THE USE OF CAPACITANCE-RESISTIVE MODELS FOR ESTIMATION OF INTERWELL CONNECTIVITY AND HETEROGENEITY IN A WATERFLOODED RESERVOIR: A CASE STUDY

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ABSTRACT

THE USE OF CAPACITANCE-RESISTIVE MODELS FOR ESTIMATION OF INTERWELL CONNECTIVITY & HETEROGENEITY IN A WATERFLOODED RESERVOIR: A CASE STUDY

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Increasing the oil recovery from the hydrocarbon reservoirs is becoming the most important issue for the oil & gas industry with the increase in energy demand and developing technologies. Waterflooding is one of the most preferable methods because of its success ratio, application ease and cost efficiency. Beside mentioned advantages, this method must be carefully planned and performed by considering reservoir heterogeneities to avoid unexpected poor recoveries.

As an alternative to the reservoir modeling and simulation studies, Capacitance-Resistive Model (CRM) has been developed which uses non-linear signal processing method and needs only production, injection and pressure data to characterize the interwell connectivities between injectors and producers. Fluid storage and connectivity coefficients, which correspond to capacitance and resistance respectively in an electrical circuit, are used in this model to convert injection signals to production responses and honor the material balance in the hydrocarbon systems.

In the light of these studies, a waterflooded carbonate reservoir has been studied to depict the connectivity between wells. Results have been checked with the initial water breakthroughs and reservoir properties which came up in a good agreement. Oil production history match has been performed by using oil fractional flow model which relates total liquid and oil rates. Finally, future prediction studies have been conducted for optimization of the rates to achieve project objectives. The results showed that CRM could be used for history matching and optimization in this carbonate reservoir and resulted in a significant change in project economics.

Keywords: waterflooding, capacitance-resistive model, interwell connectivity.

ÖZ

SU ENJEKSİYONU YAPILMIŞ SAHALARDAKİ KUYULAR ARASI ETKİLEŞİMİ VE HETEROJENLİĞİ BELİRLEMEK İÇİN KAPASİTANS-DİRENÇ MODELLERİNİN KULLANIMI

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Artan enerji talebi ve gelişen teknolojilerle birlikte hidrokarbon rezervuarlarından petrol kurtarımını arttırmak, petrol endüstrisi için en önemli mesele haline gelmiştir. Su enjeksiyonu, başarı oranı, uygulama kolaylığı ve maliyet verimi açısından en tercih edilen metotlardan birisidir. Bu bahsedilen avantajların yanında; beklenmedik düşük kurtarımlardan kaçınmak amacıyla, bu metodun rezervuar heterojenliği göz önünde bulundurularak dikkatli bir şekilde planlanması ve uygulanması gerekir.

Rezervuar modelleme ve simülasyon çalışmalarına alternatif olarak, doğrusal olmayan sinyal işleme modeli kullanan ve sadece üretim, enjeksiyon ve basınç verisine ihtiyaç duyan bir kapasitans - direnç modeli (CRM) geliştirilmiştir. Bu modelde, enjeksiyon sinyallerini üretim tepkilerine dönüştürmek ve hidrokarbon sistemlerindeki kütle korunumunu sağlamak amacıyla, elektrik devresindeki kapasitans ve dirence karşılık gelen akışkan depolama ve iletişim katsayıları kullanılmaktadır.

Bu çalışmaların ışığında; kuyular arasındaki ilişkiyi resmetmek amacıyla su enjeksiyonu yapılan bir saha çalışılmıştır. Sonuçlar kuyulardaki ilk su gelişleriyle ve rezervuar parametreleriyle kontrol edilmiş ve tutarlı bulunmuştur. Toplam akışkan üretimi ile petrol üretimini ilişkilendiren fraksiyonel petrol akış modeli kullanılarak, petrol üretim tarihçesi çakıştırılmıştır. Son olarak, proje hedeflerine ulaşmak için optimizasyon amaçlı gelecek tahminleri yapılmıştır. Sonuçlar, CRM modelinin tarihçe çakıştırma ve optimizasyon amaçlı bu karbonat rezervuarda uygulanabildiğini ve proje ekonomisinde önemli bir değişikliğe neden olabildiğini göstermiştir.

Anahtar Kelimeler: su enjeksiyonu, kapasitans-direnç model, kuyular arası etkileşim.

*To Sagittarius A**

(Supermassive Black Hole of the Milky Way)

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LIST OF ABBREVIATIONS AND SYMBOLS

Abbreviations

OOIP	: Originally Oil in Place
EOR	: Enhanced Oil Recovery
CRM	: Capacitance-Resistive Model
ICRM	: Integrated Capacitance-Resistive Model
PVT	: Pressure – Volume – Temperature
MLR	: Multivariate Linear Regression
BHP	: Bottomhole Pressure
InSAR	: Interferometric Synthetic Aperture Radar
BMLR	: Balanced Multivariate Linear Regression
IBMLR	: Instantaneous Balanced Multivariate Linear Regression
SSE	: Sum of Squared Error
GOR	: Gas-Oil Ratio
BCM	: Balanced Capacitance Model
UCM	: Unbalanced Capacitance Model
CAPEX	: Capital Expenditure
OPEX	: Operating Expense
PI	: Productivity Index
API	: American Petroleum Institute
DST	: Drill Stem Test
WOR	: Water Oil Ratio
CWI	: Cumulative Water Injection
CRMP	: Capacitance Resistive Model (Producer-Injectors)
CRMIP	: Capacitance Resistive Model (Injector-Producer Pair)
CRMT	: Capacitance Resistive Model (Total Field)
LVIR	: Linear Variation of Injection Rate
LVBHP	: Linear Variation of Bottomhole Pressure
SVIR	: Stepwise Variation of Injection Rate
cp	: Centipoise
mD	: milidarcy

Greek Symbols

α_m	: filter coefficient in CRM
$\alpha_{ij}^{(n)}$	: 12 filter coefficients in MLR
α, β	: constants in Gentil's oil fractional flow study
§	: variable of integration
λ	: weight coefficient between injector and producer
λ_{0j}	: unbalanced part of system in MLR, rbbl/d
λ_{ij}	: weight coefficient between injector i and producer j
λ_p	: weighting factor for the primary production contribution
τ	: time constant of the drainage volume, d
τ_F	: time constant of the field, d
τ_{ij}	: time constant of the drainage volume of injector i and producer j, d
τ_{ip}	: time constant when only injector i is active, d
τ_j	: time constant for drainage area of the producer in CRMP, d
τ_p	: resultant time constant of the primary production solution, d
μ	: viscosity of the fluid, cp
μ_j	: Lagrange Multiplier
$\Delta i^{(m)}$	: change in the injection for time interval of t_{m-1} to t_m, rbbl/d
Δn	: selected discretization interval
ΔP	: pressure change, psia
$\Delta p_{wf}^{(m)}$	: change in BHP for time interval of t_{m-1} to t_m, psi

Latin Symbols

a, a_1, a_2, b, c	: constants in fractional oil fractional flow models
A_{ij}	: area open to flow, m^2
$C1$ & $C2$	: proportionality constants in MLR
c_t	: total compressibility, 1/psi
c_{tij}	: total compressibility between injector i and producer j, 1/psi
CWI_i^n	: cumulative water injected into an injector i at time step n, rbbl
D	: dissipation constant in diffusivity filters
E_i	: exponential integral function
E_R	: overall recovery efficiency, %
E_V	: volumetric sweep efficiency, %
E_D	: displacement efficiency, %
$E_w^{(m)}$	: flux into the reservoir from external source, rbbl/d
f_o	: oil fractional flow or oil cut, %

i_i	: observed injection rate of injector *i*, rbbl/d
$\bar{\imath}_i$	: average injection rate, rbbls/d
$i(t)$	: injection rate at time step *t*, rbbls/d
$i^c_{ij}(t)$	: convoluted rate of injector *i* affecting producer *j* at time *t*, rbbl/d
$i'_{ij}(n)$	: convolved or filtered injection rate at step *n*, rbbl/d
I	: total number of injection wells
$I^{(m)}$	: constant injection rate during the time interval Δt_m, rrbl/d
J	: productivity index of the producer, rbbl/d/psi
J_{ij}	: productivity index when only one injector is active, rbbl/d/psi
k	: permeability, mD
$\overline{k_{\imath\jmath}}$	: average effective permeability, m^2
L_{ij}	: length of the path, m
n_0	: initial time step
N	: total number of production wells
N_T	: total number of time steps
N_p	: cumulative oil production in oil fractional flow models, rbbl
$N_{p,j}^n$	: cumulative liquid produced from a producer *j* at time step *n*, rbbl
$\bar{p}_{ij}$	: average pressure of the medium when injector *i* is only active
P_{wf}	: flowing bottomhole pressure of the producer
P_{wf}^i	: flowing bottomhole pressure of injector *i*
P_{wf}^j	: flowing bottomhole pressure of producer *j*
$P_{wfj}'(n)$	: convolved BHP at step *n* for producer *j*
$P_{wfkj}'(n)$	: convolved BHP at step *n* for producer *k*
$\bar{p}$	: average pressure, psia
p_o	: oil price, $/rbbl
p_{wo}	: disposal cost of the produced water per barrel, $
p_w	: water injection cost per barrel, $
q	: production rate, bbl/d-m-y
$\bar{q}_j$	: average production rate, bbls/d
$q(t)$	: production rate at time step *t*, bbls/d
$q_{ij}(t_0)$	: initial production rate when only injector *i* is active
$q_j(t_0)$	: pre-injection total primary production rate of producer *j* in CRM
$q_j(t)$	: total production rate at time step *t*, bbls/d
q_t	: production rate at time *t*, bbl/d
$\hat{q}_j(t)$	: estimated model production rate, rbbl/d
q_{ij}	: production rate at producer *j* that corresponds only to injector *i*
q_{0j}	: unbalanced part in CRM, rbbl/d
q_{oj}	: oil production rate of producer *j*
q_o	: oil rate, rbbl/d

q_w	: water rate, rbbl/d
q_l	: liquid rate, rbbl/d
r	: distance from any point to the well, m
R	: optimization algorithm objective function
t	: time, d-m-y
t_0	: initial time step, d-m-y
T_{ij}	: transmissibility between injector *i* and producer *j*, rbbl/d
v_j	: coefficient determining the effect of changing the BHP of producer *j*
v_{kj}	: coefficient showing the effect of changing BHP of nearby producer *k*
V_p	: pore volume being drained, rbbl
$V_{p_{ij}}$	: pore volume between injector *i* and producer *j*, rbbl
W_i	: cumulative water injection, rbbl
WOR	: cumulative water-oil ratio, %
W_p	: cumulative water production, rbbl

Subscription

i	: injector index
j	: producer index
k	: producer-BHP index
n	: time-like variable

CHAPTER 1

INTRODUCTION

As the conventional reservoir exploitation is becoming more difficult, reservoir characterization and the net present value maximization of the existing reservoirs have become very important. That is why secondary and enhanced oil recovery (EOR) methods have come into play and become very popular in oil & gas industry during the last century.

Considering the current technologies, oil recovery process can be subdivided into three stages depending on the production methods namely primary, secondary and tertiary production (Figure 1.1). Primary production is the initial stage controlled by the energy of reservoir nature itself and continues until the oil production becomes uneconomical. Secondary recovery can be achieved after primary production by waterflooding or injection of immiscible fluid (water or natural gas) for pressure maintenance. Tertiary recovery may start after either primary or secondary recovery and includes thermal, gas injection, chemical and microbial methods.

For the most of the reservoirs, it is more advantageous to study and plan a secondary or tertiary process within the early stage of production life. According to Terry and Rogers (2015), the primary production methods can recover up to 25 to 30% of the original oil in place (OOIP). The remaining 70% to 75% of the resource is large and attractive target for additional recovery.

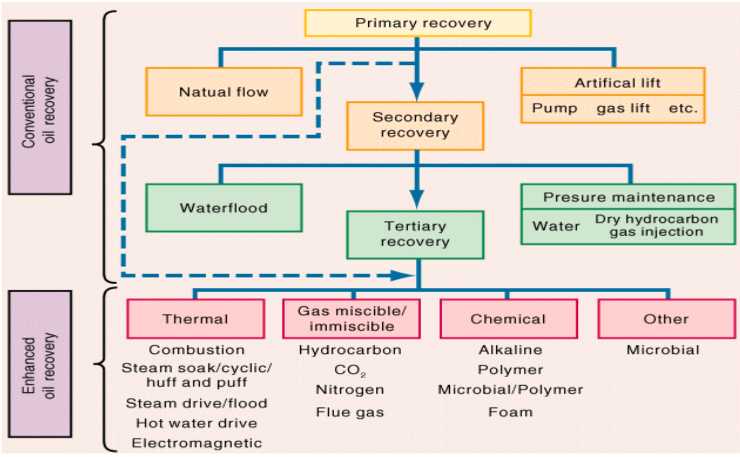


Figure 1.1 Hydrocarbon Recovery Methods (Moritis G., 1998)

The most common secondary recovery method applied all around the world is waterflooding because of its proved success ratio, application ease and cost efficiency. The recovery efficiency of a waterflood is largely a function of the sweep efficiency (success of contacting the pore space in oil-bearing zone) and the ratio of oil – water viscosities. Gross heterogeneities (fractures, high permeability streaks, faults etc.) and high viscosity ratios may lead to significant bypassing of residual oil and lower flooding efficiencies (Terry & Rogers, 2015).

Analysis of injection and production data to infer the interwell connectivity becomes more crucial in cases that the reservoir is heterogeneous or information about the reservoir is not enough. Several studies were conducted which are based on statistics, neural network, analytical and numerical calculations to infer interwell connectivities and understand the flow mechanisms.

The Capacitance-Resistive Model (CRM) is one of these studies using the most reliable data in the waterflooding projects which are “rate” and “pressure”. This method is a material balance based flow model, which considers the transmissibility and compressibility effects, to understand the interactions and their dissipations between injector-producer pairs. In this study, this method is applied to a waterflooded carbonate reservoir to characterize interwell connectivities and optimize oil production to maximize the net present value of the project economics.

CHAPTER 2

LITERATURE REVIEW

2.1 Waterflood Prediction Methods

According to Thakur and Satter (1998), the main purposes of the waterflood reservoir management studies are to estimate reserves, recovery rates and flood life for designing a project which can be done by the analysis of past and future performance. The common methods for these studies can be categorized as follows:

- Volumetric Methods
- Empirical Methods
- Classical Methods
- Performance Curve Analysis Methods
- Numerical Simulation Methods

2.1.1 Volumetric Methods

Once the oil in place prior to waterflood is calculated by using the original oil in place and cumulative production, the ultimate recovery can be estimated by using a recovery efficiency factor.

Estimated Ultimate Recovery = (Pre-Injection Oil in Place) x (Recovery Efficiency)

Recovery efficiency factor can be estimated from analog fields which show similar characteristics. It can be also estimated from the product of the volumetric sweep and displacement efficiencies as shown below (Satter and Thakur, 1994):

$$E_R = E_V \cdot E_D \tag{2.1}$$

where,

E_R : overall recovery efficiency

E_V : volumetric sweep efficiency made up of areal and vertical sweep efficiencies

E_D : displacement efficiency determined from laboratory tests

Another way of estimating the displacement efficiency and residual oil saturation is fractional flow theory (Bukley and Leverett, 1942) which requires some petrophysical parameter inputs. In addition to these methods, empirical correlations such as proposed by Croes and Schwarz (1955) can be used to calculate the displacement efficiency (Figure 2.1). From this figure, both oil recovery and water oil ratio can be determined as a function of the total liquid production (oil+water) and viscosity ratio.

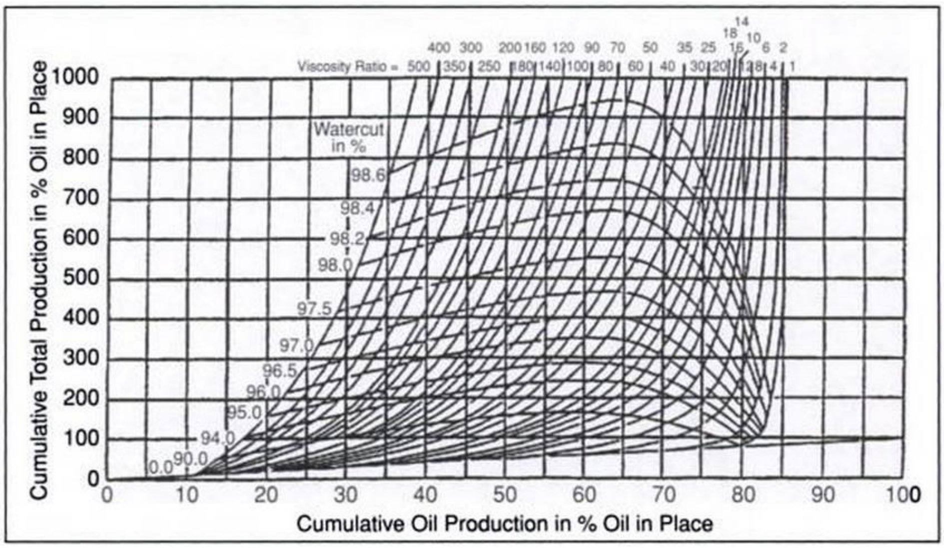


Figure 2.1 Experimental Waterflood Performance (Croes and Schwarz, 1955)

Volumetric method may be very important at early time decision making stages for the waterflooding projects. Although the volumetric method gives an estimate of the waterflood recovery, it does not provide production forecast to use in economic model of the project.

2.1.2 Empirical Methods

Empirical methods for predicting waterflood performances are mainly based on:

- Correlations with rock and fluid properties
- Rate, timing and production trend responses

Gutherie and Greenberger (1955) found that the oil recovery in water drive reservoirs was related to the some rock and fluid parameters (permeability, porosity, oil viscosity, formation thickness, connate water saturation, depth, oil reservoir volume factor, area and well spacing) and proposed an equation for recovery estimation. Schauer (1957) presented an empirical method for predicting the waterflood behavior of Illinois Basin waterfloods and constructed a plot showing percentage fill-up at first signs of an oil production response as a Lorenz coefficient. Guerrero and Earlougher (1961) presented a number of rule of thumbs for predicting performance which have limited applicability. Arps et al. (1967) conducted a statistical study (312 water-drive reservoirs) which resulted in an equation depending on porosity, connate water saturation, permeability, oil & water viscosities, initial pressure and pressure at depletion. There are two more studies proposed by Bush and Helander (1968) and Wayhan et al. (1970) which have limited usefulness in particular area being studied. Craig (1971) summarized all these empirical methods, which can provide good result when derived from and applied to the areas having similar characteristics, to show that these models could be used for estimating the performance of the projects.

2.1.3 Classical Methods

Craig (1971) summarized the published classical methods which primarily concerns with reservoir heterogeneity, areal sweep and displacement mechanism.

Reservoir Heterogenity: These studies have a common assumption of piston like displacement. Yuster and Calhoun (1944) developed equations which explain the variation in injectivity within three stages of a five spot pattern waterflood based on an assumption of equal mobilities. Muskat (1950) extended this study by increasing the mobility ratio range to 0.1 to 10 and discussed about the effects of permeability distributions. Prats et al. (1959), based on the same approach, developed a method of predicting five spot pattern waterflood performance by including combined effects of mobility ratio and areal sweep efficiency. Stiles (1949) proposed a method which accounts for the different flood-front positions in liquid filled, insulated linear layers to derive oil recovery and water cut equations by using the permeability variation of the layers and layer flow capacities. Dykstra-Parsons (1950) developed a method which uses a correlation between waterflood recovery, mobility ratios and permeability distributions by studying more than 200 flood pot tests performed.

Areal Sweep: Muskat (1946) conducted several mathematical and experimental studies to determine the streamline and isopotential distributions in various flooding patterns. Hurst (1953) developed Muskat's method to consider initial gas saturation prior to water saturation with an assumption of equal mobilities. Caudle and coworkers [Slobod and Caudle (1952), Dyes et al. (1954), Caudle and Witte (1959), Caudle and Loncaric (1960), Kimbler et al. (1964) and Caudle et al. (1968)] had many studies on areal sweep efficiencies in different flooding patterns which are four, five, seven, nine spot and line drive patterns. Aronofsky (1952) and Aronofsky and Ramey (1956) worked on the areal sweep efficiencies at breakthrough as a function of mobility ratio for five spot and line drive well arrangements. A study presented by Deppe (1961), which is about the injectivity of pattern floods as a series of linear and radial systems, used by Hauber (1964) to calculate five spot and direct line drive pattern flood performance.

Displacement Mechanism: Buckley and Leverett (1942) developed a method considering the mechanism of oil displacement by water in either a linear or radial system which was later modified by Welge (1952) to simplify its usage. Roberts (1959) and Kufus and Lynch (1959) combined the frontal drive equation with Dykstra-

Parsons method to eliminate the limitation of piston like displacement. Craig et al. (1955) developed a method, which is one of the most practical methods, based on Welge equation and correlations of areal sweep efficiency at and after water breakthrough. Wasson and Schrider (1968) proposed a method of predicting five spot waterflood performance in stratified reservoirs which combined several studies as Yuster and Calhoun (1944), Caudle and Witte (1959) and Craig et al. (1955). Rapoport et al. (1958) developed a method based on a laboratory-developed relationship between linear and five-spot flooding behavior. Higgins and Leighton (1962) performed a study based on stream tube approach at unit mobility ratio, shape factors and Buckley Leverett displacement mechanisms which can be applied for 5-spot, 7-spot, direct / staggered line drive and peripheral patterns.

Craig (1971**)** compared the developed waterflood performance prediction methods and categorized these into the four groups which consider primarily:

- Reservoir heterogeneity
- Areal sweep effects
- Numerical methods
- Empirical approaches

According to this study, “perfect method” for predicting waterflood performance must include all pertinent fluid flow, well pattern and heterogeneity effects. However, most of the methods developed, except the recent mathematical models, are weak because of their assumptions to be used in field cases where the heterogeneity has a great effect on reservoir production.

2.1.4 Performance Curve Analysis Methods

In case of enough available data for the analysis of decline in oil production rate, the past performance of the well, group of wells or field can be extrapolated to predict future performance. It seems that just rates are needed for this work but the reality is different because the production history includes different external effects caused by workover operations, production policies, surface operations, weather, market conditions etc. Hence, care must be taken in analyzing the trend of past production and studying the possible future projections which would directly affect the economics of the project. The commonly used performance curve analysis methods for waterflood projects are shown in Figure 2.2 (Satter and Thakur, 1994):

- Log of oil production rate vs time
- Oil production rate vs cumulative oil production
- Log of water or oil cut vs cumulative oil production
- Oil-water contact or gas oil contact vs cumulative oil production
- Log of cumulative gas production vs log of cumulative oil production

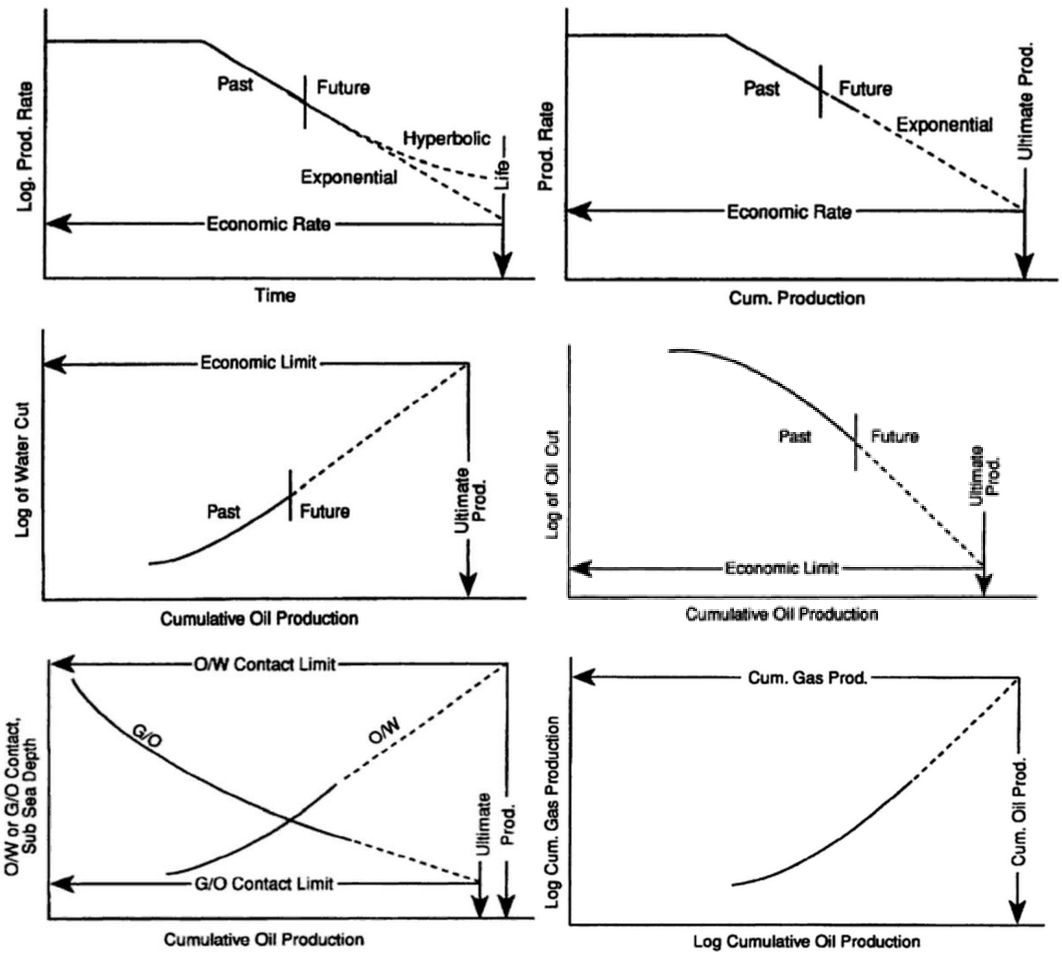


Figure 2.2 Commonly Used Performance Curve Types (Satter and Thakur, 1994)

There are three main types of trends which are used to predict the future production performance of the wells. Mathematical derivations of these hyperbolic, harmonic and exponential curves for rates and cumulative productions are expressed in the study of Arps (1945, 1956).

2.1.5 Numerical Reservoir Simulation

Numerical reservoir simulation studies are based upon material balance which also takes the reservoir heterogeneity and fluid flow direction into account by dividing reservoir into grid cells (Ertekin et al., 2001). Rock and fluid properties and their changes with time for each grid block are very important because of the calculations depending on space and time. Computations using material balance and fluid flow equations are performed for different fluid phases in each cell and time step. Numerical simulation study can be divided into three stages;

1) Data preparation
2) History matching
3) Performance prediction

Data Preparation

The data needed for simulation (expensive and time consuming) must include;

- General data for reservoir (grids, layers, maps, initial conditions)
- Rock and fluid properties (basic/special core analyses and PVT data)
- Grid data & properties (petrophysical parameters)
- Production/injection and well data (rate, pressure and completion data)

History matching

History matching of pressure and production of the well / region / field consists of optimization of the input data until the calculated results match with the observed historical data. But one must remember that these solutions are not unique. That is why uncertainty analysis of each input data must be done to be aware of the possible error ranges. History matching procedure can be summarized as follows:

- Initializing the reservoir model
- Matching pressure and original hydrocarbon in place
- Saturation matching
- Field and well rate matching

Performance Prediction

When the production history is matched, in order to determine the optimum operating conditions and maximize the economics of the project, performance prediction is done by using the same matching parameters and possible development scenarios.

2.2 Interwell Connectivity Determination and Recent Works

2.2.1 Statistical Methods

In addition to the mentioned waterflood performance prediction methods, there are statistical approaches that focus on the performance of production wells by considering their relationships with surrounding injection wells. Heffer et al. (1997) used Spearman rank correlations of well rates to find a relationship between injector-producer pairs and evaluated with geomechanics by focusing on the maximum horizontal stress. Refunjol and Lake (1996) also used Spearman analysis to analyze flow paths by adding time lag concept which corresponds to effect of compressibility of the reservoir fluids. Jansen and Kelkar (1997) studied exploratory data analysis methods on the injection and production data, considering rate and pressure versus time and spatial location analysis. De Sant'Anna Pizarro et al. (1998) used the Spearman rank technique to validate with numerical simulation and examined its benefits and limitations. Soeriawinata and Kelkar (1999) also proposed a method to analyze the superposition effect of multiple injection wells on a producing well by using cross-correlation of summation of the rates of injectors with the producer.

2.2.2 Linear Regression Models

Albertoni and Lake (2003) used a more robust multivariate linear regression method which calculates the interwell connectivity between injector-producer well pairs quantitatively by using the diffusivity filters to consider the time lag between injection and production rates. Gentil (2005) demonstrated the physical meaning of the calculated weighting factor by explaining them as the relative average transmissibility between a pair divided by summation of all pairs' transmissibilities. Dinh and Tiab (2008) developed a model based on the MLR model with the BHP's of injection and production wells instead of rates.

2.2.3 Neural Network Models

Some other studies focused on the neural networks to analyze these relationships. Panda and Chopra (1998) used artificial neural networks to analyze the interaction between injection and production wells within a pattern by using the injection rates, permeability, thickness as the input of the network and oil/water rates as the output of the model. Demiryurek et al. (2008) performed sensitivity analysis based on a real field data to quantify the connectivities of the injector/producer pairs by using trained network.

2.2.4 Capacitance – Resistive Models

Yousef et al. (2006) developed a more complicated model which uses nonlinear signal processing model to evaluate the interwell connectivity by considering not only the injection but also the primary production and bottomhole pressure effects. In this model, time constants instead of the diffusivity filters were used to characterize the time delay of injection signal at the producers. Liang et al. (2007) developed a simple CRM model to optimize oil production without using BHP data by adapting a power-law water cut prediction model. Sayarpour et al. (2007) presented analytical solutions for the continuity equation of the CRM to model three different reservoir control volumes by considering stepwise and linear variations in both injection rates and pressures. Kaviani et al. (2008) proposed segmented CRM and compensated CRM to overcome some limitations of the model. Sayarpour et al. (2009) applied these models to a CO_2 case and concluded that it is a reliable tool for performance prediction for both waterflood and CO_2 flooding. Weber et al. (2009) used the capacitance–resistive model to optimize injection allocation in large reservoirs with many variables and suggested some simplification methods. Yousef et al. (2009) studied CRM applications to detect the permeability trends and enhance the geological features by using log-log and flow capacity plots. Delshad and Paurafshary (2009) also used this model to detect the presence of fractures in a reservoir and calculate fracture permeability. Izgec and Kabir (2009) extended the use of CRM to immature fields in which the transient flow was studied and validated on a streamline simulation study. Nguyen et al. (2011) developed an integrated capacitance-resistive model (ICRM) (using cumulative volumes instead of rates) which is solved by linear regression and compared CRM model parameters with the parameters used in streamline simulation. Naseryan et al. (2011) compared the results of MLR & CRM and showed the advantages of CRM with respect to MLR. Wang et al. (2011) superimposed the CRM established producer-injector connection on InSAR satellite imagery of surface subsidence to analyze the reasons of subsidence in the study area. Kim et al. (2012) applied the ICRM to waterfloods and evaluated the uncertainty on model parameters. Bastami et al. (2012) integrated the capacitance - resistive model into operational and economic analysis of a case study. Salazar et al. (2012) presented a case study of CRM application combined with decline curve analysis to predict the behavior of a mature reservoir under gas injection.

2.3 Multivariate Linear Regression Model

Albertoni and Lake (2002), suggested a linear multivariate regression technique to predict the total liquid production of a well by just using injection and production rates (in reservoir volumes). This technique is based on the material balance which considers just only oil and water, not the gas rate. Working period must not include significant free gas production in this analysis.

In Albertoni & Lake's work, reservoir is considered as a system that processes a stimulus and returns a response. Diffusivity filters are used to take into account the time lag and attenuation that occurs between stimulus and response. Because of the fact that there are several injection and production wells acting at the same time, the input signal is affected by the location and the orientation of the each injector - producer pairs. Three regression types were suggested depending on the models' constraints;

- Multivariate Linear Regression (MLR)
- Balanced Multivariate Linear Regression (BLMR)
- Instantaneous Balanced Multivariate Linear Regression (IBMLR)

2.3.1 Multivariate Linear Regression (MLR)

When the field production rate is considerably different from the injection rate, it can be stated that waterflood is unbalanced and the MLR must be used in this case. In MLR approach, the estimated production rate of a producer j is given by;

$$\hat{q}_j(t) = \lambda_{0j} + \sum_{i=1}^{I} \lambda_{ij} i_i(t) \qquad (j = 1, 2, \ldots . N) \qquad (2.2)$$

where, N is the total number of production wells and I is the total number of injection wells. This equation states that the total production rate (q) at well j is equal to the sum of the injection rates of each injector (i_i) plus a constant term λ_{0j}. The λ_{ij} parameters are the weighting coefficients that determine the connectivity between pairs and the constant term λ_{0j} represents the unbalanced part of the system.

MLR approach is generally used for unbalanced system but this is not the only case. It can be used also for the possible cases below:

- Study of a selected portion of a waterflooded area (boundary influx)
- Production not associated with injected water (primary production or aquifer)
- Injection losses to upswept areas / layers

Yousef (2006) explained the solution of MLR weights by minimizing the sum of squared error (SSE) between the measured and estimated total liquid production rates;

$$Min\left[SSE = \sum_{n=1}^{N_T}\left(q_j(n) - \hat{q}_j(n)\right)^2\right] \tag{2.3}$$

where, N_T is the total number of data points for a time period.

2.3.2 Balanced multivariate linear regression (BMLR)

If the field injection rate is equal to the total production rate (balanced waterflood) the BMLR must be used. In this model λ_{0j} is set to zero as follows:

$$\hat{q}_j(t) = \sum_{i=1}^{I}\lambda_{ij}i_i(t) \qquad (j = 1, 2, \ldots. N) \tag{2.4}$$

This equation states that at any time (t), the total production rate at well j is equal to the sum of the injection rates of every injector.

In the BMLR approach, the balance condition below should be also satisfied;

$$\bar{q}_j = \sum_{i=1}^{I}\lambda_{ij}\bar{\iota}_i \qquad (j = 1, 2, \ldots. N) \tag{2.5}$$

Yousef (2006) explained the solution by introducing a Lagrange multiplier (μ_j), the objective function becomes

$$\left[\sum_{n=1}^{N_T}\left(q_j(n) - \hat{q}_j(n)\right)^2 - 2\mu_j\left(\bar{q}_j - \sum_{i=1}^{I}\lambda_{ij}\,\bar{\iota}_i\right)\right] \tag{2.6}$$

2.3.3 Instantaneous Balanced Multivariate Linear Regression (IBMLR)

The IBMLR approach is very similar to the BMLR approach. The production rate at producer j is described as summation of the rates of each injector with a separate balance condition (Yousef, 2006).

The balance condition in this case is more restrictive compared to the BMLR, which requires that waterflood to be in balance at every time step (t); therefore, IBMLR should be used when the waterflood is in balance at every time step. The IBMLR model for each producer j is;

$$\hat{q}_j(t) = \sum_{i=1}^{I} \lambda_{ij}\, i_i(t) \tag{2.7}$$

The instantaneous balance condition is;

$$\sum_{j=1}^{N} \hat{q}_j = \sum_{i=1}^{I} i_i \tag{2.8}$$

$$\sum_{j=1}^{N} \sum_{i=1}^{I} \lambda_{ij} i_i = \sum_{i=1}^{I} i_i \tag{2.9}$$

Equation also can be written as;

$$\sum_{i=1}^{I} \left(\sum_{j=1}^{N} \lambda_{ij} \right) = \sum_{i=1}^{I} i_i \tag{2.10}$$

thus, the balance condition for each injector is given by;

$$\sum_{j=1}^{N} \lambda_{ij} = 1 \tag{2.11}$$

In IBMLR, the sum of the weights for each input variable (injector) is equal to one.

Yousef (2006) states that the IBMLR system must be solved simultaneously for all producers while the BMLR system can be solved for each producer. The constraints (one for each injector) are introduced in the system of equations again by means of Lagrange multipliers. The objective function in this case is;

$$\left[\sum_{j=1}^{N}\sum_{n=1}^{N_T}\left(q_j(n)-\hat{q}_j(n)\right)^2-\sum_{i=1}^{I}2\mu_j\left(1-\sum_{j=1}^{N}\lambda_{ij}\right)\right] \tag{2.12}$$

2.3.4 Diffusivity Filters

In real cases, it is not very practical to observe the instantaneous effects on producer caused by the injection. According to Albertoni and Lake (2002), to represent the accurate flow behavior in the reservoir, diffusivity filters must be used to consider the time lag and attenuation of the changes. Small permeability, large pore volume, large viscosity and large total compressibility may be the possible reasons for a large dissipation in the reservoirs.

Diffusivity filters and their effects are defined by two factors: the diffusivity constant (parameter depends on the medium) and the distance between the pairs. There is one diffusivity constant for each pair and obtained after an iterative process that minimizes the error between the modeled and the observed production rates.

These diffusivity filters are applied on injection rates and their basic shapes are obtained from the impulse propagation equation (the transient solution to the radial diffusivity equation) assuming a homogeneous reservoir, which is superimposed in time. The filtered form of injection rates are given by;

$$i_{ij}^{c}(t)=\sum_{n=0}^{11}\alpha_{ij}^{(n)}i_i\,(t-n) \tag{2.13}$$

which is the effective injection rate of injector *i* affecting producer *j* at time *t*. The filters include the effects of the most recent 12 months of injection. The $\alpha^{(n)}$ are 12 filter coefficients obtained from the discretization of the filter function. In case of large dissipation, more than 12 filter coefficients may be needed (see Appendix-A for derivation). Figure 2.3 illustrates different dissipation effects of filters (no, moderate and large dissipation) on production behavior.

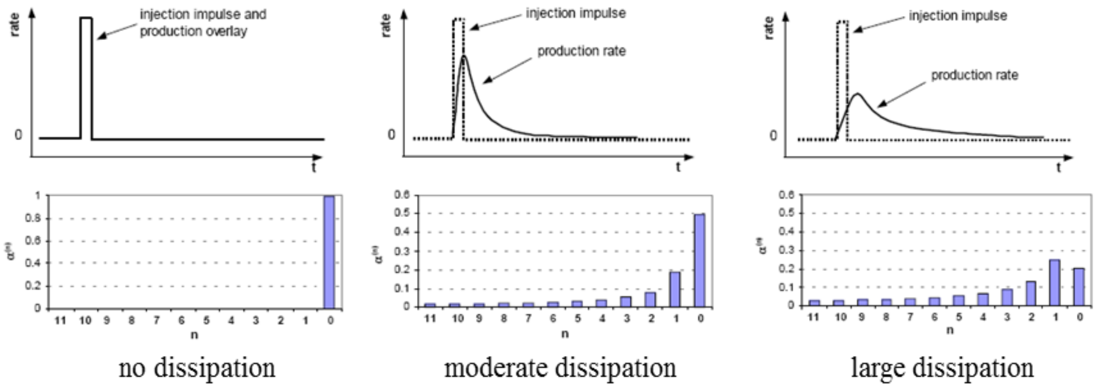


Figure 2.3 Dissipation on Injection-Production Response (Albertoni & Lake, 2002)

2.3.5 Assumptions on Multivariate Linear Regression (MLR)

The general assumption is that all the parameters that affect connectivity between wells must be constant within selected the time period for analysis. These constant parameters can be categorized as below;

Constant number of production wells: The number of the wells and corresponding locations must remain constant within the analysis period. In case of introducing new wells, it would result in a complete new set of weighting coefficients.

Constant producing bottomhole pressure: To capture the pure injection effect on production well, the well performance should be analyzed just based on the injection rates by keeping the bottomhole pressure constant. Unless these effects are decoupled, it is not possible to estimate correct representative weights.

Constant well productivity: Working with the wells which are stimulated would exhibit different production profiles even the injection rates are kept constant. That is why no major changes in wellbore and reservoir properties should occur in the production wells within the analysis period.

<u>Constant gas-oil ratio (GOR):</u> Normally, changes in water and oil saturations will not significantly affect the reservoir properties because of the low compressibility changes. But a change in gas saturation causes a change in the reservoir total compressibility and also indirectly the reservoir diffusivity. GOR should be constant and equal to the dissolved gas-oil ratio in the analyzed period.

<u>No new completions:</u> No new layers should be completed during analysis period.

<u>Constant non-waterflooding production:</u> In the MLR approach, the production accounted for by non-waterflooding reasons (primary production or aquifer support) is assumed to be constant.

2.4 Capacitance – Resistive Models

Previous studies proved that CRM is a powerful tool, which combines surrogate modeling and material balance, to estimate the interwell connectivity within a short time and practical way.

Yousef et al. (2006) introduced a procedure that uses a nonlinear signal processing model to provide information about the interwell connectivity between producer-injector pairs and possible flow barriers. This approach uses a more complex model than MLR by including capacitance (compressibility) effects as well as resistive (transmissibility) effects and does not require any prior knowledge about the reservoir properties.

The additional advantages of this model over MLR can be listed as follows:

- Applicable when wells are shut-in frequently / for long periods of time
- Capable of integrating the effect of primary production
- Use of BHP to decouple the injection effects from the pressure related ones

The name CRM is selected for this model because of its analogy to a resistor-capacitor (RC) circuit (Thompson, 2006). A production rate response to a step-change in injection rate (Figure 2.4) is analogous to voltage measurement of a capacitor in a parallel RC circuit where the battery potential is equivalent to the injection signal.

For each injector-producer pair, two parameters are determined; one parameter (the weight coefficient, λ) quantifies the connectivity and another (the time constant, τ) quantifies the degree of fluid storage between the wells. By considering the inputs and outputs, the capacitance model could be expressed as the total fluid mass balance which takes compressibility into account.

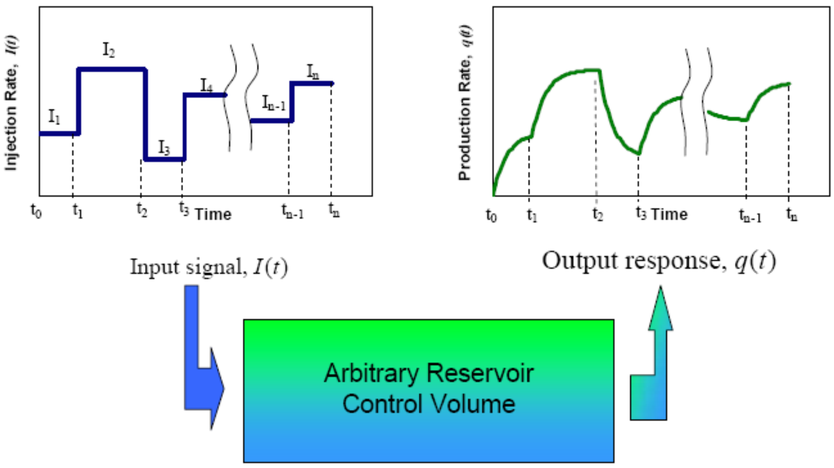


Figure 2.4 Injection Rate Signal on Production Response (Sayarpour, 2008)

The material balance differential equation for an injector-producer well pair at reservoir conditions is given by;

$$c_t \, V_p \, \frac{d\bar{p}}{dt} = i(t) - q\,(t) \tag{2.14}$$

where, c_t is the total compressibility; V_p is the pore volume being drained, $\bar{p}$ is the average pressure, $i(t)$ is the injection rate and $q(t)$ is the total production rate. This equation states that at any time, the net rate of mass change in the drainage volume can be explained by a change in the average pressure in a porous system which has constant total compressibility.

To work with just rate and bottomhole pressure data instead of average pressure parameter, which is not always easy to obtain, the following linear productivity model can be used;

$$q = J\,(\bar{p} - p_{wf}) \tag{2.15}$$

where, J and p_{wf} are the productivity index and flowing bottomhole pressure of the producer, respectively. Eliminating the average pressure by using new productivity model gives

$$\tau \frac{dq}{dt} + q(t) = i(t) - \tau J \frac{dp_{wf}}{dt} \tag{2.16}$$

where, τ is the "time constant" of the drainage volume, and is expressed as

$$\tau = \frac{c_t V_p}{J} \tag{2.17}$$

This equation is developed based on the following assumptions (Sayarpour, 2008):

- Constant temperature (isothermal)
- Instantaneous equilibrium
- Two immiscible phases
- Negligible capillary pressure effect
- Small fluid compressibility
- Darcy’s law applies
- Constant productivity index

By using integrating factor technique and integration by parts, equation becomes (for details see Appendix-B):

$$q(t) = q(t_0)e^{\frac{-(t-t_0)}{\tau}} + \frac{e^{\frac{-t}{\tau}}}{\tau} \int_{§=t_0}^{§=t} e^{\frac{§}{\tau}}\, i\,(§) d\,§ + J\left[p_{wf}(t_0)e^{\frac{-(t-t_0)}{\tau}} - p_{wf}(t) + \frac{e^{\frac{-t}{\tau}}}{\tau} \int_{§=t_0}^{§=t} e^{\frac{§}{\tau}} p_{wf}(§) d§\right] \quad (2.18)$$

where, t_0 is the initial time and $§$ is a variable of integration.

This equation states that the output signal includes three different parts. The first term on the right side of the equation is the response of primary (pre-injection) production rate. The second component is the contribution from the injection input signal. The last component is the output signal caused by changing the BHP of the producer (Yousef et al., 2006)

2.4.1 Discrete Model

Discretizing the integrals, capacitance model for one injector and one producer at constant BHP becomes;

$$q(n) = q(n_0)e^{\frac{-(n-n_0)}{\tau}} + \sum_{m=n_0}^{m=n} \alpha_m i(m) \quad (2.19)$$

where,

$$\alpha_m = \frac{\Delta n}{\tau}\, e^{\frac{(m-n)}{\tau}} \quad (2.20)$$

n is a time-like variable and Δn is the selected discretization interval. α_m is the filter coefficient which shapes the form of the output signal. For fixed Δn, the time constant, which accounts for attenuation and time lag between injector and producer pair, characterizes the filter coefficients (Yousef, 2006).

The integration and the discrete version of the model represent convolved form of the input injection signal which is also called as a filtered injection rate. Total production rate at step n is a function of the primary production component and the injection history between n and n_0. The contribution of each step in the injection history is controlled by the time constant which transforms the injection input signal to take the form of the output signal by using filter coefficients.

The time constant τ, is a direct measure of the dissipation in response between an injector and producer pair. If there were no dissipation between a well pair, τ would be small and a change in the injection rate would cause an equivalent and simultaneous change in the production rate. The main reason for a large dissipation is a large τ which can also be detailed as a large total compressibility, a large pore volume, a small productivity or permeability as stated in the formula.

Yousef et al. (2006) explained the effects of time constants on production signal by using three different values of τ, as shown in Figure 2.5. For τ < 1.0 time unit, the producer signal is very similar with the one for injection which indicates that the injection change causes a nearly instantaneous and equivalent change at the producer. For τ = 10 time units, the injection at every step n does not have its entire effect instantaneously acting on the producer. Injection from previous steps contributes to production at step n. The injection output signal at τ = 50 time units, results in larger attenuation and more time lag. From this study, it can be concluded that the larger the τ the more attenuated and delayed the production signal.

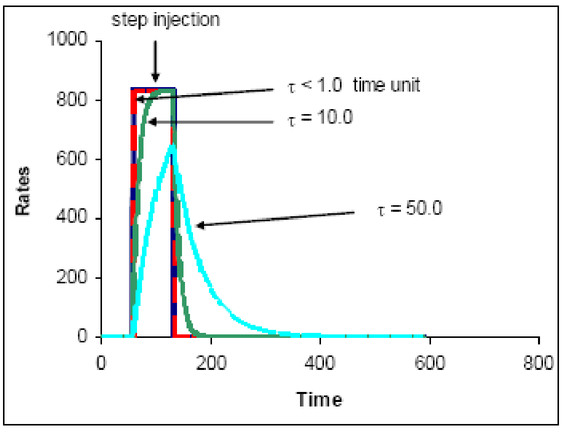


Figure 2.5 Filtered Injection Rate Responses for Different Time Constant Values (Yousef, 2006)

2.4.2 Extension to Multiple Producers and Injectors

In real world, usually there are more than one production and injection well acting simultaneously and the total production rate at one production well is usually supported by different injection wells. Thus, the Capacitance-Resistive Model must be generalized to describe a system consisting of one producer and multiple injectors.

One way is to apply this integration is assuming the corresponding injector is the only injector acting in the medium and the rate at the producer is affected only by that injector. The material balance equation for each injector-producer pair in a system consisting of producer *j* and injector *i* is;

$$c_{t_{ij}} V_{p_{ij}} \frac{d\bar{p}_{ij}}{dt} = \lambda_{ij} i_i(t) - q_{ij} \tag{2.21}$$

Then, by making use superposition in space, the governed material balance equation for producer *j* and *I* injectors is;

$$\sum_{i=1}^{I} c_{t_{ij}} V_{p_{ij}} \frac{d\bar{p}_{ij}}{dt} = \sum_{i=1}^{I} \lambda_{ij} i_i(t) - \sum_{i=1}^{I} q_{ij} \tag{2.22}$$

where, c_{tij}, V_{pij}, and $\bar{p}_{ij}$ now all represent properties in a volume drained by producer *j* when injector *i* is only active in the medium. q_{ij} is the production rate at producer *j* if there were only one injection well (*i*) affecting it.

Compared to one injector-producer pair model, Equation 2.22 suggests that the total volume drained by producer *j* and *I* injectors can be decomposed into separate pore volumes in which each pore volume is drained by the *ij* well pair when the corresponding injector is the only active well in the medium (Figure 2.6).

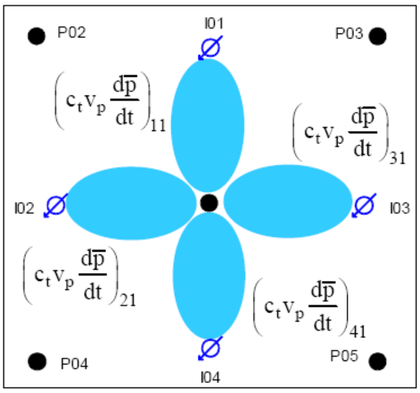


Figure 2.6 Schematic of the Pore Volumes Used by One Producer & Injectors (Yousef, 2006)

$$\sum_i^I \tau_{ij} \frac{dq_{ij}}{dt} + \sum_i^I q_{ij}(t) = \sum_{i=1}^I \lambda_{ij} i_i(t) - \frac{dp_{wf_j}}{dt} \sum_{i=1}^I \tau_{ij} J_{ij} \tag{2.23}$$

where,

$$\tau_{ij} = \frac{c_{t_{ij}} V_{p_{ij}}}{J_{ij}} \tag{2.24}$$

which provides one time constant (τ_{ij}) and weight (λ_{ij}) for each injector-producer pair.

The solution of equation will consist of three terms. The first part is for primary production (pre-injeciton) depletion. The second term accounts for the contribution from multiple injection input signals. The last term is for the changing BHP of the producer (Yousef et al., 2006).

The primary production term and the BHP term requires some mathematical manipulations and approximations. The primary production term, the first term in the solution of equation is;

$$First\ term = q_{1j}(t_0)e^{\frac{-(t-t_0)}{\tau_{1p}}} + q_{2j}(t_0)e^{\frac{-(t-t_0)}{\tau_{2p}}} + \cdots + q_{ij}(t_0)e^{\frac{-(t-t_0)}{\tau_{ip}}} \tag{2.25}$$

where, $q_{ij}(t_0)$ and τ_{ip} are the initial production rate when only injector i is active, and the corresponding time constant, respectively. Because of the fact that q_{ij} is usually not available, the primary production solution requires expression in terms of known quantities. One way is to impose the same time constant in all terms which results in;

$$First\ term = e^{\frac{-(t-t_0)}{\tau_p}} \sum_{i=1}^{I} q_{ij}(t_0) \tag{2.26}$$

$$First\ term = q_j(t_0) e^{\frac{-(t-t_0)}{\tau_p}} \tag{2.27}$$

where, $q_j(t_0)$ is the initial total production rate of producer j. τ_p is the resultant time constant of the primary production solution.

The injection term, the second term in the solution of equation needs no further approximation and is given by:

$$Second\ term = \sum_{i=1}^{I} \lambda_{ij} \left[e^{\frac{-t}{\tau_{ij}}} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau_{ij}}} i_{ij}(\S) d\S \right] \tag{2.28}$$

which, provides one time constant (τ_{ij}) and weight (λ_{ij}) for each pair.

The BHP term, the third term in the solution of Eq. 2.39, is given by;

$$Third\ term = J_{1j} \left[p_{wf_j}(t_0) e^{\frac{-(t-t_0)}{\tau_{1j}}} - p_{wf_j}(t) + \frac{e^{\frac{-t}{\tau_{1j}}}}{\tau_{1j}} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau_{1j}}} p_{wf_j}(\S) d\S \right]$$
$$+ J_{2j} \left[p_{wf_j}(t_0) e^{\frac{-(t-t_0)}{\tau_{2j}}} - p_{wf_j}(t) + \frac{e^{\frac{-t}{\tau_{2j}}}}{\tau_{2j}} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau_{2j}}} p_{wf_j}(\S) d\S \right] + \cdots +$$
$$J_{ij} \left[p_{wf_j}(t_0) e^{\frac{-(t-t_0)}{\tau_{ij}}} - p_{wf_j}(t) + \frac{e^{\frac{-t}{\tau_{ij}}}}{\tau_{ij}} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau_{ij}}} p_{wf_j}(\S) d\S \right] \tag{2.29}$$

where J_{ij}, and τ_{ij} are the productivity index when only one injector is active, and the corresponding time constant, respectively. Since J_{ij} is not known, the BHP term must be defined in terms of known quantities. As it was in the primary production term, the same time constant concept can be used here;

$$Third\ term = v_j \left[p_{wf_j}(t_0) e^{\frac{-(t-t_0)}{\tau_j}} - p_{wf_j}(t) + \frac{e^{\frac{-t}{\tau_j}}}{\tau_j} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau_j}} p_{wf_j}(\S) d\S \right] \quad (2.30)$$

where, v_j is a coefficient that determines the effect of changing the BHP of producer.

It is approximated by:

$$v_j = \sum_{i=1}^{I} J_{ij} \quad (2.31)$$

Then, the generalized capacitance model for producer j and I injectors is given by;

$$q_j(t) = \lambda_p q(t_0) e^{\frac{-(t-t_0)}{\tau}} + \sum_{i=1}^{I} \lambda_{ij} \left[e^{\frac{-t}{\tau_{ij}}} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau_{ij}}} i_{ij}(\S) d\S \right] +$$
$$v_j \left[p_{wf_j}(t_0) e^{\frac{-(t-t_0)}{\tau_j}} - p_{wf_j}(t) + \frac{e^{\frac{-t}{\tau_j}}}{\tau_j} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau_j}} p_{wf_j}(\S) d\S \right] \quad (2.32)$$

The discrete form is:

$$q_j(n) = \lambda_p q(n_0) e^{\frac{-(n-n_0)}{\tau_p}} + \sum_{i=1}^{I} \lambda_{ij} i'_{ij}(n) +$$
$$v_j \left[p_{wf_j}(n_0) e^{\frac{-(n-n_0)}{\tau_j}} - p_{wf_j}(n) + p'_{wf_j}(n) \right] \quad (2.33)$$

where,

$$i'_{ij}(n) = \sum_{m=n_0}^{m=n} \frac{\Delta n}{\tau_{ij}} e^{\frac{(m-n)}{\tau_{ij}}} i_{ij}(m) \quad (2.34)$$

$$p'_{wf_j}(n) = \sum_{m=n_0}^{m=n} \frac{\Delta n}{\tau_j} e^{\frac{(m-n)}{\tau_j}} p_{wf_j}(m) \quad (2.35)$$

λ_p and τ_p are the weighting factor and time constant for the primary production contribution. λ_{ij} is the weight coefficient between injector i and producer j; τ_{ij} is the time constant for the medium between injector i and producer j; $i'_{ij}(n)$ is the convolved or filtered injection rate at step n and $p'_{wf_j}(n)$ is the convolved BHP at step n for producer j; v_j is a coefficient that determines the effect of changing the BHP of producer j.

It is also possible to observe producer-producer interactions which can also influence production rates of the producers. By incorporating the BHP's of the other producers in the BHP term it is possible to extend the equation to account for producer-producer interactions:

$$q_j(n) = \lambda_p q\,(n_0) e^{\frac{-(n-n_0)}{\tau_p}} + \sum_{i=1}^{i=I} \lambda_{ij} i'_{ij}(n) +$$
$$\sum_{k=1}^{k=K} v_{kj} \left[p_{wf_{kj}}(n_0) e^{\frac{-(n-n_0)}{\tau_{kj}}} - p_{wf_{kj}}(n) + p'_{wf_{kj}}(n) \right] \quad (2.36)$$

where, v_{kj} is a coefficient that determines the effect of changing the BHP of producer k on the production rate of producer j; $p'_{wf_{kj}}(n)$ is the convolved BHP at step n for producer k.

In this way, the time constants in the BHP terms are changed from τ_j to τ_{kj} in order to incorporate producer-producer interactions. But on the other side, from the case studies performed, it was found that all τ's in BHP term tend to be very large. Thus, the BHP term can be simplified as,

$$BHP\ term = \sum_{k=1}^{k=K} v_{kj}\left(p_{wf_{kj}}(n_0) - p_{wf_{kj}}(n)\right) \tag{2.37}$$

According to this equation, the data used in the regression procedure to determine the v's coefficients are the differences between the BHP at initial step (n_0) and the BHP at any step (n), so if the producer BHP is constant, the BHP data will be simply zero.

In the capacitance resistive model, there are two sets of parameters that require estimation. One set is the time constants (τ_p, τ_{ij}, and τ_{kj}), and another set is the weighting coefficients (λ_p, λ_{ij}, and V_{kj}).

To determine the optimum solution of λ's and τ's, a non-linear optimization procedure is required. The weights λ_{ij} obtained from the optimization provide a quantitative expression of the connectivity between each (ij) pair; the larger the λ_{ij}, the greater the connectivity. The time constants τ_{ij} are direct measures of the dissipation between each pair; the larger the τ_{ij}, the larger the dissipation.

Kaviani et al. (2008) also studied on CRM applications and proposed the "Segmented CRM" and "Compensated CRM" to overcome some difficulties in CRM applications. Segmented CRM can be used where BHP data are unknown and Compensated CRM makes the model need less parameter when a new producer is added or an existing producer is shut-in. They can be used simultaneously if both conditions are the case.

In that study, different shifting filter is used and a discretized equation is achieved. Based on the well-known CRM equation proposed by Yousef et al. (2006)

$$q(t) = q(t_0)e^{\frac{-(t-t_0)}{\tau}} + \frac{e^{\frac{-t}{\tau}}}{\tau}\int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau}}\, i\,(\S)d\,\S + J\left[p_{wf}(t_0)e^{\frac{-(t-t_0)}{\tau}} - p_{wf}(t) + \frac{e^{\frac{-t}{\tau}}}{\tau}\int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau}}\, p_{wf}(\S)d\S\right] \tag{2.38}$$

by assuming a constant injection rate and pressure in each time step, equation becomes;

$$q_j(t) = \lambda_p q\,(t_0) e^{\frac{-(t-t_0)}{\tau_p}} + \sum_{i=1}^{i=I} \lambda_{ij} i'_{ij}(t) + \sum_{k=1}^{k=K} v_{kj} \left[p_{wf_{kj}}(t_0) e^{\frac{-(t-t_0)}{\tau_{kj}}} - p_{wf_{kj}}(t) + p'_{wf_{kj}}(t) \right] \tag{2.39}$$

where,

$$i'_{ij}(t) = \sum_{m=1}^{n} \left[e^{\frac{(t_m - t)}{\tau_{ij}}} - e^{\frac{(t_{m-1} - t)}{\tau_{ij}}} \right] i_j(t_m) \tag{2.40}$$

$$p'_{wf_{kj}}(t) = \sum_{m=1}^{n} \left[e^{\frac{(t_m - t)}{\tau_{kj}}} - e^{\frac{(t_{m-1} - t)}{\tau_{kj}}} \right] p_{wf_{kj}}(t_m) \tag{2.41}$$

By neglecting the time constant between producers in pressure contribution part as it was also done in Yousef (2006), final version of equation becomes;

$$q_j(t) = \lambda_p q\,(t_0) e^{\frac{-(t-t_0)}{\tau_p}} + \sum_{i=1}^{i=I} \lambda_{ij} \sum_{m=1}^{n} \left[e^{\frac{(t_m - t)}{\tau_{ij}}} - e^{\frac{(t_{m-1} - t)}{\tau_{ij}}} \right] i_j(t_m) + \sum_{k=1}^{k=K} v_{kj} \left[p_{wf_{kj}}(t_0) - p_{wf_{kj}}(t) \right] \tag{2.42}$$

Detailed procedure of discretization can be found in Appendix-C.

2.4.3 Types of Capacitance-Resistive Model

Two different approaches depending on the type of waterflood are proposed by Yousef et al. (2006):

- Balanced Capacitance Model (BCM)
- Unbalanced Capacitance Model (UCM)

Both approaches are based on a total material balance, using the total (oil + water + gas) production rates (in reservoir volumes/time), the injection rates (in reservoir volumes/time) and bottomhole pressures (if available / not constant) for every well in a waterflood as input data.

Balanced Capacitance Model

Waterflood is balanced when the field-wide injection rate is approximately equal to field-wide liquid production rate. In this case, the following form of the capacitance-resistive model should be used;

$$\hat{q}_j(n) = \lambda_p\, q(n_0) e^{\frac{-(n-n_0)}{\tau_p}} + \sum_{i=1}^{i=I} \lambda_{ij} i'_{ij}(n) + \sum_{k=1}^{k=K} v_{kj}\, BHP_{kj} \tag{2.43}$$

where,

$$i'_{ij}(n) = \sum_{m=n_0}^{m=n} \frac{\Delta n}{\tau_{ij}}\, e^{\frac{(m-n)}{\tau_{ij}}}\, i_{ij}(m) \tag{2.44}$$

$$BHP_{kj} = \left[p_{wf_{kj}}(n_0) e^{\frac{-(n-n_0)}{\tau_{kj}}} - p_{wf_{kj}}(n) + \sum_{m=n_0}^{m=n} \frac{\Delta n}{\tau_{kj}}\, e^{\frac{(m-n)}{\tau_{kj}}}\, p_{wf_{kj}}(m) \right] \tag{2.45}$$

This equation states that the total production rate at any step *n*, is a linear combination of the primary production, the convolved or filtered injection rates of every injector, and the BHP change of every producer.

All the coefficients mentioned above can be determined by minimizing the squared errors between measured production rates and those generated by equation:

$$Min \left[\sum_{n=1}^{T_n} \left(q_j(n) - \hat{q}_j(n) \right)^2 \right] \tag{2.46}$$

subject to average balance constraint,

$$\bar{q}_j = \bar{\hat{q}}_J \tag{2.47}$$

The final objective function is defined as follows by using Lagrange multipliers;

$$Min\left[\sum_{n=1}^{T_n}\left(q_j(n)-\hat{q}_j(n)\right)^2-2\mu_j(\bar{q}_j-\bar{\hat{q}}_j)\right] \tag{2.48}$$

By setting this equaiton's derivative with respect to each of the coefficients equal to zero, a set of *I*+*K*+2 linear equations can be solved simultaneously for $\lambda_p, \lambda_{ij}, v_{kj}$ and μ_j. As a constraint of this objective, sum of the weights of one injector should be equal to 1.

Unbalanced Capacitance-Resistive Model

A waterflood is unbalanced when the field production rate is considerably different from field injection rate. There may be different reasons for evaluating the waterflood as unbalanced;

- Study of a selected portion of a waterflooded area (boundary influx)
- Production not associated with injected water (aquifer effect)
- Injection losses to upswept areas / layers

If this is the case, a constant rate q_{0j} should be added to the model.

$$\hat{q}_j(n)= q_{0j}+\lambda_p pp+\sum_{i=1}^{i=I}\lambda_{ij}i'_{ij}(n)+\sum_{k=1}^{k=K}v_{kj}\,BHP_{kj} \tag{2.49}$$

The minimization procedure is similar to the one in the BCM. The system is solved by minimizing the squared errors;

$$Min\left[\sum_{n=1}^{T_n}\left(q_j(n)-\hat{q}_j(n)\right)^2\right] \tag{2.50}$$

Minimization proceeds as before which generates a set of *I*+*K*+1 linear equations which can be solved for $\lambda_p, \lambda_{ij}, v_{kj}\ and\ q_{0j}$. Unlike BCM, the sum of the weights for each injector can be less than or equal to 1.

In the derivations of the CRM requires several assumptions (Kaviani et al, 2008):

- Constant number of producers; i.e. no shut-in period or new production wells
- Availability of BHP data or constant and similar BHP
- Constant reservoir and well conditions
- Long period of data
- Negligible change in gas saturation
- Uncorrelated injection rates

2.5 Analytical Solutions for Different Reservoir Volumes

Sayarpour et al. (2008) proposed analytical solutions for the differential equation of the Capacitance-Resistive Model based on superposition in time. Solutions are suggested for three different reservoir-control volumes:

- CRMT - Drainage volume of the entire field
- CRMP - Drainage volume of each producer
- CRMIP - Drainage volume between each injector/producer pair

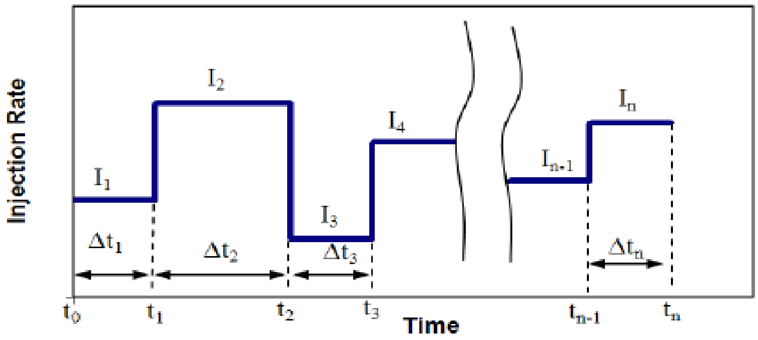
Considering the CRM equation for one injector-producer pair (Yousef et al., 2006);

$$q(t) = q(t_0)e^{\frac{-(t-t_0)}{\tau}} + \frac{e^{\frac{-t}{\tau}}}{\tau}\int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau}}\, i\,(\S)d\,\S - J\left[p_{wf}(t) - e^{\frac{-(t-t_0)}{\tau}}\, p_{wf}(t_0)\right] + J\,\frac{e^{\frac{-t}{\tau}}}{\tau}\int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau}}\, p_{wf}(\S)d\S \tag{2.51}$$

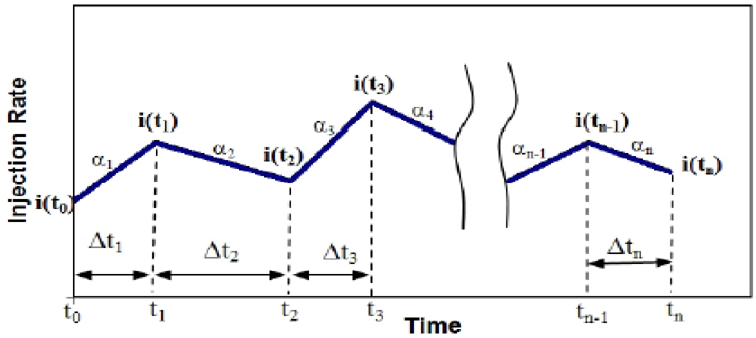
Yousef (2006) discretized the integrals in the equation over the entire production history by considering equal discretizations of time intervals. Instead of numerical solution of the CRM developed by Yousef et al. (2006) and Liang et al. (2007), integrals can be evaluated analytically by using superposition in time, in which an analytical solution at the end of each time interval can be used as initial condition for the next time interval.

Based on the assumption of linear variation of bottom-hole pressure (LVBHP), these analytical solutions were derived for two different projections which are (Figure 2.7):

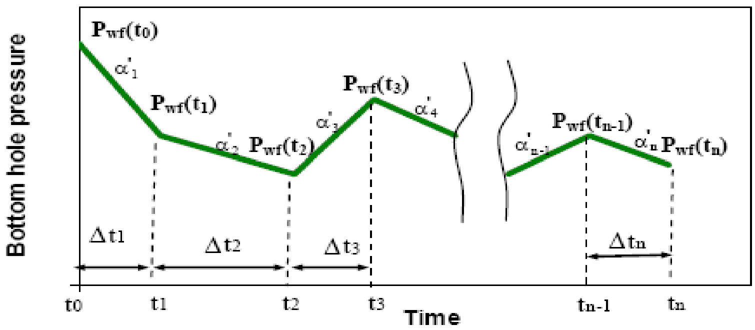
- Stepwise variation of injection rate (SVIR)
- Linear variation of the injection rate (LVIR)



Stepwise Variation of Injection Rates Between t_0 to t_n



Linear Variation of Injection Rates Between t_0 to t_n



Linear Variation of Bottomhole Pressures Between t_0 to t_n

Figure 2.7 Schematic of Rate and Pressure Changes (Sayarpour et al., 2008)

2.5.1 CRMT – One Time Constant for Field

In this type, reservoir is modeled by a single producer and a single injector (Figure – 2.8) as a tank by including the total production and injection rates which represent $q(t)$ and $i(t)$. τ_F is used as field time constant.

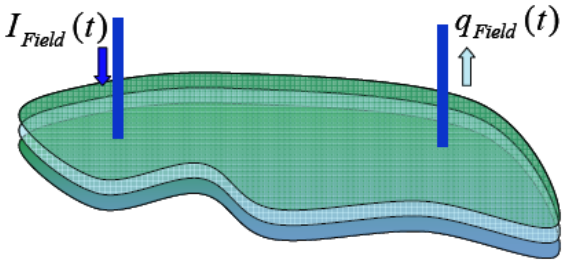


Figure 2.8 Schematic of CRMT (Sayarpour et al., 2008)

By considering the system with one injector - one producer and constant field injection rate for a time interval Δt_m, the total field-production rate can be stated as;

$$q(t) = q(t_0)\, e^{-\frac{(t-t_0)}{\tau}} + e^{\frac{-t}{\tau}} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau}}\, \frac{1}{\tau}\, i(\S) d\S - e^{\frac{-t}{\tau}} \int_{\S=t_0}^{\S=t} J\, e^{\frac{\S}{\tau}}\, \frac{dp_{wf}}{d\S} d\S \qquad (2.52)$$

By integrating the second term by parts, it becomes:

$$q(t) = q(t_0)\, e^{-\frac{(t-t_0)}{\tau}} + \left[i(t) - e^{-\frac{(t-t_0)}{\tau}}\, i(t_0)\right] - e^{\frac{-t}{\tau}} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau}}\, \frac{1}{\tau}\, \frac{di(\S)}{d\S} d\S - e^{\frac{-t}{\tau}} \int_{\S=t_0}^{\S=t} J\, e^{\frac{\S}{\tau}}\, \frac{dp_{wf}}{d\S} d\S \qquad (2.53)$$

<u>CRMT Solution for Series of SVIR (stepwise variation of injection rate)</u>

For a time series of data points (SVIR and LVBHP), by assuming constant productivity index during the time interval Δt_m, equation can be integrated from time t_{m-1} to t_m as follows:

$$q(t_m) = q(t_{m-1})\, e^{-\left(\frac{\Delta t_m}{\tau}\right)} + \left(1 - e^{-\left(\frac{\Delta t_m}{\tau}\right)}\right)\left[I^{(m)} - J\,\tau\, \frac{\Delta p_{wf}^{(m)}}{\Delta t_m}\right] \tag{2.54}$$

where, $I^{(m)}$ is the constant injection rate during the time interval Δt_m

For all time intervals from t_0 to t_n gives the superposition in time solution;

$$q(t_n) = q(t_0)\, e^{-\left(\frac{t_n - t_0}{\tau}\right)} + \sum_{m=1}^{n}\left\{\left(1 - e^{-\left(\frac{\Delta t_m}{\tau}\right)}\right)\left[I^{(m)} - J\tau\, \frac{\Delta p_{wf}^{(m)}}{\Delta t_m}\right] e^{-\left(\frac{t_n - t_m}{\tau}\right)}\right\} \tag{2.55}$$

Solution for one injector – producer pair with the assumptions of stepwise variation of injection rate and linear variation of producer's BHP are shown above where Δt_m is the difference between t_m and t_{m-1} and $q(t_0)$ is the total production rate at the end of primary recovery.

CRMT Solution for Series of LVIR (linear variation of injection rate)

For a time series of data points (LVIR and LVBHP), by assuming constant productivity index during the time interval Δt_m, equation can be integrated from time t_{m-1} to t_m as follows:

$$q(t_m) = q(t_{m-1}) e^{\left(-\frac{\Delta t_m}{\tau}\right)} + \left(i(t_m) - e^{-\left(\frac{\Delta t_m}{\tau}\right)} i(t_{m-1})\right) - \tau\left(1 - e^{-\left(\frac{\Delta t_m}{\tau}\right)}\right)\left[\frac{i(t_m) - i(t_{m-1})}{t_m - t_{m-1}} + J\left(\frac{p_{wf}(t_m) - p_{wf}(t_{m-1})}{t_m - t_{m-1}}\right)\right] \tag{2.56}$$

The equation above is developed for only one time interval Δt_m, of LVIR and LVBHP and can be extended for a series of time steps;

$$q(t_n) = q(t_0)\, e^{\left(-\frac{t_n - t_0}{\tau}\right)} + \left(i(t_n) - e^{-\left(\frac{t_n - t_0}{\tau}\right)} i(t_0)\right) - \tau \sum_{m=1}^{n} \left\{ e^{-\left(\frac{t_n - t_m}{\tau}\right)} \left(1 - e^{-\left(\frac{\Delta t_m}{\tau}\right)}\right) \left[\frac{\Delta i^{(m)}}{\Delta t_m} + J \left(\frac{\Delta p_{wf}^{(m)}}{\Delta t_m}\right)\right]\right\} \tag{2.57}$$

Solution for one injector – producer pair with the assumptions of linear variation of injection rate and BHP of producer are shown above where $\Delta i^{(m)}$and $\Delta p_{wf}^{(m)}$ represent a change in the injection and BHP for time interval of t_{m-1} to t_m.

The variation of BHP of individual wells cannot be accounted for in estimating parameters, if more than one producer exists; BHP term must be eliminated. Moreover, if a portion of the field injection is maintained in the reservoir, the field injection rate must be modified and in case of any source of support is available (aquifer influx), a new parameter must be added as shown below;

$$q_F(t_n) = q_F(t_0) \left[e^{-\left(\frac{t_n - t_0}{\tau_F}\right)}\right] + \sum_{m=1}^{n} \left\{ \left(E_W^{(m)} + \lambda_F^{(m)} I_F^{(m)}\right) e^{-\left(\frac{t_n - t_m}{\tau_F}\right)} \left(1 - e^{-\left(\frac{\Delta t_m}{\tau_F}\right)}\right)\right\} \tag{2.58}$$

where, $E_W^{(m)}$ indicates the flux into the reservoir from external source other than injectors and $\lambda_F^{(m)}$represents the weight coefficient of the portion.

2.5.2 CRMP – One Time Constant for Each Producer

For a control volume around a producer, pattern of I number of injectors and a producer, is shown in the Figure-2.9;

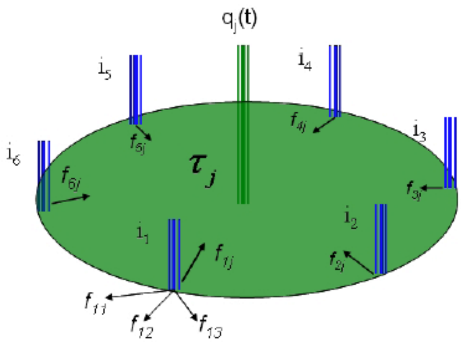


Figure 2.9 Schematic of the Control Volume of Producer j (CRMP) (Sayarpour et al., 2008)

Liang et al. (2007) presented the differential equation for the capacitance model as;

$$\tau_j \frac{dq_j(t)}{dt} + q_j(t) = \sum_{i=1}^{I} \lambda_{ij}\, i(t) - \tau_j J_j \frac{dp_{wf}}{dt} \tag{2.59}$$

where, V_P , c_t and p_{wf} are the pore volume, total compressibility and flowing bottomhole pressure, respectively.

$$\tau_j = \left(\frac{c_t V_p}{J}\right)_j \quad \text{time constant for drainage area of the producer} \tag{2.60}$$

$$\lambda_{ij} = \frac{q_{ij}(t)}{i_i(t)} \quad \text{ratio of injection rate of injector } i \text{ flowing toward producer } j \tag{2.61}$$

By considering the BHP variations, solution for this differential equation can be expressed as:

$$q_j(t) = q_j(t_0)\, e^{-\frac{(t-t_0)}{\tau_j}} + e^{\frac{-t}{\tau_j}} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau_j}} \frac{1}{\tau_j} \sum_{i=1}^{I} \lambda_{ij}\, i_t(\S)\, d\S - e^{\frac{-t}{\tau_j}} \int_{\S=t_0}^{\S=t} J_j\, e^{\frac{\S}{\tau_j}} \frac{dp_{wf,j}}{d\S} d\S \tag{2.62}$$

Instead of the numerical integration proposed by Liang et. al. (2007), analytical integration with superposition in time used for both SVIR and LVIR conditions. Integrating the equation above by parts;

$$q_j(t) = q_j(t_0)\, e^{-\frac{(t-t_0)}{\tau_j}} + \sum_{i=1}^{I} \left[\lambda_{ij} \left(i_i(t) - e^{-\frac{(t-t_0)}{\tau_j}} i_i(t_0) \right) \right]$$

$$- e^{\frac{-t}{\tau_j}} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau_j}} \left(\sum_{i=1}^{I} \lambda_{ij} \frac{di_i(\S)}{d\S} + J_j \frac{dp_{wf,j}}{d\S} \right) d\S \qquad (2.63)$$

CRMP Solution for Series of SVIR (stepwise variation of injection rate)

For a time series of data points (SVIR and LVBHP), by assuming constant productivity index during the time interval Δt_m, equation can be integrated from time t_{m-1} to t_m as follows;

$$q_j(t_m) = q_j(t_{m-1}) e^{-\frac{(\Delta m)}{\tau_j}} + \left(1 - e^{\frac{-\Delta m}{\tau_j}} \right) \left(\sum_{i=1}^{I} \lambda_{ij} I_i^{(m)} - J_j \tau_j \frac{\Delta p_{wf,j}^{m}}{\Delta t_m} \right) \qquad (2.64)$$

For the series of time interval in the model, by replacing $q(t_{n-1})$ from the previous time step solution for the all time intervals starting from t_0, equation can be expressed as;

$$q_j(t_n) = q_j(t_0) e^{-\frac{(t_n - t_0)}{\tau_j}} +$$

$$\sum_{m=1}^{n} e^{-\frac{(t_n - t_m)}{\tau_j}} \left(1 - e^{\frac{-\Delta t_m}{\tau_j}} \right) \left[\sum_{i=1}^{I} \lambda_{ij} I_i^{(m)} - J_j \tau_j \frac{\Delta p_{wf,j}^{(m)}}{\Delta t_m} \right] \qquad (2.65)$$

CRMP Solution for Series of LVIR (linear variation of injection rate)

For a time series of data points (LVIR and LVBHP), by assuming constant productivity index during the time interval Δt_m, equation can be integrated from time t_{m-1} to t_m as follows:

$$q_j(t_m) = q_j(t_{m-1})e^{-\frac{(\Delta t_m)}{\tau_j}} + \sum_{i=1}^{I} \lambda_{ij} \left[i_i(t_m) - e^{-\frac{(\Delta t_m)}{\tau_j}} i_t(t_{m-1}) \right] -$$

$$\tau_j \left(1 - e^{-\left(\frac{\Delta t_m}{\tau_j}\right)} \right) \left[\sum_{i=1}^{I} \lambda_{ij} \frac{\Delta i_i^{(m)}}{\Delta t_m} + J_j \frac{\Delta p_{wf,j}^{(m)}}{\Delta t_m} \right] \tag{2.66}$$

For the series of time interval in the model, by replacing $q(t_{n-1})$ from the previous time step solution for the all time intervals starting from t_0, equation can be expressed as;

$$q_j(t_n) = q_j(t_0)e^{-\frac{(t_n - t_0)}{\tau_j}} + \sum_{i=1}^{I} \lambda_{ij} \left[i_i(t_n) - e^{-\frac{(t_n - t_0)}{\tau_j}} i_i(t_0) \right] -$$

$$\sum_{m=1}^{n} \left\{ \tau_j \, e^{-\frac{(t_n - t_m)}{\tau_j}} \left(1 - e^{-\left(\frac{\Delta t_m}{\tau_j}\right)} \right) \left[\sum_{i=1}^{I} \lambda_{ij} \frac{\Delta i_i^{(m)}}{\Delta t_m} + J_j \frac{\Delta p_{wf,j}^{(m)}}{\Delta t_m} \right] \right\} \tag{2.67}$$

2.5.3 CRMIP – One Time Constant for Each Injector-Producer Pair

In CRMIP, the affected pore volume of any injector/producer pair is considered. The volumetric balance over the affected pore volume of any injector-producer pair is illustrated below (Figure 2.10),

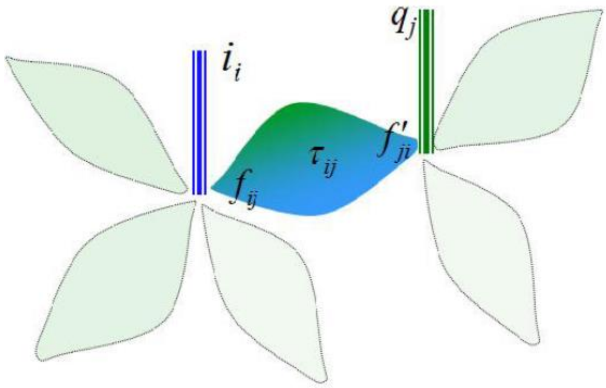


Figure 2.10 Schematic of the Pore Volumes Used by One Producer/Injector Pair (Sayarpour et al., 2008)

The equation for each producer-injector pair was stated by Yousef et al. (2006);

$$\frac{dq_{ij}(t)}{dt} + \frac{1}{\tau_{ij}}\, q_{ij}(t) = \frac{1}{\tau_{ij}}\, \lambda_{ij} i_i(t) - J_{ij}\, \frac{dp_{wf,j}}{dt} \tag{2.68}$$

where, the time constant τ_{ij} is defined as;

$$\tau_{ij} = \left(\frac{c_t\, V_p}{J}\right)_{ij} \tag{2.69}$$

Yousef et al. (2006) initially summed the CRM equation over all the injectors for the production rate of producer j in a multi-well system and presented the following equation:

$$q_j(t) = -\sum_{i=1}^{I} \tau_{ij}\, \frac{dq_{ij}(t)}{dt} + \sum_{i=1}^{I} \lambda_{ij}\, i(t) - \frac{dp_{wf,j}}{dt} \tau_j \sum_{i=1}^{I} \tau_{ij}\, J_{ij} \tag{2.70}$$

Yousef et al. (2006) initially applied superposition in space and Liang et al. (2007) numerically solved for production rate of each producer, but in this work firstly the equation of each pair is solved through superposition in time and then superposition in space to find production rate by summing up the all injectors' contribution;

$$q_j(t) = \sum_{i=1}^{I} q_{ij}(t) \qquad (\, j = 1, 2, 3, \dots, N\,) \tag{2.71}$$

where,

$$q_{ij}(t) = q_{ij}(t_0) e^{-\frac{(t-t_0)}{\tau_{ij}}} + \lambda_{ij} \left[i_i(t) - e^{-\frac{(t-t_0)}{\tau_{ij}}}\, i_t(t_0) \right] -$$

$$e^{-\frac{t}{\tau_{ij}}} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau_{ij}}} \left(\frac{di_{ij}(\S)}{d\S} - J_{ij}\, \frac{dp_{wf,j}}{d\S} \right) d\S \tag{2.72}$$

CRMIP Solution for Series of SVIR (stepwise variation of injection rate)

For a time series of data points (SVIR and LVBHP), by assuming constant productivity index during the time interval Δt_m, equation can be integrated from time t_{m-1} to t_m as follows:

$$q_{ij}(t_m) = q_{ij}(t_{m-1})e^{-\frac{\Delta t_m}{\tau_{ij}}} + \left(1 - e^{\frac{-\Delta t_m}{\tau_{ij}}}\right)\left(\lambda_{ij} I_i^{(m)} + J_{ij}\,\tau_{ij}\,\frac{\Delta p_{wf,j}^{(m)}}{\Delta t_m}\right) \quad (2.73)$$

where, $I_i^{(m)}$ and $\Delta p_{wf}^{(m)}$ are the injection rate and change in BHP of producer

For the series of time interval in the model, by replacing $q(t_{n-1})$ from the previous time step solution for all time intervals starting from t_0, equation can be expressed as;

$$q_{ij}(t_n) = q_{ij}(t_0)e^{-\frac{(t_n - t_0)}{\tau_{ij}}} + \sum_{m=1}^{n}\left[\left(1 - e^{\frac{-\Delta m}{\tau_{ij}}}\right)\left(\lambda_{ij} I_i^{(m)} + J_{ij}\,\tau_{ij}\,\frac{\Delta p_{wf,j}^{(m)}}{\Delta t_m}\right)e^{-\frac{(t_n - t_m)}{\tau_{ij}}}\right] \quad (2.74)$$

Then $q_j\,(t_n)$ can be calculated by considering each of injector contribution as;

$$q_j(t_n) = \sum_{i=1}^{I} q_{ij}(t_n) = \sum_{i=1}^{I} q_{ij}(t_0)e^{-\frac{(t_n - t_0)}{\tau_{ij}}} + \sum_{i=1}^{I}\left\{\sum_{m=1}^{n}\left[\left(1 - e^{\frac{-\Delta t_m}{\tau_{ij}}}\right)\left(\lambda_{ij} I_i^{(m)} + J_{ij}\,\tau_{ij}\,\frac{\Delta p_{wf}^{(m)}}{\Delta t_m}\right)e^{-\frac{(t_n - t_m)}{\tau_{ij}}}\right]\right\} \quad (2.75)$$

CRMIP Solution for Series of LVIR (linear variation of injection rate)

For a time series of data points (LVIR and LVBHP), by assuming constant productivity index during the time interval Δt_m, equation can be integrated from time t_{m-1} to t_m as follows:

$$q_{ij}(t_m) = q_{ij}(t_{m-1})e^{-\left(\frac{\Delta t_m}{\tau_{ij}}\right)} + \lambda_{ij}\left[i_i(t_m) - e^{-\left(\frac{\Delta t_m}{\tau_{ij}}\right)} i_i(t_{m-1})\right]$$

$$-\tau_{ij}\left(1 - e^{-\left(\frac{\Delta t_m}{\tau_{ij}}\right)}\right)\left[\lambda_{ij}\frac{\Delta i_i^{(m)}}{\Delta t_m} + J_{ij}\frac{\Delta p_{wf,j}^{(m)}}{\Delta t_m}\right] \tag{2.76}$$

where, $\Delta i_i^{(m)}$ and $\Delta\, p_{wf}^{(m)}$ are the change in injection rate of injector *i* and change in BHP of producer *j*

For a time series of data points, by superposition in time and assuming a constant productivity index during any time interval of Δ_{tm}, q_{ij} can be calculated as;

$$q_{ij}(t_n) = q_{ij}(t_0)e^{-\left(\frac{t_n - t_0}{\tau_{ij}}\right)} + \lambda_{ij}\left[i_i(t_n) - e^{-\left(\frac{t_n - t_0}{\tau_{ij}}\right)} i_i(t_0)\right]$$

$$-\tau_{ij}\sum_{m=1}^{n}\left\{e^{-\left(\frac{t_n - t_m}{\tau_{ij}}\right)}\left(1 - e^{\frac{-\Delta t_m}{\tau_{ij}}}\right)\left[\lambda_{ij}\frac{\Delta i_i^{(m)}}{\Delta t_m} + J\frac{\Delta p_{wf,j}^{(m)}}{\Delta t_m}\right]\right\} \tag{2.77}$$

Then $q_j(t_n)$ can be calculated by considering each of injectors contribution as;

$$q_j(t_n) = \sum_{i=1}^{I} q_{ij}(t_0)e^{-\left(\frac{t_n - t_0}{\tau_{ij}}\right)} + \sum_{i=1}^{I}\lambda_{ij}\left[i_i(t_n) - e^{-\left(\frac{t_n - t_0}{\tau_{ij}}\right)} i_i(t_0)\right] -$$

$$\sum_{i=1}^{I}\left\{\tau_{ij}\sum_{m=1}^{n}\left\{e^{-\left(\frac{t_n - t_m}{\tau_{ij}}\right)}\left(1 - e^{\frac{-\Delta t_m}{\tau_{ij}}}\right)\left[\lambda_{ij}\frac{\Delta i_i^{(m)}}{\Delta t_m} + J\frac{\Delta p_{wf,j}^{(m)}}{\Delta t_m}\right]\right\}\right\} \tag{2.78}$$

To match the total production history for a pattern of *I* injectors and *N* producers in different reservoir control volumes, Table 2.1 shows the parameters to be solved:

Table 2.1 Comparison Between Numbers of Unknowns in CRM (Sayarpour, 2008)

	CRMT	CRMP	CRMIP
Unknowns	$q_F(t_0), \lambda_F, \tau_F)$	$q_j(t_0)$'s, λ_{ij}'s, τ_j's and J_j's	$q_{ij}(t_0)$'s, λ_{ij}'s, τ_{ij}'s and J_{ij}'s
No. of Unknowns without BHP data	3	N x (I + 2)	3 x I x N
No. of Unknowns with BHP data	3	N x (I + 3)	4 x I x N

As a summary of comparison between the previously developed capacitance resistive models and these analytical solutions, Table 2.2 shows the advantages and disadvantages of the different models.

Table 2.2 Comparison Between Developed CRMs (Sayarpour, 2008)

Compared Criteria	Yousef et al. (2006)	Liang et al. (2007)	Sayarpour et al. (2008)
Analytical solution for only one change of injection rate and bottomhole pressure for total liquid production	CRMIP	CRMP	All CRM's
CRMIP solution approach	Summation of CRM differential equations of each injector-producer pair is solved	n/a	q_{ij} are evaluated individually and then their summation generates stable q_j's
Solution for injection rate fluctuations for total liquid solution	Numerical solution	Numerical solution	Analytical solution with superposition in time
Analytical solution for both injection rate and BHP fluctuations for total liquid production	n/a	n/a	Analytical solution with superposition in time for all CRM's
Oil production optimization	n/a	Based on maximizing net present value	Based on reallocation of fixed field-injection rate
Model validation examples	Variable injection rates and fixed BHP	Variable injection rates and fixed BHP	Variable injection rates and variable BHP
Timestep increments	fixed	fixed	variable
Block refinement	n/a	n/a	CRMT and CRMIP

2.6 Integrated Capacitance Resistive Models (ICRM)

Although the CRM models need just rates and pressures, they use nonlinear multivariate regression to estimate model parameters. If a field including lots of wells is considered, obtaining a unique solution with reliable results and establishing confidence intervals of the model parameters may be difficult because of the nonlinear nature of these models (Weber et al., 2009).

Nguyen et al. (2011) developed a model which uses linear multivariate regression on production-injection, initial reservoir and bottomhole pressure data to minimize these complex calculations. Suggested approach uses cumulative water injection and cumulative total liquid production instead of rates. Due to the simpler formulation of the ICRM formula, unique solutions are easier to obtain compared to other models and a remarkable reduction of the computation time can be achieved.

ICRM can be applied to the reservoirs which have no aquifer, no volatile oil and no gas cap initially. It is applicable to both primary and secondary recovery which can be used in large fields. A detailed analysis of use ICRM in primary recovery can be found in the study of Nguyen at al. (2011)

It is based on material balance as it is same in other models and developed from the CRMP governing differential equation as the following:

$$\frac{dq_{j(n)}}{dt} + \frac{1}{\tau_j}\, q_{j(n)} = \frac{1}{\tau_j} \sum_{i=1}^{I} \lambda_{ij}\, i_{i(n)} - J_j\, \frac{dp_{\ wf,j}^{\ n}}{dt} \tag{2.79}$$

where, τ_j is the producer j 's time constant and λ_{ij} represents the fraction of water rate from injector i flowing towards producer j.

After multiplying both sides of equation by dt and integrating from t_0 to t_n;

$$\int_{q_{j0}}^{q_{jn}} dq_{jn} + \frac{1}{\tau_j}\left(\int_{t_0}^{t_n} q_{jn}\, dt\right) = \frac{1}{\tau_j}\left[\sum_{i=1}^{I}\left\{\lambda_{ij}\left(\int_{t_0}^{t_n} i_{in}\, dt\right)\right\}\right] - J_j\left(\int_{\mathrm{p}_{wf,j}^{0}}^{\mathrm{p}_{wf,j}^{n}} dp_{\ wf,j}^{\ n}\right) \tag{2.80}$$

Rearranging the terms and integrating equation, the equation becomes;

$$N_{p,j}^{n} = \left(q_{j0} - q_{jn}\right)\tau_j + \sum_{i=1}^{I}\left(\lambda_{ij}\, CWI_i^n\right) + J_j\,\tau_j\left(p_{wf,j}^{0} - p_{wf,j}^{n}\right) \tag{2.81}$$

where, $N_{p,j}^{n}$ represents the cumulative amount of total liquid produced from a producer j at time step n. CWI_i^n accounts for the cumulative volume of water injected into an injector i at time step n and I is the total number of injectors. If producer's BHP is constant, equation can be simplified:

$$N_{p,j}^{n} = \left(q_{j0} - q_{jn}\right) \tau_j + \sum_{i=1}^{I} \left(\lambda_{ij}\, CWI_i^n\right) \tag{2.82}$$

Model parameters are estimated by linear multivariate regression that minimizes the following objective function;

$$\min z = \sum_{n=1}^{N_t} \sum_{j=1}^{N} \left(\left(N_{p,j}^{n}\right)_{obs} - \left(N_{p,j}^{n}\right)_{cal} \right)^2 \tag{2.83}$$

with the constraints which makes coefficients meaningful;

$$\sum_{j=1}^{N} f_{ij} \leq 1 \quad \text{for all } i \tag{2.84}$$

$$\lambda_{ij} \geq 0 \quad \tau_j > 0 \quad \text{for all } i \text{ and } j \tag{2.85}$$

which, represent a material balance allowing for a loss of water injected within the control volume when the sum of gains is less than one and ensures that injected water does not adversely affect the reservoir production (Weber et al., 2009).

2.7 Empirical Oil Fractional Flow Models

All CRM and regression based surrogate models mentioned up to now can estimate the total liquid production. Considering the optimization of the project economics, not only the total rates but also the oil rates as a function of time are very important.

In Figure 2.11, oil production trend of a successfully waterflooded oilfield is shown. Initially, the reservoir pressure increases as the gas-filled pore volumes are refilled with water which also results in re-dissolving free gas back into oil. The oil production response occurs after the fill-up of the gas space. As the injection continues, the peak oil production rate is reached and oil production rate declines with increase in water cut until the residual oil saturation of is reached (Thakur and Satter, 1998). Mentioned periods of the flood life can be expressed by different mathematical expressions that fit the rate/time relationship best and forecasted (Willhite, 1986).

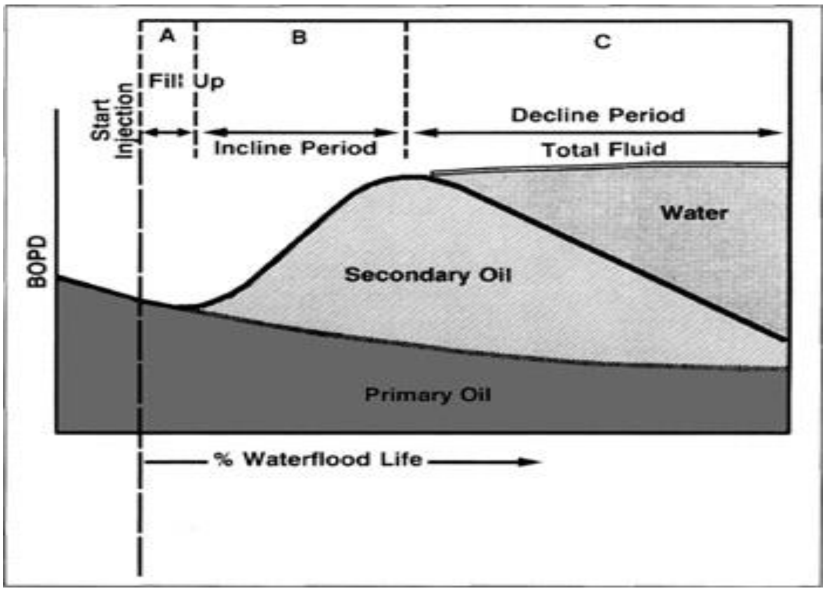


Figure 2.11 Typical Waterflood Performance (Thakur, G. C., 1991)

Total production rate during secondary or tertiary recoveries are obtained easily by CRMs and can be combined with oil fractional-flow model to estimate oil production.

Oil fractional flow models are either based on saturation front propagation or empirical models. The saturation based models are dependent on reservoir parameters which may be difficult to obtain most of the times. Therefore, empirical oil fractional-flow models are used commonly because of its ease and less data requirements. Papay (2003) summarized the most common empirical methods which use only production data as follows:

Makszimov (1959) proposed the following equation to calculate the cumulative oil and cumulative water production based on the laboratory measurements and production data of oil reservoirs (Figure 2.12):

$$\log\left(W_p\right) = \log\ (b) + N_p \log\ (a) \tag{2.86}$$

where,

W_p = cumulative water production, bbl – m^3

N_p = cumulative oil production, bbl – m^3

a, b = constant

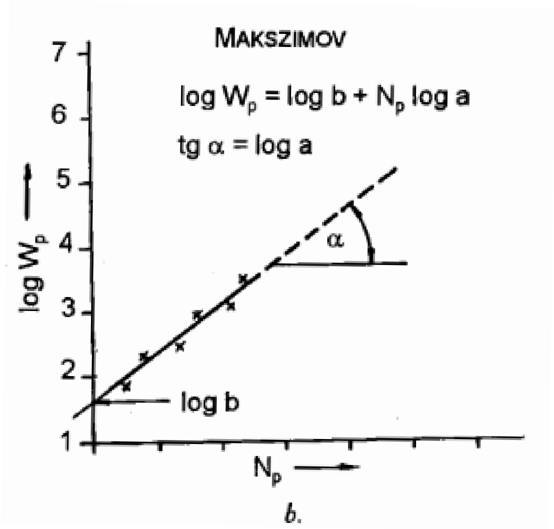


Figure 2.12 Cumulative Water Production vs Oil Production (Makszimov, 1959)

According to this relationship, the logarithm of the cumulative water production is the linear function of the cumulative oil production. By assuming $q = q_o + q_w$, equation becomes;

$$\frac{q_o}{q} = f_o = \frac{1}{1 + b\, a^{N_p}\, \ln a} \tag{2.87}$$

Von Gunkel et al. (1968) proposed an equation for the cumulative water-oil ratio and cumulative oil production (Figure 2.13):

$$WOR = \frac{W_p}{N_p} = a + b\, e^{c\, N_p} \tag{2.88}$$

where,

W_p = cumulative water production, bbl – m^3

N_p = cumulative oil production, bbl – m^3

a, b, c = constants

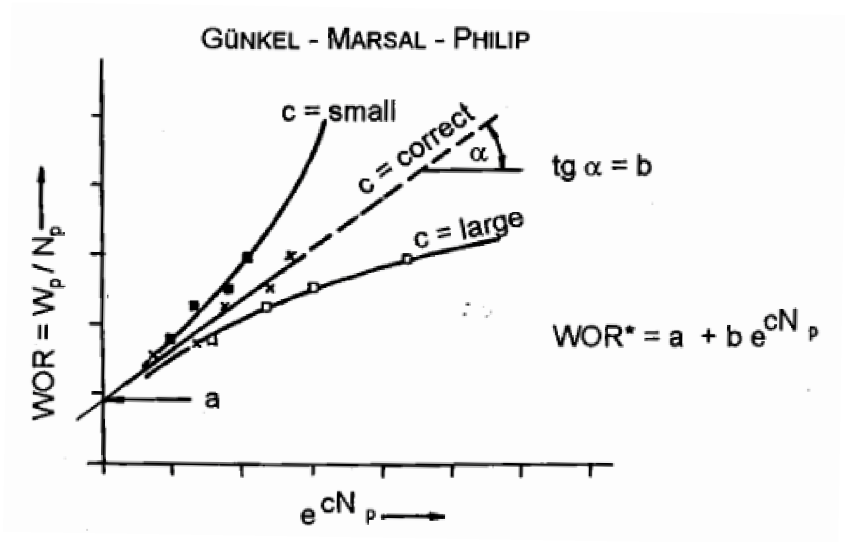


Figure 2.13 Cumulative WOR vs Oil Production (Von Gunkel et al., 1968)

According to this relationship, the cumulative water-oil ratio is a function of the cumulative oil production. By assuming $q = q_o + q_w$, equation becomes;

$$\frac{q_o}{q} = f_o = \frac{1}{1+a+(1+c\,N_p)\,b\,e^{c\,N_p}} \tag{2.89}$$

Timmerman (1971) proposed an equation based on a relationship for forecasting production for oil displacement by water (Figure 2.14):

$$\log\frac{q_o}{q_w} = a + b\,N_p \tag{2.90}$$

where

q_o = oil production rate, bbl – m^3 /time

q_w = water production rate, bbl – m^3 /time

N_p = cumulative oil production, bbl – m^3

a, b = constants

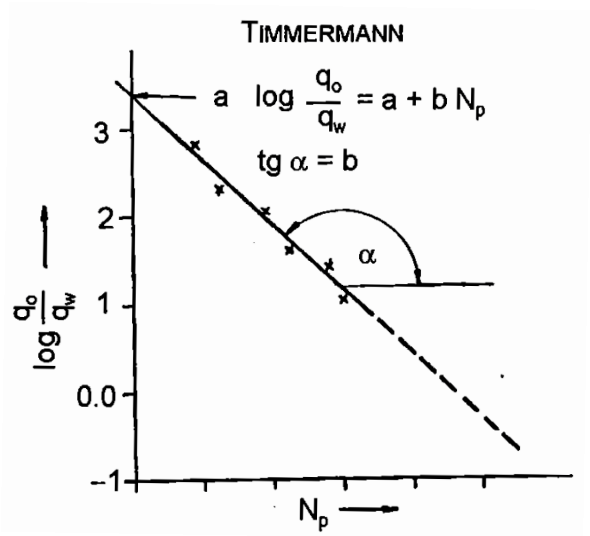


Figure 2.14 OWR vs Cumulative Oil Production (Timmerman, 1971)

According to this relationship, the logarithm of the actual oil-water ratio is the linear function of the cumulative oil production. By assuming $q = q_o + q_w$, equation becomes;

$$\frac{q_o}{q} = f_o = \frac{10^{a+bN_p}}{1+10^{a+bN_p}} \tag{2.91}$$

Kazakov (1976) proposed the following equation to calculate cumulative water-oil ratio and cumulative water production relationship based on the production data of oil fields (Figure 2.15):

$$WOR = \frac{W_p}{N_p} = (a-1) + b\,W_p \tag{2.92}$$

where,

WOR = cumulative water-oil ratio, %

W_p = cumulative water production, bbl – m^3

N_p = cumulative oil production, bbl – m^3

a, b = constants

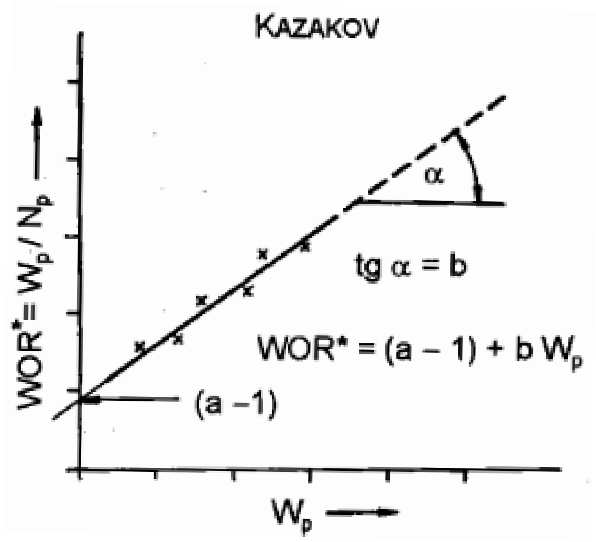


Figure 2.15 Cumulative Water-Oil Ratio vs Water Production (Kazakov, 1976)

According to this relationship, the ratio of cumulative water-oil ratio is the linear function of the cumulative water production. By assuming $q = q_o + q_w$, equation becomes;

$$\frac{q_o}{q} = f_o = \frac{(1-b\,N_p)^2}{(a-2bN_p)(1-b\,N_p)+N_p\,b\,(a-b\,N_p)} \tag{2.93}$$

Ershaghi and Omoregie (1978) proposed an equation for the cumulative oil production and water fraction relationship based on Buckley-Leverett-Welge displacement equation (Figure 2.16):

$$\frac{k_{ro}}{k_{rw}} = a_1\, e^{-a_2\, S_w} \tag{2.94}$$

where, a_1 and a_2 are constant

$$N_p = a + b\,\left[ln\left(\frac{1}{(1-f_o)} - 1\right) - \frac{1}{(1-f_o)}\right] \tag{2.95}$$

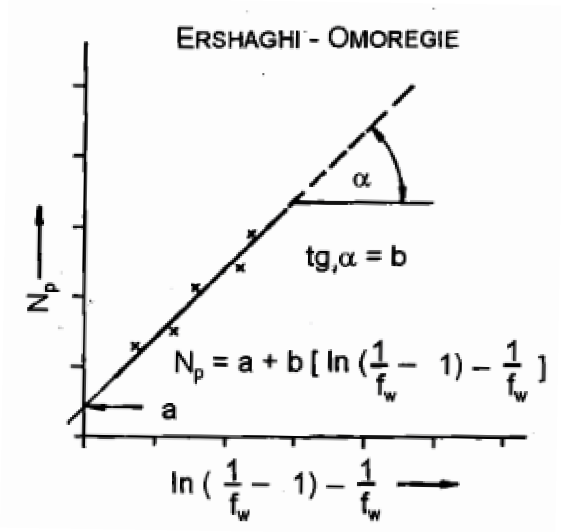


Figure 2.16 Cumulative Oil Production vs Oil Cut (Ershaghi and Omoregie, 1978)

By assuming $q = q_o + q_w$, oil production rate vs cumulative oil production equation becomes;

$$N_p = a + b\ \left[\ln\left(\frac{q_o}{q-q_o}\right) - \frac{q}{q-q_o}\right] \tag{2.96}$$

Table 2.3 summarizes all mentioned models and the proposed equations which uses only rates and cumulative volumes.

Gentil (2005) proposed an empirical power-law fractional flow model which relates water - oil ratio and cumulative water injection. Estimated water/oil ratio can be calculated from the following equation:

$$\frac{q_w}{q_o} = \alpha\, W_i^{\beta} \tag{2.97}$$

where,

W_i = cumulative water injection, bbl – m^3

α, β = constants

when the injection and production rates are in balance, equation becomes:

$$f_o = \frac{q_o}{q_o+q_w} = \frac{1}{1+\frac{q_w}{q_o}} = \frac{1}{1+F_{wo}} = \frac{1}{1+\alpha\, W_i^{\beta}} \tag{2.98}$$

All these mentioned models can be used to predict and optimize oil flow rates by combining the total production results coming from any CRM based application.

Table 2.3 Common Empirical Oil Fractional Flow Models (Papay, 2003)

Method	Basic equation	*Time	*Rate - cumulative production
TIMMERMANN	$\log\frac{q_o}{q_w} = a + b\,N_p$	$\tau = \frac{N_p+W_p}{q}$	$q_o = q\,\frac{10^{a+bN_p}}{10^{a+bN_p}}$
MAKSZIMOV	$\log W_p = \log b + N_p \log a$	$\tau = \frac{N_p+W_p}{q}$	$q_o = q\,\frac{1}{1+b\,a^{N_p}\ln a}$
ERSAGHI - OMOREGIE	$N_p = a + b\,[\ln(\frac{1}{f_w} - 1) - \frac{1}{f_w}]$	$\tau = \frac{N_p+W_p}{q}$	$N_p = a + b\,(\ln\frac{q_o}{q-q_o} - \frac{q}{q-q_o})$
KAZAKOV	$^{**}WOR^{*} = (a-1) + b\,W_p$	$\tau = \frac{N_p+W_p}{q}$	$q_o = q\,\frac{(1-b\,N_p)^2}{(a-2\,b\,N_p)(1-b\,N_p)+N_p\,b\,(a-b\,N_p)}$
GÜNKEL - MARSAL - - PHILIP	$WOR^{*} = a + b\,e^{cN_p}$	$\tau = \frac{N_p+W_p}{q}$	$q_o = q\,\frac{1}{1+a+(1+c\,N_p)\,b\,e^{cN_p}}$

$^{*}\ q = q_w + q_o = \text{const.}$

$^{**}\ WOR^{*} = W_p / N_p$

2.8 Economic Evaluation of the Waterflooding Projects

According to Satter & Thakur (1994), there are commonly used economic criteria and methods of analyzing project economics. Steps for analyzing a project economically can be seen as in Figure 2.17 and detailed as following items:

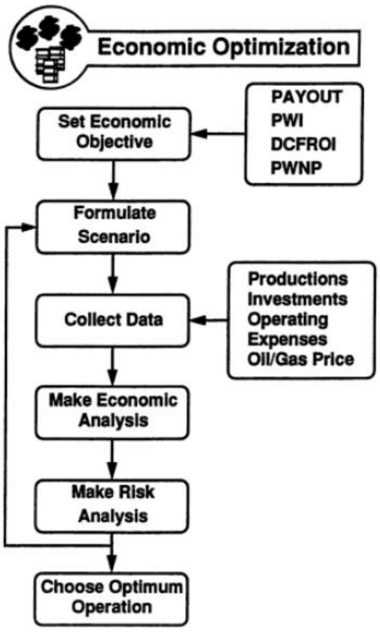


Figure 2.17 Economic Optimization Algorithm for Projects
(Satter & Thakur, 1994)

Economic Objectives: Each project has its own economic criteria to fit its strategy for doing business profitably. The main decision making items can be listed as:

- Payout Time
- Profit to Investment Ratio
- Present Worth Net Profit
- Investment Efficiency or Present Worth Index or Profitability Index
- Discounted Cash Flow Return on Investment or Internal Rate of Return

Formulating Scenarios: This stage is the decision point of the selecting best scenario for the specific project. Usually more than one scenario are studied and the most profitable of them is selected. This study may include the followings:

1) Injection type (peripheral or pattern flooding)
2) Well spacing and number of wells
3) Timing of the wells
4) Rate and pressure optimization

Collecting Data: The data required for economic analysis can include mainly production and injection rates, investment and operating costs, financial data (oil & gas prices) and economic data like shares, loyalty, taxes etc.

Economic Analysis: The procedure for economic analysis is outlined below:

1) Calculating revenues from oil and gas sales
2) Calculating total costs including the CAPEX, OPEX and taxes
3) Calculating the undiscounted and discounted cash flow

Risk and Uncertainties: Project analysis must include also risk analysis which must consider technical, economic and political conditions. Assumptions can be done in forecasting recoveries, prices, investment / operating costs and economic situation.

CHAPTER 3

STATEMENT OF PROBLEM AND SCOPE

Characterization of the hydrocarbon reservoirs and predicting future production to maximize economic return of the asset are becoming the most important issues for the companies as the amount of available resources becoming limited. As a result of the decrease in the success of new discoveries, additional oil recovery methods have become inevitable for most of the brown fields.

Waterflooding, the oldest and most common secondary recovery method, has considerable advantages in terms of high recoveries and low costs. Even an increase in the recovery is observed, most of the hydrocarbon is left behind because of unexpected poor recoveries. To avoid losing time and money because of not understanding the flow mechanisms in the field, a detailed study should be conducted in all projects.

To picture the heterogeneity and predict performance, there are different possible methods which can be categorized mainly as empirical, analytical and numerical models. Empirical models are not capable of explaining the physical logic of the problem. Analytical models are more robust, but need simplifications because of complexity of equations. Reservoir simulation studies need a large amount of reservoir data to have a "representative" model which may be challenging for a project with limited time and budget.

To offer rapid reservoir evaluation and quick estimation of the performance of the reservoirs with less and more reliable data, a wide variety of approaches have been developed. One of the approaches to complement reservoir simulation is surrogate models that rely on rate and pressure data to estimate reservoir properties. The main advantages of this approach are requirement of fewer data, less computation time and reduction of uncertainty in the geological model.

The Capacitance – Resistive Model is one of these successful methods which uses measured rate and pressure data to infer interwell connectivities between injectors and producers by taking compressibility and transmissibility effects into account. This model uses non-linear regression method and consists of three main parts which are primary production contribution of the well itself, injection contributions from other injectors and pressure change effects related to well itself and nearby producers.

The main objectives of the study are to determine the interwell connectivities between injector-producer pairs and define the flow paths of the injected water by using CRM. In this way, the management of the reservoir can be optimized by using the operational and economical decisions. Oil productions and oil fractional flows come into picture to relate the liquid rates coming from CRM with the oil rates to be used in economic analysis. Bringing all these data together with the economic parameters, maximization of the project revenue can be achieved in a long term period.

In this study, a real waterflooded carbonate reservoir is studied by using Capacitance-Resistive Model to observe if any additional economical and technical income can be achieved. By taking heterogeneity and the uncertainty into account, the answer of the following question is investigated “What could have been done and achieved in a ten year project period by characterization of the reservoir and optimization of the injection volumes?”

CHAPTER 4

FIELD OVERVIEW

4.1 General Overview of the Field

The field is a large carbonate anticlinal structure trending NE-SW, bounded by a high angle thrust fault in the south. Producing zone is limited by the major fault striking parallel to the axis of the structure and the surrounding stratigraphic boundaries.

Formation is a limestone with lateral and vertical facies changes due to lithological and grain size variations. The main formation can be subdivided into two parts; the top porous zone and the bottom tight zone (Memioglu et.al., 1983). This is also validated by the core analysis showing that the porosity distribution is bimodal which represents the mentioned two sections. Permeability distribution shows almost a log-normal distribution within moderate values. DST values (limited data) show similar results with the cores which rarely indicate fracture effects. The main disadvantage of this medium is that the vertical permeability is high as horizontal one in general (some out of trend data) which has an important effect on water breakthroughs (Figure 4.1).

The reported oil API gravity is about 26° API. The measured bubble point pressure is 197 psi which enabled only liquid phase flow in the reservoir. The differential solution gas-oil ratio, oil formation volume factor and oil viscosity at bubble point pressure was measured as 122 scf/stb, 1.1 bbl/stb and 6.75 cp, respectively (SSI, 1985).

According to pressure and production data gathered in the first four year production period, there is no aquifer support in the field. By the decrease in the pressure with the production, it was understood that reservoir had only been producing under expansion mechanism and it was decided to start peripheral water injection to pressurize the reservoir and displace oil.

Peripheral low salinity water injection project has started in 1960 and continued until today. There are 45 wells drilled up to now in the field which consists of 16 producers, 4 injectors, 18 abandoned (producer + injector) and 7 fresh water wells currently (Figure 4.2).

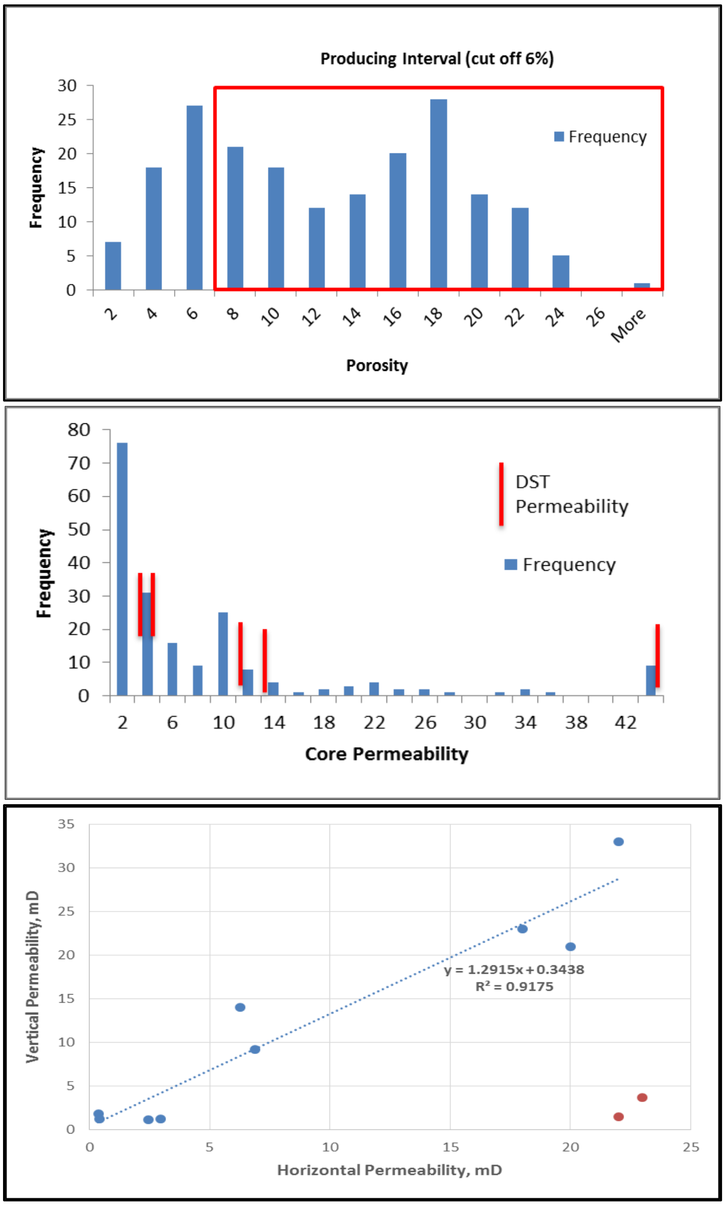


Figure 4.1 Porosity & Permeability Characteristics of the Producing Formation

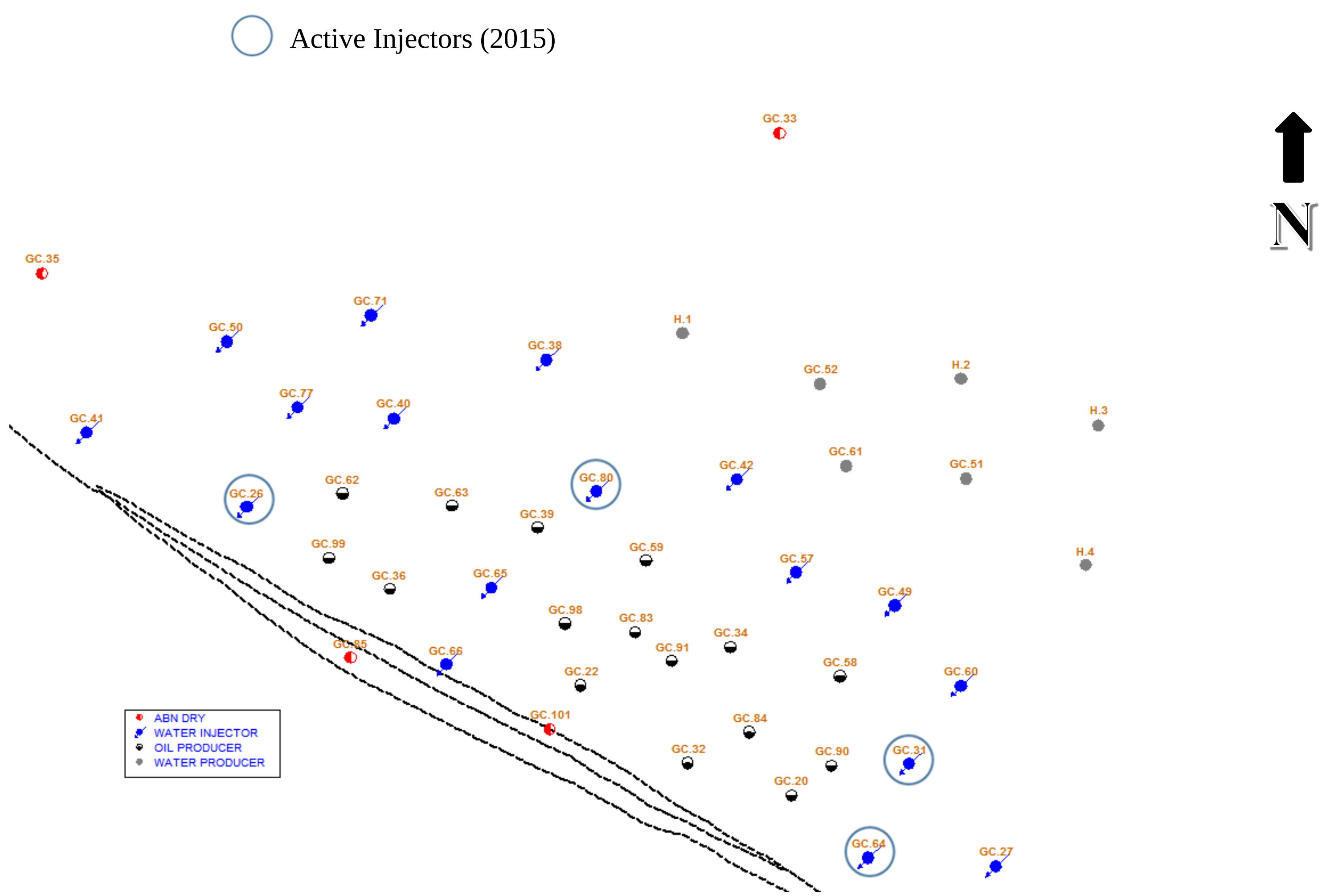


Figure 4.2 Well Locations and Reservoir Sturcture

4.2 Injection and Production History of the Field

Oil production started in 1956 and new wells were drilled to delineate the reservoir after the discovery of the structure. The increase in the number of wells and the production continued in the following three years. Following the decrease in the field pressure with the production, it was decided to start peripheral water injection for both pressure maintenance and displacement.

Water injection project had been initiated by the year of 1960 and new production and injection wells were drilled to enhance production in the following years. Also, some modifications were made in injection pattern to maximize the production. Injection continued up to the year of 1985 and was stopped to perform a simulation study to optimize the decrease in oil production as a result of increase in the water cuts.

According to the studies performed, injection pattern was changed and injectors were placed into interior parts of the reservoirs. Because of the heterogeneities and closer well distances, a worse water cut increase scenario continued in the following years and some of the wells were abandoned due to high water cuts.

By the end of 2014, the cumulative production is 13.6 million bbls of the field which corresponds to 20 % of OOIP. Average oil production is about 310 bbls/d and average water cut throughout the field is approximately 88%. Up to now, 50.2 million bbls of water injected into the reservoir which is 1.22 times of the total liquid production from the field (Figure 4.3).

Because of the pattern change in early 1990s, “before 1992” and “after 1992” pattern configurations and the related information must be taken into account to understand the flow mechanisms in the field. The mentioned patterns are shown in the following figures (Figure 4.4 & Figure 4.5).

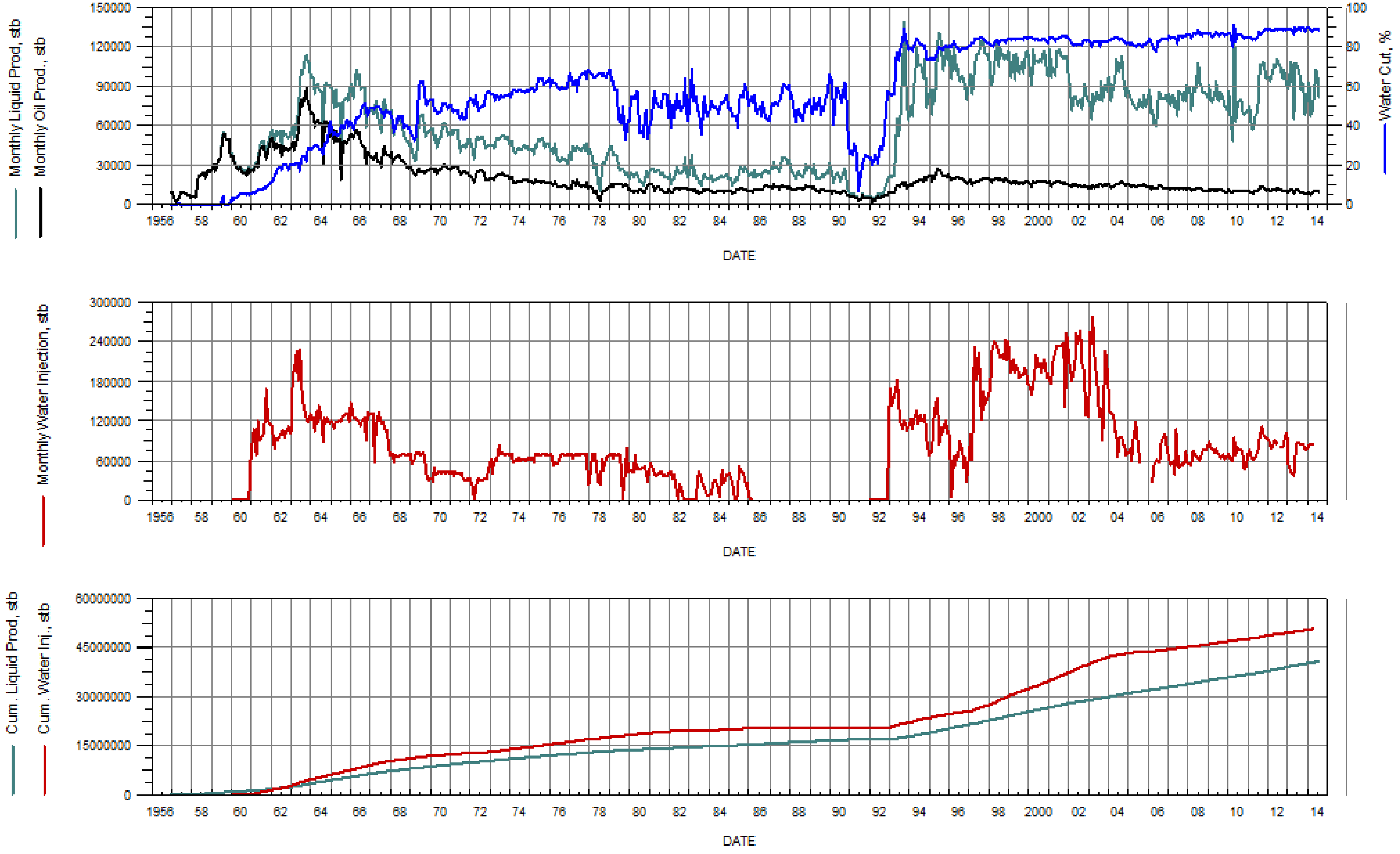


Figure 4.3 Production and Injection History of the Field

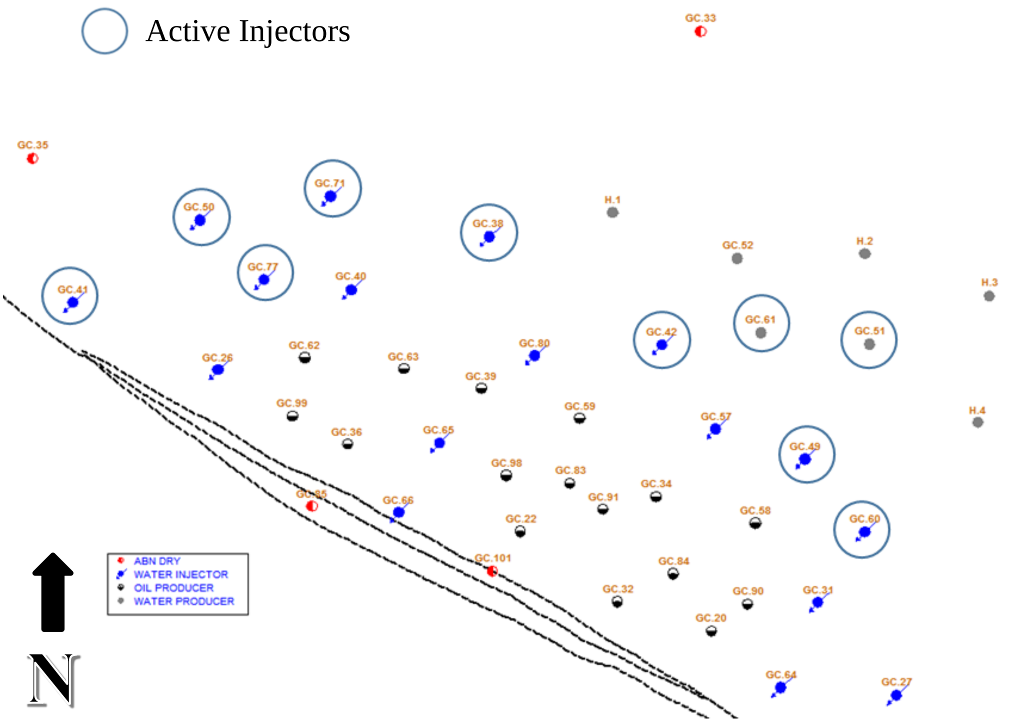


Figure 4.4 Current Situation of the Wells and Active Injectors Before 1992

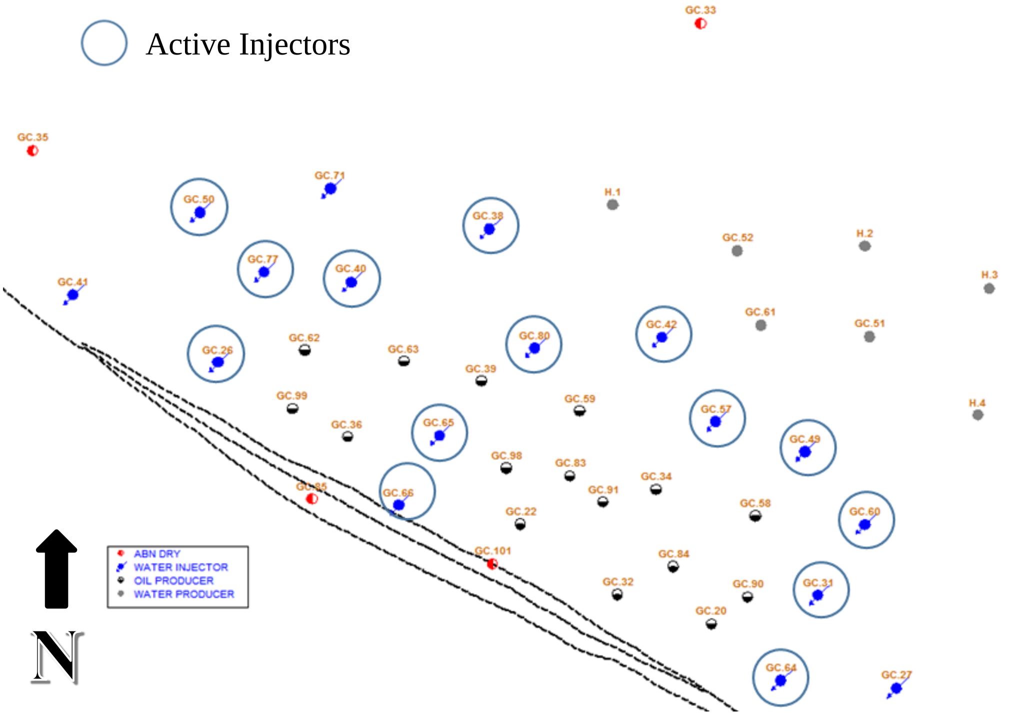


Figure 4.5 Current Situation of the Wells and Active Injectors After 1992

4.3 Early Time Water Breakthrough Analysis

One of the most helpful analysis which was used to correlate the weight coefficients with the real data is early water breakthrough analysis. This analysis was done by considering the timing of the first water production in the wells and the possible related nearby active injectors at that time.

In the first four years of the production, 14 wells (G-20, 22, 26, 27, 31, 32, 34, 35, 36, 38, 39, 40, 41 and 42) were drilled and four of them were converted to injectors as planned for waterflooding project. In the next five years, after the initiation of the waterflooding, 17 wells (G-49, 50, 51, 52, 57, 58, 59, 60, 61, 62, 63, 65, 66, 71, 80, 83 and 84) were drilled to develop the field and maximize oil production.

The active injectors (G-27, 38, 41, 42, 50, 51, 52, 60, 61 and 71) between the years 1960 and 1968 were investigated to observe possible interactions between these wells and the water breakthroughs in producers. In addition, the distances between wells and the fluctuations / responses in both injection and production rates were used to map these interactions. Figure 4.6 illustrates one of the water breakthrough analysis example in which the water production starts with the start of injection in a nearby injector. The general map of interactions which is generated by using breakthrough analysis, fluctuation similarities and well distances are shown in Figure 4.7

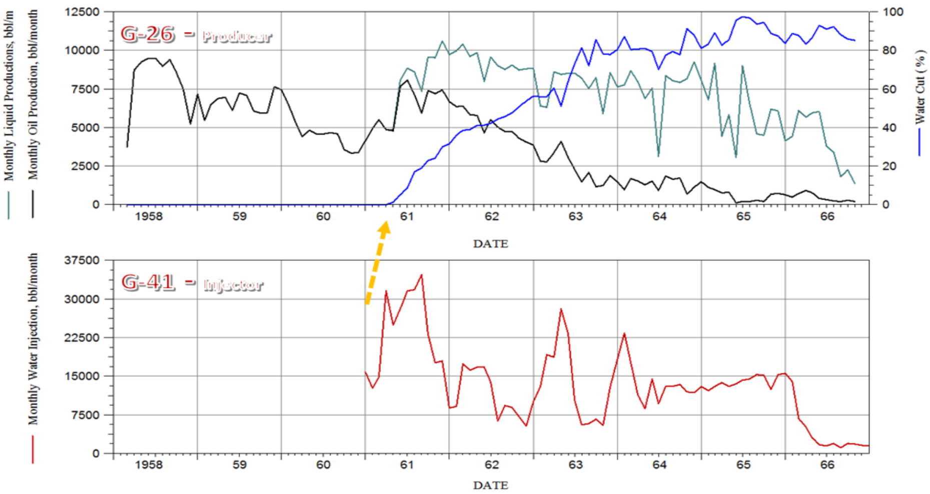


Figure 4.6 An Example of Water Breakthrough Analysis and Related Well Pairs

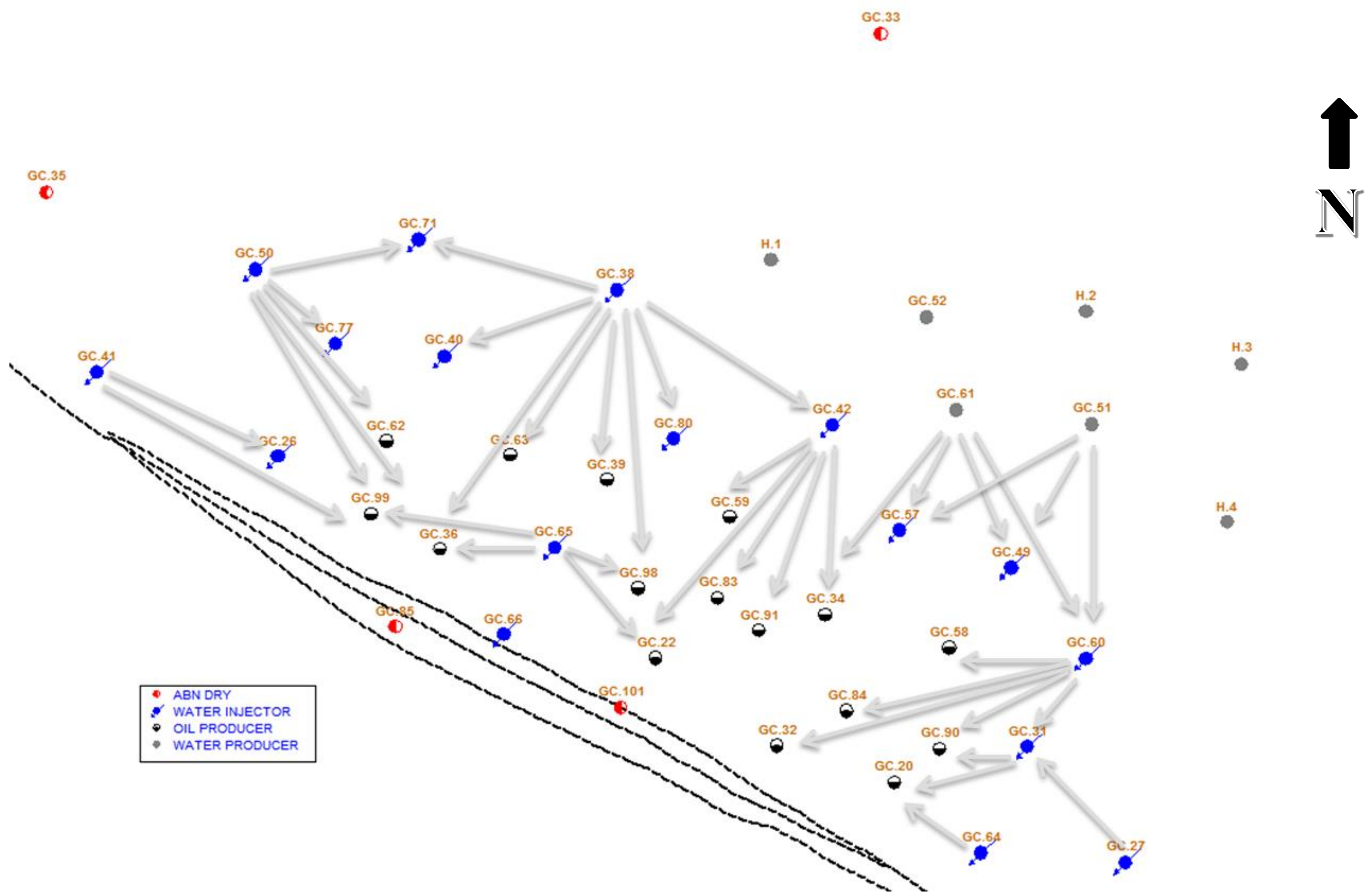


Figure 4.7 Map of the Interpreted Interwell Connectivities (no scale)

CHAPTER 5

METHODOLOGY

In Chapter-5, the methodology of the capacitance resistive model application in a real field and the optimization of the project objective are discussed. Firstly, the model which was used to match production history of the field and the preparation steps are introduced. Secondly, fractional oil flow match model selection and matching procedure are discussed. Finally, different optimization algorithms which combine the capacitance-resistive, oil fractional flow and the economic models are presented.

5.1 Capacitance-Resistive Model Application

5.1.1 Available Data and Preparation

Because of the discovery time of the field, it has a long historical data to be analyzed. As it was stated in "Injection and Production History of the Field" part of Chapter 4, the field has been waterflooded since 1960 with a seven year gap between the years 1985-1992. Available data for the field consists of monthly recorded 59 years of production and 55 years of injection history (Figure 5.1).

In addition to production and injection rate data, static bottomhole pressures and dynamic liquid levels measured from the wells are available beginning from the year of 1987. Unfortunately, flowing bottomhole pressures could not be measured because of the current technologies available at that time. Instead of these missing data, liquid levels were used to include bottom-hole pressures indirectly. The field data shows that there is no constant bottomhole pressure production. Therefore, study would result in unrealistic fitting parameters unless the pressures contributions were incorporated.

5.1.2 Working Period Selection

Focusing on the previously mentioned assumptions of CRM (Kaviani et al., 2008);

- Constant number of producers; i.e. no shut-in period or new production wells
- Availability of BHP data or constant/similar BHP
- Constant reservoir and well conditions
  - No new perforations in other zones
  - Constant productivity index
- Long period of data
- Negligible change in gas saturation
- Uncorrelated injection rates

In this case study, injection was stopped between the years 1985-1992 and new pattern was designed in which injectors became closer to the producers located in interior parts of the reservoir. After a certain time following the pattern change, increase in water cut and liquid productions became stabilized. In addition to this criteria, properly measured dynamic liquid levels are available after the year of 1987 which restricted the working period in a narrower interval.

By considering stabilized flow period, the constant number of production wells, availability of rate & liquid level data, shut-in periods and constant GOR conditions; between beginning of 1996 and middle of 2000 time interval was selected as the history match working period for this case study (Figure 5.2).

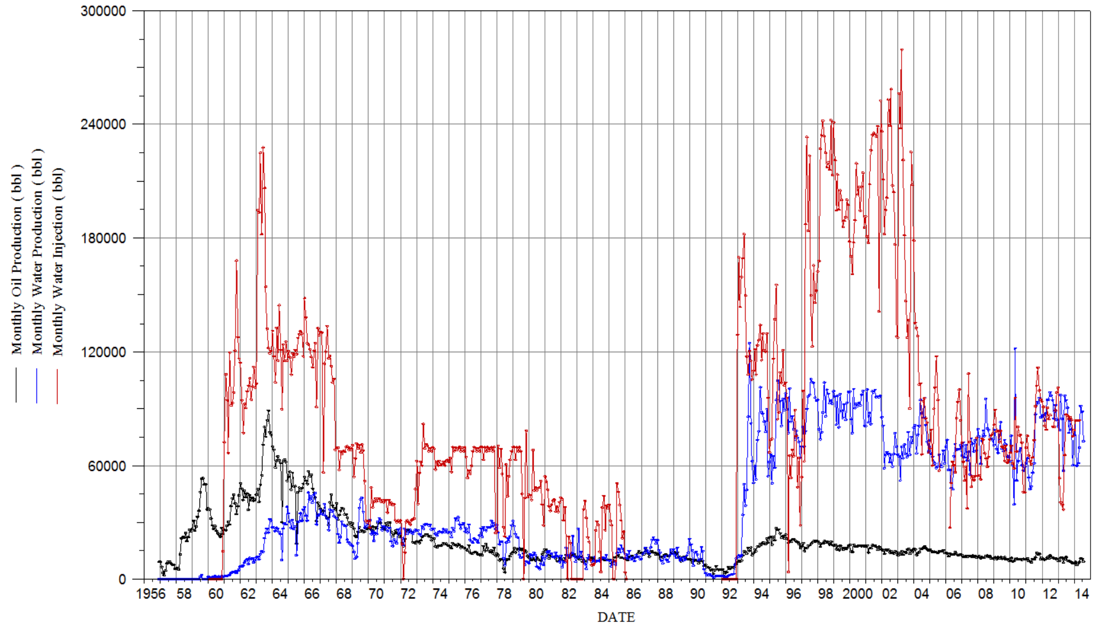


Figure 5.1 Injection and Production History of the Field

The reasons for selecting this time period can be listed as follows;

- The producers represent the current situation in the field
- There are enough fluctuations in the rates to determine the connectivities
- Rate & liquid level data were measured properly (bad data quality after 2000)
- Reservoir pressure is above bubble point pressure (constant GOR)
- No workover operations or changes during & after the period (constant PI)

Selected time intervals for different stages with their explanations are listed below;

1) 01.01.1996 – 01.12.1998 (36 months - history matching)
2) 01.01.1999 – 01.06.2000 (18 months - history match validation by forecast)
3) 01.07.2000 – 01.07.2010 (121 months - optimization period)

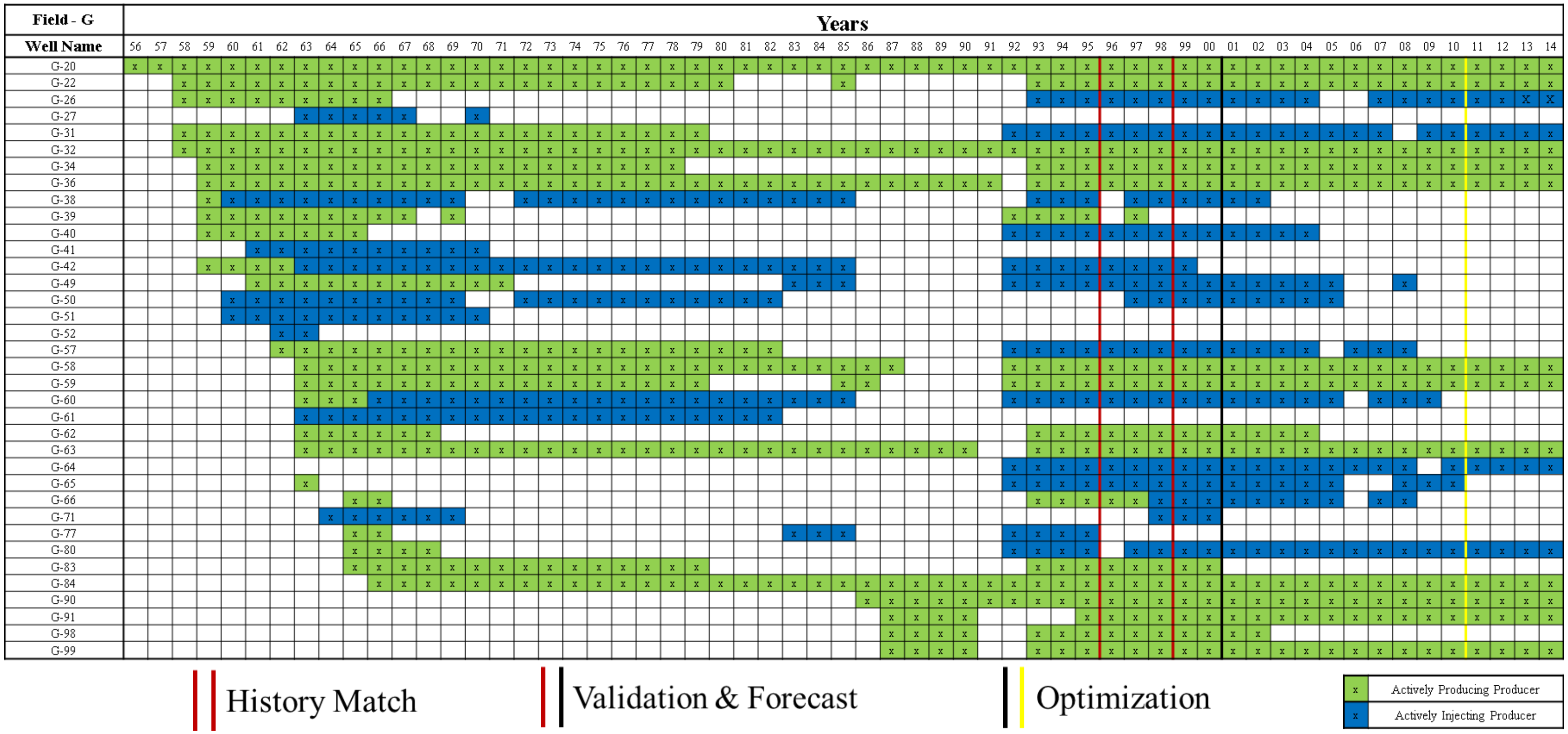


Figure 5.2 Active Time Periods of the Wells and Selected Working Period

The reason of selecting optimization period within 2000-2010 years is because of the fact that the low salinity water injection was stopped after 2010. Instead of fresh water, produced water was injected after 2010. The change in the injected liquid type resulted in a negative effect on the production behavior. Because of this performance change, it would not be meaningful to compare these periods. So, that part was excluded from the analysis.

5.1.3 Data Preparation

The production, injection and liquid level data were prepared as inputs within the working period interval. As stated in the CRM formula, rates must be in reservoir volumes which were achieved by using formation volume factor information gathered from fluid properties. 1.04 rbbl/stb and 1.01 rbbl/stb values were used to convert surface volumes into reservoir ones for producers and injectors, respectively.

There are two types of production rates calculated based on both “calendar day” and “working day”. Calendar day based rates are calculated by dividing the monthly production by the number of the corresponding months’ days and the other one just takes the exact working days into account.

The injection rates were also calculated as calendar based rates. The bottomhole pressures were calculated by using a pressure gradient to convert liquid levels to the pressures which is 1.35 psi/m with an assumption of the presence of both water and oil in the annulus.

The calendar day based rates for both production and injection were used as inputs of the model to honor the material balance in the system.

5.1.4 Capacitance-Resistive Model Selection and Generation

The equation discretized by Kaviani et al. (2008) was used in the model to capture all primary production effects of the producer itself, injection contributions from the other injectors and the pressure effects of nearby producers. The main reason to select this method instead of the other studies mentioned in the literature survey is to integrate also the pressure effects of nearby producers. The equation which was used for the CRM calculations proposed by Kaviani et al. (2008) is as follows:

$$q_j(t) = q_{0j} + \lambda_p q\,(t_0) e^{\frac{-(t-t_0)}{\tau_p}} + \sum_{i=1}^{i=I} \lambda_{ij} \sum_{m=1}^{n} \left[e^{\frac{(t_m - t)}{\tau_{ij}}} - e^{\frac{(t_{m-1} - t)}{\tau_{ij}}} \right] i_j(t_m) + \sum_{k=1}^{k=K} v_{kj} \left[p_{wf_{kj}}(t_0) - p_{wf_{kj}}(t) \right] \tag{5.1}$$

where, q_{0j} is the unbalance effect parameter

The equation above includes three important parts which are primary production, injection contribution and the pressure contribution terms. Primary production contributions of the wells were calculated from the time interval before injection started again in the year of 1992. Injection contributions of each injector-producer pair (14 x 15) for 175 months were formulated to capture continuous (convolved or filtered) effect of injection. Also, pressure contributions between producers (15 x 15) for 53 months are formulated by using the liquid level data just for the history match period. Pressure contribution formulas were not extended until the end of the optimization because optimization procedure assumes constant pressure production, no way to simulate the pressure contributions for the future in this model.

To cover all the parts of the equation, fitting parameters were tabulated to be solved as a part of the solution matrix (Figure 5.3) and listed below;

1) One weight and time constant parameter of primary production for each well
2) 14 x 15 weight coefficient and time constant matrices for producer-injectors
3) 15 x 15 weight coefficient matrix for producer-producer interactions

Microsoft® Excel based Analytic Solver Platform software was used to generate CRM model. GRG2 (Generalized Reduced Gradient) algorithm, which is better for smooth non-linear problems, was used to solve the generated matrices.

Also, the total liquid production and injection rates were checked to analyze if the field is in balanced or unbalanced condition (difference between injection and liquid production rates). Almost all months in the working period are unbalanced but liquid production is considerably more than the injection only in the first 14 months. That is why unbalanced effect q_{0j} was used in this time period. Because of the fact that injection volumes are more than the production volumes in the remaining period, there is no need to use any unbalance effect and sum of the weights for injectors are expected to be equal or lower than unity.

Λ_{pj}	G-20	G-22	G-32	G-34	G-36	G-58	G-59	G-62	G-63	G-83	G-84	G-90	G-91	G-98	G-99
τ_{pj}	G-20	G-22	G-32	G-34	G-36	G-58	G-59	G-62	G-63	G-83	G-84	G-90	G-91	G-98	G-99
Λ_{ij}	G-20	G-22	G-32	G-34	G-36	G-58	G-59	G-62	G-63	G-83	G-84	G-90	G-91	G-98	G-99
G-26															
G-31															
G-38															
G-40															
G-42															
G-49															
G-50															
G-57															
G-60															
G-64															
G-65															
G-66															
G-71															
G-80															
τ_{ij}	G-20	G-22	G-32	G-34	G-36	G-58	G-59	G-62	G-63	G-83	G-84	G-90	G-91	G-98	G-99
G-26															
G-31															
G-38															
G-40															
G-42															
G-49															
G-50															
G-57															
G-60															
G-64															
G-65															
G-66															
G-71															
G-80															
Vkj	G-20	G-22	G-32	G-34	G-36	G-58	G-59	G-62	G-63	G-83	G-84	G-90	G-91	G-98	G-99
G-20															
G-22															
G-32															
G-34															
G-36															
G-58															
G-59															
G-62															
G-63															
G-83															
G-84															
G-90															
G-91															
G-98															
G-99															

Figure 5.3 Weight Coefficient and Time Constant Matrices Used in the Model

Finally, objective function “*error between the observed and predicted liquid rates are minimum*” was set with constraints to reach best solution;

1) Weight coefficients of pairs are non-negative (≥ 0)
2) Time constants of pairs are greater than zero ($\tau_{ij} > 0$)
3) Sum of the weights coefficients is equal or less than one ($\sum_{j=1}^{N} \lambda_{ij} \leq 1$)
4) Pressure coefficients of nearby producers are less than zero while the coefficient of producer itself is greater than zero ($v_{kj} \leq 0$ & $v_j \geq 0$)

Firstly well by well and finally simultaneous non-linear regression was performed within the first 36 months period. Then procedure was extended through the next 18 months period without changing any parameters to validate the history match fitting parameters. Being sure about the fitting parameters matching the production history, they were used in optimization period for the next 121 months period.

5.2 Fractional Oil Flow Match

For the optimization algorithm, the oil rates are much more important than total liquid (oil + water) rates which leads to need for a correlation between liquid rates and oil fractional flow. An empirical model was selected for calculation of the oil rates.

The used model was selected by studying all of the mentioned oil fractional models covering the cumulative oil production and oil cut relationship. Kazakov (1976) was selected as the most promising one which fits the water cut and oil production trend reasonably well within the working period. Most of the late time production trends of the wells which are important for the optimization period were matched very well.

Remembering the model once more, the relationship between cumulative oil production and oil fractional flow can be stated as follows:

$$\frac{q_o}{q} = f_o = \frac{(1-b\,N_p)^2}{(a-2bN_p)(1-b\,N_p)+N_p\,b\,(a-b\,N_p)} \tag{5.2}$$

N_p = cumulative oil production $\qquad$ a, b = constants

To find these constants, history matching procedure by using nonlinear regression for the historical time steps is needed. In this model, the given liquid rate uses the previous time step's oil cut to determine the current time step's oil production which is used for calculating the corresponding time step's cumulative oil production. The new cumulative oil production is used again to calculate the oil cut for the next time step to repeat all these steps by using the given formula.

5.3 Optimization and Economic Analysis of the Project

Because of the fact that late time of the waterflooding project is analyzed in this case study, some of the items listed in the economic evaluations can be excluded here. That is why just only the operating costs of injection/production, rates and prices of hydrocarbons are considered for this project.

The overall optimization workflow by using capacitance model and oil fractional flow model summarized by Sayarpour (2008) is illustrated in Fig. 5.4;

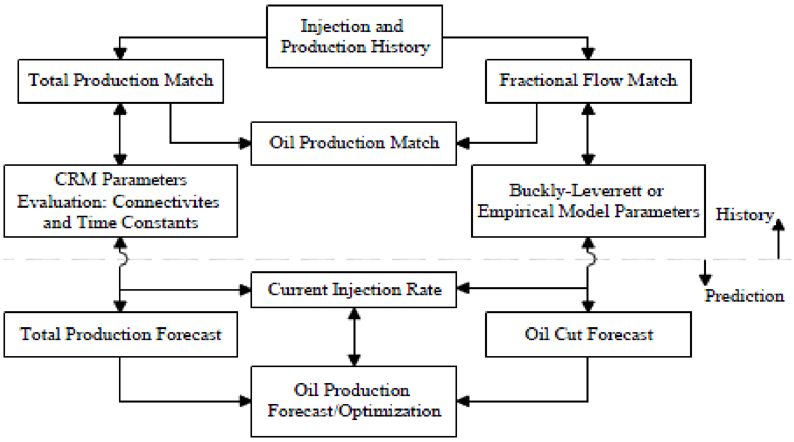


Figure 5.4 Economic Optimization Algorithm for Project (Sayarpour, 2008)

Firstly, total liquid and oil production matches are performed by using capacitance-resistive and oil fractional flow models. When the production history is matched, injection & production forecast are done by using the same parameters to optimize the both injection and production rates depending on the defined objective function.

The most important part of an optimization problem is determining the objective function. According to the study conducted by Bastami et. al. (2012), depending on the purpose of the optimization, different objective functions can be defined as follows;

- Maximizing cumulative oil production
- Minimizing cumulative water production
- Maximizing net present value of the project (incomes & costs)
- Maintaining the oil production rate while minimizing other phases' production

In this case study, two alternative options which consider either technical or economical outcomes of the project were used;

1) Maximization of the cumulative oil production during a specific time interval. The objective function is as follows:

$$R = \sum_{j=1}^{N_p} \int_{t_0}^{t} q_{o\,j}(t)dt \tag{5.3}$$

Model tries to maximize cumulative oil production technically by changing injection volumes.

2) Maximization of the profit of this waterflooding project during a specific time interval. The objective function is as follows:

$$R = (p_o - p_{wo}) \sum_{j=1}^{N_p} \int_{t_0}^{t} q_{o\,j}(t)dt - p_w \sum_{i=1}^{N_i} \int_{t_0}^{t} i_i(t)dt \tag{5.4}$$

where, p_o, p_{wo} and p_w are oil price, disposal cost of the water per produced barrel oil and the water injection cost per barrel, respectively. In this part, model tries to maximize revenue by just calculating the economical values of both production and injection volumes.

CHAPTER 6

RESULTS AND DISCUSSIONS

Chapter-6 brings all previously discussed chapters together to show the main results of the study. Chapter starts with CRM application results for this specific case study. It is followed by the analysis of fitting parameters to explain them by making use of geological explanations. Then, oil fractional flow model selection and history match results are discussed. Finally, bringing all these outcomes together, the optimization of the injection and project revenue are conducted and explained.

6.1 Capacitance-Resistive Model Application

As stated in Chapter-5, the discretized model proposed by Kaviani et al. (2008) was used to decouple the effects of the all contributing parts. Firstly, unbalance effect was investigated to correctly match the other contributions related to both injection and pressure. According to this analysis, it was observed that all monthly production rates are considerably different than the injection rates but just only the first 14 months of them are more than the injection ones. Unbalance effect parameter was used in this time period for each well.

For primary production coefficient parameters, it was found that λ_p=1 and τ_p=50,000 model parameters represent the average field decline rate and are suitable to be used for the producers in this model. In the injection contribution part, 14 x 15 weight coefficient and time constant matrices were generated. Integrating the pressure contribution part, a 15 x 15 pressure effect coefficient matrix was generated to decouple the effects of injection and pressure contribution. All these model parameters were solved simultaneously to honor the material balance of the system and model constraints in each time step. The resulting fitting parameters for injection and pressure effects are tabulated below;

Table 6.1 Calculated Weights, Time Constants and Pressure Coefficients

λ_{ij}	20	22	32	34	36	58	59	62	63	83	84	90	91	98	99
26					0.07			0.18	0.15						0.05
31	0.10					0.13					0.03	0.15			
38								0.01	0.06						
40					0.05			0.18	0.10						0.05
42		0.07		0.01			0.13			0.08			0.02	0.10	
49	0.07		0.01	0.04		0.05				0.02	0.01	0.05	0.01		
50								0.09	0.01						0.02
57	0.05	0.02	0.01	0.10		0.04	0.08			0.08	0.04	0.03	0.02	0.28	
60			0.01	0.09		0.04					0.02	0.10	0.03		
64	0.15		0.01			0.02					0.03	0.12			
65		0.03		0.02	0.07		0.08	0.14	0.08	0.05			0.02	0.50	0.01
66		0.01			0.04			0.10	0.03	0.04			0.01	0.60	0.01
71					0.04			0.09	0.01						0.01
80		0.01		0.01	0.04		0.06	0.08	0.05	0.06			0.02	0.30	0.02
τ_{ij}	20	22	32	34	36	58	59	62	63	83	84	90	91	98	99
26					150				20						57
31	160					85					155	637			
38								280	250						
40					80			20	95						46
42		20		150			30			151			130	80	
49	240		20	20		260				350	43	20	10		
50								200	150						166
57	350	350	35	25		242	30			34	30	55	25	30	
60			30	250		150					101	262	40		
64	50		16			242					245	559			
65		250		450	52		650	25	250	207			125	60	50
66		100			62			35	280	161			150	90	48
71					248			59	30						92
80		290		250	90		90	162	56	75			150	93	180
V_{kj}	20	22	32	34	36	58	59	62	63	83	84	90	91	98	99
20	0.01		-0.02									-0.04			
22		0.01	-0.01							-0.06			-0.01	-0.15	
32	-0.07	-0.03	0.02										-0.03		
34				0.02		-0.03	-0.18			-0.09	-0.01		-0.01		
36					0.07			-0.02	-0.11						-0.07
58				-0.01		0.03									
59				-0.08			0.01			-0.05			-0.04	-0.25	
62					-0.08			0.04	-0.01						-0.01
63								-0.02	0.01						-0.03
83		-0.04		-0.01						0.04				-0.40	
84	-0.05		-0.03	-0.09		-0.19					0.01	-0.04	-0.01		
90	-0.04					-0.11					-0.01	0.08			
91		-0.07	-0.01	-0.07			-0.06			-0.19			0.03	-0.15	
98		-0.05					-0.14			-0.05			-0.02	0.25	
99					-0.01			-0.02	-0.05						0.12

After the history matching period, validation & forecast part was carried out by using the same fitting parameters and injection data observed in injectors. As it was in the history match period, the total production rates coming from the model fit the observed data in an acceptable range which shows an approximately 10 % error within 53 time steps (Figure 6.1).

The figure shows two different observed liquid production rates which are "calendar day" and "working day" based. As stated in "Data Preparation" part of Chapter 5, the difference between these rates is coming from the non-productive time of the wells due to some pump failures, work-over and some surface operations. In regression process, it was aimed to keep the model results within these calculated rate ranges.

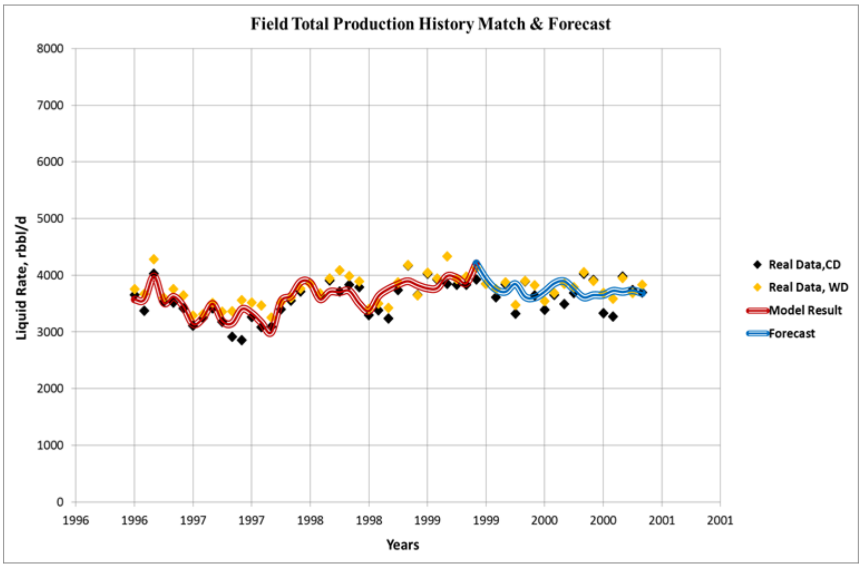


Figure 6.1 Overall Field Production History Match and Validation Forecast

One other important point in this process is the need of initial judgment by the user to prevent meaningless fitting parameters just calculated because of the mathematical approximations. That is why there is a "to do list" proposed by Weber (2008) which aims data cleaning and problem size reduction;

The problems observed in CRM applications are categorized in the following parts;

Many Constraints & Variables: The number of parameters needed to define a CRM is directly related to the number of wells. These parameters must be estimated simultaneously because of the nature of field-wide material balance.

In this case study, 2 x 15 primary production parameters, 14 x 15 x 2 injection contribution parameters and 15 x 15 pressure contribution parameters (total of 675 fitting parameters) are needed to generate this complicated model. Not only these parameters but also the constraints for each time step and total must be used the model to achieve meaningful results. For this specific case study, over 900 constraints are needed for all the time steps over a three year matching period.

<u>Shut-in / Intermittent / Changing Wells:</u> Throughout the life of a reservoir the number of active wells changes for several reasons such as economics, operational and technical. Because of the fact that every injection signal results in a production signal, these data may skew calculated connectivities from the real ones.

In this case study, deep investigation was performed to avoid shut-in periods of the producing wells and unstable period of active wells. Besides, two producers which are currently inactive and having low productions were excluded from the model.

<u>Outliers:</u> High or low production rates can affect the model calculation negatively. Observed low rates may be a result of a partial amount of monthly production (workover or shut-in) and high rates may be seen because of the change in a well productivity or equipment used in a well. Whatever the reason is, a big fluctuation causes problems because of the nature of nonlinear regression analysis.

In this case study, the time steps which have low production rates related to working days were excluded from the objective function to avoid wrong fitting. As a consequence, the model results were kept in a range of the mentioned rate values.

Weber (2009) also proposed the following data cleaning and problem size reduction methods:

<u>Reducing the Number of Gains:</u> In the case of a large scale problem, some of the wells which have negligible effect on the production response can be ignored. There are two possible ways to do this; the first one is to eliminate the wells having values less than a determined cut-off value if it is geologically reasonable. The second one is to eliminate some pairs having higher distances than the threshold values.

Because of the nature of mathematical approximations, even the furthest wells may be in interaction with each other if they can succeed to minimize the objective function. Here, the judgment of interpreter comes into picture by using the information of other sources to eliminate the pairs which do not have the possibility to interact. Also, inactive wells must be removed from the calculation to decrease computational time.

In this case study, by using well distances, static/dynamic data and breakthrough analysis, all possible well interactions were analyzed and the pairs which do not have possibility were excluded from calculation (can be seen from matrices). East and west part of the field (line crossing G-65, G-66 and G-80 can be accepted as center line) were analyzed separately. No inactive wells were used in calculations.

<u>Shut-in Logic:</u> As the theory states that all measured rates are the result of some measured injection signals, additional logic must be needed to overcome this missing production rate responses. There are two possible ways of eliminating these errors. One of them is excluding these steps from the objective function. The other one is the procedure proposed by Kaviani et al. (2008) which applies superposition with a virtual injector. In this way, production as much as the injection rate from the same point continues and the total rate becomes zero as it is in real case. But this also results in new but less additional parameters compared to the conventional CRM to be solved.

In this case study, because of the most suitable working period is selected, no elimination was needed. Only the rates of the wells not covering whole month were not included in the objective function (production days less than 28 days in a month).

<u>Outlier Classification Algorithms:</u> After all modifications before the calculation, outlier classification can be done to minimize total objective function. A given measured rate is classified as an outlier if its residual is extremely different than the tolerable range, they can be replaced by the neighboring data values.

In this case study, no outlier classification algorithms were applied. The observed data and the model are matched reasonably well in history matching and forecasting period.

Not to include the time steps having critical non-productive time (less than 28 days in a month - shown as baseline) in the objective function they were excluded from analysis. Total liquid production matches are shown in Figure 6.2 through Figure 6.17.

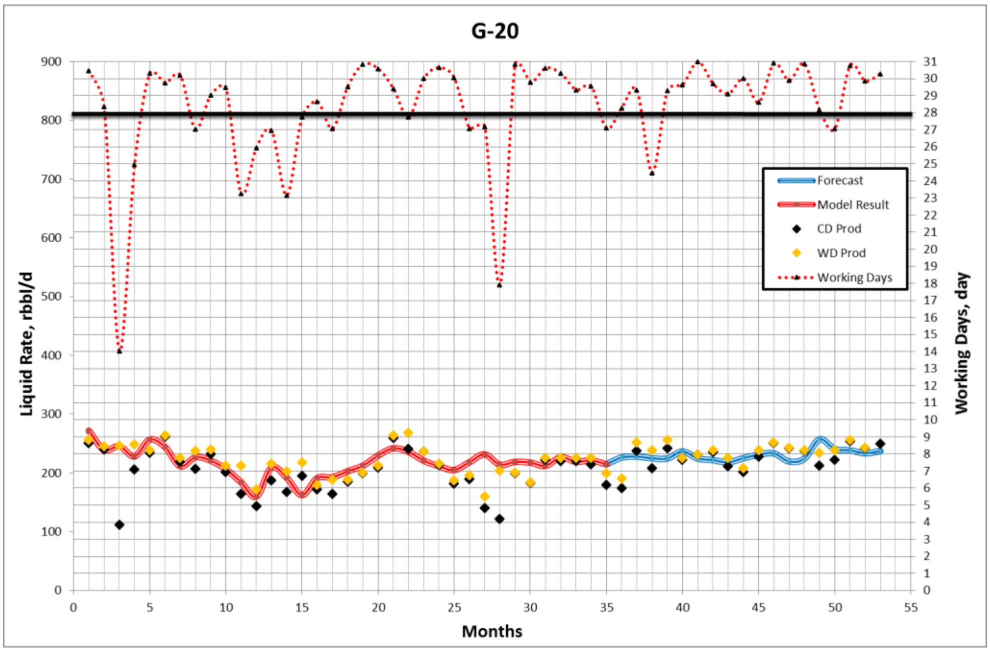


Figure 6.2 History Match and Forecast of G-20 Well

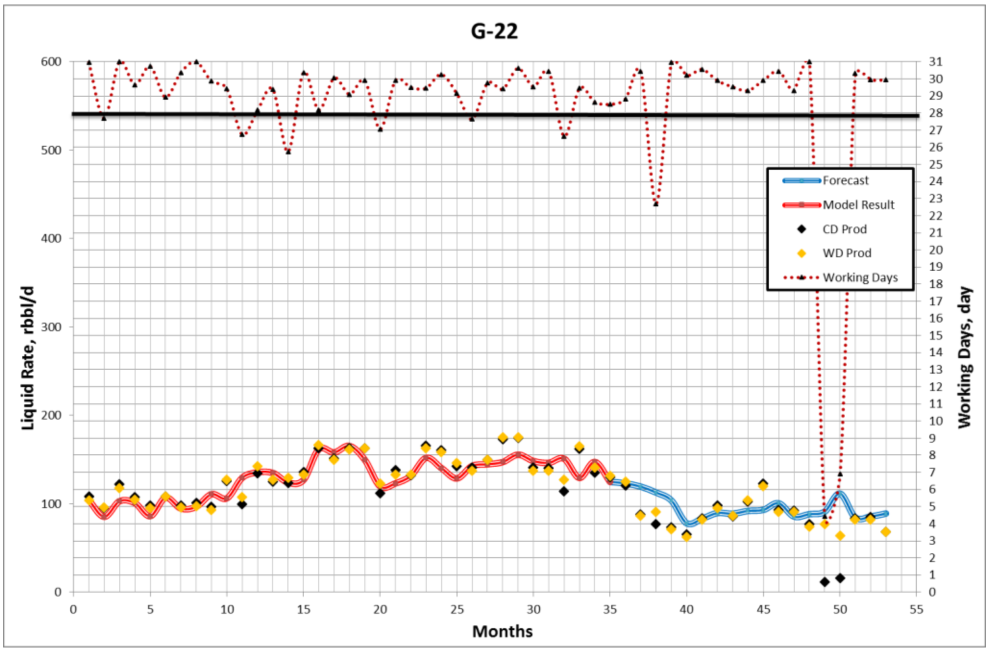


Figure 6.3 History Match and Forecast of G-22 Well

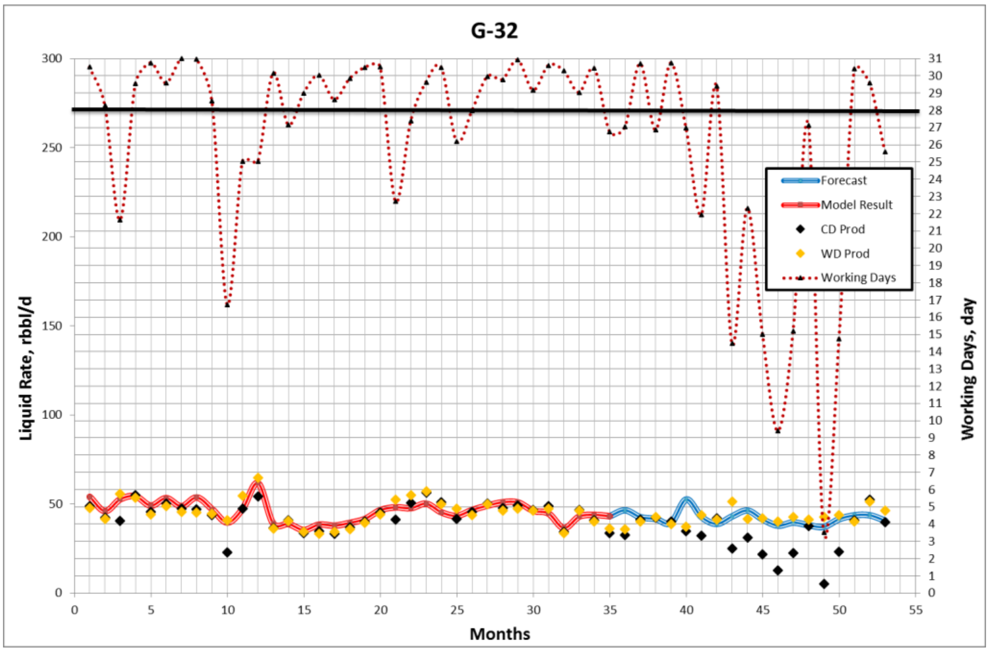


Figure 6.4 History Match and Forecast of G-32 Well

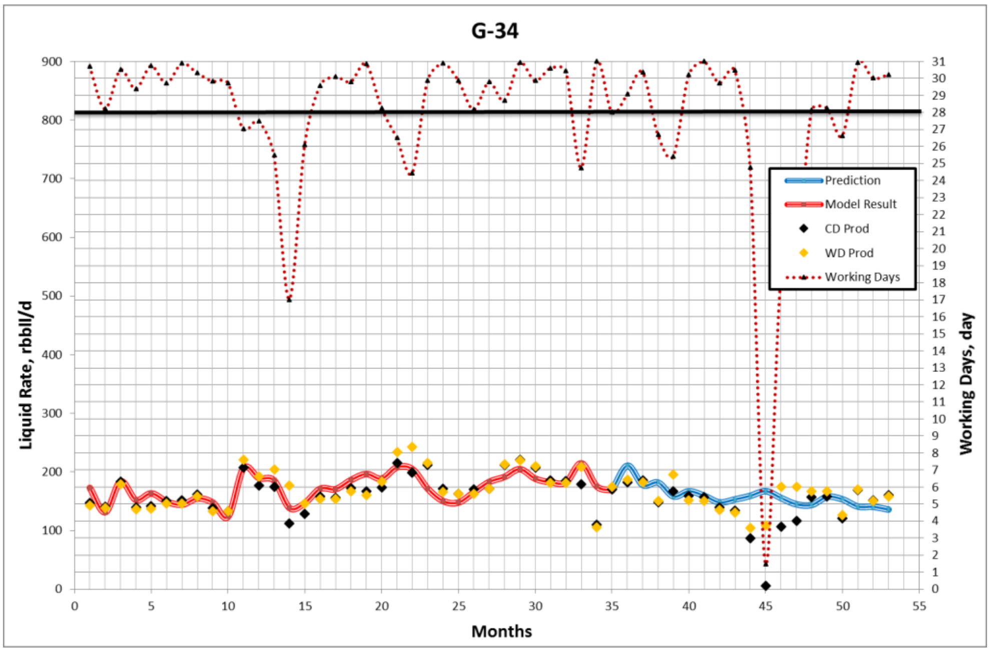


Figure 6.5 History Match and Forecast of G-34 Well

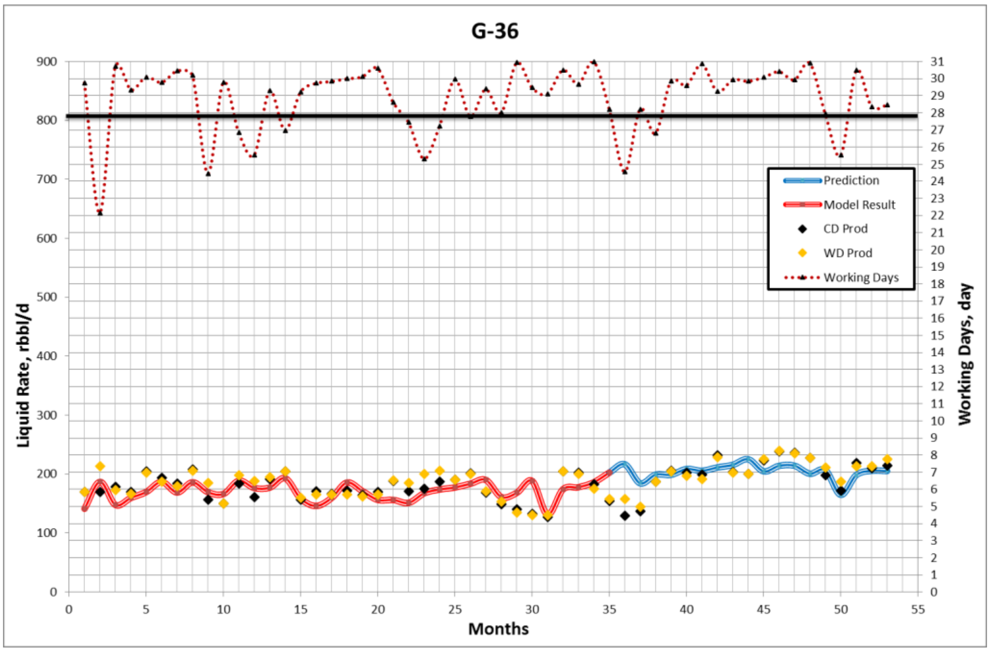


Figure 6.6 History Match and Forecast of G-36 Well

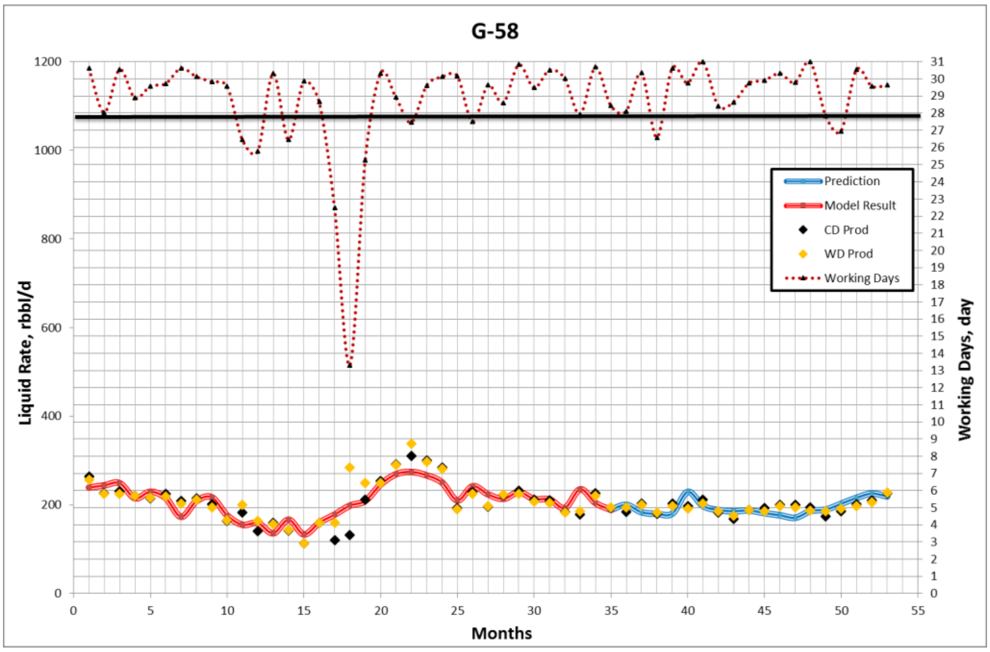


Figure 6.7 History Match and Forecast of G-58 Well

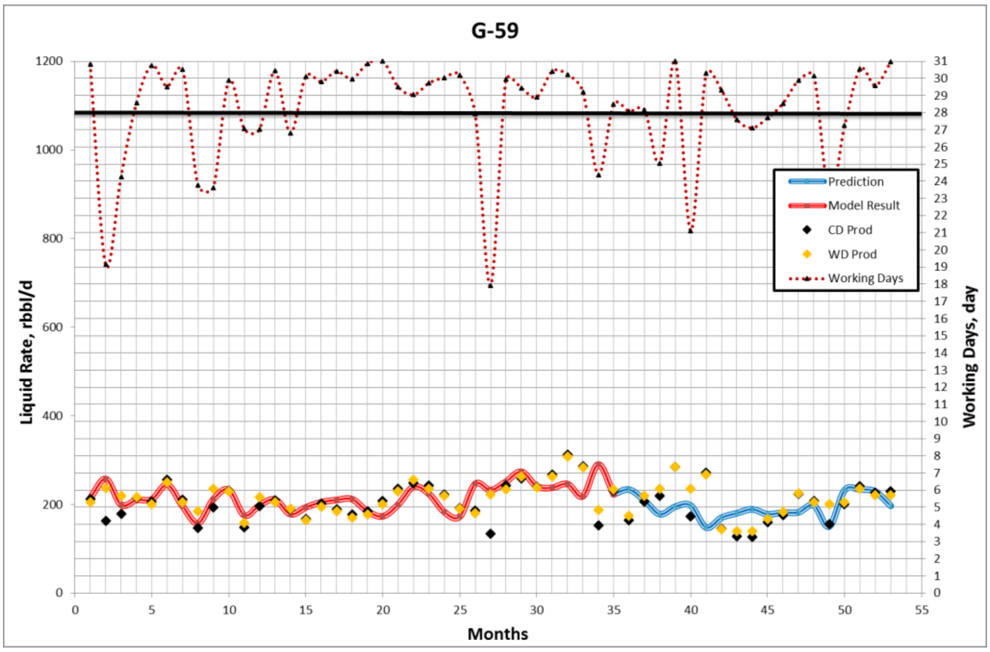


Figure 6.8 History Match and Forecast of G-59 Well

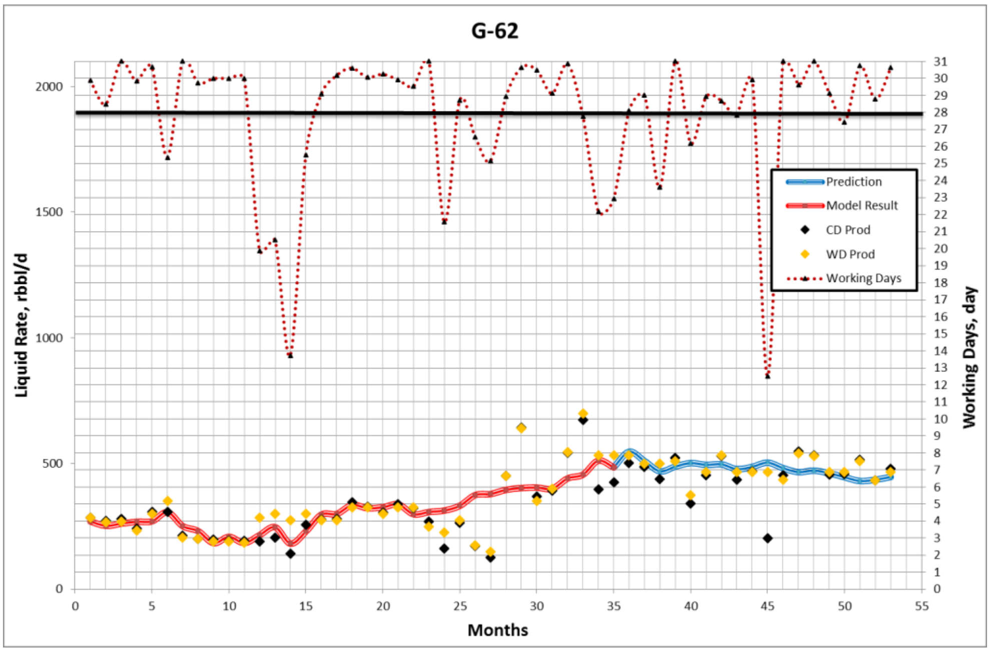


Figure 6.9 History Match and Forecast of G-62 Well

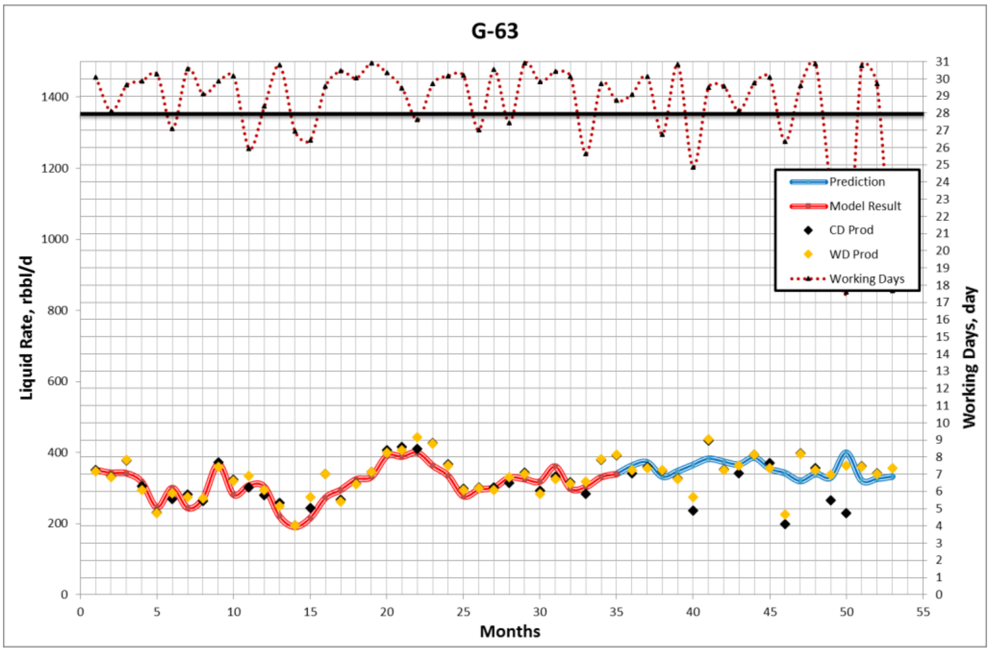


Figure 6.10 History Match and Forecast of G-63 Well

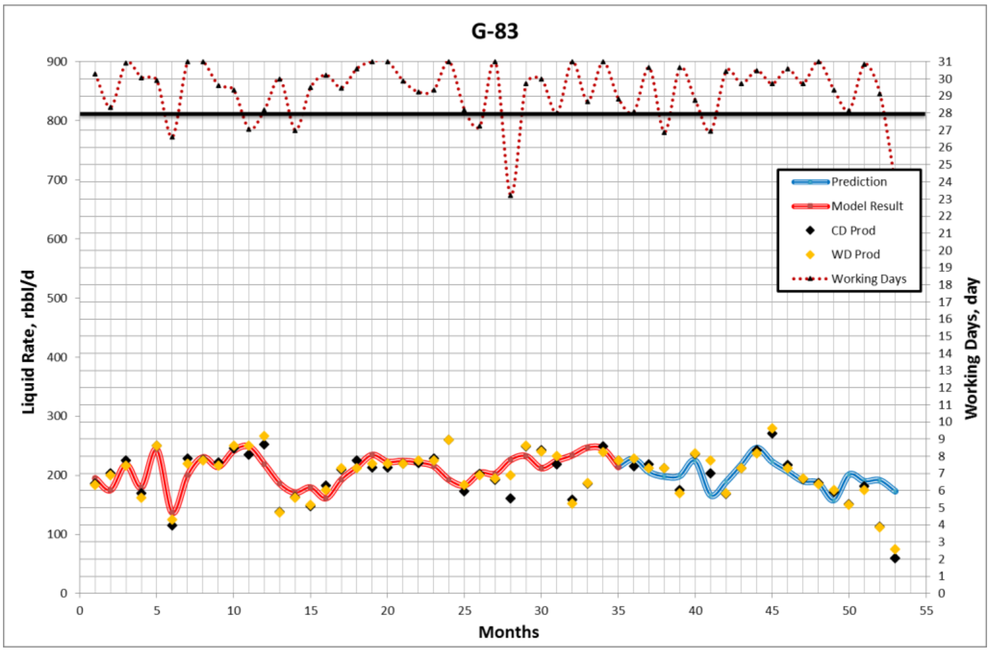


Figure 6.11 History Match and Forecast of G-83 Well

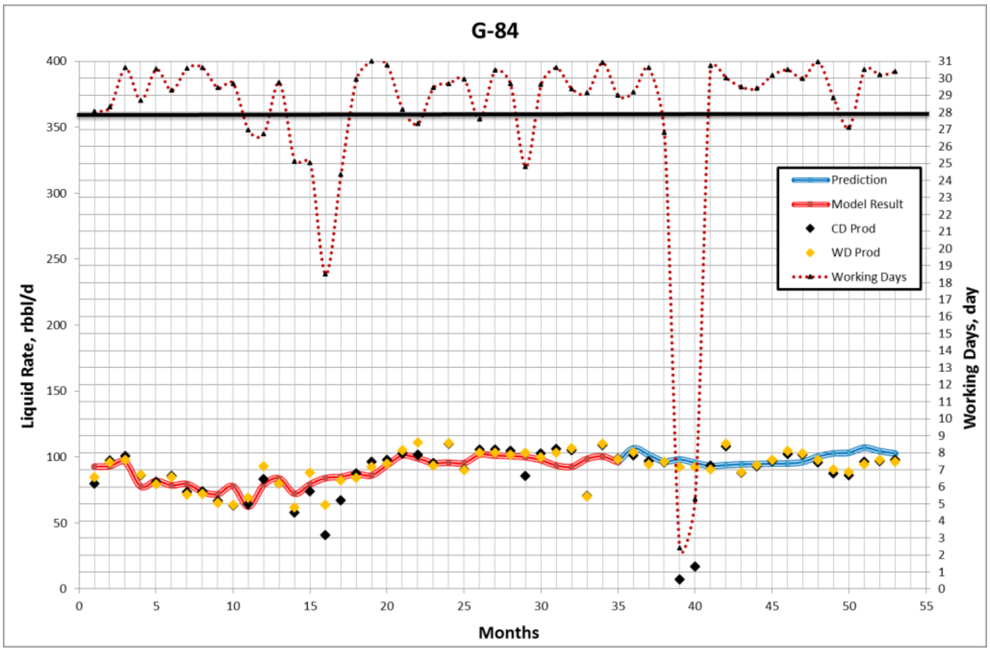


Figure 6.12 History Match and Forecast of G-84 Well

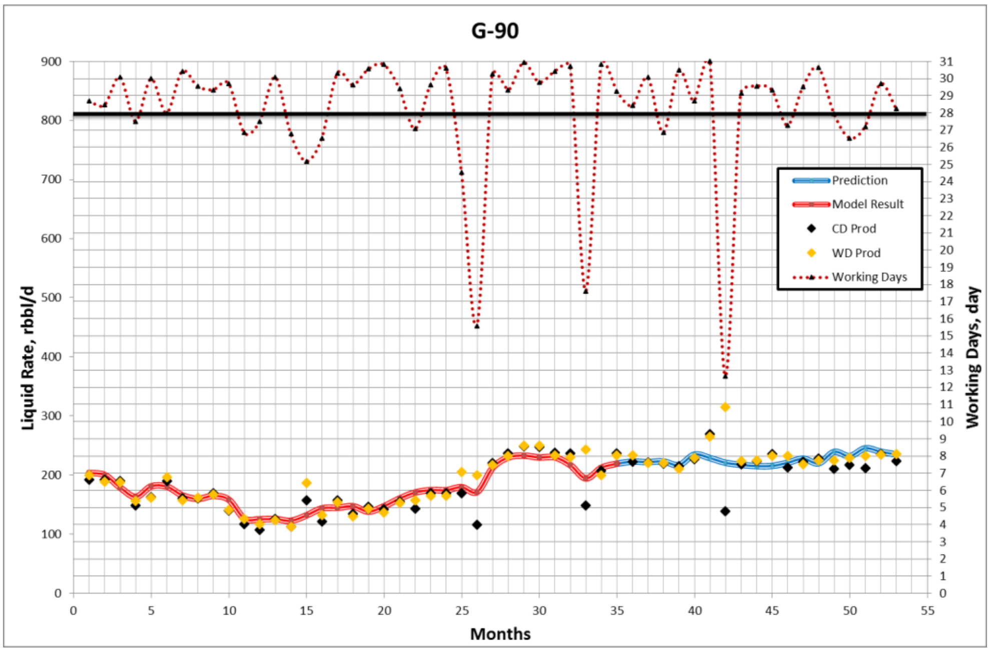


Figure 6.13 History Match and Forecast of G-90 Well

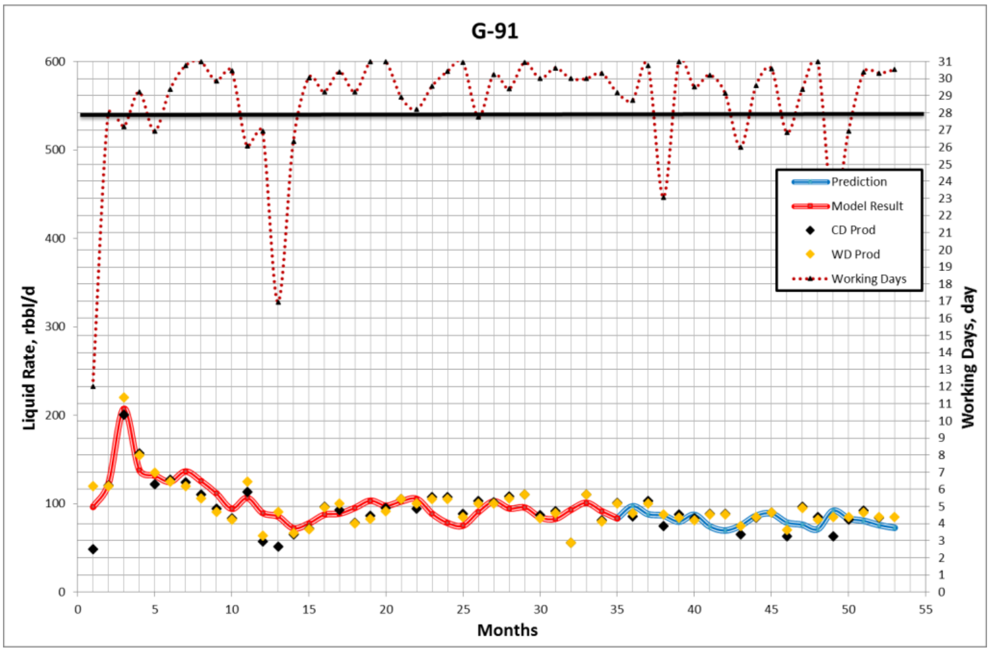


Figure 6.14 History Match and Forecast of G-91 Well

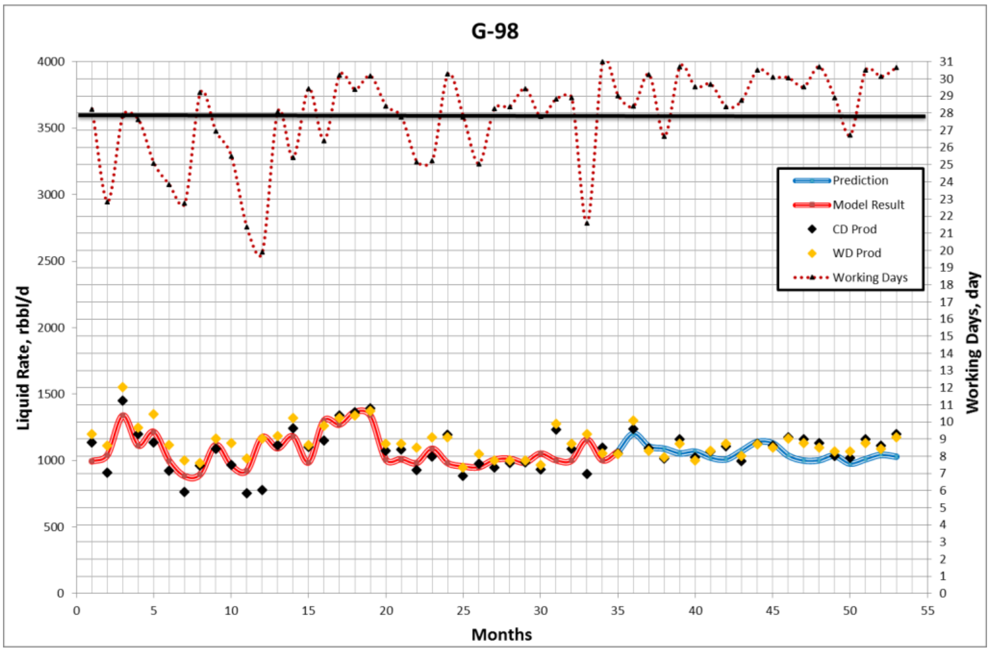


Figure 6.15 History Match and Forecast of G-98 Well

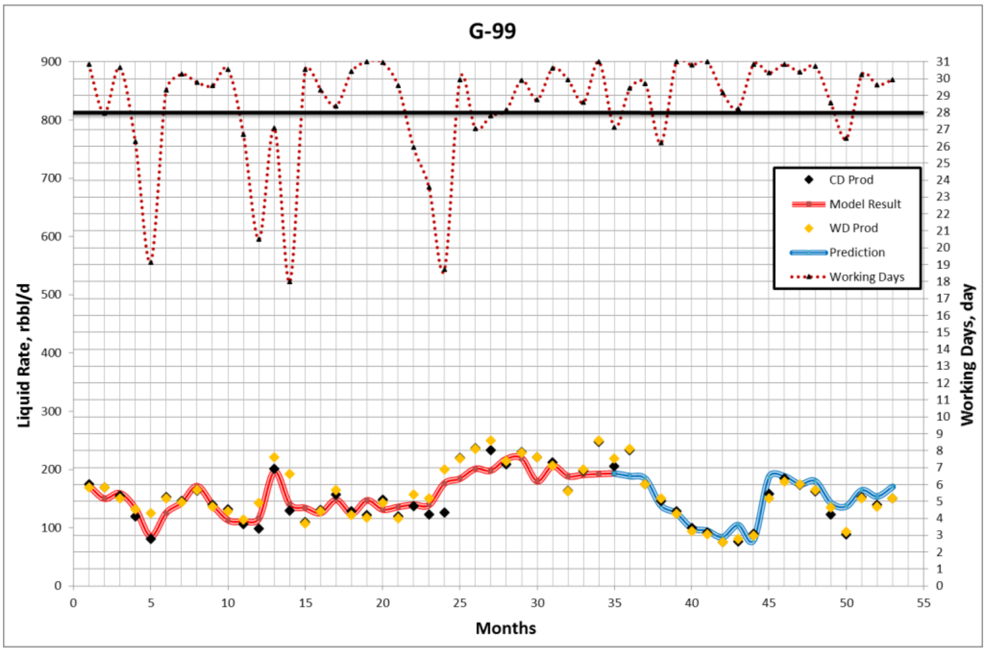


Figure 6.16 History Match and Forecast of G-99 Well

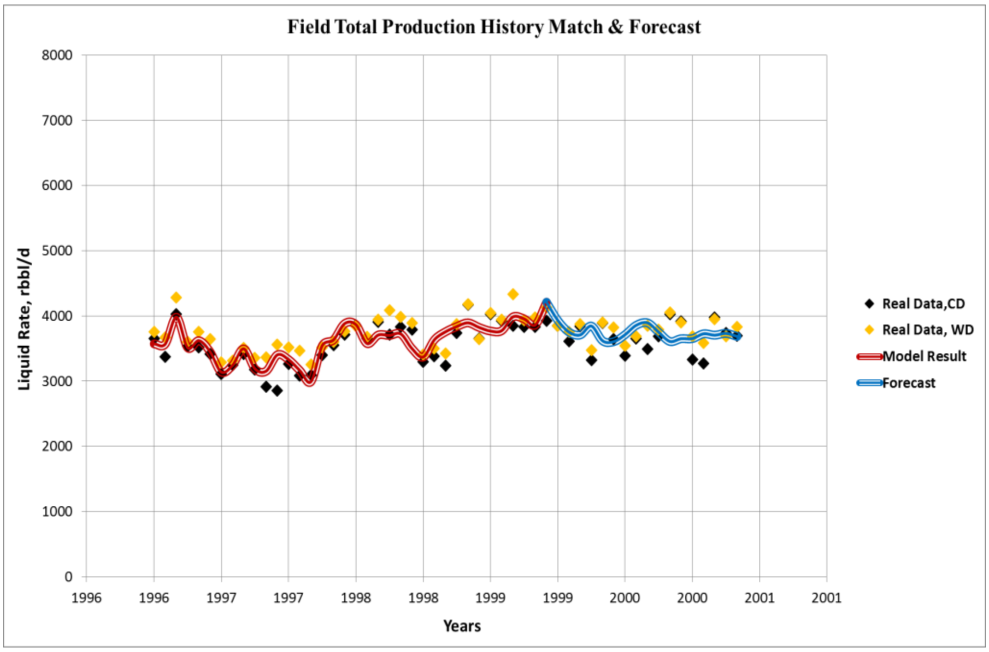


Figure 6.17 History Match and Forecast of the Field Production Data

The weight coefficients which were used to match the total production in wells are shown in Figure 6.18. As seen from the figure, there is a complex system including injectors affecting surrounding producers which will be discussed in next page.

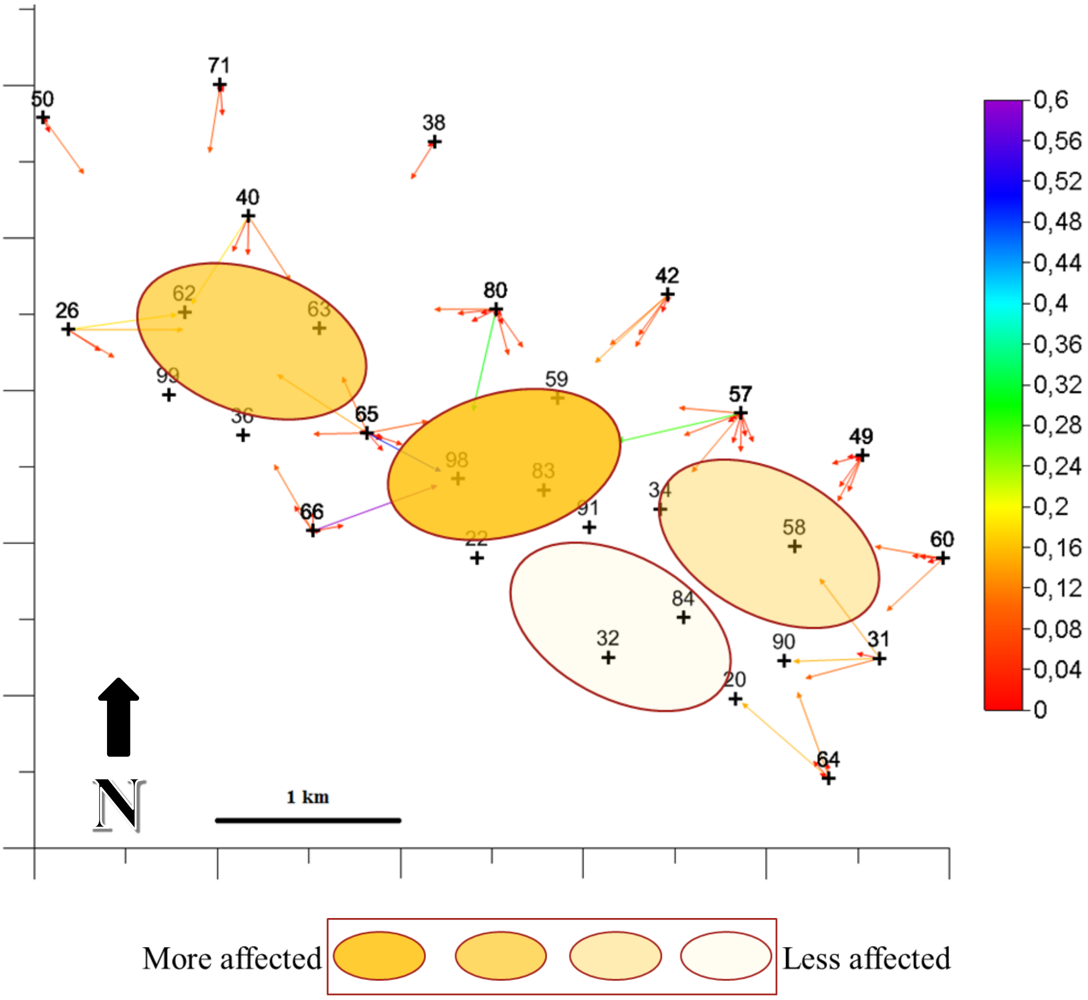


Figure 6.18 Weight Coefficients Between Injector-Producer Pairs

According to these results, all of the injectors have some interactions with surrounding producers. The weight coefficients change in a range between 0.1 and 0.6 which shows that the reservoir has regions showing different characteristics in terms of conductivity.

By comparing the weight coefficient values, the regions can be categorized in four parts depending on the degree of heterogeneity. As can be seen from graph, the most affected part by waterflooding is the middle of the field (near G-98 and G-83). Then it is followed by the northwest part (near G-62 and G-63) and east part (G-58) of the structure. The southeast part of the field is the least affected part which can be also validated by both production rates and water cuts in that region (to be explained in the following sections).

6.2 Analysis of Fitting Parameters: Weights and Time Constants

This part of the study explains the physical meanings of the fitting parameters and evaluates the results of the case study. There are two main parameters that characterize the interaction between production and injection wells; weight coefficients and time constants.

Weight Coefficients

Gentil (2005) explained the weight parameters by using transmissibility terms. By considering a single injector and producers connected by different flow paths (Figure 6.19);

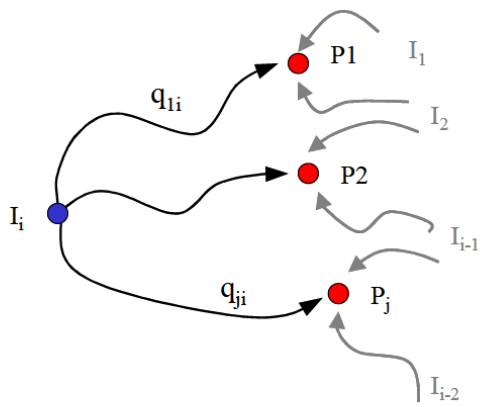


Figure 6.19 Schematic of the Interactions Between Injectors-Producers (Gentil, 2005)

It is possible to express weights in terms of rates;

$$\lambda_{ij} = \frac{q_{ij}}{I_i} \tag{6.1}$$

where,

q_{ij} is the contribution of the rate between an injector-producer pair

I_i is the total injection rate

By considering steady-state flow equations, Darcy's law can be applied for the parallel flow in the linear system:

$$q_{ij} = \frac{\overline{k_{ij}}\, A_{ij}\, (p_{wf}^{i} - p_{wf}^{j})}{\mu\, L_{ij}} = T_{ij}\, (p_{wf}^{i} - p_{wf}^{j}) \tag{6.2}$$

where,

$\overline{k_{ij}}$ is the average effective permeability

A_{ij} is the area open to flow

L_{ij} is the length of the path

μ is the viscosity of the field

p_{wf}^{i} *and* p_{wf}^{j} is the injection and producer flowing bottomhole pressures

T_{ij} is the transmissibility between injector *i* and producer *j*

Equation explaining the weights becomes:

$$\lambda_{ij} = \frac{T_{ij}\left(p_{wf}^{i} - p_{wf}^{j}\right)}{\sum T_{ij}\left(p_{wf}^{i} - p_{wf}^{j}\right)} \tag{6.3}$$

This shows that weights contain parameters related to rock and fluid properties like permeability and viscosity, flow geometry and operating conditions. This relation can give an idea about the investigated reservoir parameter if the others are known or can be approximated.

Referring to both Yousef et al. (2006) and Gentil (2005), there are three main outcomes of these explanations:

1) In a balanced waterflooding operation, summation of the λ coefficients should be equal to the one within a closed boundary system;

$$\sum_{i=1}^{I} \lambda_{ij} = 1 \tag{6.4}$$

2) In a homogenous balanced reservoir, there is correlation between λ coefficients and well locations & reservoir properties which usually exhibits an inverse relationship between the distance of pairs and corresponding weights;

$$\lambda_{ij} = \frac{\frac{1}{L_{ij}}}{\Sigma\frac{1}{L_{ij}}} \tag{6.5}$$

3) For the case of a symmetrical pattern, where the flow geometries are similar, the ratio of the weights for a given injector and two neighboring producers can be estimated by:

$$\frac{\lambda_{i1}}{\lambda_{i2}} = \frac{\frac{\overline{k_{\iota 1}}\,A_{i1}}{\mu\,L_{i1}}}{\frac{\overline{k_{\iota 2}}\,A_{i2}}{\mu\,L_{i2}}} = \frac{\overline{k_{\iota 1}}}{\overline{k_{\iota 2}}} \tag{6.6}$$

Time Constants

Yousef et al. (2006) explained that time constants as a function of total compressibility, pore volume and productivity index. The reservoir parameters such as porosity, and compressibility have important effects on these parameters. In addition to this, the distances between wells are indirectly related with time constants.

$$\tau_{ij} = \frac{c_{t_{ij}}\,V_{p_{ij}}}{J_{ij}} \tag{6.7}$$

The case studies analyzed by Yousef et al. (2006) showed that both time constants and weight coefficients do not reflect interaction between injectors. This does not mean that it is a general result for all cases, but it must be investigated for the cases.

Moreover, it was also stated that weights and time constants are not totally independent. λ_{ij} is directly proportional to the productivity index whereas the corresponding time constant is inversely proportional to the same productivity index. Thus, these two main parameters of the capacitance resistive model are inversely related. Log-log plot of these parameters shows an inverse relationship.

Yousef et al. (2006) and Delshad et al. (2009) used these plots to determine the possibility of fracture presence in porous media. In a homogenous media, drawing a log-log plot of weight coefficients and time constants results in a line with a slope of -1 while the parameters of heterogeneous system's parameters deviate from this line.

In this field case study, the mentioned outcomes were investigated to determine if the results are similar with the theoretical expectations. All the calculated weight coefficients and corresponding distances of pairs are plotted. As it can be seen from the Figure 6.20, there is a reasonable correlation between the mentioned parameters in most of the data. There are some deviations caused by the dataset related to the affected parts (regions where G-98 and G-62 are located). It can be concluded that there are some heterogeneous parts in the system but the rest of it shows similar characteristics.

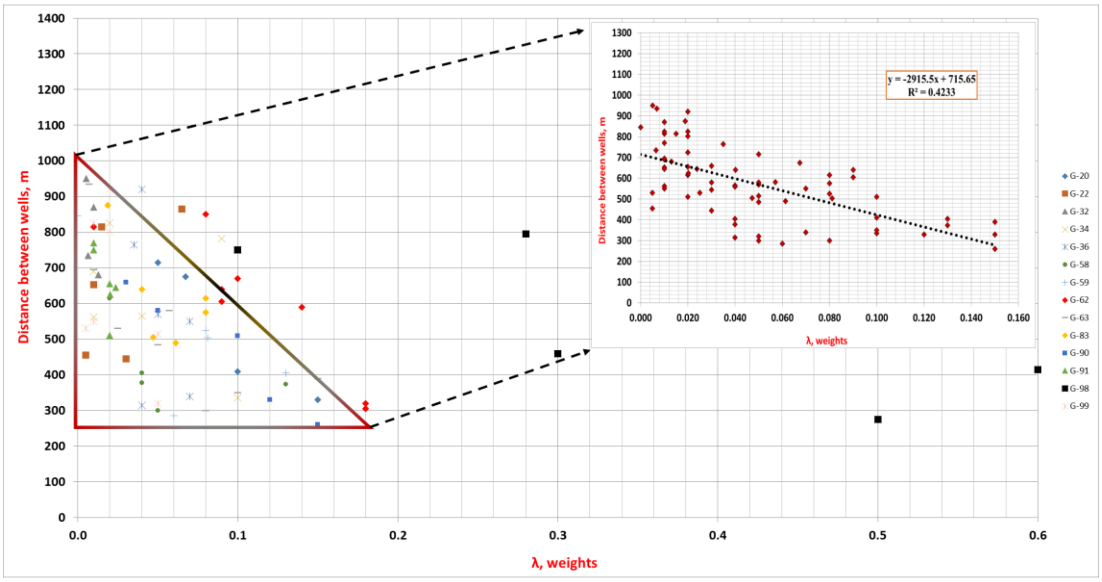


Figure 6.20 Weight Coefficients vs Distances Between Wells

As mentioned before, the weight coefficients and time constants are inversely related parameters because of the productivity index terms that they include. The weights and time constants (larger values than λ = 0.01) are plotted on a log-log plot to see this effect which resulted in as expected trend but more scattered (Figure 6.21). The main reason of these scatterings may be because of the insensitivity of these parameters (not sensitive as the weight coefficients). Little changes in these parameters do not affect the history match dramatically so it is possible to find a similar match with a theoretically expected weight coefficient – time constant relationship with some modifications.

The main idea behind this graph is about the heterogeneity of the reservoir. There are some outliers in these fitting parameters which would have been an indication of conductive zones. Some points out of this ellipse which are related to affected regions (regions where G-98, G-62 and G-58 are located) were also determined by the weight coefficient distributions.

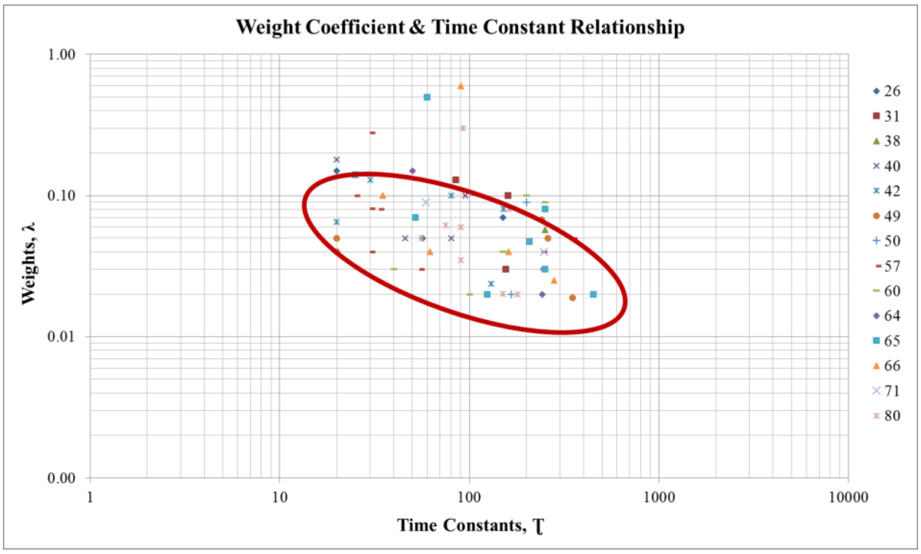


Figure 6.21 Weight Coefficients & Time Constants Relationship

To be confident about these fitting parameters, they must be investigated by using the other information groups coming from the static and dynamic data of the field. Starting from the weight coefficients, the results can be summarized as follows:

1) Almost all of the injectors are in communication with more than one producer and there are no big differences between weight coefficients except the ones in the region which G-98 and G-62 are located. These coefficients are related to high-conductive zones near these producers.

2) The weight coefficients change in a range between 0.01 and 0.6 which shows a log-normal distribution with a mean of 0.07 (Figure 6.22). This analysis indicates that there are some heterogeneous regions in which high conductive corridors dominate the flow paths. Not only by statistical analysis but also log-log plot of the weight coefficients and time constants resulted in similar outcomes.

Statistical Analysis	
Mean	0.071394243
Standard Error	0.010183053
Median	0.047204872
Mode	0.01
Standard Deviation	0.093883106
Sample Variance	0.008814038
Kurtosis	16.19052729
Skewness	3.624415187
Range	0.595
Minimum	0.005
Maximum	0.6
Sum	6.068510654
Count	85

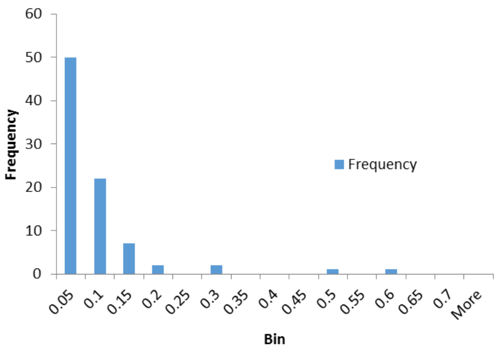


Figure 6.22 Statistical Analysis of Weight Coefficients

3) The time constants change in a range between 1 and 650 and show a log-normal distribution with a mean of 142 (Figure 6.23). This analysis shows that most of the time constant values fall into low interval which indicates a fast signal-response effect. This result can explain the existence of preferred flow paths of injected water and observed unexpected high water cuts in the system.

Statistical Analysis	
Mean	142.7468999
Standard Error	14.70870701
Median	92.85330722
Mode	150
Standard Deviation	135.6075782
Sample Variance	18389.41527
Kurtosis	3.631620985
Skewness	1.735108101
Range	649.0831156
Minimum	0.916884447
Maximum	650
Sum	12133.48649
Count	85

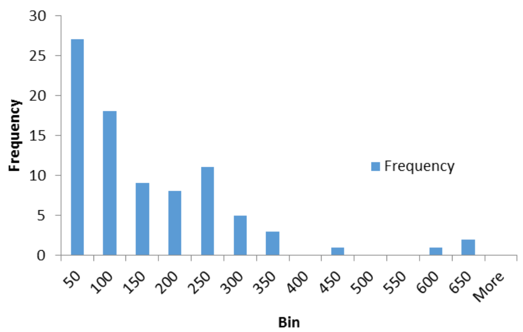


Figure 6.23 Statistical Analysis of Time Constants

4) If sum of the weight coefficients of each injector are investigated, it can be seen that they (except G-65 which is located in the center of the wells) are lower than unity which means injection outside of the drainage area of the producers or the reservoir limits (Table 6.2).

This is a common outcome of a peripheral waterflooding operation. Specifically for this field, taking the total injection and production volumes into account, almost 1.12 times amount of produced liquid (oil + water) equivalent low salinity water was already injected into the reservoir by the beginning of the year 1996. Thus, it is not surprising to see that some of the injected water leaks out of the reservoir. The total loss in the injection rates calculated by using the weight coefficients and rates is equal to 60% of the injected water. 32% of this injection rate losses is directly related to three wells (G-38, G-50 and G-40) located in the north west of the field.

Table 6.2 Weight Coefficient Table Showing the Calculation Results

Λij	G-20	G-22	G-32	G-34	G-36	G-58	G-59	G-62	G-63	G-83	G-84	G-90	G-91	G-98	G-99	Sum Λij
G-26					0.07			0.18	0.15						0.05	0.45
G-31	0.10					0.13					0.03	0.15				0.41
G-38								0.01	0.06							0.07
G-40					0.05			0.18	0.10						0.05	0.38
G-42		0.07		0.01			0.13			0.08			0.02	0.10		0.41
G-49	0.07		0.01	0.04		0.05				0.02	0.01	0.05	0.01			0.26
G-50								0.09	0.01						0.02	0.12
G-57	0.05	0.02	0.01	0.10		0.04	0.08			0.08	0.04	0.03	0.02	0.28		0.74
G-60			0.01	0.09		0.04					0.02	0.10	0.03			0.29
G-64	0.15		0.01			0.02					0.03	0.12				0.34
G-65		0.03		0.02	0.07		0.08	0.14	0.08	0.05			0.02	0.50	0.01	1.00
G-66		0.01			0.04			0.10	0.03	0.04			0.01	0.60	0.01	0.83
G-71					0.04			0.09	0.01						0.01	0.15
G-80		0.01		0.01	0.04		0.06	0.08	0.05	0.06			0.02	0.30	0.02	0.65

5) Most of the wells located in the north-west region are not effective as the others. The main reason of this situation can be explained by taking the past injection times into account. As can be seen from the Figure 6.24, there are interwell connectivity plots of both the early time water breakthroughs period in 1960s and the situation in the beginning of the year 1996. Because of the

timing and the pattern change, they do not reflect exactly the same picture but the flow paths are all in agreement with each other.

Up to the year of 1996, most of the water was injected from G-38, G-42, G-50, G-60 and G-61. In the working periods of analysis, G-38 and G-50 continued to inject the highest rates in the field but this was not same for the other wells. Because of the cumulative water injection from these wells and their locations (outside part), it can be expected to observe lower contributions compared to the wells located in other regions.

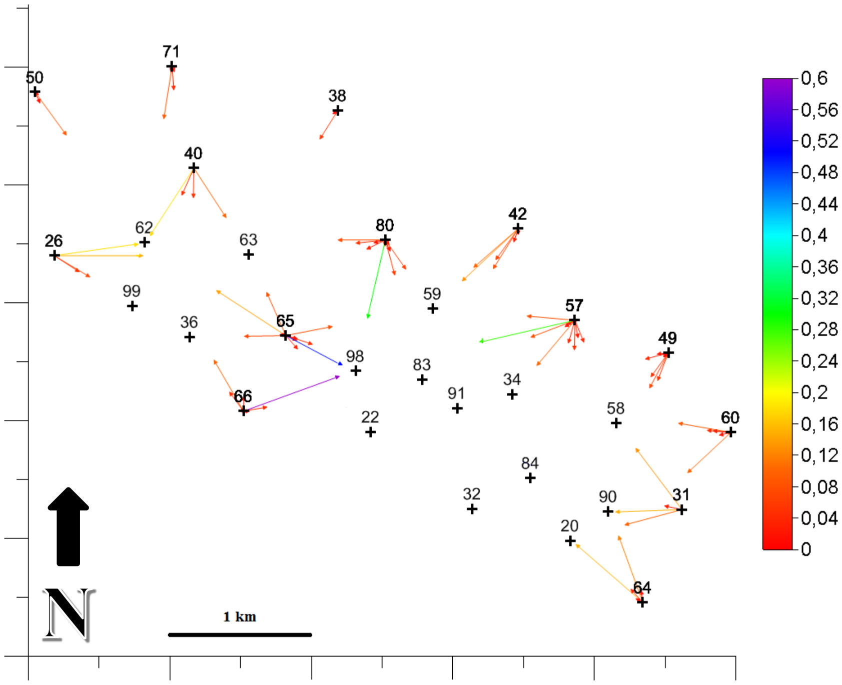
At this point, the injection rates and weights coefficients must be distinguished from each other because the main production responses are related to the injection rates not the weight coefficients.

6) To compare the weight coefficients and time constants with the expected flow directions coming from pressure analysis, initial liquid in place and cumulative volumes were used. This method is used because of the absence of the static reservoir pressure data in the working time period.

   First of all, by using the petrophysical data, original liquid in place has been distributed in 2D by krigging method. Then, again by using the same method, cumulative injection and liquid production distributions were generated. Based on some assumptions like closed system, homogenous displacement and constant compressibility, grid based distributions were used in calculations;

   Liquid in Place = Initial Liquid in Place – Liquid Production + Injection

   The result of material balance approximations are shown in Figure 6.25, which can be used as the indirect pressure distribution of the field just before the history match working period. When they are overlapped with the current weight coefficients, it shows an acceptable relationship with the main flow directions and the regions where injection losses observed (Figure 6.26).



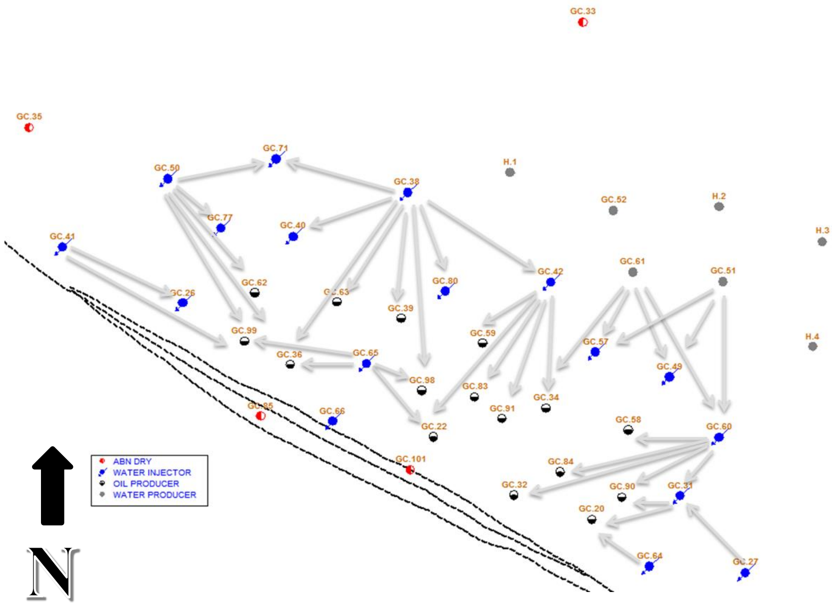


Figure 6.24 Comparison of the Initial Water Breakthroughs and CRM Results

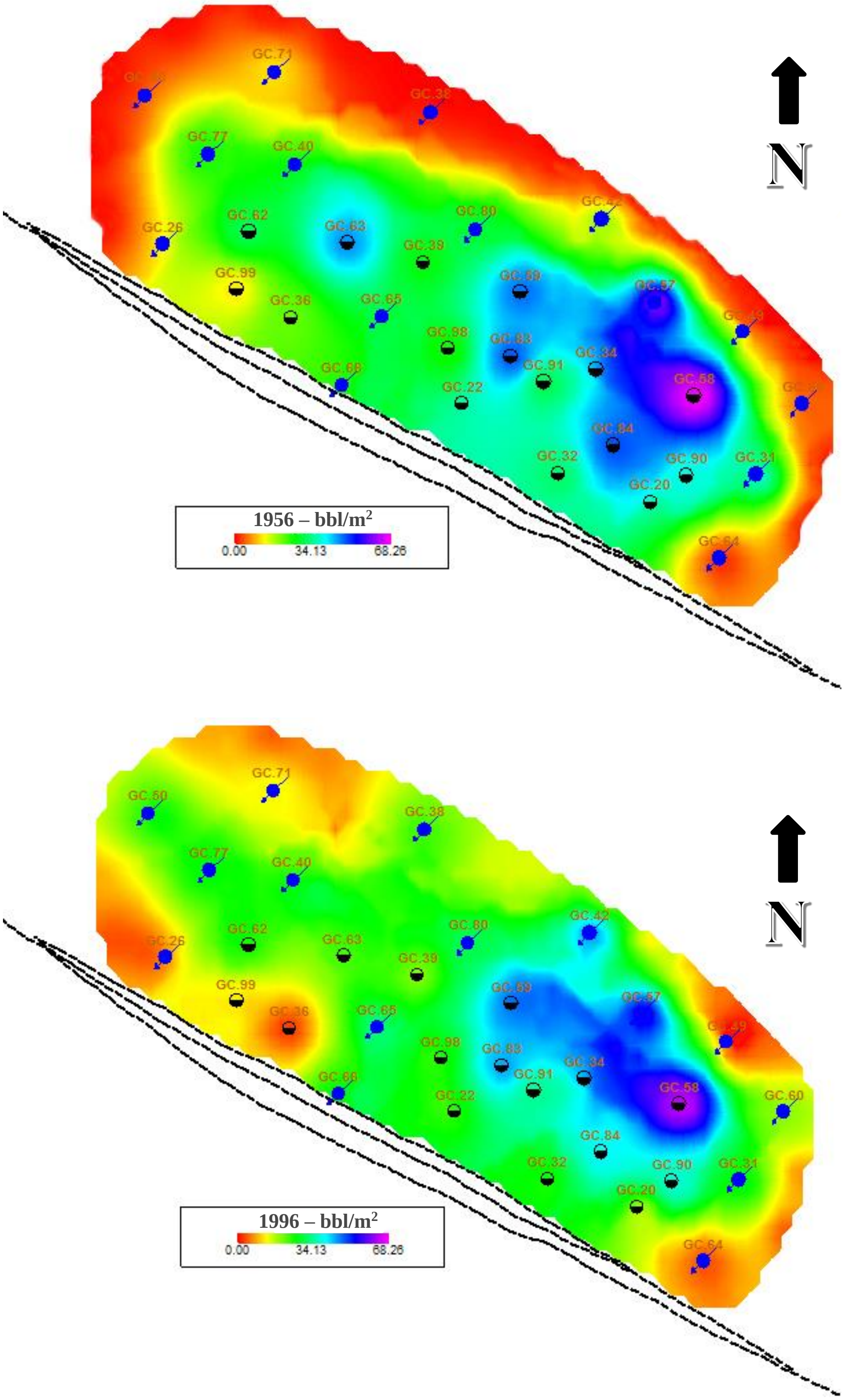


Figure 6.25 Comparison of Total Liquid in Place in 1956 & 1996

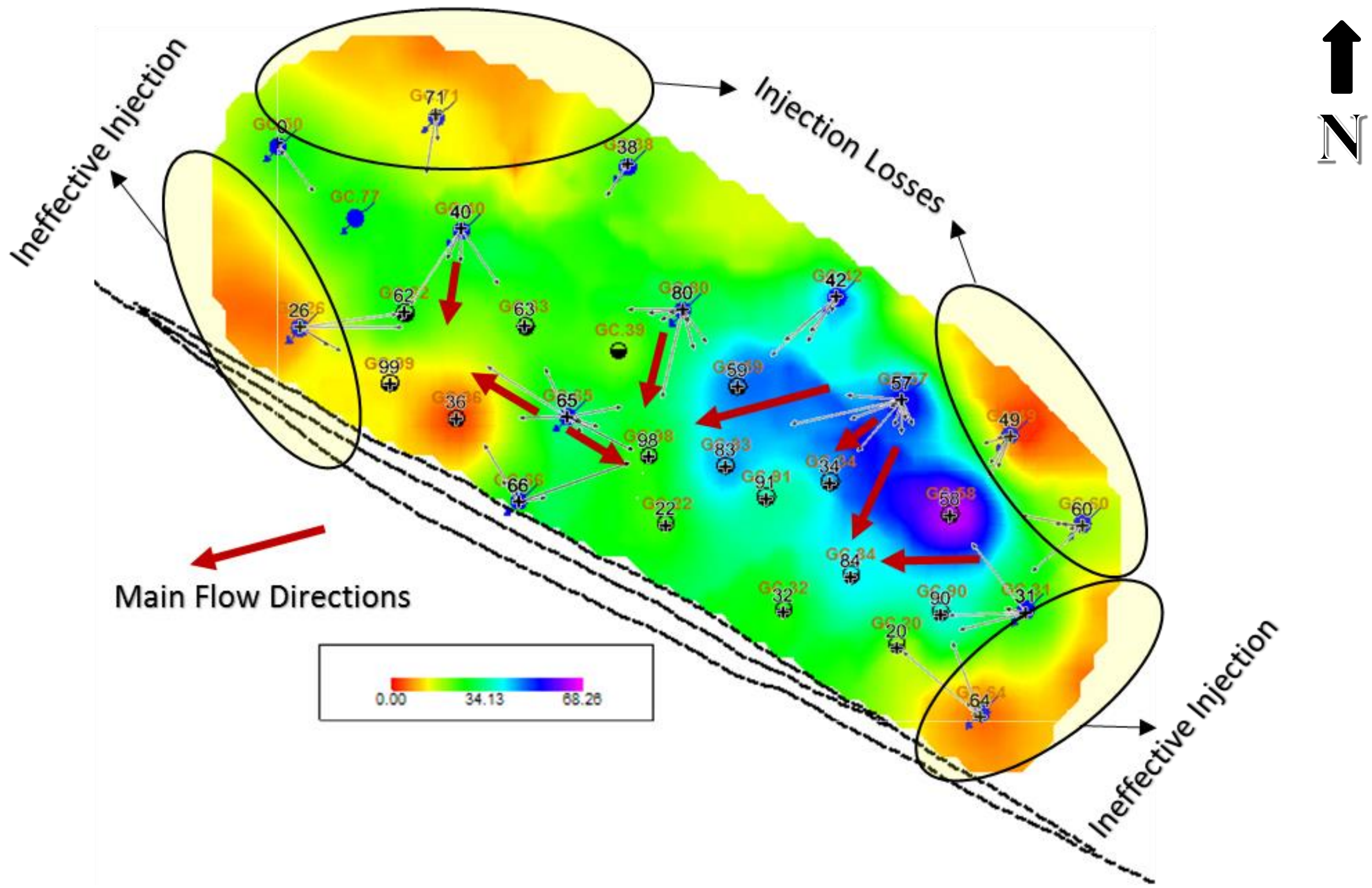


Figure 6.26 Overlapped Map of Weight Coefficients & LIP Distribution in 1996

In addition to pressure difference, permeability distribution of the system is also very important to determine the preferred flow paths in the reservoir. By using the core and DST permeabilities, a vector map (*the darker the arrow the more permeable direction*) was generated throughout the field to compare with the calculated weight coefficients (Figure 6.27).

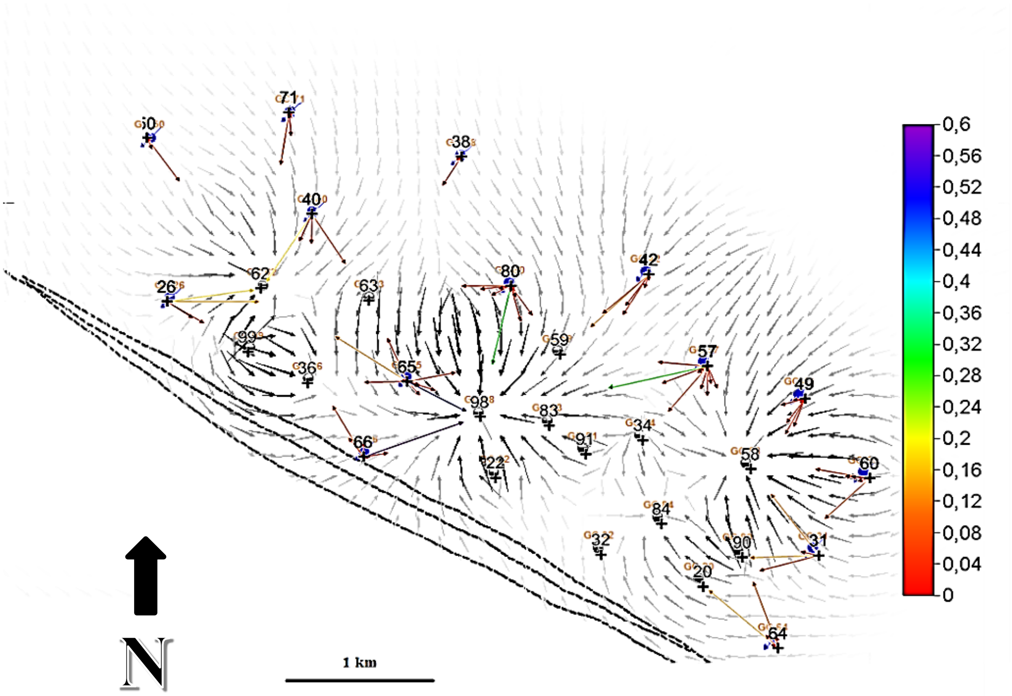


Figure 6.27 Permeability Vector Map vs Weight Coefficients

To generate a representative permeability vector map and correctly define the flow paths, average permeabilities of the wells are used in distributions. As it can be seen from the figure, the weight directions and permeability vectors are in agreement and represent the main flow directions.

There are mainly three parts representing the conductive regions of the reservoir in which G-62, G-98 and G-58 wells are located. Another important conclusion is the irresponsive parts (near G-32, G-84, G-36 production wells) of the reservoir which show low production rates and water cuts compared the other producers. As a result of this comparison, it was realized that the weight coefficients and the permeability trends are strongly in agreement.

Water cut distribution also shows less waterflooding effects in some regions which are also in agreement with the main displacement directions analyzed from pressure and permeability distributions. As can be seen from Figure 6.28, the wells with lower water cuts are located close to the reverse fault in the south east and west part of the field where the injected water prefers to flow according to the model results.

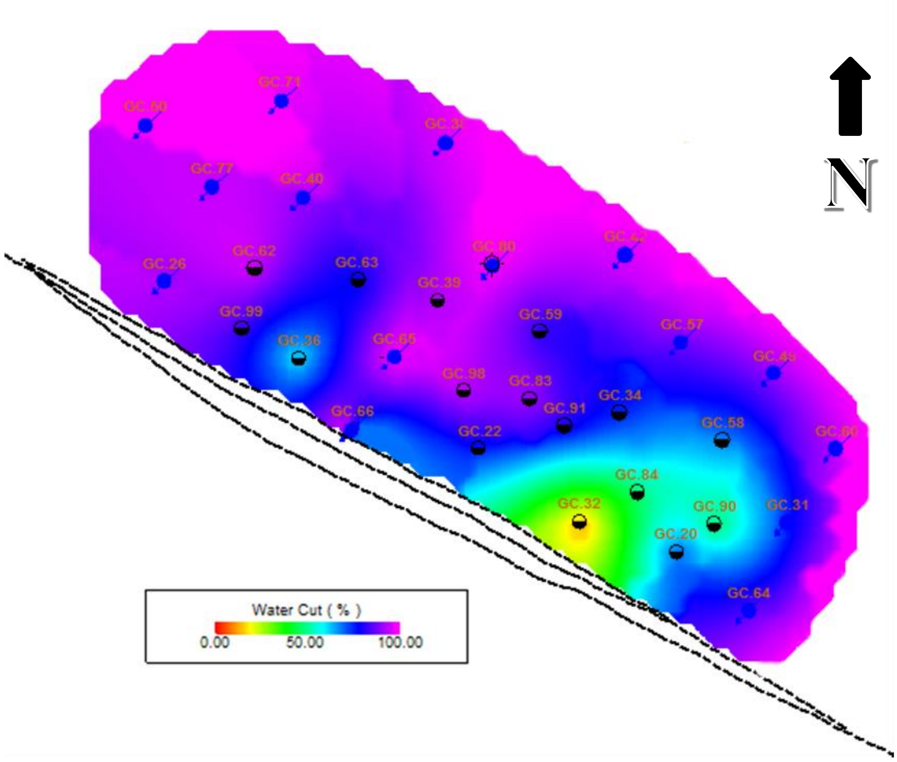


Figure 6.28 Water Cut Distribution of the Wells in the Year of 1996

7) Time constants were investigated by comparing with the relative distances between well pairs and pore volume distributions. There is not just a direct relationship between pore volume and time constant but also there is a productivity index term to be taken into account. That is why it is not very easy to explain all time constants on a just single map. Instead of that way, time constants of the same injector with their pairs must be investigated internally. Main conclusion from Figure 6.29 is that the time constants are larger where there is a large pore volume, larger distance and poor well connectivity.

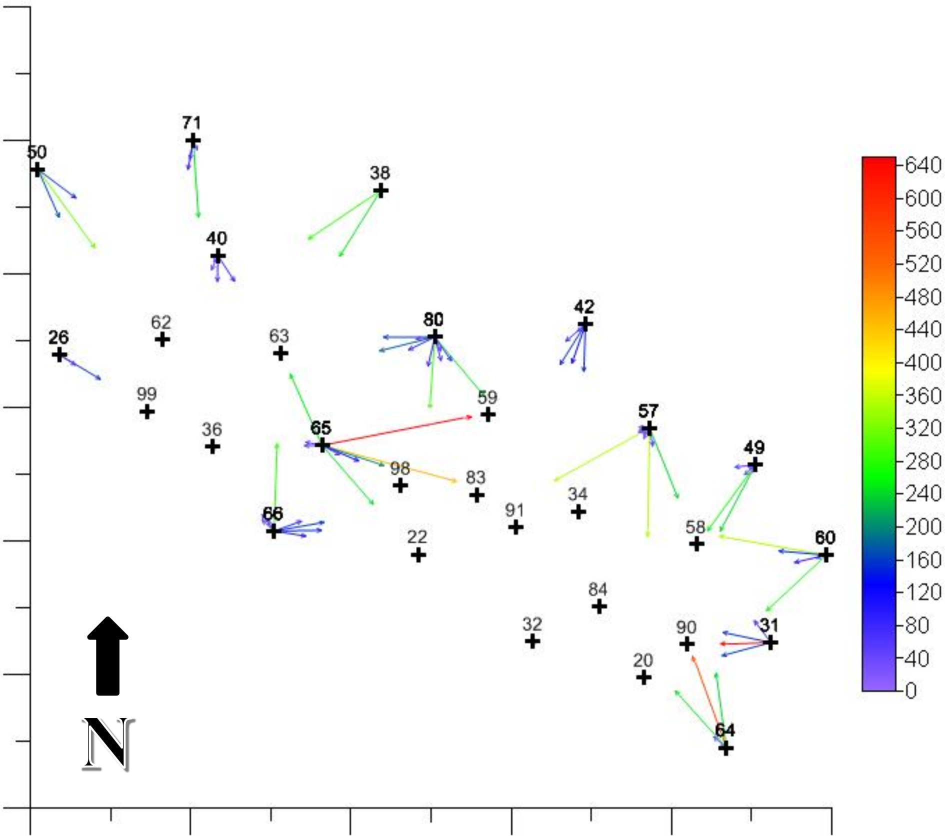


Figure 6.29 Time Constants Between Injector-Producer Pairs

8) Pressure coefficient data showed that all of the wells have pressure effects on itself and the nearby producers. According to the analysis (Figure 6.30), most of the pressures coefficients fall into -0.15 to 0.1 interval which means moderate effects on production. Outliers in this data set belongs to the well pairs of G-98 whose production is highly dependent on the surrounding wells.

Statistical Analysis	
Mean	-0.03994
Standard Error	0.010478
Median	-0.03
Mode	-0.02
Standard Deviation	0.085764
Sample Variance	0.007356
Kurtosis	5.712729
Skewness	-0.87834
Range	0.65
Minimum	-0.4
Maximum	0.25
Sum	-2.67617
Count	67

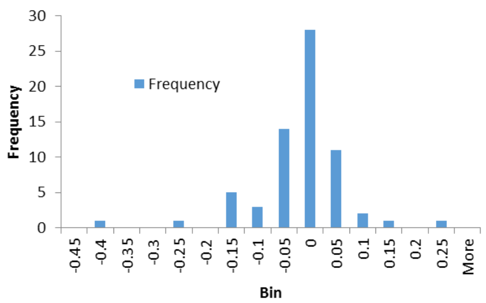


Figure 6.30 Statistical Analysis of Pressure Coefficients

6.3 Fractional Oil Flow Match

Kazakov (1976) model is the selected oil fractional flow model which fits the water cut and oil production trend very well within the optimization period (2000-2010). Remembering the model once more, the relationship between cumulative oil production and oil cut can be stated as follows;

$$\frac{q_o}{q} = f_o = \frac{(1-b\,N_p)^2}{(a-2bN_p)(1-b\,N_p)+N_p\,b\,(a-b\,N_p)} \qquad (6.8)$$

N_p = cumulative oil production, bbl – m^3

a, b = constants

To find these constants, history matching procedure was applied by using nonlinear regression. Even the early time trends (before 1992) are not very well matched in all wells because of the changes in field conditions, it is acceptable to be used for optimization period. Table 6.3 shows the calculated constants and the average matching errors for different wells between the years of 2000-2015. When these constants are plotted on a graph (Figure 6.31), it can be observed that almost all wells except G-62, G83 and G-98 have similar water cut increase trends which also honors the high conductive zones determined by CRM.

Table 6.3 Calculated Constants of the Oil Fractional Flow Model

Well	a	b	% Error (2000-2015 years)
G-20	1.24	4.2E-07	4.31
G-22	1.20	8.7E-07	2.82
G-32	1.03	8.6E-07	9.72
G-34	1.40	1.65E-06	2.36
G-36	1.25	5.74E-07	3.69
G-58	1.80	7.71E-07	3.56
G-59	1.92	1.4E-06	1.89
G-62	8.50	4.5E-06	0.4
G-63	1.20	7.13E-07	1.38
G-83	6.15	1E-06	0.16
G-84	1.18	7.43E-07	9.44
G-90	1.55	1.32E-06	3.19
G-91	2.99	2.91E-06	2.88
G-98	13.00	2.8E-06	0.18
G-99	3.80	4.00E-06	1.35

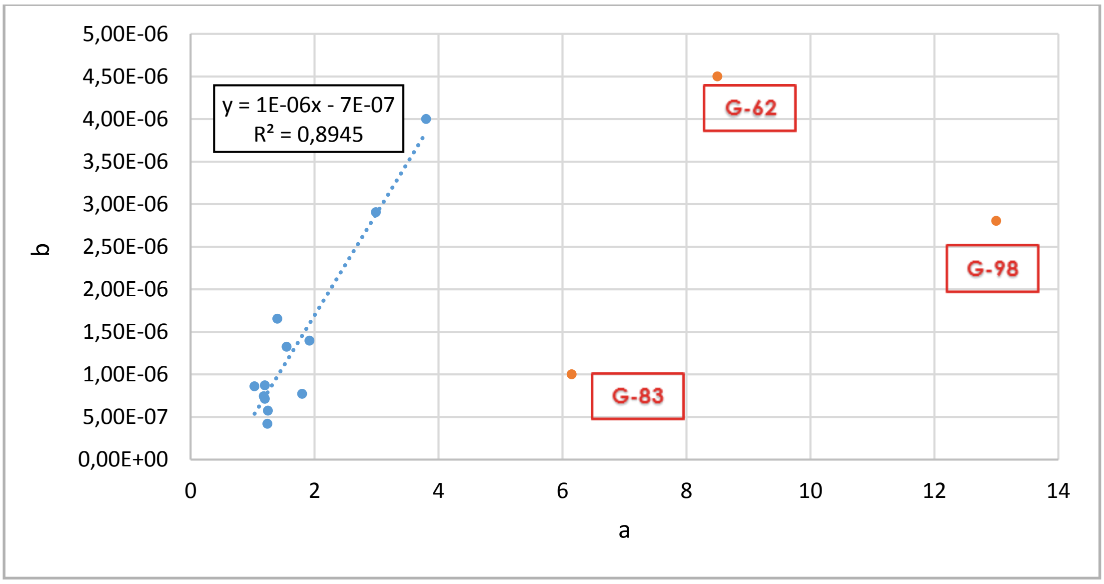


Figure 6.31 Obtained Model Constants and Water Cut Increase Profiles

Fractional flow model match results can be seen from Figure 6.32 through Figure 6.46.

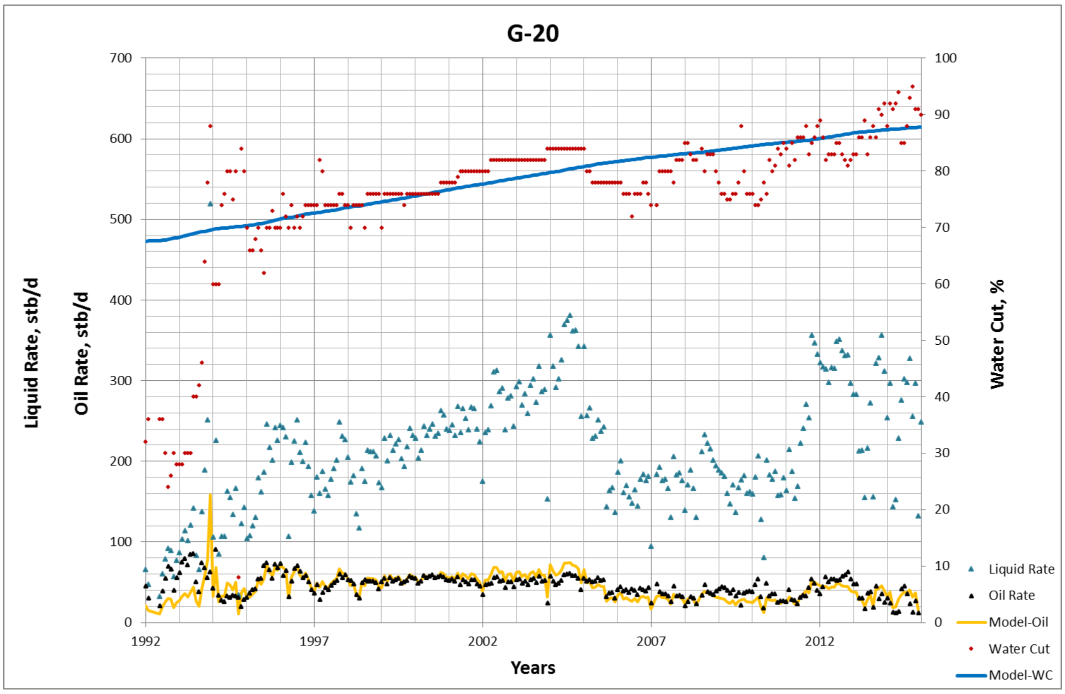


Figure 6.32 Oil Fractional Flow Match of G-20

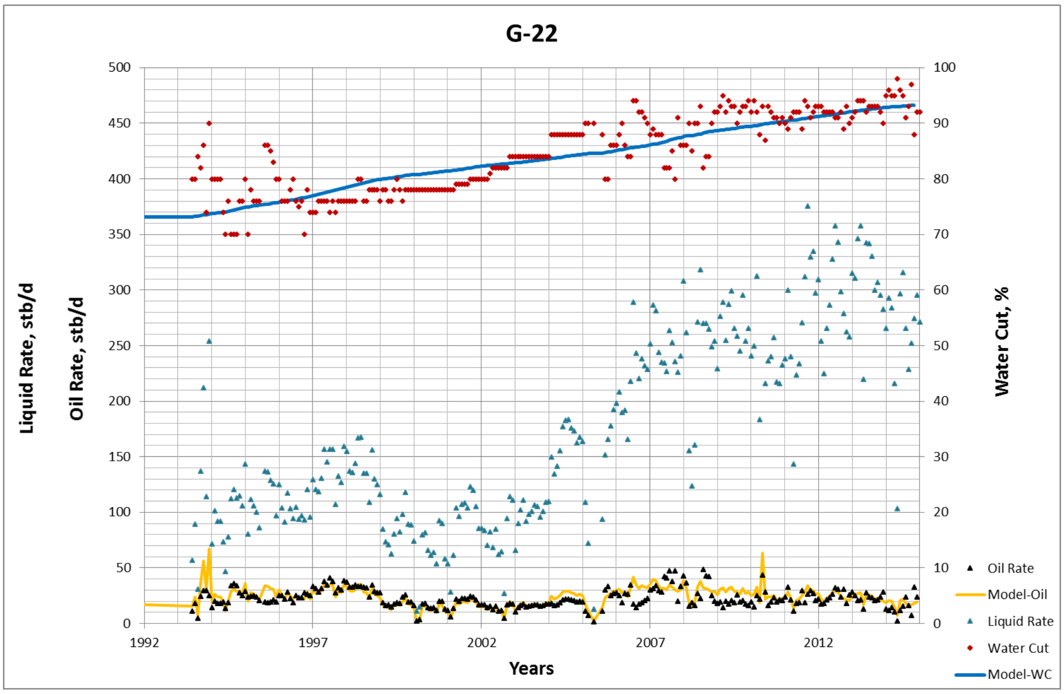


Figure 6.33 Oil Fractional Flow Match of G-22

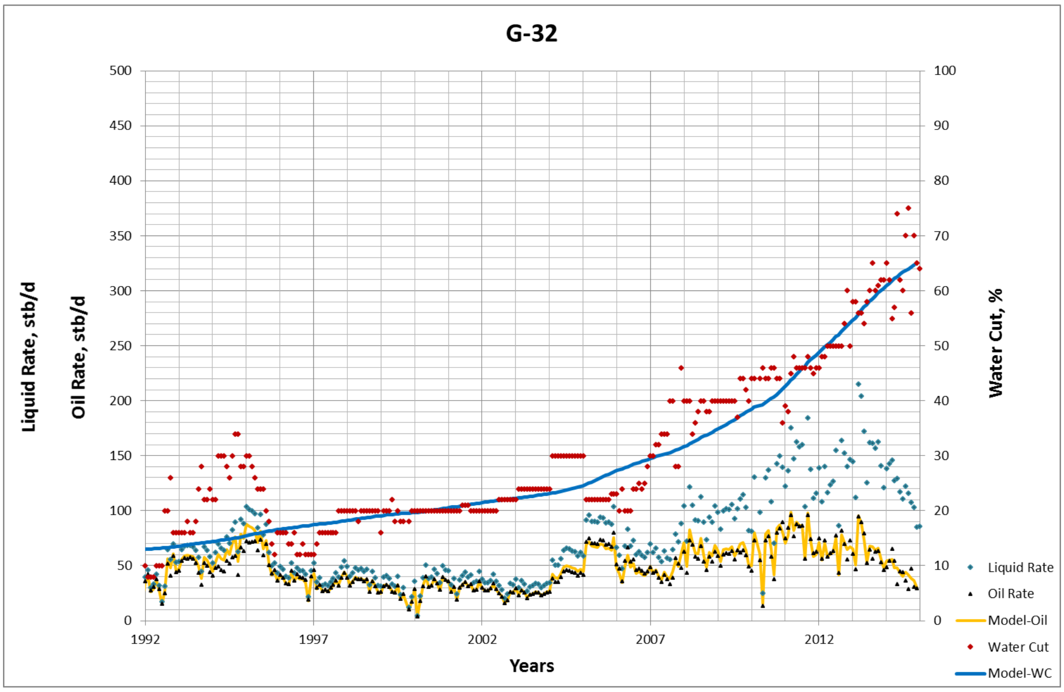


Figure 6.34 Oil Fractional Flow Match of G-32

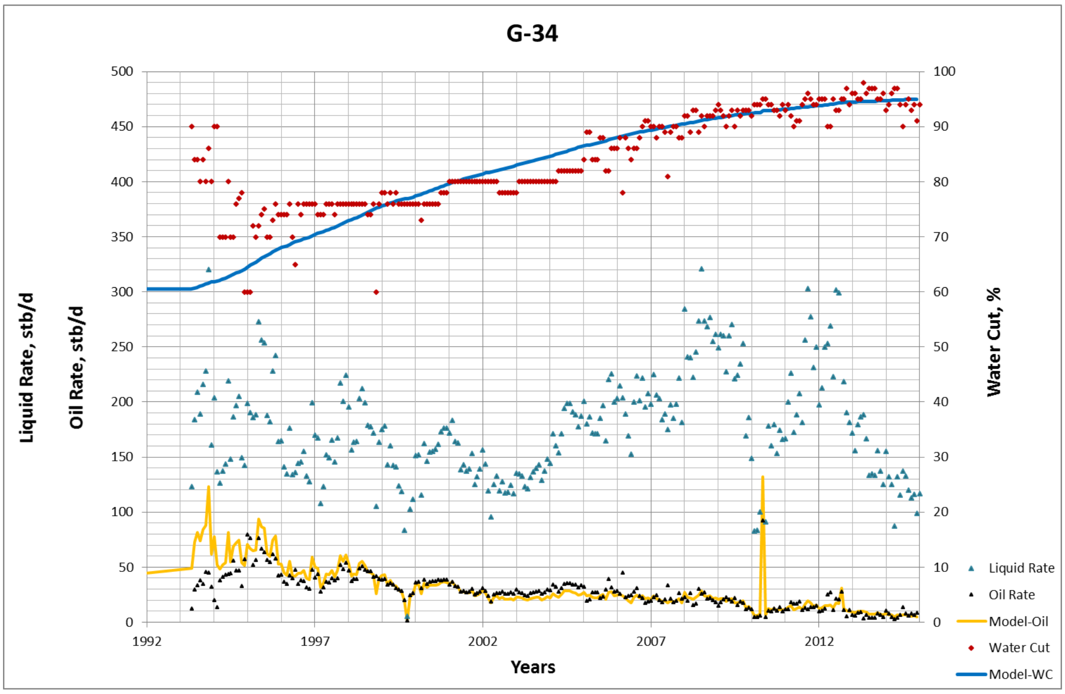


Figure 6.35 Oil Fractional Flow Match of G-34

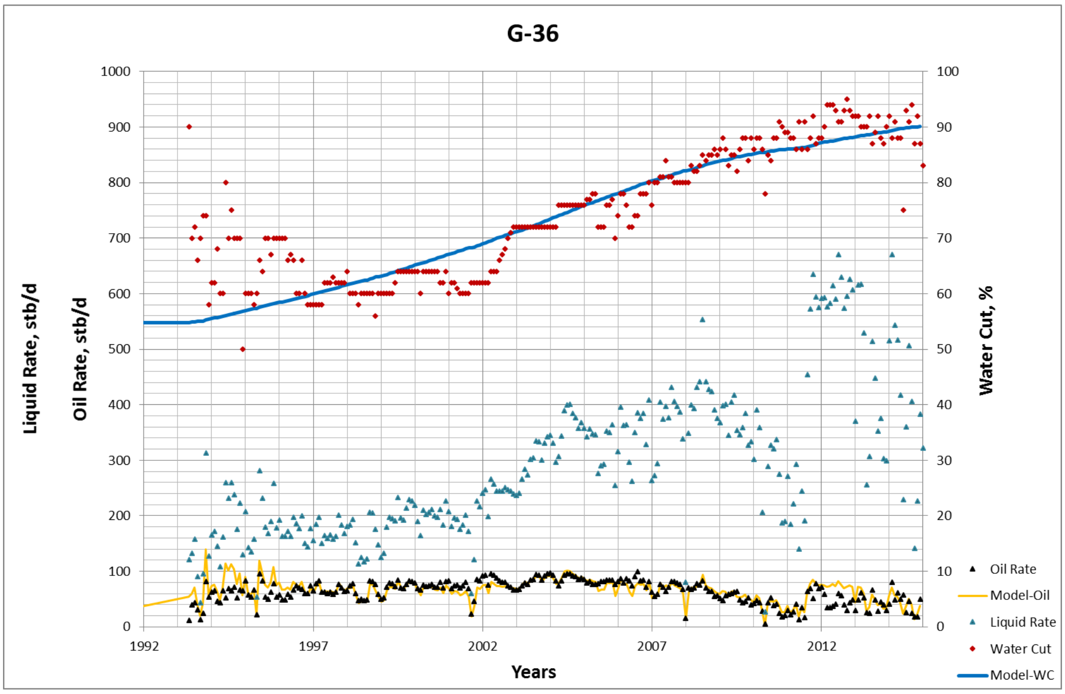


Figure 6.36 Oil Fractional Flow Match of G-36

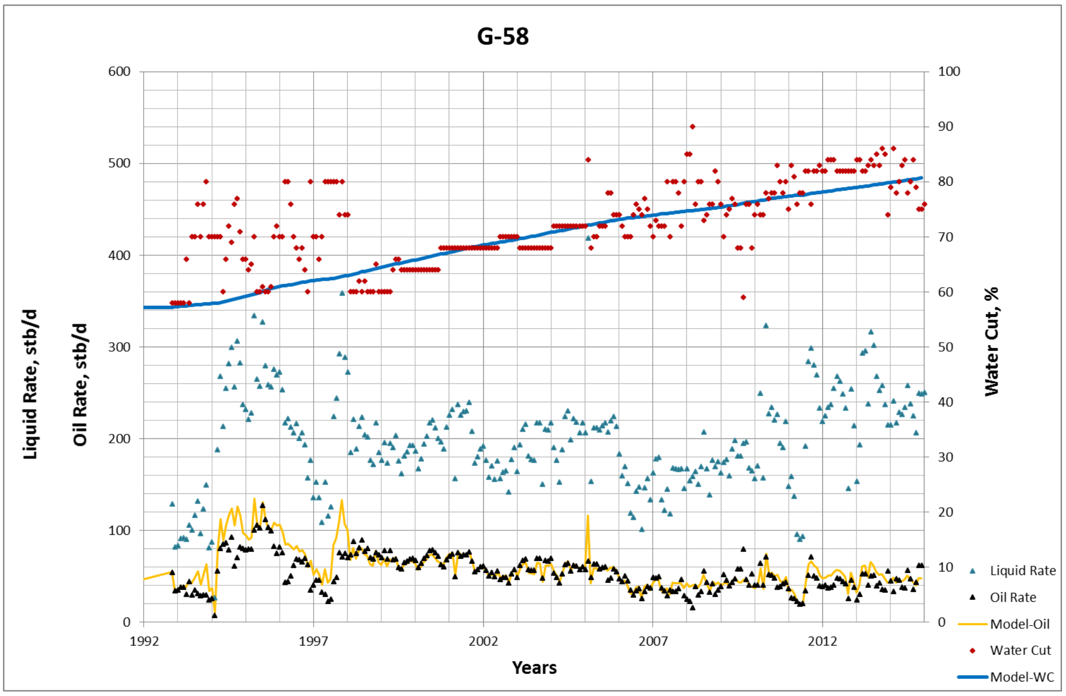


Figure 6.37 Oil Fractional Flow Match of G-58

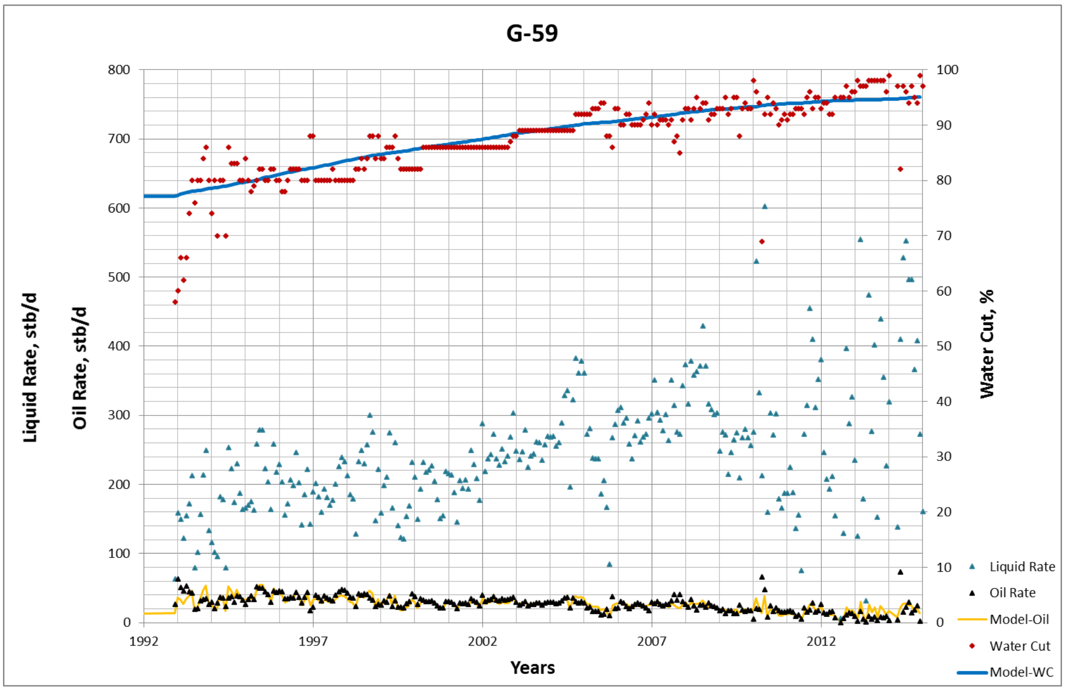


Figure 6.38 Oil Fractional Flow Match of G-59

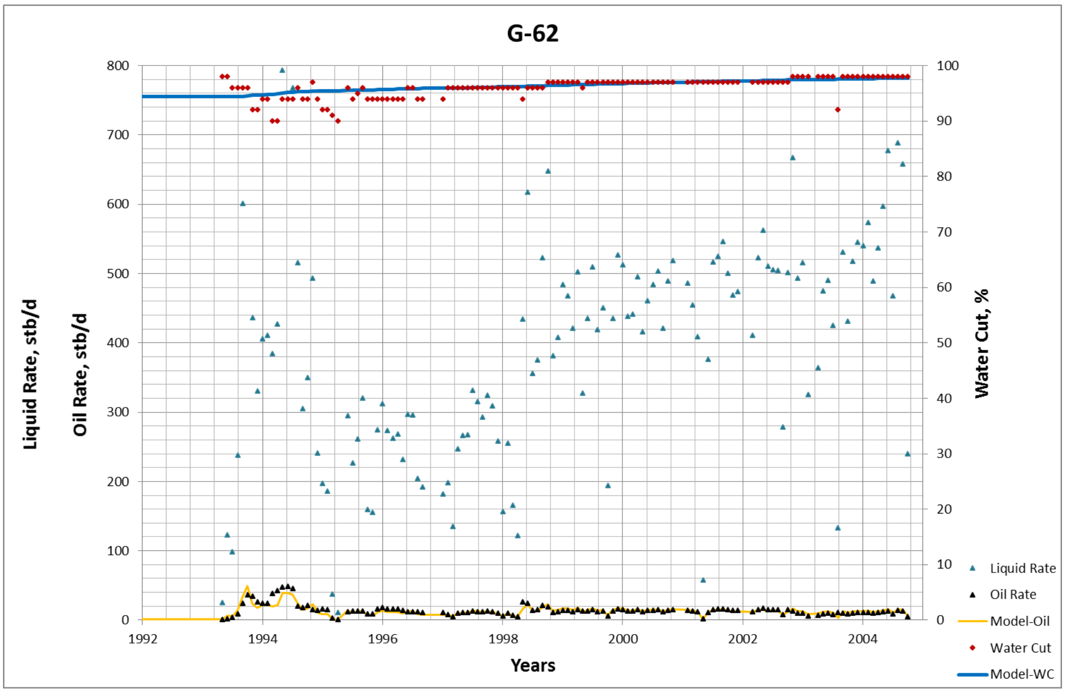


Figure 6.39 Oil Fractional Flow Match of G-62

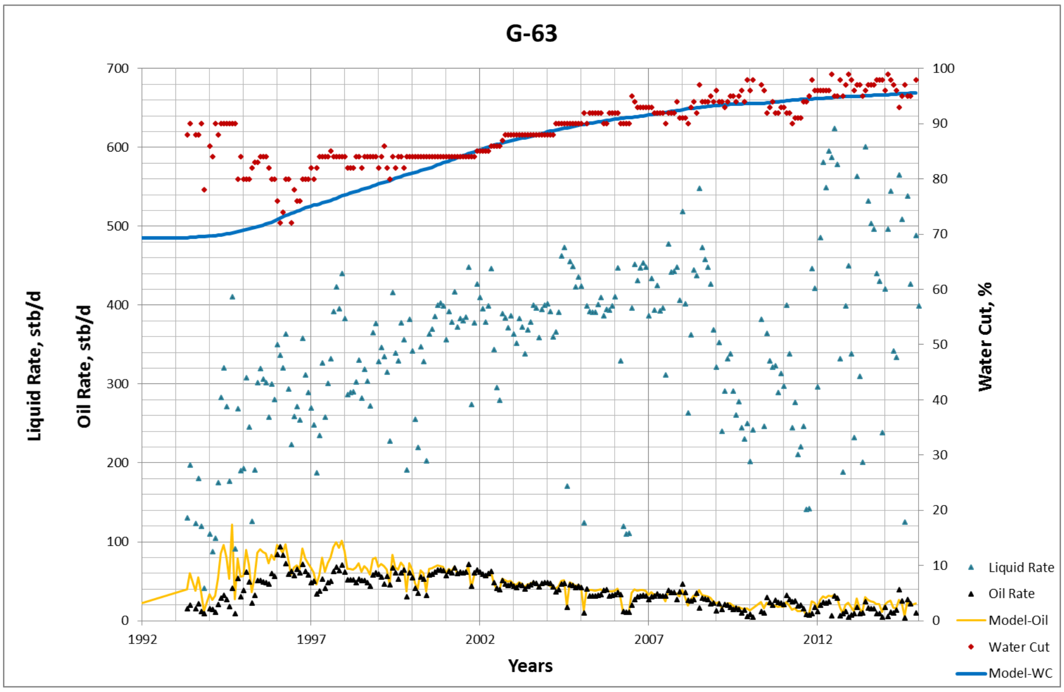


Figure 6.40 Oil Fractional Flow Match of G-63

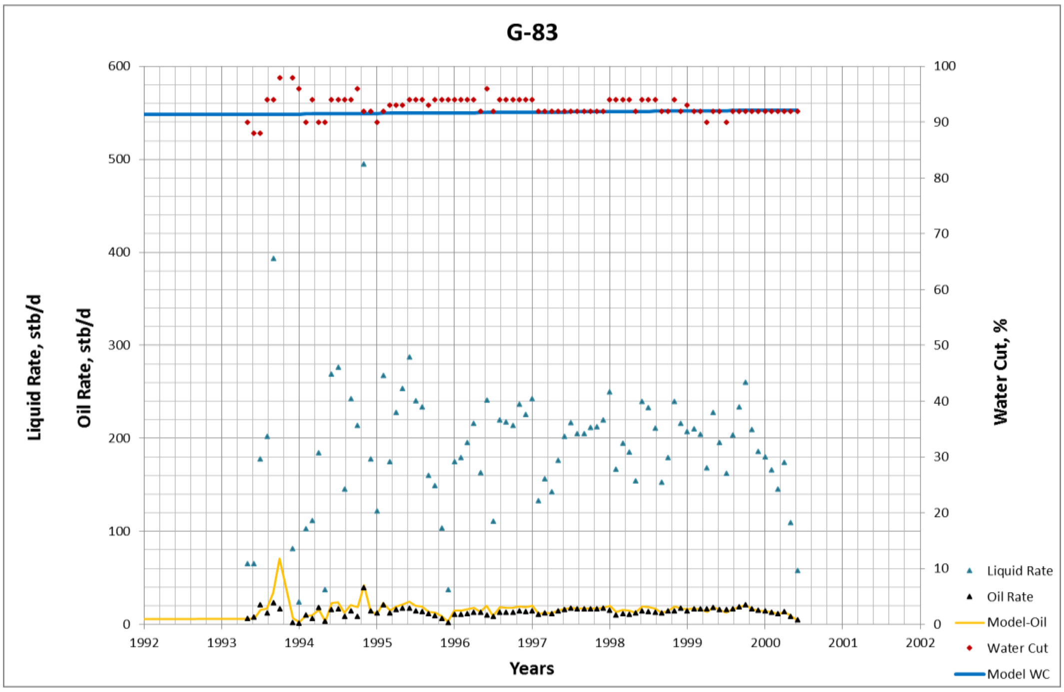


Figure 6.41 Oil Fractional Flow Match of G-83

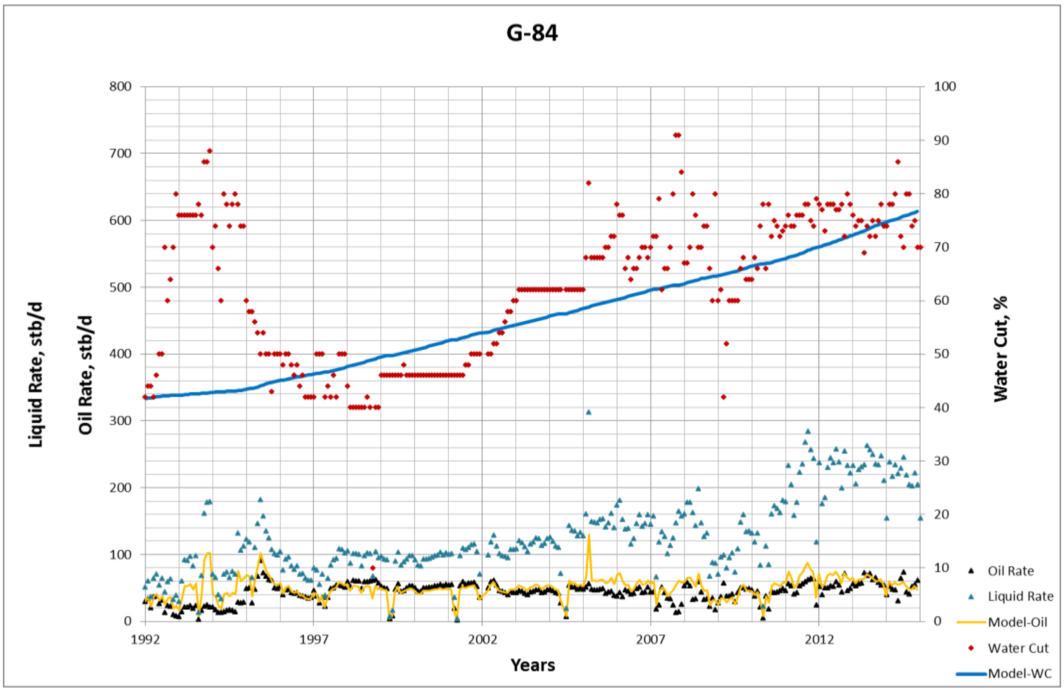


Figure 6.42 Oil Fractional Flow Match of G-84

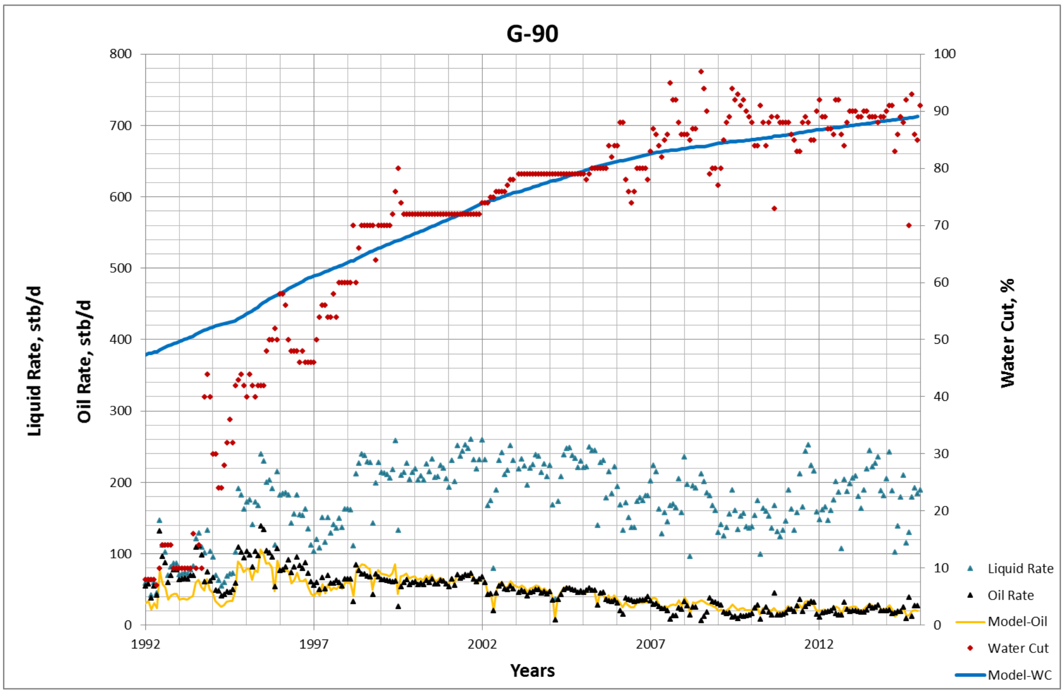


Figure 6.43 Oil Fractional Flow Match of G-90

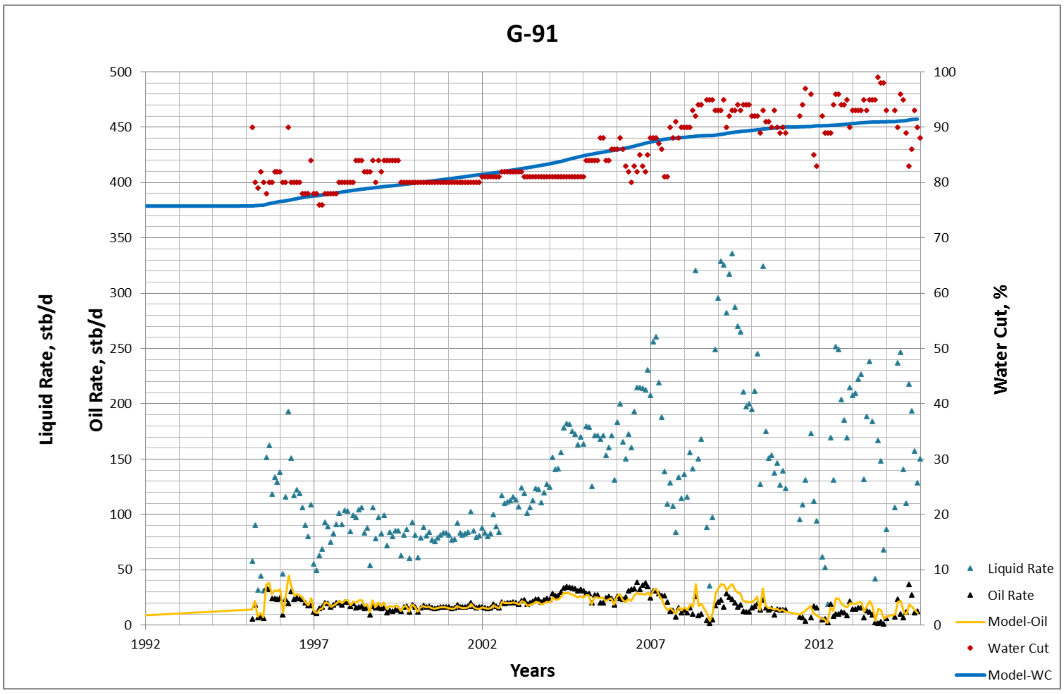


Figure 6.44 Oil Fractional Flow Match of G-91

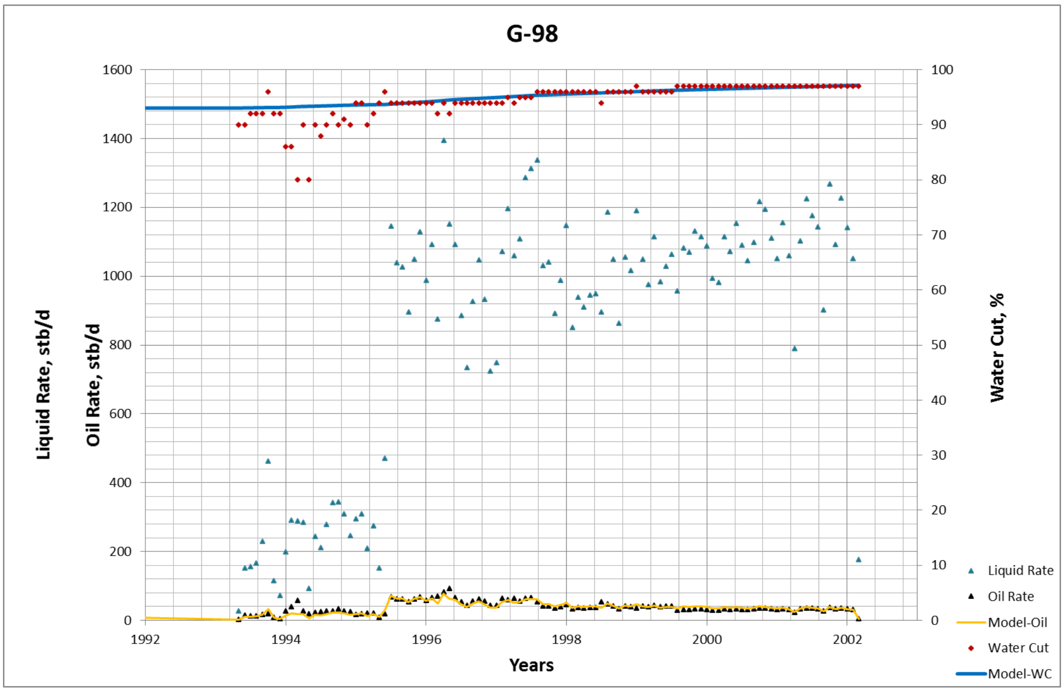


Figure 6.45 Oil Fractional Flow Match of G-98

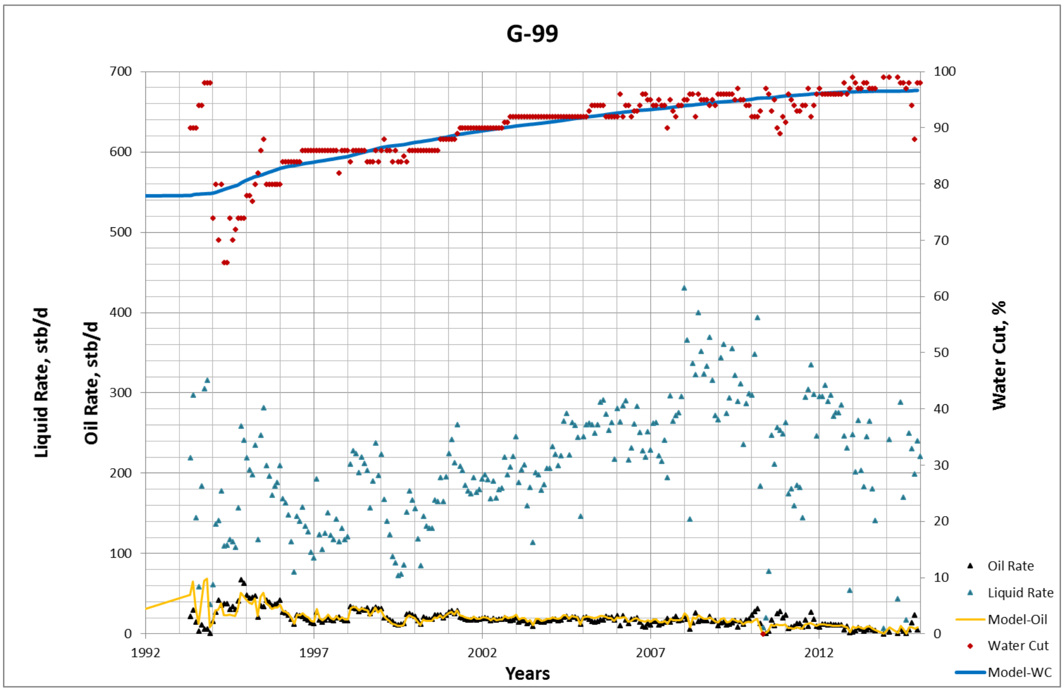


Figure 6.46 Oil Fractional Flow Match of G-99

6.4 Optimization of the Project Objective (10 year period, 2000-2010)

Starting from the "*Maximization of the Cumulative Oil Production*" objective, optimization was done by using primary production and injection contribution terms. In this part, it is assumed that the wells are under constant pressure production. Injection term was used as continuous part of the history match period to capture the filtered injection behavior coming from previous time steps. Again by using nonlinear regression, injection rates were optimized by calculating the cumulative oil production and oil cut in each time step to achieve the maximum cumulative oil production in ten year optimization period (2000-2010). A number of limitations which are obtained from the historical field data were used for the rates in injection wells;

- Maximum amount of injection rate per well is 1,300 stb/d
- Maximum amount of the injection rate for the field is 10,000 stb/d

In Figure 6.47, the result of this optimization algorithm is depicted. In the beginning of the optimization period, there is a sharp increase in the production and then water cut starts to increase with water injection which results in the decrease in oil production. Because of the multiple optimization in both rates and water cuts, some injection wells were closed in this period either to slow down the water cut increases or stop production in those wells. These are G-38, G-40, G-42, G-50, G-60, G-66 and G-71 which have less important effects on producers compared to the other injectors (G-42 and G-60 was used periodically instead of totally shut-in).

As a result of this optimization process, 711,140 bbls of extra oil production compared to the current real situation was observed which corresponds to 28.81 million $ extra production income within this ten year optimization period. By considering the 1 $/bbl injection and 3.5 $/bbl produced water disposal cost, the total revenue of the project is calculated as 65.44 million $. The quarterly calculated excess oil volumes and corresponding money incomes are tabulated in Table 6.4. This analysis was done by using the spot Brent oil prices gathered from U.S. Energy Information Administration.

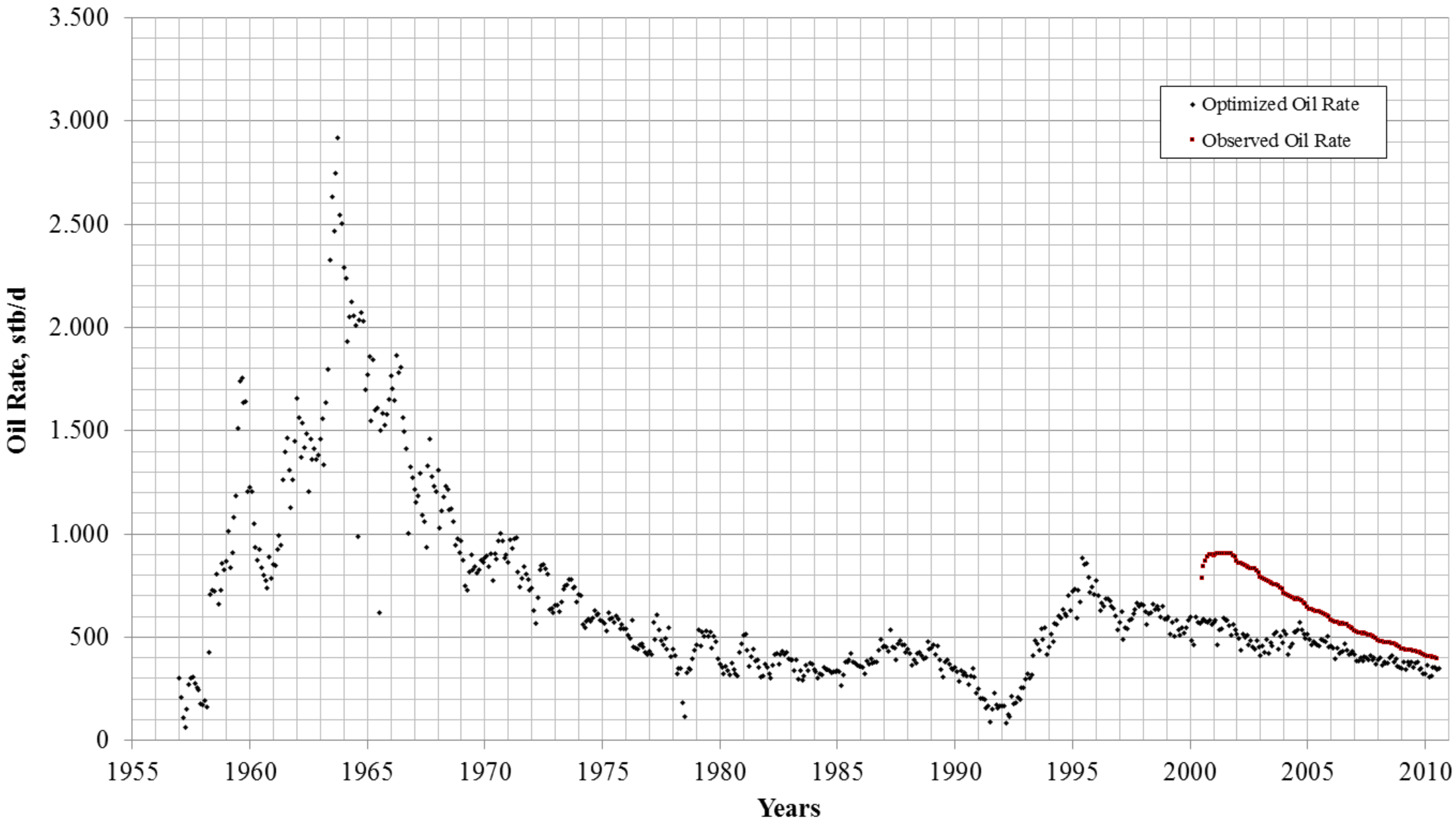


Figure 6.47 Optimized Oil Rates - “Maximum Cumulative Oil Production”

Table 6.4 Quarterly Calculated Extra Oil Production & Economics of the Project (Objective Function ➔ Maximization of the Cumulative Oil Production)

Quarter	Average Oil Rate, stb/d	Average Oil Price, $/bbl	Quarterly Excess Oil, stb	Quarterly Extra Income, $	Total Income, $	Total Revenue, $
2000- 3rd	836	30.46	23,591	729,751	2,352,189	1,169,384
2000- 4th	897	29.72	29,702	882,945	2,439,375	1,237,130
2001- 1st	898	25.87	34,139	887,087	2,126,448	923,777
2001- 2nd	905	27.27	30,519	829,857	2,258,945	1,054,059
2001- 3rd	904	25.30	32,247	816,943	2,092,271	887,863
2001- 4th	896	19.35	32,924	636,206	1,586,957	384,995
2002- 1st	861	21.13	34,581	727,049	1,663,950	473,272
2002- 2nd	848	25.05	32,743	819,328	1,943,122	756,697
2002- 3rd	835	26.93	34,283	924,649	2,056,277	874,015
2002- 4th	821	26.74	33,861	906,456	2,007,413	829,600
2003- 1st	783	31.52	29,869	943,458	2,259,680	1,093,772
2003- 2nd	768	26.17	28,963	756,315	1,838,772	677,797
2003- 3rd	756	28.45	24,057	681,034	1,968,883	811,658
2003- 4th	739	29.39	21,780	639,164	1,988,340	836,547
2004- 1st	706	31.92	22,498	718,028	2,061,409	920,363
2004- 2nd	692	35.45	16,930	596,673	2,244,125	1,107,520
2004- 3rd	683	41.39	12,434	513,298	2,587,497	1,453,678
2004- 4th	666	44.16	14,352	635,653	2,695,598	1,567,154
2005- 1st	637	47.70	14,682	699,488	2,778,224	1,659,278
2005- 2nd	625	51.63	14,881	768,407	2,953,453	1,838,208
2005- 3rd	619	61.47	12,452	765,863	3,483,586	2,370,194
2005- 4th	602	56.88	12,034	682,229	3,135,966	2,028,022
2006- 1st	577	61.75	13,429	826,971	3,257,882	2,158,242
2006- 2nd	567	69.53	12,802	890,550	3,607,816	2,511,222
2006- 3rd	562	69.62	11,028	760,018	3,584,243	2,489,108
2006- 4th	548	59.68	11,622	693,553	2,990,848	1,900,401
2007- 1st	525	57.76	12,402	713,599	2,772,542	1,689,498
2007- 2nd	519	68.58	11,161	765,432	3,254,771	2,173,676
2007- 3rd	512	74.95	10,586	795,860	3,512,660	2,433,640
2007- 4th	500	88.56	10,240	910,272	4,046,532	2,971,559
2008- 1st	479	96.94	9,008	873,787	4,250,181	3,181,694
2008- 2nd	474	121.40	8,363	1,004,994	5,269,189	4,202,268
2008- 3rd	469	114.40	6,703	761,592	4,910,037	3,844,902
2008- 4th	458	54.66	9,512	519,327	2,293,800	1,232,172
2009- 1st	440	44.43	6,508	287,809	1,788,272	732,398
2009- 2nd	437	58.70	6,548	388,166	2,346,219	1,291,302
2009- 3rd	430	68.20	5,883	399,216	2,683,074	1,630,361
2009- 4th	421	74.63	8,261	617,920	2,872,216	1,822,505
2010- 1st	408	76.25	7,182	547,174	2,847,343	1,801,638
2010- 2nd	403	77.78	4,869	382,093	2,893,953	1,849,982
2010- 3rd	397	76.82	1,512	114,246	915,546	568,148
			711,140	**28,812,459**	**110,619,604**	**65,439,700**

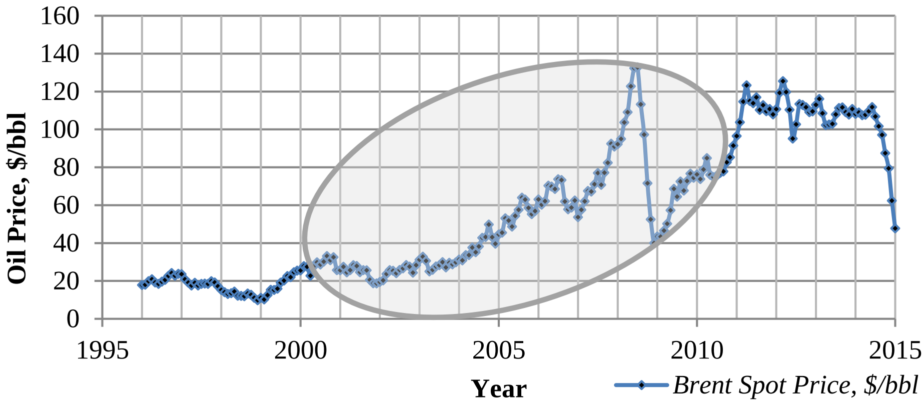


Figure 6.48 Spot Brent Oil Prices – (U.S. Energy Information Agency, 2015)

Spot Brent oil prices used in the calculations can be found in Figure 6.48. One of the most important parameter for the economics comes into picture at this point. Because the maximization of the project revenue objective function considers these fluctuations and decides on the rates of both injection and indirectly production. It decides on the volume of injection by checking the oil prices and costs in each time step.

The result of the "*Maximization of the Project Revenue*" optimization is shown in Figure 6.49 and Table 6.5. It shows similar results with the previous objective function case (also same active wells) but the main difference is the decrease and increase in the injection rates depending on the oil prices. There are multiple optimizations in both rates and water cuts which calculate the summation of the money income based on oil production, disposal cost of the produced water based on the amount of produced oil and finally water injection costs. By considering the 1 $/bbl injection cost and 3.5 $/bbl produced water disposal, the algorithm tries to maximize the net income in each step. There is a strong correlation between the oil prices trend and oil production which is also indirectly related to injection amounts.

As a result of this optimization process, 627,310 bbls of extra oil production is observed which corresponds to 28.32 million $ extra production income within this 10 year optimization period (less than the other case). But in total revenue, because of the optimization of the disposal and injection costs, it was calculated as 68.67 million $ which is 3.23 million $ (drilling cost of two more wells) more than the first case.

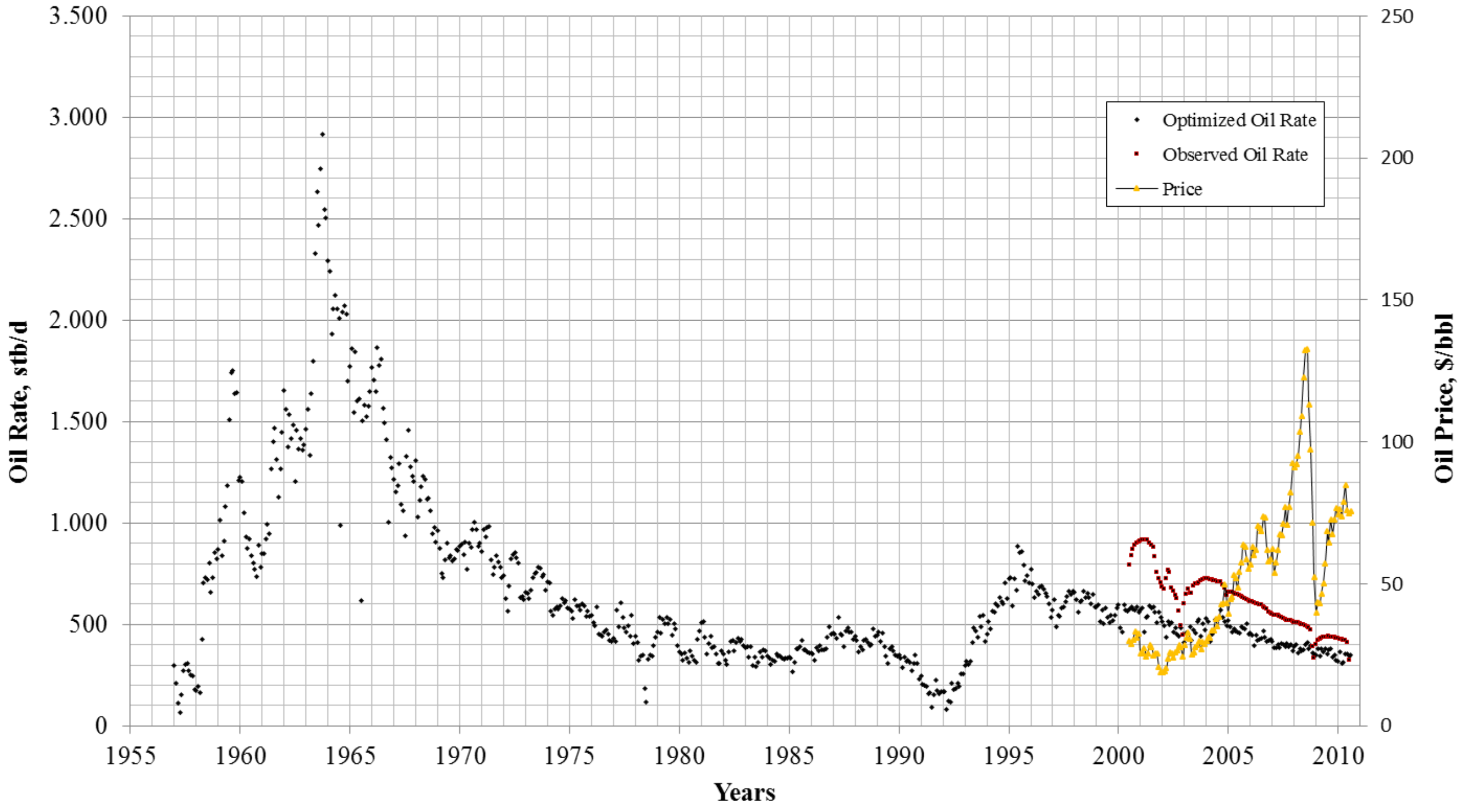


Figure 6.49 Optimized Oil Rates - “Maximum Project Revenue”

Table 6.5 Quarterly Calculated Extra Oil Production & Economics of the Project (Objective Function ➔ Maximization of the Project Revenue)

Quarter	Average Oil Rate, stb/d	Average Oil Price, $/bbl	Quarterly Excess Oil, stb	Quarterly Extra Income, $	Total Income, $	Total Revenue, $
2000- 3rd	834	30.46	23,394	723,738	2,346,176	1,164,060
2000- 4th	899	29.72	29,883	886,895	2,443,326	1,240,446
2001- 1st	916	25.87	35,765	929,247	2,168,608	960,247
2001- 2nd	912	27.27	31,101	845,338	2,274,427	1,094,954
2001- 3rd	867	25.30	28,857	730,525	2,005,853	935,311
2001- 4th	729	19.35	17,622	341,267	1,292,018	582,813
2002- 1st	695	21.13	19,389	409,922	1,346,823	559,960
2002- 2nd	734	25.05	22,353	561,210	1,685,003	775,892
2002- 3rd	644	26.93	16,862	454,347	1,585,975	896,714
2002- 4th	503	26.74	4,779	128,786	1,229,744	910,118
2003- 1st	640	31.52	16,698	528,048	1,844,270	779,359
2003- 2nd	665	26.17	19,557	512,101	1,594,557	628,155
2003- 3rd	701	28.45	18,987	536,813	1,824,662	767,531
2003- 4th	721	29.39	20,064	588,779	1,937,955	792,167
2004- 1st	724	31.92	24,194	772,413	2,115,794	968,809
2004- 2nd	717	35.45	19,181	676,531	2,323,983	1,179,500
2004- 3rd	710	41.39	14,864	613,987	2,688,186	1,545,861
2004- 4th	673	44.16	14,938	666,481	2,726,427	1,662,247
2005- 1st	658	47.70	16,646	793,867	2,872,603	1,746,785
2005- 2nd	650	51.63	17,123	884,109	3,069,155	1,946,062
2005- 3rd	637	61.47	14,075	865,611	3,583,334	2,464,263
2005- 4th	624	56.88	13,964	791,880	3,245,618	2,130,920
2006- 1st	610	61.75	16,501	1,016,641	3,447,553	2,337,162
2006- 2nd	601	69.53	15,920	1,107,325	3,824,590	2,717,082
2006- 3rd	589	69.62	13,417	927,686	3,751,911	2,648,414
2006- 4th	564	59.68	13,086	780,418	3,077,712	1,982,142
2007- 1st	547	57.76	14,458	832,845	2,891,788	1,801,547
2007- 2nd	539	68.58	13,011	891,784	3,381,123	2,293,554
2007- 3rd	526	74.95	11,846	890,361	3,607,161	2,523,730
2007- 4th	518	88.56	11,910	1,058,623	4,194,883	3,114,063
2008- 1st	509	96.94	11,709	1,136,261	4,512,655	3,434,717
2008- 2nd	501	121.40	10,811	1,299,993	5,564,187	4,488,697
2008- 3rd	488	114.40	8,460	965,109	5,113,554	4,042,268
2008- 4th	399	54.66	4,102	285,851	2,060,323	1,456,799
2009- 1st	416	44.43	4,292	190,767	1,691,230	643,110
2009- 2nd	437	58.70	6,539	389,242	2,347,295	1,292,410
2009- 3rd	439	68.20	6,694	455,534	2,739,393	1,683,839
2009- 4th	436	74.63	9,673	723,413	2,977,709	1,923,054
2010- 1st	428	76.25	9,026	687,835	2,988,004	1,935,847
2010- 2nd	417	77.78	6,217	488,822	3,000,683	1,979,444
2010- 3rd	326	76.82	-659	-49,789	751,510	637,408
			627,310	**28,320,613**	**110,127,758**	**68,667,459**

CHAPTER 7

CONCLUSIONS

This study considered the application of CRM to this specific carbonate reservoir. Although field has a peripheral water injection, which makes analysis more complicated, successful results were achieved in history matching with 10% error.

One of the most important outcomes of this study is the proof of the injection outside the reservoir from the flanks. The sum of weight coefficients and the voidage replacements are in agreement showing that almost 60% of the injected water could not reach to the drainage area of the current producers during history matching period.

Obtaining the fitting parameters, they were physically and geologically validated by using static and dynamic data derived from other sources. Weight coefficients were compared with the water breakthroughs and the theoretical flow paths determined by pressure and permeability distributions which showed very similar trends.

As it was stated in MLR and CRM sections, weight coefficients have an inverse relationship with the distances between well pairs. This phenomena was observed also in this study (larger distances ➔lower weight coefficients) indicating that most parts of the reservoir show similar characteristics except some conductive regions. There are mainly two regions which includes conductive parts resulting in heterogeneities in the system.

Another analysis were conducted to understand the heterogeneity of the reservoir is the log-log plot of the weight coefficients and time constants. Most of the data showed an inverse relationship except some regions which which were already defined as affected parts being supported by the high conductive flow paths. These regions cause some heterogeneities when compared to the rest of the structure.

To overcome difficulties of non-linear regression method such as unrealistic results just coming from mathematical approximations, unnecessary parameters were eliminated before calculations. Qualitative weight coefficient and pair distance cut-offs were applied for the well pairs which have no possible interaction with each other. In addition to this, the monthly productions which include non-productive times and have outlier fluctuations in rates were not included in objective function. Results showed that these initial judgments made the analysis more stable and robust.

In order to analyze the real outcomes of the project, a relationship between total liquid and oil rate was investigated by using historical data. As a result of different trials, a successful match between the cumulative oil production and oil cut was achieved by an empirical oil fractional flow model. Two coefficients were achieved for each well to match the model which also showed consistent results with the water cut increase trends of high conductive and homogeneous areas.

Finally, combining all these studies to reach the defined objectives for the project, an optimization algorithm was generated. Two different objective functions were studied; 1) maximization of the cumulative oil production 2) maximization of the project revenue. While the first one just considers the maximum oil production technically, the other one takes the water injection and produced water disposal costs and oil prices into account. Result showed that both models are very successful in terms of extra oil production and the second one which considers economic terms is more profitable.

As a result of all these studies, it was concluded that the Capacitance-Resistive Model was a proven useful tool to understand multiphase flow in the reservoir and optimize project economics even the field is in late time production. By using the other information coming from different sources, successful achievements can be observed in ongoing projects.

CHAPTER 8

RECOMMENDATIONS

CRM is one of the most useful surrogate models to infer interwell connectivities but there is a big uncertainty about the representativeness of the solution because of the mathematical approximation dependency. There are many variables and constraints depending on the number of wells which makes the calculations more speculative.

To decrease the uncertainty range, prior judgments which depend on the geological knowledge are essential for avoiding the unrealistic mathematical results. Unrelated injector-producer and producer-producer pairs may be eliminated to both increase accuracy of the results and decrease the computation time. Objective function of the model must be set with a great attention to simplify problem to avoid the difficulties.

The results obtained from the model should be compared with the expectations already figured out from the static and dynamic data prior to the study. Pressure and permeability distributions, initial water breakthroughs, rate fluctuations and water cut distributions are highly important to predict and compare the possible flow paths.

Understanding the multiphase flow phenomena in the reservoir, oil rates should always be integrated to predict the future performance of the field and most importantly the economics of the project. One of the big disadvantages of these models is being not able to predict oil rates and water cuts internally. But there are recent studies ongoing such as Cao et al. (2014) and Zhao et al. (2015) which have the same idea but a more complicated model with saturation functions to determine both phases' rates and water cut. These studies may be a very important step for the future of CRM applications.

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APPENDIX A

DIFFUSIVITY FILTERS

Albertoni, A., & Lake, L. W. (2002)

The pressure change caused by an injection rate at any point can be expressed as follows;

$$\Delta P = C_1 x\ Ei\ (-d\ \frac{r^2}{t}) \qquad \text{(A.1)}$$

where,

C_1 = a constant

E_i = exponential integral function

r = distance from point to the well

t = time

d = dissipation of the medium

By using superposition and a linear model of the rate-pressure relationship, equation becomes;

$$\Delta q = \left\{ \begin{array}{ll} C_2 x\ Ei\left(-d\ \dfrac{r^2}{t}\right) & for\ t \leq 1 \\ C_2 x \left[Ei\left(-d\ \dfrac{r^2}{t}\right) - Ei\left(-d\ \dfrac{r^2}{(t-1)}\right)\right] & for\ t \geq 1 \end{array} \right\} \qquad \text{(A.2)}$$

where, C_2 = new constant proportionality

Considering a fluctuating injection rate, a filter function can be generated to determine the production rate at any time and any point. To discretize the function with an assumption of montly data of a year, 12 normalized filter coefficients of discrete filter are determined;

$$\alpha^{(n)} = \frac{\int_{t=n}^{t=n+1} \Delta q \, dt}{\int_{t=0}^{t=12} \Delta q \, dt} \tag{A.3}$$

where $\alpha^{(n)}$s ≤ 1 and the sum of all coefficients is equal to one.

The discrete filter function is characterized by the distance from the injector, the time and the dissipation. When applied to an injection history, the convolved/filtered injection rate is expressed as;

$$i_{ij}^{c}(t) = \sum_{n=0}^{11} \alpha_{ij}^{(n)} i_i \, (t-n) \tag{A.4}$$

APPENDIX B

CAPACITANCE-RESISTIVE MODEL DERIVATION

Yousef, A. A. (2006)

Material balance equation of a given system;

$$\tau \frac{dq}{dt} + q(t) = i(t) - \tau J \frac{dp_{wf}}{dt} \tag{B.1}$$

Equation can also be written in a general form;

$$\frac{dq}{dt} + \frac{1}{\tau} q = r(t) \tag{B.2}$$

where,

$$r(t) = \left[\frac{i(t) - \tau J \frac{dp_{wf}}{dt}}{\tau}\right] \tag{B.3}$$

By using the integrating factor technique $f(t) = e^{\frac{t}{\tau}}$ and multiplying by the equation;

$$e^{\frac{t}{\tau}} \left[\frac{dq}{dt} + \frac{1}{\tau} q\right] = e^{\frac{t}{\tau}} [r(t)] \quad ==> \quad \frac{d}{dt}\left[e^{\frac{t}{\tau}} q\right] = e^{\frac{t}{\tau}} [r(t)] \tag{B.4}$$

By integrating this equation with respect to *t*;

$$e^{\frac{t}{\tau}}\, q = c + \int e^{\frac{t}{\tau}}\ [r(t)]\, dt \tag{B.5}$$

where, c is the constant of the integration. By dividing the both sides by $e^{\frac{t}{\tau}}$;

$$q = c\, e^{\frac{-t}{\tau}} + e^{\frac{-t}{\tau}} \int e^{\frac{t}{\tau}}\ [r(t)]\, dt \tag{B.6}$$

The constant of the integration can be estimated bu using initial condition;

$$q(t) = q(t_0) e^{\frac{-(t-t_0)}{\tau}} + \frac{e^{\frac{-t}{\tau}}}{\tau} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau}}\ i\ (\S) d\ \S + J\ \ e^{\frac{-t}{\tau}} \left[\frac{dp_{wf}}{d\S}\right] d\S \tag{B.7}$$

where, § is a variable of integration. By using integration by parts for the third part of right hand side of the equation, the final analytical solution becomes:

$$q(t) = q(t_0) e^{\frac{-(t-t_0)}{\tau}} + \frac{e^{\frac{-t}{\tau}}}{\tau} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau}}\ i\ (\S) d\S +$$

$$J\ \left[p_{wf}(t_0) e^{\frac{-(t-t_0)}{\tau}} - p_{wf}(t) + \frac{e^{\frac{-t}{\tau}}}{\tau} \int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau}}\, p_{wf}(\S) d\S\right] \tag{B.8}$$

APPENDIX C

CRM FILTERED INJECTION AND PRESSURE EFFECTS DERIVATION

Kaviani et. al. (2008)

Assuming a step function for injection signal, equation can be written as follows:

$$\frac{e^{-\frac{t}{\tau}}}{\tau}\int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau}}\, i(\S)\, d\S = \frac{e^{-\frac{t}{\tau}}}{\tau}\int_{\S=t_0}^{\S=t_1} e^{\frac{\S}{\tau}}\; i(t_1)\, d\S + \cdots + \frac{e^{-\frac{t}{\tau}}}{\tau}\int_{\S=t_{n-1}}^{\S=t} e^{\frac{\S}{\tau}}\, i(t) d\S \qquad \text{(C.1)}$$

and at each time step;

$$\frac{e^{-\frac{t}{\tau}}}{\tau}\int_{\S=t_{m-1}}^{\S=t_m} e^{\frac{\S}{\tau}}\; i(t_m)\, d\S = \; i(t_m)\left[e^{\frac{(t_m-t)}{\tau}} - e^{\frac{(t_{m-1}-t)}{\tau}}\right] \qquad \text{(C.2)}$$

By including time series;

$$i'_{ij}(t) = \; \sum_{m=1}^{n}\left[\frac{(t_m-t)}{\tau_{ij}} - \frac{(t_{m-1}-t)}{\tau_{ij}}\right]\; i_j(t_m) \qquad \text{(C.3)}$$

Assuming a step function for pressure signal, equation can be written as follows:

$$\frac{e^{-\frac{t}{\tau}}}{\tau}\int_{\S=t_0}^{\S=t} e^{\frac{\S}{\tau}}\, p_{wf}(\S)\, d\S = \frac{e^{-\frac{t}{\tau}}}{\tau}\int_{\S=t_0}^{\S=t_1} e^{\frac{\S}{\tau}}\; p_{wf}(t_1)\, d\S + \cdots$$
$$+\frac{e^{-\frac{t}{\tau}}}{\tau}\int_{\S=t_{n-1}}^{\S=t} e^{\frac{\S}{\tau}}\, p_{wf}(t) d\S \qquad \text{(C.4)}$$

and at each time step;

$$\frac{e^{-\frac{t}{\tau}}}{\tau}\int_{§=t_{m-1}}^{§=t_m} e^{\frac{§}{\tau}}\ p_{wf}(t_m)\ d§ = \ p_{wf}(t_m)\ \left[e^{\frac{(t_m-t)}{\tau}} - e^{\frac{(t_{m-1}-t)}{\tau}}\right] \qquad \text{(C.5)}$$

By including time series;

$$p_{wf}{'}_{kj}(t) = \ \sum_{m=1}^{n}\left[\frac{(t_m-t)}{\tau_{ij}} - \frac{(t_{m-1}-t)}{\tau_{ij}}\right]\ p_{wf}(t_m) \qquad \text{(C.6)}$$

APPENDIX D

OPTIMIZED INJECTION RATES

Table D.1 Optimized Injection Rates (Maximum Cumulative Oil Production)

Date	G-26	G-31	G-38	G-40	G-42	G-49	G-50	G-57	G-60	G-64	G-65	G-66	G-71	G-80
Jul-00	1211	1300	0	0	430	1300	0	1300	985	1300	1300	0	0	874
Aug-00	1300	1300	0	0	317	1300	0	1300	955	1300	1300	0	0	928
Sep-00	1300	1300	0	0	457	1300	0	1300	852	1300	1300	0	0	891
Oct-00	1300	1300	0	365	675	1300	0	1300	554	1300	1300	0	0	606
Nov-00	1300	1101	0	387	913	1300	0	1300	533	1300	1300	0	0	566
Dec-00	1300	737	0	579	964	1300	0	1300	588	1225	959	482	0	567
Jan-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	4	0	896
Feb-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Mar-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Apr-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
May-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Jun-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Jul-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Aug-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	1	0	899
Sep-01	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Oct-01	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Nov-01	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Dec-01	1300	1300	0	398	1300	1300	0	1300	502	1300	1300	0	0	0
Jan-02	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Feb-02	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	2	0	898
Mar-02	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Apr-02	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
May-02	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Jun-02	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Jul-02	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Aug-02	1300	1300	0	0	0	1300	0	1300	896	1300	1300	4	0	1300
Sep-02	1300	1300	0	0	913	1300	0	1300	0	1300	1300	0	0	1287
Oct-02	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Nov-02	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Dec-02	1300	1300	0	369	1300	1300	0	1300	531	1300	1300	0	0	0
Jan-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Feb-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Mar-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Apr-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900

Table D.1 (Continued)

May-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Jun-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Jul-03	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Aug-03	1300	1300	0	0	375	1300	0	1300	525	1300	1300	0	0	1300
Sep-03	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Oct-03	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Nov-03	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Dec-03	1300	1300	0	380	1300	1300	0	1300	520	1300	1300	0	0	0
Jan-04	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Feb-04	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Mar-04	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Apr-04	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
May-04	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Jun-04	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Jul-04	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Aug-04	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Sep-04	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Oct-04	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Nov-04	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Dec-04	1300	1300	0	0	1300	1300	0	1300	900	1300	1300	0	0	0
Jan-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Feb-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Mar-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Apr-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
May-05	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Jun-05	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Jul-05	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Aug-05	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Sep-05	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Oct-05	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Nov-05	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Dec-05	1300	1300	0	0	1300	1300	0	1300	900	1300	1300	0	0	0
Jan-06	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Feb-06	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Mar-06	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Apr-06	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
May-06	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Jun-06	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Jul-06	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Aug-06	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Sep-06	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Oct-06	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Nov-06	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Dec-06	1300	1300	0	0	1300	1300	0	1300	899	1300	1300	0	1	0
Jan-07	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Feb-07	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Mar-07	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300

Table D.1 (Continued)

Apr-07	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
May-07	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Jun-07	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Jul-07	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Aug-07	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Sep-07	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Oct-07	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Nov-07	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Dec-07	1300	1300	0	0	1300	1300	0	1300	900	1300	1300	0	0	0
Jan-08	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Feb-08	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Mar-08	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Apr-08	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
May-08	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Jun-08	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Jul-08	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Aug-08	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Sep-08	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Oct-08	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Nov-08	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Dec-08	1300	1300	0	0	1300	1300	0	1300	898	1300	1300	0	2	0
Jan-09	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Feb-09	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Mar-09	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Apr-09	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
May-09	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Jun-09	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Jul-09	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Aug-09	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Sep-09	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Oct-09	1300	1300	0	0	1300	1300	1	1300	0	1300	1300	0	0	899
Nov-09	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Dec-09	1300	1300	0	0	1300	1300	0	1300	898	1300	1300	0	2	0
Jan-10	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Feb-10	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Mar-10	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Apr-10	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
May-10	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Jun-10	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Jul-10	1300	1300	0	0	1300	1300	0	1300	900	1300	1300	0	0	0

Table D.2 Optimized Injection Rates (Maximum Project Revenue)

Date	G-26	G-31	G-38	G-40	G-42	G-49	G-50	G-57	G-60	G-64	G-65	G-66	G-71	G-80
Jul-00	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Aug-00	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Sep-00	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Oct-00	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Nov-00	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Dec-00	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Jan-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Feb-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Mar-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Apr-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
May-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Jun-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	0
Jul-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	0
Aug-01	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	0
Sep-01	0	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	0
Oct-01	0	1300	0	0	0	0	0	1300	0	1300	1300	0	0	0
Nov-01	0	1300	0	0	0	0	0	1300	0	1300	1300	0	0	0
Dec-01	0	1300	0	0	0	0	0	1300	0	1300	1300	0	0	0
Jan-02	0	1300	0	0	0	0	0	1300	0	1300	1300	0	0	0
Feb-02	0	1300	0	0	0	0	0	1300	0	1300	1300	0	0	0
Mar-02	304	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	0
Apr-02	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	0
May-02	1300	1300	0	0	0	1300	0	1300	0	1300	1300	0	0	0
Jun-02	0	1300	0	0	0	0	0	1300	0	1300	1300	0	0	0
Jul-02	235	1300	0	0	0	0	0	1300	0	1300	1300	0	0	0
Aug-02	0	1300	0	0	0	0	0	1300	0	1300	1300	0	0	0
Sep-02	0	1300	0	0	0	0	0	1300	0	1300	1300	0	0	0
Oct-02	0	0	0	0	0	0	0	1300	0	1300	0	0	0	0
Nov-02	0	0	0	0	0	0	0	1300	0	0	0	0	0	0
Dec-02	0	0	0	0	0	0	0	1300	0	0	0	0	0	0
Jan-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Feb-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	0
Mar-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	0
Apr-03	0	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	0
May-03	0	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	0
Jun-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	0
Jul-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	0
Aug-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	0
Sep-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	0
Oct-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Nov-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Dec-03	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900

Table D.2 (Continued)

Jan-04	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Feb-04	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Mar-04	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Apr-04	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
May-04	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Jun-04	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Jul-04	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Aug-04	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Sep-04	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Oct-04	1300	1300	0	0	440	1300	0	1300	460	1300	1300	0	0	1300
Nov-04	1300	1300	0	0	239	1300	0	1300	824	1300	1300	101	0	501
Dec-04	534	1300	0	0	193	877	0	1300	858	1300	1300	142	0	557
Jan-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Feb-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Mar-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Apr-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
May-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Jun-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Jul-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Aug-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Sep-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Oct-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Nov-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Dec-05	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Jan-06	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Feb-06	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Mar-06	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Apr-06	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
May-06	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Jun-06	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Jul-06	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Aug-06	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Sep-06	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Oct-06	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Nov-06	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Dec-06	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Jan-07	900	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	1300
Feb-07	900	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	1300
Mar-07	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Apr-07	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
May-07	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Jun-07	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900

Table D.2 (Continued)

Jul-07	1300	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	900
Aug-07	900	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	1300
Sep-07	900	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	1300
Oct-07	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Nov-07	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Dec-07	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Jan-08	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Feb-08	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Mar-08	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Apr-08	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
May-08	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Jun-08	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Jul-08	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Aug-08	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Sep-08	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Oct-08	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	0
Nov-08	0	1300	0	0	0	0	0	1300	0	1300	0	0	0	0
Dec-08	0	1300	0	0	0	0	0	1300	0	0	0	0	0	0
Jan-09	900	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	1300
Feb-09	900	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	1300
Mar-09	900	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	1300
Apr-09	900	1300	0	0	0	1300	0	1300	1300	1300	1300	0	0	1300
May-09	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Jun-09	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Jul-09	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Aug-09	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Sep-09	1300	1300	0	0	0	1300	0	1300	900	1300	1300	0	0	1300
Oct-09	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Nov-09	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Dec-09	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Jan-10	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Feb-10	1300	1300	0	0	900	1300	0	1300	0	1300	1300	0	0	1300
Mar-10	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Apr-10	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
May-10	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	900
Jun-10	1300	1300	0	0	1300	1300	0	1300	0	1300	1300	0	0	0
Jul-10	0	0	0	0	0	0	0	1300	0	1300	0	0	0	0