ELECTRICITY MARKET MODELING
USING STOCHASTIC AND ROBUST OPTIMIZATION

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MİRAY HANIM YILDIRIM

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submitted by **MİRAY HANIM YILDIRIM** in partial fulfillment of the requirements for the degree of **Philosophy of Doctorate in Department of Scientific Computing, Middle East Technical University** by,

Prof. Dr. Bülent Karasözen
Director, Graduate School of **Applied Mathematics** _____________

Assoc. Prof. Dr. Ömür Uğur
Head of Department, **Scientific Computing, METU** _____________

Prof. Dr. Gerhard Wilhelm Weber
Supervisor, **Institute of Applied Mathematics, METU** _____________

Assist. Prof. Dr. Özlem Türker Bayrak
Co-Supervisor, **Industrial Engineering Dept., Çankaya Univ.** _____________

Examining Committee Members:

Assoc. Prof. Dr. Sevtap Kestel
Department of Actuarial Sciences, METU _____________

Prof. Dr. İnci Batmaz
Department of Statistics, METU _____________

Prof. Dr. Gerhard Wilhelm Weber
Department of Scientific Computing, METU _____________

Assoc. Prof. Dr. Yeliz Yolcu Okur
Department of Financial Mathematics, METU _____________

Assoc. Prof. Dr. Asım Egemen Yılmaz
Department of Electrical and Electronics Engineering, Ankara Univ. _____________

Date: _____________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: MİRAY HANIM YILDIRIM

Signature :

ABSTRACT

ELECTRICITY MARKET MODELING
USING STOCHASTIC AND ROBUST OPTIMIZATION

Yıldırım, Miray Hanım
Ph.D., Department of Scientific Computing
Supervisor: Prof. Dr. Gerhard Wilhelm Weber
Co-Supervisor: Assist. Prof. Dr. Özlem Türker Bayrak

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Sustainable development relying on sustainable and renewable energy systems is becoming one of the major policies of many countries. This forces the policy makers to establish many reforms and revolutions, which evolve electricity markets into a more competitive form. The competitive environment results in surging electricity demand and supply that brings in a critical challenge: uncertainty. In this thesis, the uncertainties with respect to prices and demand in the market are explored by using stochastic portfolio optimization and robust optimization techniques.

A stochastic optimization model is developed to maximize the overall expected profit in the electricity market by generating possible stochastic electricity supply and demand curves. Stochastic electricity supply curves of prices are generated by using Ornstein-Uhlenbeck mean-reverting process and running Monte-Carlo simulations. In order to overcome the drawbacks of this model, a second model is developed by using robust optimization techniques. This model handles uncertainties both in supply-demand balance of electricity and in renewable energy resources. The supply-demand balance of electricity is explored by using a novel hybrid approach: Wavelet-Multivariate Adaptive Regression Splines (in short: W~MARS). This method forecasts day-ahead electricity prices by considering the challenges such as high volatility, high frequency, nonstationarity and multiple seasonality. Then, we refine W~MARS by a novel robust optimization model, called Robust W~MARS (in short: R~W~MARS), which ensures sustainability and renewability by projecting ellipsoidal uncertainty. The models developed in the thesis are tested by using real electricity market data. Concluding remarks on the models and an outlook to future studies are presented at the end of the thesis.

Keywords: Electricity market; stochastic portfolio optimization; Ornstein-Uhlenbeck mean-reverting process; electricity price modeling, wavelet transform, Multivariate Adaptive Regression Splines, robust optimization, ellipsoidal uncertainty, W~MARS, R~W~MARS

ÖZ

ELEKTRİK PİYASASININ RASTSAL VE GÜRBÜZ OPTİMİZASYON KULLANILARAK MODELLENMESİ

Yıldırım, Miray Hanım
Doktora, Bilimsel Hesaplama Bölümü
Tez Yöneticisi: Prof. Dr. Gerhard Wilhelm Weber
Ortak Tez Yöneticisi: Yrd. Doç. Dr. Özlem Türker Bayrak

Mart 2015, 106 Sayfa

Sürdürülebilir ve yenilenebilir enerji sistemlerine dayanan sürdürülebilir gelişme birçok ülkenin esas politikalarından biri haline gelmektedir. Bu durum politikacıları, elektrik piyasalarını daha rekabetçi bir hale getiren reformlar ve köklü değişiklikler yapmaya zorlamaktadır. Rekabetçi ortam ise dalgalı elektrik arz ve talebine neden olmakta, bu da belirsizlik gibi kritik bir zorluğu beraberinde getirmektedir. Bu tezde, piyasadaki belirsizlikler rastsal portföy optimizasyonu ve gürbüz optimizasyon yöntemleri kullanılarak incelenmiştir.

Olası rastsal elektrik arz ve talep eğrileri oluşturarak elektrik piyasasındaki kârı maksimize eden bir rastsal optimizasyon modeli geliştirilmiştir. Rastsal elektrik talep eğrileri Ornstein-Uhlenbeck ortalama gerileme süreci ve Monte-Carlo benzetimi kullanılarak oluşturulmuştur. Bu modeldeki eksiklikleri gidermek için gürbüz optimizasyon teknikleri kullanılarak ikinci bir model geliştirilmiştir. Bu model hem arz-talep dengesindeki hem de yenilenebilir enerji kaynaklarındaki belirsizlikleri ele almaktadır. Elektrik arz-talep dengesi, yeni hybrid bir yaklaşım olan Dalgacık-Çok değişkenli Uyarlanabilir Regresyon Uzanımları (kısaca W~MARS) kullanılarak incelenmiştir. Bu yöntem, yüksek dalgalanma, yüksek frekans, durağan olmama ve çok mevsimsellik gibi zorlukları göz önünde bulundurarak bir sonraki günün elektrik fiyatlarını tahmin etmektedir. W~MARS,

eliptik belirsizliklerin izdüşümünü alarak sürdürülebilirliği ve yenilenebilirliği temin eden, Gürbüz W~MARS (kısaca R~W~MARS) olarak adlandırılan yeni bir gürbüz optimizasyon modeliyle iyileştirilmiştir. Tezde geliştirilen modeler gerçek elektrik piyasası verileri kullanılarak test edilmiştir. Tez sonunda modeller ile ilgili tespitler ve gelecek çalışmalar ile ilgili öngörüler sunulmuştur.

Anahtar Kelimeler: Elektrik piyasası, Gürbüz portföy optimizasyonu, Ornstein-Uhlenbeck ortalama gerileme süreci, Elektrik fiyat modellemesi, Dalgacık dönüşümü, Çok değişkenli Uyarlanabilir Regresyon Uzanımları, Gürbüz optimizasyon, Eliptik belirsizlik, W~MARS, R~W~MARS

To my husband
and
In memory of my dearest father

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TABLE OF CONTENTS

APPENDICES

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

1.1 Scope and Motivation

A typical energy system is a complicated structure comprising of production of electricity (i.e., conversion of energy resources to electricity), distribution of electricity across a grid structure, transport of energy resources, and all other interactions with the external world, namely the policy makers, end users, environment, and the energy sources. Figure 1.1 depicts these interactions.

Regarding an energy system, sustainability and renewability are the main concerns of the policy makers in many countries. However, especially two factors – dependency on fossil fuels (mainly oil) and contribution to global warming through emission of greenhouse gases (GHG) – are restraining the countries to achieve a sustainable and renewable energy system. The contemporary approach in reducing the dependency on fossil fuels and emission of GHGs, thus achieving a sustainable energy system, is to utilize renewable electricity generation technologies.

To propose solutions for this major problem in energy systems, researches, policy makers, and other partners mainly focus on managing the energy resources under certain constraints and assisting these efforts through improving energy policies and strategies.

On the other hand, regarding to the complexity of the system (cf. Figure 1.1), it can be deduced that a single global method to handle the entire system may be infeasible. Accordingly, a reasonable approach is to develop solutions for the subsystems, which may be integrated to give a comprehensive solution.

One of the major components of an energy system is the electricity market, which deals with the production and the pricing of electricity. Therefore, the main objective is to optimize the electricity market to determine the production portfolio subject to supply and demand constraints.

These constraints are often highly volatile in electricity markets. In most of the electricity markets, many reforms and revolutions have been established by policy makers to solve this surging in electricity demands and supply. As a consequence, *uncertainty* appears as the main challenge in an electricity market optimization problem. One of the main sources of this uncertainty in demand and supply is the competitive market structure. Moreover, in practice, it is often difficult to handle uncertainties without huge data sets, distributional assumptions, lengthy computational time, and excessive computational efforts.

Thus, advanced mathematical methods and computationally efficient approaches are needed to handle these uncertainties. In the scope of this study, the uncertainties in the electricity market are explored and modeled by using stochastic and robust optimization techniques. The study especially focuses on novel computationally efficient approaches.

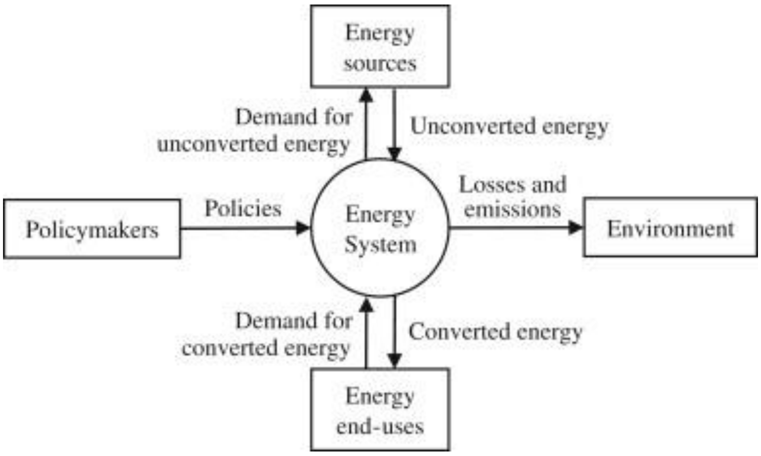

Figure 1.1: Energy system and its interaction with external world (source: [77]).

1.2 Objectives and Contributions of the Thesis

Traditionally, electricity markets are analyzed using optimization models and forecasting models. However, the uncertainties, especially in supply and demand, makes it difficult not only to optimize the diversity of the energy resources and power generation but also to determine electricity prices especially in the short-term. In addition, probability models of uncertainty and computations of multidimensional integrals related to expectations and probabilities are needed to be considered. Since chance constrained models are non-convex and generally intractable, **the main objective of the thesis** is defined as

- **Developing a dynamic and robust model for electricity markets under uncertainties**.

As the first step to achieve this objective, a stochastic model is developed. This stochastic model assumes a distribution based on the scenarios generated for supply and demand. On the other hand, we observed that the computational effort is high since the model requires the generation of a vast number of scenarios.

Robust optimization is proposed as an alternative to figure out this problem. Noting that robust optimization should involve an effective method to forecast the parameters involved, a hybrid method merging wavelet transform and multivariate adaptive regression splines (W~MARS[1]) is developed, which does not require any distribution assumption. On the other hand, the uncertainties in a system may be correlated or uncorrelated; mono-type or hybrid; single or multiple; interval, polyhedral, or ellipsoidal, etc. In our robust optimization model we propose an efficient method for modeling ellipsoidal, single/multiple, correlated/uncorrelated types of uncertainties. This approach is combined with W~MARS to give Robust W~MARS (R~W~MARS), which is capable of handling time-dependent uncertainties.

We foresee that the robust optimization model developed in this study has the potential to be an integral part of a comprehensive energy system model. However, this requires that the robust optimization model yields sustainable results. To assess the sustainability of the results, it should be examined whether the model ensures security of energy demand and supply. To establish a background for this analysis, a study on energy security indicators is conducted at European Commission Joint Research Center Institute for Energy and Transport in the Netherlands.

1.3 Organization of the Thesis

The thesis is organized as follows:

- Chapter 2 presents a review of existing approaches in electricity market optimization.

- Chapter 3 presents the models developed. Stochastic portfolio optimization model and robust optimization model, R~W~MARS, are explained.

  Stochastic optimization model requires generation of possible electricity supply curves and demand curves. Stochastic electricity supply curves thereby are modeled by Ornstein-Uhlenbeck mean-reverting process and running Monte-Carlo simulations.

  In R~W~MARS, a robust approach for handling the uncertainties is considered. As a part of R~W~MARS, W~MARS is used for estimating the electricity prices.

- In Chapter 4, applications of the models described in Chapter 3 are presented. Both models are demonstrated using a descriptive data set. Data and the results are also presented in this chapter.

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[1] In order to distinguish the abbreviation style of our method from the given notations of W-RMARS, W-RCMARS, etc., which stand for Weak RMARS, Weak RCMARS, respectively [113], here, we use a "~", which is an icon of a "wave". Therefore, we abbreviate to W~MARS, R~W~MARS, etc., in this thesis.

- Chapter 5 concludes the thesis by presenting a summary of the overall outputs and an outlook to future research.

CHAPTER 2

REVIEW OF ELECTRICITY MARKET OPTIMIZATION METHODS

Electricity market optimization models mainly aim to optimize the electricity generation portfolio, generally by maximizing the profit, while considering different parameters such as generator capacities, demand, supply and price.

The parameters included in electricity market optimization problem are typically interrelated. Among these parameters, considering a spot market, electricity price is extremely important since the balance between the price and demand should be set on the day before. Therefore, forecasting of the next-day electricity price is critical for spot markets.

This chapter is mainly dedicated to a review of existing methods in electricity market optimization and electricity price forecasting as a part of market optimization. On the other hand, considering electricity market as a major component of an energy system, we also find it useful to provide a brief review of energy system models in this chapter.

The chapter is organized to present reviews of existing energy system models, electricity market optimization models, and forecasting models for next-day electricity markets respectively in the following sections.

2.1 Energy System Models

Energy systems are tremendously large structures that include exploration and mining of energy resources, conversion into useful forms, generation, transmission and distribution of electricity, production of heat, and conversion of resources into useful energy. Table 2.1 shows types of energy models considering different aspects.

The ability to manage and control energy systems, which is very challenging especially when the geographical area is large, has both economical and environmental aspects. From oil crisis in 1970s to 2000s, many energy models were constructed regarding these concepts. Among these, Brookhaven Energy System Optimization Model (BESOM) is formulated as a linear programming model, whose objective can be defined as the minimization of system costs, the minimization of consumed resources or the minimization of emissions [36]. Although the model can be employed for regional systems, it is a static model, i.e., it comprises just one period. Another model, which is Time-stepped Energy System Optimization Model (TESOM), is a multi-period model and is based on BESOM [93]. However, TESOM does not include multi-regional or inter-regional issues. Energy Technology Assessment (ETA) is developed for the integration of energy and economic aspects. ETA minimizes overall energy system costs with multi-period perspective, but cannot be employed for multi-regional form. ETA-MACRO is derived from ETA in order to calculate energy consumption and investment. Market Allocation Model (MARKAL) is another model that integrates energy and economy and is developed by International Energy Agency (IEA). MARKAL is a dynamic linear model that minimizes overall system costs [4,128]. A variety of different versions of MARKAL, such as stochastic, multi-region, emissions trading and macroeconomic version, are developed. Stanford Research Institute (SRI) and Generalized Equilibrium Modeling System (GEMS), which include an economic equilibrium between energy demand and supply, are network and equilibrium models, respectively. Energy Flow Optimization Model (EFOM) is the basic model that is developed in and for Europe [42,67]. EFOM, which minimizes system costs and energy balances, can be utilized for calculating the investment for infrastructure. EFOM can cover single or multiple regions and its time span can be modified as static or dynamic. However, there is no integration between economical aspects and the energy system in EFOM. EFOM is modified as EFOM-ENV to analyze environmental issues. However, this new version does not consider economical aspects [21]. PRIMES is a modular model that is developed for European Union (EU) countries [48]. This model can be used for forecasting of one period, because it is a static network model. MESSAGE is a different type of energy system model developed by International Institute of Applied Systems Analysis (IIASA) [20]. MESSAGE involves sub-modules for investment analysis and energy consumptions and minimizes the total energy related costs for fifty years planning horizon. The main disadvantage of this model is that sub-modules cannot work simultaneously.

Table 2.1: Types of energy models [21].

General Purposes of Energy Models	**Specific Purposes of Energy Models**
To predict or forecast the future	Energy Demand Models
To explore the future (scenario analysis)	Energy Supply Models
To look back from the future to the present	Impact Models
(“backcasting”)	Appraisal Models

Analytical Approach	**Geographical Coverage**	**Sectoral Coverage**
Top-Down Models	Global	Single-sectoral models
Bottom-Up Models	Regional	Multi-sectoral models

Several authors have evaluated main renewable energy technologies by taking sustainability indicators into account. For instance, the paper [49] compares wind power, hydropower, photovoltaic and geothermal energy considering the price of generated electricity, greenhouse gas emissions, availability of renewable energy resources, efficiency of energy conversion, land requirements, water consumption, and social impacts. The article [98] proposes strategies for a renewable and sustainable energy system considering technological improvements. The work [146] suggests a new perspective for a renewable and sustainable energy system. The authors conclude that wind and small hydroelectricity power plants are the most sustainable sources for the electricity generation.

Constructing renewable and sustainable energy systems requires high investment costs. Therefore, a long-term planning of the investment strategies is very important. The article [34] proposes a community-scale renewable energy system's model and solves the mathematical model by using interval linear programming, chance-constrained programming, and mixed integer-linear programming (MIP). The work [89] proposes a modified EFOM model to find optimal capacities of power plants and volumes of emissions trading with minimum cost and maximum robustness. The paper [125] utilizes MIP and three artificial intelligence techniques for network planning under the carbon emission trading program. The work [91] uses multi-criteria decision making for assessing sustainability of renewable energy scenarios. The paper [105] proposes a multi-objective optimization model to choose the most cost-effective mix of renewable system by maximizing the contribution to the peak load and minimizing the combined intermittence. The article [41] proposes a modified EFOM model to analyze the policies for using renewable energy resources.

There are also studies considering Turkish energy system. Starting of these efforts on modeling Turkish energy system date back to three decades ago. However, the models developed cannot describe Turkish energy system's current structure. As a consequence, energy policies and strategies cannot be developed based on a model. The first energy model for Turkey, which was constructed by Kavrakoğlu et al. [85], is an MIP-based model and handles the country as a single bulk region. Modified version of ETA was implemented for Turkey by [16]. ETA-MACRO model was later implemented in Turkey as described in the paper [71]. In the second half of 1990s, energy-environment interaction [92], energy-environment-economy interactions are modeled [16,92]. In this progress, renewable and sustainable energy strategies and policies are not imposed on models. The analyses are made for political and strategic issues without taking mathematical planning and optimization into account. Most of the studies made until now show that only strategic aspects have been considered to analyze the conversion of clean energy. For instance, the article [86] proposes the renewable energy policies for Turkey and explains the role of political organizations that shape these policies. There also exists review studies for Turkish energy sector [112]. The article [83] proposes sustainable energy policies and utilization of the renewable energy sources for Turkey. It is also known that improving renewable energy technologies not only solves several energy related environmental problems but also helps sustaining the development. The optimization algorithms can be preferred as suitable tools for approaching complex renewable energy systems [19]. However, Turkish energy planning system is still lacking of an optimization algorithm for sustainable and renewable structure.

2.2 Electricity Market Optimization Models

Many electricity markets have been evolving into spot markets only since the last decade. Therefore, there is not many integral electricity market optimization model considering all aspects of the market.

Electricity market optimization studies in the literature utilize different approaches. In the papers [18,109] multi-stage stochastic portfolio optimization models under demand uncertainty are presented. However, in these works the authors did not cover uncertainties in production costs and their effects to electricity prices. The article [135] proposes a mixed-integer stochastic programming approach for the selection of power generation technology. However the method generates limited number of scenarios, which in turn affects the final results. The work [52] combines the management of electricity portfolios with financial risk factors. In another work, expected profit is maximized under uncertainties in inflow of hydropower plants and price [68]. However, the work excludes different types of generators. The model in the article [129] involves a multi-stage stochastic program, which considers different types of generation costs, investments, and the effect of gas market. The paper [162] develops a two-stage method, which optimizes both portfolio selection and conditional value at risk. The reader may refer to [104,149] for extensive reviews on stochastic programming of electricity and energy markets.

In addition to these studies, there exist approaches regarding electricity markets based on game theory. These approaches are especially used for wholesale markets where the power generation companies directly sell electricity to customers. Here, companies can join and stay in a market individually (non-cooperative) or collaboratively (cooperative) in order to gain more profits [82,103,172]. These approaches are also extended to dynamic games, stochastic games, games with emission restrictions and combinations of these. For instance, behaviors of the collaborative market participant are analyzed for uncertain coalition values in [5,6]. Moreover, a new dynamic game theory model is presented by [97]. This study considers emission restrictions with collaborative market participants. Therefore, one model has been developed and called as the Kyoto game. Games with Kyoto restrictions are also discussed in [151]. The time-discrete dynamic structure of Kyoto game and its theoretic background are highlighted in [154,156]. The reader may refer to [33,45] for an extensive survey on these game-theory based approaches.

Traditional methods like multistage stochastic programming for electricity planning models under dynamic and uncertain conditions result in computationally intractable models. These methods can also yield infeasible solutions even if small perturbations occur. Particularly this case can be observed in many energy planning models, since the majority of the literature assumes certain parameters for these models [17]. One of the major concerns for electricity planning is *sustainability*, which is directly related with scenario analyses. Robust and stochastic optimization approaches improve the capability of scenario analyses, because of the uncertainty sets defined for the input parameters. For instance, when the green-house gases (GHG) emission is desired to be projected through 40 years, an uncertainty set can be defined for corresponding input parameters. The most important advantage of using robust optimization is to guarantee a feasible solution even when input parameters change from scenario to scenario. There are limited number of studies that use robust optimization in this field [17,89]. None of these studies involves multiple uncertainties in their model.

2.3 Forecasting Models for Next-Day Electricity Market

In this section, existing techniques are reviewed based on the classification shown in Figure 2.1. Although these methods are commonly used for price forecasting, they can also be utilized to forecast demand. The methods are classified mainly as time-series based methods and game theory based methods. Time-series based methods are further classified based on the existence of explanatory variables in the model.

Comprehensive reviews on electricity price forecasting are available in the literature [33–35]. Among these, especially the papers [33,34] focus on short-term forecasting. However, there is no substantial review work specific to next day's electricity price forecasting.

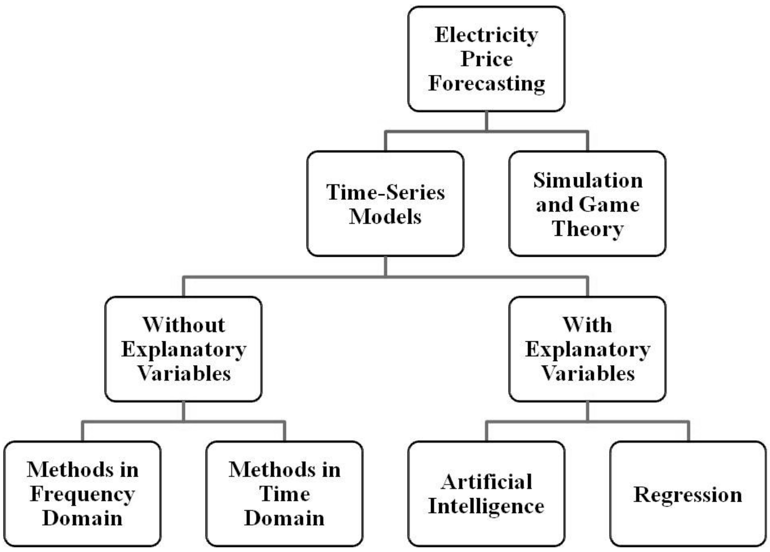


Figure 2.1: Categorization of the methods used in next day's electricity price forecasting.

2.3.1 Time-Series Models

Time-series models are especially useful in handling common characteristics of next day's electricity price data, which are seasonality and high volatility, outliers, high rate of recurrence, and non-constant mean and variance [37]. Among time-series based models, based on the existence of explanatory variables, neural networks (NN), support vector machines (SVM), data mining, generalized autoregressive conditional heteroskedasticity (GARCH), and dynamic regression (DR) are the methods with explanatory variables. On the other hand, wavelet transforms (WT) in frequency domain, autoregressive (AR) models, integrated (I) models, moving average (MA) models and their combinations (ARIMA, ARMA, ARMAX, etc.) in time domain are the methods without explanatory variables. Mathematical formulations of some of these models are given in Table A.1 for reference purposes.

2.3.1.1 Time-Series Models with Explanatory Variables

Time-series models with explanatory variables consider the factors such as electricity demand, fuel price, available generator capacities, temperature, and humidity, and identify their effects on the electricity prices. These methods are categorized as artificial intelligence (AI) and regression-based methods.

AI methods mimic human brain in order to train future prices by using electricity price history and the factors affecting the prices. Figure 2.2 schematically represents this process.

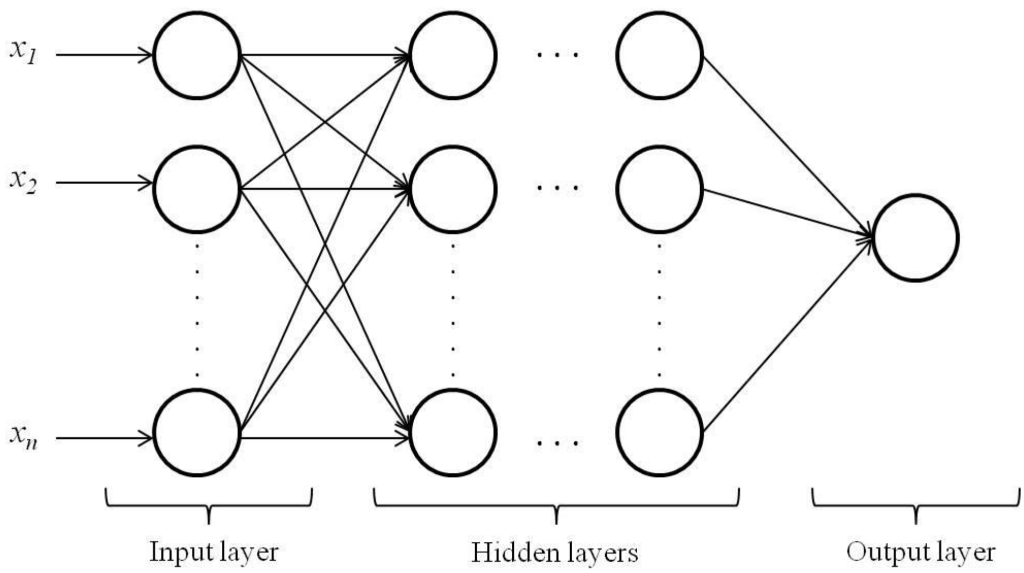


Figure 2.2: Basic representation of a Neural Network.

NN and its variations are the most commonly used methods among time-series based methods with explanatory variables. NN is used in the papers [59,150] to forecasts next day's electricity price, with a focus on the weekends and public holidays in the latter work. Artificial NN (ANN) is used in the articles [30,102,121,123,141,144] to forecasts electricity prices. An ANN-based approach - Input-Output Hidden Markov Model (IOHMM) – is used in the paper [65] to forecast next day's electricity price. Cascaded NN, which involves a chain of NN engines, is proposed in [11]. Fuzzy NN used in the work [7] is another method derived from NNs. NN yields smaller errors compared to DR, ARIMA, and transfer functions.

It is known that if there are no spikes in the prices, ANN gives better results. However, in case of spikes an enhanced probability neural network (EPNN) yields accurate results [95]. EPNN adds a new layer, summation, and a new process, orthogonal experimental design, to the network in order to decrease the forecast error.

There are hybrid approaches that use AI philosophy. Support vector machines (SVM) and evolutionary algorithms (EA) are also used to forecast next day's electricity market. The article [56] uses a SVM algorithm with particle swarm optimization (PSO) for several electricity markets. Other combinations are made by [51] and [126]. In [126], the parameters of SVM are optimized by a genetic algorithm and results are obtained with acceptable accuracy. On the other hand, [51] uses a self-organized map network (SOM) to group the input data set as an unsupervised learning mechanism and SVM to fit the training data as a supervised learning mechanism. NN and evolutionary algorithms with an iterative parameter search process are combined in article [10]. SVM, PSO and SOM are used together to forecast electricity prices in the article [107].

The forecasting methods based on regression model the relationship between electricity prices and the factors that affect the prices. Thus, electricity price can be estimated by using exogenous variables like demand. Dynamic regression and generalized autoregressive conditionally heteroskedastic (GARCH) method are the most common techniques in this category. However, in most studies, dynamic regression is used to compare the prediction performances. For instance, in [108], dynamic regression and transfer function methods are applied. The relationship between fundamental factors (such as demand, demand slope and curvature, demand volatility, excess of generation capacity, scarcity, price volatility, etc.) and their effects over time via several versions of regression are modeled in [84]. In [60], multivariate regression is used to analyze the effect of renewable energy to electricity prices. Forecasting is made by using GARCH method in [57]. GARCH seasonal dynamic factor analysis (GARCH-SeaDFA) [58] is a specific approach for the structure of electricity price data. The forecasting performance of GARCH models is especially better when volatility is included. Because of this fact, [70,72] use GARCH models for their analyses.

Generalized additive models (GAM) are used to maximize quality of prediction via involving nonlinear effects [74,140]. The paper [130] uses generalized additive models via location, scale and shape estimation of specific time instances. The estimated parameters are used as an input for dynamically changing prices. Another approach is proposed in [116] by using the logic given in [63,64]. In [116], GAM generates an initial model of next day's price which is improved by a robust optimization technique.

2.3.1.2 Time-Series Models without Explanatory Variables

Electricity prices can be predicted by the models that do not involve explanatory variables. A classification of these models can be made based on whether time domain or frequency domain is used. AR, ARIMA, and ARMA are the most frequently used time-based models. On the other hand, wavelet transform (WT) enlarges the time space to the time-frequency space. For instance, [9] applies WT as preprocessor in order to make this expansion and then to forecast prices with a better performance via combination of NN and EA. One of the main advantages of WT is that the method can decompose time-series in time and frequency. By considering this advantage, [22] proposes a WT with multi-resolution decomposition. The method performs better than single resolution forms.

In liberal markets, electricity price data generally have a high frequency, non-constant mean and variance, and multiple seasonality. Thus, AR, ARIMA and ARMA are very suitable methods for this kind of data. The ARIMA model proposed by [39] gives reasonable prediction errors for. In some studies, models without explanatory variables are combined with models with explanatory variables. For instance, [136] suggests a model by using ARMA extended by GARCH. Similarly, [38] proposes a WT-based model combined with ARIMA, and [137] presents a WT-based model combined with ARIMA and GARCH. ARIMA and its variations are used in [32]. AR models with nonparametric extensions proposed in [159]. In [84], regression and AR are equipped with time-varying parameter effects. AR and regression models give better results when they include time-varying coefficients.

2.3.2 Simulation and Game Theory

Game-theory and simulation-based methods are generally devoted to improve strategies for market participants. These methods are developed for predicting market operators' buying and selling bids. Moreover, simulation models try to imitate the real market and its conditions directly. For instance, [27] develops a market simulator for the Spanish market. The algorithm includes the following steps: (i) calculation of intersection of supply, demand and market clearing price for each hour in a day, (ii) assignment of selling bid, (iii) assignment of buying bid, (iv) acceptance of the bids if the maximum variation of the unit output between two consecutive hours is between the required limits, (v) verification of bids for non-divisible quantity rule, (vi) verification of minimum revenue rule. The algorithm used in [27] guarantees a feasible result.

Game theory is generally used to determine bidding strategies. A generation of companies' bidding strategies under operational constraints is investigated in [31]. A static game theory and a cost-minimization unit-commitment algorithm are developed for generating companies. Thus, the companies can analyze bidding strategies in the market.

There are also combined versions of game theory and simulation in order to use the advantages of both methods. For example, [69] proposes a multi-agent based simulation model for physical power exchange markets. Stochastic control theory is used by [61], where a Cournot competition model is considered for bidding strategies. In addition, simulations are made to see long-term optimal strategies.

2.3.3 Other Approaches

Forecasting next day's price is a challenging problem since the prices can be affected by many factors. As a consequence, different approaches are developed to tackle these issues and to suppress the disadvantages of classical methods. For instance, in [175], the market price is investigated for New York Independent System Operator day-ahead market with different demand values. Satisfactory predictions can be made for the Spanish electricity market by using weighted nearest neighbors [96]. A forecasting system with multi-component, which consists of a fuzzy inference system, an intelligent system, and least-squares estimation is developed by [94]. Designing the input vectors of electricity prices is an important issue that affects the forecasting performance. A hybrid NN model is proposed by [8] with a relief algorithm. This algorithm is used to select the features of the input vector. In order to handle daily seasonality, [147] uses a functional nonparametric model. Here, the electricity price is considered as the discrete-time realization of a continuous-time stochastic process.

Forecasting of spikes in the prices is another issue in day-ahead electricity markets. In [13], a specific method that consists of a probabilistic NN and a hybrid neuro evolutionary system is developed. Another hybrid neuro-evolutionary system is developed in [12] to improve forecasting accuracy. A hybrid nonlinear chaotic dynamic and evolutionary strategy-based approach is developed in [143]. The article [15] uses autoregressive-moving-average model with exogenous inputs (ARMAX), where fuzzy logic is employed. A hybrid wavelet-ARIMA and radial basis function neural model is proposed in [131] to obtain an improved accuracy with less input data. Stochastic programming is also applied in price forecasting, especially, for bidding strategies. For instance, a quadratic mixed-integer stochastic programming model is proposed in [40] for optimal-4qbid strategies. In [53], a stochastic mixed-integer linear programming model is used for the bidding problem.

Among next-day electricity price forecasting methods, NN-based ones are the most common. Errors ranges involved in these NN-based methods and traditional methods such as AR, ARIMA, GARCH, linear regression (LR), and multiple regression (MR) are presented in Tables A.2 and A.3 for reference purposes.

In some electricity markets, the explanatory variables (e.g., electricity demand, fuel price, available generator capacities, temperature, humidity) are highly affective. Especially in such systems, traditional methods yield significant errors. The most critical factors affecting these systems are price history and electricity demand. Other common factors include resource prices, generator capacities, climate effects, and time slot. Factors affecting the prices are tabulated in Table A.4. These factors vary with respect to the market of concern. Among the electricity markets, mainly European markets - especially, the Spanish - are studied, which is triggered by the early revolution into a competitive market structure (cf. Table A.5). For a detailed review of the models presented here, the reader may refer to [167].

CHAPTER 3

MODELS DEVELOPED FOR ELECTRICITY MARKET OPTIMIZATION

This chapter mainly describes two electricity market optimization models, stochastic optimization model and robust optimization model, developed in this thesis.

The stochastic optimization model is based on the generation of supply and demand scenarios for a given set of market data. In this model, Ornstein-Uhlenbeck mean-reverting process and Monte-Carlo simulations are used to model stochastic supply curves.

As a part of robust optimization model, we developed a hybrid model merging wavelet transform and multivariate adaptive regression splines (W~MARS) to forecast the parameters involved in the system. This model is combined with an efficient method for modeling uncertainties in the system to give the robust counterpart of W~MARS (called R~W~MARS).

Stochastic optimization model and robust optimization model, including a detailed explanation of W~MARS and R~W~MARS, are presented, respectively, in the following sections.

3.1 Stochastic Optimization Model

Countries aim to create economically efficient electricity portfolios considering two basic energy security indicators - affordability and availability - while preventing any energy shortage. However, due to uncertainties both in supply and demand, stochastic optimization techniques are often required in creating the portfolio.

Here, a novel stochastic and simulation based method, which utilized Ornstein-Uhlenbeck mean-reverting process and Monte-Carlo simulations, is described. The methodology involves generation of stochastic supply curves for different scenarios by considering the power-generation techniques. These scenarios are incorporated in a stochastic mixed-integer portfolio optimization model to maximize the profit and to obtain the most economic diversity of energy resources.

Figure 3.1 illustrates the approach used in the stochastic optimization model. Inputs of the model include the supply curves, demand scenarios, and the power plant capacities. This optimization model maximizes the expected profit under specified constraints to give optimal market prices and quantities.

The main contribution of the method presented here is in modeling of the electricity supply curves and their integration into a stochastic mixed-integer optimization model. The supply curves are constructed as piecewise linear functions and market prices are determined by considering electricity production costs, electricity demand, natural gas prices, exchange rates, and generator types. Modeling of the supply and the demand curves and the stochastic portfolio optimization model are described in the following subsections.

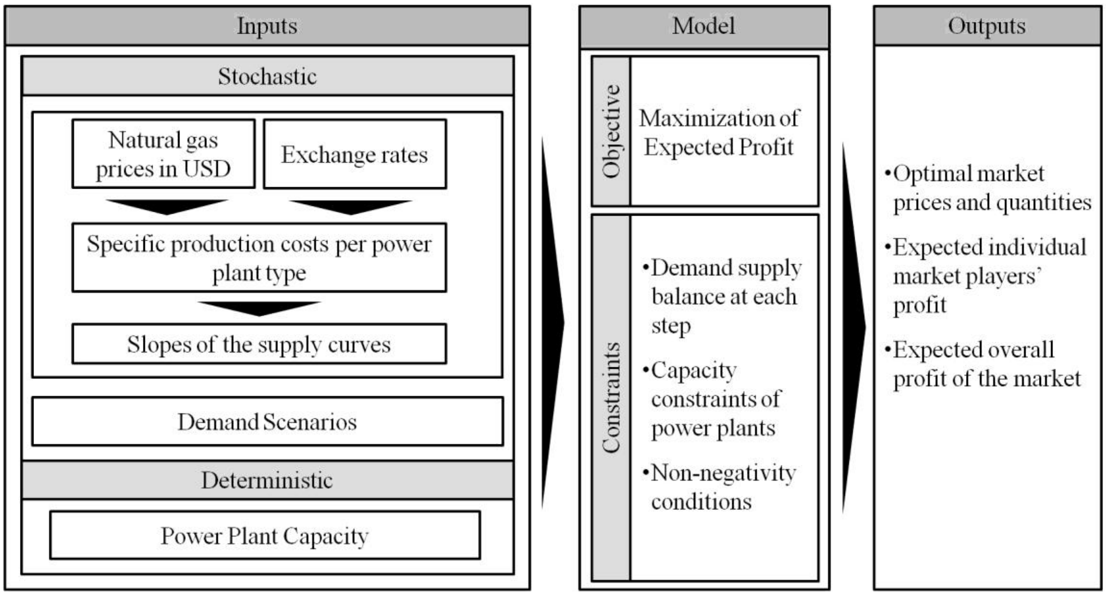


Figure 3.1: Approach in the stochastic electricity market optimization model.

3.1.1 Modeling the Supply and the Demand Curves

The supply and the demand curves especially determine the optimal market outcome, i.e., the price and quantity for a good [120,145].

A *merit-order curve* is usually employed to represent the total electricity supply when defining the market. Such a curve presents the marginal costs and capacities of all generators and ranges from the least expensive to the most expensive unit. The generation technology and the fuel used are the main causes for the marginal costs. For instance, usually gas power plants have higher production costs compared to hydropower and nuclear power plants. Figure 3.2 illustrates such a merit-order curve, where solar electricity generators have the least production cost and oil-based electricity generators have the highest production cost. In this figure, the intersection of the supply curve defined by the merit-order curve and the demand determines the optimal market outcome. Energy companies, whose production costs are lower than the market price, produce as much as possible, considering their available capacities. Since the production costs among the plants are different, any producer with the lowest production cost earns relatively more than any other producer with relatively higher production cost. Accordingly, for the demand scenario in Figure 3.2, producer 5 stands just at the breakeven point whereas producers 1-4 generate profit. Therefore, the remaining plants should not operate for the market to make a total profit. When the demand becomes low, power plants with higher operating costs standstill, which results in lower total production costs, hence, in a lower electricity price. On the contrary, when there is an increase in demand, the power plants with higher costs are needed to satisfy the demand; this results in a higher electricity price. However, the electricity production and demand are also affected by uncertainties. Therefore, different stochastic scenarios for the supply and demand curves are modeled instead of a deterministic approach to generate different possible optimal market outcomes and profits.

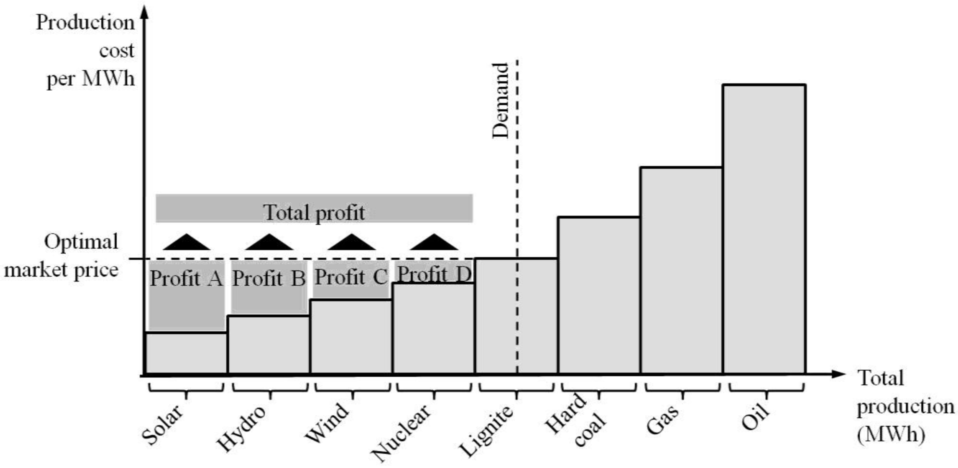


Figure 3.2: Integrated electricity supply and demand.

3.1.1.1 Modeling the Supply Curve

Considering common resources for electricity generation (cf. Figure 3.2), natural gas is especially important. Worldwide, natural gas is traded in USD and the prices are generally set by the *Henry Hub*, which is an important distribution hub on natural gas pipeline in the USA. However, natural gas prices are highly volatile [66]. Therefore, since gas-fired power plants have high production costs, the supply curve must be considered as a stochastic parameter, especially, for the markets, which are highly dependent on gas-fired power plants.

For a generic electricity market, the electricity consumption may be billed in a currency different than USD. Therefore, stochastic foreign exchange rates between USD and the local currency must also be considered when determining the supply curve. In order to obtain production cost and supply curve scenarios for each electricity producer, Monte-Carlo simulation technique is used to obtain consistent and unbiased estimators [90,111]. Each scenario is generated by considering the effect of stochasticity in natural gas prices and exchange rates. For this purpose, well-known Ornstein-Uhlenbeck mean-reverting process is used since commodities like oil and gas, and the exchange rates mostly exhibit mean reversion [142]. This implies that they tend to return to a long-term mean over time. The Ornstein-Uhlenbeck mean-reverting process is defined as

$$dS_t = \alpha\left(L - S_t\right)d_t + \sigma dW_t, \tag{3.1}$$

where $\sigma > 0$ and $\alpha > 0$. Here, $(S_t)_{t\geq 0}$ is the price process of a risky asset, namely, $(W_t)_{t\geq 0}$ is a standard Brownian motion, and σ is a constant volatility. L is the long-term mean of the process S_t, to which it reverts over time, and α “measures” the “speed” of mean-reversion. The explicit solution of this process is [62]

$$S_t = e^{-\alpha t}S_0 + \alpha\left(1 - e^{-\alpha t}\right) + \int_0^t e^{a(s-t)}\sigma dW_s. \tag{3.2}$$

Here, S_0 denotes the initial price at time $t = 0$. Through a regression analysis, the parameter α can be estimated. Similarly, if a mean-reversion process is assumed, the volatility σ may be estimated from historical data. For a detailed analysis and discussion about parameter estimation of the Ornstein-Uhlenbeck mean-reverting process, readers may refer to [157,158].

Instead of constructing scenario trees for each parameter separately, a two-dimensional scenario tree is constructed (cf. Figure 3.3) to have a computationally efficient scenario generation procedure. Natural gas prices and exchange rates are generated simultaneously for each time step. By running and combining scenarios for the natural gas prices and exchange rates, $k \times m = n$ scenarios are generated at each time step. Here, k is the number of gas price scenarios, m is the number of exchange rate scenarios, and n is the total number of scenarios per time step. Kolmogorov-Smirnov (uniform) distance is used while generating scenarios [47,75,119].

For each simulated gas price and exchange scenario, discrete supply curves are modeled as piecewise linear functions between each producer. The intersection of the supply curves with the demand curves resulted in different optimum market outcomes and profits. One illustrative piecewise linear power supply curve is shown in Figure 3.4(a). For this particular case, the intersection of the mean demand scenario and the supply curve resulted in a market price indicated by the dash lines in Figure 3.4, meaning that power producers 1-4 can provide all of their available capacities and power producer 5 only one part of his capacity. For this scenario, total profit is found by calculating the area between the dash line and the piecewise linear supply curves. By using this approach, a total number of $k \times m$ supply scenarios are generated for each time period. Figure 3.4(b) presents illustrative piecewise supply curves for all scenarios.

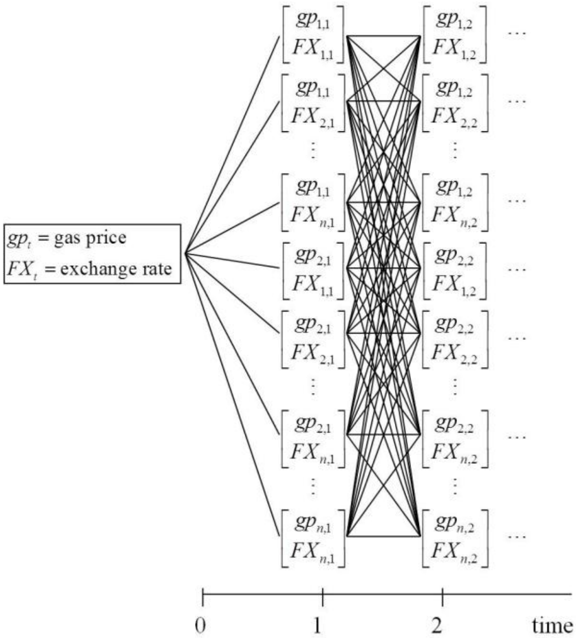


Figure 3.3: Two-dimensional scenario tree.

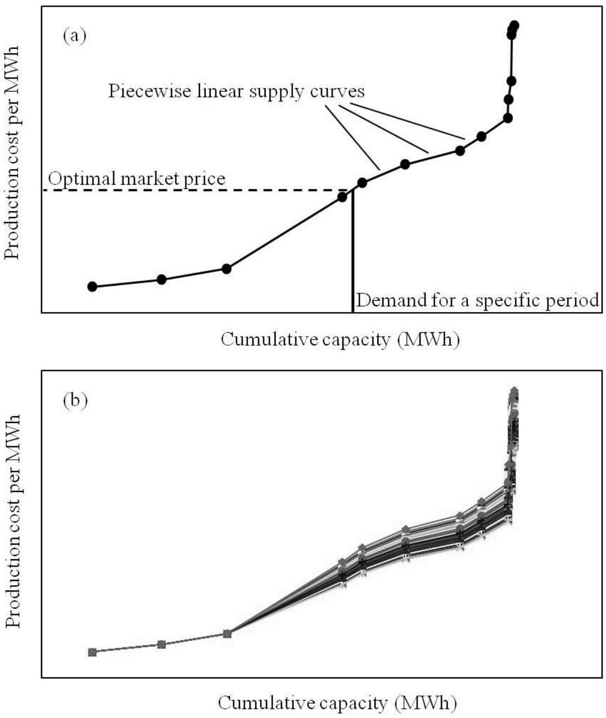


Figure 3.4: Illustrative piecewise supply curves; (a) for a particular scenario, (b) for all scenarios.

3.1.1.2 Modeling the Demand Curve

The electricity demand typically consists of a *base* load, a *middle* load and a *peak* load. The *base* load is defined as the permanently demanded power during 24 hours and 365 days per year, whereas the *middle* load is the power which is additionally demanded to the base load during some hours per day. Finally, the *peak* load represents the power which is demanded only in few hours and/or days per year. Typically the peak loads occur at 12 p.m. and 6 p.m. daily. In summary, there is high variation in the electricity demand between peak times (day) and off-peak times (night); and between the seasons (winter and summer). Daily variation is basically caused by the decrease in the need for electricity consumption by most of the companies and households. Hence, the electricity demand is the lowest in mid-night. On the other hand, seasonal variation is caused by the use of electric heaters or air conditioners during very hot or very cold periods of the year. For most of the consumers, the elasticity of the demand is very low. This is because of the lack of substitutes for energy and the costumers overrating the product.

Overall electricity consumption is modeled for each time period. To have the mean deterministic demand forecast, a growth factor is determined by using the geometric mean formula and multiplying it with the demand of the previous period. To include the variability of this forecast, the approximate standard deviations from this mean demand scenario are calculated and included. As a result, one deterministic demand, one high-demand and one low-demand scenario are obtained. Figure 3.5 presents an illustrative example showing deterministic, low- and high-demand scenarios.

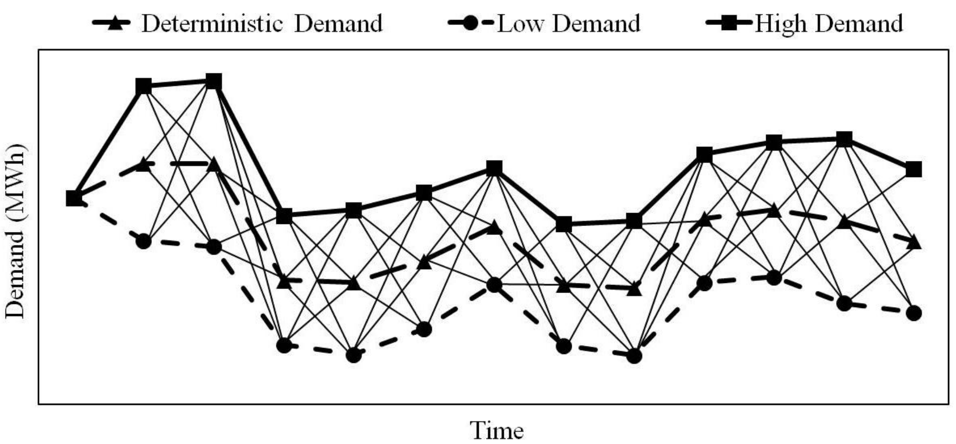


Figure 3.5: Deterministic, low-, and high-demand scenarios.

The inputs used in modeling the supply and demand scenarios are listed below for reference purposes:

- Periodic (e.g., monthly) energy consumption data for modeling demand scenarios.
- Production data of existing power plants to determine yearly effective production mean.
- Production costs of each power plant per generation type to derive the supply curves.
- Periodic Henry Hub natural gas spot prices to simulate production costs through a stochastic price process.
- Averages of periodic exchange rates to simulate production costs through a stochastic price process.

Normally, since the standard error is proportional to $1/\sqrt{\text{sample size}}$ in Monte-Carlo simulation [43], number of demand and supply scenarios should be kept high to minimize the error. However, this considerably increases the computational time. Here, reduced number of scenarios can be randomly selected to decrease the computational time. Referring to the Central Limit Theorem, the required number of scenarios can be determined by using $n = \left[\frac{z_\delta \cdot S}{\varepsilon \cdot \bar{x}}\right]^2$, where S shows the sample standard deviation, z_δ represents the confidence level and ε is the percentage error [63].

3.1.2 Optimization Model

The electricity demand and supply, along with the generation capacities serve as limiting constraints for the expected profit maximization model [99]. A mixed-integer stochastic model is given below in Equations (3.3)-(3.8) for any given supply scenario k and any demand scenario l:

maximize

$$z = \sum_{t=1}^{12}\sum_{i=1}^{14}\left(prob^k\left(\pi_{ti}^k - c_{ti}^k\right)\right)x_{ti} - \sum_{i=1}^{14} f_i y_i \tag{3.3}$$

subject to

$$\sum_{i=1}^{14} x_{ti} = d_t^l \qquad \left(t = 1,\ldots,T\right), \tag{3.4}$$

$$\sum_{t=1}^{12} x_{ti} \le s_i^k \qquad \left(i = 1,\ldots,P\right), \tag{3.5}$$

$$\sum_{i=1}^{14} y_i \le 14, \tag{3.6}$$

$$\sum_{t=1}^{12} x_{ti} \le y_i M \qquad (i=1,\dots,P), \tag{3.7}$$

$$x_{ti} \ge 0,\ y_i \in \{0,1\} \qquad (t=1,\dots,T;\ i=1,\dots,P). \tag{3.8}$$

Here, $t=1,\dots,T$ is the index for time period, T is the number of time periods (e.g., $T=12$ if the period is monthly), $i=1,\dots,P$ is the index for power plants, P is the number of power plants, k is the index for scenarios, $l=1,2,3$ is the index for demand scenarios (deterministic, low, or high), π_{ti}^k is the market price of electricity supplied, c_{ti}^k is the variable electricity production cost, f_t is the fixed operating and maintenance cost, d_t^l is the total electricity demand at time t, s_i^k is the total capacity of plant i, $prob^k$ is the probability for occurrence of each scenario k, and M is a sufficiently large positive number. Occurrence of each scenario is assumed to be uniformly distributed. The decision variable x_{ti} represents the amount of electricity that can be supplied by plant i in period t. The binary decision variable y_i shows whether the plant i is dispatched or not.

The total capacity of the plants may exceed the total load demand, and excess supply cannot be stored. Thus, Constraints (2.4) imply that all the demand can be supplied by using equality constraints. On the other hand, Constraints (2.5) imply that the total amount of electricity cannot exceed the total capacity of plants. Constraints (2.6) and (2.7) guarantee that at most P number of plants can be dispatched and if they are dispatched, then they should produce electricity. Constraints (2.8) show non-negativity conditions of decision variables. Specific technical parameters of the plants (i.e., ramp up and ramp down rate, shut down und start up costs of the power plants, etc.) are neglected due to the lack of data. Transmission costs are also excluded since the current conditions refer to an incomplete type market [122].

The mixed-integer stochastic model is solved for each of 3 demand scenarios and k number of supply scenarios. At each run, price and variable electricity cost, demand, and supply values are changed according to the outputs of scenario trees.

3.2 Robust Optimization Model

Many constraints involved in an electricity market are often highly volatile. A robust optimization model is developed as efficient method for modeling ellipsoidal, single/multiple, correlated/uncorrelated types of uncertainties. This model also requires forecasting of the parameters, such as price. Considering high volatility of the market, these parameters need to be forecasted precisely in short term. The robust optimization model developed in this study, makes use of a hybrid method merging wavelet transform and multivariate adaptive regression splines (W~MARS) for forecasting the parameters involved. This section explains W~MARS and the robust counterpart model, of W~MARS, called as R~W~MARS, in closer detail.

3.2.1 Wavelet – Multivariate Adaptive Regression Splines (W~MARS)

The forecasting methods reviewed in Chapter 2 have their own strengths and weaknesses. Therefore, researchers have developed hybrid methods to merge the strength of different methods. The article [173] combines ARIMA with NN. The article [9] combines WT-EGARCH-chaotic least squares support vector machines (LSSVM). The article [174] recommends WT-ARIMA-LSSVM-practical swarm optimization (PSO) combination. The article [137] combines WT with ARIMA and GARCH.

MARS models [55] and their variations [113–115,139,152] are not used very commonly in electricity price forecasting. As the first application, MARS model is applied for Ontario's electricity prices [171]. In [132], MARS is only used for initial parameter selection and then seasonal autoregressive integrated moving average (SARIMA) model, a SARIMA model with GARCH (SARIMA–GARCH) and its combination with regression model are presented. A nonlinear autoregressive model with exogenous inputs (NARX) is compared with MARS and wavelet NN by [14]. Two variations of MARS, namely Conic MARS (CMARS) presented in [153] and Robust Conic MARS (RCMARS) developed by [114], are used in [116] to forecast Turkish electricity prices.

W~MARS is a substantial alternative to classical time-series methods, NN, MARS models and all these methods' variations. The main motivations to combine these two methods for electricity price forecasting can be summarized as follows [55,118]:

- WT decomposes series from ill-behaved form to a more stable one. At the same time, WT traces frequency and time dimension of the data simultaneously. These properties directly serve for a nonstationary and volatile structure of electricity prices.
- Processing and computation of WT is very fast even when level of series is very high. For this purpose, pyramid and inverse pyramid algorithms are used.
- In WT, the level of constitutive series can be controlled according to the forecasting performance.
- MARS models complex nonlinear relationship between variables without assumptions and can handle multiple inputs easily.
- The relative importance of the dependent variables can be identified.
- The model can be trained and long training procedure is not required even for the large data sets.
- Outputs of MARS can easily be interpreted. Like WT, MARS is also implemented very easily and gives results very fast.
- Both methods are assumption free.

In summary, specifically WT captures multiple seasonality, unusual behaviors and volatility, whereas MARS eliminates the selection of explanatory variables problem, thus a combined method can handle the challenges introduced by the electricity market problem.

In order to comprehend W~MARS method in detail, multivariate adaptive regression splines and wavelet transform must be covered. The following subsections describe the multivariate adaptive regression splines (MARS) and the wavelet transform.

3.2.1.1 Multivariate Adaptive Regression Splines (MARS)

MARS was first introduced by Friedman in 1991 [55]. It is defined as an extension of linear models to model the interactions and non-linearities automatically. A two-stage (i.e., forward stage and backward stage) additive model is generated by the model [55]. MARS determines basis functions (BFs) and adds them to the model to construct a sufficiently large model, which usually overfits the data set, in the forward stage until maximum number of basis functions is reached, which is specified by the user. However, since the model is large and overfitting the data set, it is overcomplicated and possibly including many incorrect terms. The overfit model is trimmed to reduce the complexity of the model while regarding the fit to the data in the backward stage. BFs contributing less in the residual sum of squares (RSS) are pruned at the backward stage. As a result, an optimally estimated model is produced [73].

The form of piecewise linear one-dimensional BFs created by the data set are as follows [73]:

$$[x-\tau]_+ = \begin{cases} x-\tau, & \text{if} \quad x>\tau \\ 0, & \text{otherwise} \end{cases}, \qquad [x-\tau]_- = \begin{cases} \tau-x, & \text{if} \quad x>\tau \\ 0, & \text{otherwise} \end{cases}. \tag{3.9}$$

Here, τ is a univariate knot obtained from the data set. Functions given in Equation (3.15) are called truncated linear functions. Truncated linear functions, with a knot at the value τ, both together are termed as a reflected pair. It is aimed to construct reflected pairs for each input X_j $\left(j=1,2,\ldots,p\right)$ with p-dimensional knots $\tau_i=\left(\tau_{i,1},\tau_{i,2},\ldots,\tau_{i,p}\right)^{\mathrm{T}}$ $\left(i=1,2,\ldots,N\right)$ at each observed value x_{ij} of that input i = 1,2,…,N. As a result, collection of BFs is obtained, which can be stated as a set S:

$$S := \{[X_j-\tau]_+ \ , [\tau-X_j]_+ \mid \tau \ \in \ \{x_{1j},x_{2j},...,x_{Nj}\},\ j=1,2,3,...,p\}. \tag{3.10}$$

Here, N and p denotes the number of observations and the dimension of the input space respectively. If the input values are all distinct, then there are 2Np BFs. The model in the forward stage is generated by using the BFs in the set S through their possible products. As a result, the generated model, applied at a candidate input vector $\boldsymbol{x}$, is of the form

$$Y=\alpha_0+\sum_{m=1}^{M}\alpha_m\psi_m(\boldsymbol{x}) + \varepsilon, \tag{3.11}$$

where $\boldsymbol{x} = (x_1, x_2, ..., x_p)^T$. Here, $\varepsilon \sim N\left(0, \sigma^2\right)$ is a random noise term, which is supposed to have a normal distribution with zero mean and finite variance σ^2, M is the cardinality of the set of BFs in the current model, $\psi_m(\boldsymbol{x})$ are BFs, and α are the unknown coefficients for the constant 1 (m = 0) or for the mth BF. Given the observation $(\boldsymbol{x}_i, y_i)$ $(i = 1, 2, ..., N)$, the form of the mth multivariate BF is as follows [73]:

$$\psi_m(\boldsymbol{x}) = \prod_{j=1}^{K_m} [s_{km}.(x_{v(j,m)} - \tau_{jm})]_+, \tag{3.12}$$

where K_m is the number of truncated linear functions multiplied in the mth BF, $x_{v(j,m)}$ is the input variable corresponding to the jth truncated linear function in the mth BF, τ_{jm} is the knot value corresponding to the variable $x_{v(j,m)}$ and $s_{jm} = \pm 1$.

In forward stage, MARS starts with the constant function $\psi_0(\boldsymbol{x}) = 1$ to estimate α_0, considering all functions in the set S as candidate. Possible forms of the BFs $\psi_m(\boldsymbol{x})$ are 1, x_k, $[x_k - \tau_i]_+$, $x_k x_l$, $[x_k - \tau_i]_+ x_l$ and $[x_k - \tau_i]_+ [x_l - \tau_j]_+$.

Input variables cannot be the same for each BF. Therefore, the BFs use different input variables, x_k and x_l, and their corresponding knots, τ_i and τ_j. At each stage, with one of the reflected pair in the set S, all products of a function $\psi_m(\boldsymbol{x})$ in the model set are regarded as a new function pair and added to the model set. The term producing the largest decrease in training error has the form:

$$\alpha_{M+1}\psi_k(\boldsymbol{x}) \cdot [X_j - \tau]_+ + \alpha_{M+2}\psi_k(\boldsymbol{x}) \cdot [\tau - X_j]_+. \tag{3.13}$$

Here, α_{M+1} and α_{M+2} are the coefficients and are estimated by least square, like all other M +1 coefficient appearing in the model. These products are stepwise added to the model in forward stage. The forward stage stops when a user-specified number of terms is reached. The model generated at the end of the forward stage typically overfits the data. Therefore, a backward stage is run to prune the model.

The terms contributing less in the residual squared error are stepwise removed from the model. The iterations continue until the final models includes an optimal number of effective terms [55]. Therefore, an estimated best model $\hat{f}_\mu$ of each number of terms μ is produced at the end of this process. Generalized cross-validation (GCV) is used to find the optimal number of terms μ. GCV also shows the lack of fit when using the MARS model. The GCV is defined as [55]:

$$LOF(\hat{f}_\mu) = GCV(\mu) := \frac{\sum_{i=1}^{N} (y_i - \hat{f}_\mu(\boldsymbol{x}_i))^2}{(1 - M(\mu)/N)^2}, \tag{3.14}$$

where $M(\mu)$ is the effective number of parameters in the model, and N is the number of sample observations [73].

3.2.1.2 Filter Theory and Wavelet Transform

Fourier analysis, which localizes just frequency domain, has a remarkable impact on applied mathematics. As an improvement of Fourier analysis, wavelet method can efficiently describe the functions both in frequency domain and time domain. This time-frequency localization is one of the most important advantages in comparison to standard methods like kernel smoothers and orthogonal series [110]. Besides, even for a large data set and sharp spikes, wavelets provide a simple form with its fast algorithms in order to find a statistically significant representations [110]. Here, only discrete wavelet transform (DWT) is considered, since N electricity prices are observed at discrete time points $t = 0,1,\ldots,N-1$. Let X_t represent the electricity price at time t and $\boldsymbol{X}$ represents its corresponding N-dimensional column vector. Let D_t represent the electricity demand at time t and $\boldsymbol{D}$ represents its corresponding N-dimensional column vector. Here, the length of $\boldsymbol{X}$ is restricted to $N := 2^J$. To handle this restriction in W~MARS method; a simple modification is made before the transformation and is explained in the following section. Let Ω_n represent the nth DWT coefficient ($n = 0,1,\ldots,N-1$), $\boldsymbol{W}$ be the corresponding column vector of length $N := 2^J$, and $\boldsymbol{\Omega}$ be an $N \times N$ real-valued matrix that ensures $\boldsymbol{\Omega}^T \boldsymbol{\Omega} = I$. A wavelet transform of $\boldsymbol{X}$ is an orthonormal transform and can be written as $\boldsymbol{W} = \boldsymbol{\Omega X}$, where $\boldsymbol{\Omega} \in \mathbb{R}^{N\times N}$ and $\boldsymbol{W} \in \mathbb{R}^N$. Here, $\boldsymbol{\Omega}$ and $\boldsymbol{W}$ form time and scale coefficients, which means a multiresolution analysis of $\boldsymbol{X}$ in terms of DWT coefficients. However, in the transformation progress, boundary effects arise and cannot be eliminated [118]. Therefore, an efficient algorithm called pyramid algorithm of order $O(N)$ that is faster than fast Fourier transform is used to calculate wavelet coefficients [101]. The algorithm initially takes the original data set $\boldsymbol{X}$ and forms low-pass and high-pass parts by using filtering operations. Here, $\boldsymbol{\Omega} \in \mathbb{R}^{N\times N}$ is obtained by convolutions of the wavelet filter.

Let h_l $(l = 0,1,\ldots,L-1)$ be wavelet filter and g_l $(l = 0,1,\ldots,L-1)$ be associated scaling filter with respect to some $L \in 2\mathbb{N}$. In order to have an orthogonal wavelet matrix $\boldsymbol{\Omega}$, h_l $(l = 0,1,\ldots,L-1)$ must have the following properties:

$$\sum_{l=0}^{L-1} h_l = 0, \tag{3.15}$$

$$\sum_{l=0}^{L-1} h_l^2 = 1, \tag{3.16}$$

$$\sum_{l=0}^{L-1} h_l h_{l+2n} = 0, \text{ for all } n \in \mathbb{Z}\setminus\{0\}, \tag{3.17}$$

where $h_l = 0$ for all $l \in \mathbb{Z} \setminus [0, L-1]$. These properties respectively mean that the sum of the wavelet filters is zero; the wavelet filter has unit energy and it is orthogonal to even shifts. Especially, the last two properties show the orthonormality property of wavelet filter. The scaling filter is the quadrature mirror filter of wavelet filter and defined as $g_l = (-1)^{l+1} h_{L-1-l}$, where $l = 0, 1, \ldots, L-1$, and $g_l = 0$ for $l \in \mathbb{Z} \setminus [0, L-1]$.

Let $H(\cdot)$ be the transfer function for h_l and $\mathcal{H}(\cdot)$ be the associated squared gain function. These functions are given by $H(f) = \sum_{l=0}^{L-1} h_l e^{-i2\pi fl}$ and, hence, $\mathcal{H} \equiv H(f)^2$. Orthonormality of wavelet filters can also be defined in terms of squared gain functions as

$$\mathcal{H}(f) + \mathcal{H}\left(f + \frac{1}{2}\right) = 2, \quad \text{for all } f \in \mathbb{R}. \tag{3.18}$$

Similarly, if $G(f)$ is the transfer function for g_l and $\mathcal{G} := G(f)^2$ is the associated squared gain function, then:

$$\mathcal{G}(f) = \mathcal{H}\left(\frac{1}{2} - f\right), \quad \text{for all } f \in \mathbb{R}, \tag{3.19}$$

and

$$\mathcal{G}(f) + \mathcal{G}\left(f + \frac{1}{2}\right) = 2, \quad \text{for all } f \in \mathbb{R}. \tag{3.20}$$

Hence,

$$\mathcal{H}(f) + \mathcal{G}(f) = 2, \quad \text{for all } f \in \mathbb{R}, \tag{3.21}$$

which shows that if one of the filter is high-pass, then the other is a low-pass filter. The reader may refer to [118] for details on filter theory.

In this study, whole Daubechies (D) filter family is used in order to compare the performance of the method according to the filter types. This family includes Haar wavelet filter, Daubechies 4 (D4), D6, D8, D10, D12, D14, D16, D18, and D20 [44]. As an example, Haar and D4 filters are given in Table 3.1. Here, it should be noted that wavelet filters are high-pass filters and scaling filters are low-pass filters. The reader may also refer to [100] for the other filters' numerical values.

Implementation of wavelet transform with Daubechies filter family was done by *Mallat's pyramid and inverse pyramid algorithms* for decomposition and reconstruction, respectively [101]. The pyramid algorithm gives decomposed data series in J iterations. At the initial step, all input data, i.e., price and demand, are separately decomposed into high-pass ($\boldsymbol{W}_1$) and low-pass ($\boldsymbol{V}_1$) parts by using h_l and g_l, respectively. This also means that $\boldsymbol{W}_1$ forms $N/2$ wavelet coefficients and $\boldsymbol{V}_1$

forms $N/2$ scaling coefficients. Here, $\boldsymbol{V}_1$ is used as an input of next iteration and a second decomposition is made to calculate $\boldsymbol{W}_2$ and $\boldsymbol{V}_2$. At each iteration, scaling coefficients are subsampled and decomposed to form new wavelet and scaling coefficients. Hence, at each step, rougher (i.e., for high-pass) and smoother (i.e., for low-pass) frequencies are obtained. At the end of the algorithm, low-pass and high-pass parts are obtained where time information for both sides is still kept. In this study, keeping both time and frequency information is one of most important reason to use wavelet transform.

Table 3.1: Numerical values of Haar and Daubechies 4 wavelet and scaling filters [100].

	Wavelet Filters		Scaling Filters	
Haar	$h_0 = \frac{1}{\sqrt{2}}$	$h_1 = -\frac{1}{\sqrt{2}}$	$g_0 = \frac{1}{\sqrt{2}}$	$h_0 = \frac{1}{\sqrt{2}}$
D4	$h_0 = \frac{1-\sqrt{3}}{4\sqrt{2}}$	$h_1 = \frac{-3+\sqrt{3}}{4\sqrt{2}}$	$g_0 = \frac{1+\sqrt{3}}{4\sqrt{2}}$	$g_1 = \frac{3+\sqrt{3}}{4\sqrt{2}}$
	$h_2 = \frac{3+\sqrt{3}}{4\sqrt{2}}$	$h_3 = \frac{-1-\sqrt{3}}{4\sqrt{2}}$	$g_2 = \frac{3-\sqrt{3}}{4\sqrt{2}}$	$g_3 = \frac{1-\sqrt{3}}{4\sqrt{2}}$

In order to demonstrate pyramid algorithm mathematically, let us have time-series $X = \left(X_t :\ t = 0,1,\ldots,N-1\right)$ and $X_t = V_{0,t}$. Thus, at the jth step, $V_{j-1,t}\ \left(t = 0,1,\ldots,N_j - 1\right)$ and $N_j := N/2^j$ for $j = 1,2,\ldots,J$. Let further $h_l\ \left(l = 0,1,\ldots,L-1\right)$ be wavelet filters. Then, the jth wavelet and scaling coefficients are defined by

$$W_{j,t} := \sum_{l=0}^{L-1} h_l V_{j-1,2t+1-l \bmod N_{j-1}} \tag{3.22}$$

and

$$V_{j,t} := \sum_{l=0}^{L-1} g_l V_{j-1,2t+1-l \bmod N_{j-1}}, \tag{3.23}$$

respectively. The results for all t are

$$\boldsymbol{W}_j \left(= \left(\boldsymbol{W}_{j,t}\right)_t\right) := \left[W_{j,0}, W_{j,1}, W_{j,2}, \ldots, W_{j,N_j-1}\right]^{\mathrm{T}} \tag{3.24}$$

and

$$\boldsymbol{V}_j \left(= \left(\boldsymbol{V}_{j,t}\right)_t\right) := \left[V_{j,0}, V_{j,1}, V_{j,2}, \ldots, V_{j,N_j-1}\right]^{\mathrm{T}}. \tag{3.25}$$

Finally, a combination of all steps of pyramid algorithm yields

$$\boldsymbol{X} = \boldsymbol{\Omega}^{\mathrm{T}}\boldsymbol{W} = \left[\boldsymbol{\Omega}_1^{\mathrm{T}}, \boldsymbol{\Omega}_2^{\mathrm{T}}, \ldots, \boldsymbol{\Omega}_J^{\mathrm{T}}, \boldsymbol{\Psi}_J^{\mathrm{T}}\right]\begin{bmatrix}\boldsymbol{W}_1\\ \boldsymbol{W}_2\\ \vdots\\ \boldsymbol{W}_J\\ \boldsymbol{V}_J\end{bmatrix} = \sum_{j=0}^{J}\boldsymbol{\Omega}_j^{\mathrm{T}}\boldsymbol{W}_j + \boldsymbol{\Psi}_J^{\mathrm{T}}\boldsymbol{V}_J, \tag{3.26}$$

or

$$\boldsymbol{X} = \boldsymbol{\Omega}^{\mathrm{T}}\boldsymbol{W} = \sum_{n=0}^{N-1}\boldsymbol{\Omega}_n^{\mathrm{T}}\boldsymbol{W}_n = \sum_{j=1}^{J}\boldsymbol{\Omega}_j^{\mathrm{T}}\boldsymbol{W}_j + \boldsymbol{\Psi}_J^{\mathrm{T}}\boldsymbol{V}_J, \tag{3.27}$$

where the vector $\boldsymbol{W}$ is partitioned into sub-vectors and the matrix $\boldsymbol{\Omega}$ is formed by convolution of filters and partitioned into sub-matrices. Hence, $\boldsymbol{W}$ and $\boldsymbol{\Omega}$ are given by

$$\boldsymbol{W} = \begin{bmatrix}\boldsymbol{W}_1\\ \boldsymbol{W}_2\\ \vdots\\ \boldsymbol{W}_J\\ \boldsymbol{V}_J\end{bmatrix} \text{ and } \boldsymbol{\Omega} = \begin{bmatrix}\boldsymbol{\Omega}_1\\ \boldsymbol{\Omega}_2\\ \vdots\\ \boldsymbol{\Omega}_J\\ \boldsymbol{\Psi}_J\end{bmatrix}, \tag{3.28}$$

where $\boldsymbol{\Omega}_j$ is a $(N/2^j)\times N$ matrix, $\boldsymbol{\Omega}_J$ and $\boldsymbol{\Psi}_J$ are $1\times N$ matrices, $\boldsymbol{W}_j$ is a $(N/2^j)\times 1$ column vector, and $\boldsymbol{V}_J$ is the last element of $\boldsymbol{W}$. Since the WT localizes both time and frequency dimensions, wavelet and scale coefficients refer to these localizations on time and scale dimensions, respectively.

Similarly, reconstructions of the series are implemented by using *inverse pyramid algorithm* [118]. In this case, up-sampling is applied by using the following formula at each j:

$$V_{j-1,t} = \sum_{l=0}^{L-1} g_l V^{\uparrow}_{j,t+1-l \bmod N_{j-1}} + \sum_{l=0}^{L-1} h_l W^{\uparrow}_{j,t+1-l \bmod N_{j-1}}, \tag{3.29}$$

for $t = 0,1,\ldots,N_{j-1}-1$, where

$$V^{\uparrow}_{j,t} := \begin{cases} 0, & t = 0,2,\ldots,N_{j-1}-2,\\ V_{j,\frac{t-1}{2}}, & t = 1,3,\ldots,N_{j-1}-1, \end{cases} \tag{3.30}$$

and

$$W^{\uparrow}_{j,t} := \begin{cases} 0, & t = 0,2,\ldots,N_{j-1}-2,\\ W_{j,\frac{t-1}{2}}, & t = 1,3,\ldots,N_{j-1}-1. \end{cases} \tag{3.31}$$

The reader may refer to [118] for the pseudo code of pyramid and inverse pyramid algorithms.

3.2.1.3 Implementation of W~MARS

W~MARS method, which is illustrated in Figure 3.6, is implemented in three main steps. The input data for W~MARS include historical time data $\left(X_t, X_{t-1}, ..., X_{t-N+1}\right)$ in order to forecast X_{t+1}.

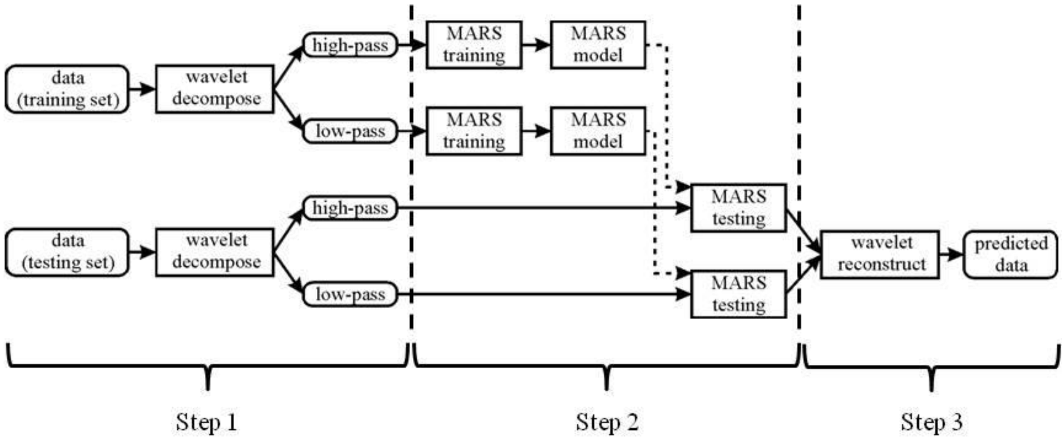


Figure 3.6: Schematic representation of W~MARS.

Step 1: This step constitutes the decomposition of the data. By using Mallat's pyramid algorithm [101], time series $\boldsymbol{X}$ of length $N=2^J$ are transformed into $N/2$ wavelet and scale coefficients, as illustrated in Figure 3.7. Here, the length of series must be the power of 2 so that the data can be decomposed into two halves, $\boldsymbol{W}_j$ and $\boldsymbol{V}_j$, at each iteration. Therefore, the initial data set is broken to have a set of length $N=2^J$, as denoted in [118].

At the end of this step, J-1 transformations, whose jth transformation results in $\boldsymbol{W}_j$ and $\boldsymbol{V}_j$, are made in order to discretize the data into low-pass and high-pass parts. Hence, spikes in the data are extracted from the series and their impact is processed separately.

Discretization processes are made by symmetric and asymmetric wavelet filters like Daubechies (D4, D6, D8, D10, D12, D14, D16, D18, and D20) and Haar, respectively. Since these filters are used for orthogonal transform of $\boldsymbol{\Omega}$, the wavelet filter h_l has a real-valued sequence.

Unlike the articles [38] and [106], which employ only D4 and Haar filters, respectively, here an appropriate filter that yields the best performance is selected by the method automatically. Haar and nine different Daubechies wavelet filters are employed to transform the time series. In total, 10 different low-pass and high-pass parts are obtained for each input variable of this step. MATLAB 2012b is used for implementation of this step.

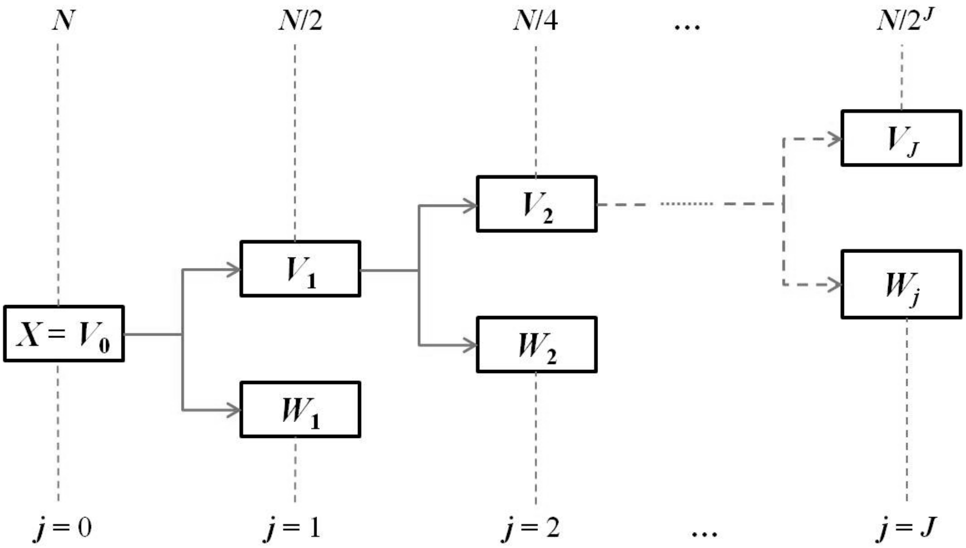


Figure 3.7: Decomposition of electricity prices at the first step of W~MARS.

Step 2: After decomposing the data into its low-pass and high-pass parts, the model is built by MARS algorithm. In the first phase of MARS, basis functions are added iteratively such that the largest reduction of training error is obtained. This phase of MARS algorithm is called as *forward selection*. Since the constructed model is large and it overfits the data, the second phase of the MARS, backward deletion, is applied. Here, the basis functions are deleted according to *generalized cross validation* (*GCV*). Both phases of MARS are applied for each low-pass and high-pass parts. The models with lowest GCVs are selected and the model is used for the testing procedure of MARS model. Finally, significant variables, their interactions and degree of interactions are determined and value at time t+1 is predicted. This step is implemented using ARESLab [80].

Step 3: Since the predicted values in *Step 2* are in the decomposed form, this step comprises reconstruction of the series. Here, *Step 1* is reversed using the same filters. Figure 3.8 illustrates this procedure, where $\boldsymbol{V}_{J^*}$ and $\boldsymbol{W}_{j^*}$ indicate predicted low-pass and high-pass parts, and $\boldsymbol{X}^*$ represents the predicted values. This step is implemented by using MATLAB R2012b.

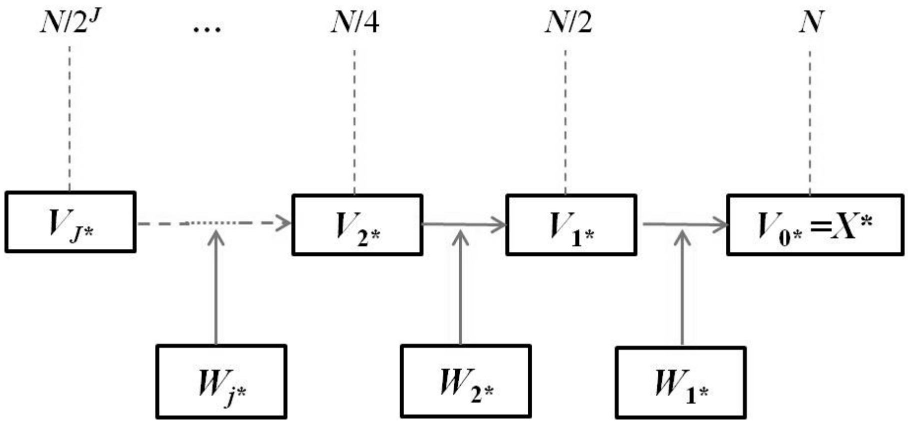


Figure 3.8: Reconstruction of predicted electricity prices.

3.2.2 Robust Wavelet – Multivariate Adaptive Regression Splines (R~W~MARS)

Data used in many energy planning models include random fluctuations. For instance, the demand, supply, investment and emission parameters are not known properly. Some of these parameters can be estimated, hence, they include estimation errors. Demand and supply are generally forecasted and they include forecasting errors as well. On the other hand, some of the parameters may not be possible to estimate (e.g., emission quotas in Turkey). In this case, the parameters are subject to subjective assessment noises, defining uncertainty sets of the parameters.

Here we develop a tractable robust electricity market optimization model. The uncertainties in the parameters, represented by uncertainty regions, are modeled by using W~MARS. The tractability property of the robust model increases the complexity. To reduce the complexity and trace time-dependent uncertainty, an efficient method utilizing a projection of uncertainties is employed.

The proposed model, which handles uncertainties in the data and tracks their dynamics, also considers renewability and sustainability of the electricity market. Renewable and sustainable electricity market model consists of two types of uncertainties. One of these uncertainties is related to the electricity demand since the demand has an adaptive structure and changes according to electricity price, temperature, etc. The other type is related to emissions since the model aims to give sustainable results. The model is illustrated in Figure 3.9.

To account for the uncertainties in the system, modeling uncertainties should be comprehended clearly. The following section presents a background for modeling of uncertainties.

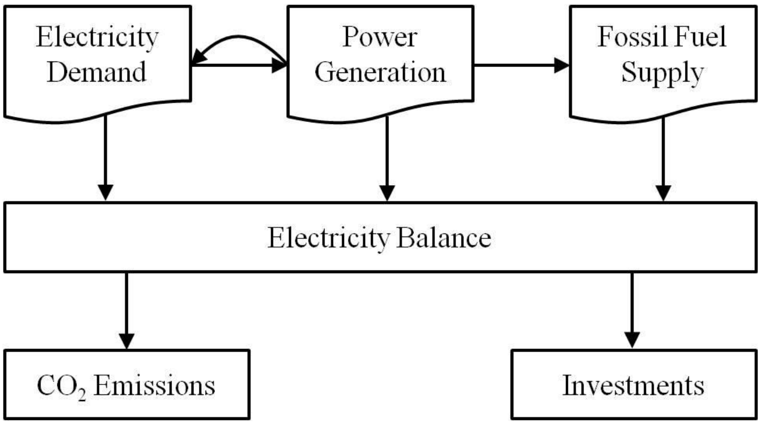


Figure 3.9: Schematic representation of robust electricity market optimization model.

3.2.2.1 Modeling Uncertainties

Modeling and optimization of real-world problems generally involves uncertain parameters because of various changing situations. Let us define a general optimization problem under uncertainty as stated below:

$$\begin{aligned} &\text{maximize } \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} \\ &\text{subject to} \\ &f_i(\boldsymbol{x}, \tilde{\boldsymbol{D}}_i) \geq 0 \quad (i \in I), \\ &\boldsymbol{x} \in X, \end{aligned} \tag{3.32}$$

where $f_i(\boldsymbol{x}, \tilde{\boldsymbol{D}}_i)$ $(i \in I)$ are given functions, X is a given set and $\tilde{\boldsymbol{D}}_i$ $(i \in I)$ are vectors of random coefficients. This problem can be reformulated by using a vector of expected values, $\boldsymbol{D}_i^0$ and a random parameter vector $\tilde{\boldsymbol{D}}_i$. In order to handle the feasibility problem, a chance-constrained model is formulated as follows, referring to a probability measure P over the event space:

$$\begin{aligned} &\text{maximize } \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} \\ &\text{subject to} \\ &\mathrm{P}\left(f_i(\boldsymbol{x}, \tilde{\boldsymbol{D}}_i) \geq 0 \right) \geq 1 - \varepsilon_i \quad (i \in I), \\ &\boldsymbol{x} \in X. \end{aligned} \tag{3.33}$$

However, chance-constrained models are non-convex and generally intractable [28]. They encounter numerical difficulties, especially, during the solution progress. Besides, they need probability models of uncertainty and computations of multidimensional integrals related with expectations and probabilities. Thus, the following robust optimization problem is proposed by [24]:

$$\begin{aligned}
&\text{maximize } \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} \\
&\text{subject to} \\
&\min_{\boldsymbol{D}_i \in U_i} f_i(\boldsymbol{x}, \boldsymbol{D}_i) \geq 0 \quad (i \in I), \\
&\boldsymbol{x} \in X,
\end{aligned} \tag{3.34}$$

where U_i $(i \in I)$ are uncertainty sets. The selection of the uncertainty is one of the most critical issue in robust optimization problems. Three main concerns should be considered in selecting the uncertainty regions [26]:

- The uncertainty region should be consistent with the uncertain parameter and its data set.
- The uncertainty region should be statistically meaningful.
- The uncertainty region should provide a tractable robust counterpart problem.

In this study, the most important decision criterion is the computational tractability. If the original form of the model can be solved in polynomial time, then the robust problem should also be solved in polynomial time. The other criterion is to guarantee the feasibility of the model within the limits of uncertainties. The decision criterion for feasibility depends on the problem type and the uncertainty type used for the problem. The most common problem and uncertainty types are given in Table 3.2 with their robust counterparts [28].

Table 3.2: The most common problem and uncertainty types.

Model/Problem Type	**Uncertainty Region**	**Constraint**	**Robust Counterpart**
LP model	Polyhedron	$\tilde{\boldsymbol{a}}^{\mathrm{T}}\boldsymbol{x} \geq \tilde{b}$	LP
LP model	Ellipsoidal	$\tilde{\boldsymbol{a}}^{\mathrm{T}}\boldsymbol{x} \geq \tilde{b}$	CQP
QCQP model	Polyhedron	$\left\Vert \tilde{\boldsymbol{A}}\boldsymbol{x} \right\Vert_2^2 + \tilde{\boldsymbol{b}}^{\mathrm{T}}\boldsymbol{x} + \tilde{c} \leq 0$	-
QCQP model	Ellipsoidal	$\left\Vert \tilde{\boldsymbol{A}}\boldsymbol{x} \right\Vert_2^2 + \tilde{\boldsymbol{b}}^{\mathrm{T}}\boldsymbol{x} + \tilde{c} \leq 0$	SDP
SOCP model	Ellipsoidal	$\left\Vert \tilde{\boldsymbol{A}}\boldsymbol{x} + \tilde{\boldsymbol{b}} \right\Vert_2 \leq \tilde{\boldsymbol{c}}^{\mathrm{T}}\boldsymbol{x} + \tilde{d}$	SDP
SDP model	Ellipsoidal	$\sum_{j=1}^{n} \tilde{\boldsymbol{A}}_j x_j \geq \tilde{\boldsymbol{B}}$	-

LP: Linear programming model, QCQP: Quadratic constrained quadratic programming model, CQP: Conic quadratic programming, SDP: Semi-definite programming model.

Table 3.3: Types of uncertainty region.

Uncertainty Region	**Mathematical Representation**	**Robust Counterpart**	**Tractability**
Box	$\lVert \boldsymbol{p} \rVert_\infty \le 1$	$\boldsymbol{a}^{\mathrm{T}}\boldsymbol{x} + \lVert \boldsymbol{B}^{\mathrm{T}}\boldsymbol{x} \rVert_1 \le \beta$	Linear Program
Ball	$\lVert \boldsymbol{p} \rVert_2 \le 1$	$\boldsymbol{a}^{\mathrm{T}}\boldsymbol{x} + \lVert \boldsymbol{B}^{\mathrm{T}}\boldsymbol{x} \rVert_2 \le \beta$	Conic Quadratic Program
Polyhedral	$\boldsymbol{Cp} + \boldsymbol{d} \ge 0$	$\boldsymbol{a}^{\mathrm{T}}\boldsymbol{x} + \boldsymbol{d}^{\mathrm{T}}\boldsymbol{y} \le \beta$ $\boldsymbol{C}^{\mathrm{T}}\boldsymbol{y} = -\boldsymbol{B}^{\mathrm{T}}\boldsymbol{x}$ $\boldsymbol{y} \ge 0$	Linear Program
Cone (closed, convex, pointed)	$\boldsymbol{Cp} + \boldsymbol{d} \in K$	$\boldsymbol{a}^{\mathrm{T}}\boldsymbol{x} + \boldsymbol{d}^{\mathrm{T}}\boldsymbol{y} \le \beta$ $\boldsymbol{C}^{\mathrm{T}}\boldsymbol{y} = -\boldsymbol{B}^{\mathrm{T}}\boldsymbol{x}$ $\boldsymbol{y} \in K^*$	Conic Optimization
Separable functions	$\sum_i f_{\ell i}(\boldsymbol{p}_i) \le 0,\ \forall \ell \in \{1,...,L\}$	$\boldsymbol{a}^{\mathrm{T}}\boldsymbol{x} + \sum_\ell \sum_i \lambda_\ell f_{\ell i}^*(\frac{s_{\ell i}}{\lambda_\ell}) \le \beta$ $\sum_\ell s_{\ell i} = \boldsymbol{b}_i^{\mathrm{T}}\boldsymbol{x} \quad (i \in \{1,...,m\})$ $\boldsymbol{\lambda} \ge \boldsymbol{0}$	Convex Optimization

In Table 3.2, $\tilde{\boldsymbol{A}}, \tilde{\boldsymbol{B}}, \tilde{\boldsymbol{a}}, \tilde{\boldsymbol{b}}, \tilde{\boldsymbol{c}}$, and $\tilde{\boldsymbol{d}}$ are uncertain parameters in the corresponding models. Tractability of the robust counterpart problems of the form minimize $\left\{\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} \middle| \ \boldsymbol{Ax} \le \boldsymbol{b},\ \forall \boldsymbol{A} \in \hat{U}\right\}$ given in Table 3.3 [26].

In Table 3.3, $\boldsymbol{x} \in \mathbb{R}^n$ is the decision variable, $\boldsymbol{c} \in \mathbb{R}^n$ and $\boldsymbol{b} \in \mathbb{R}^\ell$ are known parameters, $\boldsymbol{A} \in \mathbb{R}^{\ell \times n}$ is a matrix with uncertain parameters and $\hat{U}$ is the (estimated) uncertainty region.

When a single constraint of robust counterpart is defined, it should be in the following form:

$$(\boldsymbol{a} + \boldsymbol{Bp})^{\mathrm{T}}\boldsymbol{x} \le \boldsymbol{\beta},\quad \forall \boldsymbol{p} \in U, \tag{3.35}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the design vector, $\boldsymbol{a} \in \mathbb{R}^n$, $\boldsymbol{B} \in \mathbb{R}^{n \times m}$ and $\boldsymbol{\beta} \in \mathbb{R}^n$ are known parameters, $\boldsymbol{p} \in \mathbb{R}^m$ is the uncertain parameter and $\hat{U}$ is the uncertainty region for $\boldsymbol{p}$. If this region is a box or a ball, then a worst-case solution can be acquired by solving the optimization problem maximize $\boldsymbol{p}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{x}$ subject to $\boldsymbol{p} \in U$. If the region is polyhedral or a cone, then the duality theorem can be used to obtain a solution. Here, if we define the uncertainty region as *K*, then we have:

$$\max\left\{\boldsymbol{p}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{x} \ \middle|\ \boldsymbol{Cp} + \boldsymbol{d} \in K\right\} = \min\left\{\boldsymbol{d}^{\mathrm{T}}y \ \middle|\ \boldsymbol{C}^{\mathrm{T}}y = -\boldsymbol{B}^{\mathrm{T}}\boldsymbol{x},\ y \in K^*\right\}. \tag{3.36}$$

Here, K^* denotes the *dual cone* of K and has non-empty interior.

Uncertainty Sets in terms of Norms

Let us define uncertainty in the electricity demand as follows:

$$\tilde{\boldsymbol{D}} = \boldsymbol{D}_{Nom} + \sum_{j \in N} \Delta \boldsymbol{D}_j \tilde{Z}_j, \tag{3.37}$$

where $\boldsymbol{D}_{Nom}$ is the nominal value of the data. Furthermore, $\Delta \boldsymbol{D}_j$ $(j \in N)$ are directions of the perturbation, $\tilde{Z}_j$ $(j \in N)$ are independent and identically distributed random variables, and N is a set of finite vectors over $\mathbb{N}$. The nominal value of the electricity demand is generally assumed to be $E\left[\tilde{\boldsymbol{D}}\right] = \boldsymbol{D}_{Nom}$. In this case, the uncertainty set U is defined as

$$U = \left\{ \boldsymbol{D} \;\middle|\; \exists\, \boldsymbol{u} \in \mathbb{R}^{|N|} : \boldsymbol{D} = \boldsymbol{D}_{Nom} + \sum_{j \in N} \Delta \boldsymbol{D}_j u_j,\ \|\boldsymbol{u}\| \le \Omega \right\}, \tag{3.38}$$

where the tolerance parameter Ω guarantees feasibility. Here, $\boldsymbol{u} = (u_1, u_2, \ldots, u_{|N|})^{\mathrm{T}}$ and $\|\boldsymbol{u}\|$ is called as the *absolute norm*, which is introduced and described in the paper [28].

In general, the following norms are used in the robust optimization [28]:

- the polynomial norm: l_k $(k = 1, 2, ..., \infty)$;
- the norm $l_2 \cap l_\infty$ to model bounded and symmetrically distributed random data: $\max\left\{\|\boldsymbol{u}\|_2,\ \Omega\|\boldsymbol{u}\|_\infty\right\}$, for some $\Omega > 0$;
- the norm $l_1 \cap l_\infty$ to model bounded and symmetrically distributed random data; here, we use that the robust counterpart of a linear model is still a linear model. We refer to $\max\left\{\frac{1}{\Gamma}\|\boldsymbol{u}\|_1,\ \|\boldsymbol{u}\|_\infty\right\}$, for some $\Gamma > 0$.

The set of efficiently computable convex inequalities shows computational tractability [25]. If the uncertainty set U is restricted to an ellipsoidal form, then the set will be the intersection of finitely many ellipsoids. Thus, the set will be

$$U(\Pi, \boldsymbol{Q}) = \left\{ \Pi(\boldsymbol{u}) \;\middle|\; \|\boldsymbol{Q}\boldsymbol{u}\|_2 \le \rho \right\}. \tag{3.39}$$

Here, $\boldsymbol{u} \mapsto \Pi(\boldsymbol{u})$ is an affine embedding of $\mathbb{R}^L$ into $\mathbb{R}^{m \times n}$ and $\boldsymbol{Q} \in \mathbb{R}^{M \times L}$. This type of uncertainty can be obtained by convex quadratic inequalities that form polytopes. Hence, the final model will be a conic quadratic model constrained as follows:

$$\boldsymbol{a}_i^{\mathrm{T}}\boldsymbol{x} + \alpha_i \geq \left\| \boldsymbol{B}_i \boldsymbol{x} + \boldsymbol{b}_i \right\|_2 \qquad (i = 1,2,..., M), \tag{3.40}$$

where $\boldsymbol{a}_i$ and $\boldsymbol{b}_i$ are vectors, $\boldsymbol{B}_i$ is a matrix and α_{i} is real number. In this case, we have

$$U = \left\{ \boldsymbol{A} = \boldsymbol{P}^0 + \sum_{j=1}^{k} u_j \boldsymbol{P}^j \;\middle|\; \boldsymbol{u}^{\mathrm{T}}\boldsymbol{u} \leq 1 \right\}, \tag{3.41}$$

where $\boldsymbol{P}^j$ $(j = 0,1,...,k)$ are $m \times n$ matrices.

The simplest case of ellipsoidal uncertainty is obtained when the uncertainty is constraint-wise, e.g., every constraint uncertainty set U_i being an ellipsoid, and we define the uncertainty set in the form $U = \Pi_{i=1}^{M} U_i$.

Relaxation of Normal Distribution Assumption

The distributional assumption for uncertainties is another important issue for robust optimization problems. Since the uncertainties are generally taken as forecasting errors, it is assumed that they are normally distributed. There also exists many other distributions that are studied on this issue, e.g., bivariate or multivariate normal distributions [1,29], uniform, Poisson, binomial [79], etc. In addition, moments [46,88], kernel densities [35,161], empirical distribution and histograms [29] are used to solve for distributional assumptions.

In the papers [78,79], an uncertain mixed-integer program is proposed. Uncertainty is described as a known distribution function. Here, the corresponding deterministic model is given as follows:

$$\begin{aligned} & \text{minimize / maximize } \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + \boldsymbol{d}^{\mathrm{T}}\boldsymbol{y} \\ & \text{subject to} \\ & \boldsymbol{Ex} + \boldsymbol{Fy} = e, \\ & \boldsymbol{Ax} + \boldsymbol{By} \leq p, \\ & \underline{x} \leq \boldsymbol{x} \leq \overline{x}, \\ & y_k \in \{0,\ 1\} \ \forall k. \end{aligned} \tag{3.42}$$

In this model, nominal parameters are defined as a_{lm}, b_{lk} and p_l, and $\tilde{a}_{lm}$, $\tilde{b}_{lk}$ and $\tilde{p}_l$ are the values that uncertainty imposed on them. Here, l represents an uncertain inequality, m represents the index of the continuous terms, k is the index of the binary terms. Thus, the following inequality is obtained:

$$\sum_m \tilde{a}_{lm} x_m + \sum_k \tilde{b}_{lk} y_k \leq \tilde{p}_l. \tag{3.43}$$

For each inequality l, uncertainties are evaluated by random perturbations as given below:

$$\begin{aligned}
\tilde{a}_{lm} &= (1+\varepsilon\xi_{lm})a_{lm}, \\
\tilde{b}_{lk} &= (1+\varepsilon\xi_{lk})b_{lk}, \qquad (3.44) \\
\tilde{p}_{l} &= (1+\varepsilon\xi_{l})\,p_{l},
\end{aligned}$$

where ξ_{lm}, ξ_{lk} and ξ_l are independent random variables and $\varepsilon > 0$ is an uncertainty level. For the values of the feasible vectors of $\boldsymbol{x}$ and $\boldsymbol{y}$, the probability of violation of the inequalities should be at most of some prescribed level $\kappa \in [0,1]$:

$$P\left(\sum_m \tilde{a}_{lm}x_{lm} + \sum_k \tilde{b}_{lk}y_{lk} > \tilde{p}_l + \delta \max\{1, |p_l|\}\right) \le \kappa, \qquad (3.45)$$

where $\delta > 0$ is feasibility tolerance. Here, the counterpart of the strict inequality can be rewritten as follows:

$$\sum_m \tilde{a}_{lm}x_{lm} + \sum_k \tilde{b}_{lk}y_{lk} - \tilde{p} \le \delta \max\{1, |p_l|\}. \qquad (3.46)$$

After substituting the deterministic parameters with perturbed parameters, we obtain

$$\sum_m (1+\varepsilon\xi_{lm})a_{lm}x_m + \sum_k (1+\varepsilon\xi_{lk})b_{lk}y_k - (1+\varepsilon\xi_l)\,p_l \le \delta \max\{1, |p_l|\}. \qquad (3.47)$$

After rearrangement of the inequality, we get

$$\sum_m a_{lm}x_m + \sum_k b_{lk}y_k - p_l + \varepsilon\left(\sum_{m\in M_l} \xi_{lm}a_{lm}x_m + \sum_{k\in K_l} \xi_{lk}b_{lk}y_{lk} - \xi_l\,p_l\right) \le \delta \max\{1, |p_l|\}. \qquad (3.48)$$

Here, M_l and K_l denote the uncertainties for a_{lm} and b_{lk}, respectively. Finally, the probability of constraint violation is found as follows:

$$P\left(\sum_m a_{lm}x_m + \sum_k b_{lk}y_k - p_l + \varepsilon\left(\sum_{m\in M_l} \xi_{lm}a_{lm}x_m + \sum_{k\in K_l} \xi_{lk}b_{lk}y_{lk} - \xi_l\,p_l\right) \le \delta \max\{1, |p_l|\}\right) \le \kappa. \qquad (3.49)$$

When the probability distributions of the sum of random variables are known, then the following sum can be used for the model in order to find an uncertain term in the inequalities:

$$\xi := \sum_{m\in M_l} \xi_{lm}a_{lm}x_m + \sum_{k\in K_l} \xi_{lk}b_{lk}y_{lk} - \xi_l\,p_l. \qquad (3.50)$$

This transformation is used for uniform, normal, difference of normal, binomial, Poisson and a general discrete distribution in [79]. In our study, all the uncertainty regions are defined via W~MARS method, where the distribution assumption is relaxed as well.

3.2.2.2 Robust Counterpart of Electricity Market Model

If the uncertainties are ignored, the system can normally be modeled using a *Dynamic-Deterministic Electricity Planning Model* (*DDEPM*), which is formulated as follows:

Indices:

i: Plant type,

j: Sector type,

t: Time periods.

Parameters:

$d_{j,t}$: Electricity demand from sector type j,

$P_{i,t}$: Electricity price of plant i in period t,

$FC_{i,t}$: Fixed cost of plant type i in period t,

$VC_{i,t}$: Variable cost of plant type i in period t,

$I_{i,t}$: Investment cost of plant type i in period t,

$Cap_{i,t}$: Installed capacity of plant type i in period t,

$MaxCap_{i}$: Maximum capacity of plant i,

$E_{i,t}$: Emission rate of green house gases (GHG) from plant type i in period t,

L: Coefficient of transmission and distribution losses,

Q_{i}: Maximum GHG emission of plant type i.

Decision Variables:

$X_{i,t}$: Amount of electricity generated in plant i in period t,

$Y_{i,t}$: Capacity extension for plant i in period t.

Herewith, the optimization problem is given as follows:

$$\text{maximize} \ \sum_{i=1}^{N}\sum_{t=1}^{T}\Big(\big(P_{i,t}-VC_{i,t}\big)L\big(X_{i,t}+Y_{i,t}\big)-I_{i,t}Y_{i,t}-FC_{i,t}\Big) \tag{3.51}$$

subject to

$$\sum_{i=1}^{N} X_{i,t} \geq d_{j,t} \ \ \forall t,\, j, \tag{3.52}$$

$$\sum_{t=1}^{T} X_{i,t} \le Cap_{i,t} \ \ \forall i, \tag{3.53}$$

$$Cap_{i,t} + Y_{i,t} \le MaxCap_i \ \ \forall i,t, \tag{3.54}$$

$$\sum_{t=1}^{T} E_{i,t} X_{i,t} \le Q_i \ \ \forall i, \tag{3.55}$$

$$X_{i,t} \ge 0 \ \ \forall i,t, \tag{3.56}$$

$$Y_{i,t} \ge 0 \ \ \forall i,t. \tag{3.57}$$

The first constraint set guarantees that the total electricity produced cannot be lower than the electricity demand. The second constraint set guarantees that the total electricity produced in the plants cannot be greater than the plants' capacities. The third constraint set guarantees that new capacity expansions cannot be greater than the maximum capacity that can be installed a plant. The fourth constraint set restricts the GHG emission. Finally, sign restrictions are defined. The objective function represents the profit to be maximized.

On the other hand, our renewable and sustainable electricity market model includes the uncertainties in the electricity demand the emissions. Therefore, the model above must be modified by considering uncertainties.

The robust counterpart of our electricity planning model is constructed with single uncertainty case for each parameter. By this approach, we aim to obtain

- A computationally more tractable model,
- A model, which compensates for the price of robustness,
- A model driven by the uncertainty sets, and
- A computationally efficient dynamic robust optimization model.

Let us assume that the emission rate is uncertain:

$$\tilde{E}_{i,t}(\xi) = \bar{E}_{i,t} + \hat{E}_{i,t}\xi_{i,t}, \quad \xi_{i,t} \in [-1,1], \tag{3.58}$$

where $\xi \in \mathbb{R}^m$ is the uncertainty factor. Under ellipsoidal uncertainty, a generating or prototype-kind of uncertainty set is $\Xi = \{\xi : \ \|\xi\|_2 \le \sigma\}$, whereas under polyhedral (e.g., an axes-parallel box-shaped) uncertainty, that set is $\Xi = \{\xi : \ \|\xi\|_\infty \le \sigma\}$. Here, σ is an immunization factor. If its value is increased, then the uncertainty set is increased as well as the worst case of the uncertain parameters. The emission constraint under the polyhedral uncertainty is

$$\sum_{t=1}^{T}\left(\bar{E}_{i,t}+\hat{E}_{i,t}\xi_{i,t}\right)X_{i,t}\le Q,\quad \forall i. \tag{3.59}$$

Thus, dimension of the uncertainty factors is still linear and model complexity of the robust counterpart does not change. The robust-ellipsoid equivalent of this constraint is

$$\sum_{t=1}^{T}\left(\bar{E}_{i,t}X_{i,t}\right)+\sigma\left\|\sum_{t=1}^{T}\hat{\boldsymbol{E}}_{i,t}\cdot\boldsymbol{X}_{i,t}\right\|_2\le Q,\quad \forall i, \tag{3.60}$$

which can be regarded as a conic quadratic constraint. In this case, the model complexity increases from linear to quadratic. The next uncertain parameter implied in the model is the demand. Similarly, it can be defined as

$$\tilde{d}_{j,t}(\varsigma):=\bar{d}_{j,t}+\hat{d}_{j,t}\varsigma_{j,t},\quad \varsigma_{j,t}\in[-1,1]. \tag{3.61}$$

Since the demand is not multiplied by a decision variable, the demand constraint remains linear. A second alternative for uncertain demand has the following form [23]:

$$\tilde{d}_{j,t}\in\left[d_{j,t}-\varsigma,d_{j,t}+\varsigma\right]\ . \tag{3.62}$$

Let us also assume that electricity price is uncertain. It can be defined for polyhedral uncertainty as

$$\tilde{P}_{i,t}(\zeta):=\bar{P}_{i,t}+\hat{P}_{i,t}\zeta_{i,t},\quad \zeta_{i,t}\in[-1,1] \tag{3.63}$$

or

$$\tilde{P}_{i,t}\in\left[P_{i,t}-\zeta_{i,t},P_{i,t}+\zeta_{i,t}\right], \tag{3.64}$$

respectively and for ellipsoidal uncertainty $\tilde{P}_{i,t}(\zeta)$, where $\Xi=\left\{\zeta:\ \|\zeta\|_2\le\sigma\right\}$.

If uncertain demand, emission and price sets are substituted with the deterministic sets, the entire model will be in the following form under polyhedral uncertainty:

$$\text{maximize}\ \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\left(\left(\bar{P}_{i,t}+\hat{P}_{i,t}\zeta_{i,t}\right)-VC_{i,t}\right)L\left(X_{i,t}+Y_{i,t}\right)-I_{i,t}Y_{i,t}-FC_{i,t}\right) \tag{3.65}$$

subject to

$$\sum_{i=1}^{N}X_{i,t}\ge\bar{d}_{j,t}+\hat{d}_{j,t}\varsigma_{j,t}\ \text{and}\ \varsigma_{j,t}\in[-1,1]\quad \forall t,j, \tag{3.66}$$

$$\sum_{t=1}^{T}X_{i,t}\le Cap_{i,t}\ \ \forall i, \tag{3.67}$$

$$Cap_{i,t}+Y_{i,t}\le MaxCap_i\ \ \forall i,t, \tag{3.68}$$

$$\sum_{t=1}^{T}\left(\bar{E}_{i,t}+\hat{E}_{i,t}\xi_{i,t}\right)X_{i,t}\leq Q,\ \forall i, \tag{3.69}$$

$$X_{i,t}\geq 0\ \ \forall i,t, \tag{3.70}$$

$$Y_{i,t}\geq 0\ \ \forall i,t. \tag{3.71}$$

The same model is stated below based on ellipsoidal uncertainty:

$$\text{maximize}\ \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\left(\tilde{P}_{i,t}-VC_{i,t}\right)L\left(X_{i,t}+Y_{i,t}\right)-I_{i,t}Y_{i,t}-FC_{i,t}\right) \tag{3.72}$$

subject to

$$\sum_{i=1}^{N}X_{i,t}\geq \bar{d}_t+\sigma\left\|\hat{\boldsymbol{d}}_t\right\|_2\quad \forall t, \tag{3.73}$$

$$\sum_{t=1}^{T}X_{i,t}\leq Cap_{i,t}\quad \forall i, \tag{3.74}$$

$$Cap_{i,t}+Y_{i,t}\leq MaxCap_i\quad \forall i,t, \tag{3.75}$$

$$\sum_{t=1}^{T}\left(\bar{E}_{i,t}X_{i,t}\right)+\sigma\left\|\sum_{t=1}^{T}\hat{\boldsymbol{E}}_{i,t}\cdot\boldsymbol{X}_{i,t}\right\|_2\leq Q,\quad \forall i, \tag{3.76}$$

$$X_{i,t}\geq 0\quad \forall i,t, \tag{3.77}$$

$$Y_{i,t}\geq 0\quad \forall i,t. \tag{3.78}$$

Modeling Ellipsoidal Uncertainty

Robust counterparts of the optimization models with specified uncertainties guarantee feasibility under constraint violations. In general, uncertainty sets are determined by using probability distributions and statistical estimates. However, data from energy markets may not fit to a specific probability distribution. On the other hand, uncertainty can be handled easily when it has a specific shape, like polyhedral and ellipsoidal [50]. Although ellipsoidal uncertainty increases the complexity of the model, it relatively yields more robust results [127]. Therefore, ellipsoidal uncertainty is used and a new algorithm, R~W~MARS, is developed to overcome the increase in model complexity.

Let us define an ellipsoid $\varepsilon(\boldsymbol{q},\boldsymbol{Q})$ in $\mathbb{R}^n$, where $\boldsymbol{q}$ and $\boldsymbol{Q}$ represent the center and shape matrix, respectively. Thus, the ellipsoid takes the form

$$\varepsilon(\boldsymbol{q},\boldsymbol{Q})=\left\{\boldsymbol{x}\in\mathbb{R}^n\ \middle|\ (\boldsymbol{x}-\boldsymbol{q})^{\mathrm{T}}\boldsymbol{Q}^{-1}(\boldsymbol{x}-\boldsymbol{q})\leq 1\right\}, \tag{3.79}$$

where $\boldsymbol{Q}=\boldsymbol{Q}^{\mathrm{T}}$ and $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x}>0$ for all nonzero $\boldsymbol{x}\in\mathbb{R}^n$. Since shape matrix has to be of the form $\boldsymbol{Q}=\boldsymbol{Q}^{\mathrm{T}}$ and $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x}>0,\ \forall\boldsymbol{x}\neq 0$ (i.e., $\boldsymbol{Q}$ is positive definite), the ellipsoid could not be formed when $\boldsymbol{Q}$ is singular. Therefore, our ellipsoid is modeled in new form:

$$\varepsilon(\boldsymbol{q},\boldsymbol{Q}) = \left\{ \boldsymbol{x} \in \mathbb{R}^n \;\middle|\; \boldsymbol{l}^{\mathrm{T}}\boldsymbol{x} \le \boldsymbol{l}^{\mathrm{T}}\boldsymbol{q} + \left(\boldsymbol{l}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{l}\right)^{1/2},\ \forall \boldsymbol{l} \in \mathbb{R}^n \right\}, \tag{3.80}$$

where $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}}$ and $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x} \ge 0$ for all $\boldsymbol{x} \in \mathbb{R}^n$ (i.e., $\boldsymbol{Q}$ is positive semidefinite) and we define

$$\rho\left(\boldsymbol{l} \mid \aleph\right) := \sup_{x \in \aleph}\left(\boldsymbol{l}^{\mathrm{T}}\boldsymbol{x}\right), \tag{3.81}$$

where $\aleph \subseteq \mathbb{R}^n$, in particular,

$$\rho\left(\boldsymbol{l} \mid \varepsilon(\boldsymbol{q},\boldsymbol{Q})\right) := \boldsymbol{l}^{\mathrm{T}}\boldsymbol{q} + \left(\boldsymbol{l}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{l}\right)^{1/2}. \tag{3.82}$$

Here, the distance from $\varepsilon(\boldsymbol{q},\boldsymbol{Q})$ to a fixed point $\boldsymbol{x}$ is

$$\max_{\boldsymbol{l}^{\mathrm{T}}\boldsymbol{l}=1}\left(\boldsymbol{l}^{\mathrm{T}}\boldsymbol{x} - \rho\left(\boldsymbol{l} \mid \varepsilon(\boldsymbol{q},\boldsymbol{Q})\right)\right) = \max_{\boldsymbol{l}^{\mathrm{T}}\boldsymbol{l}=1}\left(\boldsymbol{l}^{\mathrm{T}}\boldsymbol{x} - \boldsymbol{l}^{\mathrm{T}}\boldsymbol{q} - \left(\boldsymbol{l}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{l}\right)^{1/2}\right). \tag{3.83}$$

Hence,

- if the distance $\left(\varepsilon(\boldsymbol{q},\boldsymbol{Q}),\boldsymbol{x}\right) > 0$, then the point $\boldsymbol{x}$ will be the outside of the ellipsoid;
- if the distance $\left(\varepsilon(\boldsymbol{q},\boldsymbol{Q}),\boldsymbol{x}\right) = 0$, then the point $\boldsymbol{x}$ will be on the boundary of the ellipsoid;
- if the distance $\left(\varepsilon(\boldsymbol{q},\boldsymbol{Q}),\boldsymbol{x}\right) < 0$, then the point $\boldsymbol{x}$ will be inside of the ellipsoid.

On the other hand, introducing ellipsoidal uncertainty increases the complexity of the model because of the parameters involved in modeling the uncertainty. To handle this complexity we propose a new method. Figure 3.10 demonstrates the geometric representation of our method. Here, let us assume that the ellipsoid represents the bounds of emission constraints' uncertainty. According to *Average Projected Area Theorem* [134], average projected area of a convex solid body is one quarter of the surface area. Therefore, if we project the ellipsoid on *x-y* and *y-z* planes, we obtain two ellipses with a total area equal to one half of the surface area of the ellipsoid. Hence, we can reduce the area, where we have to search for the feasible results. This approach, in turn, decreases the computational complexity. By utilizing this approach, exact values of the uncertainty in *x* and *z* axes, and corresponding interval in *y* axis can be obtained. Figure 3.11 illustrates the process of obtaining the ellipsoid by using the uncertain parameters and projection of the ellipsoid on principle planes.

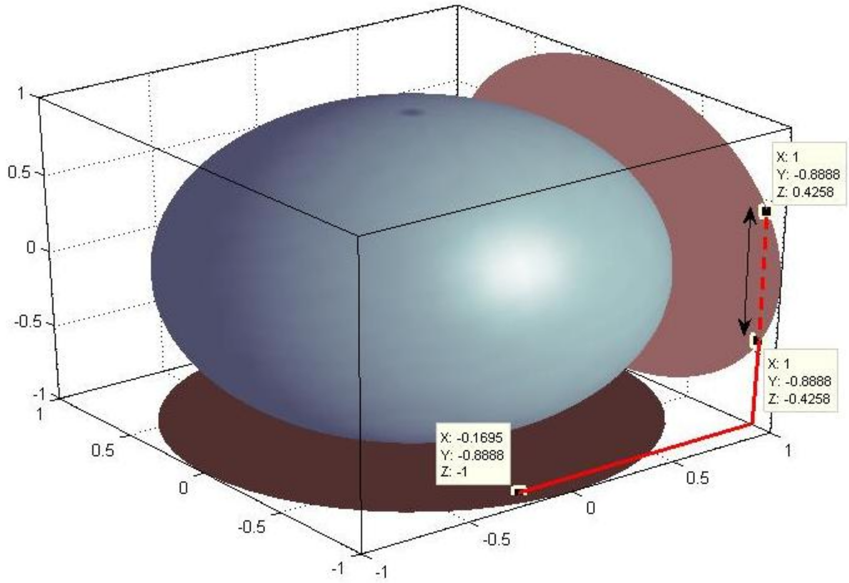


Figure 3.10: Geometric representation of the uncertainty modeling with R~W~MARS.

As a result, our R~W~MARS algorithm is constructed as follows:

Repeat for time $t = 1, 2, \ldots, T$:

Step 1: Generate uncertainty region by using W~MARS,

Step 2: Set initial ellipsoid ε_0 related to the uncertain parameters,

Step 3: Project the region and update uncertainty region $\varepsilon_1 = \varepsilon_{proj}$ (cf. Figure 3.11),

Step 4: Generate uncertain parameter and run the optimization model,

Step 5: $t \leftarrow t + 1$.

The algorithm is implemented on two platforms; MATLAB for the ellipsoidal uncertainty and GAMS for the optimization problem.

Applications of both stochastic optimization model and robust optimization model are demonstrated in the next chapter.

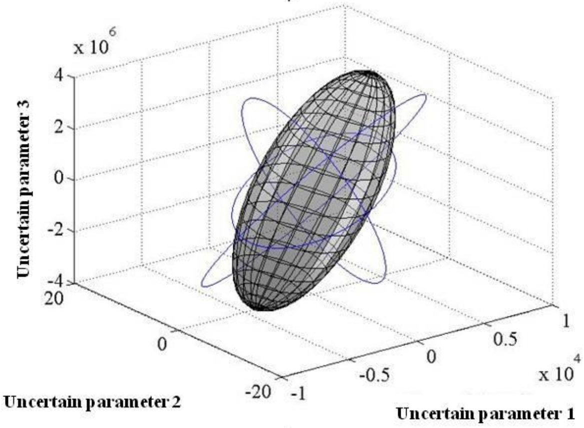
Data for the uncertain parameters

Time	Uncertain parameter 1	Uncertain parameter 2	Uncertain parameter 3
1	28763	42	7467280
2	28432	42	7414575
…	…	…	…
…	….	…	…
T-1	31575	42	6720628
T	29812	39	6061354

Covariance matrix

$$\begin{bmatrix} 2.88\times10^{7} & 3.31\times10^{4} & 8.64\times10^{9} \\ 3.31\times10^{4} & 80.44 & 1.03\times10^{7} \\ 8.64\times10^{9} & 1.03\times10^{7} & 2.89\times10^{12} \end{bmatrix}$$

Error Ellipsoid



Projections on principle planes

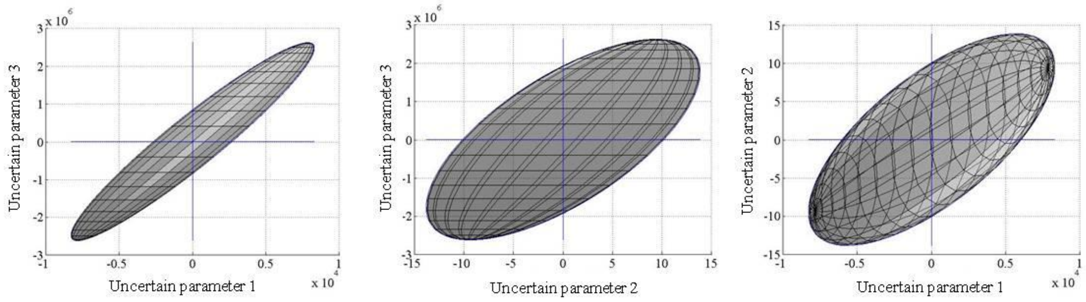


Figure 3.11: Projection of uncertainties.

CHAPTER 4

APPLICATION OF THE MODELS

The models presented in Chapter 3 are validated individually using real market data.

The stochastic optimization model is implemented for Nigerian electricity market. Nigerian electricity market is selected since it is one of the largest energy markets in the world. This is especially true due to its high population (162.5 million people) and huge energy resources (37.20 billion barrels oil and 5154 billion m^3 natural gas reserves). Although the country owns various extensive energy resources, it is seriously suffering from supply of electricity.

Electricity market data for Nigeria are available for the period between 2001 and 2007. The data include production costs and supply of each power plant, the demand, and electricity price on a monthly basis. By implementing the model, we optimized the monthly production portfolio for the year 2008. The data and the results are summarized in Appendix B. The results show that, for the given market conditions, only four plants out of six produce electricity on profit. This indicates that, to have a more profitable market, low-cost electricity generation techniques, which are mainly dependent on renewable sources, need to be implemented. Details of this study are presented in the working paper [164] and the conference proceeding [165].

The next optimization model that we develop in this study is the robust optimization model, R~W~MARS. Before implementing R~W~MARS, we test W~MARS, which is a sub-model to estimate uncertain parameters of the robust optimization model. An application of W~MARS is provided by utilizing the model to estimate the next-day electricity price in Spanish market. For this application, the Spanish electricity market is specifically selected since it is one of most commonly studied electricity market in the world, thus the results of the model can be compared with the results reported in the literature. Besides, the Spanish electricity market is one of the oldest electricity markets, which has evolved into a spot market. Spanish market data includes hourly electricity prices for the year 2002. The data and the results are summarized in Appendix C. The results show that W~MARS has a similar prediction error compared to existing models in terms of weekly forecasting errors. Details of the study are presented in the book chapter [167], conference proceedings [116,155,163], and the working paper [168].

After validating W~MARS, R~W~MARS is implemented using market data of Marmara Region of Turkey. Marmara Region is selected as the target because of both (i) considering the need for an electricity market model in Turkey, and (ii) Marmara Region has the highest density of population and industry in Turkey. The data for the Marmara Region includes electricity demand, price, generation costs, capacities, and emissions. The results show that for the planning horizon between 2011 and 2020 share of natural gas can be reduced while increasing the share of hydroelectric and renewable energies. It is shown that use of renewable energy sources can be increased from 30% to 56% when wind and solar energy resources are considered. The data and the results are presented in Appendix D. The details of the study are presented in conference proceedings [166,169,170].

Although, these applications prove the use of models with real market data individually, they do not provide information required for comparing the stochastic optimization model and robust optimization model. Therefore, for comparison and demonstration purposes, both models are applied using the same data for the United Kingdom (UK) electricity market.

This chapter presents the comparison of the models based on an application using the UK market data. The following sections describe the market data, results obtained using our stochastic optimization model, the results obtained from our robust optimization model, and a discussion on the results, respectively.

4.1 Data Structure

There are 10 groups of power suppliers in UK electricity market. These groups include natural gas burning power plants, hydroelectricity plants, open cycle gas turbine (OCGT) power plants, nuclear power plants, pumped storage hydroelectricity plants, wind power plants, biomass burning power plants, coal burning power plants, oil burning power plants, and interconnectors. These groups of plants are as follows:

- Plant#1: Natural gas burning power plants,
- Plant#2: Hydroelectricity plants,
- Plant#3: OCGT power plants,
- Plant#4: Nuclear power plants,
- Plant#5: Pumped storage hydroelectricity plants,
- Plant#6: Wind power plants,
- Plant#7: Biomass burning power plants,
- Plant#8: Coal burning power plants,
- Plant#9: Interconnectors,

- Plant#10: Oil burning power plants.

For ease of reference we used the term "plant" to indicate the groups of suppliers. Although interconnectors do not actively participate in power generation, since they are supplying electricity into the market, they are also classified as "plant".

UK data is available on half-hour basis. Half-hourly data includes the price, demand, supplied electricity and GHG emission by each power plant. Table 4.1 presents a representative half-hourly data for UK electricity market. In addition to these, capacities, investments costs, and generation costs of each power plant are also available (cf. Table 4.2).

Table 4.1: United Kingdom (UK) electricity market data per half-hourly basis [176]. Representative data for the first two hours of 1 April 2012 are presented.

Date		**01-Apr-12**			
Settlement period		**00.00 - 00.30**	**00.30 - 01.00**	**01.00 - 01.30**	**01.30 - 02.00**
Price (£/MWh)		41.82	42.35	43.99	44.51
Demand (MW per half hour)		28762.92	28431.74	28796.4	29151.46
Supply (MW per half hour)	**Gas**	4637	4401	4557	4807
	Coal	15343	15374	15277	15248
	Nuclear	6309	6305	6304	6307
	Hydro	155	155	152	152
	Net Pumped	0	0	0	0
	Wind	258	269	290	296
	OCGT	0	0	0	0
	Oil	0	0	0	0
	Biomass	44	44	44	44
	Interconnector	1566	1564	1566	1566
Emission ($kgCO_2eq$)	**Gas**	1136065	1078245	1116465	1177715
	Coal	6290630	6303340	6263570	6251680
	Nuclear	37854	37830	37824	37842
	Hydro	1860	1860	1824	1824
	Net Pumped	0	0	0	0
	Wind	1419	1479.5	1595	1628
	OCGT	0	0	0	0
	Oil	0	0	0	0
	Biomass	10780	10780	10780	10780

Table 4.2: Capacities, generation costs and investment costs of main generation technologies in UK electricity market [176,177].

	Capacity (MW)	Generation cost (£/MWh)	Investment cost (£/MWh)
Gas	15950.96	80.30	12.40
Coal	24588.83	104.50	33.40
Nuclear	9108.33	99.00	77.30
Hydro	841.42	83.20	74.20
Net Pumped	128.13	45.20	74.20
Wind	4446.25	93.90	79.20
OCGT	86.88	90.50	7.10
Oil	254.67	148.00	91.30
Biomass	809.67	93.20	46.10
Interconnector	2921.92	93.09	55.02

Using this data, optimization models are used to determine the generation portfolio and the profit in the market for May 2012. Here, data for April 2012 are used for testing purposes.

4.2 Application of the Stochastic Optimization Model

The most important aspect in an electricity market optimization problem is the uncertainty of the future returns. Here, the analysis of the electricity market is considered in terms of portfolio management and optimization. Future returns and other related variables are determined by using the portfolio management modules stated below:

- A module describing the random quantities of the model and their evolution (scenario generator),
- An optimization module for given objective function and evolution of variables.

Before starting with the first module, considering that one of the disadvantages with the stochastic optimization model is the low computational efficiency, we convert the data from a half-hourly to a daily basis to reduce the CPU time.

In the first module, Monte-Carlo simulation is used for £-USD exchange rates and natural gas prices dictated by the Henry Hub. Through simulating different price paths for the natural gas prices and the exchange rates we can build up a two-dimensional scenario tree. This scenario tree serves as input for generating different piecewise linear supply curves (cf. Figure 4.1). Here, these parameters are optimized through Ornstein-Uhlenbeck mean-reverting process (cf. Table 4.3). Each scenario creates new slopes of the piecewise linear supply curves, which serve as a stochastic input variable of the optimization model.

Table 4.3: Parameters of price process for natural gas prices and exchange rates.

	α	μ	σ	S_0 (May 1, 2012)	L
Natural Gas Prices	1.31	2.92%	24.8%	2.29	3.92
Exchange Rates	5.74	0.38%	3.0%	0.61	0.64

In the second module, together with the demand scenarios (cf. Figure 4.2) and the power plant capacities, they represent the inputs for the stochastic portfolio optimization model. The model outputs optimal quantities as well as distribution for the profits and risk measures by maximizing the overall profit. Here, MS Excel and GAMS are used for the first and the second modules, respectively.

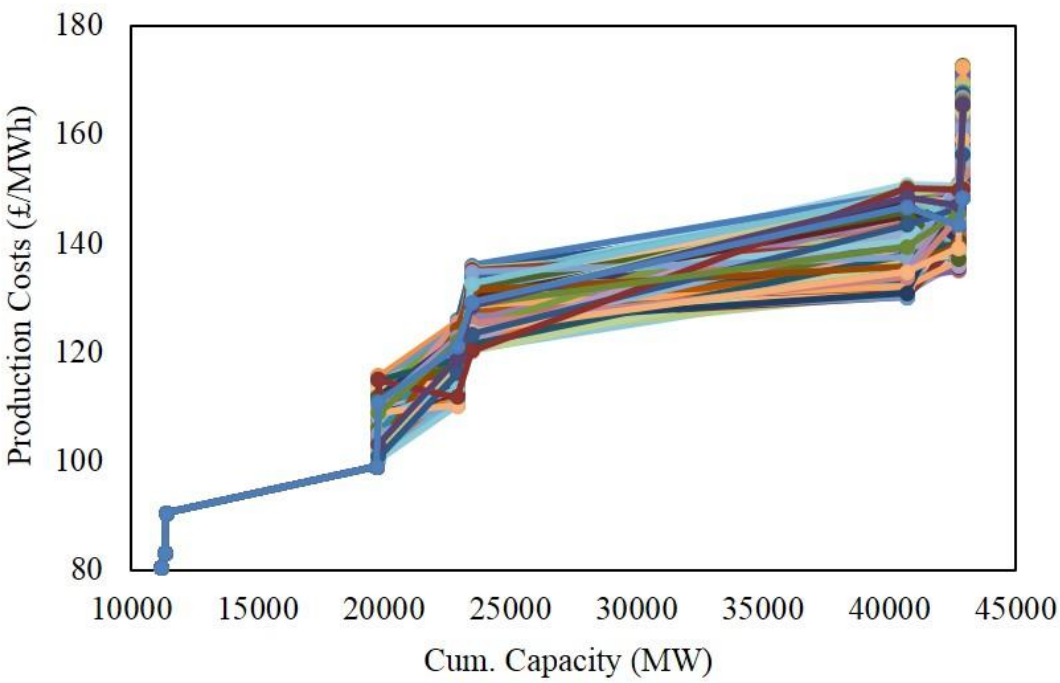


Figure 4.1: Piecewise linear supply curves for 100 scenarios in May 2012.

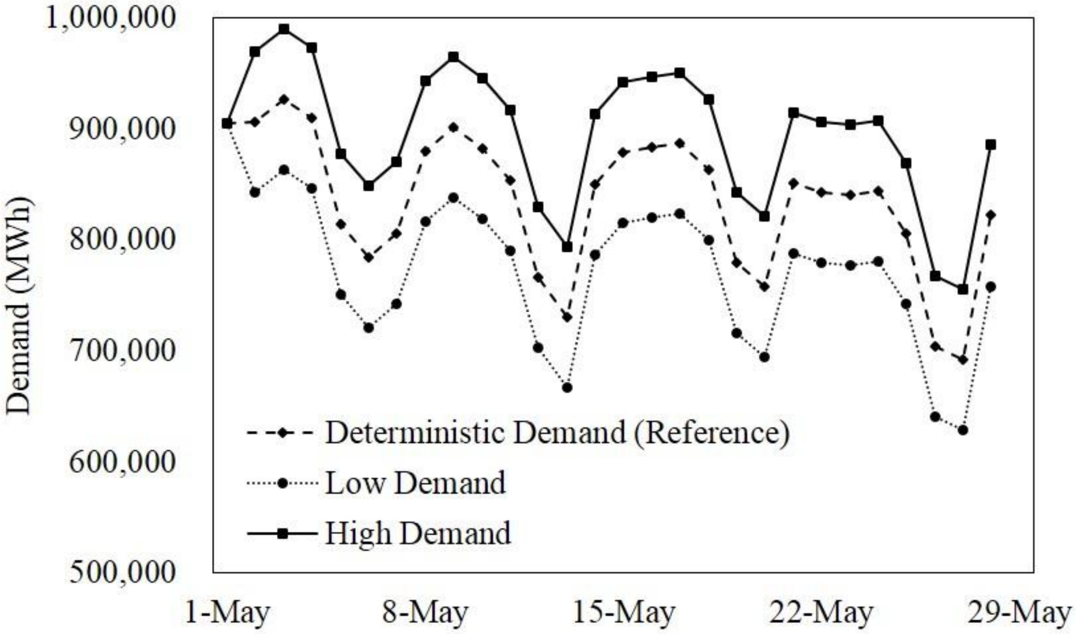


Figure 4.2: Deterministic, low-, and high-demand scenarios in May 2012.

Figures 4.3-4.5 represent the share of the total profit among the plant types in May 2012 computed by referring to deterministic, low, and high demand scenarios respectively. The results show that the plants 9 and 10, which are biomass burning power plants and interconnectors respectively, are not involved in power generation. This is simply because the production costs of these plants are high compared to that of the other plant types.

The results also show that plant 8, which represents coal burning power plants, is dominating the share of profit in all demand scenarios. The reason for this is that coal burning power plants have the highest capacity in the market while having the least generation cost. On the other hand, this specific output shows that the stochastic optimization model, which mainly considers the generation costs, does not account for the renewability and the sustainability of the market.

Comparing the effects of the demand scenarios, we can deduce that the loss is comparably high in the low demand scenario. This is caused by the high-capacity and low-generation-cost plants (mainly the plant 8, which is coal burning) producing electricity at loss.

As an obvious outcome, it can be seen that the overall profit in the market increases with increasing demand (cf. Figure 4.6).

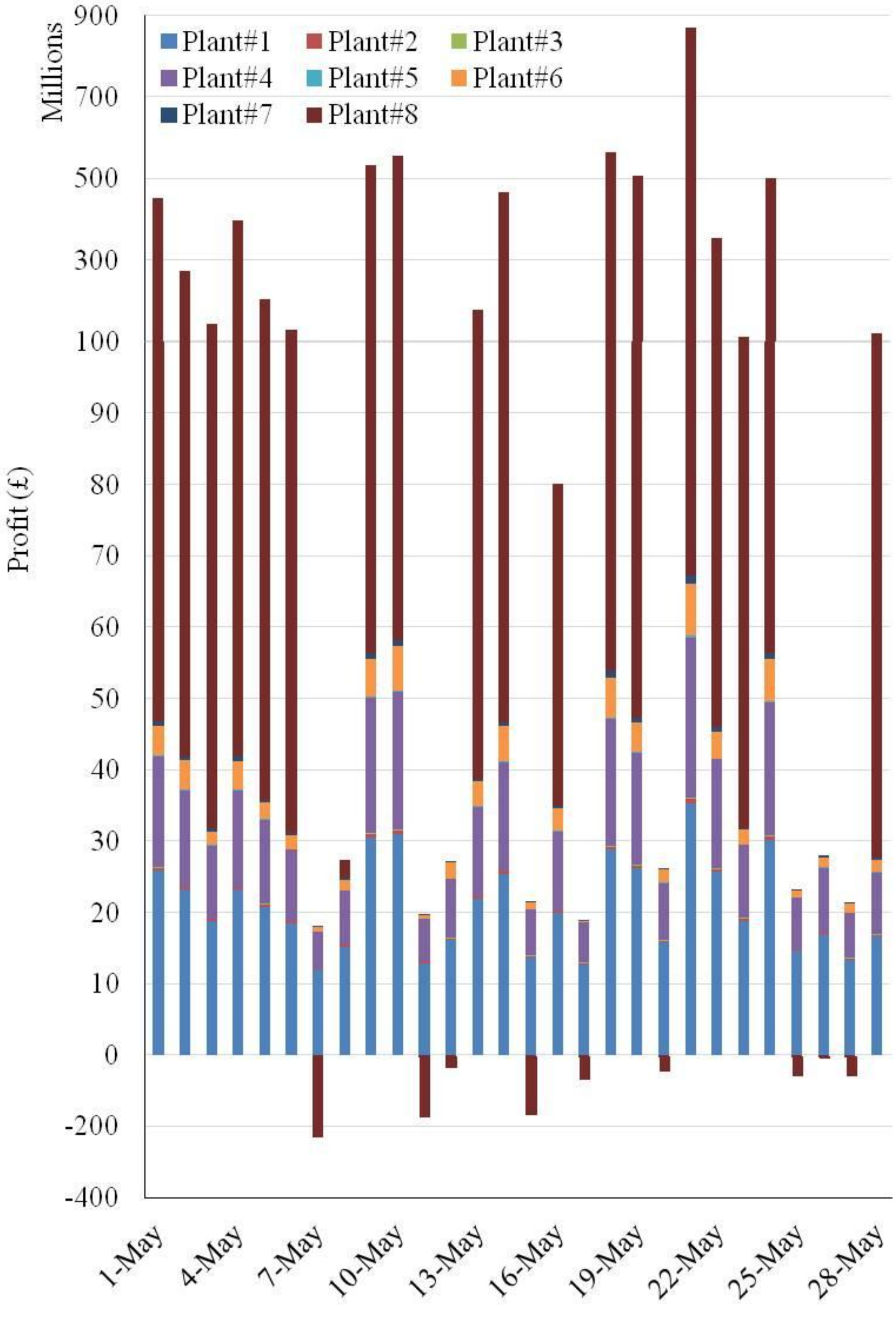


Figure 4.3: Share of total profit among 8 power plants in deterministic demand scenario.

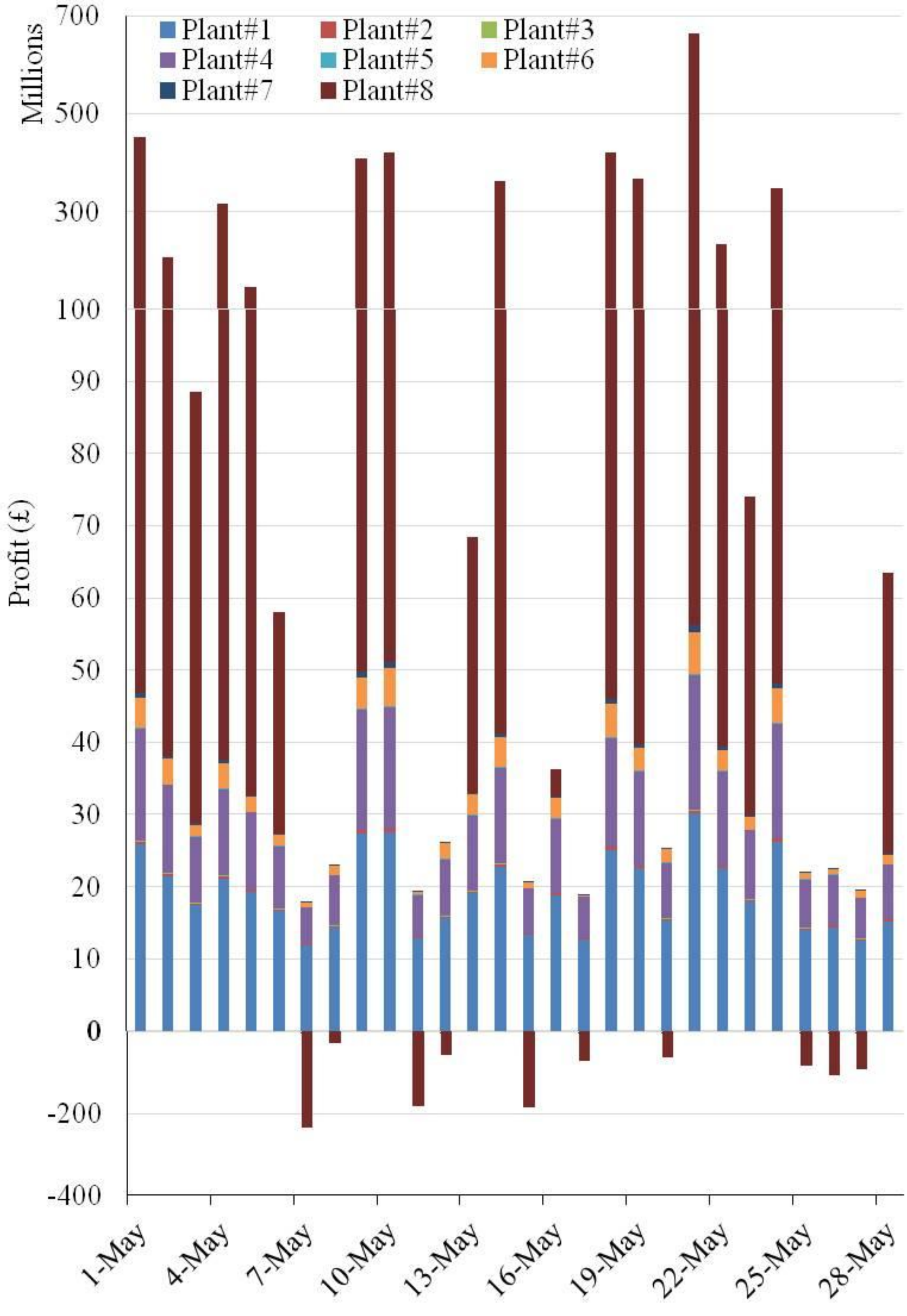


Figure 4.4: Share of total profit among 8 power plants in low-demand scenario.

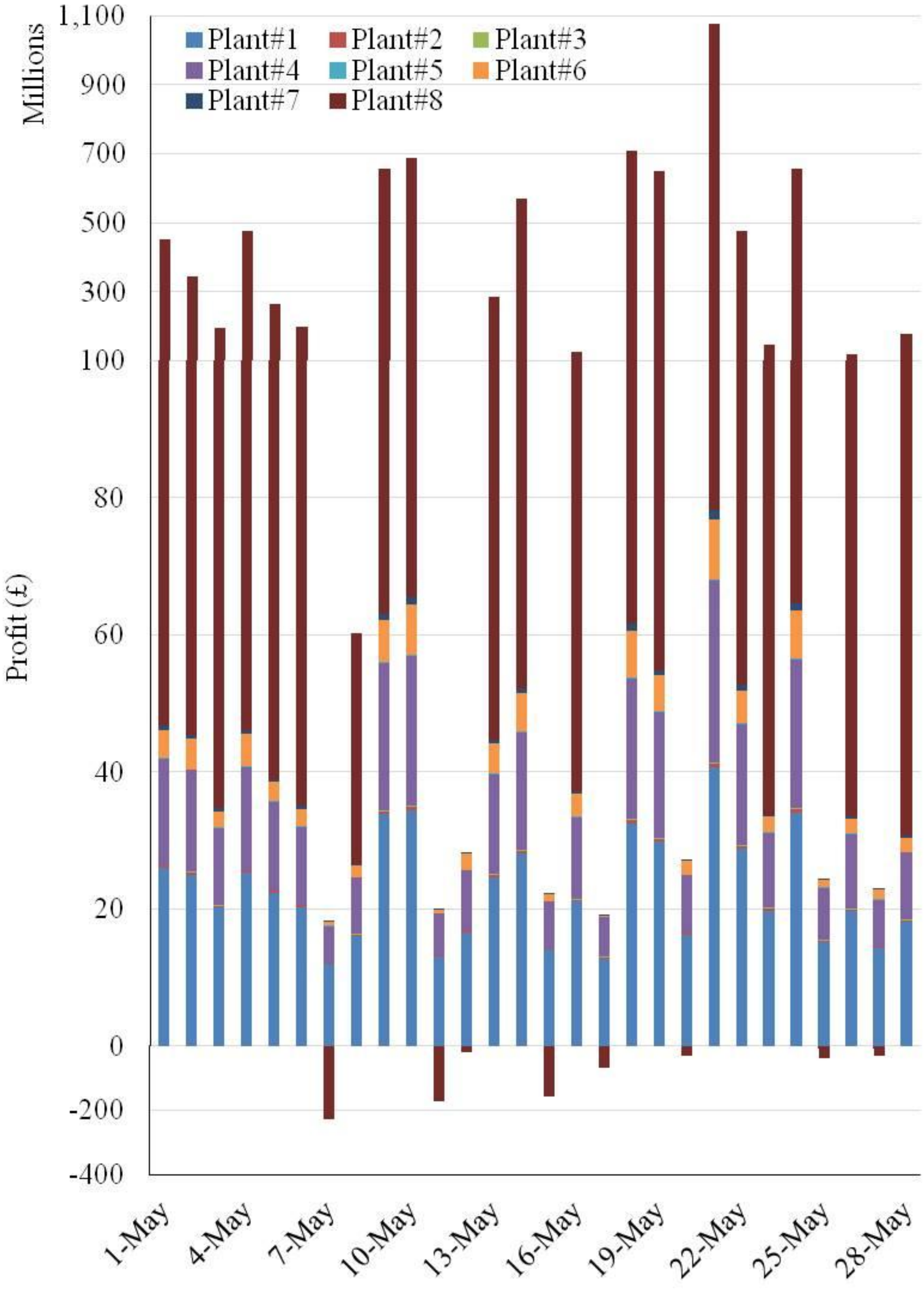


Figure 4.5: Share of total profit among 8 power plants in high-demand scenario.

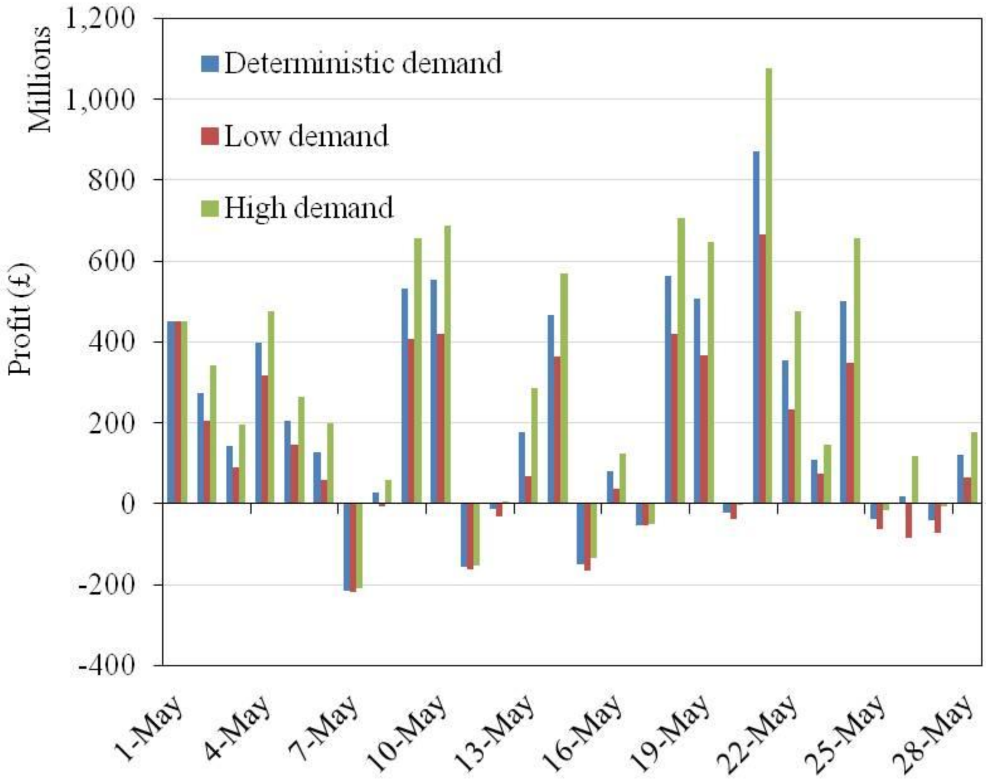


Figure 4.6: Total profits in deterministic, low- and high-demand scenarios.

4.3 Application of the Robust Optimization Model

As the first step in application of the robust optimization model, half-hourly price and demand are estimated by using W~MARS (cf. Figures 4.7 and 4.8) to determine the uncertainties in price and demand. On the other hand, price and demand are not the only uncertain parameters in the market. GHG emissions are also uncertain. However, since GHG emission depends on the amount of electricity generated, it cannot be possible to estimate emissions by using W~MARS.

As stated in Section 4.1, the data for UK market are available per half-hourly basis. This property makes the data relatively smoother. Hence, the spikes in price and demand data within a day are moderate (cf. Figures 4.9 and 4.10). As a result the errors in forecasting price and demand are significantly low.

In forecasting price and demand, we use different wavelet filters to investigate their effects. It is observed that there is no significant difference between the results obtained by different wavelet filters (cf. Figures 4.9 and 4.10). Mean absolute percentage errors (MAPE) for price and demand estimation are approximately found as %4 and %3, respectively (cf. Tables 4.4 and 4.5).

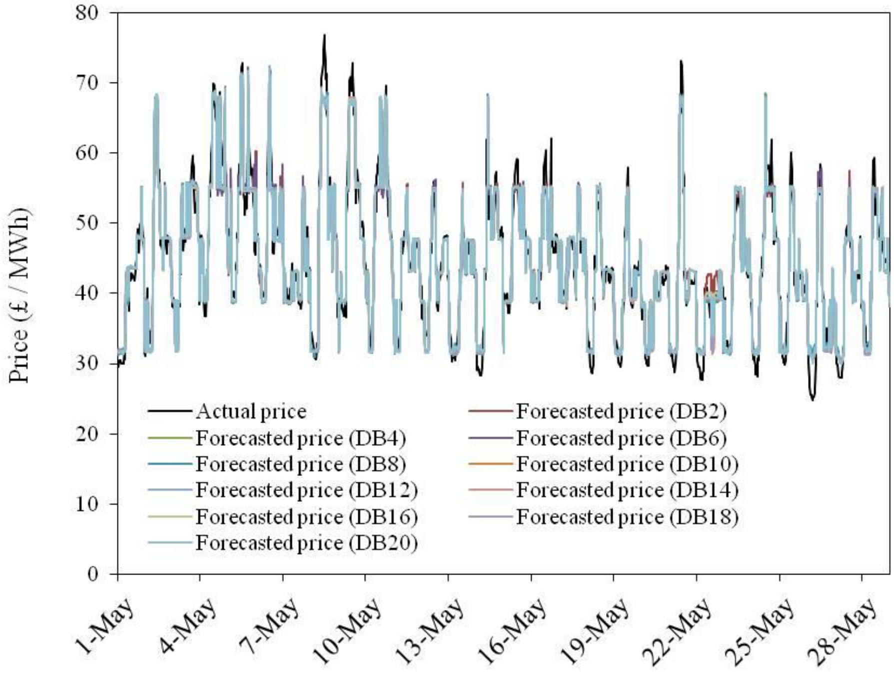


Figure 4.7: Forecasted electricity prices using different filters compared to actual prices in May 2012.

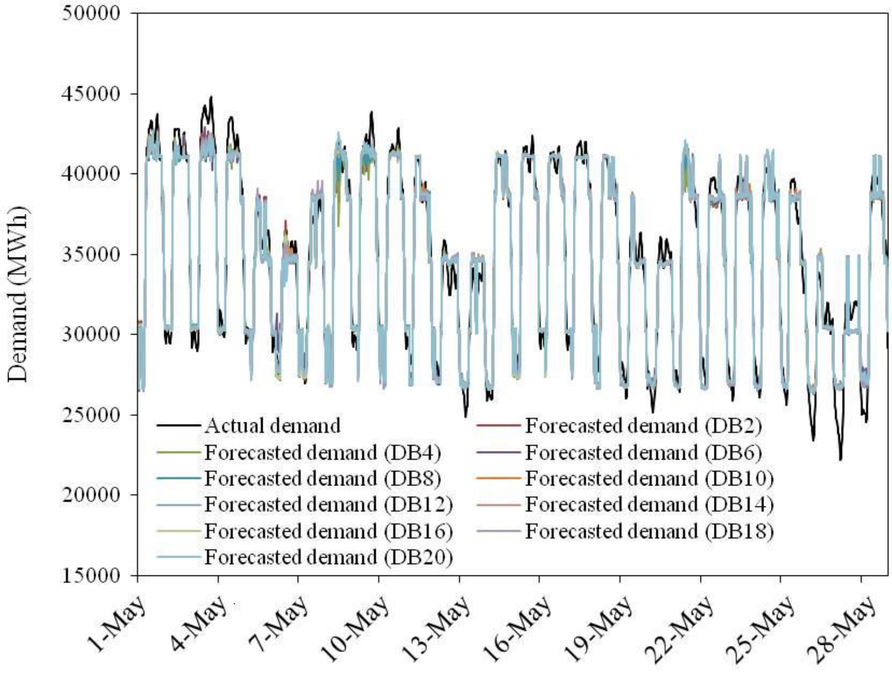


Figure 4.8: Forecasted demand using different filters compared to actual demand in May 2012.

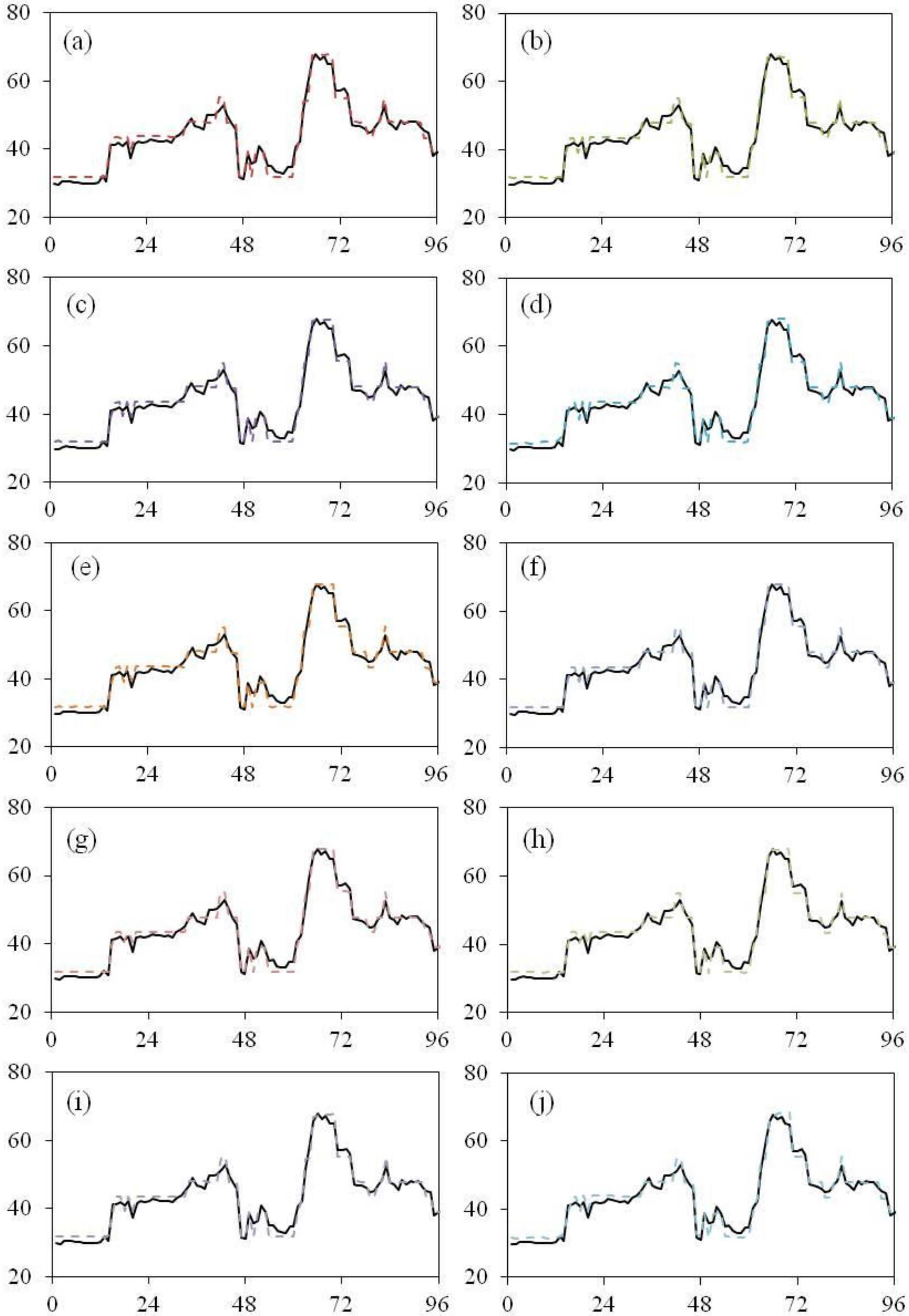


Figure 4.9: Effect of the filters on electricity price (£/MWh) forecasting. Full lines and dash lines represent actual price and forecasted price in the first two days of May 2012 with half-hour periods. Filters DB2, DB4, DB6, DB8, DB10, DB12, DB14, DB16, DB18, and DB20 are used in (a)-(j), respectively.

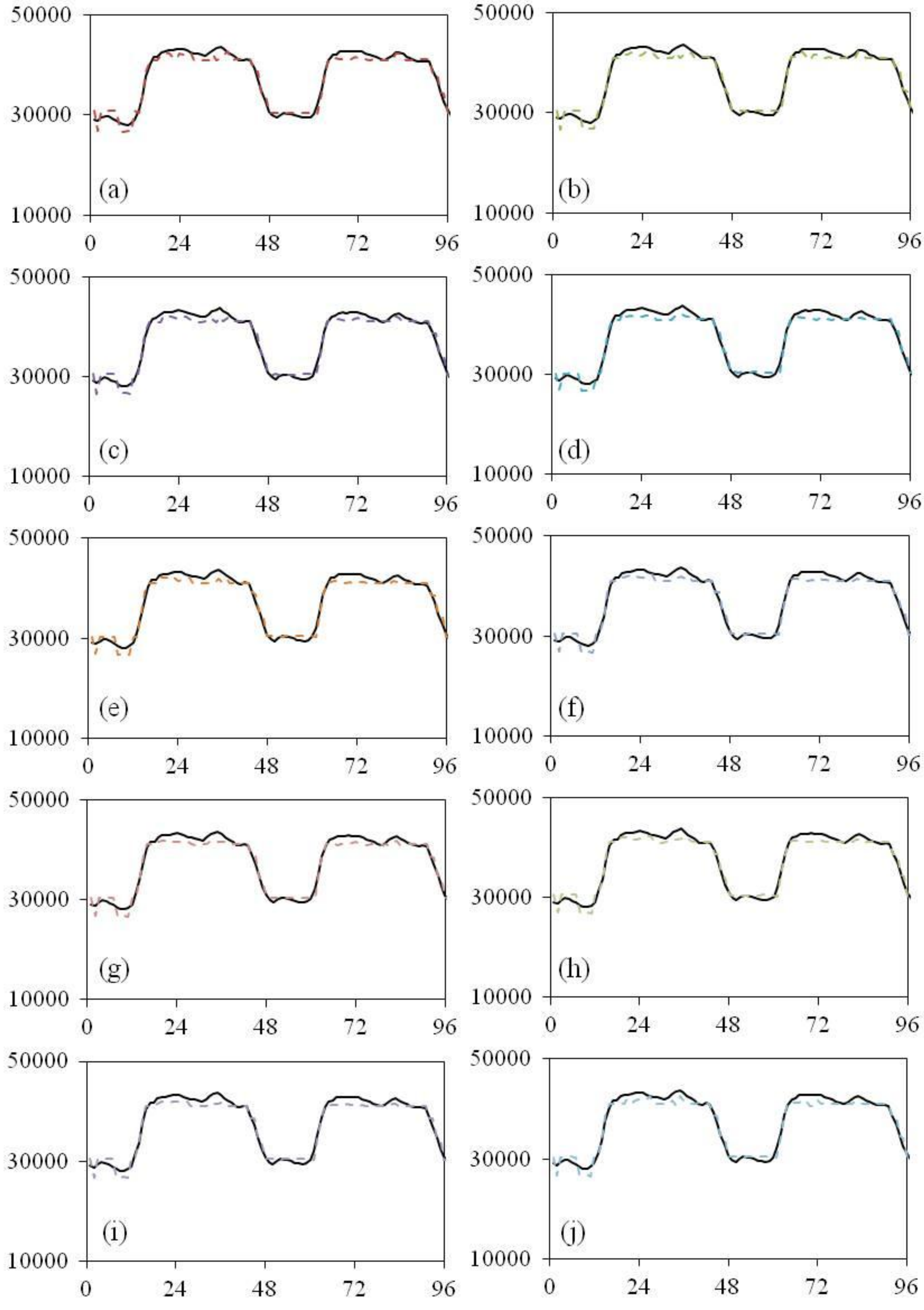


Figure 4.10: Effect of the filters demand (MWh) forecasting. Full lines and dash lines represent actual demand and forecasted demand in the first two days of May 2012 with half-hour periods. Filters DB2, DB4, DB6, DB8, DB10, DB12, DB14, DB16, DB18, and DB20 are used in (a)-(j), respectively.

Table 4.4: Errors in price estimation by using different wavelet filters. MAPE: Mean absolute percentage error, MAD: Mean absolute deviation, RMSE: Root mean square error.

	DB2	DB4	DB6	DB8	DB10	DB12	DB14	DB16	DB18	DB20
MAPE	0.0423	0.0404	0.0414	0.0404	0.0407	0.0404	0.0408	0.0409	0.0406	0.0407
MAD	1.8476	1.7637	1.8129	1.7631	1.7734	1.7622	1.7809	1.7871	1.7718	1.7778
RMSE	2.3591	2.2377	2.3251	2.2359	2.2422	2.2296	2.2578	2.2656	2.2459	2.2623

Table 4.5: Errors in demand estimation by using different wavelet filters. MAPE: Mean absolute percentage error, MAD: Mean absolute deviation, RMSE: Root mean square error.

	DB2	DB4	DB6	DB8	DB10	DB12	DB14	DB16	DB18	DB20
MAPE	0.0290	0.0288	0.0288	0.0287	0.0281	0.0281	0.0285	0.0282	0.0281	0.0285
MAD	969.3	963.0	956.7	958.0	936.4	935.6	948.3	939.4	937.0	950.2
RMSE	1173.9	1176.2	1166.1	1153.7	1136.4	1136.4	1145.4	1138.1	1139.0	1154.4

After forecasting price and demand by using W~MARS and determining the uncertainties accordingly, our robust optimization model is run to find the electricity generation portfolio. Figures 4.11-4.13 represents the portfolio results based on price and demand forecasted by using different wavelet filters. The results show that the wavelet filter used in W~MARS does not make any considerable difference in electricity the generation portfolio.

It can be observed that the distribution of the portfolio among different types of plants is relatively balanced. Besides, the results also show that the renewables (plants 2, 5, and 6, which are hydroelectricity power plants, net pumped storage hydroelectricity power plants, and wind power plants respectively) appear in the portfolio. These results verify the renewability and sustainability of R~W~MARS.

4.4 Comparison of the Models

Electricity generators in general operate by considering mainly the electricity generation costs, with low or no emphasis on renewability or sustainability. Actual generation portfolio presented in Figure 4.14(a) indicates this behavior, where the plants with low production costs tend to generate as much as possible to increase their profits while meeting the demand.

Stochastic portfolio optimization model mimics this behavior by making use of the merit-order curve (cf. Figure 3.2). Accordingly, the plants with low production costs are allowed to generate at their full capacities until the demand is met. For this reason, our stochastic optimization model better predicts the actual generation portfolio (cf. Figure 4.14(b)).

Different from the stochastic optimization model, R~W~MARS considers not only the profitability but also renewability and sustainability of the market. Renewability and sustainability is achieved by limiting GHG emissions for each plant. This approach results in a generation portfolio with more emphasis on renewable resources (cf. Figure 4.14(c)). On the other hand, R~W~MARS yields a less profitable market (computed as approximately £ 90 million in May 2012), since it does not consider the merit-order curve. The profitability may be improved by implementing the merit-order curve in R~W~MARS. This approach would yield a more realistic portfolio at the expense of reduced renewability and sustainability.

Apart from the performance of the models considering sustainability, renewability and profitability aspects, both models can meet the demand.

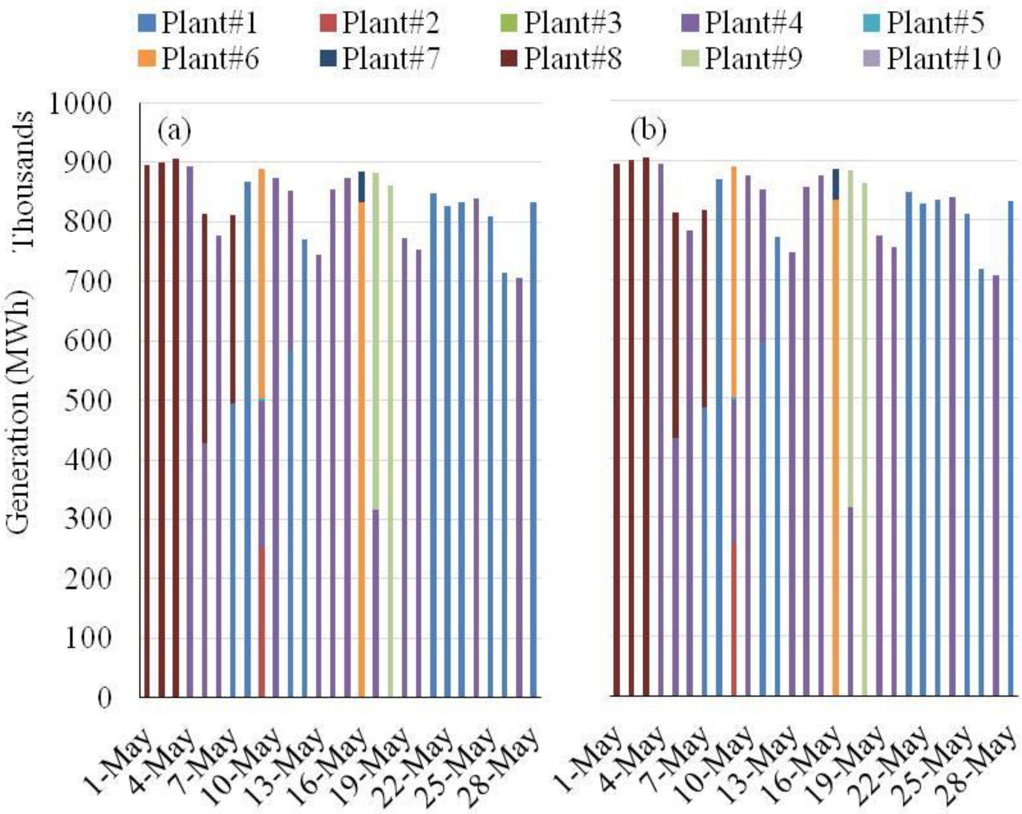


Figure 4.11: Electricity generation portfolio in May 2012, determined by R~W~MARS based on price and demand forecasted by using (a) DB2 and (b) DB4 filters.

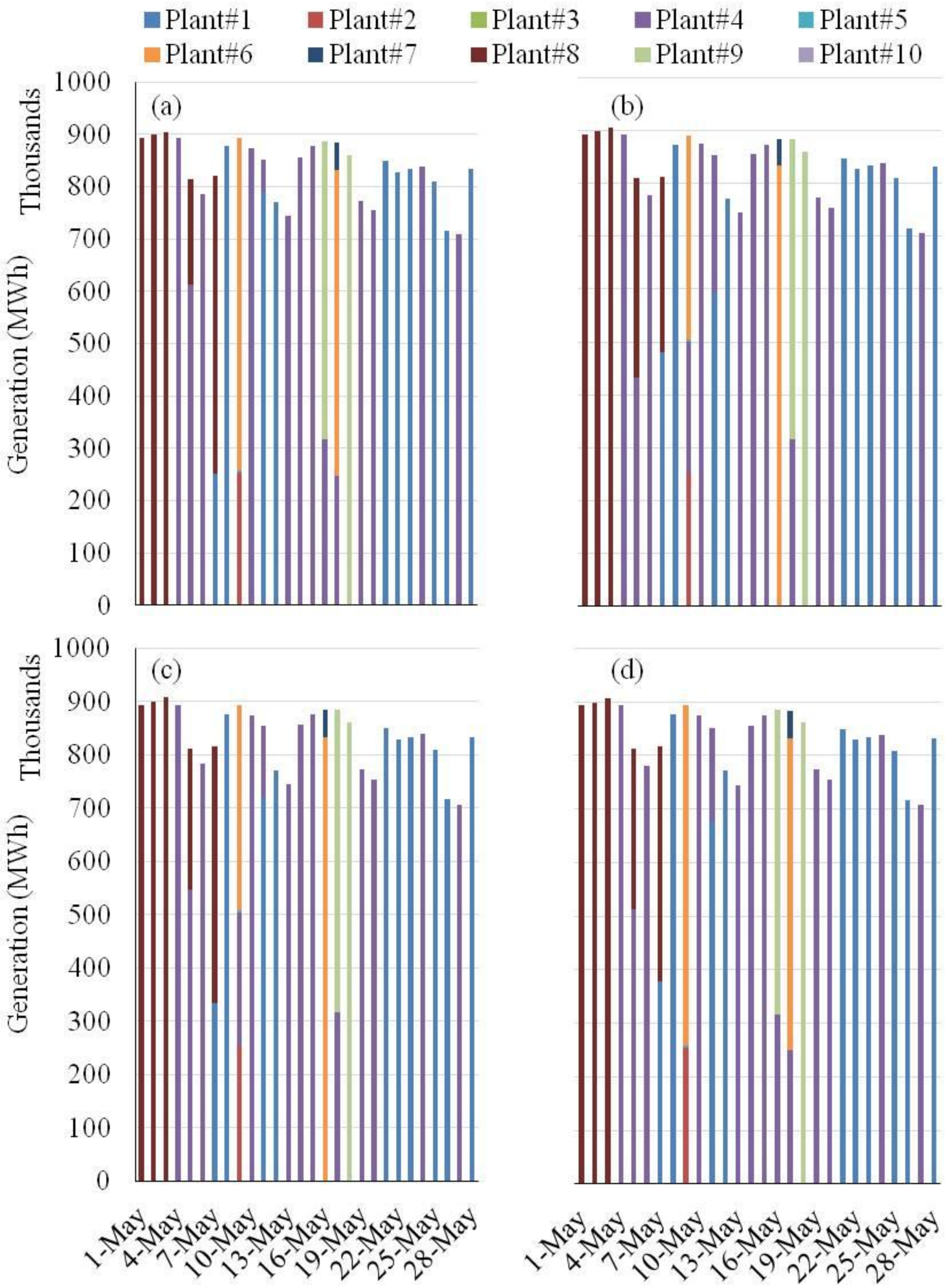


Figure 4.12: Electricity generation portfolio in May 2012, determined by R~W~MARS based on price and demand forecasted by using (a) DB6, (b) DB8, (c) DB10 and (d) DB12 filters.

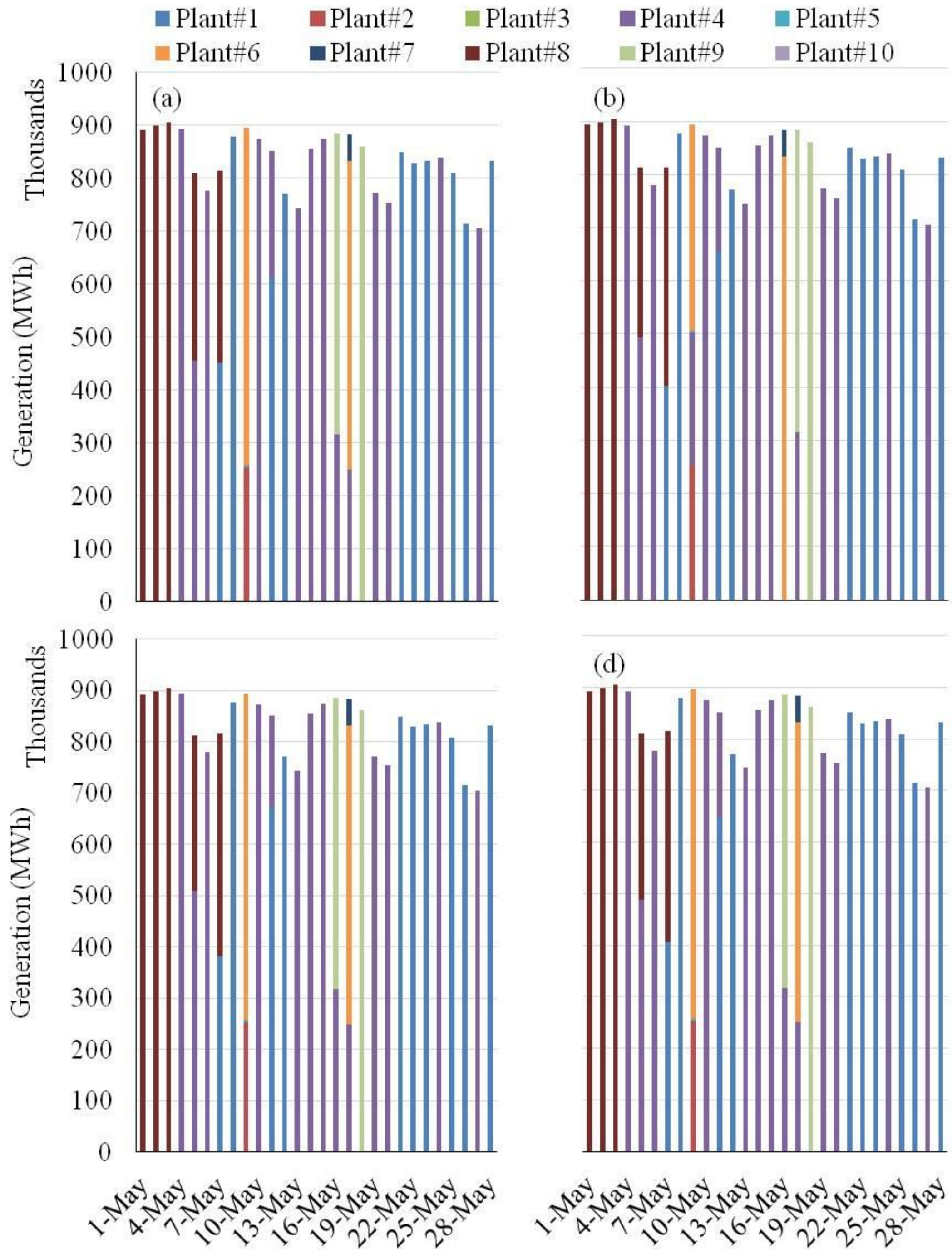


Figure 4.13: Electricity generation portfolio in May 2012, determined by R~W~MARS based on price and demand forecasted by using (a) DB14, (b) DB16, (c) DB18 and (d) DB20 filters.

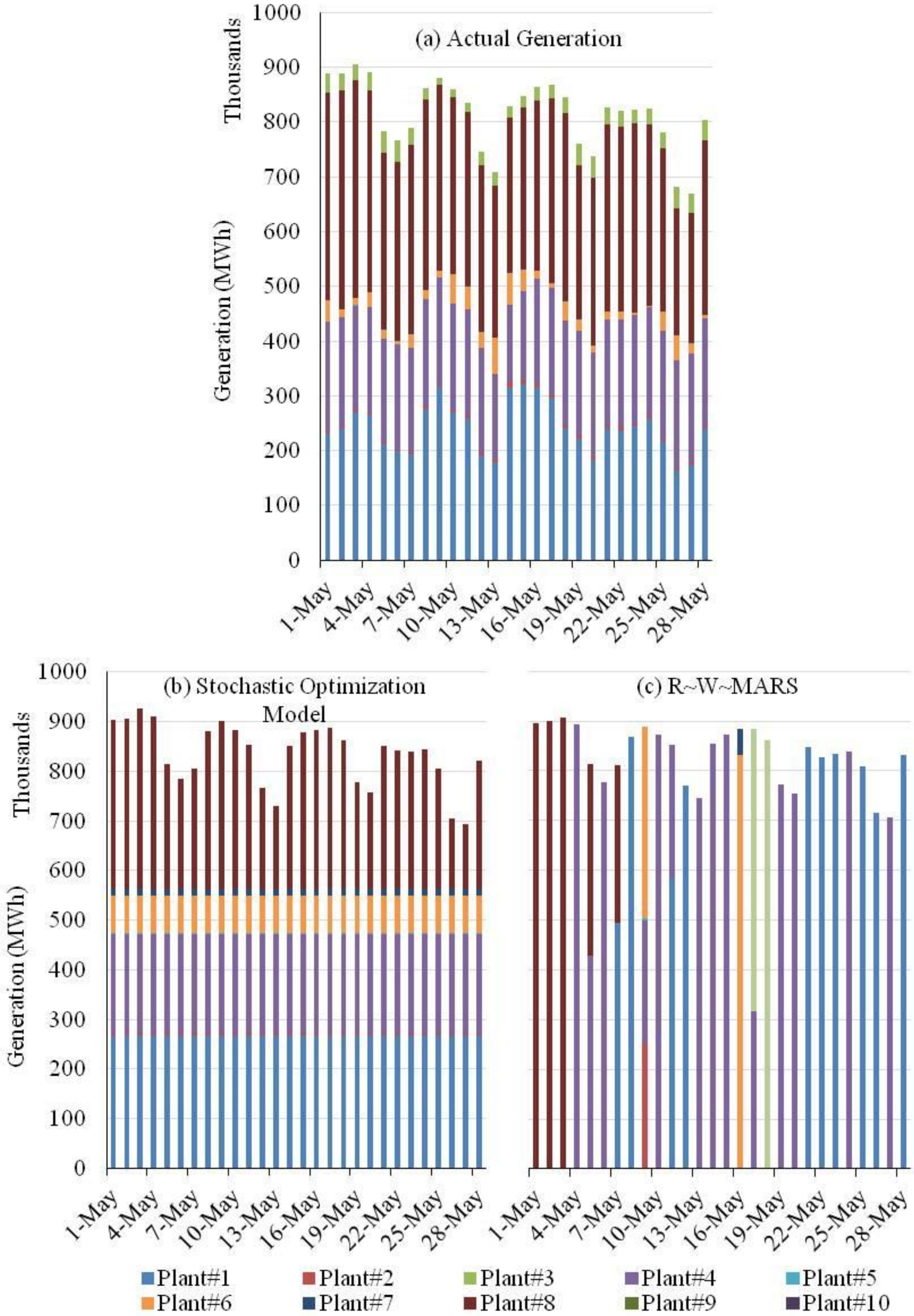


Figure 4.14: Comparison of the electricity generation portfolios, computed by using the stochastic optimization model and R~W~MARS, with the actual generation in UK in May 2012.

CHAPTER 5

CONCLUDING REMARKS AND OUTLOOK AT FUTURE WORKS

In this thesis work, various models are developed using stochastic portfolio optimization and robust optimization techniques to handle uncertainties in electricity markets. Stochastic optimization model is implemented for Nigerian electricity market, whereas W~MARS and R~W~MARS are implemented for Spanish and Turkish electricity markets, respectively. Although, these applications prove the use of models with real market data individually, they do not provide information required for comparing the stochastic optimization model and the robust optimization model. Therefore, for comparison and demonstration purposes, both models are applied using the same data for the English electricity market. Brief descriptions of the models developed in the thesis and the properties of these models are provided subsequently.

1. **Stochastic Portfolio Optimization Model:** A novel statistical and simulation-based method is developed for portfolio optimization of electricity markets to maximize the profit and to obtain the most economic diversity of energy resources. The method involves generation of stochastic supply curves and scenario trees for different power-generation techniques. The method utilizes Ornstein-Uhlenbeck mean-reverting process and Monte-Carlo simulations to generate stochastic electricity supply curves. The method is implemented using UK electricity market data to determine the power generator portfolio for base-, low-, and high-demand scenarios.

2. **W~MARS:** A novel method, which is a combination of wavelet transform and multivariate adaptive regression splines, is developed. The hybrid method merges the strengths of both methods. WT captures multiple seasonality, unusual behaviors and volatility, whereas MARS eliminates the selection of explanatory variables problem. The method is demonstrated applied on UK electricity market data to forecast next-day electricity prices.

3. **Renewable and Sustainable Electricity Market Model under Uncertainties:** A novel dynamic, region-specific and sustainable robust optimization model, which is capable of handling uncertainties, is developed and implemented for the UK electricity market. Here, uncertain parameters are determined by the use of R~W~MARS method.

The studies in the literature show that energy systems comprise not only production, conversion and distribution of energy sources, but also its interactions with the external world (cf. Figure 1.1) [77]. Therefore, security of energy supply (SES) and energy security indicators are studied. Both composite and simple forms of indicators are analyzed with their impacts to energy models. *Energy security* concept is defined by different authorities considering its different aspects and perspectives. For instance, International Energy Agency (IEA) defines energy security as "the uninterrupted physical availability at a price which is affordable, while respecting environment concerns" [81]. Various definitions of SES are reviewed in [160]. These definitions refer to availability, accessibility, affordability, technology development, sustainability, regulation, energy and economic efficiency, and environmental management as the factors affecting energy security. Regarding the sustainability of the market, energy indicators are defined in the paper [117]. These indicators are listed in Table E.1 for reference purposes.

The objective of the study has been defined as *developing a dynamic and robust model for electricity markets under uncertainties* at the beginning of the thesis. Considering the achievements mentioned above, it can be concluded that the objective of the thesis became satisfied. However, there are still some issues to be improved. These potentials and challenges are listed below.

1. Stochastic portfolio optimization model developed in the first stage of the thesis does not involve yet an evaluation of the *sensitivity* of the portfolios, which may be quite important in policy making. In article [64], *Malliavin calculus* is used in portfolio optimization problems to evaluate sensitivity. Therefore, the methods presented in [32,33] can be applied for a sensitivity analysis on portfolios in the electricity market.

2. In our stochastic portfolio optimization, UK electricity market is considered. This market is being expanded new market participants. The effects of expansion should be considered and an integrated scenario-generation mechanism can be utilized to include new market participants.

3. W~MARS method developed in the second stage of the thesis is used to forecast next day electricity price and demand in UK electricity market. However, the results indicate that W~MARS needs further improvement in tracing the sharp spikes. To improve the method, performances of least asymmetric filters, best localized filters, coiflet filters can be compared and their effects on spikes can be determined and optimized.

4. Electricity market model that we proposed is described by the demand and the electricity prices. The model can be expanded to include other effects such as temperature, humidity, natural gas price, etc., as listed in Table A.4.

5. There exist optimization-supported variations of MARS, namely, CMARS; and their robust counterparts RCMARS and RMARS [98–102]. Further hybrid methods, namely, W~CMARS, R~W~CMARS and R~W~MARS, can be developed, especially, to handle further ill-behaved data. Besides,

R~W~MARS can be modified as a multi-objective portfolio optimization model [54].

6. In W~MARS method, discrete wavelet transform (DWT) is utilized. However, DWT restricts the sample size by 2^J since low-pass and high-pass parts are decomposed into half sets in each iteration. This drawback can be overcome by utilizing a maximally overlapping discrete wavelet transform.

7. Complexity in handling elliptic uncertainties in the robust optimization model can be reduced by analyzing the contours of the ellipsoid. In order to have the contours, the ellipsoid can be divided into two parts, as shown in Figure 5.1. In this case, we again have a feasible region in two dimensions.

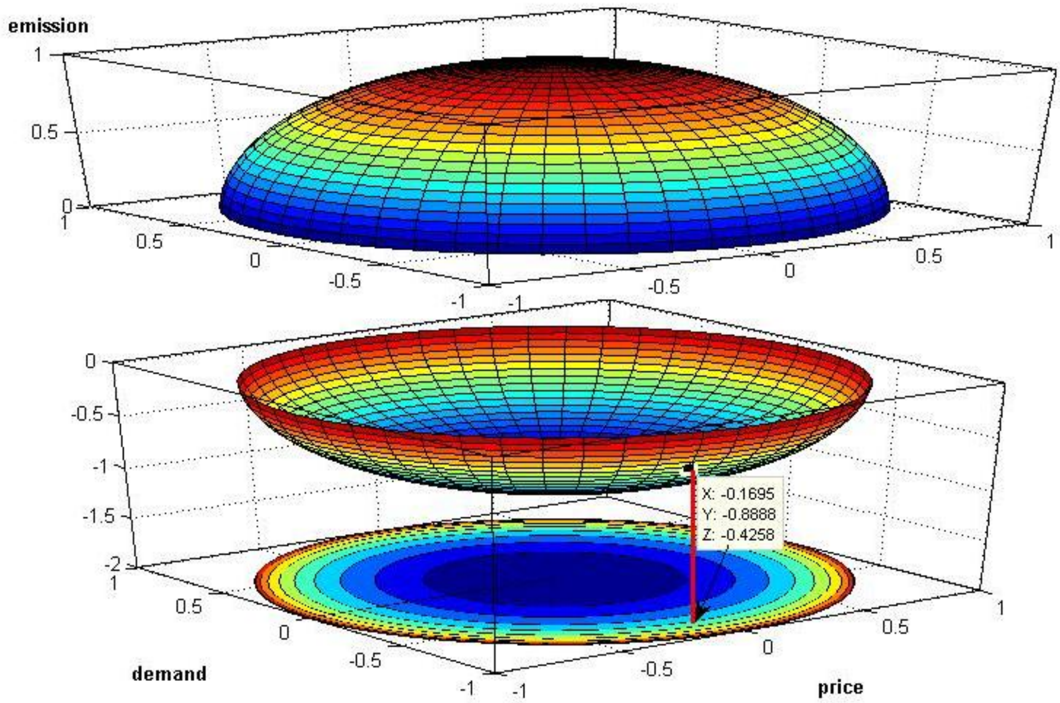


Figure 5.1: Contour plot of half-ellipsoid.

8. In our robust optimization model, the uncertainties are assumed to be independent from each other. As an extension, the model may be improved to examine the effects of intersecting uncertainties.

9. When we have more than one uncertainty in our model at the same time, e.g., price, demand, and emission, then the geometric shape of our uncertainty bound takes the form as shown in Figure 5.2. Multiple correlated uncertainties may be determined for emission, demand, and price.

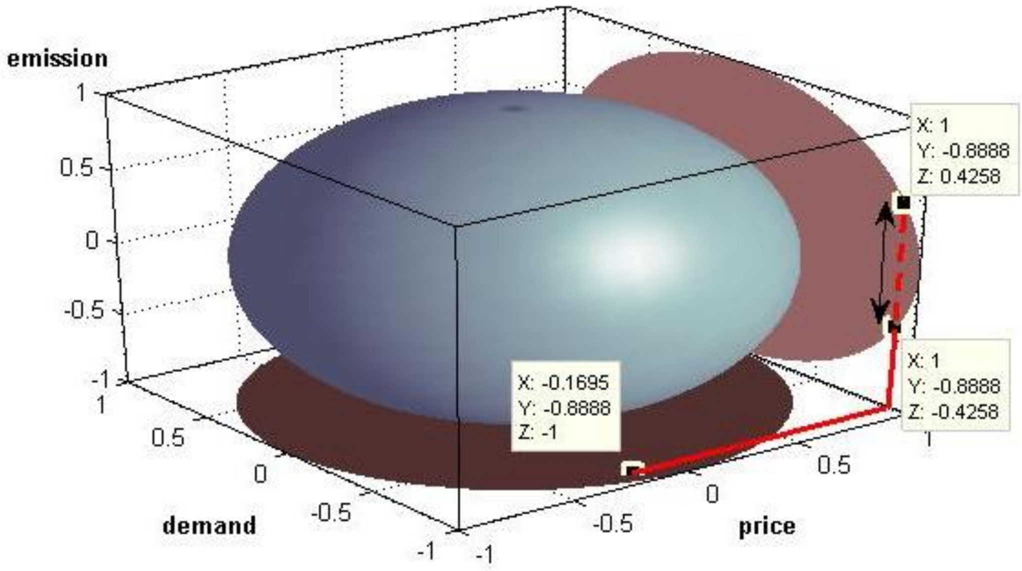


Figure 5.2: Geometric representation of the uncertainty modeling in multiple form with R~W~MARS.

10. Energy security indicators can be used for scenario generation and policy making. For instance, the article [133] integrates indicators with market allocation model (MARKAL) in order to elaborate future. The model developed in this thesis can be further enhanced by integrating energy security indicators. As a result, a comprehensive model for the entire market may be obtained.

11. Energy systems always include risk factors. Risks are especially encountered because of the uncertainty in electricity pricing and generation. Therefore, risk factors may be analyzed from the perspective of uncertainty by utilizing value at risk (V@R) and conditional value at risk (CV@R) [124].

Models and methods developed in this study along with the outlook presented above are believed to accelerate the studies for modeling of electricity spot markets.

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APPENDIX A

FORECASTING MODELS

Table A.1: Mathematical equations for time-series based forecasting models [167].

Model	Mathematical Equation
AR	$\left(1-\sum_{i=1}^{p}\phi_i \cdot L^i\right)\cdot y_t = c+\varepsilon_t$
ARMA	$\left(1-\sum_{i=1}^{p}\phi_i \cdot L^i\right)\cdot y_t = c+\left(1+\sum_{i=1}^{q}\theta_i \cdot L^i\right)\cdot \varepsilon_t$
ARIMA	$\left(1-\sum_{i=1}^{p}\phi_i \cdot L^i\right)\cdot (1-L)^d\, y_t = c+\left(1+\sum_{i=1}^{q}\theta_i \cdot L^i\right)\cdot \varepsilon_t$
GARCH	$\left(1-\sum_{i=1}^{p}\phi_i \cdot L^i\right)\cdot y_t = c+\left(1+\sum_{i=1}^{q}\theta_i \cdot L^i\right)\cdot \varepsilon_t$
DR	$\left(1-\sum_{i=1}^{p}\phi_i \cdot L^i\right)\cdot y_t = c+\sum_{i=1}^{n}\sum_{j=1}^{r_i}a_{i,j}\cdot L^j \cdot x_{i,t}\cdot \varepsilon_t$
ARMAX	$\left(1-\sum_{i=1}^{p}\phi_i \cdot L^i\right)\cdot y_t = c+\left(1+\sum_{i=1}^{q}\theta_i \cdot L^i\right)\cdot \varepsilon_t+\sum_{i=1}^{n}a_i \cdot x_{i,t}$

Table A.2: NN-based studies and their corresponding errors [167].

Ref. No	**Method**	**Error Type**	**Accuracy**
[37]	ARIMA, DR, TF, NN, and WT	FMSE, DE	1-23%
[150]	NN	DMAPE	8.93-12.19%
[59]	NN	PE	Error less than €0.01 in 85% of the cases
[144]	ANN	MAE, MAPE	1-9%
[102]	ANN	APE	1-25%
[11]	Cascaded NN	MAPE, WMAPE	4-7%
[7]	Fuzzy NN	WMAPE	7.5%
[96]	Weighted NN	MRE, MAE, MMRE	5-14%
[121]	ANN	MAPE	3-10%
[123]	ANN	MAE, RMSE	0.5-9%
[95]	Enhanced Probability NN	RMSE, MAPE, MAE	1-8%
[30]	ANN	RMSE, MAPE, MAE	0.7-11%
[141]	NN	MAPE	11-33%
[9]	WT-Hybrid forecast method (NN and EA)	WME, WPE	4-26%
[8]	Modified relief algorithm and hybrid NN	WMAPE, WMAE	4-9%
[13]	FST-probabilistic NN-HNES	MAE, MAPE	5-47%
[131]	WT-ARIMA-RBFNN	WFE	4-7%

MAE: mean absolute error, APE: absolute percentage error, MAPE: mean APE, MPE: mean percentage error, MSE: mean square error, RMSE: root mean square error, FMSE: forecast MSE, RMSFE: root mean square forecasting error, PE: prediction error, DE: daily error, DMAPE: daily MAPE, WMAPE: weekly MAPE, WMAE: weekly MAE, MRE: mean relative error, MMRE: mean error relative to mean price, WME: weekly mean error, WPE: weekly peak error, WFE: weekly forecast error.

Table A.3: Studies based on time-series methods without explanatory variables and their corresponding errors [167].

Ref. No	Method	Error Type	Accuracy
[108]	DR-TFM	FMSE	2-8%
[84]	AR, LR	RMSE, MAE, MAPE, Max APE	0-13%
[60]	MR	-	-
[57]	GARCH	FMSE	1-43%
[58]	GARCH-SeaDFA	MAPE	5-9%
[70]	GARCH	MFE, MAFE, RMSFE	1-53%
[72]	GARCH	-	-
[130]	GAM	MWE	6-23%
[9]	WT-Hybrid NN and EA	WME, WPE	4-26%
[22]	Hybrid WT	APE, MAPE, RMSE	2-44%
[39]	ARIMA	FMSE, WME	4-21%
[136]	ARMA-GARCH	MAE-MAPE	4-13%
[38]	WT-ARIMA	MAPE	5-11%
[137]	WT-ARIMA and GARCH	MAPE	0-2%
[32]	ARIMA, ARIMA-EGARCH, and ARIMA-EGARCH-M	RMSE, MAPE, MAE	10-96%
[159]	AR and its extension	WME	2-50%

Table A.4: Factors affecting electricity price [167].

Ref. No	Factors Affecting Price
[7, 13, 32, 38, 73, 97, 133, 138, 139, 145, 146, 150]	Historical prices
[8, 10–12, 22, 37, 51, 57, 58, 95, 103, 108, 109, 118, 123, 132]	Historical prices, demand
[60]	Demand, composition of electricity production by each energy resource (renewables, cogeneration, hydro, nuclear, combined cycle, fuel and natural gas), net electricity exports, pumping and distribution losses
[56]	Fuel price, market concentration index, reserve margin
[39]	Historical prices (demand and available daily production of hydro units in with explanatory variable case)
[59]	Historical prices, day and month type
[150]	Historical prices, demand, settlement period
[9]	Historical prices, demand, available generation
[123]	Historical prices, demand, change in demand, time slot of the day, day of week
[30]	Historical prices, demand, change in demand, time slot of the day, day of week
[84]	Historical prices, demand, demand volatility, demand slope and curvature, scarcity, spread, diurnal and weekly effects, seasonality, trend, excess in generation capacity
[65]	Historical prices, demand, hydro generation, nuclear generation, thermal generation
[15]	Historical prices, demand, imports
[159]	Historical prices, demand, temperature
[126]	Historical prices, demand, temperature, humidity, crude oil prices, wind speed
[70]	Historical prices, natural gas prices
[95]	Historical prices, system load and temperature
[141]	Weekly variation data of electricity price and demand

Table A.5: Paper distribution with respect to electricity market [167].

Market	**Number of Papers**	**Ref. No**
Spanish electricity market	21	[56–58, 61, 64, 67, 69, 71–73, 83, 85, 86, 89, 91, 94, 99, 100, 102, 103, 105]
PJM (Pennsylvania, New Jersey, Maryland)	11	[51, 54, 60, 63, 67, 68, 81, 86, 95, 98, 99]
Italian electricity market	4	[55, 64, 77, 96]
New England	4	[64, 66, 96, 100]
New York	4	[64, 74, 75, 93]
National Electricity Market-Victoria New South Wales	3	[56, 65, 82]
Nord Pool	3	[88, 92, 104]
United Kingdom	2	[52, 70]
European Energy Exchange	2	[53, 84]
Ontario	2	[96, 101]
Iran electricity market	1	[30]
Turkish electricity market	1	[116]
Five hubs of the Midwest Independent System Operator	1	[32]

APPENDIX B

STOCHASTIC OPTIMIZATION MODEL APPLICATION ON NIGERIAN ELECTRICITY MARKET

Table B.1: Structure of the data for the Nigerian electricity market. The figures are representative.

Year	**2007**			
Month	**Jan**	**Feb**	**Mar**	**Apr**
Demand (MWh)	1609305398	1497207856	1264121511	1229518408

		Capacity (MW)	**Generation cost (NGN/MWh)**
plants in the order of increasing generation cost	**Hydro-1**	540	2416
	Hydro-2	450	3355
	Hydro-3	540	4026
	Steam turbine-1	880	5636
	Steam turbine-2	350	6710
	Gas (Combined cycle)	450	6978
	Gas turbine-1	518	8052
	Gas turbine-2	413	8320
	Gas turbine-3	270	8723
	Gas turbine-4	100	9394
	Gas turbine-5	80	9797
	Diesel-1	10	11407
	Diesel-2	15	12078
	Diesel-3	15	12749

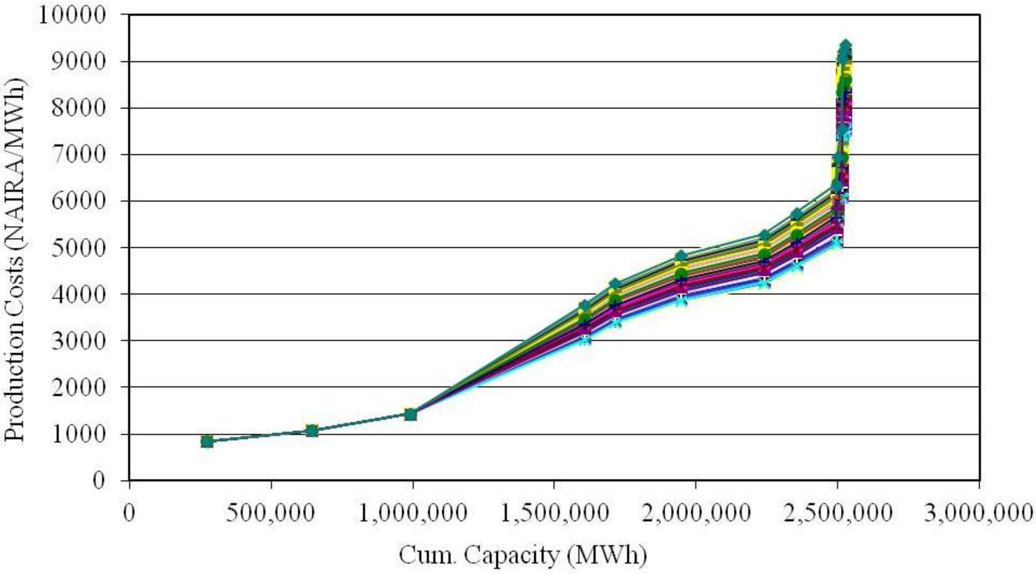

Figure B.1: Piecewise supply curves for all scenarios in January 2008.

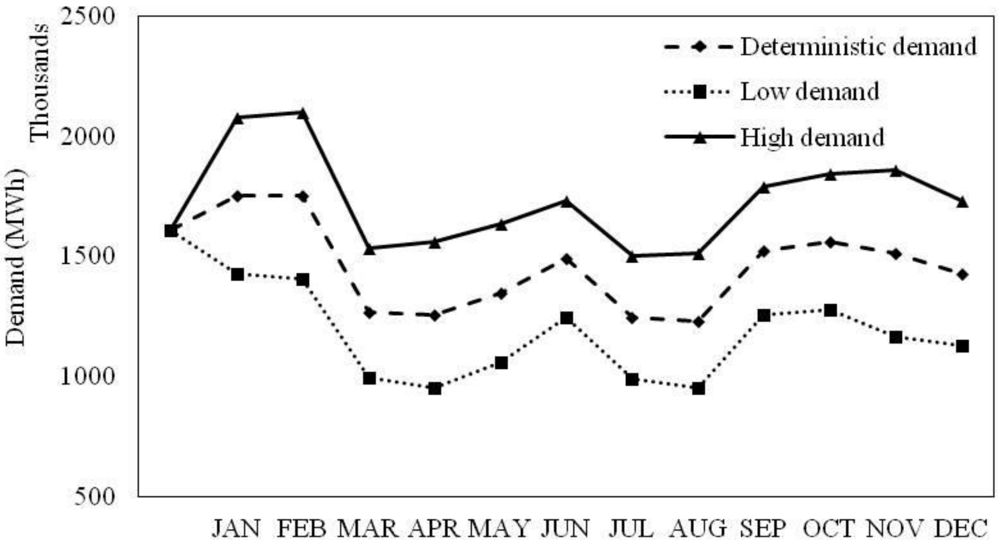

Figure B.2: Demand scenarios for the year 2008.

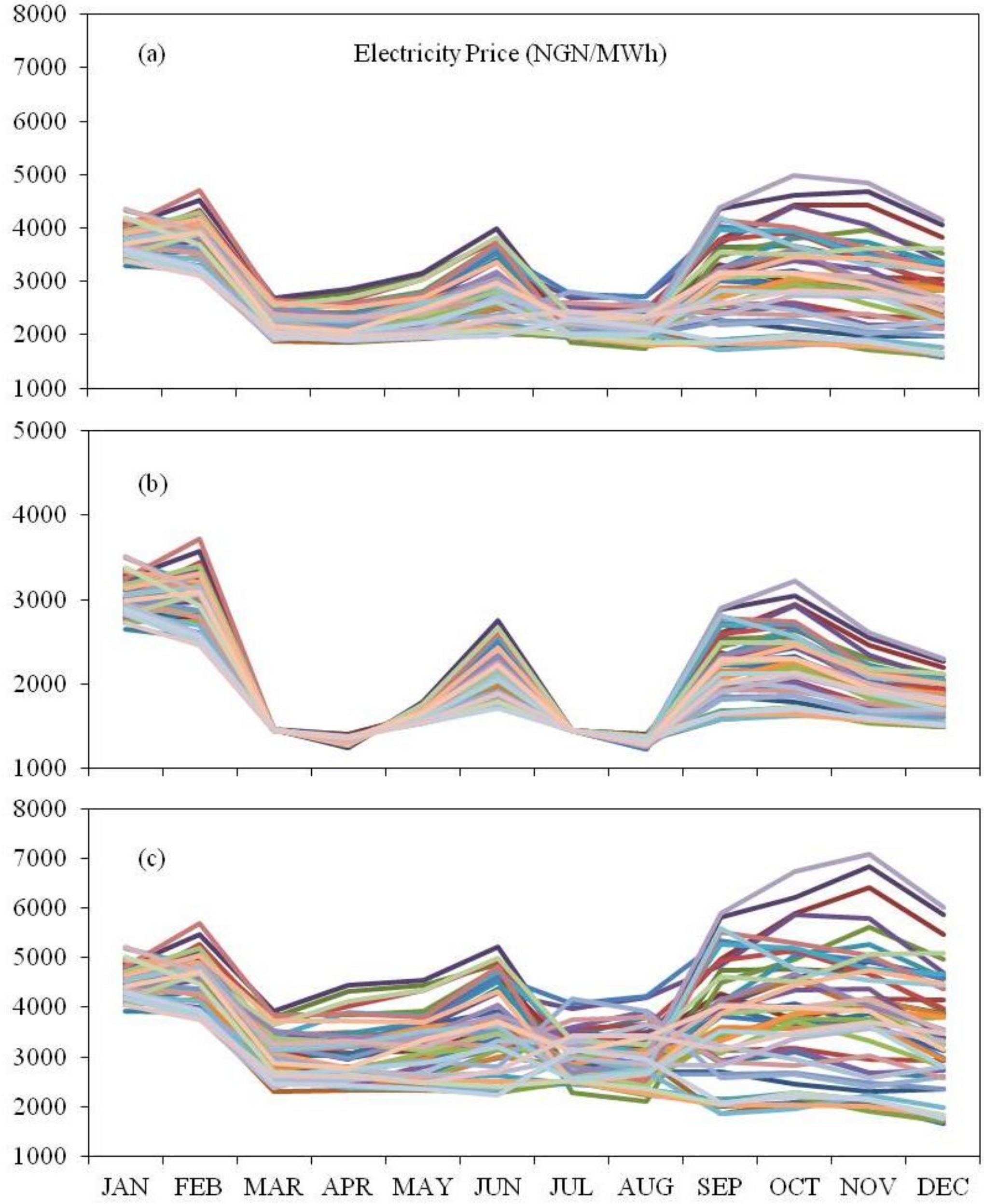

Figure B.3: Electricity price when demand is (a) deterministic, (b) low, and (c) high.

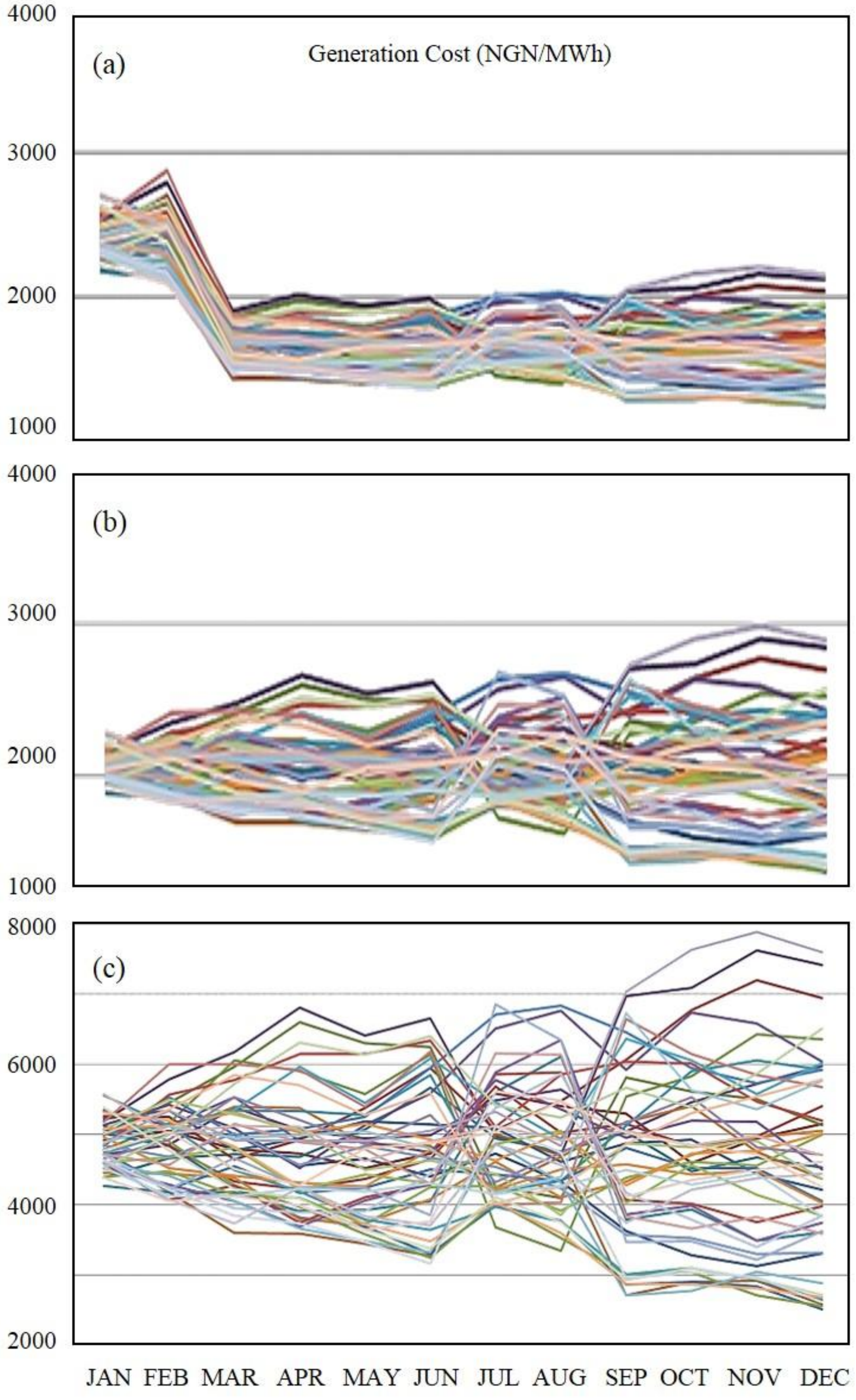

Figure B.4: Electricity production cost of when demand is (a) deterministic, (b) low, and (c) high.

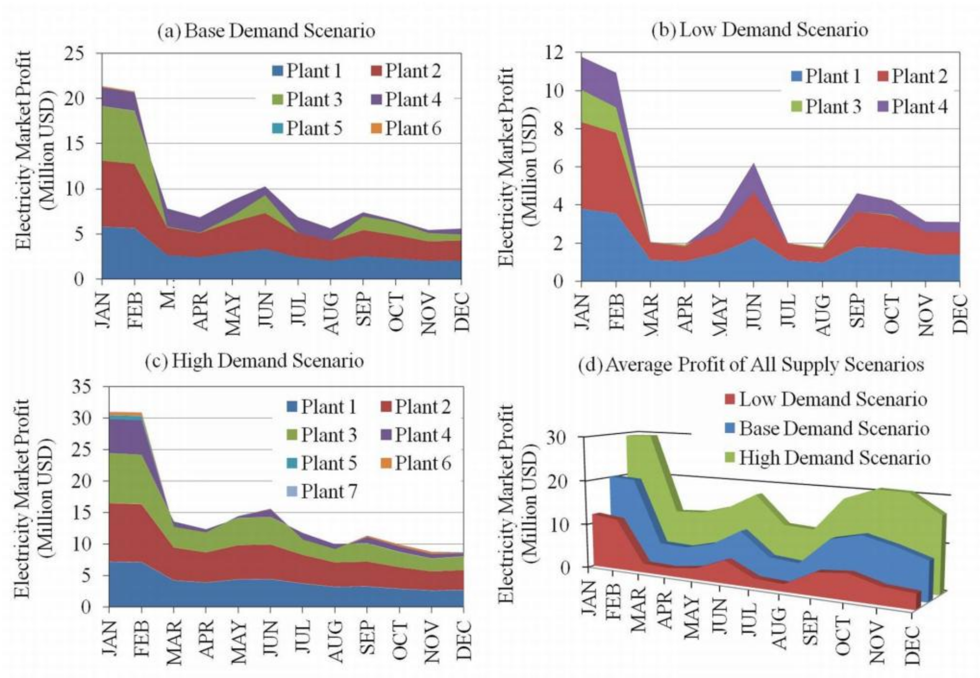

Figure B.5: Electricity generation portfolio and corresponding profits for different demand scenarios computed using the stochastic optimization model.

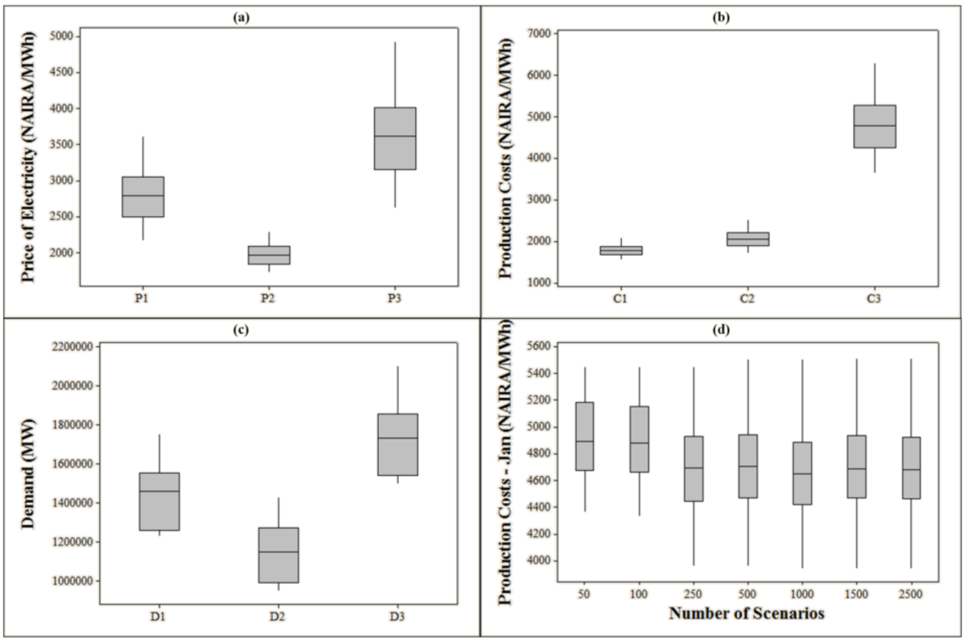

Figure B.6: Box plots of scenarios; (a) price of electricity when demand is deterministic (P1), low (P2), and high (P3); (b) production costs of electricity when demand is deterministic (C1), low (C2), and high (C3); (c) demand of electricity when demand is deterministic (D1), low (D2), and high (D3); (d) variations in all scenarios for production costs.

Table B.2: Number of scenarios required for 95% confidence level and 10% error (price of electricity when demand is deterministic (P1), low (P2), and high (P3); production costs of electricity when demand is deterministic (C1), low (C2), and high (C3)).

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN
P1	1.8	4.4	3.3	5.0	6.9	14.2	4.0	4.8	24.2	25.8	27.9	25.1	1.8
P2	1.9	4.5	0.0	0.3	0.6	6.6	0.0	0.4	10.8	12.1	7.3	5.2	1.9
P3	1.8	4.4	7.1	11.8	12.8	20.4	8.6	11.6	34.7	36.3	43.4	41.6	1.8
C1	1.1	2.6	2.0	3.0	2.9	4.2	2.6	3.3	6.5	6.6	7.7	8.2	1.1
C2	0.8	2.0	3.6	5.7	5.4	7.8	4.7	6.2	12.3	12.4	14.5	15.5	0.8
C3	1.7	4.0	7.2	11.4	11.1	15.7	9.3	12.2	25.0	25.0	29.4	31.8	1.7

APPENDIX C

W~MARS APPLICATION ON SPANISH ELECTRICITY MARKET

Table C.1: Representative hourly data for the Spanish electricity market.

Date	Time	Price (€/MWh)	Demand (MWh)
01-Jan-02	00:00-01:00	4.52	20419.00
	01:00-02:00	3.13	19574.00
	02:00-03:00	2.27	18973.00
	03:00-04:00	2.09	18079.00

Table C.2: Training and Testing Periods.

Season	Training Period	Test Period
Winter	1 Jan -17 Feb 2002	18 - 24 Feb 2002
Spring	2 Apr -19 May 2002	20 - 26 May 2002
Summer	2 July -18 Aug 2002	19 - 25 Aug 2002
Fall	1 Oct -17 Nov 2002	18 - 24 Nov 2002

Table C.3: Weekly Forecasting Errors (%).

Seasons	Wavelet-ARIMA [38]	Wavelet-ARIMA-GARCH [137]	CNN [11]	MIT-CNEA [10]	NES [12]	MARS	CMARS	W~MARS
Winter	4.78	0.63	4.32	4.88	4.28	6.52	5.90	4.18
Spring	5.69	0.65	4.31	4.65	4.39	3.32	2.94	4.79
Summer	10.70	1.19	6.37	5.79	6.53	5.42	-	6.88
Fall	11.27	2.18	6.22	5.96	5.37	5.24	-	11.37

Table C.4: Weekly Forecasting Error Variances.

Seasons	Wavelet-ARIMA [38]	Wavelet-ARIMA-GARCH [137]	CNN [11]	MIT-CNEA [10]	NES [12]	W~MARS
Winter	0.0019	0.0002	0.0020	0.0036	0.0013	0.0019
Spring	0.0025	0.0002	0.0025	0.0027	0.0015	0.0030
Summer	0.0108	0.0009	0.0049	0.0043	0.0033	0.0077
Fall	0.0103	0.0008	0.0048	0.0039	0.0022	0.0091

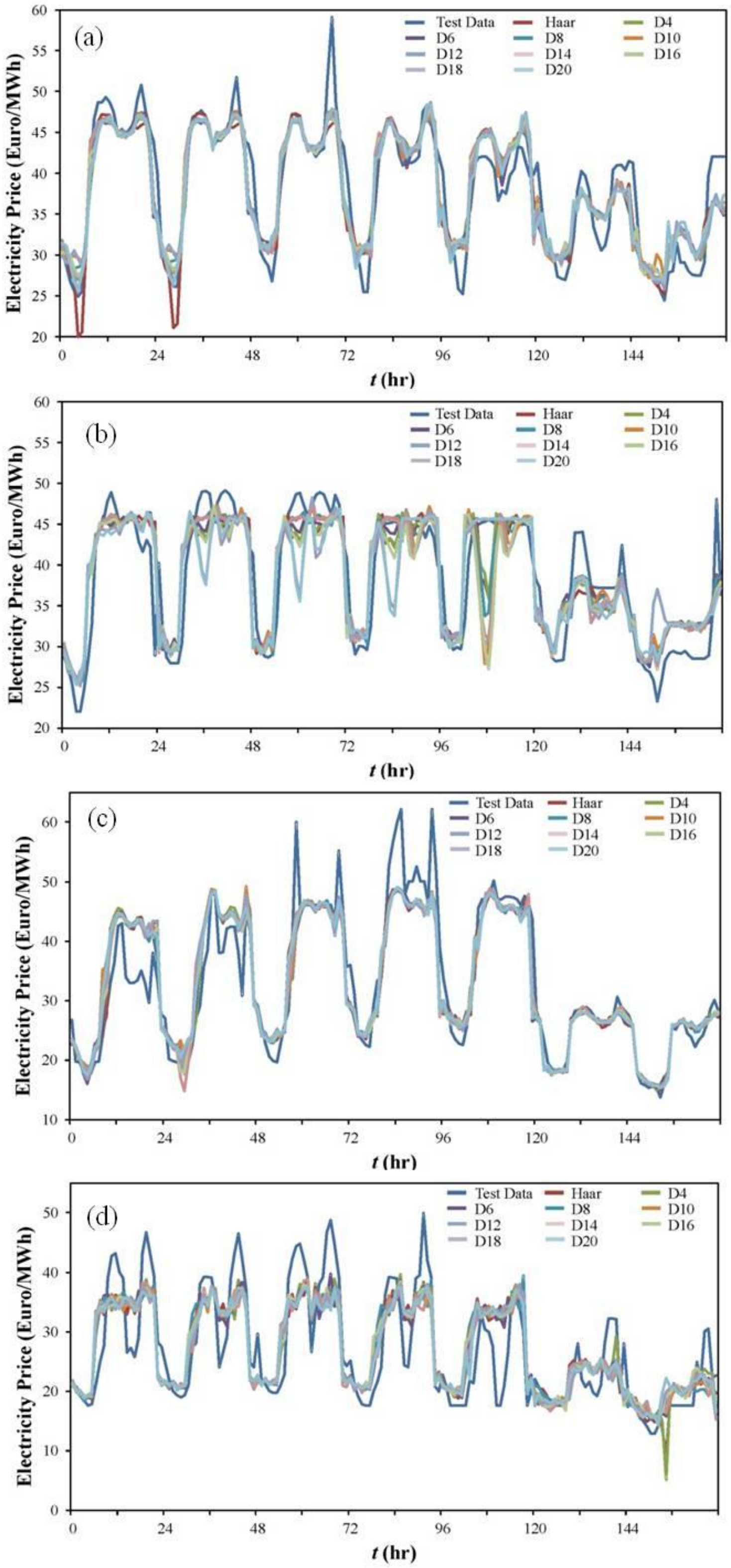

Figure C.1: Hourly electricity price forecast by using W~MARS for (a) winter, (b) spring, (c) summer, and (d) autumn.

Table C.5: Significant coefficients used in electricity price forecasting.

Variable	**February**							**May**						
	Mon	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**
$\mathbf{D_t}$	√	√	√	√	√	√	√	√	√	√	√	√	√	√
$\mathbf{P_t}$	√	√	√	√	√	√	√	√	√	√	√	√	√	√
$\mathbf{P_{t-1}}$				√	√	√	√		√		√	√	√	
$\mathbf{P_{t-2}}$	√	√	√						√	√	√			√
$\mathbf{P_{t-3}}$	√	√						√	√			√	√	
$\mathbf{P_{t-4}}$	√	√	√	√	√		√		√	√		√		√
$\mathbf{P_{t-5}}$	√				√				√			√	√	√
$\mathbf{P_{t-6}}$	√	√	√	√	√	√		√	√	√	√	√		√
$\mathbf{P_{t-7}}$	√	√	√		√		√	√		√	√	√	√	√
$\mathbf{P_{t-8}}$	√	√	√	√		√	√		√	√	√		√	
$\mathbf{P_{t-9}}$	√		√	√	√	√	√			√		√	√	√
$\mathbf{P_{t-10}}$				√		√	√	√			√	√		
Adj R^2	**0.96**	**0.95**	**0.95**	**0.96**	**0.94**	**0.95**	**0.98**	**0.95**	**0.94**	**0.94**	**0.93**	**0.95**	**0.90**	**0.92**
RMSE	**5.06**	**5.09**	**5.01**	**5.61**	**6.66**	**4.01**	**2.14**	**1.87**	**1.58**	**1.71**	**1.72**	**1.33**	**1.96**	**1.35**
Variable	**August**							**November**						
	Mon	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**
$\mathbf{D_t}$	√	√	√	√	√	√	√	√	√	√	√	√	√	√
$\mathbf{P_t}$	√	√	√	√	√	√	√	√	√	√	√	√	√	√
$\mathbf{P_{t-1}}$	√	√			√		√	√	√	√				
$\mathbf{P_{t-2}}$		√	√	√	√	√		√	√	√	√	√	√	√
$\mathbf{P_{t-3}}$	√	√	√	√		√	√		√	√	√	√	√	√
$\mathbf{P_{t-4}}$				√	√		√	√	√	√	√		√	
$\mathbf{P_{t-5}}$			√	√						√	√	√	√	√
$\mathbf{P_{t-6}}$				√		√	√		√			√		√
$\mathbf{P_{t-7}}$	√	√	√	√	√	√		√	√	√	√	√	√	√
$\mathbf{P_{t-8}}$						√	√			√				√
$\mathbf{P_{t-9}}$	√	√	√			√	√	√	√		√			
$\mathbf{P_{t-10}}$	√	√	√		√	√				√	√	√	√	√
Adj R^2	**0.97**	**0.96**	**0.95**	**0.96**	**0.93**	**0.94**	**0.94**	**0.96**	**0.95**	**0.96**	**0.95**	**0.95**	**0.92**	**0.96**
RMSE	**2.55**	**2.88**	**3.21**	**3.09**	**3.75**	**2.55**	**2.34**	**2.55**	**2.38**	**2.36**	**2.54**	**2.30**	**1.93**	**2.31**

D_t: Demand at time t, P_t: Price at time t, Adj R^2: Adjusted R^2, RMSE: Root Mean Square Error

APPENDIX D

R~W~MARS APPLICATION ON TURKISH ELECTRICITY MARKET IN MARMARA REGION

Table D.1: Structure of the data for the electricity market in Marmara Region. (The figures are representative.)

Year	**1997**	**1998**	**1999**	**2000**
Demand (MWh)	33806019	33942891	35072520	39085268
Price (TL/MWh)	33	34	40	48

	Capacity (MW)	**Generation cost (TL/MWh)**	**Emission ($kgCO_2eq/MWh$)**
Gas	105376.58	53.71	67
Lignite	4790.37	125.60	1100
Coal	20542.77	113.24	17.5
Biomass	538.74	95.94	1400
Hydro	3727.75	153.14	610
Wind	29005.14	138.59	19
Geothermal	70.00	72.93	38

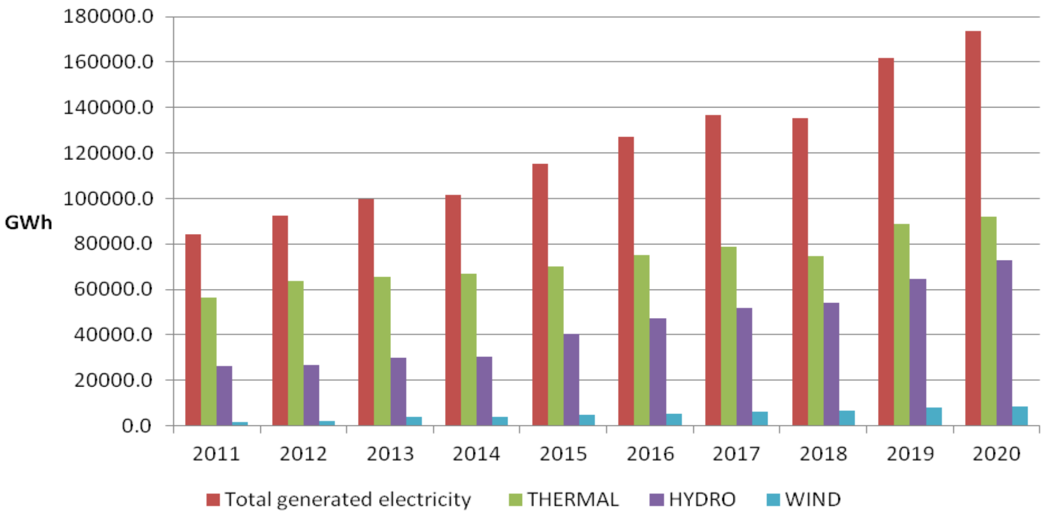

Figure D.1: Electricity generation portfolio computed by using R~W~MARS. Wind resources are included.

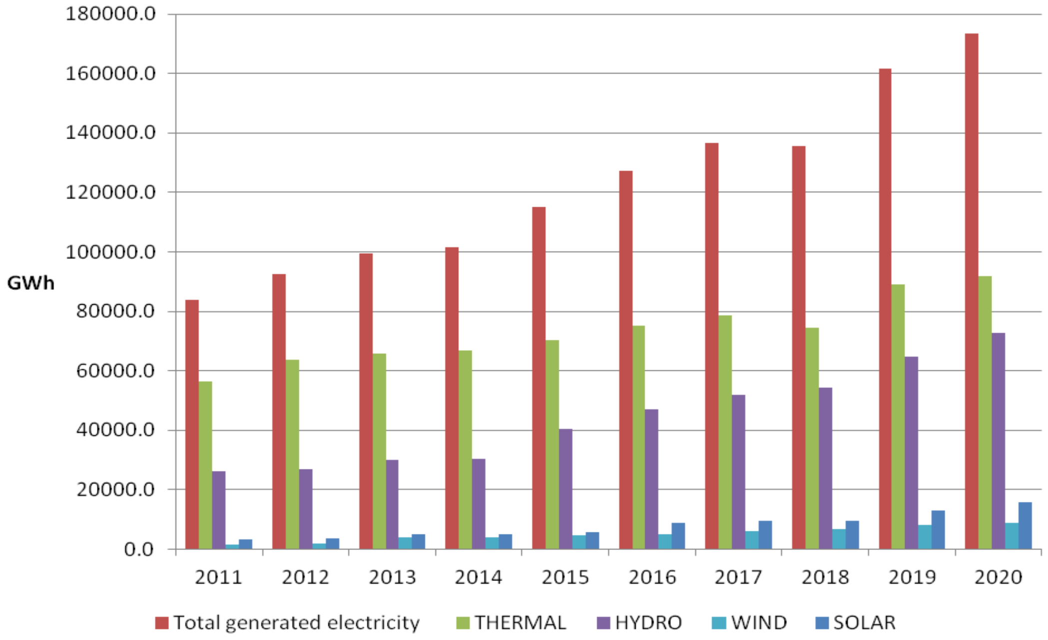

Figure D.2: Electricity generation portfolio computed by using R~W~MARS. Wind and solar resources are included.

APPENDIX E

ENERGY SECURITY INDICATORS

Table E.1: Security indicators related to sustainability.

SES Indicators	Competitive Energy Market Indicators	Environmental Protection Indicators
- Dependence on imports	- Energy intensity	- Percentage of renewable energy sources in the primary energy production
- Dependence on imports of solid fuel	- Efficiency of energy conversion	- Percentage of renewable energy sources in the electrical energy production
- Dependence on oil imports	- Efficiency of electrical energy production	- Indicators of intensity of emitted CO_2
- Dependence on natural gas imports	- Transformation of energy sector	- Emitted CO_2 per GDP
- Differentiation of primary fuel	- Independent energy regulator	- Emitted CO_2 per Gross Domestic - Energy Consumption
- Differentiation of fuel of electrical energy production	- Private participation	- Emitted CO_2 per capita
- Differentiation of energy fuel	- Dividing of public enterprise	- Emitted CO_2 per electricity and steam production
- Strategic oil supplies	- Energy law for the reforming and privatization of energy enterprises	- Application of Kyoto Protocol
	- Adjustment of energy pricelist	
	- Level of competition	
	- Per capita energy consumption	
	- Per capita electrical energy consumption	

CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: YILDIRIM, Miray Hanım
Nationality: Turkish
Date and Place of Birth: 27 May 1982, Kaman
Marital Status: Married
email: mirhanay@gmail.com

EDUCATION

Degree, Institution, Year of Graduation
M.Sc., Çankaya University Industrial Engineering Department, 2007
B.Sc., Çankaya University Computer Engineering Department, 2005
B.Sc., Çankaya University Industrial Engineering Department, 2004

WORK EXPERIENCE

Year, Place, Enrollment
2014-…, Ankara Development Agency, Expert
2012-2013, European Commission Joint Research Center Institute for Energy and Transport, Expert
2004-2014, Çankaya University Industrial Engineering Department, Teaching Assistant

FOREIGN LANGUAGES

English

PUBLICATIONS

Book Chapters, Books Edited and Book Reviewed

1. "Work and Construction Machinery Cluster '08 - Project Summaries" (in Turkish), Ed. L. Kandiller, M.H. Yıldırım, Ankara, Çankaya University (2009).

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4. M.H. Yıldırım, Ö.T. Bayrak, G.-W Weber (2012), "A Renewable and Sustainable Electricity Planning Model under Uncertainties: A Case Study for Marmara Region of Turkey", International Conference on Operations Research Book of Abstracts/2012, Hannover.

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7. M.H. Yıldırım, G.-W. Weber (2013), "A Stochastic-Global Optimization Approach to Electricity Market Model", OR 2013 -International Conference on Operations Research Book of Abstracts, Rotterdam.

8. G.-W. Weber, A. Özmen, M.H. Yıldırım (2014), "New Approaches to Day-Ahead Electricity Price Forecasting: MARS and CMARS Models", the 20th Conference of the International Federation of Operational Research Societies, Book of Abstracts, Barcelona.