MOM ANALYSIS OF LOADED PRINTED DIPOLES WITH APPLICATIONS IN THE DESIGN OF ELECTRICALLY SMALL ANTENNAS

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ABSTRACT

MOM ANALYSIS OF LOADED PRINTED DIPOLES WITH APPLICATIONS IN THE DESIGN OF ELECTRICALLY SMALL ANTENNAS

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Mixed Potential Integral Equation based Method of Moments (MoM) code is developed in MATLAB environment to analyze a loaded printed dipole antenna. Closed-form spatial domain Green's functions are used in the formulation.

The developed code is utilized in the analyses of Electrically Small Antennas (ESAs). It is known that real part of the input impedance of an ESA is very low, whereas imaginary part is negative and high. The inductive loading method is used in order to overcome this problem. Effects of load values and positions on the input impedance and on the efficiency of the antenna are investigated. The results obtained by the developed MoM code are compared to ones obtained by EMPro which is Agilent’s 3D electromagnetic solver and good agreement is observed. Then, three ESA’s with different lengths are manufactured and they are loaded with the proper load values at proper positions. The input return loss values of these antennas are measured and compared with simulations. It is seen that measurement results are in good agreement with the results obtained by the developed code.

Keywords: Loaded Printed Dipole Antenna, Electrically Small Antenna, Method of Moments, Mixed Potential Integral Equation
ÖZ

YÜKLÜ MIKROŞERİT DİPOLLERİN MOMENT METODU ANALİZİNİN ELEKTRİKSEL KÜÇÜK AN TEN TASARIMINDAKİ UYGULAMALARI

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Yüklü bir baskı dipol anten analiz edilebilmesi için Karışık Potansiyel İntegral denklemi kullanarak Moment Metodu analizi yapan bir kod MATLAB ortamında geliştirilmiştir. Formülasyonda gerçek uzaydaki kapalı form Green fonksiyonları kullanılmıştır.


Anahtar Kelimeler: Yüklü Baskılı Dipol Anten, Elektriksel Küçük Anten, Moment Metodu, Karışık Potansiyel İntegral Denklemi
To My Mother,
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CHAPTER 1

INTRODUCTION

Dipole antennas have been used in a vast number of applications for many years due to their simple structures. Generally, the length of a thin wire dipole antenna is a little bit shorter than half wavelength to obtain purely resistive input impedance of about 70-80 ohm. A dipole antenna is considered to be “thin” when its radius is much smaller than the wavelength.

A half wavelength dipole antenna operating at $f_c$ has a resonance also at $2f_c$. However, the input impedance at this frequency is very high due to the very low input currents at the terminals of the antenna. Hence, in practice, the antenna does not operate properly at $2f_c$ and this frequency is called anti-resonance frequency. In order to provide better input impedance at the anti-resonance frequency, the inductive loading method is presented in [1]. In [1], traps, parallel combination of inductor, capacitor and resistor, are used to load the antenna. The resonance frequencies of the traps are adjusted to obtain better input impedance match at the anti-resonance frequency of the dipole antenna. Hence, those antennas can be used at both $f_c$ and close to $2f_c$ frequencies. Later, trap loading approach is utilized to design broadband antennas by using Genetic Algorithm to optimize the position and values of the traps [2]-[4].

It is obvious that the physical length of the dipole should be long for low frequency applications. This creates a problem if the reserved place for the antenna is restricted such as mobile communication systems operating in the HF band. When the length of the antenna is shortened, the input impedance becomes capacitive. The input impedance of these antennas can be matched to real values by loading the dipole with lumped inductors. Inductively loaded dipole antennas are first studied in [5]. In [5], an analytical expression is obtained for the input impedance of the loaded dipole antenna by assuming an approximate current distribution on the antenna according to the value and the position of the inductive load. In 1965,
another study is published by Czerwinski [6] that reports the experimental results obtained for inductively loaded antennas. The experimental results obtained by Czerwinski support the analytical results presented in [5]. Harrison’s study is extended by Lin, Nyquist and Chen and the approximate solution of current distribution on a loaded antenna is proposed [7]. Although these approximate analytical methods provide fairly good accuracy, the full wave numerical methods started to be widely used in early 1970’s due to the improved accuracy that they provide. Inductively loaded monopoles are first investigated using Method of Moments (MoM) by Hansen [8], [9]. In [8] and [9], the efficiency and the input impedance of these antennas are studied for different load positions and the effects of load position on these parameters are presented.

In 1970, printed structures started to be more popular by the availability of good substrates with low tangent loss and attractive thermal and mechanical properties. Nowadays, these types of structures are commonly used for microwave circuits. Printed antennas and Monolithic Microwave Integrated Circuits (MMIC) are very well known applications of printed structures. Low weights, low profiles, repeatability, ease of manufacturing and low cost are some of the advantages of printed structures [10].

Printed dipole antennas have two arms of radiating elements just as free-space cylindrical dipole antennas as shown in Figure 1.1.

![Figure 1.1: The printed dipole antenna](image)

The feeding of the printed dipole antenna can be achieved in different ways. Coplanar strips (CPSs) are widely preferred due to the advantages including their uniplanar form and the ease of manufacturing [11], [12]. However, CPSs are
balanced transmission line and a transition as shown in Figure 1.2 needs to be designed to convert them into unbalanced transmission lines like microstriplines.

![Figure 1.2: CPS feed with transition [11]](image)

The parallel strip line shown in Figure 1.3 can be used as an alternative method to avoid the design of the aforementioned transition. This provides easy implementation when the strips of the dipole are printed on different sides of the substrate.

![Figure 1.3: Parallel Strip Line fed printed dipole antenna [13]](image)

In this thesis work, as another method, the three dimensional structure shown in Figure 1.4 is preferred due to the smaller size of the transition required to convert
the microstrip line to the parallel strip lines. This feeding structure provides an easy way to feed the printed dipole.

Figure 1.4: The microstrip transition [14]

As mentioned above, the resonance length of the antenna would be inconveniently long for some type of applications when operating frequency is low. Electrically small antennas would be a good solution for this type of applications. Electrically small antenna implies that the size of the antenna is much smaller than the wavelength. So these types of antennas are named as Electrically Small Antennas or commonly ESAs. The first work about ESAs is published by Wheeler in 1947 [15]. In [15], the radianlength is defined as $\lambda/2\pi$ and an antenna is classified as ESA if the maximum dimension of the antenna (a) is less than the radianlength. This corresponds to $ka < 1$ ($k$ is the wave number) which is a very well known criteria for ESAs. In order to find the relationship between antenna size and radiation properties, the radiation power factor (RPF) which is the ratio of the radiated power to the reactive power is used by Wheeler. At the end of this work, he concluded that RPF is directly related to the physical volume of the antenna. However, Wheeler’s approach was a rough approximation because it is based on the modeling of the electrically small antenna as a lumped capacitance or an inductance. The radiated spherical waves have not been taken into account in this analysis. This makes the analysis only valid for extremely small antennas. Although Wheeler’s work was not accurate enough, it has importance since it is first work related to ESAs. In 1948, another important study on ESAs is published by Chu [16]. The same relationship
between the quality factor (Q) of an electrically small antenna and its physical volume is also proposed by Chu. (Note that Q is the inverse of RPF used by Wheeler.) In [16], the derivation of the equation for the minimum theoretical Q for the electrically small antenna is achieved by using spherical wave functions. The antenna is enclosed within a sphere whose diameter equals to the maximum dimension of the antenna and during the calculation it is assumed that no energy would be stored inside this sphere. Chu’s approach was so theoretical and it was only valid for a specific type of omni directional antenna. However, Chu’s contribution to the theory of electrically small antennas is just as important as Wheeler’s work.

Although ESA has an advantage in terms of occupying area, it is observed that the input impedance of ESAs have low resistance and high capacitive reactance. To control the input impedance of the antenna, inductive loading method can be implemented to the printed monopoles like wire antennas [17]-[19].

In this thesis work, the analysis of loaded printed dipole antenna is achieved by using MoM and ESAs are chosen as the application example of these antennas. To obtain the integral equations for MoM analysis, Mixed Potential Integral Equation (MPIE) formulation is chosen. The important point of the MoM analysis for the printed structures is finding the Green’s functions in layered media. This directly determines the efficiency of the analysis. The spectral domain Green’s functions are known for layered media. Thus, the spatial domain Green’s functions can be calculated using Sommerfeld Integral. But, this integral is highly oscillatory and numerical computation of it is very expensive [20]. In order to overcome this problem, a method is proposed in 1988 [21]. In this method, the spectral domain Green’s function is approximated in terms of a summation of complex exponentials by using Prony’s Method [22]. Then using this approximation, the spatial domain Green’s functions are expressed by utilizing the Sommerfeld Identity. Since the spectral domain Green’s functions are expressed in terms of complex exponentials, this method is generally referred as Discrete Complex Image Method (DCIM) in the literature. This study is improved in [23] by extracting the quasi-static images prior to the complex exponential approximation. In [21] and [23] vector and scalar Green’s functions only for a Horizontal Electric Dipole (HED) are studied. In 1995, the scalar and the vector Green’s functions for Horizontal Electric Dipole,
Horizontal Magnetic Dipole, Vertical Electric Dipole and Vertical Magnetic Dipole are reported [24]. In [24] an alternative method, Generalized Pencil of Function (GPOF), is used to write the spectral domain Green’s function in terms of complex exponentials. GPOF method provides better accuracy and it is less sensitive to noise compared to Prony’s Method. The efficiency and accuracy of DCIM are further improved by the utilization of two level [25] and three level [26] approaches.

In this work, the closed form spatial domain Green’s functions are calculated using a MATLAB program developed by Aytaç Alparslan. This program is based on the three level approximation method presented in [26].

A program is developed in MATLAB environment to analyze the loaded printed dipole antennas. The developed code makes use of the complex exponentials and their corresponding coefficients obtained from Alparslan’s program.

Chapter 2 starts with the theoretical background of Method of Moments. The general procedure to solve an operator equation by using MoM is explained. The derivation of Mixed Potential Integral Equation (MPIE) formula for the printed dipole is presented. The calculation steps of the closed form Green’s functions are studied in detail. The entries of MoM matrix and excitation vector are expressed explicitly. Finally, how loading is incorporated into MoM analysis is discussed.

In Chapter 3, simulation and measurement results of inductively loaded printed dipoles are presented. Several inductively loaded printed dipole antennas are analyzed by using the developed code. The effects of loading the antenna at different load positions on the efficiency and the input impedance parameters are studied. The results obtained from the developed code are verified by comparing them to the results obtained from commercially available electromagnetic field solver EMP. Three different loaded antenna structures are chosen and manufactured. The input return loss of these antennas are measured and compared to the results obtained from the developed code.

The conclusions and future works are presented in Chapter 4.
CHAPTER 2

METHOD OF MOMENTS ANALYSIS OF PRINTED STRUCTURES

In electromagnetics, numerical methods are preferred to accurately analyze the complex problems. Method of Moments (MoM) is a widely used method for scattering and radiation problems. Although the name “Method of Moments” is based on the ground of Russian literature it is used by Richmond [27] in 1965 and Harrington [28] in 1967 in terms of electromagnetic theory.

The main idea of the Method of Moments is to convert an integral, differential or integro-differential equation into a matrix equation. This conversion process will be summarized by considering the following operator equation:

\[ L(f(x)) = g(x) \]  \hspace{1cm} (2.1)

where \( L \) is a linear operator, \( g(x) \) is the known source function and \( f(x) \) is the unknown function.

In electromagnetics, \( L \) is generally an integral operator. For conducting scatterers this integral equation is obtained by imposing the boundary conditions at the surface of the object. If the boundary condition on the electric field is enforced the integral equation it is referred as Electric Field Integral Equation (EFIE). On the other hand in Magnetic Field Integral Equation (MFIE), the boundary condition on the magnetic field is considered. The current induced on the conductor becomes the unknown function.

For scattering problems the incident field is the known source function whereas source models like delta gap and magnetic frill current are the known functions for radiation problems.

In the MoM analysis, first, the unknown function \( f(x) \) is expanded in terms of known “basis” or “expansion” functions \( f_a(x) \) with unknown coefficients \( \alpha_a \). It can be given in the following form:
\[ f(x) \equiv \sum_{n=1}^{N} \alpha_n f_n(x) \]  

(2.2)

Substitution of equation (2.2) into (2.1) yields

\[ L \left( \sum_{n=1}^{N} \alpha_n f_n(x) \right) \equiv g(x) \]  

(2.3)

Due to the linearity of \( L \), (2.3) can be written as:

\[ \sum_{n=1}^{N} \alpha_n L f_n(x) \equiv g(x) \]  

(2.4)

Secondly, the residual or error function is defined as:

\[ R(x) = \sum_{n=1}^{N} \alpha_n L f_n(x) - g(x) \]  

(2.5)

In order to minimize the residual in average sense, the inner product of the residual and a set of weighting function is equated to zero. Each weighting function, \( w_m \), is defined over the domain \( \Omega_m \).

At the end of this testing procedure, \( N \) equations in terms of \( N \) unknowns are obtained. This technique is called as “weighted residual technique”.

After the implementation of the testing procedure, equation (2.6) is obtained.

\[ \sum_{n=1}^{N} \alpha_n \left( w_m(x), L f_n(x) \right) = \left( w_m(x), g(x) \right) \text{ for } m = 1, 2, ..., N \]  

(2.6)

Finally, equation (2.6) can be written in matrix form as:

\[
\begin{bmatrix} Z_{mn} \end{bmatrix} \begin{bmatrix} \alpha_n \end{bmatrix} = \begin{bmatrix} g_m \end{bmatrix}
\]  

(2.7)

\( [Z_{mn}] \) is called the “MoM matrix”, \( [g_m] \) is called the “excitation vector” and \( [\alpha_n] \) is the vector of unknown coefficients.

The MoM matrix is written explicitly as:

\[
Z_{mn} = \left( w_m(x), L f_n(x) \right) = \int_{\Omega_m} w_m(x) L f_n(x) \, dx
\]  

(2.8)

\[
[Z_{mn}] = \begin{bmatrix}
\left( w_1, L f_1 \right) & \cdots & \left( w_1, L f_n \right) \\
\vdots & \ddots & \vdots \\
\left( w_m, L f_1 \right) & \cdots & \left( w_m, L f_n \right)
\end{bmatrix}
\]  

(2.9)

\( [Z_{mn}] \) is a full matrix which represents interaction between each basis and testing function.

The excitation vector is written explicitly as:
\[ g_m = \langle w_m(x), g(x) \rangle = \int_{\alpha_m} w_m(x) g(x) \, dx \]  

\[ [g_m] = \begin{bmatrix} \langle w_1, g \rangle \\ \langle w_2, g \rangle \\ \vdots \\ \langle w_m, g \rangle \end{bmatrix} \]  

Finally, matrix equation is solved to find the unknown coefficients.

### 2.1 Basis Functions

There are two types of basis functions which can be used to model the unknown quantity: Entire domain and sub-domain basis functions.

Entire domain basis functions are non-zero over the entire domain. Sinusoidal functions are an example of entire domain basis functions.

For sub-domain basis functions, the interested region is divided into \( N \) sub-domains and basis functions which are nonzero on the corresponding sub-domain are defined. Pulse basis and triangular basis functions are widely used as sub-domain basis functions in one dimensional problems like the analysis of linear antennas.

For the analysis of printed structures two dimensional basis functions (2D) are required. For arbitrarily shaped 2D conductors, Rao-Wilton-Glisson (RWG) basis functions are widely used [29]. However, in this thesis only the analysis of printed dipole antennas are considered so the shape of the conductor is restricted to be narrow rectangular. Due to the narrow width condition, the direction of the induced current can be assumed to be only along the length of the printed line and the variation of the current along the width of the line can be assumed to be constant. These assumptions make it possible to use simple basis functions which are called roof top basis functions. The roof top basis function is a triangular function along the direction of the current and constant along the transverse direction as shown in Figure 2.1.
The roof top function is expressed mathematically as:

$$\Lambda_n(x, y) = f_{1n}(x)f_2(y)$$  \hspace{1cm} (2.12)

The triangular function in the x-direction is defined as:

$$f_{1n}(x) = \begin{cases} 
\frac{1}{h_x}[(1-n)h_x + x], & (n-1)h_x \leq x \leq nh_x \\
\frac{1}{h_x}[(1+n)h_x - x], & nh_x \leq x \leq (n+1)h_x \\
0, & \text{elsewhere} 
\end{cases} \hspace{1cm} (2.13)$$

The pulse function in the y-direction is defined as:

$$f_2(y) = \begin{cases} 
\frac{1}{w}, & |y| \leq \frac{w}{2} \\
0, & \text{elsewhere} 
\end{cases} \hspace{1cm} (2.14)$$

### 2.2 Testing Functions

There are different testing methods such as point matching, collocation by sub-domains method and Galerkin’s method.

Point matching method uses dirac delta functions for testing. Generally midpoint of sub-domains is chosen as the testing point. The residual is forced to be zero at $N$ distinct points. Although this method is the simplest one, it provides least accuracy.

In collocation by sub-domains pulse functions are used as the testing function. Although analytical work increases compared to point matching method, accuracy improves. In this thesis work, Galerkin’s method is used as the testing procedure. In this method, weighting and basis functions are chosen to be same.

$$w_m = f_m \hspace{1cm} \text{for } m = 1, 2, \ldots, N \hspace{1cm} (2.15)$$
The main advantage of Galerkin’s method is to provide symmetric MoM matrix. This reduces the memory requirements and the computational time. However, computation of MoM matrix entries is more complex than point matching and collocation by sub-domains methods.

2.3 Derivation of Mixed Potential Integral Equation (MPIE) for Printed Structures

In this section, the integral equation in terms of the induced current on the printed structure will be obtained. As mentioned before since the width of the line is small compared to the wavelength, only x directed currents will be induced. The x directed electric field due to these x-directed induced currents can be written as:

\[ E_x = -j\omega A_x - \frac{\partial \phi}{\partial x} \]  \hspace{1cm} (2.16)

where \( E_x \) represents x directed electric field, \( A_x \) represents vector potential and \( \phi \) represents scalar potential.

These potentials can be expressed as in terms of the following convolution integrals:

\[ A_x = G_{xx}^\Lambda * J_x = \frac{\mu}{4\pi} \iint G_{xx}^\Lambda (x-x',y-y') J_x(x',y') dx' dy' \]  \hspace{1cm} (2.17)

\[ \phi = G_q^* \rho = \frac{1}{4\pi\epsilon} \iint G_q(x-x',y-y') \rho(x',y') dx' dy' \]  \hspace{1cm} (2.18)

\( G_{xx}^\Lambda \) is the Green’s function for vector potential and it represents the x directed vector potential due to a x directed electric dipole on the dielectric slab. Similarly, \( G_q \) is the Green’s function for scalar potential and it represents the scalar potential due to a point charge on the dielectric slab. \( \rho \) is the surface charge density and it is related to the current density in the following form through the continuity condition:

\[ \frac{\partial J_x}{\partial x} + j\omega \rho = 0 \]  \hspace{1cm} (2.19)

The x directed electric field can be written explicitly as:

\[ E_x = -j\omega \frac{\mu}{4\pi} G_{xx}^\Lambda * J_x + \frac{1}{j\omega 4\pi \epsilon} \frac{\partial}{\partial x} \left[ G_q * \frac{\partial J_x}{\partial x} \right] \]  \hspace{1cm} (2.20)

The electric field in equation (2.20) is the scattered field due to the induced currents on the conductor. The tangential component of total field which is the summation of
incident ($\vec{E}'$) and scattered ($\vec{E}''$) fields should be zero on the surface of the conductor.

This boundary condition gives rise to the following Mixed Potential Integral Equation (MPIE) for printed structures:

$$-j\omega \frac{\mu}{4\pi} G_{sx}^A * J_x + \frac{1}{j\omega 4\pi \varepsilon} \left[ G_q * \frac{\partial J_q}{\partial x} \right] = -E'_x$$  \hspace{1cm} (2.21)

**2.4 Solution of MPIE by MoM**

As stated before, rooftop functions are chosen as both basis and testing functions. For this choice the MoM matrix entries can be written as:

$$Z_{mn} = -j\omega \frac{\mu}{4\pi} \left\{ \Lambda_m(x, y), G_{sx}^A \Lambda_n(x', y') \right\} + \frac{1}{j\omega 4\pi \varepsilon} \left\{ \Lambda_m(x, y), \frac{\partial}{\partial x} \left[ G_q * \frac{\partial}{\partial x'} \Lambda_n(x', y') \right] \right\}$$  \hspace{1cm} (2.22)

Since the rooftop basis functions are piecewise differentiable, the differentiation can be transferred onto the testing function. It can be demonstrated as:

$$\left\{ \Lambda_m(x, y), \frac{\partial}{\partial x} \left[ G_q * \frac{\partial}{\partial x'} \Lambda_n(x', y') \right] \right\} = -\left\{ \Lambda'_m(x, y), G_q * \frac{\partial}{\partial x'} \Lambda_n(x', y') \right\}$$  \hspace{1cm} (2.23)

where $\Lambda' = \frac{\partial \Lambda}{\partial x}$.

The computation of each matrix entry requires the evaluation of two double integrals; one for the convolution and the other for the testing. Since those integrals are not analytically integrable, numerical integration needs to be employed.

If convolution operation had been between basis and testing functions, the integral would be calculated analytically by changing the order of integrations. So the following expressions can be obtained:

$$\iiint \Lambda_m(x, y) \int G(x-x', y-y') \Lambda_n(x', y') dx' dy' dx dy$$  \hspace{1cm} (2.24)

$$= \iiint G(u, v) \int \Lambda_m(x, y) \Lambda_n(x-u, y-v) dx dy du dv$$

Finally, MoM matrix is found as:

$$Z_{mn} = -j\omega \frac{\mu}{4\pi} \left\{ G_{sx}^A, \Lambda_m(x, y) \otimes \Lambda_q(x', y') \right\} - \frac{1}{j\omega 4\pi \varepsilon} \left\{ G_q, \Lambda'_m(x, y) \otimes \Lambda'_q(x', y') \right\}$$  \hspace{1cm} (2.25)

where $\otimes$ represents correlation between basis and testing functions.
A code is developed in MATLAB environment to numerically evaluate the inner product integrals of the Green’s functions and the correlation of basis and testing functions.

Since the correlation between the basis and the testing functions is a function of the distance between them, it is sufficient to compute only the first row of the MoM matrix to construct the overall matrix. Hence the MoM matrix can be efficiently computed as follows:

\[
\begin{pmatrix}
Z_{11} & Z_{12} & Z_{13} & \cdots & Z_{1n} \\
Z_{12} & Z_{11} & Z_{12} & \cdots & Z_{1n-1} \\
Z_{13} & Z_{12} & Z_{11} & \cdots & Z_{1n-2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
Z_{1n} & Z_{1n-1} & \cdots & \cdots & Z_{11}
\end{pmatrix}
\]  
(2.26)

Note that the self term, \( Z_{11} \), involves singularities since the source and the observation point coincide. The method used to handle these singular integrals will be presented after introducing the closed form Green’s functions used for the layered medium.

After MoM interaction matrix \( Z_{mn} \) is found, it is time to define the modeling of the source of the antenna. Delta-gap which is shown in Figure 2.2 is chosen as the source model since it can be easily implemented to MoM procedure.

![Figure 2.2: Antenna feeding](image)

In this model, the electric field between the antenna terminals could be written as:

\[
\hat{E}_x = \frac{V_{ant}}{\Delta_x} \hat{a}_x
\]  
(2.27)

where \( V_{ant} \) generally equals to 1V and \( \Delta_x \) is the gap between antenna terminals.

As \( \Delta_x \) tends to be zero, incident field takes the following form:
\[ \vec{E}_i = V_{ant} \delta(x) \hat{a}_i \]  

(2.28)

Delta-gap source is connected to the center of the antenna. Hence the entry of the excitation vector corresponding to the feed location becomes -1 and all other entries become zero. The excitation vector takes the following form:

\[
\begin{bmatrix}
0 \\
\vdots \\
-1 \\
\vdots \\
0
\end{bmatrix}
\]

(2.29)

After the calculation of the current distribution on the antenna, input impedance can be calculated using (2.30).

\[ Z_{in} = \frac{V_{ant}}{I_{ant}} \]  

(2.30)

2.5 Green’s Functions in Layered Media

In this section, the need for a closed form expressions for the spatial domain Green’s functions is discussed. Calculation of the closed form spatial domain Green’s function from its spectral domain counterpart is presented.

For a linear and time-invariant system, the response due to an arbitrary input signal can be calculated as the convolution of the impulse response with the input signal. For an electromagnetic problem, Green’s function can be considered as the impulse response of impulse source. If one knows the Green’s function, the field distribution created by an arbitrary source could be found easily. The linearity is the requirement in order to be able to apply this concept. It is known that Maxwell’s equations are linear if the medium is linear.

The scalar wave equation can be written as:

\[
(\nabla^2 + k^2) \varphi(\vec{r}) = s(\vec{r})
\]

(2.31)

where \( \varphi(\vec{r}) \) is scalar function, \( s(\vec{r}) \) is source function defined in volume \( V \), \( k \) is wave number of the medium.

The Green’s function is the solution of the wave equation for a point source as:

\[
(\nabla^2 + k^2) g(\vec{r}, \vec{r}') = -\delta(\vec{r}, \vec{r}')
\]

(2.32)
After the Green’s function is calculated, the scalar function $\varphi(\bar{r})$ can be obtained by the superposition integral as:

$$\varphi(\bar{r}) = -\int d\bar{r}^' g(\bar{r}, \bar{r}^') s(\bar{r}^')$$  \hspace{1cm} (2.33)

The free space Green’s function corresponding to unbounded medium can be derived as:

$$g(\bar{r}, \bar{r}^') = \frac{e^{-j|\bar{r} - \bar{r}'|}}{4\pi|\bar{r} - \bar{r}'|}$$  \hspace{1cm} (2.34)

When the source is positioned in a layered medium, the derivation of the Green’s function starts with a transformation into the spectral domain. Since the dielectric layers are infinite in x and y directions, transformation to the spectral domain simplifies the problem by reducing it to only one dimension. The spectral domain Green’s function can be easily obtained by considering the up going (incident or transmitted) and down going (reflected) fields at the dielectric interfaces. Then the spatial domain Green’s function can be obtained by using the following transformation.

$$G^{A,q}(\rho) = \frac{1}{4\pi} \int dk^\rho k^\rho H^{(2)}_0(k^\rho \rho) \tilde{G}^{A,q}(\rho)$$  \hspace{1cm} (2.35)

where $k^\rho$ equals to $\sqrt{k_x^2 + k_y^2}$. $\rho$ is a variable in cylindrical coordinates. $G^{A,q}$ and $\tilde{G}^{A,q}$ represent the spatial domain Green’s function and the spectral domain Green’s function, respectively. $H^{(2)}_0$ is the zeroth order Hankel function of the second kind. SIP is the Sommerfeld Integration Path, illustrated in Figure 2.3.

The integral in (2.35) can not be evaluated analytically for the Green’s function in layered media. Hence it needs to be evaluated numerically. However the numerical evaluation is computationally expensive since the integrand is highly oscillatory and slowly decaying function.

In order to avoid the numerical integration of this integral, Discrete Complex Image Method (DCIM) is proposed. This method utilizes the Sommerfeld Identity given below:
Sommerfeld Identity implies that if the spectral domain Green’s function can be approximated in terms of complex exponentials then the spatial domain Green’s function can be expressed in closed form. Hence, the first step of DCIM is to approximate the spectral domain Green’s function in terms of complex exponentials. In this thesis, Generalized Pencil of Function (GPOF) method is used to obtain this approximation. This method requires uniform samples of the complex valued function that will be approximated along a real variable. It is clear that if the sampling process is along \( k_\rho \), the exponentials will be in terms of \( k_\rho \) as well and the Sommerfeld identity can not be used.

If the SIP is deformed as shown in Figure 2.3 (denoted by \( C_{ap} \)), then the complex exponentials can be obtained in terms of \( k_z \).

\[
\frac{e^{-jk_\rho r}}{r} = -\frac{j}{2} \int_{\text{SIP}} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) e^{-jk_\rho \rho} k_z \tag{2.36}
\]

Figure 2.3: SIP and deformed path for GPOF sampling

\( k_z \) along this deformed path can be written in terms of a real valued parameter as:

\[
k_z = k \left[ -jt + \left( 1 - \frac{t}{T_0} \right) \right], 0 \leq t \leq T_0 \tag{2.37}
\]

where \( T_0 \) is named as truncation point and \( t \) is sampled between 0 and \( T_0 \).
The sampling frequency determines the accuracy of the approximation so it is an important parameter. It is known that the spectral domain Green’s functions can vary fast for \( k_{\rho} < k_{\rho_{\text{max}}} \). So the number of samples should be chosen large enough to capture this fast variation. On the other hand, \( T_0 \) should be chosen large enough to capture the asymptotic behaviour of the Green’s function. Large number of samples and large \( T_0 \) make the method inefficient. Therefore two level approximation is proposed. In this method, the sampling is performed along two different paths \( C_{\text{ap1}} \) and \( C_{\text{ap2}} \) which are shown in Figure 2.4.

![Figure 2.4: \( C_{\text{ap1}} \) and \( C_{\text{ap2}} \) together with SIP](image)

First, the spectral domain Green’s function is sampled along \( C_{\text{ap1}} \) to obtain the asymptotic part. The transformation used for \( C_{\text{ap1}} \) is as follows:

\[
C_{\text{ap1}} : k_z = -jk_z\left[T_{o2} + t\right], 0 \leq t \leq T_{o1} \tag{2.38}
\]

Since the function varies slowly in this interval, sampling frequency can be smaller. By using these samples and applying GPOF the asymptotic part is approximated in terms of complex exponentials. Then this asymptotic part is extracted from the spectral domain Green’s function and the remaining function is sampled along \( C_{\text{ap2}} \):

\[
C_{\text{ap2}} : k_z = k_z\left[-jt + \left(1 - \frac{t}{T_{o2}}\right)\right], 0 \leq t \leq T_{o2} \tag{2.39}
\]
Finally, this remainder part is also approximated in terms of complex exponentials. As a result, the spectral domain Green’s function is written as:

\[ G \approx \frac{1}{j2\varepsilon_\alpha k_i} \left[ \sum_{n=1}^{N_1} a_{1n} e^{-\alpha_{1n} k_i} + \sum_{n=1}^{N_2} a_{2n} e^{-\alpha_{2n} k_i} \right] \]  \hspace{1cm} (2.40)

where \( i \) is the source layer, \( N_1 \) and \( N_2 \) are the numbers of exponentials, \( a_{1n} \) and \( \alpha_{1n} \) are obtained from the first part of the two level approximation and \( a_{2n} \) and \( \alpha_{2n} \) are obtained from the second part.

Since spectral domain Green’s function is written in terms of exponentials, now, Sommerfeld Identity could be used to calculate spatial domain Green’s function. Spatial domain Green’s function can be written as:

\[ G \approx \frac{1}{4\pi\varepsilon_i} \left[ \sum_{n=1}^{N_1} a_{1n} \frac{e^{-jk_{1n} r_n}}{r_n} + \sum_{n=1}^{N_2} a_{2n} \frac{e^{-jk_{2n} r_n}}{r_n} \right] \]  \hspace{1cm} (2.41)

Equation (2.41) can be written in a form given below:

\[ G \approx \sum_{m=1}^{N} a_m \frac{e^{-jk_i r_m}}{r_m} \]  \hspace{1cm} (2.42)

where \( r_m \) is complex distance and equals to \( \sqrt{\rho^2 - b_m^2} \). \( k_i \) is the wave number in the source layer.

Note that there is always a complex exponential with zero exponent and coefficient equals to 1. This exponential term represents the direct term [26].

A program developed by Aytaç Alparslan in MATLAB environment is used to calculate the coefficients \( (a_m) \) and the exponents \( (b_m) \). The inputs of the software are operating frequency, number of layers, thicknesses, permittivities and permeabilities of each layer, location of the source and observation points. Two sets of coefficients and exponents are obtained: Green’s function for vector potential, \( G_A \), and Green’s function for scalar potentials, \( G_q \).

A numerical example will be presented to demonstrate the behavior of the vector and scalar Green’s functions. The geometry of the studied structure is shown in Figure 2.5 and the parameters used in the analysis are below:

\[ f = 403 \text{ MHz} \]
\[ \varepsilon_{r1} = 3.38 \]
\[ \varepsilon_{r2} = 1 \]
$h = 0.813 \text{ mm}$.

Figure 2.5: 3-layer printed structure

Magnitude of the vector and the scalar Green’s function as a function of the distance between source and observation points are shown in Figure 2.6 and Figure 2.7, respectively.

Figure 2.6: Magnitude of the Green’s function of vector potential, $G^{A}_{\text{xx}}$
2.6 Singularity Extraction

As derived before MoM matrix entries require the computation of the following inner product integrals:

$$\left\langle G_{\xi}^{A}, \Lambda_{m}\otimes \Lambda_{n} \right\rangle = \iiint dudvG_{\xi}^{A}(u,v)\iint dxdy \Lambda_{m}(x-u,y-v)\Lambda_{n}(x,y)$$

(2.43)

Recall that the Green’s function is expressed in closed form as:

$$G_{\xi}^{A} \approx \sum_{n=1}^{N} a_{n} e^{-jk_{n}r_{n}}$$

(2.44)

where $$r_{n} = \sqrt{u^{2} + v^{2} - b_{n}^{2}}$$ and $$b_{n} = 0$$ for the direct term.

During the computation of diagonal elementries since basis and testing functions are at the same location, the integration intervals over $$u$$ and $$v$$ include zero as well. When $$u$$ and $$v$$ are equal to zero, the denominator of the Green’s function becomes zero for the direct term. This causes the integrand to be singular. In the MPIE formulation, the singularity of both the scalar and vector potentials are the order of $$1/r$$. However, when the dyadic Greens’s functions for the electric field is considered the order of the singularity becomes higher due to the gradient operator. Thus, MPIE formulation is less singular than EFIE and/or MFIE.
In order to avoid numerical problems associated with this singularity, special techniques need to be employed. In this thesis, singularity extraction method is used. In this method, the Taylor series expansion of the exponential term is utilized. If the exponential $e^{-jkr}$ is expanded at around $r = 0$, it can be written as:

$$e^{-jkr} = 1 + (-jkr) + \frac{(-jkr)^2}{2!} + \ldots$$

Equation (2.46) is obtained if both sides of the equation is divided by $r$.

$$\frac{e^{-jkr}}{r} = \frac{1}{r}(-jkr) + r \left( \frac{(-jkr)^2}{2!} + \ldots \right)$$

Only the first two terms of the expansion are considered since the effects of higher order terms are so small when $r \to 0$.

The integral of the first term which represents the singularity is evaluated analytically as:

$$\int \int_{y_1 \leq y \leq y_2} \frac{1}{\sqrt{x^2 + y^2}} dx dy = y_2 \log \left[ \frac{-x_1 + \sqrt{x_1^2 + y_2^2}}{-x_2 + \sqrt{x_2^2 + y_2^2}} \right] + y_1 \log \left[ \frac{-x_2 + \sqrt{x_2^2 + y_1^2}}{-x_1 + \sqrt{x_1^2 + y_1^2}} \right]$$

$$+ x_2 \log \left[ \frac{-y_1 + \sqrt{x_2^2 + y_1^2}}{-y_2 + \sqrt{x_2^2 + y_2^2}} \right] + x_1 \log \left[ \frac{-y_2 + \sqrt{x_1^2 + y_2^2}}{-y_1 + \sqrt{x_1^2 + y_1^2}} \right]$$

### 2.7 The Effects of Lumped Loading on the MoM Interaction Matrix

When a single lumped element is loaded to the antenna, this only affects the boundary condition at the loading point. After loading, the tangential component of the electrical field is non-zero at the load position and it equals to $E_{\text{load}}$:

$$E_{\text{total}} = E_{\text{load}} = E_i + E_s = \frac{V}{d}$$

where $V$ is the potential difference across the load and $d$ is the gap in the dipole antenna where load is connected. To simplify the calculation, this gap is considered to be infinitesimal.

$$\lim_{d \to 0} E_{\text{total}} = I_g Z_a \delta(x - x_g)$$

where $x_g$ is the location of the load, $Z_a$ is the impedance of the load and $I_g$ is the total axial current at the load.

The testing procedure is implemented to the total tangential field.
The load impedance is subtracted from the corresponding diagonal element of the MoM matrix.

The new interaction matrix form is as follows:

\[
Z_{nn} = \begin{bmatrix}
Z_{11} & \cdots & Z_{1N} \\
\vdots & \ddots & \vdots \\
Z_{N1} & \cdots & Z_{NN}
\end{bmatrix}
\]

\[
[Z][I] = \left(\begin{bmatrix}Z^0 \end{bmatrix} - [Z^L]\right)[I] = [V]
\]

where \([Z^0]\) is unloaded case of \(N \times N\) MoM matrix, \([Z^L]\) is lumped load matrix.

In \([Z^L]\), only the diagonal entry corresponding to the location of the load is nonzero, all other entries are zero. Since loading only affects related self term of the MoM matrix, there is no need to calculate unloaded case MoM matrix for different loads, thus it provides computational advantage.
CHAPTER 3

SIMULATION AND MEASUREMENT RESULTS OF INDUCTIVELY LOADED PRINTED DIPOLES

In this thesis work, Electrically Small Antenna (ESA) is chosen as an application example of loaded printed dipole antennas. When the reserved place to put an antenna is restricted, ESAs could be a solution. Although ESAs have dimensional advantage due to electrically small lengths, the input impedance of ESA consists of low resistance and high capacitive reactance. It is obvious that the impedance transformation is needed in order to match these types of antennas to a 50 ohm system. Inductive loading method can be implemented to get rid of this capacitive effect. The loaded antenna is shown in Figure 3.1.

Figure 3.1: The loaded printed antenna

The inductive loading method also increases the resistance of the input impedance due to the internal resistance of the inductors. However, this method adversely affects the efficiency of the antenna, so inductors which have high quality factor (Q) should be preferred in order to reduce the negative effect of the loading.

The efficiency of the loaded antenna is calculated as [30]:

\[
\eta = \frac{R_{in} |I_{ant}|^2 - R_L |I_L|^2}{R_{in} |I_{ant}|^2}
\]  

(3.1)
where $R_m$ is the input resistance, $R_a$ is the real part of the load impedance, $I_{ant}$ is the current at the input of the antenna and $I_L$ is the current at the load location.

The first term in the numerator represents the power delivered to the antenna and the second term represents the power lost in the load. Hence, the conductor and dielectric losses are assumed to be small compared to this loss and they are ignored in this formula.

In this thesis work, Coilcraft 0603-HP series chip inductors are used since they have high Q. The equivalent circuit of the inductor is provided by the manufacturer which is shown in Figure 3.2. This equivalent circuit is used during the simulations.

![Figure 3.2: The equivalent circuit of Coilcraft HP series inductors](image)

The values of the parameters in this equivalent circuit are presented in Table 3.1.

<table>
<thead>
<tr>
<th>Inductor</th>
<th>R2(ohm)</th>
<th>Rvar(ohm)*</th>
<th>L(nH)</th>
<th>C1(pF)</th>
<th>R1(ohm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0603HP-R30 (300 nH)</td>
<td>2.5</td>
<td>8.4</td>
<td>300</td>
<td>0.040</td>
<td>120</td>
</tr>
<tr>
<td>0603HP-R33 (330 nH)</td>
<td>3.6</td>
<td>9.4</td>
<td>330</td>
<td>0.036</td>
<td>200</td>
</tr>
<tr>
<td>0603HP-R39 (390 nH)</td>
<td>4</td>
<td>10.6</td>
<td>390</td>
<td>0.037</td>
<td>140</td>
</tr>
</tbody>
</table>

* $Rvar = k \sqrt{f}$.

where $f$ is operating frequency and $k$ values are given by Coilcraft for each inductor.

All the analyses are done by using the code developed in MATLAB. The results are compared to results obtained by EMPro, which is Agilent’s 3D electromagnetic field solver, in order to verify the accuracy of the developed code. At the end of the
numerical analysis, three ESAs which have different lengths are chosen and manufactured. The return loss parameters of the antennas are measured. The measured results are compared to the results of developed code as well.

The Medical Implant Communication Service (MICS) band which is between 402-405 MHz is chosen for the design of ESAs. Half wavelength dipole at this frequency band is about 37.5 cm. However, the size of the antenna is restricted to few centimeters (2-3 cm) since it will be implanted into human body. Hence, there is a need to design an electrically small antenna.

In this work, Rogers 4003C is chosen as substrate due to its availability and low tangent loss ($\tan \delta = 0.0021$). The permittivity of the substrate ($\varepsilon_r$) is 3.38 and the thickness of the substrate is 0.813 mm.

The guided wavelength for this substrate needs to be calculated at the center frequency, 403 MHz. To calculate the guided wavelength first a single open ended microstrip line is analyzed with developed code.

Once the current distribution on the line is calculated, it is approximated in terms of two complex exponentials that represent the forward and backward propagating waves.

\[ I(x) = I^+(x) + I^-(x) \quad (3.2) \]

\[ I(x) = c_1e^{\gamma_1x} + c_2e^{\gamma_2x} \quad (3.3) \]

where $c_1$ and $c_2$ are the coefficients of incident and reflected waves, respectively.

Generally, real parts of $\gamma_1$ and $\gamma_2$ are negative and equal to each other. They represent the attenuation constant of the transmission line. On the other hand, the imaginary parts of $\gamma_1$ and $\gamma_2$ are almost same in magnitude but opposite in sign. They represent propagation constant of the line. The propagation constant of the line is taken as the average of $|\text{Imag}(\gamma_1)|$ and $|\text{Imag}(\gamma_2)|$. Then the guided wavelength $\lambda_g$ is calculated as:

\[ \lambda_g = \frac{2\pi}{\beta} \quad (3.4) \]

Prony’s method is used to write the sampled data as a linear combination of exponentials.
The guided wavelength is calculated as 66 cm for the 1 mm wide line which is printed on RO4003C.

The current distribution on 0.47λ₀ antenna which corresponds to 31 cm is shown in Figure 3.3.

![Figure 3.3: The current distribution on a 0.47λ₀ printed dipole antenna](image)

The input impedance of the 0.47λ₀ printed dipole is calculated as 56 Ω.

Since the antenna is too long, it is reduced to 4.45 cm. The current distribution of 4.45 cm printed dipole antenna is shown in Figure 3.4.

![Figure 3.4: The current distribution on a 4.45 cm printed dipole antenna](image)

It is seen that the real part of the input impedance is small and the imaginary part is high and negative. (Zᵢₙ = 0.5 – 1284 j)
Hence to make the printed dipole operate in the MICS band, the antenna needs to be inductively loaded. First, to investigate the effects of load position on the performance of the antenna 300 nH inductors are connected at four different load positions as shown in Figure 3.5 through Figure 3.8.

The load positions directly affect the current distribution on the antenna. This is shown in Figure 3.9.
It is obvious that the efficiency of the antenna changes when loads are connected at different positions. The radiation efficiency of the 4.45 cm antenna is calculated for different load positions and the results are presented in Figure 3.10.

Note that since R2 resistance in the equivalent circuit of the inductor cannot be modeled in EMPro, it is not included in the developed code as well in order to make a fair comparison.
It can be observed that the results of *EMPro* and the developed code are in good agreement. As the load position moves towards to the feed (LP4), the current passing through the load increases. Consequently, the efficiency decreases due to the increased loss.

The real and imaginary parts of the input impedance of 4.45 cm antenna are calculated for different load positions and presented in Figure 3.11 and Figure 3.12, respectively.

**Figure 3.10: Efficiency versus load position for 4.45 cm antenna**

**Figure 3.11: The real part of the input impedance versus load position for 4.45 cm antenna**
It is seen from Figure 3.11 and Figure 3.12 that the input impedance of the antenna gets closer to 50Ω, and the imaginary part becomes less capacitive as the load location moves towards the feed (LP4). This is due to the fact that the current distribution on the antenna is affected more when the load is positioned close to the feed where the current is maximum. Hence, it can be concluded that there is a trade-off between the efficiency and the impedance matching of the antenna. While improving the efficiency, input matching is becoming worse.

Another study is performed to investigate the effects of load position on the resonance frequency of the antenna. In this study again 300nH inductor is connected at the same four load positions. For each load position the input impedance of the antenna is calculated at different frequencies and the frequency at which the impedance is purely real is found. These resonance frequencies and the corresponding input resistance are presented in Table 3.2.
Table 3.2: Resonance frequency versus load position

<table>
<thead>
<tr>
<th>Load Position</th>
<th>MATLAB</th>
<th>EmPro</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Resonance Frequency (MHz)</td>
<td>Input impedance @resonance (Ω)</td>
</tr>
<tr>
<td>Unloaded</td>
<td>Out of band</td>
<td>Out of band</td>
</tr>
<tr>
<td>LP1</td>
<td>570</td>
<td>90</td>
</tr>
<tr>
<td>LP2</td>
<td>490</td>
<td>55</td>
</tr>
<tr>
<td>LP3</td>
<td>440</td>
<td>40</td>
</tr>
<tr>
<td>LP4</td>
<td>400</td>
<td>27</td>
</tr>
</tbody>
</table>

It can be observed from the table that as the load moves towards the center of the antenna, the current distribution is disturbed more and hence the resonance frequency becomes smaller. On the other hand the input resistance also changes with the position of the load. Therefore if an antenna that resonates at 403 MHz with 50Ω input resistance needs to be designed, both the position and the value of the load should be optimized.

To investigate the effects of load values on the input impedance of the antenna, three different antennas are studied. Firstly, the 4.45 cm antenna is investigated in terms of input impedance for unloaded and loaded case. LP4 is used as load position and 300nH, 330nH or 390nH is connected as load values.
Figure 3.13: Input impedance vs different load values for 4.45 cm antenna at 403 MHz

It can be easily observed that to get the antenna to the resonance, 300nH would be a correct load value.

Similar analyses are performed for dipole antennas whose lengths are 3.9 cm and 2.5 cm in order to reduce the size of the antenna even further. It is noted that the load position is kept same as the original antenna by considering the normalized position which is the ratio of load position to the length of the dipole.

As seen in Figure 3.14 and Figure 3.15, the best load value for 3.9 cm antenna is 330 nH whereas it is 390 nH for the 2.5 cm antenna.
Figure 3.14: Input impedance vs different load values for 3.9 cm antenna at 403 MHz

Figure 3.15: Input impedance vs different load values for 2.5 cm antenna at 403 MHz
Note that it was not possible to design antennas which are precisely matched to 50Ω at 403 MHz. But our aim was to choose the most proper values of the inductor among the available components.

### 3.1 The Manufacturing and Measurement of ESA

To test the antennas studied in the previous section, three different antennas with different lengths are manufactured. In order to be able to feed the dipole with a SMA connector a transition from a microstrip line is required. It is shown in Figure 3.16.

![Figure 3.16: The microstrip transition](image)

The width of a 50Ω microstrip line on this substrate 2 mm. Hence, the microstrip line starts with a width of 2 mm and it tapers down to 1 mm in order to be consistent with the width of the dipole. The photograph of the manufactured antenna together with the transition and connector can be seen in Figure 3.17.
The simulation results from the developed code and the measurement results for 4.45 cm antenna are compared in Figure 3.18. It can be seen that the resonance frequency is not 403 MHz even according to the simulation result. As discussed before, this is due to the choice of the inductor value which is commercially available.

Figure 3.17: 4.45 cm loaded printed antenna

Figure 3.18: $S_{11}$ parameter of the 4.45 cm loaded antenna
A good agreement between simulation and measurement results are observed. The frequency shift between simulation and measurement results is only 4% which quite low considering the parasitic effects introduced during the soldering of the inductors.

Now, similar comparison is performed for 3.9 cm antenna which has 330 nH loads.

For 3.9 cm antenna, the frequency shift between simulation and measurement results is only 2% which quite low considering the parasitic effects introduced during the soldering of the inductors.

Finally, 2.5 cm antenna which has 390 nH loads is measured.
Like the previous antenna, the frequency shift between simulation and measurement results is only 2% which quite low considering the parasitic effects introduced during the soldering of the inductors.

The electrically small loaded printed dipole antennas whose lengths are 4.45 cm, 3.9 cm, 2.5 cm are named as $TA_1$, $TA_2$ and $TA_3$, respectively. The efficiencies of these antennas are given in Figure 3.21. As seen in Figure 3.21, the efficiency decreases as the antenna length gets smaller. To verify this results, the gain of the antennas are measured.
To measure the antenna gain, a $\lambda/4$ length monopole antenna shown in Figure 3.22 is manufactured to be used as reference antenna (RA).

In the measurement set-up, a different antenna is also needed, thus Medical Implant Antenna (MIA) whose operating frequency is same as the testing antennas (TA1, TA2 and TA3) is used. Firstly, the power received by RA is measured when MIA is chosen as transmitting antenna. Then, the testing antenna is replaced with the reference antenna and the power received by this antenna is measured once again.
The gains for $TA1$, $TA2$ and $TA3$ with respect to RA are presented in Figure 3.23. These results are obtained by subtracting the power received by the RA from the power received by the test antenna. The gain of the reference antenna is not included in the calculations since it is not known exactly. Hence an analysis with respect to the reference antenna is performed. Note that our aim is to compare the efficiencies of the ESAs, not to measure their absolute gain values.

When the gain levels of three tested antennas are compared, it can be observed that they are in agreement with the efficiency results obtained from the developed code such that gain, consequently efficiency increases as antenna gets longer. However, when the absolute gain values are considered by assuming the gain of the RA as 2dBi, it can be seen that the measured gain values are much more than expected. Since the tested antennas are electrically small, they are expected to provide quite low gain values, but it is seen that the received power levels are comparable to the received by RA. Hence it is concluded that this measurement set-up is not suitable to measure the absolute gain values of ESAs. As reported in literature more
complex approaches like Wheeler Cap method [31] need to be utilized for accurate measurement of radiation characteristics of ESAs.
CHAPTER 4

CONCLUSIONS AND FUTURE WORKS

In this thesis work, the MoM analyses of printed dipole antennas are achieved by using a code which is developed in MATLAB environment. In the MoM formulation, Mixed Potential Integral Equation is preferred and Discrete Complex Image Method is utilized to express the spatial domain Green’s functions in closed form. Galerkin’s testing scheme is applied by using roof top basis functions. The MoM matrix entries require the evaluation of two surface integrals. By using roof top functions, correlation integral between the basis and testing function can be performed analytically. Hence, the MoM matrix entries can be efficiently computed by a single surface integration that needs to be computed numerically. Although roof top basis functions provide efficient computation, they restrict the shape of the antenna to be analyzed. As a future work, RWG basis function can be used to develop a versatile code applicable to arbitrarily shaped dipole antennas.

Electrically small antennas are chosen as application example of loaded printed dipoles and three different lengths of electrically small loaded printed dipoles are manufactured. Input return losses of these antennas are measured. EMPro which is Agilent’s 3D electromagnetic field solver is used to verify the results obtained from the developed code. It is observed that the developed code results have good agreement with both EMPro and the measurement results. However, it is observed that radiation properties of ESAs can not be measured accurately by using traditional techniques. Thus, alternative measurement techniques can be studied and/or proposed as a future work.

An ESA can be matched by properly loading a printed dipole antenna. “The proper loading” means choosing the proper load value and the load position to achieve the desired input impedance at the operating frequency. Several analyses are done using different load values and load positions. At the end of these analyses, it is observed that it is easier to match the antenna when the load is closer to the feed point. However, since the current increases approaches to the feed position, the loss on the
load increases. Thus, efficiency of the antenna decreases as the load moves towards to the feed. It can be concluded that both the load position and the load value affect both the input matching and efficiency of the antenna. As a future work, this analysis method can be used in conjunction with an optimization algorithm such as Genetic Algorithms to automatically design loaded dipole antennas that meet desired specifications.

Finally, in this work, inductive loading method is used to provide input matching of ESAs. However, it is seen that this method affects the antenna efficiency adversely. To achieve high efficiency ESAs, metamaterials can be utilized as proposed in literature. The ESA with metamaterial behaves as if it were much larger than its actual size. It changes the radiation characteristic and increases the performance of ESAs. Hence, as another future study, metamaterial loaded dipole antennas can be analyzed by applying MoM.
REFERENCES


