AERODYNAMIC OPTIMIZATION OF MISSILE EXTERNAL CONFIGURATIONS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

KIVANÇ ARSLAN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
AEROSPACE ENGINEERING

SEPTEMBER 2014

Approval of the thesis

AERODYNAMIC OPTIMIZATION OF MISSILE EXTERNAL CONFIGURATIONS

Submitted by **KIVANÇ ARSLAN** in partial fulfillment of the requirements for the degree of **Master of Science in Aerospace Engineering Department, Middle East Technical University** by**,**

Prof. Dr. Canan Özgen ________________
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Ozan Tekinalp ________________
Head of Department, **Aerospace Engineering**

Prof. Dr. Serkan Özgen ________________
Supervisor, **Aerospace Engineering Dept., METU**

Examining Committee Members:

Prof. Dr. Hüseyin Nafiz Alemdaroğlu ________________
Aerospace Engineering Dept., METU

Prof. Dr. Serkan Özgen ________________
Aerospace Engineering Dept., METU

Prof. Dr. Yusuf Özyörük ________________
Aerospace Engineering Dept., METU

Assoc. Prof. Dr. Sinan Eyi ________________
Aerospace Engineering Dept., METU

Assoc. Prof. Dr. Metin Yavuz ________________
Mechanical Engineering Dept., METU

Date: 05/09/2014

I hereby declare that all the information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Kıvanç ARSLAN

Signature :

ABSTRACT

AERODYNAMIC OPTIMIZATION OF MISSILE EXTERNAL CONFIGURATIONS

Arslan, Kıvanç
M.S., Department of Aerospace Engineering
Supervisor: Prof. Dr. Serkan Özgen

September 2014, 85 Pages

In this thesis, design optimization methods capable of optimizing aerodynamic performances of missiles and rockets are developed. Sequential Quadratic Programming (SQP) and Random Search (RS) methods are used for optimization, whereas Missile DATCOM, which is a semi-empirical aerodynamic analysis tool, is used to calculate aerodynamic coefficients of missile configurations. As the first part of the work, capabilities and limitations of SQP and RS optimization methods are investigated on a complex test function. In addition to that, a validation for aerodynamic analysis tool is done. Then, using reverse engineering approach, aerodynamic performance parameters of "NASA Tandem Control Missile" (TCM) configuration are defined as objectives to the developed design optimization method with the aim of reaching TCM configuration at the end of aerodynamic performance optimization process. Geometric properties and aerodynamic performance parameters of the optimum configurations obtained using the developed design method are compared to the ones of the TCM configuration. Moreover, an optimization case study for a generic air-to-ground missile is carried out. It is concluded that the developed design optimization method is able to determine missile external configurations for pre-defined aerodynamic performance parameters.

Keywords: Missile Aerodynamics, External Geometry Optimization, Sequential Quadratic Programming, Random Search Optimization

ÖZ

FÜZE DIŞ GEOMETRİLERİNİN AERODİNAMİK AÇIDAN EN İYİLEŞTİRİLMESİ

Arslan, Kıvanç
Yüksek Lisans, Havacılık ve Uzay Mühendisliği Bölümü
Tez Yöneticisi: Prof. Dr. Serkan Özgen

Eylül 2014, 85 Sayfa

Bu tezde, füze ve roketlerin kavramsal tasarım sürecinde kullanılabilecek aerodinamik performans en iyileştirmesi yapan bir tasarım metodu geliştirilmiştir. Bu tasarım metodunda füze konfigürasyonlarının aerodinamik katsayıların hesaplanmasında yarı ampirik bir aerodinamik analiz yazılımı olan Missile DATCOM, tasarım en iyileştirmesinde ise ardışık ikinci derece programlama ve rastgele tarama yöntemleri kullanılmıştır. Çalışma kapsamında ilk olarak, kullanılan en iyileme yöntemleri optimum değerleri bilinen bir test fonksiyonu ile doğrulanmıştır. Bunun yanı sıra, kullanılan aerodinamik analiz aracı için bir doğrulama çalışması yapılmıştır. Ardından literatürde bulunan “NASA Tandem Control Missile” (TCM) konfigürasyonuna ait aerodinamik performans parametreleri tersine mühendislik anlayışı ile, geliştirilen tasarım aracına hedef olarak tanımlanmış ve tasarım aracının TCM konfigürasyonuna ulaşması amaçlanmıştır. Tasarım aracından elde edilen konfigürasyonun geometrik özellikleri ve aerodinamik performans parametreleri TCM konfigürasyonu ile karşılaştırılmıştır. Bu çalışmaya ek olarak jenerik bir havadan karaya füze konfigürasyonu için aerodinamik tasarım optimizasyonu uygulaması yapılmıştır.

Sonuç olarak, geliştirilen tasarım metodunun önceden tanımlanan aerodinamik performans parametreleri için füze dış geometrilerini belirleyebildiği görülmüştür.

Anahtar Kelimeler: Füze Aerodinamiği, Dış Geometri En İyileştirmesi, Ardışık İkinci Derece Programlama, Rastgele Arama ile En İyileme

To My Family

ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to my supervisor Prof. Dr. Serkan Özgen for his support, guidance and advice throughout the thesis.

I would like to thank my department manager Mr. Ali Akgül for his guidance, advice and criticism during this study. I also would like to thank my colleagues in Aerodynamic Design and Analysis Department of ROKETSAN for all their help and support during the thesis.

I am very thankful to my parents Mrs. Semra Arslan, Mr. Kılıç Arslan, my sister Ms. Cansu Arslan and my brother Mr. Ahmet Emre Arslan for their help, motivation and endless love.

I want to express my best wishes to Mr. Hayri Yiğit Akargün and Mr. Mehmet Akgül for their friendship and support during this study.

TABLE OF CONTENTS

ABSTRACT .... **v**
ÖZ .... **vii**
ACKNOWLEDGEMENTS .... **x**
TABLE OF CONTENTS .... **xi**
LIST OF FIGURES .... **xiii**
LIST OF TABLES .... **xvi**
CHAPTERS
1. INTRODUCTION **1**
1.1 Aerodynamic Forces and Moments Acting on a Missile 4
1.2 Classification of Missiles 5
1.3 Missile Aerodynamic Design Parameters 8
1.3.1 Lift-to-Drag Ratio (L/D) 8
1.3.2 Stability 9
1.3.3 Maneuverability 10
1.3.4 Control Effectiveness 11
1.4 Literature Survey 12
1.5 Aim of the Thesis 13
1.6 Organization of the Thesis 14
2. METHODOLOGY **17**
2.1 Mathematical Optimization 17
2.2 Design Optimization 18
2.2.1 Random Search Method 18
2.2.2 Sequential Quadratic Programming (SQP) Method 22
2.3 Aerodynamic Analysis Methodology 25
2.4 Aerodynamic Optimization Procedure 28
3. VERIFICATION OF THE DESIGN OPTIMIZATION METHOD **33**
3.1 Goldstein and Price Function 33
3.2 Verification of the Sequential Quadratic Programming (SQP) Method 35
3.3 Verification of the Random Search Method 40
3.4 Validation of the Aerodynamic Analysis Method 43

4. MISSILE AERODYNAMIC OPTIMIZATION STUDIES 47
4.1 Aerodynamic Shape Optimization for NASA Tandem Control Missile 48
4.1.1 NASA Tandem Control Missile (TCM) Configuration 48
4.1.2 Aerodynamic Analyses of TCM 48
4.1.3 Missile Design Optimization Study for TCM 51
4.1.3.1 Objectives, Constraints and Variables of Optimization ...... Study 51
4.1.3.2 Optimization using DONLP2.. 53
4.1.3.3 Optimization using ACRS 62
4.2 Aerodynamic Shape Optimization for a Generic Air-to-Ground Missile (AGM) 72
4.2.1 Missile Design Optimization for AGM.. 72
4.2.1.1 Objectives, Constraints and Variables of Optimization ...... Study 72
4.2.1.2 Optimization using DONLP2.. 75
4.2.1.3 Optimization using ACRS.. 77
4.2.1.4 Discussion of Results 80
5. CONCLUSIONS AND FUTURE WORK 81
REFERENCES 83

LIST OF FIGURES

FIGURES

Figure 1.1 Iterative Missile Design Process [4] 3

Figure 1.2 Forces and Moments Acting on a Missile 5

Figure 1.3 Control Alternatives for Missiles.. 7

Figure 1.4 Static Stability.. 9

Figure 1.5 Statically Stable Missile Configuration [4] 10

Figure 2.1 Conventional Wing-body Missile Configuration and Coordinate System 25

Figure 2.2 Variation of Interference Factors with r/s_w or r/s_t [5] 27

Figure 2.3 Flowchart for Design Optimization with Sequential Quadratic Programming Method 29

Figure 2.4 Flowchart for Design Optimization with Random Search Method 31

Figure 3.1 Goldstein and Price Function... 34

Figure 3.2 Global Minimum Search with SQP Method, Successful 36

Figure 3.3 Global Minimum Search with SQP Method, Converged to Local Minimum.. 37

Figure 3.4 Global Minimum Search with SQP Method, Converged to Local Minimum.. 38

Figure 3.5 Change of Objective Function Values with Iterations for 15 Optimization Runs 39

Figure 3.6 Evaluated Points During Global Minimum Search with Random Search Algorithm 40

Figure 3.7 Evaluation of Iterations During Optimization 41

Figure 3.8 Change of Objective Function Values with Iterations for Best and Worst Solutions.. 42

Figure 3.9 Lift Coefficient Obtained from MD, CFD and Experiment for TCM Configuration (Mach = 1.75) 43

Figure 3.10 Drag Coefficient Obtained from MD, CFD and Experiment for TCM Configuration (Mach = 1.75) 44
Figure 3.11 Pitching Moment Coefficient Obtained from MD, CFD and Experiment for TCM Configuration (Mach = 1.75) 44
Figure 4.1 TCM B1T4C4 Configuration (Dimensions in cm) 48
Figure 4.2 Normal Force Coefficient versus Angle of Attack 49
Figure 4.3 Axial Force Coefficient versus Angle of Attack 50
Figure 4.4 Missile Center of Pressure versus Angle of Attack 50
Figure 4.5 Canard Placement Area (red) and Random Possible Canard Geometries (dashed lines) 52
Figure 4.6 Optimum Configuration Geometry (top), TCM B1T4C4 (bottom) 55
Figure 4.7 Change of Leading Edge Position throughout Optimization 56
Figure 4.8 Change of Root Chord throughout Optimization 57
Figure 4.9 Change of Taper Ratio throughout Optimization 57
Figure 4.10 Change of Semi-span throughout Optimization 58
Figure 4.11 Change of Normal Force Coefficient (M=1.75, α=16°) throughout Optimization 58
Figure 4.12 Change of Center of Pressure Location (M=1.75, α=4°) throughout Optimization 59
Figure 4.13 Change of Center of Pressure Location (M=1.75, α=16°) throughout Optimization 59
Figure 4.14 Change of Axial Force Coefficient (M=1.75, α=0°) throughout Optimization 60
Figure 4.15 Change of Objective Function throughout Optimization 60
Figure 4.16 Change of Objective Function Values with Iterations 61
Figure 4.17 Optimum Configuration Geometry (top), TCM B1T4C4 (bottom) 64
Figure 4.18 Change of Leading Edge Position throughout Optimization 65
Figure 4.19 Change of Root Chord throughout Optimization 66
Figure 4.20 Change of Taper Ratio throughout Optimization 66
Figure 4.21 Change of Semi-span throughout Optimization 67

Figure 4.22 Change of Normal Force Coefficient (M=1.75, α=16°) throughout Optimization.. 67
Figure 4.23 Change of Center of Pressure Location (M=1.75, α=4°) throughout Optimization.. 68
Figure 4.24 Change of Center of Pressure Location (M=1.75, α=16°) throughout Optimization.. 68
Figure 4.25 Change of Axial Force Coefficient (M=1.75, α=0°) throughout Optimization.. 69
Figure 4.26 Change of Objective Function throughout Optimization 69
Figure 4.27 Change of Objective Function Values with Iterations....................... 70
Figure 4.28 Change of Objective Function Values with Iterations (y axis is narrowed) 71

LIST OF TABLES

TABLES

Table 1.1 Classification of Missiles Based on Platform and Target Location [4] .. 6
Table 1.2 Advantages and Disadvantages of Missile Control Types [4] 7
Table 2.1 Indices in Equations (2.33) - (2.35) 26
Table 3.1 Variable and Objective Function Values for Best and Worst Solutions 39
Table 3.2 Variable and Objective Function Values for Best and Worst Solutions 42
Table 4.1 Flight Conditions 49
Table 4.2 Objective Aerodynamic Performance Parameters 51
Table 4.3 Missile Geometry Variables and Constraints 51
Table 4.4 Change of Configuration Geometry along Optimization 54
Table 4.5 Optimum and TCM B1T4C4 Configuration (Objective) Aerodynamic Performance Parameters 55
Table 4.6 Optimum and TCM B1T4C4 Missile Geometric Proportions 56
Table 4.7 Aerodynamic Performance Parameters of Best and Worst Configurations 61
Table 4.8 Change of Configuration Geometry along Optimization Run 63
Table 4.9 Optimum and TCM B1T4C4 Configuration (Objective) Aerodynamic Performance Parameters 64
Table 4.10 Optimum and TCM B1T4C4 Missile Geometric Proportions 65
Table 4.11 Aerodynamic Performance Parameters of Best and Worst Configurations 70
Table 4.12 Missile Geometry Variables and Limits 72
Table 4.13 Aerodynamic Performance Parameters for Sample AGM 73
Table 4.14 Flight Regime for AGM Optimization Case Study 73
Table 4.15 Optimum AGM and Objective Aerodynamic Performance Parameters 76

Table 4.16 Optimum AGM Geometric Proportions 76
Table 4.17 Change of AGM Configuration Geometry along Optimization Run. 77
Table 4.18 Optimum AGM and Objective Aerodynamic Performance Parameters
.... 78
Table 4.19 Optimum AGM Geometric Proportions 78
Table 4.20 Change of AGM Configuration Geometry along Optimization Run. 79

CHAPTER 1

INTRODUCTION

An engineering design optimization problem can be defined as obtaining the best possible engineering system under given system constraints, performance requirements and time limits. Designers generally need a tool to overcome the design optimization problem, where an analysis/simulator is coupled with an optimization method [1]. The outcomes of the optimization problem are defined as objective functions which may represent structural, aerodynamic or chemical performance parameters or properties. Objectives of the problem are functions of design variables that shape the properties of the engineering system [1].

In missile aerodynamic design, it is necessary to investigate a large design space. Complexity of a design space is dependent on the number of input variables. Missile diameter, nose, body and aft body length, number of fin sets, number of fins for each fin set, size and shape of each fin and fin cross-section can be given as some examples to the design variables. Geometric limitations are imposed to the design problem when launch platform compatibility, cost issues and other subsystem integrations are taken into account. Aerodynamic performance parameters such as maneuverability, control effectiveness, static and dynamic stability, lift-to-drag ratio are objectives for the design problem. Accordingly, in missile aerodynamic design optimization problems, there exist numerous parameters, variables and constraints that make the problem difficult to solve in terms of time and computational effort. In addition to that, performance parameters obtained as outcomes of the design process generally conflict with each other. Consequently, it is difficult to obtain the best possible solution unless the designer has enough computational resource for searching the whole design space or he/she is using mathematical optimization methods together with engineering analysis/simulation tools. In this thesis, it is aimed to develop a design optimization method that can be used to overcome difficulties of missile aerodynamic design problem and reduce time spent in conceptual design phase of missiles.

Gradient-based and stochastic optimization methods are widely used in engineering design optimization problems. Gradient-based methods determine the maxima or minima of the objective functions of the design problem by using gradient information of the function. Newton's method, steepest descent method and Sequential Quadratic Programming (SQP) are some examples to gradient-based optimization methods and are widely used in the literature for engineering problems such as aerodynamic or structural design optimization. A detailed description of SQP method is given in Chapter 2. These optimization methods are generally regarded as fast and demand low computational resources. Nevertheless, they are sometimes called local optimizers since they tend to converge to the local minimum of the design space and may produce biased results as they require an initial point to start the optimization process. In addition to that, gradient-based methods may end up with infeasible solutions since engineering optimization problems are generally highly nonlinear and discontinuous in nature. Accordingly, stochastic optimization methods that do not require gradient information of the problem are employed in design optimization problems to overcome negative effects of gradient-based methods. Random Search (RS) methods, Genetic Algorithms (GA) and Simulated Annealing (SA) are examples of stochastic optimization methods [2]. A detailed description of an RS method can be found in Chapter 2. Stochastic methods are more successful for determining global minimum, whereas they generally require more computational resources in contrast to the gradient-based methods [3].

In conceptual design of missiles, an iterative process shown in Figure 1.1 is followed. A baseline configuration that has similar mission requirements and propulsion system is selected as a starting point. Design variables that affect aerodynamics, propulsion, weight and flight trajectory of the baseline configuration are changed iteratively to meet the mission requirements established for the design process [4]. Here, as aerodynamics has priority over other design characteristics, defining a configuration that is optimized in terms of aerodynamic performance reduces the overall number of resizing iterations and decreases the time spent on the design process.

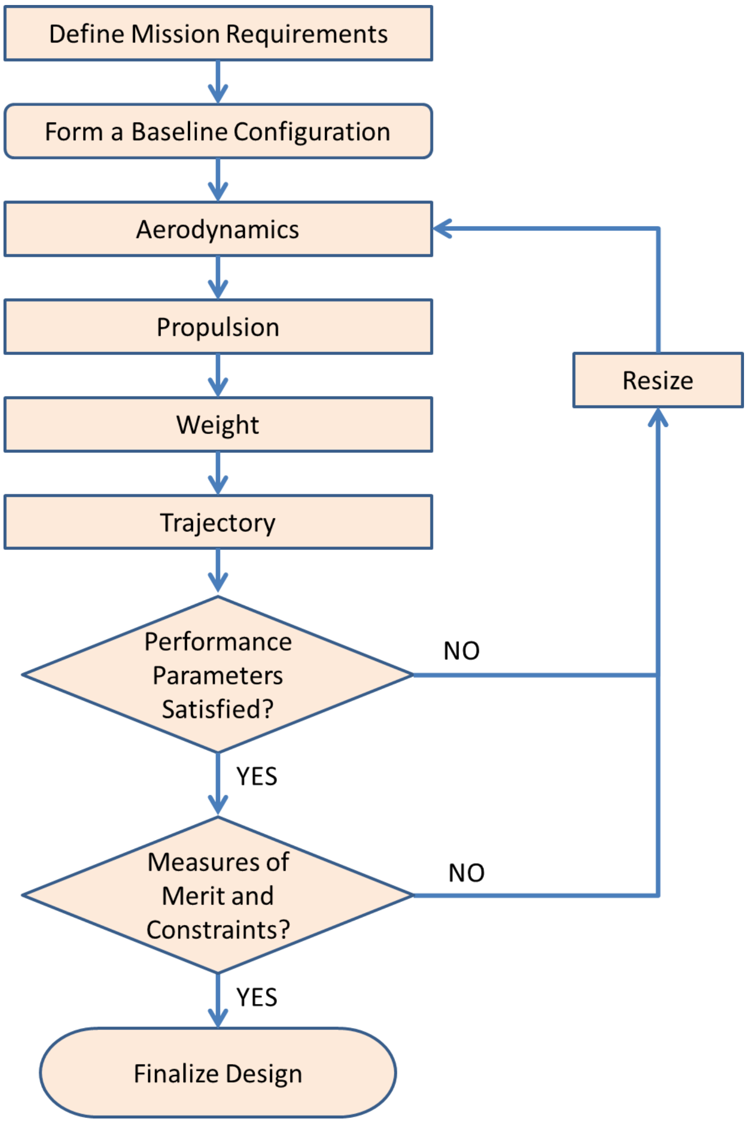


Figure 1.1 Iterative Missile Design Process [4]

Aerodynamic design of missile/rocket configurations are based on the investigation of alternative external shapes [4]. Missile aerodynamic performance parameters are generally similar to parameters that are used for aircraft design.

Since missiles/rockets are uninhabited and expendable, elevated levels of speed, altitude and maneuvering accelerations can be considered in the design process. However, these elevated levels come together with some aerodynamic problems such as nonlinearity of aerodynamic coefficients at high angles of attack and very high wing loadings in these elevated flight envelopes [5]. As missiles are uninhabited, it is possible to use extensive roll motion in flight that results in additional dynamic stability considerations. Guidance systems that compensate pilot input are necessary for missiles and some problems of stability and control that are not encountered in aircraft design are introduced with these systems. Missiles/rockets are generally slender, therefore slender-body theory is sufficient for understanding aerodynamic characteristics of missile configurations [5].

1.1 Aerodynamic Forces and Moments Acting on a Missile

Missile aerodynamics is considered under three aerodynamic forces and three moments for a six degree-of-freedom analysis and body fixed coordinate system. In Figure 1.2, directions of these forces and moments in coefficient form are given [6]. Note that, forces divided by free stream dynamic pressure (q) and reference area (S_{ref}) gives force coefficients (e.g. C_N) whereas moment coefficients (e.g. C_m) are calculated by dividing the moments with free stream dynamic pressure, reference area and reference length (L_{ref}). Here, reference length is generally taken as missile diameter and reference area is cross-sectional area of the missile body.

$$C_N = \frac{N}{q\, S_{ref}} \tag{1.1}$$

$$C_m = \frac{M}{q\, S_{ref}\, L_{ref}} \tag{1.2}$$

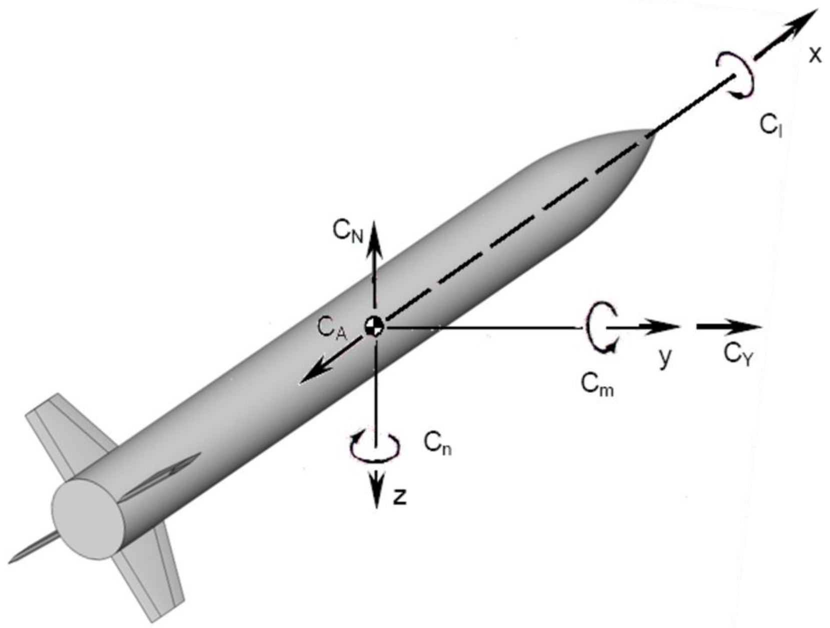


Figure 1.2 Forces and Moments Acting on a Missile

Force coefficients for a missile are shown in Figure 1.2 as C_A, C_N, and C_Y. Axial force coefficient, C_A, is positive in negative x direction whereas side force coefficient, C_Y, is positive in the y direction and normal force coefficient, C_N, is positive in the negative z direction. In Figure 1.2, moment coefficients are given as C_l, C_m, and C_n. Rolling moment, C_l, are about x-axis, pitching moment coefficient, C_m, is about y-axis and yawing moment coefficient, C_n, is about z-axis, shown in their positive senses in the figure.

1.2 Classification of Missiles

Missiles are classified depending on their launch platform and target location, mission and target type, propulsion system, design drivers, type of control etc. [4,5]. An example classification based on launch platform and target locations for state-of-the-art missiles is given in Table 1.1 [4].

Table 1.1 Classification of Missiles Based on Platform and Target Location [4]

Platform and Target Location	**Range / Mission**	**Example**
Air to Air (ATA)	Short Range ATA	AIM-9
	Medium Range ATA	AIM-120
	Long Range ATA	AIM-54
Air to Surface (ATS)	Short Range ATS	AGM-65
	Antiradar ATS	AGM-88
	Medium Range ATS	Apache
	Antitank ATS	AGM-114
	Long Range ATS	AGM-86
Surface to Surface (STS)	Long Range STS	BGM-109
	Long Range Anti-armor STS	MGM-140
	Man-portable STS	Javelin
Surface to Air (STA)	Short Range STA	FIM-92
	Medium Range STA	MIM-23
	Long Range STA	MIM-104
	Missile Defense	PAC-3

Another classification can be made based on type of control. Apart from unconventional control systems such as thrust vector or reaction jet control, missiles are controlled by deflecting aerodynamic lifting surfaces [4]. Accordingly, aerodynamic characteristics and the overall performance of a missile are hugely affected by control surface location. Mainly, missiles are classified as tail controlled, canard controlled and wing controlled (Figure 1.3) [4].

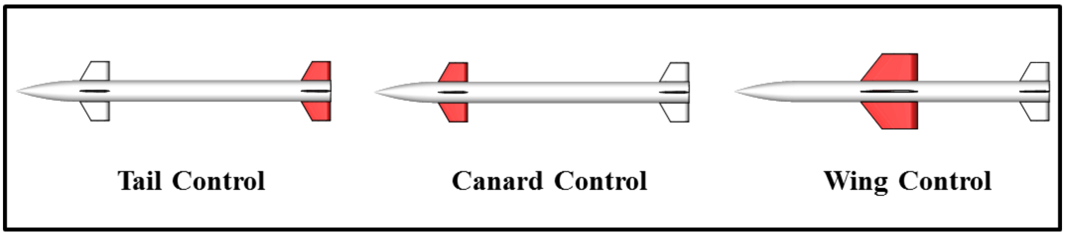
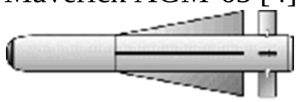
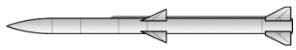
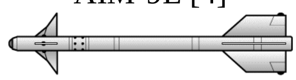
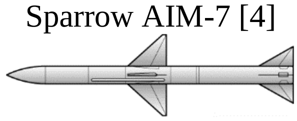
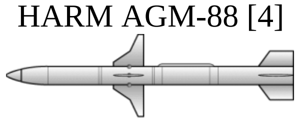


Figure 1.3 Control Alternatives for Missiles

Each type of control alternative has advantages and disadvantages that need to be considered in missile design. These advantages and disadvantages are summarized in Table 1.2 [4].

Table 1.2 Advantages and Disadvantages of Missile Control Types [4]

Control Type	**Advantages**	**Disadvantages**	**Examples**
Tail Control	• Efficient packaging • Low actuator torque • Low induced rolling moment • Efficient at high angles of attack	• Decreased lift at low angles of attack	Maverick AGM-65 [4]  AMRAAM AIM-120 [4] 
Canard Control	• Efficient packaging • Simplified manufacturing • Increased lift at low angles of attack	• Stall at high angles of attack • High induced roll	AIM-9L [4]  Gopher SA-13 [4] 
Wing Control	• Fast response • Maneuverability at low angles of attack • Small trim angle	• Poor packaging • High hinge moments • Large wing size • Large induced roll	Sparrow AIM-7 [4]  HARM AGM-88 [4] 

1.3 Missile Aerodynamic Design Parameters

Lift-to-drag ratio, stability, maneuverability and control effectiveness are the main design parameters for missile aerodynamic design problems [7].

1.3.1 Lift-to-Drag Ratio (L/D)

Lift-to-drag ratio, also known as aerodynamic efficiency, is one of most important parameters in missile design. It has a significant impact on range and also affects maneuverability [4]. It is defined as the ratio of the total lift to the total drag of a missile. It can also be written in coefficient form. In coefficient form, relation of lift and drag to the axial and normal forces can be written as;

$$\begin{aligned} C_D &= C_N \sin\alpha + C_A \cos\alpha \\ C_L &= C_N \cos\alpha - C_A \sin\alpha \end{aligned} \tag{1.3}$$

In these equations, α represents the missile angle of attack. Range of a missile is a function of lift-to-drag ratio according to the Breguet range equation [4];

$$R = \left(v\, I_{sp}\right) \cdot \left(\frac{C_L}{C_D}\right) \cdot \ln\left(\frac{W_{BC}}{W_{BC} - W_P}\right) \tag{1.4}$$

where cruise velocity is denoted as v, specific impulse denoted as I_{sp} and weight of the missile before launch and weight of fuel are denoted as W_{BC} and W_P, respectively.

1.3.2 Stability

Stability of a missile is considered in terms of static stability and dynamic stability. A statically stable missile can produce some amount of pitching moment with increasing angle of attack in opposing manner [4]. In other words, a statically stable missile weathercocks to flow direction when flight conditions are changed. In order to hold this aerodynamic characteristic, sign of the C_{m_α} (pitch stiffness derivative) must be negative as shown in Figure 1.4 [8].

$$C_{m_\alpha} \approx \frac{\Delta C_m}{\Delta \alpha} < 0 \tag{1.5}$$

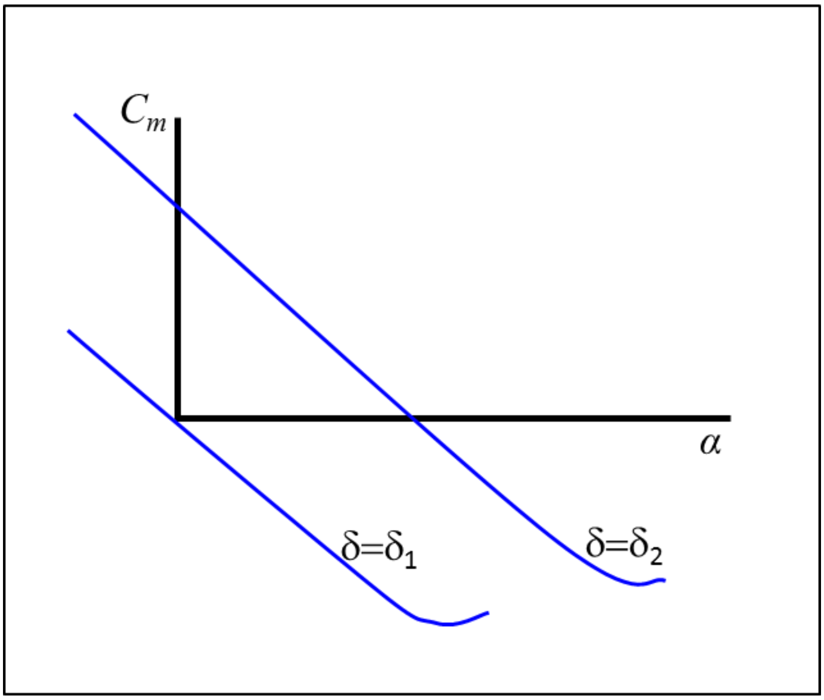


Figure 1.4 Static Stability

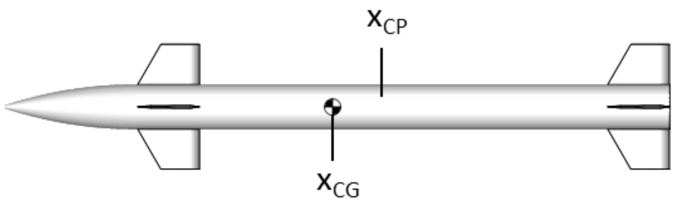


Figure 1.5 Statically Stable Missile Configuration [4]

Static stability is generally quantified with the static margin, distance between center of pressure (x_{CP}) and center of gravity (x_{CG}) divided by the missile diameter (D) [4];

$$SM = \frac{x_{CG} - x_{CP}}{D} \quad (1.6)$$

Negative, zero and positive values of static margin represents stable, marginally stable and unstable missile configurations, respectively. A smaller negative static margin means a stable missile that can be trimmed ($C_m = 0$) at high angles of attack, accordingly is able to produce high normal force and has increased maneuverability.

1.3.3 Maneuverability

Capability of a missile to change speed and direction in a given time is termed as maneuverability [9]. Load factor is a measure of this aerodynamic performance parameter and defined as the ratio of the normal acceleration due to aerodynamic force to the gravitational acceleration [8].

$$n = \frac{N/m}{g} \quad (1.7)$$

where

$$N = C_N S_{ref} \left(\frac{1}{2}\rho v^2\right). \tag{1.8}$$

At a given speed, a missile with a higher load factor is able to achieve higher normal force; accordingly, it is able to maneuver tighter (low turn radius) and faster (high turn rate) [10].

1.3.4 Control Effectiveness

Control effectiveness of a missile is defined as the ratio of the control surface deflection at trim condition to the angle of attack [8].

$$CE = \frac{\delta_{trim}}{\alpha} \tag{1.9}$$

In missile design, generally a control effectiveness value that is less than 1 is desired [7]. A missile with smaller control effectiveness can be trimmed at a higher angle of attack, accordingly has better control power and maneuverability [8].

When missile aerodynamic design parameters are considered in the pitch plane, aerodynamic coefficients such as axial force coefficient (C_A), normal force coefficient (C_N), pitching moment coefficient (C_m) and flight conditions such as velocity (Mach number), angle of attack (α), control surface deflection (δ) are the main factors that affect the aerodynamics of a missile.

1.4 Literature Survey

In literature, some studies on missile shape optimization are available. These studies are conducted for different design objectives, using both stochastic and gradient-based methods for optimization. Genetic Algorithms (GA) and Simulated Annealing (SA) methods are preferred in most of the studies. Using pareto GAs, Anderson, Burkhalter and Jenkins [11] investigated missile aerodynamic shape optimization. An aerodynamic prediction code is coupled with a GA optimizer. A number of design variables concerning the missile shape and aerodynamic performance objectives are included in his study. More recently, Tanıl [12] developed an external configuration design tool that uses GA as optimization method. The method proposed is able to size missiles depending on the flight performance objectives. Karakoç [13] worked on missile external configuration optimization using GA. Main focus for his work is maximizing flight performance and minimizing radar cross-section of missiles. Yong et al. [14] studied aerodynamic shape optimization for canard controlled missiles for maximizing flight range. A GA optimizer is used together with a 3 degree-of-freedom flight simulation code and an aerodynamic analysis tool. Tekinalp and Bingöl [15] developed a missile trajectory optimization method. SA optimization method is employed as the optimizer. A 2 degree-of-freedom flight simulation model and an aerodynamic solver are used together in the proposed method. Öztürk [2] worked on multiobjective design optimization of rockets and missiles. Objectives on range, flight time, hit angle and velocity, aerodynamic coefficients are considered together. A continuous SA optimization algorithm called Hide-and-Seek is used. In addition to these stochastic design optimization methods, there exists the work of Tanrıkulu and Ercan [16] that uses gradient-based optimization. They proposed a method to be used for external configuration optimization of unguided missiles at the conceptual design phase. A gradient-based optimization algorithm is used to determine optimal configurations. Objectives of the optimization are defined considering flight performance of the configurations.

As discussed above, most of the work on missile shape optimization is based on genetic algorithms and simulated annealing method. Popularity of these methods can be recognized in different fields of science and technology. As an alternative to these stochastic methods, Random Search (RS) approaches can be considered in optimization. Especially for functions that are discontinuous and non-smooth, RS method is easy to implement. There exist numerous applications that show RS approaches are quite robust even though convergence to the global optimum is not guaranteed [17]. Computational resource requirements for RS are also seen as acceptable [17]. Silveira Jr. et al. [18] suggested that Adaptively Random Search (ARS), which is a type of RS, is superior to the other stochastic methods such as GAs. Tsoulos and Ragaris [19] compared some RS algorithms with SA on different test functions and showed that RS yields faster solutions in most of the cases. Recent applications of RS can be found in the literature [17,18,20].

Apart from stochastic approaches, gradient-based methods are employed in design optimization problems. One of the most recently developed methods is Sequential Quadratic Programming (SQP) and it is considered as one of the most suitable for gradient-based optimization [21]. SQP methods are successfully employed for aerodynamic shape optimization in the literature [22,23].

1.5 Aim of the Thesis

The aim of this thesis is to develop a design optimization method that can be used for the conceptual design phase of missile aerodynamic design. The methodology is based on shaping external geometry of the missile under given constraints in an automated manner. It aims to determine the best possible configuration in terms of pre-defined aerodynamic design objectives inside the design space. Most importantly, the proposed design optimization method reduces the time spent for missile external configuration design in conceptual design process when compared to the conventional design approach.

In this work, a gradient-based and a random search optimization algorithm coupled with a semi-empiric missile aerodynamics solver are used to determine the optimum aerodynamic external configurations. In the literature, most of the work related to the missile design optimization use genetic algorithms as optimization method. In global optimization problems, random search methods show sufficiently successful results with acceptable computational effort. Accordingly, a random search optimization method is employed in this thesis to investigate its capabilities in missile external geometry optimization. In addition to that, a gradient-based method, Sequential Quadratic Programming (SQP) is also coupled with the same aerodynamic solver. SQP is known as one of the best performing and state-of-the-art gradient-based optimization methods. Two different design optimization procedures are developed using two different optimization approaches and compared to each other in order to understand their applicability and performances in missile aerodynamic design. FORTRAN programming language is used for the implementation of these methods.

In order to analyze each configuration during optimization, an aerodynamic solver code is used. Since obtaining aerodynamically the best performing external geometry in minimum time is a driving factor to develop this design optimization methodology, aerodynamic coefficient fast prediction tool Missile DATCOM is used for aerodynamic analyses.

1.6 Organization of the Thesis

This thesis consists of 5 chapters. Background information on missile conceptual design and missile aerodynamics are given in Chapter 1. Some methods that can be applied for missile design optimization together with existing works in literature are also discussed. Aim of the thesis is also given in Chapter 1.

Chapter 2 gives detailed information on methods that are used in missile aerodynamic optimization procedure developed in this work. SQP and RS based optimization algorithms employed in this work are discussed together with aerodynamic analysis methodology.

Verification of the developed optimization methodology is investigated in Chapter 3. A global optimization test function is used for this purpose. Results are discussed and compared.

In Chapter 4, two applications of missile aerodynamic optimization are presented. For the first application, a missile configuration for wind tunnel test cases, NASA Tandem Controlled Missile (TCM), is used as baseline. With reverse-engineering approach, canard geometry of TCM is optimized. In the second application, optimization of a generic Air-to-Ground Missile (AGM) is discussed. For these missile aerodynamic optimization applications, results are given separately for SQP and RS optimization methods employed in the aerodynamic optimization procedure.

Chapter 5 finalizes this thesis with concluding remarks and recommendations for future studies.

CHAPTER 2

METHODOLOGY

Methodology for the development of the design optimization method is described in this chapter. Governing equations of employed optimization algorithms are given first. After the description of governing equations, aerodynamic analysis method used for performance evaluation of missile configurations is explained. Finally, integration of optimization methods with aerodynamic analysis tool to form the design optimization method is discussed.

2.1 Mathematical Optimization

In general terms, mathematical optimization is defined as [24];

$$minimize\ f(x), \qquad x = [x_1, x_2, \ldots\ldots, x_n]^T \in \mathbb{R}^n \tag{2.1}$$

subject to the constraints:

$$\begin{aligned} g_j(x) &\leq 0, \qquad j = 1, 2, \ldots, m \\ h_j(x) &= 0, \qquad j = 1, 2, \ldots, r \end{aligned} \tag{2.2}$$

where $f(x)$, $g_j(x)$ and $h_j(x)$ are scalar functions of the real column vector x.

In $x = [x_1, x_2, \ldots\ldots, x_n]^T$, x_i represents the (design) variables, whereas $f(x)$ is called as the objective function. Inequality constraints and equality constraints are represented by $g_j(x)$ and $h_j(x)$, respectively [24].

2.2 Design Optimization

Design optimization is defined as a process in which design variables are updated automatically in order to achieve a better design output [25]. As discussed in the previous chapter, there exist numerous optimization methods that can be used for design optimization purposes. In this study, Random Search (RS) and Sequential Quadratic Programming (SQP) methods are employed for the development of the design optimization method for missile external configurations.

2.2.1 Random Search Method

Random Search Method is a stochastic optimization method. Design variables throughout the optimization process are selected randomly; therefore, this method does not require gradient information of the objective function [26]. Since it is not possible to define an exact objective function for the aerodynamic design problem solved in this work, using random search algorithm is advantageous.

In this study, a random search algorithm named as Derivative-Free Adaptively Controlled Random Search (ACRS) is employed in the developed design optimization method. Well known Price's algorithm is the base of ACRS; however, it improves the Price's algorithm by gathering as much as available information from the objective function evaluations during the optimization process [26]. In order the determine minimum of the objective function, ACRS follows the steps given below [26]:

Given a positive integer m such that $m \geq 2n + 1$, where n is number of variables;

Step 0: Initial set determination;

$$\text{Set } k = 0; \\ S^k = \{x_1^k, \dots, x_m^k\} \tag{2.3}$$

Points $x_i^k, i = 1, \dots, m$ are chosen randomly over the optimization domain and objective function f is evaluated at each point $x_i^k, i = 1, \dots, m$.

Step 1: Determine the maximum and minimum objective function values (f_{max}^k, f_{min}^k) and corresponding points (x_{max}^k, x_{min}^k) for the current set;

$$f_{max}^k = f(x_{max}^k) = \max_{x \in S^k} f(x) \tag{2.4}$$

$$f_{min}^k = f\left(x_{min}^k\right) = \min_{x \in S^k} f(x) \tag{2.5}$$

Stop when convergence criterion is satisfied.

Step 2: Random $n + 1$ points $x_{i_0}^k, x_{i_1}^k, \dots, x_{i_n}^k$ are chosen over S^k. Centroid c_w^k of the n points $x_{i_1}^k, \dots, x_{i_n}^k$ is determined from;

$$c_w^k = \sum_{j=1}^{n} w_j^k x_{i_j}^k \tag{2.6}$$

where

$$w_j^k = \frac{\eta_j^k}{\sum_{j=1}^{n} \eta_j^k} \quad (2.7)$$

$$\eta_j^k = \frac{1}{f\left(x_{i_j}^k\right) - f_{min}^k + \phi^k} \quad (2.8)$$

$$\phi^k = \omega \; \frac{(f_{max}^k - f_{min}^k)^2}{f_{max}^0 - f_{min}^0} \quad (2.9)$$

where ω is a sufficiently large positive constant (e.g. $\omega = 10^3$).

Step 3: A weighted reflection is performed to determine the trial point $\tilde{x}^k$; let,

$$f_w^k = \sum_{j=1}^{n} w_j^k f\left(x_{i_j}^k\right) \quad (2.10)$$

then take

$$\tilde{x}^k = \begin{cases} c_w^k - \alpha^k\left(x_{i_0}^k - c_w^k\right), if \;\; f_w^k \le f(x_{i_0}^k); \\ x_{i_0}^k - \alpha^k\left(c_w^k - x_{i_0}^k\right), if \;\; f_w^k > f(x_{i_0}^k); \end{cases} \quad (2.11)$$

with α^kgiven as;

$$\alpha^k = \begin{cases} 1 - \dfrac{f\left(x_{i_0}^k\right) - f_w^k}{f_{max}^k - f_{min}^k + \psi^k}, if \;\; f_w^k \le f\left(x_{i_0}^k\right); \\ 1 - \dfrac{f_w^k - f\left(x_{i_0}^k\right)}{f_{max}^k - f_{min}^k + \psi^k}, if \;\; f_w^k > f\left(x_{i_0}^k\right); \end{cases} \quad (2.12)$$

Compute $f(\tilde{x}^k)$ if $\tilde{x}^k \in D$; if not go to Step 2.

Step 4: If $f(\tilde{x}^k) \ge f_{max}^k$then take;

$$S^{k+1} = S^k \tag{2.13}$$

Set $k = k + 1$ and go to Step 2.

Step 5: If $f_{max}^k \leq f(\tilde{x}^k) < f_{max}^k$ then take;

$$S^{k+1} = S^k \cup \{\tilde{x}^k\} - \{x_{max}^k\}, \tag{2.14}$$

Set $k = k + 1$ and go to Step 1.

Step 6: If $f(\tilde{x}^k) < f_{min}^k$ then;

$$\tilde{S} = S^k \cup \{\tilde{x}^k\} - \{x_{max}^k\}, \tag{2.15}$$

and select the subset S_{min} of $2n + 1$ points in $\tilde{S}$ corresponding to the smallest values of f. To use a quadratic model of the objective function, diagonal matrix Q, the vector c and the scalar d are determined such that

$$f(x_i) = \frac{1}{2}x_i'Qx_i + c'x_i + d, \quad x_i \in S_{min}, \; i = 1, \dots, 2n + 1 \tag{2.16}$$

Step 7: If the diagonal entries of Q are not all positive, then take

$$S^{k+1} = \tilde{S} \tag{2.17}$$

set $k = k + 1$ and go to Step 1.

Step 8: If Q is positive definite, let

$$\tilde{x}_q^k = -Q_c{}^{-1}, \tag{2.18}$$

If $\tilde{x}_q^k \notin D$ or $f(\tilde{x}_q^k) \geq f(\tilde{x}^k)$, then take

$$S^{k+1} = \tilde{S}; \tag{2.19}$$

else take

$$S^{k+1} = \tilde{S} \cup \{\tilde{x}_q^k\} - \{\tilde{x}_{max}^k\}, \tag{2.20}$$

where $\tilde{x}_{max}^k$ determined from

$$f(\tilde{x}_{max}^k) = \max_{x \in S_{min}} f(x) \tag{2.21}$$

Set $k = k + 1$ and go to Step 1.

2.2.2 Sequential Quadratic Programming (SQP) Method

Sequential Quadratic Programming is a gradient-based optimization method. Along the optimization process, design variables are determined according to the gradient information of the objective function. Nevertheless, due to nature of the aerodynamic design problem discussed in this work, gradient information of the objective function is non-existent. Therefore, gradients need to be calculated numerically by using objective function directly.

In addition to ACRS method discussed previously, aerodynamic shape optimization is also done by using SQP method. The SQP algorithm employed in the developed design optimization method, DONLP2, is capable of locating a local minimum inside the design space quickly [27]. However, this quickness comes with an expense; locating the global minimum is very much dependent on the initial design point, especially in a complex design space containing numerous local minima. As an SQP algorithm, DONLP2 requires gradients of the objective function. Accordingly, gradients are calculated numerically using a sixth order approximation computing a Richardson extrapolation of three symmetric differences [27]. In order the determine minimum of the objective function, DONLP2 follows the methodology given below [21,27]:

For a quadratic problem that is defined as;

Find ΔX that minimizes the quadratic objective function Q;

$$Q = \nabla f^T \Delta X + \frac{1}{2}\Delta X^T [\nabla^2 L] \Delta X \tag{2.22}$$

subject to the linear equality constraints defined as;

$$h_k + \nabla h_k^T \Delta X = 0 \tag{2.23}$$

where

$$k = 1, 2, \ldots, p \quad \text{or} \quad h + [H]^T \Delta X = 0 \tag{2.24}$$

The Lagrange function, $\tilde{L}$, for the problem given in (2.22) is defined as

$$\tilde{L} = \nabla f^T \Delta X + \frac{1}{2}\Delta X^T [\nabla^2 L] \Delta X + \sum_{k=1}^{p} \lambda_k (h_k + \nabla h_k^T \Delta X) \tag{2.25}$$

where λ_k is the Lagrange multiplier associated with the kth equality constraint. The Kuhn-Tucker necessary conditions can be stated as;

$$\nabla f^T + [\nabla^2 L] \Delta X + [H]\lambda = 0 \tag{2.26}$$

and

$$h_k + \nabla h_k^T \Delta X = 0 \qquad k = 1, 2, \dots, p \tag{2.27}$$

Equation (2.22) can be identified as a standard form of the original optimization equation without inequality constraints;

$$\begin{gathered} minimize\ f(x), \\ x = [x_1, x_2, \dots\dots, x_n]^T \ \in \ \mathbb{R}^n \end{gathered} \tag{2.28}$$

subject to:

$$h_j(x) = 0, \qquad j = 1, 2, \dots, r \tag{2.29}$$

Accordingly, equation (2.22) can be solved iteratively for the solution of the problem given in (2.28). Addition of inequality constraints to the general optimization problem defined in (2.28), following can be written:

Find X which minimizes

$$Q = \nabla f^T \Delta X + \frac{1}{2} \Delta X^T [\nabla^2 L] \Delta X \tag{2.30}$$

subject to

$$\begin{gathered} g_j + \nabla g_j^T \Delta X \ \leq 0 \qquad j = 1, 2, \dots, m \\ h_k + \nabla h_k^T \Delta X = 0 \qquad k = 1, 2, \dots, p \end{gathered} \tag{2.31}$$

with Lagrange function given by

$$\tilde{L} = f(\mathrm{X}) + \sum_{j=1}^{m} \lambda_j g_j(X) + \sum_{k=1}^{p} \lambda_{m+k} k_k(X) \tag{2.32}$$

2.3 Aerodynamic Analysis Methodology

Aerodynamic coefficients and related aerodynamic performance parameters of a design configuration need to be determined during the design optimization process. Accordingly, Missile DATCOM aerodynamic coefficient estimation software is employed in the developed optimization method. Missile DATCOM is a semi-empirical tool that predicts the aerodynamic coefficients for a wide variety of missile configurations. It has adequate accuracy for preliminary design phase and outputs the results very quickly. Therefore, it is convenient to use this software for design optimization since numerous configurations need to be evaluated in a short time in order to determine the most suitable one, considering the objective performance parameters and geometric constraints.

Missile DATCOM is developed based on Component Build-Up (CBU) methods. Aerodynamic force and moment coefficients of each component of the missile together with their interactions are calculated using CBU methods. Total forces and moments acting on a conventional missile configuration shown in Figure 2.1 is the summation of coefficients of each component that form the missile [28].

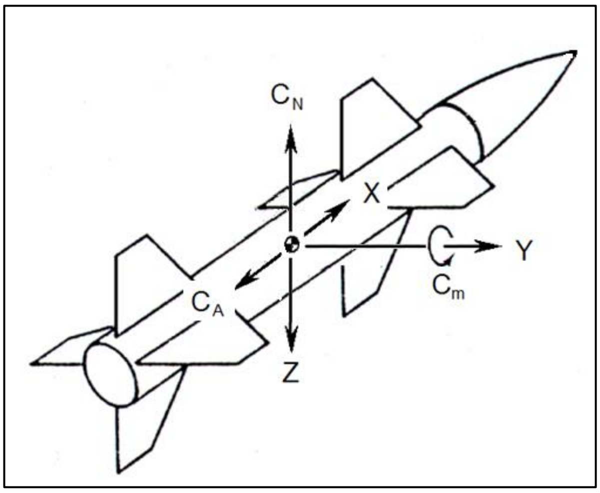


Figure 2.1 Conventional Wing-body Missile Configuration and Coordinate System

Normal force, pitching moment and axial force coefficients for a conventional missile configuration can be described as follows [28]:

$$C_{N_{BWT}} = C_{N_B} + C_{N_{W(B)}} + C_{N_{B(W)}} + C_{N_{T(B)}} + C_{N_{B(T)}} + C_{N_{T(W)}} \tag{2.33}$$

$$C_{m_{BWT}} = C_{m_B} + C_{m_{W(B)}} + C_{m_{B(W)}} + C_{m_{T(B)}} + C_{m_{B(T)}} + C_{m_{T(W)}} \tag{2.34}$$

$$C_{A_{BWT}} = C_{A_B} + C_{A_{W(B)}} + C_{A_{T(B)}} + C_{A_{T(W)}} \tag{2.35}$$

Indices in equations (2.33) - (2.35) are given in Table 2.1.

Table 2.1 Indices in Equations (2.33) - (2.35)

B	Body
$W(B)$	Wing in presence of body
$B(W)$	Body in presence of wing
$T(B)$	Tail in presence of body
$B(T)$	Body in presence of tail
$T(W)$	Tail in presence of wing

For ϕ = 0°, equation (2.33) can be expanded as [28]:

$$C_{N_{W(B)}} = \left(K_{W(B)}\alpha + k_{W(B)}\delta\right)\frac{\partial C_{N_W}}{\partial \alpha} \tag{2.36}$$

$$C_{N_{B(W)}} = \left(K_{B(W)}\alpha + k_{B(W)}\delta\right)\frac{\partial C_{N_W}}{\partial \alpha} \tag{2.37}$$

$$C_{N_{T(B)}} = \left(K_{T(B)}\alpha + k_{T(B)}\delta\right)\frac{\partial C_{N_T}}{\partial \alpha} \tag{2.38}$$

$$C_{N_{B(T)}} = \left(K_{B(T)}\alpha + k_{B(T)}\delta\right)\frac{\partial C_{N_T}}{\partial \alpha} \tag{2.39}$$

$$C_{N_{T(W)}} = \left(1 + \frac{k_{B(T)}}{k_{T(B)}}\right)\frac{\partial C_{N_T}}{\partial \alpha} \tag{2.40}$$

K terms given in above equations are carry-over interference factors between components. These interference factors are obtained from slender-body theory that is developed for wing-body configurations [5]. Relation of interference factors with body radius (*r*) and wing/tail semi-span (s_w, s_t) is given in Figure 2.2.

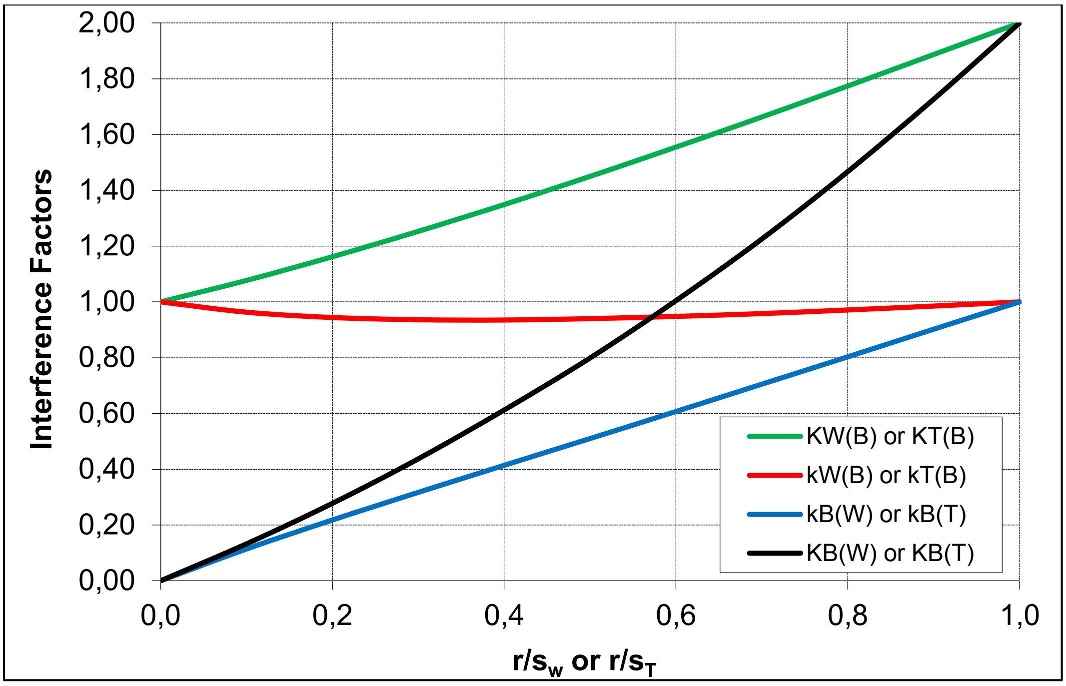


Figure 2.2 Variation of Interference Factors with r/s_w or r/s_t [5]

Note that relation between interference factors given in Figure 2.2 is;

$$\frac{k_{B(W)}}{k_{W(B)}} \approx \frac{K_{B(W)}}{K_{W(B)}}, \qquad \frac{k_{B(T)}}{k_{T(B)}} \approx \frac{K_{B(T)}}{K_{T(B)}} \tag{2.41}$$

Aerodynamic derivatives for wing/tail fins such as $\frac{\partial C_{N_W}}{\partial \alpha}$ and $\frac{\partial C_{N_T}}{\partial \alpha}$ are determined using theoretical or semi-empirical methods. Empirical methods are generally employed to predict derivatives of body aerodynamic coefficients. Equivalent angle of attack approach and vortex algorithms are also incorporated when combining body and fin aerodynamic predictions for configuration synthesis [29].

2.4 Aerodynamic Optimization Procedure

Aerodynamic optimization process consists of many sub-problems such as designation of a configuration, determining aerodynamic performance parameters of the configuration, evaluation of the configuration according to design objectives and finding a better configuration in terms of aerodynamic performance. In this study, a design optimization method that is capable of tackling these problems automatically is developed. As two different optimization algorithms are used, flowcharts and descriptions for iterative steps for both are given in the following discussion.

Design optimization method employing DONLP2 algorithm coupled with aerodynamic analysis tool follows the procedure given in Figure 2.3.

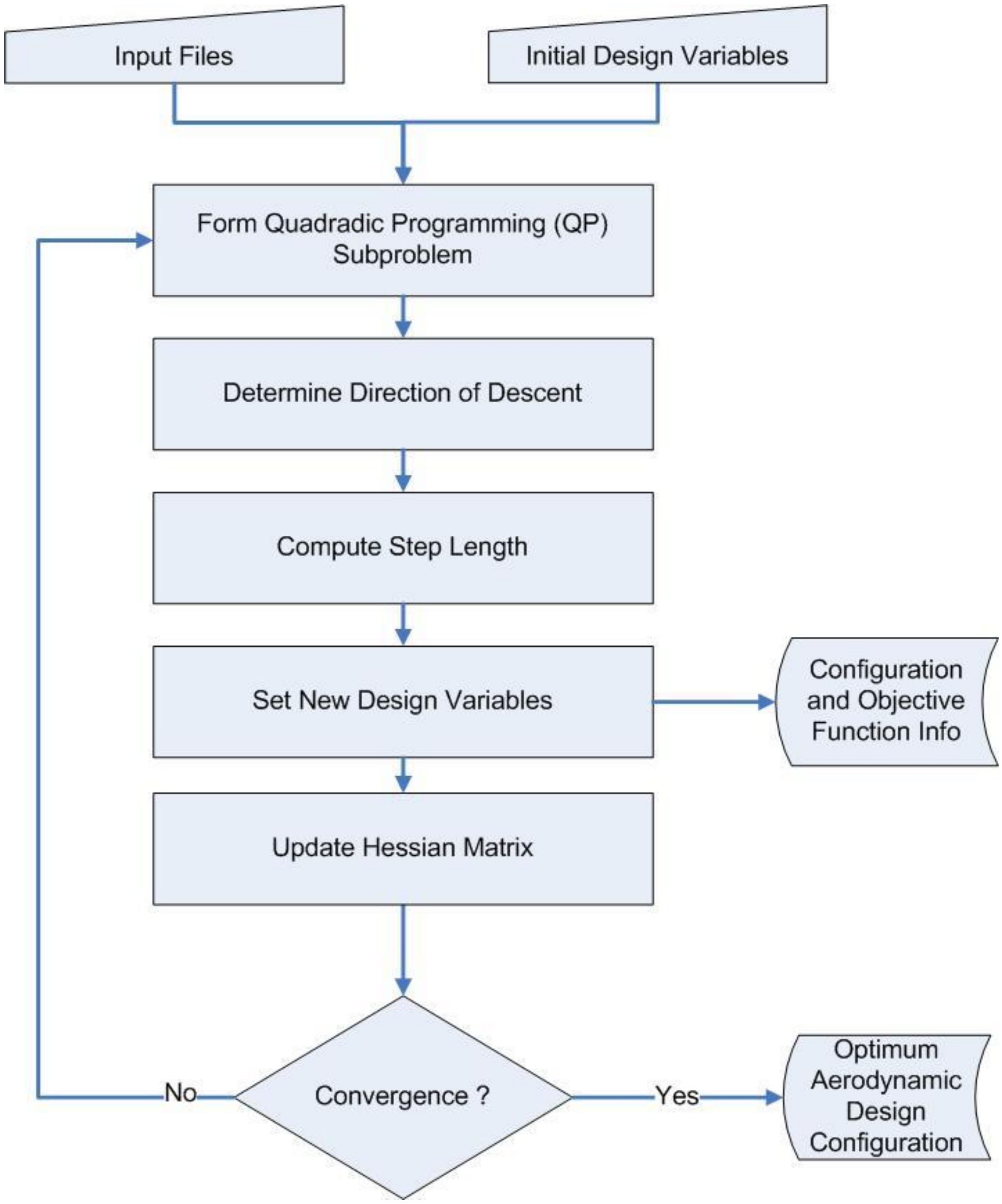


Figure 2.3 Flowchart for Design Optimization with Sequential Quadratic Programming Method

Steps of aerodynamic design optimization with DONLP2 algorithm can be described as follows:

- In order to start a design optimization, input files for baseline design configuration, geometric constraints and objective aerodynamic performance parameters need to be supplied to the design tool. Since

DONLP2 uses a SQP method, it is also necessary to initialize the design variables.

- As the first step of design optimization, DONLP2 algorithm solves the quadratic programming sub-problem formed by the objective function and design constraints. During the solution process, it is necessary to supply derivative information of the objective function. As this information is not available analytically, it is computed numerically. Aerodynamic analysis tool is called for evaluation of the objective function and its derivatives using a sixth order approximation.
- After the solution of the quadratic programming sub-problem, direction of descent is determined.
- Then, step size is calculated to determine new design variables along minimization.
- In the fourth step, a new set of design variables is determined using the descent direction and step size information. Objective function value is calculated with the aerodynamic analysis tool and stored together with the configuration information.
- Hessian matrix that consists of second-order partial derivatives of the quadratic problem is updated.
- Convergence criterion is checked. If it is not satisfied, quadratic problem is solved again using the new design variables determined in step 4. This procedure is repeated until convergence is achieved. External proportions and aerodynamic performance parameters of the optimum design configuration are provided as output.

Design optimization method employing ACRS algorithm coupled with aerodynamic analysis tool follows the procedure given in Figure 2.4.

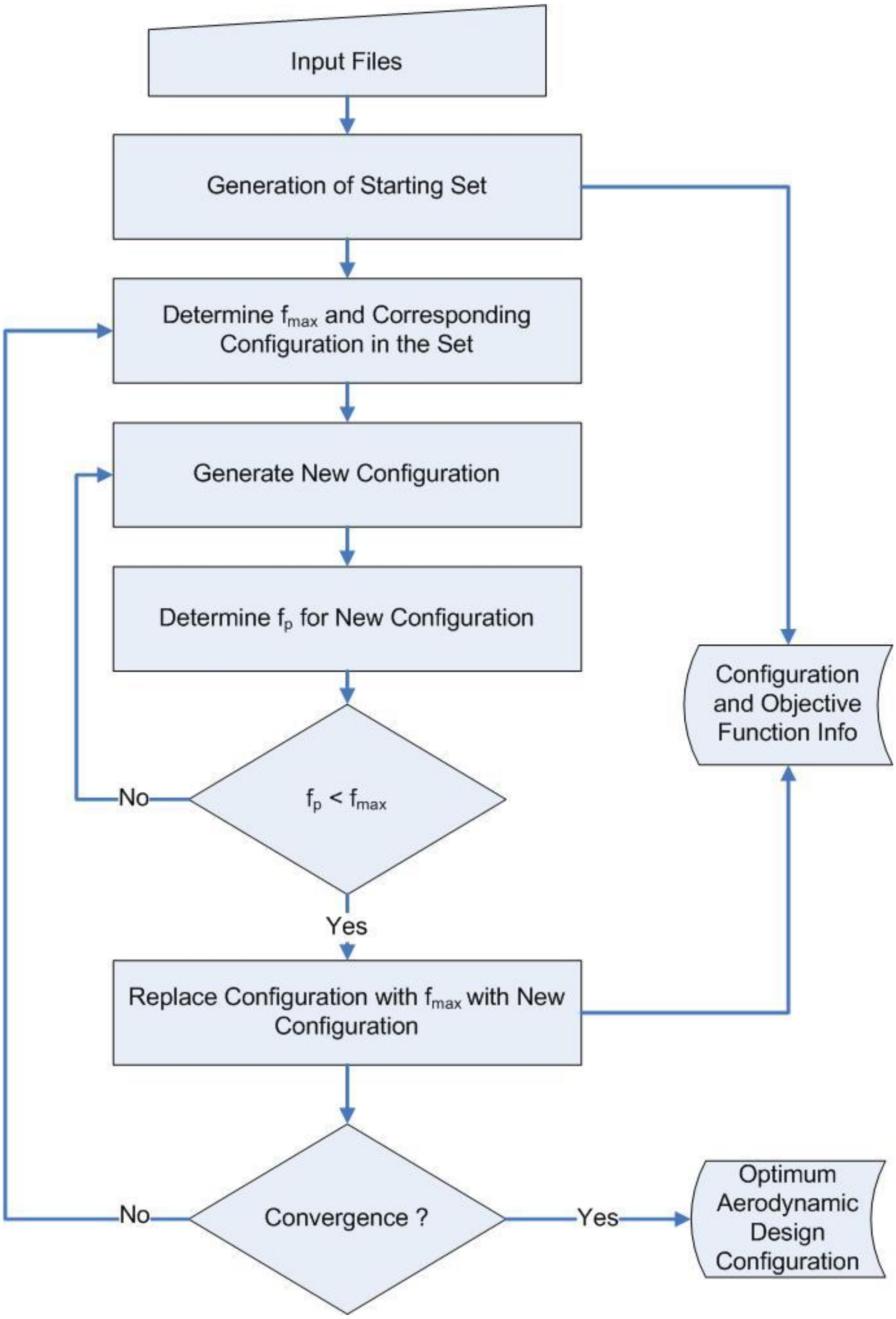


Figure 2.4 Flowchart for Design Optimization with Random Search Method

Steps of aerodynamic design optimization with ACRS algorithm can be described as follows:

- In order to start a design optimization, input files for baseline design configuration, geometric constraints and objective aerodynamic performance parameters need to be supplied.
- As the first step of design optimization, ACRS algorithm generates a set of random configurations that lie inside the design space defined by geometric constraints. Size of the set is proportional to the number of design variables.
- Aerodynamic coefficients for all of the configurations in the starting set are calculated by using aerodynamic analysis tool. Accordingly, objective function that represents the aerodynamic performance is calculated for all of the configurations. Since each configuration has a unique objective function value, they are sorted and configuration with maximum objective value (f_{max}) is found.
- After generation of starting set, ACRS generates a new random configuration based on information obtained about the design space while generating the starting set. Design parameters of this new configuration are sent to the aerodynamic analysis tool and aerodynamic coefficients of the configuration are determined. Using aerodynamic coefficients, objective function value for this configuration (f_p) is calculated.
- Since f_{max} and f_p is known, ACRS compares them in order to determine whether this new configuration is better than the configuration with f_{max} or not. If f_p is less than f_{max}, new configuration is added to solution set whereas configuration that has f_{max} is discarded from the set.
- ACRS continues to generate random configurations, calculate f_p and update the configurations in the solution set upon convergence criteria is satisfied. When generating these random configurations, information gathered from previous iterations are taken into account to direct solutions to the global minimum. This is achieved by use of a weighted centroid, a weighted reflection and a quadratic model of the objective function.
- Optimization process is stopped when the convergence criteria is met. Accordingly external proportions and aerodynamic performance parameters of the optimum design configuration are provided.

CHAPTER 3

VERIFICATION OF THE DESIGN OPTIMIZATION METHOD

A verification study on an optimization test function is done for the methods used in this work. Goldstein and Price function is chosen as the test function since it is accepted as a global optimization test function for optimization algorithms in the literature [30]. Details about this function are given in the following section. Moreover, aerodynamic analysis tool employed in developed optimization method is also validated in this chapter.

3.1 Goldstein and Price Function

Goldstein and Price function is a two variable function that has multiple local minima and defined as [26]:

$$\begin{aligned} f(x, y) = {} & [1 + (x + y + 1)^2 \\ & \cdot (19 - 14x + 3x^2 - 14y + 6xy + 3y^2)] \\ & \cdot [30 + (2x - 3y)^2 \\ & \cdot (18 - 32x + 12x^2 + 48y - 36xy + 27y^2)] \end{aligned} \tag{3.1}$$

subject to

$$-2.0 \leq (x, y) \leq 2.0 \tag{3.2}$$

Goldstein and Price function takes the global minimum value of $f(x,y) = 3$ for $(x,y) = (0,-1)$.

In Figure 3.1, Goldstein-Price function is shown. Since function values change very rapidly depending on the variables, contours are drawn logarithmically. As seen from Figure 3.1, local and global minima are located at an area where change of $f(x,y)$ is very limited, imposing a challenge to the determination of the global minimum.

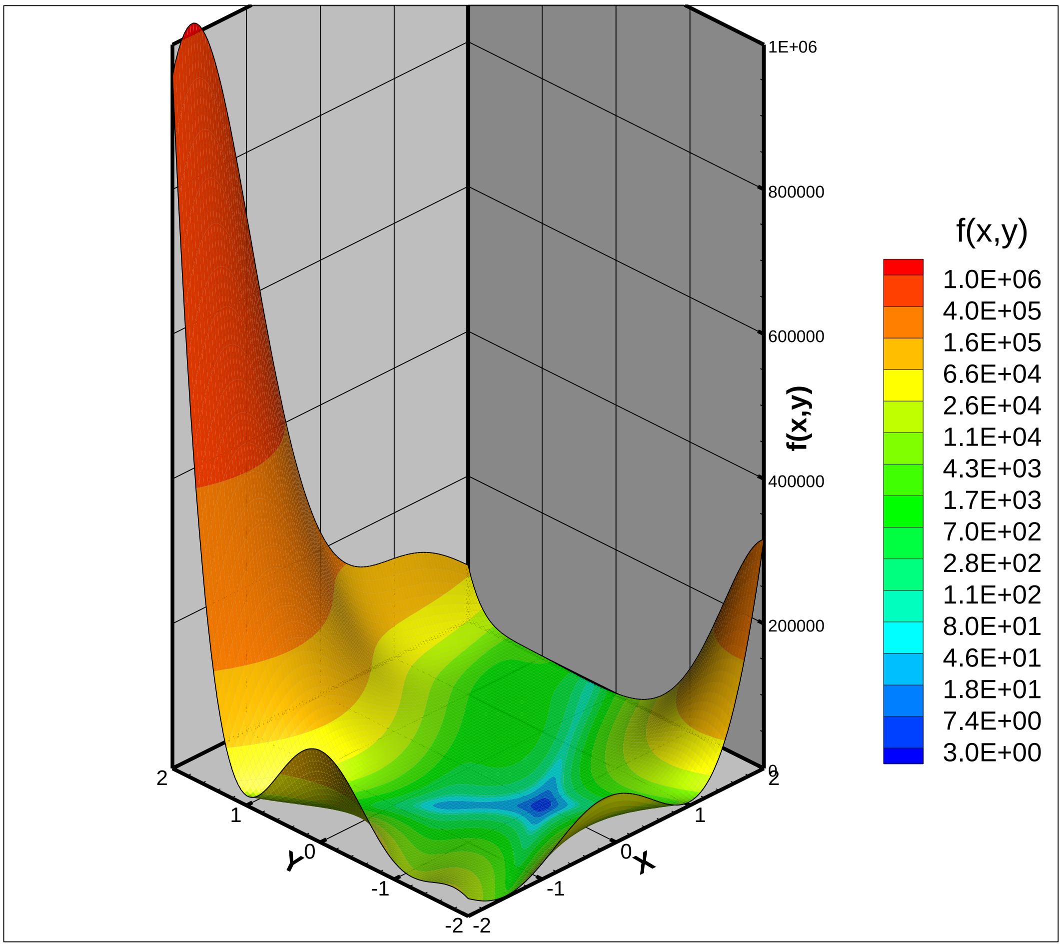


Figure 3.1 Goldstein and Price Function

In order to determine the capabilities and limitations of the design optimization methods used in this study, global minimum of the Goldstein-Price function is searched. Two different approaches under consideration in this work are tested;

stochastic, random search algorithm ACRS and gradient-based Sequential Quadratic Programming (SQP) algorithm DONLP2.

3.2 Verification of the Sequential Quadratic Programming (SQP) Method

Using the DONLP2 algorithm, global minimum point of the Goldstein and Price function is searched. Since DONLP2 is based on Sequential Quadratic Programming, a gradient-based optimization method, it is necessary to supply a starting point in the design space to begin the optimization process. In the verification study, a random starting point is selected as $(x, y) = (2,\ 1.9)$. Starting from the initial point, optimization algorithm is able to find the global minimum of the function in 15 iterations with 255 function evaluations as $(x, y) = (0, -1)$. In Figure 3.2, advance of the iterations as search marches towards the global minimum is shown.

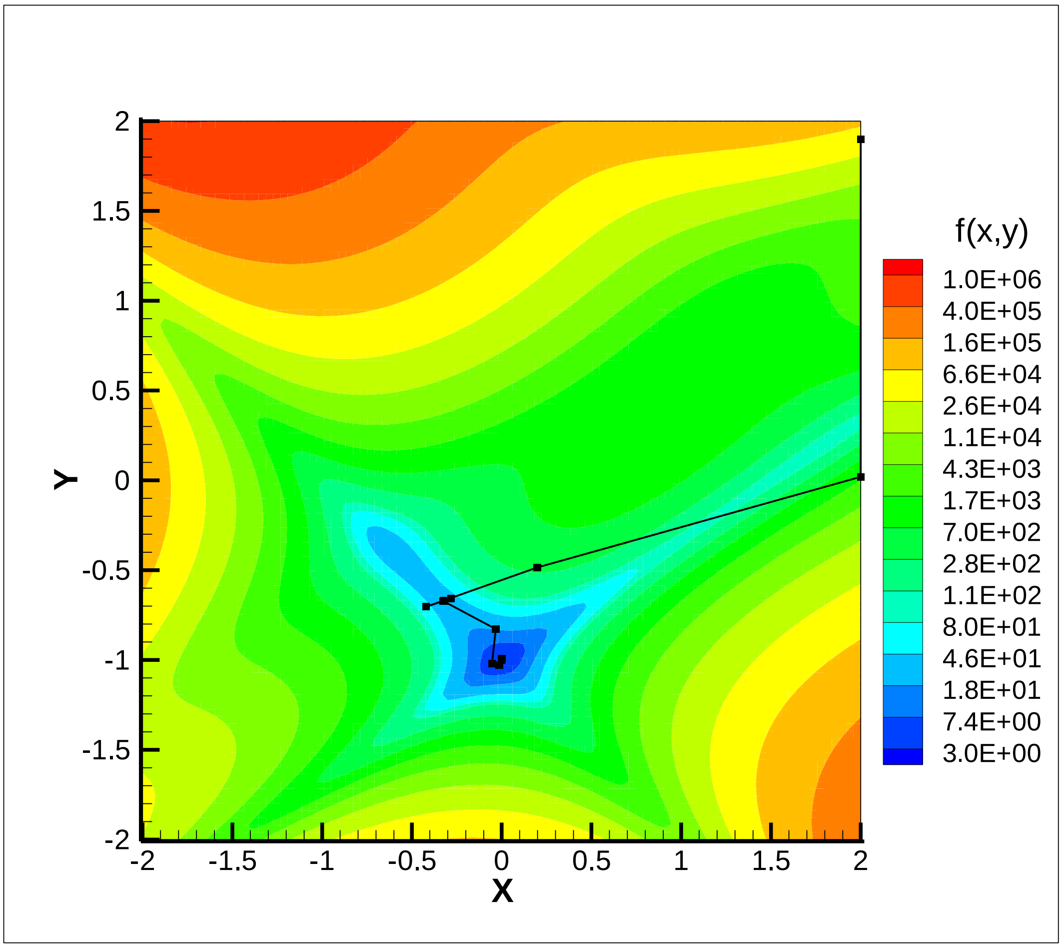


Figure 3.2 Global Minimum Search with SQP Method, Successful

Another random starting point is selected as $(x, y) = (-1.85,\ -1.35)$. Starting from the initial point, optimization run converges to a local minimum of the function in 16 iterations with 272 function evaluations as $(x, y) = (-0.6, -0.4)$ where objective function has a value of $f(x, y) = 30$. In Figure 3.3, advance of the iterations as search marches towards the local minimum is shown.

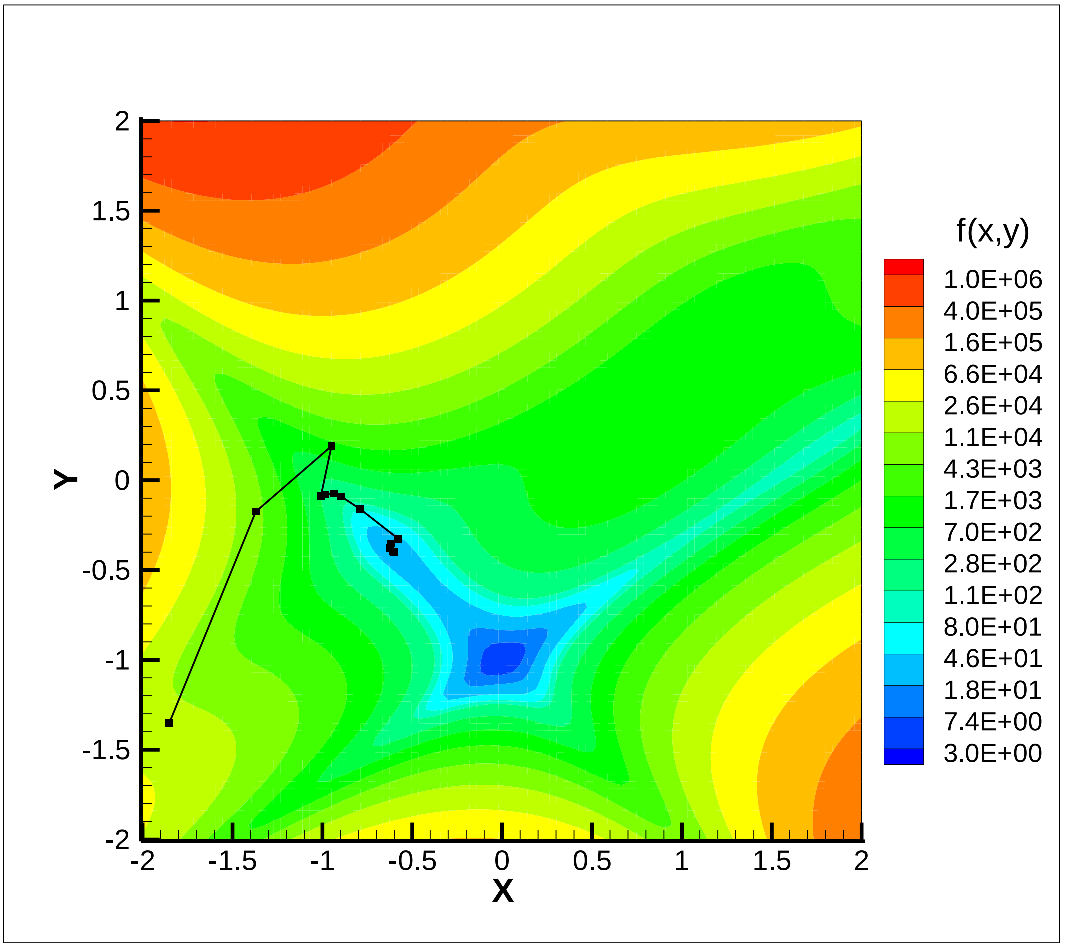


Figure 3.3 Global Minimum Search with SQP Method, Converged to Local Minimum

One more random starting point is selected as $(x, y) = (0.94,\ 0.16)$. Starting from there, optimization run converges to a local minimum of the function in 12 iterations with 197 function evaluations as $(x, y) = (1.8,\ 0.2)$, where objective function has a value of $f(x, y) = 84$. In Figure 3.4, advance of the iterations as search marches towards the local minimum is shown.

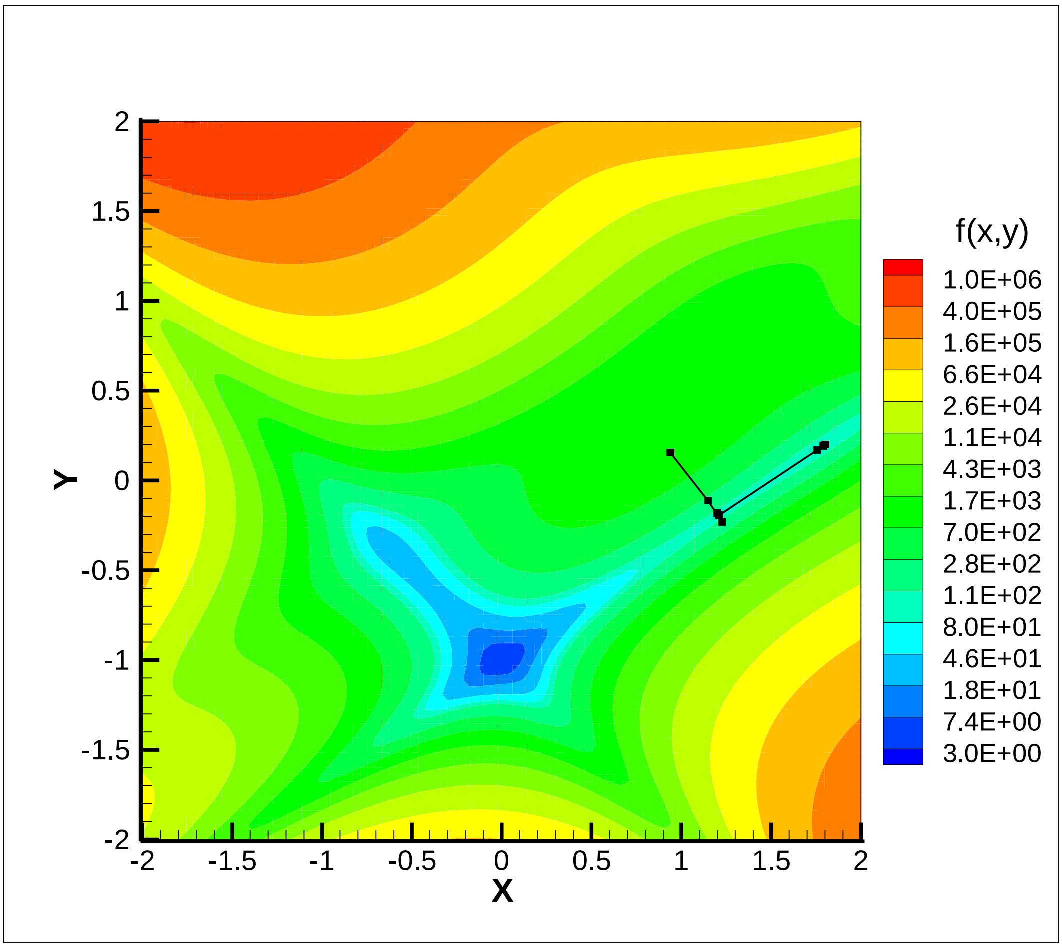


Figure 3.4 Global Minimum Search with SQP Method, Converged to Local Minimum

It is known that, during a global minimum search, accuracy of SQP methods are very dependent on the initial point and design space topology. Accordingly, Goldstein-Price function is solved 15 times to investigate repeatability and consistency of the results obtained using SQP algorithm namely DONLP2. Random initial points are generated for each solution to see the effect of the starting point on the optimization process. Change of objective function values with iterations for best, worst and remaining runs are shown in Figure 3.5. Best and worst solutions to the problem are given in Table 3.1.

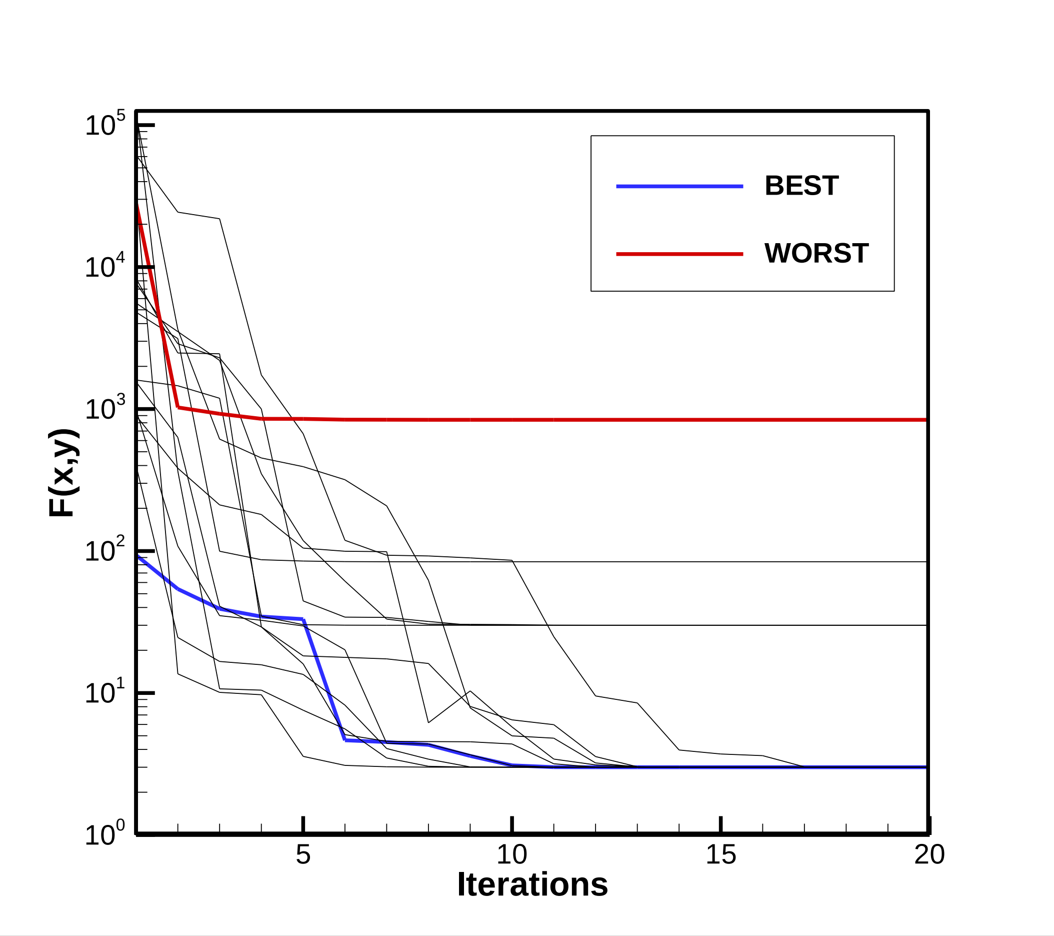


Figure 3.5 Change of Objective Function Values with Iterations for 15 Optimization Runs

Table 3.1 Variable and Objective Function Values for Best and Worst Solutions

	x	**y**	**f(x,y) – 3.0**
Best	-2.657E-09	-1.0	-7.994E-15
Worst	1.2	0.8	837.0
OBJECTIVE	0.0	-1.0	0.0

Previous discussion and Table 3.1 prove that there are solutions to the Goldstein-Price test problem that converge to global or local minima when DONLP2 algorithm is used. Therefore, this gradient-based SQP method should be used carefully when the design space is complex.

3.3 Verification of the Random Search Method

Using the derivative-free Adaptively Controlled Random Search (ACRS) algorithm, global minimum of the Goldstein-Price function is investigated. ACRS algorithm is able to find the global minima of the function in 514 iterations as $(x, y) = (0, -1)$. In Figure 3.6, all of the points used by the optimization method during global minimum search are shown. Change of these points with iterations as search marches towards the global minimum is illustrated in Figure 3.7.

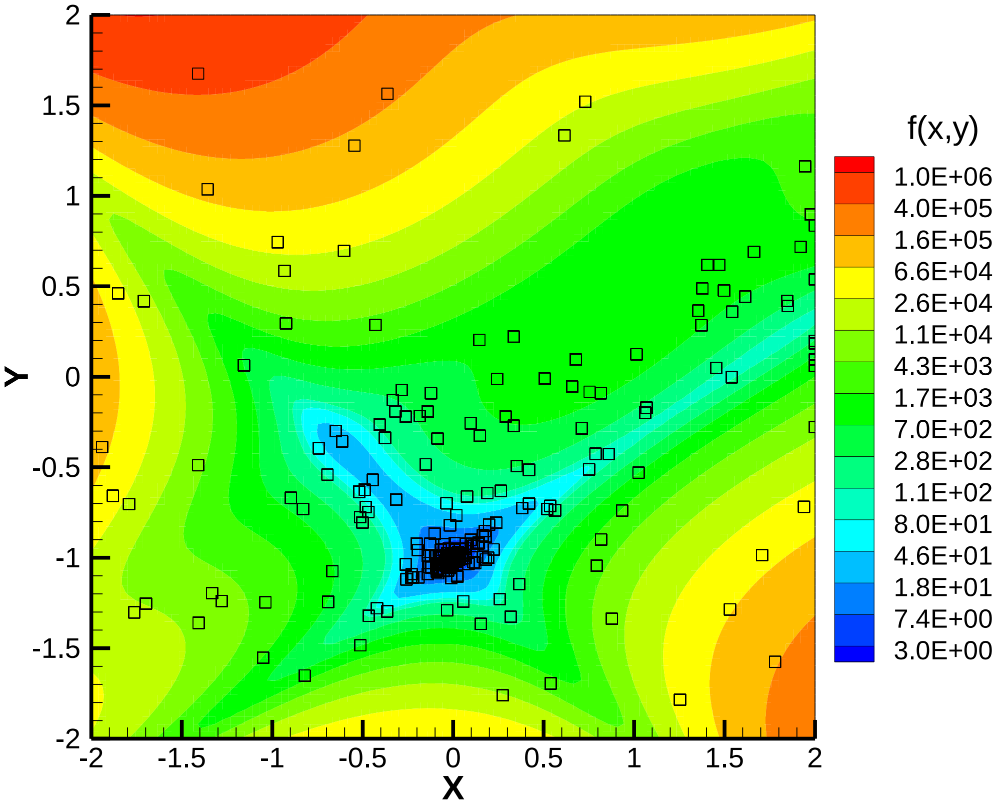


Figure 3.6 Evaluated Points During Global Minimum Search with Random Search Algorithm

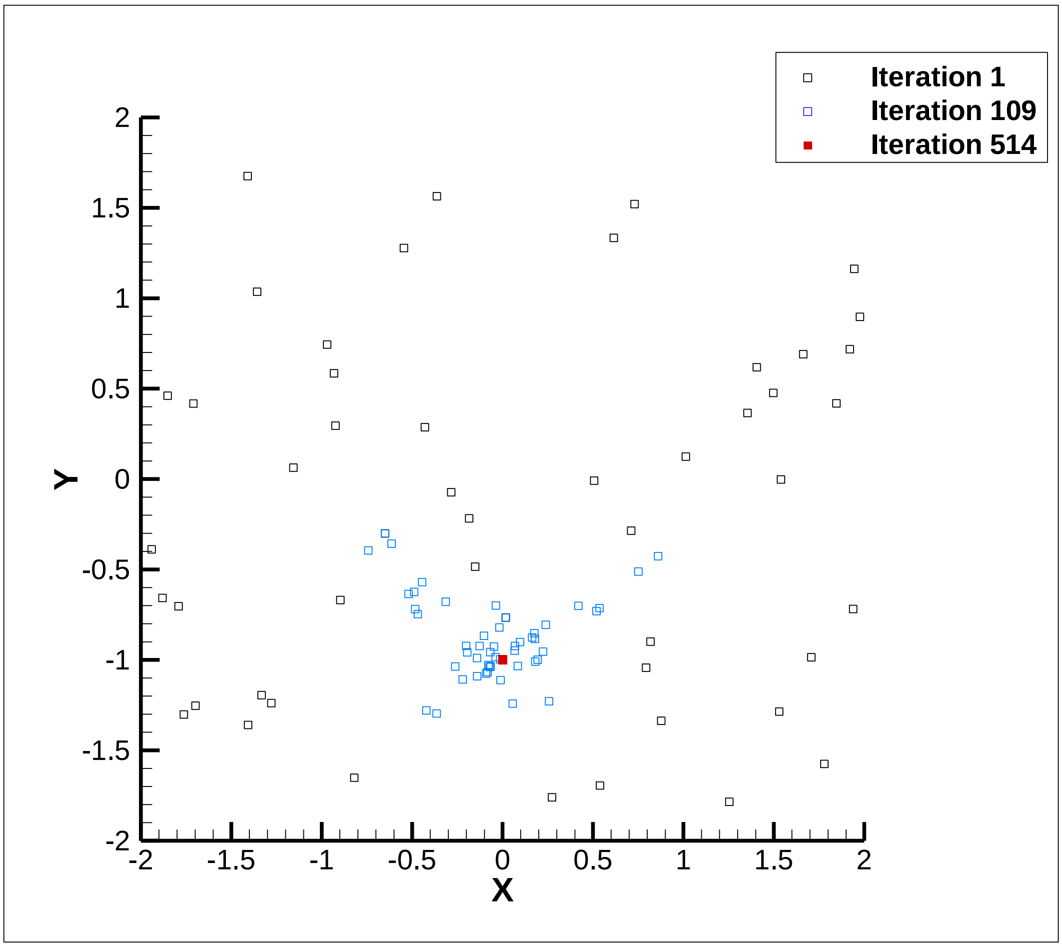


Figure 3.7 Evaluation of Iterations During Optimization

Generally, stochastic methods such as random search method are fairly good at locating global minimum in optimization problems. However, they generally require more computational time and effort, compared to gradient-based methods. Using random search optimization algorithm ACRS, Goldstein-Price function is solved 15 times to investigate repeatability and consistency of results. Note that it is not necessary to supply a starting point for optimization when using ACRS. Change of objective function values with iterations for best and worst resulting runs is shown in Figure 3.8. It can be seen that both best and worst run converge to global minimum meaning that none of the 15 runs ended up in local minima. Table 3.2 gives solutions for best and worst runs to the problem.

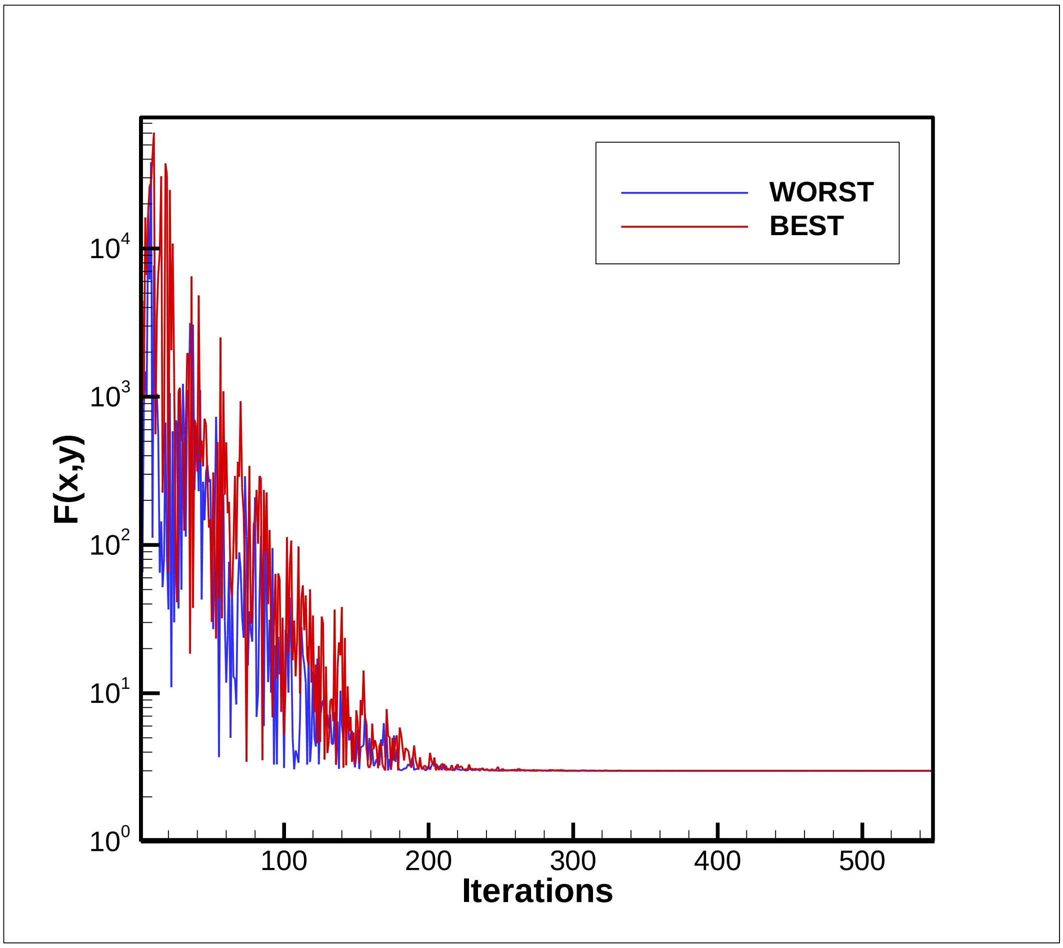


Figure 3.8 Change of Objective Function Values with Iterations for Best and Worst Solutions

Table 3.2 Variable and Objective Function Values for Best and Worst Solutions

	x	**y**	**f(x,y)-3.0**
Best	-9.982E-07	-1.0	8.350E-10
Worst	-4.130E-06	-1.0	2.173E-08
OBJECTIVE	0.0	-1.0	0.0

Previous discussion and Table 3.2 show that all of the 15 solutions to the Goldstein-Price optimization test problem ended up in the global minimum of the function, whereas SQP method discussed previously may get trapped in local minima of the function as seen in Figure 3.3, Figure 3.4 and Figure 3.5. Accordingly, it can be concluded that ACRS is more robust in locating global minimum and avoiding local minima inside a complex design space, compared to DONLP2.

3.4 Validation of the Aerodynamic Analysis Method

In this study, Missile DATCOM software is used to predict the aerodynamic coefficients of missile configurations. Results obtained from Missile DATCOM (MD) for NASA Tandem Control Missile (TCM) configuration are compared to experimental and Computational Fluid Dynamics (CFD) results found in the literature [31]. It is aimed to investigate the applicability and accuracy of the aerodynamic analysis method employed for missile shape optimization. More detail on TCM configuration can be found in Section 4.1.1.

Figure 3.9 - Figure 3.11 present lift, drag and pitching moment coefficients calculated for the TCM configuration at Mach=1.75, respectively.

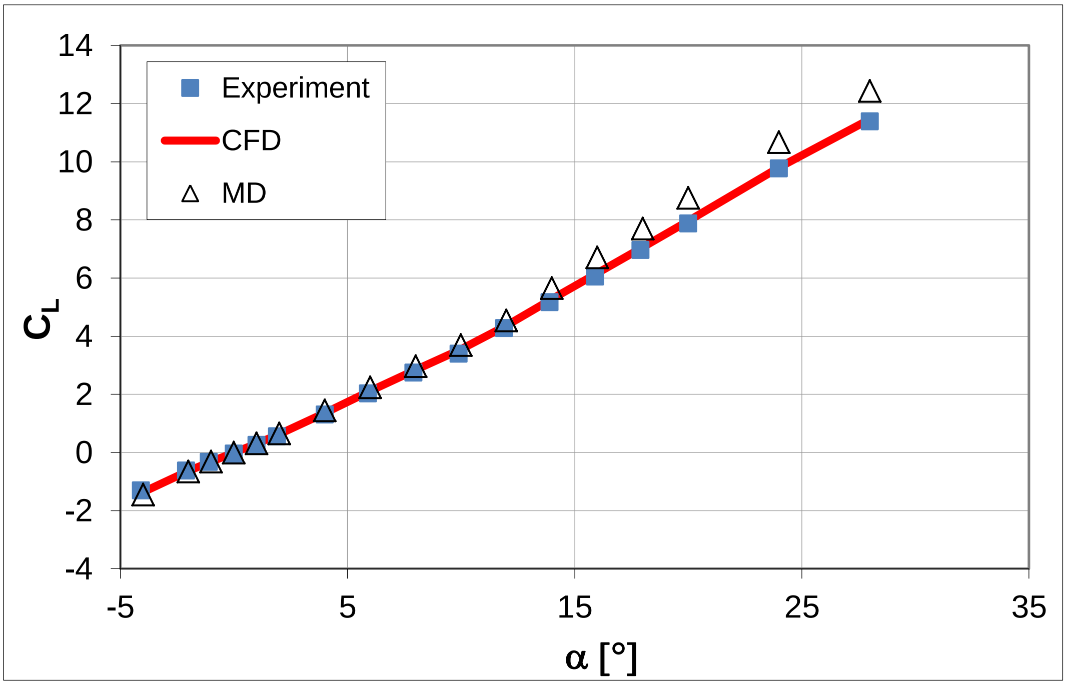


Figure 3.9 Lift Coefficient Obtained from MD, CFD and Experiment for TCM Configuration (Mach = 1.75)

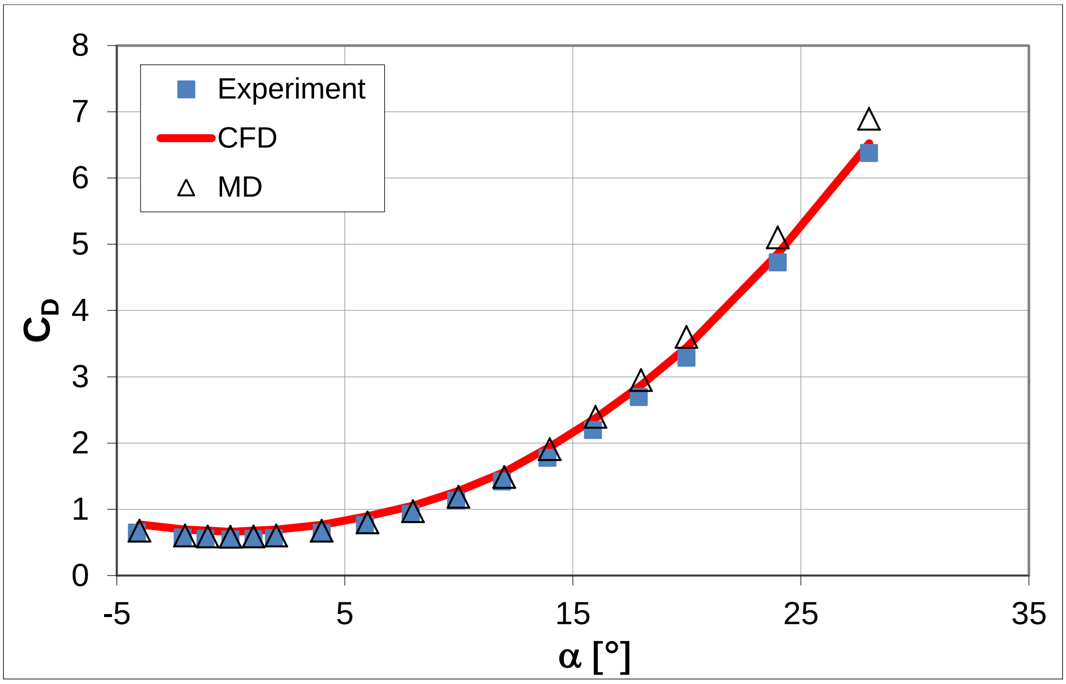


Figure 3.10 Drag Coefficient Obtained from MD, CFD and Experiment for TCM Configuration (Mach = 1.75)

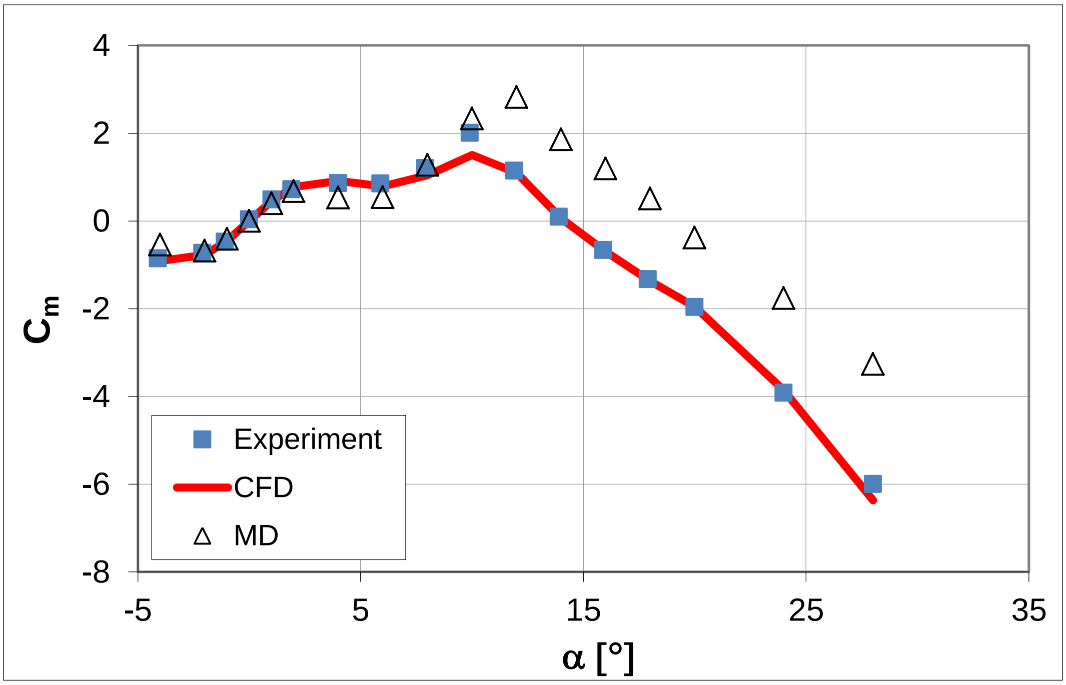


Figure 3.11 Pitching Moment Coefficient Obtained from MD, CFD and Experiment for TCM Configuration (Mach = 1.75)

From Figure 3.9, Figure 3.10 and Figure 3.11, it can be said that MD is sufficiently successful for aerodynamic coefficient prediction of conventional missile configurations. Especially for angles of attack up to 10°, results are very good and almost coincide with experiment data. Predictions start to disagree with CFD and experiment when the angle of attack is over 10°. Even then, MD has an acceptable error for conceptual and preliminary design optimization purposes.

CHAPTER 4

MISSILE AERODYNAMIC OPTIMIZATION STUDIES

In this part of the thesis, aerodynamic optimization of missile external configurations is carried out to investigate the capabilities of the developed design optimization methods. Two different optimization applications are demonstrated. In the first section, “NASA Tandem Control Missile” (TCM) configuration is considered as baseline. As a starting point, aerodynamic performance parameters of the TCM configuration are determined using aerodynamic coefficient prediction tool Missile DATCOM. Then, using the reverse engineering approach, aerodynamic performance parameters of TCM configuration are given as objectives to the design tool and it is aimed to obtain a configuration similar to the original TCM configuration. In the second section, an optimization application for a generic air-to-ground missile (AGM) configuration is demonstrated. Aerodynamic performance parameters for the AGM are defined considering typical mission requirements of such missiles. Each optimization application is done using both optimization algorithms, ACRS and DONLP2. Optimum configurations are discussed separately and compared to each other in order to investigate the performances of the algorithms.

4.1 Aerodynamic Shape Optimization for NASA Tandem Control Missile

4.1.1 NASA Tandem Control Missile (TCM) Configuration

TCM is a supersonic wind-tunnel test case configuration. In this study, a version of TCM that is known as TCM B1T4C4 in the literature is investigated. Geometry of TCM B1T4C4 is given in Figure 4.1. This configuration has the same geometric proportions for each 4 canard and tail fins and these fins are configured in X shape. It has a cylindrical body with tangent ogive nose. Wind tunnel test results and analyses done with fast-prediction aerodynamics methods for TCM are available in the literature [32,33].

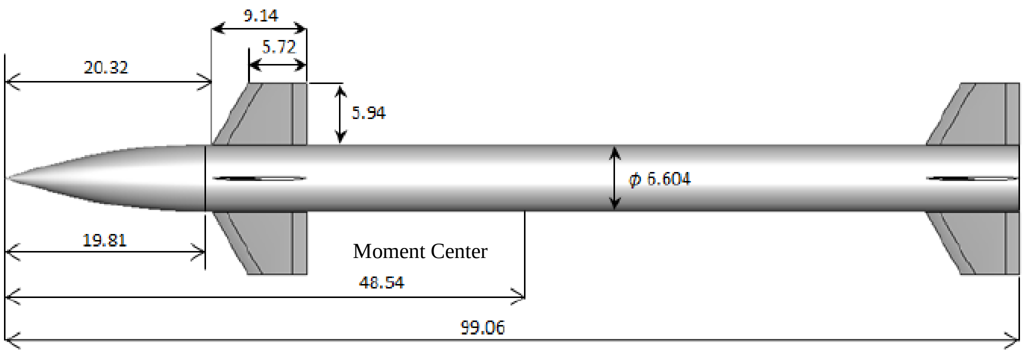


Figure 4.1 TCM B1T4C4 Configuration (Dimensions in cm)

4.1.2 Aerodynamic Analyses of TCM

Design optimization procedure employs Missile DATCOM to evaluate aerodynamic coefficients of each configuration in optimization process. Accordingly, aerodynamic analyses are done in flight conditions given in Table 4.1 to determine the aerodynamic performance parameters of the TCM configuration. From these parameters, objectives for optimization study are defined.

Table 4.1 Flight Conditions

Mach	1.75
Angle of Attack	$-4° \leq \alpha \leq 24°$
Sideslip Angle	$\beta = 0°$
Altitude	Sea Level

Change of aerodynamic coefficients versus angle of attack is given in Figure 4.2, Figure 4.3 and Figure 4.4. Center of pressure is calculated from the moment center of missile and non-dimensionalized with missile diameter. Positive values reflect a center of pressure location forward of moment center whereas negative values indicate aft of it.

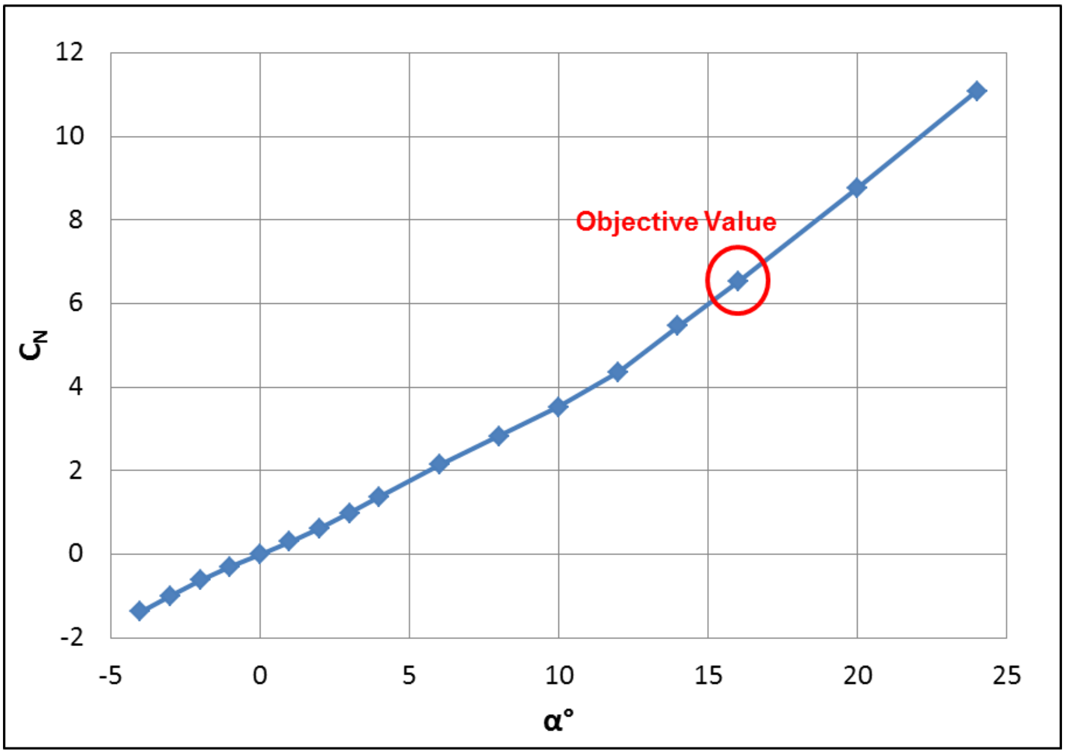


Figure 4.2 Normal Force Coefficient versus Angle of Attack

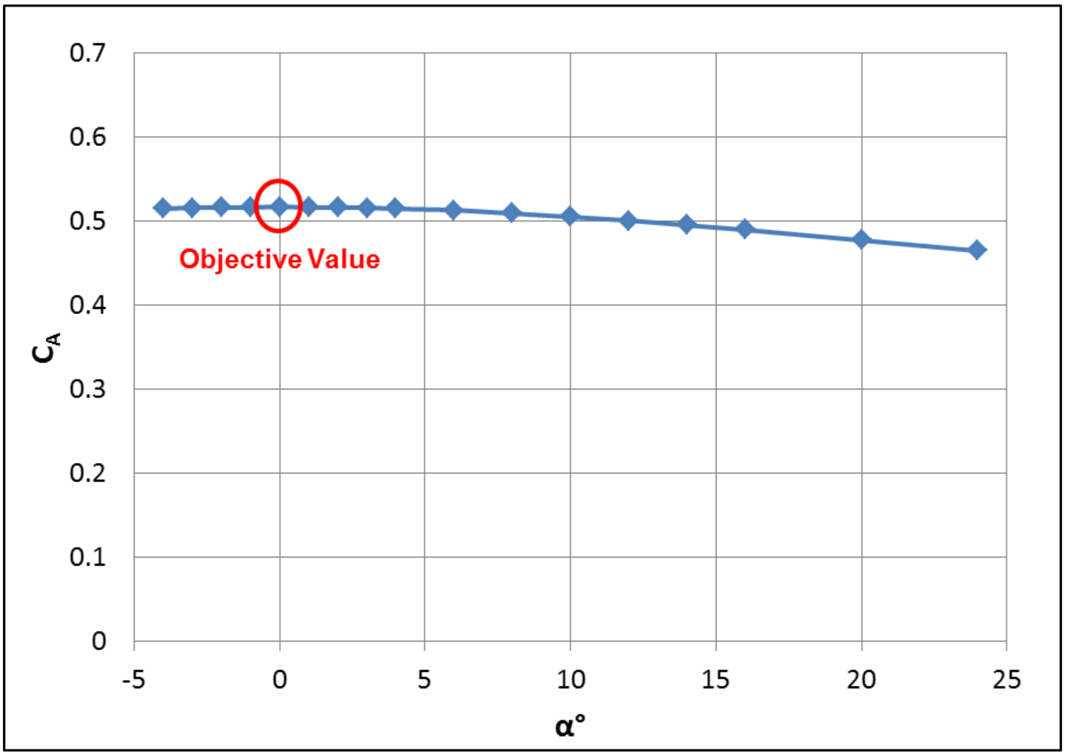


Figure 4.3 Axial Force Coefficient versus Angle of Attack

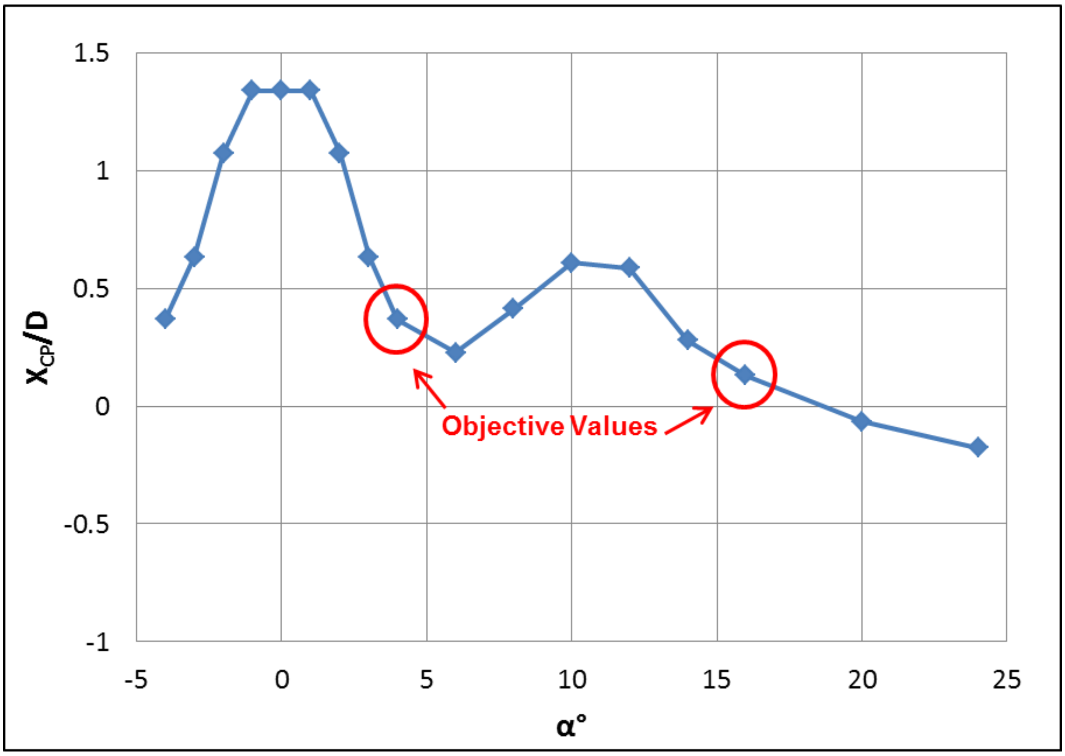


Figure 4.4 Missile Center of Pressure versus Angle of Attack

4.1.3 Missile Design Optimization Study for TCM

4.1.3.1 Objectives, Constraints and Variables of Optimization Study

In missile external configuration optimization study done for TCM, aerodynamic analysis results of TCM B1T4C4 are taken as objectives. Normal force coefficient (C_N) at 16° angle of attack, center of pressure (X_{CP}/D) at 4° and 16° angles of attack in addition to axial force coefficient (C_A) at 0° angle of attack are chosen as design objectives (Table 4.2). Note that these coefficients are calculated for Mach number of 1.75 with zero sideslip angle and at sea level. Results obtained using two different optimization algorithms are discussed in the following sections, separately.

Table 4.2 Objective Aerodynamic Performance Parameters

C_N (α=16°)	X_{CP}/D (α=4°)	X_{CP}/D (α=16°)	C_A (α=0°)
6.532	0.370	0.132	0.517

Design variables in external configuration optimization are defined for canard geometry. These variables are canard leading edge location with respect to the nose tip, semi-span, root chord and taper ratio of canard. Upper and lower limits for these variables and actual values for TCM B1T4C4 configuration are given in Table 4.3.

Table 4.3 Missile Geometry Variables and Constraints

Variable	Description	Lower Limit	Upper Limit	TCM B1T4C4
XLE [cm]	Canard leading edge position	19.0	70.0	20.32
SSPAN [cm]	Canard semi-span	3.0	15.0	5.94
RCHORD [cm]	Canard root chord	5.0	20.0	9.14
TR	Canard taper ratio	0.3	1.0	0.625

It can be seen from Table 4.3 that upper and lower limits are defined in a wide range to investigate capabilities of the developed design optimization methods. A wide range of design variables leads to increased number of possible configurations in the design space; thus, making the optimization problem more difficult. Available area for geometry and placement for canards is shown with pink in Figure 4.5. Dashed lines represent random possible canard geometries that may be encountered during optimization runs.

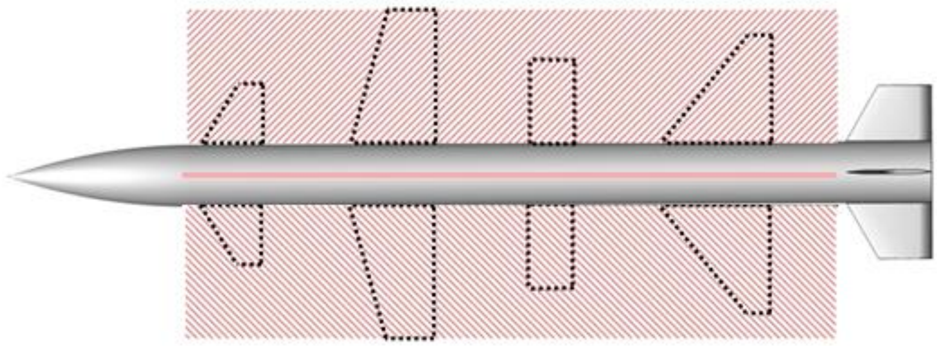


Figure 4.5 Canard Placement Area (red) and Random Possible Canard Geometries (dashed lines)

Objectives for aerodynamic optimization of TCM configuration can be given as;

$$Objective_{C_N} = [(C_{N\,(\alpha=16°)})_{TCM} - (C_{N\,(\alpha=16°)})_{OPT}\,] \, * \, P1 \tag{4.1}$$

$$\begin{aligned} Objective_{X_{CP}/_{D}} &= ([\,(X_{CP}/D_{\,(\alpha=4°)})_{TCM} - (X_{CP}/D_{\,(\alpha=4°)})_{OPT}\,] \\ &+ [\,(X_{CP}/D_{\,(\alpha=16°)})_{TCM} - (X_{CP}/D_{\,(\alpha=16°)})_{OPT}\,]) \\ &* P2 \end{aligned} \tag{4.2}$$

$$Objective_{C_A} = \left[\, C_{A\,(\alpha=0°)_{TCM}} - C_{A\,(\alpha=0°)_{OPT}} \right] \, * \, P3 \tag{4.3}$$

where indices TCM and OPT represent values regarding TCM configuration and optimum configuration, respectively. $P1, P2$ and $P3$ are penalty factors for each aerodynamic coefficient and defined relatively high compared to objective values.

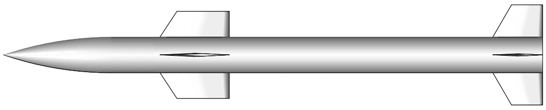
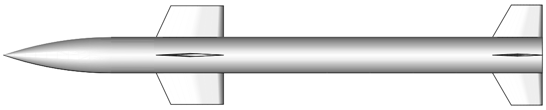
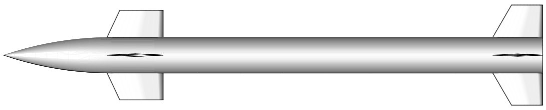
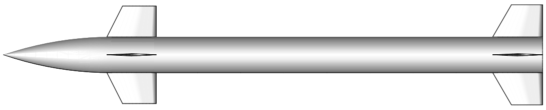
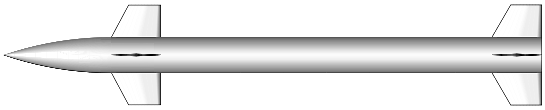
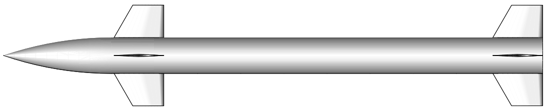
In summary, objective function to be minimized for aerodynamic design optimization is defined as;

$$F_{objective}(XLE, SSPAN, RCHORD, TR) = Objective_{C_N} + Objective_{X_{CP/D}} + Objective_{C_A} \tag{4.4}$$

4.1.3.2 Optimization using DONLP2

In this part of the study, aerodynamic design optimization is done using DONLP2, an algorithm based on Sequential Quadratic Programming optimization method, coupled with aerodynamic analysis tool. Objective aerodynamic parameters and design variable constraints for external geometry optimization case are given in Table 4.2 and Table 4.3, respectively. After 73 iterations in 2926 seconds with 2018 different configuration evaluations using aerodynamic analysis software, design tool determined a configuration that satisfies the objectives of optimization within design constraints. Some configuration geometries along the optimization run are given in Table 4.4. Optimum configuration and TCM B1T4C4 canard geometries are shown in Figure 4.6.

Table 4.4 Change of Configuration Geometry along Optimization

Iteration Step	Configuration Geometry
1	
5	
15	
30	
55	
73 (Optimum Geometry)	

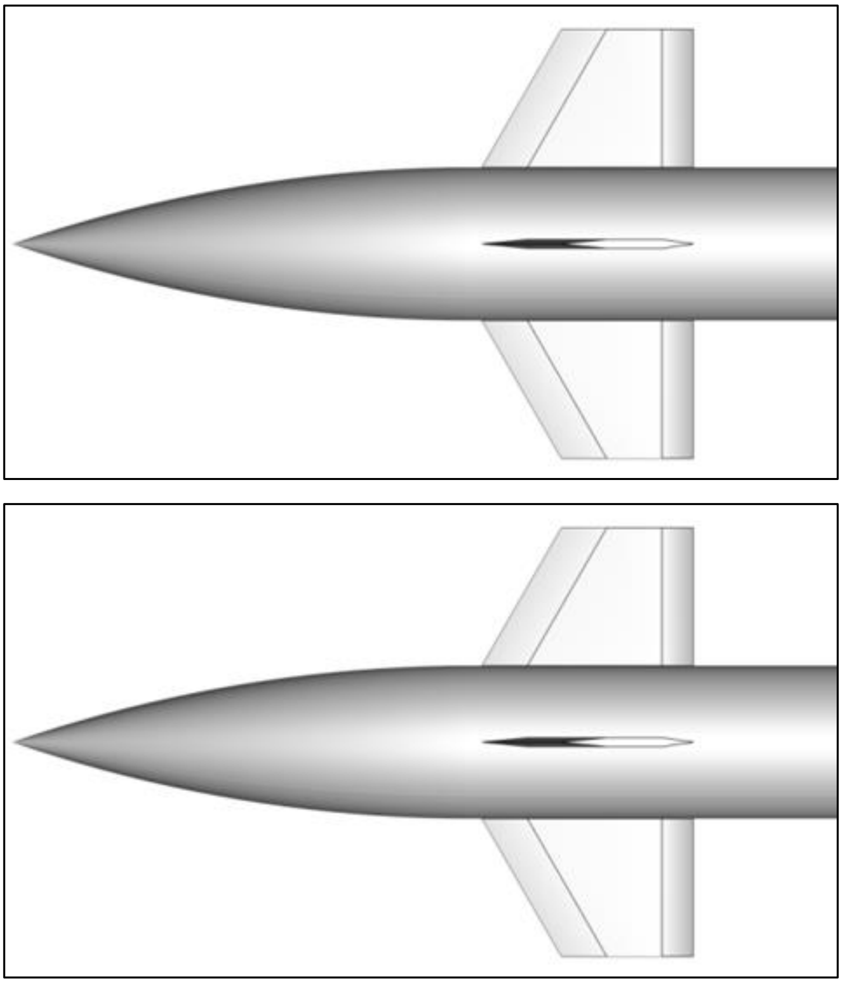


Figure 4.6 Optimum Configuration Geometry (top), TCM B1T4C4 (bottom)

Aerodynamic performance parameters and geometric proportions of optimum configuration and TCM B1T4C4 configuration are given in Table 4.5 and Table 4.6, respectively.

Table 4.5 Optimum and TCM B1T4C4 Configuration (Objective) Aerodynamic Performance Parameters

	Optimum Conf.	TCM-B1T4C4
C_N (α=16°)	6.531	6.532
X_{CP}/D (α=4°)	0.370	0.370
X_{CP}/D (α=16°)	0.132	0.132
C_A (α=0°)	0.517	0.517

Table 4.6 Optimum and TCM B1T4C4 Missile Geometric Proportions

	Optimum Conf.	TCM B1T4C4
XLE [cm]	20.32	20.32
SSPAN [cm]	5.95	5.94
RCHORD [cm]	9.12	9.14
TR	0.62	0.63

Changes of design variables and objective aerodynamic performance parameters along the optimization are shown in below figures (Figure 4.7 - Figure 4.15).

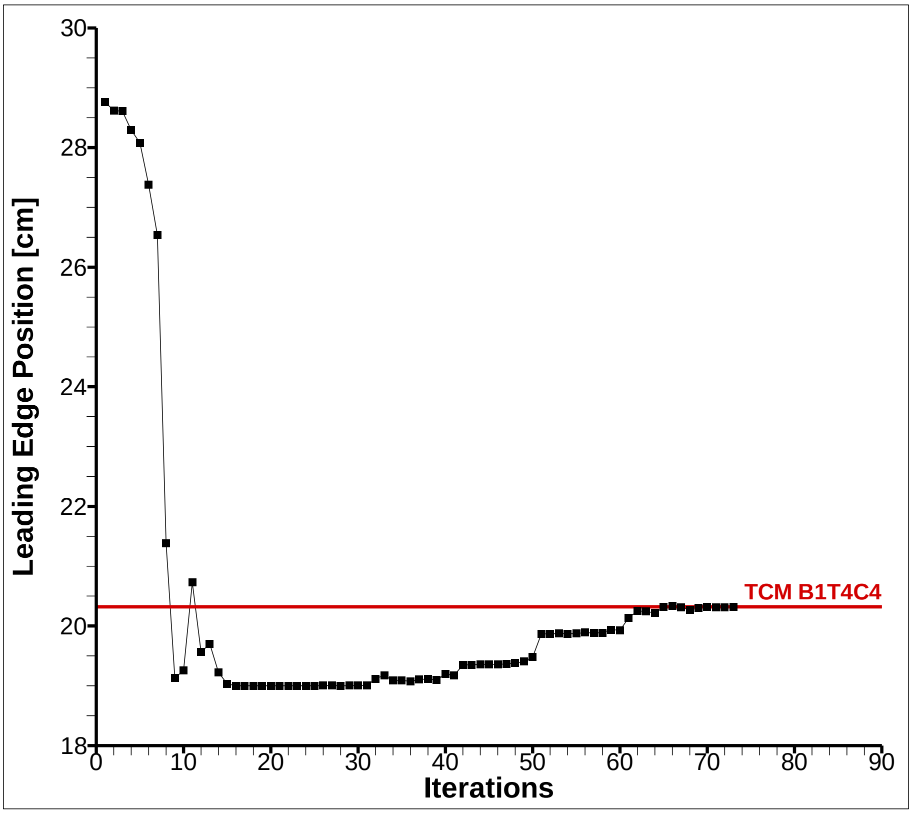


Figure 4.7 Change of Leading Edge Position throughout Optimization

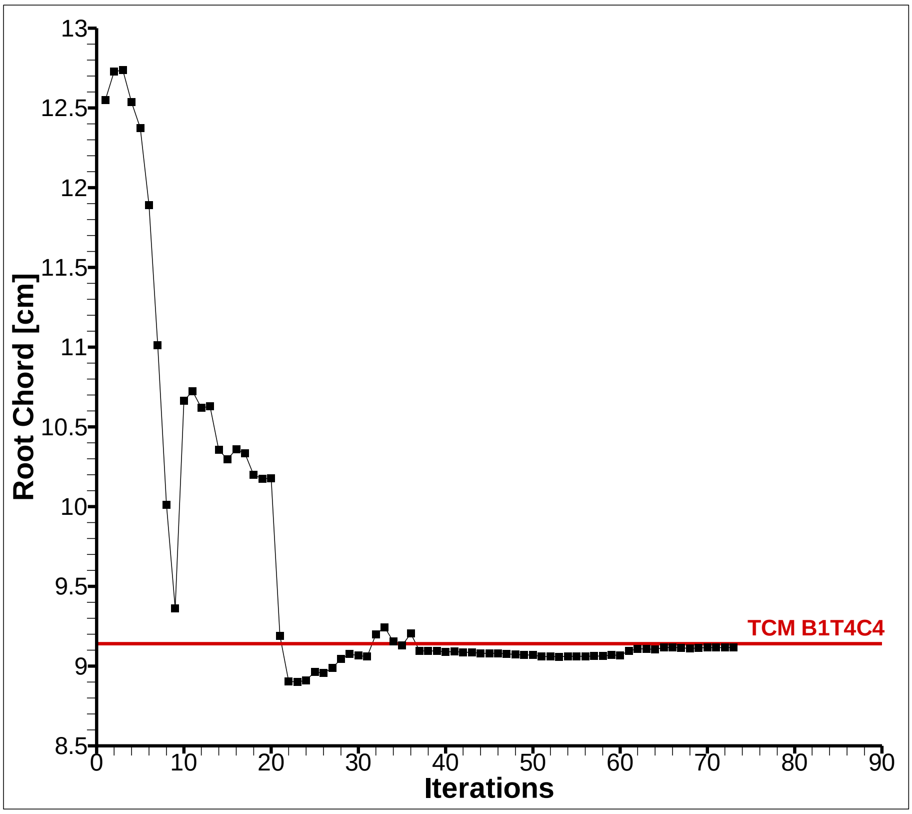


Figure 4.8 Change of Root Chord throughout Optimization

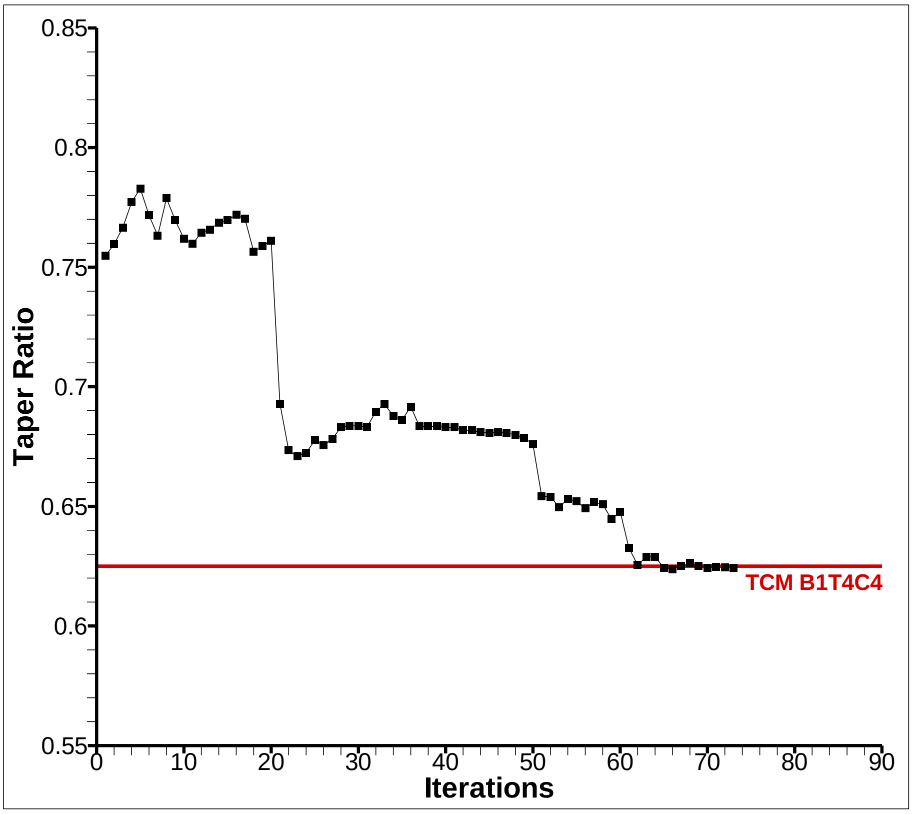


Figure 4.9 Change of Taper Ratio throughout Optimization

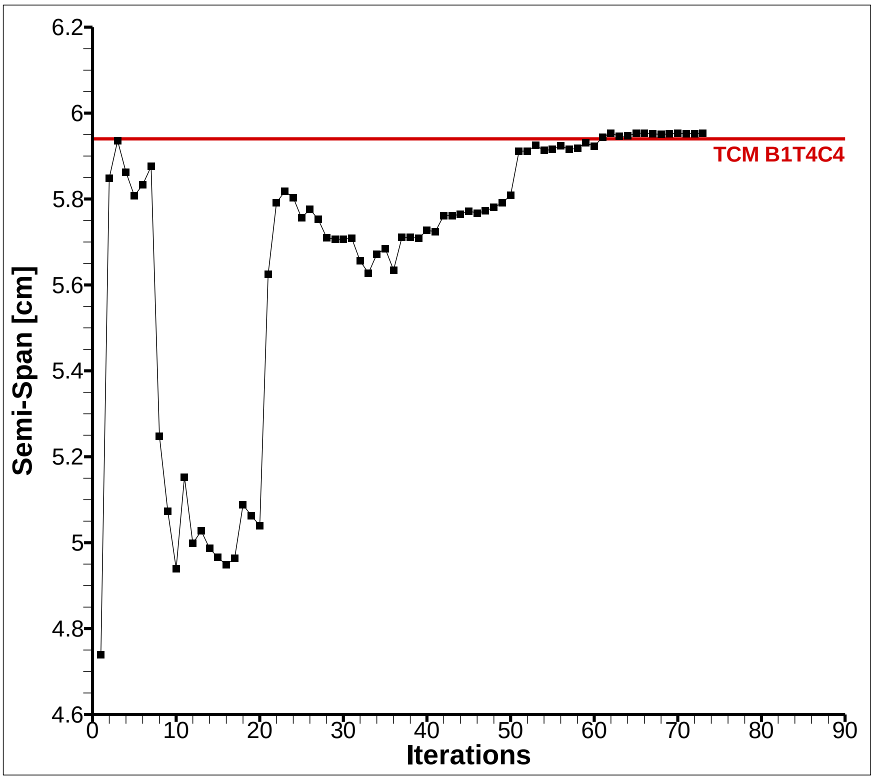


Figure 4.10 Change of Semi-span throughout Optimization

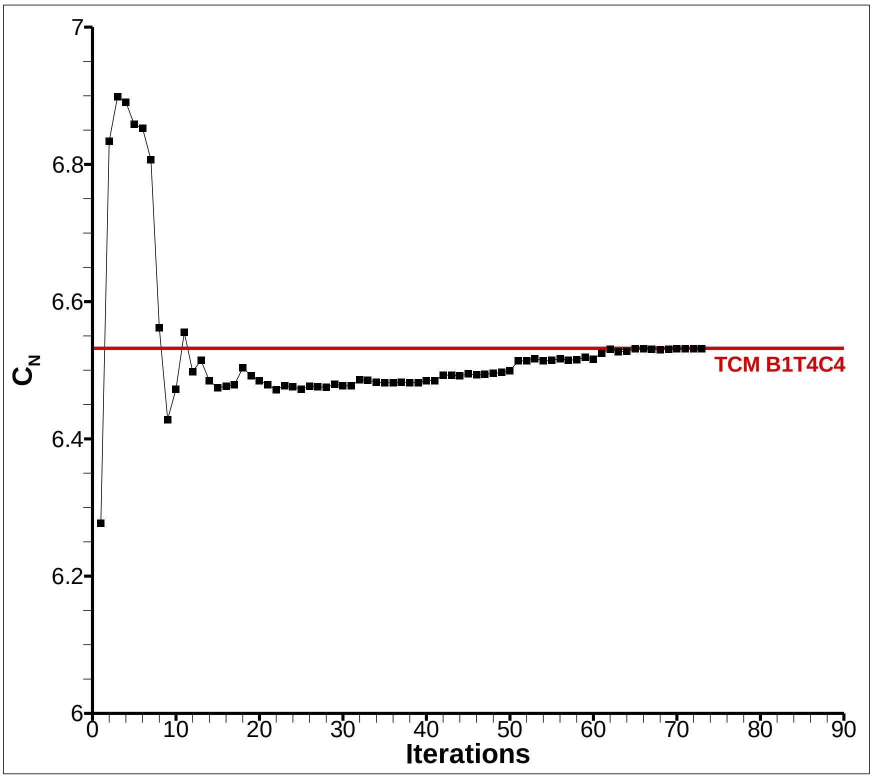


Figure 4.11 Change of Normal Force Coefficient (M=1.75, α=16°) throughout Optimization

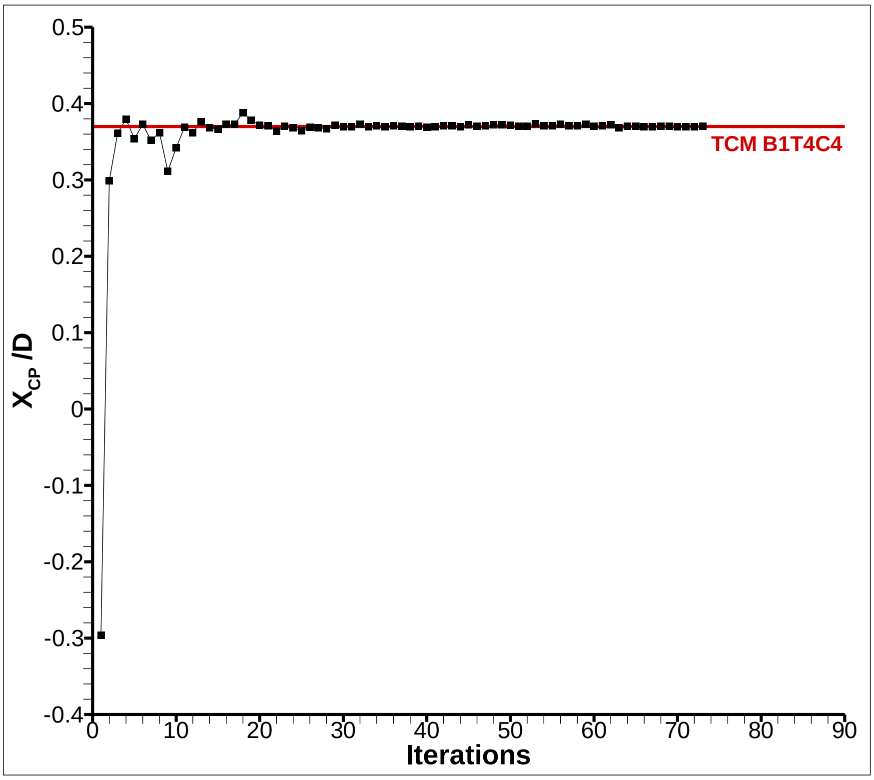


Figure 4.12 Change of Center of Pressure Location (M=1.75, α=4°) throughout Optimization

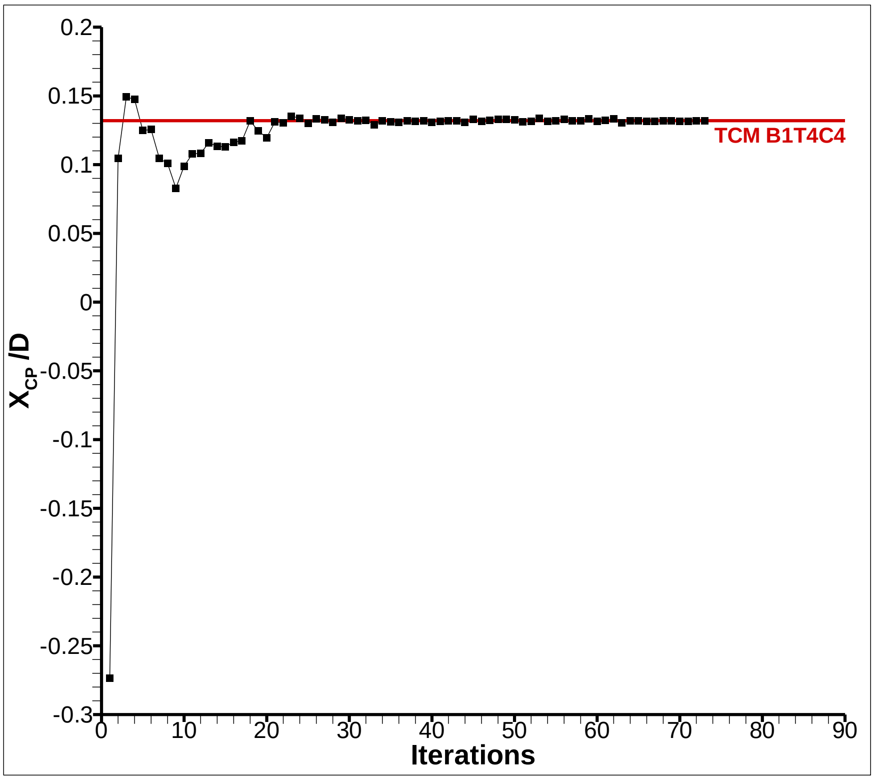


Figure 4.13 Change of Center of Pressure Location (M=1.75, α=16°) throughout Optimization

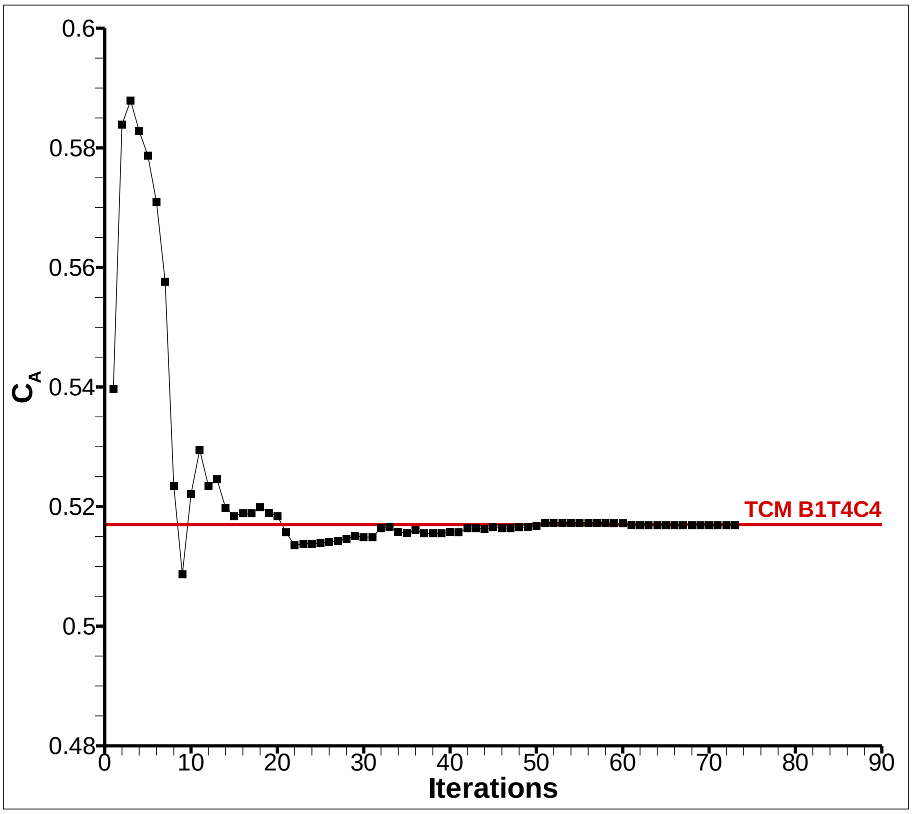


Figure 4.14 Change of Axial Force Coefficient (M=1.75, α=0°) throughout Optimization

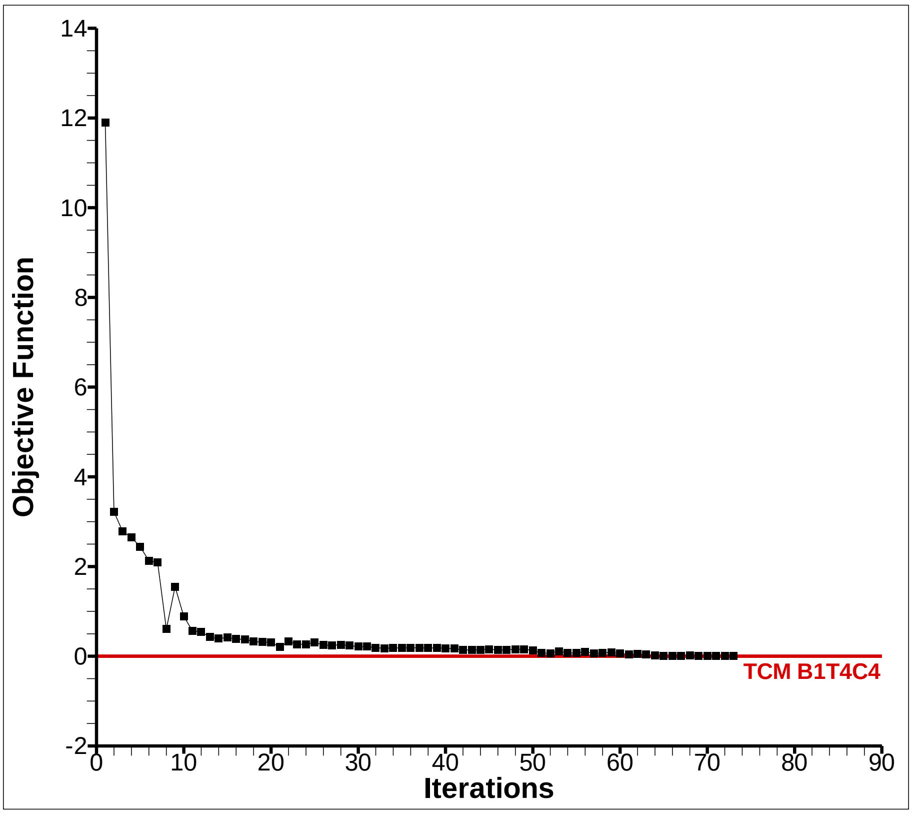


Figure 4.15 Change of Objective Function throughout Optimization

Optimization case study for TCM configuration is solved 15 times to evaluate repeatability of the results. Different initial design points are supplied for these runs. Change of objective function values with iterations for these runs is shown in Figure 4.16. Aerodynamic performance parameters of best and worst configurations according to objective function value are given Table 4.7.

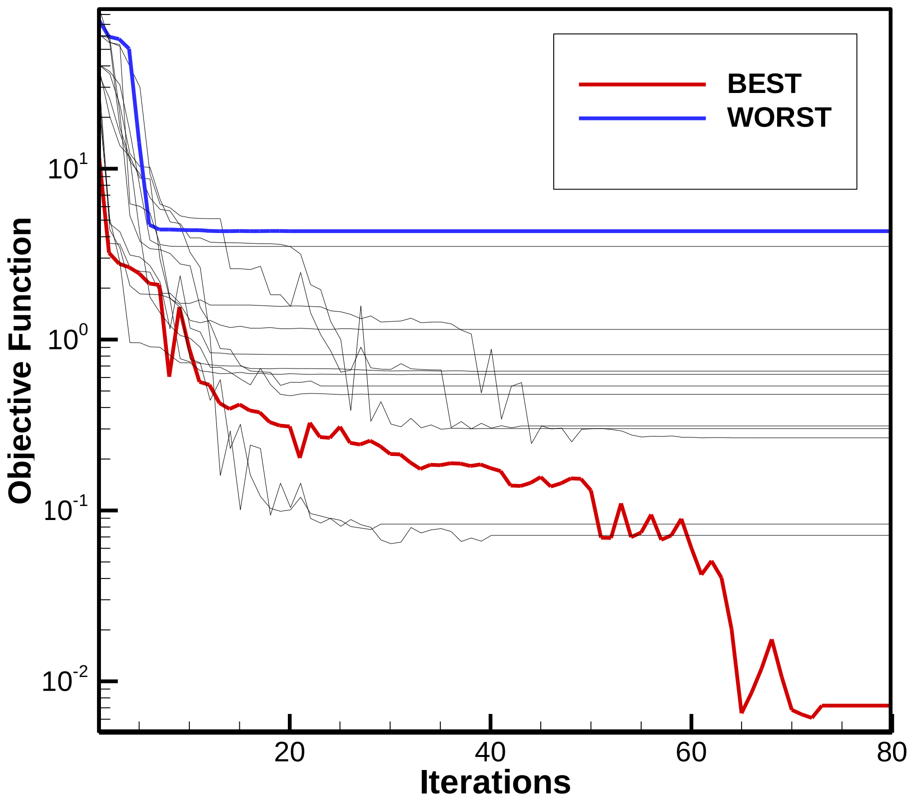


Figure 4.16 Change of Objective Function Values with Iterations

Table 4.7 Aerodynamic Performance Parameters of Best and Worst Configurations

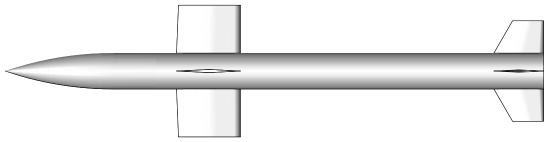
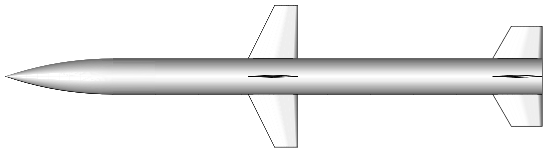
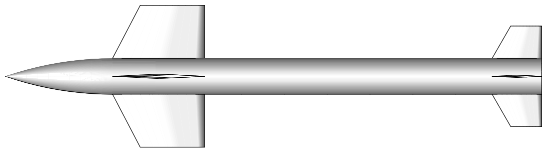
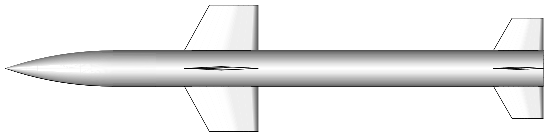
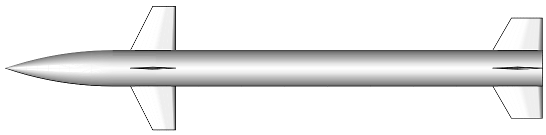
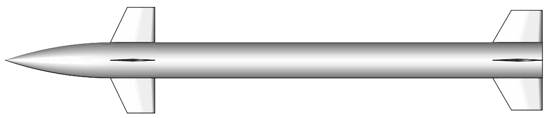
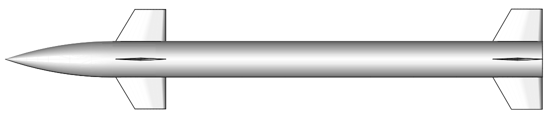
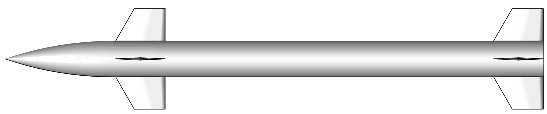
	C_N (α=16°)	X_{CP}/D (α=4°)	X_{CP}/D (α=16°)	C_A (α=0°)	f(x)
Best	6.532	0.370	0.132	0.517	2.999E-04
Worst	6.706	0.001	0.132	0.600	5.873E+00
OBJECTIVE	6.532	0.370	0.132	0.517	0.0

From Figure 4.16, it can be said that there is no guarantee for reaching to objectives of the problem when gradient-based DONLP2 algorithm is used for design optimization. In addition to that, effect of initial points on results is significant. Moreover, there is a considerable difference between best and worst configurations in terms of aerodynamic performance parameters, as seen in Table 4.7. It is necessary to consider these weaknesses of the DONLP2 algorithm for missile aerodynamic optimization problems.

4.1.3.3 Optimization using ACRS

Aerodynamic design optimization study for TCM configuration using Random Search optimization algorithm ACRS is discussed in this section. Objective aerodynamic parameters and design variable constraints for external geometry optimization case are given in Table 4.2 and Table 4.3, respectively. It takes 5539 seconds and 3819 iterations with 3819 different configuration evaluations using Missile DATCOM to determine a configuration that satisfies objectives within design constraints. Some configuration geometries along the optimization run are given in Table 4.8. Optimum configuration and TCM B1T4C4 canard geometries are shown in Figure 4.17.

Table 4.8 Change of Configuration Geometry along Optimization Run

Iteration Step	Configuration Geometry
1	
5	
50	
200	
500	
1000	
2000	
3819 (Optimum Geometry)	

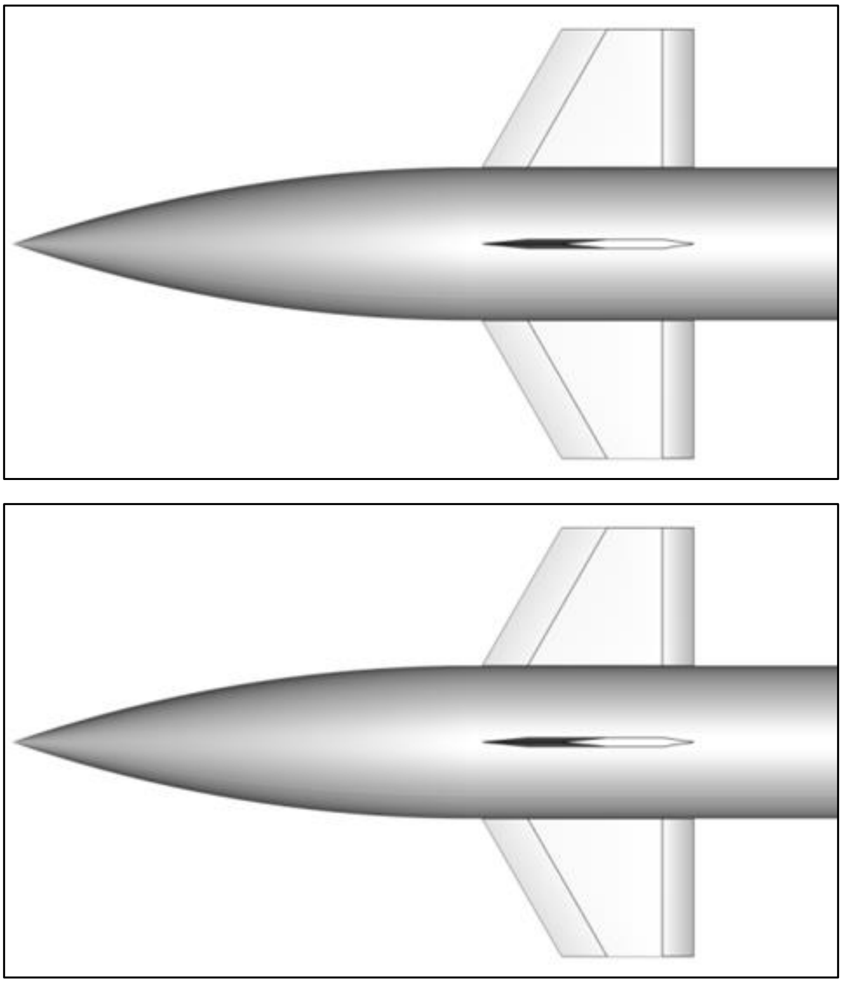


Figure 4.17 Optimum Configuration Geometry (top), TCM B1T4C4 (bottom)

Aerodynamic performance parameters and geometric proportions of optimum configuration and TCM B1T4C4 configuration are given in Table 4.9 and Table 4.10, respectively.

Table 4.9 Optimum and TCM B1T4C4 Configuration (Objective) Aerodynamic Performance Parameters

	Optimum Conf.	TCM-B1T4C4
C_N (α=16°)	6.532	6.532
X_{CP}/D (α=4°)	0.370	0.370
X_{CP}/D (α=16°)	0.132	0.132
C_A (α=0°)	0.517	0.517

Table 4.10 Optimum and TCM B1T4C4 Missile Geometric Proportions

	Optimum Conf.	TCM B1T4C4
XLE [cm]	20.33	20.32
SSPAN [cm]	5.95	5.94
RCHORD [cm]	9.13	9.14
TR	0.62	0.63

Changes of design variables and objective aerodynamic performance parameters along the optimization are shown in below figures (Figure 4.18 - Figure 4.26).

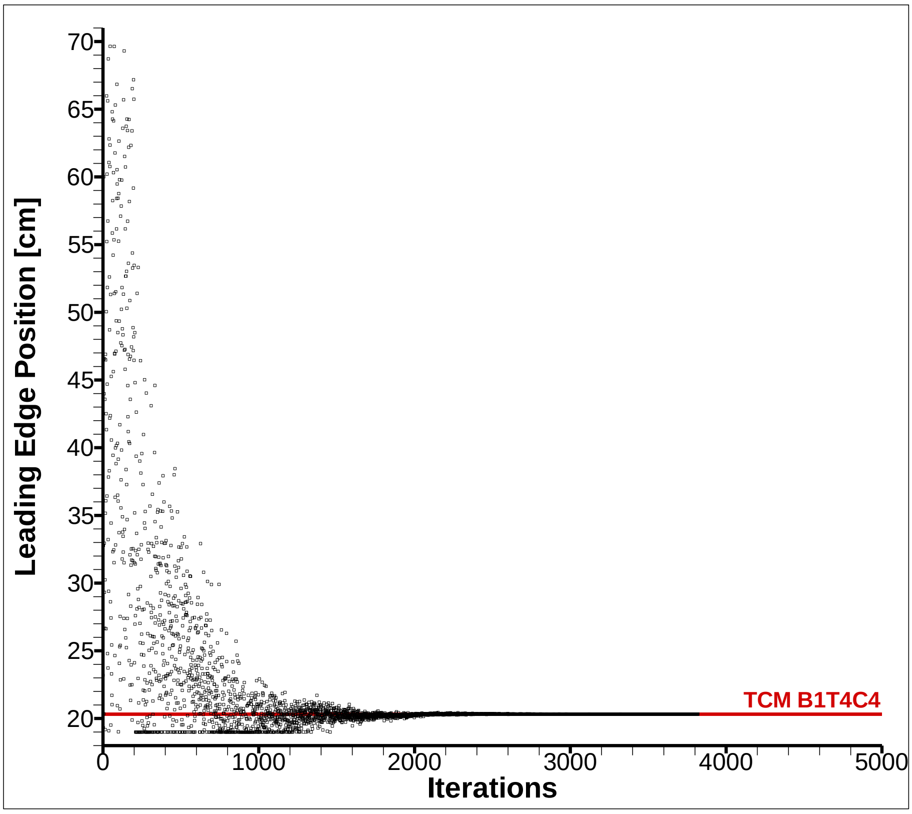


Figure 4.18 Change of Leading Edge Position throughout Optimization

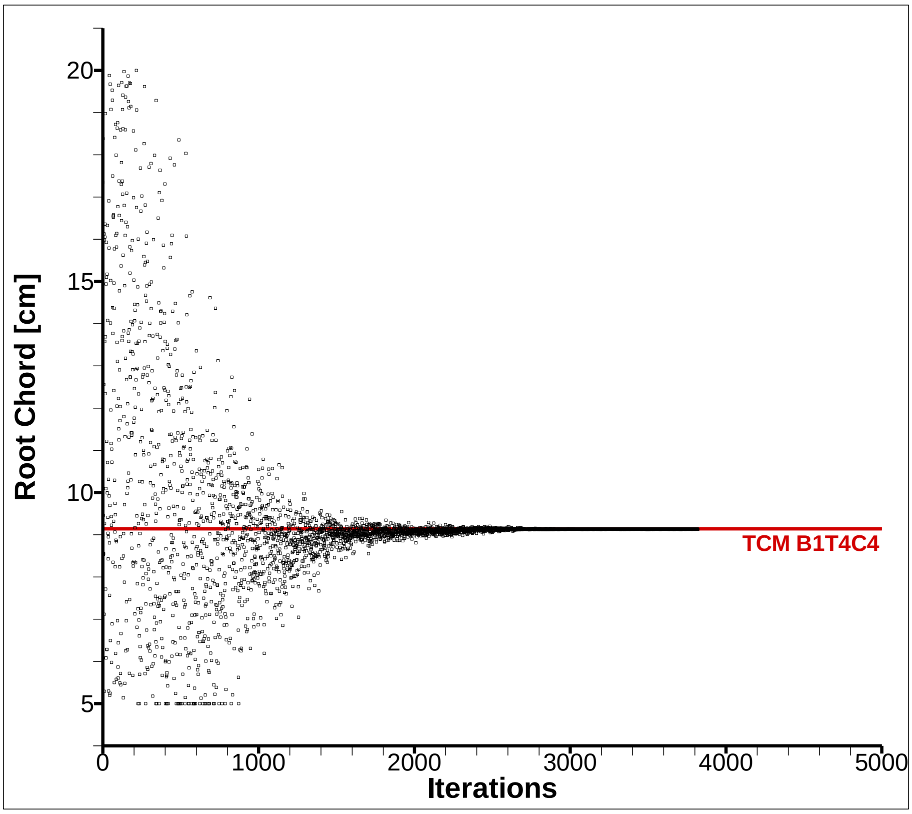


Figure 4.19 Change of Root Chord throughout Optimization

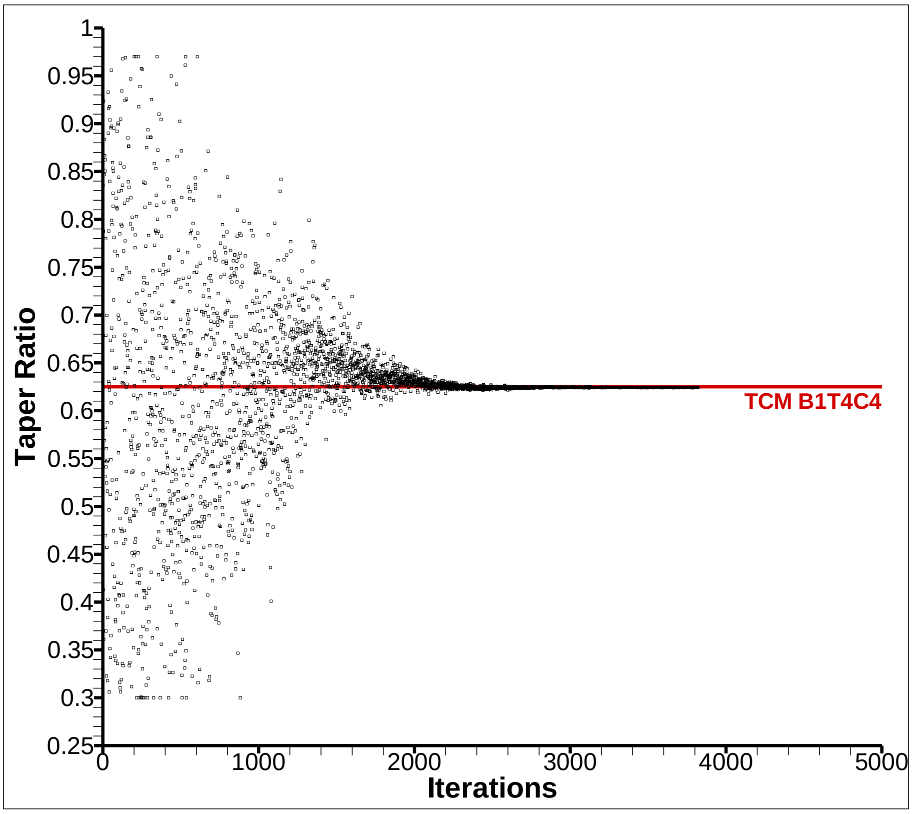


Figure 4.20 Change of Taper Ratio throughout Optimization

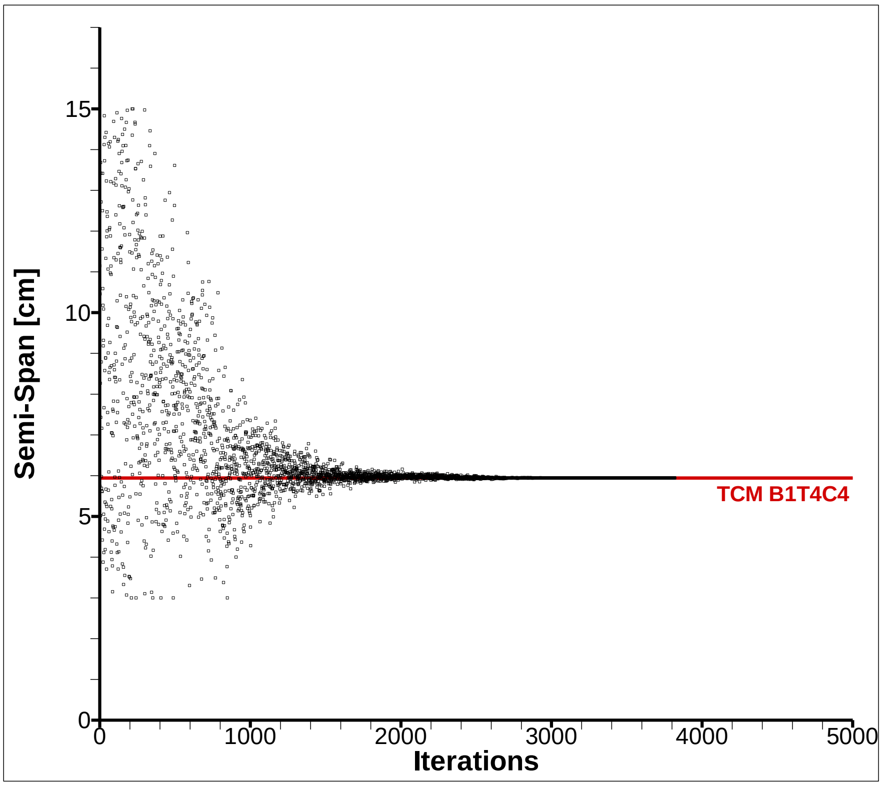


Figure 4.21 Change of Semi-span throughout Optimization

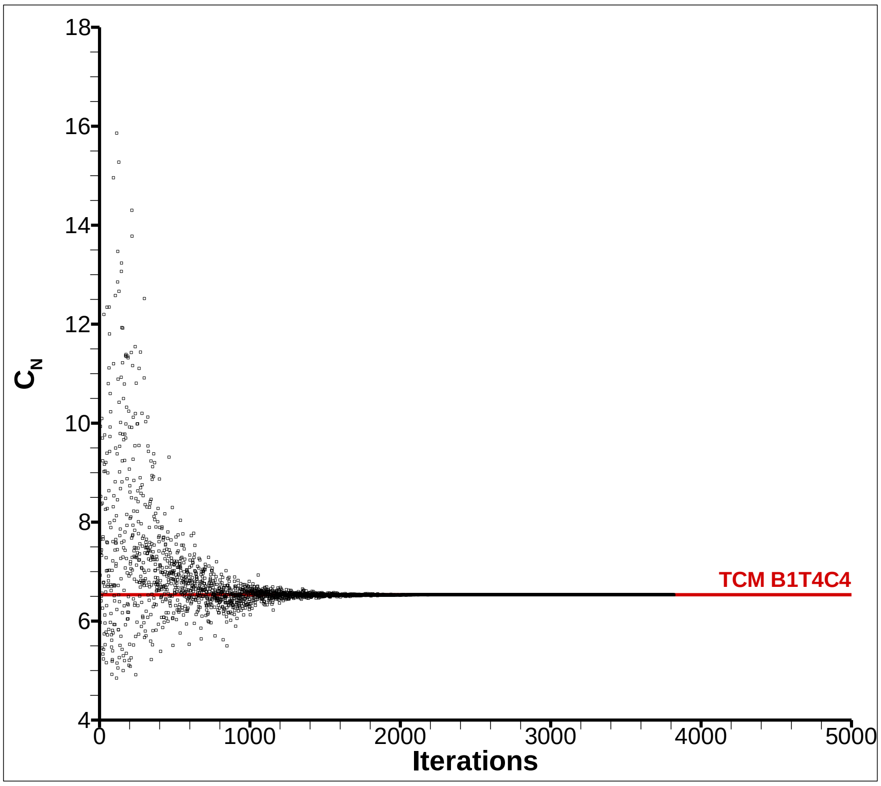


Figure 4.22 Change of Normal Force Coefficient (M=1.75, α=16°) throughout Optimization

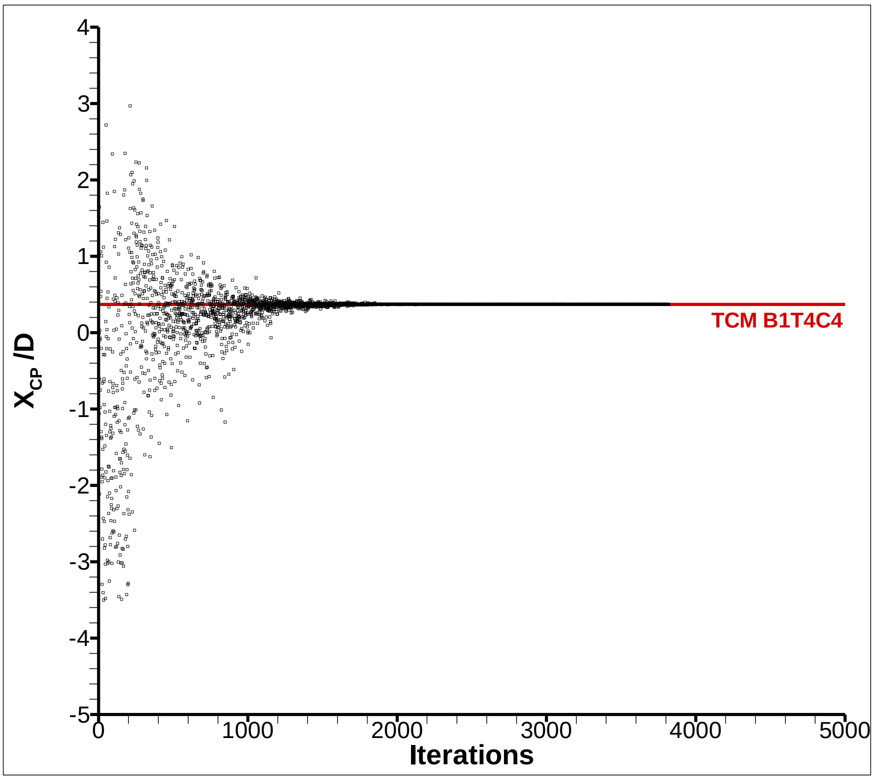


Figure 4.23 Change of Center of Pressure Location (M=1.75, α=4°) throughout Optimization

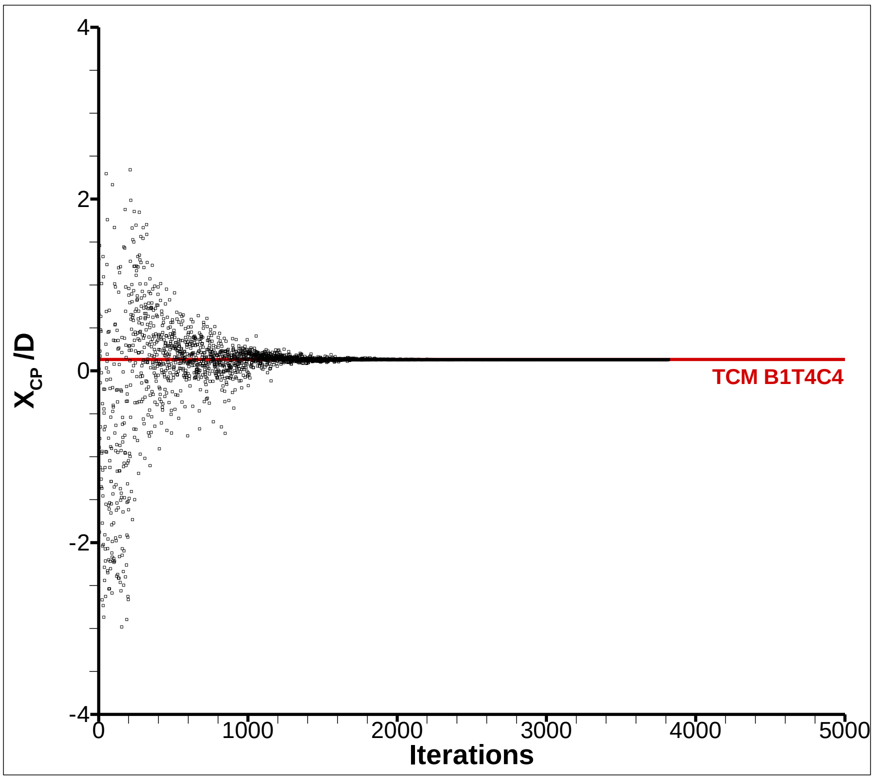


Figure 4.24 Change of Center of Pressure Location (M=1.75, α=16°) throughout Optimization

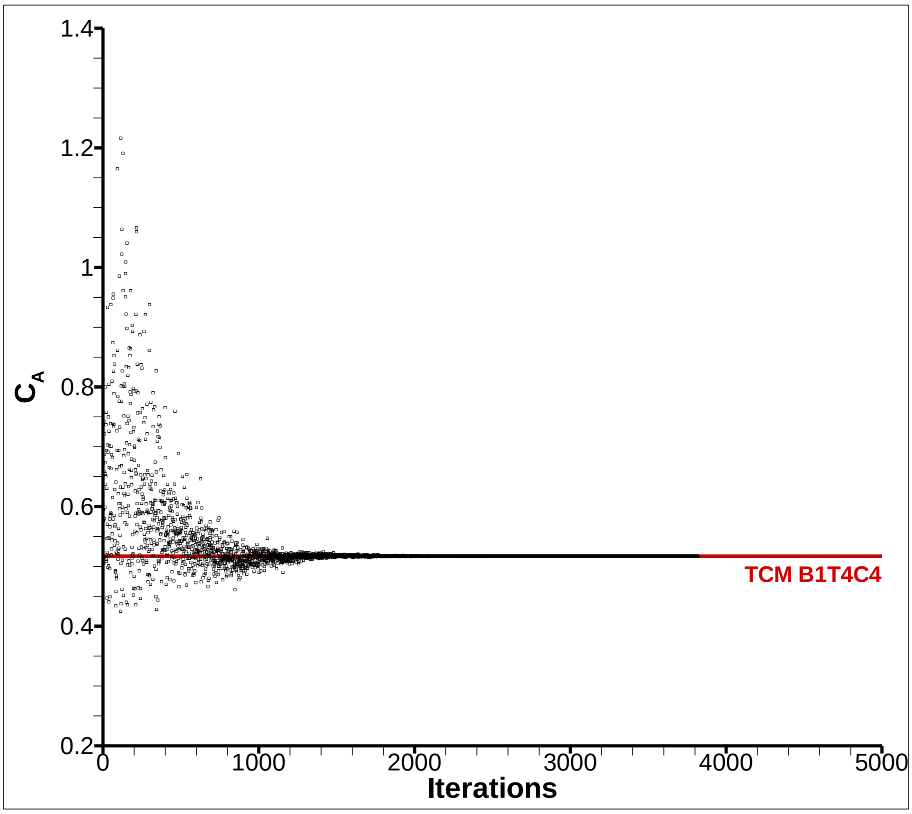


Figure 4.25 Change of Axial Force Coefficient (M=1.75, α=0°) throughout Optimization

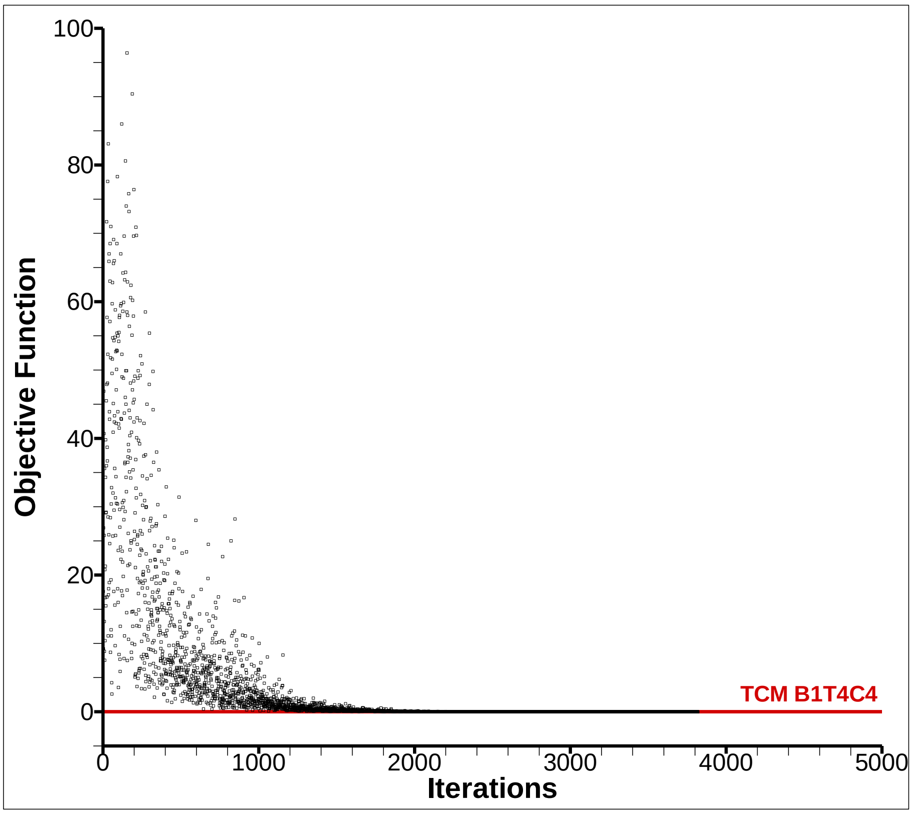


Figure 4.26 Change of Objective Function throughout Optimization

This missile optimization case study considered for TCM configuration is solved 15 times to evaluate repeatability of the results. Change of objective function values with iterations for these runs is shown in Figure 4.27 and Figure 4.28. Aerodynamic performance parameters of best and worst configurations according to objective function value are given Table 4.11.

Table 4.11 Aerodynamic Performance Parameters of Best and Worst Configurations

	C_N (α=16°)	X_{CP}/D (α=4°)	X_{CP}/D (α=16°)	C_A (α=0°)	f(x)
Best	6.532	0.370	0.132	0.517	1.215E-06
Worst	6.540	0.360	0.149	0.517	2.934E-01
OBJECTIVE	6.532	0.370	0.132	0.517	0.0

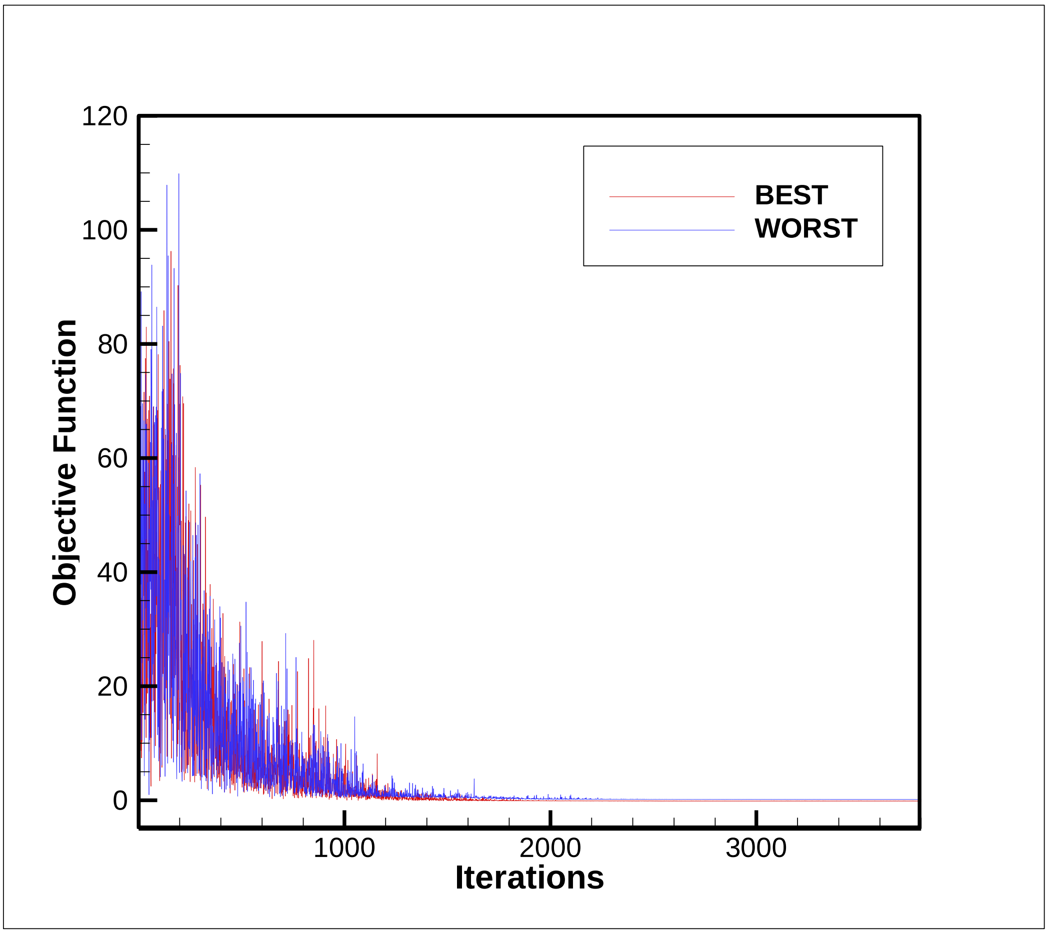


Figure 4.27 Change of Objective Function Values with Iterations

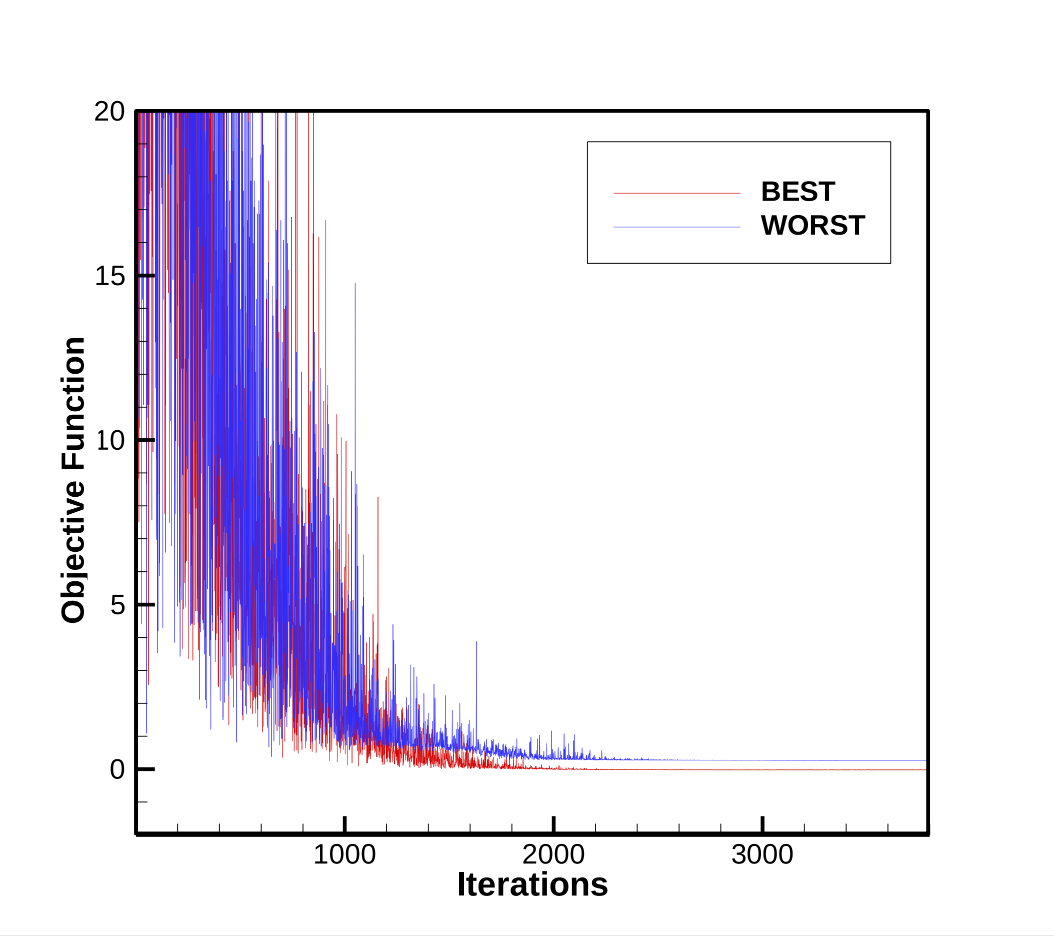


Figure 4.28 Change of Objective Function Values with Iterations (y axis is narrowed)

From Figure 4.27 and Figure 4.28, it can be seen that all of the 15 runs reach aerodynamic design objectives in the end. Accordingly, there is a slight difference between best and worst configurations in terms of aerodynamic performance parameters, as seen in Table 4.11. Moreover, ACRS algorithm does not require user defined initial points for optimization runs; therefore, there is no effect of starting point on results. Note that, DONLP2 reaches the best solution in 2018 configuration evaluations whereas ACRS algorithm evaluate 3819 configurations for best solution case, meaning that ACRS is almost twice slower than DONLP2. In the end, all of these results suggest that ACRS algorithm is more robust but slower in achieving design objectives, when compared to DONLP2.

4.2 Aerodynamic Shape Optimization for a Generic Air-to-Ground Missile (AGM)

4.2.1 Missile Design Optimization for AGM

An aerodynamic optimization case study is carried out for a generic air-to-ground missile (AGM) using developed optimization method. Both optimization algorithms, DONLP2 and ACRS, are used for this purpose. Results are presented separately.

4.2.1.1 Objectives, Constraints and Variables of Optimization Study

It is assumed that body of the AGM under consideration has a fixed geometry due to sub-system components such as rocket engine, seeker, warhead, control surface actuators etc. and missile is tail-controlled. It is also assumed that weight is constant and center of gravity does not change. Wing and tail geometries are taken as design variables. Geometric constraints defined for these design variables are given in Table 4.12.

Table 4.12 Missile Geometry Variables and Limits

Variable	Description	Lower Limit	Upper Limit
XLE1 [cm]	Wing leading edge position	30.0	120.0
RCHORD1 [cm]	Wing root chord	4.0	30.0
TR1	Wing taper ratio	0.0	1.0
SSPAN 1[cm]	Wing semi-span	3.0	16.0
XLE2[cm]	Tail leading edge position	130.0	160.0
RCHORD2 [cm]	Tail root chord	5.0	30.0
TR2	Tail taper ratio	0.0	1.0
SSPAN2[cm]	Tail semi-span	3.0	16.0

In missile external configuration optimization study for a generic AGM configuration, typical aerodynamic performance requirements and the flight regime for this type of mission are taken into account. Aerodynamic performance parameters such as load factor and lift-to-drag ratio at trim condition, control effectiveness, static margin and hinge moments are defined as objectives and constraints. These parameters are defined considering maneuverability, range, stability and weight requirements of a typical AGM and summarized in Table 4.13.

Table 4.13 Aerodynamic Performance Parameters for Sample AGM

Aerodynamic Parameter	Constraint	Objective
Trim Load Factor (n_{trim})	$n_{trim} > 3.5$	Maximize n_{trim}
Trim Lift-to-Drag Ratio ($C_L/C_{D\ trim}$)	$C_L/C_{D\ trim} > 2.5$	Maximize $C_L/C_{D\ trim}$
Static Margin (SM)	-1.0 > SM > -1.5	-
Control Effectiveness (CE)	-0.5 > CE > -0.8	-
Hinge Moment (HM)	|HM| < 2 Nm	Minimize HM

Flight regime defined for AGM external optimization case study is given in Table 4.14.

Table 4.14 Flight Regime for AGM Optimization Case Study

Mach	0.1 < Mach < 0.8
Angle of Attack	$-15° \leq \alpha \leq 15°$
Sideslip Angle	$\beta = 0°$
Altitude	Sea Level

In summary, objectives and constraints given in Table 4.13 for AGM aerodynamic optimization study is defined mathematically as;

$$Objective_{n_{trim}} = \left(\frac{3.5}{max(n_{trim})}\right)^2 * K_1 \quad (4.5)$$

$$Objective_{C_L/C_{D\,trim}} = \left(\frac{2.5}{max(C_L/C_{D\,trim})}\right)^2 * K_2 \quad (4.6)$$

$$Objective_{HM} = (\max(HM))^2 * K_3 \quad (4.7)$$

where K_1, K_2 and K_3 are relatively high numbers used to increase weight of the objectives. Constraints given in Table 4.13 are defined with penalty parameters,

$$Penalty_{n_{trim}} = [3.5 - max(n_{trim})] * P, \quad if\ max(n_{trim}) < 3.5 \quad (4.8)$$

$$Penalty_{C_L/C_{D\,trim}} = \left[2.5 - max\left(C_L/C_{D\,trim}\right)\right] * P, \ \ if\ max\left(C_L/C_{D\,trim}\right) < 2.5 \quad (4.9)$$

$$Penalty_{HM} = [|2.0 - max(|HM|)|] * P, \quad if\ max(|HM|) > 2.0 \quad (4.10)$$

$$Penalty_{SM} = \begin{cases} |\max(SM) + 1.0| * P, & if\ \max(SM) > -1.0 \\ |1.5 + \min(SM)\,| * P, & if\ \min(SM) < -1.5 \end{cases} \quad (4.11)$$

$$Penalty_{CE} = \begin{cases} |\max(CE) + 0.5| * P, & if\ \max(CE) > -0.5 \\ |0.8 + \min(CE)\,| * P, & if\ \min(CE) < -0.8 \end{cases} \quad (4.12)$$

where P is a relatively high number used to increase weight of penalty functions.

Maximum and minimum values for aerodynamic design parameters are determined using aerodynamic analysis tool for the flight regime given in Table 4.14. In this way, it is guaranteed that a search is done for an optimum configuration that satisfies aerodynamic objectives inside the given flight envelope and constraints are taken into account for the entire flight regime.

Finally, objective function to be minimized for aerodynamic design optimization is written as;

$$\begin{aligned} F_{objective}\begin{pmatrix} XLE1, RCHORD1, TR1, SSPAN1, \\ XLE2, RCHORD2, TR2, SSPAN2 \end{pmatrix} \\ = Objective_{n_{trim}} + Objective_{C_L/C_{D_{trim}}} \\ + Objective_{HM} + Penalty_{n_{trim}} + Penalty_{C_L/C_{D_{trim}}} \\ + Penalty_{HM} + Penalty_{SM} + Penalty_{CE} \end{aligned} \tag{4.13}$$

Using above objective function, aerodynamic shape optimization for sample AGM is studied. Results obtained using two different optimization algorithms are discussed separately in the following sections.

4.2.1.2 Optimization using DONLP2

Aerodynamic design optimization case study using Random Search optimization algorithm DONLP2 coupled with aerodynamic analysis tool is discussed in this section. Design variable constraints and objective aerodynamic parameters for external geometry optimization case are given in Table 4.12 and Table 4.13, respectively. After 29 iterations in 718 seconds with 1709 aerodynamic analyses for candidate configurations, an optimum configuration that satisfies the objectives of optimization within design constraints is determined. Aerodynamic performance parameters and geometric proportions of the optimum configuration are given in Table 4.15 and Table 4.16, respectively. Change of AGM geometry along optimization process is presented in Table 4.17.

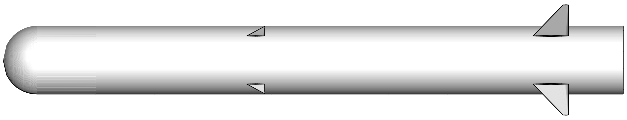
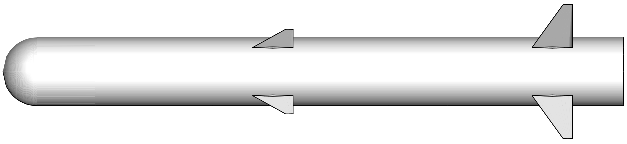
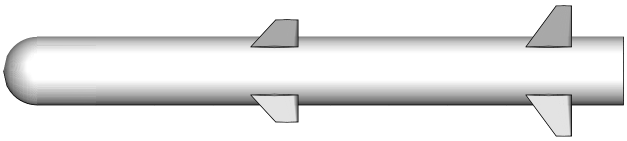
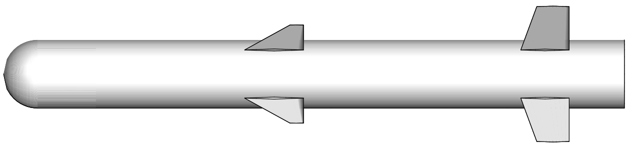
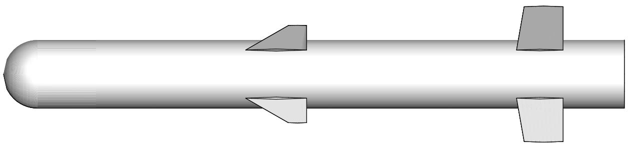
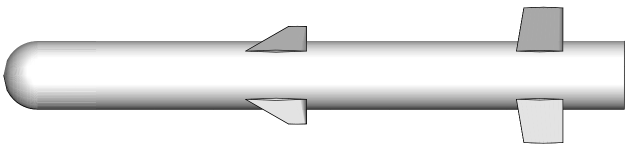
Table 4.15 Optimum AGM and Objective Aerodynamic Performance Parameters

Aerodynamic Parameter	Constraint	Objective	**Optimum AGM Configuration**
n_{trim}	$n_{trim} > 3.5$	Maximize n_{trim}	$(n_{trim})_{max} = 3.69$ g
$C_L/C_{D\ trim}$	$C_L/C_{D\ trim} > 2.5$	Maximize $C_L/C_{D\ trim}$	$(C_L/C_{D\ trim})_{max} = 3.16$
SM	-1.0 > SM > -1.5	-	-1.10 > SM > -1.48
CE	-0.5 > CE > -0.8	-	-0.50 > CE > -0.64
HM	|HM| < 2 Nm	Minimize HM	|HM| < 0.35 Nm

Table 4.16 Optimum AGM Geometric Proportions

Variable	Description	Lower Limit	Upper Limit	**Optimum AGM Configuration**
XLE1 [cm]	Wing leading edge position	30.0	120.0	63.55
RCHORD1 [cm]	Wing root chord	4.0	30.0	16.0
TR1	Wing taper ratio	0.0	1.0	0.30
SSPAN 1[cm]	Wing semi-span	3.0	16.0	9.1
XLE2[cm]	Tail leading edge position	130.0	160.0	134.7
RCHORD2 [cm]	Tail root chord	5.0	30.0	12.2
TR2	Tail taper ratio	0.0	1.0	0.84
SSPAN2[cm]	Tail semi-span	3.0	16.0	16.0

Table 4.17 Change of AGM Configuration Geometry along Optimization Run

Iteration	Configuration Geometry
1	
2	
5	
10	
20	
29 (Optimum Configuration)	

4.2.1.3 Optimization using ACRS

Aerodynamic design optimization case study using Random Search optimization algorithm ACRS coupled with aerodynamic analysis tool is discussed in this section. Design variable constraints and objective aerodynamic parameters for external geometry optimization case are given in Table 4.12 and Table 4.13, respectively. After 5678 iterations in 2328 seconds, an optimum configuration that satisfies objectives of optimization within design constraints is determined. Aerodynamic performance parameters and geometric proportions of optimum configuration are given in Table 4.18 and Table 4.19, respectively. Change of AGM geometry along optimization process is presented in Table 4.20.

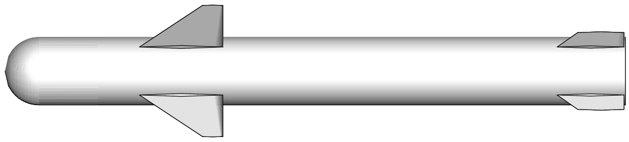
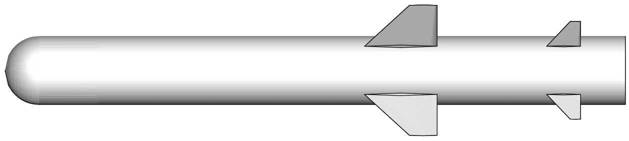
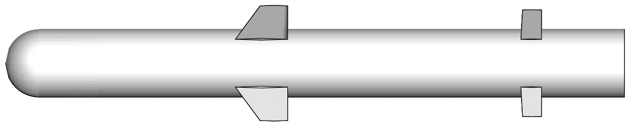
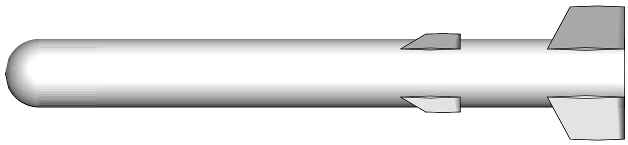
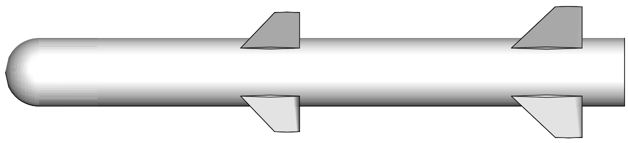
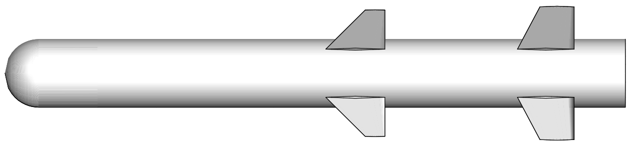
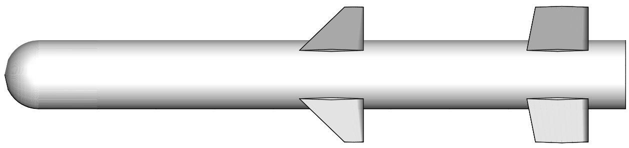
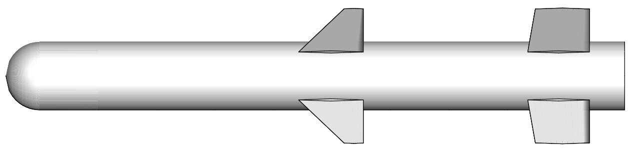
Table 4.18 Optimum AGM and Objective Aerodynamic Performance Parameters

Aerodynamic Parameter	Constraint	Objective	**Optimum AGM Configuration**
n_{trim}	$n_{trim} > 3.5$	Maximize n_{trim}	$(n_{trim})_{max} = 4.52$ g
$C_L/C_{D\ trim}$	$C_L/C_{D\ trim} > 2.5$	Maximize $C_L/C_{D\ trim}$	$(C_L/C_{D\ trim})_{max} = 3.2$
SM	-1.0 > SM > -1.5	-	-1.07 > SM > -1.46
CE	-0.5 > CE > -0.8	-	-0.50 > CE > -0.68
HM	|HM| < 2 Nm	Minimize HM	|HM| < 0.64 Nm

Table 4.19 Optimum AGM Geometric Proportions

Variable	Description	Lower Limit	Upper Limit	**Optimum AGM Configuration**
XLE1 [cm]	Wing leading edge position	30.0	120.0	77.14
RCHORD1 [cm]	Wing root chord	4.0	30.0	17.0
TR1	Wing taper ratio	0.0	1.0	0.30
SSPAN 1[cm]	Wing semi-span	3.0	16.0	16.0
XLE2[cm]	Tail leading edge position	130.0	160.0	137.5
RCHORD2 [cm]	Tail root chord	5.0	30.0	16.3
TR2	Tail taper ratio	0.0	1.0	0.88
SSPAN2[cm]	Tail semi-span	3.0	16.0	15.9

Table 4.20 Change of AGM Configuration Geometry along Optimization Run

Iteration	Configuration Geometry
1	
5	
50	
200	
500	
1000	
3000	
5678 (Optimum Geometry)	

4.2.1.4 Discussion of Results

In preceding sections, results for a sample AGM optimization case study are presented. Evolution of the AGM configuration to the optimum aerodynamic missile configuration can be seen in Table 4.17 for optimization with DONLP2 and Table 4.20 for optimization with ACRS. Table 4.15 and Table 4.18 show that both of the algorithms reach objectives and satisfy constraints. Nevertheless, a slightly better optimum configuration is obtained using ACRS in terms of final objective function value. 5678 different configurations are evaluated when ACRS algorithm is used for optimization, whereas evaluation of 1709 configurations is enough for DONLP2 algorithm to reach the optimum configuration. This is consistent with the results obtained for TCM study, discussed in the previous sections. Note that, repeatability of the results especially for the ones obtained using DONLP2 algorithm should be questioned as done previously.

Results obtained from different optimization approaches suggest similar outcomes in missile aerodynamic design point of view. Since defined aerodynamic performance objectives and constraints conflict with each other, optimization methodology seeks a balance between them to minimize the objective function. For example, areas of the wings and control surfaces (tails) increase in the end since trim load factor objective imposes a requirement as maximization of normal force of the configuration. However, due to constraints on static margin and control effectiveness, increase of wing and tail areas are limited. Lift-to-drag ratio objective has an impact on wing and tail layout, leading to some taper especially for wings. In addition to these, tail geometry and location is affected by hinge moment objective. As a result, a low tapered tail geometry that minimizes center of pressure shift is obtained for optimum configurations.

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

In this thesis, an aerodynamic design optimization methodology is developed to determine optimum external configurations of missiles. Two different optimization algorithms are employed for the development of an optimization procedure. One of the optimization approaches used is Sequential Quadratic Programming (SQP), a state-of-the-art gradient-based optimization method. An SQP algorithm known as DONLP2 is applied to the design optimization problem. In addition to that, Adaptively Controlled Random Search (ACRS) algorithm is also considered for the investigation of optimum aerodynamic missile configuration. An aerodynamic coefficient fast prediction tool based on component build-up methods, Missile DATCOM, is used for the aerodynamic analysis of missile configurations.

A verification study on a global optimization test function known as Goldstein-Price function is done to investigate the capabilities of the employed optimization algorithms. Results suggest that SQP method has difficulties on locating global optimum point in a complex search space. Due to the nature of the method, it may get stuck in local minima when solutions are repeated for different initial points. In contrast, Random Search (RS) approach is fairly good at determining global minimum. For repeated solutions to Goldstein-Price test function, RS method always converged to the global minimum. Accordingly, it can be concluded that RS approach is more suitable for design optimization problems where the solution space is considerably complex. In addition to these verification studies, a validation for the employed aerodynamic analysis tool, Missile DATCOM, is carried out. It can be understood that Missile DATCOM is sufficiently accurate for aerodynamic coefficient prediction and has acceptable accuracy to be used in developed optimization method.

To present the capabilities of the developed methodology on missile aerodynamic optimization, two different optimization applications are considered. First, a study with reverse engineering approach is discussed where NASA Tandem Control Missile (TCM) is taken as baseline configuration. For SQP and RS approaches this problem is solved separately. In the end, both optimization methods are able to reach aerodynamic objectives that yield the original geometry of TCM configuration. However, for repeated solutions, it is determined that RS approach is superior to SQP in terms of reaching the optimization objectives. The effect of the initial point on final solution and difficulties on reaching the objectives of the problem are the identified drawbacks of the SQP method. Even though SQP is faster to reach the final solution, it is hard to rely on obtaining optimum results by this method. In expense of some amount of solution time, RS approach work fairly good for aerodynamic optimization of missile configurations. As a second aerodynamic optimization application, optimization of a generic Air-to-Ground Missile (AGM) configuration is discussed. Again, both optimization methods are used to determine the optimum configuration. Results yield that ACRS algorithm determined a better aerodynamic missile configuration in terms of aerodynamic performance objectives. Note that DONLP2 algorithm is almost 4 times faster to reach the solution in this case.

For future studies, application of developed aerodynamic design optimization methodology to different types of missiles can be done. For example, an existing air-to-air missile configuration (e.g. AIM-9X) can be resized for improved aerodynamic performance with additional design variables. Moreover, different aerodynamic analysis approaches can be employed. For this purpose, some other aerodynamic fast prediction tools and CFD methods can be integrated to aerodynamic optimization methodology developed in this work. A hybrid optimization algorithm using RS and SQP method can be developed to exploit robustness of RS and computational efficiency of SQP.

REFERENCES

[1] K. M. Rasheed, "GADO: A Genetic Algorithm for Continuous Design Optimization," The State University of New Jersey, New Brunswick, New Jersey, 1998.

[2] M. Y. Öztürk, "Multiobjective Design Optimization of Missiles," Middle East Technical University, Ankara, 2009.

[3] X. Wang and M. Damodaran, "Comparison of Deterministic and Stochastic Optimization Algorithms for Generic Wing Design Problems," *Journal of Aircraft*, vol. 37, no. 5, pp. 929-932, 2000.

[4] E.L. Fleeman, *Tactical Missile Design*. Reston, Virginia: American Institute of Aeronautics and Astronautics Inc., 2001.

[5] J. N. Nielsen, *Missile Aerodynamics*. Palo Alto, California: McGraw-Hill, 1960.

[6] W. Blake, L. Auman, J. Doyle, C. Rosema, and M. Underwood, "Missile DATCOM User's Manual," United States Air Force, Ohio, 2011.

[7] B. Atak, "Enhancement of Aerodynamic Fast Prediction Methods for Slotted Rockets/Missiles," Istanbul Technical University, Istanbul, 2012.

[8] S. S. Chin, *Missile Configuration Design*. Orlando, Florida: McGraw-Hill, 1961.

[9] B. Etkin and L. D. Reid, *Dynamics of Flight: Stability and Control*. New York: John Wiley and Sons, 1995.

[10] J. D. Anderson, *Aircraft Performance and Design*. London: McGraw-Hill, 1999.

[11] M. B. Anderson, J. E. Burkhalter, and R. M. Jenkins, "Missile Aerodynamic Shape Optimization Using Genetic Algorithms," *Journal of Spacecraft and Rockets*, vol. 37, no. 5, pp. 663-669, September-October 2000.

[12] Ç. Tanıl, "Optimal External Configuration of a Missile," Middle East Technical University, Ankara, 2009.

[13] A. Karakoç, "Optimization of Aerodynamic Properties of Tactical Missiles," Middle East Technical University, Ankara, 2011.

[14] Y. R. Yang, S. K. Jung, T. H. Cho, and R. S. Myong, "Aerodynamic Shape Optimization System of a Canard-Controlled Missile Using Trajectory Dependent Aerodynamic Coefficients," *Journal of Spacecraft and Rockets*, vol. 49, no. 2, pp. 243-249, March-April 2012.

[15] O. Tekinalp and M. Bingöl, "Simulated Annealing for Missile Optimization: Developing Method and Formulation Techniques," *Journal of Guidance, Control and Dynamics*, vol. 27, no. 4, pp. 616-626, 2004.

[16] Ö. Tanrıkulu and V. Ercan, "Optimal External Configuration Design of Unguided Missiles," *Journal of Spacecraft and Rockets*, vol. 35, no. 3, pp. 312-316, 1998.

[17] J. Jezowski, R. Bochenek, and G. Ziomek, "Random Search Optimization Approach for Highly Multi-modal Nonlinear Problems," *Advances in Engineering Software*, vol. 36, pp. 504-517, 2005.

[18] N. J. Silveira, N. S. M. Cardozo, A. R. Secchi, and K. Wada, "Adaptive Random Search: A Promising Method for Determining the Stability of Mixtures," in *10th International Symposium on Process Systems Enginnering*, 2009, pp. 321-326.

[19] I. G. Tsoulos and I. E. Lagaris, "Genetically Controlled Random Search: A Global Optimization Method for Continuous Multidimensional Functions," *Computer Physics Communications*, no. 174, pp. 152-159, 2006.

[20] R. Simpson, A. Abakarov, and A. Teixeira, "Variable Retort Temperature Optimization Using Adaptive Random Search Techniques," *Food Control*, no. 19, pp. 1023-1032, 2008.

[21] S. S. Rao, *Engineering Optimization, Theory and Practice*. Hoboken, New Jersey: John Wiley and Sons, 2009.

[22] P. J. Lin and D. W. Zingg, "Comparison of Optimization Algorithms Applied to Aerodynamic Design," in *42nd AIAA Aerospace Sciences Meeting and Exhibit*, Reno, Nevada, 2004.

[23] N. B. Kuntawala, J. E. Hicken, and D. W. Zingg, "Preliminary Aerodynamic Shape Optimization of a Blended-Wing-Body Aircraft Configuration," in *49th AIAA Aerospace Sciences Meeting*, Orlando, 2011.

[24] J. A. Snyman, *Practical Mathematical Optimization: An Introduction to Basic Optimization Theory and New Gradient Based Algorithms*.: Springer, 2005.

[25] Simulia, *Application of Metal Processing Simulations*.: ASM Handbook Volume 22B, 2010.

[26] P. Brachetti, M. De Felice Ciccoli, G. Di Pillo, and S. Lucidi, "A New Version of the Price's Algorithm for Global Optimization," *Journal of Global Optimization*, no. 10, pp. 165-184, 1997.

[27] P. Spellucci, "DONLP2 Users Manual," Technical University at Darmstadt, Darmstadt, 1988.

[28] M. J. Hemsch, "Component Build-Up Method for Engineering Analysis of Missiles at Low-to-High Angle of Attack," Hampton, Virginia, 1992.

[29] S. R. Vukelich and J. E. Jenkins, "Missile DATCOM: Aerodynamic Prediction of Conventional Missiles Using Component Build-Up Techniques," in *AIAA 22nd Aerospace Sciences Meeting*, Reno, 1984.

[30] L.C.W. Dixon and G.P. Szegö, *Towards Global Optimization 2*. North-Holland, Amsterdam, 1978.

[31] A. E. Çetiner, B. Atak, A. Akgül, and D. C. Oral, "NASA Dual Control Missile Denek Taşı için Hesaplamalı Akışkanlar Dinamiği Analizleri ve Doğrulanması," in *IV. Ulusal Havacılık ve Uzay Konferansı*, İstanbul, 2012.

[32] M. R., Leavitt, L. D., Leopold, F. Mendenhall, "Experimental Test Cases," 2005.

[33] D. Lesieutre, J. Love, and M. Dillenius, "Recent Applications and Improvements to the Engineering Level Aerodynamic Prediction Software MISL3," in *40th AIAA Aerospace Sciences Meeting & Exhibit*, Reno, NV, 2002.