DESIGN AND MODELING OF A NOVEL RECTIFIER WITH CERAMIC HOLLOW FIBER MEMBRANE CONTACTOR FOR MINIATURIZED ABSORPTION COOLING DEVICES

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BY

ONUR ÖZKAN

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MECHANICAL ENGINEERING

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Approval of the thesis:

DESIGN AND MODELING OF A NOVEL RECTIFIER WITH CERAMIC HOLLOW FIBER MEMBRANE CONTACTOR FOR MINIATURIZED ABSORPTION COOLING DEVICES

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Name, Last Name  :

Signature  :

iv
ABSTRACT

DESIGN AND MODELING OF A NOVEL RECTIFIER WITH CERAMIC HOLLOW FIBER MEMBRANE CONTACTOR FOR MINIATURIZED ABSORPTION COOLING DEVICES

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Miniaturized and mobile absorption cooling devices may be used in many applications, but effective and small size heat and mass exchangers are required. Ceramic hollow fiber membranes are thermally stable so that they can be used for the rectification of ammonia in an absorption cooling device. In this study the simultaneous heat and mass transfer between the ammonia-water vapor and the reflux liquid is modeled in a rectifier and stripper with hydrophobic ceramic hollow fiber membranes. A similar model is used to simulate the same process in another rectifier with random packing. The stagnant film model is used to calculate the heat and mass fluxes. The rectifier and the stripper are divided into segments and the equations for all segments are solved simultaneously. The model is verified by comparing the results with the literature. The effects of the packing density, effective porosity, pore radius and heat and mass transfer coefficients on the distillate concentration and the flow rate are discussed. The sensitivity of the concentration profiles to the grid refinement is shown. The hollow fiber membrane module results
in a higher ammonia concentration and higher mass flow rate at the outlet compared to the random packing because it provides higher vapor-liquid interfacial area. Then, the hollow fiber membrane module model is integrated into the absorption cycle model. The integration of the new design of the stripping and rectifying sections increases the system COP from 0.44 to 0.50 at the design conditions. More importantly, the new design is able to purify the refrigerant at higher desorber heat input rate and absorber coolant temperature. Therefore, the absorption cycle may be operated at different conditions with a smaller performance loss compared to the old design. The effects of absorber coolant temperature and desorber heat input rate on the refrigerant concentration, refrigerant flow rate, system COP and cooling power are investigated. The COP trends for different operating conditions are explained in a relation with the refrigerant ammonia concentration and flow rate. The effects of the sizes and packing densities of stripping and rectifying sections are discussed in detail.

Keywords: Hollow Fiber, Ceramic Membrane, Random Packing, Absorption Cooling, Ammonia-Water.
ÖZ

KÜÇÜK ÖLÇEKLI ABSORPSİYONLU SOĞUTMA CIHAZLARI İÇİN GÖZENEKLİ SERAMİK BORULARDAN OLUŞAN BİR REKTİFİYE APARATININ TASARIMI VE MODELLENMESİ

Özkan, Onur
Yüksek Lisans, Makina Mühendisliği Bölümü

Tez Yöneticisi : Doç. Dr. Almıla Güvenç Yazıcıoğlu
Ortak Tez Yöneticisi: Doç. Dr. Derek K. Baker

Temmuz 2015, 86 sayfa

kullanılan katman sayısına göre hassasiyeti gösterilmiştir. Sağladığı yüksek ara yüzey alanı sayesinde içi boş fiber modülü ile rastgele dolgulu kolona göre daha yüksek amonyak derişimi ve debisi elde edilmiştir. İnce boşluklu membran modülü daha sonra absorbsiyon çevrimi modeline entegre edilmiştir. Dizayn koşullarında çevrimin soğutma katsayısı, yeni rektifiye ve sıyırma bölgeleri sayesinde 0.44’ten 0.50 değerine yükselmiştir. Daha da önemlisi, bu yeni tasarım sayesinde, daha yüksek ısı girdileri ve daha yüksek absorbsiyon ünitesi soğutma suyu sıcaklıklarında bile soğutucu sıvının etkili bir şekilde zenginleştirilmesi gerçekleşmiştir. Böylece, absorbsiyon çevrimi farklı çalışma koşullarında, çok fazla performans kaybı yaşamadan çalışabilecektir. Desorbsiyon ünitesindeki farklı ısı girdileri ile absorbsiyon ünitesindeki farklı soğutma suyu sıcaklıklarının; soğutucu sıvının derişimine, debisine, sistemin soğutma katsayısına ve soğutma gücünün etkileri incelenmiştir. Çevrimin soğutma katsayısının farklı çalışma koşullardaki trendleri, soğutucu sıvının derişim ve debi trendleri ile birlikte açıklanmıştır. Rektifiye ve sıyırma ünitesinin boyutlarının ile paketleme yoğunluklarının etkileri detaylıca tartışılmıştır.

Anahtar Kelimeler: İnce Boşluklu Membran, Seramik Membran, Rastgele Dolgulu, Mikro Absorbsiyonlu Soğutma, Amonyak-Su.
To My Family
ACKNOWLEDGEMENTS

I wish to express my deepest gratitude to my supervisor Assoc. Prof. Dr. Almila Güvenç Yazıcıoğlu for her guidance and advice. I greatly appreciate her help, support and constructive and motivating criticism throughout the master program.

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TABLE OF CONTENTS

ABSTRACT ........................................................................................................ v
ÖZ ..................................................................................................................... vii
ACKNOWLEDGEMENTS ................................................................................ x
TABLE OF CONTENTS ................................................................................ xi
LIST OF FIGURES ......................................................................................... xiii
LIST OF TABLES ............................................................................................ xvi
NOMENCLATURE .......................................................................................... xvii

CHAPTERS

1. INTRODUCTION ....................................................................................... 1
   1.1. Motivation .......................................................................................... 1
   1.2. Literature Review .............................................................................. 2
       1.2.1. Absorption Cooling ................................................................. 5
       1.2.2. Hollow Fiber Membranes ...................................................... 9
   1.3. Objective and Scope ......................................................................... 11

2. MATHEMATICAL MODELING .................................................................. 15
   2.1. Modeling of the Rectifier ................................................................. 15
       2.1.1. Heat and Mass Transfer Formulation .................................... 15
       2.1.2. Property Calculations ............................................................ 24
           2.1.2.1. Viscosity .................................................................... 24
           2.1.2.2. Thermal Conductivity ............................................... 26
           2.1.2.3. Diffusion Coefficients ............................................... 27
           2.1.2.4. Surface Tension .......................................................... 30
       2.1.3. Governing Equations for the Rectifier .................................. 30
           2.1.3.1. Mass Transfer .............................................................. 33
           2.1.3.2. Heat Transfer .............................................................. 37
           2.1.3.3. Balance Equations ..................................................... 39
   2.2. Absorption Cycle Model ................................................................. 40
3. RESULTS AND DISCUSSION .............................................................................................................. 47
   3.1. The Rectifier ................................................................................................................................. 47
   3.2. Overall Absorption Cycle ............................................................................................................ 59
4. CONCLUSIONS AND FUTURE WORK ........................................................................................... 73
   4.1. Summary ....................................................................................................................................... 73
   4.2. Conclusions ................................................................................................................................. 74
   4.3. Future Work ............................................................................................................................... 76

APPENDIX

SOLUTION METHOD USED BY ENGINEERING EQUATION SOLVER SOFTWARE .................................................. 85
LIST OF FIGURES

FIGURES

Figure 1. Photograph of a shim which is used to manufacture the monolithic miniaturized absorption cooling device in Determan’s thesis (retrieved from [3])......4

Figure 2. Schematic of a simple, single effect absorption cooling cycle....................6

Figure 3. Diagram of the absorption cycle used in Determan’s study [3]. ...............8

Figure 4. A distillation column with hollow fiber membranes.................................10

Figure 5. Illustration of the membrane wall.................................................................11

Figure 6. The schematic of the concentration and temperature profiles at the vapor-liquid interface.................................................................16

Figure 7. The schematic of the concentration and temperature profiles at the vapor-liquid interface with interface temperature and concentration, boundary layer thicknesses and bulk temperature and concentration..............................................17

Figure 8. The images of a few hollow fiber membranes [44] and berl saddles [45].................................................................32

Figure 9. Schematic of the heat and mass transfers at (a.) the interface in the membrane and (b.) the random packing modules...................................................33

Figure 10. Diagram of the absorption cycle used in Determan’s study [3] with numbering of the states. .................................................................40

Figure 11. Schematic of the absorption cycle with rectifier, stripper, RHEX and SHEX.................................................................46

Figure 12. Ammonia mass concentration profiles of the liquid and the vapor along the rectifier with random packing.................................................................49
Figure 13. Temperature profiles of the liquid and the vapor along the rectifier with random packing ..........................................................50

Figure 14. Ammonia mass concentration profiles of the liquid (x) and the vapor (y) along the rectifier for the random packing and the hollow fiber module with 0.06 m column diameter and 595 fibers .................................................................51

Figure 15. Temperature profiles of the liquid and the vapor along the rectifier for the random packing and the hollow fiber module with 0.06 m column diameter and 595 fibers ........................................................................................................52

Figure 16. Ammonia mass concentration profiles of the liquid (x) and the vapor (y) along the rectifier for the random packing and the hollow fiber module with 0.078 m column diameter and 1000 fibers ........................................................................................................54

Figure 17. Temperature profiles of the liquid and the vapor along the rectifier for random packing and hollow fiber module with 0.078 m column diameter and 1000 fibers ........................................................................................................54

Figure 18. The effect of packing density on the concentration profiles with 0.078m column diameter ..........................................................................................................................56

Figure 19. The individual effects of transfer coefficients and the membrane effective porosity and pore radius on the distillate ammonia concentration ..........................................................57

Figure 20. The individual effects of transfer coefficients and the membrane properties on the distillate mass flow rate ..........................................................................................................................58

Figure 21. The sensitivity of the concentration profiles to the grid refinement ......58

Figure 22. The system COP and specific cooling power for different desorber heat input rate. The state points are fixed and the required UA changes accordingly ..........................................................................................................................60

Figure 23. Temperature distribution along the rectifying and stripping sections. (Fixed state points with 800 W desorber heat input rate) ..........................................................................................63
Figure 24. Ammonia mass concentration distribution along the rectifying and stripping sections. (Fixed state points with 800 W desorber heat input rate) ..........64

Figure 25. Molar transfer rates of ammonia and water across the vapor-liquid boundary throughout the stripping and rectifying section. (Fixed state points with 800 W desorber heat input rate) .................................................................................65

Figure 26. The effect of desorber heat input rate on system COP for different absorber coolant temperatures......................................................................................................................67

Figure 27. The effect of desorber heat input rate on ammonia concentration of the refrigerant for different absorber coolant temperatures. .........................................................68

Figure 28. System COP and refrigerant ammonia concentration for different desorber heat input rates and absorber coolant temperatures for the cycle design with original components shown in Figure 1 and Table 6. ..............................................70

Figure 29. System COP with different height of stripping and rectifying sections. The total volume of the stripping and rectifying sections is kept constant..........71
LIST OF TABLES

TABLES

Table 1. Estimated weight for different mobile cooling systems. [3]..................2
Table 2. Typical COP values of different ammonia-water absorption cooling systems [7]........................................................................................................8
Table 3. The constants used in Eq. (61) [34]. ..................................................................29
Table 4. Microchannel System Design State Points [3]....................................................41
Table 5. The fixed state points of the cycle [3].................................................................42
Table 6. The $UA$ values for each component in Figure 1.............................................44
Table 7. Rectifier specifications......................................................................................48
Table 8. The vapor temperatures and concentrations at the different column positions of the membrane and random packing modules. .........................52
Table 9. Mass flow rates at the exit of the rectifier.......................................................55
Table 10. Rectifying and stripping sections specifications.......................................60
Table 11. Required $UA$ values for each component....................................................61
Table 12. The state points with new rectifying and stripping sections. .....................62
Table 13. System COP with different packing densities in the rectifying section....72
Table 14. System COP with different packing densities in the stripping section. .....72
## NOMENCLATURE

### Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>( A )</td>
<td>Area</td>
<td>( m^2 )</td>
</tr>
<tr>
<td>( a )</td>
<td>Thermodynamic Activity</td>
<td>-</td>
</tr>
<tr>
<td>( \Delta A )</td>
<td>Interfacial area</td>
<td>( m^2 )</td>
</tr>
<tr>
<td>( C_p )</td>
<td>Specific Heat for Constant Pressure</td>
<td>( J/K )</td>
</tr>
<tr>
<td>( c )</td>
<td>Molar concentration</td>
<td>mol/m(^3)</td>
</tr>
<tr>
<td>( D )</td>
<td>Diffusion coefficient, Diameter</td>
<td>( m^2/s, m )</td>
</tr>
<tr>
<td>( D_f )</td>
<td>Constant for irregularity of the fibers</td>
<td>-</td>
</tr>
<tr>
<td>( D_p )</td>
<td>Size of the berl saddles</td>
<td>m</td>
</tr>
<tr>
<td>( d )</td>
<td>Fiber diameter</td>
<td>m</td>
</tr>
<tr>
<td>( \varepsilon'_0 )</td>
<td>Combined Energy Flux</td>
<td>W/m(^2)</td>
</tr>
<tr>
<td>( Fr )</td>
<td>Froude number</td>
<td>-</td>
</tr>
<tr>
<td>( g )</td>
<td>Gravitational acceleration</td>
<td>m/s(^2)</td>
</tr>
<tr>
<td>( G )</td>
<td>Gibbs Energy</td>
<td>J</td>
</tr>
<tr>
<td>( G_\varepsilon )</td>
<td>Graetz number, ( RePrD_h / L )</td>
<td>-</td>
</tr>
<tr>
<td>( h )</td>
<td>Heat transfer coefficient for low net molar flux at the interface</td>
<td>W/m(^2)K, J/kg·K</td>
</tr>
<tr>
<td>( h' )</td>
<td>Enthalpy</td>
<td></td>
</tr>
<tr>
<td>( h'_0 )</td>
<td>Heat transfer coefficient for high net molar flux at the interface</td>
<td>W/m(^2)K</td>
</tr>
<tr>
<td>( H )</td>
<td>Enthalpy</td>
<td>J/mol·K</td>
</tr>
<tr>
<td>( J )</td>
<td>Molar flux</td>
<td>mol/m(^3)/s</td>
</tr>
<tr>
<td>( k )</td>
<td>Mass transfer coefficient for low net molar flux at the interface</td>
<td>m/s</td>
</tr>
<tr>
<td>( k'_x )</td>
<td>Mass transfer coefficient for high net molar flux at the interface</td>
<td>m/s</td>
</tr>
<tr>
<td>( M )</td>
<td>Molecular Weight</td>
<td>g/mol</td>
</tr>
<tr>
<td>( \dot{N}'_A )</td>
<td>Combined molar flux</td>
<td>mol/m(^2)/s</td>
</tr>
<tr>
<td>( P )</td>
<td>Pressure</td>
<td>bar</td>
</tr>
<tr>
<td>( P_\varepsilon )</td>
<td>Parachor</td>
<td>( cm^3/dyn^{0.25} /mol·cm^{0.25} )</td>
</tr>
<tr>
<td>( \dot{Q} )</td>
<td>Heat transfer rate</td>
<td>W</td>
</tr>
<tr>
<td>( \dot{Q}'_0 )</td>
<td>Heat Flux</td>
<td>W/m(^2)</td>
</tr>
<tr>
<td>( q )</td>
<td>Vapor Quality</td>
<td>-</td>
</tr>
<tr>
<td>( Pr )</td>
<td>Prandtl number, ( \nu / \alpha )</td>
<td>-</td>
</tr>
<tr>
<td>( R )</td>
<td>Molar rate of production of a species, Universal Gas Constant</td>
<td>mol/m(^3)/s</td>
</tr>
<tr>
<td>( Re )</td>
<td>Reynolds number, ( uD_h / \nu )</td>
<td>-</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td></td>
</tr>
<tr>
<td>$Sc$</td>
<td>Schmidt number, $v/D$</td>
<td></td>
</tr>
<tr>
<td>$Sh$</td>
<td>Sherwood number, $kD_n/D$</td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature $K$</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>Time $s$</td>
<td></td>
</tr>
<tr>
<td>$u$</td>
<td>Velocity $m/s$</td>
<td></td>
</tr>
<tr>
<td>$V$</td>
<td>Molar Volume at Normal Boiling Temperature $m^3/mol$</td>
<td></td>
</tr>
<tr>
<td>$v$</td>
<td>Molar Average Velocity, Superficial Velocity $m/s$</td>
<td></td>
</tr>
<tr>
<td>$We$</td>
<td>Weber number, $\rho v^2/l/\sigma$</td>
<td></td>
</tr>
<tr>
<td>$w$</td>
<td>Mass Fraction</td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>Ammonia molar concentration in the liquid</td>
<td></td>
</tr>
<tr>
<td>$y$</td>
<td>Ammonia molar concentration in the vapor</td>
<td></td>
</tr>
<tr>
<td>$z$</td>
<td>Ratio of the ammonia flux to the net molar flux at the interface</td>
<td></td>
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**Greek Symbols**

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<thead>
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<th>Symbol</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Thermodynamic Correction Factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Activity Coefficient</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Boundary Layer Thickness, Pore Length $m$</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Surface porosity</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Viscosity $cP, Pa\cdot s$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Correction Factor for Heat/Mass Transfer Coefficients</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Thermal conductivity $W/m\cdot K$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic Viscosity, Dipole Moment debye</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity $m^2/s$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density $mol/m^3$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Surface tension $N/m$, dyn/cm</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Tortuosity</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Modification factor for the transfer coefficients</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Rate Factor, Packing Density</td>
</tr>
<tr>
<td>$\Sigma_v$</td>
<td>Sum of Atomic Diffusion Volumes $cm^3$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Correction factor for the irregularity of the fibers</td>
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**Subscripts**

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<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$0$</td>
<td>Evaluated at Surface or Interface</td>
</tr>
<tr>
<td>$\infty$</td>
<td>Evaluated at the Bulk Fluid</td>
</tr>
<tr>
<td>$A$</td>
<td>Species A</td>
</tr>
<tr>
<td>$abs$</td>
<td>Absorber</td>
</tr>
<tr>
<td>$B$</td>
<td>Species B</td>
</tr>
<tr>
<td>$bulk$</td>
<td>For the bulk vapor or liquid</td>
</tr>
<tr>
<td>$c$</td>
<td>Critical</td>
</tr>
<tr>
<td>$CF$</td>
<td>Cooling Fluid, Coupling Fluid</td>
</tr>
<tr>
<td>$CS$</td>
<td>Concentrated Solution</td>
</tr>
<tr>
<td>$cond$</td>
<td>Condenser</td>
</tr>
<tr>
<td>$des$</td>
<td>Desorber</td>
</tr>
<tr>
<td><strong>Abbreviation</strong></td>
<td><strong>Definition</strong></td>
</tr>
<tr>
<td>----------------------------------</td>
<td>-----------------------------------------</td>
</tr>
<tr>
<td>COP</td>
<td>Coefficient of Performance</td>
</tr>
<tr>
<td>EES</td>
<td>Engineering Equation Solver</td>
</tr>
<tr>
<td>LMTD</td>
<td>Log Mean Temperature Difference</td>
</tr>
<tr>
<td>RHEX</td>
<td>Refrigerant Heat Exchanger</td>
</tr>
<tr>
<td>SHEX</td>
<td>Solution Heat Exchanger</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th><strong>Abbreviation</strong></th>
<th><strong>Definition</strong></th>
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<tr>
<td><strong>Indices</strong></td>
<td></td>
</tr>
<tr>
<td>i</td>
<td>Indices for the number of segments, Component i</td>
</tr>
<tr>
<td>in</td>
<td>Inner</td>
</tr>
<tr>
<td>int</td>
<td>Interface</td>
</tr>
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<table>
<thead>
<tr>
<th><strong>Terms</strong></th>
<th><strong>Definition</strong></th>
</tr>
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<tbody>
<tr>
<td><strong>DS</strong></td>
<td>Dilute Solution</td>
</tr>
<tr>
<td><strong>G</strong></td>
<td>Gas or Vapor</td>
</tr>
<tr>
<td><strong>h</strong></td>
<td>For heat transfer, Hydraulic (Diameter)</td>
</tr>
<tr>
<td><strong>H₂O</strong></td>
<td>Water</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>Liquid</td>
</tr>
<tr>
<td><strong>loc</strong></td>
<td>Local</td>
</tr>
<tr>
<td><strong>m</strong></td>
<td>For mass transfer</td>
</tr>
<tr>
<td><strong>mem</strong></td>
<td>Membrane</td>
</tr>
<tr>
<td><strong>mix</strong></td>
<td>Mixture</td>
</tr>
<tr>
<td><strong>NH₃</strong></td>
<td>Ammonia</td>
</tr>
<tr>
<td><strong>r</strong></td>
<td>Reduced</td>
</tr>
<tr>
<td><strong>rect</strong></td>
<td>Rectifier</td>
</tr>
<tr>
<td><strong>ref</strong></td>
<td>Refrigerant</td>
</tr>
<tr>
<td><strong>sol</strong></td>
<td>Solution</td>
</tr>
<tr>
<td><strong>vap</strong></td>
<td>Vapor</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Superscripts</strong></th>
<th><strong>Definition</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X</strong></td>
<td>Flux, Per Unit Area</td>
</tr>
<tr>
<td><strong>X°</strong></td>
<td>Infinite Dilution</td>
</tr>
<tr>
<td><strong>ex</strong></td>
<td>Excess Property</td>
</tr>
<tr>
<td><strong>sat</strong></td>
<td>Saturation</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Over lines</strong></th>
<th><strong>Definition</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ẋ</strong></td>
<td>Rate</td>
</tr>
<tr>
<td><strong>X̄</strong></td>
<td>Partial Molar</td>
</tr>
<tr>
<td><strong>X̅</strong></td>
<td>Per Mole, Molar</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.1. Motivation

Miniaturized and mobile thermally activated cooling devices can be used in several applications like automobile cooling using the thermal energy in the exhaust gas, cooling the users of nuclear, biological or chemical protective clothes and cooling of medication and vaccines in the underdeveloped regions where electricity is unavailable. The temperature in the US military chemical protective clothes rises to uncomfortable levels even if the person rests 30 minutes after every 30 minutes work [1]. Therefore, these protective clothes must be cooled by a mobile cooling device for human reliability and cognitive and motor performances. Preserving the vaccines, medications and food in the underdeveloped regions is a very important problem which is addressed by many international organizations like United Nations, UNICEF, World Health Organization etc. [2].

There are different types of mobile and miniaturized cooling devices in the literature. Determan [3] compared the mass of five different cooling systems which can provide 300W cooling for 8 hours to 20°C heat sink temperature. These systems are ice packs, a fuel cell (FC) powered, a lithium-ion battery powered and a small internal combustion engine (ICE) powered vapor compression cycles (VCC) and an absorption cycle. The estimated sizes of these systems are provided in Table 1.

The total mass of the absorption cooling cycle was found to be around 6.5 kg in spite of a low coefficient of performance (COP) of 0.4. In addition, the most important
advantage of absorption cooling is that it can utilize waste heat and it does not require substantial amount of shaft work or electricity like other systems.

Table 1. Estimated weight for different mobile cooling systems. [3]

<table>
<thead>
<tr>
<th>System</th>
<th>Weight (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice Packs</td>
<td>20.7</td>
</tr>
<tr>
<td>Lithium-Ion Battery Powered VCC</td>
<td>12</td>
</tr>
<tr>
<td>Fuel Cell Powered VCC</td>
<td>7.5</td>
</tr>
<tr>
<td>ICE Powered VCC</td>
<td>7</td>
</tr>
<tr>
<td>Absorption Cooling Cycle</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Therefore, the miniaturized and mobile absorption cooling devices can be used in several applications if the specific cooling power and the reliability of these devices are increased. The effective rectification of the refrigerant is one of the most important processes in absorption cooling cycles. The rectifier should be able to separate the refrigerant at different working conditions like different desorber heat input rate or absorber coolant temperature. Therefore, miniaturized absorption cooling devices require a relatively small and effective rectification unit. In this study, a more efficient rectifying unit using ceramic hollow fiber membranes for a miniaturized absorption cooling device is designed and simulated. In Section 1.2 the existing state-of-the art in miniaturized thermally activated cooling devices is presented through a Literature Review leading to a clear potential for further research in ceramic hollow fiber membranes to improve system performance.

1.2. Literature Review

Relatively few detailed studies exist on miniaturized thermally activated cooling devices, although they may have a wide range of usage areas as explained in the previous section. The studies on the miniaturization of absorption cooling/heat pump devices are available in the literature in recent years. These studies mainly focus on the experimental testing and modeling of the heat and mass transfer in the absorber and the desorber components [4], [5]. The first one [5] presents an experimentally validated model for the heat and mass transfer in a microchannel ammonia-water
desorber. The desorber was designed for a residential heat pump system. The required heat load was 17.5 kW and the dimension of the desorber was 178 – 178 – 508 millimeters. The heat and mass transfer was modeled in a prototype absorber which has a dimension of 140 – 140 – 508 millimeters in Garimella et al. [4]. These were the implementations of microchannel technology on the residential heat pump systems. However, the units studied are still large compared to the micro scale monolithic modular absorption heat pump developed by [6], which has an overall dimensions of 200 – 200 - 34 mm including all components of the device except the connections between the components and pumps. The channels of 304 µm hydraulic diameter were opened on 0.5 mm thick stainless steel shims by photochemical etching. These shims were bonded on each other to form the heat and mass exchanger components. A single shim and the cycle components are shown in Figure 1. The thickness of a single shim was 0.5 mm and 40 of them were used to create the module. There were two different channel configuration on shim A and shim B. They were stacked on each other one by one to obtain different flow configurations. For example, the refrigerant enters and exits from the circular ports on the condenser (flows from port 1 to port 2), while the cooling fluid enters and exits from the square ports (flows from port C2 to C1). The refrigerant flows from the channels on shim A while the cooling fluid flows from the channels on shim B. Thus, two fluids flow in a counter flow arrangement and they are not mixed. On the other hand, the absorber has a more complex design since the refrigerant is absorbed into the dilute solution and they are cooled by an external fluid. The dilute solution enters the absorber from the left square port (number 3). The refrigerant enters from the left bottom circular port (number 4). They are mixed in the channels on shim A and exit from the square port on the right hand side (number 5). The mixture is cooled by the cooling fluid which enters the absorber from the right bottom circular port (port A2) and exits from the left top circular port (port A1). The concentrated solution is heated by cylindrical heaters which are placed into the eight holes on the desorber. The solution flows upward from the four microchannels around the holes on the desorber which are shown with red arrows. The refrigerant vapor enters the rectifier from the
bottom and exits from the circular port on the right top of the rectifier (port 6). The rectifier is cooled by concentrated solution from the circular ports just on the right of the rectifier (enters from port R1, flows through the green line and exits from port R2).

Figure 1. Photograph of a shim which is used to manufacture the monolithic miniaturized absorption cooling device in Determan’s thesis (retrieved from [3]). (1: Condenser Inlet; 2: Condenser Outlet; C1: Condenser Coolant Outlet; C2: Condenser Coolant Inlet; 3: Dilute Solution Inlet; 4: Refrigerant Inlet; 5: Concentrated Solution Outlet; A1: Absorber Coolant Outlet; A2: Absorber Coolant Inlet; Red Arrows:
Refrigerant Vapor from Desorber to Rectifier; 6: Rectifier Vapor Outlet; R1: Rectifier Coolant Inlet; R2: Rectifier Coolant Outlet)

The device had a relatively low COP between 0.25 and 0.45 depending on the absorber coolant temperature and desorber heat input rate, because the rectifier could not sufficiently purify the ammonia vapor. Therefore, a new design of the rectifier is required to increase the quality of the refrigerant vapor at different desorber heat input rates. The original design uses a very simple tray column as shown in Figure 1. The performance of the rectifier can be enhanced by increasing the interfacial area between the reflux liquid and the refrigerant vapor and/or by increasing the heat and mass transfer coefficients by using different column internals, such as random/structured packing or membranes.

1.2.1. Absorption Cooling

Absorption cooling technologies can be used in various applications such as solar refrigeration and waste heat utilization in combined cooling, heating and power systems [7]–[10]. The performance of the absorption cooling cycle can be enhanced by different modifications like multi-stage and multi-effect cycles, addition of a generator-absorber heat exchanger, and cascade or hybrid cycles [7], [11]. In Figure 2 a simple absorption refrigeration cycle is shown. It is composed of 4 components which are condenser, evaporator, absorber and generator (or desorber). The condenser and the desorber are at high pressure while the evaporator and the absorber are at low pressure. The evaporator has the lowest temperature in the cycle. The refrigerant is separated from the solution by external heating in the desorber. The refrigerant vapor leaving the evaporator is absorbed into the dilute solution in the absorber. Cooling of the absorber is required since the absorption is an exothermic process.

Although several working fluid couples are suggested in the literature [12], the most common and mature pairs are ammonia-water and water-LiBr. The main advantage of the former pair is that it can be used for low temperature applications since
ammonia is the refrigerant and water is the absorbent. The latter pair cannot perform cooling applications below the freezing point of the water, because water is the refrigerant and LiBr is the absorbent in this case. However, the ammonia-water absorption cooling cycles require a rectifier to sufficiently purify the ammonia, because some amount of water also evaporates in the generator. This is not required in water-LiBr cycles since the absorbent LiBr is not volatile. If the refrigerant vapor cannot be sufficiently purified in ammonia-water absorption cooling devices, the water tends to accumulate in the evaporator. As a result the evaporator temperature rises with the increased water content in the refrigerant stream and the efficiency of the whole system deteriorates. Thus, the ammonia rectification process is one of the most important issues in these devices [13].

**Figure 2.** Schematic of a simple, single effect absorption cooling cycle.

Different modifications of the cycle can be made to increase the performance of the absorption cycle. The high temperature dilute solution which leaves the desorber can be used to heat the rich solution which leaves the absorber. In this way, less heat input is required at the desorber and the heat rejection to the environment from the absorber is reduced so that the COP of the system is improved and the irreversibility
is reduced. It is suggested that the performance of the overall system can be increased by up to 60% when a refrigerant heat exchanger (RHEX) is used [7]. Similarly, a solution heat exchanger (SHEX) can be used as condensate pre-cooler to increase the cooling capacity of the evaporator. In multi-effect absorption refrigeration cycles, there are multiple desorbers (generators) and absorbers. The heat of absorption from the second absorber is used as the heat source for the first desorber. Absorption cycle can be designed with desorber/absorber heat exchanger or absorber-heat recovery. There are also half-effect absorption, combined vapor absorption-compression, sorption-resorption, dual, combined ejector-absorption, osmotic-membrane absorption cycles etc. These numerous different cycle designs are not be discussed in this thesis.

Different distillation column configurations for ammonia-water absorption cooling systems are studied in the literature [14]. Performances of stripping and rectifying sections are compared. The effect of complete or partial condensation for the reflux liquid or using solution cooled or water cooled rectifiers are studied. The results showed that the water cooled rectifier should be avoided since the system COP decreases. Complete condensation can be used in some cases because the column design becomes simpler, but the COP becomes lower. The rectifier can be removed when the performance does not decrease significantly without a rectifier. The reason is that the efficiency of the stripping section was found to be higher than the efficiency of the rectifying section. [14]

A solution heat exchanger, a refrigerant heat exchanger, a solution cooled rectifier were also used in Determan’s study as shown in Figure 1. Figure 3 is the schematic of the cycle used in the same study [3].
In this study, slightly modified version of the cycle shown in Figure 3 is modeled using the same operating conditions. A new rectifier design is implemented into the model to increase the performance of the system and to investigate the applicability of the ceramic membranes, explained in the next section, to the rectifier of a micro absorption cooling system. The typical COP values for ammonia-water absorption cooling systems are provided in Table 2.

**Figure 3.** Diagram of the absorption cycle used in Determan’s study [3].

**Table 2.** Typical COP values of different ammonia-water absorption cooling systems [7].

<table>
<thead>
<tr>
<th>System</th>
<th>Heat Source Temperature (°C)</th>
<th>COP</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Effect</td>
<td>120-150</td>
<td>0.5</td>
<td>Commercial</td>
</tr>
<tr>
<td>Double Effect</td>
<td>120-150</td>
<td>0.8-1.2</td>
<td>Experimental</td>
</tr>
<tr>
<td>Half Effect</td>
<td>Low</td>
<td>0.2-0.3</td>
<td>Numerical</td>
</tr>
<tr>
<td>With Absorber-Heat Recovery</td>
<td>90-180</td>
<td>0.5-0.7</td>
<td>Numerical</td>
</tr>
</tbody>
</table>
1.2.2. Hollow Fiber Membranes

Membranes have certain characteristics such as high and well-defined interfacial area per unit volume that may make them appropriate for miniaturized absorption devices, but their usage in this application has not been thoroughly investigated. Although the membrane itself causes heat and mass transfer resistance, the specific interfacial area can be 5 to 20 times higher than random packing. The typical values of interfacial area per unit volume (m²/m³) for different contactor types are as follows [15]:

- Free dispersion columns: 3-30
- Packed/trayed columns: 30-350
- Mechanically agitated columns: 150-500
- Membrane modules: 1500 - 6000

In addition to the heat and mass transfer resistances because of the membrane, another disadvantage is that the heat and mass transfer coefficients depend on the packing of the fibers, which can be highly irregular because of the large number of fibers [15], [16]. Although these irregularities make the modeling heat and mass transfer more difficult, there are studies for the estimation of the irregularity effects on heat and mass transfer coefficients [17].

Hollow fiber membrane modules have been used in absorption cooling devices. For example, a novel absorber design using polymeric hollow fiber membranes is proposed for an ammonia-water absorption cooling device in the literature [18]. In that design, the vapor flowed through the lumen side of the porous fibers and the absorbent solution flowed through the shell side. In the absorbers, a cooling fluid is also required. Therefore, nonporous fibers were also included in the module so that the cooling water flowed through the lumen side of the nonporous fibers. The performance of the hollow fiber heat and mass exchanger was compared with a plate heat exchanger falling film type absorber. The new design was found to be promising because it increased the system performance significantly. However, the
application of a polymeric membrane to the rectifier of an absorption cooling system is not feasible, because the polymeric material cannot withstand the high temperatures. Ceramic membranes offer an interesting alternative, because they are stable chemically and thermally at the high operating temperature of the rectifier. The ceramic membranes are chemically and thermally stable up to 250°C. They can be made hydrophobic by making certain surface modifications [19] to ensure a non-wetted mode of operation, which is crucial for membrane contactors. In membrane modules, the porous membrane allows heat and mass transfer between the vapor and the reflux liquid. The reflux liquid flows downward through the lumen side, and the vapor flows upward through the shell side. A typical flow configuration of a distillation column with hollow fiber membranes is shown in Figure 4. The section below the feed stream is called the stripping section. The feed stream is below the rectifying section. Only the reflux liquid flows through the lumen side in the rectifying section.

![Figure 4. A distillation column with hollow fiber membranes.](image)
Simultaneous heat and mass transfer is observed through the membranes in a membrane contactor. The operation principle is shown in Figure 5. The liquid and the vapor phases are separated from each other by the membrane wall. The pores can be filled by the liquid or vapor phases depending on the hydrophobicity of the membrane and the pressure difference between the phases. However, hydrophobic membranes are modeled in this study; so the pores are assumed to be filled by vapor. The direction of the mass diffusion can be either from liquid to vapor side or from vapor to liquid side. Different species can diffuse to the same direction or opposite directions depending on the boundary conditions. In case of the distillation of ammonia-water mixture; ammonia molecules diffuse from liquid to vapor while water molecules diffuse from vapor to liquid side so that the ammonia concentration is increased in the vapor phase.

**Figure 5. Illustration of the membrane wall.**

### 1.3. Objective and Scope

The main objective of this study is to design a more efficient rectifier for a miniaturized absorption cooling device. After the literature review, ceramic hollow fiber membranes distinguish as a promising alternative because of its high interfacial
area between the phases and its thermal and chemical stability. The objectives and the scope of this study can be explained briefly in three categories.

- The hollow fiber membranes are used in the rectifier of an absorption cooling system first time in this study. However, random packing is widely used as the internal structure of the rectifiers. Therefore, the first objective of this study is to compare the performance of the random packing and the hollow fibers for ammonia rectification process in an absorption cooling system. In Chapter 2, the mathematical model for the simultaneous heat and mass transfer in a rectifier with different internal structures is described. The results are shown and compared with the literature in Chapter 3 to verify the mathematical model described in Chapter 2. The operating conditions like the pressures, temperatures, concentrations and flow rates are taken from literature so that the accuracy of the mathematical model can be verified.

- After comparing the performances of hollow fibers and random packing, the hollow fiber rectifier and the stripper models are integrated into the absorption cycle model to investigate the effect of new design to the overall system. The second objective of this study is to show that the COP of the absorption cooling system in Figure 1 is higher when hollow fibers are used to purify the ammonia. The mathematical model of the cycle is described in Chapter 2 and the results are shown in Chapter 3.

- The performance of the miniaturized absorption cycle shown in Figure 1 deteriorates considerably when the operating conditions are changed from its design point. The reason is that the rectifier is unable to purify the ammonia under different working conditions. The last objective of this study is to test the new design under different operating conditions and to prove that the new rectifier is able to purify the ammonia sufficiently for different operating conditions like different desorber heat input rate, absorber coolant
temperature etc.. Chapter 3 ends by showing the parametric results for the simulation of the absorption cycle.

The description of the mathematical model in Chapter 2 and the discussions of the results in Chapter 3 are followed by the summary, conclusions and the future work in Chapter 4.
CHAPTER 2

MATHEMATICAL MODELING

In this chapter, the modeling of the rectifier and the full cycle are discussed. The modeling of the rectifier consists of three parts which are heat and mass transfer formulation, thermodynamic property models for ammonia-water mixtures and the governing equations like the heat and mass fluxes, the mass and energy balances and the correlations for the heat and mass transfer coefficients. After modeling of the rectifier is discussed in Section 2.1, the implementation of the rectifier model to the absorption cycle modeling is presented in Section 2.2.

2.1. Modeling of the Rectifier

The modeling of the rectifier for different internal structures like random packing and hollow fiber membranes is a simultaneous heat and mass transfer problem between the liquid and the vapor phases. Equi-molar counter diffusion may be assumed for many binary diffusion problems, but there is a net molar flux between the phases for ammonia-water separation. The reason is that the molar latent heat of vaporization of ammonia and water are quite different. The boundary layers are distorted when there is a net molar flux across the boundary. As a result the heat and mass transfer coefficients should be modified according to the net molar flux at the boundary.

2.1.1. Heat and Mass Transfer Formulation

The concentration and temperature profiles at the vapor-liquid interface are shown in Figure 6. There is simultaneous heat and mass transfer between these phases. The direction of the heat and mass transfer depends on the temperatures and ammonia
concentrations of each phase, respectively. The direction of the molar flux of ammonia and water can be in opposite directions. If the net molar flux at the interface is larger than zero, then the shape of the boundary layers are distorted at the interface. Therefore, the effect of the net molar flux should be considered since the shape of the concentration and temperature profiles determines the heat and mass transfer coefficients. Figure 7 shows the boundary layer thicknesses and important temperatures and concentrations which are used in the heat and mass transfer formulation.

Figure 6. The schematic of the concentration and temperature profiles at the vapor-liquid interface.
**Figure 7.** The schematic of the concentration and temperature profiles at the vapor-liquid interface with interface temperature and concentration, boundary layer thicknesses and bulk temperature and concentration.

Equation of continuity for a multicomponent mixture can be written as follows [20]:

\[
\frac{\partial c_A}{\partial t} = -(\nabla \cdot \dot{N}_A^s) + R_A \quad \text{where } A = 1, 2, 3, \ldots, N \quad (1)
\]

\[
\frac{\partial c_A}{\partial t} = -(\nabla \cdot c_A v) - (\nabla \cdot J_A) + R_A \quad \text{where } A = 1, 2, 3, \ldots, N \quad (2)
\]

The first term in the left hand side of Eq. (1) is the rate of increase of moles of species per unit volume. This term becomes zero when for a steady state problem. The first term in the right hand side of (1) is the total molar flux of species per unit volume by convection and diffusion. The first term in the right hand side of Eq. (2) is the net rate of molar flux of species a per unit volume by convection. The second term in the right hand side of Eq. (2) is the net rate of molar flux of species a per unit volume by diffusion. The last term of Eq. (2) is the rate of production of moles per unit volume by homogenous reaction. The last term vanishes when there is no chemical reaction in the problem.
For steady, one dimensional transport, without any chemical reactions, the Eq. (1) becomes:

\[
\frac{dN^*_A}{dy} = 0
\]  

(3)

Therefore, the molar flux is constant throughout the boundary layer. The Fick’s law for binary diffusion is:

\[
J_* = -cD_{AB} \nabla x_A
\]  

(4)

The total molar flux can be obtained by adding the convective mass flux to the molecular mass flux which is defined by Fick’s law. Therefore, the combined molar flux can be written as:

\[
\dot{N}_A^* = x_A (\dot{N}_A^* + \dot{N}_B^*) - cD_{AB} \nabla x_A
\]  

(5)

The molar flux is constant throughout the boundary layer according to Eq. (3). For one dimensional transport, the molar flux at the boundary can be written as:

\[
\dot{N}_{A0}^* = x_A (\dot{N}_{A0}^* + \dot{N}_{B0}^*) - cD_{AB} \frac{dx_A}{dy}
\]  

(6)

Modifying Eq. (6), the following equation is obtained.

\[
\dot{N}_{A0}^* - x_{A0} (\dot{N}_{A0}^* + \dot{N}_{B0}^*) = (x_A - x_{A0}) (\dot{N}_{A0}^* + \dot{N}_{B0}^*) - cD_{AB} \frac{dx_A}{dy}
\]  

(7)

\[
cD_{AB} \frac{dX}{dy} = X (\dot{N}_{A0}^* + \dot{N}_{B0}^*) - (\dot{N}_{A0}^* - x_{A0} (\dot{N}_{A0}^* + \dot{N}_{B0}^*)
\]  

(8)

Let \( X = x_A - x_{A0} \). Integrating from the boundary \((y = 0 \& x = x_{A0})\) along the concentration boundary layer \((y = \delta_x \& x = x_{A0})\):
\[
\frac{\delta_x}{cD_{AB}} (\hat{N}_{A0}^* + \hat{N}_{B0}^*) = \ln \left( \frac{X_x - \hat{N}_{A0}^* x_{A0}(\hat{N}_{A0}^* + \hat{N}_{B0}^*)}{\hat{N}_{A0}^* + \hat{N}_{B0}^*} \right)
\]

(9)

\[
\exp \left( \frac{\delta_x}{cD_{AB}} (\hat{N}_{A0}^* + \hat{N}_{B0}^*) \right) = \frac{(\hat{N}_{A0}^* + \hat{N}_{B0}^*)(x_{A0} - x_{AV})}{\hat{N}_{A0}^* - x_{A0}(\hat{N}_{A0}^* + \hat{N}_{B0}^*)} + 1
\]

(10)

Eq. (6) can be written at the boundary (where \( y = 0 \)) in terms of mass transfer coefficient at high net mass flux.

\[
\hat{N}_{A0}^* = x_{A0}(\hat{N}_{A0}^* + \hat{N}_{B0}^*) - k_{s,loc}^* \Delta x_A
\]

(11)

where \( k_{s,loc}^* \) is the mass transfer coefficient at high net mass transfer. The mass transfer coefficient at low net mass transfer is defined as:

\[
k_{s,loc} = \lim_{\hat{N}_{A0}^*, \hat{N}_{B0}^* \to 0} k_{s,loc}^*
\]

(12)

Combining Eqs. (10) and (11),

\[
\exp \left( \frac{\delta_x}{cD_{AB}} (\hat{N}_{A0}^* + \hat{N}_{B0}^*) \right) = \frac{(\hat{N}_{A0}^* + \hat{N}_{B0}^*)}{k_{s,loc}^*} + 1
\]

(13)

Expanding Eq. (13) in Taylor series with respect to molar fluxes at \( (\hat{N}_{A0}^* + \hat{N}_{B0}^*) = 0 \), the definition of the limiting value of the mass transfer coefficient can be obtained.

\[
\exp \left( \frac{\delta_x}{cD_{AB}} 0 \right) + \frac{\delta_x}{cD_{AB}} \exp \left( \frac{\delta_x}{cD_{AB}} 0 \right) (\hat{N}_{A0}^* + \hat{N}_{B0}^* - 0)
\]

\[
= \frac{0}{k_{s,loc}^*} + \frac{1}{k_{s,loc}^*} (\hat{N}_{A0}^* + \hat{N}_{B0}^* - 0)
\]

(14)

\[
1 + \frac{\delta_x}{cD_{AB}} (\hat{N}_{A0}^* + \hat{N}_{B0}^*) = 1 + \frac{1}{k_{s,loc}^*} (\hat{N}_{A0}^* + \hat{N}_{B0}^*)
\]

(15)
Therefore, the local mass transfer coefficient is defined as:

\[
\frac{1}{k_{s,loc}} = \frac{\delta_i}{cD_{AB}} \quad (16)
\]

Substituting Eq. (16) into Eq. (10), the following equation is obtained.

\[
\exp \left( \frac{\dot{N}_{A0}^* + \dot{N}_{B0}^*}{k_{s,loc}} \right) = \frac{(\dot{N}_{A0}^* + \dot{N}_{B0}^*)(x_{A0} - x_{Ae})}{\dot{N}_{A0}^* - x_{A0}(\dot{N}_{A0}^* + \dot{N}_{B0}^*)} + 1 \quad (17)
\]

Dividing the numerator and the denominator of the right hand side of Eq. (17) by \((\dot{N}_{A0}^* + \dot{N}_{B0}^*)\).

\[
\exp \left( \frac{\dot{N}_{A0}^* + \dot{N}_{B0}^*}{k_{s,loc}} \right) = \frac{(x_{A0} - x_{Ae})}{\dot{N}_{A0}^* - x_{A0}(\dot{N}_{A0}^* + \dot{N}_{B0}^*)} + 1 \quad (18)
\]

\[
\exp \left( \frac{\dot{N}_{A0}^* + \dot{N}_{B0}^*}{k_{s,loc}} \right) = \frac{\dot{N}_{A0}^*}{(\dot{N}_{A0}^* + \dot{N}_{B0}^*)} - x_{Ae} \quad (19)
\]

\[
\frac{\dot{N}_{A0}^* + \dot{N}_{B0}^*}{k_{s,loc}} = \ln \left( \frac{z - x_{Ae}}{z - x_{A0}} \right) \quad (20)
\]

\[
1 = \frac{1}{\dot{N}_{A0}^* + \dot{N}_{B0}^*} k_{s,loc} \ln \left( \frac{z - x_{Ae}}{z - x_{A0}} \right) \quad (21)
\]

where \(z\) is defined as:

\[
z = \frac{\dot{N}_{A0}^*}{(\dot{N}_{A0}^* + \dot{N}_{B0}^*)} \quad (22)
\]

Finally, the molar flux of species A across the boundary can be obtained by using the following equation.
\[ \dot{N}^*_{A0} = k_{x,loc} z \ln \left( \frac{z - x_{Ac}}{z - x_{A0}} \right) \]

(23)

Similarly, the molar flux of species B can be found by the following equation.

\[ \dot{N}^*_{B0} = k_{x,loc} (1 - z) \ln \left( \frac{z - x_{Ac}}{z - x_{A0}} \right) \]

(24)

Finally, the relationship between the mass transfer coefficients with low and high net mass transfer at the boundary can be obtained by substituting Eq. (16) into Eq. (13).

\[ \exp \left( \frac{(\dot{N}^*_{A0} + \dot{N}^*_{B0})}{k_{x,loc}} \right) = \frac{(\dot{N}^*_{A0} + \dot{N}^*_{B0})}{k^*_{x,loc}} + 1 \]

(25)

The rate factor for the mass transfer is defined in Eq. (26).

\[ \phi_m = \frac{\dot{N}^*_{A0} + \dot{N}^*_{B0}}{k_{x,loc}} \]

(26)

Substituting the rate factor defined in Eq. (26) into Eq. (25), the following equations can be obtained.

\[ \exp(\phi_m) = \phi_m \frac{k_{x,loc}}{k^*_{x,loc}} + 1 \]

(27)

\[ \frac{k^*_{x,loc}}{k_{x,loc}} = \theta_m = \frac{\phi_m}{e^{\phi_m} - 1} \]

(28)

The total energy transport across the boundary consists of two terms. The first term is the energy transport associated with the mass flux at the boundary per unit area. It is calculated by using the molar flux and partial molar enthalpies of each species. The second term is the energy transport by conduction per unit area. The energy flux across the boundary can be written as:
Performing similar mathematical operations, the modified heat transfer rate for the film model can be obtained. The partial molar enthalpy is equal to the molar enthalpy of the pure substance if the mixture is ideal. Assuming ideal mixture, the partial molar enthalpies $\overline{H}_A$ and $\overline{H}_B$ can be replaced by $\tilde{C}_{pA}(T-T^o)$ and $\tilde{C}_{pB}(T-T^o)$, respectively where $T^o$ is an arbitrary temperature. The arbitrary temperature can be chosen as the interface temperature $T_0$

$$\dot{q}_0^* = (N_{A0}^* \tilde{C}_{pA} + N_{B0}^* \tilde{C}_{pB})(T-T_0) - \lambda \frac{dT}{dy}$$  \hspace{1cm} (30)$$

Integrating from the boundary $(y = 0 & T = T'_0)$ along the thermal boundary layer $(y = \delta & T = T_x)$:

$$\lambda \frac{dT}{dy} = (N_{A0}^* \tilde{C}_{pA} + N_{B0}^* \tilde{C}_{pB})(T-T_0) - \dot{q}_0^*$$  \hspace{1cm} (31)$$

$$\frac{dT}{(N_{A0}^* \tilde{C}_{pA} + N_{B0}^* \tilde{C}_{pB})(T-T_0) - \dot{q}_0^*} = \frac{dy}{\lambda}$$  \hspace{1cm} (32)$$

$$\frac{1}{(N_{A0}^* \tilde{C}_{pA} + N_{B0}^* \tilde{C}_{pB})} \ln \left( \frac{(N_{A0}^* \tilde{C}_{pA} + N_{B0}^* \tilde{C}_{pB})(T_x-T_0) - \dot{q}_0^*}{(N_{A0}^* \tilde{C}_{pA} + N_{B0}^* \tilde{C}_{pB})(T_0-T_0) - \dot{q}_0^*} \right) = \delta \frac{1}{\lambda}$$  \hspace{1cm} (33)$$

The rate factor for the heat transfer is defined in Eq. (34).

$$\phi_h = \frac{N_{A0}^* \tilde{C}_{pA} + N_{B0}^* \tilde{C}_{pB}}{\lambda / \delta} = \frac{N_{A0}^* \tilde{C}_{pA} + N_{B0}^* \tilde{C}_{pB}}{h_{loc}}$$  \hspace{1cm} (34)$$

Performing similar mathematical operations like Eqs. (11)-(15) with the limiting case for heat transfer coefficients, the definition of the heat transfer coefficient can be obtained.
The limiting case for these operations is as follow:

\[
h_{loc} = \lim_{N_A^0, C_{pA} \to 0} h_{loc}^* \quad \text{(36)}
\]

Therefore, Eq. (33) becomes:

\[
\ln \left( \frac{(N_A^* C_{pA} + N_B^* C_{pB})(T_\infty - T_0) - \dot{q}_0^*}{0 - \dot{q}_0^*} \right) = \phi_h 
\]

\[
\frac{(N_A^* C_{pA} + N_B^* C_{pB})}{h_{loc}} h_{loc} (T_\infty - T_0) = \dot{q}_0^* \left( 1 - e^{\phi_h} \right) 
\]

\[
\dot{q}_0^* = \frac{\phi_h}{1 - e^{\phi_h}} h_{loc} (T_\infty - T_0) 
\]

Similar to Eq. (28), the relationship between heat transfer coefficients with low and high net mass transfer at the boundary obtained by Eq. (40)

\[
\frac{h_{loc}^*}{h_{loc}} = \theta_h = \frac{\phi_h}{1 - e^{\phi_h}} 
\]

The heat and mass transfer equations based on the stagnant film theory have been derived from the continuity and energy equations. Since there is a net mass flux at the boundary, the boundary layers are distorted. Therefore, the heat and mass flux across the boundary depend on the net molar flux at the boundary. The distortion of the boundary layer profile can be estimated with this model, but the model is unable to predict the change of the boundary layer thickness. Although there are other models to predict the effect of net mass flow rate like the penetration model and the flat-plate boundary layer model; they are not discussed here. In fact, the results show that the rate factors \( \phi_m \) and \( \phi_h \) are close to zero and the correction factors \( \theta_m \) and \( \theta_h \) are close to unity. The effect of the molar flux at the boundary is found to be
negligible in the next chapter. When the rate factor is very small and the correction factor is close to unity, the other models give a very close result to stagnant film model. The mass flux can be calculated with Eqs. (22)-(24). The heat flux can be calculated using Eq. (39) and molar fluxes at the boundary. The implementation of these equations into the modeling of the rectifier is explained in Section 2.1.3.

2.1.2. Property Calculations

The thermodynamic properties of ammonia-water mixture are taken from the Engineering Equation Solver (EES) software which is based on the mixture equation of state described in Ibrahim and Klein’s study [21]. Three independent properties are required to fix the state of a binary mixture. Temperature, pressure, ammonia mass fraction, specific enthalpy, specific internal energy, specific entropy, specific volume and quality can be calculated using three of these properties with this software.

2.1.2.1. Viscosity

Liquid Viscosity:

The liquid viscosity is taken from a report in which the viscosity equation is based on experimental results [22]. EES software also uses this correlation to calculate the viscosity of the liquid ammonia-water mixtures.

\[
\eta_{\text{mix}} = x \ln \left( \eta_{\text{NH}_3}^{\text{sat}} \right) + (1-x) \ln \left( \eta_{\text{H}_2\text{O}}^{\text{sat}} \right) + \Delta \eta_{\text{ref-x}}
\]  

\[
\Delta \eta_{\text{ref-x}} = \left( 0.534 - 0.815 \frac{T_{\text{ref}}}{T_{c,\text{H}_2\text{O}}} \right) F(x)
\]

\[
F(x) = 6.38 \left( 1-x \right)^{1.125x} \left( 1 - \exp \left( -0.585x \left( 1-x \right)^{0.18} \right) \right) \ln \left( \eta_{\text{NH}_3}^{\text{sat}} \eta_{\text{H}_2\text{O}}^{0.5} \eta_{\text{NH}_3}^{0.5} \right)
\]

where \( \eta_{\text{NH}_3}^{\text{sat}} \) and \( \eta_{\text{H}_2\text{O}}^{\text{sat}} \) are the viscosities of saturated ammonia and saturated water at the solution temperature, respectively. \( T_{c,\text{H}_2\text{O}} \) is the critical temperature of water.
Vapor Viscosity:

The method of Reichenberg [23], [24], [25] (as cited in [26]) is used to calculate the vapor viscosity of ammonia-water mixture [26]. The mixture viscosity is calculated by Eq. (44).

\[
\eta_{\text{mix}} = K_1 \left( 1 + H_{12}^2 K_2^2 \right) + K_2 \left( 1 + 2H_{12} K_1 + H_{12}^2 K_1^2 \right) \tag{44}
\]

\[
K_1 = \frac{y_i \eta_i}{y_1 + \eta_i \left[ y_2 H_{12} \left( 3 + \left( 2M_2/M_1 \right) \right) \right]}
\]

\[
K_2 = \frac{y_2 \eta_2}{y_2 + \eta_2 \left[ y_1 H_{12} \left( 3 + \left( 2M_1/M_2 \right) \right) \right]}
\]

where \( y \) is the molar concentration of ammonia; \( \eta_i \) is the pure component viscosity and \( M \) is the molecular weight of mixture components.

\[
U_i = \left[ 1 + 0.36 T_n (T_i - 1) \right]^{1/6} \frac{T_i^{3.5} \left( 10 \mu_n \right)^2}{T_n^{3.5} \left[ 1 + \left( 10 \mu_n \right)^2 \right]}
\]

where \( T_n \) and \( \mu_n \) are the reduced temperature and reduced dipole moment of each component, respectively.

\[
T_n = \frac{T}{T_{ci}}
\]

\[
\mu_n = 52.46 \frac{\mu^2 P_{ci}}{T_{ci}^2}
\]

where \( T_{ci} \) and \( P_{ci} \) are the critical temperatures and pressures, respectively. The units of \( \mu \), \( T \) and \( P \) are debye, bar and kelvin, respectively. Finally,

\[
T_{r12} = \frac{T}{(T_r T_c)^{1/2}} \text{ and } \mu_{r12} = \left( \mu_r \mu_c \right)^{1/2}
\]

25
\[ H_{12} = \left( \frac{M_1 M_2 / 32}{(M_1 + M_2)^{3/2}} \right)^{1/2} \left[ 1 + 0.36T_{r12} (T_{r12} - 1)^{1/6} \right] (C_1 + C_2)^2 \frac{T_{r12}^{3.5}}{T_{r12}^{3.5} + (10 \mu_{r12})^7} \]  

(50)

\[ C_i = \frac{M_i^{1/4}}{(\eta U_i)^{1/2}} \]  

(51)

2.1.2.2. Thermal Conductivity

**Liquid Thermal Conductivity:**

The Filippov Equation [27], [28] (as cited in [26]) is used to calculate the liquid thermal conductivity of the mixture.

\[ \lambda_{\text{mix}} = w_1 \lambda_1 + w_2 \lambda_2 - 0.72 w_1 w_2 |\lambda_2 - \lambda_1| \]  

(52)

where \( w_1 \) and \( w_2 \) are the mass fractions and \( \lambda_1 \) and \( \lambda_1 \) are the pure component thermal conductivities of component 1 and 2. Although this is a very simple method and not suitable for multicomponent mixtures, it is very accurate for binary liquid mixtures [29].

**Vapor Thermal Conductivity:**

The Wassiljewa Equation [30] (as cited in [26]) is used with Mason and Saxena Modification [31] (as cited in [26]) to obtain the thermal conductivity of the vapor mixture.

\[ \lambda_{\text{mix}} = \sum_{i=1}^{2} \frac{y_i \lambda_i}{\sum_{j=1}^{2} y_j A_{ij}} \]  

(53)

where \( y_i \) is the mole fraction and \( \lambda_i \) is the viscosity of component \( i. \) \( A_{ij} \) is defined as follows.
\[ A_y = \left[ 1 + \left( \frac{\lambda_{tr}}{\lambda_{tr'}} \right)^{1/2} \left( \frac{M_i}{M_j} \right)^{1/4} \right] \left[ \frac{8 \left( 1 + \frac{M_i}{M_j} \right)^{1/2}}{1} \right]^{1/2} \] (54)

where \( \lambda_{tr} \) is the monatomic value of the thermal conductivity. \( M_i \) is the molecular weight of component \( i \).

\[ \frac{\lambda_{tr}}{\lambda_{tr'}} = \frac{\lambda_i}{\lambda_j} \left( \frac{M_i}{M_j} \right) \] (55)

where \( \lambda_i \) is the thermal conductivity of component \( i \).

2.1.2.3. Diffusion Coefficients

Liquid Diffusion Coefficient:

The method of Tyn and Calus [32], [33] (as cited in [26]) is used to obtain the liquid diffusion coefficients for infinite dilution.

\[ D_{AB}^\circ = 8.93 \times 10^{-8} \left( \frac{V_A}{V_B^2} \right)^{1/6} \left( \frac{P_B}{P_A} \right)^{0.6} \frac{T}{\eta_B} \] (56)

where \( V_A \) and \( V_B \) are the molar volumes of the solute and the solvent at the normal boiling temperature in \( \text{m}^3/\text{mol} \). \( P_A \) and \( P_B \) are parachors for the solute and solvent. \( T \) is the temperature in K and \( \eta_B \) is the solvent viscosity in cP. The parachor is a quantity based on the liquid surface tension.

\[ P = V \sigma^{1/4} \] (57)

where \( \sigma \) is the surface tension in dyn/cm and \( V \) is the molar volume in \( \text{cm}^3/\text{mol} \) at temperature, \( T \).
The binary diffusion coefficient for the infinite dilution of component A is calculated by Eq. (56). The binary diffusion coefficient for the infinite dilution of component B, \( D_{BA} \), can be calculated by switching the subscripts A and B. Finally, the binary diffusion coefficient for concentrated solutions can be calculated as follows.

\[
D_{AB} = \left( D_{AB}^0 x_B + D_{BA}^0 x_A \right) \alpha
\]  

(58)

where \( x_A \) and \( x_B \) are the mole fractions of component A and B. The thermodynamic correction factor, \( \alpha \), is defined in terms of the activity of components.

\[
\alpha = \left[ \frac{\left( \partial \ln a_A \right)}{\left( \partial \ln x_A \right)} \right]_{T,P}
\]  

(59)

where \( a_A \) is the activity which is defined as \( a_A = x_A \gamma_A \). \( \gamma_A \) is the activity coefficient which is defined as:

\[
RT \ln \gamma_A = \left[ \frac{\partial N \tilde{G}^{ex}}{\partial N_A} \right]_{T,P}
\]  

(60)

where \( \tilde{G}^{ex} \) is the molar excess Gibbs energy of the mixture. \( N \) is the total number of moles. For binary mixtures, \( N \) is equal to \( N_A + N_B \). The excess Gibbs energy is calculated by the function below [34].

\[
\tilde{G}^{ex} = \left( e_1 + e_2 P + (e_3 + e_4 P)T + \frac{e_5}{T} + \frac{e_6}{T^2} \right) \\
+ \left( 2x-1 \right) \left( e_7 + e_8 P + (e_9 + e_{10} P)T + \frac{e_{11}}{T} + 3 \frac{e_{12}}{T^2} \right) \\
+ \left( 2x-1 \right)^2 \left( e_{13} + e_{14} P + \frac{e_{15}}{T} + 3 \frac{e_{16}}{T^2} \right)
\]  

(61)

The constants used in Eq. (61) are listed in Table 3.
Table 3. The constants used in Eq. (61) [34].

<p>| | | | | | | | |</p>
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<tr>
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</thead>
<tbody>
<tr>
<td>$e_1$</td>
<td>$-4.626129 \times 10^4$</td>
<td>$e_9$</td>
<td>$-1.475383$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_2$</td>
<td>$2.060225 \times 10^{-2}$</td>
<td>$e_{10}$</td>
<td>$-5.038107 \times 10^{-3}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_3$</td>
<td>$7.292369$</td>
<td>$e_{11}$</td>
<td>$-9.640398 \times 10^{-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_4$</td>
<td>$-1.032613 \times 10^{-2}$</td>
<td>$e_{12}$</td>
<td>$1.226973 \times 10^{2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_5$</td>
<td>$8.074824 \times 10$</td>
<td>$e_{13}$</td>
<td>$-7.582637$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_6$</td>
<td>$-8.461214 \times 10$</td>
<td>$e_{14}$</td>
<td>$6.012445 \times 10^{-4}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_7$</td>
<td>$2.452882 \times 10$</td>
<td>$e_{15}$</td>
<td>$5.487018 \times 10$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_8$</td>
<td>$9.598767 \times 10^{-3}$</td>
<td>$e_{16}$</td>
<td>$-7.667596 \times 10$</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

The thermodynamic correction factor, $\alpha$, is unity for the infinite dilution of each component and below unity for other concentration values. In this study, although it depends on the temperature, pressure and concentration, the value of the thermodynamic factor is close to unity.

**Vapor Diffusion Coefficient:**

The Equation in Fuller, et al. [35], [36] (as cited in [26]) is used to calculate the binary diffusion coefficient.

$$D_{AB} = \frac{0.00143 T^{1.75}}{P M_{AB}^{1/2} \left[ (\Sigma_v)_A^{1/3} + (\Sigma_v)_B^{1/3} \right]^2}$$

where $D_{AB}$ is the binary diffusion coefficient in cm$^2$/s, $T$ is the temperature in K, $P$ is the pressure in bar, and $\Sigma_v$ is the sum of atomic diffusion volumes of each component. The summations of atomic diffusion volumes are equal to 20.7 and 13.1 for ammonia and water, respectively.

$$M_{AB} = 2 \left( \frac{1}{M_A} + \frac{1}{M_B} \right)^{-1}$$

where $M_A$ and $M_B$ are the molecular weights of the components in g/mol.
2.1.2.4. Surface Tension

The Macleod-Sugden correlation [37], [38] (as cited in [26]) is used to calculate the surface tension of the liquid mixture.

$$\sigma_m = \left[ P_{L,m} \rho_{L,m} - P_{V,m} \rho_{V,m} \right]^n$$

where $P_{L,m}$ and $P_{V,m}$ are the parachors of the liquid and the vapor mixtures, respectively. $\rho_{L,m}$ and $\rho_{V,m}$ are the liquid and vapor mixture densities in mol/cm$^3$, respectively. The unit of the surface tension in Eq. (68) is dyn/cm. The coefficient, $n$, can be set to 3.6 in the absence of experimental data. The pure component parachor is defined in Eq. (57). Equations (65) and (66) are suggested by Hugill and van Welsenes [39] (as cited in [26]).

$$P_{L,m} = \sum_i \sum_j x_i x_j P_j$$

$$P_{V,m} = \sum_i \sum_j y_i y_j P_j$$

where $P_j$ is defined in terms of pure component parachors:

$$P_j = \lambda_{ij} \frac{P_i + P_j}{2}$$

The binary interaction parameter is determined from experimental data. It is set to unity in this study.

2.1.3. Governing Equations for the Rectifier

The formulation of simultaneous heat and mass transfer in the packed beds for ammonia-water absorptions systems has been extensively studied [40], [41]. Numerical investigations of these packed columns have been made for different configurations and operating conditions [13], [14], [42]. There are also experimental studies for the investigation of heat and mass transfer performance of packed
columns [43]. Therefore, the rectifier is modeled with random packing in order to validate the thermodynamic model used here.

Simultaneous heat and mass transfer is modeled for ammonia-water separation using two different column designs: a packed column with berl saddles and a column with hollow fiber membranes. The images of berl saddles and hollow fiber membranes are shown in Figure 8. Equimolar counter diffusion can be assumed in the binary separation processes provided that the molar latent heats of vaporization are essentially equal. However, there is a net molar flux at the vapor-liquid interface for the separation of ammonia-water because of the large difference between their enthalpies of vaporization. The hydraulic, thermal and concentration boundary layer thicknesses and profiles are distorted because of the high net molar flux across the interface. In this study, a rate based model is used for the rectifier using the heat and mass fluxes based on the stagnant film theory [20]. The column is divided into a finite number of segments and the flux and balance equations for each segment are solved simultaneously. These sets of non-linear equations are solved by Engineering Equation Solver (EES) software, which implements Newton’s method. These equations can be solved by this method efficiently only if the initial guesses are determined very carefully. The following assumptions have been made in the model:

- Steady state and one dimensional heat and mass transfers;
- Negligible pressure losses;
- Negligible heat losses to the surrounding;
- Heat and mass transfer coefficients are corrected according to the stagnant film model;
- Thermal equilibrium exists at the interface;
- Non-wetted operation mode for the hollow fiber membrane;
- Homogenous pore geometry for the hollow fiber membrane.
Figure 8. The images of a few hollow fiber membranes [44] and berl saddles [45].

The basic species, mass, momentum, and energy balance and heat and mass flux equations are the same for the columns with random packing and hollow fiber membranes. However, the two columns have different interfacial areas and heat and mass transfer coefficients. In addition, the membrane inherently has heat and mass transfer resistances that are not present for random packing.

The mass transfer between the liquid and vapor phases are illustrated in Figure 9 for the membrane (a) and the random packing (b). The horizontal lines represent virtual boundaries between segments that are crossed by the vertical liquid and vapor flows. Heat and mass transfer between these flows occurs horizontally. The vapor-liquid interface for both cases is also indicated, which for the membrane is at the liquid side of the membrane since the membrane pores are filled with vapor.
2.1.3.1. Mass Transfer

The ammonia fluxes between bulk vapor and the interface and between the interface and the bulk liquid are calculated by Eq. (67) and (68), respectively. These are derived from the extension of the film theory developed by Colburn and Drew [46]. The water flux can be derived from the ammonia flux using Eq. (69). Detailed
discussion and application of this model can be found in the literature [13], [18], [20], [41].

\[ \hat{N}_{\text{NH}_3} = k_G^* z \rho_G \ln \left( \frac{z - y_{\text{int}}}{z - y} \right) \Delta A \]  \hspace{1cm} (67)

\[ \hat{N}_{\text{NH}_3} = k_L^* z \rho_L \ln \left( \frac{z - x}{z - x_{\text{int}}} \right) \Delta A \]  \hspace{1cm} (68)

\[ z = \frac{N_{\text{NH}_3}}{\hat{N}_{\text{NH}_3} + \hat{N}_{\text{H}_2O}} \]  \hspace{1cm} (69)

Here \( k_G^* \) is the equivalent mass transfer coefficient for the gas side and gas filled membrane pores for net high molar flux at the boundary, and \( k_L^* \) is the liquid mass transfer coefficient for high net molar flux at the boundary, and they are defined as follows: \( \hat{N}_{\text{NH}_3} = k_L^* z \rho_L \ln \left( \frac{z - x}{z - x_{\text{int}}} \right) \Delta A \)

\[ \frac{1}{k_G^*} = \frac{d_i}{k_{\text{bulk,G}} d_o} + \frac{2d_i}{k_{\text{mem}}(d_o + d_i)} \]  \hspace{1cm} (70)

\[ k_{\text{bulk,G}} = k_{\text{bulk,G}} \frac{\phi_{m,G}}{e^{\phi_{m,G}} - 1} \]  \hspace{1cm} (71)

Here \( k_{\text{bulk,G}} \) is the mass transfer coefficient on the gas side for small mass transfer rates and \( 1/k_{\text{mem}} \) is the membrane resistance. Eq. (70) is not valid when random packing is used since there is no membrane resistance. For random packing \( k_{\text{bulk,G}} \) becomes equal to \( k_G^* \). \( \phi_{m,G} \) is a correction factor for high net molar flux at the boundary and is defined as:

\[ \phi_{m,G} = \frac{N_{\text{NH}_3} + \hat{N}_{\text{H}_2O}}{k_{\text{bulk,G}} \rho_G \Delta A} \]  \hspace{1cm} (72)

Similarly, the modified mass transfer coefficient for the liquid phase is defined as:
\[ k_L^* = k_L \frac{\theta_{m,L}}{e^{\theta_{m,L}} - 1} \]  
\[ \theta_{m,L} = \frac{\dot{N}_{NH_3} + \dot{N}_{H_2O}}{k_L \rho L \Delta A} \]  

The transfer area, \( \Delta A \), is calculated differently for random packing and hollow fibers, as described in the related sections below. The correction factor defined by Eq. (72) and (74) are very close to unity for the rectification processes of ammonia in the absorption cooling devices. Therefore, this correction factor is generally neglected in the literature.

Random Packings: The mass transfer coefficients for the liquid and vapor phases for absorption and desorption processes in packed columns have been studied extensively. In this study, the empirical correlations proposed by Onda et al. [47] are used for the mass transfer coefficients and the contact area between the two phases. The wetted surface area on the packing is assumed to be equal to the interfacial area between the liquid and the gas phases, which is the heat and mass transfer area, \( \Delta A \). Ceramic berl saddles (0.25 inch) are used as the packing material. The surface area per unit volume of the packed column is 900 m²/m³. The wetted area and the liquid and gas phase mass transfer coefficients are calculated according to Eqs. (75), (76) and (77), respectively.

\[ \frac{A_{wetted}}{A_{total}} = 1 - \exp \left( -1.45 \left( \frac{\sigma_c}{\sigma} \right)^{0.75} Re^{0.1} Fr^{-0.05} We^{0.2} \right) \]
\[ = 1 - \exp \left( -1.45 \left( \frac{\sigma_c}{\sigma} \right)^{0.75} \left( \frac{v_L}{A_{total} \mu_L} \right)^{0.1} \left( \frac{v_L^2 A_{total}}{\rho_L g} \right)^{-0.05} \left( \frac{v_L^2}{\rho_L A_{total} \sigma} \right)^{0.2} \right) \]

\[ k_L \left( \frac{\rho_L}{A_{wetted} g} \right)^{1/3} = 0.0051 \left( \frac{v_L}{A_{wetted} \mu_L} \right)^{2/3} \left( \frac{\mu_L}{\rho_L D_L} \right)^{-1/2} \left( A_{total} D_p \right)^{0.4} \]
\[
\frac{k_{\text{bulk},G}}{A_{\text{total}}D_G} = C \left( \frac{v_G}{A_{\text{total}} \mu_G} \right)^{0.7} \left( \frac{\mu_G}{\rho_G D_G} \right)^{1/3} \left( A_{\text{total}}D_p \right)^{-2}
\]  

(77)

where \(v_L\) and \(v_G\) are the superficial velocities of the liquid and vapor respectively. If the nominal size of the berl saddles is larger than 15 mm, \(C = 5.23\), otherwise \(C = 2\) in Eq. (77).

**Hollow Fiber Membrane:** The heat and mass transfer area between the two phases is well defined in membrane contactor, so an area estimation like Eq. (75) is not required. The transfer area is equal to the total surface area of the membranes based on the inner diameter. However, the determination of heat and mass transfer coefficients in the shell side is a more complicated task. Some authors take the diameter/length ratio, void fraction, and the irregularity of the fiber placement into account, but others did not find a direct relationship between these parameters and the Sherwood number \((k_{\text{bulk},G}D_h/D_G)\) [17], [48], [49]. Since the main purpose of this study is to design a more efficient rectifier for a miniaturized absorption cooling device, the packing density and the Reynolds number \((u_GD_h/v_G)\) are kept as high as possible, so that the mass transfer is maximized in a small volume. Eq. (78) takes both the irregularity of the fiber placement and packing density into consideration.

\[
Sh_{\text{shell}} = \frac{k_{\text{bulk},G}D_h}{D_G} = Re_{\text{shell}}^{0.33}Sc_{\text{shell}}^{0.33} \psi_m \\
\times \left(11.25\phi^4 - 23.36\phi^3 + 18.07\phi^2 - 6.17\phi + 1.13\right)
\]

(78)

\[
\psi_m = 0.862D_f - 0.489
\]

(79)

where \(\psi_m\) is a correction factor for the irregularity of the fiber distribution and Schmidt number is defined as \(v_G/D_G \cdot D_f = 1\) and \(D_f = 2\) for highly irregular and highly regular fiber packing, respectively. Its value is between 1.6 and 1.9 for engineering applications and it is taken 1.6 in this study. Eq. (78) is valid for a large
range of packing densities \((0.1 \leq \phi \leq 0.8)\) and for laminar flow [17]. The turbulence starts at low Reynolds numbers because of the large number of fibers inside the column. The experiments generally take place below a Reynolds number of 300 when the packing density is high. The mass transfer coefficient in the lumen side can be calculated using the Leveque equation [50]:

\[
Sh = 1.615 (Gz)^{1/3} = 1.615 \left( \frac{D_h^* RePr}{L} \right)^{1/3}
\] (80)

The mass transfer coefficients for the shell and lumen sides are calculated using Eq. (78) and (80). The mass transfer coefficient inside the vapor filled pores is calculated according to Eq. (81).

\[
k_m = \frac{\varepsilon D_{mem}}{\delta \tau_{mem}}
\] (81)

where \(\varepsilon\) is the porosity, \(D_{mem}\) is the vapor diffusion coefficient in the membrane pores, \(\delta\) is the thickness of the membrane, and \(\tau_{mem}\) is the tortuosity which is defined experimentally. Both the continuum and Knudsen diffusion are considered while calculating the diffusion coefficient in the membrane [50]. The total gas side mass transfer resistance, \(k_g\), is the summation of the shell side resistance and membrane resistance.

2.1.3.2. Heat Transfer

The heat transfers between the liquid and vapor phases are illustrated in Figure 9. The heat fluxes between bulk vapor and the interface and between the interface and the bulk liquid are calculated by Eqs. (82) and (83), respectively.

\[
\dot{Q}_G = h_G^*(T_G - T_{int}) \Delta A
\] (82)

\[
\dot{Q}_L = h_L^*(T_{int} - T_L) \Delta A
\] (83)
Here $h^*_G$ and $h^*_L$ are the modified heat transfer coefficients for high net molar flux across the boundary, which are defined as follows:

$$\frac{1}{h^*_G} = \frac{1}{h^*_{bulk,G}} + \frac{1}{h_m} \quad (84)$$

$$h^*_{bulk,G} = h^*_{bulk,G} \frac{\phi_{h,G}}{e^{\phi_{h,G}} - 1} \quad (85)$$

where $h^*_{bulk,G}$ is the heat transfer coefficient on the gas side for small mass transfer rates and $1/h_m$ is the membrane resistance. $\phi_{h,G}$ is defined as:

$$\phi_{h,G} = \frac{N_{NH_3} \bar{C}_{p,NH_3} + N_{H_2O} \bar{C}_{p,H_2O}}{h^*_{bulk,G} \Delta A} \quad (86)$$

The second term on the right hand side of Eq. (84) (membrane heat transfer resistance) vanishes when random packing is used instead of hollow fiber membranes. As a result $h^*_{bulk,G}$ becomes equal to $h^*_G$. Similarly, the modified heat transfer coefficient for the lumen side is defined as:

$$h^*_L = h_L \frac{\phi_{h,L}}{e^{\phi_{h,L}} - 1} \quad (87)$$

$$\phi_{h,L} = \frac{N_{NH_3} \bar{C}_{p,NH_3} + N_{H_2O} \bar{C}_{p,H_2O}}{h_L \Delta A} \quad (88)$$

The heat transfer coefficients for the bulk liquid and vapor are obtained using the Chilton-Colburn analogy. The thermal resistance of the membrane is calculated using the thermal conductivities of the membrane ($\lambda_{mem}$) and the vapor trapped inside the pores ($\lambda_G$).

$$\frac{1}{h_m} = \left( \frac{\delta}{\varepsilon \lambda_G} \right)^{-1} + \left( \frac{\delta}{(1-\varepsilon) \lambda_{mem}} \right)^{-1} \quad (89)$$
2.1.3.3. Balance Equations

The species balances for the bulk vapor and liquid are provided in Eqs. (90)-(93).

\[ \dot{N}_{\text{G},j} y_j - \dot{N}_{\text{G},i+1} y_{i+1} = \dot{N}_{\text{NH}_3} \]  
(90)

\[ \dot{N}_{\text{L},i} x_i - \dot{N}_{\text{L},i+1} x_{i+1} = \dot{N}_{\text{NH}_3} \]  
(91)

\[ \dot{N}_{\text{G},i} (1 - y_i) - \dot{N}_{\text{G},i+1} (1 - y_{i+1}) = \dot{N}_{\text{H}_2\text{O}} \]  
(92)

\[ \dot{N}_{\text{L},i} (1 - x_i) - \dot{N}_{\text{L},i+1} (1 - x_{i+1}) = \dot{N}_{\text{H}_2\text{O}} \]  
(93)

The energy balance equations for the bulk vapor and liquid phases and interface are given in Eqs. (94)-(96), respectively.

\[ \dot{N}_{\text{G},j} \dot{H}_{\text{G},j} - \dot{N}_{\text{G},i+1} \dot{H}_{\text{G},i+1} = \dot{Q}_G + \dot{N}_{\text{NH}_3} \dot{H}_{\text{G},\text{NH}_3} + \dot{N}_{\text{H}_2\text{O}} \dot{H}_{\text{G},\text{H}_2\text{O}} \]  
(94)

\[ \dot{N}_{\text{L},i} \dot{H}_{\text{L},i} - \dot{N}_{\text{L},i+1} \dot{H}_{\text{L},i+1} = \dot{Q}_L + \dot{N}_{\text{NH}_3} \dot{H}_{\text{L},\text{NH}_3} + \dot{N}_{\text{H}_2\text{O}} \dot{H}_{\text{L},\text{H}_2\text{O}} \]  
(95)

\[ \dot{Q}_G + \dot{N}_{\text{NH}_3} \dot{H}_{\text{G},\text{NH}_3} + \dot{N}_{\text{H}_2\text{O}} \dot{H}_{\text{G},\text{H}_2\text{O}} = \dot{Q}_L + \dot{N}_{\text{NH}_3} \dot{H}_{\text{L},\text{NH}_3} + \dot{N}_{\text{H}_2\text{O}} \dot{H}_{\text{L},\text{H}_2\text{O}} \]  
(96)

The enthalpies of ammonia and water which are used in energy balance equations are calculated at the interface temperature [13].
2.2. Absorption Cycle Model

Figure 10. Diagram of the absorption cycle used in Determan’s study [3] with numbering of the states.

The operating conditions of the absorption cycle are taken from the literature to compare the performance of the cycle with different rectifier conditions. The components except the rectifier are modeled as a black box with simple energy and mass balance equations. The temperatures, pressures, ammonia mass concentrations and vapor qualities of the original design are shown in Table 4. These state properties are the actual values for the real system shown in Figure 1 and Figure 10.
Table 4. Microchannel System Design State Points [3].

<table>
<thead>
<tr>
<th>State Point</th>
<th>$T$ °C</th>
<th>$P$ kPa</th>
<th>$x$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>State points for the working fluid (Ammonia-water mixture)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Condenser Inlet</td>
<td>85</td>
<td>1600</td>
<td>0.984</td>
<td>1</td>
</tr>
<tr>
<td>2. Condenser Outlet</td>
<td>39.6</td>
<td>1600</td>
<td>0.984</td>
<td>Sub-cooled</td>
</tr>
<tr>
<td>3. RHEX High Pressure Outlet</td>
<td>30.2</td>
<td>1600</td>
<td>0.984</td>
<td>Sub-cooled</td>
</tr>
<tr>
<td>4. Evaporator Inlet</td>
<td>-1.4</td>
<td>400</td>
<td>0.984</td>
<td>0.12</td>
</tr>
<tr>
<td>5. Evaporator Outlet</td>
<td>8.6</td>
<td>400</td>
<td>0.984</td>
<td>0.94</td>
</tr>
<tr>
<td>6. RHEX Low Pressure Outlet</td>
<td>18</td>
<td>400</td>
<td>0.984</td>
<td>0.96</td>
</tr>
<tr>
<td>7. Absorber Dilute Solution Inlet</td>
<td>73.2</td>
<td>400</td>
<td>0.285</td>
<td>0.01</td>
</tr>
<tr>
<td>8. Absorber Outlet</td>
<td>50.4</td>
<td>400</td>
<td>0.370</td>
<td>Sub-cooled</td>
</tr>
<tr>
<td>9. Rectifier Coolant Inlet</td>
<td>50.8</td>
<td>1600</td>
<td>0.370</td>
<td>Sub-cooled</td>
</tr>
<tr>
<td>10. Rectifier Coolant Outlet</td>
<td>63.7</td>
<td>1600</td>
<td>0.370</td>
<td>Sub-cooled</td>
</tr>
<tr>
<td>11. SHEX High Pressure Outlet</td>
<td>109</td>
<td>1600</td>
<td>0.370</td>
<td>0.001</td>
</tr>
<tr>
<td>12. Rectifier Inlet</td>
<td>128.2</td>
<td>1600</td>
<td>0.876</td>
<td>1</td>
</tr>
<tr>
<td>13. Rectifier Reflux Outlet</td>
<td>128.2</td>
<td>1600</td>
<td>0.285</td>
<td>0</td>
</tr>
<tr>
<td>14. Desorber Dilute Solution Outlet</td>
<td>128.2</td>
<td>1600</td>
<td>0.285</td>
<td>0</td>
</tr>
<tr>
<td>15. SHEX Low Pressure Outlet</td>
<td>75.7</td>
<td>1600</td>
<td>0.285</td>
<td>Sub-cooled</td>
</tr>
</tbody>
</table>

**Flow Rates**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Refrigerant Flow Rate</td>
<td>0.33</td>
<td>g/s</td>
<td></td>
</tr>
<tr>
<td>Dilute Solution Flow Rate</td>
<td>2.37</td>
<td>g/s</td>
<td></td>
</tr>
<tr>
<td>Concentrated Solution Flow Rate</td>
<td>2.70</td>
<td>g/s</td>
<td></td>
</tr>
</tbody>
</table>

**Coolant State Points**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Condenser Coolant Inlet</td>
<td>37</td>
<td>101.3</td>
<td>-</td>
</tr>
<tr>
<td>Condenser Coolant Outlet</td>
<td>42.6</td>
<td>101.3</td>
<td>-</td>
</tr>
<tr>
<td>Evaporator Coupling Fluid Inlet</td>
<td>9</td>
<td>101.3</td>
<td>-</td>
</tr>
<tr>
<td>Evaporator Coupling Fluid Outlet</td>
<td>5.5</td>
<td>101.3</td>
<td>-</td>
</tr>
<tr>
<td>Absorber Coolant Inlet</td>
<td>37</td>
<td>101.3</td>
<td>-</td>
</tr>
<tr>
<td>Absorber Coolant Outlet</td>
<td>45.6</td>
<td>101.3</td>
<td>-</td>
</tr>
</tbody>
</table>
The overall heat transfer coefficients and the heat transfer areas for each component can be calculated by fixing state points which are shown in Table 5.

**Table 5.** The fixed state points of the cycle [3].

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condenser inlet temperature</td>
<td>85 °C</td>
</tr>
<tr>
<td>Condenser outlet temperature</td>
<td>2°C sub-cooled (39.6°C)</td>
</tr>
<tr>
<td>Evaporator outlet temperature</td>
<td>8.6°C</td>
</tr>
<tr>
<td>RHEX low pressure outlet temperature</td>
<td>18°C</td>
</tr>
<tr>
<td>SHEX high pressure outlet temperature</td>
<td>109°C</td>
</tr>
<tr>
<td>Desorber heat input rate</td>
<td>800 W</td>
</tr>
<tr>
<td>Absorber outlet temperature</td>
<td>4.8°C sub-cooled (50.4°C)</td>
</tr>
<tr>
<td>Concentrated solution mass flow rate</td>
<td>2.7 g/s</td>
</tr>
<tr>
<td>Concentrated solution ammonia concentration</td>
<td>0.37 kg/kg</td>
</tr>
</tbody>
</table>

These assumptions are enough to fix all of the states. The desorber inlet and heat input are fixed, so the desorber outlet states can be obtained. The vapor leaving the desorber enters the rectifier. The rectifier outlet state is fixed, so the rectifier heat load can be obtained. Using the fixed absorber outlet and the rectifier heat load, SHEX high pressure inlet can be obtained. SHEX high pressure outlet (desorber inlet) and SHEX low pressure inlet (desorber outlet) are already known, so SHEX low pressure outlet can be calculated. RHEX high pressure inlet, low pressure inlet and low pressure outlet are fixed. Therefore, the temperature, concentration and mass flow rate at each state can be calculated.

In addition to fixed state point of the cycle, the condenser, evaporator and absorber coolant flow rates and inlet temperatures are also required to calculate the required UA values of each component. The coolant inlet conditions are as follows:

- Absorber coolant: 101 kPa, 37°C and 20.89 g/s,
- Condenser coolant: 101 kPa, 37°C and 17.8 g/s,
- Evaporator coupling fluid: 101 kPa, 9°C and 24 g/s.

The energy balances for the heat exchange devices like condenser, evaporator, RHEX and SHEX are as follows.
The condenser:

\[ \dot{Q}_{\text{cond}} = \dot{m}_{\text{refrigerant}} \left( h_{\text{cond,in}} - h_{\text{cond,out}} \right) \]  \hspace{1cm} (97)

\[ \dot{Q}_{\text{cond}} = \dot{m}_{\text{cond,CF}} \left( h_{\text{cond,CF,out}} - h_{\text{cond,CF,in}} \right) \]  \hspace{1cm} (98)

\[ \dot{Q}_{\text{cond}} = UA_{\text{cond}} LMTD_{\text{cond}} \]  \hspace{1cm} (99)

The evaporator:

\[ \dot{Q}_{\text{eva}} = \dot{m}_{\text{refrigerant}} \left( h_{\text{eva,in}} - h_{\text{eva,out}} \right) \]  \hspace{1cm} (100)

\[ \dot{Q}_{\text{eva}} = \dot{m}_{\text{eva,CF}} \left( h_{\text{eva,CF,out}} - h_{\text{eva,CF,in}} \right) \]  \hspace{1cm} (101)

\[ \dot{Q}_{\text{eva}} = UA_{\text{eva}} LMTD_{\text{eva}} \]  \hspace{1cm} (102)

The refrigerant heat exchanger:

\[ Q_{\text{RHEX}} = \dot{m}_{\text{refrigerant}} \left( h_{\text{RHEX,high,in}} - h_{\text{RHEX,high,out}} \right) \]  \hspace{1cm} (103)

\[ Q_{\text{RHEX}} = \dot{m}_{\text{refrigerant}} \left( h_{\text{RHEX,low,out}} - h_{\text{RHEX,low,in}} \right) \]  \hspace{1cm} (104)

\[ Q_{\text{RHEX}} = UA_{\text{RHEX}} LMTD_{\text{RHEX}} \]  \hspace{1cm} (105)

The solution heat exchanger:

\[ \dot{Q}_{\text{SHEX}} = \dot{m}_{\text{CS}} \left( h_{\text{SHEX,high,in}} - h_{\text{SHEX,high,out}} \right) \]  \hspace{1cm} (106)

\[ \dot{Q}_{\text{SHEX}} = \dot{m}_{\text{DS}} \left( h_{\text{SHEX,low,out}} - h_{\text{SHEX,low,in}} \right) \]  \hspace{1cm} (107)

\[ \dot{Q}_{\text{SHEX}} = UA_{\text{SHEX}} LMTD_{\text{SHEX}} \]  \hspace{1cm} (108)

The energy and mass balances for the heat and mass exchange devices like absorber, and rectifier are as follows.
The absorber:

\[ m_{CS} = m_{\text{refrigerant}} + m_{DS} \]  \hspace{1cm} (109)

\[ m_{CS} x_{CS} = m_{\text{refrigerant}} y_{\text{refrigerant}} + m_{DS} x_{DS} \]  \hspace{1cm} (110)

\[ \dot{Q}_{\text{abs}} = m_{\text{refrigerant}} h_{\text{abs,ref,in}} + m_{DS} h_{\text{abs,DS,in}} - m_{CS} h_{\text{abs,CS,out}} \]  \hspace{1cm} (111)

\[ \dot{Q}_{\text{abs}} = m_{\text{abs,CF}} (h_{\text{abs,CF,out}} - h_{\text{abs,CF,in}}) \]  \hspace{1cm} (112)

\[ \dot{Q}_{\text{abs}} = U_{\text{abs}} LMTD_{\text{abs}} \]  \hspace{1cm} (113)

The rectifier:

\[ m_{\text{rect},\text{vap,in}} = m_{\text{refrigerant}} + m_{\text{rect,reflux,out}} \]  \hspace{1cm} (114)

\[ m_{\text{rect},\text{vap,in}} y_{\text{rect,\text{vap,in}}} = m_{\text{refrigerant}} y_{\text{refrigerant}} + m_{\text{rect,reflux,out}} x_{\text{rect,reflux,out}} \]  \hspace{1cm} (115)

\[ \dot{Q}_{\text{rect}} = m_{\text{refrigerant}} h_{\text{refrigerant}} + m_{\text{rect,reflux,out}} h_{\text{rect,reflux,out}} - m_{\text{rect,\text{vap,in}}} h_{\text{rect,\text{vap,in}}} \]  \hspace{1cm} (116)

\[ \dot{Q}_{\text{rect}} = m_{CS} (h_{\text{rect,CS,in}} - h_{\text{rect,CS,\text{out}}}) \]  \hspace{1cm} (117)

\[ \dot{Q}_{\text{rect}} = U_{\text{rect}} LMTD_{\text{rect}} \]  \hspace{1cm} (118)

By solving Eqs. (97)-(118), the required \( U_A \) values for each component to obtain the fixed state points can be calculated. These values for the original design [3] are listed in Table 6.

**Table 6.** The \( U_A \) values for each component in Figure 1.

<table>
<thead>
<tr>
<th>Component</th>
<th>( U_A ) W/K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absorber</td>
<td>38</td>
</tr>
<tr>
<td>Condenser</td>
<td>29</td>
</tr>
<tr>
<td>SHEX</td>
<td>37</td>
</tr>
<tr>
<td>RHEX</td>
<td>0.7</td>
</tr>
<tr>
<td>Evaporator</td>
<td>160</td>
</tr>
<tr>
<td>Rectifier</td>
<td>3.2</td>
</tr>
</tbody>
</table>

In this study, the rectifier is modeled in detail and the effect of different rectifier designs on the overall performance of the absorption cooling device is investigated.
Therefore, the heat and mass transfer processes are not modeled in detail for the other components. It is assumed that the $UA$ values do not change for these components during the simulation for simplicity. The mass and energy balance equations are solved for every component using these $UA$ values. The required $UA$ values are calculated by following the procedure starting from fixing the state points which are shown in Table 5. Since the heat and mass transfer process in the rectifier is modeled in detail, the condenser inlet (rectifier vapor outlet) is not assumed to be 85°C. Therefore, the new $UA$ values are calculated in the next chapter based on the fixed state points shown in Table 5 except the condenser inlet temperature and the rectifier model described in Section 2.1. Then the parametric results are obtained using new $UA$ values.

In reality the overall heat transfer coefficient, $U$, changes with mass flow rate and thermodynamic properties which depend on the concentration and temperature. Since the mass flow rates, temperatures and ammonia concentrations do not change too much to significantly alter the overall heat transfer coefficients, the effect of different rectifier designs on the overall absorption cycle can be investigated with this approximation. In a realistic simulation, the heat and mass transfer in other components, pressure drops, parasitic heat transfer between the components, and heat loss to the surrounding should be considered in detail. The purpose of this study is only to investigate the applicability of hollow fiber membranes in the rectifier and its effect on the overall system.

Instead of a simple tray column which is shown in Figure 1, a solution cooled partial condenser, hollow fiber membrane rectifier and stripper are included in the new design. The total size of these components is kept close to the size of the original tray column. The new cycle with these modifications is shown in Figure 11.
Figure 11. Schematic of the absorption cycle with rectifier, stripper, RHEX and SHEX.
CHAPTER 3

RESULTS AND DISCUSSION

In this chapter, the results are presented and discussed in detail. The chapter starts with the detailed results for the rectifier model and then continues with the results of the absorption cycle with new rectifier. In Section 3.1, the operating conditions of the rectifier are taken from the literature to be able to compare the results and verify the mathematical model. In Section 3.2, the rectifier model is integrated into the Determan’s [3] cycle. The effect of the rectifier’s new design to the overall system is simulated for different working conditions.

3.1. The Rectifier

The operating conditions and the dimension of the rectifier and the packing features are provided in Table 7. The same column dimensions are used for the hollow fiber and random packing rectifiers to compare their performance. The hydrophobic membrane properties are taken from the literature where it is reported that the membranes are thermally stable up to 250°C [19].

The details of the rectifier model and the results are also shown in detail in the previous study [51]. The profiles for ammonia mass concentration and temperature for the random packing module resulting from the present work and those from the literature are shown in Figure 12 and Figure 13. The column position is shown on the y-axis to be consistent with the vertical orientation of the column. Although this study uses the same heat and mass transfer model as [13], the temperature and concentration difference between the inlet and the exit change only about 3% which can be attributed to using different property models. On the other hand a different
heat and mass transfer model and different property models are used in [52] and these results are also shown in Figure 12 and Figure 13. In Figure 12 and Figure 13, sufficiently good agreement is seen between the present results and those in the literature to verify the present work, especially for the liquid data in Figure 13 where all data series basically overlap.

**Table 7. Rectifier specifications**

<table>
<thead>
<tr>
<th>Boundary Conditions [13]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>11.64 bar</td>
</tr>
<tr>
<td>Vapor inlet temperature</td>
<td>351 K</td>
</tr>
<tr>
<td>Vapor inlet flow rate</td>
<td>0.0049 kg/s</td>
</tr>
<tr>
<td>Vapor mass concentration</td>
<td>0.9833 -</td>
</tr>
<tr>
<td>Liquid inlet temperature</td>
<td>303 K</td>
</tr>
<tr>
<td>Liquid inlet flow rate</td>
<td>0.0019 kg/s</td>
</tr>
<tr>
<td>Liquid mass concentration</td>
<td>0.9985 -</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Column Dimension [13]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Column diameter</td>
<td>0.06 m</td>
</tr>
<tr>
<td>Column height</td>
<td>0.4 m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Random Packing [13]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Berl saddles</td>
</tr>
<tr>
<td>Dimension</td>
<td>0.25 in</td>
</tr>
<tr>
<td>Material</td>
<td>Ceramic</td>
</tr>
<tr>
<td>Surface area</td>
<td>900 m²/m³</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hollow Fiber Membrane [19]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective porosity (\varepsilon/\delta r_{\text{mem}})</td>
<td>13647 m⁻¹</td>
</tr>
<tr>
<td>Pore radius</td>
<td>0.05 µm</td>
</tr>
<tr>
<td>Outer diameter</td>
<td>2.2 mm</td>
</tr>
<tr>
<td>Inner diameter</td>
<td>1.5 mm</td>
</tr>
</tbody>
</table>
The performances of the random packing and the hollow fiber module in terms of mass concentration and temperature are compared in Figure 14 and Figure 15. The current model is constrained by the mass transfer correlation for the shell side of the fiber module, Eq. (78), which is only valid when the packing density is below 0.8. Therefore, the number of fibers is chosen as 595 which makes the packing density 0.8, so that the interfacial area is maximized.

Figure 12. Ammonia mass concentration profiles of the liquid and the vapor along the rectifier with random packing.
According to Figure 14 and Figure 15, for the assumed conditions, both the heat and mass transfer rates are larger for the fibers than for the random packing. The normalized shift in the temperature profile is larger than the normalized shift in concentration profiles. The normalization can be made by dividing the temperature or concentration change in the membrane module to the temperature or concentration change in the random packing module. The vapor temperatures and the concentrations at different column positions and their change are shown in Table 8. The vapor temperature drop in the membrane module is 2 times higher than the vapor temperature drop in the random packing module at 0.1 m column position. However, the vapor concentration increase in the membrane module is 1.5 times higher than the vapor concentration increase in the random packing module at 0.1 m column position. This result is expected because the increment in the mass transfer resistance due to the membrane is larger than the increment of the heat transfer resistance. As a result, the large increase of the interfacial area affects the
temperature profile more than the concentration profiles. The rectification process is enhanced by the membrane providing high interfacial area per unit volume, at the expense of higher heat and mass transfer resistances.

**Figure 14.** Ammonia mass concentration profiles of the liquid (x) and the vapor (y) along the rectifier for the random packing and the hollow fiber module with 0.06 m column diameter and 595 fibers.
Figure 15. Temperature profiles of the liquid and the vapor along the rectifier for the random packing and the hollow fiber module with 0.06 m column diameter and 595 fibers.

Table 8. The vapor temperatures and concentrations at the different column positions of the membrane and random packing modules.

<table>
<thead>
<tr>
<th>Column Position</th>
<th>Temperature (K)</th>
<th>Concentration (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Membrane Module</td>
<td>Random Packing</td>
</tr>
<tr>
<td>0</td>
<td>351</td>
<td>351</td>
</tr>
<tr>
<td>0.1</td>
<td>330</td>
<td>341</td>
</tr>
<tr>
<td>Change</td>
<td>21</td>
<td>10</td>
</tr>
</tbody>
</table>

The mass transfer correlations on the shell side of a hollow fiber module are generally valid for low packing densities and low Reynolds numbers [17], [48], [49]. Eq. (78) is obtained based on experiments conducted below a Reynolds number of 300. However, the operating conditions, dimensions of the column, and the number of fibers in the present work result in a Reynolds number of approximately 500 in the shell side. There are insufficient data in the literature to confidently model high Reynolds number and high packing density counter flow hollow fiber modules.
Therefore, the column diameter and the number of fibers were increased to decrease the Reynolds number on the shell side, while keeping the packing density the same. When the column diameter is 0.078 m and the number of fibers is 1000, the Reynolds number becomes lower than 300. The performances of the random packing and the hollow fiber membrane in terms of concentration and temperature profiles for the new configuration are compared in Figure 16 and Figure 17.

The concentration and temperature profiles of the rectifier with random packing do not change significantly when the diameter of the rectifier is increased according to Figure 14 and Figure 16, and Figure 15 and Figure 17. The reason is that the wetted area per unit volume also decreases with Reynolds number in the packed column, so the total interfacial area between the phases does not increase proportional to the total volume of the column. As a result, as the Reynolds number is decreased, the mass transfer coefficients also decrease which compensates for the increase in the interfacial area. Differently from packed columns, the interfacial area increases directly proportional with the number of fibers. Although the mass transfer coefficient decreases with Reynolds number, the large increase in the interfacial area enhances the heat and mass transfer rates significantly.
Figure 16. Ammonia mass concentration profiles of the liquid (x) and the vapor (y) along the rectifier for the random packing and the hollow fiber module with 0.078 m column diameter and 1000 fibers.

Figure 17. Temperature profiles of the liquid and the vapor along the rectifier for random packing and hollow fiber module with 0.078 m column diameter and 1000 fibers.
The quality and the flow rate of the refrigerant play a key role for ammonia-water absorption cooling devices. The ammonia concentration at the exit of the rectifier should be as high as possible to prevent water accumulation in the evaporator. Moreover, higher refrigerant flow rate results in a higher cooling power in the evaporator. Therefore, not only the concentration and the temperature, but also the mass flow rate at the exit of the rectifier should be considered. The mass flow rates at the exit are shown in Table 9. The mass flow rate does not change significantly when hollow fiber membranes are used instead of random packing.

**Table 9. Mass flow rates at the exit of the rectifier**

<table>
<thead>
<tr>
<th>Column Diameter</th>
<th>Packing Type</th>
<th>Mass Flow Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.060 m</td>
<td>Random packing, Hollow fiber, N=595</td>
<td>5.45 g/s</td>
</tr>
<tr>
<td>0.078 m</td>
<td>Random packing, Hollow fiber, N=1000</td>
<td>5.49 g/s</td>
</tr>
</tbody>
</table>

The effect of the packing density on the concentration profile is shown in Figure 18. The column diameter is 0.078 m and the other operating conditions are the same as in Figure 16 except for the number of fibers. The mass transfer rate increases with packing density because of two reasons. First, the interfacial area increases directly proportional with the number of fibers. Second, the vapor mass transfer coefficient increases with the packing density.
Figure 18. The effect of packing density on the concentration profiles with 0.078m column diameter.

Key design and operating parameters for the rectifier include each of the heat and mass transfer coefficients and the membrane's effective porosity and pore radius. The sensitivity of the distillate ammonia concentration (refrigerant ammonia concentration) to fractional variations in each of these key parameters is illustrated in Figure 19. In Figure 19 the Multiplication Factor is the ratio of the tested value to the nominal value for each key variable, where a value of 1 indicates nominal conditions and for example 0.8 indicates a value 20% smaller than nominal. According to Figure 18 and Figure 19, the packing density, effective porosity and the pore radius should be as high as possible to decrease the water content of the distillate. Variations in all the transfer coefficients except for the mass transfer coefficient on the vapor side do not have a significant effect on the distillate concentration.
Figure 19. The individual effects of transfer coefficients and the membrane effective porosity and pore radius on the distillate ammonia concentration.

The mass flow rate of the distilled vapor also has an important role in absorption cooling devices, since it directly affects the cooling power. The mass flow rate of the distilled vapor changes little with packing density. For example, the mass flow rate is 5.47 and 5.49 g/s for packing densities of 0.6 and 0.8, respectively. The effects of the transfer coefficients and the membrane properties on the distillate flow rate are shown in Figure 20, which is a similar representation as Figure 9. The heat transfer coefficient on the vapor side is the most important factor that affects the mass flow rate of the distillate. The mass flow rate decreases slightly with increasing effective porosity and pore radius. However, the increase in ammonia concentration with effective porosity and pore radius is more significant. Therefore, a membrane with larger effective porosity and pore radius presented in the literature can be explored to enhance the performance of the rectifier [19].
Figure 20. The individual effects of transfer coefficients and the membrane properties on the distillate mass flow rate.

The sensitivity of the results to the number of segments used in the solution procedure is shown in Figure 21. The concentration profiles are coincident for 6 or more segments according to Figure 21. Therefore, 6 segments are used in the rectifier model in this study.

Figure 21. The sensitivity of the concentration profiles to the grid refinement.
The mathematical model for the random packing module is verified by comparing the results with the literature in this section. No results were found in the literature to directly verify the membrane module, but the membrane module is assumed accurate since it is very similar to random packing module which is verified. Specifically, the heat and mass flux equations and the balance equations are the same for both internal structures. The major difference is that the heat and mass transfer coefficients are calculated with different empirical correlations. Also, there is heat and mass transfer resistances associated with the membrane. Assuming that these differences are correctly modeled, the membrane module is assumed verified.

3.2. Overall Absorption Cycle

The performances of random packing and hollow fiber membranes for the ammonia rectification process have been compared in the previous section. Hollow fiber membranes result in a higher ammonia concentration in the distillate, because of its high interfacial area. In this section parametric analyses are performed to explore trends in performance for the overall absorption cycle as key design and operating parameters are varied.

To be able to make a parametric analysis, the sizes of each component shown in Figure 1 and Figure 3 should be determined first. The required $UA$ values for each component are calculated based on the fixed state points shown in Table 5 except the condenser inlet temperature and the new rectifying and stripping section specifications which are shown in Table 10. The hollow fiber membrane specifications are given in Table 7. The number of fibers and the column dimensions result in a packing density of 70% in both stripping and rectifying sections. The total volume of the rectifying and stripping sections is very close to the volume of the rectifier shown in Figure 1. From Figure 1, the volume of the rectifier can be estimated around 0.096 liters just like the total volume of the new rectifying and stripping sections. A partial condenser has also been added to the system to obtain the reflux liquid for the rectifying section as shown in Figure 11. The $UA$ value of
The partial condenser has been selected as 9 W/K which is larger than the $UA$ value of the old rectifier design. The reason for choosing a higher value is that the flow rate of the reflux liquid should be higher because the interfacial area between the phases is also high in the rectifier. If the flow rate of the reflux liquid becomes too low, its temperature increases and the concentration decreases quickly as it flows down in the rectifier. This situation makes the rest of the rectifier useless in terms of heat and mass transfer. However, increasing the size of the partial condenser does not increase the volume of the system significantly, because the volume of the heat exchanger between the concentrated solution and the refrigerant vapor at the rectifier is very small according to Figure 1.

<table>
<thead>
<tr>
<th>Stripping Section</th>
<th>Rectifying Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>0.0322 m</td>
</tr>
<tr>
<td>Height</td>
<td>0.1 m</td>
</tr>
<tr>
<td>Number of fibers</td>
<td>150</td>
</tr>
<tr>
<td>Diameter</td>
<td>0.0195 m</td>
</tr>
<tr>
<td>Height</td>
<td>0.05 m</td>
</tr>
<tr>
<td>Number of fibers</td>
<td>55</td>
</tr>
</tbody>
</table>

The system COP, specific cooling power and the required $UA$ values for each component are calculated for different desorber heat input rates by solving every equation simultaneously using Newton’s method with EES. It is assumed that the overall heat transfer coefficients are constant, but the required heat transfer area, $A$, changes.

**Figure 22.** The system COP and specific cooling power for different desorber heat input rate. The state points are fixed and the required $UA$ changes accordingly.
The COP decreases and the specific cooling power increases as the desorber heat input rate increases according to Figure 22. The reason for increasing specific cooling power is that the mass flow rate of the refrigerant vapor increases as the desorber heat input rate is increased. Since the refrigerant flow rate is larger, the cooling capacity of the evaporator is higher. However, as the refrigerant flow rate increases, the stripping and rectifying sections start having difficulty in purifying the ammonia. Therefore, higher desorber heat input rates increase the refrigerant flow rate but decrease the ammonia concentration. As a result, the specific cooling power increases while the COP decreases.

The required $UA$ values for each component for different desorber inputs are shown in Table 11.

<table>
<thead>
<tr>
<th>$Q_{des}$ W</th>
<th>$UA_{Cond}$ W/K</th>
<th>$UA_{Eva}$ W/K</th>
<th>$UA_{RHEX}$ W/K</th>
<th>$UA_{SHEX}$ W/K</th>
<th>$UA_{Abs}$ W/K</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>36.9</td>
<td>126.1</td>
<td>0.375</td>
<td>59.8</td>
<td>33.6</td>
</tr>
<tr>
<td>700</td>
<td>40.5</td>
<td>150.7</td>
<td>0.448</td>
<td>47.9</td>
<td>37.1</td>
</tr>
<tr>
<td>800</td>
<td>43.6</td>
<td>177.2</td>
<td>0.527</td>
<td>39.4</td>
<td>40.2</td>
</tr>
<tr>
<td>900</td>
<td>46.2</td>
<td>205.8</td>
<td>0.616</td>
<td>33.0</td>
<td>42.7</td>
</tr>
<tr>
<td>1000</td>
<td>48.2</td>
<td>234.7</td>
<td>0.718</td>
<td>28.1</td>
<td>44.8</td>
</tr>
</tbody>
</table>

The required size of every component except the solution heat exchanger increases to obtain the fixed state points summarized in Table 11 for larger desorber heat input rates. This is expected because the mass flow rate of the refrigerant increases as explained in the previous paragraphs. The size of the solution heat exchanger is decreased because the temperature of the high and low pressure inlets of the solution heat exchanger is larger when the desorber heat input rate is increased. Since the high pressure outlet temperature is fixed, the heat duty and the required size of the solution heat exchanger decreases.

The cycle shown in Figure 11 is solved for the fixed state points in Table 5 except the condenser inlet temperature, with the new stripping and rectifying sections described in Table 10. The resulting sizes of each component are shown in Table 11.
The resulting state points of the cycle for 800 W desorber heat input rate are shown in Table 12.

**Table 12.** The state points with new rectifying and stripping sections.

<table>
<thead>
<tr>
<th>State Point</th>
<th>T °C</th>
<th>P kPa</th>
<th>x</th>
<th>q</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>State points for the working fluid (Ammonia-water mixture)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Condenser Inlet</td>
<td>70.1</td>
<td>1600</td>
<td>0.994</td>
<td>1</td>
</tr>
<tr>
<td>Condenser Outlet</td>
<td>39.2</td>
<td>1600</td>
<td>0.994</td>
<td>Sub-cooled</td>
</tr>
<tr>
<td>RHEX High Pressure Outlet</td>
<td>32.4</td>
<td>1600</td>
<td>0.994</td>
<td>Sub-cooled</td>
</tr>
<tr>
<td>Evaporator Inlet</td>
<td>-1.8</td>
<td>400</td>
<td>0.994</td>
<td>0.126</td>
</tr>
<tr>
<td>Evaporator Outlet</td>
<td>8.6</td>
<td>400</td>
<td>0.994</td>
<td>0.98</td>
</tr>
<tr>
<td>RHEX Low Pressure Outlet</td>
<td>18.0</td>
<td>400</td>
<td>0.994</td>
<td>0.987</td>
</tr>
<tr>
<td>Dilute Solution Inlet</td>
<td>72.3</td>
<td>400</td>
<td>0.274</td>
<td>Sub-cooled</td>
</tr>
<tr>
<td>Absorber Outlet</td>
<td>50.4</td>
<td>400</td>
<td>0.370</td>
<td>Sub-cooled</td>
</tr>
<tr>
<td>Rectifier Coolant Inlet</td>
<td>50.5</td>
<td>1600</td>
<td>0.370</td>
<td>Sub-cooled</td>
</tr>
<tr>
<td>Rectifier Coolant Outlet</td>
<td>60.9</td>
<td>1600</td>
<td>0.370</td>
<td>Sub-cooled</td>
</tr>
<tr>
<td>SHEX High Pressure Outlet</td>
<td>109</td>
<td>1600</td>
<td>0.370</td>
<td>0.001</td>
</tr>
<tr>
<td>Desorber Dilute Solution Outlet</td>
<td>130.8</td>
<td>1600</td>
<td>0.274</td>
<td>0</td>
</tr>
<tr>
<td>SHEX Low Pressure Outlet</td>
<td>72.1</td>
<td>1600</td>
<td>0.274</td>
<td>Sub-cooled</td>
</tr>
</tbody>
</table>

**Flow Rates**

- Refrigerant Flow Rate 0.36 g/s
- Dilute Solution Flow Rate 2.34 g/s
- Concentrated Solution Flow Rate 2.70 g/s

**Coolant State Points**

- Condenser Coolant Inlet 37 101.3 - Sub-cooled
- Condenser Coolant Outlet 42.8 101.3 - Sub-cooled
- Evaporator Coupling Fluid Inlet 9 101.3 - Sub-cooled
- Evaporator Coupling Fluid Outlet 5 101.3 - Sub-cooled
- Absorber Coolant Inlet 37 101.3 - Sub-cooled
- Absorber Coolant Outlet 45.8 101.3 - Sub-cooled
Comparing with Table 4, the temperatures and concentrations do not change significantly with the new rectifying and stripping sections. The reason is that the key state points are fixed according to their saturated conditions (i.e. 2°C sub-cooled). However, the required sizes of each component and the system performance depend on the desorber heat input rate as shown in Figure 22 and Table 11, respectively. The temperature and concentration distributions in the stripping and rectifying section are shown in Figure 23 and Figure 24.

**Figure 23.** Temperature distribution along the rectifying and stripping sections. (Fixed state points with 800 W desorber heat input rate)
According to Figure 23 and Figure 24, the vapor enters the stripping section from the bottom at 404 K and with 0.863 ammonia mass concentrations. The vapor exits at 387.3 K and with 0.931 ammonia mass concentration. Then, it enters the rectifying section with the same temperature and concentration. It exits from the top of the rectifier at 382.4 K and with 0.947 ammonia mass concentration. The ammonia concentration increases to 0.994 after the partial condensation according to Table 12. The liquid reflux enters the rectifier from the top at 343.3 K and with 0.583 ammonia mass concentration. It exits the rectifier at 377.7 K and with 0.478 ammonia mass concentration. Then, it is mixed with the feed stream before entering the stripping section. The liquid enters the stripping section at 382.6 K and with 0.372 ammonia mass concentration. It exits from the bottom of the stripping section at 385.1 K and with 0.360 ammonia mass concentration. The highest temperature and concentration change are in the liquid reflux in the rectifying section. The reason is that the mass flow rate of this stream is low compared to the other flow rates. The ammonia
concentration increases as the vapor flows upward in both sections. However, the rate of increase decreases as it approaches to the top of the stripping section. Therefore, a larger stripping section has no benefit in terms of ammonia concentration. The ammonia concentration of the liquid phase does not significantly change in the stripping section, but the temperature of the liquid increases as it flows downward in the stripping section. As a result, the required external heating in the desorber decreases which increases the COP of the absorption cycle.

In addition to the temperature and the ammonia concentration, the mass flow rate of the refrigerant vapor also directly affects the system performance. The mass flow rate of the vapor is 0.4 g/s at the inlet of the stripping section. It is almost constant throughout the stripping and the rectifying sections. The vapor mass flow rates at the exit of the stripping and the exit of the rectifying sections are 0.408 g/s and 0.405 g/s, respectively. The ammonia and water molar transfer rates are shown in Figure 25.

![Figure 25. Molar transfer rates of ammonia and water across the vapor-liquid boundary throughout the stripping and rectifying section. (Fixed state points with 800 W desorber heat input rate)](image-url)
According to Figure 25, the ammonia content of the vapor increases up to a certain point in the stripping section as the vapor flows upward. As the ammonia content increases in the vapor phase, the rate of ammonia diffusion from liquid to vapor decreases in the stripping section. However, the rate of ammonia diffusion from liquid to vapor phase increases in the rectifying section in the upward direction. This is because of the low temperature of the liquid reflux in the rectifying section compared to the liquid temperature in the stripping section. The direction of the water diffusion is always from vapor phase to liquid phase.

The required sizes of each component are shown in Table 11 for different desorber heat input rates based on fixed state points which are shown in Table 5 except the condenser inlet temperature. During operation under different working conditions, it is not possible to obtain a fixed temperature for a state point since the sizes of each component are constant. Therefore, the effect of different working conditions on the absorption cycle performance should be investigated for constant size of the components. It is assumed that the overall heat transfer coefficients do not change during the operation for simplicity. In fact, the overall heat transfer coefficients in the components change with different operating conditions because of the change of the temperature, flow rate and concentration. The change of the heat transfer coefficients in other components should be taken into account to obtain a more accurate result. However, it is possible to observe the effect of the rectifier and other operating conditions and make a parametric investigation because the change of flow rate, temperature and concentration of the refrigerant is around 10%. The required $UA$ values for 800 W desorber heat input rate are used for the parametric simulation. These values are shown in Table 11. The effect of desorber heat input rate on system COP for different absorber coolant temperatures are shown in Figure 26.
Figure 26. The effect of desorber heat input rate on system COP for different absorber coolant temperatures.

The COP is found between 0.25-0.45 in Determan’s study [3]. The COP is between 0.44-0.51 in Figure 26. The numerical estimation of the design point COP (800W desorber heat input and 37°C absorber coolant temperature) increases from 0.44 to 0.50 with the new design. More importantly, the COP does not significantly decrease with increasing desorber heat input rate; because the new design is able to purify the ammonia at higher desorber heat input rates. The coefficient of performance has a maximum at each absorber coolant temperature according to Figure 26. COP increases up to different desorber heat input rate for every absorber coolant temperature. The reason can be attributed to the increase of refrigerant flow rate leaving the desorber. The decline of COP as the desorber heat input rate is further increased can be understood with the ammonia concentration of the refrigerant which is shown in Figure 27.
Figure 27. The effect of desorber heat input rate on ammonia concentration of the refrigerant for different absorber coolant temperatures.

COP is higher when the absorber coolant temperature is larger because the desorber inlet temperature and the rate of evaporation in the desorber increase. However, the decline of COP is also larger as the desorber heat input rate is increased according to Figure 26. The reason is that COP starts decreasing when the refrigerant ammonia concentration is smaller than a certain value for each operating conditions. The reason of decreasing ammonia concentration in the refrigerant is that the rate of evaporation in the desorber increases and the stripping and rectifying sections becomes unable to purify the refrigerant because of high vapor flow rate.

Increasing absorber coolant temperature has also a negative effect on the refrigerant ammonia concentration, because the absorber outlet temperature increases which is also the coolant for the partial condensation. As the absorber outlet temperature increases, the heat transfer rate at the partial condensation unit decreases. As a result, less water is stripped off the refrigerant during the partial condensation and the ammonia concentration of the refrigerant increases. On the other hand, the increase of absorber coolant temperature increases the mass flow rate of the refrigerant
because of two reasons. First, as mentioned above, the amount of condensation decreases in during partial condensation. Second, the desorber inlet temperature becomes higher which results in more evaporation in the desorber. Therefore, similar to desorber heat input rate, the absorber coolant temperature has two opposite effects on COP. As desorber heat input rate and absorber coolant temperature are increased, the mass flow rate of the refrigerant increases but the ammonia concentration decreases. The peak point of COP for increasing desorber heat input rate is clearly observed in Figure 26. However, the peak point of COP for increasing absorber coolant temperature is barely observed when the desorber heat input rate is 1000 W in Figure 26. If the desorber heat input rate is further increased over 1000 W, the decrease of COP as a function of absorber coolant temperature would become clearer. According to Figure 28, COP decreases with increasing desorber heat input and absorber coolant temperature when the refrigerant ammonia concentration is low. However, it is not possible to run this simulation for higher desorber heat input rates, because the vapor temperature exceeds the critical temperature of ammonia.

In Determan’s study [3], COP decreases with increasing desorber heat input rate and absorber coolant temperature. COP did not have a maximum as a function of desorber heat input rate or absorber coolant temperature. The reason is that the rectifier was not able to sufficiently purify the refrigerant and the ammonia concentration in the refrigerant was already below 0.99 even for low desorber heat input rates. Therefore, the regions where the first derivative of COP is higher than zero in Figure 26 were not observed because of low ammonia concentration in the refrigerant stream. In contrast to Determan’s study [3], here COP does not decrease with increasing absorber coolant temperature. According to the trends in Figure 26, COP will start decreasing with increasing absorber coolant temperatures if the desorber heat input rate could be further increased. In order to observe different trends of COP in absorption cycle, the cycle is modeled without new stripping and rectifying sections, using constant $UA$ values in Table 6 including the original rectifier used in Determan’s study [3]. COP and the refrigerant ammonia concentration for different desorber heat input rates and absorber coolant
temperatures are shown in Figure 28 so that all possible trends of COP can be observed.

![Graph showing COP and refrigerant ammonia concentration](image)

**Figure 28.** System COP and refrigerant ammonia concentration for different desorber heat input rates and absorber coolant temperatures for the cycle design with original components shown in Figure 1 and Table 6.

According to Figure 28, COP increases with increasing absorber coolant temperature and desorber heat input rate, if the ammonia concentration in the refrigerant is higher than a critical value. When the ammonia concentration is high, the increase in refrigerant mass flow rate due to increasing absorber coolant temperature and desorber heat input rate is more important than the decrease in ammonia concentration. This trend becomes reversed when the ammonia concentration in the
refrigerant is low. However, this trend is not observed in this study because the stripping and rectifying sections are able to purify the refrigerant for a wide range of operating conditions. The trends in the high ammonia concentration region spread over the entire graph.

The effect of the stripper and rectifier height on system COP and cooling power are shown in Figure 29. The trends are exactly the same because the desorber heat input rate is constant and COP is calculated by dividing heat transfer rate at the evaporator (cooling power) to the desorber heat input rate. The total volume of the stripping and rectifying sections is kept constant during the simulation. The stripping and the rectifying section heights are shown in the x-axis of these figures. The heights of stripping and rectifying sections are 0.10 and 0.05 meters, respectively, for the base case. Since the diameters of each component are different, the change of height of each component is also not equal to each other so that the total volume is kept constant. According to Figure 29, there is an optimum ratio of stripping and rectifying sections. The optimum point depends on the operating conditions of the cycle and the heat and mass transfer phenomena in the distillation columns.

![Figure 29. System COP with different height of stripping and rectifying sections.](image)

The total volume of the stripping and rectifying sections is kept constant.
The system COPs for different number of fibers (or packing densities) in the rectifying and stripping sections are shown in Table 13 and Table 14, respectively.

**Table 13.** System COP with different packing densities in the rectifying section.

<table>
<thead>
<tr>
<th>Packing Density</th>
<th>Number of Fibers</th>
<th>COP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.204</td>
<td>16</td>
<td>0.489</td>
</tr>
<tr>
<td>0.306</td>
<td>24</td>
<td>0.490</td>
</tr>
<tr>
<td>0.395</td>
<td>31</td>
<td>0.492</td>
</tr>
<tr>
<td>0.496</td>
<td>39</td>
<td>0.495</td>
</tr>
<tr>
<td>0.598</td>
<td>47</td>
<td>0.498</td>
</tr>
<tr>
<td>0.700</td>
<td>55</td>
<td>0.500</td>
</tr>
<tr>
<td>0.789</td>
<td>62</td>
<td>0.500</td>
</tr>
</tbody>
</table>

**Table 14.** System COP with different packing densities in the stripping section.

<table>
<thead>
<tr>
<th>Packing Density</th>
<th>Number of Fibers</th>
<th>COP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.201</td>
<td>43</td>
<td>0.459</td>
</tr>
<tr>
<td>0.303</td>
<td>65</td>
<td>0.467</td>
</tr>
<tr>
<td>0.402</td>
<td>86</td>
<td>0.478</td>
</tr>
<tr>
<td>0.500</td>
<td>107</td>
<td>0.484</td>
</tr>
<tr>
<td>0.602</td>
<td>129</td>
<td>0.492</td>
</tr>
<tr>
<td>0.700</td>
<td>150</td>
<td>0.500</td>
</tr>
<tr>
<td>0.798</td>
<td>171</td>
<td>0.504</td>
</tr>
</tbody>
</table>

According to Table 13 and Table 14, the COP increases as the number of fibers is increased for both components. However, the rate of increase of the COP decreases as the packing density becomes larger. It is not possible to increase the number of fibers more, because of the geometrical constraints. According to Table 10, the diameters of the rectifying and stripping sections are 19.5 and 32.2 mm, respectively. The outer diameter of the fibers is 2.2 mm as shown in Table 7. The maximum numbers of fiber that can be fitted into the stripping and rectifying sections are 177 and 62, respectively. The correlations used in this study are also for the packing densities below 80%.
CHAPTER 4

CONCLUSIONS AND FUTURE WORK

4.1. Summary

The effective purification of the refrigerant is crucial for absorption cooling devices. The rectifier of the smallest absorption cooling system in the literature, which has 300W cooling capacity with a regular textbook size, is not able to sufficiently purify ammonia. As a consequence, the system has a relatively low COP and cooling power, especially at high desorber heat input rates. Therefore, a new rectifier design is essential in order to increase the COP and the cooling power of miniaturized absorption cooling systems. Hollow fiber membranes are found to be a good option for purification of ammonia because of its high interfacial area per unit volume. Ceramic hollow fiber membranes are thermally and chemically stable at the typical rectification temperatures in ammonia-water absorption cooling systems. The ammonia rectification performances of a hollow fiber membrane module and a random packing module with berl saddles are compared in this study. A mathematical model is developed based on the stagnant film model for the simulation of simultaneous heat and mass transfer. The rectifying and stripping sections are divided into finite number of segments. The heat and mass transfer rates in each segment are calculated based on the stagnant film model. The outlet temperatures, concentrations and mass flow rates are calculated using the energy, species and mass balances in each segment. Since the vapor and the liquid phases flow in opposite directions, the equations of each segment have to be solved simultaneously.
First, the detailed solution of the rectifier model is compared with the literature to verify the mathematical model. The performances of hollow fiber membranes and random packing as the internal structure of the rectifier are compared. The effects of the fiber packing density, effective porosity, pore radius and heat and mass transfer coefficients are investigated. The effect of number of segments and the grid independency of the rectifier model are shown. Then, the detailed model of the rectifying and stripping sections is integrated into the overall absorption cycle model to investigate the effect of the new design to the overall system performance. A stripping section and a rectifying section with partial condensation are used in the new design because it is recommended to use both components to purify the ammonia more effectively. The system performance with new rectifying and stripping sections is investigated. The COP and cooling power trends for different desorber heat input rate and absorber coolant temperature are shown in a relation with the ammonia concentration and mass flow rate. Finally, the effects of stripper and rectifier sizes and their packing densities on the overall system performance are examined.

4.2. Conclusions

Using hollow fiber membranes results in a higher ammonia concentration compared to the random packing. This result is expected because of the high interfacial area of the hollow fiber membranes. The results for the random packing module are compared with two different references so that the model is verified. Most of the mathematical model remains the same when the random packing is replaced by the hollow fiber membranes. The energy and mass balance and flux equations do not change when the internal structure of the rectifier is different. Therefore, the results for the random packing model are compared with the literature and it is assumed that both mathematical models are accurate. Newton’s method is used to solve the sets of non-linear equations. When the initial guesses are defined properly, the solution is obtained in a minute or less this method. However, there may be convergence problem when the initial guesses are not good enough.
The effect of packing density, membrane properties and heat and mass transfer coefficients on the distillate ammonia concentration is investigated. The packing density should be as high as possible so that the interfacial area and the distillate ammonia concentration are increased. However, uncertainty of the results because of the fiber irregularity also increases so placement of the fibers should be arranged carefully during the production. The effective porosity, mean pore radius and the vapor phase heat and mass transfer coefficients should also be as high as possible. These parameters are important because they directly affect the heat and mass transfer resistances of the membrane.

The system COP and cooling power are calculated after integration of the stripping and rectifying sections to the overall cycle. The numerical estimation of the COP is increased from 0.44 to 0.50 with the new design. The experimental value of the COP changes between 0.25-0.45 for different desorber heat input rates and absorber coolant temperatures in Determan’s study [3]. In this study, the COP is estimated between 0.45-0.52 for different desorber heat input rates and absorber coolant temperatures. The deviation is found smaller because the stripping and rectifying sections are able to purify ammonia for different working conditions. The COP is comparable with commercial single effect absorption cooling systems.

The COP and cooling power trends depend on the refrigerant ammonia concentration. The ammonia concentration does not significantly reduce when the desorber heat input rate and absorber coolant temperature are increased, because the new stripping and rectifying sections can effectively purify the refrigerant. Since the ammonia concentration is high and does not reduce with increasing desorber heat input rate and absorber coolant temperature, the COP and cooling power increases with these parameters. The reason is that the refrigerant flow rate is increased. However, this trend is reversed in the old absorption cycle, because the ammonia concentration of the refrigerant is low. When the desorber heat input rate and/or absorber coolant temperature is increased, the decrease of the refrigerant concentration causes the COP to reduce.
The effect of the stripper and rectifier sizes is investigated. The total volume of the stripping and rectifying sections is kept constant. The optimum sizes of the stripping and the rectifying sections are shown. The mass transfer is larger in the stripping section compared to the rectifier when the refrigerant concentration is low. When the refrigerant concentration is high, the mass transfer in the rectifier becomes higher than the mass transfer in the stripper. That is why there is an optimum value of the relative sizes of these components. The system COP’s for different packing densities in the stripper and the rectifier are estimated. The COP increases as the packing densities are increased, but the rate of increase reduces.

In conclusion, the new design of the rectifying and stripping sections results in a higher performance of the miniaturized absorption cooling device. The most important characteristic of the new design is that the system is able to purify the refrigerant for different operating conditions. Therefore, the absorption cooling device is more stable and its performance does not deteriorate when the operation conditions are changed.

4.3. Future Work

The future work for the hollow fiber membrane module and the absorption cycle can be discussed separately. The heat and mass transfer model in the rectifier should be accompanied by the hydrodynamic model to calculate the pressure drops which are neglected in this study. The pressure drop in the hollow fiber module may be important because the membrane pore may be wetted by the liquid if the pressure difference between the phases exceeds a critical value. Other transport mechanisms like capillary condensation and their effect on the rectifier performance should be investigated. The optimization of the pore radius is also essential because the membrane wettability and breakthrough pressure depend on the pore size. After developing a more sophisticated model, the results should be validated with experiments.
The absorption cycle model should be improved, because the current model does not take the effect of heat and mass transfer coefficients into consideration for the other components. The heat transfer model for the condenser, evaporator, RHEX and SHEX should be improved to estimate the real performance of the cycle. The heat and mass transfer in the absorber should be modeled in detail. The sizes of each component and the packing densities in the stripping and rectifying sections should be optimized with a cost analysis. The heat loss from the system and the parasitic heat transfer between the components should be calculated. Using hollow fiber membranes in the other components may be considered in the future. Finally, the overall system should be tested experimentally to validate the mathematical model. The preliminary results are promising and the ceramic hollow fiber membranes may be used in miniaturized absorption cooling devices. However, before making a commercial application, more theoretical and experimental research is needed. Using hollow fiber membranes in miniaturized absorption cooling devices is a new idea and it is open to improvements.
REFERENCES


APPENDIX

SOLUTION METHOD USED BY ENGINEERING EQUATION SOLVER SOFTWARE

The numerical solution of simultaneous non-linear equations is obtained by the extension of Newton’s method as described in the user’s guide of the Engineering Equation Solver software [53]. First, the residual of each equation is obtained by moving every term to the same side of the equation. The Jacobian matrix is obtained by differentiating each residual with respect to each unknown. The Jacobian is defined by Eq. A-1.

\[ J_{i,j} = \frac{\partial f_i}{\partial x_j} \]  

where \( J \) and \( f \) are the Jacobian and the residual, respectively. \( i \) and \( j \) represent the equation and the unknown, respectively. The Jacobian matrix is an \( N \times N \) matrix where \( N \) is the number of unknowns.

\[
\begin{bmatrix}
J_{1,1} & \cdots & J_{1,N} \\
\vdots & \ddots & \vdots \\
J_{N,1} & \cdots & J_{N,N}
\end{bmatrix}
\]  

The new value of each unknown is calculated by Eq. A-3 using the initial guesses, the inverse of the Jacobian matrix and the residual vector.
The software also groups the equations according to their number of unknowns and solves these block in sequence to increase the speed and the convergence. For example, the equations with one unknown are grouped in block zero and solved first. Then the equations in block one are solved and the procedure goes on. The iteration continues until all of the residuals become close to zero. The stopping criterion of the software is the maximum residual. The maximum relative residual has the order of magnitude of $10^{-10}$ and the maximum variable change according to Eq. A-3 is around $10^{-14}$ in this study.