# THEORY AND PRACTICE OF GEOMETRY IN MEDIEVAL ARCHITECTURE IN THE MIDDLE EAST (10th-14th CENTURIES)

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# ABSTRACT

THEORY AND PRACTICE OF GEOMETRY
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The aim of this research is to investigate the use of geometry in architecture of the Medieval Middle East considering the place of geometry in the classification of sciences in records of medieval Islamic philosophers and the practical application of geometry in the area of architecture through documents on geometric ornaments and geometric analysis of North Dome of Friday Mosque of Isfahan. This thesis discusses the arguments on the meanings of geometric ornaments and on the collaboration of mathematicians and craftsmen in designing the ornaments. The division between the theory and practice of geometry is considered with regard to philosophies of Al-Kindi, Al-Farabi, Al-Ghazali and Khayyam. It is seen that Khayyam takes a position which sees practical geometry as an antecedent to theoretical geometry counter to the general idea held by contemporary philosophers.

Keywords: Theory and practice of geometry, geometric ornament, Isfahan Friday Mosque, geometric analysis in architecture

# ÖZ

ORTADOĞU'DAKİ ORTAÇAĞ MİMARİSİNDE GEOMETRİNİN TEORİ VE PRATİĞİ (10.-14. yy.)

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Bu çalışmanın amacı Ortadoğu'daki Ortaçağ mimarisinde geometrinin kullanımını, Ortaçağ İslam filozoflarına göre geometrinin bilimlerin sınıflandırılması içindeki yeri, geometrik süslemeler üzerine olan belgeler ve Isfahan Ulu Caminin Kuzey Kubbesinin geometrik analizini ele alarak incelemektir. Bu tez ayrıca geometrik süslemelerin anlamları üzerine ve geometrik süslemelerin tasarlanmasında matematikçilerin ve zanaatkârların işbirliği üzerine olan fikirleri de tartışır. Teorik ve pratik geometri arasındaki ayrım El-Kindi, El-Farabi, El-Gazali ve Hayyam'ın felsefelerine göre ele alınmıştır. Bu konuda, çağdaşı olan filozofların aksine Hayyam'ın pratik geometriyi teorik geometriye öncül olarak ele aldığı görülür.

Anahtar Kelimeler: Teorik ve pratik geometri, geometrik süsleme, Isfahan Ulu Cami, mimaride geometrik analiz

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# INTRODUCTION

The aim of this research is to investigate the use of geometry in architecture of the Medieval Middle East considering the place of geometry in the classification of sciences in records of medieval Islamic philosophers and the practical application of geometry in the area of architecture through documents on geometric ornaments and geometric analysis of North Dome of Friday Mosque of Isfahan. In doing so, it is seen that the conceptual borders between theory and practice are challenged while reviewing the notions of medieval philosophers and mathematicians. Architecture as an area of practical geometry stands in the centre of the discussions both in its ornamental aspects and organization of spaces. The knowledge of theoretical geometry is assumed to be dominating the practical profession in the classical understanding of classification of sciences. However, the classical understanding is questioned with Khayyam's ideas on incommensurable magnitudes and irrational numbers.

Focusing on the epistemological distinctions between theory and practice of geometry in the architecture of the Medieval Middle East, this thesis also problematizes the interpretation of geometric ornaments used in architecture. The reason behind this attempt is to understand the philosophical and historical context concerning the geometric ornaments. However, due to the lack of literary evidence on the issue most of the commentaries on the subject lack sufficient proof to reveal profoundly the inherent motive underneath the visual strata. Nevertheless, by considering the geometric ornaments as a central part of the visual culture in the Islamic domain, these interpretations enriches our understanding of use of geometry in architecture.

There are two manuscripts concerning our subject. The first manuscript is the mathematician Abu Wafa Al Buzjani's (940-998) 'Book on What is Necessary for Artisans in Geometrical Construction'[1] and the second is 'On Interlocks of Similar or Corresponding Figures'[2] written by an anonymous author in the 13th century.[3] The content of the first manuscript is a compilation of ruler and compass constructions belonging to plane Euclidian geometry. It is stated that the aim of the booklet is to teach the proper way of creating geometric figures to craftsmen and no proofs were included for the sake of simplicity.[4] One of the figures explained in the manuscript were widely used in the decoration of the walls on the monumental buildings of Iran. The other manuscript is described by Özdural as composed of hastily taken notes by a mathematician who was involved in ornamental arts.[5] The relation between the two is apparent in their formal resemblance. In the latter manuscript we see a special triangle created by Khayyam. This triangle was employed by Özdural in his geometric analysis of the North Dome of the Isfahan Friday Mosque.[6] Regarding the difference between the theoretical and practical geometry used in architecture Abu Wafa's and Khayyam's approaches both imply diverse aspects of the issue. In Abu Wafa's case, we see that the methods of craftsmen fail in solving the problem proposed by Abu Wafa and he shows the craftsmen how to solve the problem in a justifiable way. The superiority of Abu Wafa's method is also a restatement of the relation between theory and practice of

[1] Five copies of the manuscript is kept in Istanbul, Ayasofya MS 2753; Cairo, D¯ar al-Kutub al-Mis.riyya MS 31024, MS 44795; Milan, Ambrosiana MS &68sup; Uppsala, Universitetbibliotek MS Tornberg 324.

[2] The only copy of the manuscript exists in Paris, Bibliotheque Nationale MS Persan 169.

[3] This thesis uses the published parts of these documents in Alpay Özdural. "Mathematics and Arts: Connections between Theory and Practice in Medieval World", Historia Mathematica, Vol. 27 (2000): 171-201 and in Alpay Özdural. "On Interlocking Similar or Corresponding Figures and Ornamental Patterns of Cubic Equations", Muqarnas, Vol. 13 (1996), pp. 191-211

[4] Jan P. Hogendijk, "Mathematics and Geometric Ornamentation in the Medieval Islamic World", *European Mathematical Society*, Newsletter no. 86, December (2012), pp. 37-43

[5] Alpay Özdural, "On Interlocking Similar or Corresponding Figures and Ornamental Patterns of Cubic Equations", *Muqarnas*, Vol. 13 (1996), pp. 191-211

[6] Alpay Özdural, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", 699-715

geometry as classified by Aristotle. However, Khayyam approaches the practical geometry as the launcher of his theoretical claims on irrational magnitudes.

Chapter two of this thesis is on the translation movement which occurred between 8th-11th centuries in Baghdad. I trace assimilation of theoretical geometry from the Greeks and explain the role of the supporting caliphs Al Mansur (reigned, 754-775) and Al Mamun (reigned 813-883) to present the motives behind the movement. The importance of the Harran School in translating the Greek sources are also discussed. Next, I explain the ongoing practical mathematical tradition in the geography by mentioning the sources on the mathematics and geometry used by surveyors and calculators. My last point in the chapter two contains a comparison of Al Kindi's, Al Farabi's and Al Ghazali's classification of sciences which they mostly developed from Aristotle's ideas on the subject. It is seen that while Al Kindi and Al Farabi privilege theoretical knowledge of geometry over practical knowledge, Al Ghazali's main concern on the classification of sciences is to differentiate worldly and other worldly sciences. Al Ghazali places the geometric knowledge as a worldly science contrary to Al Farabi and Al Kindi who acknowledge mathematical entities as intermediary between metaphysical and material world.

The third chapter contains the interpretations of geometric ornament through traditionalist, Sunni revival and classicist arguments. The traditionalist approach offers an ahistorical interpretation of geometric ornament by solely discussing the geometric ornament as manifestation of divinity neglecting the local differences. The commentaries of Ismail Al-Faruqi, Keith Critchlow, Issam Al Said, Seyyid Hossain Nasr, Nader Ardalan, Titus Burckhardt and Samer Akkach can be evaluated in this group. Sunni revival argument is held by Necipoğlu and Yasser Tabbaa. They consider the geometric ornament in relation to the religious and political atmonsphere of the 10th and 11th century Baghdad. In that respect, the complex geometric patterns are assumed to be the symbol of Sunni theology and its ideology. Terry Allen is consireded to be a classicist because he views the Medieval Islamic ornament as an extension of Late Antique ornamentation.[7] His ideas about the collaboration between crafsman and mathematicians,

[7] Barbara Genevaz, "Review of the book Five Essays on Islamic Art by Terry Allen", *Mimar Book Reviews in Mimar: Architecture in Development*, Vol: 8 edited by Hasan-Uddin Khan, (Singapore: Concept Media Ltd., 1983), pp.67-68

and his objections to interpretation of geometric ornament by referring to the philosophical views of the age are dealt with in this chapter.[8] Carol Bier and Oleg Grabar has a distinct position in my classification. Bier investigates the ornaments on the Kharraqan tomb towers and considers the inscriptions on the towers to interpret the geometric ornaments. She reaches to the conclusion that the geometric ornaments are reification of the verses in Quran. She constructs this relation through the analogy of the concept *amthal*, which means similarity, and the ornaments on the tomb.[9] Grabar considers the geometric ornament as ubiquitous and claims that there is no intrinsic meaning that belongs to the ornament unless it is specifically referred to as in the Hall of the Two Sisters in the Alhambra Palace.[10]

In the second part of the chapter three I investigate Abu Wafa's manuscripts. I discuss the collaboration of craftsmen and mathematicians referring to Özdural's and Hogendijk's arguments. While Özdural claims that the meetings which Abu Wafa points in his manuscript refers to participation of mathematicians into the work of geometric ornament,[11] Hogendijk asserts that methodological difference between the two fields hinders possible collaboration.[12]

Chapter 4 focuses on the geometric analysis of the North Dome of the Isfahan Friday Mosque. While Schroeder's analysis points to golden ratio as the ordering principle of the dome, Özdural claims that Khayyam's triangle which embodies arithmetic, geometric and harmonic ratios, is the ordering principle of the Dome in the

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[8] Terry Allen, *Islamic Art and the Argument from Academic Geometry,* (California: Solipsist Press, 2004), http://www.sonic.net/~tallen/palmtree/academicgeometry.htm (accessed 22.01.2015)

[9] Carol Bier, "Geometric Patterns and the Interpretation of Meaning: Two Monuments in Iran*", Bridges: Mathematical Connections in Art Music and Science*, ed. Reza Sarhangi, (Kansas: Bridges Conference, 2002), 74

[10] Oleg Grabar, *The Mediation of Ornament*, (Princeton: Princeton University Press, 1989), pp.145-153

[11] Alpay Özdural, "Omar Khayyam, Mathematicians and *Conversazioni* with Artisans", *Journal of Society of Architectural Historians*, Vol. 54:1, (1995), 54-71

[12] Jan P. Hogendijk, "Mathematics and Geometric Ornamentation in the Medieval Islamic World", *European Mathematical Society*, Newsletter no. 86, December (2012), pp. 37-43

section analysis.[13] According to Özdural the use of Khayyam's triangle in defining the proportions on the section of the dome might denote to Omar Khayyam as the architect of the Dome.[14] I did not investigate the correctness of the claim; however I employ Khayyam's ideas on theory and practice of geometry and the nature of mathematical entities so that a new approach on the theory and practice of geometry is revealed through the geometry used in the North Dome. This is apparent in the use of incommensurable magnitudes in the craftsmanship of the dome with regard to Özdural's analysis. Khayyam's claims on the appropriation of methods of "those who make calculations" and "measurements" in considering irrational numbers or in other words continued fractions as numbers is supporting the Aristotelian understanding of mathematics.[15] This attempt is the first step towards recognizing real numbers as it is known today and Khayyam points to the craftsmen as his inspiration source. This thesis emphasizes learning from practice in the realm of geometry as envisaged by Khayyam.

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[13] Alpay Özdural, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", Technology and Culture 39.4 (1998), 699-715

[14] Ibid.

[15] Omar Khayyam, "Commentary on the Difficulties of Certain Postulates of Euclid's Work", in *Omar Khayyam the Mathematician* by R. Rashed and B. Vahabzadeh, translated by R. Rashed and B. Vahabzadeh, (New York: Biblioteca Persical Press, 2000), pp. 235-253

# CHAPTER 2

# THEORETICAL AND PRACTICAL GEOMETRY IN ISLAMIC LANDS IN THE MIDDLE AGES

The classification of sciences in Medieval Middle Eastern geography has its roots in the Greek tradition of philosophy. Arabic philosophers developed these classifications and placed the knowledge of geometry on the same level as the Greek did before. Architectural geometry can be elaborated by considering it in the domain of practical geometry.

In this chapter, firstly I will present the translation movement which enabled the Islamic culture to meet with the classical tradition. The importance of the translation movement rests on assimilation of Euclid's Elements, Plato's *Republic*, *Theatetus* and *Timaeus*, Aristotle's *Metaphysics*, *Logic* and *Physics*, and sources of Neo-Pythagorean and Neo-Platonist philosophers by the Arabic culture with respect to our subject. Another source of geometric and mathematical knowledge comes from the ongoing practical tradition in geography. The driving forces behind the translation movement have been touched upon to present a more comprehensive understanding of the cultural atmosphere. Philosophical reflections on the knowledge of geometry has also been dealt so that the classification of sciences in theoretical and practical terms and their hierarchical attributions can be discussed through Al Kindi, Al Farabi and Al Ghazali.

## 2.1. Translation Movement in the Islamic Empire

Scientific and philosophical studies in Islamic Empire gained a new insight with the translation movement centred in Abbasid capital Baghdad between 8th – 11th centuries. The intellectual and scientific environment was mainly shaped by assimilation of exact sciences from non-Muslim traditions. Translations of the scientific works from Greek, Sanskrit, Persian and Syriac were patronized by the caliphs and their families, courtiers, officials of the state and the military, and also by the scholars and scientists.[16] These scholars and scientists were mainly the medical elite of the Nestorians in Baghdad hailing from Gundishapur and the three brothers of Musa Ibn Shakir, known as Banu Musa brothers, who were primarily concerned with sponsoring the translation of mathematical and astronomical studies.[17]

Outstanding incidences that enabled Abbasids to encounter with the ancient Greek science and philosophy were the abandonment of the Academy of Plato in East Roman Empire in 529 AD and emigration of the scientists of the academy from Athens to Sassanid Empire, which was conquered after the Battle of Nahavand by the Arabs in 642; and the conquest of Alexandria in 641 which was prominent for its Hellenistic school of philosophy.[18] So that, academies of Alexandria, Gundishapur and Marw passed to the control of Islamic Empire.

The beginning of the translation movement is attributed to Caliph Al Mansur (reigned, 754-775), who is the first Abbasid caliph and the founder of the capital Baghdad.[19] Al Masudi (d. 956), quoted after Gutas, reports that,

> He was the first caliph to have books translated from foreign languages into Arabic, among them *Kalila wa-Dimna* and *Sindhind*. There were also translated for him books by Aristotle on logic and other subjects, the *Almagest* by Ptolemy, *The Arithmetic* [by Nichomachus of Geresa], the book by Euclid [on geometry] and other ancient books from classical Greek, Pehlevi [Middle Persian], Neopersian and Syriac.[20]

Al Mansur's specific concern on astronomy and astrology is also apparent in his seeking advice from astrologers - Nawbaht (d.776-7) and Masa'allah (d.815) - in selecting the time for deposition of the foundation stone and in designing the plan of

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[16] Dimitri Gutas, *Greek Thought, Arab Culture*, (London: Routledge, 1998), 122-133.

[17] Ibid.

[18] For transfer of Alexandrian school to Antioch after the conquest see Max Meyerhof, "On the Transmission of Greek and Indian Science to the Arabs", *Islamic Culture* 11 (1937): 17-29.

[19] Gutas, *Greek Thought, Arab Culture*, 31-32.

[20] Ibid., 30-31.

Baghdad for adapting the guidance of stars.[21] The round shape of the city and Caliph's palace at the center, in addition to symbolizing centralized rule and Al Mansur's control, is also the application of Euclid's principle on forming a circle (*Elements*, 1st Book, 15th definition) in city planning which also reflects Mansur's special interest in translation and comprehension of Euclid's *Elements*.[22] The fact that Al Mansur is applying the ancient knowledge as declared in the Zoroastrian holy book *Denkard* and its restatement in Islamic terms by Abu Sahl in *Kitab an-Nahmutan* further gives a hint to understand the translation movement as an ideological act of the Abbasids which is inspired from the Zoroastrian imperial ideology.[23] The courtly patronage of the translation movement, the importance given to astrological predictions in government issues and the source of the original texts in astrological works being in Pahlavi are the actualities which point to Abbasids' nostalgic ideology for the Sassanid Zoroastrian political orientation.[24] Also Fakhry, supports the claim that the translation work done at Gundishapur and Marw in the time of Emperor Anushirwan (reigned 531-579) are the roots for the enthusiasm showed for Greek learning by the early Abbasids.[25] Franz Rosenthal holds that the driving force behind the translation movement was Mohammed's underlining the role of knowledge (*'ilm*) and placing knowledge at the center of Islamic thought. So according

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[21] Len Evan Goodman, "The Translation of Greek Materials into Arabic" in *Religion Learning and Science in the 'Abbasid Period*, ed. M. J. L. Young et al., (Cambridge, New York: Cambridge University Press, 2006), 478.

[22] Gutas, *Greek Thought, Arab Culture*, 52.

[23] Zoroastrian imperial ideology of the Sassanids is reported to us in three accounts of Pahlavi translation movement. These are Zoroastrian *Denkard* Book IV: *The Creations Book* attributed to Zarathustra and Abu Sahl ibn Nawbaht's *Kitab an-Nahmutan*. According to Nawbaht after Alexander conquered Persian lands, he ordered to copy all the documents in the archives and make them translated into Byzantine [Greek] and Coptic. Then he destroyed all the documents. However, some books, which are in China and India, remained untouched. Ardasir ibn-Babak the Sassanian got the copies of what left over and his son Sabur proceeded the work. During Kisra [Chrosroes] Anushirwan's reign all the documents were collected and put into a proper order. Moreover, since all knowledge is considered to be derived from Zoroastrian Holy Book *Avesta*, Anushirwan ordered to make translations of what can be acquired from Byzantine, Indian and Chinese sciences and based his acts on them. Abu Sahl ibn Nawbaht in Dimitri Gutas, *Greek Thought, Arab Culture*, 39-48.

[24] Ibid., 43-59.

[25] Majid Fakhry, *A History of Islamic Philosophy*, (New York: Colombia University Press, 1970), 9.

to him the translation movement would not appear without the call of Mohammed which was to seek knowledge.[26]

The tenth century biographer Ibn al Nadim also gives an account of the motive behind the translation movement which relies on a dream of Caliph Al Mamun (reigned 813-883).

> …Al Mamun saw in a dream the likeness of a man white in color, with a ruddy complexion, broad forehead, joined eyebrows, bald head, bloodshot eyes, and good qualities sitting on his bed. Al Mamun related, 'It was as though I was in front of him, filled with fear of him. Then I said, 'Who are you?' He replied, 'I am Aristotle.' Then I was delighted with him and said 'Oh sage, may I ask you a question?' He said 'Ask it.' Then I asked, 'What is good?' He replied, 'What is good in the mind.' I said again, 'Then what is next?' He answered 'What is good in the law.' I said, 'Then what is next?' He replied 'What is good with the public.' I said 'Then what more?' He answered 'More? There is no more...'[27]

According to Ibn al Nadim, this dream in which Al Mamun meets with Aristotle is the mainspring of Al Mamun's request for ancient scientific books from the Byzantine Emperor and sending Al Hajjaj, Ibn al-Batriq and Salman, who is the director of Bait al Hikma [House of Wisdom], to obtain the books for translation into Arabic. Gutas interprets this dream not as the initiator of the translation movement but as the social consequence of the translation movement.[28] In other words, according to Gutas this narrative was invented in order to canonize the Abbasid caliph and grant Aristotle a privileged position among other philosophical figures.[29]

Actually Aristotle had a distinct place in the Arabic culture since he was referred as "the philosopher" or "first teacher" in the Arabic philosophy. Among many

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[26] Franz Rosenthal, *The Classical Heritage in Islam*, (Berkeley: University of California Press, 1975), 5.

[27] Al Nadim, *Fihrist of Al Nadim Vol. II*, trans. Bayard Dodge, (New York & London, Colombia University Press,1970), 583-584.

[28] Gutas, *Greek Thought, Arab Culture*, 100.

[29] Ibid., 102.

translations of Aristotle *Metaphysics*, *Logic* and *Physics* include subjects discussing the nature of mathematical entities. *Metaphysics* was first translated by the circle of Al Kindi (801-873).[30] The translation of *Categories*, which is a part of the book *Logic,* and the translation of *Physics* has been done by Hunayn Ibn Ishaq (d. 873).[31]

House of Wisdom is generally introduced as the library founded either by Harun al Rashid or by Al Mamun. The function of this legendary library was to find scientific books to translate them into Arabic. According to Gutas, actually this fabled library is not referred with high status in reliable sources.[32] The pre-Islamic sources indicate that the Sassanian storehouses of manuscripts were also called as the houses of wisdom and accordingly the Abbasid house of wisdom may denote to a continuation of that tradition as an Arabic institution.[33]

Although the historical sources point to Al Mansur as the first patron who designated the translation of Euclid's *Elements* for the first time, there is no extant copy of the book. The first translator of the *Elements* is Al Hajjaj Bin Yusuf Bin Matar (8th-9th c.) according to sources.[34] There are two editions of this translation. One is translated during the reign of Harun Al Rashid (763-809) and the other is a corrected version presented to Caliph Al Mamun (786-833) whose names are Al-Haruni and Al Ma'muni. Unfortunately none of them are preserved until now. Nadim reports that Ishaq ibn Hunayn is the second one who translated the *Elements* and Thabit bin Qurra is recorded

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[30] Al-Kindi is one of the leading philosophers of Medieval Arabic philosophy and was the leader of a group occupied with translating classical Greek sources to Arabic. Cristina D'Ancona, , "Greek Sources in Arabic and Islamic Philosophy", *The Stanford Encyclopedia of Philosophy* (Winter 2013 Edition), Edward N. Zalta (ed.), http://plato.stanford.edu/archives/win2013/entries/arabic-islamic-greek/, accessed in 18.01.2015

[31] Cristina D'Ancona, , "Greek Sources in Arabic and Islamic Philosophy", *The Stanford Encyclopedia of Philosophy* (Winter 2013 Edition), Edward N. Zalta (ed.), http://plato.stanford.edu/archives/win2013/entries/arabic-islamic-greek/, accessed in 18.01.2015

[32] Gutas, *Greek Thought, Arab Culture,* 55-56.

[33] Ibid., 54-55.

[34] Al Nadim, *Fihrist of Al Nadim,* 634 and Al Nayrizi, *The Commentary of Al Nayrizi on Book I of Euclid's Elements of Geometry*, trans.,ed. Anthony Lo Bello, (New York: BRILL, 2003), 25.

as the corrector of it. [35] Abu al-Wafa Muhammad al Buzjani (940-998), who will be dealt extensively in the second chapter, also wrote a commentary on the book.[36]

Among the translators of classical geometry studies, Thabit bin Qurra (836-901) stands out. His edition of Euclid's *Elements* translated by Ishaq ibn Hunayn is one of the earliest available Arabic translations of the book. Moreover, he is the most renowned representative of Sabians[37] in Baghdad.[38]

Harran School, which also includes Thabit bin Qurra, is noted for its scholars who participated in the translation movement. Harran became the center of Hellenistic culture after Alexander placed Macedonian migrants to there and Greek became a language in demand in the 4th century B.C.[39] This encounter of Harranians with Hellenism laid the foundations of Harran to become one of the centers of Abbasid translation movement.[40] Thabit bin Qurra was noticed by Muhammad ibn Musa, who was one of Banu Musa, in Harran because of his fine literary style and he was brought to Baghdad by Muhammad ibn Musa as an associate.[41] Thabit's rapid promotion among other translators, his respectful position in the community and his experience to overcome the translations of most advanced mathematical and astronomical studies is a result of his education in Harran and Baghdad under the patronage of Banu Musa. One of the translations he introduced to Arabic philosophy is "Introduction to Arithmetics"

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[35] Al Nadim, *Fihrist of Al Nadim,* 634.

[36] Ibid., 635.

[37] Sabians is a religious group, which is also mentioed in the Quran, in 2:62, 5:69, 22:17, who has lived around Harran during the considered period.

[38] Ibid., 647.

[39] Mustafa Demirci, "Helen Bilim ve Felsefesinin İslam Dünyasına İntikalinde/Tercümesinde Harranlı Sabiilerin Rolü" (paper presented at the I. Uluslararası Katılımlı Bilim, Din ve Felsefe Tarihinde Harran Okulu Sempozyumu, 28-30 April, 2006), 200-214.

[40] Ibid.

[41] Al Nadim, *Fihrist of Al Nadim Vol. II*, 647.

by Nichamachus of Geresa which is important for its Neo-Pythagorean ontology. [42] This book also gave rise to flourishing of Neo-Pythagorean tradition in Arabic philosophy.

Translations of mathematical sciences also include works from Indian sources. Most notable influence of Indian mathematics is apparent in Al Khwarizmi's *Treatise on Hindu Reckoning*, Al Uqlidisi's *Kitab al-Fusul fi al-Hisab al-Hindi* [Chapters in Indian Mathematics] and *Zij al-sindhind* which is translated by Al Fazari. The transfer of Indian numerical system to Islamic science and then to the whole world is made possible by these works. Høyrup suggests that introduction of Hindu reckoning followed the way through the interaction between the Persian and Arabic practitioners.[43] He bases his claim on the absence of referential documents from Indian sources in both Al Khwarizmi's and Al Uqlidisi's works.[44] Since Al Khwarizmi's treatise is only available in Latin translation, Al Uqlidisi's work remains the earliest manuscript on Hindu numeric system in medieval Islamic world. In Al Uqlidisi's account, the use of Indian numerical system is attributed to "the scribes" and "people of this craft" who points to "Indian reckoners".[45] In addition to that, Al Uqlidisi's methodology in approaching algebraic problems is different from Indian arithmetical tradition and the Arabic word for 'dust board' which is indispensible to Hindu reckoning is a Persian word: *takht*.[46] In this respect, Høyrup claims that the transmission of Hindu numeric system should be explained by considering the sub-scientific tradition and possible contacts between Persian and Arabic craftsman of the age. [47]

### 2.2. Practical Tradition in Mathematical Sciences

Development of mathematical sciences in Islamic lands was not only a result of translation movement. The use of practical arithmetic and geometry in the Middle East

[42] Demirci, "Helen Bilim ve Felsefesinin İslam Dünyasına İntikalinde/Tercümesinde Harranlı Sabiilerin Rolü", 2006

[43] Jens Høyrup, "The Formation of 'Islamic Mathematics' Sources and Conditions", *Science in Context,* 1/2 (1987): 281-329

[44] Ibid., 286

[45] Ibid., 287

[46] Ibid., 287.

[47] Ibid., 287.

that is rooted in Babylonian, Egyptian and Persian mathematical tradition is apparent in some treatises on practical geometry, magic squares, chessboard problems and commercial calculation. According to Høyrup, the synthesis between the constructional and theoretical mathematics is actually what Islamic science has achieved by compiling different scientific methods and their results from diverse sources.[48]

There are two kinds of teaching methods which gives epistemological characteristics to mathematics in general. These are apprenticeship based systems and institutionalized schools. Considering oral and practical tradition in mathematics, learning via apprenticeship based systems, the role of recreational problems and sources which we encounter the traces of recreational mathematics will be studied in this part.

Although contemporary education system rests upon institutionalized schools, in medieval age one had a little chance to get education from schools. Instead, apprenticeship based learning was more common among youngsters. The most distinctive characteristic of apprenticeship based system is that while schools use writing and written documents, apprenticeship based teaching is completely oral.

The role of recreational problems in both methods is vital. Hermelink identifies recreational problems as “problems and riddles which use the language of everyday but do not much care for the circumstances of reality”.[49] In this sense the function of recreational problems in the mathematical tradition is “to display virtuosity, and thus, on the one hand, to demonstrate the status of the profession, as a whole consisting of expert specialist, and, on the other, to let the single members of the profession stand out, and discover themselves, as accomplished calculators, surveyors, etc.”.[50] Common recreational problems which are encountered in different sources in a broad geographical area documented in diverse times are doubling of a unity, purchase of a horse and the hundred fowls problems. Doubling of unity problem is first encountered in a Northern

[48] Ibid., 296.

[49] Heinrich Hermelink, “Arabic Recreational Mathematics as a Mirror of Age Old Cultural Relations Between Eastern and Western Civilizations”, paper presented at the First International Symposium for the History of Arabic Science, Aleppo: Aleppo University Institute for the History of Arabic Science, (1978) 44-52.

[50] Jens Høyrup, “Mathematics, Practical and Recreational” in *Encylopedia of History of Science, Technology and Medicine in Non Western Cultures*, ed. Helaine Selin, (Berlin: Springer Verlag, 2008), 1353.

Babylonian text, then in a papyrus from Roman Egypt and after in Carolingian 'Propositiones ad Acuendos Juvenes'. In the ninth century it is seen in texts from Arab world and India and after the 12th century in Latin Europe. The purchase of a horse problem is first mentioned in Book I of Plato's Republic, next it is encountered in a Chinese source: 'Nine Chapters on the Arithmetic Art' by Shuchu Jiuzhang. Diophantos' Arithmetic is the first western document which states the problem clearly. This problem was also very popular in Medieval Islamic and European Lands. The hundred fowls problem is first seen in a fifth century Chinese source and later in Carolingian 'Propositiones ad Acuendos Juvenes' and Abu Kamil's 'Book of Rare Things in Calculation'.[51]

Islamic Sources which include discussions on constructional and recreational mathematics are Al Khwarizmi's 'The Compendious Book on Calculation and Balancing' (820), Abu Bakr's 'Book on Mensuration' (translated in 12th century by Gerard of Cremona), Abu Kamil's 'Book of Rare Things in Calculation' (9th century) and Abu Wafa's 'Book on What is Necessary for Artisans in Geometrical Construction' (10th century).

Al Khwarizmi's and Abu Baqr's works are reflecting the continuous tradition rooted in Babylonian mathematics. It is apparent in their way to display the proof of algebraic problems. They use both the algebraic method and a naive cut-and-paste geometrical method namely surveyors' "algebra" in order to obtain proofs for the solutions to the problems.[52]

Al Uqlidisi and Abu Kamil mention two common recreational problems in their studies. Al Uqlidisi's 'Arithmetics' book is the earliest available book which introduces the Hindu-Arabic numerals and shows various operations from basic to complex in the new numeral system. In the last chapter he works on doubling one 64 times which is an age old recreational problem. [53] Abu Kamil's 'Book of Rare Things in Calculation'

[51] Ibid., 1354.

[52] Jens Høyrup, "Algebra, Surveyors'" in *Encylopedia of History of Science, Technology and Medicine in Non Western Cultures*, ed. Helaine Selin, (Berlin: Springer Verlag, 2008), 105-108.

[53] A.S. Saidan, "The Earliest Extant Arabic Arithmetic: Kitab al-Fusul fi al Hisab al-Hindi of Abu al-Hasan, Ahmad ibn Ibrahim al-Uqlidisi", *Isis*, 57/4, (1966), 475-490.

includes the hundred fowls problem and in 'The Algebra of Abu Kamil' geometric demonstrations of algebraic problems are encountered.

Abu Wafa's 'Book on What is Necessary for Artisans in Geometrical Construction' is the most relevant source considering the present study. Abu Wafa reports a meeting between mathematicians and artisans in this work. The diverse methods in which artisans and mathematicians approach the problem introduced in the meeting are the most important aspect of this source. Abu Wafa's awareness on the issue and his critical viewpoint on the methods reveal the differences between practitioners' and mathematicians' way of handling the problems.

### 2.3. Philosophical Reflections upon Geometry in Medieval Islam

The understanding of geometry in medieval Islam can be best investigated considering ideas of contemporary philosophers on the issue. Studies on cosmological doctrines and epistemological classifications have an important aspect in identifying the hierarchical ordering in mathematical sciences and their relation to religious sciences. Actually, after assimilation of sciences from Non-Muslim cultures, viewpoints of various Muslim communities influenced the way these sciences are understood. The differences and shared characteristics of understanding of mathematical sciences will be studied in this part.

Islamic sources which reflect philosophical evaluation of mathematical sciences are Al Amiri's 'An Exposition on the Merits of Islam', Al Farabi's 'Enumeration of Sciences', Al Khwarizmi's 'Keys of Sciences', Avicenna's 'Book of Healing', Ibn Khaldun's 'The Muqaddimah', Ikhwan Al Safa's 'Encylopedia', Al Akfani's 'Irshad al wasid ila asna almaqasid' and Taşköprüzade's 'Al Tahanawi'.[54] Although there are critical differences in classification of sciences in these sources they mainly rest upon the ideas of classical Greek philosophy. Aristotle's categorization of sciences is the main source of inspiration of these studies. Ghazali reports that al-Kindi, al-Amiri, and Ibn

[54] Ahmad Al Rabe, *Muslim Philosophers' Classifications of the Sciences: al-Kindi, al-Farabi, al-Ghazali, Ibn Khaldun*, Phd diss., Massachusetts: Harvard University, 1984, 25.

Sina were the philosophers who were mostly influenced by Aristotle's classification of sciences.[55]

The impact of Aristotelian philosophy is apparent in main branches of classification of sciences. Muslim philosophers have diversified his classification and embodied new branches of sciences to the classification. According to Aristotle's classification, sciences separate into three; theoretical sciences, practical sciences and productive sciences. In that sense theoretical sciences seeks knowledge for its own sake, practical science concerns conduct and goodness in action and productive science aims at creation of beautiful and useful objects. [56] Theoretical sciences are natural sciences, mathematics and metaphysics. Practical sciences are politics and ethics. Productive sciences are various branches of craftsmanship and rhetoric. Among these, mathematical sciences are also divided into four according to Aristotelian classification. These are arithmetic, geometry, astronomy and music.

Aristotle considers the classification of sciences in relation to closeness to the first principles. In this respect sciences which rely on fewer principles are more exact than sciences which are more conditioned. Therefore according to Aristotle arithmetic assumed more exactitude than geometry.[57] Because a unit does not have a position but a point signifies a unit and a position too. So a point needs more conditions than a unit in order to exist.

Aristotle clarifies the place of geometry by comparing it with the science of physics since both seems to study the property of physical bodies, but in different senses. The subject matter of geometry is distinguished from physics in terms of the separation between form and matter of a natural body. Accordingly, Aristotle claims that mathematicians study a natural body by considering only the form of the body, whereas a physician also investigates motion, qualitative and accidental properties of the body.[58] On the other hand, more physical branches of mathematics like optics, harmonics and

[55] Ghazali in Al Rabe, "Muslim Philosophers' Classification of Sciences: al-Kindi, al-Farabi, Ibn Khaldun", 121.

[56] Christopher Shields, "Aristotle", The Stanford Encyclopedia of Philosophy (Spring 2014 Edition), Edward N. Zalta (ed.), http://plato.stanford.edu/archives/spr2014/entries/aristotle/, (accessed, 05.03.2015).

[57] Thomas Heath, *Mathematics in Aristotle*, (Oxford: Oxford University Press, 1970), 5.

[58] Ibid., 10.

mechanics have an inverse relation to geometry considering the form and matter separation. They all treat mathematical properties as physical qualities and use mathematics as a proof method. Hence, Aristotle suggests that "we must consider natural objects neither without reference to matter nor exclusively with reference to matter".[59] In other words Aristotle thinks that mathematical objects are not separate entities from matter, but they are thought as they were separate.[60] Aristotle's understanding of mathematical objects differs from Plato at this point since according to Plato forms are distinct entities from natural bodies but in Aristotle they are embedded to the matter.

Aristotle also defined beauty on mathematical grounds. Orderly arrangement, symmetry, definiteness and mathematical sciences are forms of beautiful and in fact mathematical relations in a form make beautiful act as a cause.[61]

Plato places mathematical knowledge and mathematical entities in between the direct intuition and common sense belief in epistemological sense; and between the world of being and the world of becoming in ontological sense. In this respect, mathematical objects are not inherent in material objects rather they are perfect ideas that reflect to the material world as line, square, circle, number, etc. in the world of becoming. The hierarchy between the world of being and the world of becoming is best explained with the figure below. The source of this figure is the argument in Plato's *Republic* Book 6 (509d-511e) called the divided line theory.[62] Accordingly different degrees of truth and reality are demonstrated with a hierarchical ordering. This is also interpreted as material world is a reflection of mathematical objects and mathematical objects are reflections of the world of forms.[63]

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[59] Aristotle in Heath, *Mathematics in Aristotle*, 10.

[60] Ibid., 11.

[61] Ibid., 201.

[62] Plato, *Republic*, trans. C.D.C. Reeve, (Indianapolis: Hackett Publishing, 2004), 205-207.

[63] Stewart Shapiro, *Thinking about Mathematics The Philosophy of Mathematics,* (Oxford: Oxford University Press, 2000), 54.

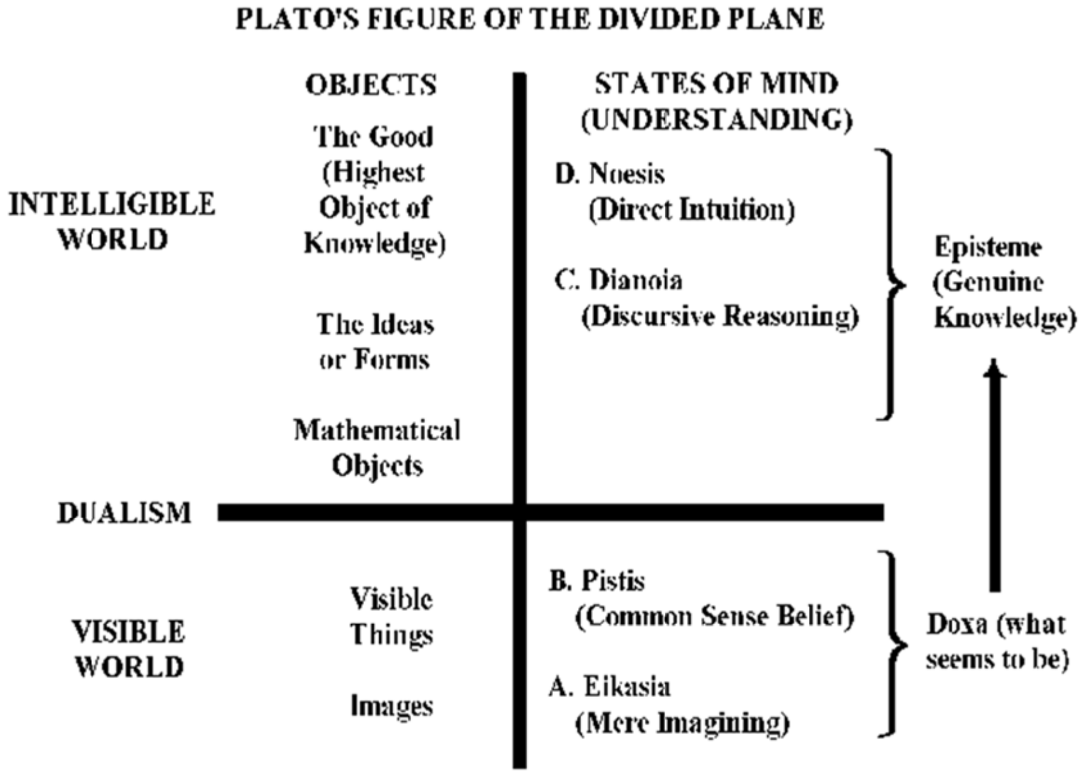


**Figure 1.1.** Plato's divided line argument

Source: http://reasonandmeaning.com/2014/10/12/the-allegory-of-the-cave-the-divided-line-the-myth-of-the-sun/

Plato's philosophical position on how to understand geometry considering the divided line is indicated as away from perceived relations between magnitudes but instead studying on principles which belong to the World of Being. Plato explains the method which geometricians use for particular problems and its relation with the reality beyond sense perception as in the following.

> You… know how [geometers] make use of visible figures and discourse about them, though what they really have in mind is the originals of which these figures are images. They are reasoning, for instance about this particular square and diagonal which they have drawn but about the Square and Diagonal; and so in all cases. The diagrams they draw and the models they make are actual things, which may have their shadows or images in water; but now they serve in their turn as images, while the student

> is seeking to behold these realities which only thought can comprehend.[64]

As we will see later, Farabi's understanding of mathematical sciences bear traces of Platonic philosophy in the sense that both consider mathematical object as an intermediary between metaphysics and material world.

#### 2.3.1. Al-Kindi

Al-Kindi is considered as the first peripatetic philosopher, which means the adherent of Aristotelian philosophy, in the Islamic tradition. His main concern was to synthesize the Islamic sciences with philosophical sciences of the ancients. *Epistle on the Fact that One Only Comes to Philosophy through Mathematics, Book on Order on the Books of Aristotle, Book on the Quiddity of Science and its Parts* and *Book on Parts of Human Science* are the sources which al-Kindi discusses classification of sciences.[65] Al-Kindi categorized sciences into two: human sciences and divine sciences. Human sciences are compiled of Aristotelian classification of sciences. On the other hand divine sciences are only for prophets and they transfer the message of God by means of them with the help of God.

Among the human sciences al-Kindi makes a separation between theoretical and practical sciences since he believes that knowledge of cause is of a higher rank than knowledge of effect.[66] On that account, theoretical sciences have a higher degree than practical sciences.

Al-Kindi divides mathematical sciences into four: arithmetic, geometry, astronomy and music. He also makes a categorization between these sciences as quantitative and qualitative sciences. Arithmetic and music are quantitative and geometry and astronomy are qualitative according to this classification. Arithmetic is quantitative in the sense that it studies addition, subtraction, multiplication and division. Science of music is quantitative since it investigates the harmonic relationship between

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[64] Plato in Shapiro, *Thinking about Mathematics The Philosophy of Mathematics,* 56-57.

[65] Jean Jolivet, "Classification of Sciences" in Vol.3 of *Encylopedia of the History of Arabic Science* ed. Roshdi Rashed, (London: Routledge, 1996), 1009.

[66] Al Rabe, "Muslim Philosophers' Classification of Sciences: al-Kindi, al-Farabi, al-Ghazali, Ibn Khaldun", 33.

numbers and their compositions; it also applies these compositions in the material world. On the other hand geometry is considered to study the immobile qualities and astronomy examines the mobile qualities of phenomena.[67] It seems that Al-Kindi's understanding of geometry has not changed much from Aristotelian viewpoint.

### 2.3.2. Al-Farabi

Al-Farabi is known as the 'Second Teacher' after Aristotle in the Islamic intellectual tradition. He has a distinctive place in the history of Islamic philosophy because he is the one who tried to create a link between Islamic thought and Ancient Greek philosophy. Farabi is also the first philosopher who dedicated a complete work on the classification of sciences namely *Enumeration of Sciences.* This study includes five chapters which he discusses science of language, logic, mathematics, natural science and metaphysics, and sciences of society. Contrary to al-Kindi he does not make a distinction between human sciences and divine sciences. His categorization of theoretical and practical sciences is encountered in his other work: *Reminder of the Way to Happiness.*[68] Mathematics, physics and metaphysics are considered as theoretical sciences and ethics, politics and economics are explained as practical sciences in this work.[69]

Farabi also distinguishes mathematical sciences into seven parts. These are arithmetic, geometry, optics, astronomy, music, the science of weights and the science of mechanical devices. Farabi considers geometry in theoretical and practical senses. In this respect practical geometry studies lines and figures in the material world either by a carpenter with wood or by a smith with iron or on a wall by a mason. On the other hand theoretical geometry deals with lines and surfaces in an abstract manner.

What is important in Farabi's understanding of mathematical sciences is that mathematics occupies an intermediary position between natural sciences and

[67] Jolivet, "Classifications of Sciences", 1010.

[68] Al Rabe, "Muslim Philosophers' Classification of Sciences: al-Kindi, al-Farabi, al-Ghazali, Ibn Khaldun", 79.

[69] Muhsin Mahdi, "Science, Philosophy, And Religıon In Alfarabi's Enumeration Of The Sciences", *The Cultural Context of Medieval Learning*, ed. J. E. Murdoch and E. D. Sylla, (Dordrecht-Holland: D. Reidel Publishing Company, 1975), 116

metaphysics. In other words the subject matter of mathematics is existent in both natural bodies and metaphysical beings.[70] Accordingly, the ontological status of arithmetic and geometry fits to this definition and considered as intermediary between metaphysical beings and natural bodies; but optics, astronomy, music, the science of weights and the science of mechanics are treated as being more related to natural sciences.[71] Endress identifies the peculiar ontological and epistemological status of mathematical sciences by regarding them as 'abstractive sciences'. [72] It is also claimed that mathematical sciences separate intellectually what is inseparable from matter in its actual existence and in this respect Farabi follows Aristotelian premises in understanding mathematical sciences.[73]

The theoretical-practical division in mathematics also has a place in hierarchy of virtues in Farabi's philosophy. Farabi conceives theoretical intellect possessing a higher rank of virtue with respect to practical intellect. Bakar explains what is meant 'theoretical intellect as a theoretical virtue' by Farabi as in the following:

> …this intellect has actually acquired knowledge of the primary intelligibles and is in a sound state to gain certain knowledge of the rest of the theoretical intelligibles or existents. The knowledge that is a theoretical virtue is certain knowledge of the existence of theoretical beings and certain knowledge of the cause of their existence. This knowledge is arrived at through demonstrative proofs, based on primary intelligibles acquired by the theoretical intellect. This knowledge therefore refers to theoretical parts of the philosophical sciences.[74]

Therefore deliberative virtue gained by practical wisdom through personal observation and experience has a lower degree. The main reason for that is practical intellect being in

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[70] Osman Bakar, *Classification of Knowledge in Islam,* (Cambridge: The Islamic Texts Society, 1998), 100.

[71] Ibid.,103.

[72] Gerhard Endress, 'Mathematics and Philosophy in Medieval Islam' in *Entreprise of Science in Medieval Islam,* ed. Jan P. Hogendijk, Abdelhamid I. Sabra, (Massachussets: MIT Press, 2003), 139.

[73] Ibid.

[74] Bakar, *Classification of Knowledge in Islam,* 110.

the service of theoretical intellect.[75] The relationship between theoretical and practical intellect is also applicable to the relation between theoretical and practical geometry.

### 2.3.3. Al-Ghazali

Ghazali's classification of sciences and understanding of geometry rests upon a different view from what we have studied so far. Ghazali classifies the sciences at first as religious sciences and intellectual sciences. According to him intellectual sciences should be classified in two methods. First, according to their relation to religious beliefs in the sense that either they are praiseworthy or objectionable or permissible.[76] Second, dividing the sciences into two: theoretical and practical; the classical Aristotelian way of categorization. However, Bakar suggests that classification sciences as religious and intellectual is more essential than the theoretical-practical division.[77]

Mathematics is considered as a praiseworthy science according to the first classification. Since mathematics can be used in service of society in practical sense and its results does not contradict with religious doctrines. The theoretical-practical distinction in sciences follows Aristotelian classification of sciences in the main branches. Theoretical sciences are mathematics, natural sciences, metaphysics; practical sciences are politics, ethics and household management[78]. Ghazali's understanding of mathematical sciences is studied in its relation with religion in the sense that mathematical sciences neither rejects nor accepts religious beliefs. Although mathematics provides clear and exact knowledge, a drawback of this science may be the result of a misunderstanding in community because people may be suspicious of religion seeing that although mathematicians arrive at such clear and exact results in their sciences, their religious beliefs may not be that strong. So, one can think that if a

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[75] Ibid., 111.

[76] Al Rabe, "Muslim Philosophers' Classification of Sciences: al-Kindi, al-Farabi, al-Ghazali, Ibn Khaldun", 128.

[77] Bakar, *Classification of Knowledge in Islam*, 217.

[78] The translation of the phrase *'ilm tadbir al-manzil* is encountered as 'household management' in various sources. Alexander Treiger, "Al-Ghazalis Classifications of the Sciences and Descriptions of the Highest Theoretical Science", *Divan Disiplinlerarası Çalışmalar Dergisi*, Vol. 16, no:30 (2011/1): 1-32; Simon Swain, *Economy, Family, and Society from Rome to Islam: A Critical Edition, English Translation, and Study of Bryson's Management of the Estate*, (Cambridge: Cambridge University Press, 2013), 357

mathematician denies religion he may be true because he is the one who knows all exactly and clearly. But according to Ghazali mathematics are merely quantitative and do not bear any connotations of metaphysical or symbolic entities as opposed to Pythagorean and Platonic philosophy of mathematics. So there is not any relationship between mathematics and religion for Ghazali at all.

Ghazali also introduces a new twofold classification of science: the worldly and other-worldly sciences. He considers mathematics and medicine to be worldly sciences and are deprived having religious and metaphysical references.

Focusing on the differences between Farabi's and Ghazali's understanding of mathematics is significant at this point. While both philosophers claim an ontological division between material and spiritual worlds. The intermediary subjects of these two worlds are different respectively. In Farabi's account mathematics occupies an intermediary position between material and metaphysical world and this is made possible by considering the abstractive characteristic of mathematical sciences. On the contrary, Ghazali assumes the intermediary entities between the two worlds to be completely spiritual. He considers that there is a difference between microcosmic and macrocosmic intermediaries. Thus, imaginative spirit is the microcosmic intermediate position between sensory and rational spirit, and macrocosmically demons serve as an intermediary between the angelic and material world.[79]

[79] Bakar, *Classification of Knowledge in Islam,* 226 note.75.

## CHAPTER 3

## HISTORIOGRAPHY ON GEOMETRIC ORNAMENT AND THE TWO DOCUMENTS ON THE USE OF GEOMETRY IN MEDIEVAL ISLAMIC ARCHITECTURE

Use of geometry in architecture can be seen as an organizer of horizontal and vertical axes and ornamental means. One who first encounters a monument from medieval period in the Middle East remarks obsessive use of geometric ornament in most of the monumental buildings. This aspect of Islamic architecture cannot be unrelated to studies on geometry in the Islamic world. Translation of Greek studies on geometry and ongoing tradition in practical geometry enabled complex geometric patterns. One of the ways that led to this achievement is pointed by Özdural as collaboration of craftsmen and mathematicians.[80] Experimentation that characterizes medieval craftsmen's method should not also be overlooked. However, modern world lacks the techniques employed by medieval craftsmen. This gap between the contemporary practice of architects and the techniques of medieval craftsmen led to different approaches in interpreting geometric ornament. We have two original documents that would have been used to surmount this difficulty of indefiniteness. They instruct us about the practice of medieval craftsmen and provides ground for discussing geometric ornament. These sources are Abu Wafa's 'Book on What is Necessary for Artisans in Geometrical Construction'[81] from 10th century, and an anonymous treatise

[80] Alpay Özdural, "Mathematics and Arts: Connections between Theory and Practice in Medieval World", *Historia Mathematica*, Vol. 27 (2000): 171-201.

[81] Five copies of the manuscript is kept in Istanbul, Ayasofya MS 2753; Cairo, D¯ar al-Kutub al-Mis.riyya MS 31024, MS 44795; Milan, Ambrosiana MS &68sup; Uppsala, Universitetbibliotek MS Tornberg 324.

called ‘On Interlocks of Similar or Corresponding Figures’[82] from 13th century. ‘Book on What is Necessary for Artisans in Geometrical Construction’ is a booklet which contains eleven chapters on ruler and compass constructions of geometric figures. Abu Wafa does not provide proofs of these figures for the sake of simplicity in order to make them more understandable by the craftsmen.[83] The chapters of the book are (1) the ruler, the compass and the gonia (i.e., a set square); (2) fundamental Euclidean ruler-and-compass constructions, and in addition a construction of two mean proportionals, a trisection of the angle, and a pointwise construction of a (parabolic) burning mirror; (3) constructions of regular polygons, including some constructions by a single compass opening; (4) inscribing figures in a circle; (5) circumscribing a circle around figures; (6) inscribing a circle in figures; (7) inscribing figures in one another; (8) division of triangles; (9) division of quadrilaterals; (10) combining squares to one square, and dividing a square into squares, all by cut-and-paste constructions; and (11) the five regular and a few semi-regular polyhedra.[84] ‘On Interlocks of Similar or Corresponding Figures’ also includes geometric figures which are accompanied by some explanations of the figures. The manuscript is identified as much higher and later stage of development than Abu Wafa’s by Chorbachi and Necipoğlu assumed that it is a work by a mathematician who was trained in practical tradition rather than a theoretical one and Özdural claims that it is compiled of hastily taken notes probably by a mathematician who is involved in ornamental arts.[85] On the other hand, the content of these documents lacks a theoretical base for evaluating geometric ornament aesthetically. Thus we are groundless in constructing a view which would help us understand the philosophical and aesthetic points about the use of geometric ornament in arts and architecture in medieval Middle East.

[82] The only copy of the manuscript exists in Paris, Bibliotheque Nationale MS Persan 169.

[83] Jan P. Hogendijk, “Mathematics and Geometric Ornamentation in the Medieval Islamic World”, *European Mathematical Society*, Newsletter no. 86, December (2012), pp. 37-43.

[84] Jan P. Hogendijk, Mathematics and Geometric Ornamentation in the Medieval Islamic World, 38.

[85] Alpay Özdural, “On Interlocking Similar or Corresponding Figures and Ornamental Patterns of Cubic Equations”, *Muqarnas*, Vol. 13 (1996), pp. 191-211.

### 3.1. Historiography on Geometric Ornament

Several viewpoints try to reveal the meaning behind the use of geometry in architecture refering to philosophical and theological ideas on mathematics. Main views of interpretation in these studies can be classified as traditionalist argument, Sunni revival argument and classicist argument.

The main point of traditionalist school on Islamic art relies on Quranic revelation and the Islamic theology which is revealed with the concept of *tawhid*. Neoplatonic and Pyhtagorean philosophies also forms the base of argument. Primarily, Sufi doctrine is adopted to interpret the 'sacred' meaning in use of geometry in art and architecture. Scholars who hold such a view are Ismail Al-Faruqi, Keith Critchlow, Issam Al Said, Seyyid Hossain Nasr, Nader Ardalan, Titus Burckhardt and Samer Akkach. [86]

Although traditional school provides us a romantic reading of Islamic art which tries to reestablish the viewpoint of the "traditional man", it is obvious that this view is ahistorical and only expresses conventional statements for its own sympathizers. To give an example it is worth to consider Ardalan's study on Islamic architecture, which he defines universal principles of traditional man in Islamic society and their preposterious relation with the circumference and the radii of a circle. He says;

> The traditional man in Islamic Society lives according to Divine Law; in addition the man with a social vocation seeks the truth through the way that exists as the inner dimension of the law. The relation between the Truth, the Way and the Law is best expressed through the symbol of the circle. The Law is the circumference, the Way is the radii leading to the the center and the center is the Truth.[87]

The nostalgia which prevails this school of thought tries to reorder the perception of the modern man and to canalize him into 'the Way of Divine Truth' instead of vanishing in the relativities of contemporary civilization. This view is nostalgic in the

[86]After The World of Islam Festival, which was held in 1976 in London, books published by authors who hold traditionalist perspective on Islamic art and architecture reached its peak. It is not surprising that the theme of the festival was "unity of Islam".

[87] Nader Ardalan, *The Sense of Unity: The Sufi Tradition in Persian Architecture*, (Chicago, London: University of Chicago Press, 1973), 5.

sense that it declares a period of time in history as perfect and constructs its narration according to the moral and religious attributes of that historical fiction.

A similiar version of imposing an abstract concept on fictive history can be found in Burckhardt's work superimposing the concept of 'Divine Unity';

> Islamic art – by which we mean the entirety of plastic arts in Islam- is essentially the projection into the visual order of certain aspects or dimensions of Divine Unity. [88]
>
> … this script is like an indefatigable attestation of the Divine Unity accompanied by joyful and serene expansion of the soul.[89]
>
> For a Muslim artist or—what comes to the same thing—a craftsman who has to decorate a surface, geometrical interlacement doubtless represents the most intellectually satisfying form, for it is an extremely direct expression of the idea of the Divine Unity underlying the inexhaustible variety of the world.[90]

The rise of publications with this approach during 1970's is related to the attempt to lure oil money from Iran by the art historians.[91] But it is best to consider Khaghani's criticisim on the issue. According to Khaghani;

> The traditionalist method is abstract in the sense that different traditions are interpreted as variations on an already formed idea, which undermines the concrete paradigm presented by particular cases. It is ahistorical since tradition is represented as the spatial ossification of an atemporal message. Based on a process of 'self-orientalisation' and a kind of platonic transendentalism, sanctified tradition is not a matter of debate but rather of understanding. [92]

[88] Titus Burckhardt, *Art of Islam: Language and Meaning*, (London: World of Islam Festival Pub.Co., 1976), 51.

[89] Ibid., 57.

[90] Ibid., 73.

[91] W. K. Chorbachi, "In the Tower of Babel: Beyond Symmetry in Islamic Design", *Computers Math. Applic.* Vol 17, No:4-6 (1989), 757.

[92] Saeid Khaghani, *Islamic Architecture in Iran Poststructural theory and Architectural History of Iranian Mosques*, (London, New York: I.B. Tauris & Co Ltd., 2011), 40-41. According to Bezci and Çiftçi "Self-orientalism means distortion of the representation of [one's] own culture by representing

In this respect, geometric art in Islamic architecture is one of the essential instruments of traditionalists who would attribute to it fictive concepts since it lacked a definite realm of denotations.

Another argument interpreting the spread of geometric ornaments on buildings relates it with the 'Sunni revival'. Necipoğlu suggests this argument and Tabaa extends her ideas to a variety of architectural and artistic elements.

Necipoğlu gives philosophical, religious and poltical context of Baghdad in 10th and 11th centuries by stating the conflicting views of Mu'tazilism and Ash'arism, and the relations of Buyids and the Caliphate and later Great Seljuks. In this sense, while Mu'tazili understanding puts together rationalism with the faith of God and explains the phenomena as accidental compositions of atoms and also suggesting that Quran is created and not co-eternal with God; the Ash'arites hold that there is a perpetual creation by God and Quran is not created. Baghdad was under the rule of Shi'i Buyid dynasty between 945-1055, which supported Shi'i and Mutazi'li views conflicting Abbasid religious policy. [93] Caliph al-Qadir (reigned 991-1031), attempted to reestablish the Sunni doctrine by abandoning Mutazi'li and Shi'i teachings. [94] Necipoğlu asserts the beginning of the Sunni Revival with the Great Seljuks reign in Baghdad in 1055. And "in this context of Sunni revival during the hegemony of the Great Seljuks" she says "that the girih[95] mode suddenly flourished".[96] In addition, she explains the former appearances of geometric ornaments in "such seemingly unrelated marginal areas as Tim, Uzgend and Kharraqan" as accidental. [97] Necipoğlu develops her view by stating that the girih mode was perceived as a symbol of the of Abbasid Caliphate by Fatimid

'itself' according to west, in western system of values." Bünyamin Bezci, Yusuf Çiftçi, "Self-Orientalization: Modernity within Ourselves or Internalized Modernization", *Akademik İncelemeler Dergisi*, Vol. 7, No.1 (2012): 139-166.

93 Gülru Necipoğlu, *Topkapı Scroll,* (Los Angeles: Getty Publications, 1996)*,* 96.

94 Ibid.

95 Girih is the Persian word for knot and used as a term to refer to the complex geometric patterns in Medieval Islamic architecture.

96 Ibid., 97.

97 Ibid., 99.

and Spanish Umayyad caliphs so that there was a notable resistance to the geometric mode in their realms.[98]

The philosophical explanations Necipoğlu brings for geometric ornament relies mainly on Ghazali's and Ibn Khaldun's views on geometry. Both attacked speculative philosophy. [99] Ghazali's view of mathematics, which he classifies in the category of intellectual science, corresponds to Ibn Khaldun's view of mathematics:

> It should be known that geometry enlightens the intellect end sets ones mind right. All its proofs are very clear and orderly. It is hardly possible for errors to enter into geometrical reasoning, because it is well arranged and orderly. Thus the mind that constantly applies itself to geometry is not likely to fall into error… Our teachers used to say that one's application to geometry does to the mind what soap does to a garment. It washes off stains and cleanses it of grease and dirt. The reason for this is geometry is well arranged and orderly, as we have mentioned.[100]

Necipoğlu also mentions Brethren of Purity, Al Jurjani, Ibn al Haytham, Ibn Sina, Ibn Rashiq in dealing with the "Geometry and Aesthetic Theory" part of the book but, since she considers geometric ornament in respect to Sunni revival, the ideas of earlier philosophers "holding emanationist cosmology with heterodox implications of 'polytheism'" were rejected but they had provided "a backdrop for the burgeoning taste for geometric abstraction".[101]

Necipoğlu's study is the most comprehensive one because she touches upon almost every work on the subject published until its publication date and tries to contextualize the the relationship between geometric ornament, mathematical sciences, philosophy and politics. George Saliba's review of the book suggests that Necipoğlu was hoisted with her own petard in searching for a new approach apart from traditionalist view since she "ends up having to justify the persistance of features of the 'rationalist'

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[98] Ibid., 101.

[99] Ibid., 103.

[100] Ibid., 104.

[101] Ibid., 192.

tradition in the bosom of orthodoxy".[102] While the aim of Sunni revival is represented as repelling speculative philosophy and rationalism for domination of orthodoxy, Saliba claims that concepts as "light of reason" and "speculative philosophy" are treated in a more tolerated way that they became compatible with Sunni orthodoxy resulting a derivation of old orientalist arguments.[103]

In comparison with Necipoğlu, Tabaa includes more artistic and architectural cases supporting his view. Mainly, his argumentation is similiar with Necipoğlu's considering Sunni revival. In the introduction part of the book he declares his intention on how to use the concept of Sunni revival:

> the very architectural and epigraphic forms that may have been inspired or mandated by the forces of Sunni revival will be discussed… It is these new forms that fleshed out the textual and verbal discourses of the new ideology, producing a symbolic language that was intended to mediate between the myth of Sunni ecumenical unity and reality of political fragmentation. [104]

Using this theoretical approach, Tabbaa first examines Ibn Muqla's geometric regularization in calligraphy in order to indicate the relation between the struggle to standardize the religious teaching under Sunnism in the 10th century and its formal reverbation in Ibn Muqla's system of proportion.[105] And later, Tabbaa introduces another canonical figure of calligraphy, Ibn al-Bawwab, who was trained by students of Ibn Muqla, in order to point to the relation between his works and the epistle of caliph al-Qadir which imposes an orthodox reading of Quran. "Perfectly cursive and easily legible script suitable for expressing the clear and true nature of the Word of God" is how

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[102] George Saliba, "Artisans and Mathematicians in Medieval Islam the Topkapi Scroll: Geometry and Ornament in Islamic Architecture by Gülru Necipoğlu Review", *Journal of the American Oriental Society*, Vol. 119, No. 4 (Oct. - Dec., 1999). 644-645.

[103] Ibid., 644.

[104] Yasser Tabbaa, *The Transformation of Islamic Art during the Sunni Revival*, (Seattle, London: University of Washington Press, 2001), 24.

[105] Ibid., 34-44.

Tabbaa constructs and expresses the relation between Ibn al-Bawwab's style and Qadiri orthodoxy.[106]

The geometric ornaments in architectural elements are also treated as a signifier of Sunni theology. The lack of geometric ornaments in Fatimid buildings in Egypt until Ayyubid period is also thought as either the isolation of the state during Fatimid period or the resistance and opposition against institutions and symbols of Sunni revival.[107] Thus, Necipoğlu and Tabbaa constructs their narrative on geometric ornament by using Sunni revival as a conceptual tool which unites and dominates the artistic tradition.

Terry Allen's argument on classicism in Islamic art worth studying in terms of our topic. Allen is consireded to be a classicist because he views the Medieval Islamic ornament as an extension of Late Antique ornamentation.[108] Allen criticises the view adopted by Necipoğlu in *Topkapı Scroll*. [109] Allen is more concerned with the arguments on the collaboration between artisans and mathematicians instead of the Sunni revival issue. In addition, the aesthetic interpretation by Necipoğlu is also considered by him. According to Allen, the roles of several groups of people should be examined in order to reach a conclusive argument. The relations between the actors such as the theoretical geometers; the artisans and architects who actually created works incorporating more sophisticated uses of geometry; their patrons and/or customers; the audience for these works; the group, if there was one, that expected that these works would operate on humans in emanationist manner described by philosophers would be basis in building historical narrative from the literary evidence.[110] In this respect, Allen argues that since none of the encyclopedias of medieval Islamic environment mentions about artists, this lack of interest on recording the occupation and works of craftsmen points to the idea

[106] Ibid., 50.

[107] Ibid., 162.

[108] Barbara Genevaz, "Review of the book Five Essays on Islamic Art by Terry Allen", *Mimar Book Reviews in Mimar: Architecture in Development*, Vol: 8 edited by Hasan-Uddin Khan, (Singapore: Concept Media Ltd., 1983), pp.67-68.

[109] Terry Allen, *Islamic Art and the Argument from Academic Geometry,* (California: Solipsist Press, 2004), http://www.sonic.net/~tallen/palmtree/academicgeometry.htm (accessed 02.09.2014).

[110] Ibid.

that encyclopedists were holding the view that "practice should be governed by theory" and artists were only interested in the practical side of their occupation.[111] On the quality of colloboration between the artists and mathematicians Allen claims that artists only consulted to theoretical geometers in designing complex geometrical patterns, and mathematicians were showing shortcut methods for their patterns.[112] Thus, according to Allen it is impossible to connect philosophers' arguments on the status of geometrical objects to the geometric patterns because of lack of written evidence on the issue. [113] In addition, the growing complexity of patterns in the 11th century is a result of evolutionary development which stemmed from antiquity.[114]

**Figure 3.1.1** Kharraqan tomb towers in Iran

Source: http://en.wikipedia.org/wiki/Kharraqan_towers

[111] Ibid.

[112] Ibid.

[113] Ibid.

[114] Ibid.

Carol Bier's approach to geometric ornament stems from her analysis of the tomb towers at Kharraqan. (Figure 3.1.1) There are two inscriptions from Qur'an on these towers. A part from Surat al Hashr is found in both towers and a single verse from Surat Al-Mu'minun is found in the later tower to the west.[115] According to Bier selections from these passages was purposeful in the sense that the choice was made to refer to the patterns on the building.[116] Especially the word *amthal* which is the plural of the word *mithal* meaning a kind of likeness or resemblance is the key to understand the meaning behind geometric patterns for Bier. Acordingly, the geometric patterns denotes to the literal images of the term *amthal* whih is referred in the passages of Quran.[117] Thus, she assumes that these patterns are used as a parable and reification of the verses of Quran which offers an allegorical interpretation of Qu'ran as a visual commentary.[118] In this respect, the geometric decoration in Kharraqan towers is declared as an early articulation of sacred geometry by Bier. Although Bier seems to reach similiar conclusions with traditionalist school her method in handling the issue is different from traditionalists. While we see a direct speculation in traditionalists method, Bier uses cultural and philosophical elements existent contemporary with the erection of the monument.

Oleg Grabar investigates the use of ornament in Islamic art and architecture through calligraphy, geometry, architectural representations and vegetal ornament. Grabar interprets the use of geometric ornament with his theory of intermediaries which he forms in first chapter of his book *Mediation of Ornament.*[119] Partly influenced by Gombrich's ideas on ornament, which defines ornament generally as a means to framing, filling and linking, Grabar points that the main reason for building, for example Mshatta

[115] Qur'an 59:21-24, 23:115.

[116] Carol Bier, "Geometric Patterns and the Interpretation of Meaning: Two Monuments in Iran", *Bridges: Mathematical Connections in Art Music and Science*, ed. Reza Sarhangi, (Kansas: Bridges Conference, 2002), 73.

[117] Carol Bier, "Art and Mitha¯ l: Reading Geometry as Visual Commentary", *Iranian Studies*, Vol. 41, Number 4, (2008), p. 507.

[118] Bier, "Geometric Patterns and the Interpretation of Meaning: Two Monuments in Iran", 74.

[119] Oleg Grabar, *The Mediation of Ornament*, (Princeton: Princeton University Press, 1989) 9-46.

facade with full of ornaments could not be solely for filling the space.[120] Rather, ornament, as the subject of the design, transforms the carrier into bearer of meaning as in the case of Mshatta facade since all of the structure was covered with a particular design. [121] Grabar gives two important features of geometric ornament in a general sense. First is the ubiquity of geometric ornament which is only a formal characteristic in the sense that semiotic or symbolic meanings of ornaments are determined according to historical context and the form remains geometric.[122] Secondly, use of geometric ornament is generally seen in illitterate, remote or popularly pious cultures since dominant classes is mostly seen as using alternate forms or signs except Islamic culture.[123] Grabar investigates geometric ornaments in Islamic architecture through Khirbat al-Mafjar, several monuments from Iran and lastly Alhambra. The geometric ornaments on Khirbat al-Mafjar belongs to Umayyad age and in this early period of Islamic art what we see is a textile effect on architecture created via geometric ornament for Grabar.[124] Thus geometric ornament serves as an intermediary to textile technique in order to express its essence.[125] Grabar also claims the same hypothesis considering Kharraqan tomb towers. (Figure 3.1.1) Moreover, he declares that use of geometry was for esthetic appreciation and not for any iconographic message.[126] His evaluation of the pattern on the North Dome of Isfahan Friday Mosque points to Omar Khayyam because of its complexity and their contemporaneousness. (Figure 3.1.2)

[120] Ibid., 41.

[121] Ibid.

[122] Ibid., 128.

[123] Ibid., 129.

[124] Ibid., 141-142.

[125] Ibid., 142.

[126] Ibid.,145.

**Figure 3.1.2** North Dome of Isfahan Friday Mosque

Source: https://www.flickr.com/photos/twiga_swala/5850931502/

The muqarnas ceiling in Hall of Two Sisters in Alhambra is refered by a poem written by Ibn Zamrak (1333-1393). (Figure 3.1.3) He describes the dome as representing heavenly spheres.[127] Yet, this attribution does not imply that all the examples of geometric ornament can be interpreted by this reference for Grabar[128]. Instead each instance can be a bearer of meaning by the external references inscribed on the artwork but such messages can not be inherent in the form.[129]

Grabar also makes an unexpected reference to M.C. Escher while discussing geometric ornament.[130] It is unexpected because of the irrelevant historical context. However, considering Escher's occupation with geometric ornament in a mind blowing complexity in his works, his ideas might reveal an approach which has not been noticed yet. Escher reports,

[127] Oleg Grabar, *The AlHambra,* (Cambridge, Massachusets: Harvard University Press, 1978), 140-150.

[128] Grabar, *Mediation of Ornament*, 148.

[129] Ibid., 149.

[130] Ibid.,153.

> Long before I discovered in Alhambra an affinity with the Moors… I had recognized this interest in myself… I tried, almost without knowing what I was doing, to fit together congruent shapes that I attempted to give the form of animals… My experience has taught me that the silhouettes of birds and fish are the most gratifying shapes of all for use in the game of dividing the plane.[131]

Grabar calls our attention to the concept which Escher uses to describe the geometric ornament - the 'game of dividing the plane'. Later although Grabar carries the issue to Escher's desire to form mimetic representations using figures like birds and fishes, the concept also has the potential to be evaluated regarding Huizinga's *homo ludens*. Huizinga disccusses the aspects of play element in culture and considers the ornamentation of an object with reference to the play element. According to Huizinga, plastic arts do not include the play element in their nature compared to poetry and music because of their dependence on material and limitations of form.[132] However the traces of the play factor in the plastic arts reveal itself best in the decorative arts because of the archaic tendency to play with lines and planes, curves and masses in a case of boredom and inanition.[133] I believe a future research which will concentrate on the play element in Islamic art and architecture would reveal much more cases concerning the 'game of dividing the plane'.

So far as we have seen, the traditionalists attempt to intepret the ornaments rely on ahistorical grounds and Necipoğlu's Sunni revival argument considering geoetric ornament as the signifier of Sunni theology is criticized by Saliba for falling into the same mistake with traditionalists. However, Necipoğlu's attempt to construct a contextual relation between the politics and ornament is a new approach . Grabar's reveals the most basic aspect of geometric ornament and claim that it is ubiquitous. In this respect any search for meaning, except there is a direct denotation in the historical

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[131] M. C. Escher in H.S.M.Coexeter, "Coloured Symmetry", *M.C. Escher, Art And Science : Proceedings Of The International Congress On M.C. Escher, Rome, Italy, 26-28 March 1985*, (Amsterdam ; North-Holland: Elsevier Science Pub. Co., 1986), 16.

[132] Johann Huizinga, *Homo Ludens; A Study Of The Play-Element İn Culture*, (Boston: Routledge&Kegan Paul, 1980), 166.

[133] Ibid., 168.

sources for a particular case, is a venture. Thus he offers to contextualize each case with external references. I am skeptical about meanings attributed subsequent to the creation of an art work and incline to put more emphasis on visual experience.

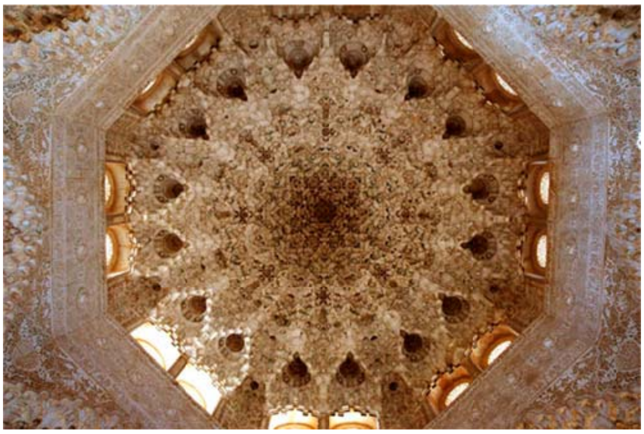

**Figure 3.1.3** Hall of Two Sisters AlHambra

Source: http://archnet.org/

## 3.2. Two Documents on the Use of Geometry in Medieval Islamic Architecture

Actually, there are three manuscripts which are related with geometric ornament. These are Abu Wafa's *On the Geometric Constructions Necessary for the Artisan* from 10th century, an anonymous treatise called *On Interlocks of Similar or Corresponding Figures* from 13th century and the *Topkapı Scroll* which is dated to late 15$^{th}$ century. But, *Topkapı Scroll* is out of the defined time scope and cultural context of this thesis for the reason that while it is possible to claim for a continuity between Abu Wafa's treatise and the anonymous Persian treatise in terms of similar forms they are considering, *Topkapı Scroll* contains no formal resemblence with either of the two.

Abu Wafa's treatise exists in a booklet which has eleven chapters. The relevant chapter is Chapter 10 mentioning combining squares to one square and dividing a square into squares.[134] In this chapter Abu Wafa reports that he has been in meetings with geometers and artisans,

[134] Jan. P. Hogendijk, "Mathematics and Geometric Ornamentation in the Medieval Islamic World", *European Mathematical Society*, Newsletter no. 86, (2012), pp. 37-43.

> I was present at some meetings in which a group of geometers and artisans participated. They were asked about the construction of a square from three squares. A geometer easily constructed a line such that the square of it is equal to three squares, but none of the artisans was satisfied with what he had done. The artisan wants to divide those squares into pieces from which one square can be assembled as we have described for two squares and five squares.[135]

In this quotation, we observe first of all that some meetings were arranged which artisans and geometers were attending but we do not see any direct reference to the design of geometric ornament. Rather, what we encounter is the methodological differentiation of geometers from artisans and craftsmen in problem solving. The main problem which may be raised in this manuscript is in what respect the geometricians involved in the design of geometric ornaments. According to Terry Allen, as we have mentioned, this collaboration did not go beyond artisans' seeking for assistance from geometers in several problems for short cut methods.[136] However, according to Özdural the relation between artisans and mathematicians occupied a closer relationship in Medieval Islamic world. Özdural uses the term *conversazioni* in order to refer to the meetings arranged between mathematicians and artisans.[137] He proposes several documents from Al Kashi and Omar Khayyam in which some sort of meeting is mentioned and interpreted as the sign of *conversazioni* in order to show that the collaboration between mathematicians and artisans is an essential part of Islamic artistic culture.[138]

Özdural's geometrical analyses of Abu Wafa's text and the anonymous Persian text draws attention to a continuity in the constructed geometric figures. This continuity seems convincing. The relevant figure is first encountered in Abu Wafa's manuscript as

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135 Abu Wafa in Alpay Özdural, "Mathematics and Arts: Connections between Theory and Practice in the Medieval Islamic World", 174.

136 Terry Allen, *Islamic Art and the Argument from Academic Geometry*, http://www.sonic.net/~tallen/palmtree/academicgeometry.htm.

137 Alpay Özdural, "Omar Khayyam, Mathematicians and *Conversazioni* with Artisans", *Journal of Society of Architectural Historians*, Vol. 54:1, (1995), 54-71.

138 Özdural, "Omar Khayyam, Mathematicians and *Conversazioni* with Artisans", 54-71.

the proof of Pythagorean theory. (Figure 3.2.1) And later a variation of the figure is apparent in the anonymous Persian treatise which is directly related with the ornamental works. (Figure 3.2.2)

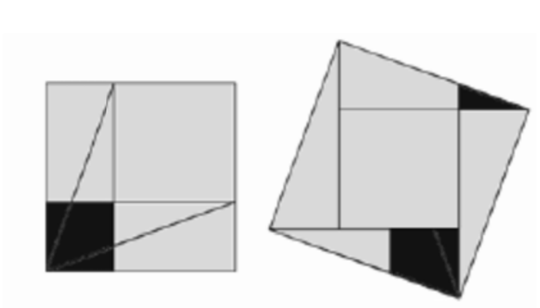

**Figure 3.2.1** Abu Wafa's proof of Pythagorean theory

Source: Jennifer L. Nielsen, "The Heart is a Dust Board: Abu'l Wafa Al-Buzjani, Dissection, Construction, and the Dialog Between Art and Mathematics in Medieval Islamic Culture", http://historyofmathematics.org/wp-content/uploads/2013/09/2010-Nielsen.pdf

Although similar proofs of Pythagorean theory have been given since Socrates, the proof of Abu Wafa is original for his squares are in different scales and convenient for practice. In it we also find the construction of a square from two different squares. The importance of square in architectural ornament is related with 15th century mathematician's work Al Kashi in the sense that a unit square was the primary module for generating muqarnas vaults and Özdural finds the same tradition in the 10th century geometric design.[139]

The relation between Abu Wafa's figure and the special triangle of Omar Khayyam is apparent in the anonymous Persian treatise, *Interlocks of Figures*. The special right triangle of Khayyam is constructed by composing a right triangle whose hypotenuse is equal to the sum of the short side and perpendicular to the hypotenuse. What is seen in *Interlocks of Figures* is the description of the problem in creating a

[139] Alpay Özdural, "Mathematics and Arts: Connections Between Theory and Practice in Medieval World", *Historia Mathematica*, Vol. 27 (2000): 171-201.

triangle in similar to Khayyam's, but the scribe mistakenly attributes the triangle to Ibn al Haytham.[140]

> Correlations involved in this drawing concern conic [sections]. The objective of it consists in constructing a right triangle in such a way that the sum of the perpendicular and the shorter side is equal to the hypotenuse. Ibn Haytham wrote a treatise on the construction of such a triangle, and there he described the conic sections...[141]

The relevant text by Omar Khayyam is the following;

> This analysis lead to a right triangle with the condition that the hypotenuse is equal to the sum of one of the sides of the right triangle and the perpendicular to the hypotenuse.[142]

Thus, by imposing the special triangle on the pattern created by Abu Wafa a synthesis of both figures is reached. (Figure 3.2.2) The scribe of the *Interlocking Figures* explains the problem:

> As mentioned above, the objective of our drawing is four conical figures [that is almonds, which he calls *turunj* (orange) at other places] with two right angles that surround an equilateral right-angled quadrilateral [that is a square].[143]

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[140] Alpay Özdural, "Omar Khayyam, Mathematicians and *Conversazioni* with Artisans", 64.

[141] "On Interlocks of Similar or Corresponding Figures" in Alpay Özdural, "Omar Khayyam, Mathematicians and *Conversazioni* with Artisans", 64.

[142] Omar Khayyam in R. Amir Moez, "A paper of Omar Khayyam" *Scripta Mathematica*, XXVI, (1963): 325-326.

[143] "On Interlocks of Similar or Corresponding Figures" in Alpay Özdural, "Omar Khayyam, Mathematicians and *Conversazioni* with Artisans", *Journal of Society of Architectural Historians*, Vol. 54:1, (1995), 64.

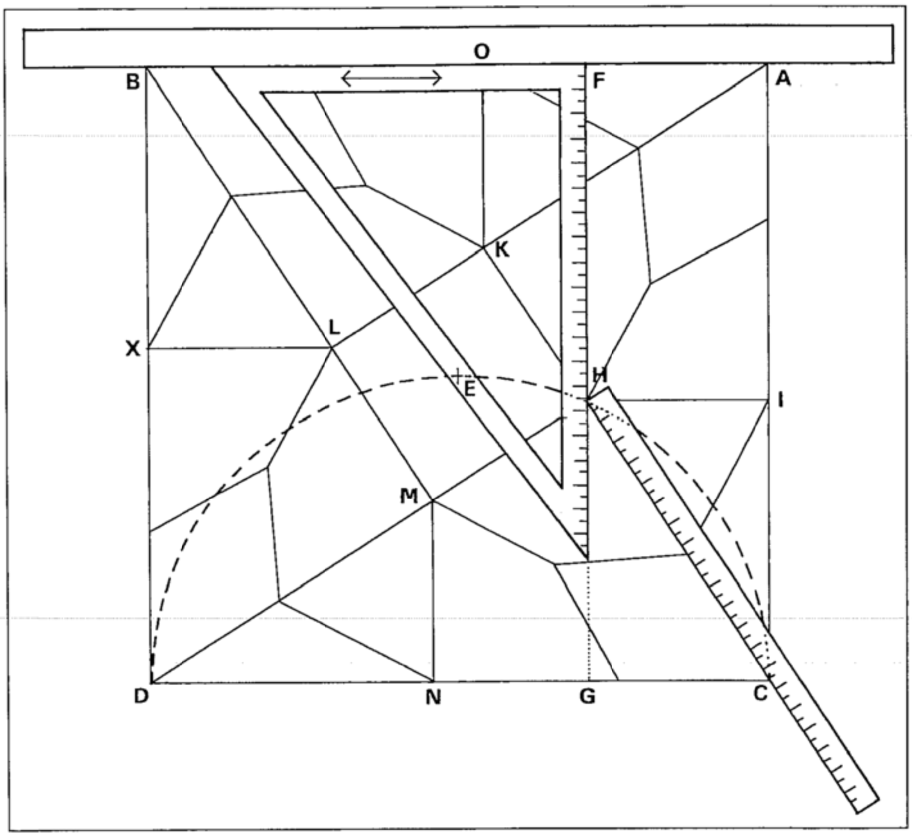


**Figure 3.2.2** Abu Wafa's proof of Pythagorean theory with Khayyam's special triangle in *Interlocks of Figures*

Source: Alpay Özdural, "Omar Khayyam, Mathematicians and *Conversazioni* with Artisans", *Journal of Society of Architectural Historians*, Vol. 54:1, (1995), 54-71

In this construction four right angle triangles are defined in such a way that each triangle's hypotenuse is equal to the sum of the short side and perpendicular to the hypotenuse, and they form Abu Wafa's proof of Pythagorean theory, HC+HG=CD. Variations of this pattern is found in the Isfahan Hakim Mosque from 17th century and in the Friday Mosque of Isfahan among the architectural decoration from Safavid period (15th century). (Figure 3.2.3, 3.2.4)

**Figure 3.2.3** Hakim Mosque Isfahan

Source: http://patterninislamicart.com/, http://kufic.info/

**Figure 3.2.4** Isfahan Friday Mosque

Source: http://patterninislamicart.com/

From these results, Özdural reaches the conclusion that *Geometric Constructions* and *Interlocks of Geometry* can be seen as evidence for claiming the existence of collaboration between artisans and mathematicians. Methodologically, the difference between artisans and mathematicians is overcame by mathematicians' use of cut-paste method in order to prove the correctness of their results in a concrete way and to create new decorative patterns for artisans.[144]

[144] Alpay Özdural, "Mathematics and Arts: Connections Between Theory and Practice in Medieval World", *Historia Mathematica*, Vol. 27 (2000), 193.

A more specific explanation on the relationship between mathematicians and artisans is given by Hogendijk.[145] According to him, the lack of evidence on the issue allows us to make only particular derivations on the methodological difference between artisans and mathematicians. First, while mathematicians were trained in the Euclidian tradition and focused on proving the problems in the Euclidian way, craftsmen, although they are familiar with Euclidian way to draw figures, are not interested in proving their designs.[146] Secondly, while texts and diagrams by mathematicians do not require any oral explanation, the texts by craftsmen usually require verbal backing to remove ambiguity.[147] Third, although for mathematicians exact and approximate constructions really differ in terms of theoretical reference, for craftsmen anything is permissible as long as it would fit their practical needs.[148] And lastly, instruments, which are not used in theoretical studies of geometry, such as set-square and compass with a fixed opening are used by craftsmen.

Thus, although we have an exact understanding of the methodological difference between mathematicians and artisans, the collaboration between them in designing geometric ornaments needs more documental evidence. The contextual continuity between the *Geometric Constructions*, *Interlocking Figures* and architectural ornaments in Medieval Iran is apparent, but in what sense mathematicians involved in the design of the patterns remains unknown.

[145] Jan P. Hogendijk, "Mathematics and Geometric Ornamentation in the Medieval Islamic World", *European Mathematical Society*, Newsletter no. 86, December (2012), pp. 37-43.

[146] Ibid., 42.

[147] Ibid.

[148] Ibid.

## CHAPTER 4

## PRACTICAL ASPECTS: A CASE STUDY FROM SELJUK ARCHITECTURE

Practical geometry can be best investigated by surveying geometry in architectural works. Geometric arabesque and the order in plan, section and elevation of particular buildings presents us evidences on the use of geometry in Islamic architecture. The term *muhandis*, which is a title of architect-planners, also justifies the link between geometry and architecture in Medieval Islam. The term refers to the one who 'geometricizes'. [149] Alpay Özdural claims that the term *muhandis* denoting architects and artisans is an outcome of the collaboration between mathematicians and artisans.[150]

Architectural practice in Seljuk Architecture offers exciting cases which reveal understanding of mathematics of the age. This thesis will first investigate Isfahan Friday Mosque focusing on the geometric constructions and ideas on the North Dome and on one of the geometric ornaments which presents the famous collaboration between mathematicians and artisans. Next, philosophical speculations on the use of geometry will be added to the investigation.

### 4.1. Isfahan Friday Mosque

Isfahan Friday Mosque manifests ruling class predilections and sectarian disruptions, which were in conflict, between 10th and 15th centuries. Mafarrukhi reports

[149] Nader Ardalan and Laleh Bakhtiar, *The Sense of Unity,* (Chicago, London: The University of Chicago Press, 1973), 9.

[150] Alpay Özdural, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", Technology and Culture 39.4 (1998), 699-715.

that a great mosque was built by Tamimi Arabs and it was restored during Muktadir's Caliphate.[151] The foundations of the current mosque dates back to the Buyid domination in Isfahan in the 10th century. As a matter of fact nothing has remained from Buyid architecture except the Jorjir portal at Hakim Mosque in Isfahan. Galdieri's excavations revealed the foundations of this hypostyle mosque in the Masjid-i Jami. The Buyid structure was built as an Abbasid style mosque with a rectangular courtyard and hypostyle composition on. The material used in this mosque was baked brick. In this sense, Buyid style employed Persian construction techniques in a traditional hypostyle congregational mosque setting.[152]

Contemporary structure of the mosque is based on the transformations applied during Seljuk rule. These transformations include the introduction of two domes in front of the mihrab and in the north end; four iwan plan and double arcaded riwaqs. This new plan defined a new building tradition of congregational mosque in Iran. (Figure: 4.1)

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[151] Mafarrukhi in Eric Schroeder "Standing Monuments of the first Period" in *A Survey Of Persian Art from Prehistoric Times to the Present,* Vol. III, ed. Arthur Upham Pope, (London, New York: Oxford University Press, 1967), 956-957.

[152] Richard Ettinghausen, Oleg Grabar, Marilyn Jenkins-Madina, *Islamic Art and Architecture 650-1250*, (New Haven, CT: Yale University Press, 2001), 110.

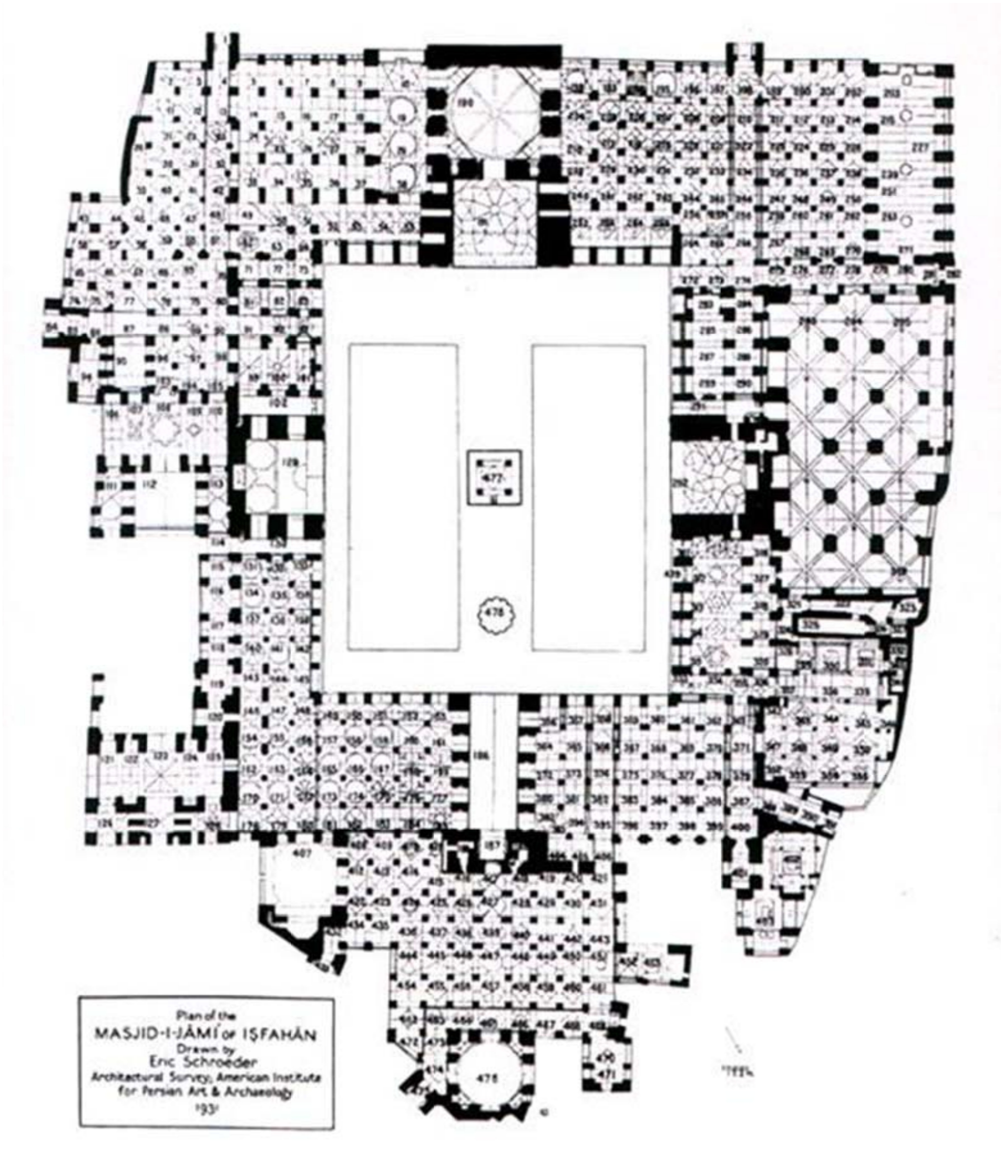


**Figure. 4.1** Eric Schroeder's plan of the Isfahan Friday Mosque

Source: [http://archnet.org/](http://archnet.org/)

The first step in transformation of the mosque was construction of a dome on the south axis of the mosque in front of the mihrab, which was sponsored by Nizam al-Mulk. Blair claims that the idea of inserting a dome into the mosque came about when Malik-Shah had seen the dome of the Umayyad Mosque in Damascus in 1086.[153] Grabar assigns the years 1086-87 to the construction of the south dome and 1088 to the north

[153] Sheila Blair, *The Monumental Inscriptions from Early Islamic Iran and Transoxiana*, (Leiden: E.J. Brill, 1992), 162.

dome. [154] Building a second dome to the opposite end of the mosque is a result of the rivalry between Nizam al-Mulk and Taj al-Mulk, the founder of the north dome. According to Blair, Nizam al-Mulk's south dome with its extraordinary vastness expresses the authority of the Seljuk rule  while Taj al-Mulk's north dome offers mathematical exquisiteness appropriate for a second-rank administrator's role.[155]

Building the four iwans to the south, east, west and north is the finishing touch in completing the structural outline of the mosque. Grabar argued that the functional use of these iwans aimed at "providing separate areas for different sects within ecumenical Islam (that) had to be accommodated".[156] In 1121 the mosque was set into fire which is the result of the conflict between Shafi and Hanafi sects.  The reason for the disagreement between the sides was that both sects demanded to be in front of the minbar area but Nizam al-Mulk's dome underlined Shafi dominion.[157] The fire destroyed the wooden parts of the mosque and the library of Taj al-Mulk which possessed more than five hundred manuscripts.[158] After the fire an entrance was built to the northeastern corner of the mosque which was addressed by an inscription dating 1121.

Construction of a mihrab by Oljeitu's order in 1310; installment of a madrasa to the southeast side of mosque by Mozaffarids; construction of a winter praying hall to the west side in the 14th century by Djahan Shah of Qaraqoyunlus; Aqqoyunlu Uzun Hasan's restorations in the south iwan that inserted glazed tile mosaics and later muqarnas vaults inside each iwan added by the Safavids are other important interventions the mosque passed through.[159]

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[154] Oleg Grabar, *The Great Mosque of Isfahan*, (New York: New York University Press, 1990), 53.

[155] Sheila Blair, *The Monumental Inscriptions from Early Islamic Iran and Transoxiana*, (Leiden: E.J. Brill, 1991), 165.

[156] Ibid., 57.

[157] David Durand-Guédy, *Iranian Elites and Turkish Rulers: A History of Isfahan in the Seljuq Period*, (London, New York: Routledge, 2010), 202-203.

[158] Durand-Guédy, *Iranian Elites and Turkish Rulers: A History of Isfahan in the Seljuq Period*, 203.

[159] André Godard, *The Art of Iran*, (New York, Washington: Frederick A. Praeger Publishers, 1965), 316.

The chronology of Friday Mosque reveals religious and political inclinations of dominant dynasties. In this respect each ruler leaved a mark that manifests contemporary political and religious idea within the current architectural movement.

In the following chapter, in considering these modifications, North Dome' geometric decoration and the overall use of geometry are interpreted through philosophical trends of the age.

### 4.2. North Dome

The use of North Dome is still a mystery. Its proportional beauty evokes admiration. British travel writer and art critic Robert Byron compares the two domes and expresses his opinion:

> The two chambers of the Friday Mosque point this distinction by their difference… In the larger, which is the main sanctuary of the mosque, twelve massive piers engage in a Promethean struggle with the weight of the dome… while the larger lacked the experience to its scale, the smaller embodies that precious moment between too little experience and too much, when the elements of construction have been refined of superfluous bulk, yet still withstand the allurements of superfluous grace; so that each element, like the muscles of a trained athlete, performs its function with winged precision, not concealing its effort, as over-refinement will do, but adjusting it to the highest degree of intellectual meaning. This is the perfection of architecture, attained not so much by the form of the elements – for this is a matter of convention – but their chivalry of balance and proportion. And this small interior comes nearer to that perfection than I would have thought possible outside classical Europe.[160]

As remarked by Byron, the North Dome owes its aesthetic appeal much to the proportional system it embodies. Eric Schroeder provides a geometric analysis of the dome attributing the use of golden section in the design and associating it with the concept of 'ideal dome'.[161]

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[160] Robert Byron, *The Road to Oxiana*, (London: Picador, 1994), 227-228.

[161] Eric Schroeder, "Seljuq Preiod", in *A Survey Of Persian Art from Prehistoric Times to the Present*, Vol. III, ed. Arthur Upham Pope, (London, New York: Oxford University Press, 1967), 1007-1009.

The general layout of the dome is composed of three parts, a square chamber in the lower section, the zone of transition in the middle section which binds the lower part with the dome by means of geometrical diversifications and the spherical dome. The square plan is first transformed to an octagone and then to a hexadecagonal base carrying a circular base for the dome. At this point the transition from square plan to a circular one is made available by eight large arches supporting a ring of sixteen arches which defines the round rim of the dome. The pointed dome is the last part which encloses the space with a brilliant five pointed star decoration on the intrados.

Schroeder points out application of golden section in the relationship between the intersections of these three parts. In the figure he created, he marks the parts which golden section is encountered by the letter "Z" indicating Adolf Zeising's studies on golden section. (Figure 4.2.1).[162]

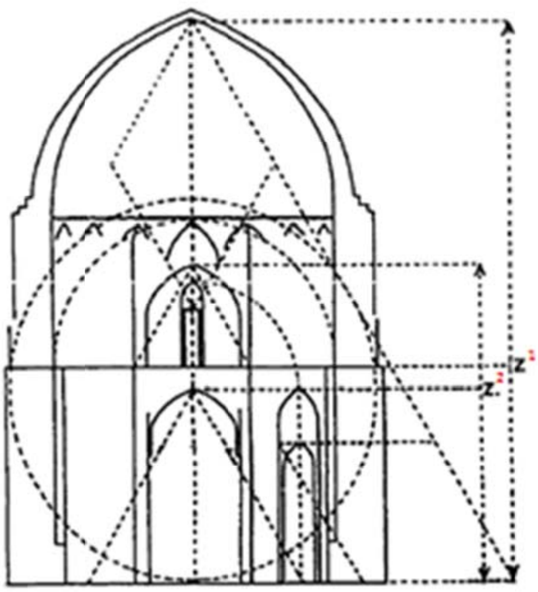



**Figure 4.2.1** Golden Section analyses in North Dome

Source: http://archnet.org/

[162]The geometrical analysis of the North Dome is based on mainly Eric Schroeder's research which was published in "A Survey of Persian Art" and Marjan Ghannad's unpublished master thesis *A Study on the Formation of the North Dome of Masjid-i-Jami Isfahan* (Master Thesis, Carleton University, 2000) which is a detailed study of Schroeder's analysis and ideas.

First of all the total inner height doubles the length of the base. When the height is divided from the beginning of the transition zone golden ratio ($Z^1$) is achieved. Golden section is also found in the relation between the height of lower main arch and height of the great octagon arch ($Z^2$). The highest end of the lower main arch is also the center of mass of an equilateral triangle whose peak is the highest end of the inner dome. (Figure 4.2.2)

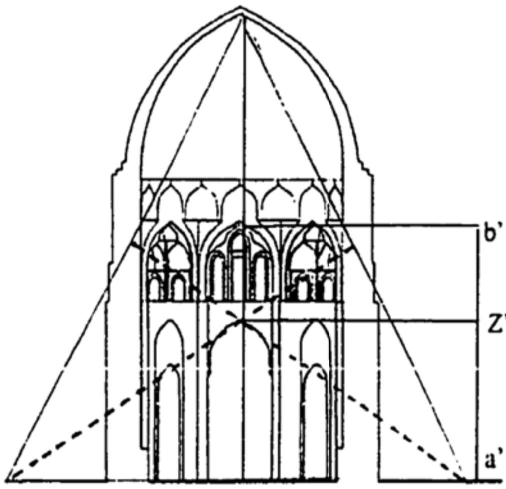



**Figure 4.2.2** Golden Section Analysis in North Dome

Source: Marjan Ghannad, *A Study on the Formation of the North Dome of Masjid-i-Jami Isfahan*, unpublished Master Thesis, (Carleton University, 2000), 24

The inner width of the octagonal space is twice the dimension of the length from the beginning of the transition zone to the rim of the dome vertically. (Figure 4.2.3)

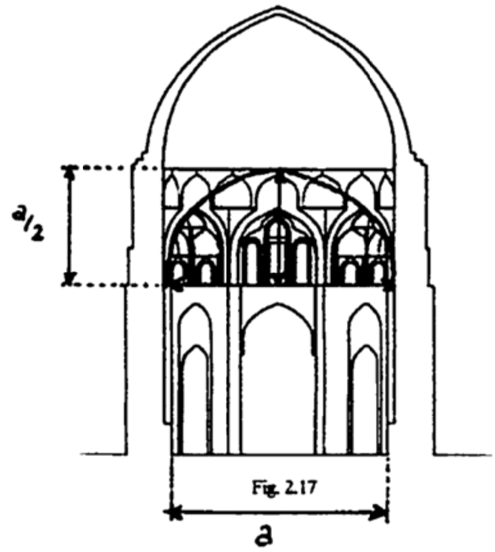



**Figure 4.2.3** Geometric analysis of North Dome

Source: Marjan Ghannad, *A Study on the Formation of the North Dome of Masjid-i-Jami Isfahan*, unpublished Master Thesis, (Carleton University, 2000), 23

An equilateral triangle based on the ground whose apex is the high end of lower middle arch determines the vertical axis for each of the lower side arches. (Figure 4.2.4)

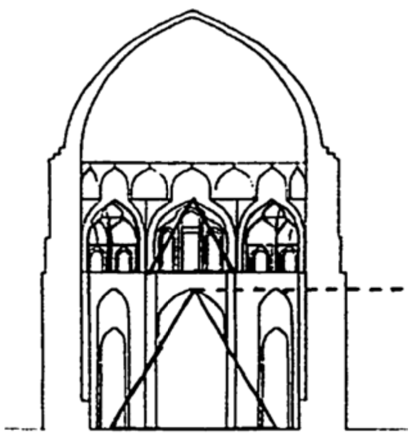


**Figure 4.2.4** Geometric analyses of North Dome

Source: Marjan Ghannad, *A Study on the Formation of the North Dome of Masjid-i-Jami Isfahan*, unpublished Master Thesis, (Carleton University, 2000), 24

Lastly, the parallelograms set whose sides are equal to quarter of the equilateral triangle peaking at the apex of interior dome and they define the height of the lower side arches and the height of the windows. (Figure 4.2.5)

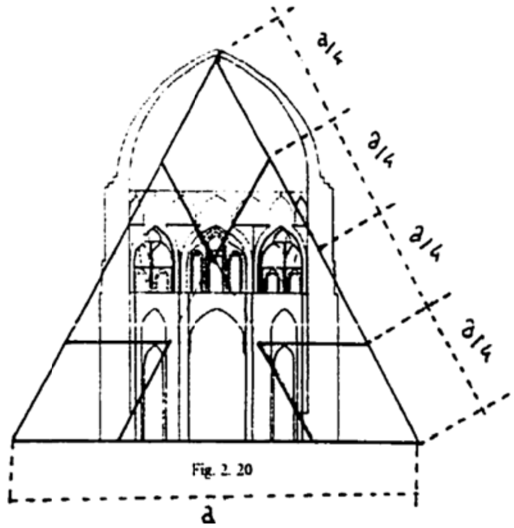



**Figure 4.2.5** Geometric analyses of North Dome

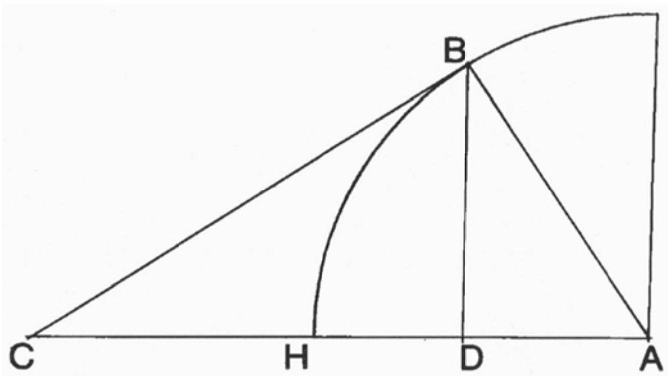
Source: Marjan Ghannad, *A Study on the Formation of the North Dome of Masjid-i-Jami Isfahan*, unpublished Master Thesis, (Carleton University, 2000), 25

Another geometrical analysis of North Dome is done by Alpay Özdural.[163] According to him, the proportional relations derived from sectional analysis points to philosopher, mathematician, astronomer and poet Omar Khayyam, who lived and worked in Isfahan in the 11th century, as the one who gave the geometrical instructions for the design of the structure. The idea is supported by superimposing Omar Khayyam's triangle to the section plan on the base, and following the harmonic, geometric and arithmetic means which fits to definite elements of the North Dome.

Omar Khayyam introduced his triangle in an untitled treatise.[164] Khayyam defines a quarter-circle with its center A and two radius lines AB and AH. When a perpendicular is drawn from B to AH namely BD, the ratio of AH/BD equals AD/DH. Khayyam proposes a right triangle ABC from this construction whose sum of perpendicular and shorter side is equivalent to the length of hypotenuse AC. He derives the longer right side is also equal to the sum of shorter right side and AD. (Figure 4.2.6)

[163] Alpay Özdural, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", 699-715.

[164] Ali R. Amir Moez, "A paper of Omar Khayyam" *Scripta Mathematica*, XXVI, (1963): 323-337.




AH/BD=AD/DH

AC=AB+BD

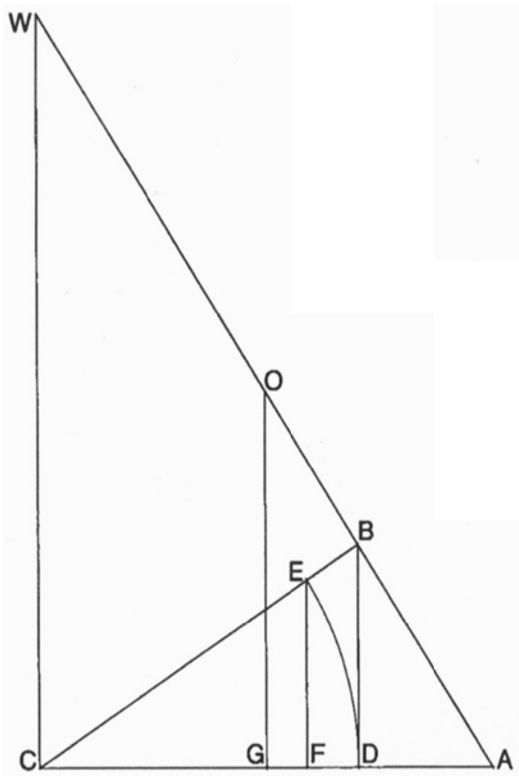
BC=AB+AD

**Figure 4.2.6** Khayyam Triangle

Source: Alpay Özdural, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", *Technology and Culture* 39.4 (1998), 699-715

The specialty of Khayyam's triangle lies in the harmonic, geometric and arithmetic means it bears. First of all the perpendicular drawn from B to AC generates a geometric mean between AB, BC, hypotenuse and the divided parts of AC from the point D. Özdural furthers Khayyam's triangle and adds the point W to the extension of AB and constructs another right triangle AWC.[165] (Figure 3.7) The perpendicular from the midpoint of AC is defined as GO and it is length is half of CW. Moreover GO is also the arithmetic mean between shorter side and hypotenuse of AWC. The ratio of AC to GO is also equal to the sum of hypotenuse and AD to AC as well as the longer segment of the hypotenuse, which is CD, to AB. Accordingly, CD is defined as the harmonic mean between the hypotenuse, AC and shorter side of the triangle, AB, i.e., (AC - CD)/AC= (AD - AB)/AB. (Figure 4.2.7)

[165] Özdural, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", 699-715.




$AC/AB=AB/AD$

$AC/BC=BC/CD$

$GO=1/2CW=1/2(AB+AC)$

$AC/GO=(AC+AD)/AC$

$AC/GO= CD/AB$

$(AC-CD)/AC=(AD - AB)/AB$

**Figure 4.2.7** Khayyam triangle advanced by Özdural

Source: Alpay Özdural, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", *Technology and Culture* 39.4 (1998), 699-715

The North Dome provides a set of ratios that are arithmetically, geometrically and harmonically bound to each other. In this respect, the base of the mosque and the dome chamber, which are marked as AC, are defined as the hypotenuses of a Khayyam triangle. The height of the square part, GM divides AC into half. NQ which is the height of the transition zone also bisects the dome chamber span. Özdural examines that the height of the square chamber, GM, which is equal to the length of the segment CD, is the result of the harmonic mean between span, AC; and height of the transition zone, NQ. [166] Since the height of the transition zone, NQ, corresponds to the shorter side, AB, of Khayyam triangle, it is derived that GO is the arithmetic mean between AC and NQ or AB where O is the point that meets the extensions of AB and GM coinciding the central window sill in the upper main arch. These set of geometric relations results in "musical

[166] Ibid.

proportion” in Iamblichus’ terms because of proportional particularities of Khayyam triangle, i.e., AC/GO=GM/NQ. (Figure 4.2.8)

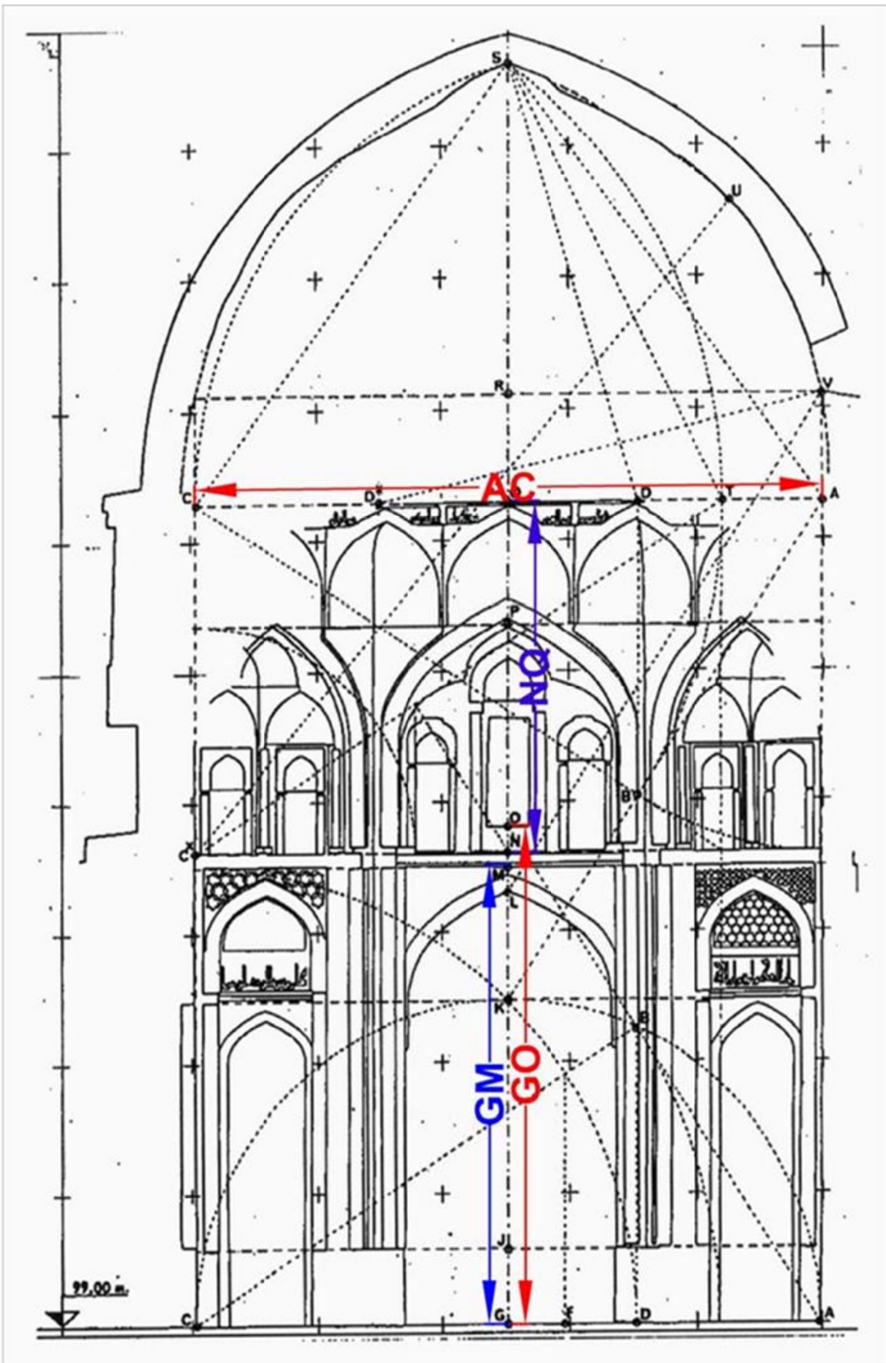



**Figure 4.2.8** Musical proportions in North Dome AC/GO=GM/NQ

Source: Proportions highlighted on analysis of Alpay Özdural, “A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan”, *Technology and Culture* 39.4 (1998), 699-715

The same arithmetical mean, that is between span and the transitional zone height, is also encountered considering LR, which is the line constructed between the peak of the central arch and the high end of the inscription band on the dome, because GO is intended to be equal with LR in order to keep the design in order. Moreover, the same harmonic mean between span and transitional zone height can be observed focusing on NR, which is the height between the beginning of the transition zone and the peak of the inscription band and which is equal to GM. Thus the harmonic relation in the square dome chamber is repeated in the transition zone. In addition the musical proportion that is seen in the square chamber can also be observed as AC/LR=NR/NQ=AC/KQ since LR, NR and KQ correspond respectively to GO, GM and GO. (Figure 4.2.9)

Further analysis reveals the musical proportion in the relationship between definitive elements as dado, springing level of central arch, central window sill in the upper main arch, peak of the central main arch and drum of the dome i.e., GK/JK= NP/KO= OQ/JK. (Figure 4.2.10)

In addition, arithmetical mean between the height below drum and square zone height equals the length of the span i.e., AC= ½ (GO+GN). The geometrical mean between these heights including the length from the peak of the dome to the rim, CS, as in the following JM/GM=KL/PQ= QR/PQ=PQ/KN=KN/KO=NR/CS=CS/AC also favors the proportional fabric inherent in the structure of the dome. (Figure 4.2.11)

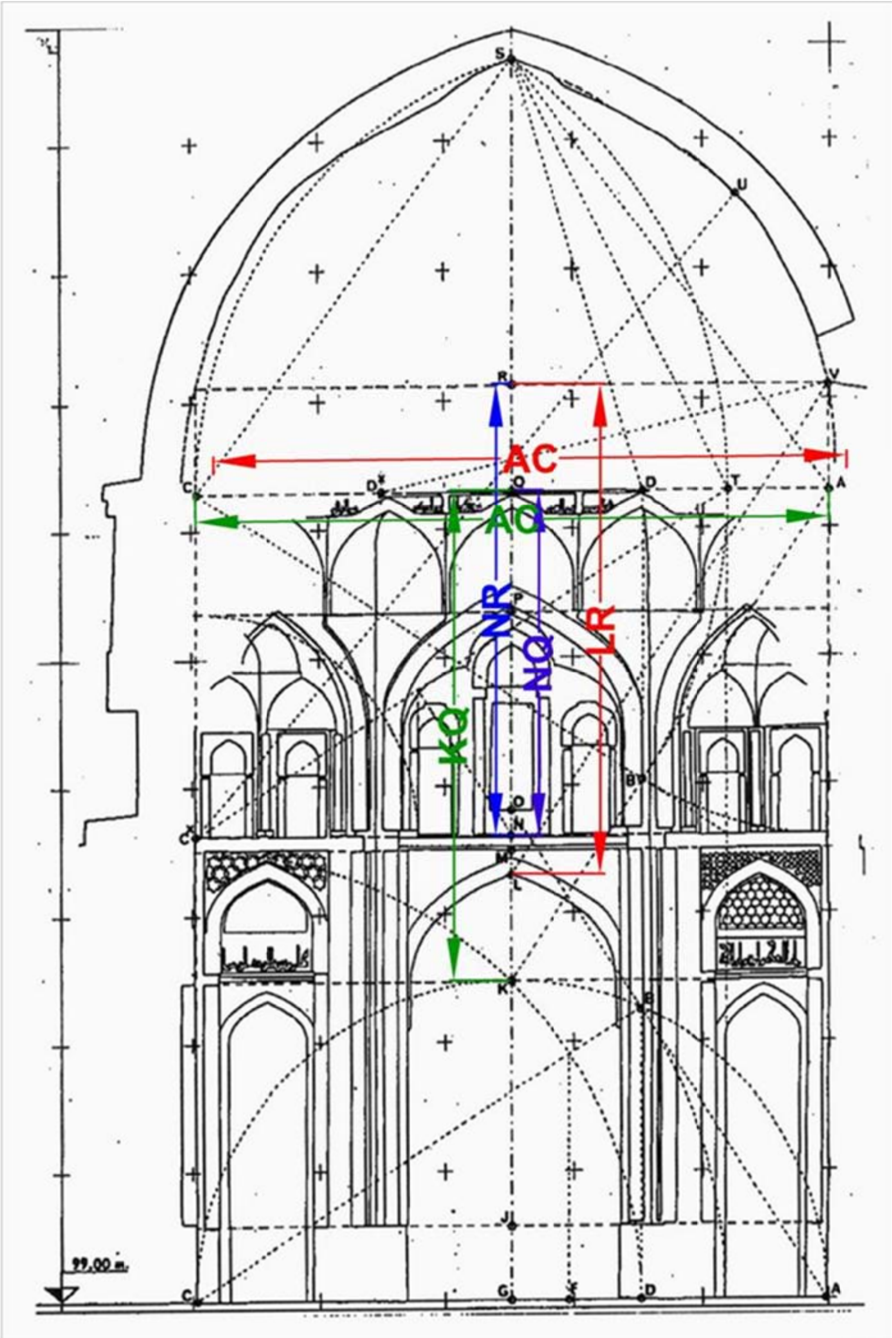



**Figure 4.2.9** Musical proportions in North Dome AC/LR=NR/NQ=AC/KQ

Source: Proportions highlighted on analysis of Alpay Özdural, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", *Technology and Culture* 39.4 (1998), 699-715

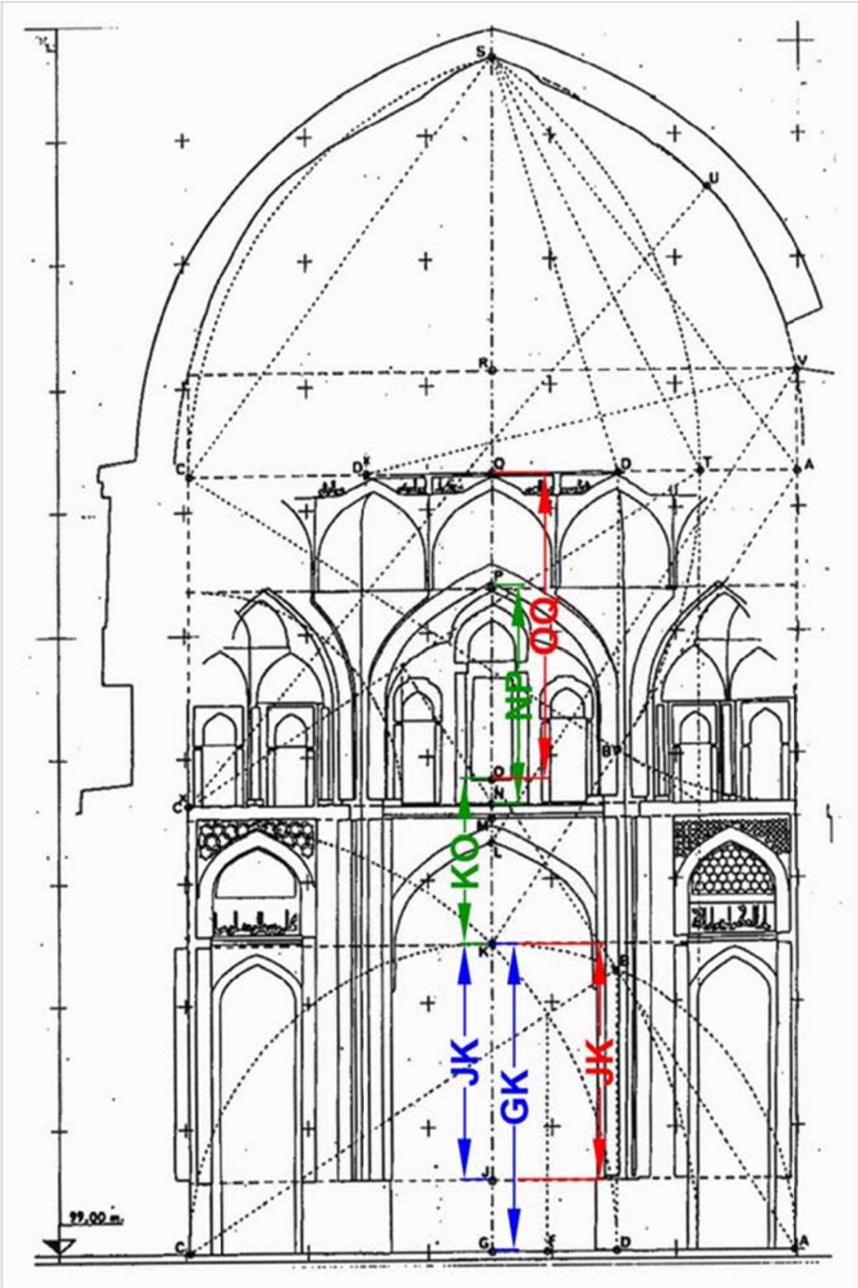



**Figure 4.2.10** Musical proportions in North Dome GK/JK= NP/KO= OQ/JK

Source: Proportions highlighted on analysis of Alpay Özdural, “A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan”, *Technology and Culture* 39.4 (1998), 699-715

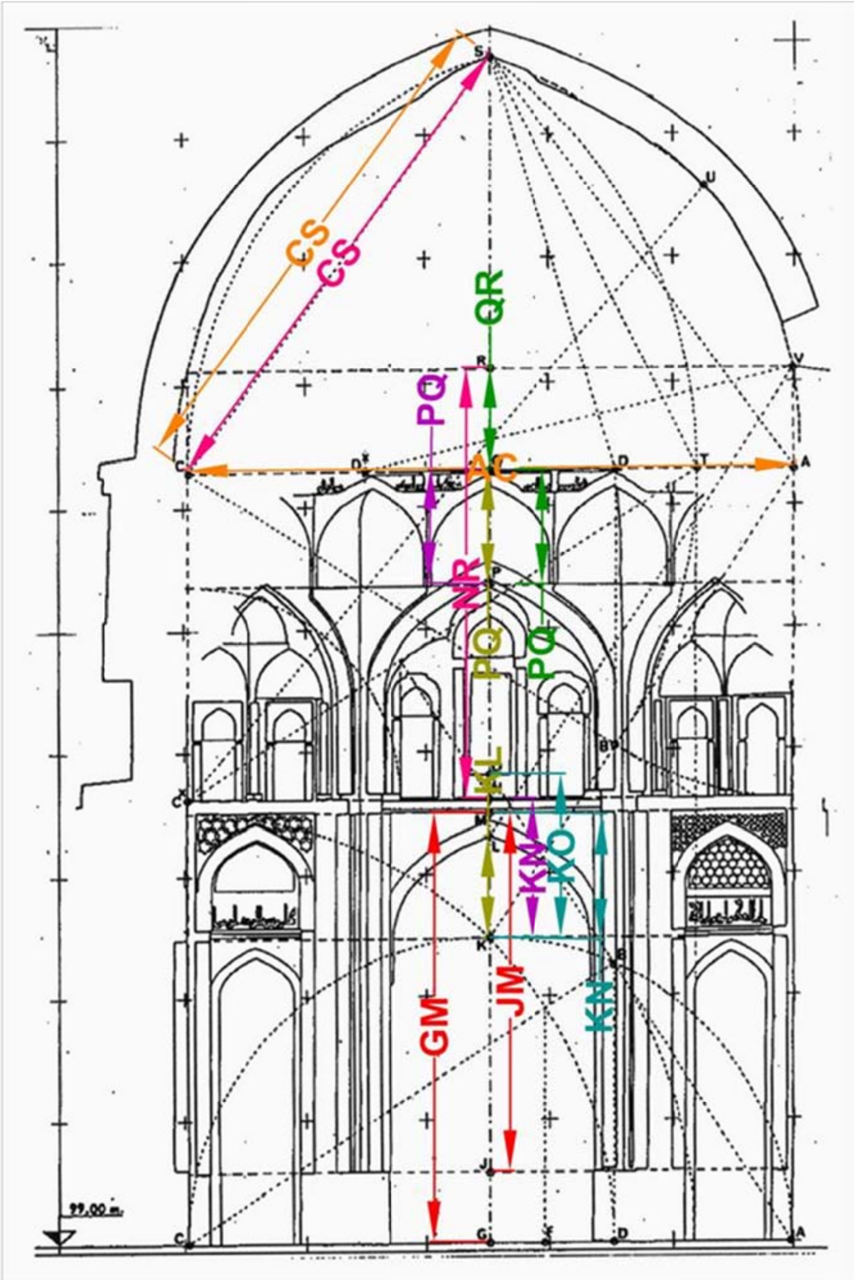



**Figure 4.2.11** Geometrical mean in North Dome
JM/GM=KL/PQ=QR/PQ=PQ/KN=KN/KO=NR/CS=CS/AC

Source: Proportions highlighted on analysis of Alpay Özdural, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", *Technology and Culture* 39.4 (1998), 699-715

#### 4.2.1. Arithmetic, Geometric and Harmonic Proportions

The use of arithmetic, geometric and harmonic proportions in the design of the dome implies a continuous tradition of practical mathematics rooted in Greek culture.

The system of proportions was discovered by Pythagoras and the three means were developed by early Pyhtagorean School. Archytas of Tarentum, who changed the name of the sub-contrary mean to harmonic mean, defines the three proportions in a fragment *On Music* as in the following,

> There are three means in music: one is the arithmetic, the second geometric and the third sub-contrary [, which they call harmonic]. The mean is arithmetic, whenever three terms are in proportion by exceeding one another in the following way: by that which the first exceeds the second, by this the second exceeds the third. And in this proportion it turns out that the interval of the greater terms is smaller and that of the smaller greater. The mean is geometric, whenever they [the terms] are such that as the first is to the second so the second is to the third. Of these [terms] the greater and the lesser make an equal interval. The mean is sub-contrary, which we call harmonic, whenever they [the terms] are such that, by which part of itself the first term exceeds the second, by this part of the third, the middle exceeds the third. It turns out that in this proportion the interval of the greater terms is greater and that of the lesser is less.[167]

A mathematical notation of the three proportions can be presented for the arithmetic mean as $c = \frac{a+b}{2}$, for the geometric mean as $c = \sqrt{ab}$ and for the harmonic mean as $c = \frac{2ab}{a+b}$. The theory of proportions is studied in Al Khwarizmi's *Keys of the Sciences* and in *Epistles of Brethren of Purity* from a Neo-Pythagorean perspective in Islamic culture.[168] The theoretical base of the understanding of proportions relies on Greek theory of means. Most of the literature concerning the three means in Islamic culture gives reference to the Neo-Pythagorean philosopher Nicomachus of Gerasa's *Introduction to Arithmetic.* He identified the most perfect proportion as possessing both arithmetic and harmonic proportions in the relation between the two extremes as in the case of 6:8=9:12. Iamblichus calls the most perfect proportion as the 'musical

[167] Archytas of Tarentum in Carl A. Huffman, *Archytas of Tarentum Pythagorean, Philosopher and Mathematician King*, (Cambridge: Cambridge University Press, 2005), 163.

[168] Al Khwarizmi "Keys of the Sciences" in *Introduction to History of Science,* Vol I., trans. George Sarton, (Baltimore: Williams and Wilkins, 1927), 659-660, Ihvan-ı Safa, *Ihvan-i Safa Risaleleri,* Vol. I, trans. Ali Avcu, ed. Abdullah Kahraman (İstanbul: Ayrıntı Yayınları, 2012), 161-173.

proportion' and adds that it has been discovered by Babylonians and used by many Pythagoreans including Plato in *Timaeus.*[169] Plato uses these proportions in explaining Craftsman's (Demiurge's) formation of world soul from portions of a mixture of both divisible and indivisible Sameness, Difference and Being in a harmonious way. [170]

Brethren of Purity deals with the three proportions presenting the structural analogy between heavenly spheres and musical intervals composed of arithmetic, geometric and harmonic means in their Epistles.[171] In this sense, the use of the three proportions in music and other crafts can be interpreted as imitation of heavenly spheres and order of cosmos. The following idea on the Epistle by Brethren of Purity also advocates this claim; "If one establishes the measure of time by the regular, harmonious and proportionate succession of motions and silences, the notes resulting will be comparable to the notes produced by the movements of the spheres and heavenly bodies and in concordance with them."[172]

Considering arithmetic, geometric and harmonic proportions used in the formation of the North Dome, the structure can be read through the Pythagorean understanding of music presented in Epistles of Brethren of Purity. Thus the notion which assumes a structural similarity between musical harmony and formation of heavenly spheres finds its architectural expression through the composition of North Dome. However this interpretation would be an argument reducing the philosophical attitude of the age to the Neo-Pythagorean and Platonic view.

According to Özdural's geometrical analysis, the structure of the dome involves the use of magnitudes which is identified with irrational numbers since Khayyam's

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[169] Iamblichus in Carl A. Huffman, *Archytas of Tarentum Pythagorean, Philosopher and Mathematician King*, 165-166.

[170]Donald Zeyl, *Plato's Timaeus.* February 17, 2012, http://plato.stanford.edu (Accessed September 12, 2012).

[171] "These sages have also said that between the relative sizes of these heavenly bodies there are various relationships of various orders arithmetical, geometrical, or musical, and similarly, such relationships also subsist between them and the body of the earth , some being noble and perfect, others less so this being a matter too long to explain." Brethren of Purity, *Epistles of the Brethren of Purity: On Music.* Translated by Owen Wright, (Oxford: Oxford University Press, 2010) 134.

[172] Amnon Shiloah, *The Epistle On Music Of The Ikhwan Al-Safa (Bagdad, 10th Century),* (Tel Aviv: Tel Aviv University, 1978), 35.

triangle has the property of involving irrational magnitudes in its very essence. Özdural also points that the ratios in Khayyam's triangle provide a tool suitable for architectural production.[173] He prefers to suggest investigating Islamic music theory in order to understand the use of irrational magnitudes in mathematical practice.[174] However, in order to grasp the essence of the subject I prefer to investigate the epistemological and ontological understanding of irrational magnitudes in Khayyam's mathematics and its relation with Platonic and Aristotelian philosophy of mathematics. Thus, philosophical foundations of Khayyam's approach will be available in order to get an idea of philosophical understanding behind proportional relations used in North Dome.

### 4.2.2. Khayyam's approach on irrational magnitudes

In architecture the use of approximations in irrational magnitudes is known since ancient Greek times. However, in this case we have a mathematician who considered the irrational magnitudes as numbers for the first time. Thus his ideas on the problem of incommensurability draws attention.

Studies on irrational numbers have started before Khayyam in Islamic mathematics. Euclid's *Elements Book* V, in which Euclid states the theory of proportions and ratios of general magnitudes, had been studied by Al Mahani and Al Nayrizi in the 9th century, and by Ibn al-Haytham in the 10th century.[175] Khayyam is the first mathematician who declared that any ratio of two magnitudes whether commensurable or incommensurable can be considered as number. The relation between arithmetical entities and geometrical magnitudes is explained considering the use of numerical ratios in music by Khayyam:

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[173] Özdural, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", 699-715.

[174] Ibid.

[175] See Jan P. Hogendijk, "Anthyphairetic Ratio Theory in Medieval lslamic Mathematics" in *From China to Paris: 2000 Years Transmission of Mathematical Ideas* edited by Yvonne Dold-Samplonius, (Stuttgart: Franz Steiner Verlag, 2002), pp. 187-202 for history of studies on Euclid's *Elements* Book V.

> The science of music is based upon combining of ratios. But the ratios used there are numerical, not geometrical. Decomposition of ratios in music is really a kind of combination. We leave it to your piercing intelligence to comprehend. We will mention a line about this idea on discussion of difficulties of books of music and science of numbers. [They say] "there is no need of geometry in it, and can be studied without geometry. This comes before geometry and there is no relation between them." But . . . the science of numbers and geometry are two sciences neither of which comes before the other.[176]

His account seems opposing to the Pythagorean and Platonic understanding of mathematics. According to Pythagorean philosophy, every phenomenon in nature can be expressed by numbers that are composed of simple fractions. Moreover, these numeric units are actually the first principle of the cosmos. However, the discovery of irrational numbers by Pythagoreans themselves posed a critical problem to their philosophy because incommensurable magnitudes cannot be expressed by simple fractions. Thus the correspondence between number and magnitude cannot be maintained with the discovery of irrational numbers.

Plato who follows Pythagoreans in his philosophy of mathematics considers the problem of incommensurability by attributing separate levels of existence to mathematical entities and by adopting a new method of classification of numbers. Plato makes an analogy between the problem of incommensurable magnitudes and the problem of knowledge, which seeks an answer for a definition of knowledge in the dialogue. Mathematician Theaetetus classifies numbers into two which are square and oblong numbers. This classification helps to separate irrational numbers from rational numbers. In that sense numbers which "can be formed by multiplying equal factors" are square numbers and numbers "which cannot be formed by multiplying equal factors" are oblong numbers.[177] This classification also presents a case for what is knowable in terms of number theory. Socrates encourages Theaeteus to find a single formula which embraces many kinds of knowledge like he did regarding rational and irrational

[176] Khayyam in Özdural, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", 699-715.

[177] Plato, *Theatetus, Sophist.* Translated by H. N. Fowler, (London: William Heinamann, 1921), 27.

numbers.[178] Theaetetus tries to form his argument on different claims such as sense perception, true judgment and true judgment with an account. However his attempts are inconclusive against Socrates' opposing arguments. But this does not bring a disappointment for the sake of knowledge theory. Instead, according to Brown, Plato reaches at a generalization that "one can find out that he does not know" as in the case of classification of mathematical entities since while square numbers denote to what is knowable, oblong numbers can only indicate approximate values.[179] Even so, Brown states that "one can proceed from a worse approximation to a better" and in that sense "'aporetic' inquiries may be profitable" following Socratic method of critical thinking.[180] Irrational numbers and incommensurable magnitudes have been declared as unknowable and excluded from epistemological study with Theaetetus' classification.

The philosophical position of Khayyam differs from the Platonic and Pythagorean perspective in his consideration of irrational numbers and incommensurable magnitudes. Aristotelian view point on incommensurability problem explains Khayyam's position on the subject. For Aristotle, some magnitudes like the quarter-tones and the diagonal of the square are expressed by measures which are more than one quantity.[181] Thus he implies that, unlike Plato and Pythagoreans, he is not concerned about the existence of irrational magnitudes. The basis of Aristotle's viewpoint on irrational magnitudes lies in his ontology of mathematical entities. According to Aristotle, the existences of mathematical entities are not distinct from sensible objects and they also do not belong to the sensible objects. Rather, they are reached by abstraction after sensible objects. Mathematical and geometrical entities are considered in relation to potentiality and actuality concepts.[182] Thus, incommensurable magnitudes

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[178] Ibid., 29.

[179] Malcolm S. Brown, "Theatetus: Knowledge as Continued Learning", *Journal of the History of Philosophy*, Vol. 7, No. 4,(1969), 362.

[180] Ibid.

[181] Aristotle, *"*Metaphisycs Book X"*,* in *The Internet Classics Archive*, translated by W. David Ross, 2009, http://classics.mit.edu/Aristotle/metaphysics.10.x.html, (Accessed September 16, 2012).

[182] "It is an activity also that geometrical constructions are discovered; for we find them by dividing. If the figures had been already divided, the constructions would have been obvious; but as it is they are present only potentially... Obviously, therefore, the potentially existing constructions are discovered by being brought to actuality; the reason is that the geometer's thinking is an actuality; so that the

locate themselves to the plurality of mathematical entities reached by abstraction. Although Aristotle does not give a specific explanation for the status of incommensurables we can identify them regarding his idea of infinite. Aristotle considers infinity as having potential existence. He claims that,

> Now, as we have seen, magnitude is not actually infinite. But by division it is infinite. (There is no difficulty in refuting the theory of indivisible lines.) The alternative then remains that the infinite has a potential existence.
>
> But the phrase 'potential existence' is ambiguous. When we speak of the potential existence of a statue we mean that there will be an actual statue. It is not so with the infinite. There will not be an actual infinite. The word 'is' has many senses, and we say that the infinite 'is' in the sense in which we say 'it is day' or 'it is the games', because one thing after another is always coming into existence. For of these things too the distinction between potential and actual existence holds. We say that there are Olympic games, both in the sense that they may occur and that they are actually occurring.[183]

The concept of infinity is explained with regard to the actualization of 'day' or 'the games' in the sense that "their parts come into existence successively one by one" as an attribute of certain sequences of individual things or individual events".[184]

Aristotle considers a different approach regarding the concept of actualization which is mental actualization. His perspective on mental actualization also applies to artificial products as building art in the sense that the form of a house that exists in the builders mind has an actual existence.[185] Accordingly, Hintikka claims that for Aristotle "conceivability implied actualizability" and "to conceive a form in one's mind was *ipso*

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potency proceeds from an actuality; and therefore it is by making constructions that people come to know them (though the single actuality is later in generation than the corresponding potency)" Aristotle, "Metaphysics Book IX", in The Internet Classics Archive, translated by W. David Ross, 2009, http://classics.mit.edu/Aristotle/metaphysics.9.ix.html, (Accessed September 16, 2012).

[183] Aristotle, "Physics Book III", in *The Internet Classics Archive*, translated by R. P. Hardie and R. K. Gaye, 2009, http://classics.mit.edu/Aristotle/physics.3.iii.html, (Accessed September 16, 2012).

[184] Jaakko Hintikka, "Aristotelian Infinity", *the Philosophical Review*, Vol. 75, No. 2. (Apr., 1966): 199.

[185] Ibid., 210.

*facto* to actualize it".[186] Thus the concept of infinity stands as potentially existent for knowledge.[187]

As for incommensurable magnitudes and irrational numbers, they share same characteristics with all geometric magnitudes and arithmetic entities.[188] As in the case of the concept of infinity, they are potentially existent and mentally actualizable entities, Aristotle also states that mathematical and geometrical entities are abstracted from sensible objects but do not exist in sensible objects either.[189]

Khayyam's philosophy of mathematics follows Aristotelian tradition. Khayyam considers the concept of quantity in respect to the definition held by Aristotle[190] in *Categories* that "Quantity is either discrete or continuous…Instances of discrete quantities are number and speech; of continuous, lines, surfaces, solids, and, besides

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[186] Ibid.

[187] "But the infinite does not exist potentially in the sense that it will ever actually have separate existence; it exists potentially only for knowledge. For the fact that the process of dividing never comes to an end ensures that this activity exists potentially, but not that the infinite exists separately." Aristotle, "Metaphysics Book IX", in The Internet Classics Archive, translated by W. David Ross, 2009, http://classics.mit.edu/Aristotle/metaphysics.9.ix.html, (Accessed September 16, 2012).

[188] Young Kee Cho observes that unlike Plato, Aristotle is not worried on the question of incommensurability which he finds his support in Aristotle's Metaphysics Book X, Part 1, "But the measure is not always one in number--sometimes there are several; e.g. the quarter-tones (not to the ear, but as determined by the ratios) are two, and the articulate sounds by which we measure are more than one, and the diagonal of the square and its side are measured by two quantities, and all spatial magnitudes reveal similar varieties of unit." *On Fictionalism in Aristotle's Philosophy of Mathematics*(PhD diss. University of Texas, 2009), 238.

[189] Aristotle states that "It is an activity also that geometrical constructions are discovered... therefore, the potentially existing constructions are discovered by being brought to actuality..." (Metaphysics Book X) which supports the mentally actualizability claim. His explanation that "As the mathematician investigates abstractions (for before beginning his investigation he strips of all the sensible qualities,... and leaves only the quantitative and continuous... and examines the relative positions of some and the attributes of these, and the commensurabilities and incommensurabilities of others, and the ratios of others; but yet we posit one and the same science of all these things--geometry)" (Metaphysics Book XI) He indicates that mathematical entities are reached by abstraction. The claim that mathematical objects does not exist in sensible objects is depicted as "It has, then, been sufficiently pointed out that the objects of mathematics are not substances in a higher degree than bodies are, and that they are not prior to sensibles in being, but only in definition, and that they cannot exist somewhere apart. But since it was not possible for them to exist in sensibles either, it is plain that they either do not exist at all or exist in a special sense and therefore do not 'exist' without qualification. For 'exist' has many senses." (Metaphysics Book XIII).

[190] Jeffrey A. Oaks, "Al-Khayyām's Scientific Revision of Algebra", *Suhayl International Journal for the History of the Exact and Natural Sciences in Islamic Civilizations*, Vol. 10, (2011): 59.

these, time and place".[191] Khayyam identifies number to share the same characteristic with magnitude under the concept of quantity with a difference that "magnitudes are not made up of indivisible parts, and there is no definite end to their division, as there is with number."[192]

Khayyam also takes the concept of number in an Aristotelian sense which defines mathematical entities as abstracted from sensible things but does not belong to sensibles;

> As to number, number is taken as abstracted in the intellect from material things; and it does not exist in sensible things, since number is a universal intelligible thing which does cannot exist except when individuated by material things.[193]

Oaks interprets Khayyam's idea of number as abstracted in the intellect but which does not belong to material objects in the sense that Khayyam is ambivalent about the concept of number. However, the parallel notions on the nature of number are apparent between Khayyam and Aristotle. And it may be the case that Khayyam does not imply ambivalence toward the concept of number but implies his adoption of Aristotelian philosophy on the nature of number.

According to Khayyam there are two types of proportionality. These are common proportionality and true proportionality. He calls common proportion to the Euclidian sense of proportion which is defined in *Elements Book V Definition 5* and *Definition 6*.[194] Khayyam argues that Euclid's definitions do not provide an exact

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[191] Aristotle, "Categories, Section I", in *The Internet Classics Archive*, translated by E. M. Edghill, 2009, http://classics.mit.edu/Aristotle/categories.1.1.html, (Accessed September 16, 2012).

[192] Omar Khayyam, "Commentary on the Difficulties of Certain Postulates of Euclid's Work", in *Omar Khayyam the Mathematician* by R. Rashed and B. Vahabzadeh, translated by R. Rashed and B. Vahabzadeh, 235. New York: Biblioteca Persical Press, 2000.

[193] Omar Khayyam, "Treatise on the division of a Quadrant of a Circle", in *Omar Khayyam the Mathematician* by R. Rashed and B. Vahabzadeh, translated by R. Rashed and B. Vahabzadeh,171. New York: Biblioteca Persical Press, 2000.

[194] Definition 5: "Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order."

understanding of proportion because it does not include incommensurables. He claims that he has the knowledge of true proportionality which also includes irrational ratios that are referred in his definition as continued fractions.[195]In this respect Khayyam agrees Euclid's definitions on proportionality but he states that "common proportionality is a necessary consequence of true proportionality".[196]

After defining the nature of true proportionality and its difference from Euclidian understanding of proportionality, namely common proportionality, considering the ratio between incommensurable magnitudes he claims that the ratio which is formed with irrationals shall be considered as number. He makes this by assuming the ratio of two magnitudes A to B as equivalent to the magnitude G which is incommensurable. Khayyam indicates his argument as in the following;

> And those who make calculations, I mean those who make measurements, often speak of one half of the unit, of one third thereof, and of other parts, although the unit be indivisible. But they do not mean by this a true absolute unit whereof true numbers are composed. On the contrary, they mean by this an assumed unit which, in

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Definition 6: "Let magnitudes which have the same ratio be called proportional." Eucllid, *The Thirteen Books of Euclid's Elements* II, trans. Thomas Heath (Cambridge: Universtiy Press, 1908), 114.

[195] "Then if it is not according to these three ways, but, that there be subtracted from the second all the multiples of the first till a remainder is left less than the first, and likewise there be subtracted from the fourth all the multiples of the third till a remainder is left less than the third, and that the number of multiples of the first contained in the second be equal to the number of multiples of the third contained in the fourth; and that there be subtracted from the first all the multiples of the remainder of the second till a remainder is left less than the remainder of the fourth till a remainder is left less than the remainder of the fourth, and that the number of multiples of the remainder of the second be equal to the number of multiples remainder of the fourth, and that likewise there be subtracted from the remainder of the second all the multiples of the remainder of the first, and there be subtracted from the remainder of the fourth all the multiples of the remainder of the third, and that their number be equal; and that likewise all the multiples of the remainders be continually subtracted one from the other as we have explained, and that the number of each remainder of the first and the second be equal to the number of the corresponding remainder of the third and the fourth ad infinitum; then, the ratio of the first to the second will inevitably be equal to the ratio of the third to the fourth. And this is true proportionality with respect to the geometrical type." (Khayyam, "Commentary on Euclid's Elements", 237). Khayyam applies antyphairetic ratio numbers, which is based on finding the greatest common measure, while considering irrational. In the case of incommensurable magnitudes as Khayyam explains the process of finding the greatest common measure continues forever. But, his argument is that common proportionality follows the rules of true proportionality which involves continuous fractions.

[196] Omar Khayyam, "Commentary on the Difficulties of Certain Postulates of Euclid's Work", 236.

> their opinion, is divisible. Then they act in whatsoever way they please in the management of the magnitudes in accordance with that divisible unit and in accordance with the numbers composed thereof; and they often speak of the root of five, of the root of ten, and of other things which they constantly do in the course of their discussions, and in their constructions and their measurements. But they only mean by this a 'five' composed of divisible units, as we have mentioned. Thus it must be known that this unit is that one which is divisible, and the magnitude G is to be considered as a number, as we have said, whatever magnitude it may be. And when we say, *We set the ratio of the unit to the magnitude G equal to the ratio of A to B*, we do not mean by this it is within our power to produce this notion in all the magnitudes, namely, that we produce what we say by a rule of the art. On the contrary, we mean by this that it is not impossible in the intellect. And our incapacity to produce that does not indicate that the thing in itself is impossible. So understand these notions.[197]

Khayyam handles the subject of philosophical aspect of irrationals by claiming that they are not "impossible in the intellect" and "our incapacity to produce them does not indicate that thing is impossible". These statements are also very close to Aristotelian conception of infinity as Aristotle suggests that infinity is potentially existent for knowledge.

Khayyam explains his theory on irrationals referring to the practitioners of mathematical arts. These must have included architects and craftsman. Although in none of his treatises he gives a hint for confirming Özdurals statement that Khayyam may be the architect of North Dome, he points to a tradition of applied mathematics used by craftsmen. This aspect of the issue helps us to understand the relation between theoretical and practical sense of mathematics. He emphasizes that various forms of numbers including irrationals are used by "those who make measurements". Accordingly, we are encountered with craftsmen applying the ignored principles of irrational numbers and magnitudes in their arts preceding Khayyam's declaration of irrationals as numbers themselves.

[197] Ibid., 253.

A further claim on the nature of the relation between practice and theory is expressed by 14th century historian Ibn Khaldun;

> In view of its origin, carpentry needs a good deal of .geometry of all kinds. It requires either a general or a specialized knowledge of proportion and measurement, in order to bring the forms (of things) from potentiality into actuality in the proper manner, and for the knowledge of proportions one must have recourse to the geometrician. Therefore, the leading Greek geometricians were all master carpenters. Euclid, the author of the Book of the Principles, on geometry, was a carpenter and was known as such. The same was the case with Apollonius, the author of the book on Conic Sections, and Menelaus, and others.[198]

Accordingly while we see necessity of the knowledge of theoretical geometry in crafts, the classical figures of the science of geometry are imagined with respect to their involvement in the practical realm. The important point in here is not the historical information which belongs to the Greek tradition but how it is reflected in the Arabic scientific tradition. The occupation of Euclid and Apollonius in crafts both reflects the application of geometric knowledge to practical area and the geometrician's learning from individual cases as highlighted by Khayyam.

Moreover, John Pennethorne's discussion on the relationship between architecture and science in Ancient Greece can also be held for the Medieval Islamic architecture especially concerning our case.[199]

> When we consider the state of the Greek astronomy at the time when both the arts and the geometry had attained their highest point, it appears natural to conclude…that the division of ancient art, at the head of which Plato has placed architecture, was the practical science of the

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[198] Ibn Khaldun, *The Muqaddimah : An Introduction To History*, vol. 2, trans. Franz Rosenthal, (New Jersey: Princeton University Press, 1967) p.322.

[199] Richard Padovan proposes that "Ideas first worked out in stone were only later translated into abstract thought and writing in the philosophers' study. Concepts born in the world of concrete things matured later in the pure abstract speculation." In arguing against Erwin Panowsky's idea that scholastic philosophy caused the emergence of gothic architecture, his interpretation of John Pennethorne that "architecture led the way for science" can also be evaluated in this case as the formative principle of Khayyam's step towards conceiving irrationals as numbers. Richard Padovan, *Proportion,*(London, New York: Spon Press, 2003)*,* 205-206.

> Greeks, and the one that chiefly excited them to the study and cultivation of the several branches of the mathematics; for it is now ascertained that all the branches of the Greek geometry were applied in the designing of the Greek works of architecture.[200]

A final note on Khayyam's approach which stems from practical uses of irrationals is his suggestion that;

> …whether the ratio between magnitudes includes numbers in its essence, or whether it is inseparable from number, or whether it is joined to number from outside its essence because of something else, or whether it is joined to number because of something inseparable from its essence without requiring an extrinsic judgment: this is a philosophical study to which geometrician must by no means devote himself.[201]

Khayyam is again emphasizing that practical mind should be dominant in the study of geometry and the philosophical questions on the existence of mathematical entities should be avoided. Thus, we encounter with an attitude which takes its background from crafts like architecture, and theorizes it for the use of scientific studies.

In summary, the investigation on the North Dome of the Isfahan Friday Mosque has changed our perception on the relation between theory and practice of geometry. The 'underdetermination' problem between the geometric analysis of Shroeder and Özdural is overcame with Özdural's use of precisely measured drawings in his anaylsis with respect to Schroeder's randomly superimposed golden ratio analysis. Özdural's analysis points to Omar Khayyam as the possible architect of the North Dome. According to his analysis the special triangle of Khayyam fits onto the section of the building with very small margin of error. I began with this assumption so as to discuss the use of geometry on the North Dome in theoretical and practical sense. Since the Khayyam triangle involves arithmetic, geometric and harmonic ratios in its organization, the existence of irrational magnitudes are inevitable. Accordingly, Khayyam's approach on the nature of incommensurable magnitudes gives us a hint for interpreting the nature of the relation

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[200] John Pennethorne in Richard Padovan, *Proportion,*(London, New York: Spon Press, 2003), pp.80-81.

[201] Khayyam, "Commentary on the Difficulties of Certain Postulates of Euclid's Work", 251.

between theoretical and practical geometry. Khayyam offers to see the units of incommensurables also as numbers referring to the tradition of craftsmen. In this sense the geometry applied on the North Dome is a launcher of Khayyam's theoretical geometry.

# CHAPTER 5

# CONCLUSION

This study is focused on the theory and practice of geometry in medieval architecture in the Middle East. The research aimed to identify the epistemological borders between theory and practice of geometry in the discipline of architecture. The primary sources in my research has been Al Kindi's, Al Farabi's and Al Ghazali's classification of sciences; Abu Wafa Al Buzjani's *Book on What is Necessary for Artisans in Geometrical Construction*; the anonymous treatise called *On Interlocks of Similar or Corresponding Figures* and Khayyam's *Commentary on the Difficulties of Certain Postulates of Euclid's Work.* In this investigation, I found that contrary to the claim that theoretical knowledge on geometry is superior to the practice of geometry, Khayyam approaches the issue in a different way in his study on incommensurable magnitudes.

Geometry in architecture reveals itself in ornamentation and formal/structural elements. Geometric ornaments have a distinctive place in medieval architecture of the Middle East and have been interpreted by scholars from various viewpoints. In this thesis, traditionalist, Sunni revivalist and classicist arguments are discussed, and discovering Grabar's reference to Escher, I noticed nonexistence of research on Islamic architecture and its ornament in relation to the theoretical framework of the 'play element' methodology.

Abu Wafa's manuscript can be regarded as an evidence of collaboration of craftsmen and mathematicians in the tradition of geometric ornament. Wafa's participation in a meeting between craftsmen and mathematicians, which Özdural calls *conversazioni,* can be read as a restatement of the hierarchical classification between theoretical and practical geometry since Wafa teaches how to construct the geometric figures in a rational way to the craftsmen in the meeting. The manuscript named *On Interlocks of Similar or Corresponding Figures* is considered to be a product of a similar meeting, but according to Özdural, the author of this is claimed to be a mathematician involved in the practice of ornament. Arithmetic, harmonic and geometric proportions are illustrated by appropriating Khayyam's triangle in the construction of a figure in the anonymous treatise named '*On Interlocks of Similar or Corresponding Figures*'. Özdural employs the Khayyam triangle to analyse the geometry of the North Dome of the Isfahan Friday Mosque. By using his analysis I discussed the philosophical aspects of arithmetic, harmonic and geometric proportions through Neo-Pythagorean and Platonic approaches. I also observed that Aristotle and Khayyam' notions on the nature of mathematical entities share many similarities. Khayyam, like Aristotle, considers mathematical entities as abstractions from material quantities. Thus, it is not a surprise that Khayyam refers to the practice of craftsmen in terms of his understanding of incommensurable magnitudes as numbers. This approach is a new step taken after Neo-Platonic and Pythagorean traditions. Khayyam's reference to the craftsmen who make measurements and calculations is a new consideration of the practical arts. According to Khayyam knowledge and practice of architecture belongs to the realm of practical geometry, and these aspects become subject to a new epistemological classification with him. Practice paved the path for theory in Khayyam's understanding of geometry.

One crucial point which needs to be elaborated in future researches is about the North Dome with regard to the underdetermination of golden ratio argument held by Schroeder and Khayyam's triangle argument held by Özdural. In spite of the fact that Özdural's study provides a precise analysis, the study can be reviewed considering the aim of the geometric analysis, which in architectural history has various uses. One of them serves to verify the 'universal aesthetic principle' which is formulized by Adolf Zeising's view of golden ratio. This is used in Schroeder's geometric analysis. On the other hand Özdural does not approach to the geometric analysis of the North Dome aesthetically; instead he uses geometric analysis in order to trace fingerprints of the architect of the building. His study on Khayyam's triangle and its use on the North Dome is like a connoisseur's search to find the artist of an art object. Özdural unearths proofs from the historic context of architecture and geometry.

A similar problem is also seen in the geometric studies on architectural history of Anatolian Seljuks. Especially the studies on Divriği Great Mosque stands out as an area awaiting further research. The investigations of Yoland Crowe[202] and Ömür Bakırer[203] tries to find a repeating unit which determines the design of the North Portal of the Divriği Great Mosque. Crowe begins her study with the assumption of a geometric design criteria and Bakırer attributes the graffiti she found on the walls of the mosque as a possible repeating unit for the design. However, although Bakırer is closer explanation of the modular system by resting upon a constructional evidence, the search for a modular system keeps going on. Maybe an evaluation of the historiographical methodology would be more useful in considering the geometric analysis method as an architectural history tool since it seems that any geometric figure actually would fit to any structure. So that the modernist motivation behind these trends of history writing would become visible.

My personal point of view considering Özdural's research is that it opens a new inquiry in the literature on proportion studies in architecture which involves Wittkower's and Viollet le Duc's views. It seems that Özdural proposes a new concept to analyse the medieval structures in addition to Viollet le Duc's.[204] This claim is supported when Özdural's geometric analysis on the Church of St. George of the Latins in Famagusta is taken into consideration.[205] Özdural's study highlights the use of arithmetic, geometric and harmonic proportions in the geometric analysis of the church. These proportions are the ones which he also uses to analyse the North Dome through Khayyam triangle since it involves all three of them. Thus, it seems that Özdural traces the arithmetic, geometric

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202 Yolanda Crowe, "Divrigi: Problems of Geography, History and Geometry," *The Art of Iran and Anatolia from the Eleventh until the Thirteenth Century*, ed. William Watson, Colloquia on Art and Archaeology in Asia 4 (London, 1974): 34-38

203 Ömür Bakırer, The Story of Three Graffiti, Muqarnas Vol. 16 (1999), pp. 42-69.

204 Viollet le Duc offers three triangles to analyse Ancient Greek, Roman, Renaissance and Gothic architecture. These triangles are 'equilateral triangle'; 'Plutarch triangle', which is a right triangle with its sides 3, 4, 5 and the 'Egyptian triangle' whose proportion between the height and base is 5/8. Çiler Buket Tosun, *Eugène Emmanuel Vıollet-Le-Duc (1814-1879) ve Etkileri*, unpublished Phd thesis, (Ankara: Hacettepe Universitesi, 2008), 43.

205 Alpay Özdural, "The Church of St. George of the Latins in Famagusta: A Case Study on Medieval Metrology and Design Techniques", *Ad Quadratum The Practical Application of Geometry in Medieval Architecture*, ed. Nancy Y. Wu (Aldershot: Ashgate, 2002) pp.217-243.

and harmonic proportions as a founding principle of medieval architecture. His studies also support Viollet le Duc's paradigm which proposes a rational system underlying medieval structures of both East and West.[206] Özdural illustrates evidences on the existence of musical proportions in medieval architecture in both of his investigations. These proportions were regarded characteristic of Renaissance architecture by Wittkower.[207] Hence, geometric analysis as an architectural history tool also works for identifying the continuity in the theory of proportions from medieval architecture to Renaissance architecture regardless of geographic and cultural differences in the case of Özdural.

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[206] Çiler Buket Tosun, *Eugène Emmanuel Vıollet-Le-Duc (1814-1879) Ve Etkileri*, unpublished Phd thesis, (Ankara: Hacettepe Universitesi, 2008), 55.

[207] Alpay Özdural, "The Church of St. George of the Latins in Famagusta: A Case Study on Medieval Metrology and Design Techniques", *Ad Quadratum The Practical Application of Geometry in Medieval Architecture*, ed. Nancy Y. Wu (Aldershot: Ashgate, 2002), 229.

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# APPENDIX A: TURKİSH SUMMARY

Bu tez Ortaçağ İslam mimarisinde geometrinin kullanımını, teorik yorumları ve pratik uygulamaları göz önünde bulundurarak ele alır. Bunu yaparken görülmüştür ki, Ortaçağ filozoflarının ve matematikçilerinin konu hakkındaki fikirleri, teori ve pratik arasındaki kavramsal sınırları sorgulatan bir literatür ortaya koyar. Pratik geometrinin bir alanı olarak mimari, gerek süslemelerdeki uygulamalarda gerekse mekânların organizasyonu açısından tartışmamızın odağında yer alır. Bilimlerin sınıflandırılmasında klasik görüşe göre teorik geometri bilgisinin pratik uygulamaya kıyasla daha üstün olduğu farz edilir. Ancak, klasik anlayış Hayyam'ın ölçülemeyen büyüklükler ve irrasyonel sayılar üzerine olan görüşleriyle irdelenmiştir.

Bu tez ayrıca, Ortadoğu'daki Ortaçağ mimarisindeki teorik ve pratik geometrinin epistemolojik ayrımına odaklanarak, mimarlıkta kullanılan geometrik süslemeler üzerine yapılan yorumları sorunsallaştırır. Bu çaba, geometrik süslemeleri felsefi ve tarihsel bir bağlamda ele alarak anlamaya çalışmaktan ileri gelir. Fakat yazınsal kanıtların eksikliğinden ötürü çoğu yorum görsel olanın altında yatan entelektüel motivasyonu ortaya çıkarmak için yeterli ispatı sunmaz. Yine de, geometrik süslemelerin İslami görsel kültürün merkezi bir parçası olduğu dikkate alınarak, bu yorumların mimarlıkta geometrinin kullanımına dair fikirlerimizi zenginleştirdiği görülür.

Konuyla ilgili olarak iki adet doküman bulunmaktadır. İlk doküman Ebu'l-Vefâ el-Bûzcânî'nin (940-998) 'Sanatkârların İhtiyaç Duyduğu Geometrik Çizimler', ikincisi ise 13. yüzyılda anonim bir kâtip tarafından hazırlanmış olan 'İçiçe Geçen Benzer veya Karşılıklı Şekiller'dir. Ebu'l-Vefâ'nın çalışması cetvel ve pergel aracılığıyla yapılan düzlemsel Öklid geometrisi çizimlerinden oluşur. Bu çalışmanın bölümleri (1) cetvel, pergel ve gönye; (2) cetvel ve pergel ile Öklitçi geometrinin temel yapıları ve ek olarak iki orta orantının konstrüksiyonu, açının üçe bölünmesi ve parabolik dev aynasının nokta tabanlı konstrüksiyonu, (3) bazıları tek pergel açıklığıyla yapılan düzgün çokgenlerin konstrüksiyonu; (4) çemberlerin içine figürler çizmek; (5) figürleri çemberle çevrelemek; (6) figürlerin içine çember çizmek; (7) figürleri birbiri içine çizmek; (8) üçgenlerin bölünmesi; (9) dörtgenlerin bölünmesi; (10) karelerden bir kare oluşturmak ve bir kareyi karelere bölmek için kes-yapıştır yöntemlerinin tümü ve (11) beş düzgün ve yarı düzgün çokgendir. Belgenin amacının sanatkârlara geometrik figürleri doğru bir şekilde kurgulamayı öğretmek olduğu söylenir ve anlaşılsın diye figürlerin geometrik kanıtlamalarından uzak durulmuştur.[208] Belgede ele alınan figürlerden bir tanesi İran'daki anıtsal binaların duvarlarında yaygın bir şekilde süsleme olarak kullanılmıştır. Diğer belgemiz ise Özdural tarafından süsleme sanatlarına dâhil olan bir matematikçinin alelacele aldığı notlar olarak değerlendirilir.[209] Bu iki belgedeki ilişki, ortaya koydukları figürlerdeki biçimsel benzerlikten doğar. İkinci belgede Hayyam'ın oluşturduğu özel bir üçgen gözümüze çarpar. Bu üçgen Özdural tarafından İsfahan Ulu Caminin Kuzey Kubbesinin geometrik analizinde kullanılır.[210] Mimaride kullanılan geometrinin teorik ve pratik ayrımına dair Ebu'l-Vefâ ve Hayyam konuyu farklı şekillerde ele alırlar. Sanatkârlar Ebu'l-Vefâ'nın sorduğu bir problemi çözememiştir ve Ebu'l-Vefâ da sanatkârlara problemin nasıl çözüleceğini ispatlanabilir bir yolla göstermiştir. Ebu'l-Vefâ'nın metodunun üstünlüğü Aristoteles'in teorik ve pratik geometri ayrımının yeniden ifade edilmesi şeklinde okunabilir. Diğer bir taraftan, Hayyam ise pratik geometriyi kendisinin irrasyonel büyüklükler üzerine olan teorik yaklaşımının kaynağı olarak görür. Bu ise bilimlerin sınıflandırılmasındaki klasik anlayışın dışında kalan bir fikir olarak karşımıza çıkar.

Bu tezin ikinci bölümü 8.-11. yüzyıllarda Bağdat'ta vuku bulmuş olan çeviri hareketini ele alır. Antik Yunan geleneğinden asimile edilmiş olan teorik geometrinin tarihçesi verilip 754-775 yılları arasında saltanatı süren Abbasi Halifesi El-Mansûr'un ve 813-883 yılları arasında hükümdarlık yapan Abbasi Halifesi El-Memun'un çeviri hareketini destekleyici rolleri, hareketin altında yatan motivasyonları incelemek için ele alınmıştır. Yunan kaynaklarının çevirisinde Harran okulunun önemi de ayrıca tartışılmıştır. Sonrasında, coğrafyada süregelen pratik geometri geleneği, ölçümcüler ve hesaplayıcıların kullandığı matematik üzerine olan kaynaklar açıklanmıştır. İkinci

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[208] Jan P. Hogendijk, "Mathematics and Geometric Ornamentation in the Medieval Islamic World", *European Mathematical Society*, Newsletter no. 86, December (2012), pp. 37-43.

[209] Alpay Özdural, "On Interlocking Similar or Corresponding Figures and Ornamental Patterns of Cubic Equations", *Muqarnas*, Vol. 13 (1996), pp. 191-211.

[210] Alpay Özdural, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", 699-715.

bölümde son olarak El- Kindî, El-Farabi ve El-Gazali'nin çoğunlukla Aristoteles'ten devşirdiği bilimlerin sınıflandırılması konusundaki fikirlerinin bir karşılaştırması yapılmıştır. Her ne kadar, Aristoteles'in ilimleri sınıflandırma yöntemi bahsedilen filozoflara kaynak olmuşsa da matematiksel nesnelerin Platon'un epistemolojisindeki yeri de El-Kindi ve El- Farabi'nin konuyu ele alışında etkili olmuştur. Aristoteles'in bilimleri sınıflandırması üçe ayrılır. Bunlar teorik, pratik ve üretken bilimlerdir. Teorik bilimler fizik, matematik ve metafizik olarak üçe ayrılır. Pratik bilimler kendi içinde ahlak ve siyaset olarak ikiye ayrılır. Üretken bilimler ise zanaatlar ve retorik bilimlerini içerir. Matematik ise kendi içinde aritmetik, geometri, astronomi ve müzik olarak dörde ayrılır. Matematiksel nesneler Aristoteles'e göre maddesel olandan ayrı değildir fakat ayrıymış gibi düşünülür.[211] Platon'un epistemolojisinde ise matematiksel bilgi, nesneler âleminin sanıları ile idealar âleminin bilgisinin arasında yer alır ve Platon matematik bilgisini aklın düşünce düzeyi ile ilişkilendirir.

El- Kindî ve El-Farabi'nin teorik geometriyi pratik geometri bilgisine göre üstün bir pozisyonda tuttukları ve El-Gazali'nin bilimlerin sınıflandırılmasındaki temel endişesinin bu dünyaya dair ve öte dünyaya dair olan bilimleri ayırmak üzerine olduğu görülmüştür. El-Gazali geometrik bilgiyi bu dünyaya dair bir bilgi olarak ele alırken, El-Kindî ve El-Farabi ise matematiksel nesnelerin metafizik ve maddi dünya arasında bulunduğunu öne sürer. Paylaştıkları bu fikir Platon'un matematiksel nesneleri ve matematiksel bilgiyi ele alışını hatırlatır.

Tezin üçüncü bölümü geometrik süslemeler üzerine olan yorumları gelenenekçi düşünce, Sünni uyanış düşüncesi ve klasikçi düşünce çerçevesinde ele alır. Gelenekçi düşünce, yerel farklılıkları yok sayarak geometrik süslemeyi sadece ilahi olanın dışavurumu olarak tartıştığından tarih dışı bir yorum getirir. Ismail Al-Faruqi, Keith Critchlow, Issam Al Said, Seyyid Hossain Nasr, Nader Ardalan, Titus Burckhardt ve Samer Akkach'ın görüşleri bu kategori içerisinde değerlendirilebilir. Gelenekçi yaklaşım konusunda Khaghani'nin eleştirisi dikkate değerdir. Khaghani'ye göre bu yaklaşımın tarih dışı olmasının sebebi, kendi içinde farklılar gösteren yapıların hepsinin tek bir kavram ile açıklanması ve süslemeleri ebedi bir ilahi mesajın somut bir göstergesi olarak

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[211] Aristotle in Thomas Heath, *Mathematics in Aristotle*, (Oxford: Oxford University Press, 1970), 11.

ele alınmasıdır.[212] Bu yaklaşım Khaghani tarafından self-oryantalist bir tutum olarak değerlendirilir.[213]

Sünni uyanış düşüncesi Gülru Necipoğlu ve Yasser Tabbaa tarafından dile getirilir. Bu fikre göre geometrik süsleme 10.-11. yüzyıllarda Bağdat'ın dini ve politik atmosferiyle ilişkilenerek şekillenmiştir. Bu bakımdan, karmaşık geometrik desenler Sünni teoloji ve ideolojinin sembolü olarak değerlendirilir. Bu dönemde hâkim olan iki farklı anlayış Kuran'ın kaynağını ve evrenin kozmolojik temellerini iki farklı şekilde yorumlar. Mutezile mezhebine göre Kuran'ın Allah gibi ebedi ve ezeli bir mevcudiyeti yoktur ve sonradan yaratılmıştır; fenomenler dünyasının ise atomların tesadüfi düzenleriyle oluştuğunu düşünürler. Eş'ariyye mezhebi ise Kuran'ın yaratılmamış ezeli bir niteliği olduğunu ve evrenin daimi bir yaratımın sonucu olduğunu öne sürer. Bağdat 945-1055 seneleri arasında Abbasi Halifeliğinin dini prensiplerine ters düşen bir yaklaşıma sahip olan Şii mezhebine bağlı Büveyhîler'in kontrolü altındaydı.[214] 991-1031 yılları arasında hüküm süren Abbasi Halifesi Kadir Mutezile ve Şii öğretileri yasaklayarak Bağdat'ta Sünni mezhebini hâkim kılar.[215] Necipoğlu, 1055 senesinde Büyük Selçukluların Bağdat'ı ele geçirmesiyle başladığı öne sürülen Sünni uyanış ile karmaşık geometrik desenlerden oluşan girih süslemenin ortaya çıktığını iddia eder.[216] Necipoğlu, bu bağlamda girih geometrik süslemenin Fatımiler ve İspanyol Emevileri tarafından Abbasilerin politik bir sembolü olarak algılandığı belirtir.[217]

Necipoğlu geometrik süslemeye dair öne sürdüğü felsefi yaklaşımı ise genel olarak Gazali ve İbn Haldun'un geometri üzerine olan görüşleriyle ortaya koyar. Ayrıca İhvan-ı Safa, Abdülkahir-i Cürcani, İbn-i Heysem, İbn-i Sina ve İbn-i Raşik'in fikirlerinden yararlanarak kitabının "Geometry and Aesthetic Theory" bölümünde

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[212] Saeid Khaghani, *Islamic Architecture in Iran Poststructural theory and Architectural History of Iranian Mosques*, (London, New York: I.B. Tauris & Co Ltd., 2011), pp. 40-41.

[213] Ibid.

[214] Gülru Necipoğlu, *Topkapı Scroll,* (Los Angeles: Getty Publications, 1996), 96

[215] Ibid., 96.

[216] Ibid., 97.

[217] Ibid., 101.

analizlerde bulunur. Fakat ele aldığı bu fikirler, öne sürdüğü Sünni uyanış düşüncesiyle geometrik süslemenin ilişkisi çerçevesinin dışında kaldığı için bu felsefelerin "gelişmekte olan geometrik soyutlama üslubuna zemin hazırladığını" düşünür.[218] Necipoğlu'nun çalışması şu ana kadar İslam mimarisindeki geometrik süslemeyle ilgili en kapsamlı kaynağı sunuyor olsa da bilim tarihçisi George Saliba'nın, Necipoğlu'nun fikirlerine olan eleştirisi dikkat çekicidir. Saliba'ya göre Necipoğlu gelenekçi yaklaşımın oryantalist fikirlerinden uzak durmaya çalışırken kendini Sunni uyanış döneminde akılcı düşüncenin sürdüğünü ispatlamaya çalışırken bulur ve bu durum gene oryantalist bir tutumun göstergesi olarak değerlendirilir. [219] Yasser Tabbaa ise Sünni uyanış döneminin sanatta ve mimaride olduğu varsayılan etkisini göstermeye çalışan sanat ve mimari alanından birçok örnek verir. Tabbaa'ya göre İbn-i Mukla ve İbnü 'l-Bevvāb'ın hat sanatında önayak olduğu geometrik standardizasyon Abbasi Halifesi Kadir'in dogmacı yaklaşımıyla ilişkilendirir.[220] Ayrıca Tabbaa, mimarideki geometrik süslemenin, Fatımilerin hâkimiyeti altındaki Mısır'da ve Eyyubilerin coğrafyasındaki yapılarda bulunmayışının sebebini ise Sünni uyanış hareketindeki kurumlar ve sembollere karşı olan tutumlarından kaynaklandığını düşünür.[221]

Terry Allen ise klasikçi bir yaklaşımla Ortaçağ İslami süslemeyi Geç Antik Dönem süslemesinin bir uzantısı olarak görür.[222] Matematikçi ve geometricilerin işbirliği hakkındaki fikirleri ve geometrik süslemelerin dönemin felsefelerine başvurularak yorumlanmasına olan itirazları bu bölümde incelenmiştir.[223] Allen'a göre eldeki yazılı

[218] Ibid., 192.

[219] George Saliba, "Artisans and Mathematicians in Medieval Islam the Topkapi Scroll: Geometry and Ornament in Islamic Architecture by Gülru Necipoğlu Review", *Journal of the American Oriental Society*, Vol. 119, No. 4 (Oct. - Dec., 1999). 644-645.

[220] Yasser Tabbaa, *The Transformation of Islamic Art during the Sunni Revival*, (Seattle, London: University of Washington Press, 2001), 50.

[221] Ibid., 162.

[222] Barbara Genevaz, "Review of the book Five Essays on Islamic Art by Terry Allen", *Mimar Book Reviews in Mimar: Architecture in Development*, Vol: 8 edited by Hasan-Uddin Khan, (Singapore: Concept Media Ltd., 1983), pp.67-68.

[223] Terry Allen, *Islamic Art and the Argument from Academic Geometry,* (California: Solipsist Press, 2004), http://www.sonic.net/~tallen/palmtree/academicgeometry.htm (accessed 22.01.2015).

kaynaklar geometrik süslemeyi dönemin felsefi akımlarıyla ilişkilendirmek için yeterli kanıt sunmaz. Carol Bier ve Oleg Grabar'ın fikirleri ise benim sınıflandırmamda ayrı bir yerde durur. Bier, Karahan İkiz Kümbetlerindeki süslemeleri ele aldığı çalışmasında yapılardaki kitabelerin içeriğini süslemelerin yorumlanmasında bir araç olarak kullanır. Böylece, geometrik süslemeleri Kuran'daki ayetlerin görsel bir somutluk kazandığı bir mecra olarak görür. Bier, bu ilişkiyi benzerlik anlamına gelen *emsal* kavramıyla kümbetlerdeki süslemeler arasında kurduğu analoji ile açıklar.[224] Bu yorum yerel ve tekil bir örnekten hareket ederek geometrik süslemeleri ele almasıyla gelenekçi düşünceden ayrılır fakat farz ettiği ilahi dışavurum ile gelenekçi düşünce ile benzer sonuçlara varır. Grabar ise geometrik süslemeyi her zaman her yerde görülebilen bir olgu olarak düşünür ve Elhamra'daki İki Kız Kardeş Salonundaki gibi süslemelerin anlamına doğrudan değinen yazıtlar yok ise geometrik süslemelere içkin bir anlam bulunmadığını iddia eder.[225] Grabar'ın ayrıca konuyla ilgili olarak işaret ettiği Escher'e ait fikirler ise geometrik süslemeyi yorumlamak açısından yeni ve alışılmışın dışında bir yaklaşım getirir. Escher'e göre geometrik süslemeler "düzlemi bölme oyunu"nun bir parçasıdır. [226] Kültür ve sanattaki oyun elemanın incelenmesine dair Huizinga'nın tartışmaları konumuza yeni bir yaklaşım getirebilme potansiyeline sahiptir.[227]

Üçüncü bölümün ikinci kısmında ise Ebu'l-Vefâ'nın 'Sanatkârların İhtiyaç Duyduğu Geometrik Çizimler' adlı çalışması aracılığıyla Özdural'ın ve Hogendijk'in sanatkârlar ve matematikçilerin işbirliği üzerine olan fikirlerini tartıştım. Bu konuda Özdural Ebu'l-Vefâ'nın belirttiği toplantıların, matematikçilerin geometrik süslemelerin

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[224] Carol Bier, "Geometric Patterns and the Interpretation of Meaning: Two Monuments in Iran*", Bridges: Mathematical Connections in Art Music and Science*, ed. Reza Sarhangi, (Kansas: Bridges Conference, 2002), 74.

[225] Oleg Grabar, *The Mediation of Ornament*, (Princeton: Princeton University Press, 1989), pp.145-153.

[226] M. C. Escher in H.S.M.Coexeter, "Coloured Symmetry", *M.C. Escher, Art And Science : Proceedings Of The International Congress On M.C. Escher, Rome, Italy, 26-28 March 1985*, (Amsterdam ; North-Holland: Elsevier Science Pub. Co., 1986), 16.

[227] Johann Huizinga, *Homo Ludens; A Study Of The Play-Element İn Culture*, (Boston: Routledge&Kegan Paul, 1980) pp.166-168.

tasarımında rol oynadığını gösterdiğini iddia etse de[228] Hogendijk, matematikçiler ile sanatkârların çalışmasındaki metodolojik farklardan ötürü bu işbirliğini varsayamayacağımızı ileri sürer.[229]

Dördüncü bölüm İsfahan Ulu Caminin Kuzey Kubbesinin geometrik analizine odaklanır. Bu konuda Schroeder'in analizi kubbenin düzenleyici prensibi olarak altın orana işaret ederken, Özdural aritmetik, geometrik ve armonik oranları barındıran Hayyam üçgenine işaret eder.[230] Bahsedilen oranlar İslam kültüründe 10. yüzyılda yaşamış olan ansiklopedist Hârizmî'nin *Mafātī□ al-□ulūm* ve İhvan-ı Safa Risalelerinde yeni Pisagorcu bir yaklaşımla işlenmiştir. İhvan-ı Safa Risalelerinde aritmetik, geometrik ve armonik oranlar, gökkürenin yapısıyla kurulan analoji ile aktarılmıştır.[231] Bu bağlamda, müzikte ve diğer sanatlarda bu oranların kullanılması gökküreye ve evrendeki düzene öykünmek şeklinde değerlendirilebilir. Kuzey Kubbesinin de bu oranları barındırdığı göz önüne alınrsa, yapının gökküredeki armoni ve düzenin bir ifadesi olduğu söylenebilir. Fakat bu yorum bahsettiğimiz geometrik oranınların sanatta ve mimaride kullanımını sadece Platoncu ve yeni Pisagorcu bir bağlamdan ele alır.

Özdural'ın kubbenin kesitindeki oranları Hayyam üçgeniyle tanımlaması, kubbenin mimarının Ömer Hayyam olabileceği fikrini doğurur.[232] Ben bu iddianın doğruluğunu araştırmaktan ziyade, Hayyam'ın teorik ve pratik geometri ve matematiksel nesnelerin doğasına dair olan fikirlerini ele alarak Kuzey Kubbesinde kullanılan geometrinin teori ve pratik ayrımında yeni bir anlayış sunduğunu ileri sürdüm. Bu iddia,

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[228] Alpay Özdural, "Omar Khayyam, Mathematicians and *Conversazioni* with Artisans", *Journal of Society of Architectural Historians*, Vol. 54:1, (1995), 54-71.

[229] Jan P. Hogendijk, "Mathematics and Geometric Ornamentation in the Medieval Islamic World", *European Mathematical Society*, Newsletter no. 86, December (2012), pp. 37-43.

[230] Alpay Özdural, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", Technology and Culture 39.4 (1998), 699-715.

[231] Brethren of Purity, *Epistles of the Brethren of Purity: On Music.* Translated by Owen Wright, (Oxford: Oxford University Press, 2010) 134.

[232] Özdural, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", 699-715.

Özdural'ın analizi çerçevesinde, ölçülemeyen büyüklüklerin kubbenin tasarımında kullanılmasıyla anlaşılabilir.

Pisagorcu evren anlayışında fenomenlerin basit kesirlerle ifade edilebileceği düşünüldüğü için irrasyonel büyüklükler çok temel bir sorun teşkil etmiştir. Pisagorcu felsefeden etkilenen Platon ise bu soruna Theatetus diyaloğunda değinir ve bilinebilir olanın eşit elemanların çarpımıyla oluşan tam kare sayılar olduğuna dikkat çeker; eşit elemanların çarpımıyla oluşturulamayan pronik sayılar ise bilinemez olarak düşünülür. Bu bağlamda irrasyonel büyüklükler bilinemez olarak sınıflandırılmış ve Platon'un bu epistemolojik çalışmasında ayrı tutulmuştur.

Hayyam ise mevzuyu Pisagorcu ve Platoncu bir bağlamın dışında ele alır. Aristoteles'in ölçülemez büyüklükler üzerine olan fikirleri Hayyam'ın konuyla ilgili yaklaşımına paraleldir. Aristoteles'e göre matematiksel nesneler maddesel olandan ayrı değildir fakat ayrıymış gibi düşünülür.[233] Yani matematiksel nesneler maddi olandan soyutlama yoluyla elde edilir. Aristoteles'in ölçülemez büyüklükler ile ilgili yek bir teorisi bulunmasa da filozofun sonsuzluk ile ilgili görüşleri, irrasyonel büyüklüklerin aktüalite ve potansiyalite ile ilişkilerini ortaya koyar niteliktedir. Aristoteles sonsuzluk kavramını potansiyel olarak var olan ve zihinsel olarak aktüelleştirilebilecek bir niteliği olduğunu söyler.[234] Bu suretle, rasyonel sayılarda olduğu gibi irrasyonel sayıların da, aynen sonsuzluk kavramında vurgu yapıldığı üzere, potansiyel olarak var olan ve zihinsel olarak aktüelleştirilebilecek elemanlar olduğunu düşünebiliriz.

Hayyam matematik felsefesinde Aristotelesçi geleneği izleyerek sayıların maddi olandan soyutlanarak elde edildiğini fakat maddi olanda bulunmadığını belirtir. [235] Oaks

[233] Aristotle in Thomas Heath, *Mathematics in Aristotle*, (Oxford: Oxford University Press, 1970), 11.

[234] "But the infinite does not exist potentially in the sense that it will ever actually have separate existence; it exists potentially only for knowledge. For the fact that the process of dividing never comes to an end ensures that this activity exists potentially, but not that the infinite exists separately." Aristotle, "Metaphysics Book IX", in The Internet Classics Archive, translated by W. David Ross, 2009, http://classics.mit.edu/Aristotle/metaphysics.9.ix.html, (Accessed September 16, 2012).

[235] Omar Khayyam, "Treatise on the division of a Quadrant of a Circle", in *Omar Khayyam the Mathematician* by R. Rashed and B. Vahabzadeh, translated by R. Rashed and B. Vahabzadeh,171. New York: Biblioteca Persical Press, 2000.

Hayyam'ın sayılar üzerine olan bu fikrini çelişkili bulduğunu ifade etse de Hayyam'ın bu beyanı Aristotelesçi bir matematik anlayışını benimsediğini ortaya koyar.[236]

Hayyam irrasyonel büyüklüklerle ilgili şunu ifade etmiştir;

> Ayrıca, hesap yapanlar (ölçüm yapanlar), birim bölünemez olsa bile genelde birimin yarısından, üçte birinden ve diğer parçalardan bahseder. Ancak, bununla, gerçek sayıların oluşturulduğu gerçek bir mutlak birimi kastetmezler. Aksine, bununla, bölünebilir olduğunu düşündükleri, varsayılan bir birimi kastederler. Bunun üzerine, söz konusu bölünebilir birime ve oluşturulan sayılara uygun olarak, büyüklükleri istedikleri gibi kullanırlar ve tartışmaları boyunca, çizimlerinde ve ölçümlerinde sürekli olarak kök beş, kök on ve yaptıkları diğer şeylerden bahsederler. Fakat bahsettiğimiz gibi bununla bölünebilir birimlerden oluşan bir 'beş'i kastederler. Dolayısıyla, bu birimin bölünebilir birim olduğu bilinmeli ve belirttiğimiz gibi, hangi büyüklük olursa olsun, G büyüklüğü sayı olarak düşünülmelidir. Bununla birlikte *birimin oranını A'nın B'ye oranına eşit olan G büyüklüğüne ayarlarız* derken, tüm büyüklüklerde bu kavramı üretmenin gücümüz dâhilinde olduğunu, yani sanatın kuralı ile söylediklerimizi ürettiğimizi kastetmiyoruz. Aksine, bunun idrakımızda imkânsız olmadığını söylüyoruz. Bunu üretmekteki yetersizliğimiz, konunun kendi içinde imkânsız olduğunu göstermez. Bu yüzden bu kavramları anlamanız gerekir.[237]

Hayyam'ın irrasyonel büyüklüklere dair olan fikirlerinin ilham kaynağının "hesap ve ölçüm yapanların" yöntemleri olduğu görülür. Bunlar elbette ki mimarları ve zanaatkârları kapsamaktaydı. Hayyam'ın elimizdeki yazılarından hiçbirinde Özdural'ın öne sürdüğü gibi Kuzey Kubbesinin mimarı olduğuna dair bir ifade yoktur. Fakat sanatkârların kullandığı matematik ve geometri uygulamalarına dikkat çekerek, klasik teori ve pratik ayrımından farklı bir anlayış ortaya koyar. Bu durum, matematiksel olarak da günümüzde kullanılan ismiyle reel sayılar kümesini oluşturan sayıları tanıyan

[236] Jeffrey A. Oaks, "Al-Khayyām's Scientific Revision of Algebra", *Suhayl International Journal for the History of the Exact and Natural Sciences in Islamic Civilizations*, Vol. 10, (2011): 59.

[237] Translated by the author from Omar Khayyam, "Commentary on the Difficulties of Certain Postulates of Euclid's Work", in *Omar Khayyam the Mathematician* by R. Rashed and B. Vahabzadeh, translated by R. Rashed and B. Vahabzadeh, (New York: Biblioteca Persical Press, 2000), p. 253.

ilk adımdır ve bu tez Hayyam'ın tasavvur ettiği şekliyle geometriyi pratik alandan öğrenmeyi vurgular.

14. yüzyılda yaşamış olan İbn-i Haldun'un Öklid'i ve Pergeli Apollonius'u marangoz olarak tahayyül etmesi de geometrinin teorik ve pratik ayrımı konusunda ayrıca dikkat çekicidir.[238] Öklid'in ve Pergeli Appollonius'un marangozlukla uğraşıyor olduğunun düşünülmesi teorik geometri bilgisinin pratik uygulamasına işaret ederken diğer bir yandan da kanımca Hayyam'ın işaret ettiği şekliyle matematik ve geometriyi pratik olandan öğrenmeyi ima eder.

Bu konu hakkında ileriki çalışmalarda göz önünde bulundurulmasının faydalı olacağını düşündüğüm iki yaklaşım var. İlki Oleg Grabar'ın, Escher'in geometrik süslemelerle ilgili değerlendirmesini ele almasıyla ortaya çıkan "düzlemi bölme oyunu" ifadesinden hareketle geometrik süslemeyi İslam kültüründeki oyun elemanın bir unsuru olarak tartışmak olabilir. İkinci ise tarihi yapılardaki oransal ilişkileri ve modülasyon biçimlerini çalışırken benimsenen metotların eleştirel bir yaklaşımla değerlendirilmesidir. Bu tezde İsfahan Ulu Caminin Kuzey Kubbesinin geometrik analizinde Schroeder'in altın oran yaklaşımından ve Özdural'ın Hayyam üçgeni yaklaşımından bahsedilmiştir. Bu iki analizden hangisinin gerçeğe tekabül ettiğine dair olan belirsizlik Özdural'ın verdiği oranları çalışırken kullandığı kesit rölevesinin Schroeder'inkine kıyasla çok daha detaylı bir kesit olmasıyla ve Özdural'ın işaret ettiği her oransal ilişkide gösterdiği hata payları ile çözüme kavuşur. Fakat gene de kubbenin tasarımında Hayyam üçgeninin mi yoksa sadece aritmetik, geometrik ve armonik oranların mı temel tasarım prensibi olarak rol aldığı muğlak kalır.

Mimarlık tarihi yazımında geometrik analizi bir yöntem olarak benimsemiş Viollet le Duc'un doğuda ve batıdaki Ortaçağ yapılarında rasyonel bir sistemin bulunduğu fikri Özdural tarafından bahsettiğimiz örnekte ve Famagusta'daki St. George Kilise'sinin geometrik analizini sunduğu makalesinde desteklenmektedir. [239] Özdural ayrıca bu makalesinde diğer bir mimarlık tarihçisi Rudolf Wittkower'ın Rönesans mimarisine

[238] Ibn Khaldun, *The Muqaddimah : An Introduction To History*, vol. 2, trans. Franz Rosenthal, (New Jersey: Princeton University Press, 1967) p.322.
[239] Alpay Özdural, "The Church of St. George of the Latins in Famagusta: A Case Study on Medieval Metrology and Design Techniques", *Ad Quadratum The Practical Application of Geometry in Medieval Architecture*, ed. Nancy Y. Wu (Aldershot: Ashgate, 2002) pp.217-243.

atfettiği oransal ilişkilerin Ortaçağ mimarisinde de bulunduğuna işaret eder. Bu bakımdan Özdural geometrik analizi bir mimarlık tarihi yazımı aracı olarak Ortaçağ mimarisinde ve Rönesans mimarisinde kullanılan oranlar teorisinin kültürel ve coğrafi bir fark gözetmeksizin süreklilik arz ettiğini göstermek için kullanır. Tüm bu bağlamlar çerçevesine geometrik analizin mimarlık tarihi yazımında ne gibi sonuçlar ortaya koymak için kullanıldığı ileriki çalışmalarda göz önünde bulundurulabilir.

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**<u>YAZARIN</u>**

Soyadı : Özden
Adı : Deniz
Bölümü : Architectural History/Mimarlık Tarihi

**<u>TEZİN ADI</u>** (İngilizce) : THEORY AND PRACTICE OF GEOMETRY IN MEDIEVAL ARCHITECTURE IN THE MIDDLE EAST (10th-14th CENTURIES)

**<u>TEZİN TÜRÜ</u>** : Yüksek Lisans Doktora

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