ANALYSIS OF PRICE-ONLY AND REVENUE SHARING CONTRACTS IN A REVERSE SUPPLY CHAIN

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ABSTRACT

ANALYSIS OF PRICE-ONLY AND REVENUE SHARING CONTRACTS IN A REVERSE SUPPLY CHAIN

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Original Equipment Manufacturers prefer to fully or partially outsource take-back activities, such as used product acquisition and handling, since dealing with reverse flow of goods is not their core competency. However, outsourcing may cause the supply chain to suffer from local optimization that results from decentralization. In such cases, different forms of contracts are offered to reduce the effects of decentralization. In this study, we consider a two-echelon reverse supply chain where a remanufacturer, facing a random demand, orders from a collector, which is in charge of used product acquisition. We analyze the decentralized decision making process under both the remanufacturer's and collector's lead considering both price-only and revenue sharing contracts. We demonstrate that there exists a wholesale price that can coordinate reverse supply chain and allocate positive profits for both parties. We also show that there exist a menu of revenue sharing contracts that can also coordinate reverse supply chain, which provides more flexibility for negotiation between parties compared to the wholesale price-only contact. Furthermore, we investigate the effects of problem parameters on the optimal decentralized decisions and coordinating contracts through an extensive numerical study.

Keywords: reverse supply chain, wholesale price, revenue sharing, Stackelberg game

ÖZ

BİR TERSİNE TEDARİK ZİNCİRİNDE FİYAT VE GELİR ORTAKLIĞI KONTRATLARININ ANALİZİ

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Orijinal Ürün Üreticileri, kullanılmış ürün toplama ve elleçleme gibi geri-alım aktivitelerini, ürünlerin tersine akışı temel faaliyet alanları olmadığından, kısmen ya da tamamen taşerona verirler. Fakat taşerona vermeyle ortaya çıkan merkezi yönetimsiz bir tedarik zinciri yapısında lokal optimizasyon bütün zincirin zarar görmesine neden olabilir. Bu tür durumlarda, merkezi yönetim eksikliğinin etkilerini azaltmak için değişik tiplerde kontratlar önerilmiştir. Bu çalışmada, rassal taleple karşı karşıya olan bir üreticiyle kullanılan ürünleri toplayan bir toplayıcıdan oluşan iki seviyeli bir tersine tedarik zinciri incelenmiştir. Sadece fiyat, ve gelir ortaklığı kontrat tipleri için, merkezi olmayan karar verme süreçleri, hem üretici hem toplayıcının liderliği durumunda analiz edilmiştir. İki firmaya da pozitif kar sağlayarak tedarik zincirini koordine eden bir fiyat kontratının varlığı gösterilmiştir. Ayrıca, zinciri koordine eden bir grup gelir ortaklığı kontratının varlığı gösterilmiştir. Bu kontrat biçimi, fiyat kontratıyla karşılaştırıldığında firmalara müzakerelerde daha fazla esneklik sağlayacaktır. Bunlara ek olarak, problem parametrelerinin optimal kararlar ve koordinasyonu sağlayan kontratlar üzerindeki etkileri, kapsamli bir sayısal çalışmayla incelenmiştir.

Anahtar Kelimeler: tersine tedarik zinciri, fiyat kontrat, gelir ortaklığı kontrat, Stackelberg oyunu

To my parents,
my sister,
and my closed friends,
without whom
none of this work
would have been possible

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Today, I finished all the requirements for the degree of Master of Science in Industrial Engineering at Middle East Technical University. Last three years have been an outstanding package of academical as well as personal life experiences for me. When I look back through, I remember all the good memories as well as intensive moments and here, I would like to reflect on the people who have supported and helped me throughout this period.

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CHAPTER 1

INTRODUCTION

With the increased environmental consciousness and stringent take-back legislations, closed-loop supply chain management (CLSCM) has been receiving growing attention from both business and academy in the last two decades (Prahinski and Kocabasoglu [1]). In the United States take-back actions are driven by market and economical benefits, while in Europe these practices are mostly legislation-driven with the purpose of fulfilling mandatory requirements, i.e. meeting a certain recovery target (Walther et al. [2], Kaya [3]). For instance, EU Commission mandates all vehicle manufacturer in EU member countries to meet 95% reuse/recovery target by 2015 (European Commission [4]). As Guide and Wassenhove [5] state, researches recently aims for potential business advantages of CLSC problem.

Closed-loop supply chain (CLSC) consists of Forward Supply Chain (FSC) and Reverse Supply Chain (RSC)(Fleischmann et al. [6]). Traditional FSC has been on interest of practitioners and researchers for more than half a century, while the oldest definition of RSC was given by Lambert et al. [7] in early 80's. In this study, we focus on reverse supply chain flows.

Many examples of RSC implementation can be found in various industries. Companies like BMW (Thierry [8]), Xerox (Kerr and Ryan [9]), Kodak (Guide et al. [10]), and Hewlett-Packard (Wu and Cheng [11]) were the pioneers that were able to benefit from their successful reverse chain. For instance, by using more than 80 percent of old camera's part, Kodak would be able to close its supply chain (Jayaraman and Luo [12]). Giants of home appliances producers such as Miele and Electrolux have developed eco-design products and involved in waste management (Miele [13], Electrolux

[14]). Apple Inc. directly purchases its customer's used iPhones, iPods and Macbook computers in order to make use of internal parts. In exchange, Apple gives a coupon to the customers as a discount for future purchases (Govindan and Popiuc [15]).

Reverse supply chains necessitate a whole new infrastructure and comprehensive approaches to handle the reverse flow of goods. A reverse supply chain initiates from the end users; potential used products (either leftovers in retailer's, end-of-use or end-of-life used products that are given up by end customers) must be purchased/collected and then, returned to the remanufacturing site. Returned products must be inspected and graded depending on their quality to see whether they are remanufacturable or scrap. Many Original Equipment Manufacturers (OEM) fully or partially outsource take back activities, such as product acquisition, used products handling and remanufacturing, since for many of them engaging in take-back activities is not their core competency. Therefore, OEMs may cooperate with some third party firms, broker or even some existing members in forward chain, for instance retailers or logistics provider, in part of RSC activities.

Since decentralization may come along with outsourcing, the alignment of decisions and objectives becomes prominent in order to prevent inefficiencies caused by decentralization. Local optimization, which is caused by decentralization, usually causes “Double marginalization", which happens when both upstream and downstream firms price a mark-up value over marginal cost.

In the literature and practice, several mechanisms such as information sharing, coordination contracts and joint decision making are introduced in order to alleviate the effects of decentralization. These mechanisms motivate separate members of the supply chain to cooperate in the maximization of total benefit (Govindan et al. [16]). As one of these mechanisms, coordination contracts has become a popular topic among researchers. Supply chain members negotiate on a set of parameters such as order quantity and wholesale price and may come up with an agreement or a "Contract". A contract is called coordinating when adequate incentive is provided to all members such that decentralized setting behaves as the integrated one (Wang [17]).

In forward supply chain contracting literature, several contracts are proposed under different contractual terms such as wholesale price, buy back, quantity discounts and revenue sharing. In a supplier-buyer supply chain, through a wholesale price contract, supplier sets a wholesale price for the product and buyer orders accordingly to replenish his inventory for the following selling season. Through a buy back contract, supplier, in addition to pricing, decides on an optimal return policy, by paying full or partial refund to the buyer for unsold, returned products (Pasternack [18]). A buyer may accept a quantity discounts contract through which the selling price charged depends on the buyer's order quantity (Jeuland and Shugan [19]). Via revenue sharing, buyer promises to share a percentage of his revenue as an exchange for the lower wholesale price supplier charges (Mortimer [20]).

However, contract mechanisms received less attention in reverse supply chains when compared to forward supply chains. In this study, we investigate a two-echelon reverse supply chain, consisting of a remanufacturer and a collector in a single-period setting. Note that the collector may be a retailer selling new products and the manufacturer might outsource the collection activities to the retailer as it is closer to end-customers. Collector is in charge of acquiring used products from end customers by paying an acquisition price per unit of used products. We assume an infinite source of used products. The collected products are then sold to the remanufacturer who faces a random demand for remanufactured products. Assuming a highly competitive market, selling price of the remanufactured items is exogenous. In this environment, we consider the decentralized setting problem under both remanufacturer's lead and collector's lead. We characterize the optimal actions of the supply chain members. We also show that there exists a wholesale price that coordinates the reverse supply chain. Furthermore, due to the low flexibility of a wholesale price contract, we also investigate a revenue sharing contract between the parties and derive a menu of revenue sharing contracts that coordinate reverse supply chain which allows for a more flexible allocation of channel profit. It is also worthwhile to show that our setting applies to a forward chain with nonlinear procurement costs as well.

The rest of the study is organized as follows: In Chapter 2, we review the literature on related topics. Chapter 3 presents the problem environment and the wholesale price-only contract. In Chapter 4, we extend our analytical model and study the implementation of revenue sharing contract. We conclude in Chapter 5 by presenting a summary of our work and future research directions.

CHAPTER 2

LITERATURE REVIEW

Closed loop supply chain management literature covers a wide variety of subjects including production scheduling, inventory management, product recovery, reverse logistics, product acquisition, channel choice and coordination. Guide and Wassenhove [5] and Souza [21] provide comprehensive reviews of Closed loop supply chain management.

In this thesis, we model a two-echelon reverse supply chain, consisting of a remanufacturer and a collector and we aim to characterize optimal decentralized decisions as well as identify coordinating contracts under wholesale price-only and revenue sharing mechanisms. In this chapter, we review the most related studies to our work, especially regarding product acquisition management, reverse channel choice and coordination contracts.

2.1 Product Acquisition Management

Reverse supply chain initiates from end customers. End-of-life or end-of-used products must be acquired through specific channels at certain quantity and quality level, which involves financial incentive for used product owner and handling reverse flow of products. Guide and Wassenhove [22] emphasize the importance of product acquisition management and state that through a well-devised acquisition of used products, reuse activity becomes economically more attractive. In this section, we point out those studies that focus on optimal product acquisition policy and pricing of remanufactured product.

V. Daniel R. Guide et al. [23] focus on product acquisition management. They study ReCellular company business case, a third-party handset remanufacturer, who buys a bulk of used handsets from intermediaries and remanufactures and sells them. Used handsets differ in quality and each quality group has the corresponding remanufacturing cost; the lower the quality of used products, the higher the remanufacturing cost. ReCellular have to pay an acquisition price per unit used handset, which depends on the quality of used handsets, that is higher acquisition price must be paid for higher quality groups. For a given acquisition price, they consider a certain return rate. Demand is assumed to be price-dependent and they define a demand rate for any given selling price. As a result, it is all ReCellular decisions to set i) acquisition price for each quality groups and ii) selling price of the remanufactured handsets so as to maximize his profit. They find the optimal selling price and acquisition price for different quality groups using a heuristic method. In this method, they define a fixed acquisition price for as-good-as-new returned product that does not need remanufacturing. As the quality gets worse, acquisition price decreases. They demonstrate that as the remanufacturing cost for each quality class increases, the acquisition price for that class decreases. As a result, total amount of returned products decreases. Since demand is equal to quantity of returned products, selling price, in turn, increases.

Bakal and Akcali [24] study the reverse logistics environment in the automobile industry. End-of-life vehicles (ELV), gathered by salvage yards or automobile parts brokers, are bought by an independent automobile remanufacturer and are remanufactured. Return function is an increasing linear function of acquisition price. Moreover, they assume demand to be a decreasing function of selling price. They solve the remanufacturer's problem to seek for optimal acquisition and selling price for ELVs and remanufactured part, respectively. This study differs from V. Daniel R. Guide et al. [23] in the sense that the remanufacturer faces uncertainty with the fraction of remanufacturable ELVs called "yield rate". They investigate the effect of random yield on on the remanufacturer decisions. They further study the case where the manufacturer has the opportunity to set the selling price after the realization of returns yield by contrasting simultaneous and postponed pricing models. It is shown that model with pricing postponement performs closer to the deterministic model than simultaneous pricing model, especially when the margin is low.

Galbreth and Blackburn [25] extend the model by V. Daniel R. Guide et al. [23] in the sense that the returns quality condition is so wide that it can be approximated as a continuous set. It is the case for products such as cell phones that used products may need a numerous combination of remanufacturing requirements. Similar to V. Daniel R. Guide et al. [23], the remanufacturer acquires used items from a collection/broker firm as much as needed. The remanufacturer faces a deterministic demand and decides on the quantity of used items to acquire. They consider remanufacturing cost as both linear and nonlinear function of quality condition. They derive the optimal acquisition lot sizing decision. They show that result of linear remanufacturing cost model resembles the case without quality uncertainty.

Above studies have focused only on the single remanufacturer decision, that is optimal acquisition price for used items, since they assume that there already exist a bulk of used items gathered by independent collection communities or broker. Whilst, we consider an OEM who investigate the implementation of a reverse channel through which it can outsource the reverse activities, i.e. remanufacturing and collection of used products. Thereby, in addition to acquisition management problem, integration of interdependencies between participating third-party firms needs to be taken into account.

In the context of product recovery, many articles like V. Daniel R. Guide et al. [23] and Bakal and Akcali [24] consider only the market for remanufactured products. Some other studies (Kaya [3]; Minner and Kiesmüller [26]; Vercraene and Gayon [27]; Bulmuş et al. [28]) extend this problem in the sense that an Original Equipment Manufacturer (OEM) uses the recovery option as a substitute to regular manufacturing process, due to relatively lower cost of remanufacturing compared to regular manufacturing.

Kaya [3] consider a hybrid manufacturing/remanufacturing setting where the manufacturer produces products from both raw material and returned used products, acquired from end customer by a third party collection agency. They assume three different scenarios in which: i) manufacturer only remanufactures the used products, ii) manufactured and remanufactured products have the same market value and iii) each of them are sold at different markets and prices. Demand in each market is stochastic.

They study return function as both deterministic and stochastic in the model. They model both centralized and decentralized settings and through a computational study, they demonstrate the inefficiency regarding local optimization of participating firms. They also put forward a contract, consisting of a wholesale price and a fixed transfer payment, that is able to coordinate supply chain.

Bulmuş et al. [28] cover the problem of product acquisition management and pricing of the new and remanufactured products. Using the same approach utilized by V. Daniel R. Guide et al. [23] for core acquisition, they model a hybrid manufacturing/remanufacturing system for a single OEM who both manufactures new product as well as remanufactures returned products. There are n different quality of used products with varying remanufacturing cost and minimal acquisition price. The return rate of each quality group is a linear function of the acquisition price. OEM must decide the quantity of used products to collect from each quality group as well as quantity of new product to manufacture in order to maximize profit. In contrast to Kaya [3], they assume a price-dependent demand function for both remanufactured and new product. There is no limitation regarding capacity and used product resource. They propose a general optimal procedure to find the most preferred quality groups to acquire used products from. They derive closed-form solution for optimal acquisition price, optimal quantity of returns for each type as well as quantity of newly manufactured units and optimal selling price of remanufactured as well as newly manufactured product. It is demonstrated that the price for new product only depends on the manufacturing cost, that is remanufacturing does not have any effect on price of new products. They also show that the profit per remanufactured item of each type only depends on the remanufacturing cost and customer's relative willingness to buy remanufactured unit. It is in contrast to V. Daniel R. Guide et al. [23] where they distribute total profit to all core types by paying the acquisition price in such a way that the sum of remanufacturing cost and acquisition price for all core types become equal. Through a sensitivity study, they find that if the customer willingness to pay for a remanufactured item increases, the OEM will acquire more used product and sell at higher prices, and decrease the quantity of newly manufactured product.

Vercraene and Gayon [27], using a queuing control framework, investigate the coordination problem of hybrid production-inventory system as well as returns acceptance control. Demand for finished product can be satisfied by either manufacturing a new product or remanufacturing returned items, that is customers do not differentiate between new and remanufactured products. Returns and demand are assumed to follow Poisson processes with independent rates. Returns may either be accepted or rejected. Hence, the problem is to decide the quantity of returns to accept/reject and to remanufacture as well as quantity of new products to manufacture in order to minimize expected cost over an infinite horizon, which is formulated as a continuous time Markov Decision Process. They suggest a control policy to manage the channel; that is when to accept returns, when to manufacture new item and when to remanufacture. They derive the optimal policy and compare it with existing heuristic approaches in the literature through an extensive numerical study.

Except Kaya [3], all above studies consider only centralized setting, that covers a single firm decision. Besides, among these articles, Kaya [3] and Vercraene and Gayon [27] assume demand to be stochastic, whereas the rest of studies consider deterministic demand and endogenous selling price for remanufactured items. Thus, none of the these studies addresses the decentralized setting decision making process, neither demand uncertainty, nor the analysis of coordination by contract. Furthermore, we also incorporate the varying channel leadership in our study, which we consider it as a gap in this part of literature.

2.2 Reverse Channel Choice

Since recovery and take back practices are not the core competency of Original Equipment Manufacturers, many enterprises undertake recovery activities by cooperating with third party service providers. Therefore, selecting the best reverse channel choice becomes an apparent problem. Savaskan et al. [29] study the case of reverse channel choice in a closed-loop supply chain environment. Remanufactured products are as good as new product and can be sold in the same market and at the same price. Demand is assumed to be a decreasing linear function of selling price. Used products can be collected either by i) the manufacturer (M-collection), ii) the retailer (R-collection) and iii) the third-party collector (3P-collection). Remanufactured items are sold through the retailer. The party in charge of collection incurs a linear acquisition cost as well as investment in promotional/advertising activities to encourage customers to return their products. Under decentralized settings, they seek for optimal return rate, selling price and wholesale price. They conclude that R-collection channel is the most generous in terms of retail price and achieves the highest return rate. Furthermore, a two-part tariff contract is put forward that enhance the channel performance.

Atasu et al. [30] investigate how the collection cost structure influences the reverse channel choice. They consider the same reverse channel choice as Savaskan et al. [29], however, under different assumptions; firstly they assume differentiable new and remanufactured products, and secondly, they also include reverse logistics cost, which is a function of collection volume, in total cost of collection. They contrast the case where reverse logistics faces economies or diseconomies of scale in the operational cost of collection. As a real example, they compare the reverse logistics cost of Kodak, which remanufactures single use cameras, with TV producer, which to the volume or size of its products, have to make excessive effort to reach used products. By relying on volume-dependent cost of collection, they demonstrate that when there are economies of scale, retailer-managed collection outperforms other channel choices, while manufacturer-managed collection is optimal when there are diseconomies of scale.

Xu and Liu [31], using similar base model to Savaskan et al. [29], studies the reverse channel choice problem when customers purchasing behavior affect the pricing of the product. They argue that the customers have their valuation for a certain product, which impact their purchasing behavior; if customer has a lower price in mind, which is called “reference price", he becomes unwilling to purchase the product and as a result, demand is decreased. They incorporate the impact of reference price in commonly used demand function, $D(p) = \phi - \beta p - \delta(p - r)$. δ denotes the sensitivity of customers to difference between reference price and selling price of the product and r is the reference price. Moreover, they show that the retailer-managed channel outperforms other options, since the retailer benefits from being closer to the customer and it is more convenient for the retailer to engage in product collection. They also state that the retailer already has the required infrastructure to involve in collection of used products, while the rest of two channel need additional necessary investment.

Although all these studies investigate the decentralized setting of reverse supply chain, they do not cover the coordination issues explicitly. Moreover, they assume that in all channels, remanufacturer has enough power to lead the channel. Thus, they lack the consideration of collector/retailer's leadership in reverse channel.

2.3 Coordinating Contracts

Supply chain coordination has been one of the popular topics in Operations Management studies and has been extensively studied in the forward supply chain for many years. Since contracting literature origins from forward supply chain, we first review some studies in this context and then, we concentrate on those works that study the reverse supply chain contracts.

2.3.1 Forward Supply Chain Contracts

Supply chain (SC) contracts are employed to integrate the interdependencies and to share the risk among supply chain members (Arshinder [32]). Several contract mechanisms have been introduced in the literature such as wholesale price-only contract, revenue sharing contract and buy back contract; each of which tries to align supply chain members' incentive through diverse set of motives (Govindan et al. [16]). For instance, through a wholesale price-only contract, a supplier sets a wholesale price and the buyer may agree and order accordingly. In a revenue sharing contract, the buyer promises to share a percentage of his revenue with the supplier in exchange for a lower wholesale price that the supplier charges compared to wholesale price-only contract. We present a brief summary of commonly used contracts in forward supply chain studied in the literature.

Lariviere and Porteus [33] studies a two-echelon supply chain in a newsvendor setting. The manufacturer sets a wholesale price per unit purchased and the retailer, facing a random demand, orders accordingly for the following selling season. Selling price is exogenous and manufacturer and retailer incur production cost per unit and marginal cost per unit, respectively. They derive the optimal production quantity by maximizing the centralized profit. Then, they analyze the decentralized setting using a game theoretic approach and argues that channel can be coordinated only if the manufacturer sets a wholesale price equal to the manufacturing cost, which results in non-positive profit for the manufacturer. Therefore, wholesale price-only contract cannot achieve coordination. However, since it is easy-to-implement and does not need further administration, firms may employ the wholesale price-only contract than a coordinating contract that may need to administer. He puts forward two performance measures to evaluate the wholesale price-only contracts; the efficiency of the contract, which is the ratio of total supply chain profit through the given wholesale price to the system-wide optimizing wholesale price, and the second is the supplier's profit share. Through an experimental study, he demonstrates as the coefficient of variation decreases, both measure increases.

Revenue sharing contract is studied by Cachon and Lariviere [34]. Studying Blockbuster Inc. real business case, they model a supplier-retailer supply chain in video rental industry. Blockbuster video rental company incurs a high cost of procurement for videocassettes which results in low availability and profitability. Blockbuster and its suppliers were the pioneers in applying revenue sharing contract in real world. Suppliers decided to decrease their selling price in exchange of a percentage of Blockbuster's revenue. Based on Warren and Peers [35], Blockbuster's market share of videos rentals increased from 24% in 1997 to 40% in 2002. Cachon and Lariviere [34] considers a supplier selling to a newsvendor facing stochastic demand. Supplier charges a wholesale price per unit and the retailer agrees to share a percentage (ϕ) of its profit with the supplier. They demonstrate that there exist a menu of revenue sharing contract, that is a pair of wholesale price and ϕ, that can coordinate the channel. They state that revenue sharing contract allows for arbitrary allocation of profit between parties.

Giannoccaro and Pontrandolfo [36] extend the model proposed by Cachon and Lariviere [34], considering three-echelon supply chain; a retailer, a distributor and a manufacturer. The retailer faces a random demand and orders to the distributor. The distributor, in turn, orders to the manufacturer and sells them to the retailer. Therefore, there are two wholesale transfer prices and two revenue sharing parameters. Through a fixed price newsvendor problem, they solve the decentralized setting as well as revenue sharing setting and they contrast it to centralized setting in order to derive a coordinating revenue sharing contract. They show that the three-stage supply chain can be coordinated through revenue sharing contract. They also discuss the desirability of contracting schemes by participating firms and through a numerical study, they show that a proper contract design can achieve a win-win condition for all parties. Linh and Hong [37] extend the newsvendor model to a two-period setting, allowing the retailer to order at the beginning of each period. The supplier is the stackelberg leader and sets the contract terms. They confirm that revenue sharing contract can coordinate the supply chain in both periods. They also discuss that their model accommodates a higher flexibility due to the utilization of different revenue sharing parameters in each period.

Buy-back contract allows the retailer to order aggressively. Through a buy-back contract, the manufacturer, in addition to a wholesale price, sets a buy-back price for the leftovers in the retailer's inventory. Pasternack [18] study the implementation of a buy-back contract in a two-echelon supply chain. The manufacturer sells products at the wholesale price per unit and allows the retailer to return a percentage of its order quantity, in case of overstock, at a buy-back price less than the wholesale price. They solve the newsvendor problem, as the manufacturer being the Stackelberg leader. They first find the system optimal solution and compare it to the hierarchical situation, i.e. decentralized setting, in order to derive a set of wholesale price, buy-back price and return policy that results in coordination, i.e. maximizes total supply chain profit. It is demonstrated that return policy through which the manufacturer allows for unlimited returns with partial credit can coordinate the channel.

2.3.2 Reverse Supply Chain Contracts

Several studies have been conducted to evaluate the application and performance of existing common contracts in the context of reverse supply chain.

Karakayalı et al. [38] consider a reverse supply chain, consisting of a remanufacturer and a collector. The demand is a deterministic, decreasing linear function of selling price as in Savaskan et al. [29]. They formulate the return as in Bakal and Akcali [24], as an increasing linear function of acquisition price paid to the customers. Their study differs from other studies in the sense that they also study the decentralized setting where the retailer is the Stackelberg leader. They first assume that used products are homogenous. Using centralized channel (CC), where both collection and remanufacturing are done by a single agent, as the benchmark, they first derive system optimal acquisition price and selling price. Then, by solving both decentralized settings, i.e. remanufacutrer-driven (RDC) and colletor-driven (CDC) channels, they find the optimal decisions of the remanufacturer and the collector. In RDC, the remanufacturer decides wholesale price and selling price and the collector sets acquisition price, while in CDC, the collector sets wholesale price and acquisition price and the remanufacturer sets selling price, considering the wholesale price set by the collector. Then, they switch to the case with heterogenous used products and repeat the same steps. Through an extensive experimental study, they investigate RDC and CDC performances. It is shown that the centralized channel attains the highest collection rate and the lowest selling price compared to RDC and CDC. Regarding the outsourcing decision, it is demonstrated that when the demand for remanufactured parts decreases, or supply of used products increases, outsourcing becomes more favorable. Furthermore, if remanufacturing cost exceeds a certain threshold, CDC outperforms RDC channel, collecting a higher quantity. In our study, we also study the decentralized setting under both parties' lead. However, we assumed demand to be random and selling price of remanufactured item to be exogenous, which is the characteristic of highly competitive markets. Moreover, we investigate the coordination by contract via a price-only contract as well as revenue sharing contract.

De Giovanni [39] study the implementation of wholesale price-only contract and revenue sharing contract in a closed-loop supply chain setting, in which the manufacturer sells the product to the retailer who is in charge of selling products and collecting used products. They incorporate the impact of green advertising effort to the model due to the fact that it influences both sales quantity and return rate. They formulate demand and return as a function of customer's goodwill, that reflects the willingness and awareness of customer to environmental-friendly activities, which is itself a function of advertising efforts by both manufacturer and retailer. They incorporate the administration cost associated with revenue sharing contracts. They formulate manufacturer's and retailer's profit function and using a game-theoretic approach, they find the optimal wholesale price and advertising efforts by the retailer and the manufacturer. They conclude that the manufacturer and the retailer is moderately conservative regarding Reverse Revenue Sharing contract (RRSC), that is the retailer and the manufacturer is willing to participate in RRSC when the return's residual value is adequately higher than the administrative costs. They also consider the case without administrative cost, which is equivalent to roughly all prior studies of revenue sharing contract and contrast their results with prior revenue sharing contract studies in the literature. It is shown that under the case without administrative cost, revenue sharing contract provides a larger Pareto-improving region, which highlights the limitation of previous studies on revenue sharing contract. We assume demand to be stochastic, analyzing a more realistic environment, and return to be a linear function of acquisition price. Moreover, we assume price to be exogenous, whereas they assume price to be a function of wholesale price. Furthermore, we consider the decentralized setting under both remanufacturer and collector.

Yoo et al. [40] study a closed-loop supply chain setting. A supplier offers a supply contract with associated transfer payment to the retailer and the retailer, in turn, decides selling price and refund price. They model the demand as a function of selling price and refund price. The return is also formulated as an increasing linear function of refund price. We use the same approach to model the returns. They assume that the supplier has adequate power over the retailer and acts as Stackelberg leader. They first solve the model as if all decisions are made by the supplier and derive optimal selling price and refund price. Then, they study the implementation of various, commonly used contracts in order to achieve system optimal solution. They first show that wholesale price-only and buy-back contract cannot practically coordinate the closed-loop supply chain. Through a computational study, it is shown that the lowest selling price and the most generous return policy is always attained by quantity discount contract. Our study differs from Yoo et al. [40] due to demand assumption. We also study the coordination under revenue sharing contract. Furthermore, we consider the decentralized setting under collector's lead as well.

Govindan and Popiuc [15] consider two-echelon and three-echelon reverse supply chain (RSC) settings. Retailer is in charge of collection of used products and forwards the returned products to the manufacturer (via distributor in three-echelon chain). The manufacturer is the leader and decides sharing parameters. In both scenarios, wholesale transfer prices are assumed to be exogenous. Demand is deterministic and return is formulated as the customers' willingness to return their used products. They solve the problem under both decentralized and centralized setting and derive an upper-bound for the retailer's and the distributor's share of revenue above which the retailer and the distributor in two- and three-echelon RSC setting, respectively, would not accept revenue sharing. Under such circumstances, they show that performance measures and total supply chain profits improve through coordination with revenue sharing contracts on both two- and three-echelon RSC. In our study, we assume wholesale price as a decision variable. Moreover, they only study the case where surplus profit gain from coordination is shared between manufacturer and retailer equally, whereas in our study profit can be arbitrarily allocated between manufacturer and retailer. Last but not the least, we assume demand to be stochastic.

As Govindan et al. [16] state, the contracting literature for open loop and closed loop supply chain is still far behind the coordination by contracts research made within forward supply chain. Hence, by summarizing all research gaps we have pointed out in this chapter, using a game-theoretic approach, we study a two-echelon reverse supply chain with a remanufacturer and a collector, who is in charge of remanufacturing and collection of used products, respectively. We assume demand for remanufactured item to be random. Supply of used product is formulated as an increasing linear function of acquisition price, results in non-linear collection cost. Assuming a highly competitive market for remanufactured products, selling price of remanufactured product is exogenous. We investigate both decentralized, either remanufacturer's lead and collector's lead, as well as centralized settings. We show that there exists a wholesale price that coordinate reverse supply chain. It is shown that optimal acquisition price remains the same for any distribution of cost parameters between the remanufacturer and the collector, as long as the sum of costs is constant. Furthermore, we extend our model and investigate a revenue sharing mechanism. We demonstrate that there exist a menu of revenue sharing contract that coordinate reverse supply chain.

CHAPTER 3

WHOLESALE PRICE-ONLY CONTRACT

As our base setting, we consider a reverse supply chain, consisting of a remanufacturer, which is in charge of product remanufacturing, and a collector as the collector of used products from the end customers. We assume demand to be a random variable X with probability distribution function $f(X)$ and cumulative distribution function $F(X)$. We also assume that demand distribution has increasing generalized failure rate (IGFR), which is defined as follows: X has an increasing generalized failure rate if $h(X) = \frac{f(X)}{\overline{F}(X)}$ is weakly increasing for all X such that $F(X) < 1$ (Lariviere and Porteus [41]), that is:

$$\frac{dh(X)}{dX} = \frac{f'(X) \cdot \overline{F}(X) + f^2(X)}{\overline{F}^2(X)} \geqslant 0$$

It is shown that most of the commonly used demand distributions such as the normal, the exponential, the Weibull and the gamma have IGFR.

We model the returns as an increasing linear function of acquisition price, a, similar to Bakal and Akcali [24], $y(a) = ra$, where $r > 0$ is acquisition price sensitivity of returns. We assume that there is no upper bound for the quantity of used product in the market. For a complete description of the notation used in our model, refer to Table 3.1.

We start our analysis by considering the centralized solution in Section 3.1. Then, we consider decentralized settings under remanufacturer's lead (RL) and collector's lead (CL) in Section 3.2 and Section 3.3, respectively. In Section 3.4, we present an extensive numerical study. It is important to note that in any decentralized setting the

Table 3.1: Notation

a	Acquisition price given to the customer by the collector
w	Wholesale price that is charged by the collector
Q	Remanufacturer's order quantity
r	Acquisition price sensitivity of return
X	Random variable denoting the demand for the remanufactured product
$f(x)$	Probability density function of random demand
$F(x)$	Cumulative distribution function of random demand
p	Selling price of the remanufactured product
c_c	Handling cost per unit for the returns
c_r	Remanufacturing cost per unit

leader of the Stackelberg game will set the transfer price between the parties since the solution would be trivial otherwise.

3.1 The Centralized Setting

For the centralized setting, the only decision variable is the acquisition price, a. It is as if there is a single firm performing all the activities from collection to remanufacturing of used products. Therefore, wholesale price is irrelevant in this problem.

Total supply chain profit is given by:

Expected supply chain profit = expected sales revenue - remanufacturing cost - total cost of handling used products - cost of used product collection

Expected sales revenue is given by:

$$\begin{aligned} S(Q) &= p\Big(\int_0^Q xf(x)dx + \int_Q^\infty Qf(x)dx\Big) \\ &= p\Big(\int_0^Q xf(x)dx + Q - QF(Q)\Big) \\ &= p\Big(QF(Q) - \int_0^Q F(x)dx + Q - QF(Q)\Big) \\ &= p\Big(Q - \int_0^Q F(x)dx\Big) \end{aligned}$$

The above follows by integration by parts. Since the quantity remanufactured is equal to the quantity collected in the optimal solution, expected centralized profit can be written as follows:

$$\Pi_{SC}(a) = p\Big(y(a) - \int_0^{y(a)} F(x)dx\Big) - c_r y(a) - c_c y(a) - a y(a) \tag{3.1}$$

where $y(a) = r.a$ is the total number of returns. By substituting it in the objective function (3.1), we have:

$$\Pi_{SC}(a) = p\Big(ra - \int_0^{ra} F(x)dx\Big) - ra(c_r + c_c) - ra^2. \tag{3.2}$$

Proposition 3.1. *The optimal acquisition price for the centralized setting is characterized by:*

$$a_{SC} = \frac{p[1 - F(ra_{SC})] - (c_r + c_c)}{2}.$$

Proof. The first and second order derivatives of the profit function (3.2) with respect to a is given by:

$$\frac{d\Pi_{SC}(a)}{da} = pr[1 - F(ra)] - r(c_r + c_c) - 2ra \tag{3.3}$$

and

$$\frac{d^2\Pi_{SC}(a)}{da^2} = -pr^2 f(ra) - 2r < 0.$$

Since the second derivative is strictly negative, $\Pi_{SC}(a)$ is concave and by setting 3.3 to zero, we derive the optimal acquisition price. □

Corollary 3.1. *The centralized acquisition price increases in p and decreases in c_r and c_c.*

Proof. We take the first derivative of centralized acquisition price with respect to p:

$$\frac{da_{SC}}{dp} = \frac{1 - F(ra_{SC}) - pr\left(\frac{da_{SC}}{dp}\right) f(ra_{SC})}{2}.$$

By rearranging the equation, we will have:

$$\frac{da_{SC}}{dp} = \frac{1 - F(ra_{SC})}{2 + pr \cdot f(ra_{SC})} > 0.$$

Next, We take the first derivative of centralized acquisition price with respect to c_r:

$$\frac{da_{SC}}{dc_r} = \frac{-pr \cdot f(ra_{SC})\frac{da_{SC}}{dc_r} - 1}{2}.$$

Then, by simplifying the equation, we have:

$$\frac{da_{SC}}{dc_r} = \frac{-1}{2 + pr \cdot f(ra_{SC})} < 0.$$

Finally, we take the derivative with respect to c_c:

$$\frac{da_{SC}}{dc_c} = \frac{-pr \cdot f(ra_{SC})\frac{da_{SC}}{dc_c} - 1}{2}.$$

By rearranging the equation, we will have:

$$\frac{da_{SC}}{dc_c} = \frac{-1}{2 + pr \cdot f(ra_{SC})} < 0,$$

which completes the proof. □

Proposition 3.2. *The acquisition price decreases, whereas the quantity of used products collected increases as r increases.*

Proof. As we take the first derivative of the centralized acquisition price with respect to r, we have:

$$\frac{da_{SC}}{dr} = \frac{p\Big(- f(ra)(a + r\frac{da_{SC}}{dr})\Big)}{2}.$$

By rearranging the equation, we have:

$$\frac{da_{SC}}{dr} = \frac{-pf(ra).a}{2 + prf(ra)} < 0.$$

Since $y_{SC} = ra_{SC}$, we can show that:

$$\frac{dy_{SC}}{dr} = a_{SC} + r\frac{da_{SC}}{dr} = \frac{2a_{SC}}{2 + prf(ra_{SC})} > 0.$$

$\square$

Thus, although the collector pays a lower acquisition price as r increases, the quantity of used products collected still increases in r.

3.2 Decentralized Setting: Remanufacturer's Lead

We continue our analysis by considering the decentralized channel under remanufacturer's lead (RL). The sequence of events under RL is given as follows:

- The remanufacturer sets w and decides its order quantity, Q.
- The collector sets a and collects $y(a) = r.a$ units and delivers $min\{y(a), Q\}$ units to the remanufacturer.

We start with the analysis of the collector's problem. Given the wholesale price, w and the order quantity, Q, the collector maximizes her own profit with respect to the acquisition price, a.

$$\begin{aligned} max \quad & \Pi_C^{RL}(a) = (w - c_c - a)y(a) \\ s.to \quad & y(a) \leqslant Q \end{aligned} \tag{3.4}$$

where $y(a) = ra$, the number of collected used products.

Proposition 3.3. *The optimal acquisition price for the collector under RL is:*

$$a_{RL} = min\{\frac{w - c_c}{2}, \frac{Q}{r}\}. \tag{3.5}$$

Proof. By taking first and second derivative of the collector's problem (3.4) with respect to a, we have:

$$\frac{d\Pi_C^{RL}(a)}{da} = wr - c_c r - 2ar \tag{3.6}$$

and

$$\frac{d^2\Pi_C^{RL}(a)}{da^2} = -2r < 0.$$

Therefore, we derive optimal unconstrained acquisition price by setting 3.6 to zero. However, since the collector would not collect more than what the remanufacturer is willing to get, Q, the acquisition price must not exceeds Q/r. As a result, the optimal response of the collector to the remanufacturer's decisions is $a_{RL} = min\{\frac{w-c_c}{2}, \frac{Q}{r}\}$.

□

Accordingly,

$$y_{RL} = min\{\frac{r(w-c_c)}{2}, Q\}.$$

Then, observing collector's strategy, the remanufacturer maximizes his own profit with respect to the wholesale price and order quantity. When formulating the remanufacturer's problem, we first note that the remanufacturer's available inventory is the minimum of its order quantity and the quantity collected by the collector:

$$min\{Q, min\{r\frac{w-c_c}{2}, Q\}\} = min\{Q, r\frac{w-c_c}{2}\}.$$

Hence, the remanufacturer's profit can be represented as:

$$\Pi_R^{RL}(Q, w) = p\Big(S - \int_0^S F(x)dx\Big) - (c_r + w)S, \tag{3.7}$$

where $S = min\{Q, r\frac{w-c_c}{2}\}$. Note that $r\frac{w-c_c}{2}$ is the maximum amount that the collector would collect. Hence, the remanufacturer cannot get more than that even if it orders more; that is even if $Q > r\frac{w-c_c}{2}$. Hence, in formulating the remanufacturer's problem, there is no need to consider the cases where $Q > r\frac{w-c_c}{2}$. As a result, the remanufacturer's problem 3.7 can be revised as:

$$\begin{aligned} &max. \quad \Pi_R^{RL}(Q, w) = p\Big(Q - \int_0^Q F(x)dx\Big) - c_rQ - wQ \qquad (3.8)\\ &s.to \quad Q \leqslant r\frac{w-c_c}{2}. \end{aligned}$$

Proposition 3.4. *The optimal wholesale price offered by the remanufacturer to the collector under RL is characterized by:*

$$w^*_{RL} = \frac{p\Big(1 - F(y^*_{RL})\Big) - c_r + c_c}{2}, \tag{3.9}$$

where $y^*_{RL} = r\frac{w^*_{RL} - c_c}{2}$.

Proof. Note that the unconstrained remanufacturer's problem (3.8) would set $w = 0$, which means that the constraint is binding, the remanufacturer's problem reduces to:

$$\Pi_R^{RL}(w) = p\Big(\frac{r(w - c_c)}{2} - \int_0^{\frac{r(w-c_c)}{2}} F(x)dx\Big) - (c_r + w)\frac{r(w - c_c)}{2}. \tag{3.10}$$

The first and second derivative of the remanufacturer's profit function (3.10) with respect to w is given by

$$\frac{d\Pi_R^{RL}}{dw} = \frac{pr}{2}\Big[1 - F\Big(r\frac{w_{RL} - c_c}{2}\Big)\Big] - \frac{r}{2}(c_r - c_c) - rw \tag{3.11}$$

and

$$\frac{d^2\Pi_R^{RL}}{dw^2} = -\frac{pr^2}{4}f\Big(r\frac{w_{RL} - c_c}{2}\Big) - r < 0.$$

Hence, we derive the optimal wholesale price by setting 3.11 to zero. □

Finally, through 3.5 and 3.9, optimal acquisition price is characterized by

$$a^*_{RL} = \frac{p[1 - F(y^*_{RL})] - (c_r + c_c)}{4}. \tag{3.12}$$

where $y^*_{RL} = r\frac{w^*_{RL} - c_c}{2} = ra^*_{RL}$.

Proposition 3.5. *Under RL, acquisition price increases, whereas the quantity of collected items increases as r increases.*

Proof. As we take the first derivative of the optimal acquisition price with respect to r, we have:

$$\frac{da^*_{RL}}{dr} = \frac{p}{4}\Big[-f(ra^*_{RL})\Big(a^*_{RL} + r\frac{da^*_{RL}}{dr}\Big)\Big].$$

Now, by rearranging the above equation, we get:

$$\frac{da^*_{RL}}{dr} = \frac{-pa^*_{RL}f(ra^*_{RL})}{4 + prf(ra^*_{RL})} < 0.$$

Thus, since $y^*_{RL} = ra^*_{RL}$, it is straight forward to show that:

$$\frac{dy^*_{RL}}{dr} = a^*_{RL} + r\frac{da^*_{RL}}{dr} = \frac{4a^*_{RL}}{4 + prf(ra^*_{RL})} > 0.$$

□

Hence, as the price sensitivity of returns, r, increases, although acquisition price paid by the collector decreases, the quantity of used items collected still increases.

Corollary 3.2. *In the decentralized case when remanufacturer is the leader, (i) acquisition price, (ii) the collector's and (iii) the remanufacturer's profit do not change when c_r and/or c_c individually changes as long as their sum remains the same.*

Proof. From 3.12, it is straight forward to prove (i). Since $y^*_{RL} = ra^*_{RL}$, as the optimal acquisition price remains the same when $c_r + c_c$ is constant, therefore y^*_{RL} also remains constant. For (ii) and (iii), we start with the collector's profit function:

$$\begin{aligned}\Pi^{RL}_{C} &= ra(w - c_c - a) = r\Big(\frac{w^*_{RL} - c_c}{2}\Big)\Big[w^*_{RL} - c_c - \Big(\frac{w^*_{RL} - c_c}{2}\Big)\Big] \\ &= r\Big(\frac{w^*_{RL} - c_c}{2}\Big)^2 = r \cdot (a^*_{RL})^2.\end{aligned}$$

Therefore, (ii) is proved. We continue with calculation of total supply chain profit in decentralized setting, sum of the collector and the remanufacturer profit function, we have:

$$\Pi^{RL}_{SC} = p\Big(ra^*_{RL} - \int_0^{ra^*_{RL}} F(x)dx\Big) - ra^*_{RL}(c_r + w) + ra^*_{RL}(w^*_{RL} - c_c - a^*_{RL})$$

$$= p\Big(ra^*_{RL} - \int_0^{ra^*_{RL}} F(x)dx\Big) - ra^*_{RL}(c_r + c_c) - r(a^*_{RL})^2.$$

Since total supply chain profit does not change as long as total cost is constant, parallel with the proof of (ii), we conclude that the difference of total profit and the collector's profit, which is the remanufacturer's profit, remains unchanged. □

The corollary indicates that no matter how the reverse channel costs are distributed between the collector and the remanufacturer, the acquisition price, and consequently, the quantity of used products collected, the collector and the remanufacturer's profits remain the same. It is interesting that a similar result is observed in Karakayalı et al. [38] as well, even if they don't state it explicitly. Briefly, they study a two-echelon, remanufacturer-collector reverse supply chain, with price-dependent demand for remanufactured products and supply of used products as a linear function of acquisition fee. Cost parameters are remanufacturing cost c^r, mandatory cleaning cost c^m and collection cost c^l. They also consider both remanufacturer-driven channel (RDC) and Collector-driven channel (CDC). It should be noted that there is no demand uncertainty and return function is a little different, they incorporate a constant return even if acquisition price is zero ($y(f) = \alpha + \beta f$). In both settings, they derive the optimal acquisition fee f and selling price p and wholesale price w. Optimal acquisition price and wholesale price for RCL are given by

$$f^*(w) = \frac{w - c^l}{2} - \frac{\alpha}{2\beta}$$

and

$$w^* = \begin{cases} \frac{\beta a + (\beta c^l - \alpha)(\beta + b) + \beta b(h - c^r - c^m)}{\beta(\beta + 2b)} & , \phi_{RDC} \geqslant 0 \\ \frac{h + s + c^l - c^m}{2} - \frac{\alpha}{2\beta} & , \phi_{RDC} < 0. \end{cases}$$

where $\phi_{RDC} = 2a - 2b(s + c^r) - \alpha - \beta(h + s - c^l - c^m)$. As they argue, if $\phi_{RDC} \geqslant 0$, the constraint in reremanufacturer's problem is binding. In our setting, ϕ_{RDC} is always greater than 0, and the corresponding optimal acquisition price in Karakayalı et al. [38] is given by

$$f^* = \frac{\frac{\beta a + (\beta c^l - \alpha)(\beta + b) + \beta b(h - c^r - c^m)}{\beta(\beta + 2b)} - c^l}{2} - \frac{\alpha}{2\beta}$$

$$= \frac{\beta a + (\beta c^l - \alpha)(\beta + b) + \beta b(h - c^r - c^m) - \beta(\beta + 2b)c^l}{2\beta(\beta + 2b)} - \frac{\alpha}{2\beta}$$

$$= \frac{\beta a - \alpha(\beta + b) + \beta bh - \beta b(c^l + c^m + c^r)}{2\beta(\beta + 2b)} - \frac{\alpha}{2\beta}.$$

We observe that, as far as summation of cost parameters (c^l, c^m, c^r) is constant, acquisition fee remains constant.

Next, we illustrate Corollary 3.2 by a numerical example. In this example, for sake of simplicity, we assume demand to be Normally distributed with mean, $\mu = 2{,}000$, and standard deviation, $\sigma = 500$. We also take $p = 1$ and $r = 15{,}000$. We calculate all optimal values for wholesale price, acquisition fee and collected amount of used products as well as the remanufacturer's and collector's profits under three different cost parameter sets. The rest of parameters remain constant in all cases. The results are given in Table 3.2. As we describe in Corollary 3.2, since acquisition fee does not change, amount of collected products as well as individual profits remain constant. It is expected that wholesale price would increase as c_c increases.

Table 3.2: Comparison of channel performance under varying remanufacturing and used products handling cost, but same total cost, under RL

c_r	c_c	$c_r + c_c$	y	w	a	Π_R	Π_C	$\Pi_R + \Pi_C$
0.5	0.1	0.6	1,249.7	**0.2666**	0.0833	277.01	104.12	381.13
0.4	0.2	0.6	1,249.7	**0.3666**	0.0833	277.01	104.12	381.13
0.25	0.35	0.6	1,249.7	**0.5166**	0.0833	277.01	104.12	381.13

Proposition 3.6. *There exists a wholesale price, w' characterized by $F^{-1}\left(\frac{p-c_r-w'}{p}\right) = r\frac{w'-c_c}{2}$, that coordinates the reverse supply chain under RL.*

Proof. Recalling that centralized acquisition price is characterized by:

$$a_{SC} = \frac{p[1 - F(ra_{SC})] - (c_r + c_c)}{2},$$

we proceed by showing that the acquisition price that would be set under RL corresponding to w', $a_{RL}|_{w=w'} = \frac{w'-c_c}{2}$, satisfies:

$$\frac{w' - c_c}{2} = \frac{p\Big(1 - F(r.\frac{w'-c_c}{2})\Big) - (c_r + c_c)}{2}.$$

Since w' satisfies $F^{-1}\Big(\frac{p-c_r-w'}{p}\Big) = r\frac{w'-c_c}{2}$:

$$\frac{w' - c_c}{2} = \frac{p\Big(1 - \frac{p-w'-c_r}{p}\Big) - (c_r + c_c)}{2}.$$

By simplifying the right hand side, the equality would follow. □

Thus, not only the collector would collect the same quantity as in centralized setting, the remanufacturer would be able to attain as many items as it is willing to.

Proposition 3.7. *Amount of collected used products in the decentralized setting under RL is less than that in the centralized case.*

Proof. When we plug w' in first derivative of the remanufacturer's profit function with respect to w, we have:

$$\frac{d\Pi_R^{RL}}{dw}|_{w=w'} = \frac{pr}{2}\Big[1 - F\Big(\frac{r(w' - c_c)}{2}\Big)\Big] - \frac{r}{2}(c_r - c_c) - rw'$$

Since w' satisfies $F^{-1}\Big(\frac{p-c_r-w'}{p}\Big) = \frac{r(w'-c_c)}{2}$:

$$\frac{d\Pi_R^{RL}}{dw}|_{w=w'} = \frac{pr}{2}\Big[1 - \Big(\frac{p - c_r - w'}{p}\Big)\Big] - \frac{r}{2}(c_r - c_c) - rw' = \frac{r}{2}(c_c - w') < 0.$$

Recall that Π_R^{RL} is strictly concave, and that $y_{RL} = \frac{r(w-c_c)}{2}$, since $\frac{d\Pi_R^{RL}}{dw}|w' < 0$, $w' > w^*_{RL}$. Hence $y_{RL}|w' > y_{RL}|w^*_{RL}$. □

Here, we provide an illustrative example to give a better insight into our results. Considering the same parameters set and demand distribution that we had for previous example, we compute the optimal acquisition fee, wholesale price, collected amount and individual as well as total profit of the remanufacturer and the collector under i) centralized, ii) coordinated channel via w' and iii) decentralized setting under RL.

Table 3.3: Comparison of centralized, coordinated and decentralized setting under RL

	y	w	a	Π_R	Π_C	$\Pi_R + \Pi_C$
Decentralized	1,249.7	0.2666	0.0833	277.01	104.12	381.13
Coordinated	1,563.8	0.3085	0.1043	246.65	163.03	409.68
Centralized	1,563.8	-	0.1043	-	-	409.68

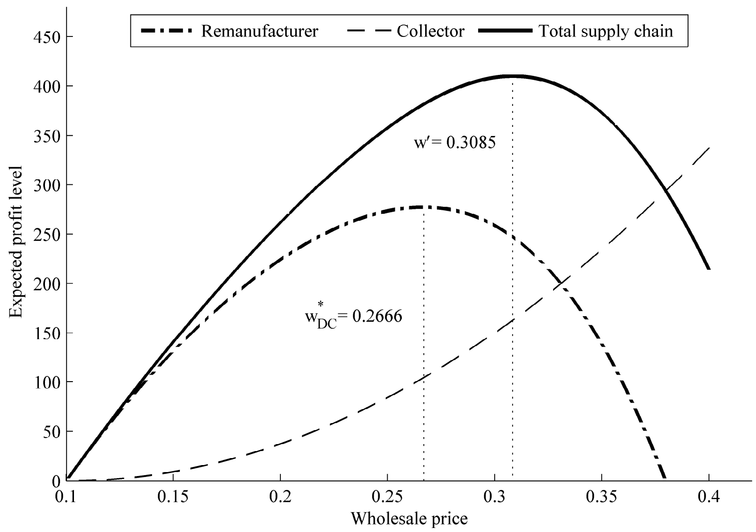


Figure 3.1: Expected profit level of the remanufacturer, collector and total supply chain with respect to the wholesale price under RL

The results are given in Table 3.3. We also illustrate the expected profit as a function of the wholesale price in Figure 3.1.

Table 3.3 clearly illustrates the characteristic of w' and how it yields coordination. As we have shown analytically, via w' we reach the centralized-level total profit and centralized-level quantity of used products collected. Figure 3.1 displays remanufacturer's, collector's and total supply chain profit level with respect to w. remanufacturer's profit reaches its maximum amount at $w^*_{RL} = 0.2666$, whilst supply chain profit is maximized at $w' = 0.3085$.

3.3 Decentralized Setting: Collector's Lead

We now consider the decentralized setting under collector's lead (CL). The sequence of events is given as follows:

- The collector sets w and a and collects accordingly.
- The remanufacturer decides on his order quantity, Q.

Using backward induction, we will first consider the remanufacturer's problem, given wholesale price and acquisition price set by the collector. Assuming that the quantity of used products collected is sufficient, the remanufacturer's expected profit is given by:

$$\Pi_R^{CL}(Q) = p\Big(Q - \int_0^Q F(x)dx\Big) - (c_r + w)Q. \tag{3.13}$$

Proposition 3.8. *The remanufacturer's optimal order quantity is:*

$$F(Q^*) = \frac{p - c_r - w}{p}. \tag{3.14}$$

Proof. By taking first and second derivatives of remanufacturer's profit (3.13) with respect to Q, we have:

$$\frac{d\Pi_R^{CL}(Q)}{dQ} = p\Big(1 - F(Q)\Big) - c_r - w \tag{3.15}$$

and

$$\frac{d^2\Pi_R^{CL}(Q)}{dQ^2} = -pf(Q) < 0.$$

Hence, Π_R is concave and by setting 3.15 to zero, we derive the optimal order quantity. □

Lemma 3.1. *In the optimal solution, the quantity collected by the collector will always match the remanufacturer's order quantity.*

Proof. Assume that $Q^* > r.a^*$ and let $\Pi_C^*(w^*, a^*)$ be the collector's expected profit. There exists $w' > w^*$ such that $Q^* > Q' > r.a^*$, $\Pi_C'(w', a^*) > \Pi_C^*(w^*, a^*)$ as the collector's sales would be $r.a^*$ in both cases. Hence, we conclude $Q^* \leqslant r.a^*$. On the other hand, when $Q^* < r.a^*$, then there exists $a' < a^*$ such that $Q^* < r.a' < r.a^*$, and $\Pi_C'(w^*, a') > \Pi_C^*(w^*, a^*)$ as the collector's sales would be Q^* under both a^* and a'. Hence, $Q^* > r.a^*$. Therefore, $Q^* = r.a^*$. □

As $F(x)$ is strictly increasing and continuous, there is a one-to-one mapping between w and Q and from 3.14, we can derive

$$w(Q) = p\Big(1 - F(Q)\Big) - c_r \tag{3.16}$$

Now, we proceed by characterizing the collector's profit function as follows:

$$\Pi_C^{CL}(Q, a) = w(Q).Q - c_c.Q - a.Q. \tag{3.17}$$

By plugging Eq. 3.16 into Eq. 3.17 and using Lemma 3.1, we revise the collector's profit (3.17) as follows:

$$\Pi_C^{CL}(Q) = \Big[p\Big(1 - F(Q)\Big) - c_r\Big]Q - c_cQ - \frac{Q^2}{r}. \tag{3.18}$$

Proposition 3.9. *The optimal quantity collected by the collector under CL is characterized by*

$$Q^*_{CL} = \frac{p\Big(1 - F(Q^*_{CL})\Big) - (c_r + c_c)}{\frac{2}{r} + pf(Q^*_{CL})}.$$

Proof. We take first and second order derivative of the collector's profit (3.18) with respect to Q:

$$\begin{aligned}
\frac{d\Pi_C^{CL}(Q)}{dQ} &= \Big(-pf(Q)\Big)Q + p\Big(1 - F(Q)\Big) - c_r - c_c - \frac{2Q}{r} \\
&= p\Big(1 - F(Q)\Big)\Big(1 - \frac{Qf(Q)}{1 - F(Q)}\Big) - c_r - c_c - \frac{2Q}{r}
\end{aligned} \tag{3.19}$$

and

$$\begin{aligned}
\frac{d^2\Pi_C^{CL}(Q)}{dQ^2} = -pf(Q)\Big(1 - \frac{Qf(Q)}{1 - F(Q)}\Big) \\
-p\Big(1 - F(Q)\Big)\Big[\frac{f(Q)}{1 - F(Q)} + Q\frac{d}{dQ}\Big(\frac{f(Q)}{1 - F(Q)}\Big)\Big] - \frac{2}{r}.
\end{aligned}$$

From 3.19, it can be shown that at optimality, $1 - \frac{Qf(Q)}{1-F(Q)} > 0$. Moreover, due to IGFR characteristic of demand distribution, $Q\frac{d}{dQ}\Big(\frac{f(Q)}{1-F(Q)}\Big) > 0$. Hence, we conclude that the collector's profit function is concave with respect to Q and we derive the optimal sales quantity of collector by setting 3.19 to zero. $\square$

Accordingly, utilizing Lemma 3.1, the optimal acquisition price for collector is characterized by:

$$a^*_{CL} = \frac{p\Big(1 - F(Q^*_{CL})\Big) - (c_r + c_c)}{2 + prf(Q^*_{CL})} \tag{3.20}$$

where $Q^* = r.a^*$.

Proposition 3.10. *Under CL, acquisition price decreases, whereas the quantity of collected items increases as r increases.*

Proof. First, we take the first derivative of $Q^*_{CL} = \frac{p\Big(1-F(Q^*_{CL})\Big)-(c_r+c_c)}{\frac{2}{r}+pf(Q^*_{CL})}$ with respect to r.

$$\frac{dQ^*_{CL}}{dr} = \frac{1}{\Big(\frac{2}{r} + pf(Q^*_{CL})\Big)^2}\Bigg\{\Big(-pf(Q^*_{CL})\frac{dQ^*_{CL}}{dr}\Big)\Big(\frac{2}{r} + pf(Q^*_{CL})\Big) \\ -\Big[p\Big(1 - F(Q^*_{CL})\Big) - (c_r + c_c)\Big]\Big(-\frac{2}{r^2} + pf'(Q^*_{CL})\frac{dQ^*_{CL}}{dr}\Big)\Bigg\}$$

By rearranging the above equation, we get:

$$\frac{dQ^*_{CL}}{dr} = \frac{\frac{2}{r^2}\tau}{\Big(\frac{2}{r} + pf(Q^*_{CL})\Big)^2\Big[\frac{2}{r}pf(Q^*_{CL}) + p^2f^2(Q^*_{CL}) + pf'(Q^*_{CL})\tau\Big]}$$

in which $\tau = p\Big(1 - F(Q^*_{CL})\Big) - (c_r + c_c) > 0$. By relying on IGFR demand distribution assumption, we conclude that $\frac{dQ^*_{CL}}{dr} > 0$. Now, by taking the first derivative of the optimal acquisition price (Eq. 3.20), we have:

$$\frac{da^*_{CL}}{dr} = \frac{1}{\Big(2 + prf(Q^*_{CL})\Big)^2}\Bigg\{\Big[-pf(Q^*_{CL})\Big(\frac{dQ^*_{CL}}{dr}\Big)\Big]\Big(2 + prf(Q^*_{CL})\Big) \\ -\Big[p\Big(1 - F(Q^*_{CL})\Big) - (c_r + c_c)\Big]\Big[pf(Q^*_{CL}) + prf'(Q^*_{CL})\Big(\frac{dQ^*_{CL}}{dr}\Big)\Big]\Bigg\}.$$

As we rearrange the above equation, we get:

$$\frac{da^*_{CL}}{dr} = \frac{\beta - \frac{dQ^*_{CL}}{dr}\left\{p^2rf^2(Q^*_{CL}) + prf'(Q^*_{CL})\Big[p\Big(1 - F(Q^*_{CL})\Big) - (c_r + c_c)\Big]\right\}}{\Big(2 + prf(Q^*_{CL})\Big)^2},$$

in which $\beta = -2pf(Q^*_{CL})\frac{dQ^*_{CL}}{dr} - pf(Q^*_{CL})\Big[p\Big(1 - F(Q^*_{CL})\Big) - (c_r + c_c)\Big] < 0$. Now, if $f'(Q^*_{CL}) \geqslant 0$, since $p\Big(1 - F(Q^*_{CL})\Big) - (c_r + c_c) > 0$ and $\frac{dQ^*_{CL}}{dr} > 0$, $\frac{da^*_{CL}}{dr} < 0$. On the other hand, if $f'(Q^*_{CL}) < 0$, by rearranging the above equation, we get:

$$\frac{da^*_{CL}}{dr} = \frac{\beta - \frac{dQ^*_{CL}}{dr}\left\{\theta + p^2rf^2(Q^*_{CL}) + p^2rf'(Q^*_{CL})\Big(1 - F(Q^*_{CL})\Big)\right\}}{\Big(2 + prf(Q^*_{CL})\Big)^2},$$

in which $\theta = -prf'(Q^*_{CL})(c_r + c_c) > 0$ and $\beta = -2pf(Q^*_{CL})\frac{dQ^*_{CL}}{dr} - pf(Q^*_{CL})\Big[p\Big(1 - F(Q^*_{CL})\Big) - (c_r + c_c)\Big] < 0$. Since, $\frac{dQ^*_{CL}}{dr} > 0$ and that, we assume demand distribution to have IGFR, we conclude that $\frac{da^*_{CL}}{dr} < 0$. □

Corollary 3.3. *Acquisition price does not change as long as sum of total handling and remanufacturing cost remains constant.*

Proof. From 3.20, since $Q^* = r.a^*$, when total remanufacturing and handling cost does not change, the acquisition price remains unchanged. □

Corollary 3.3 dictates that no matter how total cost has been shared between the remanufacturer and the collector, channel performance is constant. A similar result is observed in Karakayalı et al. [38] as well, even if they don't explicitly mention it. In collector-driven channel (CDC), they derive the optimal acquisition price for $\phi_{CDC} \geqslant 0$, which is always the case in our setting, as follows:

$$f^* = \frac{a\beta - \alpha(4\beta + b) + b\beta(h - c^r - c^m - c^l)}{2\beta(2\beta + b)}.$$

Just like what we did for decentralized setting under RL, we summarized the results given by our numerical example in Table 3.4. All parameters are the same and we calculate all optimal values of wholesale price, acquisition fee and remanufacturer's, collector's as well as total profit under three different cost parameter settings.

Table 3.4: Comparison of channel performance under varying remanufacturing and used products handling cost, but same total cost, under CL

c_r	c_c	$c_r + c_c$	y	w	a	Π_R	Π_C	$\Pi_R + \Pi_C$
0.5	0.1	0.6	1,138.4	**0.4576**	0.0759	39.66	320.66	360.32
0.4	0.2	0.6	1,138.4	**0.5576**	0.0759	39.66	320.66	360.32
0.25	0.35	0.6	1,138.4	**0.7076**	0.0759	39.66	320.66	360.32

It can be seen in Table 3.4 that, except wholesale price, acquisition price and consequently, collected amount of used products, the collector's and the remanufacturer's profit remain constant as far as sum of c_r and c_c is constant.

Proposition 3.11. *There exists a wholesale price w' characterized by $F^{-1}\Big(\frac{p-w'-c_r}{p}\Big) = r\frac{w'-c_c}{2}$, that coordinates the reverse supply chain under CL.*

Proof. When w' is fixed, Lemma 3.1 is no longer valid and we need to solve the collector's problem under such circumstances. From the remanufacturer's problem (3.13), we have the optimal order quantity given by

$$F(Q) = \frac{p - c_r - w}{p}.$$

When we plug w', the remanufacturer's order quantity becomes $Q = F^{-1}\Big(\frac{p-w'-c_c}{p}\Big)$. Now, we continue with the collector's problem as we force it to set the wholesale price equal to w'.

$$\Pi_C^{CL}(a) = (w' - c_c - a) \cdot min\Big\{F^{-1}\Big(\frac{p - w' - c_r}{p}\Big), ra\Big\}. \tag{3.21}$$

As we see, the only decision variable is the acquisition price. Since w' satisfies $F^{-1}\Big(\frac{p-w'-c_r}{p}\Big) = r\frac{w'-c_c}{2}$, we can revise the collector's profit function (3.21).

$$\Pi_C^{CL}(a) = (w' - c_c - a) \cdot min\Big\{r\frac{w' - c_c}{2}, ra\Big\}. \tag{3.22}$$

Let's first assume that $ra \geqslant r\frac{w'-c_c}{2}$. Thus, the collector's problem becomes a decreasing function of a and since $a \geqslant \frac{w'-c_c}{2}$, $a = \frac{w'-c_c}{2}$ would be the maximizer of the collector's profit function. On the other hand, when we assume $a \leqslant \frac{w'-c_c}{2}$, the profit function (3.22) would be as follows:

$$\Pi_C^{CL}(a) = ra(w' - c_c - a). \tag{3.23}$$

It is straight forward to show that this function is concave with respect to a and the optimal acquisition price would be $a = \frac{w'-c_c}{2}$. We conclude that the collector would collect the same amount that the remanufacturer orders, i.e. Lemma 3.1 is also valid for w'. As we show in Proposition 3.6, $a = \frac{w'-c_c}{2}$ can coordinate the reverse supply chain. $\square$

Proposition 3.12. *The quantity of collected used products in the decentralized setting under CL is less than that of centralized case.*

Proof. By plugging $p\overline{F}(Q^*) = w' + c_r$ and $Q^* = r\frac{w'-c_c}{2}$ into 3.19,

$$\frac{d\Pi_C^{CL}(Q)}{dQ}|_{Q=Q^*} = p\overline{F}(Q^*)\Big(1 - \frac{Q^* f(Q^*)}{\overline{F}(Q^*)}\Big) - c_r - c_c - \frac{2Q^*}{r}$$

$$= p\overline{F}(Q^*) - pQ^* f(Q^*) - c_r - c_c - \frac{2Q^*}{r},$$

we have

$$\frac{d\Pi_C^{CL}(Q)}{dQ}|_{Q=Q^*} = w' + c_r - \frac{pr}{2}(w' - c_c)f(Q^*) - c_r - c_c - w' + c_c$$

$$= -\frac{pr}{2}(w' - c_c)f(Q^*) < 0.$$

Since we have already show that the collector's profit is concave in Q, and that $\frac{d\Pi_C^{CL}(Q)}{dQ}|_{Q=Q^*} < 0$, $Q^*_{CL} < Q^*$. $\square$

We next consider the example setting we previously used under CL. Figure 3.2 depicts the optimal quantity of used products collected and optimal wholesale price under CL and centralized channel. Based on our base parameters set, coordinating quantity of used products collected is $Q' = 1563.8$ which is achieved by setting $w' = 0.3085$ in decentralized setting under CL. The rest of calculated values are summarized in Table 3.5.

It can be seen from Table 3.5 that in this particular example based on our base parameters set, remanufacturer's lead channel outperforms collector's lead channel. De-

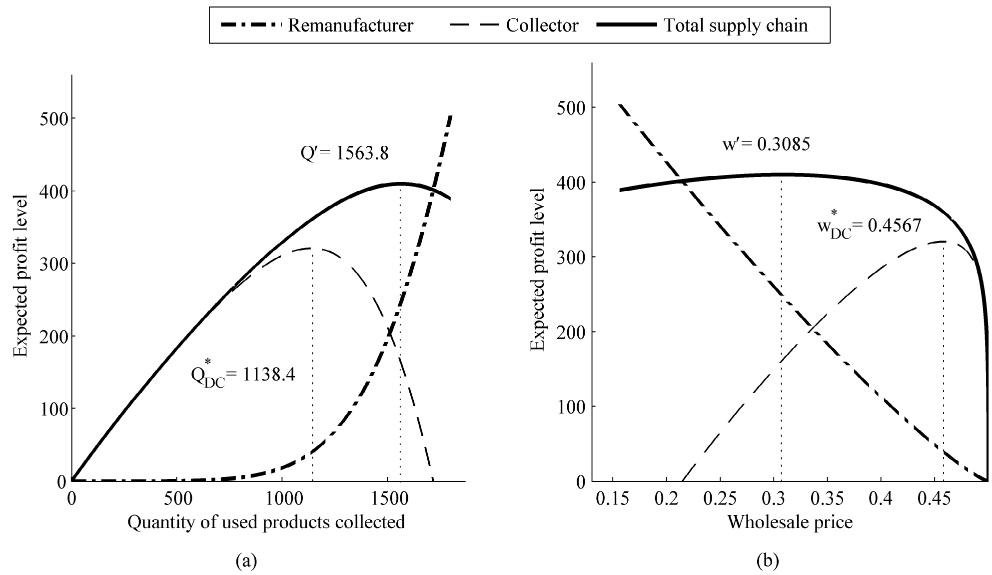


Figure 3.2: Expected profit level of remanufacturer and collector under CL channel as well as total supply chain with respect to (a) quantity of used products collected and (b) wholesale price

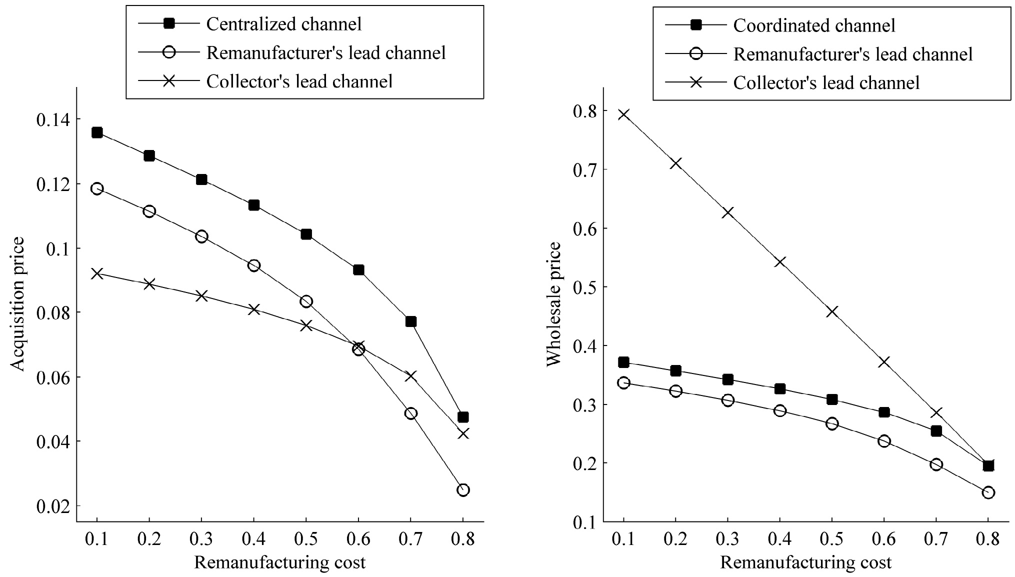
Table 3.5: Comparison of centralized, coordinated and decentralized settings

	y	w	a	Π_R	Π_C	$\Pi_R + \Pi_C$
Decentralized (CL)	1,138.4	0.4576	0.0759	39.66	320.66	360.32
Decentralized (RL)	1,249.7	0.2666	0.0833	277.01	104.12	381.13
Coordinated	1,563.8	0.3085	0.1043	246.65	163.03	409.68
Centralized	1,563.8	-	0.1043	-	-	409.68

centralized setting under RL collects $1{,}249.7$ many used items compared to $1{,}138.4$ under CL and gains higher total profit of 381.13 compared to 360.32 under CL.

3.4 Numerical Study

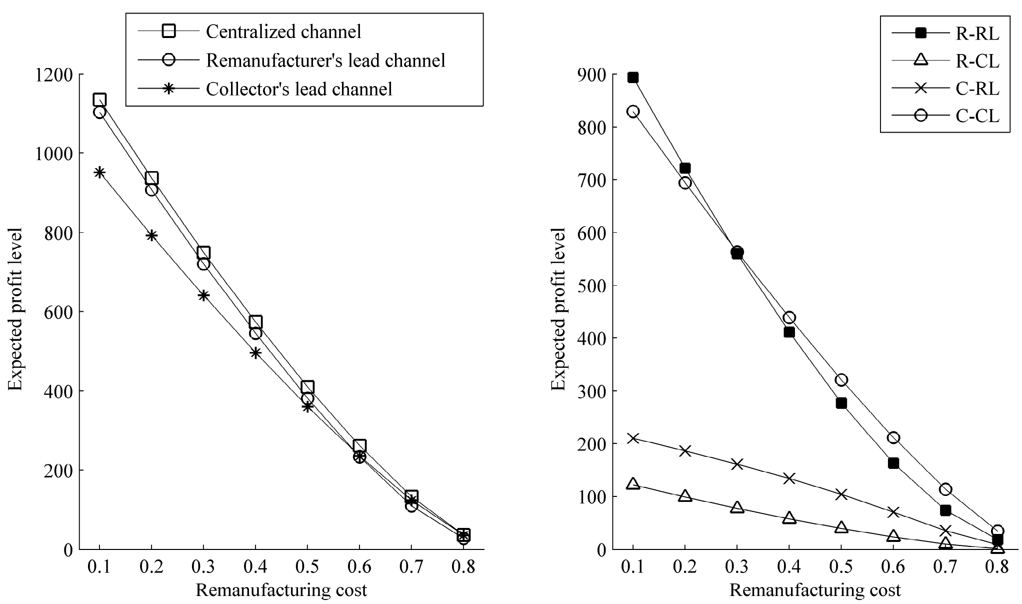
In order to get better managerial insights, we conduct an extensive numerical study. We perform experiments with different remanufacturing costs, used products handling costs, selling prices, demand standard deviations and acquisition price sensitivities to observe the behavior of channel performance under different settings. Through our study, the base parameter set is $p = 1$, $c_r = 0.5$, $c_c = 0.1$ and $r = 15{,}000$. Also, we assume demand to be normally distributed with parameter $\mu = 2{,}000$ and standard deviation $\sigma = 500$.



(a) (b)

Figure 3.3: (a) Acquisition price in centralized and decentralized settings under RL and CL vs. remanufacturing cost, (b) wholesale price in coordinated and decentralized settings under RL and CL vs. remanufacturing cost

We start our analysis with the remanufacturing cost by varying it from 0.1 to 0.8 in increments of 0.1. Increase in the remanufacturing cost decreases the remanufacturer's willingness to order a large quantity, which in turn, decreases the quantity of used product collected. The quantity of the returns collected under RL is larger compared



(a) (b)

Figure 3.4: (a) Total profit of centralized and decentralized settings under RL and CL vs. remanufacturing cost, (b) remanufacturer's (R) and collector's (C) individual profit under RL and CL vs. remanufacturing cost

to the quantity under CL for lower levels of c_r (see Figure 3.3a). However, when remanufacturing cost exceeds a certain threshold, the acquisition price is higher under CL, yielding a larger quantity of returns collected. When c_r is low, the collector is making profit through a high margin rather than a large sale, charging a higher wholesale price compared to the price under RL (see Figure 3.3b). However, when c_r is large, the margin drops sharply which prevents the collector to decrease the acquisition price as sharp as the margin. As a result, when remanufacturing cost is high, the supply chain performs better under CL (see Figure 3.4a), achieving a higher profit than the profit under RL. The observation that is worth mentioning here is that as the remanufacturing cost decreases, the leaders make the most out of it (see Figure 3.4b).

Next, we run experiments for the used product handling cost. When the handling cost is low, the collector, as the leader, sets a lower acquisition price than the price set under RL channel. Thereby, the quantity of the returns collected under RL is larger compared to the quantity under CL (see Figure 3.5a). However as used product handling cost increases, the remanufacturer under RL, in order to maintain the quantity of returns collected, should provide a higher wholesale price (see Figure 3.5b). As

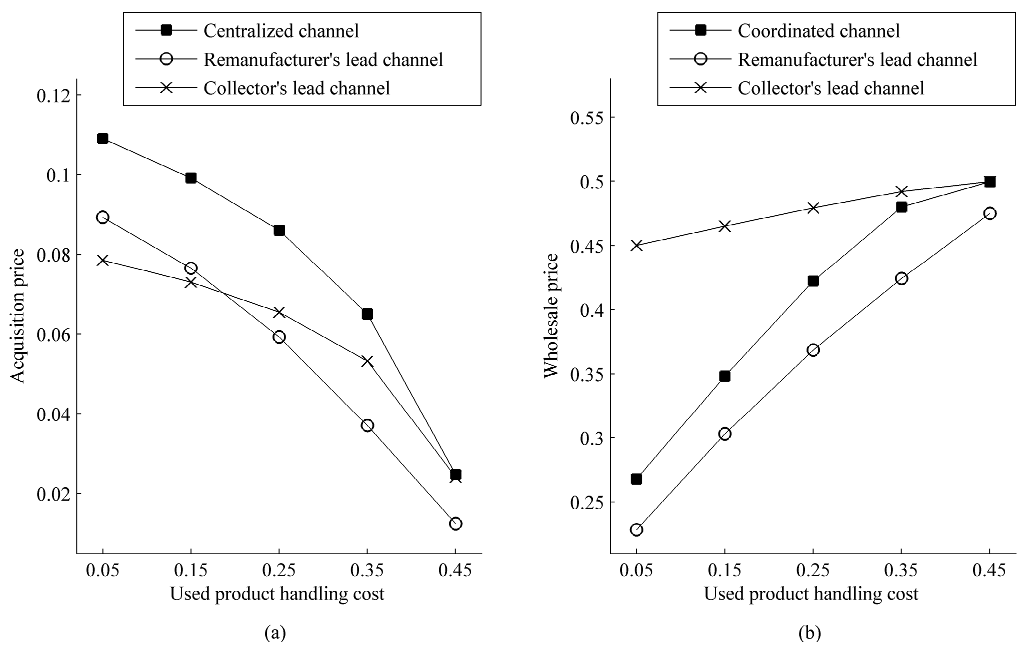


Figure 3.5: (a) Acquisition price in centralized and decentralized settings under RL and CL vs. used products handling cost, (b) wholesale price in coordinated and decentralized settings under RL and CL vs. used products handling cost

a result, the remanufacturer under RL loses interest to order a large quantity due to the decline in its margin. However, the collector, as the leader, knowing that a high value of c_c makes the remanufacturer reluctant to order a large quantity, sets such a wholesale price that the remanufacturer keeps ordering a larger quantity under CL compared to the quantity under RL.

The higher the quantity of returns, the higher the channel profit (see Figure 3.6a). As c_c exceeds a certain threshold, decentralized channel under CL achieves a higher profit than the profit under RL. When c_c is low, although the remanufacturer's profit

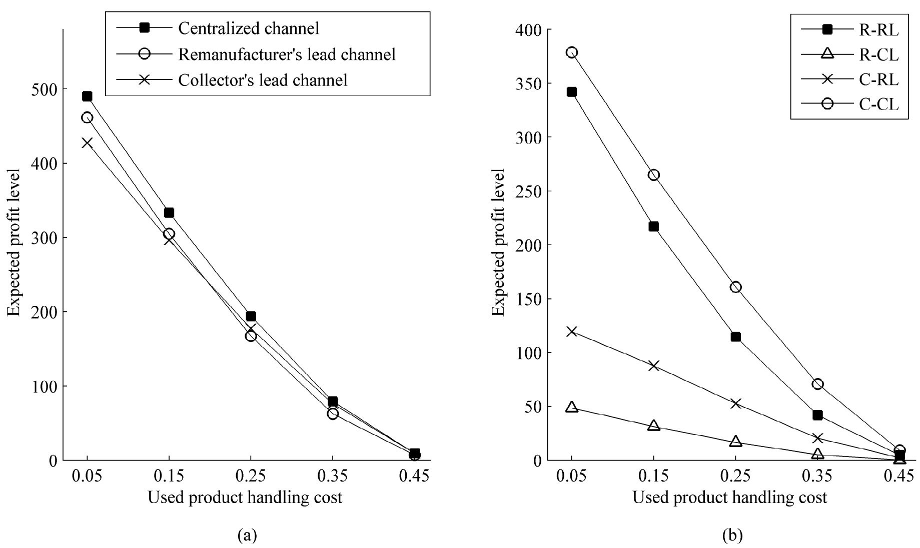


Figure 3.6: (a) Total profit of centralized and decentralized settings under RL and CL vs. used products handling cost, (b) remanufacturer's (R) and collector's (C) individual profit under RL and CL vs. used products handling cost

under RL is lower than the collector's lead under CL, the follower's profit under RL is much higher such that the total profit under RL becomes larger than total profit under CL. However, as c_c increases, the gap gets smaller and above a certain c_c, total profit under CL becomes greater than the profit under RL (see Figure 3.6a).

Next, we study the effect of selling price on the channel performance. As selling price increases, the acquisition price, and consequently, the quantity of returns collected increases for both decentralized and centralized channels (see Figure 3.7a). When selling price is high, the remanufacturer's margin under RL increases, it decides to stock more to avoid the risk of under-stocking. As a result, it pays a higher wholesale price to the collector. However, under CL, since the collector is aware of high margin, it sets a higher wholesale price (see Figure 3.7b). Therefore, the remanufacturer's margin remains low and as it can be seen in Figure 3.7a, the acquisition price under

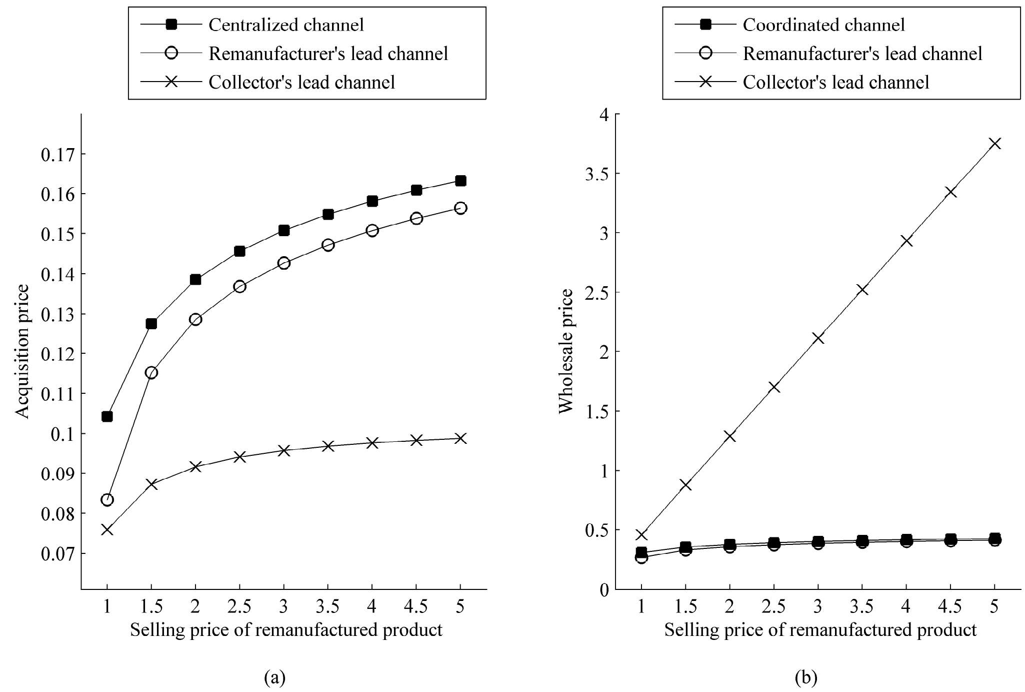


Figure 3.7: (a) Acquisition price in centralized and decentralized settings under RL and CL vs. selling price of remanufactured product, (b) wholesale price in coordinated and decentralized settings under RL and CL vs. selling price of remanufactured product

CL does not increase as sharp as the acquisition price under RL.

We can see the change in the total as well as individual profits in Figure 3.8. When the margin increases, the remanufacturer, as the leader, increases its order, paying a higher wholesale price and expects a higher profit. We can see that the collector also benefits under RL. When we compare this to the case where the collector is the leader, the increase in total profit is not as sharp as the increase in total profit under RL. As we mentioned above, the collector under CL sets a higher wholesale price, knowing that the margin is large, and leaves the remanufacturer with small margin. As we can see in Figure 3.8b, the margin is still enough for the remanufacturer to increase its order and to expect a higher profit.

Next, we analyze the effect of demand uncertainty on the acquisition price and wholesale price by varying the standard deviation from 150 to 650 in increments of 100. We run these experiments for three different selling prices $p = 1$, $p = 2$ and $p = 5$ and we summarize the results in Figure 3.9, and Tables 3.6 and 3.7. When the selling price is low, increase in demand uncertainty makes the remanufacturer not to take the risk

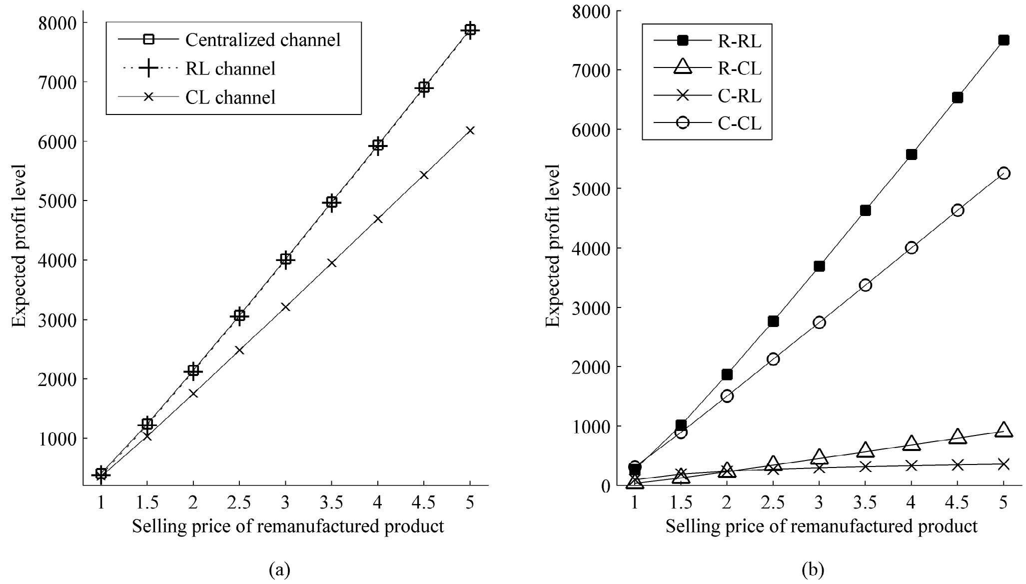


Figure 3.8: (a) Total profit of centralized and decentralized settings under RL and CL vs. selling price of remanufactured product, (b) remanufacturer's (R) and collector's (C) individual profit under RL and CL vs. selling price of remanufactured product

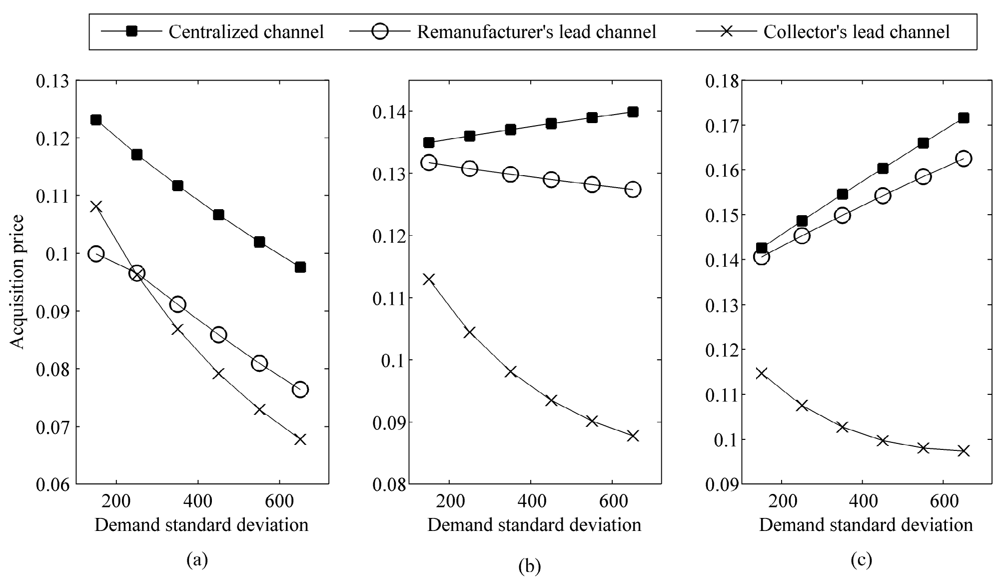


Figure 3.9: Acquisition price vs. demand standard deviation with (a) $p = 1$, (b) $p = 2$, (c) $p = 5$

of overstocking. As a result, the quantity of returns collected decreases in order to reduce the risk of overstocking (see Figure 3.9a). When we consider the case where $p = 2$, we observe that the profit margin for the centralized setting is large enough and the risk of under-stocking dominates the risk of overstocking. As a result, the remanufacturer pays a higher wholesale price (see Table 3.6) and the quantity of re-

Table 3.6: Coordinating wholesale price with respect to demand standard deviation for $p = 1, p = 2$ and $p = 5$

	w'		
σ	p=1	p=2	p=5
150	0.3463	0.3699	0.3852
250	0.3343	0.3720	0.3972
350	0.3234	0.3741	0.4091
450	0.3133	0.3760	0.4207
550	0.3039	0.3779	0.4320
650	0.2952	0.3798	0.4432

Table 3.7: Wholesale price charged by the collector under RL (w_{RL}) and CL (w_{CL}) for $p = 1, p = 2$ and $p = 5$ for different levels of standard deviation

	w_{RL}			w_{CL}		
σ	1	2	5	1	2	5
150	0.2998	0.3635	0.3813	0.4942	1.4583	4.3447
250	0.2931	0.3615	0.3906	0.4869	1.4174	4.1962
350	0.2823	0.3597	0.3996	0.4769	1.3690	4.0253
450	0.2717	0.3579	0.4084	0.4645	1.3159	3.8429
550	0.2618	0.3563	0.4169	0.4502	1.2610	3.6587
650	0.2528	0.3547	0.4251	0.4347	1.2066	3.4804

turns collected increases as uncertainty increases. However, under RL, although the margin is little larger, it does not give grounds for the remanufacturer to bear the risk of overstocking. As a result, the quantity of returns collected decreases as the demand variance increases. Under CL, since the collector charges a relatively higher wholesale price, the margin for the remanufacturer remains low and the risk of overstocking is still high. As we further increase the selling price, the remanufacturer's profit margin as the leader increases further and its order quantity increases to take advantage of possible sales as demand uncertainty increases. To induce the collector to collect more, the remanufacturer offers a higher wholesale price as demand standard deviation increases (see Table 3.7). On the other hand, under CL, even if the selling price is large, the collector takes advantage of this by setting a large wholesale price, which leaves the remanufacturer with a small margin and as a result, the quantity of returns collected decreases as demand uncertainty increases.

Through Figure 3.10 to 3.12, we study the behavior of individual profits of the remanfuacturer and the collector as well as the total profit when demand uncertainty changes. Overall, as expected, an increase in demand uncertainty decreases the expected total profit, regardless of selling price level (see Figure 3.10). A similar ar-

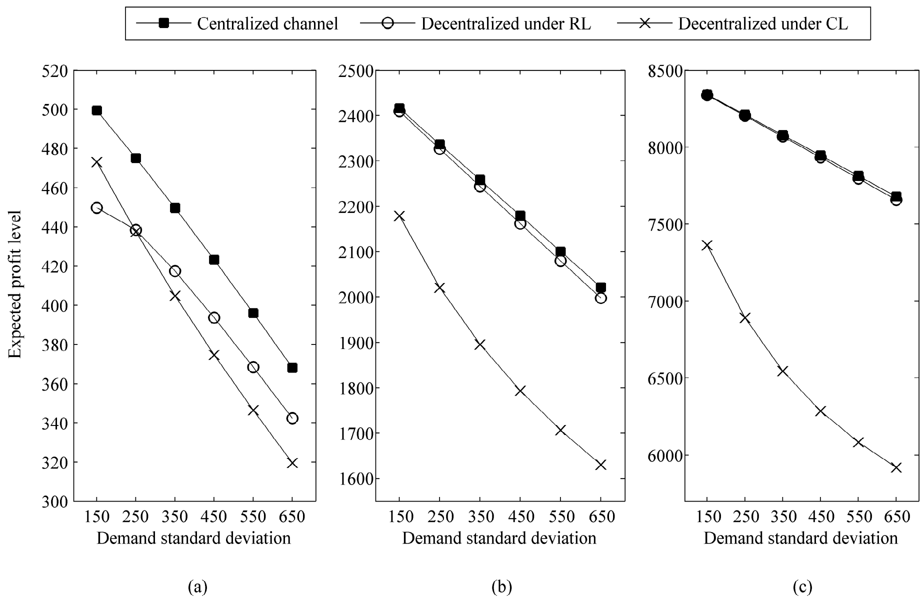


Figure 3.10: Expected profit of centralized and decentralized settings under RL and CL with respect to demand standard deviation with (a) $p = 1$, (b) $p = 2$, (c) $p = 5$

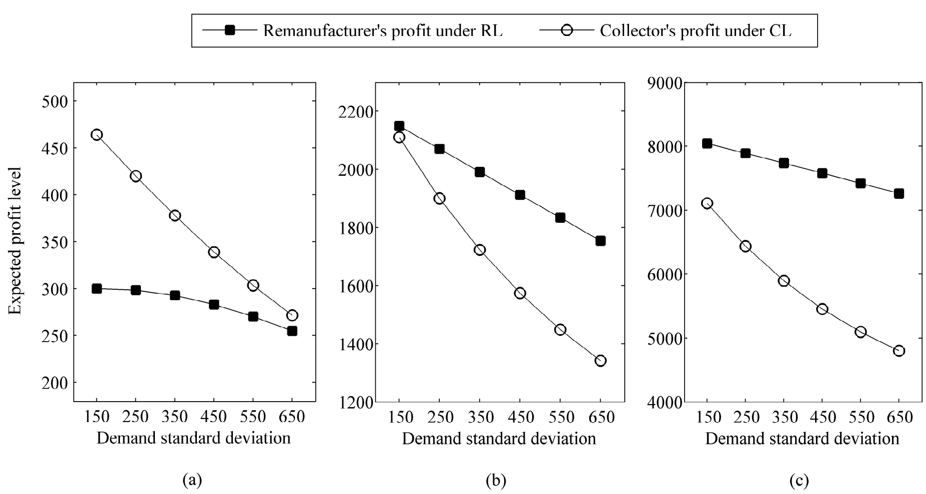


Figure 3.11: Expected profit of leaders in decentralized settings with respect to demand standard deviation with (a) $p = 1$, (b) $p = 2$, (c) $p = 5$

gument is also valid for the leader's profit (see Figure 3.11). It shall also be noted that the decline in profits (both leader's and total supply chains) is sharper under CL. However, the analysis of the followers' profits (see Figure 3.12) results in a counterintuitive observation. The remanufacturer's profit, as the follower, improves as uncertainty increases which is due to decrease in the wholesale price charged by the collector (see Table 3.7). In fact, since the collector is charging a higher wholesale price under CL, the remanufacturer would order a lower quantity as uncertainty increases and therefore, the collector loses sales. Therefore, in order to maintain an adequate level of sales, the collector charges the remanufacturer a lower wholesale price. Thereby, the remanufacturer is willing to order more and benefits as uncertainty increases (see Figure 3.12).

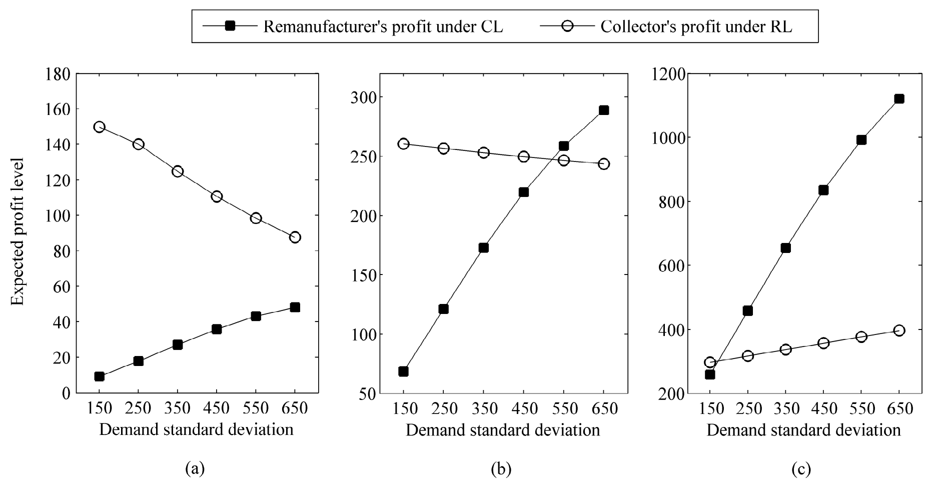


Figure 3.12: Expected profit of followers in decentralized settings with respect to demand standard deviation with (a) $p = 1$, (b) $p = 2$, (c) $p = 5$

When the collector is the follower, it loses profit by the increase in uncertainty, when selling price is low due to decrease in both order quantity and wholesale price set by the remanufacturer. The same happens when the selling price is moderate. However, as selling price increases further, the collector's profit improves as the demand standard deviation increases. The reason is twofold: firstly, when selling price is high, the remanufacturer is willing to stock more as the uncertainty increases and secondly, the remanufacturer sets a higher wholesale price as the uncertainty increases (see Table 3.7). Since increase in the wholesale price is more than the increase in the acquisition price (0.04 for wholesale price vs. 0.02 for acquisition price), the collector's profit increases with uncertainty.

We extend our analysis by studying the effect of acquisition price sensitivity on channel performances. Increase in r is equivalent to saying that larger quantity of used products is available to the collector at a given acquisition price. Therefore, less

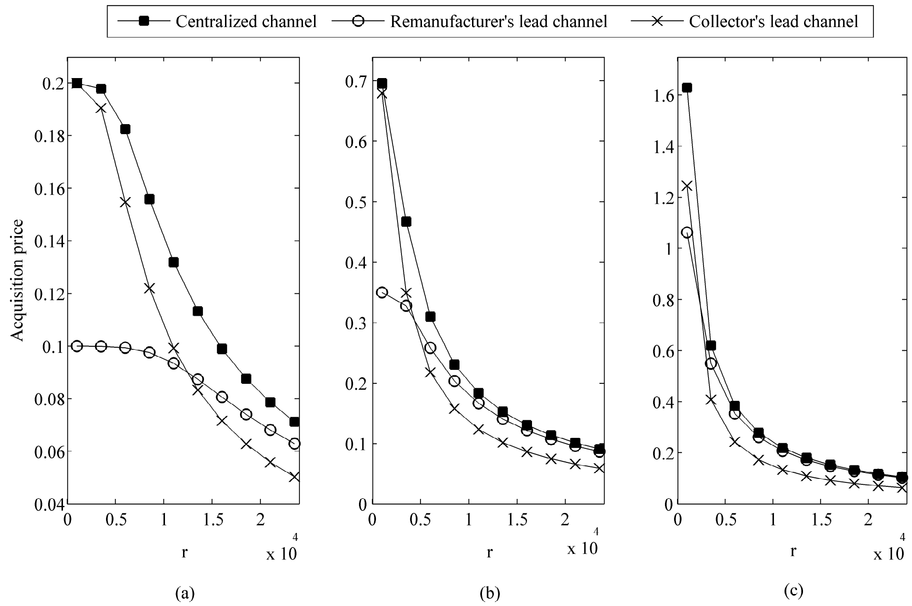


Figure 3.13: Acquisition price pays in centralized and decentralized settings under RL and CL with respect to acquisition price sensitivity, r with (a) $p = 1$, (b) $p = 2$ and (c) $p = 5$

effort, i.e. lower acquisition price, is needed when r is high (see Figure 3.13). In

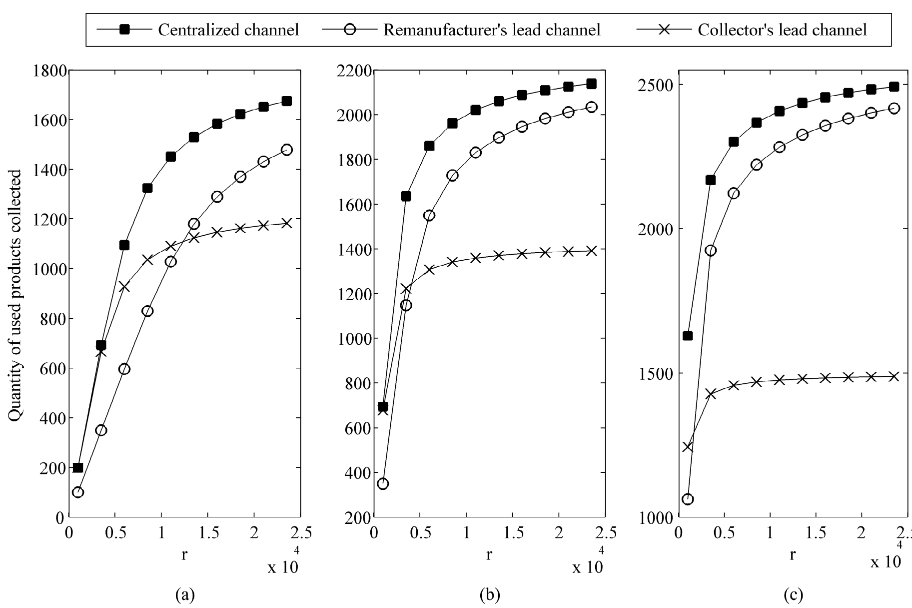


Figure 3.14: Quantity of used products collected in centralized and decentralized settings under RL and CL with respect to acquisition price sensitivity, r with (a) $p = 1$, (b) $p = 2$ and (c) $p = 5$

both decentralized and centralized settings, the optimal acquisition price decreases in r whereas the quantity of returns collected increases (see Figure 3.13 and 3.14). Similarly, the wholesale price decreases in r (see Figure 3.15). When we compare decentralized settings, we observe that CL results in a larger quantity of returns collected when r is low. As the centralized acquisition price is always larger than the decentralized settings, we may conclude that CL is better in a decentralized chain when r is low and RL is better when r is large. As a result, CL channel gains more profit when r is low (see Figure 3.16). Moreover, as the selling price increases, since RL channel would set a higher wholesale price (see Figure 3.15c) and order more, CL channel's advantage over RL channel becomes less noticeable.

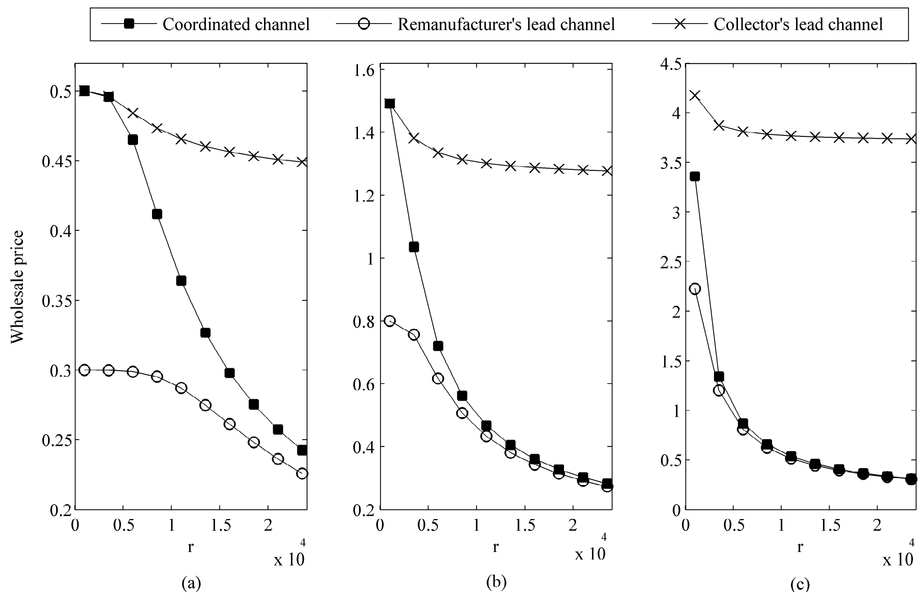


Figure 3.15: Wholesale price charged by the collector in coordinated and decentralized settings under RL and CL with respect to acquisition price sensitivity, r with (a) $p = 1$ (b) $p = 2$ and (c) $p = 5$

When it comes to individual profits, one expects that both firms benefit from a higher value of r. However, as it can be observed in Figure 3.17, the collector's profit under RL decreases as r goes beyond a certain value, which can be explained as follows: when r is low, that is lower quantity of used product is available to the collector, the remanufacturer still wants to engage the collector in collection by setting an adequate wholesale price. However, when r increases, the remanufacturer, knowing that the used products are abundant, offers a lower wholesale price. Although acquisition price also decreases in such a case, the decrease in wholesale price is larger and thereby, the collector's profit decreases. As the selling price increases, the decrease in wholesale price is more dramatic than the case for low selling price and as a result, the collector's profit starts to decrease at a lower value of r.

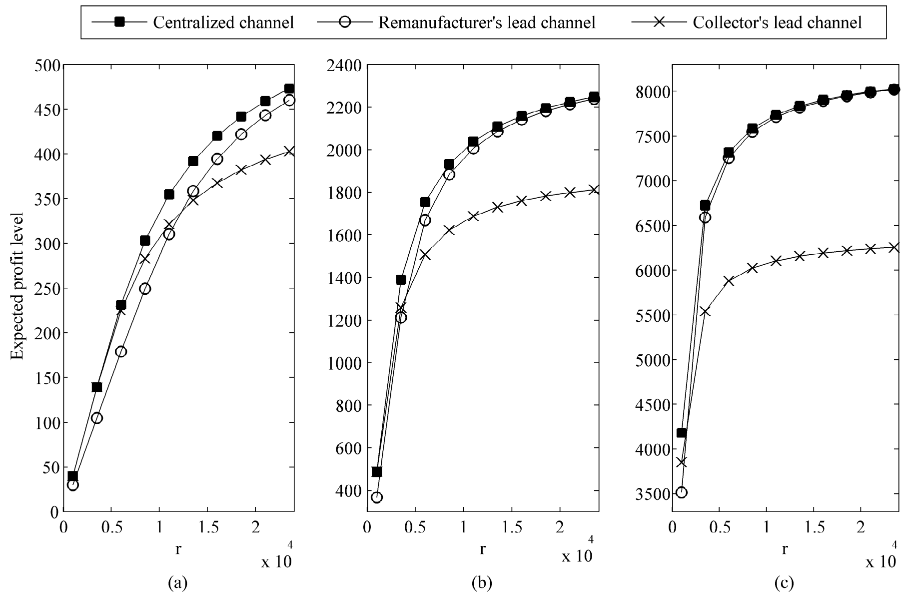


Figure 3.16: Total expected profit of centralized and decentralized settings under RL and CL with respect to acquisition price sensitivity, r with (a) $p = 1$, (b) $p = 2$ and (c) $p = 5$

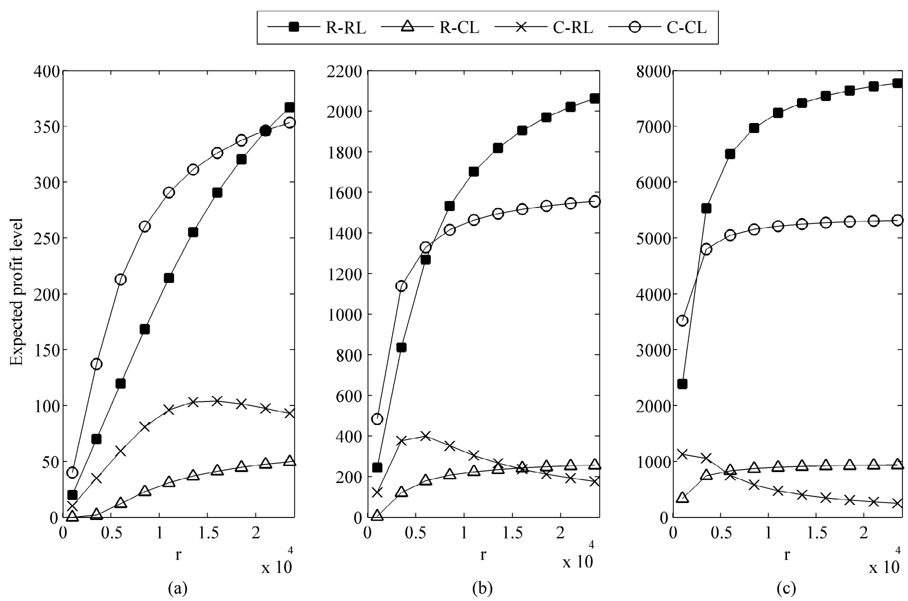


Figure 3.17: remanufacturer's and collector's expected profit under RL and CL with respect to acquisition price sensitivity, r with (a) $p = 1$, (b) $p = 2$ and (c) $p = 5$

3.5 Major Findings

In this section, we highlight our major findings through analytical observations and numerical experiments.

1. As long as the sum of remanufacturing cost and handling cost is constant, the

acquisition price and consequently, the quantity of returns collected and individual profits remain constant in the centralized setting as well as the decentralized setting under both RL and CL.

2. There exists a wholesale price that can coordinate the reverse supply chain under both RL and CL and allocates positive profit between parties.

3. The quantity of returns collected in decentralized setting under both RL and CL is less than that in centralized setting.

4. The quantity of returns collected (and the total profit) is larger under RL, when the remanufacturing cost (or used product handling cost) is low and it is larger under CL, when these costs are high (see Figure 3.3 and 3.5).

5. Under RL, when the selling price of remanufactured product is high, the quantity of returns collected increases as the demand uncertainty increases (see Figure 3.9).

6. When the collector is the leader, the remanufacturer's expected profit increases as demand uncertainty increases. Particularly, when the selling price is high, it increases significantly (see Figure 3.11).

7. When the selling price of remanufactured product is high, the collector's expected profit increases as the demand uncertainty increases (see Figure 3.12).

8. When the acquisition price sensitivity is low, the quantity of returns collected is larger under CL. It is larger under RL when the acquisition price sensitivity is high (see Figure 3.14).

9. The total expected profit under CL is higher compared to the total profit under RL, when the acquisition price sensitivity is low (see Figure 3.16).

10. Under RL, the expected profit of the collector decreases as the acquisition price sensitivity increases (see Figure 3.17).

11. The supply chain performance is better under RL when remanufacturing cost is low (see Figure 3.3), used product handling cost is low (see Figure 3.5), acquisition price sensitivity is high (see Figure 3.16) and demand uncertainty is high (see Figure 3.10).

12. The supply chain performance is better under CL when remanufacturing cost is high (see Figure 3.3), used product handling cost is high (see Figure 3.5), acquisition price sensitivity is low (see Figure 3.16) and both the selling price and demand uncertainty is low (see Figure 3.10).

CHAPTER 4

REVENUE SHARING CONTRACT

In Chapter 3, we conclude that there exists a single wholesale price that can coordinate the reverse supply chain. However, such contract does not allow for arbitrary allocation of profit between the parties (Cachon [42]). In this chapter, we demonstrate that through the implementation of a Revenue-Sharing contract it would be possible to allocate the profit so that each party has adequate incentive to participate based on the contracting parameters. Through a revenue sharing contract, the remanufacturer (buyer) promises to pay percentage of his revenue with the collector (supplier) in exchange of a lower wholesale price that the collector charges.

We incorporate a new decision variable α to our base model that indicates the fraction of revenue that the remanufacturer retains. As in Chapter 3, we study the decentralized decision making process under both remanufacturer's lead (RL) and collector's lead (CL). We assume that the leader sets the contracting parameters α and w.

4.1 Remanufacturer's lead channel

We start our analysis with considering the decentralized decision making under remanufacturer's lead (RL). The sequence of events is given as follows:

- The remanufacturer sets α, w and its order quantity, Q.
- The collector decides on the acquisition price, a.

We will first consider the collector's problem, given the wholesale price, α and Q set by the remanufacturer. The collector's expected profit is given by

$$\begin{aligned} max. \quad & \Pi_C^{RL}(a) = (1-\alpha)p\Big(y(a) - \int_0^{y(a)} F(x)dx\Big) + wy(a) - c_c y(a) - ay(a) \\ s.to \quad & y(a) \leqslant Q \end{aligned} \tag{4.1}$$

where $y(a) = ra$ is the amount that would be collected by the collector.

Proposition 4.1. *The optimal acquisition price under RL is characterized by:*

$$a_{RL} = min\Big\{\frac{(1-\alpha)p\Big(1-F(ra)\Big) + w - c_c}{2}, \frac{Q}{r}\Big\}. \tag{4.2}$$

Proof. The first and second derivatives of the collector's profit (4.1) with respect to a are given as follows:

$$\frac{d\Pi_C^{RL}(a)}{da} = (1-\alpha)p\Big(r - r{\cdot}F(ra)\Big) + wr - c_c r - 2ar \tag{4.3}$$

and

$$\frac{d^2\Pi_C^{RL}(a)}{da^2} = (1-\alpha)p\Big(-r^2 f(ra)\Big) - 2r < 0.$$

Hence, optimal unconstrained solution is characterized by setting 4.3 to zero.

$$a_{RL}(w,\alpha) = \frac{(1-\alpha)p\Big(1-F(ra)\Big) + w - c_c}{2}. \tag{4.4}$$

Taking the constraint into account, the collector's optimal response to the remanufacturer's decisions is characterized by

$$a_{RL} = min\Big\{\frac{(1-\alpha)p\Big(1-F(ra)\Big) + w - c_c}{2}, \frac{Q}{r}\Big\},$$

which completes the proof. □

Accordingly, we can derive the amount of collected products by:

$$y_{RL}(w,\alpha) = min\Big\{r{\cdot}\frac{(1-\alpha)p\Big[1-F\Big(y(w,\alpha)\Big)\Big] + w - c_c}{2}, Q\Big\}.$$

Now, we proceed by remanufacturer's problem. As in Section 3.2, the remanufacturer's available inventory is the minimum of its order quantity and the quantity collected by the collector. As a result, the remanufacturer's problem would be:

$$max. \quad \Pi_R^{RL}(w,\alpha,Q) = \alpha p\Big(S - \int_0^S F(x)dx\Big) - wS - c_r S, \tag{4.5}$$

where $S = min\{r\cdot\frac{(1-\alpha)p[1-F(ra)]+w-c_c}{2}, Q\}$. Note that y, which is characterized by $y = r\frac{(1-\alpha)p[1-F(y)]+w-c_c}{2}$, is the maximum amount that the collector would collect. Hence, the remanufacturer cannot get more than that even if it orders more; that is even if $Q > r\frac{(1-\alpha)p[1-F(y)]+w-c_c}{2}$. Hence, in formulating the remanufacturer's problem, there is no need to consider the cases where $Q > r\frac{(1-\alpha)p[1-F(y)]+w-c_c}{2}$. Hence, we revise the remanufacturer's problem (4.5) as follows:

$$\begin{aligned} max. \quad & \Pi_R^{RL}(w,\alpha,Q) = \alpha p\Big(Q - \int_0^Q F(x)dx\Big) - wQ - c_r Q, \\ s.to \quad & Q \leqslant r\cdot\frac{(1-\alpha)p[1-F(ra)]+w-c_c}{2} \end{aligned} \tag{4.6}$$

Since the unconstrained problem would set $w = -\infty$ (w can take negative values in this problem), the constraint is binding. Thereby, the remanufacturer's problem (4.6) reduces to:

$$\Pi_R^{RL}(w,\alpha) = \alpha p\Big(y - \int_0^y F(x)dx\Big) - wy - c_r y, \tag{4.7}$$

where $y(w,\alpha) = r\cdot\frac{(1-\alpha)p[1-F\big(y(w,\alpha)\big)]+w-c_c}{2}$.

Lemma 4.1. *For IGFR demand distributions,* $af'(y)\overline{F}(y) + bf^2(y)$*, is also weakly positive for* $b \geqslant a \geqslant 0$.

Proof. For an IGFR demand distribution, $f'(y)\overline{F}(y) + f^2(y) \geqslant 0$. It is straight forward to show that for $b \geqslant a \geqslant 0$:

$$0 \leqslant af'(y)\overline{F}(y) + af^2(y) \leqslant af'(y)\overline{F}(y) + bf^2(y)$$

□

Proposition 4.2. *For a given value of* $\alpha \in [0,1]$*, there is a unique* w *which maximizes the remanufacturer's profit.*

Proof. We start with the derivative of the remanufacturer's profit function (4.7) with respect to w:

$$\frac{\partial \Pi_R^{RL}(w,\alpha)}{\partial w} = \alpha p\Big[\frac{\partial y(w,\alpha)}{\partial w} - \frac{\partial y(w,\alpha)}{\partial w} F\Big(y(w,\alpha)\Big)\Big] - y(w,\alpha)$$
$$-w\frac{\partial y(w,\alpha)}{\partial w} - c_r\frac{\partial y(w,\alpha)}{\partial w}$$

$$= \frac{\partial y(w,\alpha)}{\partial w}\Big[\alpha p\overline{F}\Big(y(w,\alpha)\Big) - w - c_r\Big] - y(w,\alpha), \tag{4.8}$$

and

$$\frac{\partial^2 \Pi_R^{RL}(w,\alpha)}{\partial w^2} = \frac{\partial^2 y(w,\alpha)}{\partial w^2}\Big[\alpha p\overline{F}\Big(y(w,\alpha)\Big) - w - c_r\Big]$$
$$+\Big(\frac{\partial y(w,\alpha)}{\partial w}\Big)\Big[-\alpha p f\Big(y(w,\alpha)\Big)\Big(\frac{\partial y(w,\alpha)}{\partial w}\Big) - 1\Big] - \frac{\partial y(w,\alpha)}{\partial w}$$

$$= \frac{\partial^2 y(w,\alpha)}{\partial w^2}\Big[\alpha p\overline{F}\Big(y(w,\alpha)\Big) - w - c_r\Big]$$
$$-\alpha p f\Big(y(w,\alpha)\Big)\Big(\frac{\partial y(w,\alpha)}{\partial w}\Big)^2 - 2\frac{\partial y(w,\alpha)}{\partial w}. \tag{4.9}$$

On the other hand, since $y(w,\alpha) = r\cdot\frac{(1-\alpha)p\overline{F}\big(y(w,\alpha)\big)+w-c_c}{2}$, by taking the first and second derivate of $y(w,\alpha)$, we have

$$\frac{\partial y(w,\alpha)}{\partial w} = \frac{r}{2}\Big[1 - p(1-\alpha)\frac{\partial F\Big(y(w,\alpha)\Big)}{\partial w}\Big]$$

$$= \frac{r}{2}\Big[1 - p(1-\alpha)\frac{\partial y(w,\alpha)}{\partial w} f\Big(y(w,\alpha)\Big)\Big].$$

By rearranging the equality:

$$\frac{\partial y(w,\alpha)}{\partial w} = \frac{1}{\frac{2}{r} + p(1-\alpha)f(y(w,\alpha))} > 0. \tag{4.10}$$

Also,

$$\frac{\partial^2 y(w,\alpha)}{\partial w^2} = \frac{-p(1-\alpha)\frac{\partial f(y(w,\alpha))}{\partial w}}{\left[\frac{2}{r} + p(1-\alpha)f(y(w,\alpha))\right]^2}$$

$$= \frac{-p(1-\alpha)\left(\frac{\partial y(w,\alpha)}{\partial w}\right)f'\Big(y(w,\alpha)\Big)}{\left[\frac{2}{r} + p(1-\alpha)f\Big(y(w,\alpha)\Big)\right]^2}. \tag{4.11}$$

By substituting 4.10 and 4.11 into the second derivative equation with respect to w (4.9), we have:

$$\frac{\partial^2 \Pi_R^{RL}(w,\alpha)}{\partial w^2} = \frac{\theta \boldsymbol{f'}\Big(y(w,\alpha)\Big)\left[\alpha p\overline{\boldsymbol{F}}\Big(y(w,\alpha)\Big) - w - c_r\right] + \kappa \boldsymbol{f^2}\Big(y(w,\alpha)\Big) + \beta}{-\left[(1-\alpha)f\Big(y(w,\alpha)\Big)pr + 2\right]^3} \tag{4.12}$$

where $\theta = p(1-\alpha)r^3 > 0$, $\kappa = (1-\alpha)(2-\alpha)p^2r^3 > 0$, $\beta = (8-6\alpha)pr^2 f\Big(y(w,\alpha)\Big) + 8r > 0$.

Now, if $f'\Big(y(w,\alpha)\Big) > 0$, based on first order condition (4.8), since $\frac{\partial y(w,\alpha)}{\partial w} > 0$, $\alpha p\overline{F}\Big(y(w,\alpha)\Big) - w - c_r > 0$, then $\frac{\partial^2 \Pi_R(w,\alpha)}{\partial w^2}$ is negative. Also, if $f'\Big(y(w,\alpha)\Big) < 0$, by rearranging 4.12, we have:

$$\frac{\partial^2 \Pi_R^{RL}(w,\alpha)}{\partial w^2} =$$
$$-\frac{\alpha p\theta \boldsymbol{f'}\Big(y(w,\alpha)\Big)\overline{\boldsymbol{F}}\Big(y(w,\alpha)\Big) + \kappa \boldsymbol{f^2}\Big(y(w,\alpha)\Big) - f'\Big(y(w,\alpha)\Big)(w + c_r) + \beta}{\left[(1-\alpha)f\Big(y(w,\alpha)\Big)pr + 2\right]^3}$$

Using Lemma 4.1, since $\kappa \geqslant \alpha p\theta$, $\frac{\partial^2 \Pi_R(w,\alpha)}{\partial w^2} < 0$. Hence, for a given value of

$\alpha \in [0,1]$, if there is any extremum in the remanufacturer's profit, it is not only a local maximizer, also it is a unique profit maximizer, since Π_R is a continuous, differentiable function and it would not be possible to have multiple local maximum points without any local minimum. $\square$

Proposition 4.3. *Under RL, the remanufacturer's optimal decision is not to share revenue with the collector.*

Proof. Recall that

$$\frac{\partial \Pi_R^{RL}(w,\alpha)}{\partial w} = \frac{\partial y(w,\alpha)}{\partial w}\Big[\alpha p \overline{F}\Big(y(w,\alpha)\Big) - w - c_r\Big] - y(w,\alpha) = 0.$$

Since $\frac{\partial y(w,\alpha)}{\partial w} = \frac{1}{\frac{2}{r}+p(1-\alpha)f\big(y(w,\alpha)\big)}$, we can write optimal wholesale price as a function of α:

$$w(\alpha) = \alpha p \overline{F}\Big(y(w(\alpha),\alpha)\Big) - \Big[\frac{2}{r} + p(1-\alpha)f\Big(y(w(\alpha),\alpha)\Big)\Big] y\Big(w(\alpha),\alpha\Big) - c_r. \quad (4.13)$$

Then, by plugging 4.13 in remanufacturer's profit (4.7), the remanufacturer's profit becomes only a function of α:

$$\Pi_R^{RL}(\alpha) = \alpha p\Big(y(\alpha) - \int_0^{y(\alpha)} F(x)dx\Big) - w(\alpha)y(\alpha) - c_r y(\alpha). \quad (4.14)$$

where $y(\alpha) = \frac{r}{2}\Big[(1-\alpha)p\overline{F}\Big(y(\alpha)\Big) + w(\alpha) - c_c\Big]$ and $w(\alpha) = \alpha p \overline{F}\Big(y(\alpha)\Big) - \Big[\frac{2}{r} + p(1-\alpha)f\Big(y(\alpha)\Big)\Big] y(\alpha) - c_r$. Now, we continue with remanufacturer's problem (4.14) and take derivative with respect to α as follows:

$$\frac{d\Pi_R^{RL}(\alpha)}{d\alpha} = p\Big(y(\alpha) - \int_0^{y(\alpha)} F(x)dx\Big) + \frac{dy(\alpha)}{d\alpha}\Big(\alpha p \overline{F}\Big(y(\alpha)\Big) - w(\alpha) - c_r\Big) - y(\alpha)\frac{dw(\alpha)}{d\alpha}. \quad (4.15)$$

By substituting $w(\alpha) = \alpha p \overline{F}\Big(y(\alpha)\Big) - \Big[\frac{2}{r} + p(1-\alpha)f\Big(y(\alpha)\Big)\Big] y(\alpha) - c_r$ in 4.15, we have

$$\frac{d\Pi_R^{RL}(\alpha)}{d\alpha} = p\Big(y(\alpha) - \int_0^{y(\alpha)} F(x)dx\Big)$$
$$+y(\alpha)\Bigg[\frac{dy(\alpha)}{d\alpha}\Big(\frac{2}{r} + p(1-\alpha)f(y(\alpha))\Big) - \frac{dw(\alpha)}{d\alpha}\Bigg]. \tag{4.16}$$

Now, if we take derivative of $y(\alpha) = \frac{r}{2}\Bigg[(1-\alpha)p\overline{F}\Big(y(\alpha)\Big) + w(\alpha) - c_c\Bigg]$ with respect to α,

$$\frac{dy(\alpha)}{d\alpha}\Bigg[\frac{2}{r} + p(1-\alpha)f\Big(y(\alpha)\Big)\Bigg] = -p\overline{F}(y(\alpha)) + \frac{dw(\alpha)}{d\alpha}, \tag{4.17}$$

and take $\frac{dw(\alpha)}{d\alpha}$ to the left hand side of 4.17 and plug it to 4.16, we have:

$$\frac{d\Pi_R^{RL}(\alpha)}{d\alpha} = p\Big(y(\alpha) - \int_0^{y(\alpha)} F(x)dx\Big) + y(\alpha)\Big[-p\overline{F}(y(\alpha))\Big]$$

$$= p\Bigg[y(\alpha)F\Big(y(\alpha)\Big) - \int_0^{y(\alpha)} F(x)dx\Bigg] > 0.$$

Hence, we conclude that the remanufacturer's profit function is increasing in α in interval $[0, 1]$ and the optimal remanufacturer's share of revenue is $\alpha^* = 1$, i.e. remanufacturer does not share any revenue with the collector. $\square$

Thereby, we show that in the Stackelberg game with revenue sharing, remanufacturer, as the leader, retains all the revenue. As a result, this setting reduces to the case with no revenue sharing we studied in Section 3.2.

4.2 Collector's lead channel

In this section, we consider the case where the collector is the leader. The sequence of events is as follows:

- The collector sets α, w and a.

- The remanufacturer decides its order quantity, Q.

We have to start with the remanufacturer's problem:

$$max. \quad \Pi_R^{CL}(Q) = \alpha p\Big(Q - \int_0^Q F(x)dx\Big) - wQ - c_r Q \tag{4.18}$$

Proposition 4.4. *Under CL, remanufacturer's optimal order quantity is given by:*

$$Q = F^{-1}\Big(\frac{\alpha p - w - c_r}{\alpha p}\Big)$$

Proof. By taking the first and second derivative of the remanufacturer's profit function (4.18) with respect to Q, we have:

$$\frac{\partial \Pi_R^{CL}(Q)}{\partial Q} = \alpha p\Big(1 - F(Q)\Big) - w - c_r, \tag{4.19}$$

and

$$\frac{\partial^2 \Pi_R^{CL}(Q)}{\partial Q^2} = -\alpha p f(Q) < 0.$$

Hence, by setting 4.19 to zero, we derive the remanufacturer's optimal order. □

Now, the collector, observing the remanufacturer's decision, would maximize its profit:

$$\Pi_C^{CL}(w, \alpha, a) = (1-\alpha)p\Big(y(a) - \int_0^{y(a)} F(x)dx\Big) + wy(a) - c_c y(a) - ay(a), \tag{4.20}$$

where $y(a) = ra$.

Lemma 4.2. *In the optimal solution, the collector's collection quantity will always match the remanufacturer's order quantity.*

Proof. Assume that $Q^* > ra^*$ and let $\Pi_C^*(w^*, \alpha^*, a^*)$ be the collector's expected profit. There exists $w' > w^*$ such that $Q^* > Q' > ra^*$, $\Pi_C'(w', \alpha^*, a^*) > \Pi_C^*(w^*, \alpha^*, a^*)$ as the collector's sales would be ra^* in both cases. Hence, we conclude $Q^* \leqslant ra^*$. On the other hand, when $Q^* < ra^*$, then there exists $a' < a^*$ such that $Q^* < ra' < ra^*$, and $\Pi_C'(w^*, \alpha^*, a') > \Pi_C^*(w^*, \alpha^*, a^*)$ as the collector's sales would be Q^* under both a^* and a'. Hence, $Q^* > ra^*$. Therefore, $Q^* = ra^*$. □

Since $F(x)$ is strictly increasing and continuous, there is a one-to-one mapping between w and Q:

$$w(Q) = \alpha p \overline{F}(Q) - c_r, \tag{4.21}$$

Therefore, using Lemma 4.2 and plugging 4.21, we can proceed with the collector's problem (4.20) as follows:

$$\Pi_C^{CL}(\alpha, Q) = (1-\alpha)p\Big(Q - \int_0^Q F(x)dx\Big) + (\alpha p \overline{F}(Q) - c_r)Q - c_c Q - \frac{Q^2}{r} \tag{4.22}$$

Proposition 4.5. *For a given value of* $\alpha \in [0, 1]$*, there is a unique* Q*, which maximizes the collector's profit.*

Proof. By taking first derivative of the collector's profit function (4.22) with respect to Q, we have:

$$\frac{\partial \Pi_C^{CL}(\alpha, Q)}{\partial Q} = (1-\alpha)p\overline{F}(Q) - \alpha p f(Q)\cdot Q + \alpha p \overline{F}(Q) - c_r - c_c - \frac{2Q}{r}$$

$$= p\overline{F}(Q)\Big(1 - \frac{\alpha Q f(Q)}{\overline{F}(y)}\Big) - c_r - c_c - \frac{2Q}{r}, \tag{4.23}$$

and,

$$\frac{\partial^2 \Pi_C^{CL}(\alpha, Q)}{\partial Q^2} = -pf(Q)\Big(1 - \frac{\alpha Q f(Q)}{\overline{F}(y)}\Big) - \alpha p \overline{F}(Q)\Big[\frac{f(Q)}{\overline{F}(y)} + Q\Big(\frac{f(Q)}{\overline{F}(y)}\Big)'\Big] - \frac{2}{r}.$$

According to the first order condition, $1 - \frac{\alpha Q f(Q)}{\overline{F}(y)} > 0$. Recall that $\Big(\frac{f(Q)}{\overline{F}(y)}\Big)' > 0$, due to IGFR property, we can conclude that $\frac{\partial^2 \Pi_C^{CL}(\alpha, Q)}{\partial Q^2} < 0$. Hence, if there is any extremum in the collector's profit function, it is not only a local maximizer, but also it is a unique profit maximizer, since Π_C is a continuous, differentiable function and it is not possible to have multiple local maximizer points with no local minimum point. □

Proposition 4.6. *Under CL, the collector's optimal decision is to take all the revenue from the remanufacturer.*

Proof. First, by setting 4.23 to zero, we obtain the optimal sales quantity of the collector as a function of α:

$$Q(\alpha) = \frac{p\overline{F}\Big(Q(\alpha)\Big) - c_r - c_c}{\frac{2}{r} + \alpha p f\Big(Q(\alpha)\Big)}. \tag{4.24}$$

Then, we plug 4.24 into the collector's problem (4.22) and proceed by taking first and second derivatives of the collector's profit function with respect to α:

$$\frac{\partial \Pi_C^{CL}\Big(\alpha, Q(\alpha)\Big)}{\partial \alpha} = -p\Big(Q(\alpha) - \int_0^{Q(\alpha)} F(x)dx\Big) + p(1-\alpha)\frac{\partial Q(\alpha)}{\partial \alpha}\Big[1 - F\Big(Q(\alpha)\Big)\Big]$$

$$+\Big[p\overline{F}\Big(Q(\alpha)\Big) - \alpha p f\Big(Q(\alpha)\Big)\frac{\partial Q(\alpha)}{\partial \alpha}\Big]Q(\alpha) + \Big[\alpha p\overline{F}\Big(Q(\alpha)\Big) - c_r\Big]\frac{\partial Q(\alpha)}{\partial \alpha}$$

$$-c_c\frac{\partial Q(\alpha)}{\partial \alpha} - \frac{2Q(\alpha)}{r} \cdot \frac{\partial Q(\alpha)}{\partial \alpha}$$

$$= \frac{\partial Q(\alpha)}{\partial \alpha}\Big[p\overline{F}\Big(Q(\alpha)\Big) - \alpha p Q(\alpha) f\Big(Q(\alpha)\Big) - c_r - c_c - \frac{2Q(\alpha)}{r}\Big]$$

$$+pQ(\alpha)\Big[1 - F\Big(Q(\alpha)\Big)\Big] - p\Big(Q(\alpha) - \int_0^{Q(\alpha)} F(x)dx\Big). \tag{4.25}$$

From first derivative of remanufacturer's profit with respect to Q (4.23), we know that $p\overline{F}(Q) - \alpha p Q f(Q) - c_r - c_c - \frac{2Q}{r} = 0$. Hence, the first order derivative of the collector's profit function (4.25) reduces to:

$$\frac{\partial \Pi_C^{CL}(\alpha, Q)}{\partial \alpha} = pQ\Big(1 - F(Q)\Big) - p\Big(Q - \int_0^Q F(x)dx\Big)$$

$$= p\Big(- QF(Q) + \int_0^Q F(x)dx\Big) < 0,$$

meaning that the collector's profit is decreasing in α. Thereby, the collector would set $\alpha = 0$. $\square$

However, since $0 \leqslant \frac{\alpha p - w - c_r}{\alpha p} \leqslant 1$, α can be zero if and only if $w + c_r = 0$. We present an illustrative example based on our base parameter set ($p = 1$, $c_r = 0.5$, $c_c = 0.1$, $\mu = 2{,}000$, $\sigma = 500$ and $r = 15{,}000$) in Table 4.1 and Figure 4.1. In this setting, the collector asks for all revenue, leaving the remanufacturer with zero profit. As it can be seen in Table 4.1, in such circumstances, the collector collects centralized-level quantity of used products and pays all remanufacturer's costs ($c_r = 0.5$).

Table 4.1: Comparison of decentralized channel performance under CL with (w/ RS) and without revenue sharing (w/o RS) with centralized channel performance

	α	y	w	a	Π_R	Π_C	$\Pi_R + \Pi_C$
w/o RS	-	1,138.4	0.4576	0.0759	39.66	320.66	360.32
w/ RS	1	1,138.39	0.4576	0.0759	39.66	320.67	360.33
	0.9	1,156.38	0.3588	0.0771	39.16	324.82	363.98
	0.8	1,176.41	0.2602	0.0784	38.52	329.39	367.92
	0.7	1,198.96	0.1618	0.0799	37.71	334.48	372.19
	0.6	1,224.70	0.0637	0.0816	36.63	340.22	376.85
	0.5	1,254.57	-0.0340	0.0836	35.18	346.76	381.94
	0.4	1,289.97	-0.1311	0.0860	33.13	354.39	387.52
	0.3	1,333.12	-0.2273	0.0889	30.10	363.49	393.59
	0.2	1,387.69	-0.3221	0.0925	25.30	374.74	400.04
	0.1	1,460.32	-0.4140	0.0974	16.90	389.33	406.23
	0	1,563.80	-0.5	0.1043	0.00	409.68	409.68
Centralized	-	1,563.80	0.3085*	0.1043	246.65*	163.03*	409.68

In Figure 4.1, individual profits and total expected profit of decentralized setting under CL under the cases with and without revenue sharing is presented. The collector as the leader would set $\alpha = 0$ to maximize its profit and achieves higher profit than the case without revenue sharing ($\alpha = 1$). The remanufacturer, however, gains no profit.

4.3 Coordination under Revenue Sharing Contract

In Section 4.1 and Section 4.2, we studied the sequential games under decentralized settings. We have shown that either under RL or CL, there is no tendency to share the revenue. In Section 4.1, we demonstrate that the remanufacturer always retains the whole profit, while in Section 4.2, the collector asks for all revenue from the

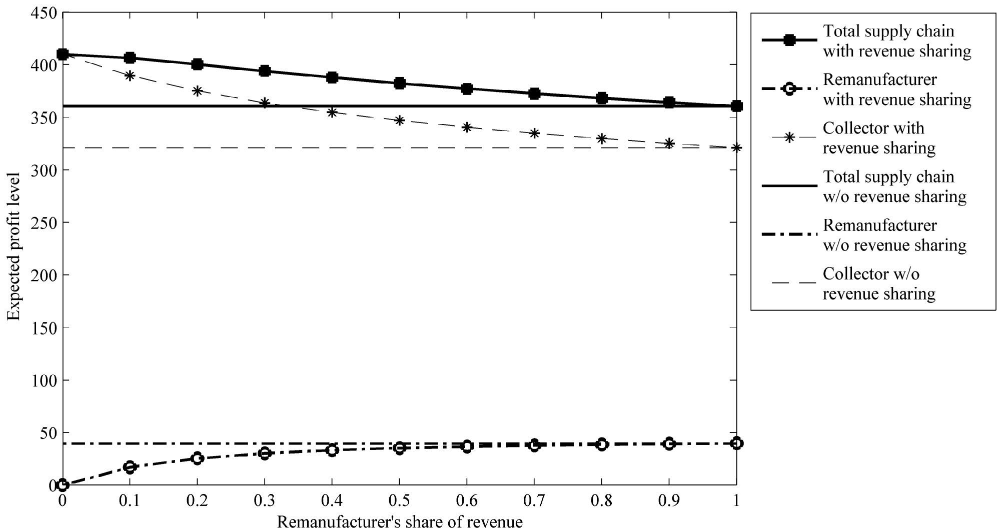


Figure 4.1: Expected profit of total supply chain, remanufacturer and collector under CL with and without revenue sharing

remanufacturer.

In this section, we put forward a menu of revenue sharing contracts, i.e. pairs of (w, α), that can coordinate the reverse channel. Under any one of the contracts in this menu, centralized total profit and recovery rate, i.e. the total quantity of collected used products, are guaranteed. However, the allocation of profit between the remanufacturer and the collector changes depending on which specific pair is used.

Proposition 4.7. *There exists a menu of revenue sharing contracts (w, α) that can coordinate the reverse supply chain under remanufacturer's lead.*

Proof. Note that through a coordination contract, acquisition price paid to the customers must be equal to the centralized acquisition price, guaranteeing the centralized-level of quantity of used products collected. When the remanufacturer is the leader, optimal acquisition price paid by the collector is given by:

$$a(w, \alpha) = \frac{(1-\alpha)p\overline{F}(ra) + w - c_c}{2}. \tag{4.26}$$

From Chapter 3, we already know that when w', which satisfies $F^{-1}\left(\frac{p-w'-c_r}{p}\right) = r\frac{w'-c_c}{2}$, is employed in decentralized setting, coordination is achieved, that is $a_C = a_{DM}|_{w=w'} = \frac{w'-c_c}{2}$. We claim that all pairs of $(\hat{w}, \hat{\alpha})$, that satisfy $(1-\hat{\alpha})p\overline{F}(ra_C) + \hat{w} = w'$, will support the collector to set $a = a_C$. Therefore, the collector would collect as many items as in centralized setting, which is $ra_C = r\frac{w'-c_c}{2}$. However, we need to check if the remanufacturer is willing to order the exact amount of used items.

By plugging $ra_C = r\frac{w'-c_c}{2} = F^{-1}\Big(\frac{p-w'-c_r}{p}\Big)$ into $(1-\hat{\alpha})p\overline{F}(ra_C) + \hat{w} = w'$, we get $\hat{w} = \hat{\alpha}(w'+c_r) - c_r$. As we plug $w = \hat{w}$, $\alpha = \hat{\alpha}$ as well as $\hat{w} = \hat{\alpha}(w'+c_r) - c_r$ in the remanufacturer's order quantity given by $Q = F^{-1}\Big(\frac{\alpha p - w - c_r}{\alpha p}\Big)$, we have:

$$\begin{aligned} Q = F^{-1}\Big(\frac{\hat{\alpha}p - \hat{w} - c_r}{\hat{\alpha}p}\Big) &= F^{-1}\Big(\frac{\hat{\alpha}p - (\hat{\alpha}(w'+c_r) - c_r) - c_r}{\hat{\alpha}p}\Big) \\ &= F^{-1}\Big(\frac{p - w' - c_r}{p}\Big). \end{aligned}$$

Since $F^{-1}\Big(\frac{p-w'-c_r}{p}\Big) = r\frac{w'-c_c}{2}$, we conclude that the remanufacturer's order quantity would be the same as quantity of used products collected by the collector. □

Hence, any pair of (w, α) that satisfies $w = \alpha(w'+c_r) - c_r$, in which w' satisfies $F^{-1}\Big(\frac{p-w'-c_r}{p}\Big) = r\frac{w'-c_c}{2}$, can coordinate the reverse supply chain under remanufacturer's lead.

Proposition 4.8. *There exists a menu of revenue sharing contracts (w, α) that can coordinate the reverse supply chain under collector's lead.*

Proof. As we proceed in Proposition 3.11 in Chapter 3, Lemma 4.2 is not valid when we force the collector and remanufacturer to employ particular pair of (w, α). Thus, we need to solve the sequential game as follows. Starting from the follower's problem, we have the optimal order quantity of the remanufacturer, which is $Q = F^{-1}\Big(\frac{\alpha p - w - c_r}{\alpha p}\Big)$. From Proposition 4.7, we already know that $(\hat{w}, \hat{\alpha})$, that satisfies $\hat{w} = \hat{\alpha}(w'+c_r) - c_r$, can coordinate the reverse supply chain under RL. As we plug $w = \hat{w}$, $\alpha = \hat{\alpha}$ and $\hat{w} = \hat{\alpha}(w'+c_r) - c_r$ into the remanufacturer's optimal order quantity, we get $Q = F^{-1}\Big(\frac{p-w'-c_r}{p}\Big)$. Since w' satisfies $F^{-1}\Big(\frac{p-w'-c_r}{p}\Big) = r\frac{w'-c_c}{2}$, $Q = r\frac{w'-c_c}{2}$. Now, we continue with the collector's problem, given the remanufacturer's decision.

$$\begin{aligned} \Pi_C^{CL}(a) = (1-\hat{\alpha})p\Big(&min\{Q, y(a)\} - \int_0^{min\{Q,y(a)\}} F(x)dx\Big) \\ &+(\hat{w} - c_c - a)min\{Q, y(a)\}, \end{aligned} \tag{4.27}$$

in which $y(a) = ra$. You notice that the collector's problem is a function of a and we have already plugged $(\hat{w}, \hat{\alpha})$. Note that the collector would never collect more than the remanufacturer's order quantity, the collector's problem (4.27) can be revised as follows:

$$max. \quad \Pi_C^{CL}(a) = (1-\hat{\alpha})p\Big(ra - \int_0^{ra} F(x)dx\Big) + ra(\hat{w} - c_c - a) \tag{4.28}$$
$$s.t \quad a \leqslant \frac{w' - c_c}{2}.$$

The unconstrained problem is concave with respect to a and by taking the only constraint into account, the optimal acquisition price is given by

$$a = min\{\frac{(1-\hat{\alpha})p\Big(1 - F(ra)\Big) + \hat{w} - c_c}{2}, \frac{w' - c_c}{2}\}. \tag{4.29}$$

We can see that both values in curly braces are equal, similar to what we discuss in Proposition 4.7. Hence, the quantity collected by the collector would be $y = r\frac{w'-c_c}{2}$ which is equal to the remanufacturer's order quantity. $\square$

Thus, any pair of (w, α) that satisfies $w = \alpha(w' + c_r) - c_r$, in which w' satisfies $F^{-1}\Big(\frac{p-w'-c_r}{p}\Big) = r\frac{w'-c_c}{2}$, can coordinate the reverse supply chain under collector's lead.

4.4 Numerical Study

So far, we show that under the Stackelberg gaming structure, the solution is trivial. However, if coordination becomes an option, there exists a menu of revenue sharing contracts that can coordinate reverse supply chain. As we show in Section 4.3, there is a linear relation between coordinating w and α. When there is no revenue sharing ($\alpha = 1$), the remanufacturer must pay wholesale price $w = w'$ to coordinate. By setting α less than 1, the remanufacturer would pay a lower wholesale price. As α goes further below a certain threshold, the collector will pay to the remanufacturer (the wholesale price becomes negative). In Figure 4.2, we see that as α becomes zero,

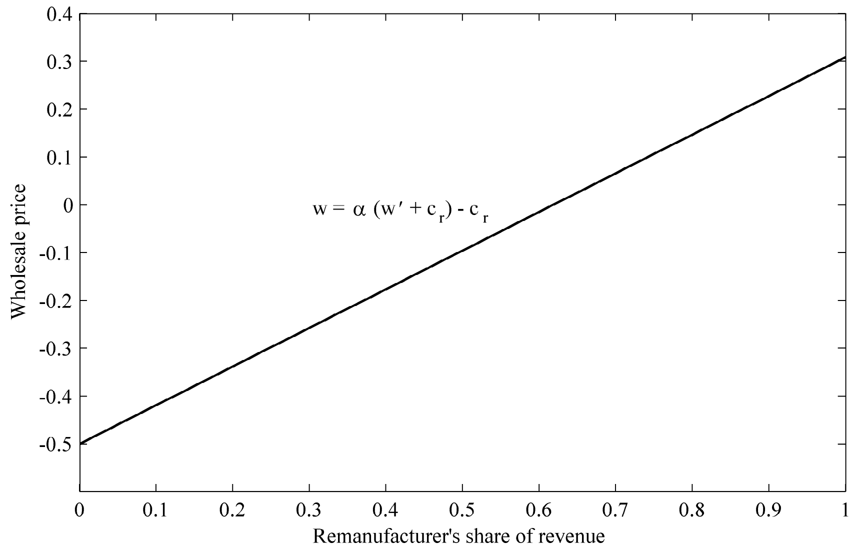


Figure 4.2: Menu of revenue sharing contract

the amount that the collector pays to the remanufacturer covers remanufacturing cost of used products.

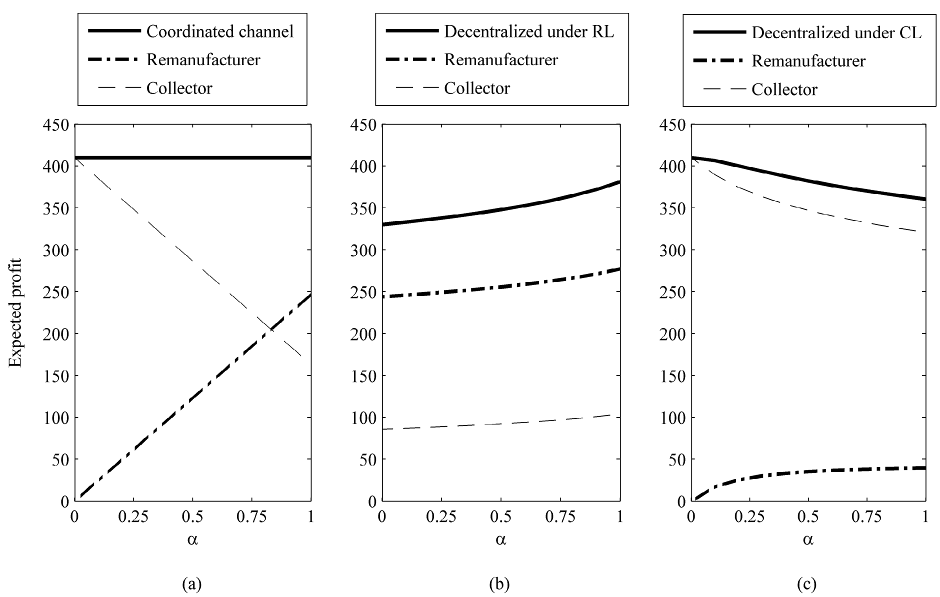


Figure 4.3: Expected profit of total supply chain, remanufacturer and collector in coordinated channel, decentralized channel under RL and CL with respect to α

As the remanufacturer's share of revenue decreases, although the wholesale price decreases as well, the decrease in the revenue is higher than the cost and therefore, its profit decreases. As a result, since total channel expected profit is constant under coordination, collector's profit increases as α decreases. We compare coordination scheme under RL and CL Stackelberg games in Figure 4.3. In Section 4.1, we show that the remanufacturer's profit function is increasing in α and the remanufacturer would set $\alpha = 1$ in order to maximize its profit. Figure 4.3b also shows that $\alpha = 1$ also maximizes total supply chain profit and collector's profit in the feasible region $0 \leqslant \alpha \leqslant 1$. The collector, as the leader, maximizing its own profit, sets $\alpha = 0$, i.e. asks for all remanufacturer's revenue. As it can be observed, decentralized setting under CL channel with revenue sharing achieves centralized-level profit, compared to Figure 4.3a and 4.3c.

We carry out a numerical study on revenue sharing contract performance by changing p, c_r, c_c, σ and r. When the selling price increases, the remanufacturer consider ordering more quantity. Since we are analyzing interaction of cooperating partners, the remanufacturer should provide higher incentive for the collector to collect more. As a result, as it can be observed in Figure 4.4a, at any level of α, the remanufacturer pays a higher wholesale price as selling price increases. When remanufacturing becomes costly, the remanufacturer is unwilling to order much and decreases the wholesale price. On the other hand, when collector's cost increases, the collector asks for higher wholesale price to collect adequately. Figure 4.4b and 4.4c depict these behaviors in revenue sharing contracts performance.

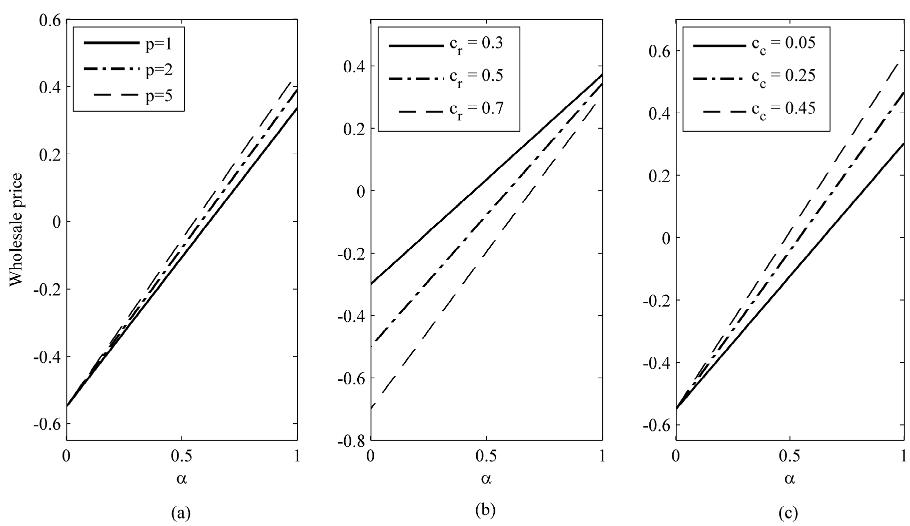


Figure 4.4: Coordinating wholesale price and remanufacturer's share of revenue (α) with respect to different (a) selling prices, (b) remanufacturing costs and (c) used product handling costs

We next consider the effects of demand uncertainty on the menu of contracts. When selling price is low, the risk of overstocking causes the system to decrease the order quantity and consequently, to decrease the wholesale price. Therefore, we can see that

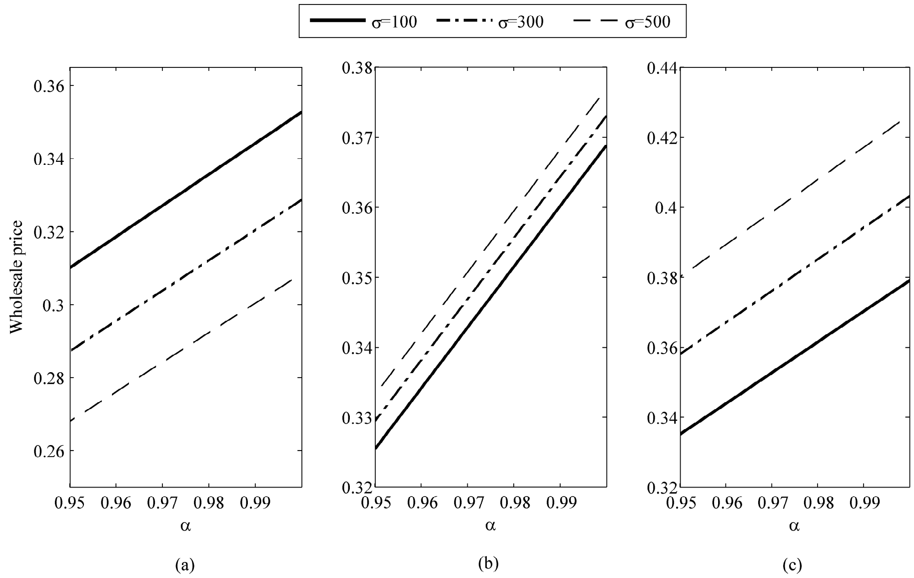


Figure 4.5: Coordinating wholesale price and remanufacturer's share of revenue with respect to different demand standard deviation for (a) $p = 1$, (b) $p = 2$, (c) $p = 5$

at a given value of α, as the demand variance increases, the coordinating wholesale price decreases. As the profit margin increases, the system decides to stock more in order to avoid the risk of under-stocking. As a result, more incentive is provided to the collector. As it can be seen in Figure 4.5c, a higher wholesale price is paid by the remanufacturer for a given value of α, when uncertainty increases.

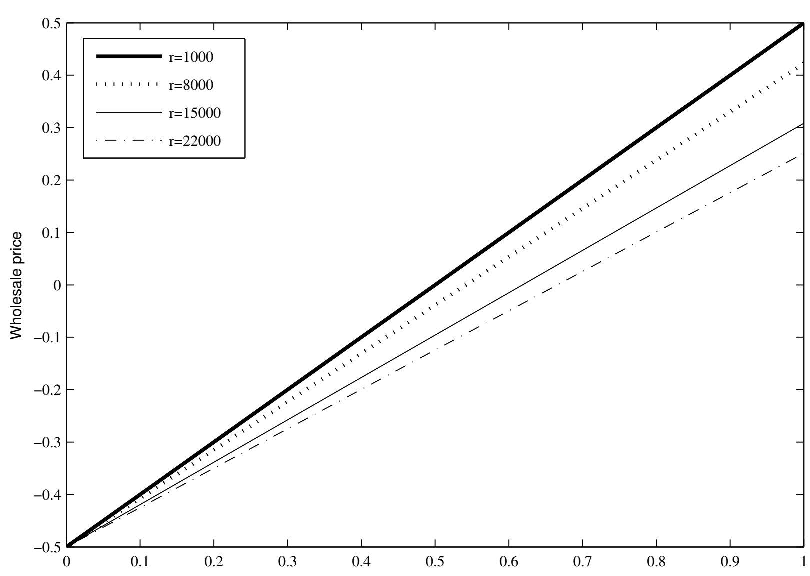


Figure 4.6: Coordinating wholesale price and remanufacturer's share of revenue with respect to price sensitivity of returns

In Figure 4.6, we illustrate behavior of the contract as price sensitivity of the returns changes. As mentioned before, increase in r is equivalent to increase in availability of used items. In other word, as r increases, at the same acquisition price, the collector can collect a larger quantity of used products. Thus, the remanufacturer would pay a lower wholesale price at any given value of α, as r increases.

CHAPTER 5

SUMMARY, CONCLUSIONS AND FUTURE WORKS

Adopting take-back activities in the form of closed-loop supply chain requires that Original Equipment Manufacturers (OEM) involve in a completely different, broad range of new activities, namely used product acquisition, inspection and remanufacturing. OEMs may partner with third-party firms in order to take full or partial responsibility of take-back action. Although outsourcing may seem a key decision, but it may initiate some inefficiencies in the performance of supply chain caused by decentralization. In order to reduce the effect of decentralization, some forms of contracts are utilized to align decentralized firms' decisions to optimize the total supply chain performance. In this thesis, we study decentralized and centralized settings of a two-echelon reverse supply chain, consisting of a remanufacturer and a collector. Under stochastic demand and price-dependent return assumptions, we derive the optimal acquisition price and wholesale price of decentralized setting for both remanufacturer's lead and collector's lead channel. We demonstrate that the optimal acquisition price is the same for different costs of remanufacturing and used products handling, as far as the summation of costs remains constant. Moreover, the quantity of used products collected and the individual profits of the remanufacturer and the collector remain unchanged. In other words, different assignments of responsibilities between remanufacturer and collector results in the same channel performance. We demonstrate that this result is not peculiar to our modeling approach.

By comparing centralized and decentralized solution, we show that there exists a wholesale price that can coordinate the reverse supply chain, which is unprecedented in the contracting literature. Beforehand, studies show that a wholesale price-only contract cannot coordinate supply chain since it results in non-positive profit for supplier in forward channel. However, in this study, we characterize the wholesale price that achieves coordination and corresponding positive profit for the remanufacturer and the collector.

Through an experimental study, we contrast remanufacturer's lead and collector's lead channel performance. It is shown that although it seems that remanufacturer may have a higher power to lead the channel, the channel would benefit more from collector leadership in some cases. For instance, when remanufacturing cost or used product handling cost is high, the collector invests more in product acquisition and collects more used products. Moreover, when price sensitivity of return is low, i.e. higher expectation level of end customer, the collector, as the leader, offers more acquisition price and collects more used product compared to remanufacturer's lead channel.

We further study the implementation of revenue sharing contract in the reverse supply chain. Although a wholesale price-only contact (WPC) is much easier to implement when compared to revenue sharing contract (RSC), WPC does not provide flexibility, particularly regarding the allocation of channel profit between the parties. We derive a menu of revenue sharing contract that yields coordination in reverse supply chain and provide more flexibility in profit allocation.

Our intentional focus in this study is to investigate the coordination issues in reverse supply chain. The assumptions and the methodology have been designed in order to attain tractable results without the loss of generality. To this end, we may partially lose the attention to a more realistic environment, namely the quality of return. As future research direction, we suggest reader to relax the homogeneity of used products assumption and to study the effect of used product differentiation in acquisition management. It would be also interesting to take forward channel activity into account and investigate the effect of our achievement in the scope of closed-loop supply chain. We base our intention on the cases where an OEM decides to delegate take-back responsibility to third-party firms and that, the remanufactured product has different valuation and are sold in different market. However, for instance, the collector may be a retailer through which the OEM sells its product. Thereby, the reverse channel performance would definitely affect the forward interactions. In this case, it is also valuable if someone takes green advertising effort into consideration.

Bibliography

[1] C. Prahinski and C. Kocabasoglu. Empirical research opportunities in reverse supply chains. *Omega*, 34(6):519 – 532, 2006.

[2] Grit Walther, Eberhard Schmid, and Thomas S. Spengler. Negotiation-based coordination in product recovery networks. *International Journal of Production Economics*, 111(2), 2008.

[3] Onur Kaya. Incentive and production decisions for remanufacturing operations. *European Journal of Operational Research*, 201(2):442 – 453, 2010.

[4] Eurostat European Commission. *Energy, transport and environment indicators*. European Commission, 2012. ISBN 978-92-79-26596-9. URL `http://goo.gl/4ocvbg`.

[5] V. D. R. Guide and Luk N Wassenhove. Or forum-the evolution of closed-loop supply chain research. *Operations research*, 57(1):10–18, 2009.

[6] Mortiz Fleischmann, Hans Ronald Krikke, Rommert Dekker, and Simme Douwe P. Flapper. A characterisation of logistics networks for product recovery. *Omega*, 28(6):653 – 666, 2000.

[7] Douglas M Lambert, James R Stock, and Richard D Irwin. *Strategic physical distribution management*. RD Irwin, 1982.

[8] Martijn Thierry. An analysis of the impact of product recovery management on manufacturing companies, 1997.

[9] Wendy Kerr and Chris Ryan. Eco-efficiency gains from remanufacturing: A case study of photocopier remanufacturing at fuji xerox australia. *Journal of cleaner production*, 9(1):75–81, 2001.

[10] V. D. R. Guide, Vaidy Jayaraman, and Jonathan D Linton. Building contingency planning for closed-loop supply chains with product recovery. *Journal of operations Management*, 21(3):259–279, 2003.

[11] Yen Chun Jim Wu and Wei Ping Cheng. Reverse logistics in the publishing industry: China, hong kong, and taiwan. *International Journal of Physical Distribution & Logistics Management*, 36(7):507–523, 2006.

[12] Vaidyanathan Jayaraman and Yadong Luo. Creating competitive advantages through new value creation: A reverse logistics perspective. *The Academy of Management Perspectives*, 21(2):56–73, 2007.

[13] Miele. Miele sustainability report, 2013. URL `http://www.miele.ca/Media/docs/about/Miele_Sustainability.en-CA.pdf`.

[14] Electrolux. Electorlux annual report 2014, 2014. URL `http://www.electroluxgroup.com/en/wp-content/uploads/sites/2/2015/02/Electrolux-Annual-Report-2014.pdf`.

[15] Kannan Govindan and Maria Nicoleta Popiuc. Reverse supply chain coordination by revenue sharing contract: A case for the personal computers industry. *European Journal of Operational Research*, 233(2):326–336, 2014.

[16] Kannan Govindan, Maria Nicoleta Popiuc, and Ali Diabat. Overview of coordination contracts within forward and reverse supply chains. *Journal of Cleaner Production*, 47:319 – 334, 2013.

[17] Charles X. Wang. A general framework of supply chain contract models. *Supply Chain Management: An International Journal*, 7(5):302–310, 2002.

[18] Barry Alan Pasternack. Optimal pricing and return policies for perishable commodities. *Marketing Science*, 4(2):166–176, 1985.

[19] Abel P. Jeuland and Steven M. Shugan. Managing channel profits. *Marketing Science*, 2(3):239–272, 1983.

[20] Julie H Mortimer. The effects of revenue-sharing contracts on welfare in vertically-separated markets: evidence from the video rental industry. *Harvard Institute Research Working Paper*, 2002.

[21] Gilvan C. Souza. Closed-loop supply chains: A critical review, and future research*. *Decision Sciences*, 44(1):7–38, 2013.

[22] V. D. R. Guide and Luk N Wassenhove. Managing product returns for remanufacturing. *Production and Operations Management*, 10(2):142–155, 2001.

[23] Jr. V. Daniel R. Guide, Ruud H. Teunter, and Luk N. Van Wassenhove. Matching demand and supply to maximize profits from remanufacturing. *Manufacturing & Service Operations Management*, 5(4):303–316, 2003.

[24] Ismail Serdar Bakal and Elif Akcali. Effects of random yield in remanufacturing with price-sensitive supply and demand. *Production and operations management*, 15(3):407–420, 2006.

[25] Michael R Galbreth and Joseph D Blackburn. Optimal acquisition quantities in remanufacturing with condition uncertainty. *Production and Operations Management*, 19(1):61–69, 2010.

[26] Stefan Minner and Gudrun P. Kiesmüller. Dynamic product acquisition in closed loop supply chains. *International Journal of Production Research*, 50 (11):2836–2851, 2012.

[27] Samuel Vercraene and Jean-Philippe Gayon. Optimal control of a production-inventory system with productreturns. *International Journal of Production Economics*, 142(2):302 – 310, 2013.

[28] Serra Caner Bulmuş, Stuart X Zhu, and Ruud H Teunter. Optimal core acquisition and pricing strategies for hybrid manufacturing and remanufacturing systems. *International Journal of Production Research*, 52(22):6627–6641, 2014.

[29] R Canan Savaskan, Shantanu Bhattacharya, and Luk N Van Wassenhove. Closed-loop supply chain models with product remanufacturing. *Management science*, 50(2):239–252, 2004.

[30] Atalay Atasu, L Beril Toktay, and Luk N Van Wassenhove. How collection cost structure drives a manufacturer's reverse channel choice. *Production and Operations Management*, 22(5):1089–1102, 2013.

[31] Jie Xu and Nan Liu. Research on closed loop supply chain with reference price effect. *Journal of Intelligent Manufacturing*, pages 1–14, 2014.

[32] K Arshinder. An integrative framework for supply chain coordination. *Unpublished doctoral thesis, Indian Institute of Technology Delhi, New Delhi*, 2008.

[33] Martin A Lariviere and Evan L Porteus. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing & service operations management*, 3(4): 293–305, 2001.

[34] Gérard P. Cachon and Martin A. Lariviere. Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Management Science*, 51 (1):30–44, 2005.

[35] A. Warren and M. Peers. Video retailers have day in court-plaintiffs say supply deals between blockbuster inc. and studios violates laws. *Wall Street Journal*, 13, 2002.

[36] Ilaria Giannoccaro and Pierpaolo Pontrandolfo. Supply chain coordination by revenue sharing contracts. *International journal of production economics*, 89 (2):131–139, 2004.

[37] Cao To Linh and Yushin Hong. Channel coordination through a revenue sharing contract in a two-period newsboy problem. *European Journal of Operational Research*, 198(3):822–829, 2009.

[38] Ibrahim Karakayalı, Hülya Emir-Farinas, and Elif Akcali. An analysis of decentralized collection and processing of end-of-life products. *Journal of Operations Management*, 25(6):1161–1183, 2007.

[39] Pietro De Giovanni. Environmental collaboration in a closed-loop supply chain with a reverse revenue sharing contract. *Annals of Operations Research*, 220(1): 135–157, 2014.

[40] Seung Ho Yoo, DaeSoo Kim, and Myung-Sub Park. Pricing and return policy under various supply contracts in a closed-loop supply chain. *International Journal of Production Research*, 53(1):106–126, 2015.

[41] Martin A. Lariviere and Evan L. Porteus. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing & Service Operations Management*, 3 (4):293–305, 2001.

[42] Gérard P Cachon. Supply chain coordination with contracts. *Handbooks in operations research and management science*, 11:227–339, 2003.