

CHANCE CONSTRAINED SCHEDULE DESIGN FOR HETEROGENEOUS  
FLEET IN LINER SHIPPING SERVICE

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FLEET IN LINER SHIPPING SERVICE**

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## ABSTRACT

### CHANCE CONSTRAINED SCHEDULE DESIGN FOR HETEROGENEOUS FLEET IN LINER SHIPPING SERVICE

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This study deals with designing a schedule for a heterogeneous fleet of liner shipping service by considering uncertainties. Shipping industry encounters with different kinds of uncertainties. Uncertainties of waiting times of the ships and handling times of the cargos might affect the actual departure times of the ships. In this study, service level is represented as the probability of on-time departure of a ship. Assuming that handling and waiting times are normally distributed, the problem is formulated as a mixed integer nonlinear stochastic program where the objective is to minimize the total fuel consumption. In formulation of the problem, three new aspects are considered. The first one is considering the heterogeneous fleet. The second one is considering the differences of the ports and the third is considering a new service level measure. The developed model is able to determine sailing times, departure times and service levels. Service levels are determined in a way to satisfy the overall service level of the service route. Overall service level could be defined by the shipping company for the entire route. The objective function of the model contains a nonlinear convex term.

For handling the nonlinearity of the objective function, the model is reformulated by applying second order conic programming. The reformulated model could be solved by commercial software such as CPLEX. Finally, several experimental factors are defined and effects of these factors on fuel consumption cost and optimal solutions are analyzed. Moreover, for showing the benefits of the model, different comparisons are done.

Keywords: Maritime Scheduling, Liner Shipping, Heterogeneous Fleet, Service Level, Second Order Conic Programming

## ÖZ

### HETEROJEN FİLOLU DÜZENLİ GEMİ SEFERLERİ İÇİN GÜRBÜZ ÇİZELGELEME

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Bu çalışma heterojen filo ile düzenli gemi seferlerinin belirsizlikler altında çizelge-  
lenmesi problemini ele almaktadır. Deniz taşımacılığında farklı belirsizlikler sözkö-  
nusudur. Gemilerin limanlarda sıra beklemeleri ve yükleme-boşaltma sürelerindeki  
belirsizlikler gerçekleşen kalkış zamanlarını etkilemektedir. Bu çalışmada servis se-  
viyesi planlanan zamanda kalkma olasılığı olarak düşünülmüştür. Yükleme-boşaltma  
ve bekleme sürelerinin Normal dağılıma uyduğu varsayılarak çizelgeleme problemi  
karışık tamsayılı doğrusal olmayan şans kısıtlı program olarak ifade edilmektedir.  
Öyle ki minimize edilmek istenen hedef fonksiyonu toplam yakıt tüketimidir. Tanım-  
lanan problemde üç yeni durum ele alınmıştır. İlki heterojen filo durumudur. İkincisi  
servis seviyeleri bakımından her limanın farklı değerlendirilmesi. Üçüncüsü ise çiz-  
zelge için yeni bir servis seviyesi ölçüsünün önerilmesidir. Geliştirilen model seyir  
sürelerini, kalkış zamanlarını ve servis seviyelerini belirlemektedir. Model liman ve  
gemilerin servis seviyeleri çizelge için düşünülen toplu servis seviyesini sağlayacak

şekilde belirlemektedir. Modelin hedef fonksiyonunda doğrusal olmayan terimler bulunmaktadır. Doğrusal olmayan terimler ikinci derece konik programlama ile ifade edilmekte ve IBM ILOG CPLEX ile çözülmektedir. Son olarak modelin oluşturduğu çizelgelerin performansı belirlenen deneysel faktörlerin farklı seviyeleri için incelendi ve karşılaştırmalar yapıldı.

Anahtar Kelimeler: Gemi Çizelgeleme, Düzenli Gemi Seferleri, Heterojen Filo, Servis Seviyesi, İkinci Derece Konik Programlama



This thesis is heartily dedicated to my father, mother and sister who have supported and inspired me to go my own way. Also this thesis is dedicated to all those who believe in the richness of learning.

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# CHAPTER 1

## INTRODUCTION

Shipping is the major international transportation mode. Liner shipping, tramp shipping and industrial shipping are the three main types of ocean shipping services. According to Yao et al. [34], among these three types, liner shipping service has increased significantly during recent years. World Shipping Council [1] mentions that “liner shipping could lay claim to being the world’s first truly global industry.” They also mention that “there are almost 6000 ships, mostly container ships operating in liner services and container ships come in a variety of sizes.” In the liner shipping service, container ships operate on closed routes. They follow published schedules and transport containers between many origins and destinations.

According to Ronen [26], bunker fuel cost constitutes three quarters of the operating cost of a larger container ship when fuel price is around 500 USD per ton. By considering this fact, shipping companies prefer slow steaming to reduce the fuel consumption cost during the journey time. However, reducing the speed of a ship is critical as there are many uncertainties around a ship’s sailing time at sea. Liner shipping companies announce fixed schedules in advance and it is important for the companies to provide a schedule which is reliable. Unreliability of the schedules poses losses for the company. Many factors such as congestion, fluctuation of container handling times and weather condition affect actual arrival and departure times of the ships and cause delays. For overcoming these uncertainties, shipping companies put buffer times in their schedules.

Recent studies show that there is a nonlinear relation between speed and fuel consumption. Small changes in speeds cause larger changes in fuel consumption rate.

This will bring more attention for minimizing the total fuel consumption along the sailing route. Minimizing bunker fuel consumption is not only beneficial for the shipping companies, but also because of environmental issues it gets of higher emphasis. In Chapter 2, we give a review of the fuel consumption functions that are used in the maritime literature.

In this thesis, we study the problem of designing a schedule for a heterogeneous fleet of liner service by considering the port time uncertainties. The objective is to minimize the total fuel consumption cost. We consider uncertainties of handling and waiting times in our study. These uncertainties might affect the schedule and cause delays. Therefore, they might also affect the departure times of the ships and cause deviation from the published departure times.

This study contributes to the literature in three ways. The first is considering the heterogeneous fleet. In liner shipping, a fleet of ships operates on a closed route. It visits port-of-calls according to published schedules. In the literature, homogeneous fleet of ships is considered to be deployed on a single route. However, in practice, different ship types could be deployed on the single route. Differences between the ships cause each to have a different fuel consumption function.

The second is considering the differences of the ports. Ports might be different with each other in terms of their importance for having on-time departure times. Congestion, demand rate and many other factors make ports different from each other. So these factors cause ports to have different importance for having on-time departure. Handling and waiting times might also be different at the ports .

The third is considering a new service level measure. We represent service level as the probability of on-time departure of a ship at a port. In addition, we define an overall service level measure for the entire route.

By considering these aspects, we develop a model to decide on sailing times, departure times and service levels in a liner shipping schedule. Service levels are determined for each port-ship type pair. However, obtained service levels guarantee that overall service level of the service route is satisfied. Overall service level could be determined by the shipping company for each service route as a degree of the relia-

bility of the schedule. Sailing times are also determined distinctly for each ship type on the sailing legs. But common departure times for the different ships types at the ports should be achieved.

The developed model is able to determine variable service levels for the port-ship type pairs because of two reasons. The first reason is the difference of the fuel consumption functions of the ship types. Lower service level could be assigned for the less efficient ship types. The lower service level for a ship means that probability of delay for departure time of that type of ship is higher. By having higher probability of delay and as whole the round-trip journey time is considered to be fixed, sailing time of the ship would increase. In other words, speed of the ship would decrease. This reduces fuel consumption and vice versa by increasing service level, sailing time of a ship would decrease. This yields in more fuel consumption. The second reason is the differences of the ports. Also obtained service levels guarantee that overall service level of the service route is satisfied. We further show that assigning variable service levels for the port-ship type pairs is more beneficial to the shipping company than assigning equal service levels while achieving the specified overall service level for the whole system.

Therefore, service levels are assigned by taking into account of the differences of the ports. Also, obtained service levels satisfy the overall service level of the company. But as another approach, delay costs at the ports could be considered in the objective function. However, considering delay function in the objective of the model might result in nonlinearity which makes the problem more difficult to be solved.

The study is organized as follows. In Chapter 2, we present literature review. In Chapter 3, we give a comprehensive statement of the problem, assumptions and mathematical model. In Chapter 4, we reformulate the model and give SOCP representation of the model. We also give SOCP representations of the fuel consumption functions that are used in the maritime literature. Finally in Chapter 5, we do several analyses and give the computational results.



## CHAPTER 2

### LITERATURE REVIEW

We classify the literature about this problem in three sections. In Section 2.1, we review the studies that consider minimizing fuel consumption by optimizing speeds. We also review the interrelated problems with speed optimization in this part. In Section 2.2, we review the fuel consumption functions that are mentioned or used in the maritime studies. In Section 2.3, we explain different kinds of uncertainties in the shipping industry and review the articles that consider uncertainty when modeling their problems.

#### 2.1 Speed optimization in maritime scheduling

Recently more attention has been devoted to reducing fuel consumption due to increase in the price of bunker fuel. According to Notteboom and Vernimmen [22], fuel consumption cost of a ship could be decreased by three main actions. The first one is using cheaper grades of bunker fuel. The second action could be taken in designing of a ship and the third one is regarded to speed of a ship. Since in this study we are considering minimization of fuel consumption of the ships by optimizing speeds, we will review the related articles in this field. There is usually a nonlinear relation between speed and fuel consumption; therefore, small changes in speeds cause larger changes in fuel consumption rate.

There is a comprehensive survey and taxonomy around speed models in maritime in the study of Psaraftis and Kontovas [23]. They have reviewed the related papers in this area and classified them according to various criteria. Important issues about

speed optimization are also studied in the work of Psaraftis and Kontovas [24]. They develop models that optimize speed of a ship for a spectrum of routing scenarios.

We should mention that since most of the derived fuel consumption functions are nonlinear, different attempts have been made for handling the nonlinearity of the functions in the maritime scheduling models. Fagerholt et al. [12] study optimizing speeds on each sailing leg with respect to time windows. They consider fixed ship route and homogeneous fleet in their study. For minimizing total fuel consumption cost along the sailing route, they present three models. In the first model, they consider speed as a primary decision variable and in the second model, they consider sailing time as a primary decision variable. In the third model, they discretize the arrival times and after that solve the model as a shortest path problem on an acyclic graph. In our model, similar to the second model of their work, we consider sailing time as a primary decision variable. However, we add other necessary constraints for the liner shipping service and service level constraints to the model. We also assume that heterogeneous fleet could be deployed on a single route, so we consider different fuel consumption functions in our model.

Hvattum et al. [13] determine optimal speeds along the sailing legs by considering nonlinear fuel consumption function. They consider fixed route and homogeneous fleet in their study. Their model is the same as the first model of the article Fagerholt et al. [12]. They consider time window constraints, speed limitation constraints and arrival time constraints in their model. They consider continuous and convex fuel consumption function and by considering that, they design a recursive algorithm for solving the model. The algorithm works in a way that at the first step, they relax time windows and calculate average speed according to the total distance and the total given voyage time. Then, they calculate arrival times at each port according to average speed that they obtained. For some ports, violation of time windows may be observed. According to maximum violation among all the ports, they fix the arrival time at that port and recalculate the speeds again. This procedure continues until feasible arrival times for all the ports are obtained. This simple algorithm could be applied to find the optimal speeds in order to satisfy the time windows.

Fagerholt et al. [12] and Hvattum et al. [13] consider a nonlinear fuel consumption



function when computing optimal speeds. However, they present general models that do not belong to a specific ocean shipping service (industrial shipping, tramp shipping and liner shipping). Wang et al. [32] consider speed optimization in designing a schedule for a liner service. They determine arrival and departure times of the ships, number of the deployed ships, berth to use at each port-of-call and speeds. They consider port and berth time windows in their model. They assume fixed route and homogeneous fleet in their problem. They formulate the problem as a mixed integer nonlinear non-convex model. For solving the model they develop a holistic solution approach. In this approach, they first relax port time windows, so the model changes to a mixed integer nonlinear model. Then, they apply piecewise linearization to linearize the model. After that, they repeatedly add the violated port time window constraints to this model until a feasible solution is obtained. Similar to this problem, we also design a schedule for the liner shipping service and we consider speeds and departure times as decision variables. Moreover, we consider port time uncertainties in designing a schedule and we assume that heterogeneous fleet could be deployed on the single route. However, we consider that numbers of the deployed ships are predetermined according to estimated amount of the demand and we do not consider berth allocation in our problem.

Wang and Meng [31] study speed optimization in a liner shipping network by considering a nonlinear fuel consumption function. They consider transshipment and container routing in their model. They decide on sailing speeds, number of deployed ships and number of containers routed on each route in order to fulfill the demand. Firstly, according to historical data, they calibrate the coefficients of fuel consumption function and determine the appropriate coefficients. Then, they develop a mixed-integer nonlinear programming model in terms of speed. In their model, there is nonlinearity in the objective function and also in one of the constraints. For solving the model, they intend to linearize the nonlinear parts. For handling the nonlinearity of the constraint, they use the reciprocal of sailing speed and consider it as a decision variable. They also consider convex and non-negative objective function and use outer-approximation method to approximate the objective function. They consider several fixed routes and single ship type on each route. Although our work differs with their work since we are designing a robust schedule, similarly to their work, we

also have a nonlinear objective function. But since we represent the model in terms of sailing time, all the constraints of the model are linear. Solution approach for handling the nonlinearity of the objective function in our study differs with their work. For handling the nonlinearity of the objective function, we reformulate the model by applying second order conic programming.

Yao et al. [34] study speed optimization jointly with bunkering port selection and bunkering amounts determination. These three decisions are important in the fuel management strategy. They first provide an empirical model to express the relation between fuel consumption and speed for different sizes of container ships. They formulate the model in a way that is able to make these three decisions simultaneously. They also highlight the importance of using the appropriate fuel consumption rate model in bunker fuel management strategy. We are studying a robust scheduling problem in our study, but similar to their work, we use different fuel consumption function for each ship type. However, they assume a homogeneous fleet on the single route, but we are considering heterogeneous fleet.

In the tramp shipping sector, we can refer to the work of Norstad et al. [20]. They consider speed optimization in applying ship routing and scheduling. They represent the formulation of the tramp ship routing and scheduling problem with speed optimization (TSRSPSO). For solving the model, they present a multi start local search heuristic. In each move of the local search for evaluating the move, they determine the optimal speeds. For finding the optimal speeds along the single route, they solve the speed optimization problem model (SOP). For solving the SOP problem, they apply the solution method of Fagerholt et al. [12]. In their method, arrival times are discretized. Rather than that method, they apply recursive algorithm. Recursive algorithm is also defined in the work of Hvattum et al. [13]. They make a comparison of these two methods. It is better to mention that along the single route, they consider homogeneous fleet.

Because of the environmental impacts of ships, speed optimization receives higher attention. Kontovas [15] studies green ship routing and scheduling problem. He clarifies that for considering emissions, three approaches could be taken. The first way is considering minimization of emissions as objective of the model. The second

one is internalizing the external cost of emissions in the objective. The third approach is adding a constraint to the model that limits the produced emissions.

We can indicate that in all of the aforementioned articles, homogeneous fleet is considered to be deployed on a single route, but in practice different ship types could be deployed on the single route. In our problem, we are considering heterogeneous fleet. Moreover by considering port time uncertainties, speeds of the ship types are optimized in a way to satisfy the overall service level of the service route. There are some difficulties around the heterogeneous fleet. Different ship types might have different fuel consumption functions, so their optimal sailing times could also be different. But common departure times at the ports should be achieved for all the ship types since the ports are visited at the same times every week. Different methods in the literature are used for solving the speed optimization problem. Discretizing arrival times, applying recursive algorithm and using a nonlinear programming solver could be mentioned as kinds of these methods for solving the SOP problem.

So far, we have reviewed the related problems that deal with optimizing speeds in order to minimize the total fuel consumption cost. In the next section, we review the fuel consumption functions that are used in the maritime literature.

## **2.2 Fuel consumption functions used in maritime literature**

Fuel consumption of a ship depends on a number of factors related to its size, speed, power plant and deadweight of a ship according to European Commission [11]. Water depth and weather condition also affect fuel consumption rate. According to fuel consumption data of ships, different fuel consumption functions with different coefficients are derived. We now summarize some of the available functions that are used or mentioned in the maritime literature. The functions are seen in Table 2.1.

Table 2.1: Fuel consumption functions used in maritime literature

---

**Barrass [6]**

---

$$*F(v) = \frac{W^{2/3}v^3}{F_c}$$

Notations:

- $F(v)$  : fuel consumption per day
  - $W$  : displacement of a ship in tones
  - $F_c$  : fuel coefficient that is dependent on the installed machinery in the ship
  - $F_c \approx 110000$  for Steam Turbine machinery  
 $F_c \approx 120000$  for Diesel machinery installation
  - Displacement is lightweight (lwt) plus deadweight (dwt).  
 The lightweight is the weight of the ship itself, when it is completely empty.  
 The deadweight is the weight that a ship carries.
- 

**Psaraftis and Kontovas [23] and Kontovas [15]**

---

$$*F(v) = A + Bv_{ij}^n$$

$$*F(v) \propto (W_{ij} + L)^{2/3}$$

$$*F(v) = (A + Bv_{ij}^n)(W_{ij} + L)^{2/3}$$

Notations:

- $F(v)$  : fuel consumption per day
  - $L$  : weight of the ship when it is empty plus consumables and fuel
  - $W_{ij}$  : payload from i to j
  - $A \geq 0, B > 0$  and  $n \geq 3$
  - These papers mention that  $n=3$  is a good approximation for tankers and bulk carriers, but it may not be a good approximation for some ship types.  
 For container ships exponent can be 4, 5 or even higher.
- 

**Schradly et al. [27]**

---

$$*F(v) = c_0 + c_1v + c_2v^2 + c_3v^3$$

$$*F(v) = p_0 + p_1e^{p_2v^3}$$

Notations:

- $F(v)$  : fuel use in gallons per hour
- 

**Kowalski [16] and Wang and Meng [31]**

---

$$*F(v) = av^b + \epsilon$$

Notations:

- $F(v)$  : daily fuel consumption of the main engine
- $\epsilon$  : the error term of power regression function
- $b$  : a parameter in the range [3,4].

---

**Mulder et al. [18] and Dun et al. [9]**

---

$$*F(v) = F^d \times \left(\frac{v}{v^d}\right)^3$$

Notations:

- $F(v)$  : actual fuel consumption rate at metrics tons per hour
- $F^d$  : designed fuel consumption
- $v^d$  : designed speed

---

**Wang and Meng [30]**

---

$$*F(v) = av^2$$

Notations:

- $F(v)$  : fuel consumption per nautical mile
- They randomly generated coefficient  $a$  in the range  $[0.02/24, 0.03/24]$ .

---

**Fagerholt et al. [12] and Norstad et al. [20]**

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$$*F(v) = 0.0036v^2 - 0.1015v + 0.8848$$

Notations:

- $F(v)$  : fuel consumption per nautical mile
- It is valid for the speed range  $[14, 20]$ .

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**Yao et al. [34]**

---

$$*F(v) = k_1v^3 + k_2$$

Notations:

- $F(v)$  : fuel consumption rate per day
- This article has obtained different values for the coefficients  $k_1$  and  $k_2$  according to different sizes of container ships.

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**Karlsson and Eriksson [14]**

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$$*F(v) = ae^{(bv^2+cv)}$$

$$*F(v) = ae^{(bv^3+cv^2+dv)} + E$$

$$*F(v) = ae^{(bv^3+cv^2+dv)} + kv + m$$

Notations:

- $F(v)$  : fuel consumption per day
- It has been studied on reefer vessels.

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**Du et al. [8]**

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$$*F(v) = c_0 + c_1.v^\mu$$

Notations:

- $F(v)$  : fuel consumption per unit time
- $\mu = 3.5$  (for feeder container ships)
- $\mu = 4$  (for medium-sized container ships)
- $\mu = 4.5$  (for jumbo container ships)

---

Note: In all the functions,  $v$  is the speed of a ship that is measured in knots (nautical miles/hour).

We can clarify that in most of the studies, fuel consumption is considered to be as a function of speed only. However, some articles have derived functions that are dependent on speed and displacement of a ship. In general, power functions and exponential functions are used in the literature. Furthermore, most studies approximate fuel consumption per day as a cubic function of speed. However, Psaraftis and Kontovas [23] indicate that this approximation is good for tankers and bulk carries, but it may not be good for container ships. They mention that for these ships, exponent 4, 5 or even higher could be considered.

In our problem, we use the function of the article Yao et al. [34]. Because we consider a heterogeneous fleet on a single route, we need to have a function with different coefficients for different container ship types. In their work, different coefficients are derived for different container ship sizes. Since we are solving our model in terms of sailing time, this function with positive coefficients is a convex function in terms of sailing time. But other bunker functions could also be used by making necessary changes. For handling the nonlinearity of the fuel consumption function, we reformulate our model as a SOCP problem. In Chapter 4, we also give SOCP representations of other fuel consumption functions that we have reviewed in this section.

### 2.3 Uncertainties in shipping operation

Shipping, like other transportation modes, encounters different types of uncertainties. Fluctuations in demand, port operations and sailing time could be mentioned as kinds of uncertainties in this sector. When shipping companies plan a fleet or design a schedule, they attempt to consider demand and port operations disruptions.

Demand uncertainty is considered in the study of Meng and Wang [17]. They deal with liner ship fleet planning problem. For handling demand disruption, they develop a chance constrained model and apply distribution based approach. They assume that demand between any two ports of the route, follows normal distribution. Their model is able to determine fleet size and mix, ship to route assignment and route service frequency. Wang et al. [33] also consider demand uncertainty in their work. For handling demand uncertainty, they develop a joint chance constrained model and use sample average approximation method in solving their problem.

We can also mention that demand fluctuations and imbalanced flows between seaports might also affect other decisions. As an example they necessitate dynamic asset management. Erera et al. [10] study asset management problem for the tank container operators. They consider routing and reposition decisions jointly in their model.

There are also uncertainties around port operations and also during the sailing times of the ships. These uncertainties could affect the schedules. Most studies consider these kinds of uncertainties in designing their schedules. Christiansen and Fagerholt [7], deal with determining a robust schedule for each ship in the fleet. They clarify that because of bad weather and unpredictable service times at the ports, ship scheduling is associated with a high degree of uncertainty. They use set partitioning approach for solving the problem. At the first step, they generate all the feasible schedules. For measuring the degree of reliability of a schedule, they assign penalty cost for each ship-schedule pair. They further bring feasible schedules into the set partitioning model and solve it in order to minimize the sum of operating costs, penalty costs and spot costs. Although they design a robust schedule in their study, they consider operating with fixed speed. They do not determine optimal speeds according to fuel consumption function in their problem.

Wang and Meng [30] deal with designing a robust schedule for a liner ship service. They consider fluctuations of waiting times of ships and handling times of cargoes in their study. They formulate the problem as a mixed-integer nonlinear stochastic programming model. For solving the model, they apply sample average approximation method and adopt several linearization techniques to linearize the nonlinear constraints and the nonlinear objective function. For improving the computational efficiency, they also propose a decomposition scheme. By considering the trade-off between delay cost and total cost (including fuel consumption cost), buffer times are assigned for each port. They consider homogeneous fleet on the route and they also determine the optimal number of the deployed ships.

Similar to their work, we also consider uncertainties of the handling and waiting times in designing a schedule. However in our model, we consider a heterogeneous fleet on the single route and we assign buffer times for each port-ship type pair. Moreover, buffer times are assigned in a way to satisfy the overall service level of the service route. We also assume that according to estimated amount of the demand during the voyage time, number of the deployed ships of each type is predetermined, so the voyage should be completed in a predetermined duration. Since the model could be solved in a reasonable time, as a trial, different numbers of the ship types could be inserted in the model. This gives us an opportunity to analyze the effects of the number of the deployed ships on total fuel consumption cost and optimal solutions. It also gives information about the feasibility of a schedule.

Mulder et al. [18] consider fixed schedule and determine an optimal recovery policy for which the total associated costs to delays and recovery actions are minimized. They use Markov decision process to formulate the problem. The states of the Markov process denote the ship's position and the amount of delay with compare to the primal schedule. In addition since a finite number of possible states are needed in the Markov process, they discretize delay. In each state of the Markov process, a decision is made about which recovery action to take in that state. Recovery actions such as increasing or decreasing the sailing speed are considered. The transition probability of the current state to any other state depends on the current delay of that state. They propose a mixed integer programming formulation and two heuristic methods to solve the problem. For small problems, the mixed integer programming model could be



solved by appropriate software. However, for larger instances, the computational time increases exponentially, so they also present two heuristic methods to solve the model.

Wang and Meng [29] consider uncertainty of port time (pilotage and container handling time) in designing a schedule for liner ship service so as to minimize the total fuel consumption and operating cost. They consider weekly frequency, several routes and homogeneous fleet on each route. For hedging against uncertainty, they consider sea contingency time on each leg. They consider sea contingency to be proportional to the distance of the voyage that is remained (residual voyage distance). For solving the problem, they first develop optimal speed problem for finding the optimal speed function. Then, they develop a mixed integer nonlinear convex stochastic problem and approximate the objective function by applying piecewise linearization. For improving computational burden, they apply cutting plane algorithm to use small subset of line segments. They also consider number of the deployed ships as a decision variable.

Therefore, in their study, they consider having more buffer times at the beginning of the voyage and as the ship approaches to its destination, they consider having less buffer times. But our model is more flexible in assigning buffer times. In our model, buffer times are assigned by considering the importance of the ports. Different ports are considered to have different degree of importance in terms of having on-time departure times. The scheduler could manage the buffer times at the ports by changing the importance degree of the ports. So if he wishes to have more buffer times for the prior ports of the voyage, he can do it by raising the importance degree for those ports.

Port uncertainty is also considered in the work of Qi and Song [25]. They design a schedule for a liner shipping service so as to minimize the total fuel consumption along the voyage. They assume weekly frequency and homogeneous fleet in their problem. They first, develop a model for determining optimal transit times. Since their developed model is difficult to solve, they classify the problem in three cases. In the first case, they consider deterministic port times. In the second case, they consider stochastic port times and on-time arrival times at all the ports. In the third case, they consider to have stochastic port times and also to have delay in arrival times of ships

at some ports. They solve the first and second case according to the propositions that they present. For solving the third case, they use simulation based stochastic approximation method.

However, all of the aforementioned articles about robust scheduling in the liner shipping consider that homogeneous fleet of ships is deployed on the single route. But in practice, different ship types in terms of having different fuel consumption functions could be deployed on a single route. Difference of the ports in terms of their importance for having on-time departures is also not considered in the literature. Differently from the literature, in this study, we consider different fuel consumption functions, distinct weights for the ports and a new service level measure.

In our study for handling the nonlinearity of the objective function, we give the SOCP representation of the model. Then, we solve the reformulated model by CPLEX. This software is able to solve SOCP constraints. In the next chapter, we give a comprehensive statement of the problem, assumptions and the mathematical model of the problem.

## CHAPTER 3

### PROBLEM DEFINITION

According to World Shipping Council [1], “Liner shipping is the service of transporting goods by means of high-capacity, ocean-going ships that transit regular routes on fixed schedules and there are approximately 400 liner services in operation today.” A service is a sequence of ports that performs a round trip. In liner shipping, fleet of ships visits ports according to a predetermined frequency. In most of the studies, deployed fleet on the single route is assumed to be homogeneous; however it might be heterogeneous in practice.

In this study, we assume that heterogeneous fleet of container ships could be deployed on a single route. Heterogeneity of a fleet might be due to the difference between capacities of the ships. However, differences in engine characteristics and physical parameters of ships result in different fuel consumption function for each ship type. Here, we assume that each ship type in the fleet can have different bunker consumption function. We assume that fuel consumption of a ship is related only to speed. There is a nonlinear relation between fuel consumption rate and sailing speed of a ship. We also assume that the numbers of the deployed ships of each type on the route is predetermined according to the estimated cargo shipment demand.

Liner shipping companies mostly provide weekly regular services. In this study, we also consider weekly service frequency. Thus, the number of the total deployed ships on the route is equal to the round-trip journey time in weeks. This means that each port-of-call on the sailing route is visited on the same day of the week. Departure time at each port is predetermined and is announced by the shipping company.

In this thesis, we study the problem of designing a schedule for a heterogeneous fleet of the liner shipping service by considering the port time uncertainties. The objective is to minimize total fuel consumption cost during the round-trip journey time. However, overall service level of the service route should be satisfied. The service level under consideration measures schedule uncertainty. The uncertainties that we consider in this study are related to fluctuations of waiting and handling times.

Handling time refers to the time that is needed for loading and unloading cargoes at a port. Waiting time for a ship is a duration that a ship has to wait after arriving at a port. For the major ports, waiting time might be higher because of the higher congestion at the port. We assume that handling and waiting times for different ship types at the same ports are equal, but they might be different at distinct ports. For characterizing the uncertainty issue, we consider distribution based approach and in our model we assume that handling and waiting times at the ports of the sailing route follow normal distribution.

Our primary decision variables are sailing time of each ship type on each leg and departure times at the ports. As we are also considering port time uncertainties, in addition to sailing times and departure times, we determine service levels. We measure service level as the probability of on-time departure of a ship at a port. The determined service levels should satisfy the overall service level of the service route. Overall service level could be defined for each route by the shipping company as a level of the reliability of the schedule.

In addition, buffer times and speeds could be computed. Buffer time is a duration that is assigned in the schedule in order to overcome the uncertainties. We clarify the problem in Figure 3.1. As an example, in Figure 3.1, there are three ports on a closed service route. Each ship visits the first port after finishing the round-trip journey. The difference between departure times of the two sequential ports gives the summation of the sailing time, buffer time and the mean of handling and waiting times. As buffer time increases, sailing time of a ship decreases. In other words, speed of a ship increases. This results in more fuel consumption. In our problem, we determine service levels, departure times and sailing times in a way that fuel consumption cost is minimized during the journey.

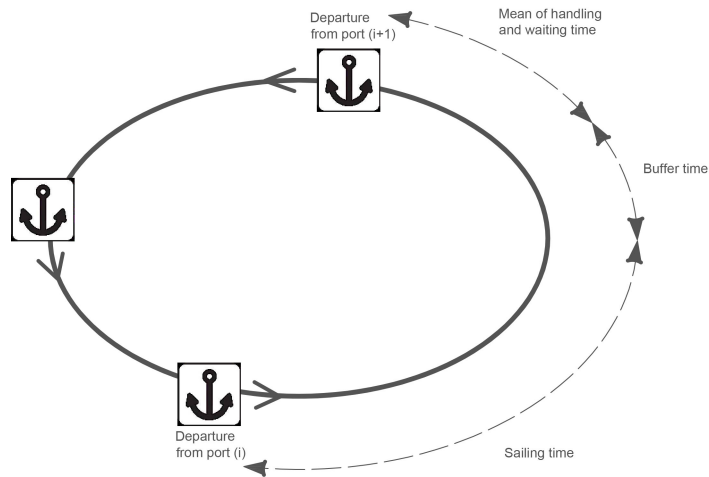


Figure 3.1: Illustration of the problem

Service levels are determined for each port-ship type pair. The reason that the service levels might be different between the ship types is the difference between fuel consumption functions of the ship types. The reason that service levels might be different between the ports is the differences between the ports. Mean and variance of handling and waiting times might be different between ports. In addition, each port might have different degree of importance for the operator of liner shipping company. At major ports or the ports with higher demand, deviation from published schedule poses more loss to the shipping company. To take into account of this fact, we define different weight for each port in our model. As another approach, service levels for the port-ship type pairs could be determined as fixed values by a shipping company. However, we determine optimal service levels according to the developed model in order to minimize the total cost during the journey.

Since service levels could be different for the port-ship type pairs, sailing times and buffer times could also be different between ship types on the sailing legs. However, overall service level of the service route should be satisfied and common departure time for the different ship types should be determined at each port.

### 3.1 Mathematical model

In this section, we present the mathematical formulation of the problem. We first give the notations below:

**Sets :**

$R$  : set of ship types;  $r \in R$  represents a ship type

$\Gamma$  : set of port-of-calls;  $i \in \Gamma$  represents a port number

$L$  : set of possible values for delay probabilities;  $l \in L$  represents a delay probability

**Indices and parameters :**

$I$  : the total number of the port-of-calls on the route

$N$  : the total number of the ships on the route

$c_{opt}^r$  : the operating cost per hour for a ship of type  $r$

$v_{min}^r$  : the minimum speed for a ship of type  $r$  (in knots)

$v_{max}^r$  : the maximum speed for a ship of type  $r$  (in knots)

$n_r$  : the number of the deployed ships of type  $r$  on the route

$l_{ij}$  : the ocean distance between the  $i$ th port-of-call and the  $j$ th port-of-call  
(in nautical miles)

$\tilde{w}_i$  : the random waiting time at the  $i$ th port-of-call with parameters  $\mu_i^w$  and  $\sigma_i^w$

$\tilde{h}_i$  : the random handling time at the  $i$ th port-of-call with parameters  $\mu_i^h$  and  $\sigma_i^h$

$p_{fuel}$  : the bunker price (USD/ton)

$W_i$  : the weight of the  $i$ th port-of-call

$1 - \alpha_i^{lr}$  : service level for a ship of type  $r$  at the  $i$ th port-of-call for the delay probability of  $l$

$1 - \beta$  : the overall service level

**Decision variables :**

$d_i$  = published departure time at the  $i$ th port-of-call  $\forall i \in \Gamma$

$s_{ij}^r$  = sailing time of a ship of type  $r$  between the  $i$ th port-of-call and the  $j$ th port-of-call  
 $\forall i, j \in \Gamma$

$y_i^{lr}$  =  $\begin{cases} 1 & \text{if service level } (1 - \alpha_i^{lr}) \text{ is selected for a ship of type } r \text{ at the } i\text{th port-of-call} \\ 0 & \text{otherwise} \end{cases}$

Model 1:

$$\min : \sum_{i \in \Gamma} \sum_{r \in R} f_{i,i+1}^r(s_{i,i+1}^r) l_{i,i+1} p_{fuel} n_r \quad (3.1)$$

Subject to:

$$\Pr \left( d_{i+1} \leq d_i + s_{i,i+1}^r + \tilde{w}_{i+1} + \tilde{h}_{i+1} \right) \leq \sum_{l \in L} \alpha_{i+1}^{lr} y_{i+1}^{lr} \quad \forall i \in \Gamma, \forall r \in R \quad (3.2)$$

$$l_{i,i+1}/v_{max}^r \leq s_{i,i+1}^r \leq l_{i,i+1}/v_{min}^r \quad \forall i \in \Gamma, \forall r \in R \quad (3.3)$$

$$d_{I+1} - d_1 = 168 \sum_{r \in R} n_r \quad (3.4)$$

$$\sum_{i \in \Gamma} \sum_{l \in L} \sum_{r \in R} W_i (n_r/N) \alpha_i^{lr} y_i^{lr} \leq \beta \quad (3.5)$$

$$\sum_{l \in L} y_i^{lr} = 1 \quad \forall i \in \Gamma, \forall r \in R \quad (3.6)$$

$$d_1 = 0 \quad (3.7)$$

$$s_{i,i+1}^r, d_{i+1} \geq 0, y_i^{lr} \in \{0, 1\} \quad \forall i \in \Gamma, \forall r \in R \quad (3.8)$$

In the objective function of the model, the term  $f_{i,i+1}^r(s_{i,i+1}^r)$  is fuel consumption function of a ship of type  $r$  on the leg  $(i, i + 1)$  and is represented in tons per nautical mile. We also assume that the number of deployed ships on the sailing route is predetermined, so we omit operating costs of the ships during the voyage in the objective function since it takes a fixed value equal to  $\sum_{r \in R} c_{opt}^r (d_{(I+1)} - d_{(1)}) n_r$ .

Constraint (3.2) ensures that by considering uncertainties of handling and waiting times at the port, the probability of delay at the port  $(i + 1)$  for a ship of type  $r$  is at most equal to value of  $(\alpha_{i+1}^{lr})$ . In other words, the probability of on-time departure at the port  $(i + 1)$  for a ship of type  $r$ , is at least equal to the service level of a ship of type  $r$  at the port  $(i + 1)$ . Constraint (3.3) satisfies the sailing time limitation on each leg of the sailing route according to minimum and maximum speed of a ship. Constraint (3.4) ensures that the schedule is able to satisfy the weekly frequency. Constraint (3.5) is necessary for satisfying the overall service level of the service route. Weights of the ports are also considered in this constraint. Constraint (3.6) guarantees that exactly

one service level is selected for each port-ship type pair and constraint (3.7) assumes that departure time from port (1) is zero. Index  $(I + 1)$  in this problem, refers to the port (1) after finishing a round-trip journey.

Before solving the model, we need to make some changes. In the next chapter, we first linearize the first constraint of the model. We assume that handling and waiting times follow normal distribution and by considering that, we transform the chance constraint in a closed form after some computations. Also the objective function of the model contains nonlinear term. For handling the nonlinearity of the objective function, we represent the model as a SOCP problem. Then, the model could be solved by commercial software such as CPLEX.



## CHAPTER 4

### REFORMULATION OF THE MODEL

Fuel consumption function that we use in this study is convex and dependent on speed of a ship. There is a nonlinear relation between fuel consumption and speed of a ship. In this chapter, we first give a linearized form of the chance constraint. However, the objective function of the model is nonlinear and since model is written in terms of sailing time, all the constraints are linear. For handling the nonlinearity of the objective function of the model, we reformulate the model by applying second order conic programming. We also give SOCP representations of some of the other fuel consumption functions that are mentioned in the literature.

#### 4.1 Linearization of the chance constraint

We assume that handling and waiting times obey normal distribution. We define a set for delay probability values which includes discrete points. The decision variable  $y_i^{lr}$  is a binary variable that is equal to one if service level  $(1 - \alpha_i^{lr})$  is selected for a ship of type  $r$  at the port  $i$ . After some computations, constraint (3.2) can be written as following (The notations were explained in Chapter 3):

$$d_i + s_{i,i+1}^r + \sum_{l \in L} \phi^{-1} (1 - \alpha_{i+1}^{lr}) y_{i+1}^{lr} \cdot \sqrt{(\sigma_{i+1}^w)^2 + (\sigma_{i+1}^h)^2} + (\mu_{i+1}^w + \mu_{i+1}^h) \leq d_{i+1},$$
$$\forall i \in \Gamma, \forall r \in R \quad (4.1)$$

We can mention that if the model was presented in terms of speed, nonlinearity in the

constraint (4.1) would arise. We will discuss it later. In the next section, we propose the SOCP representation of the model.

## 4.2 Second order conic programming (SOCP) representation of the model

In second order conic programming, a linear function is minimized over the intersection of an affine set and the product of second-order (quadratic) cones. SOCPs are nonlinear convex problems that include linear and (convex) quadratic programs as special cases (for more information see Taly and Nemirovski [28] and Alizadeh and Goldfarb [5]). For dealing with a nonlinear convex term of the objective function of the model, we reformulate the model as a SOCP problem in this section.

As mentioned before, we assume that heterogeneous fleet of container ships could be deployed on a single route. Therefore, we will use a different fuel consumption function for each ship type in our problem. The fuel consumption function that we use in our study is given by Yao et al. [34]. In their work, different coefficients for the fuel consumption function have been obtained for different types of the container ships. The function is in the form  $f(v_{ij}^r) = p_r (v_{ij}^r)^3 + q_r$  and is represented in tons per day. Therefore, fuel consumption in tons per nautical mile could be represented as

$$f(v_{ij}^r) = \frac{p_r}{24} (v_{ij}^r)^2 + \frac{q_r}{24v_{ij}^r}$$

Since  $l_{ij} = v_{ij}^r s_{ij}^r$ , fuel consumption as a function of sailing time can be reformulated as  $f(s_{ij}^r) = \frac{p_r l_{ij}^2}{24} (1/s_{ij}^r)^2 + \frac{q_r}{24l_{ij}} (s_{ij}^r)$ . After applying these changes, the objective function of the model will be as follows:

$$\min : \sum_{i \in \Gamma} \sum_{r \in R} \left( a_r (l_{i,i+1})^3 (1/s_{i,i+1}^r)^2 + b_r (s_{i,i+1}^r) \right) p_{fuel} n_r \quad (4.2)$$

For linearizing the objective function of the model, we add auxiliary variable  $t_{i,i+1}^r$ . After adding it, the objective function will change to:

$$\min : \sum_{i \in \Gamma} \sum_{r \in R} \left( a_r (l_{i,i+1})^3 t_{i,i+1}^r + b_r (s_{i,i+1}^r) \right) p_{fuel} n_r \quad (4.3)$$

and so the following constraint should be added to the model:

$$\frac{1}{(s_{i,i+1}^r)^2} \leq t_{i,i+1}^r \quad (4.4)$$

But before adding this constraint it should be represented as SOCP constraints, so first we have to transform this constraint as hyperbolic constraints. In general, hyperbolic constraints are represented as:

$$w^2 \leq xy, x \geq 0, y \geq 0 \quad (4.5)$$

and when  $w$  is vector it can be written as:

$$w^T w \leq xy, x \geq 0, y \geq 0 \quad (4.6)$$

Then, SOCP constraints can be represented as follows:

$$\left\| \begin{bmatrix} 2w \\ x - y \end{bmatrix} \right\| \leq x + y \quad (4.7)$$

By defining new variable  $h_{i,i+1}^r$ , Inequality (4.4) can be written as:

$$(h_{i,i+1}^r)^2 \leq t_{i,i+1}^r \quad (4.8)$$

$$1 \leq s_{i,i+1}^r h_{i,i+1}^r \quad (4.9)$$

So SOCP constraints are as follows:

$$\left\| \begin{bmatrix} 2h_{i,i+1}^r \\ t_{i,i+1}^r - 1 \end{bmatrix} \right\| \leq t_{i,i+1}^r + 1 \quad (4.10)$$

$$\left\| \begin{bmatrix} 2 \\ s_{i,i+1}^r - h_{i,i+1}^r \end{bmatrix} \right\| \leq s_{i,i+1}^r + h_{i,i+1}^r \quad (4.11)$$

After adding constraints (4.10) and (4.11), the model will be reformulated as following:

$$\min : \sum_{i \in \Gamma} \sum_{r \in R} (a_r (l_{i,i+1})^3 t_{i,i+1}^r + b_r (s_{i,i+1}^r)) p_{fuel} n_r$$

subject to:

$$4(h_{i,i+1}^r)^2 + (p_{i,i+1}^r)^2 \leq (g_{i,i+1}^r)^2 \quad (4.12)$$

$$4 + (q_{i,i+1}^r)^2 \leq (z_{i,i+1}^r)^2 \quad (4.13)$$

$$(t_{i,i+1}^r - 1) = p_{i,i+1}^r \quad (4.14)$$

$$(t_{i,i+1}^r + 1) = g_{i,i+1}^r \quad (4.15)$$

$$s_{i,i+1}^r - h_{i,i+1}^r = q_{i,i+1}^r \quad (4.16)$$

$$s_{i,i+1}^r + h_{i,i+1}^r = z_{i,i+1}^r \quad (4.17)$$

$$g_{i,i+1}^r, z_{i,i+1}^r \geq 0, \quad p_{i,i+1}^r, q_{i,i+1}^r \text{ free} \quad (4.18)$$

and constraints (3.3) – (3.8) , (4.1)

### 4.3 SOCP representations of other fuel consumption functions used in the maritime literature

In this section, we give SOCP representations of fuel burn functions given in the Section 2.2. The use of SOCP for the fuel consumption function is seen in the work of Du et al. [8]. In general, power functions and exponential functions are seen in the literature. For the exponent of the power functions, different values are used. We first give SOCP representations for the power functions and then for the exponential functions.

#### 4.3.1 Power functions

The power function is in the form

$$F(v) = cv^{a/b}$$

Since we have proposed the model in terms of sailing time, we reformulate the functions of the Section 2.2 in terms of sailing time. When power function is represented in terms of sailing time ( $s$ ), it changes to  $c\left(\frac{\text{distance}}{s}\right)^{a/b}$ . Power functions with positive coefficients are convex and SOCP representable. However, for the function that is used in Fagerholt et al. [12] or Norstad et al. [20], if we represent it in terms of sailing time, the second term of the function will not be convex. Therefore, if we wish to use that function, we can reformulate the model in terms of speed to be able to use that function. In Section 4.3.3, we present the model in terms of speed.

By adding auxiliary variable  $t$ , we can write the power function in terms of sailing time as  $\left(\frac{1}{s^{a/b}}\right) \leq t$  (for now, we can omit coefficient  $c$  in the computations since it takes constant value). Then:

$$1 \leq t^b s^a \quad (4.19)$$

According to Alizadeh and Goldfarb [5] and Taly and Nemirovski [28], inequality (4.19) could be represented as

$$y^{2^l} \leq s_1 s_2 \dots s_{2^l}, \quad y, s_1, \dots, s_{2^l} \geq 0 \quad (4.20)$$

inequality (4.20) could be expressed by  $2^{l-1}$  inequalities of the form  $w_i^2 \leq u_i v_i$  where  $w_i, u_i, v_i \geq 0$ . Therefore, by reformulating the power functions we can represent them as SOCP.

As an example, we can show it for  $b = 2.5$ .  $\left(\frac{1}{s^{2.5}} \leq t, s, t \geq 0\right)$  could be written as  $(1^8 \leq t^2 \cdot s^5, s, t \geq 0)$ . Therefore, it can be expressed by the following hyperbolic inequalities and we can represent them as SOCP constraints.

$$w_1^2 \leq s, w_2^2 \leq w_1 t, 1 \leq w_2 s, w_1, w_2 \geq 0$$

The general SOCP representations for power functions are also explained in the work of Aktürk et al. [4].

### 4.3.2 Exponential functions

We can represent the exponential function as a SOCP problem. We can do it by approximating the exponential functions with power series. This method is mentioned

in Nemirovski [19]. According to Nemirovski [19], exponential function could be approximated as following:

For every  $p \geq 1$ ,

$$\exp(x) = \lim_{r \rightarrow \infty} \left( 1 + \frac{x}{2^r} + \frac{1}{2} \left( \frac{x}{2^r} \right)^2 + \dots + \frac{1}{p!} \left( \frac{x}{2^r} \right)^p \right)^{2^r} \quad (4.21)$$

We can simplify expression (4.21) as

$$\exp(x) = \lim_{r \rightarrow \infty} (1 + c_1 x + c_2 x^2 + \dots + c_p x^p)^{2^r} \quad (4.22)$$

It is observed from expression (4.22) that all the terms are in the form of power function. So each term could be represented as a SOCP in a way that we explained before. Therefore, exponential functions could also be represented as SOCP by using this approximation.

### 4.3.3 Representing the model in terms of speed

As mentioned before, we can also present the model in terms of speed. The model in terms of speed would be as following:

Model 2:

$$\min : \sum_{i \in \Gamma} \sum_{r \in R} f_{i,i+1}^r(v_{i,i+1}^r) l_{i,i+1} p_{fuel} n_r \quad (4.23)$$

Subject to:

$$d_i + (l_{i,i+1}/v_{i,i+1}^r) + \sum_l \phi^{-1} (1 - \alpha_{i+1}^l) y_{i+1}^{lr} \cdot \sqrt{(\sigma_{i+1}^w)^2 + (\sigma_{i+1}^h)^2} + (\mu_{i+1}^w + \mu_{i+1}^h) \leq d_{i+1}$$

$$\forall i \in \Gamma, \forall r \in R \quad (4.24)$$

$$v_{min}^r \leq v_{i,i+1}^r \leq v_{max}^r \quad \forall i \in \Gamma, \forall r \in R \quad (4.25)$$

$$v_{i,i+1}^r, d_{i+1} \geq 0, y_i^{lr} \in \{0, 1\} \quad \forall i \in \Gamma, \forall r \in R \quad (4.26)$$

$$\text{and constraints (3.4) – (3.7)} \quad (4.27)$$

By observing Model 2 it is seen that constraint (4.24) has become nonlinear. However, we can also represent SOCP for that constraint. We can write it as  $\frac{1}{v_{i,i+1}^r} \leq t_{i,i+1}^r$ ,

which holds tight at the optimality. So, the following SOCP constraints would be added to the model.

$$2^2 + a_{i,i+1}^r{}^2 \leq b_{i,i+1}^r{}^2, \quad a_{i,i+1}^r = t_{i,i+1}^r - v_{i,i+1}^r, \quad b_{i,i+1}^r = t_{i,i+1}^r + v_{i,i+1}^r, \quad t_{i,i+1}^r, v_{i,i+1}^r \geq 0$$





## CHAPTER 5

### COMPUTATIONAL STUDY

In this chapter, we do several analyses and report the computational results. For evaluating the model, we first define different service routes. For each route, we specify the number of deployed ships. Since we did not have statistical data, we used some trial and error to determine the number of deployed ships. We determined the number of total deployed ships on each route in a way to obtain a feasible schedule in that period. If few ships are deployed on the route, the schedule will be infeasible since ships have speed limitations and cannot sail at higher speeds than their speed limits. We also assume that combination of container ships could be deployed on a single route.

As mentioned before, fuel consumption function that we consider in our study is in the form  $f(v_{ij}^r) = a_r(v_{ij}^r)^3 + b_r$  and is calculated in tons per day. The function in terms of sailing time and tons per nautical mile would be as following:  $f(s_{ij}^r) = \frac{a_r l_{ij}^2}{24} (1/s_{ij}^r)^2 + \frac{b_r}{24 l_{ij}} (s_{ij}^r)$ . We take the values of the coefficients  $a_r$  and  $b_r$  for different container ships from the article Yao et al. [34]. We consider having two different ship types in the fleet on each route. We present properties of the ship types in Table 5.1. Speeds ( $v$ ) are expressed in knots (nautical miles/hour).

Table 5.1: Properties of the container ships

| Ship type | Speed range | Bunker fuel consumption model (tons per mile) |
|-----------|-------------|---|
| A         | [13.5,21]   | $(0.000188)v^2 + (1.22/v)$                    |
| B         | [15,24]     | $(0.000281)v^2 + (2.33/v)$                    |

We consider service routes: Europe East Asia trade route (AE), Asia Europe Express

(AEX) and Atlantic Pacific Express (APX). Routes AEX and APX are taken from Yao et al. [34] and route AE is defined in Notteboom and Vernimmen [21]. Parameters of the service routes are seen in Table 5.2. In this study by the word port, we refer to the port-of-call since some ports might be revisited during the journey on each route.

Table 5.2: Parameters of the service routes

|       | Parameter | Value                                 |                           |
|-------|-----------|---------------------------------------|---------------------------|
| Route | AE        | number of port-of-calls               | 10                        |
|       |           | service frequency                     | weekly                    |
|       |           | number of deployed ships on the route | (type A: 4 and type B: 4) |
|       | AEX       | number of port-of-calls               | 15                        |
|       |           | service frequency                     | weekly                    |
|       |           | number of deployed ships on the route | (type A: 5 and type B: 4) |
|       | APX       | number of port-of-calls               | 24                        |
|       |           | service frequency                     | weekly                    |
|       |           | number of deployed ships on the route | (type A: 6 and type B: 6) |

The distances between the ports are shown in Table 5.3. We used online calculator of Port World [2] to determine the distances between ports of the service routes AEX and APX. Other port distance calculators and applications are also available. The distances between ports of the service route AE is given in the study of Notteboom and Vernimmen [21].

Table 5.3: Distances for the service routes AE, AEX and APX

| APX Route    |           | AEX Route  |           | AE Route  |           |
|--------------|-----------|------------|-----------|-----------|-----------|
| Port         | Dist (nm) | Port       | Dist (nm) | Port      | Dist (nm) |
| Chiwan       | 25        | Hakata     | 152       | Shanghai  | 576       |
| Hong Kong    | 343       | Kwangyang  | 72        | Dalian    | 280       |
| Kaohsiung    | 898       | Pusan      | 445       | Qingdao   | 512       |
| Busan        | 344       | Shanghai   | 554       | Ningbo    | 2143      |
| Kobe         | 353       | Kaohsiung  | 343       | Singapore | 8353      |
| Tokyo        | 7695      | Hong kong  | 34        | Rotterdam | 318       |
| Balboa       | 47        | Yantian    | 1444      | Hamburg   | 401       |
| Manzanillo   | 1157      | Singapore  | 8670      | Antwerp   | 8343      |
| Miami        | 300       | Rotterdam  | 267       | Singapore | 1435      |
| Jacksonvilla | 109       | Hamburg    | 383       | Hong Kong | 875       |
| Savannah     | 86        | Thamesport | 7051      |           |           |
| Charleston   | 594       | Colombo    | 1552      |           |           |
| New York     | 3304      | Singapore  | 1420      |           |           |
| Rotterdam    | 212       | Hong Kong  | 343       |           |           |
| Bremerhaven  | 300       | Kaohsiung  | 878       |           |           |
| Felixstowe   | 3236      |            |           |           |           |
| New York     | 261       |            |           |           |           |
| Norfolk      | 367       |            |           |           |           |
| Charleston   | 1556      |            |           |           |           |
| Manzanillo   | 2937      |            |           |           |           |
| San Pedro    | 347       |            |           |           |           |
| Oakland      | 4563      |            |           |           |           |
| Tokyo        | 353       |            |           |           |           |
| Kobe         | 1389      |            |           |           |           |

Note: Dist (nm), means the distance to the next port-of-call that is expressed in nautical miles

We also defined distinct weight for each port. Different factors could be considered by the scheduler in defining the weights for the ports and it might be dependent to the shipping company. In this study, we defined the weights for the ports according to congestion of the ports. For taking into account the congestion factor, we used an approximation. We considered numbers of the arrivals and the expected arrivals of the vessels at the ports at a specific time according to data of [3]. By considering that data, we approximated the weights of the ports. We divided summation of the arrivals and expected arrivals of the vessels at each port of the route to the total summation of the arrivals and expected arrivals of all the ports on the route. The weights of the

ports for the service routes are shown in Appendix A.

We also define a set that contains of possible delay probabilities. We define the set as  $\{0.01 \times i : i \text{ is an integer, } 1 \leq i \leq 40\}$  and also, we add a small number of 0.005 to the set. We assume that this set is same for all the routes. We also assume that delay probability values for all the ports of the routes could be chosen from this set.

After determining the service routes, weights of the ports and the ship types, we now consider the important parameters that can have significant effects on the results and lead to different solutions. These factors are overall service level ( $1 - \beta$ ), handling time of cargoes and waiting time of a ship. As mentioned before, uncertainties that we consider in this study are due to fluctuations of handling and waiting times. Since we assume that these random parameters follow normal distribution, mean and variance of these parameters could also be considered as separate experimental factors. For simplicity in our experimental design and as we are determining departure times, we consider combined value of handling and waiting times. So the number of the experimental factors is reduced to three in total. Experimental factors and levels of each factor are seen in Table 5.4. For each parameter, we define two different levels. We also assume the fuel price of \$550 in all the experiments. For determining the values for the different levels of  $\mu$  and  $\sigma$ , we did some trial and error and picked the values from uniform distributions. We tried different values for  $\mu$  and  $\sigma$  and run all the settings and the replications for all the routes. We selected the values that give feasible schedule for all the runs.

Table 5.4: Experimental factors

| Parameter                 | Level         |               |
|---------------------------|---------------|---------------|
|                           | L             | H             |
| $1 - \beta$               | 0.8           | 0.9           |
| $\mu$ (in hour) $\sim$    | $U(13, 17)$   | $U(18, 21)$   |
| $\sigma$ (in hour) $\sim$ | $U(3.9, 5.1)$ | $U(5.2, 6.6)$ |

We first study effects of these factors on fuel consumption cost, sailing times, departure times, service levels and buffer times. We also analyze effects of time windows on optimal solutions. Further, we study the relation between weights of the port-of-calls and service levels. Finally, we do comparisons between fuel consumption costs

of our model and other feasible methods.

### 5.1 Effects of experimental factors on fuel consumption cost

For analyzing effects of the experimental factors on fuel consumption cost, we considered 8 experimental settings ( $2 \times 2 \times 2$ ) and for each setting we solved 10 replications. We did the experiment for the route AE and in each replication, we generated random  $\mu$  and  $\sigma$  values for each port. After solving the model, for each level of the experimental factors, we computed average of the fuel consumption costs from all the runs. The results are seen in Table 5.5.

Table 5.5: Effects of experimental factors on fuel consumption cost

| Parameter   | Level | Fuel consumption cost ( $\$10^6$ ) | % Change |
|-------------|-------|------------------------------------|----------|
| $1 - \beta$ | 0.9   | 18.92                              | -0.87    |
|             | 0.8   | 18.75                              |          |
| $\mu$       | L     | 18.62                              | 2.33     |
|             | H     | 19.05                              |          |
| $\sigma$    | L     | 18.79                              | 0.43     |
|             | H     | 18.87                              |          |

It can be concluded from Table 5.5 that when the overall service level decreases, fuel consumption cost decreases since ships can sail at lower speeds. However, as overall service level decreases, unreliability of the schedule also increases. Unreliability of the schedule poses losses to the shipping company. The company can decide on the degree of service level of the schedule by comparing costs for different values of  $\beta$ .

In Table 5.5, we reported fuel consumption costs for both levels of the overall service level factor. In addition, for seeing effects of overall service level on fuel consumption cost more clearly, we plotted the fuel costs for the different values of overall service level in Figure 5.1. It is seen from Figure 5.1 that as the overall service level of the route increases total fuel consumption cost also increases. Moreover, it is seen that increasing of the service level costs more to the shipping company as it gets higher.

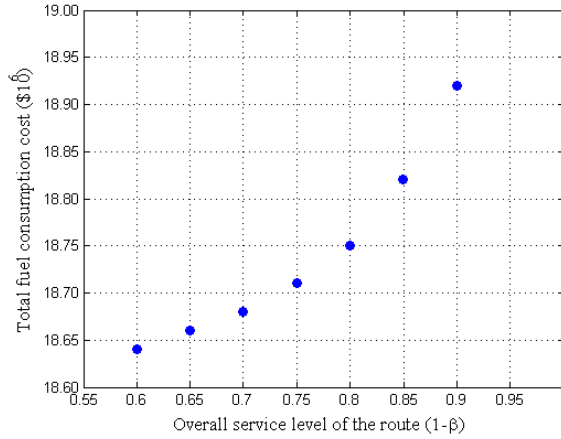


Figure 5.1: Effect of overall service level on total fuel consumption cost

From Table 5.5 it is also observed that decreasing means of the handling and waiting times have significant savings to the liner shipping company since ships can sail at lower speeds. Sailing at lower speeds results in less fuel consumption cost.

Also as uncertainty ( $\sigma$ ) of handling and waiting times increases, it is observed from Table 5.5 that fuel consumption cost increases. When  $\sigma$  increases since round-trip journey time is fixed, more buffer time should be assigned in the schedule to overcome the uncertainties. By considering more buffer time in the schedule, ships are forced to sail at higher speeds. So, this increases fuel consumption rate during the journey.

For the case with zero uncertainty, total fuel consumption cost will be 18.56 (\$10<sup>6</sup>). By considering uncertainty in the schedule, fuel consumption cost increases. We can mention that fuel cost increases by 1.25% as uncertainty level increases to ‘L’.

## 5.2 Effects of experimental factors on optimal solutions

In the previous section, we observed effects of experimental factors on fuel consumption cost. Now, we want to see how the levels of each factor affect service levels, buffer times, speeds and departure times. To analyze effects of experimental factors on optimal solutions, we considered eight settings and we replicated each setting for ten times. We did the experiment for the route AE.

### 5.2.1 Effects of overall service level ( $1 - \beta$ )

Here, we want to study effects of overall service level on optimal solutions. According to the value of overall service level, service levels for the port-ship type pairs are determined with respect to the differences of the ports and the ship types.

We did the experiment and for each level of overall service level, we computed average of the speeds for each ship type on each leg of the sailing route. We also computed average of the service levels of each ship type at the ports and also average departure times at the ports that were obtained from all the settings and replications. The results are seen in Table 5.6. In the following tables,  $L_{ijk}$  refers to the ship of type  $k$  on the leg  $(ij)$ . For example  $L_{12A}$  refers to the ship of type A on the leg (12).

Table 5.6: Effects of ( $1 - \beta$ )

| Parameter                   | $(1 - \beta)$ | $L_{12A}$ | $L_{12B}$ | $L_{23A}$ | $L_{23B}$ | $L_{34A}$ | $L_{34B}$ | $L_{45A}$ |
|-----------------------------|---------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Speed<br>(in knots)         | 0.9           | 20.67     | 20.46     | 20.70     | 20.42     | 20.68     | 20.44     | 20.54     |
|                             | 0.8           | 20.21     | 20.21     | 20.21     | 20.21     | 20.21     | 20.21     | 20.26     |
| Service level               | 0.9           | 0.72      | 0.69      | 0.77      | 0.76      | 0.69      | 0.65      | 0.97      |
|                             | 0.8           | 0.60      | 0.60      | 0.60      | 0.60      | 0.60      | 0.60      | 0.93      |
| Departure time<br>(in days) | 0.9           | 1.96      |           | 3.38      |           | 5.20      |           |           |
|                             | 0.8           | 1.94      |           | 3.29      |           | 5.10      |           |           |

| Parameter                   | $(1 - \beta)$ | $L_{45B}$ | $L_{56A}$ | $L_{56B}$ | $L_{67A}$ | $L_{67B}$ | $L_{78A}$ | $L_{78B}$ |
|-----------------------------|---------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Speed<br>(in knots)         | 0.9           | 20.49     | 20.52     | 20.50     | 20.50     | 20.50     | 20.67     | 20.46     |
|                             | 0.8           | 20.18     | 20.21     | 20.21     | 20.21     | 20.21     | 20.21     | 20.21     |
| Service level               | 0.9           | 0.97      | 0.88      | 0.85      | 0.60      | 0.60      | 0.80      | 0.78      |
|                             | 0.8           | 0.92      | 0.61      | 0.60      | 0.60      | 0.60      | 0.60      | 0.60      |
| Departure time<br>(in days) | 0.9           | 10.54     | 28.41     |           | 29.83     |           | 31.48     |           |
|                             | 0.8           | 10.44     | 28.43     |           | 29.86     |           | 31.44     |           |

| Parameter                   | $(1 - \beta)$ | $L_{89A}$ | $L_{89B}$ | $L_{910A}$ | $L_{910B}$ | $L_{101A}$ | $L_{101B}$ |
|-----------------------------|---------------|-----------|-----------|------------|------------|------------|------------|
| Speed<br>(in knots)         | 0.9           | 20.51     | 20.50     | 20.57      | 20.49      | 20.58      | 20.49      |
|                             | 0.8           | 20.23     | 20.20     | 20.31      | 20.14      | 20.33      | 20.13      |
| Service level               | 0.9           | 0.97      | 0.97      | 0.90       | 0.88       | 0.96       | 0.96       |
|                             | 0.8           | 0.93      | 0.91      | 0.67       | 0.61       | 0.90       | 0.88       |
| Departure time<br>(in days) | 0.9           | 49.45     |           | 53.26      |            | 56.00      |            |
|                             | 0.8           | 49.59     |           | 53.31      |            | 56.00      |            |

For summarizing the results, we computed averages of the speeds and buffer times on all the legs for each ship type. We also computed summation of the weighted service levels of all the ports for each ship type and take the average of them that were obtained from all the runs. The results are seen in Table 5.7.

Table 5.7: Summary table of effects of  $(1 - \beta)$

| Ship type | Speed (in knots) |       | Weighted service level |      | Buffer time (in days) |      |
|-----------|------------------|-------|------------------------|------|-----------------------|------|
|           | 0.9              | 0.8   | 0.9                    | 0.8  | 0.9                   | 0.8  |
| A         | 20.59            | 20.24 | 0.91                   | 0.81 | 0.17                  | 0.09 |
| B         | 20.48            | 20.19 | 0.89                   | 0.79 | 0.16                  | 0.09 |

By observing Tables 5.6 and 5.7 it is obvious that when the overall service level changes from 0.9 to 0.8, changes in service levels, buffer times, speeds and also small changes in departure times are seen. It is observed that speeds have decreased for each ship type when overall service level decreases. It is also observed that service levels and buffer times have also decreased for each ship type. Now, we will explain how decreasing the overall service level changes service levels, buffer times, speeds and departure times.

As overall service level decreases, service level for each ship type at each port also decreases since it will result in less fuel consumption cost. So, since service levels have decreased for each ship type at the ports, less buffer times are assigned. This is obvious from Table 5.7, where average buffer times have decreased for both of the ship types by decreasing of the overall service level.

Therefore, by decreasing overall service level according to Table 5.7, less buffer time is assigned in the schedule. This will result in lower speeds since we are minimizing the total fuel consumption cost during the voyage.

As mentioned before, by decreasing overall service level, buffer times for each ship type at the ports have decreased. Changes of buffer times and speeds might change the departure times at some ports. From Table 5.6 it is observed that departure times have shifted forward at some ports and backward at others. The direction of the movement



of the departure times by changing the overall service level depends on the amount of the changes in speeds and buffer times of each ship type at the ports.

It is also observed that speed of a type A ship is equal to or higher than speed of a ship of type B on all the legs. So, service level for a type A ship is equal to or higher than the service level of a ship of type B for each level of the overall service level on all the legs. This is the result of the difference between fuel consumption functions of the ship types. The model assigns higher service levels for the ship types with higher fuel efficiency. By the ship types with higher fuel efficiency, we refer to the ships for which increasing the speed costs less according to the slope of their fuel consumption function.

### **5.2.2 Effects of mean ( $\mu$ )**

Handling time at a port mainly depends on number of the cranes and also number of the containers handled. Waiting time at a port also depends on congestion and number of available berths. When a vessel arrives at a port if there is no free berth, it has to wait. On the major ports and the ports with higher demand, waiting times for the ships might be higher. These factors and many others make handling and waiting times different between ports. As mentioned before, we have assumed that handling and waiting times obey normal distribution and mean and variance of these random parameters could be different between ports.

Now, we study effects of the mean of the random parameters on optimal solutions. We solved the model and computed averages of the speeds and the service levels for each ship type on each leg. We also computed average of the departure times at the ports that were obtained from all the runs. We did the experiment for each level of  $\mu$  separately. The results are seen in Table 5.8.

Table 5.8: Effects of  $\mu$ 

| Parameter                   | $\mu$ | $L_{12A}$ | $L_{12B}$ | $L_{23A}$ | $L_{23B}$ | $L_{34A}$ | $L_{34B}$ | $L_{45A}$ |
|-----------------------------|-------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Speed<br>(in knots)         | L     | 20.08     | 19.87     | 20.11     | 19.85     | 20.09     | 19.87     | 20.00     |
|                             | H     | 20.80     | 20.79     | 20.80     | 20.78     | 20.80     | 20.78     | 20.80     |
| Service level               | L     | 0.67      | 0.64      | 0.69      | 0.68      | 0.65      | 0.62      | 0.95      |
|                             | H     | 0.65      | 0.65      | 0.68      | 0.68      | 0.64      | 0.63      | 0.95      |
| Departure time<br>(in days) | L     | 1.88      |           | 3.18      |           | 4.93      |           |           |
|                             | H     | 2.02      |           | 3.49      |           | 5.38      |           |           |

| Parameter                   | $\mu$ | $L_{45B}$ | $L_{56A}$ | $L_{56B}$ | $L_{67A}$ | $L_{67B}$ | $L_{78A}$ | $L_{78B}$ |
|-----------------------------|-------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Speed<br>(in knots)         | L     | 19.93     | 19.97     | 19.95     | 19.96     | 19.96     | 20.08     | 19.88     |
|                             | H     | 20.74     | 20.76     | 20.76     | 20.76     | 20.76     | 20.80     | 20.79     |
| Service level               | L     | 0.94      | 0.74      | 0.72      | 0.60      | 0.60      | 0.70      | 0.69      |
|                             | H     | 0.94      | 0.74      | 0.73      | 0.60      | 0.60      | 0.70      | 0.69      |
| Departure time<br>(in days) | L     | 10.26     | 28.43     |           | 29.78     |           | 31.31     |           |
|                             | H     | 10.73     | 28.41     |           | 29.92     |           | 31.61     |           |

| Parameter                   | $\mu$ | $L_{89A}$ | $L_{89B}$ | $L_{910A}$ | $L_{910B}$ | $L_{101A}$ | $L_{101B}$ |
|-----------------------------|-------|-----------|-----------|------------|------------|------------|------------|
| Speed<br>(in knots)         | L     | 19.97     | 19.95     | 20.04      | 19.90      | 20.05      | 19.89      |
|                             | H     | 20.77     | 20.75     | 20.84      | 20.74      | 20.85      | 20.72      |
| Service level               | L     | 0.95      | 0.94      | 0.78       | 0.74       | 0.93       | 0.92       |
|                             | H     | 0.95      | 0.94      | 0.78       | 0.75       | 0.93       | 0.92       |
| Departure time<br>(in days) | L     | 49.62     |           | 53.34      |            | 56.00      |            |
|                             | H     | 49.42     |           | 53.22      |            | 56.00      |            |

For a better understanding, we summarized the results. We computed averages of the speeds and buffer times of all the legs for each ship type. We also computed summation of the weighted service levels on all the legs and take the average of them that were obtained from all the runs. The results are seen in Table 5.9.

Table 5.9: Summary table of effects of  $\mu$

| Ship type | Speed (in knots) |       | Weighted service level |      | Buffer time (in days) |       |
|-----------|------------------|-------|------------------------|------|-----------------------|-------|
|           | L                | H     | L                      | H    | L                     | H     |
| A         | 20.04            | 20.80 | 0.86                   | 0.86 | 0.132                 | 0.129 |
| B         | 19.90            | 20.76 | 0.84                   | 0.84 | 0.120                 | 0.123 |

According to Table 5.8 as  $\mu$  increases, speeds for the ship types on the sailing legs also increase. This comes from the fact that since ships have to finish the round-trip journey in a predetermined period, by increasing mean of handling and waiting times at the ports, ships force to sail at higher speeds. This fact is also obvious from Table 5.9.

It is observed from Tables 5.8 and 5.9 that by changing  $\mu$ , service levels have slightly changed. This happens since we are imposing an overall service level in the model and increasing mean of handling and waiting times of the ports could be overcome by increasing speeds of the ships on the sailing legs rather than adjusting service levels.

According to Table 5.9, by changing the level of  $\mu$ , buffer times have still remained the same. Since service levels have not changed by changing of  $\mu$  and also the level of the  $\sigma$  is constant, buffer times are still approximately the same for the each level of  $\mu$ .

Although buffer times have remained the same, increasing of the speeds might change the departure times at the ports. Departure times at some ports have shifted forward and at some others have shifted backward.

We can deduce from Tables 5.8 and 5.9 that for the both levels of  $\mu$ , speeds for a ship of type B on the sailing legs are equal to or lower than the speeds of a ship of type A. So, assigned service levels for a ship of type A are equal to or higher than the assigned service levels for a ship of type B. This is the result of the different fuel consumption function that each ship type has. Since service levels for a ship of type A are higher than the service levels of a ship of type B, assigned buffer times for a ship of type A are higher than the assigned buffer times for a ship of type B.

### 5.2.3 Effects of standard deviation ( $\sigma$ )

In this part, we study effects of the uncertainty level of the handling and waiting times on optimal solutions. For analyzing effects of  $\sigma$ , we did the experiment as before. We computed averages of the speeds, service levels and departure times. The results are seen in Table 5.10.

Table 5.10: Effects of  $\sigma$

| Parameter                   | $\sigma$ | $L_{12A}$ | $L_{12B}$ | $L_{23A}$ | $L_{23B}$ | $L_{34A}$ | $L_{34B}$ | $L_{45A}$ |
|-----------------------------|----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Speed<br>(in knots)         | L        | 20.38     | 20.20     | 20.40     | 20.17     | 20.39     | 20.19     | 20.33     |
|                             | H        | 20.49     | 20.47     | 20.51     | 20.46     | 20.50     | 20.46     | 20.47     |
| Service level               | L        | 0.67      | 0.64      | 0.69      | 0.67      | 0.65      | 0.62      | 0.95      |
|                             | H        | 0.65      | 0.65      | 0.68      | 0.68      | 0.64      | 0.63      | 0.95      |
| Departure time<br>(in days) | L        | 1.95      |           | 3.32      |           | 5.14      |           |           |
|                             | H        | 1.95      |           | 3.35      |           | 5.16      |           |           |

| Parameter                   | $\sigma$ | $L_{45B}$ | $L_{56A}$ | $L_{56B}$ | $L_{67A}$ | $L_{67B}$ | $L_{78A}$ | $L_{78B}$ |
|-----------------------------|----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Speed<br>(in knots)         | L        | 20.26     | 20.30     | 20.28     | 20.29     | 20.29     | 20.38     | 20.20     |
|                             | H        | 20.41     | 20.43     | 20.43     | 20.43     | 20.43     | 20.49     | 20.47     |
| Service level               | L        | 0.94      | 0.75      | 0.72      | 0.60      | 0.60      | 0.70      | 0.69      |
|                             | H        | 0.94      | 0.74      | 0.73      | 0.60      | 0.60      | 0.70      | 0.69      |
| Departure time<br>(in days) | L        | 10.46     | 28.43     |           | 29.86     |           | 31.46     |           |
|                             | H        | 10.52     | 28.41     |           | 29.84     |           | 31.46     |           |

| Parameter                   | $\sigma$ | $L_{89A}$ | $L_{89B}$ | $L_{910A}$ | $L_{910B}$ | $L_{101A}$ | $L_{101B}$ |
|-----------------------------|----------|-----------|-----------|------------|------------|------------|------------|
| Speed<br>(in knots)         | L        | 20.30     | 20.28     | 20.37      | 20.23      | 20.39      | 20.22      |
|                             | H        | 20.44     | 20.42     | 20.51      | 20.40      | 20.52      | 20.39      |
| Service level               | L        | 0.95      | 0.94      | 0.79       | 0.74       | 0.93       | 0.92       |
|                             | H        | 0.95      | 0.94      | 0.78       | 0.75       | 0.93       | 0.92       |
| Departure time<br>(in days) | L        | 49.55     |           | 53.30      |            | 56.00      |            |
|                             | H        | 49.49     |           | 53.26      |            | 56.00      |            |

For summarizing the results, we computed averages of the speeds and buffer times of all the sailing legs for each ship type. We computed the averages separately for the each level of  $\sigma$ . We also computed summation of the weighted service levels on all the legs and take the average of them that were obtained from all the runs. The results are seen in Table 5.11.

Table 5.11: Summary table of effects of  $\sigma$

| Ship type | Speed (in knots) |       | Weighted service level |      | Buffer time (in days) |      |
|-----------|------------------|-------|------------------------|------|-----------------------|------|
|           | L                | H     | L                      | H    | L                     | H    |
| A         | 20.35            | 20.48 | 0.86                   | 0.86 | 0.11                  | 0.15 |
| B         | 20.23            | 20.43 | 0.84                   | 0.84 | 0.10                  | 0.14 |

According to Tables 5.10 and 5.11, it is observed that as uncertainty ( $\sigma$ ) increases, buffer times, speeds and departure times have changed and service levels have slightly changed.

As  $\sigma$  increases, service levels have remained the same since overall service level is constant. However, more buffer time is needed to be assigned in the schedule to overcome the uncertainties. This fact is seen in Table 5.11. It is obvious that by increasing  $\sigma$ , average buffer time also increases for each ship type.

By assigning more buffer time in the schedule, speeds of the ship types on all the sailing legs would increase since the round-trip journey time is fixed. This increases fuel consumption. The percentage of the increase of the fuel consumption cost was reported in Section 5.1.

By changing the level of  $\sigma$ , as mentioned, speeds and buffer times have changed. Changes in speeds and buffer times might change the departure times at the ports. Departure times at some ports have shifted forward and at some other ports have shifted backward according to Table 5.10.

We can also mention that as in the previous part, it is observed from Tables 5.10 and 5.11 that for the both level of  $\sigma$ , speed of a ship of type B on the sailing legs is equal to or lower than the speed of a ship of type A and so service level for a ship of type A is equal to or higher than the service level of a ship of type B. The model tries to assign higher service levels for the ship types with higher fuel efficiency to minimize the total fuel consumption rate during the journey.

### 5.3 Effects of time windows

In this section, we want to study effects of port time windows on fuel consumption cost and service levels. Time windows refer to the set of available times in a week that port can provide service. So in practice, time windows might be imposed in the schedule. In most of the studies, time windows are considered for arrival times of ships at the ports. However, we are determining departure times of ships in this study, so we define port time windows for departure times of ships at the ports. For including time windows in the problem, following constraint should also be added to the model. In this constraint,  $d_l$  and  $d_u$  are lower and upper bounds of time windows respectively.

$$d_l \leq d_i \leq d_u \quad i = 1, 2, \dots, I \quad (5.1)$$

For studying effects of time windows, we considered route AEX and added constraint (5.1) to the model. We generated appropriate time windows for departure times at the ports of the route AEX by considering the article of Yao et al. [34]. We did the experiment for the setting ( $\beta$ : 0.9,  $\mu$  level: H,  $\sigma$  level: L). Effect of time windows on total fuel consumption cost is seen in Table 5.12.

Table 5.12: Effect of time windows on fuel consumption cost

| Fuel consumption cost (\$10 <sup>6</sup> ) |                   | Change (\$)       |
|--|-------------------|-------------------|
| Without time windows                       | With time windows |                   |
| 20.66                                      | 20.75             | 87,601.46 (0.42%) |

It is observed from Table 5.12 that by considering time windows, fuel consumption cost increases. By including time windows, ships have to depart from the ports at the specific time intervals. This forces ships to sail at different speeds than they used to sail before.

Including time windows in the model has changed the service levels of ship types at some ports. This is obvious from Table 5.13. This happens since by considering time windows, departure times of the ships at the ports would change. Changes in departure times result in changes in speeds. Therefore, by changing speeds of the

ship types and the departure times at the ports, service levels might also change. By including time windows, service levels at some ports have increased and at some other ports have decreased.

Table 5.13: Effect of time windows on service levels

| Port       | With time windows |        | Without time windows |        |
|------------|-------------------|--------|----------------------|--------|
|            | Service level     |        | Service level        |        |
|            | Type A            | Type B | Type A               | Type B |
| Kwangyang  | 0.60              | 0.60   | 0.60                 | 0.60   |
| Pusan      | 0.91              | 0.91   | 0.86                 | 0.85   |
| Shanghai   | 0.96              | 0.95   | 0.96                 | 0.96   |
| Kaohsiung  | 0.60              | 0.60   | 0.60                 | 0.60   |
| Hong Kong  | 0.91              | 0.90   | 0.91                 | 0.90   |
| Yantian    | 0.60              | 0.60   | 0.60                 | 0.60   |
| Singapore  | 0.96              | 0.87   | 0.98                 | 0.97   |
| Rotterdam  | 0.88              | 0.83   | 0.89                 | 0.85   |
| Hamburg    | 0.60              | 0.60   | 0.60                 | 0.60   |
| Thamesport | 0.60              | 0.60   | 0.60                 | 0.60   |
| Colombo    | 0.60              | 0.60   | 0.60                 | 0.60   |
| Singapore  | 0.98              | 0.98   | 0.97                 | 0.97   |
| Hong Kong  | 0.95              | 0.94   | 0.91                 | 0.89   |
| Kaohsiung  | 0.82              | 0.80   | 0.60                 | 0.60   |
| Hakata     | 0.60              | 0.60   | 0.60                 | 0.60   |

#### 5.4 Effects of weights on service levels

Our aim in this section is to see only the effects of the weights of the ports on optimal service levels. However until now,  $\mu$  and  $\sigma$  values were considered different among ports of the service routes AE and AEX and since these parameters also have effects on optimal service levels, for overcoming their effects on optimal service levels and to see only the effects of weights, we did the experiment for the route APX and generated equal  $\mu$  and equal  $\sigma$  values for all the ports of the route. We did the experiment for the setting ( $\beta$ : 0.9,  $\mu$  (in days): 0.61,  $\sigma$  (in days): 0.19) and replicated for once. Results of the experiment are seen in Table 5.14. The first column shows the name

of the port-of-call. Second column refers to the number of the port-of-call. Third and fourth columns refer to the service levels of the ships of types A and B. The last column shows the importance weight of the port. All columns are sorted from largest to smallest according to the importance weights of the ports.

Table 5.14: Effects of weights on service levels

| Port-of-call | Call-number | Service level |        | Weight |
|--------------|-------------|---------------|--------|--------|
|              |             | Type A        | Type B |        |
| Rotterdam    | 14          | 0.98          | 0.98   | 0.1388 |
| Hong kong    | 2           | 0.98          | 0.98   | 0.1364 |
| Busan        | 4           | 0.98          | 0.97   | 0.1240 |
| Chiwan       | 1           | 0.96          | 0.95   | 0.0738 |
| Balboa       | 7           | 0.95          | 0.94   | 0.0693 |
| Kaohsiung    | 3           | 0.95          | 0.94   | 0.0626 |
| New york     | 13          | 0.93          | 0.91   | 0.0493 |
| New york     | 17          | 0.93          | 0.91   | 0.0493 |
| Kobe         | 5           | 0.89          | 0.87   | 0.0366 |
| Kobe         | 24          | 0.89          | 0.87   | 0.0366 |
| Tokyo        | 6           | 0.86          | 0.84   | 0.0312 |
| Tokyo        | 23          | 0.87          | 0.83   | 0.0312 |
| Norfolk      | 18          | 0.81          | 0.79   | 0.0260 |
| Bremerhaven  | 15          | 0.74          | 0.71   | 0.0218 |
| Oakland      | 22          | 0.71          | 0.66   | 0.0206 |
| Miami        | 9           | 0.60          | 0.60   | 0.0151 |
| Felixstowe   | 16          | 0.60          | 0.60   | 0.0142 |
| Jacksonvilla | 10          | 0.60          | 0.60   | 0.0109 |
| Savannah     | 11          | 0.60          | 0.60   | 0.0109 |
| Manzanillo   | 8           | 0.60          | 0.60   | 0.0106 |
| Manzanillo   | 20          | 0.60          | 0.60   | 0.0106 |
| Charleston   | 12          | 0.60          | 0.60   | 0.0100 |
| Charleston   | 19          | 0.60          | 0.60   | 0.0100 |
| San pedro    | 21          | 0.60          | 0.60   | 0.0003 |

It is observed from Table 5.14 that as the weight gets smaller, service level at a port gets lower. Since weight shows the level of the importance of a port, so as it decreases, importance degree of a port decreases. For the less important ports, delay would be



higher. Having more delay in departure time of a ship at a port decreases service level. Therefore, the weights of the ports play an important role in assigning the service levels.

## **5.5 Comparisons**

In this section, we first compare the fuel consumption costs between two different methods. The first method is assigning variable service levels of the model for the port-ship type pairs. The other method is assigning equal service levels for all the port-ship type pairs as same value as the overall service level. Furthermore, we study another comparison. We add a new constraint to the model which forces different ship types to have equal service level at the same port. We compare fuel consumption cost of the model before adding this constraint and after adding this constraint.

### **5.5.1 Assigning variable service levels vs. assigning equal service levels for the port-ship type pairs by considering behaviors of the functions**

By considering the overall service level of the service route, we solved the model and determined variable service levels for the port-ship type pairs. We call our method as Method 1 (Proposed model). However, there is also a method which assigns equal service levels for the port-ship type pairs as same value as the overall service level. We name this method as Method 2 (Model with fixed service levels). In this part, we will compare the results of these two methods to see how fuel consumption cost changes.

We consider container ships of types A and B in this analysis. The fuel consumption functions of both are shown in Figure 5.2. It is observed from Figure 5.2 that behaviors of the functions for both of the ship types are approximately the same in a specified speed range. In this part, we also want to analyze that how difference between functions changes the results. So in addition to main fuel consumption function of each ship type, we change the coefficients of the fuel consumption functions for both of the ship types. We change the coefficients in a way to decrease the slope of the fuel consumption function for a ship of type A and increase the slope of the func-

tion for a ship of type B in a specified range. The altered fuel consumption functions are shown in Figure 5.3. As you can see in Figure 5.3, by changing the coefficients, behaviors of the functions have become more different. In the rest of the study, we refer to the functions of Figure 5.2 as "F1" and the functions of Figure 5.3 as "F2".

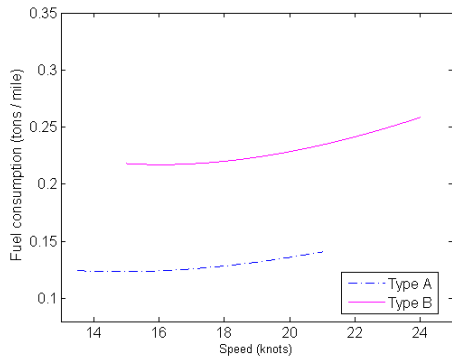


Figure 5.2: Main fuel consumption functions of the ships of types A and B. (F1)

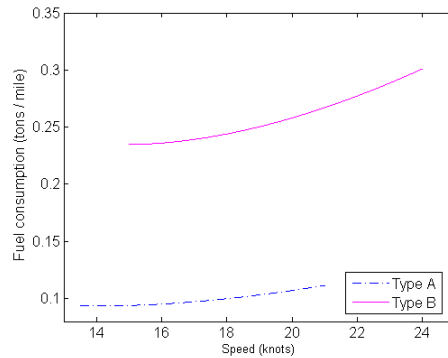


Figure 5.3: Altered fuel consumption functions of the ships of types A and B. (F2)

First, we considered route AEX. We generated eight settings and replicated each setting for ten times. In each replication, we generated random  $\mu$  and  $\sigma$  values for each port in a specific range. We applied Method 1 and solved the model for each setting and found the variable service levels and corresponding fuel consumption costs. Also, we did the experiment under both F1 and F2's fuel consumption functions to see how fuel consumption cost changes. Results of the experiment for the route AEX are shown in Table 5.15 in the next page. First column of the table shows the settings. The second column of Table 5.15, shows the fuel consumption costs by applying this method. Then, we applied Method 2 and assigned equal service levels for the port-ship type pairs as same value as the overall service level and found the corresponding fuel consumption costs. Fuel consumption costs for this method, are shown in the third column of Table 5.15. Fourth column of Table 5.15, shows the percentage of the difference of the fuel consumption costs between these two methods.

Table 5.15: Route AEX, Fuel consumption cost (\$10<sup>6</sup>)  
(Method 1 vs. Method 2)

|    | Setting<br>$1 - \beta, \mu, \sigma$ | Proposed model | Fixed service levels | % Change |
|----|-------------------------------------|----------------|----------------------|----------|
| F1 | 0.9, L, L                           | 20.14          | 20.30                | 0.79     |
|    | 0.9, L, H                           | 20.23          | 20.45                | 1.12     |
|    | 0.9, H, L                           | 20.69          | 20.92                | 1.08     |
|    | 0.9, H, H                           | 20.82          | 21.13                | 1.51     |
|    | 0.8, L, L                           | 20.04          | 20.15                | 0.53     |
|    | 0.8, L, H                           | 20.09          | 20.24                | 0.73     |
|    | 0.8, H, L                           | 20.55          | 20.70                | 0.73     |
|    | 0.8, H, H                           | 20.62          | 20.83                | 1.00     |
| F2 | 0.9, L, L                           | 19.64          | 19.85                | 1.08     |
|    | 0.9, L, H                           | 19.75          | 20.05                | 1.50     |
|    | 0.9, H, L                           | 20.34          | 20.62                | 1.39     |
|    | 0.9, H, H                           | 20.48          | 20.88                | 1.92     |
|    | 0.8, L, L                           | 19.51          | 19.66                | 0.74     |
|    | 0.8, L, H                           | 19.57          | 19.77                | 1.00     |
|    | 0.8, H, L                           | 20.16          | 20.36                | 0.96     |
|    | 0.8, H, H                           | 20.25          | 20.51                | 1.30     |

By observing Table 5.15, we can conclude that assigning variable service levels of the model is more beneficial to the liner shipping company than assigning equal service levels. This is true for both F1 and F2. It is also observed that as  $\mu$  and  $\sigma$  values increase, the percentage of change between two methods also increases. This comes from the fact that by increasing  $\mu$  and  $\sigma$  values, ships have to sail at higher speeds. When ships sail at higher speeds, according to fuel consumption functions that each has, importance of assigning variable service levels for each port-ship type pair increases. Also, percentage of change for the overall service level of 0.9 is higher than the overall service level of 0.8. This indicates that when service level requirements are tight, proposed model achieves higher improvement.

When comparing the results between F1 and F2 for each setting, we see that the percentage of change for F2 is higher than the percentage of change for F1. This

comes from the fact that as behaviors of the functions differ more from each other, assigning variable service levels of the model becomes more important. In the fuel consumption functions of F2, the difference between behaviors of the functions is more significant than F1.

Then we considered route AE for analyzing and did the same experiment as before. For each setting, we generated ten replications and computed the fuel consumption costs for each method. However in applying Method 2, infeasible solutions incurred for two of the settings. This is another disadvantage of this method and occurs because different ship types have different speed ranges and assigning equal service levels for all the ship types at all the ports might be infeasible. The results of the experiment are seen in Table 5.16. We show infeasible settings in blank in the table.

Table 5.16: Route AE, Fuel consumption cost ( $\$10^6$ )  
(Method 1 vs. Method 2)

|    | Setting<br>$1 - \beta, \mu, \sigma$ | Proposed model | Fixed service levels | % Change |
|----|-------------------------------------|----------------|----------------------|----------|
| F1 | 0.9, L, L                           | 18.64          | 18.71                | 0.35     |
|    | 0.9, L, H                           | 18.74          | 18.83                | 0.48     |
|    | 0.9, H, L                           | 19.08          | -                    | -        |
|    | 0.9, H, H                           | 19.20          | -                    | -        |
|    | 0.8, L, L                           | 18.52          | 18.59                | 0.35     |
|    | 0.8, L, H                           | 18.57          | 18.66                | 0.48     |
|    | 0.8, H, L                           | 18.93          | 19.01                | 0.43     |
|    | 0.8, H, H                           | 18.99          | 19.10                | 0.58     |
| F2 | 0.9, L, L                           | 18.66          | 18.75                | 0.50     |
|    | 0.9, L, H                           | 18.78          | 18.90                | 0.68     |
|    | 0.9, H, L                           | 19.21          | -                    | -        |
|    | 0.9, H, H                           | 19.37          | -                    | -        |
|    | 0.8, L, L                           | 18.51          | 18.60                | 0.48     |
|    | 0.8, L, H                           | 18.57          | 18.69                | 0.65     |
|    | 0.8, H, L                           | 19.02          | 19.13                | 0.57     |
|    | 0.8, H, H                           | 19.10          | 19.24                | 0.76     |

Similar to route AEX, from Table 5.16 it is observed that assigning variable service

levels of the model is more beneficial to the liner shipping company than assigning equal service levels. Also percentage of change for F2 is higher than the percentage of change for F1 in each setting.

### 5.5.2 Assigning variable service levels vs. assigning equal service levels for the ship types by considering behaviors of the functions

In the model, we assumed that different ship types could have different service levels. We now add a new constraint to the model which forces the different ship types to have equal service level at the same port. We will compare the fuel consumption costs between these two considerations.

We generated eight settings. We replicated each setting for ten times. We did the experiment for the route AEX under fuel consumption functions of the set F2. The fuel consumption costs are seen in Table 5.17.

Table 5.17: Route AEX, Fuel consumption cost ( $\$10^6$ ) (M1 vs. M2)

| Setting<br>$1 - \beta, \mu, \sigma$ | M1      | M2      | Difference (\$) |
|-------------------------------------|---------|---------|-----------------|
| 0.9, L, L                           | 19.6397 | 19.6462 | 6415.02         |
| 0.9, L, H                           | 19.7492 | 19.7584 | 9202.42         |
| 0.9, H, L                           | 20.3356 | 20.3453 | 9687.73         |
| 0.9, H, H                           | 20.4818 | 20.4948 | 13072.55        |
| 0.8, L, L                           | 19.5113 | 19.5166 | 5291.47         |
| 0.8, L, H                           | 19.5733 | 19.5801 | 6886.69         |
| 0.8, H, L                           | 20.1637 | 20.1718 | 8110.83         |
| 0.8, H, H                           | 20.2468 | 20.2570 | 10277.53        |

In the Table 5.17, second column shows total fuel consumption costs when variable service levels for different ship types are assigned. The third column shows total fuel consumption costs when different ship types are forced to have equal service levels at the same port.

It is observed from Table 5.17 that as  $\mu$  and  $\sigma$  increase, the importance of assigning

variable service levels for the ship types at the same port increases. Also, when comparing the settings for the overall service level 0.9 with 0.8, it is seen that as the schedule gets tighter, the difference between costs of the two methods increases.

## CHAPTER 6

### CONCLUSIONS AND FUTURE STUDY

In this thesis, we studied designing of a schedule for a liner ship service by considering the port time uncertainties in order to maintain an overall service level of a service route. We considered a heterogeneous fleet of container ships on a single route. The objective was minimizing total fuel consumption cost during the round-trip journey time. We considered uncertainties of handling and waiting times at the ports. Assuming that port time uncertainty is normally distributed, we developed a mixed integer nonlinear model. There is a nonlinear relation between fuel consumption and speed of a ship. For handling the nonlinearity of the objective function, we reformulated the model as a SOCP problem. The reformulated model could be solved by commercial software such as CPLEX. We also showed SOCP representations for some of the other fuel consumption functions in the maritime literature.

For analyzing the model, we defined three liner service routes and specified the numbers of the deployed ships. We defined distinct weights for the ports on each route. We then, designed an experiment and studied effects of experimental factors on total fuel consumption cost and optimal solutions. We also studied effects of time windows on fuel consumption cost and optimal service levels. Moreover, we studied the relation between service levels and weights of the ports. Finally, we showed that assigning variable service levels for the port-ship type pairs is more beneficial to the shipping company than assigning equal service levels while achieving the specified service level for the whole system. We did this experiment under different fuel consumption functions sets and observed that as the behaviors of the functions of the ship types differ more from each other, importance of assigning variable service levels in-

creases.

Different from the literature, in this study, we have considered heterogeneous fleet and a new service level measure. We have also taken into account of the differences of the ports in modeling the problem. This research can be extended by integrating demand uncertainty in the model in order to determine the optimal numbers of the deployed ships on the routes. Moreover, instead of fixed routes, routing decisions can be considered. Also, since a shipping company gives services on different routes, service levels could be assigned in a way to satisfy the overall service level of a company.



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## APPENDIX A

### WEIGHTS

Table A.1: Weights of the port-of-calls

| APX Route    |         | AEX Route    |         | AE Route     |         |
|--------------|---------|--------------|---------|--------------|---------|
| Port-of-call | Weight  | Port-of-call | Weight  | Port-of-call | Weight  |
| Chiwan       | 0.07381 | Hakata       | 0.00636 | Shanghai     | 0.17096 |
| Hong kong    | 0.13642 | Kwangyang    | 0.00404 | Dalian       | 0.04537 |
| Kaohsiung    | 0.06261 | Pusan        | 0.06093 | Qingdao      | 0.05158 |
| Busan        | 0.12402 | Shanghai     | 0.15698 | Ningbo       | 0.04234 |
| Kobe         | 0.03660 | Kaohsiung    | 0.03183 | Singapore    | 0.22843 |
| Tokyo        | 0.03116 | Hong kong    | 0.08000 | Rotterdam    | 0.07259 |
| Balboa       | 0.06927 | Yantian      | 0.01379 | Hamburg      | 0.02260 |
| Manzanillo   | 0.01059 | Singapore    | 0.21547 | Antwerp      | 0.05428 |
| Miami        | 0.01512 | Rotterdam    | 0.06996 | Singapore    | 0.22843 |
| Jacksonvilla | 0.01089 | Hamburg      | 0.02179 | Hong Kong    | 0.08341 |
| Savannah     | 0.01089 | Thamesport   | 0.00127 |              |         |
| Charleston   | 0.00998 | Colombo      | 0.01029 |              |         |
| New york     | 0.04930 | Singapore    | 0.21547 |              |         |
| Rotterdam    | 0.13884 | Hong kong    | 0.08000 |              |         |
| Bremerhaven  | 0.02178 | Kaohsiung    | 0.03183 |              |         |
| Felixstowe   | 0.01422 |              |         |              |         |
| New york     | 0.04930 |              |         |              |         |
| Norfolk      | 0.02601 |              |         |              |         |
| Charleston   | 0.00998 |              |         |              |         |
| Manzanillo   | 0.01059 |              |         |              |         |
| San pedro    | 0.00030 |              |         |              |         |
| Oakland      | 0.02057 |              |         |              |         |
| Tokyo        | 0.03116 |              |         |              |         |
| Kobe         | 0.03660 |              |         |              |         |



## APPENDIX B

### COMPUTATIONAL RESULTS

The results of all the experiments are reported here. There are some new abbreviations used in these tables. The meaning of them is as follows.

- **Var** : Refers to the method of assigning variable service levels for the port-ship type pairs (Method 1)
- **Equal** : Refers to the method of assigning equal service levels for the port-ship type pairs (Method 2)
- **F1** : Refers to applying the functions of the set F1
- **F2** : Refers to applying the functions of the set F2
- $\bar{v}_A$  : The average speed of a ship of type A on all the legs
- $\bar{v}_B$  : The average speed of a ship of type B on all the legs
- $\bar{b}_A$  : The average buffer time of a ship of type A at all the ports
- $\bar{b}_B$  : The average buffer time of a ship of type B at all the ports

Table B.1: Experimental results 1

| AE Route, Var, F1 |         |       |          |                  |             |             |             |             |
|-------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep               | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 1                 | 0.1     | L     | L        | 18.61            | 20.06       | 19.88       | 0.141       | 0.127       |
| 2                 | 0.1     | L     | L        | 18.57            | 19.97       | 19.80       | 0.148       | 0.133       |
| 3                 | 0.1     | L     | L        | 18.65            | 20.12       | 19.96       | 0.142       | 0.130       |
| 4                 | 0.1     | L     | L        | 18.64            | 20.14       | 19.93       | 0.148       | 0.132       |
| 5                 | 0.1     | L     | L        | 18.66            | 20.17       | 19.97       | 0.150       | 0.133       |
| 6                 | 0.1     | L     | L        | 18.70            | 20.25       | 20.05       | 0.152       | 0.136       |
| 7                 | 0.1     | L     | L        | 18.63            | 20.10       | 19.92       | 0.151       | 0.139       |
| 8                 | 0.1     | L     | L        | 18.62            | 20.08       | 19.88       | 0.144       | 0.126       |
| 9                 | 0.1     | L     | L        | 18.69            | 20.22       | 20.03       | 0.148       | 0.132       |
| 10                | 0.1     | L     | L        | 18.63            | 20.12       | 19.91       | 0.145       | 0.129       |
| 1                 | 0.1     | L     | H        | 18.70            | 20.27       | 20.04       | 0.185       | 0.168       |
| 2                 | 0.1     | L     | H        | 18.66            | 20.18       | 19.96       | 0.194       | 0.175       |
| 3                 | 0.1     | L     | H        | 18.74            | 20.34       | 20.11       | 0.188       | 0.171       |
| 4                 | 0.1     | L     | H        | 18.74            | 20.35       | 20.09       | 0.194       | 0.173       |
| 5                 | 0.1     | L     | H        | 18.76            | 20.40       | 20.12       | 0.196       | 0.175       |
| 6                 | 0.1     | L     | H        | 18.80            | 20.48       | 20.21       | 0.199       | 0.178       |
| 7                 | 0.1     | L     | H        | 18.73            | 20.33       | 20.08       | 0.198       | 0.181       |
| 8                 | 0.1     | L     | H        | 18.71            | 20.29       | 20.04       | 0.189       | 0.166       |
| 9                 | 0.1     | L     | H        | 18.79            | 20.43       | 20.18       | 0.193       | 0.173       |
| 10                | 0.1     | L     | H        | 18.73            | 20.33       | 20.07       | 0.190       | 0.172       |
| 1                 | 0.1     | H     | L        | 19.05            | 20.89       | 20.67       | 0.144       | 0.126       |
| 2                 | 0.1     | H     | L        | 19.02            | 20.85       | 20.61       | 0.150       | 0.131       |
| 3                 | 0.1     | H     | L        | 19.09            | 20.92       | 20.74       | 0.143       | 0.129       |
| 4                 | 0.1     | H     | L        | 19.08            | 20.93       | 20.71       | 0.149       | 0.131       |
| 5                 | 0.1     | H     | L        | 19.10            | 20.94       | 20.74       | 0.150       | 0.133       |
| 6                 | 0.1     | H     | L        | 19.14            | 20.98       | 20.80       | 0.152       | 0.136       |
| 7                 | 0.1     | H     | L        | 19.08            | 20.92       | 20.71       | 0.154       | 0.137       |
| 8                 | 0.1     | H     | L        | 19.06            | 20.90       | 20.66       | 0.144       | 0.125       |
| 9                 | 0.1     | H     | L        | 19.12            | 20.97       | 20.76       | 0.148       | 0.131       |
| 10                | 0.1     | H     | L        | 19.07            | 20.91       | 20.70       | 0.146       | 0.129       |
| 1                 | 0.1     | H     | H        | 19.16            | 21.00       | 20.81       | 0.184       | 0.169       |
| 2                 | 0.1     | H     | H        | 19.13            | 20.98       | 20.77       | 0.194       | 0.174       |
| 3                 | 0.1     | H     | H        | 19.20            | 20.99       | 21.15       | 0.172       | 0.185       |
| 4                 | 0.1     | H     | H        | 19.20            | 21.00       | 21.12       | 0.178       | 0.189       |
| 5                 | 0.1     | H     | H        | 19.23            | 21.00       | 21.36       | 0.172       | 0.199       |
| 6                 | 0.1     | H     | H        | 19.29            | 20.98       | 21.82       | 0.159       | 0.226       |
| 7                 | 0.1     | H     | H        | 19.20            | 20.99       | 21.11       | 0.185       | 0.194       |
| 8                 | 0.1     | H     | H        | 19.17            | 21.00       | 20.89       | 0.183       | 0.172       |
| 9                 | 0.1     | H     | H        | 19.26            | 20.98       | 21.60       | 0.161       | 0.210       |



Table B.1: Experimental results 1

| AE Route, Var, F1 |         |       |          |                  |             |             |             |             |
|-------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep               | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 10                | 0.1     | H     | H        | 19.19            | 20.98       | 21.06       | 0.179       | 0.182       |
| 1                 | 0.2     | L     | L        | 18.50            | 19.73       | 19.71       | 0.076       | 0.072       |
| 2                 | 0.2     | L     | L        | 18.46            | 19.65       | 19.61       | 0.083       | 0.076       |
| 3                 | 0.2     | L     | L        | 18.53            | 19.81       | 19.77       | 0.080       | 0.074       |
| 4                 | 0.2     | L     | L        | 18.52            | 19.79       | 19.75       | 0.081       | 0.076       |
| 5                 | 0.2     | L     | L        | 18.54            | 19.82       | 19.78       | 0.083       | 0.076       |
| 6                 | 0.2     | L     | L        | 18.58            | 19.91       | 19.86       | 0.087       | 0.077       |
| 7                 | 0.2     | L     | L        | 18.51            | 19.76       | 19.72       | 0.084       | 0.078       |
| 8                 | 0.2     | L     | L        | 18.50            | 19.73       | 19.71       | 0.076       | 0.071       |
| 9                 | 0.2     | L     | L        | 18.57            | 19.88       | 19.84       | 0.083       | 0.075       |
| 10                | 0.2     | L     | L        | 18.52            | 19.78       | 19.74       | 0.082       | 0.074       |
| 1                 | 0.2     | L     | H        | 18.54            | 19.83       | 19.80       | 0.101       | 0.095       |
| 2                 | 0.2     | L     | H        | 18.50            | 19.76       | 19.71       | 0.110       | 0.099       |
| 3                 | 0.2     | L     | H        | 18.58            | 19.92       | 19.87       | 0.106       | 0.097       |
| 4                 | 0.2     | L     | H        | 18.57            | 19.89       | 19.85       | 0.108       | 0.098       |
| 5                 | 0.2     | L     | H        | 18.59            | 19.93       | 19.88       | 0.109       | 0.099       |
| 6                 | 0.2     | L     | H        | 18.63            | 20.02       | 19.96       | 0.113       | 0.102       |
| 7                 | 0.2     | L     | H        | 18.55            | 19.86       | 19.82       | 0.109       | 0.102       |
| 8                 | 0.2     | L     | H        | 18.54            | 19.83       | 19.80       | 0.101       | 0.094       |
| 9                 | 0.2     | L     | H        | 18.62            | 19.98       | 19.94       | 0.108       | 0.099       |
| 10                | 0.2     | L     | H        | 18.56            | 19.89       | 19.83       | 0.107       | 0.098       |
| 1                 | 0.2     | H     | L        | 18.90            | 20.53       | 20.49       | 0.078       | 0.071       |
| 2                 | 0.2     | H     | L        | 18.87            | 20.48       | 20.42       | 0.085       | 0.075       |
| 3                 | 0.2     | H     | L        | 18.94            | 20.61       | 20.54       | 0.082       | 0.072       |
| 4                 | 0.2     | H     | L        | 18.93            | 20.58       | 20.54       | 0.082       | 0.075       |
| 5                 | 0.2     | H     | L        | 18.94            | 20.61       | 20.56       | 0.084       | 0.075       |
| 6                 | 0.2     | H     | L        | 18.98            | 20.69       | 20.62       | 0.088       | 0.076       |
| 7                 | 0.2     | H     | L        | 18.92            | 20.56       | 20.51       | 0.085       | 0.077       |
| 8                 | 0.2     | H     | L        | 18.91            | 20.53       | 20.50       | 0.076       | 0.071       |
| 9                 | 0.2     | H     | L        | 18.97            | 20.65       | 20.61       | 0.083       | 0.075       |
| 10                | 0.2     | H     | L        | 18.92            | 20.58       | 20.52       | 0.083       | 0.073       |
| 1                 | 0.2     | H     | H        | 18.96            | 20.64       | 20.59       | 0.102       | 0.094       |
| 2                 | 0.2     | H     | H        | 18.93            | 20.60       | 20.52       | 0.111       | 0.098       |
| 3                 | 0.2     | H     | H        | 19.00            | 20.72       | 20.65       | 0.107       | 0.096       |
| 4                 | 0.2     | H     | H        | 18.99            | 20.69       | 20.64       | 0.109       | 0.097       |
| 5                 | 0.2     | H     | H        | 19.01            | 20.72       | 20.67       | 0.109       | 0.099       |
| 6                 | 0.2     | H     | H        | 19.05            | 20.80       | 20.73       | 0.113       | 0.102       |
| 7                 | 0.2     | H     | H        | 18.98            | 20.67       | 20.62       | 0.111       | 0.102       |
| 8                 | 0.2     | H     | H        | 18.96            | 20.63       | 20.60       | 0.101       | 0.094       |

Table B.1: Experimental results 1

| AE Route, Var, F1 |         |       |          |                  |             |             |             |             |
|-------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep               | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 9                 | 0.2     | H     | H        | 19.03            | 20.77       | 20.71       | 0.109       | 0.098       |
| 10                | 0.2     | H     | H        | 18.98            | 20.70       | 20.62       | 0.109       | 0.096       |

Table B.2: Experimental results 2

| AEX Route, Var, F1 |         |       |          |                  |             |             |             |             |
|--------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 1                  | 0.1     | L     | L        | 20.11            | 18.84       | 18.82       | 0.109       | 0.106       |
| 2                  | 0.1     | L     | L        | 20.08            | 18.79       | 18.76       | 0.111       | 0.107       |
| 3                  | 0.1     | L     | L        | 20.17            | 18.98       | 18.96       | 0.108       | 0.107       |
| 4                  | 0.1     | L     | L        | 20.12            | 18.88       | 18.84       | 0.109       | 0.105       |
| 5                  | 0.1     | L     | L        | 20.18            | 19.01       | 18.96       | 0.113       | 0.109       |
| 6                  | 0.1     | L     | L        | 20.20            | 19.04       | 19.02       | 0.110       | 0.108       |
| 7                  | 0.1     | L     | L        | 20.13            | 18.91       | 18.87       | 0.112       | 0.107       |
| 8                  | 0.1     | L     | L        | 20.13            | 18.89       | 18.87       | 0.107       | 0.105       |
| 9                  | 0.1     | L     | L        | 20.16            | 18.96       | 18.93       | 0.112       | 0.109       |
| 10                 | 0.1     | L     | L        | 20.14            | 18.92       | 18.87       | 0.110       | 0.105       |
| 1                  | 0.1     | L     | H        | 20.19            | 19.03       | 19.00       | 0.144       | 0.139       |
| 2                  | 0.1     | L     | H        | 20.16            | 18.98       | 18.94       | 0.144       | 0.142       |
| 3                  | 0.1     | L     | H        | 20.26            | 19.17       | 19.15       | 0.143       | 0.140       |
| 4                  | 0.1     | L     | H        | 20.20            | 19.10       | 18.99       | 0.144       | 0.138       |
| 5                  | 0.1     | L     | H        | 20.27            | 19.24       | 19.12       | 0.147       | 0.142       |
| 6                  | 0.1     | L     | H        | 20.29            | 19.25       | 19.19       | 0.145       | 0.141       |
| 7                  | 0.1     | L     | H        | 20.22            | 19.10       | 19.05       | 0.147       | 0.140       |
| 8                  | 0.1     | L     | H        | 20.21            | 19.07       | 19.05       | 0.141       | 0.139       |
| 9                  | 0.1     | L     | H        | 20.25            | 19.15       | 19.11       | 0.147       | 0.142       |
| 10                 | 0.1     | L     | H        | 20.22            | 19.11       | 19.05       | 0.144       | 0.138       |
| 1                  | 0.1     | H     | L        | 20.66            | 19.97       | 19.89       | 0.110       | 0.105       |
| 2                  | 0.1     | H     | L        | 20.63            | 19.94       | 19.82       | 0.112       | 0.105       |
| 3                  | 0.1     | H     | L        | 20.72            | 20.09       | 20.00       | 0.110       | 0.105       |
| 4                  | 0.1     | H     | L        | 20.67            | 20.00       | 19.90       | 0.110       | 0.103       |
| 5                  | 0.1     | H     | L        | 20.73            | 20.12       | 20.01       | 0.114       | 0.108       |
| 6                  | 0.1     | H     | L        | 20.75            | 20.15       | 20.04       | 0.111       | 0.106       |
| 7                  | 0.1     | H     | L        | 20.68            | 20.03       | 19.92       | 0.113       | 0.105       |
| 8                  | 0.1     | H     | L        | 20.68            | 20.00       | 19.93       | 0.108       | 0.104       |
| 9                  | 0.1     | H     | L        | 20.71            | 20.09       | 19.97       | 0.113       | 0.107       |
| 10                 | 0.1     | H     | L        | 20.69            | 20.04       | 19.93       | 0.111       | 0.103       |
| 1                  | 0.1     | H     | H        | 20.78            | 20.20       | 20.07       | 0.146       | 0.138       |
| 2                  | 0.1     | H     | H        | 20.75            | 20.17       | 20.01       | 0.147       | 0.138       |
| 3                  | 0.1     | H     | H        | 20.85            | 20.31       | 20.20       | 0.145       | 0.139       |
| 4                  | 0.1     | H     | H        | 20.79            | 20.24       | 20.07       | 0.146       | 0.135       |
| 5                  | 0.1     | H     | H        | 20.86            | 20.35       | 20.20       | 0.149       | 0.141       |
| 6                  | 0.1     | H     | H        | 20.88            | 20.38       | 20.23       | 0.146       | 0.139       |
| 7                  | 0.1     | H     | H        | 20.81            | 20.28       | 20.10       | 0.149       | 0.138       |
| 8                  | 0.1     | H     | H        | 20.80            | 20.18       | 20.16       | 0.142       | 0.138       |
| 9                  | 0.1     | H     | H        | 20.84            | 20.32       | 20.16       | 0.149       | 0.140       |

Table B.2: Experimental results 2

| AEX Route, Var, F1 |         |       |          |                  |             |             |             |             |
|--------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 10                 | 0.1     | H     | H        | 20.81            | 20.26       | 20.12       | 0.146       | 0.137       |
| 1                  | 0.2     | L     | L        | 20.01            | 18.62       | 18.60       | 0.068       | 0.065       |
| 2                  | 0.2     | L     | L        | 19.99            | 18.56       | 18.54       | 0.069       | 0.065       |
| 3                  | 0.2     | L     | L        | 20.07            | 18.76       | 18.74       | 0.068       | 0.065       |
| 4                  | 0.2     | L     | L        | 20.02            | 18.65       | 18.63       | 0.067       | 0.064       |
| 5                  | 0.2     | L     | L        | 20.07            | 18.77       | 18.74       | 0.070       | 0.067       |
| 6                  | 0.2     | L     | L        | 20.09            | 18.82       | 18.79       | 0.069       | 0.065       |
| 7                  | 0.2     | L     | L        | 20.03            | 18.67       | 18.64       | 0.067       | 0.064       |
| 8                  | 0.2     | L     | L        | 20.03            | 18.68       | 18.65       | 0.067       | 0.065       |
| 9                  | 0.2     | L     | L        | 20.06            | 18.73       | 18.71       | 0.070       | 0.067       |
| 10                 | 0.2     | L     | L        | 20.04            | 18.68       | 18.65       | 0.066       | 0.064       |
| 1                  | 0.2     | L     | H        | 20.06            | 18.74       | 18.70       | 0.089       | 0.085       |
| 2                  | 0.2     | L     | H        | 20.03            | 18.68       | 18.64       | 0.090       | 0.085       |
| 3                  | 0.2     | L     | H        | 20.12            | 18.88       | 18.84       | 0.089       | 0.086       |
| 4                  | 0.2     | L     | H        | 20.07            | 18.76       | 18.73       | 0.088       | 0.084       |
| 5                  | 0.2     | L     | H        | 20.12            | 18.89       | 18.85       | 0.091       | 0.088       |
| 6                  | 0.2     | L     | H        | 20.15            | 18.94       | 18.89       | 0.090       | 0.086       |
| 7                  | 0.2     | L     | H        | 20.08            | 18.79       | 18.74       | 0.089       | 0.084       |
| 8                  | 0.2     | L     | H        | 20.08            | 18.80       | 18.75       | 0.088       | 0.085       |
| 9                  | 0.2     | L     | H        | 20.11            | 18.85       | 18.81       | 0.091       | 0.088       |
| 10                 | 0.2     | L     | H        | 20.08            | 18.80       | 18.76       | 0.087       | 0.084       |
| 1                  | 0.2     | H     | L        | 20.52            | 19.72       | 19.65       | 0.069       | 0.064       |
| 2                  | 0.2     | H     | L        | 20.50            | 19.66       | 19.60       | 0.070       | 0.063       |
| 3                  | 0.2     | H     | L        | 20.59            | 19.84       | 19.76       | 0.070       | 0.063       |
| 4                  | 0.2     | H     | L        | 20.53            | 19.73       | 19.67       | 0.069       | 0.062       |
| 5                  | 0.2     | H     | L        | 20.59            | 19.84       | 19.77       | 0.071       | 0.065       |
| 6                  | 0.2     | H     | L        | 20.61            | 19.88       | 19.80       | 0.070       | 0.064       |
| 7                  | 0.2     | H     | L        | 20.54            | 19.74       | 19.69       | 0.068       | 0.063       |
| 8                  | 0.2     | H     | L        | 20.55            | 19.76       | 19.69       | 0.068       | 0.063       |
| 9                  | 0.2     | H     | L        | 20.57            | 19.81       | 19.74       | 0.071       | 0.066       |
| 10                 | 0.2     | H     | L        | 20.55            | 19.76       | 19.69       | 0.067       | 0.062       |
| 1                  | 0.2     | H     | H        | 20.59            | 19.84       | 19.77       | 0.090       | 0.084       |
| 2                  | 0.2     | H     | H        | 20.56            | 19.79       | 19.71       | 0.092       | 0.083       |
| 3                  | 0.2     | H     | H        | 20.66            | 19.97       | 19.87       | 0.092       | 0.083       |
| 4                  | 0.2     | H     | H        | 20.60            | 19.86       | 19.79       | 0.090       | 0.082       |
| 5                  | 0.2     | H     | H        | 20.66            | 19.98       | 19.88       | 0.094       | 0.086       |
| 6                  | 0.2     | H     | H        | 20.68            | 20.01       | 19.92       | 0.091       | 0.085       |
| 7                  | 0.2     | H     | H        | 20.61            | 19.87       | 19.81       | 0.089       | 0.084       |
| 8                  | 0.2     | H     | H        | 20.61            | 19.89       | 19.80       | 0.090       | 0.083       |

Table B.2: Experimental results 2

| AEX Route, Var, F1 |         |       |          |                  |             |             |             |             |
|--------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 9                  | 0.2     | H     | H        | 20.64            | 19.94       | 19.86       | 0.093       | 0.086       |
| 10                 | 0.2     | H     | H        | 20.61            | 19.89       | 19.80       | 0.089       | 0.082       |

Table B.3: Experimental results 3

| AE Route, Var, F2 |         |       |          |                  |             |             |             |             |
|-------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep               | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 1                 | 0.1     | L     | L        | 18.62            | 20.37       | 19.72       | 0.162       | 0.111       |
| 2                 | 0.1     | L     | L        | 18.57            | 20.32       | 19.62       | 0.170       | 0.117       |
| 3                 | 0.1     | L     | L        | 18.67            | 20.44       | 19.80       | 0.163       | 0.115       |
| 4                 | 0.1     | L     | L        | 18.66            | 20.46       | 19.75       | 0.170       | 0.116       |
| 5                 | 0.1     | L     | L        | 18.69            | 20.50       | 19.78       | 0.172       | 0.117       |
| 6                 | 0.1     | L     | L        | 18.74            | 20.56       | 19.86       | 0.175       | 0.119       |
| 7                 | 0.1     | L     | L        | 18.65            | 20.44       | 19.73       | 0.174       | 0.122       |
| 8                 | 0.1     | L     | L        | 18.63            | 20.40       | 19.72       | 0.164       | 0.111       |
| 9                 | 0.1     | L     | L        | 18.72            | 20.53       | 19.83       | 0.168       | 0.117       |
| 10                | 0.1     | L     | L        | 18.65            | 20.43       | 19.74       | 0.166       | 0.115       |
| 1                 | 0.1     | L     | H        | 18.73            | 20.58       | 19.82       | 0.210       | 0.149       |
| 2                 | 0.1     | L     | H        | 18.68            | 20.54       | 19.76       | 0.219       | 0.155       |
| 3                 | 0.1     | L     | H        | 18.79            | 20.62       | 19.90       | 0.211       | 0.154       |
| 4                 | 0.1     | L     | H        | 18.78            | 20.62       | 19.88       | 0.217       | 0.156       |
| 5                 | 0.1     | L     | H        | 18.80            | 20.66       | 19.87       | 0.222       | 0.156       |
| 6                 | 0.1     | L     | H        | 18.86            | 20.71       | 20.00       | 0.225       | 0.160       |
| 7                 | 0.1     | L     | H        | 18.77            | 20.61       | 19.85       | 0.223       | 0.162       |
| 8                 | 0.1     | L     | H        | 18.74            | 20.58       | 19.82       | 0.212       | 0.150       |
| 9                 | 0.1     | L     | H        | 18.84            | 20.68       | 19.95       | 0.217       | 0.156       |
| 10                | 0.1     | L     | H        | 18.76            | 20.59       | 19.87       | 0.213       | 0.154       |
| 1                 | 0.1     | H     | L        | 19.17            | 20.94       | 20.46       | 0.156       | 0.115       |
| 2                 | 0.1     | H     | L        | 19.13            | 20.92       | 20.37       | 0.167       | 0.120       |
| 3                 | 0.1     | H     | L        | 19.21            | 20.97       | 20.51       | 0.157       | 0.119       |
| 4                 | 0.1     | H     | L        | 19.21            | 20.97       | 20.47       | 0.163       | 0.121       |
| 5                 | 0.1     | H     | L        | 19.23            | 20.98       | 20.47       | 0.165       | 0.123       |
| 6                 | 0.1     | H     | L        | 19.28            | 21.00       | 20.67       | 0.159       | 0.131       |
| 7                 | 0.1     | H     | L        | 19.20            | 20.96       | 20.44       | 0.170       | 0.125       |
| 8                 | 0.1     | H     | L        | 19.18            | 20.94       | 20.44       | 0.158       | 0.115       |
| 9                 | 0.1     | H     | L        | 19.26            | 20.99       | 20.53       | 0.161       | 0.122       |
| 10                | 0.1     | H     | L        | 19.20            | 20.96       | 20.46       | 0.160       | 0.119       |
| 1                 | 0.1     | H     | H        | 19.31            | 21.00       | 20.83       | 0.184       | 0.169       |
| 2                 | 0.1     | H     | H        | 19.27            | 21.00       | 20.61       | 0.204       | 0.166       |
| 3                 | 0.1     | H     | H        | 19.38            | 20.99       | 21.15       | 0.173       | 0.185       |
| 4                 | 0.1     | H     | H        | 19.37            | 20.98       | 21.15       | 0.177       | 0.189       |
| 5                 | 0.1     | H     | H        | 19.41            | 20.99       | 21.36       | 0.172       | 0.200       |
| 6                 | 0.1     | H     | H        | 19.51            | 21.00       | 21.85       | 0.159       | 0.225       |
| 7                 | 0.1     | H     | H        | 19.37            | 20.99       | 21.12       | 0.185       | 0.194       |
| 8                 | 0.1     | H     | H        | 19.32            | 21.00       | 20.89       | 0.183       | 0.172       |
| 9                 | 0.1     | H     | H        | 19.46            | 20.98       | 21.60       | 0.161       | 0.210       |

Table B.3: Experimental results 3

| AE Route, Var, F2 |         |       |          |                  |             |             |             |             |
|-------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep               | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 10                | 0.1     | H     | H        | 19.35            | 21.00       | 21.03       | 0.180       | 0.181       |
| 1                 | 0.2     | L     | L        | 18.48            | 19.79       | 19.67       | 0.089       | 0.064       |
| 2                 | 0.2     | L     | L        | 18.43            | 19.73       | 19.57       | 0.098       | 0.066       |
| 3                 | 0.2     | L     | L        | 18.52            | 19.88       | 19.73       | 0.095       | 0.064       |
| 4                 | 0.2     | L     | L        | 18.51            | 19.85       | 19.71       | 0.096       | 0.066       |
| 5                 | 0.2     | L     | L        | 18.53            | 19.89       | 19.74       | 0.097       | 0.067       |
| 6                 | 0.2     | L     | L        | 18.58            | 19.98       | 19.82       | 0.100       | 0.068       |
| 7                 | 0.2     | L     | L        | 18.49            | 19.83       | 19.68       | 0.098       | 0.069       |
| 8                 | 0.2     | L     | L        | 18.48            | 19.79       | 19.68       | 0.086       | 0.065       |
| 9                 | 0.2     | L     | L        | 18.57            | 19.94       | 19.80       | 0.096       | 0.066       |
| 10                | 0.2     | L     | L        | 18.50            | 19.85       | 19.70       | 0.096       | 0.065       |
| 1                 | 0.2     | L     | H        | 18.54            | 19.91       | 19.75       | 0.117       | 0.084       |
| 2                 | 0.2     | L     | H        | 18.49            | 19.86       | 19.65       | 0.129       | 0.087       |
| 3                 | 0.2     | L     | H        | 18.58            | 20.01       | 19.81       | 0.124       | 0.085       |
| 4                 | 0.2     | L     | H        | 18.57            | 19.98       | 19.80       | 0.125       | 0.087       |
| 5                 | 0.2     | L     | H        | 18.59            | 20.01       | 19.83       | 0.125       | 0.088       |
| 6                 | 0.2     | L     | H        | 18.64            | 20.10       | 19.90       | 0.130       | 0.090       |
| 7                 | 0.2     | L     | H        | 18.55            | 19.95       | 19.77       | 0.128       | 0.091       |
| 8                 | 0.2     | L     | H        | 18.54            | 19.91       | 19.76       | 0.113       | 0.086       |
| 9                 | 0.2     | L     | H        | 18.63            | 20.07       | 19.88       | 0.126       | 0.086       |
| 10                | 0.2     | L     | H        | 18.56            | 19.96       | 19.78       | 0.124       | 0.086       |
| 1                 | 0.2     | H     | L        | 18.99            | 20.60       | 20.45       | 0.089       | 0.064       |
| 2                 | 0.2     | H     | L        | 18.95            | 20.56       | 20.38       | 0.098       | 0.066       |
| 3                 | 0.2     | H     | L        | 19.04            | 20.67       | 20.50       | 0.095       | 0.064       |
| 4                 | 0.2     | H     | L        | 19.02            | 20.65       | 20.49       | 0.096       | 0.066       |
| 5                 | 0.2     | H     | L        | 19.04            | 20.68       | 20.52       | 0.097       | 0.066       |
| 6                 | 0.2     | H     | L        | 19.09            | 20.76       | 20.57       | 0.102       | 0.068       |
| 7                 | 0.2     | H     | L        | 19.01            | 20.63       | 20.47       | 0.098       | 0.069       |
| 8                 | 0.2     | H     | L        | 19.00            | 20.60       | 20.46       | 0.087       | 0.064       |
| 9                 | 0.2     | H     | L        | 19.08            | 20.72       | 20.56       | 0.096       | 0.066       |
| 10                | 0.2     | H     | L        | 19.02            | 20.64       | 20.48       | 0.096       | 0.065       |
| 1                 | 0.2     | H     | H        | 19.06            | 20.71       | 20.54       | 0.118       | 0.084       |
| 2                 | 0.2     | H     | H        | 19.02            | 20.68       | 20.45       | 0.129       | 0.087       |
| 3                 | 0.2     | H     | H        | 19.11            | 20.78       | 20.59       | 0.122       | 0.086       |
| 4                 | 0.2     | H     | H        | 19.10            | 20.76       | 20.58       | 0.125       | 0.087       |
| 5                 | 0.2     | H     | H        | 19.12            | 20.79       | 20.61       | 0.124       | 0.088       |
| 6                 | 0.2     | H     | H        | 19.17            | 20.86       | 20.66       | 0.129       | 0.091       |
| 7                 | 0.2     | H     | H        | 19.08            | 20.75       | 20.56       | 0.127       | 0.091       |
| 8                 | 0.2     | H     | H        | 19.07            | 20.71       | 20.54       | 0.115       | 0.085       |

Table B.3: Experimental results 3

| AE Route, Var, F2 |         |       |          |                  |             |             |             |             |
|-------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep               | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 9                 | 0.2     | H     | H        | 19.15            | 20.83       | 20.65       | 0.125       | 0.087       |
| 10                | 0.2     | H     | H        | 19.09            | 20.76       | 20.56       | 0.125       | 0.086       |



Table B.4: Experimental results 4

| AEX Route, Var, F2 |         |       |          |                  |             |             |             |             |
|--------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 1                  | 0.1     | L     | L        | 19.60            | 19.07       | 18.66       | 0.121       | 0.095       |
| 2                  | 0.1     | L     | L        | 19.56            | 19.02       | 18.60       | 0.122       | 0.096       |
| 3                  | 0.1     | L     | L        | 19.68            | 19.22       | 18.81       | 0.122       | 0.095       |
| 4                  | 0.1     | L     | L        | 19.61            | 19.08       | 18.70       | 0.118       | 0.096       |
| 5                  | 0.1     | L     | L        | 19.69            | 19.21       | 18.83       | 0.123       | 0.100       |
| 6                  | 0.1     | L     | L        | 19.71            | 19.28       | 18.86       | 0.122       | 0.096       |
| 7                  | 0.1     | L     | L        | 19.63            | 19.13       | 18.72       | 0.122       | 0.097       |
| 8                  | 0.1     | L     | L        | 19.62            | 19.08       | 18.74       | 0.118       | 0.095       |
| 9                  | 0.1     | L     | L        | 19.66            | 19.17       | 18.79       | 0.122       | 0.099       |
| 10                 | 0.1     | L     | L        | 19.63            | 19.14       | 18.73       | 0.121       | 0.095       |
| 1                  | 0.1     | L     | H        | 19.70            | 19.27       | 18.84       | 0.158       | 0.127       |
| 2                  | 0.1     | L     | H        | 19.67            | 19.23       | 18.76       | 0.159       | 0.127       |
| 3                  | 0.1     | L     | H        | 19.79            | 19.42       | 18.97       | 0.158       | 0.126       |
| 4                  | 0.1     | L     | H        | 19.72            | 19.28       | 18.88       | 0.154       | 0.128       |
| 5                  | 0.1     | L     | H        | 19.80            | 19.42       | 19.00       | 0.160       | 0.131       |
| 6                  | 0.1     | L     | H        | 19.83            | 19.50       | 19.02       | 0.159       | 0.127       |
| 7                  | 0.1     | L     | H        | 19.74            | 19.33       | 18.89       | 0.159       | 0.128       |
| 8                  | 0.1     | L     | H        | 19.73            | 19.28       | 18.91       | 0.154       | 0.126       |
| 9                  | 0.1     | L     | H        | 19.77            | 19.41       | 18.94       | 0.159       | 0.131       |
| 10                 | 0.1     | L     | H        | 19.74            | 19.38       | 18.87       | 0.158       | 0.125       |
| 1                  | 0.1     | H     | L        | 20.29            | 20.18       | 19.72       | 0.122       | 0.095       |
| 2                  | 0.1     | H     | L        | 20.26            | 20.15       | 19.67       | 0.123       | 0.096       |
| 3                  | 0.1     | H     | L        | 20.37            | 20.30       | 19.83       | 0.123       | 0.094       |
| 4                  | 0.1     | H     | L        | 20.31            | 20.19       | 19.74       | 0.120       | 0.095       |
| 5                  | 0.1     | H     | L        | 20.38            | 20.30       | 19.84       | 0.124       | 0.098       |
| 6                  | 0.1     | H     | L        | 20.41            | 20.35       | 19.91       | 0.123       | 0.096       |
| 7                  | 0.1     | H     | L        | 20.33            | 20.23       | 19.76       | 0.123       | 0.096       |
| 8                  | 0.1     | H     | L        | 20.32            | 20.19       | 19.78       | 0.119       | 0.095       |
| 9                  | 0.1     | H     | L        | 20.36            | 20.26       | 19.82       | 0.123       | 0.098       |
| 10                 | 0.1     | H     | L        | 20.33            | 20.23       | 19.76       | 0.122       | 0.094       |
| 1                  | 0.1     | H     | H        | 20.44            | 20.39       | 19.95       | 0.158       | 0.126       |
| 2                  | 0.1     | H     | H        | 20.40            | 20.39       | 19.83       | 0.161       | 0.126       |
| 3                  | 0.1     | H     | H        | 20.52            | 20.53       | 19.95       | 0.159       | 0.125       |
| 4                  | 0.1     | H     | H        | 20.45            | 20.41       | 19.93       | 0.156       | 0.126       |
| 5                  | 0.1     | H     | H        | 20.53            | 20.53       | 20.02       | 0.162       | 0.129       |
| 6                  | 0.1     | H     | H        | 20.56            | 20.57       | 20.04       | 0.159       | 0.127       |
| 7                  | 0.1     | H     | H        | 20.47            | 20.45       | 19.93       | 0.161       | 0.128       |
| 8                  | 0.1     | H     | H        | 20.46            | 20.42       | 19.94       | 0.156       | 0.125       |
| 9                  | 0.1     | H     | H        | 20.51            | 20.49       | 20.00       | 0.161       | 0.130       |

Table B.4: Experimental results 4

| AEX Route, Var, F2 |         |       |          |                  |             |             |             |             |
|--------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 10                 | 0.1     | H     | H        | 20.47            | 20.47       | 19.94       | 0.159       | 0.125       |
| 1                  | 0.2     | L     | L        | 19.47            | 18.73       | 18.52       | 0.076       | 0.057       |
| 2                  | 0.2     | L     | L        | 19.44            | 18.68       | 18.45       | 0.077       | 0.057       |
| 3                  | 0.2     | L     | L        | 19.55            | 18.90       | 18.64       | 0.078       | 0.056       |
| 4                  | 0.2     | L     | L        | 19.49            | 18.75       | 18.55       | 0.075       | 0.055       |
| 5                  | 0.2     | L     | L        | 19.55            | 18.89       | 18.65       | 0.080       | 0.057       |
| 6                  | 0.2     | L     | L        | 19.58            | 18.97       | 18.68       | 0.079       | 0.056       |
| 7                  | 0.2     | L     | L        | 19.50            | 18.76       | 18.58       | 0.075       | 0.057       |
| 8                  | 0.2     | L     | L        | 19.50            | 18.81       | 18.56       | 0.077       | 0.056       |
| 9                  | 0.2     | L     | L        | 19.53            | 18.86       | 18.61       | 0.080       | 0.057       |
| 10                 | 0.2     | L     | L        | 19.50            | 18.78       | 18.58       | 0.074       | 0.056       |
| 1                  | 0.2     | L     | H        | 19.53            | 18.88       | 18.60       | 0.100       | 0.075       |
| 2                  | 0.2     | L     | H        | 19.50            | 18.80       | 18.55       | 0.101       | 0.075       |
| 3                  | 0.2     | L     | H        | 19.61            | 19.03       | 18.74       | 0.101       | 0.074       |
| 4                  | 0.2     | L     | H        | 19.55            | 18.91       | 18.63       | 0.099       | 0.073       |
| 5                  | 0.2     | L     | H        | 19.62            | 19.03       | 18.74       | 0.104       | 0.076       |
| 6                  | 0.2     | L     | H        | 19.64            | 19.11       | 18.77       | 0.102       | 0.074       |
| 7                  | 0.2     | L     | H        | 19.56            | 18.89       | 18.67       | 0.098       | 0.076       |
| 8                  | 0.2     | L     | H        | 19.56            | 18.95       | 18.64       | 0.100       | 0.074       |
| 9                  | 0.2     | L     | H        | 19.60            | 19.00       | 18.70       | 0.103       | 0.076       |
| 10                 | 0.2     | L     | H        | 19.57            | 18.92       | 18.67       | 0.097       | 0.074       |
| 1                  | 0.2     | H     | L        | 20.12            | 19.83       | 19.57       | 0.077       | 0.056       |
| 2                  | 0.2     | H     | L        | 20.09            | 19.79       | 19.51       | 0.078       | 0.056       |
| 3                  | 0.2     | H     | L        | 20.20            | 19.97       | 19.67       | 0.079       | 0.056       |
| 4                  | 0.2     | H     | L        | 20.14            | 19.85       | 19.59       | 0.076       | 0.055       |
| 5                  | 0.2     | H     | L        | 20.21            | 19.96       | 19.68       | 0.081       | 0.056       |
| 6                  | 0.2     | H     | L        | 20.23            | 20.02       | 19.71       | 0.079       | 0.056       |
| 7                  | 0.2     | H     | L        | 20.15            | 19.84       | 19.62       | 0.076       | 0.056       |
| 8                  | 0.2     | H     | L        | 20.15            | 19.90       | 19.59       | 0.077       | 0.055       |
| 9                  | 0.2     | H     | L        | 20.19            | 19.94       | 19.65       | 0.081       | 0.057       |
| 10                 | 0.2     | H     | L        | 20.16            | 19.86       | 19.62       | 0.075       | 0.056       |
| 1                  | 0.2     | H     | H        | 20.21            | 19.99       | 19.67       | 0.101       | 0.075       |
| 2                  | 0.2     | H     | H        | 20.17            | 19.95       | 19.61       | 0.102       | 0.074       |
| 3                  | 0.2     | H     | H        | 20.29            | 20.12       | 19.77       | 0.103       | 0.073       |
| 4                  | 0.2     | H     | H        | 20.22            | 20.01       | 19.68       | 0.100       | 0.073       |
| 5                  | 0.2     | H     | H        | 20.29            | 20.12       | 19.78       | 0.106       | 0.074       |
| 6                  | 0.2     | H     | H        | 20.32            | 20.19       | 19.80       | 0.103       | 0.073       |
| 7                  | 0.2     | H     | H        | 20.23            | 20.00       | 19.72       | 0.100       | 0.074       |
| 8                  | 0.2     | H     | H        | 20.24            | 20.06       | 19.69       | 0.101       | 0.073       |

Table B.4: Experimental results 4

| AEX Route, Var, F2 |         |       |          |                  |             |             |             |             |
|--------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 9                  | 0.2     | H     | H        | 20.27            | 20.11       | 19.74       | 0.106       | 0.075       |
| 10                 | 0.2     | H     | H        | 20.24            | 20.02       | 19.72       | 0.099       | 0.073       |

Table B.5: Experimental results 5

| AE Route, Equal, F1 |         |       |          |                          |             |             |             |             |
|---------------------|---------|-------|----------|--------------------------|-------------|-------------|-------------|-------------|
| Rep                 | $\beta$ | $\mu$ | $\sigma$ | obj (\$10 <sup>6</sup> ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 1                   | 0.1     | L     | L        | 18.68                    | 20.09       | 20.09       | 0.165       | 0.165       |
| 2                   | 0.1     | L     | L        | 18.63                    | 19.98       | 19.98       | 0.165       | 0.165       |
| 3                   | 0.1     | L     | L        | 18.72                    | 20.17       | 20.17       | 0.169       | 0.169       |
| 4                   | 0.1     | L     | L        | 18.71                    | 20.14       | 20.14       | 0.168       | 0.168       |
| 5                   | 0.1     | L     | L        | 18.73                    | 20.19       | 20.19       | 0.173       | 0.173       |
| 6                   | 0.1     | L     | L        | 18.76                    | 20.25       | 20.25       | 0.170       | 0.170       |
| 7                   | 0.1     | L     | L        | 18.70                    | 20.13       | 20.13       | 0.176       | 0.176       |
| 8                   | 0.1     | L     | L        | 18.69                    | 20.11       | 20.11       | 0.168       | 0.168       |
| 9                   | 0.1     | L     | L        | 18.76                    | 20.24       | 20.24       | 0.170       | 0.170       |
| 10                  | 0.1     | L     | L        | 18.70                    | 20.13       | 20.13       | 0.169       | 0.169       |
| 1                   | 0.1     | L     | H        | 18.80                    | 20.31       | 20.31       | 0.218       | 0.218       |
| 2                   | 0.1     | L     | H        | 18.74                    | 20.20       | 20.20       | 0.218       | 0.218       |
| 3                   | 0.1     | L     | H        | 18.84                    | 20.39       | 20.39       | 0.221       | 0.221       |
| 4                   | 0.1     | L     | H        | 18.82                    | 20.36       | 20.36       | 0.221       | 0.221       |
| 5                   | 0.1     | L     | H        | 18.85                    | 20.41       | 20.41       | 0.226       | 0.226       |
| 6                   | 0.1     | L     | H        | 18.88                    | 20.47       | 20.47       | 0.223       | 0.223       |
| 7                   | 0.1     | L     | H        | 18.82                    | 20.35       | 20.35       | 0.229       | 0.229       |
| 8                   | 0.1     | L     | H        | 18.81                    | 20.33       | 20.33       | 0.221       | 0.221       |
| 9                   | 0.1     | L     | H        | 18.88                    | 20.46       | 20.46       | 0.222       | 0.222       |
| 10                  | 0.1     | L     | H        | 18.82                    | 20.35       | 20.35       | 0.222       | 0.222       |
| 1                   | 0.1     | H     | L        | 19.13                    | 20.91       | 20.91       | 0.165       | 0.165       |
| 2                   | 0.1     | H     | L        | 19.08                    | 20.83       | 20.83       | 0.165       | 0.165       |
| 3                   | 0.1     | H     | L        | 19.17                    | 20.98       | 20.98       | 0.169       | 0.169       |
| 4                   | 0.1     | H     | L        | 19.16                    | 20.96       | 20.96       | 0.168       | 0.168       |
| 5                   | 0.1     | H     | L        | -                        | -           | -           | -           | -           |
| 6                   | 0.1     | H     | L        | -                        | -           | -           | -           | -           |
| 7                   | 0.1     | H     | L        | -                        | -           | -           | -           | -           |
| 8                   | 0.1     | H     | L        | -                        | -           | -           | -           | -           |
| 9                   | 0.1     | H     | L        | -                        | -           | -           | -           | -           |
| 10                  | 0.1     | H     | L        | -                        | -           | -           | -           | -           |
| 1                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 2                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 3                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 4                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 5                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 6                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 7                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 8                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 9                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |

Table B.5: Experimental results 5

| AE Route, Equal, F1 |         |       |          |                  |             |             |             |             |
|---------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                 | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 10                  | 0.1     | H     | H        | -                | -           | -           | -           | -           |
| 1                   | 0.2     | L     | L        | 18.56            | 19.86       | 19.86       | 0.109       | 0.109       |
| 2                   | 0.2     | L     | L        | 18.51            | 19.75       | 19.75       | 0.109       | 0.109       |
| 3                   | 0.2     | L     | L        | 18.60            | 19.93       | 19.93       | 0.111       | 0.111       |
| 4                   | 0.2     | L     | L        | 18.58            | 19.90       | 19.90       | 0.110       | 0.110       |
| 5                   | 0.2     | L     | L        | 18.61            | 19.94       | 19.94       | 0.113       | 0.113       |
| 6                   | 0.2     | L     | L        | 18.64            | 20.00       | 20.00       | 0.111       | 0.111       |
| 7                   | 0.2     | L     | L        | 18.57            | 19.88       | 19.88       | 0.115       | 0.115       |
| 8                   | 0.2     | L     | L        | 18.57            | 19.87       | 19.87       | 0.110       | 0.110       |
| 9                   | 0.2     | L     | L        | 18.63            | 19.99       | 19.99       | 0.111       | 0.111       |
| 10                  | 0.2     | L     | L        | 18.58            | 19.89       | 19.89       | 0.111       | 0.111       |
| 1                   | 0.2     | L     | H        | 18.63            | 20.00       | 20.00       | 0.143       | 0.143       |
| 2                   | 0.2     | L     | H        | 18.58            | 19.89       | 19.89       | 0.143       | 0.143       |
| 3                   | 0.2     | L     | H        | 18.67            | 20.07       | 20.07       | 0.145       | 0.145       |
| 4                   | 0.2     | L     | H        | 18.66            | 20.04       | 20.04       | 0.145       | 0.145       |
| 5                   | 0.2     | L     | H        | 18.68            | 20.09       | 20.09       | 0.148       | 0.148       |
| 6                   | 0.2     | L     | H        | 18.71            | 20.15       | 20.15       | 0.146       | 0.146       |
| 7                   | 0.2     | L     | H        | 18.65            | 20.02       | 20.02       | 0.151       | 0.151       |
| 8                   | 0.2     | L     | H        | 18.64            | 20.01       | 20.01       | 0.145       | 0.145       |
| 9                   | 0.2     | L     | H        | 18.71            | 20.14       | 20.14       | 0.146       | 0.146       |
| 10                  | 0.2     | L     | H        | 18.65            | 20.03       | 20.03       | 0.146       | 0.146       |
| 1                   | 0.2     | H     | L        | 18.99            | 20.66       | 20.66       | 0.109       | 0.109       |
| 2                   | 0.2     | H     | L        | 18.94            | 20.57       | 20.57       | 0.109       | 0.109       |
| 3                   | 0.2     | H     | L        | 19.02            | 20.72       | 20.72       | 0.111       | 0.111       |
| 4                   | 0.2     | H     | L        | 19.01            | 20.70       | 20.70       | 0.110       | 0.110       |
| 5                   | 0.2     | H     | L        | 19.03            | 20.73       | 20.73       | 0.113       | 0.113       |
| 6                   | 0.2     | H     | L        | 19.06            | 20.78       | 20.78       | 0.111       | 0.111       |
| 7                   | 0.2     | H     | L        | 19.00            | 20.68       | 20.68       | 0.115       | 0.115       |
| 8                   | 0.2     | H     | L        | 19.00            | 20.67       | 20.67       | 0.110       | 0.110       |
| 9                   | 0.2     | H     | L        | 19.05            | 20.78       | 20.78       | 0.111       | 0.111       |
| 10                  | 0.2     | H     | L        | 19.01            | 20.69       | 20.69       | 0.111       | 0.111       |
| 1                   | 0.2     | H     | H        | 19.08            | 20.81       | 20.81       | 0.143       | 0.143       |
| 2                   | 0.2     | H     | H        | 19.02            | 20.72       | 20.72       | 0.143       | 0.143       |
| 3                   | 0.2     | H     | H        | 19.11            | 20.87       | 20.87       | 0.145       | 0.145       |
| 4                   | 0.2     | H     | H        | 19.10            | 20.85       | 20.85       | 0.145       | 0.145       |
| 5                   | 0.2     | H     | H        | 19.12            | 20.89       | 20.89       | 0.148       | 0.148       |
| 6                   | 0.2     | H     | H        | 19.15            | 20.94       | 20.94       | 0.146       | 0.146       |
| 7                   | 0.2     | H     | H        | 19.09            | 20.84       | 20.84       | 0.151       | 0.151       |
| 8                   | 0.2     | H     | H        | 19.08            | 20.83       | 20.83       | 0.145       | 0.145       |

Table B.5: Experimental results 5

| AE Route, Equal, F1 |         |       |          |                  |             |             |             |             |
|---------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                 | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 9                   | 0.2     | H     | H        | 19.14            | 20.93       | 20.93       | 0.146       | 0.146       |
| 10                  | 0.2     | H     | H        | 19.09            | 20.85       | 20.85       | 0.146       | 0.146       |

Table B.6: Experimental results 6

| AEX Route, Equal, F1 |         |       |          |                  |             |             |             |             |
|----------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                  | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 1                    | 0.1     | L     | L        | 20.27            | 19.19       | 19.19       | 0.172       | 0.172       |
| 2                    | 0.1     | L     | L        | 20.23            | 19.10       | 19.10       | 0.169       | 0.169       |
| 3                    | 0.1     | L     | L        | 20.33            | 19.30       | 19.30       | 0.167       | 0.167       |
| 4                    | 0.1     | L     | L        | 20.27            | 19.19       | 19.19       | 0.167       | 0.167       |
| 5                    | 0.1     | L     | L        | 20.34            | 19.32       | 19.32       | 0.170       | 0.170       |
| 6                    | 0.1     | L     | L        | 20.35            | 19.35       | 19.35       | 0.165       | 0.165       |
| 7                    | 0.1     | L     | L        | 20.29            | 19.23       | 19.23       | 0.172       | 0.172       |
| 8                    | 0.1     | L     | L        | 20.29            | 19.22       | 19.22       | 0.169       | 0.169       |
| 9                    | 0.1     | L     | L        | 20.33            | 19.30       | 19.30       | 0.175       | 0.175       |
| 10                   | 0.1     | L     | L        | 20.30            | 19.24       | 19.24       | 0.170       | 0.170       |
| 1                    | 0.1     | L     | H        | 20.42            | 19.49       | 19.49       | 0.226       | 0.226       |
| 2                    | 0.1     | L     | H        | 20.38            | 19.40       | 19.40       | 0.222       | 0.222       |
| 3                    | 0.1     | L     | H        | 20.48            | 19.61       | 19.61       | 0.220       | 0.220       |
| 4                    | 0.1     | L     | H        | 20.42            | 19.49       | 19.49       | 0.220       | 0.220       |
| 5                    | 0.1     | L     | H        | 20.49            | 19.62       | 19.62       | 0.223       | 0.223       |
| 6                    | 0.1     | L     | H        | 20.51            | 19.65       | 19.65       | 0.217       | 0.217       |
| 7                    | 0.1     | L     | H        | 20.45            | 19.54       | 19.54       | 0.225       | 0.225       |
| 8                    | 0.1     | L     | H        | 20.44            | 19.52       | 19.52       | 0.221       | 0.221       |
| 9                    | 0.1     | L     | H        | 20.49            | 19.61       | 19.61       | 0.229       | 0.229       |
| 10                   | 0.1     | L     | H        | 20.45            | 19.54       | 19.54       | 0.223       | 0.223       |
| 1                    | 0.1     | H     | L        | 20.89            | 20.33       | 20.33       | 0.172       | 0.172       |
| 2                    | 0.1     | H     | L        | 20.85            | 20.25       | 20.25       | 0.169       | 0.169       |
| 3                    | 0.1     | H     | L        | 20.95            | 20.42       | 20.42       | 0.167       | 0.167       |
| 4                    | 0.1     | H     | L        | 20.89            | 20.32       | 20.32       | 0.167       | 0.167       |
| 5                    | 0.1     | H     | L        | 20.95            | 20.43       | 20.43       | 0.170       | 0.170       |
| 6                    | 0.1     | H     | L        | 20.96            | 20.45       | 20.45       | 0.165       | 0.165       |
| 7                    | 0.1     | H     | L        | 20.91            | 20.36       | 20.36       | 0.172       | 0.172       |
| 8                    | 0.1     | H     | L        | 20.91            | 20.35       | 20.35       | 0.169       | 0.169       |
| 9                    | 0.1     | H     | L        | 20.95            | 20.43       | 20.43       | 0.175       | 0.175       |
| 10                   | 0.1     | H     | L        | 20.92            | 20.37       | 20.37       | 0.170       | 0.170       |
| 1                    | 0.1     | H     | H        | 21.10            | 20.67       | 20.67       | 0.226       | 0.226       |
| 2                    | 0.1     | H     | H        | 21.05            | 20.58       | 20.58       | 0.222       | 0.222       |
| 3                    | 0.1     | H     | H        | 21.16            | 20.75       | 20.75       | 0.220       | 0.220       |
| 4                    | 0.1     | H     | H        | 21.10            | 20.66       | 20.66       | 0.220       | 0.220       |
| 5                    | 0.1     | H     | H        | 21.17            | 20.77       | 20.77       | 0.223       | 0.223       |
| 6                    | 0.1     | H     | H        | 21.18            | 20.78       | 20.78       | 0.217       | 0.217       |
| 7                    | 0.1     | H     | H        | 21.13            | 20.71       | 20.71       | 0.225       | 0.225       |
| 8                    | 0.1     | H     | H        | 21.12            | 20.69       | 20.69       | 0.221       | 0.221       |
| 9                    | 0.1     | H     | H        | 21.17            | 20.77       | 20.77       | 0.229       | 0.229       |

Table B.6: Experimental results 6

| AEX Route, Equal, F1 |         |       |          |                  |             |             |             |             |
|----------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                  | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 10                   | 0.1     | H     | H        | 21.13            | 20.71       | 20.71       | 0.223       | 0.223       |
| 1                    | 0.2     | L     | L        | 20.12            | 18.86       | 18.86       | 0.113       | 0.113       |
| 2                    | 0.2     | L     | L        | 20.09            | 18.79       | 18.79       | 0.111       | 0.111       |
| 3                    | 0.2     | L     | L        | 20.18            | 18.98       | 18.98       | 0.110       | 0.110       |
| 4                    | 0.2     | L     | L        | 20.13            | 18.87       | 18.87       | 0.110       | 0.110       |
| 5                    | 0.2     | L     | L        | 20.18            | 18.99       | 18.99       | 0.112       | 0.112       |
| 6                    | 0.2     | L     | L        | 20.20            | 19.03       | 19.03       | 0.109       | 0.109       |
| 7                    | 0.2     | L     | L        | 20.14            | 18.91       | 18.91       | 0.113       | 0.113       |
| 8                    | 0.2     | L     | L        | 20.14            | 18.90       | 18.90       | 0.111       | 0.111       |
| 9                    | 0.2     | L     | L        | 20.17            | 18.97       | 18.97       | 0.115       | 0.115       |
| 10                   | 0.2     | L     | L        | 20.15            | 18.92       | 18.92       | 0.112       | 0.112       |
| 1                    | 0.2     | L     | H        | 20.21            | 19.05       | 19.05       | 0.148       | 0.148       |
| 2                    | 0.2     | L     | H        | 20.17            | 18.97       | 18.97       | 0.146       | 0.146       |
| 3                    | 0.2     | L     | H        | 20.27            | 19.18       | 19.18       | 0.144       | 0.144       |
| 4                    | 0.2     | L     | H        | 20.21            | 19.06       | 19.06       | 0.144       | 0.144       |
| 5                    | 0.2     | L     | H        | 20.27            | 19.18       | 19.18       | 0.146       | 0.146       |
| 6                    | 0.2     | L     | H        | 20.29            | 19.22       | 19.22       | 0.143       | 0.143       |
| 7                    | 0.2     | L     | H        | 20.23            | 19.10       | 19.10       | 0.148       | 0.148       |
| 8                    | 0.2     | L     | H        | 20.23            | 19.09       | 19.09       | 0.145       | 0.145       |
| 9                    | 0.2     | L     | H        | 20.26            | 19.16       | 19.16       | 0.150       | 0.150       |
| 10                   | 0.2     | L     | H        | 20.23            | 19.11       | 19.11       | 0.146       | 0.146       |
| 1                    | 0.2     | H     | L        | 20.68            | 19.96       | 19.96       | 0.113       | 0.113       |
| 2                    | 0.2     | H     | L        | 20.64            | 19.89       | 19.89       | 0.111       | 0.111       |
| 3                    | 0.2     | H     | L        | 20.73            | 20.06       | 20.06       | 0.110       | 0.110       |
| 4                    | 0.2     | H     | L        | 20.68            | 19.97       | 19.97       | 0.110       | 0.110       |
| 5                    | 0.2     | H     | L        | 20.74            | 20.07       | 20.07       | 0.112       | 0.112       |
| 6                    | 0.2     | H     | L        | 20.75            | 20.09       | 20.09       | 0.109       | 0.109       |
| 7                    | 0.2     | H     | L        | 20.70            | 20.00       | 20.00       | 0.113       | 0.113       |
| 8                    | 0.2     | H     | L        | 20.69            | 19.99       | 19.99       | 0.111       | 0.111       |
| 9                    | 0.2     | H     | L        | 20.73            | 20.05       | 20.05       | 0.115       | 0.115       |
| 10                   | 0.2     | H     | L        | 20.70            | 20.00       | 20.00       | 0.112       | 0.112       |
| 1                    | 0.2     | H     | H        | 20.80            | 20.17       | 20.17       | 0.148       | 0.148       |
| 2                    | 0.2     | H     | H        | 20.76            | 20.10       | 20.10       | 0.146       | 0.146       |
| 3                    | 0.2     | H     | H        | 20.86            | 20.27       | 20.27       | 0.144       | 0.144       |
| 4                    | 0.2     | H     | H        | 20.80            | 20.18       | 20.18       | 0.144       | 0.144       |
| 5                    | 0.2     | H     | H        | 20.86            | 20.28       | 20.28       | 0.146       | 0.146       |
| 6                    | 0.2     | H     | H        | 20.88            | 20.30       | 20.30       | 0.143       | 0.143       |
| 7                    | 0.2     | H     | H        | 20.82            | 20.21       | 20.21       | 0.148       | 0.148       |
| 8                    | 0.2     | H     | H        | 20.82            | 20.20       | 20.20       | 0.145       | 0.145       |



Table B.6: Experimental results 6

| AEX Route, Equal, F1 |         |       |          |                  |             |             |             |             |
|----------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                  | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 9                    | 0.2     | H     | H        | 20.86            | 20.27       | 20.27       | 0.150       | 0.150       |
| 10                   | 0.2     | H     | H        | 20.83            | 20.22       | 20.22       | 0.146       | 0.146       |

Table B.7: Experimental results 7

| AE Route, Equal, F2 |         |       |          |                          |             |             |             |             |
|---------------------|---------|-------|----------|--------------------------|-------------|-------------|-------------|-------------|
| Rep                 | $\beta$ | $\mu$ | $\sigma$ | obj (\$10 <sup>6</sup> ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 1                   | 0.1     | L     | L        | 18.72                    | 20.09       | 20.09       | 0.165       | 0.165       |
| 2                   | 0.1     | L     | L        | 18.65                    | 19.98       | 19.98       | 0.165       | 0.165       |
| 3                   | 0.1     | L     | L        | 18.77                    | 20.17       | 20.17       | 0.169       | 0.169       |
| 4                   | 0.1     | L     | L        | 18.75                    | 20.14       | 20.14       | 0.168       | 0.168       |
| 5                   | 0.1     | L     | L        | 18.78                    | 20.19       | 20.19       | 0.173       | 0.173       |
| 6                   | 0.1     | L     | L        | 18.82                    | 20.25       | 20.25       | 0.170       | 0.170       |
| 7                   | 0.1     | L     | L        | 18.74                    | 20.13       | 20.13       | 0.176       | 0.176       |
| 8                   | 0.1     | L     | L        | 18.73                    | 20.11       | 20.11       | 0.168       | 0.168       |
| 9                   | 0.1     | L     | L        | 18.82                    | 20.24       | 20.24       | 0.170       | 0.170       |
| 10                  | 0.1     | L     | L        | 18.75                    | 20.13       | 20.13       | 0.169       | 0.169       |
| 1                   | 0.1     | L     | H        | 18.87                    | 20.31       | 20.31       | 0.218       | 0.218       |
| 2                   | 0.1     | L     | H        | 18.79                    | 20.20       | 20.20       | 0.218       | 0.218       |
| 3                   | 0.1     | L     | H        | 18.92                    | 20.39       | 20.39       | 0.221       | 0.221       |
| 4                   | 0.1     | L     | H        | 18.90                    | 20.36       | 20.36       | 0.221       | 0.221       |
| 5                   | 0.1     | L     | H        | 18.94                    | 20.41       | 20.41       | 0.226       | 0.226       |
| 6                   | 0.1     | L     | H        | 18.98                    | 20.47       | 20.47       | 0.223       | 0.223       |
| 7                   | 0.1     | L     | H        | 18.89                    | 20.35       | 20.35       | 0.229       | 0.229       |
| 8                   | 0.1     | L     | H        | 18.88                    | 20.33       | 20.33       | 0.221       | 0.221       |
| 9                   | 0.1     | L     | H        | 18.97                    | 20.46       | 20.46       | 0.222       | 0.222       |
| 10                  | 0.1     | L     | H        | 18.89                    | 20.35       | 20.35       | 0.222       | 0.222       |
| 1                   | 0.1     | H     | L        | 19.29                    | 20.91       | 20.91       | 0.165       | 0.165       |
| 2                   | 0.1     | H     | L        | 19.22                    | 20.83       | 20.83       | 0.165       | 0.165       |
| 3                   | 0.1     | H     | L        | 19.33                    | 20.98       | 20.98       | 0.169       | 0.169       |
| 4                   | 0.1     | H     | L        | 19.32                    | 20.96       | 20.96       | 0.168       | 0.168       |
| 5                   | 0.1     | H     | L        | -                        | -           | -           | -           | -           |
| 6                   | 0.1     | H     | L        | -                        | -           | -           | -           | -           |
| 7                   | 0.1     | H     | L        | -                        | -           | -           | -           | -           |
| 8                   | 0.1     | H     | L        | -                        | -           | -           | -           | -           |
| 9                   | 0.1     | H     | L        | -                        | -           | -           | -           | -           |
| 10                  | 0.1     | H     | L        | -                        | -           | -           | -           | -           |
| 1                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 2                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 3                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 4                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 5                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 6                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 7                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 8                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |
| 9                   | 0.1     | H     | H        | -                        | -           | -           | -           | -           |

Table B.7: Experimental results 7

| AE Route, Equal, F2 |         |       |          |                  |             |             |             |             |
|---------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                 | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 10                  | 0.1     | H     | H        | -                | -           | -           | -           | -           |
| 1                   | 0.2     | L     | L        | 18.57            | 19.86       | 19.86       | 0.109       | 0.109       |
| 2                   | 0.2     | L     | L        | 18.51            | 19.75       | 19.75       | 0.109       | 0.109       |
| 3                   | 0.2     | L     | L        | 18.62            | 19.93       | 19.93       | 0.111       | 0.111       |
| 4                   | 0.2     | L     | L        | 18.60            | 19.90       | 19.90       | 0.110       | 0.110       |
| 5                   | 0.2     | L     | L        | 18.63            | 19.94       | 19.94       | 0.113       | 0.113       |
| 6                   | 0.2     | L     | L        | 18.67            | 20.00       | 20.00       | 0.111       | 0.111       |
| 7                   | 0.2     | L     | L        | 18.58            | 19.88       | 19.88       | 0.115       | 0.115       |
| 8                   | 0.2     | L     | L        | 18.58            | 19.87       | 19.87       | 0.110       | 0.110       |
| 9                   | 0.2     | L     | L        | 18.66            | 19.99       | 19.99       | 0.111       | 0.111       |
| 10                  | 0.2     | L     | L        | 18.59            | 19.89       | 19.89       | 0.111       | 0.111       |
| 1                   | 0.2     | L     | H        | 18.66            | 20.00       | 20.00       | 0.143       | 0.143       |
| 2                   | 0.2     | L     | H        | 18.59            | 19.89       | 19.89       | 0.143       | 0.143       |
| 3                   | 0.2     | L     | H        | 18.71            | 20.07       | 20.07       | 0.145       | 0.145       |
| 4                   | 0.2     | L     | H        | 18.69            | 20.04       | 20.04       | 0.145       | 0.145       |
| 5                   | 0.2     | L     | H        | 18.72            | 20.09       | 20.09       | 0.148       | 0.148       |
| 6                   | 0.2     | L     | H        | 18.76            | 20.15       | 20.15       | 0.146       | 0.146       |
| 7                   | 0.2     | L     | H        | 18.68            | 20.02       | 20.02       | 0.151       | 0.151       |
| 8                   | 0.2     | L     | H        | 18.67            | 20.01       | 20.01       | 0.145       | 0.145       |
| 9                   | 0.2     | L     | H        | 18.75            | 20.14       | 20.14       | 0.146       | 0.146       |
| 10                  | 0.2     | L     | H        | 18.68            | 20.03       | 20.03       | 0.146       | 0.146       |
| 1                   | 0.2     | H     | L        | 19.11            | 20.66       | 20.66       | 0.109       | 0.109       |
| 2                   | 0.2     | H     | L        | 19.04            | 20.57       | 20.57       | 0.109       | 0.109       |
| 3                   | 0.2     | H     | L        | 19.15            | 20.72       | 20.72       | 0.111       | 0.111       |
| 4                   | 0.2     | H     | L        | 19.13            | 20.70       | 20.70       | 0.110       | 0.110       |
| 5                   | 0.2     | H     | L        | 19.16            | 20.73       | 20.73       | 0.113       | 0.113       |
| 6                   | 0.2     | H     | L        | 19.19            | 20.78       | 20.78       | 0.111       | 0.111       |
| 7                   | 0.2     | H     | L        | 19.12            | 20.68       | 20.68       | 0.115       | 0.115       |
| 8                   | 0.2     | H     | L        | 19.11            | 20.67       | 20.67       | 0.110       | 0.110       |
| 9                   | 0.2     | H     | L        | 19.19            | 20.77       | 20.77       | 0.111       | 0.111       |
| 10                  | 0.2     | H     | L        | 19.13            | 20.69       | 20.69       | 0.111       | 0.111       |
| 1                   | 0.2     | H     | H        | 19.21            | 20.81       | 20.81       | 0.143       | 0.143       |
| 2                   | 0.2     | H     | H        | 19.15            | 20.72       | 20.72       | 0.143       | 0.143       |
| 3                   | 0.2     | H     | H        | 19.26            | 20.87       | 20.87       | 0.145       | 0.145       |
| 4                   | 0.2     | H     | H        | 19.24            | 20.85       | 20.85       | 0.145       | 0.145       |
| 5                   | 0.2     | H     | H        | 19.27            | 20.89       | 20.89       | 0.148       | 0.148       |
| 6                   | 0.2     | H     | H        | 19.30            | 20.94       | 20.94       | 0.146       | 0.146       |
| 7                   | 0.2     | H     | H        | 19.23            | 20.84       | 20.84       | 0.151       | 0.151       |
| 8                   | 0.2     | H     | H        | 19.22            | 20.83       | 20.83       | 0.145       | 0.145       |

Table B.7: Experimental results 7

| AE Route, Equal, F2 |         |       |          |                  |             |             |             |             |
|---------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                 | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 9                   | 0.2     | H     | H        | 19.30            | 20.93       | 20.93       | 0.146       | 0.146       |
| 10                  | 0.2     | H     | H        | 19.24            | 20.85       | 20.85       | 0.146       | 0.146       |

Table B.8: Experimental results 8

| AEX Route, Equal, F2 |         |       |          |                  |             |             |             |             |
|----------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                  | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 1                    | 0.1     | L     | L        | 19.82            | 19.19       | 19.19       | 0.172       | 0.172       |
| 2                    | 0.1     | L     | L        | 19.76            | 19.10       | 19.10       | 0.169       | 0.169       |
| 3                    | 0.1     | L     | L        | 19.89            | 19.31       | 19.31       | 0.167       | 0.167       |
| 4                    | 0.1     | L     | L        | 19.82            | 19.19       | 19.19       | 0.167       | 0.167       |
| 5                    | 0.1     | L     | L        | 19.90            | 19.32       | 19.32       | 0.170       | 0.170       |
| 6                    | 0.1     | L     | L        | 19.91            | 19.35       | 19.35       | 0.165       | 0.165       |
| 7                    | 0.1     | L     | L        | 19.84            | 19.23       | 19.23       | 0.172       | 0.172       |
| 8                    | 0.1     | L     | L        | 19.84            | 19.22       | 19.22       | 0.169       | 0.169       |
| 9                    | 0.1     | L     | L        | 19.89            | 19.30       | 19.30       | 0.175       | 0.175       |
| 10                   | 0.1     | L     | L        | 19.85            | 19.24       | 19.24       | 0.170       | 0.170       |
| 1                    | 0.1     | L     | H        | 20.01            | 19.49       | 19.49       | 0.226       | 0.226       |
| 2                    | 0.1     | L     | H        | 19.95            | 19.40       | 19.40       | 0.222       | 0.222       |
| 3                    | 0.1     | L     | H        | 20.08            | 19.61       | 19.61       | 0.220       | 0.220       |
| 4                    | 0.1     | L     | H        | 20.01            | 19.49       | 19.49       | 0.220       | 0.220       |
| 5                    | 0.1     | L     | H        | 20.09            | 19.62       | 19.62       | 0.223       | 0.223       |
| 6                    | 0.1     | L     | H        | 20.11            | 19.65       | 19.65       | 0.217       | 0.217       |
| 7                    | 0.1     | L     | H        | 20.04            | 19.54       | 19.54       | 0.225       | 0.225       |
| 8                    | 0.1     | L     | H        | 20.03            | 19.52       | 19.52       | 0.221       | 0.221       |
| 9                    | 0.1     | L     | H        | 20.09            | 19.61       | 19.61       | 0.229       | 0.229       |
| 10                   | 0.1     | L     | H        | 20.04            | 19.54       | 19.54       | 0.223       | 0.223       |
| 1                    | 0.1     | H     | L        | 20.59            | 20.33       | 20.33       | 0.172       | 0.172       |
| 2                    | 0.1     | H     | L        | 20.53            | 20.25       | 20.25       | 0.169       | 0.169       |
| 3                    | 0.1     | H     | L        | 20.65            | 20.42       | 20.42       | 0.167       | 0.167       |
| 4                    | 0.1     | H     | L        | 20.58            | 20.32       | 20.32       | 0.167       | 0.167       |
| 5                    | 0.1     | H     | L        | 20.66            | 20.43       | 20.43       | 0.170       | 0.170       |
| 6                    | 0.1     | H     | L        | 20.68            | 20.45       | 20.45       | 0.165       | 0.165       |
| 7                    | 0.1     | H     | L        | 20.61            | 20.36       | 20.36       | 0.172       | 0.172       |
| 8                    | 0.1     | H     | L        | 20.60            | 20.35       | 20.35       | 0.169       | 0.169       |
| 9                    | 0.1     | H     | L        | 20.66            | 20.43       | 20.43       | 0.175       | 0.175       |
| 10                   | 0.1     | H     | L        | 20.62            | 20.37       | 20.37       | 0.170       | 0.170       |
| 1                    | 0.1     | H     | H        | 20.84            | 20.67       | 20.67       | 0.226       | 0.226       |
| 2                    | 0.1     | H     | H        | 20.78            | 20.58       | 20.58       | 0.222       | 0.222       |
| 3                    | 0.1     | H     | H        | 20.91            | 20.75       | 20.75       | 0.220       | 0.220       |
| 4                    | 0.1     | H     | H        | 20.84            | 20.66       | 20.66       | 0.220       | 0.220       |
| 5                    | 0.1     | H     | H        | 20.92            | 20.77       | 20.77       | 0.223       | 0.223       |
| 6                    | 0.1     | H     | H        | 20.93            | 20.78       | 20.78       | 0.217       | 0.217       |
| 7                    | 0.1     | H     | H        | 20.87            | 20.71       | 20.71       | 0.225       | 0.225       |
| 8                    | 0.1     | H     | H        | 20.86            | 20.69       | 20.69       | 0.221       | 0.221       |
| 9                    | 0.1     | H     | H        | 20.93            | 20.77       | 20.77       | 0.229       | 0.229       |

Table B.8: Experimental results 8

| AEX Route, Equal, F2 |         |       |          |                  |             |             |             |             |
|----------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                  | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 10                   | 0.1     | H     | H        | 20.87            | 20.71       | 20.71       | 0.223       | 0.223       |
| 1                    | 0.2     | L     | L        | 19.62            | 18.86       | 18.86       | 0.113       | 0.113       |
| 2                    | 0.2     | L     | L        | 19.58            | 18.79       | 18.79       | 0.111       | 0.111       |
| 3                    | 0.2     | L     | L        | 19.69            | 18.98       | 18.98       | 0.110       | 0.110       |
| 4                    | 0.2     | L     | L        | 19.63            | 18.88       | 18.88       | 0.110       | 0.110       |
| 5                    | 0.2     | L     | L        | 19.70            | 18.99       | 18.99       | 0.112       | 0.112       |
| 6                    | 0.2     | L     | L        | 19.72            | 19.03       | 19.03       | 0.109       | 0.109       |
| 7                    | 0.2     | L     | L        | 19.65            | 18.91       | 18.91       | 0.113       | 0.113       |
| 8                    | 0.2     | L     | L        | 19.64            | 18.90       | 18.90       | 0.111       | 0.111       |
| 9                    | 0.2     | L     | L        | 19.68            | 18.97       | 18.97       | 0.115       | 0.115       |
| 10                   | 0.2     | L     | L        | 19.65            | 18.92       | 18.92       | 0.112       | 0.112       |
| 1                    | 0.2     | L     | H        | 19.73            | 19.05       | 19.05       | 0.148       | 0.148       |
| 2                    | 0.2     | L     | H        | 19.69            | 18.97       | 18.97       | 0.146       | 0.146       |
| 3                    | 0.2     | L     | H        | 19.81            | 19.18       | 19.18       | 0.144       | 0.144       |
| 4                    | 0.2     | L     | H        | 19.74            | 19.06       | 19.06       | 0.144       | 0.144       |
| 5                    | 0.2     | L     | H        | 19.81            | 19.18       | 19.18       | 0.146       | 0.146       |
| 6                    | 0.2     | L     | H        | 19.83            | 19.22       | 19.22       | 0.143       | 0.143       |
| 7                    | 0.2     | L     | H        | 19.76            | 19.10       | 19.10       | 0.148       | 0.148       |
| 8                    | 0.2     | L     | H        | 19.76            | 19.09       | 19.09       | 0.145       | 0.145       |
| 9                    | 0.2     | L     | H        | 19.80            | 19.16       | 19.16       | 0.150       | 0.150       |
| 10                   | 0.2     | L     | H        | 19.77            | 19.11       | 19.11       | 0.146       | 0.146       |
| 1                    | 0.2     | H     | L        | 20.32            | 19.96       | 19.96       | 0.113       | 0.113       |
| 2                    | 0.2     | H     | L        | 20.28            | 19.89       | 19.89       | 0.111       | 0.111       |
| 3                    | 0.2     | H     | L        | 20.39            | 20.06       | 20.06       | 0.110       | 0.110       |
| 4                    | 0.2     | H     | L        | 20.33            | 19.97       | 19.97       | 0.110       | 0.110       |
| 5                    | 0.2     | H     | L        | 20.40            | 20.06       | 20.06       | 0.112       | 0.112       |
| 6                    | 0.2     | H     | L        | 20.42            | 20.09       | 20.09       | 0.109       | 0.109       |
| 7                    | 0.2     | H     | L        | 20.35            | 20.00       | 20.00       | 0.113       | 0.113       |
| 8                    | 0.2     | H     | L        | 20.35            | 19.99       | 19.99       | 0.111       | 0.111       |
| 9                    | 0.2     | H     | L        | 20.39            | 20.05       | 20.05       | 0.115       | 0.115       |
| 10                   | 0.2     | H     | L        | 20.35            | 20.00       | 20.00       | 0.112       | 0.112       |
| 1                    | 0.2     | H     | H        | 20.48            | 20.17       | 20.17       | 0.148       | 0.148       |
| 2                    | 0.2     | H     | H        | 20.43            | 20.10       | 20.10       | 0.146       | 0.146       |
| 3                    | 0.2     | H     | H        | 20.55            | 20.27       | 20.27       | 0.144       | 0.144       |
| 4                    | 0.2     | H     | H        | 20.48            | 20.18       | 20.18       | 0.144       | 0.144       |
| 5                    | 0.2     | H     | H        | 20.55            | 20.28       | 20.28       | 0.146       | 0.146       |
| 6                    | 0.2     | H     | H        | 20.57            | 20.30       | 20.30       | 0.143       | 0.143       |
| 7                    | 0.2     | H     | H        | 20.50            | 20.21       | 20.21       | 0.148       | 0.148       |
| 8                    | 0.2     | H     | H        | 20.50            | 20.20       | 20.20       | 0.145       | 0.145       |

Table B.8: Experimental results 8

| AEX Route, Equal, F2 |         |       |          |                  |             |             |             |             |
|----------------------|---------|-------|----------|------------------|-------------|-------------|-------------|-------------|
| Rep                  | $\beta$ | $\mu$ | $\sigma$ | obj ( $\$10^6$ ) | $\bar{v}_A$ | $\bar{v}_B$ | $\bar{b}_A$ | $\bar{b}_B$ |
| 9                    | 0.2     | H     | H        | 20.55            | 20.27       | 20.27       | 0.150       | 0.150       |
| 10                   | 0.2     | H     | H        | 20.51            | 20.22       | 20.22       | 0.146       | 0.146       |