# TIME SERIES ANALYSIS OF TURKISH EXPORTS OF SELECTED ORES AND CONCENTRATES 

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# TIME SERIES ANALYSIS OF TURKISH EXPORTS OF SELECTED ORES AND CONCENTRATES 

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# ABSTRACT <br> TIME SERIES ANALYSIS OF TURKISH EXPORTS OF SELECTED ORES AND CONCENTRATES 

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This study investigates exports of chromium ores and concentrates, feldspar, copper ores and concentrates, zinc ores and concentrates, natural borates and their concentrates from Turkey. The cointegration analysis demonstrates that there exists long run relationship among the variables. The study uses Vector Error Correction (VECM) model to forecast the export amounts of aforesaid ores and concentrates based on exogenous variables, incentive certificates, fixed investment amount, total industrial turnover index, industrial production index and manufacturing purchasing managers' index of China. It is seen that incentive certificates and industrial production index are positively related to all the endogenous variables.

Keywords: Exports of Ores and Concentrates, Cointegration, Multivariate Time Series Analysis, VECM.

## ÖZ

# SEÇiLMİŞ CEVHER VE KONSANTRELERİN İHRACATININ ZAMAN SERİSİ ANALİZİ 

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Bu çalşsmada, krom cevherleri ve konsantreleri, feldispat, bakır cevherleri ve konsantreleri, çinko cevherleri ve konsantreleri ve tabii boratlar ve bunların konsantreleri ihracatı araştırılmıştır. Eşbütünleşme analizi, değişkenler arasında uzun dönemli bir ilişki bulunduğunu göstermektedir. Anılan cevher ve konsantrelerin ihracat miktarlarını tahmin etmek için teşvik belge adedini, sabit yatırım tutarını, toplam sanayi ciro endeksini, sanayi üretim endeksini ve Çin imalat sanayi satın alma yöneticileri endeksini dışsal değişken olarak kabul eden Vektör Hata Düzeltme Modeli (VHDM) kullanılmıştır. Teşvik belge adedi ile sanayi üretim endeksinin tüm içsel değişkenler ile pozitif ilişkili olduğu görülmektedir.

Anahtar Kelimeler: Cevher ve Konsantre İhracatı, Eşbütünleşme, Çok Boyutlu Zaman Serileri Analizi, Vektör Hata Düzeltme Modeli.

To My Parents

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## LIST OF ABBREVIATIONS

| ADF | : Augmented Dickey-Fuller |
| :---: | :---: |
| AIC | : Akaike Information Criterion |
| BIC | : Bayesian Information Criterion |
| CH | : Canova-Hansen |
| DPT | : Devlet Planlama Teşkilatı |
| ECT | : Error Correction Term |
| EGLS | : Estimated Generalized Least Squares |
| FPE | : Forecast Prediction Error |
| GLS | : Generalized Least Squares |
| HQ | : Hannan-Quinn |
| KPSS | : Kwiatkowski-Phillips-Schmidt-Shin |
| LSE | : Least Squares Estimation |
| MAE | : Mean Absolute Error |
| ML | : Maximum Likelihood |
| MPE | : Mean Percentage Error |
| MSE | : Mean Square Error |
| OCSB | : Osborn-Chui-Smith-Birchenhall |
| PMI | : Purchasing Managers' Index |
| PP | : Phillips-Perron |


| RMSD | : Root Mean Square Deviation |
| :--- | :--- |
| SC | : Schwarz |
| TURKSTAT: | Turkish Statistical Institute |
| VAR | : Vector Autoregressive |
| VARMA | : Vector Autoregressive Moving Average Model |
| VARMAX | : Vector Autoregressive Moving Average Processes with Exogenous |
|  | Variables |
| VARX | : Vector Autoregressive Processes with Exogenous Variables |
| VECM | : Vector Error Correction Model |
| VHDM | : Vektör Hata Düzeltme Modeli |
| WN | : White Noise |

## CHAPTER 1

## INTRODUCTION

It is known that mining in Anatolia dates back to approximately 10.000 B.C. and it leaded the civilizations developed here. It is not a coincidence that Anatolia has been a cradle for man civilizations and this situation results from many mine resources known and produced since ancient times.

In the time of Ottoman Empire, mining activities were carried out in order to meet the gun and ammunition needs and to print money. In the last periods of Ottoman Empire, mines of Anatolia were opened for foreign capital entry in the area of country in accordance with Paris Treaty dated 1862. France being in the first place, England, Germany, Italy and Russia made considerable investments to produce Anatolia mines and they performed considerable mine production. Traces of mining activities performed in this period are found in many places of the country today.

Turkey's Republic period mining dates back to İzmir Economic Congress in 1923. In this period in which liberal economy was adopted, important steps were taken in mining, "Mines and Industry High School", "Industry Mine Bank", "Mineral Research and Exploration Institute" and "Etibank" were established respectively in order to be appointed in education, finance, research and production areas.
"Mining Sector" is one of the sectors constituting national economy. Mining sector provides the basic elements needed by the other sectors of economy, primarily the industry (agriculture, services, transportation, energy etc.) and on the other hand creates new employment opportunities in rural areas and prevents migration. In these regions, it makes important infrastructure investments possible, transportation being in the first place. Modern technology, marketing and finance methods are adopted
and they become widespread in the rural areas of the country in parallel with the development of the mining sector. In another aspect, export of mining products is an important source which gains foreign currency to the country. In almost all developed industry countries of today, the mining sector undertakes the role of a "leading sector" which started economic development. In domestic incomes of such countries, the share of mining sector is a high rate such as $10-15 \%$ (Council of Chiefs of Mining Sector, 2012).

### 1.1. Purpose and Motivation

Mining sector has great importance for Turkey's economy. National industry and agriculture sector, which are developing, depend on the mining sector for main inputs. Although it is known that there are quite various and rich mine reserves in Turkey, most of these resources couldn't be commissioned yet. Share of mining sector in Turkey's domestic income is in a very low level such as $1.2 \%$. Although Turkey has the potential to become an important mine exporter, Turkey's mine export is quite limited and some mine products are imported. Briefly, the mining sector cannot make the great and important contribution to the development of the country despite its potential (Council of Chiefs of Mining Sector, 2012).

The share of the mining sector in Turkey's domestic income should be considered as an indicator that the mine resources in Turkey are not evaluated sufficiently and this study aims at making a contribution to the development of the mining sector in Turkey.

Statistical analysis of the mines, which are very important for Turkey and for the world, is important in terms of establishing future policies.

Types of ores and concentrates to be modeled in the study are listed below. Export values of these ores and concentrates are generally the highest compared to other ores and concentrates.

## Chromium Ores and Concentrates

Turkey, which is among the limited number of chromium producers in the world, acquires an important foreign currency income due to the export of crude ore, ferrochromium and chromium chemicals. In addition to the increasing consumption of crude ore in the domestic market, a considerable amount of shivered and concentrated ores are exported.

Although the production of Turkey ranked first in the world in some of the past years, it has generally preserved its place between the $3^{\text {rd }}$ and the $6^{\text {th }}$. Share of Turkey in chromium ore export is about $15 \%$.

When long term chromium ore production and export amounts of the Turkey are examined, there is a general trend increase in production and export in spite of some fluctuations.

In January 1987, the government of USA has renewed its decision that the chromium ore and ferrochromium are strategic materials based on the fact that the procurement sources of chromium are not very reliable.

The number of major countries producing chromium ore is about 30 and the largest chromium metal producer in the world is USA.

Chromium ore is used primarily in metallurgy, chemistry, refractory and molding industry. The most important area of use of chromium ore in metallurgy industry is the production of stainless steel. Chrome provides steel with hardness and resistance against fraction and impacts, it protects steel from abrasion and oxidation. In this scope, various alloys of chromium are used in automotive, submarine, ship and plane industry and in gun production. Super alloys of chromium are used in the construction of heat resistant, high productivity turbine motors. Chrome chemicals are used in chemistry industry in the production of sodium bichromate, chromic acid and dye raw materials. Chrome chemicals are consumed in metal coating, alutation,
ceramics, polisher tools, catalyst, conserved food canning (canning agents), water treatment, drilling fluid and many other areas (DPT, 2001).

## Feldspar

Feldspar is one of the basic raw materials in tile and ceramic production and welding electrode production. Also, glass, plastic and dye industry are main feldspar consumers.

USA ranks the first in terms of feldspar consumption amount.

Feldspar group minerals have an important marker because they are important raw materials for a certain level of quality in ceramic and glass industries. In recent feldspar production order of the countries; there are Italy, Turkey, USA, Thailand and other European countries (DPT, 2001).

## Copper Ores and Concentrates

Use of copper in human history started in ancient ages. People used copper in their daily lives in ornaments, guns and handcrafts and as civilization developed, the need for copper increased. Copper, the consumption of which is above 13 million tons today, is the second most used metal. Constant demand increase for copper is parallel with the developments in industrialization. The place of copper in industrialization and mechanization is an indisputable fact.

The features of high electricity and heat conductivity make the copper an essential input for power plants and conductive material. The copper has wide areas of use in cool air machines and equipments, in shipping vehicles and external coatings due to its anti-corrosive properties. In addition, the copper has an important place in welding works, metallurgy and bronze production and many other areas of use can be listed. The widest areas of use are plants regarding electricity production and transmission, construction, transportation machines and equipments, respectively.

In today's world, in which life standards constantly rise depending of the economic developments, the fact is understood that the demand for copper will increase constantly, even if a material is devised for some areas of use, the copper will always maintain its currency (DPT, 2001).

## Zinc Ores and Concentrates

Today, zinc is the most consumed metal in the world after steel, aluminum and copper. Zinc is used in the production of many alloys and compounds in the industry because it is chemically active and it can be alloyed with other metals easily. It is used in protection of other metals, especially iron and steel from abrasion due to its strong electro-positive properties. There are five areas in which the produced zinc metal is used as main product. These are galvanization, press mould alloys, brass and bronze alloys, zinc oxide and milled zinc alloys.

Base metals, especially iron and zinc are coated with zinc in order to be protected from corrosion. A large portion of zinc produced in the world is used in this way. The most important reasons for this are zinc's resistance to normal disintegration events, the property of iron protection (galvanization) and economy. Galvanization industry is the area in which zinc is used most in terms of amount. Its second area of use is the production of press mould alloys. Galvanized materials are primarily layers, strip pipes, tubes, wires, wire ropes, structural figures and many metal articles. Galvanized materials are used mostly in construction works, agriculture, construction of agriculture articles and automotive industry. There are thousands of application areas in which galvanized iron and steel products are especially used. Ceiling and wall coatings, wire and wire products, water tanks, pipes, buckets and canister, nails, screws, bolts, hinges can be listed. Zinc coatings are also used for the protection of structures under sea or in other chloride-rich solutions from corrosion. Pipelines, storage tanks, steel wave breakers, bridges and piers are this kind of places.

In accordance with the developing technology, zinc metal has acquired an important place recently as alloy input in the production of composite materials. Its area of use
is widened especially in the space industry and robot development and automatic control systems (DPT, 2001).

## Natural borates and their concentrates

Boron is one of the essential elements for all living creatures to sustain their lives in the nature. In addition, boron is the element which has the widest area of use in the world.

Areas of use of boron minerals and products which are used in a variety of sectors increase gradually. While about $10 \%$ of produced boron minerals are consumed directly as minerals, the remaining portion is used to obtain boron products.

Boron is used in industries such as window glass, bottle glass etc. Boric acid is an essential element in special glasses and they are used as refined wet/dry borax, boric acid or in natural structure as colemanite/borax. Boron is added to glass products for which heat insulation is deemed necessary as boron increases the viscosity, surface hardness and resistance of glass intermediate product when it is mixed with its molten structure.

Boric oxide is used to decrease viscosity and saturation temperature of enamels. Enamel coated with metal prevents oxidation and glamorizes its look. Steel, aluminum, copper, gold and silver can be coated with enamel. Enamel increases resistance against acids. Many kitchen tools are enamel coated. Bathrooms, chemistry industry equipments, water tanks, guns, etc. can be also coated with enamel. Boron gives ceramic resistance to scratch.

Boron compounds are added to soaps and detergents due to its germicide and water softener effects and they are also added to powder detergents to increase whitening effect.

Boron provides fire resistance. Boron compounds used in plastics in gradually increasing rates to prevent inflammableness.

Boron minerals are used in order to increase or prevent the development of flora.

Boron is the basic nutrition of many plants in varying rates.

Borates are used in non-ferrous metal industry as a material that constitutes protective clinker and accelerates melting due to its properties of constituting a smooth, adhesive, protective, clean and burr-free fluid in high temperatures. Boron compounds are used in electrolyte coating industry and in the production of electrolytes. In alloys, it is especially used to increase the hardness of steel.

Boron steel, boron carbide and titanium boron alloys are used in nuclear reactors. Boron minerals are also used in the storage of nuclear waste.

Boron used as anti-freeze material in automobiles to secure immediate swelling of airbags and also in hydraulic systems.

Boron is used for cleaning heavy metals in waste waters such as mercury, lead, silver from water.

Boron is used in cancer treatment. It is used especially in brain cancers to destroy infected cells and it is preferred because its damage to healthy cells is in minimum level.

Boron is used in energy storing and also as fuel (DPT, 2001).

### 1.2. Literature Review and Outline of the Study

Mining has received the attention of many researchers due to the fact that it has exhaustible characteristics and various social impacts. However, the studies specific to export modeling of the ores and concentrates are not available. There are very few studies which are about the relationship between mining export and economic growth. Sahoo and Sahu (2014) applied cointegration and causality analysis to find the relationship among mining export, industrial production and economic growth for

India and they showed that $\log$ run equilibrium relationship exists among the variables. This is the only study that we found in the literature considering the statistical analyses of the export of mines.

Chapter 2 provides information about the variables used in the model. Chapter 3 gives detailed information about systems of dynamic simultaneous equations and Chapter 4 is about data analysis. We provide the concluding remarks related to our study in the last section.

## CHAPTER 2

## DATA DESCRIPTION

In this section, brief information related to the variables that are used in study will be given.

Endogenous variables with Customs Tariff Statistics Position are 261000-Chromium ores and concentrates, 252910-Feldspar, 260300-Copper ores and concentrates, 260800-Zinc ores and concentrates and 2528-Natural borates and their concentrates. For simplicity, Chromium ores and concentrates are named as Chrome, Copper ores and concentrates are named as Copper, Zinc ores and concentrates are named as Zinc, Natural borates and their concentrates are named as Boron.

Exogenous variables are incentive certificates, fixed investment amount, total industrial turnover index, industrial production index and manufacturing purchasing managers' index of China.

Monthly data on these variables are obtained from January 2007 to February 2015. Each time series contains 98 observations and observations announced in 6 months after February 2015 are used for measuring forecast accuracy.

All endogenous variables are obtained from Turkish Statistical Institute (TURKSTAT). Incentive certificates, fixed investment amount are obtained from Ministry of Economy. Total industrial turnover index and industrial production index are obtained from TURKSTAT. Manufacturing purchasing managers' index of China is obtained from National Bureau of Statistics of China.

Incentive certificates and fixed investment amount belong to mining industry, extraction and processing subsector. Total industrial turnover index and industrial production index are calendar adjusted, they belong to mining and quarrying sector and base years are 2010.

Industrial turnover index is calculated in order to measure the evolution of the market of industry. It gives measure of the development of the receipts of sales. Industrial production index is calculated to measure the evolution of economy and the positive and negative effects of economical and political decisions in short term. Manufacturing purchasing managers' index is a significant indicator of business conditions and the overall economic condition. Hence, it is decided to use these variables to describe the export amounts of the selected ores and concentrates.

The time series plot of all the variables used in the study from January 2007 to February 2015 is given in Figure 1.


Figure 1. Time Series Plot of Export Amounts of the Ores and Concentrates in Tonne

From Figure 1, it is seen that export amounts of feldspar and chrome are generally the highest between January 2007 and August 2015. It is also seen that there is a decrease in export amounts of the ores and concentrates between late 2008 and early

2009 which is probably due to global financial crisis. It can be said that export amounts of the ores and concentrates of Turkey are affected by production amounts of the ores and concentrates and economic situations of the countries which demand for the ores and concentrates from Turkey. Hence, it is seen that all the variables move together through time. In the statistical analysis, we use this information to develop our time series model.

The leading countries that import selected ores and concentrates of Turkey are given in Table 1. From this table, it is seen that China is the leading importer of almost all the selected ores and concentrates of Turkey. For this reason, manufacturing purchasing managers' index of China is included in the model as an exogenous variable.

Table 1. Leading Importers of Turkey's Selected Ores and Concentrates in Tonne in 2014

| RANK | BORON | COPPER | CHROME | FELDSPAR | ZINC |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | China | China | China | Italy | Belgium |
| $\mathbf{2}$ | USA | Bulgaria | Swedish | Spain | Iran |
| $\mathbf{3}$ | Taiwan | Swedish | Belgium | Russia | China |

Descriptive statistics of export amounts of the ores and concentrates are given in Table 2. It can be said that export amounts of chrome and feldspar are generally the highest compared to other ores and concentrates. It is also seen that the mean and median values are not far from each other. Hence, we can say that the distributions of export amounts of the ores and concentrates are almost symmetric.

Table 2. Descriptive Statistics of Export Amounts of the Selected Ores and Concentrates in Tonne

|  | BORON | COPPER | CHROME | FELDSPAR | ZINC |
| :---: | :---: | :---: | :---: | :---: | :---: |
| MEAN | 51166 | 27797 | 157176 | 337910 | 28729 |
| MEDIAN | 51271 | 25469 | 156029 | 353305 | 28836 |
| ST.DEVIATION | 17655 | 15400 | 59487 | 79011 | 9444 |

The time series plot of the variables shows that the means of the series are not constant and changing over time. This is an indication of non-stationarity. The plot
also suggest that the variables are moving together and this might be the indication of cointegration relation between the variables. Hence, the proper modeling strategy for these series could be the vector error correction model (VECM). These assumptions will be checked in the data analysis part but now, the methodology and theoretical background related to the suggested modeling strategy are introduced.

## CHAPTER 3

## METHODOLOGY AND ANALYTICAL FRAMEWORK

In time series analysis, decision makers often need forecasts of a variable of interest. If observations of a variable are available and if they contain information about the future development of that variable, it is reasonable to use some function of the data collected in the past to forecast future values. In dealing with economic variables, often the value of one variable is not only related to its past values but also it depends on past values of others. Additional information which comes from other variables can be used to obtain forecasts with good properties (Lütkepohl, 2005).

Vector autoregressive models (VAR), vector autoregressive moving average models (VARMA), vector error correction models, vector autoregressive processes with exogenous variables (VARX), vector autoregressive moving average processes with exogenous variables (VARMAX) are some of the models which can be used to obtain forecasts for the variables of the system.

### 3.1. Systems of Dynamic Simultaneous Equations

In practice, the generation processes may also be affected by the variables outside of the system. Such variables are called as exogenous or unmodelled and the variables within the system are called as endogenous (Lütkepohl, 2005). A model with unmodelled variables can have the structural form:

$$
\begin{equation*}
\mathbf{A} y_{t}=A_{1}^{*} y_{t-1}+\ldots+A_{p}^{*} y_{t-p}+B_{0}^{*} x_{t}+B_{1}^{*} x_{t-1}+\ldots+B_{s}^{*} x_{t-s}+w_{t} \tag{3.1.1}
\end{equation*}
$$

where $y_{t}=\left(y_{1 t}, \ldots, y_{K t}\right)^{\prime}$ is a $K$-dimensional vector of endogeneous variables and $\mathrm{x}_{\mathrm{t}}$ $=\left(\mathrm{x}_{1 \mathrm{t}}, \ldots, \mathrm{x}_{\mathrm{Mt}}\right)^{\prime}$ is an $M$-dimensional vector of unmodelled variables, $\mathbf{A}$ is $\left(K^{\times} K\right)$ matrix and shows the relations between the endogenous variables, $A_{i}^{*}$ 's are $(K \times K)$ coefficient matrices, $B_{j}^{*}$ 's are $(K \times M)$ coefficient matrices and $w_{t}$ is a $K$-dimensional error vector. If the error term is white noise, the model described in (3.1.1) is referred to as Vector Autoregressive Model with Exogenous Variables VARX $(p, s)$. Generally, models of the form (3.1.1) are often called linear systems and in the econometrics literature they are known as dynamic simultaneous equations models (SEM).

In equation (3.1.1), $x_{t}$ is called exogenous if $x_{t}, x_{t-1}, \ldots, x_{t-s}$ are independent of the error term $w_{t} . x_{t}$ is called strictly exogenous if all its leads and lags are independent from all leads and lags of the error term $\mathrm{w}_{\mathrm{t}}$. Consider the following system:

$$
\begin{align*}
& y_{1 t}=z_{1}^{*}+\alpha_{11,1}^{*} y_{1, t-1}+\alpha_{12,1}^{*} y_{2, t-1}+\beta_{12,1}^{*} x_{1, t-1}+w_{1 t},  \tag{3.1.2}\\
& \mathrm{y}_{2 \mathrm{t}}=\mathrm{z}_{2}^{*}+\alpha_{22,1}^{*} \mathrm{y}_{2, \mathrm{t}-1}+\mathrm{a}_{21,0} \mathrm{y}_{1 \mathrm{t}}+\alpha_{21,1}^{*} \mathrm{y}_{1, \mathrm{t}-1}+\mathrm{w}_{2 \mathrm{t}} .
\end{align*}
$$

The system above can be written in terms of the representation (3.1.1) as follows:

$$
\begin{align*}
{\left[\begin{array}{cc}
1 & 0 \\
-\mathrm{a}_{21,0} & 1
\end{array}\right]\left[\begin{array}{l}
y_{1 t} \\
y_{2 t}
\end{array}\right]=} & {\left[\begin{array}{ll}
\alpha_{11,1}^{*} & \alpha_{12,1}^{*} \\
\alpha_{21,1}^{*} & \alpha_{22,1}^{*}
\end{array}\right]\left[\begin{array}{l}
y_{1, t-1} \\
y_{2, t-1}
\end{array}\right] } \\
& +\left[\begin{array}{cc}
z_{1}^{*} & \beta_{12,1}^{*} \\
z_{2}^{*} & 0
\end{array}\right]\left[\begin{array}{c}
1 \\
x_{1, t-1}
\end{array}\right]+\left[\begin{array}{l}
w_{1 t} \\
w_{2 t}
\end{array}\right] . \tag{3.1.3}
\end{align*}
$$

Here, $\mathrm{y}_{\mathrm{t}}=\left(\mathrm{y}_{1 \mathrm{t}}, \mathrm{y}_{2 \mathrm{t}}\right)^{\prime}$ and $\mathrm{x}_{\mathrm{t}}=\left(1, \mathrm{x}_{1 \mathrm{t}}\right)^{\prime}$ are both two-dimensional and the instantaneous effects are reflected in the elements of $\mathbf{A}$.

The reduced form of the system is achieved by premultiplying (3.1.1) with $\mathbf{A}^{-1}$ which gives

$$
\begin{equation*}
y_{t}=A_{1} y_{t-1}+\ldots+A_{p} y_{t-p}+B_{0} x_{t}+B_{1} x_{t-1}+\ldots+B_{s} x_{t-s}+u_{t} \tag{3.1.4}
\end{equation*}
$$

where $A_{i}:=\mathbf{A}^{-1} A_{i}^{*}(\mathrm{i}=1, \ldots, \mathrm{p}), B_{j}:=\mathbf{A}^{-1} B_{j}^{*}(\mathrm{j}=1, \ldots, \mathrm{~s})$ and $u_{t}:=\mathbf{A}^{-1} w_{t}$.

The reduced form is useful for forecasting, multiplier analysis and control purposes. For the model given in (3.1.3), the reduced form is below:

$$
\left[\begin{array}{l}
y_{1 t}  \tag{3.1.5}\\
y_{2 t}
\end{array}\right]=A_{1}\left[\begin{array}{l}
y_{1, t-1} \\
y_{2, t-1}
\end{array}\right]+B_{1}\left[\begin{array}{c}
1 \\
x_{1, t-1}
\end{array}\right]+\left[\begin{array}{l}
u_{1 t} \\
u_{2 t}
\end{array}\right]
$$

where

$$
\begin{align*}
& A_{1}=\left[\begin{array}{ll}
\alpha_{11,1} & \alpha_{12,1} \\
\alpha_{21,1} & \alpha_{22,1}
\end{array}\right]=\left[\begin{array}{cc}
\alpha_{11,1}^{*} & \alpha_{12,1}^{*} \\
\mathrm{a}_{21,0} \alpha_{11,1}^{*}+\alpha_{21,1}^{*} & \mathrm{a}_{21,0} \alpha_{12,1}^{*}+\alpha_{22,1}^{*}
\end{array}\right]  \tag{3.1.6}\\
& B_{1}=\left[\begin{array}{ll}
\beta_{11,1} & \beta_{12,1} \\
\beta_{21,1} & \beta_{22,1}
\end{array}\right]=\left[\begin{array}{cc}
z_{1}^{*} & \beta_{12,1}^{*} \\
\mathrm{a}_{21,0} z_{1}^{*}+z_{2}^{*} & \mathrm{a}_{21,0} \beta_{12,1}^{*}
\end{array}\right] \tag{3.1.7}
\end{align*}
$$

and

$$
\left[\begin{array}{l}
u_{1 t}  \tag{3.1.8}\\
u_{2 t}
\end{array}\right]=\left[\begin{array}{c}
w_{1 t} \\
a_{21,0} w_{1 t}+w_{2 t}
\end{array}\right]
$$

In lag operator notation, the reduced form (3.1.4) can be written as

$$
\begin{equation*}
A(L) y_{t}=B(L) x_{t}+u_{t} \tag{3.1.9}
\end{equation*}
$$

where $A(L):=\mathrm{I}_{K}-A_{1} L-\ldots-A_{p} L^{p}$ and $B(L):=B_{0}+B_{1} L+\ldots+B_{s} L^{s}$.

By multiplying (3.1.9) with $\mathrm{A}(\mathrm{L})^{-1}$, the effect of a change in an exogenous variable on the endogenous variables can be observed. The resulting representation:

$$
\begin{equation*}
y_{t}=D(L) x_{t}+A(L)^{-1} u_{t} \tag{3.1.10}
\end{equation*}
$$

where $D(L):=A(L)^{-1} B(L)$ and (3.1.10) is called the final form of the system. For the model given with reduced form (3.1.5), the final form is below:

$$
\begin{align*}
{\left[\begin{array}{l}
y_{1 t} \\
y_{2 t}
\end{array}\right] } & =\left(\mathrm{I}_{2}-A_{1} L\right)^{-1} B_{1} L\left[\begin{array}{c}
1 \\
x_{1 t}
\end{array}\right]+\left(\mathrm{I}_{2}-A_{1} L\right)^{-1}\left[\begin{array}{l}
u_{1 t} \\
u_{2 t}
\end{array}\right] \\
& =\left(\sum_{i=1}^{\infty} A_{1}^{i-1} B_{1} L^{i}\right)\left[\begin{array}{c}
1 \\
x_{1 t}
\end{array}\right]+\left(\sum_{i=0}^{\infty} A_{1}^{i} L^{i}\right)\left[\begin{array}{l}
u_{1 t} \\
u_{2 t}
\end{array}\right] \tag{3.1.11}
\end{align*}
$$

### 3.1.1. Cointegrated Variables

When a linear combination of two or more nonstationary, $\mathrm{I}(1)$, time series is stationary, $\mathrm{I}(0)$, then they are cointegrated. When there are $r$ cointegration relations among the endogenous variables and they are not cointegrated with the exogenous variables, the form of the model is as follows:

$$
\begin{align*}
\mathbf{A} \Delta y_{t}= & \alpha^{*} \beta^{\prime} y_{t-1}+\Gamma_{1}^{*} \Delta y_{t-1}+\ldots+\Gamma_{p-1}^{*} \Delta y_{t-p+1} \\
& +B_{0}^{*} x_{t}+B_{1}^{*} x_{t-1}+\ldots+B_{s}^{*} x_{t-s}+w_{t} \tag{3.1.1.1}
\end{align*}
$$

where $\mathbf{A}$ is a $(K \times K)$ matrix of instantaneous effects, $\alpha^{*}$ is a $(K \times r)$ matrix of structural loading coefficients, $\beta$ is the $(K \times r)$ cointegration matrix, $\Gamma_{j}^{*}(\mathrm{j}=1, \ldots, \mathrm{p}-1)$ is a $(K \times K)$ matrix of structural short-run coefficients and all other symbols are defined as in (3.1.1).

If there is cointegration between endogenous and unmodelled variables, the form of the model is:

$$
\begin{align*}
\mathbf{A} \Delta y_{t}= & \alpha^{*} \beta^{+\prime}\left[\begin{array}{l}
y_{t-1} \\
x_{t-1}
\end{array}\right]+\Gamma_{1}^{*} \Delta y_{t-1}+\ldots+\Gamma_{p-1}^{*} \Delta y_{t-p+1} \\
& +Y_{0}^{*} \Delta x_{t}+Y_{1}^{*} \Delta x_{t-1}+\ldots+Y_{s-1}^{*} \Delta x_{t-s+1}+w_{t} \tag{3.1.1.2}
\end{align*}
$$

where $\Upsilon_{\mathrm{j}}^{*}(\mathrm{j}=0,1, \ldots, \mathrm{~s}-1)$ is the coefficient matrix.

If the joint generation process of $y_{t}$ and $x_{t}$ has a reduced form vector error correction model (VECM) representation,

$$
\begin{align*}
{\left[\begin{array}{c}
\Delta y_{t} \\
\Delta x_{t}
\end{array}\right]=} & {\left[\begin{array}{c}
\alpha \\
\alpha_{x}
\end{array}\right] \beta^{+\prime}\left[\begin{array}{l}
y_{t-1} \\
x_{t-1}
\end{array}\right]+\left[\begin{array}{cc}
\Gamma_{1} & r_{1} \\
0 & \Gamma_{1}^{x}
\end{array}\right]\left[\begin{array}{l}
\Delta y_{t-1} \\
\Delta x_{t-1}
\end{array}\right]+\ldots } \\
& +\left[\begin{array}{cc}
\Gamma_{p-1} & \Upsilon_{p-1} \\
0 & \Gamma_{p-1}^{x}
\end{array}\right]\left[\begin{array}{l}
\Delta y_{t-p+1} \\
\Delta x_{t-p+1}
\end{array}\right]+\left[\begin{array}{l}
u_{t} \\
v_{t}
\end{array}\right] \tag{3.1.1.3}
\end{align*}
$$

premultiplying this model form with

$$
\left[\begin{array}{cc}
\mathbf{A} & -\Upsilon_{0}{ }^{*}  \tag{3.1.1.4}\\
0 & \mathrm{I}_{M}
\end{array}\right]
$$

gives the model where the first $K$ equations are in the structural form (3.1.1.2).

### 3.2. Estimation for Stationary Variables

Assume $\left(y_{t}^{\prime}, x_{t}^{\prime}\right)^{\prime}$ is generated from a stationary process and the parameters of the reduced form (3.1.4) will be estimated. The reduced form can be written as

$$
\begin{equation*}
y_{t}=A Y_{t-1}+B X_{t-1}+B_{0} x_{t}+u_{t} \tag{3.2.1}
\end{equation*}
$$

where $A:=\left[A_{1}, \ldots, A_{\mathrm{p}}\right], B:=\left[B_{1}, \ldots, B_{\mathrm{s}}\right], Y_{t}:=\left[\begin{array}{c}y_{t} \\ \cdot \\ \cdot \\ \cdot \\ y_{t-p+1}\end{array}\right], X_{t}:=\left[\begin{array}{c}x_{t} \\ \cdot \\ \cdot \\ \cdot \\ x_{t-s+1}\end{array}\right]$.
Furthermore, let us assume that $u_{t}$ is standart white noise with nonsingular covariance matrix $\sum_{\mathrm{u}}$ and a matrix $R$, a vector $\mp$ exist such that

$$
\begin{equation*}
\beta:=\operatorname{vec}\left[A, B, B_{0}\right]=R \Varangle . \tag{3.2.2}
\end{equation*}
$$

For a sample of size $T$, the compact form of the system is

$$
\begin{equation*}
Y=\left[A, B, B_{0}\right] Z+U, \tag{3.2.3}
\end{equation*}
$$

where $Y:=\left[y_{1}, \ldots, \mathrm{y}_{\mathrm{T}}\right], Z:=\left[\begin{array}{c}Y_{0}, \ldots, Y_{\mathrm{T}-1} \\ X_{0}, \ldots, X_{\mathrm{T}-1} \\ x_{1}, \ldots, x_{\mathrm{T}}\end{array}\right]$ and $U:=\left[u_{1}, \ldots, u_{\mathrm{T}}\right]$.

The vector form of the equation (3.2.3) is

$$
\begin{equation*}
y=\left(Z^{\prime} \otimes \mathrm{I}_{K}\right) R \mathrm{~J}+\mathbf{u}, \tag{3.2.4}
\end{equation*}
$$

where $y:=\operatorname{vec}(Y)$ and $u:=\operatorname{vec}(\mathrm{U})$. Generalized Least Squares (GLS) estimator is

$$
\begin{equation*}
\hat{\mathrm{f}}=\left[R^{\prime}\left(Z Z^{\prime} \otimes \sum_{u}^{-1}\right) R\right]^{-1} R^{\prime}\left(Z \otimes \Sigma_{u}^{-1}\right) y . \tag{3.2.5}
\end{equation*}
$$

and this estimator is not operational since $\sum_{u}$ is unknown (Lütkepohl, 2005). However, it can be estimated from the Least Squares (LS) estimator,

$$
\begin{equation*}
\hat{\mathrm{f}}=\left[R^{\prime}\left(Z Z^{\prime} \otimes \mathrm{I}_{K}\right) R\right]^{-1} R^{\prime}\left(Z \otimes \mathrm{I}_{K}\right) y \tag{3.2.6}
\end{equation*}
$$

which gives residuals $\breve{\mathrm{u}}=\mathrm{y}-\left(\mathrm{Z}^{\prime} \otimes \mathrm{I}_{\mathrm{K}}\right) \mathrm{R} \breve{\mathrm{f}}$ and the estimator of $\sum_{\mathrm{u}}$,

$$
\begin{equation*}
\overline{\sum_{u}}=\breve{U} \breve{U}^{\prime} / T \tag{3.2.7}
\end{equation*}
$$

where $\breve{U}$ is such that $\operatorname{vec}(\breve{U})=\breve{\mathbf{u}}$. The estimator of the white noise covariance matrix generates the Estimated Generalized Least Squares Estimator (EGLS),

$$
\begin{equation*}
\hat{\hat{\mathrm{H}}}=\left[R^{\prime}\left(Z Z^{\prime} \otimes \breve{\Sigma}_{u}^{-1}\right) R\right]^{-1} R^{\prime}\left(Z \otimes \breve{\Sigma}_{u}^{-1}\right) y . \tag{3.2.8}
\end{equation*}
$$

Under standard assumptions, this estimator is consistent and asymptotically normal,

$$
\begin{equation*}
\sqrt{\mathrm{T}} \hat{\hat{f}}-\mathrm{f}) \xrightarrow{\mathrm{d}} \mathrm{~N}\left(0, \Sigma_{\hat{\mathcal{p}}}\right) \tag{3.2.9}
\end{equation*}
$$

where

$$
\begin{equation*}
\sum_{\hat{f}}=\left(R^{\prime}\left[\operatorname{plim}\left(T^{-1} Z Z^{\prime}\right) \otimes \sum_{u}^{-1}\right] R\right)^{-1} \tag{3.2.10}
\end{equation*}
$$

To have this result, plim $\left(T^{-1} Z Z^{\prime}\right)$ where plim refers to weak consistency and the inverse of the matrix in (3.2.10) should exist. To guarantee the asymptotic normal distribution of the EGLS estimator, $u_{t}$ should be standard white noise, $x_{t}$ should be generated by a stationary and stable VAR process which is independent of the $u_{t}$ and the VAR part should be stable, that is,

$$
\begin{equation*}
|A(z)|=\left|\mathrm{I}_{K}-A_{1} z-\cdots-A_{p} z^{p}\right| \neq 0 \text { for }|z| \leq 1 \tag{3.2.11}
\end{equation*}
$$

Also, all the exogenous variables should be stochastic. It can be modified so as to include nonstochastic variables as well. In that case, plim in (3.2.10) becomes a nonstochastic limit in some or all components (Anderson, 1971; Harvey, 1981). An estimator for $\beta=R \mathrm{~J}$ is obtained as $\hat{\hat{\beta}}=R \hat{\mathrm{f}}$. If (3.2.9) holds, this estimator also has an asymptotic normal distribution,

$$
\begin{equation*}
\sqrt{\mathrm{T}}(\widehat{\widehat{\beta}}-\beta) \xrightarrow{\mathrm{d}} \mathrm{~N}\left(0, \Sigma_{\widehat{\beta}}=\mathrm{R} \Sigma_{\mathrm{f}_{\mathrm{f}} \mathrm{R}^{\prime}}\right) . \tag{3.2.12}
\end{equation*}
$$

Moreover, under general conditions, the estimator $\widehat{\widehat{\sum}}_{u}$ of the white noise covariance matrix is asymptotically independent of $\widehat{\hat{\beta}}$ and it has the same asymptotic distribution as the estimator $U U^{\prime} / T$ based on the unobserved true residuals.

### 3.3. Estimation of Models with I(1) Variables

When there are integrated and cointegrated variables in the model and a reduced form VECM corresponding to the structural form (3.1.1.2),

$$
\begin{align*}
\Delta y_{t}= & \alpha \beta^{+\prime}\left[\begin{array}{l}
y_{t-1} \\
x_{t-1}
\end{array}\right]+\Gamma_{1} \Delta y_{t-1}+\ldots+\Gamma_{p-1} \Delta y_{t-p+1} \\
& +\Upsilon_{0} \Delta x_{t}+\Upsilon_{1} \Delta x_{t-1}+\ldots+\Upsilon_{s-1} \Delta x_{t-s+1}+u_{t} \tag{3.3.1}
\end{align*}
$$

is formed, where $y_{t}$ is a process of dimension $K, \operatorname{rk}\left(\Pi^{+}\right)=r$ so that $\Pi^{+}=\alpha \beta^{+\prime}$, where $\alpha$ and $\beta^{+}$are $\left(K^{\times} r\right)$ matrices with $\operatorname{rk}(\alpha)=\operatorname{rk}\left(\beta^{+}\right)=r$ and $u_{t}$ is standart white noise with nonsingular covariance matrix $\sum_{u}$. Then, estimation procedure is as follows:

Assume that a sample of size $T$ and all required presample values are available, defining

$$
\begin{align*}
& \Delta Y:=\left[\Delta y_{1}, \ldots, \Delta y_{t}\right],  \tag{3.3.2}\\
& Y_{-1}^{+}:=\left[y_{0}^{+}, \ldots, y_{T-1}^{+}\right] \text {with } y_{t-1}^{+}:=\left[\begin{array}{l}
y_{t-1} \\
x_{t-1}
\end{array}\right], \tag{3.3.3}
\end{align*}
$$

$$
\Delta X^{+}:=\left[\Delta X_{0}^{+}, \ldots, \Delta X_{T-1}^{+}\right] \text {with } \Delta X_{t-1}^{+}:=\left[\begin{array}{c}
\Delta y_{t-1}  \tag{3.3.4}\\
\cdot \\
\cdot \\
\Delta y_{t-p+1} \\
\Delta x_{t} \\
\\
\Delta x_{t-1} \\
\cdot \\
\cdot \\
\cdot \\
\Delta x_{t-s+1}
\end{array}\right]
$$

and

$$
\begin{equation*}
U:=\left[u_{1}, \ldots, u_{T}\right], \tag{3.3.5}
\end{equation*}
$$

the model is achieved as follows:

$$
\begin{equation*}
\Delta Y=\alpha \beta^{+\prime} Y_{-1}^{+}+\Gamma^{+} \Delta X^{+}+U \tag{3.3.6}
\end{equation*}
$$

where

$$
\begin{equation*}
\Gamma^{+}:=\left[\Gamma_{1}: \ldots: \Gamma_{p-1}: \Upsilon_{0}: \Upsilon_{1}: \ldots: \Upsilon_{s-1}\right] \tag{3.3.7}
\end{equation*}
$$

$\beta^{+}$is a $((K+M) \times r)$ matrix and $\alpha$ is $(K \times r)$.

Because the error correction term now involves all the cointegration relations between the endogenous and unmodelled variables, it is possible that $r>K$. But, to be able to use all estimator types, $r \leq K$ is assumed. If $r=K$, the matrix $\Pi^{+}:=\alpha \beta^{+\prime}$ is of full row rank under the usual assumption, $\operatorname{rk}(\alpha)=\operatorname{rk}\left(\beta^{+}\right)=r$. Therefore, estimate of the matrix $\Pi^{+}=\alpha \beta^{+\prime}$ can be simply found by applying multivariate LS to (3.3.6). An estimator of $\beta^{+}$can then be obtained by normalizing the cointegration matrix such that

$$
\beta^{+}=\left[\begin{array}{c}
\mathrm{I}_{K}  \tag{3.3.8}\\
\beta_{(M)}^{+}
\end{array}\right]
$$

and using

$$
\begin{equation*}
\hat{\beta}^{+\prime}=\left(\widehat{\Pi}_{(1)}^{+}\right)^{-1} \widehat{\Pi}^{+} \tag{3.3.9}
\end{equation*}
$$

where $\widehat{\Pi}_{(1)}^{+}$is the $(K \times K)$ submatrix consisting of the first $K$ columns of the LS estimator $\widehat{\Pi}^{+}$of $\Pi^{+}$. From the matrix version (3.3.6), the LS estimator of $\Pi^{+}$and $\Gamma^{+}$ is seen to be:

$$
\left[\widehat{\Pi}^{+}: \hat{\Gamma}^{+}\right]=\left[\Delta Y Y_{-1}^{+\prime}: \Delta Y \Delta X^{+\prime}\right]\left[\begin{array}{cc}
Y_{-1}^{+} Y_{-1}^{+\prime} & Y_{-1}^{+} \Delta X^{+\prime}  \tag{3.3.10}\\
\Delta X^{+} Y_{-1}^{+\prime} & \Delta X^{+} \Delta X^{+\prime}
\end{array}\right]^{-1}
$$

For GLS estimation, it is assumed that $\beta^{+}$is normalized as in (3.3.8). Replacing the short-run parameters $\Gamma^{+}$by their LS estimators for a given $\Pi^{+}$gives,

$$
\begin{equation*}
\hat{\Gamma}^{+}\left(\Pi^{+}\right)=\left(\Delta Y-\Pi^{+} Y_{-1}^{+}\right) \Delta X^{+\prime}\left(\Delta X^{+} \Delta X^{+\prime}\right)^{-1} \tag{3.3.11}
\end{equation*}
$$

Hence,

$$
\begin{equation*}
\Delta Y=\Pi^{+} Y_{-1}^{+}+\left(\Delta Y-\Pi^{+} Y_{-1}^{+}\right) \Delta X^{+\prime}\left(\Delta X^{+} \Delta X^{+\prime}\right)^{-1} \Delta X^{+}+U^{*} \tag{3.3.12}
\end{equation*}
$$

Rearranging terms and defining the $(T \times T)$ matrix

$$
\begin{equation*}
\mathrm{D}:=\mathrm{I}_{\mathrm{T}}-\Delta \mathrm{X}^{+\prime}\left(\Delta \mathrm{X}^{+} \Delta \mathrm{X}^{+\prime}\right)^{-1} \Delta \mathrm{X}^{+} \tag{3.3.13}
\end{equation*}
$$

gives

$$
\begin{equation*}
R_{0}=\Pi^{+} \mathrm{R}_{1}+\mathrm{U}^{*}=\alpha \beta^{+\prime} \mathrm{R}_{1}+\mathrm{U}^{*} \tag{3.3.14}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{R}_{0}:=\Delta \mathrm{YD} \text { and } \mathrm{R}_{1}:=\mathrm{Y}_{-1}^{+} \mathrm{D} \tag{3.3.15}
\end{equation*}
$$

Let us denote the first $r$ and $K-r$ rows of $\mathrm{R}_{1}$ by $\mathrm{R}_{1}^{(1)}$ and $\mathrm{R}_{1}^{(2)}$, respectively, then (3.3.14) can be rewritten as

$$
\begin{equation*}
\mathrm{R}_{0}-\alpha R_{1}^{(1)}=\alpha \beta_{(K-r)}^{+\prime} R_{1}^{(2)}+U^{*} \tag{3.3.16}
\end{equation*}
$$

Based on (3.3.16), the GLS estimator of $\beta_{(\mathrm{K}-\mathrm{r})}^{+\prime}$ is

$$
\begin{equation*}
\hat{\beta}_{(\mathrm{K}-\mathrm{r})}^{+\prime}=\left(\alpha^{\prime} \sum_{u}^{-1} \alpha\right)^{-1} \alpha^{\prime} \sum_{u}^{-1}\left(R_{0}-\alpha R_{1}^{(1)}\right) R_{1}^{(2)^{\prime}}\left(R_{1}^{(2)} R_{1}^{(2)^{\prime}}\right)^{-1} \tag{3.3.17}
\end{equation*}
$$

and the EGLS estimator is found as follows (Ahn and Reinsel, 1990 ; Saikkonen, 1992):

$$
\begin{equation*}
\hat{\hat{\beta}}_{(\mathrm{K}-\mathrm{r})}^{+\prime}=\left(\hat{\alpha}^{\prime} \widehat{\sum}_{u}^{-1} \hat{\alpha}\right)^{-1} \hat{\alpha}^{\prime} \widehat{\sum}_{u}^{-1}\left(R_{0}-\hat{\alpha} R_{1}^{(1)}\right) R_{1}^{(2) \prime}\left(R_{1}^{(2)} R_{1}^{(2) \prime}\right)^{-1} \tag{3.3.18}
\end{equation*}
$$

where the first $r$ columns of $\widehat{\Pi}^{+}$is the estimator $\hat{\alpha}$ and

$$
\begin{equation*}
\widehat{\Sigma}_{u}:=(T-K p)^{-1}\left(\Delta Y-\widehat{\Pi}^{+} Y_{-1}^{+}-\hat{\Gamma}^{+} \Delta X^{+}\right)\left(\Delta Y-\widehat{\Pi}^{+} Y_{-1}^{+}-\widehat{\Gamma}^{+} \Delta X^{+}\right)^{\prime} \tag{3.3.19}
\end{equation*}
$$

If $u_{t} \sim N\left(0, \sum_{u}\right)$, the VECM (3.3.1) can be estimated by maximum likelihood (ML) taking the rank restriction for $\Pi^{+}:=\alpha \beta^{+\prime}$ into account (Johansen 1988, 1995). The $\log$-likelihood function for a sample of size $T$ is

$$
\begin{align*}
\ln l & = \\
& -\frac{K T}{2} \ln 2 \pi-\frac{T}{2} \ln \left|\sum_{u}\right|  \tag{3.3.20}\\
& -\frac{1}{2} \operatorname{tr}\left[\left(\Delta Y-\alpha{\beta^{+}}^{\prime} Y_{-1}^{+}-\Gamma^{+} \Delta X^{+}\right)^{\prime} \sum_{u}^{-1}\left(\Delta Y-\alpha \beta^{+^{\prime}} Y_{-1}^{+}-\Gamma^{+} \Delta X^{+}\right)\right]
\end{align*}
$$

Let us define

$$
\begin{equation*}
M:=\mathrm{I}_{T}-\Delta X^{+^{\prime}}\left(\Delta X^{+} \Delta X^{+^{\prime}}\right)^{-1} \Delta X^{+}, R_{0}:=\Delta Y M, R_{1}:=Y_{-1}^{+} M \tag{3.3.21}
\end{equation*}
$$

and

$$
\begin{equation*}
S_{i j}:=\frac{R_{i} R_{j}^{\prime}}{T} \quad i=0,1, \tag{3.3.22}
\end{equation*}
$$

$\lambda_{1} \geq \cdots \geq \lambda_{K}$ are the eigenvalues of $S_{11}^{-1 / 2} S_{10} S_{00}^{-1} S_{01} S_{11}^{-1 / 2}$ and $v_{1}, \ldots, v_{K}$ are corresponding orthonormal eigenvectors. Assuming $\operatorname{rk}\left(\Pi^{+}\right)=r$ which implies that the matrix can be represented as $\Pi^{+}=\alpha \beta^{+\prime}$, where $\alpha$ and $\beta^{+}$are $(K \times r)$ with $\operatorname{rk}(\alpha)=$ $\operatorname{rk}\left(\beta^{+}\right)=r$, the log-likelihood function in (3.3.20) is maximized for

$$
\begin{align*}
& \beta^{+}=\tilde{\beta}^{+}:=\left[\mathrm{V}_{1}, \ldots, \mathrm{~V}_{\mathrm{r}}\right]^{\prime} \mathrm{S}_{11}^{-1 / 2}  \tag{3.3.23}\\
& \alpha=\widetilde{\alpha}:=\Delta \mathrm{YMY}_{-\mathrm{p}}^{+} \tilde{\beta}^{+}\left(\tilde{\beta}^{+^{\prime}} \mathrm{Y}_{-1}^{+} \mathrm{MY}_{-1}^{+} \tilde{\beta}^{+}\right)^{-1}=\mathrm{S}_{01} \tilde{\beta}^{+}\left(\tilde{\beta}^{+^{\prime}} \mathrm{S}_{11} \tilde{\beta}^{+}\right)^{-1} \\
& \Gamma^{+}=\tilde{\Gamma}^{+}:=\left(\Delta Y-\widetilde{\alpha} \tilde{\beta}^{+^{\prime}} \mathrm{Y}_{-1}^{+}\right) \Delta{X^{+}}^{\prime}\left(\Delta X^{+} \Delta{X^{+}}^{\prime}\right)^{-1} \\
& \sum_{u}=\widetilde{\Sigma}_{u}:=\left(\Delta Y-\widetilde{\alpha} \tilde{\beta}^{+^{\prime}} \mathrm{Y}_{-1}^{+}-\tilde{\Gamma}^{+} \Delta X^{+}\right)\left(\Delta Y-\widetilde{\alpha} \tilde{\beta}^{+^{\prime}} \mathrm{Y}_{-1}^{+}-\tilde{\Gamma}^{+} \Delta X^{+}\right)^{\prime} / \mathrm{T}
\end{align*}
$$

### 3.4. Unconditional and Conditional Forecasts

If the endogenous variables are generated by the reduced form (3.1.4) where $u_{t}$ is white noise and independent of the $x_{t}$ process, the optimal $h$-step forecast of $y_{t+h}$ at origin $t$ can be obtained as follows:

$$
\begin{equation*}
y_{t}(h)=A_{1} y_{t}(h-1)+\ldots+A_{p} y_{t}(h-p)+B_{0} x_{t}(h)+\ldots+B_{s} x_{t}(h-s) \tag{3.4.1}
\end{equation*}
$$

where $\mathrm{y}_{\mathrm{t}}(j):=\mathrm{y}_{\mathrm{t}+\mathrm{j}}$ and $\mathrm{x}_{\mathrm{t}}(j):=\mathrm{x}_{\mathrm{t}+\mathrm{j}}$ for $j \leq 0$ and $h=1,2, \ldots$.

If the future values of the unmodelled variables are unknown, forecasts of $y_{t}$ is called as unconditional forecasts. In this type of forecast, future values of unmodelled variables should be estimated and then they should be used in forecast procedure. If the future values of exogenous variables are known, this type of forecast is referred to as conditional forecasts. Let us consider the reduced model (3.1.4) in VARX $(1,0)$ form,

$$
\begin{equation*}
Y_{t}=\mathbf{A} Y_{t-1}+\mathbf{B} x_{t}+U_{t} \tag{3.4.2}
\end{equation*}
$$

where

$$
Y_{t}:=\left[\begin{array}{c}
y_{t}  \tag{3.4.3}\\
\cdot \\
\cdot \\
\cdot \\
y_{t-p+1} \\
x_{t} \\
\cdot \\
\cdot \\
\cdot \\
x_{t-s+1}
\end{array}\right], \quad U_{t}:=\left[\begin{array}{c}
u_{t} \\
0 \\
\cdot \\
\cdot \\
\cdot \\
0
\end{array}\right]((K p+M s) \times 1),
$$

$\mathbf{A}$ is a $((K p+M s) \times(K p+M s))$ matrix such that $\mathbf{A}:=\left[\begin{array}{ll}A_{11} & A_{12} \\ A_{21} & A_{22}\end{array}\right]$,
where

$$
\begin{align*}
& A_{11}:=\left[\begin{array}{cccc}
A_{1} & \ldots & A_{p-1} & A_{p} \\
\mathrm{I}_{K} & & 0 & 0 \\
& & . & . \\
& . & . & . \\
0 & . . & . & . \\
0 & \ldots & 0
\end{array}\right], A_{12}:=\left[\begin{array}{cccc}
B_{1} & \ldots & B_{s-1} & B_{s} \\
0 & \ldots & 0 & 0 \\
. & & . & . \\
. & . & . & . \\
. & . & . & . \\
0 & \ldots & 0 & 0
\end{array}\right], A_{21}: 0,  \tag{3.4.4}\\
& A_{22}:=\left[\begin{array}{cccc}
0 & \ldots & 0 & 0 \\
\mathrm{I}_{M} & & 0 & 0 \\
& & . & . \\
& . & . & . \\
0 & . . . & \mathrm{I}_{M} & .
\end{array}\right]
\end{align*}
$$

and

$$
\mathbf{B}:=\left[\begin{array}{c}
B_{0}  \tag{3.4.5}\\
0 \\
\cdot \\
\cdot \\
\cdot \\
0 \\
I_{M} \\
0 \\
\cdot \\
\cdot \\
\cdot \\
0
\end{array}\right]((K p+M s) \times M) .
$$

Successive substitution for lagged $Y_{t}^{\prime}$ 's gives

$$
\begin{equation*}
Y_{t}=\mathbf{A}^{h} Y_{t-h}+\sum_{i=0}^{h-1} \mathbf{A}^{i} \mathbf{B} x_{t-i}+\sum_{i=0}^{h-1} \mathbf{A}^{i} U_{t-i} . \tag{3.4.6}
\end{equation*}
$$

Premultiplying $Y_{t}$ by the $(K \times(K p+M s))$ matrix $J:=\left[\mathrm{I}_{K}: 0: \ldots: 0\right]$ gives:

$$
\begin{equation*}
y_{t+h}=J \mathbf{A}^{h} Y_{t}+\sum_{i=0}^{h-1} J \mathbf{A}^{i} \mathbf{B} x_{t+h-i}+\sum_{i=0}^{h-1} J \mathbf{A}^{i} J^{\prime} u_{t+h-i} \tag{3.4.7}
\end{equation*}
$$

where $U_{t}=J^{\prime} J U_{t}=J^{\prime} u_{t}$. Given $x_{t+1}, \ldots, x_{t+h}$, the optimal $h$-step forecast of $y_{t}$ at origin $t$ is:

$$
\begin{equation*}
y_{t}(h \mid x):=J \mathbf{A}^{h} Y_{t}+\sum_{i=0}^{h-1} J \mathbf{A}^{i} \mathbf{B} x_{t+h-i} \tag{3.4.8}
\end{equation*}
$$

Then, the corresponding forecast error is

$$
\begin{equation*}
y_{t+h}-y_{t}(h \mid x)=\sum_{i=0}^{h-1} J \mathbf{A}^{i} J^{\prime} u_{t+h-i} . \tag{3.4.9}
\end{equation*}
$$

So, the MSE of the conditional forecast is

$$
\begin{equation*}
\sum_{y}(h \mid x):=\operatorname{MSE}\left[y_{t}(h \mid x)\right]=\sum_{i=0}^{h-1} J \mathbf{A}^{i} J^{\prime} \sum_{u} J\left(\mathbf{A}^{i}\right)^{\prime} J^{\prime} . \tag{3.4.10}
\end{equation*}
$$

(Lütkepohl, 2005). In the data analysis part, the conditional forecast technique will be used.

## CHAPTER 4

## DATA ANALYSIS

Data analysis is done using R software. Firstly, the statistical properties of the variables are examined. Then, Johansen's technique is adopted to test the long run relationship among the variables and to verify the causal relationship between the variables, Granger Causality test is applied. Moreover, structural changes are analyzed before fitting the model.

Pearson's correlation coefficients between endogenous and exogenous variables are shown in Table1.

Table 3. Pearson's Correlation Coefficients between Endogenous and Exogenous Variables

|  | Incentive <br> Certificates | Fixed <br> Investment <br> Amount | Total <br> Industrial <br> Turnover <br> Index | Industrial <br> Production <br> Index | Manufa. <br> PMI of <br> China |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Chrome | 0.338 | 0.068 | 0.197 | 0.310 | 0.172 |
| Feldspar | 0.049 | 0.108 | 0.116 | 0.134 | 0.191 |
| Copper | 0.287 | 0.303 | 0.149 | 0.154 | 0.029 |
| Zinc | 0.507 | 0.269 | 0.562 | 0.544 | -0.062 |
| Boron | 0.206 | 0.137 | 0.602 | 0.452 | -0.353 |

From Table 3, it is seen that chrome is mostly correlated with incentive certificates, feldspar is mostly correlated with manufacturing PMI of China, copper is mostly correlated with fixed investment amount, zinc and boron are mostly correlated with total industrial turnover index.

It is investigated whether there is need for power transform. Using results of BoxCox transformation and looking at the time series plots of the endogenous variables it is decided that there is no need for power transform.

### 4.1. Granger Causality Test

To investigate the causal relationship between the variables, bivariate Granger Causality test is used. Bivariate Granger causality test for $X$ and $Y$ evaluates whether the past values of $X$ are useful for predicting $Y$ once $Y$ 's history has been modeled (Hamilton, 1994). Test results are given in Table 4.

Table 4. Significant Results of Granger Causality Test

|  | F-statistic | p-value |
| :---: | :---: | :---: |
| Feldspar -> Zinc | 5.818 | 0.018 |
| Feldspar -> Boron | 1.644 | 0.0001 |
| Zinc -> Boron | 4.651 | 0.034 |

From Table 4, it is seen that past values of feldspar are useful for predicting zinc, boron and past values of zinc are useful for predicting boron. The other variables do not have significat short-term relationships at 5\% significance level.

### 4.2. Structural Change Detection

To investigate structural changes in the series, Fstats and breakpoints functions in "strucchange" package are used. Test results are given in Table 5.

Table 5. Result of Structural Change Test

|  | Breakpoints <br> Year (Month) |
| :---: | :---: |
| Chrome | $2009(5)$ |
| Feldspar | $2008(7)$ |
| Copper | $2009(8)$ |
| Zinc | $2010(5)$ |
| Boron | $2011(6)$ |

From Table 5, it is seen that there is no common breakpoint. So, we decided not to put this information into the model.

### 4.3. Integration: Unit Root and Stationarity Tests Results

To be able to model a time series, it should be stationary. Hence, before starting the analysis, we have to test whether the series is stationary. There are two types of tests: Stationarity tests and unit root tests. One of the well known stationarity test is Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test where in the null hypothesis it tests whether the series is stationary. Popular unit root tests seen in the literature are Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP) test (Hamilton, 1994). KPSS, ADF and PP test results are given in Table 6.

Table 6. Unit Root and Stationarity Test Results at 5\% Significance Level

|  | KPSS <br> Trend Stationary | KPSS <br> Level Stationary | ADF <br> Test <br> None | ADF <br> Test <br> Drift | $\begin{gathered} \text { ADF } \\ \text { Test } \\ \text { Trend } \end{gathered}$ | PP Test <br> Constant |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Chrome | Reject trend stationarity | Fail to reject level stationarity | Fail to reject Ho:I(1) | Reject <br> $\mathrm{Ho}: \mathrm{I}(1)$ | Reject <br> Ho:I(1) | Reject <br> Ho:I(1) | Reject <br> Ho:I(1) |
| Feldspar | Reject trend stationarity | Fail to reject level stationarity | Fail to reject Ho:I(1) | Reject <br> Ho:I(1) | Reject <br> $\mathrm{Ho}: \mathrm{I}(1)$ | Reject <br> Ho:I(1) | Reject <br> Ho:I(1) |
| Copper | Fail to reject trend stationarity | Fail to reject level stationarity | Fail to reject Ho:I(1) | Fail to reject <br> Ho:I(1) | Reject <br> Ho:I(1) | Reject <br> Ho:I(1) | Reject <br> Ho:I(1) |
| Zinc | Fail to reject trend stationarity | $\begin{array}{\|c\|} \hline \text { Reject } \\ \text { level } \\ \text { stationarity } \\ \hline \end{array}$ | Fail to reject <br> Ho:I(1) | Fail to reject <br> Ho:I(1) | Reject <br> Ho:I(1) | Reject <br> Ho:I(1) | Reject <br> Ho:I(1) |
| Boron | Fail to reject trend stationarity | Reject level stationarity | Fail to reject Ho:I(1) | Reject <br> Ho:I(1) | Reject <br> Ho:I(1) | Reject <br> Ho:I(1) | Reject <br> Ho:I(1) |

Table 7 gives the results of ndiffs function in "forecast" package. This function estimates the number of nonseasonal differences required to make the given series stationary. It is decided that all the series have unit root and to make them stationary differencing should be applied.

Table 7. Number of First Differences Required for the Series

|  | ndiffs <br> "KPSS" | ndiffs <br> "ADF" | ndiffs <br> "PP" |
| :---: | :---: | :---: | :---: |
| Chrome | 0 | 1 | 0 |
| Feldspar | 0 | 1 | 0 |
| Copper | 0 | 1 | 0 |
| Zinc | 1 | 0 | 0 |
| Boron | 1 | 1 | 0 |

Osborn-Chui-Smith-Birchenhall (OCSB) and Canova-Hansen (CH) tests are used to detect seasonal unit roots (Hamilton, 1994). Table 8 demonstrates the results of nsdiffs function in "forecast" package. This function estimates the number of seasonal differences required to make the given series stationary. The results suggest that there is no seasonal unit root.

Table 8. Number of Seasonal Differences Required for the Series

|  | nsdiffs <br> "OCSB" | nsdiffs <br> "CH" |
| :---: | :---: | :---: |
| Chrome | 3 | 0 |
| Feldspar | 0 | 0 |
| Copper | 0 | 0 |
| Zinc | 0 | 0 |
| Boron | 0 | 0 |

Table 6, Table 7 and Table 8 indicate that the series are not stationary and one nonseasonal difference is required to have stationary series. Thus, all the variables are integrated of order one.

### 4.4. Lag Order of the Series in VAR

In multivariate time series modeling, we need to define the number of past lag values of the series that should be in the model. Here, the aim is to use the significant past observations to describe the current values of the series. To find the number of lags VARselect function in "vars" package is used.

Table 9. Optimal Lag Number According to Each Criterium

|  | AIC | HQ | SC | FPE |
| :---: | :---: | :---: | :---: | :---: |
| Chrome | 1 | 1 | 1 | 1 |
| Feldspar | 2 | 2 | 1 | 2 |
| Copper | 3 | 3 | 1 | 3 |
| Zinc | 1 | 1 | 1 | 1 |
| Boron | 2 | 2 | 1 | 2 |

From Table 9, it is seen that maximum lag number is 3 which corresponds to the lags in the VAR representation. So, in Johansen's test the lag order of the series is taken as 3 .

When the integrated multivariate time series is under consideration, we have to control whether there exists cointegration relation between them, that is, all or some of the series are moving together or not. If so, there is a long-run relationship between them and efficient way to model such series is the VECM. One way to test the existence of cointegration is the Johansen's trace test which is a likelihood ratio test and sequentially we are trying to find the number of linearly independent equations (Hamilton, 1994). For the Johansen test, we have to decide the VAR order of the model. Based on the results given in Table 9, the lag order of the series is taken as 3 in Johansen's test.

### 4.5. Cointegration: Johansen's Test Results

To test whether the series are cointegrated, Johansen Procedure is performed and the results are given in Table 10.

Table 10. Johansen's Cointegration Test

```
######################
# Johansen-Procedure #
######################
Values of test statistic and critical values of test:
    test 10pct 5pct 1pct
r<= 4 | 9.17 7.52 9.24 12.97
r<= | 22.18 17.85 19.96 24.60
```

Table 10 (continued)

| $r<=2$ | 42.35 | 32.00 | 34.91 | 41.07 |
| :--- | ---: | ---: | ---: | ---: |
| $r<=1$ | 70.42 | 49.65 | 53.12 | 60.16 |
| $r=0$ | 105.84 | 71.86 | 76.07 | 84.40 |

From Table 10, it is seen that there are four cointegrating relations (the cointegrating rank) to explain the long run relation between the endogenous variables at $5 \%$ significance level.

### 4.6. Vector Error Correction Model

As there are cointegrating relations between the variables, VECM with exogenous variables is fitted using "tsDyn" package. VAR with 3 lags corresponds to VECM with 2 lags. Hence, number of lags in VECM should be taken as 2 . VECM with 2 lags is fitted but in this model it is seen that only one variable lagged 2 periods is significant in just one equation. For this reason, VECM with 1 lag is fitted.

VECM which is obtained using ML estimation can be written as follows:

```
Chrome}\mp@subsup{}{\textrm{t}}{}=-0.713\mathrm{ ECT1 - 0.015ECT2 + 0.025ECT3 + 1.301ECT4 +
0.136\Delta\mp@subsup{\mathrm{ Chrome }}{\textrm{t}-2}{}-0.026\Delta\mp@subsup{\mathrm{ Feldspar }}{\textrm{t}-2}{}-0.164\Delta\mp@subsup{\mathrm{ Copper }}{\textrm{t}-2}{}-0.661\Delta\mp@subsup{\mathrm{ Zinc}}{\textrm{t}-2}{}+
0.896\Delta\mp@subsup{Boron}{t-2}{2}+1142.584IC - 10.405FIA+85.311TITI+700.868IPI+
1907.395MPMIC

Feldspar \(_{\mathrm{t}}=-0.159\) ECT1 \(-0.187 \mathrm{ECT} 2+0.799\) ECT3 \(-3.308 \mathrm{ECT} 4-\)
\(0.005 \Delta\) Chrome \(_{\mathrm{t}-2}-0.361 \Delta\) Feldspar \(_{\mathrm{t}-2}-0.335 \Delta\) Copper \(_{\mathrm{t}-2}-0.160 \Delta\) Zinc \(_{\mathrm{t}-2}-\)
\(1.471 \Delta\) Boron \(_{\mathrm{t}-2}+814.658 \mathrm{IC}+35.602\) FIA \(-61.862 \mathrm{TITI}+449.751 \mathrm{IPI}+\) 5095.298MPMIC
\(\widehat{\text { Copper }}_{\mathrm{t}}=0.027 \mathrm{ECT} 1-0.047 \mathrm{ECT} 2-1.412 \mathrm{ECT} 3+0.510 \mathrm{ECT} 4-\)
\(0.046 \Delta\) Chrome \(_{\mathrm{t}-2}+0.046 \Delta\) Feldspar \(_{\mathrm{t}-2}+0.071 \Delta\) Copper \(_{\mathrm{t}-2}-0.239 \Delta\) Zinc \(_{\mathrm{t}-2}-\) \(0.031 \Delta\) Boron \(_{\mathrm{t}-2}+317.451 \mathrm{IC}+15.464\) FIA -121.038TITI \(+226.031 \mathrm{IPI}+\) 400.098MPMIC
```

\mp@subsup{\textrm{Znnc}}{\textrm{t}}{}=-0.024\textrm{ECT}1+0.011ECT2 + 0.209ECT3 - 0.991ECT4 -
0.002\Delta\mp@subsup{\mathrm{ Chrome }}{\textrm{t}-2}{}+0.0002\Delta\mp@subsup{\mathrm{ Feldspar }}{\textrm{t}-2}{}-0.154\Delta\mp@subsup{\mathrm{ Copper }}{\textrm{t}-2}{}-0.088\Delta\mp@subsup{\mathrm{ Zinc}}{\textrm{t}-2}{}
+0.031\Delta\mp@subsup{Boron}{t-2}{}+310.311IC + 0.084FIA + 60.318TITI+282.796IPI+
371.692MPMIC
$\widehat{\text { Boron }}_{\mathrm{t}}=-0.036$ ECT1 +0.119 ECT2 +0.067 ECT3 -0.050 ECT4 + $0.031 \Delta$ Chrome $_{\mathrm{t}-2}-0.041 \Delta$ Feldspar $_{\mathrm{t}-2}+0.001 \Delta$ Copper $_{\mathrm{t}-2}-0.159 \Delta$ Zinc $_{\mathrm{t}-2}-$ $0.143 \Delta$ Boron $_{\mathrm{t}-2}+118.132$ IC +0.150 FIA +155.167 TITI +142.375 IPI1364.240MPMIC
where $\Delta$ is the first difference operator, IC is incentive certificate, FIA is fixed investment amount, TITI is total industrial turnover index, IPI is industrial production index, MPMIC is manufacturing PMI of China, ECT is the error correction term coming from the long-run cointegrating relationship.

To find a better model among several trials, their information criteria values are compared. The smaller value means the better estimation. The well-known information criteria are Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The AIC and BIC of the model are 9617.104 and 9806.866, respectively.

Table 11. Cointegrating Matrix $\left(\beta^{+}\right)$of the Model

|  | r1 | r2 | r3 | r4 |
| :---: | :---: | :---: | :---: | :---: |
| Chrome | 1.000 | $-7.725 \mathrm{e}-17$ | $1.335 \mathrm{e}-18$ | $-2.067 \mathrm{e}-19$ |
| Feldspar | $-1.884 \mathrm{e}-17$ | 1.000 | $-5.550 \mathrm{e}-18$ | $-1.658 \mathrm{e}-17$ |
| Copper | $1.443 \mathrm{e}-16$ | $-2.060 \mathrm{e}-18$ | 1.000 | $6.923 \mathrm{e}-17$ |
| Zinc | 0 | 0 | 0 | 1.000 |
| Boron | 1.350 | -7.183 | 0.230 | -0.042 |
| Const | 162242.768 | 451442.135 | 3434.806 | 38781.833 |

After estimation of the model parameters, we need to check the validity of the assumptions of the model which are normality of the errors, having constant error variance and independence of the errors.

To check the normality assumptions for the residuals mvShapiro.Test function in "mvShapiroTest" package is used. This function is designed for checking multivariate normality.

Table 12. Generalized Shapiro-Wilk Test for Multivariate Normality

```
data: vecm$residuals
MVW = 0.983, p-value = 0.126
```

From Table 12, it is seen that p -value is greater than 0.05 . So, the residuals are normally distributed at $5 \%$ significance level.

To check the constant variance assumptions archTest function in "MTS" package is used. This function is designed for checking multivariate homoscedasticity.

Table 13. Test for Multivariate Homoscedasticity

```
Q(m) of squared series(LM test):
Test statistic: 15.696 p-value: 0.109
Rank-based Test:
Test statistic: 11.538 p-value: 0.317
```

From Table 13, it is seen that p-values are greater than 0.05 . So, the residuals have constant variance at $5 \%$ significance level.

To check the independence of the series BoxPierce function in "portes" package is used. This function is designed for checking multivariate independence.

Table 14. Test for Multivariate Autocorrelation

| Lags | Statistic | df | pvalue |
| :---: | :---: | ---: | :--- |
| 1 | 23.562 | 25 | 0.545 |
| 2 | 60.881 | 50 | 0.139 |
| 3 | 84.539 | 75 | 0.211 |
| 4 | 100.145 | 100 | 0.477 |
| 5 | 120.056 | 125 | 0.608 |
| 6 | 136.328 | 150 | 0.781 |
| 7 | 167.014 | 175 | 0.655 |
| 8 | 199.102 | 200 | 0.505 |
| 9 | 219.580 | 225 | 0.589 |
| 10 | 245.101 | 250 | 0.576 |
| 11 | 261.982 | 275 | 0.704 |
| 12 | 288.853 | 300 | 0.668 |

From Table 14, it is seen that all p-values are greater than 0.05 . So, the residuals are independent and there is no serial correlation problem at 5\% significance level.

From the results of the analysis, it is seen that incentive certificates and industrial production index are positively related to all the endogenous variables which are chromium ores and concentrates, feldspar, copper ores and concentrates, zinc ores and concentrates and natural borates and their concentrates. Then, manufacturing purchasing managers' index of China is positively related to all the endogenous variables except from export amount of natural borates and their concentrates and fixed investment amount is positively related to all the endogenous variables except from export amount of chromium ores and concentrates. Also, total industrial turnover index has positive relationship with chromium ores and concentrates, zinc ores and concentrates and natural borates and their concentrates.

The error correction terms have negative coefficients, as expected, which indicate that the system is stable and converges to the equilibrium track after some disturbance in the system. Large absolute values of the coefficient on the ECT show that equilibrium agents remove a large percentage of disequilibrium in each month, i.e., the speed of adjustment is very rapid. For chromium ores and concentrates, the value of ECT1 is -0.713 which shows that almost $71 \%$ of the disequilibrium is corrected within one month.

The model is fitted using the series which are from January 2007 to February 2015. Plots of the series with recently announced 6 observations and corresponding forecast values are shown below.


Figure 2. Observed and Forecasted Export Amounts of Chromium Ores and Concentrates in Tonne


Figure 3. Observed and Forecasted Export Amounts of Feldspar in Tonne

From Figure 3, it can be said that there is an unexpected increase in export amounts of feldspar after February 2015. The reason to this situation can be examined further in future studies by analysing this variable alone.


Figure 4. Observed and Forecasted Export Amounts of Copper Ores and Concentrates in Tonne


Figure 5. Observed and Forecasted Export Amounts of Zinc Ores and Concentrates in Tonne


Figure 6. Observed and Forecasted Export Amounts of Natural Borates and Their Concentrates in Tonne

As it is seen from the plots of observed and forecasted export amounts of the ores and concentrates, the model has a good fit for some variables. Detailed information about forecasts of the ores and concentrates is shown in the following tables.

Table 15. Observed and Forecasted Export Amounts of Chromium Ores and Concentrates in Tonne with Forecast Accuracy Measures

| EXPORT AMOUNTS OF CHROMIUM ORES AND CONCENTRATES IN |  |  |  |
| :---: | :---: | :---: | :---: |
| TONNE |  |  |  |
| DATE | OBSERVED | FORECASTED | RMSD |
| March 2015 | 77468.689 | 166093.900 | 50383.021 |
| April 2015 | 114504.385 | 160290.300 | MPE $(\%)$ |
| May 2015 | 146291.151 | 134818.000 | -31.347 |
| June 2015 | 196343.824 | 152571.300 | MAE |
| July 2015 | 101660.261 | 156875.600 | 43069.815 |
| August 2015 | 142341.053 | 155887.800 |  |

Table 16. Observed and Forecasted Export Amounts of Feldspar in Tonne with Forecast Accuracy Measures

| EXPORT AMOUNTS OF FELDSPAR IN TONNE |  |  |  |
| :---: | :---: | :---: | :---: |
| DATE | OBSERVED | FORECASTED | RMSD |
| March 2015 | 402116.260 | 313395.400 | 140509.755 |
| April 2015 | 532361.830 | 344490.200 | MPE (\%) |
| May 2015 | 493033.740 | 345144.600 | 28.101 |
| June 2015 | 535150.653 | 353866.200 | MAE |
| July 2015 | 428830.610 | 321111.900 | 134683.509 |
| August 2015 | 424961.060 | 330344.800 |  |

Table 17. Observed and Forecasted Export Amounts of Copper Ores and Concentrates in Tonne with Forecast Accuracy Measures

| EXPORT AMOUNTS OF COPPER ORES AND CONCENTRATES IN TONNE |  |  |  |
| :---: | :---: | :---: | :---: |
| DATE | OBSERVED | FORECASTED | RMSD |
| March 2015 | 33408.338 | 26444.440 | 10145.474 |
| April 2015 | 26305.978 | 40342.090 | MPE (\%) |
| May 2015 | 37379.815 | 21789.510 | -7.730 |
| June 2015 | 18074.791 | 25512.020 | MAE |
| July 2015 | 28972.943 | 23564.000 | 9350.582 |
| August 2015 | 20144.236 | 26811.240 |  |

Table 18. Observed and Forecasted Export Amounts of Zinc Ores and Concentrates in Tonne with Forecast Accuracy Measures

| EXPORT AMOUNTS OF ZINC ORES AND CONCENTRATES IN TONNE |  |  |  |
| :---: | :---: | :---: | :---: |
| DATE | OBSERVED | FORECASTED | RMSD |
| March 2015 | 27035.030 | 30739.760 | 6331.641 |
| April 2015 | 30202.760 | 27815.970 | MPE $(\%)$ |
| May 2015 | 29808.030 | 28092.770 | 10.160 |
| June 2015 | 40439.494 | 35120.180 | MAE |
| July 2015 | 41225.566 | 30573.210 | 5419.920 |
| August 2015 | 39710.662 | 30969.590 |  |

Table 19. Observed and Forecasted Export Amounts of Natural Borates and Their Concentrates in Tonne with Forecast Accuracy Measures

| EXPORT AMOUNTS OF NATURAL BORATES AND THEIR CONCENTRATES |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| IN TONNE |  |  |  |  |
| DATE | OBSERVED | FORECASTED | RMSD |  |
| March 2015 | 64700.720 | 61035.750 | 7781.795 |  |
| April 2015 | 68526.800 | 58097.830 | MPE $(\%)$ |  |
| May 2015 | 71516.430 | 59693.690 | 8.471 |  |
| June 2015 | 58639.150 | 61887.970 | MAE |  |
| July 2015 | 70920.700 | 62802.800 | 7045.737 |  |
| August 2015 | 66482.450 | 61491.430 |  |  |

## CHAPTER 5

## CONCLUSION

Mining sector has great importance for Turkey's economy. National industry and agriculture sector, which are developing, depend on the mining sector for main inputs. Although it is known that there are quite various and rich mine reserves in Turkey, most of these resources couldn't be commissioned yet. Share of mining sector in Turkey's domestic income is in a very low level such as $1.2 \%$. Although Turkey has the potential to become an important mine exporter, Turkey's mine export is quite limited and some mine products are imported. Briefly, the mining sector cannot make the great and important contribution it could have to the development of the country.

This study aims at making a contribution to the development of the mining sector in Turkey modeling the export amounts of chromium ores and concentrates, feldspar, copper ores and concentrates, zinc ores and concentrates, natural borates and their concentrates from Turkey based on exogenous variables, incentive certificates, fixed investment amount, total industrial turnover index, industrial production index and manufacturing purchasing managers' index of China.

It is seen that chrome is mostly correlated with incentive certificates, feldspar is mostly correlated with manufacturing PMI of China, copper is mostly correlated with fixed investment amount, zinc and boron are mostly correlated with total industrial turnover index. According to the results, past values of feldspar are useful for predicting zinc, boron and past values of zinc are useful for predicting boron.

The cointegration analysis demonstrates that there exists long run relationship among the aforesaid ores and concentrates. As there are cointegrating relations between the variables, VECM with exogenous variables is fitted.

From the achieved models, it is seen that incentive certificates and industrial production index are positively related to all the endogenous variables and the error correction terms have negative coefficients which indicate that the system is stable and converges to the equilibrium track after some disturbance in the system. One can use the suggested model to decide on the amount of export on the future months Hence, using this information the companies decide on the production level of these ores and concentrates.

Future studies on statistical analysis of the mines, which are important for Turkey and for the world, will contribute to the development of the mining sector in Turkey. As a future study, the ores and concentrates can be studied one by one.

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## APPENDIX A

## R CODE

```
Chrome=ts(read.table("clipboard",header=F),start=2007,frequency=12)
Copper=ts(read.table("clipboard",header=F),start=2007,frequency=12)
Zinc=ts(read.table("clipboard",header=F),start=2007,frequency=12)
Boron=ts(read.table("clipboard",header=F), start=2007,frequency=12)
Feldspar=ts(read.table("clipboard",header=F),start=2007,frequency=12)
install.packages("TSA")
library(TSA)
BoxCox.ar(Chrome)
BoxCox.ar(Chrome, method = c("ols"))
BoxCox.ar(Chrome, order, lambda = seq(-2, 2, 0.01), plotit = TRUE,
method = c("yule-walker"))
BoxCox.ar(Feldspar)
BoxCox.ar(Feldspar, method = c("ols"))
BoxCox.ar(Feldspar, order, lambda = seq(-2, 2, 0.01), plotit = TRUE,
method = c("yule-walker"))
BoxCox.ar(Copper)
BoxCox.ar(Copper, method = c("ols"))
BoxCox.ar(Copper, order, lambda = seq(-2, 2, 0.01), plotit = TRUE,
method = c("yule-walker"))
BoxCox.ar(Zinc)
BoxCox.ar(Zinc, method = c("ols"))
BoxCox.ar(Zinc, order, lambda = seq(-2, 2, 0.01), plotit = TRUE,
method = c("yule-walker"))
BoxCox.ar(Boron)
BoxCox.ar(Boron, method = c("ols"))
BoxCox.ar(Boron, order, lambda = seq(-2, 2, 0.01), plotit = TRUE,
method = c("yule-walker"))
install.packages("tseries")
library("tseries")
plot(Chrome, ylab='Chrome', xlab='Year')
par(mfrow=c(1,2))
acf(Chrome,lag.max=36)
pacf(Chrome,lag.max=36)
kpss.test(Chrome, null = c("Trend"))
kpss.test(Chrome,null=c("Level"))
install.packages("urca")
```

```
library(urca)
adf.chrome=ur.df(Chrome ,type=c("none"),lags=10,
selectlags=c("AIC"))
summary(adf.chrome)
adf.chromedrift=ur.df(Chrome ,type=c("drift"),lags=10,
selectlags=c("AIC"))
summary(adf.chromedrift)
adf.chrometrend=ur.df(Chrome ,type=c("trend"),lags=10,
selectlags=c("AIC")) summary(adf.chrometrend)
pp.chrome= ur.pp(Chrome,type=c("Z-tau"), model=c("constant"),
lags=c("short"))
summary(pp.chrome)
pp.chrometrend= ur.pp(Chrome,type=c("Z-tau"), model=c("trend"),
lags=c("short"))
summary(pp.chrometrend)
plot(Feldspar, ylab='Feldspar', xlab='Year')
par(mfrow=c(1,2))
acf(Feldspar,lag.max=36)
pacf(Feldspar,lag.max=36)
kpss.test(Feldspar, null = c("Trend"))
kpss.test(Feldspar,null=c("Level"))
adf.Feldspar=ur.df(Feldspar ,type=c("none"),lags=10,
selectlags=c("AIC"))
summary(adf.Feldspar)
adf.Feldspardrift=ur.df(Feldspar ,type=c("drift"),lags=10,
selectlags=c("AIC"))
summary(adf.Feldspardrift)
adf.Feldspartrend=ur.df(Feldspar ,type=c("trend"),lags=10,
selectlags=c("AIC"))
summary(adf.Feldspartrend)
pp.Feldspar=ur.pp(Feldspar,type=c("Z-tau"), model=c("constant"),
lags=c("short"))
summary(pp.Feldspar)
pp.Feldspartrend=ur.pp(Feldspar,type=c("Z-tau"),
model=c("trend"),lags=c("short"))
summary(pp.Feldspartrend)
plot(Copper,ylab='Copper',xlab='Year',type='l')
par(mfrow=c(1,2))
acf(Copper,lag.max=36)
pacf(Copper,lag.max=36)
kpss.test(Copper, null = c("Trend"))
kpss.test(Copper,null=c("Level"))
adf.Copper=ur.df(Copper ,type=c("none"),lags=10,
selectlags=c("AIC"))
summary(adf.Copper)
adf.Copperdrift=ur.df(Copper ,type=c("drift"),lags=10,
selectlags=c("AIC"))
summary(adf.Copperdrift)
adf.Coppertrend=ur.df(Copper ,type=c ("trend"),lags=10,
selectlags=c("AIC")) summary(adf.Coppertrend)
```

```
pp.Copper= ur.pp(Copper,type=c("Z-tau"), model=c("constant"),
lags=c("short")) summary(pp.Copper)
pp.Coppertrend= ur.pp(Copper,type=c("Z-tau"), model=c("trend"),
lags=c("short"))
summary(pp.Coppertrend)
plot(Zinc,ylab='Zinc',xlab='Year',type='l')
par(mfrow=c(1,2))
acf(Zinc,lag.max=36)
pacf(Zinc,lag.max=36)
kpss.test(Zinc, null = c("Trend"))
kpss.test(Zinc,null=c("Level"))
adf.Zinc=ur.df(Zinc ,type=c("none"),lags=10, selectlags=c("AIC"))
summary(adf.Zinc)
adf.Zincdrift=ur.df(Zinc ,type=c("drift"),lags=10,
selectlags=c("AIC"))
summary(adf.Zincdrift)
adf.Zinctrend=ur.df(Zinc ,type=c("trend"),lags=10,
selectlags=c("AIC"))
summary(adf.zinctrend)
pp.Zinc= ur.pp(Zinc,type=c("Z-tau"), model=c("constant"),
lags=c("short"))
summary(pp.Zinc)
pp.Zinctrend= ur.pp(Zinc,type=c("Z-tau"), model=c("trend"),
lags=c("short"))
summary(pp.Zinctrend)
plot(Boron,ylab='Boron',xlab='Year',type='l')
par(mfrow=c(1,2))
acf(Boron,lag.max=36)
pacf(Boron,lag.max=36)
kpss.test(Boron, null = c("Trend"))
kpss.test(Boron,null=c("Level"))
adf.Boron=ur.df(Boron ,type=c("none"),lags=10, selectlags=c("AIC"))
summary(adf.Boron)
adf.Borondrift=ur.df(Boron ,type=c("drift"),lags=10,
selectlags=c("AIC"))
summary(adf.Borondrift)
adf.Borontrend=ur.df(Boron ,type=c("trend"),lags=10,
selectlags=c("AIC"))
summary(adf.Borontrend)
pp.Boron= ur.pp(Boron,type=c("Z-tau"), model=c("constant"),
lags=c("shortsummary(pp.Boron)
pp.Borontrend= ur.pp(Boron,type=c("Z-tau"), model=c("trend"),
lags=c("short")) summary(pp.Borontrend)
library(forecast)
ndiffs(Chrome,test=c("kpss"), max.d=4)
ndiffs(Chrome,test=c("adf"), max.d=4)
ndiffs(Chrome,test=c("pp"), max.d=4)
ndiffs(Feldspar,test=c("kpss"), max.d=4)
```

```
ndiffs(Feldspar,test=c("adf"), max.d=4)
ndiffs(Feldspar,test=c("pp"), max.d=4)
ndiffs(Copper,test=c("kpss"), max.d=4)
ndiffs(Copper,test=c("adf"), max.d=4)
ndiffs(Copper,test=c("pp"), max.d=4)
ndiffs(Zinc,test=c("kpss"), max.d=4)
ndiffs(Zinc,test=c("adf"), max.d=4)
ndiffs(Zinc,test=c("pp"), max.d=4)
ndiffs(Boron,test=c("kpss"), max.d=4)
ndiffs(Boron,test=c("adf"), max.d=4)
ndiffs(Boron,test=c("pp"), max.d=4)
nsdiffs(Chrome, m=12, test=c("ocsb"), max.D=3)
nsdiffs(Chrome, m=12, test=c("ch"), max.D=3)
nsdiffs(Chrome, m=12, test=c("ocsb"), max.D=4)
nsdiffs(Chrome, m=12, test=c("ch"), max.D=4)
nsdiffs(Chrome, m=12, test=c("ocsb"), max.D=5)
nsdiffs(Chrome, m=12, test=c("ch"), max.D=5)
nsdiffs(Chrome, m=12, test=c("ocsb"), max.D=20)
nsdiffs(Chrome, m=12, test=c("ch"), max.D=20)
sdChrome=diff(Chrome,12)
plot(sdChrome,ylab='sdChrome (kg)',xlab='Year')
par(mfrow=c(1,2))
acf(sdChrome,48)
pacf(sdChrome,48)
nsdiffs(sdChrome, m=12, test=c("ocsb"), max.D=3)
nsdiffs(sdChrome, m=12, test=c("ch"), max.D=3)
nsdiffs(sdChrome, m=12, test=c("ocsb"), max.D=4)
nsdiffs(sdChrome, m=12, test=c("ch"), max.D=4)
nsdiffs(sdChrome, m=12, test=c("ocsb"), max.D=5)
nsdiffs(sdChrome, m=12, test=c("ch"), max.D=5)
nsdiffs(sdChrome, m=12, test=c("ocsb"), max.D=20)
nsdiffs(sdChrome, m=12, test=c("ch"), max.D=20)
ndiffs(sdChrome,test=c("kpss"), max.d=4)
ndiffs(sdChrome,test=c("adf"), max.d=4)
ndiffs(sdChrome,test=c("pp"), max.d=4)
kpss.test(sdChrome, null=c("Level"))
kpss.test(sdChrome, null = c("Trend"))
sddChrome=diff(sdChrome,12)
nsdiffs(sddChrome, m=12, test=c("ocsb"), max.D=3)
nsdiffs(sddChrome, m=12, test=c("ch"), max.D=3)
nsdiffs(sddChrome, m=12, test=c("ocsb"), max.D=4)
nsdiffs(sddChrome, m=12, test=c("ch"), max.D=4)
nsdiffs(sddChrome, m=12, test=c("ocsb"), max.D=5)
nsdiffs(sddChrome, m=12, test=c("ch"), max.D=5)
nsdiffs(sddChrome, m=12, test=c("ocsb"), max.D=20)
nsdiffs(sddChrome, m=12, test=c("ch"), max.D=20)
ndiffs(sddChrome,test=c("kpss"), max.d=4)
ndiffs(sddChrome,test=c("adf"), max.d=4)
ndiffs(sddChrome,test=c("pp"), max.d=4)
```

```
kpss.test(sddChrome, null=c("Level"))
kpss.test(sddChrome, null = c("Trend"))
sdddChrome=diff(sddChrome,12)
nsdiffs(sdddChrome, m=12, test=c("ocsb"), max.D=3)
nsdiffs(sdddChrome, m=12, test=c("ch"), max.D=3)
nsdiffs(sdddChrome, m=12, test=c("ocsb"), max.D=4)
nsdiffs(sdddChrome, m=12, test=c("ch"), max.D=4)
nsdiffs(sdddChrome, m=12, test=c("ocsb"), max.D=5)
nsdiffs(sdddChrome, m=12, test=c("ch"), max.D=5)
nsdiffs(sdddChrome, m=12, test=c("ocsb"), max.D=20)
nsdiffs(sdddChrome, m=12, test=c("ch"), max.D=20)
ndiffs(sdddChrome,test=c("kpss"), max.d=4)
ndiffs(sdddChrome,test=c("adf"), max.d=4)
ndiffs(sdddChrome,test=c("pp"), max.d=4)
kpss.test(sdddChrome, null=c("Level"))
kpss.test(sdddChrome, null = c("Trend"))
sddddChrome=diff(sdddChrome,12)
nsdiffs(sddddChrome, m=12, test=c("ocsb"), max.D=3)
nsdiffs(sddddChrome, m=12, test=c("ch"), max.D=3)
nsdiffs(sddddChrome, m=12, test=c("ocsb"), max.D=4)
nsdiffs(sddddChrome, m=12, test=c("ch"), max.D=4)
nsdiffs(sddddChrome, m=12, test=c("ocsb"), max.D=5)
nsdiffs(sddddChrome, m=12, test=c("ch"), max.D=5)
nsdiffs(sddddChrome, m=12, test=c("ocsb"), max.D=20)
nsdiffs(sddddChrome, m=12, test=c("ch"), max.D=20)
ndiffs(sddddChrome,test=c("kpss"), max.d=4)
ndiffs(sddddChrome,test=c("adf"), max.d=4)
ndiffs(sddddChrome,test=c("pp"), max.d=4)
kpss.test(sddddChrome, null=c("Level"))
kpss.test(sddddChrome, null = c("Trend"))
sdddddChrome=diff(sddddChrome,12)
nsdiffs(sdddddChrome, m=12, test=c("ocsb"), max.D=3)
nsdiffs(sdddddChrome, m=12, test=c("ch"), max.D=3)
nsdiffs(sdddddChrome, m=12, test=c("ocsb"), max.D=4)
nsdiffs(sdddddChrome, m=12, test=c("ch"), max.D=4)
nsdiffs(sdddddChrome, m=12, test=c("ocsb"), max.D=5)
nsdiffs(sdddddChrome, m=12, test=c("ch"), max.D=5)
nsdiffs(sdddddChrome, m=12, test=c("ocsb"), max.D=20)
nsdiffs(sdddddChrome, m=12, test=c("ch"), max.D=20)
ndiffs(sdddddChrome,test=c("kpss"), max.d=4)
ndiffs(sdddddChrome,test=c("adf"), max.d=4)
ndiffs(sdddddChrome,test=c("pp"), max.d=4)
kpss.test(sdddddChrome, null=c("Level"))
kpss.test(sdddddChrome, null = c("Trend"))
sddddddChrome=diff(sdddddChrome,12)
nsdiffs(sddddddChrome, m=12, test=c("ocsb"), max.D=3)
nsdiffs(sddddddChrome, m=12, test=c("ch"), max.D=3)
nsdiffs(sddddddChrome, m=12, test=c("ocsb"), max.D=4)
```

```
nsdiffs(sddddddChrome, m=12, test=c("ch"), max.D=4)
nsdiffs(sddddddChrome, m=12, test=c("ocsb"), max.D=5)
nsdiffs(sddddddChrome, m=12, test=c("ch"), max.D=5)
nsdiffs(sddddddChrome, m=12, test=c("ocsb"), max.D=20)
nsdiffs(sddddddChrome, m=12, test=c("ch"), max.D=20)
ndiffs(sddddddChrome,test=c("kpss"), max.d=4)
ndiffs(sddddddChrome,test=c("adf"), max.d=4)
ndiffs(sddddddChrome,test=c("pp"), max.d=4)
kpss.test(sddddddChrome, null=c("Level"))
kpss.test(sddddddChrome, null = c("Trend"))
library(urca)
adf.sddddddChrome=ur.df(sddddddChrome ,type=c("none"),lags=10,
selectlags=c("AIC"))
summary(adf.sddddddChrome)
adf.sddddddChromedrift=ur.df(sddddddChrome ,type=c("drift"),lags=10,
selectlags=c("AIC"))
summary(adf.sddddddChromedrift)
adf.sddddddChrometrend=ur.df(sddddddChrome ,type=c("trend"),lags=10,
selectlags=c("AIC")) #ADF Tests
summary(adf.sddddddChrometrend)
pp.sddddddChrome= ur.pp(sddddddCChrome,type=c("Z-tau"),
model=c("constant"), lags=c("short")) #PP Test
summary(pp.sddddddChrome)
pp.sddddddChrometrend= ur.pp(sddddddChrome,type=c("Z-tau"),
model=c("trend"), lags=c("short")) #PP Test
summary(pp.sddddddChrometrend)
ndiffs(diff(sddddddChrome),test=c("kpss"), max.d=4)
ndiffs(diff(sddddddChrome),test=c("adf"), max.d=4)
ndiffs(diff(sddddddChrome),test=c("pp"), max.d=4)
nsdiffs(diff(sddddddChrome), m=12, test=c("ocsb"), max.D=3)
nsdiffs(diff(sddddddChrome), m=12, test=c("ch"), max.D=3)
nsdiffs(diff(sddddddChrome), m=12, test=c("ocsb"), max.D=4)
nsdiffs(diff(sddddddChrome), m=12, test=c("ch"), max.D=4)
nsdiffs(diff(sddddddChrome), m=12, test=c("ocsb"), max.D=5)
nsdiffs(diff(sddddddChrome), m=12, test=c("ch"), max.D=5)
nsdiffs(diff(sddddddChrome), m=12, test=c("ocsb"), max.D=20)
nsdiffs(diff(sddddddChrome), m=12, test=c("ch"), max.D=20)
kpss.test(diff(sddddddChrome), null=c("Level"))
kpss.test(diff(sddddddChrome), null = c("Trend"))
library(urca)
adf.sddddddanddChrome=ur.df(diff(sddddddChrome)
,type=c("none"),lags=10, selectlags=c("AIC"))
summary(adf.sddddddanddChrome)
adf.sddddddanddChromedrift=ur.df(diff(sddddddChrome)
,type=c("drift"),lags=10, selectlags=c("AIC"))
summary(adf.sddddddanddChromedrift)
adf.sddddddanddChrometrend=ur.df(diff(sddddddChrome)
,type=c("trend"),lags=10, selectlags=c("AIC"))
summary(adf.sddddddanddChrometrend)
pp.sddddddanddChrome= ur.pp(diff(sddddddChrome),type=c("Z-tau"),
model=c("constant"), lags=c("short"))
```

```
summary(pp.sddddddanddChrome)
pp.sddddddanddChrometrend= ur.pp(diff(sddddddChrome),type=c("Z-
tau"), model=c("trend"), lags=c("short"))
summary(pp.sddddddanddChrometrend)
par(mfrow=c(1,2))
acf(sdChrome,48)
pacf(sdChrome,48)
plot(ts(sddChrome), ylab='sddChrome', xlab='Year')
par(mfrow=c(1,2))
acf(sddChrome,48)
pacf(sddChrome,48)
plot(ts(sdddChrome), ylab='sdddChrome', xlab='Year')
par(mfrow=c(1,2))
acf(sdddChrome,48)
pacf(sdddChrome,48)
plot(ts(sddddChrome), ylab='sddddChrome', xlab='Year')
par(mfrow=c(1,2))
acf(sddddChrome,48)
pacf(sddddChrome,48)
plot(ts(sdddddChrome), ylab='sdddddChrome', xlab='Year')
par(mfrow=c(1,2))
acf(sdddddChrome,48)
pacf(sdddddChrome,48)
plot(ts(sddddddChrome), ylab='sddddddChrome', xlab='Year')
par(mfrow=c(1,2))
acf(sddddddChrome,48)
pacf(sddddddChrome,48)
plot(ts(diff(sddddddChrome)), ylab='sddddddanddChrome', xlab='Year')
par(mfrow=c(1,2))
acf(diff(sddddddChrome),lag.max=36)
pacf(diff(sddddddChrome),lag.max=36)
fit=arima(Chrome, order=c(0,0,1), seasonal=list(order=c (0,0,0),
period=12))
fit
sdChrome=diff(Chrome,12)
fit=arima(sdChrome,order=c(0,0,1),seasonal=list(order=c (0,0,0),
period=12))
fit
sddChrome=diff(sdChrome,12)
fit=arima(sddChrome, order=c (0,0,1), seasonal=list(order=c (0,0,0),
period=12))
fit
sdddChrome=diff(sddChrome,12)
fit=arima(sdddChrome, order=c(0,0,1), seasonal=list(order=c(0,0,0),
period=12))
fit
sddddChrome=diff(sdddChrome,12)
fit=arima(sddddChrome,order=c (0,0,1), seasonal=list(order=c (0,0,0),
period=12))
fit
```

```
sdddddChrome=diff(sddddChrome,12)
fit=arima(sdddddChrome,order=c(0,0,1),seasonal=list(order=c(0,0,0),
period=12))
fit
sddddddChrome=diff(sdddddChrome,12)
fit=arima(sddddddChrome, order=c(0,0,1),seasonal=list(order=c(0,0,0),
period=12))
fit
sdddddddChrome=diff(sddddddChrome,12)
fit=arima(sdddddddChrome, order=c (0,0,1), seasonal=list (order=c (0, 0,0)
, period=12))
fit
fit=arima(Chrome,order=c(0,0,0),seasonal=list(order=c(0,0,1),
period=12))
fit
sdChrome=diff(Chrome,12)
fit=arima(sdChrome,order=c(0,0,0),seasonal=list(order=c (0,0,1),
period=12))
fit
sddChrome=diff(sdChrome,12)
fit=arima(sddChrome,order=c (0,0,0), seasonal=list(order=c(0,0,1),
period=12))
fit
sdddChrome=diff(sddChrome,12)
fit=arima(sdddChrome,order=c (0,0,0),seasonal=list(order=c(0,0,1),
period=12))
fit
sddddChrome=diff(sdddChrome,12)
fit=arima(sddddChrome,order=c(0,0,0),seasonal=list(order=c(0,0,1),
period=12))
fit
sdddddChrome=diff(sddddChrome,12)
fit=arima(sdddddChrome, order=c (0,0,0), seasonal=list(order=c(0,0,1),
period=12))
fit
sddddddChrome=diff(sdddddChrome,12)
fit=arima(sddddddChrome,order=c(0,0,0),seasonal=list(order=c(0,0,1),
period=12))
fit
install.packages("strucchange")
library(strucchange)
fs.chrome <- Fstats(Chrome ~ 1)
plot(fs.chrome)
plot(fs.chrome, pval = TRUE, alpha = 0.05)
sctest(fs.chrome)
plot(Chrome, ylab='Chrome', xlab='Year')
lines(breakpoints(fs.chrome))
breakpoints(fs.chrome)
```

```
bp.chrome <- breakpoints(Chrome ~ 1)
summary(bp.chrome)
plot(bp.chrome)
breakpoints(bp.chrome)
breakdates (bp.chrome)
ci.chrome <- confint(bp.chrome)
breakdates(ci.chrome)
ci.chrome
plot(Chrome, ylab='Chrome', xlab='Year')
lines(ci.chrome)
fs.feldspar <- Fstats(Feldspar ~ 1)
plot(fs.feldspar)
plot(fs.feldspar, pval = TRUE, alpha = 0.05)
sctest(fs.feldspar)
plot(Feldspar, ylab='Feldspar', xlab='Year')
lines(breakpoints(fs.feldspar))
breakpoints(fs.feldspar)
bp.feldspar <- breakpoints(Feldspar ~ 1)
summary(bp.feldspar)
plot(bp.feldspar)
breakpoints(bp.feldspar)
breakdates(bp.feldspar)
ci.feldspar <- confint(bp.feldspar)
breakdates(ci.feldspar)
ci.feldspar
plot(Feldspar, ylab='Feldspar', xlab='Year')
lines(ci.feldspar)
fs.copper <- Fstats(Copper ~ 1)
plot(fs.copper)
plot(fs.copper, pval = TRUE, alpha = 0.05)
sctest(fs.copper)
plot(Copper, ylab='Copper', xlab='Year')
lines(breakpoints(fs.copper))
breakpoints(fs.copper)
bp.copper <- breakpoints(Copper ~ 1)
summary(bp.copper)
plot(bp.copper)
breakpoints(bp.copper)
breakdates(bp.copper)
ci.copper <- confint(bp.copper)
breakdates(ci.copper)
ci.copper
plot(Copper, ylab='Copper', xlab='Year')
lines(ci.copper)
fs.zinc <- Fstats(Zinc ~ 1)
plot(fs.zinc)
plot(fs.zinc, pval = TRUE, alpha =0.05)
sctest(fs.zinc)
```

```
plot(Zinc, ylab=' Zinc ', xlab='Year')
lines(breakpoints(fs.zinc))
plot(Zinc, ylab='Zinc ', xlab='Year')
breakpoints(fs.zinc)
bp.zinc <- breakpoints(Zinc ~ 1)
summary(bp.zinc)
plot(bp.zinc)
breakpoints(bp.zinc)
breakdates(bp.zinc)
ci.zinc <- confint(bp.zinc)
breakdates(ci.zinc)
ci.zinc
plot(Zinc, ylab=' Zinc ', xlab='Year')
lines(ci.zinc)
fs.boron <- Fstats(Boron ~ 1)
plot(fs.boron)
plot(fs.boron, pval = TRUE, alpha =0.05)
sctest(fs.boron)
plot(Boron, ylab='Boron', xlab='Year')
lines(breakpoints(fs.boron))
plot(Boron, ylab='Boron', xlab='Year')
breakpoints(fs.boron)
bp.boron <- breakpoints(Boron ~ 1)
summary(bp.boron)
plot(bp.boron)
breakpoints(bp.boron)
breakdates(bp.boron)
## confidence intervals
ci.boron <- confint(bp.boron)
breakdates(ci.boron)
ci.boron
plot(Boron, ylab='Boron', xlab='Year')
lines(ci.boron)
ChromeFeldsparCopperZincBoron=cbind(Chrome,Feldspar,Copper,Zinc,Boron)
install.packages("vars")
library(vars)
VARselect(Chrome,lag.max=8,type="both")
VARselect(Feldspar,lag.max=8,type="both")
VARselect(Copper,lag.max=8,type="both")
VARselect(Zinc,lag.max=8,type="both")
VARselect(Boron,lag.max=8,type="both")
summary(ca.jo(ChromeFeldsparCopperZincBoron,type="trace",ecdet="cons
t",K=3, spec="longrun"))
IncentiveCertificates=ts(read.table("clipboard",header=F),start=2007
,frequency=12)
FixedInvestmentAmount=ts(read.table("clipboard",header=F),start=2007
,frequency=12)
```

IndustrialTurnoverIndexTotal=ts(read.table("clipboard",header=F), sta rt=2007, frequency=12)
IndustrialProductionIndex=ts(read.table("clipboard",header=F), start= 2007, frequency=12)
ManuPMIofChina=ts (read.table("clipboard", header=F), start=2007, freque ncy=12)
install.packages("tsDyn")
library(tsDyn)
vecmexochinaPMI=VECM (ChromeFeldsparCopperZincBoron [, c ("Chrome", "Feld spar", "Copper", "Zinc", "Boron")], 1, r=4, include="const", beta=NULL, estim="ML", LRinclude="const", exogen=IncentiveCertificatesF ixedInvestmentAmountIndustrialTurnoverIndexTotalIndustrialProduction IndexManuPMIofChina[, c("IncentiveCertificates","FixedInvestmentAmoun t","IndustrialTurnoverIndexTotal", "IndustrialProductionIndex", "ManuP MIofChina")])
summary (vecmexochinaPMI)
coef(vecmexochinaPMI)
install.packages("mvShapiroTest")
library(mvShapiroTest)
mvShapiro.Test(vecmexochinaPMI\$residuals)
install.packages("portes")
library("portes")
BoxPierce(vecmexochinaPMI\$residuals,lags=seq(2),order=0, SquaredQ=TRUE)
newIncentiveCertificates=ts (read.table("clipboard", header=F)) newFixedInvestmentAmount=ts (read.table ("clipboard", header=F)) newIndustrialTurnoverIndexTotal=ts (read.table("clipboard", header=F)) newIndustrialProductionIndex=ts (read.table("clipboard", header=F)) newManuPMIofChina=ts(read.table("clipboard", header=F))
newIncentiveCertificatesFixedInvestmentAmountIndustrialTurnoverIndex TotalIndustrialProductionIndexManuPMIofChina=cbind (newIncentiveCerti ficates, newFixedInvestmentAmount, newIndustrialTurnoverIndexTotal, new IndustrialProductionIndex, newManuPMIofChina)
predict(vecmexochinaPMI, exoPred=newIncentiveCertificatesFixedInvestm entAmountIndustrialTurnoverIndexTotalIndustrialProductionIndexManuPM IofChina, n.ahead=6)

