OPTIMIZATION OF FEEDBACK IN A MULTIUSER MISO COMMUNICATION DOWNLINK WITH ENERGY HARVESTING USERS

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ABSTRACT

OPTIMIZATION OF FEEDBACK IN A MULTIUSER MISO COMMUNICATION DOWNLINK WITH ENERGY HARVESTING USERS

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We study the optimization of the number of bits allocated by energy harvesting users for sending feedback to a common multiple-antenna access point (AP). The nodes need to distribute their feedback transmissions judiciously across time (and channel states) in order to maximize certain throughput goals. While the MISO channel capacity from the AP to a user is a strictly increasing function of the number of feedback bits sent by the user to the AP for providing channel state information, the energy consumption for sending this feedback is (assumed to be) directly proportional to the number of feedback bits. Considering long term throughput, the nodes need to adapt the number of bits of feedback to their energy harvesting profiles.

Keywords: Energy Harvesting, MISO downlink, Multiuser communication, Channel capacity

ÖZ

ENERJİ HARMANLAYAN ÇOK-KULLANICILI TEK-ÇIKTILI HABERLEŞME
İNİŞ YOLUNDA GERİ BESLEME ENIYILEŞTİRİLMESİ

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Bu çalışmada, enerji hasat kullanıcılarının ortak çok antenli erişim noktasına geri bildirim göndermek için kullandığı bit sayısı optimize edilmeye çalışıldı. Düğümler veri hacmini en yüksek seviyede tutmak için zaman boyunca (kanal durumuna göre) geri bildirim yayınlarını akıllıca kullanmak durumundadırlar. Erişim noktasından (AP) kullanıcıya olan MISO kanal kapasitesi kullanıcıdan erişim noktasına kanal durum bilgisini gönderen geri bildirim bit sayısının artan bir fonksiyonu olmasına rağmen, geri bildirim enerji tüketiminin kullanılan geri bildirim bit sayısıyla doğru orantılı olduğu kabul edilir. Uzun süreli veri hacmi göz önüne alındığında, düğümlerin enerji hasat profillerine göre geri bildirim bit sayılarını ayarlamaları gerekir.

Anahtar Kelimeler: enerji hasadı, ÇKTÇ iniş yolu, çok kullanıcılı iletişim, kanal kapasitesi

To my beloved family

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AP	Access Point
AWGN	Additive White Gaussian Noise
BLE	Bluetooth Low Energy
CSI	Channel State Information
CSIT	Channel State Information at the Transmitter
EH	Energy Harvesting
LP	Linear Programming
M2M	Machine to Machine
MIA	Mutual Information Accumulation
MISO	Multi Input Single Output
PFS	Proportional Fair Scheduling
Rx	Receiver
Tx	Transmitter
WSN	Wireless Sensor Network

CHAPTER 1

INTRODUCTION

For large scale distributed device networks such as sensor networks and M2M networks, Energy Harvesting (EH) is a very promising technology, especially when the networked devices use low transmit power and send at very low data rates. Consider, for example, the following scenario: in an M2M network, nodes need to be kept updated with information of other nodes by a central AP continuously. Control signals require a significant amount of data transmission in the downlink. As the Access Point (AP) is connected to the power grid, it has no major energy constraint. However the sensor nodes, designed for energy-neutral operation (spending about as much energy as they harvest from the environment) have to schedule their operations based on the state of their batteries.

One of the main reasons why energy harvesting introduces a challenge for communication networks is that operations at different networking layers, i.e. coding, power control, scheduling, etc. need to be adapted to the rate of energy harvesting, which may be sporadic and hard to predict [1–9].

In energy harvesting networks, the lifetime of the system can be extended without replacing the batteries. However, the harvested energy is non-uniform across time and hard to predict which poses a new challenge on system designing. In [10] the authors consider a point to point fading channel with EH transmitter and their approach is finding a policy to maximize the throughput by a deadline. In [11] the approach is extended to broadcast transmission [12][13] and relays transmission. Imperfections in system models such as discrete power transmission [14] and battery leakage [15] have also been studied.

In Wireless Sensor Networks(WSN), because of unpredictability of energy sources, one of the main challenging issues is designing an adaptive duty cycling procedure that lets sensor nodes to maintain their power supply at sufficient state (normal energy operation) by adapting to changes in environmental conditions [16]. In [17] the authors propose an algorithm to increase the network lifetime by arranging the duty cycling of wireless sensor nodes and they also evaluate the performance by implementing it on a new communication device, Bluetooth Low Energy (BLE). In [18] the authors implement different transmission scheduling policies on USRPs.

In all the mentioned works, authors consider perfect channel knowledge at transmitter side without any extra cost. However in case of an EH receiver, for providing the channel information to transmitter, receiver needs to manage its feedback with respect to its energy profile.

In particular, in a multiple antenna communication link where Channel State Information (CSI) is crucial for approaching channel capacity, the quantized link state information sent by users to a base station may have to be adapted to the energy budget of the users. Note that in the Multi Input Single Output (MISO) channel (with m antennas at the transmitter and one at the receiver), with full channel state information at transmitter (CSIT), maximum transmission rate can be achieved by beamforming along the $m \times 1$ channel gain vector h (whose magnitude is $\|h\|$). The ergodic capacity of the channel for a given transmitting power P is [19]

$$C_{CSIT}(P) = E_h \left[\log\left(1 + P\|h\|^2\right)\right], \tag{1.1}$$

where $E_h[.]$ denotes the expectation over h.

When the transmitter has no CSI, and only knowledge of channel statistics are available, with assumption the channels gain distributions are iid, the optimum transmission policy uses the same power at each transmitting antenna. The ergodic channel capacity for this case is

$$C_{NO-CSIT}(P) = E_h \left[\log\left(1 + \frac{P}{m}\|h\|^2\right)\right]. \tag{1.2}$$

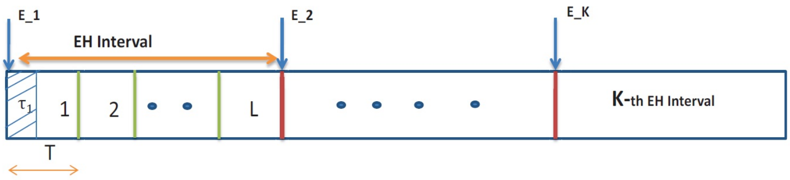

Figure 1.1: Energy harvesting time frame structure.

When the transmitter has partial CSI (gained through feedback through b bits generated by random vector quantization [20]), and the number of transmitting antennas is large, the achievable ergodic rate approximately[1] is [21]

$$R_{FB}(P) = E_h \left[\log \left(1 + P\|h\|^2 \left(1 - 2^{\left(\frac{-b}{m-1}\right)} \right) \right) \right]. \tag{1.3}$$

According to the above formula by sending more bits of feedback the achievable channel rate will be increased. However, in [22], the authors show that feedback transmission needs to be designed judiciously since it reduces the allocated transmission time in each frame and also it consumes energy at user.

The work of Gangula, Gesbert and Gunduz [23] is the closest to ours, where the authors consider a point to point MISO fading channel. In their system model a multi antenna transmitter is connected to a power source whereas the single antenna receiver scavenges energy from the environment. The total communication time is divided to K equal length EH intervals and between each pair of energy arrivals there are L data frames. The energy packets will arrive at the beginning of the EH interval and are known by Rx. The energy arrival time frame structure has been shown in Fig 1.1, where E_i denotes the amount of energy arriving in the beginning of EH interval i (E_i with the same definition also is used in some parts of this thesis).

The authors [23] assume that the receiver has complete knowledge of channel and in each data frame k, the receiver sends quantized channel information through an AWGN feedback channel, for a duration of τ_k time units. In the rest of the frame of length T, Tx sends data to Rx.

[1] Numerical results show that the approximation is tight if b and/or m are large enough.

The achievable ergodic rate in the k^{th} EH interval is

$$R_k = \left(1 - \frac{\tau_k}{T}\right) \times E_{h,w_k} \left[\log\left(1 + \frac{P}{1 - \frac{\tau_k}{T}} \|h\|^2 \cos^2\left(\angle\left(h, \widehat{h}\right)\right)\right)\right], \tag{1.4}$$

where $\widehat{h}$ is the quantized version of the h vector. The above expression is intractable so in the formulation of the problem the authors use the upper bound of ergodic rate for m transmitting antenna and a single receiving antenna

$$R_k^U = \left(1 - \frac{\tau_k}{T}\right) \log\left[1 + \frac{Pm}{1 - \frac{\tau_k}{T}} \left(1 - \left(\frac{m-1}{m}\right) 2^{\frac{-b_k}{m-1}}\right)\right], \tag{1.5}$$

where b_k is the number of bits of feedback in frame k. By using the AWGN feedback channel model, the number of bits can be written as a function of energy consumption and the number of channel usages as

$$b_k = \tau_k \log\left(1 + \frac{Q_k}{\tau_k \sigma^2}\right), \tag{1.6}$$

where Q_k is the amount of energy Rx consumes in frame k and σ^2 is the noise variance. From equations 1.5 and 1.6 the ergodic rate could be written as

$$R_k^U = \left(1 - \frac{\tau_k}{T}\right) \log\left[1 + \frac{Pm}{1 - \frac{\tau_k}{T}} \left(1 - \frac{m-1}{m} \left(1 + \frac{Q_k}{\tau_k \sigma^2}\right)^{\frac{-\tau_k}{m-1}}\right)\right]. \tag{1.7}$$

The authors [23] formulate the problem of maximizing the throughput achieved by optimizing τ_k and feedback rate in a finite amount of time under energy (harvesting) constraints at the receiver side as

$$\begin{aligned} \max_{Q_k,\tau_k} U &= \sum_{k=1}^{K} R_k^U \qquad (1.8) \\ \text{s. t.} & \\ L \sum_{i=1}^{l} Q_i &\leq \sum_{i=1}^{l} E_i, l = 1, ..., K \\ 0 \leq \tau_k \leq T \quad &, \quad \text{and } Q_k \geq 0, k = 1, ..., K, \end{aligned}$$

where U is total throughput of the system and L is the number of frames between two energy arrivals.

The authors [23] show that the optimization problem is concave function of Q_k and τ_k. Therefore by applying the algorithms which have been proposed in literature for optimal energy allocation with the energy harvesting constraint, the optimal value of Q_k an τ_k can be found.

In this thesis, we consider the optimization of the amount of feedback in a multiuser MISO system with energy harvesting users in order to maximize the expected throughput of the users. Here we assume error-free feedback channel. In our system model, in each frame a portion of time is allocated to feedback and therefore it would remain unused, if the user does not send feedback. Furthermore, the power dedicated to feedback transmission is assumed to be fixed. Hereafter, our optimization on the number of bits allocated for feedback, b, will be based on the approximate rate given in Eq. 1.3. The aim of this optimization is optimal usage of energy harvesting users with low energy harvesting rate such as piezoelectric, electromagnetic, indoor solar cell, etc. [24].

Afterward the problem of finding a proper route to let the AP to communicate with users with bad direct channel states has been studied. We study the optimal routing problem in multihop wireless sensors with energy limited users. The objective of this study is to find a policy for users to send their data to AP with cooperation of other users in a way that the system consumes minimum amount of energy.

In the past decade, there has been a sustained effort dedicated on cooperative communications and its noticeable gains with respect to the traditional wireless communication systems. The majority of the previous work on cooperative communications has focused on energy accumulation [25]. In that model, the receiver combines the signals coming from different paths using techniques such as maximal ratio combining and the receiver is able to decode the message if the sum of individual SNRs exceed a threshold level. In this case, each transmitter has to transmit the same bits using the same modulation and coding.

Rateless Codes [26] [27] facilitate accumulation of codewords (instead of energy) at the receiver. Basically, the transmitter divides the available information into K blocks, and at each time randomly chooses and XORs a subset of the blocks, such that the receiver will be able to decode the original message when it accumulates a sufficient number of coded packets. In a multihop scenario, as soon as a relay node decodes the message, it starts to retransmit the message using fountain encoding. A node on the path can accumulate coded packets from the previous hop transmissions, which improves the energy-efficiency. In the literature usually idealistic rateless codes are assumed, where the nodes can accumulate mutual information instead of packets, and the receiver can decode the message whenever the amount of received mutual information from previous transmissions exceeds the message size. In [28], [29], the authors show that in a high SNR regime the information accumulation technique works with lower energy expenditure and time latency than classical energy accumulation techniques.

Our study is on energy efficient transmission for wireless networks with a single source and a single destination. We first present a study on the unlimited energy case and present an efficient optimal solution, which is based on the results of [30]. Then we propose a heuristic based on Dijkstra's algorithm, and analyse the same problem for the limited-energy case. We present an analysis based on counter examples, that shows that this is an interesting and hard problem, and some important properties that hold for unlimited energy do not hold anymore for the limited energy case. A heuristic method is also provided.

The studies [30], [31] are the most relevant to our routing problem. In [29] Draper et al. propose a method for finding the optimal path by solving a Linear Programming (LP) problem for each subset and order of the nodes, which requires the computation of $\sum_{k=0}^{N} \binom{N}{k}$ LPs, where N is the number of relays. In [30], the authors show that for unicast routing problem, a greedy algorithm can be applied and the complexity of finding the optimal path by using greedy algorithm is 2^N. In [31], Draper et al. give a heuristic method for the optimal path in which the algorithm calculates a polynomial number of LPs. In [32], the authors consider the energy minimization problem with variable transmission duration and times, and they propose an algorithm that finds a suboptimal route, transmission times and powers for each node

The outline of this work is as follows: In Chapter 2, first, the problem statement for optimizing of feedback in a MISO downlink with energy harvesting users is given. Next, the case of fixed number of bits in feedback is studied, with the goal of optimizing the times at which a node sends feedback to the access point for achieving maximum expected throughput. Later, the case of variable size feedback is studied. A policy for reaching the maximum expected throughput is derived, and the performance of the proposed policy is evaluated. Then, we analyze the system performance in terms of throughput and fairness for users with different probabilistic characteristics. Next, in Chapter 3 the system model of routing problem of MIA wireless networks with energy harvesting users is described in detail. The case of unlimited energy is discussed and a heuristic method with complexity of $O(N^3)$ is proposed. According to simulation results, the difference in transmission time for 98% of the samples is less than 3% with respect to the optimal approach. The rest of Chapter 3 is dedicated to energy limited case. In this part, after introducing the optimal solution, a heuristic method is proposed and its performance is evaluated. More specifically, we show that the complexity of the proposed heuristic method is $O(N^2)$. Finally, the thesis conclusion and future work directions are provided in Chapter 4.

CHAPTER 2

THE MULTIUSER MISO PROBLEM

2.1 System Model

We consider a multiuser MISO system that consists of an access point with m antennas and M single-antenna users (Fig. 2.1). In each frame, in uplink part, a portion of time is allocated to each user to send its channel state information to the AP through error-free feedback channel. Feedbacks contain information of both channel gain and channel direction. Users decide whether or not to send feedback in their allocated time with respect to their energy level and channel gain. In the remaining portion of the frame, in downlink part, AP transmits data to one of the users. We assume a simple rate maximizing time division strategy, where the AP selects a user with the largest channel gain magnitude, among those that have sent channel state feedback. For the case of more than one user having highest achievable rate, AP chooses one of them randomly (Fig. 2.2).

In this scenario, users are considered to have energy harvesting capabilities (Fig. 2.3), whereas the AP is assumed to be connected to a reliable power source supplying average power P. We assume the channel gains are i.i.d. in each timeslot and take values from a discrete set of N possible states. As we will see, the complexity of the proposed algorithm in dynamic rate feedback case is a function of number of possible channel gains; therefore, the assumption of limited number of channel gains is made.

The objective of the users is to maximize their own long term throughput with respect to energy harvesting constraint. For achieving it, users need to apply an optimal policy for feedback transmission scheduling.

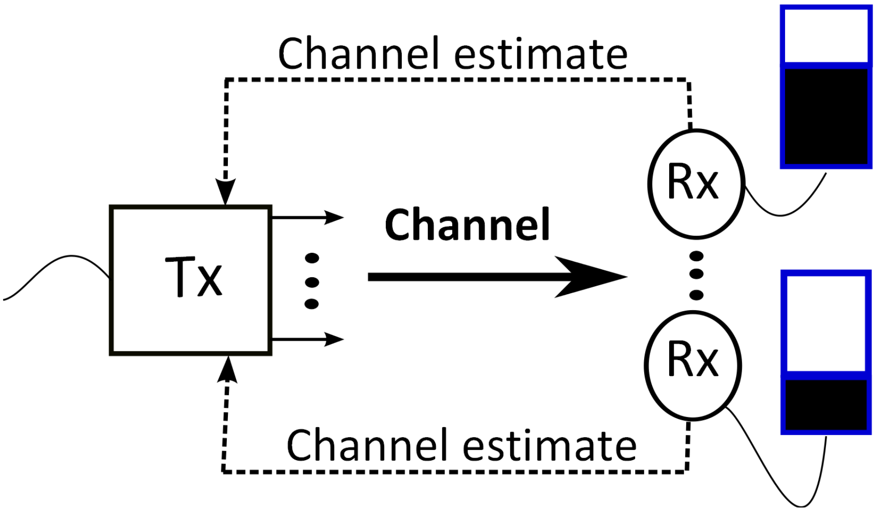

Figure 2.1: System model, multiuser MISO system with energy harvesting users.

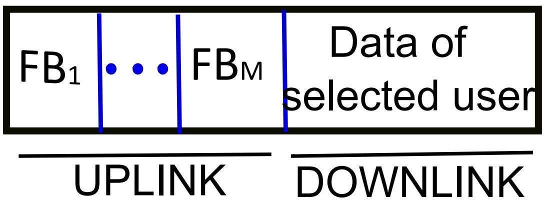

Figure 2.2: Frame structure, where FB_i refers to the dedicated time for the i^{th} user to send its feedback and the rest of frame is used for data transmission.

In the following, first we will consider fixed rate feedback transmission and later we will study the variable rate feedback transmission.

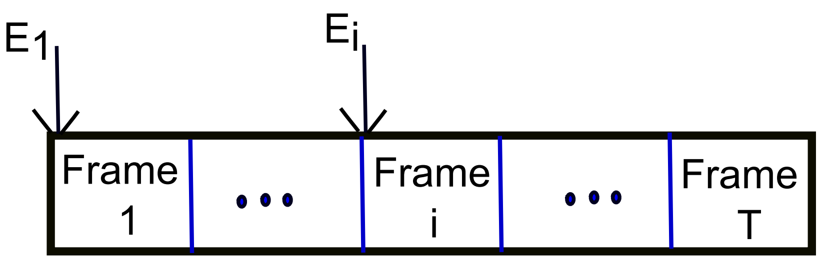

Figure 2.3: Energy harvesting time frame structure.

2.2 Fixed-Rate Feedback

We begin by studying the case that feedback contains a fixed number of bits (b bits), i.e., each user should decide to send feedback or not in a current data frame. In addition, it is assumed that all future energy arrivals and future channel conditions are known in advance at starting time by each node. In this section, in a finite time horizon we address an offline scheduling. The objective of users is to optimize their own long term throughput. As it is noted in theorem 2.2.1, if nodes act to attain this objective, they also maximize the expected long term throughput of the whole system. Afterward, we will propose a policy for all users, which maximizes the throughput of each user.

Theorem 2.2.1 *Considering all users make decision independently. Assuming that energy arrivals and channel states of users are independently and identically distributed for each user and in each frame, if each user applies a policy π that maximizes its expected throughput, then the throughput of the overall system is maximized.*

Proof:

$$\begin{aligned} \max_{\pi} E[R_{net,tot}] &= \max_{\pi} E[R_{1,tot} + R_{2,tot} + ... + R_{M,tot}] \\ &= \max_{\pi} E[R_{1,tot}] + E[R_{2,tot}] + ... + E[R_{M,tot}] \\ &\overset{iid}{=} \max_{\pi} M E[R_{i,tot}] = \max_{\pi} M E[R_{tot}], \end{aligned} \tag{2.1}$$

□

where $E[R_{net,tot}]$ and $E[R_{i,tot}]$ are the expected throughputs attained by the whole network and the i^{th} user, respectively. Here, since users are applying the same policy, their expected throughputs can be decoupled.

The expected throughput of a user over a problem horizon of T frames is

$$\max_{\omega_i \in \{0,1\}} \quad E(R_{tot}) = \max_{\omega_i \in \{0,1\}} \sum_{i=1}^{T} \omega_i R_{S_i} P_c(S_i) \tag{2.2}$$

s. t.

$$bE_b \quad \sum_{j=1}^{i} \omega_j \leq \sum_{j=1}^{i} E_j \text{ , } i = 1, 2, ..., T,$$

where $\omega_j \in \{0, 1\}$ indicates whether a user sends feedback in time frame j or not,

R_{S_i} is the rate of the node at channel state S_i,

$P_c(S_i)$ is the probability that a channel in state S_i be selected by the access point,

E_j is the amount of harvested energy by a user at the beginning of frame j, and

E_b is the feedback energy consumption per bit.

Now, by the following definitions and theorems we will find the optimal policy for feedback transmission scheduling.

Definition 2.2.1: Candidate vector of each node is a vector whose elements are channel capacities of the node at frames, in which the user sends feedback (i.e., i s.t. $\omega_i = 1$). The size of this vector is less than or equal to T. The elements of this vector are sorted in decreasing order of channel capacity.

Definition 2.2.2: For $A, B \in \mathbb{R}^n$, $A \geq B$ if

$A(i) \geq B(i) \qquad for\ i = 1, ..., n.$

In this study, it is assumed that users know their future energy harvesting rate and channel states. According to this information, users should choose a policy (which results a candidate vector) for their feedback transmission scheduling, to maximize their own expected throughput. Now, we are ready to state the following theorem which indicates the optimum candidate vector.

Theorem 2.2.2 *If candidate vector of a policy is greater than candidate vector of other policies, the policy achieves maximum expected throughput.*

Proof: Assume candidate vector A is greater than candidate vector B. As $A(i) \geq B(i)$ so $P_c(A(i)) \geq P_c(B(i))$, so $\sum_{i=1}^{n} A(i)P_c(A(i)) \geq \sum_{i=1}^{n} B(i)P_c(B(i))$, which means that the expected throughput of candidate vector A is greater than candidate vector B. $\square$

One energy packet is defined as the amount of energy which is sufficient for transmission of one feedback packet. We assume that the studied node will harvest E_{tot} energy packets until the end of the whole transmission. Hence a node should send feedback in E_{tot} of the frames, which motivates the following algorithm for the solution of the offline problem.

2.2.1 The Offline Solution

The offline algorithm assumes that energy arrivals and channel states are known ahead of time. For a given node, first the node chooses the E_{tot} best frames in terms of channel state, and sets them as the candidate instants for sending feedback. Next, it checks the first time that energy causality is disturbed and names it as T_1 (we name these instants as decision epochs). Assume that until time T_1 the node harvests E_1 energy packets, but the algorithm suggests it to consume E_1' energy packets, which is greater than E_1. The node should choose the instants the channel is in one of its E_1 best instants in the interval from the starting time to T_1. The node repeats this procedure for the time period after T_1. It can consume $E_{tot} - E_1$ energy packets for its future feedback transmission. Therefore, it selects the instants the channel is in one of its $E_{tot} - E_1$ best instants for time after T_1 and chooses them as candidate instants. Again it searches the first time after T_1 that energy causality is disturbed (name it T_2), assume that the node in the interval of T_1 to T_2 harvests E_2 energy packets but the algorithm suggests to consume E_2' packets of energy, which is greater than E_2. Again the node chooses the times the channel is in one of its E_2 best instants in the interval of T_1 to T_2. Now the node makes a decision on its feedback scheduling for time after T_2 and repeats the same procedure. The flowchart of the algorithm is given in Fig. 2.4.

In the rest, it is shown that the candidate vector chosen by the proposed policy domi-

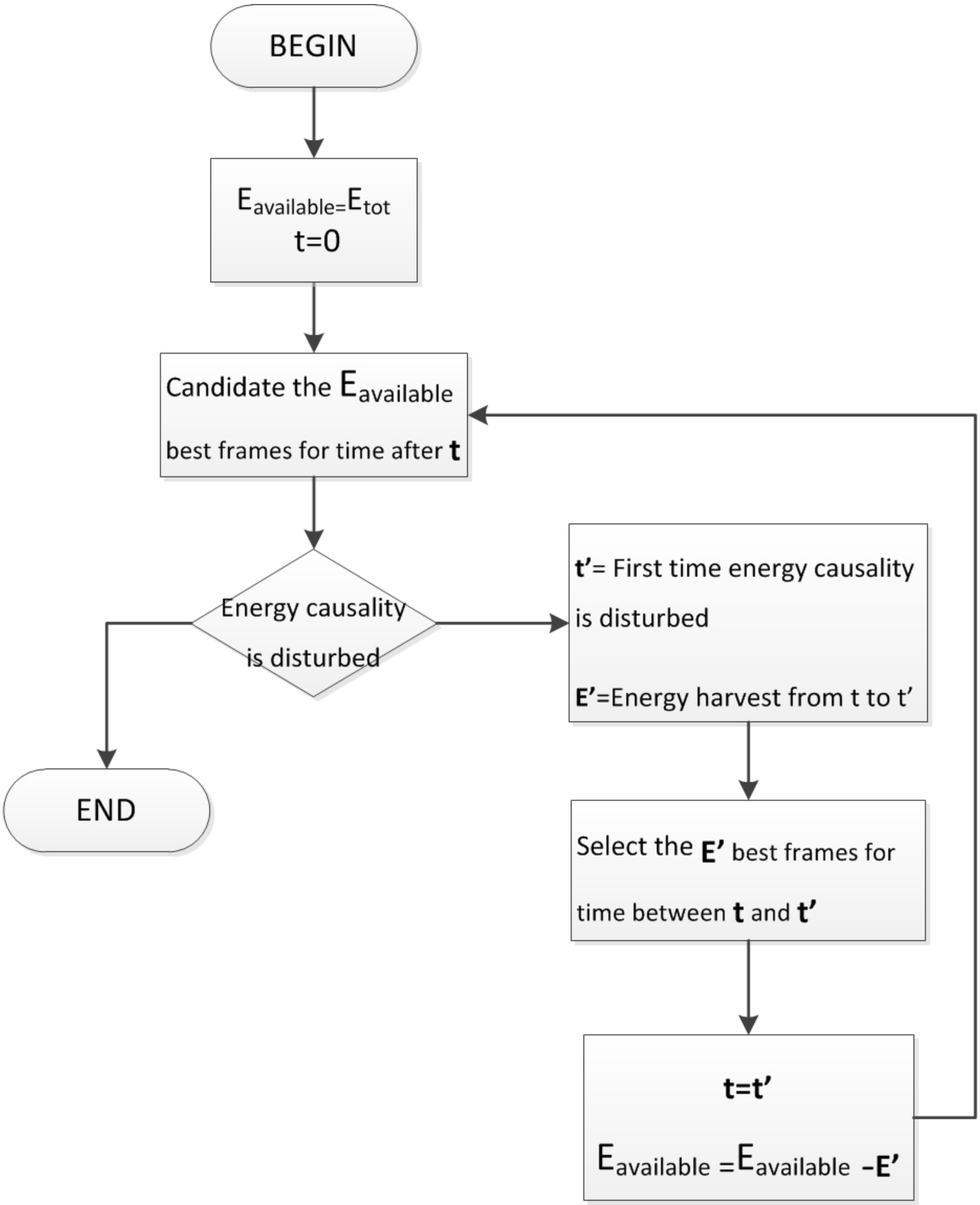

Figure 2.4: Flowchart of proposed algorithm for fixed rate feedback.

nates the candidate vectors of other policies. Therefore, according to theorems 2.2.1 and 2.2.2, the policy maximizes throughput.

2.2.2 Proof of Correctness of the Algorithm

Let A be a candidate vector achieving the highest expected throughput and B be the candidate vector determined by the proposed algorithm. Let timeslot x be the first time which candidate vector A decides to send feedback but candidate vector B does not. Let T_x be the closest future decision time to timeslot x. The proposed algorithm transmits $\sum_{j=1}^{x} E_j$ packets of feedback up to time T_x. Hence, there should be at least one time slot before T_x, where B decides to send but A does not (call it timeslot y), but the proposed algorithm chooses timeslots with the highest channel capacity, so all the chosen timeslots must observe higher channel capacity than timeslot x. This means that a timeslot which has a higher channel capacity than channel capacity in time slot x exists, and is not selected in A. Therefore A could be improved by incorporating this time slot. This improvement continues until A chooses time slots with the same capacity values as B.

2.2.3 Simulation Results

In this part, the performance of the proposed algorithm is compared with a greedy algorithm and the case in which users have access to an unlimited amount of energy (Fig. 2.5). In greedy algorithm, it is assumed that the user sends feedback if there is available energy. The channels gains are modelled by Rayleigh fading.

In the simulation, it is assumed that the system consists of 5 users and the access point will choose one of them for transmission. The results are gathered for over 10^6 timeslots for each rate of energy harvesting. Energy harvesting distribution is modelled by an exponential distribution with different mean values.

As it is observed in Fig. 2.5, for low energy harvesting rate, the proposed policy considerably outperforms the greedy policy. However, in high energy harvesting rate, both policies have the same performance.

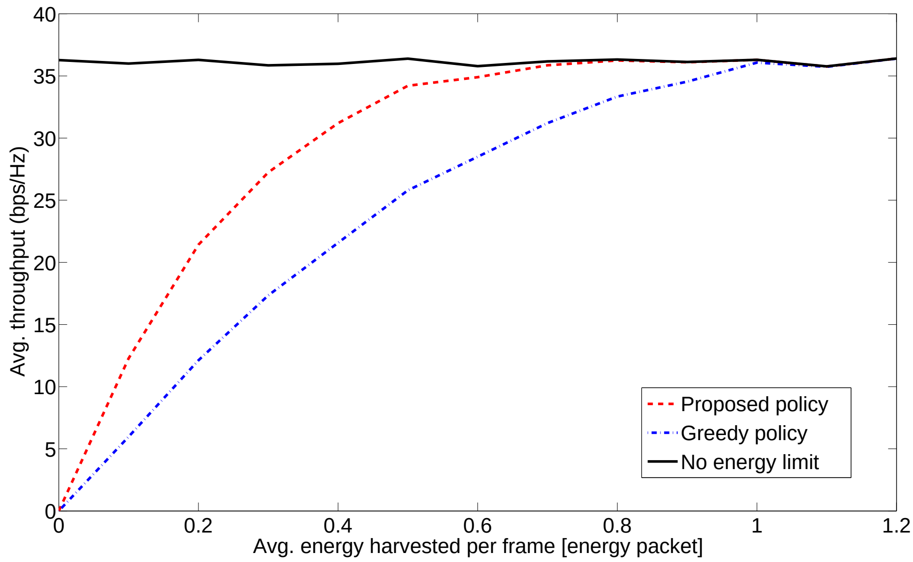

Figure 2.5: Average throughput vs. average energy harvested (energy packet per timeslot) at an SNR 1, achieved by optimum policy and greedy policy. The black level shows the average throughput if the user does not have energy limitation. The oscillation in black curve is due to the randomness in channel state realization.

2.3 Variable length feedback

In this part of the study, we consider a more general case, that is, variable length feedback. So users will decide on both time and the number of feedback bits.

In the following section, the amount of gain a user will get by increasing the number of feedback bits is derived from the literature.

2.3.1 A Review on Point-to-Point MISO Systems with Finite Rate Feedback

When transmitter has partial information of the channel state, the optimal policy to achieve maximum throughput is beamforming across the direction of quantized channel vector. The achievable rate with this policy [21] by using Random Vector Quantization (RVQ)[20] is

$$R_{FB}(P) = E_{w,h}\left[log\left(1 + P\|h\|^2 cos^2\left(\angle\left(h, \widehat{h}\right)\right)\right)\right], \tag{2.3}$$

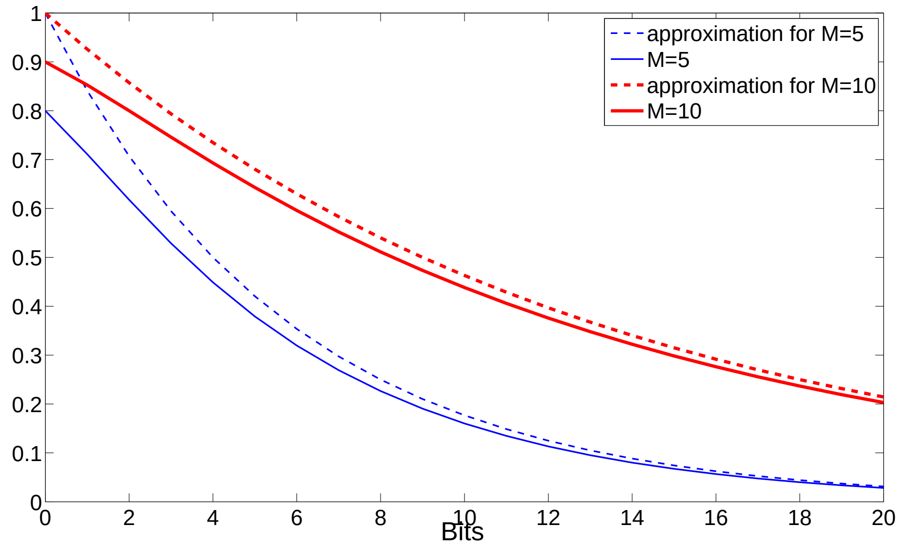

Figure 2.6: Tightness of the upper bound for Eq. 2.4. The horizontal axis is the number of feedback bits and the vertical axis is the value of function and its approximation.

where $E_{w,h}$ denotes expectation over the quantization vector w and the channel vector h,

P is the transmitting power,

$\|h\|$ is the norm of the channel gain, and

$\angle\left(h,\widehat{h}\right)$ is the angle between channel vector and its quantized vector.

As it has been proved in [33] for channels with Rayleigh fading the inequality given by

$$E_{w,h}\left[sin^2\left(\angle\left(h,\widehat{h}\right)\right)\right] < 2^{\frac{-B}{m-1}}, \tag{2.4}$$

holds, where B is the number of bits of quantization and m is the number of transmitting antennas.

Hereafter, we will use Eq. 2.4 as an upper bound. Tightness of the bound is shown in Fig. 2.6, where one observes that the bound becomes tighter as the number of feedback bits and/or number of transmitting antennas increase.

By applying the approximation to the achievable rate formula

$$R_{FB}(P) \simeq E_h \left[log \left(1 + P\|h\|^2 \left(1 - 2^{\frac{-B}{m-1}} \right) \right) \right]. \tag{2.5}$$

In the rest of this work, the optimization on the number of bits allocated for feedback will be based on the achievable rate (Eq. 2.5).

2.3.2 Bit Allocation for Same Initial Energy

In the first step, let us assume that the users are initialized with finite amounts of energy, and they do not harvest energy during the transmissions. At the moment, we assume the amount of initial energy of nodes are equal and this is known by them. We assume channel state processes are ergodic and the problem horizon is sufficiently long such that the frequency of occurrence of each channel state for each user converges to the probability of occurrence of that state, within the problem horizon. Expected reward of this system is

$$\max_{b_i} \quad E(R_{tot}) = \max_{b_i} \sum_{i=1}^{N} mR(h_i, b_i) P_c(R(h_i, b_i)) P(\|h_i\|) \tag{2.6}$$

s. t.

$$TE_b \quad \sum_{i=1}^{N} b_i P(\|h_i\|) = B \times E_b,$$

where $R(h_i, b_i)$ is the achievable rate of channel when the channel norm is $\|h_i\|$ and user sends b_i bits of feedback,

N is the number of possible quantized magnitudes of h,

m is the number of users,

T is the number of timeslots,

B refers to number of feedback bits a user can send according to its initial energy,

$P_c(R(h_i, b_i))$ is the probability that user with channel rate $R(h_i, b_i)$ be chosen by the access point,

$P(\|h_i\|)$ is the probability that the channel norm be $\|h_i\|$, and

E_b is the amount of energy for transmitting 1 bit.

Theorem 2.3.1 *In the optimal policy for maximizing the expected throughput,* $R(h, b)$ *is an increasing function with respect to* $\|h\|$.

Proof: Let us assume that by policy A, users achieve the maximum expected throughput. Assume channel states are sorted by their channel capacity with respect to the number of bits allocated to them such that $R(h_1, b_1) < R(h_2, b_2) < ... < R(h_i, b_i) < R(h_j, b_j) < ....$ To prove the theorem by contradiction, we assume that there exists an h_i with $h_i > h_j$ but $R(h_i, b_i) < R(h_j, b_j)$. We define $R(h_i, b'_i)$ and $R(h_j, b'_j)$ in a way that $R(h_i, b'_i) = R(h_j, b_j)$ and $R(h_j, b'_j) = R(h_i, b_i)$. Here we want to show that with bit allocation b'_i and b'_j for states h_i and h_j respectively, although the expected throughput remains the same, $b'_i + b'_j < b_i + b_i$. So by using those free bits, the expected throughput will be increased and this is in contrast with our first assumption that policy A is optimum.

$$\begin{aligned}
&R(h_i, b'_i) = R(h_j, b_j) \\
&\Rightarrow \log_2\left(1 + P\|h_i\|^2\left(1 - 2^{\frac{-b'_i}{M-1}}\right)\right) \\
&= \log_2\left(1 + P\|h_j\|^2\left(1 - 2^{\frac{-b_j}{M-1}}\right)\right) \\
&\Rightarrow \|h_i\|^2(1 - 2^{\frac{-b'_i}{M-1}}) = \|h_j\|^2(1 - 2^{\frac{-b_j}{M-1}}) \\
&\Rightarrow 1 - 2^{\frac{-b'_i}{M-1}} = \frac{\|h_j\|^2}{\|h_i\|^2}(1 - 2^{\frac{-b_j}{M-1}}).
\end{aligned} \tag{2.7}$$

Similarly using $R(h_j, b'_j) = R(h_i, b_j)$, one obtains

$$1 - 2^{\frac{-b'_j}{M-1}} = \frac{\|h_i\|^2}{\|h_j\|^2}(1 - 2^{\frac{-b_i}{M-1}}). \tag{2.8}$$

Notice that since $R(h_i, b_i) < R(h_j, b_j) = R(h_i, b'_i)$, one can write

$$b_i < b'_i, \tag{2.9}$$

and similarly, the inequality $R(h_j, b'_j) < R(h_j, b_j)$ implies that

$$b'_j < b_j. \tag{2.10}$$

Let's define $f(x) = 1 - 2^{\frac{-x}{M-1}}$. $f(x)$ is a concave function. Defining $L = \frac{\|h_i\|^2}{\|h_j\|^2}$ which is greater than 1, one arrives at

$$\begin{aligned} f(b'_j) &= Lf(b_i) \\ f(b_j) &= Lf(b'_i). \end{aligned} \tag{2.11}$$

Hence

$$\begin{aligned} f(b'_j) - f(b_i) &= (L-1)f(b_i) \\ f(b_j) - f(b'_i) &= (L-1)f(b'_i). \end{aligned} \tag{2.12}$$

Since $f(x)$ is a positive-definite and monotonically increasing function, Eq. 2.9 implies that $f(b_i) < f(b'_i)$. Using this in Eq. 2.12, one obtains

$$f(b_j) - f(b'_i) > f(b'_j) - f(b_i). \tag{2.13}$$

$f(x)$ is a concave function so its derivative $f'(x)$ is monotonically decreasing. Therefore with respect to Eq. 2.9 and Eq. 2.13, $b'_j - b_i < b_j - b'_i$ which means $b'_i + b'_j < b_i + b_j$. So by using those free bits, the performance of policy A can be improved, which is in contrast with our first assumption.

Also, if $P(\|h_i\|) > P(\|h_j\|)$, first we can separate channel state $\|h_i\|$ into two groups $\|h_{i_1}\|$ and $\|h_{i_2}\|$, where $\|h_{i_1}\| = \|h_{i_2}\| = \|h_i\|$ and $P(\|h_{i_1}\|) = P(\|h_j\|)$ and $P(\|h_{i_1}\|) + P(\|h_{i_2}\|) = P(\|h_i\|)$. $\square$

According to Theorem 2.3.1, $R(h, b)$ is an increasing function with respect to $\|h\|$. Therefore in optimal bit allocation $P(R(h_i, b_i) > R(h_j, b_j)) = P(\|h_i\| > \|h_j\|)$. Consequently the expected throughput of the system can be written as

$$\max_{b_i} \quad E(R_{tot}) = \max_{b_i} \sum_{i=1}^{N} mR(h_i, b_i) f_c(\|h_i\|) P(\|h_i\|) \tag{2.14}$$

s. t.

$$TE_b \quad \sum_{i=1}^{N} b_i P(\|h_i\|) = B \times E_b,$$

where $f_c(\|h_i\|) = \sum_{j=0}^{n-1} \binom{n-1}{j} \frac{1}{j+1} P(\|h_i\|)^j F(\|h_{i-1}\|)^{n-j-1}$, in which $F(\|h_i\|)$ is the Cumulative Distribution Function (CDF) of $\|h\|$. Furthermore, AP policy in choosing one of the candidate users randomly is indicated by the term $\frac{1}{j+1}$ in the expression. We have derived the optimization problem, where the objective function is concave in b_i and the constraint is linear in b_i.

2.3.3 Bit Allocation for Different Initial Energies

Now let's assume that users gain different initial energy levels. For this case, in the following, we will propose a throughput maximizing algorithm for finding optimum bit allocation policy. The algorithm is based on an exhaustive search that could be used as a benchmark for analysing the performance of heuristic methods.

In this scenario, since users do not have the same energy, the symmetry of problem is disturbed and equation $P_c(R(h_i, b_i)) = F(\|h_i\|)$ is not necessarily valid any more. Therefore, in order to find the optimum bit allocation, we need to check all possible priority lists for each system state (set of all channel states of the system at a time). For each priority list, users do bit allocation to maximize their own throughput regarding to their achievable rate, and their chance to be selected as is determined in the priority set. After doing these processes for all possible priority lists, we choose the priority list which provides the maximum expected throughput of the system. Table 2.1 is an example of one possible priority list for a system with two users and six possible channel states. The number of possible cases for the priority list is $2^{(6\times6)}$, but it can be reduced as shown in Theorem 2.3.1: In the optimal bit allocation for a case if user j with channel norm $\|h_j\|$ is chosen by the AP, the user with a higher channel gain will be selected as well. Therefore in optimal bit allocation, if one partition of table is filled by 1, all its above and left ones should be filled by 1; and if a partition is filled

Table 2.1: Example of a possible priority list for a system with 2 users and 6 possible channel states. Listed numbers show which user has higher priority for being selected by the AP.

h_6	h_5	h_4	h_3	h_2	h_1	${}^{user1}/_{user2}$
1	1	1	1	1	1	h_1
1	1	1	1	2	2	h_2
1	1	1	1	2	2	h_3
1	1	2	2	2	2	h_4
1	1	2	2	2	2	h_5
1	1	2	2	2	2	h_6

by 2, all its below and right partitions should be also filled by 2.

2.3.3.1 Heuristic Method

In this part of the study, the performance of system when users do bit allocation regardless to energy level of other users (independent decision making) is compared with optimal decision making on feedback bits allocation.

In the independent decision making method (heuristic method), we assume that users do not consider the energy level of other users and make decision on feedback policy by considering the probability of being the best channel in that timeslot. Therefore, $P_c(R(h_i, b_i))$ in Eq. 2.6 is simplified to $(F(\|h_{i-1}\|))^{n-1}$ where n is the number of users and $F(\|h_i\|) = P(\|h\| \leq \|h_i\|)$ (more accurately: $P_c(R(h_i, b_i)) = \sum_{i=0}^{n-1} \binom{n-1}{i} \frac{1}{i+1} P(\|h_i\|)^i F(\|h_{i-1}\|)^{n-i-1}$). The performance comparison of independent and optimal decision making has been shown in Fig. 2.9.

2.3.4 Feedback Bit Allocation with EH Users

In this section, let's assume users are able to harvest energy during transmission and the time period between two consequent energy packet arrivals is very large compared to timeslots. Therefore we can use the results of previous sections. As it will be shown in Section 2.3.5, the expected throughput of the system with heuristic method is close to optimal decision making. Here we will use the heuristic method as a tight lower bound of the optimal decision making. In the following, the expression of expected throughput of each user with independent decision making is considered,

$$\max_{b_i} \quad E(R_{tot}) = \max_{b_i} \sum_{i=1}^{N} R(h_i, b_i) f_c(\|h_i\|) P(\|h_i\|) \tag{2.15}$$

s. t.

$$bE_b \quad \sum_{j=1}^{t} \sum_{i=1}^{N} b_i(j) P(\|h_i\|) = \sum_{j=1}^{t} B(j) E_b, 0 < t < T,$$

where $f_c(\|h_i\|) = \sum_{i=0}^{n-1} \binom{n-1}{i} \frac{1}{i+1} P(\|h_i\|)^i F(\|h_i\|)^{n-i-1}$. As it is seen, the problem statement is a concave function of energy consumption and its constraint is a linear function of energy; so the expected throughput of the system is a concave function of the energy consumption. Concavity of expected throughput on energy consumption lets one use the common algorithm for the optimization of EH systems.

In next section, the expected throughput of the system by using offline method energy harvesting and greedy algorithm, which consumes the whole harvested energy until next energy packet arrival, will be compared. In both of them, the heuristic method is used for bit allocation.

Also the online methods, which have been studied in energy harvesting communication devices can be applied to online model of this problem set, as well.

2.3.5 Simulation Results

In this part the performance of variable size feedback is evaluated.

In Fig. 2.7, the bit allocation of feedbacks for average energy consumptions of 1, 7, and 15 energy packets in each timeslot is shown. There are two users, the squared norm of channel gain $\|h\|^2 \in \{1, 2, ..., 10\}$, and it has an exponential distribution with mean of 3 (values greater than 10, are mapped to 10) and AP has 10 transmitting antennas.

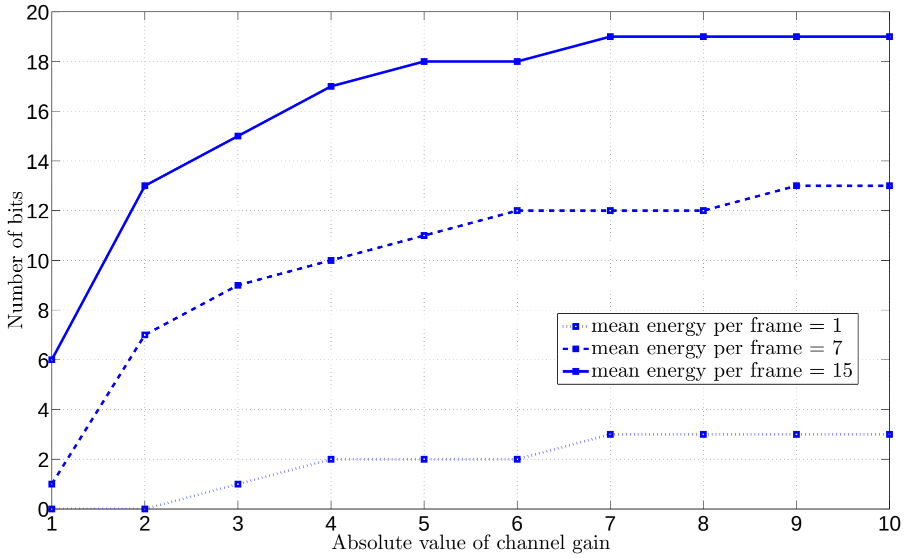

Figure 2.7: Optimum bit allocation for feedback for different mean energy harvest consumed per frame.

As it is observed, if the energy arrival rate of users are low, they may not send any feedback for low channel gains.

Next, the performance of fixed feedback size and variable feedback size are compared. The expected throughput of each user in the system with 2,3, and 4 users for two cases of fixed feedback size and variable feedback size are plotted in Fig. 2.8. AP has 10 transmitting antennas and SNR=0.1. The simulation was performed over 10^7 timeslots and the squared norm of the channel gain is assumed to be the truncated version of an exponential with mean 3, where values greater than 10 are mapped to 10. Performance is studied with respect to the average energy consumption. It appears that when the available energy for feedback is low, allowing variable size feedback improves the performance significantly, compared to the case of fixed number of feedback bits.

In Fig. 2.9, the performance of independent decision has been plotted. For a system with two users, the expected throughput of the system for independent and optimal bit allocation policy has been shown. In our simulations, $\|h\|^2 \in \{1, 2, ..., 5\}$ and it has an exponential distribution with mean of 3. The initial energy of the user 1 is 5 $E_b \times T$ and the initial energy of the other user takes different values. AP has 5 transmitting

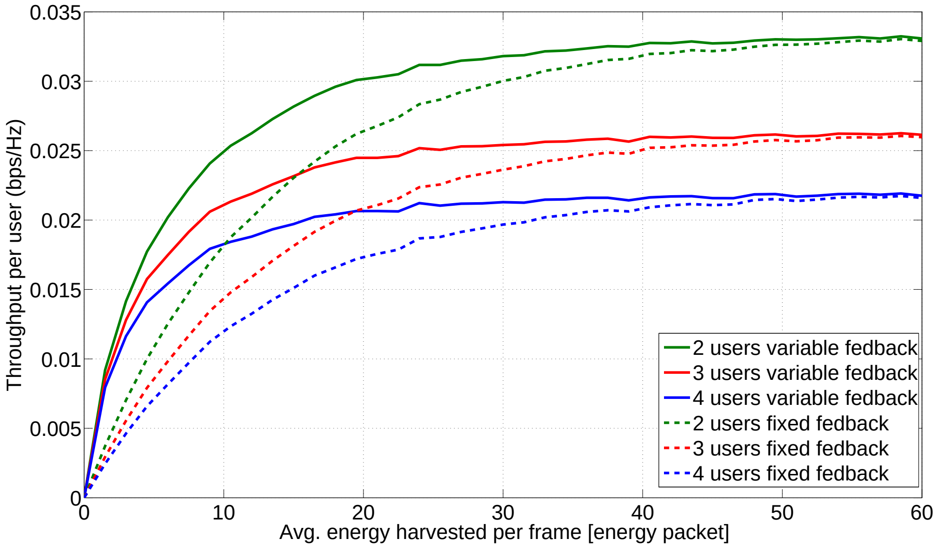

Figure 2.8: Expected throughput per user vs. average consumed energy, for fixed length feedback (dashed curves) and variable length (solid curves) at an SNR of 0.1.

antennas. As it is shown in Fig. 2.9, the performance of the heuristic method is very close to the optimal policy.

For analysing the performance of lazy scheduling, we assume a system with 2 users and the energy arrival of each user is independent with Poisson distribution, with a mean of $3 \times E_b$. Users know their future harvest profile.

In Fig. 2.10, the ratio of average throughput of each user with lazy scheduling method over greedy method is shown. In each simulation, users have 10 energy packet arrivals and the time between two energy packet arrivals is 10^5 timeslots.

As it is observed in Fig. 2.10, system performance improves up to 20% in some cases by using lazy scheduling.

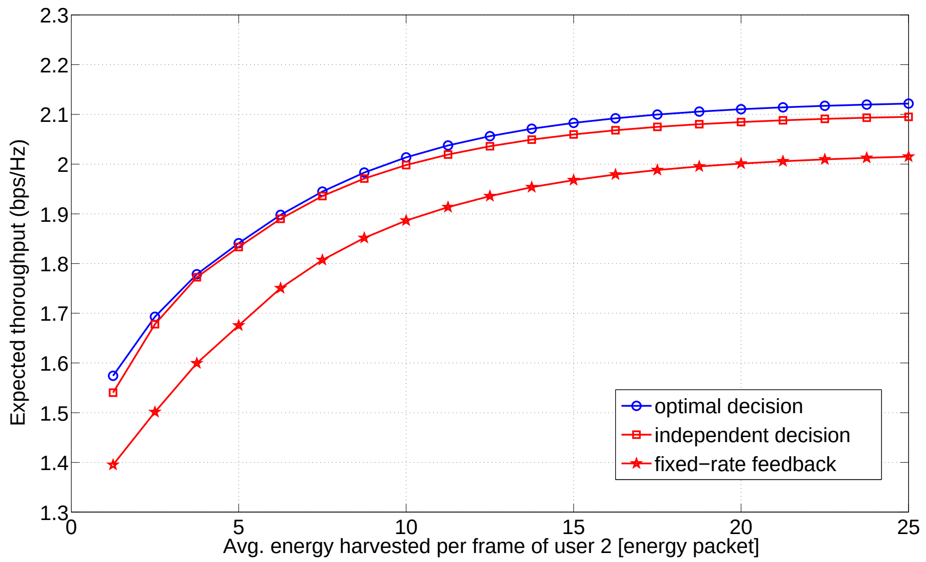

Figure 2.9: Expected throughput of system vs. the consumed energy by second user, when user 1 consumes 5 energy packets on the average in each timeslot, for independent (red squares) and optimal (blue circles) and fixed rate feedback (red stars) at an SNR of 0.1.

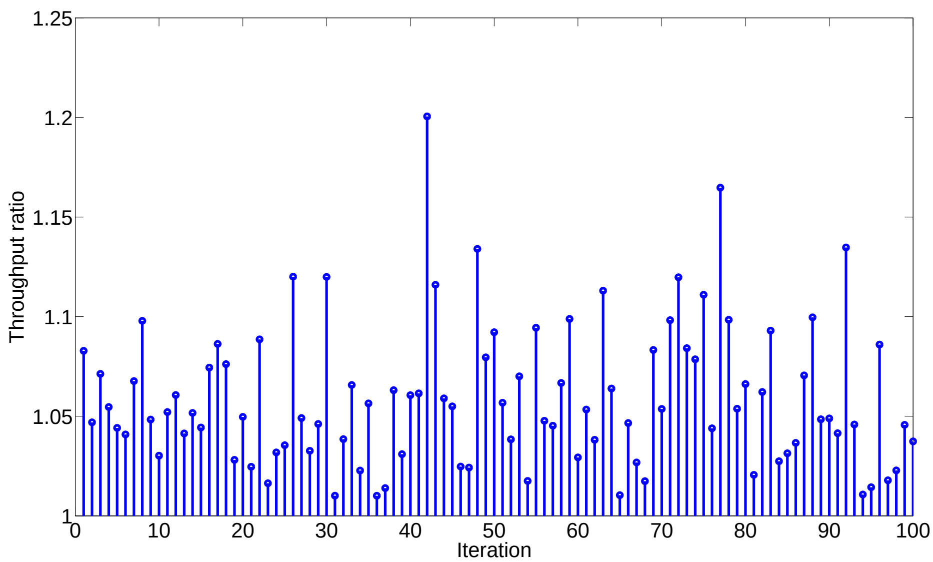

Figure 2.10: Ratio of the system throughput for lazy scheduling over greedy scheduling for 100 different samples at an SNR of 0.1.

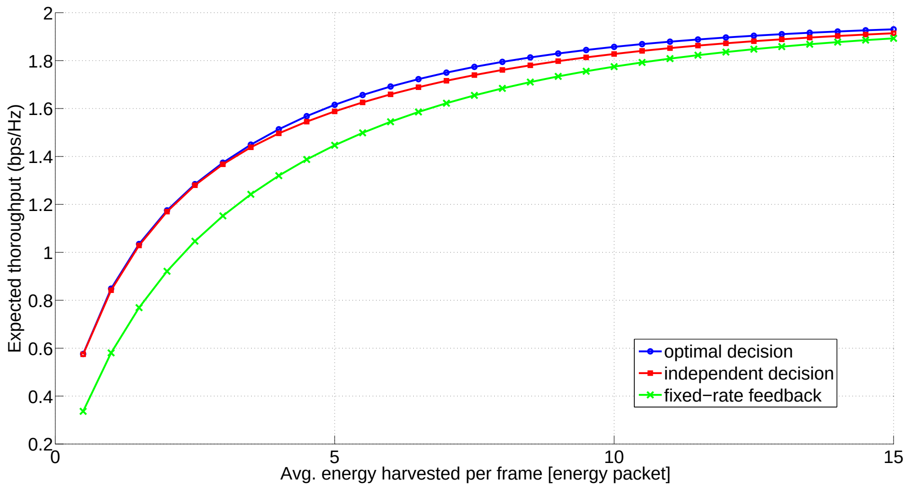

Figure 2.11: Expected throughput of system with 2 users. Mean of squared channel gains are 2 and 4 for users at SNR 0.1.

2.4 EXTENSIONS OF THE EH MULTIUSER MISO PROBLEM

2.4.1 Bit Allocation for Different Channel Distributions

In this part of the study, we consider users having different channel gain distribution (e.g., due to different distance to AP). Here we analyse the performance of the heuristic method (independent decision making) and compare it with the optimal bit allocation policy. As before, for finding the optimal bit allocation of feedback, we need to check all possible priority sets and choose the one which provides the maximum expected throughput. Later, we will evaluate in optimal bit allocation, how much the system will maintain throughput fairness across the users.

In simulations, we assume that the system consists of two users and the squared norm of the channel gain has an exponential distribution with mean of 2 and 4. Also it is assumed the channel gain gets discrete values in a way that $\|h\|^2 \in \{1, 2, ..., 5, 6\}$. As it is seen in Fig. 2.11, the performance of the system with independent decision making is very close to optimal one.

2.4.2 Fairness

One important issue for systems in which users share system resources, is maintaining throughput fairness across the users. For achieving this aim, some scheduling methods have been proposed. The methods are based on different mathematical and conceptual definitions of fairness (s.t. TCP fairness, Max-min fairness, Jain's fairness index, etc). Different scheduling algorithms have been proposed to provide fair system service to users. In the special case, when users experience the same SNR distribution, MAX SNR scheduling that chooses a user with the highest SNR will maintain fairness over long term horizon.

In MAX SNR scheduling, the AP sends data to the user, which experiences the highest channel rate at any given timeslot s. For maintaining fairness across all users which have different mean channel gains, the AP must consider the gathered throughput of all users up to timeslot s. One example of this technique is PFS [34] [35].

In this scenario, at timeslot s, the AP schedules the user $k_*(s)$, which has the highest normalized capacity, i.e.

$$k_*(s) = \max_{k=1\ldots K}\{\frac{C(k,s)}{R(k,s)}\}. \tag{2.16}$$

Here $C(k,s)$ is the channel capacity in timeslot s for user k.

$R(k,s)$ is the transmission throughput of user k over the channel up to timeslot s. The throughput is updated after each timeslot according to

$$\begin{aligned} R(k,s+1) &= R(k,s)(1-\frac{1}{t_c})\ k \neq k_* \\ R(k_*,s+1) &= R(k_*,s) + \frac{C(k_*,s)}{t_c}, \end{aligned} \tag{2.17}$$

where t_c is a time constant set to maintain fairness over a determined time horizon.

To continue, we will compare the received throughput each user when users experience different channel gain distribution.

Here, we compare the system service while users experience the same channel gain distribution but their average energy consumptions are different. In the simulation,

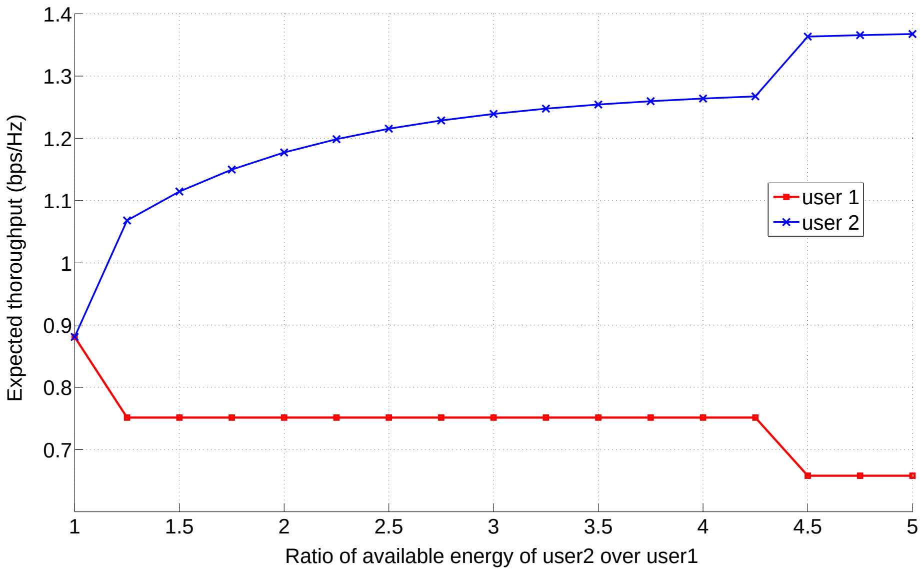

Figure 2.12: Expected throughput of users when user 1 consumes 5 energy packet for feedback transmission in average at SNR 0.1.

user 1 consumes 5 energy packets on the average. The jumps in Fig. 2.12 happen due to a change in the priority set of the system.

In Fig. 2.13, the expected throughput of users are shown. We consider a system with two users, where users experience Rayleigh fading channels with different mean values. For the first user, $E(\|h\|^2) = 1$ and for the second one, $E(\|h\|^2) = 4$. In this simulation, it is assumed that users consume the same amount of energy. The performance of MAX SNR scheduling and PFS scheduling are also shown. As observed in Fig. 2.13, by applying PFS scheduling, the system service is distributed among users with more fairness and a cost of small reduction in the expected throughput of the system.

In the following section, we will analyse the system performance in high and low SNR scenarios to see how the expected throughput of users will change by changing transmission power.

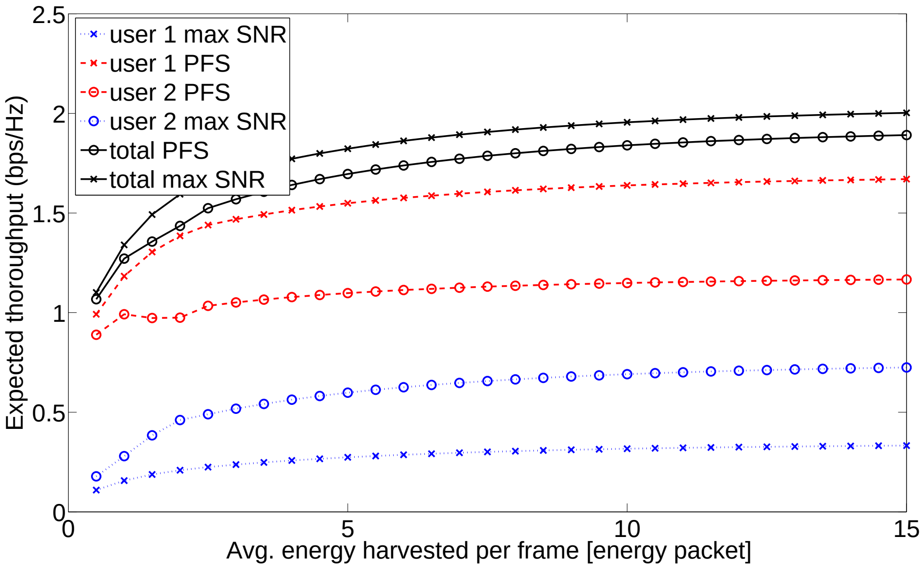

Figure 2.13: Expected throughput of users in MAX SNR scheduling and PFS scheduling at SNR 0.1. Users have the same initial energy and mean of the squared norm of the channel gains are 1 and 4 for the first and the second, respectively.

2.4.3 System Performance in High SNR and Low SNR

As it has been mentioned in Chapter 1 (see Eq. 1.3), for MISO transmission the approximated achievable rate of finite rate feedback [21] is

$$R_{FB}(P) \simeq E_h \left[log \left(1 + P\|h\|^2 \left(1 - 2^{\frac{-b}{m-1}} \right) \right) \right]. \tag{2.18}$$

In the following, the system performance in terms of the expected throughput of each user will be analysed.

2.4.3.1 High SNR

Let's assume that the transmission power is much greater than $1(P >> 1)$, then the achievable rate expression can be approximated as

$$
\begin{aligned}
R_{FB}(P) &\simeq E_h\left[\log\left(1+P\|h\|^2\left(1-2^{\frac{-b}{m-1}}\right)\right)\right]\\
&\simeq E_h\left[\log\left(P\|h\|^2\left(1-2^{\frac{-b}{m-1}}\right)\right)\right]\\
&= E_h\left[\log\left(P\right)+\log\left(\|h\|^2\left(1-2^{\frac{-b}{m-1}}\right)\right)\right]\\
&= \log\left(P\right)+E_h\left[\log\left(\|h\|^2\left(1-2^{\frac{-b}{m-1}}\right)\right)\right]. \qquad (2.19)
\end{aligned}
$$

The first term of Eq. 2.19 depends only on transmission power and the second term depends on the channel distribution and bit allocation policy for feedback transmission. Therefore, in high SNR scenario, by increasing the transmission power, the expected throughputs of users with any feedback transmission policy have the same increase as confirmed by the simulations. We compare the performances of fixed and variable feedback size. The expected throughput of each user in the system with 2,3, and 4 users for two cases of fixed feedback size and variable feedback size are plotted in Fig. 2.14 and Fig. 2.15, for SNR=10 and SNR=100, respectively. AP has 10 transmitting antennas. The simulation is performed over 10^7 timeslots and the squared norm of the channel gain is assumed to be the truncated version of an exponential with mean 3, where values greater than 10 are mapped to 10. As observed in these figures, the difference of the expected throughputs of users for two different policies are the same for SNR=10 and SNR=100.

2.4.3.2 Low SNR

Now, let's assume that the transmission power is much lower than 1 ($P << 1$). By using Taylor expansion, the achievable rate expression can be approximated as

$$
\begin{aligned}
R_{FB}(P) &\simeq E_h\left[\log\left(1+P\|h\|^2\left(1-2^{\frac{-b}{m-1}}\right)\right)\right]\\
&\simeq E_h\left[P\|h\|^2\left(1-2^{\frac{-b}{m-1}}\right)\right]\\
&= P\times E_h\left[\|h\|^2\left(1-2^{\frac{-b}{m-1}}\right)\right]. \qquad (2.20)
\end{aligned}
$$

As it is observed from Eq. 2.20 the achievable rate is the product of transmission power by a function of channel state distribution and bit allocation policy. There-

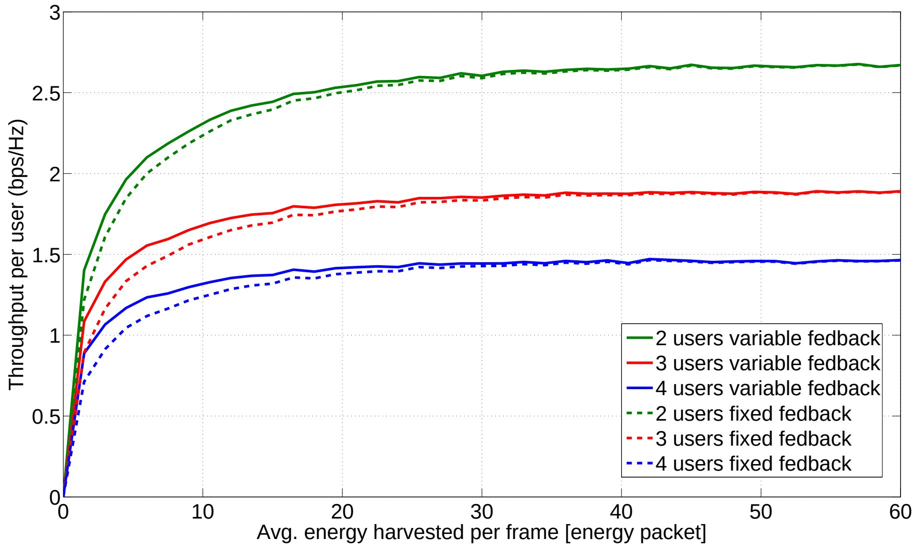

Figure 2.14: Expected throughput per user vs. consumed average energy, for fixed length feedback (dashed curves) and variable length (solid curves) and the SNR is 10.

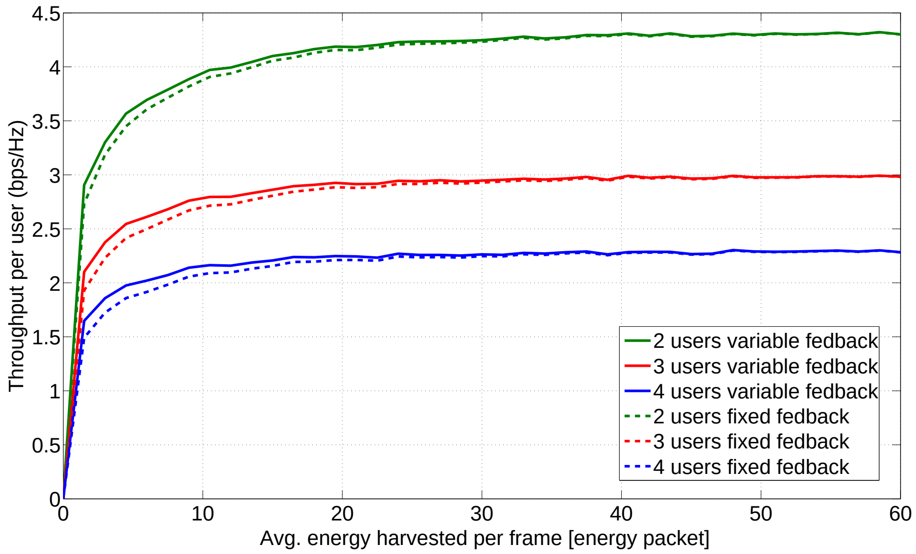

Figure 2.15: Expected throughput per user vs. average consumed energy, for fixed length feedback (dashed curves) and variable length (solid curves) and the SNR is 100.

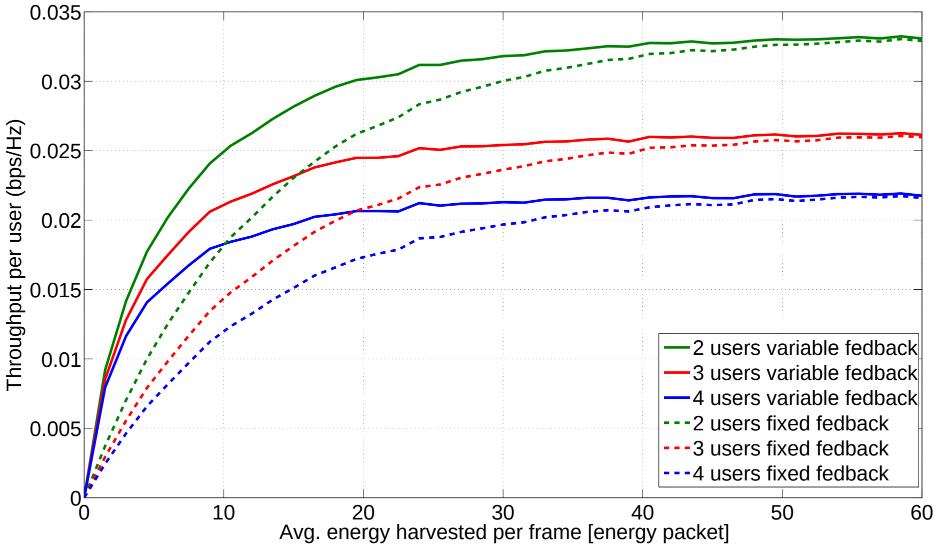

Figure 2.16: Expected throughput per user vs. average energy consumed, for fixed feedback length (dashed curves) and variable length (solid curves) and the SNR is 0.01.

fore, by increasing the transmission power, the expected throughput of users will increase but the ratio of them for different feedback policies will be kept the same, as confirmed by simulations. As observed in Fig. 2.16 and Fig. 2.17, after increasing transmission power 10 times, the expected throughput of users for both feedback scheduling policies increases 10 times. Except SNR value the simulation conditions are the same as described in Sec. 2.4.3.1.

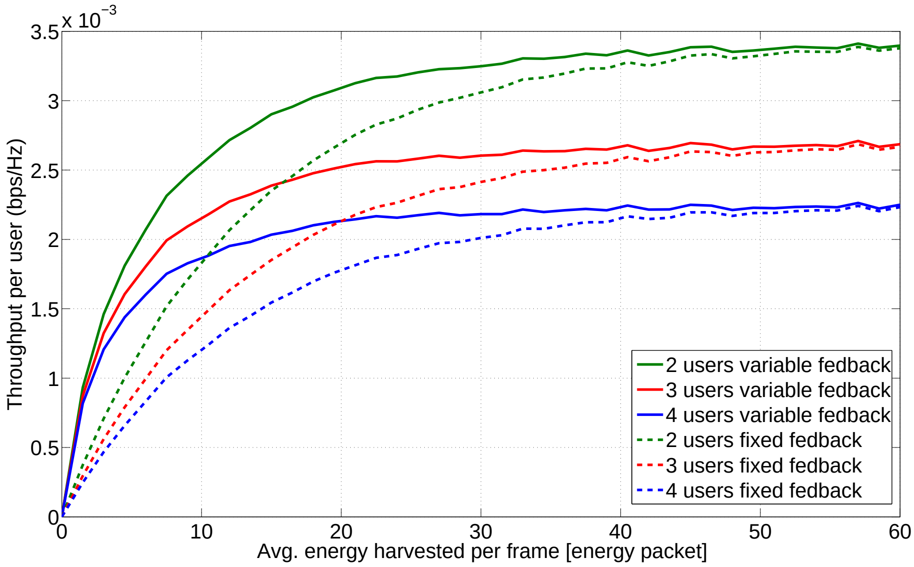

Figure 2.17: Expected throughput per user vs. average energy consumed, for fixed feedback length (dashed curves) and variable length (solid curves) and the SNR is 0.001.

CHAPTER 3

ROUTING WITH MUTUAL INFORMATION ACCUMULATION

3.1 Introduction

In wireless systems, the AP may not have direct access to the users to send data. Therefore, the AP and users can communicate with the help of other users as relays. Finding a proper path for this aim is the focus of this chapter, which is a study on the optimal routing problem in multihop wireless sensors with energy limited users. The objective of this study is to find a policy for users to send their data to the AP with the cooperation of other users, in a way that the system consumes the minimum amount of energy.

In the past decade, many studies have been done on cooperative communications and its observable gains with respect to the traditional wireless communication systems. Most of the previous works on cooperative communications have focused on energy accumulation. However, in this chapter we assume that the nodes on the path are able to accumulate mutual information from the transmissions of the previous nodes on the path.

3.2 System Model

In our system model similar to [29], we consider a network consisting $N+2$ nodes which includes a source, a destination and N potential relays as in Fig. 3.4. The channel power gain between each pair (i, j) is denoted by $h_{i,j}$. Channel conditions are assumed to be fixed throughout the end-to-end transmission. The channel capacity between two nodes (i, j) is denoted by $C_{i,j}$ (bits/sec/Hz). If node i transmits for Δt seconds the amount of information that node j will gather is $C_{i,j}\Delta t$ bits/Hz. We assume that the transmission power is equal and fixed for all nodes; therefore, minimizing energy and delay are equivalent objectives.

In this study, we focus on unicast transmission, where the source has a packet to be delivered to the destination. Finding a route with minimum energy expenditure (or equivalently delay) is the issue of this study. Here we assume that the network has just one free channel (of bandwidth W Hz) for transmission. So, just one node can transmit at each time and the others should be kept silent. During the transmission, undecoded nodes keep track of the transmissions until they gather B bits/Hz of data from previous hop transmissions. For example, if the routing path is $\overline{\pi} = [1 = \pi_1, \pi_2, \ldots, \pi_j, \ldots, N+2]$, node π_j decodes the message at the end of $j-1^{st}$ stage if

$$\sum_{i=1}^{j-1} A_{\pi_i} C_{\pi_i,\pi_j} \geq B \qquad (3.1)$$

is valid, where A_i is the duration of the transmission of node i. The problem becomes finding the optimal path that starts with the source and ends at the destination, and finding the time allocation on that path that results in minimum total energy, subject to the rate constraint of Eq. 3.1 for all nodes on the path. Although this mutual information accumulation assumption is based on idealistic rateless codes, it can easily be generalized to the practical cases by multiplying B by (1+ϵ), where ϵ accounts for the additional time/energy expenditure due to non-idealities.

3.3 Energy Efficient Routing with Unlimited Energy at Nodes

3.3.1 Finding the Optimal Path

In [30] the authors show that the optimal path should satisfy the two following conditions:

1. Just one node transmits during each timeslot. (A timeslot is defined as the duration between two consequent nodes decoding the message.)

2. Given the optimal set of transmitting nodes, each transmitter in this set starts to transmit as soon as it decodes.

Based on these results, the problem of finding the optimal route and transmission times reduces to finding the optimal set of transmitting nodes, which has a complexity of 2^N.

3.3.2 Heuristic Method

Dijkstra's well known shortest path construction algorithm using link costs could be naively applied here, as a heuristic, by taking as weights, the mutual information accumulated from a single link (disregarding the previous hop transmissions). With such link-based metrics, performance in some cases becomes far from the optimal. In this work, we want to modify Dijkstra's algorithm in a way that improves its efficiency, while keeping its desirable polynomial complexity. The following example illustrates our motivation.

Consider the three-node network in Fig. 3.1. The number written on each link is the capacity (number of bits per transmission) of that link. In this network, if we simply apply Dijkstra's algorithm, it takes link weights as the reciprocal of link capacity (corresponding to the number of transmissions per bit), without considering mutual information accumulation; and it suggests the path A, C for the completion of transmission in the shortest amount of time (with the smallest number of transmissions). But, taking into account the mutual information accumulation, we find that path A, B, C is much better.

The proposed algorithm works with a parameter k, where k is the number of nodes on the path from which each node on the path can accumulate mutual information. In other words, each node on the path can accumulate mutual information from the last k nodes on the path.

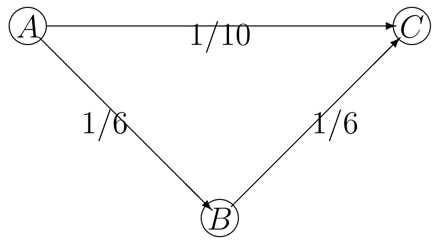

Figure 3.1: A three node topology that shows the advantages of our modification with respect to the original Dijkstra's algorithm. The weights denote the channel capacity of each link. Roughly, to send 1 bit, the number of transmissions needed on path A, C is 1/(1/10)=10. On the other hand, the number of transmissions needed on path A, B, C is: $6 + (1 - \frac{6}{10}) \times 6 = 8.4$.

3.3.2.1 Proposed Suboptimal Algorithm (Heuristic-U)

This algorithm is based on Dijkstra's algorithm but with a difference. In the classical Dijkstra's algorithm, in each stage, the cost of unvisited nodes is calculated by adding the cost-to-go of a visited node, with the link cost between the visited and unvisited nodes. In our modification, the link cost between a visited and unvisited node is calculated as the residual mutual information of the unvisited node, divided by the achievable rate of the link between the visited and unvisited nodes. We also add a parameter k, which denotes the number of previous hops from which a node can accumulate mutual information. If we set $k = 1$, the heuristic reduces to the one proposed in [30]. Our proposal is described in Algorithm 1.

The performance of this algorithm will be evaluated in Section 3.5. The complexity of the algorithm is $O(N^3)$. The complexity can be reduced if we the parameter k is reduced. If we set $k = 1$, the algorithm becomes the original Dijkstra's algorithm.

3.4 Energy Efficient Routing with Energy-limited Nodes

3.4.1 Finding the Optimal Path

In practice, most networks suffer from energy limitations related to the network lifetime duration. In this section, we present a study on the limited energy case and analyze whether the greedy algorithm in [30] holds for this case. In the end, a suboptimal algorithm will be described and its performance will be provided in Section

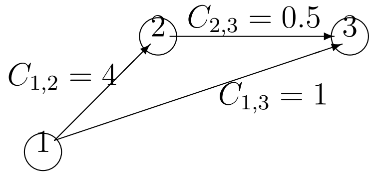

Figure 3.2: A three node topology for Remark 1.

3.5.

Draper et al. [29] presented an algorithm for optimal scheduling for minimum energy transmission, which needs to solve $N!$ linear programs (N is the number of relays). As mentioned before in [30], it was proven that for an unlimited-energy case, given the optimal set of transmitting nodes, the optimal transmission order and durations can be found using a greedy algorithm. The following examples show that the greedy algorithm of [30] cannot be generalized for the limited energy case.

Remark 1 *Even if a node is in the optimal set of transmitting nodes, it does not necessarily start to transmit as soon as it decodes.*

Proof is by a counter example. Consider the topology in Fig. 3.2. The numbers on the links denote the achievable rates. Assume that the destination has to accumulate 10 units of mutual information and node 1 has 8 units of initial energy. Node 1 starts to transmit and node 2 decodes at time 2.5. If node 2 starts to transmit, the total transmission duration becomes 17.5 time units . On the other hand if node 1 continues transmitting, it runs out of energy at time 8, and node 2 transmits for 4 units of time, which results in a total duration of 12 time units. This proves the correctness of Remark 1. It also implies that, even if we are given the optimal set of nodes, we may still have to check every possible transmission order.

In [30] it was shown for unlimited-energy case that between each two events (an event is a decoding instant), just one node transmits in the optimal scheduling. In this work, we define an event as a time instant when something happens in the network (which could be a new node decoding the message, or a node running out of its energy). Our next remark is as follows,

Remark 2 *In the energy-limited case, between two events more than one node may* *transmit in the optimal solution.*

Proof: Here we want to show that in the optimal scheduling, it is possible that more that one node transmit between two sequential events.

Let's assume that 3 nodes participate in optimal transmission. So the following equations should be satisfied

$$\begin{aligned}
\text{min} \quad & T_1 + T_2 + T_3 && \text{(3.2a)}\\
\text{s. t.} \quad & T_1 \times C_{1,2} \geq B && \text{(3.2b)}\\
& T_1 \times C_{1,3} + T_2 \times C_{2,3} \geq B && \text{(3.2c)}\\
& T_1 \times C_{1,d} + T_2 \times C_{2,d} + T_3 \times C_{3,d} = B && \text{(3.2d)}\\
& T_1 \leq E_1,\ T_2 \leq E_2,\ T_3 \leq E_3. && \text{(3.2e)}
\end{aligned}$$

Solving for T_3 from Eq. 3.2d one can write

$$T_3 = \frac{1}{C_{3,d}} \times (B - T_1 \times C_{1,d} - T_2 \times C_{2,d}). \tag{3.3}$$

Substituting Eq. 3.3 into Eq. 3.2a, we obtain

$$\begin{aligned}
\text{min} \quad & \left(1 - \frac{C_{1,d}}{C_{3,d}}\right) \times T_1 + \left(1 - \frac{C_{2,d}}{C_{3,d}}\right) \times T_2 && \text{(3.4a)}\\
\text{s.t.} \quad & T_1 \times C_{1,2} \geq B && \text{(3.4b)}\\
& T_1 \times C_{1,3} + T_2 \times C_{2,3} \geq B && \text{(3.4c)}\\
& T_1 \times C_{1,d} + T_2 \times C_{2,d} \geq B - E_3 \times C_{3,d} && \text{(3.4d)}\\
& T_1 \leq E_1 && \text{(3.4e)}\\
& T_2 \leq E_2. && \text{(3.4f)}
\end{aligned}$$

In this network, the possible events are:

$i)$ node 1 or 2 finishes its energy (which is shown in Fig. 3.3 by lines 4, 5 respectively),

$ii)$ the energy of node 3 finishes (which does not happen during the transmission of node 1 and node 2, so it is not shown in Fig. 3.3),

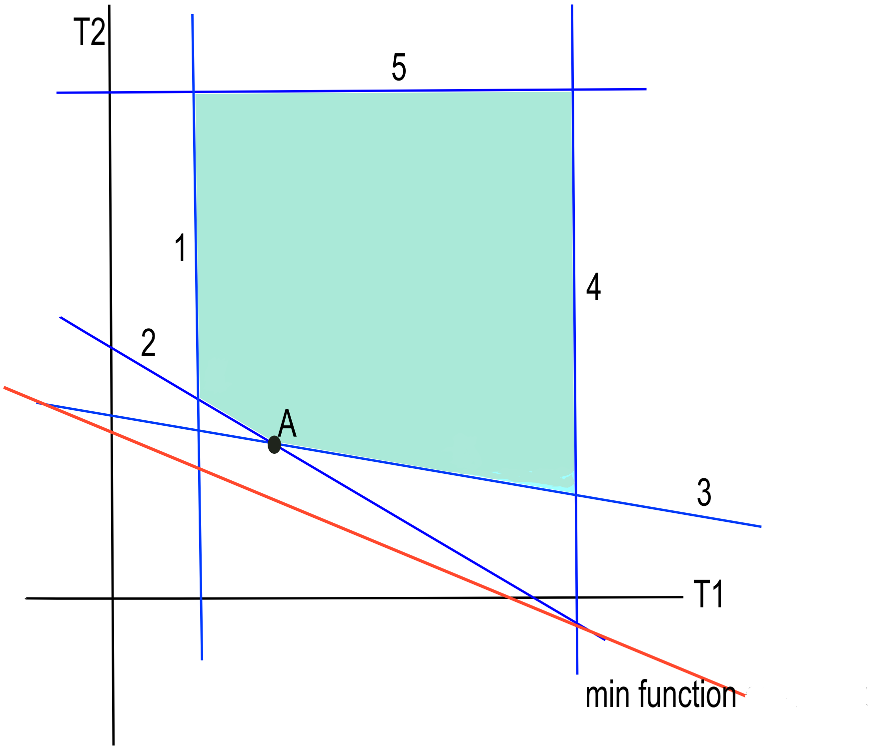

Figure 3.3: Example of a feasible region to illustrate the Remark 2.

.

$iii)$ node 2 or node 3 decode the packet (which is shown by lines 1 and 2 respectively),

$iv)$ the destination node decodes the packet which is shown by the colored area (feasible region). As seen from the above constraints, all of them except the constraint (3.4d) declare an event in the network.

In Fig. 3.3 we depict lines $1-5$ that indicate the constraints (3.4b)$-$(3.4f) and we show one of the possible feasible regions in color. The optimal time schedule is the first point of the feasible region that the line corresponding to the minimum function will touch as it is moving upward. In Fig. 3.3, if we shift the minimum function line upward, point A is the first point of the feasible region it will touch. So it is the optimal solution.

Now, if we assume that just one node will transmit between each two sequential events, we start from the origin and in each step we can move right or up until we touch a new event line. But with this process, point A is not accessible. So, for reaching node A in at least during one of the two sequential events, more than one node transmit.□

Remark 1 and Remark 2 show that in order to find the optimal routing: 1) 2^N possible transmitting sets may need to be checked as in [30], 2) One also needs to run a linear program for each order to find the transmission duration of each node (differently than [30]).

3.4.2 A Heuristic Method (Heuristic-L)

Here we present a heuristic method with a complexity of $O(N^2)$. A similar algorithm was suggested in [36] for the unlimited energy case. In this algorithm, the sender continues the transmission until the first event happens. At every event (whether a new node decoding the packet, or the transmitting node running out of energy), the algorithm chooses the node that has the best achievable rate to the destination (among the nodes that have already decoded the packet and have energy).

3.5 Simulation Results

In this section, the simulations will be described in detail first, then the numerical results will be presented. Here we consider a 2D network shown in Fig. 3.4, which consists of a source and a destination and N relays, which are distributed uniformly inside a circle with radius 10 and center at (0, 0). Source is located at point (-8, 0) and the destination is at point (8, 0).

In order to understand the basics of the problem, we ignore the effect of fading or channel variations, and assume constant channel capacities. We set the message size to an arbitrary value, 10 bits and compute the channel capacities $C_{ij} = \log_2(1 + \frac{1}{d_{ij}^2})$.

In the first set of simulations, we wish to gauge the performance of the heuristic method which has been proposed for the unlimited energy case. Keeping the locations of the source and destination fixed, the positions of relay nodes are varied over 200 instances. The number of relays is 18. The plot of cumulative distribution function (CDF) of the ratio of the transmission time of the heuristic method to the optimal

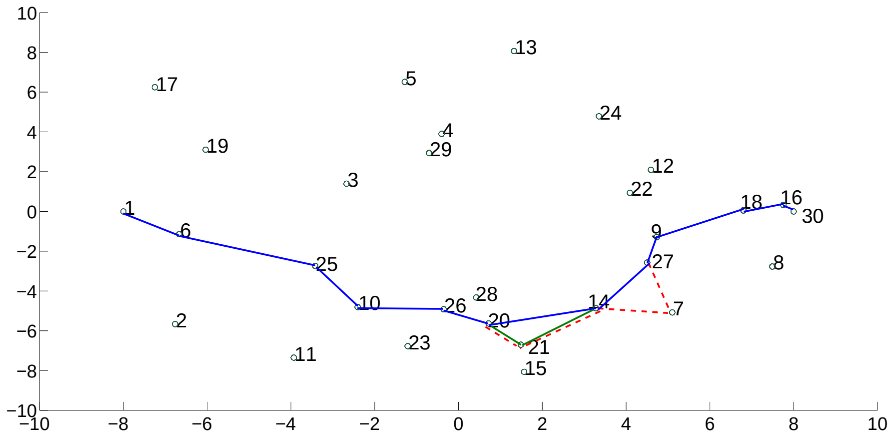

Figure 3.4: A sample network with 28 relays. Where node 1 is source and node 30 is the destination. The blue, green, and the dashed red lines are the chosen paths computed for $k = 1, 5, \infty$ respectively.

route is shown in Fig. 3.5.

As it is seen, for more than 98% of the samples, the difference between the transmission times is less than 3%. As mentioned in the heuristic method, we can set different values for k (to track the information each node gathered during the last k transmissions of that path) for the algorithm. In Fig. 3.6 the transmission times of the algorithm for different values of k are compared. Assuming the number of the nodes is 30, cumulative distributions of the ratio of transmission times for $k = 1, 2, 5$ to the transmission time for unlimited k are plotted. As one sees from Fig. 3.6, when we set $k = 5$, the performance is almost optimal. The case $k = 1$ corresponds to the heuristic algorithm in [30].

Finally to check the performance of the heuristic method (Heuristic-L) for the case of energy limited nodes, the plot of the CDF of the ratio of the transmission time of Heuristic-L to that of OPT (which is found by an exhaustive search and LP) over 100 instances is shown in Fig. 3.7. Number of potential relays in the network is 10, and therefore, 12 nodes including the source and the destination. The plot shows that for more than 90% of the samples, the difference between the transmission times is less than 7%. Considering these results and the simplicity of Heuristic-L, it proves to be a quite effective scheme.

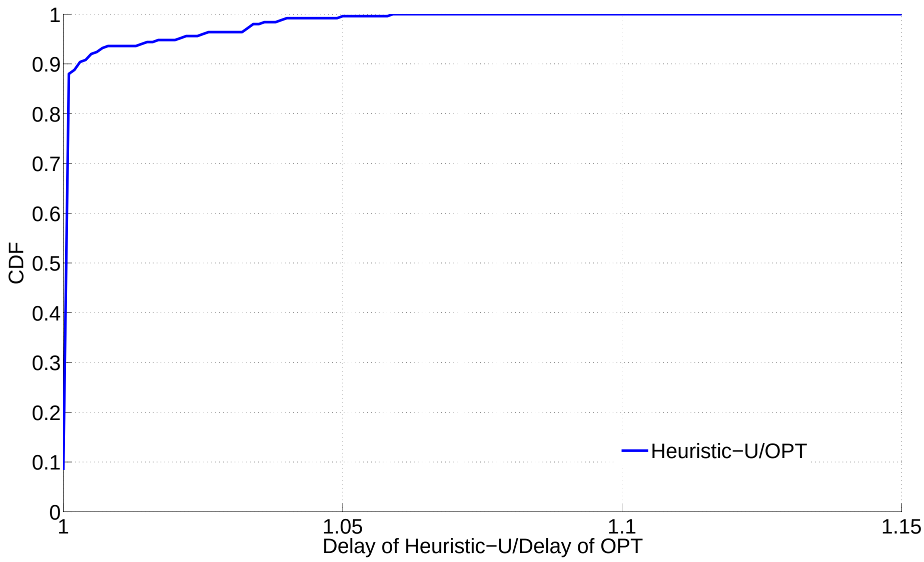

Figure 3.5: Nodes with unlimited energy: Cumulative Distribution Function plot of the ratio of transmission time of Heuristic-U to OPT over all randomly generated example cases.

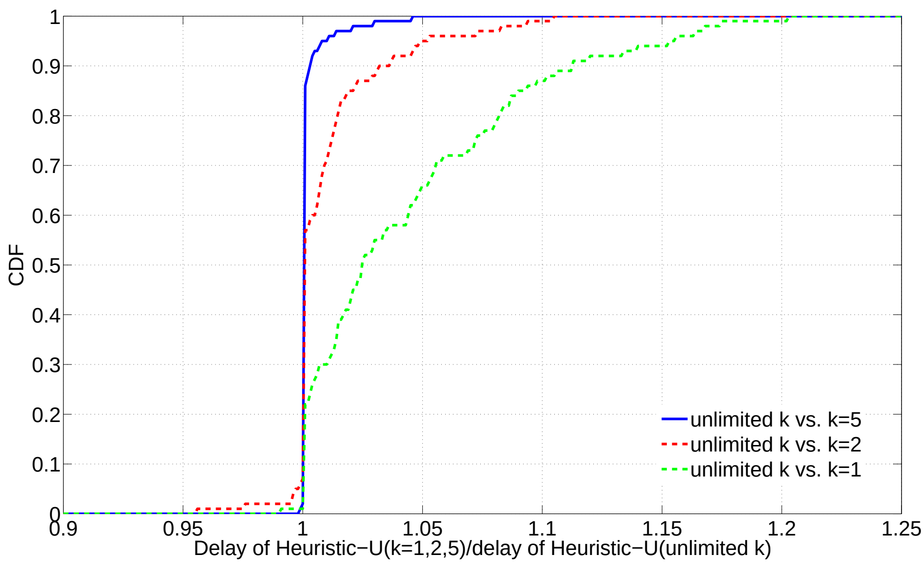

Figure 3.6: Nodes with unlimited energy: CDF plot of the ratio of transmission time of Heuristic-U with $k = 1, 2, 5$ to Heuristic-U with unlimited k, over all randomly generated example cases.

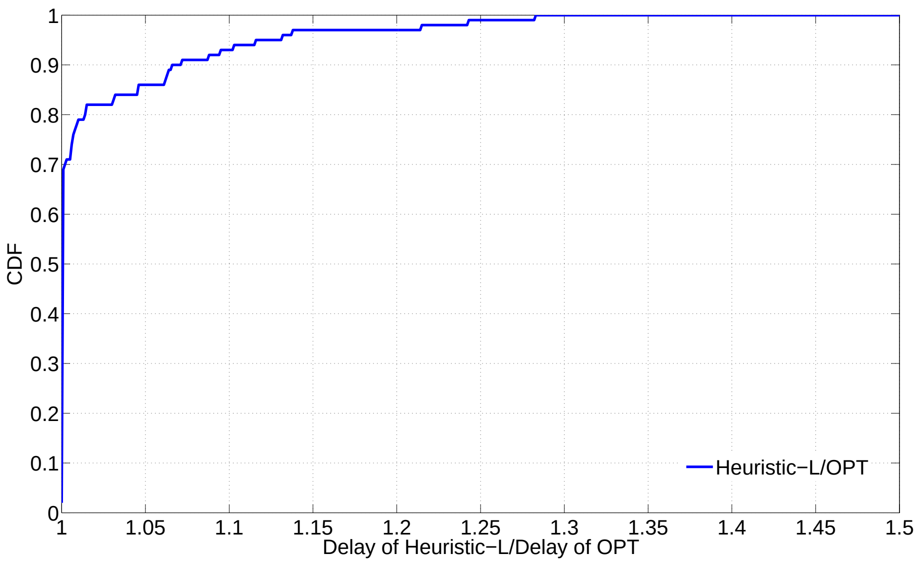

Figure 3.7: Limited Energy Case: The Cumulative Distribution Function of the ratio of the transmission time of the heuristic method over the optimal schedule (12 nodes).

Algorithm 1 Proposed Suboptimal Routing Algorithm for the Unlimited Energy Case (Heuristic-U)

Define T_i as the minimum access time of node i. And set $T_{source} = 0$ and the rest set to ∞ as the initial value

Define set of unchecked nodes Π_u and of the checked nodes Π_c

Set $\Pi_c = \emptyset$ and set $\Pi_u = \Pi$.

For all nodes, set `access path of node` $i = \emptyset$

while `destination` $\in \Pi_u$ **do**

  Choose a new node (node n) in Π_u as $\arg\min_{i \in \Pi_u}\{T_i\}$

  if n=destination **then**

    Set $\Pi_u = \Pi_u/$`destination` and $\Pi_c = \Pi_c \cup$ `destination`

  else

    Set $\Pi_u = \Pi_u/n$ and $\Pi_c = \Pi_c \cup n$

    Follow the access path of node n from the source and keep track of the amount of information the other nodes gather from the transmission of last k nodes on the path.

    Set t_i for $i \in \Pi_u$ to $T_n + ($`remaining info`$/C_{n,i})$

    if $t_i < T_i$ **then**

      $T_i = t_i$

      set `access path of node` i = (`access path of node` n)$\cup$ n

    end if

  end if

end while

CHAPTER 4

CONCLUSION

In this thesis, we have studied a system with a common multi-antenna AP and multiple single-antenna energy harvesting users. Aiming to throughput maximization, we study the optimization of the number of transmitted feedback bits by EH users. The nodes need to distribute their feedback transmissions judiciously across time. The aim of this study is optimal usage of available energy for low energy harvesting rate users.

To this end, in the first step, it is assumed that the feedback size is fixed; therefore, users decide to send a feedback packet or not. For this scenario, we propose an optimal algorithm and its optimality has been proved. The results of simulation declare that for low energy harvesting rate, the proposed algorithm dramatically outperforms the greedy algorithm.

Next, we consider variable length feedback. In this case, users decide on the number of bits to allocate for feedback, as well. The expression of expected throughput of each user is derived. For a special case in which users have the same initial amount of energy (no energy harvesting at the moment) and their channel's gain distributions are identical, it is proved that the expected throughput of users is a concave function of the feedback length. Therefore, the optimal bit allocation can be found through straightforward algebra.

Moreover, for non-identical distribution of channel gains the optimal bit allocation is found based on an exhaustive search. Afterward, we propose a heuristic method which has less computational complexity. Our simulation results confirm that the performance of the heuristic method is close to the optimal bit allocation policy. Later, based on the heuristic method, we have found that the expected throughput of users is a concave function of the energy they have consumed. This result lets use lazy scheduling for energy consumption between energy arrivals to optimize the system performance. In simulations, it is observed that when the available energy for feedback is limited, allowing variable sized feedback (as opposed to a fixed number of feedback bits) improves the performance significantly.

In Section 2.4 the fairness of giving system service to EH users with different channel gain distribution is evaluated. We use Proportional Fair Scheduling (PFS) technique, which has been used in some related works to maintain throughput fairness across the users. The simulation shows that by applying this technique, users experience more fairness in getting system service with the cost of a small decrease in system throughput.

Also, in this study we consider routing with the minimum transmissions time (equivalently, minimum energy) on a cooperative wireless network, where nodes have the ability to perform mutual information accumulation. For the case of unlimited energy at nodes, we propose a heuristic, which is based on Dijkstra's algorithm. Numerical evaluations show that it performs very close to optimum (greedy policy), and outperforms a related heuristic proposed in recent literature [30].

Next, we consider the case where nodes have limited energy. By counterexamples we prove the greedy policy for the optimal set, which is proposed before for unlimited energy case, fails in the limited energy case. The optimal solution involves enumerating all possible subsets (i.e. set of transmitting nodes) of the set of nodes, and running a Linear Programming (LP) solution for each of them, which is computationally expensive. We then exhibit a heuristic for this case, which performs close to the optimum and has less computational complexity.

This research area contains many interesting open problems which need to be studied in future. Here channel states are considered independent; however, in practice, channel state has correlation with its previous states. In correlated channel model, the AP could take the information of channels in previous frames into account instead of just focusing on received feedback at the current frame. Also multiuser selection scenario can be studied in future. In addition, our system model assumes a time portion in each frame that is allocated to each user to send feedback. However, for large networks this model would waste a large portion of frames for feedback. Therefore, this model needs to be modified for large scale systems.

Moreover, in routing problem, sensitivity analysis for an optimal routing policy with regard to estimation errors in channel quality is an interesting research topic. Also the routing for a system with energy harvesting users is an open area for future studies.

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