SECONDARY MATHEMATICS TEACHERS' MATHEMATICS RELATED BELIEFS THROUGHOUT A PROFESSIONAL DEVELOPMENT PROGRAM BASED ON MATHEMATICAL MODELLING

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

DUYGU ÖREN VURAL

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY
IN
SECONDARY SCIENCE AND MATHEMATICS EDUCATION

MARCH 2015

Approval of the thesis:

SECONDARY MATHEMATICS TEACHERS' MATHEMATICS RELATED BELIEFS THROUGHOUT A PROFESSIONAL DEVELOPMENT PROGRAM BASED ON MATHEMATICAL MODELLING

submitted by **DUYGU ÖREN VURAL** in partial fulfillment of the requirements for the degree of **Doctor of Philosophy in Secondary Science and Mathematics Education Department, Middle East Technical University** by,

Prof. Dr. Gülbin Dural Ünver _________
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Ömer Geban _________
Head of Department, **Secondary Science and Mathematics Education**

Assoc. Prof. Dr. Bülent Çetinkaya _________
Supervisor, **Secondary Science and Mathematics Education Dept., METU**

Examining Committee Members:

Assoc. Prof. Dr. Ayhan Kürşat Erbaş _________
Secondary Science and Mathematics Education Dept., METU

Assoc. Prof. Dr. Bülent Çetinkaya _________
Secondary Science and Mathematics Education Dept., METU

Assoc. Prof. Dr. Nihat Boz _________
Secondary Science and Mathematics Education Dept., GAZİ UNI.

Assoc. Prof. Dr. Ömer Faruk Özdemir _________
Secondary Science and Mathematics Education Dept., METU

Assoc. Prof. Dr. Çiğdem Haser _________
Elementary Education Dept., METU

Date: 03.03.2015

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: Duygu Ören Vural

Signature :

ABSTRACT

SECONDARY MATHEMATICS TEACHERS' MATHEMATICS RELATED BELIEFS THROUGHOUT A PROFESSIONAL DEVELOPMENT PROGRAM BASED ON MATHEMATICAL MODELLING

Ören Vural, Duygu

Ph.D. Department of Secondary School Science and Mathematics Education

Supervisor: Assoc. Prof. Dr. Bülent Çetinkaya

March 2015, 199 pages

The purpose of this qualitative study is to investigate in-service secondary school mathematics teachers' mathematics related beliefs, to explore changes in their beliefs after they participated in a one-year professional development program (PDP) based on mathematical modelling as well as teachers' perception about the effects of PDP on their belief change. Semi-structured interviews and an open-ended analogy questionnaire were the main data sources. Results showed that most of the teachers held more than one category of beliefs, such as teachers held beliefs about certainty of results and existence of different ways of solutions for a mathematical problem, or beliefs about solving too many practice questions for teaching and learning concepts and making relations between concepts simultaneously. Instrumental and problems solving beliefs, content-focused with emphasis on performance and content-focused with emphasis on conceptual understanding beliefs, and skill-mastery with passive reception of knowledge and conceptual understanding with unified knowledge beliefs were the most frequently holding beliefs among teachers. Data analysis indicated that, after PDP there were category change in some teachers' beliefs. Also, most of the teachers' beliefs about mathematical problems and problem solving, learning and teaching either changed or teachers developed new beliefs. Moreover, teachers reported PDP's influence especially on their beliefs about teaching and learning mathematics. Weekly meetings, classroom implementations as well as the collaboration occurred and support offered in meetings was found as most the influential elements of PDP for the change and development of their beliefs

Keywords: Teacher Beliefs, Belief Change, Mathematical Modelling, Professional Development Programs

ÖZ

MATEMETİKSEL MODELLEME TEMELLİ BİR MESLEKİ GELİŞİM PROGRAMI BOYUNCA ORTAÖĞRETİM MATEMATİK ÖĞRETMENLERİNİN MATEMATİKLE İLGİLİ İNANÇLARI

Ören Vural, Duygu

Doktora, Ortaöğretim Fen ve Matematik Alanları Eğitimi Bölümü

Tez Yöneticisi: Doç. Dr. Bülent Çetinkaya

Mart, 2015, 199 sayfa

Bu nitel çalışmanın amacı, matematiksel modelleme üzerine bir yıllık bir mesleki gelişim programına katılan lise matematik öğretmenlerinin matematiğe dair inançlarını, bu inançların mesleki gelişim programından sonraki değişimini ve aynı zamanda öğretmenlerin kendi inanç değişimleri üzerine algılarını araştırmaktır. Yarı yapılandırılmış görüşmeler ve açık uçlu bir analoji anketinden elde edilen veriler bu çalışmanın temel veri kaynaklarıdır. Sonuçlar öğretmenlerin çoğunluğunun aynı andan birbiriyle çelişen inançlara ("sonuçların kesinliği" ve "bir matematik probleminin farklı çözümlerinin olması" inançları ve "kavramları öğrenmek ve öğretmek için çok sayıda soru çözme" ve "kavramlar arası bağlantılar kurma"inançları) sahip olduklarını göstermiştir. Matematiğin doğası hakkında "enstrümantalist" ve "problem çözme" inançları, matematik öğretimi hakkında "içerik-performans odaklı" ve "içerik odaklı-kavramsal anlama" inançları ve matematik öğrenimi hakkında "beceri kazanımı-pasif öğrenen" ve "bilgi ile kavramsal anlama" inançlarının öğretmenlerin en sık sahip oldukları inançlar olduğu tespit edilmiştir. Veri analizi sonuçları, mesleki gelişim programından sonra, bazı öğretmenlerin inançlarında kategori değişikliği olduğunu göstermiştir. Aynı şekilde, öğretmenlerin matematiksel problem ve problem çözme ile matematik öğretimi ve öğrenimi hakkındaki inançlarının ya değiştiği ya da bu konuda öğretmenlerin yeni inançlar geliştirdiği bulunmuştur. Ayrıca, öğretmenler programın özellikle matematik öğretimi ve öğrenimi hakkındaki inançlarını etkilediğini belirtmişlerdir. Programın, inanç değişimi ve yeni inanç gelişimi üzerinde en fazla etkili olan bileşenlerinin, sınıf içi uygulamalar, haftalık toplantılar ve bu toplantılarda gerçekleşen işbirliği ve sunulan destek olduğu ortaya çıkmıştır.

Anahtar Kelimeler: Öğretmen İnançları, İnanç Değişimi, Matematiksel Modelleme, Mesleki Gelişim Programları

To my family and my friends...

ACKNOWLEDGEMENTS

I would like to present my deepest and sincere gratitude to my advisor Dr. Bülent Çetinkaya who have supported and engouraged me throughout the study. Also, I would like to thank my committee members Dr. Çiğdem Haser, Dr. Ayhan Kürşat Erbaş, Dr. Ömer Faruk Özdemir, and Dr. Nihat Boz for their valuable comments and suggestions.

I would like to thank my friends Beril Yılmaz Senem, Hanife Can Şen, Fulya Kula and Sevim Sevgi. Without their support I would never have been able to finish my dissertation. Last but not the least; I would like to thank my husband Erkan Vural and my family for supporting me throughout writing this dissertation and my life in general.

This study was based upon a research project supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under grant number 110K250. Opinions expressed are those of the author and do not necessarily represent those of TUBITAK.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVIATIONS

PDP	: Professional Development Program
SBT	: School based Training
NM	: Nature of Mathematics
T	: Teaching Mathematics
L	: Learning Mathematics
MoNE	: Ministry of National Education

CHAPTER 1

INTRODUCTION

As science and technology have been developing, the skills and abilities that individuals needed to possess have been changing critically. The past view about importance of possessing information dramatically evolved toward a view, which highlights the importance of problem solving with use of information. This view influenced the educational approaches and whether the mathematics education develops students' problem solving abilities in real life and work situations has been questioned (Blum & Niss, 1991; Greer, 1997; Kaiser, Blomhoj, & Sriraman, 2006; Lesh & Doerr, 2003; Sriraman & Lesh, 2006). Because of this concern, in many countries including Turkey, mathematics curricula were changed or revised and put more emphasis on developing students' problem solving skills. Curriculum changes in Turkey were started in 2004, and mathematical modelling has become one of the main focuses of secondary school mathematics curriculum together with other key skills; reasoning, problem solving, making connection and communicating (Ministry of National Education [MoNE] Board of Education, 2011). Mathematical modelling can be simplified to a process of finding solution to real life problems by using mathematical tools and methods. There are different approaches and descriptions about mathematical modelling (Bautista, Wilketson-Jerde, Tobin, & Brizuela, 2014). However, it is generally defined as a process of mathematizing, interpreting, verifying, generalizing real life situations or complex systems (Lesh & Doerr, 2003).

Mathematical modelling is considered as 21st century skill (English & Sriraman, 2010) and it is explicit focus of other countries' mathematics curriculum. For example, the standards proposed by NCTM (1989) highlighted use of mathematical modelling in secondary grades and suggested that students in 9-to-12 grades should be able to "apply the process of mathematical modelling to real word problem situations" (p.137), while standards proposed by Common Core State Standard Initiative (2010) stressed that mathematical modelling should be included in mathematical practice for students at every level.

The primary reason to integrate mathematical modeling in mathematics curricula is that mathematical modeling is considered to have significant role in teaching and learning mathematics. There are different orientations about the use of mathematical modelling in teaching mathematics, such as utilizing mathematical modelling as a goal and as means (Galbraith, 2007; Niss, Blum, & Galbraith, 2007) The common argument in those orientations is that mathematical modelling is recognized as a powerful tool to increase and promote students' understanding of mathematical concepts (English, 2007; Lingefjard, 2000; Lesh & Harel, 2003). Moreover, modeling is used to make students apply mathematics in variety of context and situations outside the classroom (Niss, Blum, & Galbraith, 2007). Contrary to routine mathematical problems which are specified in a particular content area, solved with predetermined or standard algorithms, and generally have a single answer, modelling problems (or model eliciting activities) provide students a realistic context in which students are required to strategize, to use their prior knowledge, to test and to revise their solutions (Greer, 1997; Lesh, Cramer, Doerr, Post, & Zowojewski, 2003; Verschaffel & Decorte, 1997; Zawolewski & Lesh, 2003). Besides, through modelling process, students can see the problem in several different viewpoints depending on their ways of thinking and use different ways of solutions. Through engaging in modelling activity, students construct mathematical ideas by themselves, and their thinking processes are explicitly revealed from their descriptions, explanations, justifications, and representations as they work on the task and present their products (Doerr, 2006). Thus, modeling problems (or model eliciting activities) are considered to promote high-level mathematical thinking (Doerr, 2006). In fact, research studies reported that when students engage in modeling tasks they develop deeper understanding of mathematics (Blum & Niss, 1991; Boaler, 2001; Schoenfeld, 2009).

Although students are expected to construct the mathematical ideas by themselves while engaging in the modeling task, teachers have crucial roles in construction of mathematical ideas (Niss, Blum, & Galbraith, 2007). Researchers portrayed teachers' roles for incorporation of modeling in mathematics teaching and for implementation of modeling in classrooms (Doerr, 2006, 2007; Lingefjärd & Meier, 2010; Niss, Blum, & Galbraith, 2007; Stillman, 2010). It was suggested that through modelling process, teachers need to attend to students' ways of thinking, listen to and appreciate students' ideas, respond to their thinking, and adjust their pedagogical strategies to support students' mathematical ideas (Chazan & Ball, 1999; Doerr, 2006; Doerr & Lesh, 2003; Lesh & Doerr, 2000; Sherin, 2002). It was also proposed that for effective implementation of modelling, teachers need to support group discussions and students working in small groups (Glabraith & Clatworthy, 1990). This implies that the implementation of modeling in mathematics classrooms requires a serious shift in the ways of traditional classroom practice which emphasize memorization and execution of rules and procedures; of teachers' roles as the controller of the whole classroom activity by transferring knowledge with step-by-step instruction; and of students' roles as passive listener and receiver of knowledge transferred by teacher (English & Watters, 2005; Zawojewski & Lesh, 2003).

Although, reformed mathematics curricula propose different content including modeling and different teacher and students' roles, there is still a gap between what is proposed and what is happening in the classroom (Lingefjärd & Mejer, 2010). It is commonly accepted that changes in classroom practice demanded by reform movements ultimately depend on teachers (Fullan & Miles, 1992). However, research studies showed that despite using the language of reform, teachers' actual practices do not reflect what they report. Shorr and Clark (2003), for example, indicated that although teachers' reported use of reform-oriented approaches in their teaching of mathematics, their teaching practice involved memorization, performing procedures, use of routine and procedural problem and emphasis on getting correct result with little or no exploration of students' solution strategies. Research studies conducted in Turkey reported similar teacher practice in such a way that many of the mathematics teachers focused students getting isolated skills through repeated practice.

Researchers proposed number of factors effecting teachers' implementations of reform (modelling). For example, insufficient knowledge and limited experience about modelling and use of modelling in teaching mathematics was found to affect teachers' inclusion of modeling in their teaching and effective use of modelling (Swan, 2007). However, research studies also showed that even if teachers gain knowledge, experiences and skills about the reform practices from teacher education programs or professional development programs, still they might not adopt and implement the reform practices in their teaching (Ernest, 1994; Törner, 2001). For example, Ernest (1994) emphasized that "knowledge is important, but alone is not account for the differences between mathematics teachers. Two teachers can have similar knowledge, but while one teaches mathematics with problem solving orientation, the other has more didactic approach" (p.249). Considerable amount of research indicated teachers' beliefs as an important factor affecting teachers' implementation of reform (Cooney, Shealy & Arwold, 1998; Swan, 2007; Wilson & Cooney, 2002) in such a way that teachers do not follow the reform recommendations unless their beliefs do not align with the reform, (Thompson, 1992). Therefore, delineating the beliefs teachers hold and change in beliefs, which are incompatible with the reform, are important considerations for the success of the reform in mathematics education.

It is commonly accepted that each teacher holds a particular belief system comprising a wide range of beliefs about learners, teachers, teaching, schooling, resources, knowledge, and curriculum (Gudmunsdottir & Shulman, 1987; Lovat & Smith, 1995). Beliefs are considered to act as a filter on one's new experiences and information (Pajares, 1992), and through which teachers make their decision (Clark & Peterson, 1986). Because of their function as a filter through which new information and experience is processed, and driver of decisions making process, beliefs are considered as an important factor effecting teachers' instructional practices (Thompson, 1992). Findings from a growing body of research supported the premise that teachers' instructional practices are shaped by the beliefs they hold (Cooney, Shealy & Arwold, 1998; Ernest, 1989a; Handal & Herington, 2003; Nespor, 1987; Pajares, 1992; Swan, 2007; Thompson, 1992; Wilson & Cooney, 2002).

Teachers' mathematics related beliefs are generally classified under three dimensions; beliefs about nature of mathematics, beliefs about teaching mathematics, beliefs about learning mathematics (Ernest, 1989). As for mathematics related beliefs, researchers suggest that teachers' beliefs about what mathematics is and what it means to teach and how to learn mathematics has a direct impact on how they teach mathematics and the way they teach mathematics (Ernest, 1989; Pajares, 1992; Thompson, 1984, 1992). For example, it was pointed out that teachers who see mathematical knowledge as static and as a set of rules and procedures which produce one correct answer, and who consider knowing mathematics as being able to perform procedures without understanding the underlying meaning, follow a teaching involving step-by-step instruction of procedures, followed by students being asked problems for practicing those procedures (Thompson, 1994; Wood, Cobb, & Yackel, 1991). Moreover, contrary to teachers who believe that mathematics is learned by receiving knowledge of operation, teachers who believe that students learn mathematics by constructing their own understanding through solving problems were found to use more problems in their teaching and to devote more time to develop student's strategies before teaching related facts (Peterson, Fennema, Carpenter, & Loef, 1989).

Research related to teachers' beliefs and their use of modelling indicated similar relationship between mathematics related beliefs and teachers' modelling practice. For example, teachers' beliefs about nature of mathematics were revealed as the factor for low realization of modeling in mathematics teaching (Kaiser & Maass, 2007). It was also found that teachers' use of applications and modelling in their teaching and their effort to overcome the barriers on the use of it, is all shaped by their instructional goals which are connected to teachers' mathematics related beliefs (Förster, 2011). Research also revealed that teachers' interpretations of the written curriculum differ according to their beliefs about mathematics and about the goals of teaching mathematics. For example, because of their beliefs about elementary geometry and goals of teaching geometry, teachers considered that the domain of geometry was not suitable for modelling (Girmat & Eichler, 2011). Besides, Maass (2011) found that teachers' beliefs about effective teaching had a key role on their intention to change their classroom practice (as proposed by professional development initiative). As studies indicated, teachers' beliefs about nature of mathematics, teaching and learning mathematics have influence on their modelling practice as well as their intention to implement modelling. Therefore, investigating teachers' mathematics related beliefs can shed light on teachers' modeling practice and can contribute to the effort for changing their beliefs.

The results of research studies highlight that in order to change teachers' traditional classroom practice into more reform oriented practices; teachers' beliefs need to change (Pajares, 1992; Thompson, 1992). Indeed, research focusing on teacher change indicated that change of teachers' beliefs is one of the three dimensions in practicing an innovation/reform, together with use of new or revised materials and use of new teaching approaches (Fullan, 2001, 2007). However, research also proposed that teachers do not automatically change the beliefs they hold. Teachers' beliefs are formed through experience and cultural transmission (Pajares, 1992). Yet, some of their beliefs are formed before they enter into teacher education or they become teachers, and those beliefs are deeply rooted and resistant to change. Therefore, their beliefs are not generally affected from reading and being asked to apply an innovation (Stipek, Givvin, Salmon, & MacGyvers, 2001). Researchers pointed out that teacher change could not be possible without support and guidance (Ball & Cohen, 1999; Putnam & Borko, 1997). For the change of teachers' beliefs, researchers also suggest that "teachers need to engage in practical inquiry to move back and forth among a variety of settings to learn about new instructional strategies, to try them out in their own classrooms and to reflect on what they observe in a collaborative setting" (Kagan, 1992; Peterson, Fennema, Carpenter, & Loef, 1989; Wood, Cobb, & Yackel, 1991). Professional development programs are considered to provide such context for teachers (Borko, 2005; Guskey, 2002).

Professional development programs (PDPs) are described as the systematic efforts for changing classroom practice of teachers, their attitudes and beliefs, and the learning outcomes of students (Guskey, 2002). There are several PDP models for teachers appeared in literature. The most common PDPs are conducted as workshops (Garet, Porter, Desimone, Birman, & Yoon, 2001), which occur outside of teachers' classrooms in a structured short period of time. This traditional type of PDPs are criticized for being ineffective since they do not provide enough time, activities and content for teachers to facilitate change (Loucks-Horsley, Hewson, Love, & Stiles, 1998). Moreover, when there can be some changes in teachers' practice, attitudes, or beliefs, these changes are not substantial and long lasting (Schorr & Lesh, 2003). NCTM (2003) proposes that professional development experiences for teachers should be transformative in such a way that they should trigger substantial changes in teachers' knowledge and beliefs. Parallel to this, there is a growing interest in untraditional type of PDPs which appeared in literature as school-based or practice-based professional development, such as lesson studies, study groups activities, etc. This type of PDPs, which are taking place in teachers' own context, are considered as more responsive to needs and goals of teachers (Darling-Hammond, 1997), and well aligned with the reform movements (NCTM, 2003).

For Guskey (2002), three outcomes can be attained by PDPs: change in classroom practice of teachers, change in their attitudes and beliefs, and change in learning outcomes of students. Guskey (2002) proposes that these three outcomes are reciprocal, yet, change in attitudes and beliefs can be obtained when teachers see improvement in their students learning. Researchers recommended important features for effective PDPs. For example, it is proposed that in order to be effective, PDP should focus on teachers' daily activities and should offer them opportunities to participate in cycles of shared and ongoing dialogue, classroom enactment, and reflection (Blumenfeld, Krajcik, Mark, & Solloway, 1994; McLaughlin, 1994 in Turner, Warzon, Christensen, 2011, p.724), and should sustain over time (Cohen & Hill, 1998). Little (2002) also added that strong professional development communities are important for effective PDP in such a way that establishment of communication norms and trust, and collaborative interactions between groups of teachers are the features of PDP considered to have important contribution to school reform. In addition, Borko (2004) suggested that teachers' own classrooms are considered as powerful context for PDPs and use of artifacts about their classroom practice such as instructional plans and assignments, videotapes of lessons, samples of students' works are powerful tools for facilitating teacher change. Additionally, continuing support for teachers is recommended throughout the PDP in order to translate new ideas into everyday practice (Lee & William, 2005).

There is a large body of research on professional development of in-service mathematics teachers and much of them are focusing on improving teachers' subject matter and pedagogical knowledge, and pedagogical content knowledge. Similarly, research on professional development of mathematics teachers about mathematical modelling are generally centering on developing teachers' knowledge, skills and abilities about modelling and modelling competencies (Garcia & Ruiz-Higueras, 2011; Maass & Gurlitt, 2011; Wake, 2011). However, there is relatively little research conducted on development or change of teachers' mathematical (mathematics related) beliefs in the context of a PDP, which is designed specifically on mathematical modelling. Moreover, research studies conducted in Turkey about mathematical modelling generally condense on teachers conceptions of mathematical modelling or change of their conception after participating short term PDPs. There is still little information about Turkish secondary school mathematics teachers' mathematics related beliefs, and whether or how their beliefs would change after participating in a PDP on mathematical modelling. In this regard, the present study is aiming to investigate mathematical related beliefs of secondary school mathematics teachers and to explore the process of changes in their beliefs.

1.1 Purpose of the Study

The purpose of this study to investigate in-service secondary school mathematics teachers beliefs about nature of mathematics, and teaching and learning mathematics and to explore changes in their beliefs after they participated in a one year professional development program on mathematical modelling as well as teacher' perception of the influence of professional development program on their beliefs

1.2 Research Questions

The research questions guided this study are;

1. What are the secondary mathematics teachers' beliefs about nature of mathematics, and teaching and learning mathematics?

2. How did teachers' beliefs about nature of mathematics, learning and teaching mathematics change after participating in a one year professional development program on mathematical modelling?

3. What do teachers think to be the influence of the professional development program on mathematical modeling on their beliefs about nature of mathematics, and teaching and learning mathematics?

1.3 Significance of the Study

Investigating secondary mathematics teachers' mathematics related beliefs and the change in their beliefs in the context of professional development program based on mathematical modelling are considered important because of the several reasons.

Teachers are considered as at the center of any educational reform (Cuban, 1990). As indicated by several researches (Thompson, 1984; Pajares, 1992) teachers' beliefs about subject matter and its teaching and learning play an important role in teachers' effectiveness. Therefore, any attempt to improve mathematics instruction would benefit from a better understanding of teachers' mathematics related beliefs.

As a part of recent reform in mathematics curriculum in Turkey, mathematical modelling is considered to have significant role in teaching and learning mathematics (Doerr & Lesh, 2003; Lesh & Doerr, 2003). Also, teachers' beliefs about what mathematics is, how mathematics can be learned and should be taught are key determinants of their classroom practice, in the end they can be considered as important factors in effective use of modelling in mathematics teaching (Blum & Niss, 1991; Ikeda, 2007; Förster, 2011; Girmat & Eschler, 2011; Veiger, 2011) . Therefore, documenting teachers' mathematics related beliefs could portray a picture about their modelling practice.

Moreover, as research studies suggested that effective implementation of modeling in mathematics classrooms requires a serious shift in the ways of traditional classroom practice that emphasize memorization and execution of rules and procedures; of teachers' roles as the controller of the whole classroom activity by transferring knowledge with step-by-step instruction; and of students' roles as passive listener and receiver of knowledge transferred by teacher. Given that, teachers' beliefs about nature of mathematics, learning and teaching of mathematics need to change. Therefore illuminating the process of teachers' belief change and the context effecting that change would be important for designing professional development programs for teachers.

Although there are plenty of studies on teachers' beliefs, much of these research studies were quantitative studies aiming to investigate beliefs through surveys or questionnaire and most of them were conducted with preservice mathematics teacher (Andrews & Hatch, 1997; Beswick, 2012; Raymond, 1997; Vacc & Bright, 1999; Van Zoest, Jones, & Thornton, 1994). There is not a great deal of information about Turkish secondary mathematics teachers' beliefs about nature of mathematics, learning and teaching of mathematics.

As indicated teachers beliefs and their belief change were investigated through a one-year professional development program based on modelling. The results of this study also would contribute to the research literature on effective professional development programs.

1.4 Definition of the Key Terms

Definitions of the important terms associated with the study are listed below:

Modelling: Modeling is defined as a process of defining the phenomenon and the relations in it with the mathematical expressions and bringing out the mathematical patterns in a phenomenon (Verschaffel, Greer, & De Corte, 2002).

Model Eliciting Activities: Model eliciting activities are described as tools designed for students' and teachers' to promote externalization of their thinking and conceptualization steps for the problem situations (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003; Lesh & English, 2005; Lesh & Sriraman, 2005). Model eliciting activities provide an opportunity to develop a model for a real-life situation, to describe, revise, and refine ideas and to explain conceptual systems by models (Lesh& Doerr, 2003a).

Beliefs: Beliefs are considered as part of conceptions and defined as "more general mental structure, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences, and the like" (Thompson, 1984).

CHAPTER 2

REVIEW OF THE RELATED LITERATURE

In this study, secondary school mathematics teachers' mathematics related beliefs (beliefs about nature of mathematics, beliefs about mathematics teaching and beliefs about mathematics learning), the changes in their beliefs after participating in a one-year professional development program on mathematical modelling, and what teachers think about the effects of PDP on their belief change were investigated. Therefore, the underpinnings of this study are based on three domains of research; "research on mathematical modelling", "research on mathematics related beliefs", and "research on teacher change and professional development programs". In the following sections, related literature about these domains of research is presented. Firstly, the literature about mathematical models and mathematical modelling will be elaborated and research on teachers modelling practice and its relationship between beliefs will be provided. Then, the theoretical aspects of belief construct and related research about teachers' mathematics related beliefs will be documented. Lastly, literature on teacher change and professional development programs and research studies on professional development program about teachers' belief changes will be presented.

2.1 Mathematical Modelling

In recent years, there has been a great interest among mathematics educators and researchers about mathematical modelling, and teaching and learning of it. Mathematical modelling is roughly described as a process of transferring a real life situation into mathematical language. Mathematical modeling and its' applications are considered to facilitate students to comprehend and learn mathematical concepts more meaningfully, to see real life applications of mathematics, and to understand the connections between mathematics and real life. For these reasons, there is a great interest among mathematics educators about mathematical modeling, teaching and learning of it, and there is a great effort in all over the world to integrate mathematical modeling into mathematics curricula (*Department for Education* [DFE]; NCTM, 1989, 2000; *Ministry of National Education* [MEB], 2011).

The term mathematical modelling is different from the terms "model" and "mathematical model". In daily language, model is commonly used as something to represent concrete objects (i.e., airplane model, car model, etc.), or seeing someone as a model (i.e., model citizen, model teacher, etc.). In scientific language, model is also used as something to represent systems with the use of symbols (i.e, flow chart, equation etc.). The word model, on the other hand, is used in mathematics as "conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notation systems, and they are used to construct, or explain the behaviors of other system(s) - perhaps, so that the other system can be manipulated or predicted intelligently" (Lesh & Doerr, 2003a, p.10). Models and modelling is also discriminated in the literature in such a way that a model can be regarded as a product, while the modelling is considered as a process of creating models (Lesh & Sriraman, 2005). In this sense, Lesh and Lehrer (2003) defined mathematical models as "purposeful mathematical descriptions of situations, embedded within particular system of practice, that feature an epistemology of model fit and revision", while the mathematical modelling is defined as "a process of developing representational descriptions for specific purposes in scientific situations" (p.109).

Although the importance of mathematical modelling has been accepted among the mathematics educators, there is no agreement about the unique definition of it. Therefore, there are different approaches and definitions for mathematical modeling in the literature (Galbraith & Stillman, 2006; Kaiser, 2006; Kaiser, Blomhøj, & Sriraman, 2006). For example, according to one perspective, modelling is considered as an implementation of mathematics in real life and work situations (Crouch & Haines, 2004; Haines & Crouch, 2001; Houston, 2002; Izard, Haines, Crouch, & Neill, 2003; Jensen, 2007; Lingefjard, 2000). In this regard, Verschaffel, Greer and De Corte (2002) delineated mathematical modeling as an application of mathematics to solve problem situations in the real world. Verschaffel et al., (2002) added that mathematical modelling is a complex process consisting of phases such as understanding the situation, constructing a mathematical model, working through the mathematical model, interpreting the outcome, evaluating the outcome in relation to the original situation, and communicating the interpreted results. Similarly, Lingefjard (2006) defined mathematical modeling by stressing the phases involved as "a mathematical process that involves observing a phenomenon, conjecturing relationships, applying mathematical analyzes (equations, symbol structures, etc.), obtaining mathematical results, and reinterpreting the model". From another perspective, mathematical modelling is considered as more than applications of real-life situations, it is seen as a conceptual system that describes and explains the behaviors of other systems (Lesh & Doerr, 2003a). It is delineated that mathematical modelling is a process of mathematizing, interpreting, verifying, generalizing real life situations or complex systems, which involves cycles of developing-revising-testing, and requires different ways of thinking and solutions steps with regard to classical problem solving approaches (Lingefjard, 2000; Lesh & Harel, 2003).

It is agreed that modelling process is cyclic rather than linear (Burkhardt, 1994; Lesh & Doerr, 2003; Haines and Crouch, 2007; NCTM, 1989; Verschaffel et al., 2002). It is also accepted that this cyclic process is iterative. Lesh and Doerr (2003) summarized the cycles that students going through during modelling process as (a) using their informal knowledge students try to understand and simplify the problem (i.e., selecting and interpreting the proper information), (b) students develop a model where they decide the relationships among the variables, construct the hypotheses and evaluate the information, (c) analyzing the model, students try to decide if their system has a gap or satisfy the goals, (d) checking the model, students reflect on the solution from different perspectives with restructuring the solution in order to make their model acceptable (Lesh& Doerr, 2003). Haines and Crouch (2007) also described cyclic nature of mathematical modeling and delineated six stages involved as statement of real life problem, formulation of the model, solving, and interpretation of solution, refining the model, reconsidering the real life problem and repeating the cycles. Lesh and Doerr (2003), on the other hand, summarized steps involved in modelling cycles as description, manipulation, translation and verification.

Researchers considered tasks or activities including mathematical modelling as a tool for identifying students thinking process. These activities are denoted as model-eliciting activities (MEAs) and they are briefly defined as the problem solving activities, which elicit a mathematical model (Lesh & Yoon, 2007; Lesh, Hoover, Hole, Kelly, & Post, 2000). Since MEAs can be used as a tool for understanding students thinking process, they are sometimes called as thought revealing activities. Students' thinking processes are revealed during their engagement with MEAs from their representations, descriptions, explanations and justifications and as they working on MEAs and producing an end product (Lesh, Hoover, Hole, Kelly, & Post, 2000). Six principles are described to design good quality of MEAs (model construction, reality, self-assessment, construct documentation, construct shareability, reusability, and effective prototype). Model construction principle denotes that designed MEA should create a model related to real life situations. Reality principle indicates that problem situation should be real, and should exist in the real life in order for students to understand the situation meaningfully. Self-assessment principle signifies that in order to evaluate the usefulness of end product and solutions, problem context should have relevant standards. Construct documentation accounts for students documenting their thinking process explicitly throughout the solution process. Construct shareability and usability denote for created constructs/models' shareability and usability for similar real life situations. Effective prototype principle indicates that for shareability of obtained solutions with others and for usability of solutions in similar situations, solutions should sufficiently be prototype (Lesh, Hoover, Hole, Kelly, & Post, 2000).

In literature on mathematical modeling, there is a stress on the similarities and distinctions between mathematical modeling and problem solving (Doerr & Lesh, 2003; Lesh & Yoon, 2007; Zawojevski & Lesh, 2003). Lingefjard (2002) claimed that modeling process includes problem solving as a sub-process. Lesh and Doerr (2003), on the other hand, considered traditional problem solving as a subset of model-eliciting activities. Yet, Yu and Chang (2011) regarded MEAs as open-ended problems. Moreover, Reusser (1995) considered word problems as simple forms of mathematical modelling. On the contrary, Lesh and Doerr (2003) proposed that traditional word problems are based on computation and symbolic manipulation; however, MEAs offer more meaningful real life contexts, which contribute students' conceptual learning.

Although, it is seen as a sub-set of problem solving activities or as a higher category including problem solving, the distinctions between mathematical modelling and traditional problem solving is strongly highlighted in the literature (Lesh & Doerr, 2003; Lesh & Yoon, 2007; Zawojewski, 2010; Zawojewski & Lesh, 2003). Based on this literature, Erbaş and his collegues (Erbaş, Kertil, Çetinkaya, Çakıroğlu, Alacacı, & Baş, 2014) compared traditional problem solving approaches and mathematical modelling as presented in Table 2.1.

Table 2.1 Comparison of Traditional Problem Solving Approaches and Mathematical Modelling

Traditional Problem Solving Approaches	**Mathematical Modelling**
The process of reaching to specific goal by using givens	Multiple cycles, different interpretations
Real or realistic life situations with idealized problem context	Authentic real life context
It is expected from students to use taught structures such as formulas, algorithms, strategies, mathematical ideas, etc.	Students experience development of significant mathematical ideas and structures, revision, and refinement steps in the modelling process
Individual working stands in the forefront	Group work is stressed. (e.g., social communication, sharing mathematical ideas, etc.)
Abstracted from real life	Associated with real life and possessing interdisciplinary nature
It is expected from students to give meaning to mathematical symbols and constructs	Students try to describe real life situations mathematically
Teaching of specific problem solving strategies (e.g. developing distinct approach, transferring it on a shape etc.) and using it in the solution of similar problems	It involves more than one inconspicuous solution strategies developed by students consciously that are specific to certain situation
There is a unique, correct solution	There are more than one solution strategies and solutions (model)

There are two approaches for the use of mathematical modelling in teaching mathematics. According to first approach, mathematical modeling is seen as a goal and it is considered that it should be taught as a subject matter (Burkhard, 2006; Lingefjard, 2006). This approach is also regarded as applications of mathematical concepts into real life situations (Niss et al., 2007). According to second approach, on the other hand, mathematical modeling is seen as a tool or vehicle (Lesh & Doerr, 2003a, 2003b; Lesh & Lehrer, 2003; Gravemeijer, 2007) a paradigm beyond constructivism, a way of teaching and learning mathematics by using real life situations (Lesh and Doerr, 2003).

It is strongly advocated that students at any grade level can be benefitted from engaging in mathematical modelling activities (Doerr & English, 2003; Lesh & Doerr, 2003). Through modelling process students are dealing with “constructing, explaining, justifying, predicting, conjecturing, and representing as well as quantifying, coordinating, and organizing the data” (English & Watters, 2005, p.58) and through these experiences students generally work in a groups. Therefore, mathematical modelling activities contribute students’ communication skills, motivations, and better understanding of real world contexts, mathematical concepts and the connections between real life and mathematics (Doerr & English, 2003; English & Watters, 2005; Ikeda & Stephens, 2001; Lesh & Doerr, 2003). Because of importance of mathematical modelling for learning mathematics, mathematical modelling is advocated to be the integral part of school mathematics curriculum (Blum & Niss, 1991; Borromeo Ferri & Blum, 2009; Doerr & Lesh, 2011; Lesh & Doerr, 2003; Lesh & Zawojewski, 2007; Lingefjard, 2006).

2.1.1 Teachers’ Implementation of Mathematical Modelling

Despite its importance and contributions to students’ learning mathematics, use of mathematical modelling in mathematics teaching is still rare (Burkhardt, 2006; Maass, 2005). Research studies indicated that there exist several factors affecting teachers’ adoption and implementation of mathematical modelling. Blum (1996) categorized such factors as organizational, pupil-related, teacher-related, and material-related. Studies indicated that role of teachers is critical for the integration and effective implementation of mathematical modeling (English & Watters, 2005; Zawojewski et al., 2003). However, there are difficulties regarding teachers about the integration of mathematical modeling into mathematics teaching (Blum & Niss, 1991; Burkhardt, 2006; Kaiser & Maass, 2007; Maass, 2005). For example teachers’ knowledge, skills and experience about implementing MEAs are considered as important factors that support or hinder their use of mathematical modelling in mathematics teaching (Blum et al., 2003; Doerr & English, 2006; Doerr & Lesh, 2003; Niss et al., 2007; Shorr & Lesh, 2003). It is delineated that knowledge of mathematical modelling (Stacey, 2008), modelling pedagogy (Blum et al., 2003; Niss et al., 2007), pedagogical content knowledge about mathematical modelling are significant for successful and effective implementation of mathematical modelling. In addition to knowledge about and of implementation of mathematical modelling, what teachers think about mathematical modelling, and teaching and learning with mathematical modelling were found as a factor that support or hinder their adoption and implementation of mathematical modelling in mathematics teaching (Blum&Niss, 1991; Kaiser & Maass, 2007). For example, in their study, Blum and Niss (1991) revealed the main difficulties regarding teachers' implementation of mathematical modelling is related to their conceptions of mathematical modeling instruction as more open-ended and requiring more effort and qualifications than their regular instruction and including less formal mathematics. In their study, Kaiser and Maass (2007) revealed that teachers' excluding mathematical modeling in their instruction is related to what they think about their students' beliefs about mathematical modeling.

Studies on teachers' mathematical modelling practice proposed that teachers' implementation of modelling as well as what they think about mathematical modelling and teaching and learning of it are influenced by what they think about mathematics, teaching and learning of it, which are called teachers' mathematics related beliefs. For example, Ikeda (2007) explored that one of the reason that teachers did not consider the mathematical modelling as primary component of school mathematics is their perception of mathematics. Förster (2011) found that teachers' use of applications and modelling in their teaching and their effort to overcome the barriers on the use of it are all shaped by their instructional goals which are connected to teachers' mathematics related beliefs. Girmat and Eichler (2011) showed that teachers' interpretations of the written curriculum differ according to their beliefs, in such a way that because of their beliefs about elementary geometry and goals of geometry, teachers considered that the domain of geometry was not suitable for modelling. Investigating the factors affecting teachers' adoption of innovative practice with technology and mathematical modelling, Veiger (2011) found that teachers' refraining to use technology and mathematical modeling is related to their beliefs that "students should learn the basis of mathematics before engaging in technology use and mathematical modeling activities since, if students used technology first they do not fully comprehend the mathematical procedures" and "technology and mathematical modeling is not appropriate for all topics in mathematics and for all students". Moreover, Ikeda (2007) stated that teachers do not accept application and modelling as an important part of mathematics curriculum because of the beliefs that" if students know the definitions and can carry out the algorithm, they will be able to apply these" (p.462).

Doerr (2003) proposed that teachers' views of mathematics teaching and learning may have effect on their modelling practice since modelling practice requires different teacher and student roles and different teaching styles and strategies than traditional ones. In modelling process, students are expected to think, evaluate and appraise their own ideas while teachers need to provide opportunities for students to think, evaluate ideas, share and discuss (Doerr, 2003). It is pointed out that teachers face with a wide range of students thinking when using modeling, and this creates new demands for teachers such as listening students, hearing unexpected approaches, responding students' ideas, offering useful representation, making connections among mathematical ideas, providing a learning environment for students, etc. (Doerr, 2003; Doerr & English, 2006). Teachers also need to pay attention to students' existing knowledge and may support the connection between students' knowledge and mathematical ideas in the task. Moreover, they need to encourage students to show their understanding while implementing the task. All of these call for a change in classical teacher and student roles. However, teaching mathematics through modelling challenges with teachers' current views of mathematics teaching and learning (Doerr, 2003). Beside, explaining the obstacles found in different studies for teaching modelling in eight different countries, Ikeda (2007) mentioned that teachers' perceptions of mathematics have roles on their implementation of modelling in classrooms. For example, because of their perceptions of mathematics as a logical and consistent construction of thinking, or views of mathematics focusing on concepts and procedural skills, teachers either do not use modelling, or use it very rare only for concept reinforcement. As literature proposed, understanding what teachers think about mathematics, teaching and learning of mathematics, namely their mathematics related beliefs, is important to shed light on their perceptions and implementations of mathematical modelling. In the next section, the literature on beliefs is presented.

2.2 Beliefs

The construct of beliefs, specifically mathematics teachers' beliefs, have been a great interest among the mathematics educational research since 1970s (Furinghetti & Pehkonen, 2002; Kagan, 1992) as an attempt to understand the mechanism underlying teachers' instructional strategies used in mathematics classroom (Leder, Pehkonen & Törner, 2002). It was accepted that teachers hold several beliefs about the subject-matter they teach, and how to teach and learn that subject-matter (Thompson, 1992) and these beliefs impact their classroom practice (Thompson, 1992; Pajares, 1992) including their teaching preferences, interaction and communication with students (Borko & Putnam, 1996), and their interpretation of students action (Thompson, 1992). Despite its importance, belief research is difficult for researcher since there is no observable indicator for belief (Kagan, 1992; Pajares, 1992). This brings a proliferation in the characterizations and definitions of belief, thus, the study of belief becomes challenging. Although it is challenging, it is imperative to clarify the theoretical positions on belief construct, therefore the following section is devoted to description of the belief including its definitions, its relationship with the other constructs and its characterizations.

2.2.1 Definition of Belief

Despite the importance and the popularity of research on beliefs, there is no consensus on the universal definition of beliefs (Cross, 2009). Pajares (1992) argued that educational research community has been unable to adopt a specific working definition since beliefs studied in diverse fields have resulted in variety of meanings. Due to lack of consensus on the definition of beliefs, researchers have often formulated their own definition, which sometimes contradicts with the others (Frunghetti & Pehkonen, 2002). For example, Schoenfeld (1992) defines belief as "an individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior" (p.358), while Lester, Garofalo and Kroll (1989) state that "beliefs constitute the individual subjective knowledge about self, mathematics, problem solving, and the topics dealt with in problem statements" (p.47). Hart (1989), on the other hand, describes beliefs as "certain types of judgments about set of objects" (p.44). Another explanation about beliefs is proposed by Ponte (1994), who delineates beliefs as "inconvertible personal 'truths' held by everyone, deriving from experience, or from fantasy, with a strong affective and evaluative component" (p.169).

In varied definitions, several terminologies such as conceptions, beliefs, attitudes, views, perspectives, perceptions etc., are used either in the same way, or differently. For example, Thompson (1992) considers beliefs as a sub-class of conceptions. She defines teachers' conceptions "as more general mental structure, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences, and the like" (p.130). However, Thompson (1992) argues that the distinction between 'beliefs' and 'conceptions' may not vital. Lloyd and Wilson (1998), on the other hand, consider beliefs as connected with conceptions. They use the word 'conception' to refer to "a person's general mental structures that encompass knowledge, beliefs, understandings, preferences, and views" (p.249). Underhill (1988, in Frunghetti & Pehkonen, 2002, p.40) thinks that beliefs are some kind of attitudes, while Bassarear (1989) sees attitudes and beliefs on the opposite extremes of a bipolar dimension (cited in Frunghetti & Pehkonen, 2002, p.40). Törner and Grigutsch (1994), on the other hand, consider beliefs as a cognitive aspect of attitude. Rokeach (1968) called beliefs as attitudes and describes that beliefs have a cognitive component representing knowledge, an affective component capable of arousing emotion, and behavioral component activated when action is required.

It is proposed that the reason for varied understanding of beliefs is the researchers' approach about the place of beliefs in affective-cognitive domain (Frunghetti & Pehkonen, 2002; Pajares, 1992). Researchers have often envisioned the place of beliefs in affective-cognitive domain in different ways. Some researchers (for example; Thompson, 1992; Bassarear, 1989) consider beliefs as a real part of cognitive-processing while others (for example; Lester et al., 1989) acknowledge that they contain some affective elements (Frunghetti & Pehkonen, 2002). On the other hand, it is argued that researchers' use of different definitions is related to pragmatic reasons. Hekimoğlu (2004) claims that although different from each other, the belief definitions that researchers use are consistent with their research interests. For example, since Hart mainly deals with gender issues and equity in math class, Hart's (1989) definition of beliefs stresses the psychological and sociological dimensions of beliefs, whereas Ponte focuses on learning and problem solving and Ponte's (1989) definition of belief emphasizes the psychological definition of beliefs.

2.2.2 The Relationship between Knowledge and Beliefs

While characterizing beliefs, researchers often relate or differentiate between beliefs and knowledge. For example, cognitive researchers subsume beliefs as a type of knowledge (Pajares, 1992), while others who advocate that beliefs include some affective parts consider knowledge as a component of belief. According to one common approach accepted in the literature, knowledge is true and justified, whereas beliefs can be held without necessarily having a base in evidence (Richardson, 1996). Thompson (1992), on the other hand, distinguishes knowledge from beliefs in such a way that she relates knowledge with truth and certainty, while sees beliefs as more associated with doubts and disputes. In another approach, knowledge is considered as a subset of beliefs or beliefs is seen as a subset of knowledge (Murphy & Mason, 2006). In his sensible system framework, Leatham (2006) describes that "of all the things we believe, there are some things that we "just believe" and other things that we "more than believe-we know" (p.92). For those things we "more than believe" he refers to as knowledge, and for those things we "just believe" he refers to as beliefs.

Plato, who sets the agenda for the theory of knowledge, states that "knowledge is justified true belief" (as cited in Schmitt, 1992). In a similar vein, Scheffler (1965) claimed the following proposition: X knows Q if and only if (i) X believes Q, (ii)

X has right to be sure Q, (iii) Q (cited in Wilson & Cooney, 2002, p.129). This claim implies that in order to know that something is the case (which is true); there must be reasonable evidence to support the existence of this. Here, belief is a necessary but not sufficient condition for knowing. Similar to this proposition, Thompson (1992) delineates that knowledge is related to truth and certainty, while belief is more associated with doubts and disputes (Frunghetti & Pehkonen, 2002). Thompson distinguishes beliefs from knowledge by the degree of intersubjective consensus, and the type of argument needed for the acceptance of beliefs and knowledge, respectively. Correspondingly, Frunghetti & Pehkonen (2002) considers two different aspects of knowledge; objective (official) knowledge that is accepted by a community and subjective (personal) knowledge that is not necessarily subject to an outsider's evaluation.

From another point of view, Hekimoğlu (2004) proposes that although knowledge requires some form of justification, evidence, or supporting reasons, this statement does not necessarily imply that a justified true belief is knowledge, since justification might be incomplete in certain crucial respects. According to Hekimoğlu (2004), beliefs and knowledge are closely connected (for example; what beliefs one is capable of are related to/restricted to the sorts of things that one is capable of knowing, and one's knowledge consists of those beliefs that one might confidently hold). Based upon these premises, he argues that "it is useless to seek answers to the question of whether it is possible to distinguish between beliefs and knowledge in mathematics education" (p.7). Indeed, the pathway from belief to knowledge is blurred and complicated in mathematics education. In research literature, there is still no clear distinction between beliefs and knowledge (Pajares, 1992; Thompson, 1992).

Having a different point of view, Op't Eynde, De Corte and Verschaffel (2002) argue that the ultimate epistemological criteria for discriminating between beliefs and knowledge are situated not in the individual, but in the social context; what is accepted as belief in one specific situation can be considered as knowledge in a different situation. For example, a mathematics teacher in Turkey can believe that using calculator in math lesson does not develop students' computational skills, since the use of calculators is not common enough in mathematics lessons in Turkey. On the other hand, teachers in another country, who is more accustomed to using calculator in math lessons, might know that the use of calculators does not hinder students' computational skills, but facilitate the development of higher order thinking skills. Therefore, in order to decide what is knowledge or beliefs in one situation, one must fully understand the conditions (physical, social, etc.) in that situation.

Philipp (2007) also argues that determination of truth is debatable, since what is considered as true in a situation in a time can be modified in another situation at other times. Philipp (2007) considers knowledge and beliefs as conceptions and he offers the following description; "a conception is a *belief* for an individual if he or she could respect a position that is in disagreement with the conception as reasonable and intelligent, and it is *knowledge* for that individual if he or she could not respect a disagreeing position with the conception as reasonable and intelligent" (p.267). For Philipp (2007), one person's belief may be another person's knowledge.

Though the belief literature consists of different characterizations about beliefs and knowledge, some researchers do not take the distinction between belief and knowledge so strict (Frunghetti & Pehkonen, 2002). For example Thompson (1992), argue that it is not important to distinguish between knowledge and belief. Similarly, Pajares (1992) argues that "it is difficult to pinpoint where the knowledge is ended and belief began", and then he suggested that "most of the constructs were simply different words meaning the same thing" (p. 309).

Although there is no agreed unique definition of beliefs, mathematics education researchers generally prefer to use some definitions to others because of the practical reasons. Among the definitions devised by researchers in mathematics education Thompson's (1992) definitions is one of the commonly used definitions as reference in the research literature in mathematics education (Furinghetti & Pehkonen, 2002). Thompson (1992) considered beliefs as part of conceptions and defined it as “more general mental structure, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences, and the like” (p.130).

2.2.3 Sources of Beliefs

Belief theorists agree on the idea that beliefs are created early in one’s life through process of enculturation, social construction and cultural transmission (Leder, 1992; Nespor, 1987; Pajares, 1992; Raths, 2001). About the formation of beliefs, Raths (2001) highlights training, reflection on experiences and socialization process in school, while Leder (1992) stresses influence of other people (teachers, peers, parents, etc), values attached to the learning, and learning-related affective and cognitive variables.

Pajares (1992) proposes two sources for the formation of beliefs; emotion packed experiences and cultural transmission. Emotion packed experiences can be a form of a vivid memory from which a particular belief emerged (Nespor, 1987). Ambrose (2004) asserts that some prospective teachers explained their difficulty when learning some mathematical concepts (e.g, learning multiplication tables) and teachers see these experiences as related to their beliefs and consider that they are incapable of learning mathematics. It is accepted that emotional component of these experiences is one feature that differentiates beliefs from other forms of knowledge. In this respect, Goodman (1988, in Ambrose, 2004) suggested that teachers’ beliefs about teaching were derived from guiding images based on positive and negative experiences that teachers had as students.

Cultural transmission, on the other hand, indicates that beliefs may be held subconsciously, resulting from a “hidden curricula” of our everyday lives, as a form of assumptions and stereotypes (Ambrose, 2004). For example, teachers’ prior experiences as students in schools or as prospective teachers observing other teachers, consisted of mostly memorizing procedure, thus many teachers assume that mathematics always requires memorization. People are not aware of the culturally transmitted belief they hold since they never examined nor discuss them. Ambrose (2004) denoted that “these implicit beliefs may guide behavior in ways that could be characterized as habits, with individual doing things in particular ways the reasons for which they are hardly cognizant" (p.93).

2.2.4 Belief System and Belief Structure

Belief system is defined as "having represented within it, in some organized psychological but not necessarily logical form, each and every one of a person's countless beliefs about physical and social reality" (Rokeach, 1986, p.2). Green (1971) describes belief system as a system including individuals' conscious and unconscious beliefs, hypothesis and expectations and the combinations of these. One of the commonly accepted assumptions in belief research, which describes the nature of beliefs system, is the notion of "belief structure" proposed by Green (1971). Green (1971) identifies three properties of belief system; quasi-logicalness, psychological strength, and belief clusters. These three properties of belief system are also considered as the characteristics that differentiate beliefs from knowledge system (Furinghetti & Pehkonen, 2002).

Green (1971) pointed out that belief system has quasi-logical structure, which means that, some beliefs are derivative and others are primary beliefs. Primary beliefs are beliefs considered as a reason or basis of other beliefs. Beliefs that are derived from other beliefs, on the other hand, are called derivative beliefs. The quasi-logicalness of the belief system signifies that there is no logical relationship between primary and derivative beliefs, that is; the order between the primary and derivative beliefs is not fixed. Besides, it denotes that each person's belief system has its own logic. On the other hand, quasi-logicalness of belief system gives rise to holding beliefs, which are not necessarily in consensus with other beliefs (Furinghetti & Pehkonen, 2002).

Beliefs also have spatial order or psychological strength, which means that the importance of beliefs depends on the person. A belief can be more important for someone than to others, and for a person some beliefs may be more important than the other beliefs. The degree of beliefs' importance is related to their psychological centrality. In the belief system, beliefs which held with greatest psychological strength are central or core beliefs, while beliefs held with less psychological strength are peripheral beliefs. Psychologically central or core beliefs are the beliefs less likely to change, while the peripheral beliefs are susceptible to change. Furthermore, a belief being central or peripheral is not related to its being primary or derivative. According to Green (1971), "a belief may be logically derivative but psychologically central, or it may be logically primary and psychologically peripheral" (p. 46). Beliefs being amenable to change are not related to their quasi-logical status, it is related to psychological strength of the beliefs. In his description of belief system, Rokeach (1968) delineated centrality of a belief as its connectedness to the other beliefs; a belief that has more functional connections with other beliefs is more central. Similar to Green, Rokeach also proposed that centrality of beliefs indicates its strength, its importance for its holder and its predisposition to action and he suggested that the more central the beliefs are, the more resistant they are to change. It was suggested that the reason of teachers not accepting a new curriculum or making only surface changes (adopting some of new materials in his/her old style of teaching) indicated that there is a change in teacher's peripheral beliefs but no change in his/her core beliefs. Similarly, when there is a discrepancy between teacher's expressed beliefs and his/her beliefs in action, this may indicate that the core beliefs are not espoused by the teacher (Furinghetti & Pehkonen, 2002).

Lastly, Green (1971) proposed that beliefs are held in clusters, which means that they are ordered into clusters, and a cluster is somehow isolated from other clusters. Clustering property of belief system also signifies that there is no belief, which is totally independent from other beliefs. Thus, an individual may hold incompatible core or central beliefs into different clusters simultaneously without any contradiction between them. Therefore, clustering property of beliefs suggests an alternative explanation for the inconsistencies in belief system and the contradiction between espoused and enacted beliefs (or beliefs and practices) (Beswick, 2006).

2.2.5 Exploring Beliefs

Rokeach (1968) emphasized that beliefs cannot be directly observed, thus, they must be inferred from people speech, intentions and actions. However, the difficulty of assessing or exploring beliefs is widely acknowledged by the researchers (Pajares, 1992; Rokeach, 1968). Although, there exists a general agreement on the idea that beliefs are connected to planned and enacted instructional practice in the classroom (Roehring & Kruse, 2005), yet, there is sometimes a discrepancy between what teachers say about what they believe (professed/espoused belief) and what they really do (practice). Therefore, it is not possible to elicit what a teacher believes by only inferring from his/her behavior, or by only relying on what he or she say. Since teachers can follow similar practice for different reasons, beliefs cannot be inferred directly from teacher behavior (Kagan, 1992). Moreover, teachers may not be aware of their own beliefs (unconscious beliefs), and they may not possess the language to expose their beliefs, or they may not be willing to expose them publicly (Cooney, 1985; Thompson, 1984). On the other hand, Thompson (1992) suggests that inconsistencies between teachers' professed beliefs and actual practices are stem from the methodology that researchers used to measure teachers beliefs. She recommends going beyond teachers' professed beliefs and examining, at least, teachers' verbal data along with observational practice or mathematical behavior. Besides, Dawson (1999) recommends asking teachers to respond to highly focussed mathematically, and pedagogically specific situations, instead of posing theoretical and decontextualized questions. Since these contextual situations are likely occur in the mathematics classrooms teachers are (or will be) operating in, they can generate significant access to teachers' beliefs and intended practice. Additionally, Pajares (1992) argued that the study of beliefs is feasible when beliefs are well defined, and the methodology and design is chosen appropriately.

2.2.6 Categorization of Teachers' Beliefs

There is a plethora of attempts to categorize teachers' beliefs in the research literature. While some researchers developed rather comprehensive categories, others focused only on a single category of belief or set of categories (Speer, 2008). Thus, many different categorizations schemes emerged in the literature (Ernest, 1988, 1989b; Kuhs & Ball, 1986; Lerman, 1990; Prawat, 1992; Thompson, 1992). However, teachers' mathematical beliefs are generally classified under three dimensions; beliefs about nature of mathematics, beliefs about teaching mathematics, beliefs about learning mathematics (Cooney, 2003; Cross, 2009; Ernest, 1988; Thompson, 1992).

Researchers devised various categories for beliefs about nature of mathematics, teaching and learning of mathematics. Cross (2009) states that researchers (Cooney, 2003; Ernest, 1988; Lerman, 1983) studying teachers' beliefs about nature of mathematics suggest categorizations ranging from viewing mathematics as a static, procedure-driven body of formulas, to a dynamic domain of knowledge based on sense making and pattern seeking. For example, Ernest (1989b) distinguished between three views about nature of mathematics as instrumental, Platonist, and problem-solving view. Instrumentalist category for the nature of mathematics beliefs sees mathematics as a collection of rules, facts, formulas, and skills, which are useful but unrelated with each other. Subsequently, mathematics is considered as a set of unrelated but utilitarian rules and facts. Platonist category of nature of mathematics beliefs, on the other hand views mathematics as unified and static body of knowledge. Instead of being created, mathematics is considered as something to be discovered. Problem solving category of nature of mathematics beliefs sees mathematics as dynamic and continually expanding field of human creation. It is considered as an invention encompassing a process of inquiry and coming to know (Cross 2009).

Apart from Ernest's (1989b), there are also other categorizations proposed in the literature. For example, Cobb and Steffe (1983) and Dionne (1984, in Cross, 2009) offered three perspectives as traditional, formalist, and constructivist perspectives. Similarly, Törner and Grigutsch (1994) proposed three categories as system, toolbox, and process aspects. Lerman (1990) suggested two conceptions of mathematics, which reside on a continuum, from absolutist to fallibilist. Thompson

(1984), on the other hand, developed three categories for mathematics related beliefs including conceptions of mathematics as Level 0, Level 1, and Level 2.

Although named differently, much of the devised categories for the nature of mathematical beliefs are considered to share similar underlying characteristics, therefore they are aligning with each other (Andrews & Hatch, 2001; Cross, 2009; Shilling, 2010). For example, Shilling (2010) argues that despite different names, the notions in Törner and Grigutsch (1994) and Dionne (1984)'s frameworks are parallel to those in Ernest's framework. Andrews and Hatch (2001) also assert that much of the early works employing belief categorizations produced results, which would have informed Ernest's (1989b) theoretical construct. Andrews and Hatch (2001) contend that Thompson (1984)'s three conceptions of mathematics are overlapping Ernest's categories in a way that who saw mathematics as prescriptive and pre-determined (Level-0) is similar to instrumentalist view; who saw mathematics as a coherent set of inter-related concepts (Level-1) is similar to Platonist view; and who saw mathematics as something to be discovered and verified (Level-2) is similar to problem-solving view. Andrews and Hatch (2001) also argue that Dionne's (1988) three forms of mathematics are informed Ernest's categories-traditionalist (instrumentalist), formalist (Platonist) and constructivist (problem solving). Moreover, Lerman's (1990) absolutist view considered as parallel to Ernest (1989b)'s Platonist view, while the fallibilist view has similar characteristics with problem solving view.

There is also a similar correspondence between researchers' categorizations for beliefs about teaching and learning mathematics (Andrews & Hatch, 2001; Beswick, 2005; Cross, 2009). For example, based on review of the literature in mathematics education Kuhs and Ball (1986) classified teachers' views of teaching mathematics into four; learner-focused, concept-focused with an emphasis on conceptual understanding, content-focused with an emphasis on performance, classroom-focused. *Learner-focused* view denotes a "mathematics teaching that focuses on the learner's personal construction of mathematical knowledge" (p.2). This view emphasizes students' active involvement in exploring and constructing mathematical knowledge, subsequently, teachers' function as facilitators and stimulators of students learning. C*ontent-focused with an emphasis on conceptual understanding* view stands for a "mathematics teaching that is driven by the content itself but emphasizes conceptual understanding" (p.2). For this view, teachers emphasize students' understanding of the logical relationships between concept and ideas. Unlike the learning-focused model in which students' ideas and interest are primary consideration, content is organized to follow some notion of scope and sequence the teacher may have. C*ontent focused with an emphasis on performance* view signifies a "mathematics teaching that emphasizes student performance and mastery of mathematical rules and procedures" (p.2). For this view, teachers should always skillfully demonstrate, explain, and define the subject material, while students should learn the rules well and practice extensively until they master the skills needed to get correct answers. C*lassroom-focused* view denotes a "mathematics teaching based on research knowledge about effective classroom" (p.2). For this view, teachers' knowledge about effective classroom is the focus in mathematics teaching. For optimal instruction, teachers should keep their lessons well-structured, material thoroughly explained, homework carefully assigned, students' work closely monitored, and classes free of discipline problems. The students' roles are to listen attentively to the teacher and to cooperate by following directions, answering questions, and completing tasks assigned by the teachers.

Ernest (1989b), on the other hand proposed six models for the beliefs about teaching mathematics; investigational model, problem-posing and solving model, the conceptual understanding enriched with problem solving model, the conceptual understanding model, the mastery of skills and facts model, the day-to-day survival model. These models are categorized under three views of mathematics teaching; instructor model, teachers as explainer, and teacher as facilitator. Parallel to these, Thompson's (1984) three categories for mathematics related beliefs also include three conceptions of mathematics teaching as Level 0, Level 1, and Level 2. It was suggested that Thompson's (1984) three level for the conceptions of mathematics teaching and Kuhs and Ball's (1986) three of four models of mathematics teaching (learner-focused, concept-focused with an emphasis on conceptual understanding, content-focused with an emphasis on performance, classroom-focused) are aligning to Ernest's (1989a) three categories of teachers' views of mathematics teaching "instructor model", "teachers as explainer", and "teacher as facilitator", while fourth category -classroom-focused- in Kuhs and Ball's (1986) categorization considers the content covered is beyond control of the teacher who only present material in a way that found to be effective by process-product research studies (Andrews & Hatch, 2001; Cross, 2009, Van Zoest, Jones, & Thornton, 1994).

For the beliefs about mathematics learning, on the other hand, Ernest (1989b) proposed six models; child's exploration and autonomous pursuit of own interest model, child's constructed understanding and interest driven model, child's constructed understanding driven model, child's mastery of skills model, child's linear progress through circular scheme model, child's complaint behavior model. These models are categorized under three views of mathematics learning "learning as active construction of understanding", "learning as reception of knowledge with unified knowledge", and "skill mastery with correct performance". Ernest's (1989b) models of learning mathematics are based on following constructs; "A view of learning as the active construction of knowledge as a meaningful connected whole, versus a view of learning mathematics as passive reception of knowledge; The development of autonomy and the child's own interests in mathematics versus a view of learner as submissive and compliant" (p.23). Thompson's (1984) three categories for mathematics related beliefs also include three conceptions of mathematics learning as Level 0, Level 1, and Level 2. It was suggested that Thompson's (1984) three level for the conceptions of mathematics teaching are aligning to Ernest's (1989b) three categories of teachers' views of mathematics learning (Andrews & Hatch, 2001; Cross, 2009, Van Zoest, Jones, & Thornton, 1994).

Among the frameworks/categorizations for teachers' mathematics related beliefs, Ernest (1989b)'s categorization on teachers' beliefs about nature of mathematics is considered as a seminal work and it is one of the widely adopted and used frameworks (Beswick, 2012). Also, Kuhs and Ball's framework for belief about mathematics teaching is utilized commonly since it gives detailed explication about teachers' teaching of mathematics. In the research literature, Van Zoest et al. (1994) combined Kuhs and Ball (1986) and Ernest (1989b) framework to investigate teachers' beliefs about nature of mathematics and mathematics teaching. Later several other researchers used this combination to investigate teachers' beliefs. Beswick (2005, 2012), on the other hand, added Ernest (1989b)'s views about mathematics learning to Van Zoest et al. (1994)'s combined framework and proposed Table 2.2 to indicate the relationship between belief categories (Beswick, 2005, p.40). Beswick denoted that "beliefs on the same row are regarded as theoretically consistent with one another, and those in the same column have been regarded by some researchers as a continuum (Anderson & Piazza, 1996; Perry et al., 1999; Van Zoest et al., 1994)" (p.40).

Table 2.2 Combination of Ernest's and Kuhs & Ball's frameworks

Nature of mathematics (Ernest, 1989b)	**Mathematics teaching** (Kuhs & Ball, 1986)	**Mathematics learning** (Ernest, 1989b)
Instrumentalist	Content-focused with emphasis on performance	Skills mastery with passive reception of knowledge
Platonist	Content-focused with emphasis on conceptual understanding	Conceptual understanding with unified knowledge
Problem-solving	Learner-focused	Autonomous exploration of own interest

Researchers proposed that what teachers think about mathematics and mathematical knowledge is closely associated with what they think about mathematics teaching and learning (Hofer & Pintrich, in Andrews & Hatch, 2000, p.37). Similarly, Ernest (1989b) suggested that "importance of the teacher's mental model of mathematics teaching is that it is the key determinant of how mathematics is taught" and "is likely to be closely related to and influenced by the teacher's conception of the nature of mathematics" (p.22–23). In their study

Andrews and Hatch (1999) also found that what teachers believes about mathematics teaching informed by what they believe about mathematics.

2.3 Relationship between Beliefs and Practice

Despite variety in focus, numerous researches focused on how teachers' beliefs and their instructional practices are related. Research studies showed that the value that some teachers placed on particular course content influenced the way they teach (Ernest, 1988; Nespor, 1987). Ball (1993) acknowledged that despite having similar mathematical knowledge, teachers might teach very differently depending on their views of the teaching and learning of mathematics (their beliefs about what constitutes effective mathematics instruction). Later, researchers noticed that beliefs do not only have a direct effect on teachers practice, but it also plays a mediating role between knowledge and practice (Wilkins, 2008). For example, Brown and Cooney (1992) found that teachers' disposition to teach in a specific way or their use/not use knowledge learned from different experiences is affected by their beliefs about mathematics. Aside from these, researches also revealed that beliefs act as a filter through which teachers screen and reorganize their new knowledge and experiences (Kagan, 1992; Pajares, 1992).

Research on teachers' beliefs indicated controversial relationship between teachers' beliefs and practices. A branch of research has been based on the idea that there is a linear relationship between teachers' beliefs and their practice and teachers' practice follows from their beliefs (Beswick 2005; Cooney, 2001; Ernest, 1989a; Pajares, 1992). On the contrary, Guskey (1986) argued that beliefs are the results of the change in teachers' classroom practice (in Cobb, Wood & Yackel, 1990). Cobb et al., (1990), on the other hand, suggested that the relationship between beliefs and practice is not unidirectional or linear, but rather, they are dialectically affecting each other and developing together. The other branch of research on teachers' beliefs focused on the consistency or inconsistency between the beliefs and practice. While some researchers reported consistencies between teachers' beliefs and their practices (Stipek et al, 2001; Thomson, 1984), others focused on the inconsistency between beliefs and practice (Cooney, 1985; Shiled, 1999 in Beswick, 2005).

Several other researchers investigating teachers' beliefs and practices focused on how context impacts beliefs and practices (Ajzen & Fishbein, 1980; Green, 1971; Hoyles, 1992; Pajares, 1992; Sullivan & Mousley, 2001). For example, Sullivan & Mousley (2001) considered the context as constraints for the enactment of the beliefs. Hoyles (1992), on the other hand, proposed that beliefs were consequences of experiences, which depended on contexts, therefore situated. Hoyles (1992) described that different contexts elicit different beliefs; therefore, she did not distinguish between espoused and enacted beliefs. Parallel to this, Leatham (2006) offered the idea of sensible system. Leatham (2006) stated that teachers are sensible instead of inconsistent beings, and sensible system does not allow contradiction. For Leatham (2006) clustering nature of belief make possible to adjust beliefs depending on contexts, so that a teacher may believe one thing in one situation and the opposite in another. Leatham (2006) claimed that inconsistencies occur because either researcher's interpretations of teachers' beliefs or teachers' abilities of articulate their beliefs are problematic.

While explaining the inconsistencies between teachers' beliefs and practice, some researchers utilized the Green's (1971) description of belief system, which signifies contextual nature of beliefs. According to the idea of belief system, beliefs are held in clusters rather than as isolated entities. Green (1971) proposed that beliefs in different clusters could develop in different context, so that there could be inconsistencies between beliefs in different clusters, which may be unnoticed. Sometimes, researchers considered context as account for the inconsistencies between beliefs and practice. For example, Beswick (2003) argued that context in which beliefs are evaluated (such as survey items or questions) could not be adequately corresponding to teachers' practice being considered (in Beswick, 2005, p.42), therefore, contexts in which articulated beliefs can be different from the observed. Moreover, Speer (2005) argued that the inconsistency between beliefs and practice are because of methodology used to infer beliefs from teachers' actions or words.

2.4 Belief Change for the Educational Reform

Fullan (2007) points out that an innovation is multidimensional rather than being unidimensional. He argues that teachers must understand the philosophy of reform to change their current educational practice. Fullan (2001) proposes three key dimensions in the implementation of any new educational practice or program; 1) use of new or revised materials, 2) use of new teaching approaches, and 3) a change in beliefs. Therefore, in order for achieving an innovation, all three dimensions should be met. Fullan (2007) claims that "changes in beliefs and understanding (first principles) are foundation of achieving lasting reform (p.37), because changes in beliefs are closely related to "the skills and material changes in actual practice along the three dimensions in materials, teaching approaches, and beliefs, in what people do and think- are essential if the intended outcome is to be achieved" (p.37).

As theory implies, in order to change mathematics instruction toward more reform oriented practices teachers need to possess beliefs about mathematics, mathematics teaching and learning which is significantly different from school mathematics tradition (Lloyd, 2001; Thompson, 1992). However, most of the teachers are not familiar with the reform; most of them never experienced reform as either students or as teachers. This lack of personal familiarity with innovation creates obstacle for teacher accept the innovation. For example, constructivist theories of learning mathematics propose that mathematics is learned through an active and social process of construction (Cobb, 1995; von Glasersfeld, 1984), and many contemporary teaching practices based on this theory. Contrary to contemporary teaching practices, many teachers experience of learning mathematics includes memorizing rules. On the other hand, they are expected to consider mathematical understanding as the capacity to use mathematics to reason, to communicate, and to pose and solve meaningful problems (NCTM, 1991, 2000). Researchers proposed that since teachers' beliefs and practices are strongly tied to school tradition, seeking ways to facilitate teachers to make change in their beliefs is critical for the success of reform in mathematics education (Lloyd, 2002).

Although change of beliefs is important for the success of any reform, belief change is not a simple process. Teachers do not change their beliefs automatically when they are being asked to implement or read the the research recomentation (Stipek et al., 2001). Researchers suggested number of ways to facilitate change of beliefs. For example, it was indicated that process of changing teachers' beliefs is better to include classroom implementations. Moreover, reflections on teachers' own classroom experiences have been found effective for changing beliefs. Stipek et al. (2001) argued that "for meaningful and lasting change to occur, teachers need to engage in practical inquiry to move back and forth among a variety of settings to learn about new instructional strategies, to try them out in their own classrooms and to reflect on what they observe in a collaborative setting" (p.224-225). Similarly, Hart (2002) denoted that beliefs are challenged through the process of reflection and shared conversation. Therefore, teachers' classroom practice and process of reflection and shared conversations are important and should take into into account while changing beliefs.

2.5 Teachers' Professional Development

Developing teachers' knowledge and skills about students' mathematical understanding is concern for many professional development programs. Researchers claim that this development creates an opportunity to shift teachers' beliefs about mathematics learning and teaching (Lloyd, 2002). Ball (1993) describes this as bifocal perspective, which denotes "perceiving mathematics through the mind of learner while perceiving the mind of learner through the mathematics" (p.159). Cognitively Guided Instruction (CGI) is a professional development project based on the idea that "teacher development involves a fundamental change in content and organization of teachers' knowledge about children's mathematical thought" (Fennema et al., 1996, in Lloyd, 2002, p.151).

Lloyd (2002) considered teachers' own classrooms as rich context for educative experiences. He denoted that as teacher implement curriculum materials in their classrooms, they may develop new mathematical and pedagogical beliefs and skills through designing instructions and interactions with their students and through use of technology, and so on. Also, Ball (1993) denoted that teachers continually develop new knowledge from their own experience as they engage with students and materials and content they are teaching. Teachers' learning from their experiences is considered to increase their reform vision, since their existing beliefs may be challenged through the process of engaging reform practice. Moreover, researchers proposed that use of/implementing innovative curriculum materials provides useful context for teachers to learn about themselves, their students, mathematics and teaching and learning mathematics (Ball & Cohen, 1996; Lloyd, 1996; in Lloyd, 2002, p.153), as they learn to teach with new materials.

Ball (2002) also indicated that teachers' engagement with the reform curriculum (and curriculum materials) as learners provides them an opportunity to reflect about challenging mathematics, nature of mathematical activity, and to reflect on the process of learning and teaching mathematics. Lloyd (2002) denoted that professional development based on curriculum could influence teachers' beliefs about mathematics, and learning and teaching of mathematics. Teachers require support for both implementation and adaptation of reform-curriculum (Lloyd, 2002) Teachers experiencing reform practices provide an opportunity for them to recognize the difference between traditional and reform curricula and to realize multiple approaches to mathematical subject matter (content) and mathematics pedagogy. As they value the difference, therefore, they may develop reform oriented beliefs.

2.6 Research about Teachers' Mathematics Related Beliefs and Belief Changes

Researchers in mathematics education suggest that teachers' conceptions about what mathematics is and what it means to teach mathematics have direct impact on how they teach mathematics and the way they teach mathematics (Ernest, 1989a; Thompson, 1992). Thompson (1992) pointed out that most of the mathematics teachers in America consider mathematical knowledge as static, and as a set of rules and procedures which produces one correct answer; knowing mathematics as being able to perform procedures without understanding the underlying meaning. Correspondingly, it was revealed that teacher holding this type of beliefs reported to follow a teaching involve step-by-step instruction of procedures followed by students being asked problems for practicing the procedures (Thompson, 1994; Wood, Cobb, & Yackel, 1991). On the other hand, Peterson, Fennema, Carpenter, & Loef (1989) explored that teachers who believe that students learn mathematics by constructing their own understanding through solving problems used more world problems in their teaching and separate more time to develop student's strategies before teaching related facts contrary to teachers believing that mathematics is learned by receiving knowledge of operation.

Additionally, a significant number of studies investigating the teachers' mathematical beliefs conducted with pre-service mathematics teachers rather than with in-service ones. Moreover, the number of studies conducted with secondary school mathematics teachers constitutes a much small portion of the studies conducted with in-service teachers.

Andrews and Hatch (1997, 1999a) conducted a quantitative study to investigate almost six hundred English teachers of secondary mathematics' beliefs about nature of mathematics and mathematics teaching. To explore teachers' beliefs, researchers used survey, and, factor analysis of the data obtained from survey showed that teachers' beliefs about mathematics and mathematics teaching were in varying proportions according to the individual's underlying disposition, of both fallibilist and absolutist influences. Andrews and Hatch (1999a) found that what teachers' beliefs about mathematics teaching informed by what they believe about mathematics.

Vacc and Bright (1999) examined change in 34 pre-service teachers' beliefs about teaching and learning mathematics through an instruction based on Cognitively Guided Instruction (CGI) as part of mathematics method course. Teachers' beliefs were evaluated through CGI Belief Scale four times through the program. Results showed significant changes in teachers' beliefs about mathematics instruction and students teaching to a more constructivist orientation.

Beswick (2005) investigated 25 secondary school mathematics teachers' beliefs using Ernest's categorization about nature of mathematics and mathematics learning and Van Zoest et al. (1994)'s categorization of beliefs about mathematics teaching. To examine teachers' beliefs, Beswick (2005) used a belief survey, including 26 items asking the extent of agreement on five-point Likert scale, and she found that "whereas many of the teachers appeared to hold beliefs consistent with Ernest's (1989b) Problem-solving view, a considerable number also held more traditional beliefs, although it would appear that very few had beliefs that could readily be classified as Instrumentalist" (p.52).

As a continuation study, Beswick (2007) investigated eight teachers for more detailed investigation and conducted semi-structured interviews about mathematics, mathematics teaching and learning with these teachers to obtain an authentic understanding of their beliefs and a series of classroom observation by focusing on the instances of the features of classroom environments as consistent with constructivist principles. Beswick's study focused on the two teachers (Jim and Andrew) those who were perceived by students as consistent with constructivist principles and observations of their classes supported this conclusion. Beswick aimed to identify the central beliefs underlying teachers' classroom practices and she found nine beliefs evident from the data. These nine beliefs were related to nature of mathematics, mathematics learning and role of the teacher. Beswick pointed out that Andrew's actions were primarily driven by the beliefs to role of teachers whereas Jim's beliefs were related to nature of mathematics and mathematics learning. Beswick's findings highlight the importance of beliefs that teachers hold about the nature of mathematics and about mathematics teaching and learning on the classroom environments that teacher create.

Van Zoest et al. (1994) conducted an experimental study and compared the beliefs about mathematics teaching held by the pre-service teachers involved in an intervention program based on socio-constructivist approach to mathematics instruction (small-group teaching experiences supported by on-going seminars and written reflections on children's thinking) with a group of their peers who did not participated in the intervention. Researchers used a belief survey to investigate pre-service teachers' beliefs and they found that at the end of intervention program most pre-service teachers started to consider children building their own knowledge through social interaction important. However, with respect to pre-service teachers who did not involve in the program, those teachers who involved in the intervention program develop significantly stronger beliefs about teachers as encourager of mathematical thinking and the child's active and personal role in learning. Van Zoest et al. (1994) argued that the strength of the beliefs developed was influenced by the intensity of intervention (experienced with children), supporting atmosphere provided in the project and opportunity provided for preservice teachers to reflect on what they are doing.

Raymond (1997) conducted a multiple case study to investigate 6 beginning elementary mathematics teachers' beliefs about nature of, teaching and learning mathematics and the level of consistency between their beliefs and practice. Teachers' beliefs were investigated through interviews and a belief survey. At the end of analysis of data gathered from beliefs surveys, interviews, observations, and teachers written documents, Raymond focused on the beliefs of one teacher who had traditional beliefs about mathematics (mathematics is fixed, predictable, absolute, certain and applicable and it is an unrelated collection of facts, rules and skills), however, had non-traditional beliefs about teaching (teachers should provide students activities, manipulatives and different views) and learning (students should learn mathematics by discovery, reasoning and group working). It was stated that, although teacher's beliefs about teaching and learning mathematics were student-centered, her practice was found as teacher-centered and traditional. However, her practice is more closely related to her beliefs about mathematics as content (nature of mathematics beliefs). Raymond (1997) reported that time concerns, classroom management concerns, lack of resources, use of testing, and students' behaviors are considered by the teacher as possible reasons for the inconsistencies between beliefs and practice. Raymond concluded that contextual factors effects teachers practice even if they held non-traditional beliefs about teaching and learning.

As part of a larger study (Beswick, 2005, 2007), Beswick (2012) aimed to reveal two (Sally and Jennifer) secondary school mathematics teachers' beliefs about nature of mathematics (beliefs about mathematics as a discipline and as a school subject) that affected their mathematics teaching. Teachers' beliefs were explored using survey and interview data and observation data was used to investigate their practice. Results showed that Sally, an experienced teacher, hold problem solving beliefs about school mathematics, but hold Platonist view of mathematics as discipline (see school mathematics and mathematics as discipline as something separate from each other). About the beliefs about teaching and learning mathematics, Sally held learner-focused beliefs of mathematics teaching and autonomous exploration of own interest orientation to mathematics learning. Jennifer, less experienced teacher, on the other hand, did not hold a single category of nature of mathematics beliefs; even some of her beliefs are contradictory. However, she held student-centered beliefs about teaching mathematics. However, it was reported that her beliefs about mathematics as a discipline evolve from largely Platonic orientation to a problem solving view.

As a part of a larger research project, Cross (2009) conducted a case study, through a professional development (PD) based on mathematical argumentation and writing, to explore high school teachers' professed mathematics related beliefs, the alignment between these beliefs and teachers instructional practice, and how these beliefs support or hinder teachers' incorporation of reform oriented classroom materials and instructional strategies. As a part of project, teachers were provided PD to help them incorporate writing and discourse tasks in their instructional activities and to develop ways to facilitate students engagements in those activities. Among teachers participated in the PD, Cross focused on five teachers and teachers' views about mathematics as a discipline, mathematics pedagogy, and students learning were explored through a semi-structured interview. Teachers were also observed and discussed about their specific actions and pedagogical decisions aroused in the observations. Qualitative data analysis revealed "in general, beliefs were very influential on the teachers' daily pedagogical decisions and that their beliefs about the nature of mathematics served as a primary source of their beliefs about pedagogy and student learning" (Cross, 2009, abstract). Specifically, Cross (2009) reported that three of the teachers possessed nature of mathematics beliefs 'mathematics is computation' and 'the goal of mathematical problem is obtaining the correct answer'. Parallel to this, during their observed practice these teachers did not engage in any group discussions or organized collaborative activity and they lectured during the lesson which was based on teacher–student interaction followed an initiate–respond–evaluate pattern. Moreover, these teachers expressed their roles as knowledge giver (providing students' mathematical knowledge, and ensuring students to store knowledge provided by repeated practice and memorization), learning as applying the correct procedure in the right context with accurate computation. Cross (2009) proposed that these teachers' beliefs about teaching and learning appeared to be derived from their beliefs about nature of mathematics. On the other hand, other two teacher's nature of mathematics beliefs included mathematics as thought processes and mental actions of individuals. However, though two teachers' nature of mathematical beliefs was aligned, how their beliefs were manifested was different. For example, one of the teachers (Mr. Simpson) adjusted his teaching to elicit students' thinking process regardless of type of students and content and focused on process rather than product while solving problem, while the other teacher (Ms. Jones) adjusted her instruction depending on the subject for which she sometimes used teacher-centered practice (for example in algebra), but other times (in geometry) she tried to elicit students thinking with group working, probing questions, or by asking students for explanations. Cross (2009) found that difference in Mr. Jones instructions depending on domains of content was resulted from her conflicting beliefs about nature of mathematics, students learning and mathematics teaching (she believed that nature of algebra and geometry was different and underachieving students learn best through direct instruction). Moreover, Cross (2009) suggested that by the end of the project teachers practice did not change dramatically because they filtered new practice through old belief system. It was also found that, 4 of the 5 teachers only began to question effectiveness of their practice, and they reported that they were not confident about using alternative methods of designing and orchestrating instruction proposed by the project because of curricular and institutional constraints. Based on the result of the study, Cross (2009) suggested that if the beliefs about the nature of mathematics change, then derivative beliefs (for which Cross claimed that teaching and learning beliefs are derivative of nature of mathematics beliefs) would begin to change, therefore the process of belief change should focus on teachers views/beliefs of mathematics.

Stipek et al. (2001) also found that five dimensions of beliefs (that were obtained from the factor analysis of the survey items) are strongly associated with each other; “(1) mathematics is a set of operations to be learned; (2) students' goal is to get correct solutions; (3) the teacher needs to exercise complete control over mathematics activities; (4) mathematics ability is "fixed and stable; and (5) extrinsic rewards and grades are effective strategies for motivating students to engage in mathematics” (p.222).

Hart (2002) conducted a follow-up research study of teachers’ beliefs after participating in a teacher enhancement project to explore the beliefs that teachers hold about their change and about factors that they thought to have most influence on them. Hart found that three factors (collaboration, colleagues in the project, and modelling of thinking and behaviors advocated) were believed to have effect on their change process. Hart (2002) argued that collaboration and the supporting context in which teachers work with colleagues is critical to teacher change. Teachers participating project expressed the belief that ideas and strategies proposed in the program, observation of colleagues while teaching, debriefing and planning together with colleagues is important for them and the reflection made contributed their change process.

In order to develop effective teacher education programs, it is important not only identify the presence of change but also teachers beliefs about their change (Hart, 2002). Pehkonen and Toerner (1999) regarded these beliefs as indicator and estimator or teachers’ experiences and important for designing of future professional development programs.

Stipek et al. (2001) said that “Several researchers have suggested that professional development programs designed to help teachers implement inquiry-oriented mathematics instruction are minimally effective, in part because teachers "filter what they learn through their existing beliefs.” (214). Cohen and Ball (1990), for example, observed in their study that teachers assimilated new practices to their more traditional beliefs about mathematics education. In their words, “New wine was poured, but only into old bottles” (p. 334), (Schram & Wilcox, 1988; Skemp, 1978).

CHAPTER 3

METHODOLOGY

In this chapter, the methodology of the study is presented. Along with the chapter; design, context and participants of the study, data collection and analysis procedures, the issues of validity and trustworthiness, researcher's roles, limitations and delimitations of the study are elaborated.

3.1 Research Questions

The purpose of this study is to investigate mathematical related beliefs of in-service secondary mathematics teachers and to explore the changes in their beliefs after teachers have participated in a one-year professional development program on mathematical modelling. The research questions guided this study are;

1. What are the secondary mathematics teachers' beliefs about nature of mathematics, and teaching and learning mathematics?

2. How did teachers' beliefs about nature of mathematics, learning and teaching mathematics change after participating in a one year professional development program on mathematical modelling?

3. What do teachers think to be the influence of the professional development program on mathematical modeling on their beliefs about nature of mathematics, and teaching and learning mathematics?

A qualitative approach was used to answer research questions of this study. In research studies, qualitative approaches are used for study of issues in depth and detail (Patton, 1990 p.14). Maxwell (1996) summarized that qualitative approaches can be used for “1) understanding the *meaning*, for the participant in the study, of the events, situation, and actions they are involved with and or the accounts that they give of their lives and experiences; 2) understanding the particular *context* within which the participants act, and the influence that this context has on their actions; 3) identifying *unanticipated* phenomena and influences, and generating new grounded theories about the latter; 4) understanding the *process* by which events and actions take place; 5) developing *causal explanations*” (p.17-19). One of the important features of qualitative research is that “qualitative researchers study things in their natural settings, attempting to make sense of, or interpret phenomena in terms of meaning people bring to them” (Denzin & Lincoln, 2005, p.3). Different from quantitative studies focusing on the measurement and causal relationships between variables, qualitative studies focus on the process and try to reveal the nature of reality concerning how the social experiences are created and made sense of (Denzin & Lincoln, 2005). Qualitative studies rely on the post-positivist perspective. Contrary to positivist tradition considering reality out there, which has to be studied, captured and understood, post-positivist perspective debate that reality can never be fully captured; it can be only approximated (Guba, 1990, cited in Denzin & Lincoln, 2005, p.11). Therefore qualitative studies use multiple ways of approximating the reality as possible.

This study is designed to explore teachers’ beliefs and process of belief change. Along with the principles proposed by Maxwell (1996) the primary focus of this study is to understand teachers’ beliefs and identifying the influence of the context (PDP) on the process of belief change.

3.2 Design of the Study

Research design denotes the set of guidelines, which address how the researcher connects the theoretical paradigms to strategies of inquiry and methods for collecting empirical materials (Denzin & Lincoln, 2005). There are different types of designs in qualitative research, such as, case study, ethnography, grounded theory, and phenomenology. Eliciting, understanding and constructing meaning is the common goal of these designs. This study used case study as a research design.

In case studies, researchers aim to seek a case or multiple cases. The case(s) can be a program, an event, an activity, a process or one or more individuals. It is either single case or multiple cases; a case study can be conducted to explain (the link between a real life intervention and intervention's results), to describe (the intervention and the context), to illustrate (certain topics within evaluation), to explore (the situation when outcomes of intervention is not clear), to do a meta-evaluation (Yin, 2002). Stake (1995, cited in Creswell, 2003, p.15) described that "the cases are bounded by time and activity, and researchers collect detailed information using variety of data collection procedures over a sustained period of time". Maxwell (1996) denoted that generalization might be taken into account while the selection of cases; however, the main concern is to develop "an adequate description, interpretation and theory of this case" (p.55). Schramm (1971, cited in Yin, 2002) described that "the essence of a case study, the central tendency among all types of case study, is that it tries to illuminate a decision or set of decisions; why they were taken, and how they were implemented, and with what result" (p.12). Yin (2002) also denoted that, in case study research, the phenomenon is investigated in real life context, especially when the phenomenon and the context are intertwined. Because of this grift situation, researchers use multiple data sources to enlighten the phenomenon and can be benefitted from the theoretical propositions in the literature while orienting data collection and data analysis.

Using case study as the research design, this study aimed to *describe* teachers' beliefs and *explore* the changes in teachers' beliefs within a professional development program, which can be considered as an intervention. The unit of analysis, which defines the case, of this study is the group of teachers attending the professional development program on mathematical modelling.

3.3 Context and Participants of the Study

This study was the part of a larger research project, supported by the Scientific and Technological Research Council of Turkey (TUBITAK) (grand no 110K250), aiming to develop pre-service and in-service teachers' knowledge and skills about mathematical modelling and use of mathematical modelling problems in their teaching. The research project was longitudinal, took three years. The project has three main purposes; (i) to develop well-designed mathematical modelling tasks, activities which are incompatible with the objectives of secondary school mathematics curriculum and can be used for pre-service teacher training and for secondary school mathematics lessons; (ii) to develop an in-service teacher training program (in-service professional development program) framed by mathematical modelling perspective and to explore how the program would affect teachers' knowledge, beliefs and practice; (iii) to design a graduate course on mathematical modelling, for pre-service mathematics teachers and to investigate how this course effect pre-service teachers' knowledge, competencies and attitudes regarding mathematical modelling and use of modeling in mathematics teaching. This study was related to the second purpose of the research project, specifically focused on the beliefs of in-service teachers and how professional development program effect teachers' beliefs.

Participants

This study involved a group of ten in-service secondary mathematics teachers who participated in the PDP on mathematical modelling. These teachers were teaching in two different high schools located in Çankaya district of Ankara. The first school is an Anatolian High School where five (3 male and 2 female) participant teachers were teaching in. The second is Anatolian Teacher High School where the remaining five participant teachers (1 male and 4 female) were teaching in. The numbers of years for teaching experiences of participating teachers are between 13 and 26. Neither of participants had a previous experience of attending a workshop or PDP on mathematical modelling. Teachers' demographic information is presented in Table 3.1 (Pseudonyms are used for participants' names).

Table 3.1 Participants' demographic information

Participant	Gender	Education Degrees	School type working in	Teaching Experience (Years)	# of years of working in this school	Grades of teaching in last 3 years	Any experience of workshop or PD on modelling	Use of technology
Sude	Female	Math Ed.	ATTHS*	24	13	9-to-12	None	None
Meltem	Female	Math	ATTHS	23	16	9-to-12	None	None
Kerim	Male	Math Law	ATTHS	26	11	9-to-12	None	None
Kadri	Male	Math Ed.	ATTHS	22	2	9th,10th	None	None
Mert	Male	Math	ATTHS	24	10	9-to-12	None	None
Ayla	Female	Math Ed.	AHS**	19	6	10th,11th	None	None
Hazal	Female	Math	AHS	16	9	9-to-12	None	None
Rengin	Female	Math	AHS	13	10	9th, 10th	None	None
Filiz	Female	Math	AHS	16	10	9-to-12	None	Projector
Alp	Male	Math Ed.	AHS	15	6	9-to-12 mostly geometry	None	Projector

*TTHS: Teacher Training High School
**AHS: Anatolian High School

3.4 Professional Development Program

The professional development program (PDP) which was designed in the scope of the research project includes both development and evaluation purposes. By means of application of PDP, it was aimed to develop in-service teachers' knowledge, competencies and attitudes about use of model eliciting activities (MEAs) in mathematics teaching and to evaluate the effectiveness of it in order to determine the dimensions that an effective PDP should have.

The PDP was based on the models and modelling perspective (Lesh & Doerr, 2003). PDP aimed to develop teachers' knowledge, skills and attitude about use of MEAs in mathematics teachers. For this aim, PDP focused on instructing and informing participant teachers about nature of MEAs and how to use MEAs for teaching mathematics and gaining experience of implementing MEAs in their teaching. For this purpose, PDP was started with a four-day workshop and continued nine months as cycles of participant teachers' implementing different MEAs in their classes.

Selection of Participants for the PDP

Participants of PDP were determined based on purposive sampling by the research project team considering the school types that teachers were teaching in. Three schools (two Anatolian Teacher High Schools and one Anatolian High School) were selected and administrators of these schools were informed about the PDP and they were asked if the mathematics teachers in their school would be interested in and volunteer to participate in the study. A total of fifteen teachers volunteered to participate in the study. None of these teachers had an experience of participating in a PDP or workshop on mathematical modelling. These teachers attended in a four-day workshop conducted in September 2011. Among the teachers attended to the workshop, 10 teachers wanted to continue participating in the PDP. 5 of these teachers (2 female and 3 male) were teaching in an Anatolian Teacher High School, and the remaining 5 of them (4 female and 1 male) were teaching in an Anatolian High School.

3.5 Implementation of PDP

PDP started with a four-day workshop conducted in September 2011. After the workshop, a school based training period began and it took nearly nine months throughout the first and second semester of 2011-2012 academic-years. The sequence of activities and events conducted throughout the PDP process is presented in Table 3.2.

Table 3.2 Sequence of activities and events conducted throughout the PDP

Time	Activity			Name of MEA
Sept-2011	Workshop (4day)			
October2011 to January 2012	School base training	1st MEA	Preliminary meeting	Bank Robbery
			Implementation of 1st MEA	
			Follow up meeting	
		2nd MEA	Preliminary meeting	Street Parking
			Implementation of 2nd MEA	
			Follow up meeting	
		3rd MEA	Preliminary meeting	The Summer Jobs
			Implementation of 3rd MEA	
			Follow up meeting	
		4th MEA	Preliminary meeting	Water Tank
			Implementation of 4th MEA	
			Follow up meeting	
February 2012		Mid-year meeting		
March 2011 to May 2012		5th MEA	Preliminary meeting	Pack Them in!
			Implementation of 5th MEA	
			Follow up meeting	
		6th MEA	Preliminary meeting	Magazine Sale
			Implementation of 6th MEA	
			Follow up meeting	
		7th MEA	Preliminary meeting	Bouncing Balls
			Implementation of 7th MEA	
			Follow up meeting	
June 2012	End of year meeting			

3.5.1 Workshop

A workshop was conducted by the research team 1) to understand teachers' current conceptions about modeling and their expectations from PDP, and 2) to introduce the goal of professional development program, 3) to provide them preliminary knowledge and information about the nature of MEAs and use of modelling in mathematics teaching, and 4) to have them review their content and pedagogical content knowledge by means of working on MEAs. Workshop was started in September 2011 and it took four days.

First day of the workshop began with a one-to-one interview with each teacher, which was conducted to understand his or her preliminary knowledge and beliefs about mathematical modelling. Then, teachers were informed about the goal of the PDP and were asked to participate in a concept mapping activity. In this activity, the aim was teachers developing an idea about mathematical modelling. At first, teachers were informed about how to do a concept map, then, they were provided concepts related with mathematical modelling and asked to connect those concepts with suitable words. After concept mapping activity, a discussion was held with teachers on the place of MEAs in mathematics teaching, and about when and how to use MEAs in mathematics teaching. By this discussion, it was aimed that teachers construct a view related to the importance of studies conducted about mathematical modeling for both their students and themselves. In the first day, teachers were also provided five different MEAs/modelling questions; they were asked to investigate these questions considering the quality of questions, mathematical concepts in the questions, and the process of solving these questions. Teachers expressed their opinions about mathematical concepts in the MEAs, nature of modelling questions, difference between modelling questions and other types of mathematical questions, use of mathematical modelling in mathematics teaching. After this period, teachers worked in groups on one of these questions as a group of 3-4 people (as if they were students). In this activity, research team demonstrated how to use a graphic calculator, so that, teachers used calculators while solving the modelling question. Then, groups presented how they had solved the question and discussed with each other about their solutions.

In the second day, teachers were presented an example of student’s solution for the modeling question that they solved in previous day (summer-job problem). They were asked to investigate this solution and compare it with their solutions considering the assumptions in the solution. In this day, research team made a presentation about how to use technology in mathematics education, particularly, kinds of technological tools and software that can be used in the process of engaging mathematical modelling and how to use these tools was introduced to teachers. After that, teachers were asked to make groups and engage in solving a modelling problem (MEA) (bouncing balls problem) as if they were students. In this period, one member of the research team introduced the modelling problem and acted as a teacher who was applying the mathematical modelling in his class. Groups read and thought about the modeling problem, the teacher walked through the class and observed the groups, asked them about how they thought on problem. Groups also asked some questions to the teacher about the solution process, where the teacher guided them to discuss within the group or with the whole class. After groups finished their works, they were asked to made posters showing their solution process. Then, they presented their works one by one and discussed on other groups' solutions. After this activity, teachers were asked to evaluate what they have been through engaging in solution of modelling problem (how they formed model, where they had difficulties, how to handle with the difficulties etc.), both from teacher and student aspects.

In the second day, teachers were also introduced the notion of "*ways of students' thinking*" and the document of "*student thinking sheet*" was presented to teachers. Teachers were asked that how would they evaluate students' solution to a problem. With this question, a discussion was held with teachers to understand how they consider about different types of students' thinking. After that, teachers were given different student-solutions on the modelling problem (bouncing ball problem), and they were asked to focus on types of student thinking and fill the student-thinking sheet. After this activity, teachers discussed their works with each other.

In the third day, teachers were presented dynamic geometry software (geogebra) and informed about how to use this software. An application activity for the use of the geogebra software was conducted using laptops provided for each teacher. In this activity, teachers were given a direction list and necessary guidance by the research team to complete the works (including understanding the use of tools in the software, investigating the area under a graph of a parabola, forming different geometrical shapes).

After geogebra activity, teacher were given a modelling problem (How to pack? problem) and asked to read and think on the question for the five minutes. Before passing to the solution of the problem, each teacher was asked what he or she understood from the problem and discussed with each other about their thinking. With this discussion, it was aimed to make each teacher to fully understand the problem. Also, different than the process followed in the first and second day, with this modeling problem, the aim was to make teacher to notice the difference between passing directly on the solution and discussing "understanding the problem" before solution. After this process, teachers formed groups and worked on the solution of the problem and presented their solution to the class with a poster they have prepared. After this activity teachers filled a document about the nature of modelling problem based on the modelling problem they worked on (How to pack?). Then, teachers were presented students solutions for the "how to pack?" problem gathered from the pilot application of this problem on students.

After the period explained above, a discussion was held with teachers on the advantages and disadvantages of group working and the roles of the teacher during application of MEAs. After discussion, a presentation was made by the members of research team about the role of teachers and questions that teacher can ask his/her students during the application of MEAs, importance of group work and how to form groups, and what should be paid attention when conducting group work.

The forth and the last day of the workshop was started with a presentation about mathematical models and modelling, nature and properties of MEAs. During the presentation teachers were asked to explain what they thought about each topic in the presentation, with clarifying the argument they propose by giving examples from the applications conducted during the workshop. After that, teachers were asked to explain that how their knowledge about mathematical modelling and expectations from the PDP has changed after attending in the workshop.

Additionally, teachers were asked to evaluate PDP in an anonymous written format.

In the last day, teachers were also asked to make a lesson plan regarding the use of one of the MEAs that they have worked on throughout the workshop. Teachers were given a list of parameters (objectives to be attained, teaching strategies that can be used while application of the MEA, student's thinking ways for this MEA, etc.) that they should consider while preparing the lesson plan. After teachers completed their lesson plans, a discussion was conducted on their plans, the importance of MEAs and how to use MEAs in their lessons. In this day, teachers were also asked to prepare a concept map similar to that they have prepared in the first day of the workshop, it is indicated that they can either make a new concept map, or change the first concept map they have prepared. After concept mapping activity, workshop was ended with a discussion on teachers' evaluation of the process of four-day workshop.

3.5.2 School-Based-Training Period of PDP

School based training period (SBTP) was started with beginning of the 2011-2012 academic semester, after the workshop conducted in September 2011, and it continued nine months. Seven MEAs, selected from a pool of MEAs, which were prepared by the research team, were covered throughout the PDP. Every month, two teachers from each school implemented a selected MEA. One week prior to the implementation, a focus group meeting (preliminary meeting) was conducted in each school with the attendance of the participant teachers and 4 members of the research team. The week after the implementation, a follow-up meeting was held with the same participants in each school. With this cycle (preliminary meeting-implementation-follow up meeting), seven MEAs were implemented throughout the PDP. Every teacher implemented three MEAs in his/her classes, and every MEA was implemented by two teachers in each school. There were also two whole-group meetings conducted with the attendance of all of the participant teachers. First meeting was in the semester break (February 2011) and the second one was at the end of the 2011-2012 academic year (June 2012). These meetings were held to discuss how the PDP was going on and to administer the data collection instruments to teachers for the data collection.

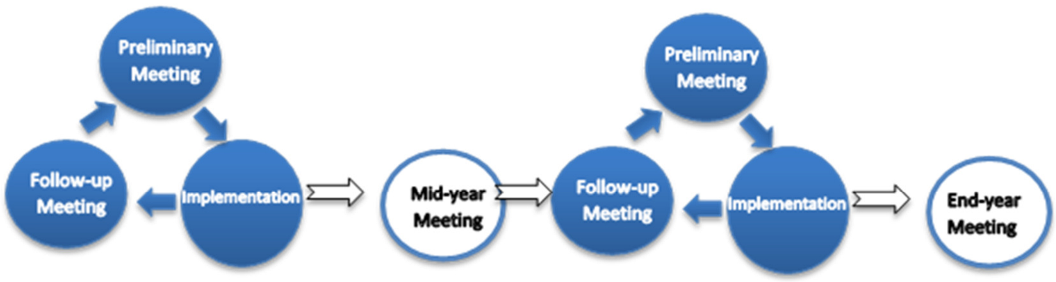

Figure 3.1 Flow of School Based Training Period

Figure 3.1 describes the flow of the SBTP. The detailed information about meetings and the implementation process of MEAs is presented in the following sections.

3.5.3 Focus Group Meetings

Before and after each implementation, there were two focus-group meetings held with the five teachers in the each school and four members of the research team (one project coordinator, three bursary students). The first focus-group meeting was conducted one week prior to the implementation week, as a preliminary meeting. Before a preliminary meeting teachers were given the problem sheet of the MEA, and were asked to solve and think on the problem considering the concept covered in the problem, preliminary knowledge and skills that students need(ed) to solve the problem, possible students' way of thinking, difficulties, and mistakes while solving problem and guiding questions that can be asked to students during the implementation, and they were asked to write their individual works on "Student Thinking Sheet" (See Appendix A) before coming to the meeting. The preliminary meeting started with each teacher's describing the concept covered in the problem and explaining his/her way(s) of solution for the problem. Then, teachers discussed about their solutions, and criticized/evaluate others' solutions (such as possible advantages and disadvantages). Sometimes there were mistakes teachers had made during the solution and he or she noticed his/her mistake during explaining to the group, or other teachers noticed (his/her) mistake(s). Moreover, in some cases, some teachers couldn't solve the problem (including the teachers who will implement the problem). In such cases, the teacher(s) expressed his/her opinions about the solutions emerged during the meeting. One of the focuses of the preliminary meeting was "ways of students' thinking" (which signifies students' ways/strategies of solutions for the modelling problem). After the group (teachers) talked about their solutions of the problem, the next thing to discuss was what they think about *students' ways of thinkin*g; how students could solve the problem, where they might have difficulties, misconceptions or mistakes, etc. While they had been discussing on these issues, they were also asked to write all of the ideas emerged in the meeting on the "Student Thinking Sheet" (see Appendix A) filled as a joint written document. Beside all these, teachers shared their ideas about planning of the implementation, such as number of students in the group, time separated for the parts of lessons (introduction, solution of problem, presentation of solution by groups, and summarizing), methods that will be used during the lesson, etc. They were also asked to write the ideas emerged about the planning of implementation as a "joint implementation plan" (see Appendix B).

Second focus-group meeting was conducted one week after the implementation week, as a follow-up meeting. After two teachers in each school implemented a MEA in their classes, members of the research team collected student groups' working sheets (groups' working sheets include students' works and poster papers) and copied them to distribute to the teachers in the same school. Each teacher got one copy of students' working sheets from two implementations. Before coming to the meeting, teachers were asked to investigate students' solutions and identify students' ways of thinking from the worksheets and fill the "ways of thinking form" based on their own investigation. At the beginning of the follow-up meeting, there was an evaluation by the teacher(s) who applied the modeling problem. Teachers who made the implementation explained how they conducted the lesson, how students solved the problem, the difficulties aroused or encountered, etc. After that, a whole group discussion about students' solution strategies was conducted. Teachers were prompted to identify students' ways of thinking, students' mistakes, difficulties, and misconceptions as much as they understand from the students' working sheets. Sometimes, teacher(s) who implemented the MEA or members of the research team tried to clarify students' solution(s) when other teachers have difficulty to understand it from students' written work. At the end of the follow-up meeting, the teachers who will be implemented the next MEA were selected and the problem sheet was distributed to each teacher and they were asked to investigate the problem (try to solve and think on possible solution strategies, ways of students' thinking, difficulties and mistakes) before coming to the meeting conducted before the implementation (preliminary meeting). In both focus-group meeting, members of research team guided and promoted teachers to express their ideas on the topic discussed by introducing the ideas, asking question, encouraging them to express their ideas, and summarizing the discussion.

3.5.4 Implementation of MEAs

Every month, two teachers in each school implemented a selected MEA. Each teacher implemented three MEAs throughout the PDP period. The schedule for implementation of MEAs and practicing teachers were presented in Table 3.3. The schedule was determined considering teachers' opinions about the alignment between MEAs, the curriculum, and the grades of classes teachers taught. Teachers who implemented the first MEA (Bank Robbery) were selected in the first preliminary meeting, while teachers who implemented one of the other MEAs were selected during the follow-up meeting of the previous MEA.

Before the preliminary meeting of each MEA, groups of teachers in two schools were asked to think about the implementation of MEA and form an individual implementation plan. During the preliminary meeting, teachers were asked to form a joint implementation form, which includes the issues described in the individual implementation plans. The teachers who will implement the MEA had the joint implementation plan. However, the practicing teachers were asked to improve the joint implementation plan when there is no consensus between teachers about the certain aspects of the plan, and clarify every aspects of plan for themselves. Beside, to understand their readiness level, teachers who would implement were interviewed individually about their implementation plan (for interview questions, see Appendix C, D, E, F) in the implementation day.

Table 3.3 The schedule for implementation of MEAs and practicing teachers

	MEAs	**Subjects/ Grade**	**Practicing Teachers**	**Time**
1st MEA	Bank Robbery	Logic/ 9th	School A- Filiz /Rengin School B- Meltem/Sude	November 2011
2nd MEA	Street Parking	Trigonometry Geometry/ 10th	School A-Ayla/Hazal School B-Kadri/Kerim	December 2011
3rd MEA	Summer Jobs	Ratio& Proportionality , Weighted average/ 9th-12th	School A- Filiz /Rengin School B- Meltem/Sude	January 2012
4th MEA	Water Tank	Functions/ 9th-12th	School A-Alp/Ayla School B-Kadri/Mert	February 2012
5th MEA	How to Store	Geometry/ 9th-12th	School A- Filiz /Rengin School B- Meltem/Sude	March 2012
6th MEA	Magazine Sales	Quadratic equations/ 10th	School A-Alp/Hazal School B-Kerim/Mert	April 2012
7th MEA	Bouncing Ball	Exponential Functions, Inequalities/ 11th	School A-Ayla/Hazal School B-Mert/Kerim	May 2012

During the implementation, three member of the research team were available in the class to support and help the practicing teacher(s) during the implementation as well as to observe and to record the implementation by video-recorders. Additionally, other teachers in the same school, generally those who had not lessons during the implementation hour, could sometimes attend in the implementation to observe. The aim of these observations is to identify the opportunities, difficulties and effective strategies while implementing MEAs.

Implementation generally started with practicing teacher's introduction of MEA to students. Before this research, students were not familiar with mathematical modelling problems and teaching with mathematical modelling, and they had no experience about becoming a part of a research project. At the beginning of the SBT teachers informed students about the research project and activities that they would participate in, so that students could adapt to the activities conducted and did not feel uncomfortable (this situation was also verified by researchers' observations and teachers' declarations) because of the existence of the research team members and video recording of their classroom activities. After the practicing teacher made an introduction, students formed groups (numbers of the group members were determined in the preliminary meeting with a collective consensus or only by the practicing teacher's decision). MEA sheets were distributed to groups. Research team members helped practicing teachers to distribute MEA sheets. Then practicing teachers asked students to read the problem (MEA) and try to understand it. After students read the problem, practicing teachers sometimes asked the class about what the problem asked for. Sometimes he/she requested any of the volunteer students to explain their ideas about what the problem asked for, or, sometimes, he/she walked around the class to ask each group about what they thought about the problem. Generally, after the period of making sure that student understood the problem, groups tried to solve the problem while the practicing teachers observed the groups, tried to understand what students were doing, or sometimes asked group members some probing or guiding questions about the problem, however, the degree of using probing/guiding questions and quality of these questions changed from teacher to teacher. Some teachers considered these probing questions as giving clue to students and were reluctant to ask, however some teachers asked questions like 'What did you do?', 'What did you think?'. Implementation of MEAs took two class hours. After groups reached to a solution, they were asked to write a whole group report in which they explain their solution with reasons (as detailed as possible) and add their working (or drafts) sheets for solution of the problem to this report. In the last quarter of the second hour, the practicing teachers asked groups to present their solution on the board. Here, sometimes the teachers asked other groups' ideas about the presenting group's solutions. Sometimes, if groups had presented their ideas but couldn't reach a certain solution, some practicing teachers explained the solution. At the end of the implementation, the research team members gathered students' reports including their written solutions and explanations. Beside distributing and gathering problem and working sheets, one of the research team members took observation notes, second member dealt with the video and audio recording of the implementation (both whole class and groups' works), while the third member walked around the class with teacher to be available when the teacher needed any help.

After the implementation, each practicing teacher was interviewed individually (see interview question in Appendix C, D, E, F), where teachers were asked to evaluate their implementation, to explain what they would prefer to keep same or change if they had a chance of making same implementation again, what part(s) of implementation (an instance or a specific case appeared in the implementation) they wanted to share and discuss them with the other teacher in the follow-up meeting. In essence, these issues that teachers pointed out were brought on the table in follow-up meetings. Moreover, in the last two modeling implementations, selected videos recorded in the class by the practicing teacher were discussed in the follow-up meeting.

3.5.5 Mid-Year and End of Year Meetings

There were also two (whole-group) meetings conducted with the attendance of all of the participant teachers. First meeting was in the semester break (February 2011) and the second one was at the end of the 2011-2012 academic year (June 2012). These meetings provided teachers opportunities to reflect on and critique the professional development activities and to administer the data collection instruments to teachers.

3.6 Instruments and Data collection

Qualitative research relies on the data gathered from multiple data sources. To answer the research questions of this study, the data was collected in nine months, through utilization of several data collection instruments. To investigate teachers' existing beliefs, their belief changes throughout the PDP, and their perception about PDP's effects on their belief changes, data gathered from semi-structured interviews, an open-ended analogy questionnaire, mid-year meetings and end of year meetings were used. Semi-structured interviews and open-ended analogy questionnaire constituted the primary data sources while weekly meetings and mid-year and end of year meeting records constituted the data sources used to support the finding explored from the primary data sources. The alignment of research questions with data sources were presented in Table 3.4. Next, each data collection instrument and data source will be explained in detail.

Table 3.4 Alignment of research questions with data sources

	Research Questions	Data Sources
1	What were the secondary school mathematics teachers' beliefs about nature of mathematics, mathematics learning and mathematics teaching?	Semi-structured interview (Mid-year) Analogy questionnaire
2	How did teachers' beliefs change after participating in a one-year PDP?	Semi-structured interviews (End of year) Analogy questionnaire
3	What do teachers think to be the influence of the professional development program on mathematical modeling on their beliefs about nature of mathematics, and teaching and learning mathematics?	Semi-structured interviews (End of year) Mid-year and year-end meetings

Open-Ended Analogy Questionnaire

An open-ended analogy questionnaire was given to teachers in the first day of the workshop and in the end of year meeting to understand their beliefs about learning and teaching of mathematics and the changes in their beliefs. This questionnaire is adapted from Chauvot's (2000) study and includes two open-ended analogy questions on mathematics teaching and learning. In these questions, teachers are provided a list of analogies/metaphors and asked to select one (or develop one) that represent 1) "mathematics teacher" and 2) "mathematics learning" best and least and to explain the reasons of their selections (for analogy questionnaire see Appendix D).

Semi-Structured Interviews

First Interview: The first semi-structured interview with teachers was conducted during PDP, in February. Each teacher was interviewed individually in his/her own school, during his/her free classes. Each interview took approximately 60-to-90 minutes and was audio-recorded. The aim of this interview was to gain insight into teachers' beliefs about nature of mathematics, learning and teaching of mathematics. Interview included five sections; background and regular teaching practice, (ideas about) nature of mathematics, mathematics learning, mathematics teaching. The aim of the questions about background and regular practice was to understand teachers' educational experiences (as a student) and their teaching practice. The remaining of the interview focused on the beliefs about nature of mathematics, mathematics learning and teaching. The interview questions were prepared based on the related literature (Beswick, 2012; Thompson, 1984). A mathematics education researcher was asked to evaluate the appropriateness (if questions could uncover the related beliefs) and clarity (language and format) of interview questions. After necessary revisions were made, the interview was conducted with a doctorate student who had an experience as a mathematics teacher in secondary schools. The aim for this pilot application (of interview) was to understand whether the questions could be understandable by a teacher, and to determine (nearly) how much time the interview would take.

As mentioned earlier, at the beginning of the PDP, teachers' beliefs about teaching and learning of mathematics were investigated with an open-ended analogy questionnaire. However, data gathered from analogy questionnaire was not deep enough to understand teachers' beliefs with associated reasons that teachers had to support their beliefs and the connections among their beliefs and their practices. Therefore, the questions in this interview had two foci, asking "what teachers think about the subject of the questions now", and, "how long they have been thinking like that/whether they were thinking like that in the past". If the teacher responded a change in his/her thinking, the latter question was followed with another sub-question; "what (specific case, instance, process, experience etc.) made you change your thinking?" The sample questions in the interview are presented at Table 3.5 (see Appendix C, D, E, F, for the whole interview questions).

Table 3.5 Sample interview questions (first interview)

1. Background and regular practice
   - Could you please explain about your past experience as students?
   - Would you describe your regular lesson (How do you start, what do you do throughout the lesson? What kinds of questions, activities, and examples do you use? What are your and students' roles? Do you use any material/how do you use? Do you use specific method(s) in your teaching, how do you conclude lesson? Do you make any evaluation throughout and/or at the end of the lesson? How do you evaluate whether students understand what you taught?)
2. Nature of mathematics
   - What is mathematics for you, how do you describe it?
   - If you chose four words to define mathematics what would they be?
   - How do you describe mathematical problem? Could you please give an example?
3. Mathematics teaching
   - What do you think about the purpose of mathematics teaching? (What is your purpose of math teaching?)
   - What are the characteristics of good math teacher?
   - How do you describe an ideal math class?

Table 3.5 (Continued)

4. Mathematics learning
- How do students learn best?
- What should students do in/out of class to learn math?
- What are your role and students' roles in their learning?

End of Year Interview: Second semi-structured interview was conducted after the PDP in June. The interview was conducted individually and it took (approximately) 60 minutes. Each interview session was audio-recorded. Aim of this interview was to understand changes in teachers' beliefs about nature of mathematics, learning mathematics, teaching mathematics, and teachers' perceptions of the effect of PDP on their beliefs. Some of the questions about nature of mathematics, teaching and learning of mathematics were the same as the questions in the interview conducted in February. There were also some additional questions about the effect of PDP on their beliefs about nature of mathematics, teaching and learning of mathematics. The sample interview questions are presented in Table 3.6 (see Appendix C, D, E, F, for the whole interview).

Table 3.6 Sample interview questions (end of year interview)

- How do you describe mathematics?
- How do students learn mathematics best?
- What should students do to understand a concept?
- What is the purpose of mathematics teaching?
- What should teacher do to make students understand a concept?

General Evaluation Interview: Third semi-structured interview was conducted at the end of the PDP with teachers. Each teacher was interviewed individually with the attendance of three members of the research team. The aim of this interview was to get teachers' evaluation about PDP and its components in terms of its contributions to their knowledge, beliefs and practices and deficiencies if they observed about any components of PDP. Each interview took 15-20 minutes and each interview session was audio-recorded. The sample questions in the interview are presented in Table 3.7 (see Appendix F for the whole interview).

Table 3.7 Sample interview questions (general evaluation interview)

- When you consider period between your first and last application of modelling problems, and the activities that you participated during the PDP, how do you evaluate yourself? If there exist any change, what are the factors contributing your change?
- At the end of PDP, was there any change in your opinions about mathematics, mathematics teaching and learning?

Mid-year and end of year meetings: After the first semester ended, a meeting was held with the attendance of all participant teachers from two schools and all members of the research team. The meeting was conducted in semester break, for a whole day. The aim of the meeting was to administer data collection instruments, and to discuss the process of PDP with teachers. At the beginning of the meeting data collection instruments of the projects (concept maps, analogy questionnaire) were applied. The remaining of the meeting was separated to a whole group discussion about the PDP. Teachers mentioned about their experience during the PDP and shared their ideas about the process of the PDP, their needs, suggestions, etc. The meeting was audio and video recorded.

At the end of the 2012 academic year, an end of year meeting was conducted as a concluding of PDP period. The meeting was held in June with the attendance of all participating teachers and members of the research team. The aim of this meeting was to apply data collection instruments of the project (including the analogy questionnaire), as well as, to get teachers' evaluations about the effectiveness of PDP in a discussion environment.

3.7 Data Analysis

Data gathered through interviews, open-ended analogy questionnaire, and meetings were analyzed to explore teachers' beliefs and change in their beliefs. To investigate teachers' beliefs about nature of mathematics, teaching and learning of mathematics, the main data sources used were semi-structured interviews and analogy questionnaire conducted with each teacher. Data gathered from meeting records (mid-year and end of year meetings) was used to support the findings. Teachers' expressions as response to the specific cluster of questions related with each category of beliefs are considered to explore teachers' beliefs about nature of mathematics, and teaching and learning mathematics (see Appendix C, D, E, F, for the questions related with nature of mathematics, teaching and learning mathematics). In order to decide the category of teachers' beliefs, the rationale proposed by the theoretical framework was used. Below, the rationale for the analysis of teachers' expressions regarded as evidences for each domain of belief is explained.

3.7.1 Beliefs about Nature of Mathematics

The Rationale for the Category of Instrumentalist View

Abbreviation used for the code of this category of belief is NM1. If the major emphasis in a teacher's expressions is on a utilitarian aspect of mathematics, his/her explanations are limited to basic computations, and there is a strong focus on following facts, rules, and procedures, then this sentence (or paragraph) is considered an evidence for an *instrumentalist view* and it is coded as NM1. Moreover, teachers' explanations including or similar to statements presented below are likely to indicate an instrumentalist view of mathematics. Belief statements presented in Table 3.8 below were taken from Thomson's (1984, 1992) and Beswick's (2012) studies.

Table 3.8 Sample belief statements for NM1 category

- Mathematics is an exact discipline-free of ambiguity and conflicting interpretations
- Certainty is an inherent quality of mathematical activity. The procedures and methods used in mathematics guarantee right answers.
- Content of mathematics is 'cut and dried'. Mathematics offers few opportunities for creative work.
- Mathematics came about as a result of basic needs that arise in everyday situations
- Mathematics is predictable, absolute and fixed. The content of mathematics has not changed much in the recent past.
- Mathematics is basically the usage of arithmetic skills in daily life.

Table 3.8 (Continued)

• Mathematical knowledge is composed of facts, rules and procedures • Mathematics is computation. • Mathematics is an exact discipline-free of ambiguity and conflicting interpretations • Certainty is an inherent quality of mathematical activity. The procedures and methods used in mathematics guarantee right answers

Based on the descriptions of instrumentalist view and statements indicating an instrumentalist view, the following list of words and phrases are used as key words when investigating teachers' expressions about nature of mathematics. After that, samples of teacher expressions indicating this view and coded as NM1 are presented.

Words/phrases that depict this view

- Computation
- Rules and facts
- Certain/absolute/fixed/exact
- Needs in daily life (like computation, arithmetical skills, etc.)
- Right answer/solution

Sample(s) of teachers' expressions regarded as indication for instrumentalist belief and coded as NM1

"Mathematics is like difficult calculations encountered in life…a set of operations…" *(Meltem, first interview).*

"Mathematics is terms, expressions, operations in everywhere…an order… rules of life…it is a rule that order our lives" *(Hazal, first interview).*

"Calculation produce correctness and this is the nature of mathematics" *(Alp, first interview).*

"Mathematics likes certain answers; math knowledge is either right wrong, like black-and-white" *(Rengin, first interview).*

"I accept mathematical knowledge, I do not judge if it is true or false… I accept that rules as correct" *(Sude, first interview).*

The Rationale for the Category of Platonist View

Abbreviation used for the code of this category of belief is NM2. If the major emphasis in a teacher's expressions is on mathematics as a collection of related knowledge that already exists, which is discovered but not created, then this sentence (or paragraph) is considered an evidence for a *Platonist view*, and it is coded as NM2. Moreover, teachers' explanations including or similar to statements presented below are likely to indicate a Platonist view of mathematics. Belief statements presented in Table 3.9 below were taken from Thomson's (1984, 1992) and Beswick's (2012) studies.

Table 3.9 Sample belief statements for NM2 category

- Mathematics is composed of rules and procedures with the principles behind them.
- Mathematics is an organized and logical system of symbols and procedures that explain ideas present in the physical world.
- Mathematics is a human creation, but mathematical ideas exist independently of human ability to discover them. Because of this, mathematics is more than a system of symbols; it is the idea as well.
- Mathematics is mysterious- its broad scope and the abstractness of some of its concepts make it impossible for person to understand it fully.
- Mathematics is accurate, precise and logical.
- Mathematics is consistent, certain and free of contradictions and ambiguities.
- Mathematical content is fixed and predetermined, as it is dictated by ideas presented in the physical world.
- Mathematics content is coherent. Its topics are interrelated and logically connected within an organizational structure or 'skeleton'
- Changes in the content of mathematics occur only at the extreme as it continues to expand.
- Mathematical ideas exist independently of human ability to discover them

Based on the description of Platonist view and statements indicating a Platonist view, following list of words and phrases are used as key words when investigating teachers' expressions about nature of mathematics.

Words/phrases that depict this view

- Organized/related/logical/coherent system of symbols and procedures
- Idea (not just symbols and procedures)
- Abstract (difficult to understand)
- Static (only in extreme cases mathematics content can change)
- Consistent/certain (free of ambiguities)
- Independent of human ability (exist in physical world)
- Discovered, not created

Since some of the key words are common between instrumentalist view and Platonist view, it is necessary to clarify the difference between the two views. Following discrimination between views was followed while coding teachers' explanations.

Difference between Instrumentalist and Platonist views

- Both instrumentalists and Platonists consider mathematics as composing of rules and procedures, but Platonists also adds that mathematics also contains principles behind the rules and procedures.
- Both instrumentalists and Platonists say that mathematics is logical, but Platonists also say that it is organized.
- Both instrumentalists and Platonists say that mathematics is certain. However, instrumentalists focus on certainty of procedures and methods used, while Platonists consider certainty of mathematical knowledge; it has no contradictions or ambiguity in it.
- Both instrumentalists and Platonists consider that content of mathematics is fixed and predetermined. Instrumentalists say that "so mathematics offer limited opportunities for creative work", but, Platonists say that

> "mathematics content is fixed because of the ideas presented in the physical world which is fixed".

Below, samples of teacher expressions for Platonist view that were coded as NM2 are presented.

Sample(s) of teachers' expressions regarded as indication for Platonist view and coded as NM2

"There is no development in mathematical knowledge…we always use the same knowledge…we do not produce something new, we use already existed" (Ayla, first interview).

"We accept the already existed mathematical knowledge, and solve new problem, but we don't find new knowledge, what is found does not contradict to past knowledge, it is maybe a new theorem but it is based on the already existed knowledge" (Sude, first interview).

"Everything in mathematics is very logical. As you take its logic, everything goes in correct way" (Ayla, first interview).

The Rationale for the Category of Problem Solving View

Abbreviation used for the code of this category of belief is NM3. If the major emphasis in a teacher's expressions is on mathematics as a dynamic and continually expanding human creation located in social and cultural context and there is a strong focus on process of getting to know then this sentence (or paragraph) is considered an evidence for a *problem solving view*, and it is coded as NM3. Moreover, explanations including statements presented in Table 3.10 are likely to indicate a problem solving view of mathematics. Statements in Table 3.10 were taken from Thomson's (1984, 1991) and Beswick's (2012) studies.

Table 3.10 Sample belief statements for NM3 category

- The primary purpose of mathematics is to serve as a tool for sciences and other fields of human endeavor.
- Mathematical content originated from two sources: from the needs of the sciences and other practical needs, and from mathematics itself.
- The study of mathematics sharpens one's ability to reason logically and rigorously.
- The validity of mathematical propositions and conclusions is established by the axiomatic method.
- Mathematics is continuously expanding its content and undergoing changes to accommodate new developments.
- Mathematics is a beautiful, creative and useful human endeavor that is both a way of knowing and a way of thinking.
- Justifying the mathematical statements that a person makes is an extremely important part of mathematics.
- The results of mathematics are tentative; subject to revision in the light of new evidence.

Based on the descriptions of problem solving view and statements indicating a problem solving view, following list of words and phrases are used as key words when investigating teachers' expressions about nature of mathematics. Next, samples of teacher expressions, which are regarded as indication for this view and coded as NM3, are presented.

Words/phrases that depict this view

- Dynamic/developing
- Human creation/product/endeavor
- Social and cultural context (shapes math)
- Tools for science
- Reasoning logically/rigorously
- Justifying/validating

- Tentative/open to revision

Sample(s) of teachers' expressions regarded as indication for instrumentalist belief and coded as NM3

"I can defend that 2x2=3, when I say this, some says you are wrong, some may be skeptical about it…then new things will be developed with another idea, with interaction" (Kerim, first interview).

"Mathematics is human's collective product…it is developing as human interact with each other" (Kerim, first interview).

"Mathematics is functional so that it is used in other sciences…mathematics is always developing because of the needs" (Kadri, first interview).

"Having the mathematical data is not important, important thing is to use data, establishing relationship between data" (Filiz, first interview)

3.7.2 Beliefs about Teaching Mathematics

The Rationale for the Category of Content-Focused with Emphasis on Performance Model

Abbreviation used for the code of this category of belief is T1. If the major emphasis in a teacher's expressions is on the content which is organized according to a hierarchy of skills and concepts, the role of the teacher such as demonstrating, explaining and presenting the content in an expository style, and the role of students which have been set by the teacher such as listening, participating and doing exercise, then, this sentence (or paragraph) is considered as an evidence *for content focused with emphasis on performance model* and it is coded as T1. Moreover, explanations including or similar to the statements presented in Table 3.11 are likely to indicate a content-focused with emphasis on performance model of mathematics teaching. Below statements were taken from studies of Beswick (2012), Van Zoest et al. (1994), Swan (1986), Kupari (2003), Kuhs and Ball (1986).

Table 3.11 Sample belief statements for T1 category

- It is the teacher's responsibility to direct and control all instructional activities including the classroom discourse.
- Telling children the answer is an effective way of facilitating their mathematics learning.
- It is not necessary to understand sources of children's error; follow-up instruction will correct their difficulties.
- It is the teachers' responsibility to provide the children with clear and concise solution methods for mathematical problems.
- Although there are some connections between different areas, mathematics mostly made up of unrelated topics.
- There are always predetermined solution methods for the mathematical problems and students' responsibility is to execute those methods when solving problems.
- Classroom activities should focus on helping students master the content of the curriculum
- Basic computational skills are sufficient for teaching junior secondary school mathematics.
- Teaching is structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstanding when students fail to 'grasp' what is taught.

Based on the descriptions of content-focused with emphasis on performance model and statements indicating this model following list of words and phrases are used as key words when investigating teachers' expressions about teaching mathematics. After that, samples of teacher expressions, which are regarded as indication for this model and coded as T1, are presented.

Words/phrases that depict this view

- Teacher as instructor
- Helping students master the content

- Teaching basic computational skills
- Correcting students error/difficulties
- Giving concise/clear explanations or ways of solution
- Teacher as dominant/director/controller
- Teacher-centered/ teacher set the roles

Sample(s) of teachers' expressions regarded as indication for this view and coded as T1

"A mathematics teacher should know everything very well, because how much s/he knows, that much s/he explains well" (Ayla, first interview).
"[group working] it is not suitable to our teaching… we are accustomed to teacher centered teaching; teacher explains and ask questions, students listens and answer questions" (Hazal, first interview).
"In our teaching, I am the active one, in every aspect…the important point is that teacher is the knowledgeable one, students are those who use my knowledge…teachers give the knowledge" (Sude, first interview).
"I don't say "work at home"…because I teach the concepts in class…if they have misconceptions or deficiencies; they can cover those by listening to me" (Sude, first interview).

The Rationale for the Category of Content-Focused with Emphasis on Conceptual Understanding Model

Abbreviation used for the code of this category of belief is T2. If the major emphasis in a teacher's expressions about the classroom activities is on the mathematical content and structure of mathematics, but, there is also an emphasis on students understanding of relations among mathematical ideas and concepts and logic behind mathematical procedures then this sentence (or paragraph) is considered as evidence for *content-focused with emphasis on conceptual understanding model*, and it is coded as T2. Moreover, explanations including or similar to the statements presented in Table 3.12 are likely to indicate a content-focused with emphasis on conceptual understanding model of mathematics teaching. Below statements were taken from studies of Beswick (2012), Van Zoest et al. (1994), Swan (1986), Kuhs and Ball (1986).

Table 3.12 Sample belief statements for T2 category

• The teacher's explanations should assist students to 'see' the relationships between the new topic and those already studied. • Teachers must be able to represent mathematical ideas in a variety of ways. • When solving a certain type of mathematics problems, firstly teachers should show one or more solution methods then students are allowed to solve problems by following or extending teachers' solution methods. • Allowing students to work individually on their solution method can be an effective way to teach mathematics. • Teacher has greater control over students' explorations. • Teaching is assessing when students are ready to learn; providing a stimulating environment to facilitate exploration; and avoiding misunderstanding by the careful sequencing of experience. • Ability to get correct answers, use algorithms and recite definitions that may have been learned by rote, is not adequate evidence of knowing mathematics

Based on the descriptions of content-focused with emphasis on conceptual understanding view and the statements for this view following list of words and phrases are used as key words when investigating teachers' expressions about teaching mathematics. Samples of teacher expressions, which are regarded as indication for this view and coded as T2, are presented at next.

Words/phrases that depict this view

- Teacher as explainer
- Assisting students to see the relationship between concepts
- Teacher represents mathematical ideas
- Teacher has control on students' exploration
- Avoiding students' misunderstanding

- First teacher shows, then students mimic/try/follow/solve
- Sequence of teaching is important
- Students' ideas and interest as not primary for determining curriculum/mathematical content

Sample(s) of teachers' expressions regarded as indication for this view and coded as T2

"I want them to make relation with the previous concepts, and attend in the lesson….by answering the questions" (Sude, first interview)
"For example when explaining function, I start with machine examples, by saying that "what are the first machine doing; multiply by two", "what are the second machine doing; multiply by two and add two", like this I express it visually then I pass to the formula" (Mert, first interview).

The Rationale for the Category of Learner-Focused Model

Abbreviation used for the code of this category of belief is T3. If the major emphasis in a teacher's expressions is on the role of teacher as facilitator of students learning, the use of learner focused approach to teach mathematics, mathematics learning as students' actively involving in exploration of mathematical ideas, then, this sentence (or paragraph) is considered as evidence for *learner focused model* and it is coded as T3. Moreover, explanations including or similar to statements presented in Table 3.13 are likely to indicate a learner-focused model of mathematics teaching. Below statements were taken from studies of Beswick (2012), Van Zoest et al. (1994), Swan (1986).

Table 3.13 Sample belief statements for T3 category

• It is important for children to be given opportunities to reflect on and evaluate their own mathematical understanding. • Ignoring the mathematical ideas that children generate themselves can seriously limit their learning. • Providing children with interesting problems to investigate in small groups is an effective way to teach mathematics. • The teacher should encourage the students to guess and conjecture and should allow them to reason things on their own rather than show them how to reach a solution or answer. The teacher must act in a supporting role. • Children always benefit by discussing their solutions to mathematical problems with each other. • A vital task for the teacher is motivating children to resolve their own mathematical problems. • A key responsibility of a teacher is to encourage children to explore their own mathematical ideas. • Teaching is a non-linear dialogue between teacher and students in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.

Based on the descriptions of learner-focused model and statements which may indicate this model, following list of words and phrases were used as key words while investigating teachers' expressions about teaching mathematics. Samples of teacher expressions regarded as indication for this model and coded as T3, are presented at next.

Words/phrases that depict this view

- Teacher as facilitator
- Providing problems
- Group working/making discussion
- Encouraging students to conjecture/reason/discuss
- Teacher as supporter/supportive role
- Motivate students to use their own methods/ways

- Students ideas are important for teaching
- Students' ideas and interest as key to determine curriculum/mathematical content

Sample(s) of teachers' expressions regarded as indication for this view and coded as T3

"Teachers should not hinder students thinking, but should encourage them to think. He stated that teachers should respond students' ways of solutions with logical explanations. I consider this as most important feature of mathematics teaching" (Kadri, first interview).

"Mathematics curricula should be based on students' needs" (Kerim, first interview).

3.7.3 Beliefs about Learning Mathematics

The Rationale for the Category of Skills Mastery with Passive Reception of Knowledge View

Abbreviation used for the code of this category of belief is L1. If the major emphasis in a teacher's expressions is on students' learning as mastery of skills with passive reception of knowledge and correct performance then this sentence (or paragraph) is considered as evidence for *skills mastery with passive reception of knowledge view* and it is coded as L1. Moreover, explanations including or similar to statements presented in Table 3.14 are likely to indicate skills mastery with passive reception of knowledge view. Below statements were taken from studies of Beswick (2012), Swan (2007), Kuhs and Ball (1986), Barktatsas &Malone (2005), Kupari (2003).

Table 3.14 Sample belief statements for L1 category

- Listening carefully to the teacher explaining a mathematics lesson is the most effective way to learn mathematics.
- Students learn mainly by attentively watching the teacher demonstrating procedures and methods for performing mathematical tasks and by practicing those procedures.
- Knowledge of mathematics is being able to get answers and do problems using the rules that have been learned.
- Learning mathematics means being able to demonstrate mastery of the skills described by instructional objectives.
- Students' performance of completing exercises and problems in textbooks and tests is an evidence of learning mathematics.
- To be good at mathematics, it is important for students to remember formulas and procedures.
- If students are having difficulty, an effective approach is to give them more practice by themselves in class.
- Learning is an individual activity based on watching, listening, and imitating until fluency is attained.
- Students learn best by being told how to do mathematics.

Based on the descriptions of skills mastery with passive reception of knowledge view and the statements which may indicate this view, following list of words and phrases are used as key words when investigating teachers' expressions about learning mathematics. Samples of teacher expressions, which are regarded as indication for this view and coded as L1 are presented at next.

Words/phrases that depict this view

- Students as listener/passive receptor
- Listening carefully what teacher demonstrate/explain
- Practicing the procedures/methods/rules that teacher demonstrate
- Memorizing/remembering formulas/rules
- Performance (on exercises) as evidence of learning
- Solving too many test questions (for mastery of skills)

- Learning mathematics as answering correctly the questions / using rules learned

Sample(s) of teachers' expressions regarded as indication for this view and coded as L1

"Students should listen very well what teacher explains on the board,…take notes, even small details…and should certainly review at home for not forgetting" (Ayla, first interview)
"Mathematics is not a subject that can be learned by studying on your own…if her/him [student's] knowledge is little, how can s/he learn?" (Sude, first interview)
"Students should listen the course well…because mathematics is a course that you need someone to learn from" (Sude, first interview)

The Rationale for the Category of Conceptual Understanding with Unified Knowledge View

Abbreviation used for the code of this category of belief is L2. If the major emphasis in a teacher's expressions is on learning as reception of unified knowledge but active construction of understanding then this sentence (or paragraph) is considered as evidence for "*conceptual understanding with unified knowledge* view, and it is coded as L2. Moreover, explanations including or similar to statements presented in Table 3.15 are likely to indicate a conceptual understanding with unified knowledge view. Below statements were taken from studies of Beswick (2012), Swan (2007), Kuhs and Ball (1986), and Swan (2007).

Table 3.15 Sample belief statements for L2 category

- Students should listen to teacher's explanations to understand the relationships between the new topic and those already studied.
- Knowing how to solve problem is as important as getting the correct answer.
- Understanding the logical relationships within the system of mathematics is important for learning.
- Learning is individual activity based on practical exploration and reflection.

Based on the descriptions of conceptual understanding with unified knowledge view and the statements, which may indicate this view, following list of words and phrases are used as key words when investigating teachers' expressions about learning mathematics. Samples of teacher expressions, which are regarded as indication for this view and coded as L2, are presented at next.

Words/phrases that depict this view

- Understanding connections/relationships between mathematical concepts
- Teachers explanations as facilitator for students to learning
- Knowing how to solve problem
- Reception of unified (connected/integrated) knowledge
- Students' exploration as important for learning

Sample(s) of teachers' expressions regarded as indication for this view and coded as L2

"I want to see the steps (process)…I do not interested in your [students'] answer, how you reach that answer is important for me… mathematics cannot be learned like this" (Ayla, first interview)
Good stduents make connections between concepts (Sude, first interview),
"As a teacher, you think about; how I explain the content, from where I should start, how I make students to comprehend better, what kind of questions I should ask" (Meltem, end of year interview)

The Rationale for the Category of Autonomous Exploration of Own Interest View

Abbreviation used for the code of this category of belief is L3. If the major emphasis in a teacher's expressions is on learning as active construction of understanding, possibly even as autonomous problem posing and solving then this sentence (or paragraph) is considered as evidence for an *autonomous exploration of own interest* view, and it is coded as L3. Moreover, explanations including or similar to statements presented in Table 3.16 are likely to indicate an autonomous exploration of own interest view. Below statements were taken from studies of Beswick (2012), Swan (2007), Barktatsas &Malone (2005).

Table 3.16 Sample belief statements for L3 category

- It is important for children to be given opportunities to reflect on and evaluate their own mathematical understanding.
- Children always benefit by discussing their solutions to mathematical problems with each other.
- Allowing a child to struggle with a mathematical problem, even a little tension, can be necessary for learning to occur.
- Ignoring the mathematical ideas that children generate themselves can seriously limit their learning.
- Learning is an interpersonal activity in which students challenged and arrive at understanding through discussion.
- Students learn mathematics as they solve problems and discuss their solutions.

Based on the descriptions of autonomous exploration of own interest view and statements that may indicate this view following list of words and phrases are used as key words when investigating teachers' expressions about teaching mathematics. Samples of teacher expressions, which are regarded as indication for this view and coded as L3, are presented at next.

Words/phrases that depict this view

- Learning by solving problems
- Discussing solutions
- Group working as a mediator for learning
- Students' reflection
- Construction of understanding

Sample(s) of teachers' expressions regarded as indication for this view and coded as L3

"Less data, more interpretation, if s/he does not produce something or add something from him/herself, s/he doesn't understand" (Kerim, first interview).
"Students learn through cooperation and exchange of ideas" (Kerim, end of year interview)
"Teacher should not be information giver, students could be actively involved in the concepts that they could do" (Ayla, end of year interview).

3.8 Validity and Reliability of the Study

Validity and reliability are important concerns of a scientific research. While, validity denotes the accuracy of inferences made from the data; reliability identify the degree of consistency of these inferences (Fraenkel & Wallen, 1996). Although, the terms "validity" and "reliability" are commonly accepted in quantitative research tradition, qualitative researchers (Cresswell & Miller, 2000; Lincoln & Guba, 1985) used the term "trustworthiness" instead, and they widely accepted an alternative criteria, proposed by Guba and Lincoln (1985), which are considered more appropriate for ensuring the accuracy and consistency of inferences in qualitative research. List of criteria along with its analogous terms used in quantitative research and methodological strategies to attain the criteria are presented in the following Table 3.17.

Table 3.17 Criteria for determining the quality of qualitative research

Criteria	**Analogous term**	**Methodological strategies to attain the related criteria**
Credibility	Internal Validity	Rigorous research methods Development of early familiarity with culture of participating organizations Random sampling of individuals serving as informants Triangulation via use of different methods, different types of informants and different sites Tactics to help ensure honesty in informants Iterative questioning in data collection dialogues Negative case analysis Debriefing sessions between researcher and superiors Peer scrutiny of project Use of "reflective commentary" Description of background, qualifications and experience of the researcher Member checks of data collected and interpretations/theories formed Thick description of phenomenon under scrutiny Examination of previous research to frame findings
Transferability	External Validity	Provision of background data to establish context of study and detailed description of phenomenon in question to allow comparisons to be made
Dependability	Reliability	Employment of "overlapping methods" In-depth methodological description to allow study to be repeated
Confirmability	Objectivity	Triangulation to reduce effect of investigator bias Admission of researcher's beliefs and assumptions Recognition of shortcomings in study's methods and their potential effects In-depth methodological description to allow integrity of research results to be scrutinized Use of diagrams to demonstrate "audit trail"

Credibility is one of the most important criteria for trustworthiness. As analogous to internal validity, it deals with whether the study measures what it is intended to measure. In other words, it is related with the congruence between findings and reality. Transferability, on the other hand, is parallel to the external validity defined as “the extent to which the findings of the study can be applied to other situation”. Dependability is parallel to reliability and it signifies the stability of findings over time and shows the extent to which the study can be repeated by a future researcher. Lastly, confirmability as analogous to objectivity denotes for the degree to which the results of the study is objective, free from investigator’s bias, assumptions and beliefs. As suggested by Lincoln and Guba (1985), as a qualitative study, the present study establishes credibility, transferability, dependability, and confirmability criteria by adopting strategies such as prolonged time in the field, data triangulation, member checking, rich and thick description, dependability audit, and confirmability audit.

Prolonged time in the field: This strategy was fulfilled by prolonged engagement in the research site. As the study took for two academic semesters, the researchers had got the chance for building an understanding about and familiarity with the research settings and participants. By this way, researcher’s effect was minimized. Also, prolonged time satisfied teachers’ trust and intimate participation in the study and explanation during the interviews. Furthermore, prolonged engagement in the field facilitated to see if the change in teachers’ beliefs is as a result of their engagement in the professional development program.

Data triangulation: Different data sources such as interviews, transcriptions of video-recorded meetings, questionnaire were utilized to satisfy the data triangulation. By this way, researcher could build a rigorous justification for the themes and codes.

Member checking: Member checking was used to eliminate researcher’s bias when establishing themes, coding data, and making interpretations. During the analysis of implemented interview transcripts, participant teachers were asked for evaluating categories obtained from the data (such as if they were talking about nature of mathematics, learning or teaching) and themes obtained from analysis of each category (such as if they were mentioning about *certainty* of mathematical knowledge, if they emphasized the *conceptual understanding* of the mathematical ideas, or if they focused on *skill mastery* of procedures for learning a mathematical concept, etc.).

Rich and thick description: For the present study, researcher tried to elucidate all research process, methodology and research context. Also, findings were supported with direct participant quotes gathered from one-to-one interview transcripts. By this way, those who read this study can draw their conclusions from the given data and interpret the findings.

Dependability audit: In order to ensure the reliability of data collection and data coding process a professor and a PhD student in mathematics education were asked to code the data gathered from interviews independently. After independent coding, inter-rater dependability was evaluated for three (researcher, professor and Phd student) different coders' coding. 85% of inter-coder agreement was reached between three coding. After that, all coders reached on a consensus on the non-agreed codes by discussing their codes and clarifying how every member established his/her codes.

Confirmability audit: In order to confirm that whether the results, interpretations, conclusions, and suggestions were concluded from the data, a colleague who was a Phd student in mathematics education examined the raw data and she confirmed that interpretations were clearly derived from the data.

3.9 Researchers' Role

In qualitative research, role of researcher should be described as clearly and detailed as possible, since data is collected, analyzed and interpreted through the researchers' eyes, which means that researcher is a kind of data collection instrument (Denzin & Lincoln, 2003). With these descriptions, others, readers of the study can be aware of the possible biases that researcher might have during data collection, analysis and interpretation procedure.

The kind of researcher's involvement in the research process differed during different phases. During classroom application of MEAs, and during meetings, as a member of research team, my presence was as passive as possible, except when teacher needed to ask for help during activity sheet distribution or gathering in implementation of MEAs. I generally used the video recording tool to record teacher's implications or focus group meetings. During one-to-one interviews, I, as the researcher, was the primary means of data collection. During teachers' regular teaching in their classroom, my role follows that of a non-participant observer (Creswell, 1994). This means that my role as researcher and observer was clearly known by all students and teachers. I did not interact as a participant in the teachers' teaching process in the class.

CHAPTER 4

RESULTS

In this chapter, results of the present study are presented along with the related research questions. The purpose of the study was to investigate secondary school mathematics teachers' mathematics related beliefs, the change in their beliefs after they have participated in a one- year PDP on mathematical modelling, and their perceptions of the influence of PDP on their change. In the light of theoretical framework, the findings were investigated across multiple teachers, and they are presented in the following sections.

4.1 Teachers' Beliefs about Nature of Mathematics, and Teaching and Learning Mathematics

This section addresses the first research question; "what were the high school mathematics teachers' beliefs about nature of mathematics, teaching and learning mathematics?" Teachers' beliefs before participating in PDP were determined based on data sources as explained in method section. The findings are presented as a cross-case analysis of the teachers' beliefs. Teachers' mathematics related beliefs before participating in PDP are presented in three sub-sections; beliefs about nature of mathematics, beliefs about teaching mathematics and beliefs about learning mathematics.

4.1.1 Teachers' Beliefs about Nature of Mathematics

Teachers' beliefs about nature of mathematics were described based on data gathered from semi-structured interviews conducted with teachers after the PDP had started. Teachers were asked several questions (see Appendix C) about definition of mathematics, nature of mathematical knowledge, nature of mathematical problems and problem solving and their expressions as response to the questions were coded considering the categories of nature of mathematical beliefs proposed by theoretical framework. Coding process included two phases; in the first phase, the themes or the idea emerged from the data (teachers' expressions) were identified, and then, in the second phase, these themes are matched with the sub-categories of nature or mathematics beliefs (instrumentalist, Platonist, problem solving). Themes or ideas for the nature of mathematics beliefs emerged from teachers' expressions are presented in Table 4.1.

Table 4.1 Themes about teachers’ beliefs about nature of mathematics

	NM1 (Instrumentalist)		**NM2 (Platonist)**		**NM3 (Problem Solving)**	
	Math & math knowledge	***Problem & Prob. Solving***	***Math & math knowledge***	***Problem & Prob.Solving***	***Math & math knowledge***	***Problem & Prob. Solving***
Meltem	Calculation Operation Collection of operation Daily-life operation Certainty of results	Problem as any question Reaching a result Use of formulas Use of basic knowledge Short, simple, practical solution	Develop intelligence Collection of ideas Human product Related knowledge Include concepts			Different ways of solution
Hazal	Operation, Expressions Rules Order Real-life Certainty of results Certain/clear answer	Any question needs answer Reaching a solution Exact/certain answer	Static Certainty of mathematical knowledge Human product			Analyzing question and using data Different ways of solutions
Ayla	Daily-life usage Correctness of methods and solutions	Reaching result Shorter and practical solution	Logical Thinking correctly Not certain but static Already existed	Use of logic Real life problems as applications of math concepts	Base of all science	Use of reasoning Different ways of solution are possible
Sude	Numbers Order Used in daily life Operations	Problems consist numbers Reaching a result Basic /operational knowledge before solving problems	Collection of systematic knowledge Certainty of math knowledge Static Human product-already existed No contradiction Clearness	Math problem as a system	Increase reasoning ability Human needs Base of other sciences	Different ways solutions Generalizing Trying to solve problems develops mathematics

Table 4.1 (Continued)

Alp	Accuracy Certainty of results Calculation Correctness	Step by step operations Finding the wanted	No ambiguity Certainty of mathematical knowledge		Relationship between object Dynamic Human product Teach thinking Tools for other sciences	Encourage own ways Different ways of solutions
Mert	Certainty of rules and results Based on rules Following rules	Applying knowledge & doing calculations Related to numbers Finding result	No ambiguity Systematical & logical thinking	Practical, useful & understandable solutions	Thinking reasonably Developing skills and intelligence Technological needs trigger development	Different ways of solution
Filiz	Results are not open to discussion Performing operation & finding result	Producing solutions Correctness of solution Exact/clear/certain result		Understanding problem Deciding/choosing operation	Analysis/ establishing relationship understanding/ interpreting Changing How to use data	Establishing the relationships between data Accept students ways
Rengin	Daily life usage Calculation Accurate Numbers	Reaching result (by algebraic operation)	Clear (precise) Certainty (not sure)	Good solutions relate different concepts	Dynamic Human needs Human product Base of all science	Logicalness of results Appreciate students solutions Different ways of solutions
Kadri	Certainty of rules, facts, formulas		Systematic thinking Certainty of math knowledge No ambiguity	Questions in math language Problem solving: understanding and solving	Human needs Human product A tool for science Dynamic	Accept students ways No good solution, stress understanding
Kerim	Certainty of rules				A tool for decision making, interpretation Dynamic Changeable Skepticism	Different ways of solutions Encourage students own ways

Results about themes for the beliefs about nature of mathematics as presented in the Table 4.1 show that (i) two teachers (Meltem and Hazal) held mainly instrumental (NM1) beliefs; (ii) four teachers (Alp, Mert, Filiz, Rengin) held mainly instrumental (NM1) and problem solving (NM3) beliefs; (iii) two teachers (Sude and Ayla) held beliefs in each category; (iv) two teachers (Kadri and Kerim) held mainly problem solving (NM3) beliefs. Teachers' beliefs about nature of mathematics are presented in Table 4.2 below.

Table 4.2 Teachers' beliefs about nature of mathematics before the PDP

	Beliefs about Nature of Mathematics		
	Instrumental	**Platonist**	**Problem Solving**
Meltem	✓		
Hazal	✓		
Alp	✓		✓
Mert	✓		✓
Filiz	✓		✓
Rengin	✓		✓
Sude	✓	✓	✓
Ayla	✓	✓	✓
Kadri		✓	✓
Kerim			✓

Based on the teachers' beliefs about nature of mathematics, teachers' beliefs were categorized into five groups;

- Group 1– Teachers holding mainly NM1 beliefs
- Group 2–Teachers holding mainly NM1 and NM3 beliefs
- Group 3–Teachers holding NM1, NM2 and NM3 beliefs
- Group 4–Teachers holding mainly NM2 and NM3 beliefs
- Group 5–Teachers holding mainly NM3 beliefs

Group 1 – Teachers Holding Mainly NM1 Beliefs

This group includes teachers (Meltem and Hazal) who mainly held instrumental beliefs before participating in the PDP. It should be indicated that Meltem and

Hazal also held beliefs in other categories (Platonist and problem solving), however, these beliefs are less in intensity with respect to their instrumentalist beliefs. The commonality among this group of teachers' responses to the questions asked in the first interview was that while defining mathematics, describing nature of mathematical knowledge, mathematical problems and problem solving these teachers focused mostly on calculation, operation, rules, formulas and results, certainty of results, daily-life usage of mathematics, reaching a result and solution.

In the first interview conducted with Meltem, it was revealed that while defining mathematics and the nature of mathematical knowledge, most of her expressions were related with "calculation". Her expressions indicated that Meltem's conception of mathematics was mostly utilitarian, which reduce the function of mathematics to daily-life computations (NM1). For example, when defining mathematics Meltem said that "*mathematics is like difficult calculations encountered in life…mathematical calculations that we do mentally develop our intelligence, like solving puzzle...a set of operations…we can use mental calculation in daily life such as when buying a bread, not only addition-subtraction we can use mathematics in ordering, placement of a room etc."* Meltem's emphasis on mathematics as mental calculations conducted in daily life also appeared in her response when she said that "*some concepts like logarithm, complex numbers, trigonometry, are not in our life. Only some are in our life, such as banking problem, shopping problem, velocity problem, etc."* Meltem declared that mathematics is important because *"we cannot calculate without mathematics.*"

While Meltem considered mathematics as calculation or a tool for calculation, Hazal focused on "rules and operations" as she defined what the mathematics is. Although she considered mathematics as related with the life, her conception of mathematics is rather formal, such as mathematics as rules, expressions, and operations (NM1). For example, Hazal said that "*Mathematics is terms, expressions, operations in everywhere…an order… rules of life…it is a rule that order our lives."* For Hazal, mathematics is important because "a*lthough some are* *not aware of it, mathematics is in our life…life is a mathematical design".* Hazal also said that most representative words that describe mathematics are "life", "reality", "formulized", and "no doubt".

One of common aspects between Meltem and Hazal's description of mathematical knowledge was their emphasis on the certainty in mathematics. Both Meltem and Hazal considered that mathematical knowledge is certain and it should be certain (NM1). While Hazal described mathematics as "*a certain science*", she added that "*because I am a teacher. I do not teach something that is not certain…it should be verified."* Hazal also stated that teachers should stand behind the knowledge presented because students trust in teachers' knowledge and mathematical formulas. For Hazal, mathematical knowledge is based on data, formulas and proofs and mathematical results are certain. Hazal declared that although there can be different interpretations in mathematics; these interpretations are based on truth. Hazal said that "*if I couldn't find an answer for a mathematical question I thought that I haven't learned or I have made a mistake while solving, because I think that there should be a certain, clear answer*". Similar to Hazal, Meltem declared that except open-ended questions, mathematical concepts and results are certain.

About the nature of mathematical problems, Meltem's and Hazal's definitions were similar. While Hazal said that a mathematical problem is "*a question that needs an answer*", Meltem stated that "*any mathematical question is a problem" (NM1)*. Hazal described a mathematical problem in such a way that "*it can be real life problem or a question includes only mathematical terms. There is a story part and a mathematical term part in a problem*", whereas Meltem described that "i*t can be real life problems like velocity or work problems or others including complex numbers, finding a square root, or calculating logarithmic functions*." Another commonality in this group is that both Hazal and Meltem focused on the results as they describe the problem solving (NM1). Hazal defined problem solving as "*reaching a solution by thinking and with analyzing formulas.*" Similarly, Meltem described problem solving as "*reaching an end result*", and "*using formulas related with the subject and obtaining the result*". For Hazal, knowing formulas is not enough for solving a problem, but she asserted that knowing procedures is necessary for solving the problem. She said that "*you should analyze and use data correctly with paying attention to the procedure…knowing procedures is important" (NM1).* For Hazal there should be a solution and an exact answer for a mathematical problem. Meltem also stated that "*the shorter solutions are good solutions"* (NM1).

Group 2 – Teachers Holding Mainly NM1 and NM3 Beliefs

This group includes teachers (Alp, Mert, Filiz, and Rengin) who mainly held instrumental (NM1) and problem solving (NM3) beliefs before participating in the PDP. When their responses are investigated, it was seen that these group of teachers focused on themes indicated both problem solving and instrumental beliefs. While defining mathematics, describing nature of mathematical knowledge, mathematical problems and problem solving, these teachers focused mostly on certainty, rules, accuracy, calculation, finding/reaching the correct results/answer of the problem, and doing operations (NM1), however, they also considered mathematics as dynamic, a human product, a base for other science, which foster systematical/logical thinking and also they appreciated different ways of solutions and students' developing their own solutions during problem solving (NM3).

For example, in the first interview conducted with Alp, he described mathematics both as a form of relationship (NM3), focused on abstractness (NM3) and calculation (NM1). He delineated that "*mathematics* is *relationships between objects, a written form of this relationship….it starts with concrete things, and then goes to abstract with its rules…you calculate on things that you don't see in abstract mathematics*". Alp considered mathematics as related with and base for other science (NM3) and at the same time, he focused on accuracy and certainity (NM1). He said that mathematics is important because "*it contributes to the development of society and other sciences...it is base for other sciences like physics, chemistry, engineering, etc. All these are application of math*". According to Alp, words such as *accuracy, beauty, certainty, calculation* describe the mathematics. He said that "*accuracy in mathematics signifies that its rules and what it do is accurate*" (NM1), "*beauty denotes for the beauty of theorems and proofs, and beauty of what is inside math*" (NM3), "*certainty means that its results are certain*" (NM1), "*calculation produce correctness, and this is the nature of mathematics*" (NM1).

Mert, on the other hand, focused on thinking systematically and reasonably, and contribution to science (NM3). He defined that mathematics is "*thinking systematically and reasonably…thinking reasonably can be used in everywhere in life"*. Mert also added that *"the purpose of mathematics is to contribute to science….mathematics is important because it contributes to thinking systematically, interpreting, and being aware of living in society"*. In addition to thinking rationally, Mert also stressed "*following rules*" (NM1) when he described the words defining mathematics beside the other words such as *thinking rationally, developing skills* and *intelligence, contributing to technology*.

Being in this group, Filiz's descriptions of mathematics also revealed instrumental and Platonist belief themes. Similar to Alp, Filiz also stated mathematics as a form of relationship (NM3). Besides, she focused on reasoning, analyzing, and beauty of mathematics (NM3) while describing mathematics. For example, Filiz declared that mathematics is "*ability to analyze", "establishing relationships between events or variables", "includes reasoning and analysis…develops human brain… beautiful and enjoyable"*. On the other hand, she also stressed the "solution" when describing the mathematics (NM1). Filiz delineated that mathematics is *"producing and reaching to a solution"* Filiz also asserted that mathematics problem solutions are fixed (NM1). She stated that said that "*I love math because a historical event can be interpreted differently by different countries, but in mathematics questions are always solved in same way, it is not shimmering, because of these reasons I admire math*". As parallel with her description of mathematics Filiz stated the words that describe mathematics as "*analysis*", "*an abstract science*", "*making relationship between data*", and "*interpreting*" (NM3). However, Filiz said that "*performing operation*" and "*finding results*" are other important words for mathematics because "*these two are about life…because in life you have always data in your hand, if you use them correctly you can reach the results…the important thing is reaching results*" (NM1).

Rengin, on the other hand, described mathematics by focusing on daily usage of it (NM1). She described that "*mathematics is meaning of everything…it is used in everywhere…everything is a chance, a probability, a calculation…matter of truth. Everything in real life has a mathematical expression".* Rengin said that mathematics is important because "*it is base for all sciences" (NM3) "even for literature and for geography…mathematics is used everywhere. Those who seems not related with mathematics use mathematics in his/her life. For example, when buying bread s/he gives money and calculates the change…when s/he looks at cholesterol level in her/his blood test results*". As seen Rengin's conception of mathematics is utilitarian (NM1). According to Rengin, words that describe mathematics were *numbers, equality, life,* and *endlessness (eternity).* Rengin said that mathematics wouldn't be existed without numbers; it is based on numbers (NM1). She stated that equality denotes for certainty in such a way that "*mathematics likes certain answers; math knowledge is either right wrong, like black-and-white*" (NM1). Rengin stressed that life is denoting for mathematics because "*the humanity's existence is depends on probability which is mathematics*". Rengin also said that eternity means that "*mathematics will be developing*" (NM3).

Their responses about the development of mathematical knowledge indicated that all four teachers believed that mathematics is dynamic (NM3). Alp stated that "*People in the past found mathematics, still people are finding...development of math occurs step by step, with small increments. Some produces a theorem, other proves a part of it…mathematics is developing with small increments now, but in the past the developments was bigger*". Similar to Alp, Mert consider math as dynamic, however, contrary to Alp, Mert stated that mathematical knowledge in the present is more detailed. He said that "*there is much knowledge than in the past…knowledge is more detailed now...as technology develops, math knowledge* *develops…since technology needs mathematics*". Like Alp, Filiz considered mathematical knowledge as developing, and she stated that "*hypothesis in math has been changing in time as they were proved. As human becomes more effective in using the knowledge in their hands, and as the technology develops, mathematics is changing gradually*". Similar to Alp, Mert and Filiz, Rengin also described math as dynamic; however, she connected dynamicity of mathematics with human needs. Rengin said that "*As a child grow older, s/he needs numbers…then s/he needs other things...math develops because of needs of humanity and as they learn…mathematics is continuing to develop*". It was also seen that both Alp and Rengin consider mathematics as human product (NM3).

One of the common aspects in Alp and Mert responses is that both teachers considered either mathematical knowledge (NM2) or rules and results are certain (NM1). Alp stated that "*mathematical rules are certain, proved and validated. There is no ambiguity or doubt...A theorem in physics, for example, gravity theorem, can be rejected but , in mathematics rules, for example, volume of a cylinder cannot be rejected I guess…Because its' base is strong, it is proved and validated*". Mert also considered mathematical knowledge, rules and results as certain, he expressed that "*there should be certainty in mathematical knowledge because it is based on rules. Certainty in results is compulsory in mathematics, because if there wouldn't been certainty, there could be contradictions in society, and this could affect life".* He also stated that *"there is no doubt in mathematics because it is science…science has no doubt, it has certainty*". Contrary to Alp and Mert, Rengin and Filiz pointed out that there may not be certainty in mathematics. Filiz told that, mathematics may not be certain because "*it is based on hypothesis and assumptions" (NM3).* Rengin, on the other hand, said that there may not be certainty in mathematics but there is clarity in it (NM2). She stated that "*there is certainty when there is equation…but, mathematical knowledge we use in one system can be different in another system…however, there is clarity in mathematics*".

From their responses to the questions related with mathematical problem and problem solving, it was emerged that some of the teachers focused on calculations, numbers, operations, and formulations while describing the mathematical problem, and stressed obtaining a result or answer while explicating problem solving (NM1). For example, Mert said that "*a problem which is related with numbers is definitely related with mathematics". He described problem solving as "*applying the knowledge you learned before on the problem, doing calculations*" (NM1). According to Filiz, a mathematical problem is "*an event that must be solved, which includes some data and unknowns*", whereas the problem solving is "*resolving the issue by using the data we have…producing solution"* (NM1). Rengin on the other hand delineated mathematical problems by stating that "*we search for a result for the events that we encounter in life by formulation and use mathematical and algebraic operation*" and described problem solving as "*reaching to a certain result by doing certain mathematical operation"(NM1).* Alp, on the other hand characterized mathematical problem as "*an issue that must be solved…a problem that its solution can be done with mathematics*" and described problem solving as "*trying to find thing that you are looking for*" (NM3). Besides, both Mert and Rengin added that mathematical problems are in life or encountered in life. Mert said that "*when you construct a building, how much iron you will use, how many floor you make, how to make column, etc. these are problems".* Rengin also exemplify a mathematical problem as "*what is done in daily life, for example; everybody who has a car calculates fuel consumption".* Alp, on the other hand delineated mathematics problems differently; he said that "*math problems are specially designed questions for students to solve. For example, in geometry there are questions asking degree of angle, trigonometric equations, cotangent of an angle etc. What is problem here is that you are trying to find thing that you are looking for*".

Although Rengin's expressions indicated a belief about mathematical problem solving as searching an end result, she also considered "understanding", "interpretation", and "logicalness of results" as important part of problem solving (NM3); she said that, "*understanding the problem, and expressing it in* *mathematical sentences is very important…calculation is less important while interpreting and checking whether it is logical or not is more important*". For Filiz, on the other hand, "deciding the operation conducted" (NM2) and "establishing the relationship between data" is critical when solving problem (NM3); she said that "*ability of doing operation…I mean deciding operation…calculators can do operation but, important thing is that students themselves should decide which operation will be performed…having the mathematical data is not important, important thing is to use data, establishing relationship between data".*

For Alp, every problem has a solution and there can be more than one solution for a problem. However Alp said that "*a good solution can be shorter or more practical or different*". But, Alp also added that "*I don't want memorized way, I want students to think on, contemplate and try their own steps*" (NM3). Mert considered that there can be more than one solution for a problem; he said that good solution depends on students; however, the practical and understandable solutions are more preferable. Filiz also said that *"I write different ways of solutions on the board and say student that you can solve the question by whichever method you want*". Filiz stated that she says students that "*if you solve correctly, I accept your way*". She also added that a student can use geometry to solve a math question; s/he doesn't have to use formulas as long as s/he solves correct (NM1). About solutions of a problem, Filiz said that sometimes she preferred the short solutions but generally told students that "*good solution is your solutions*" (NM3). Filiz stated that she did not force students to use her way. She also said that "*when we were students, teacher said that "solve this way", I was very angry with it because it prevent thinking…reasoning, this is wrong for me*". About solutions of a problem/question, Rengin said that "*for some questions there could be one solution, like polynomial or function questions…but I appreciate students developing different ways*" (NM3). For Rengin good solution is a solution that she couldn't think before. She added that "*I do not like if it is different, I like because sometimes it is short solution, for example my proof or solution can be long, then students says we can use this one…what I like it about is that student comprehend the previous content and apply it in the question. Although I didn't* *solve similar question…student had learned, and compare it in his/her mind and apply it in different question…Relating between the concepts*". Rengin pointed out that she can think that a problem has more than one answer, but she told that students want exact results. She said that "*for example in parabolas we can find two values, or in angle-side problems, we find more than one value*". However, Rengin told that "*students don't like "what can be" questions. That is students want mathematics somehow exact/clear. Questions like "which values can be for x, or how many values can be in Z" is somehow not be liked by students*".

Group 3 – Teachers Holding NM1, NM2 and NM3 Beliefs

This group includes only two teachers (Sude and Ayla) who held every category of beliefs before participating in the PDP. When describing mathematics, Sude focused mostly on utilitarian aspects of mathematics (NM1). Sude stressed the daily life usage of mathematics. She defined mathematics as "*what ease our life*", "*it is in everywhere, in our life*". However, she also added that mathematics increase *reasoning ability* and *perception (NM3)*. Sude said that mathematics is important because "*it is base of our life…a science that arranges/orders our life… we use when making daily plan…it is a job for me, my philosophy of life, I decide all my life according to mathematics*", "*in our daily life we always use math, even sometimes not aware of it, for example in daily time planning*", however she said that "*mathematics is base for other sciences*" (NM3). According to Sude, words that describe mathematics were *numbers, life, intelligence*, and *order.* Sude said that "*when I think math, the first thing comes to my mind is numbers, it is related with our life, a base, related with intelligence, and since it is a collection of systematic knowledge it brings order*" (NM1).

Ayla, on the other hand, described mathematics by emphasizing "logic" and "correctness" in her descriptions (NM2). Ayla described mathematics as "*a part of life"* and said that "*math shows how to think correctly".* Although, she considered that mathematics is everywhere in life, she declared that mathematics is important for those who are aware of it. She said that "*mathematics is everywhere in life, we use it always. Most people are not aware of it, so, math is not important for them,* *but I am aware of it, and it is my job so it is important for me. Students will use math in their future life in exams. Therefore, mathematics is important for them"*. As it is seen, for Ayla mathematics has a pragmatic function (NM1). Ayla stated that the words "*working", "logic", "thinking correctly", "correctly interpreting"*, and "*satisfaction"* are the words that describe mathematics *(NM2)*. Her descriptions revealed that Ayla consider correctness and logic as the inherent qualities of mathematics. She said that "*everything in mathematics is very logical. As you take its logic, everything goes in correct way. At the end you find one correct solution and you satisfy*" (NM2).

For Sude, mathematics is a human product (NM3). For the development of mathematical knowledge Sude said that "*mathematics is developing since the ancient times…sometimes development occur rapidly as the technology develop, sometimes it is slow…Development of mathematical knowledge is as a result of human needs and the development will continue*". Sude also added that as teacher they can only apply mathematics, they do not develop mathematics. Ayla, on the other hand, stated that mathematical knowledge is not certain (NM3). She said that "*there are theorems and their proofs, it is okay, but also there are things that are changing"*. Although Ayla said that some of the mathematical knowledge is not certain, she, however, considered that mathematical knowledge is static (NM2). Ayla said that "*there is no development in mathematical knowledge…we always use the same knowledge…we do not produce something new, we use the ones already existed*".

Sude's response to the question about truth or falsity of mathematical knowledge indicated that she considers mathematical knowledge as authoritarian (NM1). Sude stated that "*I accept mathematical knowledge, I do not judge if it is true or false… I accept that rules as correct*". Moreover, according to Sude, mathematical knowledge is certain (NM2). She explained that "*like we accept axioms, we accept mathematical knowledge as certain…we encounter with doubt only in probability concept"*. Sude also stated that the mathematical knowledge used is always same (NM2), which indicates a static conception of mathematics. She said that "*we* *accept the already exist mathematical knowledge, and solve new problem, but we don't find new knowledge, what is found does not contradict to past knowledge, it is maybe a new theorem but it is based on the already existed knowledge*".

When describing mathematical problem, it was revealed that Sude focused on numbers and results (NM1). Sude stated that "*mathematical problems are the problems consist of numbers. A system which has a target, an aim to obtain a result, has a reason and a result"*. She added that problem solving is "*reaching a positive result…If you can solve it, it means that you reach to a result, then it is positive*". On the other hand for Ayla, a mathematics problem is a real-life problem (such as, employer-pool, distance-speed). She said that questions such as finding sinus of an angle are not mathematical problems; they are application questions (NM2). Ayla emphasized "*reaching a result*" for the problem solving (NM1), but she also emphasized *"reasonin*g" (NM3). Ayla said that problem solving is "*reaching the results by using mathematics and a chain of reasoning*". Her responses showed that Sude considers the understanding problem and relationship between knowledge at hand and previous knowledge (NM2). Sude said that "*you need to understand the problem and know to make connections with their previous knowledge"*. Also Sude indicated that there is more than one way of solutions for a problem and using more than one way to check the correctness of answer is important for problem solving (NM3). Though accepting existence of more than one way of solutions, Sude indicated some types of solutions as more preferable and better than others. She explained that short solutions are more understandable and there is less chance of making mistakes in short solutions (NM2). Sude denoted that "*short and clear ways are good ways of solution, because they are comprehended better… because the longer the way, the more you make mistakes or operational error"*. Ayla, on the other hand, pointed out that "*sometimes there is one, sometimes there are more than one solution" (MN3), however*, she also added that *"there is best solution if it is practical, different, new, short and subtle*" (NM2). Apart from these, according to Sude knowing formulas and facts is important for problem solving (NM1). She said that "*students should keep in mind the mathematical formulas and facts to solve problems, because they may not have* *time to produce formula, not in course time, or in the exams"*. Sude explained that since they teach everything in math, memorizing/keeping in mind the formulas saves time.

Group 4 – Teachers Holding Mainly NM2 and NM3 Beliefs

This group includes only one teacher (Kadri) who mainly held Platonist (NM2) and problem solving (NM3) beliefs before participating in the PDP. When describing what the mathematics is, Kadri focused on mathematics being systematical (NM2). He stated that "*mathematics is a systematic way of thinking, it teaches to see life, and solve the problem in life systematically, solving step by step, evaluate, approach to problems…how to use data to obtain result*". For Kadri, mathematics is important because "*it makes life easier, although it makes education more difficult*" (NM2) and "*there is mathematics in sciences*" (NM3). According to Kadri, words that describe mathematics were *unchanging rules/formulas, trust, facilitator/functional, developing*. Kadri said that "*since the base of mathematics is strong, rules and formulas are unchanging" (NM1), "we trust rules, formulas so that we use mathematics in several areas in life such as in clocks, measurements, addition/subtraction, etc." "Mathematics is functional so that it is used in other sciences…mathematics is always developing because of the needs" (NM3).*

For the development of mathematical knowledge, Kadri said that "*mathematics has been developed because of the needs of humanity…in time; humans learned different mathematics because in their life they needed it*" (NM3). From his response about truth or falsity of mathematical knowledge, it was revealed that Kadri considers textbook as authoritarian resource for mathematical knowledge (NM1). Kadri also considered that mathematical knowledge is certain (NM2). However, he considers certainty as similar to non-ambiguity and said that mathematical knowledge is certain because "*humanity had developed mathematics" (NM3) "…and we are sure because we have developed*" (NM2).

When describing mathematical problems and problem solving, Kadri focused on structural properties; however, he also emphasized understating the problem and relating knowledge with previous knowledge (NM2). Kadri defined that a mathematics problem is "*a question that is described and designed in mathematics language…for example, "what is the remainder for division of 2011x2012 to 5" is a problem*". Kadri described problem solving as "*understanding problem asked and solving it using data given in problem*". Kadri said that "*it is important to connect/relate what you know (previous knowledge) and what the problem is asked for and the data*". Moreover Kadri argued that although not regarded in the educational system, producing different ways of solutions is important in problem solving (NM3).

Group 5 – Teachers Holding Mainly NM3 Beliefs

This group includes only one teacher (Kerim) who mainly held problem solving beliefs (NM3) before participating in the PDP. When describing mathematics Kerim focused on "interpreting", "decision making" and "problem solving" (NM3). He stated that mathematics is *"an interpretation, knowing self, struggling…a lesson that develops decision making and taking initiative", "knowing math provides you an advantage in modern society…this is the greatest aim of math", "It is not important to know integral, derivative etc. My aim is not to teach math topics. The important thing is making interpretation, taking risks, having skills of struggling, making decisions".* Kerim said that mathematics is important because "*it teaches how to make interpretation, take risk, struggle, think, and solve problems in life by themselves*". Kerim also stated that "*mathematics is a tool…people who know mathematics do not look at events unidirectional, they can learn everything easily*". Parallel to these words such as "interpreting", "making decision", "taking initiative and risks", "struggling", "trusting self", "knowing self" are the those that describe the mathematics best.

According to Kerim, mathematics is human product and mathematical knowledge has been developing because of curiosity and skepticism of human (NM3). Kerim stated that "*those who are skeptical and curious develop mathematics by applying* *trial-error methods*". For Kerim philosophy is important than mathematics because, "*theories wouldn't have been developed without thinking*". Kerim said that "*mathematics is human's collective product…it is developing as human interact with each other*". Kerim considered that mathematical knowledge is not stable; it is always changing (NM3). He explained that "*stability in mathematics is impossible because, society and people are changing*". About mathematical knowledge, Kerim denoted that mathematical knowledge may not be authoritarian (NM3). He said that "*I can decide the truth of any mathematical knowledge based on logic and rationale….and something that is a truth for me might not be correct for someone else*". For Kerim, rules in mathematics are certain, however, "*new knowledge should be verified by mathematicians*" (NM3). Kerim said that "*although there are certain things in mathematics, nothing is unchangeable…Maybe in the future rules of mathematics can be change*" (NM3). Moreover, Kerim said that mathematicians should have some doubt; "*I can defend that 2x2=3, when I say this, some says you are wrong, some may be skeptical about it…then new things will develop with another idea, with interaction."* However, when talking about solutions of problems, Kerim said that mathematical rules are certain (NM1). He said that "**y***ou cannot change the general rules of mathematics, but you can interpret it differently, the same song can be interpreted differently by different singers, similarly every teacher's solutions should not be same*".

Kerim described the mathematical problem as problematic situation. He stated that a mathematical problem is "*any issue that has to be dealt with/solved…when you read a question, if there is something that you don't understand then it is a problem*". Kerim focused on "interpreting" when describing about problem solving. For Kerim, problem solving is not applying the rules but rather it is interpreting (NM3). He described problem solving as "*dealing with the issue*", "*struggling with the problem which is either mathematical or real life, by interpreting the data in the hand*". Kerim said that "*3x+4=2 is not a problem because, there is no interpretation. You apply the rule and find the result*". Moreover, it was revealed that Kerim appreciated the different ways of solutions for problem solving (NM3). For Kerim, "*solution of a problem can change person to person…everybody has different was of solution…best way is your own way*". Kerim said that he do not encourage students to use his methods or ways, he wants them to find their ways if they are mature enough, because they can use their ways or logic, but they forget others' ways or logic. Kerim also added that he do not give much importance to solutions being short or long or practical, rather solution ways being different is more important.

4.1.2 Teachers' Beliefs about Teaching Mathematics

Teachers' beliefs about teaching mathematics were described based on data gathered from semi-structured interview conducted with teachers in the middle of the PDP and the analogy questionnaire conducted at the beginning of PDP. In the interview, teachers were asked several questions (see Appendix C, D, E, F) about teacher's roles, characteristics of good mathematics teachers, purpose of mathematics teaching, description of their own teaching and ideal mathematics class and their expressions as response to these questions were coded considering the categories of beliefs about mathematics teaching proposed by theoretical framework. Coding process included two phases; in the first phase, the themes or the idea emerged from the data (teachers' expressions) were identified, and then, in the second phase, these themes are matched with the sub-categories of beliefs about teaching mathematics (T1, T2, T3). Themes for the beliefs about teaching mathematics emerged from teachers' expressions are presented in Table 4.3.

Table 4.3 Themes about teachers' beliefs about teaching mathematics

	Beliefs about Teaching Mathematics		
	T1	**T2**	**T3**
Meltem	Transferring the content (as teacher's role) Teacher as who has larger perspective Skill development & pragmatic (as purpose of math teaching) Content focused (strict curriculum follower)	Foster conceptual understanding (reason behind procedures and relations among concepts) Explaining daily life usage of concept Explain the content broadly, enlarge students' perspective (as teacher's role) Considering students perspectives occasionally (to communicate) Considering students' thinking (to prepare herself about the answers of possible student questions) Use of materials for visualization	
Hazal	Giving content and then solving too many question to practice formulas (way of teaching) Recalling formulas Teacher as shaper (of students) Teacher as instructor /director Skill mastery (solving too many test questions) More questions, more computation, less mistake	Teacher as explainer Arranging sequence of questions Students attending in lesson (for motivation) Relating new and previous concepts/formulas (with questions) Explaining different ways (for motivation) Teaching how to use formulas	Encouraging/appreciating different ways of solutions Developing mathematical thinking
Ayla	Teacher as instructor (give content-solve example-distribute test) Solving too many questions Teacher has the control Teacher centered (teacher tells-students listen; teacher asks-students answer) Students attend but do not have control	Telling real-life usage/examples (to motivate/take attention) Teach logical thinking Teacher as explainer Pay attention to students' questions Misunderstanding/difficulties are considered after covering the content Against memorization- Pay attention to steps of solutions (how to reach an answer is important than answer)	

Table 4.3 (Continued)

Alp	Avoiding to evaluate students understanding Teacher as instructor Transferring knowledge Teaching as making students to do operations, to have background knowledge for their future Teacher dominant class (quiet, neat, everyone listens teacher, teacher explains and solves questions)	Considering students' concept image Consider students' levels (while solving questions and explaining the content) Considering students response while transferring knowledge Considering students motivation to learn Making connections (relate previous concepts with new ones) Using analogy and visual examples Showing proofs (indicate where the formulas are come from) First teacher-then student solve Using history of math occasionally (to enlarge students views)	
Sude	Basic knowledge for problem solving Misconceptions &deficiencies are covered by listening to teacher	Considering level of students during instruction Using real-life examples occasionally (to take attention) Making connection with previous concepts Encourage students to ask questions Logical and systematic thinking (as purpose of teaching math)	Making comparison and reasoning
Mert	Transferring knowledge Solve too many question for teaching	Teacher as explainer (first teacher explains and solve-then student solve) Considering students levels (while arranging the pace of lesson, types of questions, sequencing the content) Relating with previous topic (when introducing new concept) Taking students' attention/making them attend in the lesson, knowing them(for motivation) Material and visuals use (for motivation) Indicating reasons for learning (for motivation) Foster thinking logically & systematically (as purpose of teaching math)	

Table 4.3 (Continued)

Rengin	Recalling frequently Teacher dominant-students attend	Teacher as explainer (recall previous topic-explain the content-provide proofs-solve examples) Considering students' understanding (listening students ideas) Explaining the sequence-relation with other concepts-daily use of concept Paying attention to the sequence of questions Encouraging students to solve question (for motivation) Changing explanations considering students' questions Explaining the reasons behind formulas by proving Explaining the reasons of solution methods Presenting more than one way of solutions Considering why and how(s) while preparing content	
Filiz	Solving too many questions (to consolidate) Correcting students errors & misconceptions	Making connection with previous concepts (while teaching new concept) Considering the mathematics contents as related Considering students background knowledge as important Starting new concept with interesting question (to motivate) Teacher as explainer (as a person who makes concept more understandable) Arranging sequence of questions Encouraging students' explanations Paying attention to students' ideas	Providing different questions to make connections, to interpret

Table 4.3 (Continued)

Kerim		Making connections and reminding previous concepts Explaining the content (core)-supporting with understandable examples Considering students’ understanding First teacher -then students solve Considering students’ background knowledge Solving not too many but different questions Checking students’ awareness Considering students’ motivations and interests Teacher as active-students allowed to make interpretation (activeness changes according to level of class) Changing teaching considering students’ level Selecting examples considering students’ levels	Discussing with students (about solutions)-allow students’ asking questions Trying to make students develop a concept image Students’ learning is more important than curriculum Encouraging students attend in/deal with/strive for the lesson Teaching thinking and taking initiative/how to think and reason (teach fishing, not to give fish) Encouraging students develop/produce their own questions Responding students ideas Not teacher dominant-mutually asking questions
Kadri	Solving questions to consolidate Correcting students errors Teacher as controller (students as test subject)	Selecting examples considering students’ levels Trying different methods/use different ways to explain concept First teacher then students solve Encouraging to mimic solutions Using analogy and visuals Changing instruction according to students difficulties Considering students’ understanding (focused not content but students’ understanding) Making connections Representing different ways	Considering students’ needs Appreciating students effort to solve Teacher as facilitator Encouraging students to think in a different way Encouraging students aware of their errors Considering to follow a strict curriculum as contradictory to spirit of teaching math Creating discussing environment Teacher as guider

Results about themes for the beliefs about teaching mathematics as presented in the Table 4.3 show that (i) four teachers (Meltem, Hazal, Ayla, and Alp) held mainly T1 and T2 beliefs; (ii) four teachers (Sude, Mert, Rengin, and Filiz) held mainly T2 beliefs; (iii) two teacher (Kadri and Kerim) mainly T2 and T3 beliefs. Teachers' beliefs about teaching mathematics are presented in the Table 4.4 below.

Table 4.4 Teachers' beliefs about teaching mathematics before PDP

	Beliefs about Teaching Mathematics		
	T1	**T2**	**T3**
Meltem	✓	✓	
Hazal	✓	✓	
Ayla	✓	✓	
Alp	✓	✓	
Sude		✓	
Mert		✓	
Rengin		✓	
Filiz		✓	
Kadri		✓	✓
Kerim		✓	✓

Based on teaching mathematics belief categories held, teachers' beliefs are categorized into three groups;

- Group 1 – Teachers holding mainly T1 and T2 beliefs
- Group 2 – Teachers holding mainly T2 beliefs
- Group 3 – Teachers holding T2 and T3 beliefs

Group 1 – Teachers Holding Mainly T1 and T2 Beliefs

This group includes teachers (Meltem, Hazal, Ayla, and Alp) who mainly held T1 and T2 beliefs before participating in the PDP. The commonality among this group of teachers about their responses to the questions asked in the first interview is that while describing teacher roles, characteristics of good mathematics teachers, purpose of mathematics teaching, descriptions of their own teaching and ideal mathematics class, these teachers focused mainly on teacher roles as transferring the content, instructor and director who is knowledgeable and controller of the classroom activities, teaching as based on skill development, content-focused, and best way to teach or ideal teaching as solving too many-different types of questions with students (T1). On the other hand, it was seen that these teachers also focused on fostering students' understanding by motivating them, relating concept taught with the previous concepts, paying attention to the sequence of questions asked during teaching, encouraging students occasionally to attend in the lesson, ask questions, and solve questions after they were showed how to solve, explaining where the formulas are come from, etc., (T2).

For teacher roles in teaching mathematics Alp said that "*teachers are who should know everything and transfer it*". Moreover, he described ideal classroom and teacher's activities in it by saying that "*an ideal class must be quiet, everyone should listen to teacher, teacher tells the content, solves the examples, students either solves on board, or in their places*". Meltem explained teachers' role in such a way that "*my role is active; giving the content actively, explaining the content in a broad ways…teachers are more active since students can view the content in a narrow aspect*". Additionally, Hazal described teachers' role as who is authority in class. She denoted that "*the initiative belongs to teachers, teaching is teacher based, I mean everything is in our hands….you are the authority…instructor."* Also data obtained from the analogy questionnaire supported the beliefs about teacher roles emerged from the interview data. For example Ayla said that *"Mathematics teacher is like a cook trying to make the most delicious food with the ingredients, namely students"*. From Ayla's expressions, it was revealed that Ayla thought that the person directing and controlling the teaching job is the teacher. Her explanations can be considered an evidence for T1 beliefs.

It was seen that these group of teachers consider teaching as based on skill development and content-focused (T1). Additionally, they highlighted solving too many-different types of exercise questions as the best way to teach mathematics (T1). When explaining the aims considered while designing teaching Meltem denoted that "*teachers follow the contents of the curriculum, delivering the content is my responsibility*". Meltem also asserted that one of her purposes of mathematics teaching is students' skills development. Moreover, Alp said that "*purpose of the mathematics teaching is making students do computations, concepts*". Aligning with these, Hazal asserted that "*as you solve more questions in class students' subject knowledge increases and their errors reduces*". Hazal also said that "*after giving the content, it is important to consolidate it by solving too many questions.*"

Beside their T1 beliefs, results revealed that this group of teachers considered making relation among the concepts, explaining where the formulas are come from, paying attention to the sequence of questions and making students to solve questions after they were showed how to solve, and encouraging students to attend in the lesson and ask questions as important for teaching a mathematical concept (T2). All these indicated that teachers also paid attention to student' understanding (T2). When explaining the characteristics of a good mathematics teacher Meltem described that occasionally teachers should view from the students' perspective. It was understood that Meltem considered viewing from students' perspective to foster students' understanding. She said that "*a teacher should look at content from students' perspective, that is, what students think about when I explain this, what they can ask me, how I can answer to make them understand*". Hazal said that "*my purpose of mathematics teaching is not make students memorize the formula, but to make them like mathematics, make them understand and use formulas*".

Group 2 – Teachers Holding Mainly T2 Beliefs

This group includes teachers (Sude, Mert, Rengin, and Filiz) who mainly held T2 beliefs before participating in the PDP. The commonality among this group of teachers about their responses to the questions asked in the first interview is that while describing teacher roles, characteristics of good mathematics teachers, purpose of mathematics teaching, description of their own teaching and ideal mathematics class, these teachers focused mainly on roles of teacher as explainer who recall previous concepts, explain the new content, and solve questions; purpose of mathematics teaching as logical and systematical thinking; content of mathematics as being related with each other; teaching mathematics as fostering students' understanding by motivating them, relating concept taught with previous concepts, paying attention to the sequence of questions asked during teaching, encouraging students occasionally to attend in the lesson and ask questions, solving questions after teacher were showed how to solve, explaining where the formulas are come from, etc., (T2).

This group of teachers considered teachers as persons who make explanations for the mathematical concepts to make it more understandable for students. For example Mert said that "*teacher's role in the lesson is to make explanations for the concepts*". Similarly, Filiz stated that "*a teacher is who makes new concept understandable*". In a parallel way Rengin said that "*in the past my aim was to present the curriculum, and solve related question…but, then, I began to pay attention to how to make students comprehend and focused on how to solve*". Also data obtained from the analogy questionnaire supported the beliefs these group of teachers' beliefs about teaching mathematics emerged from the interview data. For example Sude said that *"mathematics teacher should take into account individual differences as a conductor and uses teaching techniques."* Sude's expression was showed that she highlighted use of teaching techniques considering students' individual differences (T2).

It was revealed that teachers attach importance to explain the reason behind the rules or to explain where the formulas are come from since they care about students understanding. For example Rengin said that "*it is important to show the reasons of formulas…both for its use and for not to be memorized"*. Another common belief among this group of teacher is their beliefs about the purpose of mathematics teaching that stress the logical and systematical way of thinking. For example Ayla told that "*the purpose of mathematics teaching is to teach logical thinking and convenances*". This group of teacher focused on the importance of relating concept taught with the previous concepts, to make it more understandable. For example, Filiz stated that "*in other courses subjects may be disconnected from each other, but it is different in mathematics…a friend of mine* *told me that you make relation with previous concept and it becomes meaningful*". In a similar vein, sequence of questions asked during the lesson, for example a sequence according to the difficulty level or a sequence based on relation with the concepts were considered important for students understanding. For example, Rengin said that after seeing students learning, she paid more attention to sequence of questions. She denoted that "*only preparing the content is not important, I realized that how important the content, you give content, but it is not enough, the questions you solve is also important, the sequence of questions*". Among this group, another common belief was related to students' attendance in the lesson. Teachers highlighted that student should be encouraged to attend in lesson by asking questions. For some teachers students' attendance was important for motivation, while for some, it is important to understand what students think about the concept taught. Ayla said that "*students direct teachers thinking with the questions that they asked to teacher, because I pay attention to students'*. Moreover, for this group of teachers' students can attend in lesson by solving questions on board, however, they should solve a question after teacher showed them how to solve it.

Group 3 – Teachers Holding Mainly T2 and T3 Beliefs

This group includes teachers (Kadri and Kerim) who mainly held T2 and T3 beliefs before participating in the PDP. The commonality among this group of teachers about their responses to the questions asked in the first interview is that while describing teacher's roles, characteristics of good mathematics teachers, purpose of mathematics teaching, and description of their own teaching and ideal mathematics class, these teachers focused mostly on making connections with previous concepts, considering students' level-understanding- difficulties-motivation-and interest during instruction or when selecting examples, explaining the content using different ways, methods, or different examples, encouraging students solving questions after teacher show how to solve, etc., (T2). However, it was revealed that Kerim and Kadir also held T3 beliefs. Their expressions to the questions related with mathematics teaching indicated that Kerim and Kadir considered mathematics teacher as facilitator and guider who should response students questions and pay attention to their needs, encourage students to think, to reason, and to develop their own ways of thinking or solutions, create a discussion environment and discuss students about their solutions.

For Kerim and Kadri, teacher's role in mathematics teaching is being a facilitator and guider. Kadri indicated that "*teacher should not be dominant in teaching, students are also set forward*". Also data obtained from the analogy questionnaire supported the beliefs emerged from the interview data. For example, Kerim denoted that mathematics teachers resemble coaches. He explained that *"a coach is addressing to a group, S/he teaches and guides them to make them successful, makes plan…produce...like a teacher."* From Kerim's expressions, it was seen that he highlighted being guide and making contribution while describing mathematics teacher. His explanations can be considered an evidence for T3beliefs.

Both Kadri and Kerem attached importance of making students to think. For example, Kadri stated that teachers should not solve everything on the board; otherwise students do not have opportunity to think. Kerim also denoted that "*teachers should not hinder students thinking, but should encourage them to think. He stated that teachers should respond students' ways of solutions with logical explanations. I consider this as most important feature of mathematics teaching*". Moreover, both Kadri and Kerem asserted that curricula and teacher must be flexible and should consider students' needs. About curriculum, Kadri said that "*mathematics curricula should be based on students' needs*" while Kerem denoted that "*teachers can use curriculum flexibly considering the levels of students".*

4.1.3 Teachers' Beliefs about Learning Mathematics

Teachers' beliefs about learning mathematics were described based on data gathered from semi-structured interview conducted with teachers in the middle of the PDP and the analogy questionnaire conducted at the beginning of PDP. In the interview, teachers were asked several questions (see Appendix C, D, E, F) about students' roles, characteristics of successful and unsuccessful students, indication and evaluation of students learning and understanding mathematical concepts, how students learn mathematics best, description of students' learning in their own classroom and learning in an ideal classroom, and their expressions as response to these questions were coded considering the categories of beliefs about learning mathematics proposed by theoretical framework. Coding process included two phases; in the first phase, the themes or the idea emerged from the data (teachers' expressions) were identified, and then, in the second phase, these themes are matched with the sub-categories of beliefs about learning mathematics (L1, L2, L3). Themes for the beliefs about learning mathematics emerged from teachers' expressions are presented in the Table 4.5.

Table 4.5 Themes about teachers' beliefs about learning mathematics

	Beliefs about learning mathematics		
	L1	**L2**	**L3**
Meltem	Mental calculations Possessing knowledge and skills Recall and practice to learn Willingness to show performance Listening in class & practice at home Making practice (students' role) Students as who have narrow perspective Solving practice questions (to understand)	Encouraging to ask (if they don't understand) Solving thought provoking questions to learn Making connections/relation with previous concepts Explaining reasons behind principles (for permanent learning) Students being active by asking questions	
Hazal	Students performance on questions (indicate understanding) Memorizing formula Saying/recalling formula (indicate listening/learning) Students as listener/follower of lesson/receptor Students performance in tests (indicate their deficiencies)	Relating concepts/formulas(checked with questions as indication for understanding) Focusing on students solutions (to understand their learning)	Developing ideas is important (develop different way/ explain it and defend it)
Ayla	Students as listener (listen & take notes & review/practice at home) Recalling to overcome learning difficulties Performance on questions (immediately solving indicate learning)	Evaluating learning from students responses Memorization is discouraged Process of solution/how to reach answer	Learning by doing/seeing
Sude	Learn from one who knows Listen & review & practice Correct performance and asking high level question (as indication of understanding) Producing short and clear answers	Not memorizing but knowing the reason behind Making connection between concepts Learning as depending on teachers, teaching methods, and encouraging students thinking in different ways Attending in lesson and expressing him/herself	Make interpretation and comparison Learn by doing/living

Table 4.5 (Continued)

Mert	Students as listener Listening, writing, reviewing, doing homework (to consolidate learning) Ability to solve question (as indication of understanding) Solving too many questions for learning Performance on homework as indication for learning	Trying to do by own Background knowledge Needing to learn Understanding where the rules are came from (reason behind) Not memorizing	Communication (for learning)
Alp	Students as passive receptor Understanding as correct performance (doing operations & solving questions) Listening carefully & taking notes & doing homework Solving too many questions for learning	Correct answer (may not indicate understanding, but rote learning) Background knowledge (for connecting new knowledge and understanding) Diagnose their deficiencies by themselves	
Rengin	Answering questions/ showing performance (evidence of learning) Rewriting (to recall and consolidate learning) Solving examples (to learn) First learning in every detail then solving questions Clear answer (evidence of learning)	Making connection with different concepts Applying what is learned in a new condition Memorizing formulas with reasons Asking questions when they don’t understand Asking how if questions	Making analysis-connections (evidence of learning)
Filiz	Practicing on solved questions (mastery of skills) Writing and listening (to consolidate learning) Computation skills	Making relations between concepts Asking questions and attending in lesson to complete deficiencies Students answers about different questions (to understand and their analysis)	

Table 4.5 (Continued)

Kerim	Following and listening to the lesson Solving similar questions (indicate understanding)	Background knowledge (to understand and to make connections) Trying to solve a question by oneself (indicate understanding or difficulty) How to use what is learnt Correct answer is not important	Interpreting and producing solutions for daily-life problems Ways of thinking about problem is more important than computation and correct answer Making their own decision Solving problem by themselves Asking why/how questions-interrogate-produce their own-effort Reasoning/interpretation-not memorizing Appreciate different ways Consider to persuading students (math is needed) Evaluation of learning by open-ended questions Communication and motivation (necessary to learn) Learning depends on learner
Kadri	Performance in exams and solving similar questions (as evidence for learning) Listen-write-recall-practice	Making connections between concepts Asking questions Expressing ideas	Students explanations of their solutions Students has responsibility for their learning Making interpretation Peer learning/group learning Discussing Make inferences (not based on knowledge) Solve problems Present and explain his/her ways to friends (as evidence for learning) Apply in a new condition (as evidence for learning) Consider memorization as hinder for learning

Results about themes for the beliefs about learning mathematics as presented in Table 4.6 show that (i) eight teachers (Meltem, Hazal, Ayla, Alp, Sude, Mert, Rengin, and Filiz) held mainly L1 and L2 beliefs; (ii) two teachers (Kadri and Kerim) held mainly L2 and L3 beliefs. Teachers' beliefs are presented in Table 4.6 below.

Table 4.6 Teachers' beliefs about learning mathematics before PDP.

	Beliefs about Learning Mathematics		
	L1	**L2**	**L3**
Meltem	✓	✓	
Hazal	✓	✓	
Ayla	✓	✓	
Alp	✓	✓	
Sude	✓	✓	
Mert	✓	✓	
Rengin	✓	✓	
Filiz	✓	✓	
Kadri			✓
Kerim			✓

Based on learning mathematics belief categories held, teachers' beliefs are categorized into two groups;

- Group 1 – Teachers holding mainly L1 and L2 beliefs
- Group 2 –Teachers holding mainly L3 beliefs

Group 1 – Teachers Holding Mainly L1 and L2 Beliefs

This group includes teachers (Meltem, Hazal, Ayla, Alp, Sude, Mert, Rengin, and Filiz) who mainly held L2 and L3 beliefs before participating in the PDP. The commonality among this group of teachers about their responses to the questions asked in the first interview is that while describing students' roles, characteristics of successful and unsuccessful students, indication and evaluation of students' learning and understanding of mathematical concepts, how students learn mathematics best, description of students' learning in their own classroom and learning in an ideal classroom, these teachers focused mainly on students roles as listener who need to listen carefully to every detail taught in class, take notes, recall and practice at home by solving questions or homework exercises; evidence of understanding and learning of mathematical concepts as performance on solving questions asked by the teacher or on homework questions, telling formulas related with the concept, producing correct and clear answers to the questions; the way of learning mathematics as solving too much questions related with the concept, etc. (L1). Beside listed L1 beliefs, these teachers also held L2 beliefs about learning mathematics such as learning mathematics as understanding the reasons behind principles or formulas, and making relations between the concepts; roles of students as being active in the class by attending in the lesson, asking questions, and expressing themselves and their difficulties; evidence of learning as students' solving different types of questions, not correct answers but the process or the way of solutions, etc.

As presented in Table 4.5, most of the teachers in this group considered that students should listen to teacher, take notes, make review and solve practice questions to learn mathematics (L1). For example, about the way of learning mathematics Hazal said that "*students should listen to and follow the lesson very well…writing is very important…since it is formula based, it is not easy to remember if you don't write*". Rengin also stated that "*no matter how successful that the students be, they should consolidate the concept seen by solving example questions*". For some of the teachers, students' performance on solving questions asked by the teacher or on homework questions, telling formulas related with the concept, producing correct and clear answers to the questions are evidence of understanding of the concept (L1). For example Ayla said that she decides whether or not students understand a concept or not by looking at their appearances and performance. She denoted that *"students those who understand immediately solve the question, you can detect it from their glances, their faces*". Similarly, Mert indicated that "w*e detect their understanding from the examples they solve, or whether or not s/he can answer the questions that we asked…or if s/he has error in solving*". Hazal also said that she determined if students understand a concept or not based on her observation of students' performance on questions that she asked.

She also added that since there is no time to have students on board to solve questions, she distributed tests to them. She stated that "*we distribute tests occasionally, the results of them are very important to us, because it shows where students have deficiencies*". Similarly, Alp said that "*if students do not understand they ask us, I guess, but the important thing is being able to solve questions related with content.*

Beside L1 beliefs teachers held, it was revealed that some of the teachers indicated that knowing the reason behind formulas and rules, making relation with the previous concepts indicate understanding (L2). For example, to explain how to check her students' understanding and learning of a concept Hazal said that "*I ask questions related with the previous topic, look at if they could make relations between new and previous topic or they could not*". Also some of teacher considered that ability to solve different types of questions indicate understanding (L2). For example Filiz stated that she asked different types of questions than what she teaches to check students understanding. Among this group, some teachers indicated that when solving questions finding correct answers do not always indicate understanding but the process or the way of solutions do (L2). For example, Alp denoted that sometimes students correctly answering the questions asked may be because of memorization of formulas. He highlighted that "*you should look at how s/he solves instead of the result*" (L2).

Also data obtained from the analogy questionnaire supported the beliefs about learning mathematics emerged from the interview data. For example Rengins stated that watching a movie do not represent learning mathematics because *"while watching a movie, everything is already given, there is no contribution. But, learning mathematics does not occur with rote learning and without interpretation"* (L2). Rengin's expression pointed out that she paid attention contributions of learner, discouraged rote learning and encouraged interpretation. Similarly Filiz denoted that "*mathematics learning cannot be represented by a passive sitting activity that does not include participation*" (L2).

Group 2 – Teachers Holding Mainly L3 Beliefs

This group includes teachers (Kadri and Kerim) who mainly held L3 beliefs before participating in the PDP. The commonality among this group of teachers about their responses to the questions asked in the first interview is that while describing students' roles, characteristics of successful and unsuccessful students, indication and evaluation of students' learning and understanding of mathematical concepts, how students learn mathematics best, description of students 'learning in their own classroom and learning in an ideal classroom these teachers focused mainly on learning as a process which depends mostly on learners rather than teachers, in which students need to make their own decisions by interpreting, reasoning and inferring, to discuss and to communicate with teachers and friends; indication of understanding and learning of mathematics concepts as interpreting and producing solutions for daily life problems, presenting and explaining his/her solution ways to his/her friends, applying what is learnt in a new condition, etc., (L3).

Kerim indicated that "*there is no such term as best teaching; here learner has the greater role than teacher*". Similarly, for Kadri, students are the leading part of the learning process. He said that "*students have responsibility for their learning*". As evidence of understanding, Kerim considers students solving different types of questions and making reasoning and interpretation as important. He said that "*less data, more interpretation, if s/he does not produce something or add something from him/herself, s/he doesn't understand*". When explaining the evidence of understanding and learning Kadri said that "*Students should present, explain visually, do his/her friends understand what s/he explains?, Does s/he apply what is learnt in a new condition, when passing from rational numbers to other concept, does s/he interpret in new concept, for example, we saw functions, does s/he relate it with the next concept?*". For Kadri students should not be those who depend on knowledge, but should make inferences. Kadri also stated that students should be active in class, should ask question to each other, offer ideas and discuss them in class. According to Kadri, mathematics can be learned by solving daily life problems. Students should produce their solutions for daily life problems.

4.2 Change in Beliefs about Nature of Mathematics, Teaching and Learning Mathematics

4.2.1. Change in Beliefs about Nature of Mathematics

As seen in Table 4.7, results indicated that, after PDP, beliefs about nature of mathematics have changed for 3 of the 10 teachers (Meltem, Hazal, and Rengin) have changed. Although, the other teachers' beliefs about nature of mathematics did not change, they developed new beliefs or their beliefs became more elaborated in the present belief categories. Close examination of the changed or newly developed beliefs, it is seen that these beliefs indicates that these beliefs were related with the nature of mathematical problem and problem solving.

Table 4.7 Change in teachers' beliefs about nature of mathematics

	Beliefs before PDP	**Beliefs after PDP**	**Specific changes or development**
Kerim	NM3	NM3	Open-ended problems
Kadri	NM (2-3)	NM (2-3)	Problem solving as process
Ayla	NM(1-2-3)	NM(1-2-3)	Problem Solving
Sude	NM(1-2-3)	NM(1-2-3)	Problem Solving
Meltem	NM1	NM1-NM2	Nature of mathematical problems
Filiz	NM(1-3)	NM(1-3)	Problem Solving
Hazal	NM1	NM2	Problem Solving Nature of mathematical problems
Rengin	NM (1-3)	NM2	Logical-Systematical thinking
Alp	NM(1-3)	NM(1-3)	Math and real life connection
Mert	NM(1-3)	NM(1-3)	Problem Solving

At the beginning of PDP several of the teachers considered that there can be several solution ways for a problem, yet, they also believed that answers or results should be certain, clear and exact. However, after attending PDP some teachers began to think that answers of the open-ended questions might not be exact. For example, Meltem denoted that "*If I didn't involve in such a project, I had not encountered with open-ended problems as we saw [modelling problem]. That is,* *we have always thought that mathematics is based on certain/clear results, however, it is not what we thought*". Meltem also expressed that although before she thought that problems have always a pre-determined solution, now she thinks that open-ended problems do not always have a predetermined solution. She stated that "*for example in open-ended problems, you [members of projects] use different methods; we use different methods, which might never be used before*". After PDP, Ayla also noted that "*you don't have to follow already given steps while solving problem, we see it in modelling, I solved in a way but students solved in a way a 180 degree opposite of my way, not using the same step, using substantially different steps*". Correspondingly, Filiz also said that "*Students can use different ways of solutions…we saw it in summer job problem…We teach different ways but students use their own ways*". Kerem also began elaborate his ideas about open-ended problems. After PDP, he said that "*some of the problems used in the class should be open-ended, because open-ended problems trigger discussions, and with discussions different ways of thinking emerges*".

Moreover, after PDP teachers beliefs about mathematical problems and problem solving have changed considerably or began to elaborate. For example, in the first interview Meltem described problem solving as *"reaching a result…with use of formulas… and basic knowledge"* on the other hand, in the interview conducted after PDP, she described problem solving as "*problems promote thinking skills, reasoning…by use of intelligence, students interpret the problem then try a solution, conclude their solution by using operations*". She also told that, understanding the problem is very important; while solving a problem, first students need to think, and then make sense, after that, use operation and solve it. While in the first interview Hazal defined the mathematical problem as "*a mathematical question that needs an answer…mathematical problems can be asked in story mood including real life names and situations, or it can be mathematical expressions*", in the second interview conducted after PDP, Hazal began to discriminate between mathematics problems and mathematics questions. She said that mathematics questions are based on operations; on the other hand mathematics problem is related to real life. For Hazal, finding value of unknown in an equation is not a problem, it is a question. Ayla also had similar focus about mathematics problems after PDP. In the interview conducted after PDP, Ayla said that question like "1x5=?" is not a problem. For Ayla a problem must be expressed in a story context. She noted that in the former what is to be done is clear/certain, but in latter, what is asked must be comprehended, so the mind works intensely. In a similar fashion, in the second interview Kadri said that "*what we did [in class] is not problem solving, it is question solving…problem solving should has a process but what we did is to ask question- and- get the answer"*. In the second interview, it was seen that Mert's beliefs about problem solving have changed too. While in the first interview Mert described problem solving as "*applying knowledge that we possess on the problem and doing mathematical operation*", in the second interview, Mert said that "p*roblem solving develops students' logical and reasonable thinking. Knowledge comes after"*. Mert also stated that "*problems should be interesting, related with content and foster thinking…Problem solving should be used to encourage students think actively, keep their mind working…it can be used to make students comprehend the content"*. After PDP, Kadri's focuses on problem solving has been elaborated too. Before PDP, Kadri defined mathematical problems as questions defined in mathematics languages and focused on understanding that problem is asked for while problem solving. However, after PDP, he began to discriminate between questions and problems and denoted that although finding answer is important in question solving, the process of solution is more important than the answer in problem solving. Furthermore, after PDP Hazal's began to think that knowing formulas and procedures is not enough to solve problem. In the first interview Hazal said that *"you have to know operations, should be careful, and for some questions you should analyze and make relations"*. In the second interview she also added that "*with knowing formulas, you can solve mathematical problems up to a point, but you also need to make connection, relate each other, express and visualize*". Hazal told that there should be interpretation and for interpreting it is needed to make connection, internalize, and think in variety of ways. Hazal stated that "*we could have used formulas unconsciously in modelling problem with making analysis and making connection to our previous knowledge including formulas*". Hazal also added that "*we cannot mention about the certain formulas for problems like modelling problems*".

4.2.2 Change in Beliefs about Teaching and Learning Mathematics

As seen in Table 4.8 results indicated that after PDP beliefs about teaching mathematics have changed for 3 of the 10 teachers (Ayla, Meltem, and Hazal). Although the other teachers' beleifs about teaching mathematics did not change, they developed new beliefs or their beliefs become more elaborated in the presert belief categories. Close examination of changed or newly developed beliefs indicates that the specific changes or developments in teachers beliefs about teaching mathematics concentrates on the role of a teacher, use of real life examples, visuals and other materials, and developing students thinking and understanding.

Table 4.8 Change in teachers' beliefs about teaching mathematics

	Beliefs before PDP	Beliefs after PDP	Specific Changes or Development
Kerim	T2-T3	T2-T3	Roles of teacher (guider) (should not give clear/exact answers)
Kadri	T2-T3	T2-T3	Using real-life problems Students own discoveries more important than presenting exact solution
Ayla	T1-T2	T2	Role of teacher (guider) More emphasis on understanding
Sude	T2	T2	Explaining by making connection Using materials and daily-life examples
Meltem	T1-T2	T2	Role of teacher (make students to think) Teachers as not information giver Students ideas as important for teacher
Filiz	T2	T2	Real-life examples, taking attention, using visualization Knowing different ways of solutions
Hazal	T1-T2	T2	Taking students' attentions Arousing curiosity Paying attention to understanding (Modelling) problems are more instructive than classical problems
Rengin	T2	T2	Using materials and real life examples Listening students ideas
Alp	T1-T2	T1-T2	Real Life examples
Mert	T2	T2	Discourage memorization

In Table 4.9, results about change in teachers' beliefs about learning mathematics are presented. As indicated, the belifes about learning mathematics has changesd for 4 of the 10 (Ayla, Sude, Hazal, and Rengin). Although, the other teachers' beliefs about learning mathematics did not change they developed new beliefs or their beliefs become more elaborated in the preset belief categories. Close examination of changed or newly developed beliefs indicates that the specific changes or developments in teachers beliefs about learning mathematics concentrates on role of group work, discussions, and discovery for learning mathematics, learning by doing, and by problem solving, use of real life examples, models and materials for students understanding and learning, students' developing their own ways of solution and students roles in learning.

Table 4.9 Change in teachers' beliefs about learning mathematics

	Beliefs before PDP	**Beliefs after PDP**	**Specific Changes or Development**
Kerim	L3	L3	Learning occurs by group work Learning by doing
Kadri	L3	L3	Learning by making discovery
Ayla	L1-L2	L2	Learning from each other Learning by problem solving
Sude	L1-L2	L2	Use of model and material for meaningful learning Students develop their own ways Discussion and sharing in groups as a tool for effective and permanent learning
Meltem	L1-L2	L1-L2	Students as active in learning process Group work is important for learning
Filiz	L1-L2	L1-L2	Student can produce their own ways
Hazal	L1-L2	L2	Benefits of discussion in group work Advantages of solving modelling problems for learning Discourage memorization
Rengin	L1-L2	L2	Real life examples for permanent learning Students can decide how to use information Discovery support learning
Alp	L1-L2	L1-L2	Group work facilitates learning from each other
Mert	L1-L2	L1-L2	Group work facilitates communication, collaboration

After PDP, it was seen that many of the teachers' beliefs about teachers' role in teaching mathematics and students' role in learning mathematics has changed or developed. At the beginning of PDP, Kerim said that teachers should not be dominant in teaching mathematics and students also should attend in lesson by asking questions and expressing their ideas, making reasoning and interpretation, and from analogy questionnaire it was seen that Kerim saw teacher as a guide. However, after PDP, his expressions about teacher roles as being guider were elaborated and included different aspects related with teachers' role in group works. About the teachers' role in modelling implementations, Kerim said that "*teachers must be guider here, that is, teachers do not interfere, s/he should answer the questions asked by the groups…s/he should give hints when students have difficulty…every group should produce something*". As parallel with Kerim, Ayla emphasized similar aspects about teacher's role. She said that students should choose their own way of solutions and make their own interpretation about problems and teacher should only guide them. When talking about modelling in the year-end interview, Ayla further emphasized that when using modelling problems, teacher should not interfere but support students when they needed, or when they have difficulty. Similarly, in the year-end interview Sude asserted that *"teacher should act like in teaching with modelling, it should be different. Teacher would work with students".* As parallel, Meltem denoted that "*teacher should not be information giver, students could be actively involved in the concepts that they could do, achieve".*

Another change or development about teachers' beliefs about teaching mathematics is about students' thinking and understanding. After PDP, some of the teachers started to pay more attention to students understanding when talking about how to teach mathematics. For example, while describing mathematics teaching Meltem stated that *"as a teacher, you think about; how I explain the content, from where I should start, how I make students to comprehend better, what kind of questions I should ask.".* Although Meltem considered teacher as who is responsible for teaching best, controlling the content, she said that teachers need to direct students to think while teaching. As connected with increasing attention to students understanding, after PDP teachers started to emphasize use of materials, visuals, concrete examples, models, and real-life examples, as well as methods such as teaching concepts by relating with each other. For example, in the first interview Sude focused on teacher aspect when describing how students learn mathematics best; she pointed out that students could learn best when there is a good method of teaching and good teachers who explain well and encourage students thinking in different ways. However, in the second interview she mostly focused on what teacher conducted during teaching and gave specific examples about it. She said that "*it would be better if the teaching practice is more tangible, such as, explaining hypotenuse by going outside, to the garden, would be better, or using an prism model to exemplify, to model…it would be better because if information given is surface, as there is so much information and formulas, then it becomes boring. It would be easier if concrete examples that are from life of students could be used*". Sude told that using material would be more meaningful than drawing the shapes on board. It was also seen that in the second interview Sude focused more on relating concepts with each other. She said that explaining a concept by relating with other concepts is meaningful (such as relating logarithm with exponential functions). In a similar vein, Kadri said that "*teachers should select examples from real life in order to teach a mathematical concept in a way that facilitates students' understanding*". Similarly, Filiz denoted that teachers giving real life examples or using visuals and materials during teaching mathematical concepts takes students' attention and this facilitates students learning mathematics. Moreover, Meltem pointed out that different strategies can be generated for teaching different concepts such as asking questions, relating with previous concepts, giving example from real-life etc., in order to make students attend in the lesson.

After PDP, teachers' expressions about what to do to teach mathematics better or to make students understand better, teachers began to include different ways of solutions for a problem as their attention to students understanding has increased. For example, after PDP, Filiz began to think that teachers should know different ways of solution for a problem. She stated that "*in the first implementation, we* *considered only our own way of solution…but later, we have understood that we should consider other ways to mediate during the implementation*". Filiz clarified that after the first implementation of her, she comprehended that "*a teacher should know different ways of solutions to facilitate and mediate when students have difficulty to solve, ask questions about their solutions*". While Filiz stressed knowing different ways of a problem, Kerim emphasized that teachers can teach ways of solution to students, but they should not always provide clear and exact solutions and answers to students. He declared that "*you teach ways of solutions, but I do not want teachers do everything…also students should contribute, teachers should not be dominant, they can interfere at some points, but then they should step back…if teachers do everything, give everything clearly and exactly, how can students discuss, interpret, ask about?"*. In a like manner, after PDP Mert denoted that presenting clear and certain, exact solutions to students lead to rote learning, thus teacher should give hints. Mert stated that *if students do not develop, solve, then you need to give hints, but you shouldn't give hints immediately, you need to wait"*.

As parallel with teaching beliefs about ways of solutions about a problem and teachers use of them in teaching, it was revealed that teachers started to attach importance to students' developing their own ways of solution for permanent learning. About this, Kadri said that students own discoveries are more important than teachers' presenting exact solutions. Kadri denoted that "*if students discover by themselves, it is easier to adopt and remember*". Sude also noted that "*students developing their own ways and defending them is meaningful for not forgetting them. Similar to what we did in modelling application. Formulas are applied, but what they find by making effort is hard to forget, because they didn't use readily made templates, they used their own ways"*. In a different aspect, Filiz denoted that "*the students who seems passive can identify different ways of solutions for problem…we can investigate students from a different point of view…a thinking that all students are same is not very correct*".

Beside students developing their own ways of solution, learning by doing is considered as an important facilitator for permanent learning. After PDP, Kerim denoted that students learning during the modeling implementations was permanent with respect to learning by solving classical problems (i.e exercise questions), since they are experiencing the activity. Kerim denoted that "*learning being permanent, natural and occurred with interpretation is helpful for students*". Similarly, after PDP, Sude began to think that students learn best by doing and experiencing. She stated that "*students learn best while doing and experiencing things that are related to daily life. They would be comfortable because every students are evaluated in his/her condition, s/he feels comfortable, because the system does not judge his/her. But in our system it is either right or wrong. In ideal, learning by doing and experiencing is better; you can learn Paris from books, or by going there, or the child who lives in Trabzon or Rize does not forget how the tea tree grows, since s/he works in it*".

After PDP, teachers started to emphasize the effects of group work for students' learning mathematics. For example, after PDP Kadri started to think that the process of learning is different in modelling because of group working. Kadri said that "*group work creates difference…students interact with each other, share their knowledge….even sharing something, since it is different than standard, make students to learn a different view*". Kerim also expressed that group work can facilitate students' learning. He stated that "*students learn through cooperation and exchange of ideas…students who has deficiency can overcome their deficiencies. Therefore, it is helpful for students*". He also added that in group work teachers' weight reduces, so that, the space in which students became active increases more. Similarly, about group work, Mert denoted that that "*group work increases the communication between students…in the group, one can solve, and others can be supported by him/her*".

As connected with the group work conducted during the modeling implementations, some of the teachers began to emphasize the role of discussion, especially that occurring between the students. Some teachers expressed that students discussing solutions, ideas, or views that they developed facilitate their learning. For example, Sude stated that *"we saw them discussing with each other while in modelling application, during group work there was different views since they have discussed with each other, sometimes they listened each other, sometimes they didn't, however, it was more meaningful than my monotonous teaching."*

Moreover, beliefs about the best way of learning mathematics have changed for some teachers, for example, after PDP Ayla's explanations about the questions related with learning mathematics. She pointed out that *"students do not learn by solving too many questions. Every student is different, maybe some of them learn by solving questions, others may need to see important points…for others seeing, feeling and writing is necessary to learn…some learn with problems like modelling.*"

4.3 Teachers' Perceptions about the Effects of PDP on Their Beliefs about Nature of Mathematics, and Teaching and Learning Mathematics

When asked about what they think about the effect of PDP on their belief neither of the teachers reported influence of PDP on their beliefs about nature of mathematics. However, some of the teachers declared there were some changes in their beliefs about teaching and learning mathematics after they have participated in PDP. The changes that teachers' reported were about teacher's and student's roles, ideal mathematics teaching, evaluation of students learning, role of materials, visuals, models, real life examples and problems in learning and teaching mathematics, learning by problem solving, and role of group work in students' learning.

Kadri said that he admitted the effect of PDP. Although in the past he was thinking about the problems of mathematics teaching, he said that after PDP he could identify what it should be and he understood that mathematics teaching should be active and must be student-based.

He said that *"Before [PDP], I was skeptic about it…but of course, you have problems in the class, you think that you are teaching, but the reports says there are many students who get zero from exams, but if you ask teachers they say that they teach very well...but we don't…I saw that if we approach students differently, then they can learn"*. Kadri said that although what he thought about how teaching should be is similar, PDP opened a new window to him. He indicated that "o*ur system is stereotyped…there is power struggle between teacher and students…but it must be students based, students should fell free. I saw this in this program…the class was more flexible and the teaching was more active…as a teacher I felt less nervous…In the past, I thought similarly about what it should be, but this program showed an example, opened a new window….It showed me that you can teach this way"*. Similar to Kadri, Ayla asserted that PDP has influences on her thoughts. She explained that "*it made a change; first of all, the method I used is that teacher explains on board and students attend, but, here it is different, students are active and teacher is guider, actually this is what must be done*." Additionally, Meltem reported that her descriptions about the roles of teachers and students, as well as her thoughts about group working have changed. Meltem declared that after PDP she was thinking teachers as who is not information giver, students as who is actively involving in the teaching process, and she was considering that group works can contribute students to produce and to view from a larger perspective. When asked about if she thought like this before she said that *"not like this, we came from a tradition where teachers were active in teaching. But after the project, our thoughts have changed. That is, in this project students both use and develop knowledge if they do not know, they could work in groups with supports of their friends. Together they produce and reach at something; they support each other with reasoning/thinking. In the past, I never thought like that because, I didn't involve in such project. My current idea is somehow different"*. Similar to Meltem, students' working in groups was found as an important focus in Sude's description for ideal teaching and learning. Sude said that PDP had influence on her thoughts about ideal teaching and learning. She reported "*like teaching with modelling, it should be different. Teacher would work with students…not everyone has to teach on their seats, seats design would be different…for example, group* *working can be used…since students would be active, you answer questions with students, by the way you also work, as well as students work, then there would be class discussion…evaluation is based on how much students involve, attend in the group works, their roles, and how they made solutions".*

After PDP, both Filiz and Ayla expressed that their views of success in mathematics and successful students have changed. Filiz asserted that in the past her view of successful students was related with students' exam grades, the correctness of their answers, or level of solutions they have made, however, with PDP she comprehended that students may have different ability as if they are seen passive in class. She declared that "*the students who seems to passive can identify different ways of solutions for problem…we can investigate students from a different point of view…thinking or interpreting that all students are same not very correct*". In a like manner, Ayla declared that after PDP her view of success in mathematics and successful students has changed. She said that students who is passive in her class showed unexpected performances in modelling implementations. Ayla denoted that "*mathematics success is not related with grades that students get, it is related with their comprehension, their approach to problems…their attendance, their being active in solving, all of them show success*".

Some teachers reported that PDP affected their ideas about use of visuals, models, materials, real life connections and real life problems in teaching and learning mathematics. For example Rengin said that "*you cannot go beyond the curriculum; however, this project showed us the importance of modelling, exemplifying, providing students rich contexts… the other day when I was explaining the prisms, the questions that students asked me made me think that we need to make lesson richer and less monotonous by giving real life examples*". Also, Kerim indicated that what he was aware of most clearly in PDP was the connection between real life and mathematics. He pointed out that they explain mathematics in an abstract way, by making only a rough relation with the current events, and with PDP he saw that mathematics teaching can be implemented by connecting mathematics with real life. Moreover, Meltem stated that in the past they had used some material for demonstration (such as using prism or cubes in solid concept to show students the solid), but they have never used materials while solving problem. She said that with this project they saw that materials can be used; to make them students work on problem and visualize it. As parallel, Filiz declared that "*in the past, the teaching approach, the way of best teaching was to increase the amount of questions by changing the numbers in the questions*". She highlighted that her view of students learning has changed. She pointed out that "*it is understood that they [students] can comprehend better when they give a model, a concrete shape, or a problem related with real-life….they [students] can see themselves with the shape in front on them…making them to draw the shape, to interpret the problem and leave them to decide the concepts used to solve the problem is good*". Similarly, Rengin said that, after PDP she thought that learning is more permanent when students use materials and work on real life applications of mathematics.

Some teachers reported a change in their beliefs about learning with problem solving. For example, Alp denoted that after PDP he began to think that mathematics can be learned with a mathematics problem. He said that "*Of course, my thoughts have changed. At first I thought that how we will teach mathematics with a problem, how it can be for students, how they deal with the problem through two-hour lesson, how can a mathematics problem include mathematical concepts…Then we saw that it can be taught".* Similar to Alp, Filiz also denoted that problem solving can be effective for students learning. Filiz stated that "*before, I thought that solving to many exercise questions is helpful. But we have seen that actually, in some concepts students can solve problem by focusing and thinking on it, and they can develop an idea about the concept*".

About components of PDP, classroom implementations and weekly meetings were found to have influences on teachers change. Kerim asserted that classroom implementations contributed him to change. He told that "*person learn so much by doing…in our implementation, as we can try once, twice, third times, there is so* *much things have been changing in your forth try…how you approach to the problem, how you interfere, what you say to students, what you don't say, what are its limitations, how you direct students are changing*". Kerim also pointed out that classroom implementation showed the applicability of teaching with modeling problems as well as its benefits. Ayla also told that she considered classroom implementations as the reason of her changes. She stated that "*you became more experienced about what you will do, how you interfere during implementation*". On the other hand, Hazal delineated that weekly meetings conducted during PDP had important contributions to him. He said that during discussions conducted in meetings, they exchanged ideas with other teachers; they made group works, and interact with each other. Because of these reasons they have developed. As connected to weekly meetings, collaboration conducted among teachers and between teachers and project members were considered important by some teachers. For example, when asked about the changes after participating in PDP, Kerim denoted that the differences he felt about himself was first of all because of the collaboration conducted with his colleagues and with project members during meeting, which resulted in exchanging of ideas between team members. He said that "*because of working in such group, there is exchange of ideas…for years, we thought and talked individually…but there was a difference occurred because of, for the first time, participating in teamwork.*" Similarly, Ayla also denoted that she benefited from the cooperation that she made with her colleagues, during PDP. She said that "*everyone shares a different view; I have benefitted from them…solutions that I have never thought about, for example Rengin's, her solutions are more practical and more effective, Hazals, Ali's were also…we had a good cooperation among us and I benefitted from them.*"

4.4 Summary of Results

The alignment between categories of teachers' beliefs about nature of mathematics (NM), teaching mathematics (T), and learning mathematics (L) before PDP are represented in Table 4.10.

Table 4.10 Teachers beliefs about nature of mathematics, teaching and learning mathematics before PDP

	NM	T	L
Hazal	1	1-2	1-2
Meltem	1	1-2	1-2
Ayla	1-2-3	1-2	1-2
Sude	1-2-3	2	1-2
Alp	1-3	1-2	1-2
Mert	1-3	2	1-2
Rengin	1-3	2	1-2
Filiz	1-3	1-2	1-2
Kadri	2-3	2	3
Kerim	3	2-3	3

According to categories of nature of mathematics beliefs held, it was found that two teachers held mainly NM1 (instrumental) beliefs; four teachers held mainly NM1 (instrumental) and NM3 (problem solving) beliefs; two teachers held in every category of beliefs (NM1, NM2, NM3); one teacher held mainly NM2 (Platonist) and NM3 (problem solving) beliefs and one teacher held mainly NM3 (problem solving) beliefs. Most frequently helf beliefs among teachers were NM1 and NM3.

- Teachers holding NM1 beliefs focused mostly on calculation, operation, rules, formulas, results, certainty of results, and utility of mathematics when they defined mathematics and described the nature of mathematical knowledge; besides they focused on reaching the correct results or answer of the problem, doing operations, use of formulas while describing mathematical problems and problem solving. Additionally, these groups of teachers did not differentiate between practice questions and mathematical problems.

- On the other hand, teachers holding NM2 beliefs mainly considered that mathematics is systematical and logical way of thinking, and collection of ideas; mathematical knowledge is already existed and static; there is no ambiguity or contradiction in mathematical knowledge; mathematical knowledge is certain and clear; and mathematical knowledge are related.

- Teachers holding NM3 beliefs mainly considered mathematics as dynamic and a human product. They considered mathematics as a base or tool for other science. For teachers holding NM3 beliefs, there is not a fixed solution for a mathematical problem, there can be several ways of solution for any mathematical problem.

According to categories of teaching mathematics beliefs held, it was found that four teachers held mainly T1 and T2 beliefs, four teachers held mainly T2 beliefs, two teacher mainly T2 and T3 beliefs. Most frequently held beliefs among teachers were T1 and T2.

- Teachers holding T1 beliefs mainly considered teacher as instructor and director who is authority for knowledge and controller of the classroom activities; teacher roles as transferring the content; mathematics teaching as content-focused and based on skill development; best way to teach (or ideal teaching) as solving too many of questions with students.

- Teachers holding T2 beliefs mainly considered teacher role as explainer. They see content of mathematics as related with each other. For these teachers, mathematical content is determinant of mathematics teaching but students understanding also considered important. To facilitate students' understanding of concepts, these teachers focused on relating concept taught with previous concepts, explaining the reasons behind the formulas, paying attention to the sequence of questions asked during teaching, encouraging students to attend, and ask questions, and to solve questions after they were showed how to solve.

- Teachers holding T3 beliefs mainly considered teacher role as facilitator and guider who should response students' questions, pay attention to their needs, encourage students to think and reason, and to develop their own way of thinking or ways of solution, create a discussion environment and discuss with students about their solutions, etc.

According to categories of learning mathematics beliefs held, it was found that eight teachers held mainly L1 and L2 beliefs, two teachers mainly L2 and L3 beliefs. Most frequently held beliefs among teachers were L1 and L2.

- Teachers holding L1 beliefs mainly considered students roles as listener who need to listen carefully to every detail taught in class, take notes, recall and practice at home by solving questions or homework exercises. The way of learning mathematics for these teachers is solving too much questions related with the concept. Moreover, for these teachers, evidence of understanding and learning of a mathematical concept is based on students' performance on solving questions asked by the teacher or on homework questions. Students' telling formulas related with the concept and producing correct and clear answers to the questions are considered as indication of learning.

- Teachers holding L2 beliefs mainly considered students roles as being active in the class by attending in the lesson, asking questions, and expressing themselves and their difficulties. However, students are expected to solve questions after teachers showing an example. For this group of teacher, learning mathematics requires understanding the reasons behind principles or formulas, and making relations between the concepts. These teachers considered students' solving different types of questions and the students' way of solutions as evidence of understanding, while the finding correct answers may not be regarded as evidence of learning.

- Teachers holding L3 beliefs mainly considered learning as a process which depends mostly on learners rather than teachers, in which students need to make their own decisions by doing interpretation, reasoning and inferences. Discussion and communication with teachers and friends is considered important for learning. For these teachers understanding and learning mathematical concepts includes interpreting and producing solutions for daily life problems, presenting and explaining his/her solution ways to his/her friends, and applying what is learnt in a new condition.

For the change of beliefs after PDP, results indicated that, there was a category change in 3 of the 10 teachers' beliefs about nature of mathematics. Although, other teachers' holding category of beliefs about nature of mathematics did not change, they developed new beliefs or their present beliefs became more elaborated. When looking at changed, newly developed, or elaborated beliefs, it was seen that these beliefs were related with the nature of mathematical problem and problem solving. Specifically, before PDP some of the teachers believed that mathematical problem should have certain and clear results, problems have only one way of solutions, and they should be solved by following predetermined rules. After PDP, teachers started to express that open-ended problems do not always have a predetermined solution, answers of the open-ended problems might not be exact, and there can be more than one way of solutions for mathematical problems. Also, teachers who did not see any difference between mathematical problems and practice questions started to differentiate between them. While at the beginning of PDP teachers' focuses about mathematical problem and problem solving were on reaching a certain-clear result, using rules and formulas, making computations and operations, at the end of the PDP, teachers started to focus on real-life context, process and ways of solutions, understanding the problem, making connections and reasoning while solving problem.

Similar to nature of mathematical beliefs, there was a category change in 3 of the 10 teachers' beliefs about teaching mathematics. Although there was no change in the holding category, there was either a change or development in nearly all of teachers' beliefs about teaching mathematics. Specifically, after PDP teachers started to expressed that students own discoveries are more important than teachers' presenting clear and exact answers for the mathematical problems; instead of presenting the answer teachers should be a guider in the process of problem solving and support students to think and develop their own solutions.

Moreover, about teaching mathematics, teachers especially who held T1 (content-focused with emphasis on performance) beliefs have started to pay more attention to developing students' understanding, a belief indicate T2 (content-focused with emphasis on conceptual understanding) category. Beside, some teachers began to elaborate their beliefs or developed new beliefs related with the use of real life problems, materials and visuals and approaches discouraging memorization while teaching mathematics.

For the beliefs about learning mathematics, there was a category change in 3 of the 10 teachers' beliefs after PDP. Although there was no change in the holding category, however there was either a change or development in nearly all of teachers' beliefs about learning mathematics. Specifically, after PDP teachers started to underline benefits of solving problems for learning mathematics, importance of discussions occurring in group work, learning by doing, making discovery through solving problem, use of models, visuals and material for meaningful learning, and students learning from each other.

About teachers' perception about the effects of PDP on their belief, neither of the teachers reported influence of PDP on their beliefs about nature of mathematics. However, some of the teachers declared there were some changes in their beliefs about teaching and learning mathematics after they have participated in PDP. The changes that teachers' reported were composed of teacher's and student's roles, ideal mathematics teaching, evaluation of students learning, role of materials, visuals, models, real life examples and problems in learning and teaching mathematics, learning by problem solving, and role of group work in students' learning. Teachers reported that classroom implementations and weekly meetings had influences on their change and development.

CHAPTER V

DISCUSSIONS, CONCLUSIONS AND IMPLICATIONS

The purpose of this study was to investigate secondary mathematics teachers' mathematics related beliefs through a one-year PDP on mathematical modelling. There were three research questions of the study; "What were the high school mathematics teachers' beliefs about nature of mathematics?", "Mathematics learning and mathematics teaching?"; "How did teachers' beliefs change after participating in a one-year PDP?"; "What do teachers think about the influence of the PDP on their beliefs about nature of mathematics, teaching and learning mathematics?". In this chapter discussions about the results obtained from the study is provided along with related research question. Implications, assumptions and limitations of the study are presented at next.

5.1 Teachers' Beliefs about Nature of Mathematics Teaching and Learning Mathematics

Primary motivation for any belief research conducted on teachers is that beliefs are considered as an important factor affecting teachers' instructional practices (Borko & Putnam, 1996; Ernest, 1989a; Thomson, 1992; Wilson & Cooney, 2002). Thus, implementing any reform could be possible by changing teachers' beliefs that are inconsistent with the reform recommendation. Given that, teachers' use of mathematical modelling in their teaching and their effort to overcome the barriers on the use of it is all shaped by their instructional goals which are connected to teachers' mathematics related beliefs (Förster, 2011). Therefore, delineating secondary mathematics teachers' mathematics related beliefs is important.

One of the findings of the study is that many of the teachers did not hold a single category of beliefs about nature of mathematics, teaching and learning mathematics. For the beliefs about nature of mathematics, two teachers held mainly instrumental (NM1), one teacher held mainly problem solving (NM3) beliefs, and four teachers held mainly instrumental (NM1) and problem solving (NM3) beliefs, and four teachers held mainly instrumental (NM1), Platonist (NM2), and problem solving (NM3) beliefs. For the beliefs about teaching mathematics, four teachers held mainly content focused with emphasis on performance (T1) and content focused with emphasis on conceptual understanding (T2) beliefs, four teachers held mainly content focused with emphasis on conceptual understanding (T2) beliefs, two teacher mainly content focused with emphasis on conceptual understanding (T2) and learner focused (T3) beliefs. For beliefs about learning mathematics eight teachers held mainly skill mastery with passive reception of knowledge (L1) and conceptual understanding with unified knowledge (L2) beliefs, and two teachers held autonomous exploration of own interest (L3) beliefs. It can be argued that it is uncommon to hold more than one category of beliefs at the same time, especially mathematics learning and teaching beliefs which are contradictory with each other, such as traditional-teacher centered beliefs and constructivist-student centered beliefs. However, Green (1971) asserted that beliefs are held in clusters so that one can hold conflicting beliefs into different clusters. Therefore, it is possible for a teacher to hold more than one category of beliefs at the same time. Also, Ernest, (1989a) proposed that social context including institutional curriculum, the system of assessment, national school system have influence on teachers' beliefs, which result in holding teacher-centered and student-centered beliefs at the same time. Although not addressed specifically, teachers in the present study sometimes referred to educational system and curricula when they justify their existed or changed beliefs. This might had effect on holding more than one category of beliefs at the same time. Moreover, lack of awareness as connected with the lack of reflection on their beliefs is proposed as an important factor effecting teachers holding conflicting beliefs (Ernest, 1989a; Thompson, 1984). Since the social context including system of national assessment known to support teacher-centered practice, teachers might reflected on traditional-teacher centered beliefs (such as mathematics can be learned and taught by solving too many practice questions). On the other hand current reform in mathematics curricula might have affected teachers reflecting more on constructivist-student centered beliefs (such as use of real life examples and materials in teaching mathematics can facilitate students' learning mathematical concepts). As known, teachers participated in PDP as a volunteer and the context of PDP supported constructivist-student centered practices. Teachers' willingness to participate in the study by knowing the aim of the study might show that teachers felt a need to have knowledge and skills about the implementation of recent reform (mathematical modelling and use of modeling in mathematics teaching). Thus, participant teachers can be considered those who might have reflected or began to reflect on constructivists-student centered beliefs. These two considerations also can explain why teachers held different categories of beliefs about nature of mathematics, teaching and learning mathematics at the same time.

Results revealed that, the most frequently held category of beliefs for nature of mathematics is NM1 and NM3, the most frequently held category of beliefs for teaching mathematics is T2, and the most frequently held category of beliefs for learning mathematics is L1 and L2. These results are aligned with the literature (Barkatsas & Malone, 2005; Beswick, 2012; Seaman, Szydlik, Szydlik, & Beam, 2005). For example, Barkatsas and Mallone (2005) reported that there were two orientations in secondary mathematics teachers' beliefs about mathematics, mathematics teaching and learning; a contemporary-constructivist orientation and a traditional-transmission-information processing orientation. Similarly, Beswick (2012) found incontradictory beliefs held by the teachers in the study, such as a teacher held problem solving beliefs for the school mathematics, but Platonist beliefs for the mathematics as discipline. Correspondingly, Seaman et al. (2005) found that teachers simultaneously held two types of beliefs; mathematics as collection of rules, facts, and formulas and mathematics as creative and flexible human endeavor. As seen these beliefs are parallel with instrumental (NM1) and problem solving (NM3) beliefs found in the present study. In the present study, the teachers held more than one category of beliefs at the same time, including NM1 and NM3 beliefs. Therefore, the finding of this study is parallel with the literature (Barkatsas & Malone, 2005; Beswicks, 2012; Seaman, et al., 2005).

5.2 Changes in Teachers' Beliefs about Nature of Mathematics Teaching and Learning Mathematics

About the change in teachers' beliefs, results revealed that 3 of the 10 teachers' beliefs about nature of mathematics have changed after the PDP. Although, the other teachers' beliefs about nature of mathematics did not change, they developed new beliefs or their beliefs became more deepened in the exixting belief categories. The teachers' newly developed beliefs were related with the nature of mathematical problem and problem solving. Specifically, before the PDP some of the teachers believed that mathematical problem should have certain and clear results, problems have only one way of solutions, and they should be solved by following predetermined rules. After the PDP, teachers started to express that mathematical problems including modelling problems do not always have a predetermined solution, their answers might not be exact, and there can be more than one ways of solutions for mathematical problems. Also, teachers who did not see any difference between mathematical problems and practice questions started to differentiate between them. While at the beginning of the PDP, the teachers' focuses about mathematical problem and problem solving were on reaching a certain-clear result, using rules and formulas, making computations and operations, at the end of the PDP teachers started to focus on real-life context, process and ways of solutions, understanding the problem, making connections and reasoning while solving problem.

Similar to nature of mathematical beliefs, for the beliefs about teaching mathematics, 3 of the 10 teachers' beliefs have changed. Although there was no change in the other teachers' beliefs about teaching mathematics, they developed new beliefs or their beliefs became more elaborated in the present belief categories. Specifically, after the PDP teachers started to expressed that students own discoveries are more important than teachers' presenting clear and exact answers for the mathematical problems, instead of presenting the answer; teachers should be a guide in the process of problem solving and support students to think and develop their own solutions. Moreover, about teaching mathematics, teachers especially who held T1 (content-focused with emphasis on performance) beliefs have started to pay more attention to developing students' understanding, a belief indicate T2 (content-focused with emphasis on conceptual understanding) category. Beside, some teachers began to elaborate or developed new beliefs related with the use of real life problems, materials and visuals and approaches discouraging memorization while teaching mathematics.

For the beliefs about learning mathematics, 4 of the 10 teachers' beliefs have changed after the PDP. Although there was the other teachers' beliefs about learning mathematics did not change, they developed new beliefs or their beliefs became more elaborated in the present belief categories. Specifically, after the PDP teachers started to underline benefits of solving (modelling) problems for learning mathematics, importance of discussions occurring in group work, learning by doing, making discovery through solving modelling problem, use of model and material for meaningful learning, and students learning from each other.

Considering the changes and development in teachers' beliefs about nature of mathematical problems, teachers' roles and ways of teaching mathematics, learning by solving problems, doing group work, making discussion and using materials, it can be deduced that teachers' belief changes and developments are well aligning with the aim and context of the PDP. As explained before, the PDP provided teachers opportunities of individual and collaborative working on modelling problems, implementing modelling problem in their classes, observing students' responses through modelling implementations, and individual and collaborative investigation of students' written responses-works obtained from modelling implementations. Also, they were provided support throughout the PDP by research team. Thus, as a context, PDP can be considered effective in influencing teachers' beliefs about mathematical problem and problem solving, teachers' roles and ways of teaching mathematics, learning by solving problem, doing group work, making discussion and using materials.

Despite changes in teachers' beliefs, lack of drastic category change in each teacher's beliefs about nature of mathematics, teaching and learning mathematics (from NM1 beliefs towards NM3, from L1 beliefs towards L3, and from T1 towards T3) might be because of several reasons. First reason might be related with the nature of the beliefs. As literature proposed that beliefs are considered as a construct resistant to change, the core beliefs are less likely to change while their peripheral beliefs are easier to change (Green, 1971; Nespor, 1987; Pajares, 1992). Therefore, it can be deduced that changes in beliefs in some respect without radical change in the category of beliefs held might be related to the changed beliefs being peripheral. Moreover, as results indicated that specific change of development in the teachers' beliefs are well aligning with the nature of context provided by the PDP. It can be inferred that the PDP did address some beliefs more than others. For example, since teachers had opportunity to work on, discuss about, and implement mathematical modeling as well as observe students' response including producing their ways of solutions through a collaborative work occurred in groups, they might have developed beliefs about nature of mathematical problem as related with modelling problem (such as open-ended problems do not always have a predetermined solution, answers of the open-ended problems might not be exact, and there can be more than one way of solutions for mathematical problems). On the other hand, there might be no apparent opportunity for teachers to reflect on other beliefs (such as mathematical knowledge is static). This case can also be valid for the beliefs about teaching and learning mathematics, when considering the specifically changed or developed beliefs after the PDP.

The results obtained from current study revealed most of the teachers relate changes or development in their beliefs with implementation of modelling problems in their classrooms. As such, Guskey (1986) argued that change in beliefs follows change in classroom practice. The reported changes in beliefs are well aligning with the literature. Also, the results obtained from the study are compatible with the findings of the other research studies (Maas, 2011; Simon & Schifter, 1991; Szydlik, Szydlik & Benson, 2003; Swan, 2007)

5.3 Teachers' Perceptions about the Influence of the PDP on Their Beliefs about Nature of Mathematics, Teaching and Learning Mathematics

In order to change the instruction that takes place in the classrooms, teachers should change their beliefs (Richardson, 1996; Thompson, 1992). To understand the process of teachers change it is important to reveal teachers' perception of their own change process (Hart, 2002).

Teachers are one of the most important elements of the education process. If the style of mathematics instruction occurred in classrooms is to be changed, then, one of the necessary steps fort his is to change teachers' beliefs (Pajares, 1992; Thompson, 1992). Nonetheless, it cannot be expected that teachers changed their thoughts and beliefs in a wink. It is known that teachers' beliefs especially the ones located in the center and deep-rooted of their belief system resist change (Green, 1971; Rokeach, 1968). Still, the findings gained form this study showed that implemented the PDP can be considered influential in changing teachers' beliefs about nature of mathematics, and teaching and learning mathematics.

Guskey (2002) depicted that in case teachers realized that applied teaching method is effective on students, they can change their mind on teaching and learning mathematics. In this study, teachers used modeling problems in their classes, and this provided them an opportunity to observe their students' responses. In the interviews conducted with them after the PDP, teachers emphasized the contribution of use of models, modelling problems and real life examples to students' understanding, developing their ways of solution, and meaningful learning. Therefore, to experience the impact of use of modeling problems in mathematics lessons on students can be considered effective in changing the teachers' beliefs. In a similar fashion, while depicting their thoughts upon the effect of PDP, teachers frequently referred to their experiences they gained during implementation. Therefore, "classroom implementation of modelling problems" can be considered as an effective component of PDP in the change of teachers' beliefs.

Elmore (2002) emphasized that providing teachers with rich-content environments like problem solving with cooperative methods is a significant characteristics of successful career development programs. Likewise, in this study, teachers depicted that participating in weekly meetings conducted before implementation, discussing about modeling question and sharing their solution ways with each other in these meetings improved their conceptions of modeling problems and their knowledge of use of modelling problems in teaching. Moreover, follow up meetings in which students' solution ways were analyzed, teachers discussed and shared students' thinking styles with each other. It is understood from the teachers' expressions that these meetings had also positive contributions on their beliefs. The findings about the change in teachers' beliefs indicate that weekly meetings conducted before and after each application is another effective component of PDP.

5.4 Implications, Suggestions and Limitations

Results obtained from the study have several implications for researchers, practitioners and program developers.

When designing teacher education programs and professional development programs teachers' beliefs can be considered as an important factor to be addressed. For example, teacher education program and professional development programs for preservice and in-service teachers can design courses and professional development programs specifically to address commonly held beliefs such as, predetermined steps to solve a problem; solving too many questions to teach and learn mathematics.

The results obtained from this study imply that in order a PDP to be effective, it should be long term and provide teachers opportunity to reflect on their beliefs, provide them realistic experiences which they can implement the proposed innovations in their own classes and observe effects of them. Moreover, PDP should provide collaborative environment in which they share their ideas and experiences with their colleagues. Furthermore, a continual support should be offered teachers to facilitate their development and change.

A change in the research design can be recommended for the future studies, such as use of mix-study design can be adopted. Also, this study is a part of research project aimed to develop in-service teachers' knowledge and skills about modelling and use of modelling in teaching mathematics. Further studies can be designed by specifically addressing possible links between teachers' beliefs about modelling and their beliefs about teaching and learning mathematics.

As a suggestion, a further study would specifically investigate the relationship between teachers' beliefs and their teaching practices. Also, relationship between the beliefs that teachers held and degree of their change can be investigated. Moreover, how teachers' beliefs changes are projected into their teaching can be investigated through a longitudinal study including observations of teachers' regular classroom practice followed by a PDP.

This study assumed that the teachers were sincere and honest in their answers since they attended the PDP voluntarily. This study was limited to 10 in-service secondary mathematics teachers participated in a one-year PDP on mathematical modelling.

REFERENCES

Ajzen, I., & Fishbein, M. (1980). *Understanding attitudes and predicting social behavior*. Englewood Cliffs, NJ: Prentice-Hall.

Ambrose, E. Y. (2004). *Stuttering: recent developments and future directions, the ASHA leader.*

Anderson, D. A., & Piazza, J. A. (1996). Changing beliefs: Teaching and learning mathematics in constructivist preservice classroom. *Action in Teacher Education, 18(2)*, 51–62.

Andrews, P. & Hatch, G. (1997). *The influence of subject conceptions on secondary mathematics teachers' pedagogic practices*. Paper presented at BERA conference 11-14 September 1997, University of York.

Andrews, P. & Hatch, G. (1999). A new look at secondary teachers' conceptions of mathematics and its teaching'. *British Educational Research Journal, 25(3),* 203-223.

Andrews, P., & Hatch, G. (2000). A comparison of Hungarian and English teachers' conceptions of mathematics and its teaching. *Educational Studies in Mathematics, 43(1),* 31-64.

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal, 9(4),* 373-397.

Ball, D. & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-

Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3-32). San Francisco: Jossey-Bass.

Ball, D. L. (1991). Research on teaching mathematics: Making subject matter part of the equation. In J. Brophy (Ed.), *Advances in research on teaching* (Vol. 2, pp. 1-48). Greenwich: JAI.

Ball, D. L. & Cohen, D. K. (1996). Reform by the book: What is — or might be — the role of curriculum materials in teacher learning and instructional reform? *Educational Researcher, 25(9)*, 6-8.

Barkatsas, A., & Malone, J. A. (2005). "A typology of mathematics teachers' beliefs about teaching and learning mathematics and instructional practices." *Mathematics Education Research Journal, 17,* 69-90.

Bassarear, T. (1989). *The dynamic interaction of cognitive and non-cognitive factors in learning mathematics.* Paper presented at the annual conference of the New England Educational Research Organization, Portsmouth, NH.

Bautista, A., Wilkerson-Jerde, M. H., Tobin, R. G., & Brizuela, B. M. (2014). Mathematics teachers' ideas about mathematical models: A diverse landscape. *PNA, 9(1)*, 1-28.

Beswick, K. (2005). The Beliefs/Practice connection in broadly defined contexts. *Mathematics Education Research Journal, 17(2)*, 39-68.

Beswick, K. (2006). The importance of mathematics teachers' beliefs. *Australian Mathematics Teacher, 62(4)*, 17-22.

Beswick, K. (2007). Teachers' beliefs that matter in secondary mathematics classrooms. *Educational Studies in Mathematics, 65(1)*, 95-120.

Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematician's mathematics and their relationship to practice. *Educational Studies in Mathematics, 79(1)*, 127-147.

Blum, W. & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects – State trends and issues in mathematics instruction. *Educational Studies in Mathematics, (22)*, pp 37-68.

Blum, W., Galbraith, P. L., Henn, H. W., & Niss, M. *Modelling and applications in mathematics education, the 14th ICMI study*. Springer, US.

Boaler, J. (2001). *Pedagogy and power: Exploring the relationship between "reform" curriculum and equity.* Paper presented at the annual meeting of the American Educational Research Association, Seattle, WA.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33(8),* 3-15.

Borko, H., & Putnam, R. T. (1996). Learning to teach. In D. C. Berliner & R. C. Calfee (Eds.), *Handbook of educational psychology* (pp. 673-708). New York: Macmillan.

Borko, H. (2005). *Professional development for all teachers: Achieving educational excellence and equity in the era of accountability.* Presidential Invited Session at the annual meeting of the American Educational Research Association, Montreal, Quebec, Canada.

Borromeo Ferri, R. & Blum, W. (2009). Mathematical modelling in teacher education - Experiences from a modelling seminar. In European Society for Research in Mathematics Education (Eds.), *Proceedings of CERME 6* (pp. 2046- 2055). Lyon , France.

Brown, S. I., Cooney, T. J., & Jones, D. (1990). Mathematics teacher education. In W. R. Houston (Ed.), *Handbook of research on teacher education* (pp. 639-656). New York: Macmillan.

Burkhardt, H. (1994). Mathematical applications in school curriculum. In T. Husén & T. N. Postlethwaite (Eds.), The international encyclopedia of education (2nd ed.) (pp. 3621-3624). Oxford/New York: Pergamon Press.

Burkhardt, H. (2006). Modelling in mathematics classrooms: reflections on past developments and the future. *Zentralblatt für Didaktik der Mathematik, 38 (2)*, 178-195.

Chauvot, J. (2000). *Conceptualizing mathematics teacher development in the context of reform.* Unpublished doctoral dissertation, University of Georgia, Athens.

Chazan, D. & Ball, D. (1999). Beyond being told not to tell. *For the Learning of Mathematics 19*, 2-10.

Clark, C. & Peterson, P. (1986). Teachers' thought processes. In M. Wittrock (Ed), *Handbook of research on teaching (3rd ed.)*, (pp. 255-296). New York: Macmillan.

Cobb, P. (1995). Continuing the conversation: A response to Smith. *Educational Researcher 24(5)*, 25-27.

Cobb, P., & Steffe, L. P. (1983). The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education, 14(2)*, 83-94.

Cobb, P., Wood, T., & Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In R. B. Davis, C. A. Maher & N. Noddings (Eds.), *Journal for Research in Mathematics Education: Constructivist views on the teaching and learning of mathematics* (Vol. 4), (pp. 125-146). Reston, VA: NCTM.

Cohen, D. K., & Ball, D. L. (1990). Relations between policy and practice: A commentary. *Educational Evaluation and Policy Analysis, 12(3)*, 331-338.

Cohen, D. K., & Hill, H. C. (1998). *Instructional policy and classroom performance: The mathematics reform in California (RR-39)*. Philadelphia: Consortium for Policy Research in Education.

Cooney, T. J. (1985). A beginning teacher’s view of problem solving. *Journal for research inmathematics education, 16 (5),* 324-336.

Cooney, T. J. (1987). Teachers' decision making. In D. Pimm (Ed.), *Mathematics, teachers and children* (pp. 273-286). London: Hodder and Stoughton.

Cooney, T. J. (2001). Considering the paradoxes, perils, and purposes of conceptualizing teacher development. In F.-L. Lin & T. J. Cooney (Eds.), *Making sense of mathematics teacher education* (pp. 9–31). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Cooney, T. J. (2003). *Mathematics teacher education in rural communities: Developing a foundation for action*. Paper presented at the meeting of the ACCLAIM Research Symposium, Ravenwood Castle, McArthur, OH.

Cooney, T. J., Shealy, B. E., & Arvold, B. (1998). Conceptualizing belief structures of preservice secondary mathematics teachers. *Journal for Research in Mathematics Education, 29(3)*, 306-333.

Creswell, J. W. & Miller, D. L. (2000). Determining validity in qualitative inquiry. *Theory into Practice*, 39(3), 124-131.

Creswell, J.W. (1994). *Qualitative and quantitative Methods.* Newbury Park, Sage.

Creswell, J.W. (1998). *Qualitative inquiry and research design: Choosing among five approaches.* Thousand Oaks, CA: Sage.

Cross, S. (2009) *Adult teaching and learning: Developing your practice*. Maidenhead: Open University Press/McGraw-Hill Education

Crouch, R. M. & Haines, C. R. (2004). Mathematical modelling: transitions between the real world and the mathematical model. *International Journal of Mathematics Education in Science and Technology, 35(2)*, 197-206.

Cuban, L. (1990). Stability and change in curriculum. In P. Jackson (Ed.), *Handbook of research on curriculum* (pp. 216-247). New York: Macmillan.

Darling-Hammond, L. (1997). *Doing what matters most: Investing in quality teaching*. New York: National Commission on Teaching and America's Future.

Denzin, N. K., & Lincoln, Y. S. (2003). Introduction: The discipline and practice of qualitative research. In N. K. Denzin & Y. S. Lincoln (Eds.), *Strategies of qualitative inquiry* (2nd ed., pp. 1 - 45). Thousand Oaks, CA: Sage.

Denzin, N. K., & Lincoln, Y. S. (2005). Introduction: The discipline and practice of qualitative research. In N. K. Denzin, & Y. S. Lincoln (Eds.), *The Sage handbook of qualitative research* (3rd ed., pp. 1-32). Thousand Oaks, CA: Sage.

Dionne, J. (1984). The perception of mathematics among elementary school teachers. In J. Moser (Ed.), *Proceedings of the 6th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME)* (pp. 223 - 228). Madison (WI): University of Wisconsin.

Doerr, H. (2007). What knowledge do teachers need for teaching mathematics through applications and modelling? In W. Blum, P. Galbraith, H. Henn, & M. Niss (Eds), *Modelling and applications in mathematics education: the 14th ICMI study* (pp.69-78), New York: Springer.

Doerr, H. M. (2006). Examining the tasks of teaching when using students' mathematical thinking. *Educational Studies in Mathematics, 62 (1)*, 3-24.

Doerr, H. M. & English, L. D. (2006). Middle grade teachers' learning through students' engagement with modeling tasks. *Journal of Mathematics Teacher Education, 9*, 5–32.

Doerr, H. M., & English, L. D. (2003). A Modeling perspective on students' mathematical reasoning about data. *Journal for Research in Mathematics Education, 34(2)*, 110-136.

Doerr, H. M., & Lesh, R. (2003). A modeling perspective on teacher development. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: A models & modeling perspective on mathematics problem solving, learning & teaching* (pp. 125–140). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Doerr, H. M., & Lesh, R. A. (2011). Models and modelling perspectives on teaching and learning mathematics in the twenty-first century. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling*, (pp. 247-268). New York: Springer.

Dossey, J. (1992). The nature of mathematics: Its role and its influence. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 39–48). New York: Macmillan.

English, L. D. (2007). Complex systems in the elementary and middle school mathematics curriculum: A focus on modeling. In B. Sriraman (Ed.), *Festschrift in Honor of Gunter Torner. The Montana Mathematics Enthusiast* (pp. 139-156). Information Age Publishing.

English, L. D. & Sriraman, B. (2010). Problem solving for the 21st century. In B. Sriraman & L. D. English (Eds.), *Theories of mathematics education: Seeking new frontiers* (pp. 263-285). Advances in Mathematics Education, Series: Springer.

English, L. D. & Watters, J. J. (2005). Mathematical modeling in the early school years. *Mathematics Education Research Journal, 16 (3),* 58-79.

Erbaş, A. K., Kertil, M., Çetinkaya, B., Çakıroğlu, E., Alacacı, C., & Baş, S. (2014). Mathematical modeling in mathematics education: Basic concepts and approaches. *Educational Sciences: Theory & Practice, 14(4)*, 1621-1627.

Ernest, P. (1988) The attitudes and practices of student teachers of primary school mathematics. In A. Borbas (Ed.), *Proceedings of PME-12* (pp. 288-295), Veszprem, Hungary.

Ernest, P. (1989a). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics teaching: The state of art* (pp. 249–254). London: Falmer.

Ernest, P. (1989b). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching, 15*, 13-34.

Ernest, P. (1991). *The philosophy of mathematics education*. London: Falmer.

Ernest, P. (1994). The philosophy of mathematics and mathematics education. In Biehler, R., Scholz, R. W., Straesser, R. & Winkelmann, B. (Eds), *The didactics of mathematics as a scientific discipline* (pp. 335-349). Dordrecht: Kluwer.

Fraenkel, J. R., & Wallen, N. E. (1996). *How to design and evaluate research in education* (3rd ed.). New York: McGraw-Hill.

Fullan M. G. (2007). *The new meaning of educational change* (4th Ed). London: Cassell.

Fullan, M. G., & Miles, M. B. (1992). Getting reform right: What works and what doesn't. *Phi Delta Kappan, 73*, 745-752.

Furinghetti, F. & Pehkonen, E. (2002). Rethinking characterizations of belief. In G. Leder, E. Pehkonen & G. Törner (Eds), *Beliefs: A hidden variable in mathematics education?* (39-57). Dordrecht: Kluwer.

Galbraith, P. (2007). Dreaming a ‘possible dream’: More windmills to conquer. In C. Haines, P., Galbraith, W., Blum, & S. Khan, (Eds.), *Mathematical modelling (ICTMA12): Education, engineering and economics* (pp. 44-62). Chichester, UK: Horwood.

Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *ZDM, 38(2)*, 143-162.

Galbraith, P.L., & Clatworthy, N.J. (1990). Beyond standard models: meeting the challenge of modelling. *Educational Studies in Mathematics, 21(2)*,137-163.

García, F. J., & Ruiz-Higueras, L. (2011). Modifying teachers' practices: the case of a European training course on modelling and applications. In G. Kaiser, W. Blum, R. Borromeo-Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 569-578). Dordrecth, Países Bajos: Springer.

Garet, M., Porter, A., Desimone, L. Birman, B., & Yoon, K. (2001). What makes professional development effective? Analysis of a national sample of teachers. *American Education Research Journal, 38(4)*, 915-945.

Gravemeijer, K. P. E. (2007). Emergent modelling as a precursor to mathematical modelling. In W. Blum, P.L. Galbraith, H-W. Henn & M. Niss (Eds.), *Modelling and applications in mathematics education. The 14th ICMI study. New ICMI study series, vol. 10* (pp. 137-144). New York: Springer.

Green, T. (1971). *The activities of teaching*. New York, NY: McGraw-Hill.

Greer, B. (1997). Modelling reality in mathematics classrooms: The case of word problems. *Learning and Instruction, 7(4),* 293-307.

Guba, E. G. (Ed.). (1990). *The paradigm dialog.* Newbury Park, CA: Sage.

Guba, E. G., & Lincoln, Y. S. (1994). Competing paradigms in qualitative research. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 105-117). Thousand Oaks, CA: Sage

Gudmundsdottir, S. & Shulman, L. (1987). Pedagogical content knowledge in social studies. *Scandinavian Journal of Educational Research, 31,* 59-70.

Guskey, T. R. (2002). Professional development and teacher change. *Teachers and Teaching: Theory and Practice, 8(3/4)*, 381-391.

Haines, C. & Crouch, R. (2001). Recognising constructs within mathematical modelling. *Teaching Mathematics and its Applications, 20(3)*, 129-138.

Haines, C.R. & Crouch, R.M. (2007) Mathematical modelling and applications: Ability and competence frameworks. In W. Blum, P. L. Galbraith, H-W. Henn & M. Niss (Eds), *Modelling and applications in mathematics education* (pp. 417-424). New York, Springer.

Handal, B., & Herrington, T. (2003). Mathematics teachers' beliefs and curriculum reform. *Mathematics Education Research Journal*, *15(1),* 59-69.

Hart, L. C. (2002). Pre-service teachers' beliefs and practice after participating in an integrated content/methods course. *School Science and Mathematics, 102(1)*, 4–15.

Houston, K. (2002). Assessing the "phases" of mathematical modeling. In W. Blum, P. L. Galbraith, H.-W. Wenn, & M. Niss (Eds.), *Modeling and applications in mathematics education* (pp. 249-256). NewYork: Springer.

Hoyles, C. (1992). Mathematics teaching and mathematics teachers: A meta-case study. *For the Learning of Mathematics, 12(3)*, 32-44.

Ikeda, T. (2007). Possibilities for, and obstacle to teaching applications and modelling in the lower secondary levels. In W. Blum, P. L. Galbraith, H.-

W. Wenn, & M. Niss (Eds.), *Modeling and applications in mathematics education* (pp. 457-462). New York: Springer.

Ikeda, T. & Stephens, M. (2001). The effects of students' discussion in mathematical modelling. In J. F. Matos, W. Blum, S. K. Houston & S. P. Carreira (Eds.), *Modelling and mathematics education*, (pp. 381-400). Chichester: Horwood Publishing.

Izard, J. F., Haines, C. R., Crouch, R. M. & Neill, N. (2003). Assessing the impact of the teaching of modelling: Some implications. In S. J. Lamon, W. A. Parker, & K. Houston (Eds.), *Mathematical modelling: A way of life: ICTMA 11* (pp. 165-177). Chichester: Horwood Publising.

Jensen, T. H. (2007). Assessing mathematical modeling competency. In C. Haines, P. Galbraith, W. Blum & S. Khan (Eds.), *Mathematical modeling (ICTMQ 12) education, engineering and economic* (pp. 141-148). Chishester: Horwood.

Kagan, D. M. (1992). Professional growth among preservice and beginning teachers. *Review of Educational Research, 62(2)*, 129–169.

Kaiser, G. (2006). The mathematical beliefs of teachers about applications and modelling-Results of an empirical study. In J. Novotna, H. Moraova, M. Kratka, & N. Stehlikova (Eds.), *Proceedings 30th conference of the international group for the psychology of mathematics education* (Vol.3., pp. 393-400). Prague: PME.

Kaiser, G., & Maass, K. (2007). Modelling in lower secondary mathematics classroom- Problems and opportunities. In W. Blum, P. Galbraith, H. W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp.99-108). New York: Springer.

Kaiser, G., Blomhoj, M., & Sriraman, B. (Eds). (2006). Mathematical modelling and applications: empirical and theoretical perspectives. *Zentralblatt für Didaktik der Mathematik, 38(2)*, 178-195.

Kuhs, T. M. & Ball, D. L. (1986). *Approaches to teaching mathematics: Mapping the domains of knowledge, skills, and dispositions (Research Memo).* East Lansing, MI: Michigan State University, Center on Teacher Education.

Kupari, P. (2003). Instructional practices and teachers' beliefs in Finnish mathematics education. *Studies In Educational Evaluation, 29(3)*, 243-257.

Leatham, K. (2006). Viewing mathematics teachers' beliefs as sensible systems, *Journal of Mathematics Teacher Education, 9(2)*, 91-102.

Leder, G. C. (1992). Mathematics and gender: Changing perspectives. In D.A. Grows (Ed.), *Handbook of research on mathematics teaching and learning* (pp.627-633). New York: McMillan Publishing Company.

Leder, G. C., Pehkonen, E., & Törner, G. (2002). *Beliefs: A hidden variable in mathematics education.* Dordrecht: Kluwer Academic Publishers.

Lee, C., & William, D. (2005). Studying changes in the practice of two teachers developing assessment for learning. *Teacher Development, 9(2)*, 265–283.

Lerman, S. (1983). Problem solving or knowledge centered: The influence of philosophy on mathematics teaching. *International Journal o/Mathematical Education in Science and Technology, 14*, 59-66.

Lerman, S. (1990). Alternative perspectives of the nature of mathematics and their influence on the teaching of mathematics. *British Educational Research Journal, 16(1),* 53-61.

Lesh, R. & Doerr, H. M. (2000). Symbolizing, communicating, and mathematizing: Key components of models and modeling. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms: Perspectives on discourse, tools, and instructional design* (pp. 361-383). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R. & English, L. D. (2005). Trends in the evolution of models & modeling perspectives on mathematical learning and problem solving. *ZDM: The International Journal on Mathematics Education, 37(6)*, 487-489.

Lesh, R. & Sriraman, B. (2005). Mathematics education as a design science. *ZDM: The International Journal on Mathematics Education, 37(6)*, 490-505.

Lesh, R. A., & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 763-804). Charlotte, NC: Information Age Publishing.

Lesh, R. A., Cramer, K., Doerr, H. M., Post, T., & Zawojewski, J. (2003). Model development sequences. In R. A. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning and teaching* (pp. 35-58). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., & Doerr, H. M. (2003). Beyond constructivism: Models and modeling perspectives on mathematics teaching, learning, and problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 3-33). Mahwah, NJ: Lawrence Erlbaum.

Lesh, R., & Harel, G. (2003). Problem solving, modeling, and local conceptual development. *Mathematical Thinking and Learning: An International Journal, 5(2/3),* 157-190.

Lesh, R., & Lehrer, R. (2003). Models and modeling perspectives on the development of students and teachers. *Mathematical Thinking and Learning: An International Journal, 5(2/3)*, 109-130.

Lesh, R., & Yoon, C. (2007). What is distinctive in (our views about) models & modelling perspectives on mathematics problem solving, learning, and teaching? In W. Blum, P. Galbraith, H. W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 161–170). New York: Springer.

Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). Principles for developing thought-revealing activities for students and teachers. In R. A. Lesh & A. Kelly (Eds.), *Handbook of research design in mathematics and science education* (pp. 591-646). Mahwah, NJ: Lawrence Erlbaum Associates.

Lester, F. K., Garofalo, J., & Kroll, D. L. (1989). Self-confidence, interest, beliefs, and metacognition: Key influences on problem-solving behavior. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 75-88). New York: Springer-Verlag

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage.

Lingefjärd, T. (2000). *Mathematical modeling by prospective teachers using technology* (Doctoral dissertation, University of Georgia). Retrieved from http://ma-serv.did.gu.se/ matematik/thomas.htm on 24.01.2014.

Lingefjard, T. (2002). Teaching and assessing mathematical modelling. *Teaching Mathematics and Its Applications, 21(2)*, 75-83.

Lingefjärd, T. (2006). Faces of mathematical modeling. *Zentralblatt für Didaktik der Mathematik-ZDM, 38(2)*, 96-112.

Lingefjard, T. & Meier, S. (2010). Teachers as managers of the modelling process. *Mathematics Education Research Journal, 22(2)*, 92-107.

Little, J. W. (2002). Locating learning in teachers' communities of practice: Opening up problems of analysis in records of everyday work. *Teaching and Teacher Education, 18(8)*, 917-946.

Lloyd, G. (2002). Mathematics teachers' beliefs and experiences with innovative curriculum materials. In G. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 149-159). Dordrecht: Kluwer Academic Publishers.

Lloyd, G. & Wilson, S. (1998). Supporting Innovation: The impact of a teacher's conceptions of functions on his implementations of a reform curriculum. *Journal for Research in Mathematics Education, 29(3)*, 248–274.

Loucks-Horsley, S., Hewson, P. W., Love, N., & Stiles, K. E. (1998). *Designing professional development for teachers of science and mathematics*. Thousand Oaks, CA: Corwin Press.

Lovat, T. J., & Smith, D. (1995). *Curriculum: Action on reflection revisited*. Australia: Social Science Press.

Maass, K. (2005). Barriers and opportunities for the integration of modeling in mathematics classes: Results of an empirical study. *Teaching Mathematics and Its Applications, 24(2-3)*, 61-74.

Maass, K. (2011). How can teachers' beliefs affect their professional development? *Zentralblatt für Didaktik der Mathematik, 43(4)*, 573-586.

Maass, K., & Gurlitt, J. (2011). LEMA – Professional development of teachers in relation to mathematics modelling. In G. Kaiser, W. Blum, R. Borromeo-Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 629-639). Dordrecth, Países Bajos: Springer.

Maxwell, J. A. (1996). *Qualitative research design: An interactive approach*. Thousand Oaks, CA: Sage.

Mewborn, D. S. (2000). *An analysis of the research on K-8 teachers' mathematical knowledge*. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA.

Murphy, P. K., & Mason, L. (2006). Changing knowledge and beliefs. In P. A. Alexander & P. H. Winne (Eds.), *Handbook of educational psychology* (pp. 305–326). Mahwah, NJ: Lawrence Erlbaum Associates.

National Council of Teacher of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, Virginia.

Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of Curriculum Studies, 19(4)*, 317-328.

Niss, M., Blum, W. & Galbraith, P. L. (2007). Introduction. In W. Blum, P. Galbraith, H. Henn & M. Niss (Eds), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 3-32), New York: Springer.

Op 'T Eynde, P., De Corte, E., & Verschaffel, L. (2002). Framing students' mathematics related beliefs: A quest for conceptual clarity and a comprehensive categorization. In G. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education* (pp. 13-38). Boston, MA: Kluwer Academic Publishing.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research 62 (3)*, 307-332.

Patton, M. Q. (1990). *Qualitative evaluation and research methods.* Newbury Park, CA: Sage.

Pehkonen E., & Toerner, G. (1999). Teachers' professional development: What are the key change factors for mathematics teachers? *European Journal of Teacher Education, 22(2,3)*, 259-275.

Perry, B., Howard, P., & Tracey, D. (1999). Head mathematics teachers' beliefs about the learning and teaching of mathematics. *Mathematics Education Research Journal, 11*, 39–57.

Peterson; P., Fennema, E., Carpenter, T., & Loef (1989). Teachers' pedagogical content beliefs in mathematics. *Cognition and Instruction, 6(1)*, 1-40.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Charlotte, NC: Information Age Publishing.

Ponte, J. P. (1994). Knowledge, beliefs, and, conceptions in mathematics teaching and learning. In L. Bazzini (Ed), *Proceedings of the fifth international conference on systematic cooperation between theory and practice in mathematics education* (pp. 169-177). Grado.

Prawat, R. S. (1992). Are changes in views about mathematics teaching sufficient? The case of a fifth-grade teacher. *The Elementary School Journal, 93(2)*, 196–211.

Putnam, R. & Borko, H. (1997). Teacher learning: Implications of new views of cognition. In B. J. Biddle, T. L. Good, & I. F. Goodson (Eds.), *The international handbook of teachers and teaching* (pp. 1223-1296). Dordrecht, The Netherlands: Kluwer.

Raths, J. (2001). Teachers' beliefs and teaching beliefs. *Early Childhood Research & Practice, 3(1)*. Retrieved July 15, 2013, from http://ecrp.illinois.edu/v3n1/raths.html.

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practices. *Journal for Research in Mathematics Education, 28(6)*, 552–575.

Reusser, K. (1995) *The suspension of reality and sense-making in the culture of school mathematics*. Paper presented at the Sixth EARLY, Nijmegen, The Netherlands.

Richardson, V. (1996). The role of attitudes and beliefs in learning to teach. In J. Sikula (Ed.), *The handbook of research in teacher education* (pp. 102–119). New York: Macmillan.

Roehrig, G. H. & Kruse, R. A. (2005). The role of teachers' beliefs and knowledge in the adoption of a reform-based curriculum. *School Science and Mathematics, 105*, 412-422.

Rokeach, M. (1968). *Beliefs, attitudes and values*. San Francisco: Jossey-Bass.

Schoenfeld, A. H. (2009). Working with schools: The story of mathematics education collaboration. *American Mathematical Monthly, 116(3)*, 197-217.

Schoenfeld, A. H. (1992). Learning to think mathematically: problem solving, metacognition, and sense making in mathematics. In A. D. Grouws (Ed.), *Handbook of research on mathematics learning and teaching* (pp. 334-370). New York: Macmillan.

Schorr, R. Y. & Clark, K. (2003). Using a modeling approach to analyze the ways in which teachers consider new ways to teach mathematics. *Mathematical Thinking and Learning, 5(2-3)*, 191-210.

Schorr, R. Y., & Lesh, R. (2003). A modeling approach for providing teacher development. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: A models & modeling perspective on mathematics problem solving, learning and teaching* (pp. 141–157). Hillsdale, NJ: Lawrence Erlbaum Associates.

Schram, P., & Wilcox, S. (1988). Changing preservice teachers' conceptions of mathematics learning. In M. Behr, C. Lacampagne, & M. Wheeler (Eds.), *PME-NA: Proceedings of the tenth annual meeting, November* (pp. 349-355). DeKalb, IL: Northern University.

Schramm, W. (1971). *Notes on case studies for instructional media projects*. Working paper for Academy of Educational Development, Washington DC.

Sherin, M. G. (2002). When teaching becomes learning. *Cognition and Construction, 20(2)*, 119-150.

Skemp, R. R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher, 26(3)*, 9-15.

Sriraman, B., & Lesh, R. (2006). Beyond traditional conceptions of modeling. *Zentralblatt fur Didaktik der Mathematik*, 38(3), 247-254.

Stake, R. E. (1995). *The Art of Case Study Research*. Sage Publications, Thousand Oaks, CA.

Stillman, G. (2010). Implementing applications and modelling in secondary school: Issues for teaching and learning. In B. Kaur & J. Dindyal (Eds.), *Mathematical applications and modelling* (pp. 300–322). Singapore: World Scientific.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*, 213-226.

Sullivan, P., & Mousley, J. (2001). Thinking teaching: Seeing mathematics teachers as active decision makers. In F.-L. Lin & T. J. Cooney (Eds.), *Making Sense of Mathematics Teacher Education* (pp. 147-163). Dordrecht: Kluwer

Swan, M. (2007). The impact of task-based professional development on teachers' practices and beliefs: A design research study. *Journal of Mathematics Teacher Education, 10(4-6),* 217–237.

Swan, M., Turner, R., Yoon, C., & Muller, E. (2007). The roles of modelling in learning mathematics. In W. Blum, P. Galbraith, H. W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 275-284). New York: Springer.

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*, 105-127.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the national council of teachers of mathematics* (pp. 147-164). New York: Macmillan.

Thompson, P. W. (1994). Images of rate and operational understanding of the fundamental theorem of calculus. *Educational Studies in Mathematics, 26*, 229-274.

Törner, G. (2001). Mentale repräsentationen-der zusammenhang zwischen 'Subject-Matter Knowledge' und 'Pedagogical Content Knowledge'-dargestellt am beispiel der exponentialfunktionen in einer fallstudie mit lehramtsstudenten. In G. Kaiser (Ed.), *Beiträge zum mathematikunterricht* (pp. 628– 631). Hildesheim: Franzbecker.

Törner, G., & Grigutsch, S. (1994). Mathematische weltbilder bei studienanfängern-eine erhebung. *Journal für Mathematikdidaktik, 15(3/4)*, 211-252.

Turner, J. C., Warzon, K. B., & Christensen, A. (2011). Motivating mathematics learning: Changes in teachers' practices and beliefs during a nine-month collaboration. *American Educational Research Journal, 48(3)*, 718-762.

Underhill, R. G. (1988). Mathematics learners' beliefs: A review. *Focus on Learning Problems in Mathematics, 10(1)*, 55–69.

Vacc, N., & Bright, G. W. (1999). Elementary preservice teachers' changing beliefs and instructional use of children's mathematical thinking. *Journal for Research in Mathematics Education, 30(1)*, 89-110.

Van Zoest, L. R., Jones, G. A., & Thornton, C. A. (1994). Beliefs about mathematics teaching held by pre-service teachers involved in a first grade mentorship program. *Mathematics Education Research Journal, 6(1)*, 37-55.

Verschaffel, L., & De Corte, E. (1997). Teaching realistic mathematical modeling and problem solving in the elementary school. A teaching experiment with fifth graders. *Journal for Research in Mathematics Education, 28(5)*, 577-601.

Verschaffel, L., Greer, B., & De Corte, E. (2002). Everyday knowledge and mathematical modeling of school word problems. In K. P. Gravemeijer, R. Lehrer,H. J. van Oers, & L. Verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 171-195). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Von Glasersfeld, E. (1984). An introduction to radical constructivism. In P. Watzlawick (Ed), *The invented reality: How do we know what we believe we know? Contributions to constructivism* (pp. 17-40). New York: Norton.

Wake, G. (2011). Teachers' professional learning: Modelling at the boundaries. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling*, (pp. 653-662). New York: Springer.

Wilson, M. & Cooney, T. (2002). Mathematics teacher change and development; The role of beliefs. In G. Leder, E. Pehkonen, G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 127-148). Dordrecht: Kluwer Academic Publishers.

Wood, T., Cobb, P., & Yackel, E. (1991). Change in teaching mathematics. *American Educational Research Journal, 28(3)*, 587-616.

Yin, R. K. (2002). *Case Study Research, Design and Methods* (3rd ed.). Newbury Park, Sage Publications.

Yu, S. Y. & Chang, C. K. (2011). What did Taiwan mathematics teachers think of model-eliciting activities and modeling teaching? In G. Kaiser, W. Blum, R. B. Ferri and G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 147-158). NewYork: Springer.

Zawojewski, J. S. (2010). Problem solving versus modelling. In R. Lesh, P. Galbraith, C. R. Haines & A. Hurford (Eds.), *Modelling Students' Mathematical Modeling Competencies: ICTMA 13* (pp. 237-244). New York: Springer.

Zawojewski, J. S., & Lesh, R. (2003). A mode and modeling perspective on problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: A models and modelling perspective on mathematics teaching, learning, and problem solving* (pp. 317-336). Mahwah, NJ: Lawrence Erlbaum.

APPENDIX A

STUDENT THINKING SHEET

EXPLANATION: The aim of this sheet is to help teachers in eliciting students' ways of mathematical thinking while solving modelling activities. These sheets, which are prepared separately for each activity, will be later assembled into a leaflet. These leaflets will guide pre-service and practicing teachers in application of the activities. In order for the sheet to serve that purpose, it is crucial that you think in depth about every single dimension listed below and express your ideas in a detailed way. Thanks for your contribution.
Name of the Activity: **Classroom of the Implementation:** **Implementing Teachers Name-Surname:**
Student Solution Strategy–1
Student Solution Strategy –2
Student Solution Strategy –3
Student Solution Strategy –4

Strategies	Mathematical concept/ skill/process	Mistakes/ Misconceptions
Solution strategy–1		
Solution strategy –2		
Solution strategy –3		
Solution strategy –4		
Please write any other thoughts you want to express regarding this sheet.		

APPENDIX B

FORMAT OF JOINT LESSON PLAN

Name-Surname:
Model Eliciting Activity:
Class:
Related Subjects:
Total Time:
Time for introducing: Time for solution process:
Time for presentations: Time for ending:
Objectives:
The skills that the students can use:
Materials:

PREPARATION

- What are the mathematical concepts and the relations between them that is embedded in the model eliciting activity?

IMPLEMENTATION

- Before the implementation of the model eliciting activity, which prerequisite knowledge are required in order to supply the mathematical concepts that are embedded in the activity?

- Other issues that the teacher can pay attention in the preparation phase?

- Class setting (What are the criteria for determining the group‟s structure and the number of the students in one group?)

- How can the implementation be introduced?

- What can be done in order to provide an understanding of the problem and to warm up the question?

- What kind of solution strategies that the students can use while working on the model eliciting activity?

- What kind of errors students might encounter in the solution process of the model eliciting activity and what kind of questions teacher can use in order to overcome these errors?

- What kind of difficulties student might encounter in the solution process of the model eliciting activity and what kind of questions teacher can use in order to overcome these difficulties?

- What kind of questions teacher can use in the solution process of the model eliciting activity and what are the aims of these questions?

- What can be the assessment criteria while the students are working on the question?

- How can teacher organize the presentations of the solutions (e.g., groups' order, groups' presentation process)?

- What can be the assessment criteria while the students are presenting the solutions?

- How can the implementation be ended?

- On what other issues, can teacher pay attention in the implementation phase?

APPENDIX C

FIRST INTERVIEW QUESTIONS

GİRİŞ

1. Geçmiş öğrencilik deneyiminizden bahseder misiniz? Nasıl bir öğrenciydiniz? Peki matematik dersinde nasıl bir öğrenciydiniz?
2. En çok anımsadığınız (sizi iyi ya da kötü en çok etkileyen) öğretmenlerinizden bahseder misiniz? Hangi yönlerden sizi etkilediler?
3. Sizi matematik öğretmeni olmaya yönelten sebepler nelerdir? (Neden matematik öğretmeni olmayı tercih ettiniz?)
4. Bir saatlik bir matematik dersinizi anlatır mısınız?

   Derse nasıl başlıyorsunuz?

   Ders boyunca neler yapıyorsunuz, ne tür sorular, etkinlikler, vb. kullanıyorsunuz?

   Derste sizin rolünüz nedir?

   Derste öğrencilerin rolü nedir?

   Derste materyal/teknoloji kullanıyor musunuz? Nasıl kullanıyorsunuz?

   Ders işlerken kullandığınız belirli bir yöntem(ler) var mı?

   Dersi nasıl bitiriyorsunuz?

   Ders sırasında ve/veya sonunda bir değerlendirme yapıyor musunuz?

   Öğrencilerinizin derste anlattığınız konuları öğrendiklerini nasıl anlıyorsunuz?

MATEMATİK

1. Biraz matematikten bahsedelim. Sizce matematik nedir? (Matematiği nasıl tanımlarsınız? Matematiğin önemi nedir?)
   Matematiksel bilgiyi nasıl tariff edersiniz? Nasıl oluşur? Nasıl gelişir?
   Matematikte bir şeyin doğru olup olmadığını nasıl anlarsınız? Neyin doğru olduğuna karar veren otorite kimdir?
   Matematiksel bilgilerin doğruluğuna nasıl karar verilir?
   Matematikte bir bilgi nasıl doğrulanır ya da desteklenir?
2. Matematiği tanımlayan 4 kelime seçmeniz istense bunlar neler olurdu? Neden?
3. Matematikten bahsettiniz. Biraz da okul matematiğinden bahsedelim. Sizce okul matematiği nedir?
4. Matematik ile okul matematiği arasında fark var mıdır? Neden?
5. Matematiksel problem nedir? Örnek verebilir misiniz?
   Sizce problem çözmek ne demektir?
   Her problemin (her zaman) doğru bir çözüm yolu var mıdır?
6. Okul matematik problemi ile günlük hayat problemi arasında fark var mıdır?
7. Matematiksel kavram nedir? Matematiksel prosedür nedir? Sizce matematikte kurallar ve prosedürlerin rolü nedir?
   "*Matematiksel kavram gelişiminden önce öğrencilerin temel prosedür bilgisine sahip olması gerekir*" (Veya "*öğrenciler günlük hayat problemlerini çözmeye geçmeden önce bazı temel prosedürel bilgileri öğrenmelidir*") ifadesi hakkında ne düşünüyorsunuz?

MATEMATİK ÖĞRENİMİ

1. Sizce iyi bir matematik öğrencisini tanımlayan özellikler nelerdir? Sınıfınızdan örnekler verir misiniz?
2. Başarılı bir matematik öğrencisi ile başarısız bir öğrenci arasındaki farklar nelerdir?
3. Daha önce matematiksel kavramın ne olduğundan bahsetmiştiniz. Öğrencinin bir konuyla ilgili matematiksel kavramları

anladığını/kavradığını/öğrendiğini nasıl anlarsınız? (Matematiksel nedir? Matematiksel düşünme nedir?)

4. Sizce öğrenciler en iyi nasıl öğrenir? Örnek verebilir misiniz? Hep bu şekilde mi düşünüyordunuz?
    a. Eğer hayırsa, ne zamandan beri bu şekilde olması gerektiğine inanıyorsunuz?
    b. Geçmişte nasıl düşünüyordunuz?
    c. (O zamandan beri) ne değişti? Neden?
    d. Eğer değiştiğinizi düşünüyorsanız, bu değişime neyin (veya nelerin) katkısı olduğunu düşünüyorsunuz?
5. Sizce bir öğrenci matematiği öğrenmek için derste ne yapmalı? Ders dışında ne yapmalı?
6. Öğretmenin öğrencinin matematik öğrenmesindeki rolü sizce nedir?
7. Öğrencinin kendi matematik öğrenimindeki rolü nedir?

MATEMATİK ÖĞRETİMİ

1. Sizce matematik öğretiminin amacı nedir? Ne olmalıdır? Sizin matematik öğretimindeki amacınız ne?
2. Size göre iyi bir matematik öğretmeninin sahip olması gereken bilgi ve özellikler/beceriler nelerdir? Örnek verebilir misiniz? Hep bu şekilde mi düşünüyordunuz?
    a. Eğer hayırsa, ne zamandan beri bu şekilde olması gerektiğine inanıyorsunuz?
    b. Geçmişte nasıl düşünüyordunuz?
    c. (O zamandan beri) ne değişti? Neden?
    d. Eğer değiştiğinizi düşünüyorsanız, bu değişime neyin (veya nelerin) katkısı olduğunu düşünüyorsunuz?
3. Derslerinizi dizayn ederken/planlarken neleri dikkate alıyorsunuz?
4. İdeal bir matematik dersi/sınıf ortamı nasıl olmalıdır?

    Bu sınıfta öğretmen ne yapıyor olurdu?

    Öğrenci ne yapıyor olurdu?

    Sınıfın fiziki şartları nasıl olmalı? vb.

Müfredatı nasıl hazırlanmış olurdu?

Öğrencilerin öğrenmelerinin değerlendirmesi nasıl yapılırdı?

5. Siz derslerinizi idealinizdeki gibi mi işliyorsunuz? (Hayır ise) Öyle olmamasının nedenleri nelerdir?

APPENDIX D

OPEN-ENDED ANALOGY QUESTIONNAIRE

Ek-4: Açık-uçlu Yansıtıcı Anket

1. Matematik öğretmeni { • Turist Rehberi • Antrenör • Kumandan • İnşaat Ustası • Haber spikeri • Bahçıvan • Orkestra Şefi • Mühendis • Doktor • Misyoner • Jokey • Komedyen • Gardiyan • Heykeltıraş • Aşçıbaşı • ________ } gibidir.

a. Yukarıda verilen analojilerden bir matematik öğretmenini en iyi şekilde temsil edeni seçiniz veya en sondaki boşluğa kendi analojinizi yazınız ve seçiminizi açıklayınız.

b. Yukarıda verilen analojilerden bir matematik öğretmenini pek temsil etmeyeni seçiniz veya en sondaki boşluğa kendi analojinizi yazınız ve seçiminizi açıklayınız.

2. Matematik öğrenmek { • ev inşa etmek • tarife göre yemek yapmak • kilden bir heykel yapmak • ağaçtan meyve toplamak • yap-boz yapmak • film seyretmek • seri üretim yapan bir fabrikanın montaj hattında çalışmak • deney yapmak • ________ } gibidir.

a. Yukarıda verilen analojilerden matematik öğrenmeyi en iyi şekilde temsil edeni seçiniz veya en sondaki boşluğa kendi analojinizi yazınız ve seçiminizi açıklayınız.

b. Yukarıda verilen analojilerden matematik öğrenmeyi pek temsil etmeyeni seçiniz veya en sondaki boşluğa kendi analojinizi yazınız ve seçiminizi açıklayınız.

APPENDIX E

END OF YEAR INTERVIEW

MATEMATİK

1. Matematiği nasıl tarif edersiniz?
2. Daha önce sizinle yaptığımız görüşmede "Matematiksel bilgi nasıl gelişiyor?" diye sormuştum. "Orada bir gelişim yok, aynı bilgiyi kullanıyoruz sürekli" demiştiniz. Burada tam olarak ne demek istediniz? Biraz açabilir misiniz?
3. Size bir ifade okuyacağım, bu ifade hakkındaki düşüncelerinizi merak ediyorum.
   a) Formüller, kurallar gibi matematiksel gerçekleri akılda tutmak matematik öğrenmek için gereklidir.
   b) Matematik problemini çözerken her zaman izlenecek bir kural vardır.
4. Matematik öğretiminde problem çözmenin yeri nedir sizce?
5. Yılsonu toplantısında "modelleme sorularının net bir cevabı olmalıdır, çok fazla açık uçlu olmamalıdır" gibi bir fikir ortaya atıldı. Siz bu konuda ne düşünüyorsunuz?

MATEMATİK ÖĞRENME

1. Sizce öğrenciler matematiği en iyi nasıl öğrenirler?
2. Ya da öğrenciler matematikteki bir konuyu mesela anlamak için ne yapmalılar?
3. Aşağıdaki ifadeler hakkındaki düşünceleriniz nelerdir?

a) Matematik öğrenmenin en iyi yolu derste öğretmenin anlattıklarını dikkatli bir şekilde dinlemekten geçer.
b) Öğrencilere kendi matematiksel fikirlerinin üzerinde düşünecekleri ve değerlendirme yapabilecekleri ortamlar sağlamak matematik öğrenmeleri için önemlidir. (Öğrencilerin matematik problemleri için üretecekleri çözümleri birbirleriyle tartışmaları onlara yarar sağlar.)
c) Matematik öğrenme öğrencilere destekleyici bir ortamda zorlayıcı aktivitelerin verilmesiyle gelişir.

MATEMATİK ÖĞRETİMİ

1. Matematik öğretiminin amacı nedir?
2. Matematik öğretiminde öğretmenin rolü nedir? Öğrencinin rolü nedir?
3. Matematik öğretmenin en etkili yolu nedir?
4. Bir matematik konusunu öğrencilerin anlaması için derste nasıl bir yol izlemek gerekir?
5. Aşağıdaki ifade hakkında ne düşünüyorsunuz?
   a) Matematik öğretmenin en etkili yolu öğrencilere küçük gruplar halinde çalışabilecekleri ilginç problemler sağlamaktır. Öğrencilere küçük gruplar halinde uğraşacakları ilgi çekici problemler sunmak matematik öğretmenin en etkili yollarından birisidir.
   b) Öğrencilere matematiksel problemlere yönelik açık ve kesin çözüm yolları sunmak matematik öğretmeninin görevidir.
6. Öğrencilerin fikir ve düşüncelerinin matematik öğretimindeki yeri nedir?

APPENDIX F

GENERAL EVALUATION INTERVIEW

Genel değerlendirme-değişime dair düşünceler

1. Sizinle yaklaşık 10 aylık bir süre boyunca çeşitli modelleme sorularının incelenmesi, düzenlenmesi, çözülmesi ve uygulanması gibi çalışmalar yaptık. Bu sürecin bireysel olarak size (varsa) katkılarını nedenleriyle birlikte açıklar mısınız? Bu çalışmanın sonunda bilgi, beceri, uygulama ve düşüncelerinizde ne gibi değişiklikler oldu?
    a. İlk uygulama ile son uygulama arasındaki süreçte yaptığınız çalışmaları (öğrenci çözümlerini yorumlayışınız, uygulamaların planlaması vb.) kendi açınızdan nasıl değerlendirirsiniz? (Varsa) farklılıkları/değişimleri neye bağlıyorsunuz?
    b. Çalışmanın sonunda matematik, matematik öğrenimi ve öğretimine ilişkin düşüncelerinizde herhangi bir değişiklik oldu mu? Varsa bu değişime neyin/nelerin etkisi/katkısı olduğunu düşünüyorsunuz?
2. Modelleme etkinliklerinin sınıf-içi uygulamalarında ne tür zorluklar yaşadınız?
    a. Bu zorlukları aşabildiniz mi? Nasıl?
3. Etkinliklerin uygulanması sürecinde sizin bir öğretmen olarak rolünüz nasıldı? Açıklayınız. (Modelleme etkinliğini/sorusunu sınıfında uygulamak isteyen bir öğretmen arkadaşınıza onun rolünü açıklamak (ya da tavsiyelerde bulunmak) isteseniz neler söylersiniz?)

a. Bir modelleme sorusunun uygulandığı bir derste öğretmenin ve öğrencilerin rolleri nasıl olur?
b. Bu problemlerde (modelleme problemleri) öğretmenin ve öğrencinin rollerinin klasik problem çözümündeki öğretmen ve öğrenci rollerinden farkı nedir?

4. Modelleme uygulamalarında öğrencilerin öğrenmelerini nasıl değerlendiriyorsunuz?
   a. Modelleme uygulamalarındaki öğrenci öğrenmeleri klasik matematik problemlerinin uygulandığı derslerdeki öğrenmelerden farklı mıydı? Farklı ise hangi yönlerden farklıydı?
5. Süreçte edindiğiniz deneyimlerinizden yola çıkarak, bu tür modelleme sorularını sınıflarında uygulamak isteyen öğretmenlerin
   a. Neleri bilmesi ve yapabilmesi gerekir?
   b. Nasıl bir matematik öğretim yaklaşımına inanıyor olması gerekir?
6. Kendi uygulamalarınızı düşündüğünüzde, sizce bu tür etkinliklerin daha etkili olması için öğretmenler nelere dikkat etmeli?
7. Uygulama planlarını geliştirdiğiniz süreçle normal zamanda yaptığınız derse hazırlık çalışmalarınızı (günlük hazırlık, yıllık plan vb.) karşılaştırır mısınız? Bu iki sürecin benzer ve farklı yönleri nelerdir?
   a. Uygulama planlarını geliştirdiğimiz süreçte; ortak olarak öğrencilerin hatalarında ve yaşadıkları zorluklarda hangi yöntemleri kullanabileceğimizi, soruda öne çıkan kavramları ve öğrencilerin sahip olması gereken önbilgileri tartıştık. Bu sürecin size katkılarını ve sınırlıklarını değerlendiriniz.
   b. Uygulama planlarını geliştirdiğimiz süreçte, *genel olarak sınıf uygulamalarının yönetimi üzerine* örneğin grupları oluşturma, sorunun anlaşılmasını sağlama ve soruya ısındırma, sunum, toparlama gibi aşamaları tartıştık. Bu sürecin size katkılarını ve sınırlıklarını değerlendiriniz.
8. Bu çalışma boyunca, öğrenci çözüm kâğıtlarını incelediniz, nasıl düşündüklerini yorumlamaya çalıştınız. Bu süreç sizin için nasıl bir deneyimdi? (Ne gibi zorlukları vardı?)

9. Öğrencilerin çözüm kâğıtlarını incelemenizle, yazılı veya ödev kâğıtlarını okuyup değerlendirme sürecini karşılaştırmanız istense, bu iki sürecin benzer ve farklı yönleri ve size sağladığı katkı ile ilgili neler söyleyebilirsiniz?
10. Çalışmanın başındaki ve sonundaki öğrenci çözüm kâğıtlarını yorumlama şeklinizi karşılaştıracak olsanız, neler söyleyebilirsiniz?
    a. Farklılık varsa, bu farklılığı neye bağlıyorsunuz?
    b. Öğrenci çözüm kâğıtlarını inceleme yönteminizi açıklar mısınız? Yönteminizde zamanla bir değişiklik oldu mu? Neden?
11. Uygulama öncesi toplantıda, öğrencilerin soruyu nasıl çözeceklerine yönelik tahminlerde bulundunuz. Tahmin ve beklentilerinizle, öğrenci kâğıtlarından çıkan sonuçları kıyaslayabilir misiniz?
12. Sizce, bir öğretmen modelleme soruları bağlamında öğrencilerin çözüm kâğıtlarını daha iyi anlayabilmek ve yorumlayabilmek için neler yapabilir? Bu süreç size öğrencilerinizin farklı çözüm yaklaşımları, hataları, zorlukları ile ilgili ne gibi bilgi ve deneyimler kazandırdı? (Bu süreç sonunda öğrencilerinizin nasıl düşündükleri ile ilgili düşüncelerinizde bir farklılık oldu mu?)

Matematiksel Modelleme ve Matematik Öğretimi

13. Çalışma süresince karşılaştığınız türden modelleme etkinliklerinin matematik öğretimindeki yeri hakkında ne düşünüyorsunuz?
14. Matematik öğretiminde modelleme etkinlikleri kullanımının
    a. Sağladığı kolaylıklar ve avantajlar nelerdir?
    b. Getirdiği zorluklar ve sınırlılıklar nelerdir?
15. Çalışma bittikten sonraki dönemlerde de bu tür etkinlikleri kendi sınıflarınızda uygulamayı devam ettirmeyi düşünür müsünüz? Neden?
    a. Bu etkinlikleri kendi öğretim yönteminizde nerede, ne amaçla ve nasıl kullanmayı düşünürdünüz?
16. Sizce iyi bir "modelleme" sorusu nasıl olmalı, ne tür özellikler taşımalıdır?
    a. Sizce modelleme sorularının klasik matematik problemlerinden farkı nedir/ ne olmalıdır?
17. Sizce "matematiksel modelleme" nedir?

18. Yaptığımız haftalık toplantılarda (ayrıca öncesinde ve sonrasında) öğretmen arkadaşlarınızla birlikte çalışma imkânı buldunuz. Bu çalışmaları (varsa) proje öncesindeki (veya çalışma sırasındaki) diğer zümre çalışmalarınızla karşılaştırır mısınız?
    a. Öğretmen arkadaşlarınızla birlikte çalışmanın sizin açınızdan olumlu ve olumsuz yönleri nelerdi?
    b. Sizce ideal bir zümre çalışması nasıl olmalı?
19. Sizlerle yürüttüğümüz bu hizmet-içi eğitim çalışmasının etkinliğini arttırmak için eğitim bileşenlerine ekleme ya da çıkarma yapmak isteseniz, ne tür değişiklikler önerirsiniz?

CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Ören Vural, Duygu

Nationality: Turkish (TC)

Date and Place of Birth: 1 January 1983

Phone: +90 312 210 36 86

E-mail: duyguoren@gmail.com

EDUCATION

Degree	Institution	Year of Graduation
MA	METU Secondary Science and Mathematics Education	2004-2007
BS	METU Elementary Mathematics Education Middle East Technical University	1999-2004
High School	Sivas Science High Shool, Sivas	1996-1999

WORK EXPERIENCE

Year	Place	Enrollment
2004 December-present	Middle East Technical University	Research Assistant
2004 September-2004 December	Mehmet Topçu Elementary School, Antalya	Mathematics Teacher

FOREIGN LANGUAGES

Advanced English, Beginner Spanish