

OPTIMIZATION OF LOCATIONS OF VORONOI GRID POINTS
IN RESERVOIR SIMULATION

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ULVI RZA-GULIYEV

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IN RESERVOIR SIMULATION**

submitted by **ULVI RZA-GULIYEV** in partial fulfillment of the requirements for
the degree of **Masters of Science in Petroleum and Natural Gas Engineering**
Department, Middle East Technical University by,

Prof. Dr. Gülbin Dural Ünver
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Mustafa Verşan Kök
Head of Department, **Petroleum and Natural Gas Eng. Dept.**

Prof. Dr. Çağlar Sınayuç
Supervisor, **Petroleum and Natural Gas Eng. Dept., METU**

Examining Committee Members:

Prof. Dr. Mustafa Verşan Kök
Petroleum and Natural Gas Engineering Dept., METU

Asst. Prof. Dr. Çağlar Sınayuç
Petroleum and Natural Gas Engineering Dept., METU

Prof. Dr. Mahmut Parlaktuna
Petroleum and Natural Gas Engineering Dept., METU

Asst. Prof. Dr. İsmail Durgut
Petroleum and Natural Gas Engineering Dept., METU

Asst. Prof. Dr. Emre Artun
Petroleum and Natural Gas Engineering Dept., METU NCC

Date: 01.09.2015

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Name, Last Name: Ulvi, Rza-Guliyev

Signature:

ABSTRACT

OPTIMIZATION OF LOCATIONS OF VORONOI GRID POINTS IN RESERVOIR SIMULATION

Rza-Guliyev, Ulvi

M.S., Department of Petroleum and Natural Gas Engineering

Supervisor: Asst. Prof. Dr. Çağlar Sınayuç

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Reservoir simulations are computer models that can imitate real world reservoir behavior under different circumstances, therefore making it possible for reservoir engineers to make sensitivity studies in order to assess different scenarios. These models discretize the reservoir into smaller blocks either using structured grids or unstructured grids. The application of regular structured grids to correctly map reservoir's geological structure can be very difficult, if not nearly impossible. Unstructured grids can be more convenient for those cases. Voronoi gridding technique creates unstructured grids such that the boundary of two grids is normal to the line connecting Voronoi particles that represents the grids. So that it would be convenient to calculate the transmissibility on the block boundaries.

In this study instead of placing the Voronoi particles randomly, or in a regular fashion, the properties of the reservoir such as permeability anisotropy, orientation of the permeability vectors, heterogeneity of the petrophysical properties, and well locations and types were taken into consideration in the placement of Voronoi particles. A three-step algorithm, created in this thesis and written using Matlab software, takes into account the high resolution petrophysical properties in a finer static mesh, together with permeability anisotropy ratio and orientation and well

location. This algorithm generates initial distribution of grid points that honors permeability anisotropy, then assigns each grid point an error value, which is dependent on grid point placement, and tries to minimize this error by moving bad points onto better locations. The error gets lower as the Voronoi grids and the background finer static mesh agrees with each other. Finally, after each grid point's location is chosen grid points related to vertical and horizontal wells and fault are added. Algorithm was implemented on six cases of different complexity and then generated Voronoi grid blocks were used in a simple, single phase simulator to show the effects of the optimized grids. It was seen that the developed code during the study can match the given input static model and can reduce the number of grid blocks required to model a hydrocarbon reservoir.

Key words: Voronoi, PEBI, reservoir simulation, optimization

ÖZ

REZERVUAR SİMÜLASYONUNDA VORONOİ IZGARA NOKTALARININ YERLERİNİN OPTİMİZASYONU

Rza-Guliyev, Ulvi

Yüksek Lisans, Petrol ve Doğal Gaz Mühendisliği Bölümü

Tez Yöneticisi: Yrd. Doç. Dr. Çağlar Sınayuç

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Rezervuar simülasyonları gerçek saha davranışlarını farklı durumlarda taklit eden ve bu sayede rezervuar mühendislerinin farklı senaryoları değerlendirmek için hassasiyet çalışması yapmasını mümkün kılan bilgisayar modelleridir. Bu modeller rezervuarı küçük bloklara yapılandırılmış bloklar halinde ya da yapılandırılmamış bloklar halinde ayırırlar. Rezervuarın jeolojik yapısını doğru şekilde tanımlamak için yapılandırılmış blokların kullanımı imkansız olmasa bile çok zordur. Yapılandırılmamış bloklar bu durumda çok daha uygun olabilir. Voronoi ızgara yöntemi ile elde edilen yapılandırılmamış bloklar arasındaki sınır, iki bloğu birleştiren ve bloğu temsil eden parçacıkları birleştiren doğruya diktir. Bu sayede blok sınırındaki iletgenliği hesaplamak daha kolay olmaktadır.

Bu çalışmada Voronoi parçacıklarını rastgele ya da düzenli şekilde yerleştirmek yerine, rezervuarın geçirgenlik eşyönsüzlüğü, geçirgenlik vektörlerinin yönelimi, petrofiziksel özelliklerin heterojenliği, kuyu yer ve tipleri gibi özellikleri göz önüne alınarak voronoi parçacıklarının yerleri belirlenmiştir. Yüksek çözünürlüklü ince statik bir ızgarada yer alan petrofiziksel özellikler, geçirgenlik eşyönsüzlük oranı ve yönelimi ile kuyu yerlerini kullanan üç aşamalı bir Matlab kodu bu amaç için yazılmıştır. Algoritma parçacıkların ilk dağılımını geçirgenlik eşyönsüzlüğü

deęerine baęlı olarak geręekleřtirmektedir. Yazılım voronoi paręacıklarının en uygun yerlerini bir hata en aza indirme yntemi ile belirlemektedir. Hata Voronoi blokları ile ince statik ızgara ile verilen zellik sınırlarının birbirleri ile rtřmesi ile azalmaktadır. Son olarak, paręacıkların yerleri belirlendikten sonra dikey ve yatay kuyular ile fay hatları eklenmektedir. Basit, tek fazlı bir simlatr kullanılarak altı farklı durum iin en uygun hale getirilmiř ızgaraların etkisi grlmřtr. alıřma sırasında geliřtirilen kodun verilen statik model ile rtřtę ve bir hidrokarbon rezervuarını modellemek iin gerekli blok sayısını azalttıęı grlmřtr.

Anahtar kelimeler: Voronoi, PEBI, rezervuar simlasyon, optimizasyon

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CHAPTER 1

INTRODUCTION

With the dramatic advancements in computers during last half of the century, reservoir modeling became one of the most powerful tools in the hands of reservoir engineers. By giving possibility to assess different ways of exploitation of reservoirs before making a final decision, it gave opportunity to correctly evaluate all possible outcomes and to produce petroleum in the most efficient way.

Reservoir modeling is a process of usage of petrophysical and geological data obtained from different studies in the field in order to predict the behavior of the fluids under different conditions (Lie and Mallison, 2010). It is done by creating a model which is a simplification of the real reservoir. This model is discretized into a great amount of grid blocks, between which flow is calculated using fundamental laws of fluid flow.

One of the factors that effectiveness of reservoir simulation depends on is a choice of gridding type. There are many different types of the gridding techniques that have been used in reservoir simulation. In the early days of reservoir simulation, only a limited amount of Cartesian grids was used because of limitations of computers' calculating power and available memory. So there was no need in creating new gridding techniques, and for some time reservoirs were simulated by using several thousand Cartesian grid blocks. The development of computers, their calculating power and memory resulted in the possibility to use greater amount of blocks, therefore resolution of models increased. With this refinement of blocks, new demand appeared to try to represent complex geological features and fluid flow in a

more accurate manner. That was the cause that resulted in the creation of new gridding techniques.

Usually, gridding techniques are separated into two broad groups: structured and unstructured gridding. Sometimes hybrid grids are taken as the third group. Group of structured gridding types include Cartesian, cylindrical, hexagonal etc, while one of the most popular type of unstructured grids is PEBI (PErpendicular BIsector) or Voronoi grids. The difference between structured and unstructured grids is that structured grid types imply same regular shape of all of the grid blocks (for example, triangles, rectangles), while unstructured ones do not require that condition (Moog, 2013). This difference means that unstructured grids are more flexible, compared to the structured ones, which means that it can be used less amount of blocks to represent some geological entity in the model without losing accuracy (Heinemann and Brand, 1989).

Majority of unstructured grids was introduced in 1980's in order to meet specifications concerning flexible modeling. The main types of grids invented during this period include Control Volume Finite Element (Forsyth, 1989), Voronoi grids (Heinemann and Brand, 1989) and hybrid grids (Pedrosa and Aziz, 1985). Voronoi grid type appeared to be useful, because it takes better sides from both structured and unstructured grids: they were flexible, allowed usage of different grid types, providing a smooth transition from Voronoi grids to other gridding types (Katzmayr and Ganzer, 2009).

However, apart from obvious advantages of unstructured grids, they also have some problems: different number of block sides, non-orthogonality to the flow (grid orientation effects) and others.

Voronoi grid blocks are areas that are closer to its grid point than to any of the other ones, and the grid consists of this type of blocks (Palagi and Aziz, 1994). This definition means that by accurate placement of Voronoi grid points in the reservoir

simulation accurate mapping of reservoir structures could be done. This study focuses on optimization of Voronoi grid blocks' locations for this reason.

Optimization problem implies choosing of one option from a group of possible solutions to the problem in order to maximize or minimize predefined function. In the case discussed in this thesis optimization problem is in obtaining of optimized locations of predefined number of grid blocks in a reservoir simulation of a field including heterogeneities and/or permeability anisotropy while minimizing sum of errors in all of the Voronoi grids. Each grid block in the simulation in the study is assigned an error value - coefficient of badness of its placement. This error depends on the match of the Voronoi grids with finer static mesh of petrophysical properties. The higher the error in the block, the higher priority it has in the line of points that will be moved. By moving of these bad points, an attempt to find better locations to minimize the error value, and therefore better placing of grid points can be obtained without increasing the amount of them.

In order to solve optimization problem, an optimization algorithm is usually required. Optimization algorithm is a number of instructions that are required to be applied to the problem in the correct order in order to reach desired results. All optimization algorithms can be divided into two broad groups: probabilistic and deterministic optimization algorithms. Probabilistic algorithms are such algorithms that have at least one process including generation of random numbers in one of the steps. This means that for the same input this algorithm will be able to produce different results. This type of optimization algorithms is usually used when approximate steps in order to reach optimized state are not known beforehand, so it is required to search for this state everywhere. However, if these steps are known, then no random generation (or searching for the correct direction) is required and deterministic algorithms can be used. As it may be understood from this, deterministic algorithms will always give the same results for the same input values. (Weise, 2011)

The algorithm created in this study shares some concepts with evolutionary optimization algorithms that are related to the probabilistic group, but itself is related to the deterministic group. It consists of three simple steps the first of which generates predefined number of uniformly distributed initial population of grid points; the second step tries to minimize sum of errors in all of the blocks by moving grid points obtained from the first step; the last step takes result obtained in the step two and adds grid points related to wells and/or faults. This algorithm is described in details in "Methodology" chapter.

Next chapters provide more detailed information on the main subjects of this study: reservoir simulation, Voronoi gridding, reservoir heterogeneities and anisotropy, and optimization.

CHAPTER 2

RESERVOIR SIMULATION

2.1. Introduction

At any particular point in geologic time, there is only one real dispensation of petrophysical properties in the reservoir. This dispensation is the result of a complicated combined work of chemical, physical, and biological processes. Notwithstanding the fact that sometimes physics of depositional processes and processes, occurring after deposition, may be realized very well, engineers do not absolutely understand each process and its interaction with the others, which in combination with the inability to get the boundary and initial conditions results in impossibility to obtain the real singular dispensation of the properties of the reservoir that change with time. So the only way is to build numerical simulations that can imitate the real change of reservoir properties with time. Therefore, engineers try to build reservoir simulations so that they would correlate with all the obtained data. They understand that usually the real dispensation of reservoir properties will not be exactly the same as in the model prediction, but they try to get the results as close as possible (Pyrcz and Deutsch, 2014).

In less words, reservoir simulation is the process of inferring the behavior of a real reservoir from the performance of a model of that reservoir (Jensen et al., 1997).

First reservoir simulations were far from what we have today. Actually, they were physical models - for example, boxes made out of glass and filled with sand, from where fluid was passing allowing scientist/engineer to look and understand what is happening there. These simulations were first used in the 1930s and were used for

getting idea of how water breakthrough occurs in wells of the reservoir that has been waterflooded.

With advancements in computers from 1960s and later, reservoir simulations changed from physical models to computer-based models. These models divided existing reservoir into a number of connecting blocks and calculated the flow that will occur between these blocks under different conditions. When computers were just introduced, they had far less efficiency and power than what we have today - this fact was limiting number of blocks that reservoir can be divided into, which resulted in not so reliable results obtained after simulator was run. Nowadays, simulators allow to create models of millions and even billions of blocks, which makes results much more reliable (Islam et al., 2010).

Figure 2.1 shows the main steps in the creation of the reservoir model as defined by Odeh in 1982. Formulation stage here includes the introduction of assumptions required to create a reservoir model in mathematical form. Then nonlinear partial differential equations describing fluid flow are introduced, which are then undergo stage of discretization and form a bunch of nonlinear algebraic equations. This discretization can be done by applying Taylor series expansion (other techniques are integral and variational methods (Aziz and Settari, 1979).

As it was already mentioned, discretization results in formation of nonlinear algebraic equations, which in most of the cases require linearization in order to be solved. Well representation is also required at this stage in order to add fluid production/injection into equations that are still nonlinear.

After all previous steps are fulfilled, solutions can be obtained. These solutions include distribution of both pressure and saturations and also flow rates of the introduced wells. Validation step is just checking that no mistakes were made in the previous step and in the source code of the simulator. After all these stages are done, the simulator is ready to be used. (Islam et al., 2010)

2.2. Motivation to use reservoir simulation

The main purpose of reservoir simulation is to imitate real life reservoir behavior and therefore allow to predict future of reservoir under different development scenarios. So, if correct assumptions are made, if the data that the model is based on is representative of reservoir and many other nuances are kept, then the reservoir model should be a very powerful tool allowing engineers to solve many complex problems and even to foresee them; create reservoir management plan years into the future (Adamson et al., 1996).

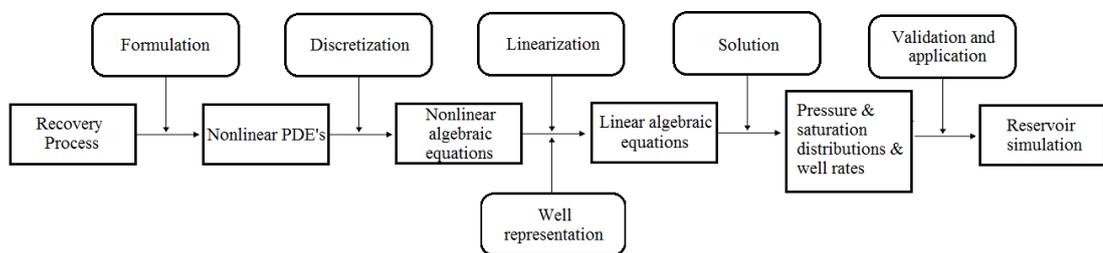


Figure 2.1. Main stages of generation of reservoir simulators (modified from Odeh, 1982).

2.3. Gridding techniques

As it was previously mentioned, simulators divide the real reservoir into a number of blocks and then calculate flow between these blocks. Therefore, it is obvious that choice of appropriate gridding technique is crucial for the effectiveness of the model being built. The choosing of the appropriate grid in reservoir simulation is based mainly on two criteria:

- It should be able to correctly map geological characteristics of the region;
- It should be able to correctly map flow of fluid governed by the flow equations. (Lake and Holstein, 2007)

Classification of gridding techniques is a difficult thing, because there are many different grid types that show absolutely different properties, however, many authors distinguish two main groups of grid types: structured and unstructured grids.

However, there are also grids that are not related to any of these groups. This subchapter will discuss these grid types one by one.

2.3.1. Structured grids

There are different definitions of structured grids in the literature, including "structured grid is a mesh type, consisting of many grid blocks of same geometrical shape" (Moog, 2013) and "structured gridding is a mesh type consisting of blocks with regular connectivity" (Castillo, 1991).



Figure 2.2. Representation of geological feature using structured Cartesian grid with refinement (a) versus unstructured grid (b) (after Moog, 2013).

Among the advantages of structured grids good convergence and high resolution is usually mentioned (Chawner, 2013), while the major drawback that is usually talked about is that regular structured grid sometimes fails in proper representation of geologically complex reservoirs (figure 2.2), which results in doubts in simulation's ability to accurately predict reservoir behavior (Moog, 2013). In the next subchapters different structured grid types are shown and discussed.

2.3.1.1. Cartesian grid

Regular Cartesian grids are the most popular gridding type used in reservoir simulation. They were used already in the first reservoir simulations used in the industry. Cartesian grids are usually represented by quadrilaterals in two dimensional models (figure 2.3 (a)) and by hexahedra in three dimensional simulations (figure 2.3

(b)). Sometimes, for better representation of geological structures, hexahedra are created by defining locations of each of its vertices. In this case, the obtained grid is called Corner Point Geometry Grids, which is also usually related to structured type.

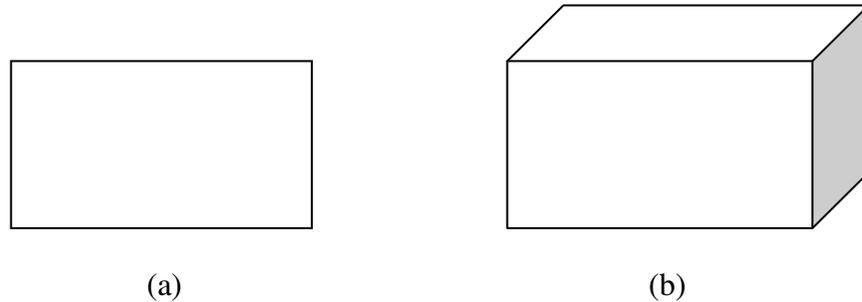


Figure 2.3. Cartesian grid in 2D (a) and 3D (b).

As it was already mentioned, sometimes reservoir that is have to be modeled has very complex structure, which usually results in necessity of locally refinement of grid blocks in the zone of increased reservoir complexity (figure 2.4). This is usually done in the fields with regular Cartesian grids and is also related to structured gridding types.

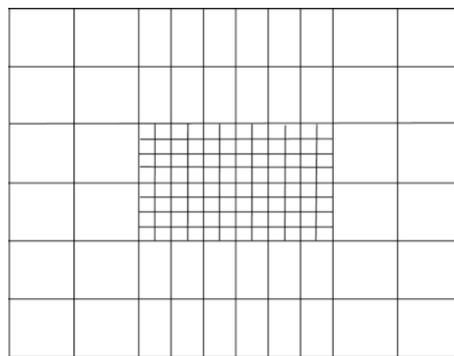


Figure 2.4. Local grid refinement in regular Cartesian grid (modified from Lake and Holstein, 2007).

2.3.1.2. Cylindrical grid

Cylindrical grid usually is used for representation of wells inside reservoir simulation. If it is used with other other gridding type, which is usually the case, then

it becomes a hybrid grid which is described in the subchapter 2.3.3.1. It can be both used in two and three dimensional reservoir simulations (figure 2.5).

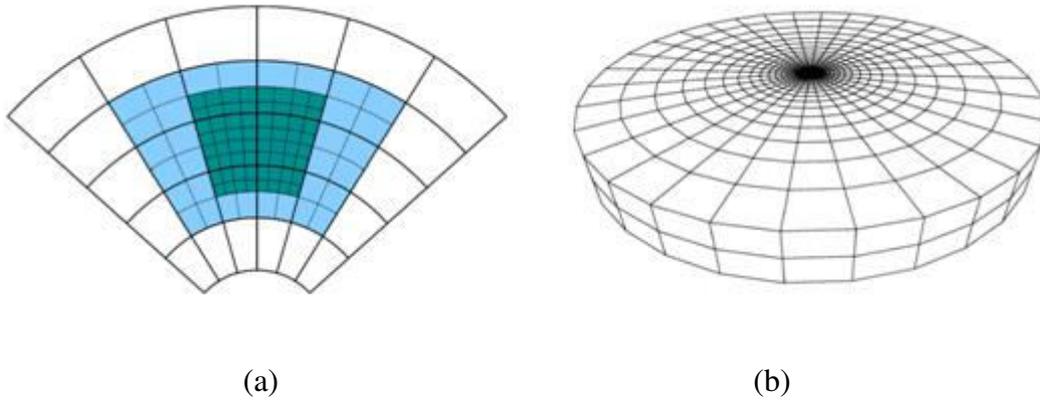


Figure 2.5. Cylindrical grid in two dimensions with local refinement (a) and three dimensions (b) (modified from Kaufmann, 2006 and Angelo et al., 2002).

2.3.1.3. Hexagonal grid

Hexagonal grid is used rarely in reservoir simulation. The first proposal of application of hexagonal grid to the reservoir simulation was in the work of Pruess and Bodvarsson (Pruess and Bodvarsson, 1983).

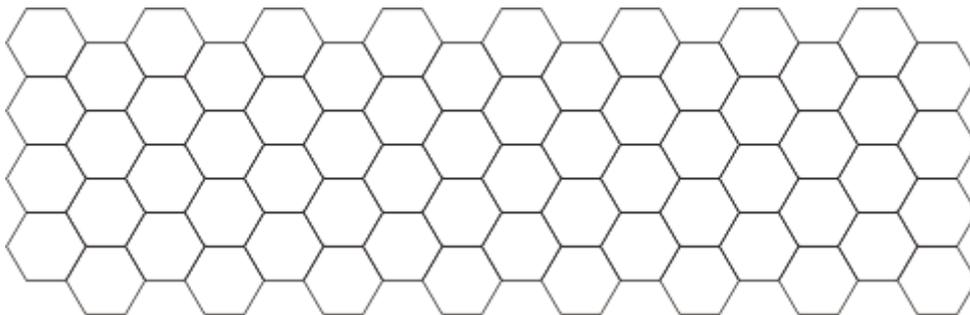


Figure 2.6. Example on hexagonal grid in two dimensions.

From the definition of structured grids, hexagonal grids must be related to them, however in reality hexagonal structure is usually obtained by applying of unstructured gridding techniques. As an example, typical shapes of Voronoi grids in

two dimensions are hexagons, while in three dimensions they are hexagonal prisms (figure 2.6).

One of the successful applications of structured hexagonal grid is described in the work of Wadsley et al. (Wadsley et al., 1990). He and his companions used hexagonal grids in order to model fluvial architecture with subsequent simulation of reservoir under production. Among the pluses of hexagonal grids, they mention the fact that hexagonal grids help to overcome grid orientation effects.

2.3.1.4. Triangular Grid

Triangular grids are used very rarely in reservoir modeling. This is due to they usually correspond to unstructured Voronoi gridding (Delaunay triangulation), which is more persistent to grid changes. Other cause of its rare usage is that they usually result in, what some authors call, "sliver" blocks that have little volume but big area of surface (Lake and Holstein, 2007).

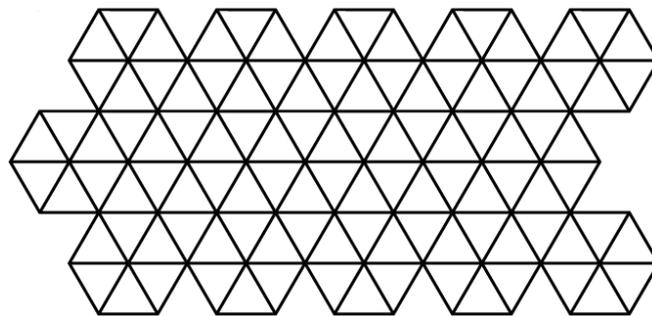


Figure 2.7. Example on triangular grid in two dimensions.

In two dimensions triangular grid is represented by triangles, while in three dimensions they exist as tetrahedra (figure 2.7).

2.3.2. Unstructured Grids

As it was already mentioned, as distinct from structured gridding types, unstructured ones do not have particular shape, which results in its flexibility that makes it

possible to more accurately represent geologic entities in the model (figure 2.2). Other differences between these types is that the unstructured grid is based on a number of grid points that have no specific indexing. After these grid points are chosen, control volumes are generated around these grid points.

One of the most popular unstructured grid types is Voronoi or PEBI grids which are the basis of the study described in this thesis.

2.3.2.1. Voronoi grid

Voronoi gridding technique is discussed in details in the next chapter, so this one only provides some basic information on them.

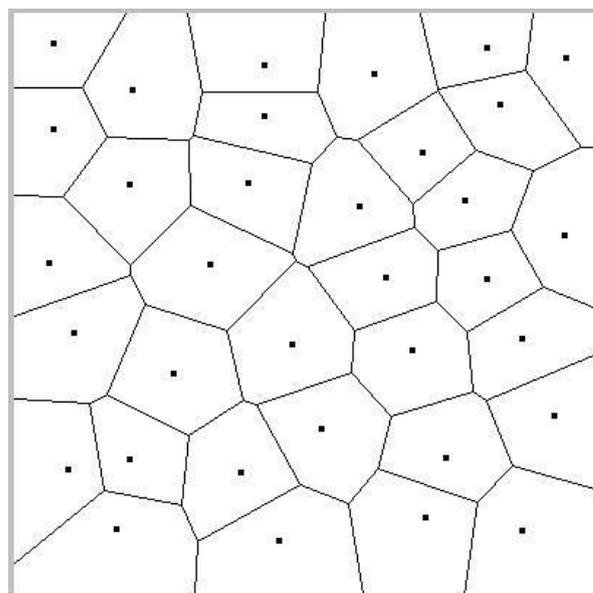


Figure 2.8. Example on Voronoi grid in two dimensions.

Voronoi grid block is an area of space that is closer to its grid point than any of the others that are present in the grid. This means that each block's sides are located in the middle of the line connecting two neighboring grid points and are perpendicular to it. Actually, that is where its second name is derived from - PERpendicular BIsector.

Voronoi grids were first proposed to be used in reservoir simulation in the paper of Heinemann and Brand in 1989 (Heinemann and Brand, 1989), and after that got some usage in reservoir simulation, however is still not very popular.

Voronoi grids can exist both in two dimensional, two and a half dimensional and three dimensional spaces. As it was already mentioned, most typical shapes than they take in two and two and a half dimensional spaces are accordingly hexagons and hexagonal prisms (Lake and Holstein, 2007).

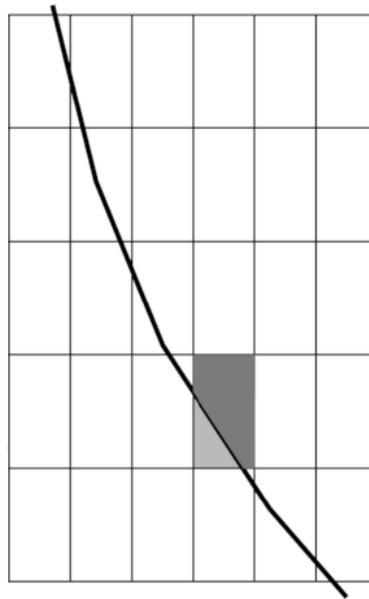


Figure 2.9. Truncated grid (modified from Lake and Holstein, 2007).

Two and a half dimension dimensional Voronoi means that Voronoi is generated for each layer of reservoir formation and then are stucked on the top of each other. So each layer has its specific thickness, which means that the structure is in three dimensions but not fully. That is why it is called two and a half dimensions. In three dimensions there are no restricting planes on the top and the bottom.

2.3.2.2. Truncated grids

Truncated grids sometimes are used with Cartesian grids in order for better representation of the faults. The grid mainly is simple Cartesian grid described in

2.3.1.1., the only difference is that if the fault passes through one of the cells, it divides this cell into two parts. This is shown on figure 2.9.

From the advantages better handling of reservoir heterogeneities can be mentioned, but this comes at great price - it may result in very sophisticated shapes of the blocks and therefore transmissibility terms between blocks will have to be calculated in a more difficult way.

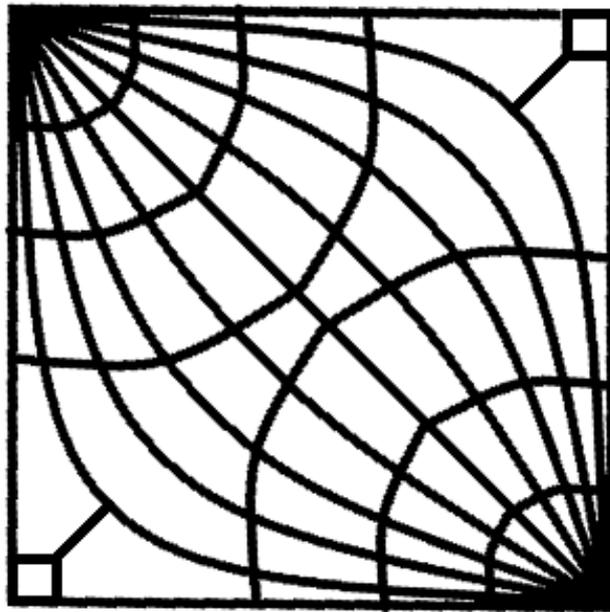


Figure 2.10. Example on curvilinear grid type.

2.3.2.3. Curvilinear grids

Discussion on application of curvilinear grids to the reservoir simulation started from the 1970s. It was mentioned in the work of Hirasaki and O'Dell (Hirasaki and O'Dell, 1970), Sonier and Chaumet (Sonier and Chaumet, 1974) and many others.

Curvilinear grid was mentioned to better simulate flow of fluids, however, by winning at representation of the fluid flow, some problems occur with representation of geological entities. So, this type of grids also did not get wide application in the industry. Figure 2.10. shows example on curvilinear geometry.

2.3.3. Hybrid grids

Hybrid grids cannot be related to any of the previous groups because it is partly structured and partly unstructured. Application of hybrid grids in reservoir simulation were first discussed in the work of Pedrosa and Aziz (Pedrosa and Aziz, 1986).

Main purpose of usage of such hybrid grids in reservoir simulation is to improve treatment of well in there. Usually, cylindrical grid type is used around the wells in order to accurately map increased pressure gradients occurring when the well is producing or injecting. These grid blocks are usually surrounded by some regular structured grids like simple Cartesian, hexagonal, triangular or others. Example on hybrid grids is shown on figure 2.11.

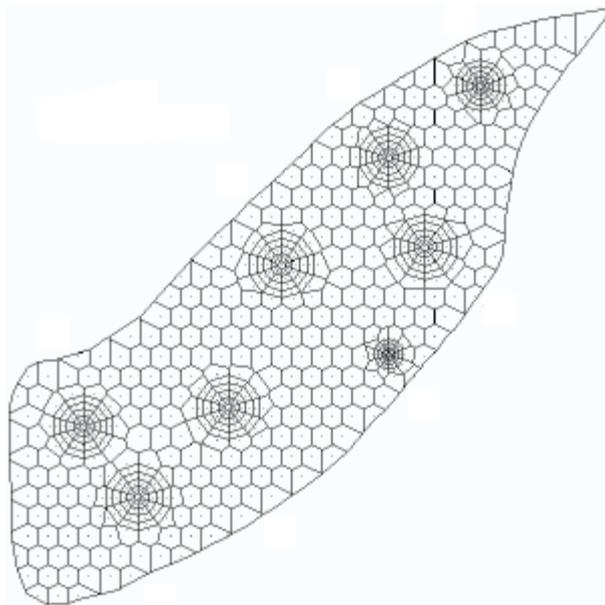


Figure 2.11. Example on hybrid grids (modified from Marcondes et al., 2009).

As it was already said, this study deals with Voronoi gridding technique which is discussed in details in chapter 3.

CHAPTER 3

VORONOI GRID BLOCKS

3.1. Introduction

Voronoi (or PEBI) grids are one of the basic geometrical structures that may be used to divide the space into small areas of ascendancy. These grids may as well be used in reservoir engineering, dividing the reservoir model into a finite number of blocks. (Aurenhammer and Klein, 2000)

Modeling of hydrocarbon reservoirs is usually done by partitioning the space occupied by reservoir into a set of fictitious blocks and applying of equations of conservation laws, such as mass conservation, on each one of them. Fluid movement from one block to another can be obtained from the discretized Darcy's law equation. The result of such modeling of flow depends on the character of the division of reservoir into blocks (placement of blocks, amount of blocks used, type of grid selected etc.) and formulation of equations of flow.

It must be mentioned at this point, that, notwithstanding the fact that different types of grids were presented and discussed in details in literature, usage of some of them together in one simulation (for example, in order to correctly handle some properties of reservoir) was always a difficult, if not impossible to solve, problem. These problems sometimes could be solved by a very special cases such as hybrid gridding techniques (Figure 3.1) or local refinement (Figure 3.2). And still, you would face up with the situation when each block depends on the placement of nearby blocks.

One of the advantages of the Voronoi gridding technique is that grid points and therefore grid blocks can be placed anywhere inside the model without taking other points into account. This results in absolute independence of placing of grid points from adjacent blocks and therefore high flexibility of Voronoi grids. Because of this property of Voronoi grids, it has been widely exploited in many different disciplines such as crystallography (Mackay, 1972), fluid mechanics (Trease, 1985), electrical engineering (McNeal, 1953), physics (Winterfield et al., 1981), biology (Richards, 1974), mathematics (Voronoi, 1908), rock characterization (Pathak et al., 1980) and many others.

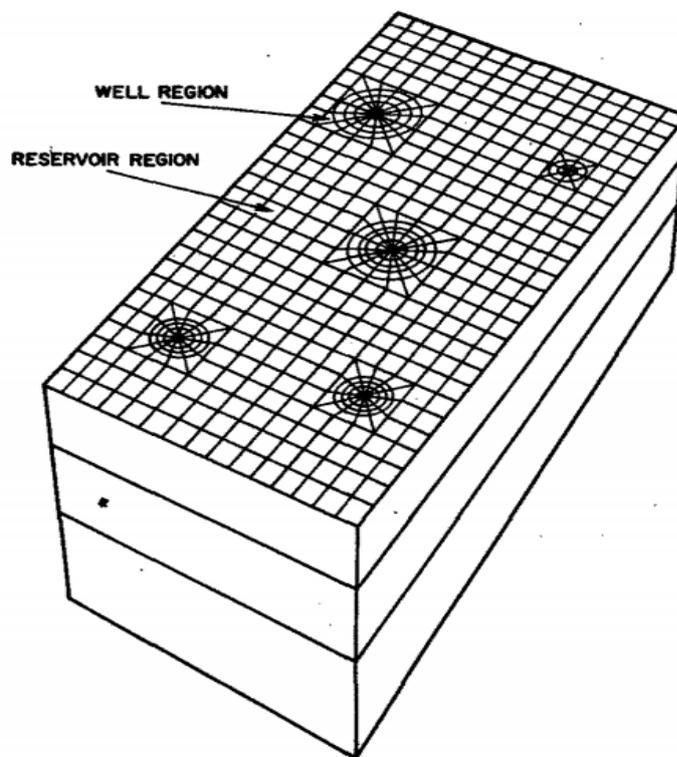


Figure 3.1. Example of usage of hybrid gridding in reservoir simulation (modified from Pedrosa and Aziz, 1985).

Voronoi grid blocks have been known under different names such as PEBI (PErpendicular BIsection) and Wigner-Seitz cells, but in the most of the papers Voronoi grid is the most widely spread name of them, which refers to the mathematician who invented them. Heinemann and Brand were the first ones who

used Voronoi gridding technique in problem of modeling fluid flow in hydrocarbon reservoirs. First of all, they depicted a way to use equations of flow for a block with an unspecified number of neighboring blocks. This was done by usage of the integral discretization technique. Then Forsyth used Voronoi to develop better accuracy of junction of fine Cartesian grid blocks with coarse ones in the process of refinement.

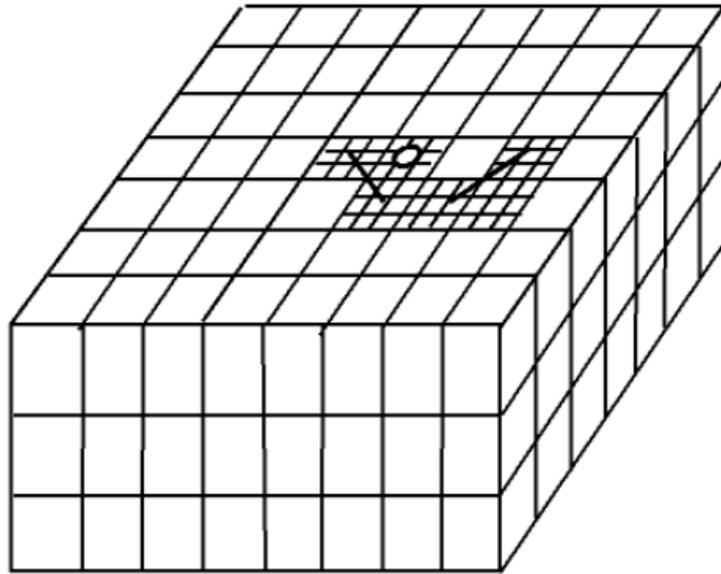


Figure 3.2. Example of local grid refinement (modified from the Kilic and Ertekin, 2003).

Voronoi grid consists of Voronoi grid block which are defined as the area around grid point that is closer to this point than to any surrounding ones (Figure 3.3.). Boundaries of grid blocks are perpendicular to the line connecting neighboring grid points and intersect this line just in the center (that is why it is also called perpendicular bisection). The latter means that Voronoi grid can be associated with point-distributed type of grids.

On the figure 3.3, dashed lines that are connecting neighboring grid points are called Delaunay mesh which consists only of triangles. If the line exists, it means that flow can occur between the points that are connected. Actually, Delaunay mesh can consist not only of triangles, but also of lines, rectangles and higher order polygons.

In most of the cases reservoir and petroleum engineers are concerned with Voronoi gridding more than with Delaunay mesh.

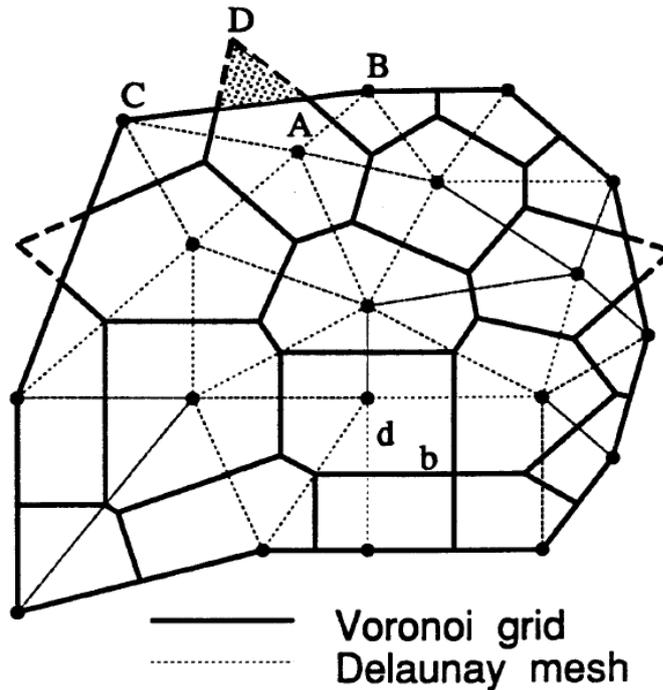


Figure 3.3. Voronoi grid and Delaunay mesh. (modified from Palagi and Aziz, 1994).

3.2. Motivation to use

Most of the grid systems that are commonly used in reservoir simulation actually are some form of Voronoi grids. Even if they are not exactly the same, they are very close to each other. Examples on such gridding techniques are shown on the figure 3.4.

Voronoi grids can connect different grid types or coarse/fine grids without applying any sophisticated algorithms. All that is required is to add grid points in required places and run grid generation algorithm as usual, all conversions will be performed automatically. The result of this is that all required gridding techniques can be used at the same time in the same grid system which develops better handling of complex structures that have to be mapped and many other problems.

Voronoi grid can also be used for simulating three-dimensional reservoirs. In this case, usually Voronoi grid is created in the same conventional way for each of the layers one by one.

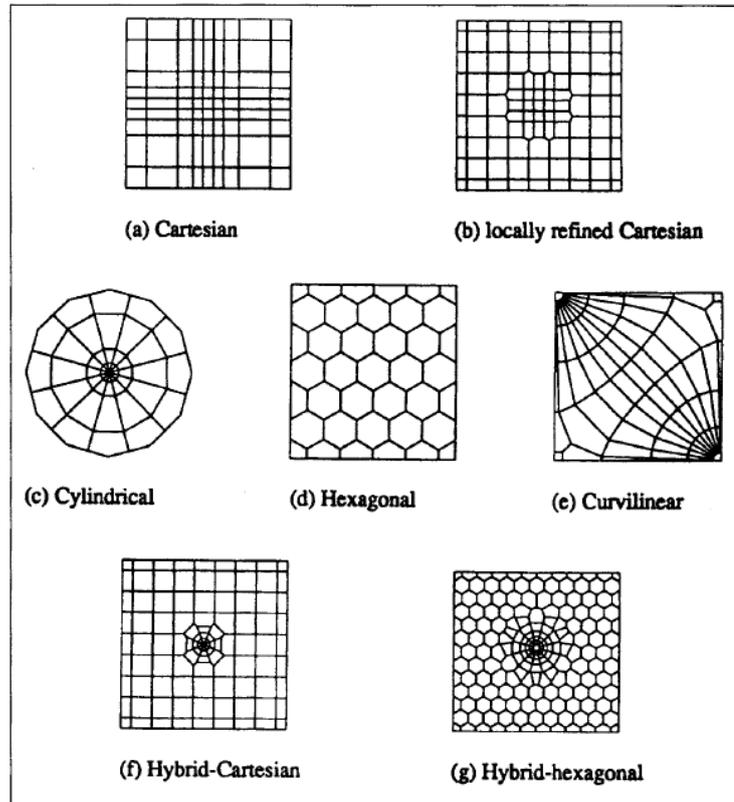


Figure 3.4. Common grid techniques that can be associated with Voronoi (modified from Palagi and Aziz, 1994).

3.3. Voronoi grid generation algorithm

There are many different grid generation algorithms. They are discussed in many literature sources, such as paper by Ho-Le (Ho-Le, 1988). In this thesis, only one grid generation algorithm will be presented in order to provide some information how it occurs. The algorithm described here was created by Frederick et al (Frederick et al., 1970).

This algorithm requires two inputs. One is the set of grid points of the blocks that will be generated, and the other one is r_{\max} - the maximum radius. This r_{\max} is used in

characterization of outer boundary. If r_{\max} is a large number, outer boundary of each block will be convex, if not, then it may be concave in some regions. Also, it must be said that the user of this algorithm is not required to explicitly identify grid points on the boundary of the region that will be divided into blocks. For more detailed discussion Palagi work (Palagi, 1992) can be referred to.

1. Choose a grid point (m).
2. Get the points that may become neighbors (n) in such a way, that the spacing between (n) and (m) would be less than the value of r_{\max} multiplied by two ($L_{ij} < 2 * r_{\max}$). After all these points are selected, all other points are stopped to be considered during the next stages.
3. Choose the point on the closest distance from the grid point (m) (minimal L_{mn}).
4. Now you have line (mn). Find the next grid point (o) moving in counter-clockwise direction, so that $m\hat{o}n$ would be maximal.
5. Next step is to generate a circle that all three points lay on and calculate radius of it. This radius is then named as r_c . Now the first vertex of Voronoi grid block with center (grid point) in (m) can be found as the center of the circle. This vertice may fall outside of the area that will be divided into blocks (e.g. point D in figure 2.3).
6. Then there are two cases: if $r_c < r_{\max}$, (o) is really a neighbor of block (m). Then you must set (n)=(o) and redo stages four and five. After some time the new neighbor is the first one, which means that all neighboring points have been processed. If this is the case, then grid block (m) is totally inside of the domain, and the other point for generation should be selected. Then everything is done from the beginning. This procedure should be performed for all points.
7. The second case is $r_c > r_{\max}$. This means that grid block intersects the outer boundary of the domain. If this is the case, continue with the next step.
8. Make (n) equal to the first grid point as in the third step. Perform stages four to seven in the clockwise direction till you reach another point outside of the domain. Then start from the beginning with the new point and continue while all the grid points are not processed.

9. After stages one to eight have been implemented for each of the points, the next step is to calculate all angles between points on the border of the domain and the corresponding grid points, such as angle $\hat{C}AB$ on figure 3.3.
10. Then there are two cases. Both of them will be discussed on an example of $\hat{C}AB$. If this angle is less than $\pi/2$, then central point of BC is a vertex of the grid block that contains points B and C.
11. Otherwise, if this angle is bigger than $\pi/2$, then some part of the grid block must be out of the domain and therefore must be deleted. After this outside part is deleted, neighboring blocks also should be adjusted.
12. And the last step is to delete all the lines that have width less than some predefined small number

(Palagi, C. L. and Aziz, K. Appendix (1994))

As it was said before, there are many other Voronoi grid generation algorithms that can be found in literature. Some of them are: Fortune's algorithm (or sweep line algorithm), Lloyd's algorithm, Bowyer-Watson algorithm etc. In this study Voronoi generation was used only for visualization of results. This visualization was performed by use of Matlab software using "Voronoi" function.

3.4. Use of Voronoi grid in reservoir simulation

As it was mentioned, use of Voronoi grids in reservoir simulation was firstly described in 1989 by Heinemann and Brand. After this introduction many scientists and engineers started to explore newly discovered horizons, perfect what was already done and tried to find additional use to them. This subchapter provides some information on how Voronoi grids were used in petroleum industry during last 26 years.

In the first years of usage of Voronoi grids one of the most productive unions was duet of Cesar Luiz Palagi and Khalid Aziz in Stanford university. In 1992 Palagi graduates from Stanford University and publishes his PhD dissertation called "Generation and application of Voronoi grid to model flow in heterogeneous

reservoirs" (Palagi, 1992). His supervisor on this work was Khalid Aziz. After graduation they publish together several more papers related to Voronoi gridding in reservoir simulation (Palagi and Aziz, 1993; Palagi et al., 1993; Palagi and Aziz, 1994). Most of these papers concentrate on general application of Voronoi to reservoir simulation, but some of them also discuss proper handling of horizontal and vertical wells using Voronoi grids.

After usage of Voronoi gridding technique in reservoir simulation proved to be efficient, several authors tried to create commercial black oil simulators that will use Voronoi grid in order to model reservoir behavior. Such type of model is discussed in the paper of Kuwauchi et al. (Kuwauchi et al., 1996). In this paper results obtained from the simulator using Voronoi grids are compared with analytical solutions and decision on effectiveness of reservoir simulator with Voronoi grids is made.

In the XXI century applications of Voronoi grid in reservoir simulation increase with more and more different applications. Some authors provide information on geological models' upscaling techniques with Voronoi (Prevost et al., 2004; Branets et al., 2009), others try to generate grid in such a way so that it would honor not only geological structures, but also flow of fluids in the reservoir (Castellini, 2001; Mlachnik et al., 2006; Merland et al., 2011; Moog, 2013); some of the authors propose new Voronoi generation algorithms (Evazi and Mahani, 2009; Katzmayer and Ganzer, 2009), others provide techniques for better handling of wells and fractures (Syihab, 2009; Li, 2011; Olorode, 2011; Fung et al., 2014).

Nowadays, Voronoi package can be found in some of the popular commercial simulators, however, usage of Voronoi grid in the industry is still not very popular. Among causes of this, Fung et al. (Fung et al., 2014) mentions extra stages that are required in order to generate Voronoi mesh, difficulties in populating of properties into Voronoi grid blocks and in the calculation of data related to well perforation. Also he mentions that in further stages of reservoir simulation generation such as history match, future predictions runs with different well locations etc. Voronoi grid requires more sophisticated and therefore less attractive reservoir modeling tools, which results in overall unattractiveness of the method. Another paper written by

Vestergaard et al. (Vestergaard et al., 2008) describes application of Voronoi grids to the problem of modeling of giant carbonate reservoir. Among the complications that they dealt with while building the model, problems with history match, inefficiency of linear solvers which were less efficient than for the case of Cartesian grid with similar grid block sizes are mentioned. Also it must be said, that before trying to apply Voronoi gridding technique to this problem, Cartesian grid simulation was performed, which was proved to be incompatible with the real data.

So, decision on whether to use or not Voronoi gridding technique in reservoir simulation is still open.

CHAPTER 4

RESERVOIR HETEROGENEITIES AND ANISOTROPY

4.1. Introduction

From the petroleum engineering point of view, definition of term "reservoir heterogeneity" would be geological intricacy of a reservoir and how this intricacy affects flow of fluid. (Alpay, 1972) In simpler terms, it is "spatial changes of reservoir properties in reservoir".

This complexity is usually a result of changes in strata that occur after deposition, for example, under compaction, tectonic distortion and cementation. There are different classifications of reservoir heterogeneities, but the most widely used are as follows: microscopic heterogeneities (less than 1mm), mesoscopic heterogeneities (up to 1m), macroscopic heterogeneities (tens of meters) and megascopic heterogeneities (hundreds of meters) (figure 4.1)

Microscopic heterogeneities are heterogeneities on scale of pores and grains of formation. Mesoscopic heterogeneities can be seen on vertical measurements, e.g. during coring and logging. They alter such properties as permeability of matrix, rock-fluid interaction, formation damage and directional fluid flow. They include bedding, changes in lithology, and others.

Macroscopic heterogeneities occur on the interwell scale. They include faults, pinchout, erosional cut-out and others. Macroscopic heterogeneities can be seen during well tests or on seismic survey results. They show great effect on sweep efficiency, patterns of flow, profitability of secondary recovery and EOR.

Megascopeic are the biggest possible reservoir heterogeneities. They occur on a fieldwide scale. They are related to depositional environment and the structure of the field. Usually megascopeic heterogeneities affect petroleum reservoir volumetrics, and therefore petroleum production trends.

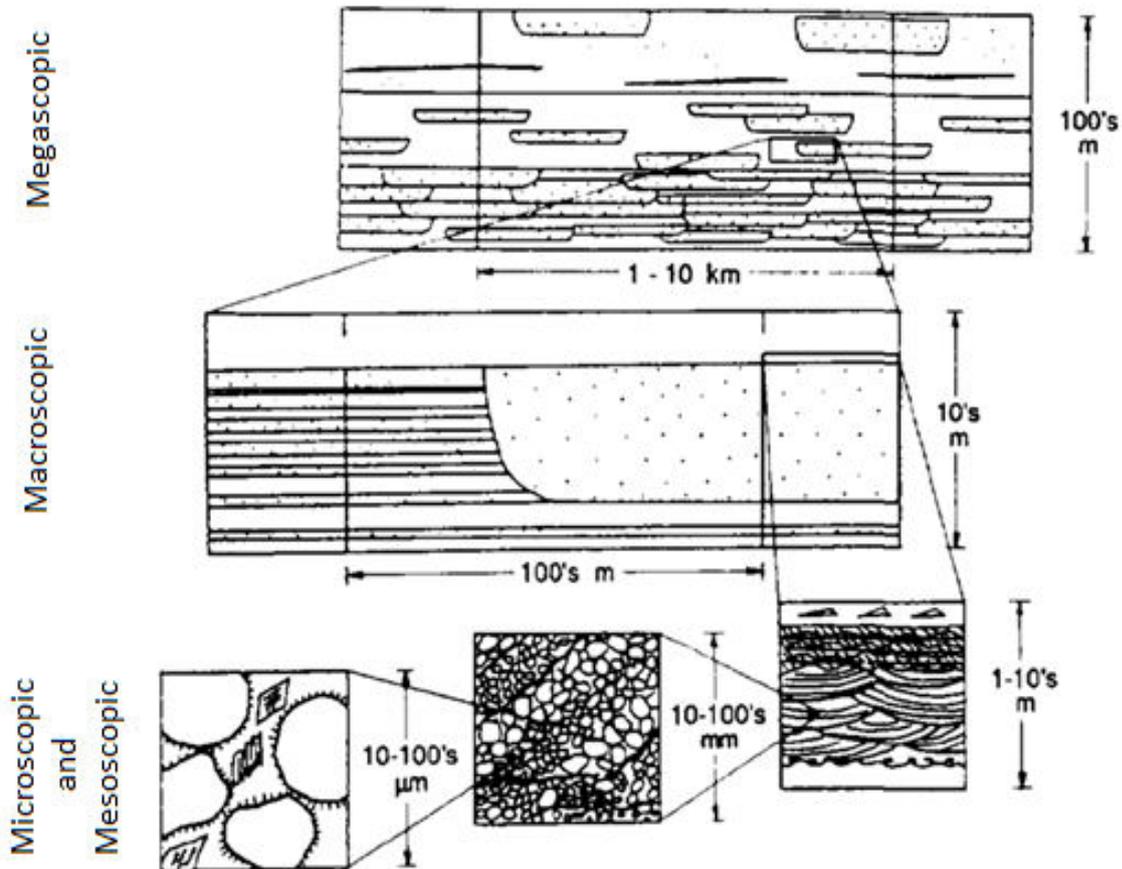


Figure 4.1. Reservoir heterogeneity classes (modified from Weber, 1986).

This study faces up with one type of reservoir heterogeneity that will be discussed further in the chapter - channeling.

4.2. Channeling

Channeling is found usually in fluvial deposit systems. This means that during some time in the history here existed flowing body of water, e.g. river. Actually, there are two types of fluvial deposit systems: braided and meandering fluvial systems.

Braided fluvial pattern usually occurs when the river does not have enough discharge to take its sediment load with itself or in the cases when the river has banks that can be easily eroded. In most of the cases braided pattern can be found in the upper parts of a fluvial deposit system. In those regions bodies of water usually have steeper gradients, mainly coarse sediments and frequent changes in discharge. These conditions result in frequent intersection of channels, as it can be seen on figure 4.2. So, this means that the channel that is created in result is a very complicated system consisting of great amount of frequently intersecting channels.

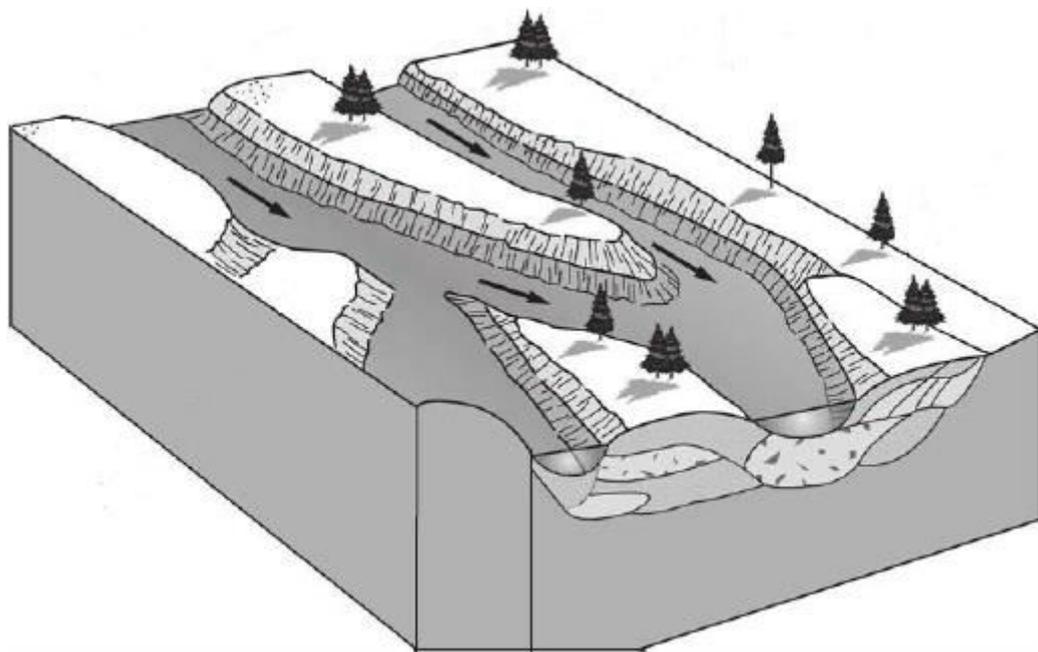


Figure 4.2. Braided fluvial deposition system (modified from Galloway and Hobday, 1996).

As it was already said, frequent discharge changes result in overloading of sediment. During flood, body of water is able to move all of its sediments. Nevertheless, usually rivers have little amount of flowing water, which results in inability to move sediments by flow. Because in the upper parts of fluvial deposit system coarse sediments are usually deposited, base of the resulting channel consists of coarse particles, which means better reservoir qualities in the future (if this structure is not affected greatly by the post-depositional conditions).

Meandering fluvial pattern (Figure 4.3) occurs in lower parts of fluvial deposition system. This is due to more gently sloping gradient than in the braided systems. The closer braided systems are to the source of the river, the straighter they are; the farther they are from the source, the more meandering character they get, until fully meandering system is not created. Here, flow has less speed, higher depth, which results in the fact that stream becomes affected by centrifugal force and bends towards the external bank. Because of this, external bank becomes severely eroded, the river is able to move towards this bank deeper in lateral direction. Therefore, the river itself becomes more and more tortuous until these sides of the river are not separated from each other by a thin layer of formations. After some time this layer is also eroded, and now the river has a better, straighter way to move, leaving one of its flanks behind. These left flanks are then called cutoffs.

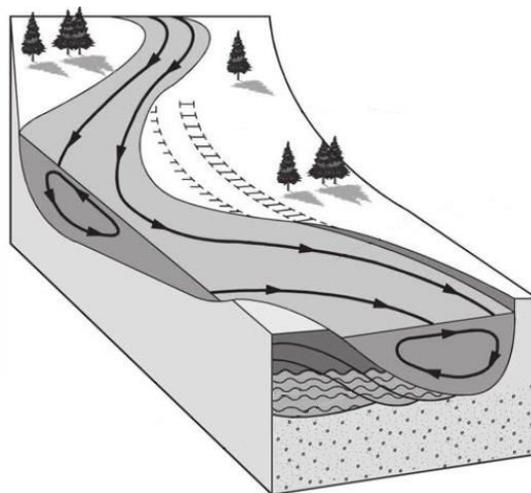


Figure 4.3. Meandering fluvial deposition system (modified from Prothero and Schwab, 2014).

Sediments accumulated here are mainly on the inner sides of the river. As in the braided fluvial deposition system, sediments closer to the source are coarser ones, while towards the end they become finer. (Prothero and Schwab, 2014)

These effects result in heterogeneities called "channeling" in reservoirs. In these channels property values may differ from the same properties of the part of reservoir

not affected by this channel. This is one of the cases, that was considered during this study.

4.3. Anisotropy

A formation is called anisotropic if the value of property in one direction differs from the value of the same property in another one. Most usual earth anisotropy is between vertical on horizontal directions, called transverse anisotropy, but it is not considered in this study. This study tries to deal with anisotropy of directional permeabilities in the horizontal plane, thus permeability in x-direction differs from permeability in y-direction. Before going further, this chapter will explain where anisotropy comes from and why it is different from reservoir heterogeneities.

As it was already mentioned, anisotropy is not the same as heterogeneities discussed previously in the chapter, however, they are usually confused with each other. There are two main differences between them.

The first one is that in anisotropy changes of reservoir properties occur at one point, but in a different direction (vector value), while heterogeneity means that there are differences in scalar or vector values in two or more different points. The second difference is that anisotropy deals mainly with physical properties, while heterogeneities may deal with anything starting from the same physical properties and ending with the composition of formation.

Anisotropy results from processes occurring during and after deposition. For example, anisotropic changes in carbonates, such as changes in directional permeabilities, may be a result of layering which affects carbonate mineralogy by changing formation diagenetic potential and texture. As opposed to carbonates, in the clastic rocks anisotropy can occur only if the rock is homogeneous or uniform to some extent. If the formation is totally heterogeneous, then no anisotropy can occur there, because in this case there will not be any directionality in the rock. Summing this up we may come up with a conclusion that anisotropy developing with the

deposition has two causes: periodic layering and grains ordering, which results from the directionality of the rock. (Rajan, 1988). This ordering is mainly performed by gravitational forces and transport.

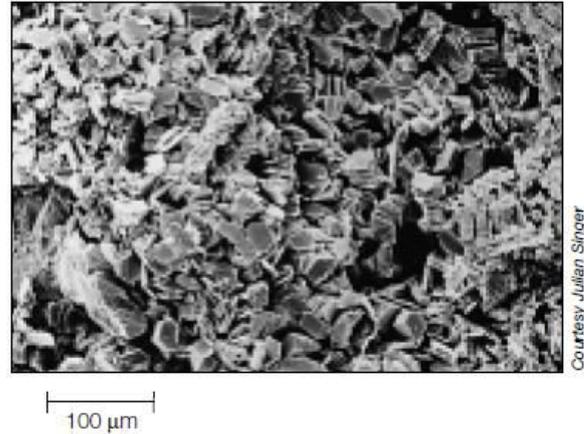


Figure 4.4. Example on diagenetic changes (Anderson et al., 1994).

After deposition, formation undergo changes due to diagenesis. Diagenesis is exposition of formation to different forces of chemical, physical and biological character after deposition. During this stage many changes can occur in formation structure: for example, when formation is buried at increasing depth, the overburden pressure increases with depth and may cause rearrangement or rotation of grains in the horizontal plane (Manrique et al., 1994). Other factors that may affect formation properties in the horizontal plane are fractures or plastic deformation and many others.

For understanding of anisotropy, processes occurring during diagenesis should be always considered, because they can dramatically change the properties of the formation, even properties already changed during deposition. For instance, alterations in ordering/packing and horizontal orientation (this also affects formation permeability) that took place during deposition may be totally demolished by the diagenetic processes.

On figure 4.4 depositional anisotropy is absolutely changed by the clay and quartz overgrowth. The point of this example is that permeability model should be based on both depositional and diagenetic alterations of rock, otherwise the representation of

formation will be incorrect, which can affect all following calculations. (Anderson et al., 1994)

This study shows attempt to carefully divide reservoir, including heterogeneity and anisotropy into Voronoi blocks in order to produce a representative result using limited amount of grid blocks.

CHAPTER 5

OPTIMIZATION

5.1. Introduction to optimization

Optimization, as a tool helping to solve different kind of problems, was accompanying humankind from the very beginning of its existence. Actually, at first this kind of optimization was absolutely primitive and was based on the instincts of early humans: they waited for most optimum conditions to plant or harvest crops, decided on whether to start a war with another tribe or used optimization when hunting animals - how many men are required to track down an animal and to kill it in as safe manner as possible.

With the introduction and development of mathematical methods, optimization methods also underwent an advancement, but still were quite primitive. The greatest advancement of optimization techniques took place in last fifty-sixty years with the development of computational technologies. After that, optimization methods had dramatic improvement that is still continuing nowadays. While optimization algorithms were developing at an enormous rate, the technologies required for implementation of this algorithm were also advancing. This created ideal conditions for optimization, and now it is difficult to imagine complex and even usual projects in various disciplines that would not use any kind of its form (Diwekar, 2008).

Optimization is the process of choosing the 'best' out of all solutions of the problem, if the "good" can be separated from the "bad" and measured. In our day-to-day life, everyone would like to have the "maximum" in some good things, like salary, health or holidays or "minimum" in bad things as expenses, over-time work and problems.

Taking this into account, the term "optimum" may be described as the "minimum" or "maximum" depending on the conditions, for example maximizing the salary while minimizing the over-time work. So the word "optimum" is much more useful than the term "best" in the same way as the term "optimize" is a way better than the word "improve". So, theory of optimization is a section of mathematics dealing with the study of optimum solutions of the problems and the procedures to obtain them.

As it was said before, the optimization is used in a wide range of various disciplines including math, physics, business and economics, social sciences, engineering and even politics. It covers all engineering disciplines, starting from chemical and petroleum engineering and ending with mechanical and electrical engineering. Most common engineering areas of usage of optimization algorithms include design of buildings, creation of tools, curve fitting, modeling of systems and many others. Almost all real optimization problems do not have only one solution. Actually, amount of solutions may be up to infinite. That is why optimization based on some of the criteria that govern the behavior of the solutions is so important. (Antoniou and Lu, 2007)

5.2. Classes of optimization algorithms

An algorithm is a collection of actions that should be performed in order to solve some problem. They are usually written with human language, not with computer code, so it would be easy to understand for humans and would not depend on the programming environment or specific computers. Optimization algorithm is an algorithm of the type described in the previous sentence, that can be used to get optimum solutions of optimization problems.

There are two broad groups of optimization algorithms: deterministic and probabilistic.

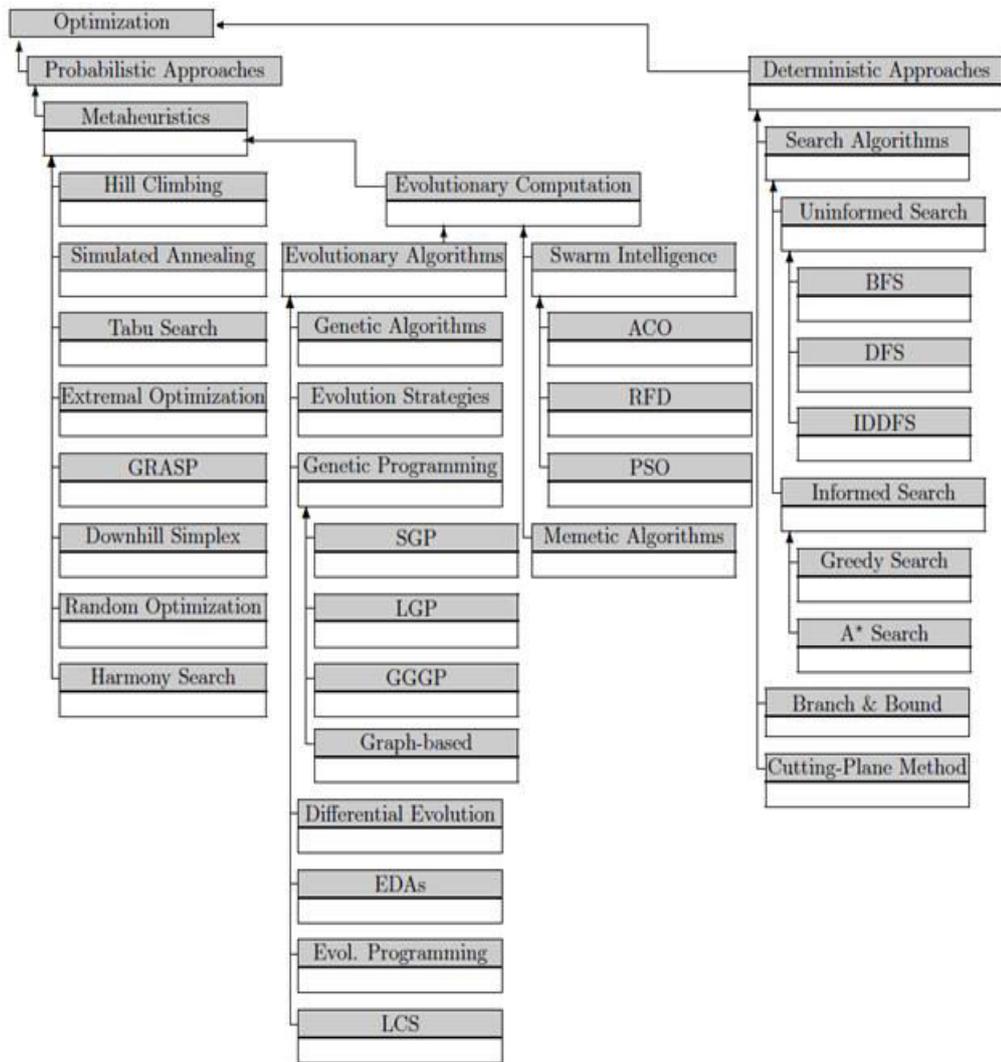


Figure 5.1. Rough classification of optimization algorithms (modified from Weise, 2011).

Further subdivision is quite a difficult job, because some of the algorithm classes take properties of both basic classes, but some rough estimation is shown on the figure 5.1.

Deterministic optimization algorithms are algorithms that during each step have only one way to move to the next one. This means that if the same set of data is used as input, this algorithm will do absolutely the same thing and the results will also be the same. So, this type of optimization algorithms is mostly suitable for the cases when the most efficient decisions on how to proceed in different situations are known and used in the algorithm. These cases occur when the dependence between the different properties of the probable solutions of the problem and their utilities are clearly understood and used.

In some cases the manner of how the deterministic algorithms approach the problem, may cause problems in getting the most optimal solution. This is the situation when the dependence between the solution and the goodness of it is not so straightforward (for example, changing or very complex), or when the size of the search space is enormous. In such kind of cases application of deterministic algorithms is not very efficient and the use of probabilistic ones is much more effective choice. Deterministic algorithms include search algorithms, which are subdivided into informed (including Greedy and A* searches) and uninformed search (including Breadth-First search (BFS), Depth-First search (DFS), and Iteratively Deepening Depth-First Search (IDDFS)).

As opposed to deterministic optimization algorithms, probabilistic ones have minimum one step in it that is based on the generation of random numbers. This means that the approach will generate random solutions, which is a very useful step if you do not know exactly how to proceed. These random approach, of course, has disadvantages - for example, if the set of input data is the same, algorithm still will produce different results, and still in many cases, they are preferable. Probabilistic optimization algorithms include metaheuristics which is further subdivided into evolutionary computation algorithms and algorithms that are not referred to that

category, including hill climbing, simulated annealing and many others. (Weise, 2011)

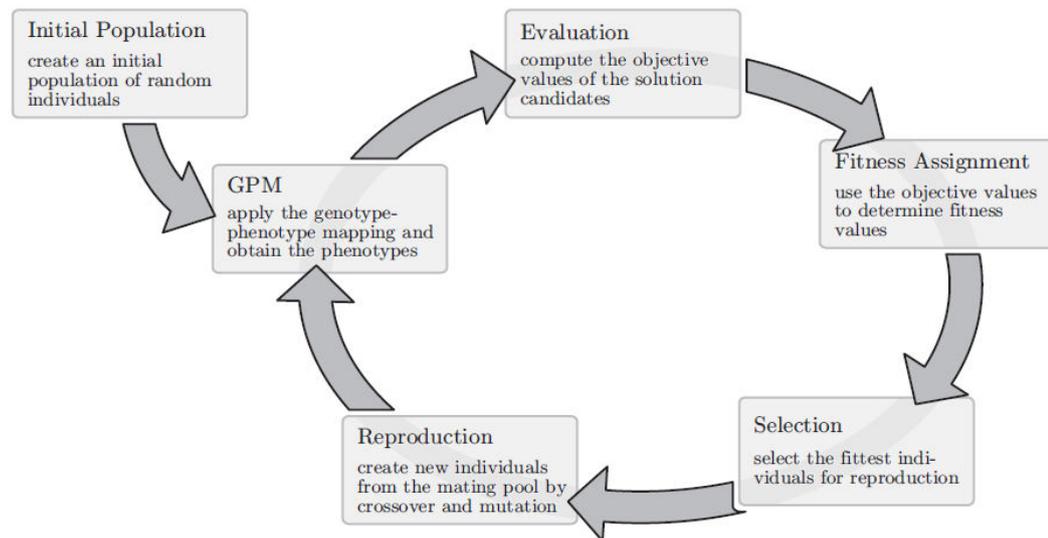


Figure 5.2. The basic cycle of evolutionary algorithms (after Weise, 2011).

As it was mentioned before, many algorithms share some properties of both probabilistic and deterministic algorithms. Algorithm created during this study is not an exception: it takes concepts of populations and fitnesses from the evolutionary algorithms which are related to the probabilistic types, however, it is purely deterministic in its nature. So, in order to provide better background on the algorithm, the next subchapter provides some preliminary information on evolutionary algorithms and their concepts shared with the algorithm described further in the thesis.

5.3. Evolutionary algorithms

Evolutionary algorithms are probabilistic optimization algorithms based on population of solutions. This means that input to these algorithm is not a single probable solution of the problem, but a set of different ones that will be processed in order to get the most optimum one. As it may be understood from their name, these algorithms are based on

analogy with some natural processes and mechanisms, such as biological evolution and survival of the fittest conceptions. These algorithms use these ideas in the processing of the population of solutions, hoping to find the one that is most suitable for the problem under consideration.

Evolutionary algorithms were not developed together by one man. Actually, they include a large set of different algorithms that were created to some extent by different scientists and researchers. Evolutionary algorithms showed high flexibility and performance in a wide range of different disciplines.

Despite that majority of evolutionary algorithms were invented by different scientists, all of them undergo similar steps when initiated. These repeating steps are called the basic cycle of evolutionary algorithms (figure 5.2). Following are the stages of this cycle for evolutionary algorithm optimization with a single objective:

1. Population of probable solutions is used as an input to the algorithm. This will be the first generation of solutions ($t=1$). This population may be random or seeded, depending on the person who uses the algorithm and the problem. If the initial population is seeded, this means that the used solutions are already preprocessed and adapted to some extent to the criterion that is used for optimization. It can even be the case when the whole first population consists of such preprocessed individuals. These adapted solutions can be obtained by using the same algorithm or by any other method. Seeding is used to reach the optimum solution in a less number of generations. The drawback of using seeding is that the fake convergence may be reached before the actual convergence is achieved. This may be the result because good solutions (or preprocessed) will have better fitness and will wipe out the random solutions. So, this type of behavior must be considered before applying of seeding.
2. Next step is to calculate objective functions for each individual solution in the population.

3. Using values obtained from the previous step, distribute the values of fitness to each of the individuals.
4. The next step is selection process - choosing individuals with higher fitness values for reproduction, therefore wiping out individuals with lower fitnesses.
5. The next step is reproduction - getting offspring from the parents selected in the previous stage. This is done by applying of specialized reproduction operations to the genotypes of the parents. Genotypes are just sets of genes. They are usually obtained by converting solutions into binary, however, there are also many other methods. Each "0" or "1" value inside of these genomes are referred to as genes. This conversion is done for easier application of reproduction operators. Most common reproduction operations are crossover and mutation. Crossover is used to combine genotypes of parents together to get genotype of children and mutation just makes changes in one genotype in order to support diversity in the solutions. After reproduction is performed, part of the last generation or the whole previous generation is swapped with the children, depending on the choice of the person using the algorithm. This will be the new generation. Number of generations is increased by one.
6. If the termination criterion is achieved, then everything stops here and solution with the highest fitness is selected as the best one. If not, then convert genotypes to phenotypes (this means converting of solutions into the form, which is better suitable for calculation of objective functions. For example, if the solutions are given in binary, convert them into system suitable for calculation, which is usually decimal), then return to step 2 and redo all the same thing for the new generations (Weise, 2011).

Now, after evolutionary algorithms were described, the last question that must be answered is why exactly algorithm created in this study is based on evolutionary optimization, but not on the other type of algorithms. The answer would be that the algorithm of the solution to the problem that will be described in the "Methodology" chapter is an iterative process, that uses population of solutions, trying to improve population's worst performing individual solutions. Actually, that is what

evolutionary optimization algorithms are all about - they are highly suitable for such kind of problems. However, the random part had to be eliminated from the process and also the process of improvement of bad solutions was changed, making the overall process deterministic, because exact direction of improvement is known from the beginning, therefore, no randomification was required.

Algorithm created during this study uses population and fitness criteria described in this chapter. More information on how exactly algorithm proceeds can be found in "Methodology" chapter in this thesis work.

CHAPTER 6

PROBLEM STATEMENT

The aim of this study is to find an algorithm that will divide the reservoir model into unstructured Voronoi grid blocks by considering the direction of permeability vectors, anisotropy ratio, permeability or porosity heterogeneity of the reservoir in such way that the defined error value in each block would be minimal. Voronoi grids are strongly connected with the locations of grid points, that is why, by adjusting the coordinates of grid points, shapes and locations of grid blocks can be altered, thus, attempt to minimize the total error can be made. Since the main pressure gradients exist around the wells, the representation of the wells is achieved by using closely placed Voronoi grids around the wells as well.

This thesis provides detailed information on algorithm created for solving of the problem described here and then shows some examples on how exactly it works.

CHAPTER 7

METHODOLOGY

7.1. Introduction

Algorithm created in this study is deterministic. As it was said before, this means that for the same input values, the results produced by the algorithm will also be the same. This algorithm consists of three major steps:

1. Generation of uniformly distributed grid points, taking into account anisotropy in the reservoir;
2. Moving of grid points that have bad locations described by fitness values (this deals with the effects of reservoir heterogeneity);
3. Adding of grid points related to vertical and horizontal wells or faults.

This chapter will cover these steps one-by-one.

Code of the algorithm written for Matlab can be found in Appendix A. Before running of the algorithm, four column vectors called permXvec, permYvec, permZvec (accordingly x, y and z coordinates of the points of petrophysical field) and permeabilitiesVec (values of petrophysical property in the locations of points assigned by vectors permXvec, permYvec and permZvec) should already be loaded into the workspace. As it was mentioned before this petrophysical property field should consist of densely and uniformly populated points with property values. For creation of the field some extrapolation/interpolation methods may be required.

7.2. Step One (generation of initial population of grid points)

As it was previously mentioned, the first step consists in the generation of uniformly distributed grid points throughout the reservoir. This distribution is created regarding directional permeability relations in the reservoir. The first step requires several input parameters: desired number of grid points (grid blocks); how many times permeability in the y-direction is higher (or less) than the permeability in x-direction, K_y/K_x (permeability in y-direction (K_y) divided by permeability in x-direction (K_x) - these directions may not align with x- and y-directions of the reservoir); angle between permeability in y-direction and y-direction of the reservoir (figure 7.1); coordinates of vertices of reservoir; some small distance and small increment that will be added to this distance at the end of each iteration.

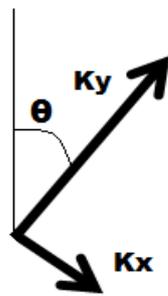


Figure 7.1. Angle between permeability in y-direction and y-direction of the reservoir.

Figure 7.2 shows an example on how the first step algorithm proceeds. The first thing that the algorithm does - it finds coordinates of the starting point (red dot at figure 7.2). If angle θ is between 0° and 90° , then it calculates red dot coordinates from:

$$startX = -b * \sin(\theta) * \cos(\theta) \quad (7.1)$$

$$startY = b * \sin(\theta) * \sin(\theta) \quad (7.2),$$

where startX - x-coordinate of starting point; startY - y-coordinate of starting point; b - reservoir width. Minus sign is used in equation (7.1), because the lower left corner of the reservoir is considered as (0;0) point, which means that starting point will have negative x-coordinate.

If angle θ is between 90° and 180° (or 180° and 270° , or between 270° and 360°), then 90° (or 180° , or 270°) is subtracted from this angle and then formulas (7.1) and (7.2) can be used. If angle θ is equal to 0° , 90° , 180° , 270° or 360° , then lower left corner of the reservoir (0;0) is chosen as the starting point. After starting point is selected, the next step is to generate grid points.

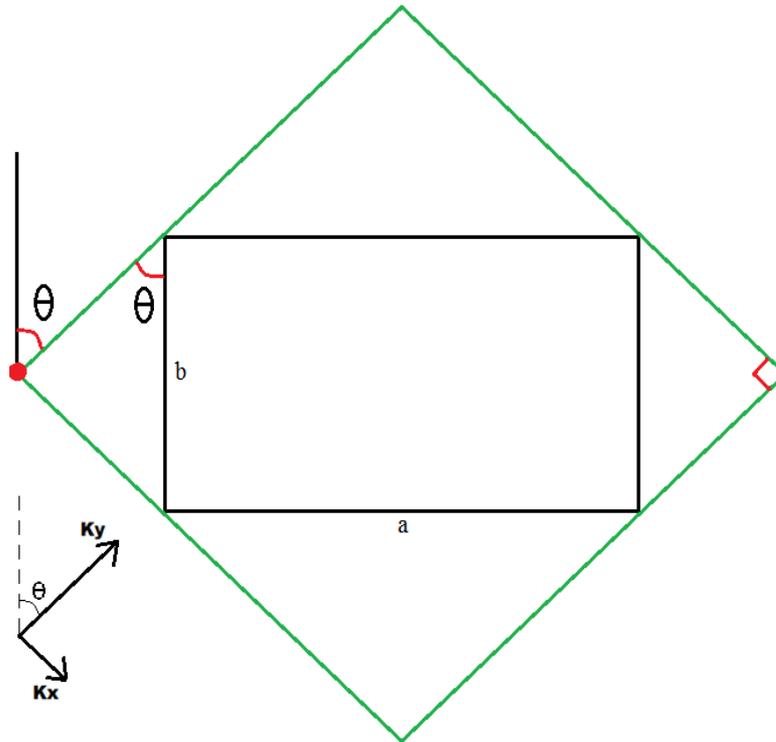


Figure 7.2. First step example. Black rectangle - reservoir; green rectangle - area, where grid points will be generated; red dot - starting point; a - reservoir length; b - reservoir width.

The main idea behind this generation is that if permeability is higher in one direction (x or y), then generated points need to be placed more densely in that direction - in other words distance from one point to the next one in the permeability y-direction divided by distance from the same point to the next one in permeability x-direction should be equal to K_y divided by K_x .

So, for example, if permeability K_y is twice as K_x , then distance between points in K_x direction is twice the distance between points in K_y direction.

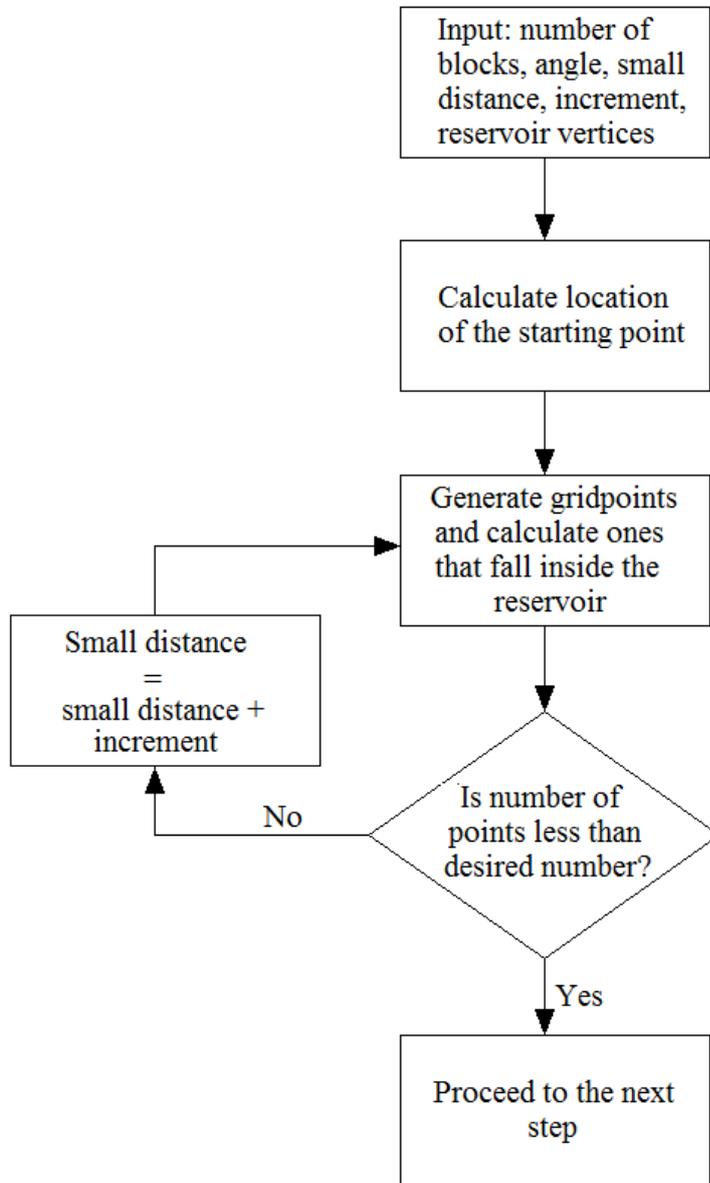


Figure 7.3. Flowchart of step one.

This is because if permeability is higher in one direction than it is in the other, then pressure disturbance generated by producing/injecting wells, when the simulation will run, will propagate at a higher rate in the direction of higher permeability.

So, in order to accurately see how much this disturbance moves, grid points should be placed more densely in that direction.

Because we know only relation of the distance in one direction to the distance in the other one, but not exact values, we need to take some random small distance for one direction and calculate distance in the other, so that we could generate these grid points - that is what small distance in the input is used for. So, the algorithm starts to generate points from starting point in K_y and K_x directions according to K_y and K_x values and the small distance from the input. As it can be seen, some of the generated points will fall outside of the reservoir (black rectangle), so after all of the points are generated these points outside of the reservoir are ignored, while all the points inside reservoir are counted. If the resulting number of grid points is higher than the desired number of points entered as an input, then small distance is increased by increment also entered as an input, then process starts from the beginning with these new distances. This process is done until the number of grid points is not less than the desired number. If distance in the beginning and increment are not very large, resulting number of blocks should be very close to desired number.

After the required number of grid points is achieved, the algorithm proceeds to the second step. For better understanding, step one flowchart is shown on figure 7.3.

7.3. Step Two (movement of the bad grid points)

After all the grid points were generated, the next step is to check if their placement is good enough. If it is bad, it would be better to move them to better places. This step also requires some input values, namely: field of petrophysical property that will be used to define different regions of the reservoir, number of iterations, number of

worst points that will be moved during each iteration, the number of times each point can be moved, required sum of errors in all of the blocks, number of regions and intervals of property values for each region.

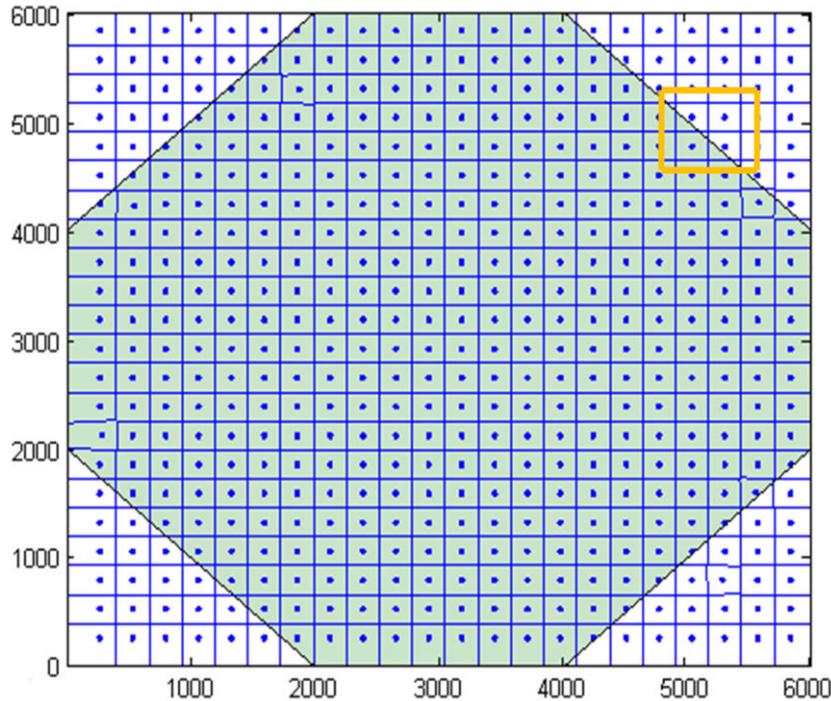


Figure 7.4. Example on results obtained from the step one. Blue area - reservoir; white area - zone outside of the reservoir.

Property field is the population of densely spaced points in the reservoir that provide some information about the reservoir property at their locations.

These property points should be representative of different regions/formations that can be met in the reservoir rock. For example, if there is channeling with better rock properties (like porosity, permeability) than the surrounding reservoir rocks' properties, then these values can be used in the algorithm.

As it was written before, this field should consist of very densely spaced points. This means that if there is not enough information about reservoir properties are obtained from previous studies in the field, some interpolation/extrapolation techniques can be used in order to spread property values all over the field. Example on petrophysical

field consisting of 56400 permeability points for the field shown on figure 7.4 is shown on figure 7.5.

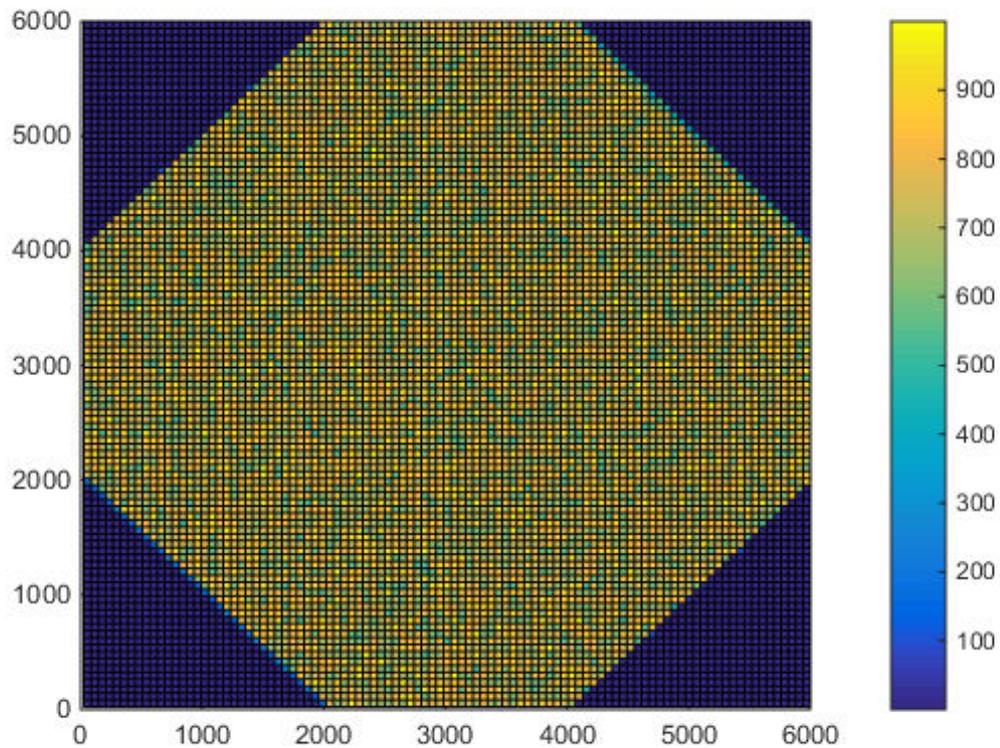


Figure 7.5. Example on petrophysical field.

It must be mentioned that this property field should also include points outside of reservoir with zero values, so that algorithm would understand where the boundaries of reservoir are located.

Imagine, that the reservoir, that has to be modeled, is not of rectangular shape (or has heterogeneities (such as channeling, etc.)). The first step has already been implemented, resulting in a scheme shown on figure 7.4. Input values for the first step were: 500 grid points, octagonal reservoir, no anisotropy or heterogeneities, angle is equal to zero. Because each grid block can only have one property value calculated as the average of all property points that fall inside of this block, gridblocks on the edges of the reservoir (or on the edges of heterogeneities if they are present) will not be representative of the area they cover.

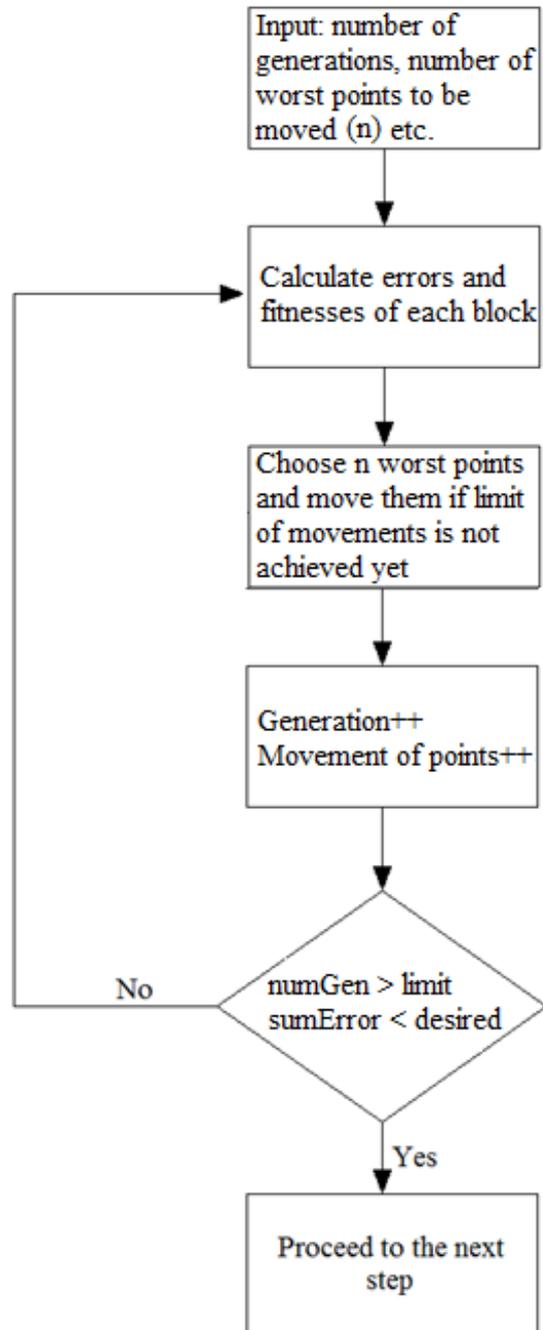


Figure 7.6. Flowchart of step two.

For example, in the figure 7.7 red dots show reservoir properties outside of the reservoir (which is zero), while the green dots show reservoir property points values (different from zero). In the figure 7.7 grid blocks 1 and 4 have property values different than zero, despite that these blocks have more than a half located outside of the reservoir.

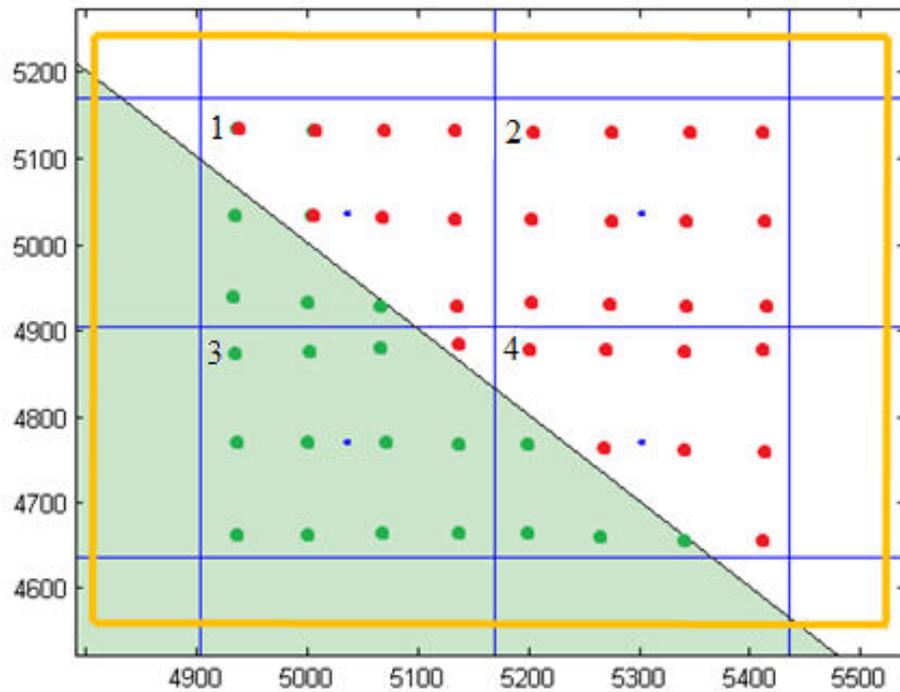


Figure 7.7. Zoom in of the orange rectangle from the figure 7.4. Green dots show property points inside reservoir; red dots show property points outside of the reservoir; blue dots are grid points.

Ideal placement of the grid points, and therefore grid blocks, would be in such way that reservoir boundary would correlate with boundaries between blocks. If it was the case, then blocks inside reservoir would have reservoir property values representative of the zone that they are located at, while blocks outside would have zero values. So the main purpose of the second step of the algorithm is to move these bad points (and consequentially block boundaries), so that the block boundaries would align with reservoir boundaries for better representation of the reservoir. This is done in the following way:

1. Calculate error and fitness values for each block. The error in the grid block is equal to the standard deviation of reservoir property values that fall inside of this grid block. For example, petrophysical field's points with permeability values equal to 500, 510, 570 and 620 md fell inside one block. Two regions of petrophysical fields were defined before: with permeabilities less than 550 md and with permeabilities higher than this values. Therefore, error of the block is equal to standard deviation of these values, which is equal to 55.98. This value is assigned to the block and is used in further calculations. However, if the block is totally inside one region, then error is equal to zero. The choice of standard deviation as an error left from very simple early runs of the algorithm, when it was very effective. Because no better alternative was found for the error function for more difficult late runs, it made its way into the final version of the algorithm.
2. Each block is also assigned a fitness value, which is its rank (or place) in a row of blocks' errors in a descending order.
3. Calculate sum of all error values.
4. For "n" (given as an input - number of points that will be moved during each iteration) least fit grid points (highest error values) do the following procedure: check if grid point can be moved at least once more; understand where grid point is located; get property points inside this block; if the grid point is inside one region, calculate mean values of the coordinates of all other property points and move the grid block in the opposite direction from the resulting mean property point at a distance given in the input.
5. Increase iteration counter by one.
6. If an iteration counter is less than a predefined number of required iterations, return to step one, do all the steps again. If the iteration counter has reached the predefined number of iterations, then take the result that has least sum of errors and use it in the next step. This means that the best result in terms of error function is chosen as an input for the next step. However, this best result is the best only for the entered input parameters, by changing them better or worse results may be achieved. This problem will be discussed in the chapter 8.3, where effects of different input values are reviewed.

Deciding on whether the obtained result is good enough for use is based mainly on error value and visual investigation of the obtained figure. If block boundaries are very far from from reservoir boundaries, then algorithm should be re-used with other input values. In order to get better results, it may be required to include less points to be moved during each iteration, but a greater number of times and therefore more generations. If these values do not help, increased number of grid points should be able to solve the problem.

In a form of flowchart this procedure would be as shown on figure 7.6. This algorithm proved to be effective in solving part of the problem described in the problem statement, however, it may encounter a problem if reservoir boundaries are too close to a rectangle, surrounding it, but isn't exactly coinciding with it.

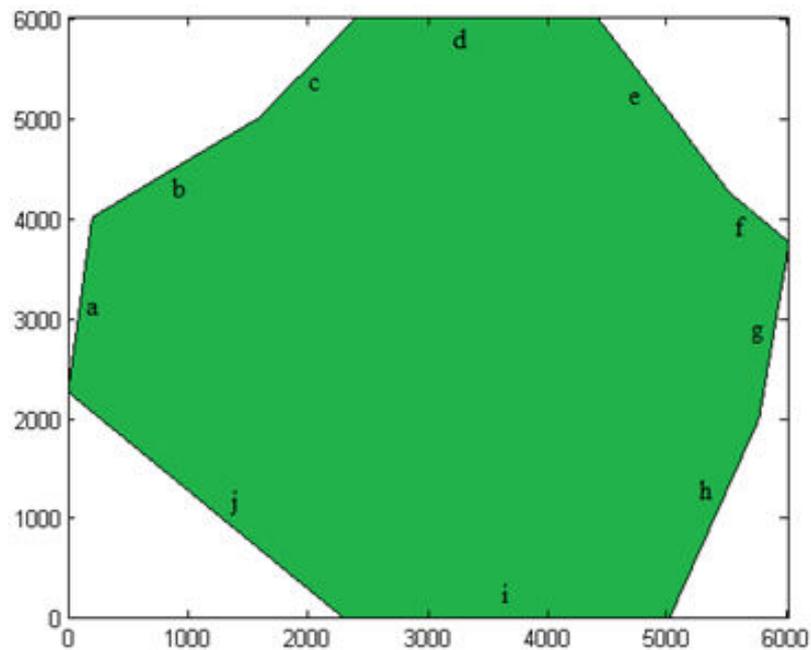


Figure 7.8. Example on reservoir.

This case is shown on figure 7.8. Reservoir sides "d" and "i" will not cause any problems, because they coincide with surrounding rectangle. However, sides "a", "f"

and "g" may cause such problem, because they are very close to the vertical sides of the rectangle.

If the distance from them to the rectangle sides is less than small distance obtained at the end of step one, then the problem may occur, because points required outside of the reservoir in order to generate block boundary coinciding with reservoir boundary, may fall outside of black rectangle and therefore will not be generated.

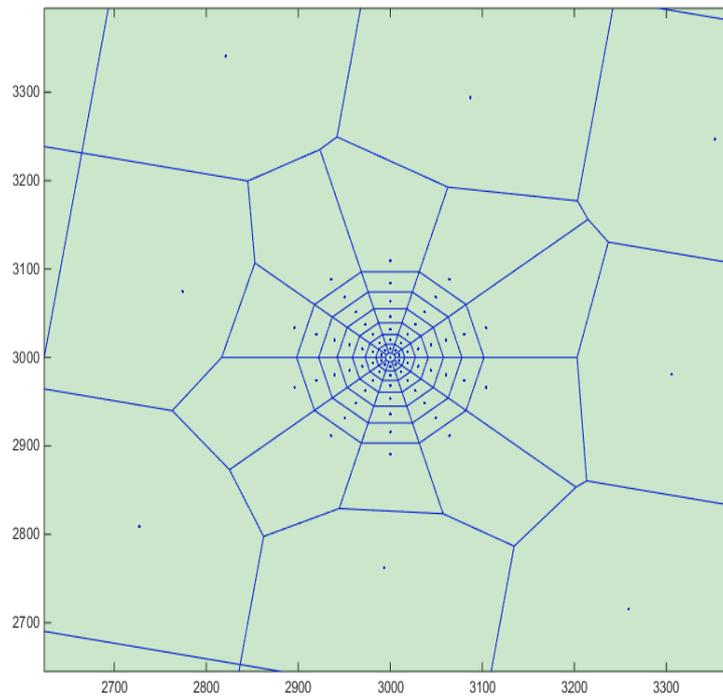


Figure 7.9. Treating of vertical wells.

This problem is solved by adding of one layer of grid points on the different sides of reservoir boundaries that have both starting and ending points closer to the rectangle sides than small distance from the end of step one. This increases the resulting number of grid blocks, but solves the problem effectively.

After handling of reservoir boundaries, points located inside rectangle but outside of reservoir can be deleted. This is done by moving of all reservoir's vertices from the center at a greater distance between the points multiplied by one and a half. Greater distance is one of the distances (in K_y or K_x direction) obtained at the end of step

one. It is multiplied by 1.5 to make sure that at least one layer of points is left outside of reservoir in order to handle all boundaries. So, reservoir vertices are pushed from the center, area of reservoir increases in all of the directions. After this all points outside of this big reservoir are deleted. These new bigger reservoir vertices are not used after this step.

As it may be understood from the algorithm, the result will strongly depend on the input parameters: how many points will be moved during each iteration; number of required iterations; distance at which least fit points will be moved. For better efficiency of the algorithm, it is proposed to use a greater number of iterations with less points to be moved during each iteration at smaller distances but for a greater number of times. However, usage of these characteristics means that calculation time will increase, so the final decision on which values to use will depend on the person using the algorithm.

After the best solution is chosen, the algorithm can proceed to the step 3, where grid points related to vertical/horizontal wells and faults can be added.

7.4. Step Three (adding of grid points related to wells and faults)

Step three adds grid points related to vertical and horizontal wells and faults. Vertical and horizontal wells are treated in different ways, while faults are treated almost in the same way as horizontal well, so discussion is divided into two blocks - vertical wells are discussed in one sub-chapter, horizontal wells and faults are discussed in the other.

7.4.1. Treatment of vertical wells

Before talking about inputs for this step, it is required to show how the vertical wells are treated. Figure 7.9 shows how grid points around vertical wells should be generated.

Here, well is located in the middle of the structure, while surrounding blocks are created in order to accurately represent a pressure drop in the area around the well. These surrounding blocks are placed very densely, because the greatest pressure drop occurs just around wells, so the more points are used there, the better representation of the real conditions can be achieved.

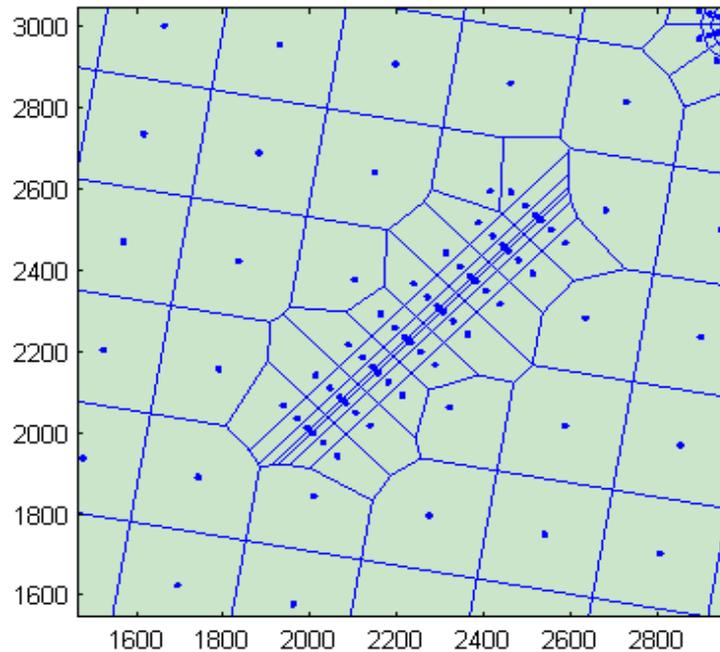


Figure 7.10. Treating of horizontal wells.

As it may be seen from the figure 7.9, new blocks should be generated around the well, and no grid points from the previous steps should interrupt their pattern. Also, the number of blocks and layers, distances between layers of points and distance from the well to the farthest layer of grid points should be considered. All these values, along with the location of the well, its radius, should be entered as an input.

After these input values are entered, the algorithm starts by deleting grid point generated during previous steps that fall into the region around well that will be repopulated. After all these points are deleted, grid point related to the well is generated, and then all surrounding points are generated layer-by-layer. This will end up with something close to the pattern shown on figure 7.9.

7.4.2. Treatment of horizontal wells and faults

As it was said, faults and horizontal wells are treated almost in the same way, so they will be discussed together. There are two main differences between them: number of layers and alternating of grid blocks' property values in the fault representation. Other than these characteristics, everything is the same. Representation of horizontal wells and faults are shown on figures 7.10 and 7.11 respectively.

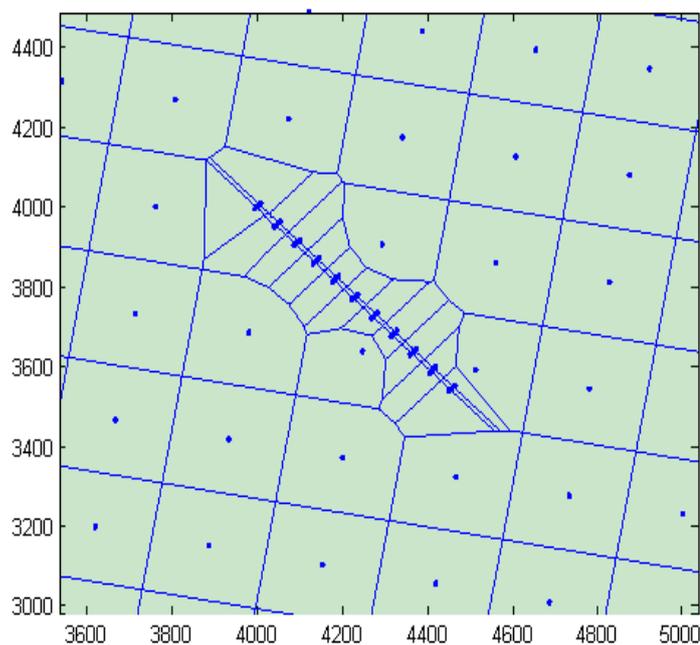


Figure 7.11. Treating of faults.

Inputs for this step are: coordinates of the fault/well; number of grid points (and number of layers for wells only) that will be added; radius of fault/well; distance from the well that will be repopulated with new points and distance between layers of grid points for wells; permeability of the fault for faults.

When all input data were entered, algorithm starts deleting grid points generated during previous steps that fall inside zone that will be repopulated.

Then it generates the layer of points related to the well/fault, and finishes with the generation of layers of points outside of well/fault. For faults, it also asks

permeability of the fault and then assigns entered value to the blocks that represent this fault. After this procedure is finished, it asks if another well/fault will be added. When all wells or faults are introduced, algorithm finishes its work by drawing result.

It must be mentioned that the algorithm described here was used only in two-dimensional problems. However, if there is a three-dimensional problem, the algorithm can be run for each layer separately. Then all results may be gathered together. If the layer is inclined and inclination angle is known, then layer can be changed into horizontal position, where the algorithm can be run, and then results can be converted back into inclined position. So, the algorithm is quite general and can be used in many cases. As it was previously mentioned, the code for running this algorithm was written in Matlab. This code can be found in Appendix A.

CHAPTER 8

RESULTS OF STUDY

8.1. Introduction

This chapter discusses the results obtained by running the algorithm for six different cases. These results will be discussed one-by-one in different sub-chapters. Table 8.1 shows what complications were added to the model in each of the cases.

Table 8.1. Description of the cases

	Case #1	Case #2	Case #3	Case #4	Case #5	Case #6
Number of points	500	500	500	500	500	600
Angle θ , °	90	120	210	45	10	90
Ky/Kx	1	3	0.4	0.5	2	2
Number of grid points moved in each iteration	5	5	5	5	5	5
Number of movements for each grid point	8	8	8	8	8	8
Distance of movement, times the distance in the direction of lowest permeability	0.07	0.07	0.07	0.07	0.07	0.07
Number of generations	450	500	450	450	450	320
Vertical well in the center	+	+	+	+	+	
Horizontal well						+
Fault						+

This chapter describes results obtained after running of the six cases described in the table and further in the chapter and makes conclusion if algorithm created for this thesis work is effective or not.

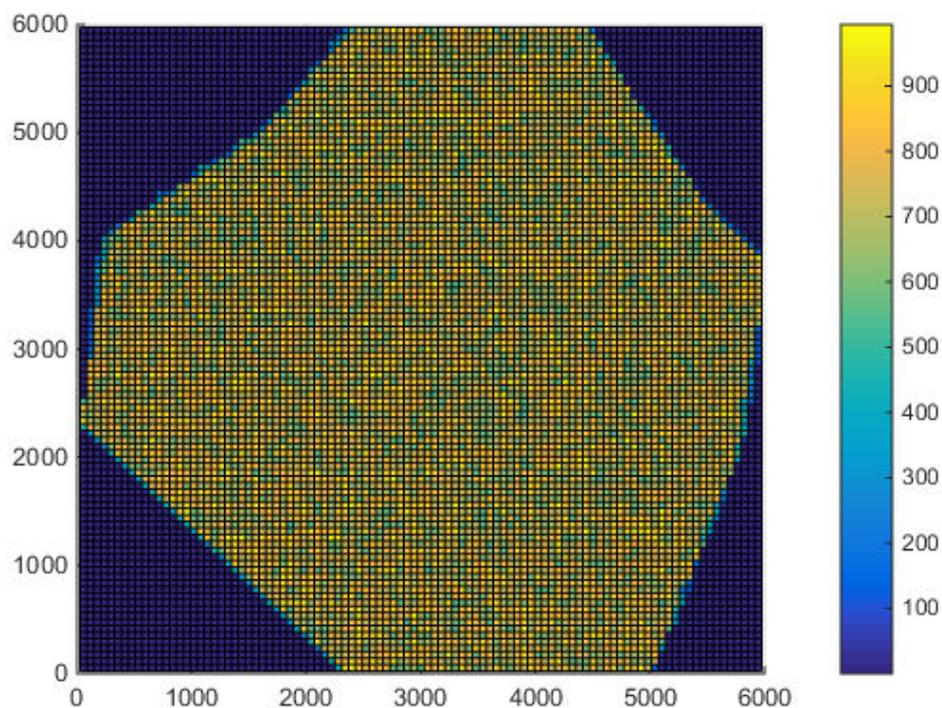


Figure 8.1. Permeability field for cases #1, #2 and #6 (plotted using MATLAB).

8.2. Cases

8.2.1. Case One (no anisotropy, no heterogeneities, one vertical well).

This case is the most simple one. This case includes absolutely homogeneous, isotropic reservoir with a vertical well in the middle. The reservoir is not rectangular. Permeability field required for running of the algorithm is shown on figure 8.1.

All values are given in milli-darcies. This permeability field was generated randomly in the required interval of permeability values. For all of the cases described in this

thesis, permeability field consists of 14400 uniformly distributed points - 120 rows and columns.

Main inputs for the first step are as follows: 500 points; 90 degrees angle between K_y and y-direction of the reservoir; no anisotropy or heterogeneities; small distance equal to 5; increment in distance also equal to 5.

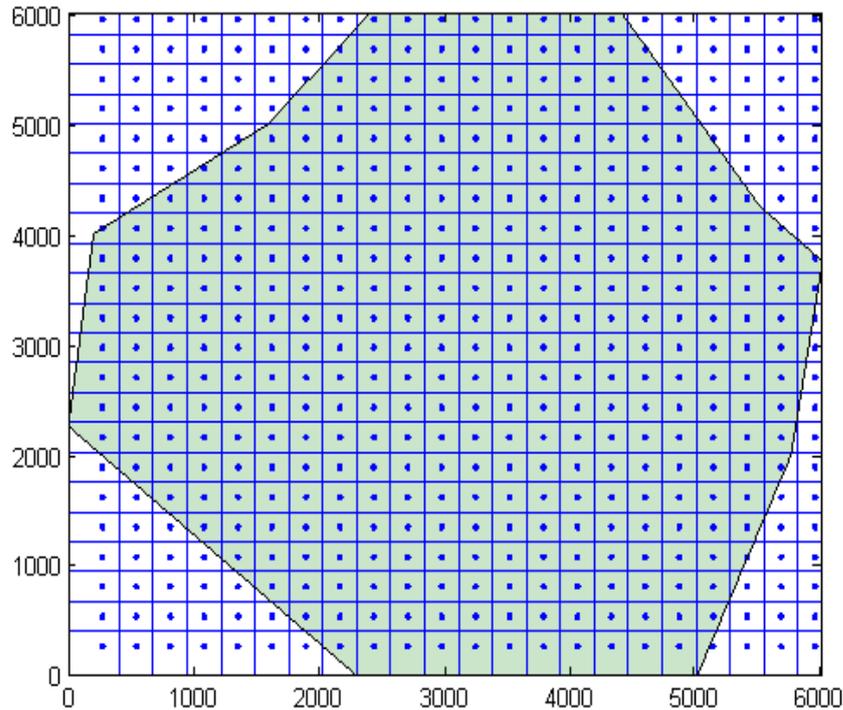


Figure 8.2. Results obtained after running of the first step for the case #1 (built in MATLAB).

At the end of the first step resulting picture is as shown on figure 8.2. Resulting sum of errors at this figure is equal to 18113. As it may be seen from the figure, grid after the first step is a regular Cartesian grid or very close to it (in the other cases).

Main inputs for the second stage are: five worst points moved during each iteration; eight movements for each point; 450 iterations; one region of permeabilities between 1 and 1000 md (entered as three column vectors - permX (x-coordinates of

permeability points), permY (y-coordinates of permeability points), permeabilities (permeability values in the points defined by vectors permX and permY)).

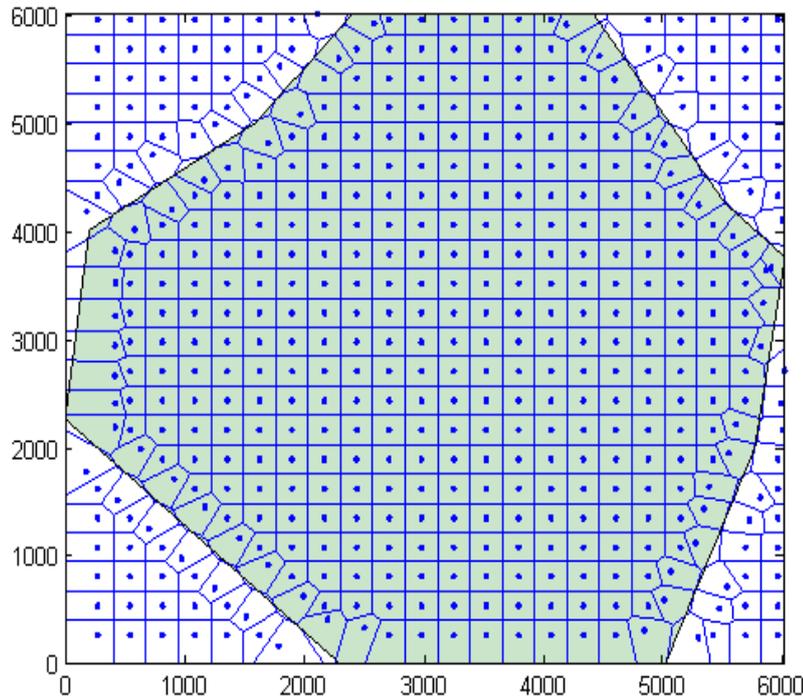


Figure 8.3. Results obtained after running of the second step of case #1 (built in MATLAB).

At the end of the second step figure 8.2 transforms into what is shown on figure 8.3. It must be mentioned that no wells or faults have been added to the model yet. Also no additional grid points related to proper handling of reservoir boundaries close to rectangle were added and no points outside of reservoir have been deleted. This means that the number of grid points in the figure 8.3 is absolutely equal to the number of grid points on figure 8.2. However, sum of errors for the case shown on the figure 8.3 is equal to 9319. This fact shows the effectiveness of the second step of the algorithm.

Figure 8.4. shows error values for all 450 generations of the first case. The curve goes rapidly down, reaches minimum in the mid-seventies and then shows upward trend.

Third stage main inputs are: vertical well in the middle of the reservoir; 60 points were added, related to this well; radius around well that was cleaned and repopulated is 100 feet; distance between the first and the second layers are 10 feet, while distance to next layers are 1.2 times the distance to the previous one.

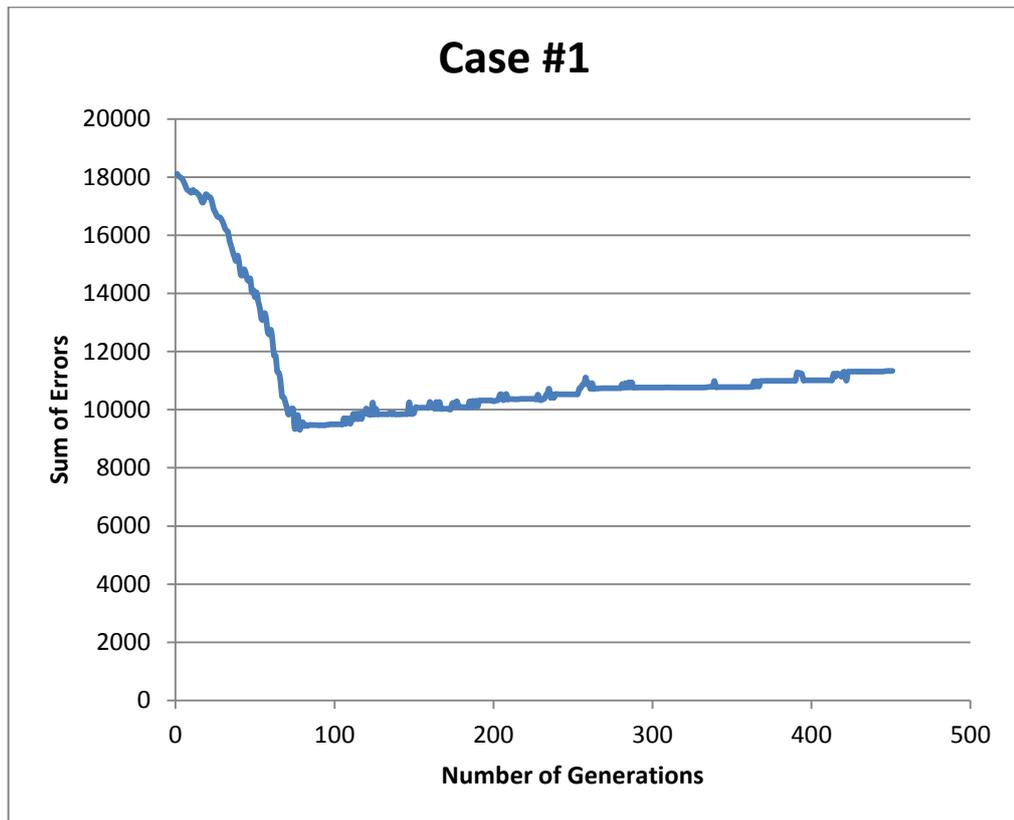


Figure 8.4. Error values for all generations of Case #1.

The final distribution of grid points (and therefore grid blocks) is shown on figure 8.5. As in similar previous figures, reservoir here is shown in blue color, while white area is a zone outside of the reservoir. The resulting number of blocks is 537. This is higher than 500 that was entered as input in the first stage because of the points added for well representation and points that were required to correctly represent the boundaries of the reservoir that are very close to the surrounding rectangle (this was discussed in details in the previous chapter). The resulting sum of errors in all of the blocks reduced from 18113 to 9319, which is almost twice.

After the dispensation of grid points shown on figure 8.3 was obtained, fluid flow simulation was created. A Python code was written based on the flow equations described in the book of Ertekin et al. (Ertekin et al., 2001) to make these simulations.

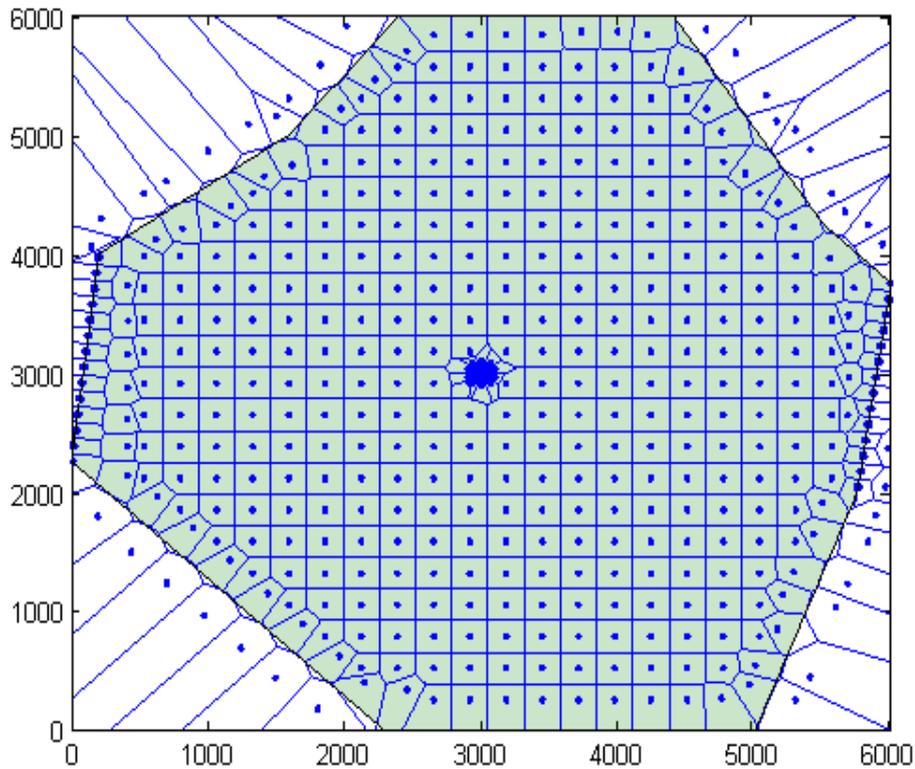


Figure 8.5. Results obtained for the case #1 (built in MATLAB).

Inputs for this fluid flow simulation run are as follows: one vertical well in the center producing 100 stb/d for 50 days with time step of 5 days. This means that data after 10 time steps was generated and analyzed. Initial reservoir pressure is equal to 3044 PSI, fluid is slightly compressible, only one phase present.

Expectations are to see pressure disturbance to propagate at the same speed in all directions, because there is no permeability anisotropy and heterogeneities in the field. This means that pressure disturbance propagation should take shape of a circle. Compare with result shown on figures 8.6-8.15. As it may be seen, pressure propagates at circular shape until it reaches reservoir boundaries.

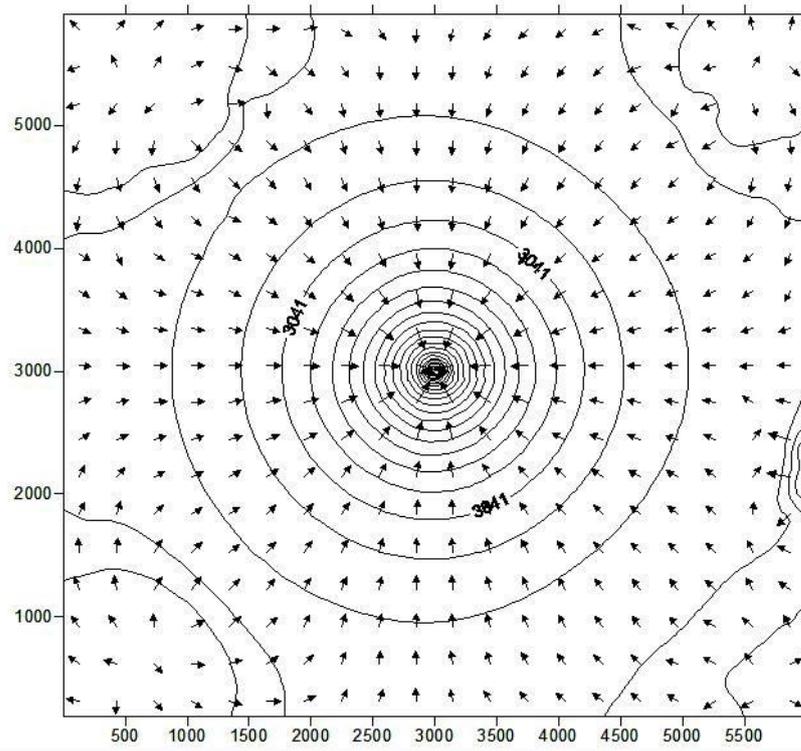


Figure 8.6. Pressure distribution after 5 days.

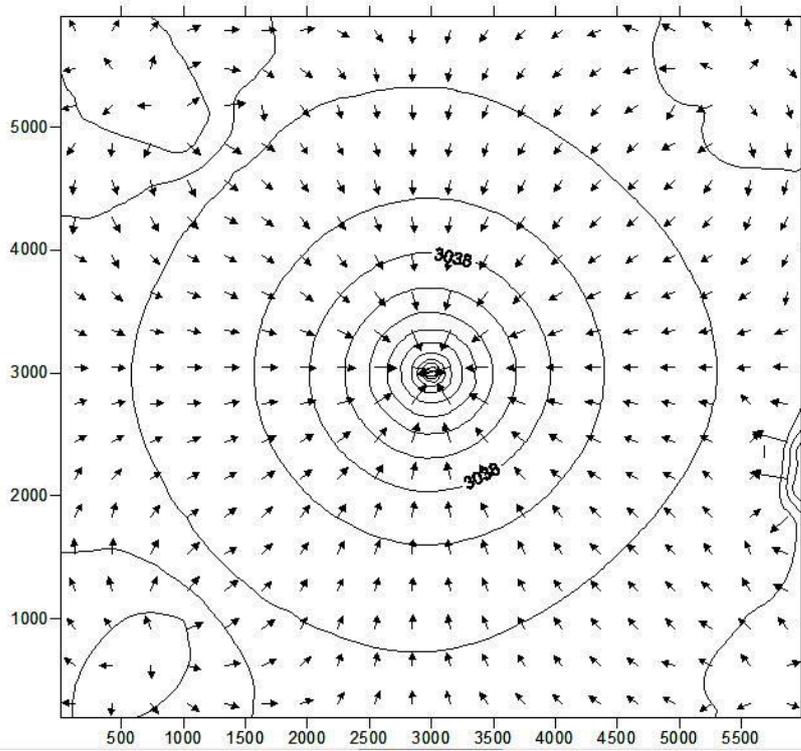


Figure 8.7. Pressure distribution after 10 days.

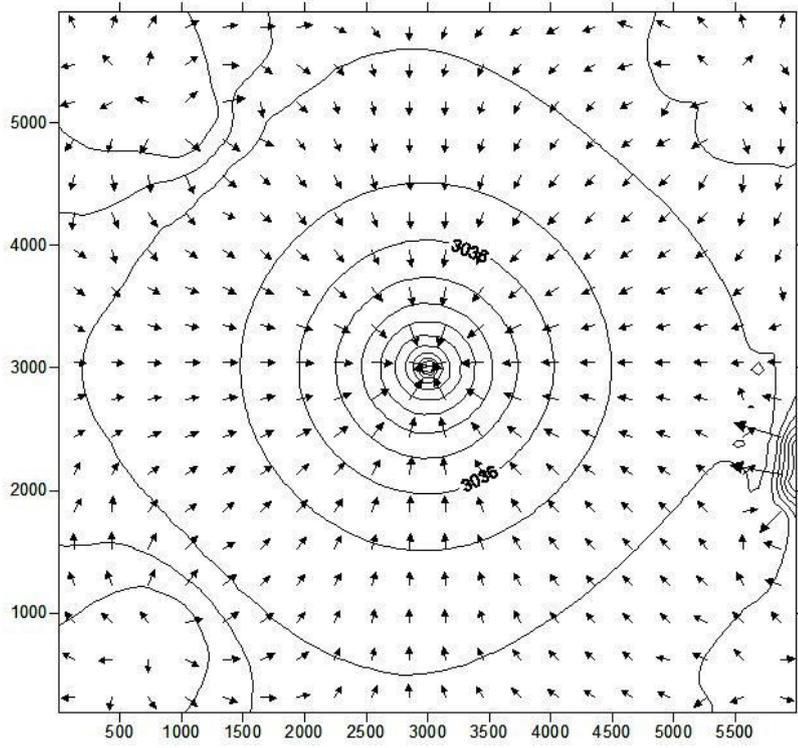


Figure 8.8. Pressure distribution after 15 days.

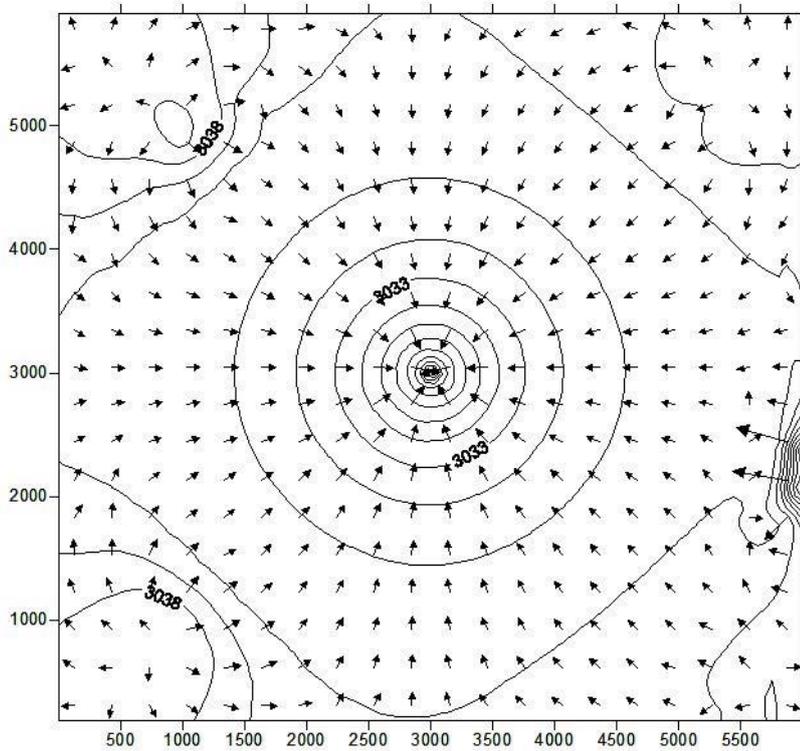


Figure 8.9. Pressure distribution after 20 days.

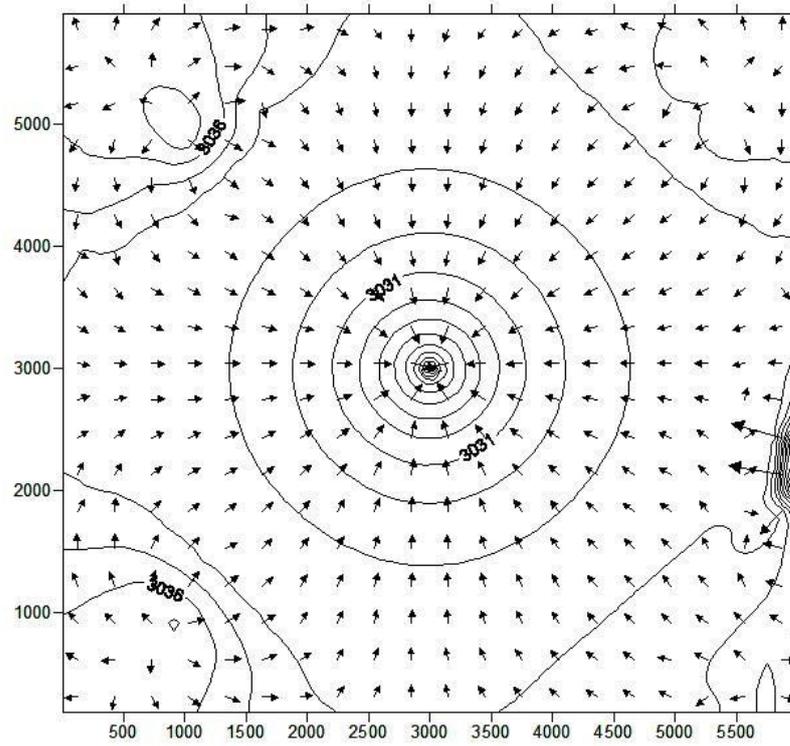


Figure 8.10. Pressure distribution after 25 days.

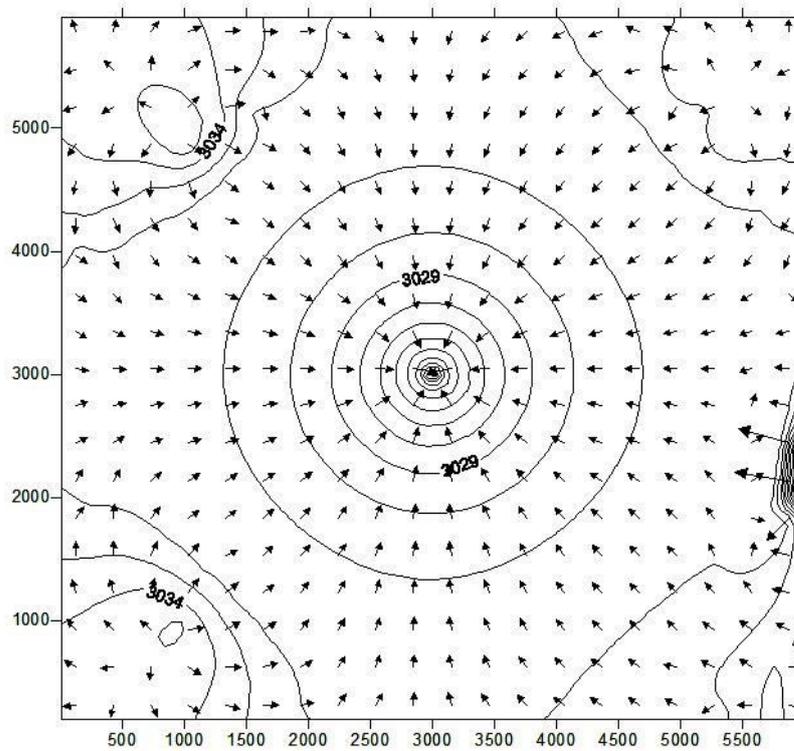


Figure 8.11. Pressure distribution after 30 days.

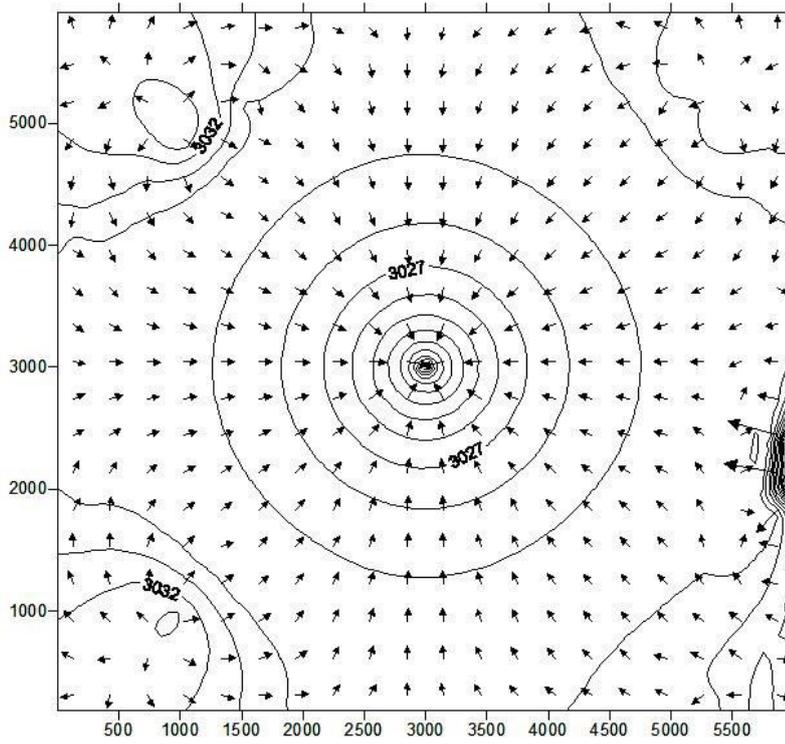


Figure 8.12. Pressure distribution after 35 days.

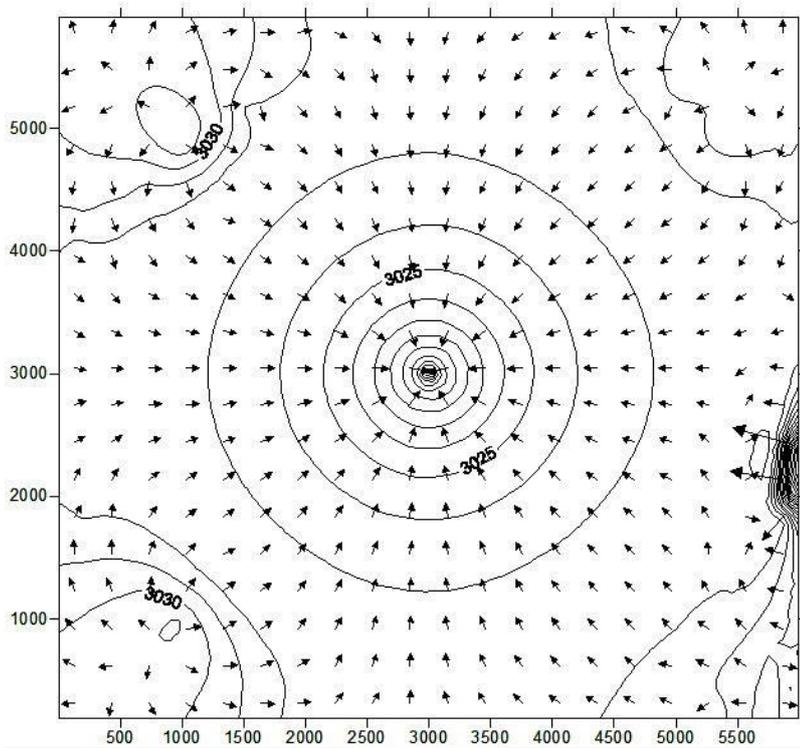


Figure 8.13. Pressure distribution after 40 days.

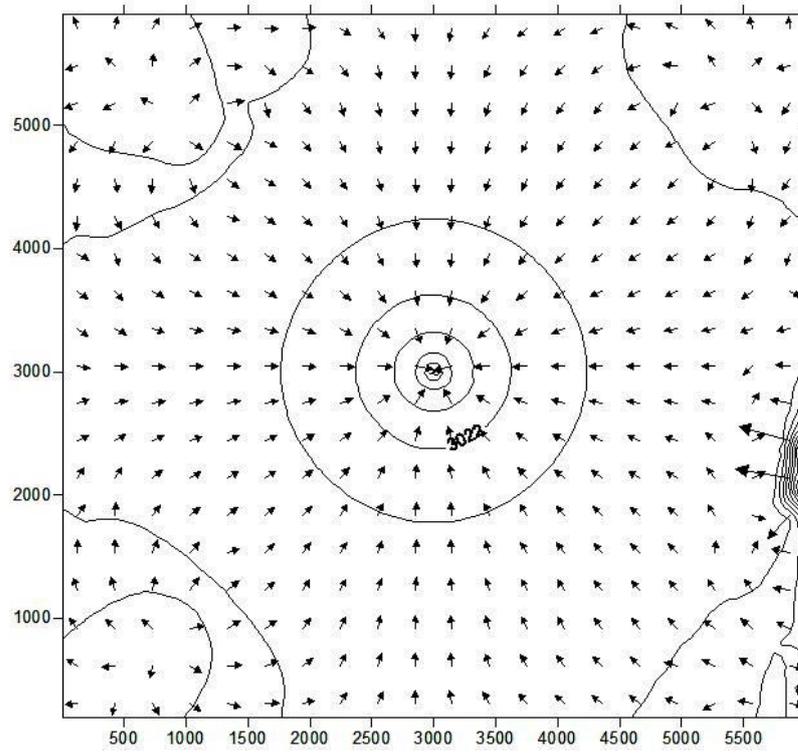


Figure 8.14. Pressure distribution after 45 days.

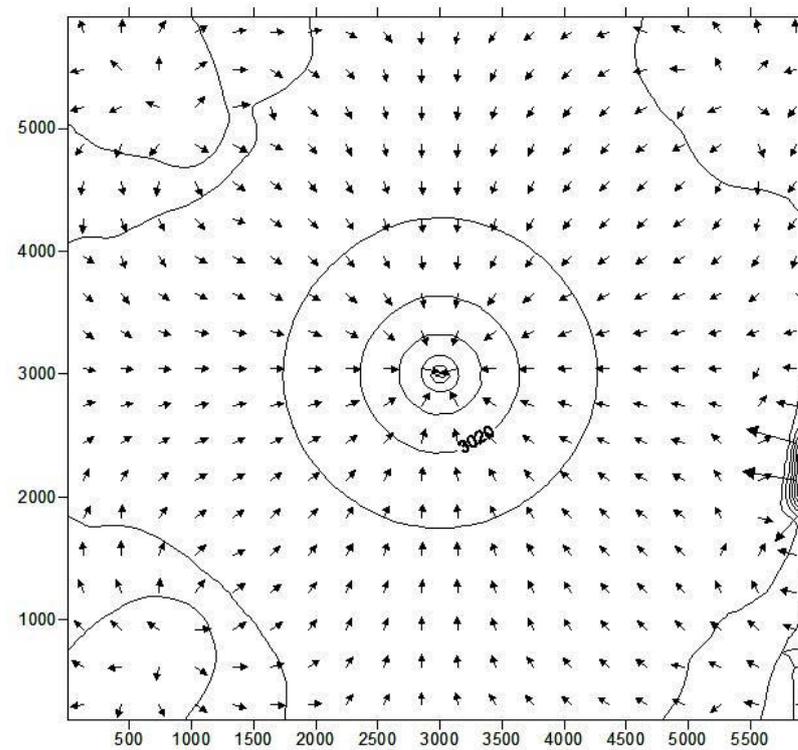


Figure 8.15. Pressure distribution after 50 days.

8.2.2. Case Two (anisotropy, no heterogeneities, one vertical well).

This case is almost identical to the previous one, with the only exception: anisotropy has been introduced. So, the reservoir is of irregular shape, with anisotropy but no heterogeneities. One vertical well is introduced in the center. Permeability field is the same as in the case #1. This means that it was also randomly generated in the required intervals - between 1 and 1000 md.

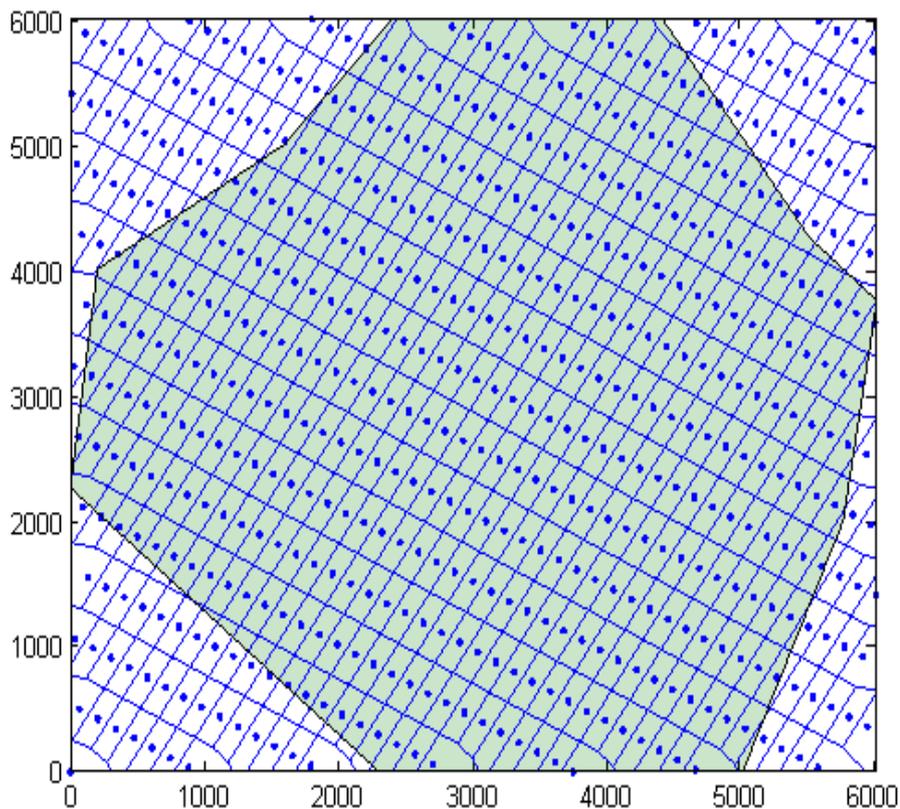


Figure 8.16. Results obtained after running of the first step for the case #2 (built in MATLAB).

Main inputs for the first step are as follows: 500 blocks; angle between K_y and y-direction of the reservoir is equal to 120 degrees; permeability in y-direction is three times higher than permeability in x-direction; small distance and increment are both equal to 5 feet.

At the end of the first step resulting picture is as shown on figure 8.16. Resulting sum of errors at this figure is equal to 19039. As it may be seen from the figure, grid after the first step is very close to Cartesian grid.

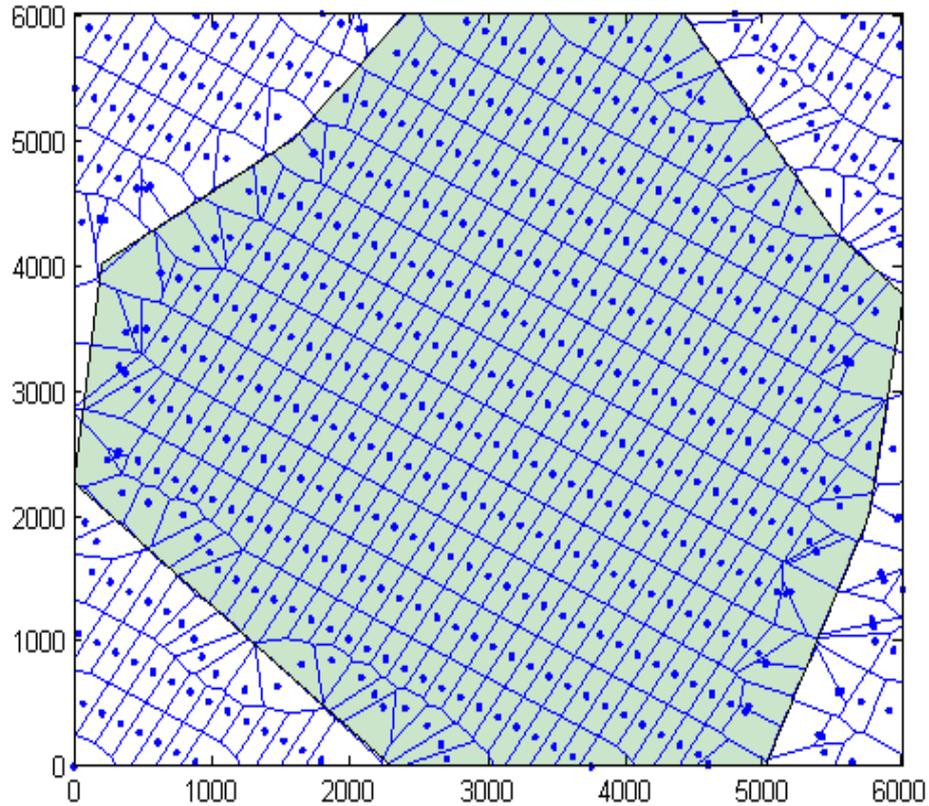


Figure 8.17. Results obtained after running of the second step of case #2 (built in MATLAB).

Main inputs for the second step are: number of points that will be moved during each iteration is equal to 5; number of required generations is equal to 450; required number of movements for each point is equal to 8; one region with permeabilities ranging from 1 to 1000 md.

Resulting locations of grid points after the second step of the case #2 are shown at figure 8.17. As in the previous case, number of blocks is still the same, so change in the sum of errors is only because of the better placement of the grid points. Sum of errors reduced here from the 19039 to 2320.

Figure 8.18. shows error values for all 500 generations of the second case. The curve goes rapidly down, reaches minimum in the eighties and then shows upward trend.

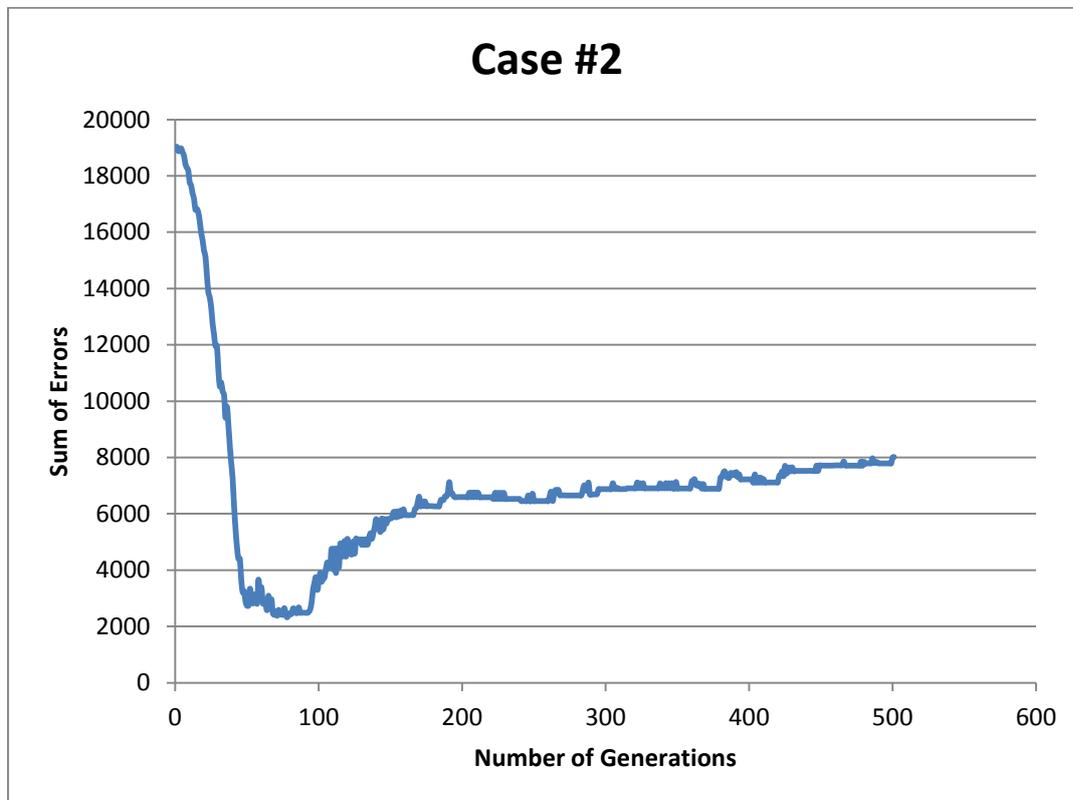


Figure 8.18. Error values for all generations of Case #2.

Main inputs for the third stage are: one vertical well in the middle; radius around the well that is cleaned and repopulated is equal to 100 feet; 60 points related to the well are added; distance between the first and the second layer of points is equal to 10 feet, while distance to next layers are 1.2 times the distance to the previous one.

The resulting distribution of grid points (and therefore grid blocks) is shown on figure 8.19. As in similar previous figures, reservoir here is shown in blue color, while white area is a zone outside of the reservoir. The resulting number of blocks is 540. This is higher than 500 that was entered as input in the first stage because of the points added for well representation and points that were required to correctly represent the boundaries of the reservoir that are very close to the surrounding rectangle (this was discussed in details in chapter 6).

The resulting sum of errors in all of the blocks reduced from 19039 to 2320, which means that sum of errors reduced more than eight times. The great effectiveness of the algorithm for the second case, compared to the case number one, may be due to input parameters entered in the step one - angle and anisotropy factor.

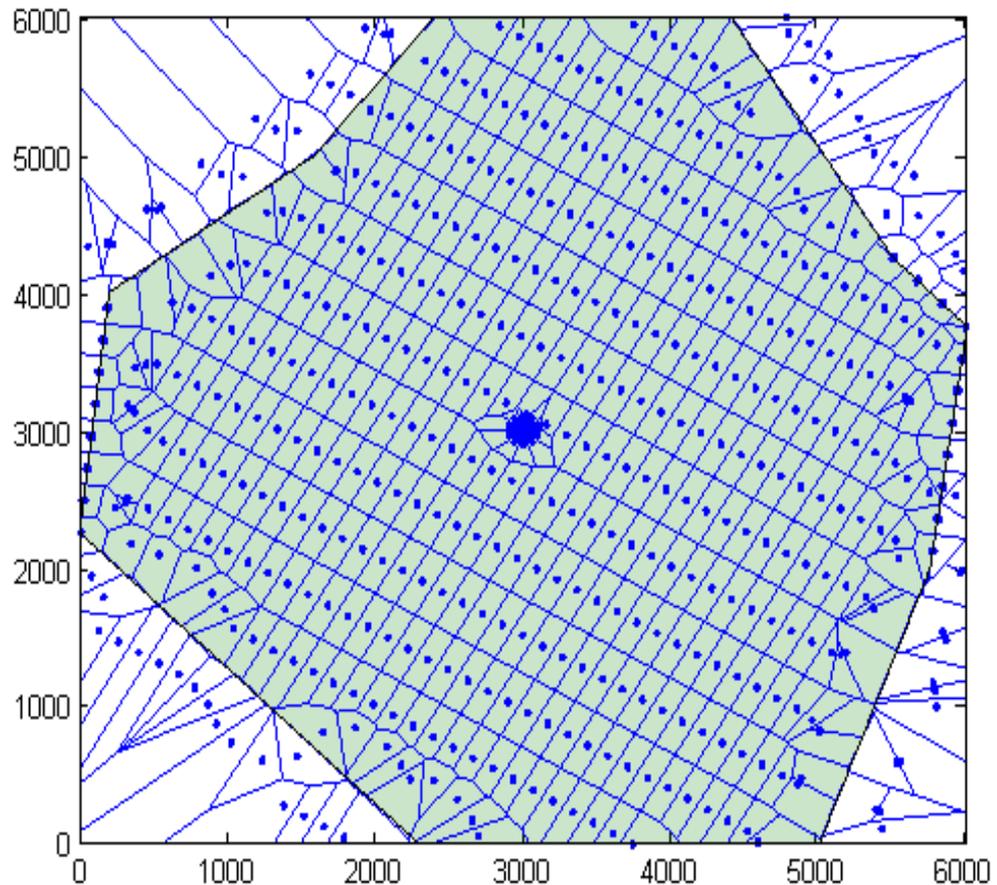


Figure 8.19. Results obtained for the case #2 (built in MATLAB).

It can also be seen from the figures 8.5 and 8.19. On the figure 8.19 block boundaries are better aligned with reservoir shape than on the figure 8.5. From this, the conclusion may be made that the algorithm's effectiveness depends on the reservoir properties.

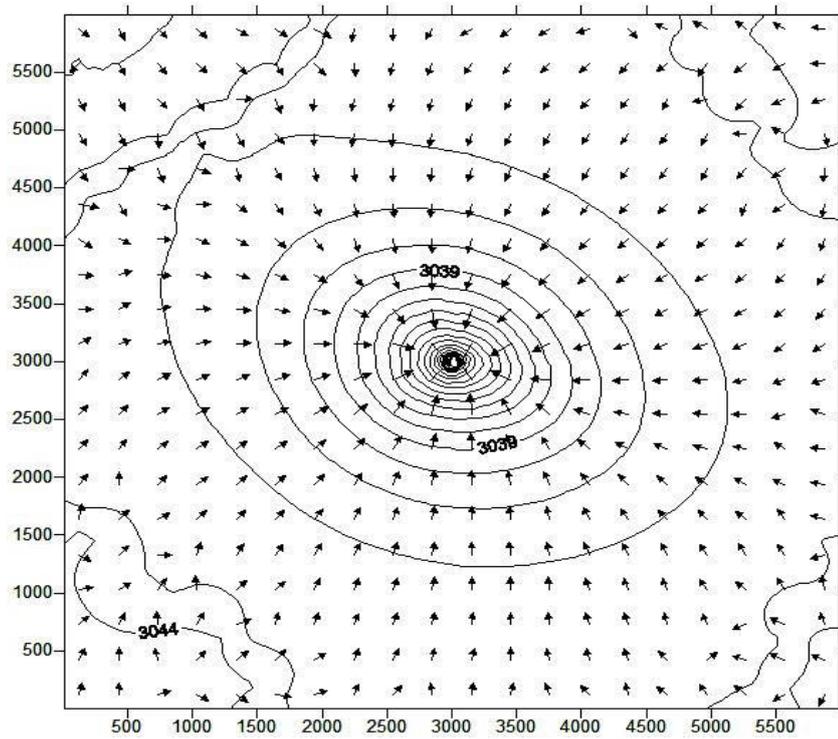


Figure 8.20. Velocity field combined with the contour map of the distribution of the pressures after 5 days of production for the second case (obtained with Surfer).

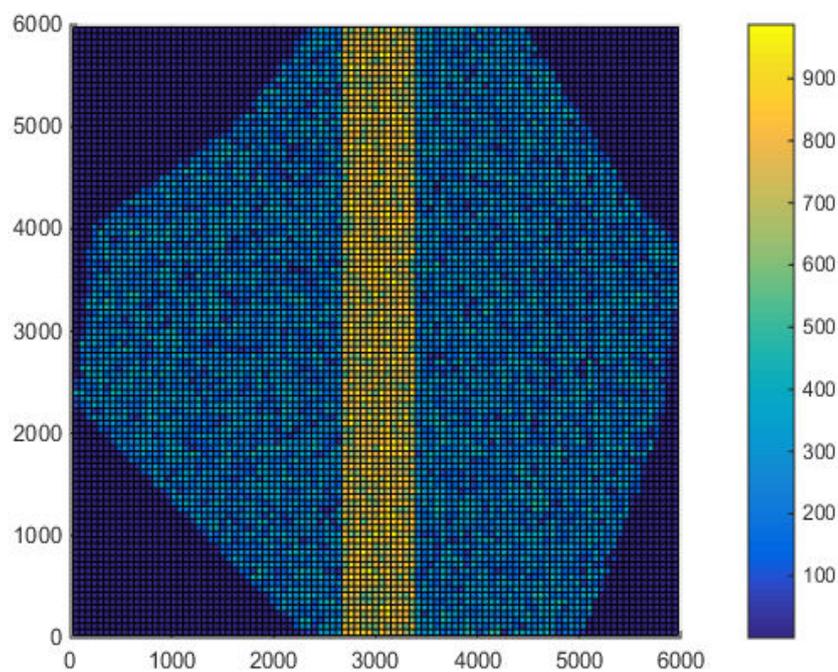


Figure 8.21. Permeability field for case #3. Generated in MATLAB.

However, by adjusting the input parameters given in the second stage effectiveness may be increased. For example, by minimizing the distance at which points are moved in each iteration and by increasing the number of movements for each point and number of iterations, slightly better results may be achieved. However, this would require more computation time for the computer. So, the choice is totally on the person using the algorithm.

For this case fluid flow simulation run was also implemented. All input parameters are just the same as in the previous case: one vertical well in the center producing 100 stb/d for 50 days with time step of 5 days. Initial reservoir pressure is equal to 3044 PSI, fluid is slightly compressible, only one phase present.

Expectations are to see pressure disturbance propagate at higher rate in the north-east and south-west directions because of the permeability anisotropy. This is what can be seen on figure 8.20.

Other figures showing results of case #2 fluid flow simulation run are in Appendix B.

8.2.3. Case Three (anisotropy, straight channel, one vertical well).

This is the first case where heterogeneity is introduced. This heterogeneity is represented by a straight channel in the middle of the reservoir.

This channel has better permeability values than the surrounding reservoir - in the channel permeability is between 500 and 1000 md, while in other parts of the reservoir they are between 1 and 500 md. The algorithm should be able to handle not only reservoir - outside of reservoir boundaries, but also reservoir - channel boundaries. This is shown on figure 8.21 - representation of permeability field.

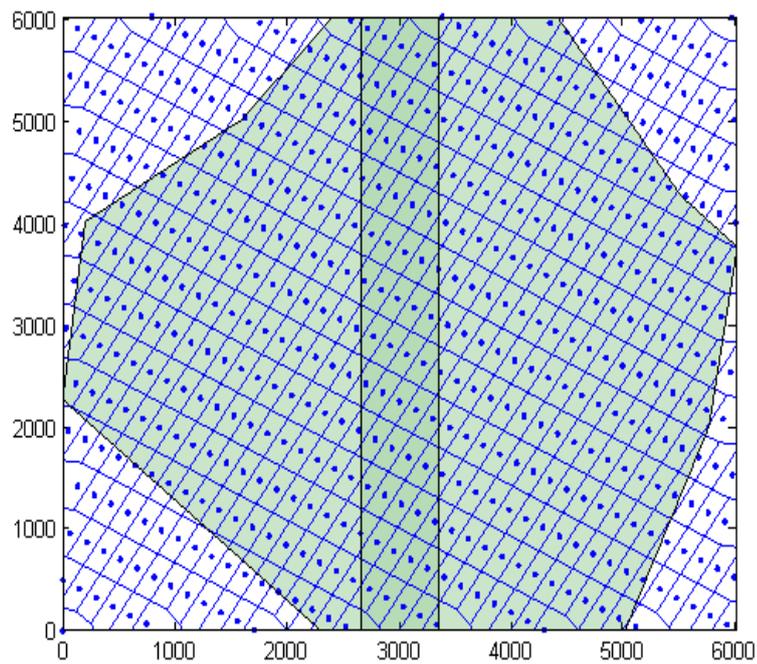


Figure 8.22. Results obtained after running of the first step for the case #3 (built in MATLAB).

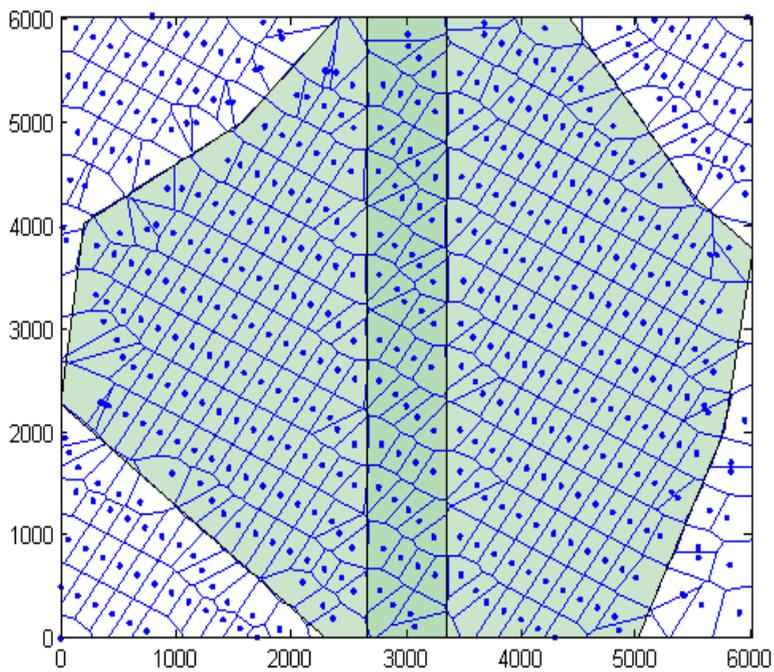


Figure 8.23. Results obtained after running of the second step of case #3 (built in MATLAB).

Main inputs for the first step are as follows: 500 grid points; angle between K_y and y-direction of the field is equal to 210 degrees; K_y to K_x relation is equal to 0.4; small distance and each iteration increment are both equal to 5.

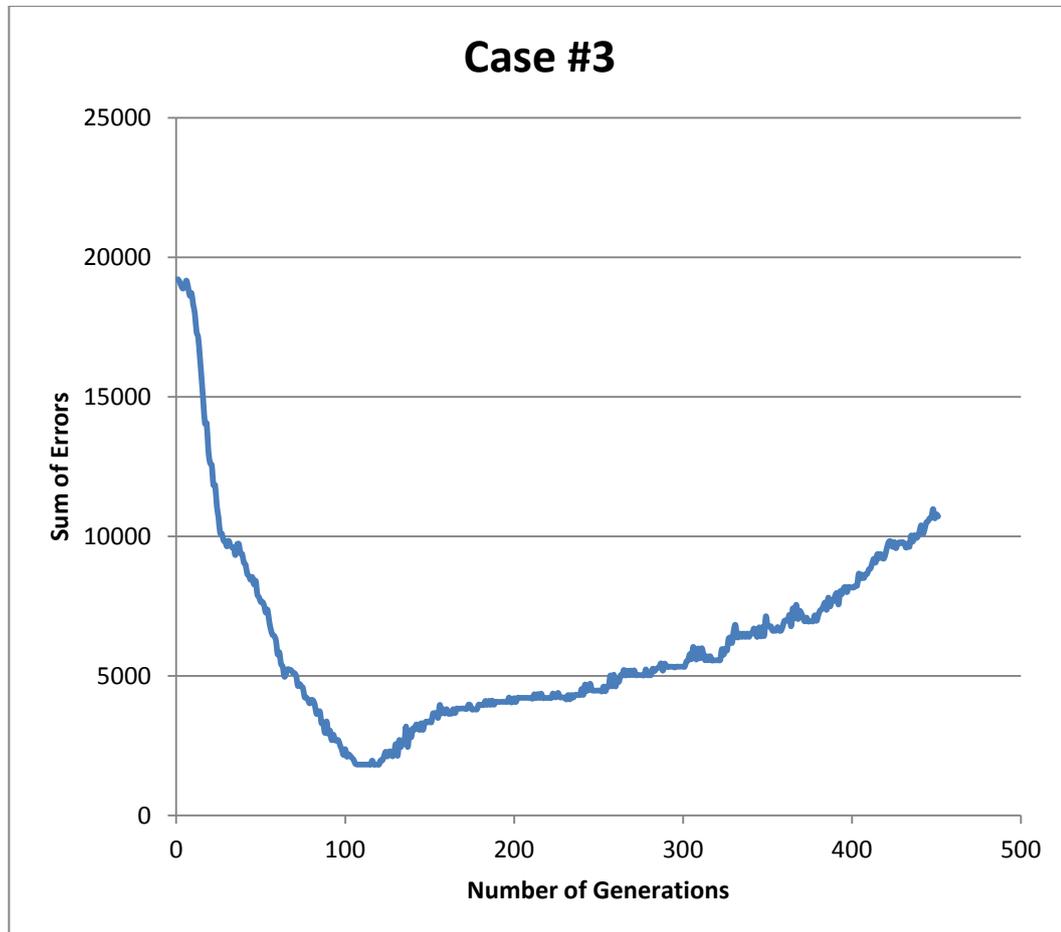


Figure 8.24. Error values for all generations of Case #3.

At the end of the first step resulting picture is as shown on figure 8.22. Resulting sum of errors at this figure is equal to 19213. As it may be seen from the figure, grid after the first step is very close to Cartesian grid.

Main inputs for the second step are: number of points that were changed during each iteration is 5; 450 iterations; each point was allowed to move 8 times; as it was said, the two regions were introduced: one with permeabilities between 1 and 500 md, the other with permeabilities between 501 and 1000 md.

Resulting locations of grid points after the second step of the case #3 are shown at figure 8.23. As in the previous cases, number of blocks is still the same as in figure 8.22, so change in the sum of errors is only because of the better placement of the grid points. Sum of errors reduced here from the 19213 to 1821.

Figure 8.24. shows error values for all 450 generations of the third case. The curve goes rapidly down, reaches minimum in the early one hundredth and then shows upward trend.

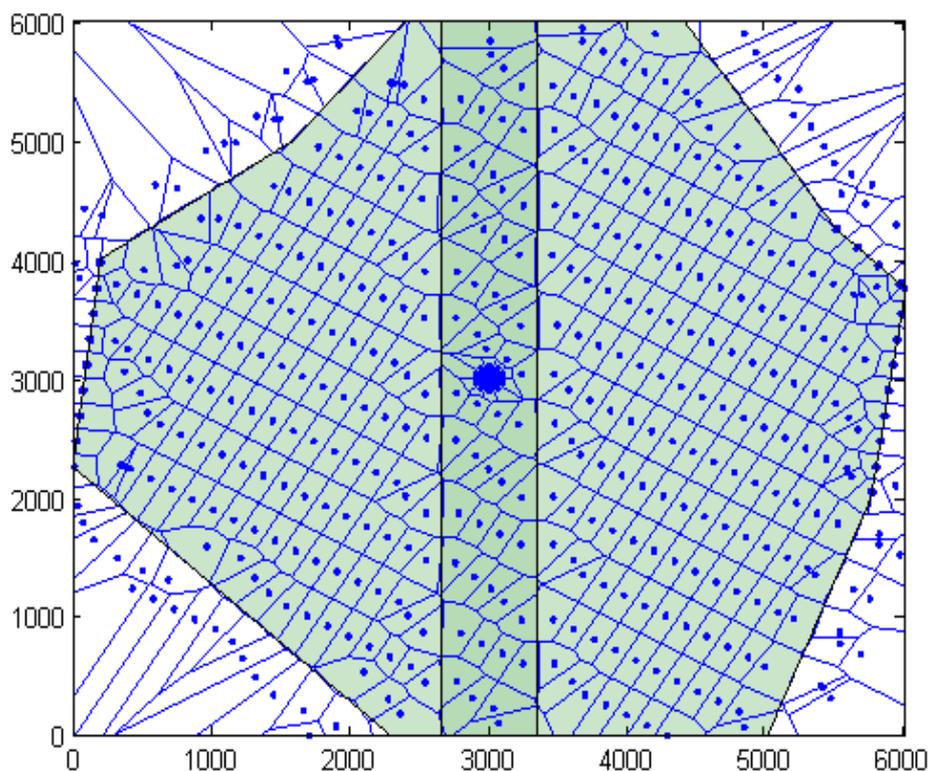


Figure 8.25. Results obtained for the case #3 (built in MATLAB).

Main inputs for the third step are: one vertical well in the center of the reservoir; radius around the well that was repopulated is equal to 100 feet; 60 points related to this well was added; distance between the first and the second layers of points around the well is equal to 10 feet; distance to the next layers are 1.2 times the distance to the previous one.

Obtained result is shown on figure 8.25. The resulting number of blocks is 527, while sum of errors in each block reduced from initial value of 19213 to the best result of 2046, which is almost 10 times! This is even beyond expectations, because it is even better than in the both previous, simpler cases.

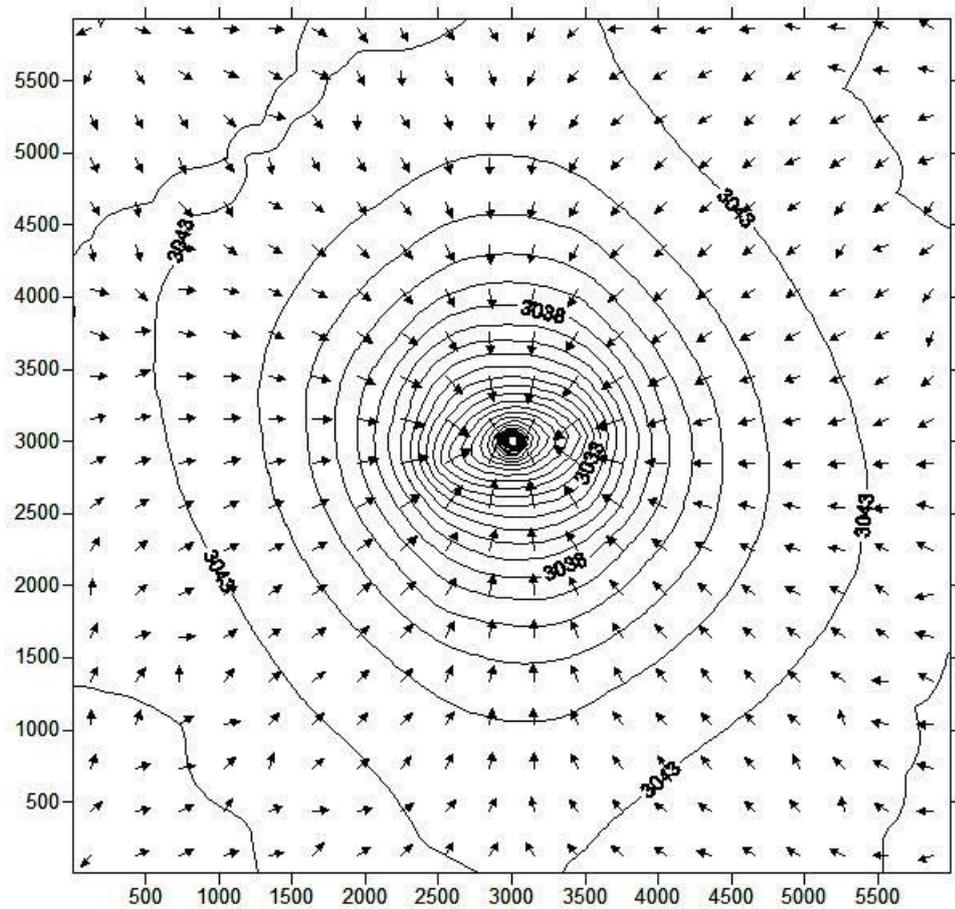


Figure 8.26. Velocity field combined with the contour map of the distribution of the pressures after 5 days of production for the third case (obtained with Surfer).

Why this happens is not very clear, but maybe this is due to the angle introduced in the first step which somehow better coincides with reservoir boundaries and makes the work of the algorithm easier. Increased number of blocks is still due to handling of reservoir boundaries close to the outside borders and adding of grid points related to the well.

For this case fluid flow simulation run was also implemented in order to see how reservoir behaves and is this behavior is close to what we expect. Inputs for the fluid flow simulation run are as follows: vertical well producing at 100 stb/d; initial reservoir pressure equal to 3044 PSI; run for 50 days; time step 5 day.

For this case the graph should show pressure disturbance propagating in the y-direction from the well at a higher speed than in the x-direction. This is shown on the figure 8.26 - velocity field combined with a contour map of pressure distribution in the field after 5 days of production from the vertical well in the center.

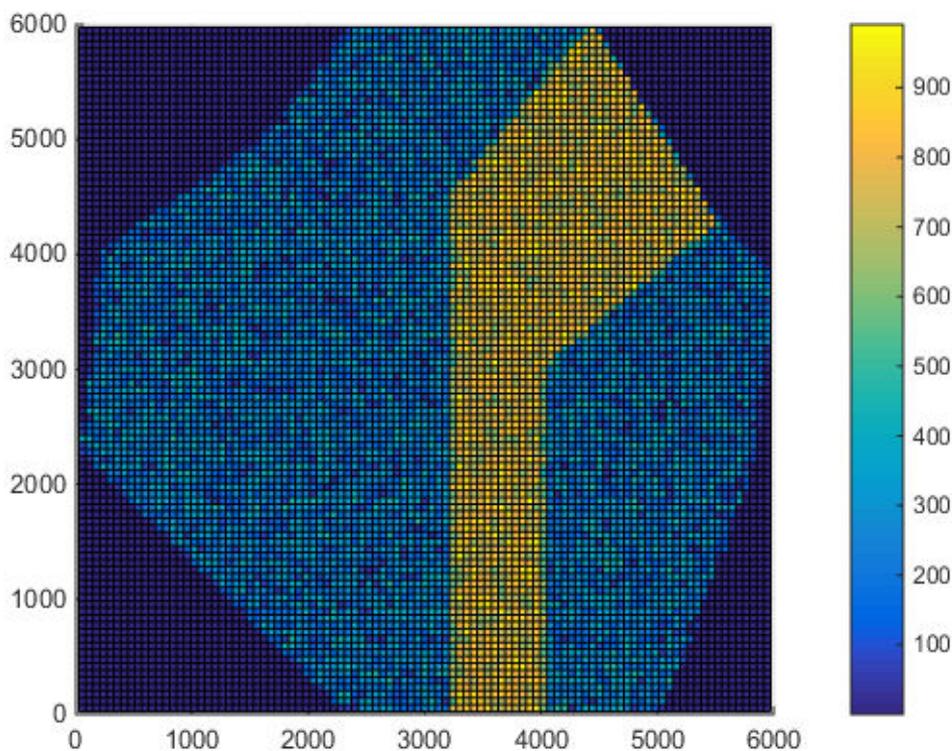


Figure 8.27. Permeability field for case #4. Plotted in MATLAB.

Other figures, for the fluid flow simulation run of this case can be found in Appendix C.

It can be seen that figure 8.26 clearly shows what was expected. Pressure disturbance reached reservoir boundaries in the upward and downward directions in the picture,

also anisotropy effect can also be seen - in the direction of higher permeability pressure disturbance propagated very far from the well.

8.2.4. Case Four (anisotropy, deviated channel, one vertical well).

This case is similar to the previous one with the only difference: now the channel is not straight, it is deviated. Other than that, everything is almost the same. Permeability field is as shown on figure 8.27.

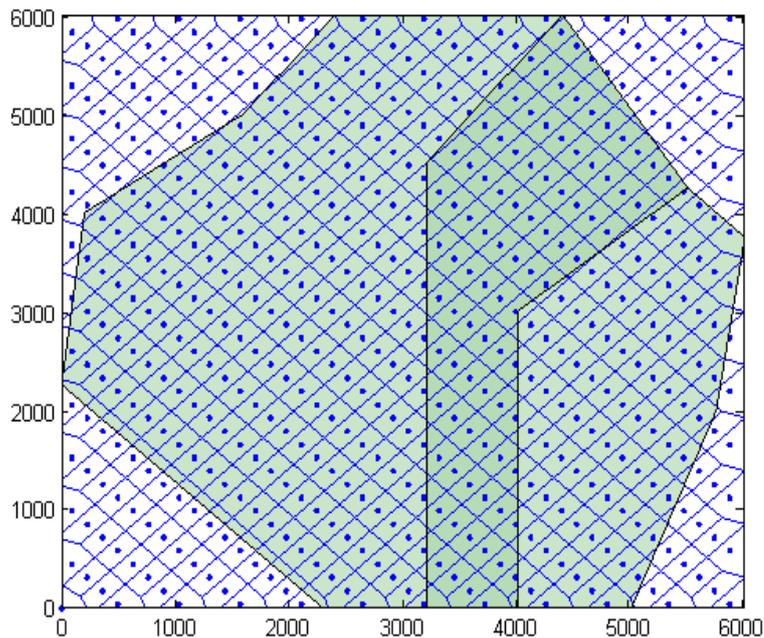


Figure 8.28. Results obtained after running of the first step for the case #4 (built in MATLAB).

Main inputs for the first step are as follows: 500 grid points; angle between K_y and y -direction of the field is equal to 45 degrees; K_y to K_x relation is equal to 0.5; small distance and each iteration increment are both equal to 5.

At the end of the first step resulting picture is as shown on figure 8.28. Resulting sum of errors at this figure is equal to 21894. As it may be seen from the figure, grid after the first step is very close to Cartesian grid.

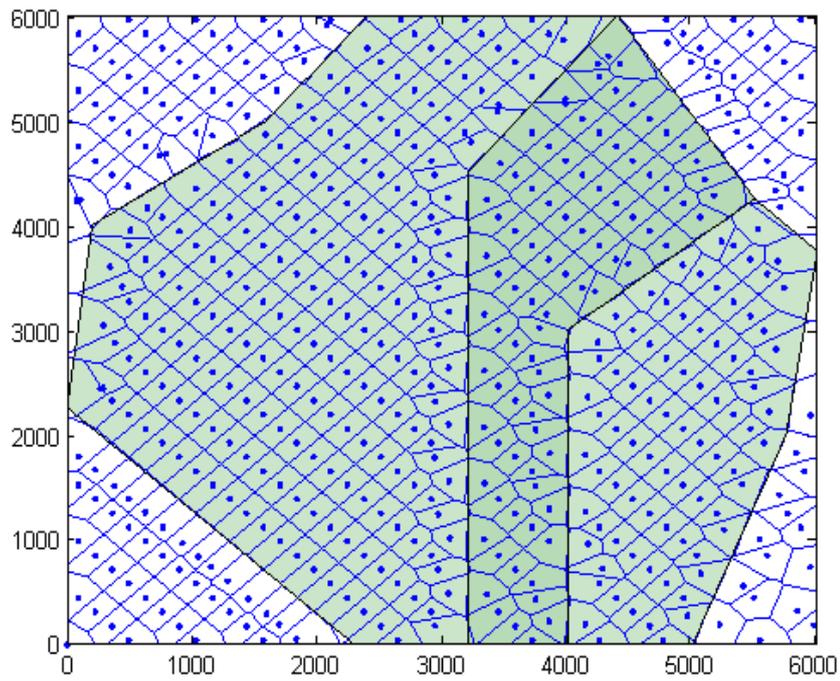


Figure 8.29. Results obtained after running of the second step of case #4 (built in MATLAB).

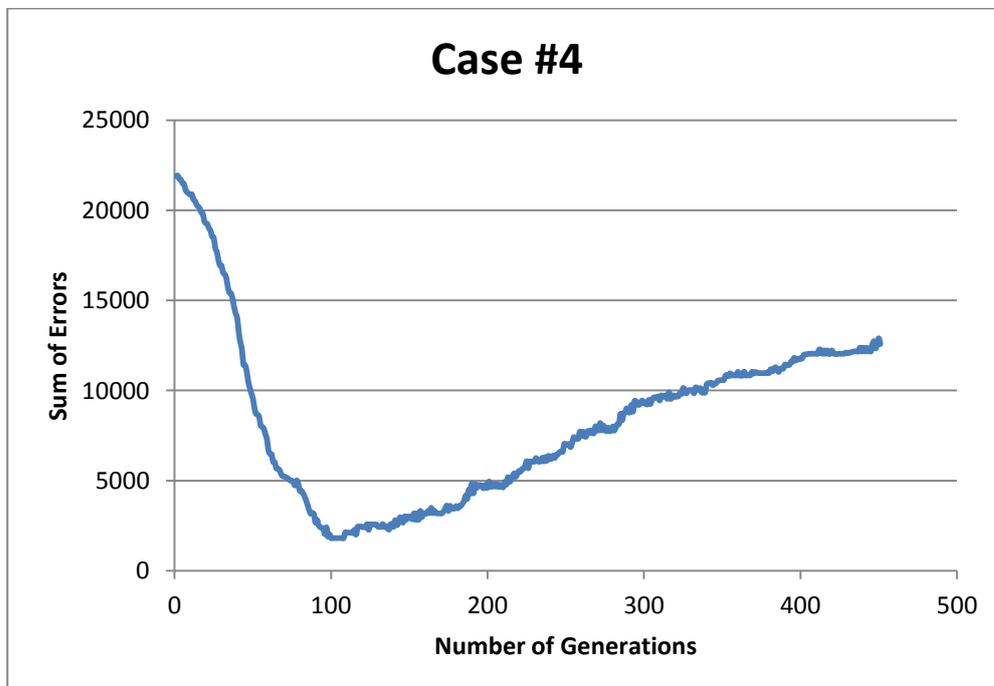


Figure 8.30. Error values for all generations of Case #4.

Main inputs for the second step are: number of points that were changed during each iteration is 5; 450 iterations; each point was allowed to move 8 times; as it was said, the two regions were introduced: one with permeabilities between 1 and 500 md, the other with permeabilities between 501 and 1000 md.

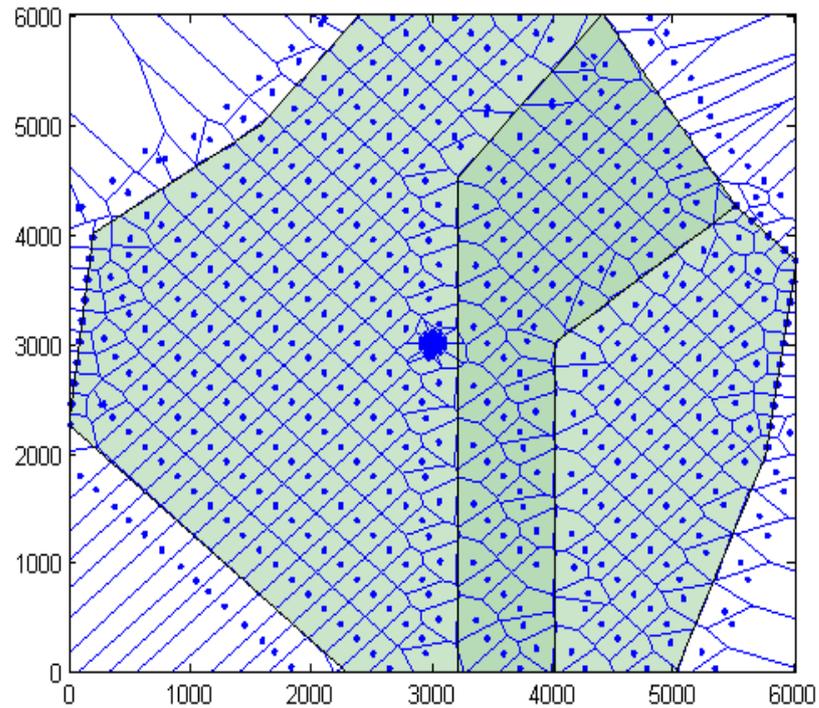


Figure 8.31. Results obtained for the case #4 (built in MATLAB).

Resulting locations of grid points after the second step of the case #4 are shown at figure 8.29. As in the previous cases, number of blocks is still the same as in figure 8.28, so change in the sum of errors is only because of the better placement of the grid points. Sum of errors reduced here from the 21894 to 1813.

Figure 8.30. shows error values for all 450 generations of the fourth case. The curve goes rapidly down, reaches minimum in the early one hundredth and then shows upward trend.

Main inputs for the third step are: one vertical well in the center of the reservoir; radius around the well that was repopulated is equal to 100 feet; 60 points related to

this well was added; distance between the first and the second layers of points around the well is equal to 10 feet; distance to the next layers are 1.2 times the distance to the previous one.

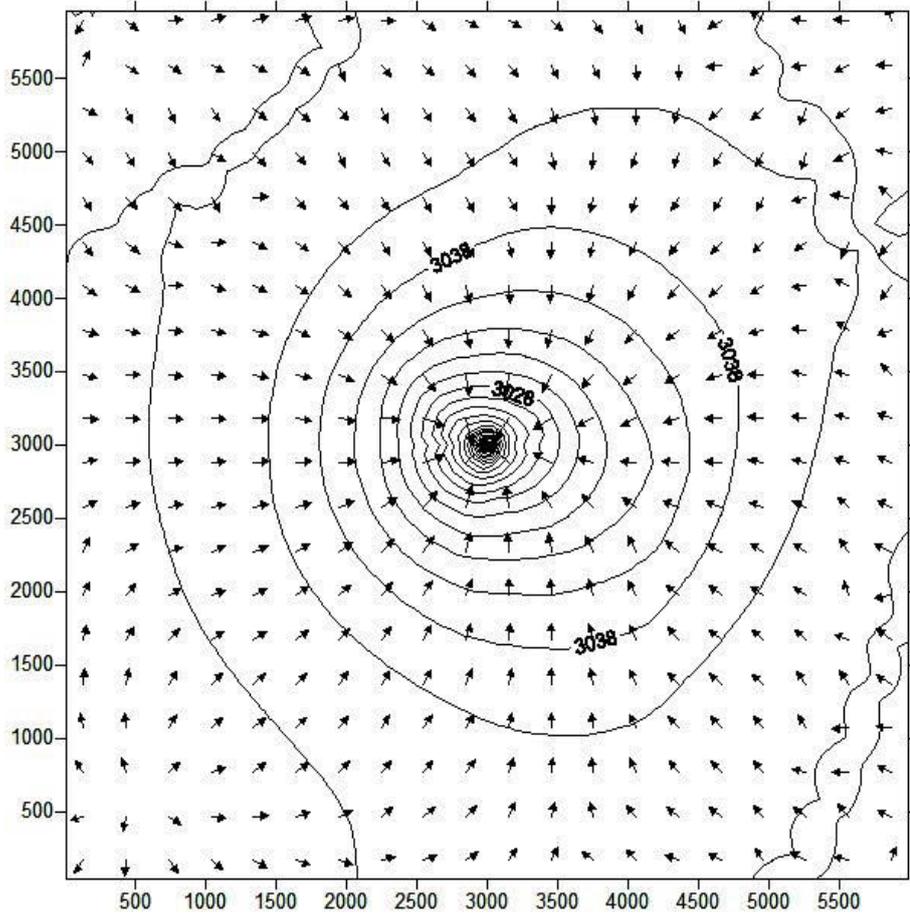


Figure 8.32. Velocity field combined with the contour map of the distribution of the pressures after 10 days of production for the fourth case (obtained with Surfer).

The final results are shown on figure 8.31. The resulting number of blocks is equal to 536, while sum of errors in all of the blocks reduced from the initial value of 21894 to 3500, which is more than 6 times.

For this case fluid flow simulation run was also implemented. Inputs for the run are still exactly as in the previous cases: vertical well producing at 100 stb/d; initial reservoir pressure equal to 3044 PSI; run for 50 days; time step 5 day. Expectations are to see pressure disturbance propagating at a higher rate towards and inside the

channel, because it has higher permeability values than the surrounding reservoir. The results are shown on the graph 8.32 - velocity field combined with a contour map of pressure distribution in the field after 10 days of production from the vertical well in the center. Other figures, obtained from the fluid flow simulation run for this case, can be found in Appendix D.

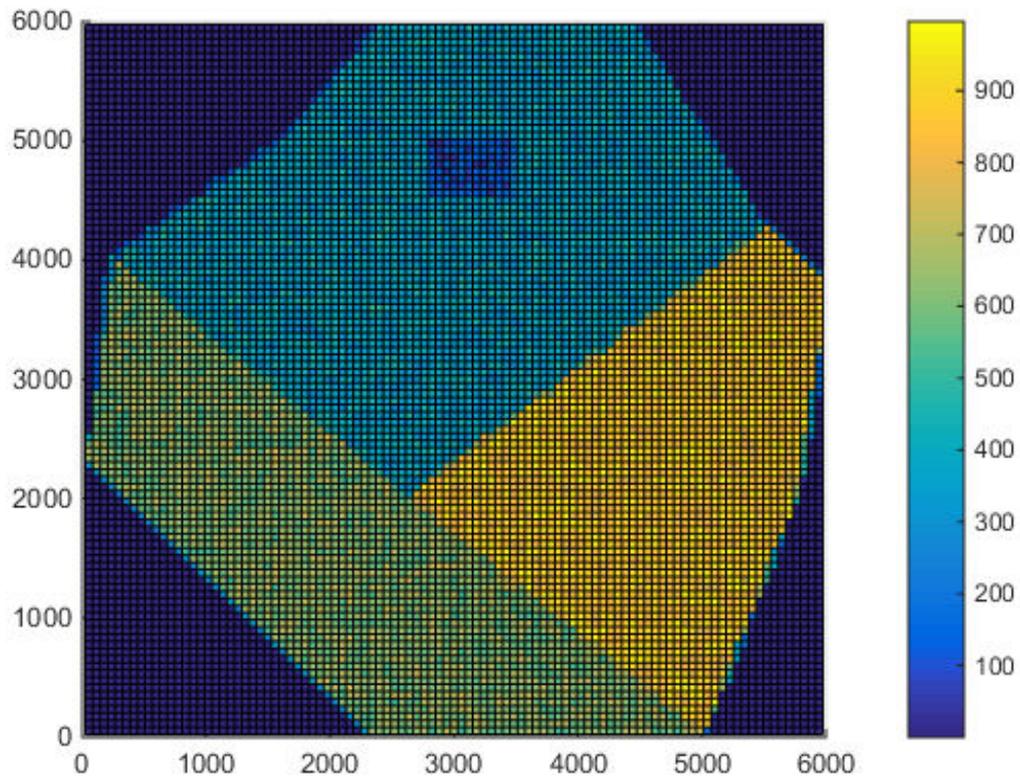


Figure 8.33. Permeability field for case #5. Plotted in MATLAB.

The picture coincides with the expectations - disturbance propagated further in the northeast and south directions, exactly where the channel is located. So, the algorithm was successful in solving this kind of problems.

8.2.5. Case Five (anisotropy, four different regions, one vertical well).

The fifth case represents more complex conditions than each of the previous cases. Now there is no channel, but some number of different regions in the field. Each

region has its own characteristics, so the purpose is to correctly represent each of this region in the model.

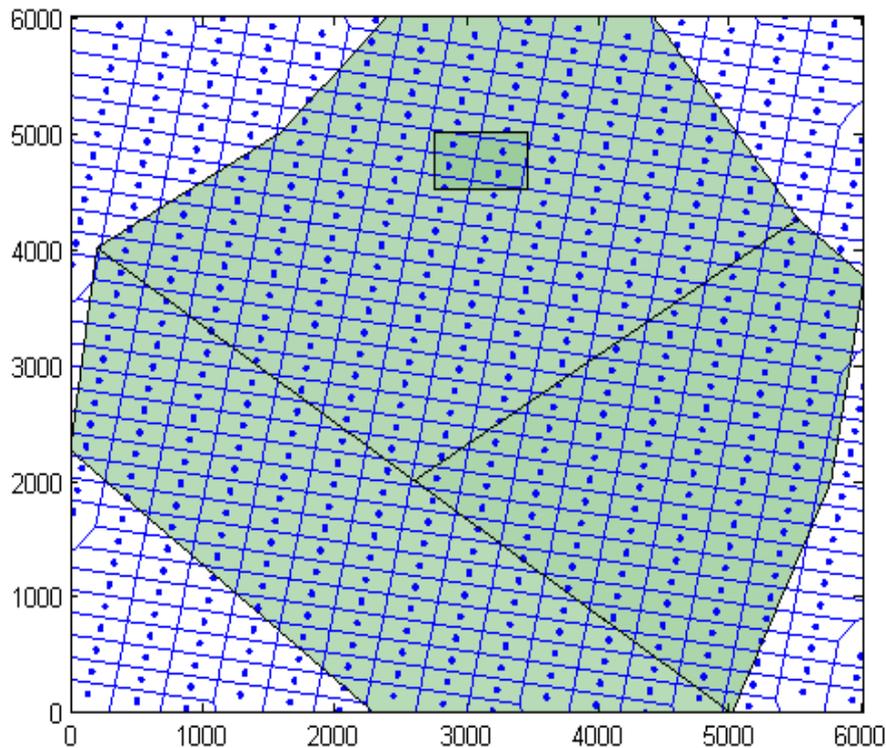


Figure 8.34. Results obtained after running of the first step for the case #5 (built in MATLAB).

This permeability field is shown on figure 8.33. As in previous cases, all permeability values are given in milli-darcies.

Main inputs for the first step are as follows: 500 grid points; angle between K_y and y-direction of the field is equal to 10 degrees; K_y to K_x relation is equal to 2; small distance and each iteration increment are both equal to 10.

At the end of the first step resulting picture is as shown on figure 8.34. Resulting sum of errors at this figure is equal to 23506. As it may be seen from the figure, grid after the first step is very close to Cartesian grid.

Main inputs for the second step are: number of points that were changed during each iteration is 5; 450 iterations; each point was allowed to move 8 times; as it was said, four regions were introduced: one with permeabilities between 1 and 250 md, the second with permeabilities between 251 and 500 md, the third with permeabilities between 501 and 750 md, and the last one with permeabilities between 750 and 1000 md. These regions were shown on the figure 8.20.

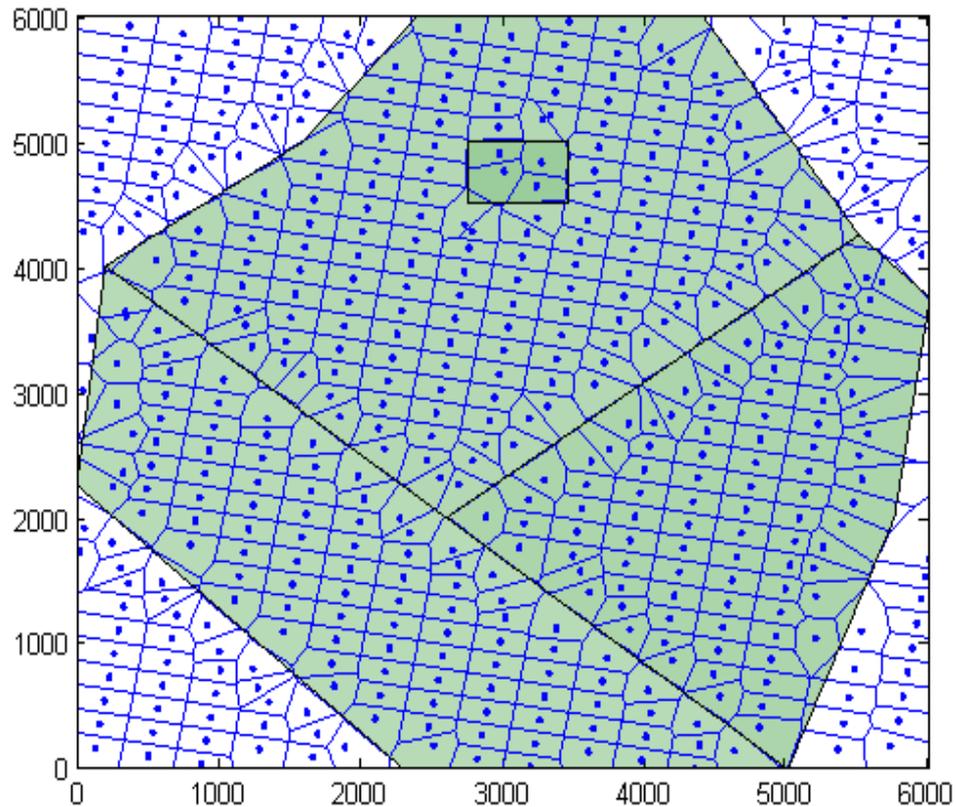


Figure 8.35. Results obtained after running of the second step of case #5 (built in MATLAB).

Resulting locations of grid points after the second step of the case #5 are shown at figure 8.35. As in the previous cases, number of blocks is still the same as in figure 8.21, so change in the sum of errors is only because of the better placement of the grid points. Sum of errors reduced here from the 23506 to 5872.

Figure 8.36. shows error values for all 450 generations of the fifth case. The curve contiouously goes down, reaches minimum in the early one hudred fourties and then shows mainly upward trend.

Main inputs for the third step are: one vertical well in the center of the reservoir; radius around the well that was repopulated is equal to 100 feet; 60 points related to this well was added; distance between the first and the second layers of points around the well is equal to 10 feet; distance to the next layers are 1.2 times the distance to the previous one.

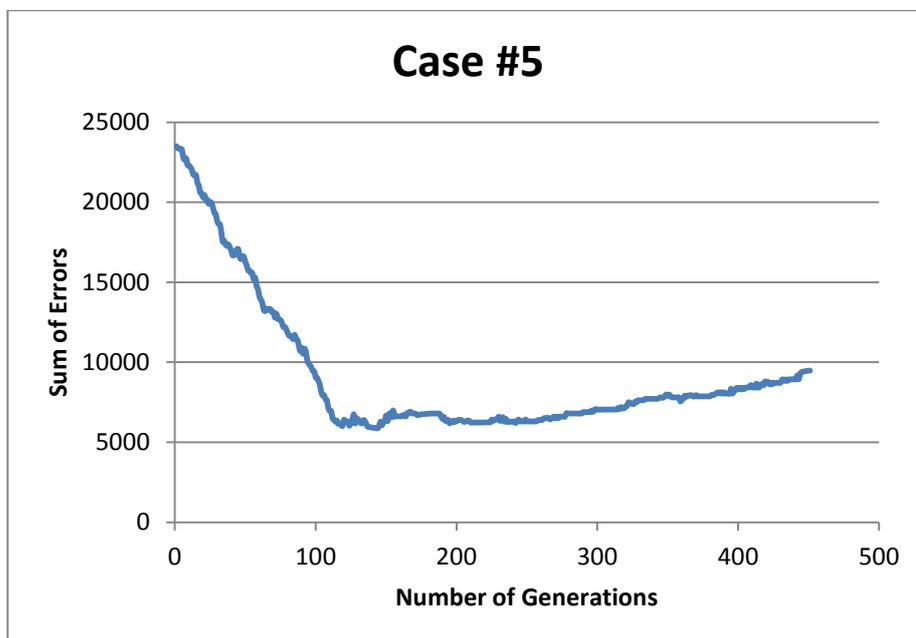


Figure 8.36. Error values for all generations of Case #5.

The obtained results are shown on figure 8.37. The resulting number of blocks is equal to 530, while the sum of errors in all of the blocks reduced from an initial value of 23506 to 5872, which is not so good as previous two cases, but still quite impressive.

For this case fluid flow simulation run was also implemented. Inputs for the run are exactly as in the previous cases: vertical well producing at 100 stb/d; initial reservoir pressure equal to 3044 PSI; run for 5 days; time step 0.5 day.

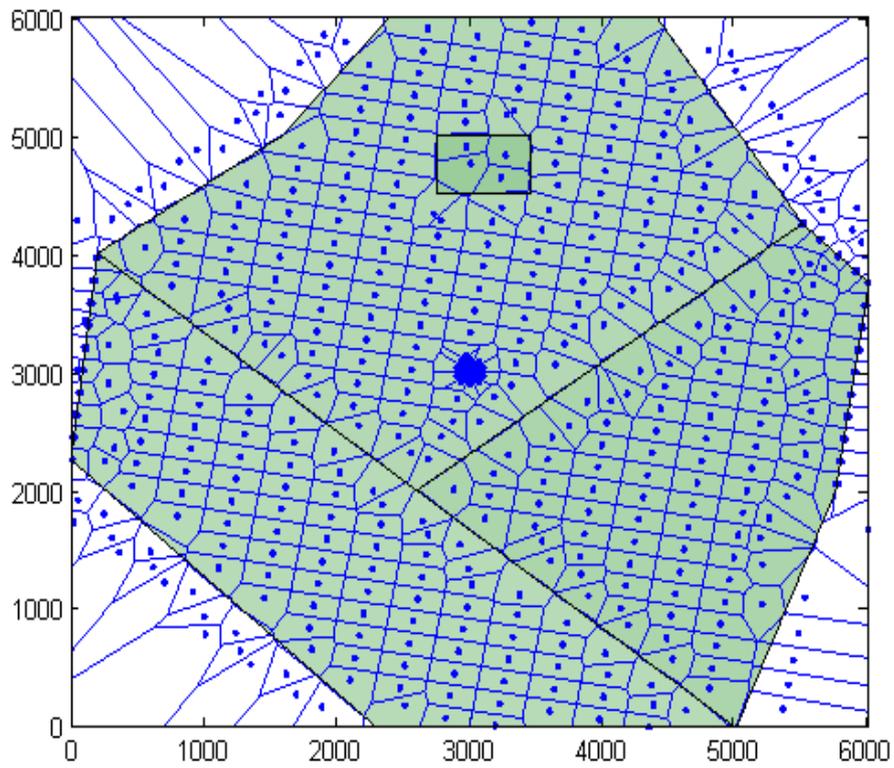


Figure 8.37. Results obtained for the case #5 (built in MATLAB).

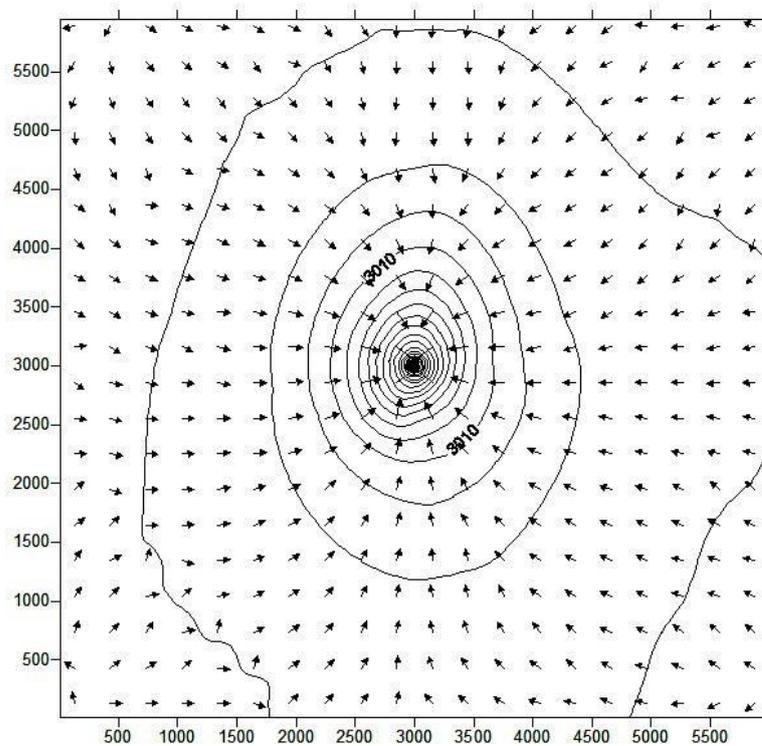


Figure 8.38. Velocity field combined with the contour map of the distribution of the pressures after 2.5 days of production for the fifth case (obtained with Surfer).

Expectations are to see pressure disturbance propagating at a higher rate towards the lower regions, because they have higher permeability values than the other regions.

The picture shown on figure 8.38 completely coincides with the expectations - disturbance propagated further in the east and south directions, exactly where the higher permeability regions are located. So, the algorithm was also successful in solving this kind of problems. Other flow simulation run's timesteps can be found in Appendix E.

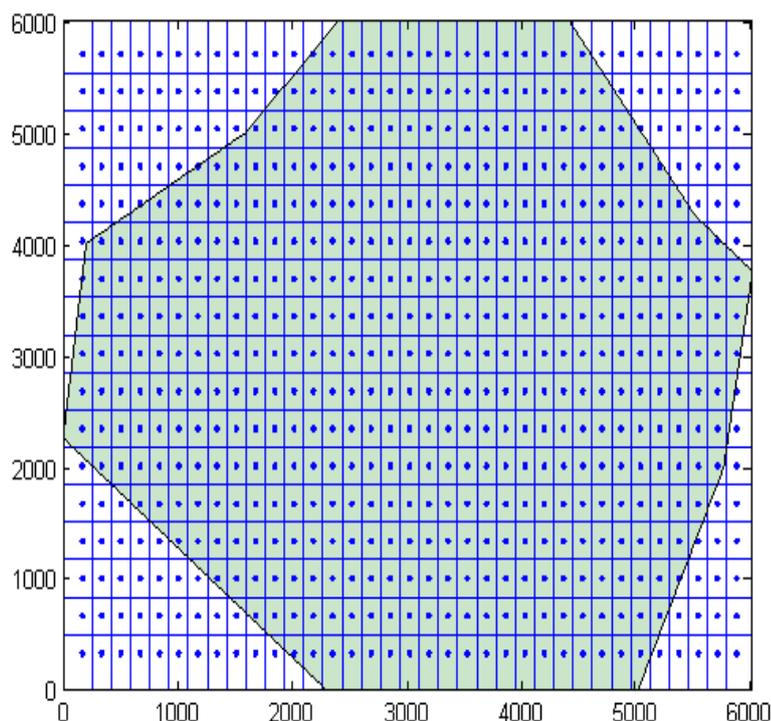


Figure 8.39. Results obtained after running of the first step for the case #6 (built in MATLAB).

8.2.6. Case Six (anisotropy, no heterogeneities, fault and horizontal well)

Case six, last case discussed in this thesis, represents conditions with compartmentalized reservoir, from where production is performed using one horizontal well.

The main purpose of showing of this case is to see, how the fault and horizontal well are added to the model. Permeability field required for running of the algorithm is the same as in cases #1 and #2.

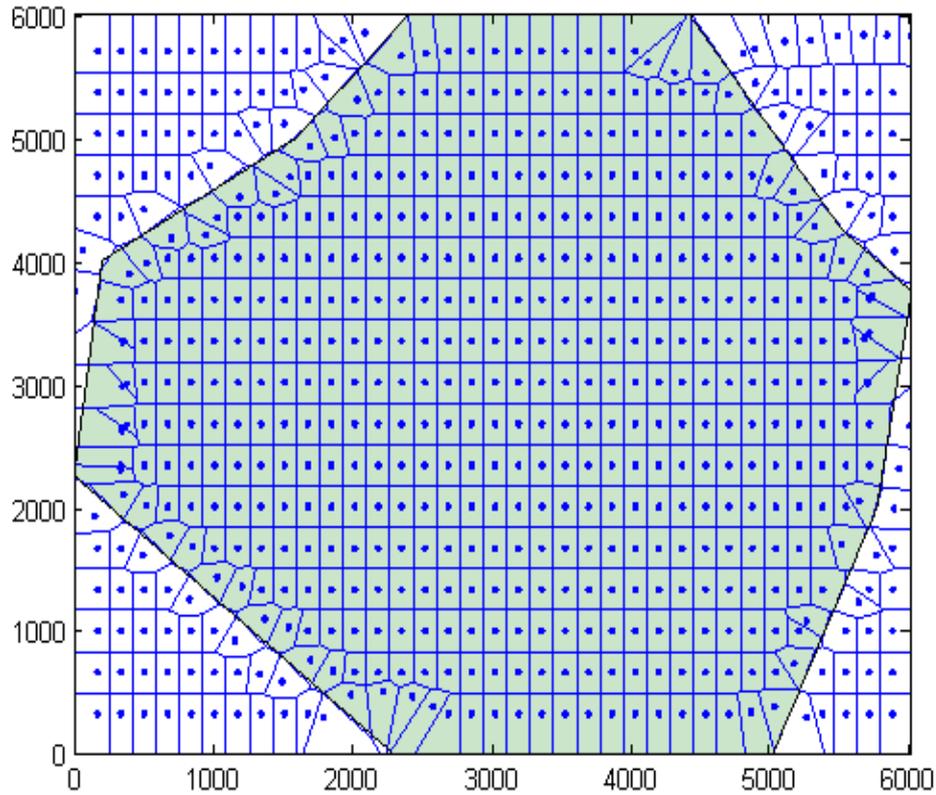


Figure 8.40. Results obtained after running of the second step of case #6 (built in MATLAB).

Main inputs for the first step are as follows: 600 grid points; angle between K_y and y -direction of the field is equal to 90 degrees; K_y to K_x relation is equal to 2; small distance is equal to 5, while each iteration distance increment is equal to 2.5. Reservoir boundaries are chosen as in all of the previous cases.

At the end of the first step resulting picture is as shown on figure 8.39. Resulting sum of errors at this figure is equal to 18317. As it may be seen from the figure, grid after the first step is very close to Cartesian grid.

Main inputs for the second step are: number of points that were changed during each iteration is 5; 320 iterations; each point was allowed to move 9 times; only one permeability region with permeabilities between 1 and 1000 md is present.

Resulting locations of grid points after the second step of the case #5 are shown at figure 8.40. As in the previous cases, number of blocks is still the same as in figure 8.39, so change in the sum of errors is only because of the better placement of the grid points. Sum of errors reduced here from the 18317 to 2811.

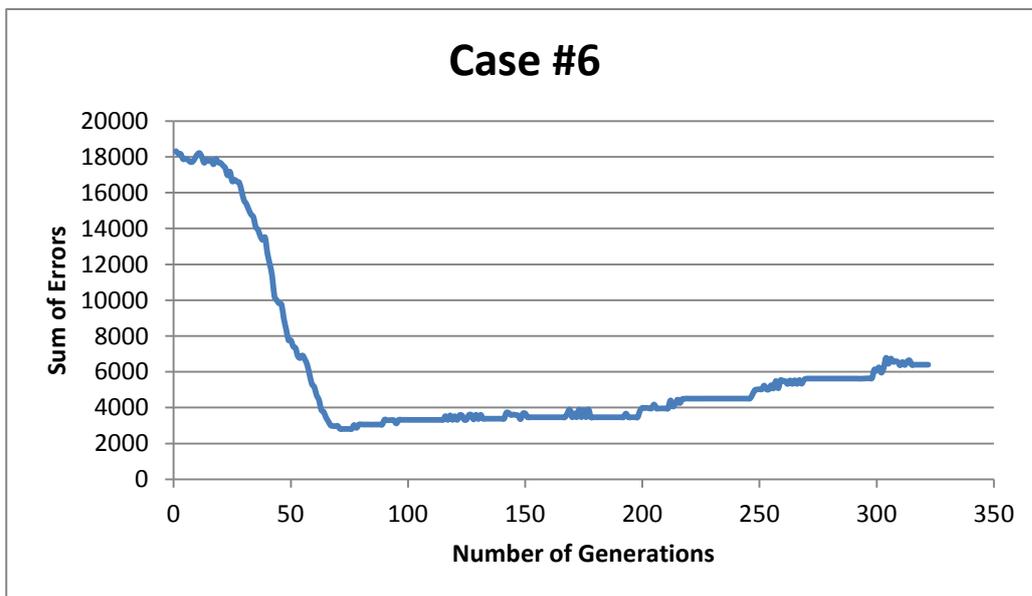


Figure 8.41. Error values for all generations of Case #6.

Figure 8.41. shows error values for all 320 generations of the fourth case. The curve continuously goes down, reaches minimum in the mid-sixties and then shows mainly upward trend.

Main inputs for the third step are: one horizontal well and one sealing fault totally dividing reservoir into two compartments (figure 8.42); radius around the well that was repopulated is equal to 50 feet; 100 points related to this well were added; distance between the first and the second layers of points around the well is equal to 5 feet; distance to the next layers are 1.2 times the distance to the previous one.

The obtained results are shown on figure 8.42. The resulting number of blocks is equal to 747, while the sum of errors in all of the blocks reduced from an initial value of 18317 to 2811, which is also pretty impressive. Resulting number of blocks is higher than the 600 entered in the first step because of the grid points related to well and fault and also points related to proper reservoir border handling.

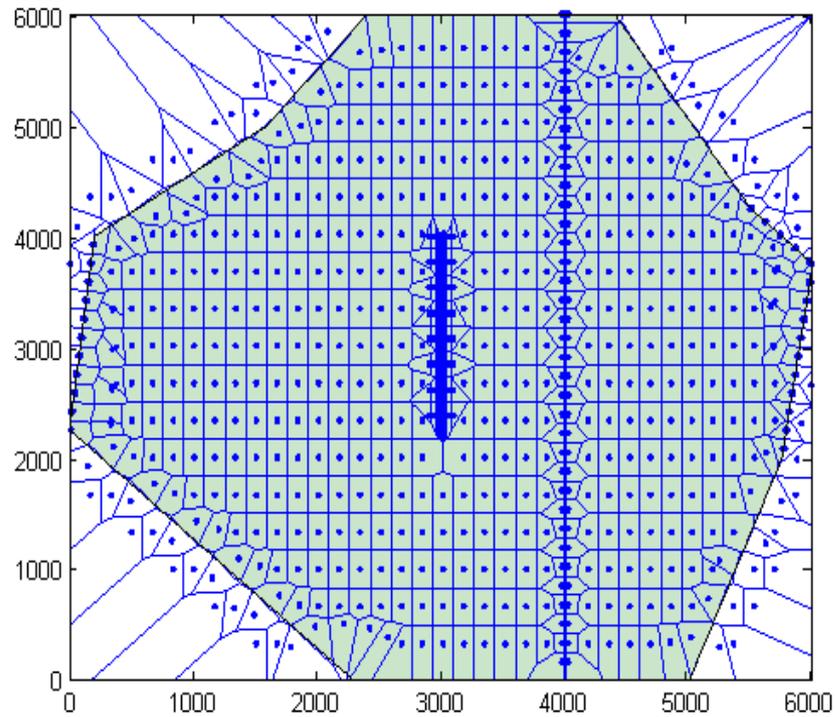


Figure 8.42. Results obtained for the case number six (built in MATLAB).

For this case fluid flow simulation run was not implemented, because the model that was used for running of previous cases is still under construction and is not able to handle horizontal wells at the moment of writing of this thesis work.

8.3. Effects of inputs on final results

This subchapter discusses effects of input values on final results by showing several cases and talking about them. At the end of it, conclusion about how parameters should be chosen in order to get less error values is made. Parameters chosen for this study are all the parameters that affect final distribution of grid points, that can be

changed. Also no sensitivity study on to what extent each of the parameters affects final result is made, so it is proposed to do it in the future studies.

8.3.1. Effect of number of grid points

It is obvious that higher amount of points should end up with less resulting error. This is shown on figure 8.43.

The only difference in the cases shown in the figure is number of grid points, other than that all input values were chosen to be absolutely the same. Background petrophysical property mesh consists of 57600 uniformly distributed permeability points with values randomly generated in the interval between 1 and 1000 md. As it may be seen from the figure, while number of blocks is four times higher on the right figure, error in there is approximately less by one-fifth. This difference is not very big, however, from the figure it may be seen that reservoir boundary handling is much better than in the left one, especially in the right lower corner.

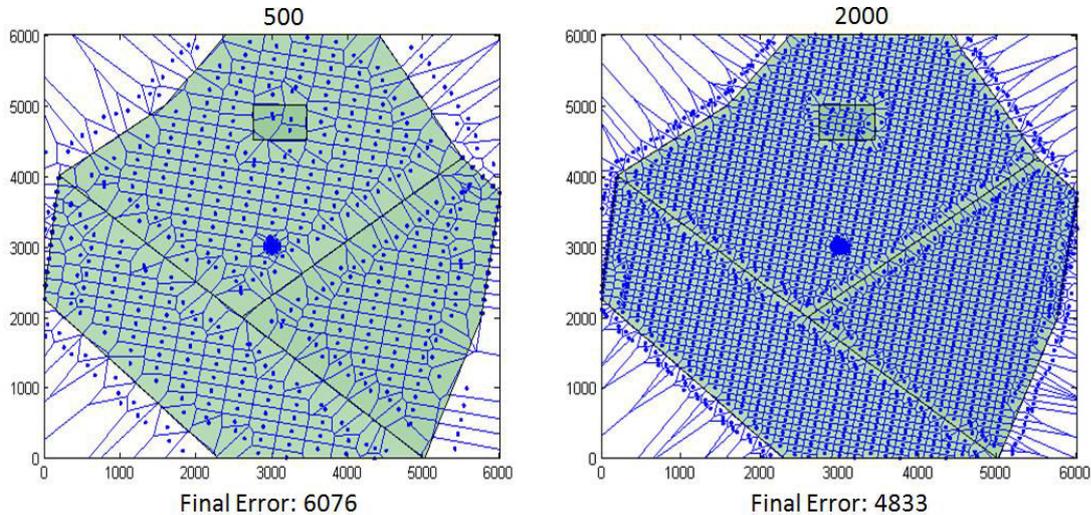


Figure 8.43. Comparing results with different number of blocks (obtained with MATLAB).

But this better accuracy comes at big price - calculation time of the second step, which is the most time consuming step in the algorithm, differs significantly for

these two cases: left one requires 200 seconds in order to perform 100 generations, while the right one requires 670 seconds for the same amount of generations. Also, because right case has higher amount of grid points, more of them will require relocation compared to the left case. This means that for the same input values, right hand side will also require higher amount of generations in order to get to the minimum error value that could be used as the final result. So, whether to use higher amount of grid points is not an easy question.

8.3.2. Effect of number of moving grid points

As it was discussed in methodology, during each generation number of grid points with highest error values are moved. This number is taken from the input given by the person using the algorithm, so there is a question whether to give big values in order to move several grid points at once, or limit them, for example, moving them one by one. Figure 8.44 shows two cases that differ from each other only by this criterion.

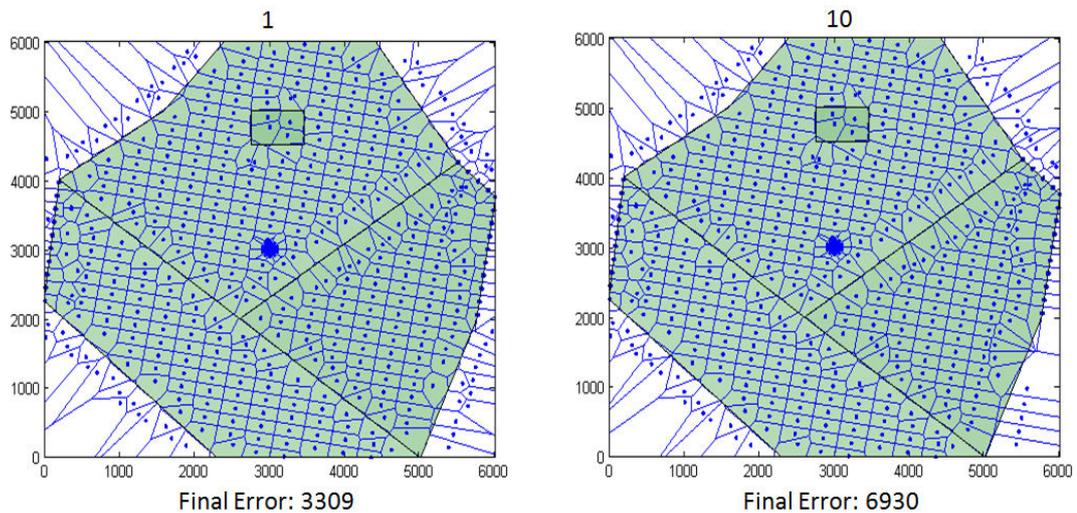


Figure 8.44. Comparing results with different number of movements for each grid point (obtained with MATLAB).

In the case shown on the left grid points were moved one by one, while in the case shown on the right ten worst grid points were chosen in each generation to be

moved. First case shows smaller final error value and that is what can be expected: when high amount of point is moved during each generation, several points that are not the worst are moved also with the worst one. The problem is that maybe better result in terms of error function may have been achieved in the middle of moving. That is why it is proposed to move less amount of grid points during each generation. However, this choice also have drawbacks. The less amount of grid points is moved during each generation, the more amount of generations may be required to get best results, which affects significantly calculation time. Also it must be said that moving several points in each generation also affects calculation time, by affecting each generation calculation time. Left hand side case requires 105 seconds in order to calculate 100 generations, while right hand side case requires 140 seconds. This difference is not very big, but also should be considered.

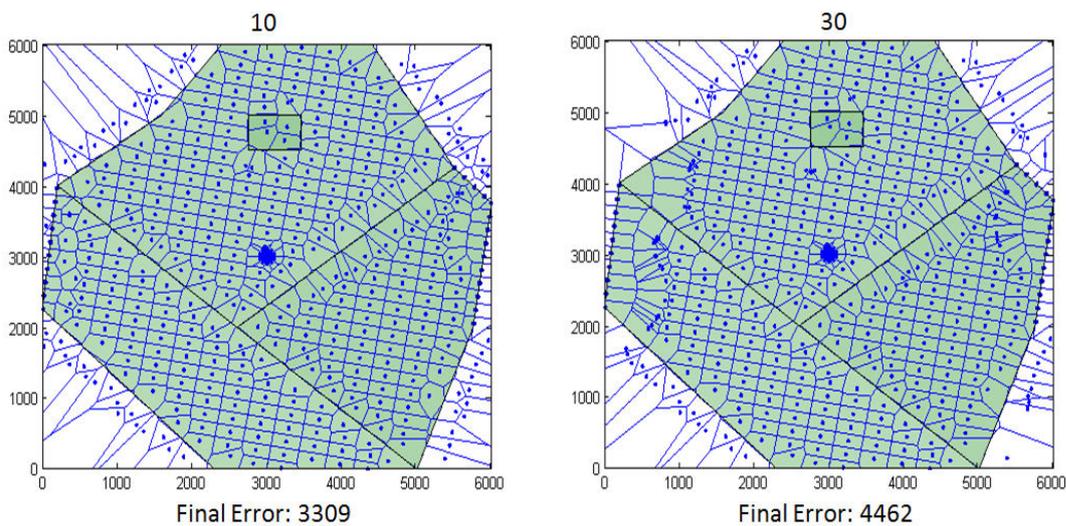


Figure 8.45. Comparing results with different limits of movements (obtained with MATLAB).

8.3.3. Effect of limit of movement of grid points

Another input criterion that affects final results is number of times each point can be moved. This is a very important values, because it lets control at what distance may point end up from its original place. The effect of this criteria is shown on figure 8.45.

As in the previous cases, these ones are also differ only in the input criterion that is discussed - all other values are absolutely the same. As it can be seen from the figure 8.45, left hand side case, using limit of movement equal to ten, has smaller error value at the end and also less distortion of the initial grid on the left and right sides of the reservoir. As it was already said, limiting movement makes it possible to control movement of points to some extent, which means that smaller value for this criterion should end up in less grid distortion. However, by limiting number of movements better results also may be missed. This criteria does not affect computation time, so it must be carefully considered whether to let points to move freely in the reservoir or to limit them.

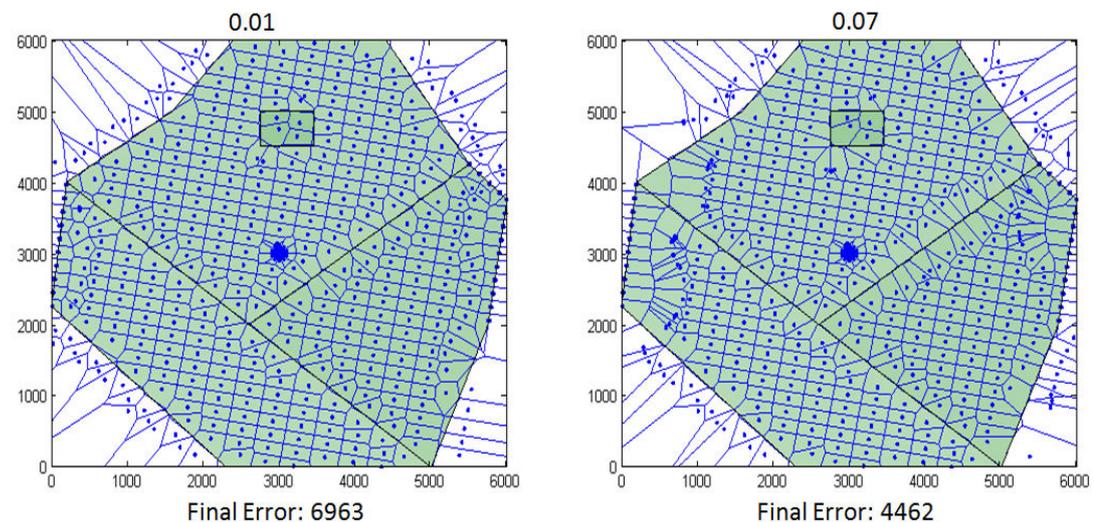


Figure 8.46. Comparing results with different distance of movement (obtained with MATLAB).

8.3.4. Effect of distance of movement of grid points

Finally, the last criterion that affects final result - distance at which points are moved. As it was mentioned in the methodology chapter, input also includes fraction of longer distance that points are moved at. Longer distance is the distance between neighboring grid points in the direction of lower permeability. The effect of this criterion is shown in the figure 8.46.

It seems like the distance at which points are moved during each iteration should be chosen as little in order to get better results. However, cases show something opposite to this supposition. Error of the case with distance of movement equal to 0.01 times the big distance has higher final error values than the case with 0.07 times the big distance. It also can be seen from the figure, right hand side case deals with reservoir properties in a more accurate way than the left hand side case. This can be seen especially in the lower left corner. The only advantage of smaller moving distance seems to be less distortion of the initial grid. So, proposition would be to use something in between of these values, but the choice totally depends on the person using the algorithm.

In the next chapter conclusion and propositions for future studies can be found.

CHAPTER 9

CONCLUSION

Based on the results obtained after running of cases of increasing complexity, algorithm described in this thesis proved to be capable of placing Voronoi grid points in reservoir simulation in order to honor geological properties of the reservoir including anisotropy orientation and ratio, and reservoir heterogeneities.

One of the main advantages of the algorithm is that it tries to obtain better locations of limited number of Voronoi grid points without making a significant increase in this number. This means that instead of increasing the number of grid points exceedingly, it provides better solutions rather than placing these points uniformly all over the field. Error values and graphs that show final distribution of the grid plots show that as the grid block boundaries coincide with background reservoir property mesh, error values decrease.

Also it must be mentioned that effectiveness of the algorithm in terms of error function can be affected by values of some input parameters required for running of the algorithm. The most effective inputs include the number of times that the points are allowed to move as less points to be moved during each iteration, but a greater number of times and therefore with more iterations. Number of grid points also affects final accuracy - the higher the number of grid points the better is the result. However, these choices will increase computation time, so they must be carefully chosen before application of the algorithm to the problem.

CHAPTER 10

PROPOSITION FOR FUTURE STUDIES

For future studies it is proposed to use the algorithm on the cases of increasing complexity and to add all necessary sophistications to the algorithm so that all the complications will be solved.

This includes:

- Trying to change algorithm, so that not only one property would be used for differentiating different geological entities. In this study only permeability was used for this purpose, proposition is to use several different properties together at one time. Maybe some weightening of these properties should be applied to each of the property points.
- Considering application of the algorithm to 3D problems, including not horizontal formations.
- Considering handling of regions with intersecting intervals of property values.
- Thinking if a better error function definition could be used.
- Checking of the results obtained for the case with horizontal wells by running flow simulation.
- Do similar flow simulation runs with Cartesian grid and compare the results for all of the cases.
- Thinking if the grid points near the reservoir boundaries can be handled better. Maybe truncation concept can be used there, which will further decrease number of grid points.

- Doing sensitivity analysis on to what extent each of the input parameters affects final result.

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APPENDIX A

SOURCE CODE

```
%{
```

This code was written as a part of a METU MSc dissertation of Ulvi Rza-Guliyev.

To run it, four column vectors named permXvec, permYvec, permZvec, permeabilitiesVec must already be loaded onto workspace of Matlab. permXvec, permYvec and permZvec must have X, Y and Z coordinates of permeability points in the field, while permeabilitiesVec must have permeability values in x direction in it. All other inputs are given in the code.

```
%}
```

```
% Start of the first step, where initial population of the gridpoints is  
% generated
```

```
n=input('Enter number of points: ');
```

```
angle=input('Enter value of Tetta, 0-360, degrees: ');
```

```
tet=pi*angle/180; % Changed to radians for trigonometric functions
```

```
if (tet < pi/2)
```

```
    w=0;
```

```
elseif (tet >= pi/2 && tet < pi)
```

```
    w=pi/2;
```

```
elseif (tet >= pi && tet < 3*pi/2)
```

```

    w=pi;
elseif (tet >= 3*pi/2 && tet < 2*pi)
    w=3*pi/2;
elseif tet == 2*pi
    tet=0;
    w=0;
else fprintf('Angle must be between 0 and 360 degrees. Start again. ');
end
m=input('Enter Ky/Kx value: ');
kykxrel=m;
R=input('Enter distance in minimum direction (should be small): ');
increment=input('Enter value of increment to the minimum distance,\nthat will be
added at the end of each iteration: ');

% Entering reservoir vertexes
resBouVert=zeros;
nprb=input('Enter number of reservoir vertexes: ');
for kl=1:1:nprb
    resBouVert(kl,1)=input('Enter x coordinate of vertex: ');
    resBouVert(kl,2)=input('Enter y coordinate of vertex: ');
end
resBouVert(nprb+1,1)=input('Enter x coordinate of first vertex: ');
resBouVert(nprb+1,2)=input('Enter y coordinate of first vertex: ');
resVertX=resBouVert(:,1);
resVertY=resBouVert(:,2);

% Calculating of rectangle boundaries
length=abs(max(resVertX)-min(resVertX));
width=abs(max(resVertY)-min(resVertY));
resVertX=resVertX-min(resVertX);
resVertY=resVertY-min(resVertY);

```

```

counter=n+1;
while counter>n

if m > 1 && tet >= 0 && tet < pi/2
    Ry=R;
    Rx=m*R;
elseif m > 1 && tet >= pi/2 && tet < pi
    Ry=m*R;
    Rx=R;
elseif m > 1 && tet >= pi && tet < 3*pi/2
    Ry=R;
    Rx=m*R;
elseif m > 1 && tet >= 3*pi/2 && tet < 2*pi
    Ry=m*R;
    Rx=R;
elseif m < 1 && tet >= 0 && tet < pi/2
    Rx=m*R;
    Ry=R;
elseif m < 1 && tet >= pi/2 && tet < pi
    Ry=m*R;
    Rx=R;
elseif m < 1 && tet >= pi && tet < 3*pi/2
    Rx=m*R;
    Ry=R;
elseif m < 1 && tet >= 3*pi/2 && tet < 2*pi
    Ry=R;
    Rx=m*R;
else
    Rx=R;
    Ry=R;
end

```

```

% Now it will calculate dimensions of matrix.
if (tet > 0 && tet < pi/2)
    overwidth=width*cos(tet)+length*sin(tet);
    overlength=width*sin(tet)+length*cos(tet);
    matrixNreal=overlength/Rx;
    matrixMreal=overwidth/Ry;
elseif (tet > pi && tet < 3*pi/2)
    overwidth=width*cos(tet-pi)+length*sin(tet-pi);
    overlength=width*sin(tet-pi)+length*cos(tet-pi);
    matrixNreal=overlength/Rx;
    matrixMreal=overwidth/Ry;
elseif (tet > pi/2 && tet < pi)
    overlength=width*cos(tet-pi/2)+length*sin(tet-pi/2);
    overwidth=width*sin(tet-pi/2)+length*cos(tet-pi/2);
    matrixNreal=overwidth/Rx;
    matrixMreal=overlength/Ry;
elseif (tet > 3*pi/2 && tet < 2*pi)
    overlength=width*cos(tet-3*pi/2)+length*sin(tet-3*pi/2);
    overwidth=width*sin(tet-3*pi/2)+length*cos(tet-3*pi/2);
    matrixNreal=overwidth/Rx;
    matrixMreal=overlength/Ry;
elseif (tet == 0 || tet == pi || tet == 2*pi)
    matrixNreal=width/Ry;
    matrixMreal=length/Rx;
elseif (tet == pi/2 || tet == 3*pi/2)
    matrixNreal=width/Rx;
    matrixMreal=length/Ry;
else fprintf('Angle must be between 0 and 360 degrees. Start again. ');
end

matrixN=ceil(matrixNreal);
matrixM=ceil(matrixMreal);

```

```

abscissa=zeros(matrixN,matrixM);
ordinate=zeros(matrixN,matrixM);

% Now it will calculate starting point, which is outside of reservoir.
if (tet > 0 && tet < pi/2)
    Sx=0-width*sin(tet)*cos(tet);
    Sy=width*sin(tet)*sin(tet);
elseif (tet > pi/2 && tet < pi)
    Sx=0-width*sin(tet-pi/2)*cos(tet-pi/2);
    Sy=width*sin(tet-pi/2)*sin(tet-pi/2);
elseif (tet > pi && tet < 3*pi/2)
    Sx=0-width*sin(tet-pi)*cos(tet-pi);
    Sy=width*sin(tet-pi)*sin(tet-pi);
elseif (tet > 3*pi/2 && tet < 2*pi)
    Sx=0-width*sin(tet-w)*cos(tet-w);
    Sy=width*sin(tet-w)*sin(tet-w);
elseif (tet == 0 || tet == pi/2 || tet == pi || tet == 3*pi/2 || tet == 2*pi)
    Sx=0;
    Sy=0;
else fprintf('Angle must be between 0 and 360 degrees. Start again.');
```

```

end

% Start of generation of probable initial population of gridpoints
if (tet > 0 && tet < pi/2 ) || (tet > pi && tet < 3*pi/2)
    for i=1:1:matrixN
        abscissa(i,1)=Sx;
        ordinate(i,1)=Sy;
        for e=2:1:matrixM
            abscissa(i,e)=abscissa(i,e-1)+Ry*sin(tet-w);
            ordinate(i,e)=ordinate(i,e-1)+Ry*cos(tet-w);
        end
        Sx=Sx+Rx*cos(tet-w);

```

```

    Sy=Sy-Rx*sin(tet-w);
end
elseif (tet > pi/2 && tet < pi) || (tet > 3*pi/2 && tet < 2*pi);
for i=1:1:matrixN
    abscissa(i,1)=Sx;
    ordinate(i,1)=Sy;
    for e=2:1:matrixM
        abscissa(i,e)=abscissa(i,e-1)+Ry*sin(tet-w);
        ordinate(i,e)=ordinate(i,e-1)+Ry*cos(tet-w);
    end
    Sx=Sx+Rx*cos(tet-w);
    Sy=Sy-Rx*sin(tet-w);
end
elseif (tet == 0 || tet == pi)
for i=1:1:matrixN
    abscissa(i,1)=Sx;
    ordinate(i,1)=Sy;
    for e=2:1:matrixM
        ordinate(i,e)=ordinate(i,e-1)+Rx*sin(tet-w);
        abscissa(i,e)=abscissa(i,e-1)+Rx*cos(tet-w);
    end
    Sy=Sy+Ry*cos(tet-w);
    Sx=Sx+Ry*sin(tet-w);
end
elseif (tet == pi/2 || tet == 3*pi/2)
for i=1:1:matrixN
    abscissa(i,1)=Sx;
    ordinate(i,1)=Sy;
    for e=2:1:matrixM
        ordinate(i,e)=ordinate(i,e-1)+Ry*sin(tet-w);
        abscissa(i,e)=abscissa(i,e-1)+Ry*cos(tet-w);
    end

```

```

    Sy=Sy+Rx*cos(tet-w);
    Sx=Sx+Rx*sin(tet-w);
end
else fprintf('Angle must be between 0 and 360 degrees');
end

v=1;
sk=matrixN*matrixM;
absc=zeros(sk,1);
ord=zeros(sk,1);

% Deleting of the points outside of rectangle
for i=1:1:matrixN
    for e=1:1:matrixM
        if abscissa(i,e)>0 && abscissa(i,e)<=length ...
            && ordinate(i,e)>0 && ordinate(i,e)<=width
            absc(v,1)=abscissa(i,e);
            ord(v,1)=ordinate(i,e);
            v=v+1;
        end
    end
end

% Means of seeing how many points were generated in current iteration
fprintf('Number of blocks on this stage is: %g\n', v);
if m>=1
    fprintf('At this stage your minimum distance is: %g\n', R);
else
    fprintf('At this stage your minimum distance is: %g\n', m*R);
end
tocontinue=input('Press 0 + enter:');

```

```

counter=v;
if v>n
    R=R+increment;
else fprintf('Number of blocks is: %g\n', counter-1);
end
end

% Saving of the final initial population in allGens
numGen=1;
allGens{numGen,1}=absc;
allGens{numGen,2}=ord;

% Inputs for the second step
lastNumPoints=input('Enter number of points that will be changed in every iteration: ');
moveFun=input('Enter fraction of distance that points will be moved (i.e. 0.07): ');
requiredNumGen=input('Enter required number of generations: ');
requiredError=input('Enter required error: ');
limitOfMovement=input('Enter required number of movements for each point: ');
numReg=input('Enter number of regions (without zero region): ');
minLim=zeros;
maxLim=zeros;
for kg=1:1:numReg
    fprintf('Enter minimum permeability in region #%g: ',kg);
    minLim(kg,1)=input("");
    fprintf('Enter maximum permeability in region #%g: ',kg);
    maxLim(kg,1)=input("");
end
minLim(kg+1,1)=0;
maxLim(kg+1,1)=abs(min(minLim)-1);

numberOfMovements=zeros(counter-1,1);

```

```

sumError=requiredError+1;
checker21=1;

permXi=size(permXvec,1);
permFieldVec=[permXvec,permYvec];
if m*R>=R
    maxDist=m*R;
    minDist=R;
else
    maxDist=R;
    minDist=m*R;
end

stopStep2=0;
minimEr=inf;
while sumError>=requiredError && numGen<=requiredNumGen+1 &&
stopStep2~=42
num=1;
number=1;
x=zeros(counter-1,1);
y=zeros(counter-1,1);
z=40*ones(counter-1,1);
absc=[absc;0];
ord=[ord;0];
while absc(num,1)~=0
    absc(num,1)=roundn(absc(num,1),-3);
    x(num,1)=absc(num,1);
    num=num+1;
end
while ord(number,1)~=0
    ord(number,1)=roundn(ord(number,1),-3);
    y(number,1)=ord(number,1);

```

```

    number=number+1;
end

xy=[x,y];
blocksOfPoints=zeros(permXi,counter-1);
distances=pdist2(permFieldVec,xy);

% Finding which block each permeability point is related to
for kk=1:1:size(distances,1)
    distancesForPoints=distances(kk,:);
    backUpDist=distancesForPoints;
    [closestDist,ind]=min(backUpDist);
    blocksOfPoints(kk,ind)=permeabilitiesVec(kk,1);
    backUpDist(1,ind)=inf;
    [closestDist2,ind2]=min(backUpDist);
    if closestDist2==closestDist
        blocksOfPoints(kk,ind2)=permeabilitiesVec(kk,1);
        backUpDist(1,ind2)=inf;
        [closestDist3,ind3]=min(backUpDist);
        if closestDist3==closestDist2
            blocksOfPoints(kk,ind3)=permeabilitiesVec(kk,1);
            backUpDist(1,ind3)=inf;
            [closestDist4,ind4]=min(backUpDist);
            if closestDist4==closestDist3
                blocksOfPoints(kk,ind4)=permeabilitiesVec(kk,1);
                backUpDist(1,ind4)=inf;
                [closestDist5,ind5]=min(backUpDist);
                if closestDist5==closestDist4
                    blocksOfPoints(kk,ind5)=permeabilitiesVec(kk,1);
                    backUpDist(1,ind5)=inf;
                    [closestDist6,ind6]=min(backUpDist);
                    if closestDist6==closestDist5

```



```

forError=0;
uu=1;
error=zeros(counter-1,1);

% Calculating of error for each block
while jj<=counter-1
    for ii=1:1:permXi
        if blocksOfPoints(ii,jj)~=0;
            forError(uu,1)=blocksOfPoints(ii,jj);
            uu=uu+1;
        end
    end
    for ht=1:1:size(minLim,1)
        if forError(1,1)>=minLim(ht,1) && forError(1,1)<=maxLim(ht,1)
            blockMin=minLim(ht,1);
            blockMax=maxLim(ht,1);
        end
    end
    ug=0;
    for hk=1:1:size(forError)
        if forError(hk,1)>=blockMin && forError(hk,1)<=blockMax
            ug=ug+1;
        end
    end
    if ug==size(forError,1)
        error(jj,1)=0;
    elseif ug<size(forError,1)
        error(jj,1)=std(forError);
    else fprintf('Something is wrong(line 346)');
    end
    checker{jj,checker21}=forError;
    jj=jj+1;
end

```

```

    uu=1;
    forError=0;
end

% Saving errors in allGens
allGens{numGen,3}=error;
sumError=sum(error);
allGens{numGen,4}=sumError;
if minimEr>sumError
    minimEr=sumError;
elseif sumError>=minimEr+2000
    stopStep2=42;
end

```

```

% Assigning of fitnesses
errorBackUp=error;
fitness=zeros(counter-1,1);
ii=1;
while ii<=counter-1
    [value,index]=min(errorBackUp);
    fitness(index,1)=ii;
    errorBackUp(index,1)=inf;
    ii=ii+1;
end

```

```

xBackUp=x;
yBackUp=y;
pp=counter-1;
dd=0;

```

```

% Moving of bad points in the required direction while checking if number
% of movements for these points is less than the limit

```

```

while dd<=lastNumPoints && pp>=2
for index=1:1:counter-1
    if fitness(index,1)==pp
        ppm=zeros(numReg+1,1);
        if numberOfMovements(index,1)<limitOfMovement;
            numberOfMovements(index,1)=numberOfMovements(index,1)+1;
            dd=dd+1;
            blm=checker{index,checker21 };
            % Calculates how many points inside block are of different
            % regions permeability.
            for zv=1:1:size(blm,1)
                for fq=1:1:numReg
                    if blm(zv,1)==0.001
                        ppm(numReg+1)=ppm(numReg+1)+1;
                    elseif blm(zv,1)<=maxLim(fq,1) && blm(zv,1)>=minLim(fq,1)
                        ppm(fq,1)=ppm(fq,1)+1;
                    else moe=42;
                end
            end
        end
        end
        xes=zeros;
        yes=zeros;
        skip=1;
        [valper,indper]=max(ppm);
        useMin=minLim(indper,1);
        useMax=maxLim(indper,1);
        for jkr=1:1:size(permeabilitiesVec,1)
            if (blocksOfPoints(jkr,index)<useMin &&
blocksOfPoints(jkr,index)>0) || (blocksOfPoints(jkr,index)>useMax)
                xes(skip,1)=permXvec(jkr,1);
                yes(skip,1)=permYvec(jkr,1);
                skip=skip+1;
            end
        end
    end
end

```

```

    end
end
centX=mean(xes);
centY=mean(yes);
myPointX=x(index,1);
myPointY=y(index,1);
bcatm1=abs(centX-myPointX);
bcatm2=abs(centY-myPointY);
xuse=myPointX-centX;
yuse=myPointY-centY;
if xuse>0 && yuse>0
    alpha=atand(bcatm2/bcatm1);
    x(index,1)=x(index,1)+maxDist*moveFun*cosd(alpha);
    y(index,1)=y(index,1)+maxDist*moveFun*sind(alpha);
elseif xuse<0 && yuse>0
    alpha=atand(bcatm2/bcatm1);
    x(index,1)=x(index,1)-maxDist*moveFun*cosd(alpha);
    y(index,1)=y(index,1)+maxDist*moveFun*sind(alpha);
elseif xuse>0 && yuse<0
    alpha=atand(bcatm2/bcatm1);
    x(index,1)=x(index,1)+maxDist*moveFun*cosd(alpha);
    y(index,1)=y(index,1)-maxDist*moveFun*sind(alpha);
elseif xuse<0 && yuse<0
    alpha=atand(bcatm2/bcatm1);
    x(index,1)=x(index,1)-maxDist*moveFun*cosd(alpha);
    y(index,1)=y(index,1)-maxDist*moveFun*sind(alpha);
elseif xuse==0 && yuse>0
    x(index,1)=x(index,1);
    y(index,1)=y(index,1)+maxDist*moveFun;
elseif xuse==0 && yuse<0
    x(index,1)=x(index,1);
    y(index,1)=y(index,1)-maxDist*moveFun;

```

```

        elseif xuse>0 && yuse==0
            x(index,1)=x(index,1)+maxDist*moveFun;
            y(index,1)=y(index,1);
        elseif xuse<0 && yuse==0
            x(index,1)=x(index,1)-maxDist*moveFun;
            y(index,1)=y(index,1);
        else fprintf('My point and point it bounces off are at the same place');
        end
    end
end
end
pp=pp-1;
end

absc=x;
ord=y;
numGen=numGen+1;
fprintf('Gen #%g\n',numGen);
allGens{numGen,1}=absc;
allGens{numGen,2}=ord;
checker21=checker21+1;
end

% Choose best result from allGens
helper=zeros;
for i=1:1:numGen-1
    helper(i,1)=allGens{i,4};
end

[value,minIndex]=min(helper);
xBest=allGens{minIndex,1};
yBest=allGens{minIndex,2};

```

```

% Remove points outside of reservoir
resVertXBackUp=resVertX;
resVertYBackUp=resVertY;
if m>=1
    moveDist=m*R;
else
    moveDist=R;
end
smt=size(resVertX,1);
for ii=1:1:smt
    if resVertX(ii,1)>=0 && resVertX(ii,1)<length/2 && resVertY(ii,1)>=0 &&
resVertY(ii,1)<width/2
        resVertXBackUp(ii,1)=resVertXBackUp(ii,1)-moveDist*cosd(45);
        resVertYBackUp(ii,1)=resVertYBackUp(ii,1)-moveDist*sind(45);
    elseif resVertX(ii,1)>=0 && resVertX(ii,1)<length/2 && resVertY(ii,1)>width/2
&& resVertY(ii,1)<=width
        resVertXBackUp(ii,1)=resVertXBackUp(ii,1)-moveDist*cosd(45);
        resVertYBackUp(ii,1)=resVertYBackUp(ii,1)+moveDist*sind(45);
    elseif resVertX(ii,1)>length/2 && resVertX(ii,1)<=length &&
resVertY(ii,1)>width/2 && resVertY(ii,1)<=width
        resVertXBackUp(ii,1)=resVertXBackUp(ii,1)+moveDist*cosd(45);
        resVertYBackUp(ii,1)=resVertYBackUp(ii,1)+moveDist*sind(45);
    elseif resVertX(ii,1)>length/2 && resVertX(ii,1)<=length && resVertY(ii,1)>=0
&& resVertY(ii,1)<width/2
        resVertXBackUp(ii,1)=resVertXBackUp(ii,1)+moveDist*cosd(45);
        resVertYBackUp(ii,1)=resVertYBackUp(ii,1)-moveDist*sind(45);
    elseif resVertX==length/2 && resVertY>=0 && resVertY<width/2
        resVertYBackUp(ii,1)=resVertYBackUp(ii,1)-moveDist;
    elseif resVertX==length/2 && resVertY>width/2 && resVertY<=width
        resVertYBackUp(ii,1)=resVertYBackUp(ii,1)+moveDist;
    elseif resVertX>=0 && resVertX<length/2 && resVertY==width/2

```

```

        resVertXBackUp(ii,1)=resVertXBackUp(ii,1)-moveDist;
elseif resVertX>length/2 && resVertX<=length && resVertY==width/2
        resVertXBackUp(ii,1)=resVertXBackUp(ii,1)+moveDist;
elseif resVertX==length/2 && resVertY==width/2
        fprintf('Point in the center. Left at its place');
else fprintf('Something is wrong with vertexes (line 584)');
end
end
end

```

```

nos=inpolygon(xBest, yBest, resVertXBackUp, resVertYBackUp);
nos=nos+0;
xk=zeros;
yk=zeros;
kew=1;
for kk=1:1:size(nos,1)
    if nos(kk,1)==1
        xk(kew,1)=xBest(kk,1);
        yk(kew,1)=yBest(kk,1);
        kew=kew+1;
    end
end
end
xBest=xk;
yBest=yk;

```

```

% Handling of boundaries close to the rectangle boundaries
ii=2;
targetZone=zeros(1,2);
while ii<=size(resVertX,1)
    if resVertX(ii,1)>=0 && resVertX(ii,1)<=1.5*maxDist && ...
        resVertX(ii-1,1)>=0 && resVertX(ii-1,1)<=1.5*maxDist
    if resVertX(ii,1)~=0 || resVertX(ii-1,1)~=0
        startX=resVertX(ii-1,1);
    end
end
end

```

```

startY=resVertY(ii-1,1);
endX=resVertX(ii,1);
endY=resVertY(ii,1);
bcat1=abs(startX-endX);
bcat2=abs(startY-endY);
boundLength=sqrt(bcat1*bcat1+bcat2*bcat2);

distForBound=maxDist/2;
numOfPoints=boundLength/distForBound;
xMove=startX-endX;
yMove=startY-endY;

if xMove>0 && yMove>0
    alpha=atand(bcat2/bcat1);
    firstX1=startX-10*sind(alpha);
    firstY1=startY+10*cosd(alpha);
    firstX2=startX+10*sind(alpha);
    firstY2=startY-10*cosd(alpha);
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)-distForBound*cosd(alpha);
        firstP1(pd,2)=firstP1(pd-1,2)-distForBound*sind(alpha);
        firstP2(pd,1)=firstP2(pd-1,1)-distForBound*cosd(alpha);
        firstP2(pd,2)=firstP2(pd-1,2)-distForBound*sind(alpha);
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove>0 && yMove<0

```

```

alpha=atand(bcat2/bcat1);
firstX1=startX+10*sind(alpha);
firstY1=startY+10*cosd(alpha);
firstX2=startX-10*sind(alpha);
firstY2=startY-10*cosd(alpha);
firstP1=[firstX1, firstY1];
firstP2=[firstX2, firstY2];
pd=2;
dd=2;
while dd<=numOfPoints+1
    firstP1(pd,1)=firstP1(pd-1,1)-distForBound*cosd(alpha);
    firstP1(pd,2)=firstP1(pd-1,2)+distForBound*sind(alpha);
    firstP2(pd,1)=firstP2(pd-1,1)-distForBound*cosd(alpha);
    firstP2(pd,2)=firstP2(pd-1,2)+distForBound*sind(alpha);
    dd=dd+1;
    pd=pd+1;
end
targetZone=[targetZone; firstP1; firstP2];
elseif xMove<0 && yMove>0
alpha=atand(bcat2/bcat1);
firstX1=startX+10*sind(alpha);
firstY1=startY+10*cosd(alpha);
firstX2=startX-10*sind(alpha);
firstY2=startY-10*cosd(alpha);
firstP1=[firstX1, firstY1];
firstP2=[firstX2, firstY2];
pd=2;
dd=2;
while dd<=numOfPoints+1
    firstP1(pd,1)=firstP1(pd-1,1)+distForBound*cosd(alpha);
    firstP1(pd,2)=firstP1(pd-1,2)-distForBound*sind(alpha);
    firstP2(pd,1)=firstP2(pd-1,1)+distForBound*cosd(alpha);

```

```

        firstP2(pd,2)=firstP2(pd-1,2)-distForBound*sind(alpha);
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove<0 && yMove<0
    alpha=atand(bcat2/bcat1);
    firstX1=startX-10*sind(alpha);
    firstY1=startY+10*cosd(alpha);
    firstX2=startX+10*sind(alpha);
    firstY2=startY-10*cosd(alpha);
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)+distForBound*cosd(alpha);
        firstP1(pd,2)=firstP1(pd-1,2)+distForBound*sind(alpha);
        firstP2(pd,1)=firstP2(pd-1,1)+distForBound*cosd(alpha);
        firstP2(pd,2)=firstP2(pd-1,2)+distForBound*sind(alpha);
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove==0 && yMove>0
    firstX1=startX-10;
    firstY1=startY;
    firstX2=startX+10;
    firstY2=startY;
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;

```

```

dd=2;
while dd<=numOfPoints+1
    firstP1(pd,1)=firstP1(pd-1,1);
    firstP1(pd,2)=firstP1(pd-1,2)-distForBound;
    firstP2(pd,1)=firstP2(pd-1,1);
    firstP2(pd,2)=firstP2(pd-1,2)-distForBound;
    dd=dd+1;
    pd=pd+1;
end
targetZone=[targetZone; firstP1; firstP2];
elseif xMove==0 && yMove<0
    firstX1=startX-10;
    firstY1=startY;
    firstX2=startX+10;
    firstY2=startY;
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1);
        firstP1(pd,2)=firstP1(pd-1,2)+distForBound;
        firstP2(pd,1)=firstP2(pd-1,1);
        firstP2(pd,2)=firstP2(pd-1,2)+distForBound;
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove>0 && yMove==0
    firstX1=startX;
    firstY1=startY+10;
    firstX2=startX;

```

```

firstY2=startY-10;
firstP1=[firstX1, firstY1];
firstP2=[firstX2, firstY2];
pd=2;
dd=2;
while dd<=numOfPoints+1
    firstP1(pd,1)=firstP1(pd-1,1)-distForBound;
    firstP1(pd,2)=firstP1(pd-1,2);
    firstP2(pd,1)=firstP2(pd-1,1)-distForBound;
    firstP2(pd,2)=firstP2(pd-1,2);
    dd=dd+1;
    pd=pd+1;
end
targetZone=[targetZone; firstP1; firstP2];
elseif xMove<0 && yMove==0
    firstX1=startX;
    firstY1=startY+10;
    firstX2=startX;
    firstY2=startY-10;
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)+distForBound;
        firstP1(pd,2)=firstP1(pd-1,2);
        firstP2(pd,1)=firstP2(pd-1,1)+distForBound;
        firstP2(pd,2)=firstP2(pd-1,2);
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];

```

```

elseif xMove==0 && yMove==0
    fprintf('Two reservoir boundary points are at the same place');
else ulvi=1992;
end
end
elseif resVertX(ii,1)>=length-1.5*maxDist && resVertX(ii,1)<=length && ...
    resVertX(ii-1,1)>=length-1.5*maxDist && resVertX(ii-1,1)<=length
if resVertX(ii,1)~=length || resVertX(ii-1,1)~=length
    startX=resVertX(ii,1);
    startY=resVertY(ii,1);
    endX=resVertX(ii,1);
    endY=resVertY(ii,1);
    bcat1=abs(startX-endX);
    bcat2=abs(startY-endY);
    boundLength=sqrt(bcat1*bcat1+bcat2*bcat2);

    distForBound=maxDist/2;
    numOfPoints=boundLength/distForBound;
    xMove=startX-endX;
    yMove=startY-endY;

if xMove>0 && yMove>0
    alpha=atand(bcat2/bcat1);
    firstX1=startX-10*sind(alpha);
    firstY1=startY+10*cosd(alpha);
    firstX2=startX+10*sind(alpha);
    firstY2=startY-10*cosd(alpha);
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;

```

```

while dd<=numOfPoints+1
    firstP1(pd,1)=firstP1(pd-1,1)-distForBound*cosd(alpha);
    firstP1(pd,2)=firstP1(pd-1,2)-distForBound*sind(alpha);
    firstP2(pd,1)=firstP2(pd-1,1)-distForBound*cosd(alpha);
    firstP2(pd,2)=firstP2(pd-1,2)-distForBound*sind(alpha);
    dd=dd+1;
    pd=pd+1;
end
targetZone=[targetZone; firstP1; firstP2];
elseif xMove>0 && yMove<0
    alpha=atand(bcat2/bcat1);
    firstX1=startX+10*sind(alpha);
    firstY1=startY+10*cosd(alpha);
    firstX2=startX-10*sind(alpha);
    firstY2=startY-10*cosd(alpha);
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)-distForBound*cosd(alpha);
        firstP1(pd,2)=firstP1(pd-1,2)+distForBound*sind(alpha);
        firstP2(pd,1)=firstP2(pd-1,1)-distForBound*cosd(alpha);
        firstP2(pd,2)=firstP2(pd-1,2)+distForBound*sind(alpha);
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove<0 && yMove>0
    alpha=atand(bcat2/bcat1);
    firstX1=startX+10*sind(alpha);
    firstY1=startY+10*cosd(alpha);

```

```

firstX2=startX-10*sind(alpha);
firstY2=startY-10*cosd(alpha);
firstP1=[firstX1, firstY1];
firstP2=[firstX2, firstY2];
pd=2;
dd=2;
while dd<=numOfPoints+1
    firstP1(pd,1)=firstP1(pd-1,1)+distForBound*cosd(alpha);
    firstP1(pd,2)=firstP1(pd-1,2)-distForBound*sind(alpha);
    firstP2(pd,1)=firstP2(pd-1,1)+distForBound*cosd(alpha);
    firstP2(pd,2)=firstP2(pd-1,2)-distForBound*sind(alpha);
    dd=dd+1;
    pd=pd+1;
end
targetZone=[targetZone; firstP1; firstP2];
elseif xMove<0 && yMove<0
    alpha=atand(bcat2/bcat1);
    firstX1=startX-10*sind(alpha);
    firstY1=startY+10*cosd(alpha);
    firstX2=startX+10*sind(alpha);
    firstY2=startY-10*cosd(alpha);
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)+distForBound*cosd(alpha);
        firstP1(pd,2)=firstP1(pd-1,2)+distForBound*sind(alpha);
        firstP2(pd,1)=firstP2(pd-1,1)+distForBound*cosd(alpha);
        firstP2(pd,2)=firstP2(pd-1,2)+distForBound*sind(alpha);
        dd=dd+1;
        pd=pd+1;
    end
end

```

```

end
targetZone=[targetZone; firstP1; firstP2];
elseif xMove==0 && yMove>0
firstX1=startX-10;
firstY1=startY;
firstX2=startX+10;
firstY2=startY;
firstP1=[firstX1, firstY1];
firstP2=[firstX2, firstY2];
pd=2;
dd=2;
while dd<=numOfPoints+1
firstP1(pd,1)=firstP1(pd-1,1);
firstP1(pd,2)=firstP1(pd-1,2)-distForBound;
firstP2(pd,1)=firstP2(pd-1,1);
firstP2(pd,2)=firstP2(pd-1,2)-distForBound;
dd=dd+1;
pd=pd+1;
end
targetZone=[targetZone; firstP1; firstP2];
elseif xMove==0 && yMove<0
firstX1=startX-10;
firstY1=startY;
firstX2=startX+10;
firstY2=startY;
firstP1=[firstX1, firstY1];
firstP2=[firstX2, firstY2];
pd=2;
dd=2;
while dd<=numOfPoints+1
firstP1(pd,1)=firstP1(pd-1,1);
firstP1(pd,2)=firstP1(pd-1,2)+distForBound;

```

```

    firstP2(pd,1)=firstP2(pd-1,1);
    firstP2(pd,2)=firstP2(pd-1,2)+distForBound;
    dd=dd+1;
    pd=pd+1;
end
targetZone=[targetZone; firstP1; firstP2];
elseif xMove>0 && yMove==0
    firstX1=startX;
    firstY1=startY+10;
    firstX2=startX;
    firstY2=startY-10;
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)-distForBound;
        firstP1(pd,2)=firstP1(pd-1,2);
        firstP2(pd,1)=firstP2(pd-1,1)-distForBound;
        firstP2(pd,2)=firstP2(pd-1,2);
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove<0 && yMove==0
    firstX1=startX;
    firstY1=startY+10;
    firstX2=startX;
    firstY2=startY-10;
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;

```

```

dd=2;
while dd<=numOfPoints+1
    firstP1(pd,1)=firstP1(pd-1,1)+distForBound;
    firstP1(pd,2)=firstP1(pd-1,2);
    firstP2(pd,1)=firstP2(pd-1,1)+distForBound;
    firstP2(pd,2)=firstP2(pd-1,2);
    dd=dd+1;
    pd=pd+1;
end
targetZone=[targetZone; firstP1; firstP2];
elseif xMove==0 && yMove==0
    fprintf('Two reservoir boundary points are at the same place');
else ulvi=1992;
end
end
elseif resVertY(ii,1)>=0 && resVertY(ii,1)<=1.5*maxDist && ...
    resVertY(ii-1,1)>=0 && resVertY(ii-1,1)<=1.5*maxDist
if resVertY(ii,1)~=0 || resVertY(ii-1,1)~=0
    startX=resVertX(ii-1,1);
    startY=resVertY(ii-1,1);
    endX=resVertX(ii,1);
    endY=resVertY(ii,1);
    bcat1=abs(startX-endX);
    bcat2=abs(startY-endY);
    boundLength=sqrt(bcat1*bcat1+bcat2*bcat2);

    distForBound=maxDist/2;
    numOfPoints=boundLength/distForBound;
    xMove=startX-endX;
    yMove=startY-endY;

```

```

if xMove>0 && yMove>0
    alpha=atand(bcat2/bcat1);
    firstX1=startX-10*sind(alpha);
    firstY1=startY+10*cosd(alpha);
    firstX2=startX+10*sind(alpha);
    firstY2=startY-10*cosd(alpha);
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)-distForBound*cosd(alpha);
        firstP1(pd,2)=firstP1(pd-1,2)-distForBound*sind(alpha);
        firstP2(pd,1)=firstP2(pd-1,1)-distForBound*cosd(alpha);
        firstP2(pd,2)=firstP2(pd-1,2)-distForBound*sind(alpha);
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove>0 && yMove<0
    alpha=atand(bcat2/bcat1);
    firstX1=startX+10*sind(alpha);
    firstY1=startY+10*cosd(alpha);
    firstX2=startX-10*sind(alpha);
    firstY2=startY-10*cosd(alpha);
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)-distForBound*cosd(alpha);
        firstP1(pd,2)=firstP1(pd-1,2)+distForBound*sind(alpha);

```

```

        firstP2(pd,1)=firstP2(pd-1,1)-distForBound*cosd(alpha);
        firstP2(pd,2)=firstP2(pd-1,2)+distForBound*sind(alpha);
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove<0 && yMove>0
    alpha=atand(bcat2/bcat1);
    firstX1=startX+10*sind(alpha);
    firstY1=startY+10*cosd(alpha);
    firstX2=startX-10*sind(alpha);
    firstY2=startY-10*cosd(alpha);
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)+distForBound*cosd(alpha);
        firstP1(pd,2)=firstP1(pd-1,2)-distForBound*sind(alpha);
        firstP2(pd,1)=firstP2(pd-1,1)+distForBound*cosd(alpha);
        firstP2(pd,2)=firstP2(pd-1,2)-distForBound*sind(alpha);
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove<0 && yMove<0
    alpha=atand(bcat2/bcat1);
    firstX1=startX-10*sind(alpha);
    firstY1=startY+10*cosd(alpha);
    firstX2=startX+10*sind(alpha);
    firstY2=startY-10*cosd(alpha);
    firstP1=[firstX1, firstY1];

```

```

firstP2=[firstX2, firstY2];
pd=2;
dd=2;
while dd<=numOfPoints+1
    firstP1(pd,1)=firstP1(pd-1,1)+distForBound*cosd(alpha);
    firstP1(pd,2)=firstP1(pd-1,2)+distForBound*sind(alpha);
    firstP2(pd,1)=firstP2(pd-1,1)+distForBound*cosd(alpha);
    firstP2(pd,2)=firstP2(pd-1,2)+distForBound*sind(alpha);
    dd=dd+1;
    pd=pd+1;
end
targetZone=[targetZone; firstP1; firstP2];
elseif xMove==0 && yMove>0
    firstX1=startX-10;
    firstY1=startY;
    firstX2=startX+10;
    firstY2=startY;
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1);
        firstP1(pd,2)=firstP1(pd-1,2)-distForBound;
        firstP2(pd,1)=firstP2(pd-1,1);
        firstP2(pd,2)=firstP2(pd-1,2)-distForBound;
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove==0 && yMove<0
    firstX1=startX-10;

```

```

firstY1=startY;
firstX2=startX+10;
firstY2=startY;
firstP1=[firstX1, firstY1];
firstP2=[firstX2, firstY2];
pd=2;
dd=2;
while dd<=numOfPoints+1
    firstP1(pd,1)=firstP1(pd-1,1);
    firstP1(pd,2)=firstP1(pd-1,2)+distForBound;
    firstP2(pd,1)=firstP2(pd-1,1);
    firstP2(pd,2)=firstP2(pd-1,2)+distForBound;
    dd=dd+1;
    pd=pd+1;
end
targetZone=[targetZone; firstP1; firstP2];
elseif xMove>0 && yMove==0
    firstX1=startX;
    firstY1=startY+10;
    firstX2=startX;
    firstY2=startY-10;
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)-distForBound;
        firstP1(pd,2)=firstP1(pd-1,2);
        firstP2(pd,1)=firstP2(pd-1,1)-distForBound;
        firstP2(pd,2)=firstP2(pd-1,2);
        dd=dd+1;
        pd=pd+1;
    end
end

```

```

    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove<0 && yMove==0
    firstX1=startX;
    firstY1=startY+10;
    firstX2=startX;
    firstY2=startY-10;
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)+distForBound;
        firstP1(pd,2)=firstP1(pd-1,2);
        firstP2(pd,1)=firstP2(pd-1,1)+distForBound;
        firstP2(pd,2)=firstP2(pd-1,2);
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove==0 && yMove==0
    fprintf("Two reservoir boundary points are at the same place");
else ulvi=1992;
end
end
elseif resVertY(ii,1)>=width-1.5*maxDist && resVertY(ii,1)<=width && ...
    resVertY(ii-1,1)>=width-1.5*maxDist && resVertY(ii-1,1)<=width
if resVertY(ii,1)~=width || resVertY(ii-1,1)~=width
    startX=resVertX(ii-1,1);
    startY=resVertY(ii-1,1);
    endX=resVertX(ii,1);
    endY=resVertY(ii,1);

```

```

bcat1=abs(startX-endX);
bcat2=abs(startY-endY);
boundLength=sqrt(bcat1*bcat1+bcat2*bcat2);
distForBound=maxDist/2;
numOfPoints=boundLength/distForBound;
xMove=startX-endX;
yMove=startY-endY;

if xMove>0 && yMove>0
    alpha=atand(bcat2/bcat1);
    firstX1=startX-10*sind(alpha);
    firstY1=startY+10*cosd(alpha);
    firstX2=startX+10*sind(alpha);
    firstY2=startY-10*cosd(alpha);
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)-distForBound*cosd(alpha);
        firstP1(pd,2)=firstP1(pd-1,2)-distForBound*sind(alpha);
        firstP2(pd,1)=firstP2(pd-1,1)-distForBound*cosd(alpha);
        firstP2(pd,2)=firstP2(pd-1,2)-distForBound*sind(alpha);
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove>0 && yMove<0
    alpha=atand(bcat2/bcat1);
    firstX1=startX+10*sind(alpha);
    firstY1=startY+10*cosd(alpha);
    firstX2=startX-10*sind(alpha);

```

```

firstY2=startY-10*cosd(alpha);
firstP1=[firstX1, firstY1];
firstP2=[firstX2, firstY2];
pd=2;
dd=2;
while dd<=numOfPoints+1
    firstP1(pd,1)=firstP1(pd-1,1)-distForBound*cosd(alpha);
    firstP1(pd,2)=firstP1(pd-1,2)+distForBound*sind(alpha);
    firstP2(pd,1)=firstP2(pd-1,1)-distForBound*cosd(alpha);
    firstP2(pd,2)=firstP2(pd-1,2)+distForBound*sind(alpha);
    dd=dd+1;
    pd=pd+1;
end
targetZone=[targetZone; firstP1; firstP2];
elseif xMove<0 && yMove>0
    alpha=atand(bcat2/bcat1);
    firstX1=startX+10*sind(alpha);
    firstY1=startY+10*cosd(alpha);
    firstX2=startX-10*sind(alpha);
    firstY2=startY-10*cosd(alpha);
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)+distForBound*cosd(alpha);
        firstP1(pd,2)=firstP1(pd-1,2)-distForBound*sind(alpha);
        firstP2(pd,1)=firstP2(pd-1,1)+distForBound*cosd(alpha);
        firstP2(pd,2)=firstP2(pd-1,2)-distForBound*sind(alpha);
        dd=dd+1;
        pd=pd+1;
    end
end

```

```

    targetZone=[targetZone; firstP1; firstP2];
elseif xMove<0 && yMove<0
    alpha=atand(bcat2/bcat1);
    firstX1=startX-10*sind(alpha);
    firstY1=startY+10*cosd(alpha);
    firstX2=startX+10*sind(alpha);
    firstY2=startY-10*cosd(alpha);
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)+distForBound*cosd(alpha);
        firstP1(pd,2)=firstP1(pd-1,2)+distForBound*sind(alpha);
        firstP2(pd,1)=firstP2(pd-1,1)+distForBound*cosd(alpha);
        firstP2(pd,2)=firstP2(pd-1,2)+distForBound*sind(alpha);
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove==0 && yMove>0
    firstX1=startX-10;
    firstY1=startY;
    firstX2=startX+10;
    firstY2=startY;
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1);
        firstP1(pd,2)=firstP1(pd-1,2)-distForBound;

```

```

    firstP2(pd,1)=firstP2(pd-1,1);
    firstP2(pd,2)=firstP2(pd-1,2)-distForBound;
    dd=dd+1;
    pd=pd+1;
end
targetZone=[targetZone; firstP1; firstP2];
elseif xMove==0 && yMove<0
    firstX1=startX-10;
    firstY1=startY;
    firstX2=startX+10;
    firstY2=startY;
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1);
        firstP1(pd,2)=firstP1(pd-1,2)+distForBound;
        firstP2(pd,1)=firstP2(pd-1,1);
        firstP2(pd,2)=firstP2(pd-1,2)+distForBound;
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif (xMove>0 && yMove==0) && (startY~=0 && endY~=0) &&
(startY~=width && endY~=width)
    firstX1=startX;
    firstY1=startY+10;
    firstX2=startX;
    firstY2=startY-10;
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];

```

```

pd=2;
dd=2;
while dd<=numOfPoints+1
    firstP1(pd,1)=firstP1(pd-1,1)-distForBound;
    firstP1(pd,2)=firstP1(pd-1,2);
    firstP2(pd,1)=firstP2(pd-1,1)-distForBound;
    firstP2(pd,2)=firstP2(pd-1,2);
    dd=dd+1;
    pd=pd+1;
end
targetZone=[targetZone; firstP1; firstP2];
elseif (xMove<0 && yMove==0) && (startY~=0 && endY~=0) &&
(startY~=width && endY~=width)
    firstX1=startX;
    firstY1=startY+10;
    firstX2=startX;
    firstY2=startY-10;
    firstP1=[firstX1, firstY1];
    firstP2=[firstX2, firstY2];
    pd=2;
    dd=2;
    while dd<=numOfPoints+1
        firstP1(pd,1)=firstP1(pd-1,1)+distForBound;
        firstP1(pd,2)=firstP1(pd-1,2);
        firstP2(pd,1)=firstP2(pd-1,1)+distForBound;
        firstP2(pd,2)=firstP2(pd-1,2);
        dd=dd+1;
        pd=pd+1;
    end
    targetZone=[targetZone; firstP1; firstP2];
elseif xMove==0 && yMove==0
    fprintf('Two reservoir boundary points are at the same place');

```

```

        else ulvi=1992;
        end
    end
end
end
ii=ii+1;
end
targetZoneX=targetZone(:,1);
targetZoneY=targetZone(:,2);
xBest=[xBest;targetZoneX];
yBest=[yBest;targetZoneY];

% Now we have best results. Next step is to introduce wells/faults.
bu=999;
fprintf('If you want to add vertical well, enter "1";\n');
fprintf('If you want to add horizontal well, enter "2";\n');
fprintf('If you want to add fault, enter "3";\n');
fprintf('If you do not want to do anything, enter "4";\n');

fault=[0,0];
faultCount=0;
horWellCount=0;
while bu~=42
    wellValue=input('Your number: ');
    if wellValue==1
        % Inputs for vertical well
        xWell=input('Enter X coordinate of location of the well: ');
        yWell=input('Enter Y coordinate of location of the well: ');
        radAroundWell=input('Enter radius around well that will be cleaned and
populated with new points: ');
        amountPoints=input('Enter amount of points to be added: ');
        wellRad=input('Enter well radius: ');
        firstLayerDist=2*wellRad;

```

```

secondLayerDist=input('Enter distance between first and second layer: ');
magnDist=input('Enter value of how much distance to the next layer will be
higher than to previous layer: ');

```

```

% Delete points near well
xy=[xBest,yBest];
xyWell=[xWell,yWell];
distancesToWell=pdist2(xy,xyWell);
for oo=1:1:size(distancesToWell,1)
    if distancesToWell(oo)<=radAroundWell
        xBest(oo,1)=0;
        yBest(oo,1)=0;
    end
end
xNew=zeros;
yNew=zeros;
tt=1;
for aa=1:1:size(xBest,1)
    if xBest(aa,1)~=0
        xNew(tt,1)=xBest(aa,1);
        yNew(tt,1)=yBest(aa,1);
        tt=tt+1;
    end
end

```

```

% Calculation of how many layers there will be
kk=secondLayerDist;
mm=firstLayerDist+secondLayerDist;
sld=secondLayerDist; % Between first layer (for well) and second
amountLayers=1; % Here
while mm<=radAroundWell
    amountLayers=amountLayers+1;

```

```

    xx=mm;
    mm=mm+kk*magnDist;
    kk=mm-xx;
end
pointsInOneLayer=amountPoints/amountLayers;
radDistBetweenPoints=360/pointsInOneLayer;
xAroundWell=zeros;
yAroundWell=zeros;
% Creating of the first layer
nn=0;
for kk=1:1:pointsInOneLayer % Was with -1, one more point was required.
check
    xAroundWell(1,kk)=xWell+firstLayerDist*sind(nn*radDistBetweenPoints);
    yAroundWell(1,kk)=yWell+firstLayerDist*cosd(nn*radDistBetweenPoints);
    nn=nn+1;
end
% Creating of other layers
qq=firstLayerDist+secondLayerDist;
ff=secondLayerDist;
for ii=2:1:amountLayers+1
    for jj=1:1:pointsInOneLayer
        xAroundWell(ii,jj)=xWell+qq*sind(jj*radDistBetweenPoints);
        yAroundWell(ii,jj)=yWell+qq*cosd(jj*radDistBetweenPoints);
    end
    yy=qq;
    qq=qq+ff*magnDist;
    ff=qq-yy;
end
% Checking if all generated points are inside of reservoir
bo1=size(xAroundWell,1);
bo2=size(xAroundWell,2);
bo=bo1*bo2;

```

```

xAroundWell=reshape(xAroundWell,bo,1);
yAroundWell=reshape(yAroundWell,bo,1);
winch=inpolygon(xAroundWell,yAroundWell,resVertX,resVertY);
winch=winch+0;
for kq=1:1:size(winch,1)
    if winch(kq,1)==0
        xAroundWell(kq,1)=0;
        yAroundWell(kq,1)=0;
    end
end
end
xf=zeros;
yf=zeros;
nm=1;
for hs=1:1:size(xAroundWell,1)
    if xAroundWell(hs,1)~=0 || yAroundWell(hs,1)~=0
        xf(nm,1)=xAroundWell(hs,1);
        yf(nm,1)=yAroundWell(hs,1);
        nm=nm+1;
    end
end
end
fprintf('Your vertical well is located at x = %g, y = %g\n', xWell, yWell);
bu=input('If you want to add another well, enter "0", if not, enter "42": ');
% Saving generated points
xNew=[xNew;xf;xWell];
yNew=[yNew;yf;yWell];
xBest=xNew;
yBest=yNew;
elseif wellValue==2
    % Input for horizontal well
    horWellCount=horWellCount+1;
    startX=input('Enter X coordinate of start (left!) of the well: ');
    startY=input('Enter Y coordinate of start (left!) of the well: ');

```

```

endX=input('Enter X coordinate of end (right! If well is parallel to y-axis start
from uppermost) of the well: ');
endY=input('Enter Y coordinate of end (right! If well is parallel to y-axis start
from uppermost) of the well: ');
amountPoints=input('Enter maximum amount of points to be added: ');
radWell=input('Enter radius of the well: ');
firstLayerDist=2*radWell;
secondLayerDist=input('Enter distance between first and second layer: ');
magnDist=input('Enter value of how much distance to the next layer will be
higher than to previous layer: ');
distFromWell=input('Enter distance from well that will be cleared and
populated with new points: ');
cat1=abs(startX-endX);
cat2=abs(startY-endY);
wellLength=sqrt(cat1*cat1+cat2*cat2);
% Checking how the well is located towards rectangle's sides
if cat1==0
    parallelWell=1;
elseif cat2==0
    parallelWell=2;
else parallelWell=0;
end
xv=zeros;
yv=zeros;
if parallelWell==0 % If well is not parallel to rectangle's sides
    alpha=atand(cat2/cat1);
    if startY > endY
        xv(1,1)=startX+distFromWell*sind(alpha);
        yv(1,1)=startY+distFromWell*cosd(alpha);
        xv(2,1)=endX+distFromWell*sind(alpha);
        yv(2,1)=endY+distFromWell*cosd(alpha);
        xv(3,1)=endX-distFromWell*sind(alpha);

```

```

yv(3,1)=endY-distFromWell*cosd(alpha);
xv(4,1)=startX-distFromWell*sind(alpha);
yv(4,1)=startY-distFromWell*cosd(alpha);
% Delete points near well
IN=inpolygon(xBest,yBest,xv,yv);
IN=IN+0;
for ii=1:1:size(IN,1)
    if IN(ii,1)==1
        xBest(ii,1)=0;
        yBest(ii,1)=0;
    end
end
xNew=zeros;
yNew=zeros;
tt=1;
for aa=1:1:size(xBest,1)
    if xBest(aa,1)~=0
        xNew(tt,1)=xBest(aa,1);
        yNew(tt,1)=yBest(aa,1);
        tt=tt+1;
    end
end
% Calculation of number of layers
kk=secondLayerDist;
mm=firstLayerDist+secondLayerDist;
amountLayers=1; % Here
while mm<=distFromWell
    amountLayers=amountLayers+1;
    xx=mm;
    mm=mm+kk*magnDist;
    kk=mm-xx;
end

```

```

pointsInOneLayer=(amountPoints/((2*amountLayers)+1))+1;
distBetweenPoints=wellLength/pointsInOneLayer;
% Creating of layer exactly on well
wellGridX=zeros;
wellGridY=zeros;
wellGridX(1,1)=startX;
wellGridY(1,1)=startY;
for ii=2:1:pointsInOneLayer+2
    wellGridX(ii,1)=wellGridX(ii-1,1)+distBetweenPoints*cosd(alpha);
    wellGridY(ii,1)=wellGridY(ii-1,1)-distBetweenPoints*sind(alpha);
end
% Creating of layers (to the upper part)
xFromWellUp=zeros;
yFromWellUp=zeros;
xFromWellUp(1,1)=startX+firstLayerDist*sind(alpha);
yFromWellUp(1,1)=startY+firstLayerDist*cosd(alpha);
for ii=2:1:pointsInOneLayer+2
    xFromWellUp(1,ii)=xFromWellUp(1,ii-
1)+distBetweenPoints*cosd(alpha);
    yFromWellUp(1,ii)=yFromWellUp(1,ii-1)-
distBetweenPoints*sind(alpha);
end
dd=firstLayerDist+secondLayerDist;
ll=secondLayerDist;
for kk=2:1:amountLayers+1
    zz=dd;
    dd=dd+ll*magnDist;
    ll=dd-zz;
    if dd<=distFromWell
        for jj=1:1:pointsInOneLayer+2
            xFromWellUp(kk,jj)=xFromWellUp(kk-1,jj)+dd*sind(alpha);
            yFromWellUp(kk,jj)=yFromWellUp(kk-1,jj)+dd*cosd(alpha);

```

```

        end
    end
end
% Creating of layers (to the down part)
xFromWellDown=zeros;
yFromWellDown=zeros;
xFromWellDown(1,1)=startX-firstLayerDist*sind(alpha);
yFromWellDown(1,1)=startY-firstLayerDist*cosd(alpha);
for ii=2:1:pointsInOneLayer+2
    xFromWellDown(1,ii)=xFromWellDown(1,ii-
1)+distBetweenPoints*cosd(alpha);
    yFromWellDown(1,ii)=yFromWellDown(1,ii-1)-
distBetweenPoints*sind(alpha);
end
ll=secondLayerDist;
dd=firstLayerDist+secondLayerDist;
for kk=2:1:amountLayers+1
    zz=dd;
    dd=dd+ll*magnDist;
    ll=dd-zz;
    if dd<=distFromWell
        for jj=1:1:pointsInOneLayer+2
            xFromWellDown(kk,jj)=xFromWellDown(kk-1,jj)-dd*sind(alpha);
            yFromWellDown(kk,jj)=yFromWellDown(kk-1,jj)-dd*cosd(alpha);
        end
    end
end
end
% Putting everything together

xFromWellUp=reshape(xFromWellUp,size(xFromWellUp,1)*size(xFromWellUp,2)
,1);

```

```
xFromWellDown=reshape(xFromWellDown,size(xFromWellDown,1)*size(xFromWellDown,2),1);
```

```
yFromWellUp=reshape(yFromWellUp,size(yFromWellUp,1)*size(yFromWellUp,2),1);
```

```
yFromWellDown=reshape(yFromWellDown,size(yFromWellDown,1)*size(yFromWellDown,2),1);
```

```
av=inpolygon(xFromWellUp,yFromWellUp,resVertX,resVertY);
```

```
av=av+0;
```

```
bv=inpolygon(xFromWellDown,yFromWellDown,resVertX,resVertY);
```

```
bv=bv+0;
```

```
for lp=1:1:size(xFromWellUp,1)
```

```
    if av(lp,1)==0
```

```
        xFromWellUp(lp,1)=0;
```

```
        yFromWellUp(lp,1)=0;
```

```
    end
```

```
end
```

```
for pr=1:1:size(xFromWellDown,1)
```

```
    if bv(pr,1)==0
```

```
        xFromWellDown(pr,1)=0;
```

```
        yFromWellDown(pr,1)=0;
```

```
    end
```

```
end
```

```
xu=zeros;
```

```
xd=zeros;
```

```
yu=zeros;
```

```
yd=zeros;
```

```
wcu=1;
```

```
wcd=1;
```

```
for hg=1:1:size(xFromWellUp,1)
```

```

    if xFromWellUp(hg,1)~=0 || yFromWellUp(hg,1)~=0
        xu(wcu,1)=xFromWellUp(hg,1);
        yu(wcu,1)=yFromWellUp(hg,1);
        wcu=wcu+1;
    end
end
for yh=1:1:size(xFromWellDown,1)
    if xFromWellDown(yh,1)~=0 || yFromWellDown(yh,1)~=0
        xd(wcd,1)=xFromWellDown(yh,1);
        yd(wcd,1)=yFromWellDown(yh,1);
        wcd=wcd+1;
    end
end
xFromWellUp=xu;
yFromWellUp=yu;
xFromWellDown=xd;
yFromWellDown=yd;
xNew=[xNew; xFromWellUp; xFromWellDown; wellGridX];
yNew=[yNew; yFromWellUp; yFromWellDown; wellGridY];
xBest=xNew;
yBest=yNew;
elseif startY < endY
    xv(1,1)=startX-distFromWell*sind(alpha);
    yv(1,1)=startY+distFromWell*cosd(alpha);
    xv(2,1)=endX-distFromWell*sind(alpha);
    yv(2,1)=endY+distFromWell*cosd(alpha);
    xv(3,1)=endX+distFromWell*sind(alpha);
    yv(3,1)=endY-distFromWell*cosd(alpha);
    xv(4,1)=startX+distFromWell*sind(alpha);
    yv(4,1)=startY-distFromWell*cosd(alpha);
    % Delete points near the well
    IN=inpolygon(xBest,yBest,xv,yv);

```

```

IN=IN+0;
for ii=1:1:size(IN,1)
    if IN(ii,1)==1
        xBest(ii,1)=0;
        yBest(ii,1)=0;
    end
end
xNew=zeros;
yNew=zeros;
tt=1;
for aa=1:1:size(xBest,1)
    if xBest(aa,1)~=0
        xNew(tt,1)=xBest(aa,1);
        yNew(tt,1)=yBest(aa,1);
        tt=tt+1;
    end
end
kk=secondLayerDist;
mm=firstLayerDist+secondLayerDist;
amountLayers=1;
% Calculate number of layers
while mm<=distFromWell
    amountLayers=amountLayers+1;
    xx=mm;
    mm=mm+kk*magnDist;
    kk=mm-xx;
end
pointsInOneLayer=(amountPoints/((2*amountLayers)+1))+1;
distBetweenPoints=wellLength/pointsInOneLayer;
% Creating of points exactly on well
wellGridX=zeros;
wellGridY=zeros;

```

```

wellGridX(1,1)=startX;
wellGridY(1,1)=startY;
for ii=2:1:pointsInOneLayer+2
    wellGridX(ii,1)=wellGridX(ii-1,1)+distBetweenPoints*cosd(alpha);
    wellGridY(ii,1)=wellGridY(ii-1,1)+distBetweenPoints*sind(alpha);
end
% Creating of layers (to the upper part)
xFromWellUp=zeros;
yFromWellUp=zeros;
xFromWellUp(1,1)=startX-firstLayerDist*sind(alpha);
yFromWellUp(1,1)=startY+firstLayerDist*cosd(alpha);
for ii=2:1:pointsInOneLayer+2%
    xFromWellUp(1,ii)=xFromWellUp(1,ii-
1)+distBetweenPoints*cosd(alpha);
    yFromWellUp(1,ii)=yFromWellUp(1,ii-
1)+distBetweenPoints*sind(alpha);
end

dd=firstLayerDist+secondLayerDist;
ll=secondLayerDist;
for kk=2:1:amountLayers+2%
    zz=dd;
    dd=dd+ll*magnDist;
    ll=dd-zz;
    if dd<=distFromWell
    for jj=1:1:pointsInOneLayer+2%
        xFromWellUp(kk,jj)=xFromWellUp(kk-1,jj)-dd*sind(alpha);
        yFromWellUp(kk,jj)=yFromWellUp(kk-1,jj)+dd*cosd(alpha);
    end
    end
end
end

```

```

% Creating of layers (to the down part)
xFromWellDown=zeros;
yFromWellDown=zeros;
xFromWellDown(1,1)=startX+firstLayerDist*sind(alpha);
yFromWellDown(1,1)=startY-firstLayerDist*cosd(alpha);
for ii=2:1:pointsInOneLayer+2
    xFromWellDown(1,ii)=xFromWellDown(1,ii-
1)+distBetweenPoints*cosd(alpha);
    yFromWellDown(1,ii)=yFromWellDown(1,ii-
1)+distBetweenPoints*sind(alpha);
end
dd=firstLayerDist+secondLayerDist;
ll=secondLayerDist;
for kk=2:1:amountLayers+2%
    zz=dd;
    dd=dd+ll*magnDist;
    ll=dd-zz;
    if dd<=distFromWell
        for jj=1:1:pointsInOneLayer+2%
            xFromWellDown(kk,jj)=xFromWellDown(kk-1,jj)+dd*sind(alpha);
            yFromWellDown(kk,jj)=yFromWellDown(kk-1,jj)-dd*cosd(alpha);
        end
    end
end

% Putting everything together

xFromWellUp=reshape(xFromWellUp,size(xFromWellUp,1)*size(xFromWellUp,2)
,1);

xFromWellDown=reshape(xFromWellDown,size(xFromWellDown,1)*size(xFrom
WellDown,2),1);

```

```
yFromWellUp=reshape(yFromWellUp,size(yFromWellUp,1)*size(yFromWellUp,2),1);
```

```
yFromWellDown=reshape(yFromWellDown,size(yFromWellDown,1)*size(yFromWellDown,2),1);
```

```
av=inpolygon(xFromWellUp,yFromWellUp,resVertX,resVertY);
```

```
av=av+0;
```

```
bv=inpolygon(xFromWellDown,yFromWellDown,resVertX,resVertY);
```

```
bv=bv+0;
```

```
for lp=1:1:size(xFromWellUp,1)
```

```
    if av(lp,1)==0
```

```
        xFromWellUp(lp,1)=0;
```

```
        yFromWellUp(lp,1)=0;
```

```
    end
```

```
end
```

```
for pr=1:1:size(xFromWellDown,1)
```

```
    if bv(pr,1)==0
```

```
        xFromWellDown(pr,1)=0;
```

```
        yFromWellDown(pr,1)=0;
```

```
    end
```

```
end
```

```
xu=zeros;
```

```
xd=zeros;
```

```
yu=zeros;
```

```
yd=zeros;
```

```
wcu=1;
```

```
wcd=1;
```

```
for hg=1:1:size(xFromWellUp,1)
```

```
    if xFromWellUp(hg,1)~=0 || yFromWellUp(hg,1)~=0
```

```
        xu(wcu,1)=xFromWellUp(hg,1);
```

```
        yu(wcu,1)=yFromWellUp(hg,1);
```

```

        wcu=wcu+1;
    end
end
for yh=1:1:size(xFromWellDown,1)
    if xFromWellDown(yh,1)~=0 || yFromWellDown(yh,1)~=0
        xd(wcd,1)=xFromWellDown(yh,1);
        yd(wcd,1)=yFromWellDown(yh,1);
        wcd=wcd+1;
    end
end
xFromWellUp=xu;
yFromWellUp=yu;
xFromWellDown=xd;
yFromWellDown=yd;
xNew=[xNew; xFromWellUp; xFromWellDown; wellGridX];
yNew=[yNew; yFromWellUp; yFromWellDown; wellGridY];
xBest=xNew;
yBest=yNew;
end
elseif parallelWell==1 % Parallel to y-axis
    xv(1,1)=startX-distFromWell;
    yv(1,1)=startY;
    xv(2,1)=endX-distFromWell;
    yv(2,1)=endY;
    xv(3,1)=endX+distFromWell;
    yv(3,1)=endY;
    xv(4,1)=startX+distFromWell;
    yv(4,1)=startY;
    % Delete points near well
    for ff=1:1:size(xBest,1)
        if xBest(ff,1)>xv(1,1) && xBest(ff,1)<xv(4,1) ...
            && yBest(ff,1)>yv(2,1) && yBest(ff,1)<yv(1,1)

```

```

        xBest(ff,1)=0;
        yBest(ff,1)=0;
    end
end
xNew=zeros;
yNew=zeros;
tt=1;
for aa=1:1:size(xBest,1)
    if xBest(aa,1)~=0
        xNew(tt,1)=xBest(aa,1);
        yNew(tt,1)=yBest(aa,1);
        tt=tt+1;
    end
end
% Calculate number of layers
kk=secondLayerDist;
mm=firstLayerDist+secondLayerDist;
amountLayers=1;
while mm<=distFromWell
    amountLayers=amountLayers+1;
    xx=mm;
    mm=mm+kk*magnDist;
    kk=mm-xx;
end
pointsInOneLayer=(amountPoints/((2*amountLayers)+1))+1;
distBetweenPoints=wellLength/pointsInOneLayer;
% Creating of points exactly on well
wellGridX=zeros;
wellGridY=zeros;
wellGridX(1,1)=startX;
wellGridY(1,1)=startY;
for ii=2:1:pointsInOneLayer

```

```

    wellGridX(ii,1)=wellGridX(ii-1,1);
    wellGridY(ii,1)=wellGridY(ii-1,1)-distBetweenPoints;
end
% Creating of layers (to the left)
xFromWellLeft=zeros;
yFromWellLeft=zeros;
xFromWellLeft(1,1)=startX-firstLayerDist;
yFromWellLeft(1,1)=startY;
for ii=2:1:pointsInOneLayer
    xFromWellLeft(1,ii)=xFromWellLeft(1,ii-1);
    yFromWellLeft(1,ii)=yFromWellLeft(1,ii-1)-distBetweenPoints;
end
dd=firstLayerDist+secondLayerDist;
ll=secondLayerDist;
for kk=2:1:amountLayers+1
    zz=dd;
    dd=dd+ll*magnDist;
    ll=dd-zz;
    if dd<=distFromWell
        for jj=1:1:pointsInOneLayer
            xFromWellLeft(kk,jj)=xFromWellLeft(kk-1,jj)-dd;
            yFromWellLeft(kk,jj)=yFromWellLeft(kk-1,jj);
        end
    end
end
% Creating of layers (to the right)
xFromWellRight=zeros;
yFromWellRight=zeros;
xFromWellRight(1,1)=startX+firstLayerDist;
yFromWellRight(1,1)=startY;
for ii=2:1:pointsInOneLayer
    xFromWellRight(1,ii)=xFromWellRight(1,ii-1);

```

```

        yFromWellRight(1,ii)=yFromWellRight(1,ii-1)-distBetweenPoints;
    end
    dd=firstLayerDist+secondLayerDist;
    ll=secondLayerDist;
    for kk=2:1:amountLayers+1
        zz=dd;
        dd=dd+ll*magnDist;
        ll=dd-zz;
        if dd<=distFromWell
            for jj=1:1:pointsInOneLayer
                xFromWellRight(kk,jj)=xFromWellRight(kk-1,jj)+dd;
                yFromWellRight(kk,jj)=yFromWellRight(kk-1,jj);
            end
        end
    end
    end
    % Putting everything together

xFromWellLeft=reshape(xFromWellLeft,size(xFromWellLeft,1)*size(xFromWellLeft,2),1);

xFromWellRight=reshape(xFromWellRight,size(xFromWellRight,1)*size(xFromWellRight,2),1);

yFromWellLeft=reshape(yFromWellLeft,size(yFromWellLeft,1)*size(yFromWellLeft,2),1);

yFromWellRight=reshape(yFromWellRight,size(yFromWellRight,1)*size(yFromWellRight,2),1);

av=inpolygon(xFromWellLeft,yFromWellLeft,resVertX,resVertY);
av=av+0;
bv=inpolygon(xFromWellRight,yFromWellRight,resVertX,resVertY);
bv=bv+0;

```

```

for lp=1:1:size(xFromWellLeft,1)
    if av(lp,1)==0
        xFromWellLeft(lp,1)=0;
        yFromWellLeft(lp,1)=0;
    end
end
for pr=1:1:size(xFromWellRight,1)
    if bv(pr,1)==0
        xFromWellRight(pr,1)=0;
        yFromWellRight(pr,1)=0;
    end
end
xu=zeros;
xd=zeros;
yu=zeros;
yd=zeros;
wcu=1;
wcd=1;
for hg=1:1:size(xFromWellLeft,1)
    if xFromWellLeft(hg,1)~=0 || yFromWellLeft(hg,1)~=0
        xu(wcu,1)=xFromWellLeft(hg,1);
        yu(wcu,1)=yFromWellLeft(hg,1);
        wcu=wcu+1;
    end
end
for yh=1:1:size(xFromWellRight,1)
    if xFromWellRight(yh,1)~=0 || yFromWellRight(yh,1)~=0
        xd(wcd,1)=xFromWellRight(yh,1);
        yd(wcd,1)=yFromWellRight(yh,1);
        wcd=wcd+1;
    end
end
end

```

```

xFromWellLeft=xu;
yFromWellLeft=yu;
xFromWellRight=xd;
yFromWellRight=yd;
xNew=[xNew; xFromWellLeft; xFromWellRight; wellGridX];
yNew=[yNew; yFromWellLeft; yFromWellRight; wellGridY];
xBest=xNew;
yBest=yNew;
elseif parallelWell==2 % Parallel to x-axis
    xv(1,1)=startX;
    yv(1,1)=startY+distFromWell;
    xv(2,1)=endX;
    yv(2,1)=endY+distFromWell;
    xv(3,1)=endX;
    yv(3,1)=endY-distFromWell;
    xv(4,1)=startX;
    yv(4,1)=startY-distFromWell;
    % Delete points near well
    for ss=1:1:size(xBest,1)
        if xBest(ss,1)>xv(1,1) && xBest(ss,1)<xv(2,1) ...
            && yBest(ss,1)>yv(4,1) && yBest(ss,1)<yv(1,1)
                xBest(ss,1)=0;
                yBest(ss,1)=0;
            end
        end
    end
    xNew=zeros;
    yNew=zeros;
    tt=1;
    for aa=1:1:size(xBest,1)
        if xBest(aa,1)~=0
            xNew(tt,1)=xBest(aa,1);
            yNew(tt,1)=yBest(aa,1);

```

```

        tt=tt+1;
    end
end
% Calculate number of layers
kk=secondLayerDist;
mm=firstLayerDist+secondLayerDist;
amountLayers=1; % Here
while mm<=distFromWell
    amountLayers=amountLayers+1;
    xx=mm;
    mm=mm+kk*magnDist;
    kk=mm-xx;
end
pointsInOneLayer=(amountPoints/((2*amountLayers)+1))+1;
distBetweenPoints=wellLength/pointsInOneLayer;
% Creating of points exactly on well
wellGridX=zeros;
wellGridY=zeros;
wellGridX(1,1)=startX;
wellGridY(1,1)=startY;
for ii=2:1:pointsInOneLayer
    wellGridX(ii,1)=wellGridX(ii-1,1)+distBetweenPoints;
    wellGridY(ii,1)=wellGridY(ii-1,1);
end
% Creating of layers (to the upper part)
xFromWellLeft=zeros;
yFromWellLeft=zeros;
xFromWellUp(1,1)=startX;
yFromWellUp(1,1)=startY+firstLayerDist;
for ii=2:1:pointsInOneLayer
    xFromWellUp(1,ii)=xFromWellUp(1,ii-1)+distBetweenPoints;
    yFromWellUp(1,ii)=yFromWellUp(1,ii-1);

```

```

end
dd=firstLayerDist+secondLayerDist;
ll=secondLayerDist;
for kk=2:1:amountLayers+1
    zz=dd;
    dd=dd+ll*magnDist;
    ll=dd-zz;
    if dd<=distFromWell
        for jj=1:1:pointsInOneLayer
            xFromWellUp(kk,jj)=xFromWellUp(kk-1,jj);
            yFromWellUp(kk,jj)=yFromWellUp(kk-1,jj)+dd;
        end
    end
end
end
% Creating of layers (to the down part)
xFromWellDown=zeros;
yFromWellDown=zeros;
xFromWellDown(1,1)=startX;
yFromWellDown(1,1)=startY-firstLayerDist;
for ii=2:1:pointsInOneLayer
    xFromWellDown(1,ii)=xFromWellDown(1,ii-1)+distBetweenPoints;
    yFromWellDown(1,ii)=yFromWellDown(1,ii-1);
end
dd=firstLayerDist+secondLayerDist;
ll=secondLayerDist;
for kk=2:1:amountLayers+1
    zz=dd;
    dd=dd+ll*magnDist;
    ll=dd-zz;
    if dd<=distFromWell
        for jj=1:1:pointsInOneLayer
            xFromWellDown(kk,jj)=xFromWellDown(kk-1,jj);

```

```

        yFromWellDown(kk,jj)=yFromWellDown(kk-1,jj)-dd;
    end
end
end
% Putting everything together

xFromWellUp=reshape(xFromWellUp,size(xFromWellUp,1)*size(xFromWellUp,2)
,1);

xFromWellDown=reshape(xFromWellDown,size(xFromWellDown,1)*size(xFrom
WellDown,2),1);

yFromWellUp=reshape(yFromWellUp,size(yFromWellUp,1)*size(yFromWellUp,2)
,1);

yFromWellDown=reshape(yFromWellDown,size(yFromWellDown,1)*size(yFrom
WellDown,2),1);
    av=inpolygon(xFromWellUp,yFromWellUp,resVertX,resVertY);
    av=av+0;
    bv=inpolygon(xFromWellDown,yFromWellDown,resVertX,resVertY);
    bv=bv+0;
    for lp=1:1:size(xFromWellUp,1)
        if av(lp,1)==0
            xFromWellUp(lp,1)=0;
            yFromWellUp(lp,1)=0;
        end
    end
    for pr=1:1:size(xFromWellDown,1)
        if bv(pr,1)==0
            xFromWellDown(pr,1)=0;
            yFromWellDown(pr,1)=0;
        end
    end

```

```

end
xu=zeros;
xd=zeros;
yu=zeros;
yd=zeros;
wcu=1;
wcd=1;
for hg=1:1:size(xFromWellUp,1)
    if xFromWellUp(hg,1)~=0 || yFromWellUp(hg,1)~=0
        xu(wcu,1)=xFromWellUp(hg,1);
        yu(wcu,1)=yFromWellUp(hg,1);
        wcu=wcu+1;
    end
end
for yh=1:1:size(xFromWellDown,1)
    if xFromWellDown(yh,1)~=0 || yFromWellDown(yh,1)~=0
        xd(wcd,1)=xFromWellDown(yh,1);
        yd(wcd,1)=yFromWellDown(yh,1);
        wcd=wcd+1;
    end
end
xFromWellUp=xu;
yFromWellUp=yu;
xFromWellDown=xd;
yFromWellDown=yd;
xNew=[xNew; xFromWellUp; xFromWellDown; wellGridX];
yNew=[yNew; yFromWellUp; yFromWellDown; wellGridY];
xBest=xNew;
yBest=yNew;
else ulvi=1992;
end
% Saving points to fprintf them at the end

```

```

horWellPoints{ horWellCount,1 }=wellGridX;
horWellPoints{ horWellCount,2 }=wellGridY;
bu=input('If you want to add another well, enter "0", if not, enter "42": ');
xBest=xNew;
yBest=yNew;
elseif wellValue==3 % Adding of fault
    faultCount=faultCount+1; % Counting faults
    startX=input('Enter X coordinate of start (left!) of the fault: ');
    startY=input('Enter Y coordinate of start (left!) of the fault: ');
    endX=input('Enter X coordinate of end (right! If well is parallel to y-axis start
from uppermost) of the fault: ');
    endY=input('Enter Y coordinate of end (right! If well is parallel to y-axis start
from uppermost) of the fault: ');
    amountPoints=input('Enter maximum amount of points to be added: ');
    firstLayerDist=input('Enter distance to the layer of points of the fault: ');
    distFromFault=input('Enter distance from fault that will be cleared and
populated with new points: ');
    cat1=abs(startX-endX);
    cat2=abs(startY-endY);
    faultPerm=input('Enter value of fault permeability: ');
    faultLength=sqrt(cat1*cat1+cat2*cat2);
    smur=(amountPoints/3)+1;
    distBetweenPoints=faultLength/smur;
    % Understanding how fault is located compared to rectangle's sides
    if cat1==0
        parallelFault=1;
    elseif cat2==0
        parallelFault=2;
    else parallelFault=0;
    end
    if parallelFault==0 % Not parallel to rectangle's sides
        alpha=atand(cat2/cat1);

```

```

if startY > endY
    firstPointUpX=startX+distFromFault*sind(alpha);
    firstPointUpY=startY+distFromFault*cosd(alpha);
    firstPointDownX=startX-distFromFault*sind(alpha);
    firstPointDownY=startY-distFromFault*cosd(alpha);
    lastPointUpX=endX+distFromFault*sind(alpha);
    lastPointUpY=endY+distFromFault*cosd(alpha);
    lastPointDownX=endX-distFromFault*sind(alpha);
    lastPointDownY=endY-distFromFault*cosd(alpha);
    polfaultX=[firstPointUpX; lastPointUpX; lastPointDownX;
firstPointDownX];
    polfaultY=[firstPointUpY; lastPointUpY; lastPointDownY;
firstPointDownY];
    % Delete points inside this region
    gs=inpolygon(xBest,yBest,polfaultX,polfaultY);
    gs=gs+0;
    for uj=1:1:size(gs,1)
        if gs(uj,1)==1
            xBest(uj,1)=0;
            yBest(uj,1)=0;
        end
    end
    xfa=zeros;
    yfa=zeros;
    hk=1;
    for ha=1:1:size(xBest,1)
        if xBest(ha,1)~=0 || yBest(ha,1)~=0
            xfa(hk,1)=xBest(ha,1);
            yfa(hk,1)=yBest(ha,1);
            hk=hk+1;
        end
    end
end

```

```

xNew=xfa;
yNew=yfa;
% Generation of points
faultUp=[firstPointUpX, firstPointUpY];
faultDown=[firstPointDownX, firstPointDownY];
faultExact=[startX,startY];
for ii=2:1:smur
    faultUp(ii,1)=faultUp(ii-1,1)+distBetweenPoints*cosd(alpha);
    faultUp(ii,2)=faultUp(ii-1,2)-distBetweenPoints*sind(alpha);
    faultDown(ii,1)=faultDown(ii-1,1)+distBetweenPoints*cosd(alpha);
    faultDown(ii,2)=faultDown(ii-1,2)-distBetweenPoints*sind(alpha);
    faultExact(ii,1)=faultExact(ii-1,1)+distBetweenPoints*cosd(alpha);
    faultExact(ii,2)=faultExact(ii-1,2)-distBetweenPoints*sind(alpha);
end
faultUpX=faultUp(:,1);
faultUpY=faultUp(:,2);
faultDownX=faultDown(:,1);
faultDownY=faultDown(:,2);
faultExactX=faultExact(:,1);
faultExactY=faultExact(:,2);
xNew=[xNew;faultUpX;faultDownX;faultExactX];
yNew=[yNew;faultUpY;faultDownY;faultExactY];
xBest=xNew;
yBest=yNew;
fault=[fault;faultExact];
elseif startY < endY
    firstPointUpX=startX-distFromFault*sind(alpha);
    firstPointUpY=startY+distFromFault*cosd(alpha);
    firstPointDownX=startX+distFromFault*sind(alpha);
    firstPointDownY=startY-distFromFault*cosd(alpha);
    lastPointUpX=endX-distFromFault*sind(alpha);
    lastPointUpY=endY+distFromFault*cosd(alpha);

```

```

lastPointDownX=endX+distFromFault*sind(alpha);
lastPointDownY=endY-distFromFault*cosd(alpha);
polfaultX=[firstPointUpX; lastPointUpX; lastPointDownX;
firstPointDownX];
    polfaultY=[firstPointUpY; lastPointUpY; lastPointDownY;
firstPointDownY];
    % Delete points inside this region
gs=inpolygon(xBest,yBest,polfaultX,polfaultY);
gs=gs+0;
for uj=1:1:size(gs,1)
    if gs(uj,1)==1
        xBest(uj,1)=0;
        yBest(uj,1)=0;
    end
end
xfa=zeros;
yfa=zeros;
hk=1;
for ha=1:1:size(xBest,1)
    if xBest(ha,1)~=0 || yBest(ha,1)~=0
        xfa(hk,1)=xBest(ha,1);
        yfa(hk,1)=yBest(ha,1);
        hk=hk+1;
    end
end
xNew=xfa;
yNew=yfa;
% Generation of points
faultUp=[firstPointUpX, firstPointUpY];
faultDown=[firstPointDownX, firstPointDownY];
faultExact=[startX,startY];
for ii=2:1:smur

```

```

    faultUp(ii,1)=faultUp(ii-1,1)+distBetweenPoints*cosd(alpha);
    faultUp(ii,2)=faultUp(ii-1,2)+distBetweenPoints*sind(alpha);
    faultDown(ii,1)=faultDown(ii-1,1)+distBetweenPoints*cosd(alpha);
    faultDown(ii,2)=faultDown(ii-1,2)+distBetweenPoints*sind(alpha);
    faultExact(ii,1)=faultExact(ii-1,1)+distBetweenPoints*cosd(alpha);
    faultExact(ii,2)=faultExact(ii-1,2)+distBetweenPoints*sind(alpha);
end
faultUpX=faultUp(:,1);
faultUpY=faultUp(:,2);
faultDownX=faultDown(:,1);
faultDownY=faultDown(:,2);
faultExactX=faultExact(:,1);
faultExactY=faultExact(:,2);
xNew=[xNew;faultUpX;faultDownX;faultExactX];
yNew=[yNew;faultUpY;faultDownY;faultExactY];
xBest=xNew;
yBest=yNew;
fault=[fault;faultExact];
else fprintf('Something is wrong');
end
elseif parallelFault==1 % Parallel to y-axis
    firstPointUpX=startX-distFromFault;
    firstPointUpY=startY;
    firstPointDownX=startX+distFromFault;
    firstPointDownY=startY;
    lastPointUpX=endX-distFromFault;
    lastPointUpY=endY;
    lastPointDownX=endX+distFromFault;
    lastPointDownY=endY;
    polfaultX=[firstPointUpX; lastPointUpX; lastPointDownX;
firstPointDownX];

```

```

    polfaultY=[firstPointUpY; lastPointUpY; lastPointDownY;
firstPointDownY];
    % Delete points inside this region
    gs=inpolygon(xBest,yBest,polfaultX,polfaultY);
    gs=gs+0;
    for uj=1:1:size(gs,1)
        if gs(uj,1)==1
            xBest(uj,1)=0;
            yBest(uj,1)=0;
        end
    end
    xfa=zeros;
    yfa=zeros;
    hk=1;
    for ha=1:1:size(xBest,1)
        if xBest(ha,1)~=0 || yBest(ha,1)~=0
            xfa(hk,1)=xBest(ha,1);
            yfa(hk,1)=yBest(ha,1);
            hk=hk+1;
        end
    end
    xNew=xfa;
    yNew=yfa;
    % Generation of points
    faultUp=[firstPointUpX, firstPointUpY];
    faultDown=[firstPointDownX, firstPointDownY];
    faultExact=[startX,startY];
    for ii=2:1:smur
        faultUp(ii,1)=faultUp(ii-1,1);
        faultUp(ii,2)=faultUp(ii-1,2)-distBetweenPoints;
        faultDown(ii,1)=faultDown(ii-1,1);
        faultDown(ii,2)=faultDown(ii-1,2)-distBetweenPoints;

```

```

    faultExact(ii,1)=faultExact(ii-1,1);
    faultExact(ii,2)=faultExact(ii-1,2)-distBetweenPoints;
end
faultUpX=faultUp(:,1);
faultUpY=faultUp(:,2);
faultDownX=faultDown(:,1);
faultDownY=faultDown(:,2);
faultExactX=faultExact(:,1);
faultExactY=faultExact(:,2);
xNew=[xNew;faultUpX;faultDownX;faultExactX];
yNew=[yNew;faultUpY;faultDownY;faultExactY];
xBest=xNew;
yBest=yNew;
fault=[fault;faultExact];
elseif parallelFault==2 % Parallel to x-axis
    firstPointUpX=startX;
    firstPointUpY=startY+distFromFault;
    firstPointDownX=startX;
    firstPointDownY=startY-distFromFault;
    lastPointUpX=endX;
    lastPointUpY=endY+distFromFault;
    lastPointDownX=endX;
    lastPointDownY=endY-distFromFault;
    polfaultX=[firstPointUpX; lastPointUpX; lastPointDownX;
firstPointDownX];
    polfaultY=[firstPointUpY; lastPointUpY; lastPointDownY;
firstPointDownY];
    % Delete points inside this region
    gs=inpolygon(xBest,yBest,polfaultX,polfaultY);
    gs=gs+0;
    for uj=1:1:size(gs,1)
        if gs(uj,1)==1

```

```

        xBest(uj,1)=0;
        yBest(uj,1)=0;
    end
end
xfa=zeros;
yfa=zeros;
hk=1;
for ha=1:1:size(xBest,1)
    if xBest(ha,1)~=0 || yBest(ha,1)~=0
        xfa(hk,1)=xBest(ha,1);
        yfa(hk,1)=yBest(ha,1);
        hk=hk+1;
    end
end
xNew=xfa;
yNew=yfa;
% Generation of points
faultUp=[firstPointUpX, firstPointUpY];
faultDown=[firstPointDownX, firstPointDownY];
faultExact=[startX,startY];
for ii=2:1:smur
    faultUp(ii,1)=faultUp(ii-1,1)+distBetweenPoints;
    faultUp(ii,2)=faultUp(ii-1,2);
    faultDown(ii,1)=faultDown(ii-1,1)+distBetweenPoints;
    faultDown(ii,2)=faultDown(ii-1,2);
    faultExact(ii,1)=faultExact(ii-1,1)+distBetweenPoints;
    faultExact(ii,2)=faultExact(ii-1,2);
end
faultUpX=faultUp(:,1);
faultUpY=faultUp(:,2);
faultDownX=faultDown(:,1);
faultDownY=faultDown(:,2);

```

```

    faultExactX=faultExact(:,1);
    faultExactY=faultExact(:,2);
    xNew=[xNew;faultUpX;faultDownX;faultExactX];
    yNew=[yNew;faultUpY;faultDownY;faultExactY];
    xBest=xNew;
    yBest=yNew;
    fault=[fault;faultExact];
else ulvi=1992;
end
bu=input('If you want to add another well, enter "0", if not, enter "42": ');
elseif wellValue==4
    xNew=xBest;
    yNew=yBest;
    bu=42;
else
    bu=input('You should have written "1","2","3" or "4". Enter "0" to choose well
type again or enter "42" to exit');
end
end
fault(1,:)=[];
% Clean all points outside of rectangle
xFinal=zeros;
yFinal=zeros;
yy=1;
gow=size(xNew,1);
for ii=1:1:gow
    if xNew(ii,1)>=0 && xNew(ii,1)<=length && yNew(ii,1)>=0 &&
yNew(ii,1)<=width
        xFinal(yy,1)=xNew(ii,1);
        yFinal(yy,1)=yNew(ii,1);
        yy=yy+1;
    end
end

```

end

% Recalculating error for final + permeabilities assignment

counterNew=size(xFinal,1);

xy=[xFinal,yFinal];

sxf=size(xFinal,1);

blocksOfPoints=zeros(sxf,counterNew);

distances=pdist2(permFieldVec,xy);

for kk=1:1:size(distances,1)

 distancesForPoints=distances(kk,:);

 backUpDist=distancesForPoints;

 [closestDist,ind]=min(backUpDist);

 blocksOfPoints(kk,ind)=permeabilitiesVec(kk,1);

 backUpDist(1,ind)=inf;

 [closestDist2,ind2]=min(backUpDist);

 if closestDist2==closestDist

 blocksOfPoints(kk,ind2)=permeabilitiesVec(kk,1);

 backUpDist(1,ind2)=inf;

 [closestDist3,ind3]=min(backUpDist);

 if closestDist3==closestDist2

 blocksOfPoints(kk,ind3)=permeabilitiesVec(kk,1);

 backUpDist(1,ind3)=inf;

 [closestDist4,ind4]=min(backUpDist);

 if closestDist4==closestDist3

 blocksOfPoints(kk,ind4)=permeabilitiesVec(kk,1);

 backUpDist(1,ind4)=inf;

 [closestDist5,ind5]=min(backUpDist);

 if closestDist5==closestDist4

 blocksOfPoints(kk,ind5)=permeabilitiesVec(kk,1);

 backUpDist(1,ind5)=inf;

 [closestDist6,ind6]=min(backUpDist);


```

jj=1;
error=zeros(counterNew,1);
%{
while jj<=counterNew
    forError=0;
    uu=1;
    for ii=1:1:permXi
        if blocksOfPoints(ii,jj)~=0;
            forError(uu,1)=blocksOfPoints(ii,jj);
            uu=uu+1;
        end
    end
    error(jj,1)=std(forError);
    checkerFinal{jj,1}=forError;
    jj=jj+1;
end
%}
% Calculating of error for each block
while jj<=counterNew
    forError=0;
    uu=1;
    for ii=1:1:permXi
        if blocksOfPoints(ii,jj)~=0;
            forError(uu,1)=blocksOfPoints(ii,jj);
            uu=uu+1;
        end
    end
    end
    for ht=1:1:size(minLim,1)
        if forError(1,1)>=minLim(ht,1) && forError(1,1)<=maxLim(ht,1)
            blockMin=minLim(ht,1);
            blockMax=maxLim(ht,1);
        end
    end
end

```

```

end
ug=0;
for hk=1:1:size(forError)
    if forError(hk,1)>=blockMin && forError(hk,1)<=blockMax
        ug=ug+1;
    end
end
if ug==size(forError,1)
    error(jj,1)=0;
elseif ug<size(forError,1)
    error(jj,1)=std(forError);
else fprintf('Something is wrong');
end
checkerFinal{jj,1}=forError;
jj=jj+1;
uu=1;
forError=0;
end
finalError=sum(error);
permMean=zeros;
% Finding of means of permeabilities in each block
for ij=1:1:counterNew
    khm=checkerFinal{ij,1};
    permMean(ij,1)=mean(khm);
end
% If mean is zero, setting block permeability as mean of surrounding
% permeability points (at 1.5*maxDist distance)
fin=inpolygon(xFinal,yFinal,resVertX,resVertY);
fin=fin+0;
for ki=1:1:size(permMean,1)
    if permMean(ki,1)<0.001 && fin(ki,1)==1
        xyFinal=[xFinal(ki,1),yFinal(ki,1)];
    end
end

```

```

permDist=pdist2(permFieldVec,xyFinal);
forMean=zeros;
kf=1;
for jah=1:1:size(permDist,1)
    if permDist(jah,1)<=1.5*maxDist
        forMean(kf,1)=permeabilitiesVec(jah,1);
        kf=kf+1;
    end
end
permMean(ki,1)=mean(forMean);
end
end
% Changing permeabilities outside of reservoir from 0.001 to 0
for ka=1:1:size(permMean,1)
    if permMean(ka,1)>0 && permMean(ka,1)<1 && fin(ka,1)~=1
        permMean(ka,1)=0;
    end
end
% Check blocks on fault and assign faultPerm values to these blocks
if faultCount>0
    for up=1:1:size(xFinal,1)
        for do=1:1:size(fault,1)
            if xFinal(up,1)==fault(do,1) && yFinal(up,1)==fault(do,2);
                permMean(up,1)=faultPerm;
            end
        end
    end
end
end
fprintf('GridPointX GridPointY GridPointZ\n');
for ik=1:1:counterNew
    fprintf('%g %g %g\n', xFinal(ik,1), yFinal(ik,1), permZvec(1,1));

```

```

end
fprintf('BlockAveragePermeability in x-direction\n');
for il=1:1:counterNew
    fprintf('%g\n',permMean(il,1));
end
fprintf('BlockAveragePermeability in y-direction\n');
for ir=1:1:counterNew
    fprintf('%g\n',permMean(ir,1)*kykxrel);
end
if horWellCount>0
    fprintf('You have %g horizontal wells\n', horWellCount);
    for ks=1:1:horWellCount
        fprintf('Horizontal well #%g gridpoints:\n', ks);
        xForPrint=horWellPoints{ks,1};
        yForPrint=horWellPoints{ks,2};
        for ru=1:1:size(xForPrint,1)
            fprintf('%g %g\n', xForPrint(ru,1), yForPrint(ru,1));
        end
    end
end
end

% Showing result
reg1verX=[3200; 3200; 4400; 5500; 4000; 4000; 3200];
reg1verY=[0; 4500; 6000; 4250; 3000; 0; 0];

% Changing region vertices to align with reservoir
reg1verX=reg1verX-min(resVertX);
reg1verY=reg1verY-min(resVertY);
boc=[0, 0.5, 0];

voronoi(xFinal,yFinal);
hold on

```

```

fill(resVertX,resVertY,boc,'FaceAlpha',0.2);
hold on
fill(reg1 verX,reg1 verY,boc,'FaceAlpha',0.1);
hold off

%{
% Main showing result
boc=[0, 0.5, 0];

voronoi(xFinal,yFinal);
hold on
fill(resVertX,resVertY,boc,'FaceAlpha',0.2);
hold off
%}

if horWellCount > 0
    save 280815case4results.mat xFinal yFinal permeabilitiesVec permXvec
permYvec allGens permMean horWellPoints finalError
else
    save 280815case4results.mat xFinal yFinal permeabilitiesVec permXvec
permYvec allGens permMean finalError
end

```


APPENDIX B

CASE 2 FLUID FLOW SIMULATION RUN

This appendix includes fluid flow simulation run pictures for case #2. Inputs for fluid flow simulation run were grid blocks and permeabilities discussed in sub-chapter 8.2.2, one vertical well in the middle of reservoir producing at 100 stb/d for 50 days. Initial reservoir pressure was chosen to be 3044 PSI. Time step was chosen as 5 days, so there are 10 pictures showing propagation of pressure disturbance after 5, 10, 15, 20, 25, 30, 35, 40, 45 and 50 days.

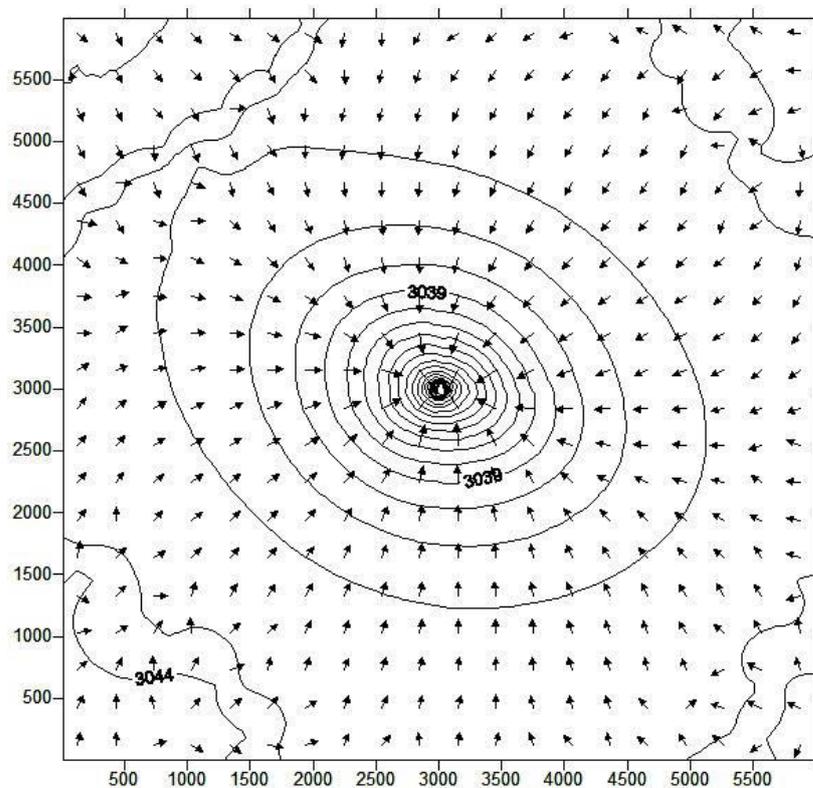


Figure B.1. Pressure distribution after 5 days.

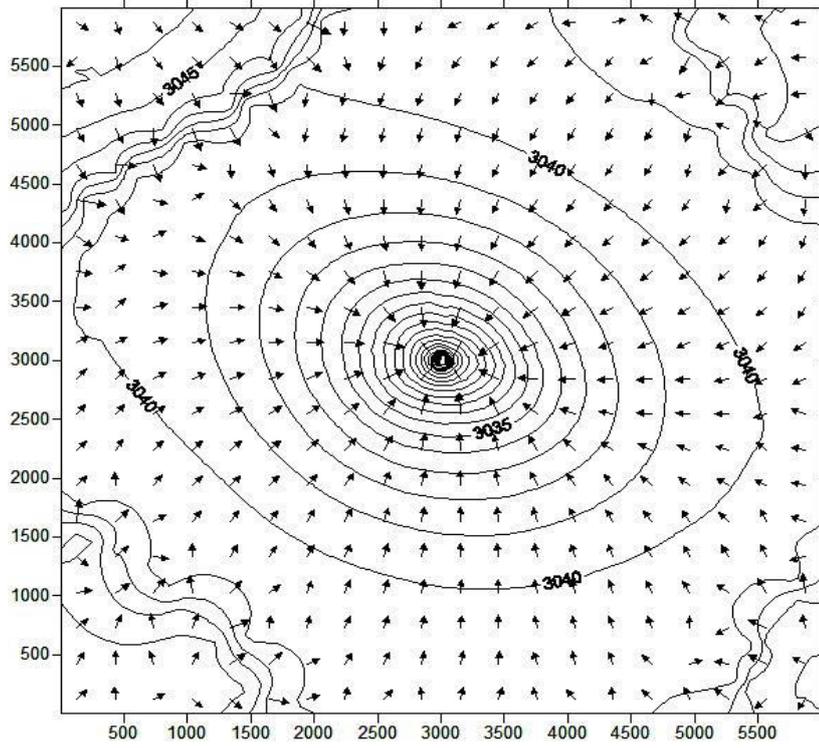


Figure B.2. Pressure distribution after 10 days.

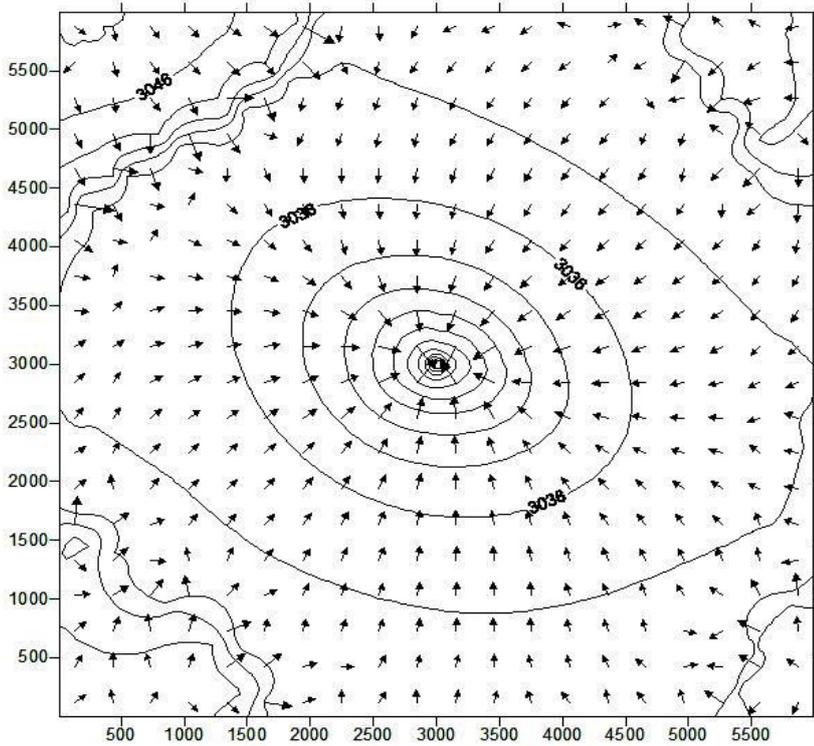


Figure B.3. Pressure distribution after 15 days.

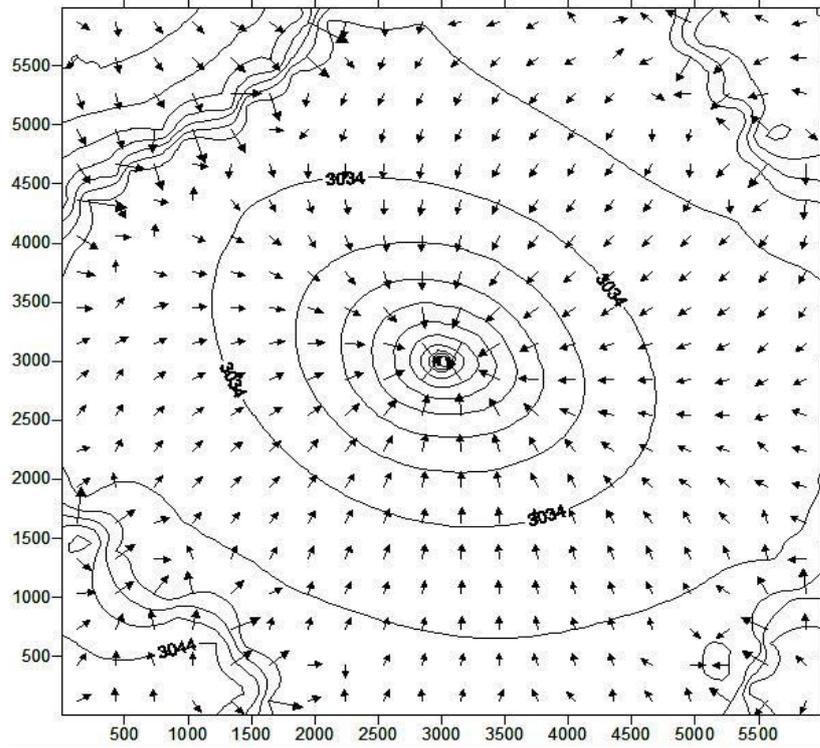


Figure B. 4. Pressure distribution after 20 days.

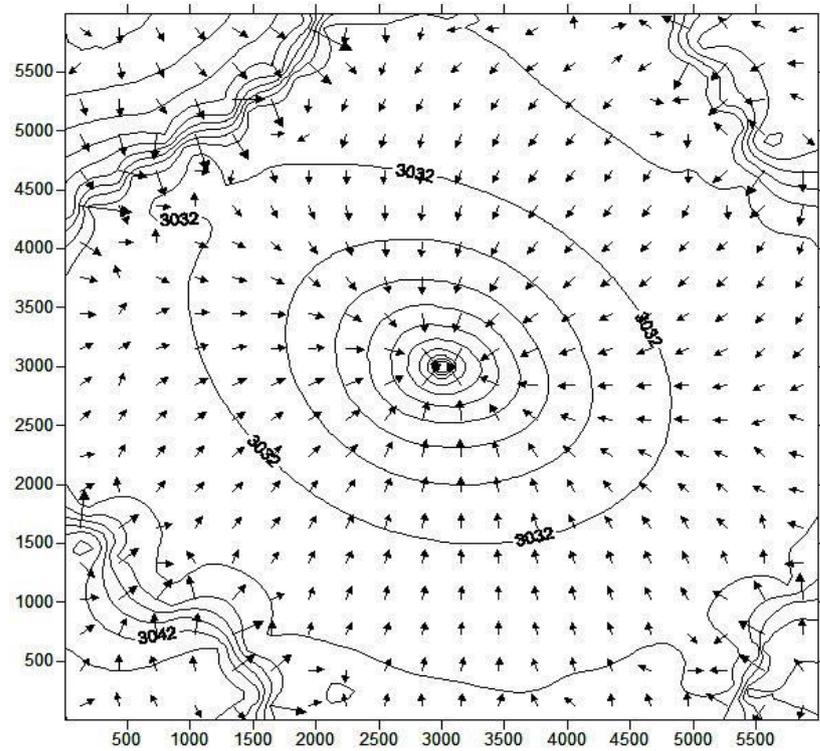


Figure B. 5. Pressure distribution after 25 days.

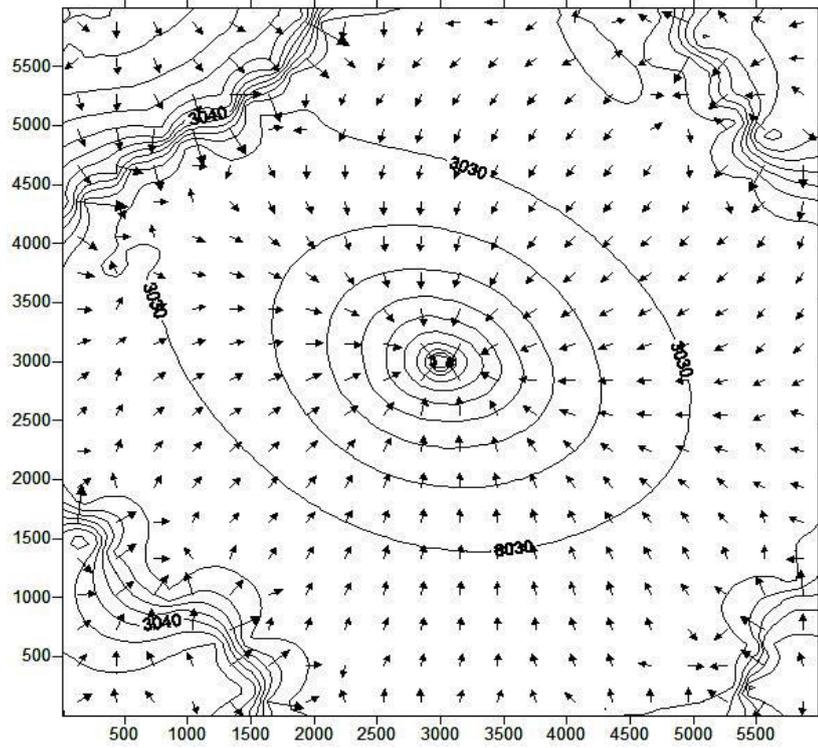


Figure B.6. Pressure distribution after 30 days.

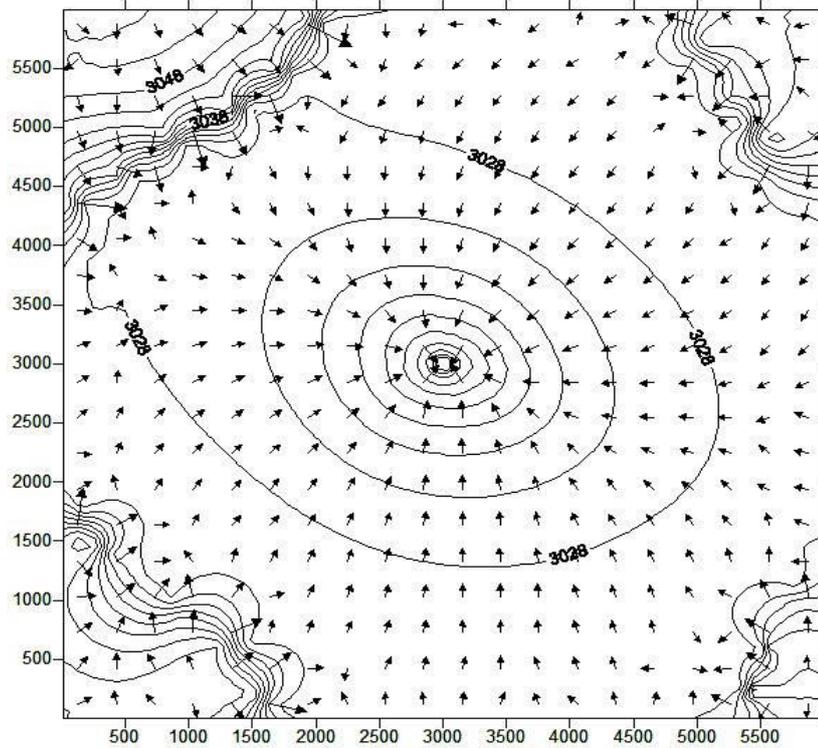


Figure B.7. Pressure distribution after 35 days.

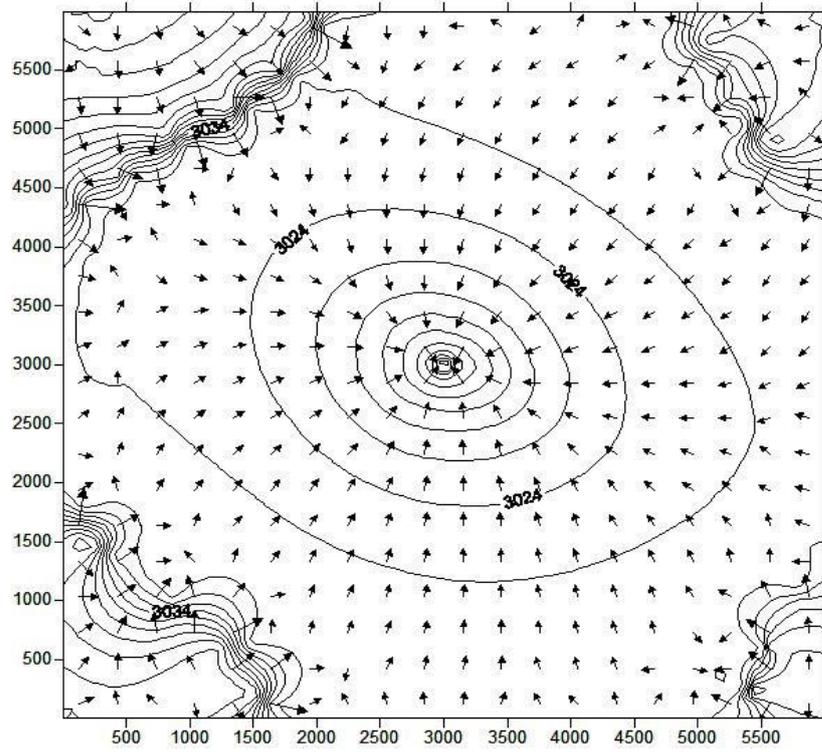


Figure B.8. Pressure distribution after 40 days.

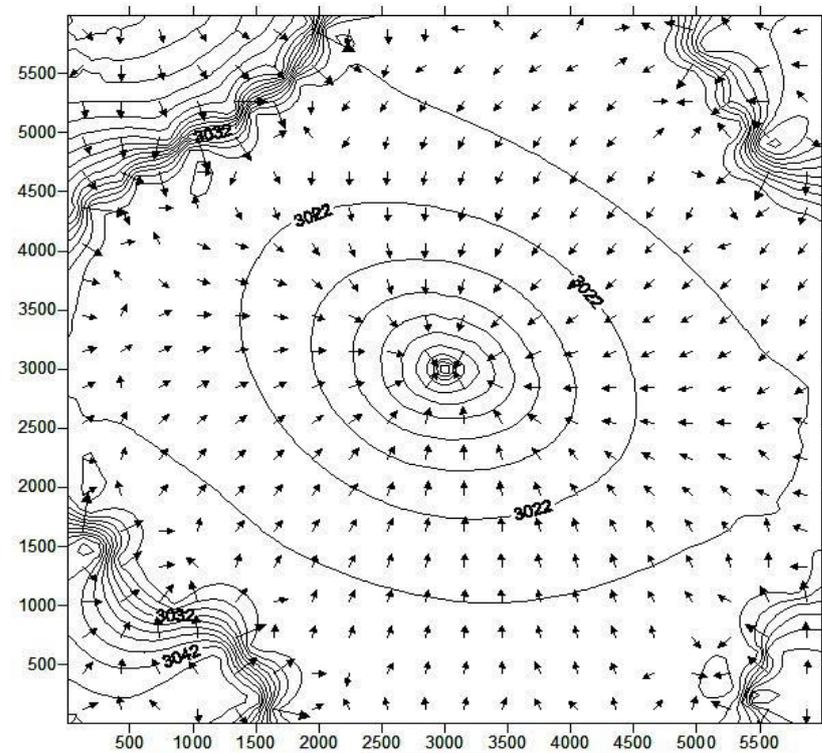


Figure B.9. Pressure distribution after 45 days.

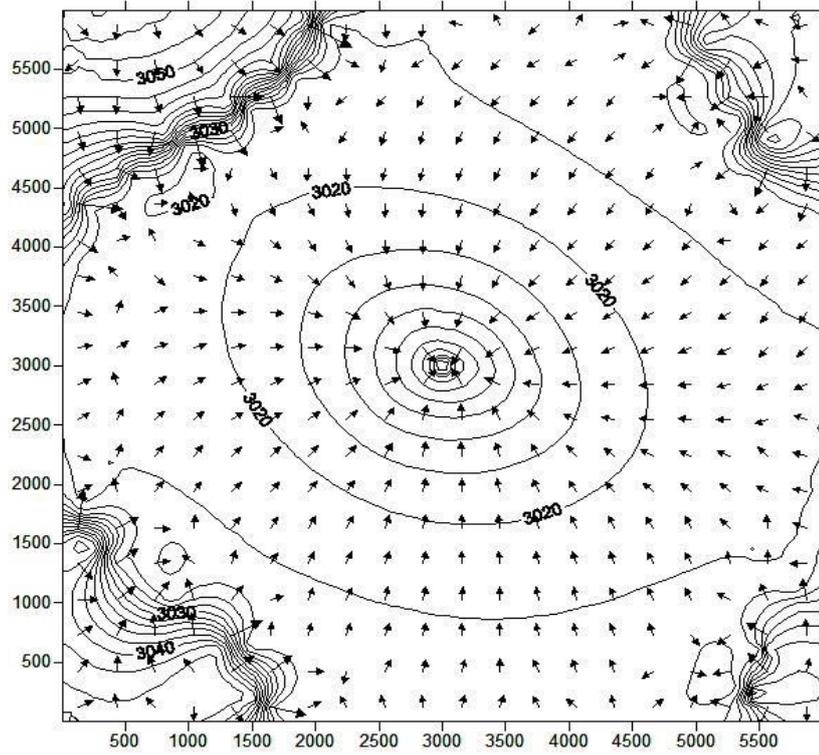


Figure B.10. Pressure distribution after 50 days.

APPENDIX C

CASE 3 FLUID FLOW SIMULATION RUN

This appendix includes fluid flow simulation run for case #3. Inputs for fluid flow simulation run were grid blocks and permeabilities discussed in sub-chapter 8.2.3, one vertical well in the middle of reservoir producing at 100 stb/d for 50 days. Initial reservoir pressure was chosen to be 3044 PSI. Timestep was chosen as 5 days, so there are 10 pictures showing propagation of pressure disturbance after 5, 10, 15, 20, 25, 30, 35, 40, 45 and 50 days.

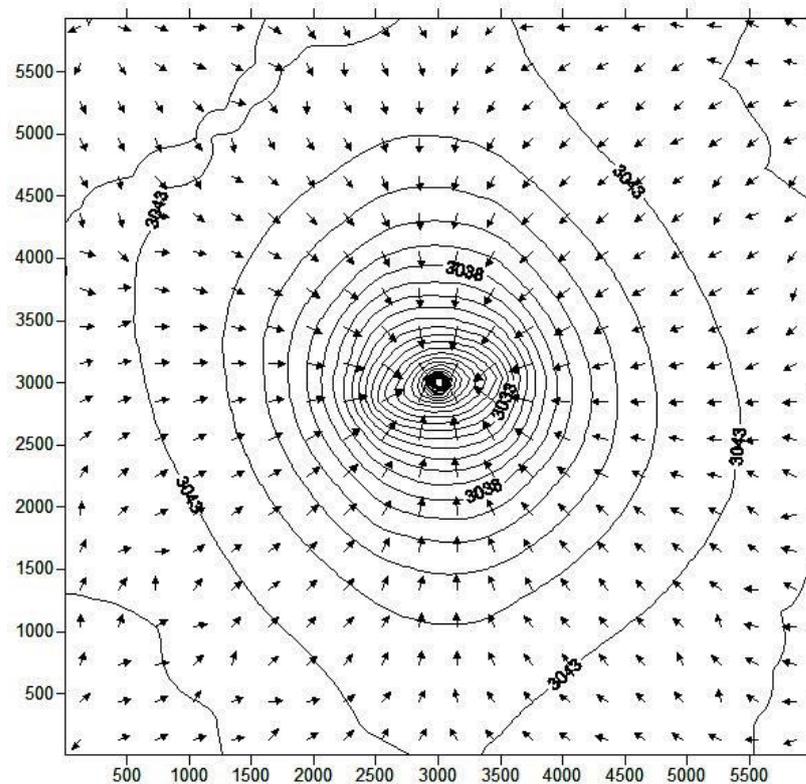


Figure C.1. Pressure distribution after 5 days.

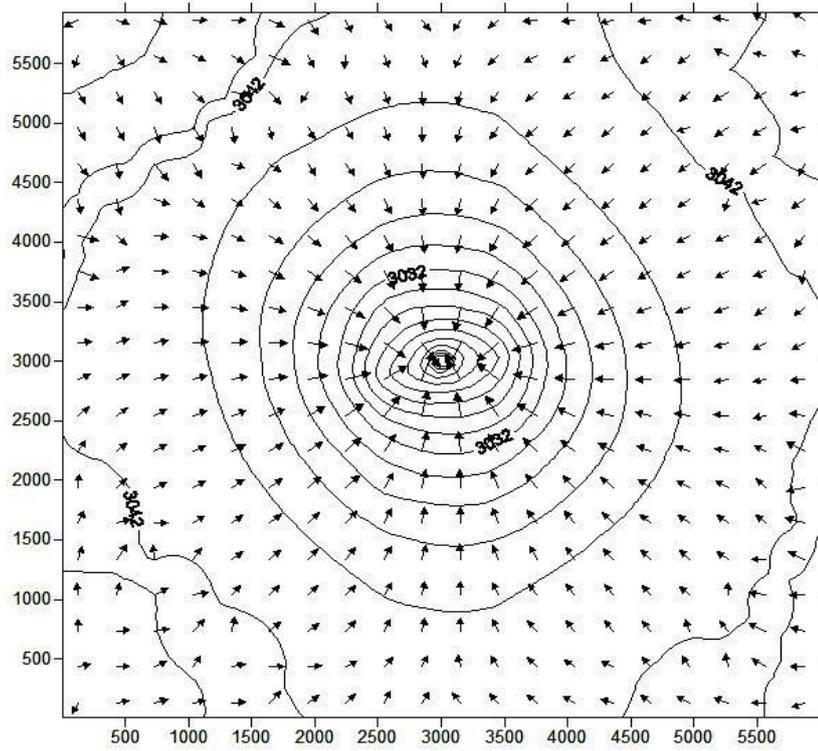


Figure C.2. Pressure distribution after 10 days.

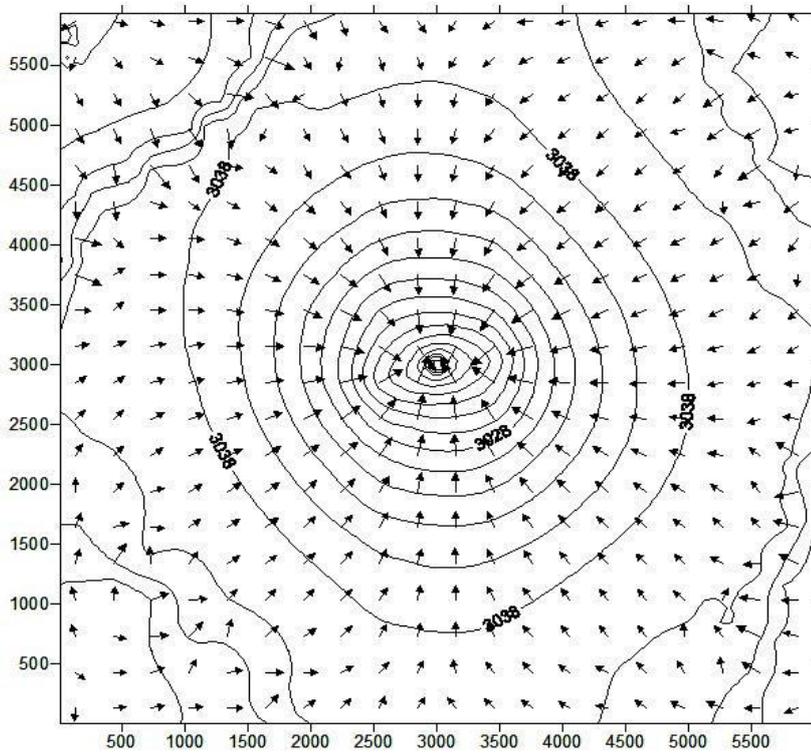


Figure C.3. Pressure distribution after 15 days.

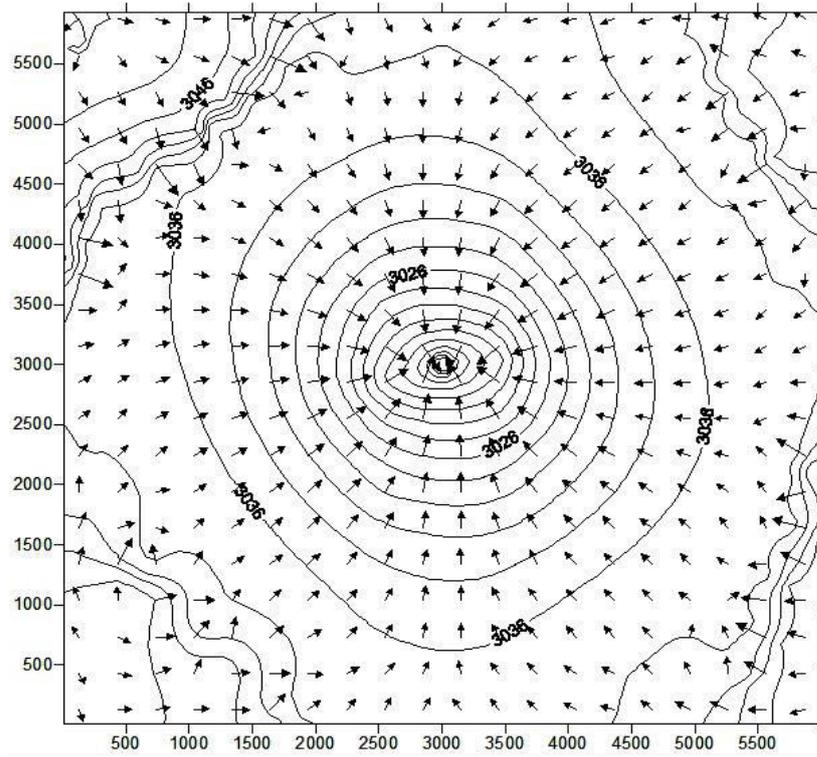


Figure C.4. Pressure distribution after 20 days.

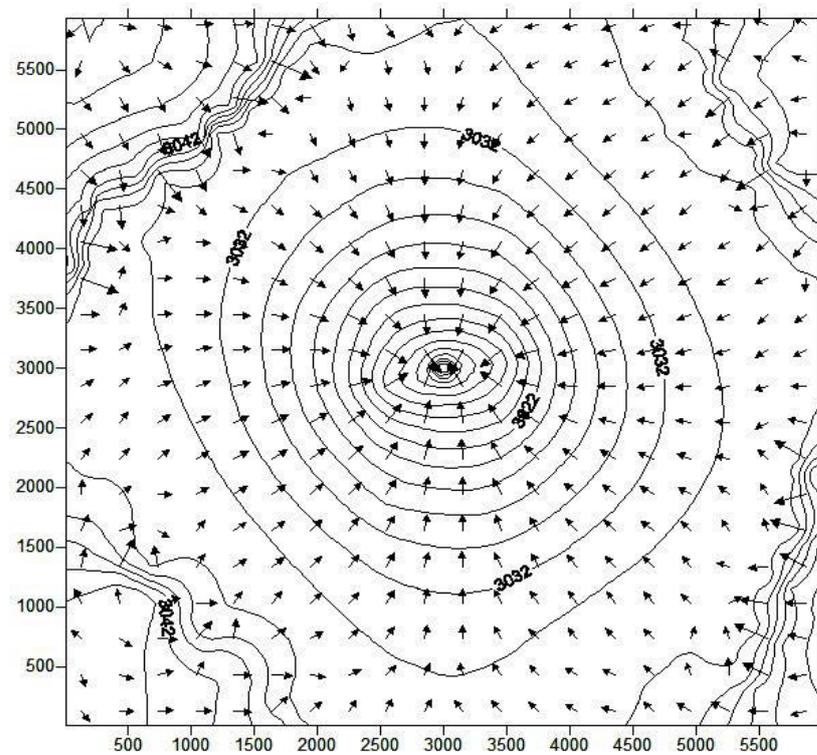


Figure C.5. Pressure distribution after 25 days.

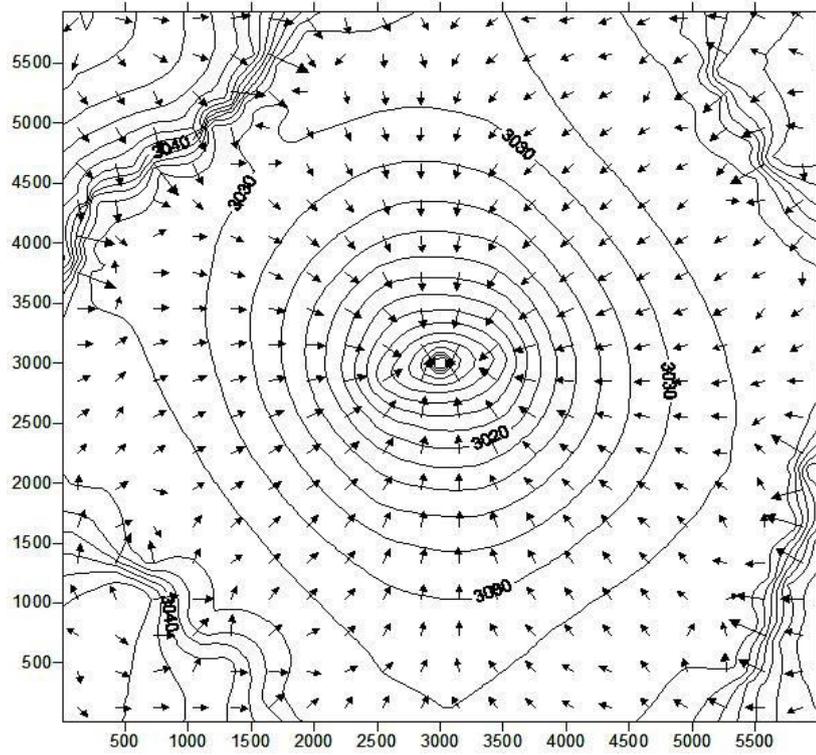


Figure C.6. Pressure distribution after 30 days.

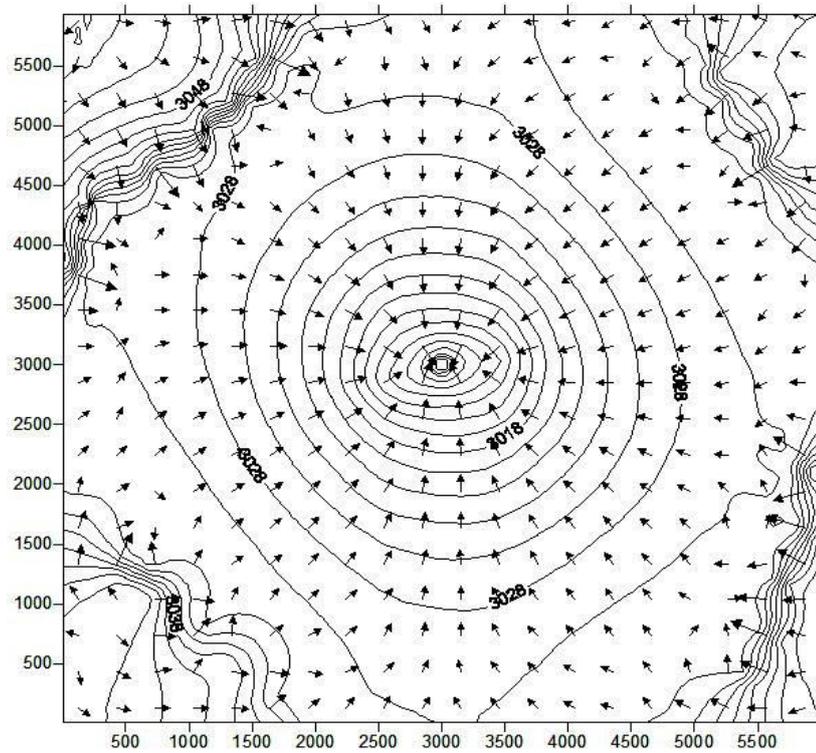


Figure C.7. Pressure distribution after 35 days.

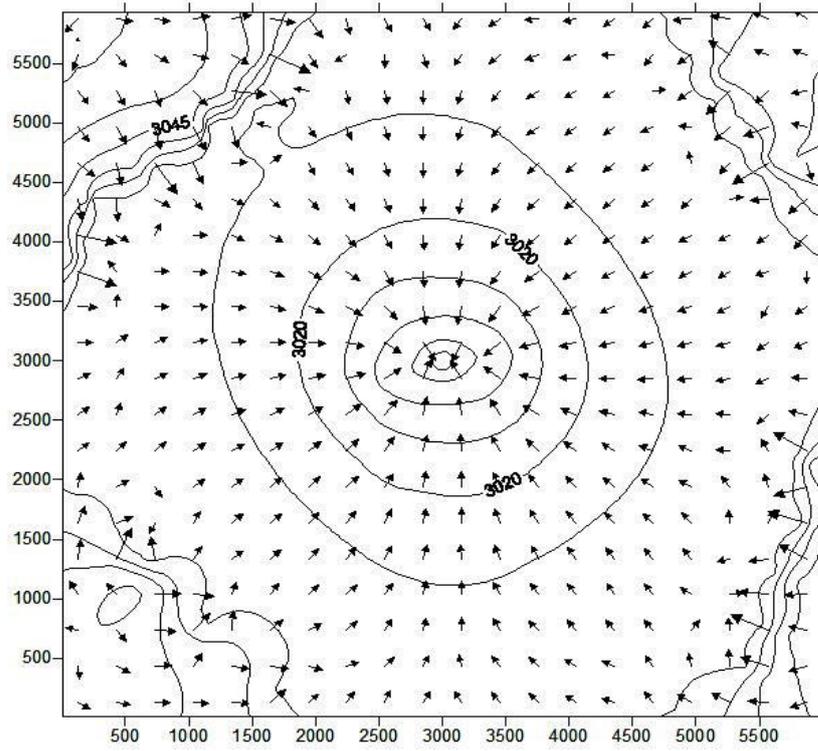


Figure C.8. Pressure distribution after 40 days.

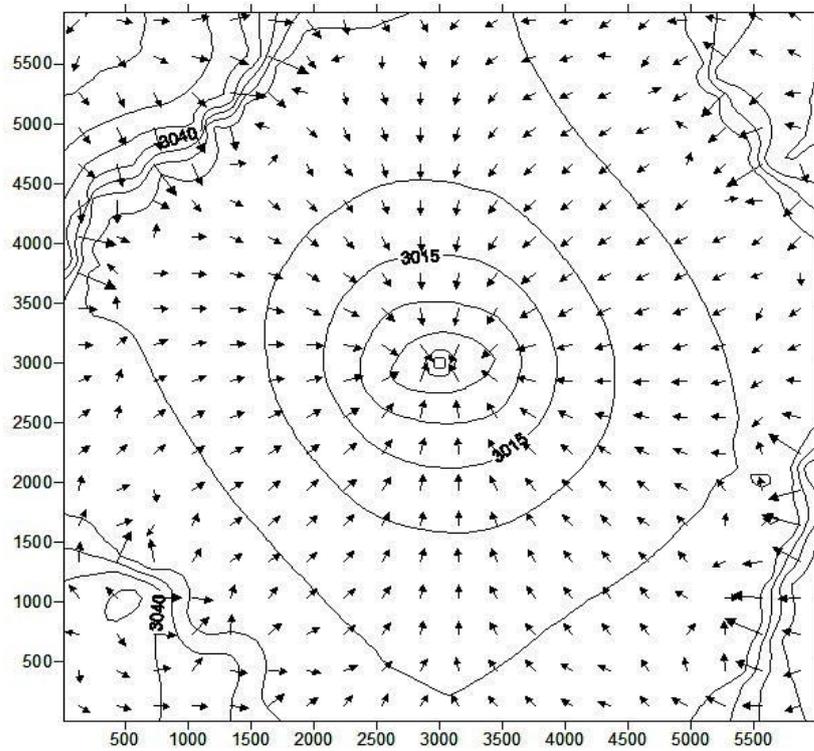


Figure C.9. Pressure distribution after 45 days.

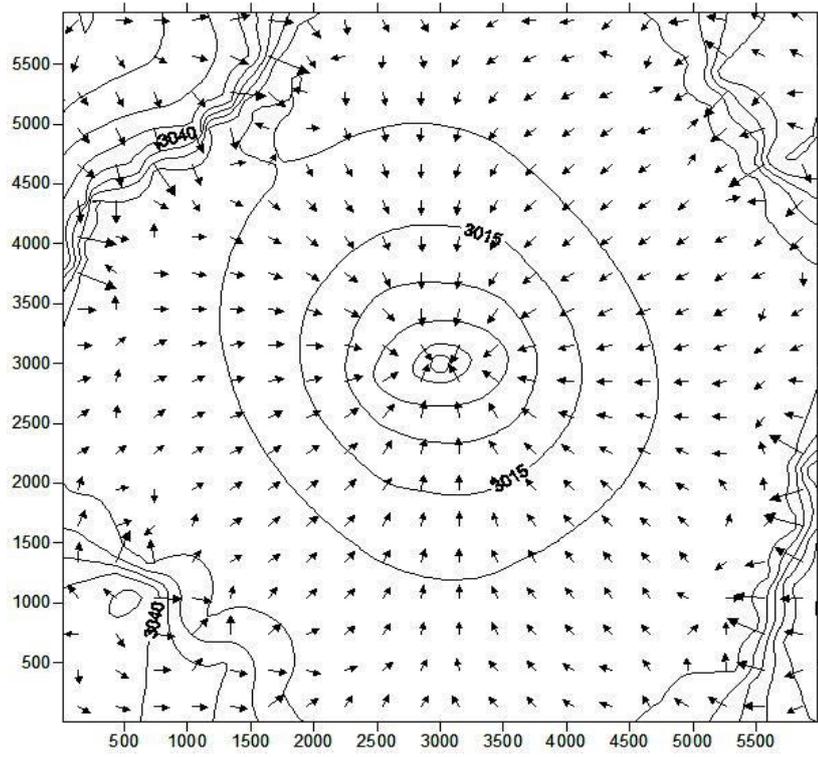


Figure C.10. Pressure distribution after 50 days.

APPENDIX D

CASE 4 FLUID FLOW SIMULATION RUN

This appendix includes fluid flow simulation run for case #4. Inputs for fluid flow simulation run were grid blocks and permeabilities discussed in sub-chapter 8.2.4, one vertical well in the middle of reservoir producing at 100 stb/d for 50 days. Initial reservoir pressure was chosen to be 3044 PSI. Timestep was chosen as 5 days, so there are 10 pictures showing propagation of pressure disturbance after 5, 10, 15, 20, 25, 30, 35, 40, 45 and 50 days.

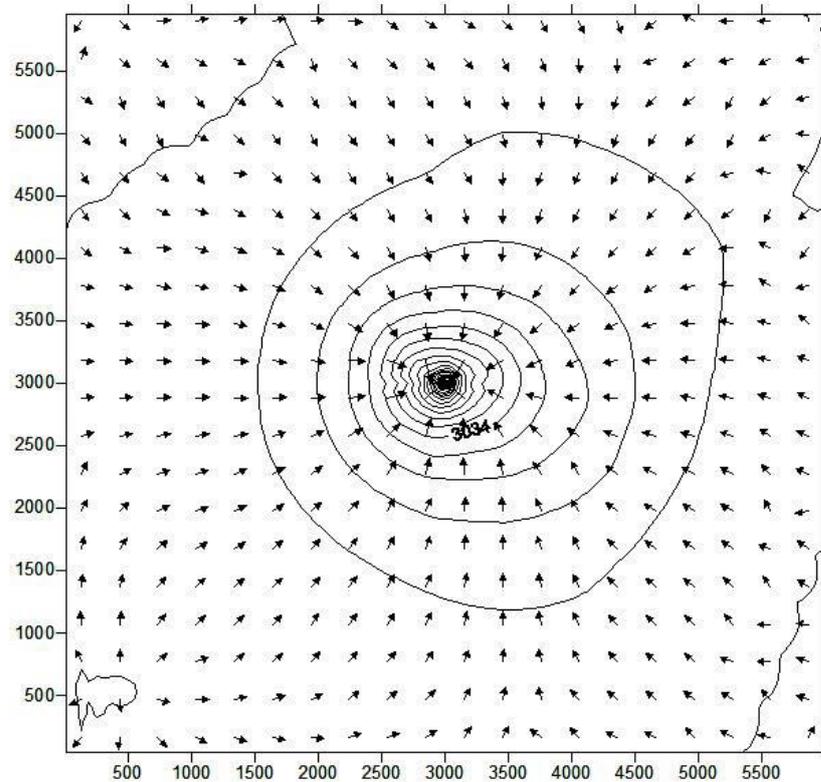


Figure D.1. Pressure distribution after 5 days.

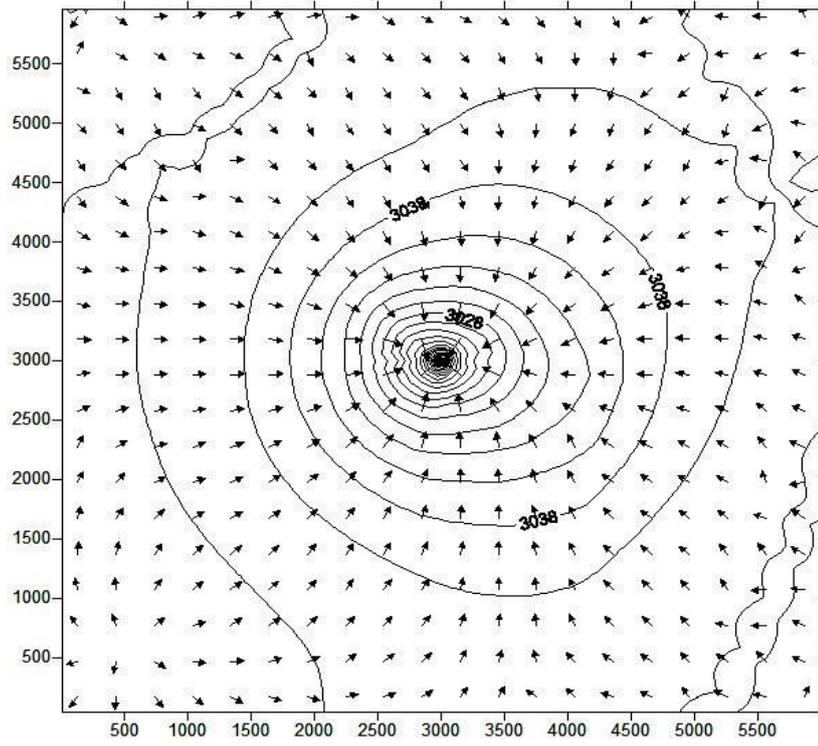


Figure D.2. Pressure distribution after 10 days.

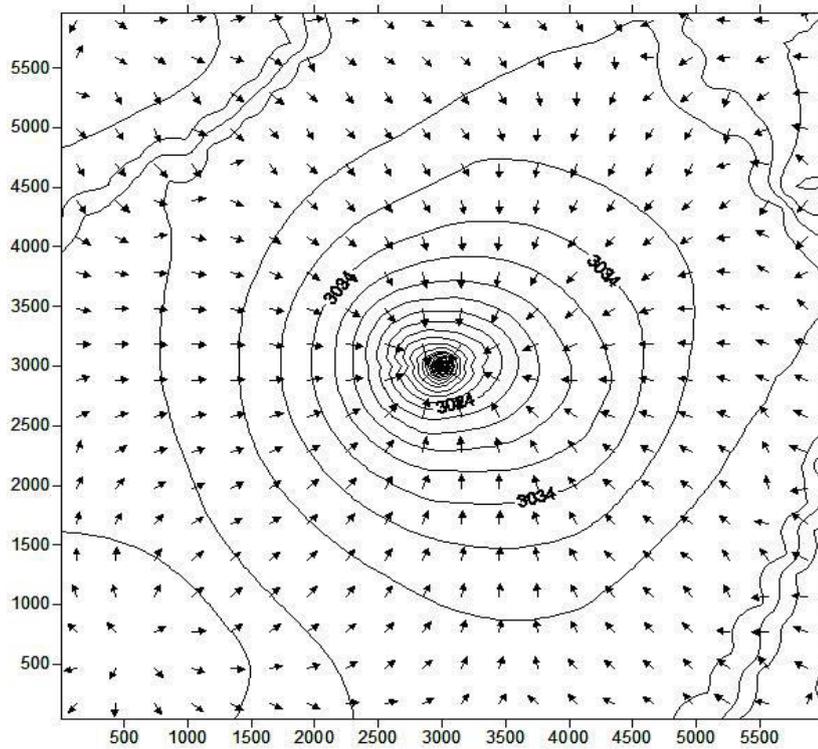


Figure D.3. Pressure distribution after 15 days.

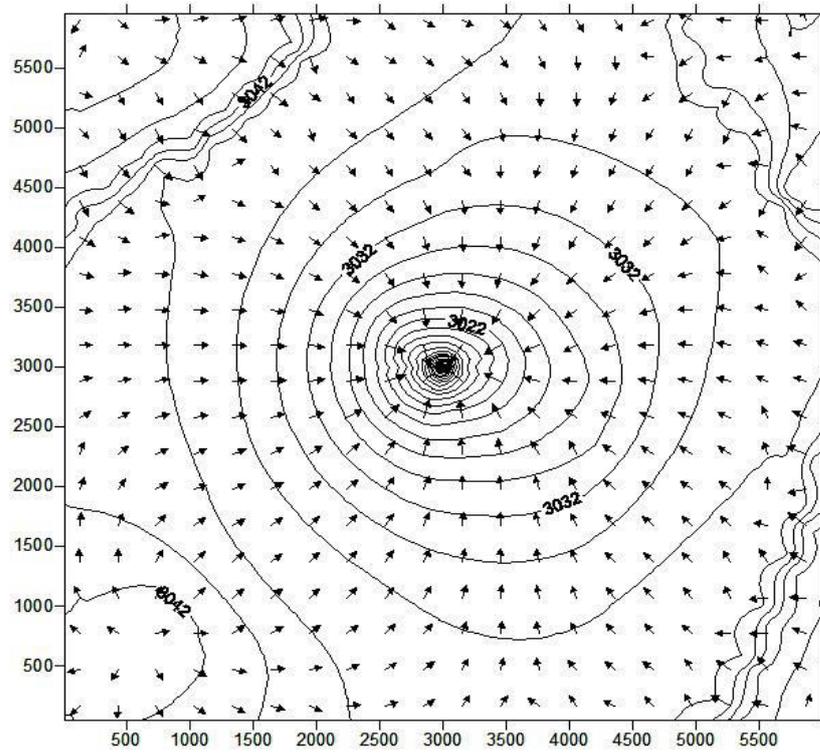


Figure D.4. Pressure distribution after 20 days.

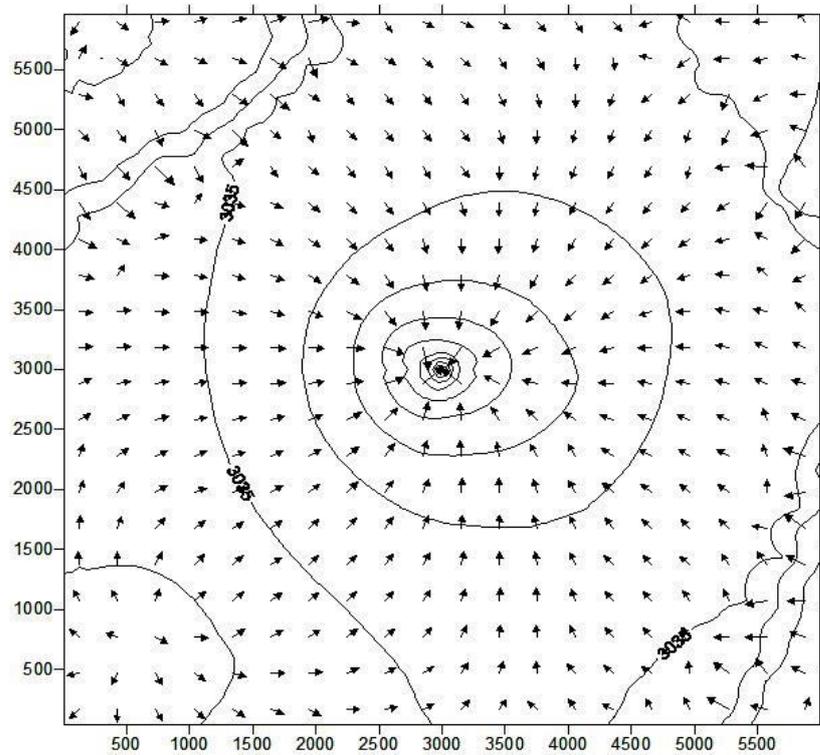


Figure D.5. Pressure distribution after 25 days.

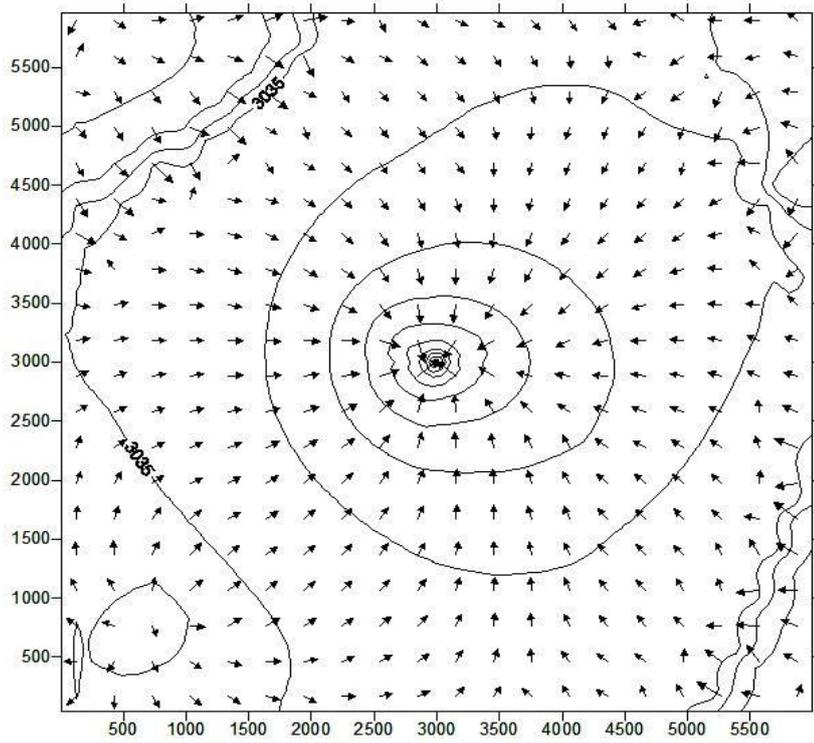


Figure D.6. Pressure distribution after 30 days.

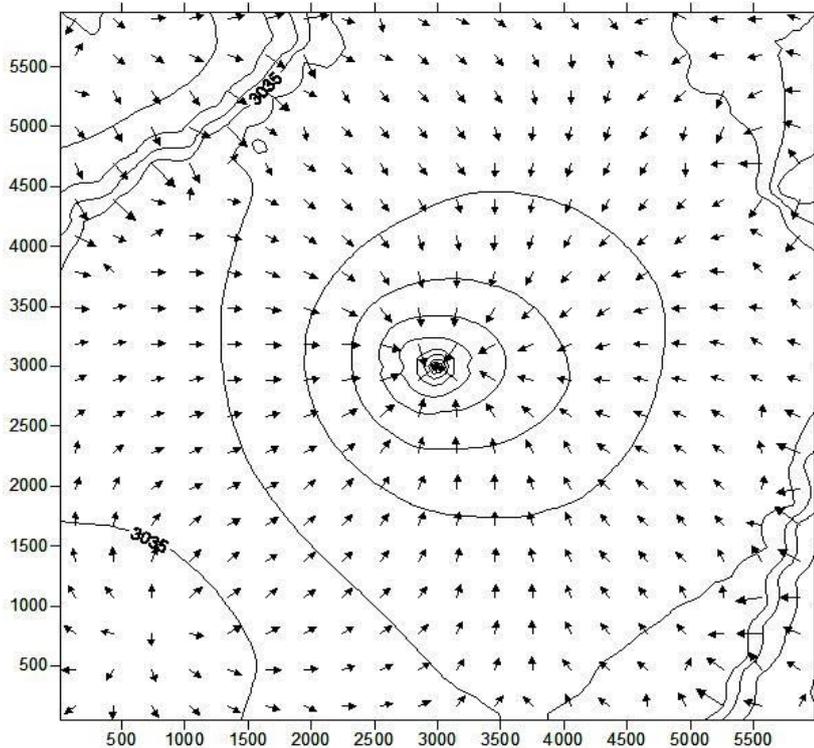


Figure D.7. Pressure distribution after 35 days.

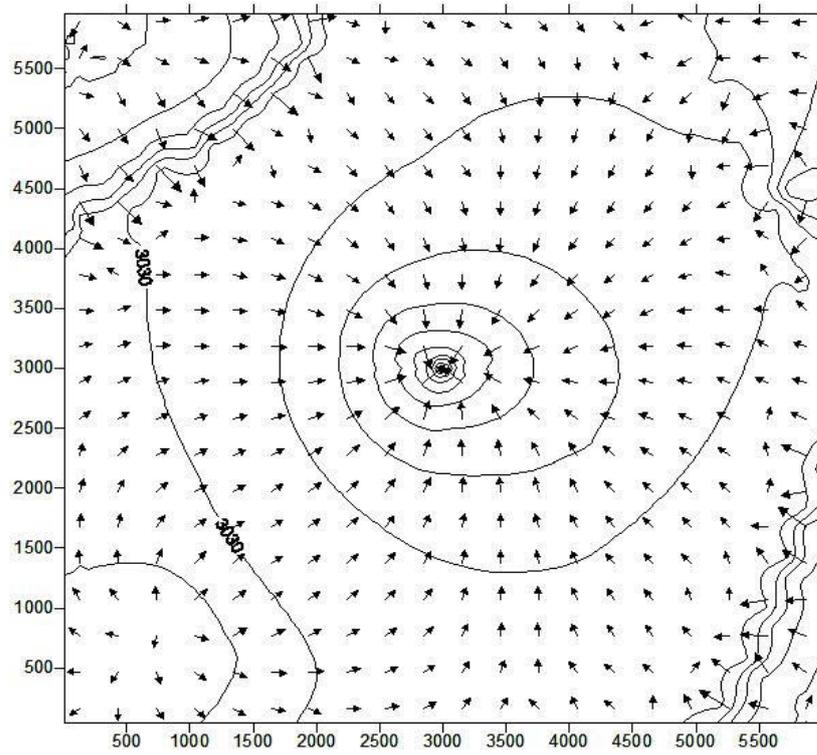


Figure D.8. Pressure distribution after 40 days.

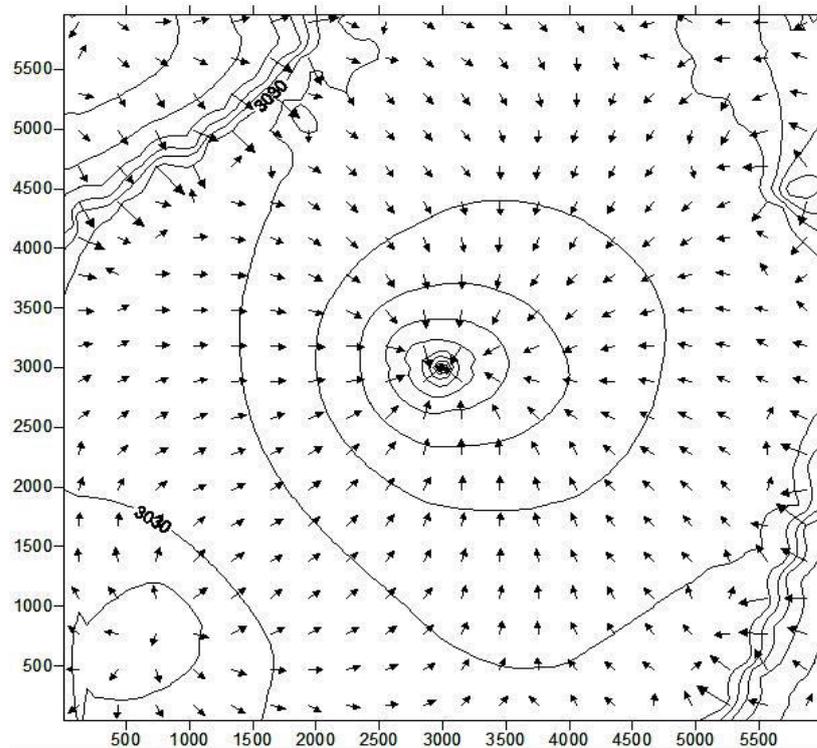


Figure D.9. Pressure distribution after 45 days.

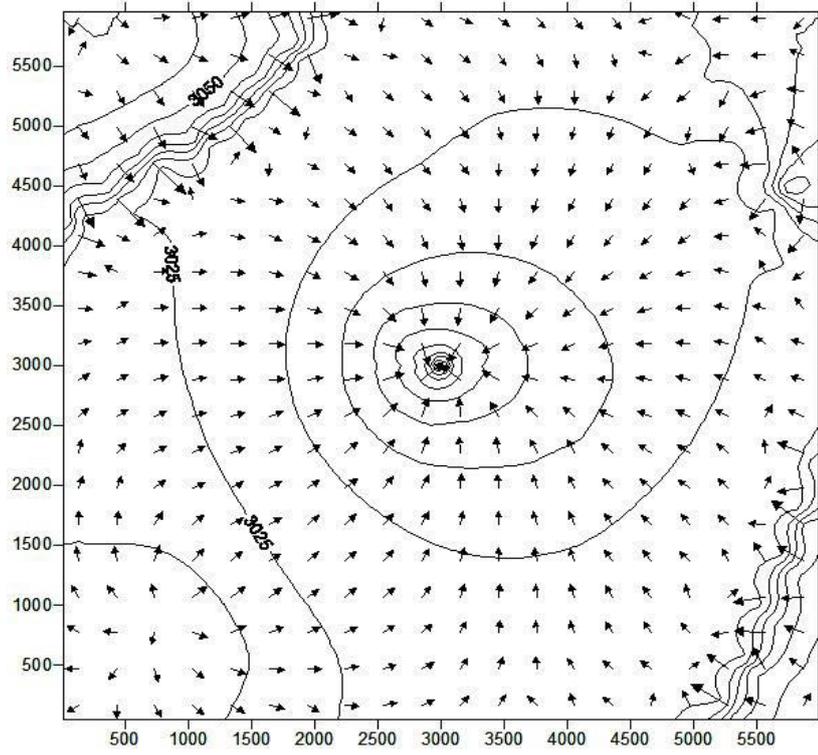


Figure D.10. Pressure distribution after 50 days.

APPENDIX E

CASE 5 FLUID FLOW SIMULATION RUN

This appendix includes fluid flow simulation run for case #5. Inputs for fluid flow simulation run were grid blocks and permeabilities discussed in sub-chapter 8.2.5, one vertical well in the middle of reservoir producing at 100 stb/d for 5 days. Initial reservoir pressure was chosen to be 3044 PSI. Timestep was chosen as 0.5 days, so there are 10 pictures showing propagation of pressure disturbance after 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5 and 5 days.

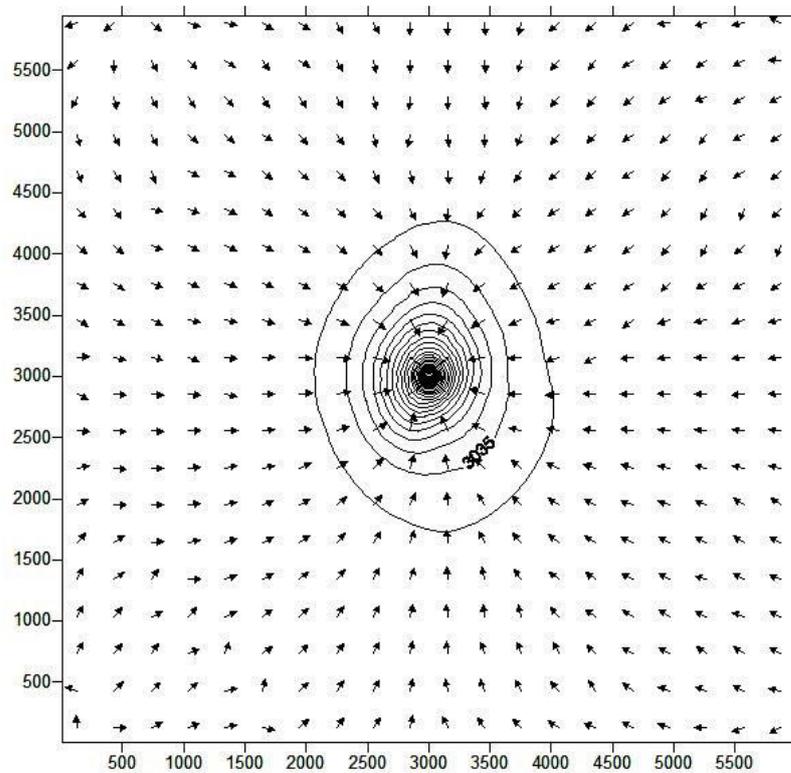


Figure E.1. Pressure distribution after 0.5 days.

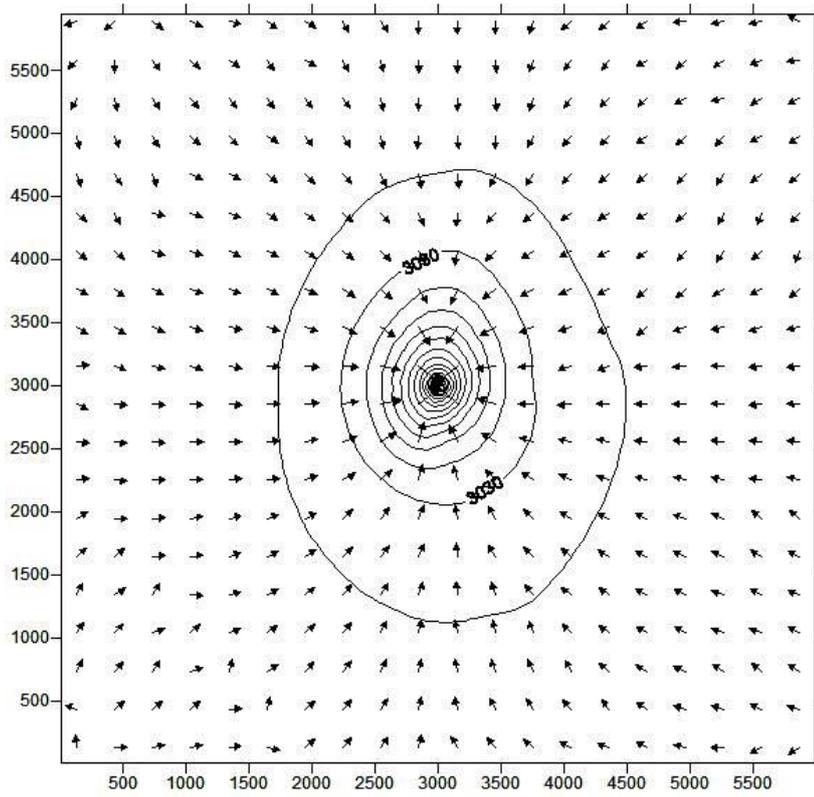


Figure E.2. Pressure distribution after 1 day.

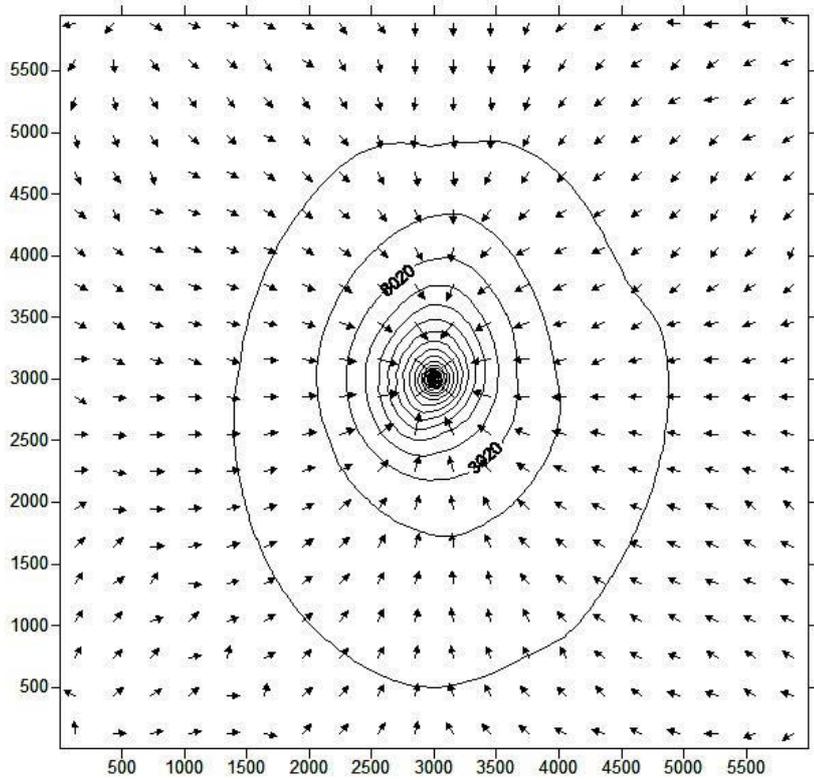


Figure E.3. Pressure distribution after 1.5 days.

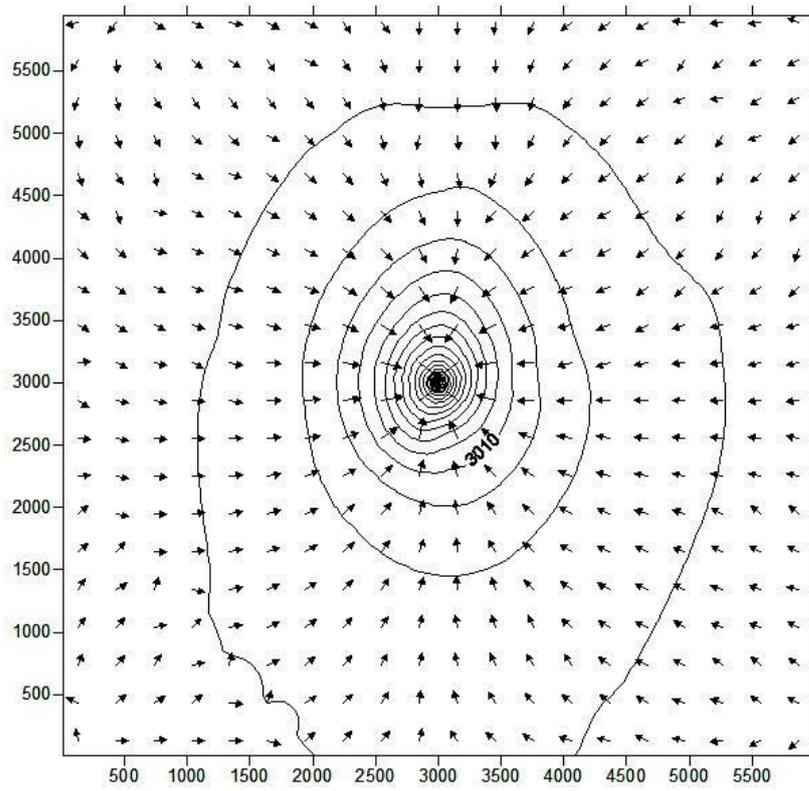


Figure E.4. Pressure distribution after 2 days.

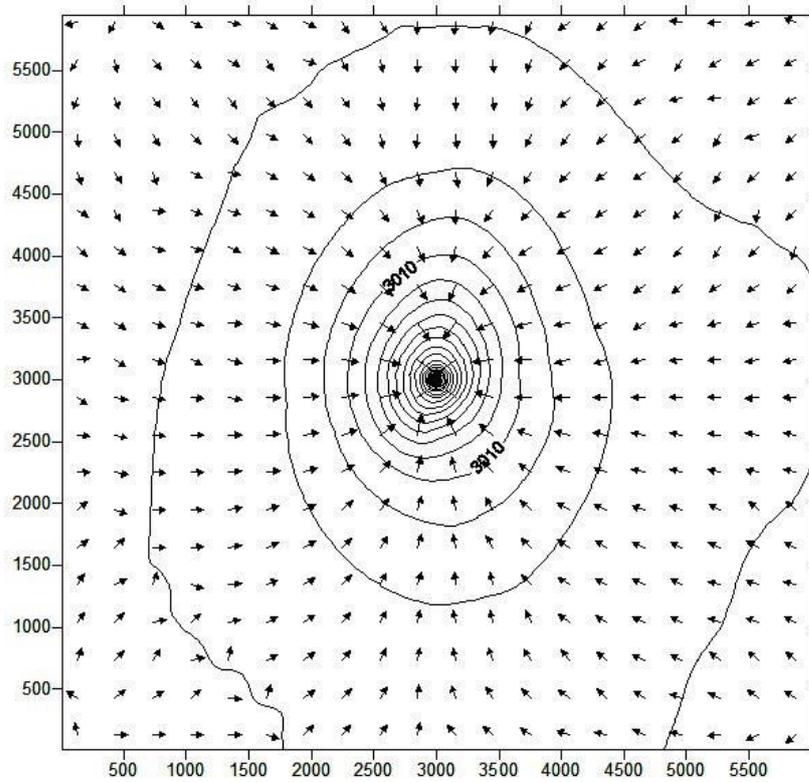


Figure E.5. Pressure distribution after 2.5 days.

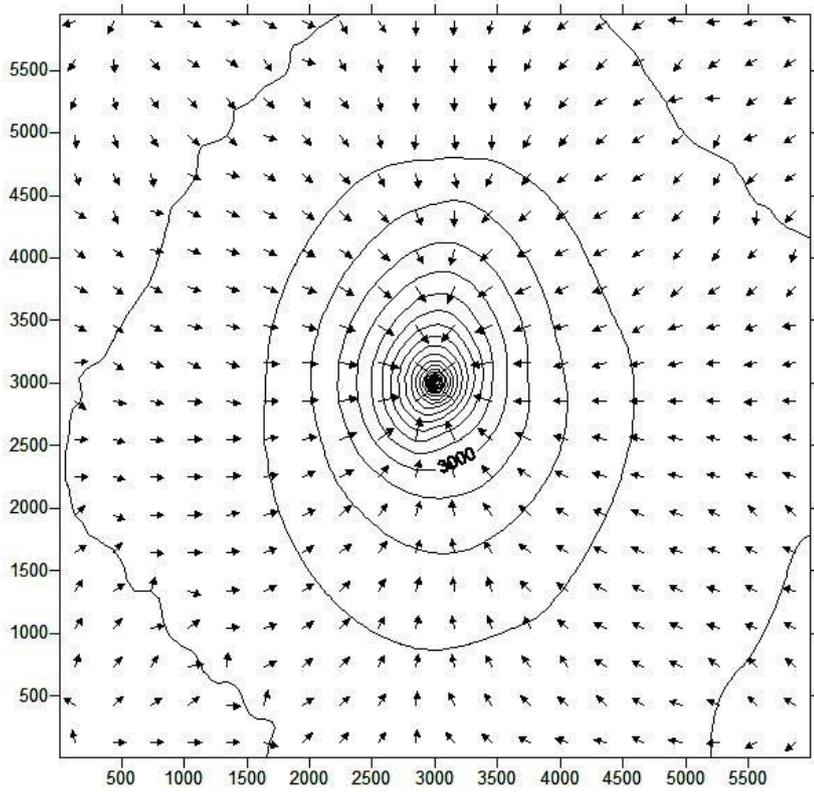


Figure E.6. Pressure distribution after 3 days.

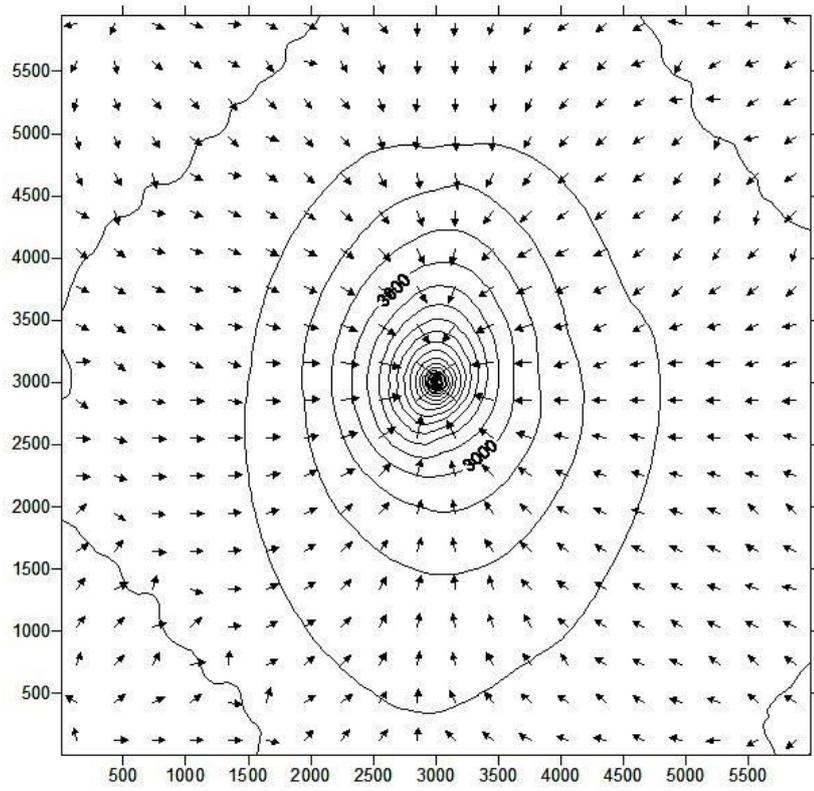


Figure E.7. Pressure distribution after 3.5 days.

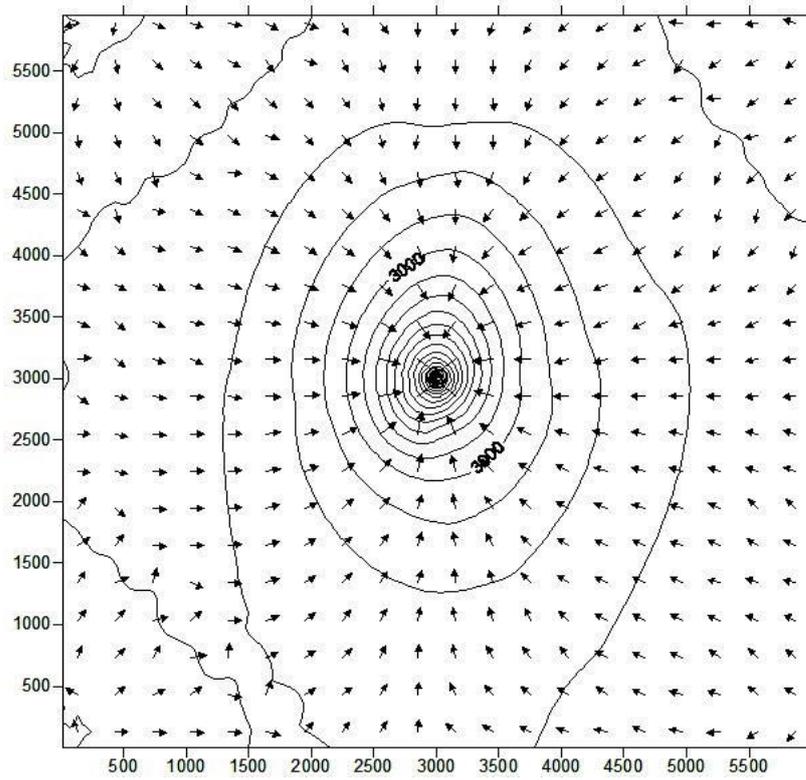


Figure E.8. Pressure distribution after 4 days.

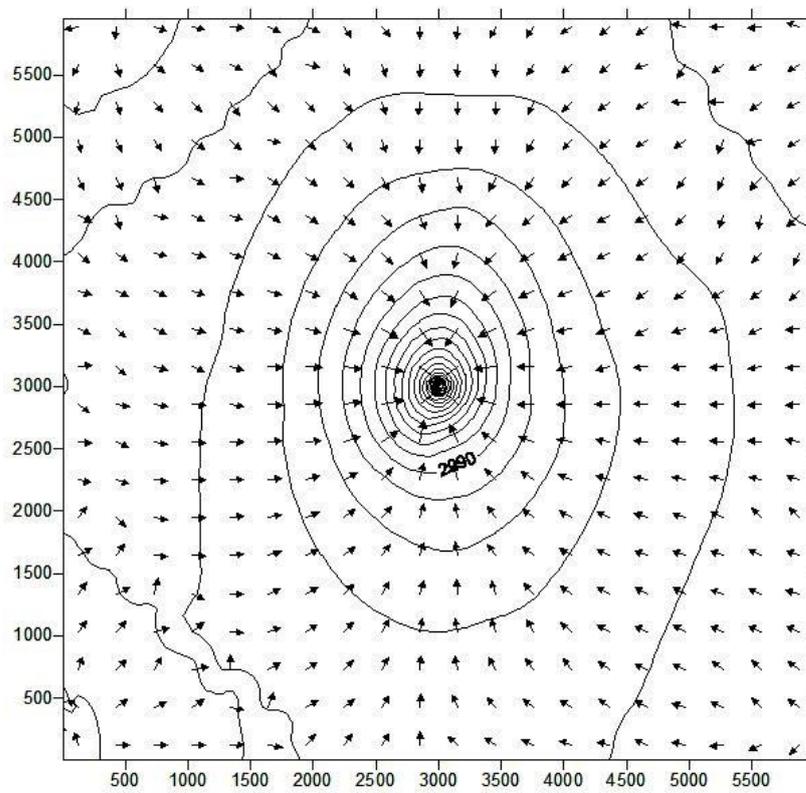


Figure E.9. Pressure distribution after 4.5 days.

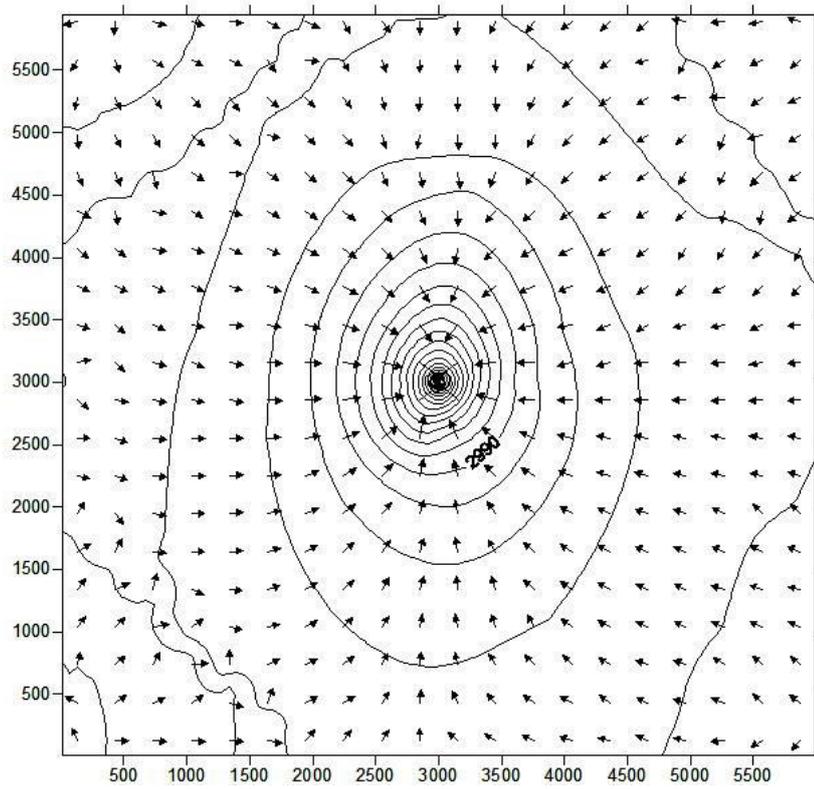


Figure E.10. Pressure distribution after 5 days.