MULTI-CHANNEL RETAILING WITH PRODUCT DIFFERENTIATION

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ABSTRACT

MULTI-CHANNEL RETAILING WITH PRODUCT DIFFERENTIATION

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In this study, we analyze a monopolist retailer's product differentiation problem in a multi-channel environment. We investigate the type of conditions that would motivate retailer to open an outlet branch, to open an online channel, and to even potentially open an online channel for the outlet branch, and how these decisions interact with each other. We use quality and price as the primary drivers in the outlet business decision in a vertical differentiation model. In the outlet business decision, specifically, we investigate the quality and price decision of the retailer for his outlet branch and whether he will be better off in terms of total profit. For the online channel, we determine the online service quality and other factors that affect the end-consumer's utility. Online service quality may involve all customer services provided by the online store, the convenience of return process, and promised delivery time windows as well as shipping charges.

We find that the retailer's decision hinges on the market expansion versus market/margin cannibalization. We show that even a direct channel for the outlet store may be preferable for the retailer, depending on the market characteristics.

ÖZ

ÜRÜN FARKLILAŞTIRMA İLE ÇOK KANALLI PERAKENDE YÖNETİMİ

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Bu çalışmada tekelci bir perakandecinin ürün farklılaştırma problemini çok kanallı satış yapılabilen bir ortamda inceliyoruz. Perakendeciyi outlet açmaya, internet gibi direkt bir kanal açmaya hatta bu kanal üzerinden üzerinden outlet ürünü satışı yapmaya teşvik eden ortamları ve bunların birbiri ile etkileşimlerini araştırıyoruz. Outlet zincirle normal mağazalar arasındaki temel fark fiyat ve kalite olarak ele alınmıştır. Bu aşamada, ürünün fiyat ve kalite açısından konumlandırılmasını, bununla birlikte perakendecinin karındaki değişikliği inceliyoruz. İnternet kanalı için internet hizmet kalitesini ve müşteri memnuniyetini etkileyecek faktörleri inceliyoruz. İnternet hizmet kalitesi internet üzerinden sağlanan müşteri hizmetleri, iade, öngörülen sürede teslim ve teslimat ücretlendirmesini de kapsamaktadır.

Perakendecinin kararının pazar payı artışı ve diğer kanallardaki pazar ve kar marjı kaybı arasındaki ödünleşmeye dayandığını bulduk. İnternet outlet mağazası perakendeci için kazançlı olabileceğini ancak pazar özelliklerine de bağlı olabileceğini gösterdik.

Anahtar Kelimeler: dikey ürün farklılaştırma, çok kanallı perakendecilik, outlet, internet üzerinden satış, internet üzerinden outlet ürünü satışı

To my dear family;

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CHAPTER 1

INTRODUCTION

Berman and Evans (2006) define a factory outlet store as a manufacturer-owned and operated store selling closeouts, excess inventory, cancelled orders. Lately, they have started to offer in-season products as well. The factory outlet as a concept has evolved over time and has become a business opportunity not for manufacturers but also specialty retailers and some third-party retailers alike.

Recently, factory outlets started to draw out attention due to different reasons. The main role of the outlet store has been to liquidate excess inventory. Although prices are below regular retail prices, outlet stores can generate handsome profits thanks to low operating cost; i.e., low rent, service standards, and plain store layout. However, nowadays, outlet stores, parallel to traditional stores, turn into alternative sales channels that offer a lower quality option of the original collection. For example, North Face manages the outlet store to liquidate inventory whereas Coach, Ann Taylor, Guess and J.Crew design its own line for the outlet business (Levy and Weitz, 2012). In J.Crew, for example, "all J.Crew Factory items are exclusive designs and based on past J. Crew collection" (accessible via www.jcrew.com/help/about jcrew.scp). Brooks Brothers (apparel), Levi's (apparel), Liz Claiborne (apparel), Samsonite (luggage) also manage their own specialty store chain alongside their outlet stores. In this respect, outlet business branch represents an opportunity to expand the market of a retailer brand through vertical differentiation.

Outlet stores are generally located far from city centers; i.e., mainly in areas of low real estate market and also in touristic regions. As of 2006 in the U.S., there were

16,000 outlet stores clustered in 225 outlet malls, with total annual revenue of $16,000. (Berman and Evans, 2006)

Direct selling channels, as an alternative to the brick-and-mortar stores, include catalog business, online stores and mobile stores. As the use of Internet and smart phones increases every day, multichannel retailing, especially the online channel, is starting to represent a significant portion of sales. From the viewpoint of the retailer, online retailing has many advantages. Online channel facilitates easy expansion of a chain, overcoming the limitations of its brick-and-mortar network – if there are any. Essentially, online channel provides a retailer means for reaching more customers, and potentially serving them through a wider assortment. The customer, in the meantime, is free of the physical inconvenience of the visit and the risk of out-of-stock that he may face at the store. However, with online purchases, the customer has to endure the risks such as those associated with the fit, color, and fabric of the product. Most important, immediate gratification is not possible anymore; the customer has to patiently wait for his product in addition to other risks associated with buying online (credit card use and risk, wrong shipments, and etc.). Thus, online channel, compared to the physical channel that is available, rids the customer of the physical inconvenience of visiting the store and hence (potentially) expands retailer's total market. Especially in apparel and general merchandise, outlet business branches and online stores are active revenue-generators for a retailer.

Gap Inc. is one of the multi-brand and multi-channel retailers in the apparel industry. The company conducts retail activities through its online channel in addition to the physical stores under the Gap, Old Navy, Banana Republic, Piperlime and Athleta brands. Additionally, Gap and Banana Republic serve their consumers not only through the physical and online store but also outlet stores under names "Gap Outlet" and "Banana Republic Factory Store", respectively. J.Crew, founded in 1983, is a multi-channel specialty retailer. Now, the firm is reaching more than 100 countries through the online channel and overseas bricks-and-mortar stores. In addition to these channels, J.Crew has been serving his consumers with outlet stores named "J.Crew Factory" since 1988. The other example in the apparel industry is Nordstrom. It serves its customers with full-line stores, outlet stores as Nordstrom Rack and online channel via shop.nordstrom.com.

Recently, we observe retailers with outlet branches and online stores making their outlet stores available online as well – basically opening a second channel for their "value" (low quality, low price) business. Nordstrom's e-commerce site Nordstromrack.com provides its consumers to off-price fashion products online. REI is a category specialist that sells outdoor gears, outdoor goods and accessories through 131 stores. Also, REI offers its consumers both options of "Shop REI" and "Shop REI Outlet" in its e-commerce site www.rei.com.

In this thesis, we study the outlet branch and online channel decisions of a retailer, and their interrelations. We also evaluate the profit implications of an online channel for the outlet branch. We build our study sequentially by analyzing the cases below.

a. outlet branch decision of a retailer

In Chapter 3, we address the outlet branch decisions of a retailer that currently has a bricks-and-mortar chain. Note that here the outlet brand represents an inferior product compared to the original brand both in terms of quality and price.

b. online channel decision of a retailer

In Chapter 4, we address the online channel decision of a retailer with a bricks-and-mortar chain devoted to his primary brand. Particularly, we investigate the online services that he provides, and whether he will be better off in terms of total profit and market expansion. Here, online services include but are not limited to the promised delivery time (and shipping services) offered by the online store.

c. online channel decision for the primary brand of a retailer that has primary and outlet branches

In Chapter 5, we are interested in the online store decision of the retailer when the retailer has already a physical chain of primary brand stores and another that belongs to the outlet branch.

d. online channel decision for the outlet branch of a retailer that has already a primary brand with the physical and online channel and an outlet physical channel

In Chapter 6, we address the outlet online channel decision of the multi-channel retailer.

In each chapter (except the last one), we analyze the optimal decisions of the retailer and the change in his total profit. In Chapter 6, we again evaluate the profit impact of opening the online outlet store.

Before we present our analysis for each case, we discuss the relevant literature in Chapter 2. Finally, we conclude in Chapter 7 summarizing our major findings and offering further research directions.

CHAPTER 2

LITERATURE SURVEY

In this work, we study a monopolist retailer's channel decisions jointly with his vertical product differentiation strategy. In this respect, our work is closely related with two streams of research: (vertical) product differentiation and retailer channel management.

Product differentiation enables a firm to identify and focus on a target consumer segment. Later, the firm distinguishes its product or service from similar goods and services which are already offered by competitors to the defined consumer segment. Therefore, by launching a distinguished good and service the firm may not only generate more profit but also expand market share. In some cases, product differentiation may be a strategic necessity for firms.

Shy (1997) categorizes product differentiation models in three main groups which are "goods-characteristics" approach, non-address approach, and address (location) approach. In "goods-characteristics" approach, each product can be defined as a sum of attributes i.e.; color, size etc. and while purchasing, the consumer prefers the product that consists of the most suitable characteristics for him.

In non-address approach, a higher level in a preferred attribute generates more demand for the provider firm. The underlying assumption here is, "all consumers gain utility from consuming a variety of products and therefore buy a variety of products." (Shy, 1997) However, in location approach, each consumer buys a maximum of one product and consumers are heterogeneous in their preferences. In this approach, location as a concept has two different meanings. One of them is the physical distance between the consumer and the firm. In this case, the consumer evaluates the prices of product in all stores and decides where to purchase, taking

distance into account as well. The other meaning is that the distance between consumer's ideal preference (taste) for the particular good and product at hand. The consumer's disutility from buying the less-than-ideal brand which is equivalent to the transportation cost in the previous case can be interpreted as distance here.

In horizontal product differentiation as a "Location Model", all consumers in the market do not have the same order preferences for products. The choice of the consumer depends on the preference of the particular consumer as well as prices. The typical example is color; the preference of product color varies in the population. Location is another example; when the firms are located in the same street, each consumer that lives on the street will rank the firms differently – depending on where they live. In the horizontal product differentiation, the consumer prefers the product closest to him (or his taste) to gain higher utility given the same prices.

In contrast to horizontal differentiation model, all consumers have the same order preferences for products in the vertical product differentiation model. For a given (equal) price, all consumers prefer the same product in vertical differentiation. Put it differently, when the firms are assumed to locate on a linear street with length of 1, ideal brand of all consumers are located at point 1. For example, holding all else constant, all consumers prefer a fuel-efficient Hybrid car to a regular car that runs on gas. Quality is a typical dimension that firms utilize to vertically differentiate. Here, quality represents any characteristic of product (or brand) which all consumers prefer more to less, ceteris paribus, such as quality of material used in the product, its reliability, durability and performance.

In our study, we use vertical differentiation to model the interaction between the primary brand and its outlet branch. Consumers in the market are assumed to be heterogeneous in their willingness to pay for quality. It is common that the outlet branch offers the product with the lower quality, provides lower services, but charges a lower price compared to the primary brand stores. The quality we use here may represent the extent the retailer invests in the material, design and originality of the product sold at the outlet store as well as the services available at its stores.

Retailing is the set of business activities that adds value to the product and services sold to consumers. (Levy and Weitz, 2012) By means of a channel of distribution, the retailer, as a final business, facilitates the coordination between manufacturers, wholesalers and end-users. Retailing is an intensely competitive industry since a retailer is easily substitutable with another one. Changing customer behavior and evolving technology, assortment planning, and demand and inventory management are a few of the major challenges that retailers face today. In this environment, a retailer can adopt a multi-channel strategy to expand his business on a national and global scale. In this strategy, a retailer utilizes multiple channels to reach the end-consumer; i.e., store and non-store retailing. The three types of non-retailing are direct selling, vending machine retailing and e-tailing. A fourth one that is recently emerging is smart phone outlets; i.e., mobile stores. Nowadays, as the Internet is immersed more and more in people's lives, e-tailing is becoming more critical. Recently, many large retailers that operate physical stores have also opened online channels to make shopping more convenient, expand their customer base, and survive the competition.

Literature on product differentiation:

Hotelling (1929) considers a simple model of horizontal differentiation. In this model, consumers are distributed uniformly on a "linear city" of length 1 and two firms compete on store location (is equivalent to product) and price. In a setting of two competing sellers he finds that locating at the centre of the market is the equilibrium strategy of the firms. This is known as "Principle of Minimum Differentiation".

 d'Aspremont, Gabszewicz and Thisse (1979) alter Hotelling's model to allow product equilibrium to exist at all product positions. They find that the equilibrium product strategy is locating at either end of the market. In other words, equilibrium occurs when the firm is maximally differentiated from its competitor.

Our model differs from Hotelling (1929) and d'Aspremont et al. (1979) in several aspects. First, we model the market dynamics by the vertically differentiated Hotelling model as opposed to horizontal differentiation that they use. The retailer differentiates himself on quality; holding all else constant, all consumers prefer a

higher quality to a lower quality. We study vertical differentiation on a line model – in a similar manner to the "linear city" that they use.

Moorthy (1988) studies vertical differentiation a la Hotelling to investigate the competitive product strategy of firms and the impact of consumer preferences, costs and price competition on these strategies. He also analyzes the impact of sequential vs. simultaneous entry on the product positions in a duopoly environment. He finds that each firm's equilibrium strategy is to differentiate its own product. He also studies a monopolist's product line decision for two products to compare with the duopoly case as a benchmark. In this sense, he points out that cannibalization has a different influence on a monopolist's product strategy compared to those of two competitors.

Moorthy (1988) is the closest paper to our work. There are similarities between his model and our model. Firstly, we both use a vertically differentiated Hotelling model to study the "quality" decisions of companies. In addition to differences in our assumptions regarding the quality investment costs, we also differ in our general approach and research questions. Moorthy (1988) focuses on the product decisions in a competitive environment whereas we focus on a two-dimensional product differentiation decision for a monopolist firm.

Moorthy (1984) works on product line design problem of a monopolist. In his model, market segmentation is implemented through consumer self-selection different from the traditional approach of market segmentation as in this thesis based on the third-degree price discrimination (or product differentiation). In traditional approach the firm can addresses segments and isolate them individually whereas the firm knowing each consumer's preferences can isolate one type of consumer form another in this model. Moorthy (1984) point outs that a monopolist has to determine the optimal product and price for the whole product line simultaneously rather than for each segment separately due to cannibalization.

Moorthy (1987) studies product line competition in a duopoly. As in this thesis, market is modelled by a vertically differentiated Hotelling model. The main research question is how firms will segment the market. In other words, he investigates whether firms will prefer full differentiation or position themselves to generate

overlapping markets. He finds that both strict segmentation and entwining strategies can have strengths and weaknesses.

Moorthy and Png (1992) focus on the timing of the product introduction strategies. While in the sequential strategy firm introduces the two differentiated products one at a time, in the simultaneous strategy the two differentiated products are launched at the same time. As in our model, consumers differ in their willingness to pay for quality. Authors mention that sequential introduction is preferable to simultaneous introduction in terms of profit when cannibalization shows up as a problem.

Purohit (1994) studies a firm planning the introduction of a new version of its currently available product. While introducing the new generation product, the firm has to mitigate the obsolescence of the old product and at the same time generate a market for the new product. With these constraints he analyzes the new product introduction strategies such as product replacement, line extension and upgrading in monopoly and duopoly settings. He finds that a line extension strategy provides higher market share whereas a product replacement strategy generates more profit. Under duopoly, the incumbent firm has to choose the higher levels of product innovation due to the threat of a competitive clone. As in our model, consumers differ in their willingness to pay for quality; however, their willingness to pay for quality involves both their current valuation of the product and expectation of future price.

Vadenbosch and Weinberg (1995) study price and product competition in a duopoly setting using a two-dimensional vertical differentiation model. They study a sequential two-stage game in which firms define their product attributes and prices in the first and second stage, respectively. Products consist of two attributes that can take nonnegative values. One attribute may be more important than the other. They find that differently from the one-dimensional vertical differentiation model, firms may not prefer maximum differentiation although this solution is possible under certain conditions. When the range of positioning options on each of the dimensions is equal, firms position the product as maximum differentiation on one dimension and minimum differentiation on the other dimension in equilibrium. The authors here, as in this thesis, study a two-dimensional vertical differentiation model. This

work is focused on competition between two firms where attributes and prices are set sequentially whereas we assume quality and price are set together, study a monopolistic environment, and study a timeline of decisions which is consistent with practice.

Lauga and Ofek (2011) similarly study a two-dimensional vertical differentiation model. Consumers are heterogeneous with respect to their willingness to pay for two product attributes in a duopolistic market. In the two-stage game, the firms define any combination of product attributes in the first stage, and after observing each other's attribute selections, firms set price simultaneously in the second stage. They find that when cost of quality is not too high, firms always choose to maximally differentiate on one dimension and minimally differentiate on the other dimension. In these equilibria, firms are maximally differentiated on the greatest attribute span of the product characteristics. In case of higher cost of quality, firms differentiate on both dimensions.

Lauga and Ofek (2011) and Vadenbosh and Weinberg (1995) study a two-dimensional vertical differentiation model as in our models in Chapter 5 and 6. However, Vadenbosh and Weinberg (1995) assume that all products have a constant marginal cost whereas Lauga and Ofek (2011) assume that marginal cost increases with quality level chosen on each attribute. Additionally, they are focused on the competitive strategies of the firms whereas we are interested in a monopolist's market expansion vs. cannibalization trade-off as differentiation opportunities emerge over time.

Desai (2001) studies whether cannibalization affects a firm's product and price decisions when consumers are heterogeneous in their willingness to pay for quality and taste preferences. He finds that when the market is covered, high-quality valued segment obtains its preferred quality whereas less-quality valued segment gets less than its preferred quality in monopoly settings. When both segments are not completely covered, the optimal strategy for the monopolist is to provide each segment its preferred quality. In a duopolistic market, both types of results can occur depending on consumer and firms attributes. Consumers differ in their willingness to pay for quality as in our model and taste preferences (transportation cost) differently

from our model. Quality and taste preferences are the dimensions of vertical and horizontal differentiation, respectively. While we model the market dynamics by the vertically differentiated Hotelling model, here, Desai (2001) studies a market that is both vertically and horizontally differentiated.

Kim, Dilip and Liu (2013) investigate whether commonality can alleviate cannibalization in product line design. They assume within an attribute vertical differentiation dynamics work whereas attributes are utilized for horizontal differentiation. They find that commonality can actually diminish cannibalization in the product line design.

Ferguson and Koenigsberg (2007) work the pricing and quantity decision of a monopoly firm offering a perishable product that deteriorates over time but does not reach a value of zero. Since leftover items are considered as lower quality product than the new product, a second selling opportunity, a product line extension to new become possible alternatives for a firm by holding it over time. The firm faces cannibalization of demand for the new products by the leftover goods. The authors find that the advantage of a second selling opportunity overcomes the loss due to cannibalization. In this work, consumers differ in their valuation of the product as in this thesis. In this thesis, product quality is also a decision variable and we further extend to two-dimensional product differentiation models. However, the authors concentrate on pricing and quantity decision of a single product and further, the stocking policy of a monopoly firm.

Literature on retail channel management:

Chen, Kaya and Özer (2008) work on manufacturer's direct online sales channel and an independently owned brick-and-mortar retailer channel when channels compete on service. The delivery lead time for the product is service measure in the direct channel whereas product availability is the service standard in the traditional retail channel. The consumers differ in their willingness to wait to receive their products. They find that time-sensitive consumers prefer the brick-and-mortar channel while others shop online. Chen et al. (2008) determine optimal dual channel strategies that depend on the channel environment affected by the cost of managing the direct channel, retailer inconvenience, and some product attributes. They identify optimal

strategies (i.e., online channel only, dual channel, and etc.) where online channel cost and retailer inconvenience cost parameters determine the thresholds.

In modeling the online channel decision, we adopt the model used by Chen et al (2008); a customer base heterogeneous in willingness to wait and a cost structure with diminishing returns in setting the online service quality (delivery time). However, we do not limit this study to the online channel decision. We further integrate the online channel decision with the outlet branch decision.

Many companies consider engaging in direct sales due to different reasons. This puts such companies in competition with their existing retail partners. Tsay and Agrawal (2004) model a supply chain that consists of a manufacturer and a reseller acting independently to investigate channel conflict and coordination under the three types of distribution scenarios; in the first only reseller sells, in the second only direct sales occur, and in the third both channels generate sales together. They find that the addition of the direct channel to the reseller channel is not necessarily adverse. Unlike this paper, there is one decision maker which is the monopolist retailer in our work. For that reason, channel conflict and coordination are out of scope whereas cannibalization remains as the focus in this study.

Zhang (2009) studies the adoption of a multichannel strategy in connection with price advertising for a retailer. More explicitly, he characterizes the conditions under which a conventional bricks-and-mortar retailer would prefer to evolve to multichannel retailer and when he would advertise his offline prices at the online store. He finds that multichannel retailing in not necessarily a profitable strategy for all retailers and the offline price information disclosure should not be used by every retailer. This paper, like in our work, characterizes when a retailer would be profitable to have the online channel available. However, Zhang (2009) focuses on the interrelation of this decision with the price advertising strategy whereas we study the connection of online channel decision with the "value" branch of a retailer.

The Internet enables the traditional retailer to acquire a new sales channel to serve its consumers. More recently, many traditional retailers have turned into clicks-and-mortar retailers to streamline their online and offline services. Clicks-and-mortar retailer can be defined as a new form of retailer type emerging with the combination

of online (Internet) channel and bricks-and-mortar retailers. Bernstein, Song and Zheng (2008) model a supply chain channel structure in a competitive oligopoly setting to investigate whether companies should adopt the "click-and-mortar" business model. As a major insight, they point out that clicks-and-mortar appears as the equilibrium channel structure. They also find that this equilibrium does not necessarily generate more profit, in some cases, it is a strategic necessity.

Coughlan and Soberman (2005) develop a model in a duopoly in which the two manufacturers serve their consumers through the primary retailers or with dual distribution (primary retailers and outlet stores). They assume that consumers are heterogeneous with respect to price and service sensitivity and service is main difference between the primary retailer and the outlet mall. Coughlan and Soberman (2005) point out that if service sensitivity is the main source of consumer heterogeneity in the market, single channel distribution through the primary retailer is superior. Otherwise, the manufacturer generates more profit with dual distribution. As in this thesis, market is modeled by a vertically differentiated Hotelling model and consumers differ in the two different dimensions. This work is focused on competition between two manufacturers on distribution types in terms of profit and market expansion. However, in this thesis, from the retailer point of view, distribution types are compared in terms of profit, market expansion and also consumer surplus in a monopolistic environment.

Liu, Gupta and Zhang (2006) study entry-deterrence decision in the context of e-tailer. Significantly, this work is focused on opportunities for e-tailer's market entry when the incumbent brick-and-mortar retailer (with or without its the online store) is active at present. They find that the incumbent is ready to cannibalize its own brick-and-mortar business by setting lower online price. Consumers differ in taste preferences (transportation cost) differently from our model. While we model the market dynamics by the vertically differentiated Hotelling model, here, authors study a market that is horizontally differentiated.

Subramanian (1998) studies competition between direct marketers and conventional retailers. The major research fields (points) are that conceptualization of competition and new variables on operational difference between direct marketers and

conventional retailers. Major insights can be categorized in the three groups: changes on the mature of competition, entry the market and market structure and lastly role on information in the multi-channel market. Subramanian (1998) points out that in market with the full information about sellers, every consumer is offered many options to shop by the direct channel. The main difference between our and his model is that author model the market with Salop's circular city.

All papers above study potential issues and business opportunities with the emergence of the online store as a secondary channel for retailers, and a direct opportunity for manufacturers. We complement this literature by studying a recent practice observed in the retail industry. We investigate how product differentiation decisions are interrelated with the online channel decisions and study what kind of market conditions would render an outlet online channel decision profitable.

CHAPTER 3

THE PHYSICAL OUTLET DECISION OF THE BRICK-AND-MORTAR RETAILER

3.1 The Physical Outlet Decision of the Brick-and-Mortar Retailer without Inconvenience Cost

In this part of Chapter 3, we are interested in a monopolist retailer's outlet business decision given its original brand position to maximize its total profit. The retailer is already selling its original brand through its primary physical channel and wants to open a second branch; i.e., the outlet chain. We characterize how the retailer positions its outlet branch in terms of quality level and price point.

A consumer's type represents his marginal willingness to pay for increments of an attribute. A higher type of consumer (i.e., with a higher taste parameter) is willing to pay more for a given quality level than a lower type. Here, quality represents any attribute of the retailer which all consumers prefer more to less, ceteris paribus, such as quality of material used in the product, store service standards and/or store design.

In this model, we assume consumers differ in their willingness to pay for quality. Thus, a consumer of type θ has the following utility function:

$$U = \begin{cases} \theta s_1 - p_1, & \text{if he purchases from the primary physical store} \\ \theta s_2 - p_2, & \text{if he puchases from the outlet store} \\ \text{reservation utility (0),} & \text{otherwise} \end{cases}$$

where s_i refers to the quality and p_i is the price of the retail branch, $i = 1, 2$. Here (s_1, p_1) refer to the already set primary brand quality and price points. (s_2, p_2) are the quality and price point of the outlet branch and are the main decision variables in this section. We assume that the outlet product is inferior to the primary brand in terms of quality and price. That is, $s_1 \geq s_2$ and $p_1 \geq p_2$.

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Consumers can observe the product qualities and prices available before they decide to buy. They buy a maximum of one product. They purchase only when their net utility is greater than or equal to their reservation utility, assumed as 0 in our study.

We assume that consumers are heterogeneous in their willingness to pay for quality; i.e., consumers are distributed uniformly on [0, b] according to their types θ, where $b > 0$. We assume that b is high enough to guarantee a high enough profit for the current brick-and-mortar chain. Thus, for integrity of Section 3.1, we assume that $(b - \frac{p_1}{s_1} - \frac{7cs_1}{16}) > 0$.(A.3.1)

The retailer's unit cost increases with its chosen quality level. We use a quadratic function to represent diminishing returns on quality, thus unit gross margin of the retailer is $(p_1 - cs_1^2)$. We assume that the unit profit margin of the primary brand is nonnegative since the retailer is profitable. That is, $(p_1 - cs_1^2) > 0$ (A.3.2). We assume that there are no fixed costs.

The parameters, decision variables and notations used in this chapter are presented in Table 3.1.

Table 3.1: Notation

Decision Variable(s)	
s_2	Quality level of the outlet branch, $s_2 > 0$
p_2	Price of the outlet branch, $p_2 > 0$
s_1	Quality level of the original brand, $s_1 \geq s_2, s_1 > 0$, (given)
p_1	Price of the original brand, $p_1 \geq p_2, p_1 > 0$, (given)
Parameters	
c	Unit cost coefficient for a given quality level, $c > 0$
θ	Quality taste parameter of consumers, $\theta \sim U[0, b]$
D_1	Demand of the original brand, $D_1 \geq 0$
D_2	Demand of the outlet branch , $D_2 \geq 0$

Before opening the outlet channel, the retailer serves the market through the physical channel of its original brand. A consumer will want to purchase from the primary

physical store if $\theta s_1 - p_1 \geq 0$. $\frac{p_1}{s_1}$ denotes the taste parameter of the last consumer who purchases the product from the primary physical store when there is only primary physical store in the market. Thus, the demand of the primary business is given by $(b - \frac{p_1}{s_1})$ and the monopolist retailer cannot cover the whole market by itself.

Lemma 3.1: Outlet branch may have on impact on the retailer's business if and only if $\frac{s_2}{p_2} \geq \frac{s_1}{p_1}$; i.e., "quality per dollar" for the outlet product is higher than that of the original product. The total market of the retailer expands by $(\frac{p_1}{s_1} - \frac{p_2}{s_2})$, consumers with θ in $[\frac{p_1}{s_1}, \frac{p_1-p_2}{s_1-s_2}]$ switch from the primary brand to the outlet branch.

Proof: Suppose now $\frac{s_1}{p_1} > \frac{s_2}{p_2}$.

A consumer of type θ prefers the primary brand to the outlet brand if and only if $(\theta s_1 - p_1) - (\theta s_2 - p_2) \geq 0$.

$$(\theta s_1 - p_1) - (\theta s_2 - p_2) = p_1\left(\frac{\theta s_1}{p_1} - 1\right) - p_2\left(\frac{\theta s_2}{p_2} - 1\right)$$

$$\geq (p_1 - p_2)\left(\frac{\theta s_2}{p_2} - 1\right) \text{ since } \frac{s_1}{p_1} > \frac{s_2}{p_2}.$$

$$\geq 0 \text{ if } \theta s_2 > p_2 \text{ and by assumption } p_1 - p_2 > 0.$$

Thus, whenever a consumer is willing to buy from the outlet branch, he prefers the primary brand to the outlet brand and thus never purchases from the outlet. Therefore, if $\frac{s_1}{p_1} > \frac{s_2}{p_2}$, then the outlet branch has zero demand $(D_2 = 0)$ and the primary branch demand is $D_1 = b - \frac{p_1}{s_1}$.

When we have $\frac{s_1}{p_1} \leq \frac{s_2}{p_2}$, we then have $\frac{p_2}{s_2} < \frac{p_1}{s_1}$. When $\frac{p_2}{s_2} \leq \theta \leq \frac{p_1}{s_1}$, the consumer will have a positive utility only if he purchases from the outlet (i.e., negative utility from the primary brand).

When $\frac{p_1}{s_1} \leq \theta$, the consumer will have positive utility from both branches and prefer the outlet if and only if $(\theta s_2 - p_2) > (\theta s_1 - p_1)$.

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$$(\theta s_2 - p_2) > (\theta s_1 - p_1) \Rightarrow \theta(s_2 - s_1) > p_2 - p_1 \to \theta < \frac{p_1 - p_2}{s_1 - s_2} = \tilde{\theta}$$

Note that $\frac{p_1}{s_1} \le \frac{p_1 - p_2}{s_1 - s_2}$ is also guaranteed by the relationship $\frac{s_1}{p_1} \le \frac{s_2}{p_2}$ and the details are left to the reader. To avoid trivial cases where $D_2 = 0$, we assume that "quality per dollar" for the outlet product is higher than that of the original product from this point on in our study. Note that whenever opening an outlet branch is not profitable, the decision variables s_2 and p_2 will be set equal to s_1 and p_1. Thus, we have $\frac{s_1}{p_1} \le \frac{s_2}{p_2}$. ∎

Figure 3.1 shows the total market of the retailer after opening the outlet store. The demand of the retailer before (a) and after (b) the introduction of the outlet channel (and its split across channels) is shown.
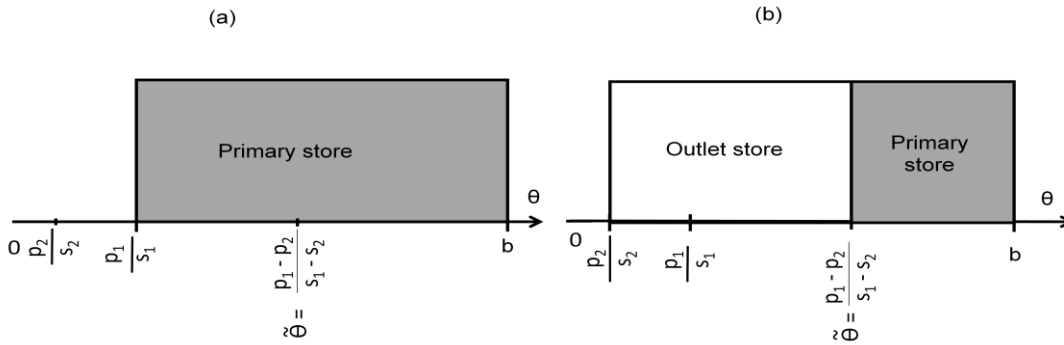
Figure 3.1: The demand of the retailer before (a) and after (b) the introduction of the outlet branch (and its split between channels)

Proposition 3.1: After opening the outlet channel, the demands of the channels are;

$D_1 = b - \frac{p_1 - p_2}{s_1 - s_2}$ for the primary business

$D_2 = \frac{p_1 - p_2}{s_1 - s_2} - \frac{p_2}{s_2}$ for the outlet business

Proof: The demand of the channels follows from Lemma 3.1 and its proof. ∎

The unit profit margins of products are $(p_1 - cs_1^2)$ and $(p_2 - cs_2^2)$ for the original brand and the outlet product, respectively.

If we take into account the costs as well, the total profit of a retailer with a functioning outlet business is

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$$\Pi = \left(b - \frac{p_1 - p_2}{s_1 - s_2}\right)(p_1 - cs_1^2) + \left(\frac{p_1 - p_2}{s_1 - s_2} - \frac{p_2}{s_2}\right)(p_2 - cs_2^2)$$

After all, the retailer's problem can be modeled as,

$$max_{s_2, p_2} \left(b - \frac{p_1 - p_2}{s_1 - s_2}\right)(p_1 - cs_1^2) + \left(\frac{p_1 - p_2}{s_1 - s_2} - \frac{p_2}{s_2}\right)(p_2 - cs_2^2) \qquad \text{(Eq.3.1)}$$

$$b - \frac{p_1 - p_2}{s_1 - s_2} \geq 0 \qquad \text{(Eq.3.2)}$$

$$\frac{s_2}{p_2} - \frac{s_1}{p_1} \geq 0 \qquad \text{(Eq.3.3)}$$

$$s_1 \geq s_2 \qquad \text{(Eq.3.4)}$$

$$p_1 \geq p_2 \qquad \text{(Eq.3.5)}$$

$$s_2 > 0 \qquad \text{(Eq.3.6)}$$

$$p_2 > 0 \qquad \text{(Eq.3.7)}$$

Constraint (3.2) and (3.3) ensures that the retailer has the nonnegative demand for respective the primary and outlet brand after opening the outlet business. Note that either can be zero. The objective function of the model, (3.1), maximizes the total profit given the retailer's original brand position. Constraints (3.4) and (3.5) are upper limits for decision variables that identify with the outlet business. Constraints (3.6) and (3.7) are non-negativity conditions for decision variables.

The retailer's problem is a nonlinear maximization problem with two decision variables.

Lemma 3.2: The profit function, $\Pi = \left(b - \frac{p_1 - p_2}{s_1 - s_2}\right)(p_1 - cs_1^2) + \left(\frac{p_1 - p_2}{s_1 - s_2} - \frac{p_2}{s_2}\right)(p_2 - cs_2^2)$ is jointly concave in p_2 and s_2 in the feasible region of p_2 and s_2.

Proof: The matrix L, Hessian matrix of profit function, is written below:

$$L = \begin{bmatrix} \dfrac{2s_1}{s_2(s_2 - s_1)} & \dfrac{2(p_1 - p_2)}{(s_1 - s_2)^2} - \dfrac{2p_2}{s_2^2} \\ \dfrac{2(p_1 - p_2)}{(s_1 - s_2)^2} - \dfrac{2p_2}{s_2^2} & \dfrac{2(p_1 - p_2)^2}{(s_1 - s_2)^3} - \dfrac{2p_2^2}{s_2^3} \end{bmatrix}$$

The leading principal minors are;

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$\Delta_1 = \frac{2s_1}{s_2(s_2-s_1)}$ (of order one)

$\Delta_2 = \frac{4(p_2s_1 - p_1s_2)^2}{(s_1-s_2)^3 s_2^3}$ (of order two)

L is negative definite if $(-1)^k . \Delta_k > 0$ where k={1,2} for all leading principal minors. $0 < s_2 < s_1$ implies that condition for the leading principal minors is satisfied. It is concluded that matrix L is negative definite, hence the profit function of the firm is jointly concave in p_2 and s_2. ■

The optimal solution of the maximization problem is given in Proposition 3.2

Proposition 3.2: The maximizers of the retailer's problem in (Eq.3.1-3.7) are $s_2^* = \frac{s_1}{2}$ and $p_2^* = \frac{(4p_1 - cs_1^2)}{8}$.

Proof: The first order partial derivatives of the objective function with respect to s_2 and p_2 are given below,

$$\frac{d\Pi(p_2, s_2)}{dp_2} = -cs_1 + \frac{2(p_1 s_2 - s_1 p_2)}{s_2(s_1 - s_2)}$$

$$\frac{d\Pi(p_2, s_2)}{ds_2} = cp_1 - \frac{(p_1 - p_2)^2}{(s_1 - s_2)^2} + \frac{p_2^2}{s_2^2}$$

Solving $\frac{d\Pi(p_2, s_2)}{\partial p_2} = 0$ and $\frac{d\Pi(p_2, s_2)}{\partial s_2} = 0$ simultaneously yields $s_2^* = \frac{s_1}{2}$ and $p_2^* = \frac{(4p_1 - cs_1^2)}{8}$, which is the optimal solution of the unconstrained optimization problem.

The quality level of the new product is equal to one half of quality level of the original brand and always nonnegative. Thus, s_2^* satisfies constraints (3.4) and (3.6). The optimal price of new product is less than one half of price of the original brand, thus satisfies (3.5). Constraint (3.7) is satisfied since $(4p_1 - cs_1^2)$ is always nonnegative with (A.3.2). After plugging s_2^* and p_2^* into constraint (3.2), it becomes $(b - \frac{p_1}{s_1} - \frac{cs_1}{4}) \geq 0$. With (A.3.1) constraint (3.2) is satisfied. Constraint (3.3) is satisfied by the optimal outlet brand positions. Thus, s_2^* and p_2^* above are also feasible (and optimal) for the problem in (Eq.3.1)-(Eq.3.7). ■

The optimal outlet price turns out to be less than one half of the primary brand price though the optimal outlet quality level is exactly equal to one half of primary product quality. Quality per dollar for the outlet is higher than quality per dollar for the primary brand. The gap between quality per dollar for the outlet and primary brand widens as unit cost (c) or the primary brand quality level increases. The outlet quality increases and the outlet price decreases with the primary brand quality level. Thus, the retailer elevates its outlet quality as s_1 increases, and shifts it to focus towards the higher consumer segments. As the primary product price increases the optimal outlet price increases. While quality level of the original brand affect the price and quality level of the outlet product, price of the original brand only affects the outlet product price.

Proposition 3.3: Opening the outlet business is preferable for both the retailer and the consumer. By opening an outlet branch, the retailer's profit increases by $\frac{c^2 s_1^3}{16}$.

Proof: Without outlet, demand and profit of the primary business are given by,

$$D_1 = \left(b - \frac{p_1}{s_1}\right) \tag{Eq.3.8}$$

$$\Pi_1 = \left(b - \frac{p_1}{s_1}\right)(p_1 - cs_1^2) \tag{Eq.3.9}$$

After rearranging (3.9), it becomes,

$$\Pi_1 = \frac{cp_1 s_1^2 + bs_1 p_1 - bcs_1^3 - p_1^2}{s_1} \tag{Eq.3.10}$$

When we plug-in s_2^* and p_2^* values into the retailer's profit function in (Eq.3.1), Π_{1+2} becomes,

$$\Pi_{1+2} = \frac{cp_1 s_1^2 + bs_1 p_1 - bcs_1^3 - p_1^2}{s_1} + \frac{c^2 s_1^3}{16} \tag{Eq.3.11}$$

Then, $\Pi_{1+2} - \Pi_1 = \frac{c^2 s_1^3}{16} > 0$ since $c > 0$ and $s_1 > 0$.

Before starting an outlet business, consumer surplus is given by,

$$CS_1 = \int_{p_1/s_1}^{b} (\theta s_1 - p_1) d\theta = \frac{b^2 s_1}{2} - bp_1 + \frac{p_1^2}{2s_1} \tag{Eq.3.12}$$

When the outlet channel is open, the formulation of the consumer surplus is given by,

$$CS_{1+2} = \int_{\frac{p_1-p_2}{s_1-2}}^{b}(\theta s_1 - p_1)d\theta + \int_{\frac{p_2}{s_2}}^{\frac{p_1-p_2}{s_1-s_2}}(\theta s_2 - p_2)d\theta$$

With s_2^* and p_2^*, it becomes ;

$$CS_{1+2} = \frac{b^2 s_1}{2} - bp_1 + \frac{32p_1^2 + 2c^2 s_1^4}{64 s_1} \qquad\qquad\qquad (Eq.3.13)$$

Then $CS_{1+2} - CS_1 = \frac{c^2 s_1^3}{32} > 0$ since $c > 0$ and $s_1 > 0$. ∎

The consumer surplus and the profit of the retailer before and after the outlet branch are summarized in Table 3.2.

Table 3.2: Comparing "only primary business" and "with an outlet branch " cases on profit and consumer surplus

	Only primary business	With an outlet branch
Total profit	$\frac{cp_1 s_1^2 + bs_1 p_1 - bcs_1^3 - p_1^2}{s_1}$	$\frac{cp_1 s_1^2 + bs_1 p_1 - bcs_1^3 - p_1^2}{s_1} + \frac{c^2 s_1^3}{16}$
Consumer Surplus	$\frac{b^2 s_1}{2} - bp_1 + \frac{p_1^2}{2s_1}$	$\frac{b^2 s_1}{2} - bp_1 + \frac{32p_1^2 + 2c^2 s_1^4}{64 s_1}$

Total demand of the retailer increases by $\frac{cs_1}{4}$ while the demand of the retailer's primary business decreases by $\frac{cs_1}{4}$ after opening the outlet business. Although the outlet business cannibalizes some of primary business demand; the retailer manages to increase its overall demand with the addition of the outlet branch.

The market expansion is $\frac{cs_1}{4}$ and its margin is $p_2^* - cs_2^{*2}$ or equivalently $\frac{4p_1 - 3cs_1^2}{8}$. The cannibalized demand from the primary brand is $\frac{cs_1}{4}$ and change in the margin is $(p_2^* - s_2^{*2} - (p_1 - cs_1^2))$ or equivalently $-\frac{4p_1 - 5cs_1^2}{8}$. The profit margins of products are equal when $p_1 = \frac{5cs_1^2}{4}$. If $p_1 > \frac{5cs_1^2}{4}$, then the profit margin of the primary business is always greater than that of the outlet product.

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As the primary product quality level increases (s_1), outlet quality level (s_2^*) increases and outlet price (p_2^*) decreases. As a result, demand of the outlet product increases.

As the unit cost coefficient (c) increases, the profit gain increases. As the unit cost coefficient increases, the optimal outlet price (p_2^*) decreases but the optimal outlet quality level (s_2^*) remains the same. Consequently, the outlet demand increases with the unit cost coefficient. The retailer achieves market expansion with increase in the outlet demand. Hence, as unit cost coefficient increases, the extra market increases. The retailer's main tradeoff is between market expansion and cannibalization of the primary store and its profit margin.

When we plug-in p_2^* and s_2^* into $\tilde{\theta}$, it becomes $\frac{p_1}{s_1} + \frac{cs_1}{4}$. If b, p_1 and s_1 remain the same, as c increases, $\tilde{\theta}$ goes up. In short, the the primary brand demand shrinks whereas the outlet brand demand becomes larger with the increase in c. The price, quality level, demand of products in the "only primary business "case and "with an outlet branch"case are summarized in Table 3.3.

Table 3.3: Comparing "only primary business" and "with an outlet branch "cases on profit, demand, price and quality level of the products

	Only primary business	With an outlet branch
Profit of the outlet product	NA	$\frac{cs_1(4p_1 - 3cs_1^2)}{16}$
Profit of the primary product	$\left(b - \frac{p_1}{s_1}\right)(p_1 - cs_1^2)$	$\left(b - \frac{p_1}{s_1}\right)(p_1 - cs_1^2) - \frac{cs_1}{4}(p_1 - cs_1^2)$
Total profit	$\frac{cp_1 s_1^2 + bs_1 p_1 - bcs_1^3 - p_1^2}{s_1}$	$\frac{cp_1 s_1^2 + bs_1 p_1 - bcs_1^3 - p_1^2}{s_1} + \frac{c^2 s_1^3}{16}$
Demand of the primary product	$b - \frac{p_1}{s_1}$	$b - \frac{p_1}{s_1} - \frac{cs_1}{4}$
Demand of the outlet product	NA	$\frac{cs_1}{2}$
Profit margin of primary product	$p_1 - cs_1{}^2$	$p_1 - cs_1{}^2$
Profit margin of outlet product	NA	$\frac{4p_1 - 3cs_1^2}{8}$
Total covered market	$b - \frac{p_1}{s_1}$	$b - \frac{p_1}{s_1} + \frac{c.s_1}{4}$
The primary price	p_1	p_1
The outlet price	NA	$\frac{4p_1 - cs_1^2}{8}$
The primary quality level	s_1	s_1
The outlet quality level	NA	$\frac{s_1}{2}$

We also investigate the retailer's initial primary brand position strategies. The retailer can position the primary brand without any consideration of the outlet business opportunity in the future. In this strategy, the retailer acts myopic. The myopic retailer positions the primary product and then the outlet product. In long term planning, the retailer also considers the future outlet product position while positioning the primary product. For that reason, we refer to that type of retailer as "non-myopic retailer." The whole reason behind this is to compare the primary brand position and profit levels under two strategies.

3.1.1 Myopic Retailer

In this part, we investigate the scenario where the retailer positions its primary brand (i.e., sets s_1 and p_1) only to maximize its primary business profit.

The demand and profit of the retailer are given by (3.8) and (3.9);

$$D_1 = \left(b - \frac{p_1}{s_1}\right) \tag{Eq.3.8}$$

$$\Pi_1 = \left(b - \frac{p_1}{s_1}\right)(p_1 - cs_1^2) \tag{Eq.3.9}$$

Lemma 3.3: The profit function of retailer, (Eq.3.9), is jointly concave in p_1 and s_1.

Proof: The matrix H, Hessian matrix of profit function, is written below:

$$H = \begin{bmatrix} \dfrac{-2}{s_1} & c + \dfrac{2p_1}{s_1^2} \\ c + \dfrac{2p_1}{s_1^2} & -2bc - \dfrac{2p_1^2}{s_1^3} \end{bmatrix}$$

The leading principal minors are;

$$\Delta_1 = \frac{-2}{s_1} \text{ (of order one)}$$

$$\Delta_2 = \frac{c(4bs_1 - cs_1^2 - 4p_1)}{s_1^2} \text{ (of order two)}$$

H is negative definite if $(-1)^k . \Delta_k > 0$ where k={1,2} for all leading principal minors. The condition for the second leading principal minor is satisfied with (A.3.1). The profit function of the retailer is jointly concave in p_1 and s_1 since the matrix H is negative definite. ∎

The total profit of the retailer can be modeled as follows;

$$max_{s_1,p_1} \left(b - \frac{p_1}{s_1}\right)(p_1 - cs_1^2) \tag{Eq.3.9}$$

$$s_1 > 0 \tag{Eq.3.14}$$

$$p_1 > 0 \tag{Eq.3.15}$$

The objective function of the model, (3.9), maximizes the profit of primary business. The constraints (3.14) and (3.15) are non-negativity conditions for decision variables.

The retailer's problem is a nonlinear maximization problem with two decision variables. The optimal solution of the unconstrained maximization problem is given in Proposition 3.4.

Proposition 3.4: The maximizers of the retailer's problem in (Eq.3.9, 3.14, 3.15) are $s_1^* = \frac{b}{3c}$ and $p_1^* = \frac{2b^2}{9c}$.

Proof: The first order partial derivatives of the objective function with respect to s_1 and p_1 are given below,

$$\frac{d\Pi(p_1, s_1)}{dp_1} = b - \frac{2p_1}{s_1} + cs_1$$

$$\frac{d\Pi(p_1, s_1)}{ds_1} = \frac{p_1^2}{s_1^2} + c(p_1 - 2bs_1)$$

Solving $\frac{d\Pi(p_1, s_1)}{dp_1} = 0$ and $\frac{d\Pi(p_1, s_1)}{ds_1} = 0$ simultaneously yields two (s_1, p_1) pairwise roots. We find that $s_1^* = \frac{b}{3c}$ and $p_1^* = \frac{2b^2}{9c}$ with objective function value of $\frac{b^3}{27c}$. Since (Eq.3.14) and (Eq.3.15) are also satisfied, the solution above is feasible (and optimal) for the problem stated. ∎

When $s_1^* = \frac{b}{3c}$ and $p_1^* = \frac{2b^2}{9c}$, the retailer generates profit of $\frac{16b^3}{432c}$ $(\frac{b^3}{27c})$. If the retailer started to serve the customer with the outlet store as an additional channel, total profit would be $\frac{17b^3}{432c}$.

3.1.2 Non-Myopic Retailer

In this part, we consider the scenario where the retailer takes into account a future outlet branch opportunity while positioning its primary brand. In other words, we want to find the primary brand positions that maximize the total profit that includes the outlet branch that will follow as well. Here, the retailer does not act myopic,

makes sequential brand position decisions in case of opening outlet branch is possible in the future. We refer this type of retailer as a non-myopic retailer.

When we plug-in s_2^* and p_2^* into (3.1), the profit function of the retailer is;

$$\Pi_{total} = \left(b - \frac{4p_1 + cs_1^2}{4s_1}\right)(p_1 - cs_1^2) + \frac{cs_1(4p_1 - 3cs_1^2)}{16} \qquad \text{(Eq.3.16)}$$

Lemma 3.4: The profit function of retailer, (3.16), is jointly concave in p_1 and s_1 .

Proof: The matrix M, Hessian matrix of profit function, is written below:

$$M = \begin{bmatrix} \dfrac{-2}{s_1} & c + \dfrac{2p_1}{s_1^2} \\ c + \dfrac{2p_1}{s_1^2} & -2bc - \dfrac{2p_1^2}{s_1^3} + \dfrac{3c^2 s_1}{8} \end{bmatrix}$$

The leading principal minors are;

$\Delta_1 = \frac{-2}{s_1}$ (of order one)

$\Delta_2 = \frac{c(32bs_1 - 32p_1 - 14cs_1^2)}{8s_1^2}$ (of order two)

M is negative definite if $(-1)^k . \Delta_k > 0$ where k={1,2} for all leading principal minors. The condition for the second leading principal minor is satisfied with (A.3.1). It is concluded that matrix M is negative definite, hence profit function of the firm is jointly concave in p_1 and s_1.   ∎

The retailer's problem is then,

$$max_{s_1, p_1} \left(b - \frac{4p_1 + cs_1^2}{4s_1}\right)(p_1 - cs_1^2) + \frac{cs_1(4p_1 - 3cs_1^2)}{16} \qquad \text{(Eq.3.16)}$$

$$s_1 > 0 \qquad \text{(Eq.3.14)}$$

$$p_1 > 0 \qquad \text{(Eq.3.15)}$$

Proposition 3.5: The maximizers of the retailer's problem in (Eq.3.14, 3.15, 3.16) are $s_1^* = \frac{2b}{5c}$ and $p_1^* = \frac{7b^2}{25c}$.

Proof: The first order partial derivatives of the objective function with respect to s_1 and p_1 are given below,

$$\frac{d\Pi(p_1,s_1)}{dp_1} = b - \frac{2p_1}{s_1} + cs_1$$

$$\frac{d\Pi(p_1,s_1)}{ds_1} = \frac{p_1^2}{s_1^2} + \frac{3c^2s_1^2}{16} + c(p_1 - 2bs_1)$$

Solving $\frac{d\Pi(p_1,s_1)}{dp_1} = 0$ and $\frac{d\Pi(p_1,s_1)}{ds_1} = 0$ simultaneously yields two (s_1, p_1) pairwise roots. We find that $s_1^* = \frac{2b}{5c}$ and $p_1^* = \frac{7b^2}{25c}$ with objective function value of $\frac{9b^3}{250c}$. Since (Eq.3.14) and (Eq.3.15) are also satisfied, the solution above is feasible (and optimal) for the problem stated. ∎

When $s_1^* = \frac{2b}{5c}$ and $p_1^* = \frac{7b^2}{25c}$, the retailer generates a profit of $\frac{9b^3}{250c}$. If the retailer started to serve the customer with the outlet channel as additional channel, the total profit would be $\frac{10b^3}{250c}$.

The findings are summarized in Table 3.4. As a result, primary brand positioning based on long term planning (non-myopic approach) is preferable for the retailer. If the retailer acts as opening outlet branch will be possible in the future, then the firm sets a higher price and a quality level for the primary brand. The profit difference between two cases exponentially increases with b and decreases with c.

Table 3.4: The primary brand position and the total profit in the "myopic" and "non-myopic" retailer cases

	Myopic Retailer	Non-myopic Retailer	$\Delta_{(M-N)}$
Price of the primary brand	$\dfrac{2b^2}{9c}$	$\dfrac{7b^2}{25c}$	$-\dfrac{13b^2}{225c} < 0$
Quality level of primary brand	$\dfrac{b}{3c}$	$\dfrac{2b}{5c}$	$-\dfrac{b}{15c} < 0$
Total profit of the retailer (without outlet)	$\dfrac{16b^3}{432c}$	$\dfrac{9b^3}{250c}$	$\dfrac{7b^3}{6750c} > 0$
Total profit of the retailer (with outlet)	$\dfrac{17b^3}{432c}$	$\dfrac{b^3}{25c}$	$-\dfrac{7b^3}{10800c} < 0$

3.2 The physical outlet decisions of a brick-and-mortar retailer with inconvenience cost

In Section 3.1, we studied a monopolist firm's decision about opening an outlet business without any consideration of an inconvenience cost regarding either channel. However, that was an idealized case. In real life there exist inconvenience costs associated with visiting a brick-and-mortar retailer. This may include the actual activity of visiting the store, time spent on the activity as well as the unavailability risk of the product. A consumer's utility decreases with this inconvenience. Note that outlet malls tend to be located far away from city centers, which translates into a higher inconvenience cost associated with visiting the outlet store.

Here we investigate the retailer's outlet branch decisions in the presence of inconvenience costs for both the primary chain and the outlet stores. We change utility functions by adding new terms; specifically k>0 for the inconvenience of visiting the primary brand chain, and m>0 for visiting the outlet chain. We assume that inconvenience cost of visiting and purchasing from the outlet store is greater

than inconvenience cost of visiting and purchasing from the primary store. In short, $m \geq k > 0.$ (A.3.3)

A consumer's type represents his marginal willingness to pay for increments of an attribute. A higher type of consumer (i.e.; with a higher taste parameter) is willing to pay more for a given quality level than a lower type. Here, quality represents any attribute of the retailer which all consumers prefer more to less, ceteris paribus, such as quality of material used in the product, store service standards and/or store design.

Consumer of type θ has the following utility function;

$$U = \begin{cases} \theta s_1 - p_1 - k, & \text{if he purchases from the primary physical store} \\ \theta s_2 - p_2 - m, & \text{if he puchases from the outlet channel} \\ \text{reservation utility } (0), & \text{otherwise} \end{cases}$$

We assume that consumers are heterogeneous in their willingness to pay for quality; i.e., consumers are distributed uniformly on [0, b] according to their types θ, where $b > 0$. We assume that b is high enough to enable a profitable business for the retailer (i.e.; for nonnegative market share and margin). Again, we assume that the outlet product is inferior to the primary brand in terms of quality and price. That is, $s_1 \geq s_2$ and $p_1 \geq p_2$.

Consumers can observe the product qualities and prices available before they decide to buy. They buy a maximum of one product. They purchase only when their net utility is greater than or equal to their reservation utility, assumed as 0 in our study.

The retailer's unit cost increases with its chosen quality level. We use a quadratic function to represent diminishing returns on quality, thus unit gross margin of the retailer is $(p_1 - cs_1^2)$. We assume that the unit profit margin of the primary brand is nonnegative since the retailer is profitable. That is, $(p_1 - cs_1^2) > 0$ (A.3.2). We assume that there are no fixed costs.

The parameters, decision variables and notations used in this chapter are presented in Table 3.5.

Table 3.5: Notation

	Decision Variable(s)
s_2	Quality level of the outlet branch, $s_2 > 0$
p_2	Price of the outlet branch, $p_2 > 0$
s_1	Quality level of the original brand, $s_1 \geq s_2, s_1 > 0$, (given)
p_1	Price of the original brand, $p_1 \geq p_2, p_1 > 0$, (given)

	Parameters
c	Unit cost coefficient for a given quality level, $c > 0$
m	Inconvenience cost of visiting and purchasing from the physical outlet, $m \geq k$
k	Inconvenience cost of visiting and purchasing from the physical store, $k > 0$
θ	Quality taste parameter of consumers, $\theta \sim U[0, b]$
D_1	Demand of the original brand, $D_1 \geq 0$
D_2	Demand of the outlet branch , $D_2 \geq 0$

Before opening the outlet channel, the retailer serves the market through the physical channel of its original brand. A consumer will want to purchase from the primary physical store if $\theta s_1 - p_1 - k \geq 0$. $\frac{p_1+k}{s_1}$ denotes the taste parameter of the last consumer who purchases the product from the primary physical store when there is only primary physical store in the market. Thus, the demand of the primary business is given by $(b - \frac{p_1+k}{s_1})$ and the monopolist retailer cannot cover the whole market by itself.

Lemma 3.5: Outlet branch may have on impact on the retailer's business if and only if $\frac{s_2}{m+p_2} \geq \frac{s_1}{k+p_1}$; i.e., "quality per dollar" for the outlet product is higher than that of the original product. The total market of the retailer expands by $\left(\frac{p_1+k}{s_1} - \frac{p_2+m}{s_2}\right)$, consumers with θ in $[\frac{p_1+k}{s_1}, \min(b, \frac{p_1+p_2+k-m}{s_1-s_2})]$ switch from the primary brand to the outlet branch.

Proof: Suppose now $\frac{s_1}{p_1+k} > \frac{s_2}{p_2+m}$. (In this case $\frac{s_1}{p_1} > \frac{s_2}{p_2}$ directly holds.)

A consumer of type θ prefers the primary brand to the outlet brand if and only if $(\theta s_1 - p_1 - k) - (\theta s_2 - p_2 - m) \geq 0$.

$$(\theta s_1 - p_1 - k) - (\theta s_2 - p_2 - m) = p_1 \left(\frac{\theta s_1}{p_1} - 1\right) - p_2 \left(\frac{\theta s_2}{p_2} - 1\right) + (m - k)$$

$$\geq (p_1 - p_2) \left(\frac{\theta s_2}{p_2} - 1\right) + (m - k) \text{ since } \frac{s_1}{p_1} > \frac{s_2}{p_2}.$$

$$\geq 0 \text{ if } \theta s_2 > p_2 \text{ and by assumption } p_1 - p_2 > 0$$

$$\text{and } m \geq k.$$

Thus, whenever a consumer is willing to buy from the outlet branch, he prefers the primary brand to the outlet brand and thus never purchases from the outlet. Therefore, if $\frac{s_1}{p_1+k} > \frac{s_2}{p_2+m}$, then the outlet branch has zero demand ($D_2 = 0$) and the primary branch demand is $D_1 = b - \frac{p_1+k}{s_1}$.

When we have $\frac{s_1}{p_1+k} \leq \frac{s_2}{p_2+m}$, we then have $\frac{p_1+k}{s_1} > \frac{p_2+m}{s_2}$. When $\frac{p_2+m}{s_2} \leq \theta \leq \frac{p_1+k}{s_1}$, the consumer will have a positive utility only if he purchases from the outlet (i.e., negative utility from the primary brand).

When $\frac{p_1+k}{s_1} \leq \theta$, the consumer will have positive utility from both branches and prefer the outlet if and only if $(\theta s_2 - p_2 - m) > (\theta s_1 - p_1 - k)$.

$$(\theta s_2 - p_2 - m) > (\theta s_1 - p_1 - k) \Rightarrow \theta < \frac{p_1 - p_2 + k - m}{s_1 - s_2} = \tilde{\theta}$$

Note that $\frac{p_1 k}{s_1} \leq \frac{p_1 - p_2 + k - m}{s_1 - s_2}$ is also guaranteed by the relationship $\frac{s_1}{p_1+k} \leq \frac{s_2}{p_2+m}$ and the details are left to the reader. To avoid trivial cases where $D_2 = 0$, we assume that "quality per dollar" for the outlet product is higher than that of the original product from this point on in our study. Thus, we have $\frac{s_1}{p_1+k} \leq \frac{s_2}{p_2+m}$. ∎

Figure 3.2 shows the total market share of the retailer after opening the outlet store. The demand of the retailer before (a) and after (b) the introduction of the outlet channel (and its split across channels) is shown.
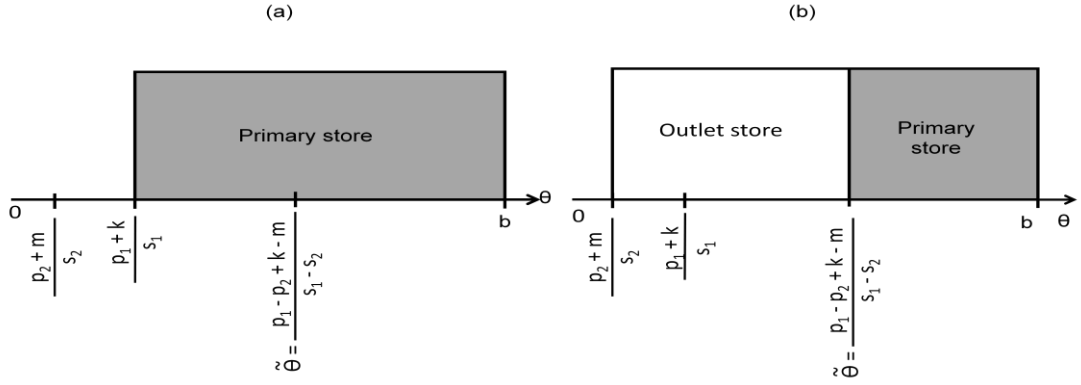
Figure 3.2: The demand of the retailer before (a) and after (b) the introduction of the outlet branch (and its split between channels) $((p_1-p_2+k-m)/(s_1-s_2)\leq b)$

Proposition 3.6: After opening the outlet channel, the demand for each channel is;

$$D_1 = \left(b - \frac{p_1-p_2+k-m}{s_1-s_2}\right)^+ \text{ for the primary business}$$

$$D_2 = \min(b, \frac{p_1-p_2+k-m}{s_1-s_2}) - \frac{p_2+m}{s_2} \text{ for the outlet business}$$

Proof: The demand of the channels follows from Lemma 3.5 and its proof. ∎

The retailer's problem can be written as follows:

$$\max \Pi_{s_2,p_2} = \left(b - \frac{p_1-p_2+k-m}{s_1-s_2}\right)(p_1 - cs_1{}^2) + \left(\frac{p_1-p_2+k-m}{s_1-s_2} - \frac{p_2+m}{s_2}\right)(p_2 - cs_2^2)$$

$$(\text{Eq.3.17})$$

s.to $b - \frac{p_1-p_2+k-m}{s_1-s_2} \geq 0$ (Eq.3.18)

$\frac{s_2}{m+p_2} \geq \frac{s_1}{k+p_1}$ (Eq.3.19)

$s_1 \geq s_2$ (Eq.3.4)

$p_1 \geq p_2$ (Eq.3.5)

$p_2 > 0$ (Eq.3.6)

$s_2 > 0$ (Eq.3.7)

Constraint (3.18) and (3.19) ensures that the retailer has the nonnegative demand for respective the primary and outlet brand after opening the outlet business. Note that

either can be zero. The objective function of the model, (3.17), maximizes the total profit given the retailer's original brand position. Constraints (3.4) and (3.5) are upper limits for decision variables that identify with the outlet business. Constraints (3.6) and (3.7) are non-negativity conditions for decision variables.

Proposition 3.7: The profit function of the retailer, (Eq.3.17), is not jointly concave in p_2 and s_2. Nor is it convex.

Proof: The matrix S, Hessian matrix of profit function, is written below:

$$S = \begin{bmatrix} \dfrac{2s_1}{s_2(s_2-s_1)} & \dfrac{k-m+2p_1-2p_2}{(s_1-s_2)^2} + \dfrac{m+2p_2}{s_2^2} \\ \dfrac{k-m+2p_1-2p_2}{(s_1-s_2)^2} + \dfrac{m+2p_2}{s_2^2} & \dfrac{2(p_2-p_1)(p_2-p_1-k+m)}{(s_2-s_1)^3} - \dfrac{2p_2(m+p_2)}{s_2^3} \end{bmatrix}$$

The leading principal minors are;

$$\Delta_1 = \frac{2s_1}{s_2(s_2-s_1)} \quad \text{(of order one)}$$

$$\Delta_2 = \left(\frac{2s_1}{s_2(s_2-s_1)}\right)\left(\frac{2(p_2-p_1)(p_2-p_1-k+m)}{(s_2-s_1)^3} - \frac{2p_2(m+p_2)}{s_2^3}\right) - \left(\frac{k-m+2p_1-2p_2}{(s_1-s_2)^2} + \frac{m+2p_2}{s_2^2}\right)^2 \quad (\text{of}$$

order two)

Before evaluation, we want to summarize positive and negative definiteness of matrix S.

- S is negative definite if $(-1)^k \Delta_k > 0$ where k= {1, 2} for all leading principal minors. Therefore, the profit function is jointly concave in p_2 and s_2.

- S is positive definite if $\Delta_k > 0$ where k= {1, 2} for all leading principal minors. Therefore, the profit function is jointly convex in p_2 and s_2.

The principal minors depend on the decision variables (s_2 and p_2) and change signs in the feasible region. We show that the concavity or convexity conditions are not satisfied by a counter example. The parameters are listed in Table 3.6.

Table 3.6: The parameters of the example

Parameter	Value
s_1	6
p_1	6
m	0.9
k	0.1

The leading principal minors are calculated with these parameter for $s_2 = \{1,2,3,4,5\}$ and $p_2 = \{1,2,3,4,5\}$. Table 3.7 shows leading principal minors for some (s_2, p_2) values that satisfy both (A.3.3) and $\frac{s_2}{m+p_2} \geq \frac{s_1}{k+p_1}$.

Table 3.7: The leading principal minors for some (s_2, p_2) values

s_2	p_2	Δ_1	Δ_2
5	1	-2.4	14.085
5	2	-2.4	6.961
5	3	-2.4	2.142
5	4	-2.4	-0.372
4	1	-1.5	1.807
4	2	-1.5	0.635
4	3	-1.5	0.026
3	1	-1.333	0.454
3	2	-1.333	0.029
2	1	-1.5	0.006

Here, the first leading principal minor is always negative. However, the second leading principal minor is sometimes negative and sometimes positive. The joint concavity requires $\Delta_1 < 0$ and $\Delta_2 > 0$. However, there is case where the second leading principal minor is negative. Thus, in our feasible range, the profit function may not be jointly concave in p_2 and s_2. The joint convexity requires $\Delta_1 > 0$. Thus, we conclude that the profit function is not jointly convex in p_2 and s_2. ∎

Since the profit function is not jointly concave or convex in s_2 and p_2, we analyze its behavior with respect to s_2 and p_2 individually next.

When the quality level of the outlet brand is given, the total profit becomes a function of p_2. The profit function of the retailer with respect to non-negative demand of outlet channel is written below,

$$\Pi = \left(b - \frac{p_1 - p_2 + k - m}{s_1 - s_2} \right) (p_1 - cs_1{}^2) + \left(\frac{p_1 - p_2 + k - m}{s_1 - s_2} - \frac{p_2 + m}{s_2} \right) (p_2 - cs_2^2)$$

$$if \ \frac{s_2}{p_2 + m} \geq \frac{s_1}{p_1 + k} \ and \ b - \frac{p_1 - p_2 + k - m}{s_1 - s_2} \geq 0$$

If the profit function is rearranged based on p_2, it becomes ;

$$\Pi = \left(b - \frac{p_1 - p_2 + k - m}{s_1 - s_2} \right) (p_1 - cs_1{}^2) + \left(\frac{p_1 - p_2 + k - m}{s_1 - s_2} - \frac{p_2 + m}{s_2} \right) (p_2 - cs_2^2)$$

$$if \max \left(0, p_1 + k - m - b(s_1 - s_2) \right) < p_2 \leq \frac{s_2(p_1 + k) - s_1 m}{s_1} \quad \text{(Eq.3.20)}$$

Proposition 3.8: For a given s_2, the retailer's profit function (Eq.3.20) is concave in p_2 in the feasible region of p_2.

Proof: The second order derivative with respect to p_2 is given below,

$$\frac{d^2\Pi}{d(p_2)^2} = \frac{2s_1}{s_2(s_2 - s_1)}$$

The ($s_2 \leq s_1$) implies that the SOC of the profit function is negative. ∎

We can find the maximizer p_2 (for a given s_2) using the FOC of the profit function as follows;

Step 0: Find p_2^+ which satisfies $\frac{\partial \Pi}{\partial p_2} = 0$.

Note that $\frac{d\Pi}{dp_2} = \frac{(k + 2p_1)s_2 - s_1[m + 2p_2 + cs_2(s_1 - s_2)]}{(s_1 - s_2)s_2}$ and $\quad p_2^+ = \frac{(k + 2p_1)s_2 - ms_1 - cs_2 s_1(s_1 - s_2)}{2s_1}$

Step 1: If $p_2^+ \in (\max(0, p_1 + k - m - b(s_1 - s_2)), \frac{s_2(p_1 + k) - s_1 m}{s_1})$, then $p_2^* = p_2^+$ and $\Pi^* = \Pi(p_2^+)$.

Step 2: If $p_2^+ \geq \frac{s_2(p_1+k)-s_1 m}{s_1}$, then set $p_2^* = \frac{s_2(p_1+k)-s_1 m}{s_1}$. When $p_2^* = \frac{s_2(p_1+k)-s_1 m}{s_1}$, the demand of outlet equals to zero and the profit function equals to $\left(b - \frac{p_1+k}{s_1} \right) \cdot (p_1 - cs_1{}^2)$.

When p_2 is given, the total profit becomes a function of s_2 as a decision variable as follows:

The profit function of the retailer based on s_2 is written below,

$$\Pi = \left(b - \frac{p_1-p_2+k-m}{s_1-s_2} \right)(p_1 - cs_1{}^2) + \left(\frac{p_1-p_2+k-m}{s_1-s_2} - \frac{p_2+m}{s_2} \right)(p_2 - cs_2^2)$$

$$if \ \frac{s_1(p_2+m)}{p_1+k} \leq s_2 < \min\left(s_1, s_1 - \frac{p_1-p_2+k-m}{b}\right) \qquad \text{(Eq.3.21)}$$

Proposition 3.9: For a given p_2, the retailer's profit function (Eq.3.21) is concave in s_2 in the feasible region of s_2.

Proof: The second order derivative with respect to s_2 is given below,

$$\frac{d^2\Pi}{d(s_2)^2} = \frac{2(p_2-p_1)(p_2-p_1-k+m)}{(s_2-s_1)^3} - \frac{2p_2(m+p_2)}{s_2^3}$$

It is known that $\tilde{\theta} > 0, s_1 \geq s_2$ and $p_1 \geq p_2$. Hence, the first term of the SOC of the profit function with respect to s_2 is negative. As a result, the SOC of the profit function is negative. ∎

We can find the maximizer s_2 (for a given p_2) using the FOC of the profit function as follows;

Step 0: There are four s_2 roots (s_2^+) to satisfy $\frac{\partial \Pi}{\partial s_2} = 0$.

Note that $\frac{d\Pi}{ds_2} = c(k+p_1) - \frac{(p_1-p_2)(k-m+p_1-p_2)}{(s_1-s_2)^2} + \frac{p_2(m+p_2)}{s_2^2}$

Step 1: If $s_2^+ \in (\frac{s_1(p_2+m)}{p_1+k}, \min(s_1, s_1 - \frac{p_1-p_2+k-m}{b}))$, then $s_2^* = s_2^+$ and $\Pi^* = \Pi(s_2^+)$.

Step 2: If there is no s_2^+ which is in that interval, put differently, $s_2^+ \leq \frac{s_1(p_2+m)}{p_1+k}$, then

set $s_2^* = \frac{s_1(p_2+m)}{p_1+k}$. When $s_2^* = \frac{s_1(p_2+m)}{p_1+k}$, the demand of outlet channel equals to zero

and the profit function equals to $\left(b - \frac{p_1+k}{s_1}\right)(p_1 - cs_1{}^2)$.

3.3 Numerical analysis on the outlet decisions with inconvenience costs

We studied the outlet business decisions in the presence of inconvenience costs. However, the total profit of the retailer, (Eq.3.17) is neither jointly concave nor jointly convex in p_2 and s_2. For that reason, in this section, we aim to analyze the effect of inconvenience cost on the optimal quality level and price of the new product, demands of the primary store and outlet channel, market size and profit of the retailer.

In our numerical experiments, we used 40 for b, 0.1 for c, tested for k in [0, 3] with the amount of increment by 0.01 and m in [k, 3] with the amount of increment by 0.01. We set 0.01 as the lower bound of both s_2 and p_2 and limited our experiments to the cases where $s_2 \leq s_1$ and $p_2 \leq p_1$.

We create two groups of trials each of which contains four combinations of s_1 and p_1. All trials are presented in Table 3.8

Table 3.8: The first and second group of trials

		The first group of trials			The second group of trials			
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8
s_1	8	8	8	8	8	6	4	2
p_1	8	10	12	14	8	8	8	8

We use 0.01 as the amount of increment for the feasible interval of s_2. In this respect, different increment is used/calculated for the p_2 to equalize the number of s_2 and p_2 values; i.e., 0.0064 for trial 1, 0.009 for trial 2, 0.0117 for trial 3, 0.0144 for trial 4, 0.0086 for trial 6, 0.0132 for trial 7, 0.0279 for trial 8. Thus, in each trial, we form the (s_2, p_2) matrix for the feasible interval of s_2 and p_2. In each trial, the

optimal outlet quality level and price is chosen from this matrix in order to maximize the retailer's total profit for a set of b, c, m and k.

We find that for a given k, as m increases, s_2^*, p_2^*, D_2^* and profit of the retailer decreases but D_1^* increases. Thus, demand and profit margin of the outlet decrease. Here, p_2^* decreases because the retailer compensates for the high outlet inconvenience. In this environment, s_2^* will be low because the retailer cannot afford a high product cost any more. When m is high, the retailer has to position the outlet business as a lower quality, lower price alternative. As a result, the total outlet demand decreases whereas the primary brand demand increases. For all trials, for a given k, as m increases, profit margin of the outlet decreases. Figure 3.3 and 3.4 show s_2^* versus m and p_2^* versus m plots for trial 2 when k=0.5, respectively. See Appendix A for D_1^* versus m, D_2^* versus m, retailer's profit versus m, total market versus m and $(p_2^* - cs_2^{*2})$ versus m plots for trial 2 when k=0.5 and related data.
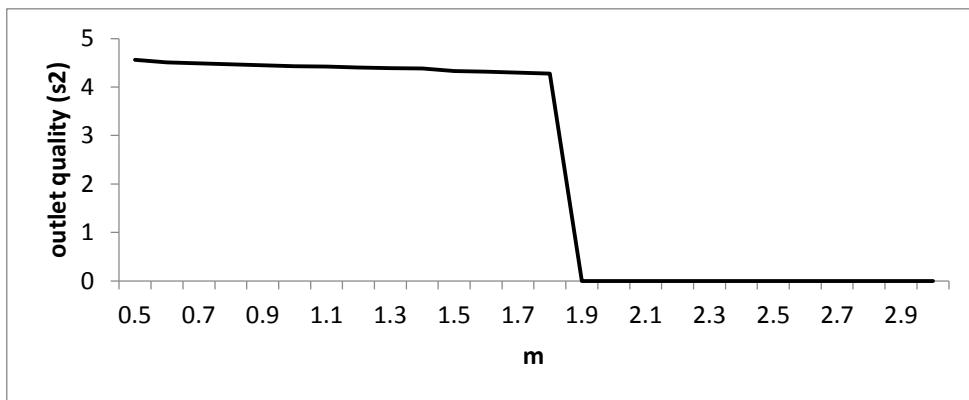
Figure 3.3: s_2^* versus m plot for trial 2 when k=0.5
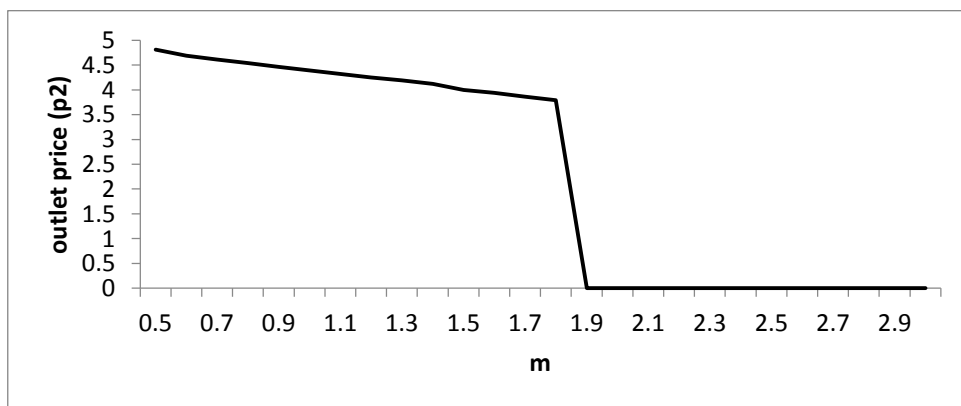
Figure 3.4: p_2^* versus m plot for trial 2 when k=0.5

When m and k are equal, as k (and m) increases, s_2^*, p_2^* increase, D_1^*, D_2^* and total profit of the retailer decrease. Here, s_2^* and p_2^* increase and the outlet position approaches that of the primary brand. The retailer starts to address the higher segment to expand its total market. Note that $\theta = \frac{p_2^* + m}{s_2^*}$ (the taste preference of the last consumer who prefers to outlet store) and $\tilde{\theta}$ go up. Thus, the total market of the retailer diminishes since the elevated outlet position is not sufficient to compensate for the inconvenience for both brands. Although the profit margin of the outlet increases, the retailer's total profit decreases. Figure 3.5 shows retailer's profit versus m plot for trial 1 when m=k. See Appendix B for s_2^* versus m, p_2^* versus m, D_1^* versus m, D_2^* versus m plots for trial 1 when m=k and related data.
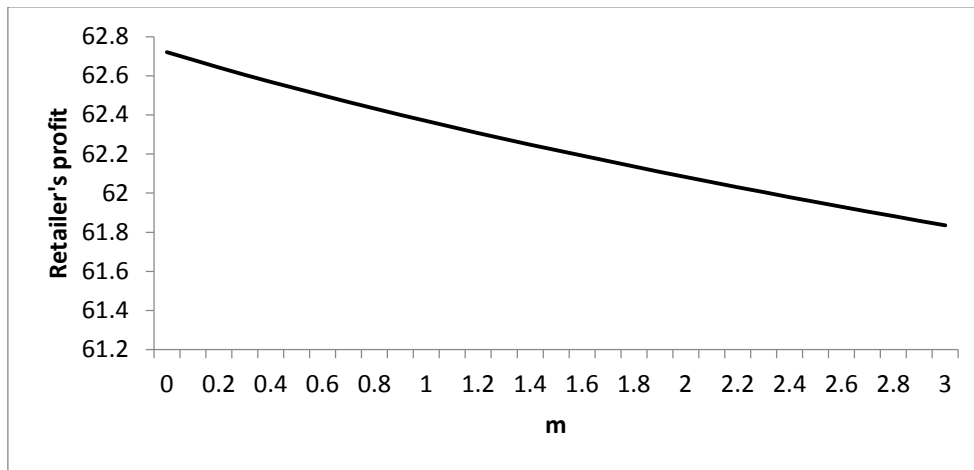
Figure 3.5: Retailer's profit versus m plot for trial 1 when m=k

In Section 3.1, we find the optimal outlet brand positions when there are no inconvenience costs. $s_2^*, p_2^*, D_1^*, D_2^*$ and the profit impact of opening the outlet store on the retailer's profit ($\Delta\Pi$) are seen in Table 3.3. In our numerical experiments, we observed that when m=k, as c increases, p_2^*, D_1^* decreases but D_2^* and $\Delta\Pi$ increases. Thus, more consumers prefer the outlet store to the primary store. The profit impact of opening the outlet store on the retailer's profit increases although the profit margin of the outlet decreases. These findings correspond to the theoretical inferences in Section 3.1.

Before evaluation the each set of trials deeply, we find that as m increases, outlet price and quality decreases. Even if the outlet quality and price decreases with

increase with m, the decrease in the demand of the outlet cannot be prevented. Thus, the outlet impact on the market and retailer's profit decreases.

After inferences with initial observations for all trials, we want to evaluate the each set of trials deeply. Firstly, we focus on the first group of trial. To make inferences, we plot s_2^* versus m, p_2^* versus m, D_1^* versus m, D_2^* versus m, $(p_2^* - cs_2^{*2})$ versus m, total market versus m and retailer's profit versus m graphs for a given k.

In the first group of trials, trial 1 has the lowest p_1 value when all trials have the same s_1 value. Thus, when there is only primary brand in the market, trial 1 has the largest market potential. It can be also that trial 1 is the most competitive one in the first set of trials.

We observe that trial 1 produces the lowest p_2^* value. As the primary brand gets more competitive in terms of price, the outlet price level tends to be lower as well. Figure 3.6 show p_2^* for the first group of trials when k=0.3.

Figure 3.6: p_2^* for the first group of trials when k=0.3

We see that trial 1 has the highest D_1^* value after opening the outlet store. Trial 1 sustains the largest market potential with the outlet store as well. After opening the outlet store, the rank of D_1^* will not be changed, but of course, D_1^* values will be decreased. See Figure C.1 for D_1^* of the first group of trials when k=0.3 in Appendix C.

We see that s_2^* value for the first group of trials is almost identical to each other. Figure 3.7 shows s_2^* for the first group of trials when k=0.3.
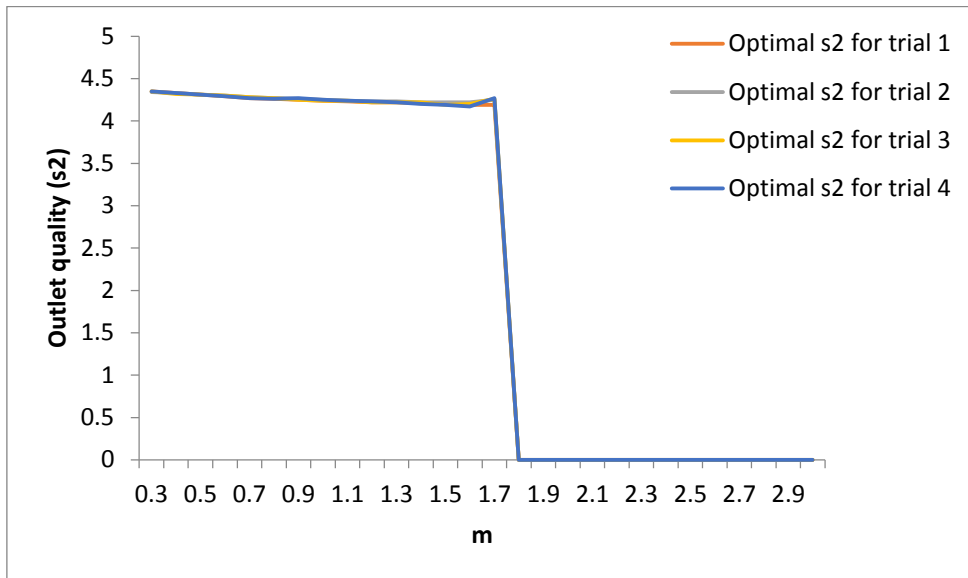
Figure 3.7: s_2^* for the first group of trials when k=0.3

We see that D_2^* value for the first group of trials is close to each other. It is the most interesting observation. Trial 1 is expected to have the highest D_2^* value since it has the lowest p_2^* value with about the same s_2^* values across the first set of trials. **Thus, the resulting s_2^* and D_2^* values are rarely impacted by the price competitiveness of the primary brand.** Figure 3.8 shows D_2^* for the first group of trials when k=0.3.
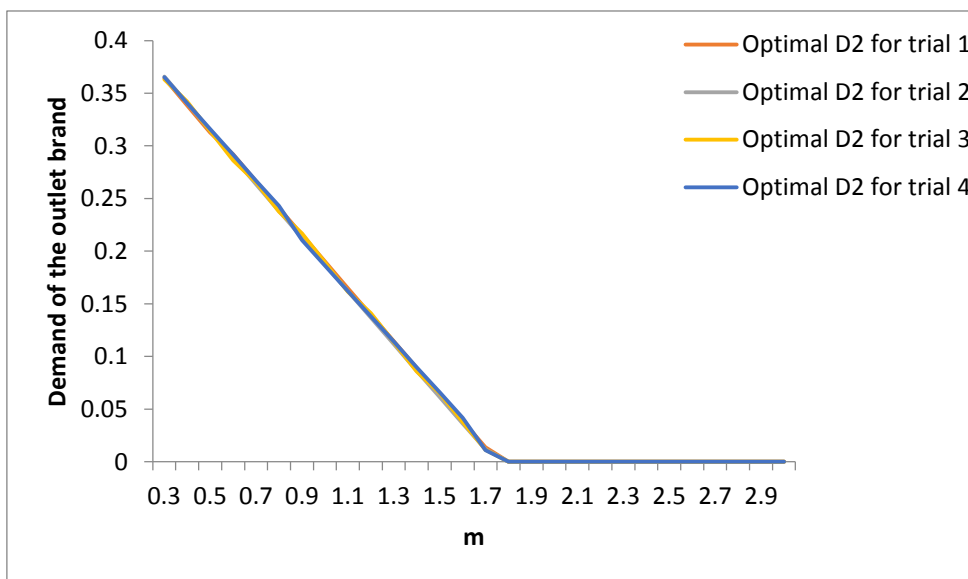
Figure 3.8: D_2^* for the first group of trials when k=0.3

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We see that trial 4 produces the highest profit. D_2^* values are close across the first set of trials whereas trial 1 has the highest D_1^* value and trial 4 has the largest profit margin of products. Thus, profit margins of the primary and outlet businesses are more significant than market share in determining the profit of the retailer.

Later, we evaluate the second group of trials. Again, we plot s_2^* versus m, p_2^* versus m, D_1^* versus m, D_2^* versus m, $(p_2^* - cs_2^{*2})$ versus m, total market versus m and retailer's profit versus m graphs for a given k.

In the second group of trials, trial 5 has the highest s_1 value when all trials have the same p_1 value. When there is only the primary brand in the market, trial 5 has the largest market potential. It can also be that trial 5 is the most competitive one in the second group of trials.

We observe that trial 5 produces the highest s_2^* and lowest p_2^*. As the primary brand gets more competitive in terms of quality, the outlet quality increases and the outlet price level decreases. Thus, trial 5 has the lowest profit margin of the outlet business. Trial 5 sustains a viable outlet business in high inconvenience costs as well. Figure 3.9 and 3.10 show s_2^* versus m and p_2^* versus m plot for the second group of trials when k=0.3.

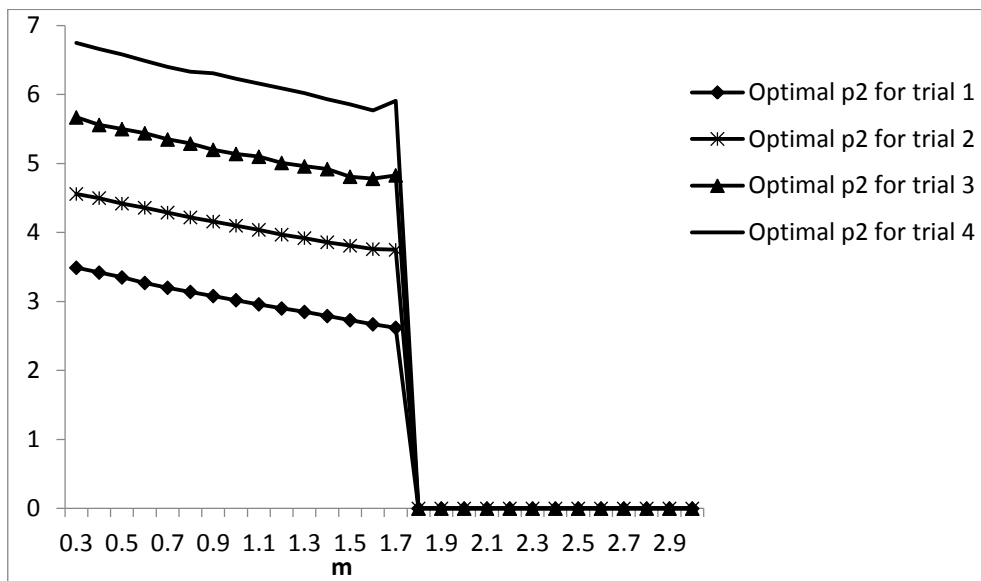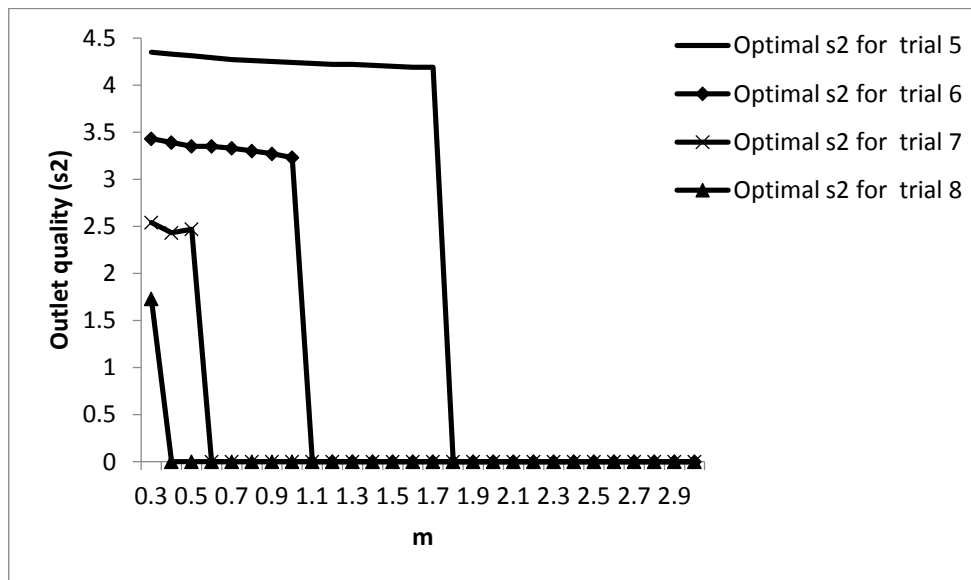Figure 3.9: s_2^* for the second group of trials when k=0.3
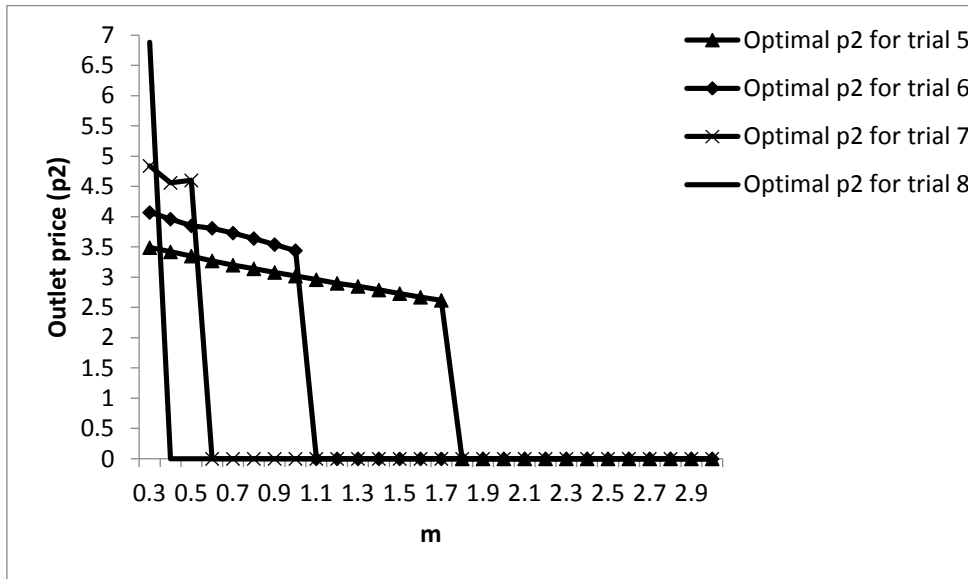
Figure 3.10: p_2^* for the second group of trials when k=0.3

We see that trial 5 produces the highest D_1^* value after opening the outlet store. We observe that trial 5 produces the largest D_2^* naturally since it produces the highest s_2^* and the lowest p_2^* value. Thus, trial 5 has the largest total market. Figure 3.11 shows D_2^* versus m plot for the second group of trials when k=0.3. See Figure D.1, D.2 for the second group of trials when k=0.3 in terms of D_1^* and retailer's total market in Appendix D.
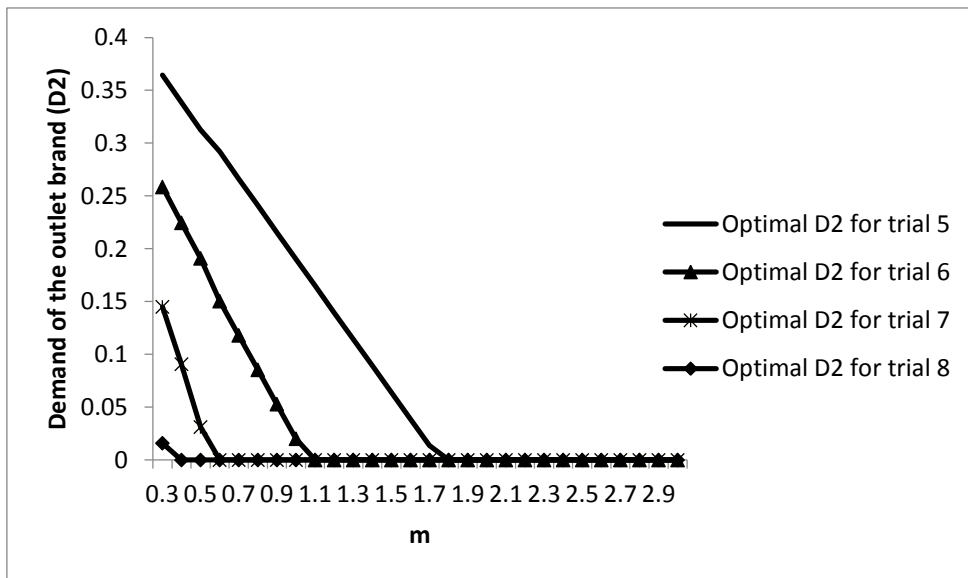
Figure 3.11: D_2^* for the second group of trials when k=0.3

We see that trial 8 produces the highest profit. Trial 5 has the largest D_1^* and D_2^* and trial 8 has the highest profit margin of product. We find that the profit margins of the primary and outlet business are more significant than market share to determine the profit of the retailer as observed in the first set of trials.

It is observed different from the first group of the trials that as the primary brand gets more competitive in terms of quality level, it gets easier to position and justify an outlet brand. Outlet becomes more competitive both in price and quality; as opposed to just price that we observed in the first group of trials.

In this chapter, we characterize the outlet brand positions and evaluate the profit impact of the outlet store. The analysis is conducted in two different settings. In the first setting, inconvenience costs are neglected. In Section 3.1, we find that opening the outlet business is preferable both the retailer and the consumer. The retailer manages to expand its total market although the outlet business cannibalizes the demand of the primary brand. Quality per dollar for the outlet is higher than quality per dollar for the primary brand. The gap between quality per dollar for the outlet and primary brand widens as unit cost (c) or the primary brand quality level increases. The outlet quality increases and the outlet price decreases with the primary brand quality level. Thus, the retailer elevates its outlet quality as primary brand quality level increases, and shifts it to focus towards the higher consumer segments. Outlet quality increases with the primary brand quality and does not depend on cost and prices at all. As the unit cost (c) increases, the profit gain and the generated market with the outlet increase. The retailer's main tradeoff is between market expansion and cannibalization of the primary store and its profit margin.

According to primary brand strategies, retailers are classified in two groups: myopic and non-myopic retailer. We find that non-myopic approach is preferable for the retailer in terms of profit since it enables firm to set higher price. Also, the consumer is offered a higher quality level of the primary brand when the retailer is non-myopic.

In the second part of the chapter, we take into account inconvenience costs associated with visiting a brick-and-mortar retailer. An extensive numerical study is conducted to evaluate the changes in the outlet product position, demand of primary and outlet business and total profit of the retailer in the presence of inconvenience cost since the analytical approach to find the optimal outlet product position is not suitable.

In Section 3.3, we create two groups of trials each and parameters set for numerical study. The two groups are different from each other in terms of competitiveness in price and quality level. As the primary brand gets more competitive in terms of quality and price, the outlet price level tends to be lower. The price competitiveness of the primary brand does not impact the quality level and demand of the outlet brand. However, as the primary brand gets more competitive in terms of quality, it produces the larger market size for the outlet brand and the outlet quality level tends to be higher. Independent of competitiveness in price and quality level, we find that the profit margins of the primary and outlet business are more significant than market share in determining the profit of the retailer. In the second set of trials, trial 5 sustains a viable outlet business longest. We find that as the primary brand gets more competitive in terms of quality level, its outlet business becomes more robust (viable) than others.

CHAPTER 4

ONLINE CHANNEL DECISION FOR A BRICK-AND-MORTAR RETAILER

In this chapter, we are interested in a monopolist retailer's online channel decision when the original brand is already available in the physical store.

In real life visiting the physical stores is associated with some inconvenience. The inconvenience cost may comprise physically going to the store; time spent on the activity as well as the stock-out risk that may send the consumer home empty-handed. Consumers are freed from the physical inconvenience cost when they purchase online. However, buying from the online channel does not offer immediate gratification; the customer now needs to wait for a pre-specified time to receive his product and now faces new risks associated with product properties such as the fit, color, and size. Hereby, an additional dimension on which consumers differ emerges; sensitivity to online services and promised delivery time.

Online service quality may involve all customer services provided by the online store, the convenience of return process, and promised delivery time windows as well as shipping charges. In the rest of this thesis, we interchangeably refer to this dimension as promised delivery time. Here, we use t to refer to this dimension in a reverse way; that is, the lower t is, the higher the online service quality. Note that t here represents all the online services offered to the customer including but not limited to the promised delivery time.

In this chapter, the retailer's major decision is to determine the online service quality (the promised delivery time, t) to maximize the retailer's total profit for a given set of original brand positions (p_1, s_1) and inconvenience cost of going to store (k).

We assume that the promised delivery time can be set in a feasible time interval $[\underline{t}, \overline{t}]$ where $\underline{t} > 0$ and all practical feasible delivery times are in this interval. In practice, online service and delivery options are limited by external factors. For example overnight delivery is the faster option available to most retailers.

In this chapter, a consumer wants to maximize the following utility function:

$$U = \begin{cases} \theta s_1 - p_1 - k, & \text{if he purchases from the primary physical store} \\ \theta s_1 - p_1 - dt, & \text{if he puchases from the primary online store} \\ \text{reservation utility } (0), & \text{otherwise} \end{cases}$$

Consumers can observe the product qualities and prices available before they decide to buy. They buy a maximum of one product. They purchase only when their net utility is greater than or equal to their reservation utility, assumed as 0 in our study.

The utility of the consumer who uses the online channel decreases as s/he waits, the utility of the consumer who purchases from the offline store decreases due to the inconvenience regarding the physical channel.

We assume that consumers differ in willingness to pay for quality and in sensitivity to online services and promised delivery time while purchasing at the online channel; i.e.; consumer types are distributed uniformly on [0, b] according to their types d and θ, where b>0. We assume that b is high enough to enable a profitable business for the retailer (i.e.; for nonnegative market share and margin). In this chapter, we assume that $(b - \frac{p_1+k}{s_1}) \geq 0$ (A.4.1) and $(b - \frac{k}{\underline{t}}) > 0$ (A.4.2) to ensure nonnegative market share.

A consumer's type represents his marginal willingness to pay for increments of an attribute. A higher type of consumer (i.e.; with a higher taste parameter) is willing to pay more for a given quality level than a lower type. Here, quality represents any attribute of the retailer which all consumers prefer more to less, ceteris paribus, such as quality of material used in the product, store service standards and/or store design.

A consumer of type d has a diminished utility by dt when he has to wait for a period t before receiving the product. As d increases, the consumer becomes more time

sensitive (i.e., impatient), and thus he starts preferring the physical store to the online store since the physical store offers instant gratification.

The retailer's unit cost increases with its chosen quality level. We use a quadratic function to represent diminishing returns on quality, thus unit gross margin of the retailer is $(p_1 - cs_1^2)$. We assume that the unit profit margin of the primary brand is nonnegative since the retailer is profitable. That is, $(p_1 - cs_1^2) > 0$ (A.3.2). We assume that there are no fixed costs.

The parameters, decision variables and notations used in this chapter are presented in Table 4.1.

Table 4.1: Notation

Decision Variable(s)	
s_1	Quality level of the original brand, $s_1 > 0$ (given)
p_1	Price of the original brand, $p_1 > 0$ (given)
t	The online service quality set for the online channel, $t \in [\underline{t}, \overline{t}\,]$
Parameters	
c	Unit cost coefficient for a given quality level, $c > 0$
k	Inconvenience cost of visiting and purchasing from the physical store, $k > 0$
θ	Quality taste parameter of consumers, $\theta \sim U[0, b]$
d	Sensitivity index to online services and promised delivery time of consumer, $d \sim U[0, b]$
z	Online channel cost coefficient, $z > 0$
D_1	Demand of the original brand, $D_1 \geq 0$
D_2	Demand of the outlet branch, $D_2 \geq 0$

Proposition 4.1: For a given positive promised delivery time at the online channel (t>0),

i. Consumers who purchase from the physical store decrease from all $\theta \in [\tilde{\theta}, b]$ and all $d \in [0, b]$ to all $\theta \in [\tilde{\theta}, b]$ and $d \in [\tilde{d}, b]$ where $\tilde{\theta} = \frac{p_1 + k}{s_1}$

ii. Consumers with $\theta \in [\tilde{\theta}, b]$ and $d \in [0, \tilde{d}]$ switch from the physical channel to the online channel. The retailer also expands its market by $\frac{k^2}{2s_1 t}$ through the online channel.

iii. The demand characterization of the channels is

$$D_{\text{physical store}} = \left(b - \frac{k}{t}\right)\left(b - \frac{p_1 + k}{s_1}\right)$$

$$D_{\text{onlinechannel}} = \frac{0.5k}{t}\left(2b - \frac{p_1 + k}{s_1} - \frac{p_1}{s_1}\right)$$

Proof: A consumer will want to purchase from the online store if $\theta s_1 - p_1 - dt \geq 0$ and from the physical store if $\theta s_1 - p_1 - k \geq 0$. He will choose the online store if

$$\theta s_1 - p_1 - dt \geq \theta s_1 - p_1 - k \ .$$

So for any consumer type with θ so that $\theta s_1 - p_1 - k \geq 0$, if $d \leq \tilde{d} = \frac{k}{t}$, then the consumer will prefer the online to offline store.

If $\theta s_1 - p_1 < k$ but $\theta s_1 - p_1 \geq 0$, then consumer will purchase from the online store if $\theta s_1 - p_1 - dt \geq 0$ ($d \leq \frac{\theta s_1 - p_1}{t}$).

Then, $D_{\text{physical store}}$ will consist of consumers that $\theta \in [\tilde{\theta}, b]$ and $d \in [\tilde{d}, b]$. Thus, $D_{\text{physical store}} = \left(b - \frac{k}{t}\right)\left(b - \frac{p_1 + k}{s_1}\right)$.

The demand of the online store will become;

$$D_{\text{onlinechannel}} = \frac{0.5k}{t}\left(2b - \frac{p_1 + k}{s_1} - \frac{p_1}{s_1}\right). \qquad\qquad \blacksquare$$

Figure 4.1 shows overall demand of the retailer after opening the online channel. The triangular area is the extra demand that the retailer generates through the online channel.
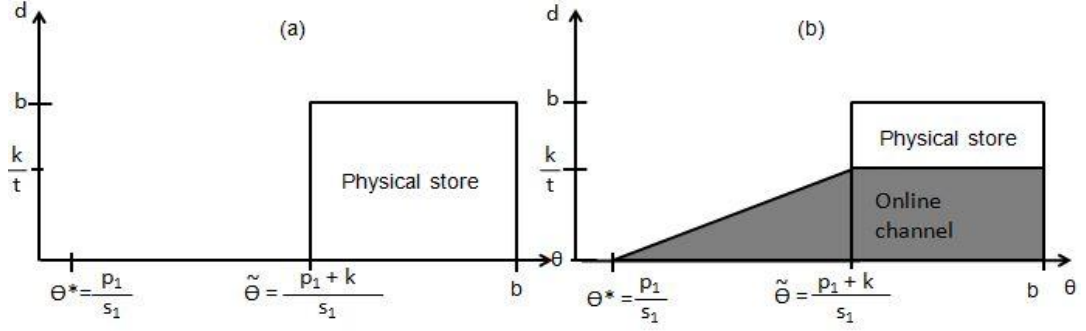
Figure 4.1: Total demand after the introduction of the online channel (and its split among channels)

Corollary 4.1: The retailer's total market increases as t decreases.

Proof: According to Proposition 4.1, the total market of retailer expands by $\frac{k^2}{2s_1t}$. As t decreases, market expansion increases. ∎

Based on Corollary 4.1, if the retailer did not have any cost for online services, he would want to set t as low as possible. However, this is not a realistic case. Generally, serving through the online channel creates additional costs for the retailer including call centre management, courier services, returns management and etc. The retailer has to incur a certain cost to make the delivery of the product in the promised time window t and provide online services. For that reason, there arises the cost versus the market expansion tradeoff associated with setting the delivery time. We assume that the firm incurs online channel cost $\frac{z}{t^2}$ as the unit cost of serving an online customer where z is the online channel cost coefficient. As online services improve; i.e., as t decreases, the cost incurred to deliver it increases exponentially. We assume that z (online channel cost coefficient) is high enough to discourage the firm to set t=0. (or serve the market perfectly.) In this chapter, we assume that

$$z > \frac{(\bar{t})^2 k(p_1 - cs_1^2)}{6(2bs_1 - 2p_1 - k)} \text{ (A.4.3).}$$

The unit profit margins of the primary and online channel are $(p_1 - cs_1^2)$ and $\left(p_1 - cs_1^2 - \frac{z}{t^2}\right)$, respectively.

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If we take into account the costs as well, the total profit of the retailer can be summarized as follows:

$$\Pi = \left(b - \frac{k}{t}\right)\left(b - \frac{p_1+k}{s_1}\right)(p_1 - cs_1^2) + \frac{0.5.k}{t}\left(2b - \frac{p_1+k}{s_1} - \frac{p_1}{s_1}\right)(p_1 - cs_1^2 - \frac{z}{t^2})$$

After all, the retailer's problem can be modeled as,

$$max_{t>0}\left(b - \frac{k}{t}\right)\left(b - \frac{p_1+k}{s_1}\right)(p_1 - cs_1^2)$$

$$+ \frac{0.5.k}{t}\left(2b - \frac{p_1+k}{s_1} - \frac{p_1}{s_1}\right)\left(p_1 - cs_1^2 - \frac{z}{t^2}\right) \qquad (\text{Eq.4.1})$$

s.to $t \in [\underline{t}, \overline{t}]$ $\qquad$ (Eq.4.2)

The objective function of the model, (4.1), maximizes the total profit when the retailer serves the customers through both channels with its original brand.

The retailer's problem is a nonlinear maximization problem.

Lemma 4.1: The profit function, $\Pi = \left(b - \frac{k}{t}\right)\left(b - \frac{p_1+k}{s_1}\right)(p_1 - cs_1^2) + \frac{k}{t}\left(\frac{2bs_1-2p_1-k}{2s_1}\right)(p_1 - cs_1^2 - \frac{z}{t^2})$ is concave in the promised delivery time t when $t \in [\underline{t}, \overline{t}]$.

Proof: The second order derivative of the objective function with respect to t is given below,

$$\frac{d\Pi^2(t)}{dt^2} = \frac{k(6z(k+2p_1-2bs_1)+kt^2(p_1-cs_1^2))}{s_1 t^5}.$$

It is clear that $\frac{k}{s_1 t^5}$ is always positive since $t > 0$. Hence, if $6z(k + 2p_1 - 2bs_1) + kt^2(p_1 - cs_1^2) < 0$, then $\frac{d\Pi^2(t)}{dt^2} < 0$. This expression is a second-degree polynomial in t. The discriminant of this expression is given below,

$$\Delta = 24zk(p_1 - cs_1^2)(2bs_1 - 2p_1 - k)$$

(A.4.1) and (A.3.2) ensure that the discriminant is always positive. Then the SOC of the profit function becomes negative when $t \in \left[-\sqrt{\frac{6z(2bs_1-2p_1-k)}{k(p_1-cs_1^2)}}, \sqrt{\frac{6z(2bs_1-2p_1-k)}{k(p_1-cs_1^2)}}\right]$.

Thus, the SOC is always negative in feasible region $[\underline{t}, \overline{t}]$. $\blacksquare$

The optimal solution of the maximization problem is given in Proposition 4.2.

Proposition 4.2: While opening an online channel, the retailer sets $t^* = \min$ $(\max(t^\circ, \underline{t}), \bar{t})$ where $t^\circ = \sqrt{\dfrac{3z(2bs_1 - 2p_1 - k)}{k(p_1 - cs_1{}^2)}}$.

i. If $z \geq z_{critic\,4\,\bar{t}}$, then $t^* = \bar{t}$.

- If $z \leq z_{limit\,4}$, then the retailer finds it profitable to open the online channel.
- Otherwise, the profit impact of the online channel on the retailer's profit ($\Delta\Pi_4$) is always negative.

ii. If $z < z_{critic\,4\,\bar{t}}$, $t^* = \max\{t^\circ, \underline{t}\}$ and profit impact of the online channel on the retailer's profit ($\Delta\Pi_4$) always nonnegative.

where $z_{critic\,4\,\bar{t}} = \dfrac{k(p_1 - cs_1^2)(\bar{t})^2}{3\,(2bs_1 - 2p_1 - k)}$ and $z_{limit\,4} = \dfrac{k(p_1 - cs_1^2)(\bar{t})^2}{(2bs_1 - 2p_1 - k)}$.

Proof: The first order derivative with respect to t is given below,

$$\frac{d\Pi(t)}{dt} = \frac{k[\,3z(2bs_1 - 2p_1 - k) + kt^2(c(s_1)^2 - p_1)]}{2s_1 t^4}$$

Solving $\dfrac{d\Pi(t)}{dt} = 0$ yields $t^\circ = \sqrt{\dfrac{3z(2bs_1 - 2p_1 - k)}{k(p_1 - cs_1{}^2)}}$. If $t^\circ \in [\underline{t}, \bar{t}]$, then t° is the maximizer of the total profit function.

The change (i.e., decrease) in the profit of the retailer with respect to drop in the profit of primary store is

$$\Delta\Pi_{primary\,store} = -\frac{k}{t^*}\left(b - \frac{p_1 + k}{s_1}\right)(p_1 - cs_1^2) \tag{Eq.4.3}$$

The increase in the profit of the retailer with respect to gain in the profit of the online channel is;

$$\Delta\Pi_{online\,channel} = \frac{0.5k}{t^*}\left(2b - \frac{2p_1 + k}{s_1}\right)\left(p_1 - cs_1^2 - \frac{z}{(t^*)^2}\right) \tag{Eq.4.4}$$

After combining (4.3), (4.4), the change in total profit is;

$$\Delta\Pi_4 = 0.5\frac{k}{t^*}\left[(p_1 - cs_1^2)\frac{k}{s_1} - \left(2b - \frac{2p_1 + k}{s_1}\right)\frac{z}{(t^*)^2}\right] \tag{Eq.4.5}$$

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The retailer will find it profitable to open then online channel if and only if $\Delta\Pi_4 > 0$.

When $t^* = \bar{t}$, then the net change in the total profit of the retailer is;

$$\Delta\Pi_4(t^* = \bar{t}) = 0.5\frac{k}{\bar{t}}\left[(p_1 - cs_1^2)\frac{k}{s_1} - \left(2b - \frac{2p_1+k}{s_1}\right)\frac{z}{(\bar{t})^2}\right] \qquad \text{(Eq.4.6)}$$

$\Delta\Pi_4(t^* = \bar{t}) \geq 0$ if and only if $z \leq z_{limit\ 4}$.

When $t^* = t°$, after plugging $t°$ into (4.5), the net change in the total profit is;

$$\Delta\Pi_4(t^* = t°) = \frac{k^2(p_1 - cs_1^2)}{3s_1\sqrt{\frac{3z(2bs_1 - 2p_1 - k)}{k(p_1 - cs_1^2)}}} \qquad \text{(Eq.4.7)}$$

When $t^* = \underline{t}$, the net change in the total profit is ;

$$\Delta\Pi_4(t^* = \underline{t}) = 0.5\frac{k}{\underline{t}}\left[(p_1 - cs_1^2)\frac{k}{s_1} - \left(2b - \frac{2p_1+k}{s_1}\right)\frac{z}{(\underline{t})^2}\right] \qquad \text{(Eq.4.8)}$$

Note that $\Delta\Pi_4(t^* = \underline{t}) > 0$ if and only if $(p_1 - cs_1^2)\frac{k}{s_1} - \left(2b - \frac{2p_1+k}{s_1}\right)\frac{z}{(\underline{t})^2}] > 0$.

Since we know that $\Delta\Pi_4(t^* = t°)$ and $\underline{t} > t°$, we must have $\Delta\Pi_4(t^* = \underline{t}) > 0$ as well. ∎

Whenever the optimal promised delivery time is $\underline{t}$ or $\bar{t}$, since they are constant, the relationships below weakly hold. Here, we analyze change in $t°$ with respect to the parameters in our model.

Corollary 4.2: As z, s_1 or c increases, $t°$ increases. If k or p_1 increases, $t°$ decreases.

Proof: The first order derivative with respect to z, k, c, p_1 and s_1 are given below,

$$\frac{dt°}{dz} = \frac{\sqrt{\frac{3z(2bs_1 - 2p_1 - k)}{(p_1 - c(s_1)^2)}}}{2z} > 0$$

$$\frac{dt°}{dk} = \frac{z(p_1 - bs_1)\sqrt{3}}{k\sqrt{zk(p_1 - cs_1^2)(2bs_1 - 2p_1 - k)}} < 0$$

$$\frac{dt°}{dc} = \frac{s_1^2\sqrt{3z(2bs_1 - 2p_1 - k)}}{2\sqrt{k}(p_1 - cs_1^2)^{3/2}} > 0$$

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$$\frac{dt°}{dp_1} = \frac{\sqrt{3}z(k - 2bs_1 + 2cs_1^2)}{2(p_1 - cs_1^2)\sqrt{zk(p_1 - cs_1^2)(2bs_1 - 2p_1 - k)}} < 0$$

$(2bs_1 - 2p_1 - k) > 0$ implies that $2bs_1 > 2p_1 + k > 0$. Then, $k - 2bs_1 + 2cs_1^2 <$

$-2p_1 + 2cs_1^2 < 0$ which makes the whole expression negative.

$$\frac{dt°}{ds_1} = \frac{\sqrt{3}z[-cs_1(2p_1 + k) + b(p_1 + cs_1^2)]}{(p_1 - cs_1^2)\sqrt{zk(p_1 - cs_1^2)(2bs_1 - 2p_1 - k)}} > 0$$

$(2bs_1 - 2p_1 - k) > 0$ implies that $2bs_1 > 2p_1 + k > 0$. The FOC of $t°$ with respect to s_1 becomes positive if the highest value of $(2p_1 + k)$ which is $2bs_1$ is plugged into denominator of $\frac{dt°}{ds_1}$. ∎

Intuitively, as k increases, utility derived from physical channel decreases. Thus, consumers who suffer from low utility value derived in physical store can begin to use the online channel. The retailer wants to capture these consumers by shortening the delivery time.

The retailer's main tradeoff is between market expansion and cannibalization of the physical channel demand and online margin loss. As k or p_1 increases, market expansion starts to dominate, hence $t°$ decreases. As z or c increases, loss in the profit margin dominates, hence $t°$ increases.

Corollary 4.3: If $z_{critic\ 4\ \bar{t}} \geq z \geq z_{critic\ 4\ \underline{t}}$, then $t^* = t°$. The net change in the retailer's profit ($\Delta\Pi_4$) is increasing in k and p_1, and decreasing in c, s_1, z and b where $z_{critic\ 4\ \underline{t}} = \frac{k(p_1 - cs_1^2)(\underline{t})^2}{3\ (2bs_1 - 2p_1 - k)}$.

Proof: The first order derivative with respect to $k, p_1, c, s_1, z\ and\ b$ are given below,

$$\frac{d\Delta\Pi}{dk} = \frac{z(5bs_1 - 5p_1 - 2k)}{3\sqrt{3}s_1(\frac{z(2bs_1 - 2p_1 - k)}{k(p_1 - cs_1^2)})^{3/2}} > 0$$

$$\frac{d\Delta\Pi}{dp_1} = \frac{kz(6bs_1 - 3k - 4p_1 - 2cs_1^2)}{6\sqrt{3}s_1(p_1 - cs_1^2)(\frac{z(2bs_1 - 2p_1 - k)}{k(p_1 - cs_1^2)})^{3/2}} > 0$$

$(2bs_1 - 2p_1 - k) > 0$ can be written $2bs_1 > 2p_1 + k$. If the lowest value of $2bs_1$ which is $(2p_1 + k)$ is plugged into the denominator of $\frac{d\Delta\Pi}{dp_1}$, then the FOC of profit with respect to p_1 becomes positive.

$$\frac{d\Delta\Pi}{dc} = -\frac{k^2 s_1}{2\sqrt{\dfrac{3z(2bs_1 - 2p_1 - k)}{k(p_1 - cs_1^2)}}} < 0$$

$$\frac{d\Delta\Pi}{dz} = -\frac{k(2bs_1 - 2p_1 - k)}{6\sqrt{3}s_1(\dfrac{z(2bs_1 - 2p_1 - k)}{k(p_1 - cs_1^2)})^{3/2}} < 0$$

$$\frac{d\Delta\Pi}{db} = -\frac{kz}{3\sqrt{3}(\dfrac{z(2bs_1 - 2p_1 - k)}{k(p_1 - cs_1^2)})^{3/2}} < 0$$

$$\frac{d\Delta\Pi}{\partial d} = \frac{kz((k + 2p_1)(p_1 + 2cs_1^2) - 3bs_1 p_1 - 3bcs_1^3)}{3\sqrt{3}s_1^2(p_1 - cs_1^2)(\dfrac{z(2bs_1 - 2p_1 - k)}{k(p_1 - cs_1^2)})^{3/2}} < 0$$

$(2bs_1 - 2p_1 - k) > 0$ implies that $2bs_1 > 2p_1 + k > 0$. If the highest value of $(2p_1 + k)$ which is $(2bs_1)$ is plugged into the denominator of $\frac{d\Delta\Pi}{ds_1}$, then the FOC of profit with respect to s_1 becomes negative. ∎

Intuitively, the better online service quality is costly. The common parameters influence $\Delta\Pi_4$ and t° in opposite way. That is, as k or p_1 increases, the online channel becomes more aggressive but $\Delta\Pi_4$ increases. As z, s_1 or c increases, the online channel gets less effective but $\Delta\Pi_4$ decreases. In the high online service cost environment, the retailer may lead to lengthen the optimal promised delivery time. Thus, $\Delta\Pi_4$ decreases. As the price of the primary brand increases and quality level of the primary brand decreases, $\Delta\Pi_4$ increases.

Table 4.2 summarizes the demand and profit measures of the "physical chain only" scenario with the "both online and physical " case.

Table 4.2: The comparisons between "only physical chain" and "both online and physical chain"

	only physical chain	both online and physical chain
Demand of physical store	$(b - \frac{p_1+k}{s_1})b$	$(b - \frac{p_1+k}{s_1})(1 - \sqrt{\frac{k^3(p_1-cs_1^2)}{3z(2bs_1-2p_1-k)}})$
Demand of online store	NA	$\frac{(2bs_1-2p_1-k)}{2s_1}\sqrt{\frac{k^3(p_1-cs_1^2)}{3z(2bs_1-2p_1-k)}}$
Profit margin of physical store	$p_1 - cs_1^2$	$p_1 - cs_1^2$
Profit margin of online store	NA	$(p_1 - cs_1^2)\frac{(6bs_1-6p_1-4k)}{(6bs_1-6p_1-3k)}$
Profit of physical store	$b\left(b - \frac{p_1+k}{s_1}\right)(p_1 - cs_1^2)$	$\left(b - \frac{p_1+k}{s_1}\right)\left(1 - \sqrt{\frac{k^3(p_1-cs_1^2)}{3z(2bs_1-2p_1-k)}}\right)(p_1 - cs_1^2)$
Profit of online store	NA	$\frac{(2bs_1-2p_1-k)}{2s_1}\sqrt{\frac{k^3(p_1-cs_1^2)}{3z(2bs_1-2p_1-k)}}(p_1 - cs_1^2)\frac{(6bs_1-6p_1-4k)}{(6bs_1-6p_1-3k)}$
Total profit	$b\left(b - \frac{p_1+k}{s_1}\right)(p_1 - cs_1^2)$	$b\left(b - \frac{p_1+k}{s_1}\right)(p_1 - cs_1^2) + \frac{k^2(p_1-cs_1^2)}{3s_1\sqrt{\frac{3z(2bs_1-2p_1-k)}{k(p_1-cs_1^2)}}}$
Total covered market	$\left(b - \frac{p_1+k}{s_1}\right)b$	$b\left(b - \frac{p_1+k}{s_1}\right) + \frac{k^2}{2s_1\sqrt{\frac{3z(2bs_1-2p_1-k)}{k(p_1-cs_1^2)}}}$

The optimal online service quality (t^*), $\Delta\Pi_4$ (i.e.; the retailer's profit change with opening the online channel) can be investigated together with respect to the online cost parameter of the market environment, z.

Figure 4.2 and 4.3 summarize the evolution of the profit impact of the online channel and the optimal promised delivery time with respect to z for Order 1 and 2, respectively. Depending on the problem parameters, both orders are feasible. See Appendix E for Lemma 4.2 and its proof.
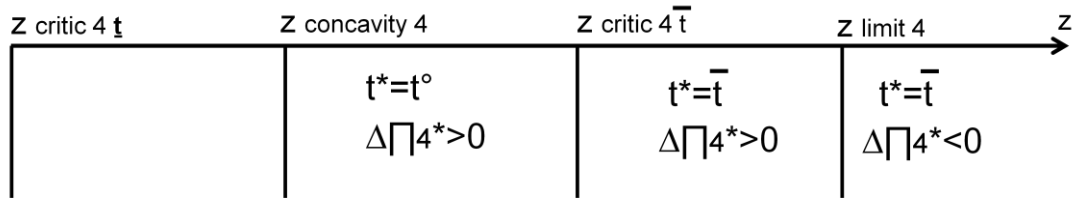
Figure 4.2: Profit change and optimal online service with respect to online service cost (Order 1)
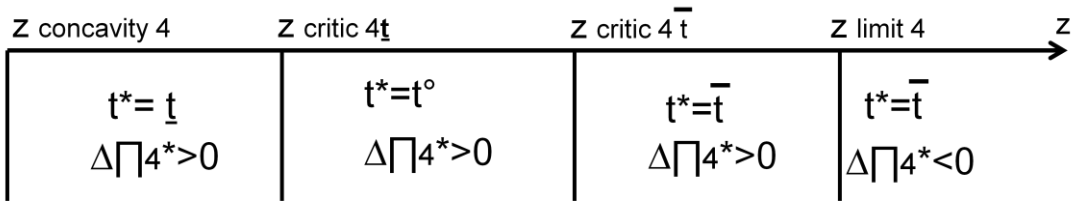
Figure 4.3: Profit change and optimal online service with respect to online service cost (Order 2)

In this chapter, we find that opening the online channel expands the retailer's market although it cannibalizes the other channel. When physically inconvenience cost of going to store is high enough, utility derived from physical channel decreases and consequently consumers who suffer from low utility value derived in physical store can begin to use the online channel. Thus, market expansion occurs through the online channel. In fact, market expansion increases as the physical channel quality decreases.

When online service cost is low enough, the online channel represents big opportunity for the retailer. As the price of the primary brand increases and quality level of the primary brand decreases, the profit impact of the online channel increases. Otherwise, generated market is small and the profit impact of the online channel can be positive or negative. When providing online services is costly, the retailer will keep the online service level as low as possible and $\Delta\Pi_4$ decreases.

CHAPTER 5

ONLINE CHANNEL DECISION FOR A BRICK-AND-MORTAR RETAILER WITH AN OUTLET BRANCH

In this chapter, we are interested in a monopolist retailer's online channel decision for the primary brand while the retailer is already managing its original and outlet brand through the respective physical channels. We assume that both business branches are viable; that is, they have positive demand and profit margin. That is, $(p_1 - cs_1^2) > 0$ (A.3.2), $\left(\frac{p_1 - p_2 + k - m}{s_1 - s_2} - \frac{p_2 + m}{s_2}\right) \geq 0$ (A.5.1) and $(p_2 - cs_2^2) > 0$ (A.5.2), $\left(b - \frac{p_1 - p_2 + k - m}{s_1 - s_2}\right) \geq 0$. (A.5.3).

The retailer's main decision is to determine the service quality to be offered at the online channel of the original brand in order to maximize the retailer's profit for a given set of original and outlet product positions $(p_1, s_1, p_2 \text{ and } s_2)$ and inconvenience cost of going to the stores (m and k). The parameters, decision variables and notations used in this chapter are presented in Table 5.1.

We assume that inconvenience cost of visiting and purchasing from the outlet stores is greater than inconvenience cost of associated with the physical brand chain since outlet malls are located far away from city centers. In short, $m \geq k > 0$ (A.3.3).

Similar to the previous chapter, the effectiveness of the online channel as an additional sales channel depends on the committed online service level (e.g., promised delivery time, t). We assume that consumers differ in sensitivity to the online service and promised delivery time while purchasing online and in willingness-to-pay for quality; i.e., consumer types are distributed uniformly on [0, b] according to their types d and θ, where b>0. We assume that b is high enough to

enable a profitable business for the retailer (i.e., for nonnegative market share and margin).

When the promised delivery time is taken into consideration, we assume that t can be set in a feasible time interval $[\underline{t}, \overline{t}]$, where $\underline{t} > 0$ and all practical delivery times are in this interval since online service and delivery options are limited by external factors.

We assume that the firm incurs online channel cost $\frac{z}{t^2}$ as the unit cost of serving an online customer where z is the online channel cost coefficient. In this chapter, again, we assume that $(b - \frac{k}{\underline{t}}) > 0$ (A.4.2) to ensure nonnegative market share for a potential online channel.

In our model, a consumer has the following utility function,

$$U = \begin{cases} \theta s_1 - p_1 - k, & \text{if he purchases from the primary physical store} \\ \theta s_2 - p_2 - m, & \text{if he purchases from the outlet store} \\ \theta s_1 - p_1 - dt, & \text{if he puchases from the primary online store} \\ \text{reservation utility (0)}, & \text{otherwise} \end{cases}$$

Consumers can observe the product qualities and prices available before they decide to buy. They buy a maximum of one product. They purchase only when their net utility is greater than or equal to their reservation utility, assumed as 0 in our study.

The utility of the consumer who uses the online channel decreases as he waits, the utility of the consumer who purchases from the offline store decreases due to the inconvenience regarding the physical channel.

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Table 5.1: Notation

Decision Variable (s)	
s_1	Quality level of the original brand, $s_1 \geq s_2$ (given)
p_1	Price of the original brand, $p_1 \geq p_2$ (given)
s_2	Quality level of the outlet branch, $s_2 > 0$ (given)
p_2	Price of the outlet branch, $p_2 > 0$ (given)
t	The online service quality set for the online channel, $t \in [\underline{t}, \bar{t}]$

Parameters	
c	Unit cost coefficient for a given quality level, $c > 0$
m	Inconvenience cost of visiting and purchasing from the physical outlet, $m \geq k$
k	Inconvenience cost of visiting and purchasing from the physical store, $k > 0$
θ	Quality taste parameter of consumers, $\theta \sim U[0, b]$
d	Sensitivity index to online services and promised delivery time of consumer, $d \sim U[0, b]$
z	Online channel cost coefficient, $z > 0$

D_1	Demand of the original brand, $D_1 \geq 0$
D_2	Demand of the outlet branch, $D_2 \geq 0$
D_3	Demand of the primary online channel, $D_3 \geq 0$

Lemma 5.1: Let θ_1 denote the quality taste parameter of the last consumer who purchases the product from the outlet store when there is only outlet store in the market; θ_2 denote the quality taste parameter of the last consumer who purchases the product from the online store when there is only online store in the market; θ^* denote the quality taste parameter of consumer that would be indifferent between purchasing the product from the online channel and the outlet store. ($\theta_1 = \frac{p_2 + m}{s_2}$, $\theta_2 = \frac{p_1}{s_1}$ and $\theta^* = \frac{p_1 - p_2 - m}{s_1 - s_2}$)

The orders of $\theta_1 \leq \theta_2 \leq \theta^*$ and $\theta_1 \geq \theta_2 \geq \theta^*$ are the only feasible ones among all possible rankings of θ_1, θ_2 and θ^*.

Proof: The number of all possible orders is six. These are $\theta_1 \leq \theta_2 \leq \theta^*$, $\theta_1 \leq \theta^* \leq \theta_2$, $\theta_2 \leq \theta^* \leq \theta_1$, $\theta_2 \leq \theta_1 \leq \theta^*$, $\theta^* \leq \theta_1 \leq \theta_2$ and $\theta^* \leq \theta_2 \leq \theta_1$.

i. If evaluating $\theta_1 \leq \theta_2 \leq \theta^*$, $\theta_2 \leq \theta^*$ simplifies into $\frac{p_2+m}{s_2} \leq \frac{p_1}{s_1}$. This expression is consistent with $\theta_1 \leq \theta_2$ in the first rank.

ii. If evaluating $\theta_1 \leq \theta^* \leq \theta_2$, $\theta^* \leq \theta_2$ simplifies into $\frac{p_2+m}{s_2} \geq \frac{p_1}{s_1}$. This expression is in contradiction with $\theta_1 \leq \theta_2$ except for the case $\theta_1 = \theta_2 = \theta^*$.

iii. If evaluating $\theta_2 \leq \theta^* \leq \theta_1$, $\theta_2 \leq \theta^*$ simplifies into $\frac{p_2+m}{s_2} \leq \frac{p_1}{s_1}$. This expression is in contradiction with $\theta_1 \geq \theta_2$ except for the case $\theta_1 = \theta_2 = \theta^*$.

iv. If evaluating $\theta_2 \leq \theta_1 \leq \theta^*$, $\theta_1 \leq \theta^*$ simplifies into $\frac{p_2+m}{s_2} \leq \frac{p_1}{s_1}$. This expression is in contradiction with $\theta_1 \geq \theta_2$ except for the case $\theta_1 = \theta_2 = \theta^*$.

v. If evaluating $\theta^* \leq \theta_1 \leq \theta_2$, $\theta^* \leq \theta_1$ simplifies into $\frac{p_2+m}{s_2} \geq \frac{p_1}{s_1}$. This expression is in contradiction with $\theta_1 \leq \theta_2$ except for the case $\theta_1 = \theta_2 = \theta^*$.

vi. If evaluating $\theta^* \leq \theta_2 \leq \theta_1$, $\theta^* \leq \theta_1$ simplifies into $\frac{p_2+m}{s_2} \geq \frac{p_1}{s_1}$. This expression is consistent with $\theta_1 \leq \theta_2$ in the sixth rank. ∎

Note that $\tilde{\theta} = \frac{p_1-p_2+k-m}{s_1-s_2}$ is always greater than θ_1 since $\frac{p_2+m}{s_2} < \frac{p_1-p_2+k-m}{s_1-s_2}$ always holds by (A.5.1). Finally, $\theta^* = \frac{p_1-p_2-m}{s_1-s_2} < \tilde{\theta}$ always holds.

Proposition 5.1: For a given positive promised delivery time (t) at the online channel, the demand of primary store, outlet store and online channel are given by;

i. For Case 1, i.e., when $\theta_1 = \frac{p_2+m}{s_2} \leq \theta_2 = \frac{p_1}{s_1} \leq \theta^* = \frac{p_1-p_2-m}{s_1-s_2}$

$$D_1^1 = \left(b - \frac{p_1-p_2+k-m}{s_1-s_2}\right)\left(b - \frac{k}{t}\right) \text{ for the primary physical store} \qquad (Eq.5.1)$$

$$D_2^1 = 0.5\left(2b - \frac{k}{t}\right)\left(\frac{k}{s_1-s_2}\right) + b\left(\frac{p_1-p_2-m}{s_1-s_2} - \frac{p_2+m}{s_2}\right) \text{ for the outlet store} \qquad (Eq.5.2)$$

$$D_3^1 = \left(2b - \frac{2p_1-2p_2+k-2m}{s_1-s_2}\right)\left(\frac{k}{t}\right)0.5 \text{ for the online store of the primary brand } (Eq.5.3)$$

ii. For Case 2, i.e., when $\theta_1 = \frac{p_2+m}{s_2} > \theta_2 = \frac{p_1}{s_1} > \theta^* = \frac{p_1-p_2-m}{s_1-s_2}$

$$D_1^2 = \left(b - \frac{p_1-p_2+k-m}{s_1-s_2}\right)\left(b - \frac{k}{t}\right) \text{ for the primary physical store} \qquad (Eq.5.4)$$

$$D_2^2 = 0.5\left(\frac{p_1-p_2+k-m}{s_1-s_2} - \frac{p_2+m}{s_2}\right)\left(2b - \frac{k}{t} - \frac{s_1p_2+s_1m-p_1s_2}{s_2t}\right) \text{ for the outlet store } (Eq.5.5)$$

$$D_3^2 = 0.5 \left(\frac{s_1 p_2 + s_1 m - p_1 s_2}{s_2 t} \right) \left(\frac{p_2 + m}{s_2} - \frac{p_1}{s_1} \right) + 0.5 \left(\frac{s_1 p_2 + s_1 m - p_1 s_2}{s_2 t} + \frac{k}{t} \right)$$

$$\left(\frac{p_1 - p_2 + k - m}{s_1 - s_2} - \frac{p_2 + m}{s_2} \right) + \left(b - \frac{p_1 - p_2 + k - m}{s_1 - s_2} \right) \frac{k}{t} \text{ for the online store of the primary brand}$$

(Eq.5.6)

Proof: A consumer will want to purchase from the online store if $\theta s_1 - p_1 - dt \geq 0$ and from the physical store for the primary brand if $\theta s_1 - p_1 - k \geq 0$. He will choose the online store if $\theta s_1 - p_1 - dt \geq \theta s_1 - p_1 - k$.

So for any consumer type with θ so that $\theta s_1 - p_1 - k \geq 0$, if $d \leq \tilde{d} = \frac{k}{t}$, then the consumer will prefer the online channel to the physical store for the primary brand.

Thus, consumers with $\theta \in [\tilde{\theta}, b]$ and $d \in [0, \tilde{d}]$ switch from the physical channel to the online channel. Then, $D_1^1 = \left(b - \frac{p_1 - p_2 + k - m}{s_1 - s_2} \right) \left(b - \frac{k}{t} \right)$.

A consumer will want to purchase from the online store if $\theta s_1 - p_1 - dt \geq 0$ and from the outlet store if $\theta s_2 - p_2 - m \geq 0$. He will choose the online store if $\theta s_1 - p_1 - dt \geq \theta s_2 - p_2 - m$.

So for any consumer type with θ so that $\theta s_2 - p_2 - m \geq 0$, if $\frac{\theta(s_1 - s_2) - p_1 + p_2 + m}{t} \geq d$, then the consumer will prefer the online channel to the outlet store.

Thus, consumer with $d \leq \frac{\theta(s_1 - s_2) - p_1 + p_2 + m}{t}$ switch from the outlet store to the online channel. Then, $D_2^1 = 0.5 \left(2b - \frac{k}{t} \right) \left(\frac{k}{s_1 - s_2} \right) + b \left(\frac{p_1 - p_2 - m}{s_1 - s_2} - \frac{p_2 + m}{s_2} \right)$

The online channel will consist consumer with $\theta \in [\tilde{\theta}, b]$ and $d \in [0, \tilde{d}]$, capture consumer with $d \leq \frac{\theta(s_1 - s_2) - p_1 + p_2 + m}{t}$ from the outlet store. Then,

$$D_3^1 = \left(2b - \frac{2p_1 - 2p_2 + k - 2m}{s_1 - s_2} \right) \left(\frac{k}{t} \right) 0.5.$$

We leave the demand derivation for Case 2 to the reader. ∎

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Corollary 5.1:

i. Under Case 1 of Proposition 5.1, opening the online channel does not change the total market of the retailer.

ii. Under Case 2 of Proposition 5.1, opening the online channel expands the total market of the retailer.

Proof: Left to the reader.

Under Case 1 of Proposition 5.1, the online channel only cannibalizes the primary and outlet store and cannot create additional demand. Figure 5.1 shows the change in the retailer's demand with the online channel under Case 1.

Figure 5.1: The demand of the retailer before (a) and after (b) the introduction of the online channel (and its split among channels) under Case 1

Under Case 2 of Proposition 5.1, although the online channel cannibalizes the primary and outlet store, it creates additional demand. Figure 5.2 shows the change in the retailer's demand with the online channel under Case 2.
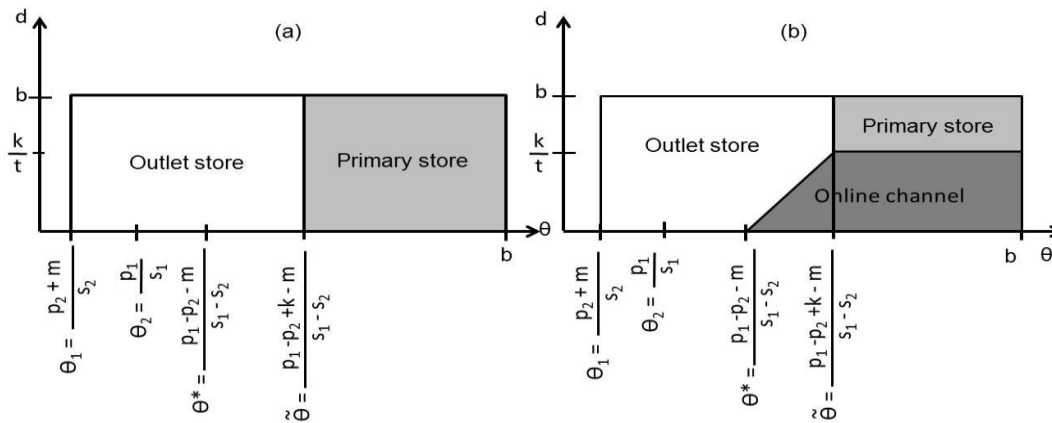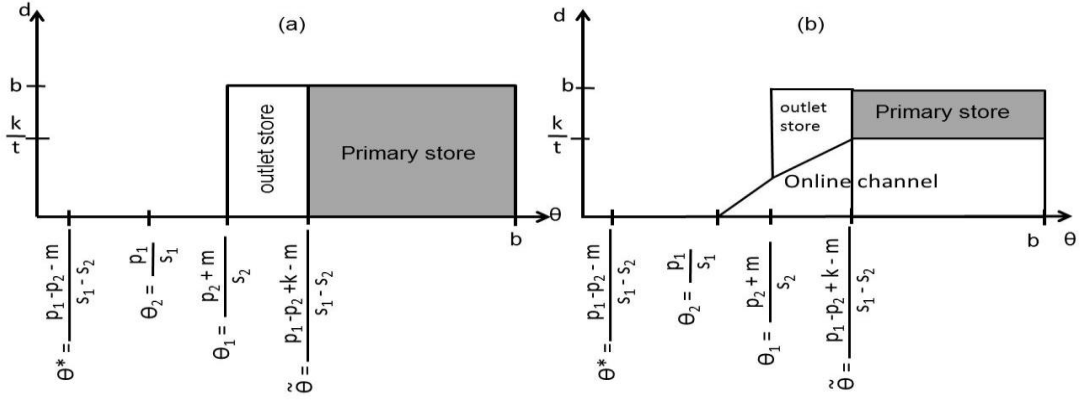
Figure 5.2: The demand of the retailer before (a) and after (b) the introduction of the online channel (and its split among channels) under Case 2

5.1 The optimal online service under Case 1 ($\theta_1 \leq \theta_2 \leq \theta^*$)

The unit profit margins of the primary and outlet store and online channel are $(p_1 - cs_1^2), (p_2 - cs_2^2)$ and $\left(p_1 - cs_1^2 - \frac{z}{t^2}\right)$, respectively.

We assume that z (online channel cost coefficient) is high enough to discourage the retailer to target the lowest t possible (or serve the market perfectly through the online channel). In this part of chapter, we assume that

$$z > \frac{(\bar{t})^2 k(p_1 - cs_1^2 - (p_2 - cs_2^2))}{6(2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k)} \text{ (A.5.4)}.$$

If we take into account the costs as well, the total profit of the retailer can be summarized as follows:

$$\Pi = D_1^1(p_1 - cs_1^2) + D_2^1(p_2 - cs_2^2) + D_3^1\left(p_1 - cs_1^2 - \frac{z}{t^2}\right)$$

After all, the retailer's problem can be modeled as,

$$max_{t>0} \ D_1^1(p_1 - cs_1^2) + D_2^1(p_2 - cs_2^2) + D_3^1\left(p_1 - cs_1^2 - \frac{z}{t^2}\right) \qquad \text{(Eq.5.7)}$$

s.to $t \in [\underline{t}, \bar{t}]$

The objective function of the model, (5.7), maximizes the total profit of the retailer if the retailer serves the online consumers with its original brand when the primary and outlet physical channels are available; i.e., when p_1, s_1, p_2, s_2, m and k are given.

The retailer's problem is a nonlinear maximization problem.

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Lemma 5.2: The profit function $\Pi = D_1^1(p_1 - cs_1^2) + D_2^1(p_2 - cs_2^2) + D_3^1\left(p_1 - cs_1^2 - \frac{z}{t^2}\right)$ is concave in delivery time t when $t \in [\underline{t}, \overline{t}]$.

Proof: The SOC of the total profit function is given below.

$$\frac{d\Pi^2(t)}{dt^2} = \frac{k[(-12m+12p_1-12p_2-12bs_1+12bs_2)z+k\left((p_1-p_2-cs_1^2+cs_2^2)t^2+6z\right)]}{(s_1-s_2)t^5}$$

It is clear that $\frac{k}{(s_1-s_2)t^5}$ is always positive since delivery time is always positive. The remaining expression is

$$g(t) = [(p_1 - cs_1^2) - (p_2 - cs_2^2)]kt^2$$

$$-z(12bs_1 - 12bs_2 - 12p_1 + 12p_2 + 12m - 6k) \qquad \text{(Eq.5.8)}$$

i. If $(p_1 - cs_1^2) - (p_2 - cs_2^2) < 0$, the total profit function is concave in t since (5.8) and also the SOC of the profit function is always negative.

ii. If $(p_1 - cs_1^2) - (p_2 - cs_2^2) > 0$, then the SOC of the profit function becomes

negative when $t \in \left[-\sqrt{\frac{6z(2bs_1-2bs_2-2p_1+2p_2+2m-k)}{k\left((p_1-cs_1^2)-(p_2-cs_2^2)\right)}}, \sqrt{\frac{6z(2bs_1-2bs_2-2p_1+2p_2+2m-k)}{k\left((p_1-cs_1^2)-(p_2-cs_2^2)\right)}} \right]$.

The SOC is always negative in feasible region $[\underline{t}, \overline{t}]$ with (A.5.4). ∎

The optimal solution of the maximization problem is given in Proposition 5.2.

Proposition 5.2: Let $\Delta\Pi_{5.1}$ denote the profit impact of the online channel on the retailer's profit under Case 1.

i. When $(p_1 - cs_1^2) - (p_2 - cs_2^2) < 0$, the retailer will set the optimal delivery time as high as possible (i.e., $\overline{t}$) and never find it profitable to open the online channel.

ii. When $(p_1 - cs_1^2) - (p_2 - cs_2^2) > 0$, the retailer sets $t^* = \min\ (\max(t^{\circ}, \underline{t}), \overline{t})$

where $t^{\circ} = \sqrt{\frac{3z(2bs_1-2bs_2-2p_1+2p_2+2m-k)}{k\left((p_1-cs_1^2)-(p_2-cs_2^2)\right)}}$.

- If $z \geq z_{critic\ 5.1}\overline{t} = \frac{(\overline{t})^2 k(p_1-cs_1^2-(p_2-cs_2^2))}{3(2bs_1-2bs_2-2p_1+2p_2+2m-k)}$, then $t^* = \overline{t}$,

> If $z \leq z_{limit\,5.1} = \frac{k[(p_1 - cs_1^2) - (p_2 - cs_2^2)](\bar{t})^2}{(2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k)}$, then the retailer

finds it profitable to open the online channel.

> Otherwise, $\Delta\Pi_{5.1} < 0$.

- If $z < z_{critic\,5.1\,\bar{t}}$, then $t^* = \max\{t^\circ, \underline{t}\}$ and $\Delta\Pi_{5.1} > 0$ always holds.

Proof: The first order derivative of the objective function with respect to t is given below,

$$\frac{d\Pi(t)}{dt} = \frac{k[k((-0.5p_1 + 0.5p_2 + 0.5cs_1^2 - 0.5cs_2^2)t^2 - 1.5z) + (3m - 3p_1 + 3p_2 + 3bs_1 - 3bs_2)z]}{(s_1 - s_2)t^4} \qquad \text{(Eq.5.9)}$$

Solving $\frac{d\Pi(t)}{dt} = 0$ yields $t^\circ = \sqrt{\frac{3z(2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k)}{k((p_1 - cs_1^2) - (p_2 - cs_2^2))}}$. If $t^\circ \in [\underline{t}, \bar{t}]$, then t° is

the maximizer of the total profit function.

The change (i.e., decrease) in the profit of the retailer in each respective channel are;

$$\Delta\Pi_{primary\,store} = -\frac{k}{t^*}(b - \frac{p_1 - p_2 + k - m}{s_1 - s_2})(p_1 - cs_1^2) \qquad \text{(Eq.5.10)}$$

$$\Delta\Pi_{outlet\,store} = -0.5\frac{k^2}{t^*(s_1 - s_2)}(p_2 - cs_2^2) \qquad \text{(Eq.5.11)}$$

The increase in the profit of the retailer with respect to gain in the profit of the online channel is;

$$\Delta\Pi_{online\,channel} = \frac{0.5k}{t^*}\left(2b - \frac{2p_1 - 2p_2 - 2m + k}{s_1 - s_2}\right)(p_1 - cs_1^2 - \frac{z}{(t^*)^2}) \qquad \text{(Eq.5.12)}$$

After combining (5.10), (5.11), (5.12), the change in total profit is;

$$\Delta\Pi_{5.1} = \frac{0.5k^2}{t^*(s_1 - s_2)}(p_1 - cs_1^2 - (p_2 - cs_2^2)) - \frac{0.5kz}{(t^*)^3}(2b - \frac{2p_1 - 2p_2 - 2m + k}{s_1 - s_2}) \qquad \text{(Eq.5.13)}$$

The retailer will find it profitable to open then online channel if and only if $\Delta\Pi_{5.1} > 0$.

i. If (5.9) is evaluated, it is positive for all $t \geq 0$ if $(p_1 - cs_1^2) < (p_2 - cs_2^2)$. To generate maximum profit, the retailer sets the optimal delivery time to the maximum feasible level; i.e.; $\bar{t}$.

(5.13) becomes ;

$$\Delta\Pi_{5.1}\left(t^* = \bar{t}\right) = \frac{0.5k^2}{\bar{t}(s_1 - s_2)}\left(p_1 - cs_1^2 - (p_2 - cs_2^2)\right)$$

$$-\frac{0.5kz}{(\bar{t})^3}\left(2b - \frac{2p_1 - 2p_2 - 2m + k}{s_1 - s_2}\right) \qquad\qquad (\text{Eq.5.14})$$

(5.14) is always negative when $(p_1 - cs_1^2) - (p_2 - cs_2^2) < 0$.

ii. When $t^* = \bar{t}$, then the net change in the total profit of the retailer is the same as (5.14). $\Delta\Pi_{5.1}(t^* = \bar{t}) \geq 0$ if and only if $z \leq z_{limit\ 5.1}$.

When $t^* = t°$, after plugging $t°$ into (5.13), the net change in the total profit is

$$\Delta\Pi_{5.1}(t^* = t°) = \frac{k^2(p_1 - cs_1^2) - k^2(p_2 - cs_2^2)}{3(s_1 - s_2)\sqrt{\dfrac{3z(2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k)}{k\cdot\left((p_1 - cs_1^2) - (p_2 - cs_2^2)\right)}}} \qquad (\text{Eq.5.15})$$

$\Delta\Pi_{5.1}(t^* = t°)$ is always positive when $(p_1 - cs_1^2) - (p_2 - cs_2^2) > 0$.

When $t^* = \underline{t}$, the net change in the total profit is ;

$$\Delta\Pi_{5.1}(t^* = \underline{t}) = \frac{1}{\underline{t}}\left[\frac{0.5k^2}{(s_1 - s_2)}\left(p_1 - cs_1^2 - (p_2 - cs_2^2)\right)\right.$$

$$\left.-\frac{0.5kz}{(\underline{t})^2}\left(2b - \frac{2p_1 - 2p_2 - 2m + k}{s_1 - s_2}\right)\right] \qquad (\text{Eq.5.16})$$

Note that $\Delta\Pi_{5.1}\left(t^* = \underline{t}\right) > 0$ if and only if $\left[\frac{0.5k^2}{(s_1 - s_2)}\left(p_1 - cs_1^2 - (p_2 - cs_2^2)\right)\right.$

$$\left.-\frac{0.5kz}{(\underline{t})^2}\left(2b - \frac{2p_1 - 2p_2 - 2m + k}{s_1 - s_2}\right)\right] > 0.$$

Since we know that $\Delta\Pi_{5.1}(t^* = t°)$ and $\underline{t} > t°$, we must have $\Delta\Pi_{5.1}\left(t^* = \underline{t}\right) > 0$ as well. ∎

$\Delta\Pi_{5.1}(t^* = t°)$ is increasing with margin difference $(p_1 - cs_1^2 - (p_2 - cs_2^2))$ and k. As quality level difference $(s_1 - s_2)$, z, m or b increases, the net gain in the profit decreases.

Whenever the optimal promised delivery time is $\underline{t}$ or $\bar{t}$, since they are constant, the relationships below weakly hold. Here, we analyze change in t° under Case 1 with respect to the parameters in our model.

Corollary 5.2: If $z_{critic\ 5.1\ \bar{t}} \geq z \geq z_{critic\ 5.1\ \underline{t}} = \frac{(\underline{t})^2 k(p_1 - cs_1^2 - (p_2 - cs_2^2))}{3(2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k)}$, then $t^* = t^{\circ}$. As z, b, m, p_2, c or s_1 increases, t° increases. If k, p_1 or s_2 increases, t° decreases.

Proof: The first order derivative with respect to z, k, b, m, p_1, p_2 , c, s_1, s_2 are given below,

$$\frac{dt^{\circ}}{dz} = \frac{1}{2z}\sqrt{\frac{3z(2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k)}{k\left((p_1 - cs_1^2) - (p_2 - cs_2^2)\right)}} > 0$$

$$\frac{dt^{\circ}}{dk} = -\frac{\sqrt{3}z(m - p_1 + p_2 + bs_1 - bs_2)}{k\sqrt{kz\left((p_1 - cs_1^2) - (p_2 - cs_2^2)\right)(2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k)}} < 0$$

$$\frac{dt^{\circ}}{db} = \frac{z(s_1 - s_2)\sqrt{3}}{\sqrt{kz\left((p_1 - cs_1^2) - (p_2 - cs_2^2)\right)(2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k)}} > 0$$

$$\frac{dt^{\circ}}{dm} = \frac{z\sqrt{3}}{\sqrt{kz\left((p_1 - cs_1^2) - (p_2 - cs_2^2)\right)(2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k)}} > 0$$

$$\frac{dt^{\circ}}{dp_1} = \frac{\sqrt{3}k[k - 2m - 2bs_1 + 2bs_2 + 2c(s_1^2 - s_2^2)]}{2(k\left((p_1 - cs_1^2) - (p_2 - cs_2^2)\right))^{3/2}\sqrt{z(2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k)}} < 0$$

$2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k > 0$ implies that $k - 2m - 2bs_1 + 2bs_2 < 2p_2 - 2p_1$. Hence, the numerator of $\frac{dt^{\circ}}{dp_1}$ is negative.

$$\frac{dt^{\circ}}{\partial d} = -\frac{\sqrt{3}k[k - 2m - 2bs_1 - 2bs_2 + 2c(s_1^2 - s_2^2)]}{2(k\left((p_1 - cs_1^2) - (p_2 - cs_2^2)\right))^{3/2}\sqrt{z(2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k)}} > 0$$

$2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k > 0$ implies that $k - 2m - 2bs_1 + 2bs_2 < 2p_2 - 2p_1$.Hence, the numerator of $\frac{dt^{\circ}}{dp_2}$ is negative and $\frac{dt^{\circ}}{dp_2}$ is positive.

$$\frac{dt^{\circ}}{dc} = \frac{k(s_1 - s_2)(s_1 + s_2)\sqrt{3z(2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k)}}{2(k\left((p_1 - cs_1^2) - (p_2 - cs_2^2)\right))^{3/2}} > 0$$

$$\frac{dt°}{ds_1} = \frac{k\sqrt{3}[b(p_1-p_2+c(s_1-s_2)^2)-cs_1(k-2m+2p_1-2p_2)]}{(k((p_1-cs_1^2)-(p_2-cs_2^2))^{3/2}\sqrt{z(2bs_1-2bs_2-2p_1+2p_2+2m-k)}} > 0$$

$2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k > 0$ implies that $2bs_1 - 2bs_2 > 2p_1 - 2p_2 - 2m + k$ Hence, the numerator of $\frac{dt°}{ds_1}$ is positive.

$$\frac{dt°}{ds_2} = \frac{\sqrt{3}kz[b(-p_1+p_2+c(s_1-s_2)^2)+cs_2(k-2m+2p_1-2p_2)]}{(k((p_1-cs_1^2)-(p_2-cs_2^2))^{3/2}.\sqrt{z(2bs_1-2bs_2-2p_1+2p_2+2m-k)}} < 0$$

$2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k > 0$ implies that the maximum value of $(2p_1 - 2p_2 - 2m + k)$ is $(2bs_1 - 2bs_2)$. With this the numerator of $\frac{dt°}{ds_2}$ is negative.

∎

We investigate increase (or decrease) in the $\tilde{\theta}$ and $t°$ with respect to common parameters which are p_1, p_2, s_1, s_2, m and k. As p_1, k or s_2 increases, $t°$ decreases and $\tilde{\theta}$ increases. Also, as p_2, m or s_1 increases, $t°$ increases and $\tilde{\theta}$ decreases. In short, while $t°$ increases (decreases), $\tilde{\theta}$ decreases (increases) with the increase in the common parameters.

The magnitude of $\tilde{\theta}$ can be interpreted as the outlet market effectiveness (competitiveness). While the outlet market effectiveness (competitiveness) increases, $t°$ decreases. Case 1 may only help the retailer if the online channel attracts more outlet customers than it cannibalizes from the primary store. Thus, as $\tilde{\theta}$ increases, it can be more competitive in the online service as well.

5.2 The optimal online service under Case 2 ($\theta_1 > \theta_2 > \theta^*$)
Here, we assume that z (online channel cost coefficient) is high enough to discourage the retailer to target the lowest t possible (or serve the market perfectly through the online channel). In this part of chapter, we assume that
$$z > \frac{(\bar{t})^2[(p_1-cs_1^2)(y(\tilde{\theta}-\theta_2)+ks_2(\tilde{\theta}-\theta_1))-(y+ks_2)(\tilde{\theta}-\theta_1)(p_2-cs_2^2)]}{6(ks_2(2b-\tilde{\theta}-\theta_1)+y(\tilde{\theta}-\theta_2))}$$ where $y = s_1p_2 + s_1m - p_1s_2$ (A.5.5).

Note that $\theta_1 = \frac{p_2+m}{s_2} \geq \theta_2 = \frac{p_1}{s_1}$ implies that $y \geq 0$.

The retailer's problem can be modeled as,

$$max_{t>0} \ D_1^2(p_1 - cs_1{}^2) + D_2^2(p_2 - cs_2) + D_3^2\left(p_1 - cs_1{}^2 - \frac{z}{t^2}\right) \qquad \text{(Eq.5.17)}$$

s.to $t \in [\underline{t}, \bar{t}]$

The objective function of the model, (5.17), maximizes the total profit of the retailer if the retailer serves the online consumers with its original brand when the primary and outlet physical channels are available; i.e., when p_1, s_1, p_2, s_2, m and k are given.

The retailer maximizes its total profit. The nonlinear maximization problem that it has can be characterized as below.

Lemma 5.3: The profit function $\Pi = D_1^2(p_1 - cs_1^2) + D_2^2(p_2 - cs_2^2) + D_3^2\left(p_1 - cs_1^2 - \frac{z}{t^2}\right)$ is concave in delivery time t when $t \in [\underline{t}, \bar{t}]$.

Proof: The SOC of the total profit function is given below.

$$\frac{d\Pi^2(t)}{dt^2} = \frac{1}{t^5} \cdot \left\{ t^2 \left[\frac{(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right)}{s_2} - \frac{(p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2)}{s_2} \right] \right.$$

$$\left. - \frac{6z}{s_2}\left[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right] \right\} \text{ where } y = s_1 p_2 + s_1 m - p_1 s_2$$

In order to understand the total profit function is concave, the conditions are listed below.

i. If $(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) > 0$ (the condition of $(p_1 - cs_1^2) - (p_2 - cs_2^2) > 0$ always holds.), then the SOC of the profit function becomes negative when

$$t \in [- \sqrt{\frac{6z[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)]}{\left[(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2)\right]}},$$

$$\sqrt{\frac{6z[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)]}{\left[(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2)\right]}}]. \text{ Thus, the SOC is always}$$

negative in feasible region $[\underline{t}, \bar{t}]$ with (A.5.5).

ii. If $(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) < 0$, the total profit function is concave in t. ∎

The optimal solution of the maximization problem is given in Proposition 5.3.

Proposition 5.3: Define $y = s_1 p_2 + s_1 m - p_1 s_2$.

Let $\Delta\Pi_{5.2}$ denote the profit impact of the online channel on the retailer's profit under Case 2.

i. If $(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) < 0$, then the retailer will set the optimal delivery time as high as possible (i.e., $\bar{t}$) and never find it profitable to open the online channel.

ii. If $(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) > 0$ holds (which is directly guaranteed when $(p_1 - cs_1^2 - (p_2 - cs_2^2) > 0)$), then the retailer sets $t^* = \min(\max(t^\circ, \underline{t}), \bar{t})$

where $t^\circ = \sqrt{\dfrac{3z[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)]}{\left[(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2)\right]}}$.

- If $z \geq z_{critic\ 5.2\ \bar{t}}$, then $t^* = \bar{t}$,
  - If $z \leq z_{limit\ 5.2}$, $\Delta\Pi_{5.2} \geq 0$
  - Otherwise, $\Delta\Pi_{5.2} < 0$.
- If $z < z_{critic\ 5.2\ \bar{t}}$, then $t^* = \max\{t^\circ, \underline{t}\}$ and $\Delta\Pi_{5.2} > 0$ always holds

where $z_{critic\ 5.2\ \bar{t}} = \dfrac{\left[(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2)\right](\bar{t})^2}{3[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)]}$ and

$z_{limit\ 5.2} = \dfrac{\left[(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2)\right](\bar{t})^2}{[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)]}$.

Proof: The first order derivative of the objective function with respect to t is given below,

$$\frac{d\Pi(t)}{dt} = \frac{1}{t^4}\Bigg\{-0.5t^2\left[\frac{(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right)}{s_2} - \frac{(p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2)}{s_2}\right] +$$

$$\frac{1.5z}{s_2}\left[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right]\Bigg\} \qquad\qquad\text{(Eq.5.18)}$$

$$t^{\circ} = \sqrt{\frac{3z[ks_2(2b-\tilde{\theta}-\theta_1)+y(\tilde{\theta}-\theta_2)]}{\left[(p_1-cs_1^2)\left(ks_2(\tilde{\theta}-\theta_1)+y(\tilde{\theta}-\theta_2)\right)-(p_2-cs_2^2)(\tilde{\theta}-\theta_1)(y+ks_2)\right]}}.$$ is obtained by solving

$\frac{d\Pi(t)}{dt} = 0$. If $t^{\circ} \in [\underline{t}, \bar{t}]$, then t° is the maximizer of the total profit function.

The change (i.e., decrease) in the profit of the retailer in each respective channel are;

$$\Delta\Pi_{primary\ store} = -\frac{k}{t^*}(b-\tilde{\theta})(p_1 - cs_1{}^2) \tag{Eq.5.19}$$

$$\Delta\Pi_{outlet\ store} = -0.5(\frac{y+ks_2}{t^*s_2})(\tilde{\theta}-\theta_1)(p_2-cs_2^2) \tag{Eq.5.20}$$

The increase in the profit of the retailer with respect to gain in the profit of the online channel is;

$$\Delta\Pi_{online\ channel} = \left(p_1 - cs_1^2 - \frac{z}{(t^*)^2}\right)\left(\frac{0.5y(\tilde{\theta}-\theta_2)}{t^*s_2} + \frac{k(b-0.5\tilde{\theta}-0.5\theta_1)}{t^*}\right) \tag{Eq.5.21}$$

After combining (5.19), (5.20), (5.21), the change in total profit is;

$$\Delta\Pi_{5.2} = \frac{0.5}{t^*s_2}\left[(p_1-cs_1^2)\left(ks_2(\tilde{\theta}-\theta_1)+y(\tilde{\theta}-\theta_2)\right)\right.$$

$$\left.-(p_2-cs_2^2)(\tilde{\theta}-\theta_1)(y+ks_2)\right] - \left(\frac{0.5y(\tilde{\theta}-\theta_2)}{t^*s_2} + \frac{k(b-0.5\tilde{\theta}-0.5\theta_1)}{t^*}\right)\frac{z}{(t^*)^2} \tag{Eq.5.22}$$

The retailer will find it profitable to open then online channel if and only if $\Delta\Pi_{5.2} > 0$.

i. If (5.18) is evaluated, it is positive for all $t \geq 0$

if $(p_1 - cs_1^2)\left(ks_2(\tilde{\theta}-\theta_1)+y(\tilde{\theta}-\theta_2)\right) < (p_2-cs_2^2)(\tilde{\theta}-\theta_1)(y+ks_2)$.

To generate maximum profit, the retailer sets the optimal delivery time to the maximum feasible level; i.e.; $\bar{t}$.

(5.22) becomes;

$$\Delta\Pi_{6.2}(t^* = \bar{t}) = \frac{0.5}{\bar{t}s_2}\left[(p_1-cs_1^2)\left(ks_2(\tilde{\theta}-\theta_1)+y(\tilde{\theta}-\theta_2)\right)\right.$$

$$\left.-(p_2-cs_2^2)(\tilde{\theta}-\theta_1)(y+ks_2)\right] - \left(\frac{0.5y(\tilde{\theta}-\theta_2)}{\bar{t}s_2} + \frac{k(b-0.5\tilde{\theta}-0.5\theta_1)}{\bar{t}}\right)\frac{z}{(\bar{t})^2} \tag{Eq.5.23}$$

(5.23) is always negative when

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$$\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) < (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2).$$

ii. When $t^* = \bar{t}$, then the net change in the total profit of the retailer is the same as (5.23). $\Delta\Pi_{5.2}(t^* = \bar{t}) \geq 0$ if and only if $z \leq z_{limit\ 5.2}$.

When $t^* = t°$, after plugged $t°$ in to (5.22), the net change in the total profit is

$$\Delta\Pi_{5.2}(t^* = t°) = \frac{2[(p_1 - cs_1^2)(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2)]}{3s_2\sqrt{\frac{3z[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)]}{[(p_1 - cs_1^2)(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2)]}}} \qquad \text{(Eq.5.24)}$$

(5.24) is always positive when

$$\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) > 0.$$

When $t^* = \underline{t}$, the net change in the total profit is ;

$$\Delta\Pi_{5.2}(t^* = \underline{t}) = \frac{1}{\underline{t}}[\frac{0.5\left((p_1 - cs_1^2)(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2)\right)}{s_2} - $$

$$\left(\frac{0.5y(\tilde{\theta} - \theta_2)}{s_2} + k(b - 0.5\tilde{\theta} - 0.5\theta_1)\right)\frac{z}{(\underline{t})^2}] \qquad \text{(Eq.5.25)}$$

Note that $\Delta\Pi_{5.2}(t^* = \underline{t}) > 0$ if and only if

$$\frac{0.5\left((p_1 - cs_1^2)(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2)\right)}{s_2} - (\frac{0.5y(\tilde{\theta} - \theta_2)}{s_2} + k(b - 0.5\tilde{\theta} - $$

$$0.5\theta_1))\frac{z}{(\underline{t})^2} > 0 .$$

Since we know that $\Delta\Pi_{5.2}(t^* = t°)$ and $\underline{t} > t°$, we must have $\Delta\Pi_{5.2}(t^* = \underline{t}) > 0$ as well. ∎

See Appendix F (Proposition 5.4 and its proof) and G (Proposition 5.5 and its proof) for profit impact of online channel on the retailer's profit under Case 1 and 2 with respect to online service cost, respectively.

As detailed above, two cases may arise depending on the problem parameters in this setting: Case 1 and Case 2. The retailer manages to expand its market under Case 2 but not under Case 1. The online channel cannibalizes the physical primary and

outlet store in both cases. We find that markets that correspond to Case 2 are more likely to produce a profit increase for the retailer through the online channel. In fact, the profit margin threshold for positive profit impact under Case 2 is less strict than that under Case 1.

The online channel profitability is determined by two factors: the difference of profit margin between the original and outlet product and the online service cost. If the profit margin of the primary brand is lower than that of the outlet brand, opening the online channel under two cases is not profitable for the retailer at all. The retailer sets the optimal promised delivery time as high as possible to avoid any profit loss. Otherwise, when the online service cost (z) is low enough, the profit impact of the online channel for all cases are always nonnegative. If the online service cost is high, profit impact of the online channel can be positive or negative.

Table 5.2 shows the summary of the retailer's total profit, market condition and the optimal delivery time under Case 1 and 2.

(where $t_{5.2}^{\circ} = \dfrac{3z[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)]}{\left[(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2)\right]}$ and

$t_{5.1}^{\circ} = \sqrt{\dfrac{3z(2bs_1 - 2bs_2 - 2p_1 + 2p_2 + 2m - k)}{k\left((p_1 - cs_1^2) - (p_2 - cs_2^2)\right)}}$)

Table 5.2: The retailer's total profit, market condition and the optimal delivery time under Case 1 and 2

		t^*	$\Delta\Pi_{5.1}$	$t°$	Change on the market
Case 2	$(p_1 - cs_1^2)\left(ks_2(\tilde\theta - \theta_1) + y(\tilde\theta - \theta_2)\right)$ $-(p_2 - cs_2^2)(\tilde\theta - \theta_1)(y + ks_2) > 0$	$\min\left(\max\left(t°, \underline{t}\right), \bar{t}\right)$	$t^* = \underline{t}$, always positive. $t^* = t°$, always positive. $t^* = \bar{t}$, depends on $z_{limit\ 6.2}$	$t°_{5.2}$	Expand
	$(p_1 - cs_1^2)\left(ks_2(\tilde\theta - \theta_1) + y(\tilde\theta - \theta_2)\right)$ $-(p_2 - cs_2^2)(\tilde\theta - \theta_1)(y + ks_2) < 0$	$\bar{t}$	always negative		
Case 1	$(p_1 - cs_1^2) - (p_2 - cs_2^2) > 0$	$\min\left(\max\left(t°, \underline{t}\right), \bar{t}\right)$	$t^* = \underline{t}$, always positive. $t^* = t°$, always positive. $t^* = \bar{t}$, depends on $z_{limit\ 6.1}$.	$t°_{5.1}$	Not expand
	$(p_1 - cs_1^2) - (p_2 - cs_2^2) < 0$	$\bar{t}$	always negative		

CHAPTER 6

JOINT ONLINE CHANNEL AND OUTLET BUSINESS

In this chapter, we want to investigate the consequences of opening an online channel for the outlet business of the retailer. Here we assume that the retailer already has the primary business with the physical and online channel and an outlet physical channel. We assume that both branches are viable, that is, they have positive demand and profit margin. That is, $(p_1 - cs_1^2) > 0$(A.3.2), $(\frac{p_1 - p_2 + k - m}{s_1 - s_2} -$

$\frac{p_2 + m}{s_2}) \geq 0$ (A.5.1) and $(p_2 - cs_2^2 > 0$ (A.5.2) and $\left(b - \frac{p_1 - p_2 + k - m}{s_1 - s_2}\right) \geq 0.$ (A.5.3).

When opening the outlet online channel, we assume that the retailer has to sustain the same standards as it sets for the primary online channel (due to reasons such as sharing the same website and etc.). Thus, the retailer will use the service quality (t) already determined when opening the primary online channel. In terms of product, the retailer will stay consistent with the quality level and price available at the outlet physical channel. Therefore, the retailer does not have any decision variables to set here; it only needs to evaluate whether it is profitable or not to open the online outlet channel.

In this model the consumer has four options to make a purchase;

$$U = \begin{cases} \theta s_1 - p_1 - k, & \text{if he purchases from the primary pyhsical store} \\ \theta s_2 - p_2 - m, & \text{if he purchases from the outlet physical store} \\ \theta s_1 - p_1 - dt, & \text{if he puchases from the online primary store} \\ \theta s_2 - p_2 - dt, & \text{if he puchases from the online outlet store} \\ \text{reservation utility (0),} & \text{otherwise} \end{cases}$$

If the consumer generates a positive utility from more than one option, he will pick the channel (business) that offers the highest utility to him.

We assume that consumers differ in sensitivity to the online service and promised delivery time (d) while purchasing at the online channel as well as willingness- to-pay for quality (θ). Consumer types are distributed uniformly on [0, b] according to their types d and θ, where b>0. We assume that b is high enough to enable a profitable business for the retailer (i.e.; for nonnegative market share and margin). In this chapter, we assume that $(b - \frac{m}{\underline{t}}) > 0$ (A.6.1) and $\left(b - \frac{p_1 - p_2}{s_1 - s_2}\right) > 0$ (A.6.2) to ensure nonnegative market share.

We assume that inconvenience cost associated with the physical outlet is greater than the inconvenience cost associated with the physical primary chain since outlet malls are located far away from city centers. In short, $m \geq k > 0$ (A.3.3). Notations used in this chapter are presented in Table 6.1.

Consumers can observe the product qualities and prices available before they decide to buy. They buy a maximum of one product. They purchase only when their net utility is greater than or equal to their reservation utility, assumed as 0 in our study.

The utility of the consumer who uses the online channel decreases because of the delivery time and the risk associated with fit, color, and etc. The utility of the consumer who purchases from the offline store decreases due to the inconvenience of physically visiting the store and risk of leaving empty-handed.

Table 6.1: Notation

Decision variable(s)	
s_1	Quality level of the original brand, $s_1 \geq s_2$ (given)
p_1	Price of the original brand, $p_1 \geq p_2$ (given)
s_2	Quality level of the outlet branch, $s_2 > 0$ (given)
p_2	Price of the outlet branch, $p_2 > 0$ (given)
t	The online service quality set for the online channel, $t \in [\underline{t}, \overline{t}]$ (given for 1st analysis, decision variable for 2nd analysis)
Parameters	
c	Unit cost coefficient for a given quality level, $c > 0$
m	Inconvenience cost of visiting and purchasing from the physical outlet, $m \geq k$
k	Inconvenience cost of visiting and purchasing from the physical store, $k > 0$
θ	Quality taste parameter of consumers, $\theta \sim U[0, b]$
d	Sensitivity index to online services and promised delivery time of consumer, $d \sim U[0, b]$
z	Online channel cost coefficient, $z > 0$
D_1	Demand of the original brand, $D_1 \geq 0$
D_2	Demand of the outlet branch, $D_2 \geq 0$
D_3	Demand of the primary online channel, $D_3 \geq 0$
D_4	Demand of the outlet online channel, $D_4 \geq 0$

In this chapter, analysis is conducted under two different frames. In the first frame, we do the analysis for a given (arbitrary) set of p_2, s_2, s_1, p_1 and t. We structure demand for all channels of the retailer, and further investigate change in the retailer's profit. In the second frame, we build on our analysis in Chapter 5 and evaluate what happens when t is set optimally (i.e., from Chapter 5) for a given set of p_2, s_2, p_1 and s_1. In this part of analysis, we evaluate the change in retailer's profit if he opens the outlet online channel with the pre-set levels p_2, s_2, p_1, s_1 and t^*.

Here, we start our analysis by characterizing the retailer's total demand and its split among channels if he opens the online outlet store for a given (arbitrary) set of p_2, s_2 and t.

Lemma 6.1: Let θ_3 denote the quality taste parameter of the last consumer who purchases the product from the online outlet store when there is only online outlet store in the market. θ_1, θ_2 and θ^* are as defined in Lemma 5.1. Then, we may have only the following orderings feasible:

i. $\theta_3 < \theta_1 \leq \theta_2 \leq \theta^*$ is a new form of Case 1 in Proposition 5.1.

ii. $\theta_3 < \theta^* < \theta_2 < \theta_1$ and $\theta^* < \theta_3 < \theta_2 < \theta_1$ are new forms of Case 2 in Proposition 5.1.

Proof: In θ x d space, if $\theta s_2 - p_2 - dt \geq 0$, then consumer may purchase the product from the online outlet. If s_2, p_2 and t are fixed, $\theta s_2 - p_2 - dt = 0$ is considered as a line in θ x d space. The line intercepts the θ axis at $\theta_3 = \frac{p_2}{s_2}$. Hence, $\theta_3 = \frac{p_2}{s_2}$ emerges as another important threshold to determine the demand of the retailer here.

i. Under Case 1 of Proposition 5.1, $\theta_1 \leq \theta_2 \leq \theta^*$ is known. It is clear that $\theta_3 < \theta_1$.

ii. Under Case 2 of Proposition 5.1, $\theta_1 > \theta_2 > \theta^*$ is known. Between θ^* and θ_3, the orders of $\theta^* > \theta_3$ and $\theta^* < \theta_3$ are both feasible. Hence, the feasible orders are $\theta_3 < \theta^* < \theta_2 < \theta_1$ and $\theta^* < \theta_3 < \theta_2 < \theta_1$. ■

Proposition 6.1: The total demand of the retailer and its split across the channels for both Case 1 and Case 2 are as below:

$$D_1 = \left(b - \frac{p_1 - p_2 + k - m}{s_1 - s_2}\right)\left(b - \frac{k}{t}\right) - \frac{0.5\,(m-k)^2}{t(s_1 - s_2)} \text{ for the primary physical store}$$

$$D_2 = \left(\frac{p_1 - p_2 + k - m}{s_1 - s_2} - \frac{p_2 + m}{s_2}\right)\left(b - \frac{m}{t}\right) \text{ for the outlet physical store}$$

$$D_3 = \left(b - \frac{p_1 - p_2}{s_1 - s_2}\right)\frac{k}{t} \text{ for the online store of the primary brand}$$

$$D_4 = \left(\frac{2(p_1 - p_2 + k - m)}{s_1 - s_2} - \frac{p_2 + m}{s_2} - \frac{p_2}{s_2}\right)\frac{0.5m}{t} + \frac{0.5(m-k)(m+k)}{t(s_1 - s_2)} \text{ for the online store of the}$$ outlet branch

Proof: In θ x d space, the online outlet captures some consumers from the outlet store when:

$$\theta s_2 - p_2 - m \le \theta s_2 - p_2 - dt \Rightarrow d \le \tilde{d} = \frac{m}{t}$$

In θ x d space, the online outlet captures some consumers from the primary online store when:

$$\theta s_1 - p_1 - dt \le \theta s_2 - p_2 - dt \Rightarrow \theta \le \frac{p_1 - p_2}{s_1 - s_2}$$

The online outlet captures some consumers from the primary store when:

$$\theta s_1 - p_1 - k \le \theta s_2 - p_2 - dt$$

$$\theta(s_1 - s_2) - p_1 + p_2 - k + dt \le 0 \qquad \text{(Eq.6.1)}$$

Any consumers placed in the region defined with (6.1) prefer the online outlet to the primary physical store. Thus, the online outlet captures some consumers from the primary store.

A consumer will be willing to purchase from the online outlet when:

$$\theta s_2 - p_2 - dt \ge 0 \qquad \text{(Eq.6.2)}$$

Any consumers placed in the region defined with (6.2) prefer the online outlet. The rest of the proof is left to the reader. ∎

Corollary 6.1: Under all cases, opening the online outlet channel expands the retailer's total market despite the cannibalization of other channels.

Proof: Before the online outlet, the retailer serves the consumers with type taste parameter greater than θ_1 and θ_2 under Case 1 and 2, respectively. In both, θ_3 is lower than these thresholds. Thus, the retailer achieves to expand its market by serving the consumers' with $\theta \epsilon [\theta_3, \theta_1]$ and $\theta \in [\theta_3, \theta_2]$ for Case 1 and 2, respectively. ∎

Figure 6.1 shows the overall demand of the retailer after opening the online outlet channel under Case 1; i.e., when $\theta_3 < \theta_1 \le \theta_2 \le \theta^*$. The extra demand generated is $\frac{0.5\, m^2}{t\, s_2}$. The total demand of the online outlet is indicated with the dashed line. The amount of market expansion increases exponentially with m whereas it decreases

with the promised delivery time (increases with the online service quality) and the quality of the outlet product.
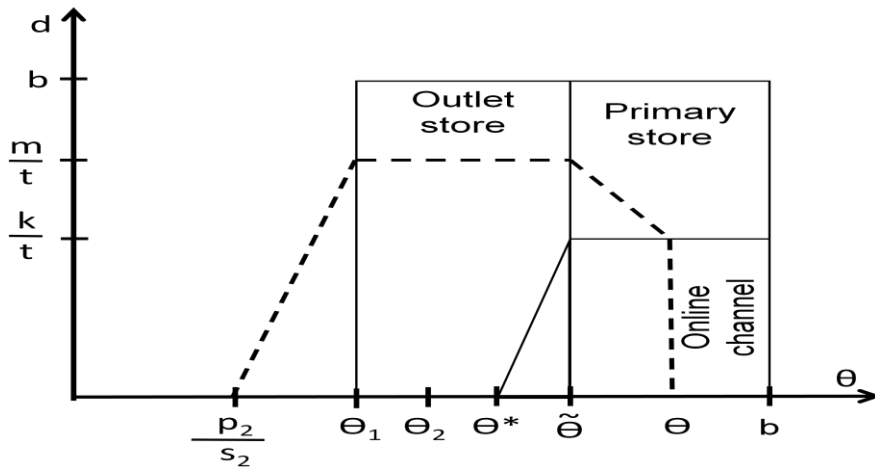

Figure 6.1: Retailer's demand after the introduction of the online outlet (and its split among channels) under Case 1 (where $\theta=(p_1-p_2)/(s_1-s_2)$)

Figure 6.2 (a) and (b) show the demand of the retailer after opening the online outlet under Case 2 for orders $\theta_3 < \theta^* < \theta_2 < \theta_1$ and $\theta^* < \theta_3 < \theta_2 < \theta_1$, respectively. Again, online outlet cannibalizes the other three channels. The total online outlet demand; i.e., cannibalized and generated demand are indicated with the dashed line. Overall, the retailer manages to expand its total demand through online outlet channel by $\dfrac{0.5m^2}{t.s_2} - \dfrac{0.5(s_1p_2+s_1m-s_2p_1)^2}{ts_1s_2^2}$.
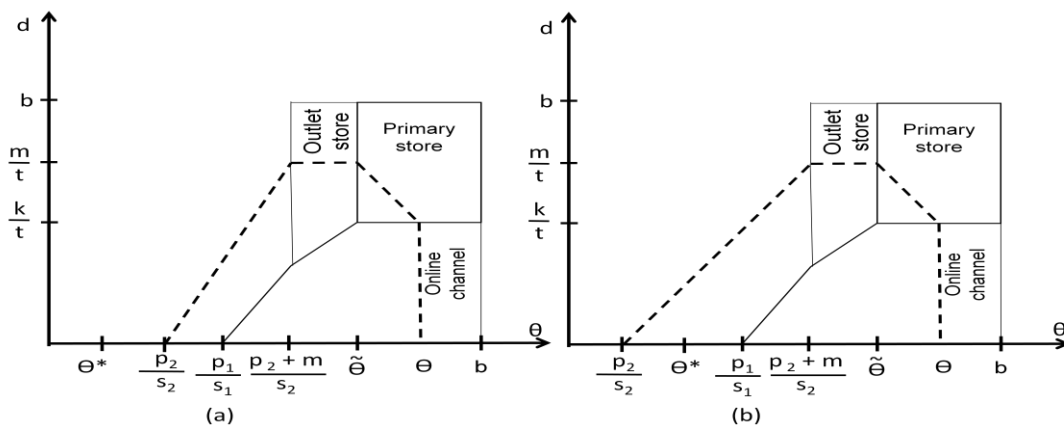

Figure 6.2: Retailer's demand after the introduction of the online outlet (and its split among channels) under Case 2 when $\theta_1>\theta_2>\theta_3>\theta^*$ (a) and $\theta_1>\theta_2>\theta^*>\theta_3$ (b) (where $\theta=(p_1-p_2)/(s_1-s_2)$)

Corollary 6.2: Let $z_{limit\ 6.1}$ and $z_{limit\ 6.2}$ denote the threshold value for the online service cost for Case1 and Case 2, where $z_{limit\ 6.1} = \frac{t^2 m[s_1(p_2 - cs_2^2) - s_2(p_1 - cs_1^2)]}{2(s_2 p_1 - s_1 p_2 - 0.5 m s_1)}$ and

$$z_{limit\ 6.2} = \frac{t^2[-\left(\frac{(m^2-k^2)}{(s_1-s_2)} + \frac{ks_2(\tilde{\theta}-\theta_1)+y(\tilde{\theta}-\theta_2)}{s_2}\right)(p_1-cs_1^2) + \left(\frac{(m^2-k^2)}{(s_1-s_2)} + \frac{m^2+(\tilde{\theta}-\theta_1)(y+ks_2)}{s_2}\right)(p_2-cs_2^2)]}{[(2m-k)(\tilde{\theta}-\theta_1) + \frac{(m-k)^2}{(s_1-s_2)} + \frac{m^2-y(\tilde{\theta}-\theta_2)}{s_2}]}.$$

Let $\Delta\Pi_{6.1}$ and $\Delta\Pi_{6.2}$ denote the profit impact of the online outlet channel on the retailer's profit under Case 1 and 2, respectively.

i. If the online service cost parameter of the market environment is smaller than $z_{limit\ 6.1}$, then opening the online outlet channel is profitable for the retailer under Case 1.

ii. If the online service cost parameter of the market environment is smaller than $z_{limit\ 6.2}$, then opening the online outlet channel is profitable for the retailer under Case 2.

Proof:

i. Under Case 1, the change (i.e.; decrease) in the profit of the retailer because of cannibalization of the current channels are;

$$\Delta\Pi_{primary\ store} = -A(p_1 - cs_1^2) \tag{Eq.6.3}$$

$$\Delta\Pi_{outlet\ store} = -B(p_2 - cs_2^2) \tag{Eq.6.4}$$

$$\Delta\Pi_{online\ store} = -C(p_1 - cs_1^2 - \frac{z}{t^2}) \tag{Eq.6.5}$$

The profit the retailer generates from the online outlet is:

$$\Delta\Pi_{online\ outlet} = (A + B + C + D)(p_2 - cs_2^2 - \frac{z}{t^2}) \tag{Eq.6.6}$$

where $A = \frac{0.5\,(m-k)^2}{t(s_1-s_2)}, B = \left(\frac{p_1-p_2-m}{s_1-s_2} - \frac{p_2+m}{s_2}\right)\frac{m}{t} + \frac{0.5k(2m-k)}{t(s_1-s_2)}, C = \frac{0.5k(2m-k)}{t(s_1-s_2)}, D = \frac{0.5m^2}{ts_2}$

After combining (6.3), (6.4), (6.5) and (6.6), the net change in the total profit of the retailer is;

$$\Delta\Pi_{6.1} = -\frac{0.5m^2}{(s_1-s_2)} \cdot (p_1 - cs_1^2) + \frac{0.5s_1m^2}{ts_2(s_1-s_2)}(p_2 - cs_2^2) - \left(\frac{m(p_1-p_2-0.5m}{(s_1-s_2)} - \right.$$

$$\left.\frac{m(p_2+0.5m)}{s_2}\right)\frac{z}{t^2} \tag{Eq.6.7}$$

The threshold z value that determines the sign of (Eq.6.7) is defined as

$$z_{limit\ 6.1} = \frac{t^2 m[s_1(p_2-cs_2^2)-s_2(p_1-cs_1^2)]}{2(s_2p_1-s_1p_2-0.5ms_1)}.$$

If $z_{limit\ 6.1} \geq z$, then $\Delta\Pi_{6.1} \geq 0$.

ii. Under Case 2, the change (i.e.; decrease) in the profit of the retailer due to cannibalization of the current channels are:

$$\Delta\Pi_{primary\ store} = -A(p_1 - cs_1^2) \tag{Eq.6.8}$$

$$\Delta\Pi_{outlet\ store} = -B(p_2 - cs_2^2) \tag{Eq.6.9}$$

$$\Delta\Pi_{online\ store} = -C\left(p_1 - cs_1^2 - \frac{z}{t^2}\right) \tag{Eq.6.10}$$

The profit generated from the online outlet is:

$$\Delta\Pi_{online\ outlet} = (A + B + C + D)\left(p_2 - cs_2^2 - \frac{z}{t^2}\right) \tag{Eq.6.11}$$

where $A = \frac{0.5\ (m-k)^2}{t(s_1-s_2)}, B = \left(\frac{2m-k}{t} - \frac{y}{ts_2}\right)0.5(\tilde{\theta} - \theta_1), C = \frac{0.5y(\theta_1-\theta_2)}{s_2t} +$

$\left(\frac{y}{s_2t} + \frac{k}{t}\right)(\tilde{\theta} - \theta_1)0.5 + \frac{k(m-k)}{(s_1-s_2)t}, D = \frac{0.5m(\theta_1-\frac{p_2}{s_2})}{t} - \frac{0.5y(\theta_1-\theta_2)}{s_2t}$ $and\ y = s_1p_2 +$

$s_1m - p_1s_2$

After combining (6.8), (6.9), (6.10) and (6.11), the net change in the total profit of the retailer is

$$\Delta\Pi_{6.2} = -\left(\frac{0.5(m^2-k^2)}{(s_1-s_2)} + \frac{0.5ks_2(\tilde{\theta}-\theta_1)+0.5y(\tilde{\theta}-\theta_2)}{s_2}\right)(p_1 - cs_1^2) + \left(\frac{0.5(m^2-k^2)}{(s_1-s_2)}\right)$$

$\frac{0.5m^2+0.5(\tilde{\theta}-\theta_1)(y+ks_2)}{s_2})(p_2 - cs_2^2) - [0.5(2m - k)(\tilde{\theta} - \theta_1) + \frac{0.5(m-k)^2}{(s_1-s_2)} +$

$\frac{0.5m^2-0.5y(\tilde{\theta}-\theta_2)}{s_2}]\frac{z}{t^2} \tag{Eq.6.12}$

The threshold z value that determines the sign of (Eq.6.12) is defined as

$$z_{limit\ 6.2} = \frac{t^2\left[-\left(\frac{(m^2-k^2)}{(s_1-s_2)}+\frac{ks_2(\tilde{\theta}-\theta_1)+y(\tilde{\theta}-\theta_2)}{s_2}\right)(p_1-cs_1^2)\right]}{\left[(2m-k)(\tilde{\theta}-\theta_1)+\frac{(m-k)^2}{(s_1-s_2)}+\frac{m^2-y(\tilde{\theta}-\theta_2)}{s_2}\right]}$$

$$+ \frac{t^2\left[\left(\frac{(m^2-k^2)}{(s_1-s_2)}+\frac{m^2+(\tilde{\theta}-\theta_1)(y+ks_2)}{s_2}\right)(p_2-cs_2^2)\right]}{\left[(2m-k)(\tilde{\theta}-\theta_1)+\frac{(m-k)^2}{(s_1-s_2)}+\frac{m^2-y(\tilde{\theta}-\theta_2)}{s_2}\right]}$$

If $z_{limit\ 6.2} \geq z$, then $\Delta\Pi_{6.2} \geq 0$. ∎

So far, all analysis is structured on for a given (arbitrary) set of p_2, s_2, p_1, s_1 and t. However, hereafter we intend to build our analysis for the rest of Chapter 6 on Chapter 5. When the retailer sets the online service quality (t) optimally (i.e.; from Chapter 5), the total demand of the retailer and its split across the channels are as given in Proposition 6.1, as well. Also, the optimal service quality (optimal promised delivery time) and its conditions under Case 1 and 2 are given in Proposition 5.2 and 5.3.

In this thesis we assume, to remain consistent with the practice, opening the primary online store and online outlet are sequential events. The optimal online service quality (t^*), $\Delta\Pi_{5.1}$ and $\Delta\Pi_{6.1}$ (i.e., the retailer's profit change with opening the online channels) can be jointly investigated with respect to z under Case 1. We assume that $\underline{t}$ is too small to enforce a real constraint on the optimal delivery time (an easy assumption that will guarantee this is to assume $\underline{t}$ is close enough to zero). Thus, the optimal delivery time will be t° or $\overline{t}$. The order of $z_{concavity\ 5.1}, z_{critic\ 5.1}, z_{limit\ 5.1}$ and $z_{limit\ 6.1}$ is significant to determine t^*, $\Delta\Pi_{5.1}$ and $\Delta\Pi_{6.1}$.

Proposition 6.2: Let $\Delta\Pi_{5.1}$ denote the profit impact of the online primary business on the retailer's profit under Case 1; $\Delta\Pi_{6.1}$ denote the profit impact of the online outlet channel on the retailer's profit under Case 1.

We have the following change in the retailer's profit with respect to the online service cost under Case 1 (i.e.; $\theta_3 < \theta_1 \leq \theta_2 \leq \theta^*$).

i. If $(p_1 - cs_1^2) - (p_2 - cs_2^2) < 0$, then $\Delta\Pi_{5.1} < 0$ for all $z \geq 0$ and the sign of $\Delta\Pi_{6.1}$ depends on $z_{limit\ 6.1}$.

ii. If $(p_1 - cs_1^2) - (p_2 - cs_2^2) > 0$, $z > z_{\text{critic 5.1}}$ and $\frac{p_1-cs_1^2}{p_2-cs_2^2} > \frac{s_1}{s_2}$, then $\Delta\Pi_{6.1} < 0$ for

all $z \geq 0$ and the sign of $\Delta\Pi_{5.1}$ depends on $z_{limit\ 5.1}$.

iii. If $(p_1 - cs_1^2) - (p_2 - cs_2^2) > 0$, $z > z_{\text{critic 5.1}}$ and $\frac{p_1-cs_1^2}{p_2-cs_2^2} < \frac{s_1}{s_2}$, then we may

have

     a) $\Delta\Pi_{5.1} < 0$ and $\Delta\Pi_{6.1} < 0$

     b) $\Delta\Pi_{5.1} < 0$ and $\Delta\Pi_{6.1} > 0$

     c) $\Delta\Pi_{5.1} > 0$ and $\Delta\Pi_{6.1} > 0$

     d) $\Delta\Pi_{5.1} > 0$ and $\Delta\Pi_{6.1} < 0$ depending on the online service cost and other parameters.

iv. If $(p_1 - cs_1^2) - (p_2 - cs_2^2) > 0$, $z_{\text{critic 5.1}} > z > z_{\text{concavity 5.1}}$, then $\Delta\Pi_{5.1} > 0$ for

all $z \geq 0$ and $\Delta\Pi_{6.1} = \begin{cases} \Delta\Pi_{6.1} > 0, \ when \ 1 < \frac{p_1-cs_1^2}{p_2-cs_2^2} < R_{case\ 1} \\ \Delta\Pi_{6.1} < 0, \ when \ \frac{p_1-cs_1^2}{p_2-cs_2^2} > R_{case\ 1} \end{cases}$.

where $R_{case\ 1} = \dfrac{\frac{0.5m^2s_1}{s_2(s_1-s_2)}+\left(\frac{m(p_1-p_2-0.5m)}{s_1-s_2}-\frac{m(p_2+0.5m)}{s_2}\right)\frac{k}{3(2bs_1-2bs_2-2p_1+2p_2+2m-k)}}{\frac{0.5m^2}{s_1-s_2}+\left(\frac{m(p_1-p_2-0.5m)}{s_1-s_2}-\frac{m(p_2+0.5m)}{s_2}\right)\frac{k}{3(2bs_1-2bs_2-2p_1+2p_2+2m-k)}}$

Proof:

i. Here, we will have $z_{limit\ 6.1} > 0 > z_{concavity\ 5.1} > z_{critic\ 5.1} > z_{limit\ 5.1}$. For all $z \geq 0$, $t^* = \bar{t}$ and $\Delta\Pi_{5.1} < 0$. However, the sign of $\Delta\Pi_{6.1}$ depends on $z_{limit\ 6.1}$. See Figure H.1 in Appendix H.

ii. Here, we will have is $z_{limit\ 5.1} > z_{critic\ 5.1} > z_{concavity\ 5.1} > 0 > z_{limit\ 6.1}$ since $z_{limit\ 6.1} < 0$. For all $z \geq 0$, $\Delta\Pi_{6.1} < 0$. The sign of $\Delta\Pi_{5.1}$ depends on $z_{limit\ 5.1}$. See Figure H.2 in Appendix H.

iii. The threshold z values lower than $z_{critic\ 5.1}$ are not taken into consideration for evaluation of orders. Also Order (3) and Order (4) become a single order (which is $z_{limit\ 5.1} > z_{critic\ 5.1}$) after rearrangement. For Order (1), $\Delta\Pi_{5.1}$ becomes negative with lower z values in comparison to $\Delta\Pi_{6.1}$. Similarly, for Order (2), $\Delta\Pi_{6.1}$ becomes negative with lower z values in comparison to $\Delta\Pi_{5.1}$. For Order (3) and (4), opening

the online outlet business is not profitable at all for all z values. However, the sign of $\Delta\Pi_{5.1}$ depends on $z_{limit\ 5.1}$.

Order (1)

Here, we will have $z_{concavity\ 5.1} < z_{critic\ 5.1} < z_{limit\ 5.1} < z_{limit\ 6.1}$. The total profit impact of the online channels is always positive when $z \in [z_{concavity\ 5.1}, z_{limit\ 5.1}]$. However, we will have $\Delta\Pi_{5.1} < 0$ but $\Delta\Pi_{6.1} > 0$ when $z \in [z_{limit\ 5.1}, z_{limit\ 6.1}]$. For all $z \geq z_{limit\ 6.1}$, both $\Delta\Pi_{5.1} < 0$ and $\Delta\Pi_{6.1} < 0$. See Figure H.3 in Appendix H.

Order (2)

Here, we will have $z_{concavity\ 5.1} < z_{critic\ 5.1} < z_{limit\ 6.1} < z_{limit\ 5.1}$. The total profit impact of the online channels is always positive when $z \in [z_{concavity\ 5.1}, z_{limit\ 6.1}]$. However, we will have $\Delta\Pi_{5.1} > 0$ but $\Delta\Pi_{6.1} < 0$ when $z \in [z_{limit\ 6.1}, z_{limit\ 5.1}]$. For all $z \geq z_{limit\ 5.1}$, we will have both $\Delta\Pi_{5.1} < 0$ and $\Delta\Pi_{6.1} < 0$. See Figure H.4 in Appendix H.

Order (3) and Order (4)

Here, we will have $z_{concavity\ 5.1} < z_{limit\ 6.1} < z_{critic\ 5.1} < z_{limit\ 5.1}$ and $z_{limit\ 6.1} < z_{concavity\ 5.1} < z_{critic\ 5.1} < z_{limit\ 5.1}$ for Order (3) and (4), respectively. Here, we always have $\Delta\Pi_{6.1} < 0$. When $z \in [z_{critic\ 5.1}, z_{limit\ 5.1}]$, we will have $\Delta\Pi_{5.1} > 0$ and total profit impact of opening the online channels may be positive or negative. For all $z \geq z_{limit\ 5.1}$, opening the online sales channels are not profitable for the retailer at all. See Figure H.5 in Appendix H.

iv. If $t°$ is plugged into (6.7) in place of optimal promised delivery time,

$\Delta\Pi_{6.1} =$

$-(p_1 - cs_1^2)\left[\frac{0.5m^2}{s_1-s_2} + \left(\frac{m(p_1-p_2-0.5m)}{s_1-s_2} - \frac{m(p_2+0.5m)}{s_2}\right)\frac{k}{3(2bs_1-2bs_2-2p_1+2p_2+2m-k)}\right] +$

$(p_2 - cs_2^2)\left[\frac{0.5m^2s_1}{s_2(s_1-s_2)} + \left(\frac{m(p_1-p_2-0.5m)}{s_1-s_2} - \frac{m(p_2+0.5m)}{s_2}\right)\frac{k}{3(2bs_1-2bs_2-2p_1+2p_2+2m-k)}\right]$

$$(\text{Eq.6.13})$$

(6.13) does not consist of z related term thus, the ratio of profit margins determines the net change in the profit.

Let $R_{case\,1}$ denote the ratio of coefficient of profit margins. Note that $R_{case\,1}$ is greater than 1.

$$R_{case\,1} = \frac{\frac{0.5m^2 s_1}{s_2(s_1-s_2)}+\left(\frac{m(p_1-p_2-0.5m)}{s_1-s_2}-\frac{m(p_2+0.5m)}{s_2}\right)\frac{k}{3(2bs_1-2bs_2-2p_1+2p_2+2m-k)}}{\frac{0.5m^2}{s_1-s_2}+\left(\frac{m(p_1-p_2-0.5m)}{s_1-s_2}-\frac{m(p_2+0.5m)}{s_2}\right)\frac{k}{3(2bs_1-2bs_2-2p_1+2p_2+2m-k)}} \qquad \text{(Eq.6.14)}\blacksquare$$

The optimal online service quality (t^*), $\Delta\Pi_{5.2}$ and $\Delta\Pi_{6.2}$ (i.e., the retailer's profit change with opening the online channels) can be jointly investigated with respect to z under Case 2. Here, we again assume that $\underline{t}$ is too small to enforce a real constraint on the optimal delivery time. Thus, the optimal delivery time will be t° or $\bar{t}$. The order of $z_{concavity\,5.2}, z_{critic\,5.2}, z_{limit\,5.2}$ and $z_{limit\,6.2}$ is significant to determine $t^*, \Delta\Pi_{5.2}$ and $\Delta\Pi_{6.2}$.

Proposition 6.3: Let $\Delta\Pi_{5.2}$ denote the profit impact of the online primary business on the retailer's profit under Case 2; $\Delta\Pi_{6.2}$ denote the profit impact of the online outlet channel on the retailer's profit under Case 2.

Define $\lambda = \dfrac{\frac{(m^2-k^2)}{(s_1-s_2)}+\frac{m^2+(\tilde{\theta}-\theta_1)(y+ks_2)}{s_2}}{\frac{(m^2-k^2)}{(s_1-s_2)}+\frac{ks_2(\tilde{\theta}-\theta_1)+y(\tilde{\theta}-\theta_2)}{s_2}}$.

We have the following change in the retailer's profit with respect to the online service cost under Case 2.

i. If $\;(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) < 0$

and $\lambda < \frac{p_1-cs_1^2}{p_2-cs_2^2}$, then $\Delta\Pi_{5.2} < 0$ and $\Delta\Pi_{6.2} < 0$ for all $z \geq 0$.

ii. If $(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) <$

0 and $\lambda > \frac{p_1-cs_1^2}{p_2-cs_2^2}$, then $\Delta\Pi_{5.2} < 0$ for all $z \geq 0$ and the sign of $\;\Delta\Pi_{6.2}$ depends on $z_{limit\,6.2}$.

iii. If $\;(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) > 0$,

$z > z_{critic\,5.2}$ and $\lambda < \frac{p_1-cs_1^2}{p_2-cs_2^2}$, then $\Delta\Pi_{6.2} < 0$ for all $z \geq 0$ and the sign of $\;\Delta\Pi_{5.2}$ depends on $z_{limit\,5.2}$.

iv. If $(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) > 0$,

$z > z_{critic\ 5.2}$ and $\lambda > \frac{p_1 - cs_1^2}{p_2 - cs_2^2}$, then we may have

      a) $\Delta\Pi_{5.2} < 0$ and $\Delta\Pi_{6.2} < 0$

      b) $\Delta\Pi_{5.2} < 0$ and $\Delta\Pi_{6.2} > 0$

      c) $\Delta\Pi_{5.2} > 0$ and $\Delta\Pi_{6.2} > 0$

      d) $\Delta\Pi_{5.2} > 0$ and $\Delta\Pi_{6.2} < 0$ depending on the online service cost and other parameters.

v. If $(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) > 0$,

$z_{critic\ 5.2} > z > z_{concavity\ 5.2}$, then $\Delta\Pi_{5.2} > 0$ for all $z \geq 0$ and

$$\Delta\Pi_{6.2} = \begin{cases} \Delta\Pi_{6.2} > 0 & when\ \frac{p_1 - cs_1^2}{p_2 - cs_2^2} < R_{case\ 2} \\ \Delta\Pi_{6.2} < 0 & when\ \frac{p_1 - cs_1^2}{p_2 - cs_2^2} > R_{case\ 2} \end{cases}$$

where $R_{case\ 2} = \dfrac{[\frac{0.5(m^2 - k^2)}{s_1 - s_2} + \frac{0.5m^2 + 0.5(\tilde{\theta} - \theta_1)(y + ks_2)}{s_2}]}{\delta}$

$$+ \dfrac{\frac{(\tilde{\theta} - \theta_1)(y + ks_2)}{3[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)]}\left[0.5(2m - k)(\tilde{\theta} - \theta_1) + \frac{0.5(m - k)^2}{s_1 - s_2} + \frac{0.5m^2 - 0.5y(\tilde{\theta} - \theta_2)}{s_2}\right]}{\delta}$$

where $\delta = [\frac{0.5(m^2 - k^2)}{s_1 - s_2} + \frac{0.5ks_2(\tilde{\theta} - \theta_1) + 0.5y(\tilde{\theta} - \theta_2)}{s_2} + \frac{((ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2))}{3[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)]}$

$$[0.5(2m - k)(\tilde{\theta} - \theta_1) + \frac{0.5(m - k)^2}{s_1 - s_2} + \frac{0.5m^2 - 0.5y(\tilde{\theta} - \theta_2)}{s_2}]]$$

Proof:

i. We have four feasible orders of the threshold z values. However, independent of these orders, $\Delta\Pi_{5.2} < 0$ and $\Delta\Pi_{6.2} < 0$ for all $z \geq 0$.

ii. Here, we will have $z_{limit\ 6.2} > 0 > z_{concavity\ 5.2} > z_{critic\ 5.2} > z_{limit\ 5.2}$. For all $z \geq 0$, $t^* = \bar{t}$ and $\Delta\Pi_{5.2} < 0$. When $z \in [0, z_{limit\ 6.2}]$, we will have $\Delta\Pi_{6.2} < 0$. See Figure I.1 in Appendix I.

iii. Here, we will have $z_{limit\ 5.2} > z_{critic\ 5.2} > z_{concavity\ 5.2} > 0 > z_{limit\ 6.2}$. For all $z \geq 0, \Delta\Pi_{6.2} < 0$. When $z \in [z_{concavity\ 5.2}, z_{limit\ 5.2}]$, total profit impact of the

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online channels may or may not be positive. For all $z \geq z_{limit\ 5.2}$, we will have both $\Delta\Pi_{5.2} < 0$ and $\Delta\Pi_{6.2} < 0$. See Figure I.2 in Appendix I.

iv. Follows similarly to proof of Proposition 6.2(iii).

v. If $t°$ is plugged into (6.12) in place of optimal promised delivery time,

$$\Delta\Pi_{6.2} = -(p_1 - cs_1^2)[\frac{0.5(m^2 - k^2)}{s_1 - s_2} + \frac{0.5ks_2(\tilde{\theta} - \theta_1) + 0.5y(\tilde{\theta} - \theta_2)}{s_2} + \frac{((ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2))}{3[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)]}$$

$$[[0.5(2m - k)(\tilde{\theta} - \theta_1) + \frac{0.5(m - k)^2}{s_1 - s_2} + \frac{0.5m^2 - 0.5y(\tilde{\theta} - \theta_2)}{s_2}]]$$

$$+(p_2 - cs_2^2)[\frac{0.5(m^2 - k^2)}{s_1 - s_2} + \frac{0.5m^2 + 0.5(\tilde{\theta} - \theta_1)(y + ks_2)}{s_2} + \frac{(\tilde{\theta} - \theta_1)(y + ks_2)}{3[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)]}$$

$$[0.5(2m - k)(\tilde{\theta} - \theta_1) + \frac{0.5(m - k)^2}{s_1 - s_2} + \frac{0.5m^2 - 0.5y(\tilde{\theta} - \theta_2)}{s_2}]] \qquad \text{(Eq.6.15)}$$

(6.15) does not consist of z related term, thus the ratio profit margins determines the net change in profit.

Let $R_{case\ 2}$ denote the ratio of coefficient of profit margins.

$$R_{case\ 2} = \frac{[\frac{0.5(m^2 - k^2)}{s_1 - s_2} + \frac{0.5m^2 + 0.5(\tilde{\theta} - \theta_1)(y + ks_2)}{s_2}]}{\delta}$$

$$+ \frac{\frac{(\tilde{\theta} - \theta_1)(y + ks_2)}{3[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)]}[0.5(2m - k)(\tilde{\theta} - \theta_1) + \frac{0.5(m - k)^2}{s_1 - s_2} + \frac{0.5m^2 - 0.5y(\tilde{\theta} - \theta_2)}{s_2}]}{\delta} \qquad \text{(Eq.6.16)}$$

where $\delta = [\frac{0.5(m^2 - k^2)}{s_1 - s_2} + \frac{0.5ks_2(\tilde{\theta} - \theta_1) + 0.5y(\tilde{\theta} - \theta_2)}{s_2} + \frac{((ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2))}{3[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)]}$

$$[0.5(2m - k)(\tilde{\theta} - \theta_1) + \frac{0.5(m - k)^2}{s_1 - s_2} + \frac{0.5m^2 - 0.5y(\tilde{\theta} - \theta_2)}{s_2}]] \qquad \blacksquare$$

The outlet online channel profitability is determined by two factors: margin ratio between the primary and the outlet business and the online service cost. When online service cost is low enough, it ceases to be the limiting factor and the margin ratio determines profitability. When providing the online service is costly, opening online channels are not profitable for the retailer at all in both Case 1 and 2. When the online service cost is moderate (medium level) and the profit margin of the original

brand is greater than that of the outlet brand, the profit impact of the primary online store is always positive, but that of outlet online store can be positive or negative. When the online service cost is low and the primary brand margin is lower, the profit impact of the primary online store is positive whereas that of the outlet online store is negative. This condition may arise when the online service cost is moderate and the primary brand margin is higher.

Under all cases, generally $z_{limit\,5}$ is larger than $z_{limit\,6}$. It translates into that the profit impact of opening online primary store is profitable for markets compared to that of the online outlet store.

Table 6.2 and 6.3 show summary of Proposition 6.2 and 6.3, respectively.

Table 6.2: Threshold values and orders of all options under Case 1 of Chapter 5 and 6

Case 1 of Chapter 5 and 6

t^* condition	t^*	$z_{limit\,6.1}$	Threshold values and order	Note
$(p_1 - cs_1^2) - (p_2 - cs_2^2) < 0$	$\bar{t}$	$z_{limit\,6.1} > 0$	$z_{limit\,6.1} > 0 > z_{concavity\,51} > z_{critic\,5.1} > z_{limit\,5.1}$	if $z_{limit\,6.1} \geq z$, then $\Delta\Pi_{6.1} \geq 0$.
$(p_1 - cs_1^2) - (p_2 - cs_2^2) > 0$	$\bar{t}$	$z_{limit\,6.1} < 0$	$z_{limit\,5.1} > z_{critic\,5.1} > z_{concavity\,5.1} > 0 > z_{limit\,6.1}$	if $z_{limit\,6.1} \geq z$, then $\Delta\Pi_{6.1} \geq 0$.
$(p_1 - cs_1^2) - (p_2 - cs_2^2) > 0$	$\bar{t}$	$z_{limit\,6.1} > 0$	Order(1),Order(2),Order(3) and Order(4)	if $z_{limit\,6.1} \geq z$, then $\Delta\Pi_{6.1} \geq 0$.
$(p_1 - cs_1^2) - (p_2 - cs_2^2) > 0$	t°	$z_{limit\,6.1} > 0$	$R_{case\,1}$	If $\frac{p_1 - cs_1^2}{p_2 - cs_2^2} < R_{case\,1}$, then $\Delta\Pi_{6.1} > 0$

Table 6.3: Threshold values and orders of all options under Case 2 of Chapter 5 and 6

		Case 2 of Chapter 5 and 6		
t^* condition	t^*	$z_{limit\ 6.2}$	Threshold values and order	Note
$(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right)$ $-(p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) < 0$	$\bar{t}$	$z_{limit\ 6.2} < 0$	four feasible orders of the negative threshold z value	$\Delta\Pi_{5.2} < 0$ and $\Delta\Pi_{6.2} < 0$ for all $z \geq 0$.
$(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right)$ $-(p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) < 0$	$\bar{t}$	$z_{limit\ 6.2} > 0$	$z_{limit\ 6.2} > 0 >$ $z_{concavity\ 5.2} > z_{critic\ 5.2} >$ $z_{limit\ 5.2}$	if $z_{limit\ 6.2} \geq z$, then $\Delta\Pi_{6.2} \geq 0$.
$(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right)$ $-(p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) > 0$	$\bar{t}$	$z_{limit\ 6.2} < 0$	$z_{limit\ 5.2} > z_{critic\ 5.2} >$ $z_{concavity\ 5.2} > 0 >$ $z_{limit\ 6.2}$	if $z_{limit\ 6.2} \geq z$, then $\Delta\Pi_{6.2} \geq 0$.

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Table 6. 3: (cont'd) Threshold values and orders of all options under Case 2 of Chapter 5 and 6

		Case 2 of Chapter 5 and 6		
t* condition	t*	$z_{limit\,6.2}$	Threshold values and order	Note
$(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right)$ $-(p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) > 0$	$\bar{t}$	$z_{limit\,6.2} > 0$	Order(1),Order(2),Order(3) and Order(4)	if $z_{limit\,6.2} \geq z$, then $\Delta\Pi_{6.2} \geq 0.$
$(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right)$ $-(p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) > 0$	$t°$	$z_{limit\,6.2} > 0$	$R_{case\,2}$	If $\frac{p_1 - cs_1^2}{p_2 - cs_2^2} < R_{case\,2}$, then $\Delta\Pi_{6.2} > 0$

CHAPTER 7

CONCLUSION

This thesis focuses on a monopolist retailer's product differentiation problem in a multi-channel environment. We investigate which conditions would motivate retailer to open an outlet branch, to open an online channel, and to even potentially open an online channel for the outlet branch, and how these decisions interact with each other.

In Chapter 3, we characterize how the retailer positions its outlet branch in terms of quality level and price point and whether he will be better off in terms of total profit. First, we neglect any inconvenience cost regarding either channel. We demonstrate that opening the outlet business is preferable for both the retailer and the consumer. Although the outlet store cannibalizes some of the primary store demand, the retailer manages to expand its market share and generate more profit through the outlet business. Also, consumer welfare increases in terms of consumer surplus. Furthermore, we analyze the type of retailer in terms of the primary brand position strategies: myopic retailer and non-myopic retailer. In the former strategy, the retailer can position the primary brand without the any consideration of the outlet business opportunity in the future. That is, the myopic retailer positions the primary product and then the outlet product. In the latter one, the retailer also considers the future outlet product position while positioning the primary product. We show that non-myopic approach is preferable for the retailer and the non-myopic retailer sets a higher price and a quality level for the primary brand compared to its myopic counterpart. In the second part of chapter, we take account inconvenience costs associated with visiting a brick-and-mortar retailer. This is closer to what happens in practice. An extensive numerical study is conducted to evaluate the changes (trend)

in the outlet product position, demand of primary and outlet business and total profit of the monopolist brick-and-mortar retailer in the presence of inconvenience cost. We also find that as the primary brand gets more competitive in terms of quality and price, the outlet price level tends to be lower. The price competitiveness of the primary brand does not impact the outlet quality. However, as the primary brand gets more competitive in terms of quality, it produces the larger market size for the outlet brand and the outlet quality level tends to be higher. We find that the profit margins of the primary and outlet business are more significant than market share in determining the profit of the retailer. We find that as the primary brand gets more competitive in terms of quality level, its outlet business becomes more robust (viable) than others.

In Chapter 4, we are interested in a monopolist retailer's online channel decision. Through the online channel, consumers are freed from the inconvenience of physically going to the store and need to wait for a pre-specified time to receive his product. With the online channel, willingness to wait for products is considered as a second dimension in addition to their differences in willingness to pay for the product quality. Under such a setting, we investigate the service quality offered at the online channel which is the retailer's main decision. Online service quality may involve all customer services provided by the online store, promised delivery time windows as well as shipping charges. We assume that the promised delivery time can be set in a feasible interval and lower bound is positive since all practically feasible delivery/service times are in this interval. We find that the retailer achieves market expansion. However, we find that the profit impact of the online channel on the retailer's total profit changes with the online service quality. Explicitly, when providing online services is not costly, opening an online channel is always preferable for the retailer. Otherwise, the extra market does not justify the costs of providing online services.

In Chapter 5, we focus on the service quality offered at the online channel while the retailer is already selling its original and outlet brand by the respective physical channels. We determine the three threshold taste parameter values (θ) and that the two feasible orders (Case 1 and Case 2) among all possible rankings of these taste parameters. We demonstrate that opening the online channel does not change the

total market of the retailer under Case 1 whereas it expands the total market of the retailer under Case 2. Furthermore, we show that the optimal promised delivery time and its conditions are different for Case 1 and 2. We find that the profit impact of the online channel on the retailer's total profit depends on the feasible interval for online services, and the online cost parameter in the market.

In Chapter 6, we focus on the profit impact of the online outlet store on the monopolist retailer's profit when the retailer already has the primary business with the physical and online channel and an outlet physical channel. The analysis on retailer's profit impact is conducted under two different settings. In the former setting, the outlet product position and the promised delivery time are taken as arbitrary; in the latter the optimal promised delivery time found in Chapter 5 is used. We find that under all cases, opening the online outlet channel expands the retailer's total market despite the cannibalization of other channels. When the optimal promised delivery time found in Chapter 5 is set, we investigate retailer's profit change with opening the online channels together with respect to the online cost parameter. We find that the profitability is determined by the two factors: margin ratio between the primary and the outlet business and online service cost. When online service cost is low enough, it ceases to be the limiting factor and margin ratio determines everything. Explicitly, for each case, when the feasible range of delivery time is not limiting for the retailer while opening the primary online channel, margin ratio becomes the critical factor to characterize the total profit impact of the online outlet channel. Otherwise, threshold value for online service cost determines the profitability of primary online, outlet online and both.

We limit our work to use the same unit cost coefficient for a given quality independent of brands. In real life, unit cost of product depends on many factors; i.e., material used in the product, dye type, etc. Thus, different unit cost coefficients may be defined for the primary and outlet brand. We also assume that there are no fixed costs. However, different fixed costs may be associated the physical and online channel.

In Chapter 6, we assume, to remain consistent with the practice, opening the primary online store and online outlet are sequential events. However, the profit impact of the

online channels on the retailer's total profit is evaluated in the same online service cost environment. This part of work can be extended to do the same analysis with a changing the online service cost parameter over time.

REFERENCES

Berman, B., Evans, J.R., Retail Management: A Strategic Approach, 10th Ed., Pearson Prentice Hall, USA. 2006

Levy, M., Weitz, B.A., Retailing Management, 3rd Ed., Richard D. Irwin Inc., USA, 2012

www.jcrew.com/help/about jcrew.jsp, Last visited on October 2014

Shy, O., Industrial Organization: Theory and Applications, 2nd Ed., The MIT Press, England, 1996

Hotelling, H., 1929," Stability in Competition", The Economic Journal, 39, 41-57.

d'Aspremont, C., Gabszewicz, J.,Thisse, J.,1979, Econometrica,On Hotelling's Stability in Competition, 47,1145-1150.

Moorthy, K.S., 1988, Marketing Science, Product and Price Competition in Duopoly, Vol. 7, No. 2, 141-168.

Moorthy, K.S., 1984, Marketing Science, Market Segmentation, Self-Selection, and Product Line Design, Vol.3, No.4, 288-307.

Moorthy, K.S., 1987, Ann. Telecommun., Product Line Competition, 42, 11-12,655-663.

Moorthy, K.S., Png, I.P.L., 1992, Management Science, Market Segmentation, Cannibalization and the Timing of the Product Introductions, Vol.38, No.3, 345-359.

Purohit, D., 1994, Marketing Science, What Should You Do When Your Competitors Send in the Clones?, Vol.13, No.4, 392-411.

Vandenbosch, M.B., Weinberg, C.B., 1995, Marketing Science, Product and Price Competition in a Two-Dimensional Vertical Differentiation Model, Vol. 14, No. 2, 224-249.

Lauga, D.O., Ofek, E., 2011, Marketing Science, Product Positioning in a Two-Dimensional Vertical Differentiation Model: The Role of Quality Costs, Vol. 30, No. 5, 903–923.

Desai, S.P., 2001, Marketing Science, Quality Segmentation in Spatial Markets: When Does Cannibalization Affect Product Line Design?, Vol. 20, No. 3, 265–283.

Kim, K., Dilip, C., Liu, Y., 2013, Marketing Science, Can Commonality Relieve Cannibalization in Product Line Design?, Vol.32, No.3, 510-521.

Ferguson, M.E., Koenigsberg, O., 2007, Production and Operations Management, How Should a Firm Manage Deteriorating Inventory?, Vol.16, No.3, 306-321.

Chen, K., Kaya, M., Özer,Ö., 2008, Manufacturing & Service Operations Management, Dual Sales Channel Management with Service Competition, Vol.10, No.4, 654-675.

Tsay, A.A., Agrawal, N., 2004, Production and Operations Management, Channel Conflict and Coordination in the E-Commerce Age, Vol.13, No.1, 93-110.

Zhang, X., 2009, Marketing Science, Retailers' Multichannel and Price Advertising Strategies, Vol.28, No.6, 1080-1094.

Bernstein, F., Song, J., Zheng, X., 2008, European Journal of Operational, ''Bricks-and-mortar'' vs. ''Clicks-and-mortar'':An Equilibrium Analysis, 187,671-690.

Coughlan, A.T., Soberman, A.D., 2005, J. of Research in Marketing, Strategic Segmentation Using Outlet Malls, Vol. 22, 61-86.

Liu, Y., Gupta,S., Zhang, Z.J.,2006, Marketing Science, Note on Self-Restraint as an Online Entry-Deterrence Strategy, Vol.52, No.11, 1799-1809.

Balasubramanian, S., 1998, Marketing Science, Mail versus Mall: A Strategic Analysis of Competition between Direct Marketers and Conventional Retailers, Vol.17, No.3, 181-195

APPENDIX A

NUMERICAL ANALYSIS FOR TRIAL 2

Table A. 1: s_2^*, p_2^*, D_1^*, D_2^* retailer's profit and total market and profit margin of the outlet for trial 2 when k=0.5

k	m	s_2^*	p_2^*	D_1^*	D_2^*	Profit	Total Market	$(p_2^* - cs_2^{*2})$
0.5	0.5	4.56	4.81	38.491	0.343	139.508	38.835	2.730
0.5	0.6	4.51	4.69	38.506	0.321	139.475	38.827	2.655
0.5	0.7	4.49	4.61	38.5217	0.295	139.444	38.817	2.593
0.5	0.8	4.47	4.54	38.537	0.269	139.416	38.806	2.541
0.5	0.9	4.45	4.46	38.552	0.242	139.391	38.795	2.479
0.5	1	4.43	4.39	38.567	0.216	139.368	38.784	2.427
0.5	1.1	4.42	4.32	38.582	0.190	139.347	38.772	2.366
0.5	1.2	4.4	4.25	38.596	0.164	139.329	38.761	2.314
0.5	1.3	4.39	4.19	38.611	0.139	139.314	38.750	2.262
0.5	1.4	4.38	4.12	38.625	0.113	139.301	38.738	2.201
0.5	1.5	4.33	4	38.637	0.092	139.291	38.730	2.125
0.5	1.6	4.32	3.94	38.651	0.067	139.283	38.718	2.073

Table A.1: (cont'd) s_2^*, p_2^*, D_1^*, D_2^* retailer's profit and total market and profit margin of the outlet for trial 2 when k=0.5

k	m	s_2^*	p_2^*	D_1^*	D_2^*	Profit	Total Market	$(p_2^* - cs_2^{*2})$
0.5	1.7	4.3	3.86	38.665	0.041	139.278	38.706	2.011
0.5	1.8	4.28	3.79	38.679	0.015	139.275	38.694	1.958
0.5	1.9	0	0	38.687	0	139.275	38.687	0
0.5	2	0	0	38.687	0	139.275	38.687	0
0.5	2.1	0	0	38.687	0	139.275	38.687	0
0.5	2.2	0	0	38.687	0	139.275	38.687	0
0.5	2.3	0	0	38.687	0	139.275	38.687	0
0.5	2.4	0	0	38.687	0	139.275	38.687	0
0.5	2.5	0	0	38.687	0	139.275	38.687	0
0.5	2.6	0	0	38.687	0	139.275	38.687	0
0.5	2.7	0	0	38.687	0	139.275	38.687	0
0.5	2.8	0	0	38.687	0	139.275	38.687	0
0.5	2.9	0	0	38.687	0	139.275	38.687	0
0.5	3	0	0	38.687	0	139.275	38.687	0
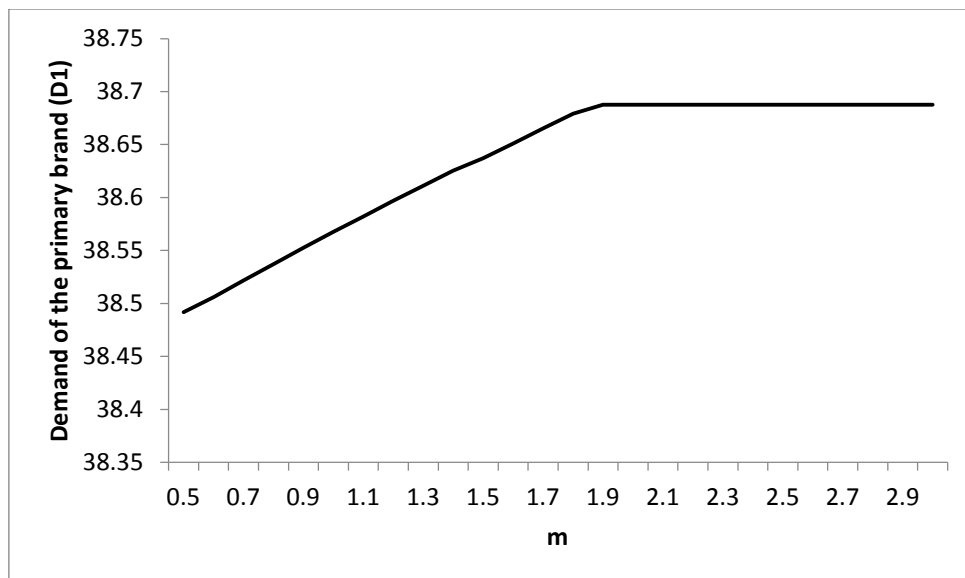
Figure A.1: D_1^* versus m plot for trial 2 when k=0.5
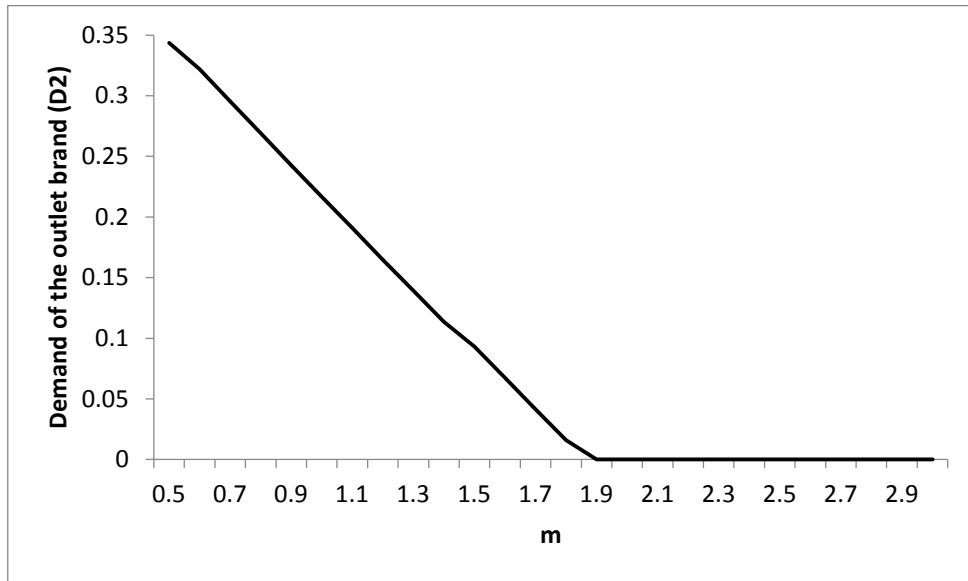
Figure A.2: D_2^* versus m plot for trial 2 when k=0.5
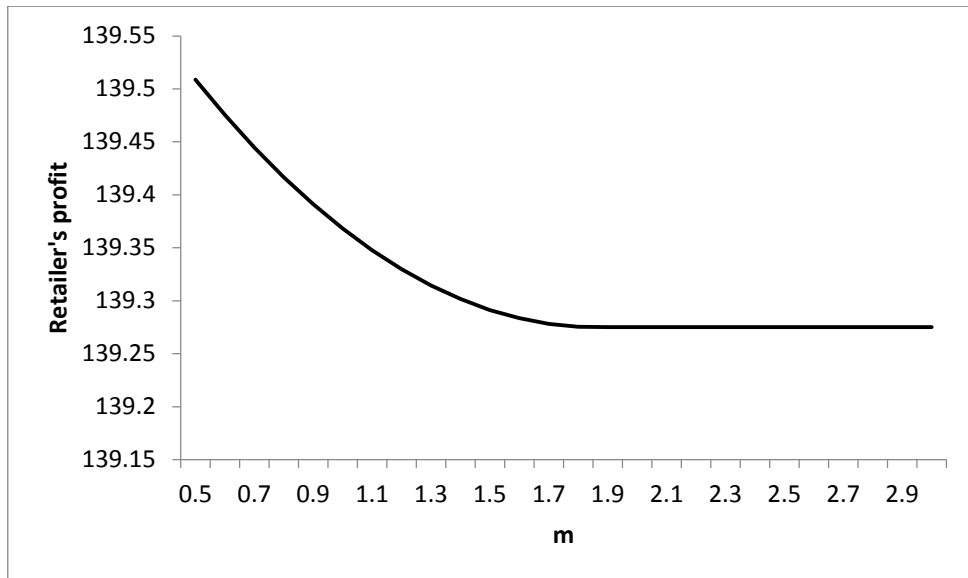
Figure A.3: Retailer's profit versus m plot for trial 2 when k=0.5
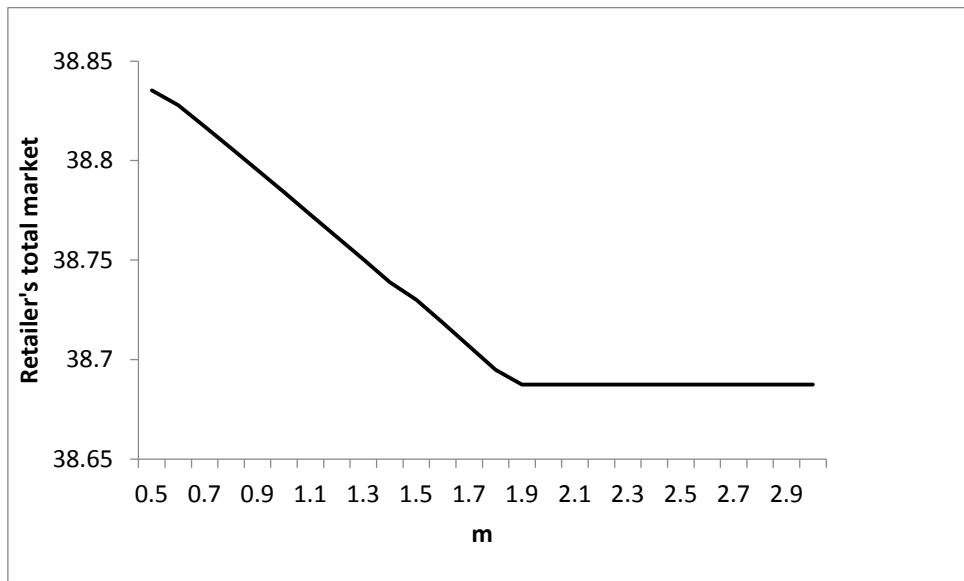
Figure A.4: Retailer's total market share versus m plot for trial 2 when k=0.5
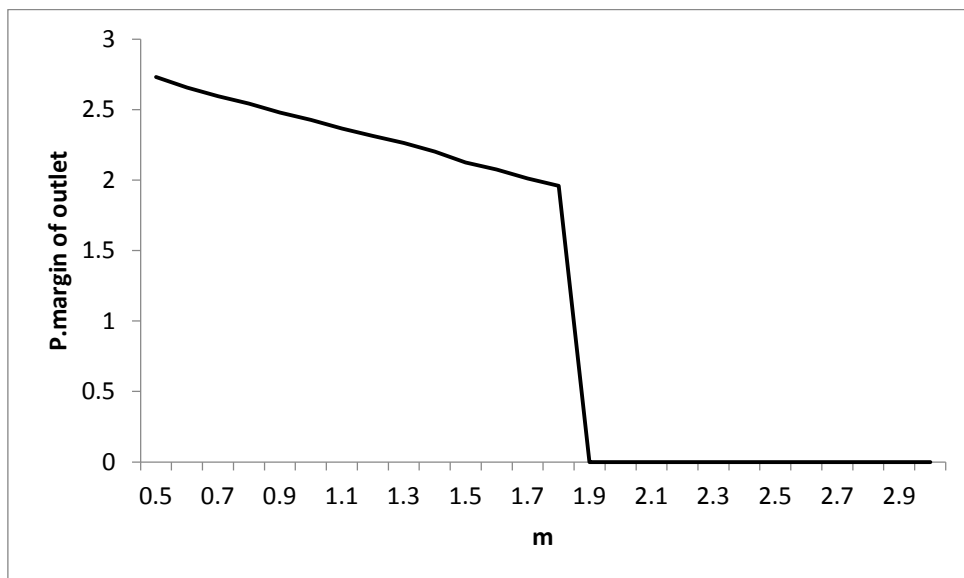
Figure A.5: $(p_2 - c(s_2^*)^2)$ versus m plot for trial 2 when k=0.5

APPENDIX B

NUMERICAL ANALYSIS FOR TRIAL 1

Table B.1: s_2^*, p_2^*, D_1^*, D_2^* ,retailer's profit for trial 1 when m=k

k	m	s_2^*	p_2^*	D_1^*	D_2^*	Profit
0	0	4	3.2	38.800	0.400	62.720
0.1	0.1	4.12	3.3	38.788	0.386	62.680
0.2	0.2	4.24	3.4	38.776	0.374	62.642
0.3	0.3	4.35	3.49	38.764	0.364	62.605
0.4	0.4	4.45	3.57	38.752	0.355	62.568
0.5	0.5	4.55	3.66	38.742	0.343	62.533
0.6	0.6	4.66	3.76	38.730	0.333	62.499
0.7	0.7	4.73	3.81	38.718	0.327	62.465
0.8	0.8	4.84	3.92	38.708	0.315	62.432
0.9	0.9	4.92	3.99	38.698	0.308	62.400
1	1	4.99	4.05	38.687	0.300	62.368
1	1	4.99	4.05	38.687	0.300	62.368
1.1	1.1	5.09	4.15	38.676	0.291	62.337

Table B.1: (cont'd) s_2^*, p_2^*, D_1^*, D_2^* retailer's profit for trial 1 when m=k

k	m	s_2^*	p_2^*	D_1^*	D_2^*	Profit
1.2	1.2	5.15	4.2	38.666	0.284	62.307
1.3	1.3	5.23	4.28	38.657	0.276	62.277
1.4	1.4	5.31	4.36	38.646	0.268	62.248
1.5	1.5	5.39	4.44	38.636	0.261	62.219
1.6	1.6	5.45	4.50	38.627	0.253	62.191
1.7	1.7	5.55	4.61	38.616	0.246	62.163
1.8	1.8	5.61	4.67	38.606	0.240	62.136
1.9	1.9	5.66	4.72	38.598	0.232	62.109
2	2	5.74	4.81	38.588	0.225	62.082
2.1	2.1	5.82	4.9	38.577	0.219	62.056
2.2	2.2	5.87	4.95	38.568	0.213	62.030
2.3	2.3	5.93	5.02	38.560	0.205	62.005
2.4	2.4	6.00	5.10	38.550	0.200	61.980
2.5	2.5	6.06	5.17	38.541	0.193	61.955
2.6	2.6	6.12	5.24	38.531	0.187	61.930
2.7	2.7	6.18	5.31	38.521	0.181	61.906
2.8	2.8	6.23	5.37	38.514	0.174	61.882
2.9	2.9	6.31	5.47	38.502	0.170	61.858
3	3	6.36	5.53	38.493	0.164	61.835
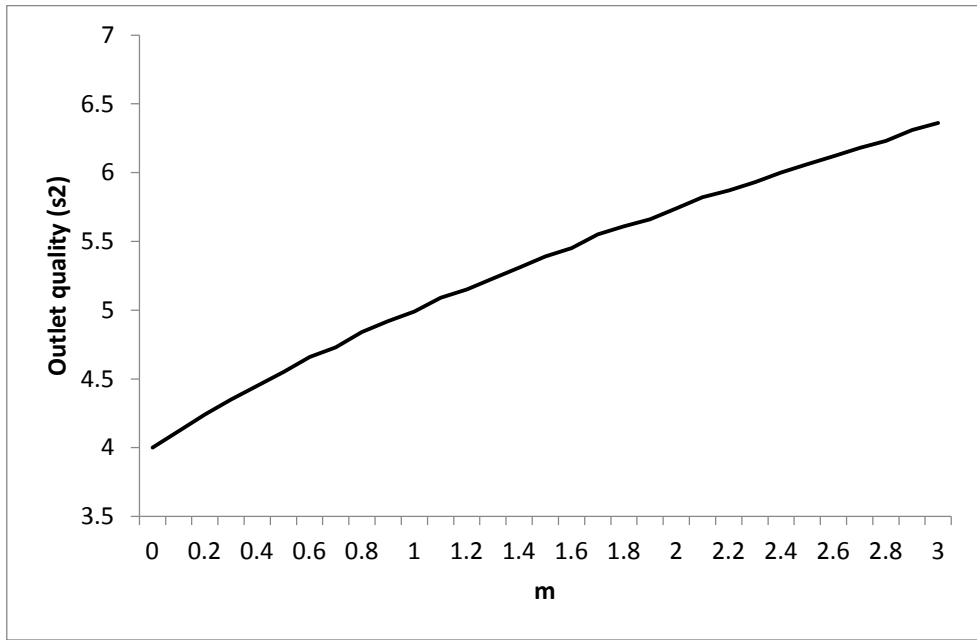
Figure B.1: s_2^* versus m plot for trial 1 when m=k
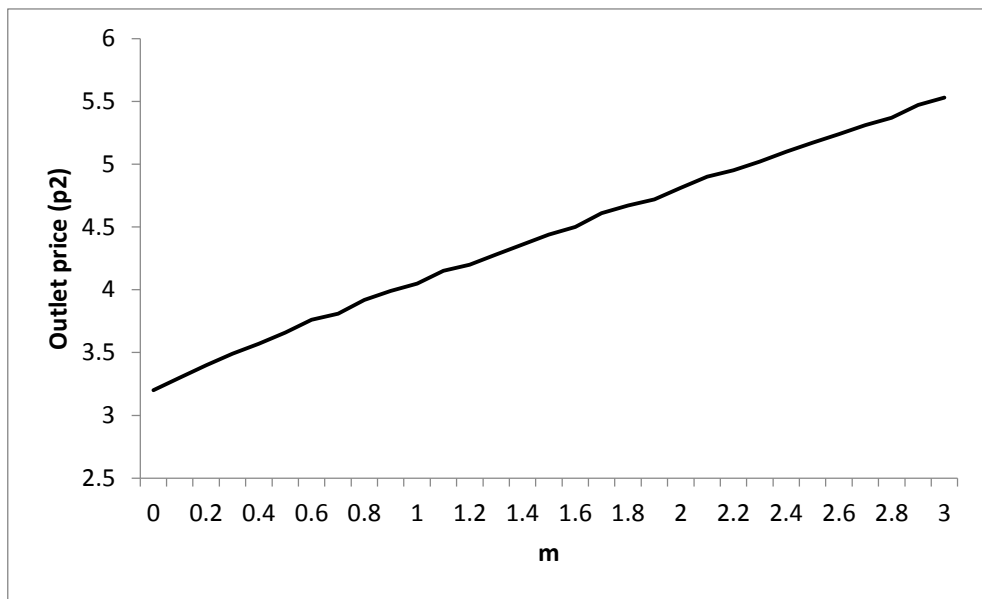
Figure B.2: p_2^* versus m plot for trial 1 when m=k
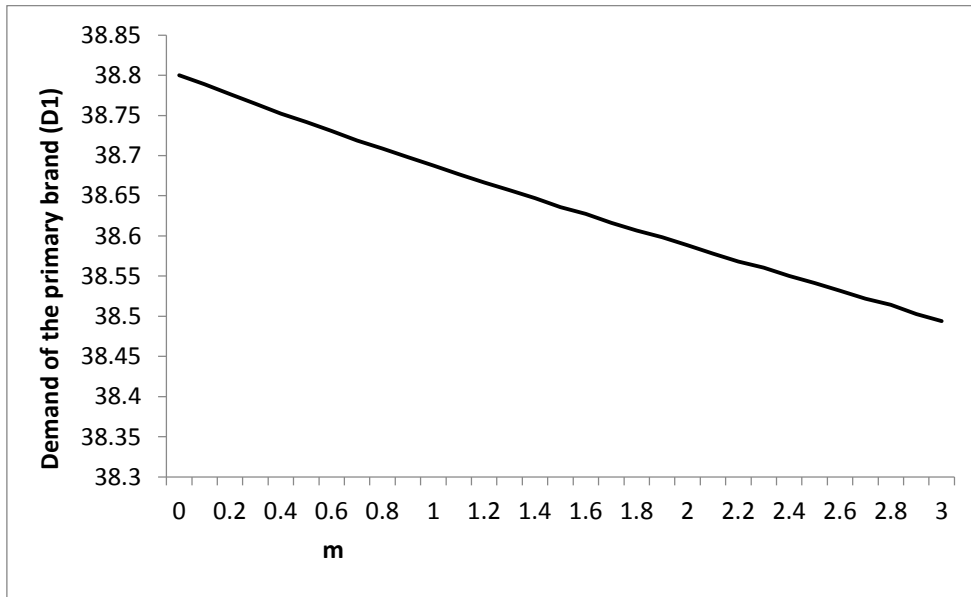
Figure B.3: $D_1{}^*$ versus m plot for trial 1 when m=k
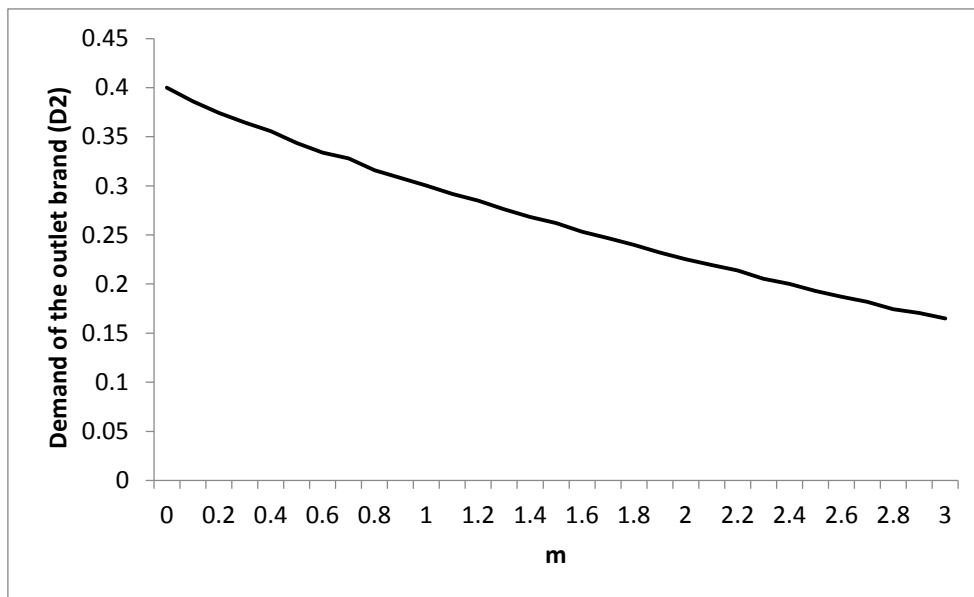
Figure B.4: $D_2{}^*$ versus m plot for trial 1 when m=k

APPENDIX C

NUMERICAL ANALYSIS FOR THE FIRST GROUP TRIALS
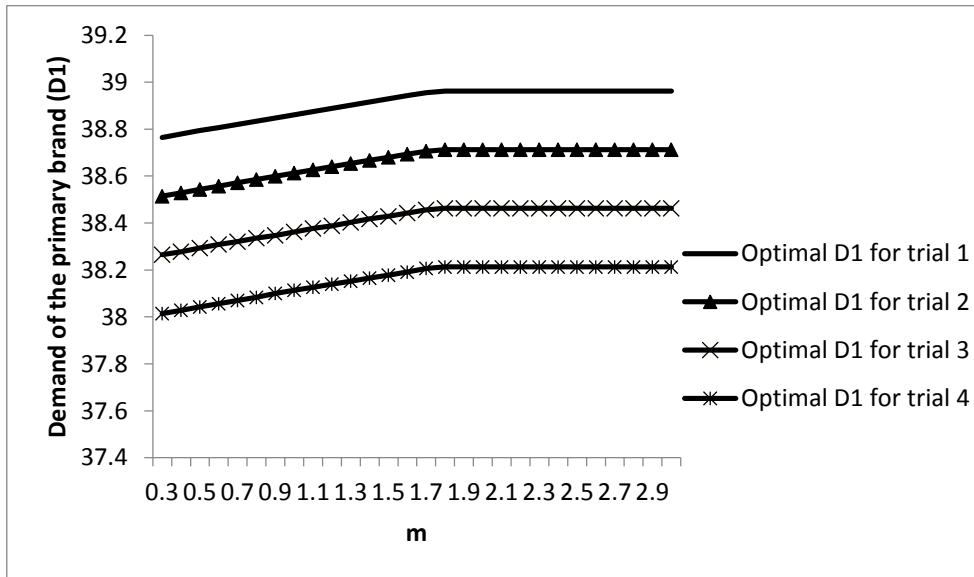

Figure C.1: D_1^* for the first group of trials when k=0.3
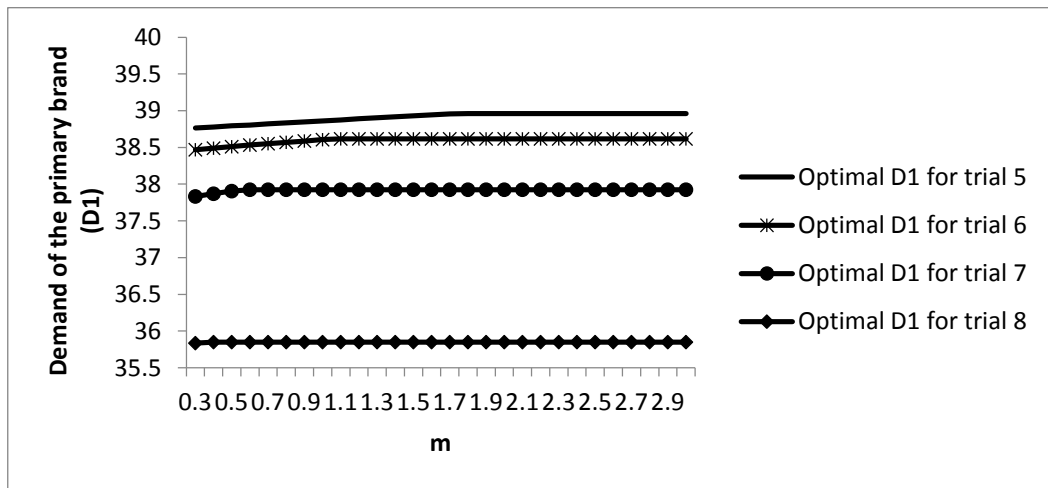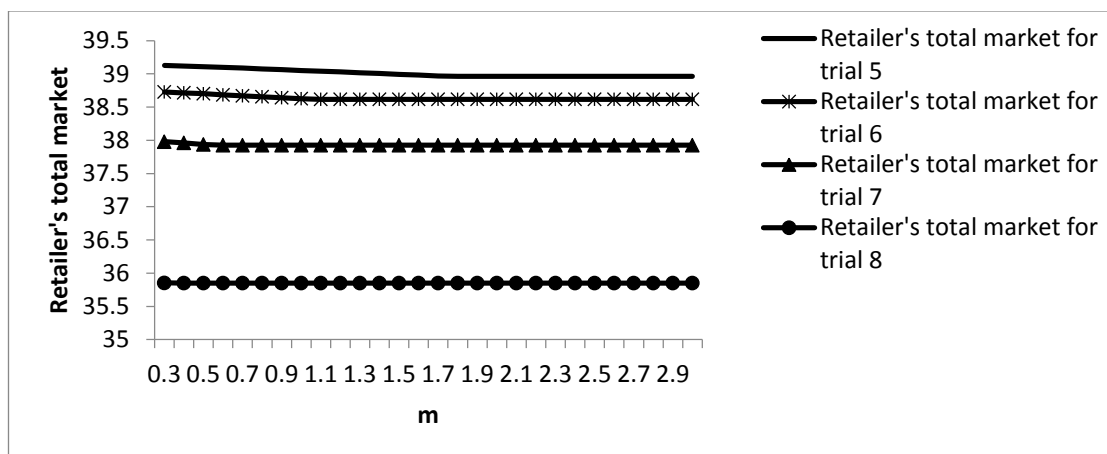
NUMERICAL ANALYSIS FOR THE SECOND GROUP TRIALS

Figure D.1: D_1^* for the second group of trials when k=0.3

Figure D.2: Retailer's market share for the second group of trials when k=0.3

LEMMA 4.2

Lemma 4.2: There are two feasible orders for the threshold z values: Order 1; i.e.; $z_{limit\ 4} > z_{critic\ 4\ \bar{t}} > z_{concavity\ 4} > z_{critic\ 4\ \underline{t}}$ and Order 2; i.e.; $z_{limit\ 4} > z_{critic\ 4\ \bar{t}} > z_{critic\ 4\ \underline{t}} > z_{concavity\ 4}$.

Proof: It is clear that $z_{limit\ 4} > z_{critic\ 4\ \bar{t}} > z_{concavity\ 4}$ is satisfied. If $\frac{(\bar{t})^2}{2} > (\underline{t})^2$ holds, then $z_{concavity\ 4} > z_{critic\ 4\ \underline{t}}$ is satisfied and consequently Order 1 occurs. Otherwise, Order 2 occurs. Note that $\underline{t}$ is used for choosing the optimal promised delivery time with min $(\max(t^\circ, \underline{t}), \bar{t})$ and calculating $z_{critic\ \underline{t}}$. ∎

APPENDIX F

PROPOSITION 5.4

Proposition 5.4: We have the following conditions in retailer's profit and optimal promised delivery time with respect to the online service cost under Case 1 (i.e., $\theta_1 \leq \theta_2 \leq \theta^* < \tilde{\theta}$).

i. If $(p_1 - cs_1^2) - (p_2 - cs_2^2) < 0$, then $\Delta\Pi_{5.1} < 0$ and $t^* = \bar{t}$ for all $z \geq 0$.

ii. If $(p_1 - cs_1^2) - (p_2 - cs_2^2) > 0$, then there are two feasible orders for the threshold z values (i.e., Order 1 and 2). For all order, the profit impact of the online channel on the retailer's profit change with $z_{limit\ 5.1}$. The optimal promised delivery time will be $\bar{t}$ or t for Order 1, however it will be $\underline{t}$, $\bar{t}$ or $t°$ for Order 2.

Proof:

i. See proof of Proposition 5.2(i).

ii. When $(p_1 - cs_1^2) - (p_2 - cs_2^2) > 0$, then there are two feasible orders for the threshold z values: Order 1; i.e., $z_{limit\ 5.1} > z_{critic\ 5.1\ \bar{t}} > z_{concavity\ 5.1} > z_{critic\ 5.1\ \underline{t}}$ and Order 2; i.e., $z_{limit\ 5.1} > z_{critic\ 5.1\ \bar{t}} > z_{critic\ 5.1\underline{t}} > z_{concavity\ 5.1}$. It is clear that $z_{limit\ 5.1} > z_{critic\ 5.1\ \bar{t}} > z_{concavity\ 5.1}$ is satisfied. If $\frac{(\bar{t})^2}{2} > (\underline{t})^2$ holds, then $z_{concavity\ 5.1} > z_{critic\ 5.1\ \underline{t}}$ holds and consequently Order 1 occurs. Otherwise, Order 2 occurs.
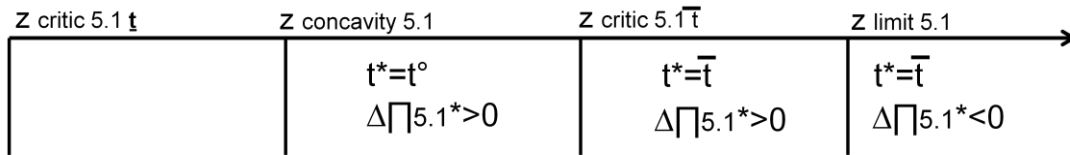
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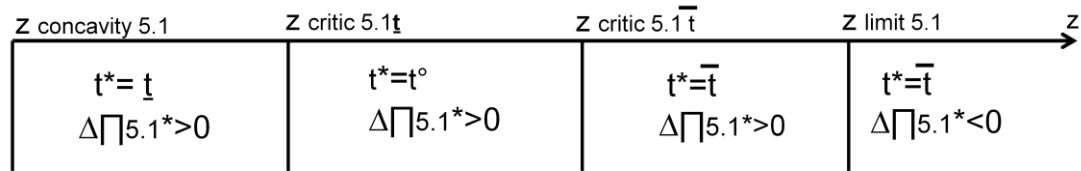Figure F.1: t^*, $\Delta\prod_{5.1}$ for Order 1 under Case 1

Figure F.2: t^*, $\Delta\prod_{5.1}$ for Order 2 under Case 1

PROPOSITION 5.5

Proposition 5.5: We have the following conditions in retailer's profit and optimal promised delivery time with respect to the online service cost under Case 2 (i.e., $\theta_1 > \theta_2 > \theta^*$)

i. If $(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) < 0$, $\Delta\Pi_{5.1} < 0$ and $t^* = \bar{t}$ for all $z \geq 0$.

ii. If $(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) > 0$, then there are two feasible orders for the threshold z values (i.e., Order 1 and 2). For all order, the profit impact of the online channel on the retailer's profit change with $z_{limit\ 5.2}$. The optimal promised delivery time will be $\bar{t}$ or t for Order 1, however it will be $\underline{t}$, $\bar{t}$ or t° for Order 2.

Proof:

i. See proof of Proposition 5.3(i).

ii. When $(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2) > 0$, then there are two feasible orders for the threshold z values: Order 1; i.e., $z_{limit\ 5.2} > z_{critic\ 5.2\ \bar{t}} > z_{concavity\ 5.2} > z_{critic\ 5.2\ \underline{t}}$ and Order 2; i.e., $z_{limit\ 5.2} > z_{critic\ 5.2\ \bar{t}} > z_{critic\ 5.2\ \underline{t}} > z_{concavity\ 5.2}$ where

$$z_{critic\ 5.2\ \underline{t}} = \frac{\left[(p_1 - cs_1^2)\left(ks_2(\tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)\right) - (p_2 - cs_2^2)(\tilde{\theta} - \theta_1)(y + ks_2)\right](\underline{t})^2}{3[ks_2(2b - \tilde{\theta} - \theta_1) + y(\tilde{\theta} - \theta_2)]}.$$

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It is clear that $z_{limit\ 5.2} > z_{critic\ 5.2\ \bar{t}} > z_{concavity\ 5.2}$ is satisfied. If $\frac{(\bar{t})^2}{2} > (\underline{t})^2$ holds, then $z_{concavity\ 5.2} > z_{critic\ 5.2\ \underline{t}}$ holds and consequently Order 1 occurs. Otherwise, Order 2 occurs.
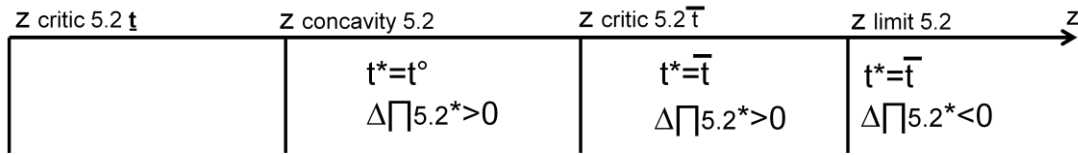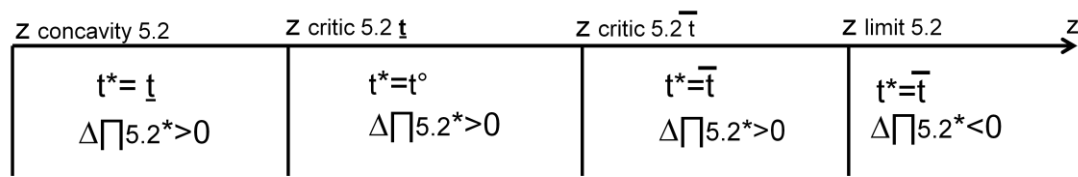
z critic 5.2 $\underline{t}$	**z** concavity 5.2	**z** critic 5.2 $\bar{t}$	**z** limit 5.2	**z**
	$t^*=t°$ $\Delta\prod 5.2^*>0$	$t^*=\bar{t}$ $\Delta\prod 5.2^*>0$	$t^*=\bar{t}$ $\Delta\prod 5.2^*<0$	

Figure G.3: t^*, $\Delta\prod_{5.2}$ for Order 1 under Case 2

z concavity 5.2	**z** critic 5.2 $\underline{t}$	**z** critic 5.2 $\bar{t}$	**z** limit 5.2	**z**
$t^*=\underline{t}$ $\Delta\prod 5.2^*>0$	$t^*=t°$ $\Delta\prod 5.2^*>0$	$t^*=\bar{t}$ $\Delta\prod 5.2^*>0$	$t^*=\bar{t}$ $\Delta\prod 5.2^*<0$	

Figure G.4: t^*, $\Delta\prod_{5.2}$ for Order 2 under Case 2

PROOF OF PROPOSITION 6.2

i.
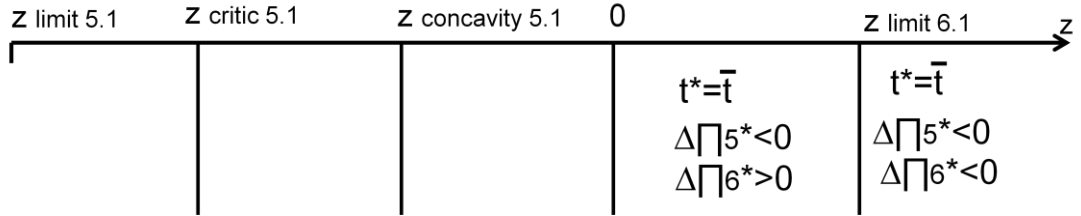
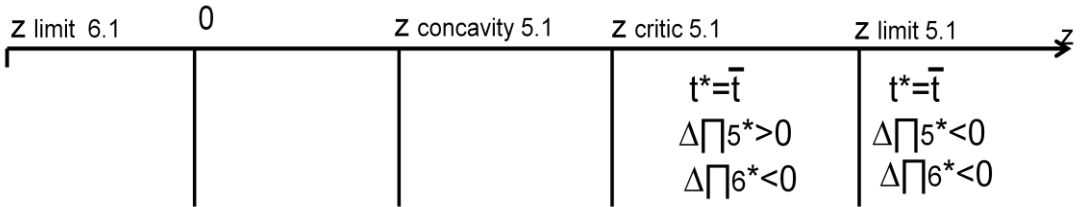Figure H.1: t^*, $\Delta\prod_{5.1}$, $\Delta\prod_{6.1}$ when $(p_1-c_1^2)-(p_2-s_2^2)<0$

ii.

Figure H.2: t^*, $\Delta\prod_{5.1}$, $\Delta\prod_{6.1}$ when $(p_1-c_1^2)-(p_2-s_2^2)>0$ and $z_{limit\ 6.1}<0$
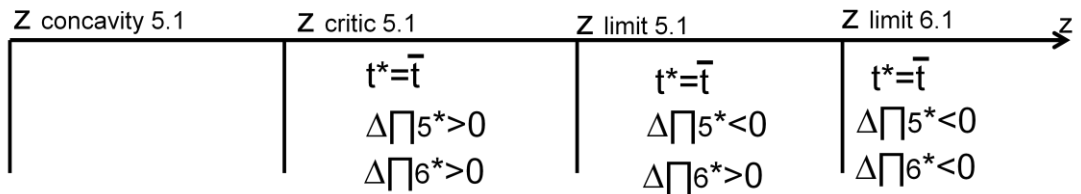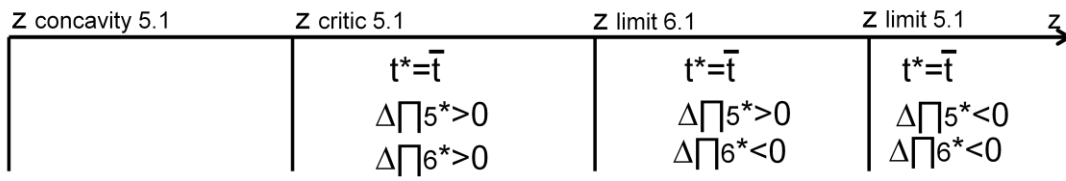
iii. Order (1)

Figure H.3: t^*, $\Delta\prod_{5.1}$, $\Delta\prod_{6.1}$ for Order (1)

Order (2)

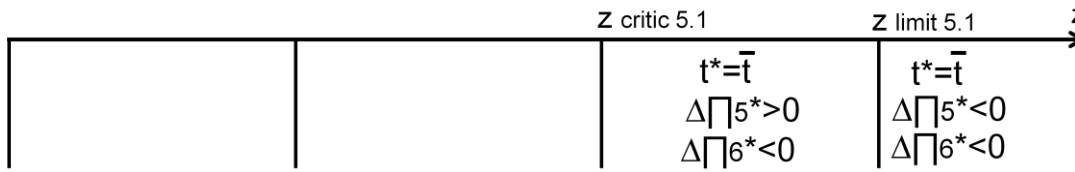Figure H.4: t^*, $\Delta\prod_{5.1}$, $\Delta\prod_{6.1}$ for Order (2)

Order (3) and (4)

Figure H.5: t^*, $\Delta\prod_{5.1}$, $\Delta\prod_{6.1}$ for Order (3) and Order (4)
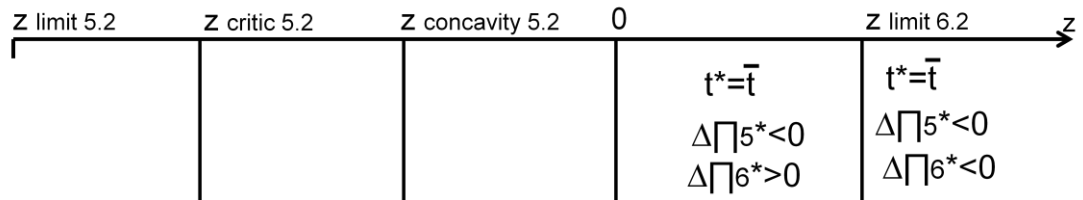
APPENDIX I

PROOF OF PROPOSITION 6.3

ii.

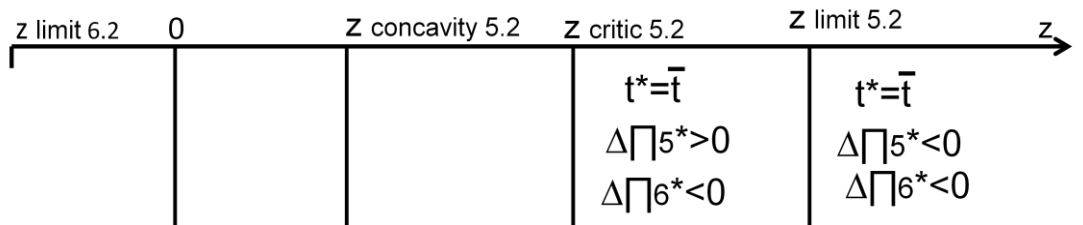Figure I.1: t^{*}, $\Delta\prod_{5.2}$, $\Delta\prod_{6.2}$ for Proposition 6.3(ii)

iii.

Figure I.2: t^{*}, $\Delta\prod_{5.2}$, $\Delta\prod_{6.2}$ for Proposition 6.3(iii)