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THE RELATIONSHIP AMONG METACOGNITION, REASONING ABILITY, AND MATHEMATICAL PROBLEM SOLVING PERFORMANCE OF NINTH GRADE STUDENTS

YÜKSEL ÖZGE ELİTAŞ

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Prof. Dr. Meliha ALTUNIŞIK
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Doctor of Philosophy.

Prof. Dr. Ceren ÖZTEKİN
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Doctor of Philosophy.

Prof. Dr. Erdinç ÇAKIROĞLU
Supervisor

Examining Committee Members

Prof. Dr. Safure Bulut	(METU,SSME)	
Prof. Dr. Erdinç Çakıroğlu	(METU,ELE)	
Assoc. Prof. Dr. Çiğdem Haser	(METU,ELE)	
Assist. Prof. Dr. Gönül Kurt Erhan	(BAŞKENT,ELE)	
Assist. Prof. Dr. Elif Yetkin Özdemir	(HU,ELE)	

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Name, Last name : Yüksel Özge Elitaş

Signature :

ABSTRACT

THE RELATIONSHIP AMONG METACOGNITION,
REASONING ABILITY, AND
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OF NINTH GRADE STUDENTS

Elitaş, Yüksel Özge
Ph.D., Department of Elementary Mathematics Education
Supervisor : Prof. Dr. Erdinç Çakıroğlu

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The purpose of the current study is to search the relationship among metacognition, reasoning ability, and the mathematical problem solving performance. Another purpose is to search the role of metacognition and reasoning ability on the prediction of mathematical problem solving performance. For this, the participants were the 578 ninth grade students who are at public Anatolian high schools in Izmir. There were three instruments. The first instrument was conducted for measuring metacognition; namely junior Metacognitive Awareness Inventory (jMAI). The second instrument was conducted for measuring reasoning ability; namely the Test of Logical Thinking (TOLT). The final instrument was conducted for measuring mathematical problem solving performance namely Mathematical Problem Solving Instrument. The design of the study is correlational model. For data analysis standard multiple linear regression was conducted. According to the results of the study, there was a significant, strong, and positive correlation between metacognition and mathematical problem solving performance. There was a significant, strong, and positive correlation between reasoning ability and mathematical problem solving performance. Also there was a significant, strong, and positive correlation between metacognition and reasoning ability. Moreover metacognition and reasoning ability significantly predicted the mathematical problem solving performance of the students. The metacognition and reasoning ability predicted and explained 54 percent of the mathematical problem solving performance of the students. Based on the result of the study, the relationship among metacognition, reasoning ability, and problem solving should be emphasized in the classrooms. Moreover, metacognition and reasoning ability should not be perceived as independent from problem solving.

Keywords: Metacognition, Reasoning Ability, Problem Solving

ÖZ

DOKUZUNCU SINIF ÖĞRENCİLERİNİN; ÜSTBİLİŞ, MANTIKSAL DÜŞÜNME YETENEĞİ VE MATEMATİKSEL PROBLEM ÇÖZME PERFORMANSI ARASINDAKİ İLİŞKİ

Elitaş, Yüksel Özge
Doktora, İlköğretim Matematik Eğitimi Bölümü
Tez Yöneticisi : Prof. Dr. Erdinç Çakıroğlu

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Bu çalışmanın amacı üstbiliş, mantıksal düşünme yeteneği ve matematiksel problem çözme performansı arasındaki ilişkiyi araştırmaktır. Çalışmanın diğer amacı da üstbiliş ve mantıksal düşünme yeteneğinin matematiksel problem çözme performansını yordama düzeyini araştırmaktır. Bu amaçlar doğrultusunda çalışmada kullanılan veriler 2014-2015 akademik yılı ilkbahar döneminin başında toplanmıştır. Çalışmanın örneklemi 578 dokuzuncu sınıf öğrencisinden oluşmaktadır. Katılımcılar batı ve kuzey Izmir bölgesindeki 17 anadolu lisesinde okuyan dokuzuncu sınıf öğrencileridir. Çalışmada 3 ölçek uygulanmıştır. Birinci ölçek olan Bilişüstü Yeti Anketi üstbiliş düzeyini ölçmek için kullanılmıştır. İkinci ölçek olan Mantıksal Düşünme Yetenek Testi öğrencilerin mantıksal düşünme yeteneklerini ölçmek için kullanılmıştır. Sonuncu ölçek olan Matematiksel Problem Çözme Ölçeği ise öğrencilerin matematiksel problem çözme performansını ölçmek için kullanılmıştır. Katılımcılardan veriler tek seferde toplanmıştır. Bu çalışma nicel bir çalışmadır ve çalışmanın deseni korelasyonel desendir. Verilerin istatistiksel analizi için çoklu regresyon analizi uygulanmıştır. Analiz sonuçlarına göre üç değişken arasında istatistiksel olarak anlamlı ilişki bulunmuştur. Hem üstbiliş ve matematiksel problem çözme performansı arasında, hem mantıksal düşünme yeteneği ve matematiksel problem çözme performansı arasında, hem de üstbiliş ve mantıksal düşünme yeteneği arasında güçlü, pozitif ve anlamlı ilişki bulunmuştur. Ayrıca üstbiliş ve mantıksal düşünme yeteneği, matematiksel problem çözme performansını istatiksel olarak anlamlı olarak yordamaktadır. Üstbiliş ve mantıksal düşünme yeteneği, matematiksel problem çözme performansının yüzde 54'ünü açıklamaktadır. Bu sonuçlara dayanarak, üstbiliş, mantıksal düşünme yeteneği ve matematiksel problem çözme performansının sınıflarda vurgulanması önerilmektedir. Ayrıca, çalışmada bulunan ilişkiye dayanılarak üstbiliş ve mantıksal düşünme yeteneğinin problem çözmeden ayrı tutulmaması önerilmektedir.

Anahtar Kelimeler: Üstbiliş, Mantıksal Düşünme, Problem Çözme

To My Parents

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LIST OF ABBREVIATIONS

ANOVA	Analysis of Variance
B	Standardized Beta Value
Df	Degree of freedom
DV	Dependent Variable
F	Frequency
IV(s)	Independent Variables
J-MAI	Junior Metacognitive Awareness Inventory
M	Mean
MoNE	Minister of National Education
MRA	Multiple Regression Analysis
N	Sample size
NCTM	National Council of Teachers of Mathematics
P	Significance level
R	Correlation Coefficient
SD	Standard deviation
TIMMS	Trend in International Mathematics and Science Study
TOLT	Test of Logical Thinking

CHAPTER 1

INTRODUCTION

In this chapter; problem statement and background information about the problem, the purpose of the study, the research problem, the hypotheses of the study, definitions of the variables, and significance of the study will be explained.

1.1 Background Information Related to Problem Solving

Problem solving had high importance in mathematics education for decades, and continues to be essential part of mathematics education (Evans, 2012; Posamentier & Krulik, 1998). According to Hembree (1992), during the 20th century, teaching and learning problem solving has gained special attention and emphasis. There was a great attention about the research area and there were two perspectives about problem solving. First perspective was that “problem solving is a basic skill, and it is a vital and required skill for students”. The second perspective was that “problem solving is a complex mental activity” (Hembree, 1992). The first perception was created after the National Council of Supervisors of Mathematics (NCSM) defined problem solving as one of the most essential ten proficiencies (1977). Later, the National Council of Teachers of Mathematics emphasized problem solving greatly in its Curriculum and Evaluation Standards for School Mathematics (NCTM, 1980). The second perspective was long standing one for years (Hemree, 1992).

According to Garofalo and Lester (1985), problem solving is a process which requires high visualization, association, abstraction, comprehension, manipulation, reasoning, analysis, synthesis, and generalization. Moreover, all of these highest faculties should be managed and coordinated appropriately. In fact, Polya's ideas about problem solving have affected the mathematics education field for decades. After Polya's famous book "How to Solve it?" created great attention to problem solving in 1945; in each new edition of the book, this attention and emphasis had increased through the years (Bahar, 2013; Donaldson, 2011; Hembree, 1992, Özalkan, 2010). After Polya published his new edition book namely "a new aspect of mathematical model" in 1973, the problem solving theme again gained great importance in the field of mathematics education. After seven years, the National Council of Teachers of Mathematics (NCTM) in "it's Agenda for Action" (1980) claimed that problem solving should be "the focus of school mathematics" in order to create a great emphasis on problem solving (NCTM, 1980, p.1). Later, the council improved this statement, and provided four process standards. The two of these four process standards were problem solving and reasoning. The council claimed that problem solving and reasoning process standards should be emphasized in all grades, and problem solving should construct a foundation for all aspects of mathematics teaching; then the students will have chance to experience the power of mathematics (NCTM, 1989). In addition to NCTM, many researchers emphasize the importance of problem solving and reasoning ability.

According to Posameinter and Krulik (1998), all of the books about problem solving and especially Polya's all books and the documents such as the NCTM standards played a vital role in creating the general acceptance that problem solving should have great importance in curriculum. It is widely accepted that problem solving must be an important and basic part of a good instructional program. Evans (2012) stated that strong problem solving abilities and skills are vital for mathematics, as well as for other subject areas, disciplines and for daily life in general. Students should be provided critical thinking and strong problem solving preparation in schools, since they need these skills in their lives. Similarly, Donaldson (2011) states that it is commonly accepted that problem solving is what mathematics is all about. So, mathematics teachers' main aim should be to help students improve their problem solving abilities. For this aim, they should teach mathematics throughout problem solving process. Then, the students will learn new mathematical concepts and integrate mathematical knowledge throughout problem solving (Donaldson, 2011).

In order to emphasize the importance of problem solving, Polya (1973) stated that the teachers should improve students' abilities to think and solve problems; and also improve abilities to connect and integrate experience of former with new one. According to Krulik and Posamentier (1998), problem solving should be a way for students to reach the beauty in the mathematics, and it should result in the integration of the mathematical experiences into a meaningful whole. For this, the teachers should introduce the students to a variety of problem solving strategies, and help them to practice and use these strategies in problem solving process. Similarly, Carson (2007) states that there are some common elements of problem solving. First one is that problem solving connects theory and practice. Secondly, problem solving teaches creativity. Next, successful problem solvers have a complete and organized knowledge base. Later, problem solving teaches transfer or how to apply conceptual knowledge. Another element is that problem solving is not an algorithm. The knowledge base and the transfer of that knowledge are vital elements of problem solving process. Also, according to Krulik and Posamentier (1998), the teachers should involve problem solving as an essential part in their regular curriculum; they need to focus on what problem solving is, how problem solving can be used in order to teach mathematical skills effectively, and how problem solving can be presented to students in an effective way. In fact, the teacher should learn that problem solving can be presented in three different ways. Firstly, "problem solving is a subject for study in and of itself". Secondly, "problem solving is an approach to a particular problem". Finally, "problem solving is a way of teaching" (Krulik & Posamentier, 1998, p. 4). First of all, the teachers should themselves be good problem solvers; before teaching problem solving to students. They should learn all of the problem solving strategies. Moreover, they should know which problem solving strategies to apply, when to apply and how to apply.

Also, they should be able to apply the problem solving strategies both to mathematics and real life experiences (Krulik & Posamentier, 1998).

Gagne (1980) stated that “the central point of education is to teach people to think, to use their rational powers, to become better problem solvers” (p.85). Also, Jonassen (2000) states that most of the psychologist and educators agree on the fact that problem solving is the most crucial learning outcome for life, because people, in fact, regularly deal with problems and solve the problems both in their daily lives and professional lives. And people are rewarded in their professional lives if they solve problems rather than memorizing information, or completing exams. But, the students in their school lives are not engaged in problem solving activities in general, and they are not required to solve meaningful problems in curriculum; learning to solve problems generally is not required in formal educational settings. Researchers and educators are inefficient to engage students in problem solving, because the researchers and educators don’t have deep knowledge about its processes, the breadth of problem solving is not understood well enough. Moreover, instructional-design research and theory have drawn little attention to the study of problem solving processes (Jonassen, 2000).

For decades, mathematicians or researchers of mathematics education provided a variety of different definitions of problem and problem solving. These differences occurred due to the different opinions of what forms a problem, and of what is important in problem solving (Donaldson, 2011). Also, Ellis (2005) stated that most of the previous research and research base related to problem solving area are lack of a common definition, so they have measurement validity problems. The researcher states that there is “no generally agreed-on set of definitions of terms” (p. 109), and thinking skills are difficult to measure. Similarly, Nickerson (1994) states that some research which aim to build up approaches to the teaching of thinking and problem solving have been directed by one or another theory, model or a conceptual framework; and also other studies have been theory free. None of the approaches to the teaching of thinking and problem solving that has yet been produced has a firm theoretical foundation. None of them seems to be based on a well-articulated theory of cognition, which is universally accepted as valid by the scientists or researchers community. This statement is correct because there seems to be no such firm and valid theory about teaching of thinking and problem solving (Nickerson, 1994). This fact explains why a wide range of opinions exist about how to teach thinking and problem solving best. Also, it points up the resistance to faster progress in the field. Instead, thinking and problem solving should be better understood; more precise, more predictive, more comprehensive and testable theories of cognition should be produced and tested. Until this required progress is achieved, studies to enhance thinking and problem solving will remain as a trial-and-error process. Researchers and educators still don't know how to teach all aspects of thinking and problem solving effectively (Nickerson, 1994). Also, Lester (1994) stated that his work from 1980 to 1994 showed that there has been little progress in problem solving research. Also, when Lester and Kehle (2003) compared the list of issues to the Lester's work in 1994, they stated that still, little development has occurred in problem solving research, and also the literature on problem solving provided little offerings to school practice. Similarly, Lesh and Zawojewski (2007) claim that there is a lack of impact and cumulativeness in the research on mathematical problem solving. This situation is not surprising because this area of research is criticized for years due to its lack of theoretical base. So, there is a great need for better theorizing in the field. Similarly, Grugnetti and Jaquet (2005) state that there are several views about the nature of problem solving in mathematics educations. In fact, in different didactical theories or in different periods, problem solving takes different identifications. As there is no unique definition of mathematics, also there is no unique or common definition of problem solving. From an epistemological point of view, we can only define a variety of facets related to problem solving. For this, more studies should be conducted related to problem solving. Also, according to Jonassen (2000), it is important to focus on problem solving, because it is at the center of practice for contemporary learning theories. Contemporary conceptions of student-centered learning environments require problem solving activity, and emphasize problem-solving outcomes. For example, open-ended learning environments, goal-based scenarios, and also problem based learning emphasize and support their explicit problem solving outcomes.

Similarly, Carlson and Bloom (2011) state that many aspects of problem-solving process still seem to be not understood deeply, and still more knowledge is needed. There is a need to understand how certain behaviors occur during problem solving, how they go through interaction with other problem solving attributes. Moreover, Nickerson (1994) explains the need for problem solving as that although people participate in problem solving naturally and spontaneously, they may fail to succeed or they may not be able to solve the problems well enough. Nickerson (1994) also states that in the past, when the students weren't taught the problem solving strategies at all levels of formal education, they were not able to do the kind of thinking and problem solving that their school-work required. Moreover, most of the students could not write wholly satisfactory explanations, and they could not defend a point of view or their perceptions about the problem solutions effectively with a persuasive argument (Nickerson, 1994). Similarly, Özsoy (2006) explains the need for problem solving as the fact that mathematical knowledge and mathematical thinking are interrelated to each other, but they are different concepts. Mathematical knowledge is required to think and solve the problem, but it is not enough. Besides the mathematical knowledge, mathematical thinking is required to understand mathematics. In order to develop mathematical thinking, instruction should make use of problem solving activities. Similarly, Schraw and Dennison (1994) state that problem solving makes students feel ready for life problems, and provide them a feeling of satisfaction and a belief about usefulness of mathematics. Also, problem solving triggers students to transfer the knowledge they have constructed in the school to the real-life conditions and to the real-life problems (Writer, Jarrett, & Robert McIntosh Mathematics Associate, 2000). According to Turkish National Ministery of Education, the students encounter many problems in their daily lives; and problem solving involves the abilities which the students will need when solving the problems in their daily lives. Also, problem solving is a vital requirement for mathematics lesson. When students engage in problem solving, then their understanding, their mathematical knowledge and mathematical abilities will be more meaningful (MoNE, 2005). In addition, Higgins (1997) emphasizes those students who have taken problem-solving instruction showed greater perseverance in solving problems, more positive attitudes about the usefulness of mathematics and deeper mathematical understanding than the students who have taken traditional mathematics instruction.

1.2 Background Information Related to Metacognition

According to Schoenfeld (1985, 1992), in order to be a successful problem solver, some elements related to problem solving should be used effectively. In his problem solving framework, these elements are resources, problem solving strategies involving heuristics, control, and beliefs and affects. Resources refer to the knowledge base, and resources involve mathematical knowledge such as facts, concepts, algorithms, and routine procedures. In fact, mathematical knowledge alone is not enough to be a competent problem solver. To make the problem solver use his resources effectively, problem solving strategies should be used. So, problem solving strategies involving heuristics are also an important element. The third element is control, which is a part of metacognition. Metacognition refers to knowledge of one's own cognition, monitoring and controlling one's own cognitive processes, and reflection. Here, control refers to resource allocation, and determination of what resources will be useful, defining which strategies will provide effective solution. It involves deciding appropriate choices and monitoring his own progress throughout problem solving process (Schoenfeld, 1985 & 1992).

Similarly, Kilpatrick (1985) also emphasized the importance of resources, strategies, and control for problem solving. The researcher stated that to be a successful problem solver, a person should have organized domain knowledge, should know techniques for representation and transformation of the problem, and metacognitive process in order to monitor and guide his own performance. According to Lester, Garofalo and Lambdin (1989), mathematics teachers define the inability of students' problem solving as a big concern for generations. The teachers state that the students are very successful at the required computational skills and algorithmic procedures; but they have inability to solve even the easiest verbal problem mostly. The reason was thought to be cognitive aspects of performance at first. But recently, there is a growing body of knowledge that much broader is needed for mathematical problem solving performance. It is the metacognition, which has a close relationship with problem solving. Also, related to the need for metacognition, Gredler (2005) states that new improvements in various disciplines, the explosion of technological changes create the need for the capabilities of managing one's learning, and learning to solve new problems. This self-directed learning occurs throughout metacognition. Thus; in addition to cognition, metacognition is required for problem solving. Similarly, Schraw and Dennison (1994) state that metacognition has a crucial role in cognitive performance on problem solving process by creating an increase in the usage of problem solving strategies. In fact, metacognition, problem solving skills and performance are all important for students.

Nickerson (1994) states that students sometimes cannot integrate the relevant and previous knowledge in their minds during the problem solving process. In other words, they cannot connect the needed knowledge with previous knowledge required for solving a new problem. Students can cope with this situation throughout the usage of metacognition which will help students manage and control their cognitive resources more effectively (Nickerson, 1994). Also, Mayer (1998) states that good problem solvers firstly need to know the basic problem solving skills and some cognitive skills for the specific subject matters. But, these are not enough to have high problem solving performance. Besides, the ability to control and monitor cognitive processes, and to be aware of the knowledge of when to use, how to coordinate, and how to monitor a variety of skills in problem solving are needed. This property is the problem solver's metacognitive skills; thus, metacognitive skill is an essential part of problem solving process (Mayer, 1998). Also, according to Baker and Brown (1980), for effective learning to occur, it is essential for a person actively to monitor one's own cognitive activities. The researchers stated that metacognitive solvers having high metacognitive ability, performed significantly better than others on problem solving (Baker & Brown, 1980).

If we desire meaningful understanding of mathematics throughout problem solving, we have to emphasize the development of students' metacognition because of the strong relation between metacognition and problem solving ability (Berardi-Coletta, Buyer, Dominowski, & Rellinger, 1995). Evans and Swan (2014) stated that if students engage in reflections of their own decisions, and their own planning actions and solution in the mathematical problem solving process, and reflect upon their thinking in the process and to focus on working on ideas, rather than working through task, then, the students find opportunities to attend to metacognitive acts by thinking on alternative approaches and evaluating these different approaches to non-routine problems. Also, metacognitive acts involve the students' engagement in arguments or discussions, compare the effectiveness of arguments, and differentiate correct logic or reasoning and explain these, and critique the reasoning of their peers. Also, by discussions, participating in arguments of others, by comparing their own ideas with others, by determining what make sense and correct reasoning, by asking questions to critique, clarify and develop the arguments, students engage in metacognitive acts. Moreover, Schoenfeld (1983, 1985, 1987, and 1992) stated that expert problem solvers frequently attend metacognitive acts by looking back and reflecting upon the strategies, solutions they use during problem solving process. The experts monitor and reflect on their thinking by seeking answers to the questions about planning. These questions are: "is this correct way?" "Is there another different representation of the problem?" By asking such questions, the experts think about alternative approaches and different strategies. Also they choose different approaches depending on their previous experiences. However, the novice problem solvers often choose one approach and they become fixed on that approach. They follow that approach relentlessly, but sometimes unprofitably (Schoenfeld, 1983; 1985; 1987; 1992).

Similarly, Carlson and Bloom (2011) mention about attributes of problem solving success where they touch the relation between problem solving success and metacognitive skills. Another study done on this relation also indicates that high metacognitive levels are associated with best performance in problem-solving by Antonietti, Ignazi and Perego (2000). Furthermore, Özsoy (2006) states that problem solving increases metacognition; and metacognition increases problem solving performance too, since they have a mutual relationship. Also, Mayer (1998) claims that metacognitive skill is an essential part of problem solving process. In addition, metacognitive skills improve students' mathematical problem solving performance. Specifically, Özsoy and Ataman (2009) claim that the students who take metacognitive strategy training showed significantly higher mathematical problem solving achievement and performed significant increase in problem solving skills than the other students who are not trained within metacognition. Özsoy (2007) states that it is not enough for students to know computational skills and strategies in order to have higher mathematical problem solving performance. They need more consciousness to be successful at problem solving process. This consciousness is acquired by metacognition. Similarly, Demircioğlu (2008) states that problem solving is very important for students. For problem solving performances to be higher, metacognition is needed. Moreover, Stillman and Mevarech (2010) state that metacognition is an ideal field for research in mathematics education nowadays and continue to be investigated in variety for many years in the future. The relationship between the problem solving in mathematics education and metacognition is a growing research field nowadays. The relationship is so close that the critical aspects of metacognition involve the degree the students value problem solving and the degree they rate themselves as problem solvers.

1.3 Background Information Related to Reasoning Ability

Tobin and Capie (1981) state that there are two important trends in reasoning ability. First trend is that many adolescents and adults have a limited formal reasoning ability, and they use the formal modes of reasoning ability limitedly. The second trend is that formal reasoning ability is a vital mediator of cognitive achievement. As a result of these two trends, most of the researchers agree on the emphasis that there is a great need for modification of instructional objectives, materials and activities. These modifications should be made according to the cognitive development level of students or learners. For this, primacy should be provided to develop the formal reasoning ability of middle and high school students, by preparing appropriate curriculum materials (Tobin & Capie, 1981).

Evans (2000) stated that although long time has passed and many changes took place in mathematics education area, in general, many commentators, researchers, and professionals still claim traditional view of ‘mathematical ability’ in the educational world. This traditional view concluded in a thought that mathematics requires only a set of abstract cognitive skills, which are performed in a variety of tasks and practical contexts. This thought concluded in a perception of relatively straightforward process of transfer of knowledge. Then, the transfer of learning and knowledge has gained vital importance in formal educational system. In this traditional view, performance was measured by the number of correct answers in test items. That concluded in rote learning rather than real understanding. In fact, English (1997) states that there is an ongoing, challenging and fascinating issue in mathematics education. This issue is how the learners reason with mathematical experiences and ideas. The issue has gained more importance in recent years due to the developments in cognitive science. Cognitive science is related to a variety of disciplines, thus it proposes rich scope for fundamental issues for mathematical learning. One of these challenging issues is how the learners form mental structures for their mathematical ideas, experiences; and how they reason with these mental structures to learn and solve problems (English, 1997).

As the time passes, the view of mathematical teaching has required a reform. The teachers are encouraged to stop teaching mathematics as a mechanical way;

instead, they are advised to start teaching mathematics based on problem solving, understanding and mathematical communication. With this reform, the teachers are advised to create a learning environment, in which students are encouraged to understand mathematics deeply, to discover mathematical ideas, and to construct relationships between mathematical ideas and daily life (McKenzie, 2001). In order to perceive mathematics as dynamic rather than static, the students should deal with activities which will encourage them to make conjectures, to search for patterns, to discover knowledge, to explain situations, to justify his ideas and to challenge his ideas (Stein et al. 1996).

Steen (1999) states that the aims of mathematics lesson involve teaching basic mathematical skills and logical thinking skills, guiding students to be productive throughout life and for work, and having literate people for future generations. In fact, mathematical reasoning is thought to develop the aims of mathematics. But to explain the relationship between the two is more problematic. In fact, reasoning is fundamental for mathematics, since mathematics is based on logic (Steen, 1999). Stenberg (1980) stated that reasoning, problem solving and intelligence have very close relationship with each other, so that it is often difficult to differentiate them. An arithmetic word problem firstly requires "problem solving" for solution. The problem also requires "reasoning". The same problem also requires "intelligence". The same close relationship between three constructs take place in most of the problems. In fact, problem solving seems to require reasoning, and reasoning seems to require problem solving (p.4). In fact, reasoning refers to combining elements of old information in order to form new information (Stenberg, 1980).

Nickerson (1994) states that the most generally accepted term which is closely related to thinking and problem solving is reasoning and decision making. In fact, mathematics is based on justifications. Epistemologically, knowledge requires a logical basis, a logical explanation and justifications. If a person can explain a situation, then he can construct understanding of knowledge on a strong basis, and then he can justify this knowledge (Johnston, 2002). According to Mueller and Maher (1996), it is commonly accepted that reasoning and also proof are fundamental for mathematical understanding. For a student to be able to learn reasoning and justification of his reasoning is vital for his mathematical knowledge growth (Mueller & Maher, 2009).

Ball and Bass (2003) defined mathematical reasoning as a fundamental part of mathematical skills, and claimed that mathematical understanding is based on reasoning. Reasoning ability is the basis for learning new mathematics; and also, the ability to reason is vital for integrating the previous mathematical knowledge to new situations. Reasoning is a process in which a person revisits and reconstructs the previous knowledge for the aim of building new arguments; when needed to construct new knowledge. Then, reasoning ability concludes in one's growth of new knowledge. That means, reasoning ability is vital for a person to build new knowledge (Ball & Bass, 2003). Mathematical reasoning is fundamental for mathematical understanding (McKenzie, 2000; Mueller & Maher, 2009). In order to understand mathematics, a person has to reason mathematically, this is why reasoning is very important for a person to construct mathematical knowledge. When a person reasons mathematically, he/she can use mathematical ideas in new conditions, and this leads to improvement in problem solving skills (Mueller &Maher, 2009).

According to Schoenfeld (1992), "in the problem solving process, a student should provide his own mathematical point of view or mathematical thinking based on basic mathematical knowledge and abstraction, or mathematization. Also student needs to apply his mathematical thinking by the help of tools of the trade which will be used for understanding the situation" (Schoenfeld, 1992, p.335). Thus, "when a student tries to solve a problem, he needs both basic mathematical knowledge and mathematical thinking which involves reasoning" (NCTM, 1991; MoNE, 2005, p.14). In a problem situation, the students cannot directly find the solution; they cannot find any obvious strategy for solution easily. In order to solve the problem, the students should use reasoning and mathematical thinking. Moreover; rather than finding the correct answer of the problem, the solution process of the problem situation have much more importance. “The solution process involves how a student approaches to the problem situation, which strategies he chooses for solution, and why he chooses this strategy, what he thinks during solution process, and which representations and contributions he makes for the solution” (MoNE, 2005, p.14-15).

Similarly, Bitner (1991) stated that five formal operational reasoning modes and critical thinking skills are the vital abilities for the success in secondary school science and mathematics courses. Also, formal operational reasoning modes are significant predictors of science and mathematics achievement. In fact, thinking processes develop throughout both declarative and procedural knowledge. So, educational settings must have a central focus of both factual knowledge and thinking processes. Since the reasoning ability is a significant predictor of mathematics achievement, instructional approaches should focus on not only declarative knowledge but also on procedural knowledge (Bitner, 1991). Similarly, Hiebert (1994) claims that conceptual knowledge should be developed with procedural knowledge. If the students apply procedures without reasoning and sense making, they tend to forget these procedures, and they cannot understand the logic or rationale of these procedures. (Pesek & Kirshner, 2000).

Mueller, Yankelewitz, and Maher (2011) state that motivation and positive dispositions toward mathematics conclude in mathematical reasoning, and then this situation concludes in understanding. The students engage in and trust in their reasoning, instead of memorized facts, or solutions of other students. Based on their reasoning, the students persuade themselves and other students about the issues that make sense. This reasoning process concludes in mathematical understanding. If a student engages in mathematical reasoning then that students get conceptual understanding (Mueller, Yankelewitz, & Maher, 2011). Mueller and Maher (2009) state that if students engage in an environment in which they explore, collaborate with each other, and defend their thinking and justify their reasoning in both small and large groups, then they develop reasoning and mathematical understanding. In a community of learners, attending in discussions, making and refuting claims, and justifying reasoning related to mathematical ideas conclude in mathematical reasoning. If the students are involved in these processes, then they are also involved in mathematical reasoning (Brodie, 2000).

According to Longman (1987), reasoning is the ability of thinking, understanding, and developing opinions, and providing judgments based on facts. Reasoning is a process in which a person forms his own opinions, judgments or makes inferences based on facts, or on a body of information. Also, reasoning may be defined as the correlation or integration of problem solving and communication (Kelly, Myllis, & Martin, 2000). Reasoning ability is formal thought or intellectual abilities of students and it refers to the stages during the thinking process (Gerber, Marek & Cavallo, 1997).

Martin and Kasmer (2010) claim that reasoning requires a person to form conclusions based on the evidence, facts or assumptions. Reasoning plays a vital and particular role in mathematics. Reasoning requires logical deduction, also formal reasoning and proof in mathematics. Moreover, it requires informal reasoning or observations, conjectures and logical explanations. The students should start development of mathematical reasoning at lower grades, and then they will understand mathematics more easily in higher grades, as reasoning is an important part in mathematics (Martin & Kasmer, 2010). Similarly, reasoning in mathematics requires a person to formulate the problem, and represent the problem, and to provide explanations about the argument in the problem, to explain and provide justifications for the solution of the problem in mathematics (Kilpatrick, Swafford & Findell, 2001). Reasoning in mathematics includes learning what the problem is, what constitutes the truth, what is correct and valid in a mathematics conjecture, also providing an explanation for the result, providing justifications to prove that the result is correct, learning and explaining why the conjecture is correct (Brodie, 2000).

Similarly, Cavallo (1996) stated that the students' reasoning ability and meaningful understanding are very important in problem solving process and in integrating the ideas. Meaningful learning and/or reasoning ability are important for overall learning in the classrooms. Both the meaningful learning and reasoning ability should be improved as much as possible, to the fullest extent, in order to maximize students' learning and understanding. Also, Mansi (2003) stated that reasoning should be emphasized throughout school mathematics. The reasoning ability is required in mathematical reasoning and proof, which is vital in mathematics (Mansi, 2003). In fact, reasoning is very important in mathematics because children's ability to learn and understand mathematics are based on reasoning. The focus and central aims for all grades should be to discover mathematical ideas, to provide conjectures, to create conclusions and to reach generalizations; rather than to focus on memorization of procedures, formulas or algorithms. Then the students will reach meaningful mathematical knowledge and apply this knowledge in different contexts. Then this will trigger their natural curiosity and will create motivation to learn more. All of this fundamental conceptual knowledge will be created by only reasoning and sense making (NCTM, 2010). Mansi (2003) stated that mathematical reasoning is the ability required in coherent and logical thinking, and making inferences or providing and forming conclusions from mathematical facts. Reasoning ability is a powerful and essential part of learning mathematics. By the help of reasoning ability, the students reason about mathematical ideas, make conjectures and connections, provide justifications and explanations about why a mathematical idea or concept make sense, or why a procedure or formula can be applied in a situation. Students should improve their reasoning and justification abilities during their mathematics learning process (Mansi, 2003).

Also, according to Nunes, Bryant, Barros and Sylva (2012), mathematical reasoning and arithmetic skills have important effect on mathematical achievement. However, there are not too many studies to support the theory and provide evidence for educational practice about this issue. So, the researchers state that mathematical reasoning is different from arithmetic skills and, development of mathematical reasoning should have importance in school curriculums. There is another reason to emphasize reasoning ability; according to Coletta, Philips and Steinert (2007), the teachers want to evaluate and improve their instructions, and compare their courses with other courses. For this, reasoning ability of the students should be assessed in order to make healthy comparisons. Also, assessing reasoning ability of students helps to identify the students who are at risk. Also, Kramarski, Mevarech and Lieberman (2001) stated that there is a direct relationship between reasoning skills and success in mathematics. Students who show better reasoning skills have good problem-solving characteristics. Also, these students define the interrelationships more, and have better communication skills.

Similarly, Gunhan (2014) states that in school curriculum, reasoning skills should be emphasized more. Especially for geometrical concepts, reasoning skills is very important. So, the teachers should provide problems which will develop students' reasoning skills. For this, the students should be provided activities in which students are encouraged to reflect their knowledge, to make logical arguments and thus to use reasoning skills more. Similarly, Battista (2007) stated that in order to present meaningful education to students, educators and teachers firstly need to understand the thinking processes that the students engage in. For meaningful understanding and to improve students' reasoning skills, it is required that conceptual understanding should be emphasized, rather than just providing procedural knowledge. Also, Usman and Musa (2013) stated that use of the thinking and reasoning patterns are very important for their mathematical performance. The teachers should measure students' formal operation levels and trigger students to use formal operation abilities in order to improve students' mathematical performance.

To conclude, as seen in the literature; problem solving, metacognition and reasoning ability are very important for students. So, it is worth to search the relationship among metacognition, reasoning ability and problem solving.

1.4 Purpose of the Study

The purpose of this research is to investigate the relationship among metacognition, reasoning ability, and problem solving performance of ninth grade students in Anatolian high schools in İzmir.

1.4.1 Research Problem

The problem is whether there is a relationship among metacognition, reasoning ability and problem solving performance of ninth grade students in İzmir.

1.4.2 The Hypotheses of the Study

The Null Hypothesis H_0 : There is no relationship between metacognition and problem solving performance.

The Null Hypothesis H_0 : There is no relationship between metacognition and reasoning ability.

The Null Hypothesis H_0 : There is no relationship between reasoning ability and problem solving performance.

The Null Hypothesis H_0 : Metacognition and reasoning ability do not predict the variability in mathematical problem solving performance.

1.4.3 The Research Questions

1) Is there a relationship between metacognition and problem solving performance of ninth grade students?
2) Is there a relationship between reasoning ability and problem solving performance of ninth grade students?
3) Is there a relationship between metacognition and reasoning ability of ninth grade students?
4) How much variance in problem solving performance scores can be explained by reasoning ability and metacognition scores?
5) Which variable in the set of variables (reasoning ability and metacognition) is the best predictor of problem solving performance?

1.5 Definitions

1.5.1 Metacognition

According to Brown (1978), development of metacognitive skills indicates efficient problem solving in various situations such as experimental, educational or in natural settings. Knowledge itself and the understanding of that knowledge are different from each other, and this distinction is very important in cognitive development. The cognitive development of children occurs through executive processes. The executive processes of modern cognitive theory are predicting, planning, checking and monitoring. These processes are vital characteristics of efficient thinking in various learning situations. Also, according to Flavell (1976), mainly metacognition is “thinking about one’s own thinking” and metacognitive knowledge is the knowledge about oneself, the task and the strategy. Metacognition requires the awareness about what a person knows, what he can do, and what he knows about his own cognition (Flavell, 1979). Lester, Garofalo and Lambdin (1989) explain metacognition as the individual’s knowledge and control of his own cognitive functioning. Metacognition requires a person to know about his cognitive performance, and to know how to regulate his cognitive actions during the task performance. Similarly, Schraw and Dennison (1994) define metacognition as the ability to reflect upon one's own learning, understand it and control his-her learning. Driscoll (2005) defines metacognition as a capability to be aware of one's own thinking and learning process.

Schraw and Dennison (1994) state that "metacognition includes two major components: knowledge of cognition and regulation of cognition. Knowledge of cognition includes three subprocesses: declarative knowledge (knowledge about self and strategies), procedural knowledge (knowledge about how to use strategies), conditional knowledge (knowledge about why and when to use strategies). Regulation of cognition includes planning, information management strategies, comprehension monitoring, debugging strategies, and evaluation" (p.460).

According to Sperling, Howard, Miller and Murphy (2002), in general, research which investigates children's metacognition emphasizes one of two frameworks despite of the other conceptions in the literature (e.g., Nelson & Narnes, 1996). The first framework, created by Flavell (Flavell, 1979; Flavell, Miller, & Miller, 1993), states that metacognition has two components: metacognitive knowledge and metacognitive experiences. Metacognitive knowledge involves task, person, and strategy components. Metacognitive experiences involve feelings of understanding and are useful for strategy selection and application (Flavell, 1979). Later, Flavell and colleagues called them as metacognitive monitoring and self-regulation. The second framework created by Brown (1978) and improved later (Baker & Brown, 1984) presents two components: knowledge of cognition and regulation of cognition. The knowledge of cognition component involves declarative, procedural, and conditional knowledge of cognition. The regulation of cognition component involves constructs such as planning, monitoring, and evaluation. This study emphasizes the Brown framework of metacognition as the theoretical foundation, and measures metacognition with a scale which was based on this foundation. In the current study, metacognition was measured with junior metacognitive awareness instrument of Sperling, Howard, Miller and Murphy (2002), which was based on Brown framework of metacognition.

Operational Definition of Metacognition:

In the current study, metacognition was measured by an instrument; namely the Junior Metacognitive Awareness Inventory (Sperling, Howard, Miller & Murphy, 2002). That instrument was based on Brown (1978) framework of metacognition. According to Brown (1978) metacognition involves two components: knowledge of cognition and regulation of cognition. The knowledge of cognition includes declarative, procedural, and conditional knowledge of cognition. The regulation of cognition includes constructs such as planning, monitoring, and evaluation.

1.5.2 Problem Solving

Problem solving is a process "to search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable aim" (Polya, 1962, p.117). For a mathematician, problem solving refers to a mathematical situation in which the solution is required; but not known. Moreover, there is no direct route or clear pathway to the solution (Polya, 1962). Problem solving is referred to an "extremely complex form of human endeavor that involves much more than the simple recall of facts or the application of well-learned procedures" (Lester 1994, p. 668). Similarly, problem is identified as "A problem is a situation that confronts a person, that requires resolution, and for which the path to the solution is not immediately known." (Krulik & Posamentier, 1998, p.1). In fact, problem is not a drill or is not a routine exercise. Problem differs from routine exercises or drilling questions. In routine exercises and drilling questions, the students already know the solution strategy, or specific solution procedures, or they only require computational skills. But in a problem, there is a challenge and it requires common knowledge, and the solution strategy is not already known (Krulik & Posamentier, 1998; Krulik & Rudnick, 1987; MoNE, 2005). Also, according to Gredler (2005); in general, problem solving refers to trying to accomplish the new and unfamiliar tasks when the person does not know the relevant solution methods. Anderson (1980) states that problem solving process is the series of cognitive operations which are held in a goal-directed manner (as cited in Jonassen, 2000). Similarly, according to Schoenfeld (1992), problem solving refers to a process, in which students act in a question, but they do not have an immediate and apparent solution for this question. Also, they do not foresee an immediate and clear algorithm or procedure to apply for the solution. Also, Jonassen (2000) states that a problem has two characteristics. Firstly, a problem should present an unknown entity meaning the difference between a goal situation and a current situation. Secondly, solving the unknown entity should have a social, cultural or intellectual value and to find the unknown entity should be worthwhile. Then, finding the unknown is the act of problem solving process. Also, in order to be a good problem solver, students are required to choose and apply the correct or appropriate cognitive strategies for tasks. Then they will be able to understand the task or the problem, represent the task, and solve the problems (Mayer, 1998; Schoenfeld, 1985). Similarly, Lester (1994) states that good problem solvers know their strengths and weaknesses related to problem solving more than the poor problem solvers. Also, good problem solvers monitor and regulate their problem solving efforts better. Moreover, good problem solvers tend to achieve sophisticated solutions to problems more than poor solvers throughout the problem solving steps. Mainly, the most known problem solving steps belong to Polya (1945); and he intended to provide these steps as a prescription of how the problem solver should proceed. These steps are: "Understanding the problem, Devising a plan, Carrying out the plan, and Looking back" (Polya, 1945).

Schoenfeld (1992) states that in all research area about problem solving, every researcher should provide his own operational definition of problem solving term. In fact, when combining of these definitions about problem solving, it can be concluded that problem solving is a situation in which there is a problematic situation, but the solution is not seen immediately. Problem solving is an act of trying to turn the unknown situation into known situation. So, in this study, the operational definition of problem solving is that problem solving is a situation in which a person confronts with an unknown situation and tries to turn the unknown into known situation throughout a series of cognitive or mental, logical or formal reasoning thinking, and metacognitive processes.

Operational Definition of Mathematical Problem Solving Performance:

In the current study, problem solving performance is measured. Problem solving performance refers to the students' scores when they are solving problems in Problem Solving Performance Test developed by Taşpınar (2011). In fact, problem solving performance is the extent to which a problem solver reaches the solution of the problem correctly (Antonietti, Ignazi & Perego, 2000).

1.5.3 Reasoning Ability

According to Longman (1987), reasoning is the ability of thinking, understanding, and developing opinions, and providing judgments based on facts. Reasoning is a process in which a person forms his own opinions, judgments or makes inferences based on facts, or on a body of information. Also, reasoning may be defined as the correlation or integration of problem solving and communication (Kelly, Myllis, and Martin, 2000). Reasoning ability is formal thought or intellectual abilities of students and it refers to the stages during the thinking process (Gerber, Marek & Cavallo, 1997).

According to Lawson (1982), "a person with high formal reasoning operation shows five reasoning modes: Controlling variables, Proportional reasoning, Probabilistic reasoning, Correlational reasoning and Combinatorial reasoning" (Lawson, 1982).

In the current study, in order to measure reasoning ability, Test of Logical Thinking instrument which was developed by Tobin and Capie (1981) was used. The instrument was developed to measure "students' formal reasoning ability that would require students to solve problems and to justify the solutions" (Tobin & Capie, 1981, p.414). The instrument was based on the framework of Lawson (1978) and it was a different version of Lawson's instrument, and a selection of ten items previously reported by Lawson (1978). In the current study, reasoning ability was measured throughout five reasoning modes: "controlling variables, proportional reasoning, probabilistic reasoning, correlational reasoning and combinatorial reasoning".

Operational Definition of Reasoning Ability:

In the current study reasoning ability is measured throughout five reasoning modes: controlling variables, proportional reasoning, probabilistic reasoning, correlational reasoning and combinatorial reasoning (Tobin & Capie, 1981).

1.6 Significance of the Study

For decades, problem solving has gained great importance and still continues to be vital for mathematics education (Evans, 2012; Hembree, 1992; Posamentier & Krulik, 2008). But professionals, mathematicians or researchers of mathematics education developed many different definitions of problem and problem solving; and there seems to be no common definition (Donaldson, 2011; Ellis, 2005; Grugnetti & Jaquet, 2005). Also, most of the previous research seems to be lack of a well-articulated or universally accepted theory (Grugnetti & Jaquet, 2005; Nickerson, 1994). Moreover, Lester and Kehle (2003) stated that little development has occurred in problem solving research from 1980 to 2003, and also the literature on problem solving provided little offerings to school practice. Similarly, Lesh and Zawojewski (2007) claim that mathematical problem solving research area seems to be lack of theoretical base. So, there is a great need for better theorizing in the field. For this, in order to better understand thinking and problem solving, more precise, more predictive, more comprehensive and testable theories should be produced and tested. Also, from an epistemological point of view, we should define a variety of relationships with problem solving. For this, more studies should be conducted related to problem solving (Donaldson, 2011; Ellis, 2005; Grugnetti & Jaquet, 2005; Lesh & Zawojewski, 2007; Lester & Kehle, 2003; Nickerson, 1994). Since there is need for further research about problem solving, the current study aims to provide a contribution to fill this gap. In the current study, it is aimed to provide a support for a network of correlations among metacognition, reasoning ability and mathematical problem solving.

In order to emphasize the importance of problem solving, decades ago, Polya (1973) stated that the teachers should improve students' abilities to think and solve problems. Decades later, Evans (2012) stated that strong problem solving abilities and skills are vital for mathematics and for daily life in general. So, the students should be provided critical thinking and strong problem solving preparation in schools. Also, according to Krulik and Posamentier (1998), the teachers should involve problem solving as an essential part in their regular curriculum; they need to focus on what problem solving is, how problem solving can be used in order to teach mathematical skills effectively, and how problem solving can be presented to students in an effective way. (Krulik & Posamentier, 1998). Since problem solving protects its importance upto now, it is still important to emphasize problem solving in the schools. So, the current study aims to gain attention to emphasize the importance of problem solving.

Mayer (1998) states that both the cognitive skills for the specific subject matters, and also the ability to control and monitor cognitive processes are needed to have high problem solving performance. Also, both the computational skills and strategies, and also consciousness about problem solving process are needed in order to have higher mathematical problem solving performance (Demircioğlu, 2008; Özsoy, 2007). Moreover, Stillman and Mevarech (2010) state that metacognition is an ideal field for research in mathematics education nowadays and continue to be investigated in variety for many years in the future. The relationship between the problem solving in mathematics education and metacognition is a growing research field nowadays. Since there is an important relationship between metacognition and problem solving; and the research area is important to search, in the current study that relationship is investigated. The current study aims to remind and emphasize the importance of metacognition for mathematical problem solving of students.

Reasoning is fundamental for mathematical understanding (McKenzie, 2000; Mueller & Maher, 2009). Similarly, Mansi (2003) stated that reasoning ability is a powerful and essential requirement for learning mathematics. Also, Ball and Bass (2003) define reasoning as a fundamental part of mathematical skills, and claim that mathematical understanding is based on reasoning. Reasoning ability is the basis for learning new mathematics; and is required for one's growth of new knowledge (Ball & Bass, 2003). In order to understand mathematics, a person has to reason mathematically, this is why reasoning is very important for a person to construct mathematical knowledge. When a person reasons mathematically, he can use mathematical ideas in new conditions, and this leads to improvement in problem solving skills (Mueller &Maher, 2009). Stenberg (1980) stated that reasoning, problem solving and intelligence have very close relationship with each other. An arithmetic word problem firstly requires "problem solving" for solution. The problem also requires "reasoning". The same problem also requires "intelligence". In fact, problem solving seems to require reasoning, and reasoning seems to require problem solving (p.4). In fact, reasoning refers to combining elements of old information in order to form new information (Stenberg, 1980). Similarly, Cavallo (1996) stated that the students' reasoning ability and meaningful understanding are very important in problem solving process and in integrating the ideas. Meaningful learning and/or reasoning ability are important for overall learning in the classrooms. Both the meaningful learning and reasoning ability should be improved as much as possible, to the fullest extent, in order to maximize students' learning and understanding. Also, Mansi (2003) stated that reasoning should be emphasized throughout school mathematics. The reasoning ability is required in mathematical reasoning and proof, which is vital in mathematics (Mansi, 2003). Similarly, according to Nunes, Bryant, Barros and Sylva (2012), mathematical reasoning and arithmetic skills have important effect on mathematical achievement. However, there are not too many studies to support the theory and provide evidence for educational practice about this issue. So, the researchers state that mathematical reasoning is different from arithmetic skills and, development of mathematical reasoning should have importance in school curriculums. There is another reason to emphasize reasoning ability; similarly, Bitner (1991) stated that formal operational reasoning modes are significant predictors of science and mathematics achievement, and are the vital abilities for the success in secondary school science and mathematics courses (Bitner, 1991). Also, Kramarski, Mevarech and Lieberman (2001) stated that there is a direct relationship between reasoning skills and success in mathematics. Students who show better reasoning skills have good problem-solving characteristics. Similarly, Gunhan (2014) states that in school curriculum, reasoning skills should be emphasized more. So, the teachers should provide problems which will develop students' reasoning skills. Similarly, Battista (2007) states that in order to present meaningful education to students, educators and teachers firstly need to understand the thinking processes that the students engage in, and to improve students' reasoning skills. Also, Usman and Musa (2013) state that use of the thinking and reasoning patterns are very important for their mathematical performance. The teachers should measure students' formal operation levels and trigger students to use formal operation abilities in order to improve students' mathematical performance. As the previous research support the importance of reasoning ability on mathematical problem solving, it is aimed to search the relationship between reasoning ability and problem solving. In the current study, it is aimed to draw attention to the importance of reasoning ability on problem solving and to the relationship between them.

As the previous research support; metacognition, reasoning ability and problem solving have relationships with each other, and all of them are important for mathematical success of students. In this study, ninth grade students are chosen as the target population. Because, in order to provide useful studies for the success of students, it is important to study with students directly. The grade of the participants is nine, because younger students show less metacognitive behaviors to measure. Younger children are not aware of their metacognitive behaviors in general (Gredler, 2005). In order to get successfully measured metacognition scores, it is important to choose higher grades. Also, the eighth grade students have a national exam, namely TEOG. So, it would be difficult to conduct instruments which take two lesson hours to eight grade students. Both the students and the teachers may be unvolunteer to participate to instruments. In order to reach higher number of participants, ninth grade students were chosen. Also, all of the instruments also are appropriate for ninth grade.

The main aim of this study is to understand the relationship among metacognition, reasoning ability and the mathematical problem solving performance of the ninth grade students. In the previous studies, the researchers generally select two of the variables; such as examining metacognition and reasoning ability; or examining metacognition and problem solving performance; or reasoning ability and problem solving performance. In contrast, in this study, these three variables; namely metacognition, reasoning ability and problem solving performance will be examined in one study. Also, in the previous studies, the correlational studies are not very common about the problem solving, reasoning and metacognition; rather, experimental designs or other designs are more common. As well as manipulation of metacognition or reasoning ability on problem solving, it is important to examine the relationship in its nature; without any intervention. So, the correlational studies are important and there is a need for the correlational study of metacognition, reasoning ability and problem solving. So, in this study, it is expected that there will be a correlation among metacognition, reasoning ability and problem solving performance as expected from the previous studies (Antonietti, Ignazi & Perego, 2000; Berardi-Coletta, Buyer, Dominowski & Rellinger, 1995; Carlson & Bloom, 2011; Higgins, 1997; Ozsoy & Ataman, 2009). The contribution of the study to the literature is that the current study examines three variables in one study; and in a correlational design. In the previous studies, the researchers generally select two of the variables; but in this study, these three variables will be examined in one study on ninth grade elementary students; which is not generally chosen by the researchers who are interested in metacognition or problem solving or reasoning ability.

Moreover, most of the studies related to metacognition or problem solving or reasoning ability, which were conducted in Turkey choose pre-service teachers or teachers (Arkan, 2011; Başaran, 2011; Çakır, 2011; Demircioğlu, 2008; Gülşen, 2012; Kasımoğlu, 2013; Kayan, 2007; Kışkır, 2011; Obay, 2009; Oğraş, 2011; Polat, 2009; Topçu, 2008). Also; in Turkey, most of the studies related to research area choose fifth grade (Özsoy, 2002 and Özsoy, 2007; Yılmaz, 2009), sixth grade (Karaoğlan, 2009; Kılıç, 2005; Yayan, 2010; Yıldız, 2008), seventh grade (Başol, 2015; Yıldız, 2008; Yılmaz, 2003); or eighth grade students as participants (Akçam, 2012; Aşık, 2009; Azak, 2014). There seems to be a few study, which is about metacognition or problem solving or reasoning ability, choose ninth grade students (Aydoğdu, 2014; Özalkan, 2010; Yavuz 2006). Also, most of the studies related to reasoning ability are about the elementary science education (Araz, 2007; Başer, 2007; Kılıç, 2009; Korkmaz, 2005; Soylu, 2006; Yenilmez, 2006). Moreover, in Turkey, it seems that there are not too many research which study the relationship between the metacognition and problem solving (Özsoy, 2007). The current study is somehow different from prior studies because the author measured all three variables in one study and explained the relationship among the three variables, in Turkey. This study may provide educators and researcher a triangular relationship about problem solving process. In fact, there are many previous studies which investigate meta-cognition, but since the metacognition is interrelated to too many other concepts: self-regulation, daily life problem solving, motivation, psychology etc., the previous studies seem to be not enough to explain the relationship among metacognitive skills, reasoning ability and mathematical problem solving performance of students. This is why there is a need to search all of these variables in one study. Although many researchers searched the metacognition, reasoning ability and problem solving independently or in couples, it seems that no prior researchers in Turkey tried to investigate the relationships among these three variables in one study. So, the current study seems to have importance for mathematics education, and to provide important contributions to the research area. So, it is important to add a new study to Turkish social sciences literature since it investigates the relationship among metacognition, reasoning ability and problem solving performance of ninth grade students, which seems to be not studied so far in Turkey. Also, for the world-wide literature, it is important to search the three variables in one study: metacognition, reasoning ability and problem solving performance of ninth grade students.

In addition, the current study aims to provide a support for the network of relationships among metacognition, reasoning ability and mathematical problem solving performance of the students. Based on the findings of the current study, it is expected and aimed to provide an emphasis on the importance of metacognition, reasoning ability and problem solving in mathematics education. Thus, the current study may contribute to the body of research that curriculum developers, professionals, educators, and teachers can benefit in designing materials, in developing curriculum, in creating classroom culture, in designing lessons. The current study provides advises for better problem solving performance of students for the contribution to the research area. Based on the findings of the current study, it is aimed to emphasize that metacognition courses or lessons which explain metacognition construct should be provided in elementary and in high schools. Metacognitive education or courses explaining metacognition construct for mathematics lessons can be designed and taught to students at all grades. In addition, lessons or courses involving metacognition construct should be provided to pre-service teachers or education faculty students at both graduate and undergraduate level. Also, the study proposes that reasoning ability should be emphasized more in the mathematics curriculums and can be the central focus at mathematics lessons. Reasoning ability should be developed throughout newly designed materials, books, activities and as a part of mathematics curriculum. Also, another advice should be the emphasis on the importance of problem solving in mathematics. Problem solving should have more importance in classrooms, and should be developed by using problem solving steps during problem solving process in the classrooms. All of these variables can also be emphasized in all departments related to mathematics or educational sciences in universities. Thus, the pre-service teachers can both know about metacognition, how to teach metacognition to students, and how to teach mathematics in metacognitive education. Also the pre-service teachers should be more aware of the problem solving steps, and they can focus on these steps more in their lessons when they become in-service teachers. Finally, they should focus on reasoning more in their prospective mathematics lessons. When the pre-service teachers learn about the importance of metacognition, reasoning ability and problem solving in their education faculty lessons, then they can emphasize these constructs in their classrooms more. Then the students can understand mathematics more meaningfully and can ease the difficulties of mathematics.

CHAPTER 2

LITERATURE REVIEW

In this chapter; firstly, metacognition, problem solving skills, and reasoning ability will be explained. Then the relationship among metacognition, problem solving and reasoning ability will be claimed. Later, previous studies which investigate the relationship among metacognition, problem solving and reasoning ability will be provided. Finally, implications of the study will be mentioned.

2.1 Background Information and Theoretical Framework

2.1.1 Metacognition

According to Gredler (2005), significant enlargement of knowledge in various disciplines, the explosion of technology into daily life, and technological changes put new demands on people. These new demands create the need for self-directed learning which is acquired by metacognition. This increases the importance of the capabilities of managing one's learning, and learning to solve new problems. Thus, in addition to cognition, metacognition is created (Gredler, 2005). Flavell, who is the founder of metacognition theory, first started with the term metamemory. Flavell (1975) used the term metamemory to refer to a person's skills for management and monitoring the input, storing, searching and retrieval of the contents in his memory. Later, Flavell identified metacognition firstly in 1976, stating that metacognition consists of both monitoring and regulation aspects.

Flavell (1976) exactly explained metacognition as follows:

> In any kind of cognitive transaction with the human or non-human environment, a variety of information processing activities may go on. Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these

> processes in relation to the cognitive objects or data on which they bear, usually in service of some concrete goal or objective. (p. 232).

Then, in 1979, Flavell emphasized the relationship between metacognition and other areas: oral skills of communication, persuasion and comprehension, reading, writing, language acquisition, memory, attention, problem-solving, social cognition, affective monitoring, and self-instruction.

According to Brown (1978), development of metacognitive skills indicates efficient problem solving in various situations such as experimental, educational or in natural settings. Knowledge itself and the understanding of that knowledge are different from each other, and this distinction is very important in cognitive development. The cognitive development of children occurs through executive processes. The executive processes of modern cognitive theory are predicting, planning, checking and monitoring. These processes are vital characteristics of efficient thinking in various learning situations. Mainly, Flavell (1976) states that metacognition is "thinking about thinking", and metacognitive knowledge is the knowledge about oneself, the task and the strategy. The researcher also explains metacognition as to be aware of how a person learns, to be able to evaluate the difficulty of the task, to monitor his own understanding, to use the information needed to reach a goal, and to assess his learning progress. Metacognition requires the awareness about what a person knows, what he can do, and what he knows about his own cognition (Flavell, 1979).

Lester, Garofalo and Lambdin (1989) define metacognition as the individual's knowledge and control of his own cognitive functioning. Metacognition requires a person to know about his cognitive performance, and to know how to regulate his cognitive actions during the task performance. Similarly, Berardi-Coletta, Buyer, Dominowski and Rellinger (1995) state that metacognitive processes trigger students to use self-observation without the hindrance of negative self-evaluation and becoming aware of what one is doing and why one is doing so. This results in learning how to learn, and in turn concludes in metacognition. Also, according to Hofstadter (1979), in the metacognitive process, one jumps out of the system and observe it, and Kluwe (1982) describes metacognition as an active, reflective process that is explicitly and exclusively directed at one's own cognitive activity (as cited in Berardi-Coletta, Buyer, Dominowski and Rellinger, 1995, p.206). Similarly, Driscoll (2005) defines metacognition as a capability to be aware of one's own thinking and learning process. Moreover, Gagne and Glaser (1987) explain metacognition as a kind of regulatory performance during learning or problem solving. Metacognition refers to knowing when or what a person knows or does not know; guessing the correctness or the results of his-her own cognitive resources and time; and controlling and monitoring the results of his-her solution or an attempt to learn (Gagne & Glaser, 1987 as cited in Driscoll, 2005).

According to Baker and Brown (1980), metacognition refers to "the knowledge and control over one's own thinking and learning activities. There are two clusters of activities in metacognition: knowledge about cognition and regulation of cognition. The first cluster knowledge of cognition involves a person's knowledge about his own cognitive resources. The second cluster of activities involves self-regulatory mechanisms that an active learner performs during problem solving process. These activities are checking the outcome, planning the next move, monitoring the effectiveness of the actions, testing, revising and evaluating the strategies for learning". For effective learning to occur, it is essential for a person actively to monitor one's own cognitive activities. A third concern about metacognition is the development and use of compensatory strategies. If a person has awareness of his own cognitive processes and monitors his progress well, then what type of remedial activities that person performs in order to solve the problem is the compensatory strategy. These strategies change according to the goal of the activity.

Gredler (2005) claims that in general, metacognition includes thinking about thinking, focusing on knowledge and regulation of cognition. In simplest form, "metacognition is individuals' knowledge about cognition and strategy use". Thus, there are two components of metacognition. The first one is the knowledge about and awareness of ones' own thinking; which involves the information about one's own capabilities and limitations, as well as being aware of the difficulties rising during learning. The other one is the knowledge of when and where to use the required strategies; that involves knowing of which particular goal-specific strategies are appropriate for the specific tasks and situations (Gredler, 2005). In a similar way, Schraw and Dennison (1994) claim that metacognition has two important components: "metacognitive knowledge which refers to knowledge of cognition; and metacognitive skillfulness which refers to regulation of cognition. According to the researchers, the knowledge of cognition component involves one's awareness of cognition in three levels: declarative level (what question-knowing about things), procedural level (how question-knowing about how to do things), and conditional level (when and why questions- knowing why and when to do things). Regulation of cognition component involves the activities and actions taken by the person with the aim of controlling his own cognition. Such actions include planning, monitoring, debugging strategies, evaluation, and information managing; which conclude in self-regulation process and improvement in problem solving performance" (Schraw & Dennison, 1994, p.460). To go one step further, Maverach and Kramarski (1997) combines all of these knowledge and suggest a method called "IMPROVE" in order to improve students' mathematical reasoning and to provide strategies to enrich students' metacognition, throughout questions which result in metacognitive process. IMPROVE "the acronym of all the steps are: Introducing new concepts, Metacognitive questioning, Practicing, Reviewing, Obtaining mastery, Verification and Enrichment and remedial. Although this method really improves students' ability to solve test-like problems and authentic tasks relating to everyday life, the important part in this method is the metacognitive questioning step. In this step, there are four kinds of self-addressed metacognitive questions: comprehension (what is the problem all about?), connection (what are the similarities and differences between the given problem and the problems you have solved in the past), strategic questions (what strategies are appropriate for solving problem and why?) and reflection questions (why am I stuck?, what am I doing here?, does the solution make sense?, can I solve it differently?). By the help of IMPROVE method and these wh- questions, we can develop students' mathematical reasoning and metacognition" (Maverach & Kramarski ,1997, p.87). Thus, briefly, metacognition is vital, because just learning goal-specific strategies is not enough to be a good strategy user (Gredler, 2005). In addition to Gredler (2005), Driscoll (2005) emphasizes that helping learners to be more aware of their thinking process is very important for the development of mindful, strategic behavior or cognitive strategies.

A model of the metacognition is provided by Flavell (1979). In this model, "there are four classes: (a) metacognitive knowledge, (b) metacognitive experiences, (c) tasks or goals, and (d) strategies or activities". In the model, "metacognitive knowledge refers to a person's knowledge or beliefs about the factors influencing cognitive activities. Metacognitive activity and cognitive activity are interrelated and mutually dependent to each other, such that metacognitive activity precedes and follows cognitive activity. Metacognitive knowledge involves three categories of metacognitive knowledge: person variables, task variables, and strategy variables. The person variable refers to a person's knowledge and beliefs about himself; how he behaves as a thinker or learner, and what he knows about other people's thinking processes. The task variable refers to all the information about the task such as the task difficulty, resources related to task etc. The strategy variable involves identification of goals and sub-goals as well as choosing the appropriate cognitive processes for achievement". Flavell added that these types of variables overlap such that the person uses the combinations and interactions of these variables. The second class of Flavell's model; metacognitive experiences refer to the internal responses of a person towards his own metacognitive knowledge, goals, or strategies. Throughout these experiences, a person gets internal feedback regarding to his current progress, as well as future expectations of development, degree of comprehension, connecting new information to old and using previous information, memory and experiences as resources of current cognitive problem solving process etc. The third class in the model is metacognitive goals and tasks which refer to the desired outcomes or objectives involving comprehension, facts from memory, production such as a written document or an answer to a math problem, or simply development of one's knowledge. The last class of model is metacognitive strategies which are used for monitoring and controlling cognitive activities, development and achieving the cognitive goal. A high metacognitive skilled person has high awareness of his own thinking, and manipulates these processes to control his own learning process, plan and monitor the cognitive activities, and to compare cognitive outcomes with internal or external standards (Flavell, 1979). Later, a model of metacognitive activities in studying is developed by Winne and Harwin (1998) and the model explains four stages. In the first stage which is "defining the task", the person generates a view about the nature of the task, available resources and constraints. In the second stage, which is "goal setting and planning", the person chooses and generates goals and plans for the task. In the third stage, which is "enacting study tactics and strategies", the person uses the selected activities in the previous stage and may change if necessary. In the last stage, which is "adapting study", "the person makes large-scale adjustments to the task, goals, plans and engagement or changes his-her conditions such as knowledge, skills, beliefs, dispositions and motivational factors for future studying. In this model, if the study task is very familiar, then the person may skip one of the stages. Also, metacognitive strategies are thought to be conscious and intentionally done". Because, a person should be aware of his thinking process and decisions related to the actions needed to take when the progress is not satisfactory (Winne & Harwin, 1998 as cited in Gredler, 2005).

In summary, there are many definitions for metacognition. To integrate these definitions; metacognition refers to a person being aware of his thoughts, understanding, learning and thinking, controlling of one's own cognition and learning, controlling and monitoring his knowledge and his performance during a task, assessing his own performance or progress, regulating the cognitive actions according to the assessment of progress, reflecting on his learning, being aware of the difficulties during the task, knowing strengths and weakness of his own thinking (Berardi-Coletta, Buyer, Dominowski & Rellinger, 1995; Driscoll, 2005; Flavell, 1976; Gredler, 2005; Lester, Garofalo & Lambdin, 1989; Schraw & Dennison, 1994). The components of metacognition are specified differently by the researchers. Mainly there are two components: "knowledge of cognition and regulation of cognition involving introducing new concepts, metacognitive questioning, practicing, reviewing, and obtaining mastery, verification, enrichment and remedial" (Flavell, 1979; Gredler, 2005; Maverach & Kramarski, 1997; Schraw & Dennison, 1994). There are different models for metacognition. Mainly the models include defining the task, goal setting and planning, enacting study tactics and strategies, and adapting study (Flavell, 1979; Winne & Harwin, 1998 as cited in Gredler, 2005).

2.1.2 Problem Solving

For decades, more than 25 years, the problem solving research has gained attention. There have been calls for research about problem solving, and the researchers draw attention on problem solving instruction (Donaldson, 2011). In fact, problem solving has been one of the basic themes in education area for decades in mathematics education. The idea and importance of problem solving has begun brilliantly with Polya in 1945, with his fundamental book of "How to Solve It?" (Bahar, 2013; Donaldson, 2011; Hembree, 1992, Özalkan, 2010). In this highly important book, the outline and framework of problem solving process, details, clues and advises of how to implement problem solving process, and the basic four steps of problem solving; the explanations and definitions of the steps were provided. Polya (1973) defined the problem solving phases and emphasized mathematical discovery and challenging the curiosity of students throughout understanding process, learning and teaching problem solving processes. He advised the teachers to challenge the curiosity of students, to arise their interest by providing them problems appropriate for their knowledge and help the students to solve the problems by asking some questions. Then the teacher gives students a chance to enjoy problem solving and promote students' independent thinking. Polya emphasized that asking questions to students when they are engaging in a problem will be the best way in order to facilitate them in problem solving. These questions may be "what are known data?" and "what is unknown?" or "could you restate the problem?", and "do you know a related problem?", "can you check each step?", "can you check result?" "can you check the argument?". Based on such questions, Polya defined four problem solving steps. In Polya's problem solving framework, there are four steps or phases in problem solving process. First step is "understanding the problem". This phase means "to see what is clearly required". This step refers to restating the problem, defining the known, given data, and defining the unknown. Polya states that trying to finding an answer to a problem without understanding it will be a meaningless action. the problem solver defines the given and wanted variables, or describes the known and unknown variables. The second step is "devising a plan". This phase requires to see "how the various items are connected? How the unknown is linked to data?". It means to conceive the idea of a solution; it requires "formerly acquired knowledge to be connected with the new situation in the problem. This step refers to reviewing is the previously learned knowledge and determining which calculations, procedures or computations to be used, and which constructions to be performed". In "devising a plan step, the problem solver tries to find a connection between the givens and wanted". "He looks for a solution strategy by using the givens to reach the wanted variables; and tries to find a solution path from the knowns to unknown. For this, the problem solver searches for the best solution strategy and makes a solution plan". The third step is "carrying out the plan". This step refers to implementing the plan in step 2. The solution plan is a general outline; but this step requires more the details about the problem, and solution strategy and procedures should be applied carefully. In "carrying out the plan, the problem solver applies his solution plan. He applies necessary computations, procedures or formulas in his plan and reaches a solution". The final step is "looking back". This phase means to look back at the completed solution, to review and discuss it. This step refers to "checking, reviewing, reconsidering and reexamining the results and the solution strategy of the problem.

In the last step, the problem solver checks his solution plan and solution strategy. He checks his computations also, and acts on the solution to reach the results" (Polya, 1973).

For decades, mathematicians or researchers of mathematics education, provided a variety of different definitions of problem and problem solving. These differences occurred due to the different opinions of what forms a problem, and of what is important in problem solving (Donaldson, 2011). Similarly, in "Research on Educational Innovations", Ellis (2005) stated that most of the previous research, and research base related to problem solving area are lack of a common definition, so they have measurement validity problems. The researcher states that there is "no generally agreed-on set of definitions of terms" (p. 109), and thinking skills are difficult to measure. Similarly, Nickerson (1994) states that some research which aim to build up approaches to the teaching of thinking and problem solving have been directed by one or another theory, model or a conceptual framework; and also other studies have been theory free. None of the approaches to the teaching of thinking and problem solving that has yet been produced has a firm theoretical foundation. None of them is based on a well-articulated theory of cognition, which is universally accepted as valid by the scientists or researchers community. This statement is correct because there is no such firm and valid theory about teaching of thinking and problem solving. This fact explains why a wide range of opinions exist about how to teach thinking and problem solving best. Also, it points up the resistance to faster progress in the field. Instead, thinking and problem solving should be better understood; more precise, more predictive, more comprehensive and testable theories of cognition should be produced and tested. Until this required progress is achieved, studies to enhance thinking and problem solving will remain as a trial-and-error process. Researchers and educators still don't know how to teach all aspects of thinking and problem solving effectively. Also, Lester (1994) stated that his work from 1980 to 1994 showed that there has been little progress in problem solving research. Also, when Lester and Kehle (2003) compared the list of issues to the Lester's work in 1994, they stated that still, little development has occurred in problem solving research, and also the literature on problem solving provided little offerings to school practice. Similarly, Lesh and Zawojewski (2007) claim that there is a lack of impact and cumulativeness in the research on mathematical problem solving. This situation is not surprising because this area of research is criticized for years due to its lack of theoretical base. So, there is a great need for better theorizing in the field. For this, more studies should be conducted related to problem solving.

Problem solving had high importance in mathematics education for decades, and continues to be essential part of mathematics education (Evans, 2012, Posamentier & Krulik, 2008). Problem solving has been one of the basic themes in education area for decades. Both of the educators and policy makers conclude and emphasize the vital role of problem solving skills on school and daily life or real life success (Bahar, 2013). Also, Nickerson (1994) explains the need for problem solving as that although people participate in problem solving naturally and spontaneously, they may fail to succeed or they may not be able to solve the problems well enough. In the past, when the students weren't taught the problem solving strategies at all levels of formal education, they were not able to do the kind of thinking and problem solving that their school-work required. Moreover, most of the students could not write wholly satisfactory explanations, and they could not defend a point of view or their perceptions about the problem solutions effectively with a persuasive argument (Nickerson,1994). Evans (2012) stated that strong problem solving abilities and skills are vital for mathematics; as well as for other subject areas, disciplines and for daily life in general. So, the students should be provided critical thinking and strong problem solving preparation in schools, since they need them for success in life. Similarly, Özsoy (2006) explains the need for problem solving as the fact that mathematical knowledge and mathematical thinking are interrelated to each other, but they are different concepts. Mathematical knowledge is required to think and solve the problem, but it is not enough. Besides the mathematical knowledge, mathematical thinking is required to understand mathematics. In order to develop mathematical thinking, the problem solving studies should take place. Problem solving has two effects on mathematics; one is to develop strategies and rules specifically to a concept, and the second one is to develop thinking styles and general approaches in order to improve a rule or formula in a concept. Students learn how to propose new strategies and to interrelate the old strategies with new types of problems (Özsoy, 2006).

Carson (2007) states that there are some common elements of problem solving. First one is that problem solving connects theory and practice. Secondly, problem solving teaches creativity. Next, successful problem solvers have a complete and organized knowledge base. Later, problem solving teaches transfer or how to apply conceptual knowledge. Another element is that problem solving is not an algorithm. In his critique, he refuses the last element: problem solving is a heuristics. The researcher finds this element problematic and states that knowledge base should now be ignored, already formed knowledge is vital for problem solving. To teach heuristics is necessary, but algorithms are also necessary. The knowledge base and the transfer of that knowledge are vital elements of problem solving process. Similarly, Stephen Krulik and Jesse Rudnick (1980) explained in "Problem Solving: A Handbook for Teachers". A problem is "a situation, quantitative or otherwise, that confronts an individual or group of individuals, that requires resolution, and for which the individual sees no apparent or obvious means or path to obtaining a solution" (p. 3). In this definition, the researchers emphasize that "the problem solver uses the formerly learned knowledge, skills and understanding in order to reach the solution of new and unfamiliar situation. The solver should integrate the previously learned knowledge into a new and unknown situation" (Krulik & Rudnick, 1980, p.3).

Also, Bransford, Sherwood, Vye, and Rieser (1986) stated that there are two general research approaches related to the goal of effectively teaching reasoning, thinking, and problem solving. In the first approach, the researchers focus on the role of domain-specific knowledge. In the second approach, the researchers focus on general strategic and metacognitive knowledge, and state that people who learn new information and monitor their learning will perform more effectively. Bransford and his colleagues stated that the programs which aim to teach thinking and problem solving should focus more on domain knowledge, as well as general skills and strategies. There is a need for both general problem solving strategies and domain-specific knowledge which is appropriately organized according to the students' needs. In addition, different ways of presenting information had important effects on reaching to a previously acquired and relevant knowledge. In order to access the previous and relevant knowledge, perceptual learning and pattern recognition are important. So, the problem solvers should be taught to differentiate the problem types, and the different solutions types to these problems. Also, the emphasis should be given to combination of general metacognitive and domain-specific knowledge (Bransford, Sherwood, Vye, & Rieser, 1986).

Jonassen (2000) states that a problem has two properties. One is that a problem should provide an unknown entity, which refers to the difference between a goal situation and a current situation. Secondly, solving the unknown entity should have a social, cultural or intellectual value and it should be worth to find the unknown entity. Then, finding the unknown is the act of problem solving process (Jonassen, 2000). Anderson (1980) states that problem solving process is a series of cognitive operations which are held in a goal-directed manner. For this, the problem solving process requires the mental representation of the state, the problem solvers construct a mental representation or mental model of the problem in their minds. These mental representations are called problem state (Anderson, 1980 as cited in Jonassen, 2000). Parallel to Anderson (1980), Jonassen (2000) claims that "in problem solving process, the most vital property is the mental construction of the problem space. Moreover, the activity-based manipulation of the problem state is the second vital property of the process. Thus, problem solving occurs by manipulation of problem space, which means making an internal mental representation or an external physical representation" (Jonassen, 2000). According to Gredler (2005), in general, problem solving refers to trying to accomplish the new and unfamiliar tasks when the person does not know the relevant solution methods. “The problem involves three components: givens, a goal, and allowable operators”. “The given component involves the elements, the relations between the elements, conditions or situations that exist in the initial form of the problem. A goal component refers to the desired outcome or solution. The allowable operators component refers to the steps or procedures which will transform the given elements to the desired goal” (Gredler, 2005).

Lesh and Jawojewski (2007) define problem solving as:

> A task, or goal-directed activity, becomes a problem (or problematic) when the “problem solver” (which may be a collaborating group of specialists) needs to develop a more productive way of thinking about the given situation (p. 782).

According to Lesh and Jawojewski (2007), the most difficult aspects of the problem solving situations include the production of useful ways to think mathematically about relationships, patterns and regularities. So, the definitions should include these characteristics, and problem solving shouldn’t be separated from concept development. In this definition, development of “productive way of thinking” requires the problem solver to engage in a process which includes mathematical interpretation of situation. So, problem solving refers to interpreting a situation mathematically, throughout various iterative cycles of expressing, testing and revising the interpretations, and also to sort out, to integrate, modify, revise or refine the mathematical concepts. Problem solving is an iterative cycle of understanding the givens and goals of the problems; when the problem solver reaches this understanding, then it is easy to link between the givens and goals.

In fact, Polya (1962) described problem solving as “finding a way out of a difficulty, a way around an obstacle, attaining an aim which was not immediately attainable” (p. v). Problem solving is a process “to search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable aim” (Polya, 1962, p.117). For a mathematician, problem solving refers to a mathematical situation in which the solution is required; but not known. Moreover, there is no direct route or clear pathway to the solution (Polya, 1962). Similarly, according to Schoenfeld (1992), problem solving refers to a process, in which students' acts in a question, but they do not have an immediate and apparent solution for this question. Also, they do not foresee an immediate and clear algorithm or procedure to apply for the solution. Moreover, according to Krulik and Posamentier (1998), problem is identified as "A problem is a situation that confronts a person, that requires resolution, and for which the path to the solution is not immediately known." (Krulik & Posamentier, 1998, p.1). In fact, problem is not a drill or is not a routine exercise. Problem differs from routine exercises or drilling questions. In routine exercises and drilling questions, the students already know the solution strategy, or specific solution procedures, or they only require computational skills. But in a problem, there is a challenge and it requires common knowledge, and the solution strategy is not already known (Krulik & Posamentier, 1998; Krulik & Rudnick, 1987; MoNE, 2005). Also, problem is a situation or a condition so that there is something that needed to be found or shown; but there is no immediate and clear way to find or show it (Grouws, 1996). Also, problem solving is a task in which a person engages in it in order to find a solution, but the solution method is not known by the solver in advance (NCTM, 2000). Moreover, problem solving is referred to an "extremely complex form of human endeavor that involves much more than the simple recall of facts or the application of well-learned procedures" (Lester 1994, p. 668). Also, in order to be a good problem solver, students are required to choose and apply the correct or appropriate cognitive strategies for tasks. Then they will be able to understand the task or the problem, represent the task, and solve the problems (Mayer, 1998; Schoenfeld, 1985). So, when combining of these definitions about problem solving, it can be concluded that problem solving is a situation in which there is a problematic situation, but the solution is not seen immediately. Problem solving is an act of trying to turn the unknown situation into known situation. Schoenfeld (1992) states that in all research area about problem solving, every researcher should provide his own operational definition of problem solving term. So, in this study, the operational definition of problem solving is that problem solving is a situation in which a person confronts with an unknown situation and tries to turn the unknown into known situation throughout a series of cognitive or mental, logical or formal reasoning thinking, and metacognitive processes.

There are two types of problems in terms of the number of answers; "well-defined and ill-defined". The "well-defined problems include the givens, desired goal and allowable operators explicitly; whereas, the ill-defined problems do not include the givens, goal and the allowable operators immediately clearly to the problem solver". In addition, the problems can be divided into "routine and non-routine problems. For routine problems, the solver has solved a familiar type in the past and now she recognizes the solution. In contrast, for non-routine problems, the solver has not solved a familiar problem in the past and now the solver cannot generate a preexisting solution" (Gredler, 2005; Jonassen, 2000).

The problem solving process has four steps according to Polya (1945); "1. Understanding the problem, 2. Devising a plan, 3. Carrying out the plan, and 4. Looking back". Similarly, Hayers (1981) provided six steps; "1. Finding the problem, 2. Representing the problem, 3. Planning the solution, 4. Carrying out the plan, 5. Evaluating the solution, 6. Consolidating gains" (as cited in Nickerson, 1994). In order to make these steps easily remembered, Bransford and Stein (1984) created "IDEAL acronym for problem solving steps: I. Identify the problem, D. Define and represent the problem, E. Explore possible strategies, A. Act on strategies, L. Look back and evaluate the effects of your activities" (as cited in Nickerson, 1994, p. 424). In a similar manner, Gredler (2005) states that there are four sub processes in problem solving: "representing the problem, planning, overcoming obstacles, executing plans. Representing the problem includes identification of key elements and creating a mental map, the restructuring of the givens, mentally redefining and clarification of problem, or reformulating the givens. Planning includes review of strategies and tactics before applying them, guessing the results of some particular approaches. Overcoming obstacles necessitates thinking about the previously unnoticed elements, combining them in a new way and exploring new relations between elements and knowledge. Executing plans include monitoring of execution of the selected strategy and changing it if needed". The researcher emphasizes that all of these problem solving steps require metacognition (Gredler, 2005).

Throughout these problem solving steps, Lester (1994) states that good problem solvers know their strengths and weaknesses as problem solvers more than poor problem solvers. Also, good problem solvers monitor and regulate their problem solving efforts better. Moreover, good problem solvers tend to get elegant solutions to problems more than poor solvers (Lester, 1994). Also, problem solving lets students transfer the knowledge they have constructed in the school to the real-life conditions and to the real-life problems. Problem solving makes students feel ready for life problems, and provide them a feeling of satisfaction and a belief about usefulness of mathematics (Writer, Jarrett, & Robert McIntosh Mathematics Associate, 2000). Moreover, Higgins (1997) confirms these benefits, and emphasizes that problem solving increases mathematical understanding. According to the writer, students who have taken 1 year of problem-solving instruction showed greater perseverance in solving problems, more positive attitudes about the usefulness of mathematics and deeper mathematical understanding than the students who have taken traditional mathematics instruction (Higgins, 1997).

According to Krulik and Posamentier (1998), the teachers should involve problem solving as an essential part in their regular curriculum, they need to focus on what problem solving is, how problem solving can be used in order to teach mathematical skills effectively, and how problem solving can be presented to students in an effective way. In fact, the teacher should learn that problem solving can be presented in three different ways. Firstly, "problem solving is a subject for study in and of itself". Secondly, "problem solving is an approach to a particular problem". Finally, "problem solving is a way of teaching" (Krulik & Posamentier, 1998, p. 4). First of all, the teachers should themselves be good problem solvers; before teaching problem solving to students. They should learn the entire problem solving strategies. Moreover, they should know which problem solving strategies to apply, when to apply and how to apply. Also, they should be able to apply the problem solving strategies both to mathematics and real life experiences (Krulik & Posamentier, 1998).

There are some problem solving strategies which are used in problem solving process. These strategies are as working backwards, finding a pattern, adopting a different point of view, solving a simpler, analogous problem, considering extreme cases, making a drawing, intelligent guessing and testing, accounting for all possibilities, organizing data, logical reasoning (Krulik & Rudnick, 1987).

In addition to problem solving steps, according to Krulik and Rudnick (1987) there are ten problem solving strategies. 1. “Working backwards: This strategy involves solving a problem from the last step to the beginning, from the back to the beginning, step by step”. 2. “Finding a pattern: This strategy involves analyzing the given numbers or data and trying to form a logical pattern of the given data”. 3. “Adopting a different point of view: This strategy involves seeing the problem in a different point of view”. Such problems cannot be solved easily by a current way. It requires being able to change the perspective or point of view and create a new one. 4. “Solving a simpler, analogous problem: This strategy involves reaching to the solution by using the solution way of a similar but much simpler problem”. By using the solution way of the similar and simpler problem, the solver reaches the solution of current problem. 5. “Considering extreme cases: This strategy involves controlling and checking the extreme cases in the current problem”. By using extreme values, the solver reaches the solution. 6. “Making a drawing: This strategy involves problem solver to visualize the given and known data in the problem”. It involves creating visualizations of the givens and wanted variables in the problem by using drawings, charts, schemes, tables, illustrations. 7. “Intelligent guessing and testing: This strategy involves guessing the answer of the problem or the solution or the exact value of the answer”. It involves making logical trials; such as guessing the answer and testing this answer if it is correct or not. 8. “Accounting for all possibilities: This strategy involves reviewing and searching all the possible answers of the problem”. 9. “Organizing data: This strategy involves organizing all the given values or knowledge or data in the problem”. 10. “Logical reasoning: This strategy involves analyzing the relationship between the given data and the asked data”. The problem solvers conduct a logical reasoning between the given values and asked value in the problem (Krulik & Rudnick, 1987).

Also, Evans (2012) examined the alternative certification of the middle and high school teachers’ mathematical problem solving abilities and perceptions. For this, the researchers provided problem solving examination to participants and wanted participants to reflect on problem solving process of both their students’ and their own. The course of semester, the teachers taught mathematics content from a problem solving perspective. The results of the study showed the teachers showed a significant development in problem solving abilities throughout the course of the semester. Also, there was a significant and direct relationship between content knowledge and problem solving ability. But, the teachers defined their students’ as weak problem solvers, who do not understand the problem, who do not know how to start a problem, who are lack of persistence, and who have poor literacy skills. Over the course of the semester, the problem solving abilities increased. Because, strong mathematics in alternative certification course concluded in stronger problem solving skills of teachers. That result emphasizes the importance of teaching mathematics from a problem solving perspective.

In summary, problem solving had high importance in mathematics education for decades, and continues to be essential part of mathematics education (Evans, 2012, Posamentier & Krulik, 2008). Also, Evans (2012) stated that strong problem solving abilities and skills are vital for mathematics; as well as for other subject areas, disciplines and for daily life in general. So, the students should be provided critical thinking and strong problem solving preparation in schools, since they need them for success in life. Based on previous studies, the problem solving can be defined as a process of finding the unknown entity throughout a series of cognitive operations within a goal-directed manner and trying to accomplish a new and unfamiliar task (Anderson, 1980, as cited in Jonassen, 2000; Gredler, 2005; Jonassen, 2000). The problem solving steps are defined differently by many researchers. Mainly, the problem solving steps involve understanding the problem, devising a plan, carrying out the plan, looking back (Bransford & Stein, 1984 as cited in Nickerson, 1994; Hayers, 1981 as cited in Nickerson, 1994; Gredler, 2005; Polya, 1945).

2.1.3 Reasoning Ability

According to Piaget, people learn through schemes which are the mental representations of thinking including objects, situations, events etc. in our life and they involve the organized patterns of thoughts or behaviors. These schemes improve throughout four stages, and these stages have a continuous pattern in cognitive development of children. People pass through four cognitive development stages. "The first cognitive stage is sensory motor" which involves 0-2 years. The second one is "preoperational stage" which occurs in 2-7 years. The third one is "concrete operational stage" which occurs in 7-11 years. Finally the last one is "formal operational stage" which occurs from 11 years old to adult. Children start understanding from concrete level through formal operational level. Students within the concrete operational level have ability to deal with concrete problems, recognize and apply conservation law, understand and apply reversibility law and able to apply classification and seriations. But they don't have ability to deal with non-observable, abstract or imaginary situations and operations. In contrast, formal operational reasoning concludes in a refinement, correction, or perfection of operations at the concrete stage. The structure of the formal stage involves and requires specific information processing abilities which trigger and improve the adolescent's ability to follow the form of logical reasoning while ignoring the content.

Similarly, Biggs and Collis (1982) stated that students at concrete operational level may have inefficiency in working memory, so they have difficulties when they deal with multiple situations or multiple concepts at the same time. During these multiple situations, they may not choose which answer is the best for the solution. Also, concrete level students generally think that a problem have only one correct solution. During open ended problems, which require multiple solutions, the concrete level students cannot identify the answers easily. The formal operational level students have efficient and deeper working memory in contrast to concrete level students. So they have ability in the production of solutions to abstract problems throughout reasoning and logical ability. The formal operational level students can think scientifically, hypothetically, and they focus on concepts and the relationships between these concepts during solution process.

In the same manner, Fuller (2001) states that "students with concrete reasoning generally have tendency to memorize the words, phrases, procedures and they have tendency to use them without deep or meaningful understanding. They need concrete objects, situations, directly experienced actions, observable situations, and step-by-step definitions and explanations to understand in long procedures or situations". They have ability to classify objects; also they have understanding of conservation, and seriation reasoning patterns. But they are not conscious about their own reasoning process. In contrast, "students with formal reasoning have ability to reason throughout relationships, abstract situations and concepts. They have ability to express themselves with symbols, their ideas with symbolism systems, and they are able to make plans throughout goals and by the help of resources in long procedures or situations. Also, they have proportional, probabilistic, combinational, correlational and controlling reasoning abilities". In contrast to concrete students, formal reasoning students have consciousness about their reasoning ability and reasoning process. They also test their solutions or conclusions in the reasoning process by the help of integrating the existing knowledge with the new knowledge. Moreover, the formal reasoning students have ability to study new subjects, or unfamiliar subjects. In order to decrease the difference between concrete and formal reasoning students, self-regulated learning methods may be used. With help of self-regulatory instructions, students in concrete reasoning level may make progress through formal reasoning level (Fuller, 2001).

According to Longman (1987), reasoning is the ability of thinking, understanding, and developing opinions, and providing judgments based on facts. Reasoning is a process in which a person forms his own opinions, judgments or makes inferences based on facts, or on a body of information. Also, reasoning may be defined as the correlation or integration of problem solving and communication (Kelly, Myllis, and Martin, 2000). Moreover, Steen (1999) claims that the literature provides some general conclusions about improving mathematical reasoning for students. The first conclusion is that in order to be successful learners, the students should be mathematically active (Anderson, Reader & Simon, 1997 quoted in Steen 1999). Because in active strategies such as discussion, projects, team-work, or collaborative learning, students develop deeper understanding and more permanent skills or conclusions. But in passive strategies such as memorization, drill, or automatic calculations or templates, students develop less meaningful understanding. The second conclusion is that in order to be successful learners, students should develop reflective thinking, or metacognitive activity (Resnick, 1987). Because, learners who reflect on their thinking, who monitor what they do and why they do so show more success than the learners who automatically provides the rules without any consciousness. The third conclusion is that there is huge variety among students. There is no single solution method, single strategy or same thinking style which will be valid and understandable for all students. Moreover, there is no single strategy which will work in all conditions for a student. Students learn differently in different conditions or situations. As supported in Howard Gardner’s multiple intelligences theory, the teachers should provide multiple strategies, solution methods or thinking styles for students, and also this diversity should occur in all different subjects. Thus, this variety may trigger students to engage in mathematical reasoning throughout the strategy which is suitable for them.

For the importance of reasoning ability in educational settings, Tobin and Capie (1981) state that "there are two important trends related to formal reasoning ability. First trend is that many adolescents and adults have a limited formal reasoning ability. Many adolescents and adults use the formal modes of reasoning ability limitedly. The second trend is that formal reasoning ability is a vital mediator of cognitive achievement". As a result of these two trends, most of the researchers agree on the emphasis that there is a great need for modification of instructional objectives, materials and activities. These modifications should be made according to the cognitive development level of students or learners. For this, primacy should be provided to develop the formal reasoning ability of middle and high school students, by preparing appropriate curriculum materials (Tobin & Capie, 1981). Also, Bitner (1991) stated that formal operational reasoning modes are significant predictors of science and mathematics achievement. Formal operational reasoning modes explained 29% of the variance in mathematics. Thinking processes develop throughout both declarative and procedural knowledge. So, educational settings must have a central focus of both factual knowledge and thinking processes. Since the reasoning ability is a significant predictor of mathematics achievement, instructional approaches should focus on not only declarative knowledge but also on procedural knowledge. Five formal operational reasoning modes and critical thinking skills are the vital abilities for the success in secondary school science and mathematics courses (Bitner, 1991). Ball and Bass (2003) defined mathematical reasoning as a fundamental part of mathematical skills, and claimed that mathematical understanding is based on reasoning. Reasoning ability is the basis for learning new mathematics; and also, the ability to reason is vital for integrating the previous mathematical knowledge to new situations. Reasoning is a process in which a person revisits and reconstructs the previous knowledge for the aim of building new arguments. Then, reasoning ability concludes in one's growth of new knowledge.

2.2 Literature Review and Related Studies

2.2.1 Relationship Between Reasoning Ability and Problem Solving

According to Hembree (1992), during the 20th century, teaching and learning problem solving has gained special attention and emphasis. There was a great attention about the research area because there were two perceptions about problem solving. First perception was that “problem solving is a basic skill, and it is a vital and required skill for students”. The second perception was that “problem solving is a complex mental activity”. The first perception was created after the National Council of Supervisors of Mathematics (NCSM) defined problem solving as one of the most essential ten proficiencies (1977). Later, the National Council of Teachers of Mathematics (NCTM) emphasized problem solving term greatly in its Curriculum and Evaluation Standards for School Mathematics (1980). The second perception was long standing one for years. According to Garofalo and Lester (1985), problem solving is a process which requires high visualization, association, abstraction, comprehension, manipulation, reasoning, analysis, synthesis, generalization. Moreover, all of these highest faculties should be managed and coordinated appropriately. Also, Jonassen (2000) states that problem solving is recognized as the most crucial cognitive activity both in everyday and professional contexts. It is required to solve problems in everyday and professional contexts mostly, and people who are able to solve problems are awarded for this ability. Despite of the importance of problem solving ability, learning to solve problems generally is not required in formal educational settings. Because, researchers and educational community don’t have deep knowledge about its processes, and instructional-design research and theory has drawn little attention to the study of problem solving processes. Researchers and educators are inefficient to engage students in problem solving. The major reason of this inefficiency is because the breadth of problem solving is not understood well enough to engage students in problem solving and to support their problem solving activities (Jonassen, 2000).

Moreover, according to Hembree (1992), there are several different abilities related to problem solving. In his meta-analysis study of experiments and relational studies in problem solving, the researcher studied four regions of problem solving. These regions are characteristics of problem solvers, conditions for difficult and easy problems, effects of instructional methods, effects of classroom-related conditions. According to the researcher, there is a direct significant relationship between problem solving and a variety of measures of basic performance, and skills in basic mathematics. These abilities are creative thinking, critical thinking, memory, perception, reasoning, skills related to analogies and inferences, spatial ability. All of these abilities have a significant correlation with problem solving.

The previous research and the developments in mathematics education have a challenging issue: how the learners construct mental structures about their mathematical experiences and how the learners reason with these structures in order to learn and solve the problems (Davis, 1992). English (1997) states that “the learners use the same reasoning mechanisms in daily life and in mathematics. The researcher claims that If we investigate our reasoning mechanisms used for communication and interaction in daily life with others, we can conclude that the same mechanisms are used in our reasoning with mathematical ideas”. Mathematical reasoning involves “reasoning with structures which are formed by our bodily experiences. These structures are formed during the interaction with environment and they are formed on propositional representations. Moreover, mathematical reasoning is imaginative because it is formed on a variety of powerful devices which structure the concrete or basic experiences and turn them into models for abstract thinking. These devices involve analogy, metaphor, metonymy and imagery” (English, 1997).

Moreover, Evans (2000) stated that although long time has passed and many changes took placed, in general, many commentators still claim traditional view of ‘mathematical ability’ in the educational world. That concluded in a thought that mathematics requires only a set of abstract cognitive skills, which are performed in a variety of tasks and practical contexts. That concluded in a perception of relatively straightforward process of transfer. Then, the transfer of learning has gained vital importance in formal educational system. In this traditional view, performance was measured by the number of correct answers in test items. That concluded in rote learning rather than real understanding. Also, according to NCTM, the students should develop reasoning throughout making sense of problems or conditions or situations. They should develop understanding by connections with prior knowledge. With making sense, reasoning creates consciousness about what is happening in a situation or develops insights about a specific situation or problem (NCTM, 2010). Mueller and Maher (1996) stated that it is commonly accepted that reasoning and also proof are fundamental for mathematical understanding. For a student to be able to learn reasoning and justification of his reasoning is vital for his mathematical knowledge growth. In a community of learners, attending in discussions, making and refuting claims, and justifying reasoning related to mathematical ideas conclude in mathematical reasoning. So, the teachers should provide students collaborative environments, in which students are triggered to explain their thinking, make their ideas public, justify and give evidence for their thinking and claims, participate in arguments and discussions (Mueller & Maher, 1996) .

In a problem solving process, mathematical thinking must occur since a student firstly needs the basic mathematical knowledge for solution (NCTM, 1991; NME, 2005, p.14). Usman and Musa (2013) stated that use of the thinking and reasoning patterns are very important for their mathematical performance. The teachers should measure students' formal operation levels and trigger students to use formal operation abilities in order to improve students' mathematical performance. According to Schoenfeld (1992), problem solving includes production of mathematical thinking based on mathematization and abstraction. It also includes the application of this mathematical view; and also, recognizing and having proficiency with the tools of the trade. Moreover, it includes the choice and use of appropriate tools for the aim of understanding the situation. (p.335).

Reasoning refers to making conclusions from assumed facts, or moving from hypothesis to conclusion. For reasoning process; other processes such as analysis, arguments and verification also should occur in the mind of a person (Lee, 1999). Reasoning refers to the ability of making logical inferences with the help of mathematical rules, formulas, relations and also mathematical representations or models. It occurs when a student try to explain his own thoughts, the reasons and logic behind choosing the solution strategy; and to make conclusions about problem, to analyze the problem situation by using and producing mathematical relations, and to makes predictions or plans for solution. It also involves a student to think and believe that mathematics is based on logic and mathematics is meaningful and understandable based on the web of logical relations (MoNE, 2005). Mueller, Yankelewitz, and Maher (2011) stated that motivation and positive dispositions toward mathematics conclude in mathematical reasoning, and then this concludes in understanding. Students engaged in and trusted in their reasoning, instead of memorized facts, or solutions of other students. Based on their reasoning, the students persuade themselves and other students about the issues that make sense. This reasoning process concludes in mathematical understanding. If a student engages in mathematical reasoning then that students get conceptual understanding. Mueller and Maher (1996) stated that if students engage in an environment in which they explore, collaborate with each other, and defend their thinking and justify their reasoning in both small and large groups, then they develop reasoning and mathematical understanding (Mueller & Maher, 1996). Reasoning is mainly the ability to monitor relations, to make connections and create conjectures, providing logical deductions by the help of assumed facts, rules and relations, and providing justifications for the created conclusions and results (TIMSS, 2003). Reasoning in general represents the process in which a person forms conclusions based on the evidences or assumptions (NCTM, 2009). Ball and Bass (2003) defined "mathematical reasoning as a fundamental part of mathematical skills, and claimed that mathematical understanding is based on reasoning. Reasoning ability is the basis for learning new mathematics; and also, the ability to reason is vital for integrating the previous mathematical knowledge to new situations. Reasoning is a process in which a person revisits and reconstructs the previous knowledge for the aim of building new arguments. Then, reasoning ability concludes in one's growth of new knowledge" (Ball & Bass, 2003). Reasoning is very important for mathematics since it requires logical deduction of conclusions driven from evidences, assumptions and information. In the high school mathematics, formal reasoning and proof has more importance. In the lower grades, mathematical reasoning includes informal observations, conjectures, justifications and explanations. Students should start development of reasoning ability in the lower grades or in elementary grades, so that they will improve it sophisticatedly in the higher grades. So, the aim of developing reasoning ability is a central focus in principles and standards for mathematics, NCTM 2000. NCTM states that, in mathematics there are important processes such as problem solving, reasoning and proof, connections, communication and representations. All these processes are the results of making sense and reasoning. Students firstly construct reasoning and make sense of ideas; and then they can solve the problems and provide proofs in mathematics. Because, for problem solving and proof, reasoning is a must. In order to develop and support reasoning and making sense, the students should choose appropriate representations, develop correct connections and provide correct communication. Moreover, in order to make those correct and appropriate decisions, the students again should construct reasoning (NCTM, 2000).

Stenberg (1980) stated that reasoning, problem solving and intelligence have very close relationship with each other, so that it is often difficult to differentiate them. An arithmetic word problem firstly requires "problem solving" for solution. The problem also requires "reasoning". The same problem also requires "intelligence". "The same close relationship between three constructs take place in most of the problems. In fact, problem solving seems to require reasoning, and reasoning seems to require problem solving" (p.4). According to Stenberg (1980), reasoning refers to combining elements of old information in order to form new information. Similarly, Kramarski, Mevarech and Lieberman (2001) stated that there is a direct relationship between reasoning skills and success in mathematics.

Students who show better reasoning skills have good problem-solving characteristics. Also, these students define the interrelationships more, and have better communication skills. Also, Gunhan (2014) states that in school curriculum, reasoning skills should be emphasized more. Especially for geometrical concepts, reasoning skills is very important. So, the teachers should provide problems which will develop students' reasoning skills. For this, the students should be provided activities in which students are encouraged to reflect their knowledge, to make logical arguments and thus to use reasoning skills more. Also, Cavallo (1996) stated that the students' reasoning ability and meaningful understanding are very important in problem solving process and in integrating the ideas. Meaningful learning and/or reasoning ability are important for overall learning in the classrooms. Both the meaningful learning and reasoning ability should be improved as much as possible, to the fullest extent, in order to maximize students' learning and understanding. For the importance of reasoning in problem solving, Mansi (2003) stated that reasoning should be emphasized throughout school mathematics. The reasoning ability is required in mathematical reasoning and proof, which is vital in mathematics. Mansi (2003) stated that mathematical reasoning is the ability required in coherent and logical thinking, and making inferences or providing conclusions from mathematical facts. Reasoning ability is a powerful and essential part of learning mathematics. Because, by the help of reasoning ability, the students reason about mathematical ideas, make conjectures, provide justifications and explanations about why a mathematical idea or concept make sense, or why a procedure or formula can be applied in a situation. Students should improve their reasoning and justification abilities during their mathematics learning process.

Battista (2007) stated that in order to present meaningful education to students, educators and teachers firstly need to understand the thinking processes that the students engage in. For meaningful understanding and to improve students' reasoning skills, it is required that conceptual understanding should be emphasized, rather than just providing procedural knowledge. Similarly, Işıksal, Koç and Osmanoğlu (2010) searched eighth grade students' reasoning skills on measurement. The researchers provided to students a task which engages students to reason and explain their thinking process. Then the students' reasoning skills on measurement with surface area and volume of a cylinder was examined. The researchers stated that eighth grade students had difficulties in solving problems which require conceptual understanding of reasoning. Students also had difficulty in solving problems which require measurement of the surface area and volume of cylinders. The results of the study showed that eighth grade students had difficulties to reason the meaning of measurement concepts separated from the symbolic manipulation of formulas. The students had difficulties to reason the relationship between surface area and volume of cylinder. Also, the students showed difficulty in solving problems which required conceptual understanding. The researchers advised that in order to improve reasoning and meaningful understanding, the teachers should emphasize both the conceptual and procedural knowledge and help students construct both the conceptual and procedural understanding. The teachers should trigger students to communicate with each other in the classroom, to discuss the mathematical concepts. Then the students will have chance to reason about the mathematical ideas. By connecting and integrating the mathematical procedures and the conceptual knowledge and ideas, the students will reach reasoning and meaningful understanding in mathematics.

According to Nunes, Bryant, Barros and Sylva (2012), mathematical reasoning and arithmetic skills have important effect on mathematical achievement. However, there are not too many studies to support the theory and provide evidence for educational practice about this issue. For this, the researchers investigate the effects of mathematical reasoning and arithmetic skills on the mathematical achievement. They prepare a longitudinal study over five years for the prediction of mathematics, science and English achievement. The researchers control age, intelligence and working memory. The results of the study show that mathematical reasoning and arithmetic skills have significant and independent effects on mathematical achievement. The effect of mathematical reasoning is higher than arithmetic skills on mathematical achievement. Reasoning and arithmetic affects mathematics more than science or English. Intelligence affects science more than mathematics. Working memory affects math and English equally. So, the researchers state than mathematical reasoning is different from arithmetic skills and, development of mathematical reasoning should have importance in school curriculums.

Evans (2000) claims that there is a relationship among mathematical thinking, reasoning ability and problem solving ability of the students. This relationship is valid for each pairs, when one of the aspect increases, the others also increase. Also, According to Tobin and Capie (1982), formal reasoning ability is the strongest predictor of process skill achievement with 36% of variance. In addition, Valanides (1997) stated that student's reasoning ability was significant predictor of school achievement. The amount of variance was highest for students' mathematics achievement with (22.8%).

Similarly, Bitner (1991) provided support for this relationship and claimed that the reasoning ability modes were the significant predictors of the achievement in mathematics and science. Bitner (1991) investigated whether the formal operational reasoning modes are the predictors of critical thinking abilities and grades assigned by teachers, or not, in science and mathematics. For this, firstly the Group Assessment of Logical Thinking (GALT) was administered to 101 rural students in grades nine through twelve. After eight months, the grades assigned by teachers were collected. The results of the study showed that the five formal operational reasoning modes in the GALT were significant predictors of critical thinking abilities and also of the grades assigned by teachers in science and mathematics. The variance in the five critical thinking abilities attributable to the five formal operational reasoning modes ranged between 28% and 70%. The five formal operational reasoning modes explained 29% of the variance in mathematics achievement and 62% of the variance in science achievement. Also, the researchers stated that since the reasoning ability is a significant predictor of mathematics achievement, instructional approaches should focus on not only declarative knowledge but also on procedural knowledge. Also, five formal operational reasoning modes are the vital abilities for the success in secondary school mathematics courses (Bitner, 1991).

Another study supporting the relationship is that Lawson (1992) investigated the relationship between the students' reasoning ability and general achievement including reading, language, social studies, art, science and mathematics. The writer conducted a study with seventy two 9th grade students and concluded that reasoning ability is an important contributor to students' general achievements in school. Reasoning ability has relationship with problem solving ability and general achievement of students. Students' formal reasoning ability has a relationship with science achievement (r=.69), and with Social studies (r=.72) and also with mathematics achievement (r=.70).

Similarly, Malik and Iqbal (2011) searched the effect of problem solving teaching strategy on the problem solving skills and reasoning ability of eight grade students. The results show that experimental groups showed higher problem solving and reasoning ability than the control group. Also, Sonnleitner, Keller, Martina and Brunner (2013) stated that (CPS) Complex problem solving was significantly related to reasoning and educational success. And reasoning ability plays a crucial role in the process of solving complex problems. Sonnleitner, Keller, Martina and Brunner (2013) searched the structure of Complex Problem Solving (CPS), which captures higher order thinking skills, and its relation to reasoning, intelligence and educational success. The researchers chose 563 secondary students and examined the different measurement models of CPS, these models are faceted or hierarchical. The results of the study showed that both of the models of Complex problem solving were significantly related to reasoning and educational success. The researchers were able to show that reasoning ability plays a crucial role in the process of solving complex problems.

In a more detailed manner, Washburn (2013) searches the relationship between mathematical ability to reasoning and academic standing. For this, the researchers selected 113 college students. The students were divided into two groups based on mathematical ability; high in mathematical ability and low mathematical ability. Then all the students filled a reasoning ability test, which is not mathematical in its nature. The results of the study showed that there is a significant relationship between mathematical ability and reasoning ability. High mathematical ability may not provide high reasoning as much as high reasoning provides high mathematical ability. Low mathematical ability excludes high reasoning more than low reasoning excludes high mathematical ability. Also, it was found that it is more difficult to have high reasoning ability than to have high mathematical ability. For a person with high reasoning to have high mathematical ability has more probability than for a person with high mathematical ability to have high reasoning. Also, for a person with low reasoning to have high mathematical ability has more probability than for a person with low mathematical ability to have high reasoning. According to the results, mathematical reasoning is abstract, and is quantitative concept. But non mathematical reasoning is more complex. Intellectual ability which represents high academic standing has a relationship with reasoning ability. High academic standing may not provide high reasoning, but high reasoning ability provides high academic standing excellently. Low reasoning ability does not exclude high academic standing, but low academic standing excludes high reasoning ability. If academic standing is low, then high reasoning ability is excluded. But, without high reasoning and with low academic standing, mathematical ability is not excluded.

Also, Jeotee (2012) searched the reasoning skills, problem solving ability and academic ability of final year university students who choose different academic programs. The researcher searched the effects of academic ability on reasoning skills and problem solving ability, and the effects of reasoning ability and problem solving on academic ability. Also, the researcher searched if students in different programs showed different levels of reasoning and problem solving. There were 333 participants who were final year student in university. The results of the study showed that reasoning skills and problem solving ability had effect on each other nearly 30 percent. But, academic ability did not have so much effect on reasoning skills and problem solving ability. The students in similar programs showed the same reasoning level and the same problem solving level. Students in different programs showed different level of reasoning and different level of problem solving. Gender created difference in reasoning skills but not in problem solving ability. The relationship between reasoning skills and the problem solving ability was approximately 28 percent. But, the relationship between reasoning skills and the academic ability was less than 3 percent. The relationship between academic ability and problem solving ability was less than one percent.

In another important study, in order to search the reasoning abilities of poor achievers vs. normal achievers using computer game tasks, Dagnino, Ballauri, Benigno, Caponetto and Pesenti (2013) conducted a study with 118 fourth grade students in primary schools, comprising 27 students as poor achievers. The researchers measured participants' logical abilities and academic skills, since the aim of the study is to search for a relationship between school performance and logical reasoning, and to analyze the major cognitive abilities of computer games. The researchers used cognitive abilities test to measure logical reasoning abilities, and used reading test, spelling test and mathematical achievement test in order to measure academic skills. Then the researchers used Logivali Test to assess the abilities in the computer games. The study also searches the emotional and behavioral aspects of poor achievers. The students who show significantly lower performance in cognitive test, reading, spelling and mathematical test form the poor achievers. In Logivali test, ability 1 refers to knowing the rules of the game. Ability 2 refers to first level reasoning, to make an inference from a single datum. Ability 3 refers to second level reasoning, to make an inference from two pieces of information. Ability 4 refers to third level reasoning, to make an inference from more than two pieces of information. Ability 5 refers to managing uncertainty, to decide if the data given is enough to decide whether a guess or given data is correct or not. Ability 6 refers to operatively apply reasoning abilities, to solve a game step by step, to reach solution. In abilities 2,3 and 4, there is a difficulty progression. The results of the study show that poor achievers have significantly lower scores in ability 3,4 and 6 when compared to normal achievers. There is also significant difference between poor and normal achievers in the ability 3 and 4, which shows that when the difficulty of task increases, this difference also increases. The results show that there is a significant relationship between school achievement and logical reasoning abilities. The poor achievers who have low school achievement show lower performance in activities which require use of logical abilities. When the difficulty of task increases such as ability 3 and 4, the difference of the performance between the two groups increases. For emotional, motivational and behavioral aspects, poor achievers are attentive and motivated despite the difficulties.

In addition, to emphasize the relationship between reasoning ability and mathematical performance, Usman and Musa (2013) searched the influence of formal operation abilities on mathematical performance. The participants were 400 senior secondary students. The formal operations test in order to measure Piagetian formal reasoning operation and mathematics performance test were conducted. According to the results, students' mathematical performance was low. Also, number of students who always use formal operations abilities and those who never use formal operations abilities were nearly equal. That supports Piaget's statement that the students who have formal operation abilities may not use these abilities always. Moreover, there was a positive and significant relationship between formal operation scores and mathematical performance of the students. Then, senior students' formal operations scores was the predictor of the students' performances in mathematics. Similarly, students' performances in mathematics identify the students' formal operation levels. In addition, formal reasoning operations significantly influenced performance of the students in mathematics. The students who always use formal operation abilities showed significantly higher mathematical performance than the students who never use formal operation abilities. Then, the students' development of reasoning ability and their formal operation levels affect their performance in mathematics. So, Usman and Musa (2013) stated that use of the thinking and reasoning patterns are very important for their mathematical performance. The teachers should measure students' formal operation levels and trigger students to use formal operation abilities in order to improve students' mathematical performance.

2.2.2 Relationship Between Reasoning Ability and Metacognition

It is commonly accepted that problem solving is what mathematics is all about. So, mathematics teachers' main aim should be to help students improve their problem solving abilities. For this aim, they should teach mathematics throughout problem solving process. Then, the students will learn new mathematical concepts and integrate mathematical knowledge throughout problem solving (Donaldson, 2011). According to Donaldson (2011), in her study, the researcher searched the teaching practices of teachers who teach mathematics throughout problem solving. The results of the study showed that these teachers commonly firstly teach problem solving strategies to students, model problem solving, limit teacher input, promote metacognition and emphasize multiple solutions of problems. So, these actions are parts of problem solving. In order to be a successful problem solver, some elements should take place. According to Schoenfeld (1985, 1992), in his problem solving framework, these elements are resources, problem solving strategies involving heuristics, control, and beliefs and affects. Resources refer to the knowledge base, and resources involve mathematical knowledge such as facts, concepts, algorithms, and routine procedures. In fact, mathematical knowledge alone is not enough to be a competent problem solver. To make the problem solver use his resources effectively, problem solving strategies should be used. So, problem solving strategies involving heuristics are also an important element. Next, the third element is control, which is a part of metacognition. Metacognition refers to knowledge of one's own cognition, monitoring and controlling one's own cognitive processes, and reflection. Here, control refers to resource allocation, and determination of what resources will be useful, defining which strategies will provide effective solution. It involves deciding appropriate choices and monitoring his own progress throughout problem solving process. Similarly, Kilpatrick (1985) also emphasized the importance of “resources, strategies, and control. The researcher stated that to be a successful problem solver, a person should have organized domain knowledge, should know techniques for representation and transformation of the problem, and metacognitive process in order to monitor and guide his own performance. The last elements; beliefs and affects are about the problem solvers’ understandings and feelings” (Kipatrick, 1985).

Mevarech and Kramarski (2003) conducted a study in order to assess the effects of metacognitive training versus worked-out examples on students’ mathematical reasoning and mathematical communication, and also to measure the long-term effects of the two methods on students’ mathematical achievement. In the study, there were two groups, one group focused on worked out examples (WE) whereas the second group focused on metacognitive training (MT). In both of the two groups, the two methods were applied in cooperative environment with emphasis on problem’s essential parts and suitable problem solving strategies. The study has continued for two years. For the first year of the study, there were 122 eighth grade Israeli students who studied algebra, as participants. Also, eight groups with 32 participants have been videotaped during problem solving process and their problem solving behaviors has been videotaped and analyzed. One year later, the participants were at the ninth grade and they were conducted the same test which was conducted in the eighth grade, in order to measure the students’ mathematical achievement. This test was used as pretest, and immediate post-test and one year later as a delayed post-test. The results of the study showed that students with metacognitive training showed better performance than the students with worked-out examples method on both the immediate and delayed post-tests in mathematical achievement. The differences between the two methods on students’ ability to explain their mathematical reasoning were gained during the discourse and in writing. Lower achievers improved more under the metacognitive training than under work-out examples method. Also, students with metacognitive training showed more performance to explain their mathematical reasoning verbally, algebraic representations of verbal situations, and algebraic solutions. Also, students with metacognitive training showed more performance to explain their mathematical reasoning.

In addition, Mevarech and Fridkin (2006) state that when the metacognitive instructional method called IMPROVE is conducted to the students, the students' mathematical knowledge, mathematical reasoning and meta-cognition significantly increases. In this study, there were 81 students who have taken a pre-college course in mathematics. With random assignment, the participants were divided into two groups. One group was control group involving traditional instruction. The second group was experimental group involving IMPROVE metacognitive training instruction. The IMPROVE group had explicit training of metacognitive process during mathematical problem solving. Results of the study showed that improve students showed significantly better performance on both mathematical knowledge and mathematical reasoning. Also, metacognitive instruction significantly improves students' general knowledge of cognition and regulation of general cognition as well as their mathematical achievement. Metacognitive instruction also affects the domain-specific meta-cognitive knowledge positively, which includes using strategies before-during-after the problem (Mevarech & Fridkin, 2006).

Also, Kramarski and Mevarech (2003) investigated the effects of cooperative learning and metacognitive training on mathematical reasoning. The writers conducted a study in order to search the effects of four instructional methods on students' mathematical reasoning and metacognitive knowledge. There were 384 eighth-grade students as participants. The first instructional method in the study was the cooperative learning combined with metacognitive training, shown as COOP+META. The second instructional method was individualized learning combined with metacognitive training, shown as IND+META. The third instructional method was cooperative learning without metacognitive training, shown as COOP. The final instructional method was individualized learning without metacognitive training, shown as IND. According to the results of the study; the COOP+META group significantly showed significantly better performance than the IND +META group. The IND+META group showed significantly better performance than the COOP and IND groups on graph interpretation and various aspects of mathematical explanations and reasoning. Moreover, the metacognitive groups (COOP+META and IND +META) showed significantly better performance than the other non-metacognitive groups (COOP and IND) on graph construction (transfer tasks) and metacognitive knowledge. In addition, Kramarski (2008) investigated the effect of metacognitive guidance on teachers' algebraic reasoning and self-regulation skills. The participants were sixty-four Israeli elementary school teachers. The participants engaged in a 3 year professional development program which aimed to enhance mathematical knowledge. One group of teachers achieved IMPROVE metacognitive questioning, the other control group achieved no metacognitive guidance. According to the results of the study, the group with metacognitive guidance showed significantly better performance than the group with no metacognitive guidance on a lot of algebraic procedural and real life tasks, which required conceptual mathematical explanations and reasoning. Moreover, the group whit metacognitive guidance showed better performance than the group with no metacognitive guidance, on the usage of self monitoring and evaluation strategies in algebraic problem solving.

Moreover, Kramarski and Hirscha (2010) investigated the differential effects of computer algebra system (CAS) and metacognitive training on mathematical reasoning. There were 83 students as participants who studied algebra in four eighth-grade classrooms. The students were randomly assigned to four instructional methods. The first instructional method was the computer algebra system CAS with metacognitive training (CAS +META). The second instructional method was metacognitive training without CAS learning (META). The third instructional method was CAS learning without metacognitive training (CAS). The final instructional method was traditional learning without CAS and without metacognitive training (CONT). According to the results, the CAS +META students showed significantly better performance that the META and the CAS students. Also, the META and CAS students showed significantly better performance than the CONT students on several aspects of mathematical reasoning. There was no significant difference between the META and CAS students. Moreover, the metacognitive students (CAS +META and META students) showed better performance than non-metacognitive students (CAS and CONT) on their metacognitive knowledge. However, there is a study which could not find a relationship between metacognition and reasoning ability.

Maqsud (1997) claimed that both metacognitive ability and nonverbal reasoning ability have significant relationship with mathematics and English performances of students. So, Maqsud (1997) searched the effect of metacognitive skills and nonverbal reasoning ability on the students'' performances in mathematics and English tests, and also the relationship between metacognitive strategies and nonverbal reasoning ability. For this, the researcher conducted two experiments with senior high school students. The results of the study showed that both metacognitive ability and nonverbal reasoning ability have significant relationship with mathematics and English performances of students. The general reasoning ability has significant effect on mathematics, and metacognitive ability also has significant effect on mathematics. But there is no interaction effect between metacognition and reasoning. The students with high reasoning ability and high metacognition showed the highest performance on mathematics. But, the students with high reasoning ability and low metacognition showed lower performance. Moreover, students with low reasoning and high metacognition showed lower performance than the former one. Finally, students with low reasoning and low metacognition showed the lowest performance on mathematics. But, when we keep high reasoning ability constant, the high metacognitive students showed higher performance than low metacognitive students. This result is the same if we keep low reasoning ability constant. Moreover, if we keep high metacognition constant, students with high reasoning ability showed higher performance in mathematics achievement than the students with low reasoning ability. Similarly, if we keep low metacognition constant, highly reasoning students showed higher performance than students with low reasoning ability. Also, the effect of reasoning ability and metacognitive ability had significant effects on English tests. Moreover, high metacognitive students showed higher performance on mathematics and English test when compared to low metacognitive students, regardless of nonverbal ability of reasoning.

Similarly, Kramarski, Mevarech and Lieberman (2001) searched the effects of multilevel versus unilevel metacognitive training on mathematical reasoning. The researchers searched the effects of three instructional methods on mathematical reasoning. The first method was the cooperative learning within multilevel metacognitive training (MMT). The second method was cooperative learning within unilevel metacognitive training (UMT). The last method was learning in the whole class with no metacognitive training as a control group. MMT students studied both mathematics and English; UMT students studied only mathematics; and the whole class with no metacognitive training students did not study with the metacognitive method. The results of the study showed that the MMT group showed significantly better performance than the UMT group, and the UMT group showed significantly better performance than the control group on mathematical achievement. The MMT and UMT group showed significantly better performance than the control group on mathematical explanations or reasoning ability. But there is no significant difference between MMT and UMT groups on mathematical explanations and reasoning ability. the second aim of the study was to search the effects of the three methods on students' ability to solve an authentic, real-life problem, which were not provided in the classroom. The result showed that there were significant differences between the three learning conditions. The MMT group showed significantly better performance than the UMT group on the total score. The UMT group showed significantly better performance than the control group on the total score. The third aim was to search the differences in metacognitive knowledge among the three methods. The MMT group showed significantly better performance than the UMT group. The MMT group showed significantly better performance than the control group on the total score. But there were no significant differences between the UMT and the control group.

Kramarski, Mevarech and Lieberman (2001) stated that metacognitive training which enhances students to focus on the similarities and differences between previous and new tasks, to comprehend a problem before trying a solution, and to think about the use of appropriate strategies for solutions improves mathematical reasoning. The metacognitive training improves metacognitive knowledge, which, in turn, improves mathematical reasoning and students' ability to transfer their previous knowledge to new situations.

2.2.3 The Relationship Between Metacognition and Problem Solving

Mayer (1998) states that good problem solvers firstly need to know the domain-specific knowledge which is called as the problem solver's skill. This domain-specific knowledge includes the basic problem solving skills and some cognitive skills for the specific subject matters. But, being good on each component skill is not enough to develop problem solving transfer and does not guarantee being good problem solver. The second crucial ingredient is the ability to control and monitor cognitive processes. They should be aware of the knowledge of when to use, how to coordinate, and how to monitor a variety of skills in problem solving. This property is the problem solver's metaskills or metacognitive knowledge, thus, metacognitive skill is an essential part of problem solving process (Mayer, 1998). Similarly Nickerson (1994) explains the relationship between metacognition and problem solving as that people including students sometimes cannot be able to apply the relevant and previous knowledge in their minds to the problems that they are trying to solve. In other words, they cannot connect the knowledge needed with previous knowledge required for solving a new problem. There are reasons for this situation. One reason is that they are not aware of the relevance of the knowledge that they have, thus they are not aware of the applicability of a specific strategy to a given problem. Second reason is that they simply cannot reach the knowledge when they need it. At this point, the metacognitive training should be used with aims to help people or students manage and control their cognitive resources more effectively (Nickerson, 1994). Moreover, Lester (1994) also emphasizes the role of metacognition on problem solving. Metacognitive actions are considered as triggering forces in problem solving process throughout beliefs and attitudes since the problem solving requires cognitive and affective actions. In fact, during the problem solving process, good solvers focus on the determination of their goals, understanding the concepts and discovering the relationships among the elements of problem, monitoring their understanding and learning, and selecting and evaluating the actions and choices to reach the goals. Also, problem solving demands being aware of both what to monitor and how to monitor performance, as well as unlearning bad habits (Lester, 1994). Also, Mevarech and Fridkin (2006) explain the relationship between metacognition and problem solving throughout problem solving steps. Metacognition and problem solving are interrelated to each other very closely. Since the problem solving process involves problem solving steps, and Polya's problem solving steps are interrelated with metacognitive skills, we cannot keep problem solving away from metacognition. If a teacher teaches Polya's steps in problem solving, then he/she supplies metacognitive training at the same time (Mevarech & Fridkin, 2006).

The claim that metacognition increases problem solving performance is approved by many researchers. Jonassen (2000) states that students with high metacognitive skills are able to encode the nature of the problem in a strategic way throughout the mental representations of the problem. Also, they are able to choose appropriate plans and select the best one to solve the problem. Moreover, they identify the possible obstacles to solve the problem and they overcome those obstacles. In fact, throughout orienting and self-judging, they show high problem solving performance (Jonassen, 2000). Also, Schoenfeld (1983, 1985, 1987, and 1992) stated that expert problem solvers frequently attend metacognitive acts by looking back and reflecting upon the strategies, solutions they use during problem solving process. The experts monitor and reflect on their thinking by seeking answers to the questions about planning. These questions are: “is this correct way?” “is there another different representation of the problem?”. By asking such questions, the experts think about alternative approaches and different strategies. Also they choose different approaches depending on their previous experiences. However, the novice problem solvers often choose one approach and they become fixed on that approach. They follow that approach relentlessly, but sometimes unprofitably.

In addition, Veenman, Wilhelm and Beishuizen (2004) searched the relationship with metacognitive skill development and intelligence. Also, the researchers searched whether the metacognitive skills are general or domain specific. There were four groups of participants; fourth grade, sixth grade, eighth grade, and university students; and they engaged in inductive learning tasks in different domains. The participants’ intelligence, metacognitive skillfulness and learning performances were measured. According to the results, metacognitive skillfulness is not domain specific. Metacognitive skillfulness is a general, domain-free and a personal characteristic which changes with age. Also, metacognitive skills play role in the development of learning, and metacognitive skills conclude in more learning performance, regardless of intelligence.

Similarly, Mevarech (1999) claims that metacognition training develops mathematical problem solving performance. The researcher provides 3 cooperative learning environments on mathematical problem solving. First one occurs in group interactions throughout metacognitive training by constructing connections and strategy application. In the second one, the interactions occur throughout direct instruction by strategy application without construction of connections. In the third one, interactions occur neither through metacognitive training nor through strategy instruction. The students trained within metacognitive training environment showed significantly higher mathematical problem solving performance than the other two groups. Also, the students trained with direct strategy instruction showed significantly higher problem solving performance than the control group, in which students trained neither metacognitive, nor direct strategy instruction (Mevarech, 1999). In addition, Mevarech and Fridkin (2006) also support this idea and add new relations. The researchers state that when the metacognitive instructional method called IMPROVE is applied to the students, the students' mathematical knowledge, mathematical reasoning and meta-cognition significantly increases. Metacognitive instruction also significantly improves students' knowledge of cognition and regulation of cognition as well as their mathematical achievement. Metacognitive instruction also affects the domain-specific meta-cognitive knowledge positively, which includes using strategies before-during-after the problem (Mevarech & Fridkin, 2006).

Similarly, Berardi-Coletta, Buyer, Dominowski and Rellinger (1995) claim that the process oriented solvers who are metacognitive solvers at the same time showed statistically significantly higher performance on problem solving than nonprocess control groups on both training and transfer tasks. Moreover, these process-oriented/metacognitive solvers showed more sophisticated problem representations and develop more complex strategies than others (Berardi-Coletta, Buyer, Dominowski & Rellinger, 1995). In addition, Swanson (1990) searched the effect of metacognitive knowledge and aptitude on problem solving, and if high levels of metacognitive knowledge on problem solving gives an equivalent to low overall aptitude. For this, think aloud protocols were conducted to 56 students who were at 5 and 6 grades. The results showed that regardless of aptitude, higher metacognitive children showed better performance than the lower metacognitive children on problem solving. In fact, high metacognitive knowledge with low aptitude students showed significantly better performance than the lower metacognitive knowledge with higher overall aptitude students. Moreover, the high aptitude with high metacognitive knowledge students used the strategy subroutine more frequently than the other groups. Also, students with higher metacognitive ability had more tendencies to depend on "hypothetico-deductive (if-then propositions) and evaluation (check the adequacy of a hypothesis) strategies" than the students with lower metacognitive ability.

Moreover, Muis (2008) searched the relationship between epistemic profiles, self-regulation or regulation of cognition and mathematical problem solving. There were two hundred sixty-eight participants taking undergraduate mathematics and statistics courses. Students' epistemic profiles were divided as rational, empirical or both. The results showed that self-reported metacognitive self-regulation and regulation of cognition in the process of problem solving, the students whose profile is rational performed the highest self-reported mean, as well as real frequency of regulation pf cognition. These students showed higher self-reported and actual regulation of cognition than the students whose profiles predominantly empirical. Also, students whose profile is predominantly rational performed better problem solving and solved more problems correctly than the other students of other profiles. Moreover, students' epistemic profiles were consistent with their approaches to problem solving.

Also, Babakhania (2011) searched whether the cognitive and meta-cognitive strategies such as self-instruction procedure teaching affect the verbal mathematical problem solving (VMPS) performance. The participants were sixty primary school students with VMPS difficulties. The experimental group has taken strategies instruction for two months. According to the results of the study, cognitive and metacognitive strategy and self-instructional procedure teaching significantly increased the verbal mathematical problem solving performance of students. Teaching the cognitive and meta-cognitive strategies such as self-instruction procedure increased the mathematical word problem solving of students with problem solving difficulty. Also, that strategies instruction affected students' knowledge and knowledge use better. In addition, the strategy instruction improved the control of mathematical word problem solving strategies and the awareness of these domains. Moreover, self-instruction procedure as a metacognitive strategy affected mathematical problem solving positively and increased the problem solving performance of the students with difficulties. In addition, Coutinho, Wiemer-Hastings, Skowronski and Britt (2005) searched how metacognition, need for cognition, and explanations or information about problem solutions influence task performance and metacognition in the process of learning and problem solving. For this, the researchers conducted two experiments. In both of the experiments, students solved analytical problems in Graduate Record Exam (GRE). Metacognitive performance which refers to accuracy calibration was measured after each problem. When the students performed in the first block of items, they got two forms of feedback on the second block. In Experiment one, after each problem, the students make a choice. They either get the solution with an explanation, or just get the solution. In Experiment two, all students were provided the solutions with explanations. In both of the experiments, the students solved the second block of problems. The students' need for cognition, their level of trait metacognition and their performance on the problems in the second block was examined to test whether students' need for cognition, level of trait metacognition, or the tendency to obtain problem explanations affected task performance. The researchers searched if high levels of task-related metacognition conclude in high task performance, if explanation feedback develops task-related metacognition and task performance more than other feedback types, and if students have willingness to look for and use feedback. The results of the study provided that when students have more experience at a task, they show better estimation of their task performance. The students who estimate their task performance better showed more success on these tasks. The students who have high need for cognition searched the problem explanations more frequently than the students who have low need for cognition. However, students who have high trait metacognition did not search for problem explanations more frequently than the students who have low trait metacognition. The students with high need for cognition showed better task performance than the students with low need for cognition. According to the results of the study, students with high metacognition and high need for cognition showed better problem solving performance than students with low need for cognition. The researchers stated that metacognitive skill conclude in higher performance for a variety of skills. Consistently, the result supported that view and showed that students who were calibrated better with high metacognition, performed better on problem tasks.

In order to support the relationship between metacognition and problem solving, Evans and Swan (2014) provided a design strategy which aim to support self and peer assessment, and to improve students' ability to think and discuss the different solution strategies of the problems in mathematics lessons. In this lesson design, the students were given problems, and time to solve these problems. After they uniquely solved the problems, they were given "sample students work" in order to engage the students in a discussion and critique. The researchers examined the use of this strategy and the outcomes in the trial periods in US and UK classrooms. The researchers claimed that this strategy will improve students 'metacognitive acts since they engage in reflections of their own decisions, and their own planning actions and solution in the mathematical problem solving process. In this design, self and peer assessment was expected to make students to look back to the process, to reflect upon their thinking in the process and to focus on working on ideas, rather than working through task. Then, the students would find opportunities to attend to metacognitive acts by thinking on alternative approaches and evaluating these different approaches to non-routine problems. In this design the students engage in arguments or discussions, compare the effectiveness of arguments, and differentiate correct logic or reasoning and explain these, and critique the reasoning of their peers. By discussions, participating in arguments of others, by comparing their own ideas with others, by determining what make sense and correct reasoning, by asking questions to critique, clarify and develop the arguments, students engage in metacognitive acts. At the end of the trial studies, the researchers could not gain clear evidence for increased solution strategies of students at the end of the trials. But, the researchers had early indications of increase in students' available solution methods, and the students started to write clearer, longer, and fuller explanations in a more detailed manner as a result of critiquing sample student work.

Similarly, Hoffman and Spatariu (2008) searched the effect of self-efficacy beliefs and metacognitive prompting upon mathematical problem solving accuracy and efficiency and mental multiplication problems. For this, the researchers chose the mathematical background knowledge and problem complexity as controlling variable. There were 81 participants who were the university students taking educational psychology courses. There were 42 mental multiplication problems. Students' correct responses as problem solving accuracy, the time passed to solve these problems as response time and problem solving efficiency as the ratio of problems divided by response time were measured. Also, students' mathematical background knowledge and their self-efficacy for mental multiplication accuracy were also measured. The participants were randomly divided into a prompting or control group. The results of the study showed that metacognitive prompting affected significantly both problem solving accuracy and problem solving efficiency. When the complexity of problem increases, metacognitive prompting concludes in more cognitive awareness and usage of unmindful problem solving strategies. The students with high self-efficacy performed more accurate and efficient problem solving independently from metacognitive prompting. Also, metacognitive prompting affected significantly the accuracy and efficiency for more complex problems. The results showed that self-efficacy and metacognitive prompting improved problem-solving performance and efficiency separately.

Moreover, Özsoy (2007) support the previous literature and searches the effects of metacognitive strategies on mathematical problem solving achievement of fifth grade primary school students. There are 47 participants who are fifth grade students. The experimental group in the study had metacognitive problem solving activities for nine weeks, and the control group had regular instruction without any metacognitive strategy. The results of the study showed that the metacognitive treatment group showed significantly higher problem solving achievement and metacognitive skills than the group without any metacognitive treatment. Also, the metacognitive treatment group showed significant improvement in "devising a plan" scores of problem solving achievement while there was no improvement in other subcategories of problem solving process. The non-metacognitive treatment group did not show any improvement in any subcategory of problem solving process (Özsoy, 2007). In addition to the previous study, Özsoy and Ataman (2009) provide another support for the effect on metacognitive strategy on problem solving achievement. The researchers investigate the influence of using metacognitive strategy training on mathematical problem solving achievement. The students who take metacognitive strategy training showed significantly higher metacognitive knowledge and metacognitive skills than the students who don't take metacognitive training. Moreover, the students trained within metacognitive strategy training showed significantly higher mathematical problem solving achievement and performed significant increase in problem solving skills than the other students who are not trained for metacognition (Özsoy & Ataman, 2009). But, Yılmaz (2003) could not support the claim that metacognitive training increases problem solving performance. The researcher investigates the effects of metacognitive training on seventh grade students' problem solving performance. There are 72 participants who are 7^{th} grade students in the study. There are three groups in the study: in one group, the students are asked questions to guide their cognition and metacognition during problem solving process throughout peer reciprocal questioning format. In the second group, students answer the same questions individually, rather than in peer format. In the third group, the students have regular instruction which does not include metacognitive questions. The results of the study showed that there was not any significant difference between the pair group, individual group and the regular group in terms of the post-test and exam problem. This result did not confirm the expectation that metacognitive training creates a difference in students' problem solving performances. But, the results showed that metacognitive training increases students' understanding of the problems and representing of the problems, independently from the completion of the solution (Yılmaz, 2003).

There are many research that support the idea that there is a significant relationship between metacognition and problem solving skills. For instance, Balcı (2007) searches the relationship between meta-cognitive skill levels and problem solving skill levels of fifth grade primary school students. The study is conducted on 269 fifth grade students (127 females and 142 males) in Adana. The results of the study show that there is a significant relationship between meta-cognitive skill levels and problem solving skill levels of the students. There is no significant difference on meta-cognitive skill levels and problem solving achievement levels of the students in terms of gender. However, in terms of socioeconomic status there is a significant difference between lower and middle class ; and also between lower and upper class students in terms of problem solving skill levels and metacognitive skill levels (Balcı, 2007). Similarly, Yıldırım (2010) searches relationships between college students' metacognitive awareness and solving similar types of mathematical problems. There are 97 participants who are at the first class at mathematics department in Tokat Gaziosmanpaşa University. The results of the study showed that there is a significant relationship between students' metacognitive awareness levels and types of mathematical problem solving levels. The relationship between the problems requiring more skill to solve and metacognitive awareness is significantly higher than the relationship between the problems requiring fewer skills to solve. Also the types of problem solving and the level of metacognitive awareness are not significantly different in terms of gender (Yıldırım, 2010). Also, Kışkır (2011) searches the relationship between metacognitive awareness and problem solving skills of the preservice teachers. There are 402 participants who are the 3rd and 4th class university students in primary education. The results of the study showed that there is significant relation between pre-service teachers' metacognitive awareness levels and their perceptions of problem solving skills. The metacognition levels do not show significant difference in terms of gender and in terms of classroom level (Kışkır, 2011).

There are some studies in literature which support the claim that metacognition has a relationship with mathematics achievement. For example, Alcı (2007) searches the relationship between the points of achievement in ÖSS, perceived problem solving abilities, self-efficacy perception and metacognitive self-regulation strategies related to mathematics achievement. There are 806 participants (208 females and 598 males) who are students in different departments in Yıldız Teknik University. The results of the study showed that there is a significant positive relationship between students' self-efficacy perception and perceived problem solving abilities; metacognitive self-regulation strategies and problem solving abilities; self-efficacy perception and metacognitive self-regulation strategies. Also, the students' self-efficacy perception, metacognitive self-regulation strategies, and ÖSS quantitative points predicts mathematics achievement significantly, while perceived problem solving abilities do not predict (Alcı, 2007). Similarly, Karaoğlan (2009) searches the relationship between 6th grade students' problem solving achievement and mathematics achievement scores after completing instruction on problem solving. There are 170 participants who are sixth grade private school students in the study. The results of the study show that there is a significant positive relation between students' problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores (Karaoğlan, 2009).

In summary, according to the previous studies, there is a relationship between metacognition and reasoning ability; between metacognition and problem solving performance; and between reasoning ability and problem solving performance (Antonietti, Ignazi & Perego, 2000; Berardi-Coletta, Buyer, Dominowski & Rellinger, 1995; Carlson & Bloom, 2011; Higgins, 1997; Kışkır, 2011; Mevarech & Fridkin, 2005; Mevarech & Kramarski, 2003; Ozsoy & Ataman, 2009).

The main aim of this study is to understand the relationship among metacognition, reasoning ability and the mathematical problem solving performance of the ninth grade students. In the previous studies, the researchers generally select two of the variables; such as examining metacognition and reasoning ability; or examining metacognition and problem solving performance; or reasoning ability and problem solving performance. In contrast, in this study, these three variables; namely metacognition, reasoning ability and problem solving performance will be examined in one study. Also, in the previous studies, the correlational studies are not very common about the problem solving, reasoning and metacognition; rather, experimental designs or other designs are more common. As well as manipulation of metacognition or reasoning ability on problem solving, it is important to examine the relationship in its nature; without any intervention. So, the correlational studies are important and there is a need for the correlational study of metacognition, reasoning ability and problem solving. So, in this study, it is expected that there will be a correlation among metacognition, reasoning ability and problem solving performance as expected from the previous studies (Antonietti, Ignazi & Perego, 2000; Berardi-Coletta, Buyer, Dominowski & Rellinger, 1995; Carlson & Bloom, 2011; Higgins, 1997; Ozsoy & Ataman, 2009). The contribution of the study to the literature is that the current study examines three variables in one study; and in a correlational design. In the previous studies, the researchers generally select two of the variables; but in this study, these three variables will be examined in one study on ninth grade elementary students; which is not generally chosen by the researchers who are interested in metacognition or problem solving or reasoning ability.

Potential implications of the study may be the emphasis on the importance of the application of metacognition education which involves the courses explaining metacognition construct for pre-service teachers. Also, the courses explaining metacognition constructs or the lessons involving metacognitive training may be given in all departments related to mathematics or educational sciences. Moreover, another implication may be the emphasis on the importance of using problem solving steps during problem solving process. Problem solving lessons or courses may be studied in the educations of pre-service elementary mathematics teachers. In addition, the emphasis on reasoning ability may be increased in classrooms, or in educational programs or in education faculties of universities.

CHAPTER 3

METHODOLOGY

In this chapter, the design of the study, the properties of population and the participants, the instruments to collect data, the data collection process, the pilot study, the reliability and validity, the statistical data analysis, limitations and assumptions of the study are explained.

3.1 Design

The design of this study was selected as a correlational study since the main aim of this study was to investigate the relationship among metacognition, reasoning ability and problem solving performance of the ninth grade students. Fraenkel and Wallen (2006) state that in a correlational research, the relationships among two or more variables are investigated. Also, there is no manipulation of variables or intervention to variables, there is not any attempt to influence the variables. Correlational study describes an existing relationship among variables (Fraenkel & Wallen, 2006). So this study is correlational.

3.2 Participants

The target population of the study is all of the ninth grade students in İzmir. The accessible population of the study is all of the ninth grade Anatolian high school students in North and West district in İzmir. The characteristics of the target and accessible population, and also sample are that the participants are the ninth grade students. The participants live in İzmir, and they are students at public Anatolian high schools in İzmir. The age, and ethnicity of the participants may change. The target population involves approximately 23000 ninth grade students including private and public high schools. The accessible population involves approximately 5780 ninth grade students. The sample of the study consists of exactly 578 ninth grade Anatolian high school students in İzmir, Turkey. In the current study, the results of the study may be generalized to West and North district of İzmir due to the accessible population number.

There are 123 public high schools and 181 total high schools including private schools in İzmir, and total 78280 students in all grades in public high schools in İzmir according to 2013-2014 school years (Minister of National Education-MoNE, 2014). Also, there are 11008 male students, and 12363 female students who are at ninth grade in public or private school in İzmir. There are 123 public and 58 private schools, and totally about 23000 ninth grade students in public or private schools. The sample of the study is 578 ninth grade students in public Anatolian high schools. The data was collected from seventeen public Anatolian high schools in North and West of İzmir. There are 47 public Anatolian high schools in West and North districts of İzmir.

There are many types of high schools in Turkey: Science High Schools, Social Sciences High Schools, Anatolian High Schools, Vocational and Technical High Schools, Plentiful Programmed High Schools and Religious High Schools. The most successful students enter to Science High Schools with a national exam. The other successful students enter to Anatolian High Schools with a national exam. There are many reasons for selecting Anatolian High Schools in the current study. First of all, it is important to study with more successful students, in order to get healthy analysis results. In addition, the more successful students answer the instruments more seriously, more willinglessly and effortfully. There is few Science High Schools in each city in Turkey, and there are more Anatolian High Schools. In order to reach more successful students in İzmir, Anatolian High School was chosen, instead of Science High Schools.

In the current study, convenient sampling is chosen. The accessibility of students, and the schools' proximity to the researcher, and the accessibility of the schools and permissions of the school administrations defined the convenience sampling. The north and west of İzmir was chosen, because the district was accessible, reaching students was inexpensive and easy; moreover, the most crowded districts in İzmir are North and West districts. The generalization threat of convenient sampling was accepted in the limitations part in this chapter.

3.3 Instruments

3.3.1 Junior Metacognitive Awareness Inventory for Metacognition

There were three instruments in this study. The first instrument was used for measuring metacognition. Firstly, the participants filled the first instrument: jMAI; namely the Junior Metacognitive Awareness Inventory (Sperling, Howard, Miller & Murphy, 2002). The instrument was developed for students from sixth grade to ninth grade in order to evaluate students' metacognition in two major constructs: knowledge of cognition and regulation of cognition. Knowledge of cognition refers to individual's knowledge about his-her own capabilities, beliefs, cognitive abilities and processes. Regulation of knowledge refers to individual's knowledge about his-her own control processes during the execution of the task (Brown, 1978). The instrument translated into Turkish and validated by Ubuz and Aydın in 2010. The instrument aims to measure students' knowledge of cognition throughout 8 items and regulation of cognition throughout 9 items. The instrument consists of 17 items and students give their answers to the each item on a 5-point Likert scale ranging from 1-never and 5-always. There is no negative statement, and no items will be recoded. The maximum point of the instrument is 85 and the minimum point is 17. For the reliability analysis, the Cronbach alpha value is .75 for knowledge of cognition which includes items 1,2,3,4,5,11,12,13 and .79 for regulation of cognition including items 6,7,8,9,10,14,15,16,17. If the alpha value is higher than .70, the results are reliable (Crocker &Algina, 1986). Thus, the instrument is valid and reliable. The instrument involves some general items: 14, 16 and 17. In these three items, the items involve highly general words and meanings. The items may not be perceived “directly” related to problem solving. Hence, during the conduction of this instrument, the students were frequently warned about that the instrument should be filled related to mathematics and the problem solving process. At the beginning of the instrument, the warning was written: the students should fill this instrument based on their mathematical lessons. Also, the researcher frequently reminded the students to fill the instrument by thinking of their mathematical lessons and their problem solving process.

3.3.2 Test of Logical Thinking for Reasoning Ability

The second instrument used in the study was the Test of Logical Thinking Inventory (TOLT). Students’ reasoning abilities were measured by TOLT which was developed by Tobin and Capie (1981), and translated into Turkish and validated by Geban, Aşkar and Özkan (1992). The test measured one major underlying dimension termed as formal thought. Each item measures on one underlying dimension; formal reasoning ability. The test involves ten items which measure five reasoning modes. The first mode which is “proportional reasoning” is measured by item 1 and 2. The second mode which is “controlling variables” is measured by item 3 and 4. The third mode which is “probabilistic reasoning” is measured by item 5 and 6. The forth mode which is “correlational reasoning” is measured by item 7 and 8. The final mode which is “combinatorial reasoning” is measured by item 9 and 10. For items from 1 to 8, students answer each item by selecting a response and providing a reason for selecting that response. The students who select both the best answer and the best justification will get 1 correct score. For the items 9 and 10, the students write all the possible combinations and get 1 correct score. The total correct score for the test is 10. The maximum point of the instrument is 10, and the minimum point is 0. The Cronbach alpha internal consistency for the test was found to be r = .85. If the alpha value is higher than .70, the results are reliable (Crocker &Algina, 1986). So the instrument is reliable.

3.3.3 Mathematical Problem Solving Instrument

Finally, the last instrument was used for measuring mathematical problem solving performance. For this purpose, the participants were asked to solve ten mathematical problems. The Mathematical Problem Solving Instrument was developed by Taşpınar (2011). There are ten problems related to mathematics, some of them are routine problems, some of them are non-routine problems. The format of items was constructed-response type where students were asked to write their responses, and there were no multiple choice items. The problems were suitable to use multiple solution strategies, and each problem has more than one solution type or more than one solution strategy. For content and construct validity, experts' opinions and advises had been taken in detail. The problem solving instrument has one construct, and the instrument measures only one construct; namely problem solving skills. Also, the instrument is suitable for both eighth and ninth grade students, since it measures the problem solving ability in one construct, and the problems measure basic problem solving skills. Moreover, the mathematical content for the problems were not from one specific topic. Instead, the problems require basic mathematical knowledge and basic problem solving strategies and skills. Also, the mathematical concepts required in the problems were covered until the end of the eighth grade curriculum. Therefore, students who are at ninth grade were assumed to have covered the mathematical concepts that were required in the problems. Thus, the instrument was considered as appropriate for the ninth grade students.

The scoring of the participants' solutions of the problems was calculated by using a rubric developed by the researcher and one mathematics teacher. According to the answers and the rubric, the participants who solved the problem with relevant approach and reached a correct solution got 3 points for each problem as the problem solving performance. The participants who solved the problem with a relevant approach but could not reach the correct answer got 2 points. The participants who wrote a partially relevant approach got 1 point, and participants who wrote totally irrelevant approach or who did not write any solution got 0 point for each problem. Then the total points of this scale for each of the problems provided the problem solving performance of the participants. The scoring process was completed by the researcher and a mathematics teacher independently. The researcher calculated one problem solving performance point for each participant based on the rubric, and the other mathematics teacher calculated her own problem solving performance points for each participant, based on the same rubric. Then, for each participant, the average of the two points was calculated. This average of the points was rounded to the closest natural number, and this rounded number was recorded as the problem solving performance for each participant. This process was handled in order to assure scoring reliability.

3.4 Data Collection Process

Firstly, the instruments of the current study were selected, and the permissions were taken from the developers of the instruments by email. Later, the permissions from the Ethic Committee of METU, and from other official committees were taken. For this, firstly Human Subjects Ethics Committee approval was taken from Middle East Technical University. The consent forms were prepared. Before collecting data from participants, the participants were given information about the study, and the consent forms were signed by participants. All of the participants were volunteer, their names and answers were confidential and no participants were damaged psychologically or physically in this study. Lastly, the Ministry of National Education (MoNE) approval was taken in order to collect data from ninth grade Anatolian high school students in İzmir. When all of the official approvals were taken, the researcher of the current study went to the Anatolian high schools, requested permissions from the school administers for the study and data collection. These approvals were presented in Appendix E.

At the first and second week of the spring semester of academic year 2014-2015, the Anatolian high schools were visited in the west and north of İzmir, the permissions were taken also from the school administers, and ninth grade students were distributed the consent forms and three instruments. Before distribution of instruments, the participants were informed about the study, and aim of the study was explained to both the school administrators, teachers, and the students. In addition, the participants were explained that their participation is on voluntary basis and they are free in deciding to answer the questions in the instruments. Moreover, they were told that their participation would not be graded. In addition, their names and their answers would be kept confidential. All the participants answered the questions on their own and independently from each other. The students answered the instruments silently, carefully and seriously. After the students signed the consent forms, the instruments were distributed. Next, the instructions of how to fill in the instruments were explained by the researcher. All of the instruments were distributed and applied by the researcher, and all data were collected by the researcher.

Data collection procedure from instruments started with the first instrument which is the Junior Metacognitive Awareness Inventory for metacognition. The participants firstly filled the metacognition instrument-jMAI and got a metacognition score. They were given 15 minutes to complete the instrument. Later, they answered the reasoning ability inventory, which is Test of Logical Thinking. For this, they were given 25 minutes, after answering the questions in the instrument-TOLT, the participants got a reasoning ability score. These two instruments took one lesson hour. After the break, finally the students answered Mathematical Problem Solving Instrument and solved ten problems and got a total problem solving performance scores, and their scores for each problem. They were given 40 minutes for Mathematical Problem Solving Instrument. The total data collection took approximately 80 minutes; namely, two lesson hours.

For the missing data, if the number of missing subjects was smaller than %5, then “replace with mean” property was conducted. If the missing data was larger than %5, “exclude pairwise” property was selected. The total number of participants is 578. The rationale argued by Tabachnick and Fidell (2007) requires that the sample size should be higher than 50+8m (where m means the number of independent variables). Since there are 2 independent variable in the multiple regression analysis (metacognition and reasoning ability points), the minimum number of sample size is 66. There might be missing data, so the sample size should be at least 66. In the current study the sample size is 578 and 578 is higher than 66, so sample size is appropriate for the study. For standard multiple regression analysis, metacognition and reasoning ability are selected as independent variables, and the problem solving performance as dependent variable. The rationale of choosing the dependent and independent variables is that the main aim of the current research is to measure the relationship the dependent variable, namely problem solving performance, has with other variables.

3.5 Pilot Study

Pilot study involved 58 ninth grade students. The participants were ninth grade students at three public high schools in the North district of İzmir. The reliability analysis was run for the pilot study data. The results of the reliability analysis of pilot study are shown in Table 3.1.

Table 3.1 Reliability Analysis for Pilot Study

Scale	Cronbach’s Alpha	Number of Items
Reasoning Ability (TOLT)	.615	10
Metacognition (jMAI)	.866	17
Problem Solving Scale	.841	10

For pilot study, reliability analysis was conducted and the Cronbach's Alpha coefficient was .615 for reasoning ability, .866 for metacognition and .841 for problem solving.

3.6 Reliability Analysis of Main Study

According to Pallant (2007), it is important to find reliable scales in a study. For reliability, the scale's internal consistency is the main issue. Internal consistency means all items to measure the same construct. Reliability refers to the degree to which the items in the scale produce consistent results (Pallant, 2007). In fact, reliability refers to "the consistency of the scores obtained—how consistent they are for each individual from one administration of an instrument to another and from one set of items to another. The term reliability, as used in research, refers to the consistency of scores or answers provided by an instrument" (Fraenkel & Wallen, 2009, p.154). The most common indicator of internal consistency is Cronbach's alpha coefficient. Ideally, the Cronbach's Alpha value of a scale should be above .7. The values above .7 are acceptable, but values above .8 are preferable (Pallant, 2007). Similarly, Fraenkel and Wallen (2009) state that for research purposes, reliability should be at least .70 and preferably higher.

The reliability analysis for reasoning ability scale-TOLT, metacognition scale-jMAI and problem solving scale were conducted, and the results were presented in Table 3.2 for the main study.

Table 3.2 Reliability Analysis of TOLT, jMAI and Problem Solving Scale

Scale	Cronbach's Alpha	Number of Items
Reasoning Ability-TOLT	.723	10
Metacognition-Junior MAI	.858	17
Problem Solving Scale	.846	10

To measure reasoning ability, the instrument TOLT was used. The instrument TOLT was translated into Turkish by Geban, Aşkar and Özkan (1992). According to Geban, Aşkar and Özkan (1992), the test of logical thinking scale has a good internal consistency, with the Cronbach's alpha coefficient reported of .85. In the current study, as seen in Table 3.2, the Cronbach's alpha coefficient was .723 for TOLT. To measure metacognition, junior MAI instrument was used. According to Ubuz and Aydın (2010), the Junior Metacognitive Awareness Inventory scale has a good internal consistency, with the Cronbach alpha coefficient reported of .75 for knowledge of cognition and .79 for regulation of cognition. In the current study, the Cronbach alpha coefficient for knowledge of cognition was .732 and the Cronbach alpha coefficient for regulation of cognition was .792. In the current study, the total Cronbach's alpha coefficient was .858 for Junior MAI. To measure problem solving performance, problem solving instrument was used. In the current study, for the problem solving instrument, the Cronbach's alpha value was .846. Since the alpha value is higher than .70, there is evidence for reliability (Crocker &Algina, 1986).

3.7 Data Analysis

The statistical program, SPSS 15.0 for windows was used for statistical analysis in this study. In the current study, quantitative data analysis methods were used for the aim of examining the research questions and testing the hypothesis. For quantitative data analysis, both descriptive and inferential statistics were conducted. For descriptive statistics; the means, standard deviations, variances, frequencies, percentages, charts and graphs were used to describe the data and for the analysis of the variables; namely metacognition, reasoning ability and problem solving performance. Descriptive statistics was used to have a general overview of the variables. As inferential statistics; correlation and standard multiple regression analysis was used to find the correlations among the variables: metacognition, reasoning ability and problem solving performance.

In SPSS, the correlation and the standard multiple regression analysis was conducted to analyze the data. The correlation analysis was also conducted in order to explain the relationship among the variables, since the aim of the study is to detect the possible relationships. Firstly, the relationship between problem solving performance and metacognition was found. Secondly, the relationship between the metacognition and reasoning ability was found. Finally, the relationship between the problem solving performance and reasoning ability was found.

The standard multiple regression analysis was conducted on problem solving performance, metacognition and reasoning ability. Here, problem solving performance was the dependent variable; whereas, metacognition and reasoning ability were the independent variables. The most important and the most affected variable was expected to be the problem solving performance in this study. Also, the main aim of the study was to find the relationship of problem solving performance with other variables. So, the problem solving performance is chosen as dependent variable. The rationale of conducting multiple linear regression is to find the partial correlation of dependent variable by taking out the contribution of the other independent variable. In partial correlation of multiple linear regression analysis, the contribution of the other independent variable is taken out of both the dependent variable and the other independent variable. Thus, there will be a chance to compare the strength of the relationships between the problem solving performance and metacognition by partial correlation of adjusted reasoning ability; and between problem solving performance and reasoning ability by partial correlation of adjusted metacognition scores. The difference between the multiple linear regression and correlation analysis will provide more meaningful and detailed information. To evaluate the strength of the correlation between the combination of predictor variables (metacognition and reasoning ability) and criterion variable (problem solving performance), the coefficient of multiple correlations - R was calculated. The coefficient of determination which is calculated as R square, was used to evaluate the percentage of variability among the problem solving performance points. R square will provide us how much of the variance in the dependent variable (problem solving performance) is explained by the independent variables (metacognition and reasoning ability). Also, Standardised Beta values of each predictor variable (metacognition and reasoning ability) were used for calculating the unique contribution of each predictor to the total variance. Standardised Beta values are need; because, “in order to compare different variables, we need to standardize the variables; in other words, the values for each of the different variables should be converted to the same scale. Lastly, Unstandardized B values were used to calculate the weights of each predictor in the regression equation; because, in order to write regression equation, unstandardized B values are needed for equation weighs” (Pallant, 2007, p.159).

3.8 Internal and External Validity of Study

Validity means the appropriateness, meaningfulness, correctness, and usefulness of the inferences a researcher makes (Fraenkel & Wallen, 2006, p.147). The instruments were translated into Turkish before the current study, and the reliability and validity of these instruments were checked before. Also, in this study, the reliability analysis was conducted, for both the pilot study and main study. Moreover, for validity, throughout the study process, and at all stages, the study was under the guidance and observation of the experts. Opinions, advises, guidelines, and permissions from the experts were taken throughout the study. Also, the experts’ opinions, guidelines and advises were taken into account and reflected in the study. Moreover, for the current study, the possible internal validity threats and external validity threats were presented.

3.8.1 Internal Validity of the Study

Internal validity represents the degree to which the differences on the dependent variable were created only and directly by the independent variable (Fraenkel & Wallen, 2006). In order to ensure that the difference on the dependent variable is created by independent variable, rather than any other variables, the internal validity threats should be taken into account. It is important to control the extraneous variables not to create any difference on the dependent variable. But, in correlational studies, some of the internal validity threats are irrelevant such as history, regression, maturation, attitude of subjects and implementation threats (Fraenkel & Wallen, 2006). In correlational study, there is no intervention, so these threats are not applicable. But, instrumentation, subject characteristics, testing, mortality and location threats are applicable.

Instrumentation threat involves three types: instrument decay, data collector characteristics and data collector bias. Instrument decay is about misinterpretation of the data results because of fatiqueness (Fraenkel & Wallen, 2006). In the current study, the results were calculated with high concentration; even so, it may be a threat. Data collector characteristics threat is about the possible different characteristics of data gatherers (Fraenkel & Wallen, 2006). In the current study, the same researcher collected all data and conducted the analysis. So, it may not be a threat. Data collector bias is about the possibility of distorting the data in order to get expected results by the researcher (Fraenkel & Wallen, 2006). It may not be a threat because the instruments were standard, the answers were recorded directly and data was not manipulated by anyone.

Subject characteristics threat is about the different characteristics of the participants to create an extraneous variable (Fraenkel & Wallen, 2006). It may not be a threat because main characteristics of the participant were controlled. All the participants have similar main characteristics; such as the same grade, similar age etc.

Testing threat is about remembering the answers of one instrument when doing the second instrument. This threat is observed mostly in pretest-posttest design (Fraenkel & Wallen, 2006). In the current study, the design was not pretest-posttest design. Also, the instruments are independent from each other, and the data collected at one time only. So it may not be a threat.

Mortality threat is about the absence of participant (Fraenkel & Wallen, 2006). In the correlational designs, when a participant is absent, it is excluded from the study, so this threat is not observed generally (Fraenkel and Wallen, 2007). Moreover, in the current study the data was collected only one time, and for a short time period; two lesson hours. So it may not be a threat.

Location threat is about the different properties of the location during data collection process (Fraenkel & Wallen, 2006, p.172). In order to control location threat, the data collection process occurred in the participants' own schools and own classrooms and in the actual lesson hours. Also the properties of the schools and classrooms were similar because all of them are public Anatolian high schools in the same district. So, it may not be a threat.

3.8.2 External Validity of Study

The external validity is about generalizability. It is about the degree of generalizing the results of the study from a current sample to a population (Fraenkel & Wallen, 2006). The target population of the study is all ninth grade students in İzmir. The target population involves approximately 23000 ninth grade students including private and public high schools. The accessible population involves approximately 5780 ninth grade students. The sample of the study consists of exactly 578 ninth grade students in İzmir, Turkey. In the current study, the results of the study may be generalized to West and North district of İzmir due to the accessible population number. There are 11008 male students, and 12363 female students who are at ninth grade in public or private school in İzmir. There are 123 public and 58 private schools, and totally about 23000 ninth grade students in public or private schools (MoNE, 2014). The sample of the study is 578 ninth grade students in public Anatolian high schools in North and West districts in İzmir. So, the accessible population involves approximately 5780 ninth grade students who are at public Anatolian high schools; since the sample size consists of %10 of the accessible population. In the current study, the results of the study may be generalized to West and North district of İzmir due to the accessible population number; rather than all districts in İzmir. Also, due to convenient sampling method, the generalizability of the results may be lower, so it may not generalize to all districts in İzmir. But when ecological generalizability is taken into account, the current study may be generalized to all ninth grade students who have similar settings, conditions and surroundings; since ecological generalizability is about the generalization of the results to which certain conditions and settings other than prevailed in a study (Fraenkel & Wallen, 2007).

3.9 Assumptions and Limitations

There were some assumptions and limitations in this study. It was assumed that the participants answered the questions in the instruments sincerely and accurately. Also, it was assumed that the instruments were completed by the participants under standard conditions in all of the high schools. Moreover, it was assumed that there was no interaction among the participants; and also between the participants and the researcher during the data collection phase.

The limitations of the study were that the study was conducted on the students in West and North districts in İzmir. Also, the convenient sampling may lead sampling bias and limitation in generalization of the results, since it may not represent the entire population. Moreover, the study was conducted to only 578 participants in İzmir. In addition, the study involved only the ninth grade students. Other limitation was that the study was conducted only to public Anatolian high school students. The private high schools and other types of public high schools were excluded.

CHAPTER 4

RESULTS

In this chapter, the results of the quantitative data analysis were provided. Firstly, the descriptive statistics was presented. Then, the inferential statistics of the quantitative data analysis was provided.

4.1 Descriptive Statistics

In this section, descriptive statistics about the students' scores on reasoning ability instrument-TOLT, metacognition instrument-Junior MAI and problem solving performance instrument-problem solving scale were provided. There were 578 ninth grade students as participants. For the missing data, according to Tabachnick and Fidell (2007, p.63), if 5% or less data points are missing in a large data set, "the problems are less serious and almost any procedure for handling missing values yields similar results." In the current study, there are a few missing data points which can be ignored. If the number of missing subjects is smaller than 5%, then "replace with mean" property may be conducted for multiple regression analysis. If the missing data was larger than 5%, "exclude pairwise" property should be selected (Pallant, 2007). In the current study missing values were less than 5% of all data, so "replace with mean" property was used for the analyses. For the descriptive data analysis; namely the mean scores, standard deviation, number of participants, and 95% confidence interval for mean bounds related to reasoning ability scale-TOLT, metacognition scale-junior MAI and problem solving scale were presented in Table 4.1.

Table 4.1 Mean Scores of TOLT, jMAI and Problem Solving Scores

Scale	Mean	SD	N	95% Confidence	Interval
				Upper bound	Lower Bound
TOLT	6.73	2.187	578	6.91	6.55
Junior MAI	57.70	11.22	578	58.62	56.78
Problem Solving	19.80	5.588	578	20.26	19.34

Note. SD=Standard Deviation. N=Number of Participants

For reasoning ability-TOLT instrument, the possible maximum value is 10 and the possible minimum value is 0. In the current study, the participants' maximum value was 10, minimum value was 1 for TOLT. For metacognition-jMAI instrument, the possible maximum value is 85 and the possible minimum value is 17. In the current study, the participants' maximum value was 84, minimum value was 21 for jMAI. For problem solving instrument, the possible maximum value is 30 and the possible minimum value is 0. In the current study, the participants' maximum value was 30, minimum value was 3.

As seen Table 4.1, the descriptive data analysis was conducted, and the standard deviation, mean, and 95% confidence interval for mean bounds of TOLT, Junior MAI and problem solving scale were presented. The mean score for TOLT was M=6.73 *(SD=2.187)*. The mean score for junior MAI was M=57.70 *(SD=11.22)*. The mean score for problem solving scale was M=19.80 *(SD=5.588)*.

4.2 The Role of Metacognition and Reasoning Ability on Predicting Problem Solving Performance

In order to investigate the role of metacognition and reasoning ability on predicting mathematical problem solving performance of ninth grade students; how well reasoning ability and metacognition are able to predict the problem solving performance; how much variance in problem solving performance scores can be explained by reasoning ability and metacognition scores; and finally, which variable in the set of variables (reasoning ability and metacognition) is the best predictor of problem solving performance; standard multiple regression analysis was conducted.

Multiple regression is not just one statistical technique; rather, it is a collection of techniques. It is used for discovery of the relationship between one dependent variable or criterion variable and a number of independent variables or predictor variables. The analysis is based on correlation, but it provides more sophisticated exploration of the interrelationship among a set of variables (Pallant, 2007). In the current study, standard multiple regression model is conducted. Because, in standard multiple regression type, all of the independent variables or the predictor variables are written into the equation simultaneously. Each independent variable is measured by its unique predictive power of dependent variable. It is used for answering how much variance in a dependent variable is explained by a set of independent variables, as a group or block. Also, it provides how much unique variance in the dependent variable is explained by each of the independent variables (Pallant, 2007).

In the current study, for the standard multiple regression analysis, problem solving performance is chosen as the dependent variable; whereas, metacognition and reasoning ability are chosen as the independent variables. The most important and the most affected variable is expected to be the problem solving performance in the current study. Also, the main aim of the study is to find the relationship of problem solving performance with other variables. So, the problem solving performance is chosen as dependent variable. Also, the criterion variable is problem solving performance for standard multiple regression analysis. In addition, the predictor variables are metacognition and reasoning ability. To evaluate the strength of the correlation between the combination of predictor variables (metacognition and reasoning ability) and criterion variable (problem solving performance), the coefficient of multiple correlations - R was calculated. The coefficient of determination which is calculated as R square, was used to evaluate the percentage of variability among the problem solving performance points. R square will provide us how much of the variance in the dependent variable (problem solving performance) is explained by the independent variables (metacognition and reasoning ability). Also, Standardised Beta values of each predictor variable: metacognition and reasoning ability, were used for calculating the unique contribution of each predictor to the total variance. In the current study, it was aimed to compare the contribution of different variables to the dependent variable. Also, it was aimed to find each of these independent variables' unique contributions. Thus, Standardised Beta values are need; because, "in order to compare different variables, we need to standardize the variables; in other words, the values for each of the different variables should be converted to the same scale so that we can compare them. Lastly, Unstandardized B values were used to calculate the weights of each predictor in the regression equation; because, in order to write regression equation, unstandardized B values are needed for equation weighs of the predictor variables" (Pallant, 2007, p.159).

4.2.1 Assumptions of Standard Multiple Regression Analysis

4.2.1.1 Normality Assumption

According to Pallant (2007), most of the statistical techniques require the assumption of "the distribution of scores on the dependent variable is normal". Normality means "a symmetrical, bell-shaped curve, which has the greatest frequency in the middle and relatively small frequencies on both extremes" (Gravetter & Wallnau, 2000, p.52). In parametric statistical techniques, normality checking is required. Normality can be assessed by using skewness and kurtosis values. For this, skewness and kurtosis values are important. Skewness and kurtosis values represent the distribution of scores on continuous variables. The skewness value presents an indication of the symmetry of the distribution. Kurtosis value presents information about the peakedness of the distribution. The skewness and kurtosis values should be between -1 and +1values for normal distributions. These values may be extended to -2 and +2 values.

For, normality checking, as well as skewness kurtosis values, the test of normality is also used. In test of normality table, the results of Shapiro-Wilk and Kolmogorov-Smirnov statistics assess the normality of the distribution of scores. Both of the values should be more than .05. If the values are smaller than .05, it suggests violation of assumption of normality. But, in larger samples, the values mostly are smaller than .05. In fact, this violation situation; due to shapiro-wilk and kolmogorov smirnov values, is quite common in larger samples. So, in larger samples, for normality, histogram and plots should be used (Pallant, 2007). The actual shape of the distribution can be seen in Histograms. The scores should be reasonably normally distributed. Also, in the normal probability plots, labelled as Normal Q-Q Plot, a reasonably straight line represents a normal distribution. Also, in the Detrended Normal Q-Q Plots, there should be no real clustering of points. The points should mostly collect around zero line.

For the normality checking of reasoning ability scores, metacognition scores and problem solving performance scores, the Skewness Kurtosis values test and test of normality checking was conducted. The results provided evidence for normality for all variables.

To conclude, for reasoning ability scale-TOLT, metacognition scale-jMAI and problem solving scale, the skewness and kurtosis values are between the required ranges. The histograms with normal curves, the normal Q-Q plots, and the Detrended Normal Q-Q plots also provided evidence for normality. In summary, normality assumption was assured for all variables. All of the results for normality checking were provided in Appendix D.

There are some assumptions of multiple regression analysis. These assumptions are: sample size, multicollinearity and singularity, outliers, normality, linearity and homoscedasticity. These assumptions should be satisfied before the analysis (Tabachnik & Fidell, 2007).

4.2.1.2 Sample Size

According to Stevens (1996, p.72), "for social science research, about 15 subjects per predictor are needed for a reliable equation". There are 2 predictor variables in the current study: reasoning ability and metacognition. So, at least 30 subjects are needed according to Stevens (1996), and there are 578 subjects in the current study. Also, Tabachnik and Fidell (2007) provided a formula for sample size: $N > 50 + 8.m$ where m means the number of predictor variables. In the current study there are two predictor variables, and by applying this formula 50+16=66, at least 66 subjects should be participants for the current study. There are 578 subjects in the current study, so the sample size assumption is satisfied.

4.2.1.3 Multicollinearity and Singularity

Multicollinearity and singularity means the degree of the relationship among the independent variables. Multicollinearity takes place when the correlations among predictor variables are high (R = .9 or above). Singularity takes place when one independent variable is a combination of other independent variables (Pallant, 2007). According to Tabachnik and Fidell (2007), "multicollinearity and singularity are problems with a correlation matrix that occur when variables are too highly correlated. With multicollinearity, the variables are very highly correlated (say, .90 and above); with singularity, the variables are redundant; one of the variables is a combination of two or more of the variables" (p. 88) . Since the multicollinearity and singularity affects the regression model, these two threats should be checked. So, the correlations among the independent variables should be less than R=.9 (Pallant, 2007).

Table 4.2 Summary of Pearson Correlation Coefficients among Variables

Scales	Problem Solving	Metacognition
Problem Solving	1.00	-
Metacognition-jMAI	.641	-
Reasoning-TOLT	.707	.686

The Pearson correlation coefficients between the independent variables and the dependent variables should be more than .3 and, the correlations between each of the independent variables should not be too high. In other words, the correlation coefficient should be less than .9 (Pallant, 2007). As seen in the Table 4.2, the Pearson correlation coefficient between reasoning ability-TOLT scores and problem solving scores is .707; and between metacognition-jMAI scores and problem solving scores is .641. These values are greater than .3. Moreover, the correlation coefficient between the reasoning ability-TOLT scores and metacognition-jMAI scores is .686 and this value is less than .9. So, multicollinearity and singularity assumption was not violated.

In addition, for checking multicollinearity, two values are needed: Tolerance and VIF. Tolerance value shows how much of the variability of one independent variable is not explained by the other independent variables. Tolerance value is measured using the formula 1- R squared for each variable. If tolerance value is very small or less than .10, it means the multiple correlation with other variable is high. This situation suggests multicollinearity. The second value VIF – variance inflation factor is the inverse of Tolerance value (1 divided by Tolerance value). VIF values more than 10 suggest multicollinearity.

Table 4.3 Tolerance and VIF Values

Scale	Tolerance	VIF
Reasoning Ability-TOLT	.529	1.890
Metacognition-JMAI	.529	1.890

As the Table 4.3 shows, the Tolerance value is .529 and higher than .10. Also, the VIF value is 1.890 and smaller than 10. So, tolerance and VIF values provided evidence for multicollinearity and singularity assumption.

4.2.1.4 Outliers

Outliers mean the extreme scores which are too high or too low when compared to the rest of a set of data. Multiple regression is very sensitive to outliers for both dependent and independent variables, since outliers affect the slope of regression line highly, and thus affects regression equation. Outliers should either be deleted, or changed with a score that is not too different from the remaining scores. Outliers can be detected from the standardized residual plot- scatterplot (Pallant, 2007). Tabachnik and Fidell (2007) state that outliers are the scores with standardised residual values above +3.3 or less than -3.3. In large samples, if there are only few outliers, than it is acceptable. Using the Tabachnick and Fidell’s guidelines, if there are two independent variables, than the critical value for outliers is 13.82; which refers to the evaluation of Mahalanobis distance values. If the maximum value is higher than this critical value, then removing these outliers is best action to take. Also, the value for Cook’s Distance should be smaller than 1 for checking outliers.

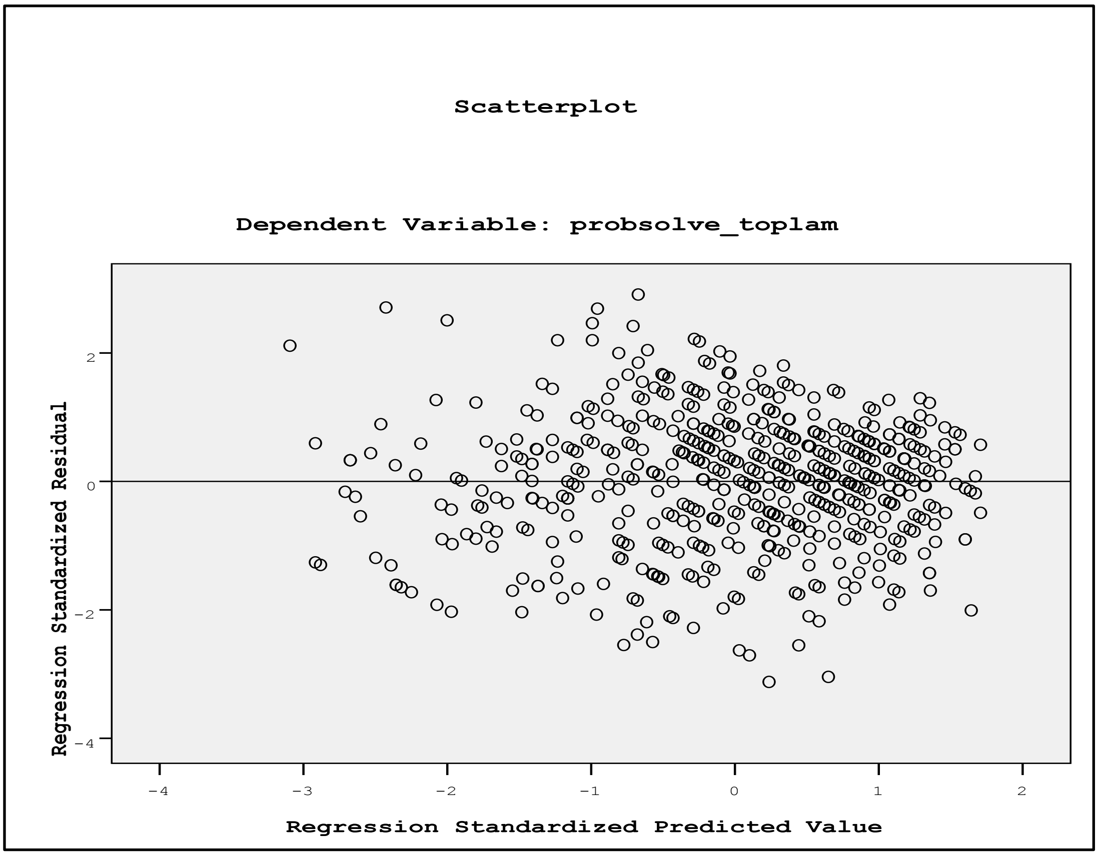

Figure 4.1 Distribution of Standardized Residual Values

In Figure 4.10 Distribution of Standardised Residual Values displayed distribution of cases' standardized residuals in scatterplot for this study. As seen in the Figure 4.10, there are three or four extreme scores with more than 3.3 or less than -3.3. Since there are only few outliers, these outliers were not excluded or changed. In large samples few outliers can be acceptable (Tabachnik & Fidell, 2007). So, outliers assumption was not violated.

Also, the Mahalanobis and Cook's values were also checked in order to provide more evidence for outliers assumption. Using the Tabachnick and Fidell's guidelines, if there are two independent variables, than the critical value for outliers is 13.82; which refers to the Mahalanobis distance values. If the maximum value is higher than this critical value, then removing these outliers is best action to take.

Table 4.4 Mahalanobis and Cook's Distance Value

	Maximum	N
Mahal. Distance	10,967	578
Cook's Distance	.032	578

As seen in the Table 4.4, the maximum Mahal. Distance value is 10.967 and this value is smaller than 13.82, so there is no need to delete the outliers. Also, the value for Cook's Distance should be smaller than 1 for checking outliers. As seen in the Figure, the Cook's Distance value is .032 and it is smaller than 1. So outlier's assumption was assured for the current study.

4.2.1.5 Normality

Normality, linearity, homoscedasticity and independence of residuals are all refer to the distribution of scores and the nature of the relationship between the variables. All of these assumptions can be controlled from the residuals scatterplots. Residuals mean the differences between the obtained value of dependent variable and the predicted value of dependent variable scores. Normality requires the residuals to be normally distributed about the predicted dependent variable scores. Linearity requires the residuals to have a straight-line relationship with predicted dependent variable scores. Homoscedasticity requires the variance of residuals about the predicted dependent variable scores to be the same for all predicted scores (Pallant, 2007).

The Normal Probability Plot (P-P) of the Regression Standardized Residual and the Scatterplot are required for checking these assumptions. In the Normal P-P Plot, the points should lie in a reasonably straight diagonal line from bottom left to top right. This means there is no major deviations from normality. In the scatterplot of standardized residuals, the residuals should be roughly rectangularly distributed, and most of the scores should be concentrated in the center or along the 0 point. Deviations from a centralized rectangle or a systematic pattern to residuals such as curvilinear or higher on one side than the other, mean violation of the assumptions.

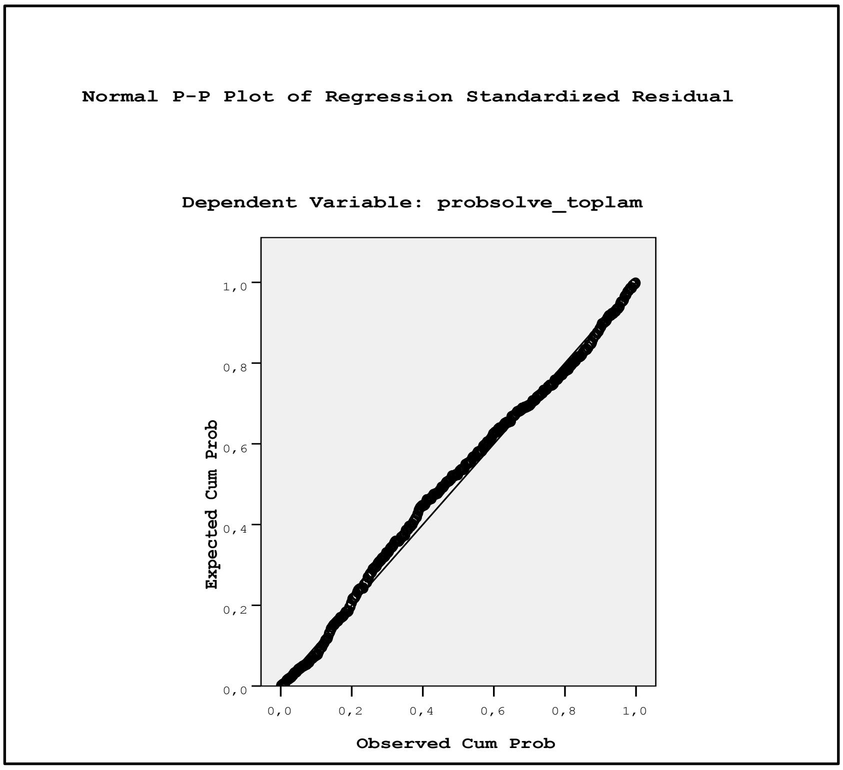

Figure 4.2 Normal P-P Plot of Regression Standardized Residual

As seen in the Figure 4.2, in the Normal P-P Plot, the points lie in a reasonably straight diagonal line from bottom left to top right. This means there are no major deviations from normality, so normality assumption as assured.

Also, as seen in the Figure 4.1, in the Distribution of Standardized Residual Values or the scatterplot of standardized residuals, the residuals were roughly rectangularly distributed. Moreover, most of the scores were concentrated in the center or along the 0 point. The scores were evenly distributed above and below the zero line. So normality assumption was assured.

4.2.1.6 Linearity

Linearity refers the residuals to have straight-line relationship with predicted dependent variable scores (Pallant, 2007). Linearity is observed when the half of the residuals stay above the zero line at some predicted values; and the other half of the residuals stay below the zero line at other predicted values on the scatterplot. The distribution of the values should be in rectangular shape instead of curved shapes (Tabachnik & Fidell, 2007).

In the Figure 4.1, the distributions of standardized residuals were shown. From Figure 4.1 it was seen that the residuals were equally and evenly distributed below and above the zero line on the scatterplot. Also, the residuals presented a rectangular shape. So linearity assumption was assured.

4.2.1.7 Homoscedasticity

Homoscedasticity requires the variance of residuals about the predicted dependent variable scores to be the same for all predicted scores (Pallant, 2007). The variability in scores for a variable should be similar at all values of another variable. Specifically, homoscedasticity assumption requires residuals to be randomly distributed around the 0 line or the horizontal line in the middle of the scatterplot, and there should be a relatively even distribution of residuals.

In the Figure 4.1, the distributions of standardized residuals was shown, and it was seen that the residuals plot had the same width approximately, for most of the values of the predicted dependent variable. Moreover, the cluster of points had approximately the same width; and the points were distributed evenly around the zero line in the scatterplot. So, the scatterplot provided support for the homoscedasticity assumption.

4.2.2 Results of Standard Multiple Regression Analysis

In order to investigate;

how well reasoning ability and metacognition are able to predict the problem solving performance;

how much variance in problem solving performance scores can be explained by reasoning ability and metacognition scores;

which variable in the set of variables (reasoning ability and metacognition) is the best predictor of problem solving performance,

standard multiple regression analysis was conducted. The results were presented in Table 4.5 Table 4.6 and Table 4.7.

Table 4.5 ANOVA

Model	Sum of Squares	Df	Mean Square	F	Sig.
Regression	9834.316	2	4917.158	345.399	.000
Residual	8185.803	575	14.236		
Total	18020.119	577			

As it can be seen from Table 4.5, the linear combination of reasoning ability and metacognition scores significantly related to problem solving performance scores, [F (2,575)=345.399, p=.000]. Thus, the provided model consisted of reasoning ability and metacognition significantly predicted the problem solving performance scores.

Also, R- square value presents how much of the variance in the dependent variable is explained by the independent variables in the model. In order to learn how much of the overall variance is explained by reasoning ability and metacognition variables, the r-square value was used in the Model Summary table. Summary of the model for the study was presented in Table 4.6.

Table 4.6 Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.739	.546	5.44	3.77309

As seen in the Table 4.6, the sample multiple correlation coefficient R=.739. Also, R-square = .546, [F (2,575)=345.399, p=.000]. So, approximately 55 % of the variance of problem solving performance scores can be explained by the linear combination of reasoning ability and metacognition. The R- square value below .4 represent a poor regression fit, and the R-square value between .4 and .7 represent moderate regression fit, and the R-square value above .7 represent strong regression fit (Tabachnik & Fidell, 2007). In the current study, the R- square value was .546 and this value is between .4 and .7. So, this value represented moderate regression fit.

Moreover, in order to search which variable is the best predictor of the problem solving performance and the strengths of predictors, Coefficients Table of multiple regression analysis was used. The standardized coefficients provide a comparison of the contribution of each independent variables on the dependent variable. Table 4.7 presented summary of coefficients.

Table 4.7 Summary of Coefficients

Model	B	Std. Error	Beta	t	Sig.	Part-R
Constant	2.663	.829		3.213	.001	
Reasoning	1.292	.099	.506	13.083	.000	.368
Metacognition	.146	.019	.294	7.604	.000	.214

As seen in Table 4.7, both the reasoning ability and metacognition provided statistically significant unique contribution to the prediction of problem solving performance scores, since p= .00 for both. By using Standardized Beta Values, it was seen that reasoning ability (Beta= .506, p=.000), and metacognition (Beta= .294, p=.000) significantly predicted problem solving performance scores. The Standardized Beta value of reasoning ability is higher than the Standardized Beta value of metacognition. Thus, it was concluded that reasoning ability made the strongest unique contribution to explaining the dependent variable: problem solving performance, when the variance explained by the other variables in the model is controlled for.

Moreover, Unstandardized B Values provided the weights of the predictor variables in the regression equation. The regression equation with reasoning ability and metacognition was significantly related to problem solving performance, R^2 = .546, [F (2,575)=345.399, p=.000].

According to these Unstandardized B weights, the regression equation occurs as:

$$\text{Problem Solving} = 1.292_{\text{reasoning}} + .146_{\text{metacognition}} + 2.663$$

Moreover, the Part Correlation Coefficient (Part-R) is used for unique contributions of predictors to the dependent variable. The square of Part-R presents unique contribution of one variable to the total R square. The square of partial correlation coefficient presents how much of the total variance in the dependent variable is uniquely explained by the specified variable; and also how much R square value differs if it is excluded from the model (Pallant, 2007, p.159).

As seen in the Table 4.7, reasoning ability had the highest part correlation coefficient, (Part-R = .368, p<.001). The square of part correlation coefficient is .135 that means reasoning ability uniquely explained 14 percent of the variance in problem solving performance scores. Moreover, metacognition had a part correlation coefficient (Part-R = .214, p<.001). The square of the part correlation coefficient is .0457 that means metacognition uniquely explained 5 percent of the variance in the problem solving performance scores.

In conclusion, standard multiple regression was used to assess the ability of two control measures (Reasoning ability and Metacognition) to predict problem solving performance. Preliminary analyses were conducted to ensure no violation of the assumptions of normality, linearity, multicollinearity and homoscedasticity. After entry of the reasoning ability and metacognition scale, the total variance explained by the model as a whole was 54.6%, F (2, 575) = 345.399, p < .001. In the final model, both of the two control measures were statistically significant, with the reasoning ability scale recording a higher beta value (beta = .506, p < .001) than the metacognition scale (beta = .294, p < .001).

In summary, the purpose of the current study is to search the relationships among reasoning ability, metacognition and problem solving performance. For this, multiple regression analysis was conducted and according to the results of the analysis, the model including reasoning ability and metacognition statistically significantly predicted the problem solving performance scores. In the next chapter, the results were discussed; implications and some recommendations were provided.

CHAPTER 5

DISCUSSION and CONCLUSION

The main purpose of the current study was to search the relationships among reasoning ability, metacognition and mathematical problem solving performance of the ninth grade students in İzmir. Also, another purpose of the study was to measure to what extend metacognition scores and reasoning ability scores predict the variance in problem solving performance scores. The other purpose was to measure which construct, metacognition or reasoning ability, was the best predictor of problem solving performance of the ninth grade students. In this chapter, firstly, the results of the analyses were discussed. Later, the limitations of the study, also implications and some recommendations for further studies were provided.

5.1 Discussion of the Results of the Analyses

For the research questions of the current study, standard multiple regression analysis was conducted and according to the results of the analysis, the model including reasoning ability and metacognition statistically significantly predicted the problem solving performance scores.

5.1.1 Discussion of the Findings for the Relationships among the Reasoning Ability, Metacognition and Problem Solving Performance

The aim of the current study was to investigate the relationship among metacognition, reasoning ability and mathematical problem solving performance of the ninth grade students. According to the results of the correlation analysis of the current study, significant relationship was found between metacognition and mathematical problem solving performance. The strength of that relationship was large. This result is consistent with the previous studies. Mayer (1998) explains this relationship as the fact that metacognitive skill is an essential part of problem solving process. Also, metacognitive skills improve students' mathematical problem solving performance (Mayer, 1998; Nickerson, 1994; Özsoy (2006). In addition, Antonietti, Ignazi and Perego (2000) state that high metacognitive levels are associated with best performance in problem-solving. Specifically, it was supported by many researchers that the students who take metacognitive strategy training showed significantly higher mathematical problem solving achievement and performed significant increase in problem solving skills than the other students who are not trained within metacognition (Berardi-Coletta, Buyer, Dominowski & Rellinger, 1995; Mevarech, 1999; Mevarech & Fridkin, 2006; Mayer, 1998; Özsoy & Ataman, 2009). As the previous studies supported, the expected result was found in the current study. The expected significant relationship was found between metacognition and problem solving.

Also, according to the results of the current study, there was a significant relationship between metacognition and reasoning ability. The strength of that relationship was large. This result was also an expected result, because the previous research studies provided support for the relationship between metacognition and reasoning ability. For example, Maverach and Kramarski (1997) found that metacognitive training increases metacognition and mathematical reasoning. Mevarech and Fridkin (2006) also support this idea and add new relations. The researchers state that when the metacognitive instructional method called IMPROVE is applied to the students, the students' mathematical knowledge, mathematical achievement, mathematical reasoning and meta-cognition significantly increases. Similarly, Kramarski and Hirscha (2010), and also Kramarski, (1998) found a support for the relationship between metacognitive training on mathematical reasoning, and stated that metacognitive training increases mathematical reasoning. In fact, Kramarski, Mevarech and Lieberman (2001) explained this relationship as the fact that metacognitive training which enhances students to focus on the similarities and differences between previous and new tasks, to comprehend a problem before trying a solution, and to think about the use of appropriate strategies for solutions improves mathematical reasoning. The metacognitive training improves metacognitive knowledge, which, in turn, improves mathematical reasoning and students' ability to transfer their previous knowledge to new situations. So, it was an expected result to find a relationship between metacognition and reasoning ability, due to the previous research.

Moreover, another result of the current study was that there is a significant relationship between reasoning ability and mathematical problem solving performance. The strength of that relationship was large. This result was also an expected result, because the evidence that support the relationship between reasoning ability and mathematical problem solving performance was found in many previous studies. Reasoning ability is a part of mathematical thinking, so the relationship between reasoning ability and mathematics is an expected result. Also, Mueller and Maher (1996) stated it is commonly accepted that reasoning and also proof are fundamental for mathematical understanding. For a student to be able to learn reasoning and justification of his reasoning is vital for his mathematical knowledge growth. To provide support for these relationships, Evans (2000) claims that there is a relationship among mathematical thinking, reasoning ability and problem solving ability of the students. This relationship is valid for each pairs, when one of the aspect increases, the others also increase. Also, According to Tobin and Capie (1982), formal reasoning ability is the strongest predictor of process skill achievement with 36% of variance. In addition, Valanides (1997) stated that student's reasoning ability was significant predictor of school achievement. The amount of variance was highest for students' mathematics achievement with (22.8%). Similarly, Bitner (1991) provided support for this relationship and claimed that formal operational reasoning modes are significant predictors of science and mathematics achievement. Formal operational reasoning modes explained 29% of the variance in mathematics. Since the reasoning ability is a significant predictor of mathematics achievement, instructional approaches should focus on not only declarative knowledge but also on procedural knowledge. Also, five formal operational reasoning modes are the vital abilities for the success in secondary school mathematics courses (Bitner, 1991). So it was an expected result for the current study to find a significant relationship between reasoning ability and problem solving, since the previous research provide support this relationship.

5.1.2 Discussion of the Findings for the Role of Reasoning Ability and Metacognition in Predicting Problem Solving Performance

In order to investigate the role of reasoning ability and metacognition in predicting mathematical problem solving performance, standard multiple regression analysis was conducted. The results of the standard multiple regression analysis showed that the provided model significantly predicted the problem solving performance of the ninth grade students. Also, both reasoning ability and metacognition made significant unique contribution in explaining problem solving performance scores of students. After entry of the reasoning ability and metacognition scale, the total variance explained by the model as a whole was 54.6%. Thus, metacognition and reasoning ability predicted and explained 54.6 percent of mathematical problem solving performance. The result of the current study is consistent with the previous studies in the literature. Since there are multiple correlations among metacognition, reasoning ability and problem solving, it is appropriate to expect that result. In order to support that relationship and prediction; Nunes, Bryant, Barros and Sylva (2012) investigated the role of mathematical reasoning and arithmetic on predicting the mathematical achievement. The results were as expected: mathematical reasoning and arithmetic made independent and unique contributions to the prediction of mathematical achievement. But, mathematical reasoning was by far the stronger predictor. Additionally, Lawson (1982) suggested that reasoning ability is related to both problem solving abilities and achievement. The results showed that students' formal reasoning ability was highly correlated with achievement in mathematics. Also, Mevarech and Kramarski (2003) claimed that students taking metacognitive training showed higher performance in mathematical achievement, and more performance to explain their mathematical reasoning. Maqsud (1997) claimed that both metacognitive ability and nonverbal reasoning ability have significant relationship with mathematics and English performances of students. Also, high metacognitive students showed higher performance on mathematics and English test when compared to low metacognitive students, regardless of nonverbal ability of reasoning. So, based on the previous studies, the result of finding 54.6 percent of explanation in mathematical problem solving by metacognition and reasoning ability was an expected and consistent result.

The results of the current study showed that reasoning ability made the highest unique contribution to mathematical problem solving performance. The reasoning ability uniquely explained 14 percent of the variance in problem solving performance scores. This result was consistent with the previous studies. Ball and Bass (2003) defined mathematical reasoning as a fundamental part of mathematical skills, and claimed that mathematical understanding is based on reasoning. Reasoning ability is the basis for learning new mathematics; and also, the ability to reason is vital for integrating the previous mathematical knowledge to new situations. Then, reasoning ability concludes in one's growth of new knowledge. Similarly, Malik and Iqbal (2011) searched the effect of problem solving teaching strategy on the problem solving skills and reasoning ability of eight grade students. The results show that experimental groups showed higher problem solving and reasoning ability than the control group. Also, Nunes, Bryant, Barros and Sylva (2012) claimed that both the mathematical reasoning and arithmetic predicted significantly the mathematical achievement independently and uniquely. But, mathematical reasoning was by far the stronger predictor.

Moreover, metacognition uniquely explained 5 percent of the variance in the problem solving performance scores. This significant and unique contribution was an expected result, since the previous studies provide support for the fact that there is relationship between metacognition and problem solving. Mevarech (1999) claims that metacognition training develops mathematical problem solving performance. Similarly, Berardi-Coletta, Buyer, Dominowski and Rellinger (1995) support this relationship.

The percent of metacognition is lower than the percent of reasoning ability in explaining the mathematical problem solving performance independently and uniquely. The result may be that the reasoning ability is more related to mathematics and mathematical problem solving, because reasoning ability is required in mathematics. In order to understand mathematics, a person has to reason mathematically, this is why reasoning is very important for a person to construct mathematical knowledge. When a person reasons mathematically, he can use mathematical ideas in new conditions, and this leads to improvement in problem solving skills (Mueller &Maher, 2009). Sonnleitner, Keller, Martina and Brunner (2013) stated that (CPS) Complex problem solving was significantly related to reasoning and educational success. And reasoning ability plays a crucial role in the process of solving complex problems. Thus, in general, there is more strong relationship between reasoning ability and problem solving than the relationship between metacognition and problem solving. Moreover, another result may bu due to the nature of metacognition instrument used in the current study. The instrument does not involve items that are directly related to problem solving. Rather, the instrument involves more general items; there are more general terms, words or sentences in the instrument. In order to emphasize mathematics and problem solving, the students were warned frequently. At the beginning of the instrument warning was added: please fill this instrument by thinking of mathematics courses, and mathematical problem solving process. The students were reminded frequently to fill the instrument based on mathematics and problem solving. Despite of these warnings, that possibility still exists. Thus, the nature of the metacognition instrument may be another reason for lower prediction power of metacognition in predicting mathematical problem solving performance.

The metacognition and reasoning ability together explained 54.6% percent of mathematical problem solving performance. This was quite respectable result, it was more than half; thus, the prediction power is very good. But, the metacognition uniquely explained 5% and reasoning ability explained 14% the variance in problem solving performance. The unique predictions of the two variables did not reach total R square; 14+5=19, not 54.6. That result occurs due to the nature of standard multiple regression analysis. Pallant (2007) explains this result as “the part correlation values respresent only the unique contribution of each variable, with any overlap or shared variance removed or partialled out. The total R square value, however; includes the unique variance explained by each variable and also that shared” (p.160). In the current study, the two predictor or independent variables: metacognition and reasoning ability was strongly correlated (r= .686). Thus, “there were a lot of shared variance that was statistically removed when they were both included in the model” (p.160). Hence, the difference occurs so much between the total variance and the sum of the unique variances.

In conclusion, standard multiple regression was used to assess the ability of two control measures (Reasoning ability and Metacognition) to predict problem solving performance. After entry of the reasoning ability and metacognition scale, the total variance explained by the model as a whole was 54.6%. Also, reasoning ability made the highest unique contribution. This result is consistent with former research’s findings supporting the influence of metacognition and reasoning ability in predicting mathematical problem solving. Also, according to the previous studies, there is a relationship between metacognition and reasoning ability; between metacognition and problem solving performance; and between metacognition and problem solving performance (Antonietti, Ignazi & Perego, 2000; Berardi-Coletta, Buyer, Dominowski & Rellinger, 1995; Bitner, 1991; Carlson & Bloom, 2011; Higgins, 1997; Kışkır, 2011; Mevarech & Fridkin, 2005; Mevarech & Kramarski, 2003; Ozsoy & Ataman, 2009; Washburn (2013).

5.2 Limitations and Recommendations

The aim of the current study was to search the relationship among metacognition, reasoning ability and problem solving performance of ninth grade students. By using the results of the current study, limitations and recommendations for future research were provided in this section.

First limitation of the current study was the grade level of the participants. In the current study, the data was collected from ninth grade students in public Anatolian high schools. So, the results might not be generalized to other populations and to all grade levels. In the future studies, the researchers should study with a larger sample size, and the participants should be from other ethnicities, socio-economic status, and the grade levels. For future research, the data can be collected from other grade levels, such as elementary schools and other grade levels in Anatolian high schools. Or the participants may be university students, or in-service or pre-service teachers etc. Also, the school types can be changed, then the participants may be selected from other types of high schools such as social sciences high schools, science high schools, vocational high schools etc. Also, for future research, data may be collected from universities or from elementary schools etc. In addition, in the current study data was collected from public high schools. Other limitation was that the study was conducted only to public Anatolian high school students. The private high schools and other types of public high schools were excluded. So, the participants from both public and private high schools can be included in the study. In order to increase generalizability, a similar study can be conducted with other grade levels, or other high school types. Then, the findings may be generalized to larger and diverse populations.

The other limitation of the study was that the study was conducted on the students in West and North districts in İzmir. The east and south districts were excluded from the study; because, the most crowded districts in İzmir are the west and north districts. Moreover, the study was conducted to only 578 participants in

İzmir. The number of participants may be increased in future research, in order to generalization of the results to larger and diverse populations.

Moreover, for generalizability, the current study had a limitation since the sampling method of the current study was convenience sampling. Also, the convenient sampling may lead sampling bias and limitation in generalization of the results, since it may not represent the entire population. For future research, instead of convenience sampling, random sampling can be used in order to reach more generalization.

Another limitation of the current study was that the design of the current study was correlational design. Since the design of the study was correlational, the current study measured the already existing constructs, and searched for a relationship among the variables. In the current study, significant relationships among variables were found, but this relationship did not provide any cause-effect relationship. In order to provide a cause-effect situation, experimental study should be conducted. So, experimental study can be conducted with the same variables, for the current study. In addition to experimental design, some other variables such as personal constructs or demographics can be added to the study for future research. Also, since the current study was a quantitative study, the study was based on inferences of the numerical data. So, qualitative study can be conducted for more detailed inferences. For qualitative data, written data such as self-reports, or spoken data such as interviews or camera-records of problem solving process can be used in order to provide a complete picture of the relationships among the variables. Or mixed research design can be used, both qualitative and quantitative study can be conducted to provide complete description of the relationship.

5.3 Recommendations and Implications for Future Research

In the current study, there are many important implications for practitioners and researchers in the psychology field, education field, and other related disciplines. The findings of the current study provided a support for the relationships among metacognition, reasoning ability and mathematical problem solving performance of the students.

In addition, based on previous studies and the findings of the current study, some implications can be provided for mathematics teachers, mathematics educators, and mathematics curriculum developers. Mathematical problem solving is the basis of mathematics. Mathematical problem solving is vital for mathematics achievement of students. The current study provided evidence for the relationship among metacognition, reasoning ability and mathematical problem solving performance of the students. So, the importance of metacognition and reasoning ability should not be ignored in mathematics education. Furthermore, the role of metacognition and reasoning ability in mathematical problem solving may be emphasized in all grades, from elementary schools to education faculties in universities.

As stated, metacognition and reasoning ability have great importance in predicting mathematical problem solving performance of the students. According to the results of the current study, metacognition and reasoning ability explained 54.6 percent of the variance in mathematical problem solving performance of the ninth grade students. This percentage is higher than half, so these concepts are important for problem solving performance. So, mathematics teachers, and educators should be explained that metacognition and reasoning ability have an important role in mathematical problem solving performance. Thus, seminars related to metacognition and reasoning ability may be provided to pre-service and if possible to in-service mathematics teachers. By the approval of social scientists, educationalist, faculties and counselors these seminars may be provided to mathematics teachers of to university students who study mathematics education in both high school level and elementary level. Also, curriculum developers may facilitate metacognition and reasoning ability in mathematics courses, by providing related activities to mathematics curriculum and textbooks. So, teachers and educators make plans and in-class activities to improve students' learning and understanding. Then the students may have chance to improve their metacognition, reasoning ability and mathematical problem solving performance all together.

5.3.1 Implications for Practice

In the current study a significant relationship was found between metacognition and mathematical problem solving performance of the students. This finding was an expected result, since the previous research found significant relationship between metacognition and mathematical problem solving (Antonietti, Ignazi & Perego, 2000; Berardi-Coletta, Buyer, Dominowski & Rellinger, 1995; Bitner, 1991; Carlson & Bloom, 2011; Higgins, 1997; Kışkır, 2011; Mevarech & Fridkin, 2005; Ozsoy & Ataman, 2009; Schoenfeld, 1992).

In fact, this relationship did not guarantee a cause effect relationship. Only based on this relationship, it would not be appropriate to tell that metacognition increases the mathematical problem solving performance. But since there was a significant relationship, it would be appropriate to emphasize the importance of metacognition in mathematical problem solving performance. Moreover, many of the previous experimental research, in which a cause and effect relationship could be stated, provided that metacognition training increased mathematical problem solving (Berardi-Coletta, Buyer, Dominowski & Rellinger, 1995; Mevarech, 1999; Mevarech & Fridkin, 2006; Mayer, 1998; Nickerson, 1994; Özsoy & Ataman, 2009). So, based on the findings of the current study and the previous studies which support the cause-effect relationship, it will be appropriate to state that in order to improve students' mathematical problem solving performance, the teachers should emphasize metacognitive behaviors in their classrooms. Also, mathematics educators, teachers, curriculum developers should design appropriate teaching and learning strategies related with metacognition. Curriculum designers and educators should provide rich learning setting and should design materials which emphasize metacognition. Moreover, teachers should provide classroom culture and design classroom environments in which students engage in metacognitive behaviors. The students should be given an opportunity to explain, monitor and defend their solutions, decisions and their thinking. Then they will have chance to improve their metacognition.

Also, in the current study a significant relationship was found between reasoning ability and mathematical problem solving performance of the students. This finding was also an expected result, since the previous research found significant relationship between reasoning ability and mathematical problem solving (Bitner, 1991; Evans, 2000; Mueller & Maher, 1996; Tobin & Capie, 1982; Valanides, 1997; Washburn (2013). Also, Mueller, Yankelewitz, and Maher (2011) state that motivation and positive dispositions toward mathematics conclude in mathematical reasoning, and then this concludes in understanding. Students engage in and trust in their reasoning, instead of memorized facts, or solutions of other students. Based on their reasoning, the students persuade themselves and other students about the issues that make sense. This reasoning process concludes in mathematical understanding. If a student engages in mathematical reasoning then that students get conceptual understanding.

As stated in the previous paragraph, only finding a significant relationship does not prove the cause-effect relationship. But, the previous studies provided support for the cause-effect relationship in experimental designs. So, since there is a significant relationship between reasoning ability and problem solving performance, then it will be proper to focus on the importance of reasoning ability in mathematical problem solving performance. Since the unique variance explained by reasoning ability was higher than the variance uniquely by metacognition was higher in the current study, then it will be proper to emphasize the importance of reasoning ability more on problem solving performance. Also, many previous studies provided evidence that reasoning ability had a positive and significant relationship with mathematical problem solving performance (Bitner, 1991; Evans, 2000; Malik & Iqbal, 2011; Mueller & Maher, 1996; Nunes, Bryant, Barros &

Sylva, 2012 Tobin & Capie, 1982; Valanides, 1997). So, based on the findings of the current study and the previous studies, it will be proper to claim that in order to develop students' mathematical problem solving performance, the teachers should focus on reasoning ability in their classrooms. Also, mathematics educators, teachers, curriculum developers should design appropriate teaching and learning strategies, as well as appropriate classroom environments in which students have chance to discuss their reasoning clearly. Also, the curriculum designers and educators should design and provide rich learning settings, classroom environments, and should design materials, activities etc. which focus on reasoning ability. Mueller and Maher (1996) emphasize that if students engage in an environment in which they explore, collaborate with each other, and defend their thinking and justify their reasoning in both small and large groups, then they develop reasoning and mathematical understanding. In a community of learners, attending in discussions, making and refuting claims, and justifying reasoning related to mathematical ideas conclude in mathematical reasoning. So, the teachers should provide students collaborative environments, in which students are triggered to explain their thinking, make their ideas public, justify and give evidence for their thinking and claims, participate in arguments and discussions (Mueller & Maher, 1996). Also, Usman and Musa (2013) state that use of the thinking and reasoning patterns are very important for their mathematical performance. The teachers should measure students' formal operation levels and trigger students to use formal operation abilities in order to improve students' mathematical performance. Thus, it is important to emphasize reasoning ability in the classrooms, and teachers should create appropriate classroom culture for discussions. Also, the teachers should design classroom environments in which the students should be given an opportunity to explain, monitor and defend their solutions, decisions and their thinking. Then, the students have chance to develop their reasoning ability.

5.3.2 Implications for Future Research

For future research, and for researchers who would like to investigate and to understand problem solving process better and in more detailed manner, there are some implications. The structure or type and content of the problems are important. Many researchers choose to measure problem solving throughout high-stakes tests, state standardized tests, or achievement tests. It is important to consider the structure (type) and content of problems. Such standardized test may not be appropriate for assessing problem solving performance, because mainly, they are not designed for this purpose. Also, multiple choice items in such test involve chance success, and if a student selects the right answer only by chance, it will be reported as problem solving performance. So, the researchers should be careful when selecting an instrument to assess problem solving performance. It is important to measure problem solving performance by using open-ended problems. Moreover, in the current study, only the answers of these problems were measured as problem solving performance. In the future research, some more questions about that problem can be asked.

Moreover, due to the nature of metacognition instrument used in the current study, the instrument seems to not involve some items that are directly related to problem solving. Instead, the instrument seems to involve more general items. Thus, for the future research, the instruments which have items directly related to problem solving may be used. Also, the instruments which provide more precise, and concrete supports for the metacognition existence may be used.

Also, as explained in chapter 4, metacognition and reasoning ability explained a significant variance of 54.6% mathematical problem solving performance. That result supports the theoretical framework of the study. But, there is no explanation for the rest of the variance in mathematical problem solving performance. Thus, there is still great need for identification of the other variables which explain the remaining variance in mathematical problem solving performance. The remaining variance may be explained by other constructs; such as personal constructs, socioeconomic status, motivational aspects, beliefs and attitudes, intelligence, reading ability, self-regulation, self-efficacy etc. Hence, for future research, problem solving may be measured by a sociocultural aspect, or motivational factors, the affect and belief context may be added to the studies. The researchers should analyze the role of demographic variables such as ethnicity, socioeconomic status, cultural variables etc. Also, the motivational aspects, belief and affect aspect of problem solving, attitudes, self-efficacy, self-regulation, aptitude etc. may be added to future research. As well as some other cognitive aspects may be added to, such as intelligence, critical thinking, creative thinking, reading skills etc. In the current study, these possible predictors were not included; because, the three instruments of the study took two lesson hours. If any other variable was added, then another lesson hour would be needed, then the teachers and students would not be volunteer to fill three lesson hours instruments, and the number participants would be less, and the study would not be feasible.

Moreover, in the current study, the problem solving performance was measured as a content-free manner. For future research, content knowledge based problem solving performance measurements may be used, or problem solving strategies may be used as a variable. Also, in the current study, problem solving was measured in mathematical domain. So, for the future studies, researchers may investigate how problem solving performance changes in different domains such as science, language, social studies, and other disciplines.

In conclusion, problem solving had high importance in mathematics education for decades, and continues to be essential part of mathematics education (Evans, 2012, Posamentier & Krulik, 2008). As the previous studies and Donaldson (2011) state, it is commonly accepted that problem solving is what mathematics is all about. So, mathematics teachers' main aim should be to help students improve their problem solving abilities. For this aim, they should teach mathematics throughout problem solving process. Then, the students will learn new mathematical concepts and integrate mathematical knowledge throughout problem solving (Donaldson, 2011).

Also, Evans (2012) state that strong problem solving abilities and skills are vital for mathematics; as well as for other subject areas, disciplines and for daily life in general. So, the students should be provided critical thinking and strong problem solving preparation in schools, since they need them for success in life. The findings of the current study provided a support for the importance of metacognition and reasoning ability on problem solving for both related research area and the practical education area. Based on the findings, the importance of metacognition and reasoning ability may be emphasized by educators, curriculum developers, and mathematics teachers on their classes. The findings may be used for the development of teaching practices in classes, mathematics curriculums, and also for teaching methods and materials that may be used for mathematical problem solving processes in the classrooms in the future. In order to facilitate students' problem solving performance, the role of metacognition and reasoning ability may be emphasized more on the classrooms. For this, new curriculum designs or changes on the curriculums focusing on the two constructs may be applied. Also, for the lessons or courses in education faculties, metacognition and reasoning ability may be more emphasized for problem solving related courses or mathematics courses.

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APPENDICES

Appendix A: TEST OF LOGICAL THINKING

MANTIKSAL DÜŞÜNME YETENEK TESTİ

AÇIKLAMA: Bu test, çeşitli alanlarda, özellikle Fen ve Matematik dallarında karşılaşabileceğiniz problemlerde neden-sonuç ilişkisini görüp, problem çözme stratejilerini ne derece kullanabileceğinizi göstermesi açısından çok faydalıdır. Bu test içindeki sorular mantıksal ve bilimsel olarak düşünmeyi gösterecek cevapları içermektedir.

NOT: Soru Kitapçığı üzerinde herhangi bir işlem yapmayınız ve cevaplarınızı yalnızca cevap kağıdına yazınız. CEVAP KAĞIDINI doldururken dikkat edilecek hususlardan birisi, 1 den 8 e kadar olan sorularda her soru için cevap kağıdında iki kutu bulunmaktadır. Soldaki ilk kutuya sizce sorunun uygun cevap şıkkını yazınız, ikinci kutucuğa yani AÇIKLAMASI yazılı kutucuğa ise o soruyla ilgili soru kitapçığındaki Açıklaması kısmındaki şıkları okuyarak sizce en uygun olanını seçiniz. Örneğin 12'nci sorunun cevabı sizce b ise ve Açıklaması kısmındaki en uygun açıklama ikinci şık ise cevap kağıdını aşağıdaki gibi doldurun:

12. [b] AÇIKLAMASI [2]

9. ve 10. soruları ise soru kitapçığında bu sorularla ilgili kısımları okurken nasıl cevaplayacağınızı daha iyi anlayacaksınız.

SORU 1: Bir boyacı, aynı büyüklükteki altı odayı boyamak için dört kutu boya kullandığına göre sekiz kutu boya ile yine aynı büyüklükte kaç oda boyayabilir?

a. 7 oda

b. 8 oda

c. 9 oda

d. 10 oda

e. Hiçbiri

Açıklaması:

1. Oda sayısının boya kutusuna oranı daima $\frac{3}{2}$ olacaktır.

2. Daha fazla boya kutusu ile fark azalabilir.

3. Oda sayısı ile boya kutusu arasındaki fark her zaman iki olacaktır.

4. Dört kutu boya ile fark iki olduğuna göre, altı kutu boya ile fark yine iki olacaktır.

5. Ne kadar çok boyaya ihtiyaç olduğunu tahmin etmek mümkün değildir.

SORU 2: On bir odayı boyamak için kaç kutu boya gerekir? (Birinci soruya bakınız)

a. 5 kutu

b. 7 kutu

c. 8 kutu

d. 9 kutu

e. Hiçbiri

Açıklaması:

1. Boya kutusu sayısının oda sayısına oranı daima $\frac{2}{3}$ dür.

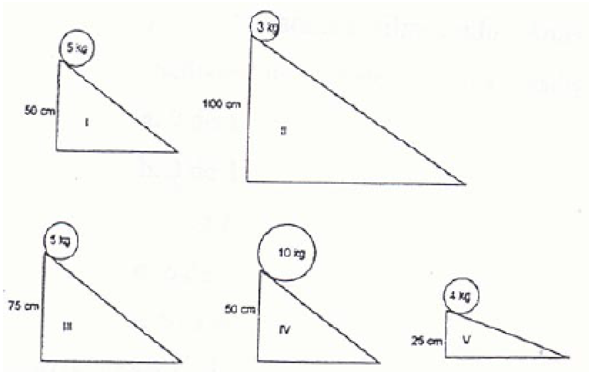
2. Eğer beş oda daha olsaydı, üç kutu boya daha gerekecekti.

3. Oda sayısı ile boya kutusu arasındaki fark her zaman ikidir.

4. Boya kutusu sayısı oda sayısının yarısı olacaktır.

5. Boya miktarını tahmin etmek mümkün değildir.

SORU 3: Topun eğik bir düzlemden (rampa) aşağı yuvarlandıktan sonra kat ettiği mesafe ile eğik düzlemin yüksekliği arasındaki ilişkiyi bulmak için deney yapmak isterseniz, aşağıda gösterilen hangi eğik düzlem setlerini kullanırdınız?



a. I ve IV

b. II ve IV

c. I ve III

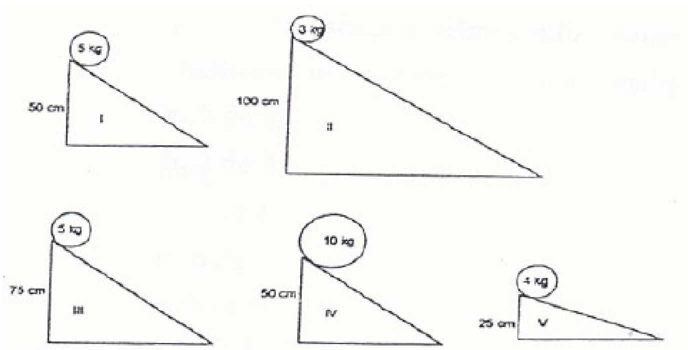
d. II ve V

e. Hepsi

Açıklaması:

1. En yüksek eğik düzlemle (rampa) karşı en alçak olan karşılaştırılmalıdır.
2. Tüm eğik düzlem setleri birbiriyle karşılaştırılmalıdır.
3. Yükseklik arttıkça topun ağırlığı azalmalıdır.
4. Yükseklikler aynı fakat top ağırlıkları farklı olmalıdır.
5. Yükseklikler farklı fakat top ağırlıkları aynı olmalıdır.

SORU 4: Tepeden yuvarlanan bir topun eğik düzlemden (rampa) aşağı yuvarlandıktan sonra kat ettiği mesafenin topun ağırlığıyla olan ilişkisini bulmak için bir deney yapmak isterseniz, aşağıda verilen hangi eğik düzlem setlerini kullanırdınız?



a. I ve IV

b. II ve IV

c. I ve III

d. II ve V

e. Hepsi

Açıklaması:

a. En ağır olan top en hafif olanla kıyaslanmalıdır.

b. Tüm eğik düzlem setleri birbiriyle karşılaştırılmalıdır.

c. Topun ağırlığı arttıkça, yükseklik azaltılmalıdır.

d. Ağırlıklar farklı fakat yükseklikler aynı olmalıdır.

e. Ağırlıklar aynı fakat yükseklikler farklı olmalıdır.

SORU 5: Bir Amerikalı turist Şark Expresi'nde altı kişinin bulunduğu bir kompartımana girer. Bu kişilerden üçü yalnızca İngilizce ve diğer üçü ise yalnızca Fransızca bilmektedir. Amerikalının kompartımana ilk girdiğinde İngilizce bilen biriyle konuşma olasılığı nedir?

a. 2 de 1

b. 3 de 1

c. 4 de 1

d. 6 da 1

e. 6 da 4

Açıklaması:

1. Ardarda üç Fransızca bilen kişi çıkabildiği için dört seçim yapmak gerekir.
2. Mevcut altı kişi arasından İngilizce bilen bir kişi seçilmelidir.
3. Toplam üç İngilizce bilen kişiden sadece birinin seçilmesi yeterlidir.
4. Kompartımandakilerin yarısı İngilizce konuşur.
5. Altı kişi arasından, bir İngilizce bilen kişinin yanısıra, üç tanede Fransızca bilen kişi seçilebilir.

SORU 6: Üç altın, dört gümüş ve beş bakır para bir torbaya konulduktan sonra, dört altın, iki gümüş ve üç bakır yüzük de aynı torbaya konur. İlk denemede torbadan altın bir nesne çekme olasılığı nedir?

a. 2 de 1

b. 3 de 1

c. 7 de 1

d. 21 de 1

e. Yukarıdakilerden hiçbiri

Açıklaması:

1. Altın, gümüş ve bakırdan yapılan nesneler arasından bir altın nesne seçilmelidir.
2. Paraların $\frac{1}{4}$ ü ve yüzüklerin $\frac{4}{9}$ u altından yapılmıştır.

3. Torbadan çekilen nesnenin para ve yüzük olması önemli olmadığı için toplam 7 altın nesneden bir tanesinin seçilmesi yeterlidir.
4. Toplam yirmi bir nesneden bir altın nesne seçilmelidir.
5. Torbadaki 21 nesnenin 7 si altından yapılmıştır.

SORU 7: Altı yaşındaki Ahmet'in şeker almak için 50 lirası vardır. Bakkaldaki kapalı iki şeker kutusundan birinde 30 adet kırmızı ve 50 adet sarı renkte şeker bulunmaktadır. İkinci bir kutuda ise 20 adet kırmızı ve 30 adet sarı şeker vardır. Ahmet kırmızı şekerleri sevmektedir. Ahmet'in ikinci kutudan kırmızı şeker çekme olasılığı birinci kutuya göre daha fazla mıdır?

a. Evet

b. Hayır

Açıklaması:

1. Birinci kutuda 30, ikincisinde ise yalnızca 20 kırmızı şeker vardır.
2. Birinci kutuda 20 tane daha fazla sarı şeker, ikincisinde ise yalnızca 10 tane daha fazla sarı şeker vardır.
3. Birinci kutuda 50, ikincisinde ise yalnızca 30 sarı şeker vardır.
4. İkinci kutudaki kırmızı şekerlerin oranı daha fazladır.
5. Birinci kutuda daha fazla sayıda şeker vardır.

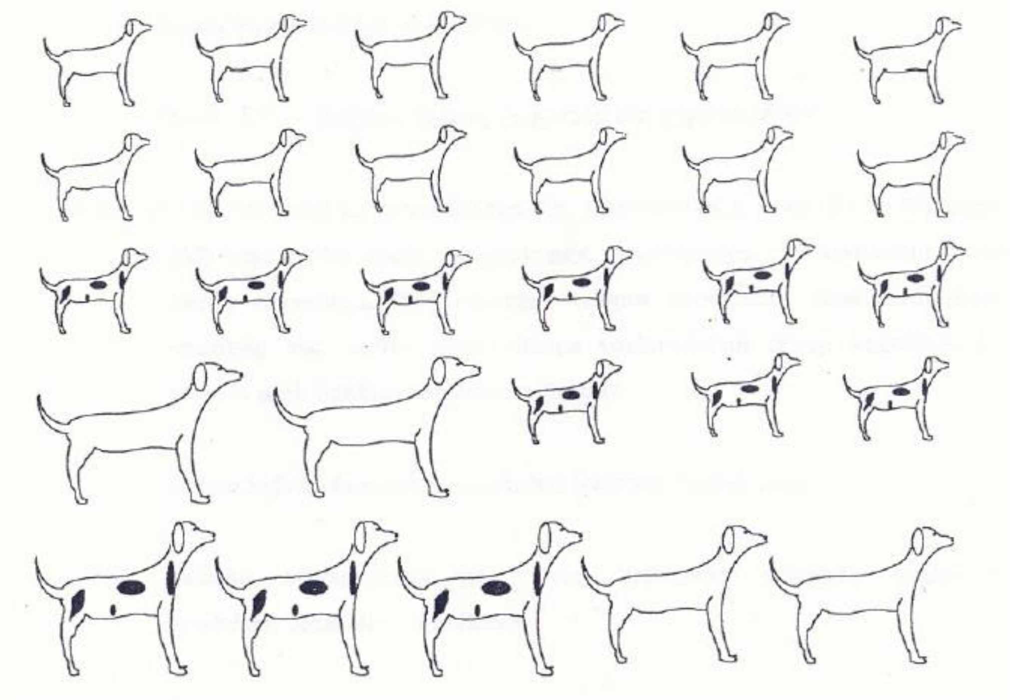
SORU 8: 7 büyük ve 21 tane küçük köpek şekli aşağıda verilmiştir. Bazı köpekler benekli bazıları ise beneksizdir. Büyük köpeklerin benekli olma olasılıkları küçük köpeklerden daha fazla mıdır?

a. Evet

b. Hayır

Açıklaması:

1. Bazı küçük köpeklerin ve bazı büyük köpeklerin benekleri vardır.
2. Dokuz tane küçük köpeğin ve yalnızca üç tane büyük köpeğin benekleri vardır.
3. 28 köpekten 12 tanesi benekli ve geriye kalan 16 tanesi beneksizdir.
4. Büyük köpeklerin $\frac{3}{7}$ si ve küçük köpeklerin $\frac{9}{21}$ i beneklidir.
5. Küçük köpeklerden 12 sinin, fakat büyük köpeklerden ise sadece 4ünün beneği yoktur.



SORU 9: Bir pastanede üç çeşit ekmek, üç çeşit et ve üç çeşit sos kullanılarak sandviçler yapılmaktadır.

Ekmek Çeşitleri	Et Çeşitleri	Sos Çeşitleri
Buğday (B)	Salam (S)	Ketçap (K)
Çavdar (Ç)	Piliç (P)	Mayonez (M)
Yulaf (Y)	Hindi (H)	Tereyağı (T)

Her bir sandviç ekmek, et ve sos içermektedir. Yalnızca bir ekmek çeşidi, bir et çeşidi kullanılarak kaç çeşit sandviç hazırlanabilir?

Cevap kağıdı üzerinde bu soruyla ilgili bırakılan boşluklara bütün olası sandviç çeşitlerinin listesini çıkarın.

Cevap kağıdında gereksiniminizden fazla yer bırakılmıştır.

Listeyi hazırlarken ekmek, et ve sos çeşitlerinin yukarıda gösterilen kısaltılmış sembollerini kullanınız.

Örnek: BSK= Buğday, Salam ve Ketçap dan yapılan sandviç

SORU 10: Bir otomobil yarışında Dodge (D), Chevrolet (C), Ford (F) ve Mercedes (M) marka dört araba yarışmaktadır. Seyircilerden biri arabaların yarışı bitiriş sırasının DCFM olacağını tahmin etmektedir. Arabaların diğer mümkün olan bütün yarışı bitirme sıralamalarını cevap kağıdında bu soruyla ilgili bırakılan boşlukalara yazınız.

Cevap kağıdında gereksiniminizden fazla yer bırakılmıştır.

Bitirme sıralamalarını gösterirken, arabaların yukarıda gösterilen kısaltılmış sembollerini kulanınız.

Örnek: DCFM yarışı sırasıyla önce Dodge'nin, sonra Chevrolet'in, sonra Ford'un ve en sonra Mercedes'in bitirdiğini gösterir.

Appendix B: JUNIOR METACOGNITIVE AWARENESS INVENTORY

Bu anketi doldururken MATEMATİK dersinde yaptıklarınızı düşünerek cevap veriniz.

Appendix A
Bilişüstü Yeti Anketi

Bu çalışmanın amacı, sizin nasıl öğrendiğiniz ve çalıştığınız hakkında bilgi edinmektir. Doğru veya yanlış cevap yoktur. Cevaplar kendi görüşlerinizi yansıtmalıdır. Her cümleyle ilgili görüş belirtirken önce cümleyi dikkatle okuyunuz, sonra cümlede belirtilen durumun size ne derecede uygun olduğuna karar veriniz. Lütfen size en uygun olan yuvarlağın içini doldurunuz. Teşekkürler!

	Hiçbir Zaman	Nadiren	Bazen	Sık Sık	Her Zaman
1. Bir şeyi anladığımı bilirim.	O	O	O	O	O
2. Gerektiğinde, öğrenmek için kendimi motive edebilirim.	O	O	O	O	O
3. Daha önce, benim için işe yaramış çalışma yollarını kullanmayı denerim.	O	O	O	O	O
4. Öğretmenin benden ne öğrenmemi beklediğini bilirim.	O	O	O	O	O
5. Konu hakkında daha önceden bilgim varsa daha iyi öğrenirim.	O	O	O	O	O
6. Öğrenirken anlamama yardımcı olacak resimler veya şemalar çizerim.	O	O	O	O	O
7. Çalışmamı bitirdiğimde kendime "Öğrenmek istediğim şeyi öğrendim mi?" diye sorarım.	O	O	O	O	O
8. Bir problemi çözmek için çeşitli çözüm yollarını denerim ve daha sonra en uygun olanını seçerim.	O	O	O	O	O
9. Çalışmaya başlamadan önce neyi öğrenmem gerektiğini düşünürüm.	O	O	O	O	O
10. Yeni bir şey öğrenirken kendime iyi gidip gitmediğime dair sorular sorarım.	O	O	O	O	O
11. Önemli bilgiye gerçekten dikkat ederim.	O	O	O	O	O
12. Konuya ilgim varsa daha çok öğrenirim.	O	O	O	O	O
13. Zihinsel açıdan güçlü olduğum noktaları, zayıf olan noktalarımı telafi etmede kullanırım.	O	O	O	O	O
14. Verilen işe bağlı olarak farklı öğrenme stratejileri* kullanırım.	O	O	O	O	O
15. Çalışmamı zamanında bitireceğimden emin olmak için ara sıra kontrol ederim.	O	O	O	O	O
16. Bir işi bitirdikten sonra kendime "Daha kolay bir yol var mıydı?" diye sorarım.	O	O	O	O	O
17. Bir işe başlamadan önce neyi tamamlamam gerektiğine karar veririm.	O	O	O	O	O

Appendix C: MATHEMATICAL PROBLEM SOLVING INVENTORY

MATEMATİKSEL PROBLEM ÇÖZME TESTİ:

1) İki torbada toplam 150 jeton vardır. 17 jeton birinci torbadan ikincisine aktarılıyor. Bu durumda birinci torba, ikinci torbanın yarısı kadar jeton içerdiğine göre ilk durumda birinci torbada kaç jeton vardı?

2) 12/15 kesrinin payından hangi sayı çıkarılıp paydasına eklenirse kesrin değeri 1/2 olur?

3) Bir kutu, sakız ve şekerlerle doludur. Sakızların sayısı şekerlerin sayısından 8 fazladır. Sakızların, kutudaki tüm sakız ve şekerlere oranı 3/5 ise, kutudaki sakız ve şekerlerin toplamı kaçtır?

4) Bir gece kral uyuyamaz. Kraliyet mutfağına gider ve orada bir tas dolusu muz bulur. Çok aç olduğundan muzların 1/6'sını alır. Aynı gece, kraliçe de uyuyamaz ve karnı acıkmıştır. Muzları görür ve kralın tasta bıraktığı muzların 1/5'ini alır. Yine aynı gece, prens uyanır, mutfağa gider ve kalan muzların 1/4'ünü yer. Bundan sonra, ikinci prens kendinden küçük olan prensin bıraktığı muzların 1/3'ünü yer. Son olarak, tahtın varisi üçüncü prens kendisinden genç olan kardeşlerinin bıraktığı muzların 1/2'sini yer ve tasta sadece üç muz kalmıştır. Kral bulduğunda tasta kaç tane muz vardı?

5) Zarifiye 6800 nüfuslu bir ilçedir. Bu ilçenin nüfusu her yıl 120 kişi azalmaktadır. Kapanca ise 4200 nüfuslu bir ilçedir. Bu ilçenin nüfusu her yıl 80 kişi artmaktadır. Kaç yıl içinde bu iki ilçenin de nüfusu birbirine eşitlenir?

6) 10 kişilik bir odada herkes kendisi hariç herkesle el sıkışmak durumundadır. El sıkışma sayısını bulunuz?

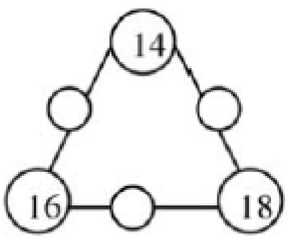
7) Bir dikdörtgenler prizmasının yan, ön ve alt yüzlerinin alanları sırasıyla 12, 24 ve 32 santimetrekaredir. Bu dikdörtgenler prizmasının hacmi kaç santimetreküptür?

8) Dört evli çift tiyatro kulübüne gitmiştir. Bayanların isimleri, Ayşe, Tuğçe, Cemile, Emine; erkeklerin isimleri ise Metin, Tekin, Çetin ve Ersin'dir. Aşağıdaki ipuçlarını kullanarak, kim kiminle evlidir, bulunuz.

- Metin, Emine'nin erkek kardeşidir.
- Emine ve Çetin daha önce bir kez nişanlanmışlardı ama Emine şimdiki kocasıyla tanışınca ayrıldılar.
- Cemile'nin bir kız kardeşi vardır ama kocasının kardeşi yoktur.
- Ayşe, Ersin'le evlidir.

9) Bir çiftlik sahibi tavuk ve tavşan satın alıyor ama hangisinden kaç tane aldığını hatırlamıyor. Kardeşinin yaşına eşit olduğu için toplamda 15 hayvan aldığını ve annesinin yaşına eşit olduğu için toplam ayak sayısının 42 olduğunu hatırlıyor. Buna göre kaç tavuk ve kaç tavşan satın almıştır?

10) Aşağıdaki şekilde, büyük daireler, onlara bağlı olan iki küçük dairenin toplamı şeklinde yerleştirilmiştir. Buna göre küçük dairelerin içindeki sayıları bulunuz.



Appendix D: NORMALITY ASSUMPTION

1. Normality Assumption for Inferential Statistics

According to Pallant (2007), most of the statistical techniques require the assumption of “the distribution of scores on the dependent variable is normal”. Normality means “a symmetrical, bell-shaped curve, which has the greatest frequency in the middle and relatively small frequencies on both extremes” (Gravetter & Wallnau, 2000, p.52). In parametric statistical techniques, normality checking is required. Normality can be assessed by using skewness and kurtosis values. For this, skewness and kurtosis values are important. Skewness and kurtosis values represent the distribution of scores on continuous variables. The skewness value presents an indication of the symmetry of the distribution. Kurtosis value presents information about the peakedness of the distribution. The skewness and kurtosis values should be between -1 and +1values for normal distributions. These values may be extended to -2 and +2 values.

For, normality checking, as well as skewness kurtosis values, the test of normality is also used. In test of normality table, the results of Shapiro-Wilk and Kolmogorov-Smirnov statistics assess the normality of the distribution of scores. Both of the values should be more than .05. If the values are smaller than .05, it suggests violation of assumption of normality. But, in larger samples, the values mostly are smaller than .05. In fact, this violation situation; due to shapiro-wilk and kolmogorov smirnov values, is quite common in larger samples. So, in larger samples, for normality, histogram and plots should be used (Pallant, 2007). The actual shape of the distribution can be seen in Histograms. The scores should be reasonably normally distributed. Also, in the normal probability plots, labelled as Normal Q-Q Plot, a reasonably straight line represents a normal distribution. Also, in the Detrended Normal Q-Q Plots, there should be no real clustering of points. The points should mostly collect around zero line.

In Table 4.8 the skewness and kurtosis values of reasoning ability scale-TOLT, metacognition scale-jMAI and problem solving scale were presented.

Table 4.8 Skewness and Kurtosis Values

	Skewness	Kurtosis
Reasoning Ability-TOLT	-.347	-.601
Metacognition-jMAI	.793	.500
Problem Solving Scale	-.727	.003

In the currents study, the skewness value was -.347 and the kurtosis value was -.601 for reasoning ability-TOLT, the skewness value was .793 and the kurtosis value was .500 for metacognition-jMAI; and the skewness value was -.727 and the kurtosis value was .003 for problem solving scale. These values are between -1 and +1. So, TOLT, jMAI and problem solving scale scores provide normal distribution.
In the Table 4.9, the Kolmogorov-Smirnov and Shapiro-Wilk values for reasoning ability scale-TOLT, metacognition scale-jMAI and problem solving scale were presented.

Table 4.9 Test of Normality

	Kolmogorov-Smirnov Sig.	Shapiro-Wilk Sig.
Reasoning Ability-TOLT	.000	.000
Metacognition-jMAI	.000	.000
Problem Solving Scale	.000	.000

In the current study, as seen in the Table 4.9, the Kolmogorov-Smirnov values and the Shapiro-Wilk values were .00 for TOLT, jMAI and problem solving scale. These values smaller than .05. These values may suggest violation of normality assumption. But this situation is quite common in large samples; so for normality, histogram and plots are also used. For this, in Figure 4.3 the histogram of mean reasoning ability-TOLT scores, in Figure 4.4 the histogram of mean metacognition-junior MAI scores, and finally in Figure 4.5 the histogram of mean problem solving scale scores were presented.

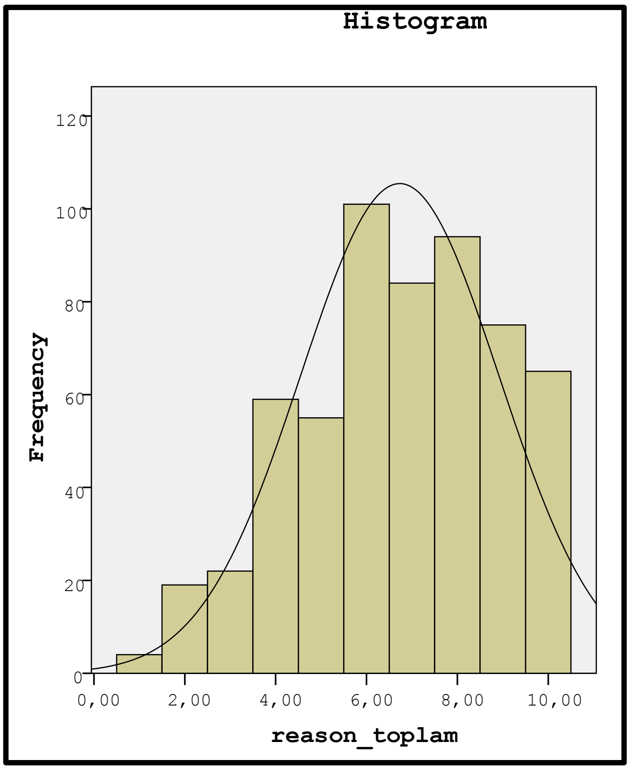


Figure 4.3 Histogram of Mean Reasoning Ability Scores

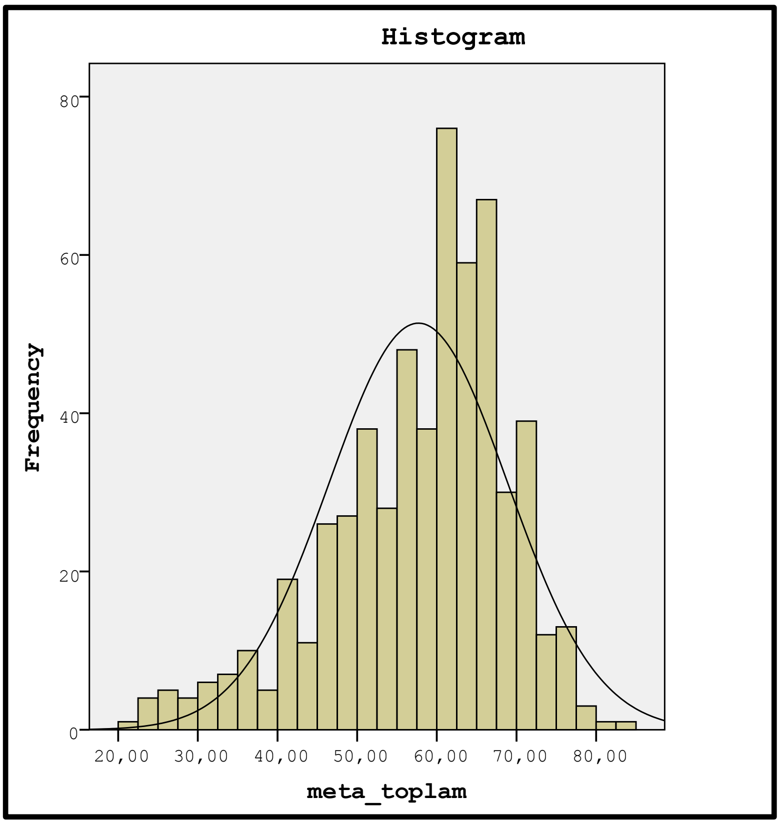


Figure 4.4 Histogram of Mean Metacognition Scores

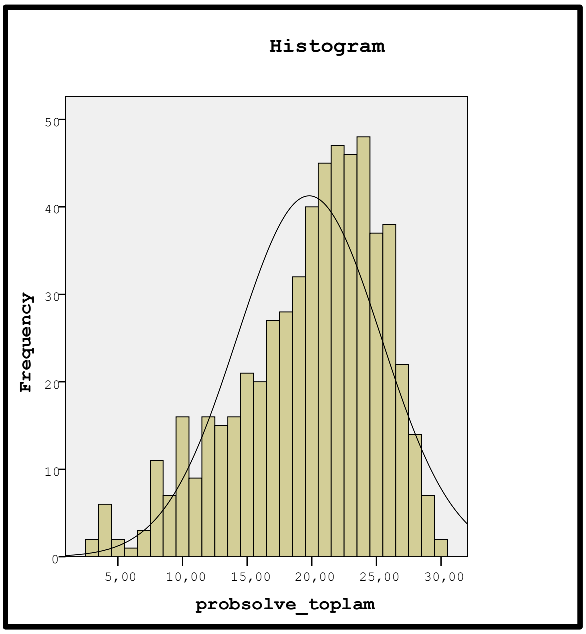


Figure 4.5 Histogram of Mean Problem Solving Scores

In Figure 4.3, in Figure 4.4 and in Figure 4.5, the histogram and the normal curve provided support for the reasonably normal distribution of TOLT, junior MAI and problem solving scale scores. So, normality assumption was assured.

Also, in the normal probability plots, labelled as Normal Q-Q Plot, a reasonably straight line represents a normal distribution. For normality assumption, Normal Q-Q Plot of TOLT, junior MAI and problem solving scale scores were also checked.

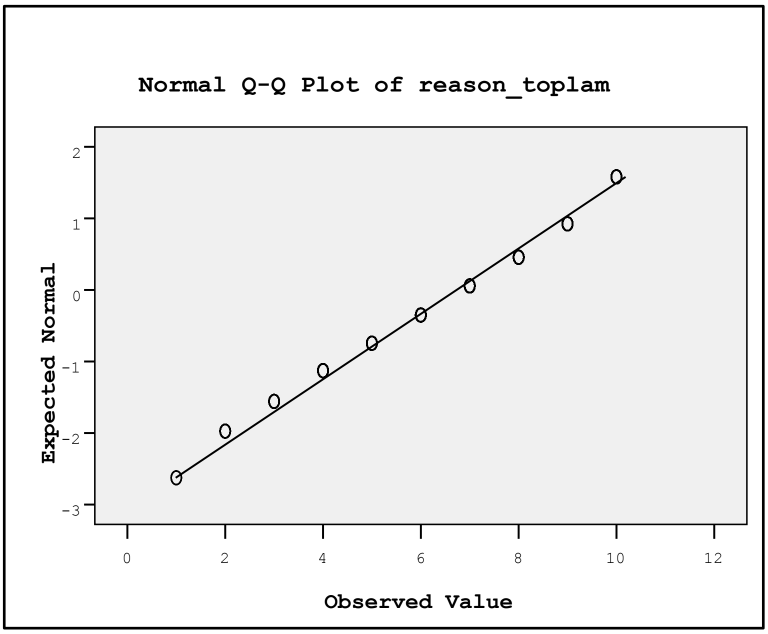


Figure 4.6 Normal Q-Q Plot of Reasoning Ability

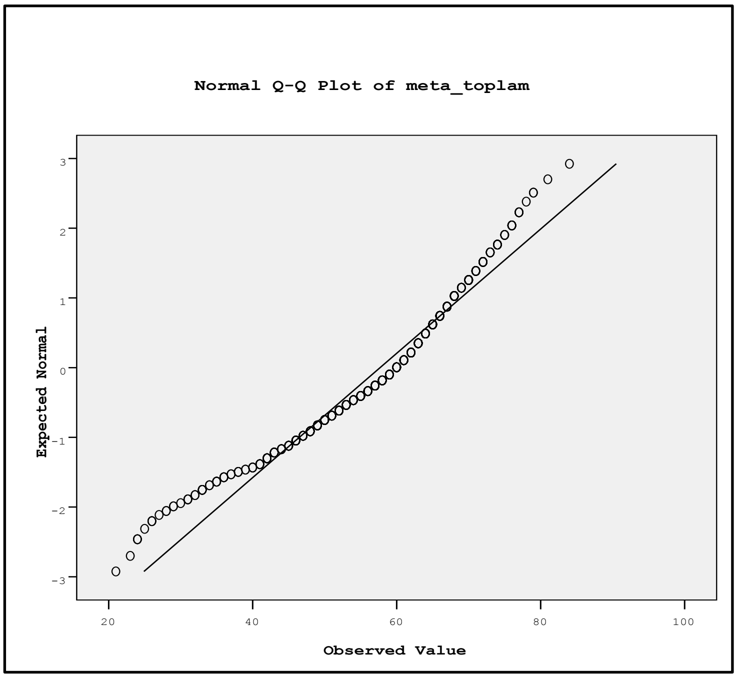


Figure 4.7 Normal Q-Q Plot of Metacognition

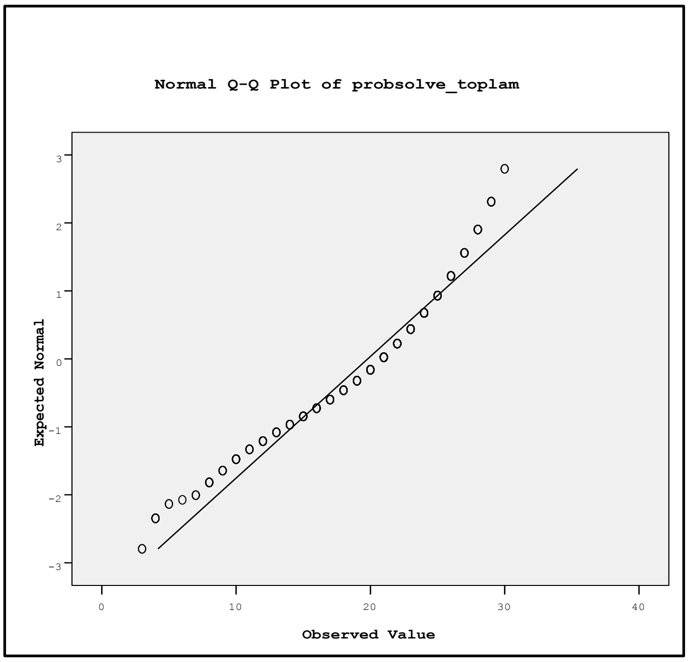


Figure 4.8 Normal Q-Q Plot of Problem Solving

Also, in the normal probability plots, labelled as Normal Q-Q Plot, a reasonably straight line represents a normal distribution. As seen in the Figure 4.6, Figure 4.7 and Figure 4.8, there is a reasonably straight line in each plot, so the plots support normality assumption for reasoning ability, metacognition and problem solving performance.

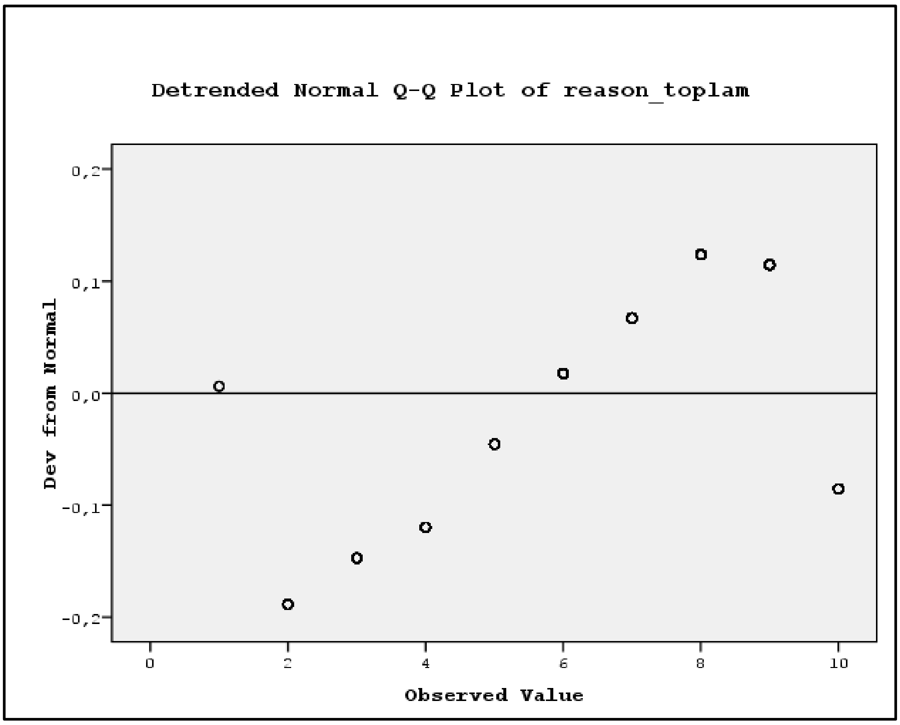


Figure 4.9 Detrended Normal Q-Q Plot of Reasoning Ability

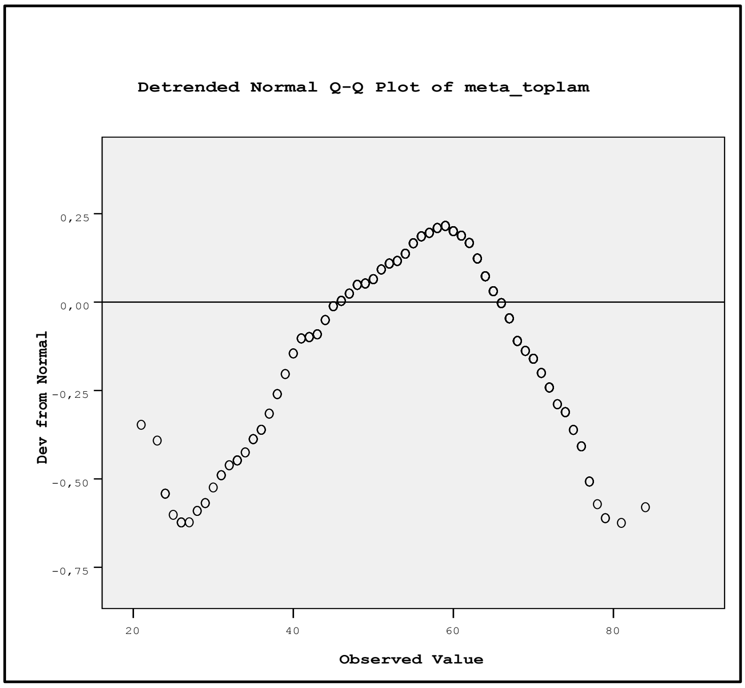


Figure 4.10 Detrended Normal Q-Q Plot of Metacognition

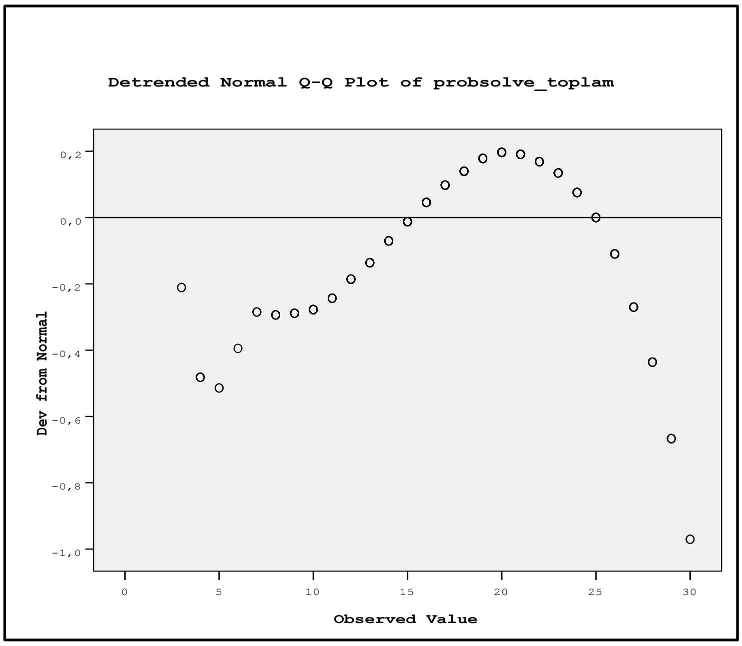


Figure 4.11 Detrended Normal Q-Q Plot of Problem Solving

Also, in the Detrended Normal Q-Q Plots, there should be no real clustering of points. The points should mostly collect around zero line, for normality. As seen in the Figure 4.9, Figure 4.10, and Figure 4.11, in each plot, there is no real clustering of points, and points mostly collect round zero line. So, the Detrended Normal Q-Q Plots for reasoning ability, metacognition and problem solving performance provided support for normality of scores.

To conclude, for reasoning ability scale-TOLT, metacognition scale-jMAI and problem solving scale, the skewness and kurtosis values are between the required range. The histograms with normal curves, the normal Q-Q plots, and the Detrended Normal Q-Q plots also provided evindence for normality. In summary, normality assumption was assured for all variables.

Appendix E: CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Elitaş, Yüksel Özge
Nationality: Turkish (TC)
Date and Place of Birth: 20 December 1983 , İzmir
Marital Status: Single
email: e133313@metu.edu.tr

EDUCATION

Degree	Institution	Year of Graduation
BS	METU Elementary Mathematics Education	2008
High School	İzmir Buca Anatolian High School, İzmir	2002

WORK EXPERIENCE

Year	Place	Enrollment
2015- Present	Manisa	Teacher

FOREIGN LANGUAGES

Advanced English

HOBBIES

Movies, Music, Reading and Cinema.

Appendix F: TURKISH SUMMARY

BÖLÜM 1

GİRİŞ

Problem çözme konusu uzun yıllardan beri matematik eğitiminde büyük bir öneme sahip olmuştur ve günümüzde hala önemli bir konu olmaya devam etmektedir (Evans, 2012; Posamentier & Krulik, 2008). Yirminci yüzyılda problem çözmeyi öğrenme ve öğretme konularına özel bir dikkat yöneltilmiş ve büyük önem verilmiştir (Hembree, 1992). Polya'nın problem çözme konusuyla ilgili fikirleri matematik eğitimi alanını uzun yıllar etkilemiştir. Polya'nın ünlü kitabı "Nasıl Çözmeli" 1945 yılında büyük bir etki uyandırmış ve dikkat çekmiştir. Kitabın her yeni baskısıyla bu buyuk önem yıllarca artmıştır (Bahar, 2013; Donaldson, 2011; Hembree, 1992, Özalkan, 2010). Özellikle Polya kitabın yeni baskısı olan "matematiksel modele yeni bakış" kitabıyla 1973'te problem çözme konusuna yeniden dikkat çekmiş ve konuyu yeniden gündeme taşımıştır. Bu kitaptan 7 yıl sonra Matematik Öğretmenleri Ulusal Konseyi (NCTM) problem çözme konusunun okuldaki matematiğin temeli ve odağı olması gerektiğini açıklamıştır (NCTM, 1980, p.1). Daha sonraki yıllarda konsey problem çözme konusunu 4 süreç standardı altında tekrar vurgulamıştır. Konseyle birlikte birçok araştırmacı yazar da problem çözme konusunun önemini vurgulamıştır. Örneğin Posameinter ve Krulik (1998) problem çözme konusunun müfredatta büyük öneme sahip olması gerektiğini belirtmiştir. Ayrıca problem çözme konusunun iyi bir öğretim programının önemli ve temel bir parçası olması gerektiğini vurgulamıştır. Matematik müfredatlarında problem çözmenin önemini vurgulamak için bir çok eyaletin ve ülkenin ulusal sınavlarda ve müfredatlarında problem çözme becerilerini ölçtüklerini belirtmiştir. Ayrıca Evans (2012) güçlü problem çözme becerisinin sadece matematik için değil; diğer disiplinler için ve de günlük hayatımız için çok önemli olduğunu belirtmiştir. Bu sebeple öğrencilerin gerçek hayatlarında başarılı olmak için de okulda güçlü problem çözme becerisi kazanması gerektiğini vurgulamıştır. Benzer olarak Donaldson (2011) problem çözmenin matematiğin temeli olduğunu öğretmenlerin temel amacının öğrencilerin problem çözme becerilerini geliştirmek olması gerektiğini belirtmiştir. Ayrıca Gagne (1985) eğitimin temel amacının insanlara düşünmeyi, kendi güçlerini kullanmayı ve böylece daha iyi problem çözücü olmalarını öğretmek olduğunu belirtmiştir. Ayrıca Jonassen (2000) de problem çözmenin hayat için en önemli kazanım olduğunu belirtmiştir.

Garofalo and Lambdin (1989) yıllardır öğrencilerin hesaplama ve algoritmik prosedürleri uygulamakta başarılı olduğunu fakat sözel problemleri çözmekte başarısız olduğunu belirtmiştir. Bunun sebebi bilişsel olarak görünse de üstbilişin de bu sorunda önemli rol oynadığını belirtmiştir. Gredler (2005) problem çözme başarısı için bilişin yanında üstbilişin de çok önemli olduğunu vurgulamıştır. Benzer olarak Schraw and Dennison (1994) üstbilişin problem çözme sürecinde çok önemli rol oynadığını belirtmiştir. Eğer matematikte problem çözmeyle birlikte anlamlı öğrenme gerçekleşsin istiyorsak üstbilişin önemini vurgulamalıyız (Berardi-Coletta, Buyer, Dominowski, & Rellinger, 1995). Çünkü etkili öğrenmenin gerçekleşmesi için bir öğrencinin kendi bilişsel aktivitelerini aktif olarak gözden geçirmesi gerekmektedir. Üstbilişsel olarak gelişmiş bireyler problem çözmede daha iyi performans sergilerler (Baker ve Brown, 1980). Uzman problem çözücüler yaptıklarını kontrol etmek, kullandıkları stratejileri ve çözüm yolları üzerinde düşünmek, kendi düşüncelerini gözlemlemek ve yansıtmak gibi üstbilişsel davranışlarda sık sık bulunurlar. Bu kişiler kendi düşüncelerini ve planlarını gözlemler ve doğru yolda olup olmadıklarını anlamak için kendilerine çeşitli sorular sorarlar. Bu süreçte problem için eski deneyimlerini bilgilerini kullanırlar ve farklı çözüm yolları bulmaya probleme farklı bakış açısıyla bakmaya çalışırlar (Schoenfeld, 1983; 1985; 1987; 1992). Üstbilişle problem çözme başarısı arasındaki bağa Carlson ve Bloom (2011) da vurgu yapmıştır. Ayrıca Antonietti, Ignazi ve Perego (2000) yüksek üstbilişsel becerilerin en iyi problem çözme performansıyla ilişkisi olduğunu belirtmiştir. Benzer olarak Özsoy (2006) da problem çözme ile üstbiliş arasındaki ilişkiyi vurgulamıştır. Mayer (1998) ise üstbilişsel becerilerin problem çözme sürecinde çok önemli bir yere sahip olduğunu belirtmiş ve üstbilişsel becerilerin öğrencilerin matematiksel problem çözme performansını artırdığını iddia etmiştir. Ayrıca Özsoy and Ataman (2009) da üstbiliş eğitimi alan öğrencilerin almayanlara göre daha yüksek matematiksel problem çözme performansı gösterdiğini ve bu öğrencilerin problem çöcme becerilerini daha çok geliştirdiklerini iddia etmiştir.

Tobin and Capie (1981) iki önemli duruma değinmiştir. Birinci durum birçok yetişkin kişinin sınırlı mantıksal düşünme yeteneğine sahip olduğu ve bu yeteneğe sahip olsalar dahi bunu sınırlı düzeyde kullandıklarıdır. İkinci durum ise mantıksal düşünme yeteneğinin bilişsel başarı için çok önemli bir aracı olduğudur. Bu sebeplerden dolayı öğretim amaçları, materyalleri ve aktiviteleri öğrencilerin bilişsel düzeyine göre düzenlenmelidir. Öğrencilerin mantıksal düşünme yeteneklerini artırmaya yönelik düzenleme yapılmalı ve düzeylerine uygun müfredat programları ve materyalleri dizayn edilmelidir (Tobin & Capie, 1981). Mueller, Yankelewitz ve Maher (2011) mantıksal düşünme yeteneğinin anlamlı öğrenmeyi ve anlamayı getirdiğini belirtir. Öğrencilerin ezberden ziyade mantıksal düşünmelerinin onlara anlamlı öğrenmeyi ve derin matematiksel anlama sürecini kazandırdığını ve kavramsal öğrenmeyi gerçekleştirdiğini belirtir. Schoenfeld (1992), problem çözme sürecinde öğrencilerin kendi bakış açılarını ve matematiksel düşüncelerini matematiksel bilgiye dayanarak mantıklı bir biçime ifade etmeleri gerektiğini belirtir. Yani öğrenci problemi çözerken hem matematiksel bilgiyi kullanmalı hem de matematiksel düşünmeli ki bu da mantıksal düşünmeyi gerektirir (NCTM, 1991; NME, 2005, p.14). Mueller ve Maher (1996) öğrencilerin kendilerini ve düşüncelerini mantıksal çerçevede açıklamalarının ve savunmalarının mantıksal düşünmeyi ve matematiksel anlamayı gerektirdiğini belirtir. Longman (1987), mantıksal düşünmenin anlama, düşünce geliştirme, aktif düşünebilme ve bilgiye dayalı çıkarım yapmayı gerektirdiğini belirtir. Mantıksal düşünme bir kişinin kendi düşününcelerini oluşturması savunması ve bilgiye gerçeklere dayanarak mantıklı çıkarımlar yapabilmesidir. Ball ve Bass (2003) mantıksal düşünmenin matematiksel beceriler için çok önemli bir temel oluşturduğunu ve matematiksel anlamanın mantıksal düşünme üzerinden gerçekleştiğini belirtmiştir. Mantıksal düşünme yeni bilgiyi öğrenmek için bir gerekliliktir, çünkü mantıksal düşünme eski ve yeni bilgiyi birbiriyle ilişkilendirir ve birbirine entegre eder. Ayrıca Steen (1999) mantıksal düşünmenin matematik için temel oluşturduğunu ve matematiğin mantık üzerine kurulduğunu belirtmiş ve mantıksal düşünmeyle matematik arasında çok sıkı bir bağ olduğunu vurgulamıştır. Nickerson (1994) düşünme ve problem çözmeyle en çok alakalı konunun mantıksal düşünme olduğunu belirtir. Mueller ve Maher (1996) mantıksal düşünmenin matematiksel anlama için temel oluşturduğunu ve matematiksel bilginin oluşması için öğrencinin mantıksal düşünmeyi öğrenmesi gerektiğini belirtir. Brodie (2000) mantıksal düşünmenin problemin ne olduğunu, neyin gerçek olduğunu, matematiksel bir bağlamda neyin doğru olduğunu, tüm bu sonuçlar için bir açıklama sunabilmeyi, ve kendi sonucunun doğru olduğunu savunabilmeyi, doğru bir mantıksal akıl yürütmeyi öğrenmeyi ve açıklayabilmeyi içerdiğini belirtmiştir. Mansi (2003) mantıksal düşünmenin uyumlu ve mantıklı düşünmeyi, çıkarımlar yapabilmeyi, matematiksel gerçeklerden sonuçlar oluşturabilmeyi ifade ettiğini belirtmiştir. Mantıksal düşünme öğrencilerin matematiği öğrenmeleri için çok önemli bir gerekliliktir. Çünkü, mantıksal düşünme sürecinde, öğrenciler matematiksel fikirler hakkında akıl yürütürler, bağlantılar ve bağlamlar oluştururlar, bu matematiksel fikirlerin neden anlamlı olduğuna dair açıklamalar ve savunmalar sunarlar (Mansi, 2003). Mantıksal düşünme bir insanın kanıtlara, gerçeklere ya da farz edilenlere dayanarak sonuçlar oluşturması ve çıkarımlarda bulunmasını gerektirir. Mantıksal düşünme matematikte çok önemli bir yere sahiptir. Mantıksal düşünme mantıksal çıkarım gerektirir, ve mantıksal akıl yürütme ve matematikte kanıt gerektirir. Ayrıca, gözlemler, akıl yürütmeler, bağlamlar ve mantıklı açıklamalar gerektirir. Öğrenciler mantıksal düşünmeyi geliştirmeye en düşük kademelerde başlamalıdır, böylece ilerleyen kademelerde matematiği daha kolay öğrenirler. Çünkü mantıksal düşünme matematikte çok önemli bir yere sahiptir (Martin & Kasmer, 2010).

Mantıksal düşünme bir kişinin problem formule edebilmesini, problemi temsil edebilmesini, problemdeki arguman hakkında açıklamalar sunabilmesini, problemin çözümü hakkında açıklamalar ve savunmalar yapabilmesini gerektirir (Kilpatrick, Swafford & Findell, 2001). Mantıksal düşünme yeteneği formal bir düşünce yapısı ya da entellektüel bir yetenektir, ve düşünme sürecindeki kademeleri ifade eder (Gerber, Marek & Cavallo, 1997).

Sonuç olarak, literaturde görüldüğü gibi, problem çözme, üstbiliş ve mantıksal düşünme yeteneği öğrenciler için çok önemlidir. Dolayısıyla bu değişkenler arasındaki ilişkiyi araştırmak önemli bir çalışma arzeder.

Çalışmanın Amacı

Bu çalışmanın temel amacı üstbiliş, mantıksal düşünme yeteneği ve matematiksel problem çözme performansı arasındaki ilişkiyi araştırmak ve incelemektir. Çalışmanın diğer bir amacı da üstbiliş ve mantıksal düşünme yeteneğinin matematiksel problem çözme performansını yordama düzeyini araştırmaktır.

Çalışmanın Hipotezleri

Çalışmadaki;

Birinci hipotez üstbiliş ve matematiksel problem çözme arasında istatiktiksel olarak anlamlı bir ilişki olduğudur.

İkinci hipotez mantıksal düşünme yeteneği ve matematiksel problem çözme performansı arasında istatiktiksel olarak anlamlı ilişki olduğudur.

Üçüncü hipotez üstbiliş ve mantıksal düşünme yeteneği arasında istatiktiksel olarak anlamlı ilişki olduğudur.

Dördüncü hipotez ise üstbiliş ve mantıksal düşünme yeteneğinin matematiksel problem çözme performansını anlamlı olarak yordadığı yönündedir.

Bu hipotezler doğrultusunda bu çalışmanın;

Birinci araştırma problemi üstbiliş ve matematiksel problem çözme arasında bir ilişki olup olmadığıdır.

İkinci araştırma problemi mantıksal düşünme yeteneği ve matematiksel problem çözme arasında bir ilişki olup olmadığıdır.

Üçüncü araştırma problemi üstbiliş ve mantıksal düşünme yeteneği arasında bir ilişki olup olmadığıdır.

Dördüncü araştırma problemi üstbiliş ve mantıksal düşünme yeteneğinin matematiksel problem çözme performansını istatistiksel olarak anlamlı biçimde yordayıp yordamadığıdır.

Beşinci araştırma problemi üstbiliş ve mantıksal düşünme yeteneğinin matematiksel problem çözme performansını ne derece yordadığıdır.

Altıncı araştırma problemi üstbiliş ya da mantıksal düşünme yeteneğinden hangisinin tek başına matematiksel problem çözme performansını daha iyi yordadığını araştırmaktır.

Tanımlar

Üstbiliş: Bu çalışmada Brown (1978) çerçevesi temel alınmıştır. Brown (1978) üstbilişin iki parçası olduğunu belirtir: biliş bilgisi ve biliş düzenlemesi. Biliş bilgisi açıklayıcı, prosedurel ve koşullu biliş bilgisini içerir. Biliş düzenlemesi ise planlama, gözden geçirme ve değerlendirmeyi içerir.

Matematiksel problem çözme performansı: Bu çalışmada matematiksel problem çözme performansı öğrencinin matematik problemlerini çözdükten sonra aldığı toplam puanı belirtir. Aslında problem çözme performansı bir problem çözücünün kaç tane problem doğru çözdüğünü belirtir (Antonietti, Ignazi & Perego, 2000).

Mantıksal düşünme: Bu çalışmada mantıksal düşünme beş mod içerir. Bunlar değişkenleri kontrol etme, kesirsel mantık, olasılıksal mantık, ilişkisel mantık ve kombinasyonel mantıktır (Tobin & Capie, 1981).

Çalışmanın önemi

Uzun yıllardır problem çözme çok büyük öneme sahip olmuştur ve hala da matematik eğitimi için çok önem arz etmektedir (Evans, 2012; Hembree, 1992; Posamentier & Krulik, 2008). Fakat matematikçiler, yazarlar, araştırmacılar ve matematik alanında çalışma yapan profesyonel kişiler problem ve problem çözme için çok farklı tanımlar yapmışlardır. Problem çözmenin ortak bir tanımı henüz kabul edilmemiştir (Donaldson, 2011; Ellis, 2005; Grugnetti & Jaquet, 2005). Ayrıca, geçmişteki çalışmalarda evrensel olarak ortak kabul görmüş bir teori de yoktur (Grugnetti & Jaquet, 2005; Nickerson, 1994). Dahası, 1980'den 2003'e kadar alanda fazla çalışma yapılamamış ve bu çalışmalar okul pratiği için çok yararlı olamamıştır (Lester & Kehle, 2003). Aslında problem çözme alanında teorik altyapı sorunu vardır diyebiliriz (Lesh & Zawojewski, 2007). Bu sebeple, problem çözme alanında teorik altyapıya katıkıda bulunacak çalışmalara ihtiyaç vardır. Bunun için, düşünme ve problem çözme konularını daha iyi anlamak için daha nitelikli, daha kesin, daha donanımlı ve daha test edilebilen çalışmalar yapılmalıdır. Ayrıca, epistemolojik olarak da problem çözmenin diğer farklı bir çok değişkenle olan ilişkileri araştırılmalıdır (Donaldson, 2011; Ellis, 2005; Grugnetti & Jaquet, 2005; Lesh & Zawojewski, 2007; Lester & Kehle, 2003; Nickerson, 1994). Problem çözme alanında çalışmalara ihtiyaç duyulduğu için, işbu çalışma bu açığı kapatmak için bir katkı sunmayı amaçlamaktadır. Bu çalışmada, üstbiliş, mantıksal düşünme ve problem çözme arasındaki ilişki ağını gösteren bir destek sunmak amaçlanmıştır.

Problem çözmenin önemini anlatmak için Polya (1973) öğretmenlerin öğrencilerinin düşünme yeteneklerini ve problem çözme becerilerini geliştirmeleri gerektiğini belirtmiş ve ısrarla yıllarca vurgulamıştır. Uzun yıllar sonra bile Evans (2012) güçlü problem çözme becerisinin matematik için çok önemli olduğunu ve günlük hayat için de çok önemli olduğunu belirtmiştir. Bu yüzden öğrencilere kritik düşünme ve problem çözme becerisi için okulun bir hazırlık kurumu olması gerektiği vurgulanmıştır (Evans, 2012). Ayrıca Krulik ve Posamentier (1998), öğretmenlerin problem çözmeyi derslerin ve müfredatın çok önemli bir parçası olarak görmesi gerektiğini, problemin ne olduğu ve problem çözmenin matematiksel becerileri etkili bir şekilde öğrenmede ne kadar önemli olduğunu vurgulaması gerektiğini, ve de problem çözmenin öğrencilere nasıl etkili bir biçimde kazandırılması gerektiğini vurgulamıştır. Problem çözme önemini günümüze kadar koruduğuna göre, problem çözmeyi okullarda ve derslerde vurgulamak hala önem arz eder. O yüzden, bu çalışma problem çözme konusunun önemini vurgulamayı ve problem çözme konusunun önemine dikkat çekmeyi amaçlamıştır.

Yüksek problem çözme performansına sahip olmak için bir konu hakkında bilişsel bilgi sahibi olmak yeterli değildir. Ayrıca bilişsel süreci gözlem ve kontrol yeteneği de gereklidir (Mayer, 1998). Ayrıca, yüksek problem çözme performansı için hem hesaplama becerileri ve stratejileri, hem de problem çözme sürecine dair farkındalık gerekmektedir (Demircioğlu, 2008; Özsoy, 2007). Dahası, üstbiliş günümüzde de araştırılmaya değer bir konudur ve ileriki yıllarda da araştırmaya değer olacaktır (Moreover, Stillman & Mevarech, 2010). Alan yazında belirtildiği üzere, üstbiliş ve problem çözme arasında önemli bir ilişki olduğu için, bu çalışmada üstbilişle problem çözmenin arasındaki ilişki araştırılmıştır. Bu çalışma, üstbilişin öğrencilerin matematiksel problem çözme performansı için ne kadar önemli olduğunu hatırlatmak ve vurgulamak amacı taşır.

Mantıksal düşünme matematiği anlamak için temel arz eder (McKenzie, 2000; Mueller & Maher, 2009). Mantıksal düşünme matematiği öğrenmek için güçlü ve gerekli bir durumdur (Mansi, 2003). Mantıksal düşünme matematiksel beceriler için gerekli bir altyapıdır. Mantıksal düşünme yeni bilgiyi oluşturmak için ve dolayısıyla yeni matematik bilgilerinin inşa edilmesi için çok gereklidir (Ball & Bass, 2003). Matematiği anlamak için bir kişi mantıksal düşünebilmelidir. Bir kişi mantıksal düşündüğünde matematiksel fikirleri yeni durumlara transfer eder ve böylece problem çözme becerileri gelişir (Mueller &Maher, 2009). Benzer olarak, öğrencilerin mantıksal düşünme ve anlamlı öğrenmeleri problem çözme becerisi ve fikirlerin bağdaştırılması için çok önemlidir. Mantıksal düşünme ve anlamlı öğrenme sınıflarda mümkün olduğunca çok geliştirilmelidir, böylece öğrencilerin anlama ve öğrenmeleri de gelişir (Cavallo, 1996). Mantıksal düşünme aritmetik becerilerden farklıdır, ve mantıksal düşünme okul müfredatların büyük öneme sahip olmalıdır (Nunes, Bryant, Barros & Sylva, 2012). Ayrıca, mantıksal düşünme ile matematik başarısı arasında direkt bir ilişki vardır ve mantıksal düşünme yeteneği yüksek öğrenciler daha iyi problem çözme karakteristiğine sahiptirler (Kramarski, Mevarech & Lieberman, 2001). Gunhan (2014) okul müfredatlarında mantıksal düşünmenin daha fazla vurgulanması gerektiğini belirtir. Ayrıca öğretmenler öğrencilerin mantıksal düşünme becerilerini geliştirmelidir der. Usman and Musa (2013) öğrencilerin mantıksal düzeylerinin sınıflarda ölçülmesi gerektiğini, ve dersin öğrencilerin mantıksal düşünme düzeylerine göre dizayn edilmesi ve işlenmesi gerektiğini belirtir. Literaturde de belirtildiği gibi, mantıksal düşünme yeteneği problem çözme üzerinde büyük öneme sahiptir. Bu sebeple, bu çalışmada mantıksal düşünme ile problem çözme arasındaki ilişkinin önemi vurgulanmak istenmiştir. Bu ilişkiye dikkat çekmek amaçlanmıştır.

Daha önceki çalışmalarda da belirtildiği gibi mantıksal düşünme, üstbiliş ve problem çözme arasında önemli bir ilişki vardır ve hepsi de öğrencilerin matematiksel başarısı için önemlidir. Daha önceki çalışmalarda genellikle bu değişkenler ikişer ikişer ele alınmıştır. Fakat bu çalışmada üç değişken tek bir çalışmada ele alınarak aralarındaki ilişki ağı araştırılmıştır. Ayrıca daha önceki çalışmalar bu değişkenler arasında genel olarak ilişkisel desenle incelenmemiştir. Genel olarak deneysel ya da diğer desenlerde incelenmiştir. Hali hazırda varolan ilişkiyi incelemek ve ortaya çıkarmak da en az deneysel çalışma kadar önemli olduğu için bu çalışma ilişkisel desen çalışmalarındaki eksikliği doldurmak için bir katkı sunmaktadır. Diğer çalışmalarda ikili olarak belirtildiği gibi, bu çalışmada üç değişken arasında da ilişki bulunması beklenmiştir (Antonietti, Ignazi & Perego, 2000; Berardi-Coletta, Buyer, Dominowski & Rellinger, 1995; Carlson & Bloom, 2011; Higgins, 1997; Ozsoy & Ataman, 2009). Bu çalışmanın alana ve literature katkısı üç değişkenin tek bir çalışmada ele alınmış olmasıdır. Genel olarak tek tek ya da ikişerli olarak ele alındığı için, dokuzuncu sınıf öğrencilerinin üstbiliş, mantıksal düşünme ve matematiksel problem çözme performansının tek bir çalışmada incelenmesinin alana katkı sağlayacağı düşünülmektedir.

Ayrıca, Türkiye'de yapılan çalışmalarda genellikle öğretmen adayları, öğretmenler, diğer branşlardaki öğretmenler ya da üniversite öğrencileri üzerinde çalışılmıştır. Üstbiliş, mantıksal düşünme ve problem çözme konularından herhangi biri yine genellikle diğer kademelerdeki öğrenciler üzerinde çalışılmıştır. Dokuzuncu sınıf öğrencileriyle çalışılan, üstbiliş ya da mantıksal düşünme ya da problem çözmeyle ilgili yapılan çalışmaların sayısı çok azdır. Bu çalışmadaki gibi üç değişkeni birden araştıran ve dokuzuncu sınıf öğrencileriyle çalışılan ve daha önce yapılmış emsal bir çalışma göründüğü kadarıyla mevcut değildir. Bu çalışma tek oluşu ve üçlü ve güçlü bir ilişki ağını tek bir çalışmada biraraya getirdiği için ve çalışılan öğrenci kademesi itibariyle diğer çalışmalardan farklıdır ve alandaki bu boşluğu doldurmak için bir katkı sağlayacağı düşünülmektedir.

Ayrıca, çalışmada elde edilen sonuçlara dayanarak üstbiliş, mantıksal düşünme ve problem çözme konularına daha çok önem verilmesi umulmaktadır. Bu konuların matematik eğitimi gündeminden düşmemesi için bu konuların önemi hakkında vurgu yapmak amaçlanmıştır. Bu çalışmanın müfredat geliştirenlere, eğitimcilere ve öğretmenlere; müfredatta değişiklik olarak, eğitim materyalleri dizaynında, sınıftaki öğretim kültüründe ve sınıf içi eğitim öğretim etkinliklerinde yararlı olacağı umulmaktadır. Bu çalışma öğrencilerin daha iyi problem çözme performansına sahip olmaları için tavsiye anlamında kullanılması anlamında alana katkı sağlayacağı umulmaktadır. Çalışma sonuçlarına dayanarak, ilköğretimde ve ortaöğretim kademelerinde üstbilişin vurgulanması ve üstbiliş eğitimi verilmesi önerilmektedir. Özellikle matematik derslerinde her kademede üstbilişi geliştirmeye yönelik etkinlikler yapılması vurgulanmakta ve önerilmektedir. Okullarda üstbilişin görmezden gelindiği, öneminin farkına varılmadığı ve yeterince üzerinde durulmadığı düşünülürse, bu çalışma matematik eğitiminde üstbilişi vurgulama anlamında alana katkı sağlayacağı umulmaktadır. Bu çalışmanın, üstbilişin hem okullarda hem de üniversitedeki öğretmen adaylarının eğitiminde vurgulanması ve öğretilmesi ve geliştirilmesi adına tetikleyici olması umulmaktadır. Ayrıca, mantıksal düşünme yeteneğinin matematik müfredatındaki yerinin vurgulanması, ve matematik derslerinde temel olarak alınması gerektiği belirtilmektedir. Mantıksal düşünme yeteneğinin yeni dizayn edilen eğitim öğretim materyallerinde, ders kitaplarında, sınıf içi aktvitelerde vurgulanması ve matematik müfredatının temel elemanlarından olması tavsiye edilmektedir. Çalışmanın diğer bir vurgusu da problem çözmenin matematikteki yeri üzerinedir. Bu çalışma, problem çözmenin sınıflarda daha önemli yer tutması, problem çözme adımlarının ve stratejilerinin derslerde kullanılması, problem çözme sürecinin öneminin ders içi aktivitelerle vurgulanması adına hatırlatıcı olması açısından önemlidir. Ayrıca, bu değişkenler üniversitelerdeki matematik ve eğitim ile ilgili tüm bölümlerde vurgulanabilmelidir. Böylece, öğretmen adayları kendileri bizzat üstbiliş eğitimi alarak, öğrencilerinin üstbiliş seviyelerini nasıl yükselteceklerini bilerek eğitime daha iyi katkı sağlayabilirler. Aynı şekilde, öğretmen adayları kendileri problem çözmenin önemini, adımlarını ve stratejilerini öğrenirlerse öğrencilerine öğretmeleri daha kolay olabilir. Mantıksal düşünmenin önemini üniversitedeyken kavrarlarsa, kendi sınıflarında da öğrencilerine daha etkili aktarabilirler. Böylece tüm bu bileşenlerle öğrenciler de matematiği daha anlamlı bir şekilde yapılandırabilirler, daha etkili bir şekilde öğrenebilirler ve hem matematikte, hem diğer derslerde hem de günlük hayatta daha iyi problem çözme performansı sergileyebilirler.

BÖLÜM 2

LITERATÜR TARAMASI

Üstbiliş

Gredler’e gore (2005) çeşitli bilim dallarındaki önemli gelişmeler ve teknolojinin günlük hayata girişi ve teknolojik değişiklikler insanlar üzerinde yeni talepler oluşturmuştur. Bu yeni teknolojik gelişmeler kendini yönlendiren öğrenme diyebileceğimiz ve üstbilişle ortaya çıkan bir süreci gerektirmiştir. Bu durum kişinin kendi öğrenmesini yönetebilmesi, ve yeni problemleri çözebilmeyi öğrenmesinin önemini arttırmıştır. Bu kendi kendine öğrenme yeteneği üstbilişsel yetenekle elde edilebilir. O yüzden bilişle birlikte üstbiliş kavramı da ortaya çıkmıştır (Gredler, 2005). Üstbiliş kavramının kurucusu Flavell ilk once meta-hafıza denen kavramı irdelemiştir. Flavell (1975) ‘te meta hafızayı çevreden gelen verilerin izlenmesi, yönetilmesi, depolanması, araştırılması ve yeri geldiğinde hatırlanması olarak kullanmıştır. Daha sonra Flawell (1976) da üstbilişi ilk olarak hem izleme hem de düzenleme süreçlerini içeren bir kavram olarak tanımlamıştır. Flawell 1976’da çevredeki akan bilişsel süreçlerin izlenmesi, düzenlenmesi ve bu süreçlerin bir amaca yönelik olarak işletilmesi olarak tanımladı. Daha sonra 1979’da Flawell üstbilişi sözlü iletişim, anlama ve ikna etme, yazma, okuma, dil öğrenme, problem çözme gibi konularla ilişkisini vurguladı.

Brown’a göre (1978) üstbiliş gerek eğitimsel gerek de diğer öğrenme süreçlerinde önemli bir problem çözme yeteneğine işaret etmektedir. Burada bilgi ile o bilginin nasıl algılandığının farkı bilişsel sürecin gelişmesi açısından önemlidir. Çocuklardaki üstbilişsel sürecin gelişiminde planlama, tahmin etme, izleme gibi kavramlar önemli yer tutar. Flawell(1976) üstbilişi kişinin kendi bilinç düzeyinin farkında olması olarak değerlendirmiştir. Lester, Garofalo and Lambdin (1989) üstbiliş, kendi bilişsel fonksiyon ve yeteneklerinin bilincinde olmak olarak tanımlamışlardır. Berardi-Coletta, Buyer, Dominowski ve Rellinger (1995) üstbilişi öğrenmeyi öğrenme diyebileceğimiz bir kavram olarak nitelendirmişlerdir.

Gredler (2005) üstbilişi biliş düzeyinin farkında olunması ve bilişsel sürecin düzenlenmesi olarak iki farklı kavramdan oluştuğunu belirtmiştir. Üstbilişin iki tane bileşeni vardır. Birincisi kendi düşünme sürecini bilmek ve kendi düşünce sürecinin farkında olmaktır. Bu bileşen kişinin kendi kapasitesini ve sınırlılıklarını bilmesini ve öğrenme gerçekleşirken karşılaşılan güçlükleri farkında olmayı içerir. Ikinci bileşen öğrenilen stratejilerin nerede, nasıl ve ne zaman kullanılacağını bilmektir. Bu bileşen hedefe özgü hangi stratejinin hangi durumlarda kullanılmasının uygun olacağı bilginin içerir. Benzer olarak, Schraw ve Dennison (1994) üstbilişin iki önemli bileşeni olduğunu belirtmiştir. Birinci bileşen üstbilişsel bilgidir ve bu biliş bilgisini üstbilişsel sürecin bilinç düzeyinin farkındalığı olarak ifade eder. Üstbilişsel bilgi bileşeni kişinin kendi bilişinin üç düzeyde farkında olmasıdır. Bu düzeyler ifade etme-açıklayıcı düzey (ne sorusu), prosedürel düzey (nasıl sorusu), ve durumsal düzey (ne zaman ve neden) düzeylerini içerir. Diğer bileşen bilişin düzenlenmesidir ve üstbilişsel becerileri ve bilişsel süreçlerin düzenlenmesi yeteneği olarak ifade edilmiştir. Bu bileşen kişinin kendi bilişini control etme amacıyla yaptığı aktiviteleri içerir. Bu aktiviteler, planlama, gözlemleme, strateji seçimi, değerlendirme, bilgi yönetimi gibi aktiviteleri içerir. Ayrıca aktiviteler öz düzenleme sürecine de girer ve problem çözme performansında gelişme getirir (Schraw & Dennison, 1994). Ayrıca, Driscoll (2005) öğrencilere kendi düşünme süreçlerinin farkında olmalarına yardım edilmesi gerektiğini, bu farkındalığın öğrencilerin daha akılcı, ve stratejik bilişsel davranışlar sergilemesi ve bilişsel stratejilerini geliştirmesi için çok önemli olduğunu vurgulamıştır.

Flavell (1979) üstbiliş modellemesini oluşmuştur .Bu modele gore dört tane sınıflandırma vardır. Bu sınıflandırmalar (a) üstbilişsel bilgi, (b) üstbilişsel tecrübe, (c) amaçlar ve görevler, and (d) strateji ve aktivitelerdir. Üstbilişsel ve bilişsel süreçler biribiriyle ilgili ve bağlantılıdır. Üstbilişsel bilinç düzeyi üç kategoriden oluşur: a)insan faktörü b)yapılması istenen iş faktörü c)nasıl yapılacağının cevabı olan strateji faktörünü içerir. Daha sonra üstbilişsel aktiviteler Winne and Harwin (1998) tarafından dört düzey olarak belirlendi. Bu düzeyler: a)görevin veya işin ne olduğunun tanımlanması veya tahlili b) amaçların belirlenmesi ve planlama c) çalışma taktik ve stratejilerini hayata geçirme d) uyarlama çalışması (yapılması istenen işe durumsal olarak değişiklikler yapılması) dır.

Özetlersek üstbilişin bir çok tanımı vardır. Bu tanımların geneli şunu ifade eder: üst biliş kişinin kendi bilinç düzeyini bilmesi, bilgilerin elde edimesi, depolanması, kendi performansını ve kapasitesini bir eylem icra ederken control edebilmesi, kendi performansını değerlendirmesi ve bu değerlendirmenin sonucunda bilinç düzeyinde gerekli düzeltmeleri yapması, zorlukların, kendi zayıf ve güçlü taraflarını bilmesi olarak nitelendirebiliriz (Berardi-Coletta, Buyer, Dominowski & Rellinger, 1995; Driscoll, 2005; Flavell, 1976; Gredler, 2005; Lester, Garofalo & Lambdin, 1989; Schraw & Dennison, 1994).

Problem Çözme

Uzun yıllardan beri, hatta 25 yılı aşkın bir süredir problem çözme araştırması önem arz etmiştir (Donaldson, 2011). Aslında problem çözme, matematik eğitiminin çok daha önceki zamanlardan başlamak üzere bir parçasıydı. Problem çözme 1945'te Polya'nın "nasıl çözebiliriz" adlı kilometre taşı niteliğindeki kitabının yayımlanması ile başladı (Bahar, 2013; Donaldson, 2011; Hembree, 1992, Özalkan, 2010). Bu kitapta, problem çözme süreci, detayları, ipuçları ve problem çözme sürecini nasıl oluşturmak gerektiği, problem çözmenin 4 temel adımı, bu adımlara dair açıklamalar ve tanımlamalar gibi bilgiler sunulmuştur. Polya (1973)'te problem çözme aşamalarını tanımlamış, matematiksel keşifi vurgulamış, öğrenme sürecinde öğrencilerin keşif duygusunu tetiklemek gerektiğini ve öğrencilerin ilgisini artırmak gerektiğini belirtmiştir. Öğrencilere bilgi düzeylerine uygun problemler sunulmasını ve problem çözme aşamasında öğrencilere çeşitli sorular sorularak yardım edilmesi gerektiğini belirtmiştir. Öğrencilerin problem çözme becerisini geliştirmek için: "bilinenler ve bilinmeyenler nelerdir?", "problem yeniden ifade eder misin?", "benzer bir problem biliyor musun?", "çözümdeki her adımını control eder misin?", "çözümünü control eder misin?", "ne mantıkla çözdüğünü ifade eder misin?" şeklinde sorular sorulması gerektiğini vurgulamıştır. Bu şekilde öğrencilerin problem çözmeyi eğlenceli bulmalarını ve öğrencilerin bağımsız düşünme yeteneklerinin artacağını belirtmiştir. Bu soruları temel alarak Polya problem çözme sürecindeki çerçevesinde dört aşama olduğunu belirtmiştir. 1) Problemi anlama: Bu aşamada ihtiyaç duyulan şeyin ne olduğu belirlenir. Eldeki verilerin ne olduğu ve bilinmeyenin ne olduğunun tespit edilmesi gibi süreçleri içerir. 2) Plan Geliştirme: eldeki verilerle istenen çözüm için plan geliştirilmesidir. 3) Planın İcra edilmesi: gerekli hesaplama, formül ve prosedürler tatbik edilerek problem çözülmeye çalışılır. 4) Değerlendirme: problem çözme teknik ve stratejileri incelenir, bunlar hakkında gerekli değişiklikler yapılır.

Donaldson(2011) yıllar boyunca matematikçiler ve matematik eğitimi uzmanları problem çözme ve problemin ne olduğu konusunda bir çok çalışma ve tanımlama yapmışlardır. Bu tanımlardakı farklılık problemin ne olduğu ve problem çözmede neyin önemli olduğu gibi konularda farklı görüşler olmasıdır. Ellis (2005)teki "eğitim metodlarında yenilik" eserinde geçmişte yapılan bir çok araştırtma ve çalışmanın ortak bir tanım ortaya çıkaramadığını ifade etmiştir. Yine Nickerson (1994) problem çözme yaklaşımlarının teorik alt yapısının eksik olduğunu ifade etmişlerdir. Lester (1994) 1980'den 1994'e kadar yaptığı çalışmada problem çözme tekniklerinde çok az bir gelişme olduğunu ifade etmiştir. Lester ve Kehle (2003), Lester"in 1994'teki çalışmalarını incelemiş ve bu tarih itibarıyla çok az gelişme olduğunu teyit etmişlerdir. Ayrıca problem çözme alanında yapılan araştımaları eğitim (okul) pratiğine pek bir katkısı olmadığını ifade etmişlerdir. Benzer olarak Lesh ve Zawojewski de (2007) matematiksel problem çözme araştırmalarında herhangi bir değişiklik yapacak etki ve kümülatif brikimin olmadığını ifade etmişlerdir. Bu şaşırtıcı bir durum değildir. Bu araştırma konusu yıllarca teorik bir temelden yoksun olduğu için eleştirilimiştir. Bu sahada; teorik çalışmalar yapmak gereklmektedir ve bunun için daha fazla çalışma yapılmalı ki teori oluşmalıdır.

Uzun yıllar boyunca problem çözme matematik eğitiminde önemli bir yere sahiptir (Evans, 2012; Posamantier ve Krulik, 2008). Evans (2012) ve Nickerson (1994) problem çözme yetenek ve kabiliyetlerinin matematiğin yanı sıra diğer disiplinler ve günlük hayat içinde genel olarak önemli olduğunu ifade etmişlerdir. Bu nedenle; öğrenciler eleştirel düşünce yapısına sahip olmalı ve problem çözme teknikleri ile ilgili eğitilmelidir ki bu onların hayatta başarılı olmasını etkileyecek bir faktördür. Benzer olarak Özsoy (2006) matematiksel bilgi ve düşüncenin birbirleriyle ilgili olduğunu ifade etmiş ve bu yüzden problem çözme tekniklerinin gerekliliğini vurgulamıştır. Fakat bu iki kavram birbirleri ile iligili olmalarına rağmen birbirlerinden farklıdır. Özsoy (2006) problem çözmenin matematik üzerinde iki etkisi olduğunu ifade etmiştir.Biri herhangi bir kavram için strateji ve kurallar geliştirme, bir diğeri ise bir kavram için düşünce tarzlarının ve genel yaklaşımların geliştirilerek kural veya formülün daha sağlam bir zemine oturmasını sağlamaktır. Lesh ve Jawoweski (2007) problem çözmeyi şu şekilde tanımlamışardır: bir görev veya amaç odaklı faaliyet, problem çözücü ya da problem çözen kimsenin daha verimli bir çözüm yoluna ihtiyaç duyduğu zaman meydana gelen durumdur.

Aslında Polya (1962) problem çözmeyi zorluklar arasından bir yol bulma , bir engeli aşma olarak tanımlamıştır. Krulik ve Posamentier (1998) de problem çözüme giden yolun hemen oraya çıkmadığı bir durum olarak tanımlamışlardır. Onlara gore problem çözme rutin çalışma veya sorunsallardan farklıdır. Rutin faaliyetlerde formül ve prosedürler zaten biliniyordur. Tek ihtiyaç duyulan şey hesaplama kabiliyetidir. Fakat problem çözme denen olguda hemen akla gelebilecek formül, standart set ve yöntemler yoktur. Schoenfeld (1992) bütün problem çözme aşamalarında problem çözme teriminin kendi operasyonel tanımlamasının gerekli olduğunu ifade etmiştir. Dolayısıyla bu çalışmada problem çözmenin operasyonel tanımı şudur; insanın daha once bilmediği bir durumla karşılaşması, bu durumu bilinir kılmak maksadıyla çeşitli bilişsel, mantıksal ve metabilişsel süreçlerin tatbik edilmesidir.

Sonuç olarak, problem çözme matematik eğitiminde önemli bir yere sahiptir (Evans, 2012; Posmantier ve Krulik, 2008). Evans'a göre (2012) problem çözme matematik yanında diğer dispilinler içinde ve hatta günlük yaşam içinde gerekli olan bir olgudur. Dolayısıyla problem çözme öğrencilerin okul müfredatlarında öğretilmelidir.Daha önceki çalışmalara dayanarak problem çözme; bilinmeyenin amaç odaklı bir aktivite ile ve çeşitli zihinsel süreçlerin işletilmesi suretiyle bilinir hale getirilmeye çalışılmasıdır.

Mantıksal Düşünme

Piaget'nin teorisine gore; insanlar, durumlar, olaylar ve nesneler hakkında belirli kalıplar rehberliğinde öğrenme işlemini gerçekleştirir. Bu öğrenme kalıpları dört aşamada gelişir. Birincisi duyusal devinim aşamasıdır. Bu aşamada esas olan motor yeteneneklerinin gelişmesidir, bu da 0-2 yaş aralığına tekabül eder. İkinci aşama işlem öncesi dönemdir. Bu aşama formel işlem uygulama yeteneklerinin tam olarak kazanılamadığı 2-7 yaş arası dönemdir. Üçüncü aşama somut işlemler dönemidir. Bu aşamada işlem uygulama yeteneğinin belirgin hale (elle tutulur da diyebiliriz) gelmesidir ve 7-11 yaş arası dönemi kapsar. Son aşama soyut işlemler dönemidir. Bu aşamada formel işlem uygulama yeteneğinin yanı sıra soyut ve analitik yeteneklerin oluştuğu (11 yaş-yetişkinlik) zaman dilimidir. 7-11 yaş arası sürecinde yani somut işlemler döneminde birey problemleri sınıflandırmayı ve onlarla mücadele etme yeteneğine sahiptir. 7-11 yaş arasında birey soyut, hayali ve gözlemlenmesi mümkün olmayan problemleri çözme yeteneğine sahip değildir. 11 yaş-yetişkinlik döneminde; yani soyut işlemler döneminde; doğrulama, mükemmelleştirme ve bir durumu süzgeçten geçirip rafine etme yeteneğini kazanırlar. Biggs ve Collins (1982) 7-11 yaş arasında yani somut işlemler döneminde hafızada eksiklikler olabileceğini , aynı anda birden çok değişiklik ve kavramlarla ilgilenmenin zor olacağını ifade etmişlerdir. 7-11 yaş arasında problemlerin bir tek çözüm yolu olduğuna inanılır. Açık uçlu problem ve durumlarda, bu durumlar birden fazla çözüm gerektirebildiğinden, gerekli tanımlama ve sınıflandırmanın kolaylıkla yapılamadığı ve cevapların kolay bulunamadığı bir durum ortaya çıkar. Fakat 11 yaş-yetişkinlik seviyesi yani soyut işlemler döneminde neden sonuç ilişkisi ve soyut kavramlara hakimiyet bakıkmından çok daha iyidir. Bu dönemde formel mantıksal düşünme gerçekleşir. Fuller (2001) somut işlemler dönemindeki bireylerin kelime prosedür ve kavramları ezberlemeye yatkın olduğunu, elle tutulabilen , direct tecrübe edilebilen, adım-adım çözüm ve açıklamalarla uzun prosedürlerin üstesinden gelebileceğini belirtmiştir. Bu bireyler nesne ve kavramları sınıflandırma yeteneğine sahiptirler. Fakat kendilerinin bu neden-sonuç ilişkisini kurabildiklerinin farkında değillerdir. Soyut işlemler dönemindeki bireyler neden sonuç ilişkisi kurabilir ve mantıksal düşünme yeteneğini kazanırlar. Soyut kavram ve tanımlamalarla gerekli çözümlemeleri yaparlar. Kendilerini sembollerle ifade edebilirler. Bu bireyler neden sonuç ilişkisi ve bireysel mantıksal süreçlerinin bilincindedir ve mantıksal düşünme yeteneğine sahiptirler.

Longman(1987) mantıksal düşünme, anlama ve gerçeklere dayalı fikir üretebilme becerisidir demiştir. Ayrıca Kelly, Myliss ve Martin (2000) mantıksal düşünmenin problem çözmeyi, problem çözme ve iletişim arasında korelasyon kurma ve birbirine entegre etme becerisi olarak açıklamıştır. Ayrıca, Anderson, Reader ve Simon (1997) ve Steen (1999) öğrenciler iyi bir öğrenci olmak için matematiksel olarak aktif olmalıdır diye belirtmiştir. Resnick'e göre (1987) tartışma, proje çalışması ve takım çalışması gibi aktiviteler öğrencilerin daha kapsamlı bir anlama ve idrak etme yeteneğini kazanmasını ve de kalıcı yetenekler kazanmasını sağlar. Bu tür aktiviteleri aktif öğrenme stratejileri olarak adlandırabiliriz. Fakat ezbere dayalı, bilinen set ve yöntemlerle, hesaplamaya dayalı stratejiler ortaya pozitif sonuçlar çıkartmazlar ki bunlar aynı zamanda pasif yöntemlerdir. Başarılı bir öğrenci olmak için öğrenciler üstbilişsel zihin süreçlerini işletmelidir. Ne ve neden yaptığını bilen kimseler ezbere dayalı otomatik şartlandırılmış kimselerden daha başarılıdır. Tek bir çözüm , düşünce tarzı , yöntem veya sonuç yoktur. Bireylerin zihinsel kapasiteleri arasında büyük bir fark vardır. Değişik koşul ve şartlarda değişik düzeyde öğrenme seviyesi elde edilir. Howard Gardner'ın belirrtiği gibi öğretmenler değişik strateji, ve yöntemler geliştirmelidir. Neden sonuç ilişkisi ve mantıksal düşünme matematik dalında başarı için önemli olduğundan öğrenim teknikleri sadece ifade etme seviyesinde değil prosedürel seviyede de yoğunlaşmalıdır (Resnick, 1987).

Beş formel mantıksal düşünme seviyesi ve kritik düşünce yetenekleri ortaokul seviyesindeki matematik ve fen dersleri için önemlidir (Bitner 1991). Ayrıca Ball ve Bass (2003) matematiksel neden sonuç ilişkisinin, mantıksal düşünmenin matematiksel yeteneklerin önemli bir kısmına tekabül ettiğini, matematiksel öğrenmenin neden sonuç ilişkisine ve mantıksal düşünme yeteneğine dayandığını ifade etmişlerdir. Neden sonuç ilişkisi, mantıksal düşünme becerisi matematik öğreniminin temeli olduğu ve yine daha once elde edilmiş matematik bilgisinin yeni durumlara uyarlanmasında öneml olduğunu ifade etmişlerdir. Ayrıca mantıksal düşünmenin daha önceki bilgilerin sentezlenerek yeni bir bilgi oluşturulması olarak da ifade edilebilir.

İlişkiler

Mantıksal Düşünme Yeteneği ve Problem Çözme Arasındaki İlişki

Mantıksal düşünme, anlamlı çıkarımlar yapma olarak tanımlanabilir. Matematiksel modeller kullanılarak, bir öğrenci kendi düşüncelerini ve çözüm stratejisinin seçimindeki sebepleri ve mantığı belirtmesi problem hakkında çözümlemeler yapması, bir durumun matematiksel ilişkileri kullanrak ve üreterek analizini yapması ve bütün bunların sonucunda çözüm için matematiksel şablon ve plan oluşturmasıdır (NME, 2005). Mantıksal düşünme matematiğin belli bir mantığa dayanması ve mantıksal ilişkilerin bir sentezlenmesi sonucu olduğunun bilinmesidir (Ball & Bass, 2003). Matematiksel mantığın ve mantıksal düşünmenin, matematiğin temel bir parçasını oluşturduğunu ve bunun yeni matematik kuramlarını öğrenmek için gerekli olduğunu vurgulamışlardur (NCTM, 2009). Matematikte problem çözme, neden sonuç ilişkisi kurma, mantıksal düşünme, kanıt oluşturmanın ve bütün bunları anlamlı hale getirmenin önemli aşamalar olduğunu belirtmişlerdir. Bütün bu aşamalar mantıksal bir işletim sürecinin bir sonucudur. Öğrenciler ilk once mantıksal süreci inşa etmeli, problem çözmek için kanıt oluşturmalı ve bunu mantıksal bir alt yapıya dayandırmalıdır (NCTM, 2010). Stenberg'e göre (1980) mantıksal düşünme, problem çözme ve zekanın birbirleriyle çok yakın ilişki içinde olduğunu tespit etmiştir. Kamarski, Lieberman ve Mevarech'e göre (2001) matematikteki başarı ile mantıksal düşünme arasında bir bağlantı olduğunu ifade etmiştir. Mantıksal düşünme yeteneği yüksek olan öğrenciler daha etkili problem çözme stillerine sahiptirler. Aynı zamanda bunlar daha iyi tanımlama ve iletişim becerisine de sahiptirler. Nunes, Bryant, Barros ve Sylva (2012) matematiksel akıl yürütmenin ve aritmetik yeteneklerin başarı üzernde önemli bir etkiye sahip olduğunu belirtmiştir. Evans (2000) matematiksel düşünce, mantıksal düşünme ve problem çözme arasında önemli bir ilişkiye sahip olduğunu belirtmiştir.

Ayrıca, Tobin ve Capie (1982) formel mantıksal düşünme yeteneğinin, beceri kazanmada %36 lık bir varyansla en güçlü gösterge olduğunu ifade etmişlerdir. Valanides (1997) mantıksal düşünme yeteneğinin okul başarısı için önemli bir gösterge olduğunu ifade etmiştir. Bitner (1991) bu ilişkiye destek vermiş, mantıksal düşünmenin matematik ve fendeki başarı için önemli bir gösterge olduğunu ifade etmişlerdir. Bitner (1991) beş adet olan operasyonel formel mantısal düşünme yeteneğinin ortaoluldaki matematik başarısı için önem arzettmektedir demiştir. Lawson (1992) genel okul başarısı ve mantıksal düşünme yeteneği arasındaki ilişkiyi araştırmış 72 tane 9ncu sınıf öğrencisi ile yaptığı calışmada mantıksal yeteneklerin öğrencilerin okul başarısına önemli katkılar sunduğunu tespit etmiştir. Ayrıca, Someithner, Keller, Martin, ve Bruner (2013) karmaşık problem çözme yeteneğinin mantıksal yetenek ve okul başarısı ile yakın ilişkisi olduğunu tespit etmişlerdir. Kunchon'a göre (2012) mantıksal düşünme yeteneği, problem çözme ve akademik yeteneklerle iligili üniversite son sınıf öğrencileri arasında araştırma yapmıştır. Buna gore cinsiyet farklılıkları mantıksal düşünme yeteneğinde farka neden olmakta fakat problem çözmede herhangi bir etkiye yol açmamaktadır. Mantıksal düşünme yeteneği ile problem çözme kabiliyeti arasında %28 lik bir korelasyon vardır. Mantıksal düşünme yeteneği ve akademik başarı arasında %3 ten az , akademik başarı ve problem çözme arasında %1 den az ilişki vardır.

Mantıksal Düşünme ile Üstbiliş Arasındaki İlişki

Donaldson (2011) matematiğin aslında bir problem çözme faaliyeti olduğu yaygın bir görüş olduğunu belirtmiştir. Matematik öğretmenin de görevi doğal olarak öğrencilerin problem çözme yeteneklerini geliştirmektir demiştir. Mevarech ve Kramarski (2008) yaptığı çalışma üstbilişsel eğitim alan öğrencilerin matematik alanında daha başarılı olduğunu ortaya koydu. Maverech ve Fridkin (2006) IMPROVE adı verilen üstbilişsel metodun öğrencilerin matematik yeteneğini ve bilgi seviyesini artırıdığını ortaya koymuştur. Üstbilişsel eğitimin öğrencilerin genel zihinsel gelişimlerininde de olumlu bir rol oynadığı tespit edilmiştir. Benzer şekilde, Kramarski ve Mevarech (2003) üstbilişsel eğitimin matematiksel neden-sonuç ilişkisi ve mantıksal düşünme yeteneği üzerindeki etkisini araştırmıştır. Araştırmacılar dört öğretim metodunun öğrenciler üzerindeki etkisini araştırmışlardır. COOP META (işbirliği metodu ve üstbilişsel eğitim), İND META (bireysel öğrenim ve üstbiliş eğitimi), COOP (sadece işbirliği metodu, üstbilişsel eğitim yok) ve IND (bireysel öğrenim ve üstbilişsel eğitim yok). Sonuçlar sentezlendiğinde; matematiksel açıklamalar ve mantıksal düşünme performansında CCOP+ META öğrencileri IND+META'dan daha iyi, IND+META öğrencileri de COOP ve IND den daha iyi performans ortaya koymuştur. Kramarski (2008) de üstbilişsel rehberlik eğitimi almış öğretmenlerin mantıksal düşünmelerinin daha iyi olduğunu ortaya koymuştur. Üstbilişsel rehberlik eğitimi alan ve almayanlar incelendiğinde kendini değerlendirme, izleme ve cebir problemlerini çözmede eğitim alanlar daha iyi çıkmıştır. Üstbilişsel eğitim metabilişsel zihin süreçlerini geliştirmiştir. Bu da matematiksel beceri ve mantığı optimum seviyeye çıkarıp, daha once öğrenilen metodların yeni durumlara uyarlanmasını sağlamıştır (Kramarski, 2008; Kramarski, Mevarech and Lieberman, 2001).

Üstbiliş ve Problem Çözme Arasındaki İlişki

Mevarch (1999) üstbiliş ile matematik problem çözme performans arasında ilişki bulunduğunu ve üstbilişsel eğitimin matematiksel problem çözme performansını artırdığını iddia etmiştir. Bu sebeple yaptığı çalışma sonucuna göre, üstbilişsel eğitim alanlar, almayanlara göre daha yüksek matematiksel problem çözme performansı göstermiştir. Direkt strateji öğrenme eğitimi alanlar ve hiç eğitim almayanlara göre daha iyi problem çözme performansı göstermiştir. Mevarech ve Fridkin (2006) yaptıkları çalışmada üstbilişsel eğitimin matematiksel performansa olan etkisini incelemiştir. Araştırma sonuçlarına göre, üstbilişsel eğitim alanlar, hem matematiksel bilgide hem mantıksal düşünmede hem de üstbilişsel yetenekte artış ve gelişme göstermişlerdir. Üstbilişsel eğitimin hem öğrencilerin bilişsel bilgi hem de bilişsel düzenleme hem de matematik başarısına olumlu katkıda bulunduğu belirtilmiştir (Mevarech & Fridkin, 2006). Ayrıca, Similarly, Berardi-Coletta, Buyer, Dominowski ve Rellinger (1995) yaptıkları çalışma sonucunda, üstbilişsel problem çözücülerin problem çözmede daha yüksek performans gösterdiklerini belirtmişlerdir. Benzer şekilde, Swanson (1990) üstbilişsel bilginin ve kapasitenin problem çözme üzerindeki etkisini araştırmıştır. Araştırma sonuçlarına göre kapasiteden bağımsız olarak yüksek üstbilişe sahip öğrencilerin daha iyi problem çözme performansı sergilediği görülmüştür. Babakhania (2011) bilişsel ve üstbilişsel stratejilerin öğrencilerin matematiksel problem çözme performansına etkisini incelemiştir. Sonuçlara göre, bilişsel ve üstbilişsel strateji öğretimi öğrencilerin matematiksel problem çözme performansını anlamlı bir şekilde yükseltmiştir. Ayrıca, Özsoy (2007) kendi çalışmasında üstbilişsel eğitimin öğrencilerin problem çözme performansını arttırdığını belirtmiştir. Yine Özsoy ve Ataman (2009) üstbilişsel strateji eğitiminin matematiksel problem çözme performansını artırdığını araştırmalarının sonucuna dayanarak belirtmiştir.

Özetle bir çok çalışma ve inceleme üstbiliş ile mantıksal düşünme yeteneği ve problem çözme performansı arasındaki yakın ilişkiyi desteklemiştir (Antonietti, Ignazi & Perego, 2000; Berardi-Coletta, Buyer, Dominowski & Rellinger, 1995; Carlson & Bloom, 2011; Higgins, 1997; Kışkır, 2011; Mevarech & Fridkin, 2005; Mevarech & Kramarski, 2003; Ozsoy & Ataman, 2009).

BÖLÜM 3

YÖNTEM

Bu çalışmada hipotezleri test etmek için ve araştırma problemlerini incelemek ve cevaplayabilmek için nicel veriler toplanmış ve istatistiksel analizler uygulanmıştır.

Çalışmanın modeli ilişkisel çalışmadır. Bu çalışmada her hangi bir deney uygulanmadığı için ve hali hazırda varolan özellikler ölçüldüğü için çalışmanın modeli ilişkisel ya da korelasyonel çalışmadır.

Katılımcılar

Bu çalışmada kullanılan veriler 2014-2015 akademik yılı ilkbahar döneminin ilk haftasında bizzat araştırmacı tarafından toplanmıştır. Çalışmanın örneklemi 578 dokuzuncu sınıf öğrencisinden oluşmaktadır. Katılımcılar batı ve kuzey İzmir bölgesindeki 17 anadolu lisesinde okuyan dokuzuncu sınıf öğrencileridir. Öğrencilerin etnik kimliği, yaşı, cinsiyeti, sosyo-ekonomik statusleri, demografik özellikleri, karakteristik ve kişisel özellikleri değişkenlik gösterebilir. Fakat hem hedef populasyonun, hem ulaşılabilir populasyonun, hem de örneklemin ortak özelliği öğrencilerin İzmir ilindeki devlet anadolu liselerinde okuyan dokuzuncu sınıf öğrencileri olmasıdır. Çalışmadaki hedef populasyon tüm İzmir ili iken ulaşılabilir populasyon batı ve kuzey İzmir bölgesidir. Örneklem ise batı ve kuzey İzmir bölgesindeki 17 anadolu lisesinde okuyan 578 dokuzuncu sınıf öğrencisidir.

Veri toplama süreci ve veri toplama araçları

Bu çalışmada katılımcılara 3 ölçek uygulanmıştır.

1. Bilişüstü Yeti Anketi

Birinci ölçek olan Bilişüstü Yeti Anketi üstbiliş düzeyini ölçmek için kullanılmıştır. Bu ölçek 1'den 5'e kadar numaralandırılmış Likert tipi bir ölçektir ve 17 sözel maddeden oluşmaktadır. Ölçekten alınacak minimum puan 17 ve maksimum puan 85'dir.

2. Mantıksal Düşünme Yeteneği Testi

İkinci ölçek olan Mantıksal Düşünme Yeteneği Testi öğrencilerin mantıksal düşünme yeteneklerini ölçmek için kullanılmıştır. Bu testte öğrencilere 10 matematik sorusu sorulmuştur. Her soru iki alt sorudan oluşmuştur. Bu alt sorular sorunun direkt cevabını ve bu cevabın neden seçildiğini açıklayan diğer sorudan oluşmaktadır. Hem soruyu doğru cevaplayan hem de cevabını doğru şekilde açıklayan katılımcılar 1 puan almıştır. İki durumdan biri eksikse 0 puan verilmiştir. testten alınacak minimum puan 0'dır ve maksimum puan 10'dur.

3. Matematiksel Problem Çözme Ölçeği

Sonuncu ölçek olan Matematiksel Problem Çözme Ölçeği ise öğrencilerin matematiksel problem çözme performansını ölçmek için kullanılmıştır. Bu testte katılımcılara 10 matematik problemi sorulmuştur. Problemi doğru yolla ve doğru sonuca vararak çözen katılımcıya 3 puan verilmiştir. Doğru yolla çözen fakat doğru cevaba ulaşamayan katılımcıya 2 puan verilmiştir. Doğru bir başlangıç yapan fakat sonuca ulaşamayan katılımcıya 1 puan verilmiştir. Yanlış başlangıç yapan veya boş bırakan katılımcıya 0 puan verilmiştir. Testten alınacak minimum puan 0'dır ve maksimum puan 30'dur.

Bu üç ölçeğin doldurulması için katılımcılara 2 ders saati süre verilmiştir. katılımcılar ölçekleri tek seferde doldurmuştur ve katılımcılardan veriler tek seferde toplanmıştır. Bu çalışma niceliksel veri analizi sonuçlarına dayanan nicel bir çalışmadır ve çalışmanın deseni korelasyonel desendir. Bu çalışmada kaıtlımcılara herhangi bir deney yapılmamıştır. Katılımcıların ölçülen özelliklerine her hangi bir müdahalede bulunulmamıştır. İşbu sebeplerle ve katılımcıların halihazırda varolan özellikleri ölçüldüğü için çalışma korelasyonel bir çalışmadır.

Analiz

Bu çalışmada toplanan verilerin istatistiksel analizi için hem betimsel hem de çıkarımsal data analizi uygulanmıştır. Betimsel data analizi için aritmetik ortalama standart sapma 95 güvenilirlik aralığı yüzdelik dilimler frekans tablo grafikler hesaplanmıştır. Çıkarımsal data analizi için korelasyon ve çoklu regresyon analizi uygulanmıştır.

BÖLÜM 4

SONUÇLAR

İstatistiksel analiz sonuçlarına göre çalışmadaki üç değişken arasında beklendiği gibi istatistiksel olarak anlamlı ilişkiler bulunmuştur. Sonuçlara göre üstbiliş ve matematiksel problem çözme performansı arasında orta derecede pozitif ve istatistiksel olarak anlamlı ilişki bulunmuştur. Mantıksal düşünme yeteneği ve matematiksel problem çözme performansı arasında da orta derecede pozitif ve istatistiksel olarak anlamlı ilişki bulunmuştur. Ayrıca üstbiliş ve mantıksal düşünme yeteneği arasında da orta derecede pozitif ve istatistiksel olarak anlamlı ilişki bulunmuştur. Araştırmada elde edilen diğer bir sonuç ise üstbiliş ve mantıksal düşünme yeteneği, matematiksel problem çözme performansını istatiksel olarak anlamlı biçimde yordamaktadır. Ayrıca üstbiliş tek başına ve mantıksal düşünme yeteneğinden bağımsız olarak da matematiksel problem çözme performansını istatistiksel olarak anlamlı biçimde yordamaktadır. Benzer bir sonuç olarak mantıksal düşünme yeteneği de tek başına ve üstbilişden bağımsız olarak matematiksel problem çözme performansını istatistiksel olarak anlamlı biçimde yordamaktadır. Fakat tek başına mantıksal düşünme yeteneği matematiksel problem çözme performansını tek başına üstbilişten daha iyi yordamakta ve açıklamaktadır. Üstbiliş ve mantıksal düşünme yeteneği birlikte matematiksel problem çözme performansının yüzde 54'ünü açıklamaktadır. Üstbiliş tek başına mantıksal düşünme yeteneğinden bağımsız olarak matematiksel problem çözme performansının yüzde 4'ünü açıklamaktadır. Ayrıca mantıksal düşünme yeteneği tek başına ve üstbilişten bağımsız olarak matematiksel problem çzöme performansının yüzde 14'ünü açıklamaktadır.

BÖLÜM 5

YORUM VE TARTIŞMA

Bu çalışmada üstbiliş mantıksal düşünme yeteneği ve matematiksel problem çözme performansı arasında istatistiksel olarak anlamlı bir ilişki olduğu hipotezi kurulmuş ve beklenen sonuçlar elde edilmiştir. Bu sonuçlara dayanarak matematiksel problem çözme performansını artırmak için üstbiliş ve mantıksal düşünme yeteneğinin sınıflarda vurgulanması önerilmektedir. Ayrıca çalışmada bulunan ilişkiye dayanılarak üstbiliş ve mantıksal düşünme yeteneğinin problem çözmeden ayrı tutulmaması ve görmezden gelinmemesi önerilmektedir. Ayrıca matematik öğretmenleri için problem çözme performansını artırabilmeleri için üstbiliş ve mantıksal düşünme yeteneğini artıracak önlemler alınması önerilmektedir.

Bu çalışmanın amacı İzmirde dokuzuncu sınıflarda okuyan öğrencilerin mantıksal düşünme yeteneği, üstbiliş ve matematiksel problem çözme performansı arasındaki ilişkileri araştırmaktır. Ayrıca bir diğer amacı üstbiliş ve mantıksal düşünme yeteneği skorlarının problem çözme performansını nasıl etkilediğidir. Yine bir amaç da üstbilişin mi ya da mantıksal düşünme yeteneğinin mi problem çözme performansının en iyi göstergesi olduğunu araştırmaktır. Bu bölümde önce analiz sonuçlarını ele alacağız. Daha sonra çalışmanın kısıtlılıklarını ve gelecekteki araştırmalar için önerileri ele alacağız.

Araştırma soruları için, korelasyon ve standart çoklu regresyon analizi yapılmıştır. Analiz sonuçlarına gore mantıksal düşünme yeteneği ve üstbiliş istatistiksel olarak anlamlı bir şekilde problem çözme performansını açıklamış ve yordamıştır.

5.1.1 Mantıksal Düşünme, Üstbilişin ve Problem Çözme Performansıyla Aralarındaki İlişkilerine Yönelik Çalışmanın Sonuçlarının Tartışılması

Bu çalışmanın amacı İzmirde dokuzuncu sınıflarda okuyan öğrencilerin mantıksal düşünme yeteneği, üstbiliş ve problem çözme performansı arasındaki ilişkileri araştırmaktır. Bu çalışmanın korelasyon analizine gore üstbiliş ile problem çözme performansı arasında önemli bir ilişki mevcuttur. Bu sonuç önceki araştırmalarla uyumludur. Üstbilişin problem çözmede temel bir öneme sahip olduğunu vurgulamıştır (Mayer, 1998). Üstbiliş, matematiksel problem çözme performansını da artırmaktadır (Mayer, 1998; Nickerson, 1994; Özsoy, 2006). Antonietti, Ignazi ve Perego (2000) bunlara ilaveten üstbilişin üst seviyelerinin problem çözmedeki performans ile yakın ilişkili olduğunu ortaya çıkarmıştır. Spesifik olarak, üstbilişsel eğitime tabi tutulan öğrenciler matematik problem çözmede eğitim görmeyenlere gore önemli başarılar elde ettiği başka çalışmalarla da kanıtlanmıştır (Berardi-Coletta, Buyer, Dominowski & Rellinger, 1995; Mevarech, 1999; Mevarech & Fridkin, 2006; Mayer, 1998; Özsoy & Ataman, 2009).

Çalışmadaki diğer bir sonuç ise çalışmanın korelasyon analizine gore üstbiliş ile mantıksal düşünme arasında önemli bir ilişki mevcuttur. Bu sonuç önceki araştırmalarla uyumludur. Örneğin, Maverach ve Kramarski (1997) üstbilişsel eğitimin matematiksel ve mantıksal düşünmeyi artırdığını belirtmiştir. Ayrıca, birçok araştırmada üstbilişsel eğitimin mantıksal düşünmeyi artırdığı ve aralarında bir ilişki olduğu belirtilmiştir (Mevarech & Fridkin, 2006; Kramarski, 1998; Kramarski & Hirscha, 2010; Kramarski, Mevarech & Lieberman, 2001).

Diğer bir sonuç ise çalışmanın korelasyon analizine gore matematiksel problem çözme performansı ile mantıksal düşünme arasında önemli bir ilişki mevcuttur. Bu sonuç önceki araştırmalarla uyumludur ve beklenen bir sonuçtur. Bitner (1991) formel operasyonel mantık modlarının matematik ve fen derslerindeki başarının önemli göstergelerinden biri duğunu ifade etmiş, bu modlar matematikteki değişkenlerin % 29 unu açıklamıştır. Düşünme süreçleri ifade etme ve prosedürel bilgi çerçevesinde gelişir. Dolayısıyla eğitim yöntemleri hem gerçek bilgi hem de düşünce süreçleri üzerinde odaklanmalıdır. Mantıksal düşünme matematik dalındaki başarının önemli bir göstergesi olduğundan, öğretim yaklaşımları sadece ifade etme değil aynı zamanda prosedürel biligiye de odaklanmalıdır. Beş formel operasyonel mantık modları ve kritik düşünce stillleri ortaokul seviyesindeki fen ve matematik derslerindeki başarı için önemlidir (Bitner, 1991). Geçmişte yapılan bir çok araştırma problem çözme ile mantıksal düşünme arasındaki ilişkiyi desteklemiştir ve mantıksal düşünmenin matematik ve problem çözmede önemli bir yere sahip olduğunu belirtmiştir (Evans, 2000; Mueller & Maher, 1996; Tobin & Capie, 1982; Valanides, 1997).

5.1.2 Mantıksal Düşünme ile Üstbilişin Matematiksel Problem Çözmeyi Açıklaması ve Yordaması Üzerine Sonuçların Tartışılması

Mantıksal düşünme ve üstbilişin matematiksel problem çözmeyi açıklaması ve yordaması üzerine yapılan araştırmada standart çoklu regresyon analizi uygulanmıştır. Çoklu regresyon analizi sonuçlarına göre üstbiliş ve mantıksal düşünme yeteneği matematiksel problem çözme performansını istatiksel olarak anlamlı biçimde yordamaktadır. Ayrıca üstbiliş tek başına ve mantıksal düşünme yeteneğinden bağımsız olarak da matematiksel problem çözme performansını istatistiksel olarak anlamlı biçimde yordamaktadır. Benzer bir sonuç olarak mantıksal düşünme yeteneği de tek başına ve üstbilişden bağımsız olarak matematiksel problem çözme performansını istatistiksel olarak anlamlı biçimde yordamaktadır. Fakat tek başına mantıksal düşünme yeteneği matematiksel problem çözme performansını tek başına üstbilişten daha iyi yordamakta ve açıklamaktadır. Üstbiliş ve mantıksal düşünme yeteneği birlikte matematiksel problem çözme performansının yüzde 54.6'sını açıklamaktadır. Üstbiliş tek başına mantıksal düşünme yeteneğinden bağımsız olarak matematiksel problem çözme performansının yüzde 4'ünü açıklamaktadır. Ayrıca mantıksal düşünme yeteneği tek başına ve üstbilişten bağımsız olarak matematiksel problem çzöme performansının yüzde 14'ünü açıklamaktadır. Daha önceki çalışmaların da gösterdiği gibi, bu sonuçlar beklenen sonuçlardır (Lawson, 1982; Maqsud, 1997; Mevarech & Kramarski, 2003; Nunes, Bryant, Barros & Sylva, 2012).

Çalışmanın Sınırlılıkları ve Öneriler

Çalışmadaki ilk sınırlılık katılımcıların sınıf kademesidir. Bu çalışmada sadece dokuzuncu sınıf öğrencileriyle çalışılmıştır. Diğer sınırlılık, kaıtlımcıların sadece anadolu lisesinde okuyor olmalarıdır. Ayrıca örneklem sayısının 578 öğrenci olması diğer sınırlılıktır. Ilerdeki çalışmalar için lise dışındaki kademelerde de çalışılması, diğer sınıflardan katılımcı seçilmesi, katılımcı sayısının yükseltilmesi, okul türünün değiştirilmesi önerilmektedir. Ayrıca çalışma İzmirin Kuzey ve Batı bölgesinde uygulanmıştır, diğer çalışmaların farklı illerde ya da İzmir'in farklı bölgelerinde yapılması önerilebilir. Ayrıca bu çalışma korelasyonel desendedir, diğer olası çalışmalar farklı desenlerde olabilir, deneysel veya karma çalışmaların yapılması önerilir. Çalışma nicelikseldir, ve niteliksel çalışmaların yapılması da önerilir. Çalışma sadece üç değişkenle yapılmıştır, olası çalışmalar için farklı değişkenlerin eklenmesi de önerilir.

Öneriler

Şu anki çalışmada psikoloji, eğitim ve diğer ilgili disiplinlerle alakalı çalışma yapanlar için bir çok muhtemel önemli noktalar vardır. Çalışma üstbiliş, mantıksal düşünme ve matematiksel problem çözme arasındaki güçlü ilişkiye dahil önemli destekler sunmaktadır. Ek olarak geçmişteki çalışmalara ve bu çalışmanın bulgularına dayanılarak, bazı önerme veya varsayımlar matematik öğretmenleri ve müfredat geliştiricileri için sağlanabilir.

Matematik problemi çözme matematiğin temelidir. Matematik problemi çözme matematikteki başarı için önemlidir. Bu çalışma üstbiliş, mantıksal düşünme yeteneği ve matematik problem çözme becerisi arasındaki ilişkiye dair önemli kanıtlar sunmaktadır. Dolayısıyla üstbiliş ve mantıksal düşünme yeteneği matematik eğitiminde unutulmamalı ve bu konulara gerekli önem verilmelidir. Ayrica üstbiliş ve mantıksal düşünme yeteneğinin matematik problemi çözmedeki rolü ilk okuldan eğitim fakültelerine kadar her süreçte vurgulanabilir.

İfade edildiği gibi üstbiliş ve mantıksal düşünme, matematiksel problem çözme performansının tahmin edilmesinde ve açıklanmasında önemli bir yere sahiptir. Şu anki çalışmaya göre üstbiliş ve mantıksal düşünme, dokuzuncu sınıf öğrencilerinin matematiksel problem çözme performansında %54.6lık bir varyansa sahiptir. Bu oran yarıdan fazladır. Dolayısıyla matematik öğretmenlerine ve eğitimcilere üstbiliş, mantıksal düşünmenin matematiksel problem çözme performansında önemli rol oynadığı aktarılmalıdır. Dolayısıyla üstbiliş hakkındaki eğitim göreve başlamamış ve hatta şu an görevde olan öğretmenlere seminerler yoluyla aktarılabilir. Sosyal bilimcilerin ve eğitim fakültelerinin onayıyla üniversitelerdeki matematik bölümündeki ve eğitim fakültesindeki öğretim üyeleri de seminer verilebilir. Ayrıca müfredat geliştiriciler üstbiliş ve mantıksal düşünme konularını ders içi aktivitelere, ders kitaplarına ve matematik müfredatına da aktarabilirler. Bu sayede öğretmen ve eğiticiler ders planlarını buna gore yapar ve sınıf içi aktivitelerde öğrencilerini geliştirirler. Bu sayede öğrenciler üstbiliş, mantıksal düşünme ve matematiksel problem çözme performanslarını geliştirme fırsatına sahip olabilirler.

Bu çalışmada üstbiliş, mantıksal düşünme ve matematiksel problem çözme performansı hakkında önemli bağlantılar bulunmuştur. Bu daha önceki uzmanların çalışmaları da göz önüne alındığında beklenen sonuçtur. Fakat bu ilişki sebep sonuç ilişkisini garantileyemez. Sadece buna dayanarak üstbiliş ve mantıksal düşünmenin matematiksel problem çözme performansını artıracağını söylemek pek de uygun olmaz. Fakat önemli de bir ilişki olduğundan üstbilişin ve mantıksal düşünmenin matematiksel problem çözme performansındaki önemini vurgulamak doğrudur.

Bu çalışmanın ve diğer geçmiş çalışmaların da sonuçları göz önüne alındığında matematiksel problem çözme performansını artırmak için öğretmenlerin sınıfta üst bilişsel davranışları ve mantıksal düşünmeyi vurgulaması önemldir. Aynı zamanda, matematik öğretmenleri, eğitimciler, müfredat geliştiriciler üstbiliş, mantıksal düşünme ve problem çözme ile ilgili uygun öğrenme tekniklerini tasarlayabilirler. Müfredat tasarlayıcılar ve eğitimciler üstbiliş, mantıksal düşünmeyi ve problem çözmeyi geliştirecek ve vurgulayacak uygun eğitim öğretim materyalleri hazırlayabilirler. Öğretmenler de sınıflarında üst bilişsel davranışların, mantıksal düşünmenin ve problem çözmenin pekişmesine yönelik uygun sınıf ortamı hazırlamalıdırlar. Öğrenciler kendi çözüm, karar ve düşüncelerini açıklamaya ve savunmalarına imkan verilmelidir ki üstbilişsel ve mantıksal düşünme yeteneklerini geliştirebilsinler. Bu sayede öğrenciler ezber yerine kendi mantıklarına güvenirler. Bu durum da matematikte kolay anlamayı ve anlamlı öğrenmeyi sağlar.

Appendix G: TEZ FOTOKOPİSİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü ☐

Sosyal Bilimler Enstitüsü ☒

Uygulamalı Matematik Enstitüsü ☐

Enformatik Enstitüsü ☐

Deniz Bilimleri Enstitüsü ☐

YAZARIN

Soyadı : Elitaş
Adı : Yüksel Özge
Bölümü : İlköğretim Matematik Eğitimi-Elementary Mathematics Education (ELE)

TEZİN ADI (İngilizce) : The relationship among metacognition, reasoning ability and mathematical problem solving performance of ninth grade students.

TEZİN TÜRÜ : Yüksek Lisans ☐ Doktora ☒

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
3. Tezimden bir bir (1) yıl süreyle fotokopi alınamaz. ☒

TEZİN KÜTÜPHANEYE TESLİM TARİHİ: