AN INVESTIGATION OF PRE-SERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS' ABILITY TO CONNECT THE MATHEMATICS IN CONTENT COURSES WITH THE MIDDLE SCHOOL MATHEMATICS

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ABSTRACT

AN INVESTIGATION OF PRE-SERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS' ABILITY TO CONNECT THE MATHEMATICS IN CONTENT COURSES WITH THE MIDDLE SCHOOL MATHEMATICS

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This study investigated if and how preservice middle school mathematics teachers related the mathematical knowledge addressed in general mathematics content courses in a four-year teacher education program to their future teaching of middle school mathematics. The study involved two interrelated sections. On one hand, preservice middle school mathematics teachers' views on the issue were gathered through asking open-ended questions via a semi-structured interview protocol. On the other hand, their performance on a structured task-based interview was observed in order to find out how they utilized their mathematical knowledge of number theory concepts developed in the specific course Basic Algebraic Structures in conducting *mathematical tasks of teaching*. Semi-structured interview protocol and structured task-based interview protocol were prepared by the researcher.

Participants of the study were 14 preservice middle school mathematics teachers who were enrolled in 3rd and 4th years of the teacher education program. Findings revealed that preservice teachers had conflicting views about the content courses. They considered the mathematics learned in general content courses as *higher level*, *irrelevant to middle school mathematics* and *not applicable to teaching of middle school mathematics,* but also as *constituting the base for middle school mathematics.* Participants' work on the four mathematical tasks of teaching provided several perspectives on the extent to which they were able to use their knowledge from Basic Algebraic Structures course in the teaching of middle school mathematics. Although the participants were selected from among the most competent ones in the Basic Algebraic Structures course and also in teaching related courses, many of them had difficulties with relating their mathematical knowledge from the course to given teaching tasks.

Keywords: Pre-service Middle School Mathematics Teachers, Content Knowledge, Mathematical Tasks of Teaching, Elementary Number Theory

ÖZ

ORTAOKUL MATEMATİK ÖĞRETMENİ ADAYLARININ ALAN DERSLERİNDEKİ MATEMATİK İLE ORTAOKUL MATEMATİĞİNİ İLİŞKİLENDİRME BECERİLERİNİN İNCELENMESİ

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Bu çalışma ortaokul matematik öğretmeni adaylarının, dört yıllık bir öğretmen yetiştirme programında, genel matematik dersleri kapsamında öğrendikleri matematik bilgisini, gelecekte ortaokul matematiğini öğretme ile ilişkilendirme durumunu incelemiştir. Çalışma birbirini tamamlayan iki kısımdan oluşmuştur. İlk kısımda, öğretmen adaylarının konu ile ilgili görüşleri açık uçlu soruları içeren bir yarı-yapılandırılmış görüşme protokolü yardımıyla elde edilmiştir. İkinci kısımda, katılımcıların yapılandırılmış göreve-dayalı görüşmede göstermiş oldukları performanslar, *matematik öğretiminin görevleri* yerine getirmeleri sırasında Temel Cebirsel Yapılar dersinde öğrendikleri sayılar teorisi bilgisini nasıl kullandıklarına dair bilgi edinmek amacıyla gözlemlenmiştir. Yarı yapılandırılmış görüşme protokolü ve yapılandırılmış göreve-dayalı görüşme protokolü araştırmacı tarafından hazırlanmıştır.

Çalışmanın katılımcıları, öğretmen yetiştirme programının üçüncü ve dördüncü sınıf öğrencileri arasından seçilen 14 ortaokul matematik öğretmeni adayıdır. Bulgular, öğretmen adaylarının alan dersleri hakkında karmaşık fikirlere sahip olduğunu göstermiştir. Öğretmen adayları, bir yandan bu derslerde öğretilen matematiğin *yüksek düzeyde, ortaokul matematiği ile ilgisiz* ve *ortaokul matematiğinin öğretiminde uygulanamaz* olduğunu düşünürken, aynı zamanda bu matematiğin, *ortaokul matematiğinin temelini oluşturduğunu* da ileri sürmüştür. Katılımcıların, *matematik öğretiminin görevlerinden* dört tanesini içeren testte göstermiş oldukları performanslar, Temel Cebirsel Yapılar dersinde edindikleri bilgileri ortaokul matematiğini öğretmek amacıyla ne düzeyde kullanabildikleri konusunda farklı bakış açıları sağlamıştır. Çalışmaya katılan öğretmen adayları hem Temel Cebirsel Yapılar dersinde, hem de öğretimle ilgili derslerde en yetkin öğrenciler arasından seçilmiş olmalarına rağmen, çoğu öğretmen adayı bu temel derste öğrendikleri matematik bilgisini, matematik öğretiminin gerekleri ile ilişkilendirmede zorluklar yaşamıştır.

Anahtar Kelimeler: Ortaokul Matematik Öğretmeni Adayları, Alan Bilgisi, Matematik Öğretiminin Görevleri, Sayılar Teorisine Giriş

To memory of

Zişan Güner Alpaslan & Mustafa Alpaslan

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CHAPTER 1

INTRODUCTION

It is widely stated that elementary and middle school mathematics teachers should go through a serious mathematical preparation (Conference Board of the Mathematical Sciences (CBMS), 2001); National Mathematics Advisory Panel (NMAP), 2008; Shulman, 1986). They should not only know the concepts and procedures they are supposed to teach, but also develop a profound understanding of mathematics from a much broader perspective (NMAP, 2008). It is important for teachers to maintain an integrated conception of mathematics as a discipline beyond conceptually understanding the subject matter to be taught (Shulman, 1986). Teachers should possess the knowledge of how core concepts and principles can be organized in multiple ways within the unitary discipline of mathematics; what makes a particular proposition valid, and valuable and sometimes more central than another plausible one to the discipline (Shulman, 1986). This conception of mathematics should also include the knowledge of interconnections between theory, procedures and applications; which enables teachers to flexibly arrange essential mathematical ideas while planning instruction for their students' learning of mathematics as a logical activity and also appreciate the sophistication and power of the subject (CBMS, 2001).

Teacher education programs undertake a significant role in raising effective teachers, who are well informed about how mathematics is connected over the span of curriculum from primary school to university (NMAP, 2008). Thus, prospective middle school mathematics teachers are suggested to take university level courses focusing on fundamentals of the mathematics they are going to teach. These courses are suggested to be taught by mathematics experts who are genuinely concerned with professional training of mathematics teachers (CBMS, 2001). Coursework in subject matter currently constitutes an important part of mathematics teachers' studies in the teacher education programs. However, its effects on prospective teachers' knowledge, skills and dispositions are not substantiated by sufficient empirical evidence. There exist quite a few number of research in the accessible literature investigating the relationship between mathematics teachers' coursework in teacher education programs and their students' achievement (Goldhaber & Brewer, 1997, 2000; Hawkins, Stancavage, & Dossey, 1998; Monk; Monk, & King, 1994; Rowan, Chiang, & Miller, 1997; Wenglinsky, 2002) some with promising results of the existence of a positive relationship between the two (Floden & Meniketti, 2005), while others resulted inconclusive (Ferrini-Mundy, Burrill, Floden, & Sandow, 2003). One of the main reasons for the inconsistency in research findings might be the indirect relationship between teachers' knowledge and students' learning (Ferrini-Mundy et al., 2003). Indeed, the unclarified relationship is mediated by how teachers teach, as well as how their students learn. The main assumption for conducting such investigations, on the other hand, is the expectation that teachers' knowledge has an influence on their actions during teaching (Ferrini-Mundy et al., 2003). This suggests taking a closer look at what exactly teachers gain from mathematics courses, rather than investigating its reflections only on students' achievement. Therefore, there is a need to focus on understanding teachers' gains in mathematics courses and how these gains are connected to actual practice of teaching mathematics (Floden & Meniketti, 2005).

This study investigated, in the broader sense, how preservice middle school mathematics teachers viewed the general mathematics coursework in terms of its relevance to their future teaching; and in particular, how they considered the potential use of their mathematical knowledge of number theory concepts developed in mathematics content coursework in the actual work of teaching. Number theory concepts are chosen for the study because of the significant role it plays in enhancing students' reasoning, argumentation, and proof skills and therefore suggested to be incorporated into all grade levels within a mathematics curriculum (Campbell & Zazkis, 2002). Turkish middle school mathematics curriculum emphasizes studying this content as well. One of the foci of the curriculum (Ministry of National Education (MoNE), 2013) is middle school students' learning of prime and relatively prime numbers, factors, multiples, and divisibility concepts. Solving of problems that require calculating the greatest common divisor or the least common multiple of numbers is given special importance. Along with choosing the domain of number theory for this study, the content course to be studied was determined as Basic Algebraic Structures course in which the fundamental number theory concepts were covered in the particular mathematics teacher education program.

1.1. Theoretical Framework

Based on the need for understanding the ways in which teachers must know their content, specifying the correct amount and range of that knowledge, and also for promoting effective use of such knowledge in the real classroom setting, Ball and her colleagues (Ball et al., 2008; Ball, Hill & Bass, 2005) focused their studies on the work of teaching. They have qualitatively analyzed what teachers actually do in the course of teaching and what type of mathematical knowledge is required in performing this task. As a result of their observations, Ball and her colleagues proposed a detailed outline of "*mathematical knowledge for teaching*" (Ball, Thames & Phelps, 2008; Ball, Hill & Bass, 2005).

Mathematical Knowledge for Teaching (MKT) is a refinement of Shulman's (1986) initial categorization of content knowledge for teaching (Ball, Thames & Phelps, 2008). MKT divides Shulman's category of Subject Matter Knowledge into three sub-domains; which are Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), and Horizon Content Knowledge. On the other hand, it defines two other sub-domains, Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT), under Shulman's second category of Pedagogical Content Knowledge (PCK). Shulman's third category of Curricular Knowledge (CK) is also relocated under PCK as Knowledge of Content and Curriculum within this new framework (Ball, Thames & Phelps, 2008).

Among the knowledge types identified within the framework, Specialized Content Knowledge (SCK) is the most important one for Ball and her colleagues. It is considered as the knowledge base that defines mathematics teaching as a profession because it "is the mathematical knowledge and skill unique to teaching" (Ball, Thames & Phelps, 2008, p.400). In other words, although other mathematics-related professionals may know the mathematical content well, they do not need to have this kind of special understanding of mathematics. It is characterized as the ability to satisfy *mathematical demands of teaching*, the terminology Ball et al. (2008) used for describing particular pieces of mathematically challenging work contained in the teaching of mathematics. Some of these mathematical demands of teaching are identified as communicating the reasoning that underlies an algorithm and what it implies, assessing the correctness of unusual mathematical claims, and evaluating the applicability of student-generated methods to other conditions (Ball, Thames & Phelps, 2008; Hill & Ball, 2004). The framework is explained in the next chapter in detail.

By means of using particular tasks of teaching identified by Ball et al. (2008), the framework provided the theoretical basis for this study, on which disciplinary knowledge learned in the mathematics content coursework was purposefully connected to practice of teaching middle school mathematics. For this study, four of the mathematical tasks of teaching were selected; which were *responding to students' "why" questions, evaluating the plausibility of students' claims, recognizing what is involved in using a particular representation,* and *inspecting equivalencies*. While describing the characteristics of content knowledge for teaching, Ball et al. (2008) valued teachers' being able to explain why the number 1 is not considered as a prime. A mathematical explanation for this question had already been studied within the Basic Algebraic Structures course which participant preservice teachers of this study attended as a part of their studies in the teacher education program. However, whether preservice teachers were aware of the usability of this information in the context of teaching, or even if they were aware of the reason themselves or not, were not known. This specific *why* question fostered the selection of *responding to students' "why" questions* as one of the four mathematical tasks of teaching for this study.

The two tasks, *evaluating the plausibility of students' claims* and *inspecting equivalencies* were utilized with the purpose of understanding how preservice teachers used their critical thinking, reasoning, and proof skills in the particular content domain. Number theory content is highlighted in the mathematics education literature for its facilitating the development of such skills in individuals (Campbell & Zazkis, 2002), which was also aimed in the Basic Algebraic Structures course. For this reason, the two mathematical tasks of teaching constituted a good base for designing the two tasks of the study. While one of these two tasks was about determining the correctness of a hypothetical student's use of the divisibility rules, the other one involved evaluating the equivalency of two different uses of the same algorithm, the one for calculating the greatest common divisor of two numbers.

Lastly, *recognizing what is involved in using a particular representation* was combined with the case of explaining how the algorithm for calculating the least common multiple of three given numbers works. Characteristic of this task was its requiring the preservice teachers to consider two different definitions of the term least common multiple. One of the definitions was their definition of the concept (which was the same as how it was defined in middle school mathematics curriculum) and the other one was from the Basic Algebraic Structures Course.

1.2. Research Questions

This study investigated if and how preservice middle school mathematics teachers built the relationship between the mathematical knowledge addressed in general mathematics content courses in a specific teacher education program and their future teaching of middle school mathematics. The study involved two interrelated sections. On one hand, preservice middle school mathematics teachers' views on the issue were gathered through asking open-ended questions. On the other hand, preservice teachers' performance on four mathematical tasks was observed with the purpose of finding out how they used their mathematical knowledge of number theory concepts developed in the Basic Algebraic Structures course in conducting mathematical tasks of teaching. The following research questions guided the study:

1. How do preservice middle school mathematics teachers perceive the general mathematics content courses in the teacher education program in terms of their relevance to their future teaching?
    a. How do preservice middle school mathematics teachers perceive the specific mathematics content course “Basic Algebraic Structures” in terms of its relevance to their future teaching?
2. How do preservice middle school mathematics teachers use their mathematical knowledge from Basic Algebraic Structures course in conducting mathematical tasks of teaching?

1.3. Significance of the Study

The global tendency in mathematics teacher education has been the study of content and methodology courses, which created a discrepancy between knowing and teaching (Bair & Rich, 2011; Potari, 2001). One reason for the disconnection is claimed to be that the content courses are scholarly, irrelevant, and remote from classroom teaching (Ball, Thames & Phelps, 2008; NMAP, 2008). On the contrary, existing research suggests that mathematics coursework is not much irrelevant for middle school mathematics teachers. Although not always consistent across studies, a positive relationship has often been reported to exist between teachers’ mathematical knowledge and their students’ achievement (Goldhaber & Brewer, 1997, 2000; Hawkins et al., 1998; Monk; Monk & King, 1994; Rowan et al., 1997; Wenglinsky, 2002). In a similar sense, CBMS (2012) urged prospective teachers to develop complete proficiency in mathematics several grades beyond the level they are assigned to teach.

However, effects of disciplinary content coursework on preservice teachers’ knowledge for teaching have not been studied much. Understanding what prospective teachers gain from these courses in relation to their future teaching remains to be a critical concern for both teacher educators and educational researchers (Floden & Meniketti, 2005). For this reason, the current study focused on specialized content knowledge component of the broader framework of Mathematical Knowledge for Teaching, which was associated with the particular usage of mathematical knowledge for the purposes unique to teaching (Ball et al., 2008). Two main purposes of the study were to investigate how preservice middle school mathematics teachers perceived their general mathematics content coursework in terms of its relevance to their future teaching and how they considered the potential use of their mathematical knowledge of number theory concepts developed in mathematics content coursework in the actual work of teaching. Results of the study may provide teacher educators with new perspectives on whether and how their instruction provides preservice middle school mathematics with usable content knowledge for teaching in the area of number theory.

Number theory content plays an important role in middle school mathematics education (CBMS, 2001; Campbell & Zazkis, 2002). Beyond learning the topic "of historical interest" (Campbell & Zazkis, 2002, p.592) itself, studying this content provides potential avenues for students to develop reasoning, critical thinking and generalizing skills. It helps students with developing a connected understanding of the number system, together with its structure and patterns (Zazkis, 1999). Launching recommendations for the professional education of middle grades mathematics teachers, CBMS (2001) argues in favor of designing courses that emphasize basic number theory concepts, in which prospective teachers are suggested to experience conjecturing and justifying their ideas about even and odd, and prime and composite numbers. Making sense of the Prime Factorization Theorem and its extension to algebra is stated among the many purposes of such courses.

Despite its great importance to prospective teachers' workspace (CBMS, 2001), number theory content has been the focus of relatively little research found in the literature (Bair & Rich, 2011) and extremely little research on Turkish preservice middle school mathematics teachers' understanding of number theory concepts. Therefore, the results of this study are likely to contribute to the research literature on teaching of this specific content area.

1.4. Definition of Important Terms

Preservice Middle School Mathematics Teachers: They are the third and fourth year students of a four-year teacher education program, Elementary Mathematics Education (EME) Program, at the university that the study was conducted. They are trained to teach grades 5 to 8 in middle schools. All of them had completed the nine content courses offered to them by the Department of Mathematics at the time of the study.

Mathematical Tasks of Teaching: They are the repetitive tasks of teaching that require teachers to organize their mathematical knowledge in a specialized way while teaching mathematics (Ball et al., 2008). Ball and her colleagues examined what teachers actually do in the course of teaching and as a result of this they identified 16 mathematical tasks of teaching. Four of them are used in this study: *responding to students' "why" questions, evaluating the plausibility of students' claims, recognizing what is involved in using a particular representation,* and *inspecting equivalencies.* All of the sixteen mathematical tasks of teaching are given in the next section, while presenting the broader framework MKT. In this study, phrase of "mathematical tasks of teaching" refers to either all, or four of the tasks used in this study, conditionally.

Basic Algebraic Structures Course: The course is one of the nine mathematics content courses offered by the Department of Mathematics to the students of Elementary Mathematics Education program. Preservice middle school mathematics teachers take this course together with their mathematics major counterparts in their first year of the program. The course content covers basics of algebra and number theory. Main topics included in the course content are related to binary operations, groups, rings, integral domains, ideals, fields, the concept of isomorphism, division and Euclidean algorithms, prime factorization and the greatest common divisor and the least common multiple concepts, and Quotient structures (Middle East Technical University, Academic Catalog, 2005). A syllabus of the course is presented in Appendix A.

CHAPTER 2

LITERATURE REVIEW

This chapter documents research related to goals of the study. The study investigated preservice middle school mathematics teachers' views on the mathematics coursework in terms of its relevance to their future teaching; and how they used their mathematical knowledge of number theory concepts developed in the Basic Algebraic Structures course in conducting mathematical tasks of teaching. Three research topics were critical for launching this study on. In particular, research in teacher knowledge, how this knowledge is affected by the study of mathematics content courses in teacher education programs, and mathematical knowledge for teaching number theory concepts are reviewed in this chapter.

2.1. Theoretical Background for Teacher Knowledge

The notion of teacher knowledge, that is, the nature of the knowledge required for teaching and how this knowledge develops during teacher education, have changed over the last several decades (van den Kieboom, 2013). While in the mid-1980s, most research on teaching were investigating pedagogical aspects of teaching such as classroom management and wait-time, Shulman (1986) addressed that key questions about teaching were lacking in the available literature of the time. Shulman's work, then, attempted to answer the questions of "Where do teacher explanations come from? How do teachers decide what to teach, how to represent it, how to question students about it and how to deal with problems of misunderstanding?" (p.8). Asking a variety of similar questions, Shulman offered new directions for the development of teacher knowledge and teacher education (van den Kieboom, 2013).

According to Shulman (1986) teachers are supposed to know their content at least as deeply as a mathematician does, but it is functional only when accompanied by a sound pedagogical skill. Proficiency in doing mathematics is not sufficient. Teachers need to know ways of presenting the mathematical substance to their students in an understandable way, selecting appropriate illustrations of mathematical ideas and anticipating difficulties that students in different developmental stages may experience in learning of one particular topic. They should also be familiar with what students already know and which misconceptions they may hold, together with ways to overcome. Shulman (1987) entitled this type of knowledge in which content and pedagogy are melted together as *pedagogical content knowledge* (PCK).

Subsequent to introducing the notion of PCK, Shulman (1986) divided content knowledge for teaching into three major domains: subject matter content knowledge, curricular knowledge and PCK. Content knowledge refers to "the amount and organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p.9). Beyond the knowledge of basic concepts and principles of the discipline, subject matter content knowledge deals with the ways in which they are organized, established and validated. Curricular knowledge, is the knowledge of entire programs designed for the teaching of particular topics, placement of topics in consecutive grade levels, variety of materials available for teaching those topics and the knowledge of how to select and use them in particular circumstances.

This initial categorization of Shulman (1986) is recognized by many researchers as a framework for teacher knowledge and further developed for diverse purposes. For instance, Even (1990) built an analytic framework for subject matter knowledge used in teaching a particular mathematical subject. She applied it to the concept of functions and used this framework for understanding interrelationships between teachers' subject matter knowledge and PCK in functions (Even, 1993). Ball (1991) distinguished between the knowledge *about* mathematics and the knowledge *of* mathematics. She proposed this categorization as a new framework for teachers' subject matter knowledge. More recently, Ball et al. (2008) built a practice-based theory of content knowledge for teaching, expanding Shulman's (1986) introductory work. The framework is called Mathematical Knowledge for Teaching (MKT). In this study, MKT was employed as the theoretical framework for teacher knowledge. Therefore, the next section is devoted to MKT. Also, the reason why MKT was selected from among many of the frameworks available for teacher knowledge is justified within the next section.

2.2. Mathematical Knowledge for Teaching (MKT)

MKT is one of the several teacher knowledge frameworks built upon Shulman's (1986) notion of PCK (Ball, Thames, & Phelps, 2008). Its underlying principle is the centrality of using of the mathematical content knowledge in teaching, rather than having it (Ball, Bass & Hill, 2004). By MKT, Ball and her colleagues addressed "the mathematical knowledge used to carry out the *work of teaching mathematics"* (Hill, Rowan & Ball, 2005, p.373). *Teaching* here refers to any one of the actions that teachers take in order to enhance student understanding in the interactive classroom setting and meet the needs that appear in the meantime. It also includes preparation of lesson plans, instructional materials and homework, assigning grades to students' performances, treating every member of the classroom equally and many other responsibilities of a teacher (Ball, Thames, & Phelps, 2008). By analyzing recurrent tasks of teaching, Ball et al. re-partitioned content knowledge for teaching as in Figure 2.1.

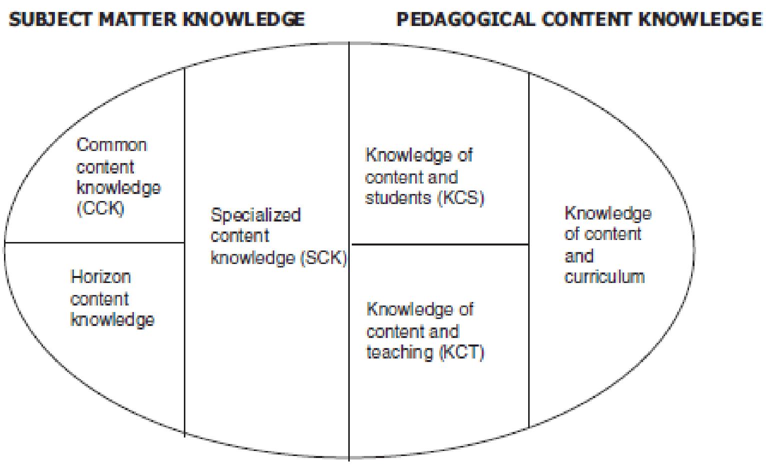

Figure 2.1 Domains of Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008, p.403).

They suggested three distinct domains under each of the subject matter knowledge and pedagogical content knowledge categories. Subject matter knowledge consists of common content knowledge (CCK), specialized content knowledge (SCK) and horizon content knowledge (HCK).

CCK is defined as the mathematical knowledge and skill that is used by anyone who knows mathematics, without a purpose of teaching others. On the contrary, SCK is the "mathematical knowledge and skill unique to teaching" (Ball et al., 2008, p.400). It is associated with everyday tasks of teaching that are distinctive to the profession-teaching. Ball et al. (2008) summarized these tasks under the name "*mathematical tasks of teaching*" as:

Table 2.1

Mathematical Tasks of Teaching

Presenting mathematical ideas
Responding to students' "why" questions
Finding an example to make a specific mathematical point
Recognizing what is involved in using a particular representation
Linking representations to underlying ideas and to other representations
Connecting a topic being taught to topics from prior or future years
Explaining mathematical goals and purposes to parents
Appraising and adapting the mathematical content of textbooks
Modifying tasks to be either easier or harder
Evaluating the plausibility of students' claims (often quickly)
Giving or evaluating mathematical explanations
Choosing and developing useable definitions
Using mathematical notation and language and critiquing its use
Asking productive mathematical questions
Selecting representations for particular purposes
Inspecting equivalencies

Note: Taken from Ball, Thames & Phelps, 2008, p.400.

The third domain within the subject matter knowledge is “horizon content knowledge” (Ball, 1993). It is an “awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (Ball et al., 2008, p.403). Teachers should encourage their students to build new mathematical learning on their previous knowledge, and do this in a way that facilitates students’ learning of forthcoming topics. For this reason, teachers should be familiar with the broad picture of mathematics that reveals how mathematics is connected from one grade level to the next. Horizon content knowledge (HCK) emphasizes making sense of these connections between mathematical ideas from a broader perspective.

Similarly, MKT divides Shulman’s pedagogical content knowledge into three other domains. They are knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of content and curriculum. KCS is a combination of knowledge about students and about mathematics. It represents teachers’ work of predicting student thinking, adjusting mathematical work according to students’ ability levels, and being familiar with what entertains and motivates students. Knowing about students is a crucial aspect of this type of knowledge. KCT, on the other hand, combines knowledge about mathematics with the knowledge about teaching. KCT mainly deals with planning of instruction, and consists of many decision making tactics for how to introduce a concept as the first time, which illustrations of it to rely on more than some others, and when and how to engage students in thinking more deeply about the topic.

Among the six domains of MKT, this study particularly focuses on SCK domain. Mathematics content courses’ relevance to teaching middle school mathematics and its use in the actual work of teaching are controversial issues among researchers (Ball, 2008). Because this study aims to gain insights about possible usage of university level mathematics knowledge in the teaching of middle school mathematics, this study is built on “*mathematical tasks of teaching*”, which is addressed in the domain of SCK.

2.3. Role of Content Courses on Teacher's Mathematical Knowledge for Teaching

Mathematics teacher candidates are often required to complete a number of advanced mathematics courses as part of their preparation in teacher education programs (Potari, 2001). Underlying assumptions are stated in two common ways: either as studying formal disciplinary mathematics contributes to effective teaching (Davis & Simmt, 2006) or as the knowledge acquired is influential on students' learning (Ball, Lubienski, & Mewborn, 2001). However, research investigating the relationship between teacher knowledge and the two outcome variables, student achievement or teaching effectiveness, did not certainly verify the two widely accepted conjectures about teachers' knowledge (Ferrini-Mundy, Burrill, Floden, & Sandow, 2003). Indeed, individual reviews of the numerous research studies on the issue (e.g., Floden & Meniketti, 2005; Wilson & Floden, 2003; Wilson, Floden, & Ferrini-Mundy, 2001) came to an agreement that the results were inconclusive.

Within the different content areas studied, a greater consistency for a positive relationship shows up in the field of mathematics, across studies (Floden & Meniketti, 2005; Wilson & Floden, 2003). However, still contradictory results are reported in each of the research reviews for this particular content area.

Majority of research findings, for the case of mathematics content area, support the common belief that teacher effectiveness and student achievement are positively influenced by teachers' mathematical preparation (Wilson & Floden, 2003). Chaney (1995) reported that 8^{th} grade students whose teachers had majored in mathematics scored highest in the National Education Longitudinal Study of 1988 (NELS:88) in which 24,599 students took the mathematics achievement test. Rowan, Chiang and Miller (1997) used the same data in their research and concluded a weak but positive relationship between the number of mathematics items teachers responded to correctly and their students' mathematics achievement. In a more recent study Telese (2012) investigated the effects of middle school mathematics teachers' content knowledge and pedagogical knowledge on 8^{th} grade students' achievement, using the data set from National Association of Educational Progress's (NAEP) 2005 assessment. Results indicated that teachers' advanced mathematical courses influenced their students' achievement, more than their pedagogical preparation did.

While there exist many other studies in the literature suggesting similar positive effects of teacher knowledge (Goldhaber & Brewer, 1997, 2000; Hawkins et al., 1998; Monk, Monk & King, 1994; Rowan et al., 1997; Wenglinsky, 2002), one particular research reported the opposite. Rowan, Correnti and Miller (2002) detected that students whose teachers had an advanced degree in mathematics showed less progress in mathematics achievement. Authors reasoned, in the case that results reflected the reality; two potential explanations might be suggested for the unexpected negative effect. In particular, teachers' mathematical preparation might either have substituted for their pedagogical preparation, or had not helped them with the ability to adjust their own mathematical understanding to students' level. In another study, Monk (1994) determined that number of advanced mathematics courses taken was associated positively with student achievement only up to five courses. When their number exceeded five, increasing number of content courses had smaller effect on student achievement. On the other hand, some research found neither positive nor negative relationship between the variables in question (Eisenberg, 1977; Rowan et al, 2002).

Making precise inferences out of complicated findings would be misleading (Wilson, Floden, & Ferrini-Mundy, 2001). Researchers instead preferred to discuss about the reasons yielding the inconsistency in results and eliminate them from the future research on teacher knowledge. One of the main reasons commonly agreed on was using substitute variables for measuring teachers' mathematical knowledge, such as teachers' pathway to the profession, and both the number and type of courses teachers have taken in teacher education programs (Hill, Rowan, & Ball, 2005). Later on, both of these approaches were argued as far from representing the actual knowledge used in teaching elementary and middle school mathematics (NMAP, 2008). Ferrini-Mundy et al. (2003) framed this source of uncertainty as inadequateness of the theory that would define the substantial mathematical knowledge required for teaching. Another source for the unstable results is claimed to be the indirect relationship between teachers' knowledge and students' learning.

The two variables' interaction is largely affected by how teachers teach and how their students learn, as well as it is mediated by the curriculum followed and the ways in which instructional materials are used (Ferrini-Mundy, et al. 2003). In other words, most studies investigating the effects of teacher knowledge have concentrated only on having the mathematics knowledge, without making a point of usability of this knowledge in teaching (Adler & Ball, 2009).

Not all of the available studies in this research area will be examined here, because they do not contain what prospective teachers learn from the content courses in relation to their future teaching performance (Floden & Meniketti, 2005), which would provide more useful information for the conduct of this study. For this reason, the next section is devoted to description of the studies that are interested in prospective teachers' experiences in the content courses and teachers' relevant experiences in teaching.

2.4. Prospective Teachers' Experiences in the Mathematics Content Courses

Contrary to much of the previous research investigating teacher knowledge, which concentrated on the degree of mathematics knowledge attained, few of the studies were more interested in how this knowledge was used in the work of teaching (Adler & Ball, 2009; Floden & Meniketti, 2005). Two of the studies with practicing teachers are critical to review here in line with the purposes of this study.

Zazkis and Leikin (2010) investigated secondary school teachers' views on the usefulness of advanced mathematical knowledge in teaching. The study was built on 42 teachers' responses to a written questionnaire and interview data with ten teachers who preferred to verbally answer the same questionnaire. Four questions of the questionnaire were asking about the usability of participants' advanced mathematical knowledge individually first, and then by considering the secondary school curriculum, personal experiences of a teaching situation, such as inspecting a students' work, and mathematical problems or tasks that would necessitate an advanced level of mathematics knowledge; through providing examples. Most of the participants commented on usefulness of the knowledge in general terms, rather than providing concrete examples. Zazkis and Leikin (2010) interpreted this finding as a confirmation for the disconnectedness between university mathematics and secondary school mathematics, based on the difficulty participants experienced in generating examples. Analysis of purposes and benefits participating teachers perceived in having an advanced level of mathematics knowledge yielded a distinction between these benefits as teacher-self-oriented and student-oriented. Teacher-self-oriented benefits referred to improvements personally experienced in skills, such as problem solving and logical thinking. On the other hand, student-oriented benefits implied advancement in teacher behaviors such as facilitating students' learning, representing the knowledge in multiple ways, connecting the topics to future curriculum, and increasing students' motivation and interest. Also teachers' mathematics knowledge was found to increase their confidence in teaching. This factor was considered as both teacher-self-oriented, as teachers' confidence with their knowledge of mathematics; and student-oriented benefit, as their confidence in teaching mathematics. Since asking for specific examples did not work well in this research study, Zazkis and Leikin (2010) suggested future studies to explore explicit contextual connections between university mathematics and secondary school content.

Wiley (2014) studied three middle school mathematics teachers' experiences in the grade levels 7th and 8th to identify how their pure mathematical knowledge was incorporated into their teaching and its extent. All three teachers were the graduates of teacher preparation programs in which pure mathematics coursework was heavily weighted. Although the participants acknowledged certain benefits of content courses to their own understanding of mathematics and their habits of mind; they perceived the courses as "too abstract" to be used with the middle school content. When they were asked to give examples of where their teaching was assisted by their pure knowledge of mathematics, they had difficulty with pinpointing specific instances. While prioritizing the connectedness of their extensive knowledge to middle school mathematics, they also highlighted the lack of applicability of this knowledge to teaching practice. Consequently, Wiley (2014) suggested future research to search for formal experiences that would let the teachers use their pure knowledge of mathematics in teaching practice.

Other research investigated prospective teachers' experiences in the content coursework (Hart & Swars, 2009; Hart, Oesterle, & Swars, 2013). Hart and Swars (2009) recognized absence of research on prospective teachers' perspectives on the mathematics content coursework literature, and argued that such study would inform future research. Hart and Swars (2009) conducted a phenomenological study with 12 elementary prospective teachers, with the main focus on their lived experiences in the mathematics content courses which were taught jointly by the departments of elementary education and of mathematics. Data were collected through interviews, in which participants were directed open-ended questions such as "After taking the math courses, do you feel confident that your content knowledge is sufficient in understanding PreK-5 math? Why or why not?" (p.163) Open-coding process on the transcribed data resulted in three themes representing participants' experience in content courses. In particular, the three themes involved participants' ideas about the content of the courses, feelings about the coursework, and the ways in which the courses were delivered.

Hart and Swars's (2009) participants considered their experiences in the content courses as discrepant from what they went through in the overall teacher education program. They perceived the courses as emphasizing procedural knowledge more rather than conceptual knowledge, not including material related to elementary mathematics, and lacking the activities paying attention to elementary students' thinking. These are the main findings addressed under the first theme. Reported under the second theme was participants' negative feelings for their experiences in the coursework, such as the emotions of stress, discourage, struggle and frustration. Lastly, in the third theme, how the courses were taught was reported from the participants' perspective. Participants mostly criticized the teacher-centered nature of the teaching, consisting of instructors' lecturing and showing PowerPoint presentations, and students' taking notes. Lack of illustrations, hands-on activities and revisit of non-understood points were the other deficiencies participants highlighted. Consequently, Hart and Swars (2009) concluded the importance of determining proper mathematics curricula to be taught in teacher education programs, familiarizing instructors of these courses with the elementary mathematics education, and improving their pedagogical practices for better teaching.

2.5. Learning and Teaching of Number Theory

Number theory is basically the inspection of number systems with respect to their essential characteristics and structure (Verschaffel, Greer & De Corte, 2007; Zazkis, 1999). Contents include "figurate numbers, whole number patterns and sequences, multiples, factors, divisors, primes, composites, prime decomposition, relatively prime numbers, divisibility, and divisibility rules" (Campbell & Zazkis, 2002b, p.3). Beougher (1966) regarded number theory among as one of the most fertile topics to be included in early grades curriculum; for the benefit of both students and teachers. It is argued to promote both parties' appreciation and attitude towards mathematics. Moreover, Beougher (1966) considered the topic as a source for individuals to notice the many interrelated, structural features of mathematics. Similarly, Campbell and Zazkis (2002a) recommended that number theory content be given more emphasis within the broader curriculum; because studying in this specific domain has the potential to provide individuals with reasoning, argumentation, proof and algebraic thinking skills; besides structural awareness. However, important aspects of the domain that should be studied in the primary school level are not yet identified through empirical research (Prediger, Stehlikova, Torbeyns, & van den Heuvel-Panhuizen, 2011).

Much of the research utilized elementary number theory as a context for investigating teachers' or students' problem solving (Toh, Leong, Toh, Dindyal, Quek, Tay, & Ho, 2014) and proof skills (Dreyfus, Hershkowitz, & Schwarz, 2001; Edwards, 1998; Lee & Wheeler, 1987; Martin & Harel, 1989; Miyakawa, 2002; Tabach, Levenson, Barkai, Tsamir, Tirosh, & Dreyfus, 2011). However, relatively little research is found on teaching and learning of the content itself (Bair & Rich, 2011). Among the research studies reviewed by the researcher, Bair and Rich (2011)'s work is the most pertinent to present study.

Bair and Rich (2011) developed a conceptual framework characterizing the development of specialized content knowledge for teaching in algebraic reasoning and number theory. The two mathematics courses they taught Algebraic Reasoning and Number Theory enabled them to study a large sample of K-8 teacher education students, who were taking either one or both of the courses within three consecutive years. Participants had differing mathematical knowledge and background. The sample included mostly undergraduate students and sometimes graduate students, some of which were at the same time practicing teachers or were holding master's degree in mathematics. Data were collected simultaneously as teaching of the two courses took place. Lived experiences of students in the classroom, their responses to verbal or written tasks, and also authors' individual observations and reflections constituted the data for the study. Authors met weekly to review past week's data and also to prepare for the next class. A grounded theory approach to data analysis was employed on the data, and resulted in a five-level (*Level 0 to Level 4*) developmental framework for specialized content knowledge for teaching algebraic reasoning and number theory. The framework treated four integral components of teachers' capabilities to (1) solve problems and justify his/her reasoning, (2) use multiple representations, (3) recognize, use, and generalize conceptually similar tasks, and (4) pose problems.

Bair and Rich (2011) indicated that the resulting framework did not include hierarchical levels where students progress from one level to another orderly; instead participating students were stated to move back and forth within the five developmental levels. Moreover, four dimensions were interconnected to each other. For instance, lack of ability to recognize conceptually similar or dissimilar tasks could be the reason for a lack of ability to explain and justify relationships, as authors illustrated. The framework had the potential to guide mathematics teacher education, specifically in monitoring and enhancing teacher education students' progress (Bair & Rich, 2011).

2.6. Summary of the Literature Review

Despite the wide acceptance of the idea that teachers need to know their content from a more advanced perspective, the exact nature and scope of that knowledge is not certainly defined (Ball, Hill & Bass, 2005). Based on the need for understanding the ways in which teachers must know their content, specifying correct amount and range of that knowledge and promoting effective use of such knowledge in the real classroom setting, Ball and her colleagues (Ball et al., 2008; Ball et al., 2005) focused their studies on the work of teaching. They qualitatively analyzed what teachers actually do in the course of teaching and what type of mathematical knowledge is required in performing this task. As a result of their observations, Ball and her colleagues proposed a detailed outline of "*mathematical knowledge for teaching*" (Ball, Thames & Phelps, 2008; Ball, Hill & Bass, 2005).

Initially, much of the research on teachers' mathematical knowledge concentrated on its relation to either teacher effectiveness or student achievement. The common trend have been criticized for using substitute variables for teacher knowledge, such as the number of mathematics courses taken or having a degree in mathematics. Although the criticisms are in point, since these characteristics of teachers may not truly reflect their knowledge for teaching; majority of findings support the existence of a positive relationship in between, but only in the field of mathematics. Then, the following question is raised: How teachers' MKT is affected by their mathematics coursework? This study attempts to address this void within the content domain of number theory. Number theory is of great importance to mathematics education at all levels, because it lends itself to many opportunities for students' understanding of the nature and structure of mathematics. While most research use the topic of number theory as a context for studying problem solving and proving skills, research on learning and teaching of the topic itself is scant. Hence, the main focus of this study is on preservice teachers' using of their knowledge of number theory in relation to teaching middle school mathematics.

CHAPTER 3

METHODS

This study investigated preservice middle school mathematics teachers' views on the general mathematics coursework in terms of its relevance to their future teaching; and how they used their mathematical knowledge of number theory concepts developed in the Basic Algebraic Structures course in conducting mathematical tasks of teaching at the middle school level. In this chapter, method of inquiry will be explained in detail. First, the design and participants of the study will be introduced and each of the instrumentation, data collection and data analysis procedures will be described. Next, the issues related to trustworthiness of the study will be addressed.

3.1. Design of the Study

In this study, qualitative research methodology was employed. Qualitative research methods are appropriate when the researcher is interested in how individuals generate their own conceptions of life events and situations, as they personally encounter or take part in (Merriam, 2009). While conducting qualitative studies, researchers aim to present a comprehensive overview of the issue they are investigating, as a final product (Fraenkel & Wallen, 2006). The first research question was investigated through phenomenology and the second research question was investigated through basic qualitative research.

Phenomenology refers to the studying how people consciously experience their life and/or the world and understand the essence of these experiences (Merriam, 2009). This also frames investigating how preservice middle school mathematics teachers make sense of their experiences about mathematics content courses in the teacher education program and a specific mathematics course for their future career, through their responses to the semi-structured interview questions.

Merriam (2009) describes basic qualitative research as a design which investigates the reality individuals construct as they interact with their social worlds. The focus is to understand the meaning of a phenomenon for the individuals (Merriam, 2009), such as using the mathematical content knowledge of number theory concepts covered in the Basic Algebraic Structures course for conducting the mathematical tasks of teaching in middle school classes.

3.2. Context

The context for this study was an Elementary Mathematics Education program (EME) at an English-medium public university in Ankara. The four-year undergraduate program was training pre-service middle school mathematics teachers to teach mathematics at grade levels 5 to 8. Courses offered in the program ranged from mathematics, physics, statistics, and mathematics teaching methods courses, to educational sciences, research methods, history, language and elective courses. The first two years of the program mostly focused on the study of university level mathematics courses. Together with the two other content courses given in the third year of the program, preservice teachers were to take a total of nine mathematics content courses, all of which were taught by the Department of Mathematics. Four of the content courses, including Basic Algebraic Structures, were offered to EME students and their mathematics major counterparts concurrently in their first year. The last two years of preservice teachers' studies in the program included the study of educational courses related to teaching of mathematics, whereas school experience and practice teaching courses were placed at the fourth year. Detailed list of courses is given in Appendix B.

3.3. Participants

Participants of the study were 14 preservice middle school mathematics teachers who were enrolled in the particular EME program, in the Spring semester of 2013-2014 academic year. Five of the preservice teachers (all females) were in their 3^{rd}- year, and nine of them (7 females and 2 males) were in their 4^{th}- year in the program.

Preservice middle school mathematics teachers were selected by means of purposive and convenient sampling strategies. As qualitative studies are conducted with the major purpose of eliciting detailed information; and generalization of findings is not intended, selection of a purposive sample was favorable for the study (Merriam, 2009). Merriam (2009) defines a purposive sample to be the one that has the greatest potential to enlighten the study. In order to obtain a purposive sample, researchers rely on their already existing information and use their judgment to select participants that they believe will serve the purpose of the study best (Fraenkel & Wallen, 2006). Mathematics courses in the EME program were offered in the first three years of the program; and each of the two courses on methods of teaching mathematics were offered within the third year. Since the study was conducted at the end of the Spring semester, both 3rd- and 4th-year preservice teachers had completed mathematics courses and methods courses, and hence expected to have the ability to establish relationships between them.

During the 2013-2014 Spring semester, about 45 preservice middle school mathematics teachers were enrolled in each of 3rd and 4th year levels of the EME program. In order to select participants from among them, the following procedure was employed. First, the course grades that preservice teachers have taken from Basic Algebraic Structures course were accessed through the student affairs information system and listed from highest to lowest. Participants were selected from among those who were placed in the upper part of the list. This preliminary criterion was determined based on the finding that a reasonable correlation exists between middle school mathematics teachers' mathematics coursework and their mathematical knowledge (Hill, 2007). That is to say, the participants selected by this means were considered as the most successful ones in the course, hence in number theory concepts, compared to their peers. Next, the second criterion of selection was applied to the preservice teachers who ranked relatively higher on the list. Those who were also competent in the courses related to mathematics teaching, especially in the methods of mathematics teaching course, were identified by the instructor of the methods course in the EME program at the time of the study and hence possessed sufficient knowledge of students. In this way, a homogeneous sample of preservice middle school mathematics teachers was obtained. A homogeneous sample is the one in which all of the members are selected because they possess a certain trait or characteristic (Fraenkel & Wallen, 2006). In the case of present study, participants' common characteristic was being more competent both in the mathematical content course and in the mathematics education courses, compared to their peers in the same year of the program. Participants' having adequate knowledge of the two types of mathematics (the one they were taught in Basic Algebraic Structures Course and the one they will teach in middle schools) was critical for this study in the sense that they were expected to establish sound relationships between the two, in line with the purpose of this study.

The obtained sample was also a convenient sample for the researcher since it provided her with certain advantages in the allocation of resources such as time, energy, money, location, and availability of individuals (Merriam, 2009; Fraenkel & Wallen, 2006). Although convenience sampling strategy is suggested not be employed as a basis alone for selecting participants, any sampling strategy involves some sort of convenience for the researcher (Merriam, 2009). In this study, the selected sample consisted of preservice teachers studying at the same university with the researcher who was also working as a research assistant. Thus, the researcher was able to reach the participants conveniently throughout the study.

3.4. Instrument

Two distinct interview protocols were implemented to the participants as the main data collection tools for this study. Table 3.1 shows the research questions that each of the protocols was used for answering. Both of the interview protocols were designed by the researcher in the Fall semester of 2013-2014 academic year and they were administered to the participants within the next semester, in a single session, consecutively. Detailed descriptions of the protocols are provided below.

Table 3.1

Interview Protocols in Relation to Research Questions

Research Question	Interview Protocol
1. How do preservice middle school mathematics teachers perceive the general mathematics content courses in the teacher education program in terms of their relevance to their future teaching? a. How do preservice middle school mathematics teachers perceive the specific mathematics content course "Basic Algebraic Structures" in terms of its relevance to their future teaching?	Semi-Structured Interview Protocol for Preservice Teachers' Ideas
2. How do preservice middle school mathematics teachers use their mathematical knowledge from Basic Algebraic Structures course in conducting *mathematical tasks of teaching*?	Structured, Task-Based Interview Protocol

3.4.1. Semi-Structured Interview Protocol for Preservice Teachers' Ideas

The semi-structured interview protocol including 17 open-ended questions was implemented first to understand preservice teachers' perceptions of if/how the general mathematics content courses offered in their program were related to their training, before they experienced the task-based interview. The questions were about mathematical content courses in general and Basic Algebraic Structures course as illustrated in Table 3.2. Probing questions were asked based on obtained responses.

3.4.2. Structured, Task-Based Interview Protocol

The second interview was a structured task-based interview. Goldin (2000) states that structured task-based interviews are used in qualitative studies in the field of mathematics education to portray and make sense of mathematical behavior. By conducting task-based interviews, qualitative researchers take the opportunity to make systematic and thorough observations of subjects' mathematical thought (Goldin, 2000). One or more tasks of the subject are presented by the interviewer and the interviewee is interacted with the interviewer and the given task simultaneously

(Goldin, 2000). The amount and nature of the intervention are previously determined by the researcher, but may also be adjusted at the time of interviewing (Goldin, 2000).

Table 3.2

Examples of interview questions.

1. What kind of new mathematical knowledge have you learned at the university?
2. Have you learned any mathematical knowledge related to the mathematics that you will be teaching in the future to middle school students?
3. In what kind of courses have you learned such knowledge?
4. Do the general mathematics content courses you have taken from the Department of Mathematics contribute to your teaching profession?
5. Is the mathematical content that you have learned in Basic Algebraic Structures course related to the mathematics that you will be teaching in the middle school level? How?
6. Do the knowledge and skills you acquired through Basic Algebraic Structures course contribute to your teaching of mathematics? In which cases?

In the current study, participants were presented four mathematical tasks previously developed by the researcher. Each task involved a hypothetical classroom event and participants were asked a number of related questions, and respond by pretending the role of a middle school mathematics teacher who experienced these specific events in his/her own classroom. Such *as if* experiments are referred to as *role-playing* in the related literature (Aronson & Carlsmith, 1968). By combining the two research instruments of structured task-based interviews and role-playing, this interview of the study aimed to how preservice teachers established relationships between number theory concepts they learned in Basic Algebraic Structures course and their future teaching of these concepts to middle school students.

Interview tasks were designed based on three primary considerations. The first was the number theory concepts covered in the Middle School Mathematics Curriculum (MoNE, 2013). In Turkish curriculum, teaching of number theory concepts takes place within the sub-learning area “Factors and Multiples” of the broader learning area “Numbers and Operations” (MoNE, 2013). There are eight learning objectives listed under Factors and Multiples, five at the 6th grade level and the rest are at the 8th grade level. The eight objectives are listed below in Table 3.3, accompanied by the important notes specified for teachers in the curriculum guide.

Table 3.3

Learning Objectives Addressing Number Theory Concepts in Middle School Mathematics Curriculum (MoNE, 2013)

Grade	Objective
6	Identifies factors and multiples in natural numbers.
6	Explains and uses rules for divisibility by 2, 3, 4, 5, 6, 9 and 10.
	Consider that rule for divisibility by 6 can be developed by making use of the rules of divisibility by 2 and 3.
6	Identifies prime numbers with their properties.
	Prime numbers up to 100 are found with the use of Sieve of Eratosthenes.
6	Identifies prime factors of natural numbers.
6	Identifies the common factors and multiples of two natural numbers; solves related problems.
	Problems that require finding greatest common divisor (gcd) and least common multiple (lcm) of two natural numbers are not mentioned at this grade level.
8	Finds factors of given positive integers; writes positive integers in the exponential form or as a product of exponential factors.
	Practices for identifying prime factors of a positive integer are also included.
8	Computes greatest common divisor (gcd) and least common multiple (lcm) of two natural numbers; solves related problems.
8	Determines if two given natural numbers are relatively prime or not.

Note: Translated by the researcher.

The above objectives were taken as a fundamental basis in determining the mathematical content of hypothetical classroom events and events that were likely to occur in a real classroom setting.

The second consideration in designing of interview tasks was the number theory concepts taught in mathematics content courses offered in the EME program. Primary number theory concepts that underlie the hypothetical classroom events were addressed in the Basic Algebraic Structures course at the University in which the study was conducted. This course was offered to EME students and their mathematics major counterparts concurrently in their first year, by the Department of Mathematics. The course content covered basics of algebra and number theory. A tentative syllabus of the course, which had been in use for the last five years, is given in Appendix A.

The third consideration guided the researcher in preparing hypothetical classroom events was that the events would require the participants to use their mathematics knowledge from Basic Algebraic Structures course in conducting the *mathematical tasks of teaching* as a middle school mathematics teacher. In other words, it was the integration of “Specialized Content Knowledge” component of the general framework “Mathematical Knowledge for Teaching” (Ball, Thames & Phelps, 2008) into the interview tasks. SCK is mainly concerned with how teachers, unlike other mathematics-related professionals, need to organize their mathematical knowledge in an attempt to satisfy the *mathematical demands of teaching* (Ball, Thames & Phelps, 2008; Hill & Ball, 2004). Ball and her colleagues (Ball, Thames & Phelps, 2008; Hill & Ball, 2004) summarized those mathematical tasks of teaching that are distinctive to SCK (See Table 2.1 in Literature Review section). There are 16 teacher behaviors listed in this summary. Four of these behaviors constituted the basis upon which the four hypothetical classroom events of this study were created. They are *responding to students’ “why” questions, evaluating the plausibility of students’ claims, recognizing what is involved in using a particular representation,* and *inspecting equivalencies.* Each of the tasks exemplified a different *mathematical task of teaching* as summarized in Table 3.4.

Table 3.4

Tasks in Interview Protocol in Relation to Mathematical Tasks of Teaching

Mathematical Tasks of Teaching	Place in the Interview Protocol
Responding to students' "why" questions	Task 1
Evaluating the plausibility of students' claims	Task 2
Recognizing what is involved in using a particular representation	Task 3
Inspecting equivalencies	Task 4

After the interview tasks were given their primitive forms by the researcher, three experts' opinion were obtained. Two of the experts were subject matter specialists from the Department of Mathematics at the university the study was conducted, one of which had been involved in the work of curriculum development for middle schools in Turkey. They examined the tasks in terms of mathematical correctness and decided whether the researcher made valid or invalid connections between the concepts. The other expert was the supervisor of the researcher who was a mathematics education researcher. She examined the tasks not only in terms of their mathematical correctness, but also with two additional perspectives: likeliness of the hypothetical classroom events to take place in a middle school classroom, and compatibility of the tasks with the research purpose of the study. By this way, content-related evidence of validity was ensured, and the interview protocol was revised according to three experts' reviews and recommendations. In addition to experts' opinions, findings from the pilot study (explained in the next section) were used in finalizing the research instrument. Task 1 and corresponding questions are presented in the Figure 3.1 as an example. The entire interview protocol is given in Appendix C.

TASK 1
(Responding to Students "Why" Questions)

You are teaching "Prime Numbers". You presented below definition of a prime number to your students.
Definition:
A number greater than 1 is called a prime number if its only divisors are 1 and itself.
After a while, one of your students asked:

- Teacher, the number "1" is also divisible only by 1 and itself. Why do not we take it as a prime number, then?

(Above part was given to the participant in written form, and following questions were asked verbally:

1. What do you think about this issue? Why do not we take "1" as a prime number?
2. How do you explain this to your 6^{th} grade students?
3. Have you learn anything about this issue in the courses you taken from the faculty of education? / Do you remember anything related to this?
4. Can you find an answer to this question by using the course book of Basic Algebraic Structures?
5. How do you explain this to your 6^{th} grade students?)

Figure 3.1 An illustrative task from the task-based interview

3.4.3. The Pilot Study

The pilot study was conducted with the purpose of reviewing the interview questions, and enhancing the procedures to be followed in conducting the interviews. Two 3^{rd}-year and two 4^{th}-year preservice teachers participated in the pilot study voluntarily. Participants of the pilot study were selected from among the preservice teachers who were determined to be relatively successful in the Basic Algebraic Structures course and in educational courses, just as those who were selected for the actual study. After completing the interview, participants were requested to comment on understandability and clarity of the mathematical tasks presented to them and the open-ended questions asked. The pilot study yielded three important changes with data collection tools and procedure. Both interview protocols were revised according to feedbacks received from participants. One important modification was made with the wording of an open-ended question. Since the participants evaluated two successive questions to be the equivalent of each other, although they responded to the former one as expected, they did not interpret the latter one as intended by the researcher. For this reason, at the end of the pilot study, the prior question was kept as it was, but the subsequent question was re-worded. Table 3.5 shows the two questions, and the revisions made. Also, a number of follow-up questions were added to the interview protocol.

Table 3.5

Two successive questions and the revisions made

Question	Initial Version	Revised Version
6^{th}	Does the mathematical content that you have learned in Basic Algebraic Structures course relate to the mathematics that you will be teaching in the middle school level? How?	-
7^{th}	Do the knowledge and skills that you acquired through Basic Algebraic Structures course contribute to teaching profession? In which cases?	Do you think that the knowledge and skills you acquired through Basic Algebraic Structures course will help you in your teaching of mathematics? In which cases?

Based on the results of the pilot study, materials provided to the participants during the interviews were revised as well. In Task 3 and Task 4, participants were presented some mathematical statements from the course book. They were expected to examine the statements and select an appropriate one to their situation. However, the pilot study revealed that these statements were too much in number to examine at a time. Participants had difficulty in examining those statements consecutively, as each one necessitated a considerable amount of thinking. In addition, they were distracted by the variety of formats in which the statements were worded. With the participants' agreement, the researcher reduced the number of statements from 21 to 12, and re-worded some of them to obtain a single simple format such as "Prove that" and proposition.

Furthermore, all four participants of the pilot study indicated that the idea of using the course book (while working on the mathematical tasks) caused them to feel anxious, because long time had passed since they had taken the course. For this reason, the researcher prepared a summary booklet which consisted only of the related content. Still, participants of the actual study were provided with the whole book in the case they preferred to use.

3.5. Data Collection Procedure

The data were collected from preservice middle school mathematics teachers towards the end of Spring semester in 2013-2014 academic year, after necessary permissions were gathered from the university (See Appendix D). Purposively selected preservice teachers were contacted directly by the researcher and kindly requested to participate in the study. All of the selected preservice teachers volunteered to participate. Interviews were conducted separately in one-to-one settings by the researcher and lasted between 60 to 100 minutes. Interviews were audio- and video-recorded by the researcher with the participants' permission. Their worksheets were also collected to be analyzed.

At the beginning of the interview, participants were informed that no grading would be made out of their performance. The researcher explained to the participants that they would mainly work on four mathematical tasks as if they were middle school mathematics teachers; but they would also be asked some verbal questions. The tasks were printed on separate sheets of paper and presented to the participants one by one. For each of the tasks, participants were given time to think about and respond to related questions either verbally or in written form. Although participants were provided with paper and pencil during the whole interview, they used them only in the case they needed. They mainly worked on the short booklet prepared by the researcher. However, in some parts of the interview, they were also permitted to use the course book of Basic Algebraic Structures.

3.6. Data Analysis

In this study, two types of data were collected and analyzed separately. After all interviews were transcribed by the researcher, content analysis technique was followed to analyze the semi-structured interview data. Content analysis "process involves the simultaneous coding of raw data and the construction of categories that capture relevant characteristics of the document's content" (Merriam, 2009, p.205). In this study, the analysis of the semi-structured interview data included careful observation of the transcribed data with the purpose of capturing any statement or word coming from participants which indicated their views about the relevance of general mathematics content courses to their future teaching of middle school mathematics. Statements describing the courses were categorized into six groups: *higher level*, *irrelevant to middle school mathematics* and *not applicable to teaching of middle school mathematics, unnecessarily extensive, too abstract,* and *constituting the base for middle school mathematics.* Each of the categories are explained in the results section for answering the first research question of the study. A similar analysis was followed for answering the subquestion concerning the specific content course Basic Algebraic Structures. Parts of the content that were related, by participants of the study, to middle school mathematics content, and those who considered the course useful for their future work of teaching were reported in frequencies. Moreover, those who considered the course useful were asked to specify where, in which situations could this happen. Responses were categorized by using content analysis technique and compared with the 16 mathematical tasks of teaching identified by Ball, Thames and Phelps (2008).

Another type of analysis was conducted for answering the second research question. Participants' written work on the mathematical tasks of teaching was analyzed, together with the supporting explanations they made. Correctness and the depth of the mathematical ideas were the focus in this part of the data analysis. Frequencies of correct responses were reported for each of the four tasks, while alternative responses from participants were also explained in detail.

3.7. Trustworthiness of the Study

Many interpretivist researchers argue that there is no *fact of the matter* in determining the criteria for evaluating the quality of conclusions drawn from a qualitative research study (Miles & Huberman, 1994). However, still efforts to specify shared standards of evaluation continue (Howe & Eisenhart, 1990). Being one of the many entrepreneurs, Merriam (2009) suggested that research results are "trustworthy to the extent that there has been some rigor in carrying out the study" (p.209). She explained trustworthiness and rigor in qualitative research with reference to quantitative terms. She referred the concepts *validity*, *reliability* and *external validity* as *credibility*, *dependability* and *transferability* respectively, as Lincoln and Guba (1985) also did. The following is a discussion of the three criteria for this study.

3.7.1. Internal Validity or Credibility

Merriam (2009) defined credibility of a qualitative research study as the consistency between the actual situation and the way it is interpreted in research findings. She suggested six ways of ensuring credibility, which are triangulation, member checks, adequate engagement in data collection, negative case analysis, researcher position, and peer review. Three of them were employed in this study; which are peer review, researcher position and adequate engagement in data collection.

Researchers' providing information about their preconceptions, tendencies and hypotheses regarding the research conducted is desirable in qualitative studies (Merriam, 2009). Such information enables the reader to understand "how a *particular* researcher's values and expectations influence the conduct and conclusions of the study" (Maxwell, 2005, p.108).

In this study, the researcher was the only instrument to collect and analyze data. As she was novice in conducting qualitative research, she tried to read and learn about specific types of qualitative research and critical issues affecting the quality of research. The first concern applying to the current study was the researcher's past experiences with the mathematics content course Basic Algebraic Structures she had taken in the same undergraduate program at the same university with participants of the study, previously. Especially for the task based interview, the researcher had her own answers to mathematical tasks of teaching, but kept them away while conducting the interviews. She was extremely careful for not directing the participants towards a specific response that she had anticipated while developing the interview tasks. She was also equally rigorous in analyzing participants' responses with an objective perspective.

The second important concern was the researcher's relationship with the participants of the study. The data were collected from two different groups of preservice teachers. While 4th year participants were already in touch with the researcher, 3rd year participants were contacted by the researcher for the first time within the scope of this study. Since the researcher was a graduate assistant for the 4th year participants' several courses over the last two semesters before the study, it was easier to motivate them to take the interviews seriously.

The 3rd year participants were also willing to participate in the study. The reason for their motivation might be that they were told that a set of selection criteria was applied to select them. In addition, the researcher tried to ensure mutual trust and comfort with the participants, at the beginning of each interview through casual conversations. These conditions seemed to help 3rd year participants to feel free to express their thinking and show their mathematical work without hesitation.

Peer review process includes a knowledgeable peer's reviewing some excerpts from the raw data and evaluating the plausibility of conclusions made depending on this data (Merriam, 2009). In this study, the researcher asked a graduate student working in the mathematics education field, who was competent in qualitative research, to assess the consistency of her findings with the actual data. The data collection and analysis process were also monitored by the supervisor of the researcher to ensure peer review.

Adequate engagement of data is a strategy for saturating the findings, by adjusting the number and length of observations until they start to become consistent repetitions of each other (Merriam, 2009). Although the data for this study were collected from participants in a single session each, they were given plenty of time both for the semi-structured and the tasks-based interview within this single session.

Questions were, at times, re-directed to participants for better understanding of their thinking. Especially in the task-based interview they were given time to study the tasks several times in their own, until they felt ready to explain their thinking. Further questions were always asked; and originating from the nature of the task-based interview itself, researcher spent ample time with each participant, observing their work carefully. Moreover, data collection process showed the researcher that 14 participants were sufficient to be studied within the context of this study, as most of the time, findings included frequent repetitions.

3.7.2. Reliability or Dependability

Reliability, in general terms, is about replicability of research results. However, as it is not expected to obtain the same results when a qualitative study is replicated, *reliability* here refers to the consistency between the data collected and inferences made based on this data (Merriam, 2009).

Merriam (2009) suggested four methods for increasing reliability of a qualitative research study. These methods are triangulation, peer examination, researcher's position, and the audit trial. In this study, researcher's position and peer examination were attained. Both of these concepts were discussed previously for ensuring also the credibility of the study.

In addition to these, various interview scripts were provided throughout the results section to illustrate the interview tasks, the questions asked, interview contingencies showed up, and also the researcher's decisions. This important strategy of providing interview scripts was suggested by Goldin (2000) as a way of allowing replicability in task-based interview research.

3.7.3. External Validity or Transferability

The most widely used way of ensuring transferability is *rich and thick description* (Merriam, 2009) which is a detailed portraying of the study conducted. It includes the design, participants, and the procedures followed for concluding findings. Therefore, this strategy enables the reader to compare important features of a qualitative study to their own conditions and transfer the findings of the study in case of similarity (Merriam, 2009). While reporting the present study, the researcher tried to provide sufficient description of each detail with paying careful attention to issues that are taken for granted in order not to miss any important point. Rich and thick description of the context was also used as a validation strategy, as suggested by Creswell (2007).

3.8. Limitations of the Study

There are four substantial limitations of this study. First of all, the researcher was inexperienced in conducting interviews and carrying out a qualitative research study. By the help of useful readings and that of conducting a pilot study, she gained valuable insights about critical issues regarding qualitative research. After evaluating the quality of the pilot study with some of her colleagues and also with her supervisor, the researcher reflected on herself in order to better perform the appropriate procedures in the actual study. Furthermore, aiming to reduce potential biases, the researcher provided a detailed explanation of her role in conducting this research while discussing the *researcher's position* also for increasing credibility.

The second important limitation of the study was about the selection of participants from a single teacher education program. Interview data were collected from 14 preservice middle school mathematics teachers, all studying at the same university in which the medium of instruction was English. Thus, participants of the study might not be representative of 3rd and 4th year preservice middle school mathematics teachers in Turkey. It should be noted that generalization was not a concern in this study.

The third, and probably the most important limitation of the study was concerned with the instrumentation process. The task-based interview included very specific instances from each of the three bases: middle school mathematics, university mathematics and mathematical tasks of teaching. First, the content of the tasks was restricted to the knowledge and teaching of basic number theory concepts, and a single mathematics content course. Also, the study explored participants' behaviors in only four of the 16 mathematical tasks of teaching. More of them could have been integrated into the interview tasks to reach more general conclusions about preservice teachers' Specialized Content Knowledge.

Last, although it would have been more informative for the research community to support the findings of this study with other aspects of mathematics teaching, the interview data were analyzed with respect to a single perspective: the concrete mathematical relationships provided by preservice teachers. Participants' beliefs, attitudes or values, or other factors that might have an effect on their handling the mathematical tasks of teaching, were not incorporated into this study. Additionally, the findings were limited to the instruments that the study used for data collection.

CHAPTER 4

FINDINGS

This chapter summarizes findings of the research under two main sections. The first section includes information regarding preservice middle school mathematics teachers' views on the nine mathematics content courses offered in their program, as identified from their responses to semi-structured interview questions. In this section, preservice teachers' views regarding the specific content course, "Basic Algebraic Structures", and how they perceived this course in terms of its relevance to their future teaching are reported as well; as the course was the focus of the present study. The second section is devoted to describing how participants used their mathematical knowledge from Basic Algebraic Structures course in conducting *mathematical tasks of teaching*. Detailed analysis of participants' work on four illustrative mathematical tasks of teaching is presented on the basis of correctness and depth of the mathematical ideas they proposed.

Responses obtained in each section are documented mostly through summaries or direct quotations of participants' claims. In order to convey the actual meaning more correctly while translating from Turkish to English, some modifications were made on participants' quotes. Parentheses and brackets were used for indicating the modifications made as illustrated in Table 4.1. Moreover, some of the statements were supported by illustrative pictures of participants' written work.

Table 4.1

Examples of Modifications on Participants' Quotes

Modification	Purpose
one thinks that they (the content courses) are not that necessary	Clarifying the meaning
taking the "abstract" course (preservice teachers refers to two of the basic algebra courses by this name)	Researcher's explanation
did not understand why [they had been] taking these course at all	Tense adjustment
[L]et us say we write a prime number in the form of a x b.	Sentence adjustment
we check if [the number] is divisible by both 2 and 3	Completing the meaning Increasing readability
[...]	Excluded parts from the quote
...	Indicating pauses in speech

4.1. Preservice Middle School Mathematics Teachers' Views on General Mathematics Content Courses

Preservice middle school mathematics teachers participated in this study mainly considered the mathematics they learned in general content courses as *higher level* (n=8), *irrelevant to middle school mathematics* (n=6) and *not applicable to teaching of middle school mathematics* (n=7). Participants frequently expressed "*In those courses we have studied higher level (mathematics) than the middle school mathematics*" (P14, 3rd-year), "*The things we have learned there had no relation to the things with the middle school*" (P4, 3rd-year), and "*When we start teaching in grades 5, 6, 7, 8, we won't use any one of that knowledge we have learned here*" (P13, 4th-year). Some other participants pointed out that the content covered in these courses were *unnecessarily extensive* (n=3) and *abstract* (n=2). P3 (4th-year) argued that she did not need to know such extensive mathematics as a middle school mathematics teacher:

To me, they (content courses) have nothing for us. I mean, if we think of the middle grades education. Well, calculus, linear algebra and the like.... I do not know, will it ever come into use in middle grades education? I mean, we won't teach yet that much extensive things to our students.

P2 (4^{th}-year) explained the abstractness of mathematical content covered in the courses by giving an example from her past experiences:

For me to teach a concept, I myself need to know it first, to be able to teach it. But, it (the mathematics addressed in content courses) seems to me kind of, you now... too abstract. We cannot teach it to the student in that form. Let me give an example. I have taken geometry concepts course (an elective course offered by the Faculty of Education). In the course, you know, the transformation geometry, translation and the like... (are covered). In fact, we have seen those subjects previously in (those courses), in the geometry course, in the course 201 (one of the nine content courses); but I could not remember any one of those things (while taking the elective course later). I am trying to say, if we attempt to give (those concepts) to the student with those definitions (from the content course), we cannot make it understood. There I have realized that even I had not understood those things. [...] Consider the symmetry (topic), what is symmetry? Well, a mirror-image and such things, we present it to the child in that way. But, it has lots of mathematical functions, explanations and other things in (the course 201), you know.

On the other hand, four of the participants regarded the mathematics they learned in the content courses as "*constituting the base*" (P2, 4^{th}-year) for middle school mathematics. P7 (4^{th}-year) expressed it as in the following:

Well, actually as I said before, the courses like introduction to the basics of mathematics, those introduction courses such as algebra and the like... Of course they are the fundamentals, what comes from where, how does it come, it may be a bit more like ... has given ideas to us about what is there at the base of mathematics. (In those courses) we had attended more to the essence of mathematical concepts.

However, 3 of the 4 participants, who perceived the content of these courses a base for middle school mathematics, still did not think that taking these courses was beneficial for their career. Constituting a good example for this approach, P7 continued her words with:

... but that part still does not convince me at all about why to know (that mathematics). [...] Maybe I am wrong at this moment, but I do not find them necessary, let me say this for not all of them, let me say for the most of them.

On the contrary, P6 indicated positive views about the need for learning the basics of mathematical knowledge:

We need to know mathematics. Okay. We will not teach these exactly, but I think we should now this. I believe we are required to know ... the basis. [...] Well, if we are considering ourselves as mathematics teachers, then it should not be merely let's say... solving equations for instance. I should know everything that, you know, underlies … I mean, where does it come from, we need to know this. If I know these things, then I think I can be a more effective teacher. (P6, 3rd-year)

Although not all of the participants used the word "irrelevant" explicitly in their statements, their responses to two consecutive interview questions revealed that they generally did not relate the mathematical content covered in the mathematics courses in the teacher education program to the mathematics taught at the middle school. Participants were asked first "What kind of new mathematical knowledge have you learned at the university, during your undergraduate education?" This question was followed by: "Have you learned any mathematical knowledge related to the mathematics that you will be teaching in the future to middle school students?" While all of the 14 participants provided concrete examples of their learning from both the content courses and the methods of teaching mathematics courses for answering the first question, this was not the case for the subsequent question. When the second question was directed to participants, most of them (11) spoke only of the methods courses and/or some of the elective courses offered by the Faculty of Education, without making reference to any one of the nine content courses. Besides, some of the participants added to their comments that "*the others, content courses were not even close to the field of middle school mathematics education*" (P10, 4th-year). For example, P5 (4th-year) and P7 (4th-year) answered the latter question as:

One of the must courses (is related to the middle school mathematics): Method. None of the other courses I have taken from the Department of Mathematics is related. But four of the elective courses I have taken up to now, I think, are very relevant. I could mention: GeoGebra, problem solving, *hmm… geometry concept, and then hands-on. These were all more useful. But among the must courses, it is only the methods course I think. (P5, 4th-year)*

What have not I learned! I think I have learned lots of things... But, I think, rather than the mathematics courses, the courses I took, generally that are related to my field (mathematics teaching) have more contribution to my profession. You know the methods courses, especially the methods courses are to me the most essential courses of the department of mathematics education; I believe they are the ones that should be given from the very beginning to end.(P7, 4th-year)

Participants were pointedly asked about the rationale behind requiring preservice middle school mathematics teachers to take those mathematics content courses. Almost half of them (n=6) referred to its contributions to their personal growth, rather than its professional benefits. They commonly pointed out improvements they had experienced in terms of their "*intellectual development*," (P5, 4th-year) "*brain function, […] analytical thinking skills"* (P8, 4th-year), and ability to "*look from different perspectives*" (P1, 3rd-year). Below are some illustrations of participants' related statements.

Well, since we study middle school mathematics education, well the mathematics (courses) we take here ... I can consider them only as ... in terms of their widening our viewpoints, the dimension, I mean, broadening our horizons. Otherwise, take either calculus courses, or diff (differential equations), in these courses we have studied much higher level, well, higher level than the middle school mathematics. […] Relevant to that (mathematics), I mean at that level, we did not learn anything at that low level. (P14, 3rd-year)

Well, when we were taking the "abstract" course (preservice teachers refers to two of the basic algebra courses by this name), I felt more that you know ... our analytical thinking skills improved. There, while writing proofs at most, especially establishing cause and effect relations between those things ... you know either proving or refuting something. I mean, they are really improving our thinking skills. (P8, 4th-year)

P5: *There are many people in our department who strictly object to this (taking advanced content courses). You know, [the sayings]: "But, I am going to be a middle school mathematics teacher, why am I learning this kind of abstract mathematics?" As for me, I am against them. This is something essential.*
Researcher (R): *Why do you think it is essential?*

> ***P5:*** *After all, we are at the university… Of course, they will not teach us adding or subtracting. Even that calculus is beneficial ... for the completion of that cognitive development. For us to be challenged more, these courses need to be given to us. (P5, 4th-year)*

In addition to these non-teaching-specific perspectives, P14 touched upon an important subject of the field of teacher education. She pointed out that taking the content courses might have positive effects on preservice teachers' attitudes towards mathematics:

> *At the first sight for someone who studies, I mean who is going to be a mathematics teacher, one thinks that they (the content courses) are not that necessary, but as you take them you know one's ... how should I say? Her attitude towards mathematics and also thoughts about mathematics get better. Maybe we are taking these courses for this reason. (P14, 3rd-year)*

Apart from P14, four other participants considered that content courses were aimed at teaching related benefits such as preservice teachers' learning of mathematical "*concepts […] more deeply,*" (P2, 4th-year) "*what underlies mathematical knowledge, and where does it come from*" (P6, 3rd-year). For example, P12 and P8 explained that they were in favor of taking the content courses, by referring to connectedness of mathematics.

> *Mathematical topics of course are not all disconnected from each other; they are not the topics that are independent. They are all interwoven subjects with each other. And, for a mathematics teacher to teach well, one of the foremost prerequisite characteristics is probably his knowing of the mathematical topics in the best way and also his ability to make sense of them in the best way. Well, for him to make sense of mathematics, he needs to take mathematics courses that are at a high cognitive level, and from various areas. Okay, I am going to teach adding and subtracting, but… In my first year, I was thinking the exact opposite; I mean, they seemed to be very unnecessary. In the fourth year, my opinion has completely changed. I mean, it definitely needs to be known. (P12, 4th-year)*

> *In fact, some of the subjects are related, you know, equations, algebraic expressions… Well, we are actually getting some of the things started without making the students noticed. I mean, all the topics in mathematics are related to each other, but we are teaching a simpler version of it. Not too much related but it provides benefits. I would say that. […] For example, let me give an example from the "abstract" course. There we use the method of*

> *induction you know, but well, to the students also, we teach patterns. Actually, for finding the things with the patterns, like the relations and such things, to be able to find the general formula, it is required that we know and that we master the method of induction. (P8, 4th-year)*

On the other hand, three of the participants indicated that they "*did not understand why [they had been] taking these course at all*" (P3, 4th-year). P7 (4th-year) expressed her thinking as "*I am also really wondering very much the key rationale for this. Someone should tell me why we are taking these courses; there are times when I think this way. […] I ask myself 'What will I do with…?'*" She also underlined how she had been questioning herself about the issue as follows:

> *I am also thinking like ... we have taken these courses with engineers and others... Every one of us takes the same courses. Now, the mathematics that an engineer needs to know is that (mathematics), and mine is also that. I mean, they (the stakeholders) think that I am required to know those things, if I am taking these courses, this means they are thinking like this. Well… considering they are requiring us to take, I am speaking for the calculus and the differential equations, their (must mathematics courses given to students of both departments) being the same is confusing me. Then I say: "This means, rather than using this knowledge, anyone who receives an undergraduate education at a university is required to know it." These are the courses to be learned at the university. Because otherwise, when we think of its use, consider an engineer and consider me. But, if I consider 111, 112 and so on, they are more for us, and to the mathematicians, the pure field. […] Of course, there must be a purpose in it, but maybe since I am prejudiced, I am thinking this way. But, what exactly is the purpose ... for the case of our department, I have not figured it out yet.*

Moreover, P9 (3rd-year) criticized the mainstream that content courses were defined as must courses in the EME program, because she considered them as "*too much*" for the teaching profession. Instead, she suggested replacing these courses with some other "*practical courses related to mathematics education, such as a (mathematical) modeling course.*" P9 also highlighted the difference between being a mathematics teacher and a mathematics major as: "*The two fields are separated from each other. They (stakeholders) are offering us much more of it (mathematics). They are more ... for the mathematicians, not for us. I mean, there is no need for this much, I think, for the educational part.*"

Nevertheless, regardless of holding positive or negative views about the necessity of content courses for their professional development, 11 participants of the study other than P9 were still pleased with taking these courses. Participants mostly asserted that they "*have taken these courses with great pleasure*" (P1, 3rd-year) and "*did not have much trouble with them*" (P3, 3rd-year). They also favored the courses being offered to them by the mathematics department. P1's and P8's below statements exemplify most participants thinking:

> *I think that's a good thing, because experts are giving the instruction. Well, this might be challenging for many people, but while learning something... Oh, this is my opinion but, if I have the intention to learn something, then I think doing the best or I mean learning in the best way is vital. As for who can teach the best, I think they are the mathematicians. (P8, 4th-year)*

> *I think they should be the ones who offer [these courses]. All in all, you know ... that is the mathematics department. I mean, they are more expert in mathematics. Therefore, in my opinion, it is better that they are giving the instruction to us. (P1, 3rd-year)*

Two of the participants considered what would be different in the case when these courses were offered by the Faculty of Education. The two participants indicated quite the opposite ideas. While P7 indicated that it could be helpful for making the learned knowledge applicable to teaching of middle school mathematics, P12 did not regard it as quite possible:

> *I think mathematics department's offering these courses is good. Well, now if they were given by our own department, maybe we could somehow combine the things with the education better, we would better connect them with our own field. Maybe, the things that we could make more use of... you know it came out to be like "Will it ever come into use?" for us, but in that case, maybe we could be able to integrate them in a more purposeful manner. But, at the Department of Mathematics, it seems like at the center of everything, there is mathematics. (P7, 4th-year)*

> *Well, in terms of the professional development, if our department was offering these courses, to what extent it could be … there may be existing strategies for this, but I cannot imagine right now. I need to know first about the way they would be taught, if adapted to our department. But, I suppose it would be like so … remote. Might it be because I cannot see the relation of those topics anyway? Maybe this is the reason. But, you know still there*

would not be much difference. […] But, if the purpose for our department's offering was about relating it to the curriculum of the target audience that we ourselves will teach, to what extent can this be achieved? […] Even if they are given by our own department, things will remain same; I somehow feel this way. (P12, 4th-year)

P11 (4th-year) suggested the following change in the chronological order of the courses taken, for increasing content courses' potential benefits for their professional development:

It might have been more efficient for me to take those courses now. Because, we had not known the profession, having graduated from the high school and entered to the university entrance exam. You know, we did not know about what we would teach, which topics were included, the objectives for instance, what is an objective and the like… We have learned about them here, our goals and as such. Maybe it would be better to learn those (content courses) after these (concepts related to teaching) were given to us.

Thorough analysis of the interview data also revealed that some of the participants held conflicting views simultaneously. To illustrate, P2 (4th-year) indicated first:

I honestly do not think they do (contribute) much, I mean to the things with the middle school, or related to the things I will teach…I think they do not have any contribution. None of them… While taking those courses, I have passed them by studying for only getting over that moment, I have learned only for passing the courses. I did not think of it like it would contribute me in the future. […] We will not teach anything about those (subjects). […] I think I will not use them in the middle school.

But when the researcher asked her about the reason behind her being required to take these courses as a preservice middle school mathematics teacher, she replied:

***P2:** Hmm… It may be for us to learn the concepts and the like more deeply. Maybe they are given to us for this reason.*
***R:** Why do you think that we need to know them more deeply?*
***P2:** Actually, now it is like I am refuting myself, but for me to be able to teach a concept, I have to know it first so that I can teach it. […] I mean if I know those things deeply, it is like I can transfer them to the students more eligibly.*

In other words, while P2 at first considered the content courses as not contributing to her professional development, later she defended that she needed to have this kind of knowledge for delivering a better teaching.

Similarly, P6 (3rd-year), who emphasized the need for knowing sources of mathematical knowledge which was dealt with in the content courses, indicated at the same time that the content courses were "*not much contributing to her professional development.*" A similar inconsistency was apparent in P7's below expressions:

> *Surely, I believe it is required that we know; but as a middle school mathematics teacher, to tell the truth, I am one of those who does not think that the courses I have taken from the Department of Mathematics provides me a great benefit. Since, what we have seen here is a kind of different, a more, more advanced version. [...] Here we concentrated on where things come from, how they come, you know, the theorems, and so on, we concentrated on them. But as I said, speaking for myself, I mean, of course we should know; we should know, that's quite another story; but I still do not think that it contributes me much. (P7, 4th-year)*

In summary, participants of the study mainly considered the mathematics learned in general content courses as *higher level*, *irrelevant to middle school mathematics* and *not applicable to teaching of middle school mathematics*. Some of them also emphasized that the contents was *unnecessarily extensive* and *abstract*. In addition to these negative views, there existed participants who regarded the mathematical knowledge learned in these courses as *constituting the base for middle school mathematics*.

Among the 14 participants of the study, only 5 of them (two 3rd-year and three 4th-year) considered content courses as an essential component of middle school mathematics teachers' professional education. The rest of the participants either perceived them only as providing personal growth benefits or indicated that they could make no reason for why they were required to take these courses. Still, most of the participants (n=11) were pleased with taking the courses; and also favored that the courses were offered by the mathematics department. In addition, data analysis showed that some of the participants maintained contradicting views about the necessity of content courses in their program. Particularly, they indicated both that the courses were not contributing to their professional development, and also that they needed to know that mathematics for a better teaching.

Consequently, results showed that preservice middle school mathematics teachers participated in this study were not informed about underlying purposes of the content courses they took. The need to be notified initially about their purpose was framed by one of the participants of the study as follows:

> *If we were informed by a preliminary explanation like "We are giving these courses to you and you may need these in this or that situation later on"; maybe this could have increased our motivation and we could consider that "it works" and study more on it. Maybe later on we could be able to establish the relationship between them in our own minds. But we need to understand that these courses are serving some purpose first. (P2, 4th-year)*

4.1.1. Preservice Middle School Mathematics Teachers' Views on the Mathematics Content Course "Basic Algebraic Structures" in Terms of its Relevance to Their Future Teaching

Participants' views about the relevance of the Basic Algebraic Structures course to their future teaching were identified from their responses to two main interview questions. They were requested first to evaluate the content covered in the course in terms of its relation to middle school mathematics curriculum, and then to evaluate usability of the knowledge of that content in the teaching of middle school mathematics.

Participants were not able to decide whether the course content was related to the middle school curriculum or not, "*because [they did] not remember the content of the course well*" (P1, 3rd-year). Syllabus of the course which was in use over the years participants of the study took the course was presented. Examining the syllabus, they mostly addressed the following topics as the most related ones to the middle school mathematics curriculum: divisibility (n=10), prime factors (n=4), greatest common divisor and least common multiple concepts (n=4), congruence of integers (n=3), and binary operations (n=2). Some of the participants, while naming these topics as related ones, included also in their statements that they could "*not say the same thing for the rest of the topics*" (P11, 4th-year). P3 and P7's below statements illustrate the case:

Maybe I cannot say that all of them are related. Or even if they are related, it is probable that I do not know how to associate them. [...] In the simplest term, I can specify divisibility, prime factors, greatest common divisors... these are all the things we are going to use, you know, the least common multiple and so on ... the things we will use in the (middle school). Apart from these topics, for the others, I do not know how to relate them. (P7, 4th-year)

Hmm... Okay. I remember there were divisibility and such things... These are very related in fact, but the others were really ... so nonsense to me. Groups, isomorphic groups and such things… I mean, for those topics... I was not even able to understand the logic of learning these topics at all. [...] Well, only the divisibility issues seem to be related right now, they seem to me like they are the things will be used. (P3, 3rd-year)

One of the participants made a connection with one of the topics that most of the participants did not even consider; *groups*. However, she did not correctly remember the information that the set of rational numbers form a group under multiplication operation only when zero was excluded from the set.

I will not teach these to the students, but hmm… For instance, we are doing an operation with integers; we can do that operation because of the characteristics (addressed) here. In fact, it is like ... somehow we are teaching these. We do not name it as a group. We do not say that rational numbers form a group (under multiplication), but still every element has an inverse there, and we find it. We use multiplicative inverses. I mean, we use them even if we do not explicitly mention their names. (P6, 3rd-year)

Three of the participants made their comments in general terms, without making reference to particular topics. They all indicated that the course content was a base for middle school mathematics although they did not know how to relate the two in the course of teaching.

In fact, the subjects here (reviews the syllabus) constitute the base for many of the topics. Normally, we teach the basic things to the students, these (subjects) are related to them in any case, but while teaching those (basic things) to the children how can we make use of these (topics listed in the syllabus)? You see…I do not know that. I cannot know what they (the topics in the course) will help with? (P8, 4th-year)

After evaluating the course content in terms of its relation to middle school mathematics curriculum, participants were asked whether they were expecting the knowledge and skills acquired through this course to contribute their future teaching of middle school mathematics or not. Eleven of the 14 participants gave positive answers to this question such as "*This course does (contribute)*" (P10, 4th-year) and "*Yes, I think it will*" (P4, 3rd-year). However, their answers did not go beyond being a superficial "yes" to a yes-no question because when they were asked about how they would use this knowledge, they could not produce explanations or examples for where and how having this kind of knowledge could be helpful in the work of teaching:

> *In the course of teaching ... Now, first of all, any knowledge is good knowledge ... as long as you do not know wrong, I think anything you know will necessarily help you in somewhere with something. This is my belief. I mean, it will help me without a doubt, but I had always been having difficulty about when will it provide benefits. I have already had this difficulty about the content courses all the time. "Now I am learning this but, am I going to use this? Where?" (P7, 4th-year)*

> *I think it contributes. I think it will be helpful, because you never know the level of the students you will meet. Therefore, the higher we keep our knowledge of our field, the better (we are). (P9, 3rd-year)*

In contrast, three of the 14 participants anticipated "*no benefit out of having the knowledge"* (P2, 4th-year) of the mathematical content covered in the course. P8 (*4th-year*) explained her concerns about applicability of the knowledge in question to the work of teaching middle school mathematics as follows:

> *I do not think it (contribution) is likely to happen. I may encounter things that I myself can relate in terms of content, but not while teaching. For instance, divisibility or greatest common divisor. Here, we are going to teach the least common multiple to children, maybe with this topic it can be connected mathematically, but while I will be giving instruction on this topic, how will it come into use? Well, I cannot imagine at the moment. I cannot relate it with the course of teaching... How deeply can I teach to the children about these topics? I do not think that I should teach something so deeply to middle school students.*

While examining the course content, four of the participants made additional comments on the usability of the related knowledge with teaching purposes. They all highlighted that even if some of the course content was related to middle school mathematics; it was too abstract to be used in teaching of it.

> *Okay. Here (in the syllabus) are topics that are quite useful for us. Especially, divisibility, prime factors ... Well, actually these are the fundamentals of middle school mathematics, not new things, but the version we see here is a more broadened form, and hence too abstract. We will not be using them in this form, but in a more concrete form. (P9, 3^rd-year)*

> *But, I think these are too abstract. We cannot transfer these to our students, these are the concepts that remain to be abstract even in us, hence its transfer is really difficult, I mean, even we had difficulty while learning these things. Okay, maybe we have to learn them, but I do not know how much it is needed. (P10, 4^th-year)*

Additionally, the researcher asked participants to assume that they were middle school mathematics teachers and to try to imagine circumstances in which they would make use of their knowledge gained from Basic Algebraic Structures course. Nine of the participants addressed the case of having "*very curious children*" (P10, 4^th-year) in the class "*who ask really strange questions*" (P3, 4^th-year). Participants generated the following examples to such student questions: "*Why, teacher?*" (P1, 4^th-year) "*Where did this come from? How did this happen? How come?"* (P13, 4^th-year), or "*Why does not it hold here, in this case?"*(P6, 3^rd-year) Also, some of the participants indicated that they *"had encountered such events in [their] school experience*" (P10, 4^th-year).

Other predictions about where to use aforementioned content knowledge in teaching were observed less frequently. Three of the participants referred to the case of dealing with their students' unusual works.

> *If he finds another answer, something different from mine. At that moment, he may make a logical explanation, and it may also sound logical to me, and also to his friends. In such a case I should be able to evaluate and clarify to him why that thing he did cannot be accepted, maybe. I may encounter such a case. (P14, 3^rd-year)*

Two of the participants stated that they can make use of this knowledge for their own "*making sense of mathematics*" (P12, 4^th-year). P7 (4^th-year) explained this perspective as in the below excerpt:

Rather than students' asking questions, while I am preparing for a class (something) may confuse me. I may consider (asking) "What if it was something like this, instead of that, what we do normally?" In that case also, I can look up this book (course book for Basic Algebraic Structures course), not only for their questions, but for my own preparation.

Two other participants mentioned potential use of the knowledge in question while "*writing algebraic statements and making generalizations […] instead of (depending on) specific examples*" (P3, 4th-year). However, for such skills to be needed, they both considered encountering *"cases that would call for a proof"* (P11, 4th-year) without naming the situation which would require them to do so. Still another participant, P4 (3th-year), thought that she "*would find alternative examples for those (students) who did not understand"* some points, since she "*would be able to look from a broader perspective than that of the students by having such extended knowledge.*"

Apart from the unnamed situations that would require the participants to conduct proofs, each of the four cases specified by participants of the study corresponded to four of the mathematical tasks of teaching identified by Ball, Thames and Phelps (2008). They are responding to students' "why" questions (n=9), evaluating the plausibility of students' claims (n=3), giving (or evaluating) mathematical explanations (n=2), and finding an example to make a specific mathematical point (n=1) in the order they were represented above.

In summary, participants of the study related only a limited section of the course content to the middle school mathematics curriculum. However, when they were asked about use of the general content covered in the teaching of middle school mathematics, they mostly considered it as useful without specifying any potential instances. But, the analysis of the additional question revealed that participants in fact expected to use this content knowledge in conducting some, or at least few of the *mathematical tasks of teaching.*

4. 2. Participants' Work on the Four Mathematical Tasks of Teaching

4.2.1. Task I

In the first task of the interview, participants worked on a widely-used definition of prime number: *A number greater than 1 is called a prime number if its* *only divisors are 1 and itself*. The hypothetical classroom event included the participants' introducing this definition to their 6th grade students and after a while one of the students' asking: *Teacher, the number "1" is also divisible only by 1 and itself. Why do not we take it as a prime number?*

Once the participants read this scenario, they were asked to explain how they would respond to this *why* question of the student. Participants' first interpretation of the case represented their initial state in *responding to students' why questions*, from among the sixteen mathematical tasks of teaching identified by Ball, Thames and Phelps (2008). In the next step, participants looked for an answer for this question from Basic Algebraic Structures course book under the guidance of the researcher. Last, they were re-asked to formulate an answer to the student's question, with the help of the ideas they gained from studying the course book.

4.2.1.1. Participants' responding to students' why questions

None of the 14 participants of the study had an immediate answer to the student's question, but their first interpretation of the situation notably differed. Nine of the participants found this question quite reasonable, but they felt inadequate in providing an acceptable answer. Some of the participants' first reactions to the task were indicative of the case. For instance, reading the scenario, P6 (3rd-year) thought out loud: *"S/he rings true. The student is right. Now, okay… How do I answer this?"* P9 (3rd-year) pointed out that *"it [was] something quite open to asking … logical, and highly probably [would] be asked"* and she questioned herself: *"Oh, what will we do now?!"* P13 (4th-year) asked herself repeatedly *"Why do not we take 1? Why not?"* and she reflected her amazement as *"You know what, now I realize that I had made no account of this before."* In addition, P8 (4th-year) indicated that she "*ha[d] always been wondering this, too.*"

Some other participants considered this question as a "*very difficult"* (P2, 4th-year; P9, 3rd-year), "*nice*" (P11, 4th-year) and "*familiar"* (P5, 4th-year; P12, 4th-year; P14, 3rd-year) one. Two of them connected the issue with their own past experiences to elucidate its familiarity:

> *Kids have asked this question when I was doing my practice teaching. I also remember the teacher's explanation … To me … It did not convince me at all,*

> *but still may be true, I do not know. She said "As it is divisible only by 1 and itself, 1 is divisible by only itself," but she did not see the other (1). She made this kind of an explanation, but it had not been any convincing. How do we do this? I am having difficulty… To be honest, I would not be able to answer this. How would I pass over? I am thinking of this now. (P12, 4th-year)*
>
> *My students are already asking this now, too... Well, for example a die is rolled. In fact, we are teaching probability now. 1, 2, 3, 4, 5, 6, you now. Probability of obtaining a prime number is asked. They start counting from 1. Then [they ask] like "No, not 1. We did not use to start from 1, did we?" Then, I direct them to 2. Yes, but, what kind of a thing should I tell them, I do not know. Okay, I can think about this right now. (P14, 3rd-year)*

The remaining 5 participants of the study regarded the student's question as nonsense, because they perceived this definition to be something adopted without reasoning or questioning. They simply relied on the idea that the definition itself "*says 'prime' to the numbers that are greater than 1"* (P4, 3rd-year). For instance, P7 (4th-year) expressed her thinking in the following way: "*Okay, I would say (to the student): 'You should check the definition, there is no such thing in this definition.*'" Besides, P10 (4th-year) wondered *"[Would] it work if I emphasize that the least prime number is 2?"* Her emphasis showed that P10 excluded the number 1 from the set of primes without engaging in any mathematical reasoning process; and hence she regarded the number 2 as the least prime number straight-forwardly.

In order to encourage participants to think deeply and elaborate on their ideas, the researcher explained to them that defining in mathematics was arbitrarily naming mathematical concepts (Çakıroğlu, 2013; Vinner, 1991). For example, a trapezoid can be defined as a quadrilateral having *at least* one pair of parallel sides or *exactly* one pair of parallel sides. Depending on the definition we choose, all parallelograms are either included in the class of trapezoids, or excluded completely. Only when it is acknowledged that definitions are arbitrary, aforementioned difference withdraws from leading to confusion (Vinner, 1991). The important thing is about how to make a selection from among the alternative definitions of one concept (Çakıroğlu, 2013).

As participants grasped the above characteristics of mathematical definitions, they also recognized that stating this definition for the numbers greater than 1 might be due to an underlying rationale. From that moment on, the task turned out to be a more meaningful one for them; so they agreed on finding an answer to the student's question.

Despite having indicated that they had no idea about why to make this preference for the definition of prime number initially, almost all of the participants tried to make sense of the underlying reason(s) by reading the definition itself for several times and making their own interpretations. Four of the participants thought that each prime number must have a factor other than 1. P4 (3rd-year) stated this criterion of her as: "*[L]et us say we write a prime number in the form of a x b. These (a and b) must be different from each other.*" P7 (4th-year) maintained the same position as follows:

> *1 and itself... 1 and … other than 1... Now, "1 and itself" does not work here. It is like we say that there must be two divisors. […] Yes. I divided 1 by 1. Okay. But now, I am… like dividing again by 1, not by any other number. It is like... There must be two divisors.*

On the other hand, P6 (3rd-year) and P8 (4th-year) attributed the exclusion of 1 from the set of primes to a specific characteristic of "1" as a number. In particular, P6 claimed: "*We say prime (emphasizes the word) number. But now, 1 … I mean 1 divides everything. But what is its specialty (distinguishing feature) then*?" P8 also asserted: "*Well, if the factors are only 1 and itself… actually all the numbers have a factor of 1.*" Although P6 and P8's observations could be a good starting point to find an answer, the two participants could not take this idea to any further. While remaining participants could not suggest any opinion about the issue, some of them remarked that they would "*look for an answer, and get back to the student later*" (P2, 4th-year).

4.2.1.2. Relating mathematical ideas to Basic Algebraic Structures Course

A mathematical clarification for the hypothetical student question was mentioned in the course book of Basic Algebraic Structures as given in Figure 4.1

Definition 2.15 Prime Integer

An integer p is a prime integer if $p > 1$ and the only divisors of p are ± 1 and $\pm p$.

Notice that the condition $p > 1$ makes p positive and ensures that $p \neq 1$. The exclusion of 1 from the set of primes makes possible the statement of the Unique Factorization Theorem. […]

Figure 4.1 Clarification note for the underlying rationale for excluding the number 1 from the set of primes (Gilbert & Gilbert, 2000, p.73).

The underlying annotation, as placed immediately after the statement of the definition, gained participants' attention as intended. However, although all of the participants read this note, only 6 of them recognized that it was pointing out the information they were searching for. Six participants instantly deduced that the answer "*must be related to the Unique Factorization Theorem*" (P2, 4th-year) and they tried to remember what the theorem was about. Once they figured out that the theorem was stating the prime factorization they were well accustomed to, participants simply relied on their existing knowledge of prime factorization, rather than following the booklet. Their comments all concentrated on the functionality of the given definition, which was an important criterion for selecting definitions from among different alternatives (Çakıroğlu, 2013). Although participants did not mention the term *functionality* itself, they discussed this criterion from three different but interrelated perspectives. Namely, including 1 in the set of primes, would be useless, would require replacing existing applications of Unique Factorization Theorem, and prevent prime factorization from being unique. For example, two of the participants tried to clarify the reason why taking the number 1 as a prime would serve no useful purpose. P13 (4th-year) expressed this opinion as:

> *[The theorem] says: Every integer can be represented in the form of product of prime integers, it says. Like the way we do prime factorization, it tells this. Well, while we do prime factorization also, we start from 2 … because we do not accept 1 since … if we accept 1; it will repeatedly come (as a factor) again. This is the reason. […] We do not know how many times, because we can divide infinitely many times by 1. It would be useless.*

P1 (3^{rd}-year) and P5 (4^{th}-year) combined the above reasoning with one specific use of the Unique Factorization Theorem; namely computing the number of positive divisors of an integer. These two participants' reasoning was quite different from the one mentioned above, in the sense that P1 and P5 considered including the number 1 in the set of primes could create an effect on some other applications of the Unique Factorization Theorem, rather than being of no use. However, while describing their thoughts, both P1 and P5 regarded the number 1 to be found only once as a factor in the number being factorized. Below is an exemplification of their argumentation:

> *Hmm... For example take 6. Normally, prime factors are 2 and 3. If I add 1 also to this (factorization) ... Well (normally) the number of positive divisors becomes 4 (obtains by calculating (1+1)(1+1)) but if I put a 1 also, it becomes 8 (calculates the result of operation: (1+1)(1+1)(1+1)). Again here will be some ... thing, incongruity. I mean, we were multiplying the exponents increased by 1 each. You know, it is not meaningful to take 1... for this case. (P5, 4^{th}-year)*

P6 (3^{rd}-year) also referred to the usage of prime factorization, but without specifying an example explicitly as P1 and P5 did. The major point she stressed was the uniqueness aspect of prime factorization. She asserted: *In other cases ... in cases where we use this (prime factorization), if other representations would exist which one would we use? If it is unique, then we all use that one* (P6, 3^{rd}-year). P3 (4^{th}-year) further clarified the *uniqueness* aspect as illustrated below, together with the reason why excluding 1 enables us to uniquely do prime factorization:

> *Now I have re-discovered this! I mean, I have really understood now why 1 is not a prime number. Well, normally we prime ... when doing prime factorization of a number, you know, we write in the form of exponents of 2, 3, or 5. But, the reason why we do not count 1 is that any exponent of 1 will give the same result. This means, it will not have a single representation. [...] Well, these are not wrong too, these expressions (points to the expressions she wrote previously: $12 = 1^2x2^2x3$ and $12 = 1^3x2^2x3$) ... but when, if we are in the need of a ... some standard thing (expression) it is okay too for me to exclude 1.*

Although the remaining eight participants had read the underlying note attentively, they passed through the following pages of the booklet without giving sufficient thought to the main idea argued in this note. They were later re-directed to reading this clarification by the researcher and further encouraged to work on the task. Below dialog is an example of how P7 (4^{th}-year) reached a conclusion with little help:

> ***P7:*** *Let us take 36. It is 2 to the exponent 2 and 3 to the exponent 2 (writes as 2^2x3^2). These are the prime factors. The theorem says this way.*
> *... (P7 does not react.)*
> ***R:*** *What would be the difference if we have taken the number 1 also as a prime?*
> ***P7:*** *If we took 1, 1 to the exponent… but kids do not know the concept of infinity. We are in the 6^{th} grade. Well, infinity may create some confusion, but I would say like "I am multiplying a lot of 1's here, I do not know how many times. Since here, you have to write 1 as many times as you can divide (that number). But, dividing with 1 has no end. I mean, you can divide as many times as you want. Therefore, you can write here 1 as many times you like, too." and then I would link this to … that 1 is of no effect.*

The researcher asked questions similar to the one she asked to P7 above to other participants in order to facilitate their thinking. Some of the questions were: "How excluding 1 may serve our stating of the Unique Factorization Theorem?" or "What may be the relationship between excluding 1 and stating the Unique Factorization Theorem?" Dialogs between the researcher and the participants followed similar patterns as the above illustrated one. Most of the participants concluded that excluding the number 1 from the set of primes was promoting *functionality* of the definition, but they communicated this idea through proposing any one of the three interpretations demonstrated before, without using the term functionality itself. Namely, the participants concluded that including 1 in the set of primes would be useless, would require replacing existing applications of Unique Factorization Theorem, and prevent prime factorization from being unique. Still, some of the participants avoided engaging in the task and reflected their discomfort about their performances. For instance, P8 (4^{th}-year) indicated that she "*understand[s] the theorem, but… still can't see how to relate it to the (student's) question.*" Similarly, P12 (4^{th}-year) insisted: "*I have just realized how far I am from those things.*"

The participants who formulated an answer for the student's question by examining the mathematical argumentation provided in Basic Algebraic course book were re-asked to explain how they would respond to a student posing such a question. Most of the participants indicated that "*clarifying this (mathematical) rationale to a 6th grade student [was] a hard task*" (P13, 4th-year) and they did not know "*how to simplify this explanation"* (P12, 4th-year) to their students' level. P11 (4th-year) expressed her difficulty in communicating her ideas to the student by saying "*At this moment, it is difficult for me to state this. I mean, I have things composed in my mind but… well, I cannot do … could not make it up."* Similarly, P5 (4th-year) asserted: "*This is now making sense to me, but now the student… How can I get the students understand this?*" Even so, participants tried to formulate a final answer to the student's question. Most of them re-verbalized their own understanding of the issue as an explanation to be presented directly to the student. These explanations were mainly the restatements of participants' own perceptions of the underlying rationale, which were based on the functionality of the given definition.

Three of the participants (P1, 3rd-year; P5, 4th-year; and P9, 3rd-year), on the other hand, preferred to get the student himself work on prime factorization of an arbitrarily chosen number. However, they did not pay attention to the sequence of related learning objectives included in the curriculum guideline. Assuming that the students learn prime factorization before defining the prime number, they suggested having the student factorize any number himself, by accepting 1 also as a prime. Then, they would ask to the student "*What would you write over the 1 (as an exponent) here?"* (P9, 3rd-year) or "*How many times would you divide 15 (the number to be factorized) by 1, in this case?"*(P5, 4th-year) P14 (3rd-year) also ignored the same information about middle school curriculum, and indicated that she would involve more participants in the discussion of the topic, in the following way:

> *I would write several factorizations of a number, let us say 20, on the board. Actually, I would write $20 = 1x2^2 x5$ several times on the board, and then ask students "What number can I write over 1?" Everybody tells a number, and I write some of them. Then I emphasize that we are all reaching the same result. "Then, why taking 1?" I would conclude.*

In brief, preservice teachers participated in this study were not able to answer a hypothetical 6^{th} grade student's question immediately at the beginning of the interview. Yet, through their work on the Task 1 and the Basic Algebraic Structures course book, most participants reached valuable insights into the reason why the definition of prime number might be stated for the numbers greater than 1. Participants reached these conclusions either by their own effort or with the researcher's help. Nevertheless, their final responses to the student were not simpler than their own understandings of the underlying mathematical ideas. In other words, they could not suggest any perspectives appropriate to the sixth grade level. These findings showed that the Basic Algebraic Structures course included the study of an important mathematical idea for preservice middle school mathematics teachers. However, it seemed that it was disconnected from participating preservice mathematics teachers' teaching concerns and did not immediately help them in responding students' needs.

4.2.2. Task II

In Task II of the interview, participants were given a homework sheet that was assumed to be previously answered by a 6^{th} grade student in order for them to evaluate the correctness of student's responses. In the case they detected a response to be incorrect, they were asked to determine the reason behind this mistake of the student, and propose a way to overcome it. Next, participants were requested to find a relevant mathematical statement to the situation from the given list of statements derived from Basic Algebraic Structures course book. Last, they were asked to rethink about how to overcome the student's mistake by taking the advantage of the mathematical idea central to the statement they have selected.

The homework sheet included a general direction statement: "For the number 3264, fill in the blanks with either 'divisible' or 'not divisible'. Explain your reasoning." Seven items listed below this statement were asking students to determine whether the number 3264 is divisible by each of the numbers 2, 3, 4, 5, 6, 8 and 9 or not, respectively. Among the hypothetical student responses provided for each of the items, only the response to the fifth item was based on an erroneous reasoning as follows:

The number 3264 is _*divisible*__by 8.

Reasoning: *3264 is divisible by both 2 and 4.*

Other six responses were correct and based on a correct reasoning. Participants were given time to read and evaluate the correctness of student's responses. Then, they were asked the subsequent interview questions.

4.2.2.1. Participants' evaluation of the plausibility of the student's claims

Among the 14 preservice teachers who participated in the study, 12 of them evaluated the student's reasoning for the fifth item as "incorrect". However, only two of them were able to provide an accurate justification for their judgment. They asserted that although the number in this case, 3264, was divisible by 8, the student's path to "*making this inference [was] wrong, since 2 and 4 are not relatively prime [to each other]*" (P5, 4th-year). These two participants demonstrated a complete understanding of the concept "relatively prime-ness". For instance, P6 (3rd-year) explained: The student should have checked, at first "*if the two numbers [2 and 4] have another (other than 1) common divisor. Actually, this is what [being] relatively prime means.*"

Apart from the correct justifications, three other participants also mentioned the need for factors which were relatively prime to each other. However, their explanations revealed that they had an inaccurate understanding of relatively prime numbers. Specifically, they considered two relatively prime numbers also as individually prime numbers, as in the following comparison P8 (4th-year) made between the two cases of dividing the number 3264 by 8 and by 6:

> *Now, in the case of 6, we check if [the number] is divisible by both 2 and 3. Because only when [it is] in the form of a product of two prime numbers, we can use such a rule. Since 2 and 4 are not prime, relatively to each other, for this reason we cannot apply this with 8. [...] Hereby, in fact, we should check its prime divisors, that is, prime factors. If divisible by those [numbers], then that number is also divisible.*

Four of the participants suggested that the student should have found two prime factors of 8 in order to be able to apply such a rule, as illustrated in P3 (4^{th}-year)'s interpretation:

> *In the case of 6, he is correct. 2 and 3. He had made prime factorization. Since it (3264) is divided by these two (2 and 3), he reasoned it is also divisible by 6. But for 8; 2 is prime but 4 is not a prime. Here the student has a difficulty. I think [the student] does not know what a prime number is. [S/he] just thought about multiplying what numbers [give 8]. That is, with 6, he thought 6 is the product of 2 and 3, with 8 also, the product of 2 and 4. I think the student has a problem with prime numbers.*

Three other participants focused solely on the use of divisibility rule for 8 without seeking any mathematical justification for their judgment. P10 (4^{th}-year) claimed:

> *[The student] should take the last three digits. After checking whether the number 264 is divisible by 8 or not, [s/he] can give an answer to me. Let me check. […] Her/his answer is correct. [It is] divisible by 8, since when I check the last three digits, divisible by 8. But… The expression "both 2 and 4", such an expression would be wrong. On what basis, [s/he] says this? No such rule exists.*

On the other hand, 2 of the preservice teachers were unsure about the correctness of the student's response to the item and remained undecided until the end of the task. Below excerpt of P7 (4^{th}-year)'s thinking out loud illustrates how uncertain she was while evaluating the student's work:

> *It is like … [the student] cannot say that if divisible by 2, divisible by 4, then [it is] also divisible by 8. No, may [the student] say? In fact [it is] divisible. I am thinking of the numbers that are divisible by 8. It may be because [they are] divisible by both 2 and 4, but is not that a mistake to generalize it this way? [...] No. Is there any mistake here?*

After participants evaluated the student's response, they were asked the questions "Why do you think the student may have done this mistake?" and "How would you help this student overcome her/his mistake?" Independent of their diverse replies to the previous questions of the Task II, all of the participants indicated that the student *"must have overgeneralized the rule of "divisibility by 6"* (P12, 3^{rd}-year) to the case of 8. Their methods for overcoming the student's mistake are classified under three categories: Presenting a counterexample, comparing the cases of 8 and 6, and telling students to check prime factors of the number.

Most of the participants (ten of the 14 preservice teachers) preferred using a counterexample. For instance, P10 (4th-year) explained her own counterexample as: "*A number, for example 100. I have just checked. Yes. I simply give the example of 100. Can we divide it by 2? Yes. Can we divide it by 4? Yes. But, can we divide it by 8? No*." Another participant, P14 (4th-year) picked up the number 12 as a counterexample to students claim.

> *Hmm ... I can make a number that is divisible by both 2 and 4, but not divisible by 8. Let us take 12. Now, this is divisible by 2, and also divisible by 4. The student will see this. But he will see that (it is) not divisible by 8. For this reason, then, for [a number to be] divisible by 8, being divisible by both 2 and 4 is not sufficient.*

Two of the participants considered comparing conditions of divisibility by 6 and divisibility by 8. However, they did not clarify how to implement this activity explicitly. When P6 (3rd-year) responded as "*Maybe, I would provide a comparison with the case of divisibility by 6,*" she planned merely to ask to the student about "*the relationship between (couples) 2 and 3, and 2 and 4?*" but she did not produce any further explanations about the thinking process that she would have the student gone through.

Remaining two participants simply stated that they would tell the student to check for prime factors of the divisor number 8 without mentioning the reason why to do so. P3 (4th-year) stated her strategy as "*I would say to the student: 'We do the prime factorization, and these are not prime factors of 8.*'"

Up to this point, the interview questions were asked in order to understand participants' initial state in *evaluating the plausibility of students' claims* - one of the mathematical tasks of teaching (Ball, Thames, & Phelps, 2008). Analyses showed that even if most of the preservice teachers (12 of the 14 participants) were able to identify the incorrect response of the student, they were not totally competent in providing a mathematical justification for their judgment, and helping the student to overcome this mistake. Only two of the participants could provide an accurate mathematical justification for their judgment. In addition, most of the participants contented themselves with convincing the student that his/her response was erroneous (e.g. by giving counterexample), but none of them explained underlying mathematical reason explicitly. Only P1 (3rd-year) mentioned its importance from the student's perspective, but her attempt was also superficial:

> *Maybe I can give a counterexample. A number that is divisible by 2 and 4, but not by 8. For example, 20 is divisible by both 2 and 4, but it is not divisible by 8. This may be helpful. But, it is more important [for the student] to learn the actual rationale. I do not know that.*

4.2.2.2. Relating the mathematical ideas to the Basic Algebraic Structures Course

After participants worked on the student's response, they were provided with a list of mathematical statements from Basic Algebraic Structures course book (see Appendix E). They were requested to find a statement from the list, the one that was most related to the important mathematical idea that the student might have ignored while answering the fifth item. Twelve of the participants selected Statement 3, which directly stated the relevant proposition:

> If $a \mid c$ and $b \mid c$, and $(a,b) = 1$, prove that ab divides c.

This time, they all made correct interpretations of both the statement and the concept of "relatively prime-ness". It might be due to the reason that relatively prime-ness, in this statement, was expressed by means of greatest common divisor of two numbers. One of these participants, P3 (4th-year), reached the same idea by the help of another statement, Statement 10:

> Let a and b be positive integers. If $d = (a,b)$ and m is the least common multiple of a and b, prove that $dm = ab$.

While trying to make sense of the statement on her own, P3 concluded: *"[R]ight, okay. They must be relatively prime. When they are relatively prime, [their] greatest common divisor must be 1. [Then] the least common multiple becomes the product of those two numbers."* Then, she reviewed the list again and arrived at Statement 3. On the other hand, only two participants could not find any statement related to the condition. However, after they were told by the researcher (Statement 3), they also made accurate interpretations of it.

Last, preservice teachers were asked to think about how to overcome the student's mistake one more time, but this time, by taking the advantage of the idea central to the statement they have selected from the list. Participants' responses to this question were analyzed in an attempt to understand how they related the specific mathematical knowledge addressed in this second task of the interview with their teaching. Responses revealed that participants comprehended and appreciated the significant the mathematical idea "two numbers must be relatively prime to each other, in order for their product to divide any number that is divisible by both of them." However, their strategies to overcome student's mistake did not go beyond direct explanation of the idea to the student. Below quotations from participants' statements illustrate the case.

> *I can make the student think of multiples. But… obviously, it will not always be a multiple. Maybe both of them are divisible by 2. For example, one of them may be 6 and the other may be 8, or so. Well, it may be this way. I can tell that they should not have a common divisor. (P1, 3rd-year)*

> *Relatively prime-ness, or whether a multiple of each other or not or ... how should I say ... if [they are] divisible by the same number. If they are divisible by the same number; for example, here both of them are divisible by 2. Since it is a number different from 1… In here, we check if [they are] divisible by a different number at the same time... What else can we do? We cannot do anything else. (P2, 4th-year)*

> *Then, in fact something like when we examine the divisibility rule, we should check two relatively prime numbers. At the beginning, [I] could not figure out it … Even using relatively prime-ness may be something that would make it clear for the student immediately. For example, it may be confusing here. Okay, I am not confused, we say that [while dividing with 18], we can check for 2 and 9; but the student may [ask] here: "Well teacher, why cannot we check for 3 then?" But only telling them that 2 and 9 are relatively prime to each other will be enough for them. (P14, 4th-year)*

In summary, preservice teachers were able to comprehend the mathematical reason why the student's reasoning was erroneous after studying the important point as it is addressed in Basic Algebraic Structures course. However, they could not integrate this specific piece of knowledge into their teaching effectively. Without exception, all of the preservice teachers focused on explaining their own understanding of the issue directly to the student, instead of integrating it into an instructional activity that would help the student overcome his/her mistake.

4.2.3. Task III

Before starting to work on Task III, participants were asked to state their own definitions of the term *the least common multiple*. This act was aimed at building a common ground for the discussion of the ideas between the participant and the researcher; and more specifically for making sure that the participants had a correct understanding of what purpose the forthcoming algorithm was to serve. Next, participants were presented a step-by-step description of the standard algorithm to be used for calculating the least common multiple of two positive integers (see Figure 4.2), which was adapted from teachers' guidebook for 6th grades (MoNE, 2010).

The least common multiple of the numbers 24 and 36 are found by creating a factors list as in the following demonstration. In this technique, the two numbers are divided continuously, by beginning with the least prime number. Multiplication of the numbers contained in this factor list gives the least common multiple of the numbers 24 and 36.

24	36	2
12	18	2
6	9	2
3	9	3
1	3	3
	1	

2x2x2x3x3 =72

Figure 4.2 An algorithm for calculating the least common multiple of two numbers. (Adapted from MoNE, 2010, p. 107).

Participants were required to explain why the given algorithm worked for obtaining the least common multiple of any two numbers. After they finished their initial explanations, a definition of the term was given to them from the Basic Algebraic structures course book. Last, they were re-asked to explain why the given algorithm works, depending on this particular definition.

Participants all clarified the term least common multiple in very similar ways. Some of their statements included: "*when the two numbers are given, that which is a multiple of both of these numbers, but the least of them*" (P1),"*the least number that is divisible by both of the two given numbers without any remainder"* (P12), and "*two numbers have lots of common multiples, but the least of them*" (P14). In addition to these brief explanations, P7 (4^{th}-year) further clarified the meaning of the term from a different perspective:

> *I ask a question like "What is the common multiple that both numbers meet first?" But, how do I decide that the number is the least of the common multiples [...] For the both numbers I am thinking separately: by which number should I multiply these numbers to get there?*

P8 expressed the same idea algebraically as "*the least number c we can write in the form* $a.k = b.l = c$". While writing this expression, she assumed *a* and *b* as the two numbers whose least common multiple was being searched for. As all the participants demonstrated a correct understanding of the term least common multiple, no remediation was required before moving on to the interview questions of Task III.

4.2.3.1. Participants' recognizing what is involved in using a particular representation: The case of standard algorithm used for calculating the least common multiple of two positive integers

Participants were expected to clarify why the given algorithm for calculating the least common multiple of two positive integers worked. Ten of the 14 participants of the study started their work by writing individual prime factorizations of the two numbers, 24 and 36. They applied another procedure for computing the least common multiple (*lcm*), by using this representation. Six of these preservice teachers produced partial explanations for how using this alternative method gave the number intended to calculate. Then, they went back to the original task and tried to make use of their ideas gained from applying this particular procedure as a facilitator in answering the original question. Their explanations were considered as partial, because they concentrated only on explaining why the resulting number was a common multiple of both numbers, but they overlooked the reason why obtained number was the least one of all common multiples. An illustrative description of the procedure adopted by the ten participants is given below with a quotation from P1 and a representation from P13's written work in Figure 4.3:

> *Actually, I have been doing like this, I mean, I write them individually, one by one. Let me write. From 8 times 3, 24 (is equal to) 2^3x3; and 36 is 4 times 9; 2^2x3^2. Then, the thing that I do with these, since it is the lcm, I take the greater ones (exponents of each of the factors 2 and 3). (P1, 3rd-year)*

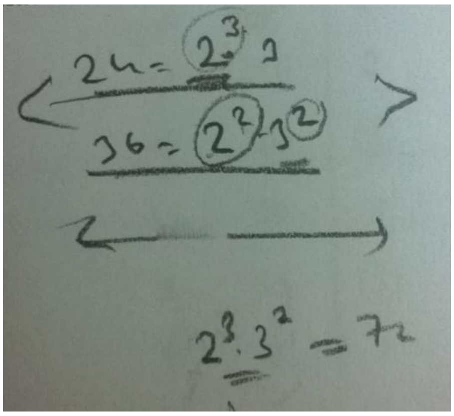

Figure 4.3 P13's representation of the alternative procedure applied by ten of the fourteen participants

Being one of the six participants who made a reference to the same intermediate step: applying their own procedure, and ended up with the same partial explanation; P14 described how this procedure works as in the following excerpt:

> *[B]ecause if I take the square of 2, as it would be less than the 2's part of this (points to the exponent 3 in the expression 24 = 2^3x3); you know, if I consider the 2's part only, for this reason, I need to take the cube of 2 from here. Here also, the same way. Here is 3. Here is 3^2. I mean, I need to take the square of 3, so that it (the lcm) can be divided, ensuring for both (24 and 36). If I do like this (her own method), it will give the same like this (The result obtained from the standard algorithm 2x2x2x3x3 =72). (P14, 3rd-year)*

P14's above reasoning was also representative of the other five participants' reasoning. However, three of the six participants, including P14, had difficulty in transferring their ideas gained from using this representation to the case of standard algorithm. In particular, when the researcher asked P14 "*How do you relate these two representations*?" she was not able to state explicitly how the original algorithm was satisfying the same conditions as the alternative one did. P14 responded to this question as follows:

> *If we express these two (individual prime factorizations of the two numbers she used in her own method) in the same thing (representation), it turns out to be such kind of a method. At this moment, I cannot think of it any other way. Yes. It is a memorization for me right now. You know, write these (numbers in the factors list) orderly, and multiply them. (P14, 3rd-year*)

Another example was when P4 (3rd-year) compared the situation with the case of finding the greatest common divisor (*gcd*) of two numbers. Through making this comparison, P4 expressed the difficulty she encountered while transferring her ideas:

> *Hmm… It (the question) makes sense now… Now, if you were asking for the gcd it would be simple, because for the gcd we look for the common factors. For the both numbers, for instance, 2, 2 again, and 3 are common. They give the gcd. But, why do we also take the product for lcm? … Now, here we are to find the common multiple. We separate these (24 and 36) to prime numbers, prime factors. […]. But, I do not know that thing, that why do we multiply the all. (P4, 3rd-year)*

On the contrary, after applying the same procedure and reasoning as above, two of the participants carried over this understanding directly to the standard algorithm:

> *In that case (of standard algorithm) I do like this; I close over one of them (covers the whole column including the number 36 with her finger). I do not see these. I find the same thing. When I do it for this (number 24) some of these (the numbers in the factors list) won't come. For instance, this (3) won't come. Here comes, 2^3 and 3. When I do the same for this (number 36), these factors come (points to the numbers 2, 2, 3, and 3 in the factors list one by one). Well, I mean, all of these (factors) are into this product (72). (P1, 3rd-year)*

Because we are trying to find a multiple of both (numbers). For instance, if I were to take only … Let us do not take this (the last number 3 in the factors list) since it only divides this (36) let us do not take this. What happens? (The final product becomes) 24. Okay, it is a multiple of this (24) but not of 36. For it (the lcm) to be a "common" (emphasizes the word) multiple, we need to take both number's divisors. We need to take their product. It is required. (P6, 3rd-year)

P7 also formulated the same reasoning as above, regarding the working principle of her own procedure; but she altered her perspective to another one while switching to the case of standard algorithm. Here, she basically took the advantage of the greatest common divisor concept, which she had not referred before, as in the following manner:

What I have said here (in the alternative way) is actually… Since we have formulized this, we do it sequentially like this (the standard algorithm). I have already found the common divisors here (points to each of the common divisors in the factors list that she has circled previously). Actually, here we pass to gcd too. I mean these (prime factors of gcd) already exist in this product; and I want it (the lcm) to be divisible by 2 and 3 at the same time, one more time. […] For instance, here this (24) was divided by 2, but this (36) was not. Or, here this (36) was divided by 3, but there is no division thing here (for 24). Therefore, I am thinking like it (the lcm) is required to be divisible by both 2 and 3 again … that the number I will find. For this reason, I am writing these (factors, 2 and 3) also subsequently (as factors of the lcm). (P7, 4th-year)

Besides these partial explanations, two of the ten participants formulated more complete explanations for how this alternative method of their own works. To state explicitly, in addition to the condition that the outcome number must be divisible by each of the numbers 24 and 36, they also clarified the reason why the resulting number was *the least* of all common multiples. They were also able to relate the two representations. For instance, P12 (4th-year) made below interpretation about how this method works:

I am trying to find the least number that is divisible by both of these numbers. If I want to find the least one, then I have to consider the factors that are common. I look for which are the common ones (finds out the common factors as 2^2 and 3). These exist in both. I take them directly. Now... Here remains 2 *(in 24) and 3 (in 36). I also have to take these both (2^2 and 3), for ensuring a multiple for both of them. (P12, 4th-year)*

When the researcher asked P12 about how he related his explanation to the standard algorithm, he replied: "*Because we divided the common ones in the same row ... for obtaining the least one, we have taken the same (common) factors only once."* Similarly, P5 (4th-year) drew below conclusion about the standard algorithm, after applying the usual separate prime factorizations of the two numbers:

By writing side by side, I am eliminating factors of those numbers (24 and 36), I mean, if commonly exist in both of them. If not (common), I take (it) once. Well, after all, the number that I will obtain by taking the product of these numbers (pointing to the factors list), is […] a number divisible by both of these numbers we obtain, but that the least number we obtain.

Remaining two participants, who calculated the *lcm* of the two numbers in their own way, indicated that the standard algorithm had been originated from this alternative representation. However, they were not able to express explicitly how it might have turned out to be that specific procedure later on. P3's below trial illustrates these two participants' work:

We are separating them to their prime factors, in fact. Then … the greater one. I mean, when we write it individually, we always did this way until now. For 24, for instance 2^3x3 and for 36, 2^2x3^2. By writing this… hmm… For finding the lcm, they always said like "You take 2^3 and also 3^2." I mean, we take the greater … the ones that have the greater multitudes. But, it has not any meaning really, right now. […] I cannot figure it out.

Four participants of the study generated their understandings of the algorithm without making a reference to the alternative way used by the previous ten participants. Still, two of them maintained the same rationale as mentioned before, which was referred to as "partial". In other words, they addressed only the reason why outcome number was a common multiple of the two numbers, as illustrated below:

Why does it work? Well, when I do the prime factorizations of these two, I obtain the numbers that both divide this (24), and divide this (36). I mean, as I find all the prime factors inside these two numbers, this number (the final *product) is divisible by this (24), I mean, because I have multiplied all; and also divisible by this (36) … Well, it is like… That (the product) becomes a multiple of these, the least. (P11, 4th-year)*

P9 (3rd-year) adopted a very different representation from all other participants of the study, and explained her reasoning by the help of a Venn diagram:

Here…We firstly do prime factorization. By this way (the standard algorithm), well, we have taken the factors that are common only once, we do not divide separately and take (them) two times; like when dividing them both, 2 divides them both, I will take one 2 from here because it belongs to both. […] If we represent by means of sets, it is like we take their union. […] Since these are common (points to common factors in the factors list), if I take these twice, my number will increase. But … (draws the Venn diagram) I write the common ones (points to intersection of the two sets), and also each one of those (factors) that are not common to both (points non-intersecting parts each)… when I multiply, I obtain the least number in the form of product. […] For instance, when trying to find the common multiples by dividing them (24 and 36) separately and multiplying the factors, I mean, I would be taking factors common to both, twice. Normally, what we were doing with the sets (while finding their union)? More… I mean, as we add (the common elements) for one extra time, we used to subtract the intersection part at the end. (It is) the same rationale.

Like P9, P8 also touched upon the condition that *lcm* must be the least one of all common multiples, but she did not clarify how the given algorithm was serving to this end.

Here, in $a.k = b.m = c$, these k and m have to be the least numbers for us to obtain the least c. For this reason, it is something like, here (in the standard algorithm) we have multiplied 24 by 3 (points to the number 3 in the factor list, which did not divide 24), we also have multiplied 36 by 2 (points to the number 2, which did not divide 36). I mean, this product (72) contains the both (24 and 36). (P8, 4th-year)

4.2.3.2. Relating the mathematical ideas to the Basic Algebraic Structures Course

After participants explained their initial reasoning about why the given algorithm works, they were presented in Figure 4.4 below definition from Basic Algebraic Structures course book.

A *least common multiple* of two non-zero integers a and b is an integer m that satisfy the conditions

1. m is a positive integer
2. $a|m$ and $b|m$
3. $a|c$ and $b|c$ imply $m|c$.

Figure 4.4 Definition of the term least common multiple from Basic Algebraic Structures course book (Gilbert & Gilbert, 2000, p.77).

The researcher first checked the participants' understanding of the above definition. Making sense of this definition was quite a troublesome process for most of the participants. The main difficulty they encountered was about the third condition that the least common multiple *m* has to satisfy. They perceived this statement to be an obvious conclusion about the least common multiple concept, rather than being an indispensable condition that the *lcm* has to satisfy. P7 (4^{th}-year) articulated this common perception of theirs as given below:

> *It says: this number, suppose that I have a number (c). It says like, if it is divided by both a and b, this means that it is also divided, in any case, by their lcm. What is there for me to explain about this? […] Why do we state this in here (in the definition)?*

In such cases, the researcher intervened in, as illustrated in the following example:

> ***P7:*** *Why do we say like this? Why do we need the third one? There cannot be any case where it (the lcm) does not divide (the number c).*
> ***R:*** *Well, what would be missing in this definition, if we were to exclude the third condition?*
> ***P7:*** *Hmm. What would be missing, if I exclude the third one? It must divide both this and this (re-reads the third condition), we have said … But, at that case one thing may not be … it may not be the least. It restricts to the least one, this means. Yes! I have just noticed at this instant. If I exclude this, I can choose m as 144 also; but when I write this (the third condition), it is right, I am restricting to that thing, it becomes the least. […] Well, indeed what makes this definition "the definition" is this part (the third condition); because […] without this (the third condition), it (the lcm) could be anything. That was a must, that means. (P7, 4^{th}-year)*

Some of the participants questioned themselves by asking questions such as:

If m is the lcm of a and b, then it is said to must satisfying the three things, but I cannot any ... understand what function does (the number) c perform here ... I mean, what is the purpose of that number like c being here? [...] Is it this complicated or I wonder is it me who cannot do the thing? (P3, 4th-year)

But, as what, did it assume c here? Why did it bring c in? (P10, 4th-year)

The researcher helped these participants figure out the role played by the variable *c* in stating of this definition. Below is given her dialog with P10, with the purpose of demonstrating the frequently occurred process with these participants:

R: *Can you find an example for c, for instance here, for the case of 24 and 36?*
P10: *Hmm… It, then, must be a multiple of 72.*
R: *Then, what does it mean for m to divide c?*
P10: *Okay. I suppose something like, well … as follows: […] This (72) is the least multiple I have reached. […] I mean I might have increased this 72, by multiplying it by 2, by 3 … it goes upwards like this, but the least common multiple I can reach becomes 72, since it divides all the others (common multiples). (P10, 4th-year)*

On the other hand, six of the participants made correct interpretations of the given definition without any help, and appreciated the way it was stated, as P14 (3rd-year) did:

Now, the lcm must be positive and must be divided by the both […] in order to ensure for both. When we come to the third one, it says, this (the number c) will be a number different from 72. It may also be 72 but, again it will be a multiple of both 24 and 36; and 72 will be divided this (c). I mean… well, there may be some numbers greater than 72 or different from 72, but 72 must be the least one among them for itself to divide the others (other multiples) Yes, I am also enlightened!

Verifying that each of the participants accepted the given definition of the term, they were called back to the work of explaining why the standard algorithm for finding the *lcm* of two numbers works. This time, they were required to consider the alternative definition they had been just exploring, instead of depending on their own definition, which they had stated at the very beginning of Task III.

Compared to their initial performances, in this second observation, four more participants provided complete explanations. Each of the four participants, having proposed partial clarifications previously, concentrated on the missing aspect of their initial responses, and clarified the reason why the resulting number was the least one of all common multiples.

> *If I take more and more of them, I mean all the factors of 24 and all the factors of 36, I cannot reach the least of common multiples. Here, we decrease, I mean, the product; like we take their intersection (prime factors common to each number), and take it out. (P11, 4th-year)*

> *Well, this inference (points to the standard algorithm) already comes from there (the definition), I mean, why am I doing this (following these steps)? For decreasing the multipliers, numbers of them. I mean, these factors (point to common ones) exists in both this (24), and this (36). Then, I say that, using it once is enough. Let me take one of them. […] It is something like I am getting rid of including them as a second time. But, what happens if I take them twice; I reach a number like c. In that case again, I need to eliminate them later. (P10, 4th-year)*

Together with those who had already provided complete explanations in their first attempts, the number of complete explanations increased to 7 in the second trial. In addition to these accurate clarifications, P6 (3rd-year) further questioned herself:

> *Why do we divide them by primes? Because… hmm… well we do not have to write prime numbers in fact. What if we write here 6 for instance (in the factors list), it is the same as 2x3 (points to the two factors in the factors list) But, if we write prime numbers, you know… What was a prime number? A number that is not divisible by anything, except 1 and itself. Ah-hah. I think, that is why. For instance, let us think about writing 6 there. 6 is divisible by both 2 and 3. Maybe one of these numbers (points to dividends found in the whole algorithm), this 9 for example […] is not divisible by 2, but is divisible by 3. For this reason it would be problematic. That is why taking primes is easier. (P6, 3rd-year)*

Three of the participants replicated their initial partial explanations. P8 (4th-year) also, who previously could not make any sound reasoning, provided the same partial explanation as her counterparts this time:

> *If we move in the reverse direction, now, if we move backwards; I mean, we have multiplied here, as you know; but if we take the lcm and divide it. If I*

divide it by 24, it means I will divide it by 2, for three times, and also by 3 for once. But, it must also be divisible by this 36. Now, for me to be able to divide it by 36, I need to divide by 2 twice, and by 3 also twice. I had already divided by 2 twice (while dividing by 24). But, I need to divide by one more 3 (points to last 3 in the factors list). For this reason, we multiply all of these (factors). (P8, 4th-year)

On the other hand, three participants indicated that they were not able to connect the given definition to the algorithm in any way. P7 repeatedly asked:

I do not know how to do this. How they may have thought of it? How they may have transferred this (definition) to this (algorithm)? Here, I have had multiplied 24 by 3, and 36 by 2; but how come? There is no meaning, I mean. How they have connected this? (P7, 4th-year)

P2 thought also about the reason why she was having this difficulty as follows:

But I am not able to relate to this (the standard algorithm). Should I go backwards? […] Now, I need to reach 72 from here (by using the standard algorithm), but in here (in the definition), you know… it is something like we already start from 72. I think, that's why, I cannot connect them. (P2, 4th-year)

In summary, although they were all able to define the term the least common multiple in their own words, most preservice teachers could not provide complete explanations for how the standard algorithm used for its calculation works. While they concentrated on the reason why it gives a number that is a common multiple of any two numbers, they missed the point that the outcome number was the least of all common multiples at the same time. Participants also struggled with understanding the alternative definition of the term presented to them from Basic Algebraic Structures course book. Yet, after comprehending the definition correctly, some of the participants improved their explanations to more complete ones. However, half of the participants still could not reach complete explanations, since they were not able to relate their ideas gained from studying this alternative definition to the working principles of the standard algorithm.

4.2.4. Task IV

The fourth task of the interview was associated with the standard algorithm for calculating the greatest common divisor (*gcd*) of any two positive integers. Participants were introduced an alternative use of the algorithm as a student-generated method. In this alternative use, the standard algorithm was executed twice for finding the greatest common divisor of three numbers, which were 450, 180 and 420. The algorithm was applied first to only two of the given numbers (450 and 180), and then was applied for the second time to the resulting number (90) together with the third one (420), in a pairwise manner. Participants were asked to determine if this particular usage of the algorithm was generalizable to the case of any three numbers; that is, it would give the greatest common divisor of three numbers correctly all the time. Participants were required to provide the rationale behind their decision and also state the idea they were supposed to test as a mathematical statement to be either proven or refuted. A parallel statement was indeed placed in the exercises section of the Basic Algebraic Structures course book for the reader to practice proving on one's own. In the case that participants were not able to state it correctly themselves; they were allowed to select one from the list of statements presented to them. However, they were not expected to write a mathematical proof of the statement, since such enforcement would go beyond the purpose of this study.

Before moving on the interview questions, participants were requested to describe the student's work in their own words in order to make sure that they understood the applied procedure correctly. There were some participants who had difficulty in comprehending the steps followed at the first glance, but they all grasped the correct meaning with a little effort.

4.2.4.1. Participants' inspecting equivalencies

All the participants except P2 (4^{th}-year) regarded the student's work as a generalizable method for calculating the greatest common divisor of any three numbers, but they were not equally confident in expressing their final decisions. Two of the participants were very confident of their judgment that they did not refrain from praising the student with sayings such as "*the clever student*" (P5, 4^{th}-year) and "*good for him to be able to think in this way*" (P10, 4^{th}-year). On the contrary, other participants avoided making such outright assertions and preferred to use softer words like "*to me, it seems to be true*" (P6, 3rd-year), "*it may also have held by chance, but it seems to me like … it always works*" (P12, 4th-year) or "*probably it will work in all cases*" (P3, 4th-year). On the other hand, P2 (4th-year) could not figure out if the student's method was generalizable or not, until the researcher clarified to her.

Five of the participants relied on empirical justifications for accepting the student's algorithm. They merely checked whether obtained result was the actual greatest common divisor of the three numbers or not. To illustrate, after applying the standard algorithm itself (applying to all three of the numbers at a time) with the original numbers given (450, 180, and 420) as 30, P11 (4th-year) concluded: "*Yes, 30. I guess the student invented a different, a nice way.*" P13 (4th-year) calculated the greatest common divisor of the same numbers by changing the order of the numbers used in the two steps. Namely, she calculated greatest common divisor of the numbers 180 and 420 in the first step, as 60; and then she performed the same procedure for the numbers 60 and 450 in the second step. She compared the two results and made her decision as "*Yes. We can generalize this.*" P8 (4th-year) in a similar manner, compared prime factorizations of the three numbers and highlighted that "*the prime factors common to each of the three numbers [were] again 2, 3, and 5,*" which gave the true greatest common divisor, when multiplied. On the other hand, two of the participants emphasized the need for making this comparison for other triples of numbers. However, each of them executed the comparison for only one different triple. Below dialog between the researcher and P4 (3rd-year) illustrates P4's depending on just one example to make her decision:

> ***P4:*** *Hmm. Wait a minute. I need to try it with some other numbers. (She picks up 16, 32, and 42; calculates greatest common divisor of 16 and 32, as 8; and then calculates that of 8 and 42.) It is 2. So, can we say that the greatest common divisor of these numbers is 2? The greatest common divisor? … Yes, we can say this, because it is 2 indeed.*
> ***R:*** *How did you make this decision?*
> ***P4:*** *Because all the three numbers are divisible by only 2.*
> ***R:*** *How about your final decision? Do you think that it will always work?*
> ***P4:*** *Yes, because it holds every time.*

Four of the participants focused on providing explanations for the reason why continuing with the result of the first step (90) sounded logical to them. P7 (4^{th}-year) framed this viewpoint as "*I think, here the primary concern is just why we take 90. I would explain this as… all the numbers that divide 90 already divides 450, and also 180.*" Similarly, P3 (4^{th}-year) claimed that: "*These numbers* (points to the numbers 450 and 180) *have also been divided anyway; their prime factors also have been counted. For this reason, it is appropriate to continue with 90."* Although participants P3 and P7 touched upon one of the important mathematical points by this claim, their explanations were considered as partial, because they both did not pay attention to the necessity of explaining how the outcome number becomes the greatest of all common divisors, as P6 (3^{rd}-year) did. In fact, P6's following explanation was the most outstanding one compared to each of her 13 counterparts:

> *Because here the student had found for the two (450 and 180), 90… I mean the greatest for these two can be 90 at most. Plus, the third number comes. We had already eliminated these (points to the prime factors that divide only 450 or only 180), in fact. These would be useless … even if one of these numbers were to divide 420 … because it would still not be common to all the three of them. Therefore, it is meaningful to reduce to 90. Then, we also check for common divisors of 90 and 420. It sounds logical.*

Three other participants pointed out that the number calculated at the end of second step (30) was less than the one obtained in the first step (90), and they regarded this detail as a way of justification as follows:

> *That the number gradually lessens is desirable. Logically correct because ... if it (the factors list) is like this for the first two numbers (450 and 180), then the third number may be divisible by only some of these (points to the circled numbers which are common divisors of 450 and 180) but not by the others. This means, a lesser number will result from here (the second step). The last number cannot be greater. Maybe we can check it in this way. (P12, 4^{th}-year)*

> *Here (at the end of first step) I had found the common divisor of the two numbers, 90. But when it comes with 420 (the third number)... it can reduce. If it does decrease, it works ... It must be somehow reduced. It (420) is not a multiple of 90 anyway. If it were ... 540 for instance, it would give the same 90 again. But 420. I think it is working. (P1, 3^{rd}-year)*

Unlike the other 12 participants of the study, P1 (3rd-year) and P5 (4th-year) referred to consulting an algebraic argumentation that would examine the correctness of student's claim, but P1 did not think of this act as a serious option. She even laughed at herself: *"Think about if I were to start proving this right now, by using a, b, c!"* and she ignored using her higher level mathematical knowledge while working with middle school mathematics. On the other hand, P5 took this idea seriously and provided a mathematical argumentation that would be an integral part of an algebraic proof showing that:

If $d_1 = (a, b)$ and $d_2 = (d_1, c)$, then $d_2 = (a, b, c)$.

In particular, assuming the given conditions, P5 showed that d_2 was a common divisor of the three numbers *a*, *b*, and *c* algebraically; but instead of writing a formal mathematical argument, he preferred to explain his ideas verbally as follows:

> *For instance, let the greatest common divisor of a and b be* d_1*. Then, let the greatest common divisor of* d_1 *and cbe*d_2*. That is,* d_2 *divides* d_1*;* d_2 *divides c. Now, if* d_2*divides* d_1*, then* d_2 *divides a and* d_2 *divides b … because* d_1 *divides both a and b. Since* d_2 *also divides c, it is like … the common divisor of a, b and c becomes* d_2*.*

However, for proving the other aspect; namely showing that d_2 was the greatest of the all common divisors, he could not formulate an equally valid algebraic argumentation. P5 explained his reasoning without any reference to algebraic inferences as he did in the above while proving that d_2 was a common divisor of the three numbers *a*, *b*, and *c*.

> *Hmm… now does it become the greatest? […] Okay. The greatest number dividing* d_1 *and c is* d_2*. Then, since* d_1 *divides a and b, then* d_2 *also divides the two, a and b. For this reason, it is just because of this c (points to c in the equation* $d_2 = (d_1, c)$*). This c restricts this.*

In other words, although P5 had some insight about the mathematical reason, he was not able to explicitly state why the number d_2 became the *greatest* of the all common divisors.

4.2.4.2. Relating the mathematical ideas to the Basic Algebraic Structures Course

After participants completed their initial decision-making processes, they were required to state the mathematical idea they had been testing in this last task of the interview as an algebraic statement to be either proved or disproved. Most of the participants' attempts did not result in complete statements. Only three participants stated it correctly as:

$(a, b, c) = ((a, b), c)$ (P6, 3rd-year)
$gcd\ (a, b, c) = gcd\ (gcd\ (a, b), c)$ (P8, 4th-year)
$gcd\ (a, c) = x \wedge gcd\ (x, b) = y \Rightarrow gcd\ (a, b, c) = y$ (P12, 4th-year)

P10 also produced a one-directional statement for the relation, as P12 did above, but since she stated it for the reverse direction, her statement, if proved (or refuted), would not answer the question.

$k = gcd\ (a, b, c) \Rightarrow gcd\ (a, b) = d, gcd\ (d, c) = k$ (P10, 4th-year)

For making sure that other participants also have understood what kind of a statement they were supposed to formulate, the researcher wanted them to state the idea verbally first. Although all of them verbalized the main idea correctly, they could not transform it into an algebraic statement to be tested. The following is an illustration of two participants' failed attempts, together with their written expressions given in Figure 4.5 and Figure 4.6 and supporting verbal explanations:

> *Let the three numbers' ... greatest common divisor of a, b, and c be m. Hmm... How do we do with the gcd? Let ... a equals m times k_1 for instance, b equals m times k_2, c equals m times k_3. Now, if I were to find first the greatest common divisor of a and b ... let us say this is x for instance. I wonder if I can reach this m again? I will check for this, will the gcd of the three numbers give m. Now I will find another letter for a and x. Let n_1 (writes $a = xn_1$) and n_2 (writes $b = xn_2$). Now I will check whether the gcd of x and c is m or not. (P3, 4th-year)*

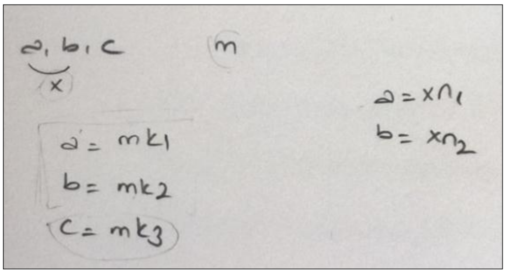

Figure 4.5 P3's trial of writing an algebraic statement

> *What is our first statement? d that divides a, that divides b and that divides c. The greatest integer we have, now, when we look at this… we will take the two things, a and b together, then we will examine c. Hmm…What comes from these two is different. For instance, what that may be… let e. e divides a; e divides b. Then the resulted thing is d, in fact it is our gcd. It both divides e and c. (P13, 4th-year)*

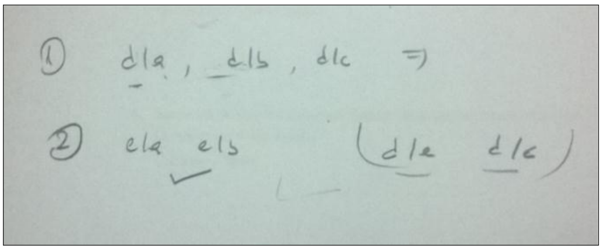

Figure 4.6 P13's trial of writing an algebraic statement

Those participants who could not construct a statement themselves were provided with the list of statements from Basic Algebraic Structures course book (see Exercises in Appendix E). Each of the participants easily determined that among the 12 statements in the list, the last one was stating the proposition they were trying to formulate:

> Let *a*, *b*, and *c* be three nonzero integers. If *d* is the greatest common divisor of *a*, *b* and *c*, show that $d = ((a, b), c)$.

At this stage, P5 (4th-year) was kept out of these processes of writing an algebraic statement, or selecting an appropriate one from among the alternatives, because he already had applied such a procedure previously himself. Although he did not write it as an individual statement, what he tried to prove in his decision-making process was the desired statement itself.

Unlike the previous three tasks of the interview, in the 4th task, participants were not taken back to the work of conducting the specific mathematical task of teaching-*inspecting equivalencies*-, on purpose. Specifically, in Task I, participants were re-asked to respond to the student's question, after referring to Basic Algebraic Structures course book. Similarly, at the end of Task II, they were requested to think about how to overcome the student's mistake again, by taking the advantage of the mathematical ideas studied from the book. Also in Task III, they were asked to explain why the given algorithm works, as a second time, depending on the definition adopted by the course book. However, in the 4th task, participants' decision making processes had already been unfolded before visiting the content of Basic Algebraic Structures course by the researcher's intervention.

The primary objective of this task was not learning about participants' ability to conduct mathematical proofs for showing that the two different applications of the given algorithm were equivalent. Instead, participants' preferences for consulting a formal mathematical proof (or refutation) or their application of some other decision making criteria was the primary concern for this task. Otherwise, asking them to prove or refute the statement would be indifferent from dictating them about how to use this specific way as a decision-making tool: if the statement is proved, then the alternative method is generalizable; otherwise not. This would be of no additional use in enhancing our understanding of how participants of the study conducted the specific mathematical task of teaching-*inspecting equivalencies*.

In conclusion, most preservice teachers could not base their decisions regarding the equivalency of two uses of the same algorithm on solid foundations. Only one participant tried using his higher level mathematical knowledge for decision making purpose. Other participants mostly accepted the new two-step algorithm depending on various inadequate reasons they put forward to. Although only a few of the participants were able to state the mathematical idea they were testing as an algebraic statement themselves; when the alternative statements were presented to them, all the participants successfully determined the correct one.

Hence, analysis of this task revealed that participants of this study also had difficulty in making use of their higher level mathematical knowledge about formulating algebraic statements that would be useful for answering the questions of middle school mathematics, besides not intending to do so.

4.2.5. Summary of the Four Tasks

Within the task-based interview participants worked on four different tasks. Each of the tasks represented one *mathematical task of teaching* from the list of sixteen items identified by Ball, Thames, and Phelps (2008). In particular, the four mathematical tasks of teaching integrated in this study were *responding to students' "why" questions*, *evaluating the plausibility of students' claims, recognizing what is involved in using a particular representation*, and *inspecting equivalencies*.

In the first task, participants were exposed to the hypothetical student question why the number 1 was not regarded as a prime number although it was also divisible by 1 and itself. Participants were expected to formulate a convincing answer to the students' question. In the second task, they were presented a homework sheet assumed to be previously completed by an anonymous student. One of the responses included in the sheet exemplified a typical implicit student claim to be judged: If two numbers divides another number, their product also divides that number. In the third task, participants were required to explain why the standard algorithm for calculating the least common multiple of two numbers worked. Participants conducted each of the first three mathematical tasks of teaching twice. First, they were given time to respond to researcher's questions with their existing knowledge. Next, after studying related mathematical ideas from Basic Algebraic Structures course book, they were called back to conduct the same mathematical task of teaching as a second time. Unlike the first three tasks, in the fourth one, participants performed corresponding mathematical task of teaching, inspecting equivalencies, only once. They went through a decision making process in which they were expected to evaluate equivalency of two different applications of the same algorithm, the standard algorithm for calculating the greatest common divisor of three positive integers. The main concern causing such a dissimilar design for the fourth task was unfolding the decision making process participants employed without any interference.

Preservice teachers showed different levels of competencies in conducting the above mathematical tasks of teaching both before and after visiting the related content from Basic Algebraic Structures course. Quite a limited number of participants efficiently dealt with the mathematical tasks of teaching before visiting their knowledge from the course. For instance, only 3 of the participants provided an accurate justification for rejecting the erroneous student claim in Task II; and only 2 of them were able to formulate complete explanations for how the given algorithm worked in Task III. On the other hand, no participants had an immediate answer to the students' question in the first task. Also in the fourth task, no participants relied on a completely valid decision making process. Only one participant provided a verbal mathematical argumentation showing partially that this new use of the algorithm was equivalent to the standard use of it.

However, the sources for participants' initial mathematical knowledge for conducting these mathematical tasks of teaching were unknown. At this point, researcher asked each of the participants if they had employed any kind of knowledge they had learned in the courses they took from the Faculty of Education. Although most of them replied "*If I have used any, it must be coming from the methods course,*" (P14, 3rd-year, Task II) and none of them specified a certain knowledge. The rest of them answered this question by a simple "no". Moreover, the researcher had the information that mathematical material addressed in the four tasks was not directly handled in the methods courses verified, by consulting one of the instructors of the course. This enabled the researcher to interpret participants' second work on the first three tasks, as a result of using of the mathematical knowledge studied in Basic Algebraic Structures course. In the fourth task, participants conducted the mathematical task of teaching inspecting equivalencies only once, and it was even more difficult to understand in which courses participants developed their strategies for inspecting mathematical equivalencies.

Participants' not specifying any knowledge from the courses they had taken in the Faculty of Education made following inferences possible. Some of the participants benefited from studying the related content from Basic Algebraic Structures course while conducting given mathematical tasks of teaching as a second time. More than half of the participants gained new perspectives on the reason why the number 1 might had been excluded from the set of primes. Participants who relied previously on erroneous reasoning processes for rejecting the students claim in Task II reached more correct justifications. Similarly, in Task III, four more of the participants provided complete explanations for why the given algorithm worked after studying the definition of the term the least common multiple as it was addressed in the course.

Still, there have been participants who could not make sense of the mathematical ideas studied in the course, or those who even if understood the important mathematical points, could not use them efficiently in the tasks of teaching. For instance, in the first task, some of the participants avoided trying to relate ideas discussed in the key paragraph they read to the student’s why question, although they had figured out that the answer was hidden in there. In the second task, all the preservice teachers were able to comprehend the mathematical reason for why the student’s reasoning was erroneous after studying the related content from the course. However, almost all of them preferred explaining their own understanding of the issue directly to the student, instead of using this information in an instructional activity for promoting students’ conceptual understanding. In the third task, half of the participants were not able to relate the ideas they gained from studying the alternative definition of the term the least common multiple to the working principles of the standard algorithm used for calculating it. Their explanations for why the algorithm worked remained the same as in their initial trial.

On the other hand, stating and proving algebraic propositions, what is expected from participants in the last task of the interview, was not an instructional objective specific to Basic Algebraic Structures course; but was also aimed in other content courses. Even so, participants’ practices in the fourth task revealed that they did not consider using their knowledge about writing algebraic statements and proving them with decision making purposes in middle school mathematics education. While inspecting equivalency of two given uses of the same algorithm, the one for calculating the greatest common divisor of three positive integers, they mostly employed informal and mathematically inadequate reasoning methods.

In conclusion, participants' work on the four mathematical tasks of teaching provided various perspectives on the extent to which they were able to use their mathematical knowledge from Basic Algebraic Structures course in the teaching of middle school mathematics. While some of the participants were already able to efficiently deal with the four mathematical tasks of teaching initially, some others could do this only after visiting the course content, or with the researcher's help. Furthermore, even if the participants of the study were selected from the most successful ones in the specific content course and also teaching related courses, the number of those who could not meet the satisfactory efficiency in teaching tasks was considerable.

CHAPTER 5

DISCUSSION AND IMPLICATIONS

This study investigated preservice middle school mathematics teachers' views on the general mathematics coursework in terms of its relevance to their future teaching; and how they used their mathematical knowledge of number theory concepts developed in the Basic Algebraic Structures course in conducting mathematical tasks of teaching at the middle school level. The study was composed of two consecutive sections in which a semi-structured interview protocol and a task based interview protocol were administered to participants. Through the semi-structured interview questions, preservice middle school mathematics teachers' perceptions of the relevance of general mathematics content courses to the work of teaching middle school mathematics were investigated. Perceptions and views regarding the Basic Algebraic Structures course were addressed individually as the course was central to the study. In the task-based interview, participants' work on four mathematical tasks of teaching was observed with the purpose of finding out the connections they made in the specific domain of number theory.

Findings of the study are explained in detail in the previous chapter. This chapter presents discussion of the findings, potential implications of the study, and suggestions for future research.

5.1. Views on General Mathematics Content Courses

Findings from the semi-structured interview questions revealed that preservice teachers considered the mathematics learned in general content courses as *higher level*, *irrelevant to middle school mathematics* and *not applicable to teaching of middle school mathematics*. These characterizations from the preservice teachers' perspective were very similar to how Ball, Thames and Phelps (2008) characterized the content courses studied in teacher education programs. Ball and her colleagues referred to these courses as *scholarly, irrelevant, and remote from classroom teaching,* each of the terms corresponding to a similar meaning with the mentioned findings of the study respectively. These common ideas of preservice teachers were previously anticipated by the proponents of content coursework in mathematics teacher education and they were not unexpected findings. Along with the suggestions of comprehensive coursework for middle school mathematics teachers, CBMS (2001) acknowledged that teacher education students may question the rationale behind learning things that were not included in the middle grades mathematics. Findings of the study confirmed the prediction. Participants hold serious concerns regarding usability of the learned knowledge while teaching middle school mathematics. CBMS attributed such questioning to preservice teachers' unfamiliarity with the learning expectations of mathematics content across the grades levels; which are prerequisites of preparing all students to be ready for the college and workplace (Rivera, 2014). Because middle grades mathematics teachers' work begins after the elementary school and involves preparation of students for the high school and beyond, their professional education should provide them with the knowledge of how mathematical content is connected over the span of curriculum (CBMS, 2013). In this study, only a small portion of participants addressed the connectedness of mathematics. While some of them proposed *connectedness* as a justification for taking the courses, it did not suffice for some others. Moreover, literature points that content courses were considered as the "backstage of mathematics" by both preservice and in-service teachers (Wiley, 2014, p.104). Similar to that, few participants of the present study appreciated taking the content courses because they learned the fundamentals of mathematics there.

Participants were not always consistent in their statements. Some of them held conflicting views about necessity of studying these courses in the teacher education program they are enrolled in. While at the same time approving that the content courses were about fundamentals of the mathematical knowledge and they needed to have this kind of knowledge as middle school mathematics teachers, they did not consider the courses as contributing to their professional development. This might be due to the lack of knowledge and experience about how to connect the mathematics studied to middle school teaching, as some of the participants already indicated in their statements.

Participants generally did not relate the mathematical content covered in these courses to the mathematics taught at middle school. Wiley (2014) studied three practicing middle school teachers' use of their advanced mathematical knowledge in teaching and found that they considered their advanced level of mathematics knowledge as "too abstract" to be used with the middle school content. As in the case of present study, those teachers had difficulty with pinpointing specific instances in which their knowledge could come into use. Findings from the studies investigating prospective elementary teachers' views were also consistent with the findings of this study to a certain extent. Hart and Swars (2009) reported in their study that preservice teachers tended to consider content courses as concerned with *adult higher thinking* and as far from elementary school mathematics content. In a more recent study conducted by Hart, Oesterle and Swars (2013), prospective elementary teachers highlighted the disconnections between their experiences in *Mathematics for Teachers* courses and teaching of elementary school mathematics. Their questioning for the usefulness and relevance of the courses are also in agreement with the current study's findings. Similar to the case of preservice middle school mathematics teachers participated in the present study, participants of Hart et al.'s (2013) study explained the difficulty they encountered in identifying the role of content courses in their professional education.

On the other hand, Hart et al.'s (2013) findings differed from that of the current study in one aspect; preservice teachers' emotions related to their experiences in the content courses. Elementary preservice teachers participated in Hart et al.'s (2013) study described negative emotions about their experiences in the content courses such as stress, discourage, and frustration. In contrary, participants of the current study were pleased with taking these courses; and even with that the courses were offered to them by mathematicians instead of teacher educators. One reason for this might be that the participants of this study were purposely selected from the group of high achievers in one of the content courses. This might have caused the participants to be the ones those did not experience those negative feelings, but the comfort of success. It could be the case that if participants were selected from students who did not have higher success in mathematics content courses, they would have expressed similar negative feelings.

Participants were openly asked about the reason why they might have been required to take the content courses. The only benefit they associated with the study of content courses was its contributions to individuals' intellectual development. Most of the participants referred to improvements they experienced in terms of intellectual development, analytical thinking skills and ability to look from different perspectives. This shows that participants of the study were somehow aware of the role of content courses in opening up their mathematical horizon as suggested by CBMS (2001). Wiley (2014) reported that practicing teachers also perceived similar contributions of studying mathematics in an advanced level, such as its facilitating logical thinking and making the individuals more "math-minded".

5.2. Views on the Mathematics Content Course "Basic Algebraic Structures" in Terms of its Relevance to Their Future Teaching

From among the topics listed in the Basic Algebraic Structures course syllabus, participants identified number theoretical concepts as related content to middle school mathematics. But, they perceived little application of this knowledge to teaching middle school students, and when asked to provide imaginary examples of situations in which the particular knowledge could come into use, they had difficulty with pinpointing specific instances. On the contrary, in Wiley's (2014) study one of the three participating teachers who had two years of experience in teaching considered it as quite useful for practice, together with talking about its use.

Number theory content is highlighted for its simplicity, practicality and accessibility from a pedagogical perspective (Campbell & Zazkis, 2011). The names of the topics listed in the syllabus were exactly the same as how they were termed in the middle school. Participants might have taken the advantage of this, while specifying this content as related to middle school, but when it came to think about its use, they kept silent. This suggests two interpretations of the case: either participants did not consider (or remember) the material studied under "number theory" topics and made their decisions based on the familiarity of concepts' names; or, if they remembered, and carefully considered, this means they perceived no direct use of them in teaching middle school students. Or, one might ask "What would be the case if other topics names were similar to those used in middle school?"

When the general course content was considered, participants specified cases matching well with the four of the *mathematical tasks of teaching* identified by Ball et al. (2008): responding to students' "why" questions, evaluating the plausibility of students' claims, giving (or evaluating) mathematical explanations, and finding an example to make a specific mathematical point. However, almost none of their examples included a reference to use of mathematical concepts studied in the course, evidencing again that the course, although including some content conceptually related to middle school mathematics, was disconnected from participants' teaching concerns.

5.3. Using the mathematical knowledge from Basic Algebraic Structures course in conducting mathematical tasks of teaching basic number theory concepts

The purpose of conducting the task-based interview was to understand how preservice teachers used their mathematical knowledge from Basic Algebraic Structures course in conducting *mathematical tasks of teaching* number theory concepts. Four of the sixteen *mathematical tasks of teaching* were selected from the list of major tasks encountered in teaching that required the teachers to have a unique mathematical understanding and reasoning (Ball, Thames & Phelps, 2008). The four mathematical tasks of teaching were *responding to students' "why" questions*, *evaluating the plausibility of students' claims, recognizing what is involved in using a particular representation,* and *inspecting equivalencies*. Each of the four mathematical tasks of teaching was combined with a different concept of the number theory content covered in the national middle school mathematics curriculum (MoNE, 2013) to create the four main tasks participants worked on through the task-based interview. Respectively the number theory concepts studied were the definition of a prime number, divisibility of a number by 8, and the greatest common divisor and the least common multiple concepts.

The first three of the tasks involved preservice teachers' conducting the given mathematical task of teaching with their existing knowledge and skills, visiting the related mathematical information from the course, and then re-conducting the mathematical tasks of teaching by taking the advantage of the studied knowledge. Findings showed that only a limited portion of the participants efficiently dealt with the mathematical tasks of teaching in their first trial; and those who benefited from the studied information in the course without the researcher's help was also little in number. These two kinds of performances observed in participants may be considered as relatively the best performances emerged in this study compared to those of who responded to the task correctly only after the researcher's scaffolding or who could not arrive at a conclusion at all. Careful review of the participants' individual performances over the four tasks pointed out two of them performed best. The two participants were in different grade levels in their program (P5, 4th-year; P6, 3rd-year), but they possessed a common characteristic that other 12 participants of the study did not. Both of them were pursuing a minor degree in the Department of Mathematics and they had already completed more than half of the courses in the minor program. Although making an inference about a causal relationship between their additional mathematics coursework experience and their outstanding performance in the tasks would be misleading, the two participants' interest in collegiate mathematics was well worth considering within the findings of this study. The finding reminds us Shulman's statement that "[w]e expect that the subject matter content understanding of the teacher be at least equal to that of his or her lay colleague, the mere subject matter major" (1986, p.6) and raises again the many researchers' questions about the appropriate extent of content preparation needed by teachers of mathematics (Ball, Hill & Bass, 2005; Ball, Thames & Phelps, 2008; Davis & Simmt, 2006; Rowland, Huckstep, & Thwaites, 2005; Wilson, Floden, & Ferrini-Mundy, 2001; Zazkis, 1999).

Performances other than P5 and P6's were very similar to each other in that the twelve participants either conducted only one of the four mathematical tasks of teaching accurately without the researchers' help, and/or succeeded in some of the tasks by the researcher's help. However, distribution of their correct moves over the different phases of the four tasks was so diversified that it was not possible to differentiate between their performance levels. In other words, participants' overall works on the four tasks were each consisting of very different combinations of some correct and incorrect understandings of the mathematical content.

Although the participants of the study were selected from among the most successful ones in Basic Algebraic Structures course and also in teaching related courses, most participants could not meet the satisfactory efficiency in teaching tasks. Moreover, in the second part of semi structured interview, participants were provided with the syllabus of the Basic Algebraic Structures course and they were asked to specify the topics that were related to middle school mathematics curriculum. Examining the syllabus, participants addressed divisibility, prime factors, the greatest common divisor and the least common multiple concepts. In fact, these topics constituted exactly the content domain of the four mathematical tasks of teaching studied within the task-based interview. This means, even if the participants of the study were able to realize the connections between the course and middle school mathematics in terms of content, they were not much successful in using this knowledge from the course in conducting the four mathematical tasks of teaching as evidenced by findings from the task-based interview.

Findings of the study also showed that some of the participants benefited from studying mathematical ideas in the Basic Algebraic Structures course in conducting the given mathematical tasks of teaching as a second time. During the interviews researcher asked probing questions to get the participants think deeply on the mathematical issues, which helped them end up with better results many times. In other words, the study showed how preservice middle school mathematics teachers may benefit from the study of mathematical tasks of teaching in a mathematically challenging context. Ball and Bass (2003) considered teaching as a "mathematically-intensive work" that involves significant and challenging mathematical reasoning and problem solving. For this reason, they valued designing professional education of teachers to include more intellectually and mathematically challenging tasks, as it would make teachers' mathematical knowledge more deeply useful and practical. This study confirmed that following Ball and Bass's (2003) suggestion regarding creating opportunities for preservice teachers to experience such mathematical work in the teacher education program may provide them important benefits.

5.4. Implications and Recommendations

The findings of the study addresses that mathematics content courses in teacher education programs should be connected to the mathematics that preservice teachers are to teach in the future in teacher education programs. This connection might be emphasized by creating learning activities through which preservice teachers may relate the two mathematics, and make their own mathematical knowledge more usable while teaching middle school students. Definitions should be given ample importance in preservice teachers' studies related to teaching mathematics. They should be given opportunities to compare alternative definitions of mathematical concepts and reasons about why the one is preferred within the curriculum they are supposed to teach, over the other available ones. Discussions on what would be different if another definition was employed may also be useful. This kind of activity may enhance preservice teachers' understanding of the mathematical concepts more deeply and fosters applicability of the more general ideas behind those concepts into different situations. Formal definitions should also be a part of such studies. Preservice teachers should experience analyzing and using formal mathematical definitions of the concepts, stating them in their own words and making them understandable to lower grade students.

The idea of generalization should be well-transmitted to preservice teachers, through their teaching-related studies in their program. The study showed that most participants relied on superficial generalizations while conducting the mathematical tasks of teaching. Although they were familiar with general arguments, mathematical proofs, and counterexamples, they did not employ this knowledge efficiently in conducting the tasks. This suggests the need for combining such mathematical activities of preservice teachers, with their practices related to teaching middle school mathematics. In other words, preservice teachers should be challenged with tasks of teaching, in which they would be encouraged to employ their higher level mathematical argumentation skills.

For both type of activities, methods of teaching mathematics courses might be a good place to integrate such work in, where a considerable amount of preservice teachers' experiences related to teaching of mathematics take place, and also where teachers' knowledge needs are of primary importance. Moreover, informing preservice teachers about how and why these studies -more generally the mathematics they learn at the university- are connected to their future work of teaching, would facilitate both their considering of this knowledge as useful and their using of this knowledge more efficiently with teaching purposes.

The study has some implications for future research. Findings of this study are restricted to the specific content area-number theory, the specific content course-Basic Algebraic Structures, and four mathematical tasks of teaching studied in the task-based interview-*responding to students' "why" questions, evaluating the plausibility of students' claims, recognizing what is involved in using a particular representation*, and *inspecting equivalencies*. Further research is needed with other mathematics content and other content courses, and involve other mathematical tasks of teaching.

A limited number of preservice teachers from a single teacher education program participated in this study. Both 3rd and 4th-year participants were supposed to imagine the situations of teaching they had never faced with, as a teacher alone. This might have prevented them from making their higher level knowledge of mathematics compatible with that of children and hence employ it in the tasks of teaching middle school mathematics. For this reason, conducting the study with practicing teachers as well as the students of other teacher education programs, may provide additional insights.

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APPENDIX A: SYLLABUS OF THE BASIC ALGEBRAIC STRUCTURES COURSE

Math 116
2012-2013 Spring Syllabus

	Tentative Outline of the Course	
Week	Topics	Section
1	Divisibility Prime Factors and G.C.D.	2.3 2.4
2	Congruence of Integers Congruence Classes	2.5 2.6
3	Binary Operations Matrices	1.4 1.5
4	Groups Subgroups	3.1 3.2
5	Cyclic Groups Homomorphisms Isomorphisms	3.3 3.5 3.4
6	Permutation Groups	4.1
7	Normal Subgroups Quotient Groups	4.4 4.5
8	Rings and Subrings	5.1
9	Ideals and Quotient Rings Ring Isomorphisms	6.1 6.2
10	Integral Domains and Fields The Field of Real Numbers Complex Numbers and Quaternions	5.2 7.1 7.2
11	Polynomials over a Ring	8.1
12	Divisibility and G.C.D.	8.2
13	Factorization in F[x]	8.3
14	Zeros of a Polynomial	8.4

Exams and Grading:
30 pts-Midterm 1: April 4, 2013 on Thursday at 17:40
30 pts-Midterm 2: May 9, 2013 on Thursday at 17:40
40 pts-Final : TBA

Textbook: Elements of Modern Algebra, Jimmie Gilbert & Linda Gilbert, 5th. Edition, Books/Cole 2000.
(Five copies are available in the reserve section of the library. Call no: QA162 G527 2000.)

APPENDIX B: DEPARTMENT OF ELEMENTARY MATHEMATICS EDUCATION UNDERGRADUATE CURRICULUM

First Year, First Semester MATH 111 Fundamentals of Mathematıcs (3) MATH 115 Analytic Geometry (3) MATH 117 Calculus I (5) EDS 200 Introductıon to Educatıon (3) ENG 101 English for Academic Purposes I (3) IS 100 Introduction to Information Technologies and Applications (0)	**First Year, Second Semester** MATH 112 Discrete Mathematics (3) MATH 116 Basic Algebraic Structures (3) MATH 118 Calculus II (5) CEIT 100 Computer Applications in Education (3) ENG102 English for Academic Purposes II (4)
Second Year, First Semester PHYS 181 Basic Physics I (5) MATH 219 Introduction to Differential Equations (4) STAT 201 Introduction To Probability & Statistics I (3) ELE 221 Instructional Principles and Methods (3) EDS 220 Educational Psychology (3) Any 1 of the following set ... HIST 2201 Principles of Kemal Atatürk I (0) HIST 2205 History of The Turkish Revolution I (0)	**Second Year, Second Semester** PHYS 182 Basic Physics II (5) MATH 201 Elementary Geometry (3) STAT 202 Introduction to Probability & Statistics II (3) ELE 225 Measurement and Assessment (3) ENG 211 Academic Oral Presentation Skills (3) Any 1 of the following set … HIST 2202 Princıples of Kemal Atatürk II (0) HIST 2206 History of The Turkish Revolution II (0)
Third Year, First Semester MATH 260 Basic Linear Algebra (3) ELE 341 Methods of Teaching Mathematics I (3) Any 1 of the following set … TURK 201 Elementary Turkish (0) TURK 305 Oral Communication (2) Elective Elective	**Third Year, Second Semester** ELE 310 Community Service (2) ELE 329 Instructional Technology and Material Development (3) ELE 342 Methods of Teaching Mathematics (3) EDS 304 Classroom Management (3) Any 1 of the following set … TURK 202 Intermediate Turkish (0) TURK 306 Written Expression (2) Restricted Elective
Fourth Year, First Semester ELE 301 Research Methods (3) ELE 419 School Experience (3) ELE 465 Nature of Mathematical Knowledge for Teaching (3) Restricted Elective Elective	**Fourth Year, Second Semester** ELE 420 Practice Teaching in Elementary Education (5) EDS 416 Turkish Educational System and School Management (3) EDS 424 Guidance (3) Elective

APPENDIX C: INTERVIEW PROTOCOL

Semi-Structured Interview

1. What are the types of knowledge that a middle school mathematics teacher must have?
2. At the time you entered this university did you know the mathematical knowledge that a middle school mathematics teacher needs to have?
3. What kind of new mathematical knowledge have you learned at the university, during your undergraduate education?
    a. In which course(s) have you learned such knowledge?
4. Have you learned any mathematical knowledge related to the mathematics that you will be teaching in the future to middle school students?
5. Do the general mathematics content courses you have taken from the Department of Mathematics contribute to your teaching profession? Why /why not? (Participants were reminded of the nine courses they had taken.)
6. What may be the reason(s) behind requiring preservice middle school mathematics teachers to take the content courses?"
7. What do you think about the content courses' being offered to you by the department of mathematics?
8. Do you think the mathematics you learned in the content courses is related to the mathematics you are going to teach in the middle school? How?
9. Which one of these courses is related most with the mathematics you will be teaching in the future? Can you give examples?
10. What do you think about Basic Algebraic Structures course? Do you remember the content of this course? (Course syllabus was provided to participants.)
11. Is the mathematical content that you have learned in Basic Algebraic Structures course related to the mathematics that you will be teaching in the middle school?
    a. If yes- How they are related?
    b. If no- How they are different from each other?
12. Do the knowledge and skills you acquired through Basic Algebraic Structures course contribute to your teaching of middle school mathematics?
    a. In what kind of situations having this kind of knowledge may help you with your teaching middle school mathematics?
    b. Considering yourself as a middle school mathematics teacher, what kind of situations may require you to have this kind of advanced mathematical content knowledge?

Task Based Interview

Task I

You are teaching “Prime Numbers”. You presented below definition of a prime number to your students.

Definition:

A number greater than 1 is called a prime number if its only divisors are 1 and itself.

After a while, one of your students asked:

- Teacher, the number “1” is also divisible only by 1 and itself. Why do not we take it as a prime number, then?

(Above part was given to the participant in written form, and following questions were asked verbally:

1. What do you think about this issue? Why do not we take “1” as a prime number?

2. How do you explain this to your 6^{th} grade students?

3. Have you learn anything about this issue in the courses you taken from the faculty of education? / Do you remember anything related to this?

4. Can you find an answer to this question by using the course book of Basic Algebraic Structures?

5. How do you explain this to your 6^{th} grade students?)

Task II

You are assessing the correctness of students' responses to the homework that you assigned about divisibility rules. Below is one of your students' homework sheets.

HOMEWORK

Fill in the blanks with either "divisible" or "not divisible". Explain your reasoning.

1. The number 3264 is _*divisible*__by 2.
   Reasoning: *4, the last digit of the number is even.*
   Even numbers are divisible by 2.

2. The number 3264 is _*divisible*__by 3.
   Reasoning: *2+3+6+4 = 15*
   15 is a multiple of 3.

3. The number 3264 is _*divisible*__by 4.
   Reasoning: *the last two digits compose 64, which is divisible by 4.*

4. The number 3264 is _*divisible*__by 6.
   Reasoning: *because it is divisible by both 2 and 3.*

5. The number 3264 is _*divisible*__by 8.
   Reasoning: *3264 is divisible by both 2 and 4.*

6. The number 3264 is _*not divisible*__by 9.
   Reasoning: *3+2+6+4 = 15*
   15 is not a multiple of 9.

(Above part was given to the participant in written form, and following questions were asked verbally:

1. Is there a mistake here?
2. Why do you think the student might have done this mistake?
3. What do you do for eliminating student's mistake?
4. Have you learn anything about this issue in the courses you taken from the faculty of education? / Do you remember anything related to this?
5. One of the statements from the list of Exercises 2.4 in the course book addresses the condition the student ignored while responding to this item. Can you find which one is it?
6. How can you make use of the specific mathematical point addressed in the statement you found for eliminating the student's mistake?)

Task III

You are planning your next class' instruction for teaching the concepts of gcd and lcm. While reviewing the curriculum guideline you came across the below explanation.

The least common multiple of the numbers 24 and 36 are found by creating a factors list as in the following demonstration. In this technique, the two numbers are divided continuously, by beginning with the least prime number. Multiplication of the numbers contained in this factor list gives the least common multiple of the numbers 24 and 36.

24	36	2
12	18	2
6	9	2
3	9	3
1	3	3
	1	

2x2x2x3x3 =72

(Above part was given to the participant in written form, and following questions were asked verbally:

1. Can you explain why this algorithm works? How?
2. Have you learn anything about this issue in the courses you taken from the faculty of education? / Do you remember anything related to this?

(Below definition was given to the participant.)

A *least common multiple* of two non-zero integers a and b is an integer m that satisfy the conditions

1. m is a positive integer
2. $a|m$ and $b|m$
3. $a|c$ and $b|c$ imply $m|c$.

3. Do you notice any relationship between this definition and the above algorithm?
4. Can you explain why the algorithm works by depending on the given definition? How?

Task IV

You are evaluating your students' homework. One of the problems you asked required students to calculate the greatest common divisor of three numbers, 450, 180 and 420. You see some of the students used a different method, one that you have not taught in the class; which is given below.

450	180	②
225	90	2
225	45	③
75	15	③
25	5	⑤
5	1	5
1		

2.3.3.5 = 90

90	420	②
45	210	2
45	105	③
15	35	3
5	35	⑤
1	7	7
	1	

EBOB (450,180,420) = 2.3.5 =30

EBOB (450,180) =2.3.3.5 = 90

(Above part was given to the participant in written form, and following questions were asked verbally:

1. Do you think this method gives the correct result when calculating the greatest common divisor of any three numbers? Why?
2. How did you make your decision? Which idea you tested? Can you write it as an algebraic statement to be proved or refuted?
3. One of the statements in the list of Exercises 2.4 in the course book states the idea you tested here. Can you find which one is it?

APPENDIX D: ETİK KURUL İZİN BELGESİ

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
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30.05.2014

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İlköğretim Bölümü

Gönderen : Prof. Dr. Canan Özgen
IAK Başkanı

İlgi : Etik Onayı

Danışmanlığını yapmış olduğunuz İlköğretim Bölümü öğrencisi Merve Arslan'ın "Preservice Teachers' Ideas Concerning Basic Algebraic Structures Course in Terms of its Use in Their Future Teaching" isimli araştırması "İnsan Araştırmaları Komitesi" tarafından uygun görülerek gerekli onay verilmiştir.

Bilgilerinize saygılarımla sunarım.

Etik Komite Onayı

Uygundur

30/05/2014

Prof.Dr. Canan Özgen
Uygulamalı Etik Araştırma Merkezi
(UEAM) Başkanı
ODTÜ 06531 ANKARA

APPENDIX E: ALGEBRAIC STATEMENTS FROM THE COURSEBOOK

Exercises 2.4

In this set of exercises, all variables represent integers.

1. If c is a divisor of a and b, prove that c is a divisor of $ax + by$ for all $x, y \in Z$.
2. Give an example where $a \mid (bc)$, but $a \nmid b$ and $a \nmid c$.
3. If $a \mid c$ and $b \mid c$, and $(a,b) = 1$, prove that ab divides c.
4. If $b > 0$ and $a = bq + r$, prove that $(a,b) = (b,r)$.
5. Let $a$and b be integers, at least one of them not 0. Prove that an integer c can be expressed as a linear combination of a and b if and only if $(a,b) \mid c$.
6. Prove that $(ab,c) = 1$ if and only if $(a,c) = 1$ and $(b,c) = 1$.
7. Prove that if $m > 0$ and (a,b) exists, then $(ma, mb) = m. (a,b)$.
8. Prove that if $d = (a,b)$, $a = a_0d$, and $b = b_0d$, then $(a_0,b_0) = 1$.
9. Prove that if $d = (a,b)$, $a \mid c$, and $b \mid c$, then $ab \mid cd$.
10. Let a and b be positive integers. If $d = (a,b)$ and m is the least common multiple of
    a and b, prove that $dm = ab$.
11. Let a and b be positive integers. Prove that if $d = (a, b)$, $a = a_0d$, and $b = b_0d$ then the least common multiple of a and b is a_0b_0d.
12. Let $a, b,$ and c be three nonzero integers. If d is the greatest common divisor of $a, b,$ and c, show that $d = ((a,b), c)$.

APPENDIX F: TÜRKÇE ÖZET

ORTAOKUL MATEMATİK ÖĞRETMENİ ADAYLARININ ALAN DERSLERİNDEKİ MATEMATİK İLE ORTAOKUL MATEMATİĞİNİ İLİŞKİLENDİRME BECERİLERİNİN İNCELENMESİ

Giriş

Ortaokul matematik öğretmenlerinin, öğretecekleri sınıf düzeyinin birkaç seviye ilerisinde matematiksel yeterlilik göstermesi (CBMS, 2012) ve ilkokul seviyesinden üniversite seviyesine kadar matematik müfredatında kapsanan konuların birbirleri ile olan ilişkileri hakkında bilgi sahibi olması sıklıkla önerilmektedir (NMAP, 2008). Bu nedenle, öğretmen adaylarının, öğretmen yetiştirme programlarında gerçekleştirdikleri çalışmaların önemli bir kısmını üniversite seviyesinde aldıkları matematik dersleri oluşturur (Bair ve Rich, 2011; Potari, 2001). Ancak bu derslerin, öğretmen adaylarının bilgi ve becerilerinin gelişmesinde nasıl bir rol oynadığı deneysel olarak yeterince araştırılmamıştır. Öğretmen adaylarının bu derslerden, ilerideki öğretmenlik mesleklerine ilişkin olarak ne kazandığını belirlemek, hem öğretmen eğitimcileri, hem de eğitim araştırmacıları için büyük önem taşımaktadır (Floden ve Meniketti, 2005).

Bu çalışma, genel anlamda, ortaokul matematik öğretmeni adaylarının genel olarak öğretmen eğitimi programında aldıkları matematik alan derslerini öğretmenlik mesleği ile olan ilişkileri açısından nasıl değerlendirdiklerini ve özel olarak, bir matematik alan dersinde öğrendikleri sayılar teorisi bilgilerini, matematik öğretimi sırasında kullanabilme durumlarını incelemiştir.

Kuramsal Çerçeve

Bu çalışmada, Ball, Thames ve Phelps (2008) tarafından geliştirilen “Matematik Öğretmek için Gereken Bilgi” kuramsal çerçevesi kullanılmıştır. Bu çerçeve, Ball ve diğerlerinin (2008), öğretmenlerin sahip olması gereken matematik bilgisinin doğasını ve kapsamını belirlemek amacıyla yürüttükleri çalışmaların bir sonucu olarak ortaya çıkmıştır (Ball vd., 2008; Ball, Hill ve Bass, 2005).

Matematik Öğretmek için Gereken Bilgi (Ball vd., 2008), ilk olarak Shulman (1986)'ın sınıflandırdığı Öğretmek için Gereken Alan Bilgisi'nin yeniden düzenlenmiş halidir. Bu kuramsal çerçevede yer alan bilgi türleri arasından Özelleşmiş Alan Bilgisi, Ball ve diğerleri için en önemlisidir. Çünkü, Özelleşmiş Alan Bilgisi, matematik öğretimini bir meslek olarak tanımlayan bilgi tabanı olarak görülmektedir ve matematik öğretmeye özgü bilgi ve beceriyi ifade eder (Ball vd., 2008, s.400).

Araştırmacılar, halihazırda mesleğini icra etmekte olan öğretmenleri gözleyerek, matematik öğretimi sırasında yapılan her bir işin nasıl bir matematiksel bilgi gerektirdiğini incelemiş, ve sonuç olarak *matematik öğretiminin görevlerini* belirlemiştir (Ball vd., 2008; Ball, Hill ve Bass, 2005). Toplam sayıları on altı olan bu görevlerden dört tanesi, öğrenilen üniversite matematiği ile ortaokul matematiğinin öğretimi arasında anlamlı ilişkiler kurabilmek amacı ile bu çalışmada kullanılmıştır. Bu dört görev sırasıyla şöyledir: *öğrencilerin "neden" sorularına cevap vermek, öğrenci fikirlerinin doğruluğunu değerlendirmek*, *özel bir gösterimin altında yatan düşünceyi kavramak* ve *eşitlik/eşitsizlikleri irdelemek.*

Araştırma Soruları

Bu çalışma ortaokul matematik öğretmeni adaylarının, genel matematik dersleri kapsamında öğrendikleri matematik bilgisini gelecekte ortaokul matematiğini öğretme ile ilişkilendirme durumunu incelemiştir. Çalışma birbirini tamamlayan iki kısımdan oluşmuştur. İlk kısımda, öğretmen adaylarının konu ile ilgili görüşleri açık uçlu sorular yardımıyla elde edilmiştir. İkinci kısımda, katılımcıların yapılandırılmış göreve-dayalı görüşmede göstermiş oldukları performanslar, *matematik öğretiminin görevlerini* yerine getirmeleri sırasında Temel Cebirsel Yapılar dersinde öğrendikleri sayılar teorisi bilgisini nasıl kullandıklarına dair bilgi edinmek amacıyla gözlemlenmiştir. Aşağıda belirtilen araştırma soruları bu çalışmaya yön vermiştir:

3. Ortaokul matematik öğretmeni adayları, öğretmen yetiştirme programında aldıkları genel matematik derslerini, gelecekteki matematik öğretimleri ile ilişkisi açısından nasıl değerlendirmektedir?

a. Ortaokul matematik öğretmeni adayları, genel matematik derslerinden biri olan Temel Cebirsel Yapılar dersini, gelecekteki matematik öğretimleri ile ilişkisi açısından nasıl değerlendirmektedir?

4. Ortaokul matematik öğretmeni adayları, Temel Cebirsel Yapılar dersinde öğrendikleri matematiksel bilgiyi matematik öğretiminin görevlerini yerine getirmede nasıl kullanırlar?

Alanyazın Taraması

Öğretmenlerin, öğretecekleri matematiği çok daha kapsamlı bir şekilde bilmesi gerektiği konusunda fikir birliği olmasına rağmen, bu bilginin kesin niteliği ve sınırları tam olarak tanımlanmamıştır (Ball vd., 2005). Önceleri, öğretmen bilgisini araştıran çalışmaların çoğu, bu bilginin ya öğretmen etkinliği ile ya da öğrenci başarısı ile olan ilişkisine odaklanmıştır. Bu akım daha sonra öğretmen bilgisi değişkeninin yerini tutması için farklı değişkenlerin kullanıldığı gerekçesiyle eleştirilmiştir (Hill, Rowan ve Ball, 2005). Çalışmaların birçoğunda (Chaney, 1995; Goldhaber ve Brewer, 1997, 2000; Hawkins, Stancavage ve Dossey, 1998; Monk, 1994; Monk ve King, 1994; Rowan, Correnti ve Miller, 2002; Telese, 2012) karşımıza çıkan, *üniversitede alınan matematik dersleri* gibi değişkenlerin, öğretmenlerin öğretmek için gerekli olan bilgisini doğrulukla yansıtmıyor olabileceği düşünüldüğüne, eleştiriler yerindedir. Ancak, matematik alanında yapılan çalışmaların çoğu, öğretmenlerin matematiksel geçmişi ve öğretmek için bilgileri arasında pozitif bir ilişkinin var olduğunu desteklemektedir (Floden ve Meniketti, 2005; Wilson ve Floden, 2003). Bu sonuç bilgi bizi şu soruya götürür: Öğretmenlerin aldıkları matematik alan dersleri, onların öğretmek için olan bilgilerini nasıl etkiler? Bu çalışma, bu soruyu sayılar teorisi alanında yanıtlamayı amaçlamaktadır.

Sayılar teorisi matematik eğitiminin bütün seviyelerinde önemlidir; çünkü bu konunun çalışılması öğrencilerin matematiğin doğasını ve yapısını anlamaları için olanak sağlar (Beougher, 1966; Campbell ve Zazkis, 2002). Yapılan çalışmaların çoğu, sayılar teorisini problem çözme (Toh, Leong, Toh, Dindyal, Quek, Tay ve Ho, 2014) ve ispat yapma (Dreyfus, Hershkowitz ve Schwarz, 2001; Edwards, 1998; Lee ve Wheeler, 1987; Martin ve Harel, 1989; Miyakawa, 2002; Tabach, Levenson, Barkai, Tsamir, Tirosh ve Dreyfus, 2011) becerilerini incelemek için bir alan olarak kullanırken, konunun kendisinin öğrenilmesi ve öğretilmesi hakkındaki çalışmalar yetersizdir (Bair ve Rich, 2011).

Yöntem

Bu çalışmada nitel araştırma yöntemleri kullanılmıştır. Birinci araştırma sorusunun cevaplanması için olgubilim, ikinci araştırma sorusunun cevaplanması için ise temel nitel çalışma yöntemi (Merriam, 2009) uygulanmıştır.

Katılımcılar

Çalışmanın katılımcılarını, 14 ortaokul matematik öğretmeni adayı oluşturmaktadır. Katılımcılar, ilgili öğretmen yetiştirme programının üçüncü ve dördüncü sınıf öğrencileri arasından, amaçlı ve kolay ulaşılabilir örnekleme yöntemleri aracılığıyla seçilmiştir. Öncelikle öğretmen adaylarının Temel Cebirsel Yapılar dersinden aldıkları harf notları yüksekten düşüğe doğru sıralanmıştır. Bu sıralamada üst sıralarda yer alan öğrenciler arasından, matematik öğretimi ile ilgili derslerde, özellikle Matematik Öğretim Yöntemleri dersinde de başarılı olanlar çalışmaya katılımcı olarak seçilmiştir. Öğretmen adayların iki alanda da başarılı olması, bu çalışmanın amacına uygun olarak, ilişkilendirme becerilerini gözlemek açısından önem taşır.

Ayrıca çalışmanın örneklemi araştırmacı için kolay ulaşılabilir bir örneklemdir, çünkü çalışma araştırmacının araştırma görevlisi olarak çalıştığı üniversitede gerçekleştirilmiştir. Çalışma süresince araştırmacı katılımcılara kolayca ulaşma imkânına erişmiştir.

Veri Toplama Araçları

Çalışmanın verileri araştırmacı tarafından hazırlanan iki ayrı görüşme protokolü aracılığıyla toplanmıştır. Bunlardan birincisi yarı-yapılandırılmış görüşme protokolü, ikincisi yapılandırılmış göreve-dayalı görüşme protokolüdür.

Yarı-Yapılandırılmış Görüşme Protokolü

Yarı-Yapılandırılmış Görüşme Protokolü öğretmen adaylarının matematik alan dersleri hakkındaki görüşlerini incelemeye yönelik açık uçlu soruları içermektedir. Bu protokoldeki soruların çoğu genel anlamda matematik dersleri ile ilgilidir. Fakat protokol, çalışmanın merkezinde olması nedeniyle, Temel Cebirsel

Yapılar dersine yönelik görüşleri ayrıca ele alan soruları da içermektedir. Sorulardan bazıları örnek olarak aşağıda verilmiştir:

1. Üniversitede ne tür matematiksel bilgiler öğrendin?
2. Gelecekte ortaokul öğrencilerine öğreteceğin matematik ile ilgili matematiksel bilgi edindin mi?
3. Bu bilgileri hangi derslerde öğrendin?
4. Matematik Bölümü'nden aldığın genel matematik dersleri mesleki gelişimine katkı sağlıyor mu?
5. Temel Cebirsel Yapılar dersinde öğrendiğin matematik, gelecekte öğreteceğin ortaokul matematiği ile ilişkili midir? Nasıl?
6. Temel Cebirsel Yapılar dersinde öğrendiğin bilgi ve beceriler mesleki gelişimine katkı sağlıyor mu? Hangi durumlarda?

Yapılandırılmış Göreve-Dayalı Görüşme Protokolü

Yapılandırılmış göreve-dayalı görüşmeler nitel çalışmalarda matematiksel davranışları gözlemek ve anlamak amacıyla kullanılır. Goldin (2000)'e göre bu tür görüşme, araştırmacıların katılımcıların matematiksel düşüncesini sistematik olarak incelemesine ve derinlemesine anlamasına olanak sağlar. Görüşme sırasında katılımcıya bir ya da birkaç tane görev verilir. Katılımcı hem bu görevle hem de görüşmeyi yapan kişiyle etkileşim halinde olur. Görüşmeyi yapan kişinin nerede ve ne kadar müdahalede bulunacağı önceden belirlenir, ancak görüşe sırasında ortaya çıkan etkenlere bağlı olarak değiştirilebilir (Goldin, 2000).

Bu çalışmadaki göreve-dayalı görüşme kapsamında katılımcılara dört görev sunulmuştur. Her bir görev sınıf ortamında karşılaşılması olası bir durumu ve arkasından bu durumla ilgili katılımcıya sorulacak soruları içermiştir. Katılımcılardan, verilen durumlarla karşı karşıya kalan bir öğretmen gibi düşünerek sorulara cevap vermeleri istenmiştir.

Görüşmede kullanılan görevler hazırlanırken üç temel esas dikkate alınmıştır. Bunlardan ilki, ortaokul matematik müfredatında yer alan, sayılar teorisi konusu dahilinde hedeflenen kazanımlardır. Ortaokul seviyesinde karşılaşılması olası durumların tasarlanması için, kullanılan matematiksel içeriğin sınırlarını belirlemede kazanımlar etkin rol oynamıştır. İkinci olarak, ilgili öğretmen yetiştirme programında verilen matematik alan dersleri kapsamında ele alınan sayılar teorisi konuları dikkate alınmıştır. Verilen görevlerin üstesinden gelebilmek için bilinmesi gereken temel sayılar teorisi kavramlarının, çalışmanın gerçekleştirildiği üniversitede Temel Cebirsel Yapılar dersi içerisinde işlendiği görülmüştür. Bu nedenle, çalışma bu ders üzerinden yürütülmüştür. Bu şekilde, ortaokul ve üniversitedeki sayılar teorisinin uygun kısımları bir araya getirildikten sonra, görevler "Matematik Öğretmek için Gereken Bilgi" (Ball vd., 2008) kuramsal çerçevesinin Özelleşmiş Alan Bilgisi bileşenine göre şekillendirilmiştir. Ball ve diğerleri (2008) tarafından belirlenen *matematik öğretiminin görevleri,* öğretmen adaylarının matematik öğretimi sırasında karşılaşabilecekleri ve onların Temel Cebirsel Yapılar dersinde öğrendikleri sayılar teorisi bilgisini kullanmalarını gerektirecek durumların tasarlanmasında yol göstermiştir.

Bu çalışmada matematik öğretiminin 16 görevinden dört tanesi seçilerek kullanılmıştır. Katılımcılara sunulan her bir görev, *matematik öğretiminin görevlerinden* birisini temsil etmektedir. Bu görevler sırasıyla *öğrencilerin "neden" sorularına cevap vermek, öğrenci fikirlerinin doğruluğunu değerlendirmek, özel bir gösterimin altında yatan düşünceyi kavramak* ve *eşitlik/eşitsizlikleri irdelemek*tir.

Verilerin Analizi

Görüşmeler yoluyla elde edilen veriler, iki ayrı şekilde analiz edilmiştir. Yarı-yapılandırılmış sözlü görüşmede elde edilen veriler içerik analizi yöntemi kullanılarak analiz edilmiştir. Katılımcıların, aldıkları matematik derslerinin gelecekteki öğretmenlikleriyle olan ilişkisine dair görüşünü belirten her bir kelime ya da cümle dikkatlice not edilmiş ve kendi aralarında sınıflandırılmıştır. Temel Cebirsel Yapılar dersine özgü sorulara verilen cevaplar da aynı şekilde işlem görmüştür.

Yapılandırılmış göreve-dayalı görüşmeden elde edilen veriler daha farklı bir analiz yöntemine tabi tutulmuştur. Katılımcıların verilen görevler üzerinde yaptıkları yazılı ve sözlü çalışmalar birbirini destekler şekilde değerlendirilmiştir. Bu kısımda, öne sürülen fikirlerin doğruluğu ve derinliği analizin temel noktasını oluşturmuştur. Her bir görev için verilen kabul edilebilir yanıtlar sıklık ifadesi ile birlikte belirtilmiş, alternatif yanıtların açıklamasına da yer verilmiştir.

Bulgular

Genel Matematik Dersleri Hakkında Görüşler

Bulgular, öğretmen adaylarının alan dersleri hakkında karmaşık fikirlere sahip olduğunu göstermiştir. Çalışlmanın katılımcıları bir yandan bu derslerde öğretilen matematiğin *yüksek düzeyde, ortaokul matematiği ile ilgisiz* ve *ortaokul matematiğinin öğretiminde uygulanamaz* olduğunu düşünürken, aynı zamanda bu matematiğin, *ortaokul matematiğinin temelini oluşturduğunu* da ileri sürmüştür.

On dört katılımcı arasından, sadece beş öğretmen adayı (iki 3.sınıf ve üç 4.sınıf) alan derslerini ortaokul matematik öğretmenlerinin mesleki eğitiminin gerekli bir parçası olarak görmüştür. Diğerleri bu dersleri sadece kişisel gelişimlerine faydalı dersler olarak değerlendirmiş veya bu dersleri hangi sebepten dolayı aldıklarına anlam veremediklerini belirtmiştir. Yine de katılımcıların çoğu (f=11) bu dersleri almaktan ve hatta bu derslerin Matematik Bölümü tarafından verilmesinden memnun olduklarını belirtmiştir.

Katılımcıların bazıları alan derslerinin gerekliliği konusunda çelişen fikirler sunmuştur. Bir yandan bu derslerin öğretmenlerin mesleki gelişimine katkısı olmadığını ileri sürerken, diğer yandan buradaki matematik bilgisine sahip olurlarsa daha iyi bir öğretim yapabileceklerini söylemişlerdir. Sonuç olarak, bulgular öğretmen adaylarının matematik alan derslerinin işlevi hakkında bilgi sahibi olmadığını göstermiştir. Görüşmeler sırasında bu eksikliği açıkça ifade eden katılımcılar da olmuştur.

Temel Cebirsel Yapılar Dersi Hakkında Görüşler

Çalışmada yer alan öğretmen adayları, Temel Cebirsel Yapılar Dersinin sadece sınırlı bir kısmının ortaokul matematik müfredatı ile ilişkili olduğunu ifade etmiştir. Genel olarak bu derste edinilen bilgi ve becerilerin, ortaokul matematiğinin öğretimi sırasında kullanılma potansiyeli sorulduğunda, öğretmen adayları bunun yararlı olacağını belirtmiş, ancak örnek durumlar gösterememiştir. Katılımcıların konu üzerinde daha somut düşünmesine yardımcı olmak için, kendilerini ortaokul öğretmeni olarak hayal etmeleri ve Temel Cebirsel Yapılar Dersinde edindikleri bilgilerden herhangi birini kullanmalarını gerektirebilecek durumlar hakkında tahmin yürütmeleri istenmiştir. Bu ilave sorunun analizi katılımcıların aslında bu dersten edindikleri bilgileri matematik öğretiminin bazı gereklerini yerine getirmede kullanmayı öngördükleri ortaya çıkmıştır. Bu durumlar, Ball ve diğerlerinin (2008) tanımladığı *öğrencilerin "neden" sorularına cevap vermek, öğrenci fikirlerinin doğruluğunu değerlendirmek, matematiksel açıklamalar yapmak veya değerlendirmek* ve *önemli bir matematiksel noktaya değinmek için özel bir örnek bulmak* görevleriyle örtüşmektedir.

Katılımcıların Matematik Öğretimin Görevleri Üzerine Çalışması

Göreve-dayalı görüşmede katılımcılara dört görev sunulmuştur. Her bir görev, Ball ve diğerleri (2008) tarafından belirlenen *matematik öğretiminin görevlerin*den bir tanesini temsil etmektedir. Bu görevler sırasıyla *öğrencilerin "neden" sorularına cevap vermek, öğrenci fikirlerinin doğruluğunu değerlendirmek, özel bir gösterimin altında yatan düşünceyi kavramak* ve *eşitlik/eşitsizlikleri irdelemek*tir.

Birinci görevde katılımcılara bir ortaokul öğrencisinin sorduğu varsayılan şu soru yöneltilmiştir: "1 de sadece 1'e ve kendisine bölünüyor. Neden 1'i asal sayı olarak kabul etmiyoruz?" Öğretmen adaylarından bu soruya ikna edici bir cevap oluşturmaları beklenmiştir. İkinci görevde katılımcılara, önceden bir öğrencinin doldurduğu kabul edilen bir ödev kâğıdı sunulmuştur. Buradaki yanıtlardan bir tanesi, öğrencinin üstü kapalı olarak öne sürdüğü matematiksel bir iddiayı içermektedir. Şöyle ki, öğrenci soruya herhangi iki sayıya bölünen bir sayının, o sayıların çarpımına da bölündüğünü varsayarak cevap vermiştir. Bu görevde katılımcılardan beklenen, öğrencinin hatasını, sebebiyle birlikte belirlemek ve duruma nasıl müdahale edeceklerini açıklamaktır. Üçüncü görevde, katılımcılardan iki sayının en küçük ortak katını hesaplamak için kullanılan standart algoritmanın neden çalıştığını açıklamaları istenmiştir. Katılımcılar bu üç görevi ikişer kez gerçekleştirmiştir. Katılımcılara ilk önce araştırmacının sorularına var olan bilgileriyle cevap vermeleri için süre verilmiştir. İkinci aşamada ise, Temel Cebirsel Yapılar ders kitabının ilgili kısmı üzerinde araştırmacı ile birlikte çalıştıktan sonra tekrar aynı soruları cevaplamaları istenmiştir. Bu üç görevden farklı olarak dördüncü görevde, katılımcılar verilen görevi sadece bir kez gerçekleştirmiştir. Burada, herhangi üç sayının en büyük ortak bölenini hesaplamak için kullanılan standart algoritmanın farklı bir kullanımı katılımcılara sunulmuş ve onlardan bu kullanımın her zaman doğru sonucu verip vermeyeceğini değerlendirmeleri istenmiştir. Bu görevin diğerlerinden farklı tasarlanmasının nedeni, katılımcıların yürüttüğü karar verme sürecinin herhangi bir müdahale olmaksızın incelemesi amacıdır.

Katılımcılar yukarıda açıklanan görevleri yerine getirirken farklı düzeylerde yetkinlik göstermiştir. Verilen görevleri Temel Cebirsel Yapılar dersine değinmeden önce başarıyla yerine getiren öğretmen adaylarının sayısı oldukça sınırlıdır. Örneğin, katılımcılardan sadece üç tanesi ikinci görevdeki hatalı öğrenci yanıtını reddetmek için doğru bir gerekçe sunabilmiştir. Benzer şekilde, üçüncü görevde verilen algoritmanın neden çalıştığını tam olarak açıklayabilen katılımcıların sayısı sadece ikidir. Bunun yanı sıra, katılımcılardan hiçbiri birinci görevdeki öğrenci sorusuna doğru yanıt verememiştir. Yine dördüncü görevde matematiksel olarak geçerli bir karar verme süreci uygulayan katılımcı yoktur.

Temel Cebirsel Yapılar ders kitabından ilgili kısımların çalışılması, bazı katılımcıların verilen görevi ikinci kez gerçekleştirmesinde faydalı olmuştur. Öğretmen adaylarının yarıdan fazlası neden 1'in asal sayı olarak kabul edilmediği konusunda yeni fikirler edinmiş; hatalı öğrenci yanıtının neden kabul edilemeyeceğine dair yanlış yorum yapan katılımcıların çoğu daha doğru açıklamalara ulaşmıştır. Üçüncü görevde, verilen algoritmanın neden çalıştığını tam olarak açıklayan katılımcıların sayısı dört artmıştır. Yine de, ders kitabından çalışılan temel matematiksel fikirleri anlayamayan; ya da kendisi anlasa bile *matematik öğretiminin görevlerinde* etkin bir şekilde kullanamayan birçok katılımcı olmuştur.

Dördüncü görevde katılımcılardan beklenen cebirsel ifadeleri yazma ve kanıtlama becerisi sadece Temel Cebirsel Yapılar dersine özgü hedefler değildir; aksine birçok alan dersinin hedefleri arasındadır. Buna rağmen, katılımcıların bu görevde yaptıkları çalışmalar, onların bu becerilerini ortaokul matematiği ile ilgili kararlarını vermek amacıyla kullanmayı göz ardı ettiğini göstermiştir. En büyük ortak bölen hesaplamak için kullanılan standart algoritmanın verilen alternatif uygulamasını doğru olarak kabul ederken, öğretmen adayları genellikle matematiksel olarak yetersiz muhakeme yöntemlerine dayanarak karar vermiştir.

Sonuç olarak, öğretmen adaylarının verilen dört görev üzerindeki çalışmaları, Temel Cebirsel Yapılar dersinde edindikleri bilgi ve becerileri ortaokul matematik öğretiminde ne kadar etkin kullanabildikleri konusunda çeşitli bakış açıları kazandırmıştır. Katılımcılardan bazıları verilen görevi ilk denemelerinde başarıyla yerine getirirken, bazıları bunu ancak Temel Cebirsel Yapılar dersinin ilgili kısmını çalıştıktan sonra ya da araştırmacının yardımıyla yapabilmiştir. Ayrıca, katılımcıların hem Temel Cebirsel Yapılar dersinde, hem de öğretimle ilgili derslerde en yetkin olan öğrenciler arasından seçilmiş olmasına rağmen, içlerinden çoğu verilen görevleri yerine getirmede yeterince başarılı olamamıştır.

Tartışma ve Öneriler

Genel Matematik Dersleri Hakkında Görüşler

Çalışmaya katılan öğretmen adayları matematik alan derslerini *yüksek düzeyde, ortaokul matematiği ile ilgisiz* ve *ortaokul matematiğinin öğretiminde uygulanamaz* olarak değerlendirmiştir. Bu betimlemeler Ball ve diğerlerinin (2008) değerlendirmesiyle benzerlik göstermektedir. Öğretmen adaylarının alan dersleri hakkında bu şekilde düşünüyor olması beklenmeyen bir sonuç değildir. Matematik öğretmenlerinin eğitiminde, alan derslerinin önemini vurgulayan CBMS (2001) öğretmen adaylarının ortaokulda öğretilmeyen bu dersleri kendilerinin neden aldıklarını sorgulayacaklarını öngörmüştür. Bu çalışma bu tahmini doğrulamıştır. Katılımcılar öğrendikleri bilgilerin kullanımı hakkında ciddi endişeler taşımaktadır. Diğer yandan, katılımcılardan bazıları bu derslerin ortaokul matematiğin temelini oluşturduğunu savunmuştur. Benzer bulgular Wiley (2014)'in öğretmenler ve öğretmen adaylarıyla yaptığı çalışmasında görülmüştür.

Katılımcıların alan derslerinin gerekliliği hakkında çelişkili fikirlere sahip olmasının nedeni, üniversitede öğrendikleri matematik ile ortaokulda öğretecekleri matematik arasında nasıl bağlantı kuracaklarını bilmemelerinden ve bunu deneyimlememiş olmalarından kaynaklanıyor olabilir. Hart, Oesterle ve Swars (2013)'ın çalışmasında olduğu gibi, öğretmen adayları alan derslerinin ve matematik öğretimine ilişkin derslerin birbirinden ayrık olduğunu, ancak bu derslerin öğretmenlerin zihinsel becerilerinin gelişmesi açısından faydalı olduğunu vurgulamıştır. Her iki çalışmada da katılımcılar alan derslerini öğretmenlerin mesleki gelişimiyle ilişkilendirmekte zorlanmıştır. Ancak Hart ve diğerlerinin (2013) çalışmasına katılan öğretmen adayları bu derslere karşı korku ve stres gibi olumsuz hislere sahipken; bu çalışmada yer alan öğretmen adayları dersleri almaktan memnun olduklarını belirtmişlerdir. Bunun sebebi katılımcıların alan derslerinden biri olan Temel Cebirsel Yapılar dersinde başarı gösteren öğrenciler arasından seçilmiş olması olabilir.

Temel Cebirsel Yapılar Dersi Hakkında Görüşler

Katılımcılar, Temel Cebirsel Yapılar dersinde işlenen konular arasından sayılar teorisinin ortaokul matematiği ile ilişkili olduğunu söyleseler de, burada öğrendikleri bilgilerin matematik öğretimi sırasında uygulanamayacağını düşündükleri ortaya çıkmıştır. Bunun aksine, Wiley (2014)'in çalışmasında yer alan üç öğretmenden biri sayılar teorisi konusun uygulama açısından oldukça elverişli olduğunu söylemiş ve bu konuda uygun örnekler sunabilmiştir.

Katılımcılara sunulan ders izlencesinde yer alan sayılar teorisi konularının ortaokul matematiğindeki haliyle tamamen aynı dil ile ifade ediliyor olması, öğretmen adaylarının bu konuları kolayca belirlemesine neden olmuş olabilir. Belirledikleri konuların matematik öğretimi sırasında hangi durumlarda kullanılabileceği sorulduğunda, öğretmen adayları fikir yürütememişlerdir. Bu durum iki şekilde yorumlanabilir: Katılımcılar konuların içeriğini düşünmeden kendilerine tanıdık gelen konu başlıklara bakarak karar vermiş olabilir. Ya da, aksi durum varsayılırsa, gerçekten bu bilginin ortaokulda matematik öğretiminde kullanılamaz olduğunu düşünüyor olabilirler.

Temel Cebirsel Yapılar Dersinde Edinilen Matematiksel Bilginin Sayılar Teorisi Konusunun Öğretilmesinde Kullanımı

Çalışma, sadece sınırlı sayıda katılımcının verilen *matematik öğretiminin görevleri*ni başarılı bir şekilde yerine getirebildiğini göstermiştir. Katılımcılardan bazıları tamamen kendi çabasıyla doğru sonuçlara ulaşırken, bazıları da araştırmacının yardımıyla bunu başarmıştır. Öğretmen adaylarının her bir görevdeki performansları bireysel olarak değerlendiğinde, iki öğretmen adayının diğerlerine göre daha başarılı olduğu kanısına varılmıştır. Bu iki öğretmen adayı farklı sınıf seviyelerinde olmalarına rağmen (P5, 4.sınıf; P6, 3.sınıf), diğer adayların sahip olmadığı bir ortak özelliklerinin olduğu ortaya çıkmıştır. İki aday da Matematik Bölümü'nde yandal programına kayıtlıdır ve tamamlanması gereken derslerin yarısını tamamlamış durumdadır. Buradan, adayların ilaveten aldıkları matematik dersleri ile verilen görevlerde gösterdikleri başarı arasında bir sebep sonuç ilişkisinin çıkarılması yanıltıcı olabilir. Ancak, bu bulgu yine de dikkate almaya değer bir sonuçtur ve birçok araştırmacının yanıt aradığı soruyu yeniden ortaya koymuştur: Matematik öğretmenlerinin sahip olması gereken alan bilgisinin ideal sınırı ne olmalıdır? (Ball, Hill ve Bass, 2005; Ball, Thames ve Phelps, 2008; Davis ve Simmt, 2006; Rowland, Huckstep ve Thwaites, 2005; Wilson, Floden ve Ferrini-Mundy, 2001; Zazkis, 1999).

Dersin izlencesini değerlendirirken, katılımcıların çoğu sayılar teorisi konularının ortaokul matematiği ile ilişkili olduğunu söylemiştir. Ayrıca, katılımcıların hem Temel Cebirsel Yapılar dersinde, hem de öğretimle ilgili derslerde en yetkin olan öğrenciler arasından seçilmiş olmasına rağmen, içlerinden çoğu verilen görevleri yerine getirmede yeterince başarılı olamamıştır. Başka bir deyişle, öğretmen adayları içerik olarak iki matematik arasında ilişki kurabilseler de, Temel Cebirsel Yapılar dersinde edindikleri bilgilerini *matematik öğretiminin görevleri*ni yerine getirmede etkili kullanamamışlardır.

Öneriler

Çalışmanın bulguları öğretmen yetiştirme programlarında verilen matematik alan derslerinin, öğretmen adaylarının gelecekte öğretecekleri matematik ile ilişkilendirilmesinin önemini ortaya koymaktadır. Öğretmen adaylarının iki matematik arasında ilişki kurmasını sağlayacak ve sahip oldukları matematiksel bilgiyi matematik öğretiminde etkin kullanılabilir hale getirmelerine yardımcı olacak öğrenme aktiviteleri tasarlanabilir. Tanımlar öğretmen adaylarının matematik öğretimi hakkındaki çalışmaları arasında önemle ele alınmalıdır. Öğretmen adaylarına değişik tanımları karşılaştırabilecekleri, öğretecekleri matematik müfredatındaki bir tanımın neden tercih edildiğinin üzerinde düşünebilecekleri fırsatlar verilmelidir. Eğer tanımlardan birinin yerine farklı bir tanım seçilirse ne olurdu, ne değişirdi? Öğretmen adaylarının bu gibi sorular üzerinde tartışmaları faydalı olabilir. Bu tür aktiviteler öğretmen adaylarının kavramları daha derinlemesine anlamalarına yardımcı olabilir. Formel matematiksel tanımlar da bu tür aktivitelerin bir parçası olmalıdır.

Ayrıca, öğretmen adaylarının matematik öğretimi ile ilgili deneyimleri arasında matematiksel genelleme aktivitelerine de yer verilebilir. Bu çalışmada katılımcıların büyük bir bölümünün *matematik öğretiminin görevleri*ni yerine getirirken yüzeysel genellemelere başvurduğu gözlenmiştir. Genel argümanlar, ispatlar ve ters örneklere üniversitede aldıkları matematik dersleri sayesinde aşina olan katılımcıların, bu bilgilerini matematik öğretimi sırasında kullanmamış olmaları ilgi çekicidir. Öğretmen adayları için, bu tür bilgilerini matematik öğretimi ile ilgili olan deneyimleriyle birleştirebilecekleri aktivitelerin tasarlanması önerilebilir.

APPENDIX G: TEZ FOTOKOPİSİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü ☐

Sosyal Bilimler Enstitüsü ☐

Uygulamalı Matematik Enstitüsü ☐

Enformatik Enstitüsü ☐

Deniz Bilimleri Enstitüsü ☐

YAZARIN

Soyadı : DİLBEROĞLU
Adı : Merve
Bölümü : İlköğretim Fen ve Matematik Alanları Eğitimi

TEZİN ADI (İngilizce): An investigation of pre-service middle school mathematics teachers' ability to connect the mathematics in content courses with the middle school mathematics

TEZİN TÜRÜ : Yüksek Lisans ☐ Doktora ☐

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
3. Tezimden bir bir (1) yıl süreyle fotokopi alınamaz. ☐

TEZİN KÜTÜPHANEYE TESLİM TARİHİ: