows the basic principle of a band-gap reference [50]. The idea behind this principle is to compensate the negative temperature dependence of the base-to-emitter voltage of a bipolar transistor with the positive temperature dependence of the thermal voltage [49], [51].

Figure 44 The triangle wave generator with the current reference
The fundamental circuit topology for the voltage-mode current reference is shown in Figure 46.

For this circuit, the output expression is given as

$$V_{\text{ref}} = V_{BE2} + (V_T \ln n) \left(1 + \frac{R_2}{R_3}\right).$$

In (23), the positive temperature coefficient (TC) comes from the thermal voltage $V_T$ while the base-to-emitter voltage $V_{BE}$ has a negative TC. Thus, zero TC can be obtained by arranging the term $(1+R_2/R_3)\ln(n)$ [49]
Since a voltage-mode band-gap reference is unable to provide a output voltage larger than 1.2 V, it is crucial to employ a current-mode band-gap circuit, which can be designed to satisfy the output voltage of 3.5 V [51]. The current-mode band-gap reference circuit implemented in the PWM IC can be seen in Figure 47 [52].

![Figure 47 The implemented current-mode band-gap reference](image)

The circuit shown in Figure 47 consists of two parts; the band-gap core, and the start-up circuit. The start-up circuit prevents setting the voltage of node X to zero when the supply voltage is given and the operation is initialized for the first time. After the initialization, the band-gap core will be activated and generate the desired output voltage.

The start-up circuit is designed in such a way that it will be active when the voltage of node X is zero, and it will be disabled from the band-gap core when the voltage of node X is greater than the turn-on voltage of the transistor Q1. The start-up circuit has four diode-connected transistors M1-M4 so that a fixed voltage will be created in the gate of M5 through M1-M4. This voltage is set to a specific value, which will be enough to make M5 start conducting current in case of the voltage of node X being zero. Thus, the band-core will be forced to start-up even if the voltage of node X is zero.
The operation of the band-gap core is based on the cancellation of the negative temperature dependence of the base-to-emitter voltage of a bipolar transistor with the positive temperature dependence of the thermal voltage [52]. The expression of the reference voltage $V_{\text{ref}}$ is given as

$$V_{\text{ref}} = R_4 \left( \frac{V_{BE1} - V_{BE2}}{R_1} + \frac{V_{BE1}}{R_2} \right), R_2 = R_3,$$  \hspace{1cm} (24)

where $V_{BE1}$ and $V_{BE2}$ are the base-to-emitter voltages of $Q_1$ and $Q_2$ vertical npn bipolar transistors, respectively, and $R1-R4$ are the on-chip resistors.

As in the feedback circuit of the constant sink/source currents generator shown in Figure 36, the OPAMP is a critical part of the band-gap reference circuit as well and it also requires high open-loop gain since it is in a negative feedback configuration. According to the work in [53], an open-loop gain in the order of 50 dB are enough to satisfy good temperature compensation. For the OPAMP of the band-gap reference, the OPAMP circuit used in the feedback circuit of the constant sink/source currents generator is also employed with minor changes since high open-loop gain with stability is provided.

The input offset voltage of an OPAMP is another parameter, which seriously affects the performance of a band-gap reference. The main reason is that the input offset voltage is process-dependent. If a mismatch occurs in fabrication of the differential input transistors, the bias currents flowing through these transistors will differ creating an offset voltage between the inputs. To be able to minimize the effect of the offset voltage, the transistors in the differential input pair should have large widths as much as possible. Also, device matching should be satisfied by applying required layout techniques such as common-centroid method [49].

For analyzing the input offset voltage of the OPAMP, a Monte Carlo simulation has been run for 200 samples taking impact of both process and device mismatches. As the result in Figure 48 suggests, the mean value of the input offset voltage is -96 µV, where the standard variation is 4.64 mV.
Figure 48 Monte Carlo simulation results of the input offset for the OPAMP used in the band-gap core

The accuracy of the band-gap reference circuit is very critical since its output is used to generate the current-reference. Figure 49 shows the Monte Carlo simulation result of the band-gap reference. This simulation includes both process and device mismatches for 200 samples. As can be seen in Figure 49, the mean value is 3.49481 V with 0.115 V of standard variation.

Figure 49 Monte Carlo simulation results of the band-gap reference
After the design of the band-gap reference circuit is finished, we can conclude the design of the current reference shown in Figure 43. The selection of 35 KΩ resistor is critical in terms of obtaining more accurate current reference. The lowest TC resistor available in the process used for the design of the PWM IC is 170 ppm/°C. On the other hand, a high precision and stable off-chip resistor can be used for more accurate operation. An off-chip resistor with a TC of 25 ppm/°C and 0.1 % tolerance is chosen for this purpose [54]. Figure 50 shows the results of the PVT variation analysis of the current reference, where the off-chip resistor is modeled with respect to the selected product. The reference current exhibits a TC of 35.3 ppm/°C over a temperature range from -40°C to +125°C as suggested by simulation results shown in Figure 50.

![Figure 50 Variation of the reference current over different PVT conditions](image)

It is critical to check the stability performance of the OPAMP of the current generator similar to the OPAMPs of the feedback circuits employed in the constant source/sink currents generator circuit. For this purpose, a Monte Carlo analysis has been applied to the OP4. The worst phase margin is obtained with a mean value of 45.34° and a deviation of 0.97° for 125°C as can be seen in Figure 51.
After the design of the current-reference circuit is finished, the design of the triangle wave generator is completed. The full schematic of the triangle wave generator is shown in Figure 52, where the OPAMPs used in the circuit remain as a symbol due to limited illustration area.

Figure 52 The final version of the triangle wave generator

Figure 53 and 54 show the simulation results of the feedback circuits participated in the triangle wave generator. In this simulation, the frequency of the clock applied to the circuit is 5 MHz. The circuit includes all parasitic effects, including the wire-bond...
inductors. For the upper current mirrors, the voltages of nodes $B$, $C$, and $D$ are kept at 3.5 V level, which is the output of the band-gap reference. Also, the lower current mirror is achieved to equalize the source-to-drain voltages of $M_6$ and $M_7$ by the help of its feedback circuit.

![Figure 53](image1)

Figure 53 Equalization of the voltages of nodes $B$, $C$, and $D$ by the feedback circuit

![Figure 54](image2)

Figure 54 Equalization of the voltages of nodes $E$ and $F$ by the feedback circuit

Figures 55 and 56 show the simulation results for the clock-controlled triangle wave generation for 1 MHz and 5 MHz frequencies, respectively. As the results suggest, the charging of the capacitor occurs when the clock signal is low while the capacitor
discharges when the clock signal is high. Thus, the method developed for the external clock controlled triangle wave generation is verified on the simulation level.

Figure 55 External-clock controlled triangle wave generation with 1 MHz frequency

Figure 56 External-clock controlled triangle wave generation with 5 MHz frequency
4.2.2 Comparator

The comparator used in the PWM IC is required to compare the reference input signal with the generated triangle wave. Two of the most used topologies used in the literature are the static comparators and dynamic comparators. The static comparators can be considered as continuous-mode comparators while the dynamic comparators as clocked-mode comparators. The dynamic comparators are mostly used in A/D converters and memory circuits, where continuous operation is not required [55]–[57] [58]. In dynamic comparators, the comparison is only started when they are triggered by a clock signal. By this way, low power circuits can be designed. On the other hand, The static comparators are open-loop based implementations [59], [60].

Propagation delay of a comparator is a critical parameter that defines the speed of a comparator. For a comparator used in PWM generation, the propagation delay should be much smaller than the period of the PWM. As a rule of thumb, a ratio of (1/10) yields a good design [8]. Another critical parameters are the open-loop gain, the slew rate, and the input offset [61].

The two-stage OPAMP without compensation is a good candidate for a high speed comparator if the power consumption is not a concern. It can provide high DC gain and low input offset [58], [59], [62]. This circuit can tolerate small capacitive loads but it will face slew-rate limitation when it is required to drive larger capacitive loads. For this purpose, a further improvement can be applied to the two-stage comparator so that larger capacitive load can be driven. Improvement of driving large capacitive loads is provided with the tapered buffer topology [63], [64], [65]. The tapered buffer is a circuit with a chain of inverter stages. Jaeger in [66] stated that minimum propagation delay can be achieved for a tapered buffer if each transistor in a stage has a width, where it is fixed multiple of that of transistors participated in the previous stage. The ratio between the widths of the transistors in each stage is constant and it is called as taper factor denoted as $F$. The idea of the $N$-stage tapered buffer is shown in Figure 57.
The equation for the taper factor is expressed as

\[ F = \left( \frac{C_L}{C_i} \right)^{1/N}, \]  

(25)

where \( C_i \) is the input capacitance of the first inverter (minimum size), \( C_L \) is the load capacitance, and \( N \) is the number of inverter stages. The tapered buffer is designed so that it can drive capacitive loads up to 10 pF, where \( N=4 \) and \( F=6 \).

The schematic of the comparator with tapered buffer is illustrated in Figure 58. The comparator has a tapered inverter at its output since the PWM and its complement are required as the outputs of the PWM IC.

Figure 58 The circuit schematic of the comparator

Figure 59 shows the propagation delay simulation of the comparator. The circuit is simulated for the worst case scenario, which is 5 MHz operation when it drives a capacitive load of 10 pF. The delay is the time interval between the time instants of \( (V_{IH}+V_{IL})/2 \) and \( (V_{OH}+V_{OL})/2 \), where \( V_{IH} \) and \( V_{IL} \) are the high and low levels of the input voltage, and \( V_{OH} \) and \( V_{OL} \) are the high and low levels of the output voltage,
respectively. The propagation delay occurs at both low-to-high transition and high-to-low transitions of the output. The propagation delay can be calculated by taking average of the rising and the falling values. The average propagation delay is calculated as 3.48 ns, which is much smaller than the lowest period of the desired PWM operation (200 ns for 5 MHz PWM operation).

Figure 59 Propagation delay of the comparator

4.3 Layout

The full-chip layout of the presented PWM IC including the pad-frame is shown in Figure 60.

Figure 60 The microphotograph of the PWM IC
In the layout of the PWM IC, common standard analog layout techniques have been followed and implemented for obtaining better results. The device matching using common-centroid layout techniques has been applied for the differential pair of the OPAMPs, the transistors in the current mirror, etc. For improved performance, dummy transistors and resistors are used in the layout design. In the band-gap reference, the vertical npn transistors is selected as 1:15 (1:n) so that 4x4 array can be obtained for better layout.

4.4 Top-level Simulations

The top-level simulations of the PWM IC have been performed with full-chip parasitic extraction including pad frame and all bond-wire effects. The performance of the PWM IC for 5 MHz operation is illustrated in this chapter since it is the most challenging scenario for the PWM IC.

Figure 61 shows the 5 MHz PWM operation of the presented PWM IC with 500 kHz sinusoidal input signal.

Figure 61 5 MHz operation of the PWM IC with 500 kHz sinusoidal input signal
Another simulation performed is the duty cycle variation of the PWM signal with respect to the adjustment of the input signal. For this analyzation, a DC input signal has been applied to the PWM IC, and its value has been changed to determine the lowest and highest duty cycle obtained. As the simulation result in Figure 62 suggest, the PWM IC can generate 5 MHz PWM signal with its duty cycle changing from 95 % to 5 %.

![5 MHz operation of the PWM IC with 50 %, 95 %, and 5 % duty cycles](image)

In this section, the design process of the PWM IC has been covered by considering the design specifications. Many simulations including Monte Carlo and process corner analysis have been extensively carried out to be able to ensure its proper operation before the chip has been fabricated. The next chapter covers the measurement results that have been applied to the PWM IC so that its functionality can be verified.
CHAPTER V

PERFORMANCE EVALUATION BASED ON EXPERIMENTAL RESULTS

The PWM IC presented in Chapter 4 has been fabricated in a commercial 0.35 µm CMOS process. Figure 63 shows the microphotograph of the fabricated chip. The full-chip area is 1.8 mm x 1.4 mm including the pad frame. All pads are ESD protected. ESD rating of the chip is yet to be evaluated, but expected to be Class-2A or higher.

Figure 63 The microphotograph of the PWM IC

The integrated circuit packaging is an important step considering its effect on the performance of the integrated circuit. The bonding wires, connecting the pads on the die to the package pins, result in parasitic inductances. These bond-wires have large impact on the performance of the circuits, especially when carrying high frequency, high current switching signals. To minimize the parasitic effects of the wire-bonds, the length of them should be kept as small as possible [67], [68]. Another parasitic
influence results from the leads of the package whose length should be reduced as well. For the package of the presented PWM IC, Quad Flat No-Leads (QFN) package was chosen. QFN packages offer minimized parasitics thanks to their leadless structure. Another advantage of the QFN package is its exposed paddle, which is generally connected to the ground plane of a PCB by metal vias. It brings improved thermal performance acting as a heat sink by decreasing the thermal resistance [69]. Besides, through the use of wire-bonds directly connected to the exposed paddle (a.k.a. down-bonds), the QFN package helps reduce the total parasitic inductance of the circuit ground plane. To keep other wire-bonds as short as possible, the smallest QFN package that can accommodate the die was chosen. For this work, a 5 mm x 5 mm QFN package with 16 pins shown in Figure 64 is used.

![The fabricated PWM IC in the QFN package](image)

Figure 64 The fabricated PWM IC in the QFN package

In order to test the functionality of the presented PWM IC, a test printed circuit board (PCB) was designed and fabricated. A picture of the test PCB populated with the PWM IC and all necessary components is shown in Figure 65. It is a two-layer board with the dimension of 5 cm x 5 cm x 1.6 mm. As its schematic can be seen in Figure 66, the
test PCB contains the supply voltage filtering part to filter out the AC components from the DC voltage supply, and the PWM IC.

Figure 65 The test PCB for the PWM IC

Figure 66 The schematic of the test PCB
To correctly validate the performance of the PWM IC, it is important to design the test PCB carefully by applying some design and layout rules. One is the decoupling on the PCB level in addition the decoupling on the IC level. To decouple the high-frequency noise in the supply pins of the PWM IC, decoupling ceramic capacitors $C_8$, $C_{10}$, $C_{11}$, and $C_{12}$ have been used. It is critical to locate these capacitors very close to the IC’s supply pins to minimize the self-inductance of the PCB trace. Another issue is the ground connection. It is also required to make short connections from the ground pins to the ground plane to be able to minimize the self-inductance due to the PCB traces. A large ground plane prevents large voltage drops and satisfies all ground connections having the same reference potential.

In order to measure the PWM IC, various tests were performed as will be detailed in the following sub-sections. In all of these tests, the PWM IC was used to drive either a single or a multitude of class D power amplifiers. These PAs are populated on the same PCB as shown in Figure 67. As can be seen from the figure, Class D PAs are proceeded by appropriate filtering constructed by L type networks. The design of these networks is beyond the scope of this work and will not be explained. However, it is sufficient to say that, they help to reconstruct the input signal by filtering out the harmonics at the output of the PAs.

Figure 67 The test setup with required hardware
Figure 68 shows all the test and measurement equipment for the measurement setup. The DC supplies are used to provide power to the PWM IC and Class D PAs, the function generators are used to generate the external clock and the PWM input coding signal, and the scope is used to plot the output waveforms.

Figure 68 The test setup with the required equipment

5.1 Triangle Wave Generation

The presented PWM IC has been tested for the evaluation of the clock-controlled triangle wave generation. The measurement results for 1 MHz and 5 MHz operation are shown in Figure 69 and 70, respectively. These results are taken by using a scope and replotted on MATLAB software for better visualization. These measurement results illustrate the external clock signal applied to the PWM IC, and the generated triangle wave. As can be seen in Figure 69 and 70, the method developed for the generation of the triangle wave with adjustable frequency is verified. When the external clock signal is low, the charging of the off-chip capacitor occurs while the discharging is enabled when the external clock signal is high. The measured peak-to-peak amplitude of 1 MHz triangle wave is 980 mV and its DC offset is deviated by 30
mV from the ideal operation (2.5 V offset). On the other hand, 5 MHz triangle wave has been generated with a peak-to-peak amplitude of 940 mV and its DC offset is deviated by 80 mV from the ideal operation.

Figure 69 Triangle generation for 1 MHz operation

Figure 70 Triangle wave generation for 5 MHz frequency

Another test applied to the PWM IC is the validation of the synchronized triangle wave generation. For this purpose, an external clock signal is applied to two PWM ICs and the generated triangle waves of two chips is probed. Figure 71 shows two synchronized
triangle waves with a frequency of 5 MHz, and the time delay between two signals is measured as 800 ps. Please note that a DC offset variation of 50 mV and a peak-to-peak amplitude variation of 40 mV occur between two generated triangle waves, which is related to component tolerances and process variations.

![Graph showing synchronized triangle wave generation of two chips for 5 MHz](image)

**Figure 71** Synchronized triangle wave generation of two chips for 5 MHz

### 5.2 PWM Generation

The PWM generation has been tested for both AC and DC input signals. The PWM operation with an AC input signal measurements have been performed by considering the class D power amplifier applications. As we have covered in Chapter 3, DC-DC converters use the PWM in their control loop so that they compare a DC error voltage with a carrier signal to regulate their output voltage. Thus, the PWM operation with a DC input signal has also been experimented. Besides, the operation with a DC input signal has been used to determine the range of the duty cycle variation by adjusting the DC input signal. The results are demonstrated for frequencies of 1 MHz and 5 MHz.

Figures 72 and 73 show the duty cycle variation of the PWM signal with respect to a DC input signal. The top-level simulation of the PWM IC have suggested a duty cycle variation range from 5% to 95% for up to 5 MHz frequency. On the other hand, the measured applicable range of duty cycle variation for 1 MHz PWM operation is
between 8% to 92% while the measured values for 5 MHz PWM operation is between 20% to 80% as can be seen in Figures 72 and 73, respectively. The discrepancy is related to the slew rate of the comparator. There are additional capacitive loading effects at the output of the comparator, such as PCB traces, cable connections, and the input capacitances of the probe. They all limit the capacitive load driving capability of the comparator.

**Figure 72** The duty cycle variation range for 1 MHz PWM operation

**Figure 73** The duty cycle variation range for 5 MHz PWM operation
The PWM operation has also been tested for an AC input signal. Figure 74 shows the results of 5 MHz PWM operation including a sinusoidal input signal with a frequency of 1 MHz and a peak-to-peak amplitude of 200 mV. The PWM operation can be roughly verified by this figure in such a way that the PWM signal switches from low level to high level when the value of the triangle wave is greater than the value of the input signal, and vise-versa. However, more detailed analysis on this data is required to strengthen the verification of the PWM operation.

\[ V_{PWM,pp} \times \left( R_1 \right) \times \left( \frac{1}{C_m} \right) \]  

where \( R_1 \) is the amplitude of the input signal and \( C_m \) is the peak-to-peak amplitude of the generated triangle wave. Considering the case in Figure 74, the ideal operation requires a fundamental component with an amplitude of 0.5 V since \( C_m \) is set to 1 V, \( V_{PWM,pp} \) is 5 V, and \( R_1 \) is equal to 0.1 V. For better evaluation, the measured result is compared with the ideal case implemented in MATLAB software. Figure 75 shows
the amplitude spectrum comparison of the generated and the ideal PWM signals for the case in Figure 74. The amplitude of the fundamental component is 0.53 V, which is larger than the 0.5 V because the generated triangle wave has a peak-to-peak amplitude less than 1 V compared to the ideal case. Another important observation is related to the IM harmonics at the frequency of 4 MHz. For the ideal operation, the IM at the frequency of 4 MHz is zero as expected while there is a harmonic component in practice due to the deviation in the peak-to-peak amplitude of the generated triangle wave compared to the ideal case. However, these IM harmonics are not problematic in the operation since they are away from the fundamental component thanks to the high frequency operation. Thus, they can be filtered out with appropriate filtering.

![Amplitude Spectrum Comparison](image)

Figure 75 Spectral analysis of the PWM operation in Figure 74

Another measurement has been configured to be able to validate the synchronized performance of the PWM IC. For this purpose, an external clock signal with a frequency of 5 MHz and a coding input signal of 500 kHz have been connected to two PWM ICs while the generated PWM signals have been probed. To analyze the results,
the generated PWM signals have been digitally filtered in MATLAB software and the time delay between the modulated inputs signals have been measured. Figure 76 illustrates the filtered PWM signals generated by two synchronized PWM ICs. The time delay between the two signals is measured as 4.8 ns, where this value is specified as 5.5 ns in the design specifications. Please note that there is a deviation of 13 mV in the peak-to-peak amplitudes of two signals, which is related to the component tolerances and the process variations.

![Figure 76 The filtered PWM signals of two PWM ICs in synchronized operation](image)

The propagation delay of the PWM IC is another parameter that have been measured. Figure 77 shows the result of propagation delay measurement, where the PWM frequency is set to 5 MHz and the input signal is arranged to a DC value of 2.5 V. The rising-edge propagation delay is measured as 15.6 ns and the falling-edge propagation delay is measured as 22 ns, where the simulation values are 2.98 ns and 3.98 ns, respectively for 10 pf capacitive load. This deviation is related to the extra capacitive loading, which limits the slew rate of the comparator. On the other hand, the average propagation delay is measured as 18.8 ns, which is much smaller (1/10) than the period of the 5 MHz PWM signal.
Another design specification for the PWM IC is the generation of the PWM signal and its complement up to 5 MHz frequency. Figures 78 and 79 show the PWM and its complement generation for 5 MHz and 1 MHz, respectively. The major difference between the 1 MHz and 5 MHz operations is that the PWM IC falls short in slew rate for 5 MHz operation compared to 1 MHz operation.

Figure 77 Propagation delay measurement of the PWM IC for 5 MHz operation

Figure 78 PWM signal and its complement for 5 MHz frequency
5.3 Integration with the Class D Power Amplifier

The integration of the PWM IC with a class D power amplifier has been established and a resistive load has been connected to the output of the class D power amplifier. The class D power amplifier has an L type network, which is a matching circuit consisting of an inductor and a capacitor. It can provide filtering since it is a resonant circuit and it has a band-pass filter shape frequency response. The design of the L type network and the class D power amplifier are not covered in this work.

The PWM frequency is set to 5 MHz and a single-tone sinusoidal input signal with a frequency of 500 kHz is applied to the PWM IC. The operation of the class D power amplifier has been observed for the input signals with the peak-to-peak amplitudes of 100 mV, 200 mV, and 400 mV. It is verified that the output of the class D power amplifier performs a linear increase as the peak-to-peak amplitude of the input signal is doubled as can be seen in Figure 80.

Figure 79 PWM signal and its complement for 1 MHz frequency
Figure 80 The output of the Class D power amplifier for an input signal with different amplitudes

To be able to validate the external synchronization capability of the PWM IC in a class D power amplifier, a measurement setup has been arranged with two identical PWM ICs and class D power amplifiers as can be seen in Figure 67. The outputs (both PWM signal and its complement) of the two PWM ICs, which are both driven by the same external clock signal are connected to the PWM inputs of the two identical class D power amplifier circuits. The PWM frequency is set to 5 MHz by adjusting the external clock signal. A sinusoidal input signal with a frequency of 500 kHz is applied to the both PWM ICs. The class D power amplifiers have two identical L type networks so that the filtered input signals can be obtained and the time-delay between them can be measured.

Figure 81 shows the outputs of the two class D power amplifiers, which are synchronized by the PWM IC. The time delay between these two signals is measured as 8.9 ns. In addition to the time delay related to the PWM IC, the sources for the time delay between the outputs of the class D power amplifiers can be listed as component tolerances on both L-networks and class D power amplifiers, PCB trace mismatches, and cable connections.
Figure 81 The outputs of the two synchronized Class D power amplifiers

5.4 Power Consumption

The PWM IC operates with a single supply voltage of 5 V. The maximum power consumption of the PWM IC is measured in 5 MHz operation. Table 6 tabulates the comparison of the measurement results and the simulation results of the power dissipation for 5 MHz operation.

<table>
<thead>
<tr>
<th></th>
<th>Power dissipation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation</td>
<td>20 mW</td>
</tr>
<tr>
<td>Measurement</td>
<td>25 mW</td>
</tr>
</tbody>
</table>

In this chapter, the measurements performed to measure the PWM IC have been covered in detail. The tests performed includes the validation of the triangle wave generation, the PWM generation and its spectral response, the integration with a Class D PA, synchronization capability. Next chapter includes the conclusion of this work and the possible contribution that can be added in the future.
CHAPTER VI

CONCLUSION AND FUTURE WORK

6.1 Conclusion

In this thesis, the design and implementation of an analog high frequency pulse width modulation integrated circuit is presented. The chip is fabricated in a commercial 0.35 μm CMOS process. The selection of sampling method as the natural sampling offers excellent spectral response. Besides, this architecture uses an analog triangle wave as the carrier signal to provide small harmonic distortion. Its analog nature also offers higher resolution and less quantization error compared to the digital implementations.

The PWM IC generates a maximum of 5 MHz user adjustable PWM signal and its complement. It employs an external clock controlled architecture so that it allows the multitude of chips to operate in synchronization. Based on the results of the presented work, the following conclusions and comments can be drawn:

1. Generating the triangle wave with respect to an external clock signal is verified. The charging and the discharging cycles of an off-chip capacitor is successfully controlled by the external clock signal. By this way, the triangle wave with adjustable frequency is obtained, which results in frequency adjustable PWM operation.

2. In synchronized operation at 5 MHz frequency, the skew error between the triangle waves is measured as 800 ps, which includes variation due to the component tolerances, mismatches in the cable connections, and PCB related deviations.

3. The time delay between two coded AC input signals with up to 500 kHz frequency is measured as 4.8 ns at the output of the PWM IC. This value is
lower than the specified value, which is 5.5 ns corresponding to a phase shift of 1° in a phased array system at 500 kHz operation frequency.

4. The integration of the PWM IC and the class D PA has been verified for 5 MHz PWM and 500 kHz sinusoidal input signal. In the first test configuration, a single PWM IC has driven a single class D PA. In this test, the amplitude of the 500 kHz input signal is doubled for three different values and, the rise in the output voltage amplitude of the PA has been successfully observed. For the second test, two PWM ICs have been synchronized by an external clock signal so that the skew error between the output signals of the class D PAs have been observed. The skew is measured as 8.9 ns, which can further be reduced by minimizing the mismatch sources in the system such as cable connections and PCB layout of the class D PAs.

5. The measured average propagation delay at the output of the PWM for 5 MHz operation is 18.8 ns. This value is much smaller than the period of 200 ns (5 MHz frequency), which corresponds to the most challenging condition of the PWM IC. The measured values are larger than the simulation results due to capacitive loading effects mostly in the PCB traces and cables, which are not taken into account during simulations.

6. The measured applicable duty cycle variation range is from 20% to 80% for 5 MHz PWM operation, and from 8% to 92% for 1 MHz PWM operation. The decrease in the ranges between 1 MHz and 5 MHz occurs due to the slew-rate limitation of the comparator. In practice, the output of the comparator faces with extra capacitive loading effects, such as PCB traces, cables, the measurement signal probe. Minimizing these capacitive sources will yield better results.

7. In the triangle wave generation, there is around 50 mV deviation in the peak-to-peak amplitude, and the DC offset of the triangle wave from the ideal case due to the process variations and the component tolerances. Due this discrepancy, the PWM IC falls short in terms of cancellation of the first order IM harmonics \((m=1, n=\pm1)\). However, this situation can be overcome thanks to the high frequency operation. These harmonics can easily be filtered out so that they do not result in the harmonics distortion at the output of the class D PA.
A performance comparison between the presented PWM IC and other analog-based implementations can be made. The digital implementations are excluded from the comparison since they suffer from low resolution.

As can be seen in Table 7, LTC6692 of Linear Technology, USA [9] is the closest alternative of the PWM IC presented in this work. Its major advantage over the proposed PWM IC is its duty cycle variation range. On the other hand, it suffers from the frequency error since the frequency is set by an off-chip resistor. Besides, its maximum PWM frequency is 1 MHz, and it falls short compared to the PWM IC in terms of the external synchronization ability.

Table 7 The comparison of analog-based pulse width modulators

<table>
<thead>
<tr>
<th>Pulse Width Modulator</th>
<th>Circuit Type</th>
<th>Operating Frequency</th>
<th>External Synchronization</th>
<th>Duty Cycle Variation (1 MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[8]</td>
<td>Discrete</td>
<td>500 kHz</td>
<td>None</td>
<td>-</td>
</tr>
<tr>
<td>[9]</td>
<td>Integrated</td>
<td>1 MHz</td>
<td>None</td>
<td>0% - 100%</td>
</tr>
<tr>
<td>Proposed Work</td>
<td>Integrated</td>
<td>5 MHz</td>
<td>Yes</td>
<td>8% - 92%</td>
</tr>
</tbody>
</table>

In conclusion, the analog high frequency PWM generation, and the external clock-controlled architecture of the presented PWM IC make it be a strong candidate in PWM-based applications such as the switch-mode power supplies and the class D power amplifiers.

6.2 Future Work

In this thesis, an analog high frequency pulse width modulation integrated circuit with external clock synchronization capability is implemented by satisfying many of its specifications. As a future work, the design of the comparator can be revised to be able to increase its slew rate so that the duty cycle variation range can be increased. Besides, a trimming structure can be added to the circuit to minimize the process variation dependence of the generated triangle wave.

For further improvement of the presented PWM IC, a digital-to-analog converter (DAC) circuit can be designed as the digital front-end. Most of the SONAR systems
employ special signals such as linear frequency modulated (LFM) and hyperbolic frequency modulated (HFM) to locate and classify the underwater targets. These signals are usually generated in an FPGA or a DSP. Thus, with the development of an integrated DAC, the PWM IC will have a digital interface, which brings an important simplicity to the systems.
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