EXPLORING SECONDARY MATHEMATICS TEACHERS' CONCEPTIONS OF MATHEMATICAL LITERACY

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

EXPLORING SECONDARY MATHEMATICS TEACHERS' CONCEPTIONS OF MATHEMATICAL LITERACY

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The purpose of this research study was to investigate secondary mathematics teachers' conceptions of mathematical literacy. In doing so, the following three crucial interrelated issues were addressed: (i) teachers' conceptions of what the notion of mathematical literacy means, (ii) teachers' conceptions of how the development of mathematical literacy could be facilitated, and (iii) teachers' conceptions of what a mathematics curriculum emphasizing mathematical literacy should look like.

The data gathered for this study were done by means of semi-structured interviews in three distinct stages with 16 in-service mathematics teachers from various types of secondary schools during the spring semester of 2014-2015. In order to capture more comprehensive understanding of teachers' conception of mathematical literacy, each interview was also supplemented by researcher-designed tasks. The content analysis of data began with the data collection process with the help of the qualitative data analysis software NVivo10. The coding scheme was initially developed relying on the conceptual framework and research questions and then was expanded, and refined, in accordance with an ongoing analysis of the data.

The results of this study indicated that teachers' conceptions of the concept of mathematical literacy were identified as the nature of mathematical literacy, fundamental mathematical capabilities for mathematical literacy, and relationships between mathematics and mathematical literacy. Secondly, teachers' conceptions of the effective development of mathematical literacy were classified as barriers to development of mathematical literacy, and central domains for mathematical literacy development. Finally, teachers' conceptions of mathematics curriculum emphasizing mathematical literacy were categorized as challenges in curriculum implementation in the context of mathematical literacy, and recommended curriculum modifications in relation to mathematical literacy. Based on the research findings, teachers' competence and disposition toward mathematical literacy, early mathematical literacy development, and mathematics curriculum emphasizing mathematical literacy understanding were reported to be of paramount importance for effective mathematical literacy development.

Keywords: Mathematics Education, Mathematical Literacy, Teachers' Conceptions, Secondary Mathematics Teachers

ÖZ

LİSE MATEMATİK ÖĞRETMENLERİNİN MATEMATİK OKURYAZARLIĞINA İLİŞKİN KAVRAYIŞLARININ İNCELENMESİ

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Bu çalışmanın amacı lise matematik öğretmenlerinin matematik okuryazarlığına ilişkin kavrayışlarını araştırmaktır. Bunu yaparken, birbiriyle ilişkili üç önemli husus ele alınmıştır. Bu hususlar sırasıyla; (i) matematik okuryazarlığı kavramına ilişkin öğretmen kavrayışlarının incelenmesi, (ii) etkili matematik okuryazarlık gelişimine ilişkin öğretmen kavrayışlarının incelenmesi ve (iii) matematik okuryazarlığına vurgu yapan matematik öğretim programına ilişkin öğretmen kavrayışlarının incelenmesidir.

Bu çalışmada veriler 2014-2015 bahar döneminde çeşitli okul türlerinde görev yapan 16 lise matematik öğretmeni ile üç farklı aşamada yapılan yarı-yapılandırılmış görüşmeler yoluyla elde edilmiştir. Öğretmenlerin matematik okuryazarlığı hakkındaki kavrayışlarını daha derinlemesine incelemek amacıyla, her görüşme araştırmacı tarafından tasarlanmış görevlerle desteklenmiştir. Veri analizi nitel veri analizi yazılımı olan NVivo 10 yardımıyla içerik analizi yöntemi kullanılarak veri toplama süreci ile birlikte başlatılmıştır. Kodlama düzeni başlangıçta kavramsal çerçeve ve araştırma sorularına dayanarak geliştirilmiş daha sonra veri analizinin ilerleyen süreçlerinde daha ayrıntılı bir işleme tabi tutularak genişletilmiştir.

Yapılan analizler sonucunda, matematik okuryazarlığı kavramına ilişkin katılımcı öğretmenlerin kavrayışları; matematik okuryazarlığının doğası, matematik okuryazarlığı için gerekli temel matematik yetenekleri ve matematik ve matematik okuryazarlığı arasındaki ilişki olarak üç kategori altında toplanmıştır. İkinci olarak, etkili matematik okuryazarlığı gelişimine ilişkin katılımcı öğretmenlerin kavrayışları; matematik okuryazarlığı gelişimi önündeki ana engeller ve matematik okuryazarlığı gelişimini sağlayan merkezi alanlar olarak iki kategori altında irdelenmiştir. Son olarak, matematik okuryazarlığına vurgu yapan matematik öğretim programına ilişkin katılımcı öğretmenlerin kavrayışları; matematik okuryazarlığına vurgu yapan matematik öğretim programı önündeki zorluklar ve matematik okuryazarlığı bağlamında önerilen öğretim programı değişiklikleri olarak iki kategori altında ele alınmıştır. Bu çalışmanın bulguları, öğretmenlerin matematik okuryazarlık eğiliminin ve yeterliliklerinin, erken matematik okuryazarlık gelişiminin ve matematik okuryazarlığına vurgu yapan matematik öğretim programının etkili matematik okuryazarlık gelişimi için büyük önem taşıdığını göstermiştir.

Anahtar Kelimeler: Matematik Eğitimi, Matematik Okuryazarlığı, Öğretmen Kavrayışları, Lise Matematik Öğretmenleri

This work is dedicated to my beloved family

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ABBREVIATIONS

AAMTV	:	Australian Association of Mathematics Teachers
AMESA	:	Association for Mathematics Education of South Africa
BSA	:	Basic Skills Agency
DBE	:	Department of Basic Education.
DES	:	Department of Education and Skills
DfES	:	Department for Education and Skills
DoE	:	Department of Education
HMI	:	Her Majesty's Inspectorate
KOM	:	Competencies and the Learning of Mathematics
MEB	:	Milli Eğitim Bakanlığı
NCTM	:	National Council of Teachers of Mathematics
NIACE	:	National Institute of Adult Continuing Education
OECD	:	Organization for Economic Cooperation and Development
PDF	:	Phase Dimension Framework
PISA	:	Programme for International Student Assessment
QCA	:	Qualifications and Curriculum Authority
TALIS	:	Teaching and Learning International Survey
TIMSS	:	Trends in International Mathematics and Science Study
UK	:	United Kingdom
UNESCO	:	United Nations Educational, Scientific and Cultural Organization

CHAPTER I

INTRODUCTION

Mathematics has become increasingly important the modern life and its usefulness in all its aspects has always been perceived as important for our personal, educational, occupational, social life and, in general, everyday activities (DfES, 2004). Mathematical skills needed in daily activities are something that no one can avoid. It is indeed very vital for any person to count, estimate, weigh, measure, or recognize numbers in order to sustain life or survive in society. This follows the necessity for any individual to have the ability to appreciate and understand information which is presented in mathematical terms. Mathematical skills, for example, enable people to pay for any purchase, to shop from the supermarket, to pay the bills or to make a shopping list at the minimum amount of cost (Evans, 2000a; Willis, 1992). To take another example, for making travel planning or booking reservation, mathematical skills allow people to read, understand and translate simple graphs, charts, geometric figures and timetables, tell the time, work out journey times and duration (Coben et al., 2003; Cockcroft, 1982). Moreover, individuals have to develop practical skills to deal effectively with time and money by using their mathematical and reasoning ability. They should be able to communicate information on taxes, interest rates and expenditure as well as to calculate discounts and interest rates in order to handle their own finances (Benn, 1997; Dingwall, 2000; Ness et al., 2007; Paulos, 1988). The list provided above continues to expand by including almost all the basic skills needed for an individual to survive in this modern world. These skills can vary over time and places, but what certainly remains unchanged and essential through all the time and places is to have enough confidence needed to apply mathematics in practical everyday situations in the right manner and to give the necessary response to any encountered problem in context (Benn, 1997; Cockcroft, 1982; Kemp, 2005; Steen, 2001a). Otherwise, if people show lack of understanding in certain mathematical ideas and procedures faced with, then they can simply feel helpless and incapacitated to some extent in their daily lives. Besides, although there are many developments connected with technological advances in every part of the world, basic mathematical qualifications and the ability to apply them in working life seem as inevitable as ever (Bynner & Parsons, 2000; Hoyles et al., 2002; Morgan et al., 2004). Therefore, mathematics also provides one with the ability to reason from given data, to cope with probabilistic situations, to think algorithmically and discretely, and, in general, to take part in decisions involving quantitative matters in an informed and effective way (Mavugara-Shava, 2005). Moreover, mathematics is a powerful means to communicate information and ideas in an unambiguous and concise way. It provides the ability to manipulate numbers, symbols and other mathematical objects despite its special symbolic notations in which abstract ideas are not always ready to obtain easily (Cockcroft, 1982; Kemp, 2005; Orton, 1994). In addition, mathematics clearly helps people enhance their capacity of reasoning so that they can think more logically and independently in making decisions (Cockcroft, 1982; Heymann, 2003). As Close and Oldham (2005) argue, mathematics can also be approached more as a tool and medium for managing or solving problems in people's personal, educational, and occupational lives, and generally for making sense of the world around them. It is thus not unreasonable to suggest that understanding mathematics develops understanding the world around us to a large extent.

Mathematics is useful not only for citizens individually but also for a society as a whole. It is thus very important to look at mathematics education from a critical perspective for reinforcing the democracy into the society (Niss, 1994). In a paper concerning mathematical literacy, Steen (1999) contends that "to develop an informed citizenry and to support a democratic government, schools must graduate students who are [mathematically literate] as well as literate" (p.8). Hence, the modern society asserts that the purpose of mathematics education is to produce people whose mathematics is meaningfully embedded in real-life contexts, that is to say, the focus of mathematics instruction is to educate people to become mathematically literate (Woodbury, 1998). In this sense, the Department of Education (DoE) (2003) emphasizes that mathematical literacy provides people with awareness and understanding of the role that mathematics plays in the world. It enables learners to develop both competence and confidence to think numerically and spatially in order to interpret and critically analyze everyday situations and solve problems. So, internationally, mathematical literacy understanding has begun to be considered as an integral part of the mathematics curriculum to provide the ability to apply mathematical knowledge and skills in everyday life. Accordingly, many countries have attempted to make an extensive mathematics curriculum review in an effort to shape the mathematics instruction to the extent that individuals can meaningfully construct their own personal mathematical understanding through investigating, communicating and discussing their own ideas (Botha, 2011; Mavugara-Shava, 2005). For example, in the USA, the National Council of Teachers of Mathematics (NCTM) (1989) held that instruction in mathematics should have been considerably revised. They pointed out that this revision would bring about mathematics education that would likely to make people to become mathematically literate “both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied to diverse fields” (NCTM 1989, p.1). Another example of this major mathematics curriculum review may be taken from UK. The National Curriculum in UK has adapted the general aims of education in the mathematics context by considering the fact that mathematics is very important in the modern society. This has formed the basic structure of the outline of the general aims of the National Curriculum (QCA, 2009). The paper of the National Curriculum (QCA, 2009) describes mathematics as an international language that provides learners with necessary qualifications and opportunities to organize and plan their lives. Thus, it has a clear objective to develop secure people, who communicate confidently and believe in themselves. Namely, its purpose is to create well-informed and confident individuals that become mathematically literate and can effectively undertake problems in real-life situations as well as in their workplaces. In other words, the general idea behind these curriculum revisions is to produce citizens who are well-prepared to use their mathematical knowledge and skills in real-life situations and who are supplied with criticality and financial capabilities to be able to survive in today’s technologically-based information society (QCA, 2009).

Correspondingly, many countries are currently rising to the challenge of shaping their mathematics education to generate students who are adequately prepared to apply mathematics for understanding important issues of life and solving daily problems efficiently, namely, who are mathematically literate. This emphasizes the importance of the cultivation of a strong understanding of mathematics in everyday life as well as the concern for investigations of the abstract world of mathematics (OECD, 2013a). The construct of mathematical literacy lays particular stress on the need to improve students' capacity to make use of mathematics in context, and it is thus essential that instruction of mathematics in schools must be significantly revised so that students can have rich experiences in their mathematics classrooms to accomplish to become mathematically literate (OECD, 2013a). In this sense, the *Programme for International Student Assessment* (PISA) was set up to create a trend for looking at how well students could use mathematics to deal with a range of problems and situations encountered in daily life, rather than how much they have learned of their school mathematics curricula (OECD, 1999). This trend has led to a gradual move away from a more elitist and traditional perspective of mathematics as something at which only the more talented few can be successful, to one in which mathematics is considered as a subject at which everyone needs to become proficient to some degree (Mavugara-Shava, 2005). However, implementing such a mathematics curriculum perspective naturally involve some challenges due to the instructional practices, knowledge, beliefs and, in general, understanding of teachers who for a long time have considered the teaching of mathematics to be the competence of a few (Sidiropoulos, 2008). For example, early indications of teachers' understanding suggest that mathematical literacy is being seen as downgrading or inferior to mathematics (Steen et al., 2007; Tout, 2001) or as equivalent to with less mathematics (Gal, 2000). Even those who claim a deeper understanding of the nature of mathematics curriculum that attempts to create citizens who are prepared to use their mathematical knowledge and skills in order to cope with everyday issues so that it could assure critical, effective and full participation in real-life situations often do not reflect this in their practice (Fullan, 2007; Richardson, 1998; Webb & Webb, 2004; Zimmerman, 2004, 2006). Therefore, ensuring opportunities for all students to become mathematically literate requires investigations of how teachers make sense and perceive the concept of mathematical literacy and its place in mathematics education.

1.1. Statement of the problem

Mathematics seems to become more and more pervasive in our daily lives. This raises the importance of the ability to understand and interpret arguments depending on mathematical arguments in order to evaluate many of the issues that people encounter daily in advertisements, forecasts, and public policies such as armament spending, international aid, environmental issues, crime rates and health statistics. Such ability to reason with numbers and other mathematical concepts is also important to become intelligent consumers in order to make informed decisions. When people perform calculations and estimate quantities in their daily lives, they rely on this ability to tell the time and read timetables, to be able to work with common measures of length, weight, speed, acceleration, temperature, density and capacity, to buy food, clothing or season tickets for sports, play, or music, to deal with basic money matters, to check a shopping bill, to be able to read and interpret charts, diagrams, simple graphs and pictorial representations commonly shown in media, to understand statistical data and ideas of chance, to plan trips and estimate expenses, to handle funds for saving and investing, to use and understand variety of geometrical shapes in practical settings and so on (Benn, 1997; HMI, 1985; OECD, 2010; Ojose, 2011; Steen, 2001a; Venkat, 2013; Willis, 1990a). Mathematical skills are also important in the workplace, and most jobs at all levels today require people to work with basic number concepts and to apply higher-order reasoning about quantities. For example, nurses use unit conversions to verify accuracy of drug dosages; sociologists draw inferences from data to understand human behavior; biologists develop computer algorithms to map the human genome; lawyers use statistical evidence and arguments involving probabilities to convince jurors, farmers use computers to find markets, analyze soil, and deliver controlled amounts of seeds and nutrients. It seems clear that people need to apply their knowledge of mathematics to a wide range of problems that frequently occur in their everyday situations. Without some familiarity with this knowledge, everyday surroundings of people would remain incomprehensible to them. Indeed, all of the activities mentioned above fall under the heading of mathematical literacy (Bynner & Parsons, 2000; Cockcroft, 1982; Hoyles et al., 2002). It is, therefore, argued that mathematical literacy provides a perspective and a means of understanding certain mathematical ideas and procedures encountered in everyday living (OECD, 2013a; Orton, 1994; Quantitative Literacy Design Team, 2001; Stacey & Turner, 2015). For example, Evans (2000a) states that mathematical literacy is the ability to collect, employ, process and communicate numerical, quantitative, spatial, statistical information presented in various mathematical ways and to judge in accordance with the nature of the activity, thereby enabling a typical member of the culture or subculture to participate effectively and efficiently in activities that they value. It consists of being able to make a rational response to a wide range of problem situations by establishing the relations and using the appropriate mathematics in order cope with them well (Dumont et al., 2012; Kemp, 2005; Westwood, 2008). To summarize, mathematical literacy is the ability to formulate, employ, and interpret mathematical knowledge in a variety of contexts. It enables learners to develop the ability and confidence to deal with everyday issues. So, it assures critical, effective and full participation in real life (OECD, 2013a). For this reason, the search for meaningful mathematics education aiming at producing an individual who is in every way mathematically competent and literate is of main concern in many countries today. However, the significant challenge that these countries are often rising up is how to shape the mathematics instruction in schools in order to create citizens who acquire meaningful mathematical skills and are thus mathematically literate in every way (OECD, 2013a). In this respect, educational policy makers and curriculum planners in Turkey are also continuously trying to translate the educational expectations and policies into instructional practices that will eventually produce mathematically literate citizens who have necessary mathematical competence needed to engage with life's diverse contexts and situations (PISA National Preliminary Report [MEB], 2013b). However, the fact that educational reforms may not be implemented as intended is a phenomenon which is widely known (Morris & Adamson, 2010). For example, it is clear that teachers have a central role in implementing the new curriculum, but they always have a large spectrum of curriculum interpretations with respect to its goals and teaching practices (Cohen & Hill, 2000). In this respect, Jablonka (2003) looked into different international discussions on mathematical literacy and observed that the discussions mainly differ according to the user who is involved in or affected by a course of action. On the one hand, there are researchers who emphasize the formal application of mathematics by mathematicians to real-world contexts, which calls for a high level of mathematical knowledge and the ability to use and apply that knowledge (Gellert et al., 2001; Hope, 2007; Jablonka, 2003; Skovsmose, 2007). On the other hand, other researchers place more emphasis on some basic level of mathematics to allow people to take rational and well informed decisions in their everyday lives to care for their families or to contribute positively and substantially to their workplace or society (McCrone & Dossey, 2007; McCrone et al., 2008; Powell & Anderson, 2007). Therefore, teachers seem to be unsure with respect to how it can be taught and assessed, and their practices in most cases do not reflect the principles of the intended educational reforms and policies in mathematics (Handal & Herrington, 2003). Moreover, teachers' prior held beliefs and understandings may also lead to a multitude of new educational policy interpretations (Cohen & Ball, 1990; Franke et al., 2007; Romberg, 1997). Furthermore, textbook writers may have different interpretations of educational innovations and reforms for mathematical literacy understanding. That is to say, while some textbooks are structured only around the power of abstraction offering absolute truths about relations among mathematical statements or assertions, other textbooks are organized to some extent with respect to the mathematical literacy learning outcomes, which are concrete and contextual, offering contingent solutions to problems about real-life situations (DoE, 2003). Clearly, teachers receive mixed messages from policy documents, guidelines and textbook writers as to what is expected from them in their classrooms with respect to mathematical literacy. On the whole, it can be said that teachers' conceptions of the nature of mathematics are usually shaped and reinforced by "teachers' conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences" about mathematics (Thompson, 1992, p.132). Similarly, we could perhaps go a little further and ask whether teachers' conceptions of the nature of mathematical literacy are also formed and developed through their beliefs, views, and preferences about the concept of mathematical literacy. There is, therefore, increasing concern about how secondary mathematics teachers conceive or understand mathematical literacy in general because their conceptions of mathematical literacy play a significant role in forming their instructional behaviors and practices in the classrooms in order to reach the desired outcomes and meet the requirements of mathematics education in the context of mathematical literacy to a great extent (Askew et al., 1997; Botha, 2011; Fransman, 2010; Sidiropoulos, 2008).

1.2. Purpose of the study

The purpose of this research study is to explore secondary mathematics teachers' conceptions of mathematical literacy. In doing so, the following three crucial areas are addressed: (i) teachers' conceptions about what the notion of mathematical literacy means, (ii) teachers' conceptions about how effective development of mathematical literacy occurs, and (iii) teachers' conceptions about what a mathematics curriculum emphasizing mathematical literacy should look like. Accordingly, the following research questions have guided this study:

1) What are secondary mathematics teachers' conceptions of the notion of mathematical literacy?

2) What are secondary mathematics teachers' conceptions of how the development of mathematical literacy could be facilitated?

3) What are secondary mathematics teachers' conceptions of what a mathematics curriculum emphasizing mathematical literacy should look like?

1.3. Rationale and significance of the study

The literature shows us that any description of the competence in various levels of mathematical literacy is in some way based on the individuals' level of mathematical knowledge and skills (Benn, 1997; Coben, 2000; Manaster, 2001; Niss, 2015). In other words, with necessary mathematical knowledge and understanding, people can easily acquire desirable power to deal effectively with their everyday problems (OECD, 2013a; Steen, 2001b; Turner, Blum, & Niss, 2015). Mathematics is, therefore, powerful and crucial for future citizens (Hoyles & Noss, 2000). However, Benn (1997) observes that the majority of adults have probably had a traumatic school experience with mathematics. Thus, if a real-life problem is thought to involve mathematics, then it can be simply avoided and rather than performing school mathematics various alternative strategies are tried to be employed. In that respect, Ashcraft and Kirk (2001) report that there are many people who are not able to deal confidently and competently with many everyday life situations which force them to effectively use mathematics. Indeed, Noss (1991) acknowledges that many people are motivated to learn mathematics in order not only to help themselves cope with life's difficulties but also to enable themselves to make sense of their life, but:

> …mathematics, at least the mathematics of the school classroom, is typically seen as hard-edged, as a subject in which meaningless problems are posed at best about real but material objects but often about unreal and meaningless objects (Noss, 1991, p.81-82).

Moreover, Hilton (1980) argues that the features of the mathematics curriculum experienced by many people are much conducive to the spread of mathematics anxiety and avoidance. He specifies these features, as 'rote calculations', 'memory dependence', 'unmotivated problems', 'spurious applications', 'tests and authoritarianism'. These are all very real factors that inhibit the learning of mathematics in school. Therefore, as in the example quoted from a series of reflection on schooldays by Margaret Drabble, "most people leave school as failures at math, or at least feeling like failures. Some students are not even given a chance to fail" (Tobias, 1978, p.26). As a consequence, it has been accepted for a long period of time that "some people do not do well in mathematics and that some people end up not liking mathematics" (Gagne, 1983, p.10). Willis (1992) argues that while the necessary call for higher levels of mathematical literacy is based on assumptions about the intrinsic usefulness of mathematics, narrowly traditional ways of defining the subject and an implicit acceptance of the naturalness of the mathematical meritocracy, prevent any real change. In other words, traditional school mathematics curricula unfortunately do not deal consistently with all aspects of mathematical literacy (Steen, 1990). Moreover, the Quantitative Literacy Design Team (2001) observes that the problem of a disconnection of school mathematics from meaningful contexts is particularly acute and one of the major impediments to mathematical literacy in today's schools. Unfortunately, many students suffer from this disconnection, whereby it causes a striking absence of common number sense. This makes teachers' use of school-learned methods in the real world difficult because the structure of the mathematics instruction implemented in the classrooms and students' everyday lives are too disparate. As a result, students start to believe that adopting classroom practices in the real world is inappropriate, so they also even do not attempt to use school mathematics in real life. However, one of the most important things that students should gain from school mathematics is the attitude that mathematics really can help as this builds the essence of mathematical literacy which is about improving people's use of mathematics in their everyday life situations (Chapman et al., 1990). It is actually in these situations where they will be required to use mathematics in order to effectively function in today's modern society. In other words, mathematics is clearly seen to be relevant to the life of people by responding their natural curiosity and desire to understand and master world around them (Benn, 1997; OECD, 2013a; Steen, 2001a).

On the other hand, it is also important to note that the relevance of mathematics to real life is understandably important for people to do their work successfully, but placing it into real contexts is frequently seen as a general panacea (Barr, 1993) as well as the difficulty in finding how to teach mathematics so as to support adults' functioning adequately in their work and everyday lives. We can develop a variety of activities that the typical individual takes part in, but in the usual basic education or college pre-calculus courses, people's activities that they are involved in change over their lifetime because of the differences between people (Evans, 2000b). As argued by Quadling (1982), this also presents us with a curriculum problem in a way that only some people will ever use any particular piece of mathematics. For example, in his words, "engineers and navigators obviously need to know some trigonometry, a subject that is of no use whatsoever for pharmacists and bank employees. Economists need to understand statistics, but not electricians" (p.412). One should, however, not forget that obviously "few children at school can be sure what work they will do in later life" (ibid, 412). This clearly poses a big challenge for teachers who attempt to relate the curriculum to learners' contexts (Coben et al., 2003). However, learning about mathematics in the classroom is only meaningful if reflection upon mathematical relations is accompanied by personally and socially relevant situations where, as in everyday situations, mathematics becomes a tool to achieve new relevant goals (Schliemann, 1999). Accordingly, "the situation in which mathematics is used outside school gives it meaning, making mathematics outside school a process of modelling rather than a mere process of manipulation of numbers" (Nunes et al., 1993, p.30). However, mathematics curricula and pedagogy are often described as not responding adequately to the need of the citizens of today and tomorrow to meet personal and social goals (National Numeracy Review Report, 2008). That is, the process of schooling seems to encourage the idea that there is not supposed to be much continuity between what one knows outside school and what one learns in school (Resnick, 1987). Therefore, here the knowledge of mathematical literacy becomes important, not just for utilitarian or abstract purposes or goals, but within the context of students' attempts to understand their own individual and collective lives and to make their lives more meaningful and functional (Benn, 1997; OECD, 2013a).

In this sense, there is growing concern that existing mathematics education of many countries inadequately equips their citizens to use and apply mathematics effectively in different phases of their lives. This concern has resulted in a move away from a more elitist and traditional view of mathematics education to one assessing how well students can use school mathematics in realistic situations, which is referred as mathematical literacy (OECD, 2003). This move has been accelerated due to the international comparative studies such as PISA and TIMSS that measure students' performance in mathematics and the resulting pressure on nations to improve their position in international league tables (Smith, 2004; Wake, 2005). In this regard, PISA is concerned with how one becomes mathematically literate and, therefore, how to assess levels of mathematical literacy of individuals. The focus of the PISA assessment programme is on how well young people have been prepared to meet challenges and demands of everyday life or how well they can adapt their learning to the needs of daily life (OECD, 2013a). In this context, even though mathematical literacy has not been clearly and explicitly described in our existing national secondary school mathematics curriculum (MEB, 2013a), its importance has been implicitly accepted and emphasized. For example, mathematical modeling and problem solving including understanding a problem, making a plan, applying the plan, checking the accuracy and validity of the solution, generalizing the solution and establishing a new and original problem are among the most essential mathematical skills and competences that the current mathematics curriculum aims to develop. Besides, mathematical processes and the underlying mathematical capabilities such as using mathematical language and terminology correctly and effectively (communication), mathematical reasoning and proof, associating the subjects/concepts within mathematics with other areas are also highlighted to be important to give students. In addition, valuing mathematics and its learning, developing psychomotor skills, using information and communication technologies (ICT) in place and effectively are also given great priority in the existing secondary school mathematics curriculum. Despite all these highlights, according to PISA National Report (MEB, 2015), regretfully, 67.5% of our students achieved Level 2 or a lower level, compared with an OECD average of 45.5%. Level 2 is a basic minimum level of mathematics required to succeed in adult life and in future education. Students at Level 1 can complete tasks involving familiar contexts where all the relevant information is provided and questions are clearly defined. They can identify and carry out routine procedures. Students below Level 1 are unable to complete successfully the most basic PISA mathematics tasks. In Turkey, 42% of students unfortunately failed to attain Level 2, indicating, according to the OECD standards, a lack of adequate mathematical literacy skills. These students failed to demonstrate that they have baseline mathematical skills, such as the capacity to interpret and recognize situations in contexts that basically require no more than direct inference, use a single representation to help explore and understand a situation, use basic algorithms, formulae and procedures, and the capacity to make literal interpretations and apply direct reasoning (OECD, 2013a). In the PISA 2012 results, Turkey's mean mathematics score was 448 points, which was significantly different from the OECD country average of 494 points. Turkey became the 44^{th} in mathematics among 65 countries. Obviously, we are far behind in comparison with the OECD average and other participating countries. Although Turkey has involved in PISA test since 2003, we constantly witness and experience that the results have been vehemently debated more each time in the country.

Thus, there is substantial agreement among curriculum planners and educators that it is critical to have an understanding of how students comprehend mathematics and apply it to real life to provide them with the most appropriate environment and program for mathematical literacy development (Ontario Ministry of Education, 2005). In fact, some of the skills required to be mathematically literate seem to be embedded into the existing national secondary school mathematics curriculum (MEB, 2013a) and teachers need to be able to foster those skills within their teaching practices as the implementation of these practices in the classroom is the major factor influencing learning outcomes of being mathematically literate (Askew et al., 1997). There is, therefore, a great expectation from teachers to have an adequate understanding and know how to incorporate mathematical literacy understanding into their instructional practices when and where necessary (Milton et al., 2007). This brings us the importance of how crucial is the role of teachers in the planning and implementation of the mathematics curriculum for cultivating the knowledge of mathematical literacy among learners as "teachers figure as a key connection between policy and practice…and what the policy implies for instruction are both a crucial factor on their practice and at least an indirect influence on student achievement" (Cohen & Hill, 2000, p.329). However, teachers do not only implement the curriculum, but they also develop, define and reinterpret it (Thompson, 1992). Thus, teachers' beliefs, knowledge and their practices occurred within the classroom will all significantly influence students' development of mathematical literacy (Askew et al., 1997). This is "what teachers think, what teachers believe, and what teachers do at the level of the classroom that ultimately shapes the kind of learning that young people get" (Hargreaves, 1994, p.ix). Indeed, there is considerably much research on the influence of teachers' knowledge and professed beliefs on their instructional practices (Artzt et al., 2008; Ball, 1988; Ford, 1994; Franke et al., 2007; Hill et al., 2008; Liljedahl, 2008; Pajares, 1992; Peterson, 1998; Thompson, 1992), but remarkably little research exists on the influence of teachers' knowledge and beliefs on mathematical literacy development (Askew et al., 1997; Botha, 2011; Fransman, 2010; Mavugara-Shava, 2005; Sidiropoulos, 2008). On the other hand, in Turkey, despite the existence of some research studies on the comparative analysis of the PISA mathematics results (e.g., Özmusul & Kaya, 2014) and the determination of students' and pre-service teachers' mathematical literacy self-efficacy and achievement levels, and the factors affecting these self-efficacy and achievement levels (Akyüz & Pala, 2010; Anıl, 2009; Güneş & Gökçek, 2013; Özgen & Bindak, 2011; Uysal & Yenilmez, 2011; Yenilmez & Ata, 2013), there has not yet been any research done to gain some general insight into secondary school teachers' conceptions of mathematical literacy. In this sense, it is important to recognize that teachers and their instructional practices have a significant influence on the development of mathematical knowledge and skills that enable learners to effectively tackle problems encountered in daily life, the workplace, and the social, civil, political and cultural environment (Department of Basic Education, 2011). Thus, the quality of teaching received by students has the greatest impact on their outcomes and consequently on their acquisition of mathematical literacy (Aaronson, et al., 2007; Doyle, 2007; Leigh & Ryan, 2011; OECD, 2005). On the other hand, in spite of various changes and improvements in the principles and standards for mathematics instruction from rote learning to meaningful learning with critical thinking, discovery, and insight in order to better internalize and apply mathematical concepts in real life, most of the mathematics teachers still demonstrate strong resistance to integrate mathematical literacy strategies into their instructional methods and practices (Siebert & Draper, 2012). This resistance exists because many teachers may find themselves uneasy, incompetent, or unqualified about mathematical literacy and feel that they lack the necessary skills and confidence to implement mathematical literacy strategies and methods for teaching and learning of mathematics (Draper, 2008). Moreover, mathematics curriculum reforms in many countries around the world have been constantly gaining momentum to call for the development of mathematical literacy for all citizens (NCTM, 2000), but any innovation in curriculum or math instruction has hardly been effectively implemented as intended and most of the curriculum reforms have sadly failed to reach the desired success (Anderson & Piazza, 1996; Cuban, 1993; Feldman, 2000; Fullan, 2007; Smith & Southerland, 2007; Sowell & Zambo, 1997; Wilson, 1990). In other words, the success of any curriculum reform is mainly based on giving the necessary attention to teachers' conceptions of this reform or innovation movement (Handal & Herrington, 2003). However, policymakers and education authorities who are responsible for educational reforms and initiatives unfortunately fail to pay enough attention to teachers' conceptions about these reform movements and most of the innovations have been introduced or enforced through a top-down approach without consultation with teachers who are required to implement these innovative strategies (Kyeleve & Williams, 1996; Martin, 1993; Norton et al., 2002). Therefore, the findings of this research study are expected to suggest some important implications for educational strategists, curriculum planners and teacher training programs regarding the development of mathematical literacy. Besides, in approaching this issue, it is also worth noting that teachers' understanding of mathematical literacy is important as they educate mathematically literate citizens of today and tomorrow of the nation in a way that all teachers have a crucial role to play in establishing and developing the mathematical literacy skills of their students. Thus, this lack of existential research in the literature has motivated to conduct this study.

1.4. Definitions of terms

Mathematical literacy

Mathematical literacy is an individual's capacity to efficiently respond to mathematical demands of personal, social and working life as well as the capacity to adapt and conform easily to new demands and requirements in a constantly changing society that is entirely embedded with quantitative information and controlled by modern technology (OECD, 2013a). Thus, it requires an ability to formulate, employ, and interpret problem situations presented in a range of different contexts by analyzing, reasoning and communicating mathematical ideas accurately and in a satisfactory manner as a constructive, engaged and reflective citizen (OECD, 2013a).

Teachers' conceptions

The conceptions are considered as "a more general mental structure, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences, and the like" (Thompson, 1992, p.130). In this sense, teachers' conceptions of mathematical literacy are characterized as what teachers regard to be the central values and feasible goals of the secondary school mathematics curriculum, the role of students and teachers in the teaching and learning processes, effective and efficient classroom practices, suitable and viable teaching approaches and emphases, rigorous and legitimate mathematical constructs and procedures, and optimal and sustainable outcomes of instruction in the context of mathematical literacy.

CHAPTER II

LITERATURE REVIEW

The aim of this study is to investigate secondary mathematics teachers' conceptions of mathematical literacy by addressing three critical issues related to their conceptions about what the notion of mathematical literacy means, how effective development of mathematical literacy occurs, and what a mathematics curriculum emphasizing mathematical literacy should look like. This chapter is accordingly organized into four sections. The first section begins with the description of the research studies related to the notion of mathematical literacy. It is followed by the analysis of the previous studies on the development of mathematical literacy. Then, research on mathematics curriculum with emphasis on mathematical literacy is reviewed. Finally, the description of the conceptual framework of the study is given.

2.1. The notion of mathematical literacy

The first section includes (i) research studies pertaining to the meaning of mathematical literacy, (ii) international perspectives with respect to numeracy, quantitative literacy, and mathematical literacy, (iii) the review of the PISA mathematics framework (iv) previous research studies on the fundamental mathematical capabilities associated with mathematical literacy, and (v) the analysis of studies on the relationships between mathematics and mathematical literacy.

2.1.1. The meaning of mathematical literacy

The view put forward in Chapter I argues that people need to become mathematically literate to process, communicate and interpret mathematical information in a variety of contexts in order to survive in today's modern society. However, it is also argued that there is no universally accepted definition of mathematical literacy in the research literature. The concept of mathematical literacy has been much debated over many years by educators and researchers, because there is an issue of meaning, that is to say, it is not yet clear that what we mean when we talk about mathematical literacy (American Institutes for Research, 2006; Brooks et al., 2001; Castle, 1992; Coben et al., 2003; Goldenberg, 2014; Jablonka, 2003; NIACE, 2011; O'Donoghue, 2002; Sfard, 2014; Steen, 2001a; Westwood, 2008; Withnall, 1995a). For example, Withnall (1995a) argues that the term 'mathematical literacy' (what she often calls as 'numeracy') is widely used in adult basic education, but there seems to be little or no general agreement among practitioners as to what it actually means. Deciding what forms mathematical literacy skills is a considerably difficult task. This is due to the fact that no matter what level of attainment is reached there always becomes a proportion of people who are already in need of certain amount of mathematics at each level of skills. Therefore, it can be said that mathematical literacy "must remain a fluid term capable of re-conceptualization according to the contexts in which it is used and by whom" (Withnall, 1995a, p.16). Similarly, Jablonka (2003) argues that there are a number of different approaches to mathematical literacy that vary in accordance with "the culture and the context of the stakeholders who promote it" (p.76). It can be considered, on the one hand, "as the ability to use basic computational and geometrical skills in everyday contexts, as the knowledge and understanding of fundamental mathematical notions" (Jablonka, 2003, p.76). On the other hand, it is regarded "as the ability to develop sophisticated mathematical models, or as the capacity for understanding and evaluating another's use of numbers and mathematical models" (Jablonka, 2003, p.76). Coben et al. (2003), also arguing in a similar fashion, assert that mathematical literacy (what is often called as 'numeracy') has become a personal attribute very much dependent on the context in which the individual is operating and it means different things to different people according to their interest and lifestyles. Accordingly, as Coben et al. (2003) state, the issue of mathematical literacy is generally a debatable one and it is being seriously under-researched as it is "a deeply contested and notoriously slippery concept" (p.9). It can be simply seen that there exists confusion and ambiguity about the meaning of mathematical literacy. Indeed, "we have a definition, but no clear meaning" (Goldenberg, 2014, p.140). Nonetheless, although the definition of mathematical literacy is elusive, it can be said that all attempts to define mathematical literacy come to the conclusion that it is a valuable skill or competence an individual possesses to use mathematics in solving real-life contextual problems, thereby implying the empowerment of learners to meet the general demands of living in the 21st century (Education Queensland, 2007; Gellert et al., 2001; OECD, 2013a; Skovsmose, 2007).

2.1.2. International perspectives: Numeracy, quantitative literacy, or mathematical literacy

It is important to note that mathematical literacy is commonly used synonymously with numeracy and quantitative literacy (Jablonka, 2003). The term 'numeracy' was first coined in a report of the Central Advisory Council for Education known as the Crowther Report (1959), which dealt with the education of boys and girls between the ages of 15 to 18. The term was introduced in a section of the report regarding the curriculum of the sixth form so that it could help remedy some shortcomings of groups of science and arts specialists. For this reason, in this report, numeracy is considered as a way of improving communication between these two groups. So, while the arts specialists acquire the skills to become more numerate, the science specialist should also acquire the skills and techniques to become more literate. In this way, it would secure both numeracy and literacy for each group. Moreover, the Crowther Committee (1959) defines "numerate" as the mirror image of being "literate" (paragraph 398, p.269). The Crowther Committee (1959) continues to argue that "by 'numeracy' we mean not only the ability to reason quantitatively but also some understanding of scientific method and some acquaintance with the achievement of science" (paragraph 419 (e), p.282). In this respect, numeracy is regarded as "the minimum knowledge of mathematics and scientific subjects which any person should possess in order to be considered educated" (quoted in Withnall, 1995a, p.11). It is, therefore, seen that the Crowther Report (1959) lays a particular emphasis on the necessity of mathematical and scientific understanding as well as the ability to think quantitatively and avoid statistical fallacies in order to be considered 'numerate'. However, Castle (1992) points out that there is no agreement about what constitutes a minimum level of competence in mathematics which people should reach to be considered numerate in this definition. The problem arises from the vast continuum of mathematical knowledge and extensive range of contexts in which a satisfactory level of mathematical understanding and skills are required for being numerate. Castle (1992) goes on to say that, as written in the Crowther Report, familiarity with the mathematical and scientific methods in order to be considered educated “is a value-laden criterion for adult numeracy which says nothing about individual mathematical competence. Many people who are educated are also mathematically incompetent” (Castle, 1992, p.226). In a similar manner, Withnall (1995a) argues that an approach to numeracy developed by the Crowther Committee ascribes it to a discrete set of abilities in mathematics. In other words, the mastery of numeracy equally depends not only on a particular level of formal education in a specific length of time, but also on some standard methods of teaching mathematics. Therefore, mathematics teachers serve as gatekeepers of mathematical knowledge. However, such an approach to numeracy comes with its own complications “to judge what ‘fully educated’ really means?” (Withnall, 1995a, p.11). The term ‘numeracy’ is also discussed in a report on mathematics education by Cockcroft and his Committee of Inquiry (1982) called Mathematics Counts. He identifies the source of the concept of numeracy as the Crowther Report (1959) and acknowledges that the word has changed its meaning considerably since it was first coined by Crowther (1959). Cockcroft (1982) offers two dictionary definitions as evidence of this change. While the Oxford English Dictionary defines the meaning of the word ‘numerate’ as “acquainted with the basic principles of mathematics and science” (paragraph 37, p.11), Collins Concise Dictionary gives the term as “able to perform basic arithmetic operations” (paragraph 37, p.11). Cockcroft (1982) contrasts these two definitions and says that the second definition is in the spirit of the evidence reviewed by his committee. He argued that if numeracy is to be equated with “an ability to cope confidently with the mathematical demands of adult life” (paragraph 38, p.11), this definition would be too limited because it only implies an ability to perform basic arithmetic operations not an ability to make use of them confidently in commonly encountered situations. Then, probably the most quoted definition is given by the Cockcroft and his Committee of Inquiry (1982), which uses numeracy to mean the possession of two particular attributes:

- an ‘at-homeness’ with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his everyday life;
- some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease (paragraph 39, p.11).

It is worth pointing out here that these attributes must be read in the context of what Cockcroft (1982) generally considers mathematics as "a powerful means of communication" (paragraph 3, p.1). Willis (1990b) states that this is slightly ironic, because on the one hand, it is true that the capacity to critically interpret ideas and arguments either involving mathematical concepts or presented in mathematical forms is of great importance for effective participation in society, but on the other hand, it is also true that mathematical expression of ideas may indeed prevent communication for a great number of people. However, Withnall (1995a) argues that the implication of Cockcroft's definition of numeracy is that "a numerate person should have sufficient confidence to be able to appreciate and understand some of the ways in which mathematics can be used as a means of communication" (p.13). This is also evident from Cockcroft's inquiry into the mathematical needs of adult life in that his main emphasis is on "the need to have sufficient confidence to make effective use of whatever mathematical skill and understanding is possessed whether this be little or much" (paragraph 34, p.10). Similarly, Evans (1989) highlights that there are several remarkable features of Cockcroft's definition of numeracy, which are its special emphasis on "confidence, practicality and its critical potential" (p.204). Moss (1984) also agrees that numeracy is a part of people's everyday life. The author argues that each individual pursues his/her own self-interest, and thus people's needs are various and also change over time. Changes in lifestyles may, accordingly, affect people's daily routines, so numeracy is something more than basic or everyday mathematics (Moss, 1984). However, Steen (1990) points out the fact that "public emphasis on numeracy can too easily lead to specifications for minimum performance, which in turn lead to minimum accomplishment. Sometimes such campaigns feature a 'back-to-basics' approach" (p.228). For this reason, Cockcroft (1982) argues that "our concern is that those who set out to make their pupils 'numerate' should pay attention to the wider aspects of numeracy and not be content merely to develop the skills of computation" (paragraph 39, p.11). It is, therefore, important to note that the view of numeracy in the Cockcroft Report (1982) is clearly "beyond learning the arithmetic operations (although it obviously does not exclude this learning) and, consequently… does not fit at all well with any 'back to basics' movement in the teaching of numeracy" (Nunes & Bryant, 1996, p.3). Furthermore, it is also important to note that the Cockcroft report came at a time when there was an urgent need to help 'the bottom half', a group which had traditionally been given a very raw deal. The report was unreservedly utilitarian, focusing on the practical mathematical demands of employment and adult life generally (Gardiner, 2004). This is also evident from Cockcroft (1982)'s "Foundation list of mathematical topics" which includes number, money, percentages, use of calculator, time, measurement, graphs and pictorial representation, spatial concepts, ratio and proportion and statistical ideas (Cockcroft, paragraph 458, p.135-140). Cockcroft (1982), in particular, argues that the curriculum must be designed from the bottom up based on his 'Foundation list' in a sense that "school mathematics can be conceived in terms of a single curriculum 'ladder' up which all students climb, at different speeds and to different heights, with pragmatic 'numeracy-for-all' first, followed later by 'mathematics-for-those-who-insist'" (Gardiner, 2004, p.5). On the other hand, Willis (1990b) observes that the Foundation list is well-defined and consists of the mathematical knowledge that is necessary for everyone to manage real mathematical demands in everyday life, but:

> it is not at all clear that mastery of the topics in the Cockcroft foundation list produces numerate people…Nor is it clear that people would be numerate to an extent commensurate with their acquisition of that mathematical content (Willis, 1990b, p.6-7).

Nonetheless, it is important to recognize that the Cockcroft's (1982) report has been one of the most influential and wide ranging documents published in the field of mathematics education, and thereby its 'Foundation list of mathematical topics' provides the basis for the national curriculum for mathematics in schools and the adult numeracy core curriculum (BSA, 2001; Coben et al., 2003). The term 'numeracy' is therefore largely used in adult mathematics education programs because it is not only accompanied by an individual's familiarity with basic ideas in everyday mathematics and adequate number sense in order to handle data and to interpret problem statements involving mental processing in real-world contexts (de Lange, 2003; Jablonka, 2003), but it also connotes more than functional use of numerical skills to process, communicate, and evaluate the numerical information (Benn, 1997; FitzSimons et al., 1996; Gal, 2000). On the other hand, the National Council on Education and the Disciplines (Steen, 2001a) favors to use the term 'quantitative literacy' rather than mathematical literacy. It is represented by "a cluster of phenomenological categories:

quantity, change and relationships, and uncertainty" (de Lange, 2003). However, mathematical literacy can be thought of as the encompassing literacy that consists of both numeracy and quantitative literacy (de Lange, 2003). For example, Jablonka (2003) prefers to speak of the term 'mathematical literacy' "to focus attention on its connection to mathematics and to being literate" (p.77), namely it pertains "to a mathematically educated and well-informed individual" (p.77).

There are accordingly some important approaches to the concept of mathematical literacy (what is often called as 'numeracy' or 'quantitative literacy' by some researchers). For example, Johnston (1994) argues that mathematical literacy is actually more than being able to recognize, comprehend, represent and handle numbers, or even being able to achieve school or university mathematics. Mathematical literacy is critical awareness which promotes and establishes connections between mathematics and the real world. So, according to Johnston (1994), mathematical literacy is the ability "to situate, interpret, critique, use, and perhaps even create mathematics in context, taking into account all the mathematical as well as social and human messiness which comes with it" (p.34). Similarly, Tout (2001) comments that the approach to mathematical literacy is very different from the one which is just about number skills. Tout (2001) maintains that mathematical literacy is not only to interpret textual representations or symbolic notations confidently, but also to communicate, use and apply mathematics efficiently and critically in practical everyday situations to fully participate in a wide range of life roles. Besides, the following view of mathematical literacy from the Australian Association of Mathematics Teachers' report (1997) also seems to represent the arguments put forward above. In this report, it is proposed that being mathematically literate is to use mathematics effectively to meet the general demands of life at home, at work, and through participating in community and social life. Hence, as Hogan (2002) emphasizes, mathematical literacy is not about acquiring numerous decontextualized mathematical rules, principles, concepts, operations and algorithms nor about gaining mathematics knowledge for its own sake. It is about understanding and transferring mathematics to other areas of learning, life and work to get involved effectively in community and civic life. Accordingly, mathematical literacy is regarded as "contributing to the empowerment, effective functioning, economic status, and well-being of citizens and their communities" (Gal, 2000, p.ix). To put it simply, it is not only essential for survival but also for civilization (Robbins, 2002). Accordingly, mathematical literacy is in general regarded as the use of mathematics to choose, interpret, apply and communicate mathematical knowledge and skills in order to cope with everyday issues encountered so that it could assure critical, effective and full participation in civic life and decision making (Department of Education and Early Childhood Development, 2009; NIACE, 2011). Perhaps we should also point out the fact that it is not necessarily just commonly encountered situations that require mathematically literate behavior, but also new situations. Therefore, as Withnall (1995b) argues, being mathematically literate requires to "access, interpret and respond, sometimes critically, to mathematically-based information both in the immediate and in the wider environment" (p.14). This seems to confirm the idea that a mathematically literate individual is the one who responds freely by oneself to a range of situations in which mathematical ideas are dominated, and "actively using the power of mathematics rather than delegating or ignoring quantitative issues" (Gal et al., 2005, p.151). Therefore, whilst recognizing that currently the concept of mathematical literacy is heavily contested, all of the above discussions point to the conclusion that mathematical literacy consists of the knowledge and skills required to efficiently respond to mathematical demands of individuals in personal, social and working life with the ability to adapt and conform easily to new demands and requirements in a constantly changing society that is entirely embedded by quantitative information and controlled by modern technology (National Numeracy Review Report, 2008).

Moreover, Steen (1997) states that mathematical literacy has five dimensions, which are practical, civic, professional, recreational, and cultural. Steen (2001a) highlights that context is an integral part of mathematical literacy. Context and mathematical literacy are certainly indivisible and highly interrelated. Hence, mathematical literacy can be seen in every issue that is related to people's personal, social and working life. Therefore, according to him, everyone is responsible for gaining sufficient level of mathematical literacy. Steen (2001a) also points out that there a number of different definitions of mathematical literacy. While some focus on basic skills, others focus on higher order thinking. In order to narrow and clarify these various definitions, Steen (2001a) suggests more straightforward and comprehensive portrait of mathematical literacy involving the following elements: (i) Confidence with Mathematics, (ii) Cultural Appreciation, (iii) Interpreting Data, (iv) Logical Thinking, (v) Making Decisions, (vi) Mathematics in Context, (vii) Number Sense, (viii) Practical Skills, (ix) Prerequisite Knowledge, and (x) Symbol Sense.

Furthermore, mathematical literacy is categorized as critical and functional literacy by Skovsmose (2007). Functional literacy is described by competencies that an individual possesses to perform a particular job function whereas critical literacy is defined through skills in order to identify and assess the issues such as working conditions and political issues. Skovsmose emphasizes that the functional and critical literacy could take on very different meanings and interpretations depending on what context we are considering or on the context of the learner. Skovsmose also prefers to mention reflective knowledge as for mathematics instead of critical literacy. Reflective knowledge pertains to competence in figuring out how mathematics is used or could be used in simple and complex procedures or systems.

As the above arguments indicate, the meaning of mathematical literacy varies according to the purpose and context being used (Gellert et al., 2001; Goldenberg, 2014; Hope, 2007; Jablonka, 2003; McCrone & Dossey, 2007; Powell & Anderson, 2007; Sfard, 2014; Skovsmose, 2007; Westwood, 2008). Clearly, varying, yet overlapping, perceptions of mathematical literacy are held by the researchers ranging from informal mathematics involving basic mathematical skills (Department of Basic Education, 2011; McCrone et al., 2008; Powell & Anderson, 2007) to formal mathematics requiring higher-order thinking skills (Gellert et al., 2001; Hope, 2007; Jablonka, 2003). While some researchers assert that mathematical literacy involves formal application of mathematics to real-world contexts requiring a high level of mathematics knowledge and the competence to use and apply it (Gellert et al., 2001; Hope, 2007; Jablonka, 2003; Pugalee, 1999), other researchers contend that it involves some basic level of mathematics to empower people both personally and as citizens to make better informed decisions when dealing with problem situations occurring in their daily living and the workplace (Department of Basic Education, 2011; McCrone & Dossey, 2007; McCrone et al., 2008; Powell & Anderson, 2007). For example, the model of mathematical literacy constructed by Pugalee (1999) is sharply based on students' level of mathematical knowledge, which is "valuing mathematics, becoming confident in one's ability to do math, becoming problem solvers, communicating mathematically, and reasoning mathematically" (p.19). Similarly, Gellert et al. (2001) argue that mathematical literacy cannot be explained in terms of basic skills only, as it consists of mathematical problems in contexts that require attributes such as conceptual understanding of formal mathematical knowledge and problem-solving skills (Gellert et al., 2001). They believe that mathematical literacy involves "a level of mathematical understanding that goes beyond the minimal abilities of calculating, estimating, and gaining some number sense, and basic geometrical understanding" (p.59). They further state that such abilities can be promoted "by experiencing mathematical modes of thinking, such as searching for patterns, classifying, formalizing and symbolizing, seeking implications of premises, testing conjectures, arguing, and thinking propositionally, and creating proofs" (p.59), which all require sufficiently advanced levels of mathematical abstraction. Therefore, mathematical literacy is sometimes viewed just as 'survival mathematics', but "this view underestimates the importance and the power of the concept of abstraction" (Gellert et al., 2001, p.62). In a like manner, Jablonka (2003) accepts mathematical literacy in terms of higher-order mathematical skills that are applicable to all kinds of contexts. According to her, "it emphasizes higher-order thinking (developing and applying general problem solving skills) rather than basic mathematical skills" (p.81). However, Jablonka (2003) also argues that when we try to define mathematical literacy, it is certain that "it cannot be conceptualized exclusively in terms of mathematical knowledge, because it is about an individual's capacity to use and apply this knowledge" (p.78). This conception of mathematical literacy helps to create predisposition towards seeing the world through mathematical eyes. In this sense, there are also some researchers who view that mathematical literacy is essential for all individuals to meet the basic demands of everyday living. For example, McCrone and Dossey (2007) argue that the focus of mathematical literacy is on "bringing relevance and deeper understanding to mathematical learning situations that empower individuals relative to their present and envisioned needs" (p.32) rather than studying higher levels of more formal mathematics. McCrone and Dossey (2007) continue to argue that this emphasis of mathematical literacy applies to all people, not just to those who are expected to become scientists, bankers or engineers. In a similar manner, Powell and Anderson (2007) contend that mathematical literacy is important and necessary for every individual to sustain informed, functional, and effective daily living. Moreover, Department of Basic Education (2011) depicts the essential components of mathematical literacy as follows: (i) the use of elementary mathematical content, (ii) authentic real-life contexts, (iii) solving familiar and unfamiliar problems, (iv) decision making and communication, (v) the use of integrated content and/or skills in solving problems. According to Department of Basic Education (2011), mathematical literacy is limited to mathematical content which involves basic mathematical principles, procedures and skills in order to interpret numerically and statistically based events encountered in the daily living, the workplace, and the social, political and global lives of people as critical citizens. Therefore, abstract mathematical concepts are not the central focus of mathematical literacy. So, it should simply ensure that people can be "a self-managing person, a contributing worker and a participating citizen in a developing democracy" (Department of Basic Education, 2011, p.8).

2.1.3. The review of the PISA mathematics framework

The Organization for Economic Cooperation and Development [OECD] (2013a) reports that the PISA 2012 Assessment and Analytical Framework has several important sections. The first section, 'Definition of mathematical literacy', describes the formal definition of the mathematical literacy and some supplementary information to underline and analyze the aspects of this definition that are especially essential for the model of mathematical literacy in practice. The second section, 'Organizing the domain', outlines three aspects: (i) the mathematical processes and the fundamental mathematical competencies controlling those processes, (ii) the mathematical content that is intended to utilized in the assessment items, and (iii) the contexts in which the assessment items are organized. The final section, 'Assessing mathematical literacy', identifies structural issues about the mathematics assessment, including the proficiency levels for mathematics and the attitudes to be investigated that relate to mathematical proficiency.

2.1.3.1. Definition of mathematical literacy in view of the PISA mathematics framework

The focus of OECD/PISA mathematical literacy domain is concentrated upon the capacities of learners to analyze, reason and communicate mathematical ideas accurately and in a satisfactory manner while they pose, handle and evaluate problem situations in a range of different contexts. For example, under PISA (2012), mathematical literacy is defined as:

> Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens (OECD, 2013a, p.25).

According to this definition, learners as active problem solvers are required to competently deal with the three crucial processes of problem solving, which suggests the verbs 'formulate,' 'employ,' and 'interpret'.

2.1.3.2. Organizing the domain

PISA assesses the degree to which 15-year-old students can deal with mathematics efficiently and competently when faced with a challenge or a problem given in the context of real-life events and situations. In order to be able to assess mathematical literacy, PISA 2012 identifies three interconnected and complementary criteria to be used, which are the mathematical processes, the mathematical content knowledge and contexts (OECD, 2013a).

As mentioned above, the definition of mathematical literacy assumes an individual's ability to *formulate*, *employ*, and *interpret* mathematics. These three mathematical actions allow a helpful and worthwhile basis for classifying the mathematical processes. First of all, the process of 'formulate' in the mathematical literacy definition is pertaining to identifying opportunities and constraints to apply mathematical knowledge and skills to construct mathematical structure for the problem situation provided in some contextualized way. It refers to accurately diagnosing the given problem in order to administer and put formal mathematics into use for the mathematical model building of the problem situation. Thus, in this process, the translation or representation of a problem situation from a real-life system to the domain of mathematics is of paramount importance. Secondly, the process of 'employ' in the mathematical literacy definition is related to the implementation and manipulation of mathematical concepts, structures, rules, algorithms, and tools to handle mathematically-formulated problems for finding mathematical results. Employing mathematics is thus associated with performing mathematical reasoning, procedures, facts, concepts and tools to obtain a mathematical solution to the formulated model. Finally, the process of 'interpret' used in the mathematical literacy definition addresses the abilities of individuals to contemplate and think deeply about the mathematical solutions received in order to evaluate and make sense of them in the precise nature and scope of a presented problem or challenge (OECD, 2013a).

PISA 2012 Assessment and Analytical Framework also outlines mathematical content for mathematical literacy with regard to four categories that comprise diverse and various challenges and problems emerging through interaction with everyday phenomena. As an estimation of mathematical literacy level, PISA aims to assess the levels and kind of mathematics that are convenient for 15-year-old students to be able to produce firm judgment and comprehensive reasoning as well as to make well-founded decisions in order to become constructive, concerned, involved and reflective citizens. Therefore, in order to cover the mathematics domain, to enclose the significant strands of school mathematics curricula, and to satisfy the requirements for historical development in mathematics, four main content categories which are (i) change and relationships, (ii) space and shape, (iii) quantity, and (iv) uncertainty and data are used in PISA. These are called as four overarching ideas and have a strong connection with well-known curricular strands such as numbers, algebra, and geometry in intricate and overlapping ways. In other words, these four content categories with many diverse areas take a large number of topics across the mathematics curriculum into coverage. However, at the same time it is important to recognize that there is not a one to one correspondence between the topics of the curriculum and these content categories because a fine division between categories is contrary to the complex and complicated situation of any problem based on real-life context (OECD, 2013a).

An essential facet of mathematical literacy is indeed an engagement with mathematics by using and applying mathematics in a variety of contexts to deal with the daily life problems. In that respect, PISA mathematics framework categorizes contexts according to distance from individual' interests. It can therefore include contexts from directly related to private life, school life, work life and leisure sports to local community and society, scientific issues and any other more general interests. Contexts for assessment items, therefore, are selected regarding individuals' interests and lives, and the aspect of their world in which the everyday problems are placed upon them as they become involved in society as constructive, concerned, engaged and reflective citizens. Accordingly, mathematical problem situations and assessment items developed for the PISA mathematics framework are classified into the following four main contexts: (i) Personal, (ii) Occupational, (iii) Societal, and (iv) Scientific. Problems categorized in the personal context category concentrate on activities related to people's own interests and their immediate surroundings. Problems sorted for the occupational context category center on the areas of working life. Problems labelled as the societal context category concern with the issues surrounding the society and culture. Problems located into the scientific category pertain to the real-life application of mathematics and the matters about science and technology (OECD, 2013a).

2.1.3.3. Assessing mathematical literacy

The PISA framework also specifies the directions to be taken in the assessment. In addition to the paper-based components, PISA 2012 includes a computer-based assessment of mathematics, but it is optional for participating countries depending on countries' varied technological capacities. The PISA 2012 consists of open constructed-response, closed constructed-response and selected-response items. Open constructed response items require more extensive writing, or showing a calculation, and frequently include some explanation or justification from a student. Closed constructed-response items involve a more structured setting for the problem solutions that can be readily evaluated to be either correct or incorrect. Selected response items necessitate the selection of one or more responses from several response options that can be simply processed. In general, items include the application of essential mathematical knowledge and skills which are conveniently relevant for 15-year-old students' level. These items are connected with one of three mathematical processes: (i) formulate, (ii) employ, and (iii) interpret. They are also assigned to represent the mathematical content knowledge defined for the PISA mathematics framework mentioned above. Additionally, each item is placed in one of four context categories: (i) personal, (ii) occupational, (iii) societal, and (iv) scientific. The items selected for the PISA 2012 mathematics survey represent a broad spectrum of difficulties to match the wide ability range of students participating in the assessment. In this way, all the major categories of the assessment are exemplified with the items of a wide variety of difficulties. Finally, PISA mathematics framework also expresses a number of proficiency levels with the descriptions of the corresponding degree of mathematical literacy in each level. Table 2.1 below shows the descriptions of these proficiency levels reported for PISA 2012 mathematics scale.

Table 2.1: Proficiency scale descriptions for mathematics in OECD PISA Study (OECD, 2013a, p.41)

6	***At Level 6*** students can conceptualize, generalize and utilize information based on their investigations and modelling of complex problem situations. They can link different information sources and representations and flexibly translate among them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply their insight and understandings along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations. Students at this level can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments and the appropriateness of these to the original situations.
5	***At Level 5*** students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterizations and insight pertaining to these situations. They can reflect on their actions and formulate and communicate their interpretations and reasoning.
4	***At Level 4*** students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic, linking them directly to aspects of real-world situations. Students at this level can utilize well-developed skills and reason flexibly, with some insight, in these contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments and actions.

3	***At Level 3*** students can execute clearly described procedures, including those that require sequential decisions. They can select and apply simple problem-solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They can develop short communications when reporting their interpretations, results and reasoning.
2	***At Level 2*** students can interpret and recognize situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and making literal interpretations of the results.
1	***At Level 1*** students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are obvious and follow immediately from the given stimuli.

2.1.4. Fundamental mathematical capabilities associated with mathematical literacy

Mathematical competence is defined by Niss (2003a) as the ability to comprehend, evaluate, perform, and apply mathematics in a range of intra-and extra-mathematical contexts. Based on the work (known as the KOM project) of Mogens Niss and his Danish colleagues (Niss & Højgaard, 2011; Niss & Jensen, 2002; Niss, 2003b), Niss (2003a) classifies eight competencies as (i) Mathematical thinking competency which involves the ability to put forward questions and awareness of the kinds of answers that mathematics provides. It also includes the ability to recognize and manage the extent and limits of the presented mathematical concepts as well as the ability to differentiate between different kinds of mathematical expressions; (ii) Problem handling competency which consists of specifying, posing, formulating, and tackling a variety of mathematical problems in a number of different ways; (iii) Modelling competency which contains the ability to translate and interpret model components with respect to reality. It also involves the ability to work with a model by analyzing the elements of the model critically; (iv) Reasoning competency which includes the ability to follow and judge the chains of reasoning. It also involves the ability to comprehend what a mathematical proof is and how it varies from mathematical argument as well as the ability to create and implement informal and formal arguments; (v) Representation competency which consists of the ability to interpret, differentiate and translate between various forms of representations of mathematical facts, entities and ideas; (vi) Symbols and formalism competency involves the ability to handle and interpret symbolic, formal, and technical language of mathematics as well as the ability to switch between the formal mathematical system and the natural language; (vii) Communication competency which comprises the ability to express oneself about mathematical facts in either oral, written, or visual forms as well as the ability understand and interpret someone else's expressions and texts represented in a variety of ways; (viii) Aids and tools competency which includes the ability to make use of diverse types of aids and tools by recognizing their limitations and possibilities. These competencies are also divided into two major groups of four competencies. The first group refers to capability in asking and answering questions within, and through mathematics, and includes the first four competencies whereas the last four representing the second group are the ones that pertain to understanding and using mathematical language and tools.

In conjunction with the KOM project (Niss & Højgaard, 2011; Niss & Jensen, 2002; Niss, 2003a), similar ideas have surfaced elsewhere in the world (Niss, 2015). In the well respected and substantial report named as "Adding It Up: Helping Children Learn Mathematics" (National Research Council, 2001), this is perhaps obvious. The report is written by the Mathematics Learning Study Committee and edited by Kilpatrick, Swafford, and Findell. In this seminal report, it is stated that acknowledging the difficulty to find the appropriate term referring to all facets of competence, skill, expertise, and sophisticated knowledge in mathematics, the Mathematics Learning Study Committee decided to prefer mathematical proficiency to represent the characteristics of anyone to succeed in learning mathematics. According to this committee, mathematical proficiency includes five essential elements, or strands that provide one way of describing mathematical literacy:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- Strategic competence: ability to formulate, represent, and solve mathematical problems
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (National Research Council, 2001, p.116).

The evidence of the effect of KOM project around the world is even more obvious in the case of PISA. Starting from the initial published version till now (OECD 2000, 2003, 2006, 2009a, 2013a, & 2016), the PISA mathematics framework places special stress on the eight mathematical competencies of the KOM project (Niss, 2003a, 2015). For example, PISA 2012 Assessment and Analytical Framework gives a particular emphasis to this set of fundamental mathematical competencies (referred to as capabilities) that lays a solid foundation for each of the above stated mathematical processes of 'formulate', 'employ', and 'interpret', and mathematical literacy in practice. Indeed, mathematical processes that are supported by these mathematical capabilities shape the central part of the mathematical literacy description in PISA (OECD, 2013a). In other words, these mathematical capabilities through comprehensive factual knowledge form the main and actual basis for mathematical literacy (Niss, 2015). Accordingly, the seven fundamental mathematical capabilities used in PISA framework are as follows: (i) Communication, (ii) Mathematizing, (iii) Representation, (iv) Reasoning and argument, (v) Devising strategies for solving problems, (vi) Using symbolic, formal and technical language and operations, and (vii) Using mathematical tools. According to de Lange (2003, 2006), individuals need to possess the above-mentioned capabilities with varying levels in order to become mathematically literate as well as the feeling of confidence and comfort to apply mathematics and quantitative ideas. However, it does not mean that a mathematically literate person also necessarily becomes mathematically competent as all the capabilities may not be reflected in a precise manner (Niss, 2015). PISA 2012 Assessment and Analytical Framework depicts different extent of these capabilities within each of the three mathematical processes in Table 2.2 below (OECD, 2013a).

Table 2.2: Relationship between mathematical processes and fundamental mathematical capabilities (OECD, 2013a, p.32)

	Formulating situations mathematically	**Employing mathematical concepts, facts, procedures and reasoning**	**Interpreting, applying and evaluating mathematical outcomes**
Communicating	Read, decode, and make sense of statements, questions, tasks, objects, images, or animations (in computer-based assessment) in order to form a mental model of the situation	Articulate a solution, show the work involved in reaching a solution and/or summarize and present intermediate mathematical results	Construct and communicate explanations and arguments in the context of the problem

Mathematizing	Identify the underlying mathematical variables and structures in the real world problem, and make assumptions so that they can be used	Use an understanding of the context to guide or expedite the mathematical solving process, e.g. working to a context-appropriate level of accuracy	Understand the extent and limits of a mathematical solution that are a consequence of the mathematical model employed
Representation	Create a mathematical representation of real-world information	Make sense of, relate and use a variety of representations when interacting with a problem	Interpret mathematical outcomes in a variety of formats in relation to a situation or use; compare or evaluate two or more representations in relation to a situation
Reasoning and argument	Explain, defend or provide a justification for the identified or devised representation of a real-world situation	Explain, defend or provide a justification for the processes and procedures used to determine a mathematical result or solution Connect pieces of information to arrive at a mathematical solution, make generalizations or create a multi-step argument	Reflect on mathematical solutions and create explanations and arguments that support, refute or qualify a mathematical solution to a contextualized problem
Devising strategies for solving problems	Select or devise a plan or strategy to mathematically reframe contextualized problems	Activate effective and sustained control mechanisms across a multi-step procedure leading to a mathematical solution, conclusion, or generalization	Devise and implement a strategy in order to interpret, evaluate and validate a mathematical solution to a contextualized problem
Using symbolic, formal and Technical language and operations	Use appropriate variables, symbols, diagrams and standard models in order to represent a real-world problem using symbolic/formal language	Understand and utilize formal constructs based on definitions, rules and formal systems as well as employing algorithms	Understand the relationship between the context of the problem and representation of the mathematical solution. Use this understanding to help interpret the solution in context and gauge the feasibility and possible limitations of the solution
Using mathematical tools	Use mathematical tools in order to recognize mathematical structures or to portray mathematical relationships	Know about and be able to make appropriate use of various tools that may assist in implementing processes and procedures for determining mathematical solutions	Use mathematical tools to ascertain the reasonableness of a mathematical solution and any limits and constraints on that solution, given the context of the problem

2.1.5. Mathematics and mathematical literacy

It is understandably difficult to define precisely what we mean when we talk about mathematical literacy but what is clear and certain is that mathematics is explicitly mentioned in most of the definitions of mathematical literacy. For example, Benn (1997) assumes that mathematical literacy is so closely related to school mathematics in a sense that “[mathematical literacy] is to mathematics as literacy is to language” (Steen, 1990, p.211). However, mathematical literacy and mathematics are also distinguished from each other as each has a distinctive feature in terms of their nature and aims (Spangenberg, 2012). For example, Steen (2001b) points out that there are also notable differences between mathematics and mathematical literacy (what he calls 'numeracy') in a way that mathematics reveals the power of abstraction, suggesting universal and infallible truths about relations among ideal objects, but mathematical literacy reveals the power of practicality, which is factual and contextual, suggesting practical solutions to real world problems. Steen (2001b) continues to argue that mathematics is also arranged in a systematic way by categories inherited from the past, but mathematical literacy is based on the way mathematical knowledge is applied in the global information society. Besides, mathematics is seen and experienced mostly in school, but mathematical literacy is mostly confronted and experienced in the real world situations. Similarly, Manaster (2001) argues that mathematical statements or assertions are connected with relationships among abstractions, whereas inferences of mathematical literacy are mostly related to something real. Manaster (2001) also emphasizes that proofs and reasoning in mathematics are of paramount importance. In addition to this, one of the most important characteristics of mathematics is to understand why assertions based on assumptions must be true. In contrast, mathematical literacy necessarily draws on approximations and incomplete or sometimes inaccurate data to make inferences. Another argument in support of the above stated arguments is also made by another researcher stating that mathematical literacy "is less formal and more intuitive, less abstract and more contextual, less symbolic and more concrete" (de Lange, 2003, p.77). According to de Lange (2003), the focus of mathematical literacy is based on critical reasoning, sound judgment, effective data interpretation, complete reflection and as well as on other mathematical capabilities. Venkat (2007) offers similar support for the arguments given above. She highlights that although the emphasis of mathematics as a subject on its applications in real life is usually done in most of the mathematics curricula, the real emphasis of mathematics is indeed placed:

> …on abstract rather than concrete concepts, on intra-mathematical connections rather than mathematics-real-world connections, on rigor and logic rather than interpretation and critique, and on knowledge itself, as well as applications of knowledge (Venkat, 2007, p.77).

Therefore, while mathematics is considered as one of the scholarly and theoretical branches of abstract science with decontextualized ideas and mathematical formalization that require higher order cognitive skills and robust comprehension of mathematical concepts, operations, and relations, mathematical literacy is not considered as the application of pure mathematical concepts, but rather mainly the promotion of investigative and creative skills in elementary mathematical content to deal with challenges involving statistics and numeric or spatial data encountered in daily life (Department of Basic Education, 2011; Spangenberg, 2012). Unlike mathematics which is a purely academic subject, mathematical literacy gives much importance to data handling and statistics (Frith, 2011).

Moreover, in approaching the relationship between mathematical literacy and school mathematics, one should also consider that mathematical literacy does not appear to come up as an invariable and automatic consequence after a certain number of years of compulsory education. For example, O'Donoghue (2003) states that "mathematics and [mathematical literacy] are not congruent. Nor is [mathematical literacy] an accidental or automatic by-product of mathematics education at any level" (p.4). O'Donoghue (2003) argues that the tension between mathematics and mathematical literacy in mathematics education needs to be acknowledged and the relationship between them should be clarified. According to him, there is a general expectation that the school mathematics experience should deliver mathematically literate individuals, but it usually fails to do so for a significant percentage of the school-going population. This failure partly results from both a lack of clarity in the goals of mathematics education and a clear understanding of the meaning of mathematical literacy. Furthermore, O'Donoghue (2003) also proposes that mathematical literacy and mathematics are not interchangeable. He considers mathematical literacy as involving some ideas and concepts of mathematics, rather than vice versa. That is, "when the goal is [mathematical literacy], some mathematics will be involved but mathematical skills alone do not constitute numeracy" (O'Donoghue, 2003, p.4). Moreover, Australian Association of Mathematics Teachers (AAMT) (1997) observes that mathematical literacy is not a synonym for school mathematics, but they are highly interrelated in that mathematical literacy gains strength by school mathematics. In other words, school mathematics plays an important role in the development of mathematical literacy. Therefore, it is often assumed that if people acquire the necessary mathematical knowledge and skills then it may somehow help them to become mathematically literate more or less whether they apply such knowledge and skills or not (National Numeracy Review Report, 2008). However, the widely held assumption that is not precisely true is that students who have done enough mathematics necessarily become mathematically literate (Frith, 2011). In this regard, AAMT (1997) points out that knowledge of mathematics is clearly essential for the development of mathematical literacy, but having solely that knowledge does not adequately guarantee that people become mathematically literate enough. This seems to be difficult to accept, but there has been considerable research effort indicating that knowledge alone is hardly sufficient for problem solving or further learning (Resnick & Klopfer, 1989). Thus, it is perhaps counterintuitive but more mathematics does not automatically lead to higher level of mathematical literacy, because there are a large number of examples of individuals who maintain minimal mathematical literacy level despite more advanced and complex mathematics course work in their backgrounds. Conversely, there are also numerous examples of individuals with significant levels of mathematical literacy but having little formal mathematical background (Hughes-Hallett, 2001). There is therefore not yet clarity or enough clarity over whether mathematical literacy skills can be gained by means of the acquisition of standard problem-solving skills and formal mathematics (Willis, 1990b).

In this sense, Steen (2001b) underlines the fact that mathematical literacy requires many years of education and training to achieve. Rather than pertaining to the understanding of pure mathematical concepts, it is so much more relevant to applying elementary mathematical facts in uncertain and complex settings. Thus, mathematical literacy and mathematics should become inseparable and integral aspects of the school curriculum. They are crucial to success in personal life and work performance, and empower each other, but both are not the same subject. Similarly, Hughes et al. (2000) also point out that it will be meaningless to teach students mathematics in order to make them mathematically literate if they do not assuredly operate and translate their knowledge and skills of mathematics to a range of diverse circumstances in their future working life and employment as well as social and domestic life. Therefore, as

Gal (2009) asserts, the focus of mathematical literacy is on the applicability of learned knowledge to daily life by associating it with various real world contexts. There is no doubt that this view is true, but arguably the most important point when considering this issue is that associating mathematics to real-life contexts necessitates to maintain fine and careful balance. That is to say, while contextual details provide necessary connections and relations that are essentially vital for many students to enhance learning and long-term retention, these details at the same time can conceal important motifs that form the core and spirit of mathematics (Quantitative Literacy Design Team, 2001). With this in mind, it would also be interesting to see another argument from Betty Johnston (1994), whose notion of mathematical literacy is not simply equivalent to mathematics, that is, "just as to be literate is to have a capacity for dealing with language in some way, so to be [mathematically literate] could be seen as having capacity for dealing with mathematics in some way" (p.32). Johnston (1994) proposes that there are five strands of meaning-making in mathematics: (i) Meaning supported by ritual, where meaning is gained with the assistance of rote-learning of particular content categories; (ii) Meaning supported by conceptual engagement, where mathematical meaning is established with the help of problem-solving heuristics, and cognitive processes; (iii) Meaning supported by the use, where meaning is enhanced by means of the real-life applications; (iv) Meaning supported by historical and cultural understanding, where meaning is improved by the agency of a strong comprehension of the origin and artistic use of mathematics in the past; (v) Meaning supported by critical engagement, where meaning is created by way of inquiring about 'in whose interest' and also about the suitability and adequacy of the mathematical model of the real world situation. So, in general all five strands of meaning are all together weaved to make, not mathematics, but what Johnston (1994) calls mathematical literacy. Therefore, as Tout (2001) argues, it is important to recognize that mathematical literacy is not downgrading or inferior to mathematics. More importantly, mathematical literacy is a social activity because it concerns about making meaning in mathematics and being critical about mathematics. In other words, mathematical literacy develops awareness about the significant role of mathematics by making sense of it in the modern world around us (DoE, 2003; Spangenberg, 2012).

Moreover, many mathematics problems that students are encouraged to discuss and work on are presented in artificial, pretentious and unrealistic storytelling ways. They are posed generally to make students practice over learned techniques and memorized rules. On the contrary, the problems embedded in mathematical literacy are usually not well defined, they usually require to gather, evaluate or interpret data and may not accordingly have a single and uniform answer (Frith, 2011). As mentioned above, this is due to the fact that mathematical literacy always demands situations set into a real world context that has some particular intimacy to the individuals involved (Coben & Chanda, 2000). In other words, in comparison to the abstract structure of mathematics, mathematical literacy constantly functions within real life and it is firmly and tightly interviewed with real context (Hughes-Hallett, 2001). That is why mathematical literacy is not less than mathematics, but more (O'Donoghue, 2002). However, it cannot be ignored that mathematics is certainly an important part of what constitutes mathematical literacy. In other words, as Coben (2000) argues, being mathematically literate means to become:

> …competent, confident, and comfortable with one's judgments on whether to use mathematics in a particular situation and if so, what mathematics to use, how to do it…and what the answer means in relation to the context (p.35).

Accordingly, mathematical literacy must be underpinned by mathematical knowledge of an appropriate kind (Coben et al., 2003). Indeed, the relationship between mathematics and mathematical literacy is plainly depicted according to how mathematics is defined. If it is defined as an abstract, theoretical and decontextualized, academic subject, mathematical literacy cannot be simply reducible to mathematical knowledge and skills. Such knowledge and skills are the fundamental basis and indispensable precondition to acquire mathematical literacy but they are still not enough (Niss, 2003a; Stacey & Turner, 2015). On the other hand, if it is conceived "as a pure, fundamental science; as an applied science; as a system of tools for societal and technological practice; as an educational subject; and as a field of aesthetics" (Niss, 2003a, p.216), the knowledge and competence of mathematics goes far beyond and much deeper than the mere ability to deal with only its pure and theoretical aspect. In this case, mathematical literacy is regarded as more or less the same as the mastery of mathematics (Niss, 2003b; Stacey & Turner, 2015). It then follows that people actually need both mathematics and mathematical literacy all together (Steen, 2001a). It is, however, quite true that the questions like 'How is mathematical literacy different from or related to mathematical knowledge and skills? 'How does one promote mathematical literacy within the school mathematics context?', and 'What is the role of context and life experience in school mathematics and mathematical literacy?' are quite difficult and there are also no standard answers to them (O'Donoghue, 2003).

2.2. The development of mathematical literacy

This section includes (i) challenges faced in the development of mathematical literacy and (ii) effective instructional practices for mathematical literacy development.

2.2.1. Challenges faced in the development of mathematical literacy

Mathematical literacy seems to be essential if people are to achieve their full potential, but many people do not feel prepared to use their mathematical literacy skills in their personal, working, social and educational life (Roohr, Graf, & Liu, 2014). In this respect, a number of significant barriers in the way of people developing mathematical literacy understanding are documented in the literature. For example, Rosnick (1981) argues that the perception that mathematics is theoretical, unreal and hard to do adversely affects people's success in the subject as well as their inclination towards it. Hence, one of the main reasons for many people to stay away from mathematics is its inherent difficulty and abstractness. That is to say, "unfamiliarity with mathematical symbols and the abstract concepts to which they refer breeds contempt for mathematics" (Rosnick, 1981, p.418). Coben (2003) supports this view that the much emphasis on the abstract nature of mathematics indeed paves the way for many people to depreciate the importance of daily mathematics to apply. Thus, negative attitude toward mathematics gives rise not only to refrain from learning mathematics properly and successfully but also to use it in everyday life. Benn (1997) also explains this well that, for most of the adults, memories of school mathematics are sadly memories of failures in a sense that their problems with mathematics originate in the school classrooms. It actually seems to start when the teacher fails to communicate effectively. The learner then fails to understand what is being taught. At the same time, the learner's embarrassment prevents him/her from asking for any clarification. Therefore, the lack of understanding is never remedied. So, it leads to further misunderstandings and undermining confidence due to the hierarchical structure in which mathematics is taught. On the other hand, mathematical qualifications are basically needed to access to further education or employment. This automatically produces pressure on the learner. Later on, this pressure may turn into feelings of anxiety and anger. Hence, the learner deals with such pressure by showing no interest or understanding of mathematics. Thus, many students also develop the feeling of anxiety and fear towards mathematics in consequence of the negative perception and poor image of mathematics in mind because of the discomfort in events or situations that require mathematical knowledge (Ma & Kishor, 1997). These feelings of anxiety and negative attitudes towards mathematics are primary obstacles to the development of mathematical literacy when compared to any lack of specific mathematical knowledge and skills (Atweh, 2001), because mathematics anxiety has been found to be especially crucial for predicting achievement in mathematics (Ashcraft & Moore, 2009). For example, recent research with young children by Galla and Wood (2012) demonstrates that too much anxiety is accompanied with poor performance in mathematics. Similarly, Tobias (1993) asserts that rather than a lack of ability, it is math anxiety that hinders a great number of people from realizing their mathematical potential. She believes that this is still a promising case since while nothing can be done about the lack of ability, a lot can be done to overcome math anxiety. For example, as Parsons et al. (2009) put it, people can manage their math anxiety by feeling confident in their ability in mathematics. People with lack of confidence in mathematics and negative attitudes towards it may easily develop anxiety. In other words, students with lack of confidence in their mathematical skills can easily give up studying mathematics, but a little confidence in mathematics may provide them the necessary motivation to gradually improve their performance in mathematics (Tobias, 1993). It is worth stating at this point that the understanding that mathematics expresses itself in almost every aspect of life seems to be ignored by many people (Backhouse et al., 1992). The lack of awareness about the use and application of mathematics in real life and other disciplines engenders low motivation, prejudices and negative attitudes towards mathematics learning, and this ultimately gives rise to the thought that no need to learn if it is useless or offers nothing besides abstract concepts (Olson, 1998). However, gaining the idea of the usefulness or relevance of mathematics to real life clearly acts as a motivating force for learning mathematics as well as coping with and eliminating math anxiety (Coben, 2003). Therefore, the fact of the matter is surely that the lack of confidence in mathematics (Parsons et al., 2009), negative attitudes towards the subject (Shaw & Shaw, 1997), the lack of motivation in mathematics (Kariuki & Wilson, 2002), and mathematics anxiety (Ashcraft & Moore, 2009) can be considered as important barriers to achievement in mathematics, as well as the development of mathematical literacy.

Despite many modifications and enhancements in the principles and standards for mathematics instruction from rote learning to meaningful learning with critical thinking, inquiry, and insight in order to better internalize and apply mathematical concepts, most of the mathematics teachers still show firm resistance and much struggle to incorporate mathematical literacy strategies into their instructional methods and practices (Siebert & Draper, 2012). This resistance may exist because integration of mathematical literacy practices into mathematics teaching requires a major shift from the traditional mathematics instructional routines with which a large number of teachers are indeed pleased, familiar and confident enough (Draper, 2002). However, the implementation of such teaching and learning routines creates a great barrier for building up mathematical literacy understanding among people (Colwella & Enderson, 2016). The development of mathematical literacy skills is simply based on the instruction involving better use and more applications of mathematical knowledge and skills into everyday life (Yore, Pimm, & Tuan, 2007), but such an instructional approach is potentially less stimulating to secondary school teachers, and it plants the seed of doubt and confusion into their mind about how to integrate this kind of instruction into their teaching (Shanahan & Shanahan, 2008). Therefore, many teachers may become resistant to it because they find themselves uncomfortable, inexperienced, or unqualified about mathematical literacy and feel that they lack the necessary skills and confidence to implement mathematical literacy strategies and methods for teaching and learning of mathematics (Draper, 2008; Moje, 2008). Similarly, the view of providing mathematical literacy understanding is not difficult that anybody can easily do it is also rejected by Julie (2006). Indeed, it is generally agreed that many teachers have not sufficiently experienced learning mathematics in relation to mathematical literacy understanding. For example, Madison (2004) holds the view that many teachers usually have much practice in illustrating mostly artificial applications of abstract mathematics, but they clearly lack in specific qualifications to derive abstract mathematics from various applications. Many mathematics teachers are not prepared to effectively provide mathematical literacy understanding to their students. Their mathematics learning experiences were mostly based on the rote computation and memorization of algorithms or formulas and some drill and practice activities (Stigler & Hiebert, 2004). Obviously, majority of mathematics teachers were educated with traditional methods and practices that do not reflect any concern in real life (Schoenfeld, 2001). Accordingly, they often tend to teach mathematics in the same way as they were taught with worksheets and textbooks (Westwood, 2008), but it clearly does not much contribute to mathematical literacy development (Colwella & Enderson, 2016). Therefore, the quality of teacher education is of great importance to provide students with higher academic standards (Ejiwale, 2013) and consequently to improve their mathematical literacy understanding because the level of the student achievement in mathematical knowledge and skills is closely related to teachers' qualifications in the process of teaching and learning the subject matter (Hill et al., 2005).

Similarly, from the UNESCO' point of view, a great number of studies indicates that even though teachers struggle to adjust their classroom practices so as to promote the potential for effective classroom activities necessitated by mathematical literacy understanding of students, for example allowing students to practice learning through inquiry, problem posing, investigations and open problems, the result generally does not become satisfactory as expected (UNESCO, 2012). This is because of the fact that teachers are not been specifically trained in response to the requirements of mathematical literacy development. The required teaching expertise or experience in this area is much higher than that required in traditional teaching and learning approach. However, the mathematical and pedagogical expertise demanded by mathematical literacy is usually underrated in pre-service teacher education programs as well as in traditional in-service training programs for teachers. This makes it difficult for teachers to perceive the advantages and effectiveness associated with recommended changes in the modes of instruction related to mathematical literacy and they are accordingly often reluctant to put these new practices into the operation. The fact is that mathematics education that devotes itself to the development of non-routine mathematical problem solving skills and inquiry learning activities that enable students to become mathematically literate is not easy to implement (Romberg, 2001). Indeed, as Monteiro and Pinto (2005) emphasize, there exists a big gap between the teacher education provided in universities and the multifaceted real-life events with which such education is supposed to be consistent. Besides, many teachers who are poorly trained to teach mathematics are unfortunately not able to develop sufficient knowledge and skills to teach mathematics effectively in the context of mathematical literacy (Ball, Hill, & Bass, 2005; Ginsburg et al., 2008). Therefore, students' poor performance in mathematical literacy can be attributed to weak teacher education programs (Sawchuk, 2011) and the lack of necessary professional development of teachers (Ejiwale, 2013). An Australia-wide survey conducted as a part of a national teacher education research project also supports the view put forward above (Milton et al., 2007). In this survey, 303 secondary school teachers in their first or second years of teaching were asked to respond to the questions about the effectiveness of their pre-service preparation for teaching literacy and numeracy to a range of students. The survey showed that beginning teachers did not feel well prepared to integrate mathematical literacy understanding into their teaching when and where necessary and largely reacted negatively in response to the preparation provided by their pre-service courses. All these above-mentioned arguments draw many researchers and educators' attention to the importance of adequate teacher training or education programs in order to prepare teachers for the specific challenges they face in and responsibilities they are required to take on when teaching mathematics in a productive way with respect to mathematical literacy (Tatto, 2012).

In addition to the lack of sufficient teacher training for mathematical literacy, the lack of appropriate textbooks and other instructional materials to guide teachers are also among the obstacles for the development of mathematical literacy (Tatto, 2012). Textbooks used in the teaching and learning of mathematics do not adequately promote mathematical literacy. The emphasis of textbooks content is usually on rote memorization and recall of vast amounts of facts (Pia, 2015). They generally fail to contain enough number of worked examples and relevant illustrations about mathematical literacy. Even though many teachers are willing to use the textbooks, they do not find these textbooks helpful and effective tools to promote mathematical literacy. They are considered as useless and inappropriate or even sometimes detrimental to the development of mathematical literacy (Westwood, 2008).

Moreover, students learn best when teachers respect student diversity and develop varied instruction in accordance with their students' strengths and weaknesses as all students learn at different paces (Hall, 2002; Murray & Jorgensen, 2007; Tomlinson, 1999). Differentiated learning increases the possibility of meeting the needs of students who find mathematical literacy challenging (Tomlinson, 2003). Similarly, the Canadian research project, Differentiated Instruction to Improve Students' Math Literacy, by Olson and Larsen (2012), supports the use of differentiated instruction to improve students' mathematical literacy level, because differentiation provides students with many opportunities for active engagement and meaningful learning (Santangelo & Tomlinson, 2009). However, according to Tobin and Tippett (2014), teachers always experience some challenges when they try to address the needs of diverse learners. For example, teachers may experience some fears and insecurities about the new expectations in differentiated learning. It is perhaps, they no longer see themselves in the seat of classroom authority. They are probably afraid that they are not able to manage the classroom and consequently get out of control (Tomlinson, 1995). Besides, the lack of enough time, basic resources, expertise and administrative support are other potential challenges when implementing differentiated instruction (Carolan & Guinn, 2007; Tobin & Tippett, 2014).

Furthermore, teaching and learning mathematics in crowded classes is another important reason for students' low performance in mathematical literacy (Darling-Hammond, 2014). Many studies indicate that class sizes have a direct impact on learning and retention abilities in mathematics (Angrist & Lavy, 1999; Ehrenberg et al., 2001; Finn & Achilles, 1999; Lee & Smith, 1995; Pong & Pallas, 2001). For example, crowded classes are found to result in a very difficult classroom environment for learning (Smith & Glass, 1980), and that large classes hinder positive attitudes and relationships between teachers and students (Blatchford & Mortimore, 1994).

Crowded classes can influence the interaction and communication among students and may cause excessive noise and disruptive classroom behaviors, which consecutively influence the lesson plans and kinds of activities the teachers implement (Ehrenberg et al., 2001). Besides, the time allocated to each individual student becomes much limited due to the large class size. However, low performing students especially benefit most from small classes, as teachers in smaller classes have more time for individualized instruction (Stasz & Stecher, 2000). Moreover, teachers have less time and so cannot cover enough number of topics which in turn affects the number of curriculum materials covered. In crowded classrooms, teachers may also adopt different instructional practices and assessment that underestimate discovery learning and problem based learning (Pong & Pallas, 2001), which are however effective teaching approaches for mathematical literacy development (Askew et al., 1997).

2.2.2. Effective instructional practices for developing mathematical literacy

There are many rigorous studies finding that the quality of teaching received by the students has the greatest impact on their outcomes and consequently on their acquisition of mathematical literacy (Aaronson, et al., 2007; Doyle, 2007; Leigh & Ryan, 2011; OECD, 2005). In this sense, teachers and their instructional practices play a pivotal role in helping students to gain confidence and desire to apply and use their mathematics knowledge and logic in order to make sense of the world (Hope, 2007). Thus, mathematics teaching should lay particular stress on the development of mathematical knowledge and skills that allow learners to effectively tackle problems encountered in daily life, the workplace and the social, civil, political and cultural environment (Department of Basic Education, 2011). The success of this simply depends on the learning environments teachers create, the lessons they design and implement, the use of technology resources and other educational resources they bring into the classroom, their teaching methods and strategies, and the quality of the assessment and evaluation in the educational process (National Numeracy Review Report, 2008). For example, Boaler (2000) compared the learning of mathematics in two schools where mathematics was taught in completely different ways. One had a content-based mathematical approach and the other had a process-based mathematical approach. Students in the content-based mathematical environment held the view that mathematics was all about memorizing a vast number of rules, formulas, and equations, and this view appeared to negatively influence students' mathematical behavior. On the other hand, students in the process-based mathematical environment were encouraged to develop their own ideas, formulate and extend problems, and use their own mathematics. Namely, the approach in this environment was based on the philosophy that students need to use mathematics in situations that were realistic and meaningful to them. Findings indicated that the students who were taught in the process-based approach expressed great enjoyment of open-ended work and much interest in their lessons, whereas the students who were taught in the content-based approach showed their lack of understanding and dislike of textbooks. Despite being highly motivated and hardworking, many of the students in the content-based approach found mathematics lessons tedious and boring. However, since the environments in the classrooms with process-based approach were much closer to the real world, students were generally more talented and very eager to use school-learned methods in their life than students in the classrooms with more procedural and school bound approach.

The instructional practices are defined as teachers' qualitative dimensions which consist of teachers' abilities to implement proper cognitive strategies, techniques, and classroom practices in the teaching and learning processes in order to develop effective classroom dialogues and regulate the instruction as desired, and build classroom environments in which learners cooperatively and collaboratively take part in inquiry-related activities (Englert et al., 1992; Goe, 2007). For example, the dimensions described in Artzt' (2008) framework for the examination of instructional practices that focus on teaching for student learning with understanding help us better analyze and appreciate the roles and responsibilities of teachers that are fundamental for developing mathematical literacy of students. Phase-Dimension Framework (PDF) by Artzt et al. (2008) is built on three fundamental components of mathematics lessons, including tasks, discourse and the learning environment. Artzt et al. (2008) emphasize that teachers in general use tasks in the teaching and learning process to allow students to construct meaningful knowledge by associating their existing knowledge with the new information and interest through active engagement in classroom activities. In order to stimulate higher student involvement, “tasks must be motivational, at an appropriate level of difficulty, and sequenced in a meaningful way” (Artzt et al., 2008, p.12). Primarily, it is significant to determine and clarify the learning goals of teaching mathematics in order to select or create appropriate tasks to satisfy these goals. Being aware of the cognitive demands of tasks is a central consideration in this satisfaction (Arbaugh & Brown, 2005). If rational and relevant tasks are provided by the teachers, mathematical thinking and cognition will be developed in the most efficient way (Harel & Sowder, 2005). Thus, it is proposed that teachers and students should work jointly on challenging and worthwhile mathematical tasks that strengthen sophisticated mathematical reasoning and sense-making (Stein et al., 1996). Similarly, as Henningsen and Stein (1997) indicate, the tasks teachers implement in their lessons should acquire qualities that draw the learners’ interest and sustain high level of cognitive processing. It is unreasonable to present some simple activities continuously in a routine manner, or to prepare a difficult task which makes learners feel hopeless to achieve it. Students’ attention in mathematics classrooms is more likely to move away from the task in hand, if there is an inappropriate or unchallenging task in discussion. Namely, “if a task is not adequately focused the pupils are likely to flounder, but if it is too focused there will be too little scope for exploration” (Orton, 2004, p.168). In a similar way, Brufee (1993) writes about the appropriateness of the difficulty level of collaborative learning tasks. He stresses that if a task is too simple, learners easily get bored. In this case, there is no need to concentrate much on the task, because it is monotonous and unchallenging, and the learners handle the problem too quickly. On the other hand, if a task is too difficult, it presents a big obstacle to learners from the beginning and puts them into dependency on the teacher’s authority. Then both learners and the teacher have no choice but to be completely dependent on the direct instruction. Therefore, the careful design or relevant choice of tasks is really important to encourage mathematical activity and sustain attention among learners (Bransford et al., 2000). In that respect, mathematical tasks embedded in real-life contexts clearly promote interest and motivation which are two inseparable parts of learning mathematics (Stylianides & Stylianides, 2008). Hence, it is recommended that the tasks for mathematical literacy development should be implemented with its purpose of offering opportunities for learners to experience how mathematics relates to the real world by allowing the learners to apply mathematical ideas to produce valuable decisions influencing their personal, working and social life (OECD, 2013a). The tasks with which students become engaged in the classroom should offer many opportunities for developing mathematical literacy. In other words, the emphasis should be on the content being taught in context and making the subject applicable to real-life situations (DoE, 2003).

Utilizing multiple mathematical representations (e.g., tables, diagrams, graphs, figures, charts, symbols, equations, computers, software) to model and develop mathematical ideas and arguments is also of great importance for mathematical literacy development (de Lange, 2003; Niss & Højgaard, 2011). It is important to recognize that the effective use of manipulatives is mostly based on a teacher's background knowledge and interpretation of different kinds of mathematical representations (Moyer, 2001). Therefore, incorporating and translating between different representations when teaching mathematics are basically expected from teachers in order to enhance their students' proficiency in communicating mathematical ideas with more precision (Whitin & Whitin, 2002). The other thing expected from teachers is to use a range of instructional aids and tools, including technology in the mathematics classroom in order to provide learners much time and flexibility to focus on problem solving, reasoning, decision making and evaluation. Although the increased access and use of technology in the learning and doing of mathematics do not automatically guarantee the improvement in the quality of teaching and learning in mathematics classrooms, it is inevitable fact that technology has a considerable impact on the mathematics that teachers present or how they teach it when used properly (Ontario Ministry of Education, 2005). Bransford et al. (2000) hold the parallel view that when computer technology is used appropriately, it is likely to improve student achievement. For example, students can use computer to verify accuracy of drug dosages or they can develop computer algorithms to map the human genome or they can use computers to find markets, analyze soil, and deliver controlled amounts of seeds and nutrients (Hoyles et al., 2002; Quantitative Literacy Design Team, 2001). Since technology clearly helps to improve the learners' knowledge and comprehension of real-life situations that require them to work with basic number concepts as well as to apply higher-order reasoning about quantities (Botha, 2011), it is therefore highly recommended that teachers need to keep up with technology trends to develop students' mathematical literacy understanding (Ontario Ministry of Education, 2005).

Moreover, it is important for teachers to present opportunities for students to articulate and explain their ideas by questioning, listening and responding to all of the ideas and arguments in class (Artzt et al., 2008). Becoming a mathematically literate individual requires the capacity to interact with members of the community, and this interaction usually becomes in the form of discourse (Doyle, 2007). Discourse refers to sharing and comparing of one another's ideas and perceptions to establish, describe, explain, evaluate, and justify mathematical relations (Lesh & Doerr, 2003), which have all a significant role in understanding the meaning behind mathematical literacy (Doyle, 2007). Clearly, teachers should be willing to participate in a mathematical discussion with students, because they have a key role in coordinating discourse in the classroom, encouraging students to attend to and stay engaged in the task at hand, and ensuring contributions from everyone (Artzt et al., 2008). Even though the quality of teachers' interactions with students varies according to three domains: instructional support, organizational and emotional (Pianta & Hamre, 2009), positive interactions between the teacher and students have always a substantial influence on the academic achievement and learning outcomes in mathematics (Reyes et al., 2012). Thus, teachers are supposed to be active mediators and guides of classroom discourse, reinforcing students' discussion on worthwhile ideas as well as their reflections on their own thinking about these ideas (Schoenfeld, 2001). Accordingly, learning mathematics should not be thought as only receiving pure and abstract mathematical information from the teacher, but rather as involving in sense-making processes in a way that the teacher and students are working collaboratively to advance mathematical understanding (Franke et al., 2007). So, teachers are the chief director to create a context in which effective mathematical discussions happen to help students construct their own mathematics that naturally promotes their mathematical literacy understanding (Doyle, 2007).

Accordingly, students need to interact with each other when approaching any problem from a mathematical point of view in order to effectively work with mathematical ideas and to incorporate them into their existing knowledge (Richards, 1991). They need such interaction to assimilate what they are learning in relation to what they already know. Namely, they relate new information to other previous information they already possess. Thus, to build their own mathematical constructions, students have to interact with each other in such ways that they can stimulate, promote and challenge their ideas (Artzt et al., 2008). However, teachers are likely to be concerned that interaction among students might take up too much time. Therefore, many mathematics classes do not provide sufficient opportunities for interaction among students to develop better mathematical understanding. In the long run, however, time lost will be somehow compensated if mathematical ideas are digested more thoroughly (Orton, 2004). That is to say, as the students confidently and effectively interact with each other in class, they gradually adapt their ideas that are temporary and uncertain into more clear and steady ideas, which is in itself is the central component of the effective learning process (Lee, 2006). Since encouraging student participation in classroom discussions truly acts as a facilitator to better grasp mathematical ideas, students must be given enough time and the opportunity to talk about and reflect on their own ideas (Franke et al., 2007). Listening to what students talk is also highly valuable as they uncover a great deal about their understanding or misconception in mathematics. In other words, allowing students to talk mathematically might also offer the teacher a continuous and detailed means of assessing their students' learning and progress, thereby deciding where to go next in order to keep going the development of that learning (Lee, 2006). Therefore, in developing mathematical literacy understanding of students, researchers place great stress on the importance of meaningful and constructive classroom interactions where students are actively involved in the lesson, participating in discussions and group activities (National Numeracy Review Report, 2008; Venkat & Graven, 2008; Venkat, 2007).

The teacher's questioning skills are also of great importance to support the students' sense-making of the situation and to guide their development of higher-order thought (Doyle, 2007; Widjaja et al., 2010). Teachers always tend to use questioning method to provoke students to clarify their ideas regarding the problem under investigation (Artzt et al., 2008). Through questioning, teachers identify how well students make connections or the kinds of connections among mathematical ideas that require students to explain, analyze, reflect, and justify their thinking and reasoning (Reinhart, 2000). Based on the cognitive demands of the questions and the student's level of understanding, the teacher decides how much time needed for students to respond to the questions. Accordingly, when students' responses are received, the teacher may decide to adjust the pace of teaching, reteach, or strengthen the mathematical concepts (Ontario Ministry of Education, 2005). Thus, the critical role of teachers in the classroom as part of everyday learning and teaching begins when they move away from the transmission of a body of knowledge from the teacher to students towards listening, questioning, and probing for better understanding of mathematics (NCTM, 1991; Welsh Government [DfES], 2012).

Creating a positive learning environment is also crucial for developing positive attitudes among students towards mathematics which, in turn, shapes students' performance in mathematical literacy (OECD, 2013b). Learning environment represents the context in which the teaching and learning process occurs (OECD, 2009b). In other words, as Artzt et al. (2008) state, it describes the kind, tone, and style of interpersonal interactions between students and a teacher through appropriate discourse and task engagement to foster the development of mathematics among all students within the social and intellectual climate of the classroom. Artzt et al. (2008) add that it also refers to instructional modes and amount of time assigned for them as well as classroom management and organizational issues in order to promote student learning in mathematics. Therefore, according to Artzt et al. (2008), a learning environment includes an appropriate social and intellectual climate, the effective use of modes of instruction and pacing of the content and the application of certain administrative routines. The social and intellectual climate implies the forms of the interpersonal interactions in class and promotes learners' social, emotional and cognitive development. Learning environment should make a contribution to the discourse and task engagement to support the growth of mathematical understanding among all learners. Generating such an environment requires trust and mutual respect between the teacher and the learners and among the learners themselves. In this sense, the effective and productive development of mathematical literacy is strongly related to students' sense of comfort, safety and confidence within the learning environment

(Franke et al., 2007; Ontario Ministry of Education, 2005). This reveals the importance of the negotiation, acceptance and development of rules in a classroom community. They should be interactively established by both the teacher and the students in the course of classroom activity (Lampert, 1990). Therefore, the teacher has a special role to establish an organized and supportive classroom environment to help students construct mathematical ways of knowing that are compatible with those of wider society (Yackel & Cobb, 1996).

Moreover, teachers have a direct and full responsibility to set a good pace and diverse modes of strategies so that students have sufficient time to discuss and effectively engage in the construction of new knowledge. Modes of strategies are the instructional methods and activities that teachers implement in the classroom to allow students to realize the lesson objectives and learning outcomes. Therefore, instructional activities must stimulate students to explore mathematical ideas and prompt students' thought processes in ways that establish and reinforce better mathematical comprehension (Artzt et al., 2008). Student achievement is more influenced by the underlying structures and quality of teaching strategies than those of the school environment (Scheerens & Bosker, 1997). However, it is generally accepted that there is no specific and precise way to teach mathematics. The efficiency of instructional strategies "is domain specific as well as goal-specific; it depends on the cultural context and professional traditions" (OECD, 2009b, p.97). Accordingly, rather than a single best type of practice, the OECD TALIS study suggests a variety of teaching practices including structuring practices, student-oriented practices, and enhanced activities (OECD, 2009b). In general, many researchers suggest powerful strategies for better acquisition of mathematical literacy such as pair work, whole class or group work and discussions (Fuchs et al., 1998; Linchevski & Kutscher, 1998; Venkat & Graven, 2008); cooperative and collaborative learning approaches (Davidson et al., 1991; Davidson, 1990; Frith & Prince, 2006); differentiated instruction (Pettig, 2000; Tomlinson, 2000); project work, guided discovery and inquiry learning (Borasi, 1992, 1996; Vithal, 2006); problem solving (de Lange, 2003; OECD, 2013a, 2014b; Ontario Ministry of Education, 2005); and mathematical modelling (Brown & Schäfer, 2006; Steen, Turner, & Burkhardt, 2007).

In addition, research projects such as the Effective Teachers of Numeracy Study (Askew et al., 1997) make important contributions to the understanding of what strategies are required for effective teaching or quality teaching of mathematical literacy. Askew et al. (1997) note that through the analysis of teachers' instructional practices, knowledge and beliefs, teacher's instructional approaches can be described according to their orientations towards mathematics teaching. These approaches are described as either connectionist orientation, transmission orientation, discovery orientation or a combination of all or any two. Askew et al. (1997) describe characteristics of each of these orientations and set out the key distinctions between these orientations according to the three aspects of beliefs. According to Askew et al. (1997), connectionist teachers are concentrated on assisting students to develop efficient conceptually based strategies by using discussion and challenge to introduce links between different meanings and representations. Thus, connectionist teachers normally have a wider knowledge involving both practical and formal methods and representations and also knowledge of informal strategies that students with different learning styles find helpful and supportive. On the other hand, transmission teachers lay a great deal of emphasis on students' acquisition of a set of classic methods for solving a limited range of routine problems by providing certain classical methods and ensuring that students practice with them. Thus, transmission teachers seem to limit themselves to a narrow range of written methods and representations, using pictorial representations for an initial introduction and justification before moving quickly to a formal method. Finally, Askew et al. (1997) add that discovery teachers place particular stress on students' own development of concepts and strategies by using practical activities and experience provided by the teacher as a basis. Hence, they have knowledge of wide range of practical equipment which helps students to obtain answers, and of standard formal methods, but are not always clear how best to assist students to bridge between them.

In another study about the teaching practices of mathematical literacy, Graven and Venkat (2007) identify four pedagogic agendas which differ according to "the nature and degree of integration of context with mathematics within pedagogic situations" (p.74). These agendas are the context-driven agenda in order to explore contexts with regard to learners' daily life demands and meet them by using mathematics; the content and context driven agenda in order to explore a context for the purpose of consolidating the understanding of mathematics and the context; the mainly content-driven agenda in order to learn mathematics and then to apply it to a variety of contexts; and the content agenda in order to offer second chance to learn the basics of mathematics. Graven and Venkat (2007) call these four agendas a spectrum rather than a continuum, because these agendas are not strictly bounded or totally distinct from each other. Teachers may move along and adopt several agendas at different points in time across a year in order to satisfy the core requirements of the curriculum. Indeed, as Graven and Venkat (2007) point out, teachers' agendas are shaped by their conceptions of mathematical literacy. Namely, if teachers see mathematical literacy as watered-down mathematics for low-performing students, they will probably pursue the content agenda trend. On the other hand, if teachers regard mathematical literacy as an efficient way of developing a clear understanding of mathematically-based information, they will most likely apply classroom activities with special attention to mathematical reasoning and arguments about statistics and uncertainty, change and relationships, spatial relationships, and quantities.

2.3. The mathematical literacy emphasis in the curriculum

In this section of the chapter, the paper has concentrated on two important issues which merit a brief mention here: (i) curriculum reforms towards the development of mathematical literacy and (ii) obstacles to the implementation of curriculum reforms

2.3.1. Curriculum reforms towards the development of mathematical literacy

Another important dimension in the quality of mathematical literacy development refers to curriculum issues. If we agree to embrace a multifaceted and wider view of mathematics, and to promote mathematical literacy understanding within mathematics teaching and learning, new effective ways of describing and assessing mathematics curricula centered mostly around mathematical understanding and skills rather than factual information, rules and procedures will become very important (Niss, 2003b). However, it is often indicated that learning in mathematics is generally restrained by and limited to the procedures, and transmission and acquisition of a large amount of mathematical content (Venkat, 2007) with little focus on an active and continual processes of sense making, meaning construction (Schoenfeld, 1992) and thinking mathematically through problem solving (Schoenfeld, 2013). One major point of debate is related to whether the main focus of the curriculum is the development of the learning and understanding of mathematical content or the cultivation of students' confidence and capability to use critical and creative thinking skills to solve problems and communicate mathematically in a wide range of contexts (AMESA, 2003). In this regard, some researchers seem to give some support to the shift towards to the practice of mathematics teaching and learning with much emphasis on the relationship between real life and mathematics, but they also attract attention to the potential risk of failure to establish a sound balance between the traditional mathematical procedures and the applications of mathematics in real-world settings (Sethole, 2003). Or even more speculatively, they reveal that mathematics can lose its own specific importance due to real-life scenarios in mathematics teaching and learning (Adler, Pournara, & Graven, 2000). In this sense, Adler et al. (2000) persuasively argue that although associating mathematics with everyday real world knowledge and the day to day experiences of learners (what he calls integration within and across mathematics) is something to be much desired in the mathematics curriculum, the possibility of the implementation of such curriculum might be groundless or even very difficult because of a practical or a theoretical point of view. Adler et al. (2000) suggest that successful implementation is primarily based on establishing more realistic time frames for teaching and learning mathematics through the increased use of integrated mathematical tasks for assessment practices as well as comprehensive teacher assistance and counselling. Moreover, Venkat (2007) points out that the nature of the curriculum specification should highlight the development of both mathematics and mathematical literacy in a balanced and integrated manner. In other words, if students are expected to develop mathematical literacy habits or attributes, contexts or everyday events should be used to uncover the hidden mathematical patterns and structures while simultaneously mathematics should be used to help students to make sense of contexts or events around them (DoE, 2006). Venkat (2010) further indicates that a distinguishing characteristic of the curriculum with a focus on mathematical literacy is the emphasis given to everyday contextual and authentic learning situations. A curriculum that aims at producing participating citizens, contributing workers and self-managing people should foster skills at collecting, analyzing, justifying, organizing and critically evaluating mathematical information in order to effectively engage in and meet the general demands of a broad range of situations in real life (DoE, 2003). This, in turn, creates a general expectation that students should not be felt threatened and held back when they encounter a wide array of contexts and everyday life problems in their current or future lives but rather help them make more informed decisions (Bansilal, Webb, & James, 2015).

2.3.2. Obstacles to the implementation of curriculum reforms

Mathematics curriculum reforms in many countries around the world have been steadily gaining impetus in order to call for more mathematical literacy for all citizens by providing them many diverse problem posing activities at different educational levels and a lot of authentic opportunities to explore mathematical ideas (NCTM, 2000). However, many researchers have identified that curriculum innovations have hardly been effectively implemented as intended and most of the curriculum reforms have unfortunately failed to achieve the desired success (Anderson & Piazza, 1996; Cuban, 1993; Feldman, 2000; Fullan & Miles, 1992; Fullan, 2007; Smith & Southerland, 2007; Sowell & Zambo, 1997; Wilson, 1990). There are many different factors affecting the reform process or the harmony between curricular principles and teachers' classroom practices. For example, Clarke (1997) summarizes twelve factors that affect reform trends and ideas in teacher roles in the following way: (i) the reform movement in general, (ii) the principal and school community, (iii) internal support personnel, (iv) the spirit of collegiality, collaboration, and experimentation, (v) the grade-level team of teachers, (vi) innovative curriculum materials, (vii) the in-service program, (viii) external support personnel, (ix) the researcher as audience and critical friend, (x) outcomes valued by the teacher, (xi) day-to-day conditions under which teachers work, and (xii) teacher knowledge. Crowded classrooms and the lack of professional support for teachers are also important obstacles to achieve the curriculum innovations (Ryan et al., 2009). Other significant factors influencing successful implementation of any new curriculum are time limitations, expectations from parents and learners, a rigorous public examination system, a limited access to required instructional resources, a lack of specificity and common understanding about curriculum reform, teachers' deficiencies in content knowledge, pedagogy and skills, and the lack of consistency between teachers' beliefs and the ideology underlying the new curriculum reform (Bennie & Newstead, 1999). Actually, most of the teachers primarily show hidden or sometimes open ignorance to curriculum innovations and they develop very limited and shallow understanding of any reform movement in education. According to many teachers, the reform idea refers to a particular set of policies and practices. Such confusion or misunderstanding of the curriculum reform generally stems from their beliefs and initial knowledge and that also influences their instructional practices (Spillane et al., 2002). Many research studies, in this sense, have highlighted that teachers are the main actors for a successful curriculum reform and innovation movements in mathematics (Battista, 1994; Kirk & McDonald, 2001; Little, 1993; Spillane, 1999) and their pedagogical knowledge, practices, beliefs and perceptions accelerate or delay the intended change in the reform process (Fullan & Stegelbauer, 1991; Koehler & Grouws, 1992; Pajares, 1992; Sosniak, Ethington, & Varelas, 1991). In other words, if teachers have positive beliefs and perceptions about the new curriculum, this will correspondingly facilitate its implementation. Otherwise, the implementation of the curriculum will be very difficult and probably take a very prolonged time to be successful enough (Burkhardt, Fraser, & Ridgway, 1990; Prawat, 1990). That is to say that the success of any curriculum reform or innovation adoption is largely attributed to sufficiently taking into account teachers' beliefs and perceptions about this reform or innovation movement (Handal & Herrington, 2003). However, many educational reforms in curriculum and instruction ignore teachers' conceptions about these reforms and most of the innovations have been introduced or imposed by policymakers and education authorities through a top-down approach without consultation with teachers who are required to implement these innovations (Kyeleve & Williams, 1996; Martin, 1993; Norton et al., 2002). Therefore, the lack of enough consideration of teachers' beliefs also seems to be among the most basic reasons for the failure of the successful implementation of any educational reform (Knapp & Peterson, 1995).

2.4. Conceptual framework of the study

The focus of the study reported here was to investigate secondary mathematics teachers' conceptions about mathematical literacy. First of all, in view of the arguments mentioned in Section 2.1, it is clear that there has been a wide range of discussions and debates about mathematical literacy in international reports and literature, but all these arguments collectively support the idea that mathematical literacy is, on the whole, elusive and varies according to the purpose and context being used. In such a case, the definition of mathematical literacy as conceptualized in the *Programme for International Student Assessment* (PISA) (2012) is more comprehensive and broad as it covers all main aspects of other definitions. Among others, it clearly resorts to Freudenthal's (1983) realistic mathematics education approach and describes mathematical literacy in high relation to the authentic real-world contexts in which mathematics is applied and thereby contributing to a better understanding of mathematics (OECD, 2013a). In other words, it can be easily seen that the emphasis is not just on the application of the basic principles and standard procedures but also on the engagement of mental processes or cognitive functions through the use and implementation of formal mathematics to solve various problems involving real world situations representing people's personal, occupational and social lives in order to enable them to become constructive, concerned and reflective citizens of their current and future life (OECD, 2013a).

Moreover, the definition of mathematical literacy by OECD (2013a) suggests a model of mathematical literacy, which has generally been a vital element of the PISA mathematics framework. Figure 2.1 below demonstrates an overview of the principle components of this model as well as the significant and close relationships with each other. The outer-most box of the figure indicates that mathematical literacy occurs within the problem situations or challenges that exist in the real world context. The model portrays these challenges in two different categories. The first one is the category of contexts representing different spheres of life from which the problems emerge, such as personal, societal, occupational, and scientific contexts.

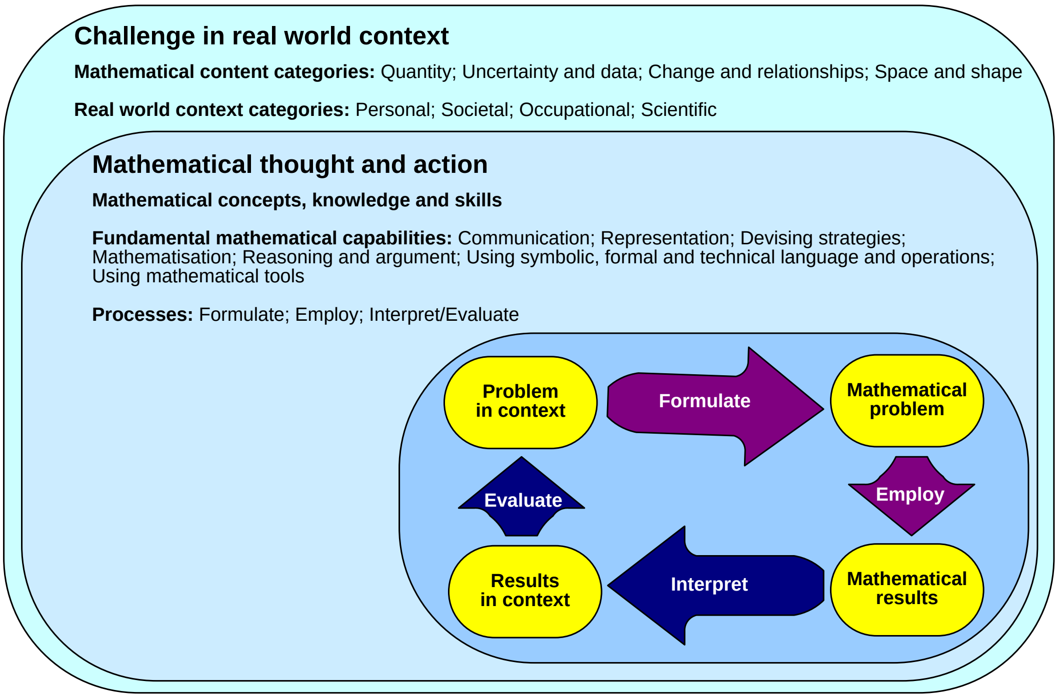

Figure 2.1: Model of Mathematical Literacy in Practice (OECD, 2013a, p.26)

The second one is the nature of the mathematical facts or reality located or existing within the essence of the problem or challenge. The four mathematical content categories which include quantity, uncertainty and data, change and relationships, and space and shape encompass a wide spectrum of mathematical phenomena to analyze and tackle the challenges or problems arising from the real-life contexts. In order to handle such contextualized problems, individuals must also employ mathematical thought and action represented by three different ways. Firstly, the model recognizes that people need to use a variety of mathematical knowledge and skills while they express and represent mathematical ideas, plan and carry out solution strategies, formulate and put forward informal and formal arguments, develop and justify a reasoning, and so forth. Secondly, these mathematical processes and dispositions are presented in the model in terms of seven fundamental mathematical capabilities which are (i) Communication, (ii) Mathematizing, (iii) Representation, (iv) Reasoning and argument, (v) Devising strategies for solving problems, (vi) Using symbolic, formal and technical language and operations, and (vii) Using mathematical tools. Thirdly and finally, as an individual is committed to and engaged with the problem, these fundamental mathematical capabilities are activated and implemented respectively or together by using relevant mathematical content to produce a convenient and reliable solution. This accordingly requires the formulation of the problem situation, the employment of mathematical ideas, constructs, procedures and reasoning, and the interpretation of a mathematical solution and final result. Thus, the mathematical modelling cycle in the inner-most box of Figure 2.1 represents a basic and conceptual portrait of the phases through which a problem solver navigates when manifesting mathematical literacy. It begins with the 'problem in context'. Correspondingly, the problem solver attempts to determine the appropriate mathematics in the problem situation and formulates it mathematically with respect to the concepts and relationships determined. The 'problem in context' is accordingly converted into a 'mathematical problem' which is convenient and ready for mathematical interpretation and construction of the model. The problem solver then employs mathematical ideas, concepts, facts, processes, and tools to receive 'mathematical results'. This phase naturally requires a problem solver to perform mathematical reasoning, manipulation, transformation and calculation. Next, the problem solver needs to interpret the 'mathematical results' on the basis of the original problem context. This necessitates to evaluate the mathematical results and their rationality as well as practicality in the real-life contexts. These phases of formulating, employing, and interpreting mathematics are principal and crucial elements of both the mathematical modelling cycle and the definition of mathematical literacy (OECD, 2013a). Hence, it can be said that the definition given the *Programme for International Student Assessment* (PISA) (2012) best reflects the idea of mathematical literacy. For this reason, it also best identifies the requirements of this study.

Besides, teachers' conceptions for this research study are interpreted or considered as "a more general mental structure, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences, and the like" (Thompson, 1992, p.130). In this sense, although the distinction may not be a remarkably significant in essence, it will be more appropriate for us to use teachers' conceptions of mathematical literacy for this study rather than to simply speak of the teachers' beliefs about mathematical literacy. Thompson (1992) characterizes teachers' conceptions of mathematics teaching in terms of what teachers regard to be the central values and feasible goals of the mathematics curriculum, the role of students and teachers in the teaching and learning processes, effective and efficient classroom practices, suitable and viable teaching approaches and emphases, rigorous and legitimate mathematical constructs and procedures, and optimal and sustainable outcomes of instruction. Moreover, Ernest (1989) states that beliefs contains "the teacher's system of beliefs, conceptions, values and ideology also referred to…as the teacher's dispositions" (p.20). Artzt et al. (2008) define beliefs "as an integrated system of internalized assumptions about the subject, the students, the learning, and teaching" (p.20). Even though some attempts to describe teachers' beliefs are made by a number of researchers, there is no clear agreement on how beliefs are to be explored, because beliefs cannot be precisely observed or measured, but must be evaluated from what individuals say, intend and do (Pajares, 1992).

Moreover, despite the difficulty of showing highly complex and dialectic relationships, many researchers acknowledge that teaching styles of teachers, modes of learning and instructional activities in the classroom are strongly related to what teachers believe about the nature of mathematics and its teaching in the classroom (Ernest, 1989; Richardson et al., 1991; Steinberg et al., 1985; Thompson, 1984, 1992; Thompson, 1985). Pajares (1992) admits the complexity involved in the evaluation of beliefs, but through his comprehensive research study, he indicates a close and rigorous relationship between the beliefs teachers hold about the teaching and learning process and their planning a learning event for attaining the desired goals, instructional decisions that form the basis of this planning, and classroom strategies and practices. So, it is generally agreed that beliefs about the nature of mathematics largely impact teachers' preferences of teaching approaches. For example, teachers holding a traditional belief most probably view mathematics as an abstract concept and far removed from tangible reality. These teachers generally find it difficult to link mathematics to real-life situations and tend to believe that mathematics involves a body of facts and formal procedures that must be studied mechanically with little or no relation to each other and hardly any correlation with people's daily lives (Mason, 2003; Schoenfeld, 1998; White & Mitchelmore, 2002). On the other hand, teachers who believe that the students must actively take part in their own learning tend to develop social and intellectual climates that provide every student an equal opportunity to participate into classroom discourse. These teachers see themselves as guides or facilitators of student learning rather than a single authority or a mere power in class. They accordingly tend to use instructional strategies that cultivate strong communication among students and stimulate students to develop higher order thinking and mathematical reasoning skills (Artzt et al., 2008). Teachers' beliefs are therefore regarded as the driving force behind teachers' lessons, in general, their subject matter knowledge, knowledge about how students learn mathematics, knowledge about how to teach mathematics, and knowledge of the curriculum (Thompson, 1992).

To put it more clearly, teachers who know their students well provide tasks that are more encouraging and at an appropriate level of difficulty. It is indeed the teacher's ability to understand what students already know, how they think about, how they learn the particular topic, why certain topics are more comprehensible than others, what misconceptions learners have and how to implement different strategies to rectify and reorganize those misconceptions (Ball, 1990; Shulman, 1986; Sowder, 2007). Therefore, teachers must be able to see what students do, hear what they state and then be able to behave accordingly as a right guide to promote the learning process (Hill et al., 2008). In a similar sense, Askew et al. (1997) identify eight key areas of knowledge of students as students' attitudes, students' ability in general, students' knowledge and understanding of mathematical literacy, students' ability in mathematics, students' approaches to mathematics, students' needs, students' background and experiences, and students' interpersonal skills. The same researchers add that beliefs about students and how they learn to become mathematically literate involve (i) whether or not some students are naturally more mathematical (ii) the type of experiences that best bring about learning, and (iii) the role of the students in lessons. Thus, if a teacher does not have good knowledge of students in terms of what they already know or how they personally approach tasks, the students may be forced to study either something that they already know or something that is too complicated for their present knowledge state (Askew et al., 1997).

Additionally, Artzt et al. (2008) state that teachers who have an extensive knowledge of pedagogical methods and know how these methods satisfy the needs of diverse groups of students and various content areas intend to implement a wide variety of instructional strategies that could develop the learning opportunities for their students. Teachers who have a broad knowledge of the content and know its connections with past and future areas of study intend to ask a range of questions of distinct levels that emphasize both procedural and conceptual issues. This accordingly helps students build the connections essential for them to construct new understandings. Knowledge about teaching refers to “the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students” (Shulman, 1987, p.15). Teachers need to be able to present a subject matter in multiple ways by using and adapting different representations based on the content and the needs of the learners to facilitate the learning and then to make the subject more comprehensible (Ball, 1990). Thus, in order to apply the meaningful pedagogical strategy and to use appropriate instructional materials for a lesson, knowledge of teaching is essential for teachers (Koellner et al., 2007). Similarly, Hill et al. (2008) support that teachers need to know various ways of how to establish students’ mathematical thinking or how to identify their misunderstandings and misconceptions and to rectify them through different teaching strategies. However, developing mathematical literacy skills of students is simply complicated for some teachers as they lack the knowledge and competence to contextualize their teaching. Namely, their knowledge of teaching concerning different teaching methods or approaches may not be satisfactory (Sidiropoulos, 2008). The success of mathematical literacy development is, however, mostly based on the teachers’ ability to use the most relevant and convenient teaching strategies such as whole-class discussions, inquiry-based instruction, and problem solving (Brown & Schäfer, 2006). Venkat (2007) also reports that learners are pleased with mathematical literacy acquisition, feel positive about it, and find mathematical literacy functional, helpful and challenging when their teachers use appropriate pedagogical strategy and instructional materials for a lesson. In other words, mathematical literacy development enables students much more communication and involvement in the class activities, as well as making sense of the mathematics being used. Moreover, in Sidiropoulos’ (2008) study, mathematics teachers expressed their beliefs about how they understood and responded to the value of teaching mathematical literacy. They regarded it as “lesser maths” (Sidiropoulos, 2008, p.225) and stated that “it is not real maths; it is the beginning of maths; it is a maths only better than nothing; it is the maths of oranges and bananas; it is a subject for the doffies” (ibid, p.225). In general, they believe that mathematical literacy is an ability to perform basic arithmetic operations rather than the ability to make use of them confidently in commonly encountered situations. However, according to Withnall (1995a), mathematical literacy is something more than basic or everyday mathematics. More importantly, mathematical literacy is seen as a social activity since it shows concerns for making meaning in mathematics and being critical about mathematics (Johnston, 1994). Therefore, knowledge of mathematics teaching approaches together with beliefs about how best to teach students to become mathematically literate are especially important. This naturally requires understanding and implementation of various teaching styles and different ways of presenting mathematical literacy ideas to learners through range of diagrammatic and verbal representations (Askew et al., 1997). For this reason, if we are concerned about making students mathematically literate, we should also pay enough attention to the teacher’s conceptions about the effective teaching of mathematics in the context of mathematical literacy.

Furthermore, teachers’ curricular knowledge involves the knowledge of the detailed information of programs designed for the teaching of different topics at given levels in a subject area. Teachers need to be familiar with the objectives, goals, learning outcomes and assessment criteria of the subject. They also need to be familiar with the level of complexity of the curricular content being taught during the previous and the following years in school (Borko & Putnam, 1996; Shulman, 1986). Thus, mathematics teachers require to be informed about both the intended new curriculum and the nature of mathematical literacy in order to facilitate what is expected of them when teaching mathematics emphasizing mathematical literacy. However, it is also well known fact that teachers’ beliefs about any specific curricular innovations clearly influence the quality of their implementations in schools (Handal & Herrington, 2003). For example, in her study, Sidiropoulos (2008) observed that the purpose of the curriculum with respect to mathematical literacy had not been well accepted by the teachers and as a result they did not adequately place value on it. It was attributed to the fact that since teachers held beliefs that were incompatible with the curriculum innovation, there existed a clear gap between curriculum as intended and curriculum as implemented. In fact, there are numerous research studies demonstrating that a mismatch between teachers' beliefs and the beliefs underpinning any particular curriculum reform can create a negative impact on the extent of the success of the reform movement together with the teachers' enthusiasm and motivation to implement it (Anderson & Piazza, 1996; Burkhardt, Fraser, & Ridgway, 1990; Cuban, 1993; Hart, 1992; Prawat, 1990). In other words, teachers' beliefs can play either a contributory or a repelling role to incorporate curriculum requirements directly into instruction (Haynes, 1996; Koehler & Grouws, 1992; Sosniak, Ethington, & Varelas, 1991). Therefore, any approach to curriculum reform implementation that do not take teachers' beliefs into account is not viable or sustainable for a long time (Knapp & Peterson, 1995; Martin, 1993).

Perhaps we should also point out the fact that although beliefs are undeniably a driving force behind the nature of a teacher's instructional practice, teachers' alleged beliefs may not always be reflected in their instructional practices (Artzt et al., 2008). For example, it was observed that although several teachers' acknowledged beliefs about teaching mathematics were highly compatible with the view recommended by the NCTM (1989), their classroom practices put forth otherwise (Artzt & Armour-Thomas, 1998). A possible explanation for this is "the readiness of the pre-service teacher's existing cognitive structure to accommodate experiences with perplexities" (Artzt et al., 2008, p.23). Other researchers have also observed similar contrast between teachers' professed beliefs and their practices in the classroom (Cohen & Ball, 1990; Cooney et al., 1998; Franke et al., 1997). Nevertheless, any discussion on the development of mathematical literacy cannot be considered separately from the discussion over teachers' beliefs or, more generally, teachers' conceptions about mathematical literacy. Accordingly, the following three aspects of teachers' conceptions about mathematical literacy were explored in this study while analyzing the data: (i) teachers' conceptions about what the nature of mathematical literacy means, (ii) teachers' conceptions about how effective development of mathematical literacy could be facilitated, and (iii) teachers' conceptions about what a mathematics curriculum emphasizing mathematical literacy should look like.

CHAPTER III

METHODOLOGY

This chapter provides a description of the research methodology that was used to explore secondary mathematics teachers' conceptions of mathematical literacy. The chapter begins with the overview and rationale for the research design taken in this study. It then continues with the description of the study participants and the criteria that were used in selecting them. Next, it describes the data collection methods that were used in the study. Later, a detailed explanation of the analysis of research data is given. Finally, the chapter concludes with a discussion to establish trustworthiness and ethical issues of the study.

3.1. Research Design

Qualitative research is more convenient to investigate people's views, emotions, beliefs and mental structures in depth that emerge through their experiences (Hogan et al., 2009). As the purpose of the research study requires to analyze the understanding of secondary school teachers' conceptions regarding mathematical literacy, qualitative research in an exploratory vein is, accordingly, well convenient for this study as it allows for rich descriptions and understanding of the phenomena that best represent the essence of teachers' conceptions (Patton, 2002). In other words, this type of research design enables the researcher to discover and identify more accurate and realistic perceptions and opinions about mathematical literacy, support inductive exploratory strategies for examining these opinions, and finally produce a richly descriptive end product (Creswell, 2009). Hence, to realize the purpose of the study, an exploratory case study of qualitative nature where the focus of the study had already been stated and described above was used. The exploratory case study design was well-suited to this study because of its ability to answer the research questions appropriately by allowing "a fine-tuned exploration of complex sets of inter-relationships" (Edwards & Talbot, 1999, p.51). Specifically, it is generally preferred strategy to characterize, evaluate and interpret the complexity of beliefs or conceptions when ‘what’ questions are being posed (Yin, 2009). According to Cohen et al. (2007), an exploratory case study design unfolds and reports the complex interactions of beliefs and other factors in unique and dynamic contexts. As Merriam (2002) puts it simply, it refers to intensive analysis of the phenomenon in order to acquire a comprehensive and detailed understanding of the event by developing more explicit theoretical statements about the phenomenon under investigation. This directed me to select the exploratory case study methodology for this research study in order to excel at bringing us to an understanding of a complex issue of conceptions of mathematical literacy through the eyes of the secondary mathematics teachers by providing detailed contextual analysis of their stated beliefs, thereby allowing us to extend the field or adding strength to what is already known through previous research studies. As a result, the value of such exploratory research is twofold. Firstly, it formalizes hypotheses about the conceptions of mathematical literacy that can be further investigated and tested by quantitative methods with other researchers. In this manner, it serves to provide evidence for further research opportunities through rich descriptions and understanding. Secondly, it produces tentative explanations and interpretations of the poorly understood phenomenon of the secondary mathematics teachers’ conceptions about mathematical literacy, and then its results can provide extensive implications for researchers and policymakers worldwide. Therefore, this research study lends itself well to the use of exploratory case study design, where the case of interest is the secondary mathematics teachers’ conceptions about mathematical literacy.

3.2. Participants

A qualitative research study entails large volumes of data collection as well as high quality data. This, therefore, necessitates in-depth study with appropriate participants (Cohen et al., 2007). For this study, a convenience and purposive sampling method was used to select 16 mathematics teachers from 9 different secondary schools in various types in Eregli such as Science High Schools, Anatolian High Schools, Vocational and Technical High Schools, Anatolian Imam Hatip High

Schools and Private High Schools. Major demographic characteristics of the participants are given in Table 3.1 below in more detail.

Table 3.1: A Demographic Profile of the Participants

Name	Gender	Graduation	Type of School Taught	Years in Teaching	Master Degree
Tchr1	Male	Faculty of Education	Vocational and Technical High School	23	Not yet
Tchr2	Male	Faculty of Science	Private High School	8	Not yet
Tchr3	Female	Faculty of Science	Vocational and Technical High School	13	Not yet
Tchr4	Male	Faculty of Education	Anatolian High School	24	Continued
Tchr5	Male	Faculty of Education	Vocational and Technical High School	18	Continued
Tchr6	Female	Faculty of Science	Anatolian High School	19	Received
Tchr7	Male	Faculty of Science	Anatolian Imam Hatip High School	14	Not yet
Tchr8	Male	Faculty of Education	Science High School	28	Not yet
Tchr9	Male	Faculty of Education	Vocational and Technical High School	18	Not yet
Tchr10	Male	Faculty of Science	Science High School	23	Not yet
Tchr11	Female	Faculty of Education	Anatolian High School	17	Not yet
Tchr12	Male	Faculty of Science	Anatolian High School	14	Continued
Tchr13	Male	Faculty of Science	Anatolian High School	18	Continued
Tchr14	Male	Faculty of Education	Anatolian High School	12	Continued
Tchr15	Male	Faculty of Education	Anatolian High School	22	Not yet
Tchr16	Female	Faculty of Education	Anatolian Imam Hatip High School	5	Continued

The sampling was approvingly convenient as sixteen teachers were both a fairly manageable and accessible sample for one researcher and offer a sufficient opportunity to gather in-depth qualitative information. Through purposive sampling, from almost every type of school, only Grade 9 or 10 teachers were chosen to participate with the prerequisite that the teacher had at least five years of teaching experience. The rationale for using the Grade 9 or 10 teachers in this study was that about 93% of students who participated in PISA 2012 study in Turkey were Grade 9 or 10 students (MEB, 2013b). Accordingly, as of the beginning of 9^{th} or 10^{th} grade teaching, it was important to ascertain what beliefs, meanings and preferences mathematics teachers held about the idea of mathematical literacy. Therefore, from this sample, it was expected that very valuable and useful information would be gathered regarding secondary mathematics teachers' conceptions of mathematical literacy.

3.3. Data Collection

Secondary mathematics teachers' conceptions of mathematical literacy, basically their views, were the focus of this research study. As mentioned in Chapter I, Thompson (1992) refers to teachers' conceptions as "a more general mental structure, encompassing beliefs, meanings, concepts, rules, mental images, preferences, and the like" (p.130). Since the purpose of this study is to see the world through the eyes of the participants and to understand more about their views, mental structures and beliefs, as well as meanings and interpretations they make out of their experiences, one-to-one interviews were thought to be the most appropriate source of data collection for this research (Merriam, 2002). This source was also supplemented with task-based interviews. The use of task-based interviews as data collection techniques allowed me to improve the quality of data and increased the trustworthiness of the study (Merriam, 2002).

3.3.1. Pilot study

Prior to collecting data for the main study, a pilot study was also conducted with two voluntary secondary mathematics teachers who were not involved in the actual study in order to assess the extent to which the interview questions could elicit responses that would address the study's research questions. After the pilot interviews, I found that there were several things that needed to be adjusted in terms of the interview schedule as well as my performance and interactions with respondents as an interviewer. First, it was noticed that there were too many questions for each interview scheduled for about an hour. In order to use the interview time wisely and give the participants enough time to think about and reflect on their responses, I realized that interview questions were required to be simplified and reduced in number so that they could best represent the purpose of the study. For example, one question, “What needs to be done in order to make students gain the fundamental skills and competences for mathematical literacy?”, was actually the same as another question, “What is the role of mathematics teachers in the development of mathematical literacy?”. The second question apparently implies the educational strategies for mathematical literacy development, assessment methods for mathematical literacy, professional responsibilities for mathematical literacy development, teachers’ disposition towards mathematical literacy understanding, planning and preparation for mathematical literacy development, and the appropriate classroom environment for mathematical literacy development. Both pilot study participants answered these two questions in a quite similar way, so the former one was eliminated and the latter one was decided to be asked in the actual interviews as it could considerably offer a better opportunity for in-depth and descriptive responses from teachers. Another thing I learned from the pilot study was that participants were undecided or reluctant to give answers to some interview questions. Accordingly, these questions which were cumbersome and inhibiting were rephrased or reviewed for greater clarity. For example, I realized that they did not understand what I meant by the question, “What might be the similarities or differences of a separate curriculum emphasizing mathematical literacy from a current mathematics curriculum?”. The participants’ reactions showed me that I had to revise this question to “Is it necessary to prepare a separate curriculum in the context of mathematics literacy? Why?” in order to avoid any confusion and misunderstanding. During the pilot study, it was also realized that presenting a series of interview tasks proved to be successful in stimulating participant discussion by challenging teachers’ beliefs related to mathematical literacy. Hence, it also gave me another opportunity to better structure the set of interview tasks based on the results obtained in the pilot study. Finally, a pilot study also provided me a suitable platform to become familiar with interviewing techniques including such practical issues as how to inform the participants sufficiently about the purpose of the research at the beginning, their role within it, and the procedures to be followed during the interview as well as handling the audiotaping equipment.

3.3.2. Interviews

Three semi-structured interviews, each lasting approximately one hour, were conducted with each participant. The interview schedule was semi-structured with open-ended questions to acquire rich-descriptive data regarding the purpose of the study in a flexible manner during the interviews. Accordingly, these interviews had provided me an insight into "what a person knows (knowledge or information), what a person likes and dislikes (values and preferences), and what a person thinks (attitudes and beliefs)" (Tuckman, 1994, p.216). Moreover, more than one interview session with each participant was much more effective to get a relatively true account of teachers' conceptions of mathematical literacy as it further explores and uncovers more about "what is inside a person's head" (Tuckman, 1994, p.216). In other words, by meeting individually with each teacher three times, issues were revisited and understandings were much clarified. Interviews with any participant were also spaced a minimum of one week apart to allow both the interviewer and the participant some time for reflection. However, in order to develop and identify relations between interview sessions, interviews were also tried to be scheduled no further apart than two weeks. All teachers selected their respective school as their interview site and all interviews were conducted during the leisure time of participants in the school day.

As mentioned above, the data was gathered in three distinct stages with a different focus. The first interviews focused on teachers' conceptions about the notion of mathematical literacy. In particular, the teachers were asked to comment about the nature of mathematical literacy, the fundamental mathematical capabilities for mathematical literacy, and the relationships between mathematics and mathematical literacy in general. The second interviews focused on teachers' conceptions about how effective development of mathematical literacy could be facilitated. In particular, the teachers were asked to comment about the barriers to development of mathematical literacy, and the central domains for mathematical literacy development. As a result of the initial analysis of the first set of data, the second interviews also included follow-up questions with regard to the notion of mathematical literacy. The last interviews focused on teachers' conceptions about what a mathematics curriculum emphasizing mathematical literacy should look like. Similarly, after the initial analysis of the set of data from the first and second interviews, the third interviews also involved follow-up questions relevant to how effective development of mathematical literacy could occur.

Three semi-structured interview guides were also developed so that it could focus on the purpose of the study (see Appendix A). The first interview guide involved a series of open-ended questions and appropriate probes based on the teachers' conceptions about the notion of mathematical literacy. For example, one sample question and its probe from the first interview guide asked respectively: "What do you understand from mathematical literacy? e.g., what can be the purpose of mathematical literacy?". Second interview guide included a set of questions each with several probes regarding the teachers' conceptions about how effective development of mathematical literacy could be facilitated. Here is one sample question with its probe from the second interview guide: "What might be the main barriers encountered in the development of mathematical literacy? e.g., what might be the possible barriers due to teacher qualifications?". The third interview guide consisted of a list of main questions and prompts related to the teachers' conceptions about what a mathematics curriculum emphasizing mathematical literacy should look like. For instance, one sample question and its prompt from the third interview guide asked respectively: "What should mathematics curriculum emphasizing mathematical literacy look like? e.g., what should be the overall aims of mathematics curriculum emphasizing mathematical literacy?". Using a semi-structured interview guide, accordingly, enabled me to cautiously search for recurring phrases, themes and commonalities across participants. Interview questions were also crafted general enough to allow participants the freedom to answer in whatever direction they chose, and probes were used here to seek more details, clarifications, or examples (Merriam, 1998). This enabled to gather more information from the participants related to the purpose of the research. Questions on the protocol were organized categorically, but changes in ordering during the interview were always possible to maintain conversational flow. In total, 48 interviews, each lasting about one hour, were recorded by an audio recorder and the recordings were transcribed verbatim and coded afterwards. When necessary, notes were taken during the interviews so as not to interrupt the respondent at an inappropriate time, and any statement or idea that required further probing was written down to pace the interview. Any observation or summary point that came to mind was also noted immediately after each interview. Finally, all the data collected from interviews were integrated to interpret and explain the research phenomenon.

3.3.2.1. Interview tasks

During each interview, participants were also handed one task, and were asked to think aloud when performing the tasks. Accordingly, three interview tasks were designed to reveal a rich, more comprehensive and increasingly nuanced understanding of the teachers' conception of mathematical literacy. The first task during the first interview, 'Definition of Mathematical Literacy', included various types of definitions of mathematical literacy from the literature to probe teachers' conceptions of the notion of mathematical literacy from a variety of standpoints in order to ascertain how they see the concept of mathematical literacy (e.g., Benn, 1997; Cockcroft, 1982; Department of Basic Education, 2011; FitzSimons et al., 1996; Gal, 2000; Hope, 2007; Jablonka, 2003; Johnston, 1994; Marsh, 2011; McCrone et al., 2008; National Numeracy Review Report, 2008; OECD, 2103a; Powell & Anderson, 2007; Tout, 2001). Here are some sample definitions taken from Task 1: "Mathematical literacy is the basic level of mathematical knowledge and skills", "Mathematical literacy is the ability to express a problem situation mathematically", and "Mathematical literacy is the ability to use mathematics in a functional way in everyday life". The intent of the second task during the second interview, 'Appropriate Types of Problems for Mathematical Literacy Development', was to uncover how teachers conceive the most preferred types of problems in our mathematics education system as well as the types of problems most appropriate for gaining necessary mathematical literacy skills. One sample type of problem from Task 2 requires students to know and apply certain mathematical formulas and rules in order to solve it. Here are two examples: "Solve 2x + 3 = 7" and "Find the volume of a box with sides 3 meters, 4 meters and 5 meters" (OECD, 2014a). Participants were asked to answer how often they use these types of problems in their mathematics classes as well as the extent to which they use these types of problems in their assessments. Finally, the last task during the third interview, 'Curriculum Approach for Mathematical Literacy Development', aimed to surface how teachers viewed the implemented curriculum approach in the present mathematics education system in the context of mathematical literacy as well as the appropriate curriculum approach for a strong and sustainable mathematical literacy development. In this context, two sample curriculum approaches were presented and each participant was asked to discuss on these approaches or on their own approaches. These two approaches given in Task 3 were: "the curriculum with emphasis on definitions, formulas, computations, examples, and applications" and "the curriculum with emphasis on problems, discovery, hypothesis, verification, generalization, association, and inference" (MEB, 2013a). Depending on the participant's evaluation, each task lasted approximately between 5-10 minutes. Every teacher was asked to read each task aloud and to talk out loud while completing the task. The teacher and the researcher engaged in a continuous conversation rather than the participant works alone and talks to oneself. The text and flowcharts of each task appear in Appendix B.

3.4. Data Analysis

The data analysis of the study consisted of "organizing, accounting for and explaining the data; in short, making sense of data in terms of the participants' definitions of the situation, noting patterns, themes, categories and regularities" (Cohen et al., 2007, p.461). In this sense, the coding scheme was initially guided by the conceptual framework and then was expanded based on an ongoing analysis of the data. Prior to the data collection, a list of external codes that were derived from the review of the literature and the conceptual framework of this study was developed (e.g., Concept of Mathematical Literacy, Effective Development of Mathematical Literacy, Mathematics Curriculum Emphasizing Mathematical Literacy). It was then intended to open-code the data by examining and organizing the material and identifying and labeling the patterns and themes that emerged from the data. Accordingly, as the data were being examined, the external codes were then supplemented with a set of internal codes (e.g., Nature of mathematical literacy, Fundamental mathematical capabilities for mathematical literacy, Relationships between mathematics and mathematical literacy, Barriers to development of mathematical literacy, Central domains for mathematical literacy development, Challenges in curriculum implementation in the context of mathematical literacy, Recommended curriculum modifications in relation to mathematical literacy). After proposing these specific internal codes, the data sets were then re-examined, and classified into new sub-categories (e.g., Possession of mathematical knowledge and skills, Functional mathematics, Problem solving, Mathematical thinking, Innate mathematical ability, Conceptual understanding, and Motivation to learn mathematics). These internal codes were generated by following the steps set forth by Cohen et al. (2007) for a high-quality case study analysis. These steps are set out below as the following. Firstly, building units of analysis of the data, which demonstrates how these units are similar to and different from each other. Secondly, creating a domain analysis, which emphasizes to arrange items and units into related clusters, themes and patterns. Thirdly, constructing connections between the domains by searching principal associations and relations between data subsets. Fourthly, making significant inferences, on the basis of the evidence, to hypothesize some key statements related to crucial elements of the event, and possibly even their causes, which is indeed the process of hypothesis generation that fosters theory generation. Fifthly, summarizing, which involves an overview and initial report of the central issues, major specifications, principal concepts, constructs and ideas obtained so far in the research. Finally, generating theory, which is grounded in the data and derives from it.

The qualitative data analysis software NVivo10 was also used to aid in coding and categorizing information to generate the common themes and patterns of practice in the study as it had the "ability to combine efficient management of non-numerical, unstructured data, with powerful processes of indexing, searching, and theorizing into relevant themes" (Creswell, 2005, p.237). Uploading each participant interview transcript data into NVivo10 not only helped me organize and interpret the data into thematic representations but also provided an opportunity for cross-participant data analysis. Possible influences in the interpretations of the data were also prevented by comparing the data in the context of the interviewees' responses. The use of this software allowed me to eliminate bias and ensure the accuracy of the research study. In this sense, the data was primarily analyzed according to the main themes that had been identified in the conceptual framework of this study. As said before, first of all, an initial list of codes was developed based on the conceptual framework, but as the research continued, inductive codes generated by the data were also be added and used in order to explore "undiscovered patterns and emergent understandings" (Patton, 2002, p.454). Through inductive analysis, emerging new patterns, themes and categories in the data were discovered which had also contributed towards possible implications of the research study. All audio data were transcribed verbatim to text data immediately after the data were collected. Having assigning the initial list of codes to all transcripts, an audio recording of each interview was listened again while reviewing all transcripts and codes. In this way, a content analysis of the transcripts was completed to identify common themes and ideas. The initial coding list with the emerging new patterns were then revised so that they could align with the actual data. This process enabled me to verify the accuracy and internal consistency of the coding system either by combining similar codes when redundancies occurred or by removing codes assigned to passages that were tangential to the study. Participants were also asked to elaborate or disagree with anything in the transcriptions in order to identify and minimize any possible bias. In this sense, 14 of them provided feedback orally and expressed no disagreements with their transcripts and my interpretations from these transcripts.

Furthermore, while coding all transcripts independently, it was important to verify that the codes assigned were meaningful, logical, and consistent with those that other readers could assign by inviting a researcher who was familiar with the same issue. In that respect, the ratio of the number of agreements to the sum of the number of agreements and disagreements was used as a measure of inter-rater reliability (Miles & Huberman, 1994). Therefore, to facilitate inter-coder reliability, one mathematics education researcher with more than 10 years of experience in analyzing qualitative data was asked to act as an external rater. Thus, during the final stage of analysis, in addition to expert review, this mathematics education researcher and I double-checked codes on all transcripts and reexamined emerged themes and categories. The comparison of the two codings resulted in an average inter-rater reliability of about 75%, which was fairly strong enough. We resolved all disagreements by examining the mismatched coding, making several changes to code definitions in order to reach 100 percent inter-coder agreement for the categories and confer thoroughness and credibility to the analysis. For example, my external rater advised me to merge

‘Meaning of mathematical literacy’, ‘Importance of mathematical literacy’, and ‘Purpose of mathematical literacy’ categories into a single category as there were significant overlaps among them. Accordingly, these three categories were merged into a single category of ‘Nature of mathematical literacy’. Similarly, the categories ‘Role of students in acquisition of mathematical literacy’ and ‘Role of teachers in acquisition of mathematical literacy’ were merged into a single category and assigned to the ‘Educational strategies for mathematical literacy development’. The external coder also advised to insert the sub-categories ‘Educational strategies for mathematical literacy development’, ‘Assessment for mathematical literacy’, ‘Professional responsibilities for mathematical literacy development’, ‘Disposition towards mathematical literacy understanding’, ‘Planning and preparation for mathematical literacy development’, and ‘Classroom environment for mathematical literacy development’ into the new main category name of ‘Central Domains for Mathematical Literacy Development’ in order to make these sub-categories more organized and readable. Moreover, the category ‘Mathematical Literacy in Current Curriculum’ was removed as the focus of the inquiry was on the general views of teachers about the mathematics curriculum emphasizing mathematical literacy rather than their specific perceptions about the current mathematics curriculum. In that respect, the expert rater advised the inclusion of two new category names related to mathematics curriculum emphasizing mathematical literacy including ‘Challenges in curriculum implementation in the context of mathematical literacy’, and ‘Recommended curriculum modifications in relation to mathematical literacy’. As a result, both coders agreed upon the revised coding matrix that was used for the final analysis. It is clear therefore that this also enhanced the reliability of research data during the analysis process, which was necessary and indispensable to this qualitative inquiry.

3.5. Trustworthiness

Validity and reliability are two key areas in order to judge and validate the quality, adequacy, or goodness of any quantitative research. While validity is connected with “the appropriateness, meaningfulness, and usefulness of the inferences, reliability is pertinent to “the consistency of these inferences over time, location, and circumstances" (Fraenkel, Wallen, & Hyun, 2012, p.458). These two concepts are normally very important in quantitative research studies, but their meanings vary in naturalistic work. These traditionally used terms of validity and reliability are replaced with the alternative concept of trustworthiness in qualitative research studies. Therefore, in place of a discussion of such positivist criteria of internal validity, external validity, reliability, and objectivity, the use of four parallel naturalistic terms of credibility, transferability, dependability, and confirmability is respectively considered as more appropriate for establishing the trustworthiness of this qualitative research study (Lincoln & Guba, 1985; Shenton, 2004).

First of all, the parallel criterion credibility (vs. internal validity) is one of the most important factors for ensuring the trustworthiness of a study. Credibility refers to whether or not the research findings represent an honest and authentic interpretation of the participants' original views through depth and rich information derived from the data of the study (Anney, 2014; Lincoln & Guba, 1985). This is the term used in place of internal validity. A series of strategies can be used to accomplish the credibility of a naturalistic study including prolonged engagement in the field or research site, triangulation, peer debriefing, and member checking. First of all, in the current study, three semi-structured interviews, each lasting approximately one hour, were conducted with each participant. Interviews with any participant were also spaced a minimum of one week apart to allow both me and the participant some time for reflection. Before the first interviews take place, several preliminary visits to participants were done to learn and develop an adequate understanding of their culture and context. In these visits, they were invited to take part in the study by giving information about the purpose of the research, their role within it, the procedures to be followed during the investigation. Thus, participants also had the opportunity to get to know more about the researcher. Accordingly, having close and continuous dialogue with participants throughout the research data collection process in about three months or even more helped to establish a relationship of trust and rapport between the researcher and the participants. Accordingly, "as rapport increases, informants may volunteer different and often more sensitive information than they do at the beginning of a research project" (Krefting, 1991, p.217-218). Hence, this period of prolonged engagement allowed me to develop more sense about the nature of the research data. Secondly, in addition to the prolonged engagement, another most well-known method that used as a way to ensure the credibility was the triangulation. There are four different modes of triangulation which are the use of multiple methods of data collection, multiple sources of data, multiple investigators, or multiple theories (Lincoln & Guba, 1985). In this study, a wide range of participants were involved. 16 secondary mathematics teachers were used for the informants as different data sources. Thus, a wide spectrum of experiences and perspectives of participants were validated against each other and, eventually, "a rich picture of the attitudes, needs or behavior of those under scrutiny may be constructed based on the contributions of a range of people" (Shenton, 2004, p.66). Moreover, site triangulation may also be achieved by the participation of mathematics teachers from 9 different secondary schools in various types such as Science High Schools, Anatolian High Schools, Vocational and Technical High Schools, Anatolian Imam Hatip High Schools and Private High Schools in order to reduce or prevent the effect of typical local factors specific to one school type. Hence, "where similar results emerge at different sites, findings may have greater credibility in the eyes of the reader" (Shenton, 2004, p.66). The other crucial process that used for improving the credibility of the qualitative data was member checking technique (Guba & Lincoln, 1989). In that respect, all participants were asked to review their transcripts and my interpretations from these transcripts for accuracy after all interviews were conducted. 14 of them expressed their willingness in this regard. Here the purpose was to give them an opportunity to compare what they said during the interviews was the same as what they actually intended and if necessary suggest changes, clarify, or expand their words. Participants provided feedback orally and expressed no disagreements with their transcripts and the interpretations from these transcripts. In this way, the data were crosschecked by participant feedback that allowed the teachers an open and honest discussion on my interpretations of their views. Thus, such checks enhanced the verification of the findings and helped ensure that the study's results were an accurate reflection of participants' views at the time of each interview. It also helped me to eliminate the researcher bias when analyzing and interpreting the results (Merriam, 2002). Finally, the peer debriefing technique was used for ensuring the credibility of the research data (Lincoln & Guba, 1985). Apart from my advisor and the other dissertation committee members, one academic colleague who was familiar with the research interest and has had experience in the field of mathematics education for more than 10 years served as the peer debriefer. Through the questioning and scholarly guidance of this debriefer, the quality of the data collection and analysis was improved. During the research process, his feedback and opinions on the research progress were always taken into consideration and this also helped me ensure that alternative theoretical explanations for the study were not ignored.

Another parallel criterion transferability, which replaced the positivist measure of external validity or generalizability, for establishing trustworthiness of a qualitative study addresses the core issue that the findings and conclusions of qualitative research can be applied to other contexts with other participants (Linconln & Guba, 1985). In fact, transferability or external validity, focusing on the generalizability of the findings, was not a priority of this study as generalizing the data was not an objective of qualitative research, but rather that of obtaining a good in-depth knowledge of the events in the context in which they occurred (Cohen et al., 2007). Nonetheless, it is the responsibility of the researcher to ensure "as complete a data base as humanly possible in order to facilitate transferability judgments on the part of others who may wish to apply the study to their own situations" (Guba & Lincoln, 1989, p.242). Thus, in this study, the thick description and purposive sampling techniques were used to facilitate the transferability of the study (Erlandson et al., 1993; Guba, 1981). In this sense, the typicality of the research context was established by explaining all the processes in detail from data collection to production of the final report so that others could replicate the study with similar conditions in other settings. Additionally, by setting definitive sample criteria based on the specific purpose that emphasized the uniqueness of the participants in this context, other researchers could have a strong foundation for comparisons with similar situations. Accordingly, transferability of this study was ensured through 'thick description' and 'purposeful sampling' by providing sufficient contextual information about the research setting and participants for the readers or practitioners who might intend to apply the study's findings to similar situations and contexts (Miles & Huberman, 1994; Creswell, 2009).

The other parallel criterion dependability, which is used in place of the reliability, for addressing trustworthiness of a qualitative study refers to the consistency and repeatability of a research study across time. It is the extent to which similar results would be obtained as much as possible if the inquiry was repeated with the same or similar methods, with the same or similar participants under the same or similar conditions (Linconln & Guba, 1985). Lincoln and Guba (1985) argue that the many techniques used to accomplish credibility issue may also serve for ensuring dependability of the study as there are close ties between credibility and dependability. However, there are also some additional techniques including using an audit trail, a code-recode strategy, and peer examination (Anney, 2014; Guba & Lincoln, 1989). In the current study, through maintaining an audit trail, that is, "a detailed chronology of research activities and processes; influences on the data collection and analysis; emerging themes, categories, or models; and analytic memos" (Morrow, 2005, p.278), the threats to dependability of the study were tried to be addressed. In other words, the researcher kept a detailed record of how and when data were collected, how the data gathered were labelled and the revisions made during each step of the analysis, as well as how many times and under what circumstances participants were contacted. In such a case, the objective was to maximize the likelihood that subsequent researchers following the same procedures in similar domains could obtain similar data and arrive at similar conclusions. Another method to improve the dependability of the study was to apply code-recode strategy suggesting that whether another rater with the same theoretical framework could have interpreted the same phenomena in the same way, which is called the degree of agreement for coding between the users (Anney, 2014). This strategy is also referred to as inter-coder reliability (Cohen et al., 2007). That is to say, "rather than demanding that outsiders get the same results, one wishes outsiders to concur that, given the data collected, the results make sense they are consistent and dependable" (Merriam, 1988, p.172). Thus, as mentioned in the data analysis section, one external coder who also acted as my peer debriefer from mathematics education department and I cross-checked the emerged categories and patterns against the interview transcripts during the final stage of analysis; we resolved all discrepancies in coding through discussion, and reached a consensus in order to maximize the reliability of the coding procedures. In that respect, using the qualitative analysis software NVivo allowed me to easily review and rearrange the codes by identifying the missing areas and refining the interpretation of the research data. Hence, it can be said that the use of NVivo10 software also helped to ensure the dependability of the study by increasing the accuracy of the data analysis process. The other technique for establishing the dependability of this study was to use the peer examination strategy. In this sense, one doctoral student who was doing qualitative research in education helped me become honest about the study during the development of research process. For example, the interview schedules were carefully examined by this doctoral student and some questions that were difficult to understand were rephrased for greater clarity. Besides, he also helped me to identify some categories not covered by me during the data analysis process.

Finally, confirmability issue (vs. objectivity) is concerned with the acceptance of the fact that research is "judged in terms of the degree to which its findings are the product of the focus of its inquiry and not of the biases of the researcher" (Lincoln & Guba, 1985, p.290). In quantitative studies, investigators are expected to draw inferences in an objective manner. However, in naturalistic inquiries, ensuring real objectivity is very difficult as some biases are eventually inevitable. Many of the strategies used to achieve dependability are also applicable for ensuring confirmability (Bowen, 2009). In this sense, a number of important strategies were adopted to accomplish the goal of addressing the confirmability issue of the current study. Particularly, the audit trail used for establishing dependability also helped to ensure conformability of the study. The confirmability audit for this study involved the audiotaped interviews and their transcriptions, data reduction and analysis notes and the results of the pilot study. In this context, the role of the audit trail in promoting confirmability was essential for managing the researcher subjectivity. Another strategy for minimizing the effect of researcher bias and achieving maximum objectivity was to consult with the peer debriefer who helped me ensure as far as possible that the research findings were the results of the participants' perceptions rather than my rigorous characteristics and preferences when analyzing and interpreting research data. The other strategy for limiting or controlling subjectivity was to engage in member checks to learn from the interviewees how well the analysis and interpretation of the data reflected their meanings. They accordingly justified that the results and conclusion of the study represented an honest interpretation of the research data which was based on their voices and not on the researcher's bias.

3.6. Ethical Considerations

Some certain issues which raised several concerns needed to be addressed during the conduct of this inquiry. In that respect, one of the major issues to deal with before starting any research study is clearly the informed consent, that is, individuals should be allowed to agree or refuse to take part in research in the light of comprehensive information concerning the nature and purpose of the research (Cohen et al., 2007). Accordingly, informed consent was orally obtained from each participant, in addition to request for a written permission from all the relevant authorities and schools to conduct interviews in schools before the study proceeded (see Appendix C). In this respect, all teachers were invited to take part in the study and the amount of information was given to ensure that each was sufficiently informed about the purpose of the research, their role within it, the procedures to be followed during the investigation, the foreseen benefits as well as information regarding confidentiality, anonymity and possible risks or harms involved in taking part in the study. Understandably, voluntary participation in research must be based on full understanding of the possible risks involved, that is to say, "its intention is to ensure that human subjects are aware that they are taking part in research with all its hazards and that their participation is voluntary" (Homan, 2002, p.25). In this sense, it was highly unlikely that any of the participants was physically or psychologically harmed during this research study. The only possible harm participants might have experienced was the feeling of anxiety or discomfort in sharing their knowledge and beliefs during the interviews that were audiotaped. Accordingly, all related aspects of what was to occur or what could occur were disclosed to the participants. In this way, the agreement to take part in the study became voluntary, free from coercion and any influence (Homan, 1991). In other words, "good practice should include a continual review of consent to ensure that people remain happy with their involvement" (France, 2004, p.184) and the potential usage of "the right to withdrawal has to be emphasized regardless of how uneasy we might feel about losing our cohort" (ibid, p.184). Correspondingly, the teachers were also not obliged to take part until the end of the study but instead had a choice to participate knowing that they could withdraw at any stage. Such honesty was necessary for establishing a trusting relationship with the participants, which was essential for this qualitative research study in order to express the concern for truthful and respectful exchanges between me and the participants.

Ethical standards also require to preserve confidentiality which means that "although researchers know who has provided the information or are able to identify participants from the information given, they will in no way make the connection known publicly" (Cohen et al., 2007, p.65). In other words, "the boundaries surrounding the shared secret will be protected" (ibid, p.65). Hence, in order not to violate participants' right to privacy during the course of an investigation, the promise of confidentiality was also maintained throughout the study. For the purpose of ensuring confidentiality, names of the participants were not mentioned during the dissemination phase of the study, but instead the teachers were numbered. The interviews were also conducted in a private environment. The audiotaped interviews by means of which the participants could be identified were accessible only to me. Furthermore, before finalizing research report, teachers were also asked to review it in order to guarantee confidentiality.

CHAPTER IV

RESULTS

This chapter reported the results of the study with respect to the secondary mathematics teachers' conceptions about mathematical literacy. Firstly, with various data visualization possibilities available in NVivo, the identified categories were shown in figures in order to make research data much easier to interpret. Secondly, all teachers together were also listed on a single chart in order to display who commented on which specific emergent categories with their corresponding frequency counts based on the number of study participants (N=16) who expressed their thoughts about these particular categories. This informs us about which category the participants talked about the most at each moment, which also shows the strength of this category in a given context. Finally, an explication of each category by inclusion of direct quotes and paraphrases from interview participants were provided in the presentation of the findings. The quotations from interviews were written by assigning a number to each participant.

To remind again, the following research questions guided this study:

1) What are secondary mathematics teachers' conceptions of the notion of mathematical literacy?

2) What are secondary mathematics teachers' conceptions about how the development of mathematical literacy could be facilitated?

3) What are secondary mathematics teachers' conceptions about what a mathematics curriculum emphasizing mathematical literacy should look like?

The research questions were answered through analyzing the responses to the interview questions about secondary mathematics teachers' conceptions of mathematical literacy. In this regard, three major themes addressed in this study were: (i) Concept of Mathematical Literacy, (ii) Effective Development of Mathematical Literacy, and (iii) Mathematics Curriculum Emphasizing Mathematical Literacy. Accordingly, the findings of the study were organized around the three aforementioned research questions. The opening section described teachers' conceptions about what the notion of mathematical literacy means. Next, teachers' conceptions about how the effective development of mathematical literacy occurs were described. The third section focused on teachers' conceptions about what a mathematics curriculum emphasizing mathematical literacy should look like. In order to elaborate and expand on these major themes, the relevant categories and their subcategories which were placed under each theme were also discussed in more detail later on the paper below.

4.1. Characterizing teachers' conceptions of the concept of mathematical literacy

The teachers' conceptions regarding what the notion of mathematical literacy means were discussed in the following three sections, with the first section focusing on their conceptions about the nature of mathematical literacy, the second section focusing on their conceptions about the fundamental mathematical capabilities for mathematical literacy and the third section focusing on their conceptions about the relationships between mathematics and mathematical literacy.

4.1.1. Teachers' conceptions of the nature of mathematical literacy

In analyzing the theme of the nature of mathematical literacy, the following seven categories were emerged from participants' responses: (i) Possession of mathematical knowledge and skills, (ii) Functional mathematics, (iii) Problem solving, (iv) Mathematical thinking, (v) Innate mathematical ability, (vi) Conceptual understanding, and (vii) Motivation to learn mathematics.

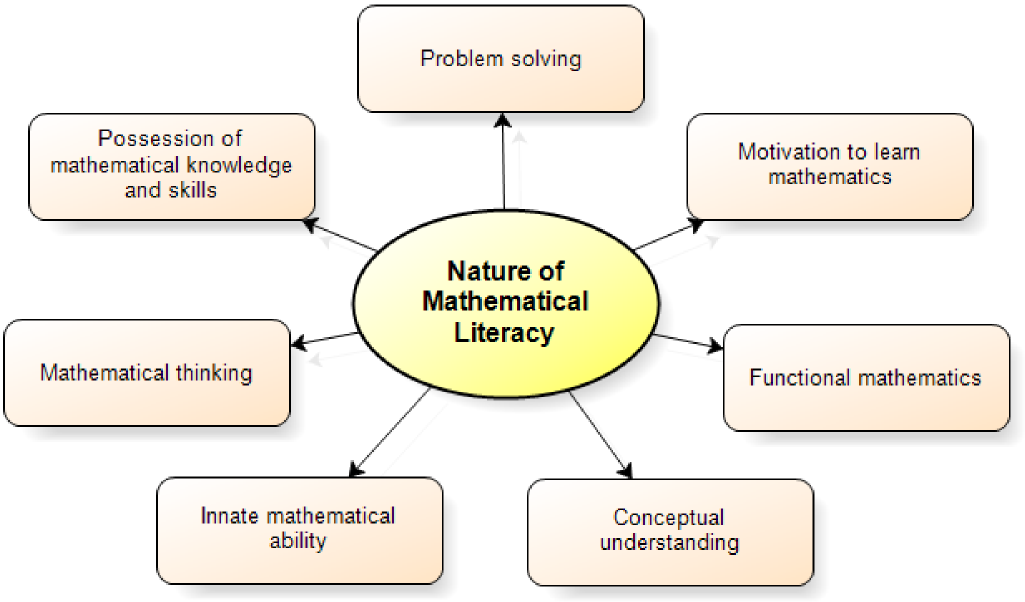

Figure 4.1: A Visual Depiction of Teachers' Conceptions of the Nature of Mathematical Literacy

In this sense, Figure 4.1 displays the visual depiction of the seven categories identified across the teachers' interviews. Moreover, in Figure 4.2 below, all teachers were listed on a single chart in order to indicate the strength or intensity of each category that was determined from their responses to interview questions regarding the nature of mathematical literacy. The numbers in parentheses next to each category represent the corresponding total number of the participants talking about this particular category. As presented in the figure, all of the teachers considered the nature of mathematical literacy as (i) Possession of mathematical knowledge and skills, and (ii) Functional mathematics. Besides, nearly all of the teachers based their overall view of the nature of mathematical literacy on (i) Problem solving, and (ii) Mathematical thinking. Moreover, quite a large number of teachers identified the nature of mathematical literacy as (i) Innate mathematical ability. For most of the teachers, the nature of mathematical literacy was also perceived as (i) Conceptual understanding. Furthermore, over half of the teachers viewed the nature of mathematical literacy as (i) Motivation to learn mathematics.

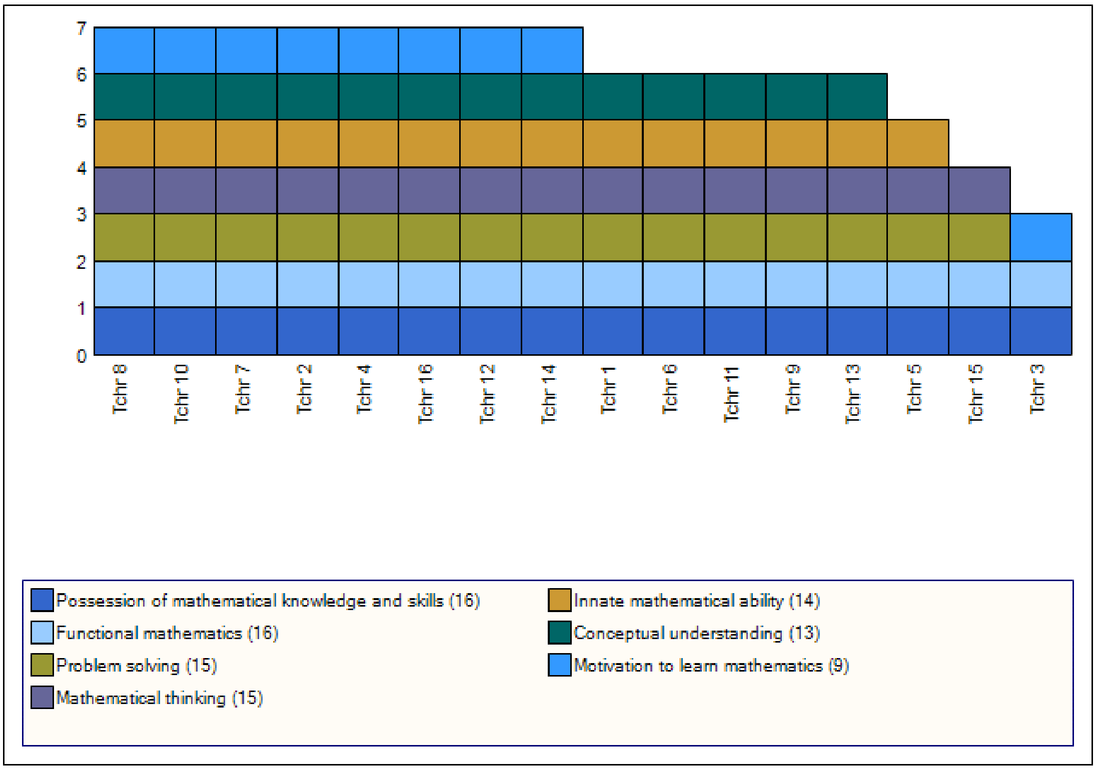

Figure 4.2: A Chart for Teachers' Conceptions of the Nature of Mathematical Literacy by Teachers

The following seven sections, accordingly, provided the results with respect to the emergent categories mentioned above in more detail.

4.1.1.1. Possession of mathematical knowledge and skills

In the analysis of the theme of the possession of mathematical knowledge and skills, several categories surfaced in participants' responses. These categories included: (i) Varying from low levels of mathematics to high levels of mathematics, (ii) Basic mathematical knowledge and skills, and (iii) Advanced mathematical knowledge and skills. In order to elaborate and expand on the above-mentioned categories, the discussions with the interview excerpts which were representative of these particular categories were provided below respectively.

4.1.1.1.1. Varying from low levels of mathematics to high levels of mathematics

Half of the teachers (n=8) stated that mathematical literacy is the possession of mathematical knowledge according to people's interests and desires in daily life. Correspondingly, they thought that while some people consider mathematical literacy as a basic level of mathematics required for the continuation of life, some others perceive it as advanced mathematics needed for scientific and technological developments. This means that mathematical knowledge needed for mathematical literacy changes according to needs and demands of people's personal, vocational and social lives. So, mathematical literacy is indeed for everyone. That is to say, *"…everyone may not read the same number of books but ultimately everyone has a certain level of literacy. Mathematical literacy is something like this"* (Tchr12). Therefore, teachers expressed that mathematically literate individuals need to know mathematics according to their needs and interests varying from the basic level to the upper level. This situation is related to the diversity of problems people facing in their life. The following excerpt is also typical of this particular point:

> …the problems of an astronaut facing are of course not the same as those of a cashier confronting in life. Clearly, the level of mathematical knowledge of the astronaut should be much higher than that of the cashier in order to perform scientific studies. This astronaut may become mathematically literate according to his level and needs. On the other hand, any municipal street sweeper who tries to clear the streets as soon as possible he can or any waitress who tries to meet the multiple demands of a lot of customers in the shortest time may also both become mathematically literate (Tchr4).

Therefore, according to this teacher, there should happen a mathematical literacy at all levels. Moreover, teachers also stated that mathematical literacy ensures the use of our basic knowledge and skills in advanced stages later on. More precisely, after receiving this basic knowledge and skills, mathematical literacy helps us to use this knowledge at an advanced level in our daily life. Therefore, as one of the participants said, *"…mathematical literacy is very important for those who want to improve their skills in mathematics to higher levels as well as for those who know little mathematics"* (Tchr1).

Hence, the participants generally define mathematical literacy according to people's needs and desires and there is surely not a certain mathematical literacy level for everyone. Depending on its usage, mathematical literacy can be an advanced or a basic level of mathematical knowledge and skills. Actually, both of them are essential for mathematical literacy. More clearly, the definition of mathematical literacy emphasizing basic knowledge and skills may be an accurate description for a part of the community, but it may become insufficient for the other parts. In other words, further mathematics level is quite necessary for some professions or branches. They use it for performing advanced mathematical operations or calculations, but ordinary citizens normally do not need to have advanced mathematics knowledge and skill. We do not expect them to know too much mathematics knowledge. What is important for the ordinary citizens is to know enough mathematics that they can use in their daily life, but if necessary they can also take steps toward more advanced mathematics knowledge and skills.

4.1.1.1.2. Basic mathematical knowledge and skills

Some of the teachers (n=6) held the view that mathematical literacy is a basic level of mathematical knowledge and skills necessary to solve the daily life problems in a variety of situations. According to them, mathematical literacy does not require any advanced knowledge of mathematics and skills because advanced mathematical knowledge is not much needed in everyday life. Otherwise, mathematical literacy becomes very distinct from everyday life. Surely, there are always some situations in which advanced mathematics is needed, but they think that it is not mathematical literacy. So, teachers do not believe that mathematical literacy has knowledge of advanced mathematics. Whenever somebody mentions mathematical literacy, the first thing that comes to their mind is the basic level of mathematical knowledge and skills that everyone should know in order to handle daily life problems. What they mean by the basic level here is the ability to employ basic mathematical algorithms and procedures. Students at this level are supposed to effectively perform basic arithmetical operations such as addition, subtraction, multiplication and division. For example, one of the teachers commented that mathematical literacy is to use basic mathematical knowledge to solve everyday life problems. In other words, as she stated, *"…it would be enough to know basic mathematical operations to become mathematically literate…there is no need to know advanced mathematical knowledge"* (Tchr3). Thus, according to these teachers, being mathematically literate means to have basic knowledge of mathematics. Namely, they considered mathematical literacy as the possession of basic level of mathematical knowledge and skills in order to meet the general demands and needs of daily living.

4.1.1.1.3. Advanced mathematical knowledge and skills

Finally, a few teachers (n=2) commented that mathematical literacy is an advanced level of mathematical knowledge and skills. They thought that we do not necessarily need to perform mathematics at the basic level in daily life since today's technological tools simply provide us with many facilities for performing it. For example, calculators, computers, mobile phones, and tablets already do many mathematical operations on our behalf. Therefore, these teachers do not believe that mathematical literacy is a basic level of mathematical knowledge, because everyone in a way learns this basic level of mathematics. They asserted that mathematical literacy is a different area of expertise in advanced mathematics. For instance, one of the teachers said that *"…it does not matter whether you know it or not, because you may somehow learn basic mathematics from your experience in everyday life. However, everyone does not learn mathematics at the advanced level"* (Tchr9). Accordingly, as another teacher remarked, *"…mathematical literacy is the ability to translate advanced mathematical structures into mathematical expressions by setting up complex mathematical equations"* (Tchr13). Therefore, according to these teachers, a good level of mathematical literacy requires people to have a higher level of mathematical knowledge and skills.

4.1.1.2. Functional mathematics

As teachers talked about theme of functional mathematics, several categories emerged from the analysis of their responses. These categories included: (i) Use of mathematics in everyday life, (ii) Use of mathematics in societal life, (iii) Use of mathematics in further education, and (iv) Use of mathematics in occupational life. In order to elaborate and expand on these categories mentioned above, the discussions with the interview excerpts which were representative of these particular categories were provided below respectively.

4.1.1.2.1. Use of mathematics in everyday life

Not surprisingly, all of teachers (n=16) expressed that mathematical literacy is being able to use mathematics in a functional way in everyday life. They all stated that mathematical literacy gives people to be able to effectively interpret daily life. For example, one of the participants mentioned, *"…mathematical literacy of course provides all people with the ability to use mathematics in everyday life functional as well as the ability to interpret life in advanced level"* (Tchr6). Teachers commented that mathematical literacy is to use mathematics to solve some of the challenges which take place in people's everyday life. For instance, one teacher expressed that *"…many things happen in our lives that we call daily life and mathematics is used to solve the difficulties that occur in those daily things"* (Tchr1). According to these teachers, mathematics is apparently a crucial part of our daily life. They all argued that using mathematics in day-to-day activities is vital in order to effectively function in today's modern society. If people fail to apply mathematics in their daily activities, they will possibly experience major problems in life. Hence, mathematical literacy requires individuals to use mathematics functional in everyday life. For example, one of the participants said that *"…why do we need mathematical literacy if we do not use it in our daily living? ...I think math literacy must have a knowledge of mathematics which is used functional in daily life"* (Tchr14). Similarly, another participant stated that it is called mathematical literacy that can help us in our daily living. She put it simply that *"…when somebody mentions mathematical literacy, what comes to our mind is the mathematics used in the bazaars, in the marketplace, at shopping centers, outdoors, or in general everyday math"* (Tchr3). This participant stated that there might be people who became high school graduates or primary school graduates. Or, there might be even other people who had never attended any school and they might be illiterate. Nonetheless, all people always try to reach the right or appropriate solution to any problem encountered in real life in practical ways by using their own knowledge of mathematics as possible as they can. Therefore, what she called mathematical literacy is to use our own mathematics knowledge and skills as much as we can in order to handle everyday life problems appropriately. Similarly, one representative excerpt from the other participant subscribing to this view is given below:

> …I believe that mathematical literacy is being able to use mathematics functional in daily life…but how much or how can we use mathematics effectively and efficiently in everyday life? How smart can we always go through every business in life? I do not know the answers to these challenging questions. What I simply know if we perform our mathematical knowledge and skills competently we will be fine in life (Tchr16).

Therefore, teachers seemed to view that mathematical literacy helps to make daily life easier for people. In this regard, they mentioned that mathematical literacy prepares people for daily living and make their everyday life more comfortable. For instance, one of the teachers pointed out that *"...mathematical literacy is the right way to realize what you can do by using mathematics…it makes you feel more comfortable in your life"* (Tchr4). In a similar manner, another teacher maintained that *"...mathematical literacy improves the quality of our life we spend as much as possible…it opens our world to new possibilities…and make life easier for us"* (Tchr10). This teacher asserted that that in order to make life easier and comfortable, people need mathematical literacy understanding. In other words, if people are familiar with numbers in daily life, if they use and apply mathematics appropriately and adequately, they can save much time in life. The following quote from the other teacher also captures this phenomenon:

> …thanks to mathematical literacy, people may save from everything in life. For example, even if a farmer writes agricultural products harvested each year on a piece of paper, he can specify on the idea about his debts and earnings from these agricultural products and this can make his life more comfortable. Very simple basic numerical data to save his business every year make his future planning easier. Similarly, an engineer might build more energy efficient and environmentally friendly constructs based on his mathematical literacy knowledge and make other people feel comfortable…However, we unfortunately need to accept that since most of the people in our country right now misuse the credit cards, they undoubtedly face with major problems. We must know that this is because of basic calculation errors or in general the lack of mathematical literacy understanding (Tchr14).

Moreover, it is also important to report here that one of the teachers (n=1), although claiming to believe that mathematical literacy is the ability to use mathematics in a functional way in daily life, argued that we should not always try to relate directly all issues in mathematics to real life. According to him, it is not necessary to strive for finding or showing the real-life applications of every topic of mathematics in class. As he commented, *"…it should not be like that after studying mathematics, then we must immediately use it on the way home or at work…there is no such thing and also there should not be"* (Tchr15). Nonetheless, all of the teachers, in general, believed that mathematical literacy is about people's ability to implement mathematical ideas or structures properly and functional in everyday life. That is to say, it is to know both where and how to use mathematics in their daily living in order to handle many different troubles in life effectively. Accordingly, these teachers thought that if we talk about the quality of human life, each person should have enough mathematical knowledge and skills to understand the basic statistics, credit accounts and interest accounts calculations in daily life. The people, in a way, encounter them in their everyday life or whenever involving any talk or discussion in daily life, they should at least feel the time in which mathematics is being used wrong. Therefore, gaining enough mathematical literacy understanding is always very important to improve the quality of human life.

4.1.1.2.2. Use of mathematics in societal life

In discussing the theme of functional mathematics, the majority of teachers (n=14) laid more emphasis on the use of mathematics in societal life. They argued various perspectives concerning the use of mathematics in societal life. For example, many teachers (n=11) indicated that mathematically literate individuals can look at things around them with a more critical eye. Lack of mathematical literacy gets people being noncritical to the events around them and such people easily accept the information presented to them without criticizing and analyzing. This is unfortunately the case for both individuals and the communities. Therefore, mathematical literacy is important in order to make people capable of critical thinking as a whole. For instance, one of the teachers expressed that *"…if we want to educate our children as being questioning or critical in their social life, we need to implement the education system emphasizing mathematical literacy"* (Tchr1). Similarly, another teacher commented that mathematical literacy enables people to have critical look at the events. As he said, *"…whenever one guy says something, it should be questioned. It must not be accepted immediately. It is really important to criticize the things said before we accept them as they are presented to us"* (Tchr8). So, the teachers in general pointed out that these questioning, criticizing and inquiry abilities, in fact, are the cornerstones of being mathematically literate.

Moreover, the responses from a number of participants (n=7) also suggested that thanks to mathematical literacy, people can express their thoughts and opinions easily and confidently in society. They said that it gives us the power of self-expression in society. For example, one participant said that *"...I think math literacy is very important. It provides people the ability to reveal themselves as individuals"* (Tchr2). A parallel view can also be found in the following response from another participant arguing that *"...math literacy, what to say? This is the person's ability to express himself in daily life"* (Tchr9). Similarly, the other participant thought that the self-confidence of mathematically literate individuals is much higher than others. They can keep her feet firmly on the ground. He said that he could easily observe this from his lessons because *"...students who are good at math are more at peace with themselves and how can I say that they are more comfortable for expressing themselves in class"* (Tchr15). Therefore, as these participants stated, mathematically literate individuals are particularly able to express themselves more comfortably in society. They can look at events rationally rather than emotionally and they can stay calm, cool and in control when dealing with potentially stressful events.

Furthermore, about half of the teachers (n=7) stated that mathematical literacy in a general sense is very important to improve the welfare and development of the society. They believe that since mathematical literacy makes a direct positive impact on the community, its reflections in society would be considerably very significant. For example, one of the teachers commented, *"…if we give the children mathematical literacy well in this country, many good things will happen in our society"* (Tchr6). Another teacher also held the similar view concerning the same issue. This teacher mentioned that lack of mathematically literate people in any community clearly paves the way for underdeveloped societies and, that is why, *"…we can live in a society in a more prosperous way if somehow most of us gain mathematical literacy"* (Tchr8).

Finally, few teachers (n=3) considered mathematical literacy as people's ability to manage themselves competently in order not to be misled or deceived by someone else. For instance, one of the teachers perceived that it is very easy to manipulate or mislead communities that lack in mathematical literacy skills. So, as he stated, *"…even though we in most cases are not aware of being misled or deceived by information around us, mathematical literacy will help us feel it right away"* (Tchr4). Similarly, another teacher explained that mathematically illiterate people are generally open to be deceived. He added that *"…they [mathematically illiterate people] accept what someone says as just right, because they do not have any criteria to assess what is being said"* (Tchr5). So, these teachers believe that people who are lacking mathematical literacy skills can be easily manipulated by someone else. Thus, mathematical literacy enables us to be informed citizen by taking our own decisions independently. Namely, it is our ability to manage ourselves rather than someone else is managing us. Simply, it is the basis of our self-confidence.

4.1.1.2.3. Use of mathematics in further education

In the analysis of the theme of functional mathematics, several teachers (n=6) also reflected on the importance of mathematical literacy which plays a significant role in the individuals' future education. For example, one of the teachers mentioned that *"…I personally believe that mathematical literacy moves our students to the upper levels in all branches of science"* (Tchr16). Similarly, another teacher also supported that mathematical literacy is very important for the future education, because one of the factors for being able to be more successful in other subjects is especially based on a good level of mathematical literacy understanding. He said that *"…real success in all branches of science like chemistry, physics or biology depends on mathematical literacy understanding"* (Tchr8). Therefore, these teachers thought that if people gain mathematical literacy understanding in the early years, this will positively affect their future education which is especially based on mathematics or related fields. For instance, thanks to mathematical literacy, they can advance in the field of engineering, physics or biology. On the other hand, mathematical literacy is also important for social science. For example, it may help students to become a good author or a talented poet. Teachers, accordingly, believe that they must help every student to develop mathematical literacy understanding as much as she or he can up to a certain level in order to easily take steps toward better future education.

4.1.1.2.4. Use of mathematics in occupational life

Finally, some of the teachers (n=4) argued with the view that mathematical literacy is the functional use of mathematics in vocational or professional life. For example, one of the teachers responded that *"...gaining mathematical literacy understanding certainly provides people with many facilities in their vocational life"* (Tchr2). Similarly, another teacher responded that people need to have enough mathematical literacy understanding in order to be successful in their vocational life. He illustrated this case:

> … For example, let's say that one man is a man of trade and if this guy has to be the guy to invest or trade, he should be good at mathematical literacy to make much profit… or if a construction foreman is not mathematically literate, it will be very hard for him to adjust the height or level of the bricks during the construction (Tchr9).

So, the teachers, in general, conceived that thanks to mathematical literacy understanding, people can receive many benefits in their professions, and that is why there should be attached too much importance to the provision of the students with better mathematical literacy understanding for their success and efficiency in their future occupations.

4.1.1.3. Problem solving

Not surprisingly, almost all of the teachers (n=15) viewed the nature of mathematical literacy as problem solving. The teachers noted that the primary aim of mathematical literacy is to develop the problem-solving skills of individuals in order to solve problems that we have not faced much in the traditional curriculum. They commented that mathematical literacy helps us understand the given information by exploring the problem situation. It is also very conducive to both formulating the authentic problem situations and constructing its mental representation. Moreover, it provides us to create a most appropriate solution to the given problem by evaluating all possible ways. Mathematical literacy is also helpful for reflecting on our solutions to the given problem. Therefore, as these teachers argued, mathematical literacy basically involves problem solving skills. For example, one of the teachers stated that the acquisition of mathematical literacy means knowing how to solve a given problem. That is to say, *"if you translate the given problem into mathematical language, and develop alternative solutions to this problem, it can be said that you are mathematically literate"* (Tchr13). Hence, the teachers in general asserted that mathematical literacy provides the ability to recognize a problem situation. It is the ability to do the necessary steps to handle the given problem. For example, if we have a problem situation, we should first be able to gather data related to the problem. Next, we should be able to convert the problem situation into algebraic expressions and equations. Finally, we should be able to apply mathematical rules to find the solution. That is to say, we should be able to make a good analysis from the beginning to the end so as to get the final result for the given problem. Another teacher put it this way, *"…mathematical literacy is firstly to understand the given problem, next to represent the problem situation mathematically, then to interpret it, and finally to apply mathematical procedures for finding solutions"* (Tchr15). Therefore, these teachers defined mathematical literacy as the ability to find solutions to a given problem. In other words, the main objective of mathematical literacy is to analyze the given information better and identify the best strategy to reach the right and appropriate solution for the given problem. Thus, the teachers firmly believe that mathematical literacy builds strong problem-solving skills for their students who are supposed to be competent problem solvers in mathematics. This in turn means that *"…students' problem-solving skills in mathematics could be strengthened through providing them better mathematical literacy understanding in our lessons"* (Tchr8).

Some of the teachers (n=4) who agreed with the view that mathematical literacy essentially refers to problem solving skills also held the view that mathematical literacy is the ability to transfer our mathematics knowledge to everyday life in order to find solutions to daily life problems we face. For example, one teacher mentioned, *"…mathematical literacy is not only the ability to try to solve any calculus problem but also the ability to get rid of our real-life problems by finding appropriate solutions to them"* (Tchr13). Therefore, he explained that we simply cannot only solve pure mathematical problems but also real-life problems through mathematical literacy understanding. Besides, the teachers thought that people with problem-solving skills could produce faster, more reasonable and more logical solutions to the challenges they face in life. The following excerpt illustrates one of these teachers' view on this issue:

> …for example, a doctor, a lawyer, a police officer or a judge need to apply these skills on any challenge encountered in their workplace. I mean that they need to decide quickly and produce the most appropriate and the most reasonable solution to problems faced in their working life…they can achieve this by the help of mathematical literacy understanding because such an understanding triggers and develops their problem solving ability (Tchr9).

So, these teachers emphasized that it is definitely mathematical literacy which allows them to be successful in life. According to them, some people can be good at mathematics but if they do not interpret the problems in their life very well or more generally if they do not use their mathematics knowledge to guide them through their life, in which case their knowledge will mean nothing. Thus, mathematical literacy provides them to interpret and solve the problems in their life in the best way as much as they can. Therefore, the teachers in general believe that if we properly teach mathematical literacy to our students, they can easily handle difficult problems in their future lives. To put it differently, when confronted with a problem instead of waiting for what they will do or who will help them, they can immediately produce the most efficient solution to any problem encountered in life.

4.1.1.4. Mathematical thinking

The majority of the teachers (n=15) regarded the nature of mathematical literacy as a kind of mathematical thinking. They expressed that mathematical literacy allows students to develop their logic, reasoning and mathematical thinking skills. In other words, mathematical literacy is to learn thinking mathematically. Indeed, when we mention mathematical literacy, the first thing that comes to participants' mind is the ability to make decisions quickly, which is clearly based on the ability to think quickly. In this regard, teachers stated that mathematical literacy is so magical and so beautiful thing that broadens people's horizons, and develops their quick thinking skills. It provides people with mathematical reasoning and logical thinking skills. It is simply something about thought or logic. Thanks to mathematical literacy, people are actually doing mental gymnastics. Hence, they may trigger their logical thinking. They may improve their brains. They can at least resurrect their mind from mental laziness. This accordingly allows them to think fast, think practical or reason on any issue in everyday life. That is to say, their intelligence improves and they do mental gymnastics through mathematical literacy. So, *"…the more we ponder over things, the newer healthy become our neurons. This automatically gives us a clear chance to think mathematically more about the things around us"* (Tchr14). In support of the above-mentioned view, one teacher, for example, commented that mathematical literacy is important for the development of mathematical thinking. He stated that *"…it [mathematical literacy] of course improves the mathematical thinking skills of children"* (Tchr1). Similarly, another teacher mentioned that *"…mathematical literacy is a complex concept that requires reasoning and thinking skills in addition to basic mathematical knowledge and skills"* (Tchr11). The third teacher, also arguing in a similar fashion, emphasized that *"…mathematical literacy teaches us how to think mathematically"* (Tchr6). Hence, these teachers believe that thanks to mathematical literacy, people clearly acquire a mathematical reasoning and argumentation skills. They mean that when people start to engage in mathematics or when their mathematical knowledge increases slightly, their mathematical literacy level also increases. This automatically leads to the development of their mathematical thinking skills and it naturally enables them to sufficiently and efficiently interpret events in life. Accordingly, when they face with problems in life, they think about how to solve them. They then also think about how to apply these solutions to other parts of real-life problems.

Some of the teachers (n=4) who viewed the nature of mathematical literacy as mathematical thinking also supported the view that mathematical literacy, in fact, involves higher-order thinking skills in addition to mathematical knowledge and skills. According to these teachers, mathematical literacy is a basic level of mathematics in the first phase but then it requires a higher level of mathematical thinking. For example, one of the teachers noted that *"mathematical literacy fully supports the power of mathematical thinking and creative thinking skills at higher* *levels"* (Tchr9). So, the teachers generally believe that mathematical literacy helps us to establish cause and effect relationships by thinking about the different alternatives in daily life. That is to say, it is the ability to think of all the possibilities carefully and in detail so as to cautiously analyze all of the parameters in cases from different angles. Therefore, according to these participants, *"...mathematical literacy clearly includes higher-order thinking skills"* (Tchr4).

4.1.1.5. Innate mathematical ability

A large number of teachers (n=14) argued that mathematically literate people have some sort of innate mathematical intelligence. They commented that some students may clearly exhibit more talent than others in mathematics, so individual genetic differences are claimed to be important for the issue of the development of mathematical literacy skills. For example, one teacher responded that *"...innate mathematical skills are something that should be considered for better mathematical literacy development"* (Tchr1). Another response in support of the above view was also taken by another teacher reporting that mathematical literacy is acquired faster with inborn mathematical competence. As she said, *"…if we want our children to grow up to be mathematically literate, we need to stimulate their innate mathematical skills. Otherwise, unused skills will regrettably soon decay over time"* (Tchr11). Similarly, the third teacher put it this way, *"...the innate mathematical ability has a significant influence over the development of mathematical literacy. So, mathematically intelligent people can easily develop mathematical literacy"* (Tchr4). He maintained that for most of the people, innate mathematical skills, on the whole, form the basis of mathematical literacy. So, the participants held the view that everyone naturally has a certain level of inborn mathematical intelligence that is often inherited from his or her parents. In their opinion, mathematical literacy is a kind of talent like some talents such as writing, drawing, singing, dancing, or playing any musical instrument. In other words, *"if you do not have the necessary talent, you cannot be a talented drawer, writer, or singer. Similarly, if you do not have mathematical competence, you also cannot adequately become a mathematically literate person"* (Tchr9). Thus, these participants claimed that everyone may have different skills and talents, but inborn mathematical intelligence is inevitably needed for better mathematical literacy acquisition. They pointed out that it is difficult for everyone to have the same mathematical literacy competence because it is a bit based on genetic factors. It can clearly be observed that some people have more mathematical intelligence than others or they are simply better at mathematics than others. Or, some people have numerical intelligence, whereas others do not. They may have verbal intelligence. The participants attribute this to genetic factors. They said that we can also try to give the higher level of mathematical literacy skills as much as we want to students with lower mathematical ability, but this is the same as watering the tree that does not bear any fruit. They, therefore, believe that our intelligence, that is our mathematical literacy level, is said to be in one way or another shaped by our genetic factors.

4.1.1.6. Conceptual understanding

For most of the teachers (n=13), the nature of mathematical literacy was perceived as conceptual understanding. They pointed out that mathematical literacy involves the understanding of mathematical concepts and making connections between these concepts. For example, one teacher viewed that *"...mathematical literacy is to recognize and understand the mathematical concepts, and then being able to give life to these concepts by relating them to previous concepts"* (Tchr11). In a similar fashion, another teacher offered further support for the view given above. This teacher said that *"...mathematical literacy is to learn new mathematical concepts by connecting them to what you already know and understand"* (Tchr13). In general, these teachers perceived mathematical literacy as the organization of mathematical knowledge we already know and the integration of this knowledge to our understanding of new mathematical ideas and structures. The following excerpt from the other teacher helps to illustrate this issue:

> …Can we effectively use our existing knowledge of mathematics or the knowledge we have already learned? Can we simply activate our prior knowledge? Can we easily transfer this knowledge to another learning? Can we efficiently apply what we have learned in mathematics? The answer to each of these questions is unfortunately negative. Those are the biggest challenges we face in our math lessons. One of the things that our students tell us the most is, 'I know it, but I do not know how to use it' (Tchr12).

Thus, he believes that mathematical literacy plays an important role here to remedy this trouble. It surely makes us understand mathematical concepts or problems and automatically strengthens the conceptual understanding. Therefore, the teachers emphasized that students should firstly know what basic mathematics such as four basic operations in mathematics mean to them in life. They should know what the concept of fractions, the concept of percentages or the concept of decimal numbers means in everyday life. If their conceptual understanding is not good enough, the language used by the teacher when explaining the lesson seems like a foreign language which is hard to understand for them. So, once they get the knowledge of mathematics, they need to use, interpret and transfer it, otherwise they would not be mathematically literate enough.

4.1.1.7. Motivation to learn mathematics

Over half of the teachers (n=9) commented that mathematical literacy is very conducive to enhancing motivation and interest in learning mathematics. They stated that mathematical literacy is simply the individuals' capacity to understand and recognize the role that mathematics plays in the world. Thus, according to them, mathematical literacy helps to motivate students to engage in mathematics. For example, one teacher held the view that *"...mathematical literacy makes us enjoy learning mathematics by making use of mathematics in our daily life"* (Tchr4). In the same manner, another teacher stated that *"...everyone primarily needs to know and understand mathematical literacy in order to endear and effectively teach mathematics to students"* (Tchr12). The perception of the other teacher is also quite similar to that of the above teachers. This teacher mentioned that *"...mathematical literacy inspires your mind to love math and it then arouses your enthusiasm for studying mathematics"* (Tchr7). Therefore, teachers perceived that mathematical literacy is used as a tool to increase interest in learning mathematics. It helps to reveal people's aptitude in mathematics. Perhaps, it is something that must be acquired before starting to learn mathematics. If it really happens like that, it may lead to an increase in the number of people who like mathematics.

Moreover, one of the teachers (n=1) who mentioned that mathematical literacy is useful for increasing interest and engagement in mathematics also supported the view that mathematical literacy is helpful for dealing with math anxiety or fear by building self-confidence. For instance, this teacher put it this way, *"...mathematical literacy is a kind of ability that prevents or reduces your math anxiety"* (Tchr14). He explained that mathematics is often considered as just a subject taken by the people with inborn mathematical talent. It is generally seen too difficult and unattainable. Besides, for most of the people, math anxiety may begin as early as elementary school years. They may have to carry the burden of math anxiety at very young ages. They may sometimes become much more depressed and unhappy. Therefore, this teacher believes that when dealing with such anxiety, mathematical literacy is of paramount importance as it presents mathematics in more realistic, fun and diverse ways.

4.1.2. Teachers' conceptions of the fundamental mathematical capabilities for mathematical literacy

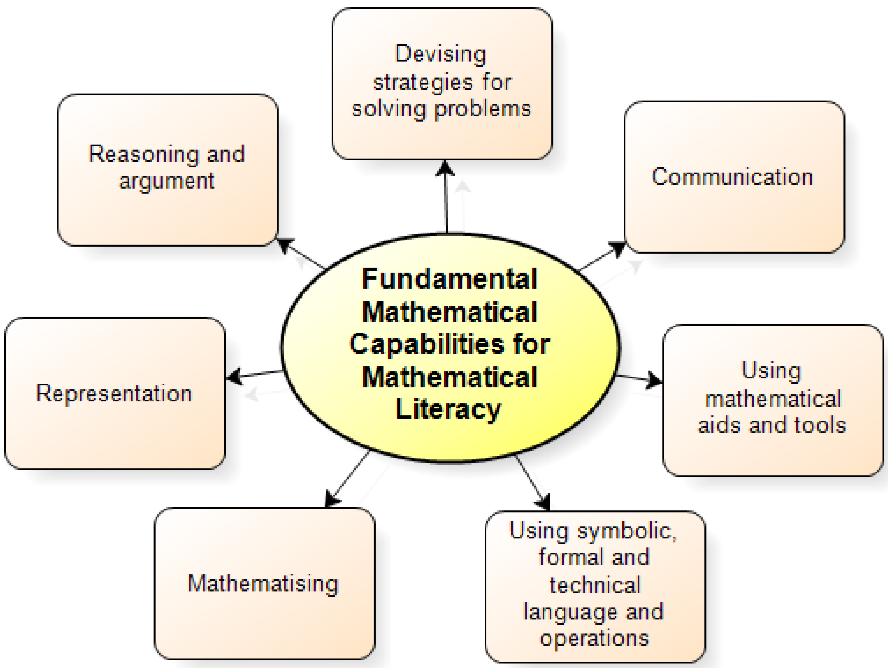

Figure 4.3: A Visual Depiction of Teachers' Conceptions of the Fundamental Mathematical Capabilities for Mathematical Literacy

In analyzing the theme of the fundamental mathematical capabilities for mathematical literacy, the following seven categories were discerned from teachers' responses: (i) Communication, (ii) Reasoning and argument, (iii) Using symbolic, formal and technical mathematical language and operations, (iv) Mathematizing, (v) Representation, (vi) Devising strategies for solving problems, and (vii) Using mathematical aids and tools. In this sense, Figure 4.3 above displays the visual depiction of these seven emergent categories identified across the interviews.

Moreover, in Figure 4.4, all teachers were listed on a single chart in order to indicate the strength or intensity of each category that surfaced in their responses to interview questions about the fundamental mathematical capabilities for mathematical literacy. The numbers in parentheses next to each category represent the corresponding total number of the participants talking about this particular category.

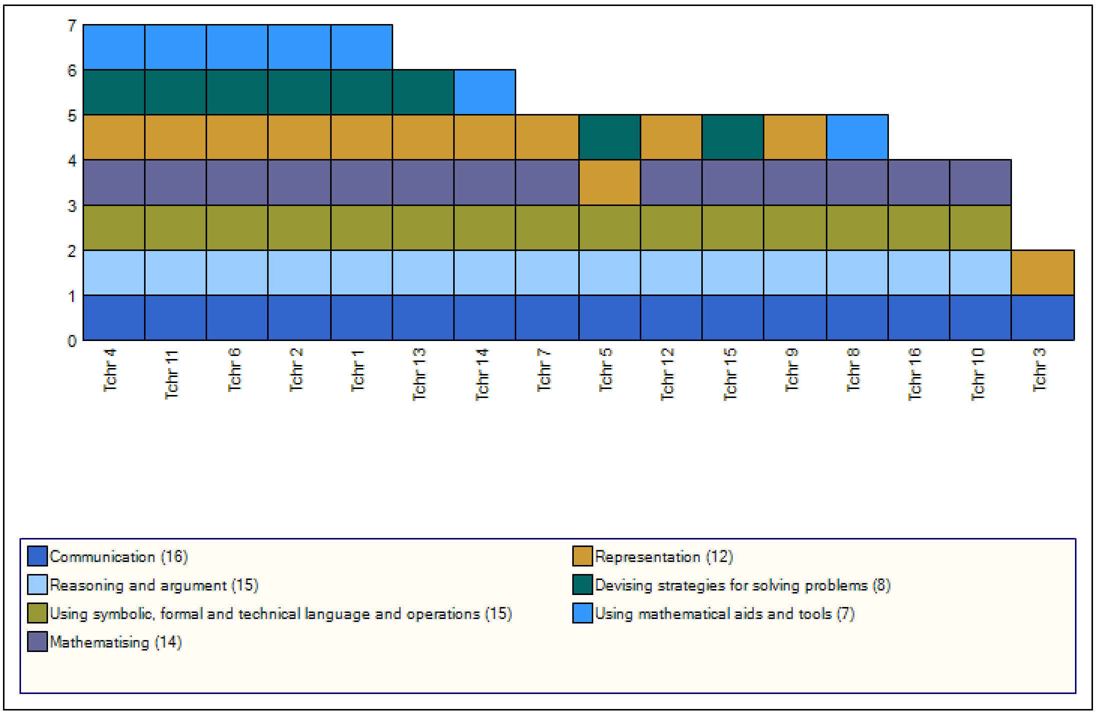

Figure 4.4: A Chart for Teachers' Conceptions of the Fundamental Mathematical Capabilities for Mathematical Literacy by Teachers

As clearly shown in Figure 4.4, all of the teachers regarded the fundamental mathematical capabilities for mathematical literacy as (i) Communication. Besides, nearly all of the teachers based their overall view of the fundamental mathematical capabilities for mathematical literacy on (i) Reasoning and argument, and (ii) Using symbolic, formal and technical mathematical language and operations. In addition, quite a large number of teachers identified the fundamental mathematical capabilities for mathematical literacy as (i) Mathematizing. Most of the teachers also identified the fundamental mathematical capabilities for mathematical literacy as (i) Representation. For half of the teachers, the fundamental mathematical capabilities for mathematical literacy were perceived as (i) Devising strategies for solving problems. Furthermore, about half of the teachers considered the fundamental mathematical capabilities for mathematical literacy as (i) Using mathematical aids and tools. The following seven sections, accordingly, provided the results with respect to the emergent categories mentioned above in more detail.

4.1.2.1. Communication

All sixteen teachers (n=16) mentioned that mathematical literacy requires the ability to clearly present and logically express oneself to others about mathematical matters both in oral and written forms. Indeed, this is the communication capability which means being able to clearly understand and explain mathematical expressions, representations or texts. For example, one teacher commented that mathematical literacy requires the ability to make sense of mathematical symbols and terms. As he added, *"...it [mathematical literacy] is of course very important to understand mathematical expressions...Perhaps, it is the most important thing for problem solving"* (Tchr1). In the same manner, another teacher noted that mathematical literacy is the ability to understand a given problem as well as the ability to express it clearly and effectively in a simple way. She said that "*…I mean it [mathematical literacy] is your capacity to explain what you understand from the problem situation"* (Tchr11). In general, teachers emphasized that since the students sometimes have a clear difficulty in understanding the mathematical expressions or texts given in the problem, they do not precisely know how to tackle the problem given. Unfortunately, this creates a big challenge for both teachers and students in the teaching and learning process. However, it is also well known fact that good understanding or interpretation of any mathematics problem is the first and foremost step for the students to be successful in mathematics. That is why teachers asserted that the acquisition of mathematical literacy is critically important for their students' success in mathematics because becoming mathematically literate firstly requires students' clear understanding and interpretation of the given problem. In this regard, one teacher highlighted that some of his students do not have any difficulty in performing mathematical operations or at least the basic mathematical operations such as addition, subtraction, multiplication, and division, but they have a clear difficulty in understanding the mathematical problems they have read. He expressed that *"…these students are not mathematically literate enough because mathematical literacy primarily means reading and understanding the given problem"* (Tchr13). In this sense, the participants expressed that when teachers ask any question, some students do not really notice what has been asked in the question. It is strange but true that there are also some students who do not even realize that there is a problem even they are asked a very obvious mathematics problem. Participants also reported that sometimes there were moments in which they gave the answer in the question, but some of their students did not even understand it. Therefore, these participants, in general, believe that mathematical literacy can help their students to clearly comprehend and explain any mathematics problem given in the class, because, as one of these participants contented, *"…mathematical literacy is our ability to understand what we have read and then easily express it verbally in addition to putting it into writing"* (Tchr15).

4.1.2.2. Reasoning and argument

Almost all of the teachers (n=15) stated that mathematical literacy requires the mathematical reasoning and argumentation competency which involves the justification of the mathematical claims, calculations or inferences based on the information and instructions given in the problem. The same teachers also stated that mathematical literacy is the ability to establish chains of reasoning and argument in order to assess whether the given mathematical solution put forward by others is correct and adequate. For instance, one teacher expressed that the ability of reasoning and argument is indeed a kind of mathematical literacy skills. She continued to express that *"…I wish we could consistently get our students to create a sequence of reasoning to support their answers to any given math question"* (Tchr11). Another response in support of the above view was also taken by another teacher reporting that *"…being mathematically literate requires my students to establish the cause and effect relationship in problem solving and make a chain of inferences by drawing on logical grounded thought processes"* (Tchr7). Participants explained that whilst tackling with any mathematics problems, their students generally try to draw inferences to make sense of their mathematical work. These mathematical inferences then allow them to reach mathematical conclusions. Thus, teachers noted that mathematical literacy helps students defend their thoughts and feelings about their mathematical inferences. It enables them to evaluate their conclusions and inferences obtained from the information given in the problem. The other teacher also held the similar view. This teacher asserted that:

> …the first thing that our students should be able learn in math classes or more generally in life is to justify their arguments or solutions to any problem encountered by creating their own reasoning and logical thinking process. In fact, the success of this process shows their mathematical literacy level (Tchr9).

Therefore, the teachers believe that mathematical literacy is the ability to find a solution for any problem through reasoning. It is being able to reason mathematically when confronted with the problem situation. More clearly, they supported the view that mathematical literacy fundamentally provides individuals with the competency to establish and develop logical relations in their everyday problem solving activities or in different arguments suggested by others.

4.1.2.3. Using symbolic, formal and technical mathematical language and operations

Nearly all of the teachers (n=15) responded that mathematical literacy requires the ability of using symbolic, formal and technical mathematical language and operations. These teachers highlighted that mathematical literacy focuses on decoding and handling formal mathematical statements. They also mentioned that one of the minimally acceptable standards of being mathematically literate calls for at least both understanding and making use of elementary mathematical systems, including simple arithmetic calculations and basic algebraic equations. For instance, one teacher stated that the minimum level for mathematical literacy is to carry out short and basic arithmetic operations. He continued to state that *"…clearly, children who are not able to do these simple mathematical operations cannot move forward to more advanced levels of mathematical literacy"* (Tchr1). In a similar manner, another teacher maintained that *"…the ability of using and manipulating mathematical expressions is very important for mathematical literacy. Our students primarily need to get this ability for better mathematical literacy acquisition"* (Tchr10). These teachers, in general, emphasized that mathematical literacy involves the understanding and implementation of mathematical expressions as well as the use of mathematical language and terminology correctly and effectively. The view of the other teacher is also quite similar to that of the above teachers. He said that the students need to know elementary mathematical procedures and language very well for better mathematical literacy development. Namely, *"…they are supposed to perform four basic mathematical operations or short arithmetic and algebraic calculations effectively and efficiently in order to be mathematically literate enough"* (Tchr13). Otherwise, even if they set up the equations of their mathematical models of real-life problems, they may have many difficulties to carry out calculations correctly and find appropriate solutions to them. Hence, the teachers, in general, perceived that using symbolic operations adequately is very crucial for mathematical literacy development. In other words, if we aim to make students mathematically literate, they firstly need to manipulate arithmetic and algebraic expressions comfortably such as basic operations, exponents, radicals or polynomials. If they fail to use formal language of mathematics or if they have weak computational and operational skills, the process of learning new mathematical knowledge or skill will become very confusing and slow for most of them.

4.1.2.4. Mathematizing

Many teachers (n=14) held the view that mathematical literacy requires mathematizing competency which refers to, on the one hand, being able to translate a problem defined in the real world to mathematical language, and on the other hand, being able to evaluate the mathematical results in relation to the original problem context. They thought that the most important thing we should know about mathematical literacy is the ability to transform a problem given in everyday life to a strictly mathematical structure, but at the same time the most challenging steps for students about problem solving is also to translate the significant variables of the problem situation into the mathematical structure. However, students who cannot formulate a given problem mathematically do not possibly succeed in problem solving. They also generally do not succeed in any mathematics course taken. On the other hand, it is also well known fact that the problem situation well expressed in a mathematical form is a problem half solved. That is why most of the teachers suggested that we need to provide students with mathematical literacy skills. In support of this, one teacher pointed out that *"...mathematical literacy involves the ability to put any mathematics problem or more generally any real-life problem into the mathematical equation and then interpret the outcomes relative to problem situation being modelled"* (Tchr16). Another teacher suggested a parallel view. She noted that *"...problems encountered at every stage of life are never seen in the form of mathematical equations… mathematical literacy enables us to transform these problems into mathematical terms for finding solutions"* (Tchr11). The other teacher commented similarly, *"...if you translate real-life problems into numerical and algebraic expressions or equations for solutions, this will be one of the significant signs for qualifying you as a mathematically literate person"* (Tchr14). Therefore, the teachers viewed that mathematical literacy simply allows individuals to express any problem situation from a real-life context to the domain of mathematics. If they properly translate the problem situation into a mathematical form by using appropriate variables presented in the situation, then they will be on the right track to become mathematically literate. After writing the mathematical equation, they are almost done with the problem. Thus, according to these teachers, the most important thing in problem solving is to understand the mathematical structure of the problem and then represent it mathematically for finding solutions. In fact, this is the real meaning of mathematical literacy.

4.1.2.5. Representation

More than half of the teachers (n=12) remarked that mathematical literacy requires the ability to understand and make use of a variety of representations including verbal, visual, algebraic, geometric, tabular, pictorial and graphical, as well as the ability to select and switch among these representations in relation to the problem situation. For example, one teacher stated that *"…it [mathematical literacy] is being able to make smooth transition between different mathematical representations of the problem situation"* (Tchr11). Similarly, another teacher reported that being mathematically literate basically involves *"…the ability to express mathematical concepts, processes and situations by using different forms of representations, such as concrete models, figures, pictures, graphics, tables, symbols, etc., as well as the ability to switch between them"* (Tchr12). The third teacher also asserted a similar view. This teacher noted that *"…mathematical literacy is an individual's capacity to interpret and translate among multiple representations of the same mathematical idea"* (Tchr13). Thus, these teachers in general think that mathematical literacy involves being able to represent problem situations algebraically, graphically, arithmetically or verbally in order to make sense of the given mathematical information. The following extract also describes what generally other teachers said about the same issue:

> . …many people regularly tune in to weather forecasts on TV or look them up on the internet. As you know, weather forecast values are usually presented in tables or charts. Similarly, if you want to follow daily business and financial news in mass media, different types of graphs, diagrams, charts and tables will be all around you. For this reason, I attach great importance to the representation skill as it forms the basis of mathematical literacy (Tchr9).

Thus, according to the teachers, representing mathematical situation in multiple ways is one of the necessary skills that we need the most for being mathematically literate. Actually, this is also a much needed skill in everyday life, as well as in many different types of professions including law, engineering, medicine, and accounting. For example, when we see any chart, table, and graph in mass media such as television, magazines, newspapers and the internet, this skill facilitates our understanding or interpretation of the information imposed on us through this mass communication media. Therefore, the teachers asserted that students' representation competency is something that is usually very much desired for mathematical literacy.

4.1.2.6. Devising strategies for solving problems

Half of the teachers (n=8) mentioned that mathematical literacy consists of the ability to devise strategies for solving problems. This ability is one of the fundamental competencies needed for mathematical literacy development, which refers to selecting or constructing a satisfactory plan or strategy for dealing with the problem, as well as careful controlling and evaluation of its implementation. In support of this, one of the teachers stated that mathematical literacy entails people to struggle against the real-life problems by reasoning and creating alternative strategies for solving them. He added that *"...it [mathematical literacy] is indeed to consider the different perspectives for the problem solution that others could not think of"* (Tchr13). So, mathematical literacy surely helps people to develop many different solution strategies in order handle any real-life problem. Similarly, another teacher contended that *"...mathematical literacy allows students to capture different solution strategies from a range of different angles"* (Tchr15). The third teacher also held the similar view regarding the same issue. This teacher emphasized that *"...it is actually very important for being mathematically literate to devise and implement many different solution strategies while tackling with the problems encountered"* (Tchr5). He continued to emphasize that *"...it requires you to be a little quick-witted at times in which you are for a short while stuck with the problem posed"* (Tchr5). Thus, these teachers viewed that mathematical literacy requires us the ability to identify many different strategies and decide on a reasonable one for drawing a generalized conclusion to any given problem. Starting from childhood, the development of this ability at certain stages of life gains very importance. According to them, it should be known that mathematics is not just about numbers or formulas, but it is also a lot about devising and implementing more effective strategies for finding solutions, as well as comparing and checking the implementations of these strategies. Therefore, instead of teaching students to memorize many mathematics formulas, we have to teach them the ability to produce different solutions by using these formulas. Hence, the teachers, in general, believe that it is mathematical literacy that truly helps students to produce different solution strategies to any problem from a variety of different angles.

4.1.2.7. Using mathematical aids and tools

About half of the teachers (n=7) commented that mathematical literacy in a way involves the ability to make use of various instructional aids and tools e.g. computers or calculators in order to initiate and foster learning processes in mathematics. These teachers strongly emphasized that students' ability to handle such aids and tools in a broad sense provides them to actively deal with mathematical activities and problems. For instance, one of the teachers responded that *"...knowing about and making use of information and communication technology tools and resources have a great influence on the development of mathematical literacy"* (Tchr4). Another response in support of the above view was also taken by another teacher stating that, *"...there exists an important role of the effective use of technological aids and tools on accelerating the acquisition of mathematical literacy skills"* (Tchr14). The following excerpts from the other teacher's interview also exhibits the same phenomenon, *"...students in accounting programs of this school [trade school] may well understand and interpret the meaning of the square root or percent of any number by using the calculator in their classes"* (Tchr1). In this respect, he said that we could use more technological tools, i.e. computers, tablets, projection devices, mathematical software, and interactive whiteboards in order to make students gain more mathematical literacy understanding in vocational high schools. Thus, according to these teachers, a core idea behind mathematical literacy undoubtedly includes making use of mathematical tools and materials in order to activate more efficient learning opportunities for students.

4.1.3. Teachers' conceptions of the relationships between mathematics and mathematical literacy

In the analysis of the theme of the relationships between mathematics and mathematical literacy, a number of categories surfaced in participants' responses.

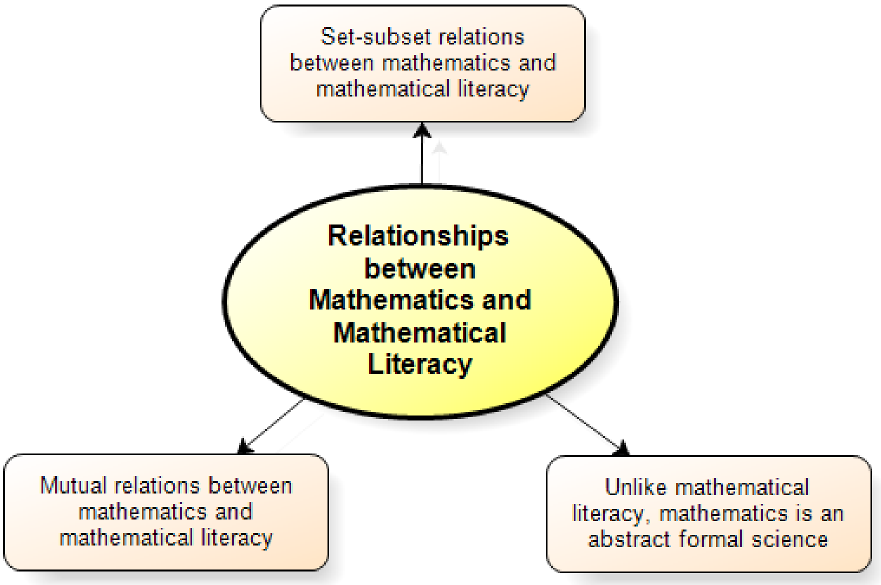

Figure 4.5: A Visual Depiction of Teachers' Conceptions of the Relationships between Mathematics and Mathematical Literacy

In this sense, Figure 4.5 shows the visual depiction of the three emergent categories identified across the interviews regarding the relationships between mathematics and mathematical literacy. These categories included: (i) Mutual relations between mathematics and mathematical literacy, (ii) Set-subset relations between mathematics and mathematical literacy, and (iii) Unlike mathematical literacy, mathematics is an abstract formal science.

Moreover, in Figure 4.6 below, all teachers were listed on a single chart in order to indicate the strength or intensity of each category that was discerned from their responses to interview questions about the relationships between mathematics and mathematical literacy. The numbers in parentheses next to each category represent the corresponding total number of the participants talking about this particular category. As presented in the figure, all of the teachers considered the relationships between mathematics and mathematical literacy as (i) Mutual relations between mathematics and mathematical literacy. Besides, for over half of the teachers, the relationships between mathematics and mathematical literacy were perceived as (i) Set-subset relations between mathematics and mathematical literacy. Furthermore, several teachers also based their overall view of the relationships between mathematics and mathematical literacy on (i) Unlike mathematical literacy, mathematics is an abstract formal science.

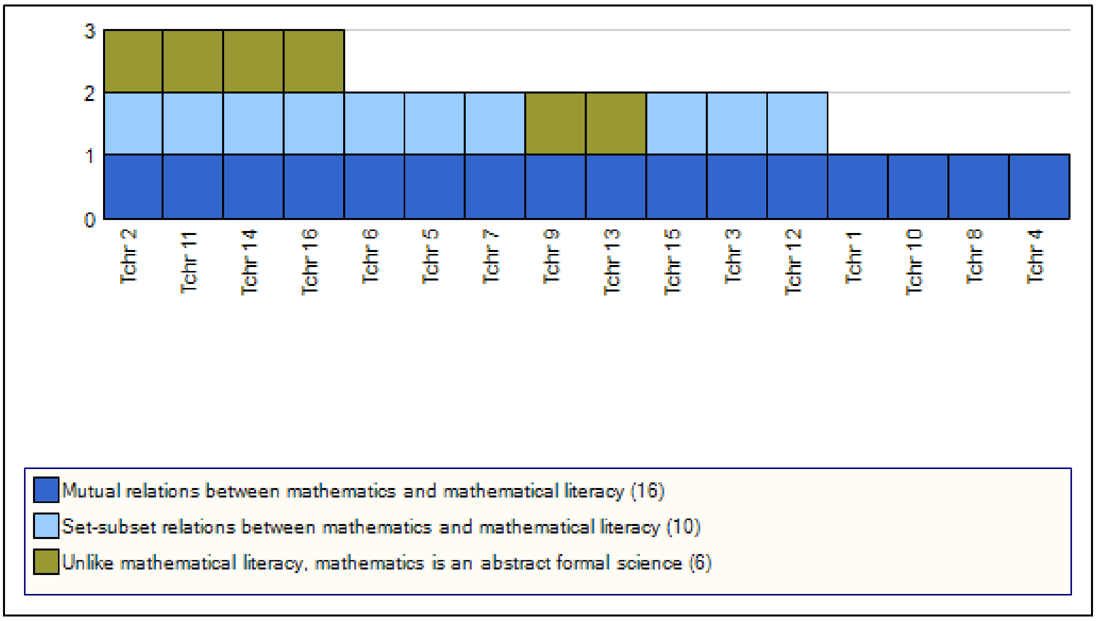

Figure 4.6: A Chart for Teachers' Conceptions of the Relationships between Mathematics and Mathematical Literacy by Teachers

The following three sections, accordingly, provided the results with respect to the emergent categories mentioned above in more detail.

4.1.3.1. Mutual relations between mathematics and mathematical literacy

All of the teachers (n=16) expressed that there exist mutual relations between mathematics and mathematical literacy. They remarked that mathematical literacy and mathematics fully complement each other. They reported that mathematics and mathematical literacy should not be considered separately as they truly support each other. For example, one of the teachers expressed that *"...the knowledge of mathematics helps to develop the level of mathematical literacy or vice versa. To put it more simply, they indeed trigger each other's development simultaneously"*

(Tchr11). Another response in support of the above perception was also taken by another teacher stating that *"…I think that mathematical literacy and mathematics are absolutely intertwined concepts and they both include one another"* (Tchr13). According to these teachers, there is no such thing that mathematics and mathematical literacy are distinctly separate in specific areas or they only intersect with each other in certain areas. They maintained that although mathematics and mathematical literacy do not exactly correspond to each other, they definitely strengthen each other by supporting and building one another up. While sometimes mathematical literacy is needed for mathematical understanding, other times, mathematics is required for mathematical literacy development. In this regard, teachers remarked that mathematical literacy helps us to gain better mathematical understanding. They emphasized that mathematical literacy is in reality fundamental for the efficient learning and the powerful application of mathematics to solve and interpret diverse problems set in a real-life context. For instance, one teacher stated that *"…mathematical literacy requires an active and effective use of mathematics in life"* (Tchr10). Accordingly, people can easily comprehend and develop mathematical knowledge that they use and apply in life. In other words, they normally do not accept the unused mathematical knowledge as their own knowledge and it does not become their own mathematical knowledge until they apply it. Therefore, as another teacher commented, *"…mathematical literacy certainly presents our students many opportunities for learning mathematics with deep understanding"* (Tchr11). According to her, it clearly provides students with the power to interpret problem situations and cautiously handle them for finding the most appropriate solutions. In this sense, mathematical literacy requires people to understand the problem and then to put the necessary steps into practice for finding solutions. In fact, many people get to learn mathematics in the same way. Hence, a person who is mathematically literate can learn and make sense of mathematics much easier than the others. The other teacher also held the similar view. This teacher noted that since mathematical concepts are usually presented in a more formal way, mathematical literacy understanding can simply facilitate individuals' understanding in mathematics. She stated that *"…we can make sense of mathematics through mathematical literacy, thereby making it easier to learn"* (Tchr16). In this respect, the teachers expressed that mathematical literacy can be a wonderful vehicle to teach mathematics effectively. It may increase the number of people who like mathematics or the number of people who have at least positive attitudes towards mathematics. Therefore, these teachers firmly believe that strong mathematical literacy understanding can automatically push their students to higher levels in mathematical understanding.

The teachers also held the view that a satisfactory knowledge of mathematics is essential for better mathematical literacy development. They mentioned that at least a basic knowledge of mathematics is necessary for having satisfactory mathematical literacy understanding. For example, one of the teachers commented that *"…students must be somewhat familiar with simple mathematical concepts and formulas to decide on the most appropriate strategy for dealing with real-life problems"* (Tchr2). A similar explanation can also be found in the following response from another teacher. This teacher asserted that *"…mathematical literacy level of students with good knowledge of mathematics is certainly developed faster than those who have a poor background in mathematics"* (Tchr4). Thus, being a mathematically literate individual is basically based on a sound foundation in a wide range of mathematical knowledge and skills. That is to say, mathematically literate individuals primarily need to develop a good level of mathematics. The view of the third teacher is also quite parallel to that of the above teachers. This teacher proposed that *"…whenever we talk about a very good mathematical literacy level of our students, the first thing to be done is to ensure that they develop a strong basis in mathematics"* (Tchr7). The teachers emphasized that the most important prerequisite for being mathematically literate is to be able to translate the given problem into mathematical expressions. For this translation to take place, people should clearly know about certain mathematical concepts very well. They should competently use these concepts for their problem solving. In fact, what these teachers simply mean is that a mathematically literate individual needs to understand and develop certain level of mathematics to use in everyday problems according to their demands and necessities.

Above-mentioned conception of mutual relations between mathematics and mathematical literacy was also further developed by many participants. They mentioned that although mathematics and mathematical literacy are somehow interrelated and influence each other's levels, they are nevertheless two different notions, and having no linear relationship between them. That is to say, being good at one does not necessarily mean being good at the other. For instance, one of the participants perceived that there is not a direct relationship between mathematical literacy and mathematics. People who know mathematics very well may not sometimes have a good mathematical literacy level. As he noted, *"...it is not always true that the more mathematics having, the better becomes mathematically literate…but nonetheless they affect each other"* (Tchr15). Another participant also argued with the similar view. This teacher pointed out that, being mathematically literate anyway requires mathematical knowledge, however, *"…there is no such thing that as the level of mathematics increases, mathematical literacy level also automatically increases…this is something different acquisition"* (Tchr6). Another view in support of the above perception was also taken by the third participant stating that *"...you may be a math genius, but it might not work. I mean if you are not able to reflect your intelligence to daily living, you will not have any good profit on your hands?"* (Tchr16). So, she believes that somebody may be a very gifted man and he may even solve many incredibly difficult mathematical problems, but he may not be successful in solving simple problems of everyday life or he may not show the same gifted mathematical skills to tackle the real-world problems. Thus, the participants supported that people who study mathematical sciences or even people who are mathematicians may not be able to offer any good solution to a very simple problem faced in everyday life. Their level of mathematical literacy may stay underdeveloped despite even being an expert in mathematics. Moreover, considering the issue from another angle, using mathematics effectively in life without having studied much mathematics was also perceived by several teachers as a practical skill or a learned habit. The following extract is typical of the comments made by one of these teachers about this issue:

> …I think that despite having no mathematical education an old peasant woman who excellently weaves many carpets with complex geometric motifs can just have a practical skill or habit to be learned. What I am saying that once she has learned it, she does not forget it. That is, if someone always does the same work, this person will be naturally specialized in this work. However, it should also not to be forgotten that if an unusual geometric motif is desired, this old peasant woman possibly might not weave these geometric motifs on a carpet. In fact, this might be explained by only a learned habit or, it is simply said that this might be a practical skill inherited from her parents or ancestors (Tchr5).

4.1.3.2. Set-subset relations between mathematics and mathematical literacy

Many teachers (n=10) also organized their thoughts about the relationships between mathematics and mathematical literacy according to set-subset relations. In that respect, the following three key issues were developed: (i) mathematical literacy is a subset of mathematics, (ii) mathematics is a subset of mathematical literacy, and (iii) mathematical literacy and mathematics intersect each other. First of all, half of the teachers mentioned that mathematical literacy is a subset of mathematics. According to these teachers, we can be mathematically literate by knowing and understanding a small part of mathematics, but we can never know or comprehend all aspects of mathematics. We can only know a certain areas of mathematics at a certain level. In support of this view, one of the teachers emphasized that *"…unlike mathematical literacy, mathematics is very wide field and it contains everything"* (Tchr12). Another teacher also suggested a similar view. He stated that *"…it seems to me that mathematics as a broader field includes mathematical literacy"* (Tchr5). Therefore, these teachers in general believe that mathematics as a universal set totally covers mathematical literacy. In other words, mathematics is a very broad concept and it includes every mathematical construct, including mathematical literacy.

On the other hand, some other teachers thought of mathematics as a subset of mathematical literacy because mathematics is apparently embedded in all aspects of everyday life. For example, one of the teachers responded that mathematical literacy makes sense of mathematics with real-life applications. Thus, *"…it looks like that mathematical literacy as a broad real phenomenon involves mathematics"* (Tchr16). A similar view can also be found in the following response from another teacher. This teacher stated that mathematical literacy obviously includes all knowledge of mathematics and that is why *"…mathematics is in general regarded as one of the subsets of mathematical literacy"* (Tchr2). Moreover, some teachers also believed that mathematics and mathematical literacy obviously intersect at some common points while diverging at others. For instance, one teacher noted that *"…it sounds to me like mathematical literacy and mathematics have an intersection region, as well as having the disjoint region"* (Tchr7). Thus, it was also seen that some of the teachers held the perception that mathematical literacy and mathematics intersect at some common grounds but are also distinguishable in other respects.

4.1.3.3. Unlike mathematical literacy, mathematics is an abstract formal science

Several teachers (n=6) commented that, in contrast to mathematical literacy, mathematics is generally perceived as an abstract formal science. For instance, one of the teachers contended that *"…mathematics is a branch of science whereas mathematical literacy is its applications to real life"* (Tchr9). This teacher mentioned that mathematics is much regarded as an abstract science while mathematical literacy makes mathematics come to life. Another teacher also held the similar view concerning the same issue. He pointed out that *"…contrary to mathematical literacy, mathematics is a science that has a purely abstract structure"* (Tchr15). Therefore, these teachers in general viewed that mathematics is a positive science dealing with abstract concepts. In other words, it is a kind of science which is based on proof, logical arguments and justifications of abstract mathematical statements. On the other hand, mathematical literacy is the ability to become well aware of mathematical structures all around us. In other words, it is to explore and apply mathematics in real world contexts in order to handle daily life problems.

4.2. Characterizing teachers' conceptions of the effective development of mathematical literacy

The teachers' conceptions regarding how effective development of mathematical literacy could be facilitated were discussed in the following two sections, with the first section focusing on their conceptions about the barriers to the development of mathematical literacy, and the second section focusing on their conceptions about the central domains for mathematical literacy development.

4.2.1. Teachers' conceptions of the barriers to development of mathematical literacy

In the analysis of the theme of the barriers to development of mathematical literacy, several categories surfaced in participants' responses.

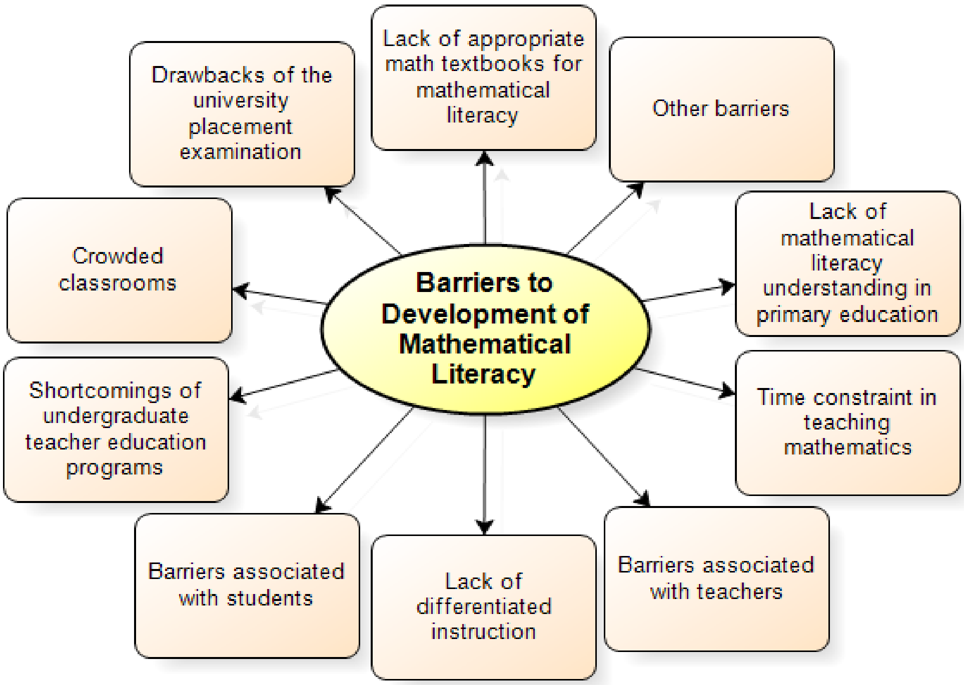

Figure 4.7: A Visual Depiction of Teachers' Conceptions of the Barriers to Development of Mathematical Literacy

In this regard, Figure 4.7 displays the visual depiction of the ten emergent categories identified across the interviews about the barriers to development of mathematical literacy. These categories consisted of: (i) Barriers associated with students, (ii) Barriers associated with teachers, (iii) Drawbacks of the university placement examination, (iv) Shortcomings of undergraduate teacher education programs, (v) Lack of appropriate math textbooks for mathematical literacy, (vi) Lack of differentiated instruction, (vii) Lack of mathematical literacy understanding in primary education, (viii) Time constraint in teaching mathematics, (ix) Crowded classrooms, and (x) Other barriers.

Moreover, in Figure 4.8, all teachers were listed on a single chart in order to indicate the strength or intensity of each category that was identified from their responses to interview questions regarding the barriers to development of mathematical literacy. The numbers in parentheses next to each category represent the corresponding total number of the participants talking about this particular category.

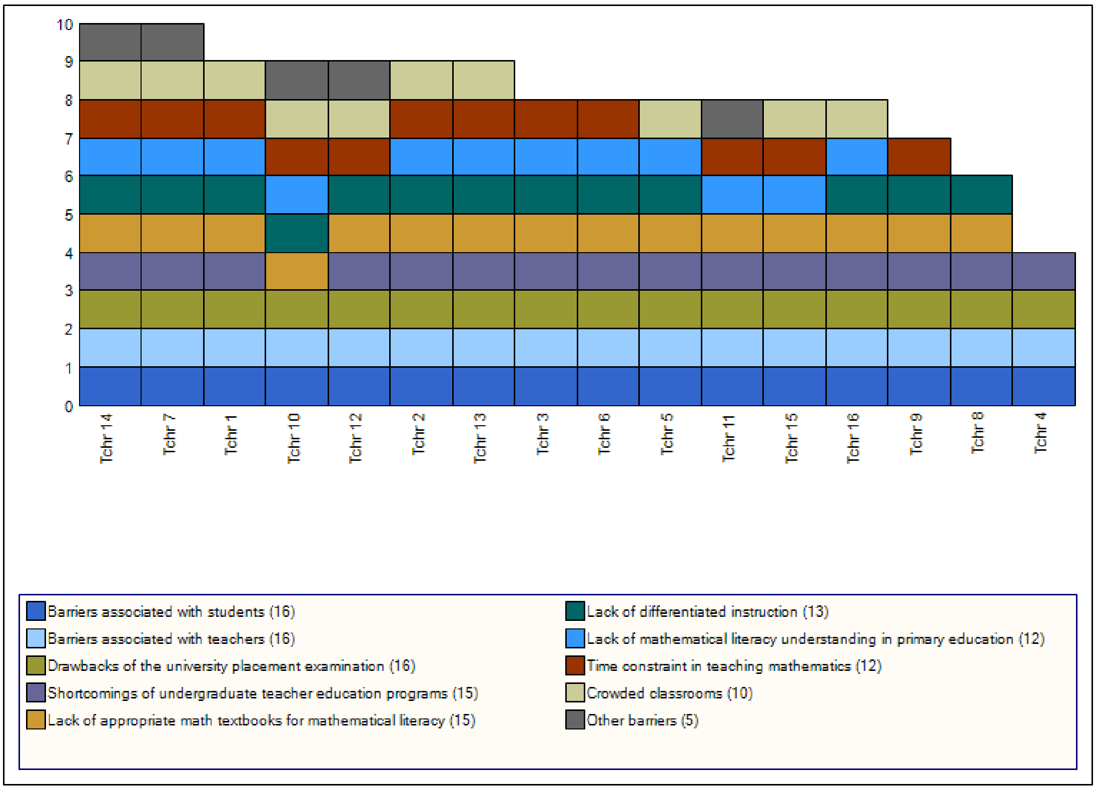

Figure 4.8: A Chart for Teachers' Conceptions of the Barriers to Development of Mathematical Literacy by Teachers

As presented in Figure 4.8, all of the teachers considered the barriers to development of mathematical literacy as (i) Barriers associated with students, (ii) Barriers associated with teachers, and (iii) Drawbacks of the university placement examination. Besides, nearly all of the teachers regarded the barriers to development of mathematical literacy as (i) Lack of appropriate math textbooks for mathematical literacy, and (ii) Shortcomings of undergraduate teacher education programs. Moreover, quite a large number of teachers identified the barriers to development of mathematical literacy as (i) Lack of differentiated instruction. For most of the teachers, the barriers to development of mathematical literacy were perceived as (i) Lack of mathematical literacy understanding in primary education, and (ii) Time constraint in teaching mathematics. Over half of the teachers also viewed the barriers to development of mathematical literacy as (i) Crowded classrooms. Furthermore, several teachers based their overall view of the barriers to development of mathematical literacy on (i) Other barriers. The following ten sections, accordingly, provided the results with respect to the emergent categories mentioned above in more detail.

4.2.1.1. Barriers associated with students

In the analysis of the theme of the barriers associated with students, several categories emerged in teachers' responses. These categories included: (i) Mathematics anxiety, (ii) Students' lack of interest in mathematics, (iii) Lack of basic arithmetic knowledge, (iv) Excessive concern about mathematics grades, and (v) Inability to search for information independently. In order to elaborate and expand on the categories mentioned above, the following five sections provided the results with the interview excerpts which were representative of these particular categories respectively.

4.2.1.1.1. Mathematics anxiety

Almost all of the teachers (n=15) mentioned that mathematics anxiety is one of the significant factors that negatively influences the development of mathematical literacy. For example, one of these teachers said that *"…mathematics anxiety is a serious obstacle to being mathematically literate"* (Tchr12). In the same manner, another teacher emphasized that mathematics anxiety in some people prevents their interest in mathematical literacy. That is to say, *"…there are some people who are afraid of or shy away from using mathematics in their daily life due to mathematics* *anxiety"* (Tchr2). Moreover, teachers told me that they usually witness a lot in their classes how math anxiety affects their students' performance in mathematics in a negative way. The following excerpt highlighted one teacher's perception regarding to this issue:

> …I have had many students in this case. For example, one of my students who is preparing for the university entrance examination continuously says to me that, 'I cannot do mathematics'. One day we sat together and solved previously asked geometry questions about the university placement exam. Next day she received a very good grade in geometry and saw that she could indeed achieve in math (Tchr4).

In addition, the teachers also noted that some students' mathematics anxiety has possibly its roots in their elementary school years. When they hear or see anything about mathematics in the lesson, their mathematics anxiety is immediately triggered. They are sometimes much more depressed by their high degree of mathematics anxiety. Therefore, according to these teachers, math anxiety actually builds a high wall against gaining mathematical literacy skills.

Besides, more than half of the teachers (n=10) who viewed mathematics anxiety as one of the factors that negatively affects the development of mathematical literacy also held the view that mathematics can be very fearful and intimidating for some of the students, which also influences their mathematical literacy development in a negative way. For example, one teacher commented that "*…in our school, most of the students are afraid of and perform poorly in mathematics. This naturally affects their mathematical literacy development"* (Tchr8). Teachers thought that students cannot do mathematics, because *"…many of our students believe that mathematics is horrible and success in it is unreachable for them since it is a very special subject"* (Tchr12). Thus, according to these teachers, a strong fear of mathematics is a reality among many students. In this regard, a lot of students unfortunately believe that they are not capable of doing mathematics. This math phobia can easily translate into students' negative thoughts about mathematics. This automatically leads to the fact that many hate or fear the subject. Therefore, their mathematical literacy level remains at a very low level.

Moreover, several teachers (n=6) mentioned that some students' lack of mathematical literacy understanding may also result from their low level of self-confidence in learning mathematics. For example, one of these teachers responded that *"…being mathematically literate primarily requires people to have a good self-confidence in their mathematical capability, but our students' self confidence in mathematics is at the lowest level*" (Tchr1). The teachers observed that most of their students do not enter the math lesson by feeling confident. According to them, this later leads to math anxiety or fears of mathematics. Hence, in order to become mathematically literate enough and easily handle everyday problems, the first thing should be done is to put this lack of self confidence in mathematics aside. That is to say, *"...our students primarily should have a strong self confidence in themselves to be able to achieve mathematical literacy"* (Tchr8).

Finally, some of the teachers (n=5) also reflected that some of their students already have negative prejudices against learning mathematics, which results in achieving a minimum level of mathematical literacy development. For instance, one of these teachers pointed out that *"...some students display a bias against mathematics and this unquestionably hampers their mathematical literary proficiency"* (Tchr9). In general, these teachers stated that students from young ages are repeatedly said that mathematics is very important for their life, but this may lead to the creation of negative pressure on students against studying mathematics or related fields. Thus, *"...perhaps, we are not aware of, but it gradually gives rise to a move away from studying mathematics and consequently mathematical literacy understanding"* (Tchr13).

4.2.1.1.2. Students' lack of interest in mathematics

Nearly all of the teachers (n=15) commented that some of their students clearly show disinterest in mathematics and this is a major stumbling block on the development of their mathematical literacy skills. They explained that students' interests are very scattered. The school or mathematical literacy is not in the interest of their students and it does not appeal to them. For example, one of these teachers expressed that:

> …I am just today's supervisor in the school's main hallway. I have observed a lot of students who do not have any mathematics course book. This means they have leisurely walks to school every day and they do nothing at all or they do not look at anything related to mathematics at home (Tchr15).

Thus, the teachers generally expressed that students do not really have any goal or purpose for coming to school. Similarly, another teacher asserted that they always try to do something to encourage their students by modelling examples from the real world, but they just sadly encounter their students' indifference to mathematics:

> …students do not take much interest of real-life examples given in class. They usually say to me, 'why do you give such examples?'…They just want me to give them the relevant formulas or rules and then ask ordinary routine mathematics questions. For example, I have recently showed them how to measure the height of any tree by using some basic trigonometry. I have also showed simple mathematical modelling about how to calculate this tree height in order to motivate them. Despite all this effort, regrettably they just said to me, 'why did you do this?' (Tchr16).

The perception of the other teacher is also quite parallel with those of the above teachers. This teacher stated that, students in vocational high schools are quite indifferent to mathematics. They are dealing with other different things rather than learning mathematics. When teachers try to teach them mathematics emphasizing mathematical literacy understanding, students immediately tell teachers that they feel like teachers do not know their real concerns which are totally distinct from those what their teachers expect from them. This issue is illustrated by the following excerpt:

> …mathematical literacy does not really make any sense for our students. They usually ask me questions like 'where could I use it in life? I say, for example, 'you will use it to calculate the slope of the road'. However, he says back to me that, 'I am not going to be an engineer and I will have no use in my life. I will become anything else different than an engineer' (Tchr7).

Hence, these teachers in general believe that students have very little or no interest in mathematics. According to them, in the absence of curiosity or interest about mathematics, we cannot talk about mathematical literacy. Therefore, as one teacher suggested *"…we should struggle to keep students interested in mathematics, if we want to make them mathematically literate"* (Tchr6).

4.2.1.1.3. Lack of basic arithmetic knowledge

A large number of teachers (n=13) reported that students' lack of basic arithmetic knowledge is one of the major factors that negatively affects their mathematical literacy performance. They believe that some of the students clearly exhibit a deficiency in their basic arithmetic skills. They expressed that this can be a clear hindrance on the way of mathematical literacy acquisition. For instance, one of the teachers mentioned that unlike students in science high schools, there is no time to give vocational high school students mathematical literacy understanding as they still learn basic facts of addition, subtraction, multiplication and division. The following excerpt captures what he said about this issue:

> …I have a lot of students who even do not know addition and subtraction operations. Unfortunately, many are missing the basic mathematical skills required for mathematical literacy understanding. Even we explain mathematical facts several times in class, they still do not understand them. For example, I am not simply able to get them to put 3 into x in order to find f (3) for f (x) = 3x -1. They obviously lack in basic mathematics knowledge (Tchr1).

So, the teachers emphasized that students who do not have a very good level of basic arithmetic knowledge already go to vocational high schools in this country. Since teachers still have many troubles to teach these students high school mathematics, they do not have time to explain other things, such as mathematical literacy. That is to say, *"...I have many students asking how to add 3 to -5 or there are those who add 5 and 2x, and then find the result as 7"* (Tchr3). Clearly, students' deficiency in basic mathematical facts is indisputable because they are even unable to implement the basic mathematical operations used in everyday life. Another teacher suggested a parallel view. He stated that many students are still not at the desired level to solve basic mathematical equations. For example:

> …in one of my quizzes, one angle was given three times the other one in a right triangle. In this case, it is clear that 4x = 90, then x = 22.5. However, most of my students could not solve it, because 22.5 was a decimal number. Many unfortunately do not even know or recognize basic mathematical procedures. However, I try to teach them this mathematics, but it is just a waste of my time (Tchr11).

Thus, the teachers declared that since students' deficiencies in certain basic arithmetic skills are already evident at all levels, it is really hard for them to gain mathematical literacy. They noted that their students surely fall below their expected level of mathematical literacy. The other teacher also held the similar view concerning the same issue. This teacher contended that:

> …You know that my profession is a teacher. However, I am just acting as caretaker in class, because I do not have any opportunity in class to teach something regarding mathematics…They do not know basic mathematical facts, procedures, rules, or formulas. I mean they do not know a multiplication table or four basic mathematical operations. They are not able to solve simple mathematical equations. Even they recall the formulas, most of them are not able to implement them (Tchr7).

Therefore, the teachers generally viewed that there is a noticeable deficiency in most of their students' core arithmetic skills. They cannot even perform basic mathematical operations in simple equations because of their deficiency in basic mathematical facts. However, according to these teachers, a basic arithmetic knowledge is a major priority for mathematical literacy development.

4.2.1.1.4. Excessive concern about mathematics grades

Less than half of the teachers (n=5) mentioned that students are usually extremely concerned about their mathematics grades, that serves as another obstacle for the development of mathematical literacy skills. For example, one of these teachers said that *"…if we as a teacher reduce our students' excessive concern about getting better grades in mathematics, we may simply direct their attention to the development of mathematical literacy skills"* (Tchr8). The following excerpt is also typical of the comments made by another teacher about the same issue:

> …In our education system, students are extremely concerned with their mathematics grades as their high school diploma final grades are considerably significant for their university placements. They, in this sense, just make an effort to earn full marks from the exams. I regret to say that both understanding of mathematics and using it in daily living are always of secondary importance for them (Tchr10).

This participant further added that teachers always say to their students that they are indeed not much concerned about their taking good grades on the exam, but they are much more concerned about how much they have learned and applied it in their life. Nonetheless, students' only concern is to receive better grades in mathematics. Namely, as the other teacher pointed out, *"...students' excessive concern about getting higher mathematics grades is never declined"* (Tchr11). Teachers emphasized that when they practice any example in class, *"…the first immediate question students are always curious about is that 'do you ask this question in the examination?'"* (Tchr12). So, the teachers, in general, believe that since achievement in mathematics is always associated with earning higher grades in mathematics, this causes an extreme concern about receiving good grades. In other words, everything is gaining value based on the possibility whether it will be asked in the exam or not. Otherwise, it is worthless, including mathematical literacy.

4.2.1.1.5. Inability to search for information independently

Finally, a small number of the teachers (n=4) held the perception that students' lack of searching for information independently is the other major barrier preventing the solid acquisition of mathematical literacy. For instance, one of these teachers commented that *"...regretfully, most of the students lack the necessary skills in searching information independently in today's age of technology"* (Tchr14). However, as he added, it is the first and foremost step for becoming mathematically literate. The teachers also expressed that they always ask for students to investigate how mathematics that they have already taught is used in everyday life, or in some certain professions. However, the result is that students are generally not able to search for and reach the desired information independently as expected from any mathematically literate individual. For example, the following excerpt demonstrates what one teacher said about this issue:

> I have recently requested students to search for how 90 degrees is used in the furniture industry. I have wanted them to have talks with carpenters in their workplaces. Despite all of my requests, many of the students downloaded everything ready from the internet and brought to the class (Tchr9).

So, the teachers believe that students do not really want to look for information or they do not know how to search for information independently. They usually say to teachers that they do not have enough time for independent information seeking. According to the participants, their students are simply accustomed to the transmission of ready-made knowledge. However, one of the most important things for their mathematical literacy acquisition is to know how to reach or search for information for accurate evaluation of events around them.

4.2.1.2. Barriers associated with teachers

In analyzing the theme of the barriers associated with teachers, several categories emerged in teachers' responses. These categories included: (i) Lack of teachers' qualifications about mathematical literacy, (ii) Teaching mathematics by rote, (iii) Difficulty of making major changes in teaching practices, (iv) Drawbacks of continuous use of routine problems, and (v) Misunderstanding of mathematical literacy. In order to elaborate and expand on the categories mentioned above, the following five sections provided the results with the interview excerpts which were representative of these particular categories respectively.

4.2.1.2.1. Lack of teachers' qualifications about mathematical literacy

Most of the teachers (n=15) perceived that the lack of teachers' qualifications about mathematical literacy is one of the serious barriers to mathematical literacy development. These participants commented that mathematics teachers do not actually set mathematical literacy higher priority in their lessons, because *"…teaching mathematics in the context of mathematical literacy is a completely different thing and it requires qualified professionals to give*" (Tchr1). Thus, the participants remarked that teachers' qualifications and competencies about mathematical literacy are of great importance when the focus is on efficient implementation of the curriculum with emphasis on mathematical literacy. The teachers argued that in order to ensure proper and effective implementation of any curriculum standards emphasizing mathematical literacy understanding, teachers first need an orientation and support through intensive training programs about mathematical literacy before implementing such curriculum. For example, one of the teachers put it this way, *"…it will be hard for us to implement curriculum emphasizing mathematical literacy* *because we are not trained about how to teach mathematical literacy skills to our students"* (Tchr11). Another teacher subscribing to the same view highlighted that if mathematics teachers are given in-service training courses about how to implement new curriculum in the context of mathematical literacy, their mathematics teaching will accordingly subject to change in accordance with mathematical literacy understanding. Thus, as he asserted, *"…for that to happen, I need to gain some certain qualifications about how to teach mathematics in the context mathematical literacy"* (Tchr13). In a similar fashion, the other teacher offered further support for the perception given above. This teacher pointed out that it is very hard for teachers to teach in mathematical literacy understanding, because they are all used to rote-learning and expository teaching from the primary school. If they were so educated in the context of mathematical literacy understanding, then they could think of different things. Their capacity to generate new ideas about mathematical literacy would be much better than now. However, *"…we were not educated to have such an understanding. I mean we are the original and real products of our rote-learning education system*" (Tchr3). They clearly remain stuck in teaching a course emphasizing mathematical literacy as it is based on both thinking and reasoning. Most of the mathematics teachers do not have a qualification such as teaching mathematics with its real world applications. As one teacher expressed, *"…like other mathematics teachers, when my students ask that where they will use the mathematics learned in class, I just say that it will be on the university entrance exam"* (Tchr10). The perception of another teacher is also quite parallel with those of the above teachers. This teacher contended that *"…we teach mathematics, but we do not even know how to teach where it is applied in life…We really lack in this regard as we are not qualified to do so"* (Tchr8). So, the teachers asserted that they really do not know how they can explain the use of mathematics in everyday life. They said that they have often heard the word 'everyday' for the last ten years, but *"…what we understand from everyday life is simply to include apples or pears into problems rather than building mathematical modelling of real-life problems"* (Tchr14). However, according to the teachers, solving problems by using mathematical modelling requires different experience and qualification like reasoning and arguments, but they frankly confessed that they do not have such an experience or qualification. Thus, the teachers believe that even if the curriculum is planned and changed to align with mathematical literacy understanding, many teachers will still not able to implement it effectively because of their lack of qualifications and competencies when planning their instruction in order to ensure to cover necessary curriculum principles and objectives. Therefore, the participants proposed that if we want teachers to accept and apply mathematics curriculum emphasizing mathematical literacy, they should primarily be informed and trained about it by conducting several professional development events and workshops. In other words, through teacher training seminars, conferences and workshops, teachers should first be informed about new curriculum principles and strands emphasizing mathematical literacy in order to make them aware of new changes. Then, teachers' qualifications should be developed in accordance with mathematical literacy understanding in order to implement these principles and strands effectively.

4.2.1.2.2. Teaching mathematics by rote

A large number of teachers (n=12) mentioned that teaching mathematics by rote is another major factor that negatively influences students' mathematical literacy development. For example, one teacher responded that *"…in class, we only teach mathematical rules and processes by rote, as well as practicing those rules and processes in order not to forget them very quickly"* (Tchr13). According to him, mathematical literacy has no importance at all. He sadly guessed that almost all our mathematics education including university rely too much on rote. Another perception in support of the above view was also taken by another teacher emphasizing that *"…many students continue to memorize a lot of mathematical rules and formulas without conceptual understanding as long as we have a sense of rote education in mathematics"* (Tchr8). Hence, the teachers frankly said that they are forced to fill their students' heads with difficult mathematical theorems and foolish rules. They can very seldom teach mathematical thinking and reasoning in class. They just prepare students for the university placement test by rote and memorization. For example, one participant reported that students should manage their time in order to compete with other students in the university entrance exam. That is why, *"…we usually tend teach mathematics by rote formulas as such a method generally results in higher scores in the entrance exam"* (Tchr7). Therefore, the teachers expressed that they teach high school mathematics by rote. They honestly said that they unfortunately keep teaching mathematics by memorization and repetition because our education system simply requires it. Moreover, they all emphasized that they were all grown up in rote education system. Thus, even though some might be open to new teaching ideas and opinions, such education received before prevents them produce other creative and new ways to teach mathematics emphasizing the development of mathematical literacy skills.

4.2.1.2.3. Difficulty of making major changes in teaching practices

More than half of the teachers (n=12) commented that the challenge of major changes in teaching practices is the other significant obstacle that prevents the acquirement of mathematical literacy. They noted that most of the teachers generally apply traditional teaching approaches in their classes. On the other hand, teachers with mathematical literacy emphasis should try to apply new effective teaching approaches in their classrooms including problem-based learning, inquiry-based learning, and mathematical modelling. In such a case, these teachers are usually supposed to be much more dedicated in order to provide their students with skills needed for mathematical literacy. For instance, one of these teachers asserted that *"…the teaching that emphasizes mathematical literacy requires a serious lesson preparation, as well as many radical shifts in my teaching approaches"* (Tchr16). In a similar manner, another teacher maintained that *"…mathematics teachers who try to put some mathematical literacy emphasis on their mathematics teachings need to use more challenging and diverse teaching activities and methods"* (Tchr5). The view of the third teacher is also quite similar to those of the above teachers. This teacher said that *"…frankly, nobody bothers about mathematics teaching practices emphasizing mathematical literacy, because they take a lot of time, and much energy*" (Tchr6). Therefore, the participants believe that if teachers are forced to lay much emphasis on mathematical literacy in class, they will probably not stand up under all pressures or loads regarding the major shift of teaching practices from their current teaching methods towards mathematical literacy teaching methods. Nonetheless, in spite of these challenges faced at the beginning, in the long run, the teachers believe that it will be more sustainable and convenient for students' achievement in mathematics.

4.2.1.2.4. Drawbacks of continuous use of routine problems

Over half of the teachers (n=9) mentioned that continuous use of routine problems in class is one of the main barriers to the acquirement of mathematical literacy skills. For example, one of these teachers viewed that *"...I always try to ask my students basic and similar types of questions in order to make them practice or apply what they learn in class"* (Tchr11). Otherwise, as she said, many students have all the time a real difficulty and confusion with unfamiliar types of questions. Similarly, another teacher responded that *"...we do not much care about non-routine problem solving skills in class, as such skills are usually not assessed in the university admission test"* (Tchr8). Another response in support of the above view was also taken by the other teacher saying that students usually get used to solving routine problems as the methods of tackling them are immediately obvious. He continued to explain that *"...whenever I transform routine tasks into non-routine problems, they have surely hard time with handling them"* (Tchr2). Hence, teachers believe that when given problems do not involve routine computations or algebraic expressions, equations and operations, most of their students are really confused and reluctant to solve them. For example, the following excerpt captures the above-mentioned view:

> …when my students are expected to solve any algebraic equation like 2x+10=30, they are all able to solve it. However, when I ask the same question by translating algebraic equation into verbal phrases in order to model a real-world situation, most of the students begin to be puzzled and discomforted by the same problem (Tchr10).

Thus, most of the teachers' teaching philosophy in class basically goes like that: In the beginning they explain step by step solutions of some related problems of any new mathematics topic in front of all students, then they give three or more identical problems regarding this topic for students' practice in a similar way.

Moreover, some of the teachers (n=4) pointed out that preparing non-routine problem activities for mathematical literacy is a big challenge for most of the teachers. For example, one of these teachers commented that *"...frankly speaking preparing lessons using non-routine problems in problem-solving activities is rather difficult for me as it takes much time after school"* (Tchr2). Another teacher, also arguing in a similar fashion, said that *"…in our mathematics textbooks, we can find a few everyday non-routine examples about some topics such as logarithms and modular arithmetic. However, if I am asked for more, I will not able to do it"* (Tchr9). Therefore, the teachers believe that although teaching non-routine everyday problems has much importance for students' mathematical literacy development, *"…placing mathematics into their daily life context is a terrible and great dilemma for me"* (Tchr5).

4.2.1.2.5. Misunderstanding of mathematical literacy

About half of the teachers (n=7) regarded the teaching of mathematics emphasizing mathematical literacy as solving as many mathematics questions as in a limited period of time. This misunderstanding of mathematical literacy is clearly one of the major factors that negatively affects students' mathematical literacy performance. For example, one of the teachers said that *"…mathematical literacy development of my students is entirely based on the level of my authority stemming from my knowledge of subject area which depends on solving any mathematics question quickly and practically"* (Tchr7). Another teacher also held the similar view. This teacher noted that *"…the good and competent mathematics teacher is the teacher who effectively tackles many questions in a minimum of time"* (Tchr14). Accordingly, she or he is the teacher who can best give mathematical literacy to her or his students. The third teacher also suggested a parallel view. He stated that *"…students' mathematical literacy skills are inevitably based on their teachers' mathematical literacy skills that focus on the ability of handling a lot of mathematics problems in less time as possible"* (Tchr8). So, the teachers believe that teachers should clearly use the authority of their knowledge in order to provide mathematical literacy skills for their students. More clearly, if students receive full and adequate answers to all questions asked in class, they will clearly trust their teacher's knowledge of mathematics. That is to say, students begin to respect their teacher's authority in class. In fact, this time is the perfect moment to develop their mathematical literacy understanding. In other words, the more mathematics questions their teachers solve in class, the better mathematically literate they become.

Moreover, a small number of the participants (n=3) also considered the teaching of mathematics emphasizing mathematical literacy as the teaching of mathematical rules and theorems. For example, one of these participants said that, *"Do you have any doubts that learning mathematical theorems and proofs well indeed allows our students to have a strong mathematical literacy knowledge base?"* (Tchr6). Therefore, according to these participants, mathematical literacy development is better achieved by teaching a lot of definitions, proofs, and theorems in mathematics.

4.2.1.3. Drawbacks of the university placement examination

In analyzing the theme of the drawbacks of the university placement examination, several categories emerged in teachers' responses. These categories included: (i) University entrance exam-oriented education, (ii) Lack of mathematical literacy emphasis on the university entrance exam, and (iii) University entrance examination anxiety. In order to elaborate and expand on the categories mentioned above, the following three subsections provided the results with the interview excerpts which were representative of these particular categories respectively.

4.2.1.3.1. University entrance exam-oriented education

A great deal of teachers (n=15) perceived that university entrance exam-oriented education is one of the dominant barriers to the development of students' mathematical literacy skills. For example, one of these teachers perceived that *"…in a system where nearly 2 million people take the university placement test, you only prepare students for this test"* (Tchr10). Thus, as he pointed out, teachers have no time to devote to teaching mathematics in the context of mathematical literacy. Another view in support of the above-mentioned perception was also taken by another teacher stating that *"…right now, my responsibility is not to teach mathematical literacy, but to show all types of questions that they will possibly be asked in the university placement examination"* (Tchr11). Similarly, the third teacher maintained that mathematical literacy acquisition is very important, but *"…the university entrance examination entirely inhibits us to give priority to mathematical literacy"* (Tchr2). Moreover, the teachers said that their success is not measured according what their students score from PISA, but it is generally measured according the number of students who are admitted to top ranking universities. The success of their school is also assessed in the same way. Hence, they believe that their responsibility is to well prepare their students for the university placement test. Namely, *"...no matter what people say about me, but my teaching should be university entrance exam oriented teaching"* (Tchr4). Thus, many teachers just teach to prepare their students for the university entrance examination. That is, they simply teach mathematics according to the types of questions which probably come up in the university placement test. They all absolutely prefer university exam oriented lessons in their classes. For example, one participant illustrated that:

> …when I solve any multiple test question related to factoring algebraic expressions, I say to my students that they should look at each choice respectively and substitute 1 for x, and if it satisfies, it will be the right answer, if it does not, try another choice. Clearly, if they want to be successful in the placement test, they should think very fast and practical (Tchr6).

Furthermore, one teacher said something more interesting than the above-mentioned views. She claimed that regretfully the teachers of the future are also trained with the test type of approach, but not with mathematical literacy approach because *"...even our intern teacher candidates are usually supervised by the teachers who have more experience and more knowledgeable about test type of questions in mathematics"* (Tchr16). Accordingly, the teachers thought that even if curriculum is changed or revised according to mathematical literacy understanding, many teachers and students may not take this change seriously enough to keep up with. These teachers continue to teach mathematics in the same way as long as the student selection and placement system continues in the same fashion as it exists before. According to the participants, the important thing for students as well as teachers, school administers and parents is to be successful in the university placement exam by getting satisfactory scores. Hence, *"...all things related to teaching mathematics solely focus on the success from the entrance examination"* (Tchr8). In this regard, the participants believe that teachers are just supposed to give necessary mathematical definitions, theories, rules and formulas and practice with them as much as they can in order to make their students to achieve or solve many multiple choice questions in the shortest possible time. For instance, one of these participants viewed that if teachers are literally expected to implement the curriculum emphasizing mathematical literacy understanding, *"…our student selection and placement system should be aligned and adjusted according to the acquirement of mathematical literacy understanding"* (Tchr11). Another participant also held the similar view regarding the same issue. This teacher said that even though some curriculum goals and learning outcomes are partially appropriate to mathematical literacy acquisition, we cannot say the same for the university placement exam. So, according to her, *"…these curriculum goals and learning outcomes are not reflected in the university placement exam and always pushed back by teachers*" (Tchr16). Similarly, the other participant mentioned that there are certain incompatibilities between the requirements of mathematical literacy and the student selection exam. As she put it, *"…the only thing we do in class is just to prepare our students like a racehorse to compete with others for the upcoming university placement exam"* (Tchr6). This teacher clearly stated that being able to gain mathematical literacy understanding is very opposite approach to our current student selection and placement system for undergraduate education. According to her, the existing education system is basically based on getting as much success as from the university placement examination. In order to be successful and maximize the score from the exam, students are forced to solve as many multiple choice questions as possible in the allotted time or earlier. In other words, success clearly depends on being practical and quick thinking. However, the acquirement of mathematical literacy understanding requires much time and effort as well as in-depth and lengthy contemplation. For this reason, the teachers in general believe that in such education system in which the student selection and placement examination for higher education is well known and major fact, the implementation of new curriculum emphasizing mathematical literacy is quite difficult because the focus of the most of the students continuously stays on this exam. The teachers emphasized that even they try to provide mathematical literacy by giving examples from daily life, after a while their students kindly remind them that they need to prepare for the university entrance examination. All mathematics teachers probably happen to agree with their students because they are getting prepared for a very challenging exam on which a lot of students compete against with each other. Therefore, according to the participants, it is quite meaningless to talk about the mathematical literacy acquisition in the existence of such a selection examination.

4.2.1.3.2. Lack of mathematical literacy emphasis on the university entrance exams

Many teachers (n=12) commented that the lack of enough mathematical literacy emphasis on the university admission test is another important obstacle that negatively influences mathematical literacy improvement. For instance, one of these teachers said that *"…the emphasis of our university placement examination has been gradually shifting towards mathematical literacy, but it is still not as it should be"* (Tchr14). Similarly, another teacher explained that *"…the university entrance examination content should be changed to make it more relevant to mathematical literacy understanding if we want students to better focus on such an understanding"* (Tchr16). The third teacher, also arguing in a similar fashion, mentioned that the content of the university admission examination is entirely problematic and has no relevance to mathematical literacy. As he added, *"…our students are being held responsible for too much useless knowledge for their future life"* (Tchr7). Thus, the teachers reported that even though not seen in the previous university entrance examinations, at least for the last few years it has been started to ask some questions in the context of mathematical literacy in the university admission exams. They said that it is from now possible to face with such non-traditional questions in the exam. However, they also claimed that there is still no clarity about this issue because almost everybody, including parents, students, and educators, are really confused of predicting how much weight will be given to mathematical literacy understanding in the content of the next entrance examination.

4.2.1.3.3. University entrance examination anxiety

Finally, over half of the teachers (n=9) expressed that the university admission examination anxiety is the other possible factor that affects students' mathematical literacy acquisition in a negative way. For example, one of these teachers emphasized that university placement examination creates incredible anxiety and sometimes leaves a serious trace in students' lives. Therefore, according to him *"…it [the university placement examination] is a big problem for most of the students to gain mathematical literacy skills"* (Tchr1). Another teacher, in a similar manner, mentioned that the concern of students and parents about the university entrance exam is very normal, because our education system is completely based on this exam. She added that *"...the important thing is not to develop mathematical literacy, but to increase the total number of correct answers in the entrance exam"* (Tchr6). Moreover, according to these teachers, mathematical literacy education needs a great amount of time to develop and complete. That is to say, it takes much time for students to reflect on the real-life problems given in class and make some comments on it for finding the solution. In such a case, maybe there will be some times that only one problem will be able to be solved in just one class hour. However, *"...if our students get used to solving problems in this manner, they cannot look at more than twenty questions in the university entrance exam"* (Tchr13). So, the teachers believe that the university entrance exam anxiety clearly has a negative impact on students. Unless they overcome this exam anxiety or unless there is another way to enter university, they will always look at life among the five options. Namely, the important thing is to make as many as correct answers out of fifty-two questions of mathematics section of the test. The more correct answers they have, the higher score they receive from the test. Then, the higher score they have, the better university they can enter. Therefore, *"...they do not care so much about real-life problems or let say mathematical literacy understanding"* (Tchr8).

4.2.1.4. Shortcomings of undergraduate teacher education programs

In the analysis of the theme of the shortcomings of undergraduate teacher education programs, two categories emerged in participants' responses. These categories included: (i) Lack of mathematical literacy understanding in undergraduate programs, and (ii) Lack of mathematical literacy course in undergraduate programs. In order to elaborate and expand on the categories mentioned above, the following two sections below provided the results with the interview excerpts which were representative of these particular categories respectively.

4.2.1.4.1. Lack of mathematical literacy understanding in undergraduate programs

Most of the teachers (n=13) mentioned that the lack of mathematical literacy understanding in undergraduate teacher education programs is one of the primary factors that negatively influences the acquirement of mathematical literacy. For example, one of these teachers said that *"…I did not see anything about mathematical literacy in my undergraduate education. I also think that there is probably now nothing much about it in undergraduate teacher education programs"* (Tchr12). In the same manner, another teacher expressed that sometimes, at the national teacher training seminars, they were told that it was important to implement group works in class in order to engage students in better learning and understanding of mathematics. However, *"…I did not make any group work with my classmates in my education, including university education. So, I really do not know how important this is to my students*" (Tchr15). He added that *"…we were very busy with theorems and their proofs during the four years of university life. I wish I could also be given other things in the context of mathematical literacy"* (Tchr15). Similarly, the other teacher emphasized that mathematical literacy certainly is not something in the foreground in mathematics teaching. He noted that *"…everything started at the university. We were educated by the professors who asked us to prove very difficult mathematical theorems in the exams. We now do the same to our own students"* (Tchr2). The following passage may be taken as typical of this kind of evidence:

> …I had chosen the mathematics teacher education program so willingly, but nothing happened as I expected. For example, there were some moments when I was really struggled and disappointed with some of my professors…I also did not complete my study program at the university on time. I got an extension due to the topology course. Our head of department was giving this course and the only thing he always said to me that why I was trying to understand this course. He said that everybody passed it by simply memorizing all theorems and proofs that he gave. I was very amazed by such talk. I said to myself how a mathematics education professor who educated future teachers of mathematics could tell such things to his students (Tchr11).

Thus, as far as she remembers, education given in the universities was unfortunately like in this format. Prospective teachers were given everything ready. They were not allowed to ask why. Whenever they asked they were always told that they did not need to know it because they would just transfer the knowledge from mathematics textbooks to students' minds.

The teachers also said that they did not take such an education emphasizing mathematical literacy at the university because mathematical literacy was usually associated with basic mathematical knowledge and skills. It seems different from or inferior to mathematics. Thus, they were only given advanced and very difficult mathematics courses at the university. For example, one teacher said that *"...I mean we took many complex and difficult courses such as Vector Spaces, Topology, Differential Equations, and Linear Algebra at the university"* (Tchr3). Hence, according to the teachers, there is no relationship between mathematics they learned at the university and the one that they teach their students in high school. Therefore, these teachers also pointed out that they have been teaching mathematics through with both their high school mathematics knowledge and the things learned after starting their teaching career. Namely, *"...If you ask me what mathematics knowledge I used when I started my first teaching, I can tell you that I used the knowledge that I learned in high school"* (Tchr14).

Some of the teachers (n=4) also added that the lack of research and discussion oriented undergraduate teacher education clearly results in the lack of mathematical literacy understanding. For example, one teacher put it this way, *"...we got whole knowledge ready at the university and we were only responsible and asked from this knowledge"* (Tchr11). He maintained that pre-service mathematics teachers are generally not encouraged to learn by discussion and discovery learning at the university. So, the participants believe that the education system in universities should be based on research and discussion oriented courses if we want to get prospective teachers to emphasize mathematical literacy understanding among their students. In these courses, pre-service teachers can discuss or investigate how better they teach students any specific topic of mathematics. If this kind of education is provided at the universities for prospective teachers, such education will also be automatically reflected in their teaching mathematics in high schools.

Moreover, a few of the teachers (n=2) remarked that due to the lack of teaching practice-oriented undergraduate teacher education, most of the in-service teachers lack in basic mathematical literacy understanding. For example, one teacher responded that *"…expert or experienced teachers in high schools need to guide pre-service teachers toward awareness of how to teach mathematics according to the implicit demands of mathematical literacy"* (Tchr13). Thus, these teachers noted that during whole university education period there should also be practice-oriented teacher education programs to engage the prospective teachers in the work of teaching mathematics emphasizing mathematical literacy understanding.

4.2.1.4.2. Lack of mathematical literacy course in undergraduate programs

Finally, more than half of the teachers (n=11) commented that the lack of mathematical literacy course in undergraduate programs is another major factor that negatively affects the development of mathematical literacy skills of students. For example, one of these teachers expressed that *"...in undergraduate teacher education programs, it would be a good thing if there was an elective mathematical literacy course which emphasizes mathematical literacy"* (Tchr12). Another teacher offered similar support for the expression given above. This teacher asserted that *"…it is, of course, very helpful for pre-service teachers to have elective mathematical literacy courses at the university level in order to practice and acquire mathematical literacy skills needed for better teaching of mathematics"* (Tchr9). So, these teachers thought that an elective or must mathematical literacy course would be very nice and supportive for prospective mathematics teachers as it provides them new visions and perceptions for developing some principles of and creative approaches to teaching and learning mathematics in the context of mathematical literacy.

4.2.1.5. Lack of appropriate math textbooks for mathematical literacy

Nearly all of the teachers (n=15) mentioned that the lack of appropriate math textbooks for mathematical literacy is one of the significant barriers that influences students' mathematical literacy performance. They openly said that no teacher thoroughly or properly uses mathematics textbooks given by the Ministry of National

Education. Students also do not use these textbooks at all. Actually, it was claimed that mathematics textbooks do not meet the needs of both teachers and students. For example, one of the teachers commented that, in the beginning of the education year, students generally take mathematics textbooks distributed by the Ministry of National Education, but most of them keep these books either at home or under the desks, because as she put it simply, *"…the mathematics textbooks are very boring and not much helpful"* (Tchr6). Hence, the teachers suggested that textbooks must be different for each school level to satisfy the demands from both teachers and students, because mathematical literacy needs of students attending different types of schools are also different. However, currently, all school types including both Science High Schools and Vocational High Schools use the same textbook for mathematics. So, as one teacher put it, *"…even though I want to use the textbook given by the Ministry of National Education in class, the examples given in it are not only insufficient in number but also inappropriate in level"* (Tchr1). Thus, these teachers thought that math textbooks are not satisfactory for mathematical literacy development. On the other hand, some of the teachers argued that we should put away the thought that whether our math textbooks are satisfactory for mathematical literacy or not. Primarily, we should think of the issue that when students pick up or put any math textbook into their hands, they do not understand anything from this textbook. They just use the textbook to copy examples from it. They use it no other way. For example, one teacher said that:

> …one day I said to my students to study pages from their math textbooks before coming to school next day. I especially asked them for reading mathematical definitions very well. Next school day, they all said to me that they did not understand anything from what they read from the textbook (Tchr16).

So, these teachers believe that the textbooks are very symbol laden and complicated to understand. They pointed out that the math textbooks are not really helpful for students to develop their mathematical skills as well as mathematical literacy skills.

The teachers also thought that there is nothing satisfactory in the content of the present mathematics textbooks on behalf of mathematical literacy. They said that they may find some examples from everyday life in these books, but most of them are not authentic mathematics problems or real-world problems. According to them, the events described in these problems are impossible to occur or take place in real life. As one teacher alleged, *"I am sorry but by putting a few pictures or photographs at the top or bottom of the problem, you cannot create or develop authentic, real-life examples"* (Tchr5). Moreover, another teacher offered further support for the perception given above. This teacher expressed that it is really difficult for mathematics teachers to relate whole mathematics to everyday issues. He complained that *"…I have a big trouble to find an appropriate book to help me give motivating real-life examples for every topic in class…Our math textbooks still lack in this regard"* (Tchr9). So, the teachers in general criticized that there is an inadequate number of examples in the math textbooks for mathematical literacy development. Some of the mathematical topics like trigonometry, logarithms and quadratics are explained by their everyday use, but for other topics it is difficult to find any sample problem to show students. According to them, adequate knowledge is indeed provided by these textbooks, but with regard to diversity of examples they are not enough. Therefore, the content and examples that they see in the books are really not satisfactory for mathematical literacy development of their students.

4.2.1.6. Lack of differentiated instruction

The majority of the teachers (n=13) stated that the lack of differentiated instruction is one of the possible limitations to students' mathematical literacy development. For instance, one teacher mentioned that mathematics needed for mathematical literacy differs according to students' mathematical skills and knowledge, but what is generally requested from teachers is to give all students the same mathematics in spite of classes with mixed levels. He noted that *"…regardless of classes where students differ greatly in terms of mathematics level, we try to give all of the students the same mathematical literacy skills"* (Tchr14). Another teacher suggested a further support for the view mentioned above. He claimed that students who are weak in mathematics more likely prefer to attend vocational high schools, because they are not sufficiently proficient in mathematical skills and knowledge of mathematics. However, teachers are simply requested to teach these students the same mathematical topics that are taught to the students of science high schools. On the other hand, a good mathematical literacy development begins with learning basic mathematical facts effectively and efficiently. However, mathematics that teachers in these vocational high schools already try to give their students is not a basic level of mathematics that they use in their daily lives. It sounds like a higher level mathematics for those students. Although these students have some difficulties in doing even basic addition and subtraction operations of mathematical facts, teachers are forced to give difficult topics of the mathematics curriculum according to the recommended sequence. For example, *"…we try to teach them trigonometric functions such as sinus, cosines, tangents, cotangents, but they do not even perform four basic mathematical operations"* (Tchr3). Therefore, he personally believes that teaching mathematics in this way only results in strong dislike and anxiety of mathematics rather than contributing to development of students' mathematical literacy skills. Hence, as he stated, vocational high school curriculum should be different from any other high school curriculum. Teaching the same mathematics in different types of schools containing students who are clearly of distinct levels in mathematics is a real obstacle to mathematical literacy development. That is to say, for a satisfactory mathematical literacy acquisition, *"…we should not be forced to teach the same mathematics to students at vocational schools due to their different mathematical abilities"* (Tchr9). The other teacher also expressed the similar view concerning the same issue. This teacher suggested that *"…if we teach mathematics by considering classes consisting of students with varying levels of mathematical abilities, their mathematical literacy performance will be much higher than before"* (Tchr8). Therefore, the teachers believe that the primary purpose should be giving mathematics education according to the students' needs and abilities. When we try to give them mathematics education on a lower or higher level than what they are actually able to achieve, we cannot create any willingness and desire for learning mathematics. This lack of willingness and desire accordingly results in ineffective development of mathematical literacy. So, it is very important for mathematical literacy development to endear mathematics to students by ensuring that mathematics is not only about abstract ideas and theoretical constructions but also about real-life applications. However, the things taught in high school mathematics provide mathematics knowledge needed for higher educational institutions. Indeed, as it is particularly stressed by the participants, the problem arises from the fact that the same mathematics curriculum is offered for all students regardless of their mathematics concentration. Teachers are allowed very small flexibility for changing their students' minds about mathematics. As a result, many students unfortunately do not gain necessary skills to function effectively in daily life.

4.2.1.7. Lack of mathematical literacy understanding in primary education

A slightly higher number of teachers (n=12) commented that the lack of mathematical literacy understanding in primary education is one of the potential barriers to students' negative attitudes toward mathematical literacy. They stated that we have great problems about students' mathematical literacy understanding arising from their primary education. For example, one of these teachers expressed that it is pretty hard for many students to gain mathematical literacy understanding, because as he said, *"...there is a lack of direct intention to give mathematical literacy understanding to students starting from their primary school years"* (Tchr1). Another teacher also held the similar view concerning the same issue. This teacher proposed that *"…our students should be accustomed to mathematical literacy understanding at all levels in education, especially at primary levels"* (Tchr13). In this way, as he added, most of the students can develop much confidence in mathematical literacy skills. Similarly, the other teacher suggested that *"...the principal basis of mathematical literacy understanding of our students should begin to be built in their elementary school years"* (Tchr3). She also noted that this will accordingly help many students not only to eagerly learn more advanced level of mathematics but also to automatically gain higher level of mathematical literacy understanding. So, the teachers pointed out that if they are expected to give their students a lesson emphasizing mathematical literacy, these students must be educated from the beginning of their early ages in a manner to become mathematically literate enough. They said that by engaging students in learning mathematics through a process of inquiry and discovery in the early years of education, we can basically introduce mathematical literacy understanding to students. Namely, *"...our students should be prepared, organized and familiar with a lesson in the context of mathematical literacy when they come to high school"* (Tchr11). In other words, the students should know in advance what specific types of mathematical activities or methods such as modelling, discovery learning, inquiry-based learning, open-ended problem solving, group work, discussions, and real-life applications will possibly be implemented in their lessons. Otherwise, as is now it is quite difficult to implement such a lesson approach in only high school education. Teachers as well as students all become stuck and do nothing about it. For this reason, the participants believe that the importance of students' early mathematical literacy development cannot be underestimated. If mathematical literacy is really thought to be essential for an individual and society, it should be acquired in early years of education and it needs to be further supported in later years of life.

4.2.1.8. Time constraint in teaching mathematics

Many teachers (n=12) expressed that the time constraint in mathematics teaching is another possible factor having a negative impact on students' mathematical literacy performance. For example, one of these teachers explained that *"...mathematics lessons emphasizing mathematical literacy will take a great amount of time and effort to plan and implement in a busy school-time schedule"* (Tchr9). Similarly, another teacher noted that what mathematical literacy requires us is the understanding, formulation and modeling of the real-life problem and then comprehensive evaluation of results, but it is also important to recognize that all these steps take so much time and dedication. That is why, he assumed that *"...mathematics teaching based on mathematical literacy understanding will challenge mathematics teachers as never before as it takes much of their time"* (Tchr10). The perception of the other teacher is also very similar to those of the above teachers. This teacher asserted that mathematics teachers are not able to afford getting out of the mathematics curriculum timeline given to them. Hence, as he said *"...in the existing system, it is very hard for us to spend much time on any activity in class"* (Tchr13). So, the teachers said that the time allocated to them in the curriculum is not enough for engaging students in activities emphasizing mathematical literacy such as building and testing models of real-life situations. Such activities usually require teachers to allocate a separate time for each student. However, they cannot devote enough time to each student in order to ensure satisfactory progress in these activities. Otherwise, there is a certain risk to slow down the teaching pace. Namely, *"...the possible consequence of this is that they may stay* *behind the timeline of the mathematics curriculum. Of course, no teacher wants to take that risk"* (Tchr15). Hence, the teachers emphasized that the teachers normally give higher priority to the mathematics curriculum in the schools. Their only business is to teach each topic in mathematics curriculum in suggested time limit. Indeed, they fully agree with the idea to support the acquisition of mathematical literacy understanding by all students, but often it happens that *"…I do not find enough time to even ask any student to explain the given problem in front of class. I really want to do it but I cannot due to time constraints in teaching"* (Tchr11). Therefore, teachers believe that teachers obviously need extensive time period in order to engage students in a wide range of activities in the context of the mathematical literacy.

4.2.1.9. Crowded classrooms

More than half of the teachers (n=10) also commented that the crowded classrooms hinder the implementation of teaching strategies emphasizing mathematical literacy development. For example, one teacher responded that *"…it is certainly impossible to deal with everyone separately in classes consisting of thirty-five students"* (Tchr12). However, as he further pointed out, one of the crucial requirements for better gaining mathematical literacy skills is to provide the individualized attention to every student in class. Another response in support of the above view was also taken by another teacher reporting that crowding in classroom learning environments has been a great challenge for providing necessary mathematical literacy to students. He added that *"…although group work activity is one necessity for mathematical literacy acquisition, how can I competently and efficiently manage any group work activity in crowded classes?"* (Tchr15). Similarly, the other teacher pointed out that mathematics teachers generally have about twenty-five lessons per week to teach with about thirty-five students in a classroom. Moreover, some mathematics teachers even teach more than six different classes in a week. That means they have more than about two-hundred students to observe and assess their needs and guide them toward fulfilling those needs. Therefore, under such busy work environment conditions, teaching in the context of mathematical literacy requires them great effort and determination to perform. Speaking for himself, *"…I do not have such dedication and energy"* (Tchr5). Hence, as one teacher expressed, teachers cannot easily implement discovery, inquiry and problem-based approaches as they all require more individual encouragement and support for each student. Thus, as she expressed, *"…class size surely affects our teaching approaches that we offer our students for mathematical literacy development"* (Tchr16). Therefore, the teachers emphasized that it is very difficult for teachers to devote enough time to each student in class with more than thirty. They cannot observe each student's learning style and behavior carefully in order to develop appropriate learning opportunities for mathematical literacy. That is to say, crowded classrooms clearly lower teacher performance in teaching mathematics. Accordingly, it would be much easier for them to provide mathematics education emphasizing mathematical literacy in small class sizes.

4.2.1.10. Other barriers

In the analysis of the theme of the other barriers, several categories emerged in participants' responses. These categories included: (i) Discontinuity in primary-secondary transition, (ii) Lack of consultation with teachers, (iii) Drawbacks of joint mathematics exams, and (iv) Drawbacks of multiple-choice tests for appointing teachers. In order to elaborate and expand on the categories mentioned above, the following four sections provided the results with the interview excerpts which were representative of these particular categories respectively.

4.2.1.10.1. Discontinuity in primary-secondary transition

Some of the teachers (n=3) mentioned that the transition from primary to secondary school in relation to mathematical literacy understanding is a big problem that prevents the development of mathematical literacy understanding of students. These teachers believe that students have more likely been taught mathematics emphasizing some mathematical literacy understanding in the primary school, but *"…they unfortunately encounter a different mathematics teaching based on formulas, rules, theorems and proofs in high school"* (Tchr11). In addition, as stated by one of the teachers, *"...most of the secondary mathematics teachers basically lack in necessary understanding of their students' level of mathematical literacy in the first* *years of secondary school"* (Tchr12). However, the teachers asserted that in order to ensure the smooth continuity from primary to secondary school, the secondary mathematics teachers should have a clear idea or detailed understanding of what skills their students have acquired with regard to mathematical literacy after they have been transferred from primary to secondary school.

4.2.1.10.2. Lack of consultation with teachers

For a small number of the participants (n=2), a lack of consultation with mathematics teachers who are in direct contact with the learners over the design of mathematics curriculum content and the formulation of new educational policies in relation to mathematical literacy is another significant barrier to students' mathematical literacy development. In support of the above view, one of these participants stated that, *"…we are generally not consulted at all about educational reforms. Therefore, we will probably not be consulted when it comes to setting up the new mathematics curriculum emphasizing mathematical literacy"* (Tchr7).

4.2.1.10.3. Drawbacks of joint mathematics exams

Moreover, a few teachers (n=2) pointed out the negative impact of joint mathematics exams on students' mathematical literacy development. For example, one of the teachers commented that, *"...sitting the same mathematics exam at the same time forces teachers to use the same method of teaching and assessment in classrooms, which is usually based on traditional teacher-centered methods"* (Tchr12). In other words, all classes simultaneously take the same mathematics exam jointly prepared by teachers of mathematics in school. Hence, even though some teachers try to teach mathematics in the context of mathematical literacy, some others may not teach mathematics by considering mathematical literacy understanding. However, the types of questions they ask in the exam vary accordingly. In such a case, it is easy for teachers to adapt themselves to others who like traditional ways of teaching mathematics. Therefore, these teachers believe that joint mathematics exams form a real barrier to progress in mathematical literacy understanding.

4.2.1.10.4. Drawbacks of multiple-choice tests for appointing teachers

Finally, only one of the participants (n=1) expressed that the preparation for multiple-choice tests for appointing teachers is the other factor which affects the acquirement of mathematical literacy skills as it inevitably directs teachers' philosophy of teaching mathematics towards heavily test-based teaching and learning. This participant remarked that pre-service teachers receive almost their whole education in multiple-choice testing system. In addition, they even have to seriously prepare for the multiple-choice test to be eligible for appointment as a math teacher in one public school. Therefore, as he put it simply:

> …What do you expect from teachers who grow up in such an educational system to develop mathematical literacy among their students? I will tell you what will then happen. They will just continue to provide mathematics education with the same educational system which is mostly based on test score gains (Tchr14).

4.2.2. Teachers' conceptions of the central domains for mathematical literacy development

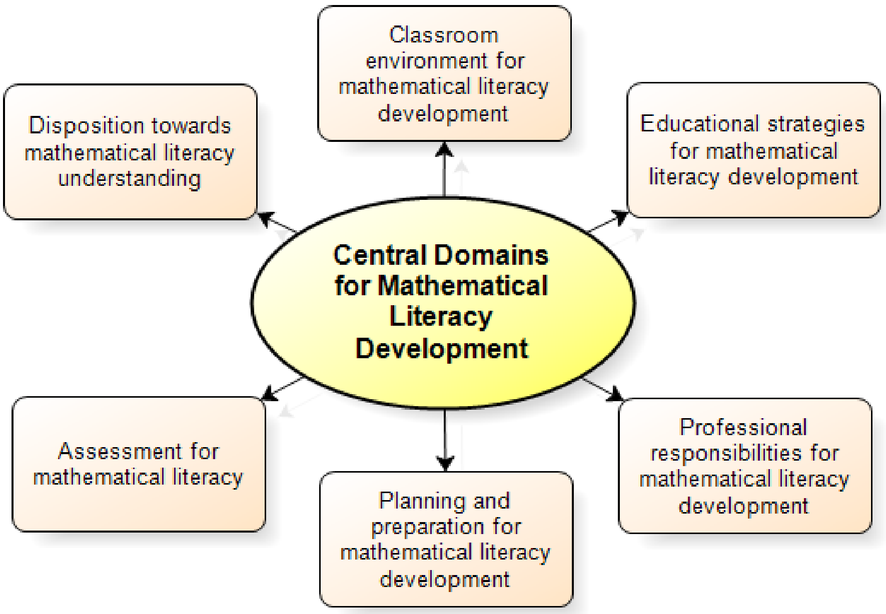

Figure 4.9: A Visual Depiction of Teachers' Conceptions of the Central Domains for Mathematical Literacy Development

In the analysis of the theme of central domains for mathematical literacy development, several categories surfaced in participants' responses to interview questions. In this sense, Figure 4.9 above displays the visual depiction of the six emergent categories identified across the interviews regarding the central domains for mathematical literacy development. These categories included: (i) Educational strategies for mathematical literacy development, (ii) Assessment for mathematical literacy, (iii) Professional responsibilities for mathematical literacy development, (iv) Disposition towards mathematical literacy understanding, (v) Planning and preparation for mathematical literacy development, and (vi) Classroom environment for mathematical literacy development.

Moreover, in Figure 4.10, all teachers were listed on a single chart in order to indicate the strength or intensity of each category that was discerned from their responses to interview questions about the central domains for mathematical literacy development. The numbers in parentheses next to each category represent the corresponding total number of the participants talking about this particular category.

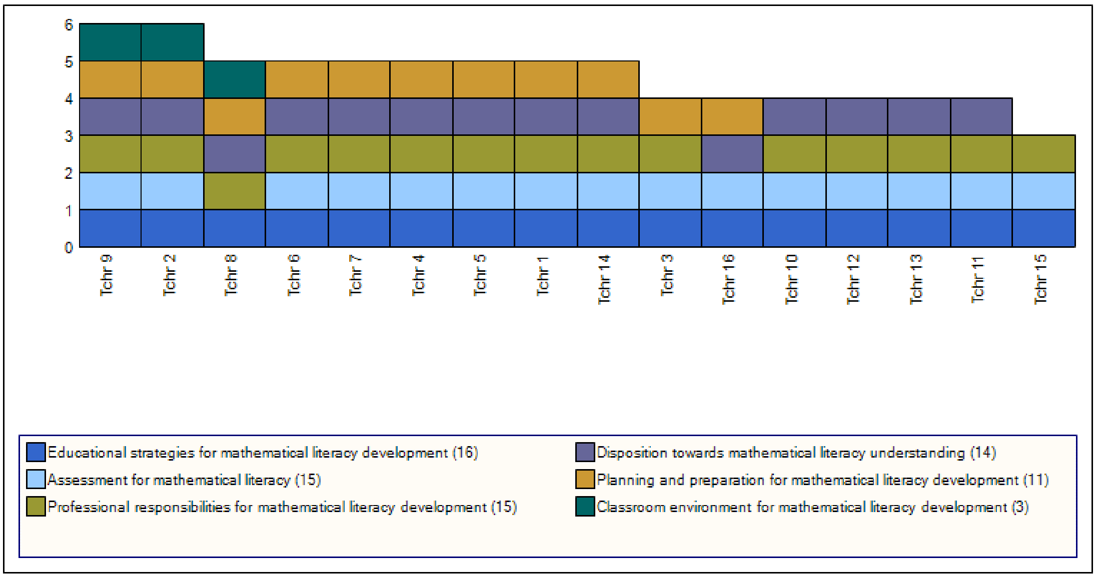

Figure 4.10: A Chart for Teachers' Conceptions of the Central Domains for Mathematical Literacy Development by Teachers

As presented in Figure 4.10, all of the teachers considered the central domains for mathematical literacy development as (i) Educational strategies for mathematical literacy development. Besides, for the majority of teachers, the central domains for mathematical literacy development were considered as (i) Assessment for mathematical literacy, and (ii) Professional responsibilities for mathematical literacy development. Moreover, a large number of teachers viewed the central domains for mathematical literacy development as (i) Disposition towards mathematical literacy understanding. In addition, over half of the teachers also identified the central domains for mathematical literacy development as (i) Planning and preparation for mathematical literacy development. Furthermore, a few teachers based their overall view of the central domains for mathematical literacy development on (i) Classroom environment for mathematical literacy development. The following six sections, accordingly, provided the results with respect to the emergent categories mentioned above in more detail.

4.2.2.1. Educational strategies for mathematical literacy development

In the analysis of the theme of the educational strategies for mathematical literacy development, several categories emerged in participants' responses. These categories included: (i) Importance of appropriate problems for mathematical literacy development, (ii) Importance of engaging students in mathematics learning for mathematical literacy development, (iii) Importance of associating mathematics with real life for mathematical literacy development, (iv) Importance of teaching according to streamed classes for mathematical literacy development, (v) Importance of creating group learning activities for mathematical literacy development, (vi) Importance of ensuring active participation of students for mathematical literacy development, (vii) Importance of implementing inquiry-based instruction for mathematical literacy development, (viii) Importance of encouraging mathematical discussion for mathematical literacy development, (ix) Importance of improving positive teacher-student relationship for mathematical literacy development, (x) Importance of using visual and technological aids and tools for mathematical literacy development, and (xi) Importance of using variety of teaching methods for mathematical literacy development. In order to elaborate and expand on the categories mentioned above, the following eleven sections provided the results with the interview excerpts which were representative of these particular categories respectively.

4.2.2.1.1. Importance of appropriate problems for mathematical literacy development

All sixteen teachers (n=16) pointed out the importance of the types of problems for better mathematical literacy improvement when implementing problem solving approach. Two key arguments were developed. The first argument suggested the preferred types of problems in mathematics education whilst the second suggested the types of problems most appropriate for gaining necessary mathematical literacy skills. In this regard, on the one hand, there was the firm belief among nearly all of the teachers (n=15) that specifically formal mathematics problems and word problems were the most preferred types of problems in the existing mathematics education. On the other hand, most of the teachers (n=13) were also convinced that exposure to all types of mathematical problems but particularly to real world problems were the most appropriate for mathematical literacy acquisition. In analyzing the most preferred types of problems in mathematics education, the following four key areas were addressed: (i) formal mathematics problems, (ii) word problems, (iii) real-world problems, and (iv) applied problems in mathematics.

First of all, a great deal of teachers (n=15) pointed out that most of the teachers prefer the mathematical tasks which involve formal mathematics contents for mathematics teaching. Saying more clearly, in order for students to handle such tasks, they should know some specific mathematical formulas and rules and effectively perform certain mathematical procedures based on these formulas and rules. For example, one of the teachers put it in the following way, *"…our students are usually accustomed to using mathematical formulas and rules as they are generally expected to apply such formulas and rules in order to solve given mathematical problems"* (Tchr16). So, these teachers believe that the formal mathematics problems are probably the most widely used type of problems in mathematics education. The equations of these problems are readily given to students and they are just supposed to solve them. The reason why this type of problems is mostly preferred in our education system is the fact that it is so much easier for teachers to receive and evaluate students' solutions to such type of problems. In other words, *"...if students listen to their teachers' directives well and study the night before the exam, it will be enough for most of them to easily solve such kind of problems in the exam"* (Tchr5).

Secondly, more than half of the teachers (n=11) stated that another preferred type of problems in the mathematics education system are the problems placed in a kind of word problems mostly requiring the same computations. In their view, this type of problems can be easily solved by applying the same procedural processes. Numbers mentioned in these problems are generally made up. Besides, all of the information that students need to solve these standard and contrived problems is always provided by the teachers. For instance, one of the teachers noted that *"...world problems are the type of problems we often use in our math classes"* (Tchr8). The teachers asserted that this type of problems can be simply handled by direct application of previously learned algorithms. Students can also easily gain practice by applying these algorithms. Therefore, in order to perform the solutions and reach final conclusions of such kind of problems, it is enough for students to understand and establish standard numerical relationships in the written texts of the given problems. Thus, as highlighted by another teacher, *"...we, in general, give importance to world problems having a conventional way and certain strategies to solve"* (Tchr13).

The third type of mathematics problems used for mathematics teaching includes real-world problems. However, for many teachers (n=10), this type of problems is rarely or never preferred among mathematics teachers as it takes a larger amount of energy and effort to prepare and solve on behalf of both teachers and students. For example, one of the teachers put it this way, *"...we have a real difficulty to prefer the kind of problems that arise in daily life and work"* (Tchr6), because real-world problems require students to exert so much energy and effort in order to represent and formulate real-life problem situations suitably and competently for finding appropriate solutions.

Finally, half of the teachers (n=8) expressed that applied problems with a primarily mathematical context are also rarely or never preferred among teachers in our mathematics education as it necessitates the use of mathematical theorems, proofs or definitions confidently. For instance, one of the teachers mentioned that, *"…we do not much use the problems which are based on theory and proof as well as on higher mathematical thinking and reasoning"* (Tchr7).

In the analysis of the types of problems which are most convenient for mathematical literacy development in mathematics education, only one key area was addressed, this concerned all types of mathematical problems but especially real world problems. In this regard, many teachers (n=13) indicated that exposure to all types of mathematical problems, particularly real world problems, are most conducive to gaining mathematical literacy understanding and skills. For example, one of the teachers responded that *"…all types of problems are very necessary and useful for mathematical literacy acquisition…all of these are like the floors of mathematical literacy…we gradually move from the bottom floor to the top floor"* (Tchr12). Another teacher, also responding in a similar fashion, said that students who correctly transfer and implement formal mathematical knowledge to solve problems encountered in everyday life can also already solve other types of problems easily and effectively. Thus, as he put it simply, *"…all kind of problems are essential, but real world problems are perhaps the most favorable and ideal for mathematical literacy development"* (Tchr5).

4.2.2.1.2. Importance of engaging students in mathematics learning for mathematical literacy development

Almost all of the teachers (n=15) mentioned the importance of engaging students in mathematics learning for better mathematical literacy development. They pointed out that endearing or arousing students' curiosity in mathematics have a significant effect on the acquisition of mathematical literacy. They thought that we usually present the subject of mathematics as something painful and very difficult to achieve. However, for the development of mathematical literacy, teachers should first be committed to motivate and engage students in learning mathematics by giving the impression that mathematics is not actually difficult subject. For example, one of the teachers said that *"…if we want our students to become mathematically literate, we* *first need to eliminate their prejudice against mathematics by awakening interest and enthusiasm in mathematics"* (Tchr13). Hence, the teachers supported the view that teachers should build and boost students' self-confidence in mathematics by arousing and maintaining their interest in mathematics. For example, instead of teaching only logarithm rules and formulas with some examples, teachers may also bring everyday life issues regarding logarithms to the classroom and pose some real-life problems to students in order to stir their interest and motivation in learning mathematics. Indeed, as one teacher put it simply, *"…it is mostly our responsibility to create positive attitudes and a strong desire for mathematics. Otherwise, our students' mathematical literacy understanding will not be activated"* (Tchr2). Similarly, another teacher emphasized that if the main goal of education is to improve students' mathematical literacy level, *"…we should basically make mathematics fun and meaningful for our students by showing the usefulness of it in the real world"* (Tchr3). In general, the teachers remarked that in mathematics classes teachers generally use abstract language of mathematics and frequently try to impose abstract mathematical procedures on students. However, forcing students to memorize pages of mathematical formulas, rules, and theorems does not surely make them become mathematically literate enough. That is why, *"...rather than using an abstract language, we as a mathematics teacher should give our attention to find out how our students can receive pleasure or enjoyment from mathematics"* (Tchr5). Simply, teachers need to teach mathematics by giving specific examples that can capture their students' interest in mathematics.

4.2.2.1.3. Importance of associating mathematics with real life for mathematical literacy development

Quite a large number of teachers (n=14) also pointed out the importance of associating mathematics with real life in order to provide better mathematical literacy development for students. For example, one of these teachers stated that most of the students usually complain about learning mathematics as it is boring and difficult. However, as she stated further, *"…teaching mathematics through examples of real-life applications helps us avoid complaint about mathematics, thereby giving strong mathematical literacy understanding"* (Tchr11). Similarly, another teacher asserted that *"...when I bring some real world problems to class, our class becomes very entertaining and efficient...they can even comprehend abstract mathematical concepts more easily and quickly"* (Tchr16). The perception of the other teacher is also quite parallel with those of the above teachers. This teacher expressed that their main purpose for teaching mathematics is to prepare their students to meet the daily demands of life. She continued to express that *"...in fact what it is simply needed for students is mathematical literacy understanding since it basically requires them to know the use of mathematics in everyday life"* (Tchr3). So, the teachers generally believe that whenever they teach any mathematics lesson by giving examples from everyday life, their students can understand it much better. In such a case, even the topics which are considered to be very difficult to learn and teach such as trigonometry, derivatives and antiderivatives can be easily taught. Hence, as one of the participants suggested:

> …when we mention some of real-life applications of trigonometry in radar and navigation technology, students can clearly become more interested in learning mathematics. Similarly, the topic of integrals can be taught through its applications in the calculation of both the area of the region under the curve and the volume of irregular objects in addition to some real-world applications of optimization in industry (Tchr12).

In this regard, he argued that if problems are modelled by using trigonometry or integral-based functions, students will be able to much realize how important to use mathematics in life, even in the most basic life functions. The teachers noted that their students normally wonder about where they will use the certain issues learned such as determinants, matrices, integrals and derivatives. So, as one of the teachers stated *"...the real-life applications of mathematics should be particularly emphasized when teaching mathematics"* (Tchr8). That is to say, in order to lessen and even eliminate such wonder, teachers should regularly mention the use of mathematics learned in daily life. Otherwise, students' intensive exposure to a lot of mathematics problems which are not associated with a real-life context apparently results in rapid and continual decrease in their motivation and interest to learn mathematics. This automatically brings about a steady decrease in their level of mathematical literacy understanding.

4.2.2.1.4. Importance of teaching mathematics according to streamed classes for mathematical literacy development

The majority of the teachers (n=14) reported that embracing mathematics teaching according to differentiated classes creates the best possible learning experience for students in mathematical literacy. They thought that if they teach mathematics according to their students' career goals and academic achievements, they will mostly become more successful in their education and social life. Otherwise, if their mathematics class is moving at too fast of a pace, some of their students will get disappointed and stuck. On the other hand, if it is moving too slowly, some others will become bored and disinterested. Thus, assigning students into different classes based on their mathematical abilities allows them to have a better and more convenient way of providing mathematical literacy skills to students. For instance, one teacher mentioned that *"…the practice of setting students into classes based upon their performance in mathematics really helps us to give better and efficient mathematical literacy skills for our students"* (Tchr1). Another teacher suggested a parallel view. He stated that *"…mathematical literacy understanding of students can be increased to higher levels provided that we stream them into classes of similar levels of mathematics ability"* (Tchr8). The other teacher also held the similar view concerning the same issue. This teacher contended that *"…most of the students are not able to acquire the necessary mathematical literacy skills until we teach mathematics according to their educational needs and achievement levels"* (Tchr9). Thus, the teachers generally stated that they truly should not teach everybody all topics including trigonometry, logarithms, complex numbers, limits, derivatives and integrals even we want to do. This is something that cannot be achieved anywhere in the world. It is also opposing to general structure and philosophy of education. According to these teachers, the logic or philosophy here should be giving mathematics education in order to meet the demands of the students from life. Otherwise, students do not have the chance to learn necessary mathematics to use in life. Therefore, the participants believe that the idea of placing students into streamed classes and teaching according to their needs and skills gain growing importance in order to provide better mathematical literacy understanding for them.

4.2.2.1.5. Importance of creating group learning activities for mathematical literacy development

Most of the teachers (n=11) commented that creating group learning activities in class supports the development of mathematical literacy skills of students. For example, one of these teachers asserted that performing group learning activities in mathematics classrooms has many benefits, including *"…many and diverse opportunities for students to gain necessary skills regarding mathematical literacy"* (Tchr1). Similarly, another teacher emphasized that *"…group learning activities enhance students' mathematical literacy understanding as they learn from and to teach each other in groups"* (Tchr15). In the same manner, the other teacher contended that *"...students who work together can foster their own and each other's mathematical literacy understanding by creating collaborative learning environment"* (Tchr16). On the other hand, one of the teachers claimed that it is much easier and very beneficial to make group learning activities with students who are good at mathematics. Otherwise, he finds cooperative learning and group work very time-consuming and pointless to implement in class. In such a case, teachers should consistently provide extra help and support to make students to achieve. However, according to him, it is so difficult to provide such help and support in crowded classrooms. Therefore, *"...I find group learning activities helpful only for high performing students to gain better mathematical literacy understanding"* (Tchr13). So, the participants thought that group learning activities require lots of work on teachers' part. Nevertheless, they highlighted that although group work sometimes becomes very hard to implement in class, there are many potential learning benefits in group work when teaching and learning mathematics. According to them, it is a powerful classroom technique to acquire mathematical literacy skills because students are encouraged to learn a great deal from each other. In other words, students easily share their ideas and explain their reasons as they are more positive about each other when they learn mathematics.

4.2.2.1.6. Importance of ensuring active participation of students for mathematical literacy development

A large number of teachers (n=11) held the perception that ensuring active participation of students provides them with the solid acquisition of mathematical literacy because students are more motivated to learn mathematics when they are involved in lessons. For instance, one of the teachers viewed that, *"…our students can gain mathematical literacy skills best when they are encouraged to stay actively involved in their learning"* (Tchr1). Another view in support of the above view was also taken by the other teacher reporting that active participation clearly helps students to think about the things that they learn in mathematics. Therefore, as he indicated *"…we should always keep our students consistently engaged in mathematics lessons by promoting a student-centered approach in class"* (Tchr4). So, this teacher thought that one of the effective ways of developing mathematical literacy skills of students is to ensure active participation of students by implementing student-centered approach rather than teacher-centered one. In general, the teachers believe that it seems to be a big challenge for most of the teachers to promote active participation in class, as they thoroughly lack time to make every student an active learner. However, they also believe that when students are actively involved in their own mathematics learning rather than passive recipients of transferred knowledge of mathematics, their learning is optimized and proceeds towards the acquisition of mathematical literacy skills. As one teacher noted, *"…active participation of students in classroom teaching encourages them to critically think about the things they do in math"* (Tchr8). Therefore, according to participants, teachers should investigate the ways in which students take an active role in their own mathematics learning. They should ensure active involvement of students in their own learning by designing activities which attract their attention and interest. In this way, teachers can make a considerable positive contribution to students' mathematical literacy understanding.

4.2.2.1.7. Importance of implementing inquiry-based instruction for mathematical literacy development

More than half of the teachers (n=10) remarked that mathematical literacy development requires teachers to implement inquiry-based instruction in which students discover the meaning of mathematical ideas through active engagement with their own learning process. For example, one teacher stated that *"…we must mostly use discovery based learning methods instead of direct or teacher-centered methods if we want to better develop our students' mathematical literacy skills"* (Tchr12). Another teacher also held the similar view. This teacher noted that, *"…for effective acquisition of mathematical literacy skills, students should be specifically oriented towards inquiry learning"* (Tchr14). So, the teachers really would like their students to explore the mathematical facts by themselves rather than memorizing them. Clearly, people never forget what they have discovered in life. Similarly, when students learn something through investigation and exploration, they can comfortably comprehend and accept it. As one teacher expressed, *"…the things that they learn become more permanent through discovery learning"* (Tchr11). Thus, the teachers, in general, believe that when teachers directly give or teach mathematical knowledge to their students, they have a real difficulty to assimilate this knowledge. In such a case, students also do not acquire necessary mathematical literacy skills. Therefore, the teachers should actually prefer to facilitate inquiry-based learning in their classes as possible as they can in order to develop mathematical literacy understanding of students.

4.2.2.1.8. Importance of encouraging mathematical discussion for mathematical literacy development

Over half of the teachers (n=9) responded that encouraging mathematical discussion in class provides learners with a robust understanding of mathematical literacy. For instance, one teacher stated that *"…I hate to solve many similar mathematics problems in front of classroom.* He continued to state that, *"…I wish I could talk about and discuss the mathematical issues with my students within a broader time in order to provide them with better and healthy mathematical literacy* *understanding"* (Tchr12). So, this teacher asserted that mathematical discussion is an important contributory factor for mathematical literacy development. He maintained that he wishes he could have more time to let his students discuss with each other for the better development of mathematical literacy. The view of another teacher is also quite similar to that of the above teacher. This teacher expressed that discussion in mathematics classrooms is really a better way to gain mathematical literacy skills because, *"…mathematical discussion among students leads to higher order mathematical thinking…and it offers opportunities for deeper learning of mathematics"* (Tchr14). So, the teachers in general argue that students are supposed to freely express their thoughts even they are not great or flawed. However, they also sadly argued that in our educational system, from elementary school to university, a brainstorming or discussion is rarely done. Nonetheless, according to the participants, *"…the process of brainstorming and discussion is an essential component of mathematics learning"* (Tchr2). Hence, if it is really aimed to promote students' mathematical literacy understanding in mathematics learning, as noted by one teacher, *"…students should freely and comfortably discuss their mathematical ideas with other students and their teachers"* (Tchr6). Therefore, the teachers believe that students need to feel comfortable in order to easily express their mathematical thoughts in a classroom environment. For that to happen, teachers should encourage a democratic classroom environment. Preparing such a classroom environment for the students not only allows them to feel safe and confident in themselves but also automatically creates and improves their motivation in mathematics learning in the context of mathematical literacy.

4.2.2.1.9. Importance of improving positive teacher-student relationship for mathematical literacy development

Many teachers (n=9) mentioned that improving positive and strong teacher-student relationship is another fundamental factor that supports the development of mathematical literacy among learners. For instance, one of these teachers proposed that *"…ensuring positive communication and dialogue with students is a critical thing for establishing the basis for mathematical literacy skills"* (Tchr9). Another teacher argued with the similar view. This teacher pointed out that a supportive teacher-student relationship undoubtedly influences students' physiology, academic achievement and attitudes towards learning mathematics. Thus, in his view, *"...the high quality of teacher-student relationships significantly contributes to students' mathematical literacy understanding"* (Tchr14). So, the participants suggested that teachers should always take their students' mathematical thoughts into consideration when they express them to their teachers even different than their teachers' thoughts. In this way, teachers can make their students feel that their opinions are very important and valuable for them. In fact, the essence of the matter, as one participant pointed out, *"...the most basic principle of effective teaching mathematics is simply to give teachers' affection and sympathy to their students when teaching something to them"* (Tchr8). Therefore, teachers believe that developing strong and positive teacher-student relationship simply lets the students emotionally feel safe in learning mathematics. Accordingly, students can give their all energy and attention to learning mathematics. As a result, their development of mathematical literacy skills is also accelerated.

4.2.2.1.10. Importance of using visual and technological aids and tools for mathematical literacy development

Several teachers (n=6) remarked that using visual and technological aids and tools clearly deepens students' mathematical literacy understanding by supporting their learning process of mathematics. For instance, one of the teachers said that using information and communication technologies makes mathematical concepts visual and interactive for their students. For example, mathematical images in three dimensions can be easily presented and shifted on the internet by using interactive whiteboard. Teachers can also offer their students several real-life applications of mathematics by building interactive modelling and simulations on computer screens. Accordingly, as he added, "*...it [using information and communication technologies] effectively facilitates the development of mathematical literacy skills"* (Tchr14). Another response in support of the above view was also taken by another teacher responding that *"...utilizing rich mathematical aids and visual materials according to lesson requirements supports sustained and positive change in the development of mathematical literacy skills of students"* (Tchr8). So, this teacher emphasized that taking advantage of those mathematical tools and aids certainly makes students feel pleased and happy in response to learning mathematics as well as developing mathematical literacy skills. In general, the teachers believe that since we are in an age of technology, effective use of technology is an indispensable part of daily life. Therefore, they contended that using visual and technological tools in mathematics education is very important for more effective mathematical literacy development of students.

4.2.2.1.11. Importance of using variety of teaching methods for mathematical literacy development

Finally, some of the teachers (n=5) held the perception that implementing variety of teaching strategies is significant for better acquisition of mathematical literacy understanding. For instance, one of these teachers commented that *"…I certainly do not agree with the claim that there is only one method for mathematical literacy development…the truth of the matter is simply that, there are many and varied"* (Tchr10). The perception of another teacher is also quite parallel with that of the above teacher. This teacher suggested that *"…if we want to firmly foster mathematical literacy understanding of students, we should consistently differentiate our teaching practices and instructional methods according to their learning styles and needs"* (Tchr14). So, the teachers underlined that using different teaching approaches and techniques certainly helps teachers not only to keep their students' interest in learning mathematics, but also to enable them to interact with mathematical literacy content in multiple ways. Therefore, the participants firmly believe that teachers should always give their students enough chance to get involved in different learning styles such as problem-based learning, modelling, group working, or inquiry-based learning for their effective mathematical literacy development.

4.2.2.2. Assessment for mathematical literacy

In the analysis of the theme of the assessment for mathematical literacy, two categories emerged in participants' responses. These categories included: (i) Expected mathematical literacy proficiency levels, and (ii) Assessment methods for mathematical literacy. In order to elaborate and expand on the categories mentioned above, the following two sections below provided the results with the interview excerpts which were representative of these particular categories respectively.

4.2.2.2.1. Expected mathematical literacy proficiency levels to be assessed

Almost all of the teachers (n=15) argued about the expected proficiency levels that need to be assessed for mathematical literacy. Two key arguments were developed. The first argument suggested the expected lower level of proficiency whilst the second suggested the expected higher level of proficiency for mathematical literacy. In analyzing the expected proficiency at the lower levels, the following three key areas were addressed: (i) performing basic mathematical operations, (ii) understanding problem situations, and (iii) knowing mathematical definitions. First of all, for a large number of teachers (n=12), performing basic mathematical operations was considered as one of the lower levels for mathematical literacy proficiency. For example, one of the teachers highlighted that, at the lowest level, mathematics teachers generally expect their students to do at least basic mathematical operations. Thus, as he put it, *"...I guess it would be very absurd to expect anything more than knowing basic mathematical operations for the minimum level of mathematical literacy understanding"* (Tchr7). So, the teachers believe that performing basic mathematical calculations and manipulations is probably the lowest level of mathematical ability for mathematical literacy. Moreover, any student who cannot do even basic mathematical operations is not supposed to go through the advanced mathematical stages of mathematical literacy. Thus, according to participants, in the first stage students must gain the ability to perform basic mathematical operations such as addition, subtraction, multiplication and division as they mostly need to perform those simple mathematical operations in their daily life effectively and efficiently.

Secondly, a few of the participants (n=2) argued that understanding mathematical structures of problem situations also lies at the lowest level for mathematical literacy. They said that it is one of the basic requirements for satisfactory mathematical literacy understanding. A representative excerpt subscribing to this view was given by one of the participants saying that, *"...the ability to understand mathematical aspects or expressions of a given problem situation is the first and lowest step on the ladder of mathematical literacy"* (Tchr6). Otherwise, as she further pointed out, *"...our students cannot formulate and interpret the problem situations unless they have this ability"* (Tchr6).

Finally, only one of the teachers (n=1) maintained that knowing mathematical definitions is of fundamental and immediate importance to the minimum level of mathematical literacy proficiency. This teacher put it this way, *"...I suppose that, students firstly need to know the mathematical definitions very well, as it is one significant prerequisite for the minimum level of mathematical literacy understanding"* (Tchr4).

In the analysis of the expected proficiency at the higher levels for mathematical literacy, the following three key areas were addressed: (i) mathematical thinking, reasoning and argument, (ii) transferring knowledge of mathematics to unfamiliar contexts, and (iii) formulating problem situations mathematically. First of all, over half of the teachers (n=9) asserted that students are especially expected to develop mathematical thinking, reasoning and argument skills for a good level of mathematical literacy proficiency. For example, one of these teachers explained that *"...I definitely regard that mathematical reasoning and thinking skills are among the higher level skills for mathematical literacy"* (Tchr7). He explained that reasoning and thinking skills positively influence and rapidly raise the level of mathematical literacy of students. The same issue is also illustrated by the following view from another teacher stating that mathematical reasoning and thinking allow students develop convincing mathematical arguments. Therefore, according to him, *"...mathematical reasoning and thinking skills are undoubtedly ideal competencies to reach higher level of mathematical literacy"* (Tchr5).

Secondly, several teachers (n=4) mentioned that transferring of what has been learned in one mathematical context to either different or unfamiliar contexts is thought as a higher level of mathematical literacy. One response in support of the above view was given by one of these teachers perceiving that, *"...relating or* *associating old knowledge of mathematics with new one through identifying mathematical structures and establishing relations in problem situations falls into a higher category of mathematical literacy"* (Tchr15).

Finally, a small number of teachers (n=2) commented that formulating problem situations mathematically is classified as higher level of proficiency in mathematical literacy. For example, one of these teachers suggested that mathematical literacy at the highest level requires translating real-world problem situations into mathematical expressions and symbols in order to employ necessary operations. As he put it simply, *"…what else do I want from my students other than representing problem situations mathematically?"* (Tchr3).

4.2.2.2.2. Assessment methods for mathematical literacy

Most of the teachers (n=13) expressed their thoughts about the assessment methods for mathematical literacy. Two key areas were developed, including assessment through students' overall performance, and no way to assess mathematical literacy. First of all, for many teachers (n=12), assessing students' overall performance requires to take a broad range of mathematical literacy skills and competencies into considerations. For example, one of these teachers mentioned that *"…assessment of mathematical literacy should be spread over a longer period of time in order to evaluate students' overall performance"* (Tchr14). He also added that this overall performance should rely on multiple forms of assessment rather than using solely a standard single type of assessment. Similarly, another teacher pointed out that *"…comprehensive and overall performance of students in mathematics should be monitored over a reasonable time-frame"* (Tchr2). According to her, using only written exams or teacher-made tests as a basis for mathematical literacy assessment instrument always remains insufficient. So, the teachers highlighted that overall performance of students in mathematics should be taken into account for reliable and fair assessment of mathematical literacy. Using only midterm or end of semester exams can measure only a small part of students' knowledge and skills. It may not accurately reflect students' ability to reason deeply or creatively or the ability to perform non-routine tasks. Therefore, participants suggested that teachers should not be restricted to assessing isolated skills and knowledge that require students solely to recall some specific facts or information if their purpose in education system is to develop a good level of mathematical literacy among students.

Moreover, for only one of the participants (n=1), it was highlighted that although the above arguments about the assessment methods for mathematical literacy have great merit, there are indeed no reliable and valid ways to assess mathematical literacy level of students. The following describes a representative comment from this participant emphasizing that teaching mathematics in the context of mathematical literacy is plausibly one of the most desirable things in today's modern education system, but, *"...it is in fact almost impossible to find honest, appropriate and accurate ways to assess mathematical literacy"* (Tchr16).

4.2.2.3. Professional responsibilities for mathematical literacy development

In analyzing the theme of the professional responsibilities of teachers for mathematical literacy development, three categories emerged in participants' responses. These categories included: (i) Importance of appropriate teacher training programs for mathematical literacy development, (ii) Need for a revolutionary change in teacher training programs for mathematical literacy development, and (iii) Not believing in benefit of teacher training programs for mathematical literacy development. In order to elaborate and expand on the categories mentioned above, the following three subsections below provided the results with the interview excerpts which were representative of these particular categories respectively.

4.2.2.3.1. Importance of appropriate teacher training programs for mathematical literacy development

A slightly higher number of teachers (n=11) perceived that appropriate teacher training programs are crucial for the emergence and sufficient development of mathematical literacy understanding among teachers. For example, one of these participants commented that *"...most of us have a great difficulty to teach mathematics with emphasis on mathematical literacy, because we did not get any* *education or training about it"* (Tchr1). Therefore, he argued that one of the most primary things is to train mathematics teachers about mathematical literacy. In a similar fashion, another teacher offered that mathematics teachers have great deficiencies in teaching mathematics emphasizing mathematical literacy. He added that *"…mathematical literacy can be given a priority in the context of our math education, provided that we get regular in-service trainings on mathematical literacy"* (Tchr10). The perception of the other teacher is also quite parallel with those of the above participants. This teacher stated that mathematics teachers cannot sufficiently provide students with mathematical literacy skills in teaching mathematics *"…unless there are compulsory teacher development trainings on how to achieve mathematical literacy acquisition for students"* (Tchr12). So, the participants believe that most of the teachers clearly need in-service teacher trainings on how to teach mathematics in the context of mathematical literacy. Thus, teachers need to be informed about teaching techniques of certain mathematical topics in mathematical literacy context through in-service training programs. At least, in order to create awareness and attract attention of teachers about mathematical literacy, teacher trainings such as symposiums and seminars should be done for mathematics teachers. It may be the best alternative practice that can be made to the existing teachers. Otherwise, they do not have any opportunity and facility to develop themselves in a way that matters mathematical literacy.

4.2.2.3.2. Need for a revolutionary change in teacher training programs for mathematical literacy development

About half of the teachers (n=7) mentioned that the radical change in teacher training programs is necessary for the optimum and sustainable teachers' progress in mathematical literacy understanding. For example, one of these teachers pointed out that traditional in-service teacher trainings are monotonous and have no impact on teachers' mathematical literacy understanding. Therefore, as he alleged, *"…we need to promote both the quality of and the equality of access to in-service training opportunities for all mathematics teachers"* (Tchr12). Similarly, another teacher viewed that *"…people who provide in-service training to us on mathematical literacy should be competent and suitably qualified in order to answer disturbing and* *challenging question marks in our heads"* (Tchr11). Another view in support of the above responses was also taken by the third teacher emphasizing that instead of wasting time in boring and repetitive teacher training programs, *"...we should talk over what other countries do for mathematical literacy achievement. I mean the reasons behind their success in PISA should be discussed in such training programs"* (Tchr14). In general, the teachers believe that teacher training programs are necessary for teachers' ongoing professional development, but their content and concepts must be changed to provide teachers with the necessary mathematical literacy understanding. In this regard, experts in this area may show them how to teach mathematics in different ways through practical knowledge rather than theoretical knowledge. They may also show them many specific real-life examples and different types of resources that they can present in their lessons. The teachers also suggested that training programs may be done in groups of four or five teachers for two or more months under the supervision of experts in the field of mathematical literacy. Trainers may present sample lessons emphasizing mathematical literacy. They may also mention possible problems encountered in the classroom, as well as diverse instructional methods in order to orient students towards better mathematical literacy understanding. Besides, resource sharing can also be done among teachers and trainers within such programs. In such a case, one teacher put it briefly in a humorous way by saying that *"...if there is any such quality training even requiring a larger registration fee, I will certainly go by running"* (Tchr6).

4.2.2.3.3. Not believing in benefit of teacher training programs for mathematical literacy development

Exceptionally, a number of the teachers (n=6) reported that they do not believe in any benefit of teacher training programs to develop enough mathematical literacy understanding for mathematics teachers. For instance, one of these teachers asserted that *"...it seems difficult to believe but more than eighty percent of the participants in teacher training programs say that such kind of programs look like holiday time to most of their colleagues"* (Tchr15). In a similar manner, another teacher maintained that mathematics teachers in general do not believe any benefit of traditional type of teacher trainings for mathematical literacy development, because as he indicated,

"…what people usually say for these kind of programs are repetitive and useless for me…teaching environment is totally different in real classroom atmosphere" (Tchr3). The teachers remarked that it is indeed very boring and completely waste of time for teachers to repeatedly listen the same things that are known to them. Thus, many teachers mostly do not like training programs and see them as a chore. The other teacher also held the similar view. This teacher noted that *"…it is generally of the form that one who has nothing or less to do with mathematics education gives us in-service training…What can that person give me about mathematical literacy?"* (Tchr7). Hence, the teachers do not believe that teacher training programs add something to their understanding of mathematical literacy. Therefore, according to them, keeping teachers regularly to receive in-service training does not provide any good result for the teaching of mathematics emphasizing mathematical literacy.

4.2.2.4. Teachers' disposition towards mathematical literacy understanding

A large number of participants (n=14) indicated that teachers' positive disposition towards mathematical literacy understanding have a profound and significant impact on their students' mathematical literacy understanding and skills. For instance, one of the participants stated that teachers are role models of their students. That is why, *"…if my students realize that I am in love with mathematics, they will also fall in love with mathematics"* (Tchr12). Similarly, another participant expressed that although the use of technology in class is important for mathematical literacy acquisition, teachers themselves play more important role than technology. He added that *"…mathematical literacy understanding of a teacher greatly affects mathematical literacy understanding of her or his students"* (Tchr4). Therefore, the participants believe that teachers' disposition and competency in teaching mathematics emphasizing mathematical literacy understanding play a vital and unique role in the development of students' mathematical literacy skills.

Moreover, it would be interesting to report that one of these participants (n=1) also considered the same issue from another angle. This participant said that he could agree up to a certain point that teachers' disposition and ability to teach mathematics in the context of mathematical literacy have a real and great influence on their students' mathematical literacy development. However, he also agreed that most of his students clearly lack in basic mathematical knowledge and skills. They could not do even simple mathematical calculations. Therefore, as he put it simply, *"…it does not so much matter whether or not their teachers have disposition and attitudes towards teaching mathematics in the context of mathematical literacy"* (Tchr7).

4.2.2.5. Planning and preparation for mathematical literacy development

In analyzing the theme of planning and preparation for mathematical literacy development, several categories emerged in teachers' responses. These categories included: (i) Importance of lesson preparation for mathematical literacy development, (ii) Importance of subject knowledge of mathematics for mathematical literacy development, and (iii) Importance of setting instructional outcomes for mathematical literacy development. In order to elaborate and expand on the categories mentioned above, the following three subsections provided the results with the interview excerpts which were representative of these particular categories respectively.

4.2.2.5.1. Importance of lesson preparation for mathematical literacy development

About half of the teachers (n=7) mentioned that students' better development of mathematical literacy is highly correlated to their teacher's effective planning for mathematics lessons emphasizing mathematical literacy understanding. For example, one of these teachers considered that developing an effective lesson plan clearly keeps teachers well organized and on track while catering for different interest areas and abilities of students. Thus, she stated that *"…a good level of lesson preparation is an essential component of teaching mathematics in the context of mathematical literacy"* (Tchr16). Similarly, another teacher proposed that successful development of mathematical literacy requires teachers to prepare themselves for many different classroom activities and focus on their implementations. He said that *"…teachers have to prepare well for and think so much about what their students need in order to better provide mathematical literacy skills for them"* (Tchr5). The other teacher also held the parallel view regarding the same issue. According to him, in order to give mathematical literacy skills to students, teachers need to make the transition from rote learning to mathematical reasoning and conceptual understanding. However, it is more difficult for teachers to make this transition because they have to work much more than before to be prepared for the lesson. For example, *"...instead of just substituting 1 for x to find f (1), we must bring more interesting and useful real-life tasks and applications of the concept of functions to class in order to demonstrate the relevance of mathematics to real world"* (Tchr1). They need to explore various and efficient ways to introduce students to mathematical literacy understanding, because *"...higher student achievement in mathematical literacy simply depends on imagining the lesson before it happens regarding students' interests, needs and levels"* (Tchr6). Therefore, the teachers believe that although it takes much time and efforts for teachers, a serious and effective lesson preparation based on mathematical literacy emphasis is essential to help their students master mathematical literacy skills.

4.2.2.5.2. Importance of subject knowledge of mathematics for mathematical literacy development

Less than half of the teachers (n=5) expressed that in-depth subject knowledge is fundamental not only to teach mathematics effectively and creatively, but also to integrate it with the other areas for guiding students to discover its real-life applications. For example, one of these teachers said that *"...a good subject knowledge in mathematics is indeed a crucial prerequisite for mathematics teachers to create and support required mathematical literacy understanding of their students"* (Tchr8). On the other hand, another teacher reflected on the same issue from different perspectives. She remarked that *"...as you can appreciate, if teachers have well subject knowledge, their students will naturally much respect and value them"* (Tchr6). So, this participant claimed that good subject knowledge is essential for getting students to have confidence in their teacher's knowledge and skills in mathematics. Thus, according to her, if students do not trust and believe in their teachers, teachers definitely suffer and have trouble in providing adequate skills and understanding including mathematical literacy skills and understanding for their students. Therefore, the teachers, in general, thought that a satisfactory level of subject knowledge is essential to teach mathematics skillfully and confidently in order to develop clear mathematical literacy understanding of students.

4.2.2.5.3. Importance of setting instructional outcomes for mathematical literacy development

A small number of the participants (n=2) explained that establishing instructional outcomes is significant as it provides both teachers and students with a clear purpose to direct teaching and learning efforts towards mathematical literacy development. The participants expressed that unfortunately we generally do not talk about the instructional outcomes that describe essential skills needed for mathematical literacy. However, as one participant emphasized, *"…our students need to be well aware of those outcomes for being mathematically literate enough"* (Tchr4). Otherwise, many students rightly wonder about what they will gain from learning uninteresting and complicated mathematical facts in class. Hence, these participants believe that if teachers want their students to gain necessary mathematical literacy skills, they should exactly identify and state what their students will be expected to learn from their instruction.

4.2.2.6. Classroom environment for mathematical literacy development

A few of the teachers (n=3) commented that good classroom management is an important aspect of ensuring effective teaching and learning mathematics to take place in the context of mathematical literacy. They stated that it is not surprising to find that implementing effective classroom management always correlates with high achievement in mathematics since it always offers a conducive environment for effective teaching and learning of mathematics. This accordingly contributes to development of mathematical literacy skills of students as they are eager to learn mathematics. For example, one of these teachers emphasized that:

> …if we really want to increase mathematical literacy level of our students we should always try to create a learning environment in which students safely and freely express their thoughts and feelings even they are meaningless and trivial (Tchr8).

Thus, the participants supported the view that it is very crucial for teachers to provide learning opportunities in well-organized and pleasant atmosphere to all of the students in order to allow them to develop necessary mathematical literacy understanding. To put it more clearly, establishing an effective classroom management allows teaching and learning of mathematics to proceed in an efficient and safe manner. This undoubtedly results in optimizing students' acquisition of mathematical literacy understanding as they feel comfortable and focused on learning mathematics.

4.3. Characterizing teachers' conceptions of the mathematics curriculum emphasizing mathematical literacy

The teachers' conceptions regarding mathematics curriculum emphasizing mathematical literacy were discussed in the following two sections, with the first section focusing on the teachers' conceptions of challenges in curriculum implementation in the context of mathematical literacy, and the second section focusing on the teachers' recommendations about curriculum modifications in relation to mathematical literacy.

4.3.1. Teachers' conceptions of the challenges in curriculum implementation in the context of mathematical literacy

In the analysis of the theme of the challenges in curriculum implementation in the context of mathematical literacy, several categories surfaced in teachers' responses to interview questions. In this sense, Figure 4.11 below displays the visual depiction of the three emergent categories identified across the teachers' interviews. These categories included: (i) Challenges associated with intensity of mathematics curriculum, (ii) Challenges associated with teachers' perspectives and practices, and (iii) Challenges associated with lack of mathematical literacy emphasis in mathematics curriculum.

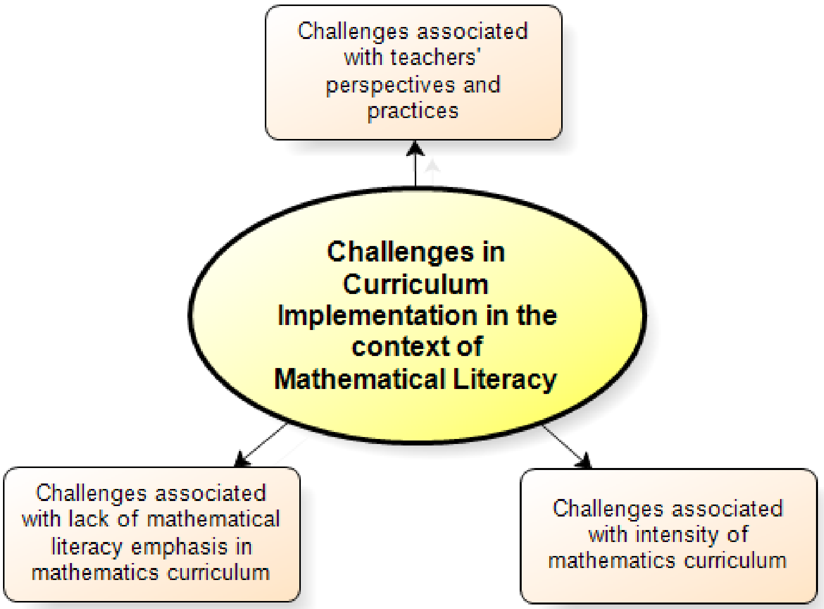

Figure 4.11: A Visual Depiction of Teachers' Conceptions of the Challenges in Curriculum Implementation in the context of Mathematical Literacy

Moreover, in Figure 4.12, all participants were listed on a single chart in order to indicate the strength or intensity of each category that was determined from their responses to interview questions related to the challenges in curriculum implementation in the context of mathematical literacy. The numbers in parentheses next to each category represent the corresponding total number of the participants talking about this particular category. As presented in the table, all of the teachers considered the challenges in curriculum implementation in relation to mathematical literacy as (i) Challenges associated with intensity of mathematics curriculum. Besides, for nearly all of the teachers, the challenges in curriculum implementation in the context of mathematical literacy were viewed as (i) Challenges associated with teachers' perspectives and practices. Furthermore, over half of the teachers also based their overall view of the challenges in curriculum implementation in relation to mathematical literacy on (i) Challenges associated with lack of mathematical literacy emphasis in mathematics curriculum.

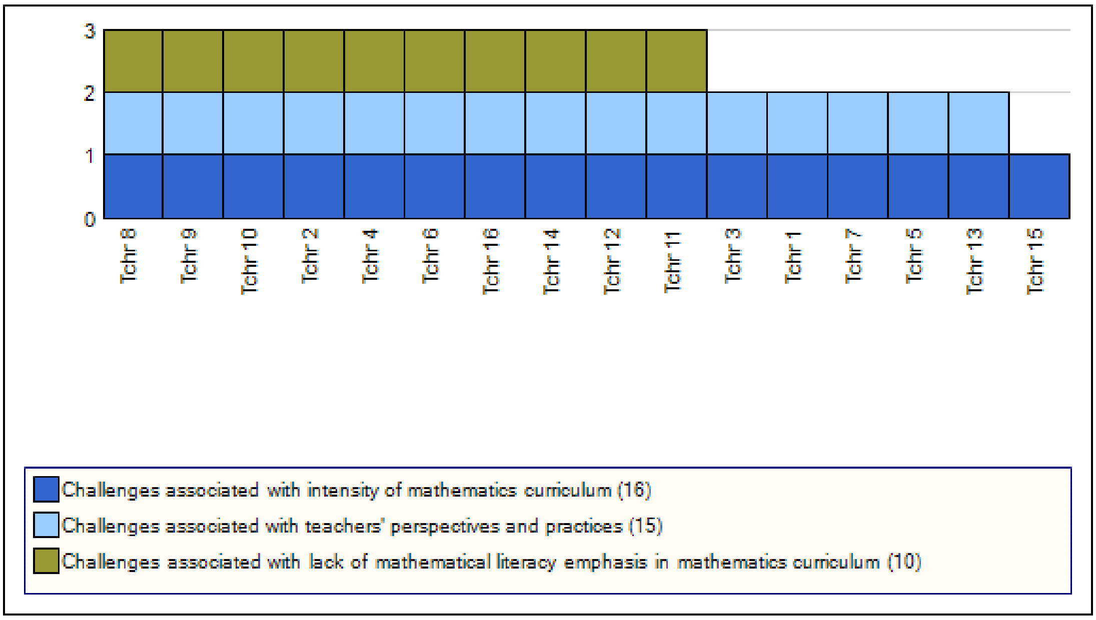

Figure 4.12: A Chart for Teachers' Conceptions of the Challenges in Curriculum Implementation in the context of Mathematical Literacy by Teachers

The following three sections, accordingly, provided the results with respect to the emergent categories mentioned above in more detail.

4.3.1.1. Challenges associated with intensity of mathematics curriculum

All of the teachers (n=16) mentioned that congested mathematics curriculum is one of the significant factors that negatively influences mathematical literacy development of students. They reported that mathematical literacy skills can be gained in a flexible and long period of learning time. This in turn means that any time constraint in mathematics curriculum may cause problems in the future. That is to say, if necessary, real-life applications of one or more specific learning objectives can be freely performed in one or more lessons. However, the mathematics curriculum plan is so congested to put such a teaching strategy into life as it requires substantial commitment of time. For example, in the tenth grade curriculum, *"…teachers have a lot of topics to go round and discuss with students in a limited period of time"* (Tchr12). They said that in their six hours of math lessons per week for each class, they must unceasingly teach mathematics topics respectively which are recommended in the intensive curriculum plan. They emphasized that there is not sufficient time for learning through investigation, inquiry or active involvement. Thus, they believe that rather than reducing the number of mathematics topics to cover, *"...our mathematics curriculum content must be reduced as reasonable as possible for allowing students enough time to gain necessary mathematical literacy skills"* (Tchr14). For example, one of the teachers explained that due to an overly intense mathematics curriculum it is very hard to find enough time for teaching activities such as modelling, group study and discussion needed for mathematical literacy development. Indeed, as he stated, *"...we seemingly teach many things to our students, but the fact is that we teach nothing"* (Tchr7). Similarly, another teacher expressed that providing mathematical literacy understanding to students requires a lot of teaching and learning time. However, as he pointed out, *"...we have very congested and rigorous mathematics curriculum which makes it very difficult for us to teach anything outside of the curriculum content"* (Tchr10). The view of the other teacher is also quite similar to those of the above teachers. This teacher noted that extensive and intense mathematics curriculum allows mathematics teachers little time or no time at all to try to explore other things like mathematical literacy. As he said, in the school, *"...we just try to keep up with other math teachers in terms of the curriculum topic being taught next"* (Tchr15). Indeed, according to the participants, all instructional strategies and practices as well as the course content, learning objectives and outcomes are clearly described and specified in the mathematics curriculum. In other words, *"...literally everything looks so pretty and well organized, but we are in a lot of trouble with respect to time to achieve these specified learning outcomes"* (Tchr6). So, the teachers in general supported the view that although many teachers believe that mathematical literacy understanding is essential for students, most of the teachers are not able achieve to provide it because there is no time or energy. Teachers' class schedules are quite busy to implement necessary teaching and learning techniques for mathematical literacy such as problem and inquiry based learning. Generally, they are forced to teach or deliver too fast in order to cover all the curriculum topics. They have no other choice than to struggle to keep pace with the congested curriculum targets. As they progress through this way, a great deal of their students unfortunately learn very little or even nothing. Therefore, these participants recommended that the most and primary thing in mathematics education is to simplify the curriculum in order to facilitate students' learning in the context of mathematical literacy.

4.3.1.2. Challenges associated with teachers' perspectives and practices

Almost all of the participants (n=15) argued about teachers' perspectives and practices for the applicability of curriculum with emphasis on mathematical literacy. Two key arguments were developed. The first argument suggested the resistance of teachers to some changes whilst the second suggested the possible ways for acceptance of new curriculum among teachers. First of all, for most of the teachers (n=14), even if mathematics curriculum is changed in a manner in which it emphasizes mathematical literacy understanding, most of the teachers will have no desire to get involved in prescribed changes and they will simply choose to resort to former instructional strategies and methods. In other words, any curriculum modification in accordance with mathematical literacy understanding will probably bring teacher resistance as well. For example, one of the teachers explained that it might be more comfortable and easy to maintain teaching in the familiar ways rather than striving to develop new teaching practices and skills. Therefore, according to him, *"...some will instantly demonstrate resistance to any change in curriculum with more emphasis on mathematical literacy"* (Tchr11). Similarly, another teacher pointed out that many teachers acknowledge that they lack the necessary knowledge and competence to adequately perform what they are expected to do in the guidelines of any new curriculum. Hence, as he put it simply, *"…What do we expect from teachers who are mostly used to teaching mathematics in traditional methods? …They are more likely to resist any curriculum change"* (Tchr12). The view of the third teacher is also quite parallel to those of the above teachers. This teacher expressed that *"...today, since everything is changing so fast in education, teachers naturally develop a formidable defense mechanism to any change"* (Tchr13). Thus, in his opinion, even the curriculum is changed along with mathematical literacy, most of the mathematics teachers still continue teaching in the same ways as before. Hence, the participants remarked that since most of the teachers are inclined to teach mathematics without sufficiently focusing on the learning objectives, principles and outcomes, many are unfortunately not unaware of these learning objectives, principles and outcomes within mathematics curriculum guidelines. Teachers generally do not read and implement principles, objectives, aims, goals and learning outcomes recommended to them by that curriculum, because everything is not as it is written in the text of curriculum. When they enter class to teach any topic, they generally try to finish the topic on time. They are usually not much concerned about goals, objectives or learning outcomes of this topic. Thus, as one participant stated, *"…we just keep mathematics curriculum into the bag and carry it with us in order to check and write down what we should teach weekly into the class register"* (Tchr9). So, the participants in general asserted that most of the teachers resist curriculum change because they would always like and prefer to keep doing things in the same way as they have in the past mathematics teaching. This is somewhat due to human nature. However, what is worse that *"…while some of them usually become unaware of modifications made in curriculum, some others even can complete the whole year without looking at one page of curriculum documents"* (Tchr5). For this reason, the teachers believe that even curriculum is changed and improved to help students with the opportunity to gain mathematical literacy skills, it is highly probable that many teachers will not much care about it.

Secondly, about half of the teachers (n=7) contended that if mathematics curriculum is changed and revised to align with mathematical literacy understanding, its acceptance should be expected in the long run through the persuasion of teachers to such change and revision. For example, one of the teachers noted that the effects of curriculum change according to mathematical literacy understanding should not be expected in the short term. Otherwise, as he posited, *"…the possibility of the applicability of such curriculum is almost close to zero because it does not make any sense to put it in front of us to implement"* (Tchr1). Another view in support of the above explanation was also taken by another teacher stating that in order to make teachers to implement the curriculum reform emphasizing mathematical literacy, *"…the first thing to be done is to convince teachers of the importance of such curriculum for people to be productive and informed citizens"* (Tchr5). Moreover, the participants thought that young teachers are generally more open to new ideas and alternative approaches. In other words, *"…new generation of teachers are different and more ready and available for new perspectives and innovations"* (Tchr4). In addition to this, the teaching process of older generation is also changing too. They have started to keep pace with new approaches and innovation. Thus, the participants believe that in spite of some difficulties to implement it in the first few years, any curriculum modification towards mathematical literacy understanding will at least create awareness among teachers about the importance of mathematical literacy. Therefore, if there becomes curriculum change or reform emphasizing mathematical literacy understanding, most of the teachers will gradually take that change into consideration after a specified period of time.

4.3.1.3. Challenges associated with lack of mathematical literacy emphasis in mathematics curriculum

More than half of the teachers (n=10) reported that lack of enough mathematical literacy emphasis in mathematics curriculum is the other significant obstacle that prevents the development of mathematical literacy understanding. They frankly said that whenever they look at the curriculum, they do not feel that the curriculum emphasizes them to teach mathematics in the context of mathematical literacy. According to them, *"…it is difficult for us to find real-life teaching and learning activities or examples that require reasoning and logical thinking skills in the curriculum"* (Tchr6). Indeed, the teachers actually believe that the curriculum slightly emphasizes the development of mathematical literacy skills of students, but what emphasis takes higher priority and precedence for students in curriculum is to carry out routine mathematical operations and calculations efficiently in order to find the final result of given mathematical problems. Hence, when presenting any mathematics problem to their students, most of the teachers usually skip the step of formulating and representing it mathematically because the problems are readily given in equations or symbols. Hence, as one participant maintained that if mathematics teachers are expected to provide necessary mathematical literacy understanding to their students, *"…our mathematics curriculum emphasis should be mainly based on teaching mathematics in the context of mathematical literacy"* (Tchr11). The other response in support of the above view was also taken by another participant stating that mathematics teachers generally stay at the level of knowing and applying mathematical rules and formulas when teaching mathematics. He continued to state that *"...the level of reasoning and argument is usually ignored and is of second importance to us as they are not rigorously and sufficiently imposed by the curriculum"* (Tchr12). Thus, the participants pointed out that although mathematics curriculum enforces teachers to give a lot of mathematics knowledge to their students in a very short period of time, it does not exactly describe and emphasize where their students will or must use this knowledge of mathematics in daily life. They honestly stated that the mathematics curriculum lacks in this regard. Since teachers use the curriculum as a major guide in teaching mathematics, they accordingly also stay lacking in teaching mathematics emphasizing mathematical literacy understanding. However, as the participants highlighted, if we adapt or modify the curriculum according to real-life demands of students, we have no doubt that great progress will be made in students' mathematical literacy understanding.

Moreover, there is clearly a natural temptation among the participants to view that although there is a lack of enough mathematical literacy emphasis in existing curriculum, it is too much to claim that there is not any emphasis on mathematical literacy. In this context, some of the teachers (n=3) also pointed out the fact that there is an apparent shift towards mathematical literacy understanding in curriculum in recent years. For example, one of the teachers subscribing to this view indicated that *"...although not a hundred percent, some of the learning outcomes and goals of present mathematics curriculum are said to be clearly consistent with learning outcomes of mathematical literacy"* (Tchr16). In her views, the one and only problem is the proper implementation of these learning outcomes and goals in mathematics classes.

4.3.2. Teachers' recommendations for the curriculum modifications in relation to mathematical literacy

In the analysis of the theme of the recommended curriculum modifications in relation to mathematical literacy, several categories surfaced in participants' responses.

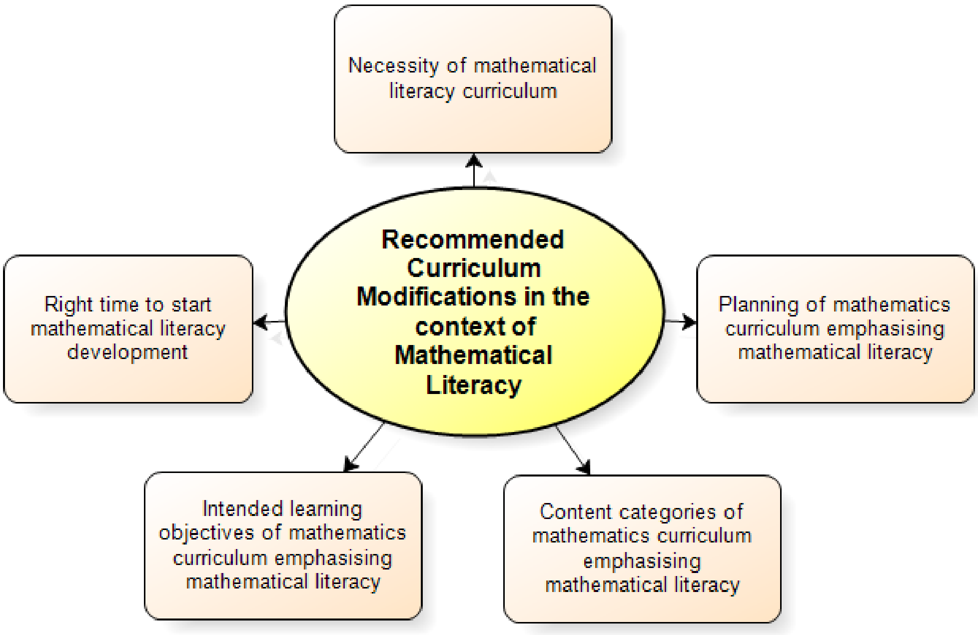

Figure 4.13: A Visual Depiction of the Recommended Curriculum Modifications in relation to Mathematical Literacy

In this sense, Figure 4.13 displays the visual depiction of these emergent categories identified across the interviews. As can be simply understood from the figure, five emergent categories were determined from the extracted findings about the recommended curriculum modifications in the context of mathematical literacy. These categories included: (i) Right time to start mathematical literacy development, (ii) Planning of mathematics curriculum emphasizing mathematical literacy, (iii) Necessity of mathematical literacy curriculum, (iv) Content categories of mathematics curriculum emphasizing mathematical literacy, and (v) Intended learning objectives of mathematics curriculum emphasizing mathematical literacy.

Moreover, in Figure 4.14 below, all teachers were listed on a single chart in order to indicate the strength or intensity of each category that was discerned from their responses to interview questions in connection with the recommended curriculum modifications in the context of mathematical literacy. The numbers in parentheses next to each category represent the corresponding total number of the participants talking about this particular category.

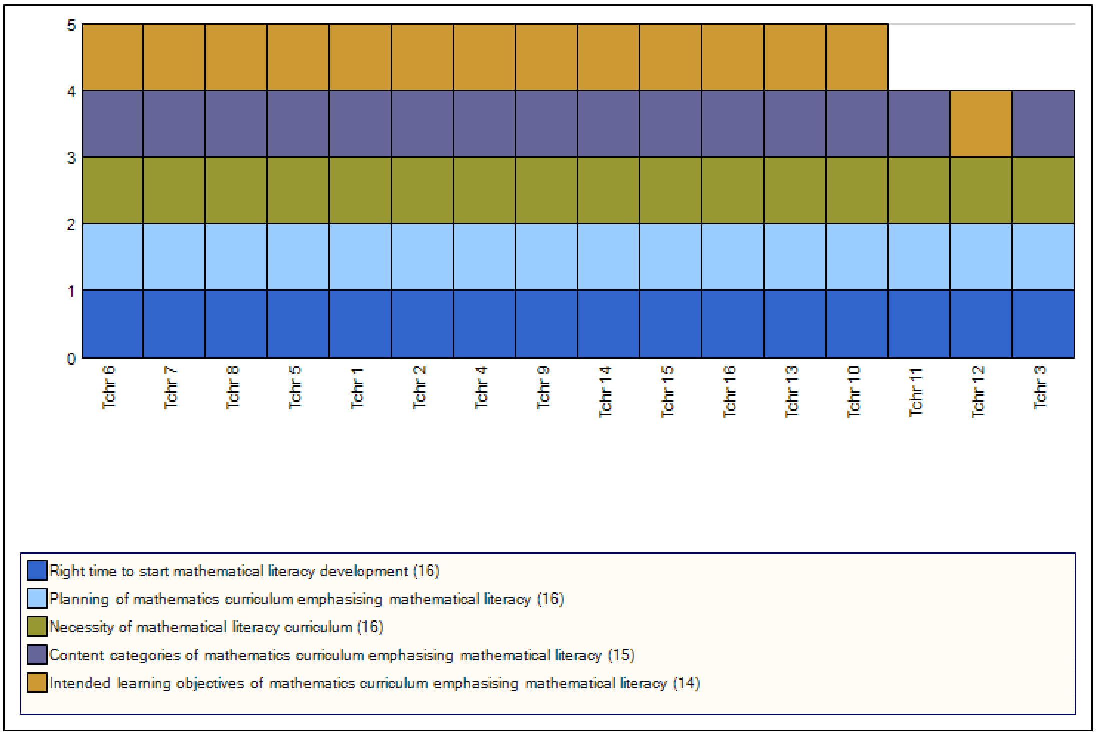

Figure 4.14: A Chart for the Recommended Curriculum Modifications in relation to of Mathematical Literacy by Teachers

As clearly demonstrated in Figure 4.14, all of the teachers reflected on (i) Right time to start mathematical literacy development, (ii) Planning of mathematics curriculum emphasizing mathematical literacy, and (iii) Necessity of mathematical literacy curriculum. Besides, quite a large number of teachers made some recommendations about (i) Content categories of mathematics curriculum emphasizing mathematical literacy. Furthermore, most of the teachers also offered some suggestions about (v) Intended learning objectives of mathematics curriculum in the context of mathematical literacy. The following five sections, accordingly, provided the results with respect to the emergent categories mentioned above in more detail.

4.3.2.1. Right time to start mathematical literacy development

All of the teachers (n=16) laid a great deal of emphasis on the right time to start mathematical literacy acquisition. A common view of all of these sixteen teachers is that it becomes too late for students to acquire mathematical literacy understanding at high school. They are all assured that firm and sustainable mathematical literacy development begins before high school. Four arguments in equal importance were developed by the teachers to support the view mentioned above. First of all, quite a large number of teachers (n=14) suggested that mathematical literacy acquisition could begin with elementary school. For instance, one of the teachers responded that *"...I certainly believe that the development of the essential basis for mathematics knowledge that underpins mathematical literacy should start from elementary school"* (Tchr7). Otherwise, according to him, it becomes too late for most of the students to start to gain mathematical literacy skills at high school. Similarly, another teacher noted that students' attitudes and interest towards mathematics are usually formed in elementary school years. Therefore, he pointed out *"...mathematical literacy understanding needs to start from the first class of elementary school in a structured way and gradually advance through all education levels"* (Tchr3). So, these teachers suggested that if we do not begin to give mathematical literacy skills such as problem solving, communication, representation, critical thinking, reasoning, reflection, and making connections in elementary school years, it will be very hard and wearisome for teachers to make any valuable or meaningful contribution to their students' mathematical literacy understanding at high school levels.

Interestingly enough, although one participant (n=1) seemed not to reject the view that mathematical literacy development may begin in elementary school, he indeed much more strongly supported the view that it is in fact more efficient to begin mathematical literacy development of students at high school because they necessarily develop more reasoning and argument skills to interpret data as they get older. This participant put it this way, *"...mathematical literacy can actually be better acquired by students in high school as they make more use of higher-order thinking and reasoning"* (Tchr1). Nonetheless, according to him, one cannot ignore the fact that *"...first steps in mathematical literacy understanding needs to be taken in primary school years"* (Tchr1).

Secondly, for many teachers (n=14), the appropriate time to start mathematical literacy development could also go back to preschool years. For example, one of the teachers subscribing to this view highlighted that *"…acquiring mathematical literacy skills is quite simply like learning a new language"* (Tchr11). She continued to explain that starting to build such skills early in preschool creates the largest possible set of benefits and opportunities for substantial mathematical literacy understanding in the high school years. The same issue is further illustrated by the following excerpt from another teacher insisting that, *"…I think that the earlier children begin to gain mathematical literacy skills the better they will develop it in their education throughout life"* (Tchr8). So, these teachers believe that if rigorous and systematic development of mathematical literacy is required, mathematical literacy skills should be provided in early childhood. Thus, starting to acquire mathematical literacy skills at preschool takes critical importance. In other words, establishing solid foundation for mathematical literacy understanding at preschool not only increases school readiness but also has longer-term effects on further development of mathematical literacy skills.

Thirdly, many teachers (n=13) proposed that the development of mathematical literacy skills should begin with the moment a person first meets with mathematics and continue to progress throughout life. For example, one of the teachers mentioned that, *"…mathematical literacy skills are truly set to be developed when an individual first becomes acquainted with the magnificent and mysterious world of mathematics"* (Tchr16). She also added that, *"…its development never ends, by contrast, it goes on to improve as long as she or he is in this world"* (Tchr16). The perception of another teacher is also quite similar to the above-mentioned view. This teacher stated that we live in a world in which we undoubtedly need mathematical literacy skills every time from birth to death. That is to say, as long as we live in this world, we always need to criticize or support someone else's ideas or thoughts with mathematical reasoning as well as expressing and defending our own ideas or thoughts. With this in mind, she bluntly said that it is very simple and clear for her to guess the correct time to start mathematical literacy development of any individual. Namely, *"…mathematical literacy should begin from the moment that a person is first introduced to mathematics and continue to grow exponentially until the last day of his or her life"* (Tchr6).

Finally, more than half of the teachers (n=10) recommended that starting mathematical literacy understanding with parental involvement or home learning environment is of great importance and absolute necessity for its steady and successful development. For example, the following expression from one of the teachers is typical of the comments made on this issue, *"...parents are the first educators of their children to help them gain self-confidence in mathematical skills"* (Tchr14). Therefore, as the same teacher pointed out, young people's growth and development of mathematical literacy greatly depends on their parents' ability to support them. A parallel expression about the same issue can also be found in the following view from another teacher saying, *"...parents have a profound impact on their children's mathematical literacy skills as their earlier learning experiences thoroughly occur in the context of the home environment"* (Tchr13). So, these teachers in general believe that parental education is a valuable primary source for effective mathematical literacy development of children.

4.3.2.2. Planning of mathematics curriculum emphasizing mathematical literacy

In the analysis of the theme of planning of mathematics curriculum in the context of mathematical literacy, several categories emerged in participants' responses. These categories included: (i) Curriculum approach for mathematical literacy development, (ii) Emphasis on the mathematical modelling cycle, (iii) Modifications according to diverse needs and mathematical skills of students, and (iv) Integrated curriculum approach. In order to elaborate and expand on the categories mentioned above, the following four sections below provided the results with the interview excerpts which were representative of these particular categories respectively.

4.3.2.2.1. Curriculum approach for mathematical literacy development

All of the teachers (n=16) pointed out the importance of curriculum approach based on mathematical literacy. Two key arguments were developed. The first argument suggested the implemented curriculum approach in the present mathematics education system whilst the second suggested the recommended curriculum approach for a strong and sustainable mathematical literacy development. First of all, there is the firm belief among most of the teachers (n=13) that mathematics in the present curriculum is usually taught in the following sequence: definitions, formulas, computations, examples, and applications. For example, one of the teachers mentioned that *"…we every time follow the same process of mathematics teaching steps: incomprehensible definitions, copious formulas, routine computations, similar examples, and familiar applications"* (Tchr10). According to him, most of the mathematics teachers are generally not interested in theorems and proving stuff. The view of another teacher is also quite similar to that of the above teacher. This teacher expressed that *"…after giving the theoretical information or formulas, a number of structurally similar problems are usually presented and solved"* (Tchr2). So, according to him, mathematics curriculum is mostly delivered in the following manner: definitions, formulas, computations, examples, and applications. The following perception from the other teacher can also be taken as typical of this kind of evidence. This teacher contended that teachers are so accustomed to traditional methods of teaching mathematics. That is to say, *"…we usually first give definitions and formulas, then perform computations by using them in the provided examples, and finally practice routine problems for more comprehension"* (Tchr12). He said that mathematical theorems and their proofs by discovery are usually not given. They are always separated out from the main teaching process as they are difficult and time consuming to teach and learn. According to him, in most of the classes, teachers mainly teach their students shortest paths to solve the given mathematics problems in order to tackle them in the minimum time possible. Accordingly, students are supposed to be well prepared for the university entrance examination. So, in general, participants believe that because of time restraints as well as the intensity of mathematics curriculum, most of the teachers have no other way out of implementing the curriculum in the following sequence: definitions, formulas, computations, examples, and practices. According to them, teachers clearly do not have any time and even desire to make students discover the underlying facts or logic that they need to learn before they solve the problem by themselves.

Secondly, almost all of the teachers (n=15) were convinced that although it apparently looks difficult to implement, the curriculum with emphasis on problems, discovery, hypothesis, verification, generalization, association, and inference is much appropriate for a good mathematical literacy acquisition. For instance, one of the teachers held the view that mathematics curriculum presented in the following sequence: problems, discovery, hypothesis, verification, generalization, association, and inference seems more suitable and effective for mathematical literacy, but the question *"...how feasible is it to implement such curriculum approach?"* (Tchr16) requires further exploration. Similarly, another teacher mentioned that student-centered learning activities in general focus on the following steps of mathematics teaching process to be successful: problems, discovery, hypothesis, verification, generalization, association, and inference. She then acknowledged that *"...it seems difficult to use and follow but in reality any mathematics curriculum based on student-centered learning activities much more supports mathematical literacy development"* (Tchr6). So, the teachers thought that the ideal way towards sufficient and consistently growing mathematical literacy development is truly based on the efficient application of each step of the curriculum approach involving problems, discovery, hypothesis, verification, generalization, association, and inference. In so doing, students make the necessary associations between prior and new knowledge, and they become actively engaged in their mathematics learning. For example, as one of the participants illustrated, *"...when teaching the concept of derivative, students can simply make connections...it is indeed instantaneous rate of change or the slope of the tangent line to the graph of the function at a given point"* (Tchr4). Therefore, despite certain difficulties and potential challenges, the participants generally believe that achieving to implement such curriculum approach surely creates and facilitates the development of mathematical literacy skills.

4.3.2.2.2. Emphasis on the mathematical modelling cycle

Many teachers (n=12) perceived that the mathematical modelling process is a central and vital component of any mathematics curriculum emphasizing mathematical literacy. For example, one teacher stated that mathematical modelling is the core and essence of mathematical literacy. In his view, *"...if mathematics curriculum aims to educate mathematically literate individuals, the mathematical modelling cycle should deserve a special place in this curriculum at all levels"*

(Tchr5). In a similar fashion, another teacher mentioned that the mathematical modelling cycle is essential for better mathematical literacy development. As he indicated, *"...thanks to clear and firm emphasis on every stage of the mathematical modelling cycle in mathematics curriculum, we can easily foster our students' mathematical reasoning and thinking skills"* (Tchr9). The third teacher also suggested a parallel view. This teacher expressed that each step of such a cycle fully and effectively contributes to development of mathematical literacy skills of students. That is why, in her opinion, *"...sufficient emphasis on the mathematical modelling cycle is what I want from any mathematics curriculum"* (Tchr16). So, the teachers highlighted that the mathematical modelling cycle is at the center of mathematical literacy understanding. They noted that mathematical modelling requires the ability to formulate a given problem in its context as well as the ability to competently employ the standard arithmetic and algebraic operations in order to solve the formulated problems. It also requires the ability to interpret and evaluate these results in the same context. Accordingly, it provides people with a sense of security and assurance to confidently formulate, employ and interpret any real-life problem rather than asking for immediate help when encountering them. Thus, the participants firmly believe that the mathematical modelling cycle is very important and it should be an integral aspect of any curriculum emphasizing mathematical literacy.

4.3.2.2.3. Modifications according to diverse needs and mathematical skills of students

More than half of teachers (n=11) suggested that better mathematical literacy development of students can be achieved as long as continuous adaptations and modifications in mathematics curriculum are done to conform to their diverse mathematical abilities and needs. For example, one of the teachers explained that mathematics curriculum is still too difficult to make it more accessible to vocational high school students. In these schools, teachers struggle to teach mathematics to students who experience great difficulty to understand mathematical concepts. In other words, students in vocational schools are generally afraid of mathematics and they do not already perform well in mathematics. He said even worse that many of his students lack basic mathematics knowledge and skills needed in everyday life.

However, without basic knowledge and skills of mathematics, it is very difficult to obtain mathematical literacy because students do not see its applicability to their life. Hence, according to him, if we desire maximum efficiency from mathematics curriculum to improve mathematical literacy understanding of students, *"...different types of mathematics curricula should be developed and used for students in different types of schools"* (Tchr1). Similarly, another teacher contended that if we want to promote mathematical literacy level of students performing poorly in mathematics, *"...mathematics curriculum content should be certainly narrowed to accommodate students' mathematical skills and needs in order to teach mathematics at their pace"* (Tchr7). The other teacher also held the parallel view regarding the same issue. This teacher believed that, *"...good level of mathematical literacy acquisition highly depends on such a mathematics curriculum which is planned and implemented to meet students' different demands and expectations from life"* (Tchr8). So, the teachers believe that low performing students regretfully receive minimum or almost no benefit from mathematics curriculum. Hence, it truly does not support their mathematical literacy development. Teachers also have many difficulties to teach even the most basic things about mathematics to low performing students. It follows that in order not to make students afraid of and feel bored with mathematics, teachers usually give students as minimum knowledge as possible. Otherwise, it is very difficult for teachers to complete the curriculum on time. That is why, according to the participants, if we want to provide effective and efficient mathematical literacy development for each student, mathematics curriculum should be designed or adapted according to diverse math skills and needs of students.

4.3.2.2.4. Integrated curriculum approach

Several teachers (n=5) recommended that integrated curriculum approach is important for creating and improving mathematical literacy skills of students as it makes learning mathematics more meaningful to them. For instance, one teacher stated that it is clearly unreasonable to suggest that mathematical literacy is only the responsibility of mathematics. According to him, we should break down the barriers between mathematics and different disciplines such as chemistry, physics, biology, history, geography, and economics in order to develop mathematical literacy skills through making meaning of knowledge. He illustrated that students can see how trigonometry is applied in physics or they can see how the topic of rate and ratio is used in chemistry. Thus, he believes that *"…integrated curriculum approach can be more effective for mathematical literacy development, as it simply helps us to see the life as a whole"* (Tchr4). Similarly, another teacher mentioned that *"...integrated curriculum approach is a way to effectively develop students' skills in mathematical literacy, for it makes mathematics much more sensitive to their needs"* (Tchr14). In general, the teachers believe that mathematics and the other subjects unquestionably have numerous content areas in common. Hence, according to them, learning in mathematics happens faster when we integrate mathematics into other areas and make it part of our daily life. In this regard, curriculum which is integrated with other disciplines and many everyday events is of critical importance. For this reason, the teachers asserted that in addition to focusing on abstract ideas or concepts within mathematics itself, we should also make its connections to other domains or subjects as well as to real-life applications outside the classroom.

4.3.2.3. Necessity of mathematical literacy curriculum

In analyzing the theme of the necessity of mathematical literacy curriculum, two categories emerged in participants' responses. These categories included: (i) Mathematical literacy approach embedded within mathematics curriculum, and (ii) Need for separate mathematical literacy curriculum. In order to elaborate and expand on the categories mentioned above, the following two subsections provided the results with the interview excerpts which were representative of these particular categories respectively.

4.3.2.3.1. Mathematical literacy approach embedded within mathematics curriculum

A great deal of teachers (n=14) pointed out that mathematical literacy understanding should be embedded within mathematics curriculum as mathematical literacy and mathematics are justifiably not distant and independent from each other. For example, one of the teachers held the view that, *"...I, of course, support the* *effective delivery of mathematical literacy within mathematics curriculum in order for students to develop both mathematics and mathematical literacy skills altogether across the same curriculum"* (Tchr11). The other view in support of the above perception was also taken by another teacher stating that mathematical literacy and mathematics are highly interrelated and share many common areas. He emphasized that we should not think of mathematics and mathematical literacy as separate and distinct from each other. The only trouble is to find teachers who teach mathematics in the context mathematical literacy. If we achieve to train in-service and pre-service teachers to support mathematical literacy development, mathematical literacy will become an integral part of mathematics curriculum in the long run. Therefore, according to him, *"…rather than thinking of mathematical literacy as a separate thing, it should be fully integrated into mathematics curriculum"* (Tchr15). Similarly, the third teacher proposed that *"…mathematical literacy must be transmitted through mathematics curriculum in order to make mathematics more tangible and easy to understand"* (Tchr16). Otherwise, as she contended, mathematics becomes more abstract and boring, as well as difficult to comprehend for students. Moreover, one teacher also warned that if mathematics and mathematical literacy are introduced as separate curriculum subjects, *"…mathematical literacy curriculum will be unavoidably perceived as a curriculum for lower-performing students in mathematics"* (Tchr14). So, the teachers in general believe that mathematics curriculum which emphasizes mathematical literacy is really of paramount importance. In their opinion, mathematical literacy and mathematics must go hand in hand in order to associate mathematics to real world circumstances. For this reason, they believe that mathematical literacy understanding can be nicely placed into mathematics curriculum.

4.3.2.3.2. Need for separate mathematical literacy curriculum

A large number of teachers (n=12) commented that mathematics and mathematical literacy can also be planned and taught as separate curriculum areas if some required conditions are not available for teaching mathematics in the context of mathematical literacy. For instance, one of these teachers said that because of the time constraint to complete mathematics curriculum, *"…it is necessary and even* *unavoidable to implement different curriculum for mathematical literacy acquisition"* (Tchr4). In this way, as he noted, many students can easily realize the use of mathematics in science, technology and other fields as well as workplaces and everyday life. Similarly, another teacher pointed out that *"…in fact it would be very nice to have separate mathematical literacy curriculum to freely discuss the value of mathematics by demonstrating its applications in many crucial areas of life without time restrictions"* (Tchr9). The third teacher, also arguing in a similar fashion, mentioned that mathematics teachers do not have any extra time to do something different than what they usually do in mathematics class. She continued to argue that *"…a separate mathematical literacy curriculum of course makes it easy for us to talk about math and life through many diverse activities over an extensive duration of time"* (Tchr6). So, thanks to this separate curriculum, teachers can get the opportunity to effectively and flexibly develop problem solving skills of their students through various learning techniques such as real-life problem based learning, discovery-based learning, project-based learning and collaborative learning. Moreover, the teachers also noted that mathematics and mathematical literacy can be planned and given as a single curriculum, but it should not be forgotten that such a curriculum just becomes appropriate for students in moderate and especially high performing schools. For example, one teacher said that *"...since low-performing math students are usually tracked or streamed into vocational secondary schools, we have real difficulty to complete even math curriculum on time in this type of schools"* (Tchr3). Hence, according to her, if mathematics and mathematical literacy are designed and implemented as a separate curriculum, it will be more realistic and efficient for students to gain mathematical literacy skills. In general, the participants emphasized that teachers are not able to allocate any time or effort toward teaching mathematics in the context of mathematical literacy because of their busy teaching schedule and seriously intensive curriculum. Most of the time, teachers even could not make their students involved and engaged in classroom activities. Therefore, the teachers believe that if mathematical literacy is taken as a different curriculum, it will definitely lessen and relax teachers' heavy teaching workload.

4.3.2.4. Content categories of mathematics curriculum emphasizing mathematical literacy

Nearly all of the teachers (n=15) expressed their thoughts about the content categories of mathematics curriculum required for better mathematical literacy development of students. Accordingly, the following four key areas were developed: (i) quantity, (ii) uncertainty and statistics, (iii) change and relationships, and (iv) space and shape. First of all, quite a large number of teachers (n=14) highlighted that numbers are everywhere in life. Wherever we look at, it is highly possible that we can face with different numbers and various quantities. Hence, they recommended that the quantity concept should be the primary and essential content category of mathematics curriculum emphasizing mathematical literacy. For example, the following perception from one of the teachers is typical of the comments made on this issue. He said that the concept of quantity may be the most important and indispensable aspect of mathematics curriculum based on mathematical literacy understanding. That is, in his view, *"…knowledge and understanding of the number and number operations always take precedence over others"* (Tchr3). So, the teachers generally pointed out that the number sense and quantity are of fundamental and paramount importance for mathematical literacy. To put it more explicitly, mathematically literate individuals should have the ability to clearly identify numbers and quantities around them. Besides, they should be able to understand their different representations as well as relations between those quantities. They should also have the ability to make strong evaluations on any given argument based on numbers and arithmetic calculations. Therefore, according to these participants, all these naturally require the fact that *"…the notion of quantity should be the central part of mathematics curriculum in the context of mathematical literacy"* (Tchr8).

Secondly, more than half of the teachers (n=10) proposed that mathematics curriculum inspiring learners to become mathematically literate should include the uncertainty and data content category as this category is intimately intertwined with real life. For example, one of the teachers noted that being mathematically literate is, in a way, based on the knowledge and understanding of basic probability and statistics and their applications to different areas of daily life. Therefore, as he stated that, *"…I* *would like to give much importance and priority to uncertainty and data in the curriculum"* (Tchr14). In general, the teachers remarked that people inherently use probability and data analysis in their everyday lives to solve many of their decision-making problems. That is to say, *"…uncertainty and statistics have a large applicability in a wide variety of fields of life from opinion polls and stock markets to weather forecasting"* (Tchr5). This in turn means that if students are required to interpret, assess and judge conclusions involving uncertainty, the content category of basic probability and statistics should surely be taught to them. That is why, these participants firmly believe that uncertainty and data analysis should be an integral part of mathematics curriculum emphasizing mathematical literacy.

Thirdly, again the same number of the teachers (n=10) highlighted that understanding of change and relationships is another crucial content category of mathematics curriculum that significantly influences students' mathematical literacy level, because real life itself always involves the process of relationships and their change over time. They stated that, thanks to mathematical literacy, we can interpret and translate these relationships into different representations in addition to creating their symbolic and graphical representations. For example, one of the teachers subscribing to this view commented that the ability to express consistent change and relationships in everyday life with patterns and functions is taken as a sign of being mathematically literate. So, as he added, *"…we should put particular stress on change and relationships in mathematics curriculum"* (Tchr7). According to the participants, change in one part of life may sooner or later influence all relationships in other parts of life. This suggests that mathematical literacy understanding requires people to identify logical and mathematical relationships in daily life and continuous change in those relationships. Change in relationships can also be suitably modelled with appropriate functions. Therefore, these teachers clearly accepted the fact that *"…no one will question if change and relationships are placed at the heart of mathematics curriculum"* (Tchr13).

Finally, over half of the participants (n=9) suggested that more emphasis could be placed on space and shape in mathematics curriculum as everyday life is surrounded by space and various geometric shapes in different sizes. For instance, one of the participants responded that *"...geometry and spatial reasoning skills such as translation, rotation and reflection of objects defined in a geometric space are of great importance to mathematical literacy development of students"* (Tchr6). According to the teachers, the ability to understand and apply the knowledge of space and shapes is the main source for logical reasoning and three-dimensional thinking skills. They noted that these thinking and reasoning skills simply allow us to handle many serious real-life problems easily such as finding the areas and volumes of irregular shapes and solids by using dynamic geometry applications. Thus, these participants in general believe that *"...when we learn to effectively use the geometry and spatial sense in our daily life we also learn to act mathematically literate"* (Tchr1).

4.3.2.5. Intended learning objectives of mathematics curriculum emphasizing mathematical literacy

In analyzing the theme of the intended learning objectives of mathematics curriculum in the context of mathematical literacy, two categories emerged in teachers' responses. These categories included: (i) Aiming to build problem-solving skills, and (ii) Aiming to increase interest and engagement in mathematics. In order to elaborate and expand on the categories mentioned above, the following two sections provided the results with the interview excerpts which were representative of these particular categories respectively.

4.3.2.5.1. Aiming to build problem-solving skills

A great deal of teachers (n=14) mentioned that the acquisition of problem solving competency is one of the central aims of mathematics curriculum emphasizing mathematical literacy, because mathematical literacy mainly refers to an individual's capacity to formulate, employ and evaluate mathematical problems in a variety of situations. For example, one of the teachers stated that *"...more emphasis should be given to the development of problem solving skills in the mathematics curriculum because these skills allow individuals to model, interpret, and solve real-life problems in different contexts"* (Tchr13). The other view in support of the above-mentioned statement was also taken by another teacher expressing that *"...mathematical literacy* *always goes hand in hand with problem solving. I cannot think of them as separate entities"* (Tchr6). Hence, as she proposed, problem solving should always be incorporated into the mathematics curriculum. Similarly, the third teacher noted that *"…problem solving should be the initial and major objective of mathematics curriculum as it provides the ability to use and apply mathematics we know in different contexts"* (Tchr1). So, the teachers attached special importance to problem solving skills for developing mathematical literacy skills of their students. According to them, problem solving process roughly involves understanding, formulating, employing and evaluating. That is to say, we should first understand each piece of information presented in the problem. Next, we should represent it mathematically. Then, we should devise a plan or strategy as well as carrying out the devised plan for finding a proper solution to the given problem. Finally, we should reflect on the solutions to the problem. In fact, that is what we are supposed to do when we face with any real-life problem. Therefore, these participants believe that problem solving should be the central focus of mathematics curriculum as it develops students' skills in reasoning and allows them to gain a much clear understanding of any problem encountered in life.

4.3.2.5.2. Aiming to increase interest and engagement in mathematics

More than half of the teachers (n=10) commented that one of the most important goals of mathematics curriculum in accordance with better mathematical literacy acquisition is to keep students interested in and engaged with mathematics. For instance, one of these teachers argued that if there is a high demand to develop mathematical literacy understanding and skills among students, *"…mathematics curriculum should of course aim to make students value mathematics by demonstrating the beauty and power of mathematics in solving real-world problems"* (Tchr4). Another teacher, also arguing in a similar fashion, said that most of his students sadly have a strong bias towards mathematics as well as lacking motivation and interest to learn it. This in turn directly negatively affects their mathematical literacy understanding and skills. Hence, he stated that *"What I desire from any mathematics curriculum is simply to guide me to increase my students' enthusiasm and engagement in mathematics"* (Tchr12). Otherwise, as he pointed out, the less excitement students have in learning mathematics, the less understanding they show for mathematical literacy. Thus, these teachers in general highlighted that the strongest aspect of any curriculum emphasizing mathematical literacy is to aim to foster and maintain students' interest and enjoyment in learning mathematics by manifesting its benefits and excellence in everyday life. According to them, only, in doing so, we can really make students appreciate the importance and value of mathematics in their surroundings. This is also the most basic and powerful path to better mathematical literacy acquisition.

4.4. Summary of the results

Overall, this chapter attempted to present the data in such a way to provide the complete picture of the secondary mathematics teachers' conceptions of mathematical literacy as possible as possible. Specifically, three main themes and different categories for each theme were identified from teachers' responses to interview questions about mathematical literacy. Accordingly, based on the research data, the teachers' conceptions of mathematical literacy were mainly categorized into (i) Concept of mathematical literacy, (ii) Effective development of mathematical literacy, and (iii) Mathematics curriculum emphasizing mathematical literacy. First of all, three themes were defined from teachers' responses to the concept of mathematical literacy. These themes included (i) Nature of mathematical literacy, (ii) Fundamental mathematical capabilities for mathematical literacy, and (iii) The relationships between mathematics and mathematical literacy. In analyzing the theme of the nature of mathematical literacy, seven categories were classified from participants' responses, which involved (i) Possession of mathematical knowledge and skills, (ii) Functional mathematics, (iii) Problem solving, (iv) Mathematical thinking, (v) Innate mathematical ability, (vi) Conceptual understanding, and (vii) Motivation to learn mathematics. Besides, in the analysis of the theme of the fundamental mathematical capabilities for mathematical literacy, seven categories were discerned from participants' responses including (i) Communication, (ii) Reasoning and argument, (iii) Using symbolic, formal and technical mathematical language and operations, (iv) Mathematizing, (v) Representation, (vi) Devising strategies for solving problems, and (vii) Using mathematical aids and tools. Moreover, in analyzing the theme of the relationships between mathematics and mathematical literacy, three categories were developed from participants' responses consisting of (i) Mutual relations between mathematics and mathematical literacy, (ii) Set-subset relations between mathematics and mathematical literacy, and (iii) Unlike mathematical literacy, mathematics is an abstract formal science.

Secondly, two themes were created from teachers' responses to how effective development of mathematical literacy occurs. These themes included (i) Barriers to development of mathematical literacy, and (ii) Central domains for mathematical literacy development. In the analysis of the theme of the barriers to development of mathematical literacy, several categories surfaced in participants' responses. These categories involved: (i) Barriers associated with students, (ii) Barriers associated with teachers, (iii) Drawbacks of the university placement examination, (iv) Shortcomings of undergraduate teacher education programs, (v) Lack of appropriate math textbooks for mathematical literacy, (vi) Lack of differentiated instruction, (vii) Lack of mathematical literacy understanding in primary education, (viii) Time constraint in teaching mathematics, (ix) Crowded classrooms, and (x) Other barriers. Besides, in analyzing the theme of the central domains for mathematical literacy development, six categories were determined from participants' responses including (i) Educational strategies for mathematical literacy development, (ii) Assessment for mathematical literacy, (iii) Professional responsibilities for mathematical literacy development, (iv) Disposition towards mathematical literacy understanding, (v) Planning and preparation for mathematical literacy development, and (vi) Classroom environment for mathematical literacy development.

Finally, two themes were emerged from teachers' responses about mathematics curriculum emphasizing mathematical literacy. These themes included (i) Challenges in curriculum implementation in the context of mathematical literacy, and (ii) Recommended curriculum modifications in relation to mathematical literacy. In the analysis of the theme of the challenges to mathematical literacy emphasis in mathematics curriculum, three categories were generated from participants' responses involving (i) Challenges associated with intensity of mathematics curriculum, (ii) Challenges associated with teachers' perspectives and practices, and (iv) Challenges associated with lack of mathematical literacy emphasis in mathematics curriculum. Moreover, in analyzing the theme of the recommended curriculum modifications in the context of mathematical literacy, five categories were specified from participants' responses consisting of (i) Right time to start mathematical literacy development, (ii) Planning of mathematics curriculum emphasizing mathematical literacy, (iii) Necessity of mathematical literacy curriculum, (iv) Content categories of mathematics curriculum emphasizing mathematical literacy, and (v) Intended learning objectives of mathematics curriculum emphasizing mathematical literacy. Accordingly, Figure 4.15 below displays the visual depiction of all of the emergent categories identified across the interviews regarding the secondary school teachers' conceptions of mathematical literacy.

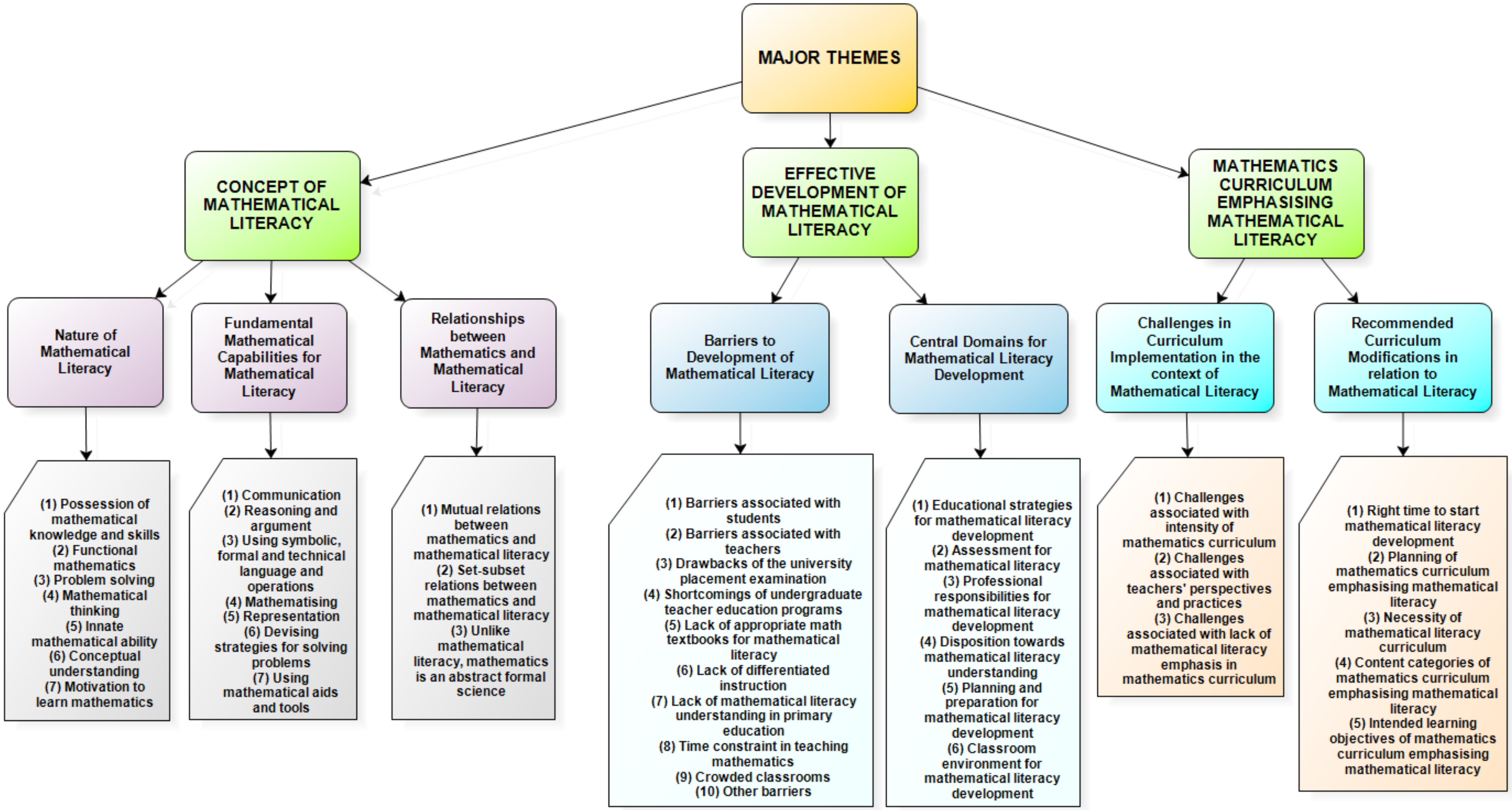

Figure 4.15: A Visual Depiction of Secondary Mathematics Teachers' Conceptions of Mathematical Literacy

CHAPTER V

DISCUSSION, CONCLUSION, AND IMPLICATIONS

The purpose of the current study was to examine secondary mathematics teachers' conceptions of mathematical literacy by identifying three issues related to their conceptions about what the notion of mathematical literacy means, how effective development of mathematical literacy could be facilitated, and what a mathematics curriculum emphasizing mathematical literacy should look like. Accordingly, teachers' conceptions of mathematical literacy were composed of three main principle components involving the concept of mathematical literacy, the effective development of mathematical literacy, and the mathematics curriculum emphasizing mathematical literacy. This chapter discusses the key findings of this study in relation to these three critical issues by comparing and contrasting them with the existing body of literature. Summary conclusions and major implications of the research's findings for teacher education and professional development are also addressed along with limitations and suggestions for future research so that mathematics education community may benefit from this study and continue to progress in the area of research on mathematical literacy.

5.1. Teachers' conceptions of the concept of mathematical literacy

Secondary school teachers' conception about the concept of mathematical literacy involved three central dimensions, which were the nature of mathematical literacy, fundamental mathematical capabilities for mathematical literacy, and the relationships between mathematics and mathematical literacy. The discussions of each of these three dimensions were presented below in the following sub-sections.

5.1.1. Conceptions of the nature of mathematical literacy

The data of this study revealed different ways of conception while analyzing the teachers' conceptions about the nature of mathematical literacy, which included the possession of mathematical knowledge and skills, functional mathematics, problem solving, mathematical thinking, innate mathematical ability, conceptual understanding, and motivation to learn mathematics. First of all, the nature of mathematical literacy was considered as the possession of mathematical knowledge and skills including basic mathematical knowledge and skills, advanced mathematical knowledge and skills or various levels of mathematical knowledge and skills from lower to higher (Gardiner, 2004; Gellert et al., 2001; Goldenberg, 2014; Hope, 2007; Jablonka, 2003; McCrone & Dossey, 2007; Powell & Anderson, 2007; Pugalee, 1999; Sfard, 2014; Skovsmose, 2007). In this regard, many teachers stated that mathematical literacy requires the possession of mathematical knowledge at diverse levels. They stated that it is not clear what mathematical knowledge and skills we should possess in order to become mathematically literate because everyone's interests and desires in daily life is different from each other. This conception of the teachers aligns with those revealed by several research studies (Gardiner, 2004; Hoogland, 2003; Sfard, 2014). For example, in the study by Sfard (2014), she suggests that mathematical literacy should not be considered as a single level; it can be of various levels depending on the people's needs and requirements. That is to say, everyone may not be mathematically literate at the same level, but everyone can be more or less mathematically literate according to their own needs (Gardiner, 2004). Moreover, in the current study, mathematical literacy was also perceived as a basic knowledge of mathematics that helps us to meet our demands in daily life. As might be expected, there can always be a group of people such as engineers, scientists and economists who use advanced mathematics for scientific and technological developments, but when looking at this issue through the logic or eyes of ordinary citizens, advanced mathematics has no place or practical value in their lives. This finding is also concurrent with the findings obtained in various studies (McCrone & Dossey, 2007; McCrone et al., 2008; Powell & Anderson, 2007). According to these researchers, mathematical literacy involves a basic level of mathematical knowledge and skills. They do not think that mathematical literacy requires people to have mathematical knowledge and skills at an advanced level. In a like manner, Curriculum and Assessment Policy Statement introduced by the South African Department of Basic Education in 2011 also considers mathematical literacy as the basic mathematical knowledge and skills which enable people to make simple calculations to tackle everyday life problems. Furthermore, contrary to the above-mentioned view, some teachers do not consider mathematical literacy as a basic level of mathematical knowledge. According to them, today's technological tools simply help us with everyday things and people do not need to gain competence in basic mathematical knowledge. It is possible to explain this conception of teachers by saying that everyone in a way learns this basic level of mathematics, but mathematical literacy is something different that requires expertise in advanced mathematics. This finding also provides qualitative confirmation of the results that have been previously obtained by some researchers (Gellert et al., 2001; Hope, 2007; Jablonka, 2003; Pugalee, 1999). For example, the model of mathematical literacy formed by Pugalee (1999) is strongly based on students' level of mathematical knowledge. Gellert et al. (2001) believe that mathematical literacy involves a level of mathematical understanding that goes beyond the basic level of mathematics skills. Similarly, Jablonka (2003) accepts mathematical literacy in terms of higher-order mathematical skills rather than minimum level of mathematical ability.

The nature of mathematical literacy was also regarded as functional mathematics including the use of mathematics in everyday life, the use of mathematics in societal life, the use of mathematics in further education, and the use of mathematics in occupational life. First of all, teachers believe that mathematics is not just numbers and figures as many people regard it. They view mathematical literacy as the effective use of mathematics in daily living for tackling real-life problems, because relating mathematics to everyday experiences automatically improves the overall quality of life. This result is in line with the findings shown by many earlier studies in the literature (Benn, 1997; Gal et al., 2005; Kemp, 2005; National Numeracy Review Report, 2008; OECD, 2013a; Stacey & Turner, 2015; Steen, 2001a; Venkat, 2013). These researchers, in general, indicate that mathematical literacy is obviously important for daily life and everyone should be sufficiently mathematically literate to meet their own basic needs in everyday life. According to them, mathematical literacy is an efficient use of mathematics in order to facilitate the lives of people and maybe this is one of the primary goals of mathematical literacy. Many teachers also laid more emphasis on the use of mathematics in societal life. They argued several perspectives with respect to the use of mathematics in societal life. For example, some teachers stated that we need to question and criticize events in our society as well as the events that are particularly in direct relation to our lives. We should not immediately accept them as they are presented to us. However, for that to happen, our level of mathematical literacy understanding is supposed to be at the sufficient level. In such a case, the importance of mathematical literacy becomes much more apparent. The study conducted by Johnston (1994) echoes the same point. She points out that mathematical literacy provides individuals an ability to manage themselves competently in order not to be misled or deceived by someone else. Similarly, Tout (2001) argues that mathematical literacy allows us to be informed citizen by taking our own decisions independently. Otherwise, as he warns us, people who lack in mathematical literacy skills can be easily manipulated by someone else. Besides, the responses from several teachers also suggested that people must necessarily acquire the mathematical literacy skills because those skills clearly help people to express themselves articulately and fluently to others in many places of life. This result is also voiced by Withnall (1995b). She states that mathematically literate individuals are continually gaining importance in any society because mathematical literacy allows people to feel comfortable with themselves and to express their thoughts and opinions easily and confidently in society. Thus, as literature shows us, mathematical literacy in a general sense is very important not only for an individual level but also for a societal level because gaining mathematical literacy understanding brings wide-ranging benefits to the society and it improves the community to better levels (Coben et al., 2003; Department of Education and Early Childhood Development, 2009; Evans, 2000a; Hogan, 2002; NIACE, 2011). Some of the teachers also commented on the importance of mathematical literacy which plays a significant role in future education of people. This result of the current study resonates with the findings of several studies (Kemp & Hogan, 2000; Kemp, 1995; Quantitative Literacy Design Team, 2001; Steen, 1990). For example, Steen (1990) argues that as long as students do not develop mathematical literacy understanding, they cannot completely understand mathematics and they cannot pass to the next level in the basic sciences. Similarly, as Kemp and Hogan (2000) point out, no matter under what circumstances, if students are mathematically literate, they can go to further education, they can be more successful in those areas they just prefer to go, such as economics, medicine or social sciences. That is to say, mathematical literacy takes them to the point where they want for their future education. Moreover, several teachers also perceived mathematical literacy as the functional use of mathematics in vocational life. They stated that access to jobs, rewards of income, promotion and careers may be restricted by low levels of mathematical literacy. This finding is clearly parallel with those of a number of studies (Bynner & Parsons, 2000; Hoyles et al., 2002; Morgan et al., 2004; Noss, 1997). According to these studies, necessary mathematical skills and knowledge are often among the prerequisites needed in a variety of workplaces. It is thus a commonly held view that poor numeracy reduces employment opportunities. For instance, Hoyles et al. (2002) argue that mathematical literacy clearly allows people to effectively progress in their professions or in their businesses. Similarly, Bynner and Parsons (2000) contend that mathematical skills and qualifications are not only demanded as an essential element for numerous jobs but also have an important impact on people's employment prospects. However, the thought-provoking thing here is that although all of the teachers put more emphasis on the use of functional mathematics in everyday activities, it seems really difficult for most of them to design a mathematics lesson in order to give necessary functional mathematical skills effectively to their students due to a number of obstacles, particularly related to the lack of teachers' qualifications for mathematical literacy, the shortcomings of undergraduate teacher education programs, the drawbacks of the university placement examination, the provision of appropriate math textbooks for mathematical literacy development, and the curriculum implementation in relation to mathematical literacy.

Unsurprisingly, the majority of the teachers also considered the nature of mathematical literacy as problem solving. In other words, students' problem-solving skills in mathematics can be firmly built and improved through providing them better mathematical literacy understanding in their lessons (Brown & Schäfer, 2006; de Lange, 2003; Jablonka, 2003; OECD, 2013a; Ontario Ministry of Education, 2005; Venkat, 2007). For example, Jablonka (2003) asserts that mathematical literacy is to use the set of abstract concepts learned in mathematics in the best way in order to solve mathematical problems. Similarly, OECD (2013a) emphasizes that mathematical literacy requires both clear understanding and simplification of the given problem situation. It also requires to produce different ideas for a solution by evaluating the given information in order to set up an algorithm based on this evaluation to solve the given problem. We can get more or less problem solving ability in our educational system, but what is important to recognize here is that how do we use that ability to solve problems in our daily life. The more we use it, the more mathematically literate we are. That is to say, mathematical literacy, in many ways, gives people perfect vision for their life. This vision understandably helps them generate the most appropriate and effective solutions to real-life problems and allows them to be successful in life (Ontario Ministry of Education, 2005). In that respect, some of the teachers who agreed with the view that mathematical literacy mainly refers to problem solving skills in mathematics also held the view that mathematical literacy is the ability to transfer mathematics knowledge to everyday life in order to solve daily life problems (Venkat, 2007). Namely, once people accept mathematics as an effective problem-solving skill and use it to find solutions to the problems encountered in life, they will probably make their lives easier and more comfortable. That is why, mathematical literacy is also naturally regarded as the ability to understand and solve the real-life problems in a more effective and efficient way.

Mathematical thinking was also perceived to constitute an important part of mathematical literacy. Many teachers thought that the purpose of mathematical literacy is to encourage people to think mathematically. As explained in the literature, some researchers also support the same point (Niss, 2003a; Steen, 2001a). Steen (2001a) indicates that one great aspect of mathematical literacy is to improve students' thinking and interpretation skills. Hence, it is acknowledged that mathematical literacy fosters mathematical reasoning and rational thinking skills and it allows us to establish causal relationships between events in order to cope with the difficulties we encounter in our personal, vocational and social life. In this way, we see what we can do by using mathematics in life. However, it is also important to bear in mind that in addition to mathematical thinking and reasoning in general, higher order thinking skills in particular also promote better mathematical literacy understanding. In this regard, like various research studies (Gellert et al., 2001; Jablonka, 2003; Hope, 2007), only a few teachers who regarded the nature of mathematical literacy as mathematical thinking also supported the view that mathematical literacy indeed involves higher-order thinking skills in addition to mathematical knowledge and skills. The difficulty to describe mathematical literacy as the power of higher-order thinking skills for most of the teachers can be attributed to their conception of mathematical literacy involving a level of mathematical understanding that does not go beyond the basic level of mathematical skills.

Not expectedly, a large number of teachers viewed that a genetic predisposition for mathematics is an important path to develop better mathematical literacy understanding. That is to say, mathematically literate people have some sort of innate mathematical intelligence. For this reason, mathematically intelligent people are always one step ahead of others in terms of mathematical literacy acquisition or understanding. In other words, inborn mathematical intelligence is a significant contributory factor to an effective mathematical literacy development. It is important to point out that this finding contradicts to what PISA 2012 results say to us. The PISA 2012 assessment dispels the common thought that mathematics achievement is predominantly a product of the innate ability rather than hard work (OECD, 2014a). Similarly, Boaler (2009) emphasizes that nature of high achievement in mathematics is a result of hard work and not of the innate ability. This conception of teachers can be explained in a way that overall intellectual capacity is seen as very influential over the mathematical ability which has the dominant influence on high achievement in mathematics. However, it does not literally mean that people without innate mathematical competence can never gain or develop mathematical literacy skills. What is meant to be emphasized here is that innate mathematical intelligence appreciably facilitates the acquisition or promotion of mathematical literacy skills. This indeed means that the acquirement or development of mathematical literacy skills can be ultimately achieved through hard work and persistent effort even if it takes a longer time to accomplish this to happen (Hobden, 2007). Yet, the important point sometimes overlooked is the fact that mathematical literacy actually ensures people to use their inborn mathematical intelligence at the maximum level.

It is promising to find that the nature of mathematical literacy was also considered by many teachers as conceptual understanding. This result aligns with the findings revealed by the report entitled as ‘Adding It Up: Helping Children Learn Mathematics’ (National Research Council, 2001). According to this report, mathematical proficiency includes five essential components that provide one way of describing mathematical literacy. One of these components is conceptual understanding which refers to comprehension of mathematical ideas by making connections between mathematical concepts. However, what is interesting is that on one hand this component is viewed as crucial by many teachers, on the other hand we are sadly gradually becoming a society that is loaded with much mathematics knowledge rather than conceptualizing and applying that knowledge. This brings with it the result that most of the students have a real difficulty to make sense of the information on a deeper level by establishing relationships between classroom learning and everyday experiences. Thus, in order to comprehend why a mathematical idea is important, students need to connect this idea to what they already know. They simply build it on their prior knowledge. They also need to know exactly where they can use their mathematical knowledge in life. At least, when they encounter basic concepts of mathematics such as addition, subtraction, multiplication, or division, they must be able to apply these concepts somewhere in life. For this reason, as the PISA 2012 mathematics framework explains, mathematical literacy is gaining more importance as it provides conceptual understanding of formal mathematical knowledge by allowing us to organize our mathematical knowledge into a coherent whole and transfer it to daily life (OECD, 2013a).

As was anticipated, some teachers also remarked that mathematical literacy is very conducive to increasing motivation and interest in learning mathematics. According to them, all people should learn the value of mathematics in life in order to become productive, creative and critical thinking citizens by using mathematical decision-making processes. In this sense, it is particularly important to motivate students to study mathematics by underlying its real-life applications in different curriculum areas (Doctorow, 2002; Martin, 2007; OECD, 2013b, 2013c). However, what was not anticipated here that fewer participants than expected considered that mathematical literacy clearly increases the interest of students towards mathematics and makes them like to study mathematics more, but as emphasized by (OECD, 2013b), mathematical literacy is indeed our capacity to understand the critical role of mathematics in the world around us and helps us endear mathematics to students. In other words, it simply helps us to overcome students' math anxiety or fear of failure by teaching them realistic mathematics in fun and diverse ways (Martin, 2007). Therefore, one of the primary purposes of mathematical literacy is to show that mathematics is a subject where everyone can do regardless of his or her background.

Overall, it is clearly seen that varying, but mutually supportive and sometimes overlapping conceptions of mathematical literacy mentioned in the literature also seem to be held by the teachers of this study. The fact that each teacher already has almost all of these views is perceived at first sight that teachers have confusing and ambiguous conceptions toward the nature of mathematical literacy. However, mathematical literacy has several dimensions as the meaning of mathematical literacy varies according to the purpose and context being used and it means different things to different people according to their interests and lifestyles (Hope, 2007; Jablonka, 2003; McCrone & Dossey, 2007; Sfard, 2014; Skovsmose, 2007; Steen, 1997; Westwood, 2008). Therefore, the fact that each teacher apparently holds most of the views simultaneously to some extent actually reflects richness in understanding of the various aspects of mathematical literacy.

5.1.2. Conceptions of the fundamental mathematical capabilities for mathematical literacy

The data of this study showed different ways of conception while analyzing the teachers' conceptions about the fundamental mathematical capabilities for mathematical literacy, which consisted of communication, reasoning and argument, using symbolic, formal and technical mathematical language and operations, mathematizing, representation, devising strategies for solving problems, and using mathematical aids and tools. This result resonates with those revealed by several studies in the literature particularly by OECD (2013a). In that respect, when mentioning the mathematical literacy, the first thing definitely needed is the ability to understand and express the given problem very well. Students' failure to understand what they have read or failure to interpret what they have understood is a very big problem in the process of teaching and learning mathematics. Actually, this shows their lack of mathematical literacy, because mathematical literacy is something that basically aims to make people understand the data presented to them (OECD, 2013a). Besides, for many teachers, mathematical literacy is to struggle to find a solution for any problem through reasoning. It is being able to reason mathematically. In fact, mathematics is a kind of cause and effect relationships and it is based on logical ideas. If these ideas are proven, we will accordingly accept them as absolute truths. Similarly, when we face a problem in life, we try to solve it and then convince others of the viability of our solution through reasoning ability. There is actually always such a thing at every stage of life. It is also known that mathematical literacy is the ability to effectively apply mathematics to daily life problems. That is why, in teachers' opinion, mathematical literacy essentially consists of mathematical reasoning and argumentation in order to use mathematics efficiently for finding solution to everyday problems. Many teachers also mentioned that the ability of using symbolic, formal and technical mathematical language and operations is another crucial element for being mathematically literate. That is to say, mathematical literacy requires students to use arithmetic calculations involving decimals, fractions, percentages and ratios competently. If they lack the ability to handle formal mathematical systems such as the basic signs used in arithmetic as well as special symbols of advanced mathematics, every new thing learned in math class will mostly remain incomprehensible and mysterious for them. For this reason, students' ability to work flexibly with mathematical symbols or their effective use of formal mathematical language is essentially important for better mathematical literacy development (OECD, 2013a). Otherwise, even if students establish the equations of their mathematical models of real-life problems, they may face with various difficulties to perform calculations correctly and find suitable solutions to them. Moreover, most of the teachers talked about the fundamental steps that we expect from students to take when solving any math problem. Accordingly, the first step expected from students is to understand the problem situation and then identify the variables situated in the given problem. Next, they are supposed to represent this problem situation mathematically by using these variables identified in the context. This step is very essential for being able to apply mathematical procedures to reach mathematical results. Finally, we expect students to interpret or evaluate their results in respect to problem situation or context. This process involves their justification of mathematical results regarding the problem situation. Indeed, all these steps taken to solve the given mathematical problem also constitute a firm basis for being mathematically literate. This is called as a mathematizing competency mentioned by OECD (2013a). Additionally, many teachers also emphasized that mathematical literacy requires some skills to understand and make use of various representations including verbal, visual, algebraic, geometric, tabular, pictorial and graphical, as well as the skills to translate among these representations regarding the problem situation. These skills are rarely observed and not very common skills for many students in our education system because representing any function algebraically or graphically, creating different tables or charts to visualize mathematical data, and reading, interpreting or drawing inferences from graphs, tables or charts are usually regarded as the upper level skills for students. However, the ability to read different types of tables, charts, diagrams and graphs certainly helps students to understand and interpret the mathematical data given in the problem situation efficiently. Actually, this is also a much needed ability in everyday life. Therefore, the representation competency is considered by OECD (2013a) as one of the essential competencies for better mathematical literacy development of students. Furthermore, according to some teachers, finding the final answer to any problem is important, but what is a far more important is to be able to devise multiple strategies to reach the final answer. Hence, the most important thing that we need to give students in mathematics education is to be able to create alternative strategies for any given problem in order to reach the most accurate and reliable solution. It is in fact mathematical literacy that surely helps students to produce a range of solution strategies to any problem situation encountered in life (OECD, 2013a). Finally, several teachers highlighted that the development of mathematical literacy is closely linked with the ability to make use of different instructional aids and tools. This is because of the fact that students' ability to use such aids and tools truly motivates them to actively get involved in classroom activities. It provides students a lot better in the mathematical interpretation of many problems (OECD, 2013a). Therefore, it is always important to make students to be aware of the benefits as well as the limitations of being able to use relevant mathematical aids and tools in order to facilitate the comprehension of complex mathematical structures.

In general, all of the above-mentioned findings are also consistent with the results of several research studies (de Lange, 2006; Niss & Højgaard, 2011; Niss & Jensen, 2002; Niss, 2003a, 2003b, 2015). According to these studies, mathematical literacy requires communication capability which means the ability to clearly present and logically express oneself to others about mathematical matters both in oral and written forms. It is highly related to people's ability to understand and interpret mathematical statements, representations or texts. Mathematical literacy also requires mathematical reasoning and argumentation competency which consists of the justification of our mathematical claims, calculations or inferences based on the information and instructions given in the problem. These studies emphasize that mathematical literacy comprises, on the one hand, the ability to implement mathematical operations. On the other hand, it comprises the understanding of the logic behind these operations. Mathematical literacy also demands mathematizing competency which refers to, on the one hand, being able to translate a problem defined in the real life to mathematical statements, and on the other hand, being able to interpret the mathematical results in relation to the original problem context. In addition to mathematizing competency, representing mathematical patterns in tables, graphs and equation models, making connections among different representations based on these patterns, and translating between among them are all also very fundamental skills for being mathematically literate. Moreover, these studies underline that mathematical literacy includes the ability to devise multiple strategies for problem solving. This is one of the important abilities that needs to be acquired for solid mathematical literacy development, which refers to choosing and establishing a satisfactory plan or strategy for dealing with the problem, as well as considerate monitoring and assessment of its implementation. Furthermore, in addition to the ability to devise multiple strategies for problem solving, it is highlighted that mathematical literacy also consists of the ability to make use of different instructional aids and tools e.g. computers or calculators in order to improve learning processes in mathematics.

Overall, most of the teachers mentioned almost all of the competencies for better mathematical literacy development. There is no doubt that skills required for mathematical literacy mentioned in the literature are also considered by the teachers of this study. However, it is too much to claim that this also leads many students to automatically gain or systematically develop most of these skills in their classes. Regretfully, according to PISA National Report (MEB, 2015), about half of the students failed to demonstrate consistently that they have a basic minimum level of mathematics required to succeed in adult life. Although Turkey has involved in PISA test since 2003, we are still far behind in comparison with the OECD average and other participating countries. So, this is not only a clear indication of a lack of necessary mathematical literacy skills in many of the students, but also evident sign of teachers' difficulties in finding as much opportunity as possible for their students to acquire these skills. Another noteworthy point to mention here is that making use of mathematical tools and materials was the least emphasized competency among the teachers. Since this competency requires students to make use of various instructional aids and tools e.g. computers or calculators in order to initiate and foster learning processes in mathematics, the teachers also have to demonstrate a reasonable or adequate level of proficiency in this area. Therefore, the underlying reason for why this competency is the least emphasized conception compared to others could be a number of drawbacks experienced by some teachers such as insufficient time to use these instructional aids and tools in class, inadequate training on their use, lack of access to these tools, or teachers' beliefs and attitudes toward the use of them in a classroom.

5.1.3. Conceptions of the relationships between mathematics and mathematical literacy

The findings of the current study demonstrated several ways of conception while analyzing the teachers' perceptions about the relationships between mathematics and mathematical literacy, which included, mutual relations between mathematics and mathematical literacy, set-subset relations between mathematics and mathematical literacy, and unlike mathematical literacy, mathematics is an abstract formal science. In that respect, majority of teachers commented that mathematical literacy helps people to gain better mathematical understanding. That is to say that whenever teachers provide students with more examples from everyday life for teaching mathematics, they can really improve their understanding in mathematics. This result is in line with the findings mentioned by many studies in the literature (DoE, 2003;

Gal, 2009; Johnston, 1994; OECD, 2013b; Quantitative Literacy Design Team, 2001; Stacey & Turner, 2015; Tout, 2001). For example, Johnston (1994) states that mathematical literacy is really important for the effective learning and the efficient application of mathematics to handle various problems placed in a real-life context. Accordingly, the more mathematical understanding people have, the better they are able to interpret and implement mathematics. As Tout (2001) puts it, mathematical literacy is one of the important contributory factors for meaningful learning of mathematics. Namely, it is really an efficient way to make sense of mathematics. Otherwise, when people do not get make sense of mathematics, most of them are unfortunately lost in the beginning of their mathematics course and probably do not have a chance to recover later. In other words, strong mathematical understanding is not exactly developed as long as people show a lack of mathematical literacy (Stacey & Turner, 2015). That is why, mathematical literacy seems to be an important factor for building deeper mathematical understanding. On the other hand, many teachers expressed that knowledge of mathematics also plays a crucial role in the development of mathematical literacy. As explained in the literature, many researchers also support the same point (AAMT, 1997; Coben & Chanda, 2000; Coben, 2000; Coben et al., 2003; National Numeracy Review Report, 2008; Niss, 2015). For instance, National Numeracy Review Report (2008) emphasizes that becoming a good mathematically literate individual mainly depends on solid mathematical knowledge and skills. Similarly, Coben et al. (2003) state that mathematically literate individuals primarily need to understand and develop certain level of mathematical proficiency according to their needs. Thus, in general, as teachers indicated, although mathematics and mathematical literacy do not exactly overlap, they clearly much support each other's development (Quantitative Literacy Design Team, 2001; Steen, 1990, 2001a).

The relationships between mathematical literacy and mathematics were also identified as the basic set or subset relations. On one hand, some teachers noted that mathematical literacy is a subset of mathematics. That is to say, unlike mathematical literacy, mathematics is very wide field and it includes every mathematical construct. On the other hand, some other teachers held the perception that mathematics is a subset of mathematical literacy. In addition to this, as mentioned in the research by Australian Association of Mathematics Teachers (AAMT) (1997), some teachers also supported the view that mathematical literacy and mathematics intersect at some common grounds while diverging at other points. This finding is indeed concurrent with the research of Stacey and Turner (2015) who emphasize that for those who consider mathematics as an abstract and theoretical discipline, mathematics as a broader field completely involves mathematical literacy whereas for others who think that mathematics is clearly embedded in daily life in many respects, mathematical literacy clearly includes all knowledge of mathematics.

Moreover, as remarked by several teachers, contrary to mathematical literacy, mathematics was often viewed as purely abstract science. A number of studies also support the same idea that mathematics is a branch of science dealing with proof, logical arguments and justifications of abstract mathematical structures whereas mathematical literacy is in general regarded as the application of mathematics in real world contexts in order to tackle everyday problems (de Lange, 2003; Department of Basic Education, 2011; Frith, 2011; Manaster, 2001; Orrill, 2001; Spangenberg, 2012; Steen, 2001b; Venkat, 2007).

Overall, what stands out here is the fact that real-life situations or scenarios are considered as important to make use of teaching and learning mathematics. Hence, mathematical literacy and mathematics should not be considered quite distinct from each other because they clearly complement each other. However, what is more prominent to report here that although mathematics and mathematical literacy are related in a way and affect each other's levels, they are nonetheless considered as two different concepts, and having no linear relationship between them. Namely, even people who do not take much mathematics education can be very good at mathematical literacy because of the life experience they bring with them. The teachers believe that using mathematics effectively in life without having studied much mathematics might be a learned habit or a practical skill inherited from her parents or ancestors. On the other hand, we can also see around us that people who are very good at mathematics may not become mathematically literate enough. Such people sometimes have a great difficulty to associate the abstract concepts with their daily life problems. This might be because of the fact that they continuously focus on abstract concepts to solve mathematical problems. Therefore, they may not easily concentrate on solving problems in their life. The view put forward above indeed explains why being good at mathematics or mathematical literacy does not mean that we are also entitled to be automatically good at the other.

5.2. Teachers' conceptions of the effective development of mathematical literacy

Secondary school teachers' conception about the effective development of mathematical literacy involved two central dimensions, which were the barriers to development of mathematical literacy, and the central domains for mathematical literacy development. The discussions of each of these two dimensions were presented below in the following sub-sections.

5.2.1. Conceptions of the barriers to development of mathematical literacy

The data of this study revealed various conceptions while analyzing the teachers' conceptions about the barriers to development of mathematical literacy, which included the barriers associated with students, barriers associated with teachers, drawbacks of the university placement examination, shortcomings of undergraduate teacher education programs, lack of appropriate math textbooks for mathematical literacy, lack of differentiated instruction, lack of mathematical literacy understanding in primary education, time constraint in teaching mathematics, crowded classrooms, and other barriers. First of all, the barriers associated with students were regarded as mathematics anxiety, disinterest of students in mathematics, lack of basic arithmetic knowledge, excessive concern about mathematics grades, and the inability to search for information independently. In this sense, nearly all of the teachers reported that mathematics anxiety is one of the important factors that truly prevents the acquirement of mathematical literacy. As mentioned by several studies, mathematics can be very intimidating for many students (Galla & Wood, 2012; Parsons et al., 2009; Rosnick, 1981; Tobias, 1993), which naturally affects their mathematical literacy development in a negative way. Ashcraft and Moore (2009) state that once mathematics anxiety is activated, it is really difficult to overcome it, and such mathematics anxiety adversely affects students' learning of mathematics, as well as the mathematical literacy gains.

In this sense, perhaps the first prerequisite for gaining mathematical literacy is to defeat mathematics anxiety and develop the confidence needed for learning mathematics. Besides, Coben (2003) attributes people’s lack of mathematical literacy understanding to their low level of self-confidence in learning mathematics. Coben (2003) also indicates that students’ negative prejudices against learning mathematics may result in achieving a lower or inadequate level of mathematical literacy development. Moreover, students’ lack of interest in mathematics was also regarded as another major obstacle for the development of mathematical literacy. Mathematics do not really make any sense for these students. They usually deal with other things rather than learning mathematics. Actually, the reason for this is the fact that many students have very little or no interest in mathematics. They are quite indifferent to mathematics. However, in the absence of curiosity or interest in mathematics, it is quite clear that the development of mathematical literacy cannot be satisfactorily achieved. Therefore, as Parsons et al. (2009) put it simply, increasing students’ interest in mathematics is one of the important steps to make them mathematically literate. Moreover, it was also reported that some students exhibit a clear deficiency in their basic arithmetic skills. They are even unable to implement the basic mathematical operations used in everyday life. However, the knowledge of basic arithmetic facts is an essential priority for mathematical literacy development (DoE, 2003; UNESCO, 2012). Here, teachers’ view of mathematical literacy as a basic level of mathematical knowledge and skills to solve the daily life problems in a variety of situations in fact accounts for their conception of seeing students’ lack of basic arithmetic knowledge to be an interruptive obstacle on the way of the mathematical literacy acquisition. Some teachers also commented that students’ extreme concern about their mathematics grades is another major factor that negatively influences students’ mathematical literacy performance. Namely, most of the students generally think that it is enough for them as long as they take a good grade in the exam. They just study to get a better grade from mathematics. There is almost nobody who try to learn mathematics in order use or apply it efficiently in everyday life. Besides, as expressed by some teachers, many students have also a great difficulty for how to search for information independently. Unfortunately, they are simply accustomed to the transmission of ready-made knowledge. However, knowing how to reach or search for information for accurate evaluation of events is really important for better mathematical literacy acquisition (Department of Basic Education, 2011). Thus, students' lack of ability to search for information independently also seems to be the other significant factor that prevents the development of mathematical literacy.

Moreover, the barriers associated with teachers were considered as the lack of teachers' qualifications about mathematical literacy, teaching mathematics by rote, challenge of making major changes in teaching practices, drawbacks of consistent use of routine problems, and misunderstanding of mathematical literacy. In this regard, most of the teachers frankly acknowledged that they were much aware of their lack of qualifications about teaching mathematics emphasizing mathematical literacy. Actually, it seems very difficult for many teachers to teach in mathematical literacy context, because they are all used to rote-learning and expository teaching from the beginning of primary school (Colwella & Enderson, 2016; Draper, 2008; Hill et al, 2005; Julie, 2006; Moje, 2008; Schoenfeld, 2001). For example, as mentioned by Hill et al. (2005), it seems simple at first glance, but indeed it requires teachers to get ultimate qualifications in order to implement mathematical literacy successfully in their classrooms. However, most of them do not see themselves as having such qualifications and remain stuck in teaching mathematics emphasizing mathematical literacy (Shanahan & Shanahan, 2008; Westwood, 2008). Therefore, teachers' qualifications and competencies about mathematical literacy are of great importance when the focus is on efficient implementation of the curriculum with emphasis on mathematical literacy (Battista, 1993; Kirk & McDonald, 2001; Little, 1993; OECD, 2009b, 2014c; Spillane, 1999). Accordingly, teachers first and foremost need training about how to give mathematical literacy understanding to their students before revising or designing mathematics curriculum requirements and standards to implement according to such understanding. In other words, teachers must be prepared for this revision or change in advance through teacher training seminars, conferences and workshops. Moreover, a large number of teachers commented that teaching mathematics by rote is another significant factor that negatively affects students' mathematical literacy development. As explained in the literature, several researchers support that what teachers generally do in class is to teach mathematics by drill and computation without conceptual understanding because they were all grown up in rote education system (Draper, 2002; Siebert & Draper, 2012; Stigler &

Hiebert, 2004). Our mathematics teaching profile in the education system is also regrettably based on strict memorization and rote learning. Such education received before becoming a teacher prevents many teachers to implement creative and new ways to teach mathematics emphasizing mathematical literacy. Hence, most of the students generally memorize many mathematical theorems, formulas and rules but they do not really know why they work. They have no idea about how to apply what they learn in class to real world problems. Obviously, with such an educational system, it seems very hard for any individual to become mathematically literate. Besides, as many teachers underlined, the challenge of making major changes in teaching practices is the other significant barrier that inhibits the development of mathematical literacy. As reported by Ontario Ministry of Education (2005), teachers with mathematical literacy emphasis should try to implement diverse teaching approaches in their classes including problem-based learning, discovery learning, mathematical modelling, and cooperative learning. However, most of the teachers generally apply traditional teaching approaches in their classes. In such a case, many teachers are not very much willing to have a teaching style laying particular stress on mathematical literacy as they initially need to make a serious effort to develop or change their present teaching practices toward this new teaching style. Nevertheless, despite the difficulties faced at the beginning, in the long run, teaching with mathematical literacy emphasis is expected to be very effective to improve students' achievement in mathematics. In addition, the preference of routine problems in class was also considered to be one major obstacle to the acquirement of mathematical literacy skills. In fact, preparing non-routine problem activities for mathematical literacy seems to be a big challenge for most of the teachers. As pointed out by Madison (2004), many teachers continuously use traditional types of problems involving routine calculations and procedures in class because non-routine problems daunt or confuse their students a lot. This might be due to the fact that non-routine ones can usually take up much time or sometimes the entire mathematics lesson (Romberg, 2001). Thus, most of the teachers accept the ordinary fact that dealing with only routine problems in class is enough for their students' achievement in mathematics. Accordingly, while some teachers view mathematics teaching emphasizing mathematical literacy as handling many routine mathematics problems in less time as possible, others often consider it as only the teaching of many mathematical rules, theorems and proofs by rote. Hence, these kinds of misunderstandings or incomplete understandings of mathematical literacy also appear to be important factors that negatively influence students' mathematical literacy performance.

Besides, the university entrance exam-oriented education was perceived as another dominant barrier to the development of students' mathematical literacy skills. The reason for being seen as a barrier could be the fact that most of the mathematics teachers could not educate mathematically literate individuals in our education system, but they are just forced to prepare their students for the university admission test as many students' concentration is mostly focused on this entrance examination. Thus, even if we plan and implement mathematics curriculum in accordance with mathematical literacy understanding, we will hardly get the desired effectiveness and efficiency from the implementation of such curriculum. So, the effective and efficient applicability of such curriculum is particularly based on the adaptation of the university admission examination according to mathematical literacy standards or requirements. However, as many teachers reported, the content of the university entrance examination is not generally helpful for the development of mathematical literacy understanding of students. This can be attributed to their view of mathematical literacy as functional mathematics or the ability to effectively solve problems encountered in life. Schorr et al. (2003) point out that in order to be successful in this type of state examinations, various rules and procedures are in advance forced to be repeated by students many times with little or no understanding. Similarly, as mentioned by Elmore (1996), these tests often lack intellectual challenge and real-life applications of knowledge and skills for students, but in order to improve mathematical literacy, the content of the exam should encourage discovery learning and inquiry based teaching approaches that would allow an opportunity for students to establish deeper understanding by exploration and investigation (Baron & Wolf, 1996; Corbett & Wilson, 1991; McNeil, 2000; Resnick & Resnick, 1992; Schorr et al., 2003). On the other hand, the university placement test is understandably one of the well known facts of this country. The students are admitted to the university according to their test scores mostly based on the number of correct answers in different sections of this placement examination. In order to be successful in this test, students have to regularly solve many questions every day. So, the primary purpose of teaching mathematics becomes to prepare the students for the university admission test rather than providing the mathematical literacy to them. However, this naturally leads to the university admission examination anxiety which is an absolute factor that influences the development of mathematical literacy in a negative way. In other words, unless students handle this exam anxiety competently or unless there is another way to enter university, they unfortunately always look at life among the five options. Regrettably, as stated by Firestone & Mayrowetz (2000), the important thing is not to develop mathematical literacy, but to increase the total number of correct answers in the admission exam. The more correct answers they have, the higher score they receive from the exam. Then, the higher score they have, the better university they can enter. Therefore, providing mathematical literacy understanding unfortunately plays a secondary role in our education system. However, the essential point of the matter should not be missed and the university admission test precisely must be one of the means for gaining necessary mathematical literacy skills. Nevertheless, it would be unfair not to mention the fact that we have recently begun to observe the kind of questions with mathematical literacy understanding, but according to the teachers, their number is so few to force mathematics teachers to give much importance to this understanding in our education system. Thus, the university placement examination content should also be modified to make it more relevant to mathematical literacy if teachers are required to focus their emphasis much on such an understanding.

Moreover, the lack of mathematical literacy understanding in undergraduate teacher education programs seems to be another primary factor that negatively affects the development of mathematical literacy. Many teachers expressed that they did not take enough education in the context of mathematical literacy at the university. Education given in undergraduate programs had nothing much to do with mathematical literacy. The fact that teachers have such an idea can be ascribed to their one of the perceptions of mathematical literacy as the ability to use mathematics in a functional way in everyday life. In this sense, the lack of research or discovery oriented undergraduate teacher education was regarded as one of the causes of the lack of mathematical literacy understanding. However, as voiced by Ball, Hill, and Bass (2005), the education system based on research or discovery in universities is extremely significant if we desire to produce prospective teachers who are inclined to lay a great emphasis on mathematical literacy understanding among their students. Moreover, the lack of teaching practice-oriented undergraduate teacher education was also thought as another reason for many in-service teachers to lack in mathematical literacy understanding. However, as emphasized by Milton et al. (2007), practice-oriented teacher education programs play a very important role in engaging the prospective teachers in the work of teaching mathematics emphasizing mathematical literacy understanding. With the help of practice-oriented teacher education programs, prospective teachers can act competently and diligently according to the implicit demands of mathematical literacy. Furthermore, the lack of mathematical literacy courses in undergraduate programs was considered as the other major factor that negatively influences the development of mathematical literacy skills of students. Thus, it is expected that such an elective or must mathematical literacy course in undergraduate teacher education studies possibly helps to equip the pre-service teachers with the basic principles and skills in designing and developing mathematics teaching in the context of mathematical literacy.

In addition, the lack of appropriate math textbooks for mathematical literacy was also acknowledged as an important challenge that influences students' mathematical literacy performance. In that respect, textbooks are supposed to be different for each school level to meet the demands from both teachers and students, because students who attend different types of schools may have different mathematical literacy needs. However, teachers have a clear shortage of appropriate resources for teaching mathematics emphasizing mathematical literacy understanding (Tatto, 2012). Moreover, as pointed out by Pia (2015), the math textbooks are very symbol laden and complicated to understand, which prevents to develop students' mathematical literacy skills. Besides, as Westwood (2008) noted, teachers also complained about inadequate number of examples in the math textbooks for mathematical literacy development. They highlighted the fact that even they have very few examples from everyday life, most of them unfortunately become not authentic mathematics problems. So, teachers generally do not find the textbooks sufficient enough for mathematical literacy development in terms of both their contents and the number of appropriate examples that they involve. Teachers' defense of such a view can also be associated with their perception of conceiving mathematical literacy mostly as the ability to use mathematics in everyday life functional.

The lack of differentiated instruction was also considered one of the possible limitations to students' mathematical literacy development. Understandably, it actually does not make any sense to teach everyone the same mathematics because students are of different levels of proficiency in mathematics. However, teachers struggle to apply the same mathematics curriculum to all students regardless of their mathematics concentration. Many times what teachers do in class is simply to teach mathematics for intermediate level learners. In this case, while students who are good at mathematics and enjoy solving challenging math problems may feel held back and frustrated, students who need additional help in mathematics may feel hopeless and anxious. Teaching mathematics in this way only results in strong dislike and anxiety of mathematics rather than contributing to development of students' mathematical literacy skills. This is an apparent barrier to mathematical literacy development of students in especially low performing schools. The teacher may also feel stressed for teaching these students at a time who have all varying levels of mathematical ability. However, what mathematical literacy requires teachers is to teach mathematics according to the students' expectations and capabilities. Therefore, teaching mathematics by considering classes consisting of students with varying levels of mathematical abilities is important (Hall, 2002; Murray & Jorgensen, 2007; Olson & Larsen, 2012; Tomlinson, 1999, 2003). This creates much willingness and desire for learning mathematics. Otherwise, any lack of willingness or desire for learning mathematics automatically results in ineffective development of mathematical literacy. In other words, effective development of mathematical literacy skills necessarily depends on giving prime importance to differentiated instruction in mathematics classrooms. The fact that teachers claim that differentiated instruction has a very important role in the development of mathematical literacy skills can also be explained by their understanding of mathematical literacy as the possession of mathematical knowledge according to people's interests and desires in daily life.

Moreover, the lack of mathematical literacy understanding in primary education was thought as another potential barrier to students' positive attitudes toward mathematical literacy development. This is in fact related to teachers' conceptions of mathematical literacy as the possession of basic mathematical knowledge and skills, problem-solving skills and conceptual understanding. They often supported the view that the development of the essential basis for mathematical literacy such as having basic arithmetic and problem solving skills and conceptual understanding should start from elementary school. However, the education given in primary schools is unfortunately not sufficiently based on the principles that underline the importance of mathematical literacy development. In other words, students' basic mathematical skills that they need in real life are not exactly established in elementary school years. Most of the students lack the ability to understand new mathematical ideas and structures by relating them to what they already know and understand. They also have much difficulty in understanding, formulating and solving basic math problems that are presented in high school classes as they probably do not sufficiently receive basic problem-solving skills in primary education. Namely, many students suffer from discontinuity problems between primary and secondary schooling (Anderson, 2003; Clarke et al., 2000; Clarke, 2000; National Council of Teachers of Mathematics, 2000; Treacy & Willis, 2003). This situation inevitably prevents the development of mathematical literacy understanding of students. That is why, one of the most important things that the students need in primary education is to gain mathematical literacy skills, if it is desired to easily impose mathematical literacy understanding upon them in high school years.

Furthermore, the time constraint in mathematics teaching was also recognized as the other possible factor having negative impacts on students' mathematical literacy performance. The main reason for teachers to consider it as a barrier can be their perceptions of mathematical literacy as problem solving and mathematizing. Building strong problem-solving skills to formulate any problem situation from a real-life context to the domain of mathematics for evaluation requires a great amount of time and effort to plan and implement. However, it is very difficult for teachers to allocate extra time for each of their students to be engaged in the activities requiring them a clear understanding, formulation and modeling of the real-life problems and then comprehensive evaluation of the results, as all these steps take so much time and dedication for both teachers and students. Hence, it is unfortunate that most of the teachers are forced to prefer not to give necessary mathematical literacy skills to their students because of the lack of more flexible time line for achieving a wide range of activities in the context of mathematical literacy. In addition to the time constraint, the crowded classrooms also hinder the implementation of teaching strategies emphasizing mathematical literacy development. Some teachers thought that in crowded classes teachers cannot do what they need to do for their students' mathematical literacy acquisition. Most of the teachers really find it very hard to implement some teaching practices involving inquiry-based and investigative classroom activities with more students as they typically require much time to apply. Understandably, they need to pay more attention to each student and to monitor his or her progress in the learning activities. Accordingly, all such teaching practices and the learning activities become more difficult in crowded classes (Angrist & Lavy, 1999; Darling-Hammond, 2014; Ehrenberg et al., 2001; Lee & Smith, 1995). Hence, if teachers are supposed to provide their students with mathematical literacy understanding, first and foremost their classes should not be crowded. Besides, a lack of consultation with mathematics teachers who are in direct contact with the learners over the design of mathematics curriculum content and the implementation of new educational policies and innovations with regard to mathematical literacy was regarded to be another important obstacle to mathematical literacy understanding, because, as indicated by Martin (1993), any approach to curriculum reform implementation that does not take teachers' opinions into account is not viable or sustainable for a long time. Moreover, the preparation for multiple-choice tests for appointing teachers appeared to be the other significant challenge for the development of mathematical literacy skills of students as it inevitably directs teachers' philosophy of teaching mathematics towards heavily test-based teaching and learning. Overall, what is really remarkable here is that the identification of a number of obstacles to the development of mathematical literacy has been made from different perspectives by the teachers. It cannot be denied that the emergence of so many opinions about this issue can indeed be explained by the fact that each teacher has already almost all of the diverse but convergent conceptions of mathematical literacy, and thereby this clearly helps to address the issue from various viewpoints.

5.2.2. Conceptions of the central domains for mathematical literacy development

The data of the current study demonstrated various conceptions while analyzing the teachers' conceptions about the central domains for mathematical literacy development, which consisted of the educational strategies for mathematical literacy development, assessment for mathematical literacy, professional responsibilities for mathematical literacy development, disposition towards mathematical literacy understanding, planning and preparation for mathematical literacy development, and classroom environment for the progress in mathematical literacy. First of all, the types of problems when implementing problem solving approach were regarded as of great importance to better mathematical literacy improvement. This can be simply attributed to teachers' conception of mathematical literacy as problem solving. Two key arguments were developed. The first argument suggested that specifically formal mathematics problems and word problems are the most preferred types of problems in the existing mathematics education whilst the second suggested that exposure to all types of mathematical problems but particularly to real world problems are the most appropriate for mathematical literacy acquisition (OECD, 2014b). The second suggestion is also statistically supported by OECD (2014a). Moreover, engaging students in mathematics learning was also considered as important for better mathematical literacy development. Students' engagement in their own learning process is crucial for gaining mathematical literacy skills as it promotes critical thinking and reasoning skills (Arbaugh & Brown, 2005). It accordingly empowers students to establish their own learning process (Harel & Sowder, 2005; Henningsen & Stein, 1997). For this reason, teachers should always try to motivate the students through active engagement in mathematical activities to increase the quality of their mathematics learning emphasizing mathematical literacy understanding. In other words, teachers should ensure active involvement of students in their own learning by designing activities which attract their attention and interest in order to make a considerable positive contribution to their students' mathematical literacy understanding. As put it by Tobias (1993), the main reason of people not being engaged by mathematics is to dislike or even sometimes hate mathematics. What usually people remember in the past that their teachers often just gave them meaningless mathematical rules and theorems, and asked for them to apply these abstract mathematical facts in problem solving. However, better acquirement of mathematical literacy understanding largely depends on teachers' attempts to help students overcome their fear of mathematics by building and developing their self confidence in it (Artzt et al., 2008; Coben, 2003; Franke et al., 2007). Therefore, if teachers intend to educate their students to become mathematically literate citizens of future, they should always stimulate and sustain their interest in mathematics learning by raising their awareness about the application of mathematical knowledge in everyday lives. In other words, they should primarily change their students' minds about the idea of mathematics by showing that mathematics is not only about abstract but also much about real life. Besides, it was reported that mathematical literacy development requires teachers to apply inquiry-based instruction in which students discover the meaning of mathematical ideas through active involvement with their own learning process. As explained in the literature review, several researchers also support the same point (Englert et al., 1992; Borasi, 1992, 1996; Vithal, 2006). Indeed, one of the major impediments to the development of mathematical literacy understanding is the classroom instruction which is mostly based on teacher-centered approach. However, mathematical literacy skills are such skills that students solely gain through inquiry-based learning. Therefore, teachers should continuously create learning opportunities for their students to explore mathematical ideas and their applications in real life by themselves. They should avoid rote learning and memorization in mathematics. In other words, learning mathematics should be through inquiry in order to enhance the development of critical thinking and promote deep understanding of mathematical literacy. Additionally, associating mathematics with real life was considered as essential to provide better mathematical literacy development for students. That is to say, students should get the idea that if they learn mathematics, it will somehow work for them in daily living. Hence, being mathematically literate requires well understanding of the importance of mathematics in daily life (DoE, 2003; OECD, 2013a). So, as mentioned by Stylianides and Stylianides (2008), associating mathematics with real-life tasks truly motivates students to enjoy and learn mathematics. Accordingly, teachers should in a way attract students' attention to mathematics by demonstrating the relevance of mathematics to real life. Hence, connecting mathematics to real-life situations basically provides the students better mathematical literacy understanding. Moreover, embracing mathematics teaching according to streamed classes was also regarded as influential to create the best possible learning experience for students in mathematical literacy. More precisely, we unfortunately try to teach the same mathematics to all students. However, in such a case, some of them do well enough while others struggle in the subject as they have different needs and mathematical abilities. When we try to teach higher level of mathematics to students who are not good enough at mathematics, they are most likely getting stuck and unsuccessful. We may even give rise to create a potential barrier for them to learning mathematics. Then, most of them usually choose to stay away from mathematics in their future education. Therefore, as revealed by a number of studies (Olson & Larsen, 2012; Pettig, 2000; Santangelo & Tomlinson, 2009; Tomlinson, 2000, 2003), in order to provide students quality and proper mathematics education based on mathematical literacy understanding, delivering differentiated instruction according to students' mathematical abilities is very essential. In addition to differentiated instruction, creating group learning activities in class was also seen as important to support the development of mathematical literacy skills of students. The results of many studies support this finding (Fuchs et al., 1998; Linchevski & Kutscher, 1998; National Numeracy Review Report, 2008; Venkat & Graven, 2008; Venkat, 2007). These studies emphasize that the benefits of group learning are significant for developing mathematical literacy understanding of students. Clearly, students may better explain things to each other as they well know and understand one another. Thus, it sometimes becomes much easier to learn mathematical facts from other students. Therefore, it is claimed that it is very effective to encourage group work among students in order to gain the necessary mathematical literacy skills. Besides, encouraging mathematical discussion in class was considered as much valuable to provide learners with a strong understanding and reinforcement of mathematical literacy. As documented by a number of studies in the literature (Brown & Schäfer, 2006; Franke et al., 2007; Venkat, 2007; Venkat & Graven, 2008), mathematical discussion in a classroom environment automatically promotes students' motivation in mathematics learning. Thus, if we want to establish and improve mathematical literacy understanding of students, we should particularly create a classroom environment in which students argue and discuss their ideas with the other members of the class. We should especially allow students to share and defend their opinions in front of the class to find the most appropriate solutions to mathematical problems asked in the classroom. Moreover, improving positive and strong student-teacher relationships was regarded as one fundamental factor that supports the development of mathematical literacy among learners. Not surprisingly, when teachers have difficulties in maintaining positive relationships with their students, it can negatively affect classroom performance in mathematics learning (Hill & Rowe, 1996; Hill & Rowe, 1998; OECD, 2010, 2013c; Pianta & Hamre, 2009; Rowe, 1997). That is to say, the poor teacher-student relationship is in fact a clear sign of poor performance in mathematical literacy. Therefore, teachers' effort to establish and foster effective and appropriate relationships between the teacher and students is very vital for their students' mathematical literacy skills. Using visual and technological aids and tools was also perceived to clearly facilitate students' mathematical literacy development as it promotes interactive learning process in mathematics. This result is also supported by several research studies (de Lange, 2006; Niss & Jensen, 2002; Niss, 2003a, 2003b, 2015; OECD, 2013a; Ontario Ministry of Education, 2005). Furthermore, implementing variety of teaching strategies was thought as of great importance to efficient development of mathematical literacy understanding. Therefore, as much emphasized in the literature, it is important for teachers to use a wide range of instructional strategies and practices in order to engage as many students as possible while teaching mathematics in the context of mathematical literacy (Artzt et al., 2008; Askew et al., 1997; Graven & Venkat, 2007; OECD, 2009b, 2010).

Considering the assessment for mathematical literacy, two perspectives are at the forefront of the discussion, which are expected mathematical literacy proficiency levels and assessment methods for mathematical literacy. The expected proficiency levels were classified as the lower and highest levels of proficiency in mathematical literacy. As also revealed by PISA 2012 Mathematics Framework (OECD, 2013a), on the one hand, performing basic mathematical operations, understanding problem situations, and knowing mathematical definitions were considered as a lower level for mathematical literacy proficiency. On the other hand, mathematical thinking, reasoning and argument, transferring knowledge of mathematics to unfamiliar contexts, and formulating problem situations mathematically were thought as a higher level of proficiency in mathematical literacy. Different important views were also developed in relation to the assessment methods for mathematical literacy. As reported by a number of studies (Department of Basic Education, 2011; OECD, 2013a; Roohr, 2014), while some views emphasized that the overall performance of students in mathematics should be taken into account for reliable and fair assessment of mathematical literacy, the others highlighted that there are indeed no reliable and accurate ways to assess mathematical literacy level of students. On balance, it can safely be said that due to having diverse conceptions of mathematical literacy, various views regarding the educational strategies for mathematical literacy development and the assessment for mathematical literacy were presented by the teachers. However, what is really worth to recognize here is the fact that each view put forward above emphasizes the importance of the development of mathematical knowledge and skills. This can also be closely associated with teachers' conception of the relationships between mathematical literacy and mathematics emphasizing that mathematically literate individuals need to understand and develop certain level of mathematics to use in everyday problems according to their demands and needs.

Moreover, effective classroom management and firm control of organizational issues are basically important aspects of ensuring effective teaching and learning to take place in mathematics (Artzt et al., 2008). Thus, teachers' planning and preparation for teaching and learning in a well-organized and well-controlled classroom environment were also seen as crucial for mathematical literacy development. In that respect, much emphasis was placed on the importance of lesson preparation for mathematical literacy development. As mentioned by Stone et al. (2002), the better development of mathematical literacy of students is highly correlated to their teacher's effective planning and preparation for mathematics lessons emphasizing mathematical literacy understanding. Another emphasis was given to the importance of subject knowledge of mathematics for mathematical literacy development. In this regard, in-depth subject knowledge is indispensable not only for teaching mathematics effectively and creatively, but also for integrating it with the other areas for guiding students to discover its real-life applications (Coben et al., 2003; Niss, 2015). The other emphasis was laid upon the importance of setting instructional outcomes for mathematical literacy development. In this sense, as stated by Ontario Ministry of Education (2005), setting instructional outcomes is important as it provides both teachers and students with a clear purpose to direct teaching and learning efforts towards mathematical literacy development. What is actually intended to be emphasized here is that teachers have a strong impact on the development of their students' mathematical literacy understanding and skills because students are always influenced by their teachers' preferred teaching styles. However, in reality, except for a few inquisitive and exploratory teachers, almost all of the mathematics teachers continue to teach mathematics in traditional ways and have no idea about the mathematical literacy. Hence, appropriate teacher training programs were also perceived as substantial for the emergence and adequate development of mathematical literacy understanding of students. In other words, if teachers are desired to give students mathematics in the context of mathematical literacy, they should certainly have special in-service teacher training sessions about how to teach mathematics with emphasis on mathematical literacy (Ejiwale, 2013; Tatto, 2012; Thompson, 1992). On the other hand, in-service training of teachers is also often assumed to be very boring and completely waste of time for the teachers to repeatedly listen the same things that are known to them. Therefore, the revolutionary modifications in teacher training programs are unavoidable for the optimum and sustainable progress in teachers' mathematical literacy understanding. In fact, it is plausible to suggest that contents of teacher training programs must be changed in order to motivate and orient teachers toward the necessary mathematical literacy understanding. Yet, other evidence suggests that even these programs' content and concepts are changed, some teachers still do not believe in any benefit of such teacher training programs to develop enough mathematical literacy understanding for mathematics teachers. The view that in-service teacher training does not provide any good result for teachers to give mathematics in the context of mathematical literacy can indeed be ascribed to these teachers' conceptions of the barriers to the development of mathematical literacy involving the drawbacks of the university placement examination system and the lack of mathematical literacy understanding in undergraduate teacher education programs. Therefore, unless these two barriers are recognized and addressed in a rational way, in-service teacher training programs seem to have little chance to become effective to provide necessary mathematical literacy understanding for teachers.

5.3. Teachers' conceptions of mathematics curriculum emphasizing mathematical literacy

Secondary school teachers' conception about the mathematics curriculum emphasizing mathematical literacy involved two central dimensions, which were challenges in curriculum implementation in the context of mathematical literacy, and recommended curriculum modifications in relation to mathematical literacy. The discussions of each of these two dimensions were presented below in the following sub-sections.

5.3.1. Conceptions of the challenges in curriculum implementation in the context of mathematical literacy

The data of this study revealed different ways of conception while analyzing the teachers' challenges in curriculum implementation in the context of mathematical literacy, which include challenges associated with intensity of mathematics curriculum, challenges associated with teachers' perspectives and practices, and challenges associated with lack of mathematical literacy emphasis in mathematics curriculum. First of all, for some teachers, a lack of mathematical literacy emphasis in mathematics curriculum was thought to be an important challenge that inhibits the development of mathematical literacy understanding. Whenever teachers look at the curriculum, they claim that teachers do not feel that the curriculum emphasizes them to teach mathematics in the context of mathematical literacy. However, some of the learning outcomes and goals of present mathematics curriculum are apparently consistent with the learning outcomes of mathematical literacy, but the one and only problem is the proper implementation of these learning outcomes and goals in mathematics classes. In this regard, the congested mathematics curriculum was considered as one significant factor that negatively influences the successful implementation of these learning outcomes and goals. It is important to realize that successful implementation of any curriculum reform is primarily based on establishing more realistic time frames for teaching and learning (Adler et al., 2000). However, some teachers criticized that curriculum is very intense and requires a strong commitment of time and effort to meet its requirements. This automatically produces a great stress for teachers, which naturally has a negative impact on students' development of mathematical literacy skills. That is to say, an intense mathematics curriculum leaves teachers no room for anything else than following the curriculum more tightly to meet its content standards. Even no matter how much teachers want to teach mathematics by using discovery and discussion based techniques, because the comprehensive and intensive curriculum allows very little or no time to implement such diverse techniques. That is why, teachers generally do not believe that they teach mathematics in a way that is beneficial to their students' life unless they have a more flexible timeline for the implementation of mathematics curriculum. Moreover, as also reported by many teachers, any curriculum modification in accordance with mathematical literacy understanding probably brings teacher resistance as well. There are indeed several studies in the literature supporting that teachers demonstrate hidden or sometimes open ignorance to curriculum innovations and they develop very limited and shallow understanding of any reform movement in education (Fullan & Stegelbauer, 1991; Handal & Herrington, 2003; Knapp & Peterson, 1995; Koehler & Grouws, 1992; Pajares, 1992; Spillane et al., 2002). Accordingly, even the curriculum is changed and adapted through mathematical literacy understanding, most of the teachers may not appreciate and welcome it unless their beliefs and perceptions about teaching mathematics change. In such a case, teachers usually intend to maintain their existing teaching practices. Thus, unless a supportive environment is created to improve the professional qualities of teachers in order to enhance their knowledge and skills needed by the modified curriculum, it is always easier for most of them to resort to old knowledge and familiar teaching and learning methods (Clarke, 1997). In fact, if mathematics curriculum is changed and revised to align with mathematical literacy understanding, its acceptance should be expected in the long run through the persuasion of teachers to such change and revision. Therefore, despite some potential difficulties to implement it in the first few years, curriculum change towards mathematical literacy understanding will ultimately create awareness among teachers about the importance of mathematical literacy. In such a case, most of the teachers will gradually accept and implement the curriculum modifications or adaptations made in accordance with mathematical literacy understanding within a certain period of time.

5.3.2. Recommended curriculum modifications in relation to mathematical literacy

The data of this study revealed different ways of conception while analyzing the teachers' recommendations about the curriculum modifications in relation to mathematical literacy including the right time to start mathematical literacy development, planning of mathematics curriculum emphasizing mathematical literacy, necessity of mathematical literacy curriculum, content categories of mathematics curriculum emphasizing mathematical literacy, and intended learning objectives of mathematics curriculum emphasizing mathematical literacy. First of all, a great deal of emphasis was placed on the fact that it becomes too late for students to start to acquire mathematical literacy understanding at high school. Four arguments in equal importance were developed by the teachers to support the view mentioned above. Firstly, it was proposed that mathematical literacy development should begin with elementary school. As reported in the literature (Department of Education and Skills, 2011; Doig et al., 2003; French, 2013; Sawyer, 2005), the firm and sustainable mathematical literacy development indeed begins in the early school years. Secondly, for some teachers, the appropriate time to start mathematical literacy development should go back to preschool years. Building solid foundation for mathematical literacy understanding at preschool not only increases school readiness but also has longer-term effects on further development of mathematical literacy skills (Dooley et al., 2014; Dunphy et al., 2014). Therefore, starting to acquire mathematical literacy skills at preschool is of great importance. Thirdly, parental education was also considered as a valuable primary source for mathematical literacy acquisition. Hence, starting mathematical literacy understanding with the parental involvement or home learning environment is a crucial and absolute necessity for its steady and successful development (Anthony & Walshaw, 2009). Finally, it was suggested that the development of mathematical literacy skills should begin with the moment a person first meets with mathematics and continue to progress throughout life. Actually, all these four arguments demonstrate how the development of the basic level of mathematical knowledge and skills is really important for the development of mathematical literacy understanding. Therefore, teachers' conception of mathematical literacy as the possession of some basic knowledge and skills of mathematics in order to meet the general demands of daily life can be influential for their argument that a solid foundation of mathematical literacy should begin in early childhood.

In this context, the content categories of mathematics curriculum required for better mathematical literacy development of students were also recommended. Accordingly, the following four key areas were developed: (i) quantity, (ii) uncertainty and statistics, (iii) change and relationships, and (iv) space and shape. First of all, as also suggested by (OECD, 2013a), the number sense and quantity are of paramount importance for mathematical literacy. That is, mathematically literate individuals should have the ability to clearly identify numbers and quantities around them. Thus, the quantity concept should be the fundamental content category of mathematics curriculum emphasizing mathematical literacy. Secondly, uncertainty and data analysis are also integral parts of mathematics curriculum emphasizing mathematical literacy as these two concepts are closely related to real life (OECD, 2013a). Thirdly, understanding of change and relationships is another crucial content category of mathematics curriculum that significantly influences students' mathematical literacy level (OECD, 2013a), because real life always involves the process of relationships and their change over time. Finally, as put it by OECD (2013a), knowledge of space and shapes is apparently very necessary in many different fields of real life. Thus, much emphasis could also be laid upon space and shape in mathematics curriculum as everyday life is surrounded by space and various geometric shapes in different sizes. In such a case, it is probably true that the fact that each teacher evidently holds most of the conceptions of mathematical literacy mentioned in the literature simultaneously to some degree actually gives rise to the consideration of content categories in a broader sense. However, in reality the most important factor is that the curriculum designed with these content categories typically equips students with knowledge and skills to handle problems and make decisions by interpreting situations in personal, occupational, societal and scientific contexts, which indeed underlines the essence of mathematical literacy.

There were also some recommendations about the importance of planning of mathematics curriculum in the context of mathematical literacy. In this regard, it was recommended that, on the one hand, mathematics and mathematical literacy could be planned and taught as separate curriculum areas if the necessary conditions are not available for teaching mathematics in the context of mathematical literacy. On the other hand, mathematical literacy understanding could also be embedded within mathematics curriculum as mathematical literacy and mathematics are admittedly not distant and independent from each other. In other words, mathematics and mathematical literacy could be given together as mathematical literacy understanding is thought to be an important and essential part of mathematics curriculum. However, there was the firm belief that due to time restraints and the intensity of mathematics curriculum, most of the teachers have no other way except implementing the curriculum in the following sequence: definitions, formulas, computations, examples, and practices, whereas it is quite likely that mathematics presented in the following sequence: problems, discovery, hypothesis, verification, generalization, association, and inference can provide students necessary mathematical literacy understanding. Indeed, it is not easy to take such curriculum approach and apply it convincingly to all classes. Teachers must first be trained and motivated in this regard. Otherwise, they are always inclined to continue to present curriculum in traditional ways. Nonetheless, despite certain difficulties and potential challenges, the curriculum with emphasis on problems, discovery, hypothesis, verification, generalization, association, and inference seems to be much appropriate for the development of mathematical literacy skills (Milli Eğitim Bakanlığı, 2015). In that respect, mathematical literacy is actually considered as the ability to identify, analyze and develop solutions to problems encountered in many different areas of life. It mainly refers to an individual's capacity to formulate, employ and evaluate mathematical problems in a variety of situations. Namely, it provides people with the ability to solve many problems in real-life situations. Or, to put it more simply, it helps people throughout their life. Accordingly, problem solving and mathematical literacy are seen as closely related two concepts as they truly foster each other (de Lange, 2003; Jablonka, 2003; OECD, 2013a, 2014a, 2014b; Venkat, 2007). Hence, for better acquisition of mathematical literacy, primary concern in mathematics curriculum was taken as the development of problem solving skills of students. Correspondingly, the mathematical modelling process was also regarded as a central and vital component of any mathematics curriculum emphasizing mathematical literacy as it basically involves the phases of formulating, employing, and interpreting any problem situation. Thus, as supported by many research studies (Brown & Schäfer, 2006; Henning & Keune, 2006; Stacey, 2015; Steen, Turner, & Burkhardt, 2007; Winter & Venkat, 2013), when planning any mathematics curriculum, it seems very crucial to lay particular emphasis on the mathematical modelling process. Additionally, it was suggested that a better mathematical literacy development of students can be achieved as long as continuous adaptations and modifications in mathematics curriculum are done to conform to students' diverse mathematical abilities and needs. Teachers expressed that some of their students' math knowledge and skills do not seem suitable to implement the current curriculum precisely and regularly in class. That is to say, students' interests and abilities are very different from what their teachers want to see and hear. In such a case, it is quite difficult for students to gain mathematical literacy skills. This also produces a great difficulty for teachers. Thus, as reported by many studies (Gregory & Chapman, 2002; Heacox, 2002; Murray & Jorgensen, 2007; Olson & Larsen, 2012; Tomlinson, 1999, 2003, 2006), in order to overcome this difficulty, mathematics curriculum needs to planned and taught according to students' diverse abilities and interests. In other words, it should be adjusted in response to needs and interests of students with different mathematical skills in such a way that it helps teachers keep their students actively involved in their learning. It is also important to remember that keeping students interested in and actively engaged with their own learning provides better and meaningful learning in mathematics (Arbaugh & Brown, 2005; Harel & Sowder, 2005; Henningsen & Stein, 1997; OECD, 2013b). For this reason, mathematics curriculum that aims to instill in students a desire and enthusiasm for learning mathematics by developing higher appreciation of its elegance and power in many real-life situations and events is also of great importance for promoting mathematical literacy understanding among students. Furthermore, it was recommended that integrated curriculum approach is important for creating and improving mathematical literacy skills of students as it makes learning mathematics more meaningful to them. Since the integrated curriculum approach deals with relations between mathematics and the other areas of science as well as the relations between mathematics and real life, being able to establish those relations between mathematics and other subject areas is really helpful for the development of strong mathematical literacy understanding (Quantitative Literacy Design Team, 2001;

Richardson & McCallum, 2003; Steen, 2004). So, aside from providing a strong basis for mathematical understanding, such curriculum approach can also make a great contribution to mathematical literacy development of students.

5.4. Conclusions and Implications

This study investigated secondary mathematics teachers' conceptions about mathematical literacy by identifying three issues related to their conceptions about the notion of mathematical literacy, the effective methods for fostering mathematical literacy, and the aspects of mathematics curriculum emphasizing mathematical literacy. In the light of the main findings received from this study, the major conclusions and implications can be expressed from various perspectives as follows.

First of all, teachers' conceptions about the notion of mathematical literacy were composed of three central components, which were the nature of mathematical literacy, the fundamental mathematical capabilities for mathematical literacy, and the relationships between mathematics and mathematical literacy. Since the nature of mathematical literacy is a multidimensional concept, it is still a very controversial issue among researchers (Coben et al., 2003; Jablonka, 2003; Sfard, 2014). Hence, the nature of mathematical literacy was also interpreted in different ways by mathematics teachers. For example, it was seen as the possession of mathematical knowledge and skills including various levels from the basic mathematical knowledge and skills to the advanced mathematical knowledge and skills depending on the people's needs and requirements. The nature of mathematical literacy was also viewed as functional mathematics involving the use of mathematics in everyday life, the use of mathematics in societal life, the use of mathematics in further education, and the use of mathematics in occupational life. In addition to above-mentioned conceptions, there are also different arguments which refer to the nature of mathematical literacy such as problem solving, mathematical thinking, innate mathematical ability, conceptual understanding, and motivation to learn mathematics. Moreover, the notion of mathematical literacy was also conceived in terms of the fundamental mathematical capabilities, which consist of communication, reasoning and argument, using symbolic, formal and technical mathematical language and operations, mathematizing, representation, devising strategies for solving problems, and using mathematical aids and tools. As mentioned in the literature particularly by OECD (2013a), these capabilities are fundamental for mathematical literacy development (de Lange, 2006; Niss, 2003b; 2015). Furthermore, the conception of the notion of mathematical literacy was also analyzed according to the relationships between mathematics and mathematical literacy. In that respect, mathematical literacy was considered as one of the significant contributory factors for meaningful learning of mathematics. Conversely, knowledge of mathematics was also regarded to play a crucial role in the development of mathematical literacy. Besides, mathematical literacy and mathematics were also perceived to intersect at some common grounds while diverging at other points. Namely, it was regarded that although mathematics and mathematical literacy do not exactly overlap, they much support one another. Although mathematics and mathematical literacy were viewed to be related in a way and influence one other's levels, for some teachers they were nonetheless conceived as two different concepts, and having no linear relationship between them. Additionally, mathematics was considered as a branch of science dealing with proof, logical arguments and justifications of abstract mathematical structures whereas mathematical literacy was perceived as real-life applications of mathematics. In other words, mathematical literacy was viewed as the application of mathematics in real world contexts in order to tackle everyday problems. In this regard, for those who considered mathematics as an abstract and theoretical discipline, mathematics as a broader field completely involves mathematical literacy whereas for others who thought that mathematics was clearly embedded in daily life in many respects, mathematical literacy clearly includes all knowledge of mathematics. But what is really important to note here is that mathematics and mathematical literacy definitely strengthen each other by supporting and building one another up (Steen, 1990, 2001a). Understandably, people can easily comprehend and develop their own mathematical knowledge that they use and apply in real life. In this sense, mathematical literacy certainly offers students many opportunities for effective and active learning of mathematics with deep understanding (Stacey & Turner, 2015). Mathematical literacy is in fact necessary for the practical application of mathematics in order to handle or evaluate diverse problems set in a real-life context. Since mathematical concepts are usually presented in a more formal way in class, mathematical literacy clearly provides students with much understanding to interpret problem situations, and then cautiously handle these problems by putting the necessary steps into practice for finding the most appropriate solutions to them. Hence, a person who is mathematically literate can surely learn and make sense of mathematics much easier than the others. That is to say, mathematical literacy understanding can simply facilitate students' understanding in mathematics (OECD, 2013b). For this reason, mathematical literacy can be a wonderful vehicle in class to teach mathematics in an effective and productive manner. On the other hand, a mathematically literate person also needs to understand and develop certain level of mathematics to use in everyday problems according to their demands and necessities (Coben et al., 2003). Therefore, a satisfactory knowledge of mathematics is essential for better mathematical literacy development (Niss, 2015). In other words, being a mathematically literate individual basically requires the development of a solid foundation in a certain range of mathematical knowledge and skills.

Secondly, teachers' conceptions about the effective development of mathematical literacy involved two central dimensions, which were the barriers to the development of mathematical literacy, and the central domains for mathematical literacy development. Major obstacles to the development of mathematical literacy were enumerated as the barriers associated with students, barriers associated with teachers, drawbacks of the university placement examination, shortcomings of undergraduate teacher education programs, lack of appropriate math textbooks for mathematical literacy, lack of differentiated instruction, lack of mathematical literacy understanding in primary education, time constraint in teaching mathematics, and crowded classrooms. In this respect, while the barriers associated with students included mathematics anxiety, disinterest of students in mathematics, lack of basic mathematics knowledge, excessive concern about mathematics grades, and the inability to search for information independently, the barriers associated with teachers consisted of the lack of teachers' qualifications about mathematical literacy, teaching mathematics by rote, challenge of making major changes in teaching practices, drawbacks of consistent use of routine problems, and misunderstanding of mathematical literacy. Moreover, the teachers' conceptions about the central domains for mathematical literacy development included their views about educational strategies for mathematical literacy development, assessment for mathematical literacy, professional responsibilities for mathematical literacy development, disposition towards mathematical literacy understanding, planning and preparation for mathematical literacy development, and classroom environment for better mathematical literacy acquisition. Educational strategies for effective mathematical literacy development were considered as the importance of types of problems chosen for mathematical literacy improvement, the importance of engaging students in mathematics learning, the importance of associating mathematics with real life, providing differentiated instruction according to students' mathematical abilities, creating group learning activities, encouraging mathematical discussion, improving positive and strong student-teacher relationships, using visual and technological aids and tools, and implementing variety of teaching strategies including inquiry-based instruction in which students discover the meaning of mathematical ideas through active involvement with their own learning process. Hence, the real-life applications of mathematics should be particularly emphasized when teaching mathematics by giving students enough chance to get involved in different learning styles such as pair work, whole class or group work and discussions (Fuchs et al., 1998; Venkat & Graven, 2008), cooperative and collaborative learning approaches (Davidson, 1990; Frith & Prince, 2006), differentiated instruction (Tomlinson, 2000, 2003), project work, guided discovery and inquiry learning (Borasi, 1992, 1996; Vithal, 2006), problem solving (de Lange, 2003; OECD, 2013a, 2014b), mathematical modelling (Brown & Schäfer, 2006; Steen, Turner, & Burkhardt, 2007), or technology-based learning de (Lange, 2006; Niss & Jensen, 2002; Niss, 2003a, 2003b) for their effective mathematical literacy development. In such a case, teachers' disposition and competency in mathematical literacy play a vital role in teaching mathematics in the context of mathematical literacy. Therefore, appropriate teacher training programs were also considered as vital for the emergence and adequate development of mathematical literacy understanding among teachers. Teachers clearly need in-service teacher trainings on how to teach mathematics in relation to mathematical literacy (Ejiwale, 2013; OECD, 2009b, 2014c; Tatto, 2012; Thompson, 1992). Accordingly, in order to create awareness and attract teachers' attention to mathematical literacy, teacher training programs such as symposiums and seminars should be done for mathematics teachers. In these programs, experts in this area may demonstrate teachers how to teach mathematics in different ways through practical knowledge rather than theoretical knowledge. They may present sample lessons emphasizing mathematical literacy. They may also show teachers many diverse instructional methods in order to orient students towards mathematical literacy understanding. Besides, resource sharing in the context of mathematical literacy can also be done among teachers and trainers within such programs. Moreover, the assessment for mathematical literacy was regarded to involve the expected mathematical literacy proficiency levels and the assessment methods for mathematical literacy. While performing basic mathematical operations, understanding problem situations, and knowing mathematical definitions were considered as a lower level for mathematical literacy proficiency, mathematical thinking, reasoning and argument, transferring knowledge of mathematics to unfamiliar contexts, and formulating problem situations mathematically were thought as a higher level of proficiency in mathematical literacy. Furthermore, for some teachers, overall performance of students in mathematics was taken into account for reliable and fair assessment of mathematical literacy, whereas for others, there were indeed no reliable and accurate ways to assess mathematical literacy level of students. Effective classroom management in order to promote student learning in mathematics was also of great importance for ensuring effective teaching and learning to take place in the context of mathematical literacy. In this sense, the lesson planning, subject knowledge of mathematics, and setting instructional outcomes were perceived to be very important for mathematical literacy development.

Finally, teachers' conception about the mathematics curriculum emphasizing mathematical literacy included two central dimensions, which were challenges in curriculum implementation in the context of mathematical literacy, and recommended curriculum modifications in relation to mathematical literacy. The congested mathematics curriculum, teachers' resistance to any change in curriculum, inappropriate difficulty level of mathematics curriculum for some students, and lack of enough mathematical literacy emphasis in mathematics curriculum were taken as major challenges to mathematical literacy emphasis in mathematics curriculum. The conceptions about the recommended curriculum modifications in the context of mathematical literacy involved views about the right time to start mathematical literacy development, planning of mathematics curriculum emphasizing the development of mathematical literacy skills, necessity of mathematical literacy curriculum, content categories of mathematics curriculum regarding mathematical literacy understanding, and intended learning objectives of mathematics curriculum emphasizing mathematical literacy. The appropriate time to start mathematical literacy development was perceived to be before high school. In other words, starting mathematical literacy understanding in preschool years was viewed to be crucial and absolute necessity for the firm and sustainable mathematical literacy development. Accordingly, the development of the essential basis for mathematics knowledge that underpins mathematical literacy should start from early childhood in a structured way and gradually advance through all education levels. Otherwise, it becomes too late for most of the students to start to gain mathematical literacy skills at high school. Hence, providing solid foundation for mathematical literacy understanding at preschool not only increases school readiness but also has longer-term effects on further development of mathematical literacy skills (Department of Education and Skills, 2011; Doig et al., 2003; Dooley et al., 2014; Dunphy et al., 2014; French, 2013; Sawyer, 2005). In general, mathematical literacy understanding was thought to be embedded within mathematics curriculum as mathematical literacy and mathematics are certainly not distant and independent from each other. Therefore, mathematical literacy understanding should be nicely placed into mathematics curriculum as mathematical literacy and mathematics always go hand in hand in order to associate mathematics to real world circumstances. In this regard, despite certain difficulties, the curriculum with emphasis on problems, discovery, hypothesis, verification, generalization, association, and inference was considered to be much appropriate for the promotion of mathematical literacy skills. Indeed, the ideal way towards sufficient and consistently growing mathematical literacy development should be based on the efficient application of each of these steps, because achieving to implement such curriculum approach clearly provides individuals with a sense of security and assurance to confidently formulate, employ and interpret any real-life problem (Milli Eğitim Bakanlığı, 2015). In that respect, the problem solving and the mathematical modelling process were also regarded as a vital component of any mathematics curriculum emphasizing mathematical literacy. Problem solving and mathematical literacy are closely related two concepts as they truly foster and interact with each other (Brown & Schäfer, 2006; de Lange, 2003; Jablonka, 2003; OECD, 2013a, 2014a, 2014b; Venkat, 2007). The mathematical modelling cycle is also seen an integral aspect of any curriculum emphasizing mathematical literacy (Brown & Schäfer, 2006; Henning & Keune, 2006; Stacey, 2015; Steen, Turner, & Burkhardt, 2007; Winter & Venkat, 2013). In this regard, the problem solving and the mathematical modelling cycle should be the central focus of any mathematics curriculum as both of them clearly improve students' skills in reasoning and allow them to gain much strong understanding of any problem encountered in life. In doing so, we can really make students appreciate the importance and value of mathematics in their surroundings. Some teachers also suggested that a better mathematical literacy development of students can be achieved as long as continuous adaptations or modifications in mathematics curriculum are done to conform to students' diverse mathematical abilities and interests. Therefore, designing or adapting mathematics curriculum according to diverse math skills and needs of students should also be taken into consideration in order to provide effective and efficient mathematical literacy development for each student (Gregory & Chapman, 2002; Heacox, 2002; Murray & Jorgensen, 2007; Olson & Larsen, 2012; Tomlinson, 1999, 2003, 2006). Moreover, integrated curriculum approach was considered as important for creating and improving mathematical literacy skills of students as mathematics and the other subjects unquestionably have numerous content areas in common. Hence, learning in mathematics happens effective when we integrate mathematics into other areas and make it part of our daily life. In this regard, curriculum which is integrated with other disciplines and many everyday events is of critical importance (Quantitative Literacy Design Team, 2001; Richardson & McCallum, 2003; Steen, 2004). For this reason, in addition to focusing on abstract ideas or concepts within mathematics itself, we should also make its connections to other domains or subjects as well as to real-life applications outside the classroom.

5.5. Limitations and Future Research Suggestions

Attempting to characterize secondary mathematics teachers' conceptions of mathematical literacy over the course of three one-hour interviews warrants some considerations. First of all, this study could have been strengthened with classroom observations which could also have led to additional questions or issues during the interviews in order to reveal unspoken thoughts and hidden conceptions of mathematical literacy compared to what was reported in the interviews. Hence, in addition to investigating secondary mathematics teachers' conceptions of mathematical literacy, this study could also have examined to what extent these conceptions could impact these teachers' instructional decisions. Therefore, this can be considered as a limitation since how these teachers' conceptions of mathematical literacy could influence their teaching of mathematics was not assessed in a real classroom environment. Secondly, based on their willingness, sixteen teachers were selected to participate in the study. Although this study does not aim to generalize the findings across all secondary mathematics teachers, the number of participants could be considered as a limitation since a limited number of participants might not have represented a wide range of views in order to obtain all different perspectives with respect to teachers' conceptions of mathematical literacy. The results can only be assumed true in similar contexts. Accordingly, the following paragraphs present some research suggestions that can further extend the present study in a number of dimensions while simultaneously addressing the above-mentioned limitations.

The results of this research study provide a picture of how teachers perceive mathematical literacy in secondary school mathematics contexts. However, how teachers talk about their conceptions may be different than how their conceptions actually play out in their instructional practices. The successful integration of mathematical literacy throughout secondary school mathematics curricula may also be based on teachers' adequate appreciation and continued reflection of these conceptions in their instructional practices. Thus, a next step could be to examine the consistencies between teachers' conceptions of mathematical literacy and their day to day classroom practices. Conducting observations of the mathematical literacy practices in their classrooms could provide insight into the possible reasons for tensions between their held beliefs and attempted practices. This investigation could also be extended to shed light on the matter of the nature of the conceptions of mathematical literacy that these teachers' students are likely to develop as a result of the influence of teachers' conceptions on their students' mathematical literacy understanding. Therefore, investigating students' conceptions of mathematical literacy in comparison to those of their teachers could be enlightening to understand the potential effects of teachers' conceptions of mathematical literacy on students' beliefs and performance in mathematical literacy.

Another area of pursuit could be to investigate the nature of the mathematical literacy practices in the university mathematics courses that prospective teachers are required to take. If it is in these courses that their conceptions of mathematical literacy are being developed, or at least influenced, then it seems worth examining what messages about mathematical literacy are actually being sent and what messages about mathematical literacy the prospective teachers are actually receiving. Therefore, future research could explore how pre-service teachers understand the nature and roles of mathematical literacy from the perspective of the prospective mathematics teachers as they might hold quite different conceptions than those of in-service teachers considering mathematical literacy in the secondary school setting. In this sense, a longitudinal study to trace the development of pre-service teachers' conceptions of mathematical literacy during their undergraduate studies could also be undertaken. Interviewing each participant more frequently over the course of the undergraduate years and conducting classroom observations might provide expanded, and possibly different, data. Studies of this kind could deliver a more accurate and complete picture of the factors that impact teachers' conceptions of mathematical literacy. Further, they may also provide mathematics educators with the necessary information to target particular areas within the teacher education process in which the desired changes in the prospective teachers' conceptions of mathematical literacy may be feasible.

Finally, the role of teachers' conceptions of mathematical literacy as a tool for teaching and learning mathematics is perhaps the most important from a pedagogical perspective that needs an investigation. Accordingly, the other area worth examining could be the importance of teachers' conceptions of mathematical literacy for promoting mathematical understanding in the classroom. Thus, additional research to investigate how teachers' conceptions of mathematical literacy influence or foster their teaching of mathematics would shed further light onto this matter.

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APPENDIX A

A.1: FIRST INTERVIEW GUIDE

MATEMATİK OKURYAZARLIĞINA İLİŞKİN ÖĞRETMEN KAVRAYIŞLARI HAKKINDAKİ GÖRÜŞME FORMU (Yarı Yapılandırılmış Görüşme)	
GENEL BİLGİLER	
Okul Adı	:
Öğretmen Adı	:
Görüşme Tarihi	:
Başlangıç Saati – Bitiş Saati	:

I. Matematik okuryazarlığı kavramı öğretmenler tarafından nasıl tanımlanıyor?

1. Matematik okuryazarlığı ifadesinden ne anlıyorsunuz?

[Matematik okuryazarılığının amacı sizce ne olabilir?]
[Matematik okuryazarlığı sizce ne kadar önemlidir?]
[Matematik okuryazarlığı bireye ve topluma ne kazandırır?]

2. Matematik okuryazarı olan bireyin sahip olması gereken temel beceri ve yeterlilikler nelerdir?

[Matematik okuryazarı birey olmak neleri gerektirir?]
[Matematik okuryazarlığını oluşturan temel unsurlar neler olabilir?]

3. Matematik okuryazarlığı ile matematik arasında sizce nasıl bir ilişki vardır?

[Matematik bilgisi matematik okuryazarlığını nasıl etkiler?]
[Matematik okuryazarlığı matematik bilgisini nasıl etkiler?]

4. Eklemek istediğiniz başka bir şey var mı?

A.2: SECOND INTERVIEW GUIDE

MATEMATİK OKURYAZARLIĞINA İLİŞKİN ÖĞRETMEN KAVRAYIŞLARI HAKKINDAKİ GÖRÜŞME FORMU (Yarı Yapılandırılmış Görüşme)	
GENEL BİLGİLER	
Okul Adı	:
Öğretmen Adı	:
Görüşme Tarihi	:
Başlangıç Saati – Bitiş Saati	:

II. Etkili matematik okuryazarlık gelişimine ilişkin öğretmen kavrayışları nelerdir?

1. Matematik okuryazarlığı gelişimi hangi süreçte başlar?

[Matematik okuryazarlığı kazanımı erken yaşlarda başlayabilir mi?]
[Matematik okuryazarlığı kazanımı hayat boyu devam eder mi?]

2. Matematik okuryazarlığı gelişiminde karşılaşılan sorunlar neler olabilir? Bu sorunlar nasıl aşılabilir?

[Öğrencilerden kaynaklanan sorunlar]
[Öğretmen yeterliliklerinden kaynaklanan sorunlar]
[Öğretim programlarından kaynaklanan sorunlar]
[Ders kitapları ve materyal kullanımından kaynaklanan sorunlar]
[Merkezi sınav sisteminden kaynaklanan sorunlar, vs.]

3. Matematik okuryazarlığı gelişiminde öğretmenin rölü nedir?

[Öğretmenin uygulaması gereken öğretim stratejileri nelerdir?]
[Öğretmenin ölçme değerlendirme yöntemi nasıl olmalıdır?]
[Öğretmenin sahip olması gereken bilgi, beceri ve yeterlilikler nelerdir?]
[Öğretmen yetiştiren lisans programlarına ilişkin önerileriniz nelerdir?]
[Öğretmenlerin mesleki eğitim çalışmaları ve uygulamalarına ilişkin önerileriniz nelerdir?]

4. Eklemek istediğiniz başka bir şey var mı?

A.3: THIRD INTERVIEW GUIDE

MATEMATİK OKURYAZARLIĞINA İLİŞKİN ÖĞRETMEN KAVRAYIŞLARI HAKKINDAKİ GÖRÜŞME FORMU (Yarı Yapılandırılmış Görüşme)	
GENEL BİLGİLER	
Okul Adı	:
Öğretmen Adı	:
Görüşme Tarihi	:
Başlangıç Saati – Bitiş Saati	

III. Matematik okuryazarlığına vurgu yapan matematik öğretim programına ilişkin öğretmen kavrayışları nelerdir?

1. Matematik öğretim programında matematik okuryazarlığına yeteri kadar vurgu yapılıyor mu? Eğer yoksa bunun nedenleri neler olabilir?

2. Matematik okuryazarlığına vurgu yapan matematik öğretim programı sizce nasıl olmalıdır?

[Genel amaçları neler olmalıdır?]
[Geliştirmeyi hedeflediği matematiksel beceri ve yeterlilikler neler olmalıdır?]
[Ölçme değerlendirme yaklaşımı nasıl olmalıdır?]
[Öğrenme alanları neler olmalıdır?]

3. Matematik okuryazarlığına ilişkin ayrı bir öğretim programının hazırlanması gerekli midir? Neden?

4. Eklemek istediğiniz başka bir şey var mı?

APPENDIX B

B.1: INTERVIEW TASK 1

MATEMATİK OKURYAZARLIĞI TANIMINA İLİŞKİN ÖĞRETMEN KAVRAYIŞLARI	
Öğretmen Adı	:
Okul Adı	:
Görüşme Tarihi	:

Kendinizi aşağıdaki matematik okuryazarlığı tanımlarından hangisine yada hangilerine daha yakın hissediyorsunuz?

Eklemek istediğiniz ifadeleri mevcut tanımların altına yada yeni bir tanım olarak sayfanın sonuna ekleyebilirsiniz.

1. Matematik okuryazarlığı temel seviye matematik bilgi ve becerisidir.

2. Matematik okuryazarlığı matematik kavramlarını öğrenmektir.

3. Matematik okuryazarlığı matematiksel düşünmeyi öğrenmektir.

4. Matematik okuryazarlığı bir durumu matematiksel olarak ifade edebilmektir.

5. Matematik okuryazarlığı matematiği günlük hayatta işlevsel olarak kullanabilmektir.

6. Matematik okuryazarlığı ileri seviyede matematik bilgi ve becerisidir.

7. Matematik okuryazarlığı üst düzey düşünme becerisidir.

8. Matematik okuryazarlığı matematik dilindeki ifadeleri anlayabilme ve aktarabilme becerisidir.

9. Matematik okuryazarlığı bireyin dünyada matematiğin oynadığı rolü anlama ve tanıma kapasitesidir.

10.

Eklemek istediğiniz yeni tanımı buraya ekleyebilirsiniz.

11.

Eklemek istediğiniz yeni tanımı buraya ekleyebilirsiniz.

B.2: INTERVIEW TASK 2

MATEMATİKSEL PROBLEM TİPLERİ	
Öğretmen Adı	:
Okul Adı	:
Görüşme Tarihi	:

Matematik okuryazarlığına vurgu yapan matematiksel problem tipi yada tipleri sizce nasıl olmalıdır?

PROBLEM 1

Öğrencilerin aşağıda verilmiş problemlerinin çözümlerini gerçekleştirip bir sonuca varabilmesi için yazılı metni ve problemde var olan sayısal ilişkileri anlayıp bunlar arasındaki ilişkiyi kurmaları gerekmektedir. İhtiyaç duyulan tüm bilgiler problemde verilmiştir.

1) Arda, Ali' den iki yaş büyüktür. Ali' nin şimdiki yaşı Cemil' in şimdiki yaşının 4 katına eşittir. Buna göre Cemil' in şimdiki yaşı kaçtır?

2) Samet etiket fiyatı 1300 TL olan bir televizyon ve etiket fiyatı 400 TL olan bir yatak alıyor. Televizyonun etiket fiyatı üzerinden %10 indirim yapılıyor. Nakliye ve montaj hizmeti için de ödenen tutar 50 TL olduğuna göre, Samet' in bu alışverişte harcadığı toplam para ne kadardır?

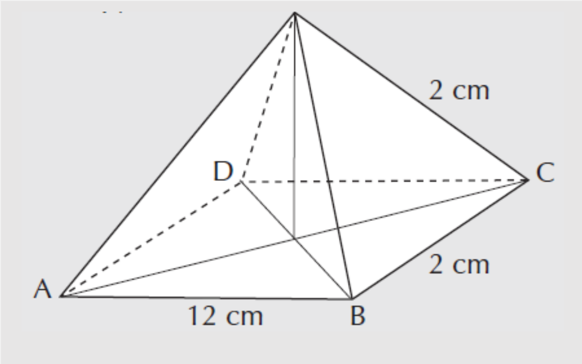
Problemleri çözmeyiniz!	Çok sık	Düzenli aralıklarla	Bazen	Hiç yapmam
a) Matematik dersi anlatılırken yukarıdaki gibi problem tipi hangi sıklıkta kullanılır?				
b) Ölçme ve değerlendirme yöntemlerinde yukarıdaki gibi problem tipi hangi sıklıkta kullanılır?				

PROBLEM 2					
	Öğrencilerin aşağıda verilmiş problemlerinin çözümlerini gerçekleştirip bir sonuca varabilmesi için belli matematiksel formül ve kuralları bilip uygulaması gerekmektedir.				
	1) $2x + 3 = 7$ denklemini sağlayan x değeri kaçtır?				
	2) Boyutları 3m, 4m ve 5m olan bir kutunun hacmi kaç m^3 tür?				
	Problemleri çözmeyiniz!	**Çok sık**	**Düzenli aralıklarla**	**Bazen**	**Hiç yapmam**
	a) Matematik dersi anlatılırken yukarıdaki gibi problem tipi hangi sıklıkta kullanılır?				
	b) Ölçme ve değerlendirme yöntemlerinde yukarıdaki gibi problem tipi hangi sıklıkta kullanılır?				

PROBLEM 3

Öğrencilerin aşağıda verilmiş problemlerinin çözümlerini gerçekleştirip bir sonuca varabilmesi için belli matematik teoremlerini bilip uygulaması gerekmektedir.

1) Bu soruda geometrik teoremleri kullanmanız gerekir:



Yukarıdaki şekilde verilenlere göre, piramitin yüksekliğini bulunuz.

2) n bir tam sayı olmak üzere, $(n+1)^2$ sayısının bir asal sayı olması mümkün müdür?

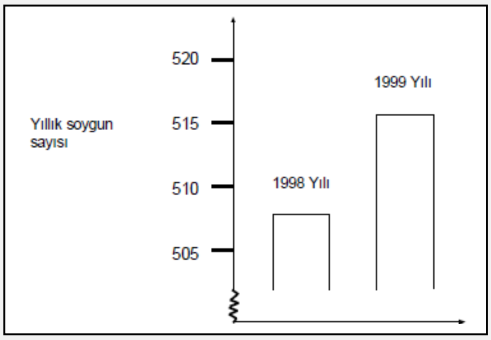
Problemleri çözmeyiniz!	**Çok sık**	**Düzenli aralıklarla**	**Bazen**	**Hiç yapmam**
a) Matematik dersi anlatılırken yukarıdaki gibi problem tipi hangi sıklıkta kullanılır?				
b) Ölçme ve değerlendirme yöntemlerinde yukarıdaki gibi problem tipi hangi sıklıkta kullanılır?				

PROBLEM 4

Öğrencilerin aşağıda verilmiş problemlerinin çözümlerini gerçekleştirip bir sonuca varabilmesi için öğrenilen formal matematiksel bilgiyi günlük yaşamda karşılaşılan problemlerin çözümüne doğru bir şekilde transfer edip uygulayabilmesi gerekmektedir.

1) Bir televizyon muhabiri, aşağıdaki grafiğe bakarak şunları söylemektedir:

"Bu grafik 1998 yılından 1999"a kadar soygunların sayısında çok büyük bir artış olduğunu göstermektedir."



Muhabirin sözlerinin grafiğe uygun bir yorum olduğunu düşünüyor musunuz? Yanıtınızı desteklemek için bir açıklama yapınız.

2) İnsanlar, sağlık nedenleriyle (örneğin spor yaparken), belirli bir kalp atış sayısını geçmemek için yaptıkları işleri sınırlamalıdır. Kişinin tavsiye edilen en yüksek kalp atış hızı ve kişinin yaşı arasındaki ilişki yıllarca aşağıdaki formül ile tanımlanmıştır:

Tavsiye edilen en yüksek kalp atış hızı= 220 – yaş

Son araştırmalar göstermiştir ki bu formülde küçük bir değişiklik yapılmalıdır. Yeni formül aşağıdaki gibidir:

Tavsiye edilen en yüksek kalp atış hızı= 208 – (0,7 × yaş)

Bir gazete makalesinde şu ifade geçmektedir: *"Eski formül yerine yeni formülün kullanılmasıyla, gençlerde dakika başına tavsiye edilen en yüksek kalp atışı küçük bir düşüş, yaşlılarda ise küçük bir artış göstermektedir."*

Yeni formülün kullanılmasıyla tavsiye edilen en yüksek kalp atış hızı hangi yaştan başlayarak artar? İşleminizi gösteriniz.

Problemleri çözmeyiniz!	**Çok sık**	**Düzenli aralıklarla**	**Bazen**	**Hiç yapmam**
a) Matematik dersi anlatılırken yukarıdaki gibi problem tipi hangi sıklıkta kullanılır?				
b) Ölçme ve değerlendirme yöntemlerinde yukarıdaki gibi problem tipi hangi sıklıkta kullanılır?				

B.3: INTERVIEW TASK 3

MATEMATİK OKURYAZARLIĞINA VURGU YAPAN MATEMATİK ÖĞRETİM PROGRAMININ BENİMSEYECEĞİ GENEL ÖĞRENME YAKLAŞIMINA İLİŞKİN ÖĞRETMEN KAVRAYIŞLARI	
Öğretmen Adı	:
Okul Adı	:
Görüşme Tarihi	:

Matematik okuryazarlığına vurgu yapan matematik öğretim programının benimseyeceği genel öğrenme yaklaşımı sizce nasıl olmalıdır?

Eklemek istediğiniz ifadeleri mevcut yaklaşımların altına yada yeni bir yaklaşım olarak sayfanın sonuna ekleyebilirsiniz.

1. Tanım → Teorem → İspat → Uygulamalar → Test

---

2. Problem → Keşfetme → Hipotez Kurma → Doğrulama → Genelleme → İlişkilendirme → Çıkarım

---

3.

Eklemek istediğiniz yeni yaklaşımı buraya ekleyebilirsiniz.

4.

Eklemek istediğiniz yeni yaklaşımı buraya ekleyebilirsiniz.

APPENDIX C

C.1: REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN SCHOOLS

17/02/2015

T.C.
BÜLENT ECEVİT ÜNİVERSİTESİ
Ereğli Eğitim Fakültesi Dekanlığına

Fakültenizde .... sicil numarası ile araştırma görevlisi olarak görev yapmaktayım. '*Lise Matematik Öğretmenlerinin Matematik Okuryazarlığına İlişkin Görüşlerinin İncelenmesi*' adlı tez çalışmamla ilgili olarak Zonguldak ili Ereğli ilçesine bağlı liselerde 2014-2015 eğitim-öğretim yılı ikinci dönem için görüşme faaliyetlerinde bulunmam gerekmektedir. Bu faaliyetler için ilçe milli eğitim müdürlüğünden izin alınması gerekmektedir. Tez önerisi ile ilgili belgeler ekte sunulmuştur.

Gereğini bilgilerinize ve olurlarınıza saygılarımla arz ederim.

Adres: Ereğli Eğitim Fakültesi
Kdz. Ereğli / Zonguldak   Arş. Gör. Murat GENÇ

Ekler:
Ek-1: Tez Önerisi
Ek-2: Görüşme Formu

C.2: LETTER OF APPROVAL FOR THE RESEARCH FROM THE MINISTRY OF EDUCATION

T.C.
KDZ.EREĞLİ KAYMAKAMLIĞI
İlçe Milli Eğitim Müdürlüğü

Sayı : 88136896/100/2212316 27/02/2015
Konu: Tez Çalışması İçin Gözlem Talebi.

KDZ. EREĞLİ KAYMAKAMLIK MAKAMINA

İlgi: Bülent Ecevit Üniversitesi Ereğli Eğitim Fakültesi Dekanlığının 19/02/2015 tarihli ve 65719904-622/3009 sayılı yazıları.

Bülent Ecevit Üniversitesi Ereğli Eğitim Fakültesi Dekanlığının İlgi yazılarında; Fakülteleri Ortaöğretim Fen ve Matematik Alanlar Eğitim Bölümü Öğretim elamanı araştırma görevlisi Murat GENÇ' in "Lise Matematik Öğretmenlerinin Matematik Okuryazarlığına İlişkin Görüşlerinin İncelenmesi" adlı tez çalışmasını 2014-2015 Öğretim Yılı 2. Dönem içerisinde gözlem ve görüşme faaliyetlerini İlçemizde bulunan Liselerde yapılabilmesi talep edilmektedir.

Bülent Ecevit Üniversitesi Ereğli Eğitim Fakültesi Dekanlığının talebi Müdürlüğümüzce uygun görülmekte ise de;

Makamlarınızca da uygun görüldüğü takdirde Olurlarınıza arz ederim.

Numan KORKMAZ
İlçe Milli Eğitim Müdürü

OLUR
…/02/2015

İbrahim ÇAY
Kaymakam

Güvenli Elektronik İmzalı
Aslı ile Aynıdır
27 Şubat 2015

Devrim Bulvarı Hükümet Konağı Kat 5 Kdz.Ereğli
Elektronik Ağ: www.eregli67@meb.gov.tr
e-posta:210975@meb.k12.tr

Ramazan USLU VHKİ
Tel: (0 372) 3237370
Faks: (0 372) 3237372

Bu evrak güvenli elektronik imza ile imzalanmıştır. http://evraksorgu.meb.gov.tr adresinden fceb-05aa-32b2-b60e-c9d2 kodu ile teyit edilebilir.

CURRICULUM VITAE

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EDUCATION

Doctor of Philosophy, Mathematics Education, Middle East Technical University, Ankara. January, 2017

Master of Arts, Mathematics Education, University of Leeds, Leeds, United Kingdom. November, 2010.

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WORKING EXPERIENCE

Years	Place	Enrollment
2011-2017	Bülent Ecevit University, Zonguldak	Research Assistant

PUBLICATIONS

Gül, Ç., Akademir, Ö., & Genç, M. (2014). Experiences of graduate students: Using Cabri as a visualization tool in math education. *World Journal on Educational Technology, 6*(3), 265-272.

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