# RISK PREMIUM ESTIMATION IN MTPL INSURANCE USING COPULA: TURKEY CASE

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# ABSTRACT

RISK PREMIUM ESTIMATION IN MTPL INSURANCE USING COPULA:
TURKEY CASE

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Motor third part liability (MTPL) insurance has a significant share among non-life or property- casualty insurance businesses in Turkey. Like most countries, it is compulsory while the premium is determined in the competitive market by companies contrary to the few other countries where regulator sets the rates. In this study, the present value of the mean pure premium per policy is estimated based on the simulations using Clayton copula probability distribution function which also defines the dependence structure. We define also development factor, which determines the time required till the ultimate claim settlement takes place. Afterwards, loss severity-frequency method is applied to pure premium obtained using the joint distribution function of claim size and development factor defined by Clayton copula to find the present value of pure premium per policy. In addition, in case of underwritten premium without loadings and taxes are lower than the pure premium, an alternative technical reserve type called, premium risk reserve, is introduced to aid the regulators.

*Keywords* : MTPL insurance, copula, pure premium, premium risk reserve.

# ÖZ

ZORUNLU TRAFİK BRANŞİNDA RİSK PRİMİ TAHMİNİ: TURKİYE UYGULAMASI

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Trafik sigortası Türkiye ve dünyada hayat dışı sigortacılık alanında önemli bir yer tutmaktadır. Birçok dünya ülkesinde olduğu gibi Türkiye'de de bu sigortanın yaptırılması zorunlu olup, bu branşın poliçe primi ise bazı ülkelerde serbest olarak belirlenirken çok az ülkede ise sigorta otoritesi tarafından oluşturulan tarifler ile belirlenmektedir. Bu çalışmada Türkiye sigortacılık alanı zorunlu trafik branşında poliçe başına ortalama risk priminin bugünkü değerinin tahmin edilmesi çalışması yapılmaktadır. Bu amaçla, hayat dışı sigortacılık sektörünün ağırlıklı olduğu zorunlu trafik branşında copula yöntemiyle ortak dağılımları elde edilen hasar büyüklüğü ve hasar gelme süresi rastgele değişkenleri kullanılarak ortalama risk priminin bugünkü değeri tahmin edilmekte ve hasar frekans yöntemi ile de poliçe başına ortalama risk priminin bugünkü değeri hesaplanmaktadır. Ayrıca şirketlerce yazılan poliçe priminin risk priminden düşük olması poliçe kapsamındaki hasarların karşılanmasında rezerlerin yetersiz kalmasına neden olabilecektir. Bunun engellenmesi adına prim riski rezerv çalışması ile ülkemiz teknik karşılıklar mevzuatına ilave bir karşılık eklenmesi önerilmektedir.

*Anahtar Kelimeler*: Trafik sigortası, teknik karşılık, risk primi, copula, risk prim karşılığı

*To My Family*

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I am also thankful to my family for their unlimited supports and understanding. This study could not have been done without their support especially, my dear wife and my elder brother.

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| | |
|---|---|
| $\mathbb{R}$ | Real Numbers |
| IBNR | Incurred But Not Reported |
| MTPL | Motor Third Party Liability |
| LoB | Line Of Business |
| IAT | Insurance Association of Turkey |
| EU | Eurepean Union |
| IMC | Insurance Information and Monitoring Center |
| PVRP | Present Value of Risk Premium |
| DF | Development Factor |
| SCR | Solvency Capital Requirement |
| VaR | Value at Risk |
| CTE | Conditional Tail Expectation |
| $\mu$ | Mean |
| $\sigma$ | Standart Deviation |
| $\rho$ | Pearson Correlation Coefficient |
| $\tau$ | Kendall's Correlation Coefficient |
| $i$ | Risk-free rate |
| t | time |
| $X_i$ | Development factor for $i^{th}$ policy |
| $Y_i$ | Claim size for $i^{th}$ policy |
| UPR | Unearned Premium Reserve |
| URR | Unexpired Risks Reserve |
| OCP | Outstandings Claim Reserve |
| UPR | Catastrophe Reserve |
| URR | Provision for Bonus/Rebates |
| $F(.)$ | Distribution Function |
| $\theta$ | Clayton Copula Parameter |
| $\overline{PVRP}$ | Average Present Value of Risk Premium |

# CHAPTER 1

# INTRODUCTION

An insurer is required by regulation where the company operates to keep certain reserves on its financial statements. Premiums are collected by the insurer at underwriting time while possible liabilities come in the future. Therefore, insurance companies must establish appropriate level of reserves so that they are enough to cover possible future claims.

The monitoring and controlling of compulsory MTLP insurance by the regulator is very important since losses resulting from the uncontrolled or non-actuarial practices in MTPL insurance may deteriorate the insurer's financial stability not only for MTPL branch but also the whole of the company. Therefore, insurers must set aside enough reserves to keep themselves insolvent and regulator must control if the insurers have enough reserves to compensate the policyholders current and future losses in case of a wind-up.

The primary function of MTPL insurance is on behalf of the insured to pay damages to third parties that arise with the use of a motor vehicle because of the fault of the insured. In order the insurer to perform for this in feature timely and continual, he must pay to the users of the insurance a certain amount of MTPL premium [36].

One of the regulatory objectives for premiums is to be adequate, which means that rates charged by insurers should be high enough to pay all losses and expenses. Risk premium, which is the portion of the premium needed to pay losses, should be determined so that the premium must be adequate for insurer and also affordable for insureds [17].

Underwriting refers to the process of selecting, classifying, and pricing applicants for insurance [17]. It is also an important process emphasized in the framework of Solvency II which mainly deals with the risks of underwriting, market, credibility, operational and liquidity risks. Underwriting risk is measured by dividing written premium to technical reserve of each firm and it determines risk of the insurer by issuing policies. And, market risk considers the financial assets of the company and its investment incomes [32]. Solvency II provides practical tools to evaluate the Solvency Capital requirement (SCR) for insurance companies in order to manage their risks (reserve risk, premium risk, catastrophe risk etc.). Two models were proposed: the standard model and the internal model. These models take the advantage of evaluating liabilities stochastically, contrary to Solvency I, which evaluates liabilities with deterministic methods [5].

Risks that insurance companies are exposed to and are necessary to identify, monitor, track and manage appropriately are numerous. One group of such risks are underwriting risk arising from the policy that is taken under coverage and is guaranteed the payment of claims. The risk of any insurance policy is that a risk event covered by the policy will occur and the under certainty about the amount of losses which arise from that event. Underwriting risk refers to the risk that could arise if the actual claims and indemnity exceed the net book value of the insurance liability due to the accidents, errors and/or changes in circumstances. It includes the risk of determining the premium (pricing risk), the risk of setting reserves (reserve risk), reinsurance risk and occurrence risk [23].

Compulsory MTPL insurance is a long-tailed distribution in actuarial analyses. In other words, claim settlement between policyholder and insurer may be finalized years after the loss occurrence mostly because of the lawsuits. For this reason, ultimately settled losses are taken into account in risk premium calculation as well as the duration that it takes till the final claim settlement is done.

In MTPL insurance, the claims are mainly divided into two part as property damages and bodily injured claims. Guarantee limits of the property damages are low, but the loss frequency is higher; at the same time, guarantee limits of the bodily claims are high, but the loss frequency is lower.

In long-tailed insurances like MTPL insurance, insurer will have a discounting advantage in a high-return investment environment when premium estimation calculation considers loss payment duration, in other words cash out-flow time. Besides, the estimation of cash duration distribution may increase the efficiency of cash management and may enable the reserve actuary to calculate reserves more truly.

## 1.1 Literature Survey

In the literature, there are many methods to calculate the risk premium of the insurance product. One of them is distance-based insurance pricing using telematics and GPS systems on auto insurance. Vickrey (1968) [39] first proposes pay as you go premium then followed by studies done by Bordoff and Noel (2008) [7], Litman (2005) [26], Boucher et al. (2013) [9], Ayuso et al. (2016) [1] and Ippisch (2010) [22].

There are vast amount of studies in parametric claim modeling and pricing on MTPL. Many of those take into account linear and nonlinear models to estimate the risk premium and aggregate losses. A crucial assumption of the classical compound Poisson model of Lundberg (1903) for assessing aggregate losses incurred in an insurance portfolio is the independence between the occurrence of a claim and its claims size. However, this independence assumption may not always hold. Song et al. (2009) [34] study dependence between the number of claims and its corresponding average claim. Gschlößl and Czado (2007) [19] use full bayesian approach to analyze a car insurance data set assuming dependence between claim size and number of claims.

In Song (2000) [41] a large class of multivariate dispersion models are constructed by linking univariate dispersion models (e.g., Poisson, Normal, Gamma) using a Gaussian copula. These models are marginally closed, their marginals belong to the same distribution class as the multivariate model and readily yield a flexible class to model error distributions of generalized linear models (GLM's) which are widely used for actuarial data modeling. For an overview and discussion of several applications Haberman and Renshaw (1996) [20] build a model for premium rating in non-life insurance using models for average claim size and claim frequency. A more detailed analysis on this issue can be found in Renshaw (1994) [31] who considers the influence of covariates on average claim size and claim frequency. Taylor (1989) [35] and Boskov and Verrall (1994) [8] fit adjusted loss ratios with spline functions and a spatial Bayesian model, respectively. However, Boskov and Verrall (1994) conclude that the separate modeling of claim size and claim frequency is preferable. Based on the compound Poisson model, Jørgensen and de Souza (1994) and Smyth and Jørgensen (2002) [24] use a non-separate approach to model the claim rate. On the other hand, Dimakos and Frigessi (2002) [13] model claim size and claim frequency separately, but rely on the independence assumptions of the classical model by Lundberg (1903). Gschlößl and Czado (2007) relax this assumption by allowing the number of claims to enter as a covariate into the model for average claim size.

Copula as a tool for modeling different dependence structures has been broadly applied to different fields ranging from finance, insurance to environmental studies. The study done by Czado at al (2012) [11] presents a mixed copula approach suggested by Song at al (2009) [34] to measure for dependence between the number of claims and its corresponding average claim size using Gaussian copula. Erntell [14] uses copula approach to model the number of claims made by a customer who bought three insurances. Copulas are also widely used in the modeling of dependency structure between insured risks in Solvency 2 internal model calculation. Frees and Valdez (1998) [15] propose copula function to measure dependence between risks of insurance, and to evaluate the loss of life mortality, the loss of adjustment expenses and reinsurance contract pricing. Moreover, Frees and Wang (2006) [16] use copulas for estimating the credibility of aggregate loss. In addition, Kaishev and Dimitrova (2006) [25] show the importance of copulas in reinsurance. Furthermore, Belguise and Levi (2002) [4], Faivre (2002) propose that the model with copulas allows for an aggregation of risks and evaluate a capital higher than when assuming independence. In addition, Barges et al. (2009) [2] evaluate the capital allocation for the overall portfolio using the TVaR as a measure of risk and a copula. Zhao and Zhou (2010) [42] apply semi parametric copula models to individual level insurance claims data to forecast loss reserves. Shi and Frees (2011) [33] investigate the aggregate loss reserving data with bivariate copulas and linear models. Besides, Diers et al. (2012) [12] use Berntein copula to model a several lines of business. All studies mentioned above consider copula functions as a powerful tool to resolve the problem of dependence between insured risks.

In general, MTPL premium to be charged by insurer is calculated based on bonus-malus or rate making systems which take into account the past claim history of driver, state, miles driven, gender of the insured. But to our knowledge, there are very limited number of study about premium calculation on MTPL insurance using claim settlement time and claim size dependency structure captured by copula method.

However, distributional assumptions on claim modeling may not cope with the realized data, as the structure of losses is long-tailed and complicated due to the IBNR claims. Pettere and Collo (2006) [30] and Weke and Ratemo (2013) [40] use copula to estimate IBNR reserve.

Pettere and Collo (2006) [30] estimate the IBNR reserves using copula on twelve hundred claim observations of a Litvaian insurance company. Copula is used to determine the joint cumulative probability distribution of claim size and development factor which measures the time between claim reporting and settlement time. They study the most appropriate copula for the sample data among many copulas and conclude that the Clayton copula is the most appropriate for such data. Then IBNR reserves are estimated using the bivariate Clayton copula. Pareto, t-distribution, lognormal and Wald distributions are tested to find best fit distribution to the sample data and best fit distribution for claim size variable is obtained with the lognormal distribution. The development factor ranging from 0 to 1100 day is found to follow a lognormal distribution. Correlation coefficient obtained by Kendall's $\tau$ between claim size and development factor is found to be 0.3. Having tested many copulas to determine the best fit copula for sample data, three classes of Archimedean copulas are examined and among them best fit is obtained by Clayton copula. After that, with the estimation on number of claims, IBNR reserves is estimated through claim size and development factor joint distribution is found by Clayton copula. Usta (2016) [38] implements the methodology used in the Pettere and Collo's study to Turkish insurance data consisting of 760 ultimately settled losses. Different from the Pettere and Collo's study, the sample data is reorganized to include only claims that makes IBNR. Also, development factor is measured in years and appropriate discrete distributions are investigated. Logaritmic distribution is found to be the appropriate for this variable, while claim size variable is modeled with lognormal distribution. Kendal's $\tau$ statistics as a measure of correlation between variables is found to be 0.36 and statistically significant. In his work, Clayton copula is also found as the most appropriate to determine the cumulative joint probability distribution. In the IBNR estimation part of the study, number of losses that would come in future periods are estimated using past claim data and finally corresponding IBNR reserve is estimated.

### 1.2 Aim of The Study

The main motivation behind this study is the lack of the information of adequate level of premium for MTPL insurance in Turkey for the regulator, which may result in the insurer to set aside inadequate reserves to cover his possible future losses. Turkey MTPL insurance is examined since the financial and technical structure of this type of insurance in Turkey highly differ from those in emerged countries. As it will be stated later, insurance companies operating in MTPL insurance have been facing high technical losses on their financial statements over years, mostly because of the inadequate level of premium charged. As a consequence of this situation, average premium level have been increased dramatically as of the second half of 2015 in Turkey. But this sharp increase in prices questioned the affordability of this insurance, created social chaotic situation among customers and resulted to be a politic issue since millions of people who are going to buy this insurance are affected. Having this experience, the regulator should take the necessary precautions for premiums not to be higher than affordable in the following years, since the rate of uninsured may increase which is the opposite of the mandatory insurance concept. But also, the regulator has to be sure that the amount of policy premium is enough for insurer not to face difficulty on payment of losses. Besides, reserves should be enough in case of wind-up of insurer where the regulator takes place of the insurer and pay losses. If the insurer does not set aside enough reserve to cover future losses, the regulator and the government pay the losses from its own sources. Therefore, the inadequate premium charged by the insurer not only affects its own financial situation but also the whole economy at the end of the day. Therefore, it's crucial to determine adequate level of premium and determine the amount of premium risk reserves.

For the reasons stated above, this thesis aims to set a methodology which quantifies adequate level of premium and amount of reserves per policy in case of the premium is underestimated compared to the risk it is associated with. It also propose an equilibrium set an additional indicator for the regulators to consider measuring the technical reserves in Turkey.

To achieve this aim, we estimate the present value of risk premium per policy (PVRP) which is expected to be sufficiently enough to cover the risks of both parties, insurer and regulator. The required components in estimating the target indicator are jointly analyzed using Clayton copula. Employing simulation analyses on the parametric distributions of claim size and modified development factor and a regulatory risk-free rate PVRP is estimated which is taken as an input for risk premium reserve estimation. Mostly acknowledged risk measures such as value at risk (VaR) and conditional tail expectation (CTE) are implemented to determine critical values of PVRP whose results are utilized to find the required capital to be invested for attaining the additional risk capital for solvency capital requirement calculations. Based on the historical data collected from Insurance Information and Monitoring Center (IMC, SBM in Turkish), the analyses of the methodology are performed using Matlab and Easy Fit.

This study is inspired from the studies done by Pettere and Collo (2006) [30] and Usta (2016) [38]. However, it differs in the sense that the common definition of development factor is extended to the duration between original year and the ultimate loss settlement and contribute to the literature as it formulates the possible future claims in terms of the contractual obligation. It contributes to the literature in the aspects of setting an equilibrium risk premium indicator for all parties in concern.

The outcomes of the study can be utilized by both parties to have a control on the probability of the ruin especially for the newly operating insurers having their business heavily in MTPL insurance which has a recognizable market share in insurance business in Turkey.

The rest of the study is organized as follows. Chapter 2 presents a brief review of the Turkey Insurance market especially on MTPL branch and Turkey technical reserves regulation. Chapter 3 describes the components, their statistical properties and copulas in modelling the proposed indicators. Substantive empirical results based on the data analysis are presented and discussed in Chapter 4 and Chapter 5 finalizes the study.

# CHAPTER 2

# INSURANCE MARKET FACTS IN TURKEY

Before going into compulsory MTPL insurance concept, it is useful to mention the definition of risk. Economists, behavioral scientists, risk theorists, statisticians and actuaries each have their own concept of risk. However, risk historically is defined in terms of uncertainty. Based on this concept, risk is defined as uncertainty concerning the occurrence of a loss. Because the term risk is ambiguous and has different meanings, many authors and corporate risk managers use the term "loss exposure" to identify potential losses. A loss exposure is any situation or circumstance in which a loss is possible, regardless of whether a loss occurs [17].

Much actuarial research in recent years has focused on the solvency of the insurance companies. Indeed, insurance companies must have a level of liability (equities and technical reserve), which allows it to be solvent in future years. Historically, the insurance companies were sufficiently capitalized compared to their engagements. However, the markets were controlled and less volatile. Furthermore, the correlation between the risks associated to the insurance business was not explicitly considered. Recently, the claim frequency and severity are increased and the legal framework became more uncertain. For example, The Lothar storm in 1999, and the disaster of the World Trade Center in 2001, were kept responsible for an explosion of the number of insolvency, and led to an exceptional conjunction with the disasters within the most various lines of business: damage, catastrophes, industrial accidents, trading losses, civil responsibility. Also, some situations are observed almost every day, as for example, in auto insurance, accidents may involve several insured at once in a collision. These events proved that the risks of insurance could be dependent, and this dependence can bind liabilities or assets of insurance. Thus, control of risk in terms of the dependence has become essential [5].

To guarantee the risks such as life, health or pension that bear upon the whole society or referring to the general part of society generated the social insurance concept, on the other hand, the risks that covers the individuals' own security generated the private insurance. From this point of view, automobile insurance is a type of private insurance. The private insurances, the contract to compensate the financial loss against risks that the real or legal entities carry, can be divided into two through the will for setting up insurance relation [10];

(i) Facultative Insurance: The contract is not compulsory and decided with free will.

(ii) Compulsory Insurance: The contract is made as a result of legal obligation.

To mention about the history of traffic insurance shortly, the drivers and the third parties are affected recognizably as a result of increase in the usage of automobile after the industrial revaluation. The difficulty in investigation on the faulty party in an insurance claim and financial affordability of the car owners triggered the countries to make legal regulations. The thought of social benefit underlying these regulations, protect the third parts against the material results of car accidents [10].

MTPL business plays key role in insurer's portfolio running on the non-life insurance. Like many countries across the world, motor liability insurance is mandatory in Turkey and provides compensation for property damages and bodily injuries caused by vehicles on highway.

The legally ending period of responsibility of the companies derived from the policy should be taken into account in insurance premium calculations. Because, even compensation notice is not reported during the valid period of the policy, the claim for compensation for the loss occurred during the valid period can be reported later. IBNR reserve is set apart by insurers to cover such claims. In relation to the protection period covered by the MTPL policy in Turkey, insurers are obligated to compensate losses from MTPL policy for a ten-year of time period enacted by Highway Traffic Law numbered 2918.

In the next sections of the thesis, as seen in the statistical indicators for insurance market of Turkey, the most part of total premium production is generated on non-life area. The share of compulsory traffic branch is very high in non-life branches. At the same time, for the loss compensation, since legal procedures take long time and the problems in specifying the price of loss compensation arise, MTPL insurance has a basis to produce IBNR. In summary, because of having respectively huge portion in non-life insurer's portfolio and enabling insurer to set aside huge amount and volatile IBNR reserves, our study is focused on MTPL insurance.

Since the study suggests regulator to put a level of premium per policy in MTPL insurance, which is mainly related to tariff regime in MTPL insurance, we give short information about the tariff regimes applied in Turkey.

Tariff is a study that informs about variables regarding to pricing a insurance production. Various tariff systems are implemented to specify the level of premium and indemnity in insurances. Tariffs are divided into two systems as free or compulsory according to inclusion of public authority. While there is no effect of public authority for the insurances that free tariff regime is valid in specifying the premium and indemnity, the public authority is effective when the compulsory tariff regime is valid [10].

Considering the tariff liberalization in Europe, in all member states, MTPL is mandatory for all users of motor vehicles. This class of insurance in EU, as a whole, is regulated under the five Directives (72/166/EEC, 84/5/EEC, 90/232/EEC, 2000/26/EC, 2005/14/EC and changes in 2009/103/EC) which give guidance on the approximation of the laws of the member states relating to insurance against civil liability in respect of the use of motor vehicles, and three Council Directives (73/239/EEC, 88/357/EEC and 92/49/EEC) with which the coordination of laws, regulations and administrative provisions, are brought out [36].

The market for MTPL insurance in the Member States of the EU is deregulated, mainly in the period from 1968 to 1994. Due to the complexity of this change, the insurance industry applies several necessary steps and fully coordinating the laws, regulations and administrative provisions before passage of the liberalized market for MTPL. EU Directive 88/357/EEC and 92/49/EEC with amendments, affirm the principle of free formation of tariffs for MTPL. Before this date, in 1984 the freedom to set prices in the insurance sector was established in Spain. In the United Kingdom, tariffs have not been subject to any type of control for several decades. In Germany, before 1994, tariffs were approved on the basis of the previous year's technical results of marginal companies, with the objective of sustaining of their existence in the market. Complete liberalization was achieved in 1994. Italy was the last major Western European country to move toward a liberalized regime following the EU directive in 1994 [18].

One of the biggest problems facing insurance companies in transition to a liberalized market, is the challenge to establish a fair price for them, and that is not so easy due to lack or complete absence of reliable data. MTPL in case of state regulated market has created system with common data base. In most cases, before switching to a fully liberalized MTPL market, the State is offering the insurers a set of allowed charges that may participate in the market [36].

To give an idea for the maximum limit that policyholder experiences by buying third party insurance, it is useful to mention indemnity put forward by the regulator. Indemnity types bringing with the third party liability insurance according to the General Conditions of MTPL regulation for personal vehicles as to per accident and per person (in TL) as of 2015 is listed below (Official Gazette dated 14 May 2016,numbered 29355);

a. Property Damage Indemnity ( $12,375 - $24,751 )

b. Health expenses indemnity ($123,757 - $1,237,570 )

c. Permanent disability indemnity ($123,757 - $1,237,570 )

d. Loss of support (death) indemnity ($123,757 - $1,237,570 )

## 2.1 General Overview of Turkey Insurance Market

Among 60 insurance, reinsurance and pension companies operating actively in Turkish Insurance Sector. 56 of them are corporate companies, two are mutuals companies and two are branches of international companies. The number of active companies was 63 at the end of 2014, declined to 60 at the end of 2015 due to two cancellations of licenses and one company exiting from the sector by ceding of its portfolio [29].

Turkish insurance sector, in line with the economic development has shown a significant progress in recent years. During 2015, the sector achieved 31.1 billion gross premiums with a 19.5% increase in nominal term and 11.2% increase in constant prices of 1998 compared to previous year. At the year, non-life written premiums increased by 11.8% and life premiums increased by 6.7% in real terms. Experiencing a premium growth in real terms, the sector also continues to grow rapidly according to the indicators showing the importance of the industry. The sector issued 69.9 million TL policies and provided 86.1 trillion TL insurance coverage in return for premium payment during the year. The sum of the coverage is about 44 times higher than the GDP, and indicates the importance of the insurance sector for the Turkish economy and steady growth. The sector has expanded its contribution to the employment, as well. In the Sector as of 2015, 60 insurance, pension and reinsurance companies operated, of which 36 have been non-life insurance, 4 have been life insurance and 19 have been pension and one has been reinsurance companies. Four companies, two of which non-life insurance, one life insurance and one reinsurance companies have withdrawn from the Sector and placed their business into run-off. Insurance companies have generated 30.3 billion TL of gross premium as a result of direct insurance activity, while 759 million TL was produced through reinsurance activities. Non-life insurance has dominated the market for years and produced substantially higher premium than life insurance. Non-life insurance business has generated 87.9% of gross premium in 2015 and life insurance has just produced 12.1%. While life premiums share in total premiums was about 20% in 2003, with the start of private pension system, it went down and experienced with 12% in 2007. Due to life insurance related to the individual loans, it had risen since 2008 but fall again to the level of 12% in 2015. The number of insurance policies issued by the companies in non-life insurance in 2015, almost 30% of them is belonged to motor third party liability insurance and 10% of them is issued in motor own damage insurance. On the other hand, gross premium of motor vehicle insurance has 20% share of the gross non-life premium. Given the non-life insurance, 80% of the number of policies issued and 87% of gross premium have been achieved by five LoB (motor vehicles, motor vehicles liability, illness / health, fire and natural disasters, general damages) as shown in Figure 2.1. The share of non-life gross premium per each line of business also illustrates the high influence of Automobile insurance in market [29].

### 2.2 Motor Vehicles Liability Insurance

Motor vehicles liability insurance accounted for 28% of direct premium volume in non-life insurance. 32 insurance companies have been operating in the business and issued 16,561,983 policies during 2015. The number of policies including land vehicle liability coverage rose to 21,731,298 along with the voluntary third party liability coverage and compulsory passenger transportation insurance given within motor own damage line of business. 99,178 of them were in compulsory passenger transportation insurance, 5,393,338 were in voluntary third party liability insurance, 108,657 in green card insurance and the rest 16,130,126 in compulsory motor third party liability insurance. Compulsory MTPL insurance which is sub LoB of motor vehicle liability insurance accounted for approximately 74% of the policies issued in motor vehicles liability insurance in 2015. The share of the sub LoB in direct premium volume and claim payments are 92%and 97%, respectively. Voluntary motor third party liability insurance accounted for 25% of the total number of policies issued in that line of busi-

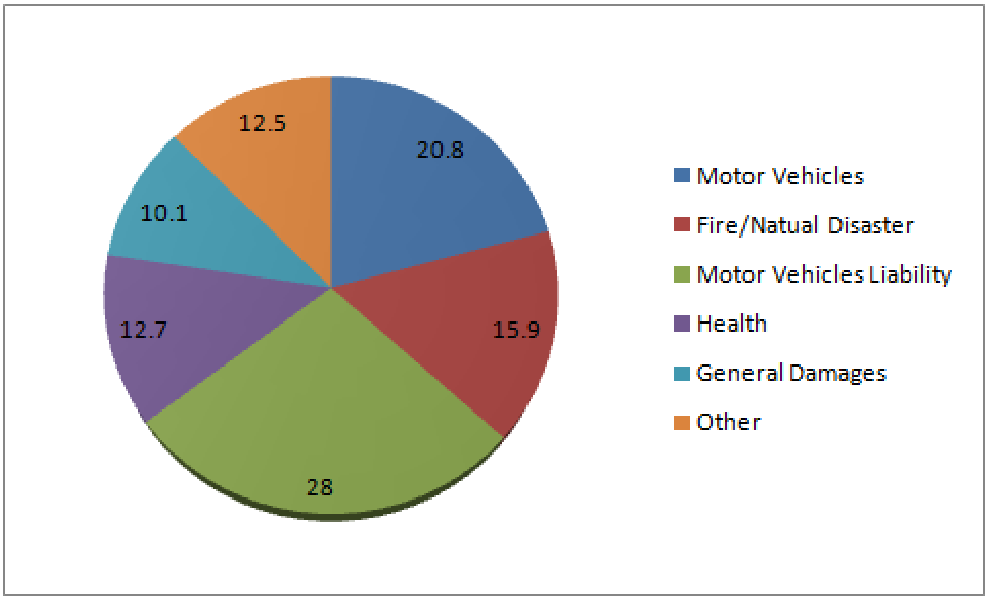


Figure 2.1: Direct Non-life premiums share by line of business as of 2015 in Turkey [29].

ness. However, its share in premium volume and total claim payments were 6.3% and 1.6%, respectively [29].

As for the technical results over the last five years shown in Table 2.1, gross premium has increased significantly until 2013. Increase rate of written premium stayed on 2% in 2014, but in 2015 increase rate reaches to the previous years' rates. However, 46% rise in incurred loss affected the technical profitability more adversely according to the previous year.

Table 2.1: Technical Results of Turkey Insurance Market between 2011 and 2015 [29].

| (Thousand USD) | 2011 | 2012 | 2013 | 2014 | 2015 |
|---|---|---|---|---|---|
| Premium Income | 1,390,954 | 1,976,578 | 2,286,886 | 2,132,061 | 2,341,655 |
| Intermediary Commissions | 244,132 | 326,251 | 359,164 | 33,262 | 351,963 |
| Ceded Premiums | 95,346 | 84,224 | 58,693 | 38,857 | 68,889 |
| Earned Premiums (Net) | 10,671,679 | 1,427,563 | 1,604,802 | 1,766,296 | 1,548,311 |
| Paid Losses | 1,167,145 | 1,369,772 | 1,229,520 | 1,505,032 | 1,477,650 |
| Outstanding Claims | 901,546 | 1,363,695 | 1,706,851 | 2,010,667 | 2,342,542 |
| Claims Incurred (Net) | 1,080,732 | 1,683,320 | 1,703,932 | 1,849,114 | 2,146,679 |
| Operating Expenses | 365,636 | 474,667 | 478,124 | 515,921 | 434,901 |

The liability side of the balance sheet is mostly populated by the amount aggregated from the technical reserves. As seen from Table 2.1, unearned premium reserves which has increased by 35% has 41% share in technical reserves accounts. Unearned premium reserves which has second biggest share in provisions has risen by 23% and reached to 39% of provisions. Although profit loaded actuarial reserves constitute 14% of the amount in total, its share in provisions has been decreasing for recent years.

Technical results of the MTPL insurance shown in Figure 2.2. Considering the last five years, loss ratio which is gross underwritten premiums over incurred losses has been over 100% reaching the top level at 2015 with 129.28%. The result shows actuarial imbalance position of insurers with respect to their claim payments. When adding the expense ratio consisting of the personal expenses, intermediary commissions the combined ratio goes up above 120%. The results of technical ratio clearly show that technical losses on the MTPL insurance mainly result from non-actuarial prices charged by insurers. High competition in this insurance may pull the premium its adequate level that is necessary to compensate aggregate losses to the level below actuarial risk premium. In this case, it is almost impossible for the insurer to keep its business solvent position injecting extra capital.

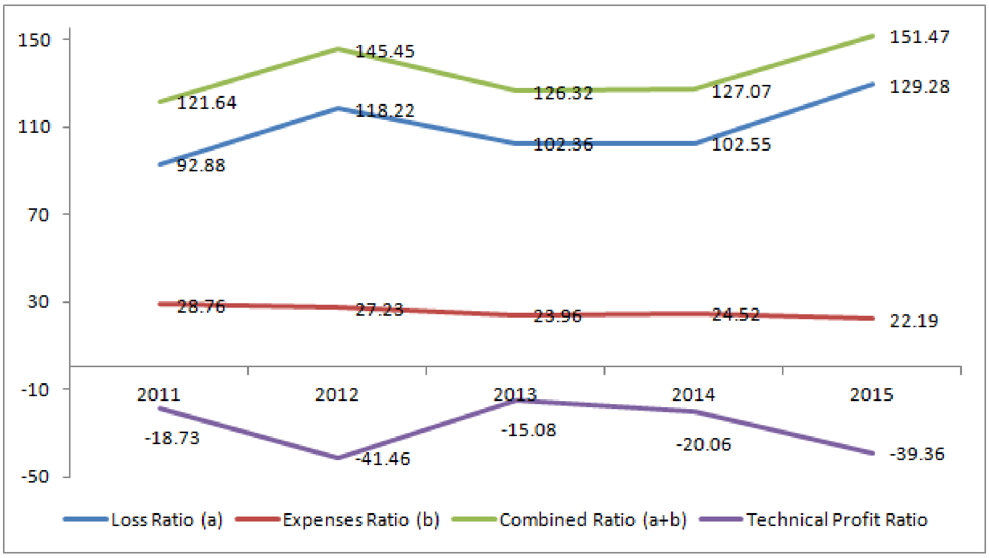


Figure 2.2: Technical Ratios (Gross,%) in MTPL Business in Turkey [29].

Having realized the deteriorating effect of the MTPL insurance for the portfolio as a whole, insurance companies raised immensely premiums collected from customers in recent years. Figure 2.3 shows that the average premium per policy has dramatically risen to the highest historical level even there is not any structural breaks over years like inflation rates, economical conditions or supreme court decisions.

As of the end of 2015, MTPL insurance has a share of 28% among non-life insurance, which is the first line of business as to market share. In Turkey, 32 insurance companies have been operating in the business and issued 16,561,983 policies during 2015. The number of policies including land vehicle liability coverage rose to 21,731,298 along with the voluntary third party liability coverage and compulsory passenger transportation insurance given within motor own damage line of business. Compulsory MTPL insurance which is sub LoB of motor vehicle liability insurance accounted for approximately 74% of the policies issued in motor vehicles liability insurance in 2015. The share of the sub LoB in direct premium volume and claim payments are 92% and 97%, respectively. Voluntary motor third party liability insurance accounted for 25% of the total number of policies issued in that line of business, however its share in premium volume and total claim payments were 6.3% and 1.6%, respectively.

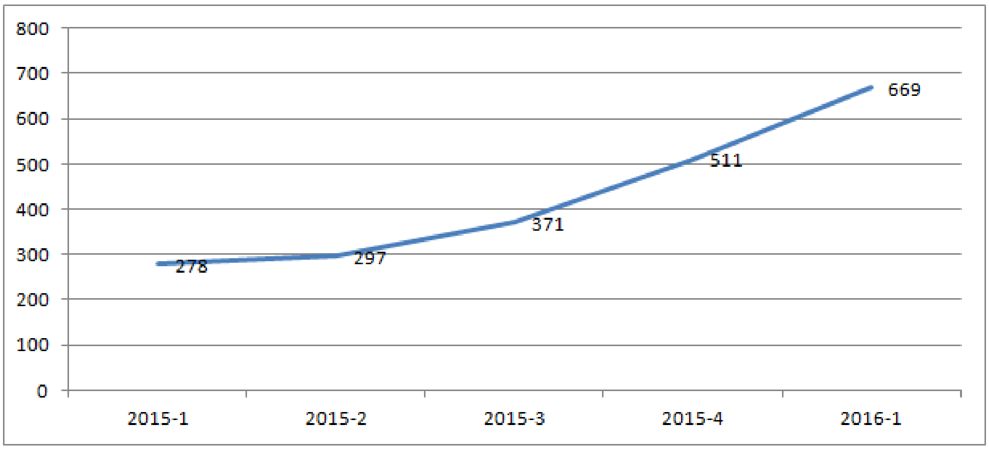


Figure 2.3: Average written gross premiums per policy (TL) in 2015 and 2016 (IMC).

In general, the premium calculation consists of three stages; firstly, pure premium, which aims to estimate the aggregate losses, is determined, secondly loadings including personal expenses, commissions, share holder's expected rate of return on capital are added and finally with the addition of taxes, the gross premium calculation is completed. From the regulatory objective of the premiums, the rates charged by insurer's should be adequate enough to cover the aggregate losses but should not be excessive for customers not to be able to purchase the policy since MTPL insurance is mandatory [17].

## 2.3 Technical Reserves in Turkish Insurance Market

It will be useful to give a short information about the technical reserves of Turkey insurance market as a preparation to the reserve part of the study. According to the article 16th of Insurance Law numbered 5684 insurance and reinsurance companies are obligated to set technical reserves arising from insurance policies. Also in the same article, technical reserves are determined and specifically calculation methods and descriptions for these reserves left to sub regulations to be explained in detail.

The technical reserves that must be set by insurers are:

i. Unearned Premiums Reserves (UPR)

ii. Unexpired Risk Reserves (URR)

iii. Outstanding Claim Provision (OCP)

iv. Profit Loaded Actuarial Reserves (PLAR)

v. Catastrophe Reserve (CR)

vi. Provision for Bonus/Rebates (PFB)

UPR, URR, OCP and PLAR are the dominating part of the liabilities of insurers on financial statements. Between 2009 and 2015, the share of the technical reserves over total liabilities lies between 31% and 40%, a total of 29,94 billion TL have been set aside by insurers [38].

The liability side of the balance sheet is mostly populated by the amount rosed from the technical reserves. As seen from Figure 2.4, UPR which has increased 35% has 41% share in technical reserves accounts. Unearned premium reserves which has second biggest share in provisions has risen 23% and reached 39% of provisions. Although profit loaded actuarial reserves constitute 14% of the amount in total, its share in provisions has been decreasing for recent years.

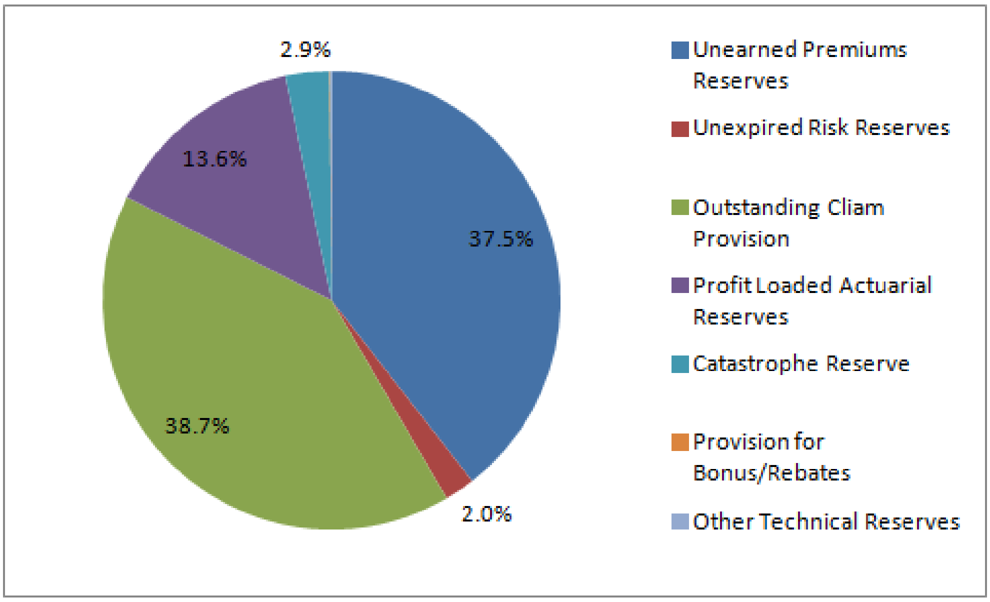


Figure 2.4: Share of Technical Reserves on Liabilities [29].

The distribution of technical reserves as of end of 2015 illustrates that, unearned premium reserves has share of 37.5%, outstanding claim provision has 38.7%, profit loaded actuarial reserves has 13.6% and the rest for other reserves.

The premium risk reserve study suggested in this study is expected to fulfill the deficiency of the unexpired risk reserve calculation. According to the current technical reserve regulation, unexpired risk reserve is calculated based on loss ratio. The insurer has to populate this reserve if it's incurred loss ratio exceeds 95%. But as mentioned previously, loss ratio method does not adequately measure the pricing risk and may result in unexpected consequences. We can say that there is no reserve type in the regulation to guarantee that insurer will have enough reserve to cover future losses in case of inadequate level of premiums. For these reasons, a proposed methodology implementing copulas to measure the dependence by fitting statistical distribution to the claim amounts is presented whose results are expected to lead practitioners and regulators to find the equilibrium premium rates.

# CHAPTER 3

# METHODOLGY

Loss frequency and loss severity method is mostly used to determine the pure premium in the actuarial literature. This method is consisted of two components, one is claim frequency and the other one is claim severity. The determination of the claim severity distribution plays key role in calculating the risk premium since the parameters derived from this distribution enables us to determine statistics and probabilities. For MTPL insurance, especially bodily injured claims, claims keep coming even long after underwriting time with excessive claim size compared to property damage claims. Additionally, the duration till the claim file is closed may need longer time than expected due to the reason stated above. The claim amount and the time it is completely paid show positive correlation. As settlement duration increases the amount of the claim also increases. This seems logical since the most of the claims with larger amounts in MTPL branch result from bodily injured losses.

When determining the underwriting based average risk premium per policy, the estimation of ultimate loss frequency is also required to be estimated. Loss frequency simply is the estimation of the number of claims over the policies written within a certain time period. For instance, it is expected to have 10 claims from 100 policies when the expected loss frequency becomes 0.1. Ultimate loss frequency defines the total number of claims within the ending time of the policy, which is legally ten years in Turkey for MTPL insurance. The policyholder buys an MTPL policy which mostly provides a protection for one year, but the liability of the insurer from the policy lasts much longer. Therefore, this fact is considered in ultimate loss frequency calculation. Gross premium per policy is finalized after adding tax and compensation costs like personal costs, intermediary commissions and share holder's rate of return expectation to the risk premium.

Risk premium estimation method in MTPL insurance is developed based on underwriting year because it is mandatory to be purchased by all drivers. Insurer's past financial losses or increasing outstanding reserves should not negatively affect the newly licensed drivers.

## 3.1 Components of Premium Estimation

In the premium estimation part of the study, claim size and development factor variables are taken into account.

As it stated previously that MTPL insurance claims mainly divided into two parts as property damages and bodily injured claims. These two claim types have different claim settlement times with different amount of losses. Therefore in order to maintain a more accurate analysis in the estimation of risk premium, these two claim types should be analyzed separately. However, the sample data used in this thesis, does not contain the claims recorded according to this specifications. Therefore, the main shortcomings of the study is to combine these claim types in a single claim analysis.

### 3.1.1 Development factor

Development factor (DF) variable is designed to encapsulate the time between underwriting of the policy and the ultimate loss settlement time measured in daily basis. For example; for a one year policy on third party liability insurance underwritten in 01/01/2010 yielded an claim with amount of 9,500 TL at 06/30/2010, reported to the insurer at 09/30/2011 and ultimate settlement loss was paid and closed by the insurer with amount of 12,500 TL at 06/30/2012. In this case, development factor is equal to 900 days.

The reason for this variable is measured in daily basis is to compile with the current unearned premium risk reserve calculation according to the technical reserve regulation since unearned premium risk reserve is also calculated in daily basis.

### 3.1.2 Claim Size

Claim size variable defines the value of ultimately settled and paid amount to the insured.

In general, claim process for insurer after an accident happened starts with the reporting of the claim to the insurer. Following the opening of loss file record of an accident, estimated loss amount based on historical claim size statistics is considered. The actual value of the claim is only be determined by the expert report for the property damages and actuarial report for bodily injured claims. Having evaluated the expertise report, the insured or third party related to insured is compensated by the insurer. But in some cases, some part of the claim amount is paid separately at different dates. Therefore, we use the final settlement amount as the sum of all payments made by insurer with respect to the claim.

Modeling the joint probability distribution function of these random variables are done by Clayton copula method based on an earlies study. Usta (2016) find that Clayton copula is the best fitted copula to Turkish MPTL claims among bivariate Archimedean copulas by comparing the estimation results with realized data. The mathematical background of copula method is given in the following section in details.

### 3.2 Copulas

In many practical applications researchers are often required to analyze multiple risks of the same group, similar risks from different groups, or different aspects of a risk group. Thus, techniques for modeling multivariate distributions are required. While modeling the joint distribution directly may be an answer, this approach may face some difficulties in practice. First, there may not be an appropriate standard multivariate distribution that is suitable for the problem at hand. Second, the standard multivariate distributions usually have marginals from the same family, and this may put severe constraints on the solution. Third, researchers may have an accepted marginal distribution that fits to the data, and would like to maintain the marginal model while extending the analysis to model the joint distribution. The use of copula provides a flexible approach to model multivariate joint behaviors. [37].

Copula is used to model cumulative joint probability distribution of more than one random variables expected to be correlated with each other. There are many forms of copulas in the literature and these are generally investigated under "copula families". For example, Gaussian copula, one of the family of copulas, is based on multivariate normal distribution. When using Gaussian copula, one needs to check whether variables considered are normally distributed. Archimedean copulas, another copula family, create dependency structure with one parameter and indifferent from the distribution of random variables. Frank, Gumbel and Clayton copulas are from the Archimedean copula family. Every copula has a unique mathematical form and the aim is same; to obtain a closed form of the joint probability function.

The use of copulas in applied mathematics fields like finance, insurance and credibility theory has an increasing popularity, especially in the last 20 years. After the regulations brought by Basel and Solvency on banking and insurance sectors, the use of copulas gain importance [6] .

While copulas are fundamental to modeling dependence, covariance is more often used in discussion of risk. The American Academy of Actuaries reports similarly use the term covariance, but probably do not intend to exclude other kinds of dependence. The covariance concept cannot capture all aspects of dependence, but it seems that the term covariance is often used in a general, nontechnical way to describe dependence in general [3].

Assume that $X_1, X_2, \dots, X_n$ are random variables with distribution functions, $F_X(x)$

$$F_1(x_1) = P(X_1 \leq x_1), \dots, F_n(x_n) = P(X_n \leq x_n),$$

respectively, and a joint distribution function of the variables is defined as

$$H(x_1, \dots, x_n) = P[X_1 \leq x_1, \dots, X_n \leq x_n].$$

The joint distribution function of the random variables contains both a description of the marginal behavior of the individual variables as well as information about the dependency structure between them. Copulas allow for a bottom-up approach, separating the marginal distribution from the dependence structure and modeling these separately. The flexibility of choosing marginal distributions free of choice and the extensive collection of copulas with various properties give the ability to model joint distributions at a deeper level. Additionally, copulas allow random variate generation using the properties of uniform inverse transformation. Therefore, once the type of copula is determined, Monte Carlo simulation enables us to generate variates from the specified distribution easily [6]. The essentials of copulas in studying the multivariate distribution functions are presented in the following theorem.

**Theorem 3.1** (Sklar's theorem [21]). *Let $H$ be a joint distribution function with marginals $F_1, \ldots F_n$. Then there exists a copula which is mapping the unit hypercube into the unit interval, $C : [0,1]_n \to [0,1]$, such that for all $x_1, \ldots, x_n \in \mathbb{R}$.*

$$H(x_1, \ldots, x_n) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)) \tag{3.1}$$

*If the marginals are continuous, then $C$ is unique; otherwise, $C$ is uniquely determined on $RanF_1 \times RanF_2 \times, \ldots, \times RanF_n$ where $RanF_i = F_i(\mathbb{R})$ denotes the range of $F_i$, $i = 1, 2, \ldots, n$. Conversely, if C is a copula and $F_1, \ldots, F_n$ are univariate distribution functions, then the function H is a joint distribution function with margins $F_1, \ldots, F_n$ [27].*

*A 2-dimensional copula is a function C : $I \times I \to$ I with the following properties:*

*i. For all $x, y \in I$,*
*C(u, 0) = 0 = C(0, v) and C(u, 1) = u and C(1, v) = v.*

*ii. For $u_1, u_2, v_1, v_2 \in I with u1 \leq u_2 and v_1 \leq v_2$,*
$C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0.$

*The condition (ii) above is equivalent to the non-negativity of the probability associated with $[u_1, u_2] \times [v_1, v_2]$. In high dimensions, the analogous condition is much more complicated.*

We will also use the fundamental result in the theory of copulas, Sklar's theorem [28]:

If $F(x, y)$ is a joint distribution function with marginals $F_1(x)$ and $F_2(y)$, then there is a copula $C(u, v)$ such that $F(x, y) = C(F_1(x), F_2(y))$ [3].

Pettere ve Kollo (2006) use bivariate Gumbel, Clayton and Frank copulas to estimate IBNR reserves. Based on the research done by Usta (2016) where Clayton copula is found to be the most appropriate copula for the sample data gathered from Turkish insurance companies' loss history, Clayton copula has been used in this study.

The mathematical formula of Clayton copula along with the Kendall's $\tau$ parameter is shown Table 3.1 [28].

Table 3.1: Details of Clayton copula.

| Copula type | Formula | Relation with Kendall's $\tau$ and $\theta$ |
|---|---|---|
| Clayton | $(u^{-\theta}+v^{-\theta}-1)^{-1/\theta}$ | $\theta=\frac{2\tau}{1-\tau}$ |

### 3.3 Correlation Structure

In actuarial literature, relationship between claim size and development factor is measured by Kendall's $\tau$ coefficient and Pearson linear correlation, $\rho$, coefficient [30]. Therefore, we will examine the properties of these coefficients here. x Pearson linear correlation coefficient $\rho$ is mostly used correlation coefficient in statistical studies. If X and Y are two random variables then the correlation coefficient $\rho$ is

$$\rho(X,Y)=\frac{Cov(X,Y)}{\sqrt{Var(X)\times Var(Y)}}=\frac{E[X-\mu_X)\times(Y-\mu_Y)]}{\sqrt{Var(X)\times Var(Y)}} \tag{3.2}$$

In the copula framework, Pearson's correlation depends on the copula of a bivariate distribution as well as the marginal distributions. An obvious weakness with Pearson's correlation is that it only measures linear dependence. Furthermore, it is only invariant in the case of strictly increasing linear transformations and not in the case of nonlinear strictly increasing transformations [6].

Kendall's tau correlation coefficient ($\tau$) ;

Let $(x_1;y_1),\ldots,(x_n;y_n)$ denote a random sample of n observations from vectors $(X;Y)$ of continuous random variables. There are $\binom{n}{2}$ distinct pairs $(x_i,y_i)$ and $(x_j,y_j)$ of observations in the sample.

Let $c$ denote the number of concordant pairs and $d$ denote the number of discordant pairs. Then Kendall's $\tau$ for the sample is defined as

$$\tau_K(X,Y)=\frac{c-d}{c+d}=\frac{c-d}{\binom{n}{2}} \tag{3.3}$$

This is equivalent with $\tau_K$ being equal to the probability of concordance minus the probability of discordance for a pair of observations $(x_i,y_i)$ and $(x_j,y_j)$ that are chosen randomly from the sample [6].

In this study, Kendall's $\tau$ coefficient which is a nonparametric and independent form distribution type is used instead of Pearson correlation coefficient because of its flexibility on linearity assumption.

# CHAPTER 4

# IMPLEMENTATION: MTPL INSURANCE IN TURKEY

The premiums in MTPL insurance is mostly far away from actuarial perspective in Turkey in practice. Insurers experienced huge losses on their financial statements in last four years in this branch. From the second half of 2015, the average premium per policy sharply increased so that losses resulting from previous years became unbearable by insurer companies. As a result of this, even though its obligation the rate of uninsureds are increased. Besides, in case of wind up of the insurer, the regulator takes over the his place and pays all claims arising from MTPL insurance. If the insurer's reserves are not enough to cover these losses, regulator has to use the governmental sources to compensate the financial deficits. Therefore, inadequate reserves resulting from faulty politics in insurance firm may finally have an impact on every single people in the country.

The general method for pricing risk is based on technical ratios, especially combined or loss ratios which are mostly used to evaluate the balance between pricing and losses. Jakovčević and Žaja (2014) studied underwriting risks as determinants of insurance cycles which are the fluctuations on premiums because of high or low profit waves within a time period. They find among underwriting risks (reserve, pricing, occurrence and reinsurance), pricing risk explains the most of insurance cycle after third quarter of financial statements. However, these ratios are not good indicators for newly started insurers since losses appear later whereas the premiums are taken in advance. For instance, branches with long tail like MTPL, huge losses resulting from bodily injured claims mostly create outflow far later than underwriting time. Such cases have a property to create IBNR. Therefore, when combined ratio is used in the calculation of pricing risk, loss part of the ratio will likely be far lower than premium written which makes the ratio to be around acceptable levels even if the insurer charges less premium than the amount based on actuarially calculated risk premium, which creates pricing risk.

## 4.1 Data

All the MTPL insurance losses are required to be reported to the Insurance Monitoring System (IMS) by regulation. The data set taken from IMS contain the information related to the losses reported from 39 non-life insurance companies issuing also MTPL policies. Over million occurrences the data selection is framed over the top 5 companies which build the claim density in the Turkey MTPL insurance market. A simple random sample scheme is applied to the data, which results in 3,309 loss records on policy base for the year 2015. It is collected randomly since insurer's portfolios may differ from each other as to car, region and risky driver types. Each loss file with no outstanding claims is considered, in other words, the loss amount ultimately paid by insurer is used. Then, aggregate losses per policy are identified and ultimate payment date is calculated to find development factor.

As we consider the present value of the future loss occurrences the discounted values are evaluated based on the regulatory risk-free rate, which is $9\%$. We consider that discounting factor must be taken into account in determination of underwriting year basis premium calculation since the interest or risk free rate of Turkey is high enough for insurers to get higher yields from the policy. Easy-Fit and Matlab are employed to run the analysis.

## 4.2 Risk Premium Estimation

The first part of the study includes the estimation methodology of the present value of the pure premium per policy (PVRP) which is expected to be adequate for insurers and planed to be set by the regulator as the level rate. The proposed real life data including claim size and development factor variables is used to estimate the pure premium using Clayton copula method. Then, based on the parameters obtained we simulate risk premium from the joint distribution by generating probabilistically weighted cash outflows duration (development factor) and appropriate claim size which is discounted with respect to annual risk-free interest rate of regulatory value of $9\%$. Additionally, ultimate claim frequency based on the historical observations is determined and taken as an estimate for 2015. These steps lead us to estimate mean PVRP per policy by multiplying resultant mean claim frequency by simulated mean risk premium. Moreover, SCR risk measures VaR and CTE are estimated using claims generated.

The algorithm of the risk premium estimation is presented in Figure 4.1 which summarizes the steps described above.

## 4.3 Descriptive Statistics and Parametric Distribution Estimation

In this part, descriptive statistics in the sample data for claim size and development factor are given and their marginal probability distributions are estimated by using Easy-fit software.

It is seen from the Table 4.1, ultimate average payment loss amount equals to 3,044.78 TL, while standard deviation is 11,489.14 TL. The distribution of claim size is heavily skewed considered the median and mean values. The minimum claim size in the sample data 19.08 TL and the maximum is 265.228 TL which is believed to appear from bodily injured claims.

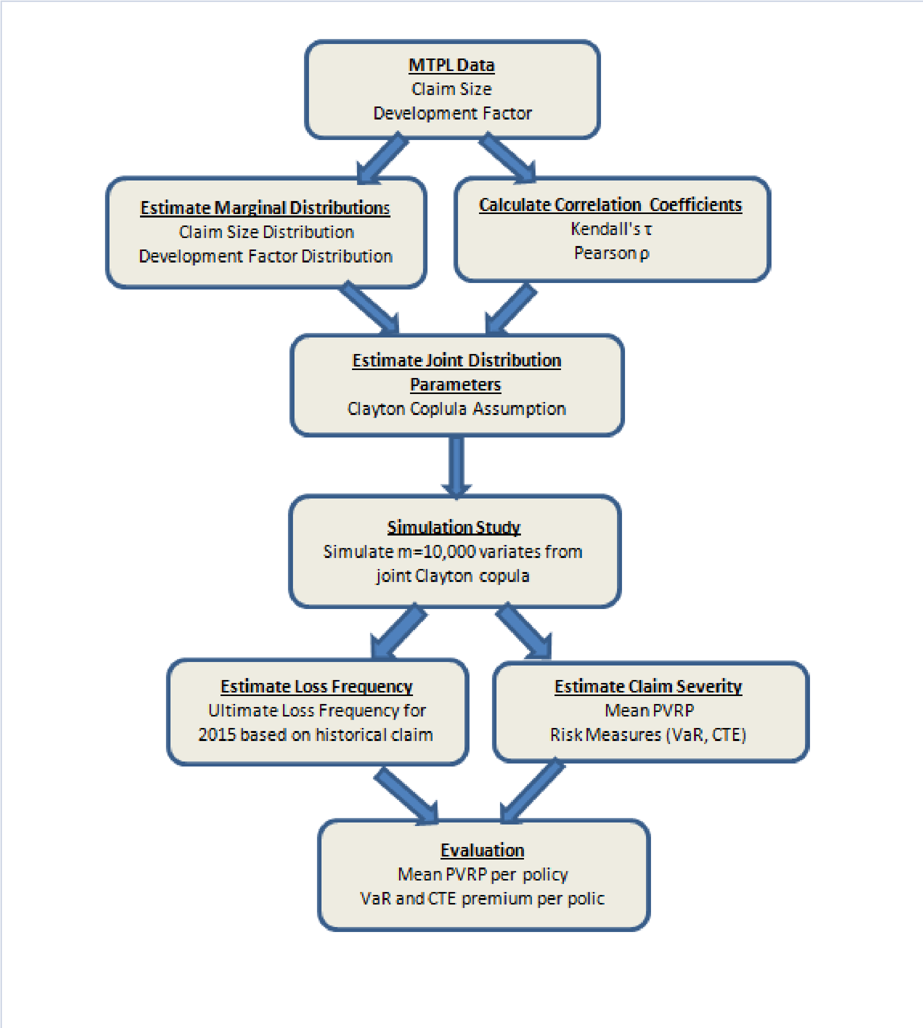


Figure 4.1: Algortihm of the Risk Premium Calculation.

The development factor which is measured daily has mean of ultimate settlement for a claim is 146.90 days, standard deviation is 291.59 days in Table 4.1 . The maximum settlement date in the sample data is 3,169 days while the minimum is 9 days. Skewness coefficient is found to be 5.58 and the distribution of development factor is estimated to be right skewed since the mean is slightly over the twice of the median. $95\%$ of the claims is seen to be closed shorter than 575 day, whereas the median is 65 days.

Easy-Fit software is used to find the appropriate fitted claim size distribution. Lognormal distribution with $\mu = 6.885$ and $\sigma = 1.2531$ with Kolmogorov-Smirnov test value of 0.066 (p value=0.001) is chosen as the claim size distribution and fitted distribution with sample data is shown in Figure 4.2. Since being used by many actuarial applications and easy for its parameters to update with such as change in inflation, the lognormal distribution is decided to be used in the analysis.

Similarly development factor follows Lognormal distribution with $\mu$=4.344 and $\sigma$=0.9596

Table 4.1: Descriptive Statistics for Claim Size and Development Factor

| Statistic | Claim Size (TL) | Development Factor (day) |
|---|---|---|
| Sample Size | 3,309 | 3,309 |
| Range | 265,208.92 | 3,160 |
| Mean | 3,044.78 | 146.9 |
| Median | 872.6 | 65 |
| Variance | 132,004,000 | 85,027.78 |
| Std. Deviation | 11,489.14 | 291.59 |
| Coef. of Variation | 3.77 | 1.98 |
| Std. Error | 199.72 | 5.06 |
| Skewness | 11.92 | 5.58 |
| Excess Kurtosis | 199.98 | 38.85 |
| Min | 19.08 | 9 |
| Max | 265,228 | 3,169 |

with respect to Kolmogorov-Smirnov test (p value ¡0.01).

## 4.4 Dependence Analysis

### 4.4.1 Empirical Correlation

The correlation between claim size and development factor variables is calculated from the sample data and the Kendall's $\tau$ coefficient is found to be 0.1720 while Pearson linear correlation $\rho$ coefficient is 0.4538. We think that the extreme values in the sample data cause the Pearson $\rho$ coefficient being much higher than Kendall's $\tau$ coefficient.

As it can be seen from the coefficients that there exists a positive relation between variables even though many literature assume these variables to be independent. This is believed to be true for overall analysis for MTPL structure by considering that bodily injured claims takes much longer to be ultimately closed because of the long lawsuit processes. Also, those claim amounts generally become much higher than property damages which are mostly finalized in short time period. Therefore, the sample data is believed to capture the general property of claims for MTPL policies.

Table 4.2: Dependence measures between claim size and development factor

| Kendall's $\tau$ | Pearson correlation |
|---|---|
| 0.1720 | 0.4538 |

### 4.4.2 Clayton Copula Parameter Estimation

The determination of the Clayton copula parameter, $\theta$, is done using Kendal's $\tau$ correlation coefficient. The corresponding parameter value, $\theta$, of Clayton copula for given

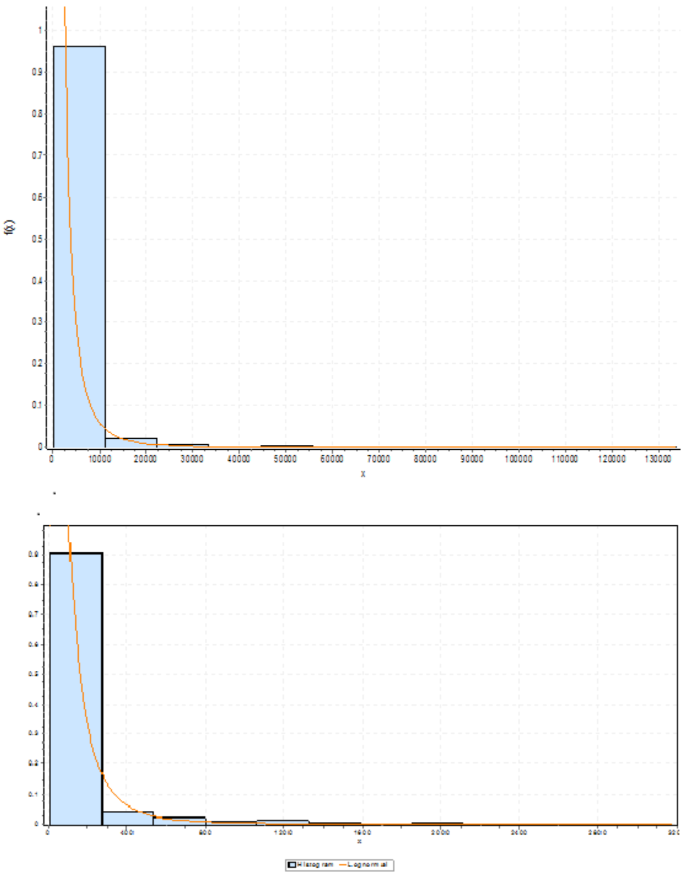


Figure 4.2: Claim Size and Development Factor Distributions.

$\tau$ is found to be 0.4155.

## 4.5 Simulation on the Joint Probability Distribution Using Copula

A-100 sample each of which consists of 10.000 data point is simulated by Matlab with the generation of Uniform (0,1) random variates, $u$ and $v$. Then the values of development factor is calculated with the inverse function of lognormal distribution with $\mu$=4.344 and $\sigma$=0.9596 and the values of claim size variable is calculated with the inverse function of lognormal distribution with $\mu$=6.885 and $\sigma$=1.2531.

The scatter plots of the joint relationship between the claim size along with development factor variables and the simulated scatter plot are shown Figure 4.3 and Figure 4.4, respectively. The joint distribution calculated from the simulation study is seen to approximate the sample data with exception of extreme values especially in the tail part of the joint distribution. As an alternative, these data points in the sample can be calculated separately with extreme value or tail value analysis. But in the study, these points have been kept in the data. Also, as mentioned in Chapter 2, the structure of bodily injured claims and property claims differ in both loss amount and development of the claims. We believe that the calculation should be done by separating those claims accordingly, and to be applied the same procedure mentioned in this study to find risk premium.

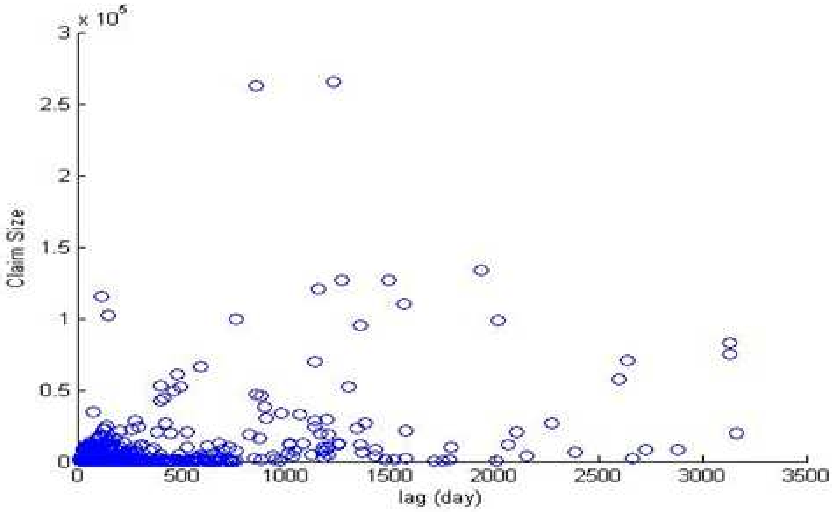


Figure 4.3: Scatter Plot of Simulated Lognormally Distributed Claims.

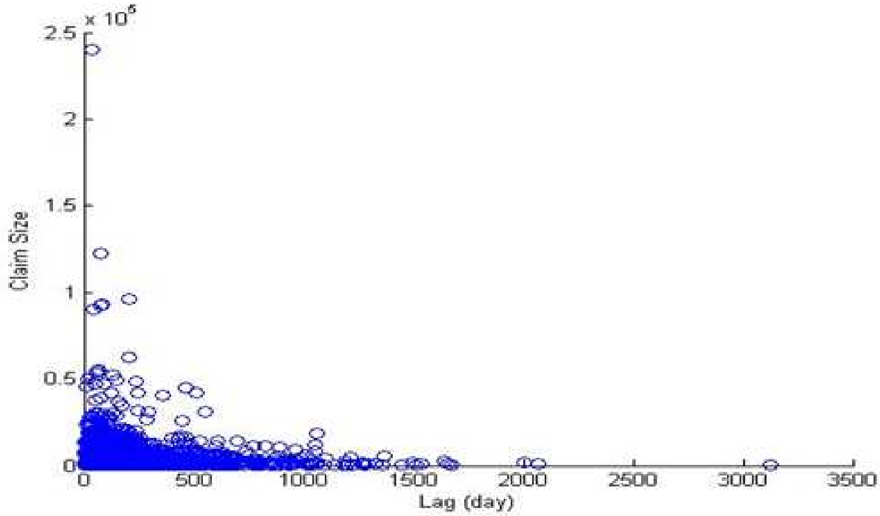


Figure 4.4: Scatter Plot from Simulation Data.

### 4.6 Empirical Loss Frequency Distribution

The underwriting year basis number of policies and number of incurred losses at the stated period is given in Table 4.3.

Table 4.3: Loss Frequency by years (IAT,2016)

| Year | Number of Policy | Number of claims | Claim Frequency |
|---|---|---|---|
| 2010 | 11,745,633 | 1,332,870 | 0.1135 |
| 2011 | 12,942,474 | 1,436,048 | 0.1109 |
| 2012 | 13,862,901 | 1,505,141 | 0.1086 |
| 2013 | 14,111,306 | 1,408,221 | 0.0998 |
| 2014 | 15,062,936 | 1,381,967 | 0.0918 |
| 2015 | 15,522,432 | 1,471,656 | 0.0948 |

In the time period between 2010 and 2015, the number of underwritten policy in year 2010 is 11.7 million, number of claims is 1.33 million yielding a loss frequency for 2010 is around 11.35%. As the number of policy increases every year between 2010 and 2015, the number of losses do not increase linearly. This is because of that the table is prepared to be based on underwriting year results and loss claims that belongs to policies in most recent years like 2014, 2015 will most likely come in following years. Therefore, when estimating the ultimate loss frequency in MTPL insurance, one should consider at least 4 or 5 year historical occurrences. As a result, we use the ultimate loss frequency rate of the years 2011 and 2010 and estimate the ultimate loss frequency in2015 is to be 11.47%. We do not apply any estimation method. It is also possible to use a range for loss frequency rate like between 10% and 12%. In addition, loss frequency is mostly affected by driver's claim history, region, car type even the usage of car measured by miles. Since our aim is to set the level risk premium, regulator may let insurers to launch risk rates depending on the risk factors over the risk premium.

As it is mentioned previously, it is important to determine the risk factors affecting the number of losses in risk premium calculation. Gender, miles driven, state, car types are commonly used as risk factors in the literature. In this study, we apply only car types risk factor as an example to show how average risk premium per policy differs with respect to car types. Our suggestion to the regulator is that other risk factors which are proven to be effective in the determination of number of losses should be left to pricing actuary while the methodology of calculating risk premium per policy is the same for every insurer.

Table 4.4: Loss Frequency by years and car types (IMS,2016)

| **Vehicle Type** | **2010** | **2011** | **2012** | **2013** | **2014** | **2015** |
|---|---|---|---|---|---|---|
| **Truck** | 0.17 | 0.18 | 0.16 | 0.14 | 0.13 | 0.07 |
| **Pickup Truck** | 0.14 | 0.12 | 0.09 | 0.08 | 0.07 | 0.03 |
| **Minibus** | 0.11 | 0.09 | 0.07 | 0.07 | 0.08 | 0.04 |
| **Autobus (18-30 seat)** | 0.18 | 0.18 | 0.14 | 0.13 | 0.14 | 0.07 |
| **Autobus(over 31)** | 0.28 | 0.29 | 0.24 | 0.19 | 0.19 | 0.10 |
| **Private Vehicle** | 0.10 | 0.09 | 0.05 | 0.05 | 0.05 | 0.02 |
| **Commercial Vehicle** | 0.28 | 0.26 | 0.17 | 0.17 | 0.19 | 0.10 |

Table 4.4 shows that loss frequency highly differs given vehicle types. While private

car drivers has the lowest loss frequency between 2010 and 2015, commercial (taxi) drivers and autobus (over 31 seat) drivers has the most. Therefore, risk factor from car types must be considered and analyzed separately in risk premium calculation.

### 4.7 Estimation of the Present Value of Pure Premium per Policy

To estimate the distribution of the present value of claim size, simulated claim size values are discounted according to the development factor values for each data point produced with the regulatory risk free rate of 9%.

The discounting formula used in the analysis is:

$$PV(Y_i) = Y_i \times (1 + 0.09 \times \frac{X_i}{365})^{-1} \tag{4.1}$$

where $X_i$ denotes development factor for $i^{\text{th}}$ policy and $Y_i$, denotes the claim size for $i^{\text{th}}$ policy.

Some statistics and risk measures about the mean PVRP and the mean PVRP per policy based on the simulation results is given in Table 4.5 and Figure 4.5. Figure 4.5 shows that MPVRP ranges between 1.990 TL and 2.160 TL.

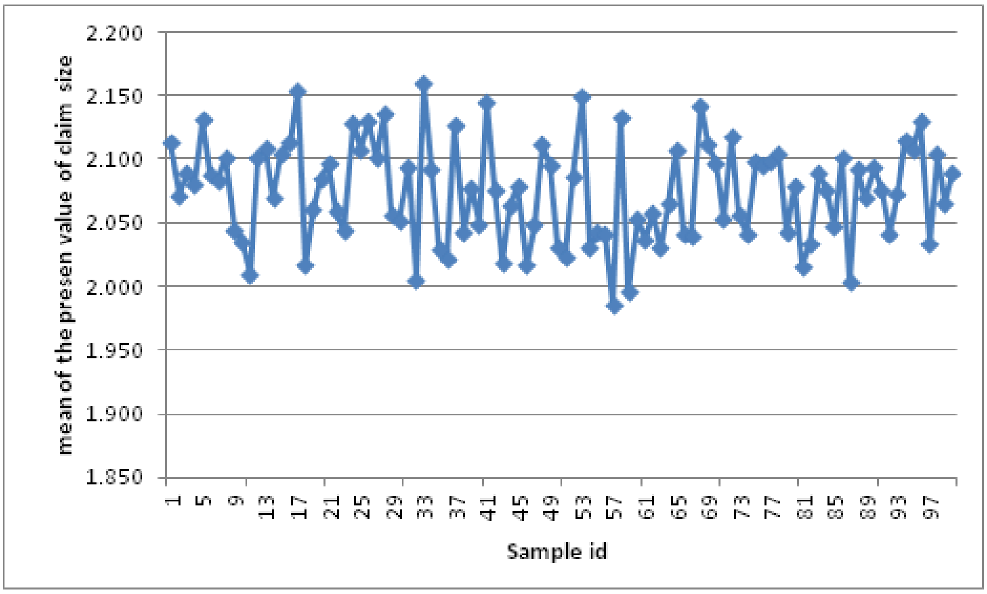


Figure 4.5: Simulation results for the mean present value of pure premium.

According to the simulation results, the mean PVRP is 2,074.85 TL, standard error of the average PVRP is calculated to be 39.27 TL. The estimated mean PVRP per policy is 237.99 TL given estimated ultimate loss frequency is 0.1147 for the year 2015.

VaR and CTE at 95% confidence level risk measures on the PVRP is 2,136.22 TL and 2,150.45 TL respectively. The estimated mean PVRP per policy with 3 $\sigma$ is 245.37 TL

Table 4.5: Statistics and risk measures values (TL).

| **Indicator** | **Values** |
|---|---|
| average | 2,074.85 |
| standard error for mean | 39.27 |
| $VaR_{95\%}$ | 2,136.22 |
| $CTE_{95\%}$ | 2,150.45 |
| Loss frequency | 0,1147 |
| Number of policy | 260,000 |
| Expected pure premium per policy | 237,99 |
| Risk Capital based on CTE | 3,206,540 |
| CTE risk per policy | 12.33 |
| 3 $\sigma$risk premium | 245,37 |

and the maximum policy risk premium is calculated to be 2,159.57 TL.

To give an example for risk capital study based on CTE risk measure, when it is assumed for insurer to underwrite 260,000 policy in a year, the calculated risk capital at 95% confidence is expected to be 3,206,540 TL. This type of risk capital calculation can also be easily done by following the methodology given in this study. Besides, the starting capital in MTPL business in order to be given working permission by the regulator to the insurer can be determined by this way according to the insurer's willing to produce policy in following years. In addition to that, it would be enough for insurers who already has running business on MTPL branch to charge an extra 12.33 TL premium to set aside risk capital for unexpected losses in the future.

Table 4.6: Estimated Risk Premium of 2015 by car types (TL)

| Vehicle Type | Ultimate Loss Frequency | Average PVRP | Average PVGP |
|---|---|---|---|
| Truck | 0.174 | 361.56 | 560.42 |
| Pickup Truck | 0.13 | 269.73 | 418.08 |
| Minibus | 0.103 | 214.28 | 332.14 |
| Autobus (18-30 seat) | 377.99 | 0.18 | 585.90 |
| Autobus(over 31) | 0.287 | 596.47 | 924.53 |
| Private Vehicle | 0.094 | 195.04 | 302.306 |
| Commercial Vehicle | 0.27 | 560.21 | 868.33 |

Table 4.6 shows the premium per policy amounts by car types given as a risk factor. Average present value of gross premium per policy (PVGP) is calculated with assumed loading factor of 1.55 over average present value of risk premium (PVRP). The maximum policy premium is taken from Autobus drivers with 924.53 TL, while private car drivers are assumed to pay 302 TL for MTPL policy.

## 4.8 Premium Risk Reserve Value

Another contribution of the study exposes a proposal to the regulator to revise its technical reserves regulation. Especially in case the companies newly operating in MTPL business may charge less premium than actuarial premium which may result in cash inflow easily in order to increase the market share. This pricing risk reserve set by the company reduces the probability of the insolvency of the insurer. Aiming to keep premium calculation as simple as possible, the regulator and insurers can easily understand and implement the method proposed here, reinsurance capacity and possible investment incomes of insurers are excluded in the premium risk reserve calculation. To sum up, insurer will have to set additional reserve for each policy at the underwriting time if the premium without loadings is less than the pure premium set by the regulator.

In case that the insurer charge less policy premium (PP) (without loadings) than the level pure premium, the premium risk reserve for a policy is proposed to be calculated as follows:

$$\text{Pricing Risk Reserve} = max(0, (\overline{PVRP} - \text{PP}))] \tag{4.2}$$

where $\overline{PVRP}$ defines the average of PVRP.

Based on the outcomes of the previous steps, we calculate the pricing risk reserve. Its value is found to be 238 TL per policy. When we consider the commonly used loading factor as 1.55 (33% company loadings plus 22% for taxes), this premium grows up to approximately 370 TL.

To illustrate the utilization of this equivalance price, assume that the gross premium is set to 300 TL for a policy, the insurer has to allocate 70 TL (23%of gross premium) as premium risk reserve. The proposed risk reserve value can be used as a threshold to control the risk as it can be seen from Figure 4.6.

In this case, the insurer will be required to put additional regulatory reserve in order to avoid being insolvent and also protect the policyholder. Besides, companies willing to charge far less premium compared to the market price can only keep their business if they have enough capital. This protection system starts at the underwriting time.

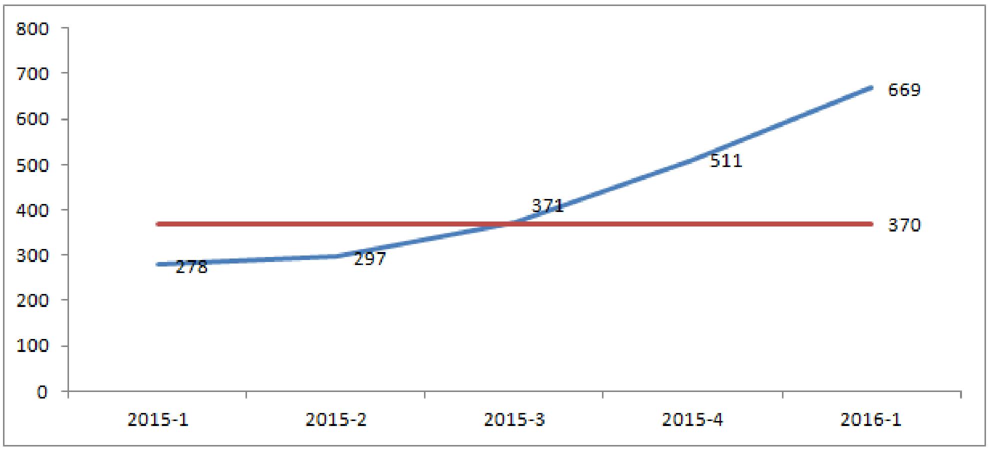


Figure 4.6: Average Policy Premium with Premium Risk Reserve Threshold.

# CHAPTER 5

# CONCLUSION

This study aims that the proposed premium risk reserve calculation method to be a guidance to the regulator to monitor and measure the premium risk reserve of the insurance companies resultant from MTPL insurance. Even the insurer insists on charging inadequate premiums, the regulator will be confident as the additional reserve is required to set aside at the duration of underwriting. Therefore, the regulator does not impose the company to put additional capital in case of insolvency.

Often experienced in Turkey, the regulator give permission to put additional capital, the insurer continue producing policies at a much lower rate in order to accumulate high amount of income as the insurer needs to create cash flow as much as possible. This devilish cycle ends with the bankruptcy in most cases.

Based on this initiatives this study targets to obtain an adequate risk premium in MTPL business and to propose as an alternative premium risk reserve. MTPL premium estimation using copula approach will contribute to the literature. The main guideline Pettere and Collo (2010) is modified and applied to the Turkish MTPL data.

In the risk premium estimation part, marginal distributions of claim size and development factor are found to follow Lognormal distribution (Development factor ∼LN ($\mu = 4.344$, $\sigma = 0.9596$); Claim Size∼LN ($\mu = 6.885$, $\sigma = 1.2531$)). Incorporating this to quantified dependence parameter Clayton copula is employed to explain the joint behavior. The simulation on the resultant copula distribution is used to generate claim size and development factor whose values are utilized to calculate the mean present value of the pure premium and the important risk measures such as mean error of standard deviation, VaR and CTE. In order to calculate premium per policy amount, ultimate loss frequency rate is estimated over past 5-year data including the whole sector claim history and number of policy underwritten in MTPL the ultimate loss frequency rate for 2015 is estimated. Finally, the average present value of pure premium per policy is quantified. It should be noted that the results reflect the realization in the data set and it will be beneficiary to update these results by taking into account the economic indicators such as inflation rate, GDP growth and loss inflation. In this study these variables are kept constant to make the premium model simple and easily applicable by the regulator and insurers.

We conclude that that the level of premium per policy pronounced as a result of the study is affordable for customers and adequate for insurers to cover their possible future losses even if they may appear many years later. In addition, the regulator may let companies to specify risk premium according to their own portfolio structure by setting their premium level and upper-lower premium limits according to the value proposed in this thesis.

In addition to pure premium calculation, the other outcome of the study is to present a threshold to the regulator to revise its technical reserve regulation. Insurance technical reserves that is determined by the regulator ensure to guarantee the obligations that emerges with the insurance contract, and protects the insured party against the economical inadequacy of the companies can face. To make the model being simple and easily implementable, reinsurance protection and possible investment return of insurer have been excluded.

The proposed risk premium calculation in this study is designed to check each policy premium without loadings underwritten by the insurer with the pure premium level set by the regulator as a guide. In case that the insurer's premium becomes lower than the officially announced pure premium, insurer will be obligated to set aside the difference amount between pure premium and policy premium without loadings. It is believed that this method will compensate the deficiency of unexpired risk reserves set by the regulator with the technical reserves regulation.

The premium risk reserve that is set for a policy under pure premium defined by the regulator should be calculated on daily basis as to the loss information during the policy period. For instance, probability of having an claim from a policy with a month left to it's expiration would most of the time be higher than the policy that newly underwritten. In the current technical regulation in Turkey, this concept is being captured in unearned premium reserve calculation. But in the regulation assumes that losses are distributed uniformly during the policy period. However, there are some LoB like health insurance where losses have different parametric distributions, especially within the year. Therefore, the pure premium calculation method proposed here can be elaborated including the loss history of the policy and seasonal frequencies. So that, unearned premium reserve can also be calculated with this way. Therefore as a future study, we will modify the premium risk reserve given in equation (4.2) as follows.

$$\text{Risk Reserve}_t = [max(0, (\text{risk premium}_t - \text{policy premium} \times (1 + i \times \frac{t}{365})))] \quad (5.1)$$

For instance, the reserve amount calculated based on the loss history and development factor at time $t = 365$ is compared to the risk reserve amount set aside at the underwriting time of that policy, Although it requires for insurers to track each of their policies on daily basis and to make calculation each time period, we believe the increasing IT technology will enable the insurer to see their reserve position and solvency situation daily and more correctly.

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