MESON SPECTRUM IN NON-RELATIVISTIC QUARK MODELS

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ABSTRACT

MESON SPECTRUM IN NON-RELATIVISTIC QUARK MODELS

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In this thesis, meson spectroscopy has been studied in a non-relativistic quark model framework. Possible meson molecules and their mass spectrum have been identified. Mass spectrum that had been constructed in earlier studies have been updated.

Keywords: Meson Spectroscopy, Meson Molecules, Non-Relativistic Quark Model

ÖZ

RÖLATİSTİK OLMAYAN KUARK MODELLERDE MEZON TAYFI

ÇEBİ, FULYA DİDEM
Yüksek Lisans, Fizik Bölümü
Tez Yöneticisi : Prof. Dr. Altuğ Özpineci

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Bu tezde, mezon tayfı rölativistik olmayan kuark model çerçevesinde çalışıldı. Mezon molekülü olabilecek mezonların bağlı durumları ve kütle tayfı belirlendi. Daha önceki çalışmalarda belirlenen kütle tayfı güncellendi.

Anahtar Kelimeler: Mezon Tayfı, Mezon Molekülleri, Rölativistik Olmayan Kuark Model

dedicated to my mother with love and gratitude...

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INTRODUCTION

1.1 Mesons and spectroscopy

In nature, there are six types of quarks named as up, down, strange, charm, bottom and top quarks, and they all have a corresponding anti-quark. These are grouped into three families; up and down, strange and charm, bottom and top. In Table 1.1, the symbols and the electric charges of the quarks are shown. Quarks are also charged under the color interactions. They can have three different colors which are named red, green and blue.

The quarks can also be classified according to their masses as light quarks or heavy quarks. This decision depends on Quantum Chromodynamics (QCD). QCD is nonperturbative below the scale Λ_{QCD} which is around $0.2GeV$. The quarks that are lighter than this scale are defined as light quarks (u, d and s[1] quarks), whereas the quarks that are heavier than this scale are defined as heavy quarks (c, b and t quarks) [3].

Up until 1960s and 1970s, although quarks were seen to be good mathematical instruments to explain the internal structure of protons and neutrons, most of the physicists did not believe they existed in nature because of lack of experimental evidence. In late 1960s, scientists performed electron-proton scattering experiments in which protons were bombarded by electrons which exchanged energy with the proton. When electrons collided with protons, either protons

[1] Strange quark falls between the two definitions because its mass is close to the scale factor. But it is generally included in the light quark definition.

moved to a higher energy excited state because of the additional energy or they shattered completely and new mesons and baryons were formed. These experiments are known as deep inelastic scattering experiments (DIS) [4]. The observations in these experiments could be explained by treating the nucleon as a collection of non-interacting point particles named partons.

Table1.1: Table of quarks

Quark	**Symbol**	**Charge**
up	u	+ $\frac{2}{3}$
down	d	- $\frac{1}{3}$
charm	c	+ $\frac{2}{3}$
strange	s	- $\frac{1}{3}$
top	t	+ $\frac{2}{3}$
bottom	b	- $\frac{1}{3}$

Although, in DIS experiments, the nuclei were observed to be made of non-interacting point partons, it was impossible to observe partons individually in nature. Because as soon as the protons disintegrated, new mesons or baryons appeared. To explain this puzzling development, physicists suggested the color confinement. It was postulated that color charged objects cannot be observed individually and they only occur in colorless groups in nature [5, 6].

Any theory of the interactions of partons should explain these observations. Now, it is known that nuclei are made of quarks and gluons and they interact with each other by a strong force. This force is created by force carriers that are exchanged between the quarks, i.e they mediate the interactions. Their interactions are described by QCD. In QCD, the exchanged particles are gluons.

When analyzing a certain process, a simple superposition law is used and the contributions from all the interactions are added together to find the final values. The coupling constant arises as a multiplication for each quark-gluon interaction. As an example, the contributions to the $q\bar{q} \rightarrow q\bar{q}$ scattering are shown in Fig. 1.1. Fig. 1.1(a) is the only process at O(α_s) while Fig. 1.1(b) is one of the processes that contribute to O(α_s^2). Although α_s is called a constant, it changes with the energy scale of the process. Therefore, this number is called as 'running

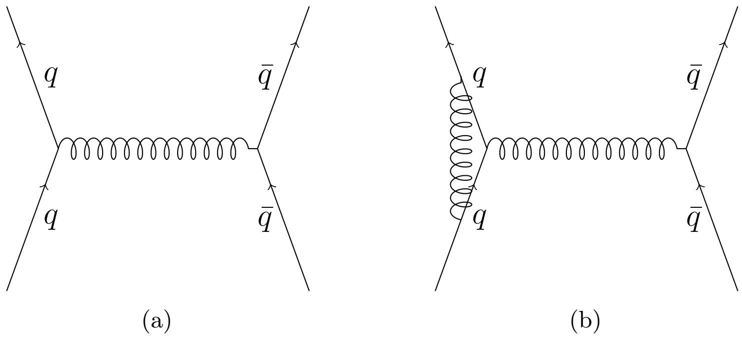

Figure 1.1: Some diagrams contributing to $q\bar{q}$ scattering

coupling constant' [7].

α_s depends on the separation of the color sources and it increases as the separation increases. In lowest-order QCD, running coupling constant is calculated as [2]

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f)ln(Q^2/\Lambda^2_{QCD})} \tag{1.1}$$

Here N_f represents the number of quark flavors that have a mass below the energy Q^2. The behavior of α_s when Q changes is shown in the Fig. 1.2. As can be seen from the figure, for small separations i.e. large Q^2, α_s becomes very small. Hence interactions described by QCD are asymptotically free [8, 9, 10, 11].

Note that Eqn. 1.1 is obtained as a result of a perturbative calculation. Hence it is valid as long as $\alpha_s \ll 1$. When $Q^2 = \Lambda^2_{QCD}$, Eqn. 1.1 diverges. This shows that Eqn. 1.1 cannot be valid as Q^2 approaches Λ^2_{QCD}.

Regarding confinement, there is not yet any proof that QCD is confining. It is believed that when two color sources in a colorless system are separated, a flux tube forms betwen them. Hence, to separate them, infinite energy is necessary. When the separation is sufficiently large, this flux tube breaks by the creation of a quark-anti-quark pair forming two (or more) colorless objects. Although this picture of confinement is not proven in QCD there are strong hints, especially

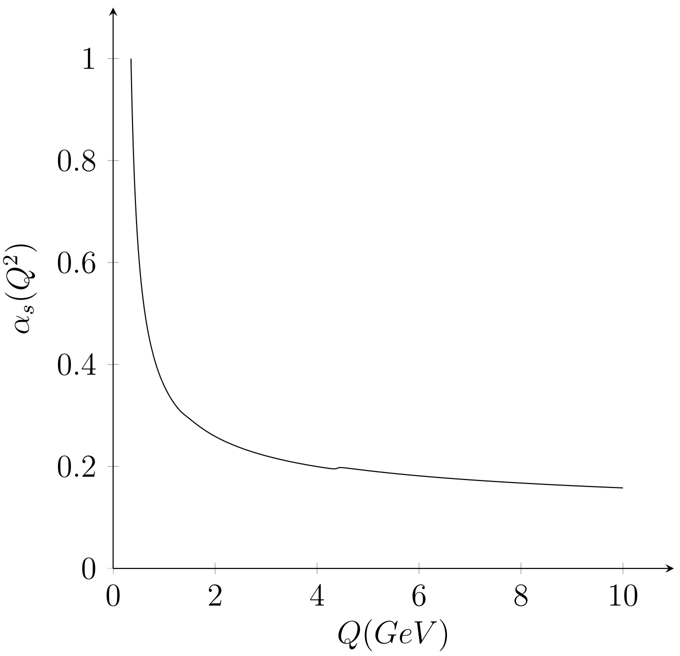

Figure 1.2: The $\alpha_s(Q^2)$ for lowest-order QCD with $\Lambda = 200$ MeV [2]

from lattice QCD, that it actually happens. For further information see [12].

1.2 Symmetries of QCD

QCD is a gauge theory based on the gauge group SU(3). Besides this gauge symmetry, QCD has other (approximate) continuous and discrete symmetries. Presence of symmetries allows us to group the spectrum into multiplets having (approximately) conserved quantum numbers, simplifying their study.

The strength of strong interactions is flavor blind, i.e all quark flavors interact with the same strength. The differences between the contributions of different quarks are all due to their different masses. If quarks had the same mass, strong interactions would not be able to distinguish the quark type, i.e QCD would have a symmetry under rotations in flavor space. In nature, the u and d quark masses can be assumed to be approximately equal. Hence QCD has an approximate SU(2) symmetry which is called the isospin symmetry [7].

Some of the discrete symmetries of QCD are parity (P), C-parity (C), G-parity

(G) and time reversal (T).

Parity is a spatial transformation in which a coordinate changes its sign. In three dimensions, all three coordinates would change their signs under parity. Electromagnetic and strong interactions are symmetric under parity but weak interactions do not conserve it. Basic representation of parity can be shown as

$$(x^0, \vec{x}) \xrightarrow{\hat{\mathrm{P}}} (x^0, -\vec{x}) \tag{1.2}$$

C-parity is a transformation that changes one particle into its antiparticle. When a C-parity operator is applied on a $q\bar{q}$ state, q and $\bar{q}$ transform into $\bar{q}$ and q, respectively.

$$(q\bar{q}) \xrightarrow{\hat{\mathrm{C}}} (\bar{q}q) \tag{1.3}$$

Note that only particles that are their own anti-particles can be eigenstate of C-parity.

G-parity is a more generalized form of C-parity. It exists in the mesons that consist of a quark and its own antiquark ($I_z = 0$) and charged states that consist of $u\bar{d}$ and $d\bar{u}$ with $I = 1$.

Time reversal transformation is a transformation in which a state Ψ changes into a state Ψ' where its all linear and angular momenta are reversed in sign but all the other properties stays the same [7]. The effects of T transformation on the space-time coordinates can be represented as

$$(x^0, \vec{x}) \xrightarrow{\hat{\mathrm{T}}} (-x^0, \vec{x}) \tag{1.4}$$

1.3 Quantum Numbers of $q\bar{q}$ States

The term hadrons were introduced to define the bound states formed by quarks and gluons. They are defined as composite particles consisting of quarks held together by the strong force. There are two types of hadrons: mesons and baryons. In the traditional picture of hadrons, mesons contain one quark and one antiquark, whereas (anti)baryons contain three (anti)quarks. Mesons have integer spins and therefore are bosons, as opposed to baryons and quarks which are spin-half particles i.e they are fermions [7, 13].

The mesons which include a heavy quark and its own anti-heavy quark, for example $c\bar{c}$ or $b\bar{b}$, are called heavy quarkonium systems. Although there are some hypothetical $t\bar{t}$ states calculated in paper [2], they are not observed in the experiments both because top quark is too heavy, therefore requires too much energy to create and because its lifetime is too short to form a meson, it immediately decays into other particles.

There are theories to classify hadrons and obtain a hadron spectroscopy. They are classified according to their types, masses, quantum numbers, etc. While constructing hadron spectroscopy, the quark contents are determined based on the quantum numbers of the mesons and baryons. Isospin, parity, C-parity and G-parity quantum numbers constitute some of them.

To theoretically determine parity quantum number, a parity operator is applied on a state

$$\mid M >= \int d^3r \Psi_{ss'}(\vec{r}) \mid q_s(\vec{r}) \bar{q}_{s'}(-\vec{r}) > \tag{1.5}$$

In Hilbert space, the applied parity does not operate on the wavefunction Ψ, it only operates on the state.

$$\begin{aligned} \hat{P} \mid M > &= \int d^3r \Psi_{ss'}(\vec{r}) \hat{P} \mid q_s(\vec{r}) \bar{q}_{s'}(-\vec{r}) > \\ &= \int d^3r \Psi_{ss'}(\vec{r}) P_q P_{\bar{q}} \mid q_s(-\vec{r}) \bar{q}_{s'}(\vec{r}) > \\ &= \int d^3r \Psi_{ss'}(-\vec{r}) P_q P_{\bar{q}} \mid q_s(\vec{r}) \bar{q}_{s'}(-\vec{r}) > \end{aligned} \tag{1.6}$$

Substituting

$$\Psi_{ss'}(-\vec{r}) = (-1)^l \Psi_{ss'}(\vec{r}) \tag{1.7}$$

where l is the orbital angular momentum quantum number, into the equation 1.6,

$$\hat{P} \mid M >= (-1)^l P_q P_{\bar{q}} \mid M > \tag{1.8}$$

So, the formula for the parity quantum number is found as

$$P_M = (-1)^l P_q P_{\bar{q}} \tag{1.9}$$

Quarks and anti-quarks have opposite parity, i.e $P_q P_{\bar{q}} = -1$. Hence

$$P_M = (-1)^{l+1} \tag{1.10}$$

To determine C-parity quantum number, one acts on the state in Eqn. 1.5 by the $\hat{C}$ operator

$$\begin{aligned}
\hat{C} \mid M > &= \hat{C} \int d^3r \Psi_{ss'}(\vec{r}) \mid q_s(\vec{r}) \bar{q}_{s'}(-\vec{r}) > \\
&= \int d^3r \Psi_{ss'}(\vec{r}) \mid \bar{q}_s(\vec{r}) q_{s'}(-\vec{r}) > \\
&= \int d^3r \Psi_{ss'}(\vec{r})(-1) \mid q_{s'}(-\vec{r}) \bar{q}_s(\vec{r}) > \\
&= (-1) \int d^3r \Psi_{s's}(\vec{r}) \mid q_s(-\vec{r}) \bar{q}_{s'}(\vec{r}) > \\
&= (-1) \int d^3r \Psi_{s's}(-\vec{r}) \mid q_s(\vec{r}) \bar{q}_{s'}(-\vec{r}) > \qquad (1.11)
\end{aligned}$$

Then, using Eqn. 1.7,

$$\hat{C} \mid M >= (-1)^{l+1} \int d^3r \Psi_{s's}(\vec{r}) \mid q_s(\vec{r}) \bar{q}_{s'}(-\vec{r}) > \tag{1.12}$$

To express $\hat{C} \mid M >$ in terms of $\mid M >$, a relation between $\Psi_{s's}(\vec{r})$ and $\Psi_{ss'}(\vec{r})$ needs to be established. For this purpose, consider the total spin states of two spin-$\frac{1}{2}$ particles.

$$\begin{aligned}
\mid s=0, s_z = & \quad 0> = \frac{1}{\sqrt{2}}(|\uparrow\downarrow> - |\downarrow\uparrow>) \\
\mid s=1, s_z = & \quad 1> = |\uparrow\uparrow> \\
\mid s=1, s_z = & \quad 0> = \frac{1}{\sqrt{2}}(|\uparrow\downarrow> + |\downarrow\uparrow>) \\
\mid s=1, s_z = & -1> = |\downarrow\downarrow>
\end{aligned} \tag{1.13}$$

Note that $s = 1$ ($s = 0$) states are (anti)symmetric under the exchange of the two spins. Hence under spin exchange, one obtains;

$$\mid ss_z > \xrightarrow{\text{spin exchange}} (-1)^{S+1} \mid ss_z > \tag{1.14}$$

where S is total spin. Thus,

$$\Psi_{s's}(\vec{r}) = (-1)^{S+1}\Psi_{ss'}(\vec{r}) \tag{1.15}$$

Using this result in Eqn. 1.12, C-parity is found as

$$\hat{C} \mid M >= (-1)^{l+S} \mid M > \tag{1.16}$$

G-parity is defined as

$$\hat{G} = \hat{C}e^{i\pi I_2} \tag{1.17}$$

It can be shown that if a $q\bar{q}$ system is an eigenstate of $\hat{G}$ parity, its eigenvalue is given by

$$G = (-1)^{I+l+S} \tag{1.18}$$

where I is the isospin of the system.

The fundamental aim of any hadron model is to describe the mass spectroscopy of hadrons from the ground state to the highest mass and to the highest angular momentum, the electroweak properties and the strong decays and interactions of hadrons [14].

1.4 Constituent Quark Model

There are several methods to obtain hadron spectroscopy such as quark model [8], sum rules [15] and lattice approaches [16]. The constituent quark model is widely used and known as the most extensive approach. In this thesis, the meson spectroscopy will be studied in a constituent quark model framework. One of the most important qualities of this model is that it successfully describes both the light flavored and heavy flavored hadrons [14].

In constituent quark model, it is assumed that the quarks are non-relativistic inside a hadron. Therefore, the total mass of a hadron, take mesons in this case, is approximately equal to the sum of the masses of the quarks that the meson consists of. Hence the constituent quark[2] masses of quarks can be described as effective masses that are chosen to make the constituent quark model work [17].

The spectroscopy has been studied for more than 40 years, starting with the discovery of the mesons. In [2], the constituent quark model is proposed to study the spectroscopy in great detail when the experimental results were not too vast. 30 years later, when updated, the spectroscopy that had been predicted in the paper still seems valid despite the minor differences and occasional deviations.

Throughout the years, numerous mesons have been theoretically predicted and also they were discovered in the experiments. Some still have not been found. Some other states are observed in the experiments but cannot be predicted by the theories [18, 19, 20]. It is puzzling because even though the theoretical predictions seem accurate for the other mesons, and the calculations look precise in general, not all the theoretical mesons have found their place in the experimental

[2] A constituent quark means a quark with dressing i.e. a current quark surrounded by gluons and virtual quarks.

spectrum. Hence, some experimentally found mesons can be a match for more than one theoretically predicted mesons. Those mesons have close mass values and same quantum numbers.

To explain these puzzles, new theories have been developed. Some of these theories are focused on the existence of exotic mesons [21, 22, 23], glueballs [24, 25, 26], tetraquarks [27, 28, 29], molecular states [30, 31, 32], and hybrids [33, 34, 35, 36]. Exotic mesons are defined as particles that have quantum numbers which are not normally allowed in quark-antiquark picture of the mesons, where glueballs are composite particles that do not have any valence quarks and consist of only gluons. Tetraquarks have two quark - anti-quark pairs just as molecular states do. The difference between them is that tetraquarks can consist of any two quark- antiquark pairs while in molecular states quarks and antiquarks are arranged in colorless mesons inside the molecule. So, while each molecular state is also grouped under tetraquarks, not all tetraquarks can be adressed as molecular states. And finally hybrids are composed of a quark-antiquark pair and a gluon. All five are called mesons because they are hadrons and they have baryon number of zero.

The main purpose of this thesis is to update the meson spectroscopy and to determine candidates for molecular states. Therefore, a comparison of current experimental and theoretical spectrum is presented. The thesis is organized as follows. In Chapter 2, quark model will be explained[3] and the method of identifying possible molecular states by using experimental data will be studied. In Chapter 3, the results will be stated and discussed. In Chapter 4, conclusions will be drawn.

[3] The quark model discussions will mainly follow [2, 1]

CHAPTER 2

QUARK MODEL

2.1 Quark Model

Non-relativistic Quark Model sufficiently describes the masses and quantum numbers of mesons and baryons. In order to do so, the wave function of hadrons should be stated. In the quark model, in the non-relativistic limit, the wave function of the meson can be written as the product of four wave functions each describing the space, spin, flavor and color parts respectively:

$$\psi = \psi^{space} * \psi^{spin} * \psi^{flavor} * \psi^{color} \tag{2.1}$$

The flavor part contributes to the wave function as constants which are determined according to the number of flavors in the quark model in general, in the systems in particular.

The color part is fixed by the requirement that the hadron is colorless. For mesons

$$\psi^{color} = \sum_{i,j=r,g,b} \delta^{ij} \frac{1}{\sqrt{3}} \tag{2.2}$$

where i is the color of the quark and $\bar{j}$ is the anti-color of the anti-quark.

The space and the spin parts can be determined by solving the Schrödinger equation using the appropriate potential energy term. For most part, it is easier and common to start with a known potential and a wave function and modify them into the cases that are being studied by changing the potential.

In this model, it is assumed that the contribution to the wave function coming from other states such as $q\bar{q}$g states would be so small that $q\bar{q}$ wave function would dominate over said contributions and the normalizations of the wave functions would yield to unity which simplifies the Hamiltonian and its solution.

In order to calculate the masses and determine the wave functions, both Schrödinger equation and spinless Bethe-Salpeter equation, which is a simple relativistic version of Schrödinger equation, can be used. Schrödinger equation can be expressed as

$$H\Psi_n = E_n\Psi_n \tag{2.3}$$

The Hamiltonian can be written as

$$H = T + V \tag{2.4}$$

where T is relativistic kinetic energy part of the Hamiltonian with rest mass energy and V is the potential energy. For a two particle system T can be written as

$$T(\vec{p}) = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} \tag{2.5}$$

which in the non-relativistic limit becomes

$$T(\vec{p}) = \frac{p^2}{2\mu} + m_1 + m_2 \tag{2.6}$$

where $\vec{p}$ is the momentum of the (anti)quark in the rest frame of the meson and μ is the reduced mass defined as

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} \tag{2.7}$$

The potential energy part of the Hamiltonian is defined as

$$V = V(r) + V_{SD}(r) \tag{2.8}$$

$V_{SD}(r)$ represents all the spin-dependent potentials. If one assumes $V_{SD}(r) = 0$, spinless Bethe-Salpeter equation is obtained.

Quark-antiquark potential has been analyzed in various works both analytically and numerically [37, 38, 39, 40, 41]. At short distances, one gluon exchange should be sufficient to describe the interactions of quarks. Hence, the potential is expected to be

$$V(\vec{r}) = -\frac{4}{3}\frac{\alpha_s}{r} \tag{2.9}$$

when $r \to 0$. At long distances flux-tube model predicts the potential to increase linearly, i.e

$$V(\vec{r}) \to kr \tag{2.10}$$

when $r \to \infty$.

The simplest interpolation between these two regimes gives the Cornell potential:

$$V(\vec{r}) = \left[kr - \frac{4}{3}\frac{\alpha_s}{r} + C\right]\vec{F}_q \cdot \vec{F}_{\bar{q}} \tag{2.11}$$

where C is an arbitrary constant which is chosen to be fit the model to experimental data and $\vec{F}_i$'s are the color factors and are given as

$$\vec{F}_i = \begin{cases} \lambda_i/2 & \text{for quarks} \\ \lambda_i^c/2 = -\lambda_i^*/2 & \text{for antiquarks} \end{cases} \tag{2.12}$$

Since the color wavefunction of all the mesons are identical, the $\vec{F}_q \cdot \vec{F}_{\bar{q}}$ factor in the potential can be replaced by its expectation value

$$< \vec{F}_i \cdot \vec{F}_j >= -\frac{4}{3} \tag{2.13}$$

One problem that needs to be addressed when using this potential for all values of r is which value of α_s to be used. In [1], α_s is assumed to be a constant to be fitted to experimental spectrum. In [2], running coupling constant is used. In this case, $\alpha_s(r)$ should be regularized at $r \to \infty$. In [2], $\alpha_s(Q)$ has been fitted to a sum of Gaussians. The running coupling constant and the obtained fit are shown in Fig. 1.2.

2.2 Non-relativistic Schrödinger and Spinless Bethe-Salpeter Solutions

By solving the Schrödinger equation, one could calculate the energy eigenvalues and wave functions of the mesons and baryons theoretically. Since the Hamiltonian is written in the the rest frame of the hadron, the energy eigenvalues correspond to the masses of the hadrons. Most of the quark model potentials tend to be similar, having a coulomb term and a linear term added together. Since there are different types of effects on the particles such as relativistic effects and spin-orbit interactions etc., the terms that represent those effects can be added to the potential depending on the importance of the said effect for that particular study. Since most of them are really small compared to the Coulomb and linear potentials, even by neglecting those effects one can find results that are close to the experimental data. In this section, the results that have been obtained from the potential in [1], which is known as Cornell potential, will be studied by using two different approximations of the kinetic energy.

Using the kinetic energy terms for both cases and the Cornell potential, the most basic solutions will be found for the D mesons, B mesons and heavy quarkonia [1]. The masses of the quarks in these calculations will be treated as free parameters whose values will be determined by fitting to experimental results. The resulting masses are called constituent quark masses and in general are larger than their pole masses.

Since the potential that is used is a central potential after all the spin-dependent potentials (V_{SD}) are neglected, the eigenfunctions will be eigenstates of the angular momentum operator. Hence the eigenstates can be written as

$$\psi(\vec{r}) = \Psi(r) Y_{lm}(\Omega) \tag{2.14}$$

where $Y_{lm}(\Omega)$ are the spherical harmonics and $\Psi(r)$ is the radial part.

To solve for the radial part, first a suitable basis will be chosen. Then, the Hamiltonian matrix will be obtained in this basis. Finally, eigenstates will be obtained by diagonalizing this matrix.

An exactly solvable system that has confined solutions is the simple harmonic oscillator (SHO) system. To make use of this similarity, SHO eigenstates will be used as the basis.

The radial part of the SHO eigenstates are [42]

$$R_{nl}(r) = N_{nl} \beta^{3/2} (2\beta r)^l e^{-\beta r} L_n^{2l+2}(2\beta r) \tag{2.15}$$

where

$$N_{nl}^2 = \frac{8(n!)}{\Gamma(n+2l+3)} \tag{2.16}$$

n is the principal quantum number[1], l is the orbital angular momentum quantum number and β is a scale factor which corresponds to the SHO frequency.

The radial functions, $R_{nl}(r)$, satisfy the orthogonality relation

$$\int_0^\infty \mathrm{r}^2 R_{nl}^*(r) R_{n'l}(r) \, \mathrm{d}r = \delta_{nn'} \tag{2.17}$$

which can be proven using

$$\int_0^\infty \mathrm{e}^{-x} x^a L_n{}^\alpha(x) L_{n'}{}^\alpha(x) \, \mathrm{d}x = \delta_{nn'} \frac{\Gamma(n+\alpha+1)}{n!} \tag{2.18}$$

The wave functions in momentum space is found using Fourier transformation. They can be obtained from coordinate space wavefunctions using

[1] It represents the energy level of the wave function

$$\Phi(\vec{p}) = \int \frac{\mathrm{d}^3 r}{(2\pi)^{3/2}} \mathrm{e}^{-\mathrm{i}\tilde{\mathrm{p}}\cdot\tilde{\mathrm{r}}} \psi(\vec{r}) \tag{2.19}$$

Another advantage of using the SHO eigenstates is that, their momentum space representation has the same functional form as the coordinate representation. The momentum space representation can be obtained from Eqns. 2.15 and 2.16 by substitutions $\vec{r} \rightarrow \vec{p}$ and $\beta \rightarrow \frac{1}{\beta}$.

To solve the Schrödinger equation, the Hamiltonian matrix is written in the form of

$$H = \begin{pmatrix} H^{l=0} & & & & \\ & H^{l=1} & & 0 & \\ & & \ddots & & \\ & 0 & & \ddots & \\ & & & & \ddots \end{pmatrix} \tag{2.20}$$

where H^l itself is formed as blocks as follows

$$H^l = \begin{pmatrix} H^{m=-l} & & & & \\ & H^{m=-l+1} & & 0 & \\ & & \ddots & & \\ & 0 & & \ddots & \\ & & & & H^{m=l} \end{pmatrix} \tag{2.21}$$

The matrix elements are defined as

$$H^{lm}_{nn'} =< nlm \mid H \mid n'l'm' >= \delta_{ll'}\delta_{mm'} < n \mid H \mid n' > \tag{2.22}$$

where $< n \mid H \mid n' >$ is the reduced matrix element.

Since the Hamiltonian is a sum of potential and kinetic terms, Eqn. 2.22 can be calculated individually for both of them. So for the potential and the kinetic terms, the integral formulas used are

$$
\begin{aligned}
< nl'm' \mid V(\hat{r}) \mid n'lm > &= \int \mathrm{d}^3r < nl'm' \mid V(\hat{r}) \mid \vec{r} >< \vec{r} \mid n'lm > \\
&= \int \mathrm{d}^3r V(r) < nl'm' \mid \vec{r} >< \vec{r} \mid n'lm > \\
&= \int \mathrm{d}r \mathrm{d}\Omega r^2 V(r) \Psi^*_{nl'm'}(\vec{r}) \Psi_{n'lm}(\vec{r}) \\
&= \int_0^\infty \mathrm{d}r \mathrm{d}\Omega r^2 V(r) R_{nl'}(r) Y_{l'm'}(\vec{r}) R_{n'l}(r) Y_{lm}(\vec{r}) \\
&= \int_0^\infty \mathrm{d}r r^2 V(r) R_{nl'}(r) R_{n'l}(r) \delta_{ll'} \delta_{mm'} \\
&= \int_0^\infty \mathrm{r}^2 R_{nl}(r) V(r) R_{n'l}(r) \delta_{ll'} \delta_{mm'} \, \mathrm{d}r \qquad (2.23)
\end{aligned}
$$

Similarly, the kinetic energy term can be found applying the same steps for the momentum space,

$$
< T >_{nn'} = \int_0^\infty \mathrm{p}^2 R'_{nl}(p) T(p) R'_{n'l}(p) \delta_{ll'} \delta_{mm'} \, \mathrm{d}p \qquad (2.24)
$$

where $R'_{nl}(p)$ has the same functional form as $R_{nl}(r)$ and can be obtained from $R_{nl}(r)$ by replacing $\vec{r} \to \vec{p}$ and $\beta \to \frac{1}{\beta}$.

Substituting Eqns. 2.5, 2.8 and 2.15 into these two equations, matrix elements can be found individually.

Table2.1: The parameters that are used in the calculations of heavy quarkonium systems [1].

Parameters	Schrödinger	Bethe-Salpeter
$m_b(GeV)$	4.7485	4.731
$m_c(GeV)$	1.3205	1.321
$\alpha_s(GeV^2)$	0.191	0.203
k	0.472	0.437
$C = 0$		

The $\infty \times \infty$ matrix can not be numerically evaluated. For numerical calculations, an $N \times N$ matrix is formed by taking the first N basis states. For a sufficiently large N, properties of the low lying states are expected to be independent of N. After finding the matrix elements of this $N \times N$ matrix, the eigenvalues, i.e. energies in this case, will be found using the formula

Table2.2: The parameters that are used in the calculations of heavy-light systems. For all the Bethe-Salpeter results, $\beta = 1.5GeV$. The parameters C_c and C_b are additive constants for the charmed and B-flavor mesons [1].

Parameters	Schrödinger	Bethe-Salpeter
$m_u(MeV)$	325	150
$m_s(MeV)$	603	366
C_c	-442	-246
C_b	-470	-214

$$det \mid H - I\lambda \mid = 0 \tag{2.25}$$

where I is the identity matrix and λ represents the eigenvalues of the Hamiltonian. Note that the scale factor β is introduced into the problem by the choice of the basis states. Since it is not a parameter in the Hamiltonian, its eigenvalues and eigenstates should be independent of β. But by approximating H by an $N \times N$ matrix, a residual dependence on β is introduced. To eliminate this residual dependence on β, a region of β should be found such that the eigenstates are independent on the value of β.

The dependence of the S-states' eigenvalues on the scale factor β is shown in Fig. 2.1. When $1.5 \leq \beta \leq 2.5$, the eigenvalues behave as they are not affected from the change in the scale factor. For P-states, a similar graph had been observed in [1].

For the numerical calculations of heavy quarkonium systems, the parameters that are used are listed in Table 2.1. And, for the numerical calculations of heavy-light systems, the parameters that are chosen are listed in Table 2.2.

The results of this calculation are shown in Tables 3.5, 3.6, 3.7, 3.8, 3.9, 3.10, 3.12, 3.13, in columns 6, 7, 8. Since the spin dependent interactions are ignored, the eigenvalues obtained in this method are expected to be averages over the different spin states. From this perspective, a good agreement with the experimental observations is seen.

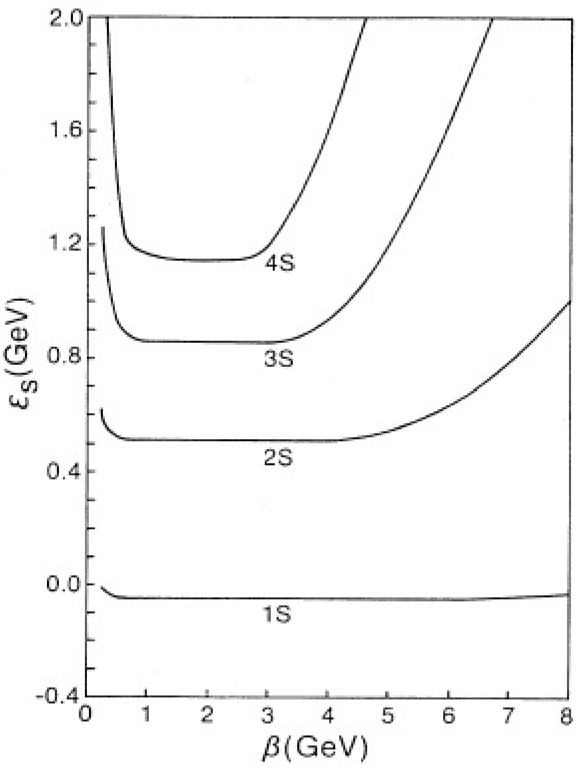

Figure 2.1: S-state eigenvalues as a function of the scale parameter β from the Schrödinger equation. The results were obtained from 9×9 matrix. [1]

2.3 Effects of Spin-dependent Interactions

In the model of the previous section, all the spin dependent effects are ignored, hence it fails to take into account the splittings due to spin. As an example, one such splitting exists between the $\eta_c(1S)$ and J/Ψ mesons, whose masses differ by $\Delta m = 115,916 MeV$. Both mesons are the ground states in the $l = 0$ sector. In $\eta_c(1S)$, the $c\bar{c}$ quarks are in the $s = 0$ state whereas in J/Ψ, they are in the $s = 1$ state. Hence their mass difference should be caused by spin dependent interactions.

To explain this splitting and also other spin dependent phenomenon, spin dependent interactions should be added to the potential. These interactions can be obtained by the taking the non-relativistic limit of Dirac equation [43]. In this section, the model developped in [2] will be summarized.

The spin-dependent potential can be written as the sum of two terms:

$$V_{SD} = H_{ij}^{hyp} + H_{ij}^{so} \tag{2.26}$$

where H_{ij}^{hyp} represents the hyperfine interaction, H_{ij}^{so} is spin-orbit interaction. The potential that describes the hyperfine interactions is

$$H_{ij}^{hyp} = \frac{4\alpha_s(r)}{3m_i m_j}\left[\frac{8\pi}{3}\vec{S}_i \cdot \vec{S}_j \delta^3(\vec{r}) + \frac{1}{r^3}\left[\frac{3\vec{S}_i \cdot \vec{r}\vec{S}_j \cdot \vec{r}}{r^2} - \vec{S}_i \cdot \vec{S}_j\right]\right] \tag{2.27}$$

Spin-orbit interaction effects are divided into two parts as

$$H_{ij}^{so} = H_{ij}^{so(cm)} + H_{ij}^{so(tp)} \tag{2.28}$$

where

$$H_{ij}^{so(cm)} = \frac{4\alpha_s(r)}{3r^3}\left[\frac{1}{m_i} + \frac{1}{m_j}\right]\left[\frac{\vec{S}_i}{m_i} + \frac{\vec{S}_j}{m_j}\right] \cdot \vec{L} \tag{2.29}$$

represents the color magnetic part of the spin orbit interaction and,

$$H_{ij}^{so(tp)} = -\frac{1}{2r}\frac{\partial H_{ij}^{conf}}{\partial r}\left[\frac{\vec{S}_i}{{m_i}^2} + \frac{\vec{S}_j}{{m_j}^2}\right] \cdot \vec{L} \tag{2.30}$$

represents the Thomas-precession term.

The terms H^{hyp} and H^{so} have factors of r^{-3} in them. They are more singular than r^{-2} term, and this makes them illegal operators. As a result, an inconsistency occurs in the Hamiltonian equation. To solve this problem, smearing is introduced into the model. The coordinate $\vec{r}$ is accepted to smear out over the distances of the order of inverse quark masses rather than being defined as $\vec{r} = \vec{r_1} - \vec{r_2}$ as done in the non-relativistic limit. Also, the coefficients of the different effects to the potential in non-relativistic limit are dependent on the interacting quarks' momentum. In this way, the smearing deals with all the singularities in the potential and turns them into legal operators. Details of the smearing procedure is beyond the scope of this thesis, so it will not be studied. For further details see [2].

The Hamiltonian equation defined above is solved for mesons by diagonalization method.

$$\tilde{H} = (p^2 + m_1^2)^{1/2} + (p^2 + m_2^2)^{1/2} + \tilde{H}_{12}^{conf} + \tilde{H}_{12}^{hyp} + \tilde{H}_{12}^{so} \tag{2.31}$$

Here $\tilde{H}$ represents the Hamiltonian that is modified by relativistic effects.

Once the spin effects are taken ito account, the basis that is used in the previous part have to be modified. One immediate generalization can be to use the basis states

$$\psi(\vec{r}) = R_{nl}(r)Y_{lm}(\Omega)u_q u_{\bar{q}} \tag{2.32}$$

where $u_q(u_{\bar{q}})$ is the spin wavefunction of the (anti)quark. Since the Hamiltonian has rotational symmetry, rather than using states of definite l, m_l, s^q, s_z^q, $s^{\bar{q}}$, $s_z^{\bar{q}}$, it is better to use states of definite j, m_l, l, s. These states can be defined as follows. Let χ_{sm_s} denote the wavefunction of states that have definite total spin. These states are given in Eqn. 1.13. Then the states of definite $j\ m_l\ l\ s$ can be written as

$$\Psi_{njm_j;ls}(\vec{r}) = R_{nl}(r) \sum < lm; sm_s \mid jm_j > Y_{lm}(\Omega)\chi_{sm_s} \qquad (2.33)$$

where $< lm; sm_s \mid jm_j >$ are the Clebsch-Gordon coefficients. $\Psi_{njm_j;ls}$ basis is used to study spin-dependent effects.

After finding the matrix elements, the Hamiltonian matrix is diagonalized. The diagonal entries correspond to the eigenvalues of the Hamiltonian which are the masses of the mesons.

For most mesons, these calculations are complete. But for self-conjugate isoscalar mesons [2], there is also the annihilation effects. These effects arise when the quarks in the meson annihilate into gluons which later form a quark-anti-quark pair. They are not included in the context of this thesis [2].

2.4 Identifying possible molecules in the meson spectrum

The puzzles in the meson spectroscopy are already discussed in a previous chapter. One possible solution is the possible molecular states scenario. Suppose that there are two mesons with masses m_1 and m_2. These two mesons come together, forming a new meson with a mass, M,

$$M \leq m_1 + m_2 \qquad (2.34)$$

It is less than the total mass value because of the binding energy which is used to bind these two mesons together. To identify an observed meson $\mathcal{M}$ as the bound state of mesons $\mathcal{M}_1$ and $\mathcal{M}_2$, the mesons $\mathcal{M}_1$ and $\mathcal{M}_2$ should be in a state whose quantum numbers match the quantum numbers of $\mathcal{M}$.

Possible quantum numbers I_T, J_T, P_T, C_T of a molecule made of $\mathcal{M}_1$ and $\mathcal{M}_2$ can be obtained from

$$\mid I_2 - I_1 \mid \leq I_T \leq \mid I_1 + I_2 \mid \qquad (2.35)$$

[2] A self-conjugate isoscalar meson is a meson that is identical with its anti-meson and is a scalar under transformation of the meson under the SU(2) group of isospin. Its total isospin (I) and the third component of isospin (I_3) are both zero.

$$| J_2 - J_1 | \leq J_T \leq | J_1 + J_2 | \tag{2.36}$$

$$P_T = P_1 \cdot P_2 \cdot (-1)^l \tag{2.37}$$

$$C_T = C_1 \cdot C_2 \tag{2.38}$$

where I_i, J_i, P_i and C_i are the quantum numbers of the meson $\mathcal{M}_i$. Since the quantum numbers are experimentally observable, and quantum numbers of most of the mesons are already known, it is easy to use these formulas to check the possible molecular states.

As an example, consider η and $\eta'(958)$. Their J^{PC} quantum numbers are both 0^{-+}. The total mass,

$$M_T = M_\eta + M_{\eta'} = 547.853MeV + 957.78MeV \tag{2.39}$$

$$M_T = 1505.633MeV \tag{2.40}$$

$$| 0 - 0 | \leq J_T \leq | 0 + 0 | \tag{2.41}$$

$$J_T = 0 \tag{2.42}$$

$$| 0 - 0 | \leq I_T \leq | 0 + 0 | \tag{2.43}$$

$$I_T = 0 \tag{2.44}$$

$$P_T = (-1) \cdot (-1) \cdot (-1)^0 \tag{2.45}$$

$$P_T = + \tag{2.46}$$

$$C_T = (+1) \cdot (+1) \tag{2.47}$$

$$C_T = + \tag{2.48}$$

So, if there is a meson with mass less than 1505.633 MeV up to 50 MeV with J^{PC} numbers of 0^{++} and isospin 0, then that meson is a possible molecular state. When the experimental data is scanned, there indeed seems to be a possible molecular state for η and $\eta'(958)$ with the appropriate quantum numbers and mass, which is $f_0(1500)$.

Some of the molecular state candidates have more than one prospect of being a molecular state, i.e. different meson pairs can bound together to form the same meson.

70 possibilities of molecular states are detected by the algorithm and they are all shown in the tables in the next chapter. These possibilities should be studied in detail as in [18, 30, 31, 32] to determine if they really are molecular states and if so, whether or not the found molecular state possibilities are consistent with the reality.

CHAPTER 3

RESULTS AND DISCUSSIONS

In this chapter, the meson spectrum is discussed by using the results from [2, 1] and experimental data. The theoretical calculations yield results that are close to each other. When [2] was published, few mesons had been observed in the experiments[1] and ever since physicists have been looking for the theoretically predicted states. In the last three decades there have been significant progress in experimental hadronic physics and a lot of states have been observed experimentally. The results obtained are listed in the tables below[2].

In the tables, the states are represented using the atomic notation;

$$n^{2S+1}L_J \tag{3.1}$$

where n is the principal quantum number, S is the total spin, L indicates the orbital angular momentum of the particle and J is the total angular momentum. This state has $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$.

In the tables the "Mass" and "J^{PC}" columns shows the states that have been obtained from the calculations of [2]. The "Particles (Exp.)" column shows the names given to the experimentally observed particles and "J^{PC} (Exp)", "I^G (Exp.)" and "Mass (MeV)" columns are their experimentally measured J^{PC}, I^G and mass values respectively.[3]

[1] The * sign before the particles in column six in the tables shows the particles that have been already found experimentally when the original paper was published in 1985.

[2] Only the mesons in PDG are considered.

[3] All the experimental data have been taken from [44]

In heavy quark meson tables there are three additional columns named as "Average (MeV)", "Shrödinger (Fulcher)" and "Salpeter (Fulcher)". In these columns the results that have been obtained in [1] are listed. The average column represents the average of the Schrödinger and Salpeter columns. As mentioned before, the results shown in these columns do not include spin effects in the potential. Therefore, for each L state from Eqn. 3.1, the solutions from [1] has only one mass value. This mass value is believed to be an average of all the masses in the respective state.

All the experimentally observed particles have been analyzed to find out if they can be expressed as a molecule of two other mesons. The names of the mesons that can be used to construct the observed meson are written in the "Bound State Candidate (BSC)" column. The criteria used to decide how these molecules are identified have been discussed in section 2.4.

As seen in the tables, some of the theoretically predicted particles are also observed by the experiments while some of them are not yet observed. And some experimentally found particles are not predicted by the theories. Those particles that are only observed experimentally, not theoretically indicate that there is something missing with the theory. They may also indicate the effects that were neglected due to their supposed lack of importance compared to the other effects.

In the following sections possible meson molecule candidates will be listed.

3.1 Light Mesons (S=C=B=0)

This group of mesons have the most number of molecular candidates. In the isoscalar sector, i.e $I = 0$, there are fourty three experimentally observed mesons and ten of them are meson molecule candidates. $\eta(1475)$, $f_0(1500)$, $\omega(1650)$, $\eta_2(1870)$, $f_0(2020)$, $f_2(2150)$, $\eta(2225)$, $f_2(2300)$, $f_0(2330)$, $f_2(2340)$ are possible molecules. In this sector, most molecular candidates have been gathered around higher masses.

$\eta(1475)$ is a possible molecular state of η and $f_0(980)$. Also, $f_0(1500)$ is a possible molecular state of η and $\eta'(958)$. $\omega(1650)$ is a possible molecular state of η and $h_1(1170)$. $\eta_2(1870)$ is a possible molecular state of η and $\eta(1295)$.

For $f_0(2020)$ there are more than one possibilities of being a molecular state. It can be a molecular state of η and $\eta(1295)$ or $f_0(980)$ and $f_0(980)$ or $\Phi(1020)$ and $\Phi(1020)$.

$f_2(2150)$ is a possible molecular state of η and $\eta_2(1645)$. $\eta(2225)$ is a possible molecular state of η and $f_0(1710)$. For $f_2(2300)$ there are also more than one possibilities of being molecular state. It can be both a molecular state of $h_1(1170)$ and $h_1(1170)$ and $f_0(980)$ and $f_2(1270)$.

$f_0(2330)$ is also on the same position. It can be a molecular state of η and $\eta(1760)$ or $\eta'(958)$ and $\eta(1405)$ or $h_1(1170)$ and $h_1(1170)$. The same thing goes for $f_2(2340)$. η and $\eta_2(1645)$, η and $\eta_2(1870)$, $h_1(1170)$ and $h_1(1170)$ and $f_0(980)$ and $f_2(1270)$ are all possibilities.

In the scalar isovector sector, i.e $I = 1$, there are twenty seven experimentally observed mesons and eight of them are possible molecular candidates. They are $\pi_1(1400)$, $a_0(1450)$, $\pi_1(1600)$, $\pi_2(1670)$, $a_2(1700)$, $\pi_2(1880)$, $\rho(1900)$ and $\pi_2(2100)$.

$\pi_1(1400)$ is a candidate for being a molecular state of $a_1(1260)$ and π^0. $a_0(1450)$ is a candidate for being a molecular state of $\pi(1300)$ and π^0. Similarly, $\pi_1(1600)$ is a candidate for being a molecular state of $f_1(1510)$ and π^0.

For $\pi_2(1670)$, there are two possibilities. It can be a molecular state of $f_2(1565)$ and π^0 or $f'_2(1525)$ and π^0. $\rho(1700)$, similarly, can be made of $h_1(1595)$ and π^0 or $\rho(770)$ and $a_0(980)$. $a_2(1700)$ is a possible molecular state of $\eta_2(1645)$ and π^0. $\pi_2(1880)$ is a possible molecular state of $a_2(1700)$ and π^0. $\rho(1900)$ is a candidate for being a molecular state of $\rho(770)$ and $a_1(1260)$. For $\pi_2(2100)$, again there are two possibilities which are $f_2(1950)$ and π^0, and $\rho(770)$ and $a_2(1320)$.

Table3.1: Light-Unflavored Mesons (S=C=B=0) with $I = 0(\eta, \eta', h, h', \omega, \phi, f, f') \to c_1(u\bar{u} + d\bar{d}) + c_2(s\bar{s})$.
The * sign before the particles in column six shows the particles that have been already found experimentally when the original paper was published in 1985.

States	*Mass* (GeV)	J^{PC}	J^{PC}/J^P (Exp.)	I^G (Exp.)	*Particle* (Exp.)	*Mass* (MeV)	*BSC*
1^1S_0	0.52	0^{-+}	0^{-+}	0^+	$*\eta$	547.853 ± 0.024	
			1^{--}	0^-	$*\omega(782)$	782.65 ± 0.12	
1^1S_0	0.96	0^{-+}	0^{-+}	0^+	$*\eta'(958)$	957.78 ± 0.06	
1^3P_0	1.09	0^{++}	0^{++}	0^+	$f_0(980)$	990 ± 20	
1^3S_1	1.02	1^{--}	1^{--}	0^-	$*\phi(1020)$	1019.455 ± 0.020	
1^1P_1	1.22	1^{+-}	1^{+-}	0^-	$*h_1(1170)$	1170 ± 20	
1^3P_1	1.24	1^{++}	1^{++}	0^+	$*f_1(1285)$	1282.1 ± 0.6	
1^3P_2	1.28	2^{++}	2^{++}	0^+	$*f_2(1270)$	1275.1 ± 1.2	
				0^+	$\eta(1295)$	1294 ± 4	
1^3P_0	1.36	0^{++}		$0^?$			
2^1S_0	1.44	0^{-+}	0^{-+}	0^+	$\eta(1405)$	1408.9 ± 2.4	
				0^+	$\eta(1475)$	1476 ± 4	η & $f_0(980)$
2^3S_1	1.46	1^{--}		$0^?$			
1^1P_1	1.47	1^{+-}		$0^?$			
1^3P_1	1.48	1^{++}	1^{++}	0^+	$f_1(1420)$	1426.4 ± 0.9	
			2^{++}	0^+	$f_2(1430)$	1436^{-16}_{+26}	
			0^{++}	0^+	$f_0(1500)$	1505 ± 6	η & $\eta'(958)$
			1^{++}	0^+	$f_1(1510)$	1518 ± 5	
1^3P_2	1.53	2^{++}	2^{++}	0^+	$*f_2{}'(1525)$	1525 ± 5	
			2^{++}	0^+	$f_2(1565)$	1562 ± 13	
2^1S_0	1.63	0^{-+}		$0^?$			
1^3D_1	1.66	1^{--}	1^{--}	0^-	$*\phi(1680)$	1680 ± 20	
			1^{+-}	0^-	$h_1(1595)$	1594^{-60}_{+18}	
			2^{++}	0^+	$f_2(1640)$	1639 ± 6	
			2^{-+}	0^+	$\eta_2(1645)$	1617 ± 5	
1^1D_2	1.68	2^{-+}		$0^?$			
2^3S_1	1.69	1^{--}	1^{--}	0^-	$*\omega(1650)$	1670 ± 30	η & $h_1(1170)$
1^3D_3	1.68	3^{--}	3^{--}	0^-	$*\omega_3(1670)$	1667 ± 4	
1^3D_2	1.70	2^{--}		$0^?$			
2^1P_1	1.78	1^{+-}		$0^?$			
2^3P_0	1.78	0^{++}	0^{++}	0^+	$f_0(1710)$	1720 ± 6	
			0^{-+}	0^+	$\eta(1760)$	1756 ± 9	
2^3P_2	1.82	2^{++}	2^{++}	0^+	$*f_2(1810)$	1815 ± 12	
2^3P_1	1.82	1^{++}		$0^?$			
			2^{-+}	0^+	$\eta_2(1870)$	1842 ± 8	η & $\eta(1295)$
1^3D_1	1.88	1^{--}		$0^?$			
1^3D_3	1.90	3^{--}	3^{--}	0^-	$\phi_3(1850)$	1854 ± 7	
1^3D_2	1.91	2^{--}		$0^?$			
			2^{++}	0^+	$f_2(1910)$	1903 ± 9	
			2^{++}	0^+	$f_2(1950)$	1944 ± 12	

Table3.2: Table 3.1 continued...

States	*Mass* (GeV)	J^{PC}	J^{PC} (Exp.)	I^G (Exp.)	*Particle* (Exp.)	*Mass* (MeV)	*BSC*
2^3P_0	1.99	0^{++}	0^{++}	0^+	$f_0(2020)$	1992 ± 16	η & $\eta(1475)$ $f_0(980)$ & $f_0(980)$ $\phi(1020)$ & $\phi(1020)$
1^3F_4	2.01	4^{++}	4^{++}	0^+	$^*f_4(2050)$	2018 ± 11	
2^1P_1	2.01	1^{+-}		$0^?$			
2^3P_1	2.03	1^{++}		$0^?$			
1^1F_3	2.03	3^{+-}		$0^?$			
2^3P_2	2.04	2^{++}		$0^?$			
1^3F_3	2.05	3^{++}		$0^?$			
1^3F_2	2.05	2^{++}	2^{++}	0^+	$f_2(2010)$	2011^{-80}_{+60}	
			0^{++}	0^+	$f_0(2100)$	2103 ± 8	
			2^{++}	0^+	$f_2(2150)$	2157 ± 12	η & $\eta_2(1645)$
			1^{--}	0^-	$\phi(2170)$	2175 ± 15	
			0^{++}	0^+	$f_0(2200)$	2189 ± 13	
1^3F_2 1^3F_4	2.24 2.20	2^{++} 4^{++}	$2^{++}(4^{++})$	0^+	$f_J(2220)$	2231.1 ± 3.5	
1^1F_3	2.22	3^{+-}		$0^?$			
1^3F_3	2.23	3^{++}		$0^?$			
			0^{-+}	0^+	$\eta(2225)$	2226 ± 16	η & $f_0(1710)$
1^3F_2	2.24	2^{++}	2^{++}	0^+	$f_2(2300)$	2297 ± 28	$h_1(1170)$ & $h_1(1170)$ $f_0(980)$ & $f_2(1270)$
1^3F_4	2.20	4^{++}	4^{++}	0^+	$f_4(2300)$	$2330 \pm 20 \pm 40$	
			0^{++}	0^+	$f_0(2330)$	2314 ± 25	η & $\eta(1760)$ $\eta'(958)$ & $\eta(1405)$ $h_1(1170)$ & $h_1(1170)$
1^3F_2	2.24	2^{++}	2^{++}	0^+	$f_2(2340)$	2339 ± 60	η & $\eta_2(1645)$ η & $\eta_2(1870)$ $h_1(1170)$ & $h_1(1170)$ $f_0(980)$ & $f_2(1270)$
			6^{++}	0^+	$f_6(2510)$	2469 ± 29	
1^1G_4	2.33	4^{-+}		$0^?$			
1^3G_4	2.34	4^{--}		$0^?$			
1^3G_5	2.47	5^{--}		$0^?$			
1^1G_4	2.51	4^{-+}		$0^?$			
1^3G_4	2.52	4^{--}		$0^?$			
1^3G_3	2.54	3^{--}		$0^?$			

Table3.3: Same as Table 3.1 but for mesons with $I = 1(\pi, b, \rho, a) \to u\bar{d}, (u\bar{u} - d\bar{d})/\sqrt{2}, d\bar{u}$.

States	*Mass* (GeV)	J^{PC}	J^{PC}/J^P (Exp.)	I^G (Exp.)	*Particle* (Exp.)	*Mass* (MeV)	*BSC*
			0^-	1^-	$\pi^\pm$	139.57018 ± 0.00035	
1^1S_0	0.15	0^{-+}	0^{-+}	1^-	$*\pi^0$	134.9766 ± 0.0006	
1^3S_1	0.77	1^{--}	1^{--}	1^+	$*\rho(770)$	775.49 ± 0.34	
1^3P_0	1.09	0^{++}	0^{++}	1^-	$a_0(980)$	980 ± 20	
1^1P_1	1.22	1^{+-}	1^{+-}	1^+	$*b_1(1235)$	1229.5 ± 3.2	
1^3P_1	1.24	1^{++}	1^{++}	1^-	$*a_1(1260)$	1230 ± 40	
2^1S_0	1.30	0^{-+}	0^{-+}	1^-	$*\pi(1300)$	1300 ± 100	
1^3P_2	1.31	2^{++}	2^{++}	1^-	$*a_2(1320)$	$1318.3^{-0.6}_{+0.5}$	
			1^{-+}	1^-	$\pi_1(1400)$	1354 ± 25	$a_1(1260)$ & π^0
2^3S_1	1.45	1^{--}	1^{--}	1^+	$*\rho(1450)$	1465 ± 25	
			0^{++}	1^-	$a_0(1450)$	1474 ± 19	$\pi(1300)$ & π^0
1^3D_1	1.66	1^{--}	1^{--}	1^+	$\rho(1570)$	1570 ± 70	
			1^{-+}	1^-	$\pi_1(1600)$	1662^{-9}_{+8}	$f_1(1510)$ & π^0
			1^{++}	1^-	$a_1(1640)$	1647 ± 22	
1^1D_2	1.68	2^{-+}	2^{-+}	1^-	$*\pi_2(1670)$	1672.2 ± 3.0	$f_2(1565)$ & π^0 $f_2'(1525)$ & π^0
1^3D_3	1.68	3^{--}	3^{--}	1^+	$*\rho_3(1690)$	1688.8 ± 2.1	
1^3D_2	1.70	2^{--}		$1^?$			
			1^{--}	1^+	$\rho(1700)$	1720 ± 20	$h_1(1595)$ & π^0 $\rho(770)$ & $a_0(980)$
			2^{++}	1^-	$a_2(1700)$	1732 ± 16	$\eta_2(1645)$ & π^0
2^1P_1	1.78	1^{+-}		$1^?$			
2^3P_1	1.82	1^{++}		$1^?$			
3^1S_0	1.88	0^{-+}	0^{-+}	1^-	$\pi(1800)$	1812 ± 12	
1^1D_2	1.89	2^{-+}	2^{-+}	1^-	$\pi_2(1880)$	1895 ± 16	$a_2(1700)$ & π^0
			1^{--}	1^+	$\rho(1900)$	1910 ± 10	$\rho(770)$ & $a_1(1260)$
			3^{--}	1^+	$\rho_3(1990)$	1982 ± 14	
1^1F_3	2.03	3^{+-}		$1^?$			
1^3F_3	2.05	3^{++}		$1^?$			
			4^{++}	1^-	$a_4(2040)$	1996^{-9}_{+10}	
2^1D_3	2.13	2^{-+}	2^{-+}	1^-	$*\pi_2(2100)$	2090 ± 29	$f_2(1950)$ & π^0 $a_2(1320)$ & $\rho(770)$
2^3D_2	2.15	1^{--}	1^{--}	1^+	$\rho(2150)$	2149 ± 17	
1^3G_5	2.30	5^{--}	5^{--}	1^+	$*\rho_5(2350)$	2330 ± 35	
1^1G_4	2.33	4^{-+}		$1^?$			
1^3G_4	2.34	4^{--}		$1^?$			
			6^{++}	1^-	$a_6(2450)$	2450 ± 130	

3.2 Strange Mesons

In strange mesons table, twenty seven mesons have been observed experimentally and eleven of them have been identified as molecular state candidates. There are twenty eight possibilities. $K^*(1410)$, $K_0^*(1430)$, $K(1460)$, $K^*(1680)$, $K_2(1770)$, $K(1830)$, $K_0^*(1950)$, $K_2^*(1980)$, $K_3(2320)$, $K_2(2250)$ and $K_4(2500)$ are candidates of meson molecules.

$K^*(1410)$ is a possible molecular state of $K_1(1270)$ and π^0. $K_0^*(1430)$ can be a molecular state of $\eta'(958)$ and $K^{\pm}$ or $\eta'(958)$ and K^0 or $K_0^*(800)$ and $K_0^*(800)$. $K(1460)$ is a possible molecular state of $a_0(980)$ and $K^{\pm}$. For $K^*(1680)$, there are five possibilities of being a molecular state. It can be a molecular state of $K_1(1650)$ and π^0 or $a_1(1260)$ and $K^{\pm}$ or $a_1(1260)$ and K^0 or $K_1(1270)$ and $K^{\pm}$ or $K_1(1270)$ and K^0. $K_2(1770)$ can be a molecular state of both $f_2(1270)$ and $K^{\pm}$, and $f_2(1270)$ and K^0. Similarly, $K(1830)$ is a possible molecular state of both $K^{\pm}$ and $K_0^*(1430)$, or K^0 and $K_0^*(1430)$. $K_0^*(1950)$ can be a molecular state of $K^{\pm}$ and $K_0^*(1460)$ or K^0 and $K(1460)$.

For $K_2^*(1980)$, there are three possibilities. It can be a molecular state of $f_2(1270)$ and $K_0^*(800)$ or π^0 and $K_1(1270)$ or π^0 and $K_2(1820)$. $K_3(2320)$ is a possible molecular state of η and $K_3^*(1780)$. For $K_2(2250)$, there are four possibilities. It can be a molecular state of $a_2(1700)$ and $K^{\pm}$ or $a_2(1700)$ and K^0 or $K_1(1400)$ and $K^*(892)$ or $K_0^*(800)$ and $K_2(1580)$.

Finally, for $K_4(2500)$ there are four different possibilities. It can be a molecular state of $f_4(2050)$ and $K^{\pm}$ or $f_4(2050)$ and K^0 or $K_4^*(2045)$ and $K^{\pm}$ or $K_4^*(2045)$ and K^0.

Table3.4: Strange Mesons ($K^+ = u\bar{s}, K^0 = d\bar{s}, \bar{K}^0 = s\bar{d}, K^- = s\bar{u}$ similarly for K^*'s).

States	*Mass* (GeV)	J^P	$I^G(J^P)$ (Exp.)	*Particle* (Exp.)	*Mass* (MeV)	*BSC*
1^1S_0	0.47	0^-	$1/2(0^-)$	$*K^\pm$	493.677 ± 0.016	
				$*K^0$	497.614 ± 0.024	
			$1/2(0^+)$	$K_0{}^*(800)$	682 ± 29	
1^3S_1	0.90	1^-	$1/2(1^-)$	$*\ K^*(892)$	891.66 ± 0.26	
1^3P_0	1.24	0^+				
1^3P_1	1.38	1^+	$1/2(1^+)$	$*K_1(1270)$	1272 ± 7	
1^1P_1	1.34	1^+	$1/2(1^+)$	$*K_1(1400)$	1403 ± 7	
			$1/2(1^-)$	$K^*(1410)$	1414 ± 15	$K_1(1270)$ & π^0
			$1/2(0^+)$	$K_0{}^*(1430)$	1425 ± 50	$\eta'(958)$ & $K^\pm$
						$\eta'(958)$ & K^0
						$K_0{}^*(800)$ & $K_0{}^*(800)$
1^3P_2	1.43	2^+	$1/2(2^+)$	$*K_2{}^*(1430)^\pm$	1425.6 ± 1.5	
				$*K_2{}^*(1430)^0$	1432.4 ± 1.3	
2^1S_0	1.45	0^-	$1/2(0^-)$	$K(1460)$	1460	$a_0(980)$ & $K^\pm$
			$1/2(2^-)$	$K_2(1580)$	1580	
			$1/2(?^?)$	$K(1630)$	1629 ± 7	
			$1/2(1^+)$	$K_1(1650)$	1650 ± 50	
1^3D_1	1.78	1^-	$1/2(1^-)$	$K^*(1680)$	1717 ± 27	$K_1(1650)$ & π^0
						$a_1(1260)$ & $K^\pm$
						$a_1(1260)$ & K^0
						$K_1(1270)$ & $K^\pm$
						$K_1(1270)$ & K^0
1^1D_2	1.78	2^-	$1/2(2^-)$	$*K_2(1770)$	1773 ± 8	$f_2(1270)$ & $K^\pm$
						$f_2(1270)$ & K^0
1^3D_3	1.79	3^-	$1/2(3^-)$	$*K_3{}^*(1780)$	1776 ± 7	
1^3D_2	1.81	2^-	$1/2(2^-)$	$K_2(1820)$	1816 ± 13	
			$1/2(0^-)$	$K(1830)$	1830	$K^\pm$ & $K_0{}^*(1430)$
						K^0 & $K_0{}^*(1430)$
2^3P_0	1.89	0^+	$1/2(0^+)$	$K_0{}^*(1950)$	1945 ± 22	$K^\pm$ & $K(1460)$
						K^0 & $K(1460)$
2^1P_1	1.90	1^+				
2^3P_1	1.93	1^+				
2^3P_2	1.94	2^+	$1/2(2^+)$	$K_2{}^*(1980)$	1973 ± 26	$f_2(1270)$ & $K_0{}^*(800)$
						π^0 & $K_1(1270)$
						π^0 & $K_2(1820)$
3^1S_0	2.02	0^-				
3^3S_1	2.11	1^-				
1^3F_4	2.11	4^+	$1/2(4^+)$	$*K_4{}^*(2045)$	2045 ± 9	
1^3F_2	2.15	2^+				
1^3F_3	2.15	3^+	$1/2(3^+)$	$K_3(2320)$	2324 ± 24	η & $K_3{}^*(1780)$
1^1F_3	2.12					
2^1D_2	2.23	2^-	$1/2(2^-)$	$*K_2(2250)$	2247 ± 17	$a_2(1700)$ & $K^\pm$
						$a_2(1700)$ & K^0
2^3D_2	2.26					$K_1(1400)$ & $K^*(892)$
						$K_0{}^*(800)$ & $K_2(1580)$
2^3D_3	2.24	3^-				
2^3D_1	2.25	1^-				
1^3G_5	2.39	5^-	$1/2(5^-)$	$K_5{}^*(2380)$	2382 ± 24	
1^3G_4	2.44	4^-	$1/2(4^-)$	$K_4(2500)$	2490 ± 20	$f_4(2050)$ & $K^\pm$
						$f_4(2050)$ & K^0
1^1G_4	2.41					$K_4{}^*(2045)$ & $K^\pm$
						$K_4{}^*(2045)$ & K^0
1^3G_3	2.46	3^-				
			?	$K(3100)$	3100 ± 11	

3.3 Charmed Mesons

There are fifteen observed mesons and two molecular state possibilities, $D_0^*(2400)^{\pm}$ and $D_1^0(2430)$, in the Charmed Mesons table. The reason few candidates have been detected may be because the more massive states have not been observed yet. The first one, $D^*{}_0(2400)^{\pm}$, can be a molecular state of $D^{\pm}$ and η. The second one, $D_1^0(2430)$, can be a molecular state of D^0 and η.

Table3.5: Same as Table 3.1 but for Charmed Mesons ($D^+ = c\bar{d}, D^0 = c\bar{u}, \bar{D}^0 = u\bar{c}, D^- = d\bar{c}$ similarly for D^*'s).

States	*Mass* *(GeV)*	J^P	$I^G(J^P)$ *(Exp.)*	*Particle* *(Exp.)*	*Mass* *(MeV)*	*Average* *(MeV)*	*Shrodinger* *(Fulcher)*	*Salpeter* *(Fulcher)*	*BSC*
			$1/2(0^-)$	D^0	1864.86 ± 0.13				
1^1S_0	1.88	0^-	$1/2(0^-)$	$*D^\pm$	1869.62 ± 0.15	1938.3	1973	1973	
1^3S_1	2.04	1^-	$1/2(1^-)$	$*D^*(2007)^0$	2006.98 ± 0.15				
			$1/2(1^-)$	$D^*(2010)^\pm$	2010.28 ± 0.13				
			$1/2(0^+)$	$D_0^*(2400)^0$	2318 ± 29				
			$1/2(?^?)$	$D_1(2420)^\pm$	2423.4 ± 3.1				
1^3P_0	2.40	0^+	$1/2(0^+)$	$D_0^*(2400)^\pm$	2403 ± 40	2428.58	2474	2456	$D^\pm$ & η
1^1P_1	2.44	1^+	$1/2(1^+)$	$D_1(2420)^0$	2421.3 ± 0.6				
1^3P_1	2.49	1^+	$1/2(1^+)$	$D_1(2430)^0$	2427 ± 40				D^0 & η
1^3P_2	2.50	2^+	$1/2(2^+)$	$D_2^*(2460)^0$	2462.6 ± 0.7				
				$D_2^*(2460)^\pm$	2464.4 ± 1.9				
2^1S_0	2.58	0^-	$1/2(0^-)$	$D(2550)^0$	2539 ± 8	2539.4	2801	2618	
2^3S_1	2.64	1^-							
1^3D_1	2.82	1^-							
1^3D_3	2.83	3^-							
			$1/2(?^?)$	$D(2600)$	2612 ± 6				
			$1/2(?^?)$	$D^*(2640)^\pm$	2637 ± 6				
			$1/2(?^?)$	$D(2750)$	2761 ± 5				
1^3F_4	3.11	4^+							

3.4 Bottom-Charmed, Charmed-Strange, Bottom, Bottom-Strange Mesons

In the meson spectroscopy, bottom-charmed (1)[4], charmed-strange (9), bottom (6) and bottom-strange (5) mesons have been scantily observed. Therefore there are no expected meson molecules in these sectors.

[4] Number of experimentally observed mesons in the relative meson table are given in the parentheses

Table3.6: Bottom-Charmed Mesons, ($B_c^+ = c\bar{b}, B_c^- = b\bar{c}$ similarly for B_c^*'s).

States	*Mass* (*GeV*)	J^P	$I^G(J^P)$ (*Exp.*)	*Particle* (*Exp.*)	*Mass* (*MeV*)	*Average* (*MeV*)	*Shrodinger* (*Fulcher*)	*Salpeter* (*Fulcher*)	*BSC*
1^1S_0	6.27	0^-	$0(0^-)$	$B_c^\pm$	6227 ± 6	6277	6338	6314	
1^3S_1	6.34	1^-							
1^3P_2	6.77	2^+							
2^1S_0	6.85	0^-							
2^3S_1	6.89	1^-							
1^3D_3	7.04	3^-							
1^3F_4	7.27	4^+							

Table3.7: Same as Table 3.1 but for Charmed-Strange Mesons, $(D_s^+ = c\bar{s}, D_s^- = s\bar{c}$ similarly for D_s^*'s).

States	*Mass* (*GeV*)	J^P	$I^G(J^P)$ (*Exp.*)	*Particle* (*Exp.*)	*Mass* (*MeV*)	*Average* (*MeV*)	*Shrodinger* (*Fulcher*)	*Salpeter* (*Fulcher*)	*BSC*
1^1S_0	1.98	0^-	$0(0^-)$	$^*D_s{}^{\pm}$	1968.49 ± 0.32	2040.4	2075	2076	
1^3S_1	2.13	1^-	$0(?^?)$	$D_s^{*\pm}$	2112.3 ± 0.5				
			$0(1^+)$	$D_{s1}(2460)^{\pm}$	2459.6 ± 0.6				
1^3P_0	2.48	0^+	$0(0^+)$	$D_{s0}^*(2317)^{\pm}$	2317.8 ± 0.6	2474.94.4	2535	2545	
1^1P_1	2.53	1^+	$0(1^+)$	$D_{s1}(2536)^{\pm}$	2535.12 ± 0.13				
1^3P_1	2.57	1^+							
1^3P_2	2.59	2^+	$0(2^+)$	$D_{s2}^*(2573)$	2571.9 ± 0.8				
2^1S_0	2.67	0^-				2709.3	2774	2714	
2^3S_1	2.73	1^-	$0(1^-)$	$D_{s1}^*(2700)^{\pm}$	2709.3^{-6}_{+9}				
1^3D_1	2.90	1^-	$0(?^?)$	$D_{sJ}^*(2860)^{\pm}$	$2862^{-2.8}_{+5.4}$	2862 ± 2	2871		
1^3D_3	2.92	3^-							
1^3D_3	2.92	3^-	$0(?^?)$	$D_{sJ}(3040)^{\pm}$	3044^{-9}_{+31}				
1^3F_4	3.19	4^+							

Table3.9: Bottom-Strange Mesons, ($B_s^0 = s\bar{b}, \bar{B}_s^0 = b\bar{s}$ similarly for K^*'s).

States	*Mass* (*GeV*)	J^P	$I^G(J^P)$ (*Exp.*)	*Particle* (*Exp.*)	*Mass* (*MeV*)	*Average* (*MeV*)	*Shrodinger* (*Fulcher*)	*Salpeter* (*Fulcher*)	*BSC*
1^1S_0	5.39	0^-	$0(0^-)$	B_s^0	5366.77 ± 0.24	5391.08	5383	5405	
1^3S_1	5.45	0^-	$0(1^-)$	B_s^*	$5415.4^{-2.1}_{+2.4}$				
			$0(1^+)$	$B_{s1}(5830)^0$	5829.4 ± 0.7				
1^3P_2	5.88	2^+	$0(2^+)$	$B_{s2}^*(5840)^0$	5839.7 ± 0.6	5839.7	5823	5859	
2^1S_0	5.98	0^-	$?(?^?)$	$B_{sJ}^*(5850)$	5853 ± 15	5853 ± 15	6149	5982	
2^3S_1	6.01	1^-							
1^3D_3	6.18	3^-							
1^3F_4	6.43	4^+							

Table3.8: Same as Table 3.1 but for Bottom Mesons, ($B^+ = u\bar{b}, B^0 = d\bar{b}, \bar{B}^0 = b\bar{d}, B^- = b\bar{u}$ similarly for B^*'s).

States	*Mass* (*GeV*)	J^P	$I^G(J^P)$ (*Exp.*)	*Particle* (*Exp.*)	*Mass* (*MeV*)	*Average* (*MeV*)	*Shrodinger* (*Fulcher*)	*Salpeter* (*Fulcher*)	*BSC*
1^1S_0	5.31	0^-	$1/2(0^-)$	$*B^{\pm}$	5279.25 ± 0.17	5302.22	5313	5313	
1^3S_1	5.37	1^-	$1/2(1^-)$	B^*	5325.2 ± 0.4				
			$1/2(0^-)$	B^0	5279.58 ± 0.17				
			$1/2(1^+)$	$B_1(5721)^0$	5723.5 ± 2.0				
1^3P_2	5.80	2^+	$1/2(2^+)$	$B_2^*(5747)^0$	5743 ± 5	5743	5798	5779	
2^1S_0	5.90	0^-	$?(?^?)$	$B_J^*(5732)$	5698 ± 8	5698 ± 8	6177	5892	
2^3S_1	5.93	1^-							
1^3D_3	6.11	3^-							
1^3F_4	6.36	4^+							

3.5 Charmonium Mesons

In the charmonium sector, twenty five mesons have been observed by experiments. Most of them have been predicted by theories and five of them are candidate molecular states; $X(3872)$, $\Psi(4160)$, $X(4360)$ and $X(4660)$. There are two possibilities for $X(3872)$. It can be a possible molecular state of D^0 and $D^*(2007)$ or $J/\Psi(1S)$ and $\omega(782)$. $\Psi(4160)$ is a possible molecular state of $h_1(1170)$ and $\eta_c(1S)$. $X(4360)$ is a possible molecular state of $a_1(1260)$ and $J/\Psi(1S)$. Finally, $X(4660)$ is a possible molecular state of $f_2(1565)$ and $J/\Psi(1S)$.

Table3.10: Charmonium Mesons 1, $(c\bar{c})$.

The * sign before the particles in column five shows the particles that have been already found experimentally when the original paper was published in 1985.

States	*Mass* (GeV)	J^{PC}	$I^G(J^{PC})$ (Exp.)	*Particle* (Exp.)	*Mass* (MeV)	*Average* (MeV)	*Shrodinger* (*Fulcher*)	*Salpeter* (*Fulcher*)	*BSC*
1^1S_0	2.97	0^{-+}	$0^+(0^{-+})$	$*\eta_c(1S)$	2981.0 ± 1.1	3038.96	3068	3070	
1^3S_1	3.10	1^{--}	$0^-(1^{--})$	$*J/\psi(1S)$	3096.916 ± 0.011				
1^3P_0	3.44	0^{++}	$0^+(0^{++})$	$*\chi_{c0}(1P)$	3414.75 ± 0.31	3501.75	3498	3507	
1^3P_1	3.51	1^{++}	$0^+(1^{++})$	$*\chi_{c1}(1P)$	3510.66 ± 0.07				
1^1P_1	3.52	1^{+-}	$?^?(1^{+-})$	$h_c(1P)$	3525.41 ± 0.16				
1^3P_2	3.55	2^{++}	$0^+(2^{++})$	$*\chi_{c2}(1P)$	3556.20 ± 0.09				
2^1S_0	3.62	0^{-+}	$0^+(0^{-+})$	$*\eta_c(2S)$	3638.9 ± 1.3	3692.50	3694	3670	
2^3S_1	3.68	1^{--}	$0^-(1^{--})$	$*\psi(2S)$	$3686.109^{-0.014}_{+0.012}$				
1^3D_1	3.82	1^{--}	$0^-(1^{--})$	$\psi(3770)$	3773.15 ± 0.33	3822.42	3807		
1^1D_2	3.84	2^{-+}							
1^3D_2	3.84	2^{--}							
1^3D_3	3.85	3^{--}							
2^3P_0	3.92	0^{++}	$0^+(?^{?+})$	$X(3915)$	3917.5 ± 2.7	3928	3992	3972	
2^1P_1	3.96	2^{++}	$0^+(2^{++})$	$\chi_{c2}(2P)$	3927.2 ± 2.6				
2^3P_2	3.98	2^{++}	$?^?(?^{??})$	$X(3940)$	3942 ± 9				
2^3P_1	3.95	1^{++}	$0^+(1^{++})$	$X(3872)$	3871.68 ± 0.17	3928	3992	3972	D^0 & $D^*(2007)^0$
									$J/\psi(1S)$ & $\omega(782)$

Table3.11: Table 3.10 continued...

States	*Mass* (GeV)	J^{PC}	$I^G(J^{PC})$ (Exp.)	*Particle* (Exp.)	*Mass* (MeV)	*BSC*
1^1F_3	4.09	3^{+-}				
1^3F_3	4.10	3^{++}				
1^3F_4	4.09	4^{++}				
3^3S_1	4.10	1^{--}	$0^-(1^{--})$	*$\psi(4040)$	4039 ± 1	
3^1S_0	4.06	0^{-+}	$?^?(?^?)$	$\psi(4050)^{\pm}$	4051^{-40}_{+24}	
1^3F_2	4.09	2^{++}				
			$0^+(?^{?+})$	$X(4140)$	4143.0 ± 3.1	
			$?^?(?^{??})$	$X(4160)$	4156^{-25}_{+29}	
			$?^?(?^?)$	$X(4250)^{\pm}$	4248^{-50}_{+190}	
2^3D_1	4.19	1^{--}	$0^-(1^{--})$	*$\psi(4160)$	4153 ± 3	$h_1(1170)$ & $\eta_c(1S)$
			$?^?(1^{--})$	$X(4260)$	4263^{-9}_{+8}	
2^1D_2	4.21	2^{-+}				
2^3D_2	4.21	2^{--}				
2^3D_3	4.22	3^{--}				
			$0^+(?^{?+})$	$X(4350)$	4351 ± 5	
			$?^?(1^{--})$	$X(4360)$	4361 ± 13	$a_1(1260)$ & $J/\psi(1S)$
4^3S_1	4.45	1^{--}	$0^-(1^{--})$	*$\psi(4415)$	4421 ± 4	
			$?(?^?)$	$X(4430)^{\pm}$	4443^{-18}_{+24}	
3^3D_1	4.52	1^{--}				
			$?^?(1^{--})$	$X(4660)$	4664 ± 12	$f_2(1565)$ & $J/\psi(1S)$

3.6 Bottomonium Mesons

In this sector, nineteen mesons have been observed experimentally and there are seven possibilities of molecular states in the bottomonium spectra. The three meson molecule candidates are $\Upsilon(4S)$, $\Upsilon(10860)$ and $\Upsilon(11020)$. $\Upsilon(4S)$ is a possible molecular state of $h_1(1170)$ and $\eta_b(1S)$. $\Upsilon(10860)$ is a possible molecular state of $\Phi(1020)$ and $\chi_{b0}(1P)$.

For $\Upsilon(11020)$, there are five possibilities. It can be a molecular state of $\rho(770)$ and $\Upsilon(2S)$ or $\chi_{b0}(2P)$ and $\rho(770)$ or $f_0(980)$ [45, 46] and $\Upsilon(2S)$ or $a_0(980)$ and $\Upsilon(2S)$ or $f_2(1565)$ and $\Upsilon(1S)$.

In total, seventy possibilities for molecular states are found in the context of this thesis.

Table3.12: Bottomonium Mesons 1, $(b\bar{b})$

The * sign before the particles in column five shows the particles that have been already found experimentally when the original paper was published in 1985.

States	*Mass* (GeV)	J^{PC}	$I^G(J^{PC})$ (*Exp.*)	*Particle* (*Exp.*)	*Mass* (MeV)	*Average* (MeV)	*Shrodinger* (*Fulcher*)	*Salpeter* (*Fulcher*)	*BSC*
1^1S_0	9.40	0^{-+}	$0^+(0^{-+})$	$\eta_b(1S)$	9391.0 ± 2.8	9425.65	9447	9449	
1^3S_1	9.46	1^{--}	$0^-(1^{--})$	$*\Upsilon(1S)$	9460.30 ± 0.26				
1^3P_0	9.85	0^{++}	$0^+(0^{++})$	$*\chi_{b0}(1P)$	$9859.44 \pm 0.42 \pm 0.31$	9890.76	9900	9901	
1^3P_1	9.88	1^{++}	$0^+(1^{++})$	$*\chi_{b1}(1P)$	$9892.78 \pm 0.26 \pm 0.31$				
1^1P_1	9.88	1^{+-}	$?^?(1^{+-})$	$h_b(1P)$	9898.6 ± 1.4				
1^3P_2	9.90	2^{++}	$0^+(2^{++})$	$*\chi_{b2}(1P)$	$9912.21 \pm 0.26 \pm 0.31$				
2^1S_0	9.98	0^{-+}				10023.26	10006	10000	
2^3S_1	10.00	1^{--}	$0^-(1^{--})$	$*\Upsilon(2S)$	10023.26 ± 0.31				
1^3D_1	10.14	1^{--}				10163.7 ± 1.4	10147.1		
1^1D_2	10.15	2^{-+}							
1^3D_2	10.15	2^{--}	$0^-(2^{--})$	$\Upsilon(1D)$	10163.7 ± 1.4				
1^3D_3	10.16	3^{--}							
2^3P_0	10.23	0^{++}	$0^+(0^{++})$	$*\chi_{b0}(2P)$	$10232.5 \pm 0.4 \pm 0.5$	10254.06	10260	10263	
2^3P_1	10.25	1^{++}	$0^+(1^{++})$	$*\chi_{b1}(2P)$	$10255.46 \pm 0.22 \pm 0.5$				
2^1P_0	10.25	1^{+-}	$?^?(1^{+-})$	$h_b(2P)$	$10259.8^{-1.2}_{+1.5}$				
2^3P_3	10.26	2^{++}	$0^+(2^{++})$	$*\chi_{b2}(2P)$	$10268.65 \pm 0.22 \pm 0.50$				
3^1S_0	10.34	0^{-+}				10355.2	10356	10352	
3^3S_1	10.35	1^{--}	$0^-(1^{--})$	$*\Upsilon(3S)$	10355.2 ± 0.5				

Table3.13: Table 3.12 continued...

States	*Mass* (*GeV*)	J^{PC}	$I^G(J^{PC})$ (*Exp.*)	*Particle* (*Exp.*)	*Mass* (*MeV*)	*Average* (*MeV*)	*Shrodinger* (*Fulcher*)	*Salpeter* (*Fulcher*)	*BSC*
1^3F_2	10.35	2^{++}							
1^1F_3	10.35	3^{+-}							
1^3F_3	10.35	3^{++}							
1^3F_4	10.36	4^{++}							
2^3D_1	10.44	1^{--}							
2^1D_2	10.45	2^{-+}							
2^3D_2	10.45	2^{--}							
2^3D_3	10.45	3^{--}							
			$?^?(?^{?+})$	$\chi_b(3P)$	10530 ± 10				
4^3S_1	10.63	1^{--}	$0^-(1^{--})$	$*\Upsilon(4S)$	10579.4 ± 1.2	10579.4	10642	10640	$h_1(1170)$ & $\eta_b(1S)$
			$?^+(1^+)$	$X(10610)^{\pm}$	10607.2 ± 2.0				
			$?^?(1^+)$	$X(10650)^{\pm}$	10652.2 ± 1.5				
3^3D_1	10.70	1^{--}							
5^3S_1	10.88	1^{--}	$0^-(1^{--})$	$\Upsilon(10860)$	10876 ± 11	10876	10908	10907	$\phi(1020)$ & $\chi_{b0}(1P)$
6^3S_1	11.10	1^{--}	$0^-(1^{--})$	$\Upsilon(11020)$	11019 ± 8	11019	11216	11159	$\rho(770)$ & $\Upsilon(2S)$
									$\chi_{b0}(2P)$ & $\rho(770)$
									$f_0(980)$ & $\Upsilon(2S)$
									$a_0(980)$ & $\Upsilon(2S)$
									$f_2(1565)$ & $\Upsilon(1S)$

CHAPTER 4

CONCLUSIONS

In this work, meson spectra and the possible molecular states have been investigated.

In Chapter 2, quark model was generally described. Then two different approaches to the calculations of particle masses have been studied. In order to do this, two papers [1] and [2] have been discussed. In the context of these two papers, three different Hamiltonians have been used. They have been solved by diagonalization method. The first two had the same central potential and two different kinetic energy terms. These two equations with their different approach of kinetic energy were called Schrödinger and spinless Bethe-Salpeter equations. Since the spin-dependent potential term had been neglected, the mass splitting had not been observed in [1]. The third Hamiltonian had a different potential that included the relativistic effects. This way, the masses of the mesons have been calculated by using three different Hamiltonians and the results have been listed and compared with the experimental results in the tables in Chapter 3.

The molecular state possibilities also have been calculated by using theoretical methods to predict the possible quantum numbers and masses of states that consist of two different known mesons. They are also listed in the tables in Chapter 3.

Note that most of the identified molecule candidates do not correspond to a state predicted by the quark model. Although such a criteria was not imposed in the selection of the candidates, this strengthens the claims that they are actually molecular states.

Note also that there are still states that are not predicted by the theory, and they are not identifiable as a molecule. Such states can be other types of exotic mesons, or even molecules of three mesons.

Lastly there are mesons that can be identified by a state predicted by the quark model and also can be identified by one or more molecular states. In such cases, it is likely that the physical meson is a superposition of all these possibilities.

All mesons should be investigated in more detail as studied in [30, 31, 32] to determine their real internal structure.

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