PROCUREMENT AND INFORMATION SHARING GAMES IN GROUP PURCHASING

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ABSTRACT

PROCUREMENT AND INFORMATION SHARING GAMES IN GROUP PURCHASING

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In this thesis, the value of collaboration and information sharing is analyzed for a market with two asymmetric competitive buyers. On one side of the chain, the supplier offers quantity discount. On the other side of the chain, the buyers may or may not engage in collaboration on purchasing quantity. The price is inverse linear function of the quantity supplied in the market. Buyers may receive a signal about uncertain market demand. Each buyer decides whether to share the signal with the other buyer and the supplier. First, four different cases are analyzed: (i) information is shared between the buyers and with the supplier, (ii) information is shared only between the buyers, (iii) neither the buyers nor the supplier shares the information, and (iv) buyers share the information only with the supplier. For each case optimum order quantity and whole sale price are calculated for both collaboration and no collaboration setting. Second, computational analysis is conducted by using the deducted results in order to determine the profit values of the parties. This leads to the following insights. First, if the market base is low for a buyer, it is substituted by other buyer. Second, the whole supply chain, supplier and weak buyer are better under collaboration setting however, the strong buyer is better only if the competition is low. Last, the supplier prefers to receive the signal, while the strong buyer is willing to share the signal with the supplier only if the difference between market bases is low.

Key words: Collaboration, information sharing, group purchasing, game theory, non-linear pricing

ÖZ

GRUPÇA SATIN ALMA SÜRECİNDE TEDARİK VE BİLGİ PAYLAŞIMI OYUNLARI

Ünel, Sevgi
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Bu tezde iki rekabet eden alıcının ve bir tedarikçinin bulunduğu pazarda alıcıların işbirliği ve bilgi paylaşımı kararları incelenmektedir. Tedarikçinin uyguladığı birim fiyat, artan satın alma miktarı ile düşmektedir. Alıcılar ise satın alma kararları konusunda işbirliğinde bulunabilirler. Pazardaki ürünün fiyatı arz edilen miktar artıkça azalmaktadır. Piyasadaki talep belirsiz olmakla beraber alıcılar talep hakkında öngörüde bulunabilirler ve bu bilgiyi diğer alıcı ve/veya tedarikçi ile paylaşabilirler. Bu tezde ilk olarak dört farklı durum analiz edilmiştir: i) talep bilgisinin alıcılar ve tedarikçi ile paylaşılması, ii) talep bilgisinin sadece alıcılar arasında paylaşılması, iii) talep bilgisinin alıcılar veya tedarikçi ile paylaşılmaması, iv) talep bilgisinin sadece tedarikçi ile paylaşılması. Her durum için ideal sipariş miktarı ile toptan satış fiyatı da belirlenmiştir. Daha sonra, farklı değişken değerleri için tedarik zincirinin genelinin ve üyelerinin kar değerleri hesaplanmıştır. Elde edilen sonuçlar şu şekildedir; pazar payı düşük olan alıcı satın alma kararı konusunda işbirliğinde bulunmayı tercih etmektedir, aksi takdirde pazar payı sıfırlanmaktadır. Diğer yandan pazar payı yüksek olan alıcı işbirliğinde bulunmayı sadece rekabetin düşük olduğu zamanlarda tercih etmektedir. Son olarak, tedarikçi talep bilgisinin paylaşılmasını tercih ederken güçlü alıcı bilgi paylaşımını alıcıların pazar payları arasındaki fark az olduğunda gerçekleştirmektedir.

Anahtar Sözcükler: İşbirliği, bilgi paylaşımı, oyun teorisi, doğrusal olmayan fiyatlandırma

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CHAPTER 1

INTRODUCTION

The market place has become a real challenge for companies in the last few decades. Product life cycles are getting shorter, competition has intensified greatly and new technologies are becoming obsolete much quicker than it was the case in the past (Langerak and Hultink, 2005) [1]. Collaboration is one of the main tools that play an important part in the minds of many companies as a solution to this problem. According to Prakash and Deshmukh (2010) [2], “collaboration is a negotiated cooperation between independent parties by exchanging capabilities and sharing burdens to improve collective responsiveness and profitability.” Furthermore, in the literature, collaboration is defined as being either horizontal or vertical. Horizontal collaboration means that companies with similar characteristics (potential competitors, same level at the supply chain) collaborate. Vertical collaboration is coordination between the buyers and the suppliers in a supply chain.

According to Dyer and Sing (1998) [3], the main reason in the end for collaboration is the call synergies or relational rents. This is defined as “a supernormal profit jointly generated in an exchange relationship that cannot be generated by either firm in isolation and can only be created through the joint idiosyncratic contributions of the specific alliance partners”. These relational rents can either be “tangible” in the form of cost savings and money or more “intangible” in the form of knowledge sharing or learning. Cruijssen et al. (2007) [4] make a distinction between four different categories which could be a motive for companies to participate in a form of horizontal cooperation, which are listed in Table 1. Horizontal cooperation is here defined as an active cooperation between two or more firms that operation on the same level of the supply chain.

Table 1 Motives for horizontal cooperation (Cruijssen et al. 2007)

Cost and Productivity	**Customer Service**	**Market Position**	**Other**
Cost reduction	Complementary goods and services	Penetrating new markets	Accessing superior technologies
More skilled labor force	Specialization	Faster speed to market	Developing technical standards

The focus of this thesis is foremost on the cost and productivity pillar as has been identified by Cruijssen et al. (2007) [4]. More specifically we focus on cost reduction by the use of group purchasing by specific alliance partners.

Group purchasing has been around for quite a while and represents a principal strategy of companies working together to realize cost containment and improve the quality of goods and services (Schneller, 2009) [5]. It provides especially the smaller players in the market like startup companies or other small and medium side enterprises to purchase various types of goods or services for a lower price than they could have achieved by acting alone. Since this kind of behavior is not special to any one industry it soon became interesting to provide this kind of organization as a service to smaller companies in the form of a so called Group Purchasing Organization (GPO).

As an example, the first GPO in the healthcare industry was already created in the late 1800’s and provided hospitals and other healthcare institutes with the help to pool their individual purchasing needs together and use this new and powerful bargaining power to achieve significant cost savings related to operation cost of the companies and thus increasing net profit (Hu et al., 2011) [6]. The service of the GPO come with a certain price tag, but if this price tag is lower than the cost reduction achieved via joining forces towards a supplier, this initial investment may be well worth the trouble.

GPO's can be used as an initial and quick solution of saving cost for smaller companies who do not want to invest too much in creating such a cooperation themselves. However, creating cooperation on their own might be more beneficial for several reasons. First, the GPO is frequently a large organization with a lot of companies within it that might not be sufficiently tuned to the specific needs of the company. After all cooperation for group purchase may have a narrow scope aimed at specific kind of products or widely oriented to reduce operational cost in all aspects. Second, most likely competitors, which one wishes not to cooperate, are also part of the GPO and make use of the same benefits which prevents the company of creating a competitive advantage via the strategy of operational effectiveness (Porter, 1980) [7]. Last, although finding the right partner(s) might come with an initial investment, it might be more profitable on the long run by not paying additional fees for third party service providers.

Group purchases are not only beneficial to the buyer's side but also vendors seem to profit. After all, it also provides the vendors with the opportunity of getting in touch with different kind of customers quickly and easily, which creates a win-win situation for both of them. However, many variables play a role in the success of an effective collaboration and will have an effect on the net profit to be reached by both the supplier as well as the buyers.

Besides group purchasing, there is also the possibility of information sharing. Companies usually engage in the act of information sharing to reduce uncertainty. In case of volatile markets every bit of information can be helpful for estimating the market demand. If all members of chain are willing to share information, each member will have less uncertainties and more information about other parts of supply chain. However, sharing information with the competitor must be a well-considered decision since both companies will then have the same market information at hand. The main reason for the lack of sharing information is the issues of confidentiality and firms may not have the incentive to share information with their partners because of the concern that they may abuse the information.

The focus of this thesis is to investigate the values of collaborative procurement among buyers and information sharing with parties in the supply chain. The rest of the thesis is structured as follows. In Chapter 2, related literature is reviewed. Then, the structures of the models for four different cases are characterized in Chapter 3. Next, in the following four chapters the models are explained in detail and analysis is done to determine the optimal order quantity and whole sale price for (i) information is shared between the buyers and with the supplier (Chapter 4), (ii) information is shared only between the buyers (Chapter 5), (iii) neither the buyers nor the supplier shares the information (Chapter 6), and (iv) buyers share the information only with the supplier (Chapter 7). The findings of computational analysis are presented in Chapter 8 and the conclusion is given in Chapter 9.

CHAPTER 2

LITERATURE REVIEW

The general issues that are studied in this thesis are collaboration, group purchasing, competition, and information sharing. In this chapter, we present the ideas from existing literature on similar subjects and point out the similarities and differences in their concepts and findings. The previous research can be categorized under two groups; collaboration and group purchasing and information sharing.

2.1 Collaboration and group purchasing

It is stated in literature that effective coordination is an important issue for both practical concerns and theoretical research. According to Chan and Roma (2011) [8] the opportunity for demand aggregation is increased due to the rapid development of information technology since it enables the cooperation and transaction among geographically distributed firms. Keskinocak and Savasaneril (2006) [9] states that many companies are intended to have more collaborative relationships with their business partners instead of having adversarial ones. In order to obtain mutual benefits, independent companies are working together and modifying their business processes.

The main initiative for group purchasing is the quantity discount offered by the manufacturer. Two widely used quantity discounts are the all unit (AQD) and the incremental quantity discounts (IQD). Under AQD scheme the discount is applied to all the units in a given order and under IQD scheme the discount applies only to additional units beyond pre specified breakpoints. When companies agree on group purchasing, then quantity discounts are offered based on their aggregated purchasing quantity instead of individual purchasing quantities.

Group purchasing for different systems is analyzed in many papers and some of them are reviewed in this section.

Weng (1995) [10] considers the system of a supplier and a group of homogenous buyers and analyzes the impact of joint decision policies on channel coordination. The research studies the supplier and buyer coordination problem by considering both the price sensitive demand and quantity depended transaction costs. Their model mainly combines the channel coordination and operation cost minimization which is new in literature because previous research considered the two issues separately. In the model the demand faced by the buyer is a decreasing function of the selling price and operating costs (i.e. ordering cost, holding cost etc.) are function of the order quantity. They claim that quantity discounts alone are not sufficient to guarantee maximum profit when demand is price sensitive and transaction costs depend on order quantity.

Two different scenarios are analyzed by Weng (1995) [10]; in the first one the supplier and the buyer maximize their own profits and in the second one the objective is to maximize the joint profit function. In scenario one, the optimum order quantity for the buyer is EOQ and this maximizes the suppliers profit only if the ratio of ordering cost and unit inventory cost of the supplier and the buyer are the same. In scenario two the joint profit function is defined as the sum of supplier's and buyer's profit and using this function joint EOQ, which results in a profit increase, is found. Then, the increase in joint profit brings the question of division of it to parties in a way that both parties choose the policy that maximizes their individual profits and joint profit simultaneously. In the previous studies (when ordering and inventory holding costs are not considered in the model explicitly) quantity discount was found to be sufficient to achieve channel coordination. However, in Weng (1995) [10] it is found that for one supplier and a group of homogenous buyers system quantity discounts alone are not enough to guarantee joint profit maximization, hence each period the buyer should make a fixed payment to supplier and then the supplier should set the average selling price to joint selling price. Moreover, it is shown that both discount policies (AQD or IQD) perform by providing the same profit to the buyer and supplier.

Chen and Roma (2011) [8] present a different perspective in group buying, even though previous studies focus on the benefits of the it, Chen and Roma (2011) [8] claim that buyers can get hurt from the cooperation in group purchasing. By considering price sensitive demand, direct competition in the market and general quantity discount schedules, Chen and Roma (2011) [8] is the first that studies group buying in distribution channel. A two level distribution channel with one supplier and two retailers who compete for the end customers are analyzed in the study in order to determine whether the retailers purchase together to obtain lower wholesale process or not. Linear demand function is used in the model by focusing on the difference in retailers' access to customers and operational efficiency, which are the two common factors in the literature that examines competing retailers. And in order to focus on the effects of competition, it is assumed that all model parameters are deterministic and common knowledge.

It is found that for identical buyers if the demand is linear, then group buying is beneficial. Moreover, while under individual purchasing the retailer's profit decreases with competition, under group purchasing higher competition level may increase the retailer's profit as long as the discount level is not low and competition is not fierce. Under IP, retailers may not always prefer deep discount on the other hand this never happens under GB and for symmetric buyers GB is always preferable. Moreover, under GB when the competition is high, this may result in less retail quantity and higher retail process and might hurt the end customers. GB softens the competition effect and improves retailer's profit. On the other hand, GB leads to a lower wholesale price and lower manufacturer's revenue.

Manufacturer would prefer symmetric retailers to asymmetric ones when they purchase individually but under group purchase manufacturer is indifferent whether the retailers are alike or different. If the quantity discount is linear, then under IP the stronger retailer may or may not set a higher price than the other retailer (deep discount may motivate the retailer to set lower price) and under GB the retail price, quantity and profit is always higher for the strong one. Moreover, the small retailer will always prefer group purchasing but strong retailer can get hurt by GB. Hence, GB is preferable if retailers are similar and if they are not alike GB may hurt the strong retailer.

It is also stated in the article that, even if these results are found by using linear demand function they are robust and applicable to other demand functions as well. Furthermore, if there are more than two retailers, then if the asymmetry level is high then the big retailer will not cooperate and small retailers will group buy. For medium or low asymmetry levels all players would form a group, however sometimes the big retailer only cooperate with one but not both small retailers.

The first analytical model of group buying is proposed by Anand and Aron (2003) [11] which focuses on online group buying model with different kinds of demand uncertainty. In their model quantity discounts are offered to the total of all customer orders. The keys issues analyzed in the paper are, the market and product characteristics that initiate the use of group buying, the optimal group buying schedule or a firm that uses group buying policy to sell the products and the performance of this group buying schedule compared to posted price. The main motivation for the supplier to offer group buying is the belief that with the monotonically decreasing price scheme supplier can create higher customer demand and increase the revenue. Then, individual customers or small to medium-sized business those have low bargaining power are targeted for group buying. Production postponement, which happens when supplier produces or procures the exact amount of product after demand realization, leads to higher supplier profit.

The analysis is done for monopolistic seller who estimates the demand for his product within two demand regimes (high and low) and it depends on product attributes and consumer preferences, the availability of complementary or substitute products and a host of macroeconomic factors. Then the monopolist's problem is to decide a price quantity schedule that maximizes his revenue without knowing which demand regime is realized. The optimal group buying scheme is simply the same as optimal posted price hence seller's revenue from group buying can never exceed the revenue from simple posted price. On the other hand, when on demand regime does not dominate the other universally (when demand curves intersect) group buying outperforms posted process and the gain increases as the demand heterogeneity increases.

The other issue discussed in Anand and Aron (2003) [11] is the effect of production procurement and production postponement. Production postponement is feasible production or procurement lead times are sufficiently small. Here the firm determines its production/procurement quantity after observing the demand as a function of its price or price quantity schedule. Under production pre commitment the firms pricing decisions are made after and are constrained by commitment to production/procurement quantities.

Production postponement is beneficial for the seller under both posted price and group buying. Under production pre commitment the equilibrium solution and seller's profit are the same under group buying and posted pricing. On the other hand, under production postponement group buying dominates posted pricing. Generally, if there is not any demand uncertainty, then group buying cannot outperform posted prices.

Game theoretical approach is used in Keskinocak and Savasaneril (2006) [9] in order to examine the effects of collaboration on buyer and supplier profits. They focus on horizontal collaboration in procurement for two firms which are competitors at the end market and can be both similar size (symmetric) and different size (asymmetric). They distinguish the market conditions which lead to different type of collaboration and the situations whether a buyer prefers a larger buyer to smaller one to collaborate. The conditions when the buyers and supplier benefit from collaboration are also analyzed.

The main motivation behind the group purchasing is the quantity discount for the buyers and price discrimination against buyers to pass part of supply chain cost to buyers and increase overall system efficiency for the supplier.

It is stated that when the buyer is uncapacitated, which means there is no limitation on procurement quantity, then collaboration is always attractive. On the other hand, if the buyers are capacitated then the small buyer may be left out of the market. Therefore, it may not be beneficial to collaborate with the bigger buyer for some cases. In general, in order to have willingness for collaboration, the procurement quantity of buyer should increase under group buying. The supplier is more willing to offer discounts when the number of buyers increases and they form a bigger market. Moreover, supplier prefers to sell smaller and equal quantities to buyers instead of selling larger quantity to same buyers. Finally, the end customer is better of due to group purchasing because total quantity produced/procured under group buying is higher and it results in a lower market price.

2.2 Information sharing

It is well know that information sharing can improve the supply chain efficiency. According to Ha et al. (2011) [12] information about product demand is obtained by retailers and many large retailers started sharing such information with their supplier. On the other hand, this may bring up the issues of confidentiality and firms may not have the incentive to share information with their partners because of the concern that they may abuse the information.

Information sharing is analyzed in many papers for different systems and some of them are reviewed in this section.

A supply chain with one manufacturer and two competing retailers are analyzed in Zhang (2002) [13] and manufacturer's optimal strategy is found to be independent of type of competition. The manufacturer is always better off by receiving information from more retailers and each retailer is always worse off by sharing his demand information. Therefore, no information sharing is found to be the unique equilibrium for both types of competition and for any degree of product interaction. On the other hand, if the supply chain is better off with information sharing, then the manufacturer may offer a side payment in order to compensate the loss of the retailers and initiate the information sharing. In this case, both retailers agree on it and a symmetric equilibrium is achieved.

Ha et al. (2011) [12] defines three different effects of information sharing; direct, competitive and spillover. "The direct effect of information sharing is the impact on itself as if its information sharing activities are not recognized, or responded to, by the rival supply chain. The competitive effect is the additional impact on the supply chain if the reaction of the rival supply chain is taken into consideration. The spillover effect is the impact on the rival supply chain." Ha et al. (2011) [12] considers two different supply chains each with one retailer and one manufacturer. The retailers are engaged either in Cournot or Bertrand competition. The effects of production diseconomies of scale, information accuracy, competition intensity and types of competition are investigated.

If the retailers are in Cournot competition, the manufacturer always benefits from information sharing while the retailer is always worse off. In other words, the direct effect is positive and the competitive effect is negative, and information sharing benefits the supply chain when direct effect dominates competitive effect. This happens either when the production diseconomy is large or when the competition is not intense or retailer's information is not accurate. Then, if the information sharing is beneficial for whole supply chain, the manufacturer may use a side payment to initiate the action. If production costs are linear, no information sharing is the unique equilibrium.

On the other hand, if there is Bertrand competition between retailers, then information sharing benefits the supply chain when the production diseconomy is large or when competition is not intense and retailer's information is more accurate. Unlike Cournot competition case, under Bertrand competition the manufacturer can be worse off due to the information sharing. The direct effect is positive and competitive effect can be positive when production diseconomy is small or both demand signals are accurate, which is different from the Cournot competition. The retailer is worse off with information sharing and not volunteer in the process, moreover, the manufacturer can be worse off because of the competitive reaction, which is different than Cournot case.

Li (1985) [14] focuses on the incentives under Cournot competition to share information about a common parameter and firm specific parameter for oligopolistic industry, where the market is controlled by small group of firms. They consider the possibility of incomplete information sharing; each firm decides whether to share the information and how much to share.

The type of uncertainty is divided into two; uncertainty about common parameter (which is demand) and uncertainty about firm specific parameter (which is cost). It is found that if the uncertainty is related to common parameter, then firms never reveal the information while if the uncertainty is related to firm specific parameter, complete information sharing is the unique equilibrium. Moreover, information sharing is always socially desirable and results in higher expected total social welfare. It is also suggested that efficiency is achieved if the number of competitive firms increases.

Game theoretical approach is used by Parlar (1988) [16] to analyze the inventory problem for two substitutable products with random demand. It is claimed that if the substitutable products are sold by different retailers, the profit function of a decision maker is influenced not only by his own order decision but also by the decision of his competitors. Hence, the problem cannot be analyzed in isolation from other's decision.

The concepts from two-person continuous games are used in the analysis and it is assumed that the players have information about demand densities, substitution rates and other parameter values. Three different cases are analyzed; the Nash solution where each player acts rationally (no player risks of lowering his own objective function for the purpose of damaging his competitor's) and wants to maximize his own objective function, maximin solution where one of the player acts irrationally and wants to inflict maximum damage to other player, and players cooperate to maximize a joint objective function. The first two models are more realistic when there is a competition between players. It is found that the profit under maximin strategy is lower than Nash strategy and cooperation increases the well-being of both players.

The incentives for information sharing among firms when they face uncertain demand under oligopolistic market are analyzed by Gar-Or (1985) [17]. When the firm observes a low demand signal and shares it, then the competitor's risk to overproduce decreases. On the other hand, if the demand signal reveals a high demand then it reduces the likelihood of the competitor to underproduce. If the private signals are highly correlated, then a firm can easily guess the competitor's signal based on its own. Hence, even if the information is not shared, the likelihood of under production (if signal is low) or over production (if signal is high) for the competitor is relatively small. This reduces both the benefits and loss from information sharing. On the other hand, if there is no correlation between signals and information is not shred, then competitors cannot predict the signal of other party easily and are more likely to be mistaken in choosing output levels. No information sharing is found to be unique equilibrium regardless of the correlation between the signals.

Shang et al. (2011) [18] are the first ones that study the information sharing in a supply chain with two manufacturers which are competitors and selling substitutable products through a common retailer. Information sharing is triggered by large production diseconomy or high competition between the manufacturers. Three different cases are analyzed; RSC (retailer stackelberg with concurrent offers) where retailer makes concurrent and identical offers of selling information for a fixed payment to the manufacturers, RSS (retailer stackelberg with sequential offers) where retailer makes sequential offers of selling information to the first manufacturer then to the second one and MS (manufacturer stackelberg) where the manufacturers simultaneously offer fixed payments for buying information from the retailer. It is found that information sharing is always beneficial to retailer and the benefit is larger when the retailer offers the contracts sequentially. On the other hand, it is beneficial to the manufacturer when production diseconomy is large and they are the leaders in offering the contracts. The other finding is that partial information sharing does not maximize the supply chain profit it may occur in equilibrium and in this case the manufacturer is willing to pay more in order to be the only informed manufacturer. Ha et al. (2011) [12] claims that when two supply chains compete at retailer level more intense competition results in less information sharing and retailer cannot benefit from a larger production diseconomy. However, Shang et al. (2011) [18] shows that more intense competition at the manufacturer level induces more information sharing and the common retailer benefit from production diseconomy.

2.3 Contribution of the thesis

As given in the previous sections, the related literature is grouped under two categories; group purchasing and information sharing. To our knowledge, this research is the first one which combines the procurement game and information sharing game for group purchasing. The situation is analyzed for one supplier two buyers where buyers can share their private information with other players and supplier offers quantity discount hence buyers can be included in group purchasing.

CHAPTER 3

THE MODEL

In our model, we consider a supply chain with one supplier and two asymmetric buyers. The buyers have their own market but they are competitors since the final products that they offer are substitutes for each other. It is assumed that the total quantity supplied to the market affects the market price; that means if more quantity is supplied market price will decrease. The market price is different for each buyer which is modeled as follows;

$$P_1 = A_1 + \theta_1 - q_1 - \beta q_2$$

$$P_2 = A_2 + \theta_2 - q_2 - \beta q_1$$

where

P_i : market price of buyer *i*

A_i : maximum reservation price in the market *i*

θ_i : uncertainty about reservation price

q_i : quantity that buyer *i* procures

β : market sensitivity to quantity

Each buyer experiences only one cost which is procurement cost from supplier and orders the exact amount that it will sell in the end market. The supplier offers a quantity discount to the buyers; then the unit wholesale price is of the form $w(q_1, q_2) = c_1 - c_2(q_1 + q_2)$ in collaboration setting and $w(q_1, q_2) = c_1 - c_2 q_i$ in no-collaboration setting.

where

c_1 : supply price for the first unit

c_2 : coefficient of discount in wholesale price

The unit wholesales price decreases depending on both the unit purchased by the buyer itself and other buyers.

It is assumed that there is uncertainty regarding the base reservation price of the consumers which is denoted by the random variable θ_i. For the generic model,

$$P_i = A_i + \theta_i - q_i - \beta q_j$$

θ_i denotes the population parameter, where the population consist of $\theta_{i,L} < 0$ and $\theta_{i,H} > 0$ values. The population is Bernoulli distributed, with parameter p, and the parameter θ_i is defined as:

$$\theta_i = (1 - p)\theta_{i,L} + p\theta_{i,H} \qquad (3.1)$$

Here both p and θ_i are random variables. A prior distribution is known for p. Each retailer receives a signal on θ, e.g., a sample from the population that parameter θ_i defines, and a posterior distribution is obtained based on the sample. It is assumed that $E[\theta_i] = 0$. The signal for θ_i is denoted with Y_i which is the sample mean from independent sampling. Note that the signal is a random variable, and an unbiased estimator of θ_i. $E[Y_i|\theta_i] = \theta_i$. Moreover, a linear expectation information structure is assumed; $E[\theta_i|Y_i]$ is a weighted average of prior mean $E[\theta_i]$, if θ_i and Y_i follows certain distributions (Ericson, 1969). It is further assumed that the parameter p in Eqn. 3.1 is $Beta(a, b)$ distributed. (More specifically, p is Beta(1,1) i.e., Uniform[0,1] distributed). Then, it is shown in Ericson (1969) that,

$$E[\theta_i|Y_i] = \frac{1}{1+\alpha_i\sigma_i^2}E[\theta_i] + \frac{\alpha_i\sigma_i^2}{1+\alpha_i\sigma_i^2}Y_i = \delta_i(\alpha_i,\sigma_i)Y_i \qquad (3.2)$$

Here, α denotes the signal accuracy $\alpha_i = \frac{1}{E[Var[Y_i|\theta_i]]}$.

The signal is assumed to be imperfect and $E[\theta_i|Y_i] = \delta_i(\alpha_i,\sigma_i)Y_i$ where $\delta_i(\alpha_i,\sigma_i)$ is used instead of $\frac{\alpha_i\sigma_i^2}{1+\alpha_i\sigma_i^2}$ for simplicity of notation.

An example about determining the posterior distribution of p is given below.

3.1 Example for the determination of posterior distribution

Suppose p is distributed $Beta(1,1)$. This is the prior distribution for p. Note that the probability density function for $Beta(a,b)$ is as follows:

$$f(x) = \begin{cases} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}, & 0 < x < 1 \\ 0 \quad , & o/w \end{cases}$$

where $\Gamma(n) = (n-1)!$ for $n \geq 1$.

For $a = 1$ and $b = 1$ the probability density function (pdf) of beta distribution becomes equivalent to that of uniform distribution, $f(x) = 1$ for $0 < x < 1$.

Suppose a sample is drawn from the population. Suppose the sample consists of 2 low and 1 high values after three trials. Then the posterior pdf evolves as follows (given $a = 1$ and $b = 1$):

After trial 1

$$f(x|1\ low) = \frac{\Gamma(a+1+b)}{\Gamma(a)\Gamma(1+b)}x^{a-1}(1-x)^{1+b-1} = 2(1-x)$$

After trial 2

$$f(x|2\ low) = \frac{\Gamma(a+2+b)}{\Gamma(a)\Gamma(2+b)}x^{a-1}(1-x)^{2+b-1} = 2(1-x)^2$$

After trial 3

$$f(x|2\,low, 1\,high) = \frac{\Gamma(1+a+2+b)}{\Gamma(1+a)\Gamma(2+b)} x^{1+a-1}(1-x)^{2+b-1} = 12x(1-x)^2$$

Figure 1 below shows the prior and posterior pdf on *p*:

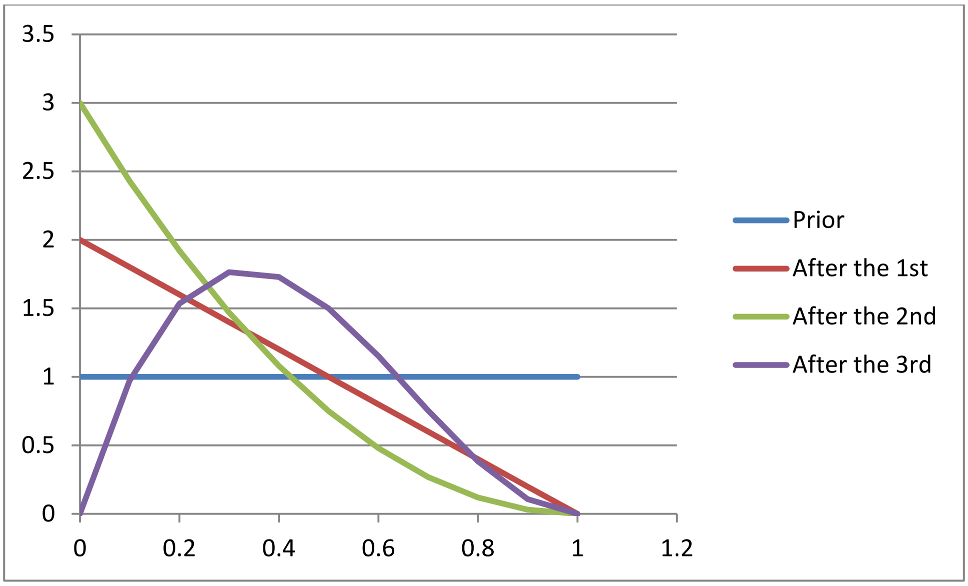

Figure 1 The prior and posterior pdf on *p*

Given that under prior distribution $E[\theta] = 0$, possible values for θ_L and θ_H can be determined as follows:

$$E[\theta] = (1-p)\theta_L + p\theta_H$$

$$E[\theta] = 0 \rightarrow \int_0^1 \left((1-x)\theta_L + x\theta_H\right)dx = \frac{\theta_L + \theta_H}{2}$$

If $\theta_L = -\theta_H$, then this results in $E[\theta] = 0$. Let $\theta_L = -1$ and $\theta_H = 1$.

After the sample, $E[\theta]$ is determined using Eqn. 3.2 as:

$$E[\theta_i|Y_i] = \frac{\alpha_i \sigma_i^2}{1 + \alpha_i \sigma_i^2} Y_i$$

where $\alpha_i = \frac{1}{E[Var[Y_i|\theta_i]]}$. Here *Y* is the sample mean of three trials:

$$Y = \frac{X_1 + X_2 + X_3}{3} = \frac{(-1) + (-1) + (1)}{3}$$

Note that $Var(Y|\theta) = \frac{Var(X|\theta)}{n} = \frac{p(1-p)(\theta_H - \theta_L)^2}{n}$. $E[Var(Y|\theta)] = \frac{(\theta_H - \theta_L)^2}{n}(E[p] - E[p^2]) = \frac{(\theta_H - \theta_L)^2}{6n}$.

Then $\alpha = \frac{(6)(3)}{(\theta_H - \theta_L)^2} = 4.5$. It is possible to determine σ^2 as follows;

$Var(\theta) = Var\big((1-p)(-1) + p(1)\big) = Var(2p-1) = E[(2p-1-0)2] = 1/3$

This results in $E[\theta|Y] = -1/5$, equivalently $E[p|Y] = 0.4$.

Check whether this is equal to what is obtained from the posterior pdf (after a sample of three)

$E[p|Y] = \int_0^1 12x^2(1-x)^2 dx = 0.4$

3.2 Probability distribution of Y_i

A sample is drawn from the population and X denotes the result of sampling. Since the population consist of low and high values only, then

$$P\{X = \theta_H\} = p$$

$$P\{X = \theta_L\} = 1 - p$$

The signal Y is defined as the average of n random variables taken from the population.

$$Y = \frac{X_1 + X_2 + \cdots X_n}{n}$$

Let $Z = X_1 + X_2 + \cdots X_n$. Then the probability distribution of Z will be similar to that of Binomial distribution and can be expressed below for a given $p = p_1$;

$$P\{Z = n\theta_L \big| p_1\} = \binom{n}{0} p_1^0 (1-p_1)^n$$

Note that p is a random variable with the probability density function $f_p(p)$. Thus, the probability that $Z = n\theta_L$ is;

$$P\{Z = n\theta_L\} = \int_0^1 \binom{n}{0} {p_1}^0 (1-p_1)^n f_p(p_1) dp_1$$

For p, with prior distribution $Beta(1,1) \equiv Uniform[0,1]$,

$$P\{Z = n\theta_L\} = \binom{n}{0} 0.5^0 0.5^n = \binom{n}{0} 0.5^n$$

The probability distribution of Z can be obtained as

$$P\{Z = m\theta_L + (n-m)\theta_H\} = \binom{n}{m} 0.5^m 0.5^{n-m} = \binom{n}{m} 0.5^n$$

Since $Y = Z/n$, probability distribution of Y can be obtained accordingly.

3.3 Sequence of events

The sequence of the events is as follows;

1. Buyers obtain a signal Y_i on the unknown parameter θ_i, and decide whether to share it with the supplier and the other buyer. It is assumed that both buyers simultaneously decide and take the same decision as to whether to share the information or not. Buyers also decide whether to collaborate or not.
2. The supplier offers c_1 and c_2 to the buyers.
3. Buyers decide on the corresponding quantities to purchase from the supplier.

This is a three-stage game and in this thesis, stage 2 and 3 are studied as a two-stage Stackelberg game, called subgame, whereas stage 1 is analyzed under combination of scenarios. There are four different scenarios depending on information sharing between the buyers and the supplier, and each scenario is composed of two subscenarios, namely; collaboration and no collaboration. Under each scenario the game is analyzed as follows; the supplier determines the wholesale price (i.e. c_1 and

c_2 values) in the first stage of the subgame and in the second stage for given c_1 and c_2 values the buyers get engaged in a game and Nash equilibrium is determined.

The supplier is assumed to be uncapacitated (i.e. able to produce the total amount purchased by the buyers) and the profit function consists of only the revenue obtained from the buyers. The buyers are also assumed to be uncapacitated (i.e. no restriction on the order quantity) and the profit function of buyer *i* is a function of q_i and q_j and composed of (Revenue - Cost), where the cost is due to the wholesale price of the supplier.

In the two-stage subgame, first the decisions under the second stage are determined and then the decisions under the first stage are concluded. Under each scenario, the profit of the supplier and the buyers are as follows.

3.3.1 Imperfect information for the buyers and the supplier (IBIS)

Under IBIS case, buyers obtain an imperfect signal and share it with the supplier and other buyer.

1) Collaboration:

$$\pi_s|Y_1, Y_2 = (c_1 - c_2(q_1 + q_2))(q_1 + q_2)$$

$$E_\theta[\pi_1|Y_1, Y_2] = E[(A_1 + \theta_1 - q_1 - \beta q_2 - c_1 + c_2(q_1 + q_2))q_1|Y_1, Y_2]$$

$$E_\theta[\pi_2|Y_1, Y_2] = E[\left(A_2 + \theta_2 - q_2 - \beta q_1 - c_1 + c_2(q_1 + q_2)\right)q_2|Y_1, Y_2]$$

where

π_s : profit of supplier

π_i : profit of buyer *i*

The supplier offers quantity discount and under collaboration setting, discount is done over total quantity ordered buyers. The profit of supplier is found by multiplying the wholesales price with the total order quantity. The profit of buyer is found by subtracting the cost from revenue and then multipliying it with the order quantity of the corresponding supplier. Moreover, under IBIS case, the signals received by each buyer are shared with all parties, hence expectation is taken over θ, and q_1 and q_2 are functions of Y_1 and Y_2.

2) No collaboration:

$$\pi_s|Y_1, Y_2 = (c_1 - c_2q_1)q_1 + (c_1 - c_2q_2)q_2$$

$$E_\theta[\pi_1|Y_1, Y_2] = E[(A_1 + \theta_1 - q_1 - \beta q_2 - c_1 + c_2q_1)q_1|Y_1, Y_2]$$

$$E_\theta[\pi_2|Y_1, Y_2] = E[(A_2 + \theta_2 - q_2 - \beta q_1 - c_1 + c_2q_2)q_2|Y_1, Y_2]$$

When there is no collaboration between buyers, quantity discount offered by the supplier is done only on the order quantity that each buyer orders. The profits of supplier and buyers are found by following the same way as in collaboration case.

3.3.2 Imperfect information for the buyers, no information for the supplier (IBNS)

Under IBNS case, buyers obtain an imperfect signal and share it with the other buyer but do not share it with the supplier.

1) Collaboration:

$$E_{\theta,Y_1,Y_2}[\pi_s] = E_{\theta,Y_1,Y_2}\big[\big(c_1 - c_2(q_1 + q_2)\big)(q_1 + q_2)\big]$$
$$= c_1E_{\theta,Y_1,Y_2}[q_1 + q_2] - c_2E_{\theta,Y_1,Y_2}[(q_1 + q_2)^2]$$

$$E_\theta[\pi_1|Y_1, Y_2] = E[(A_1 + \theta_1 - q_1 - \beta q_2 - c_1 + c_2(q_1 + q_2))q_1|Y_1, Y_2]$$

$$E_\theta[\pi_2|Y_1, Y_2] = E[\big(A_2 + \theta_2 - q_2 - \beta q_1 - c_1 + c_2(q_1 + q_2)\big)q_2|Y_1, Y_2]$$

2) No collaboration:

$$E_{\theta,Y_1,Y_2}[\pi_s] = E_{\theta,Y_1,Y_2}[(c_1 - c_2q_1)q_1 + (c_1 - c_2q_2)q_2]$$
$$= c_1E_{\theta,Y_1,Y_2}[q_1 + q_2] - c_2E_{\theta,Y_1,Y_2}[q_1^2 + q_2^2]$$

$$E_\theta[\pi_1|Y_1, Y_2] = E[(A_1 + \theta_1 - q_1 - \beta q_2 - c_1 + c_2q_1)q_1|Y_1, Y_2]$$

$$E_\theta[\pi_2|Y_1, Y_2] = E[(A_2 + \theta_2 - q_2 - \beta q_1 - c_1 + c_2q_2)q_2|Y_1, Y_2]$$

3.3.3 No information for the buyer, no information for the supplier (NBNS)

Under NBNS case, buyers obtain an imperfect signal and do share it with neither the supplier nor the other buyer.

1) Collaboration:

$$E_{\theta,Y_1,Y_2}[\pi_s] = E_{\theta,Y_1,Y_2}[(c_1 - c_2(q_1 + q_2)(q_1 + q_2)]$$
$$= c_1 E_{\theta,Y_1,Y_2}[q_1 + q_2] - c_2 E_{\theta,Y_1,Y_2}[(q_1 + q_2)^2]$$

$$E_{\theta,Y_2}[\pi_1|Y_1] = E[(A_1 + \theta_1 - q_1 - \beta q_2 - c_1 + c_2(q_1 + q_2))q_1|Y_1]$$

$$E_{\theta,Y_1}[\pi_2|Y_2] = E[(A_2 + \theta_2 - q_2 - \beta q_1 - c_1 + c_2(q_1 + q_2))q_2|Y_2]$$

2) No collaboration:

$$E_{\theta,Y_1,Y_2}[\pi_s] = E_{\theta,Y_1,Y_2}[(c_1 - c_2 q_1)q_1] + E[(c_1 - c_2 q_2)q_2]$$
$$= c_1 E_{\theta,Y_1,Y_2}[q_1 + q_2] - c_2 E_{\theta,Y_1,Y_2}[q_1^2 + q_2^2]$$

$$E_{\theta,Y_2}[\pi_1|Y_1] = E[(A_1 + \theta_1 - q_1 - \beta q_2 - c_1 + c_2 q_1)q_1|Y_1]$$

$$E_{\theta,Y_1}[\pi_2|Y_2] = E[(A_2 + \theta_2 - q_2 - \beta q_1 - c_1 + c_2 q_2)q_2|Y_2]$$

3.3.4 No information for the buyer, imperfect information for the supplier (NBIS)

Under NBIS case, buyers obtain an imperfect signal and share it only with the supplier.

1) Collaboration:

$$\pi_s|Y_1, Y_2 = \big(c_1 - c_2(q_1 + q_2)\big)(q_1 + q_2)$$
$$E_{\theta,Y_2}[\pi_1|Y_1] = E_{\theta,Y_2}[(A_1 + \theta_1 - q_1 - \beta q_2 - c_1 + c_2(q_1 + q_2))q_1|Y_1]$$

$$E_{\theta,Y_1}[\pi_2|Y_2] = E_{\theta,Y_1}[(A_2 + \theta_2 - q_2 - \beta q_1 - c_1 + c_2(q_1 + q_2))q_2|Y_2]$$

2) No collaboration:

$$\pi_s|Y_1, Y_2 = (c_1 - c_2 q_1)q_1 + (c_1 - c_2 q_2)q_2$$

$$E_{\theta,Y_2}[\pi_1|Y_1] = E_{\theta,Y_2}[(A_1 + \theta_1 - q_1 - \beta q_2 - c_1 + c_2 q_1)q_1|Y_1]$$

$$E_{\theta,Y_1}[\pi_2|Y_2] = E_{\theta,Y_1}[(A_2 + \theta_2 - q_2 - \beta q_1 - c_1 + c_2 q_2)q_2|Y_2]$$

Assumptions

The following assumptions are made throughout the analysis:

Asm1. $A_1 > A_2$. The maximum reservation price in the market is assumed to be higher for buyer 1.

Asm2. $0 \leq \beta < 1$. The items are substitutes.

Asm3. $c_2 \leq \beta - \varepsilon$ to ensure that under collaboration and quantity discount the competition among the buyers is preserved. It is assumed that ε is an arbitrarily small value agreed on between the supplier and the buyers.

CHAPTER 4

IMPERFECT INFORMATION SHARING AMONG THE BUYERS AND THE SUPPLIER (IBIS)

In this section the equilibrium points and the supplier's optimal c_1 and c_2 values are determined under the strategy that the buyers share their signals on the market demand with each other and with the supplier. The analysis is made under collaborating and non-collaborating buyers. In the analysis, first, the equilibrium quantities are determined, and then the optimum wholesale price is calculated. The assumptions which are given in the previous chapter are valid throughout the analysis.

4.1 Collaborating buyers

When the buyers are collaborating, the profit of supplier and buyers are expressed as follows;

$$\begin{aligned}E_\theta[\pi_1|Y_1,Y_2] &= E[(A_1+\theta_1-q_1-\beta q_2-c_1+c_2(q_1+q_2))q_1|Y_1,Y_2]\\ &= \big(A_1+E[\theta_1|Y_1]-q_1-\beta q_2-c_1+c_2(q_1+q_2)\big)q_1\\ &= \big(A_1+\delta_1(\alpha_1,\sigma_1)Y_1-q_1-\beta q_2-c_1+c_2(q_1+q_2)\big)q_1\end{aligned}$$

$$\begin{aligned}E_\theta[\pi_2|Y_1,Y_2] &= E[(A_2+\theta_2-q_2-\beta q_1-c_1+c_2(q_1+q_2))q_2|Y_1,Y_2]\\ &= \big(A_2+E[\theta_2|Y_2]-q_2-\beta q_1-c_1+c_2(q_1+q_2)\big)q_2\\ &= \big(A_2+\delta_2(\alpha_2,\sigma_2)Y_2-q_2-\beta q_1-c_1+c_2(q_1+q_2)\big)q_2\end{aligned}$$

As discussed in Chapter 3, Y_1 and Y_2 which are the signals received by each buyer are unbiased estimators of θ_1 and θ_2. Note $E[Y_1|\theta_1] = \theta_1$ and $E[Y_2|\theta_2] = \theta_2$, $E[\theta_1] = E[\theta_2] = 0$.

The supplier's ex-post profit function does not include any uncertainty given Y_1 and Y_2.

$$\pi_s|Y_1, Y_2 = (c_1 - c_2(q_1 + q_2))(q_1 + q_2)$$

Here, $E_\theta[\pi_1|Y_1, Y_2]$, $E_\theta[\pi_2|Y_1, Y_2]$ and $[\pi_s|Y_1, Y_2]$ denote the ex-post profit functions for the buyers and the supplier. In the expressions when q_1 and q_2 are equilibrium quantities, q_1 and q_2 are functions of c_1, c_2, Y_1 and Y_2. Ex-ante profits for the buyers and the supplier are given in Section 4.3.

In the following we discuss how the equilibrium quantities for the buyers and optimal c_1 and c_2 values for the supplier are determined. In Section 4.1.3 examples are given.

4.1.1 The Buyers' Problem

In this section, for a given value of c_1 and c_2 values the equilibrium quantities of the buyers are determined. In the analysis $A_1^{'}$ denotes $A_1 + \delta_1(\alpha_1, \sigma_1)Y_1$ and $A_2^{'}$ denotes $A_2 + \delta_2(\alpha_2, \sigma_2)Y_2$. The analysis is performed for $A_1^{'} > A_2^{'}$. It follows the same steps for $A_2^{'} > A_1^{'}$.

Proposition 1: Under IBIS setting with two collaborating buyers, for a given c_1 and c_2, the equilibrium points are expressed as follows;

Case 1: When

$$c_1 < \frac{2A_2^{'}(1 - c_2)}{2 - \beta - c_2} - \frac{A_1^{'}(\beta - c_2)}{2 - \beta - c_2}$$

Unique equilibrium with positive responses for buyer 1 and buyer 2:

$$q_1 = \frac{2(A_1^{'} - c_1)(1 - c_2) - (\beta - c_2)\left(A_2^{'} - c_1\right)}{4(1 - c_2)^2 - (\beta - c_2)^2}$$

$$q_2 = \frac{2(A_2^{'} - c_1)(1 - c_2) - (\beta - c_2)\left(A_1^{'} - c_1\right)}{4(1 - c_2)^2 - (\beta - c_2)^2}$$

Case 2: When

$$c_1 \geq \frac{2A_2^{'}(1 - c_2)}{2 - \beta - c_2} - \frac{A_1^{'}(\beta - c_2)}{2 - \beta - c_2}$$

Unique equilibrium with positive response only for buyer 1 and zero for buyer 2.

$$q_1 = \frac{A_1^{'} - c_1}{2(1 - c_2)}$$

$$q_2 = 0$$

Proof: To find the best response of buyer *i*, first derivative of profit function, π_i, is taken with respect to q_i.

For buyer 1

$$\frac{\partial E[\pi_1|Y_1]}{\partial q_1} = A_1 + \delta_1(\alpha_1, \sigma_1)Y_1 - 2q_1 - \beta q_2 - c_1 + 2c_2 q_1 + c_2 q_2 = 0$$

$$q_1(q_2) = \left(\frac{A_1 + \delta_1(\alpha_1, \sigma_1)Y_1 - c_1}{2 - 2c_2} - \frac{q_2(\beta - c_2)}{2 - 2c_2}\right)^+ = \left(\frac{A_1^{'} - c_1}{2 - 2c_2} - \frac{q_2(\beta - c_2)}{2 - 2c_2}\right)^+$$

For buyer 2

$$\frac{\partial E[\pi_2|Y_2]}{\partial q_2} = A_2 + \delta_2(\alpha_2, \sigma_2)Y_2 - 2q_2 - \beta q_1 - c_1 + 2c_2 q_2 + c_2 q_1 = 0$$

$$q_2(q_1) = \left(\frac{A_2 + \delta_2(\alpha_2, \sigma_2)Y_2 - c_1}{2 - 2c_2} - \frac{q_1(\beta - c_2)}{2 - 2c_2}\right)^+$$
$$= \left(\frac{A_2^{'} - c_1}{2 - 2c_2} - \frac{q_1(\beta - c_2)}{2 - 2c_2}\right)^+$$

Increasing q_2 decreases the response quantity of the first buyer, q_1; if buyer 2 purchases more, since the products are substitutes, buyer 1 will purchase less. Similarly, increasing q_1 decreases the response of second buyer, q_2.

The extremes of first buyer's response;

If $q_2 = 0 \; then \; q_1 = \frac{A_1^{'} - c_1}{2(1-c_2)}$

If $q_2 \geq \frac{A_1^{'} - c_1}{\beta - c_2} \; then \; q_1 = 0$

The extremes of second buyer's response;

If $q_1 = 0 \; then \; q_2 = \frac{A_2^{'} - c_1}{2(1-c_2)}$

If $q_1 \geq \frac{A_2^{'} - c_1}{\beta - c_2} \; then \; q_2 = 0$

The best responses for buyer 1 and 2 are presented in Figure 2 below.

With the assumptions made in Asm1 and Asm2, the following are the possible cases for the best responses

1) $A_1^{'} > c_1$ and $A_2^{'} < c_1$, then

$$q_1 = \frac{A_1^{'} - c_1}{2(1 - c_2)} \; and \; q_2 = 0$$

2) $A_1^{'} > c_1$ and $A_2^{'} > c_1$, then there are two possibilities.

a) Unique equilibrium with $q_1 = \frac{A_1^{'} - c_1}{2(1-c_2)}$ and $q_2 = 0$ if the following inequalities hold

$$\frac{A_2^{'} - c_1}{2(1 - c_2)} < \frac{A_1^{'} - c_1}{\beta - c_2} \qquad (4.1)$$

$$\frac{A_2^{'} - c_1}{\beta - c_2} \leq \frac{A_1^{'} - c_1}{2(1 - c_2)} \qquad (4.2)$$

If the 4.2 holds, then 4.1 also holds since $2(1 - c_2) > (\beta - c_2)$.

Then from Eqn. 4.2,

$$c_1 \geq \frac{2A_2(1 - c_2)}{2 - \beta - c_2} - \frac{A_1(\beta - c_2)}{2 - \beta - c_2} \qquad (4.3)$$

Let RHS of (4.3) be noted with $c_1^{IBIS,c}(c_2)$. This expression constitutes the boundary of the equilibrium regions $q_1 > 0$ and $q_1 = 0$ (see Figure 3)

b) Unique equilibrium with $q_1 > 0$ and $q_2 > 0$ if the following inequalities hold

$$\frac{A_2^{'} - c_1}{2(1 - c_2)} < \frac{A_1^{'} - c_1}{\beta - c_2} \qquad (4.4)$$

$$\frac{A_1^{'} - c_1}{2(1 - c_2)} < \frac{A_2^{'} - c_1}{\beta - c_2} \qquad (4.5)$$

From Eqn. 4.5

$$c_1 < \frac{2A_2^{'}(1 - c_2)}{2 - \beta - c_2} - \frac{A_1^{'}(\beta - c_2)}{2 - \beta - c_2} \qquad (4.6)$$

Note that Eqn. 4.3 and Eqn. 4.6 are reversed inequalities and RHS of these inequalities is a function of c_2.

Then, intersecting the best response functions, one obtains:

$$q_1 = \frac{2A_1^{'}(1 - c_2) - A_2^{'}(\beta - c_2) - 2c_1 + c_1\beta + c_1c_2}{4(1 - c_2)^2 - (\beta - c_2)^2}$$

$$q_2 = \frac{2A_2^{'}(1-c_2) - A_1^{'}(\beta - c_2) - 2c_1 + c_1\beta + c_1c_2}{4(1-c_2)^2 - (\beta - c_2)^2}$$

Note that $q_2 > 0$ and $q_1 = 0$ is not a possible equilibrium outcome due to assumptions Asm1-Asm2. ■

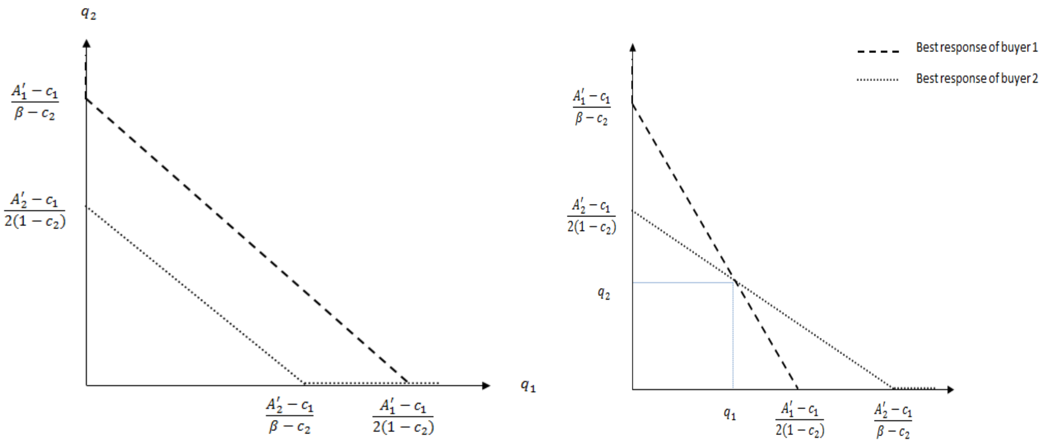

Figure 2 Best response functions for the buyers

Using the equilibrium quantities, next the supplier's problem is addressed.

4.1.2 The Supplier's Problem

The supplier knows Y_1 and Y_2, and is able to infer q_1 and q_2 equilibrium values for a given c_1 and c_2. In Figure 3 below, we present the partitions of the feasible region for decision variables c_1 and c_2. In the figure, Region 1 denotes the (c_1, c_2) values that lead to the equilibrium $(q_1, 0)$ and Region 2 denotes the (c_1, c_2) values that lead to the equilibrium $(q_1, q_2) > 0$. Note that π_s is different under Region 1 and Region 2.

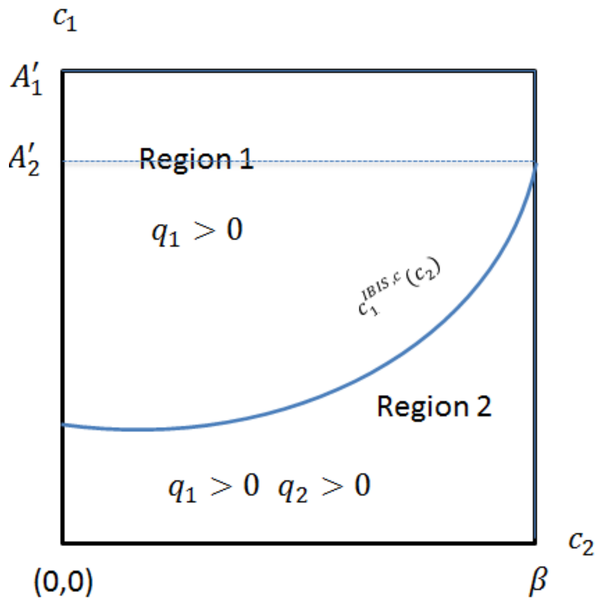

Figure 3 Feasible regions

The feasible region is divided into two by $c_1^{IBIS,c}(c_2)$, which is defined in (4.3). If we take the derivative of $c_1^{IBIS,c}(c_2)$ with respect to c_2

$$\frac{\partial c_1^{IBIS,c}(c_2)}{\partial c_2} = \frac{-[2(A_1' - A_2')(\beta - 1)]}{(\beta + c_2 - 2)^2} > 0$$

Observe that $c_1^{IBIS,c}(c_2)$ is increasing and not linear in c_2

Proposition 2: Under the IBIS setting with two collaborating buyers, if the equilibrium quantities are $q_1 > 0$ and $q_2 = 0$, then the optimal c_1 and c_2 values that maximize the supplier's profit function are denoted with

$$argmax_{c_1,c_2}\left(\pi_s^{Region1}(c_1, c_2)\right)$$

and obtained as follows;

Case A: $2A_2' < A_1'$

$$(c_1^*, c_2^*) = \left(\frac{A_1'}{2 - \beta + \varepsilon}, \beta - \varepsilon\right)$$

Case B: $\boldsymbol{2A_2' > A_1'}$

1) **Case 1:** $\frac{2A_1'-4A_2'+A_1'\beta}{A_1'-2A_2'} > \beta$

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,c}\left(c_2^h\right), c_2^h\right)$$

where $c_2^h = min\{c_{2,IBIS}, \beta - \varepsilon\}^+$

2) **Case 2:** $\frac{2A_1'-4A_2'+A_1'\beta}{A_1'-2A_2'} < 0$

$$(c_1^*, c_2^*) = \left(\frac{A_1'}{2-\beta+\varepsilon}, \beta - \varepsilon\right)$$

3) **Case 3:** $0 < \frac{2A_1'-4A_2'+A_1'\beta}{A_1'-2A_2'} < \beta$

$$(c_1^*, c_2^*) = \ argmax_{c_1,c_2}\{\pi_s(c_1(\beta-\varepsilon), \beta-\varepsilon), \pi_s\left(c_1^{IBIS,c}\left(c_2^h\right), c_2^h\right)\}$$

where $c_2^h = min\{c_{2,IBIS}, \bar{c}_2\}^+$

Proof: To find optimal c_1 and c_2 values that maximize the supplier's profit function, the first order conditions (FOC) and the second order conditions (SOC) are analyzed.

In Region 1 (i.e., when $q_1 > 0$, $q_2 = 0$), the profit function of the supplier is

$$\pi_s = (c_1 - c_2 q_1)q_1$$

For a given c_1 and c_2, the equilibrium quantities are

$$q_1 = \frac{A_1' - c_1}{2(1-c_2)}$$

$$q_2 = 0$$

Taking the derivative of π_s with respect to c_1

$$\frac{\partial \pi_s}{\partial c_1} = \frac{A_1' - 2c_1 + c_1 c_2}{2(c_2-1)^2}$$

$$\frac{\partial^2 \pi_s}{\partial c_1^2} = \frac{c_2 - 2}{2(c_2 - 1)^2} < 0 \qquad (4.7)$$

Denominator of Eqn. 4.7 is always positive and numerator is always negative. Hence, π_s is concave in c_1 for any c_2.

Taking the derivative of π_s with respect to c_2

$$\frac{\partial \pi_s}{\partial c_2} = \frac{{A_1'}^2/4 - c_1^2/4}{(c_2 - 1)^2} + \frac{(A_1' - c_1)^2}{2(c_2 - 1)^3}$$
$$= \frac{(A_1' - c_1)}{4(1 - c_2)^3}[(1 - c_2)(A_1' + c_1) - 2(A_1' - c_1)] \qquad (4.8)$$

For a given c_2, for small values of c_1, Eqn. 4.8 is negative and for high values of c_1 Eqn. 4.8 is positive. Depending on c_1, $\frac{\partial \pi_s}{\partial c_2}$ is either negative for all c_2, or positive for all c_2, or positive for low values of c_2 and negative for high values if c_2.

$$\frac{\partial^2 \pi_s}{\partial c_2^2} = \frac{-{A_1'}^2/2 - c_1^2/2}{(c_2 - 1)^3} - \frac{3(A_1' - c_1)^2}{2(c_2 - 1)^4} \qquad (4.9)$$

From (4.9) it is not possible to conclude the concavity of π_s with respect to c_2. However, structure of $\frac{\partial \pi_s}{\partial c_2}$ implies π_s is unimodal.

The FOC results in

$$\frac{\partial \pi_s}{\partial c_1} = 0 \rightarrow \frac{A_1' - 2c_1 + c_1 c_2}{2(c_2 - 1)^2} = 0$$

$$\frac{\partial \pi_s}{\partial c_2} = 0 \rightarrow \frac{(A_1' - c_1)}{4(1 - c_2)^3}[(1 - c_2)(A_1' + c_1) - 2(A_1' - c_1)] = 0$$

Thus equation system obtained from FOC does not give a solution.

The profit function of supplier is neither convex nor concave when c_1 and c_2 are considered jointly. However, since π_s is unimodal in c_1 and c_2, it is possible to find profit maximizing in c_1 and c_2 by analyzing the structure of $c_1(c_2)$ and $c_2(c_1)$ and the boundary conditions of the feasible region.

Let $c_1(c_2)$ be the function that maximizes π_s for a given c_2 and let $c_2(c_1)$ be the function that maximizes π_s for a given c1. To obtain $c_1(c_2)$ and $c_2(c_1)$,

$$\frac{\partial \pi_s}{\partial c_1} = 0 \rightarrow c_1(c_2) = \frac{A_1'}{2 - c_2} \qquad (4.10)$$

$$\frac{\partial \pi_s}{\partial c_2} = 0 \rightarrow c_2(c_1) = \frac{-(A_1' - 3c_1)}{A_1' + c_1} \qquad (4.11)$$

Step 1: Note that $c_1(c_2)$, expressed as in Eqn. 4.10, is an increasing function of c_2. Also $c_1^{IBIS,c}(c_2)$ is an increasing function of c_2. There exist two c_2 values at which $c_1(c_2)$ and $c_1^{IBIS,c}(c_2)$ intersect, obtained as follows:

At the intersection $c_1(c_2)$ is set to $c_1^{IBIS,c}(c_2)$;

$$\frac{A_1'}{2 - c_2} = \frac{2A_2'(1 - c_2) - A_1'(\beta - c_2)}{2(1 - c_2) - (\beta - c_2)}$$

Note that $c_1(c_2)$ and $c_1^{IBIS,c}(c_2)$ are undefined for some values of c_2. Specifically, $c_1(c_2)$ and $c_1^{IBIS,c}(c_2)$ behave differently in the regions $c_2 \in [0,2-\beta)$, $c_2 \in (2-\beta, 2)$ and $c_2 \in (2, \infty)$. Since $\beta < 1$, we focus only on the region $c_2 \in [0,2-\beta)$. Analyzing the c_2 value that equates $c_1(c_2)$ and $c_1^{IBIS,c}(c_2)$, we obtain:

$$[A_1'][2(1 - c_2) - (\beta - c_2)] = [2A_2'(1 - c_2) - A_1'(\beta - c_2)][2 - c_2]$$

$$[A_1'(\beta - c_2) + 2A_2'(c_2 - 1)](c_2 - 2) + A_1'(\beta + c_2 - 2) = 0 \qquad (4.12)$$

The roots of Eqn. 4.12 are $c_2^1 = 1$ and $c_2^2 = \frac{2A_1' - 4A_2' + A_1'\beta}{A_1' - 2A_2'}$. Note that for $2A_2^1 < A_1'$, $c_2^2 > 2 + \beta$ and for $2A_2^1 > A_1'$, $c_2^2 < 2 - \beta$. Since c_2^1 is outside the boundary, we only focus on c_2^2. Let c_2^2 be denoted as $\bar{c}_2$.

Case A: $2A_2' < A_1'$.

It is possible to show that for $c_2 \in [0,2-\beta)$, $c_1(c_2) > c_1^{IBIS,c}(c_2)$. The analysis follows exactly the same steps as in Case B.2 below. Optimal c_1^*, c_2^* is:

$$(c_1^*, c_2^*) = \left(\frac{A_1'}{2-\beta+\varepsilon}, \beta - \varepsilon\right) \qquad (4.13)$$

Case B: $2A_2' > A_1'$.

Here depending on the $\bar{c}_2$, the analysis changes. We make the analysis under the three mutually exclusive cases: $\bar{c}_2 > \beta$, $\bar{c}_2 < 0$, and $0 < \bar{c}_2 < \beta$.

Case 1: $\bar{c}_2 > \beta$

This case occurs if $\bar{c}_2 = \frac{2A_1' - 4A_2' + A_1'\beta}{A_1' - 2A_2'} > \beta$. It holds that for $c_2 \in [0,2-\beta)$, $c_1^{IBIS,c}(c_2) > c_1(c_2)$.

The feasible region is given in Figure 4 below.

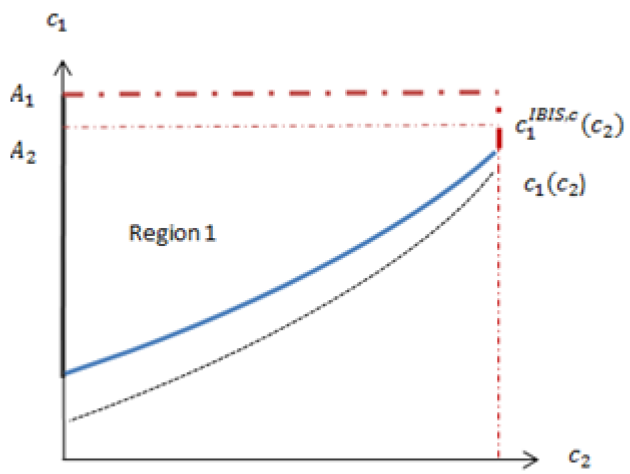

Figure 4 Feasible region for $\bar{\mathbf{c}}_2 > \boldsymbol{\beta}$

Since $c_1(c_2)$ is outside Region 1, note for a given c_2, π_s is decreasing in c_1. In other words, in Region 1 for a given c_2, π_s is maximized at $c_1 = c_1^{IBIS,c}(c_2)$. Then, we check whether π_s is increasing on the boundary, $c_1^{IBIS,c}(c_2)$. In order to do so, first $c_1^{IBIS,c}(c_2)$ (given in Eqn. 4.3) is embedded in π_s and the derivative with respect to c_2 is taken;

$$\frac{\partial \pi_s(c_1^{IBIS,c}(c_2))}{\partial c_2} = \frac{-[(2A_2' - A_2' c_2)(A_1' - A_2') - \beta(A_1' - A_2')(2A_1' - A_2')]}{(\beta + c_2 - 2)^3}$$

Analysis shows that there exists a unique root for $\frac{\partial \pi_s(c_1^{IBIS,c}(c_2))}{\partial c_2} = 0$. The function $\pi_s(c_1^{IBIS,c}(c_2))$ could be convex or concave depending on the parameters, however the uniqueness of the root, together with the fact that for small values of c_2, $\frac{\partial \pi_s(c_1^{IBIS,c}(c_2))}{\partial c_2}$ being (+) implies $\pi_s(c_1^{IBIS,c}(c_2))$ is unimodal. Thus the maximizing $c_2$can be found as follows:

$$\frac{\partial \pi_s(c_1^{IBIS,c}(c_2))}{\partial c_2} = 0 \rightarrow c_{2,IBIS} = \frac{2A_2' - 2A_1'\beta + A_2'\beta}{A_2'}$$

Let $c_2^h = min\{c_{2,IBIS}, \beta - \varepsilon\}^+$. Then the optimal c_1 and c_2 are

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,c}\left(c_2^h\right), c_2^h\right) \qquad (4.14)$$

Case 2: $\bar{c}_2 < 0$

This case occurs if $\bar{c}_2 = \frac{2A_1' - 4A_2' + A_1'\beta}{A_1' - 2A_2'} < 0$

If $\bar{c}_2 < 0$, then $c_1(c_2) \geq c_1^{IBIS,c}(c_2)$ is effective while finding c_1^*. The feasible region is given in Figure 5.

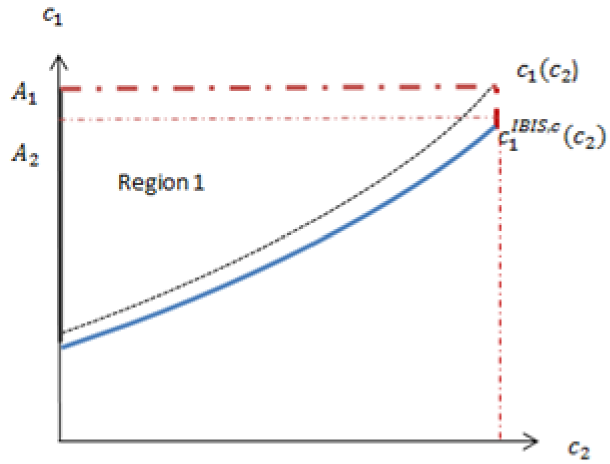

Figure 5 Feasible region for $\bar{\mathbf{c}}_2 < \mathbf{0}$

For a given c_2, the profit function of the supplier, π_s, is increasing in c_1 till it reaches $c_1(c_2)$. Moreover, π_s is increasing in c_2 along the line $c_1(c_2)$. Hence, optimal c_1 and c_2 will be;

$$(c_1^*, c_2^*) = \left(\frac{A_1'}{2-\beta+\varepsilon}, \beta - \varepsilon\right)$$

Case 3: $0 < \bar{c}_2 < \beta$

This case occurs if $0 < \frac{2A_1' - 4A_2' + A_1'\beta}{A_1' - 2A_2'} < \beta$. The feasible region is given in Figure 6 below.

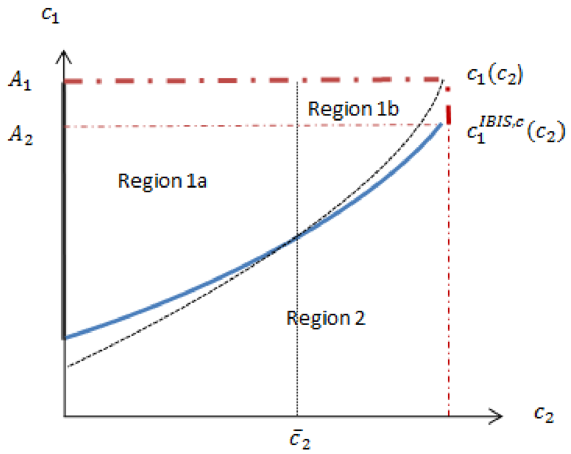

Figure 6 Feasible region for $\mathbf{0 < \bar{c}_2 < \beta}$

In Region 1a, $c_1^{IBIS,c}(c_2)$ is effective while finding c_1^* and in Region 1b, $c_1(c_2)$ is effective while finding c_1^*.

π_s is increasing in c_2 and along the line $c_1(c_2)$. In Region 1b, optimal c_1 and c_2 are

$$(c_1^*, c_2^*) = (c_1(\beta - \varepsilon), \beta - \varepsilon) = \left(\frac{A_1'}{\varepsilon - \beta + 2}, \beta - \varepsilon\right)$$

In Region 1a optimal c_1 and c_2 are $(c_1^*, c_2^*) = \left(c_1^{IBIS,c}\left(c_2^h\right), c_2^h\right)$ where $c_2^h = min\{c_{2,IBIS}, \bar{c}_2\}^+$.

Then, optimal c_1 and c_2 are

$$(c_1^*, c_2^*) = argmax_{c_1,c_2}\{\pi_s(c_1(\beta - \varepsilon), \beta - \varepsilon), \pi_s\left(c_1^{IBIS,c}\left(c_2^h\right), c_2^h\right)\}$$

■

Proposition 3: Under the IBIS setting with two collaborating buyers, if the equilibrium quantities are $q_1 > 0$ and $q_2 > 0$, then the optimal c_1 and c_2 values that maximizes the supplier's profit function are denoted with

$$argmax_{c_1,c_2}\left(\pi_s^{Region2}(c_1, c_2)\right)$$

and expressed as follows;

Case A: $3A_2' < A_1'$

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,c}\left(c_2^h\right), c_2^h\right)$$

Case B: $3A_2' > A_1'$

1) **Case 1:** $\frac{2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta}{A_1' - 3A_2'} > \beta$

$$(c_1^*, c_2^*) = \left(\frac{(A_1' + A_2')(2\beta - \varepsilon + 2)}{4\varepsilon + 8}, \beta - \varepsilon\right)$$

2) **Case 2:** $\frac{2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta}{A_1' - 3A_2'} < 0$

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,c}\left(c_2^h\right), c_2^h\right)$$

where $c_2^h = min\{c_{2,IBIS}, \beta - \varepsilon\}^+$

3) **Case 3:** $0 < \frac{2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta}{A_1' - 3A_2'} < \beta$

$$(c_1^*, c_2^*) = argmax_{c_1,c_2}\{\pi_s(c_1(\bar{c}_2), \bar{c}_2), \pi_s\left(c_1^{IBIS,c}\left(c_2^h\right), c_2^h\right)\}$$

where $\bar{c}_2 = \frac{2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta}{A_1' - 3A_2'}$ and $c_2^h = max\left\{\min\left\{c_{2,IBIS}, \beta - \varepsilon\right\}, \bar{c}_2\right\}$

Proof. Since it is assumed that $q_1 > 0$ and $q_2 > 0$, the best responses are

$$q_1 = \frac{2A_1'(1 - c_2) - A_2'(\beta - c_2) - 2c_1 + c_1\beta + c_1c_2}{4(1 - c_2)^2 - (\beta - c_2)^2}$$

$$q_2 = \frac{2A_2'(1 - c_2) - A_1'(\beta - c_2) - 2c_1 + c_1\beta + c_1c_2}{4(1 - c_2)^2 - (\beta - c_2)^2}$$

To find optimal c_1 and c_2 values that maximize the supplier's profit function, the first order condition (FOC) and the second order conditions (SOC) are analyzed.

Note that π_s is continuous and differentiable in Region 2.

Taking the derivative of π_s with respect to c_1

$$\frac{\partial \pi_s}{\partial c_1} = \frac{A'_1 + A'_2 - 4c_1}{\beta - 3c_2 + 2} + \frac{4c_2(A'_1 + A'_2 - 2c_1)}{(\beta - 3c_2 + 2)^2} \qquad (4.18)$$

Taking the derivative of π_s with respect to c_2

$$\frac{\partial \pi_s}{\partial c_2} = \frac{-X[Y + \beta Z]}{(\beta - 3c_2 + 2)^3} \qquad (4.19)$$

where

$$X = A'_1 + A'_2 - 2c_1$$

$$Y = 2A'_1 + 2A'_2 - 10c_1 + 3A'_1c_2 + 3A'_2c_2 + 3c_1c_2 = 2Z + 3c_2(A'_1 + A'_2 + c_1)$$

$$Z = A'_1 + A'_2 - 5c_1$$

Observation 1. In Eqn. 4.18, the second term is always positive. The denominator of the first term is also positive, but numerator can be negative for high values of c_1. Then, the derivative is positive for small values of c_1, and negative for large values of c_1. This implies that for a given c_2, the profit function of the supplier is unimodal in c_1. ■

Observation 2. In Eqn. 4.19 above, the denominator and the value of X in the numerator are always positive for a given c_1. Moreover, the value of Y is an increasing function of c_2. Hence, there exists a threshold level for c_2 where for small values of c_2, the derivate possibly takes positive values and for larger values than the threshold the derivate takes negative values. Whether the derivate can take positive values depend on the value of c_1.

i) For lower values of c_1 ($c_1 < (A'_1 + A'_2)/5$), Z is always positive. As a result Y is positively increasing in c_2. Since X is also positive, this implies that the final value of Eqn. 4.19 (the derivate value) is always negative in c_2. For a given c_1, maximizing c_2 value is $c_2(c_1) = 0$.

ii) For higher values of c_1 ($c_1 > (A'_1 + A'_2)/5$), Z is always negative. Thus, Y is negative for small c_2, but positive for large c_2. This implies the derivate is positive for small values of c_2, and negative for large values of c_2. The maximizing c_2 value, $c_2(c_1)$ is non-zero.

This implies that for a given c_1, the profit function of the supplier is unimodal in c_2. ■

FOC yield:

$$\frac{\partial \pi_s}{\partial c_1} = 0 \rightarrow c_1(c_2) = \frac{(A_1' + A_2')(\beta + c_2 + 2)}{4\beta - 4c_2 + 8} \qquad (4.20)$$

$$\frac{\partial \pi_s}{\partial c_2} = 0 \rightarrow c_2(c_1) = \frac{-(\beta + 2)(A_1' + A_2' - 5c_1)}{3A_1 + 3A_2 + 3c_1} \qquad (4.21)$$

When the equations 4.20 and 4.21 are solved together c_1 and c_2 are found as,

$$c_1 = \frac{A_1' + A_2'}{2}$$

$$c_2 = \frac{\beta + 2}{3}$$

Observation 3. Note that when $c_1 = (A_1' + A_2')/2 > (A_1' + A_2')/5$, the derivate with respect to c_2 takes both positive and negative values. As a result, maximizing c_2 is greater than 0, (it is indeed $(\beta + 2)/3$). c_2 value is beyond the feasible region, hence, we know that optimal c_1 and c_2 are on the boundary of the feasible region. To determine the maximizing c_1 and c_2, further analysis is required.

π_s is not jointly concave in c_1 and c_2, but is unimodal with respect to c_1 and with respect to c_2. It is possible to find profit maximizing c_1 and c_2, by analyzing the structure of $c_1(c_2)$ and $c_2(c_1)$ and the boundary conditions of the feasible region.

Step 1: Note that $c_1(c_2)$ an increasing function of c_2. As stated before $c_1^{IBIS,c}(c_2)$ is also an increasing function of c_2. We check below whether $c_1(c_2)$ and $c_1^{IBIS,c}(c_2)$ intersect.

Note that $c_1(c_2)$ and $c_1^{IBIS,c}(c_2)$ are undefined for some values of c_2. Specifically, $c_1(c_2)$ and $c_1^{IBIS,c}(c_2)$ behave differently in the regions $c_2 \in [0,2-\beta)$, $c_2 \in (2-\beta,2)$ and $c_2 \in (2,\infty)$. Since $\beta < 1$, we focus only on the region $c_2 \in [0,2-\beta)$. Analyzing the c_2 value that equates $c_1(c_2)$ and $c_1^{IBIS,c}(c_2)$, we obtain:

At the intersection $c_1(c_2)$ is set to $c_1^{IBIS,c}(c_2)$

$$\frac{(A_1' + A_2')(\beta + c_2 + 2)}{4\beta - 4c_2 + 8} = \frac{2A_2'(1-c_2) - A_1'(\beta - c_2)}{2(1-c_2) - (\beta - c_2)}$$

$$[(A_1' + A_2')(\beta + c_2 + 2)][2(1-c_2) - (\beta - c_2)] = [2A_2'(1-c_2) - A_1'(\beta - c_2)][4\beta - 4c_2 + 8]$$

$$-(\beta - 3c_2 + 2)(2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta - A_1'c_2 + 3A_2'c_2) = 0 \qquad (4.22)$$

The roots of Eqn. 4.22 are $c_2^1 = (2+\beta)/3$ and $c_2^2 = \frac{2A_1' - 4A_2' + A_1'\beta}{A_1' - 2A_2'}$. Note that for $3A_2' < A_1'$, $c_2^2 > 2+\beta$, and for $3A_2' > A_1'$, $c_2^2 > 2-\beta$. Since c_2^1 is outside the boundary, we only focus on c_2^2. Let c_2^2 be denoted as $\bar{c}_2$.

Case A: $3A_2' < A_1'$

It is possible to show that for $c_2 \in [0,2-\beta)$, $c_1(c_2) > c_1^{IBIS,c}(c_2)$. The analysis follows exactly the same steps as in Case B.2 below. Optimal c_1^*, c_2^* is,

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,c}\left(c_2^h\right), c_2^h\right)$$

Case B: $3A_2' < A_1'$

Depending on the values of $\bar{c}_2$ the analysis changes. We make the analysis under the three mutually exclusive cases: $\bar{c}_2 > \beta$, $\bar{c}_2 < 0$, and $0 < \bar{c}_2 < \beta$.

Case 1: $\bar{c}_2 > \beta$

This case occurs if $\bar{c}_2 = \frac{2A'_1 - 6A'_2 + 3A'_1\beta - A'_2\beta}{A'_1 - 3A'_2} > \beta$ (or equivalently given $A'_1 < 3A'_2$ when $\frac{A'_1}{A'_2} < \frac{3-\beta}{1+\beta}$). It holds that for $c_2 \in [0,2-\beta)$, $c_1^{IBIS,c}(c_2) > c_1(c_2)$ (see Figure 7)

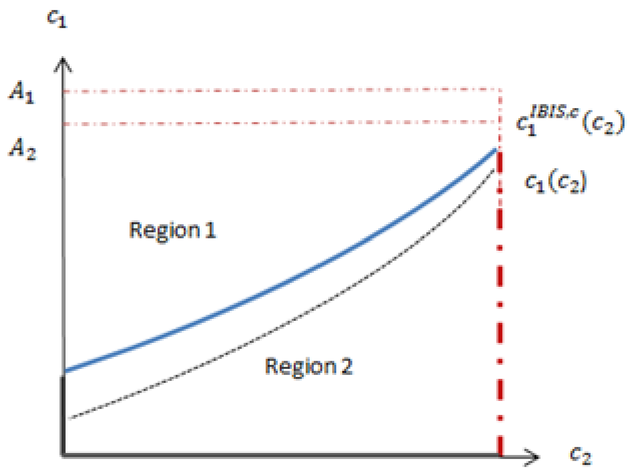

Figure 7 Feasible region for $\bar{c}_2 > \beta$

We embeded $c_1(c_2)$ in π_s, take derivative with respect to c_2 and observe that the derivate is positive. Note that $c_1(c_2)$ never attains A'_2.

Then, for $\bar{c}_2 = \frac{2A'_1 - 6A'_2 + 3A'_1\beta - A'_2\beta}{A'_1 - 3A'_2} > \beta$ optimal c_1 and c_2 will be

$$(c_1^*, c_2^*) = (c_1(c_2), c_2) = (c_1(\beta - \varepsilon), \beta - \varepsilon)$$
$$= \left(\frac{(A'_1 + A'_2)(2\beta - \varepsilon + 2)}{4\varepsilon + 8}, \beta - \varepsilon\right) \qquad (4.23)$$

Case 2: $\bar{c}_2 < 0$

This case occurs if $\bar{c}_2 = \frac{2A'_1 - 6A'_2 + 3A'_1\beta - A'_2\beta}{A'_1 - 3A'_2} < 0$. If $\bar{c}_2 < 0$, then $c_1(c_2) \geq c_1^{IBIS,c}(c_2)$ and $c_1^{IBIS,c}(c_2)$ is effective while finding c_1^*. The feasible region is given in Figure 8.

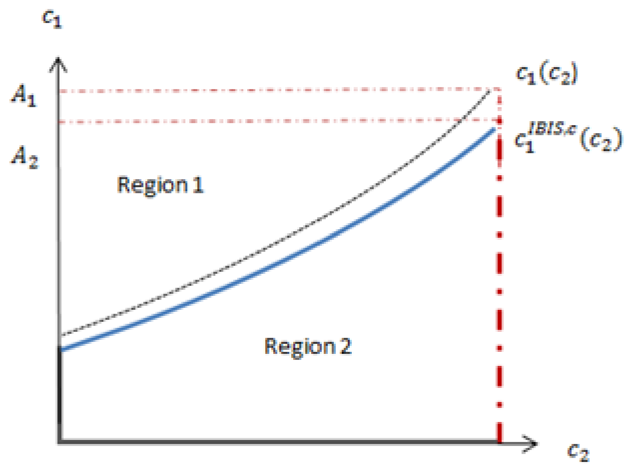

Figure 8 Feasible region for $\bar{\mathbf{c}}_2 < 0$

Since $c_1^{IBIS,c}(c_2)$ is effective while finding c_1^*, the optimal c_1 and c_2 are

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,c}(c_2^h), c_2^h\right) \qquad (4.14)$$

where $c_2^h = min\{c_{2,IBIS}, \beta - \varepsilon\}^+$.

Case 3: $0 < \bar{c}_2 < \beta$

This case occurs if $0 < \frac{2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta}{A_1' - 3A_2'} < \beta$. Whether $c_1(c_2)$ or $c_1^{IBIS,c}(c_2)$ is effective is shown in Figure 9 below.

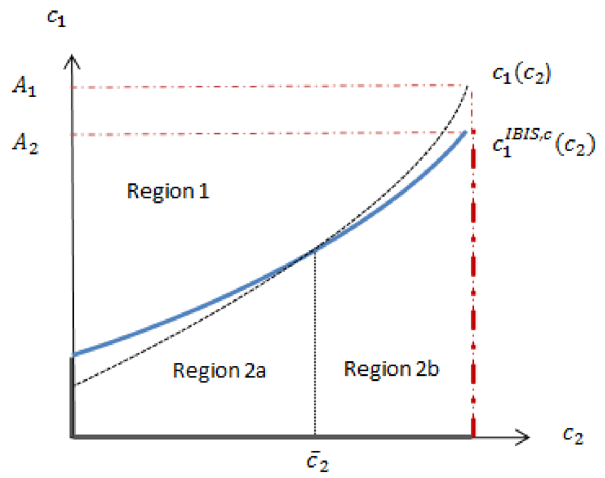

Figure 9 Feasible region for $\mathbf{0} < \bar{\mathbf{c}}_2 < \boldsymbol{\beta}$

In the Figure 9, in Region 2a, $c_1(c_2)$ is effective while finding c_1^* and in Region 2b, $c_1^{IBIS,c}(c_2)$ is effective while finding c_1^*.

π_s is increasing in c_2 and along the line $c_1(c_2)$, thus in Region 2a, optimal c_1 and c_2 are

$$(c_1^*, c_2^*) = (c_1(\bar{c}_2), \bar{c}_2) = \left(\frac{(A_1' + A_2')(\beta + \bar{c}_2 + 2)}{4\beta - 4\bar{c}_2 + 8}, \bar{c}_2\right)$$

In Region 2b optimal c_1 and c_2 are

$(c_1^*, c_2^*) = \left(c_1^{IBIS,c}(c_2^h), c_2^h\right)$ where $c_2^h = max\{\min\{c_{2,IBIS}, \beta - \varepsilon\}, \bar{c}_2\}$.

Then, optimal c_1 and c_2 are

$$(c_1^*, c_2^*) = argmax_{c_1,c_2}\{\pi_s(c_1(\bar{c}_2), \bar{c}_2), \pi_s\left(c_1^{IBIS,c}(c_2^h), c_2^h\right)\}$$

■

Proposition 4: Under the IBIS setting with two collaborating buyers, the optimal c_1 and c_2 values that maximize the supplier's profit function are

$$argmax_{c_1,c_2}\left(\max\{\pi_s^{Region1}(c_1, c_2), \pi_s^{Region2}(c_1, c_2)\}\right)$$

where c_1^* and c_2^* in Region 1 and Region 2 are given in Proposition 2 and Proposition 3.

■

4.1.3. Examples

The following 3 examples presented show how the equilibrium quantities, the optimal π_s and the corresponding $E[\pi_1|Y_1, Y_2]$ and $E[\pi_2|Y_1, Y_2]$ are determined. In all examples $\varepsilon = 0.01$

Example 1

$A_1' = 10000,\ A_2' = 9000,\ \beta = 0.8$

In Region 1, it is the Case B.1 ($\bar{c}_2 > \beta$) and $c_1^{IBIS,c}(c_2)$ is effective.

$c_{2,IBIS} = 1.022$. Then $c_2^h = min\{c_{2,IBIS}, \beta - \varepsilon\} = 0.79$

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,c}\left(c_2^h\right), c_2^h\right) = (8975.61, 0.79)$$

Then $q_1 = 2439$, $q_2 = 0$ and $\pi_s = 1.72 \times 10^7$

In Region 2, it is the Case B.1 ($\bar{c}_2 > \beta$) and $c_1(c_2)$ is effective.

$$(c_1^*, c_2^*) = (c_1(c_2), c_2) = (c_1(\beta - \varepsilon), \beta - \varepsilon) = \left(\frac{(A_1' + A_2')(2\beta - \varepsilon + 2)}{4\varepsilon + 8}, \beta - \varepsilon\right)$$
$$= (8483.83, 0.79)$$

Then $q_1 = 3582.7$, $q_2 = 1143.7$ and $\pi_s = 2.245 \times 10^7$

Example 2

$A_1' = 15000,\ A_2' = 4000,\ \beta = 0.8$

In Region 1, it is the Case 1 ($\bar{c}_2 > \beta$) and $c_1(c_2)$ is effective.

$$(c_1^*, c_2^*) = \left(\frac{A_1'}{2 - \beta + \varepsilon}, \beta - \varepsilon\right) = (12396.7, 0.79)$$

Then $q_1 = 6198$, $q_2 = 0$ and $\pi_s = 4.65 \times 10^7$

In Region 2, it is the Case 1 ($\bar{c}_2 > \beta$) and $c_1^{IBIS,c}$ is effective.

$c_{2,IBIS} = -3.2$. then, $c_2^h = min\{c_{2,IBIS}, \beta\}^+ = 0$

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,c}(c_2^*), c_2^*\right) = (0,0)$$

Then $q_1 = 7976$, $q_2 = 0$ and $\pi_s = 0$

Example 3

$A_1' = 10000,\ A_2' = 9000,\ \beta = 0.9$

In Region 1, Case 3 is valid since $\bar{c}_2 = 0.875$

In Region 1a, $c_1^{IBIS,c}(c_2)$ is effective while finding c_1^* and in Region 1b, $c_1(c_2)$ is effective while finding c_1^*.

π_s is increasing in c_2 and along the line $c_1(c_2)$, in Region 1b, optimal c_1 and c_2 are

$$(c_1^*, c_2^*) = (c_1(\beta - \varepsilon), \beta - \varepsilon) = \left(\frac{A_1'}{\varepsilon - \beta + 2}, \beta - \varepsilon\right) = (9009, 0.89)$$

Then $q_1 = 4504$, $q_2 = 0$ and $\pi_s = 2.25 \times 10^7$

In Region 1a optimal c_1 and c_2 are

$c_{2,IBIS} = 0.9$. Then, $c_2^h = min\{c_{2,IBIS}, \bar{c}_2\} = 0.875$

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,c}(c_2^h), c_2^h\right) = (8889, 0.875)$$

Then $q_1 = 4444$, $q_2 = 0$ and $\pi_s = 2.22 \times 10^7$

Then, optimal c_1 and c_2 are

$$(c_1^*, c_2^*) = argmax_{c_1, c_2}\{\pi_s^{Region1a}, \pi_s^{Region1b}\} = (9009, 0.89)$$

In Region 2, Case 3 is valid since $\bar{c}_2 = 0.888$

In Region 2a, $c_1(c_2)$ is effective while finding c_1^* and in Region 2b, $c_1^{IBIS,c}(c_2)$ is effective while finding c_1^*.

π_s is increasing in c_2 and along the line $c_1(c_2)$, in Region 2a, optimal c_1 and c_2 are

$$(c_1^*, c_2^*) = (c_1(\bar{c}_2), \bar{c}_2) = \left(\frac{(A_1' + A_2')(\beta + \bar{c}_2 + 2)}{4\beta - 4\bar{c}_2 + 8}, \bar{c}_2\right) = (8943, 0.888)$$

Then $q_1 = 4722$, $q_2 = 0$ and $\pi_s = 2.243 \times 10^7$

In Region 2b optimal c_1 and c_2 are

$c_{2,IBIS} = 0.9$. Then, $c_2^h = max\{\min\{c_{2,IBIS}, \beta - \varepsilon\}, \bar{c}_2\} = 0.89$

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,c}(c_2^*), c_2^*\right) = \left(A_2' - A_1' + \frac{A_2'}{\beta}, \frac{2A_2' - 2A_1'\beta + A_2'\beta}{A_2'}\right) = (9000, 0.9)$$
$$= (8952, 0.89)$$

Then $q_1 = 4761$, $q_2 = 0$ and $\pi_s = 2.244 \times 10^7$

Then, optimal c_1 and c_2 for Region 2 are

$$(c_1^*, c_2^*) = argmax_{c_1,c_2}\{\pi_s^{Region2a}, \pi_s^{Region2b}\} = (8952, 0.89)$$

4.2 Non-Collaborating Buyers

When the buyers are non-collaborating, the profit of supplier and buyers are expressed as follows;

$$\begin{aligned} E_\theta[\pi_1|Y_1, Y_2] &= E[(A_1 + \theta_1 - q_1 - \beta q_2 - c_1 + c_2 q_1)q_1|Y_1, Y_2] \\ &= (A_1 + E[\theta_1|Y_1] - q_1 - \beta q_2 - c_1 + c_2 q_1)q_1 \\ &= (A_1 + \delta_1(\alpha_1, \sigma_1)Y_1 - q_1 - \beta q_2 - c_1 + c_2 q_1)q_1 \end{aligned}$$

$$\begin{aligned} E_\theta[\pi_2|Y_1, Y_2] &= E[(A_2 + \theta_2 - q_2 - \beta q_1 - c_1 + c_2 q_2)q_2|Y_1, Y_2] \\ &= (A_2 + E[\theta_2|Y_2] - q_2 - \beta q_1 - c_1 + c_2 q_2)q_2 \\ &= (A_2 + \delta_2(\alpha_2, \sigma_2)Y_2 - q_2 - \beta q_1 - c_1 + c_2 q_2)q_2 \end{aligned}$$

Similar to collaborating buyers case, Y_1 and Y_2 which are the signals received by each buyer are unbiased estimators of θ_1 and θ_2. Note $E[Y_1|\theta_1] = \theta_1$ and $E[Y_2|\theta_2] = \theta_2$, $E[\theta_1] = E[\theta_2] = 0$.

The supplier's ex-post profit function does not include any uncertainty given Y_1 and Y_2.

$$\pi_s|Y_1, Y_2 = (c_1 - c_2 q_1)q_1 + (c_1 - c_2 q_2)q_2$$

The ex-post profit functions for the buyers and the supplier are denoted as, $E_\theta[\pi_1|Y_1, Y_2]$, $E_\theta[\pi_2|Y_1, Y_2]$ and $[\pi_s|Y_1, Y_2]$. In the expressions when q_1 and q_2 are equilibrium quantities, q_1 and q_2 are functions of c_1, c_2, Y_1 and Y_2. Ex-ante profits for the buyers and the supplier are given in Section 4.3.

In the following we discuss how the equilibrium quantities for the buyers and optimal c_1 and c_2 values for the supplier are determined. In Section 4.2.3 examples are given.

4.2.1. The Buyers' Problem

In this section, for a given value of c_1 and c_2 values the equilibrium quantities of the buyers are determined. Similar to collaborating buyers' case in the previous section, A_1' denotes $A_1 + \delta_1(\alpha_1, \sigma_1)Y_1$ and A_2' denotes $A_2 + \delta_2(\alpha_2, \sigma_2)Y_2$. The following analysis is performed for $A_1' > A_2'$ and same steps can be used for $A_2' > A_1'$.

Proposition 5: Under IBIS setting with two non-collaborating buyers, for a given c_1 and c_2, the equilibrium points are expressed as follows;

Case 1: Unique equilibrium with positive responses for buyer 1 and buyer 2

$$q_1 = \frac{2A_1'(1 - c_2) - A_2'\beta - 2c_1 + c_1\beta + 2c_1c_2}{4(1 - c_2)^2 - (\beta)^2}$$

$$q_2 = \frac{2A_2'(1 - c_2) - A_1'\beta - 2c_1 + c_1\beta + 2c_1c_2}{4(1 - c_2)^2 - (\beta)^2}$$

Case 2: Unique equilibrium with positive response only for buyer 1 and zero for buyer 2

$$q_1 = \frac{A_1' - c_1}{2(1 - c_2)}$$

$$q_2 = 0$$

Case 3: Multiple equilibria either with positive responses for buyer 1 and buyer 2

$$q_1 = \frac{2A_1'(1-c_2) - A_2'\beta - 2c_1 + c_1\beta + 2c_1c_2}{4(1-c_2)^2 - (\beta)^2}$$

$$q_2 = \frac{2A_2'(1-c_2) - A_1'\beta - 2c_1 + c_1\beta + 2c_1c_2}{4(1-c_2)^2 - (\beta)^2}$$

or with positive response only for buyer 1 and zero for buyer 2

$$q_1 = \frac{A_1' - c_1}{2(1-c_2)}$$

$$q_2 = 0$$

or with positive response only for buyer 2 and zero for buyer 2

$$q_1 = 0$$

$$q_2 = \frac{A_2' - c_1}{2(1-c_2)}$$

Proof: To find the best response of buyer i, first derivative of profit function, π_i, is taken with respect to q_i.

For buyer 1

$$\frac{\partial E[\pi_1|Y_1,Y_2]}{\partial q_1} = A_1 + \delta_1(\alpha_1,\sigma_1)Y_1 - 2q_1 - \beta q_2 - c_1 + 2c_2q_1 = 0$$

$$q_1(q_2) = \left(\frac{A_1 + \delta_1(\alpha_1,\sigma_1)Y_1 - c_1}{2-2c_2} - \frac{q_2\beta}{2-2c_2}\right)^+ = \left(\frac{A_1' - c_1}{2-2c_2} - \frac{q_2\beta}{2-2c_2}\right)^+$$

For buyer 2

$$\frac{\partial E[\pi_2|Y_1,Y_2]}{\partial q_2} = A_2 + \delta_2(\alpha_2,\sigma_2)Y_2 - 2q_2 - \beta q_1 - c_1 + 2c_2q_2 = 0$$

$$q_2(q_1) = \left(\frac{A_2 + \delta_2(\alpha_2,\sigma_2)Y_2 - c_1}{2-2c_2} - \frac{q_1\beta}{2-2c_2}\right)^+ = \left(\frac{A_2' - c_1}{2-2c_2} - \frac{q_1\beta}{2-2c_2}\right)^+$$

Increasing q_2 decreases the response of first buyer, q_1; if buyer 2 purchases more, since the products are substitutes, buyer 1 will purchase less. Similarly, increasing q_1 decreases the response of second buyer, q_2.

The extremes of first buyer's response;

If $q_2 = 0\ then\ q_1 = \frac{A_1' - c_1}{2(1-c_2)}$

The extremes of second buyer's response;

If $q_1 = 0\ then\ q_2 = \frac{A_2' - c_1}{2(1-c_2)}$

If $q_1 \geq \frac{A_2' - c_1}{\beta}\ then\ q_2 = 0$

The best responses for buyer 1 and 2 are presented in Figure 10 below.

With the assumptions made in A1 and A2, there are two possible cases for the best responses;

1) $A_1' > c_1$ and $A_2' < c_1$, then

$$q_1 = \frac{A_1' - c_1}{2(1-c_2)}\ and\ q_2 = 0$$

2) $A_1' > c_1$ and $A_2' > c_1$, then there are three possibilities;

a) Unique equilibrium with $q_1 = \frac{A_1' - c_1}{2(1-c_2)}$ and $q_2 = 0$ if the following inequalities hold

$$\frac{A_2' - c_1}{2(1-c_2)} < \frac{A_1' - c_1}{\beta} \qquad (4.31)$$

$$\frac{A_2' - c_1}{\beta} \leq \frac{A_1' - c_1}{2(1-c_2)} \qquad (4.32)$$

From Eqn. 4.31

$$c_1 < \frac{2A_1' - 2A_1'c_2 - A_2'\beta}{2 - 2c_2 - \beta} \qquad (4.33)$$

From Eqn. 4.32

$$c_1 \geq \frac{A_1'\beta - 2A_2' + 2A_2'c_2}{2c_2 - 2 + \beta} \qquad (4.34)$$

Let RHS of (4.33) be noted with $c_1^{IBIS,NC,1}(c_2)$ and RHS of (4.34) with $c_1^{IBIS,NC,2}(c_2)$. These expressions constitute the boundary of the equilibrium regions $q_2 > 0$ and $q_2 = 0$ and multiple equilbria. (see Figure 11)

b) Unique equilibrium with $q_1 > 0$ and $q_2 > 0$ if the following inequalities hold

$$\frac{A_2' - c_1}{2(1 - c_2)} < \frac{A_1' - c_1}{\beta} \qquad (4.31)$$

$$\frac{A_1' - c_1}{2(1 - c_2)} < \frac{A_2' - c_1}{\beta} \qquad (4.35)$$

Then from Eqn. 4.35

$$c_1 > \frac{A_1\beta - 2A_2 + 2A_2c_2}{2c_2 - 2 + \beta}$$

Note that Eqn. 4.35 and Eqn. 4.32 are reversed inequalities and RHS of these inequalities is a function of c_2.

Then, intersecting the best response functions, one obtains:

$$q_1 = \frac{2A_1'(1 - c_2) - A_2'\beta - 2c_1 + c_1\beta + 2c_1c_2}{4(1 - c_2)^2 - (\beta)^2}$$

$$q_2 = \frac{2A_2'(1 - c_2) - A_1'\beta - 2c_1 + c_1\beta + 2c_1c_2}{4(1 - c_2)^2 - (\beta)^2}$$

c) Multiple equilibrium if the following inequalities hold

$$\frac{A_1' - c_1}{\beta} < \frac{A_2' - c_1}{2(1 - c_2)} \qquad (4.36)$$

$$\frac{A_2' - c_1}{\beta} < \frac{A_1' - c_1}{2(1 - c_2)} \qquad (4.32)$$

with three equilibrium outcomes

a. $q_1 > 0$ and $q_2 = 0$

b. $q_1 = 0$ and $q_2 > 0$

c. $q_1 > 0$ and $q_2 > 0$

■

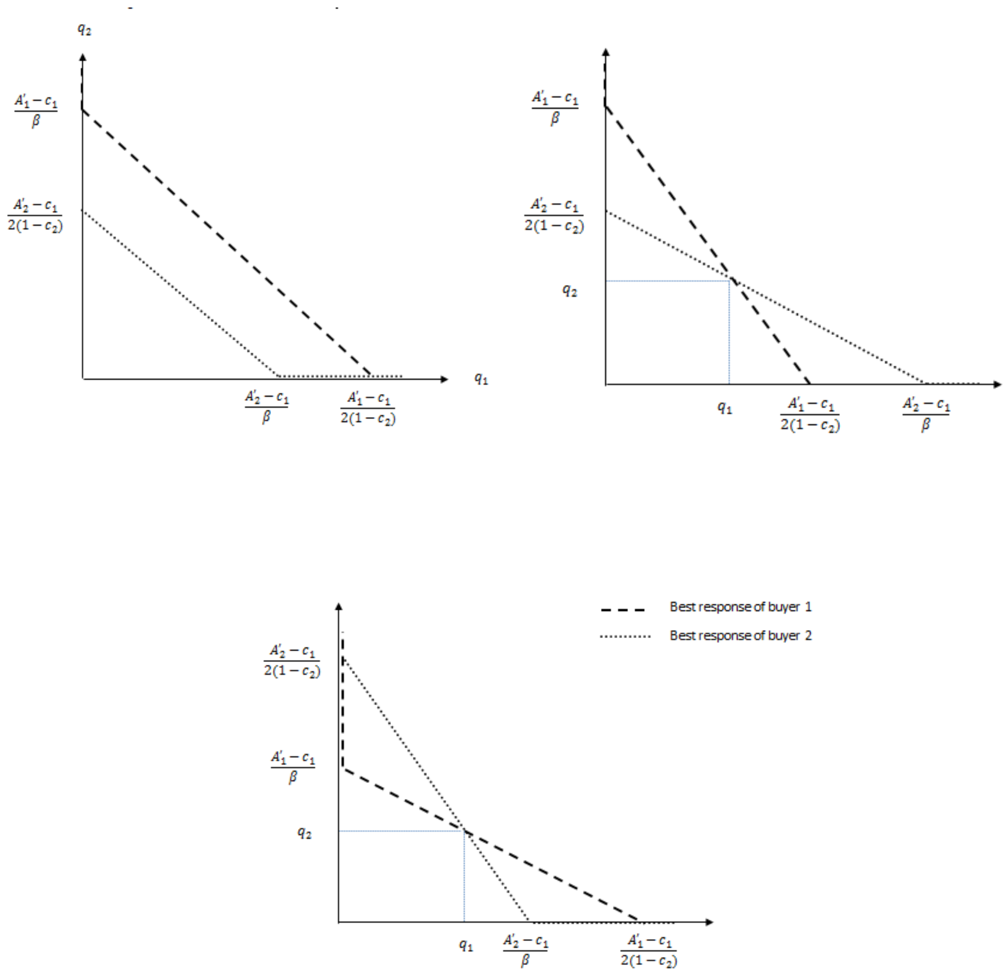

Figure 10 Best responses for the no-collaborating buyers

Using the equilibrium quantities, next the supplier's problem is addressed.

4.2.2 The supplier's problem

Similar to collaborating buyers' case, the supplier knows Y_1 and Y_2, and is able to infer q_1 and q_2 equilibrium values for a given c_1 and c_2. In Figure 11(a) below, we present the partitions of the feasible region for decision variables c_1 and c_2.

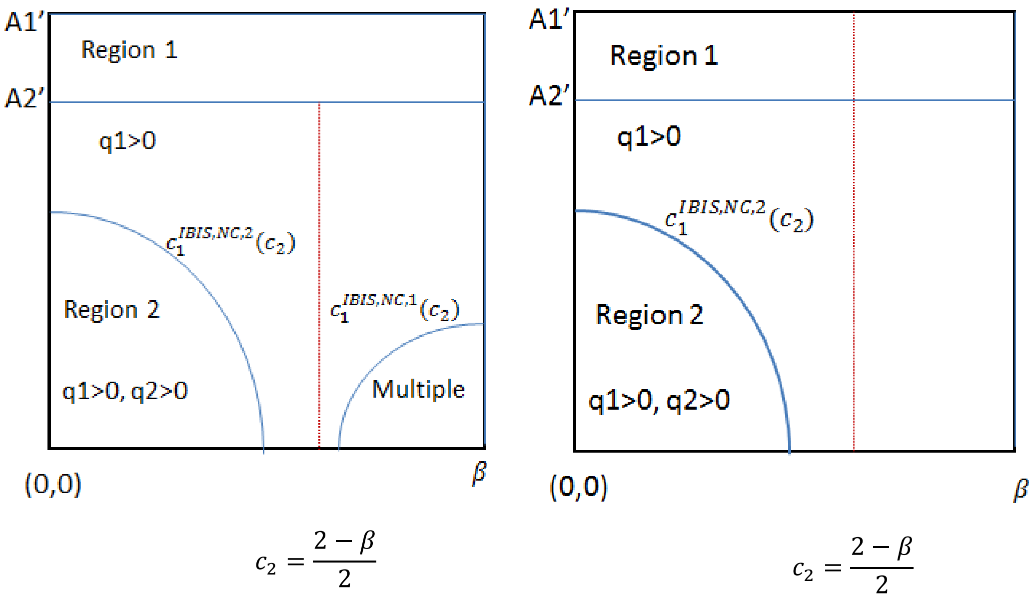

Figure 11 Feasible regions

The feasible region is divided into three by $c_1^{IBIS,NC,1}(c_2)$ and $c_1^{IBIS,NC,2}(c_2)$ which are defined in (4.33) and (4.34) (see Figure 11(a)).

For multiple equilibria, we will assume $q_1 > 0$ and $q_2 = 0$ in order to have continuity. Note $q_1 > 0, q_2 = 0$ is still part of the equilibria. Then, $c_1^{IBIS,NC,1}(c_2)$ becomes redundant and $c_1^{IBIS,NC,2}(c_2)$ divides the feasible region into two, in Region 1 only buyer 1 has positive order quantity and in Region 2 both buyers have positive order quantities (see Figure 11(b)).

Observation 4. For $0 < c_2 < \beta$, $c_1^{IBIS,NC,1}(c_2)$ is increasing in c_2 and $c_1^{IBIS,NC,2}(c_2)$ is decreasing in c_2. Furthermore, for $c_2 < 1 - \beta/2$, $c_1^{IBIS,NC,1}(c_2) > A_1'$ and for $c_2 > 1 - \beta/2$, $c_1^{IBIS,NC,2}(c_2) > A_1'$ in the feasible region.

Proof. If we take the derivative of $c_1^{IBIS,NC,1}(c_2)$ and $c_1^{IBIS,NC,2}(c_2)$ with respect to c_2

$$\frac{dc_1^{IBIS,NC,1}}{dc_2} = \frac{2\beta(A_1 - A_2)}{(\beta + 2c_2 - 2)^2} > 0$$

$$\frac{dc_1^{IBIS,NC,2}}{dc_2} = \frac{-2\beta(A_1 - A_2)}{(\beta + 2c_2 - 2)^2} < 0$$

Hence, $c_1^{IBIS,NC,1}(c_2)$ is increasing in c_2 and $c_1^{IBIS,NC,2}(c_2)$ is decreasing in c_2.

Furthermore, if $A_1' = A_2'$, then $c_1^{IBIS,NC,1}(c_2) = c_1^{IBIS,NC,2}(c_2)$. Moreover, the numerators of $c_1^{IBIS,NC,1}(c_2)$ and $c_1^{IBIS,NC,2}(c_2)$ are equal when $c_2 = 1 - \beta/2$, while the denominators are zero. This causes a discontinuity in $c_1^{IBIS,NC,1}(c_2)$ and $c_1^{IBIS,NC,2}(c_2)$, which is shown by the red dashed line in Figure 11 above.

$c_1^{IBIS,NC,1}(c_2)$ is negative when $1 - \frac{\beta}{2} < c_2 < 1 - \frac{A_2\beta}{2A_1}$, and positive otherwise. It is zero when $c_2 = 1 - \frac{A_2\beta}{2A_1}$.

$c_1^{IBIS,NC,2}(c_2)$ is negative when $1 - \frac{A_1\beta}{2A_2} < c_2 < 1 - \frac{\beta}{2}$, and positive otherwise. It is zero when $c_2 = 1 - \frac{A_1\beta}{2A_2}$.

For $c_2 < 1 - \beta/2$, it is possible to observe that $c_1^{IBIS,NC,1}(c_2) > A_1'$, and for $c_2 < 1 - \beta/2$ $c_1^{IBIS,NC,2}(c_2) > A_1'$. Thus under those case, the functions are not shown within the feasible region in Figure 11(a). ■

Proposition 6: Under IBIS setting with non-collaborating buyers if the equilibrium quantities are $q_1 > 0$ and $q_2 = 0$, then the optimal c_1 and c_2 values that maximizes the supplier's profit function are denoted with

$$argmax_{c_1,c_2}\left(\pi_s^{Region1}(c_1,c_2)\right)$$

Case A: $\mathbf{2A_2' < A_1'}$

$$(c_1^*, c_2^*) = \left(\frac{A_1'}{2-\beta+\varepsilon}, \beta-\varepsilon\right)$$

Case B: $\mathbf{2A_2' > A_1'}$

1) **Case 1:** $\frac{-(2A_1'-4A_2'+A_1'\beta)}{2A_2'} > \beta$

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,NC,2}\left(c_2^h\right), c_2^h\right)$$

where $c_2^h = min\{c_{2,IBIS,NC}, \beta-\varepsilon\}^+$

2) **Case 2:** $\frac{-(2A_1'-4A_2'+A_1'\beta)}{2A_2'} < 0$

$$(c_1^*, c_2^*) = \left(\frac{A_1'}{2-\beta+\varepsilon}, \beta-\varepsilon\right)$$

3) **Case 3:** $0 < \frac{-(2A_1'-4A_2'+A_1'\beta)}{2A_2'} < \beta$

$$(c_1^*, c_2^*) = argmax_{c_1,c_2}\{\pi_s(c_1(\beta-\varepsilon), \beta-\varepsilon), \pi_s\left(c_1^{IBIS,NC,2}\left(c_2^h\right), c_2^h\right)\}$$

where $c_2^h = min\{c_{2,IBIS}, \bar{c}_2\}^+$

Proof: To find optimal c_1 and c_2 values that maximize the supplier's profit function, the first order conditions (FOC) and the second order conditions (SOC) are analyzed.

In Region 1 (i.e., when $q_1 > 0$, $q_2 = 0$), the profit function of the supplier is

$$\pi_s = (c_1 - c_2 q_1)q_1$$

For a given c_1 and c_2, the equilibrium quantities are

$$q_1 = \frac{A_1' - c_1}{2(1-c_2)}$$

$$q_2 = 0$$

Taking the derivative of π_s with respect to c_1

$$\frac{\partial \pi_s}{\partial c_1} = \frac{A_1' - 2c_1 + c_1 c_2}{2(c_2 - 1)^2}$$

$$\frac{\partial^2 \pi_s}{\partial c_1^2} = \frac{c_2 - 2}{2(c_2 - 1)^2} < 0 \qquad (4.7)$$

Denominator of Eqn. 4.7 is always positive and numerator is always negative. Hence, π_s is concave in c_1 for any c_2.

The FOC results in

$$\frac{\partial \pi_s}{\partial c_1} = 0 \rightarrow \frac{A_1' - 2c_1 + c_1 c_2}{2(c_2 - 1)^2} = 0$$

$$\frac{\partial \pi_s}{\partial c_2} = 0 \rightarrow \frac{(A_1' - c_1)}{4(1 - c_2)^3}[(1 - c_2)(A_1' + c_1) - 2(A_1' - c_1)] = 0$$

Thus equation system obtained from FOC does not give a solution.

The profit function of supplier is neither convex nor concave when c_1 and c_2 are considered jointly. However, since π_s is unimodal in c_1 and c_2, it is possible to find profit maximizing in c_1 and c_2 by analyzing the structure of $c_1(c_2)$ and $c_2(c_1)$ and the boundary conditions of the feasible region.

Let $c_1(c_2)$ be the function that maximizes π_s for a given c_2 and let $c_2(c_1)$ be the function that maximizes π_s for a given c1. To obtain $c_1(c_2)$ and $c_2(c_1)$,

$$\frac{\partial \pi_s}{\partial c_1} = 0 \rightarrow c_1(c_2) = \frac{A_1'}{2 - c_2} \qquad (4.10)$$

$$\frac{\partial \pi_s}{\partial c_2} = 0 \rightarrow c_2(c_1) = \frac{-(A_1' - 3c_1)}{A_1' + c_1} \qquad (4.11)$$

Step 1: Note that $c_1(c_2)$, expressed as in Eqn. 4.10, is an increasing function of c_2. $c_1^{IBIS,NC,2}(c_2)$, expressed in Eqn. 4.35 is decreasing function of c_2. There exist two c_2 values at which $c_1(c_2)$ and $c_1^{IBIS,NC,2}(c_2)$ intersect, obtained as follows:

At the intersection $c_1(c_2)$ is set to $c_1^{IBIS,NC,2}(c_2)$;

$$\frac{A_1'}{2-c_2} = \frac{A_1'\beta - 2A_2' + 2A_2'c_2}{2c_2 - 2 + \beta}$$

Analyzing the c_2 value that equates $c_1(c_2)$ and $c_1^{IBIS,NC,2}(c_2)$, we obtain:

$c_2^1 = 1$ and $c_2^2 = \frac{-(2A_1' - 4A_2' + A_1'\beta)}{2A_2'}$. Since c_2^1 is outside the boundary, we only focus on c_2^2. Let c_2^2 be denoted as $\bar{c}_2$.

Case A: $2A_2' < A_1'$.

It is possible to show that for $c_2 \in [0, 2-\beta)$, $c_1(c_2) > c_1^{IBIS,c}(c_2)$. The analysis follows exactly the same steps as in Case B.2 below. Optimal c_1^*, c_2^* is:

$$(c_1^*, c_2^*) = \left(\frac{A_1'}{2-\beta+\varepsilon}, \beta - \varepsilon\right) \qquad (4.13)$$

Case B: $2A_2' > A_1'$.

Here depending on the $\bar{c}_2$, the analysis changes. We make the analysis under the three mutually exclusive cases: $\bar{c}_2 > \beta$, $\bar{c}_2 < 0$, and $0 < \bar{c}_2 < \beta$.

Case 1: $\bar{c}_2 > \beta$

This case occurs if $\bar{c}_2 = \frac{-(2A_1' - 4A_2' + A_1'\beta)}{2A_2'} > \beta$. It indicates that $c_1^{IBIS,NC,2}(c_2) > c_1(c_2)$.

Since $c_1(c_2)$ is outside Region 1, note for a given c_2, π_s is decreasing in c_1. In other words, in Region 1 for a given c_2, π_s is maximized at $c_1 = c_1^{IBIS,NC,2}(c_2)$. Then, we check whether π_s is increasing on the boundary, $c_1^{IBIS,NC,2}(c_2)$. In order to do so, first

$c_1^{IBIS,NC,2}(c_2)$ (given in Eqn. 4.35) is embedded in π_s and the derivative with respect to c_2 is taken;

$$\frac{\partial \pi_s(c_1^{IBIS,NC,2}(c_2))}{\partial c_2} = \frac{(A_1' - A_2')(2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta + 2A_1'c_2 + 2A_2'c_2)}{(\beta + 2c_2 - 2)^3}$$

Analysis shows that there exists a unique root for $\frac{\partial \pi_s(c_1^{IBIS,NC,2}(c_2))}{\partial c_2} = 0$. The function $\pi_s(c_1^{IBIS,NC,2}(c_2))$ could be convex or concave depending on the parameters, however the uniqueness of the root, together with the fact that for small values of c_2, $\frac{\partial \pi_s(c_1^{IBIS,NC,2}(c_2))}{\partial c_2}$ being (+) implies $\pi_s(c_1^{IBIS,NC,2}(c_2))$ is unimodal. Thus the maximizing $c_2$can be found as follows:

$$\frac{\partial \pi_s(c_1^{IBIS,NC,2}(c_2))}{\partial c_2} = 0 \rightarrow c_{2,IBIS,NC} = \frac{-(2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta)}{2A_1' + 2A_2'}$$

Let $c_2^h = min\{c_{2,IBIS,NC}, \beta - \varepsilon\}^+$. Then the optimal c_1 and c_2 are

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,NC,2}\left(c_2^h\right), c_2^h\right) \qquad (4.14)$$

Case 2: $\bar{c}_2 < 0$

This case occurs if $\bar{c}_2 = \frac{-(2A_1' - 4A_2' + A_1'\beta)}{2A_2'} < 0$

If $\bar{c}_2 < 0$, then $c_1(c_2) \geq c_1^{IBIS,NC,2}(c_2)$ is effective while finding c_1^*.

For a given c_2, the profit function of the supplier, π_s, is increasing in c_1 till it reaches $c_1(c_2)$. Moreover, π_s is increasing in c_2 along the line $c_1(c_2)$. Hence, optimal c_1 and c_2 will be;

$$(c_1^*, c_2^*) = \left(\frac{A_1'}{2 - \beta + \varepsilon}, \beta - \varepsilon\right)$$

Case 3: $0 < \bar{c}_2 < \beta$

This case occurs if $0 < \frac{-(2A_1'-4A_2'+A_1'\beta)}{2A_2'} < \beta$.

In Region 1a, $c_1^{IBIS,NC,2}(c_2)$ is effective while finding c_1^* and in Region 1b, $c_1(c_2)$ is effective while finding c_1^*.

π_s is increasing in c_2 and along the line $c_1(c_2)$. In Region 1b, optimal c_1 and c_2 are

$$(c_1^*, c_2^*) = (c_1(\beta-\varepsilon), \beta-\varepsilon) = \left(\frac{A_1'}{\varepsilon-\beta+2}, \beta-\varepsilon\right)$$

In Region 1a optimal c_1 and c_2 are $(c_1^*, c_2^*) = \left(c_1^{IBIS,NC,2}\left(c_2^h\right), c_2^h\right)$ where $c_2^h = min\{c_{2,IBIS}, \bar{c}_2\}^+$.

Then, optimal c_1 and c_2 are

$$(c_1^*, c_2^*) = argmax_{c_1,c_2}\{\pi_s(c_1(\beta-\varepsilon), \beta-\varepsilon), \pi_s\left(c_1^{IBIS,c}\left(c_2^h\right), c_2^h\right)\}$$

■

Proposition 7: Under IBIS setting with non-collaborating buyers if the equilibrium quantities are $q_1 > 0$ and $q_2 > 0$, then the optimal c_1 and c_2 values that maximizes the supplier's profit function are denoted with

$$argmax_{c_1,c_2}\left(\pi_s^{Region2}(c_1, c_2)\right)$$

and obtained as

a) **Case 1:** $\frac{2A_1'-6\quad {}_2'+3A_1'\beta-A_2'\beta}{-4A_2'} > \beta$

$$(c_1^*, c_2^*) = \left(c_1\left(\hat{c}_2^h\right), \hat{c}_2^h\right)$$

where $\hat{c}_2^h = min\{c_2^3, \beta-\varepsilon\}^+$

b) **Case 2:** $\frac{2A_1'-6A_2'+3A_1'\beta-A_2'\beta}{-4A_2'} < 0$

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,NC,2}\left(\hat{c}_2^h\right), \hat{c}_2^h\right)$$

where $\hat{c}_2^h = min\left\{\hat{c}_{2,IBIS}, \beta - \varepsilon\right\}^+$

c) **Case 3:** $0 < \frac{2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta}{-4A_2'} < \beta$

In Region 2a,

$$(c_1^*, c_2^*) = \left(c_1\left(\hat{c}_2^h\right), \hat{c}_2^h\right)$$

where $\hat{c}_2^h = min\{c_2^3, \hat{c}_2\}^+$

In Region 2b,

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,NC,2}\left(\hat{c}_2^h\right), \hat{c}_2^h\right)$$

where $\hat{c}_2^h = max\left\{\min\left\{\hat{c}_{2,IBIS}, \beta\right\}, \hat{c}_2\right\}^+$

Then optimal c_1 and c_2 for Region 2 are

$$(c_1^*, c_2^*) = argmax_{c_1,c_2}\left\{\pi_s\left(c_1\left(\hat{c}_2^h\right), \hat{c}_2^h\right), \pi_s\left(c_1^{IBIS,NC,2}\left(\hat{c}_2^h\right), \hat{c}_2^h\right)\right\}$$

Proof. Since it is assumed that $q_1 > 0$ and $q_2 > 0$, the best responses are

$$q_1 = \frac{2A_1'(1 - c_2) - A_2'\beta - 2c_1 + c_1\beta + 2c_1c_2}{4(1 - c_2)^2 - (\beta)^2}$$

$$q_2 = \frac{2A_2'(1 - c_2) - A_1'\beta - 2c_1 + c_1\beta + 2c_1c_2}{4(1 - c_2)^2 - (\beta)^2}$$

To find optimal c_1 and c_2 values that maximize the supplier's profit function, the first order condition (FOC) and the second order conditions (SOC) are analyzed. Note that, similar to collaboration case, π_s is continuous and differentiable inside the boundaries.

Taking the derivative with respect to c_1

$$\frac{\partial \pi_s}{\partial c_1} = \frac{(\beta+2)(A_1' + A_2' - 2c_1)}{(\beta - 2c_2 + 2)^2} - \frac{2c_1}{\beta - 2c_2 + 2} \qquad (4.41)$$

$$\frac{\partial^2 \pi_s}{\partial c_1^2} = -\frac{4(\beta - c_2 + 2)}{(\beta - 2c_2 + 2)^2} < 0 \qquad (4.42)$$

First order conditions yield;

$$\frac{\partial \pi_s}{\partial c_1} = 0 \to c_1(c_2) = \frac{(A_1' + A_2')(\beta + 2)}{4(\beta - c_2 + 2)} \qquad (4.40)$$

Observation 5. Since in Region 2 $c_2 < 1 - \beta/2$, the SOC in Eqn. 4.42 implies for a given c_2, the profit function of the supplier is concave in c_1. Thus, for a given c_2, $c_1(c_2)$ maximizes π_s. ■

Taking the derivative with respect to c_2 finding the c_2 that satisfies the FOC involves difficulties, since the partial derivative of π_s with respect to c_2 is a quadratic function of c_2 and it is difficult to be solved by radicals.

$$\frac{\partial \pi_s}{\partial c_2} = 0 \to c_2 = see\ Appendix$$

Then, all c_1's in the profit function (π_s) is replaced by $c_1(c_2)$ given in Eqn. 4.40 and derivative with respect to c_2 is taken;

$$\frac{\partial \pi_s(c_1(c_2))}{\partial c_2} = \frac{c_2\{A + \beta B + \beta^2 C\} - 10A_1A_2 + \beta D - c_2^2\{E + \beta F\} + G + c_2^3 H - \beta^2 J - \beta^3 K}{(\beta - c_2 + 2)^2(\beta + 2c_2 - 2)^3} \qquad (4.43)$$

where

$$A = 6A_1A_2 + 3A_1^2 + 3A_2^2$$

$$B = A_1^2 - 14A_1A_2 + A_2^2$$

$$C = 11A_1^2/4 - 5A_1A_2/2 + 11A_2^2/4$$

$$D = 7A_1^2/2 - A_1A_2 + 7A_2^2/2$$

$$E = 6A_1^2 + 6A_2^2$$

$$F = A_1^2 - 8A_1A_2 + A_2^2$$

$$G = 3A_1^2 + 3A_2^2$$

$$H = 2A_1^2 + 2A_2^2$$

$$J = 7A_1^2/4 - A_1A_2/2 + 7A_2^2/4$$

$$K = 3A_1^2/8 - 5A_1A_2/4 + 3A_2^2/8$$

Observation 6. For a given c_1, π_s is unimodal in c_2.

Proof. In Eqn. 4.43 above, the denominator is always negative. Eqn. 4.43 can be rewritten as;

$$\frac{\partial \pi_s(c_1(c_2))}{\partial c_2} = \frac{c_2M + c_2^2N + c_2^3P + A_1'A_2'R + {A_1'}^2S + {A_2'}^2S}{(\beta - c_2 + 2)^2(\beta + 2c_2 - 2)^3}$$

where

$$M = A_1'A_2'(6 + 5\beta^2/4 - 14\beta) + {A_1'}^2(3 + \beta + 11\beta^2/4) + {A_2'}^2(3 + \beta + 11\beta^2/4)$$

$$N = 8A_1'A_2'\beta - {A_1'}^2(6 + \beta) - {A_2'}^2(6 + \beta)$$

$$P = 2{A_1'}^2 + 2{A_2'}^2$$

$$R = -10 - \beta + \beta^2/2 + 5\beta^3/4$$

$$S = 3 + 7\beta/2 - 7\beta^2/4 - 3\beta^3/8$$

The numerator can take positive and negative values. Hence, there exist a threshold level for c_2 where for small values of c_2, the derivate possibly takes negative values and for larger values than the threshold, the derivate takes positive values.

The FOC of $\pi_s(c_1(c_2))$ with respect to c_2 gives three roots,

$$\frac{\partial \pi_s(c_1(c_2))}{\partial c_2} = 0 \rightarrow three\ roots, c_2^1, c_2^2, c_2^3, see\ Appendix$$

Two of these roots are imaginary and c_2^3 is real.

Note that, when $c_2 = -\infty$, the value of Eqn. 4.43 is positive. This implies that for a given c_1, π_s is unimodal in c_2.■

$$\frac{\partial \pi_s}{\partial c_1} = 0 \rightarrow c_1(c_2) = \frac{(A_1' + A_2')(\beta + 2)}{4(\beta - c_2 + 2)} \qquad (4.40)$$

To determine the maximizing c_1 and c_2, further analysis is required.

π_s is unimodal with respect to c_1 and c_2. The c_1 and c_2 values that maximize the supplier's profit can be found by analyzing the structure of $c_1(c_2)$ and $c_2(c_1)$ and boundary conditions of the feasible region.

Step 1. Note $c_1(c_2)$ is monotonically increasing in c_2 and on the other hand as c_2 increases, $c_1^{IBIS,NC,2}(c_2)$ decreases. Note that there exists at most a single c_2 at which $c_1(c_2)$ and $c_1^{IBIS,NC,2}(c_2)$ intersect. Let this point be called $\hat{c}_2$.

If $c_2 < \hat{c}_2 = \frac{2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta}{-4A_2'}$, then $c_1^{IBIS,NC,2}(c_2) > c_1(c_2)$ and c_1^* in Region 2 is determined by $c_1(c_2)$

If $c_2 > \hat{c}_2 = \frac{2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta}{-4A_2'}$, then $c_1^{IBIS,NC,2}(c_2) < c_1(c_2)$ and c_1^* in Region 2 is determined by $c_1^{IBIS,NC,2}(c_2)$

Step 2. Similar to collaboration case how c_1^* and c_2^* is determined is largely affected by the value of $\hat{c}_2$ with respect to lower bound on c_2 (which is 0) and upper bound on

c_2 (which is $\bar{\beta} = \min\{\beta, 1 - \beta/2\}$). We make the analysis under the three mutually exclusive cases: $\hat{c}_2 > \bar{\beta}$, $\hat{c}_2 < 0$, and $0 < \hat{c}_2 < \bar{\beta}$.

Case 1: $\hat{c}_2 > \bar{\beta}$

This case occurs if $\frac{2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta}{-4A_2'} > \bar{\beta}$. Then, c_2 is always less than $\hat{c}_2$, $c_1^{IBIS,NC,2}(c_2) > c_1(c_2)$ and $c_1(c_2)$ determines c_1^*.

There are three possibilities for this case;

- If $c_2^3 > \bar{\beta}$, optimal c_1 and c_2 will be

$$(c_1^*, c_2^*) = \left(c_1\left(\bar{\beta} - \varepsilon\right), \bar{\beta} - \varepsilon\right) \qquad (4.44)$$

- If $0 < c_2^3 < \bar{\beta}$, then optimal c_1 and c_2 will be

$$(c_1^*, c_2^*) = (c_1(c_2^3), c_2^3) \qquad (4.45)$$

- If $c_2^3 < 0$, then optimal c_1 and c_2 will be:

$$(c_1^*, c_2^*) = (c_1(0), 0) \qquad (4.46)$$

Case 2: $\hat{c}_2 < 0$ This case occurs if $\frac{2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta}{-4A_2'} < 0$. Since $\hat{c}_2 < c_2$ $c_1^{IBIS,NC,2}(c_2) < c_1(c_2)$ and $c_1^{IBIS,NC,2}(c_2)$ determines c_1^*.

For a given c_2, π_s is maximized at $c_1 = c_1^{IBIS,NC,2}(c_2)$. Embedding $c_1^{IBIS,NC,2}(c_2)$ into π_s,

$$\pi_s\left(c_1^{IBIS,NC,2}(c_2)\right) = \frac{(2A_2' - A_1'\beta)(A_1' - A_2') - c_2(A_1' + A_2')(A_1' - A_2')}{(\beta + 2c_2 - 2)^2} \qquad (4.47)$$

and taking the derivative of Eqn. 4.47 with respect to c_2

$$\frac{\partial \pi_s\left(c_1^{IBIS,NC,2}(c_2)\right)}{\partial c_2} = \frac{(A_1' - A_2')(2A_1' - 6A_2' + 2A_1'c_2 + 2A_2'c_2) + \beta(A_1' - A_2')(3A_1' - A_2')}{(\beta + 2c_2 - 2)^3}$$

Then, optimal c_2 which maximizes the supplier's profit

$$\frac{\partial \pi_s\left(c_1^{IBIS,NC,2}(c_2)\right)}{\partial c_2} = 0 \to c_2 = \hat{c}_{2,IBIS} = \frac{-(2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta)}{2A_1' + 2A_2'} \quad (4.48)$$

Note that $\pi_s\left(c_1^{IBIS,NC,2}(c_2)\right)$ is unimodal in c_2. Hence, optimal c_1 and c_2 will be

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,NC,2}(\hat{c}_{2,IBIS}), \hat{c}_{2,IBIS}\right) = \left(\frac{A_2'}{2} - \frac{A_1'}{2} + \frac{A_1' + A_2'}{\beta + 2}, \frac{-(2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta)}{2A_1' + 2A_2'}\right) \quad (4.49)$$

If $\hat{c}_{2,IBIS}$ happens to be out of the boundary (i.e. $\hat{c}_{2,IBIS} < 0$ or $\hat{c}_{2,IBIS} > \beta$), optimal c_2 is set to the nearest boundary point.

If $\hat{c}_{2,IBIS} < 0$, then optimal c_1 and c_2 will be

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,NC,2}(0), 0\right) = \left(\frac{A_1'\beta - 2A_2'}{\beta - 2}, 0\right) \quad (4.50)$$

If $\hat{c}_{2,IBIS} > \beta$, then optimal c_1 and c_2 will be

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,NC,2}(\beta - \varepsilon), \beta - \varepsilon\right) = \left(\frac{A_1'\beta - 2A_2' + 2A_2'(\beta - \varepsilon)}{3\beta - 2\varepsilon - 2}, \beta - \varepsilon\right) \quad (4.51)$$

In conclusion, if $\hat{c}_2 < 0$, optimal c_1 and c_2 will be determined by Eqns. 4.49, 4.50 and 4.51 depending on the value of $\hat{c}_2^h$.

Case 3: $0 < \hat{c}_2 < \bar{\beta}$

This case occurs if $0 < \frac{2A_1' - 6A_2' + 3A_1'\beta - A_2'\beta}{-4A_2'} < \bar{\beta}$

Then, the feasible region will be divided into two sub-regions. In Region 2a, $c_2 < \hat{c}_2$ and this is the same situation in Case 1; $c_1^{IBIS,NC,2}(c_2)$ is greater than $c_1(c_2)$ and $c_1(c_2)$ determines c_1^*. In Region 2b, $c_2 > \hat{c}_2$ and this requires the same analysis as in Case 2; $c_1^{IBIS,NC,2}(c_2)$ is less than $c_1(c_2)$ and $c_1^{IBIS,NC,2}(c_2)$ determines c_1^*.

In Region 2a,

$$(c_1^*, c_2^*) = \left(c_1\left(\hat{c}_2^h\right), \hat{c}_2^h\right)$$

where $\hat{c}_2^h = min\{c_2^3, \hat{c}_2\}^+$

In Region 2b,

$$(c_1^*, c_2^*) = \left(c_1^{IBIS,NC,2}\left(\hat{c}_2^h\right), \hat{c}_2^h\right)$$

where $\hat{c}_2^h = max\left\{\min\left\{\hat{c}_{2,IBIS}, \beta\right\}, \hat{c}_2\right\}^+$

Then optimal c_1 and c_2 for Region 2 are

$$(c_1^*, c_2^*) = argmax_{c_1,c_2}\left\{\pi_s\left(c_1\left(\hat{c}_2^h\right), \hat{c}_2^h\right), \pi_s\left(c_1^{IBIS,NC,2}\left(\hat{c}_2^h\right), \hat{c}_2^h\right)\right\}$$

■

Proposition 8: Under the IBIS setting with two non-collaborating buyers, the optimal c_1 and c_2 values that maximizes the supplier's profit function are

$$argmax_{c_1,c_2}\left(\max\{\pi_s^{Region1}(c_1, c_2), \pi_s^{Region2}(c_1, c_2)\}\right)$$

where c_1^* and c_2^* in Region 1 and Region 2 are given in Proposition 6 and Proposition 7. ■

4.2.3. Examples

The following 3 examples presented show how the equilibrium quantities, the optimal π_s and the corresponding $E_\theta[\pi_1|Y_1, Y_2]$ and $E_\theta[\pi_2|Y_1, Y_2]$ are determined. In all examples $\varepsilon = 0.01$

Example 1

$A_1' = 10000,\ A_2' = 9000,\ \beta = 0.8$

In Region 1

$c_1^* = 8264.5,\ c_2^* = 0.79,\ q_1 = 4132,\ q_2 = 0,\ \pi_s = 20661157.02$

In Region 2, Case 3 is valid since $\hat{c}_2 = 0.47777$

In Region 2a, $c_1(c_2)$ is effective while finding c_1^* and in Region 2b, $c_1^{IBIS,NC,2}(c_2)$ is effective while finding c_1^*.

In Region 2a, $c_2^3 = 0.35$, then $\hat{c}_2^h = min\{c_2^3, \hat{c}_2\} = 0.35$

$c_1^* = 5428.5,\ c_2^* = 0.35,\ q_1 = 2958.7,\ q_2 = 938.7,\ \pi_s = 17718367.35$

In Region 2b, $\hat{c}_{2,IBIS} = 0.453$, $\hat{c}_2^h = max\{\min\{\hat{c}_{2,IBIS}, \beta\}, \hat{c}_2\} = 0.4777$

$c_1^* = 5666,\ c_2^* = 0.4777,\ q_1 = 4166,\ q_2 = 0,\ \pi_s = 15277777.78$

Example 2

$A_1' = 15000,\ A_2' = 4000,\ \beta = 0.8$

Region 2 does not exist. Only Region 1

$c_1^* = 12396.7,\ c_2^* = 0.79,\ q_1 = 6198,\ q_2 = 0,\ \pi_s = 46487603$

Example 3

$A_1' = 10000,\ A_2' = 9000,\ \beta = 0.9$

In Region 1

$c_1^* = 0.89,\ c_2^* = 9009,\ q_1 = 4504,\ q_2 = 0,\ \pi_s = 22522522$

In Region 2, Case 3 is valid since $\hat{c}_2 = 0.42$

In Region 2a, $c_2^3 = 0.2985$, then $\hat{c}_2^h = min\{c_2^3, \hat{c}_2\} = 0.2985$

$c_1^* = 5295,\ c_2^* = 0.2985,\ q_1 = 2820,\ q_2 = 832,\ \pi_s = 16755862$

In Region 2b, $\hat{c}_{2,IBIS} = 0.3974$, $\hat{c}_2^h = max\{\min\{\hat{c}_{2,IBIS}, \beta\}, \hat{c}_2\} = 0.42$

$c_1^* = 5538,\ c_2^* = 0.42,\ q_1 = 3846,\ q_2 = 0,\ \pi_s = 15088757.4$

4.3 Determining the ex-ante profits

In order to find the ex-ante profits for the buyers and the supplier we need to take expectation the realization of the signals. Under equilibrium, q_1 and q_2 are functions of Y_1 and Y_2. Thus, expectations should be taken over both Y_1 and Y_2.

Ex-ante profits of the buyers and the supplier

We are interested in $E_{Y_1,Y_2}[E[\pi_i|Y_1,Y_2]]$ where $i \in \{1,2,s\}$

$$\begin{aligned} E_{Y_1,Y_2}&[E[\pi_i|Y_1,Y_2]] \\ &= \sum_{k_1 \in S_1} \sum_{k_2 \in S_2} \pi_i(k_1,k_2) P\{Y_1 = k_1\} P\{Y_2 = k_2\} \\ &= \sum_{k_1 \in S_1} \sum_{k_2 \in S_2} \pi_i(k_1,k_2) p_1^{k_1} p_2^{k_2} \qquad (4.52) \end{aligned}$$

where $p_j^{k_j}$ denotes the probability that signal Y_j takes values k_j, $k_j \in S_j$ and $\pi_i(k_1,k_2)$ is the profit of the player i when signal Y_i takes value of k_i and Y_j takes value of k_j.

Note that Y_1 and Y_2 are independent random variables. Then, for a given k_1 and k_2, ex-ante profits are determined as follows;

1. Let $A_1' = \max\{A_1 + \delta_1 k_1, A_2 + \delta_2 k_2\}$ and define A_2' accordingly.
2. Follow the analysis in Section 4.1.2 for collaboration and in Section 4.2.2 for no-collaboration setting. Use the equilibrium quantities defined in Section 4.1.1 and Section 4.2.1 to determine $c_1^*(k_1, k_2)$ and $c_2^*(k_1, k_2)$.
3. Use $q_1(k_1, k_2)$, $q_2(k_1, k_2)$, $c_1^*(k_1, k_2)$, $c_2^*(k_1, k_2)$ to obtain $\pi_i(k_1, k_2)$ for the corresponding realization of Y_1 and Y_2.
4. Ex-ante profit for the buyers and the supplier is obtained in Eqn. 4.52 above.

CHAPTER 5

IMPERFECT INFORMATION FOR THE BUYERS AND NO INFORMATION FOR THE SUPPLIER (IBNS)

The equilibrium points and the supplier's optimal c_1 and c_2 values are determined under the strategy that the buyers share their signals on the market demand with each other but do not share it with the supplier. The analysis is made under collaborating and non-collaborating buyers. In the analysis, first, the equilibrium quantities are determined, and then the optimum wholesale price is calculated. The assumptions done during the analysis are same as previous section.

5.1 Collaborating buysers

The equilibrium quantities and the profit functions of the buyers are the same as IBIS case. The profit function of the buyers and the supplier are expressed as follows;

$$
\begin{aligned}
E_\theta[\pi_1|Y_1,Y_2] &= E[(A_1+\theta_1-q_1-\beta q_2-c_1+c_2(q_1+q_2))q_1|Y_1,Y_2] \\
&= \big(A_1+E[\theta_1|Y_1]-q_1-\beta q_2-c_1+c_2(q_1+q_2)\big)q_1 \\
&= \big(A_1+\delta_1(\alpha_1,\sigma_1)Y_1-q_1-\beta q_2-c_1+c_2(q_1+q_2)\big)q_1
\end{aligned}
$$

$$
\begin{aligned}
E_\theta[\pi_2|Y_1,Y_2] &= E[(A_2+\theta_2-q_2-\beta q_1-c_1+c_2(q_1+q_2))q_2|Y_1,Y_2] \\
&= \big(A_2+E[\theta_2|Y_2]-q_2-\beta q_1-c_1+c_2(q_1+q_2)\big)q_2 \\
&= \big(A_2+\delta_2(\alpha_2,\sigma_2)Y_2-q_2-\beta q_1-c_1+c_2(q_1+q_2)\big)q_2
\end{aligned}
$$

$$E_{\theta,Y_1,Y_2}[\pi_s] = E\big[\big(c_1 - c_2(q_1 + q_2)\big)(q_1 + q_2)\big]$$
$$= c_1 E_{\theta,Y_1,Y_2}[q_1 + q_2] - c_2 E_{\theta,Y_1,Y_2}[(q_1 + q_2)^2]$$

Similar to IBIS case, $E_\theta[\pi_1|Y_1, Y_2]$, $E_\theta[\pi_2|Y_1, Y_2]$ and $E_{\theta,Y_1,Y_2}[\pi_s]$ denote the ex-post profit functions for the buyers and the supplier. In the expressions when q_1 and q_2 are equilibrium quantities, q_1 and q_2 are functions of c_1, c_2, Y_1 and Y_2. Ex-ante profits for the buyers are given in Section 5.3. For the supplier, under IBNS, ex-ante and ex-post profits do not differ. Since under equilibrium, quantities q_1 and q_2 are a function of c_1, c_2, Y_1 and Y_2, expectation is taken over Y_1 and Y_2. Then optimal c_1 and c_2 is seeked to maximize $E[\pi_s]$.

In the following we discuss how the equilibrium quantities for the buyers and optimal c_1 and c_2 values for the supplier are determined.

5.1.1 The Buyers' Problem

For a given value of c_1 and c_2, the equilibrium quantities for the buyers are obtained following the same steps as in IBIS case.

5.1.2 The Supplier's Problem

Under IBNS case the buyers do not share the demand information with the supplier, hence the supplier does not know Y_1 and Y_2.

Proposition 9: Under the IBNS setting with two collaborating buyers, the optimal c_1 and c_2 values that maximize the supplier's profit function are denoted with

$$E_{Y_1,Y_2}[\pi_s(c_1,c_2)] = \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < \frac{m_1 k_1 + c_1 - m_4}{m_2}}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq \frac{c_1 - A_1}{\delta_1}}} \pi_s(c_1, c_2, q_1, 0) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ \frac{m_1 k_1 + c_1 - m_4}{m_2} \leq k_2 < \frac{m_4' + m_1' k_1 - c_1}{m_2'}}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq \frac{c_1 - A_1}{\delta_1}}} \pi_s(c_1, c_2, q_1, q_2) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 \geq \frac{m_4' + m_1' k_1 - c_1}{m_2'}}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq \frac{c_1 - A_1}{\delta_1}}} \pi_s(c_1, c_2, 0, q_2) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 \geq \frac{c_1 - A_2}{\delta_2}}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 < \frac{c_1 - A_1}{\delta_1}}} \pi_s(c_1, c_2, 0, q_2) p_1^{k_1} p_2^{k_2}$$

where $m_1 = \frac{\delta_1(\beta - c_2)}{m_3}$, $m_2 = \frac{2\delta_2(1 - c_2)}{m_3}$, $m_3 = 2(1 - c_2) - (\beta - c_2)$, $m_4 = \frac{2A_2(1-c_2) - A_1(\beta - c_2)}{m_3}$ and $m_1' = \frac{2\delta_1(1-c_2)}{m_3'}$, $m_2' = \frac{\delta_2(\beta - c_2)}{m_3'}$, $m_3' = 2(1 - c_2) - (\beta - c_2)$, $m_4' = \frac{2A_1(1-c_2) - A_2(\beta - c_2)}{m_3'}$. In the expression S_1 and S_2 are the set of that random variables Y_1 and Y_2 can take.

Proof: For a given c_1 and c_2, the profit function of the supplier can be written under changing Y_1 and Y_2. In the following Y_1 and Y_2 denote a realization of Y_1 and Y_2. In other words, let $Y_1 = k_1$ and $Y_2 = k_2$. When partitioning the space that Y_1 and Y_2 can take, the separating lines are defined next to the corresponding inequalities. The lines are shown in Figure 12 below.

Case I: If $A_1 + \delta_1 Y_1 > A_2 + \delta_2 Y_2$ or equivalently if

$$Y_2 < \frac{A_1 + \delta_1 Y_1 - A_2}{\delta_2} \qquad Line 1$$

Case a: If

$$c_1 > A_1 + \delta_1 Y_1$$

$$c_1 > A_2 + \delta_2 Y_2$$

Equivalently, if

$$Y_1 < \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 < \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

Then, $q_1 = 0$ and $q_2 = 0$

Case b: If

$$c_1 < A_1 + \delta_1 Y_1$$

$$c_1 > A_2 + \delta_2 Y_2$$

Equivalently, if

$$Y_1 > \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 < \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

Then, $q_1 > 0$ and $q_2 = 0$

Case c: If

$$c_1 < A_1 + \delta_1 Y_1$$

$$c_1 < A_2 + \delta_2 Y_2$$

Equivalently, if

$$Y_1 > \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 > \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

Case 1:

If

$$c_1 < \frac{2(A_2 + \delta_2 Y_2)(1 - c_2) - (A_1 + \delta_1 Y_1)(\beta - c_2)}{2(1 - c_2) - (\beta - c_2)} = c_1^{IBIS,c}(c_2)$$

$$Y_2 > \frac{m_1 Y_1 + c_1 - m_4}{m_2} \qquad Line2$$

where

$$m_1 = \frac{\delta_1(\beta - c_2)}{m_3}$$

$$m_2 = \frac{2\delta_2(1 - c_2)}{m_3}$$

$$m_3 = 2(1 - c_2) - (\beta - c_2)$$

$$m_4 = \frac{2A_2(1 - c_2) - A_1(\beta - c_2)}{m_3}$$

Then, $q_1 > 0$ and $q_2 > 0$

Case2:

If

$$Y_2 < \frac{m_1 Y_1 + c_1 - m_4}{m_2}$$

Then, $q_1 > 0$ and $q_2 = 0$

Case II: If $A_1 + \delta_1 Y_1 < A_2 + \delta_2 Y_2$ or equivalently if

$$Y_2 > \frac{A_1 + \delta_1 Y_1 - A_2}{\delta_2} \qquad Line1$$

Case a: If

$$c_1 > A_1 + \delta_1 Y_1$$

$$c_1 > A_2 + \delta_2 Y_2$$

Equivalently, if

$$Y_1 < \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 < \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

Then, $q_1 = 0$ and $q_2 = 0$

Case b: If

$$c_1 > A_1 + \delta_1 Y_1$$

$$c_1 < A_2 + \delta_2 Y_2$$

Equivalently, if

$$Y_1 < \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 > \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

Then, $q_1 = 0$ and $q_2 > 0$.

Case c: If

$$c_1 < A_1 + \delta_1 Y_1$$

$$c_1 < A_2 + \delta_2 Y_2$$

Equivalently, if

$$Y_1 > \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 > \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

Case 1:

If

$$c_1 < \frac{2(A_1 + \delta_1 Y_1)(1 - c_2) - (A_2 + \delta_2 Y_2)(\beta - c_2)}{2(1 - c_2) - (\beta - c_2)}$$

$$c_1 < \frac{2A_1(1 - c_2) - A_2(\beta - c_2) + 2\delta_1 Y_1(1 - c_2) - \delta_2 Y_2(\beta - c_2)}{2(1 - c_2) - (\beta - c_2)}$$

$$c_1 < m_4' + m_1' Y_1 - m_2' Y_2$$

Equivalently,

$$Y_2 < \frac{m_4' + m_1' Y_1 - c_1}{m_2'} \qquad Line\ 3$$

where

$$m_4' = \frac{2A_1(1 - c_2) - A_2(\beta - c_2)}{m_3'}$$

$$m_3' = 2(1 - c_2) - (\beta - c_2)$$

$$m_2' = \frac{\delta_2(\beta - c_2)}{m_3'}$$

$$m_1' = \frac{2\delta_1(1 - c_2)}{m_3'}$$

Then, $q_1 > 0$ and $q_2 > 0$

Case 2:

If

$$Y_2 > \frac{m_4' + m_1' Y_1 - c_1}{m_2'}$$

Then, $q_1 = 0$ and $q_2 > 0$

The lines 1, 2, 3, 4 and 5 intersect at the same point, point A, $Y_1 = \frac{-(A_1 - c_1)}{\delta_1}$ and $Y_2 = \frac{-(A_2 - c_1)}{\delta_2}$

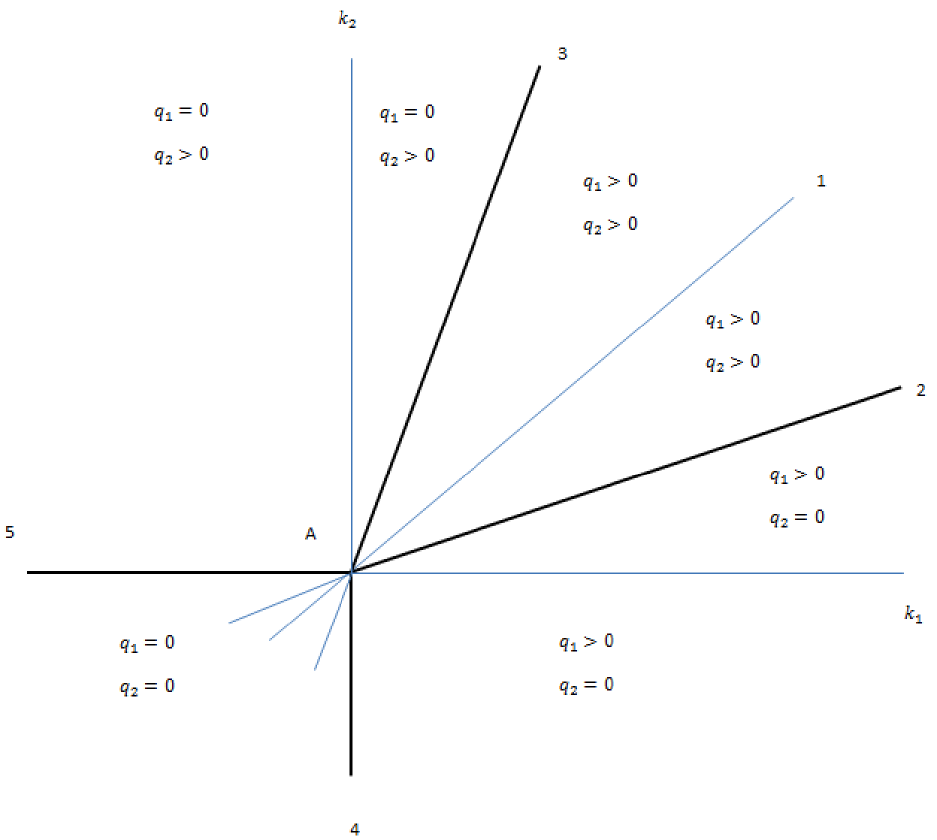

Figure 12 Equilibria under partitions of $(\boldsymbol{Y_1}, \boldsymbol{Y_2})$ space

Let $E_{Y_1,Y_2}[\pi_s(c_1, c_2)]$ denote the ex-ante profit of the supplier, where the equilibrium quantities of the buyers are $q_1(c_1, c_2, Y_1, Y_2)$ and $q_2(c_1, c_2, Y_1, Y_2)$. For a given c_1 and c_2, $E_{Y_1,Y_2}[\pi_s(c_1, c_2)]$ is obtained as;

$E_{Y_1,Y_2}[\pi_s(c_1,c_2)]$

$$= \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < \frac{m_1 k_1 + c_1 - m_4}{m_2}}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq \frac{c_1 - A_1}{\delta_1}}} \pi_s(c_1, c_2, q_1, 0) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ \frac{m_1 k_1 + c_1 - m_4}{m_2} \leq k_2 < \frac{m_4' + m_1' k_1 - c_1}{m_2'}}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq \frac{c_1 - A_1}{\delta_1}}} \pi_s(c_1, c_2, q_1, q_2) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 \geq \frac{m_4' + m_1' k_1 - c_1}{m_2'}}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq \frac{c_1 - A_1}{\delta_1}}} \pi_s(c_1, c_2, 0, q_2) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 \geq \frac{c_1 - A_2}{\delta_2}}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 < \frac{c_1 - A_1}{\delta_1}}} \pi_s(c_1, c_2, 0, q_2) p_1^{k_1} p_2^{k_2}$$

■

5.2 Non-Collaborating Buysers

When the buyers are non-collaborating, the profit of supplier and buyers are expressed as follows;

$$\begin{aligned} E_\theta[\pi_1|Y_1,Y_2] &= E[(A_1 + \theta_1 - q_1 - \beta q_2 - c_1 + c_2 q_1) q_1 | Y_1, Y_2] \\ &= (A_1 + E[\theta_1|Y_1] - q_1 - \beta q_2 - c_1 + c_2 q_1) q_1 \\ &= (A_1 + \delta_1(\alpha_1,\sigma_1) Y_1 - q_1 - \beta q_2 - c_1 + c_2 q_1) q_1 \end{aligned}$$

$$\begin{aligned} E_\theta[\pi_2|Y_1,Y_2] &= E[(A_2 + \theta_2 - q_2 - \beta q_1 - c_1 + c_2 q_2) q_2 | Y_1, Y_2] \\ &= (A_2 + E[\theta_2|Y_2] - q_2 - \beta q_1 - c_1 + c_2 q_2) q_2 \\ &= (A_2 + \delta_2(\alpha_2,\sigma_2) Y_2 - q_2 - \beta q_1 - c_1 + c_2 q_2) q_2 \end{aligned}$$

Here, $E_\theta[\pi_1|Y_1,Y_2]$, $E_\theta[\pi_2|Y_1,Y_2]$ denote the ex-post expected profits for the buyers. In the expressions when q_1 and q_2 are equilibrium quantities, q_1 and q_2 are functions of c_1, c_2, Y_1 and Y_2. Ex-ante profits for the buyers and the supplier are given in Section 5.3.

$$
\begin{aligned}
E_{\theta,Y_1,Y_2}[\pi_s] &= E_{\theta,Y_1,Y_2}[(c_1 - c_2 q_1)q_1 + (c_1 - c_2 q_2)q_2] \\
&= c_1 E_{\theta,Y_1,Y_2}[q_1 + q_2] - c_2 E_{\theta,Y_1,Y_2}[q_1^2 + q_2^2]
\end{aligned}
$$

Similar to collaborating buyers case, ex-ante and ex-post profits do not differ, expectation is taken over Y_1 and Y_2. Then optimal c_1 and c_2 is seeked to maximize $E[\pi_s]$.

In the following we discuss how the equilibrium quantities for the buyers and optimal c_1 and c_2 values for the supplier are determined.

5.2.1. The Buyers' Problem

For a given value of c_1 and c_2, the equilibrium quantities for the buyers are obtained following the same steps as in IBIS case.

5.1.2 The Supplier's Problem

Under IBNS the supplier has the knowledge of demand signals Y_1 and Y_2.

Proposition 10: Under the IBNS setting with two no-collaborating buyers, the optimal c_1 and c_2 values that maximize the supplier's profit function are denoted with

Case A: $c_2 < 1 - \beta/2$

$$E_{Y_1,Y_2}[\pi_s(c_1,c_2)] = \sum_{k_2<\frac{m_4'+m_1'k_1-c_1}{m_2'}} \sum_{k_1\geq c_1-A_1/\delta_1} \pi_s(c_1,c_2,q_1,0)p_1^{k_1}p_1^{k_2} + \sum_{\frac{m_4'+m_1'k_1-c_1}{m_2'}\leq k_2<\frac{m_4+m_1k_1-c_1}{m_2}} \sum_{k_1\geq c_1-A_1/\delta_1} \pi_s(c_1,c_2,q_1,q_2)p_1^{k_1}p_1^{k_2} + \sum_{k_2\geq\frac{m_4+m_1k_1-c_1}{m_2}} \sum_{k_1\geq c_1-A_1/\delta_1} \pi_s(c_1,c_2,0,q_2)p_1^{k_1}p_2^{k_2} + \sum_{k_2\geq c_1-A_2/\delta_2} \sum_{k_1<c_1-A_1/\delta_1} \pi_s(c_1,c_2,0,q_2)p_1^{k_1}p_1^{k_2}$$

Case B: $c_2 > 1-\beta/2$

$$E_{Y_1,Y_2}[\pi_s(c_1,c_2)] = \sum_{k_2<A_1+\delta_1k_1-A_2/\delta_2} \sum_{k_1\geq c_1-A_1/\delta_1} \pi_s(c_1,c_2,q_1,0)p_1^{k_1}p_1^{k_2} + \sum_{A_1+\delta_1k_1-A_2/\delta_2\leq k_2} \sum_{k_1\geq c_1-A_1/\delta_1} \pi_s(c_1,c_2,0,q_2)p_1^{k_1}p_1^{k_2} + \sum_{k_2\geq c_1-A_2/\delta_2} \sum_{k_1<c_1-A_1/\delta_1} \pi_s(c_1,c_2,0,q_2)p_1^{k_1}p_1^{k_2} + \sum_{k_2<c_1-A_2/\delta_2} \sum_{k_1<c_1-A_1/\delta_1} \pi_s(c_1,c_2,0,0)p_1^{k_1}p_1^{k_2}$$

where $m_1 = \frac{2\delta_1(1-c_2)}{m_3}$, $m_2 = \frac{\delta_2\beta}{m_3}$, $m_3 = 2(1-c_2)-\beta$, $m_4 = \frac{2A_1(1-c_2)-A_2\beta}{m_3}$ and $m_1' = \frac{\delta_1\beta}{m_3'}$, $m_2' = \frac{2\delta_2(1-c_2)}{m_3'}$, $m_3' = \beta-2(1-c_2)$, $m_4' = \frac{A_1\beta-2A_2(1-c_2)}{m_3'}$.

Proof: Similar to collaborating buyers case for a given c_1 and c_2, the profit function of the supplier can be written under changing Y_1 and Y_2. Let $Y_1 = k_1$ and $Y_2 = k_2$, then the space can be portioned and separating lines are defined next to the corresponding inequalities. The lines are shown in Figure 13 below.

Case A: $c_2 < 1-\beta/2$

Case I: If $A_1 + \delta_1 Y_1 > A_2 + \delta_2 Y_2$ or equivalently if

$$Y_2 < \frac{A_1 + \delta_1 Y_1 - A_2}{\delta_2} \qquad Line1$$

Case a:

$$c_1 > A_1 + \delta_1 Y_1$$

$$c_1 > A_2 + \delta_2 Y_2$$

Equivalently,

$$Y_1 < \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 < \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

Then $q_1 = 0, q_2 = 0$

Case b:

$$c_1 < A_1 + \delta_1 Y_1$$

$$c_1 > A_2 + \delta_2 Y_2$$

Equivalently,

$$Y_1 > \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 < \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

Then $q_1 > 0, q_2 = 0$

Case c:

$$c_1 < A_1 + \delta_1 Y_1$$

$$c_1 < A_2 + \delta_2 Y_2$$

Equivalently,

$$Y_1 > \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 > \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

Case 1:

If

$$c_1 < \frac{2A_1(1-c_2) + 2\delta_1 Y_1(1-c_2) - A_2\beta - \delta_2 Y_2 \beta}{2 - 2c_2 - \beta} = c_1^{IBIS,NC,1}(c_2) \qquad (4.33)$$

and

$$c_1 \leq \frac{A_1\beta + \delta_1 Y_1 \beta - 2A_2 - 2\delta_2 Y_2 + 2A_2 c_2 + 2\delta_2 Y_2 c_2}{2c_2 - 2 + \beta} = c_1^{IBIS,NC,2}(c_2) \qquad (4.34)$$

From Eq. 4.33

$$Y_2 < \frac{m_4 + m_1 Y_1 - c_1}{m_2} \qquad Line2$$

where

$$m_1 = \frac{2\delta_1(1-c_2)}{m_3}$$

$$m_2 = \frac{\delta_2 \beta}{m_3}$$

$$m_3 = 2(1-c_2) - \beta$$

$$m_4 = \frac{2A_1(1-c_2) - A_2\beta}{m_3}$$

From Eq. 4.34

$$Y_2 \geq \frac{m_4' + m_1' Y_1 - c_1}{m_2'} \qquad Line3$$

where

$$m'_1 = \frac{\delta_1 \beta}{m'_3}$$

$$m'_2 = \frac{2\delta_2(1-c_2)}{m'_3}$$

$$m'_3 = \beta - 2(1-c_2)$$

$$m'_4 = \frac{A_1\beta - 2A_2(1-c_2)}{m'_3}$$

Then $q_1 > 0, q_2 > 0$

Case 2:

If

$$c_1 < \frac{2A_1(1-c_2) + 2\delta_1 Y_1(1-c_2) - A_2\beta - \delta_2 Y_2 \beta}{2 - 2c_2 - \beta} = c_1^{IBIS,NC,1}(c_2) \qquad (4.33)$$

and

$$c_1 > \frac{A_1\beta + \delta_1 Y_1 \beta - 2A_2 - 2\delta_2 Y_2 + 2A_2 c_2 + 2\delta_2 Y_2 c_2}{2c_2 - 2 + \beta} = c_1^{IBIS,NC,2}(c_2) \qquad (4.34)$$

$$Y_2 < \frac{m_4 + m_1 Y_1 - c_1}{m_2} \qquad Line2$$

$$Y_2 < \frac{m'_4 + m'_1 Y_1 - c_1}{m'_2} \qquad Line3$$

Then $q_1 > 0, q_2 = 0$

Case 3:

If

$$c_1 > \frac{2A_1(1-c_2) + 2\delta_1 Y_1(1-c_2) - A_2\beta - \delta_2 Y_2 \beta}{2 - 2c_2 - \beta} = c_1^{IBIS,NC,1}(c_2) \qquad (4.33)$$

and

$$c_1 \leq \frac{A_1\beta + \delta_1 Y_1 \beta - 2A_2 - 2\delta_2 Y_2 + 2A_2 c_2 + 2\delta_2 Y_2 c_2}{2c_2 - 2 + \beta} = c_1^{IBIS,NC,2}(c_2) \qquad (4.34)$$

$$Y_2 > \frac{m_4 + m_1 Y_1 - c_1}{m_2} \qquad Line2$$

$$Y_2 < \frac{m_4' + m_1' Y_1 - c_1}{m_2'} \qquad Line3$$

Multiple equilibria. Note that Case 3 is not possible under Case A-I-c.

Case II: If $A_1 + \delta_1 Y_1 < A_2 + \delta_2 Y_2$ or equivalently if

$$Y_2 > \frac{A_1 + \delta_1 Y_1 - A_2}{\delta_2} \qquad Line1$$

Case a:

$$c_1 > A_1 + \delta_1 Y_1$$

$$c_1 > A_2 + \delta_2 Y_2$$

Equivalently,

$$Y_1 < \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 < \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

Then $q_1 = 0, q_2 = 0$

Case b:

$$c_1 > A_1 + \delta_1 Y_1$$

$$c_1 < A_2 + \delta_2 Y_2$$

Equivalently,

$$Y_1 < \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 > \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

Then $q_1 = 0, q_2 > 0$

Case c:

$$c_1 < A_1 + \delta_1 Y_1$$

$$c_1 < A_2 + \delta_2 Y_2$$

Equivalently,

$$Y_1 > \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 > \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

<u>*Case 1:*</u>

If

$$c_1 < \frac{2A_2(1-c_2) + 2\delta_2 Y_2(1-c_2) - A_1\beta - \delta_1 Y_1 \beta}{2 - 2c_2 - \beta} \qquad 5.1$$

and

$$c_1 < \frac{A_2\beta + \delta_2 Y_2 \beta - 2A_1 - 2\delta_1 Y_1 + 2A_1 c_2 + 2\delta_1 Y_1 c_2}{2c_2 - 2 + \beta} \qquad 5.2$$

From Eq. 5.1

$$Y_2 > \frac{m_4' + m_1' Y_1 - c_1}{m_2'} \qquad Line3$$

From Eq. 5.2

$$Y_2 < \frac{m_4 + m_1 Y_1 - c_1}{m_2} \qquad Line2$$

Then $q_1 > 0, q_2 > 0$

Case 2:

$$Y_2 > \frac{m_4' + m_1' Y_1 - c_1}{m_2'} \qquad Line3$$

$$Y_2 > \frac{m_4 + m_1 Y_1 - c_1}{m_2} \qquad Line2$$

Then $q_1 = 0, q_2 > 0$

Case 3:

$$Y_2 < \frac{m_4' + m_1' Y_1 - c_1}{m_2'} \qquad Line3$$

$$Y_2 > \frac{m_4 + m_1 Y_1 - c_1}{m_2} \qquad Line2$$

Multiple equilibria. Note that under Case A-II-c Case 3 is not possible.

For a given c_1 and c_2, how the equilibrium quantities change is presented in the Figure 13 below.

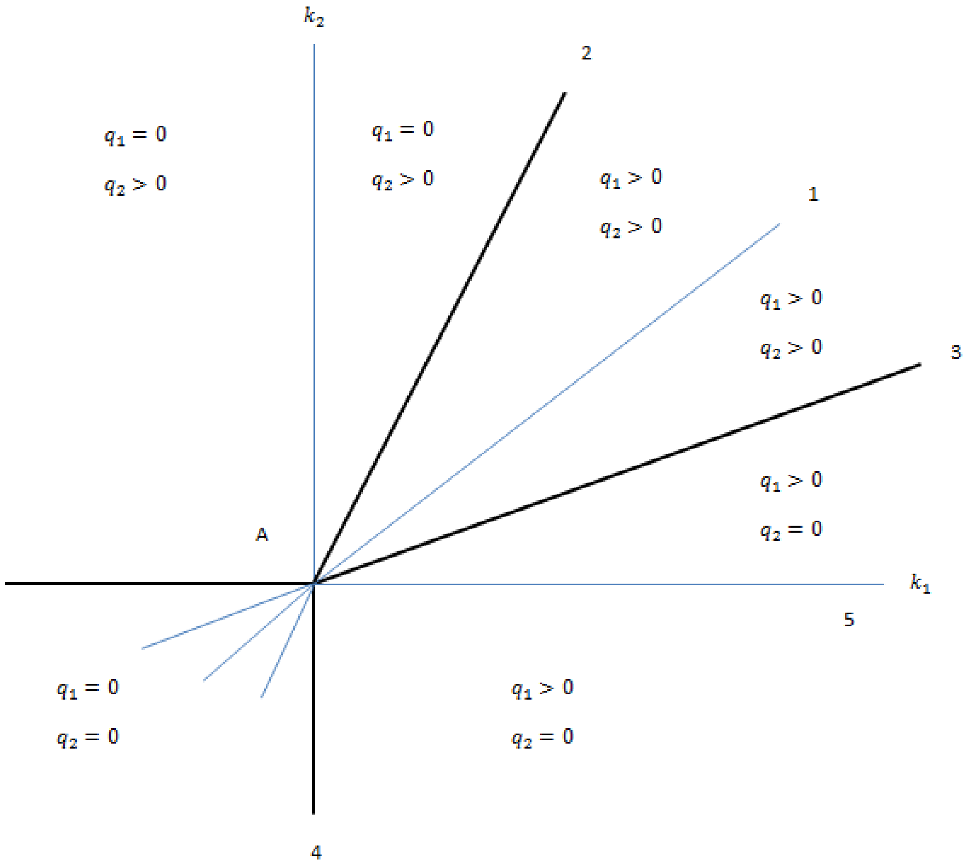

Figure 13 Equilibria under partitions of the (Y_1, Y_2) space under $\boldsymbol{c_2 < 1 - \beta/2}$

Case B: $c_2 > 1 - \beta/2$

Under this case the analysis follows similar lines with the analysis under $c_2 < 1 - \beta/2$. The equilibria under varying Y_1 and Y_2 values are presented in Figure 14.

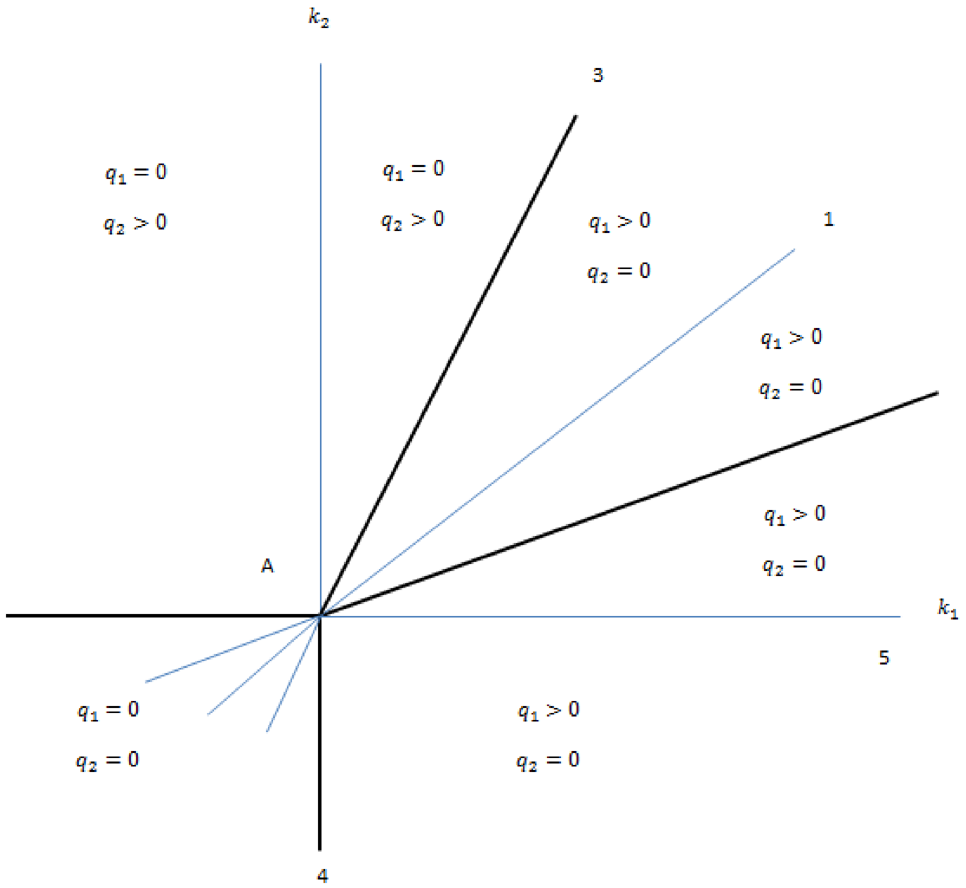

Figure 14 Equilibria under partitions of the $(\boldsymbol{Y_1}, \boldsymbol{Y_2})$ space under $\boldsymbol{c_2 > 1 - \beta/2}$

Let $E_{Y_1,Y_2}[\pi_s(c_1, c_2)]$ denote the ex-ante profit of the supplier, where the equilibrium quantities of the buyers are $q_1(c_1, c_2, Y_1, Y_2)$ and $q_2(c_1, c_2, Y_1, Y_2)$. For a given c_1 and c_2, $E_{Y_1,Y_2}[\pi_s(c_1, c_2)]$ is expressed below.

Case A: $c_2 < 1 - \beta/2$

$$
\begin{aligned}
E_{Y_1,Y_2}[\pi_s(c_1,c_2)] \\
&= \sum_{k_2<\frac{m_4'+m_1'k_1-c_1}{m_2'}} \sum_{k_1\geq c_1-A_1/\delta_1} \pi_s(c_1,c_2,q_1,0)p_1^{k_1}p_1^{k_2} \\
&+ \sum_{\frac{m_4'+m_1'k_1-c_1}{m_2'}\leq k_2<\frac{m_4+m_1k_1-c_1}{m_2}} \sum_{k_1\geq c_1-A_1/\delta_1} \pi_s(c_1,c_2,q_1,q_2)p_1^{k_1}p_1^{k_2} \\
&+ \sum_{k_2\geq\frac{m_4+m_1k_1-c_1}{m_2}} \sum_{k_1\geq c_1-A_1/\delta_1} \pi_s(c_1,c_2,0,q_2)p_1^{k_1}p_2^{k_2} \\
&+ \sum_{k_2\geq c_1-A_2/\delta_2} \sum_{k_1<c_1-A_1/\delta_1} \pi_s(c_1,c_2,0,q_2)p_1^{k_1}p_1^{k_2}
\end{aligned}
$$

Case B: $c_2 > 1-\beta/2$

$$
\begin{aligned}
E_{Y_1,Y_2}[\pi_s(c_1,c_2)] \\
&= \sum_{k_2<A_1+\delta_1k_1-A_2/\delta_2} \sum_{k_1\geq c_1-A_1/\delta_1} \pi_s(c_1,c_2,q_1,0)p_1^{k_1}p_1^{k_2} \\
&+ \sum_{A_1+\delta_1k_1-A_2/\delta_2\leq k_2} \sum_{k_1\geq c_1-A_1/\delta_1} \pi_s(c_1,c_2,0,q_2)p_1^{k_1}p_1^{k_2} \\
&+ \sum_{k_2\geq c_1-A_2/\delta_2} \sum_{k_1<c_1-A_1/\delta_1} \pi_s(c_1,c_2,0,q_2)p_1^{k_1}p_1^{k_2} \\
&+ \sum_{k_2<c_1-A_2/\delta_2} \sum_{k_1<c_1-A_1/\delta_1} \pi_s(c_1,c_2,0,0)p_1^{k_1}p_1^{k_2}
\end{aligned}
$$

5.3 Determining the ex-ante profits

Under IBNS case, the ex-ante profits for the buyers are the as IBIS case. For the supplier, under IBNS, ex-ante and ex-post profits do not differ. Since under equilibrium, quantities q_1 and q_2 are a function of c_1, c_2, Y_1 and Y_2, expectation is taken over Y_1 and Y_2. Then optimal c_1 and c_2 is seeked to maximize $E[\pi_s]$.

Ex-ante profit of the supplier

We are interested in $E_{Y_1,Y_2}[\pi_s]$

Note that Y_1 and Y_2 are independent random variables. Then, for a given k_1 and k_2, ex-ante profits are determined as follows;

1. Let $A_1' = \max\{A_1 + \delta_1 k_1, A_2 + \delta_2 k_2\}$ and define A_2' accordingly.
2. Follow the analysis in Section 5.1.2 for collaboration and in Section 5.2.2 for no-collaboration setting. Use the equilibrium quantities defined in Section 5.1.1 and Section 5.2.1.
3. For a given c_1 and c_2, the supplier determines its profit for every possible k_1 and k_2 values and expectation is taken over Y_1 and Y_2 . Possible values for c_1 is in the range $[0, \max\{A_1 + \delta_1 \max(Y_1), A_2 + \delta_2 \max(Y_2)\}]$ and for c_2 in the range of $[0, \beta - \varepsilon]$. Then, c_1 and c_2 values which maximize the expected profit are set as the optimal c_1 and c_2. The corresponding profit is the ex-ante profit for the supplier.
4. After determining $c_1^*(k_1, k_2)$ and $c_2^*(k_1, k_2)$ use $q_1(k_1, k_2)$, $q_2(k_1, k_2)$, $c_1^*(k_1, k_2)$, $c_2^*(k_1, k_2)$ to obtain $\pi_i(k_1, k_2)$ for the corresponding realization of Y_1 and Y_2 for the buyers. Ex-ante profits for the buyers are found as in IBIS case.

CHAPTER 6

NO INFORMATION FOR THE BUYERS AND NO INFORMATION FOR THE SUPPLIER (NBNS)

Under the strategy NBNS, the buyers share their signals neither with each other buyer nor with the supplier. The equilibrium points and the supplier's optimal c_1 and c_2 values are determined. The analysis is made under collaborating and non-collaborating buyers. In the analysis, first, the equilibrium quantities are determined, and then the optimum wholesale price is calculated. The assumptions done during the analysis are same as previous section. The buyer with higher A_i is called buyer 1. Note that buyer 1 of Chapter 4 and Chapter 6 might be different.

In this chapter we use the following notation:

Table 2: Notation for Chapter 6

6.1 Collaborating Buyers	6.2 Non-collaborating Buyers
$J_1 = \dfrac{A_1 - c_1}{\beta - c_2}$	$K_1 = \dfrac{A_1 - c_1}{\beta}$
$J_2 = \dfrac{A_2 - c_1}{\beta - c_2}$	$K_2 = \dfrac{A_2 - c_1}{\beta}$
$J_1' = \dfrac{A_1 - c_1 + \delta_1 Y_1}{\beta - c_2}$	$K_1' = \dfrac{A_1 + \delta_1 Y_1 - c_1}{\beta}$

$$J_2' = \frac{A_2 - c_1 + \delta_2 Y_2}{\beta - c_2} \qquad K_2' = \frac{A_2 + \delta_2 Y_2 - c_1}{\beta}$$

$$JJ1 = \frac{A_1 - c_1}{2 - 2c_2} \qquad JJ_1 = KK_1 = \frac{A_1 - c_1}{2 - 2c_2}$$

$$JJ2 = \frac{A_2 - c_1}{2 - 2c_2} \qquad JJ_2 = KK_2 = \frac{A_2 - c_1}{2 - 2c_2}$$

6.1 Collaborating buyers

The profit function of the buyers and the supplier are expressed as follows;

$$E_{\theta,Y_2}[\pi_1|Y_1] = E[(A_1 + \theta_1 - q_1 - \beta q_2 - c_1 + c_2(q_1 + q_2))q_1|Y_1]$$

$$= (A_1 + \delta_1(\alpha_1, \sigma_1)Y_1 - q_1 - \beta E_{Y_2}[q_2] - c_1 + c_2(q_1 + E_{Y_2}[q_2]))q_1$$

$$E_{\theta,Y_1}[\pi_2|Y_2] = E[(A_2 + \theta_2 - q_2 - \beta q_1 - c_1 + c_2(q_1 + q_2))q_2|Y_2]$$

$$= (A_2 + \delta_2(\alpha_2, \sigma_2)Y_2 - q_2 - \beta E_{Y_1}[q_1] - c_1 + c_2(q_2 + E_{Y_1}[q_1]))q_2$$

$$E_{\theta,Y_1,Y_2}[\pi_s] = E_{\theta,Y_1,Y_2}[(c_1 - c_2(q_1 + q_2))(q_1 + q_2)]$$
$$= c_1 E_{\theta,Y_1,Y_2}[q_1 + q_2] - c_2 E_{\theta,Y_1,Y_2}[(q_1 + q_2)^2]$$

Here, $E_{\theta,Y_2}[\pi_1|Y_1]$ and $E_{\theta,Y_1}[\pi_2|Y_2]$ denote the ex-post expected profits for the buyers. In the expression when q_1 and q_2 correspond to equilibrium quantities, q_1 is a function of c_1, c_2 and Y_1 and q_2 is a function of c_1, c_2 and Y_2. When finding expected profit, $E[\pi_i|Y_i]$, buyer *i* takes expected quantity of buyer *j* to evaluate the profit value. This is due to the fact that Y_j is not shared with buyer *i*. Supplier, on the other hand, knows neither Y_1 nor Y_2. Thus, to evaluate $E[\pi_s]$ for a given c_1 and c_2, the expected equilibrium quantities are considered and expectation taken over all possible Y_1 and Y_2 values.

In the following we discuss how the equilibrium quantities for the buyers and optimal c_1 and c_2 values for the supplier are determined.

6.1.1 The Buyers' Problem

In this section, for a given value of c_1 and c_2 values the equilibrium quantities of the buyers are determined. In this chapter, without loss of generality we call the buyer with higher A_i as buyer 1.

Property 1: Under NBNS setting with two collaborating buyers, for a given c_1 and c_2, $c_1 < A_2$, the Bayesian Nash equilibria are obtained as follows:

i) If $\frac{A_2-c_1+\delta_2 Y_2}{\beta-c_2} < \frac{A_1-c_1}{2-2c_2}$ then

$$q_1 = \frac{A_1 - c_1}{2-2c_2} + \frac{1}{2-2c_2}[\delta_1]Y_1$$

$$q_2 = 0$$

ii) If $\frac{A_2-c_1+\delta_2 Y_2}{\beta-c_2} > \frac{A_1-c_1}{2-2c_2}$ and $\frac{A_1-c_1+\delta_1 Y_1}{\beta-c_2} < \frac{A_2-c_1}{2-2c_2}$ then

$$q_1 = 0$$

$$q_2 = \frac{A_2 - c_1}{2-2c_2} + \frac{1}{2-2c_2}[\delta_2]Y_2$$

iii) If $\frac{A_2-c_1+\delta_2 Y_2}{\beta-c_2} > \frac{A_1-c_1}{2-2c_2}$ and $\frac{A_1-c_1+\delta_1 Y_1}{\beta-c_2} > \frac{A_2-c_1}{2-2c_2}$ then

$$q_1 = \frac{2A_1(1-c_2) - 2c_1 + \beta c_1 + c_1 c_2 - A_2(\beta - c_2)}{(2-2c_2)^2 - (\beta - c_2)^2} + \frac{1}{2-2c_2}[\delta_1]Y_1$$

$$q_2 = \frac{2A_2(1-c_2) - 2c_1 + \beta c_1 + c_1 c_2 - A_1(\beta - c_2)}{(2-2c_2)^2 - (\beta - c_2)^2} + \frac{1}{2-2c_2}[\delta_2]Y_2$$

Proof: To find the best response of buyer i, first the derivative of profit function, π_i, is taken with respect to q_i.

For buyer 1

$$\frac{\partial E_{\theta,Y_2}[\pi_1|Y_1]}{\partial q_1} = A_1 + \delta_1(\alpha_1,\sigma_1)Y_1 - 2q_1 - \beta E[q_2|Y_1] - c_1 + 2c_2q_1 + c_2E[q_2|Y_1]$$
$$= 0$$

$$q_1(q_2) = \left(\frac{A_1 + \delta_1(\alpha_1,\sigma_1)Y_1 - c_1 - \beta E[q_2|Y_1] + c_2E[q_2|Y_1]}{2 - 2c_2}\right)^+$$
$$= \left(\frac{A_1' - c_1}{2 - 2c_2} - \frac{E[q_2|Y_1](\beta - c_2)}{2 - 2c_2}\right)^+ \qquad (6.1)$$

$$\frac{\partial E_{\theta,Y_1}[\pi_2|Y_2]}{\partial q_2} = A_2 + \delta_2(\alpha_2,\sigma_2)Y_2 - 2q_2 - \beta E[q_1|Y_2] - c_1 + 2c_2q_2 + c_2E[q_1|Y_2]$$
$$= 0$$

$$q_2(q_1) = \left(\frac{A_2 + \delta_2(\alpha_2,\sigma_2)Y_2 - c_1 - \beta E[q_1|Y_2] + c_2E[q_1|Y_2]}{2 - 2c_2}\right)^+$$
$$= \left(\frac{A_2' - c_1}{2 - 2c_2} - \frac{E[q_1|Y_2](\beta - c_2)}{2 - 2c_2}\right)^+ \qquad (6.2)$$

If q_1 and q_2 were unrestricted in sign, then it is known that at equilibrium q_1 and q_2 are in the following forms:

$$q_1 = D_0^1 + D_1^1 Y_1$$

$$q_2 = D_0^2 + D_1^2 Y_2$$

Even though q_1 and q_2 are restricted to take (+) values, and expressed as in (6.1) and (6.2), we still assume they take the form $q_i = D_0^i + D_1^i Y_i$.

Then, Eqns. 6.1 and 6.2 can be rewritten as

$$q_1(q_2) = \left(\frac{A_1 + \delta_1(\alpha_1,\sigma_1)Y_1 - c_1 - \beta E\left[D_0^2 + D_1^2 Y_2|Y_1\right] + c_2E\left[D_0^2 + D_1^2 Y_2|Y_1\right]}{2 - 2c_2}\right)^+$$

Note $E[Y_2|Y_1] = 0$

Suppose $q_1(q_2)$ does not necessarily take (+) values. Then,

$$q_1(q_2) = D_0^1 + D_1^1 Y_1 = \frac{1}{2-2c_2}\left[A_1 + \delta_1(\alpha_1,\sigma_1)Y_1 - c_1 - \beta\left(D_0^2\right) + c_2\left(D_0^2\right)\right]$$

Then,

$$D_0^1 = \frac{1}{2-2c_2}\left[A_1 - c_1 - \beta D_0^2 + c_2 D_0^2\right] \qquad (6.3)$$

$$D_1^1 = \frac{1}{2-2c_2}[\delta_1(\alpha_1,\sigma_1)] \qquad (6.4)$$

Similarly,

$$q_2(q_1) = \frac{A_2 + \delta_2(\alpha_2,\sigma_2)Y_2 - c_1 - \beta E\left[D_0^1 + D_1^1 Y_1|Y_2\right] + c_2 E[D_0^1 + D_1^1 Y_1|Y_2]}{2-2c_2}$$

Since, $E[Y_1|Y_2] = 0$

$$q_2(q_1) = D_0^2 + D_1^2 Y_2 = \frac{1}{2-2c_2}\left[A_2 + \delta_2(\alpha_2,\sigma_2)Y_2 - c_1 - \beta\left(D_0^1\right) + c_2\left(D_0^1\right)\right]$$

$$D_0^2 = \frac{1}{2-2c_2}\left[A_2 - c_1 - \beta D_0^1 + c_2 D_0^1\right] \qquad (6.5)$$

$$D_1^2 = \frac{1}{2-2c_2}[\delta_2(\alpha_2,\sigma_2)] \qquad (6.6)$$

Note that in $q_i(q_j)$ coefficient of Y_i is independent of the action taken by other player.

This implies one could equivalently analyze best response functions in terms of D_0^i

Considering that $q_i(q_j)$ must always be non-negative, best response of D_0^i and D_1^i can be expressed as follows.

$D_0^1(D_0^2) = \frac{A_1 - c_1 - (\beta - c_2)D_0^2}{2-2c_2}$ if $D_0^2 < \frac{A_1 - c_1 + \delta_1 Y_1}{\beta - c_2}$, and 0 otherwise.

$D_1^1(D_0^2) = \frac{\delta_1}{2-2c_2}$ if $D_0^2 < \frac{A_1 - c_1 + \delta_1 Y_1}{\beta - c_2}$, and 0 otherwise.

Similarly, for buyer 2 best response function for D_0^2 can be expressed as

$D_0^2(D_0^1) = \frac{A_2 - c_1 - (\beta - c_2)D_0^1}{2 - 2c_2}$ if $D_0^1 < \frac{A_2 - c_1 + \delta_2 Y_2}{\beta - c_2}$, and 0 otherwise.

$D_1^2(D_0^1) = \frac{\delta_2}{2 - 2c_2}$ if $D_0^1 < \frac{A_2 - c_1 + \delta_2 Y_2}{\beta - c_2}$, and 0 otherwise.

Note that even if $D_0^i < 0$, $q_i(q_j)$ could be (+). In other words D_0^i may possibly take (-) values. When best response functions are expressed in terms of D_0^1 and D_0^2 discontinuity may exist, which may result in non-existence of pure strategy equilibrium.

Best response functions and equilibrium points may change with the parameter values and the signal Y_1 and Y_2. In the analysis, if equilibrium quantities q_1 and q_2 are assumed unrestricted in sign, then they can be obtained solving (6.3) and (6.5) simultaneously. However, q_1 and q_2 cannot take negative values. In the following analysis, we assume that the buyers make use of the signal of the other buyer to determine whether q_1, q_2 will be zero or (+). In other words, when determining q_1 and q_2, the signal of the other buyer is ignored, however that information is assumed to be still used to determine whether qi is (+) or not. This assumption is made to keep compatibility with the non-negative equilibrium quantities of IBIS analysis.

If $c_1 < A_2$, by definition, it holds that $J1 > J2 > JJ2$, $JJ1 > JJ2$. Depending on the values of J_1, J_2, $J_1^{'}$, $J_2^{'}$, $JJ1$, $JJ2$ and Y_1 and Y_2, 18 possible cases may exist, as discussed below. We first present a few examples to show how equilibrium might shift from a parameter setting to the next. Then we present the 18 conditions, and the corresponding cases (equilibria).

Example 1. (Case 1) Suppose $Y_1 > 0, Y_2 > 0$. Then, $J_1^{'} > J1$ and $J_2^{'} > J2$. Suppose furthermore that $JJ1 > JJ2$, $J_2^{'} > JJ1$ and $J_1^{'} > JJ2$. Under this setting the best response function D_0^1 and D_0^2 are shown in Figure 15.

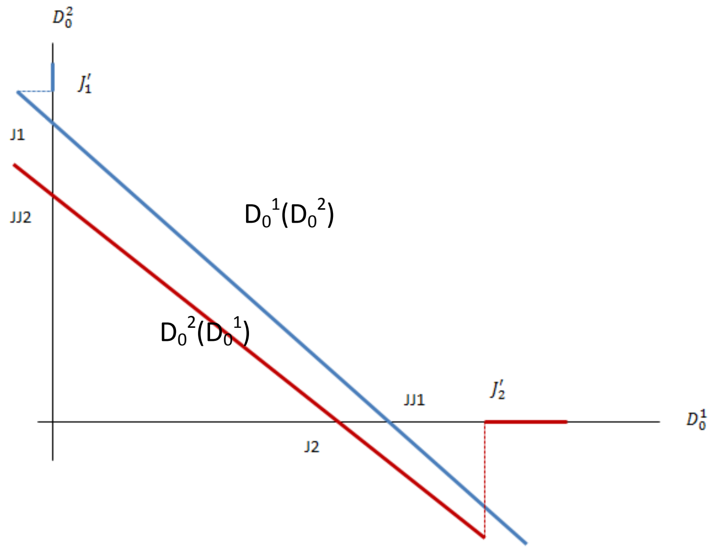

Figure 15 Case 1.

In this setting, D_0^1 and D_0^2 may intersect either at $D_0^1 < J_2'$ or for $D_0^1 > J_2'$. For this case, intersection point is assumed to be the equilibrium point even if it is negative.

If Eqns. 6.3 and 6.5 are solved simultaneously

$$D_0^1 = \frac{2A_1(1-c_2) - 2c_1 + \beta c_1 + c_1 c_2 - A_2(\beta - c_2)}{(2-2c_2)^2 - (\beta - c_2)^2} \qquad (6.7)$$

$$D_0^2 = \frac{2A_2(1-c_2) - 2c_1 + \beta c_1 + c_1 c_2 - A_1(\beta - c_2)}{(2-2c_2)^2 - (\beta - c_2)^2} \qquad (6.8)$$

Equilibrium quantities are:

$$q_1 = D_0^1 + D_1^1 Y_1$$

$$q_2 = D_0^2 + D_1^2 Y_2$$

Example 2. (Case 2) Suppose $Y_1 > 0$, $Y_2 > 0$, $JJ1 > J2$, $J_2' < JJ1$ and $J_1' > JJ2$. Under this setting the best response functions D_0^1 and D_0^2 are shown in Figure 16.

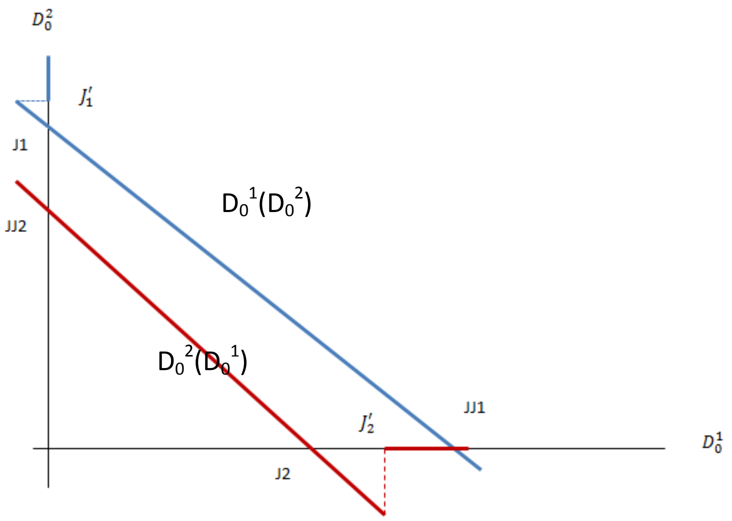

Figure 16 Case 2

Under Case 2, D_0^1 and D_0^2 intersect at the point where $D_0^1 = JJ1$ and $D_0^2 = 0$ (see Figure 16). Equilibrium quantities are:

$$q_1 = JJ1 + D_1^1 Y_1$$

$$q_2 = 0$$

Example 3. (Case 1') Suppose $Y_1 > 0$, $Y_2 > 0$, $JJ1 < J2$, $J_2' > JJ1$ and $J_1' > JJ2$. Under this setting the best response functions D_0^1 and D_0^2 are shown in Figure 17.

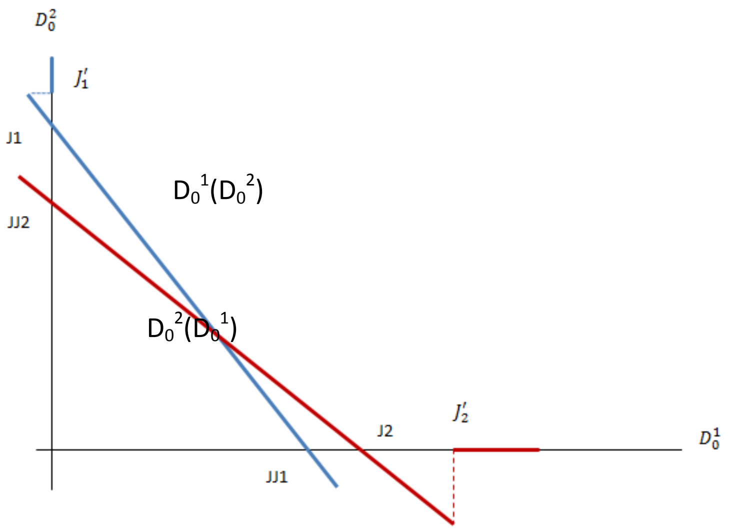

Figure 17 Case 1'

Solving Eqns. 6.3 and 6.5 are solved simultaneously, D_0^1 and D_0^2 are obtained as:

$$D_0^1 = \frac{2A_1(1-c_2) - 2c_1 + \beta c_1 + c_1 c_2 - A_2(\beta - c_2)}{(2-2c_2)^2 - (\beta - c_2)^2}$$

$$D_0^2 = \frac{2A_2(1-c_2) - 2c_1 + \beta c_1 + c_1 c_2 - A_1(\beta - c_2)}{(2-2c_2)^2 - (\beta - c_2)^2}$$

Equilibrium quantities are:

$$q_1 = D_0^1 + D_1^1 Y_1$$

$$q_2 = D_0^2 + D_1^2 Y_2$$

Depending on the parameter values, 18 possible settings are identified. And for each of the setting the corresponding equilibria are defined. Assumption of $A_1 > A_2$ implies $J1 > J2 > JJ2, \quad JJ1 > JJ2$.

Setting A. If $Y_1 > 0$ and $Y_2 > 0$ then $J_1' > J1, \quad J_2' > J2$.

1) $JJ1 > J2$
   a) $J_2' > JJ1$
      i) $J_1' > JJ2$ → Case 1

ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$

b) $J_2' \leq JJ1$

i) $J_1' > JJ2$ → Case 2

ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$

2) $JJ1 < J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 1'

ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$

b) $J_2' < JJ1$ → does not exist since $J_2' > J2 > JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → does not exist

Setting B. If $Y_1 > 0$ and $Y_2 < 0$ then $J_1' > J1$, $J_2' < J2$.

1) $JJ1 > J2$

a) $J_2' > JJ1$ → does not exist since $J_2' < J2 < JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → does not exist

b) $J_2' \leq JJ1$

i) $J_1' > JJ2$ → Case 3

ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$

2) $JJ1 < J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 2'

ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$

b) $J_2' \leq JJ1$

i) $J_1' > JJ2$ → Case 3'

ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$

Setting C. If $Y_1 < 0$ and $Y_2 > 0$ then $J_1' < J1$, $J_2' > J2$.

1) $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 4

ii) $J_1' \leq JJ2$ → Case 5

b) $J_2' \leq JJ1$

i) $J_1' > JJ2$ → Case 6

ii) $J_1' \leq JJ2$ → Case 7

2) $JJ1 < J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 4'

ii) $J_1' \leq JJ2$ → Case 5'

b) $J_2' < JJ1$ does not exist since $J_2' > J2 > JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → does not exist

Setting D. If $Y_1 < 0$ and $Y_2 < 0$ then $J_1' < J1$, $J_2' < J2$.

1. $JJ1 > J2$

a) $J_2' > JJ1$ does not exist since $J_2' < J2 < JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → does not exist

b) $J_2' \leq JJ1$

i) $J_1' > JJ2$ → Case 8

ii) $J_1' \leq JJ2$ → Case 9

2. $JJ1 < J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 6'

ii) $J_1' \leq JJ2$ → Case 7'

b) $J_2' \leq JJ1$

i) $J_1' > JJ2$ → Case 8'

ii) $J_1' \leq JJ2$ → Case 9'

Setting E. If $Y_1 > 0$ and $Y_2 = 0$ then $J_1' > J1, \quad J_2' = J2$.

1) $JJ1 > J2$
   a) $J_2' > JJ1$ → not possible
      i) $J_1' > JJ2$
      ii) $J_1' < JJ2$
   b) $J_2' \leq JJ1$
      i) $J_1' > JJ2$ → Case 2
      ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$
2) $JJ1 < J2$
   a) $J_2' > JJ1$
      i) $J_1' > JJ2$ → Case 1'
      ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$
   b) $J_2' < JJ1$ → does not exist since $J_2' = J2 > JJ1$
      i) $J_1' > JJ2$ → does not exist
      ii) $J_1' < JJ2$ → does not exist

Setting F. If $Y_1 < 0$ and $Y_2 = 0$ then $J_1' < J1, \quad J_2' = J2$.

1) $JJ1 > J2$
   a) $J_2' > JJ1$ → not possible
      i) $J_1' > JJ2$
      ii) $J_1' < JJ2$
   b) $J_2' \leq JJ1$
      i) $J_1' > JJ2$ → Case 6
      ii) $J_1' \leq JJ2$ → Case 7
2) $JJ1 < J2$
   a) $J_2' > JJ1$
      i) $J_1' > JJ2$ → Case 4'
      ii) $J_1' \leq JJ2$ → Case 5'
   b) $J_2' \leq JJ1$ does not exist since $J_2' = J2 > JJ1$
      i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → does not exist

Setting G. If $Y_1 = 0$ and $Y_2 > 0$ then $J_1' = J1, \quad J_2' > J2.$

1) $JJ1 > J2$
   a) $J_2' > JJ1$
      i) $J_1' > JJ2$ → Case 1
      ii) $J_1' < JJ2$ → does not exist since $J_1' = J1 > JJ2$
   b) $J_2' \leq JJ1$
      i) $J_1' > JJ2$ → Case 2
      ii) $J_1' < JJ2$ → does not exist since $J_1' = J1 > JJ2$
2) $JJ1 < J2$
   a) $J_2' > JJ1$
      i) $J_1' > JJ2$ → Case 1'
      ii) $J_1' < JJ2$ → does not exist since $J_1' = J1 > JJ2$
   b) $J_2' < JJ1$ → does not exist since $J_2' > J2 > JJ1$
      i) $J_1' > JJ2$ → does not exist
      ii) $J_1' < JJ2$ → does not exist

Setting H. If $Y_1 = 0$ and $Y_2 < 0$ then $J_1' = J1, \quad J_2' < J2.$

1) $JJ1 > J2$
   a) $J_2' > JJ1$ → does not exist since $J_2' < J2 < JJ1$
      i) $J_1' > JJ2$ → does not exist
      ii) $J_1' < JJ2$ → does not exist
   b) $J_2' \leq JJ1$
      i) $J_1' > JJ2$ → Case 3
      ii) $J_1' < JJ2$ → does not exist since $J_1' = J1 > JJ2$
2) $JJ1 < J2$
   a) $J_2' > JJ1$
      i) $J_1' > JJ2$ → Case 2'
      ii) $J_1' < JJ2$ → does not exist since $J_1' = J1 > JJ2$

b) $J_2' \le JJ1$

i) $J_1' > JJ2$ → Case 3'

ii) $J_1' < JJ2$ → does not exist since $J_1' = J1 > JJ2$

Setting I. If $Y_1 = 0$ and $Y_2 = 0$ then $J_1' = J1$, $J_2' = J2$.

1) $JJ1 > J2$

a) $J_2' > JJ1$ → not possible

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' \le JJ1$

i) $J_1' > JJ2$ → Case 2

ii) $J_1' < JJ2$ → does not exist since $J_1' = J1 > JJ2$

2) $JJ1 < J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 1'

ii) $J_1' < JJ2$ → does not exist since $J_1' = J1 > JJ2$

b) $J_2' < JJ1$ → does not exist since $J_2' = J2 > JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → does not exist

The equilibrium quantities for each case are as follows. Whenever $q_i = D_0^i + D_1^i Y_i$, D_0^i is obtained by intersecting Eqns. 6.3 and 6.5.

Cases	**Equilibrium quantities**
1-4-1'-2'-4'-6'	$q_1 = D_0^1 + D_1^1 Y_1$ $q_2 = D_0^2 + D_1^2 Y_2$
2-3-6-7-8-9-3'-8'-9'	$q_1 = J1 + D_1^1 Y_1$ $q_2 = 0$
5-5'-7'	$q_1 = 0$ $q_2 = JJ2 + D_1^2 Y_2$

Note that it is possible to categorize all possible cases into three major cases. There are cases where equilibrium quantities are obtained by solving Eqns 6.3 and Eq 6.5 (if $J_2' > JJ1$ and $J_1' > JJ2$), cases where $q_1 > 0, q_2 = 0$ (if $J_2' < JJ1$), and the cases where $q_1 = 0, q_2 > 0$ (if $J_2' > JJ1$ and $J_1' < JJ2$). Thus, Property 1 follows. ■

The analysis for $A_2 < c_1 < A_1$ and $c_1 > A_1$ is given in Appendix.

Using the equilibrium quantities, next the supplier's problem is addressed.

6.1.2 The supplier's Problem

Under NBNS neither the supplier have the knowledge of demand signals Y_1 and Y_2 nor the buyers have the knowledge of other buyer's signal. In Property 2, for a given c_1 and c_2 we express the supplier's profit function. To determine the optimal c_1 and c_2 values that maximize the supplier's profit function, an exhaustive search over possible values of c_1 and c_2 should be made. Possible values for c_1 is in the range $[0, \max\{A_1 + \delta_1 \max(Y_1), A_2 + \delta_2 \max(Y_1)\}]$ and for c_2 in the range of $[0, \beta - \varepsilon]$.

Property 2: Assuming that $c_1 < A_2$, for a given c_1 and c_2, the supplier's profit function is expressed as below:

(i) If $\frac{A_1 - c_1}{2 - 2c_2} > \frac{A_2 - c_1}{\beta - c_2}$ (i.e., if JJ1>J2)

$$E_{Y_1,Y_2}[\pi_s(c_1,c_2)]$$

$$= \sum_{\substack{k_2\epsilon S_2\\ k_2>Line2}} \sum_{\substack{k_1\epsilon S_1\\ k_1\geq 0}} \pi_s(Case1(k_1,k_2))p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2\\ 0\leq k_2\leq Line2}} \sum_{\substack{k_1\epsilon S_1\\ k_1\geq 0}} \pi_s(Case2(k_1,k_2))p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2\\ k_2<0}} \sum_{\substack{k_1\epsilon S_1\\ k_1\geq 0}} \pi_s(Case3(k_1,k_2))p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2\\ k_2>Line2}} \sum_{\substack{k_1\epsilon S_1\\ Line1<k_1<0}} \pi_s(Case4(k_1,k_2))p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2\\ 0\leq k_2\leq Line2}} \sum_{\substack{k_1\epsilon S_1\\ Line1<k_1<0}} \pi_s(Case6(k_1,k_2))p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2\\ k_2<0}} \sum_{\substack{k_1\epsilon S_1\\ Line1<k_1<0}} \pi_s(Case8(k_1,k_2))p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2\\ k_2>Line2}} \sum_{\substack{k_1\epsilon S_1\\ k_1\leq Line1}} \pi_s(Case5(k_1,k_2))p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2\\ 0\leq k_2\leq Line2}} \sum_{\substack{k_1\epsilon S_1\\ k_1<Line1}} \pi_s(Case7(k_1,k_2))p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2\\ k_2<0}} \sum_{\substack{k_1\epsilon S_1\\ k_1\leq Line1}} \pi_s(Case9(k_1,k_2))p_1^{k_1}p_2^{k_2}$$

ii) If $\frac{A_1-c_1}{2-2c_2} < \frac{A_2-c_1}{\beta-c_2}$ (i.e., if JJ1<J2)

$$
\begin{aligned}
&E_{Y_1,Y_2}[\pi_s(c_1,c_2)] \\
&\quad = \sum_{\substack{k_2\epsilon S_2\\ k_2\geq 0}}\sum_{\substack{k_1\epsilon S_1\\ k_1\geq 0}} \pi_s(Case1'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\quad + \sum_{\substack{k_2\epsilon S_2\\ Line2<k_2<0}}\sum_{\substack{k_1\epsilon S_1\\ k_1\geq 0}} \pi_s(Case2'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\quad + \sum_{\substack{k_2\epsilon S_2\\ k_2\leq Line2}}\sum_{\substack{k_1\epsilon S_1\\ k_1\geq\geq 0}} \pi_s(Case3'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\quad + \sum_{\substack{k_2\epsilon S_2\\ k_2\geq 0}}\sum_{\substack{k_1\epsilon S_1\\ Line1<k_1<0}} \pi_s(Case4'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\quad + \sum_{\substack{k_2\epsilon S_2\\ Line2<k_2<0}}\sum_{\substack{k_1\epsilon S_1\\ Line1<k_1<0}} \pi_s(Case6'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\quad + \sum_{\substack{k_2\epsilon S_2\\ k_2\leq Line2}}\sum_{\substack{k_1\epsilon S_1\\ Line1\leq k_1<0}} \pi_s(Case8'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\quad + \sum_{\substack{k_2\epsilon S_2\\ k_2\geq 0}}\sum_{\substack{k_1\epsilon S_1\\ k_1\leq Line1}} \pi_s(Case5'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\quad + \sum_{\substack{k_2\epsilon S_2\\ Line2<k_2<0}}\sum_{\substack{k_1\epsilon S_1\\ k_1\leq Line1}} \pi_s(Case7'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\quad + \sum_{\substack{k_2\epsilon S_2\\ k_2\leq Line2}}\sum_{\substack{k_1\epsilon S_1\\ k_1<Line1}} \pi_s(Case9'(k_1,k_2))p_1^{k_1}p_2^{k_2}
\end{aligned}
$$

where k_1 and k_2 denote possible realizations of Y_1 and Y_2, $p_i^{k_i}$ is the probability that k_i value is realized (as derived in Chapter 3) and $Line1 = \frac{(\beta-c_2)(A_2-c_1)-(2-2c_2)(A_1-c_1)}{(2-2c_2)\delta_1}$ and $Line2 = \frac{(\beta-c_2)(A_1-c_1)-(2-2c_2)(A_2-c_1)}{(2-2c_2)\delta_2}$.

Proof. Similar to IBNS case, for a given c_1 and c_2, the profit function of the supplier can be written under changing Y_1 and Y_2. When partitioning the space considering the values that Y_1 and Y_2 can take, the separating lines are defined next to the corresponding inequalities. The lines are shown in Figure 18 and 19 below. Given $c_1 < A_2$,

$J_1' > JJ2$ implies

$$Y_1 > \frac{(\beta - c_2)(A_2 - c_1) - (2 - 2c_2)(A_1 - c_1)}{(2 - 2c_2)\delta_1} \qquad Line1$$

Note for $c_1 < A_2$ it always holds that $Line1 < 0$ because

$$(\beta - c_2)(A_2 - c_1) - (2 - 2c_2)(A_1 - c_1) < 0$$

$$(\beta - c_2) < (2 - 2c_2)$$

$J_2' > JJ1$ implies

$$Y_2 > \frac{(\beta - c_2)(A_1 - c_1) - (2 - 2c_2)(A_2 - c_1)}{(2 - 2c_2)\delta_2} \qquad Line2$$

If $JJ1 > J2$, then $Line2 > 0$ since

$$\frac{A_1 - c_1}{2 - 2c_2} > \frac{A_2 - c_1}{\beta - c_2}$$

If $JJ1 < J2$, then $Line2 < 0$ since

$$\frac{A_1 - c_1}{2 - 2c_2} < \frac{A_2 - c_1}{\beta - c_2}$$

If $JJ1 > J2$ then the sample space can be partitioned as in Figure 18 below.

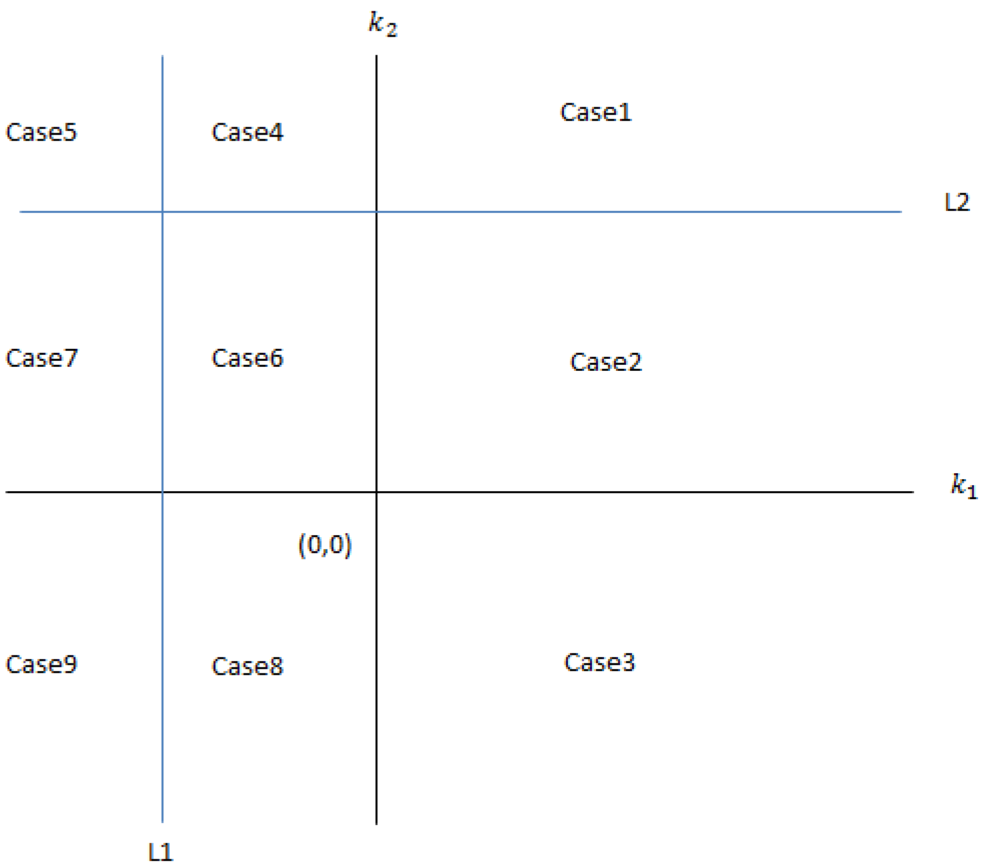

Figure 18 How equilibrium cases change as $\boldsymbol{Y_1}$ and $\boldsymbol{Y_2}$ realizations change, given $\boldsymbol{c_1 < A_2}$ and JJ1>J2

Let $E_{Y_1,Y_2}[\pi_s(c_1,c_2)]$ denote the profit of the supplier, where the equilibrium quantities of the buyers are $q_1(c_1,c_2,Y_1)$ and $q_2(c_1,c_2,Y_2)$. Note that ex-ante and ex-post profits are the same for the supplier. For a given c_1 and c_2, $E_{Y_1,Y_2}[\pi_s(c_1,c_2)]$ is obtained as;

$$E_{Y_1,Y_2}[\pi_s(c_1,c_2)]$$

$$= \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s(Case1(k_1,k_2)) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ 0 \leq k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s(Case2(k_1,k_2)) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s(Case3(k_1,k_2)) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s(Case4(k_1,k_2)) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ 0 \leq k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s(Case6(k_1,k_2)) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s(Case8(k_1,k_2)) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \leq Line1}} \pi_s(Case5(k_1,k_2)) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ 0 \leq k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 < Line1}} \pi_s(Case7(k_1,k_2)) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \leq Line1}} \pi_s(Case9(k_1,k_2)) p_1^{k_1} p_2^{k_2}$$

If $JJ1 < J2$ then the sample space can be partitioned as in Figure 19 below.

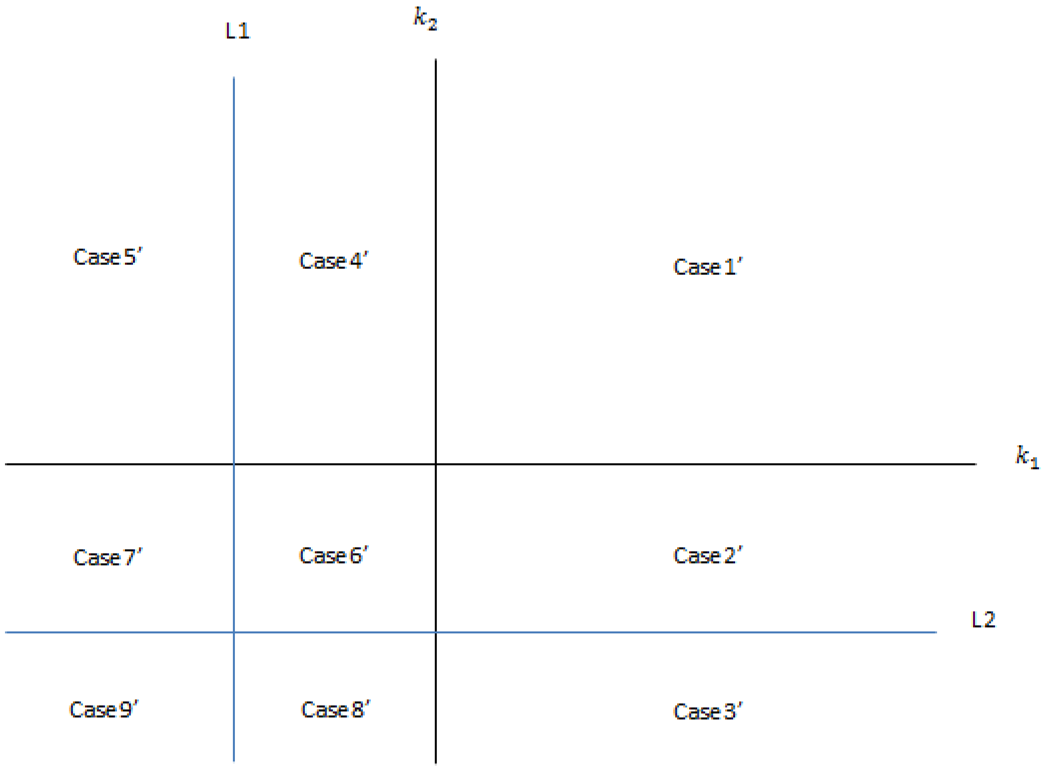

Figure 19 How equilibrium cases change as $\boldsymbol{Y_1}$ and $\boldsymbol{Y_2}$ realizations change, given $\boldsymbol{c_1 < A_2}$ and JJ1<J2.

$$
\begin{aligned}
&E_{Y_1,Y_2}[\pi_s(c_1,c_2)] \\
&\qquad = \sum_{\substack{k_2\epsilon S_2\\ k_2\geq 0}} \sum_{\substack{k_1\epsilon S_1\\ k_1\geq 0}} \pi_s(Case1'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\qquad + \sum_{\substack{k_2\epsilon S_2\\ Line2<k_2<0}} \sum_{\substack{k_1\epsilon S_1\\ k_1\geq 0}} \pi_s(Case2'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\qquad + \sum_{\substack{k_2\epsilon S_2\\ k_2\leq Line2}} \sum_{\substack{k_1\epsilon S_1\\ k_1\geq\geq 0}} \pi_s(Case3'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\qquad + \sum_{\substack{k_2\epsilon S_2\\ k_2\geq 0}} \sum_{\substack{k_1\epsilon S_1\\ Line1<k_1<0}} \pi_s(Case4'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\qquad + \sum_{\substack{k_2\epsilon S_2\\ Line2<k_2<0}} \sum_{\substack{k_1\epsilon S_1\\ Line1<k_1<0}} \pi_s(Case6'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\qquad + \sum_{\substack{k_2\epsilon S_2\\ k_2\leq Line2}} \sum_{\substack{k_1\epsilon S_1\\ Line1\leq k_1<0}} \pi_s(Case8'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\qquad + \sum_{\substack{k_2\epsilon S_2\\ k_2\geq 0}} \sum_{\substack{k_1\epsilon S_1\\ k_1\leq Line1}} \pi_s(Case5'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\qquad + \sum_{\substack{k_2\epsilon S_2\\ Line2<k_2<0}} \sum_{\substack{k_1\epsilon S_1\\ k_1\leq Line1}} \pi_s(Case7'(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&\qquad + \sum_{\substack{k_2\epsilon S_2\\ k_2\leq Line2}} \sum_{\substack{k_1\epsilon S_1\\ k_1<Line1}} \pi_s(Case9'(k_1,k_2))p_1^{k_1}p_2^{k_2}
\end{aligned}
$$

Thus Property 2 follows. ■

The analysis for $A_2 < c_1 < A_1$ and $c_1 > A_1$ is given in Appendix.

6.2 Non-Collaborating buyers

The profit function of the buyers and the supplier are expressed as follows;

$$E_{\theta,Y_2}[\pi_1|Y_1] = E[(A_1 + \theta_1 - q_1 - \beta q_2 - c_1 + c_2 q_1)q_1|Y_1]$$

$$E_{\theta,Y_2}[\pi_1|Y_1] = (A_1 + \delta_1(\alpha_1,\sigma_1)Y_1 - q_1 - \beta E_{Y_2}[q_2] - c_1 + c_2 q_1)q_1$$

$$E_{\theta,Y_1}[\pi_2|Y_2] = E[(A_2 + \theta_2 - q_2 - \beta q_1 - c_1 + c_2 q_2)q_2|Y_2]$$

$$E_{\theta,Y_1}[\pi_2|Y_2] = (A_2 + \delta_2(\alpha_2, \sigma_2)Y_2 - q_2 - \beta E_{Y_1}[q_1] - c_1 + c_2 q_2)q_2$$

$$\begin{aligned} E_{\theta,Y_1,Y_2}[\pi_s] &= E_{\theta,Y_1,Y_2}[(c_1 - c_2 q_1)q_1] + E_{\theta,Y_1,Y_2}[(c_1 - c_2 q_2)q_2] \\ &= c_1 E_{\theta,Y_1,Y_2}[q_1 + q_2] - c_2 E_{\theta,Y_1,Y_2}[q_1^2 + q_2^2] \end{aligned}$$

Similar to collaboration case, $E_{\theta,Y_2}[\pi_1|Y_1]$ and $E_{\theta,Y_1}[\pi_2|Y_2]$ denote the ex-post expected profits for the buyers. When finding expected profit, $E[\pi_i|Y_i]$, buyer *i* takes expected quantity of buyer *j* to evaluate the profit value. This is due to the fact that Y_j is not shared with buyer *i*. Supplier, on the other hand, neither knows Y_1 nor Y_2. Thus, for evaluate $E[\pi_s]$ for a given c_1 and c_2, consider the expected equilibrium quantities, expectation taken over all possible Y_1 and Y_2 values.

In the following we discuss how the equilibrium quantities for the buyers and optimal c_1 and c_2 values for the supplier are determined.

6.2.1 The Buyers' Problem

In this section, for a given value of c_1 and c_2 values the equilibrium quantities of the buyers are determined.

Property 3: Under NBNS setting with two non-collaborating buyers, for a given c_1 and c_2, if $c_1 < A_2$, then the Bayesian Nash equilibria are obtained as follows:

i) If $\frac{A_2 - c_1 + \delta_2 Y_2}{\beta} < \frac{A_1 - c_1}{2 - 2c_2}$ then

$$q_1 = \frac{A_1 - c_1}{2 - 2c_2} + \frac{1}{2 - 2c_2}[\delta_1]Y_1$$

$$q_2 = 0$$

ii) If $\frac{A_2 - c_1 + \delta_2 Y_2}{\beta} > \frac{A_1 - c_1}{2 - 2c_2}$ and $\frac{A_1 - c_1 + \delta_1 Y_1}{\beta} < \frac{A_2 - c_1}{2 - 2c_2}$ then

$$q_1 = 0$$

$$q_2 = \frac{A_2 - c_1}{2 - 2c_2} + \frac{1}{2 - 2c_2}[\delta_2]Y_2$$

iii) If $\frac{A_2 - c_1 + \delta_2 Y_2}{\beta} > \frac{A_1 - c_1}{2 - 2c_2}$ and $\frac{A_1 - c_1 + \delta_1 Y_1}{\beta} > \frac{A_2 - c_1}{2 - 2c_2}$ then

$$q_1 = \frac{2A_1(1 - c_2) - 2c_1 + \beta c_1 + 2c_1 c_2 - A_2\beta}{(2 - 2c_2)^2 - \beta^2} + \frac{1}{2 - 2c_2}[\delta_1]Y_1$$

$$q_2 = \frac{2A_2(1 - c_2) - 2c_1 + \beta c_1 + 2c_1 c_2 - A_1\beta}{(2 - 2c_2)^2 - \beta^2} + \frac{1}{2 - 2c_2}[\delta_2]Y_2$$

Proof: To find the best response of buyer i, first derivative of profit function, π_i, is taken with respect to q_i.

For buyer 1

$$\frac{\partial E_{\theta,Y_2}[\pi_1|Y_1]}{\partial q_1} = A_1 + \delta_1(\alpha_1, \sigma_1)Y_1 - 2q_1 - \beta E[q_2|Y_1] - c_1 + 2c_2 q_1 = 0$$

$$q_1(q_2) = \left(\frac{A_1 + \delta_1(\alpha_1, \sigma_1)Y_1 - c_1 - \beta E[q_2|Y_1]}{2 - 2c_2}\right)^+$$

$$= \left(\frac{A_1' - c_1}{2 - 2c_2} - \frac{\beta E[q_2|Y_1]}{2 - 2c_2}\right)^+ \qquad (6.11)$$

$$\frac{\partial E_{\theta,Y_1}[\pi_2|Y_2]}{\partial q_2} = A_2 + \delta_2(\alpha_2, \sigma_2)Y_2 - 2q_2 - \beta E[q_1|Y_2] - c_1 + 2c_2 q_2 = 0$$

$$q_2(q_1) = \left(\frac{A_2 + \delta_2(\alpha_2, \sigma_2)Y_2 - c_1 - \beta E[q_1|Y_2]}{2 - 2c_2}\right)^+$$

$$= \left(\frac{A_2' - c_1}{2 - 2c_2} - \frac{\beta E[q_1|Y_2]}{2 - 2c_2}\right)^+ \qquad (6.12)$$

If q_1 and q_2 were unrestricted in sign, then it is known that at equilibrium q_1 and q_2 are in the following forms:

$$q_1 = D_0^1 + D_1^1 Y_1$$

$$q_2 = D_0^2 + D_1^2 Y_2$$

Even though q_1 and q_2 are restricted to take (+) values, and expressed as in Eqn. 6.11 and Eqn. 6.12, we still assume they take the form $q_i = D_0^i + D_1^i Y_i$. Then, Eqns. 6.11 and 6.12 can be rewritten as

$$q_1(q_2) = \frac{A_1 + \delta_1(\alpha_1, \sigma_1)Y_1 - c_1 - \beta E[D_0^2 + D_1^2 Y_2 | Y_1]}{2 - 2c_2}$$

Note $E[Y_2|Y_1] = 0$

Suppose $q_1(q_2)$ does not necessarily take (+) values. Then,

$$q_1(q_2) = D_0^1 + D_1^1 Y_1 = \frac{1}{2 - 2c_2}[A_1 + \delta_1(\alpha_1, \sigma_1)Y_1 - c_1 - \beta(D_0^2)]$$

Then,

$$D_0^1 = \frac{1}{2 - 2c_2}[A_1 - c_1 - \beta D_0^2] \qquad (6.13)$$

$$D_1^1 = \frac{1}{2 - 2c_2}[\delta_1(\alpha_1, \sigma_1)] \qquad (6.14)$$

Similarly,

$$q_2(q_1) = \frac{A_2 + \delta_2(\alpha_2, \sigma_2)Y_2 - c_1 - \beta E[D_0^1 + D_1^1 Y_1 | Y_2]}{2 - 2c_2}$$

Since, $E[Y_1|Y_2] = 0$

$$q_2(q_1) = D_0^2 + D_1^2 Y_2 = \frac{1}{2 - 2c_2}[A_2 + \delta_2(\alpha_2, \sigma_2)Y_2 - c_1 - \beta(D_0^1)]$$

$$D_0^2 = \frac{1}{2 - 2c_2}[A_2 - c_1 - \beta D_0^1] \qquad (6.15)$$

$$D_1^2 = \frac{1}{2 - 2c_2}[\delta_2(\alpha_2, \sigma_2)] \qquad (6.16)$$

Note that in $q_i(q_j)$ coefficient of Y_i is independent of the action taken by other player.

This implies one could equivalently analyze best response functions in terms of D_0^i

Considering that $q_i(q_j)$ must always be non-negative, best response of D_0^i and D_1^i can be expressed as follows.

$D_0^1(D_0^2) = \frac{A_1 - c_1 - \beta D_0^2}{2 - 2c_2} > 0$ if $D_0^2 < \frac{A_1 - c_1 + \delta_1 Y_1}{\beta}$, and 0 otherwise.

$D_1^1(D_0^2) = \frac{\delta_1}{2 - 2c_2}$ if $D_0^2 < \frac{A_1 - c_1 + \delta_1 Y_1}{\beta}$, and 0 otherwise.

Similarly, for buyer 2 best response function for D_0^2 can be expressed as

$D_0^2(D_0^1) = \frac{A_2 - c_1 - \beta D_0^1}{2 - 2c_2} > 0$ if $D_0^1 < \frac{A_2 - c_1 + \delta_2 Y_2}{\beta}$, and 0 otherwise.

$D_1^2(D_0^1) = \frac{\delta_2}{2 - 2c_2}$ if $D_0^1 < \frac{A_2 - c_1 + \delta_2 Y_2}{\beta}$, and 0 otherwise.

Note that even if $D_0^i < 0$, $q_i(q_j)$ could be (+). In other words D_0^i may possibly take (-) values. When best response functions are expressed in terms of D_0^1 and D_0^2 discontinuity may exist, which may result in non-existence of pure strategy equilibrium.

Best response functions and equilibrium points may change with the parameter values and the signal Y_1 and Y_2. We make the following definitions (see also Table 2 in Chapter 6):

$$K_1 = \frac{A_1 - c_1}{\beta}$$

$$K_2 = \frac{A_2 - c_1}{\beta}$$

$$JJ_1 = KK_1 = \frac{A_1 - c_1}{2 - 2c_2}$$

$$JJ_2 = KK_2 = \frac{A_2 - c_1}{2 - 2c_2}$$

$$K_1' = \frac{A_1 + \delta_1 Y_1 - c_1}{\beta}$$

$$K_2' = \frac{A_2 + \delta_2 Y_2 - c_1}{\beta}$$

We assume $c_1 < A_2$. Therefore, it holds that $K1 > K2$, $KK1 > KK2$. Depending on the values of K_1, K_2, K_1', K_2', $KK1$, $KK2$ and Y_1 and Y_2, 36 possible cases may exist, as discussed below. Similar to collaboration case, we first present one example to show how equilibrium might shift from a parameter setting to the next. Then we present the 36 conditions, and the corresponding cases (equilibria).

Example 1. (Case 1). Suppose $Y_1 >, Y_2 > 0$. Then $K_1^{'} > K1$ and $K_2^{'} > K2$. Under the following setting $KK1 < K2$, $K1 > KK2$, $K_2^{'} > KK1$ and $K_1^{'} > KK2$, the best response function D_0^1 and D_0^2 are shown in Figure 20.

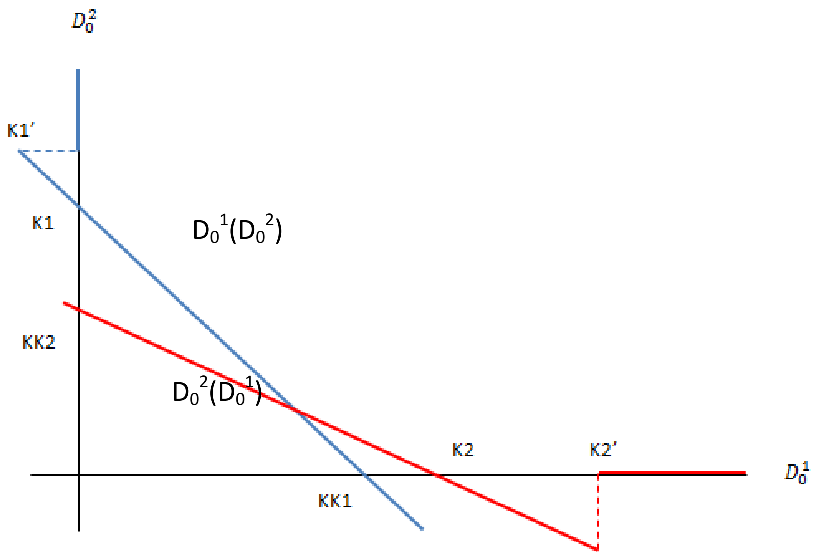

Figure 20 Case 1

If Eqns. 6.13 and 6.15 are solved simultaneously, D_0^1 and D_0^2 are obtained as:

$$D_0^1 = \frac{2A_1(1-c_2) - 2c_1 + \beta c_1 + 2c_1c_2 - A_2\beta}{(2-2c_2)^2 - \beta^2}$$

$$D_0^2 = \frac{2A_2(1-c_2) - 2c_1 + \beta c_1 + 2c_1c_2 - A_1\beta}{(2-2c_2)^2 - \beta^2}$$

Equilibrium quantities are:

$$q_1 = D_0^1 + D_1^1 Y_1$$

$$q_2 = D_0^2 + D_1^2 Y_2$$

Example 2. (Case 7). Suppose $Y_1 < 0$, $Y_2 < 0$. Then $K_1^{'} < K1$ and $K_2^{'} < K2$. Under the following setting $KK1 < K2$, $K1 > KK2$, $K_2^{'} > KK1$ and $K_1^{'} < KK2$, the best response function D_0^1 and D_0^2 are shown in Figure 21.

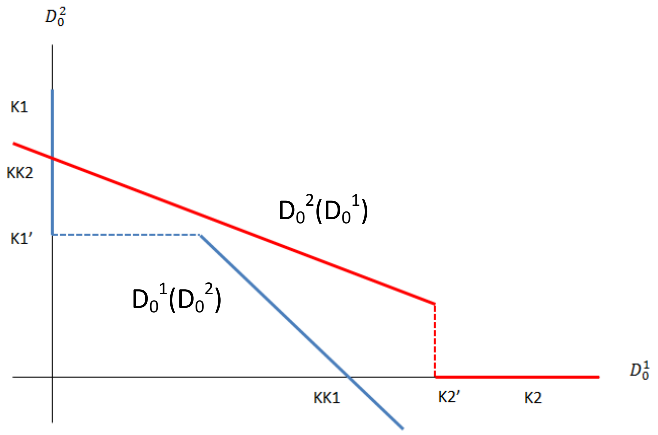

Figure 21 Case 7

Under Case 7, D_0^1 and D_0^2 intersect at the point where $D_0^1 = 0$ and $D_0^2 = KK2$ (see Figure 21). Equilibrium quantities are:

$$q_1 = D_1^1 Y_1$$

$$q_2 = KK2 + D_1^2 Y_2$$

Example 3. (Case 8). Suppose $Y_1 < 0$, $Y_2 < 0$. Then $K_1^{'} < K1$ and $K_2^{'} < K2$. Under the following setting $KK1 < K2$, $K1 > KK2$, $K_2^{'} < KK1$ and $K_1^{'} > KK2$, the best response function D_0^1 and D_0^2 are shown in Figure 22.

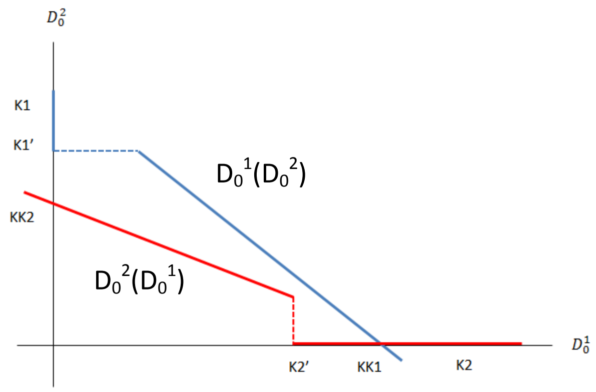

Figure 22 Case 8

Under Case 8, D_0^1 and D_0^2 intersect at the point where $D_0^1 = KK1$ and $D_0^2 = 0$ (see Figure 22). Equilibrium quantities are:

$$q_1 = KK1 + D_1^1 Y_1$$

$$q_2 = D_1^2 Y_2$$

Depending on the values of K_1, K_2, K_1', K_2', $KK1$, $KK2$ and Y_1 and Y_2, the possible settings are identified. And for each of the setting the corresponding equilibria are defined. Assumption of $c_1 < A_2$ and $A_1 > A_2$ implies $K1 > K2$, and $KK1 > KK2$.

Setting A. If $Y_1 > 0$ and $Y_2 > 0$, then $K_1' > K1$, $K_2' > K2$

A) $KK1 < K2$
  1) $K1 > KK2$
    a) $K_2' > KK1$
      i) $K_1' > KK2$ → Case 1
      ii) $K_1' < KK2$ → does not exist since $K_1' > K1 > KK2$
    b) $K_2' < KK1$ → does not exist since $KK1 < K2 < K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1'

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → Case 2'

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 3'

ii) $K_1' \leq KK2$ → Case 4'

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → Case 5'

ii) $K_1' \leq KK2$ → Case 6'

Setting B. If $Y_1 > 0$ and $Y_2 < 0$, then $K_1' > K1$, $K_2' < K2$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 2

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → Case 3

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

2) $K_1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ →does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → Case 7'

ii) $K_1' < KK2$ → does not exist $K_1' > K_1 > KK2$

2) $K_1 < KK2$

a) $K_2' > KK1$ → does not exist since $K1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → Case 8'

ii) $K_1' \leq KK2$ → Case 9'

Setting C. If $Y_1 < 0$ and $Y_2 > 0$, then $K_1' < K1$, $K_2' > K2$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 4

ii) $K_1' \leq KK2$ → Case 5

b) $K_2' < KK1$ → does not exist since $KK1 < K2 < K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 10'

ii) $K_1' \leq KK2$ → Case 11'

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → Case 12'

ii) $K_1' \leq KK2$ → Case 13'

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > K2$ → does not exist

ii) $K_1' \leq KK2$ → Case 14'

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → does not exist

ii) $K_1' \leq KK2$ → Case 15'

Setting D. If $Y_1 < 0$ and $Y_2 < 0$ then $K_1' < K1$, $K_2' < K2$.

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 6

ii) $K_1' \le KK2$ → Case 7

b) $K_2' \le KK1$

i) $K_1' > KK2$ → Case 8

ii) $K_1' \le KK2$ → Case 9

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ → does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' \le KK1$

i) $K_1' > KK2$ → Case 16'

ii) $K_1' \le KK2$ → Case 17'

2) $K1 < KK2$

a) $K_2' > KK1$ → does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' \le KK1$

i) $K_1' > KK2$ → does not exit

ii) $K_1' \leq KK2$ → Case 18'

Setting E. If $Y_1 > 0$ and $Y_2 = 0$, then $K_1' > K1$, $K_2' = K2$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1

ii) $K_1' < KK2$ → does not exist since $K_1' > K1 > KK2$

b) $K_2' < KK1$ → does not exist since $KK1 < K2 = K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ → not possible since $KK1 > K2 = K_2'$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → Case 2'

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

2) $K1 < KK2$

a) $K_2' > KK1$ → not possible since $KK1 > K2 = K_2'$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → Case 5'

ii) $K_1' \leq KK2$ → Case 6'

Setting F. If $Y_1 < 0$ and $Y_2 = 0$, then $K_1' < K1$, $K_2' = K2$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 4

ii) $K_1' \leq KK2$ → Case 5

b) $K_2' < KK1$ → does not exist since $KK1 < K2 = K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ → not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 12'

ii) $K_1' \leq KK2$ → Case 13'

2) $K1 < KK2$

a) $K_2' > KK1$ → not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → does not exist since $K_1' < K1 < KK2$

ii) $K_1' \leq KK2$ → Case 15'

Setting G. If $Y_1 = 0$ and $Y_2 > 0$, then $K_1' = K1, \quad K_2' > K2$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1

ii) $K_1' < KK2$ → does not exist since $K_1' = K1 > KK2$

b) $K_2' < KK1$ → does not exist since $KK1 < K2 < K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1'

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → Case 2'

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → not possible

ii) $K_1' \leq KK2$ → Case 4'

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → not possible

ii) $K_1' \leq KK2$ → Case 6'

Setting H. If $Y_1 = 0$ and $Y_2 < 0$, then $K_1' = K1$, $K_2' < K2$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 2

ii) $K_1' < KK2$ → does not exist since $K_1' = K_1 > KK2$

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → Case 3

ii) $K_1' < KK2$ → does not exist since $K_1' = K_1 > KK2$

2) $K_1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ →does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → Case 7'

ii) $K_1' < KK2$ → does not exist $K_1' = K_1 > KK2$

2) $K_1 < KK2$

a) $K_2' > KK1$ → does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → does not exist

ii) $K_1' \leq K2$ → Case 9'

Setting I. If $Y_1 = 0$ and $Y_2 = 0$, then $K_1' = K1$, $K_2' = K2$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1

ii) $K_1' < KK2$ → does not exist since $K_1' = K1 > KK2$

b) $K_2' < KK1$ → does not exist since $KK1 < K2 = K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ → not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → Case 2'

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

2) $K1 < KK2$

a) $K_2' > KK1$ → not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' \leq KK1$

i) $K_1' > KK2$ → not possible

ii) $K_1' \leq KK2$ → Case 6'

The equilibrium quantities for each case are as follows. Whenever $q_i = D_0^i + D_1^i Y_i$, D_0^i is obtained by intersecting Eqns. 6.13 and 6.15.

Cases	**Equilibrium quantities**	
1-2-4-6-1'-3'-10'	$q_1 = D_0^1 + D_1^1 Y_1$	$q_2 = D_0^2 + D_1^2 Y_2$
3-8-9-2'-5'-6'-7'-8'-9'-12'-13'-15'-16'-17'-18'	$q_1 = KK1 + D_1^1 Y_1$	$q_2 = 0$
5-7-4'-11'-14'	$q_1 = 0$	$q_2 = KK2 + D_1^2 Y_2$

Note it is possible to categorize all possible cases into three major cases. There are cases where both quantities are positive (if $K_2' > KK1$ and $K_1' > KK2$), cases where $q_1 > 0, q_2 = 0$ (if $K_2' < KK1$), and the cases where $q_1 = 0, q_2 > 0$ (if $K_2' > KK1$ and $K_1' < KK2$). Thus, the Property 3 follows. ■

The analysis for $A_2 < c_1 < A_1$ and $c_1 > A_1$ is given in Appendix.

Using the equilibrium quantities, next the supplier's problem is addressed.

6.2.2 The Supplier's Problem

Under NBNS neither the suppliers does have the knowledge of demand signals Y_1 and Y_2 nor the buyers have the knowledge of other buyer's signal.

Property 4: Under the NBNS setting with two non-collaborating buyers, the optimal c_1 and c_2 values that maximize the supplier's profit function are denoted with

If $\frac{A_1-c_1}{2-2c_2} < \frac{A_2-c_1}{\beta}$ and $\frac{A_1-c_1}{\beta} > \frac{A_2-c_1}{2-2c_2}$

$$E_{Y_1,Y_2}[\pi_s(c_1,c_2)]$$

$$= \sum_{\substack{k_2 \epsilon S_2 \\ k_2 \geq 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s(Case1(k_1,k_2))p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ Line2 < k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s\big(Case2(k_1,k_2)\big)p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s\big(Case3(k_1,k_2)\big)p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 \geq 0}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s\big(Case4(k_1,k_2)\big)p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ Line2 < k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s\big(Case6(k_1,k_2)\big)p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s\big(Case8(k_1,k_2)\big)p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 \geq 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \leq Line1}} \pi_s\big(Case5(k_1,k_2)\big)p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ Line2 < k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \leq Line1}} \pi_s\big(Case7(k_1,k_2)\big)p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon \\ k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \leq Line1}} \pi_s(Case9(k_1,k_2))p_1^{k_1} p_2^{k_2}$$

If $\frac{A_1 - c_1}{2 - 2c_2} > \frac{A_2 - c_1}{\beta}$ and $\frac{A_1 - c_1}{\beta} > \frac{A_2 - c_1}{2 - 2c_2}$

$$E_{Y_1,Y_2}[\pi_s(c_1,c_2)] = \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s(Case1'(k_1,k_2)) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ 0 \leq k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s\big(Case2'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s\big(Case7'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s\big(Case10'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ 0 \leq k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s\big(Case12'(k_1,k_2)\big) p_1^{k_1} p_1^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s\big(Case16'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \leq Line1}} \pi_s\big(Case11'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ 0 \leq k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \leq Line1}} \pi_s\big(Case13'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \leq Line1}} \pi_s(Case17'(k_1,k_2)) p_1^{k_1} p_2^{k_2}$$

If $\frac{A_1-c_1}{2-2c_2} > \frac{A_2-c_1}{\beta}$ and $\frac{A_1-c_1}{\beta} < \frac{A_2-c_1}{2-2c_2}$

$$
\begin{aligned}
&E_{Y_1,Y_2}[\pi_s(c_1,c_2)] \\
&\quad = \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ 0 \le k_1 \le Line1}} \pi_s\big(Case4'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2} \\
&\quad + \sum_{\substack{k_2 \epsilon S_2 \\ 0 \le k_2 \le Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ 0 \le k_1 \le Line1}} \pi_s\big(Case6'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2} \\
&\quad + \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ 0 \le k_1 \le Line1}} \pi_s\big(Case9'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2} \\
&\quad + \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 > Line1}} \pi_s\big(Case3'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2} \\
&\quad + \sum_{\substack{k_2 \epsilon S_2 \\ 0 \le k_2 \le Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 > Line1}} \pi_s\big(Case5'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2} \\
&\quad + \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 > Line1}} \pi_s\big(Case8'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2} \\
&\quad + \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 < 0}} \pi_s\big(Case14'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2} \\
&\quad + \sum_{\substack{k_2 \epsilon S_2 \\ 0 \le k_2 \le Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 < 0}} \pi_s\big(Case15'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2} \\
&\quad + \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 < 0}} \pi_s(Case18'(k_1,k_2)) p_1^{k_1} p_2^{k_2}
\end{aligned}
$$

where $Line1 = \frac{\beta(A_2-c_1)-(2-2c_2)(A_1-c_1)}{(2-2c_2)\delta_1}$ and $Line2 = \frac{\beta(A_1-c_1)-(2-2c_2)(A_2-c_1)}{(2-2c_2)\delta_2}$

Proof. Similar to collaboration case, for a given c_1 and c_2, the profit function of the supplier can be written under changing Y_1 and Y_2. When partitioning the space considering the values that Y_1 and Y_2 can take, the separating lines are defined next to the corresponding inequalities. The lines are shown in Figure 23 and 24 below. We analyze the cases $c_1 < A_2$, $A_2 < c_1 < A_1$, $c_1 > A_1$ separately.

Given $c_1 < A_2$

A) $KK1 < K2$ implies

$$\beta(A_1 - c_1) < (A_2 - c_1)(2 - 2c_2) \rightarrow (2 - 2c_2) > \beta$$

$K1 > KK2$ implies

$$(2 - 2c_2)(A_1 - c_1) > (A_2 - c_1)\beta$$

$K1 < KK2$ is not possible when $KK1 < K2$.

$K_2' > KK1$ implies

$$Y_2 > \frac{\beta(A_1 - c_1) - (2 - 2c_2)(A_2 - c_1)}{(2 - 2c_2)\delta_2} \qquad Line2$$

If $KK1 < K2$ then $Line2 < 0$

$K_1' > KK2$ implies

$$Y_1 > \frac{\beta(A_2 - c_1) - (2 - 2c_2)(A_1 - c_1)}{(2 - 2c_2)\delta_1} \qquad Line1$$

If $K1 > KK2$ then $Line1 < 0$

Then, only one case with $KK1 < K2$ and $K1 > KK2$ (where $Line2 < 0$ and $Line1 < 0$)

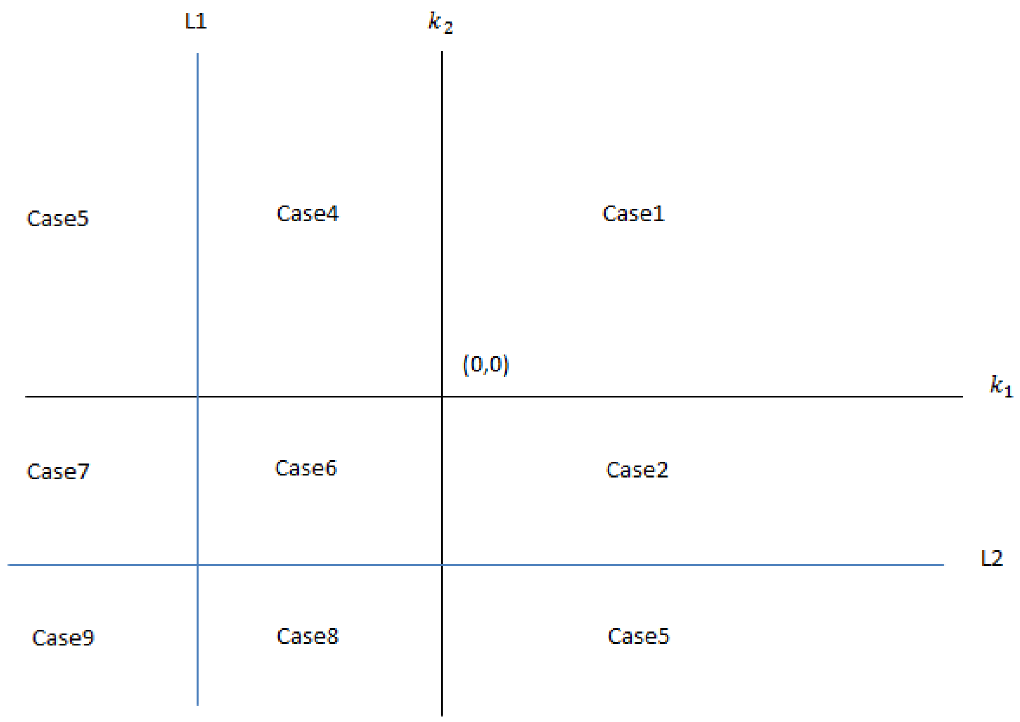

Figure 23 How equilibrium cases change as $\boldsymbol{Y_1}$ and $\boldsymbol{Y_2}$ realizations change, given $\boldsymbol{c_1 < A_2}$, $\boldsymbol{KK1 < K2}$ and $\boldsymbol{K1 > KK2}$

$$E_{Y_1,Y_2}[\pi_s(c_1,c_2)]$$

$$= \sum_{\substack{k_2 \epsilon S_2 \\ k_2 \geq 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s(Case1(k_1,k_2)) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ Line2 < k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s\big(Case2(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s\big(Case3(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 \geq 0}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s\big(Case4(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ Line2 < k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s\big(Case6(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s\big(Case8(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 \geq 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \leq Line1}} \pi_s\big(Case5(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ Line2 < k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \leq Line1}} \pi_s\big(Case7(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon \\ k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \leq Line1}} \pi_s(Case9(k_1,k_2)) p_1^{k_1} p_2^{k_2}$$

B) $KK1 > K2$ implies

$$\beta(A_1 - c_1) > (A_2 - c_1)(2 - 2c_2)$$

$K1 > KK2$ implies

$$(2 - 2c_2)(A_1 - c_1) > (A_2 - c_1)\beta$$

$K1 < KK2$ implies

$$(2 - 2c_2)(A_1 - c_1) < (A_2 - c_1)\beta$$

If $KK1 > K2$ then $Line2 > 0$

If $K1 > KK2$ then $Line1 < 0$ and if $K1 < KK2$ then $Line1 > 0$

Then, Case (a) with $KK1 > K2$ and $K1 > KK2$ where $Line2 > 0$ and $Line1 < 0$ and Case (b) with $KK1 > K2$ and $K1 < KK2$ where $Line2 > 0$ and $Line1 > 0$.

Case (a):

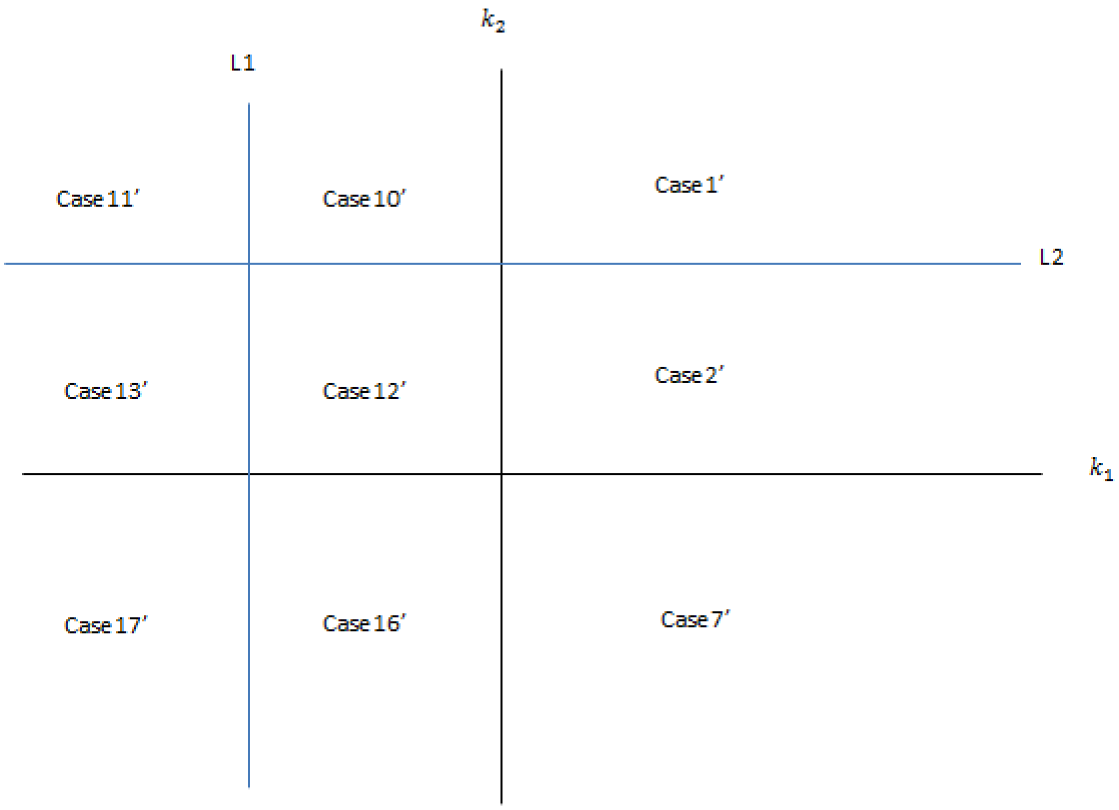

Figure 2424 How equilibrium cases change as $\boldsymbol{Y_1}$ and $\boldsymbol{Y_2}$ realizations change, given $\boldsymbol{c_1 < A_2}$, $\boldsymbol{KK1 > K2}$ and $\boldsymbol{K1 > KK2}$

$$E_{Y_1,Y_2}[\pi_s(c_1,c_2)]$$

$$= \sum_{\substack{k_2\epsilon S_2 \\ k_2>Line2}} \sum_{\substack{k_1\epsilon S_1 \\ k_1\geq 0}} \pi_s(Case1'(k_1,k_2))p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2 \\ 0\leq k_2\leq Line2}} \sum_{\substack{k_1\epsilon S_1 \\ k_1\geq 0}} \pi_s\big(Case2'(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2 \\ k_2<0}} \sum_{\substack{k_1\epsilon S_1 \\ k_1\geq 0}} \pi_s\big(Case7'(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2 \\ k_2>Line2}} \sum_{\substack{k_1\epsilon S_1 \\ Line1<k_1<0}} \pi_s\big(Case10'(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2 \\ 0\leq k_2\leq Line2}} \sum_{\substack{k_1\epsilon S_1 \\ Line1<k_1<0}} \pi_s\big(Case12'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2 \\ k_2<0}} \sum_{\substack{k_1\epsilon S_1 \\ Line1<k_1<0}} \pi_s\big(Case16'(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2 \\ k_2>Line2}} \sum_{\substack{k_1\epsilon S_1 \\ k_1\leq Line1}} \pi_s\big(Case11'(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2 \\ 0\leq k_2\leq Line2}} \sum_{\substack{k_1\epsilon S_1 \\ k_1\leq Line1}} \pi_s\big(Case13'(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\epsilon S_2 \\ k_2<0}} \sum_{\substack{k_1\epsilon S_1 \\ k_1\leq Lin1}} \pi_s(Case17'(k_1,k_2))p_1^{k_1}p_2^{k_2}$$

Case (b):

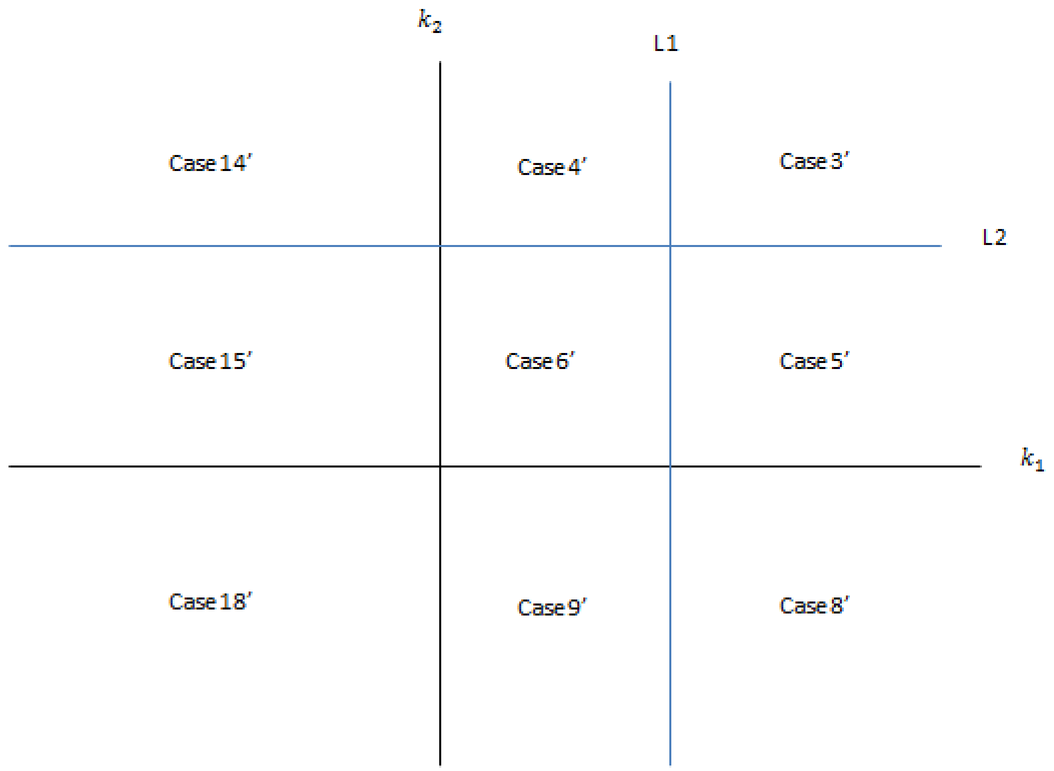

Figure 25 How equilibrium cases change as $\boldsymbol{Y_1}$ and $\boldsymbol{Y_2}$ realizations change, given $\boldsymbol{c_1 < A_2}$, $\boldsymbol{KK1 > K2}$ and $\boldsymbol{K1 < KK2}$

$$E_{Y_1,Y_2}[\pi_s(c_1,c_2)]$$

$$= \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ 0 \le k_1 \le Line1}} \pi_s\big(Case4'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ 0 \le k_2 \le Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ 0 \le k_1 \le Line1}} \pi_s\big(Case6'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ 0 \le k_1 \le Line1}} \pi_s\big(Case9'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 > Line1}} \pi_s\big(Case3'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ 0 \le k_2 \le Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 > Line1}} \pi_s\big(Case5'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 > Line1}} \pi_s\big(Case8'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 < 0}} \pi_s\big(Case14'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ 0 \le k_2 \le Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 < 0}} \pi_s\big(Case15'(k_1,k_2)\big) p_1^{k_1} p_2^{k_2}$$

$$+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 < 0}} \pi_s(Case18'(k_1,k_2)) p_1^{k_1} p_2^{k_2}$$

Thus Property 2 follows. ■

The analysis for $A_2 < c_1 < A_1$ and $c_1 > A_1$ is given in Appendix.

6.3 Determining the ex ante profits

Under NBNS case, ex-ante and ex-post profits do not differ. Since under equilibrium, quantities q_1 and q_2 are a function of c_1, c_2, Y_1 and Y_2, expectation is taken over Y_1 and Y_2. Then optimal c_1 and c_2 is seeked to maximize $E[\pi_s]$.

Ex-ante profits of the buyers and the supplier

Note that Y_1 and Y_2 are independent random variables. Then, for a given k_1 and k_2, ex-ante profits are determined as follows;

1. Let $A_1' = \max\{A_1 + \delta_1 k_1, A_2 + \delta_2 k_2\}$ and define A_2' accordingly.
2. Follow the analysis in Section 6.1.2 for collaboration and in Section 6.2.2 for no-collaboration setting. Use the equilibrium quantities defined in Section 6.1.1 and Section 6.2.1.
3. For a given c_1 and c_2, the supplier determines its profit for every possible k_1 and k_2 values and expectation is taken over Y_1 and Y_2 . Possible values for c_1 is in the range $[0, \max\{A_1 + \delta_1 \max(Y_1), A_2 + \delta_2 \max(Y_2)\}]$ and for c_2 in the range of $[0, \beta - \varepsilon]$. Then, c_1 and c_2 values which maximize the expected profit are set as the optimal c_1 and c_2. The corresponding profit is the ex-ante profit for the supplier.
4. After determining $c_1^*(k_1, k_2)$ and $c_2^*(k_1, k_2)$ use $q_1(k_1, k_2)$, $q_2(k_1, k_2)$, $c_1^*(k_1, k_2)$, $c_2^*(k_1, k_2)$ to obtain $\pi_i(k_1, k_2)$ for the corresponding realization of Y_1 and Y_2 for the buyers. The ex-ante profit of buyer i is calculated by finding profit for all possible Y_i value by taking expectation over Y_j for all possible q_j values for each, and taking expectation over Y_i .

CHAPTER 7

NO INFORMATION FOR THE BUYERS AND IMPERFECT INFORMATION FOR THE SUPPLIER (NBIS)

The equilibrium points and the supplier's optimal c_1 and c_2 values are determined under the strategy that the buyers do not share their signals on the market demand with each other buyer but only share it with the supplier. The analysis is made under collaborating and non-collaborating buyers. In the analysis, first, the equilibrium quantities are determined, and then the optimum wholesale price is calculated. The assumptions done during the analysis are same as previous section.

7.1 Collaborating buyers

The equilibrium quantities and the profit functions of the buyers are the same as NBNS case. The profit functions of the buyers are expressed as follows;

$$E_{\theta,Y_2}[\pi_1|Y_1] = E[(A_1 + \theta_1 - q_1 - \beta q_2 - c_1 + c_2(q_1 + q_2))q_1|Y_1]$$

$$= (A_1 + \delta_1(\alpha_1, \sigma_1)Y_1 - q_1 - \beta E_{Y_2}[q_2] - c_1 + c_2(q_1 + E_{Y_2}[q_2]))q_1$$

$$E_{\theta,Y_1}[\pi_2|Y_2] = E[(A_2 + \theta_2 - q_2 - \beta q_1 - c_1 + c_2(q_1 + q_2))q_2|Y_2]$$

$$= (A_2 + \delta_2(\alpha_2, \sigma_2)Y_2 - q_2 - \beta E_{Y_1}[q_1] - c_1 + c_2(q_2 + E_{Y_1}[q_1]))q_2$$

The supplier's ex-post profit function does not include any uncertainty given Y_1 and Y_2.

$$\pi_s|Y_1, Y_2 = (c_1 - c_2(q_1 + q_2))(q_1 + q_2)$$

Similar to NBNS case, $E_{\theta,Y_2}[\pi_1|Y_1]$ and $E_{\theta,Y_1}[\pi_2|Y_2]$ denote the ex-post expected profits for the buyers. Expected profit of the buyer i, $E[\pi_i|Y_i]$, is calculated by taking expected quantity of buyer *j* because Y_j is not shared with buyer i. Supplier, on the other hand, knows both Y_1 and Y_2. Ex-ante profits for the buyers and the supplier are given in Section 7.3.

In the following we discuss how the equilibrium quantities for the buyers and optimal c_1 and c_2 values for the supplier are determined.

7.1.1 The Buyers' Problem

For a given value of c_1 and c_2, the equilibrium quantities for the buyers are obtained following the same steps as in NBNS case.

7.1.2 The Supplier's Problem

Under NBIS case the buyers do share the demand information with the supplier, hence the supplier knows Y_1 and Y_2.

Proposition 17: Under the NBIS setting with two collaborating buyers, the optimal c_1 and c_2 values that maximize the supplier's profit function are denoted with for given Y_1 and Y_2

$$max_{c_1,c_2}\{[c_1 - c_2(q_1(c_1,c_2,k_1,k_2) + q_1(c_1,c_2,k_1,k_2))](q_1(c_1,c_2,k_1,k_2) + q_1(c_1,c_2,k_1,k_2))\}$$

Proof. As stated in section 7.1.1, best response functions for buyers are the same as NBNS case. The feasible region is partitioned as follows;

$JJ1 > J2$ implies

$$\frac{A_1 - c_1}{2 - 2c_2} > \frac{A_2 - c_1}{\beta - c_2}$$

Then

$$c_1 > \frac{2A_2 - 2A_2c_2 - A_1\beta + A_1c_2}{2 - \beta - c_2} \qquad Line1$$

If $Y_1 > 0$ and $Y_2 > 0$, then Line 1 is always smaller than A_2.

$J_2' > JJ1$ implies

$$\frac{A_2 + \delta_2 Y_2 - c_1}{\beta - c_2} > \frac{A_1 - c_1}{2 - 2c_2}$$

Then

$$c_1 < \frac{2A_2' - 2A_2' c_2 - A_1 \beta + A_1 c_2}{2 - \beta - c_2} \qquad Line2$$

If $Y_2 > 0$ then Line 2 is always greater than A_2'. If $Y_2 < 0$ then Line 2 is always less than A_2'.

$J_1' > JJ2$ implies

$$\frac{A_1 + \delta_1 Y_1 - c_1}{\beta - c_2} > \frac{A_2 - c_1}{2 - 2c_2}$$

Then

$$c_1 < \frac{2A_1' - 2A_1' c_2 - A_2 \beta + A_2 c_2}{2 - \beta - c_2} \qquad Line3$$

Line 3 is always greater than A_1'.

$J1 > JJ2$ implies

$$\frac{A_1 - c_1}{\beta - c_2} > \frac{A_2 - c_1}{2 - 2c_2}$$

Then

$$c_1 < \frac{2A_1 - 2A_1 c_2 - A_2 \beta + A_2 c_2}{2 - \beta - c_2} \qquad Line4$$

Line 4 is always greater than A_1.

An example of the partitions of feasible region can be seen in Figure 26 below.

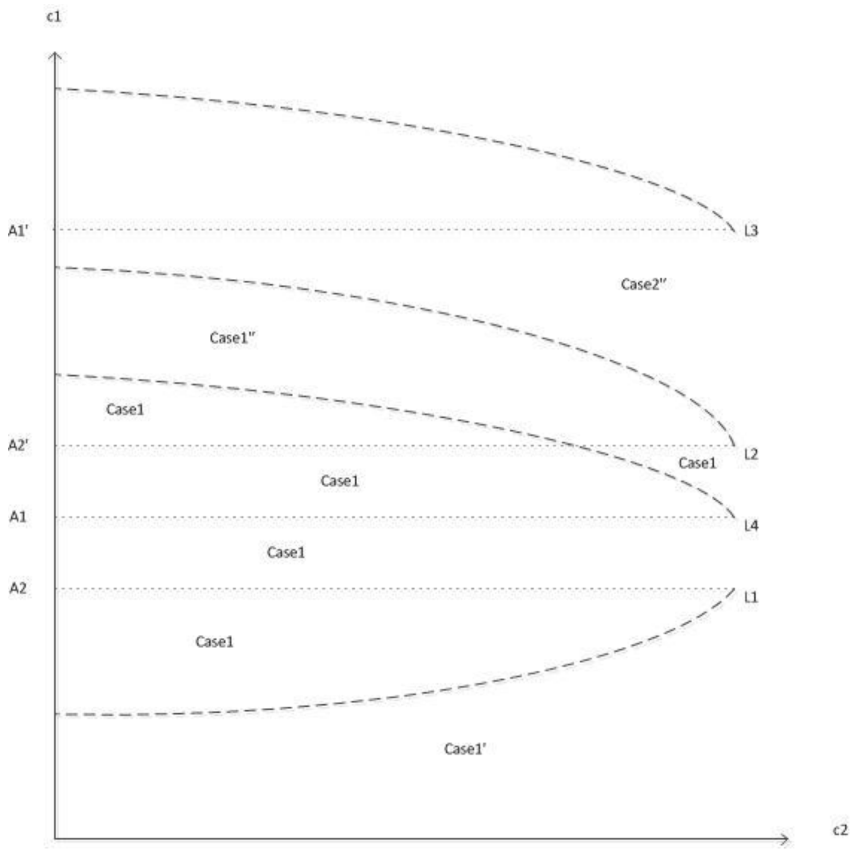

Figure 26 Example of feasible region

The optimal c_1 and c_2 is found with exhaustive search method. For every combination of c_1 and c_2 corresponding q_1 and q_2 values are found and the profit is calculated. Then, the ones which give maximum profit are set to be optimal values.

■

7.2 Non-Collaborating buyers

The profit function of the buyers are expressed as follows;

$$E_{\theta,Y_2}[\pi_1|Y_1] = E[(A_1 + \theta_1 - q_1 - \beta q_2 - c_1 + c_2 q_1)q_1|Y_1]$$

$$= (A_1 + \delta_1(\alpha_1, \sigma_1)Y_1 - q_1 - \beta E_{Y_2}[q_2] - c_1 + c_2 q_1)q_1$$

$$E_{\theta,Y_1}[\pi_2|Y_2] = E[(A_2 + \theta_2 - q_2 - \beta q_1 - c_1 + c_2 q_1)q_2|Y_2]$$

$$= (A_2 + \delta_2(\alpha_2, \sigma_2)Y_2 - q_2 - \beta E_{Y_1}[q_1] - c_1 + c_2 q_2)q_2$$

The supplier's ex-post profit function does not include any uncertainty given Y_1 and Y_2.

$$\pi_s|Y_1, Y_2 = (c_1 - c_2 q_1)q_1 + (c_1 - c_2 q_2)q_2$$

Similar to NBNS case, $E_{\theta,Y_2}[\pi_1|Y_1]$ and $E_{\theta,Y_1}[\pi_2|Y_2]$ denote the ex-post expected profits for the buyers. Ex-ante profits for the buyers and the supplier are given in Section 7.3.

In the following we discuss how the equilibrium quantities for the buyers and optimal c_1 and c_2 values for the supplier are determined.

7.2.1 The Buyers' Problem

For a given value of c_1 and c_2, the equilibrium quantities for the buyers are obtained following the same steps as in NBNS case.

7.2.2 The Supplier's Problem

Under NBIS case the buyers do share the demand information with the supplier, hence the supplier knows Y_1 and Y_2.

Proposition 17: Under the NBIS setting with two non-collaborating buyers, the optimal c_1 and c_2 values that maximize the supplier's profit function are denoted with for given Y_1 and Y_2.

$$max_{c_1,c_2}\{[c_1 - c_2(q_1(c_1, c_2, k_1, k_2))](q_1(c_1, c_2, k_1, k_2)) + [c_1 - c_2(q_2(c_1, c_2, k_1, k_2))](q_2(c_1, c_2, k_1, k_2))\}$$

Proof. As stated in section 7.2.1, best response functions for buyers are the same as NBNS case. The feasible region is partitioned as follows;

$KK1 < K2$ implies

$$\frac{A_1 - c_1}{2 - 2c_2} < \frac{A_2 - c_1}{\beta}$$

Then, if $\beta < 2 - 2c_2$

$$c_1 < \frac{2A_2 - 2A_2c_2 - A_1\beta}{2 - \beta - 2c_2} \quad Line1$$

If $\beta > 2 - 2c_2$,

$$c_1 > \frac{-2A_2 + 2A_2c_2 + A_1\beta}{\beta - 2 + 2c_2} \quad Line1'$$

$K_2' > KK1$ implies

$$\frac{A_2 + \delta_2 Y_2 - c_1}{\beta} > \frac{A_1 - c_1}{2 - 2c_2}$$

If $\beta < 2 - 2c_2$

$$c_1 < \frac{2A_2 + 2\delta_2 Y_2 - 2A_2c_2 - 2\delta_2 Y_2 c_2 - A_1\beta}{2 - 2c_2 - \beta} \quad Line2$$

Then, if $\beta > 2 - 2c_2$

$$c_1 > \frac{2A_2 + 2\delta_2 Y_2 - 2A_2c_2 - 2\delta_2 Y_2 c_2 - A_1\beta}{2 - 2c_2 - \beta} \quad Line2'$$

$K_1' > KK2$ implies

$$\frac{A_1 + \delta_1 Y_1 - c_1}{\beta} > \frac{A_2 - c_1}{2 - 2c_2}$$

If $\beta < 2 - 2c_2$

$$c_1 < \frac{2A_1 + 2\delta_1 Y_1 - 2A_1c_2 - 2\delta_1 Y_1 c_2 - A_2\beta}{2 - 2c_2 - \beta} \quad Line3$$

Then, if $\beta > 2 - 2c_2$

$$c_1 > \frac{2A_1 + 2\delta_1 Y_1 - 2A_1c_2 - 2\delta_1 Y_1 c_2 - A_2\beta}{2 - 2c_2 - \beta} \quad Line3'$$

$K1 > KK2$ implies

$$\frac{A_1 - c_1}{\beta} > \frac{A_2 - c_1}{2 - 2c_2}$$

If $\beta < 2 - 2c_2$

$$c_1 < \frac{2A_1 - 2A_1c_2 - A_2\beta}{2 - 2c_2 - \beta} \qquad Line4$$

Then, if $\beta > 2 - 2c_2$

$$c_1 < \frac{2A_1 - 2A_1c_2 - A_2\beta}{2 - 2c_2 - \beta} \qquad Line4'$$

Moreover, when $c_2 = \frac{2-\beta}{2}$ lines are not defined.

The optimal c1 and c2 is found with exhaustive search method. For every combination of c1 and c2 corresponding q1 and q2 values are found and the profit is calculated. Then, the ones which give maximum profit are set to be optimal values.

■

7.3 Determining the ex-ante profits

The ex-post expected profits are denoted as $E_{\theta,Y_2}[\pi_1|Y_1]$ and $E_{\theta,Y_1}[\pi_2|Y_2]$ in Section 7.1 and 7.2. Under NBIS strategy, information is shared with the supplier but not with the buyers. Hence, in order to find the ex-ante profits for the buyers we need to take expectation the realization of the signals. When finding expected profit, $E[\pi_i|Y_i]$, buyer *i* takes expected quantity of buyer *j* to evaluate the profit value. This is due to the fact that Y_j is not shared with buyer *i*. On the other hand, supplier knows Y_1 and Y_2. Thus, to evaluate $E[\pi_s]$ for a given Y_1 and Y_2, exhaustive search is done and corresponding expected equilibrium quantities are considered and expectation taken over all possible Y_1 and Y_2 values.

Ex-ante profits of the buyers and the supplier

We are interested in $E_{\theta,Y_j}[\pi_i|Y_i]$ where $i,j \in \{1,2\}$ and $E_{Y_1,Y_2}[\pi_s]$

Note that Y_1 and Y_2 are independent random variables. Then, for a given k_1 and k_2, ex-ante profits are determined as follows;

1. Let $A_1' = \max\{A_1 + \delta_1 k_1, A_2 + \delta_2 k_2\}$ and define A_2' accordingly.

2. Follow the analysis in Section 7.1.2 for collaboration and in Section 7.2.2 for no-collaboration setting. Use the equilibrium quantities defined in Section 7.1.1 and Section 7.2.1. For a given k_1 and k_2, supplier does an exhaustive search, determines the corresponding $q_1(c_1, c_2, k_1, k_2)$, $q_2(c_1, c_2, k_1, k_2)$, and profit values, expectation is taken over Y_1 and Y_2 . Then, the ones which maximize the expected profit are set the optimal c_1 and c_2.

3. The ex-ante profit of buyer i is calculated by finding profit for all possible Y_i value by taking expectation over Y_j for all possible q_j values for each, and taking expectation over Y_i .

CHAPTER 8

COMPUTATIONAL STUDY

In the previous sections the analysis is done to determine the optimal order quantity and whole sales price for four different information sharing strategies with collaborative and non-collaborative buyers. In this chapter we present the results of computational study, the decisions taken by the buyers and the supplier under changing parameter settings and the decision that benefits the supply chain most.

The analysis is done to observe the following effects;

i) collaboration
ii) information sharing
iii) competition
iv) quantity discount
v) signal quality

In the following sections, first general information about parameter setting is given and then the results are presented.

8.1 Experimental design

The parameter setting is done as follow;

For the base demand three different cases are analyzed;

i) Buyer 1 $\gg$ Buyer 2. $A_1 = 2000$ and $A_2 = 750$
ii) Buyer 1 $>$ Buyer 2. $A_1 = 2000$ and $A_2 = 1300$
iii) Buyer 1 $\sim$ Buyer 2. $A_1 = 2000$ and $A_2 = 1600$

For the demand signal, the population parameters for both buyers are set as;

$\theta_L = -200$ and $\theta_H = 200$

The population is Bernoulli distributed, with parameter p and p is assumed to be uniformly distributed.

N1 and N2 show the sample size sample which is used to estimate market base. In order to observe the effect of collaboration, information sharing, competition and quantity discount $N1 = 7$ and $N2 = 3$ is used, while $N1 \in \{1,10\}$ and $N2 \in \{1,10\}$ is used to see the effect of signal quality.

Decreasing competition effect (β value) decreases the collaboration effect ($c_2 < \beta$). Hence, lowest β is set to be 0.5 and to see the effect of competition moderate and high values are also used which are 0.75 and 0.9. Moreover, $\varepsilon = 0.01$ is assumed.

The results are presented in the following sections.

8.2 Effect of collaboration

During the following analysis completion level is set to lowest value ($\beta = 0.5$)

If Buyer 1 $\gg$ Buyer 2

Table 3: The effect of collaboration for Buyer 1 >> Buyer 2

$A_1 = 2000$ $A_2 = 750$ N1 = 7 N2 = 3 $\beta = 0.5$		
	Collaboration	**No Collaboration**
$\boldsymbol{\pi_{TSC}: \pi_s + \pi_1 + \pi_2}$	NBIS	NBIS
$\boldsymbol{\pi_s}$	IBIS	IBIS
$\boldsymbol{\pi_1}$	IBNS/NBNS	IBNS/NBNS
$\boldsymbol{\pi_2}$	0	0

When the market base of Buyer 2 is much lower than Buyer 1 (Buyer 1 $\gg$ Buyer 2), it is substituted by the stronger market. Then, the optimal order quantity and profit of Buyer 2 is zero. This results in no difference between collaboration and no collaboration, because only Buyer 1 remains in the supply chain, no other party to collaborate. This is valid for all four strategies.

If Buyer 1 $>$ Buyer 2

Table 4: The effect of collaboration for Buyer 1 > Buyer 2

$A_1 = 2000$ $A_2 = 1300$ $N1 = 7$ $N2 = 3$ $\beta = 0.5$		
	Collaboration	**No Collaboration**
$\pi_{TSC}: \pi_s + \pi_1 + \pi_2$	IBIS	NBIS
π_s	NBIS	IBIS
π_1	IBNS	IBNS/NBNS
π_2	NBIS	0

When the market base of Buyer 2 is lower than Buyer 1 (Buyer 1 > Buyer 2) (the difference is not as big as the previous case), collaboration is beneficial for all parties for IBIS, IBNS, and NBNS strategies, but under NBIS strategy it is harmful for Buyer 1. The competition is not fierce and the market base of Buyer 1 is still greater than Buyer 2, so he prefers to collaborate in order to benefit from quantity discount. However, under NBIS case, supplier receives the information about the demand base and sets the whole sales price accordingly but the information is not shared with the buyers. Then, strong buyer (Buyer 1) does not want to collaborate in order not to lose the market power. The supplier, whole supply chain and the weak buyer (Buyer 2) always benefit from collaboration.

If Buyer 1 ~ Buyer 2

Table 5: The effect of collaboration for Buyer 1 ~ Buyer 2

$A_1 = 2000$ $A_2 = 1600$ $N1 = 7$ $N2 = 3$ $\beta = 0.5$		
	Collaboration	**No Collaboration**
$\pi_{TSC}: \pi_s + \pi_1 + \pi_2$	NBIS	NBNS
π_s	NBIS	NBIS
π_1	NBIS	NBNS
π_2	NBIS	NBIS

When the market base of Buyer 2 is similar to Buyer 1 (Buyer 1 ~ Buyer 2), the effect of the collaboration is still beneficial for the whole supply chain, supplier and weak buyer but harmful for the strong buyer. Since the market base of the weak buyer is high compared to the previous cases, even if the buyers are not engaged in collaboration, his market is not substituted entirely.

8.3 Effect of information sharing

During the following analysis completion level is set to lowest value ($\beta = 0.5$)

If Buyer 1 $\gg$ Buyer 2

Table 6: The effect of information sharing for Buyer 1 >> Buyer 2

$A_1 = 2000$ $A_2 = 750$ N1 = 7 N2 = 3 β = 0.5				
	IBIS	**IBNS**	**NBNS**	**NBIS**
π_{TSC}: $\pi_s + \pi_1 + \pi_2$	Coll/No Coll	Coll/No Coll	Coll/No Coll	Coll/No Coll
π_s	Coll/No Coll	Coll/No Coll	Coll/No Coll	Coll/No Coll
π_1	Coll/No Coll	Coll/No Coll	Coll/No Coll	Coll/No Coll
π_2	0	0	0	0

As mentioned before, when the market base of Buyer 2 is much lower than Buyer 1 (Buyer 1 $\gg$ Buyer 2), it is substituted by the stronger market and this is valid for all four strategies. Hence, collaboration and no collaboration strategies will result in same profit values for the supplier and the strong buyer. Moreover, the supplier is better off by receiving information about market base which is in line with the results from Zhang (2002) and Ha et al. (2011). Then, IBIS and NBIS strategies results in higher profit for the supplier and whole supply chain. On the other hand, Buyer 1 is worse off by information sharing and IBNS and NBNS are the preferred strategies for him.

If Buyer 1 $>$ Buyer 2

Table 7: The effect of information sharing for Buyer 1 > Buyer 2

$A_1 = 2000$ $A_2 = 1300$ N1 = 7 N2 = 3 β = 0.5				
	IBIS	**IBNS**	**NBNS**	**NBIS**
π_{TSC}: $\pi_s + \pi_1 + \pi_2$	Coll	Coll	Coll	Coll
π_s	Coll	Coll	Coll	Coll
π_1	Coll	Coll	Coll	No Coll

π_2	Coll	Coll	Coll	Coll

When the market base of Buyer 2 is moderately lower than Buyer 1 (Buyer 1 > Buyer 2) the supplier and weak buyer are always better under collaboration. If the buyers are not engaged in collaboration, Buyer 2 is substituted as like in the previous case. Information sharing increases the total supply chain profit and it is beneficial for the supplier and weak buyer. On the other hand, strong buyer does not prefer to share his demand information.

If Buyer 1 ~ Buyer 2

Table 8: The effect of information sharing for Buyer 1 ~ Buyer 2

$A_1 = 2000$ $A_2 = 1600$ N1 = 7 N2 = 3 β = 0.5				
	IBIS	**IBNS**	**NBNS**	**NBIS**
$\pi_{TSC}: \pi_s + \pi_1 + \pi_2$	Coll	Coll	Coll	Coll
π_s	Coll	Coll	Coll	Coll
π_1	No Coll	No Coll	No Coll	Coll
π_2	Coll	Coll	Coll	Coll

As mention before, the collaboration is beneficial for the whole supply chain, supplier and weak buyer when the market base of Buyer 2 is similar to Buyer 1 (Buyer 1 ~ Buyer 2). However, the strong buyer prefers not to collaborate in order not to lose his market power. Then, NBNS strategy with no collaboration is the preferred strategy for the strong buyer. Moreover, since information sharing is beneficial for the supplier and weak buyer NBIS with collaboration is preferred.

8.4 Effect of competition

If Buyer 1 ≫ Buyer 2

Table 9: The effect of competition for Buyer 1 >> Buyer 2

$A_1 = 2000$ $A_2 = 750$ N1 = 7 N2 = 3			
	β = 0.5	**β = 0.75**	**β = 0.9**
$\pi_{TSC}: \pi_s + \pi_1 + \pi_2$	NBIS Coll/No Coll	NBIS Coll/No Coll	IBIS Coll/No Coll
π_s	IBIS	IBIS	IBIS

	Coll/No Coll	Coll/No Coll	Coll/No Coll
π_1	IBNS/NBNS Coll/No Coll	IBNS/NBNS Coll/No Coll	IBNS/NBNS Coll/No Coll
π_2	0	0	0

Independent from the competition level, if the market base of Buyer 2 is much lower than Buyer 1, it is substituted by the stronger market and collaboration and no collaboration result in same profit values. Since supplier is better off by receiving information and strong buyer is worse off by sharing information IBIS and NBNS/IBNS are the preferred strategies for supplier and strong buyer respectively.

Buyer 1 $>$ Buyer 2

Table 10: The effect of competition for Buyer 1 > Buyer 2

A_1 = 2000 A_2 = 1300 N1 = 7 N2 = 3			
	β = 0.5	**β = 0.75**	**β = 0.9**
$\pi_{TSC}: \pi_s + \pi_1 + \pi_2$	IBIS Coll	NBIS Coll/No Coll	IBIS Coll/No Coll
π_s	NBIS Coll	IBIS Coll/No Coll	IBIS Coll/No Coll
π_1	IBNS Coll	IBNS/NBNS Coll/No Coll	IBNS/NBNS Coll/No Coll
π_2	NBIS Coll	0	0

When the market base of Buyer 2 is lower than Buyer 1, its market is substituted by the strong one under moderate and high competition. Then, if the competition is low, collaboration is beneficial for all parties, and if the competition is moderate or high collaboration or no collaboration gives the same profit values for the supplier and strong buyer.

Buyer 1 $\sim$ Buyer 2

Table 11: The effect of competition for Buyer 1 ~ Buyer 2

A_1 = 2000 A_2 = 1600 N1 = 7 N2 = 3			
	β = 0.5	**β = 0.75**	**β = 0.9**
$\pi_{TSC}: \pi_s + \pi_1 + \pi_2$	NBIS	NBIS	IBIS

	Coll	Coll	Coll
π_s	NBIS Coll	NBIS Coll	IBIS Coll
π_1	NBNS No Coll	IBNS/NBNS No Coll	IBNS/NBNS Coll/No Coll
π_2	NBIS Coll	NBIS Coll	NBIS Coll

When the market base is similar for both buyers, the weak one still exists in the market even if the competition is moderate and almost substituted when the competition is high.

8.5 Effect of quantity discount

If $c_2 = 0$ then this means there is no quantity discount and collaboration and no collaboration strategies gives same results. During the following analysis completion level is set to lowest value ($\beta = 0.5$)

If Buyer 1 $\gg$ Buyer 2

Table 12: The effect of quantity discount for Buyer 1 >> Buyer 2

$A_1 = 2000$ $A_2 = 750$ N1 = 7 N2 = 3 $\beta = 0.5$		
	$C_2 = 0$	**C_2 = Optimal**
π_{TSC}: $\pi_s + \pi_1 + \pi_2$	IBNS/NBNS Coll/No Coll	IBIS/NBIS Coll/No Coll
π_s	IBIS/NBIS Coll/No Coll	IBIS/NBIS Coll/No Coll
π_1	IBNS/NBNS Coll/No Col	IBNS/NBNS Coll/No Coll
π_2	0	0

When the difference between market bases is high, weak buyer is substituted and collaboration and no collaboration results in same values for profit functions. If there is quantity discount, it is beneficial for the supplier but not for the strong buyer.

Buyer 1 $>$ Buyer 2

Table 13: The effect of quantity discount for Buyer 1 > Buyer 2

A_1 = 2000 A_2 = 1300 N1 = 7 N2 = 3 β = 0.5		
	C_2 = 0	**C_2 = Optimal**
π_{TSC}: $\pi_s + \pi_1 + \pi_2$	IBNS Coll/No Coll	IBIS Coll
π_s	NBIS Coll/No Coll	NBIS Coll
π_1	IBNS Coll/No Coll	IBNS/NBNS No Coll
π_2	NBIS Coll/No Coll	NBIS Coll

If there is quantity discount, weak buyer prefers to collaborate otherwise he is out of competition. However, if there is no quantity discount, he still remains in the market.

Buyer 1 ~ Buyer 2

Table 14: The effect of quantity discount for Buyer 1 ~ Buyer 2

A_1 = 2000 A_2 = 1600 N1 = 7 N2 = 3 β = 0.5		
	C_2 = 0	**C_2 = Optimal**
π_{TSC}: $\pi_s + \pi_1 + \pi_2$	NBIS Coll/No Coll	NBIS Coll
π_s	NBIS Coll/No Coll	NBIS Coll
π_1	NBIS Coll/No Coll	NBNS No Coll
π_2	NBIS Coll/No Coll	NBIS Coll

Similar to the previous cases, supplier is better off with quantity discount. This time strong buyer is better off with quantity discount but no collaboration and weak buyer is better off without quantity discount.

8.6 Effects of signal quality

For following analysis the parameter setting is

A1 = 2000   A2 = 1300 beta 0.5 with the possibility of quantity discount (c2 is set to optimum value). Under this setting, collaboration is beneficial for all parties. Hence, the profit values under collaboration are used in the analysis. The changes in the profits of all parties are plotted and some examples are given in the Appendix.

The signal quality does not affect the choice of the strategies of the parties. The supplier prefers to receive information under collaboration. The strong buyer prefers not to share the information, but he benefits from collaboration because he still has the market power and benefits from quantity discount.

Under IBIS, increasing N2 or N1 for a given value of N1 or N2 decreases the profits of supplier and weak buyer. However, this increases the profit of strong buyer. Then, increasing signal quantity increases the profit of strong buyer however decreases the profits of supplier and weak buyer.

Under IBNS, increasing the signal quality of weak buyers benefits the buyer and harms the supplier. On the other hand, increasing the signal quality of strong buyer benefits the supplier but does not affect the buyers significantly.

NBNS results same as IBNS; increasing the signal quality of strong buyer is beneficial to supplier and increasing the quality of weak buyer is beneficial to the buyers.

Under NBIS, collaboration is beneficial for supplier and weak buyer and the results are same as IBIS; increasing signal quality decreases the profits of supplier and weak buyer. However, no collaboration is beneficial for strong buyer, and weak buyer is substituted in this case. Under no collaboration signal of weak buyer is not relevant, and increasing signal quality of strong buyer decreases the profit of supplier and strong buyer.

CHAPTER 9

CONCLUSION

In this thesis, a market with one supplier and two asymmetric competitive buyers is analyzed for the value of collaboration and information sharing. The supplier offers quantity discount and the buyers may or may not engage in collaboration on purchasing quantity. The market base is uncertain but buyers may receive a signal about it and decide to share the signal with the other parties of the supply chain. Four different cases with two alternatives such as collaboration and no collaboration [(i) information is shared between the buyers and with the supplier, (ii) information is shared only between the buyers, (iii) neither the buyers nor the supplier is shared the information, and (iv) buyers shared the information only with the supplier] are analyzed for optimal order quantity and whole sales price and computational analysis is conducted. The following results are obtained;

- When the market base of Buyer 2 is much lower than Buyer 1 (Buyer 1 $\gg$ Buyer 2), it is substituted by the stronger market under all strategies.
- When the market base of Buyer 2 is lower than Buyer 1 (Buyer 1 $>$ Buyer 2), if buyers do not engage in collaboration, weak buyer is substituted by the stronger one.
- If Buyer 1 $>$ Buyer 2, strong buyer benefits from collaboration.
- When the market base of Buyer 2 is similar to Buyer 1 (Buyer 1 $\sim$ Buyer 2), weak buyer exists in the market even if there is no collaboration.
- If Buyer 1 $\sim$ Buyer 2, strong buyer prefers not to engage in collaboration.

- When the difference in the market base of the buyers decrease and/or they engage in collaboration, the weak buyer enters to the whole supply chain. Then, the profit of the supplier always increases. On the other hand, the profit of the strong buyer increases first but if the difference in the base market gets smaller, then its profits starts to decrease.
- Sharing the market base information with the supplier benefits the whole supply together with the supplier and weak buyer, which is harmful for the strong buyer.
- As the competition increases, weak buyer is substituted by the strong one more easily.
- Quantity discount is beneficial to the supplier independent from information sharing strategy and difference in the market base.
- If the difference in the market base is high, strong buyer is better off without quantity discount and if the market bases are similar he is better off with quantity discount.
- If the market bases are similar then weak buyer is better off without quantity discount.
- Under IBIS, increasing signal quantity increases the profit of strong buyer however decreases the profits of supplier and weak buyer.
- Under IBNS, increasing the signal quality of weak buyers benefits the buyer and harms the supplier. On the other hand, increasing the signal quality of strong buyer benefits the supplier but does not affect the buyers significantly.
- NBNS results same as IBNS; increasing the signal quality of strong buyer is beneficial to supplier and increasing the quality of weak buyer is beneficial to the buyers.
- Under NBIS, collaboration is beneficial for supplier and weak buyer and the results are same as IBIS; increasing signal quality decreases the profits of supplier and weak buyer. However, no collaboration is beneficial for strong buyer, and weak buyer is substituted in this case. Under no collaboration

signal of weak buyer is not relevant, and increasing signal quality of strong buyer decreases the profit of supplier and strong buyer.

In our models, we consider a supply chain with one supplier and two asymmetric buyers. For the future work, the model can be generalized for *n* number of buyers. Moreover, a more general supply chain with *m* number of supplier and *n* number of buyers can be considered. The buyers are assumed to be uncapacited, this assumption can be relaxed for further analysis. Finally, buyers experience only procurement cost. In the future research additional cost (i.e. inventory cost) can be added to the model.

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APPENDIX A

APPENDIX TO CHAPTER 4

Collaboration

$\frac{\partial \pi_s}{\partial c_1}$ = (((c2 - 1)*(2*a1 - 2*c1) + (a2 - c1)*(b - c2))/(4*(c2 - 1)^2 - (b - c2)^2) + ((c2 - 1)*(2*a2 - 2*c1) + (a1 - c1)*(b - c2))/(4*(c2 - 1)^2 - (b - c2)^2))*((2*c2*(b + c2 - 2))/(4*(c2 - 1)^2 - (b - c2)^2) - 1) + (2*(c1 + c2*(((c2 - 1)*(2*a1 - 2*c1) + (a2 - c1)*(b - c2))/(4*(c2 - 1)^2 - (b - c2)^2) + ((c2 - 1)*(2*a2 - 2*c1) + (a1 - c1)*(b - c2))/(4*(c2 - 1)^2 - (b - c2)^2)))*(b + c2 - 2))/(4*(c2 - 1)^2 - (b - c2)^2)

$\frac{\partial \pi_s}{\partial c_2}$ = (c1 + c2*(((c2 - 1)*(2*a1 - 2*c1) + (a2 - c1)*(b - c2))/(4*(c2 - 1)^2 - (b - c2)^2) + ((c2 - 1)*(2*a2 - 2*c1) + (a1 - c1)*(b - c2))/(4*(c2 - 1)^2 - (b - c2)^2)))*((a1 - 2*a2 + c1)/(4*(c2 - 1)^2 - (b - c2)^2) + (a2 - 2*a1 + c1)/(4*(c2 - 1)^2 - (b - c2)^2) + (((c2 - 1)*(2*a1 - 2*c1) + (a2 - c1)*(b - c2))*(2*b + 6*c2 - 8))/(4*(c2 - 1)^2 - (b - c2)^2)^2 + (((c2 - 1)*(2*a2 - 2*c1) + (a1 - c1)*(b - c2))*(2*b + 6*c2 - 8))/(4*(c2 - 1)^2 - (b - c2)^2)^2) - (((c2 - 1)*(2*a1 - 2*c1) + (a2 - c1)*(b - c2))/(4*(c2 - 1)^2 - (b - c2)^2) + ((c2 - 1)*(2*a2 - 2*c1) + (a1 - c1)*(b - c2))/(4*(c2 - 1)^2 - (b - c2)^2))*(((c2 - 1)*(2*a1 - 2*c1) + (a2 - c1)*(b - c2))/(4*(c2 - 1)^2 - (b - c2)^2) + ((c2 - 1)*(2*a2 - 2*c1) + (a1 - c1)*(b - c2))/(4*(c2 - 1)^2 - (b - c2)^2) - c2*((a1 - 2*a2 + c1)/(4*(c2 - 1)^2 - (b - c2)^2) + (a2 - 2*a1 + c1)/(4*(c2 - 1)^2 - (b - c2)^2) + (((c2 - 1)*(2*a1 - 2*c1) + (a2 - c1)*(b - c2))*(2*b + 6*c2 - 8))/(4*(c2 - 1)^2 - (b - c2)^2)^2 + (((c2 - 1)*(2*a2 - 2*c1) + (a1 - c1)*(b - c2))*(2*b + 6*c2 - 8))/(4*(c2 - 1)^2 - (b - c2)^2)^2))

No collaboration

$\frac{\partial \pi_s}{\partial c_1}$ = (((c2*(b + 2*c2 - 2))/(4*(c2 - 1)^2 - b^2) - 1)*((c2 - 1)*(2*a1 - 2*c1) + b*(a2 - c1)))/(4*(c2 - 1)^2 - b^2) + (((c2*(b + 2*c2 - 2))/(4*(c2 - 1)^2 - b^2) - 1)*((c2 - 1)*(2*a2 - 2*c1) + b*(a1 - c1)))/(4*(c2 - 1)^2 - b^2) + ((c1 + (c2*((c2 - 1)*(2*a1 - 2*c1) + b*(a2 - c1)))/(4*(c2 - 1)^2 - b^2))*(b + 2*c2 - 2))/(4*(c2 - 1)^2 - b^2) + ((c1 + (c2*((c2 - 1)*(2*a2 - 2*c1) + b*(a1 - c1)))/(4*(c2 - 1)^2 - b^2))*(b + 2*c2 - 2))/(4*(c2 - 1)^2 - b^2)

$\frac{\partial \pi_s}{\partial c_2}$ = ((8*c2 - 8)*(c1 + (c2*((c2 - 1)*(2*a1 - 2*c1) + b*(a2 - c1)))/(4*(c2 - 1)^2 - b^2))*((c2 - 1)*(2*a1 - 2*c1) + b*(a2 - c1)))/(4*(c2 - 1)^2 - b^2)^2 - (((c2 - 1)*(2*a2 - 2*c1) + b*(a1 - c1))*(((c2 - 1)*(2*a2 - 2*c1) + b*(a1 - c1))/(4*(c2 - 1)^2 - b^2) + (c2*(2*a2 - 2*c1))/(4*(c2 - 1)^2 - b^2) - (c2*(8*c2 - 8)*((c2 - 1)*(2*a2 - 2*c1) + b*(a1 - c1)))/(4*(c2 - 1)^2 - b^2)^2))/(4*(c2 - 1)^2 - b^2) - ((c1 + (c2*((c2 - 1)*(2*a1 - 2*c1) + b*(a2 - c1)))/(4*(c2 - 1)^2 - b^2))*(2*a1 - 2*c1))/(4*(c2 - 1)^2 - b^2) - ((c1 + (c2*((c2 - 1)*(2*a2 - 2*c1) + b*(a1 - c1)))/(4*(c2 - 1)^2 - b^2))*(2*a2 - 2*c1))/(4*(c2 - 1)^2 - b^2) - (((c2 - 1)*(2*a1 - 2*c1) + b*(a2 - c1))*(((c2 - 1)*(2*a1 - 2*c1) + b*(a2 - c1))/(4*(c2 - 1)^2 - b^2) + (c2*(2*a1 - 2*c1))/(4*(c2 - 1)^2 - b^2) - (c2*(8*c2 - 8)*((c2 - 1)*(2*a1 - 2*c1) + b*(a2 - c1)))/(4*(c2 - 1)^2 - b^2)^2))/(4*(c2 - 1)^2 - b^2) + ((8*c2 - 8)*(c1 + (c2*((c2 - 1)*(2*a2 - 2*c1) + b*(a1 - c1)))/(4*(c2 - 1)^2 - b^2))*((c2 - 1)*(2*a2 - 2*c1) + b*(a1 - c1)))/(4*(c2 - 1)^2 - b^2)^2

Partial derivative after imposing c1 in the function

The roots of this equation;

$c_2^1 =$ (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)/(3*(16*a1^2 + 16*a2^2)) - ((22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))/((((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 - 10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^2 + ((22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))^3)^(1/2) + (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 - 10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^(1/3) + ((((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 - 10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^2 + ((22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))^3)^(1/2) + (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 - 10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^(1/3)

$c_2^2 =$ (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)/(3*(16*a1^2 + 16*a2^2)) +
((22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 +
8*a2^2*b + 24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 -
64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))/(2*((((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 -
64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 -
10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b -
24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 -
64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 +
22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^2 + ((22*a1^2*b^2 + 8*a1^2*b +
24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b +
24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 -
64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))^3)^(1/2) + (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 -
64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 -
10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b -
24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 -
64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 +
22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^(1/3)) - ((((8*a1^2*b + 8*a2^2*b +
48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 -
28*a1^2*b - 24*a1^2 - 10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 +
14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 +
48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b +
48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^2 + ((22*a1^2*b^2 +
8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b +
24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 -
64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))^3)^(1/2) + (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 -
64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 -
10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b -
24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 -
64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 +
22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^(1/3)/2 + (3^(1/2)*(((22*a1^2*b^2 +
8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b +
24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 -
64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))/((((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 -
64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 -
10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b -
24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 -
64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 +
22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^2 + ((22*a1^2*b^2 + 8*a1^2*b +
24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b +
24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 -
64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))^3)^(1/2) + (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 -
64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 -
10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b -

24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^(1/3) + ((((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 - 10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^2 + ((22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))^3)^(1/2) + (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 - 10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^(1/3))*i)/2

$c_2^3 =$ (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)/(3*(16*a1^2 + 16*a2^2)) + ((22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))/(2*((((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 - 10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^2 + ((22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))^3)^(1/2) + (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 - 10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^(1/3)) - ((((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 - 10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^2 + ((22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))^3)^(1/2) + (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 -

10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^(1/3)/2 - (3^(1/2)*(((22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))/((((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 - 10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^2 + ((22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))^3)^(1/2) + (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 - 10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^(1/3) + ((((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 - 10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^2 + ((22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2)/(3*(16*a1^2 + 16*a2^2)) - (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^2/(9*(16*a1^2 + 16*a2^2)^2))^3)^(1/2) + (8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)^3/(27*(16*a1^2 + 16*a2^2)^3) + (3*a1^2*b^3 + 14*a1^2*b^2 - 28*a1^2*b - 24*a1^2 - 10*a1*a2*b^3 - 4*a1*a2*b^2 + 8*a1*a2*b + 80*a1*a2 + 3*a2^2*b^3 + 14*a2^2*b^2 - 28*a2^2*b - 24*a2^2)/(2*(16*a1^2 + 16*a2^2)) - ((8*a1^2*b + 8*a2^2*b + 48*a1^2 + 48*a2^2 - 64*a1*a2*b)*(22*a1^2*b^2 + 8*a1^2*b + 24*a1^2 - 20*a1*a2*b^2 - 112*a1*a2*b + 48*a1*a2 + 22*a2^2*b^2 + 8*a2^2*b + 24*a2^2))/(6*(16*a1^2 + 16*a2^2)^2))^(1/3))*i)/2

APPENDIX B

APPENDIX TO CHAPTER 5

No collaboration

We present the analysis under c2>1-beta/2.

Case B: $c_2 > 1 - \beta/2$

Case I: $A_1 + \delta_1 Y_1 > A_2 + \delta_2 Y_2$ or equivalently

$$Y_2 < \frac{A_1 + \delta_1 Y_1 - A_2}{\delta_2} \qquad Line1$$

Case a:

$$c_1 > A_1 + \delta_1 Y_1$$

$$c_1 > A_2 + \delta_2 Y_2$$

$$Y_1 < \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 < \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

Then, q1=0, q2=0

Case b:

$$c_1 < A_1 + \delta_1 Y_1$$

$$c_1 > A_2 + \delta_2 Y_2$$

$$Y_1 > \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 < \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

Then, q1>0, q2=0

Case c:

$$c_1 < A_1 + \delta_1 Y_1$$

$$c_1 < A_2 + \delta_2 Y_2$$

$$Y_1 > \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 > \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

<u>*Case 1:*</u>

If

$$c_1 < \frac{2A_1(1 - c_2) + 2\delta_1 Y_1(1 - c_2) - A_2\beta - \delta_2 Y_2 \beta}{2 - 2c_2 - \beta} = c_1^{IBIS,NC,1}(c_2) \qquad (5.14)$$

and

$$c_1 \leq \frac{A_1\beta + \delta_1 Y_1 \beta - 2A_2 - 2\delta_2 Y_2 + 2A_2 c_2 + 2\delta_2 Y_2 c_2}{2c_2 - 2 + \beta} = c_1^{IBIS,NC,2}(c_2) \qquad (5.15)$$

From Eq. 5.14

$$Y_2 < \frac{m_4 + m_1 Y_1 - c_1}{m_2} \qquad Line2$$

From Eq. 5.15

$$Y_2 < \frac{m_4' + m_1' Y_1 - c_1}{m_2'} \qquad Line3$$

Then q1>0, q2=0

<u>Case 2:</u>

$$Y_2 < \frac{m_4 + m_1 Y_1 - c_1}{m_2} \qquad Line2$$

$$Y_2 > \frac{m_4' + m_1' Y_1 - c_1}{m_2'} \qquad Line3$$

Under Case B-I-c Case 2 is not possible.

<u>Case 3:</u>

$$Y_2 > \frac{m_4 + m_1 Y_1 - c_1}{m_2} \qquad Line2$$

$$Y_2 < \frac{m_4' + m_1' Y_1 - c_1}{m_2'} \qquad Line3$$

Multiple equilibria. To preserve the consistency with IBIS analysis, under case B-I-c-3 we assume q1>0, q2=0.

Case II: $A_1 + \delta_1 Y_1 < A_2 + \delta_2 Y_2$

Then

$$Y_2 > \frac{A_1 + \delta_1 Y_1 - A_2}{\delta_2} \qquad Line1$$

Case a:

$$c_1 > A_1 + \delta_1 Y_1$$

$$c_1 > A_2 + \delta_2 Y_2$$

$$Y_1 < \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 < \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

and q1=0, q2=0

Case b:

$$c_1 > A_1 + \delta_1 Y_1$$

$$c_1 < A_2 + \delta_2 Y_2$$

$$Y_1 < \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 > \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

and q1=0, q2>0

Case c:

$$c_1 < A_1 + \delta_1 Y_1$$

$$c_1 < A_2 + \delta_2 Y_2$$

$$Y_1 > \frac{c_1 - A_1}{\delta_1} \qquad Line4$$

$$Y_2 > \frac{c_1 - A_2}{\delta_2} \qquad Line5$$

<u>*Case 1:*</u>

If

$$c_1 < \frac{2A_2(1-c_2) + 2\delta_2 Y_2(1-c_2) - A_1\beta - \delta_1 Y_1 \beta}{2 - 2c_2 - \beta} \qquad 4.24'$$

and

$$c_1 < \frac{A_2\beta + \delta_2 Y_2 \beta - 2A_1 - 2\delta_1 Y_1 + 2A_1 c_2 + 2\delta_1 Y_1 c_2}{2c_2 - 2 + \beta} \qquad 4.25'$$

From Eq. 4.24'

$$Y_2 > \frac{m_4' + m_1' Y_1 - c_1}{m_2'} \qquad Line3$$

From Eq. 4.25'

$$Y_2 > \frac{m_4 + m_1 Y_1 - c_1}{m_2} \qquad Line2$$

Then q1=0, q2>0

<u>*Case 2:*</u>

$$Y_2 > \frac{m_4' + m_1' Y_1 - c_1}{m_2'} \qquad Line3$$

$$Y_2 < \frac{m_4 + m_1 Y_1 - c_1}{m_2} \qquad Line2$$

Under Case B-II-c Case 2 is not possible.

Case 3:

$$Y_2 < \frac{m_4' + m_1' Y_1 - c_1}{m_2'} \qquad Line3$$

$$Y_2 > \frac{m_4 + m_1 Y_1 - c_1}{m_2} \qquad Line2$$

Multiple equilibria. To preserve the consistency with IBIS analysis, we assume q1=0, q2>0.

APPENDIX C

APPENDIX TO CHAPTER 6

Collaboration

Buyer's problem

1) If $\boldsymbol{A_2} < c_{\mathbf{1}} < \boldsymbol{A_1}$**:**

Under this setting, it holds that $J1 > JJ2 > J2$, $JJ1 > JJ2 > J2$. Depending on the values of $J_1, J_2, J'_1, J'_2, JJ1, JJ2$ and Y_1 and Y_2, 9 possible cases may exist, as discussed below. And for each of the setting the corresponding equilibria are defined.

Setting A. If $Y_1 > 0$ and $Y_2 > 0$ then $J'_1 > J1$, $J'_2 > J2$.

1) $JJ1 > J2$
   a) $J'_2 > JJ1$
      i) $J'_1 > JJ2$ → Case 1
      ii) $J'_1 < JJ2$ → does not exist since $J'_1 > J1 > JJ2$
   b) $J'_2 \leq JJ1$
      i) $J'_1 > JJ2$ → Case 2
      ii) $J'_1 < JJ2$ → does not exist since $J'_1 > J1 > JJ2$
2) $JJ1 < J2$ → does not exist since $JJ1 > J2$
   a) $J'_2 > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

Setting B. If $Y_1 > 0$ and $Y_2 < 0$ then $J_1' > J1$, $J_2' < J2$.

1) $JJ1 > J2$

a) $J_2' > JJ1$ → does not exist since $J_2' < J2 < JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → does not exist

b) $J_2' \leq JJ1$

i) $J_1' > JJ2$ → Case 3

ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$

2) $JJ1 < J2$ → does not exist $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

Setting C. If $Y_1 < 0$ and $Y_2 > 0$ then $J_1' < J1$, $J_2' > J2$.

1) $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 4

ii) $J_1' \leq JJ2$ → Case 5

b) $J_2' \leq JJ1$

i) $J_1' > JJ2$ → Case 6

ii) $J_1' \leq JJ2$ → Case 7

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

Setting D. If $Y_1 < 0$ and $Y_2 < 0$ then $J_1' < J1$, $J_2' < J2$.

1) $JJ1 > J2$

a) $J_2' > JJ1$ does not exist since $J_2' < J2 < JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → does not exist

b) $J_2' \leq JJ1$

i) $J_1' > JJ2$ → Case 8

ii) $J_1' \leq JJ2$ → Case 9

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

Setting E. If $Y_1 > 0$ and $Y_2 = 0$ then $J_1' > J1$, $J_2' = J2$.

1) $JJ1 > J2$

   a) $J_2' > JJ1$ → not possible

      i) $J_1' > JJ2$

      ii) $J_1' < JJ2$

   b) $J_2' \leq JJ1$

      i) $J_1' > JJ2$ → Case 2

      ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

   a) $J_2' > JJ1$

      i) $J_1' > JJ2$

      ii) $J_1' < JJ2$

   b) $J_2' < JJ1$

      i) $J_1' > JJ2$

      ii) $J_1' < JJ2$

Setting F. If $Y_1 < 0$ and $Y_2 = 0$ then $J_1' < J1,\ \ J_2' = J2$.

1) $JJ1 > J2$

   a) $J_2' > JJ1$ → not possible

      i) $J_1' > JJ2$

      ii) $J_1' < JJ2$

   b) $J_2' \leq JJ1$

      i) $J_1' > JJ2$ → Case 6

      ii) $J_1' \leq JJ2$ → Case 7

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

   a) $J_2' > JJ1$

      i) $J_1' > JJ2$

      ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

Setting G. If $Y_1 = 0$ and $Y_2 > 0$ then $J_1' = J1$, $J_2' > J2$.

1) $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 1

ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$

b) $J_2' \leq JJ1$

i) $J_1' > JJ2$ → Case 2

ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

Setting H. If $Y_1 = 0$ and $Y_2 < 0$ then $J_1' = J1$, $J_2' < J2$.

1) $JJ1 > J2$

a) $J_2' > JJ1$ → does not exist since $J_2' < J2 < JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → does not exist

b) $J_2' \leq JJ1$

i) $J_1' > JJ2$ → Case 3

ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$

2) $JJ1 < J2$ → does not exist $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

Setting I. If $Y_1 = 0$ and $Y_2 = 0$ then $J_1' = J1$, $J_2' = J2$.

1) $JJ1 > J2$

a) $J_2' > JJ1$ → not possible

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' \le JJ1$

i) $J_1' > JJ2$ → Case 2

ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

The equilibrium quantities for each case are as follows. Whenever $q_i = D_0^i + D_1^i Y_i$, D_0^i is obtained by intersecting Eqns. 6.3 and 6.5.

Cases	Equilibrium quantities
1-4	$q_1 = D_0^1 + D_1^1 Y_1 \quad q_2 = D_0^2 + D_1^2 Y_2$
2-3-6-7-8-9	$q_1 = JJ1 + D_1^1 Y_1 \quad q_2 = 0$
5	$q_1 = 0 \quad q_2 = JJ2 + D_1^2 Y_2$

2) If $\boldsymbol{A_2 < A_1 < c_1}$

By definition, it holds that $J1 > J2$, $JJ1 > JJ2$. Depending on the values of J_1, J_2, $J_1^{'}$, $J_2^{'}$, $JJ1$, $JJ2$ and Y_1 and Y_2, 14 possible cases may exist, as discussed below. And for each of the setting the corresponding equilibria are defined.

Setting A. If $Y_1 > 0$ and $Y_2 > 0$

AA. If $J_1' > 0$ and $J_2' > 0$, then $J_1' > JJ2$ and $J_2' > JJ1$

- If $J2 < JJ1$ and $J1 < JJ2$

1) $JJ1 > J2$
   a) $J_2' > JJ1$
      i) $J_1' > JJ2$ → Case 1
      ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$
   b) $J_2' < JJ1$ → does not exist since $J_2' > JJ1$
      iii) $J_1' > JJ2$
      iv) $J_1' < JJ2$
2) $JJ1 < J2$ → does not exist since $JJ1 > J2$
   a) $J_2' > JJ1$
      i) $J_1' > JJ2$
      ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 1

ii) $J_1' < JJ2$ → does not exist since $J_1' > J1 > JJ2$

b) $J_2' < JJ1$ → does not exist since $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

AB. If $J_1' > 0$ and $J_2' < 0$

- If $J2 < JJ1$ and $J1 < JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 1''

ii) $J_1' < JJ2$ → does not exist since $J_1' > JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → Case 2''

ii) $J_1' < JJ2$ → does not exist since $J_1' > JJ2$

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 1

ii) $J_1' < JJ2$ → does not exist since $J_1' > JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → Case 2

ii) $J_1' < JJ2$ → does not exist since $J_1' > JJ2$

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

BA. If $J_1' < 0$ and $J_2' > 0$

- If $J2 < JJ1$ and $J1 < JJ2$
    1) $JJ1 > J2$
        a) $J_2' > JJ1$
            i) $J_1' > JJ2$ → Case 3''
            ii) $J_1' < JJ2$ → Case 4''
        b) $J_2' < JJ1$ → does not exist $J_2' > JJ1$
            i) $J_1' > JJ2$
            ii) $J_1' < JJ2$
    2) $JJ1 < J2$ → does not exist since $JJ1 > J2$
        a) $J_2' > JJ1$
            i) $J_1' > JJ2$
            ii) $J_1' < JJ2$
        b) $J_2' < JJ1$
            i) $J_1' > JJ2$
            ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$
    1) $JJ1 > J2$
        a) $J_2' > JJ1$
            i) $J_1' > JJ2$ → Case 1
            ii) $J_1' < JJ2$ → does not exist since $J_1' > JJ2$
        b) $J_2' < JJ1$ → does not exist since $J_2' > JJ1$
            i) $J_1' > JJ2$
            ii) $J_1' < JJ2$
    2) $JJ1 < J2$ → does not exist since $JJ1 > J2$
        a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

BB. If $J_1' < 0$ and $J_2' < 0$

- If $J2 < JJ1$ and $J1 < JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 5''

ii) $J_1' < JJ2$ → Case 5''

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → Case 5''

ii) $J_1' < JJ2$ → Case 5''

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 5''

ii) $J_1' < JJ2$ → does not exist since $J_1' > JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → Case 5''

ii) $J_1' < JJ2$ → does not exist since $J_1' > JJ2$

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

Setting B. If $Y_1 > 0$ and $Y_2 < 0$ then $J_2' < 0$.

AA. is not possible since $J_2' < 0$.

AB. If $J_1' > 0$ and $J_2' < 0$

- If $J2 < JJ1$ and $J1 < JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$ → does not exist since $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → Case 6''

ii) $J_1' < JJ2$ → does not exist $J_1' > JJ2$

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$ →does not exist $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → Case 3

ii) $J_1' < JJ2$ → does not exist since $J_1' > JJ2$

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

BA. is not possible since $J_2' < 0$

BB. If $J_1' < 0$ and $J_2' < 0$

- If $J2 < JJ1$ and $J1 < JJ2$
    1) $JJ1 > J2$
        a) $J_2' > JJ1$ → does not exist since $J_2' < JJ1$
            i) $J_1' > JJ2$
            ii) $J_1' < JJ2$
        b) $J_2' < JJ1$
            i) $J_1' > JJ2$ → Case 7''
            ii) $J_1' < JJ2$ → Case 7''
    2) $JJ1 < J2$ → does not exist since $JJ1 > J2$
        a) $J_2' > JJ1$
            i) $J_1' > JJ2$
            ii) $J_1' < JJ2$
        b) $J_2' < JJ1$
            i) $J_1' > JJ2$
            ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$
    1) $JJ1 > J2$
        a) $J_2' > JJ1$ →does not exist $J_2' < JJ1$
            i) $J_1' > JJ2$
            ii) $J_1' < JJ2$
        b) $J_2' < JJ1$
            i) $J_1' > JJ2$ → Case 7''
            ii) $J_1' < JJ2$ → does not exist since $J_1' > JJ2$
    2) $JJ1 < J2$ → does not exist since $JJ1 > J2$
        a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

Setting C. If $Y_1 < 0$ and $Y_2 > 0$ then $J_1' < 0$

AA. is not possible since $J_1' < 0$

AB. is not possible since $J_1' < 0$

BA. If $J_1' < 0$ and $J_2' > 0$

- If $J2 < JJ1$ and $J1 < JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → Case 8''

b) $J_2' < JJ1$ → does not exist

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 4

ii) $J_1' < JJ2$ → Case 5

b) $J_2' < JJ1$ → does not exist

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

BB. If $J_1' < 0,\ J_2' < 0$

- If $J2 < JJ1$ and $J1 < JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → Case 8''

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → Case 8''

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 8''

ii) $J_1' < JJ2$ → Case 8''

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → Case 8''

ii) $J_1' < JJ2$ → Case 8''

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

Setting D. If $Y_1 < 0$ and $Y_2 < 0$ then $J_1' < 0,\ J_2' < 0$.

AA, **AB** and **BA** are not possible since $J_1' < 0,\ J_2' < 0$

BB If $J_1' < 0,\ J_2' < 0$

- If $J2 < JJ1$ and $J1 < JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$ does not exist since $J_2' < J2 < JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → does not exist

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → Case 9’’

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$ does not exist since $J_2' < J2 < JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → does not exist

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → Case 9’’

ii) $J_1' < JJ2$ → Case 9’’

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

Setting E. If $Y_1 > 0$ and $Y_2 = 0$

AA. Not possible

AB. If $J_1' > 0$ and $J_2' < 0$

- If $J2 < JJ1$ and $J1 < JJ2$
  1) $JJ1 > J2$
     a) $J_2' > JJ1$ → not possible
        i) $J_1' > JJ2$
        ii) $J_1' < JJ2$
     b) $J_2' < JJ1$
        i) $J_1' > JJ2$ → Case 2''
        ii) $J_1' < JJ2$ → does not exist since $J_1' > JJ2$
  2) $JJ1 < J2$ → does not exist since $JJ1 > J2$
     a) $J_2' > JJ1$
        i) $J_1' > JJ2$
        ii) $J_1' < JJ2$
     b) $J_2' < JJ1$
        i) $J_1' > JJ2$
        ii) $J_1' < JJ2$
- If $J2 < JJ1$ and $J1 > JJ2$
  1) $JJ1 > J2$
     a) $J_2' > JJ1$ → not possible
        iii) $J_1' > JJ2$
        iv) $J_1' < JJ2$
     b) $J_2' < JJ1$
        iii) $J_1' > JJ2$ → Case 2
        iv) $J_1' < JJ2$ → does not exist since $J_1' > JJ2$
  2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

BA. Not possible

BB. If $J_1' < 0$ and $J_2' < 0$

- If $J2 < JJ1$ and $J1 < JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$ → not possible

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → Case 5''

ii) $J_1' < JJ2$ → Case 5''

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$ → not possible

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → Case 5''

ii) $J_1' < JJ2$ → does not exist since $J_1' > JJ2$

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

Setting F. If $Y_1 < 0$ and $Y_2 = 0$ then $J_1' < 0$

AA. is not possible since $J_1' < 0$

AB. is not possible since $J_1' < 0$

BA. Is not possible

BB. If $J_1' < 0,\ J_2' < 0$

- If $J2 < JJ1$ and $J1 < JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$ → not possible

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → Case 8''

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$ → not possible

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → Case 8''

ii) $J_1' < JJ2$ → Case 8''

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

Setting G. If $Y_1 = 0$ and $Y_2 > 0$

AA. is not possible since $J_1' < 0$

AB. is not possible since $J_1' < 0$

BA. If $J_1' < 0$ and $J_2' > 0$

- If $J2 < JJ1$ and $J1 < JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → Case 8''

b) $J_2' < JJ1$ → does not exist

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$ → Case 4

ii) $J_1' < JJ2$ → not possible

b) $J_2' < JJ1$ → does not exist

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

BB If $J_1' < 0$, $J_2' < 0$

- If $J2 < JJ1$ and $J1 < JJ2$

  1) $JJ1 > J2$

     a) $J_2' > JJ1$

        i) $J_1' > JJ2$ → does not exist

        ii) $J_1' < JJ2$ → Case 8''

     b) $J_2' < JJ1$

        i) $J_1' > JJ2$ → does not exist

        ii) $J_1' < JJ2$ → Case 8''

  2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

     a) $J_2' > JJ1$

        i) $J_1' > JJ2$

        ii) $J_1' < JJ2$

     b) $J_2' < JJ1$

        i) $J_1' > JJ2$

        ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$

  1) $JJ1 > J2$

     a) $J_2' > JJ1$

        i) $J_1' > JJ2$ → Case 8''

        ii) $J_1' < JJ2$ → Case 8''

     b) $J_2' < JJ1$

        i) $J_1' > JJ2$ → Case 8''

        ii) $J_1' < JJ2$ → Case 8''

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

Setting H. If $Y_1 = 0$ and $Y_2 < 0$

AA. is not possible since $J_1' < 0$

AB. is not possible since $J_1' < 0$

BA. Is not possible

BB If $J_1' < 0,\ J_2' < 0$

- If $J2 < JJ1$ and $J1 < JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$ → not possible

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → Case 8''

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$
    1) $JJ1 > J2$
        a) $J_2' > JJ1$ → not possible
            i) $J_1' > JJ2$
            ii) $J_1' < JJ2$
        b) $J_2' < JJ1$
            i) $J_1' > JJ2$ → Case 8''
            ii) $J_1' < JJ2$ → Case 8''
    2) $JJ1 < J2$ → does not exist since $JJ1 > J2$
        a) $J_2' > JJ1$
            i) $J_1' > JJ2$
            ii) $J_1' < JJ2$
        b) $J_2' < JJ1$
            i) $J_1' > JJ2$
            ii) $J_1' < JJ2$

Setting H. If $Y_1 = 0$ and $Y_2 = 0$

AA. is not possible since $J_1' < 0$

AB. is not possible since $J_1' < 0$

BA. Is not possible

BB. If $J_1' < 0$, $J_2' < 0$

- If $J2 < JJ1$ and $J1 < JJ2$
    1) $JJ1 > J2$
        a) $J_2' > JJ1$ → not possible
            i) $J_1' > JJ2$
            ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → does not exist

ii) $J_1' < JJ2$ → Case 8''

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

- If $J2 < JJ1$ and $J1 > JJ2$

1) $JJ1 > J2$

a) $J_2' > JJ1$ → not possible

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$ → Case 8''

ii) $J_1' < JJ2$ → Case 8''

2) $JJ1 < J2$ → does not exist since $JJ1 > J2$

a) $J_2' > JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

b) $J_2' < JJ1$

i) $J_1' > JJ2$

ii) $J_1' < JJ2$

The equilibrium quantities for each case are as follows. Whenever $q_i = D_0^i + D_1^i Y_i$, D_0^i is obtained by intersecting Eqns. 6.3 and 6.5.

Cases	Equilibrium quantities	
1-4-1''-3''	$q_1 = D_0^1 + D_1^1 Y_1$	$q_2 = D_0^2 + D_1^2 Y_2$
2-3-2''-6''	$q_1 = JJ1 + D_1^1 Y_1$	$q_2 = 0$
5-4''	$q_1 = 0$	$q_2 = JJ2 + D_1^2 Y_2$
5''-7''-8''-9''	$q_1 = 0$	$q_2 = 0$

Supplier's problem

Similar to Property 2, we derive the supplier's profit expression for the cases $A_2 < c_1 < A_1$ and $c_1 > A_1$.

1) If $A_2 < c_1 < A_1$:

$J_1' > JJ2$ implies

$$Y_1 > \frac{(\beta - c_2)(A_2 - c_1) - (2 - 2c_2)(A_1 - c_1)}{(2 - 2c_2)\delta_1} \qquad Line1$$

Note $Line1 < 0$ because

$$(\beta - c_2)(A_2 - c_1) - (2 - 2c_2)(A_1 - c_1) < 0$$

$$(\beta - c_2) < (2 - 2c_2)$$

$J_2' > JJ1$ implies

$$Y_2 > \frac{(\beta - c_2)(A_1 - c_1) - (2 - 2c_2)(A_2 - c_1)}{(2 - 2c_2)\delta_2} \qquad Line2$$

If $JJ1 > J2$, then $Line2 > 0$ since

$$\frac{A_1 - c_1}{2 - 2c_2} > \frac{A_2 - c_1}{\beta - c_2}$$

$Line2 < 0$ is not possible since it always holds $JJ1 > J2$.

Then the sample space can be partitioned as in Figure 27 below.

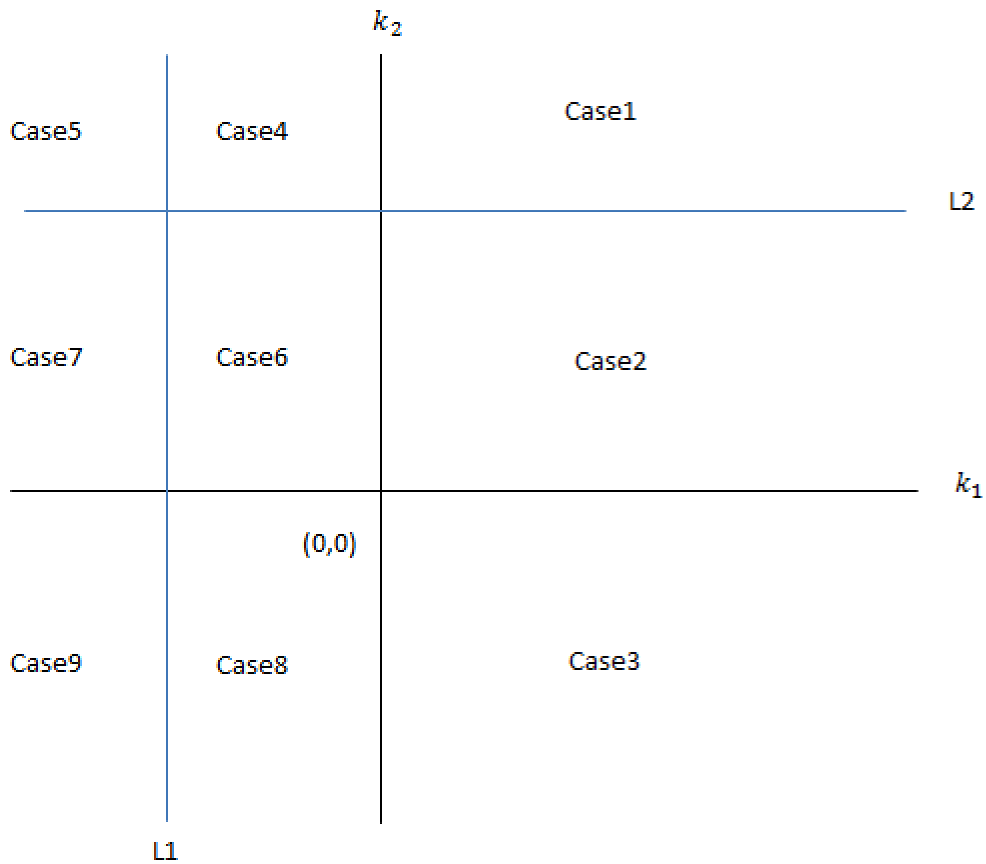

Figure 27 Given $\boldsymbol{A_2 < c_1 < A_1}$, and JJ1>J2, how the equilibrium cases change with the realization of $\boldsymbol{Y_1}$ and $\boldsymbol{Y_2}$.

Let $E_{Y_1,Y_2}[\pi_s(c_1, c_2)]$ denote the ex-ante profit of the supplier, where the equilibrium quantities of the buyers are $q_1(c_1, c_2, Y_1)$ and $q_2(c_1, c_2, Y_2)$. Note that ex-ante and ex-post profits are the same for the supplier. For a given c_1 and c_2, $E_{Y_1,Y_2}[\pi_s(c_1,c_2)]$ is obtained as;

$$
\begin{aligned}
E_{Y_1,Y_2}[\pi_s(c_1,c_2)]
&= \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s(Case1(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2 \epsilon S_2 \\ 0 \leq k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s(Case2(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \geq 0}} \pi_s(Case3(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s(Case4(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2 \epsilon S_2 \\ 0 \leq k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s(Case6(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ Line1 < k_1 < 0}} \pi_s(Case8(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 > Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \leq Line1}} \pi_s(Case5(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2 \epsilon S_2 \\ 0 \leq k_2 \leq Line2}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \leq Line1}} \pi_s(Case7(k_1,k_2))p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2 \epsilon S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \epsilon S_1 \\ k_1 \leq Line1}} \pi_s(Case9(k_1,k_2))p_1^{k_1}p_2^{k_2}
\end{aligned}
$$

2) If $\boldsymbol{A_2 < A_1 < c_1}$**:**

$J_1' > JJ2$ implies

$$Y_1 > \frac{(\beta - c_2)(A_2 - c_1) - (2 - 2c_2)(A_1 - c_1)}{(2 - 2c_2)\delta_1} \qquad Line1$$

If $J1 < JJ2$ then $Line1 > 0$ because

$$(\beta - c_2)(A_2 - c_1) - (2 - 2c_2)(A_1 - c_1) > 0$$

$$(\beta - c_2) < (2 - 2c_2)$$

If $J1 > JJ2$ then $Line1 < 0$

$J_2' > JJ1$ implies

$$Y_2 > \frac{(\beta - c_2)(A_1 - c_1) - (2 - 2c_2)(A_2 - c_1)}{(2 - 2c_2)\delta_2} \qquad Line2$$

If $JJ1 > J2$, then $Line2 > 0$ since

$$\frac{A_1 - c_1}{2 - 2c_2} > \frac{A_2 - c_1}{\beta - c_2}$$

$Line2 < 0$ is not possible since $JJ1 > J2$.

Moreover, $J_1' > 0$ implies

$$Y_1 > \frac{c_1 - A_1}{\delta_1} \qquad Line3$$

Note Line3>Line1 .

Similarly, $J_2' > 0$ implies

$$Y_2 > \frac{c_1 - A_2}{\delta_2} \qquad Line4$$

Note Line4>Line2.

Then the sample space can be partitioned as in Figure 28 and 29 below.

If $J1 < JJ2$

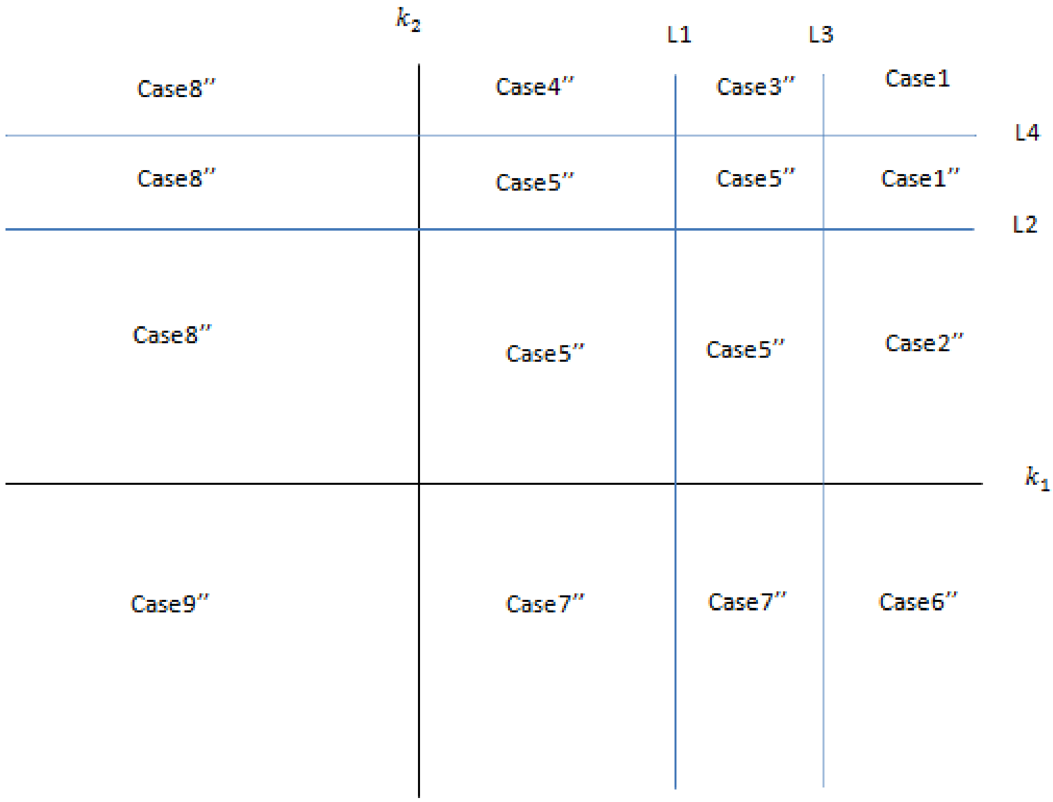

Figure 28 Given $\boldsymbol{c_1 > A_1}$, and J1<JJ2, how the equilibrium cases change with the realization of $\boldsymbol{Y_1}$ and $\boldsymbol{Y_2}$.

For a given c_1 and c_2, $E_{Y_1,Y_2}[\pi_s(c_1, c_2)]$ is obtained as;

$$
\begin{aligned}
&E_{Y_1,Y_2}[\pi_s(c_1,c_2)] \\
&= \sum_{\substack{k_2\in S_2\\ k_2\geq Line4}} \sum_{\substack{k_1\in S_1\\ k_1>Line3}} \pi_s\big(Case1(k_1,k_2)\big)p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2\in S_2\\ Line2<k_2<Line4}} \sum_{\substack{k_1\in S_1\\ k_1>Line3}} \pi_s\big(Case1''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2\in S_2\\ 0<k_2<Line2}} \sum_{\substack{k_1\in S_1\\ k_1>Line3}} \pi_s\big(Case2''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2\in S_2\\ k_2<0}} \sum_{\substack{k_1\in S_1\\ k_1>Line3}} \pi_s\big(Case6''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2\in S_2\\ k_2>Line4}} \sum_{\substack{k_1\in S_1\\ Line1<k_1<Line3}} \pi_s\big(Case3''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2\in S_2\\ Line2<k_2<Line4}} \sum_{\substack{k_1\in S_1\\ Line1<k_1<Line3}} \pi_s\big(Case5''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2\in S_2\\ 0<k_2<Line2}} \sum_{\substack{k_1\in S_1\\ Line1<k_1<Line3}} \pi_s\big(Case5''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2\in S_2\\ k_2<0}} \sum_{\substack{k_1\in S_1\\ Line1<k_1<Line3}} \pi_s\big(Case7''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2\in S_2\\ k_2>Line4}} \sum_{\substack{k_1\in S_1\\ 0<k_1<Line1}} \pi_s\big(Case4''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2\in S_2\\ Line2<k_2<Line4}} \sum_{\substack{k_1\in S_1\\ 0<k_1<Line1}} \pi_s\big(Case5''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2\in S_2\\ 0<k_2<Line2}} \sum_{\substack{k_1\in S_1\\ 0<k_1<Line1}} \pi_s\big(Case5''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2} \\
&+ \sum_{\substack{k_2\in S_2\\ k_2<0}} \sum_{\substack{k_1\in S_1\\ 0<k_1<Line1}} \pi_s\big(Case7''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}
\end{aligned}
$$

$$+ \sum_{\substack{k_2 \in S_2 \\ k_2 > Line4}} \sum_{\substack{k_1 \in S_1 \\ k_1 < 0}} \pi_s\big(Case8''(k_1, k_2)\big) p_1^{k_1} p_2^{k_2}$$
$$+ \sum_{\substack{k_2 \in S_2 \\ Line2 < k_2 < Line4}} \sum_{\substack{k_1 \in S_1 \\ k_1 < 0}} \pi_s\big(Case8''(k_1, k_2)\big) p_1^{k_1} p_2^{k_2}$$
$$+ \sum_{\substack{k_2 \in S_2 \\ 0 < k_2 < Line2}} \sum_{\substack{k_1 \in S_1 \\ k_1 < 0}} \pi_s\big(Case8''(k_1, k_2)\big) p_1^{k_1} p_2^{k_2}$$
$$+ \sum_{\substack{k_2 \in S_2 \\ k_2 < 0}} \sum_{\substack{k_1 \in S_1 \\ k_1 < 0}} \pi_s\big(Case9''(k_1, k_2)\big) p_1^{k_1} p_2^{k_2}$$

If $J1 > JJ2$

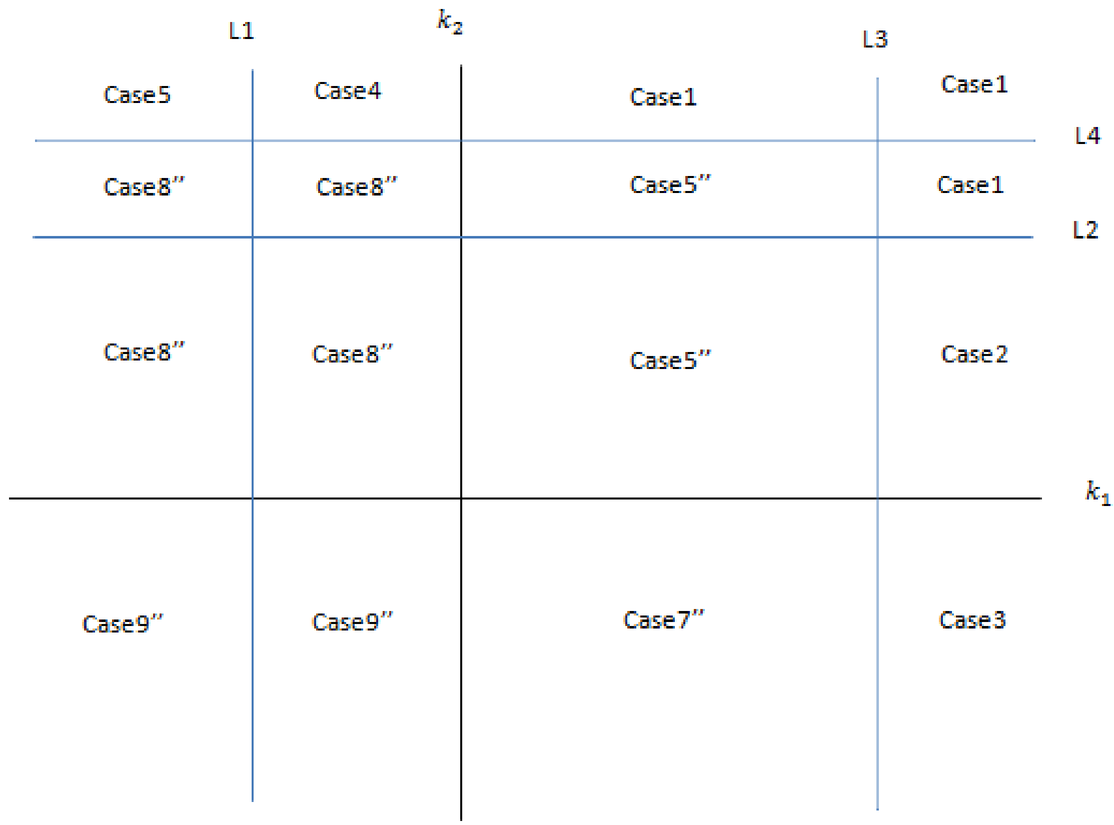

Figure 29 Given $\boldsymbol{c_1 > A_1}$, and J1>JJ2, how the equilibrium cases change with the realization of $\boldsymbol{Y_1}$ and $\boldsymbol{Y_2}$

For a given c_1 and c_2, $E_{Y_1,Y_2}[\pi_s(c_1, c_2)]$ is obtained as;

$$E_{Y_1,Y_2}[\pi_s(c_1,c_2)]$$

$$= \sum_{\substack{k_2\in S_2\\ k_2>Line4}} \sum_{\substack{k_1\in S_1\\ k_1>Line3}} \pi_s\big(Case1(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\in S_2\\ Line2<k_2<Line4}} \sum_{\substack{k_1\in S_1\\ k_1>Line3}} \pi_s\big(Case1(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\in S_2\\ 0<k_2<Line2}} \sum_{\substack{k_1\in S_1\\ k_1>Line3}} \pi_s\big(Case2(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\in S_2\\ k_2<0}} \sum_{\substack{k_1\in S_1\\ k_1>Line3}} \pi_s\big(Case3(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\in S_2\\ k_2>Line4}} \sum_{\substack{k_1\in S_1\\ 0<k_1<Line3}} \pi_s\big(Case1(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\in S_2\\ Line2<k_2<Line4}} \sum_{\substack{k_1\in S_1\\ 0<k_1<Line3}} \pi_s\big(Case5''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\in S_2\\ 0<k_2<Line2}} \sum_{\substack{k_1\in S_1\\ 0<k_1<Line3}} \pi_s\big(Case5''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\in S_2\\ k_2<0}} \sum_{\substack{k_1\in S_1\\ 0<k_1<Line3}} \pi_s\big(Case7''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\in S_2\\ k_2>Line4}} \sum_{\substack{k_1\in S_1\\ Line1<k_1<0}} \pi_s\big(Case4(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\in S_2\\ Line2<k_2<Line4}} \sum_{\substack{k_1\in S_1\\ Line1<k_1<0}} \pi_s\big(Case8''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+ \sum_{\substack{k_2\in S_2\\ 0<k_2<Line2}} \sum_{\substack{k_1\in S_1\\ Line1<k_1<0}} \pi_s\big(Case8''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+\sum_{\substack{k_2\in S_2\\k_2<0}}\sum_{\substack{k_1\in S_1\\Line1<k_1<0}}\pi_s\big(Case9''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+\sum_{\substack{k_2\in S_2\\k_2>Line4}}\sum_{\substack{k_1\in S_1\\k_1<Line1}}\pi_s\big(Case5(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+\sum_{\substack{k_2\in S_2\\Line2<k_2<Line4}}\sum_{\substack{k_1\in S_1\\k_1<Line1}}\pi_s\big(Case8''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+\sum_{\substack{k_2\in S_2\\0<k_2<Line2}}\sum_{\substack{k_1\in S_1\\k_1<Line1}}\pi_s\big(Case8''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

$$+\sum_{\substack{k_2\in S_2\\k_2<0}}\sum_{\substack{k_1\in S_1\\k_1<Line1}}\pi_s\big(Case9''(k_1,k_2)\big)p_1^{k_1}p_2^{k_2}$$

The supplier searches exhaustively over c_1, c_2 region to find c_1 and c_2 that maximizes $E_{Y_1,Y_2}[\pi_s]$.

No collaboration

Buyer's problem

If $A_2 < c_1 < A_1$, then K1>K2, KK1>KK2, KK1>K2

Setting A. If $Y_1 > 0$ and $Y_2 > 0$, then $K_1' > K1$, $K_2' > K2$

A) $KK1 < K2$ → not possible
   1) $K1 > KK2$
      a) $K_2' > KK1$
         i) $K_1' > KK2$
         ii) $K_1' < KK2$
      b) $K_2' < KK1$
         i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1'

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

b) $K_2' < KK1$

iii) $K_1' > KK2$ → Case 2'

iv) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

2) $K1 < KK2$ → not possible

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

Setting B. If $Y_1 > 0$ and $Y_2 < 0$, then $K_1' > K1$, $K_2' < K2$

A) $KK1 < K2$ → not possible

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K_1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ →does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 7'

ii) $K_1' < KK2$ → does not exist $K_1' > K_1 > KK2$

2) $K_1 < KK2$ →not possible

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

Setting C. If $Y_1 < 0$ and $Y_2 > 0$, then $K_1' < K1$, $K_2' > K2$

A) $KK1 < K2$ → not possible

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 10'

ii) $K_1' < KK2$ → Case 11'

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 12'

ii) $K_1' < KK2$ → Case 13'

2) $K1 < KK2$ → not possible

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

Setting D. If $Y_1 < 0$ and $Y_2 < 0$ then $K_1' < K1$, $K_2' < K2$.

A) $KK1 < K2$ → not possible

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ → does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 16'

ii) $K_1' < KK2$ → Case 17'

2) $K1 < KK2$ → not possible

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

Setting E. If $Y_1 > 0$ and $Y_2 = 0$, then $K_1' > K1$, $K_2' = K2$

A) $KK1 < K2$ → not possible

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ → not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 2'

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

2) $K1 < KK2$ → not possible

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

Setting F. If $Y_1 < 0$ and $Y_2 = 0$, then $K_1' < K1$, $K_2' = K2$

A) $KK1 < K2$ → not possible

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ → not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 12'

ii) $K_1' < KK2$ → Case 13'

2) $K1 < KK2$ → not possible

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

Setting G. If $Y_1 = 0$ and $Y_2 > 0$, then $K_1' = K1$, $K_2' > K2$

A) $KK1 < K2$ → not possible

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1'

ii) $K_1' < KK2$ → does not exist since $K_1' = K_1 > KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 2'

ii) $K_1' < KK2$ → does not exist since $K_1' = K_1 > KK2$

2) $K1 < KK2$ → not possible

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

Setting H. If $Y_1 = 0$ and $Y_2 < 0$, then $K_1' = K1$, $K_2' < K2$

A) $KK1 < K2$ → not possible

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K_1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ →does not exist since $KK1 > K2 = K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 7'

ii) $K_1' < KK2$ → does not exist $K_1' = K_1 > KK2$

2) $K_1 < KK2$ →not possible

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

Setting I. If $Y_1 = 0$ and $Y_2 = 0$, then $K_1' = K1$, $K_2' = K2$

A) $KK1 < K2$ → not possible

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ → not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 2'

ii) $K_1' < KK2$ → does not exist since $K_1' = K_1 > KK2$

2) $K1 < KK2$ → not possible

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

The equilibrium quantities for each case are as follows. Whenever $q_i = D_0^i + D_1^i Y_i$, D_0^i is obtained by intersecting Eqns. 6.13 and 6.15.

Cases	**Equilibrium quantities**
1'-10'	$q_1 = D_0^1 + D_1^1 Y_1$ $q_2 = D_0^2 + D_1^2 Y_2$
2'-7'-12'-13'-16'-17'	$q_1 = KK1 + D_1^1 Y_1$ $q_2 = 0$
11'	$q_1 = 0$ $q_2 = KK2 + D_1^2 Y_2$

If $\boldsymbol{A_2 < A_1 < c_1}$, then K1>K2, KK1>KK2

Setting A. If $Y_1 > 0$ and $Y_2 > 0$, then $K_1' > K1$, $K_2' > K2$

AA. if $K_1' > 0$ and $K_2' > 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1

ii) $K_1' < KK2$ → does not exist since $K_1' > K1 > KK2$

b) $K_2' < KK1$ → does not exist since $KK1 < K2 < K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1'

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

b) $K_2' < KK1$→ not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 3'

ii) $K_1' < KK2$ → not possible

b) $K_2' < KK1$ → not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

AB. if $K_1' > 0$ and $K_2' < 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1

ii) $K_1' < KK2$ → does not exist since $K_1' > K1 > KK2$

b) $K_2' < KK1$ → does not exist since $KK1 < K2 < K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1'

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 2'

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 3'

ii) $K_1' < KK2$ → not possible

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 5'

ii) $K_1' < KK2$ → not possible

BA. if $K_1' < 0$ and $K_2' > 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1

ii) $K_1' < KK2$ → does not exist since $K_1' > K1 > KK2$

b) $K_2' < KK1$ → does not exist since $KK1 < K2 < K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1'

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

b) $K_2' < KK1$ → does not exist

i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 3'

ii) $K_1' < KK2$ → Case 4'

b) $K_2' < KK1$ → does not exist

i) $K_1' > KK2$

ii) $K_1' < KK2$

BB. if $K_1' < 0$ and $K_2' < 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1''

ii) $K_1' < KK2$ → does not exist since $K_1' > K1 > KK2$

b) $K_2' < KK1$ → does not exist since $KK1 < K2 < K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1''

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 1''

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1''

ii) $K_1' < KK2$ → Case 1''

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 1''

ii) $K_1' < KK2$ → Case 1''

Setting B. If $Y_1 > 0$ and $Y_2 < 0$, then $K_1' > K1$, $K_2' < K2$

AA. not possible

AB. If $K_1' > 0$ and $K_2' < 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 2

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 3

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

2) $K_1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ →does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 7'

ii) $K_1' < KK2$ → does not exist $K_1' > K_1 > KK2$

2) $K_1 < KK2$

a) $K_2' > KK1$ → does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 8'

ii) $K_1' < KK2$ → does not exist

BA. Not possible

BB. If $K_1' < 0$ and $K_2' < 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 2''

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 2''

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

2) $K_1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ →does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 2''

ii) $K_1' < KK2$ → does not exist

2) $K_1 < KK2$

a) $K_2' > KK1$ → does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 2''

ii) $K_1' < KK2$ → Case 2''

Setting C. If $Y_1 < 0$ and $Y_2 > 0$, then $K_1' < K1$, $K_2' > K2$

AA. is not possible

AB. is not possible

BA. If $K_1' < 0$ and $K_2' > 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 4

ii) $K_1' < KK2$ → Case 5

b) $K_2' < KK1$ → does not exist since $KK1 < K2 < K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 10'

ii) $K_1' < KK2$ → Case 11'

b) $K_2' < KK1$ → not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → does not exist since $K_1' < K1 < KK2$

ii) $K_1' < KK2$ → Case 14'

b) $K_2' < KK1$→ not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

BB. If $K_1' < 0$ and $K_2' < 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 3''

ii) $K_1' < KK2$ → Case 3''

b) $K_2' < KK1$ → does not exist

i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 3''

ii) $K_1' < KK2$ → Case 3''

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 3''

ii) $K_1' < KK2$ → Case 3''

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → not possible

ii) $K_1' < KK2$ → Case 3''

b) $K_2' < KK1$

i) $K_1' > KK2$ → not possible

ii) $K_1' < KK2$ → Case 3''

Setting D. If $Y_1 < 0$ and $Y_2 < 0$ then $K_1' < K1$, $K_2' < K2$.

AA, **AB**, and **BA** not possible

BB. if $K_1' < 0$ and $K_2' < 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 4''

ii) $K_1' < KK2$ → Case 4''

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 4''

ii) $K_1' < KK2$ → Case 4''

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ → does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 4''

ii) $K_1' < KK2$ → Case 4''

2) $K1 < KK2$

a) $K_2' > KK1$ → does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' < KK1$

i) $K_1' > KK2$ → does not exit since $K_1' < K1 < KK2$

ii) $K_1' < KK2$ → Case 4''

Setting E. If $Y_1 > 0$ and $Y_2 = 0$, then $K_1' > K1$, $K_2' = K2$

AA. not possible

AB. if $K_1' > 0$ and $K_2' < 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1

ii) $K_1' < KK2$ → does not exist since $K_1' > K1 > KK2$

b) $K_2' < KK1$ → does not exist since $KK1 < K2 = K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$→ not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 2'

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

2) $K1 < KK2$

a) $K_2' > KK1$ → not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 5'

ii) $K_1' < KK2$ → not possible

BA. Not possible

BB. if $K_1' < 0$ and $K_2' < 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1''

ii) $K_1' < KK2$ → does not exist since $K_1' > K1 > KK2$

b) $K_2' < KK1$ → does not exist since $KK1 < K2 = K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ → not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 1''

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

2) $K1 < KK2$

a) $K_2' > KK1$ → not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 1''

ii) $K_1' < KK2$ → Case 1''

Setting F. If $Y_1 < 0$ and $Y_2 = 0$, then $K_1' < K1$, $K_2' = K2$

AA. is not possible

AB. is not possible

BA. Not possible

BB. If $K_1' < 0$ and $K_2' < 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 3''

ii) $K_1' < KK2$ → Case 3''

b) $K_2' < KK1$ → does not exist

i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ → not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 3''

ii) $K_1' < KK2$ → Case 3''

2) $K1 < KK2$

a) $K_2' > KK1$ → not possible

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → not possible

ii) $K_1' < KK2$ → Case 3''

Setting G. If $Y_1 = 0$ and $Y_2 > 0$, then $K_1' = K1$, $K_2' > K2$

AA. not possible

AB. not possible

BA. if $K_1' < 0$ and $K_2' > 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1

ii) $K_1' < KK2$ → does not exist since $K_1' = K1 > KK2$

b) $K_2' < KK1$ → does not exist since $KK1 < K2 < K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1'

ii) $K_1' < KK2$ → does not exist since $K_1' = K_1 > KK2$

b) $K_2' < KK1$ → does not exist

i) $K_1' > KK2$

ii) $K_1' < KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → not possible

ii) $K_1' < KK2$ → Case 4'

b) $K_2' < KK1$ → does not exist

i) $K_1' > KK2$

ii) $K_1' < KK2$

BB. if $K_1' < 0$ and $K_2' < 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1''

ii) $K_1' < KK2$ → does not exist since $K_1' = K1 > KK2$

b) $K_2' < KK1$ → does not exist since $KK1 < K2 < K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

2) $K1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 1''

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 1''

ii) $K_1' < KK2$ → does not exist since $K_1' > K_1 > KK2$

2) $K1 < KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → not possible

ii) $K_1' < KK2$ → Case 1''

b) $K_2' < KK1$

i) $K_1' > KK2$ → not possible

ii) $K_1' < KK2$ → Case 1''

Setting H. If $Y_1 = 0$ and $Y_2 < 0$, then $K_1' = K1$, $K_2' < K2$

AA. not possible

AB. not possible

BA. Not possible

BB. If $K_1' < 0$ and $K_2' < 0$

A) $KK1 < K2$

1) $K1 > KK2$

a) $K_2' > KK1$

i) $K_1' > KK2$ → Case 2''

ii) $K_1' < KK2$ → does not exist since $K_1' = K_1 > KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 2''

ii) $K_1' < KK2$ → does not exist since $K_1' = K_1 > KK2$

2) $K_1 < KK2$ → not possible when $KK1 < K2$

a) $K_2' > KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

b) $K_2' < KK1$

i) $K_1' > KK2$

ii) $K_1' < KK2$

B) $KK1 > K2$

1) $K1 > KK2$

a) $K_2' > KK1$ →does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' < KK1$

i) $K_1' > KK2$ → Case 2''

ii) $K_1' < KK2$ → does not exist

2) $K_1 < KK2$

a) $K_2' > KK1$ → does not exist since $KK1 > K2 > K_2'$

i) $K_1' > KK2$ → does not exist

ii) $K_1' < KK2$ → does not exist

b) $K_2' < KK1$

i) $K_1' > KK2$ → not possible

ii) $K_1' < KK2$ → Case 2''

The equilibrium quantities for each case are as follows. Whenever $q_i = D_0^i + D_1^i Y_i$, D_0^i is obtained by intersecting Eqns. 6.13 and 6.15.

Cases	**Equilibrium quantities**	
1-2-4-1'-3'-10'	$q_1 = D_0^1 + D_1^1 Y_1$	$q_2 = D_0^2 + D_1^2 Y_2$
3-2'-5'-7'-8'	$q_1 = KK1 + D_1^1 Y_1$	$q_2 = 0$
5-4'-11'-14'	$q_1 = 0$	$q_2 = KK2 + D_1^2 Y_2$

1’’-2’’-3’’-4’’	$q_1 = 0$	$q_2 = 0$

Supplier’s problem

If $\boldsymbol{A_2 < c_1 < A_1}$

$KK1 > K2$ implies

$$\beta(A_1 - c_1) > (A_2 - c_1)(2 - 2c_2)$$

$K1 > KK2$ implies

$$(2 - 2c_2)(A_1 - c_1) > (A_2 - c_1)\beta$$

$K1 < KK2$ implies

$$(2 - 2c_2)(A_1 - c_1) < (A_2 - c_1)\beta$$

Note that this case is not possible. If $KK1 > K2$ then $Line2 > 0$

If $K1 > KK2$ then $Line1 < 0$

Then,

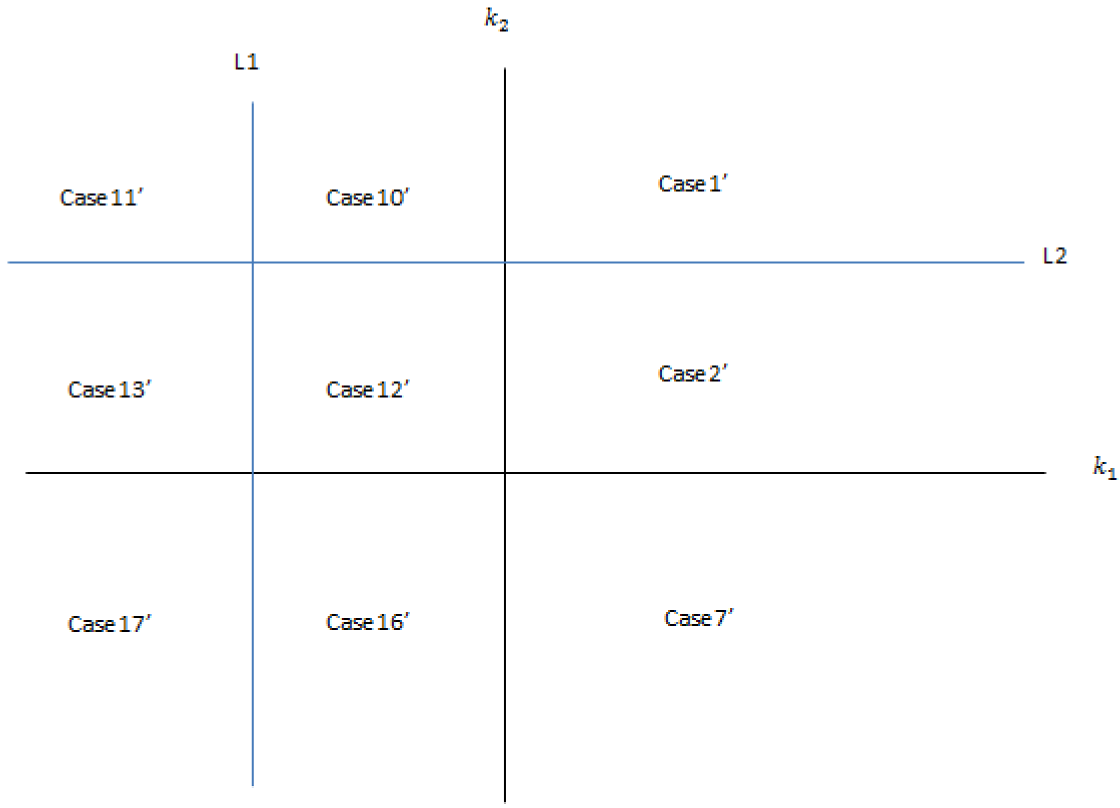

Figure 30 How equilibrium cases change as $\boldsymbol{Y_1}$ and $\boldsymbol{Y_2}$ realizations change, given $\boldsymbol{A_2 < c_1 < A_1}$, $\boldsymbol{KK1 > K2}$ and $\boldsymbol{K1 > KK2}$

$$E_{Y_1,Y_2}[\pi_s(c_1,c_2)]$$

$$= \sum_{k_2>line2}\sum_{k_1\geq 0}\pi_s\big(Case1'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{0\leq k_2\leq line2}\sum_{k_1\geq 0}\pi_s\big(Case2'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{k_2<0}\sum_{k_1\geq 0}\pi_s\big(Case7'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{k_2>line2}\sum_{line1<k_1<0}\pi_s\big(Case10'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{0\leq k_2\leq line2}\sum_{line1<k_1<0}\pi_s\big(Case12'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{k_2<0}\sum_{line1<k_1<0}\pi_s\big(Case16'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{k_2>line2}\sum_{k_1\leq line1}\pi_s\big(Case11'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{0\leq k_2\leq line2}\sum_{k_1\leq line1}\pi_s\big(Case13'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{k_2<0}\sum_{k_1\leq line1}\pi_s\big(Case17'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

If $\boldsymbol{A_2 < A_1 < c_1}$

A) $KK1 < K2$ implies

$$\beta(A_1 - c_1) < (A_2 - c_1)(2 - 2c_2) \rightarrow (2 - 2c_2) < \beta$$

$K1 > KK2$ implies

$$(2 - 2c_2)(A_1 - c_1) > (A_2 - c_1)\beta$$

$K1 < KK2$ is not possible when $KK1 < K2$.

$K_2' > KK1$ implies

$$Y_2 > \frac{\beta(A_1 - c_1) - (2 - 2c_2)(A_2 - c_1)}{(2 - 2c_2)\delta_2} \qquad Line2$$

If $KK1 < K2$ then $Line2 < 0$

$K_1' > KK2$ implies

$$Y_1 > \frac{\beta(A_2 - c_1) - (2 - 2c_2)(A_1 - c_1)}{(2 - 2c_2)\delta_1} \qquad Line1$$

If $K1 > KK2$ then $Line1 < 0$

$K_1' > 0$ implies

$$Y_1 > \frac{c_1 - A_1}{\delta_1} \qquad Line3$$

$K_2' > 0$ implies

$$Y_1 > \frac{c_1 - A_2}{\delta_2} \qquad Line4$$

Then, only one case with $KK1 < K2$ and $K1 > KK2$ (where $Line2 < 0$ and $Line1 < 0$)

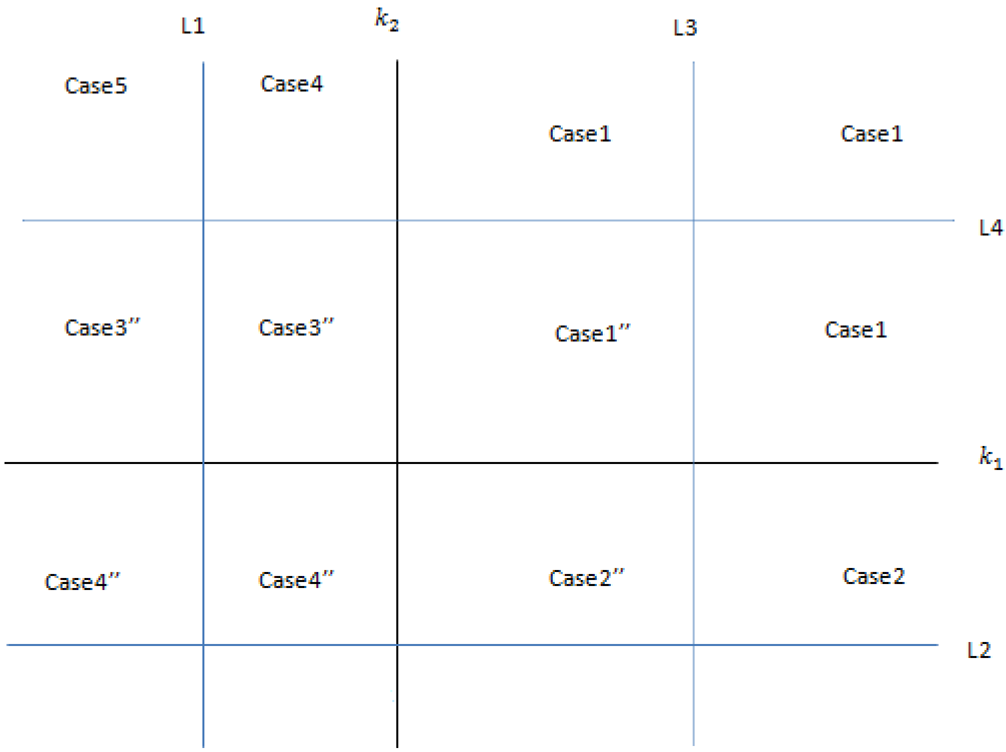

Figure 31 How equilibrium cases change as $\boldsymbol{Y_1}$ and $\boldsymbol{Y_2}$ realizations change, given $\boldsymbol{c_1 > A_1}$, $\boldsymbol{KK1 < K2}$ and $\boldsymbol{K1 > KK2}$

$$
\begin{aligned}
&E_{Y_1,Y_2}[\pi_s(c_1,c_2)] \\
&\quad = \sum_{k_2 \geq line4} \sum_{k_1 > line3} \pi_s\big(Case1(k_1,k_2)\big) p_1^{k_1} p_1^{k_2} \\
&\quad + \sum_{0 \leq k_2 < line4} \sum_{k_1 > line3} \pi_s\big(Case1(k_1,k_2)\big) p_1^{k_1} p_1^{k_2} \\
&\quad + \sum_{line2 < k_2 < 0} \sum_{k_1 > line3} \pi_s\big(Case2(k_1,k_2)\big) p_1^{k_1} p_1^{k_2} \\
&\quad + \sum_{k_2 \leq line2} \sum_{k_1 > line3} \pi_s\big(Case3(k_1,k_2)\big) p_1^{k_1} p_1^{k_2} \\
&\quad + \sum_{k_2 \geq line4} \sum_{0 \leq k_1 \leq line3} \pi_s\big(Case1(k_1,k_2)\big) p_1^{k_1} p_1^{k_2} \\
&\quad + \sum_{0 \leq k_2 < line4} \sum_{0 \leq k_1 \leq line3} \pi_s\big(Case1''(k_1,k_2)\big) p_1^{k_1} p_1^{k_2} \\
&\quad + \sum_{k_2 < 0} \sum_{0 \leq k_2 \leq line3} \pi_s\big(Case2''(k_1,k_2)\big) p_1^{k_1} p_1^{k_2} \\
&\quad + \sum_{k_2 \geq line4} \sum_{line1 < k_1 < 0} \pi_s\big(Case4(k_1,k_2)\big) p_1^{k_1} p_1^{k_2} \\
&\quad + \sum_{0 \leq k_2 < line4} \sum_{line1 < k_1 < 0} \pi_s\big(Case3''(k_1,k_2)\big) p_1^{k_1} p_1^{k_2} \\
&\quad + \sum_{k_2 < 0} \sum_{line1 < k_1 < 0} \pi_s\big(Case4''(k_1,k_2)\big) p_1^{k_1} p_1^{k_2} \\
&\quad + \sum_{k_2 \geq line4} \sum_{k_1 \leq line1} \pi_s\big(Case5(k_1,k_2)\big) p_1^{k_1} p_1^{k_2} \\
&\quad + \sum_{0 \leq k_2 < line4} \sum_{k_1 \leq line1} \pi_s\big(Case3''(k_1,k_2)\big) p_1^{k_1} p_1^{k_2} \\
&\quad + \sum_{k_2 < 0} \sum_{k_1 \leq line1} \pi_s\big(Case4''(k_1,k_2)\big) p_1^{k_1} p_1^{k_2}
\end{aligned}
$$

B) $KK1 > K2$ implies

$$\beta(A_1 - c_1) > (A_2 - c_1)(2 - 2c_2)$$

$K1 > KK2$ implies

$$(2 - 2c_2)(A_1 - c_1) > (A_2 - c_1)\beta$$

$K1 < KK2$ implies

$$(2 - 2c_2)(A_1 - c_1) < (A_2 - c_1)\beta$$

If $KK1 > K2$ then $Line2 > 0$

If $K1 > KK2$ then $Line1 < 0$ and if $K1 < KK2$ then $Line1 > 0$

Then, Case (a) with $KK1 > K2$ and $K1 > KK2$ where $Line2 > 0$ and $Line1 < 0$ and Case (b) with $KK1 > K2$ and $K1 < KK2$ where $Line2 > 0$ and $Line1 > 0$.

Case (a):

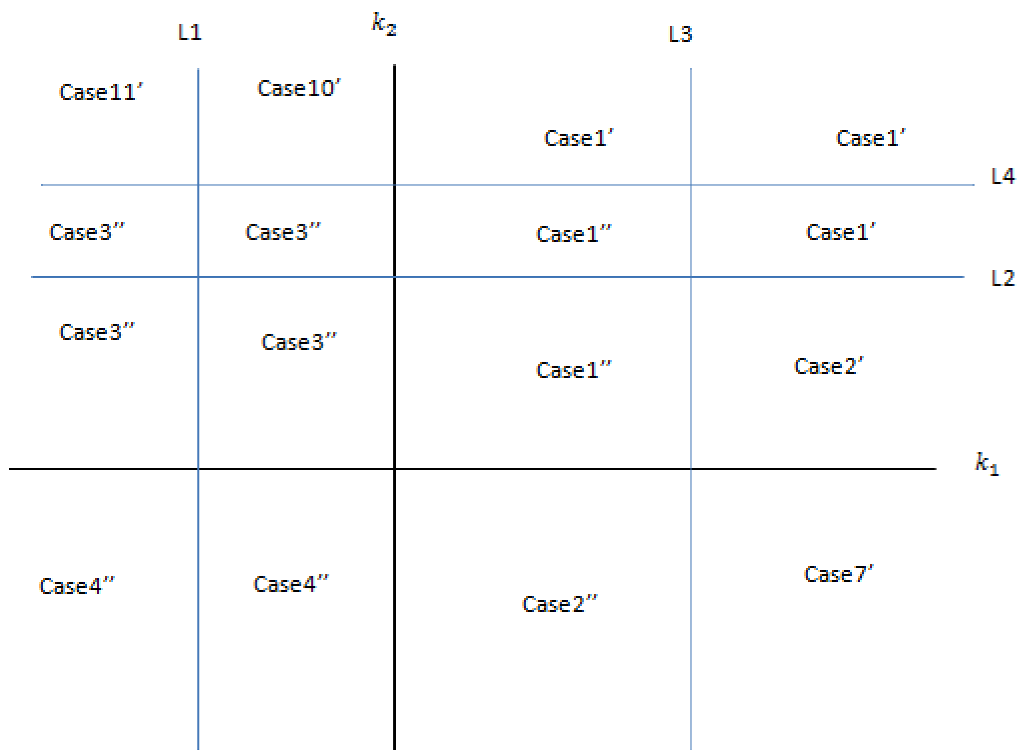

Figure 32 How equilibrium cases change as $\boldsymbol{Y_1}$ and $\boldsymbol{Y_2}$ realizations change, given $\boldsymbol{c_1 > A_1}$, $\boldsymbol{KK1 > K2}$ and $\boldsymbol{K1 > KK2}$

$$
\begin{aligned}
&E_{Y_1,Y_2}[\pi_s(c_1,c_2)] \\
&\quad = \sum_{k_2 \geq line4} \sum_{k_1 > line3} \pi_s\big(Case1'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2} \\
&\quad + \sum_{line2 < k_2 < line4} \sum_{k_1 > line3} \pi_s\big(Case1'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2} \\
&\quad + \sum_{0 \leq k_2 \leq line2} \sum_{k_1 > line3} \pi_s\big(Case2'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2} \\
&\quad + \sum_{k_2 < 0} \sum_{k_1 > line3} \pi_s\big(Case7'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2} \\
&\quad + \sum_{k_2 \geq line4} \sum_{0 \leq k_1 \leq line3} \pi_s\big(Case1'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2} \\
&\quad + \sum_{0 \leq k_2 < line4} \sum_{0 \leq k_1 \leq line3} \pi_s\big(Case1''(k_1,k_2)\big)p_1^{k_1}p_1^{k_2} \\
&\quad + \sum_{k_2 < 0} \sum_{0 \leq k_1 \leq line3} \pi_s\big(Case2''(k_1,k_2)\big)p_1^{k_1}p_1^{k_2} \\
&\quad + \sum_{k_2 \geq line4} \sum_{line1 < k_1 < 0} \pi_s\big(Case10'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2} \\
&\quad + \sum_{0 \leq k_2 < line4} \sum_{line1 < k_1 < 0} \pi_s\big(Case3''(k_1,k_2)\big)p_1^{k_1}p_1^{k_2} \\
&\quad + \sum_{k_2 < 0} \sum_{line1 < k_1 < 0} \pi_s\big(Case4''(k_1,k_2)\big)p_1^{k_1}p_1^{k_2} \\
&\quad + \sum_{k_2 \geq line2} \sum_{k_1 \leq line1} \pi_s\big(Case11'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2} \\
&\quad + \sum_{0 \leq k_2 < line4} \sum_{k_1 \leq line1} \pi_s\big(Case3''(k_1,k_2)\big)p_1^{k_1}p_1^{k_2} \\
&\quad + \sum_{k_2 < 0} \sum_{k_1 \leq line1} \pi_s\big(Case4''(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}
\end{aligned}
$$

Case (b):

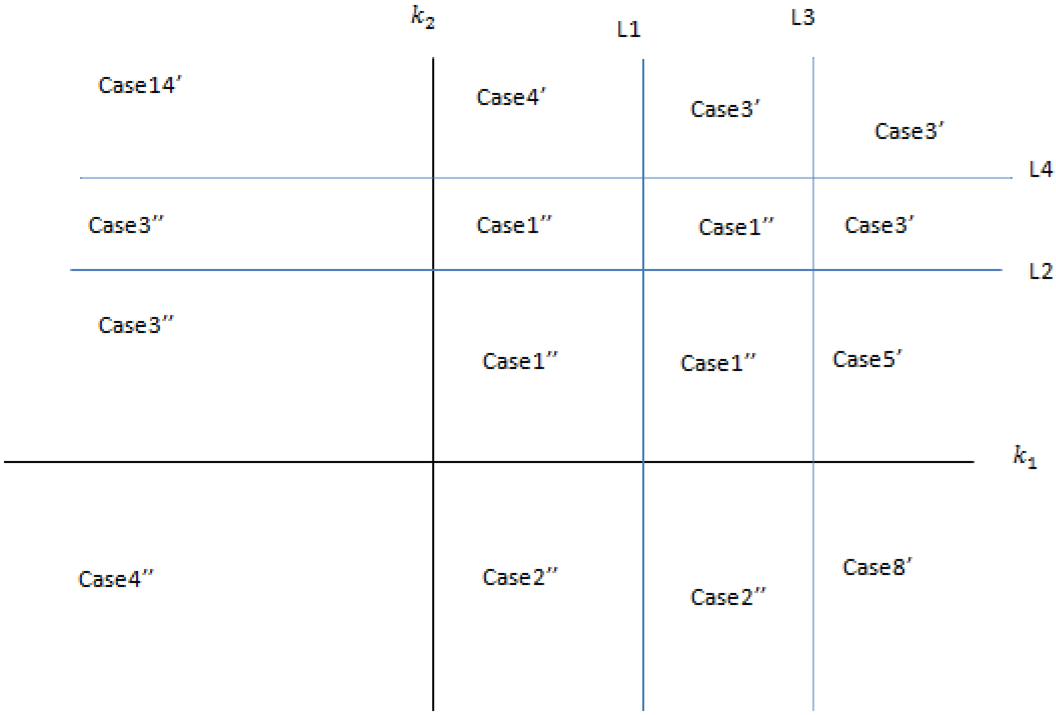

Figure 33 How equilibrium cases change as $\boldsymbol{Y_1}$ and $\boldsymbol{Y_2}$ realizations change, given $\boldsymbol{A_2 < c_1 < A_1}$, $\boldsymbol{KK1 > K2}$ and $\boldsymbol{K1 < KK2}$

$$E_{Y_1,Y_2}[\pi_s(c_1,c_2)]$$

$$= \sum_{k_2>line2}\sum_{k_1>line3}\pi_s\big(Case3'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{0\le k_2\le line2}\sum_{k_1>line3}\pi_s\big(Case5'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{k_2<0}\sum_{k_1>line3}\pi_s\big(Case8'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{k_2\ge line4}\sum_{line1<k_1\le line3}\pi_s\big(Case3'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{0\le k_2<line4}\sum_{line1<k_1\le line3}\pi_s\big(Case1''(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{k_2<0}\sum_{line1<k_1\le line3}\pi_s\big(Case2''(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{k_2\ge line4}\sum_{0\le k_1\le line1}\pi_s\big(Case4'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{0\le k_2<line4}\sum_{0\le k_1\le line1}\pi_s\big(Case1''(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{k_2<0}\sum_{0\le k_1\le line1}\pi_s\big(Case2''(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{k_2\ge line4}\sum_{k_1<0}\pi_s\big(Case14'(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{0\le k_2<line4}\sum_{k_1<0}\pi_s\big(Case3''(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

$$+ \sum_{k_2<0}\sum_{k_1<0}\pi_s\big(Case4''(k_1,k_2)\big)p_1^{k_1}p_1^{k_2}$$

The supplier searches exhaustively over c_1, c_2 region to find c_1 and c_2 that maximizes $E_{Y_1,Y_2}[\pi_S]$.

APPENDIX D

APPENDIX TO CHAPTER 8

Results IBIS cases

Table 15 Results IBIS A1 = 2000 A2 = 750 β = 0.5 C2 = 0

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	5.0051*10^5	5.0047*10^5	5.0043*10^5	5.0040*10^5	5.0037*10^5	5.0035*10^5
supplier expected profit (no-coll)	5.0051*10^5	5.0047*10^5	5.0043*10^5	5.0040*10^5	5.0037*10^5	5.0035*10^5
buyer 1 expected profit (coll)	2.5026*10^5	2.5023*10^5	2.5022*10^5	2.5020*10^5	2.5019*10^5	2.5017*10^5
buyer 1 expected profit (no-coll)	2.5026*10^5	2.5023*10^5	2.5022*10^5	2.5020*10^5	2.5019*10^5	2.5017*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 16 Results IBIS with A1 = 2000 A2 = 1300 β = 0.5 C2 = 0

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	5.4514*10^5	5.451785*10^5	5.451833*10^5	5.4509*10^5	5.4489*10^5	5.4488*10^5
supplier expected profit (no-coll)	5.4514*10^5	5.451785*10^5	5.451833*10^5	5.4509*10^5	5.4489*10^5	5.4488*10^5
buyer 1 expected profit (coll)	3.1581*10^5	3.1518*10^5	3.1481*10^5	3.1393*10^5	3.1697*10^5	3.1745*10^5
buyer 1 expected profit (no-coll)	3.1581*10^5	3.1518*10^5	3.1481*10^5	3.1393*10^5	3.1697*10^5	3.1745*10^5
buyer 2 expected profit (coll)	1.0321*10^4	1.0367*10^4	1.0414*10^4	1.0419*10^4	1.0327*10^4	1.0323*10^4
buyer 2 expected profit (no-	1.0321*10^4	1.0367*10^4	1.0414*10^4	1.0419*10^4	1.0327*10^4	1.0323*10^4

coll)

Table 17 Results IBIS with A1 = 2000 A2 = 1600 β = 0.5 C2 = 0

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	6.484081*10^5	6.484097*10^5	6.484128*10^5	648410	6.4837*10^5	6.4836*10^5
supplier expected profit (no-coll)	6.484081*10^5	6.484097*10^5	6.484128*10^5	648410	6.4837*10^5	6.4836*10^5
buyer 1 expected profit (coll)	2.4437*10^5	2.4432*10^5	2.4429*10^5	2.4425*10^5	2.4418*10^5	2.4414*10^5
buyer 1 expected profit (no-coll)	2.4437*10^5	2.4432*10^5	2.4429*10^5	2.4425*10^5	2.4418*10^5	2.4414*10^5
buyer 2 expected profit (coll)	5.2366*10^4	5.2417*10^4	5.2467*10^4	5.2491*10^4	5.2374*10^4	5.2364*10^4
buyer 2 expected profit (no-coll)	5.2366*10^4	5.2417*10^4	5.2467*10^4	5.2491*10^4	5.2374*10^4	5.2364*10^4

Table 18 Results IBIS with A1 = 2000 A2 = 750 β = 0.75 c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5.0051*10^5	5.0047*10^5	5.0043*10^5	5.0040*10^5	5.0037*10^5	5.0035*10^5
supplier expected profit (no-coll)	5.0051*10^5	5.0047*10^5	5.0043*10^5	5.0040*10^5	5.0037*10^5	5.0035*10^5
buyer 1 expected profit (coll)	2.5026*10^5	2.5023*10^5	2.5022*10^5	2.5020*10^4	2.5019*10^5	2.5017*10^5
buyer 1 expected profit (no-coll)	2.5026*10^5	2.5023*10^5	2.5022*10^5	2.5020*10^4	2.5019*10^5	2.5017*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0

buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 19 Results IBIS with A1 = 2000 A2 = 1300 β = 0.75 c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5.0745*10^5	5.0767*10^5	5.0783*10^5	5.0792*10^5	5.0808*10^5	5.0803*10^5
supplier expected profit (no-coll)	5.0745*10^5	5.0767*10^5	5.0783*10^5	5.0792*10^5	5.0808*10^5	5.0803*10^5
buyer 1 expected profit (coll)	2.7322*10^5	2.7474*10^5	2.7637*10^5	2.7546*10^5	2.7834*10^5	2.7959*10^5
buyer 1 expected profit (no-coll)	2.7322*10^5	2.7474*10^5	2.7637*10^5	2.7546*10^5	2.7834*10^5	2.7959*10^5
buyer 2 expected profit (coll)	1.4214*10^3	1.4839*10^3	1.5411*10^3	1.5538*10^3	1.5169*10^3	1.5164*10^3
buyer 2 expected profit (no-coll)	1.4214*10^3	1.4839*10^3	1.5411*10^3	1.5538*10^3	1.5169*10^3	1.5164*10^3

Table 20 Results IBIS with A1 = 2000 A2 = 1600 β = 0.75 c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5.894619*10^5	5.894634*10^5	5.894662*10^5	5.894636*10^5	5.894281*10^5	5.894192*10^5
supplier expected profit (no-coll)	5.894619*10^5	5.894634*10^5	5.894662*10^5	5.894636*10^5	5.894281*10^5	5.894192*10^5
buyer 1 expected profit (coll)	2.3881*10^5	2.3876*10^5	2.3873*10^5	2.3868*10^5	2.3858*10^5	2.3853*10^5
buyer 1 expected profit (no-coll)	2.3881*10^5	2.3876*10^5	2.3873*10^5	2.3868*10^5	2.3858*10^5	2.3853*10^5

buyer 2 expected profit (coll)	2.9354*10^4	2.9410*10^4	2.9467*10^4	2.9491*10^4	2.9335*10^4	2.9317*10^4
buyer 2 expected profit (no-coll)	2.9354*10^4	2.9410*10^4	2.9467*10^4	2.9491*10^4	2.9335*10^4	2.9317*10^4

Table 21 Results IBIS with A1 = 2000 A2 = 750 β = 0.9 c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5.0051*10^5	5.0057*10^5	5.0043*10^5	5.0040*10^5	5.0037*10^5	5.0035*10^5
supplier expected profit (no-coll)	5.0051*10^5	5.0057*10^5	5.0043*10^5	5.0040*10^5	5.0037*10^5	5.0035*10^5
buyer 1 expected profit (coll)	2.5026*10^5	2.5023*10^5	2.5022*10^5	2.5020*10^5	2.5019*10^5	2.5017*10^5
buyer 1 expected profit (no-coll)	2.5026*10^5	2.5023*10^5	2.5022*10^5	2.5020*10^5	2.5019*10^5	2.5017*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 22 Results IBIS with A1 = 2000 A2 = 1300 β = 0.9 c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5.0134*10^5	5.0137*10^5	5.0140*10^5	5.0135*10^5	5.0076*10^5	5.0075*10^5
supplier expected profit (no-coll)	5.0134*10^5	5.0137*10^5	5.0140*10^5	5.0135*10^5	5.0076*10^5	5.0075*10^5
buyer 1 expected profit (coll)	2.5317*10^5	2.5334*10^5	2.5432*10^5	2.5479*10^5	2.5240*10^5	2.5498*10^5
buyer 1 expected profit (no-coll)	2.5317*10^5	2.5334*10^5	2.5432*10^5	2.5479*10^5	2.5240*10^5	2.5498*10^5
buyer 2 expected profit (coll)	155,8814	166,7708	183,0001	175,3102	70,2809	91,0749

buyer 2 expected profit (no-coll)	155,8814	166,7708	183,0001	175,3102	70,2809	91,0749

Table 23 Results IBIS with A1 = 2000 A2 = 1600 β = 0.9 c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5.590009*10^5	5.590047*10^5	5.589958*10^5	5.589798*10^5	5.589405*10^5	5.58932*10^5
supplier expected profit (no-coll)	5.590009*10^5	5.590047*10^5	5.589958*10^5	5.589798*10^5	5.589405*10^5	5.58932*10^5
buyer 1 expected profit (coll)	2.4350*10^5	2.4337*10^5	2.4327*10^5	2.4375*10^5	2.4370*10^5	2.4364*10^5
buyer 1 expected profit (no-coll)	2.4350*10^5	2.4337*10^5	2.4327*10^5	2.4375*10^5	2.4370*10^5	2.4364*10^5
buyer 2 expected profit (coll)	1.8264*10^4	1.8325*10^4	1.8388*10^4	1.8415*10^4	1.8222*10^4	1.8196*10^4
buyer 2 expected profit (no-coll)	1.8264*10^4	1.8325*10^4	1.8388*10^4	1.8415*10^4	1.8222*10^4	1.8196*10^4

Table 24 Results IBIS with A1 = 2000 A2 = 750 β = 0.5

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	6.6293*10^5	6.628725*10^5	6.6282*10^5	6.6278*10^5	6.6274*10^5	6.6271*10^5
supplier expected profit (no-coll)	6.6293*10^5	6.628725*10^5	6.6282*10^5	6.6278*10^5	6.6274*10^5	6.6271*10^5
buyer 1 expected profit (coll)	2.2390*10^5	2.2388*10^5	2.2387*10^5	2.2385*10^5	2.2384*10^5	2.2383*10^5
buyer 1 expected profit (no-coll)	2.2390*10^5	2.2388*10^5	2.2387*10^5	2.2385*10^5	2.2384*10^5	2.2383*10^5

buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 25 Results IBIS with A1 = 2000 A2 = 1300 β = 0.5

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	6.8415*10^5	6.8429*10^5	6.8466*10^5	6.8442*10^5	6.8462*10^5	6.8456*10^5
supplier expected profit (no-coll)	6.6293*10^5	6.6287*10^5	6.6282*10^5	6.5103*10^5	6.6274*10^5	6.6271*10^5
buyer 1 expected profit (coll)	2.6190*10^5	2.6643*10^5	2.6246*10^5	2.6950*10^5	2.5779*10^5	2.5467*10^5
buyer 1 expected profit (no-coll)	2.2390*10^5	2.2388*10^5	2.2387*10^5	2.3834*10^5	2.2384*10^5	2.2383*10^5
buyer 2 expected profit (coll)	3.0554*10^3	3.1495*10^3	3.1703*10^3	3.2508*10^3	3.0857*10^3	3.0409*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 26 Results IBIS with A1 = 2000 A2 = 1600 β = 0.5

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	8.064778*10^5	8.064797*10^5	8.064836*10^5	8.064801*10^5	8.0643*10^5	8.0642*10^5
supplier expected profit (no-coll)	7.1482*10^5	7.1628*10^5	7.1552*10^5	7.1666*10^5	7.1410*10^5	7.1505*10^5
buyer 1 expected profit (coll)	2.1377*10^5	2.1373*10^5	2.1370*10^5	2.1366*10^5	2.1358*10^5	2.1354*10^5

buyer 1 expected profit (no-coll)	2.6078*10^5	2.6484*10^5	2.6001*10^5	2.6341*10^5	2.5697*10^5	2.6179*10^5
buyer 2 expected profit (coll)	3.2894*10^4	3.2942*10^4	3.2991*10^4	3.3012*10^4	3.2887*10^4	3.2873*10^4
buyer 2 expected profit (no-coll)	1.4460*10^5	1.5330*10^5	1.4473*10^5	1.5157*10^5	1.3702*10^5	1.4596*10^5

Table 27 Results IBIS with A1 = 2000 A2 = 750 β = 0.75

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	7.9446*10^5	7.9439*10^5	7.9434*10^5	7.9429*10^5	7.9424*10^5	7.9420*10^5
supplier expected profit (no-coll)	7.9446*10^5	7.9439*10^5	7.9434*10^5	7.9429*10^5	7.9424*10^5	7.9420*10^5
buyer 1 expected profit (coll)	1.6394*10^5	1.6392*10^5	1.6391*10^5	1.6390*10^5	1.6389*10^5	1.6388*10^5
buyer 1 expected profit (no-coll)	1.6394*10^5	1.6392*10^5	1.6391*10^5	1.6390*10^5	1.6389*10^5	1.6388*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 28 Results IBIS with A1 = 2000 A2 = 1300 β = 0.75

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	7.9446*10^5	7.9439*10^5	7.9434*10^5	7.9429*10^5	7.9424*10^5	7.9420*10^5
supplier expected profit (no-coll)	7.9446*10^5	7.9439*10^5	7.9434*10^5	7.9429*10^5	7.9424*10^5	7.9420*10^5

buyer 1 expected profit (coll)	1.6394*10^5	1.6392*10^5	1.6391*10^5	1.6390*10^5	1.6389*10^5	1.6388*10^5
buyer 1 expected profit (no-coll)	1.6394*10^5	1.6392*10^5	1.6391*10^5	1.6390*10^5	1.6389*10^5	1.6388*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 29 Results IBIS with A1 = 2000 A2 = 1600 β = 0.75

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	8.1595*10^5	8.1597*10^5	8.1620*10^5	8.1630*10^5	8.1618*10^5	8.1613*10^5
supplier expected profit (no-coll)	7.9446*10^5	7.9439*10^5	7.9434*10^5	7.9429*10^5	7.9424*10^5	7.9420*10^5
buyer 1 expected profit (coll)	1.6509*10^5	1.6551*10^5	1.6609*10^5	1.6331*10^5	1.6453*10^5	1.6350*10^5
buyer 1 expected profit (no-coll)	1.6394*10^5	1.6392*10^5	1.6391*10^5	1.6390*10^5	1.6389*10^5	1.6388*10^5
buyer 2 expected profit (coll)	2.5393*10^3	2.5774*10^3	2.6271*10^3	2.6307*10^3	2.5272*10^3	2.4992*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 30 Results IBIS with A1 = 2000 A2 = 750 β = 0.9

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	9.0182*10^5	9.0175*10^5	9.0168*10^5	9.0162*10^5	9.0157*10^5	9.0153*10^5
supplier expected profit (no-	9.0182*10^5	9.0175*10^5	9.0168*10^5	9.0162*10^5	9.0157*10^5	9.0153*10^5

coll)						
buyer 1 expected profit (coll)	8.9370*10^4	8.9362*10^4	8.9356*10^4	8.9350*10^4	8.9345*10^4	8.9340*10^4
buyer 1 expected profit (no-coll)	8.9370*10^4	8.9362*10^4	8.9356*10^4	8.9350*10^4	8.9345*10^4	8.9340*10^4
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 31 Results IBIS with A1 = 2000 A2 = 1300 β = 0.9

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	9.0182*10^5	9.0175*10^5	9.0168*10^5	9.0162*10^5	9.0157*10^5	9.0153*10^5
supplier expected profit (no-coll)	9.0182*10^5	9.0175*10^5	9.0168*10^5	9.0162*10^5	9.0157*10^5	9.0153*10^5
buyer 1 expected profit (coll)	8.9370*10^4	8.9362*10^4	8.9356*10^4	8.9350*10^4	8.9345*10^4	8.9340*10^4
buyer 1 expected profit (no-coll)	8.9370*10^4	8.9362*10^4	8.9356*10^4	8.9350*10^4	8.9345*10^4	8.9340*10^4
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 32 Results IBIS with A1 = 2000 A2 = 1600 β = 0.9

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	9.0190*10^5	9.0182*10^5	9.0175*10^5	9.0167*10^5	9.0157*10^5	9.0153*10^5

supplier expected profit (no-coll)	9.0182*10^5	9.0175*10^5	9.0168*10^5	9.0162*10^5	9.0157*10^5	9.0153*10^5
buyer 1 expected profit (coll)	8.9333*10^4	8.9336*10^4	8.9340*10^4	8.9357*10^4	8.9342*10^4	8.9341*10^4
buyer 1 expected profit (no-coll)	8.9370*10^4	8.9362*10^4	8.9356*10^4	8.9350*10^4	8.9345*10^4	8.9340*10^4
buyer 2 expected profit (coll)	4,486	4,9841	3,9661	1,2291	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Results IBNS Cases

Table 33 Results IBNS with A1 = 2000 A2 = 750 β = 0.5 c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5
supplier expected profit (no-coll)	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5
buyer 1 expected profit (coll)	2.5102*10^5	2.5094*10^5	2.5086*10^5	2.5080*10^5	2.5074*10^5	2.5069*10^5
buyer 1 expected profit (no-coll)	2.5102*10^5	2.5094*10^5	2.5086*10^5	2.5080*10^5	2.5074*10^5	2.5069*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 34 Results IBNS with A1 = 2000 A2 = 1300 β = 0.5 c2 = 0

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	5.445*10^5	5.445*10^5	5.445*10^5	5.445*10^5	5.445*10^5	5.445*10^5
supplier expected profit (no-coll)	5.445*10^5	5.445*10^5	5.445*10^5	5.445*10^5	5.445*10^5	5.445*10^5
buyer 1 expected profit (coll)	3.1858*10^5	3.1849*10^5	3.1841*10^5	3.1834*10^5	3.1827*10^5	3.1821*10^5
buyer 1 expected profit (no-coll)	3.1858*10^5	3.1849*10^5	3.1841*10^5	3.1834*10^5	3.1827*10^5	3.1821*10^5
buyer 2 expected profit (coll)	1.0578*10^4	1.0675*10^4	1.0771*10^4	1.0824*10^4	1.0662*10^4	1.0658*10^4
buyer 2 expected profit (no-coll)	1.0578*10^4	1.0675*10^4	1.0771*10^4	1.0824*10^4	1.0662*10^4	1.0658*10^4

Table 35 Results IBNS with A1 = 2000 A2 = 1600 β = 0.5 c2 = 0

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	6.48*10^5	6.48*10^5	6.48*10^5	6.48*10^5	6.48*10^5	6.48*10^5
supplier expected profit (no-coll)	6.48*10^5	6.48*10^5	6.48*10^5	6.48*10^5	6.48*10^5	6.48*10^5
buyer 1 expected profit (coll)	2.4461*10^5	2.4452*10^5	2.4445*10^5	2.4438*10^5	2.4430*10^5	2.4425*10^5
buyer 1 expected profit (no-coll)	2.4461*10^5	2.4452*10^5	2.4445*10^5	2.4438*10^5	2.4430*10^5	2.4425*10^5
buyer 2 expected profit (coll)	5.2611*10^4	5.2709*10^4	5.2805*10^4	5.2857*10^4	5.2695*10^4	5.2691*10^4
buyer 2 expected profit (no-coll)	5.2611*10^4	5.2709*10^4	5.2805*10^4	5.2857*10^4	5.2695*10^4	5.2691*10^4

Table 36 Results IBNS with A1 = 2000 A2 = 750 β = 0.75 c2 = 0

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5
supplier expected profit (no-coll)	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5
buyer 1 expected profit (coll)	2.5102*10^5	2.5094*10^5	2.5086*10^5	2.508*10^5	2.5074*10^5	2.5069*10^5
buyer 1 expected profit (no-coll)	2.5102*10^5	2.5094*10^5	2.5086*10^5	2.508*10^5	2.5074*10^5	2.5069*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 37 Results IBNS with A1 = 2000 A2 = 1300 β = 0.75 c2 = 0

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	5.0253*10^5	5.0289*10^5	5.0309*10^5	5.0363*10^5	5.0389*10^5	5.0389*10^5
supplier expected profit (no-coll)	5.0253*10^5	5.0289*10^5	5.0309*10^5	5.0363*10^5	5.0389*10^5	5.0389*10^5
buyer 1 expected profit (coll)	2.8252*10^5	2.8074*10^5	2.8822*10^5	2.7427*10^5	2.9413*10^5	2.9360*10^5
buyer 1 expected profit (no-coll)	2.8252*10^5	2.8074*10^5	2.8822*10^5	2.7427*10^5	2.9413*10^5	2.9360*10^5
buyer 2 expected profit (coll)	331,8673	345,1259	483,8421	318,1178	500,9974	485,7106
buyer 2 expected profit (no-coll)	331,8673	345,1259	483,8421	318,1178	500,9974	485,7106

Table 38 Results IBNS with A1 = 2000 A2 = 1600 β = 0.75 c2 = 0

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	5.8909*10^5	5.8909*10^5	5.8909*10^5	5.8909*10^5	5.8909*10^5	5.8909*10^5
supplier expected profit (no-coll)	5.8909*10^5	5.8909*10^5	5.8909*10^5	5.8909*10^5	5.8909*10^5	5.8909*10^5
buyer 1 expected profit (coll)	2.3901*10^5	2.3892*10^5	2.3883*10^5	2.3876*10^5	2.3865*10^5	2.3859*10^5
buyer 1 expected profit (no-coll)	2.3901*10^5	2.3892*10^5	2.3883*10^5	2.3876*10^5	2.3865*10^5	2.3859*10^5
buyer 2 expected profit (coll)	2.9556*10^4	2.9663*10^4	2.9770*10^4	2.9825*10^4	2.9626*10^4	2.9617*10^4
buyer 2 expected profit (no-coll)	2.9556*10^4	2.9663*10^4	2.9770*10^4	2.9825*10^4	2.9626*10^4	2.9617*10^4

Table 39 Results IBNS with A1 = 2000 A2 = 750 β = 0.9 c2 = 0

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5
supplier expected profit (no-coll)	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5
buyer 1 expected profit (coll)	2.5102*10^5	2.5094*10^5	2.5086*10^5	2.508*10^5	2.5074*10^5	2.5069*10^5
buyer 1 expected profit (no-coll)	2.5102*10^5	2.5094*10^5	2.5086*10^5	2.508*10^5	2.5074*10^5	2.5069*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 40 Results IBNS with A1 = 2000 A2 = 1300 β = 0.9 c2 = 0

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	5.001*10^5	5.001*10^5	5.0007*10^5	5.0002*10^5	5*10^5	5*10^5
supplier expected profit (no-coll)	5.001*10^5	5.001*10^5	5.0007*10^5	5.0002*10^5	5*10^5	5*10^5
buyer 1 expected profit (coll)	2.5294*10^5	2.5285*10^5	2.5331*10^5	2.5179*10^5	2.5074*10^5	2.5069*10^5
buyer 1 expected profit (no-coll)	2.5294*10^5	2.5285*10^5	2.5331*10^5	2.5179*10^5	2.5074*10^5	2.5069*10^5
buyer 2 expected profit (coll)	3,7721	3,5985	2,1532	0,2847	0	0
buyer 2 expected profit (no-coll)	3,7721	3,5985	2,1532	0,2847	0	0

Table 41 Results IBNS with A1 = 2000 A2 = 1600 β = 0.9 c2 = 0

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	5.5862*10^5	5.5862*10^5	5.5862*10^5	5.5862*10^5	5.5862*10^5	5.5862*10^5
supplier expected profit (no-coll)	5.5862*10^5	5.5862*10^5	5.5862*10^5	5.5862*10^5	5.5862*10^5	5.5862*10^5
buyer 1 expected profit (coll)	2.4415*10^5	2.4405*10^5	2.4397*10^5	2.4388*10^5	2.4375*10^5	2.4367*10^5
buyer 1 expected profit (no-coll)	2.4415*10^5	2.4405*10^5	2.4397*10^5	2.4388*10^5	2.4375*10^5	2.4367*10^5
buyer 2 expected profit (coll)	1.8448*10^4	1.8565*10^4	1.8681*10^4	1.8739*10^4	1.8503*10^4	1.8487*10^4
buyer 2 expected profit (no-coll)	1.8448*10^4	1.8565*10^4	1.8681*10^4	1.8739*10^4	1.8503*10^4	1.8487*10^4

Table 42 Results IBNS with A1 = 2000 A2 = 750 β = 0.5

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	6.6033*10^5	6.6049*10^5	6.6062*10^5	6.6074*10^5	6.6085*10^5	6.6094*10^5
supplier expected profit (no-coll)	6.6033*10^5	6.6049*10^5	6.6062*10^5	6.6074*10^5	6.6085*10^5	6.6094*10^5
buyer 1 expected profit (coll)	2.2535*10^5	2.2518*10^5	2.2504*10^5	2.2491*10^5	2.2480*10^5	2.2471*10^5
buyer 1 expected profit (no-coll)	2.2535*10^5	2.2518*10^5	2.2504*10^5	2.2491*10^5	2.2480*10^5	2.2471*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 43 Results IBNS with A1 = 2000 A2 = 1300 β = 0.5

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	6.7703*10^5	6.7781*10^5	6.7733*10^5	6.7923*10^5	6.7942*10^5	6.7951*10^5
supplier expected profit (no-coll)	6.6033*10^5	6.6049*10^5	6.6062*10^5	6.6074*10^5	6.6085*10^5	6.6094*10^5
buyer 1 expected profit (coll)	2.8248*10^5	2.75*10^5	2.6263*10^5	2.7762*10^5	2.6095*10^5	2.6085*10^5
buyer 1 expected profit (no-coll)	2.2535*10^5	2.2518*10^5	2.2504*10^5	2.2491*10^5	2.2480*10^5	2.2471*10^5
buyer 2 expected profit (coll)	2.9731*10^3	2.5929*10^3	1.9767*10^3	2.9958*10^3	1.8782*10^3	1.8782*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 44 Results IBNS with A1 = 2000 A2 = 1600 β = 0.5

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	8.022*10^5	8.0219*10^5	8.0216*10^5	8.0218*10^5	8.0254*10^5	8.0263*10^5
supplier expected profit (no-coll)	7.0707*10^5	7.0706*10^5	7.0704*10^5	7.0705*10^5	7.0731*10^5	7.0737*10^5
buyer 1 expected profit (coll)	2.1456*10^5	2.1440*10^5	2.1426*10^5	2.1413*10^5	2.1402*10^5	2.1392*10^5
buyer 1 expected profit (no-coll)	2.9281*10^5	2.9267*10^5	2.8997*10^5	2.8985*10^5	2.9226*10^5	2.9215*10^5
buyer 2 expected profit (coll)	3.3762*10^4	3.3940*10^4	3.4115*10^4	3.4213*10^4	3.3940*10^4	3.3940*10^4
buyer 2 expected profit (no-coll)	2.0901*10^4	2.1067*10^4	2.2392*10^4	2.2476*10^4	2.1006*10^4	2.0991*10^4

Table 45 Results IBNS with A1 = 2000 A2 = 750 β = 0.75

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	7.8248*10^5	7.8339*10^5	7.8419*10^5	7.8489*10^5	7.8551*10^5	7.8605*10^5
supplier expected profit (no-coll)	7.8248*10^5	7.8339*10^5	7.8419*10^5	7.8489*10^5	7.8551*10^5	7.8605*10^5
buyer 1 expected profit (coll)	1.6793*10^5	1.6761*10^5	1.6733*10^5	1.6709*10^5	1.6687*10^5	1.6668*10^5
buyer 1 expected profit (no-coll)	1.6793*10^5	1.6761*10^5	1.6733*10^5	1.6709*10^5	1.6687*10^5	1.6668*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 46 Results IBNS with A1 = 2000   A2 = 1300   β = 0.75

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	7.8248*10^5	7.8339*10^5	7.8419*10^5	7.8489*10^5	7.8551*10^5	7.8605*10^5
supplier expected profit (no-coll)	7.8248*10^5	7.8339*10^5	7.8419*10^5	7.8489*10^5	7.8551*10^5	7.8605*10^5
buyer 1 expected profit (coll)	1.6793*10^5	1.6761*10^5	1.6733*10^5	1.6709*10^5	1.6687*10^5	1.6668*10^5
buyer 1 expected profit (no-coll)	1.6793*10^5	1.6761*10^5	1.6733*10^5	1.6709*10^5	1.6687*10^5	1.6668*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 47 Results IBNS with A1 = 2000   A2 = 1600   β = 0.75

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	7.9933*10^5	7.9981*10^5	8.0249*19^5	8.0113*10^5	8.0685*10^5	8.0738*10^5
supplier expected profit (no-coll)	7.8248*10^5	7.8339*10^5	7.8419*10^5	7.8489*10^5	7.8551*10^5	7.8605*10^5
buyer 1 expected profit (coll)	1.6825*10^5	1.7271*10^5	1.6605*10^5	1.7376*10^5	1.6320*10^5	1.6301*10^5
buyer 1 expected profit (no-coll)	1.6793*10^5	1.6761*10^5	1.6733*10^5	1.6709*10^5	1.6687*10^5	1.6668*10^5
buyer 2 expected profit (coll)	2.2933*10^3	2.8069*10^3	2.5534*10^3	3.2136*10^3	2.2123*10^3	2.2122*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 48 Results IBNS with A1 = 2000 A2 = 750 β = 0.9

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	8.3243*10^5	8.3669*10^5	8.4047*10^5	8.4397*10^5	8.4740*10^5	8.5042*10^5
supplier expected profit (no-coll)	8.3243*10^5	8.3669*10^5	8.4047*10^5	8.4397*10^5	8.4740*10^5	8.5042*10^5
buyer 1 expected profit (coll)	1.0958*10^5	1.0894*10^5	1.0838*10^5	102075	1.0161*10^5	1.0120*10^5
buyer 1 expected profit (no-coll)	1.0958*10^5	1.0894*10^5	1.0838*10^5	102075	1.0161*10^5	1.0120*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 49 Results IBNS with A1 = 2000 A2 = 1300 β = 0.9

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	8.3243*10^5	8.3669*10^5	8.4047*10^5	8.4397*10^5	8.4740*10^5	8.5042*10^5
supplier expected profit (no-coll)	8.3243*10^5	8.3669*10^5	8.4047*10^5	8.4397*10^5	8.4740*10^5	8.5042*10^5
buyer 1 expected profit (coll)	1.0958*10^5	1.0894*10^5	1.0838*10^5	102075	1.0161*10^5	1.0120*10^5
buyer 1 expected profit (no-coll)	1.0958*10^5	1.0894*10^5	1.0838*10^5	102075	1.0161*10^5	1.0120*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-	0	0	0	0	0	0

coll)

Table 50 Results IBNS with A1 = 2000 A2 = 1600 β = 0.9

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	8.3243*10^5	8.3669*10^5	8.4047*10^5	8.4397*10^5	8.4740*10^5	8.5042*10^5
supplier expected profit (no-coll)	8.3243*10^5	8.3669*10^5	8.4047*10^5	8.4397*10^5	8.4740*10^5	8.5042*10^5
buyer 1 expected profit (coll)	1.0958*10^5	1.0894*10^5	1.0838*10^5	102075	1.0161*10^5	1.0120*10^5
buyer 1 expected profit (no-coll)	1.0958*10^5	1.0894*10^5	1.0838*10^5	102075	1.0161*10^5	1.0120*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Results NBNS cases

Table 51 Results NBNS with A1 = 2000 A2 = 750 β = 0.5 c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5
supplier expected profit (no-coll)	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5
buyer 1 expected profit (coll)	2.5102*10^5	2.5094*10^5	2.5086*10^5	2.5080*10^5	2.5074*10^5	2.5069*10^5
buyer 1 expected profit (no-coll)	2.5102*10^5	2.5094*10^5	2.5086*10^5	2.5080*10^5	2.5074*10^5	2.5069*10^5

buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 52 Results NBNS with A1 = 2000 A2 = 1300 β = 0.5 c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5.445*10^5	5.445*10^5	5.445*10^5	5.445*10^5	5.445*10^5	5.445*10^5
supplier expected profit (no-coll)	5.445*10^5	5.445*10^5	5.445*10^5	5.445*10^5	5.445*10^5	5.445*10^5
buyer 1 expected profit (coll)	3.1836*10^5	3.1828*10^5	3.1821*10^5	3.1814*10^5	3.1809*10^5	3.1804*10^5
buyer 1 expected profit (no-coll)	3.1836*10^5	3.1828*10^5	3.1821*10^5	3.1814*10^5	3.1809*10^5	3.1804*10^5
buyer 2 expected profit (coll)	1.0365*10^4	1.0456*10^4	1.0544*10^4	1.0594*10^4	1.0456*10^4	1.0456*10^4
buyer 2 expected profit (no-coll)	1.0365*10^4	1.0456*10^4	1.0544*10^4	1.0594*10^4	1.0456*10^4	1.0456*10^4

Table 53 Results NBNS with A1 = 2000 A2 = 1600 β = 0.5 c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	6.48*10^5	6.48*10^5	6.48*10^5	6.48*10^5	6.48*10^5	6.48*10^5
supplier expected profit (no-coll)	6.48*10^5	6.48*10^5	6.48*10^5	6.48*10^5	6.48*10^5	6.48*10^5
buyer 1 expected profit (coll)	2.444*10^5	2.4432*10^5	2.4424*10^5	2.4418*10^5	2.4412*10^5	2.4407*10^5
buyer 1 expected profit (no-coll)	2.444*10^5	2.4432*10^5	2.4424*10^5	2.4418*10^5	2.4412*10^5	2.4407*10^5

buyer 2 expected profit (coll)	5.2398*10^4	5.2489*19^4	5.2578*10^4	5.2628*10^4	5.2489*10^4	5.2489*104
buyer 2 expected profit (no-coll)	5.2398*10^4	5.2489*19^4	5.2578*10^4	5.2628*10^4	5.2489*10^4	5.2489*104

Table 54 Results NBNS with A1 = 2000 A2 = 750 β = 0.75 c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5
supplier expected profit (no-coll)	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5
buyer 1 expected profit (coll)	2.5102*10^5	2.5094*10^5	2.5086*10^5	2.5080*10^5	2.5074*10^5	2.5069*10^5
buyer 1 expected profit (no-coll)	2.5102*10^5	2.5094*10^5	2.5086*10^5	2.5080*10^5	2.5074*10^5	2.5069*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 55 Results NBNS with A1 = 2000 A2 = 1300 β = 0.75 c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5.0545*10^5	5.0609*10^5	5.0680*10^5	5.0686*10^5	5.0801*10^5	5.0801*10^5
supplier expected profit (no-coll)	5.0545*10^5	5.0609*10^5	5.0680*10^5	5.0686*10^5	5.0801*10^5	5.0801*10^5
buyer 1 expected profit (coll)	2.8754*10^5	2.5729*10^5	2.7831*10^5	2.7205*10^5	2.9316*10^5	2.9312*10^5
buyer 1 expected profit (no-coll)	2.8754*10^5	2.5729*10^5	2.7831*10^5	2.7205*10^5	2.9316*10^5	2.9312*10^5

buyer 2 expected profit (coll)	204,4274	49,4252	171,9669	133,2975	251,4233	251,4233
buyer 2 expected profit (no-coll)	204,4274	49,4252	171,9669	133,2975	251,4233	251,4233

Table 56 Results NBNS with A1 = 2000   A2 = 1600   β = 0.75   c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5.8909*10^5	5.8909*10^5	5.8909*10^5	5.8909*10^5	5.8909*10^5	5.8909*10^5
supplier expected profit (no-coll)	5.8909*10^5	5.8909*10^5	5.8909*10^5	5.8909*10^5	5.8909*10^5	5.8909*10^5
buyer 1 expected profit (coll)	2.3846*10^5	2.3837*10^5	2.3830*10^5	2.3823*10^5	2.3818*10^5	2.3813*10^5
buyer 1 expected profit (no-coll)	2.3846*10^5	2.3837*10^5	2.3830*10^5	2.3823*10^5	2.3818*10^5	2.3813*10^5
buyer 2 expected profit (coll)	2.9001*10^4	2.9091*10^4	2.9180*10^4	2.9230*10^4	2.9091*10^4	2.9091*10^4
buyer 2 expected profit (no-coll)	2.9001*10^4	2.9091*10^4	2.9180*10^4	2.9230*10^4	2.9091*10^4	2.9091*10^4

Table 57 Results NBNS with A1 = 2000   A2 = 750   β = 0.9   c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5
supplier expected profit (no-coll)	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5	5*10^5
buyer 1 expected profit (coll)	2.5102*10^5	2.5094*10^5	2.5086*10^5	2.5080*10^5	2.5074*10^5	2.5069*10^5
buyer 1 expected profit (no-coll)	2.5102*10^5	2.5094*10^5	2.5086*10^5	2.5080*10^5	2.5074*10^5	2.5069*10^5

buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 58 Results NBNS with A1 = 2000 A2 = 1300 β = 0.9 c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5.0116*10^5	5.0179*10^5	5.0248*10^5	5.0227*10^5	5*10^5	5*10^5
supplier expected profit (no-coll)	5.0116*10^5	5.0179*10^5	5.0248*10^5	5.0227*10^5	5*10^5	5*10^5
buyer 1 expected profit (coll)	2.5756*10^5	2.6667*10^5	2.7910*10^5	2.9832*10^5	2.5074*10^5	2.5069*10^5
buyer 1 expected profit (no-coll)	2.5756*10^5	2.6667*10^5	2.7910*10^5	2.9832*10^5	2.5074*10^5	2.5069*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 59 Results NBNS with A1 = 2000 A2 = 1600 β = 0.9 c2 = 0

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5.5862*10^5	5.5862*10^5	5.5862*10^5	5.5862*10^5	5.5862*10^5	5.5862*10^5
supplier expected profit (no-coll)	5.5862*10^5	5.5862*10^5	5.5862*10^5	5.5862*10^5	5.5862*10^5	5.5862*10^5
buyer 1 expected profit (coll)	2.4324*10^5	2.4316*10^5	2.4309*10^5	2.4302*10^5	2.4297*10^5	2.4292*10^5
buyer 1 expected profit (no-coll)	2.4324*10^5	2.4316*10^5	2.4309*10^5	2.4302*10^5	2.4297*10^5	2.4292*10^5

buyer 2 expected profit (coll)	1.7540*10^4	1.7630*10^4	1.7719*10^4	1.7769*10^4	1.7630*10^4	1.7630*10^4
buyer 2 expected profit (no-coll)	1.7540*10^4	1.7630*10^4	1.7719*10^4	1.7769*10^4	1.7630*10^4	1.7630*10^4

Table 60 Results NBNS with A1 = 2000 A2 = 750 β = 0.5

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	6.6033*10^5	6.6049*10^5	6.6062*10^5	6.6074*10^5	6.6085*10^5	6.6094*10^5
supplier expected profit (no-coll)	6.6033*10^5	6.6049*10^5	6.6062*10^5	6.6074*10^5	6.6085*10^5	6.6094*10^5
buyer 1 expected profit (coll)	2.2535*10^5	2.2518*10^5	2.2504*10^5	2.2491*10^5	2.2480*10^5	2.2471*10^5
buyer 1 expected profit (no-coll)	2.2535*10^5	2.2518*10^5	2.2504*10^5	2.2491*10^5	2.2480*10^5	2.2471*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 61 Results NBNS with A1 = 2000 A2 = 1300 β = 0.5

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	6.7755*10^5	6.7789*10^5	6.7745*10^5	6.7930*10^5	6.7956*10^5	6.7966*10^5
supplier expected profit (no-coll)	6.6033*10^5	6.6049*10^5	6.6062*10^5	6.6074*10^5	6.6085*10^5	6.6094*10^5
buyer 1 expected profit (coll)	2.8248*10^5	2.7500*10^5	2.6263*10^5	2.9953*10^5	2.6095*10^5	2.6085*10^5
buyer 1 expected profit (no-coll)	2.2535*10^5	2.2518*10^5	2.2504*10^5	2.2491*10^5	2.2480*10^5	2.2471*10^5

buyer 2 expected profit (coll)	2.9727*10^3	2.5925*10^3	1.9763*10^3	2.9953*10^3	1.8779*10^3	1.8779*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 62 Results NBNS with A1 = 2000 A2 = 1600 β = 0.5

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	8.0213*10^5	8.0211*10^5	8.0208*10^5	8.0211*10^5	8.0248*10^5	8.0257*10^5
supplier expected profit (no-coll)	7.0805*10^5	7.0804*10^5	7.0803*10^5	7.0804*10^5	7.0820*10^5	7.0824*10^5
buyer 1 expected profit (coll)	2.1456*10^5	2.1440*10^5	2.1426*10^5	2.1413*10^5	2.1402*10^5	2.1392*10^5
buyer 1 expected profit (no-coll)	2.9190*10^5	2.9178*10^5	2.9166*10^5	2.9157*10^5	2.9148*10^5	2.9140*10^5
buyer 2 expected profit (coll)	3.3762*10^4	3.3940*10^4	3.4114*10^4	3.4212*10^4	3.3940*10^4	3.3940*10^4
buyer 2 expected profit (no-coll)	1.9988*10^4	2.0128*10^4	2.0265*10^4	2.0342*10^4	2.0128*10^4	2.0128*10^4

Table 63 Results NBNS with A1 = 2000 A2 = 750 β = 0.75

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	7.8248*10^5	7.8339*10^5	7.8419*10^5	7.8489*10^5	7.8551*10^5	7.8605*10^5
supplier expected profit (no-coll)	7.8248*10^5	7.8339*10^5	7.8419*10^5	7.8489*10^5	7.8551*10^5	7.8605*10^5
buyer 1 expected profit (coll)	1.6793*10^5	1.6761*10^5	1.6733*10^5	1.6709*10^5	1.6687*10^5	1.6668*10^5
buyer 1 expected profit (no-coll)	1.6793*10^5	1.6761*10^5	1.6733*10^5	1.6709*10^5	1.6687*10^5	1.6668*10^5

buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 64 Results NBNS with A1 = 2000 A2 = 1300 β = 0.75

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	7.8248*10^5	7.8339*10^5	7.8419*10^5	7.8489*10^5	7.8551*10^5	7.8605*10^5
supplier expected profit (no-coll)	7.8248*10^5	7.8339*10^5	7.8419*10^5	7.8489*10^5	7.8551*10^5	7.8605*10^5
buyer 1 expected profit (coll)	1.6793*10^5	1.6761*10^5	1.6733*10^5	1.6709*10^5	1.6687*10^5	1.6668*10^5
buyer 1 expected profit (no-coll)	1.6793*10^5	1.6761*10^5	1.6733*10^5	1.6709*10^5	1.6687*10^5	1.6668*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 65 Results NBNS with A1 = 2000 A2 = 1600 β = 0.75

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	7.9930*10^5	7.9964*10^5	8.0251*10^5	8.0093*10^5	8.0698*10^5	8.0752*10^5
supplier expected profit (no-coll)	7.8248*10^5	7.8339*10^5	7.8419*10^5	7.8489*10^5	7.8551*10^5	7.8605*10^5
buyer 1 expected profit (coll)	1.6749*10^5	1.7271*10^5	1.6527*10^5	1.7376*10^5	1.6320*10^5	1.6301*10^5
buyer 1 expected profit (no-coll)	1.6793*10^5	1.6761*10^5	1.6733*10^5	1.6709*10^5	1.6687*10^5	1.6668*10^5

buyer 2 expected profit (coll)	2.2353*10^3	2.8042*10^3	2.4910*10^3	3.2108*10^3	2.2101*10^3	2.2101*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 66 Results NBNS with A1 = 2000 A2 = 750 β = 0.9

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	8.3243*10^5	8.3669*10^5	8.4047*10^5	8.4397*10^5	8.4740*10^5	8.5042*10^5
supplier expected profit (no-coll)	8.3243*10^5	8.3669*10^5	8.4047*10^5	8.4397*10^5	8.4740*10^5	8.5042*10^5
buyer 1 expected profit (coll)	1.0958*10^5	1.0894*10^5	1.0838*10^5	102075	1.0161*10^5	1.0120*10^5
buyer 1 expected profit (no-coll)	1.0958*10^5	1.0894*10^5	1.0838*10^5	102075	1.0161*10^5	1.0120*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 67 Results NBNS with A_1 = 2000 A_2 = 1300 β = 0.9

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	8.3243*10^5	8.3669*10^5	8.4047*10^5	8.4397*10^5	8.4740*10^5	8.5042*10^5
supplier expected profit (no-coll)	8.3243*10^5	8.3669*10^5	8.4047*10^5	8.4397*10^5	8.4740*10^5	8.5042*10^5
buyer 1 expected profit (coll)	1.0958*10^5	1.0894*10^5	1.0838*10^5	102075	1.0161*10^5	1.0120*10^5
buyer 1 expected profit (no-coll)	1.0958*10^5	1.0894*10^5	1.0838*10^5	102075	1.0161*10^5	1.0120*10^5

buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 68 Results NBNS with $A_1 = 2000$ $A_2 = 1600$ $\beta = 0.9$

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	8.3243*10^5	8.3669*10^5	8.4047*10^5	8.4397*10^5	8.4740*10^5	8.5042*10^5
supplier expected profit (no-coll)	8.3243*10^5	8.3669*10^5	8.4047*10^5	8.4397*10^5	8.4740*10^5	8.5042*10^5
buyer 1 expected profit (coll)	1.0958*10^5	1.0894*10^5	1.0838*10^5	102075	1.0161*10^5	1.0120*10^5
buyer 1 expected profit (no-coll)	1.0958*10^5	1.0894*10^5	1.0838*10^5	102075	1.0161*10^5	1.0120*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Results NBIS cases

Table 69 Results NBIS with $A_1 = 2000$ $A_2 = 750$ $\beta = 0.5$ $c_2 = 0$

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5.0051*10^5	5.0047*10^5	5.0043*10^5	5.004*10^5	5.0037*10^5	5.0035*10^5
supplier expected profit (no-coll)	5.0051*10^5	5.0047*10^5	5.0043*10^5	5.004*10^5	5.0037*10^5	5.0035*10^5
buyer 1 expected profit (coll)	2.5026*10^5	2.5023*10^5	2.5022*10^5	2.502*10^5	2.5019*10^5	2.5017*10^5

buyer 1 expected profit (no-coll)	2.5026*10^5	2.5023*10^5	2.5022*10^5	2.502*10^5	2.5019*10^5	2.5017*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 70 Result NBIS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ $c_2 = 0$

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	5.4557*10^5	5.4573*10^5	5.4577*10^5	5.4536*10^5	5.4508*10^5	5.4506*10^5
supplier expected profit (no-coll)	5.4557*10^5	5.4573*10^5	5.4577*10^5	5.4536*10^5	5.4508*10^5	5.4506*10^5
buyer 1 expected profit (coll)	2.2307*10^5	2.3177*10^5	2.4549*10^5	2.9165*10^5	3.2955*10^5	3.2932*10^5
buyer 1 expected profit (no-coll)	2.2307*10^5	2.3177*10^5	2.4549*10^5	2.9164*10^5	3.2955*10^5	3.2932*10^5
buyer 2 expected profit (coll)	1.4150*10^4	1.4304*10^4	1.4356*10^4	1.4717*10^4	1.4896*10^4	1.4992*10^4
buyer 2 expected profit (no-coll)	1.4150*10^4	1.4304*10^4	1.4356*10^4	1.4717*10^4	1.4896*10^4	1.4992*10^4

Table 71 Results NBIS with $A_1 = 2000$ $A_2 = 1600$ $\beta = 0.5$ $c_2 = 0$

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	6.4864*10^5	6.4864*10^5	6.4864*10^5	6.4864*10^5	6.4858*10^5	6.4856*10^5
supplier expected profit (no-coll)	6.4864*10^5	6.4864*10^5	6.4864*10^5	6.4864*10^5	6.4858*10^5	6.4856*10^5

buyer 1 expected profit (coll)	2.6606*10^5	2.6446*10^5	2.6237*10^5	2.5924*10^5	2.5412*10^5	2.5403*10^5
buyer 1 expected profit (no-coll)	2.6606*10^5	2.6446*10^5	2.6237*10^5	2.5924*10^5	2.5412*10^5	2.5403*10^5
buyer 2 expected profit (coll)	6.2113*10^4	6.2652*10^4	6.3046*10^4	6.3416*10^4	6.3559*10^4	6.3785*10^4
buyer 2 expected profit (no-coll)	6.2113*10^4	6.2652*10^4	6.3046*10^4	6.3416*10^4	6.3559*10^4	6.3785*10^4

Table 72 Results NBIS with A_1 = 2000 A_2 = 750 β = 0.75 c_2 = 0

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	5.0051*10^5	5.0047*10^5	5.0043*10^5	5.004*10^5	5.0037*10^5	5.0035*10^5
supplier expected profit (no-coll)	5.0051*10^5	5.0047*10^5	5.0043*10^5	5.004*10^5	5.0037*10^5	5.0035*10^5
buyer 1 expected profit (coll)	2.5026*10^5	2.5023*10^5	2.5022*10^5	2.502*10^5	2.5019*10^5	2.5017*10^5
buyer 1 expected profit (no-coll)	2.5026*10^5	2.5023*10^5	2.5022*10^5	2.502*10^5	2.5019*10^5	2.5017*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 73 Results NBIS with A_1 = 2000 A_2 = 1300 β = 0.75 c_2 = 0

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected	5.0924*10^5	5.0952*10^5	5.1045*10^5	5.0983*10^5	5.1175*10^5	5.1173*10^5

profit (coll)						
supplier expected profit (no-coll)	5.0924*10^5	5.0952*10^5	5.1045*10^5	5.0983*10^5	5.1175*10^5	5.1173*10^5
buyer 1 expected profit (coll)	2.4169*10^5	2.4294*10^5	2.4043*10^5	2.4310*10^5	2.3990*10^5	2.3987*10^5
buyer 1 expected profit (no-coll)	2.4169*10^5	2.4294*10^5	2.4043*10^5	2.4310*10^5	2.3990*10^5	2.3987*10^5
buyer 2 expected profit (coll)	1.9208*10^3	1.8184*10^3	2.1305*10^3	1.9451*10^3	2.2703*10^3	2.2936*10^3
buyer 2 expected profit (no-coll)	1.9208*10^3	1.8184*10^3	2.1305*10^3	1.9451*10^3	2.2703*10^3	2.2936*10^3

Table 74 Results NBIS with $A_1 = 2000$ $A_2 = 1600$ $\beta = 0.75$ $c_2 = 0$

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5.8979*10^5	5.8980*10^5	5.8980*10^5	5.8980*10^5	5.8973*10^5	5.8971*10^5
supplier expected profit (no-coll)	5.8979*10^5	5.8980*10^5	5.8980*10^5	5.8980*10^5	5.8973*10^5	5.8971*10^5
buyer 1 expected profit (coll)	2.6196*10^5	2.6032*10^5	2.5804*10^5	2.5462*10^5	2.4906*10^5	2.4893*10^5
buyer 1 expected profit (no-coll)	2.6196*10^5	2.6032*10^5	2.5804*10^5	2.5462*10^5	2.4906*10^5	2.4893*10^5
buyer 2 expected profit (coll)	3.6776*10^4	3.7259*10^4	3.7604*10^4	3.7889*10^4	3.7971*10^4	3.8137*10^4
buyer 2 expected profit (no-coll)	3.6776*10^4	3.7259*10^4	3.7604*10^4	3.7889*10^4	3.7971*10^4	3.8137*10^4

Table 75 Results NBIS with $A_1 = 2000$ $A_2 = 750$ $\beta = 0.9$ $c_2 = 0$

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	5.0051*10^5	5.0047*10^5	5.0043*10^5	5.004*10^5	5.0037*10^5	5.0035*10^5
supplier expected profit (no-coll)	5.0051*10^5	5.0047*10^5	5.0043*10^5	5.004*10^5	5.0037*10^5	5.0035*10^5
buyer 1 expected profit (coll)	2.5026*10^5	2.5023*10^5	2.0522*10^5	2.502*10^5	2.5019*10^5	2.5017*10^5
buyer 1 expected profit (no-coll)	2.5026*10^5	2.5023*10^5	2.0522*10^5	2.502*10^5	2.5019*10^5	2.5017*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 76 Results NBIS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.9$ $c_2 = 0$

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	5.0232*10^5	5.0228*10^5	5.0294*10^5	5.0325*10^5	5.0068*10^5	5.0061*10^5
supplier expected profit (no-coll)	5.0232*10^5	5.0228*10^5	5.0294*10^5	5.0325*10^5	5.0068*10^5	5.0061*10^5
buyer 1 expected profit (coll)	2.4956*10^5	2.4943*10^5	2.4945*10^5	2.4908*10^5	2.4961*10^5	2.4980*10^5
buyer 1 expected profit (no-coll)	2.4956*10^5	2.4943*10^5	2.4945*10^5	2.4908*10^5	2.4961*10^5	2.4980*10^5
buyer 2 expected profit (coll)	69,9174	86,0114	97,0139	77,7892	21,0298	16,7205
buyer 2 expected profit (no-coll)	69,9174	86,0114	97,0139	77,7892	21,0298	16,7205

Table 77 Results NBIS with $A_1 = 2000$ $A_2 = 1600$ $\beta = 0.9$ $c_2 = 0$

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	5.5950*10^5	5.5947*10^5	5.5939*10^5	5.5936*10^5	5.5929*10^5	5.5928*10^5
supplier expected profit (no-coll)	5.5950*10^5	5.5947*10^5	5.5939*10^5	5.5936*10^5	5.5929*10^5	5.5928*10^5
buyer 1 expected profit (coll)	2.0207*10^5	2.3840*10^5	2.5904*10^5	2.6050*10^5	2.5458*10^5	2.5443*10^5
buyer 1 expected profit (no-coll)	2.0207*10^5	2.3840*10^5	2.5904*10^5	2.6050*10^5	2.5458*10^5	2.5443*10^5
buyer 2 expected profit (coll)	2.3538*10^4	2.4020*10^4	2.4435*10^4	2.4679*10^4	2.4732*10^4	2.4910*10^4
buyer 2 expected profit (no-coll)	2.3538*10^4	2.4020*10^4	2.4435*10^4	2.4679*10^4	2.4732*10^4	2.4910*10^4

Table 78 Results NBIS with $A_1 = 2000$ $A_2 = 750$ $\beta = 0.5$

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	6.6293*10^5	6.6287*10^5	6.6282*10^5	6.6278*10^5	6.6274*10^5	6.6271*10^5
supplier expected profit (no-coll)	6.6293*10^5	6.6287*10^5	6.6282*10^5	6.6278*10^5	6.6274*10^5	6.6271*10^5
buyer 1 expected profit (coll)	2.2391*10^5	2.2378*10^5	2.2387*10^5	2.2377*10^5	2.2384*10^5	2.2375*10^5
buyer 1 expected profit (no-coll)	2.2391*10^5	2.2378*10^5	2.2387*10^5	2.2377*10^5	2.2384*10^5	2.2375*10^5

buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 79 Results NBIS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	6.8419*10^5	6.8433*10^5	6.8472*10^5	6.8495*10^5	6.8469*10^5	6.8465*10^5
supplier expected profit (no-coll)	6.6293*10^5	6.6287*10^5	6.6282*10^5	6.6278*10^5	6.6274*10^5	6.6271*10^5
buyer 1 expected profit (coll)	2.2196*10^5	2.2144*10^5	2.2235*10^5	2.2407*10^5	2.4055*10^5	2.3436*10^5
buyer 1 expected profit (no-coll)	2.2388*10^5	2.2378*10^5	2.2387*10^5	2.2377*10^5	2.2384*10^5	2.2375*10^5
buyer 2 expected profit (coll)	6.0534*10^3	6.4503*10^3	6.5465*10^3	6.7457*10^3	6.5155*10^3	6.4443*10^3
buyer 2 expected profit (no-coll)	20.95	0	0	0	0	0

Table 80 Results NBIS with $A_1 = 2000$ $A_2 = 1600$ $\beta = 0.5$

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	8.0649*10^5	8.0649*10^5	8.0649*10^5	8.0649*10^5	8.0644*10^5	8.0643*10^5
supplier expected profit (no-coll)	7.1523*10^5	7.1559*10^5	7.1621*10^5	7.1714*10^5	7.1399*10^5	7.1386*10^5

buyer 1 expected profit (coll)	2.4821*10^5	2.4564*10^5	2.4251*10^5	2.3772*10^5	2.2939*10^5	2.2941*10^5
buyer 1 expected profit (no-coll)	1.7521*10^5	1.7416*10^5	1.7496*10^5	1.8251*10^5	2.4348*10^5	2.5725*10^5
buyer 2 expected profit (coll)	4.6032*10^4	4.6759*10^4	4.7429*10^4	4.7779*10^5	4.8104*10^4	4.8359*10^4
buyer 2 expected profit (no-coll)	2.4207*10^4	2.5298*10^4	2.5236*10^4	2.3110*10^4	2.4494*10^4	2.6099*10^4

Table 81 Results NBIS with $A_1 = 2000$ $A_2 = 750$ $\beta = 0.75$

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	7.9446*10^5	7.9439*10^5	7.9434*10^5	7.9429*10^5	7.9424*10^5	7.9420*10^5
supplier expected profit (no-coll)	7.9446*10^5	7.9439*10^5	7.9434*10^5	7.9429*10^5	7.9424*10^5	7.9420*10^5
buyer 1 expected profit (coll)	1.6390*10^5	1.6390*10^5	1.6393*10^5	1.6386*10^5	1.6380*10^5	1.6392*10^5
buyer 1 expected profit (no-coll)	1.6390*10^5	1.6390*10^5	1.6393*10^5	1.6386*10^5	1.6380*10^5	1.6392*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 82 Results NBIS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.75$

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**

supplier expected profit (coll)	7.9446*10^5	7.9439*10^5	7.9434*10^5	7.9429*10^5	7.9424*10^5	7.9420*10^5
supplier expected profit (no-coll)	7.9446*10^5	7.9439*10^5	7.9434*10^5	7.9429*10^5	7.9424*10^5	7.9420*10^5
buyer 1 expected profit (coll)	1.6390*10^5	1.6390*10^5	1.6393*10^5	1.6386*10^5	1.6380*10^5	1.6392*10^5
buyer 1 expected profit (no-coll)	1.6390*10^5	1.6390*10^5	1.6393*10^5	1.6386*10^5	1.6380*10^5	1.6392*10^5
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 83 Results NBIS with $A_1 = 2000$ $A_2 = 1600$ $\beta = 0.75$

	N1 = 5 N2 = 5	**N1 = 6 N2 = 4**	**N1 = 7 N2 = 3**	**N1 = 8 N2 = 2**	**N1 = 9 N2 = 1**	**N1 = 10 N2 = 1**
supplier expected profit (coll)	8.1598*10^5	8.1606*10^5	8.1629*10^5	8.1641*10^5	8.1632*10^5	8.1631*10^5
supplier expected profit (no-coll)	7.9446*10^5	7.9439*10^5	7.9434*10^5	7.9429*10^5	7.9424*10^5	7.9420*10^5
buyer 1 expected profit (coll)	1.6332*10^5	1.6365*10^5	1.6343*10^5	1.6390*10^5	1.6895*10^5	1.6669*10^5
buyer 1 expected profit (no-coll)	1.6390*10^5	1.6390*10^5	1.6393*10^5	1.6386*10^5	1.6380*10^5	1.6392*10^5
buyer 2 expected profit (coll)	6.2428*10^3	6.4975*10^3	6.9099*10^3	6.9927*10^3	7.0871*10^3	7.1583*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 84 Results NBIS with $A_1 = 2000$ $A_2 = 750$ $\beta = 0.9$

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	8.9377*10^5	8.9369*10^5	8.9363*10^5	8.9357*10^5	8.9352*10^5	8.9348*10^5
supplier expected profit (no-coll)	8.9377*10^5	8.9369*10^5	8.9363*10^5	8.9357*10^5	8.9352*10^5	8.9348*10^5
buyer 1 expected profit (coll)	9.5953*10^4	9.5791*10^4	9.5638*10^4	9.5779*10^4	9.5863*10^4	9.5803*10^4
buyer 1 expected profit (no-coll)	9.5953*10^4	9.5791*10^4	9.5638*10^4	9.5779*10^4	9.5863*10^4	9.5803*10^4
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 85 Results NBIS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.9$

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	8.9377*10^5	8.9369*10^5	8.9363*10^5	8.9357*10^5	8.9352*10^5	8.9348*10^5
supplier expected profit (no-coll)	8.9377*10^5	8.9369*10^5	8.9363*10^5	8.9357*10^5	8.9352*10^5	8.9348*10^5
buyer 1 expected profit (coll)	9.5953*10^4	9.5791*10^4	9.5638*10^4	9.5779*10^4	9.5863*10^4	9.5803*10^4
buyer 1 expected profit (no-coll)	9.5953*10^4	9.5791*10^4	9.5638*10^4	9.5779*10^4	9.5863*10^4	9.5803*10^4
buyer 2 expected profit (coll)	0	0	0	0	0	0
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 86 Results NBIS with A1 = 2000 A2 = 1600 β = 0.9

	N1 = 5 N2 = 5	N1 = 6 N2 = 4	N1 = 7 N2 = 3	N1 = 8 N2 = 2	N1 = 9 N2 = 1	N1 = 10 N2 = 1
supplier expected profit (coll)	8.9377*10^5	8.9369*10^5	8.9363*10^5	8.9357*10^5	8.9352*10^5	8.9348*10^5
supplier expected profit (no-coll)	8.9377*10^5	8.9369*10^5	8.9363*10^5	8.9357*10^5	8.9352*10^5	8.9348*10^5
buyer 1 expected profit (coll)	9.5953*10^4	9.5791*10^4	9.5638*10^4	9.5779*10^4	9.5863*10^4	9.5803*10^4
buyer 1 expected profit (no-coll)	9.5953*10^4	9.5791*10^4	9.5638*10^4	9.5779*10^4	9.5863*10^4	9.5803*10^4
buyer 2 expected profit (coll)	9.3844	11.1323	8.7956	2.2851	0.0606	0.0544
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Full results IBIS cases

Table 87 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 1

	N1 = 1 N2 = 1	N1 = 1 N2 = 2	N1 = 1 N2 = 3	N1 = 1 N2 = 4	N1 = 1 N2 = 5
supplier expected profit (coll)	6.8527*10^5	6.8486*10^5	6.8559*10^5	6.8440*10^5	6.8448*10^5
supplier expected profit (no-coll)	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5
buyer 1 expected profit (coll)	2.6677*10^5	2.4729*10^5	2.6194*10^5	2.5716*10^5	2.6037*10^5
buyer 1 expected profit (no-coll)	2.2392*10^5	2.2392*10^5	2.2392*10^5	2.2392*10^5	2.2392*10^5
buyer 2 expected profit (coll)	3.2639*10^3	3.0988*10^3	3.2683*10^3	3.0866*10^3	3.0792*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 88 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 2

	N1 = 1 N2 = 6	N1 = 1 N2 = 7	N1 = 1 N2 = 8	N1 = 1 N2 = 9	N1 = 1 N2 = 10
supplier expected profit (coll)	6.8394*10^5	6.8379*10^5	6.8330*10^5	6.8327*10^5	6.8294*10^5
supplier expected profit (no-coll)	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5
buyer 1 expected profit (coll)	2.5701*10^5	2.6544*10^5	2.5695*10^5	2.6476*10^5	2.6108*10^5
buyer 1 expected profit (no-coll)	2.2392*10^5	2.2392*10^5	2.2392*10^5	2.2392*10^5	2.2392*10^5
buyer 2 expected profit (coll)	2.9611*10^3	3.0015*10^3	2.8414*10^3	2.8933*10^3	2.8041*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 89 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 3

	N1 = 2 N2 = 1	N1 = 2 N2 = 2	N1 = 2 N2 = 3	N1 = 2 N2 = 4	N1 = 2 N2 = 5	N1 = 2 N2 = 6
supplier expected profit (coll)	6.8574*10^5	6.8575*10^5	6.8496*10^5	6.8528*10^5	6.8434*10^5	6.8369*10^5
supplier expected profit (no-coll)	6.6308*10^5	6.6308*10^5	6.6308*10^5	6.6308*10^5	6.6308*10^5	6.6308*10^5
buyer 1 expected profit (coll)	2.5699*10^5	2.57*10^5	2.5534*10^5	2.5952*10^5	2.6977*10^5	2.6850*10^5
buyer 1 expected profit (no-coll)	2.2395*10^5	2.2395*10^5	2.2395*10^5	2.2395*10^5	2.2395*10^5	2.2395*10^5
buyer 2 expected profit (coll)	3.2248*10^3	3.2780*10^3	3.1755*10^3	3.1966*10^3	3.1901*10^3	3.0930*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 90 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 4

	N1 = 2 N2 = 7	N1 = 2 N2 = 8	N1 = 2 N2 = 9	N1 = 2 N2 = 10
supplier expected profit (coll)	6.8393*10^5	6.8362*10^5	6.8348*10^5	6.8314*10^5
supplier expected profit (no-coll)	6.6308*10^5	6.6308*10^5	6.6308*10^5	6.6308*10^5
buyer 1 expected profit (coll)	2.6195*10^5	2.6020*10^5	2.6251*10^5	2.6804*10^5
buyer 1 expected profit (no-coll)	2.2395*10^5	2.2395*10^5	2.2395*10^5	2.2395*10^5
buyer 2 expected profit (coll)	2.9941*10^3	2.9197*10^3	2.9046*10^3	2.9134*10^3
buyer 2 expected profit (no-coll)	0	0	0	0

Table 91 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 5

	N1 = 3 N2 = 1	N1 = 3 N2 = 2	N1 = 3 N2 = 3	N1 = 3 N2 = 4	N1 = 3 N2 = 5	N1 = 3 N2 = 6
supplier expected profit (coll)	6.8521*10^5	6.8553*10^5	6.8530*10^5	6.8474*10^5	6.8462*10^5	6.8413*10^5
supplier expected profit (no-coll)	6.6305*10^5	6.6305*10^5	6.6305*10^5	6.6305*10^5	6.6305*10^5	6.6305*10^5
buyer 1 expected profit (coll)	2.5277*10^5	2.6465*10^5	2.6232*10^5	2.6160*10^5	2.5832*10^5	2.6757*10^5
buyer 1 expected profit (no-coll)	2.2394*10^5	2.2394*10^5	2.2394*10^5	2.2394*10^5	2.2394*10^5	2.2394*10^5
buyer 2 expected profit (coll)	3.1342*10^3	3.3358*10^3	3.2669*10^3	3.1706*10^3	3.0804*10^3	3.0984*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 92 Full results IBIS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 6

	N1 = 3 N2 = 7	N1 = 3 N2 = 8	N1 = 3 N2 = 9	N1 = 3 N2 = 10
supplier expected profit (coll)	6.8381*10^5	6.8358*10^5	6.8330*10^5	6.8311*10^5
supplier expected profit (no-coll)	6.6305*10^5	6.6305*10^5	6.6305*10^5	6.6305*10^5
buyer 1 expected profit (coll)	2.5837*10^5	2.6636*10^5	2.6280*10^5	2.6355*10^5
buyer 1 expected profit (no-coll)	2.2394*10^5	2.2394*10^5	2.2396*10^5	2.2395*10^5
buyer 2 expected profit (coll)	2.9397*10^3	2.9739*10^3	2.8856*10^3	2.8532*10^3
buyer 2 expected profit (no-coll)	0	0	3.29	1.67

Table 93 Full results IBIS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 7

	N1 = 4 N2 = 1	N1 = 4 N2 = 2	N1 = 4 N2 = 3	N1 = 4 N2 = 4	N1 = 4 N2 = 5	N1 = 4 N2 = 6
supplier expected profit (coll)	6.8532*10^5	6.8534*10^5	6.8496*10^5	6.8469*10^5	6.8430*10^5	6.8347*10^5
supplier expected profit (no-coll)	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.5074*10^5
buyer 1 expected profit (coll)	2.5965*10^5	2.5577*10^5	2.5550*10^5	2.5921*10^5	2.6426*10^5	2.6623*10^5
buyer 1 expected profit (no-coll)	2.2392*10^5	2.2392*10^5	2.2392*10^5	2.2392*10^5	2.2392*10^5	2.3902*10^5
buyer 2 expected profit (coll)	3.2038*10^3	3.2145*10^3	3.1547*10^3	3.1275*10^3	3.1053*10^3	3.0281*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 94 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 8

	N1 = 4 N2 = 7	N1 = 4 N2 = 8	N1 = 4 N2 = 9	N1 = 4 N2 = 10
supplier expected profit (coll)	6.8373*10^5	6.8323*10^5	6.8321*10^5	6.8285*10^5
supplier expected profit (no-coll)	6.63*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5
buyer 1 expected profit (coll)	2.6609*10^5	2.5751*10^5	2.6521*10^5	2.6551*10^5
buyer 1 expected profit (no-coll)	2.2396*10^5	2.2394*10^5	2.2393*10^5	2.2393*10^5
buyer 2 expected profit (coll)	3.0009*10^3	2.8391*10^3	2.8899*10^3	2.8410*10^3
buyer 2 expected profit (no-coll)	6.55	3.34	1.7	0.86

Table 95 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 9

	N1 = 5 N2 = 1	N1 = 5 N2 = 2	N1 = 5 N2 = 3	N1 = 5 N2 = 4	N1 = 5 N2 = 5	N1 = 5 N2 = 6
supplier expected profit (coll)	6.8513*10^5	6.8528*10^5	6.8500*10^5	6.8459*10^5	6.8415*10^5	6.8379*10^5
supplier expected profit (no-coll)	6.6293*10^5	6.6293*10^5	6.6293*10^5	6.6293*10^5	6.6293*10^5	6.6294*10^5
buyer 1 expected profit (coll)	2.5505*10^5	2.6043*10^5	2.6168*10^5	2.6079*10^5	2.6190*10^5	2.6481*10^5
buyer 1 expected profit (no-coll)	2.2390*10^5	2.2390*10^5	2.2390*10^5	2.2390*10^5	2.2390*10^5	2.2393*10^5
buyer 2 expected profit (coll)	3.1289*10^3	3.2456*10^3	3.2104*10^3	3.1234*10^3	3.0554*10^3	3.0164*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0	6.55

Table 96 Full results IBIS with A_1 = 2000 A_2 = 1300   β = 0.5 part 10

	N1 = 5 N2 = 7	N1 = 5 N2 = 8	N1 = 5 N2 = 9	N1 = 5 N2 = 10
supplier expected profit (coll)	6.8339*10^5	6.8328*10^5	6.8271*10^5	6.8280*10^5
supplier expected profit (no-coll)	6.6293*10^5	6.6293*10^5	6.6293*10^5	6.6293*10^5
buyer 1 expected profit (coll)	2.6126*10^5	2.6502*10^5	2.7101*10^5	2.6435*10^5
buyer 1 expected profit (no-coll)	2.2392*10^5	2.2391*10^5	2.2391*10^5	2.2390*10^5
buyer 2 expected profit (coll)	2.9106*10^3	2.9083*10^3	2.9026*10^3	2.8096*10^3
buyer 2 expected profit (no-coll)	3.35	1.71	0.87	0.44

Table 97 Full results IBIS with A_1 = 2000 A_2 = 1300   β = 0.5 part 11

	N1 = 6 N2 = 1	N1 = 6 N2 = 2	N1 = 6 N2 = 3	N1 = 6 N2 = 4	N1 = 6 N2 = 5	N1 = 6 N2 = 6
supplier expected profit (coll)	6.8486*10^5	6.8501*10^5	6.8470*10^5	6.8429*10^5	6.8397*10^5	6.8322*10^5
supplier expected profit (no-coll)	6.6287*10^5	6.6287*10^5	6.6287*10^5	6.6287*10^5	6.6288*10^5	6.5268*10^5
buyer 1 expected profit (coll)	2.6113*10^5	2.6362*10^5	2.5768*10^5	2.6643*10^5	2.6546*10^5	2.7043*10^5
buyer 1 expected profit (no-coll)	2.2388*10^5	2.2388*10^5	2.2388*10^5	2.2388*10^5	2.2392*10^5	2.3648*10^5
buyer 2 expected profit (coll)	3.1647*10^3	3.2474*10^3	3.1329*10^3	3.1495*10^3	3.0674*10^3	3.0259*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	6.45	3.33

Table 98 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 12

	N1 = 6 N2 = 7	**N1 = 6 N2 = 8**	**N1 = 6 N2 = 9**	**N1 = 6 N2 = 10**
supplier expected profit (coll)	6.8330*10^5	6.8296*10^5	6.8283*10^5	6.8245*10^5
supplier expected profit (no-coll)	6.6288*10^5	6.6287*10^5	6.6287*10^5	6.6287*10^5
buyer 1 expected profit (coll)	2.6476*10^5	2.6401*10^5	2.6422*10^5	2.7040*10^5
buyer 1 expected profit (no-coll)	2.2389*10^5	2.2389*10^5	2.2389*10^5	2.2389*10^5
buyer 2 expected profit (coll)	2.9280*10^3	2.8607*10^3	2.8235*10^3	2.8362*10^3
buyer 2 expected profit (no-coll)	1.71	0.87	0.44	0.22

Table 99 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 13

	N1 = 7 N2 = 1	**N1 = 7 N2 = 2**	**N1 = 7 N2 = 3**	**N1 = 7 N2 = 4**	**N1 = 7 N2 = 5**	**N1 = 7 N2 = 6**
supplier expected profit (coll)	6.8488*10^5	6.8495*10^5	6.8466*10^5	6.8425*10^5	6.8382*10^5	6.8342*10^5
supplier expected profit (no-coll)	6.6282*10^5	6.6282*10^5	6.6282*10^5	6.6282*10^5	6.6283*10^5	6.6283*10^5
buyer 1 expected profit (coll)	2.5662*10^5	2.5953*10^5	2.6246*10^5	2.6053*10^5	2.5980*10^5	2.6472*10^5
buyer 1 expected profit (no-coll)	2.2387*10^5	2.2387*10^5	2.2387*10^5	2.2387*10^5	2.2388*10^5	2.2388*10^5
buyer 2 expected profit (coll)	3.1070*10^3	3.1878*10^5	3.1703*10^3	3.0715*10^3	2.9843*10^3	2.9638*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	3.27	1.69

Table 100 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 14

	N1 = 7 N2 = 7	N1 = 7 N2 = 8	N1 = 7 N2 = 9	N1 = 7 N2 = 10
supplier expected profit (coll)	6.8310*10^5	6.8287*10^5	6.8258*10^5	6.8240*10^5
supplier expected profit (no-coll)	6.6283*10^5	6.6282*10^5	6.6282*10^5	6.6282*10^5
buyer 1 expected profit (coll)	2.6712*10^5	2.6379*10^5	2.6614*10^5	2.6368*10^5
buyer 1 expected profit (no-coll)	2.2387*10^5	2.2387*10^5	2.2387*10^5	2.2387*10^5
buyer 2 expected profit (coll)	2.9275*10^3	2.8397*10^3	2.8141*10^3	2.7477*10^3
buyer 2 expected profit (no-coll)	0.86	0.44	0.22	0.11

Table 101 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 15

	N1 = 8 N2 = 1	N1 = 8 N2 = 2	N1 = 8 N2 = 3	N1 = 8 N2 = 4	N1 = 8 N2 = 5	N1 = 8 N2 = 6
supplier expected profit (coll)	6.8470*10^5	6.8442*10^5	6.8445*10^5	6.8388*10^5	6.8372*10^5	6.8338*10^5
supplier expected profit (no-coll)	6.6278*10^5	6.5103*10^5	6.6278*10^5	6.5661*10^5	6.6278*10^5	6.6278*10^5
buyer 1 expected profit (coll)	2.5365*10^5	2.6950*10^5	2.5866*10^5	2.6530*10^5	2.6495*10^5	2.6426*10^5
buyer 1 expected profit (no-coll)	2.2385*10^5	2.3834*10^5	2.2385*10^5	2.3149*10^5	2.2386*10^5	2.2386*10^5
buyer 2 expected profit (coll)	3.0534*10^3	3.2508*10^3	3.1046*10^3	3.0881*10^3	3.0222*10^3	2.9459*10^3
buyer 2 expected profit (no-coll)	0	0	0	3.16	1.65	0.853

Table 102 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 16

	N1 = 8 N2 = 7	N1 = 8 N2 = 8	N1 = 8 N2 = 9	N1 = 8 N2 = 10
supplier expected profit (coll)	6.8303*10^5	6.8272*10^5	6.8248*10^5	6.8224*10^5
supplier expected profit (no-coll)	6.6278*10^5	6.6278*10^5	6.6278*10^5	6.6278*10^5
buyer 1 expected profit (coll)	2.6383*10^5	2.6413*10^5	2.6565*10^5	2.6739*10^5
buyer 1 expected profit (no-coll)	2.2386*10^5	2.2385*10^5	2.2386*10^5	2.2386*10^5
buyer 2 expected profit (coll)	2.8765*10^3	2.8223*10^3	2.7909*10^3	2.7652*10^3
buyer 2 expected profit (no-coll)	0.437	0.223	0.936	0.969

Table 103 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 17

	N1 = 9 N2 = 1	N1 = 9 N2 = 2	N1 = 9 N2 = 3	N1 = 9 N2 = 4	N1 = 9 N2 = 5	N1 = 9 N2 = 6
supplier expected profit (coll)	6.8462*10^5	6.8471*10^5	6.8441*10^5	6.8397*10^5	6.8358*10^5	6.8317*10^5
supplier expected profit (no-coll)	6.6274*10^5	6.6274*10^5	6.6274*10^5	6.6275*10^5	6.6275*10^5	6.6275*10^5
buyer 1 expected profit (coll)	2.5779*10^5	2.6463*10^5	2.6214*10^5	2.6549*10^5	2.6142*10^5	2.6380*10^5
buyer 1 expected profit (no-coll)	2.2384*10^5	2.2384*10^5	2.2384*10^5	2.2385*10^5	2.2384*10^5	2.2384*10^5
buyer 2 expected profit (coll)	3.0857*10^3	3.2063*10^3	3.1309*10^3	3.0863*10^3	2.9650*10^3	2.9167*10^3
buyer 2 expected profit (no-coll)	0	0	0	1.6	0.83	0.43

Table 104 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 18

	N1 = 9 N2 = 7	N1 = 9 N2 = 8	N1 = 9 N2 = 9	N1 = 9 N2 = 10
supplier expected profit (coll)	6.8287*10^5	6.8261*10^5	6.8236*10^5	6.8212*10^5
supplier expected profit (no-coll)	6.6274*10^5	6.6275*10^5	6.6275*10^5	6.6275*10^5
buyer 1 expected profit (coll)	2.6803*10^5	2.6705*10^5	2.6568*10^5	2.6755*10^5
buyer 1 expected profit (no-coll)	2.2384*10^5	2.2385*10^5	2.2384*10^5	2.2384*10^5
buyer 2 expected profit (coll)	2.9009*10^3	2.8366*10^3	2.7730*10^3	2.7491*10^3
buyer 2 expected profit (no-coll)	0.22	1.04	0.53	0.52

Table 105 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 19

	N1 = 10 N2 = 1	N1 = 10 N2 = 2	N1 = 10 N2 = 3	N1 = 10 N2 = 4	N1 = 10 N2 = 5	N1 = 10 N2 = 6
supplier expected profit (coll)	6.8456*10^5	6.8467*10^5	6.8427*10^5	6.8393*10^5	6.8351*10^5	6.8315*10^5
supplier expected profit (no-coll)	6.6271*10^5	6.6271*10^5	6.6271*10^5	6.6271*10^5	6.6271*10^5	6.6271*10^5
buyer 1 expected profit (coll)	2.5467*10^5	2.6102*10^5	2.5897*10^5	2.6133*10^5	2.6436*10^5	2.6383*10^5
buyer 1 expected profit (no-coll)	2.2383*10^5	2.2383*10^5	2.2383*10^5	2.2383*10^5	2.2383*10^5	2.2383*10^5
buyer 2 expected profit (coll)	3.0409*10^3	3.1565*10^3	3.0789*10^3	3.0302*10^3	2.9840*10^3	2.9070*10^3
buyer 2 expected profit (no-coll)	0	0	0	0.80	0.42	0.22

Table 106 Full results IBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 20

	N1 = 10 N2 = 7	**N1 = 10 N2 = 8**	**N1 = 10 N2 = 9**	**N1 = 10 N2 = 10**
supplier expected profit (coll)	6.8282*10^5	6.8250*10^5	6.8224*10^5	6.8202*10^5
supplier expected profit (no-coll)	6.6271*10^5	6.6271*10^5	6.6271*10^5	6.6271*10^5
buyer 1 expected profit (coll)	2.6341*10^5	2.6302*10^5	2.6542*10^5	2.6732*10^5
buyer 1 expected profit (no-coll)	2.2384*10^5	2.2383*10^5	2.2383*10^5	2.2383*10^5
buyer 2 expected profit (coll)	2.8392*10^3	2.7766*10^3	2.7531*10^3	2.7308*10^3
buyer 2 expected profit (no-coll)	1.14	0.58	0.51	0.28

Full results IBNS cases

Table 107 Full results IBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 1

	N1 = 1 N2 = 1	**N1 = 1 N2 = 2**	**N1 = 1 N2 = 3**	**N1 = 1 N2 = 4**	**N1 = 1 N2 = 5**	**N1 = 1 N2 = 6**
supplier expected profit (coll)	6.7874*10^5	6.7865*10^5	6.7687*10^5	6.7749*10^5	6.7686*10^5	6.7628*10^5
supplier expected profit (no-coll)	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5
buyer 1 expected profit (coll)	2.6167*10^5	2.7823*10^5	2.6311*10^5	2.7534*10^5	2.8266*10^5	2.7391*10^5
buyer 1 expected profit (no-coll)	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5
buyer 2 expected profit (coll)	1.8783*10^3	2.9958*10^3	1.9767*10^3	2.5930*10^3	2.9732*10^3	2.2430^10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 108 Full results IBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 2

	N1 = 1 N2 = 7	N1 = 1 N2 = 8	N1 = 1 N2 = 9	N1 = 1 N2 = 10
supplier expected profit (coll)	6.7631*10^5	6.7595*10^5	6.7567*10^5	6.7567*10^5
supplier expected profit (no-coll)	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5
buyer 1 expected profit (coll)	2.8120*10^5	2.8563*10^5	2.8048*10^5	2.8490*10^5
buyer 1 expected profit (no-coll)	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5
buyer 2 expected profit (coll)	2.6237*10^3	2.8476*10^3	2.3820*10^3	2.6185*10^3
buyer 2 expected profit (no-coll)	0	0	0	0

Table 109 Full results IBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 3

	N1 = 2 N2 = 1	N1 = 2 N2 = 2	N1 = 2 N2 = 3	N1 = 2 N2 = 4	N1 = 2 N2 = 5	N1 = 2 N2 = 6
supplier expected profit (coll)	6.7848*10^5	6.7840*10^5	6.7661*10^5	6.7723*10^5	6.7660*10^5	6.7602*10^5
supplier expected profit (no-coll)	6.5990*10^5	6.5990*10^5	6.5990*10^5	6.5990*10^5	6.5990*10^5	6.5990*10^5
buyer 1 expected profit (coll)	2.6194*10^5	2.7851*10^5	2.6339*10^5	2.7561*10^5	2.8293*10^5	2.7418*10^5
buyer 1 expected profit (no-coll)	2.2580*10^5	2.2580*10^5	2.2580*10^5	2.2580*10^5	2.2580*10^5	2.2580*10^5
buyer 2 expected profit (coll)	1.8783*10^3	2.9959*10^3	1.9767*10^3	2.5930*10^3	2.9732*10^3	2.2430*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0	0

Table 110 Full results IBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 4

	N1 = 2 N2 = 7	N1 = 2 N2 = 8	N1 = 2 N2 = 9	N1 = 2 N2 = 10
supplier expected profit (coll)	6.7605*10^5	6.7570*10^5	6.7541*10^5	6.7541*10^5
supplier expected profit (no-coll)	6.5990*10^5	6.5990*10^5	6.5990*10^5	6.5990*10^5
buyer 1 expected profit (coll)	2.8148*10^5	2.8590*10^5	2.8075*10^5	2.8517*10^5
buyer 1 expected profit (no-coll)	2.2580*10^5	2.2580*10^5	2.2580*10^5	2.2580*10^5
buyer 2 expected profit (coll)	2.6238*10^3	2.8476*10^3	2.3820*10^3	2.6186*10^3
buyer 2 expected profit (no-coll)	0	0	0	0

Table 111 Full results IBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 5

	N1 = 3 N2 = 1	N1 = 3 N2 = 2	N1 = 3 N2 = 3	N1 = 3 N2 = 4	N1 = 3 N2 = 5
supplier expected profit (coll)	6.7857*10^5	6.7849*10^5	6.7671*10^5	6.7733*10^5	6.7669*10^5
supplier expected profit (no-coll)	6.5999*10^5	6.5999*10^5	6.5999*10^5	6.5999*10^5	6.5999*10^5
buyer 1 expected profit (coll)	2.6184*10^5	2.7841*10^5	2.6329*10^5	2.7551*10^5	2.8284*10^5
buyer 1 expected profit (no-coll)	2.2570*10^5	2.2570*10^5	2.2570*10^5	2.2570*10^5	2.2570*10^5
buyer 2 expected profit (coll)	1.8783*10^3	2.9959*10^3	1.9767*10^3	2.5930*10^3	2.9732*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 112 Full results IBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 6

	N1 = 3 N2 = 6	N1 = 3 N2 = 7	N1 = 3 N2 = 8	N1 = 3 N2 = 9	N1 = 3 N2 = 10
supplier expected profit (coll)	6.7612*10^5	6.7615*10^5	6.7579*10^5	6.7551*10^5	6.7550*10^5
supplier expected profit (no-coll)	6.5999*10^5	6.5999*10^5	6.5999*10^5	6.5999*10^5	6.5999*10^5

buyer 1 expected profit (coll)	2.7408*10^5	2.8138*10^5	2.8581*10^5	2.8066*10^5	2.8507*10^5
buyer 1 expected profit (no-coll)	2.2570*10^5	2.2570*10^5	2.2570*10^5	2.2570*10^5	2.2570*10^5
buyer 2 expected profit (coll)	2.2430*10^3	2.6237*10^3	2.8476*10^3	2.3820*10^3	2.6185*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 113 Full results IBNS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 7

	N1 = 4 N2 = 1	**N1 = 4 N2 = 2**	**N1 = 4 N2 = 3**	**N1 = 4 N2 = 4**	**N1 = 4 N2 = 5**
supplier expected profit (coll)	6.7874*10^5	6.7865*10^5	6.7687*10^5	6.7749*10^5	6.7686*10^5
supplier expected profit (no-coll)	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5
buyer 1 expected profit (coll)	2.6167*10^5	2.7823*10^5	2.6311*10^5	2.7534*10^5	2.8266*10^5
buyer 1 expected profit (no-coll)	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5
buyer 2 expected profit (coll)	1.8783*10^3	2.9958*10^3	1.9767*10^3	2.5930*10^3	2.9732*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 114 Full results IBNS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 8

	N1 = 4 N2 = 6	**N1 = 4 N2 = 7**	**N1 = 4 N2 = 8**	**N1 = 4 N2 = 9**	**N1 = 4 N2 = 10**
supplier expected profit (coll)	6.7628*10^5	6.7631*10^5	6.7595*10^5	6.7567*10^5	6.7567*10^5
supplier expected profit (no-coll)	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5
buyer 1 expected profit (coll)	2.7391*10^5	2.8120*10^5	2.8563*10^5	2.8048*10^5	2.8490*10^5
buyer 1 expected profit (no-coll)	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5
buyer 2 expected profit (coll)	2.2430*10^3	2.6237*10^3	2.8476*10^3	2.3820*10^3	2.6185*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 115 Full results IBNS with A_1 = 2000 A_2 = 1300   β = 0.5 part 9

	N1 = 5 N2 = 1	N1 = 5 N2 = 2	N1 = 5 N2 = 3	N1 = 5 N2 = 4	N1 = 5 N2 = 5
supplier expected profit (coll)	6.7891*10^5	6.7882*10^5	6.7704*10^5	6.7766*10^5	6.7703*10^5
supplier expected profit (no-coll)	6.6033*10^5	6.6033*10^5	6.6033*10^5	6.6033*10^5	6.6033*10^5
buyer 1 expected profit (coll)	2.6149*10^5	2.7806*10^5	2.6294*10^5	2.7516*10^5	2.8248*10^5
buyer 1 expected profit (no-coll)	2.2535*10^5	2.2535*10^5	2.2535*10^5	2.2535*10^5	2.2535*10^5
buyer 2 expected profit (coll)	1.8783*10^3	2.9958*10^3	1.9767*10^3	2.5929*10^3	2.9731*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 116 Full results IBNS with A_1 = 2000 A_2 = 1300   β = 0.5 part 10

	N1 = 5 N2 = 6	N1 = 5 N2 = 7	N1 = 5 N2 = 8	N1 = 5 N2 = 9	N1 = 5 N2 = 10
supplier expected profit (coll)	6.7645*10^5	6.7648*10^5	6.7612*10^5	6.7584*10^5	6.7584*10^5
supplier expected profit (no-coll)	6.6033*10^5	6.6033*10^5	6.6033*10^5	6.6033*10^5	6.6033*10^5
buyer 1 expected profit (coll)	2.7373*10^5	2.8103*10^5	2.8545*10^5	2.8030*10^5	2.8472*10^5
buyer 1 expected profit (no-coll)	2.2535*10^5	2.2535*10^5	2.2535*10^5	2.2535*10^5	2.2535*10^5
buyer 2 expected profit (coll)	2.2430*10^3	2.6237*10^3	2.8476*10^3	2.3820*10^3	2.6185*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 117 Full results IBNS with A_1 = 2000 A_2 = 1300   β = 0.5 part 11

	N1 = 6 N2 = 1	N1 = 6 N2 = 2	N1 = 6 N2 = 3	N1 = 6 N2 = 4	N1 = 6 N2 = 5

supplier expected profit (coll)	6.7906*10^5	6.7898*10^5	6.7720*10^5	6.7781*10^5	6.7718*10^5
supplier expected profit (no-coll)	6.6049*10^5	6.6049*10^5	6.6049*10^5	6.6049*10^5	6.6049*10^5
buyer 1 expected profit (coll)	2.6133*10^5	2.7789*10^5	2.6277*10^5	2.75*10^5	2.8232*10^5
buyer 1 expected profit (no-coll)	2.2518*10^5	2.2518*10^5	2.2518*10^5	2.2518*10^5	2.2518*10^5
buyer 2 expected profit (coll)	1.8782*10^3	2.9958*10^3	1.9767*10^3	2.5929*10^3	2.9731*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 118 Full results IBNS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 12

	N1 = 6 N2 = 6	**N1 = 6 N2 = 7**	**N1 = 6 N2 = 8**	**N1 = 6 N2 = 9**	**N1 = 6 N2 = 10**
supplier expected profit (coll)	6.7660*10^5	6.7663*10^5	6.7628*10^5	6.7599*10^5	6.7599*10^5
supplier expected profit (no-coll)	6.6049*10^5	6.6049*10^5	6.6049*10^5	6.6049*10^5	6.6049*10^5
buyer 1 expected profit (coll)	2.7356*10^5	2.8086*10^5	2.8529*10^5	2.8014*10^5	2.8456*10^5
buyer 1 expected profit (no-coll)	2.2518*10^5	2.2518*10^5	2.2518*10^5	2.2518*10^5	2.2518*10^5
buyer 2 expected profit (coll)	2.2430*10^3	2.6237*10^3	2.8475*10^3	2.3820*10^3	2.6185*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 119 Full results IBNS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 13

	N1 = 7 N2 = 1	**N1 = 7 N2 = 2**	**N1 = 7 N2 = 3**	**N1 = 7 N2 = 4**	**N1 = 7 N2 = 5**
supplier expected profit (coll)	6.7920*10^5	6.7911*10^5	6.7733*10^5	6.7795*10^5	6.7732*10^5
supplier expected profit (no-coll)	6.6062*10^5	6.6062*10^5	6.6062*10^5	6.6062*10^5	6.6062*10^5
buyer 1 expected profit (coll)	2.6118*10^5	2.775*10^5	2.6263*10^5	2.7485*10^5	2.8218*10^5
buyer 1 expected profit (no-coll)	2.2504*10^5	2.2504*10^5	2.2504*10^5	2.2504*10^5	2.2504*10^5

buyer 2 expected profit (coll)	1.8782*10^3	2.9958*10^3	1.9767*10^3	2.5929*10^3	2.9731*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 120 Full results IBNS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 14

	N1 = 7 N2 = 6	**N1 = 7 N2 = 7**	**N1 = 7 N2 = 8**	**N1 = 7 N2 = 9**	**N1 = 7 N2 = 10**
supplier expected profit (coll)	6.7674*10^5	6.7677*10^5	6.7641*10^5	6.7613*10^5	6.7613*10^5
supplier expected profit (no-coll)	6.6062*10^5	6.6062*10^5	6.6062*10^5	6.6062*10^5	6.6062*10^5
buyer 1 expected profit (coll)	2.7342*10^5	2.8072*10^5	2.8515*10^5	2.8000*10^5	2.8441*10^5
buyer 1 expected profit (no-coll)	2.2504*10^5	2.2504*10^5	2.2504*10^5	2.2504*10^5	2.2504*10^5
buyer 2 expected profit (coll)	2.2430*10^3	2.6237*10^3	2.8475*10^3	2.3820*10^3	2.6185*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 121 Full results IBNS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 15

	N1 = 8 N2 = 1	**N1 = 8 N2 = 2**	**N1 = 8 N2 = 3**	**N1 = 8 N2 = 4**	**N1 = 8 N2 = 5**
supplier expected profit (coll)	6.7932*10^5	6.7923*10^5	6.7745*10^5	6.7807*10^5	6.7743*10^5
supplier expected profit (no-coll)	6.6074*10^5	6.6074*10^5	6.6074*10^5	6.6074*10^5	6.6074*10^5
buyer 1 expected profit (coll)	2.6106*10^5	2.7762*10^5	2.6250*10^5	2.7473*10^5	2.8205*10^5
buyer 1 expected profit (no-coll)	2.2491*10^5	2.2491*10^5	2.2491*10^5	2.2491*10^5	2.2491*10^5
buyer 2 expected profit (coll)	1.8782*10^3	2.9958*10^3	1.9767*10^3	2.5929*10^3	2.9731*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 122 Full results IBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 16

	N1 = 8 N2 = 6	N1 = 8 N2 = 7	N1 = 8 N2 = 8	N1 = 8 N2 = 9	N1 = 8 N2 = 10
supplier expected profit (coll)	6.7686*10^5	6.7689*10^5	6.7653*10^5	6.7625*10^5	6.7624*10^5
supplier expected profit (no-coll)	6.6074*10^5	6.6074*10^5	6.6074*10^5	6.6074*10^5	6.6074*10^5
buyer 1 expected profit (coll)	2.7330*10^5	2.8059*10^5	2.8502*10^5	2.7987*10^5	2.8429*10^5
buyer 1 expected profit (no-coll)	2.2491*10^5	2.2491*10^5	2.2491*10^5	2.2491*10^5	2.2491*10^5
buyer 2 expected profit (coll)	2.2429*10^3	2.6237*10^3	2.8475*10^3	2.3820*10^3	2.6185*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 123 Full results IBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 17

	N1 = 9 N2 = 1	N1 = 9 N2 = 2	N1 = 9 N2 = 3	N1 = 9 N2 = 4	N1 = 9 N2 = 5
supplier expected profit (coll)	6.7942*10^5	6.7934*10^5	6.7756*10^5	6.7817*10^5	6.7754*10^5
supplier expected profit (no-coll)	6.6085*10^5	6.6085*10^5	6.6085*10^5	6.6085*10^5	6.6085*10^5
buyer 1 expected profit (coll)	2.6095*10^5	2.7751*10^5	2.6239*10^5	2.7462*10^5	2.8194*10^5
buyer 1 expected profit (no-coll)	2.2480*10^5	2.2480*10^5	2.2480*10^5	2.2480*10^5	2.2480*10^5
buyer 2 expected profit (coll)	1.8782*10^3	2.9958*10^3	1.9767*10^3	2.5929*10^3	2.9731*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 124 Full results IBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 18

	N1 = 9 N2 = 6	N1 = 9 N2 = 7	N1 = 9 N2 = 8	N1 = 9 N2 = 9	N1 = 9 N2 = 10
supplier expected profit (coll)	6.7696*10^5	6.7699*10^5	6.7663*10^5	6.7635*10^5	6.7635*10^5

supplier expected profit (no-coll)	6.6085*10^5	6.6085*10^5	6.6085*10^5	6.6085*10^5	6.6085*10^5
buyer 1 expected profit (coll)	2.7318*10^5	2.8048*10^5	2.8491*10^5	2.7976*10^5	2.8418*10^5
buyer 1 expected profit (no-coll)	2.2480*10^5	2.2480*10^5	2.2480*10^5	2.2480*10^5	2.2480*10^5
buyer 2 expected profit (coll)	2.2429*10^3	2.6237*10^3	2.8475*10^3	2.3820*10^3	2.6185*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 125 Full results IBNS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 19

	N1 = 10 N2 = 1	**N1 = 10 N2 = 2**	**N1 = 10 N2 = 3**	**N1 = 10 N2 = 4**	**N1 = 10 N2 = 5**
supplier expected profit (coll)	6.7951*10^5	6.7943*10^5	6.7765*10^5	6.7827*10^5	6.7763*10^5
supplier expected profit (no-coll)	6.6094*10^5	6.6094*10^5	6.6094*10^5	6.6094*10^5	6.6094*10^5
buyer 1 expected profit (coll)	2.6085*10^5	2.7742*10^5	2.6230*10^5	2.7452*10^5	2.8185*10^5
buyer 1 expected profit (no-coll)	2.2471*10^5	2.2471*10^5	2.2471*10^5	2.2471*10^5	2.2471*10^5
buyer 2 expected profit (coll)	1.8782*10^3	2.9958*10^3	1.9767*10^3	2.5929*10^3	2.9731*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 126 Full results IBNS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 20

	N1 = 10 N2 = 6	**N1 = 10 N2 = 7**	**N1 = 10 N2 = 8**	**N1 = 10 N2 = 9**	**N1 = 10 N2 = 10**
supplier expected profit (coll)	6.7706*10^5	6.7709*10^5	6.7673*10^5	6.7645*10^5	
supplier expected profit (no-coll)	6.6094*10^5	6.6094*10^5	6.6094*10^5	6.6094*10^5	6.6094*10^5
buyer 1 expected profit (coll)	2.7309*10^5	2.8039*10^5	2.8481*10^5	2.7966*10^5	
buyer 1 expected profit (no-coll)	2.2471*10^5	2.2471*10^5	2.2471*10^5	2.2471*10^5	2.2471*10^5
buyer 2 expected profit (coll)	2.2429*10^3	2.6237*10^3	2.8475*10^3	2.3819*10^3	

buyer 2 expected profit (no-coll)	0	0	0	0	0

Full results NBNS cases

Table 127 Full results NBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 1

	N1 = 1 N2 = 1	**N1 = 1 N2 = 2**	**N1 = 1 N2 = 3**	**N1 = 1 N2 = 4**	**N1 = 1 N2 = 5**
supplier expected profit (coll)	6.7887*10^5	6.7872*10^5	6.7699*10^5	6.7756*10^5	6.7688*10^5
supplier expected profit (no-coll)	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5
buyer 1 expected profit (coll)	2.6167*10^5	2.7823*10^5	2.6311*10^5	2.7534*10^5	2.8266*10^5
buyer 1 expected profit (no-coll)	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5
buyer 2 expected profit (coll)	1.8779*10^3	2.9953*10^3	1.9763*10^3	2.5925*10^3	2.9727*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 128 Full results NBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 2

	N1 = 1 N2 = 6	**N1 = 1 N2 = 7**	**N1 = 1 N2 = 8**	**N1 = 1 N2 = 9**	**N1 = 1 N2 = 10**
supplier expected profit (coll)	6.7635*10^5	6.7635*10^5	6.7596*10^5	6.7571*10^5	6.7568*10^5
supplier expected profit (no-coll)	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5
buyer 1 expected profit (coll)	2.7390*10^5	2.8120*10^5	2.8563*10^5	2.8048*10^5	2.8490*10^5
buyer 1 expected profit (no-coll)	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5
buyer 2 expected profit (coll)	2.2426*10^3	2.6233*10^3	2.8471*10^3	2.3816*10^3	2.6181*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 129 Full results NBNS with A_1 = 2000 A_2 = 1300   β = 0.5 part 3

	N1 = 2 N2 = 1	N1 = 2 N2 = 2	N1 = 2 N2 = 3	N1 = 2 N2 = 4	N1 = 2 N2 = 5
supplier expected profit (coll)	6.7861*10^5	6.7846*10^5	6.7673*10^5	6.7730*10^5	6.7662*10^5
supplier expected profit (no-coll)	6.5990*10^5	6.5990*10^5	6.5990*10^5	6.5990*10^5	6.5990*10^5
buyer 1 expected profit (coll)	2.6194*10^5	2.7851*10^5	2.6339*10^5	2.7561*10^5	2.8293*10^5
buyer 1 expected profit (no-coll)	2.2580*10^5	2.2580*10^5	2.2580*10^5	2.2580*10^5	2.2580*10^5
buyer 2 expected profit (coll)	1.8779*10^3	2.9953*10^3	1.9763*10^3	2.5925*10^3	2.9727*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 130 Full results NBNS with A_1 = 2000 A_2 = 1300   β = 0.5 part 4

	N1 = 2 N2 = 6	N1 = 2 N2 = 7	N1 = 2 N2 = 8	N1 = 2 N2 = 9	N1 = 2 N2 = 10
supplier expected profit (coll)	6.7609*10^5	6.7608*10^5	6.7570*10^5	6.7545*10^5	6.7542*10^5
supplier expected profit (no-coll)	6.5990*10^5	6.5990*10^5	6.5990*10^5	6.5990*10^5	6.5990*10^5
buyer 1 expected profit (coll)	2.7418*10^5	2.8148*10^5	2.8590*10^5	2.8075*10^5	2.8517*10^5
buyer 1 expected profit (no-coll)	2.2580*10^5	2.2580*10^5	2.2580*10^5	2.2580*10^5	2.2580*10^5
buyer 2 expected profit (coll)	2.2426*10^3	2.6233*10^3	2.8471*10^3	2.3816*10^3	2.6181*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 131 Full results NBNS with A_1 = 2000 A_2 = 1300   β = 0.5 part 5

	N1 = 3 N2 = 1	N1 = 3 N2 = 2	N1 = 3 N2 = 3	N1 = 3 N2 = 4	N1 = 3 N2 = 5
supplier expected profit (coll)	6.7870*10^5	6.7855*10^5	6.7682*10^5	6.7740*10^5	6.7672*10^5
supplier expected profit (no-coll)	6.5999*10^5	6.5999*10^5	6.5999*10^5	6.5999*10^5	6.5999*10^5

buyer 1 expected profit (coll)	2.6184*10^5	2.7841*10^5	2.6329*10^5	2.7551*10^5	2.8284*10^5
buyer 1 expected profit (no-coll)	2.2570*10^5	2.2570*10^5	2.2570*10^5	2.2570*10^5	2.2570*10^5
buyer 2 expected profit (coll)	1.8779*10^3	2.9953*10^3	1.9763*10^3	2.5925*10^3	2.9727*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 132 Full results NBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 6

	N1 = 3 N2 = 6	**N1 = 3 N2 = 7**	**N1 = 3 N2 = 8**	**N1 = 3 N2 = 9**	**N1 = 3 N2 = 10**
supplier expected profit (coll)	6.7618*10^5	6.7618*10^5	6.7579*10^5	6.7554*10^5	6.7551*10^5
supplier expected profit (no-coll)	6.5999*10^5	6.5999*10^5	6.5999*10^5	6.5999*10^5	6.5999*10^5
buyer 1 expected profit (coll)	2.7408*10^5	2.8138*10^5	2.8580*10^5	2.8066*10^5	2.8507*10^5
buyer 1 expected profit (no-coll)	2.2570*10^5	2.2570*10^5	2.2570*10^5	2.2570*10^5	2.2570*10^5
buyer 2 expected profit (coll)	2.2426*10^3	2.6233*10^3	2.8471*10^3	2.3816*10^3	2.6181*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 133 Full results NBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 7

	N1 = 4 N2 = 1	**N1 = 4 N2 = 2**	**N1 = 4 N2 = 3**	**N1 = 4 N2 = 4**	**N1 = 4 N2 = 5**
supplier expected profit (coll)	6.7887*10^5	6.7872*10^5	6.7699*10^5	6.7756*10^5	6.7688*10^5
supplier expected profit (no-coll)	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5
buyer 1 expected profit (coll)	2.6167*10^5	2.7823*10^5	2.6311*10^5	2.7534*10^5	2.8266*10^5
buyer 1 expected profit (no-coll)	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5
buyer 2 expected profit (coll)	1.8779*10^3	2.9953*10^3	1.9763*10^3	2.5925*10^3	2.9727*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 134 Full results NBNS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 8

	N1 = 4 N2 = 6	N1 = 4 N2 = 7	N1 = 4 N2 = 8	N1 = 4 N2 = 9	N1 = 4 N2 = 10
supplier expected profit (coll)	6.7635*10^5	6.7635*10^5	6.7596*10^5	6.7571*10^5	6.7568*10^5
supplier expected profit (no-coll)	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5	6.6016*10^5
buyer 1 expected profit (coll)	2.7390*10^5	2.8120*10^5	2.8563*10^5	2.8048*10^5	2.8490*10^5
buyer 1 expected profit (no-coll)	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5	2.2552*10^5
buyer 2 expected profit (coll)	2.2426*10^3	2.6233*10^3	2.8471*10^3	2.3816*10^3	2.6181*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 135 Full results NBNS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 9

	N1 = 5 N2 = 1	N1 = 5 N2 = 2	N1 = 5 N2 = 3	N1 = 5 N2 = 4	N1 = 5 N2 = 5
supplier expected profit (coll)	6.7904*10^5	6.7889*10^5	6.7716*10^5	6.7773*10^5	6.7705*10^5
supplier expected profit (no-coll)	6.6033*10^5	6.6033*10^5	6.6033*10^5	6.6033*10^5	6.6033*10^5
buyer 1 expected profit (coll)	2.6149*10^5	2.7806*10^5	2.6294*10^5	2.7516*10^5	2.8248*10^5
buyer 1 expected profit (no-coll)	2.2535*10^5	2.2535*10^5	2.2535*10^5	2.2535*10^5	2.2535*10^5
buyer 2 expected profit (coll)	1.8779*10^3	2.9953*10^3	1.9763*10^3	2.5925*10^3	2.9727*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 136 Full results NBNS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 10

	N1 = 5 N2 = 6	N1 = 5 N2 = 7	N1 = 5 N2 = 8	N1 = 5 N2 = 9	N1 = 5 N2 = 10

supplier expected profit (coll)	6.7652*10^5	6.7652*10^5	6.7613*10^5	6.7588*10^5	6.7585*10^5
supplier expected profit (no-coll)	6.6033*10^5	6.6033*10^5	6.6033*10^5	6.6033*10^5	6.6033*10^5
buyer 1 expected profit (coll)	2.7373*10^5	2.8103*10^5	2.8545*10^5	2.8030*10^5	2.8472*10^5
buyer 1 expected profit (no-coll)	2.2535*10^5	2.2535*10^5	2.2535*10^5	2.2535*10^5	2.2535*10^5
buyer 2 expected profit (coll)	2.2426*10^3	2.6233*10^3	2.8471*10^3	2.3816*10^3	2.6181*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 137 Full results NBNS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 11

	N1 = 6 N2 = 1	**N1 = 6 N2 = 2**	**N1 = 6 N2 = 3**	**N1 = 6 N2 = 4**	**N1 = 6 N2 = 5**
supplier expected profit (coll)	6.7920*10^5	6.7904*10^5	6.7731*10^5	6.7789*10^5	6.7721*10^5
supplier expected profit (no-coll)	6.6049*10^5	6.6049*10^5	6.6049*10^5	6.6049*10^5	6.6049*10^5
buyer 1 expected profit (coll)	2.6133*10^5	2.7789*10^5	2.6277*10^5	2.7500*10^5	2.8232*10^5
buyer 1 expected profit (no-coll)	2.2518*10^5	2.2518*10^5	2.2518*10^5	2.2518*10^5	2.2518*10^5
buyer 2 expected profit (coll)	1.8779*10^3	2.9953*10^3	1.9763*10^3	2.5925*10^3	2.9727*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 138 Full results NBNS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 12

	N1 = 6 N2 = 6	**N1 = 6 N2 = 7**	**N1 = 6 N2 = 8**	**N1 = 6 N2 = 9**	**N1 = 6 N2 = 10**
supplier expected profit (coll)	6.7668*10^5	6.7667*10^5	6.7629*10^5	6.7604*10^5	6.7601*10^5
supplier expected profit (no-coll)	6.6049*10^5	6.6049*10^5	6.6049*10^5	6.6049*10^5	6.6049*10^5
buyer 1 expected profit (coll)	2.7356*10^5	2.8086*10^5	2.8529*10^5	2.8014*10^5	2.8456*10^5
buyer 1 expected profit (no-coll)	2.2518*10^5	2.2518*10^5	2.2518*10^5	2.2518*10^5	2.2518*10^5

buyer 2 expected profit (coll)	2.2426*10^3	2.6233*10^3	2.8471*10^3	2.3816*10^3	2.6181*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 139 Full results NBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 13

	N1 = 7 N2 = 1	**N1 = 7 N2 = 2**	**N1 = 7 N2 = 3**	**N1 = 7 N2 = 4**	**N1 = 7 N2 = 5**
supplier expected profit (coll)	6.7934*10^5	6.7918*10^5	6.7745*10^5	6.7803*10^5	6.7735*10^5
supplier expected profit (no-coll)	6.6062*10^5	6.6062*10^5	6.6062*10^5	6.6062*10^5	6.6062*10^5
buyer 1 expected profit (coll)	2.6118*10^5	2.7775*10^5	2.6263*10^5	2.7485*10^5	2.8218*10^5
buyer 1 expected profit (no-coll)	2.2504*10^5	2.2504*10^5	2.2504*10^5	2.2504*10^5	2.2504*10^5
buyer 2 expected profit (coll)	1.8779*10^3	2.9953*10^3	1.9763*10^3	2.5925*10^3	2.9727*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 140 Full results NBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 14

	N1 = 7 N2 = 6	**N1 = 7 N2 = 7**	**N1 = 7 N2 = 8**	**N1 = 7 N2 = 9**	**N1 = 7 N2 = 10**
supplier expected profit (coll)	6.7682*10^5	6.7681*10^5	6.7642*10^5	6.7617*10^5	6.7614*10^5
supplier expected profit (no-coll)	6.6062*10^5	6.6062*10^5	6.6062*10^5	6.6062*10^5	6.6062*10^5
buyer 1 expected profit (coll)	2.7342*10^5	2.8072*10^5	2.8515*10^5	2.8*10^5	2.8441*10^5
buyer 1 expected profit (no-coll)	2.2504*10^5	2.2504*10^5	2.2504*10^5	2.2504*10^5	2.2504*10^5
buyer 2 expected profit (coll)	2.2426*10^3	2.6233*10^3	2.8471*10^3	2.3816*10^3	2.6181*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 141 Full results NBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 15

	N1 = 8 N2 = 1	N1 = 8 N2 = 2	N1 = 8 N2 = 3	N1 = 8 N2 = 4	N1 = 8 N2 = 5
supplier expected profit (coll)	6.7946*10^5	6.7930*10^5	6.7757*10^5	6.7815*10^5	6.7747*10^5
supplier expected profit (no-coll)	6.6074*10^5	6.6074*10^5	6.6074*10^5	6.6074*10^5	6.6074*10^5
buyer 1 expected profit (coll)	2.6106*10^5	2.9953*10^5	2.6250*10^5	2.7473*10^5	2.8205*10^5
buyer 1 expected profit (no-coll)	2.2491*10^5	2.2491*10^5	2.2491*10^5	2.2491*10^5	2.2491*10^5
buyer 2 expected profit (coll)	1.8779*10^3	2.9953*10^3	1.9763*10^3	2.5925*10^3	2.9727*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 142 Full results NBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 16

	N1 = 8 N2 = 6	N1 = 8 N2 = 7	N1 = 8 N2 = 8	N1 = 8 N2 = 9	N1 = 8 N2 = 10
supplier expected profit (coll)	6.7694*10^5	6.7693*10^5	6.7654*10^5	6.7630*10^5	6.7627*10^5
supplier expected profit (no-coll)	6.6074*10^5	6.6074*10^5	6.6074*10^5	6.6074*10^5	6.6074*10^5
buyer 1 expected profit (coll)	2.7329*10^5	2.8059*10^5	2.8502*10^5	2.7987*10^5	2.8429*10^5
buyer 1 expected profit (no-coll)	2.2491*10^5	2.2491*10^5	2.2491*10^5	2.2491*10^5	2.2491*10^5
buyer 2 expected profit (coll)	2.2426*10^3	2.6233*10^3	2.8471*10^3	2.3816*10^3	2.6181*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 143 Full results NBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 17

	N1 = 9 N2 = 1	N1 = 9 N2 = 2	N1 = 9 N2 = 3	N1 = 9 N2 = 4	N1 = 9 N2 = 5
supplier expected profit (coll)	6.7956*10^5	6.7941*10^5	6.7768*10^5	6.7825*10^5	6.7758*10^5
supplier expected profit (no-coll)	6.6085*10^5	6.6085*10^5	6.6085*10^5	6.6085*10^5	6.6085*10^5

buyer 1 expected profit (coll)	2.6095*10^5	2.7751*10^5	2.6239*10^5	2.7462*10^5	2.8194*10^5
buyer 1 expected profit (no-coll)	2.2480*10^5	2.2480*10^5	2.2480*10^5	2.2480*10^5	2.2480*10^5
buyer 2 expected profit (coll)	1.8779*10^3	2.9953*10^3	1.9763*10^3	2.5925*10^3	2.9727*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 144 Full results NBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 18

	N1 = 9 N2 = 6	**N1 = 9 N2 = 7**	**N1 = 9 N2 = 8**	**N1 = 9 N2 = 9**	**N1 = 9 N2 = 10**
supplier expected profit (coll)	6.7704*10^5	6.7704*10^5	6.7665*10^5	6.7640*10^5	6.7637*10^5
supplier expected profit (no-coll)	6.6085*10^5	6.6085*10^5	6.6085*10^5	6.6085*10^5	6.6085*10^5
buyer 1 expected profit (coll)	2.7318*10^5	2.8048*10^5	2.8491*10^5	2.7976*10^5	2.8418*10^5
buyer 1 expected profit (no-coll)	2.2480*10^5	2.2480*10^5	2.2480*10^5	2.2480*10^5	2.2480*10^5
buyer 2 expected profit (coll)	2.2426*10^3	2.6233*10^3	2.8471*10^3	2.3816*10^3	2.6181*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 145 Full results NBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 19

	N1 = 10 N2 = 1	**N1 = 10 N2 = 2**	**N1 = 10 N2 = 3**	**N1 = 10 N2 = 4**	**N1 = 10 N2 = 5**
supplier expected profit (coll)	6.7966*10^5	6.7950*10^5	6.7777*10^5	6.7835*10^5	6.7767*10^5
supplier expected profit (no-coll)	6.6094*10^5	6.6094*10^5	6.6094*10^5	6.6094*10^5	6.6094*10^5
buyer 1 expected profit (coll)	2.6085*10^5	2.7742*10^5	2.6230*10^5	2.7452*10^5	2.8185*10^5
buyer 1 expected profit (no-coll)	2.2471*10^5	2.2471*10^5	2.2471*10^5	2.2471*10^5	2.2471*10^5
buyer 2 expected profit (coll)	1.8779*10^3	2.9953*10^3	1.9763*10^3	2.5925*10^3	2.9727*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 146 Full results NBNS with A_1 = 2000 A_2 = 1300 β = 0.5 part 20

	N1 = 10 N2 = 6	N1 = 10 N2 = 7	N1 = 10 N2 = 8	N1 = 10 N2 = 9	N1 = 10 N2 = 10
supplier expected profit (coll)	6.7714*10^5	6.7713*10^5	6.7674*10^5	6.7649*10^5	6.7646*10^5
supplier expected profit (no-coll)	6.6094*10^5	6.6094*10^5	6.6094*10^5	6.6094*10^5	6.6094*10^5
buyer 1 expected profit (coll)	2.7309*10^5	2.8039*10^5	2.8481*10^5	2.7966*10^5	2.8408*10^5
buyer 1 expected profit (no-coll)	2.2471*10^5	2.2471*10^5	2.2471*10^5	2.2471*10^5	2.2471*10^5
buyer 2 expected profit (coll)	2.2426*10^3	2.6233*10^3	2.8471*10^3	2.3816*10^3	2.6181*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Full results NBIS cases

Table 147 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 1

	N1 = 1 N2 = 1	N1 = 1 N2 = 2	N1 = 1 N2 = 3	N1 = 1 N2 = 4	N1 = 1 N2 = 5
supplier expected profit (coll)	6.8528*10^5	6.8490*10^5	6.8564*10^5	6.8439*10^5	6.8453*10^5
supplier expected profit (no-coll)	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5
buyer 1 expected profit (coll)	2.5836*10^5	2.2288*10^5	2.2199*10^5	2.2215*10^5	2.2216*10^5
buyer 1 expected profit (no-coll)	2.2392*10^5	2.2392*10^5	2.2392*10^5	2.2392*10^5	2.2392*10^5
buyer 2 expected profit (coll)	4.5787*10^3	4.2976*10^3	4.5817*10^3	4.3352*10^3	4.3442*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 148 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 2

	N1 = 1 N2 = 6	N1 = 1 N2 = 7	N1 = 1 N2 = 8	N1 = 1 N2 = 9	N1 = 1 N2 = 10
supplier expected profit (coll)	6.8395*10^5	6.8380*10^5	6.8331*10^5	6.8328*10^5	6.8293*10^5
supplier expected profit (no-coll)	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5
buyer 1 expected profit (coll)	2.2226*10^5	2.2171*10^5	2.2232*10^5	2.2179*10^5	2.2197*10^5
buyer 1 expected profit (no-coll)	2.2392*10^5	2.2392*10^5	2.2392*10^5	2.2392*10^5	2.2392*10^5
buyer 2 expected profit (coll)	4.1778*10^3	4.2500*10^3	4.0478*10^3	4.1320*10^3	4.0031*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 149 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 3

	N1 = 2 N2 = 1	N1 = 2 N2 = 2	N1 = 2 N2 = 3	N1 = 2 N2 = 4	N1 = 2 N2 = 5
supplier expected profit (coll)	6.8580*10^5	6.8581*10^5	6.8504*10^5	6.8531*10^5	6.8437*10^5
supplier expected profit (no-coll)	6.6308*10^5	6.6308*10^5	6.6308*10^5	6.6308*10^5	6.6308*10^5
buyer 1 expected profit (coll)	2.3834*10^5	2.2211*10^5	2.2237*10^5	2.22*10^5	2.2121*10^5
buyer 1 expected profit (no-coll)	2.2379*10^5	2.2379*10^5	2.2379*10^5	2.2379*10^5	2.2379*10^5
buyer 2 expected profit (coll)	5.1641*10^3	5.2955*10^3	5.0750*10^3	5.1785*10^3	5.2814*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 150 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 4

	N1 = 2 N2 = 6	N1 = 2 N2 = 7	N1 = 2 N2 = 8	N1 = 2 N2 = 9	N1 = 2 N2 = 10
supplier expected profit (coll)	6.8436*10^5	6.8393*10^5	6.8364*10^5	6.8347*10^5	6.8314*10^5
supplier expected profit (no-coll)	6.6308*10^5	6.6308*10^5	6.6308*10^5	6.6308*10^5	6.6308*10^5

buyer 1 expected profit (coll)	2.2211*10^5	2.2172*10^5	2.2201*10^5	2.1171*10^5	2.2130*10^5
buyer 1 expected profit (no-coll)	2.2379*10^5	2.2379*10^5	2.2375*10^5	2.2377*10^5	2.2378*10^5
buyer 2 expected profit (coll)	4.9712*10^3	4.9652*10^3	4.8090*10^3	4.8418*10^3	4.9060*10^3
buyer 2 expected profit (no-coll)	0	0	18,38	9,48	4,85

Table 151 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 5

	N1 = 3 N2 = 1	**N1 = 3 N2 = 2**	**N1 = 3 N2 = 3**	**N1 = 3 N2 = 4**	**N1 = 3 N2 = 5**
supplier expected profit (coll)	6.8531*10^5	6.8558*10^5	6.8535*10^5	6.8480*10^5	6.8464*10^5
supplier expected profit (no-coll)	6.6305*10^5	6.6305*10^5	6.6305*10^5	6.6305*10^5	6.6305*10^5
buyer 1 expected profit (coll)	2.3016*10^5	2.3026*10^5	2.2196*10^5	2.2204*10^5	2.2222*10^5
buyer 1 expected profit (no-coll)	2.2394*10^5	2.2394*10^5	2.2394*10^5	2.2394*10^5	2.2394*10^5
buyer 2 expected profit (coll)	5.4642*10^3	5.9140*10^3	5.8074*10^3	5.6547*10^3	5.5059*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 152 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 6

	N1 = 3 N2 = 6	**N1 = 3 N2 = 7**	**N1 = 3 N2 = 8**	**N1 = 3 N2 = 9**	**N1 = 3 N2 = 10**
supplier expected profit (coll)	6.8415*10^5	6.8383*10^5	6.8360*10^5	6.8329*10^5	6.8311*10^5
supplier expected profit (no-coll)	6.6305*10^5	6.6305*10^5	6.6305*10^5	6.6305*10^5	6.6305*10^5
buyer 1 expected profit (coll)	2.2154*10^5	2.2232*10^5	2.2167*10^5	2.2187*10^5	2.2184*10^5
buyer 1 expected profit (no-coll)	2.2388*10^5	2.2391*10^5	2.2393*10^5	2.2394*10^5	2.2394*10^5
buyer 2 expected profit (coll)	5.6289*10^3	5.2650*10^3	5.4606*10^3	5.2785*10^3	5.2418*10^3
buyer 2 expected profit (no-coll)	39	20,35	10,58	5,4	2,61

Table 153 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 7

	N1 = 4 N2 = 1	**N1 = 4 N2 = 2**	**N1 = 4 N2 = 3**	**N1 = 4 N2 = 4**	**N1 = 4 N2 = 5**
supplier expected profit (coll)	6.8537*10^5	6.8539*10^5	6.8501*10^5	6.8471*10^5	6.8433*10^5
supplier expected profit (no-coll)	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5
buyer 1 expected profit (coll)	2.4392*10^5	2.2623*10^5	2.2232*10^5	2.2197*10^5	2.2163*10^5
buyer 1 expected profit (no-coll)	2.2380*10^5	2.2380*10^5	2.2380*10^5	2.2380*10^5	2.2380*10^5
buyer 2 expected profit (coll)	5.9936*10^3	5.9670*10^3	5.8507*10^3	5.9054*10^3	5.9360*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	0

Table 154 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 8

	N1 = 4 N2 = 6	**N1 = 4 N2 = 7**	**N1 = 4 N2 = 8**	**N1 = 4 N2 = 9**	**N1 = 4 N2 = 10**
supplier expected profit (coll)	6.8397*10^5	6.8374*10^5	6.8325*10^5	6.8323*10^5	6.8284*10^5
supplier expected profit (no-coll)	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5	6.6299*10^5
buyer 1 expected profit (coll)	2.2206*10^5	2.2150*10^5	2.2217*10^5	2.2160*10^5	2.2145*10^5
buyer 1 expected profit (no-coll)	2.2376*10^5	2.2378*10^5	2.2379*10^5	2.2379*10^5	2.2379*10^5
buyer 2 expected profit (coll)	5.6723*10^3	5.7947*10^3	5.4605*10^3	5.6406*10^3	5.5757*10^3
buyer 2 expected profit (no-coll)	21,11	11	5,66	2,76	1,41

Table 155 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 9

	N1 = 5 N2 = 1	**N1 = 5 N2 = 2**	**N1 = 5 N2 = 3**	**N1 = 5 N2 = 4**	**N1 = 5 N2 = 5**

supplier expected profit (coll)	6.8521*10^5	6.8535*10^5	6.8505*10^5	6.8464*10^5	6.8419*10^5
supplier expected profit (no-coll)	6.6293*10^5	6.6293*10^5	6.6293*10^5	6.6293*10^5	6.6293*10^5
buyer 1 expected profit (coll)	2.3491*10^5	2.2397*10^5	2.2404*10^5	2.2205*10^5	2.2196*10^5
buyer 1 expected profit (no-coll)	2.2391*10^5	2.2391*10^5	2.2391*10^5	2.2391*10^5	2.2388*10^5
buyer 2 expected profit (coll)	5.9923*10^3	6.3089*10^3	6.2538*10^3	6.1058*10^3	6.0534*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	20,95

Table 156 Full results NBIS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 10

	N1 = 5 N2 = 6	**N1 = 5 N2 = 7**	**N1 = 5 N2 = 8**	**N1 = 5 N2 = 9**	**N1 = 5 N2 = 10**
supplier expected profit (coll)	6.8381*10^5	6.8343*10^5	6.8329*10^5	6.8274*10^5	6.8281*10^5
supplier expected profit (no-coll)	6.6293*10^5	6.6293*10^5	6.6293*10^5	6.6293*10^5	6.6293*10^5
buyer 1 expected profit (coll)	2.2168*10^5	2.2205*10^5	2.2169*10^5	2.2160*10^5	2,2178*10^5
buyer 1 expected profit (no-coll)	2.2389*10^5	2.2390*10^5	2.2391*10^5	2.2390*10^5	2.2390*10^5
buyer 2 expected profit (coll)	6.0530*10^3	5.8234*10^3	5.8787*10^3	5.8458*10^3	5.7367*10^3
buyer 2 expected profit (no-coll)	11,11	5,79	2,82	7,67	3,93

Table 157 Full results NBIS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 11

	N1 = 6 N2 = 1	**N1 = 6 N2 = 2**	**N1 = 6 N2 = 3**	**N1 = 6 N2 = 4**	**N1 = 6 N2 = 5**
supplier expected profit (coll)	6.8491*10^5	6.8509*10^5	6.8477*10^5	6.8433*10^5	6.8401*10^5
supplier expected profit (no-coll)	6.6287*10^5	6.6287*10^5	6.6287*10^5	6.6287*10^5	6.6287*10^5
buyer 1 expected profit (coll)	2.4705*10^5	2.2260*10^5	2.2317*10^5	2.2144*10^5	2.2156*10^5
buyer 1 expected profit (no-coll)	2.2378*10^5	2.2378*10^5	2.2378*10^5	2.2378*10^5	2.2376*10^5

buyer 2 expected profit (coll)	6.3600*10^3	6.5709*10^3	6.2560*10^3	6.4503*10^3	6.3033*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	10,86

Table 158 Full results NBIS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 12

	N1 = 6 N2 = 6	**N1 = 6 N2 = 7**	**N1 = 6 N2 = 8**	**N1 = 6 N2 = 9**	**N1 = 6 N2 = 10**
supplier expected profit (coll)	6.8365*10^5	6.8331*10^5	6.8299*10^5	6.8284*10^5	6.8246*10^5
supplier expected profit (no-coll)	6.6287*10^5	6.6287*10^5	6.6287*10^5	6.6287*10^5	6.6287*10^5
buyer 1 expected profit (coll)	2.2165*10^5	2.2154*10^5	2.2170*10^5	2.2161*10^5	2.2111*10^5
buyer 1 expected profit (no-coll)	2.2377*10^5	2.2377*10^5	2.2376*10^5	2.2377*10^5	2.2377*10^5
buyer 2 expected profit (coll)	6.1733*10^3	6.1094*10^3	5.9790*10^3	5.9552*10^3	6.0789*10^3
buyer 2 expected profit (no-coll)	5,71	2,97	9,42	4,83	2,47

Table 159 Full results NBIS with $A_1 = 2000$ $A_2 = 1300$ $\beta = 0.5$ part 13

	N1 = 7 N2 = 1	**N1 = 7 N2 = 2**	**N1 = 7 N2 = 3**	**N1 = 7 N2 = 4**	**N1 = 7 N2 = 5**
supplier expected profit (coll)	6.8495*10^5	6.8501*10^5	6.8472*10^5	6.8430*10^5	6.8387*10^5
supplier expected profit (no-coll)	6.6282*10^5	6.6282*10^5	6.6282*10^5	6.6282*10^5	6.6282*10^5
buyer 1 expected profit (coll)	2.3818*10^5	2.2635*10^5	2.2235*10^5	2.2200*10^5	2.2207*10^5
buyer 1 expected profit (no-coll)	2.2387*10^5	2.2387*10^5	2.2387*10^5	2.2387*10^5	2.2386*10^5
buyer 2 expected profit (coll)	6.3202*10^3	6.5582*10^3	6.5465*10^3	6.3683*10^3	6.2295*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	5,55

Table 160 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 14

	N1 = 7 N2 = 6	N1 = 7 N2 = 7	N1 = 7 N2 = 8	N1 = 7 N2 = 9	N1 = 7 N2 = 10
supplier expected profit (coll)	6.8344*10^5	6.8313*10^5	6.8289*10^5	6.8261*10^5	6.8242*10^5
supplier expected profit (no-coll)	6.6282*10^5	6.6283*10^5	6.6283*10^5	6.6282*10^5	6.6282*10^5
buyer 1 expected profit (coll)	2.2164*10^5	2.2150*10^5	2.2177*10^5	2.2160*10^5	2.2170*10^5
buyer 1 expected profit (no-coll)	2.2386*10^5	2.2385*10^5	2.2386*10^5	2.2386*10^5	2.2387*10^5
buyer 2 expected profit (coll)	6.2960*10^3	6.2884*10^3	6.0893*10^3	6.0971*10^3	6.0031*10^3
buyer 2 expected profit (no-coll)	2,91	11,16	5,64	2,89	1,48

Table 161 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 15

	N1 = 8 N2 = 1	N1 = 8 N2 = 2	N1 = 8 N2 = 3	N1 = 8 N2 = 4	N1 = 8 N2 = 5
supplier expected profit (coll)	6.8479*10^5	6.8495*10^5	6.8452*10^5	6.8418*10^5	6.8376*10^5
supplier expected profit (no-coll)	6.6278*10^5	6.6278*10^5	6.6278*10^5	6.6278*10^5	6.6278*10^5
buyer 1 expected profit (coll)	2.3220*10^5	2.2407*10^5	2.2230*10^5	2.2209*10^5	2.2166*10^5
buyer 1 expected profit (no-coll)	2.2377*10^5	2.2377*10^5	2.2377*10^5	2.2377*10^5	2.2376*10^5
buyer 2 expected profit (coll)	6.3070*10^3	6.7457*10^3	6.5065*10^3	6.5142*10^3	6.4713*10^3
buyer 2 expected profit (no-coll)	0	0	0	0	2,83

Table 162 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 16

	N1 = 8 N2 = 6	N1 = 8 N2 = 7	N1 = 8 N2 = 8	N1 = 8 N2 = 9	N1 = 8 N2 = 10
supplier expected profit (coll)	6.8343*10^5	6.8308*10^5	6.8276*10^5	6.8251*10^5	6.8226*10^5
supplier expected profit (no-coll)	6.6278*10^5	6.6278*10^5	6.6278*10^5	6.6278*10^5	6.6278*10^5

buyer 1 expected profit (coll)	2.2163*10^5	2.2170*10^5	2.2165*10^5	2.2152*10^5	2.2135*10^5
buyer 1 expected profit (no-coll)	2.2375*10^5	2.2376*10^5	2.2376*10^5	2.2376*10^5	2.2376*10^5
buyer 2 expected profit (coll)	6.3844*10^3	6.2638*10^3	6.2054*10^3	6.1773*10^3	6.1668*10^3
buyer 2 expected profit (no-coll)	12,46	6,34	3,28	6,25	3,18

Table 163 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 17

	N1 = 9 N2 = 1	**N1 = 9 N2 = 2**	**N1 = 9 N2 = 3**	**N1 = 9 N2 = 4**	**N1 = 9 N2 = 5**
supplier expected profit (coll)	6.8469*10^5	6.8479*10^5	6.8447*10^5	6.8403*10^5	6.8363*10^5
supplier expected profit (no-coll)	6.6274*10^5	6.6274*10^5	6.6274*10^5	6.6274*10^5	6.6274*10^5
buyer 1 expected profit (coll)	2.4055*10^5	2.2292*10^5	2.2195*10^5	2.2174*10^5	2.2194*10^5
buyer 1 expected profit (no-coll)	2.2384*10^5	2.2384*10^5	2.2384*10^5	2.2383*10^5	2.2384*10^5
buyer 2 expected profit (coll)	6.5155*10^3	6.8877*10^3	6.7023*10^3	6.6920*10^3	6.4283*10^3
buyer 2 expected profit (no-coll)	0	0	0	2,83	1,43

Table 164 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 18

	N1 = 9 N2 = 6	**N1 = 9 N2 = 7**	**N1 = 9 N2 = 8**	**N1 = 9 N2 = 9**	**N1 = 9 N2 = 10**
supplier expected profit (coll)	6.8323*10^5	6.8290*10^5	6.8264*10^5	6.8238*10^5	6.8215*10^5
supplier expected profit (no-coll)	6.6274*10^5	6.6274*10^5	6.6274*10^5	6.6274*10^5	6.6274*10^5
buyer 1 expected profit (coll)	2.2179*10^5	2.2140*10^5	2.2149*10^5	2.2159*10^5	2.2145*10^5
buyer 1 expected profit (no-coll)	2.2383*10^5	2.2383*10^5	2.2383*10^5	2.2383*10^5	2.2383*10^5
buyer 2 expected profit (coll)	6.3952*10^3	6.4823*10^3	6.3567*10^3	6.2369*10^3	6.245710^3
buyer 2 expected profit (no-coll)	7,08	3,61	7,95	4,03	2,5

Table 165 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 19

	N1 = 10 N2 = 1	**N1 = 10 N2 = 2**	**N1 = 10 N2 = 3**	**N1 = 10 N2 = 4**	**N1 = 10 N2 = 5**
supplier expected profit (coll)	6.8465*10^5	6.8475*10^5	6.8434*10^5	6.8399*10^5	6.8357*10^5
supplier expected profit (no-coll)	6.6271*10^5	6.6271*10^5	6.6271*10^5	6.6271*10^5	6.6271*10^5
buyer 1 expected profit (coll)	2.3436*10^5	2.2248*10^5	2.2207*10^5	2.2189*10^5	2.2161*10^5
buyer 1 expected profit (no-coll)	2.2375*10^5	2.2375*10^5	2.2375*10^5	2.2374*10^5	2.2374*10^5
buyer 2 expected profit (coll)	6.4443*10^3	6.8270*10^3	6.6066*10^3	6.6003*10^3	6.6227*10^3
buyer 2 expected profit (no-coll)	0	0	0	1,43	0,72

Table 166 Full results NBIS with A_1 = 2000 A_2 = 1300 β = 0.5 part 20

	N1 = 10 N2 = 6	**N1 = 10 N2 = 7**	**N1 = 10 N2 = 8**	**N1 = 10 N2 = 9**	**N1 = 10 N2 = 10**
supplier expected profit (coll)	6.8319*10^5	6.8286*10^5	6.8254*10^5	6.8226*10^5	6.8204*10^5
supplier expected profit (no-coll)	6.6271*10^5	6.6271*10^5	6.6271*10^5	6.6271*10^5	6.6271*10^5
buyer 1 expected profit (coll)	2.2164*10^5	2.2168*10^5	2.2174*10^5	2.2151*10^5	2.2135*10^5
buyer 1 expected profit (no-coll)	2.2374*10^5	2.2374*10^5	2.2374*10^5	2.2374*10^5	2.2374*10^5
buyer 2 expected profit (coll)	6.4946*10^3	6.3759*10^3	6.2474*10^3	6.2988*10^3	6.2909*10^3
buyer 2 expected profit (no-coll)	3,98	2,02	4,99	2,54	1,52

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IBIS

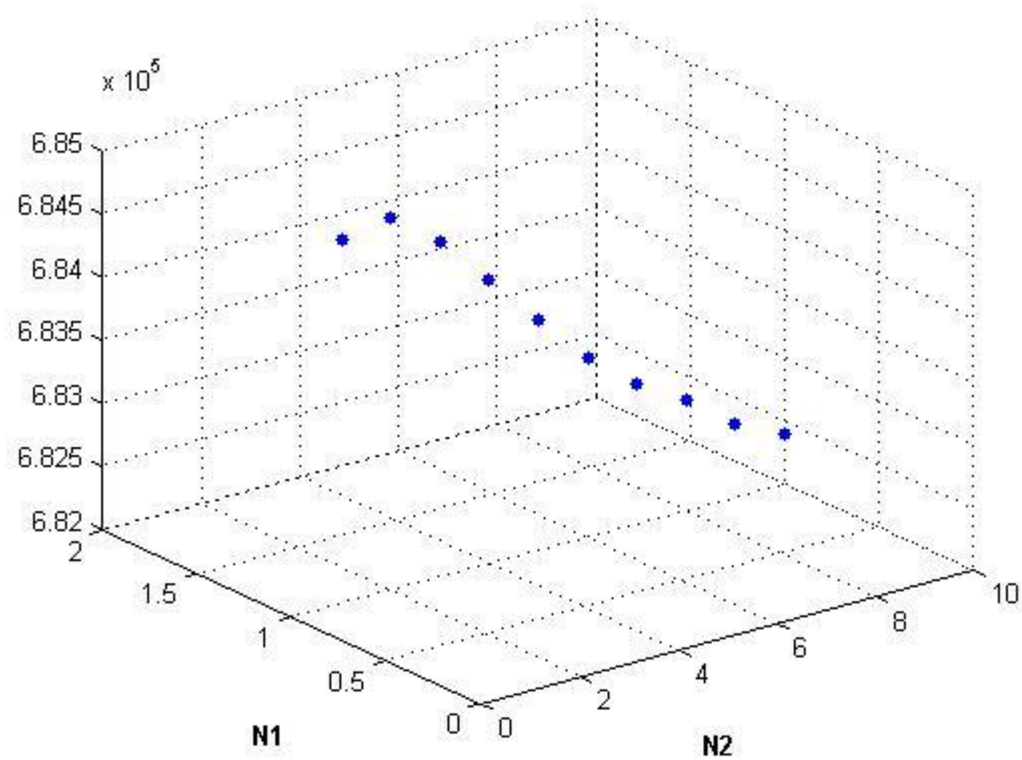

Figure 34 Supplier profit vs N2 for a given N1 (N1= 7)

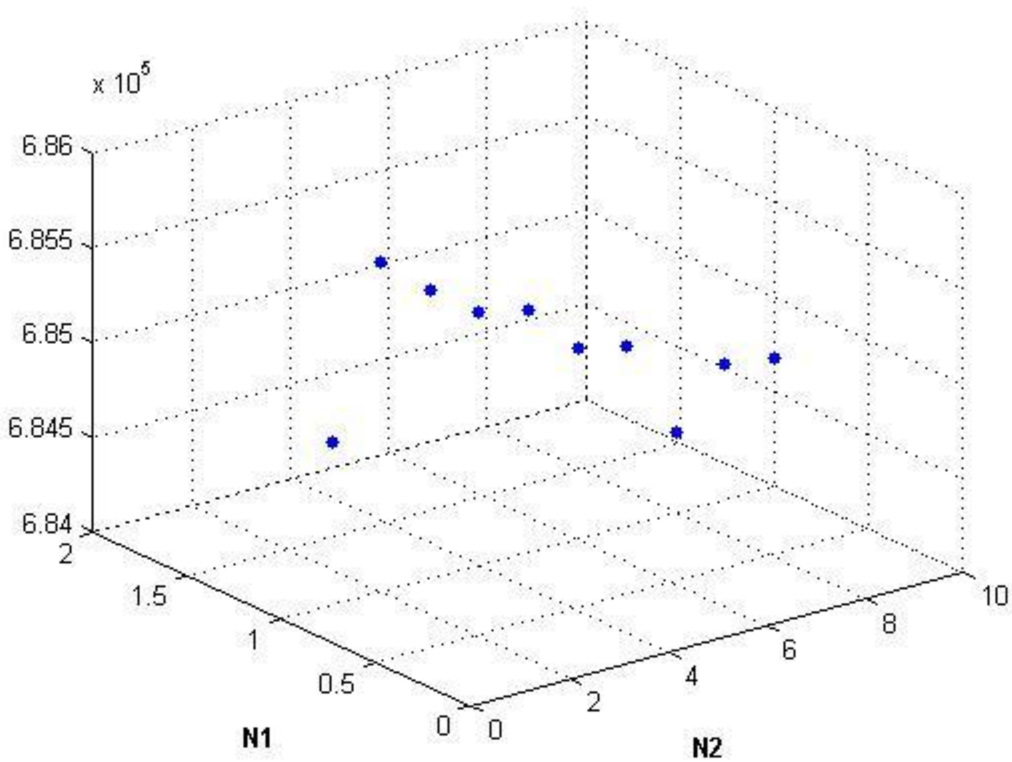

Figure 35 Supplier profit vs N1 for given N2 (N2 = 2)

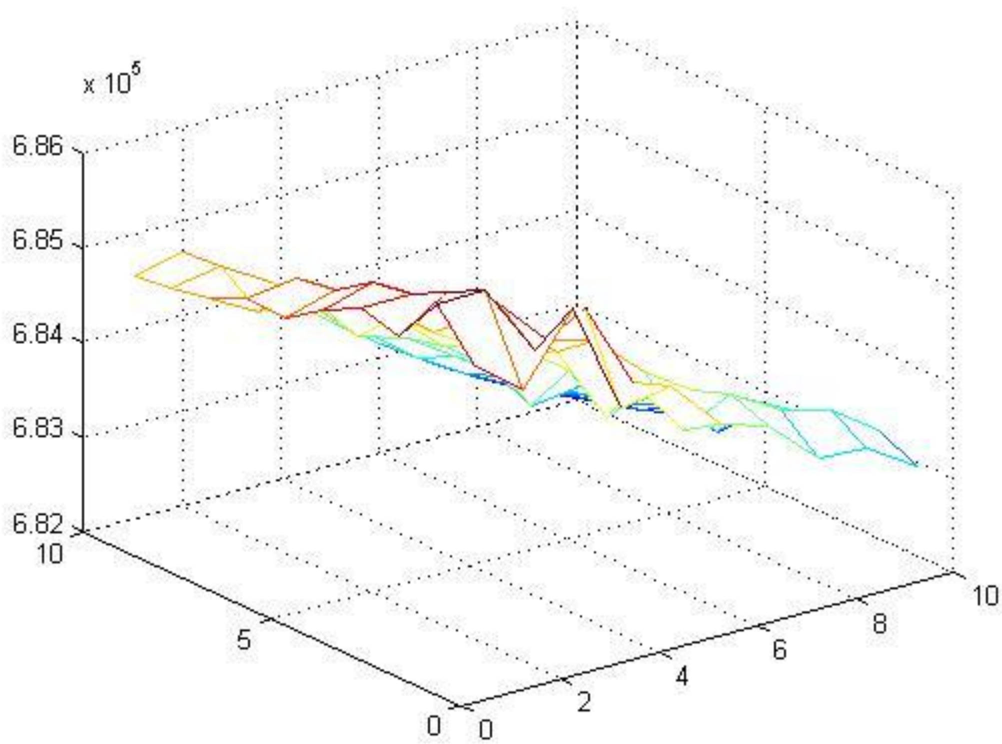

Figure 36 Supplier profit vs N1 and N2

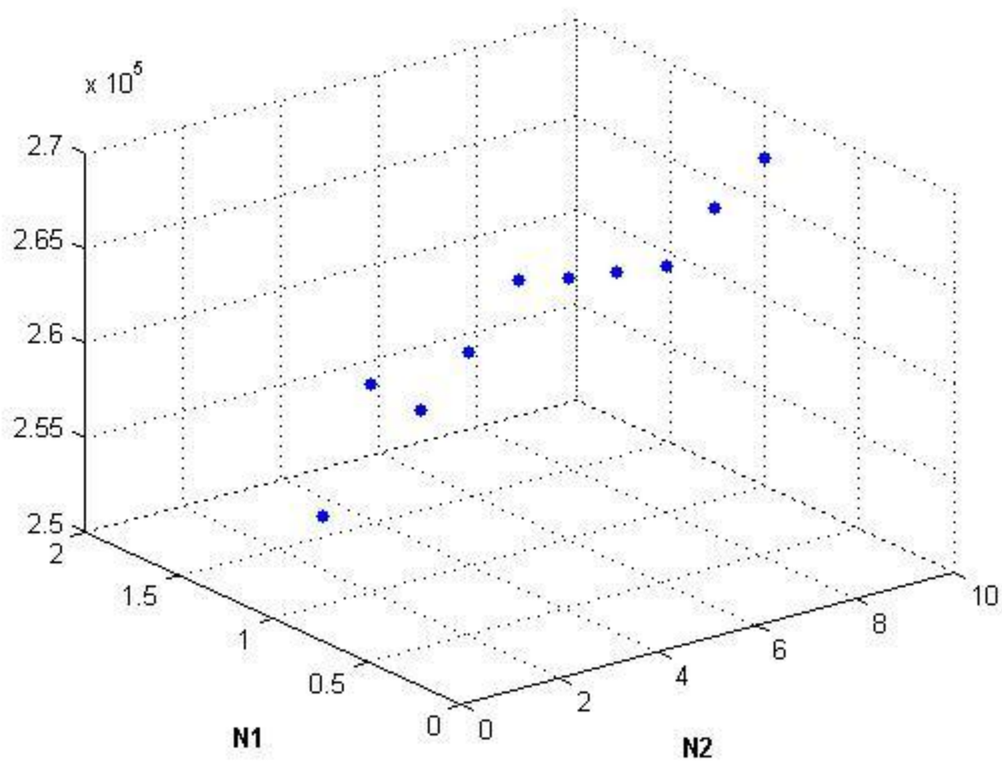

Figure 37 Buyer 1 profit vs N2 for given N1 (N1 =10)

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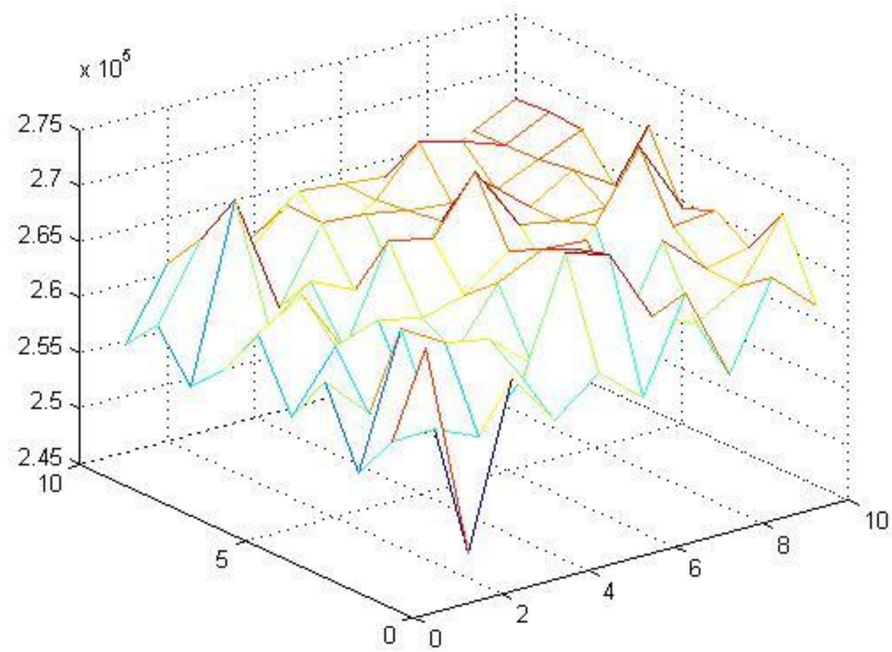

Figure 38 Buyer 1 profit vs N1 and N2

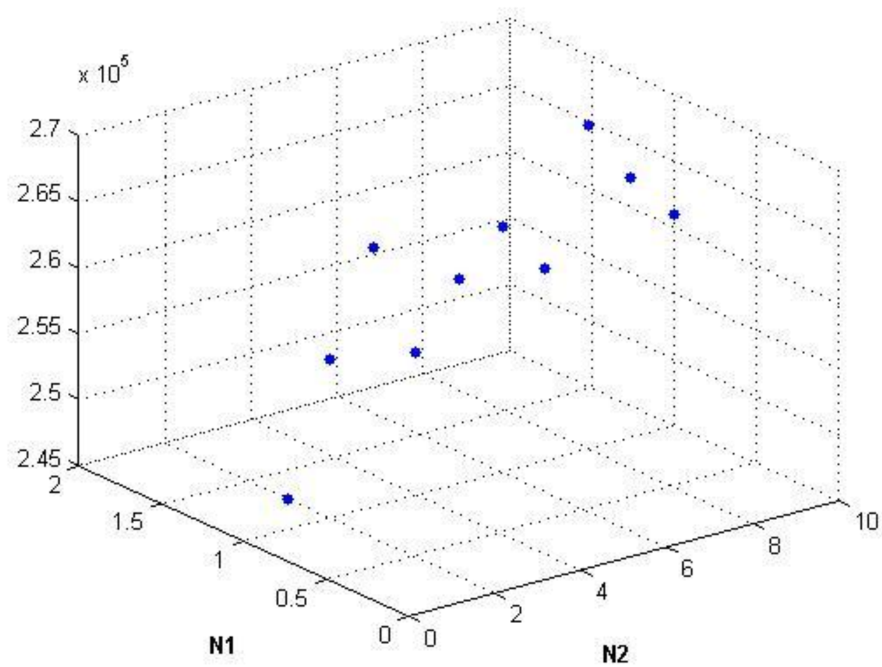

Figure 39 Buyer 1 profit vs N1 for given N2 (N2 = 3)

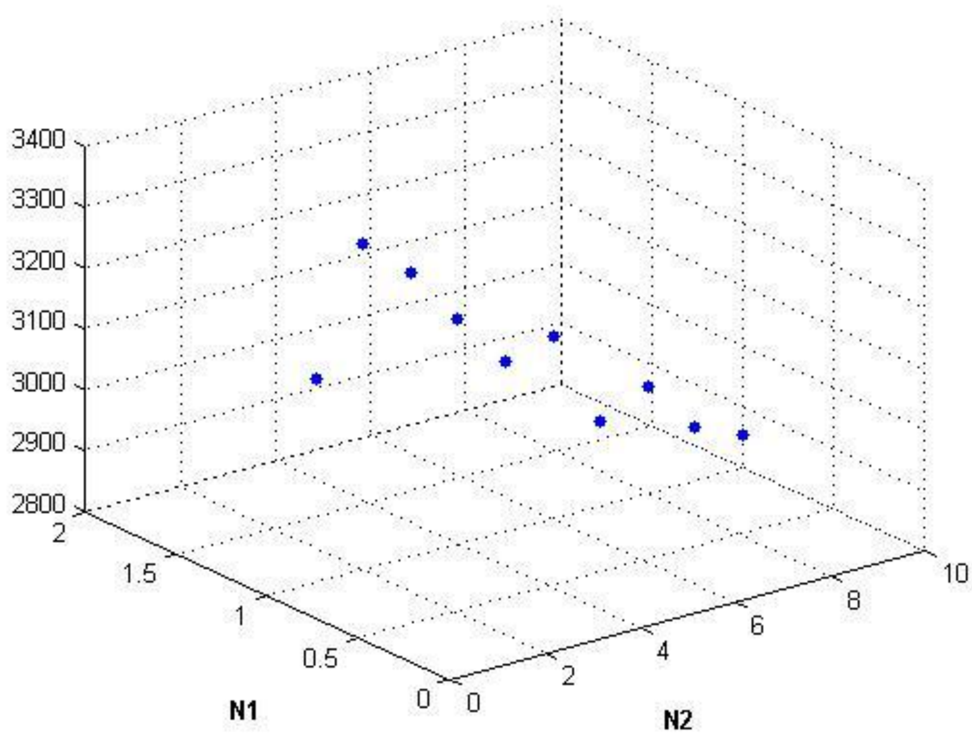

Figure 40 Buyer 2 profit vs N2 given N1 (N1 =3)

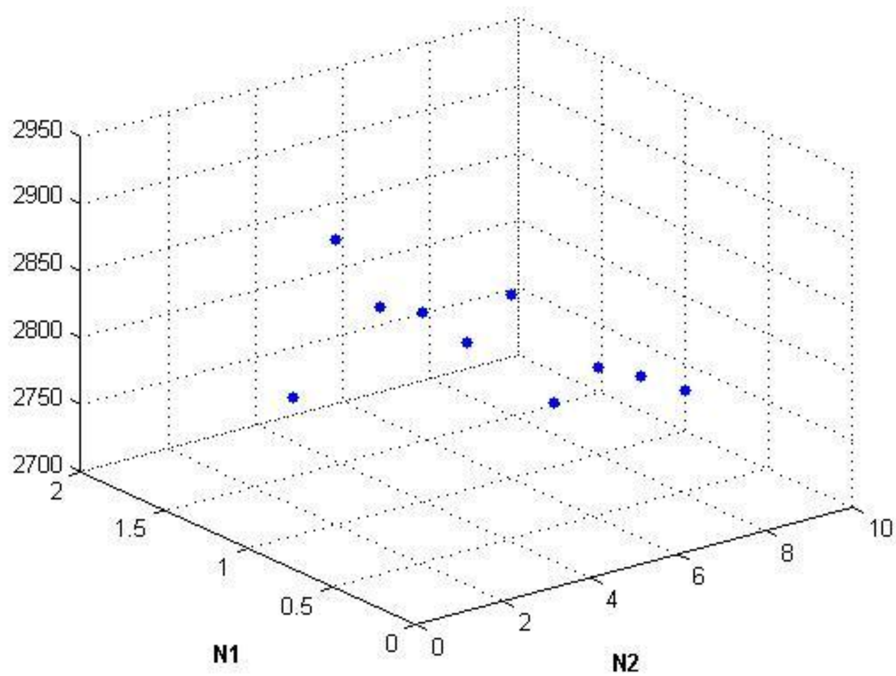

Figure 41 Buyer 2 profit vs N1 (N2 =10)

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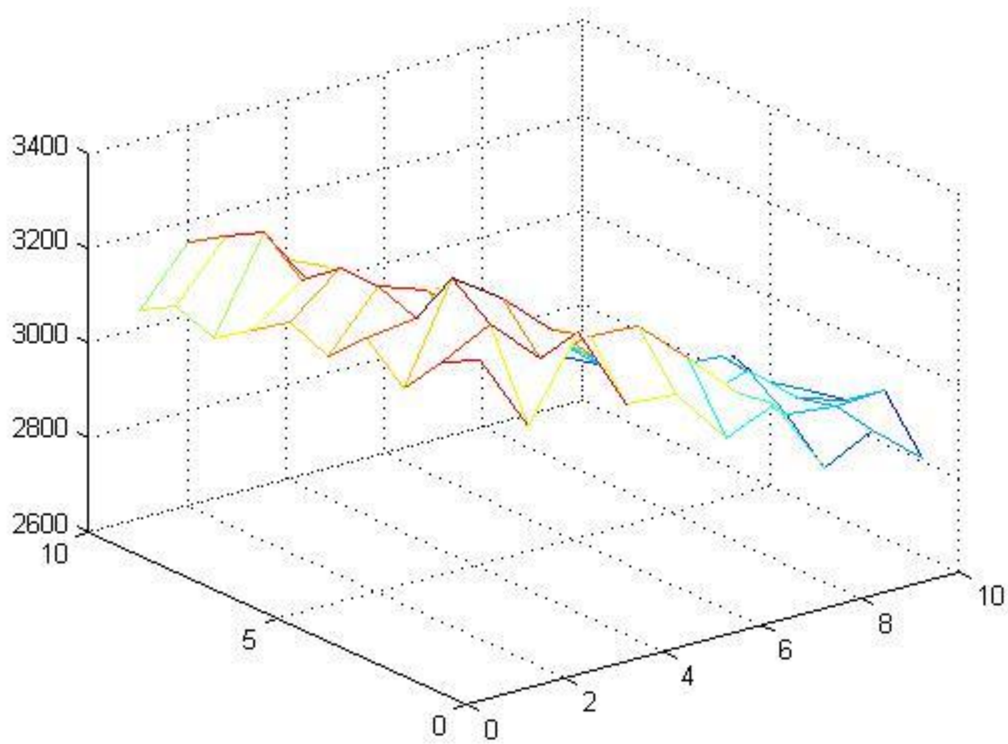


Figure 42 Buyer 2 profit vs N1 and N2

IBNS

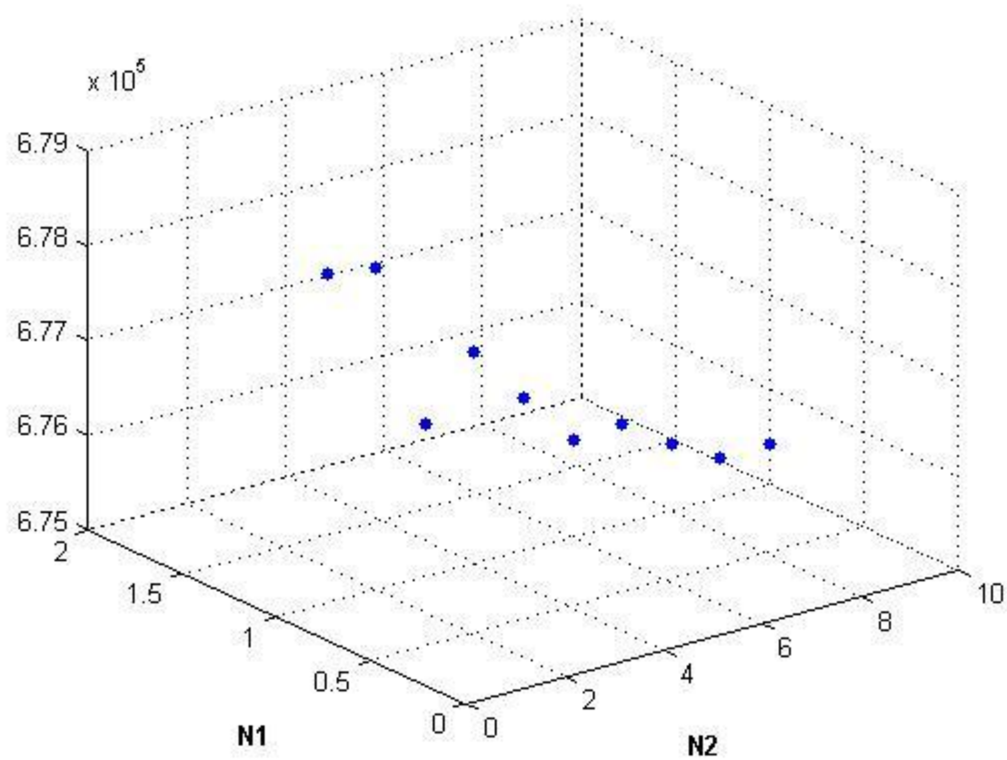


Figure 43 Supplier vs N2 (N1=2)

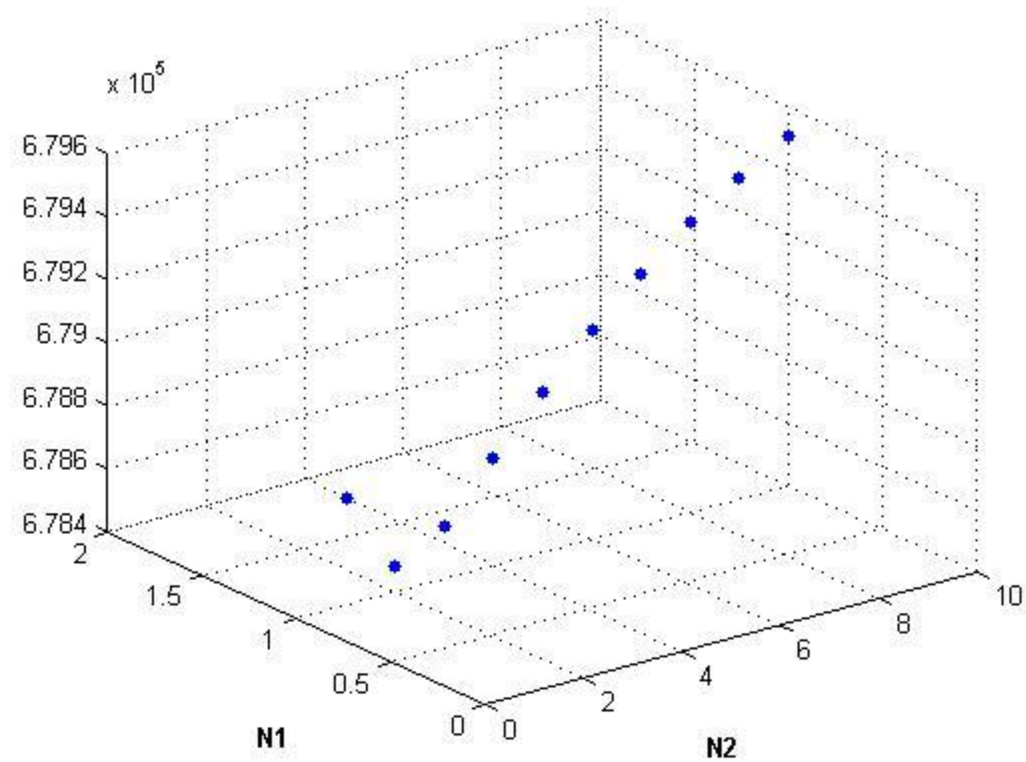


Figure 44 Supplier vs N1 (N2=1)

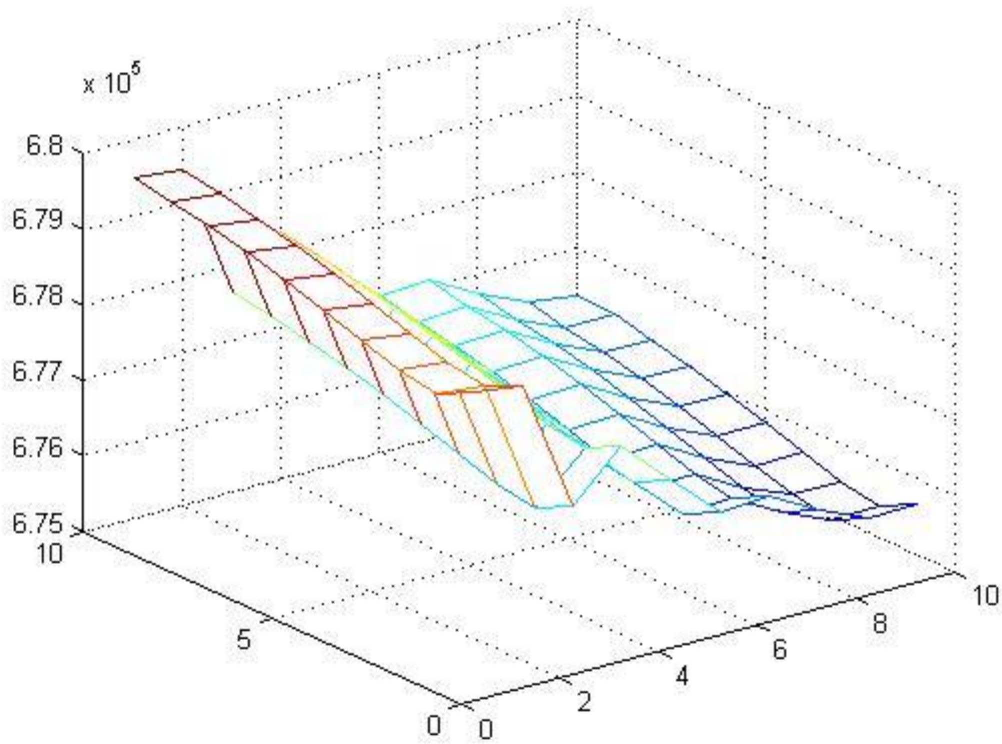


Figure 45 Supplier profit vs N1 and N2

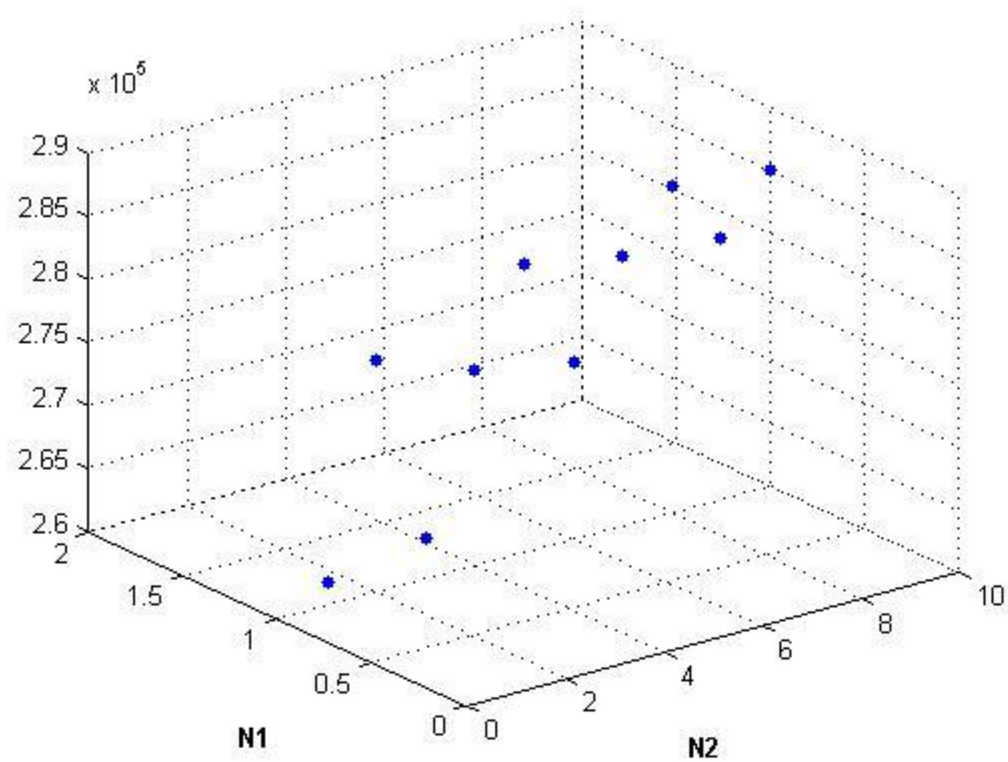


Figure 46 Buyer 1 vs N2 (N1=3)

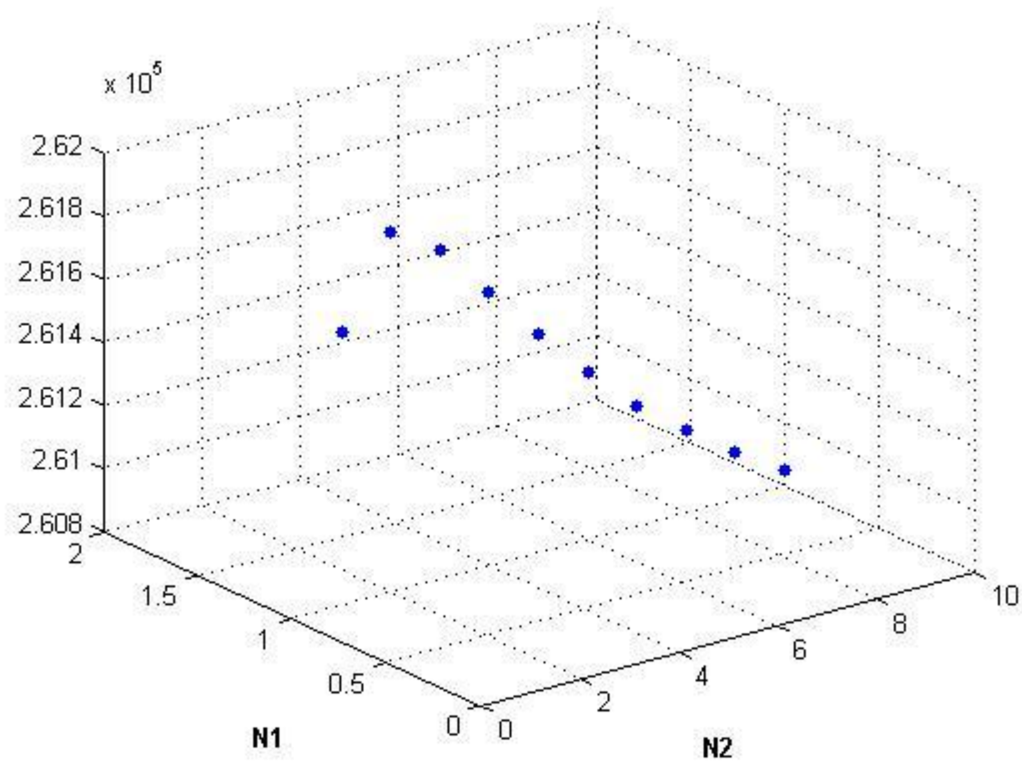
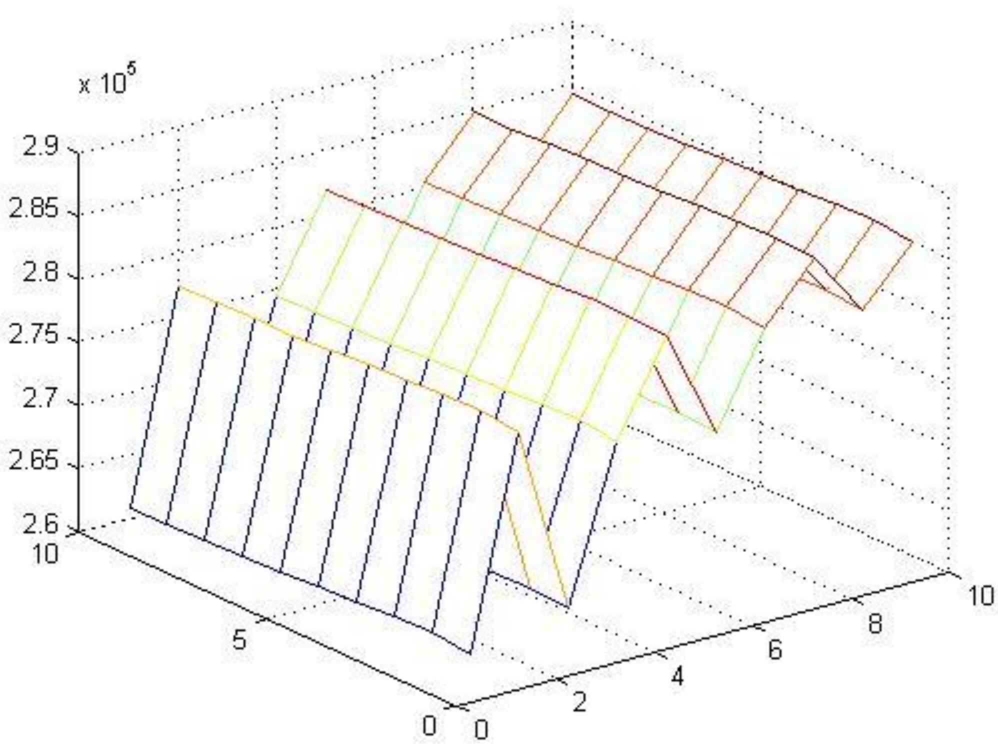


Figure 47 Buyer 1 vs N1 (N2 =1)



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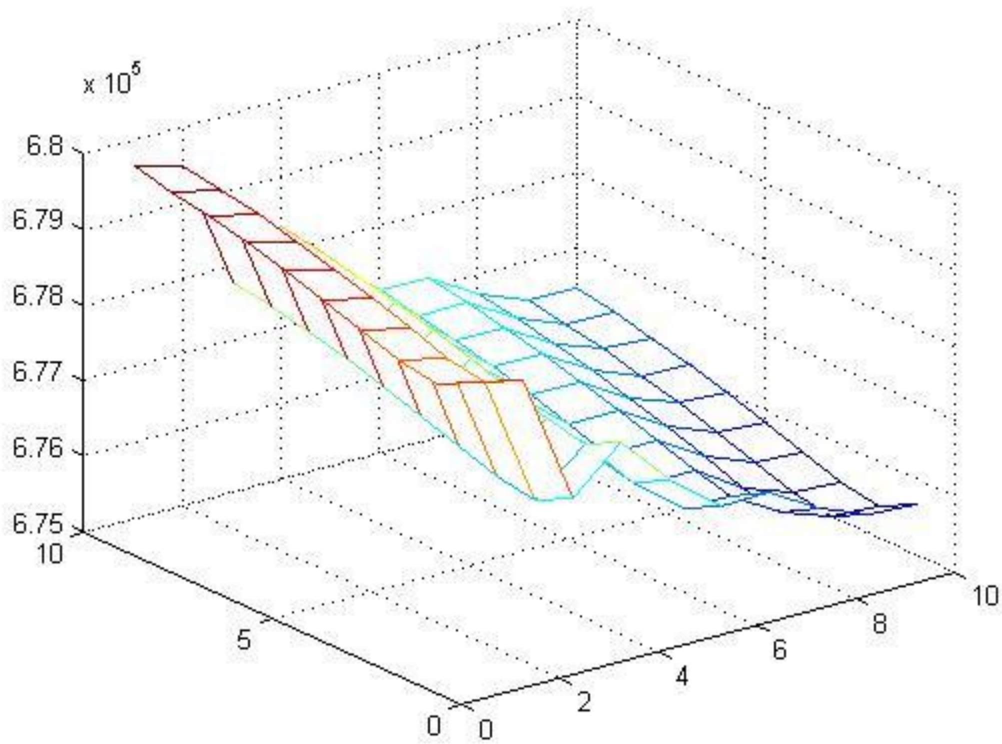

Figure 48 Supplier vs N1 and N2

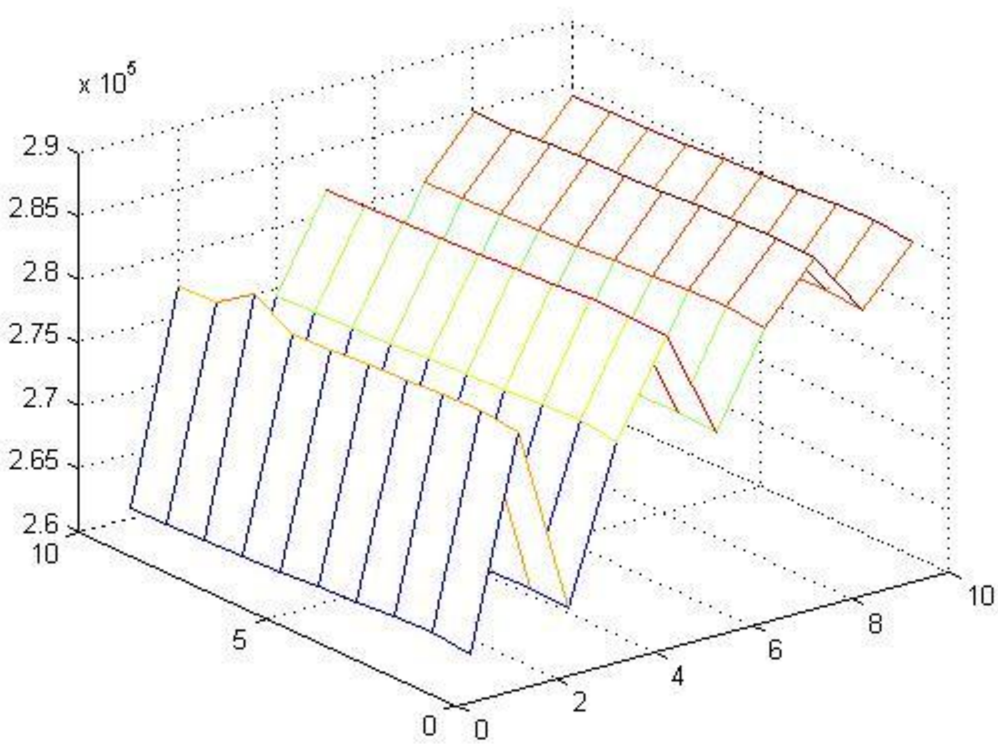

Figure 49 Buyer 1 vs N1 and N2

NBIS

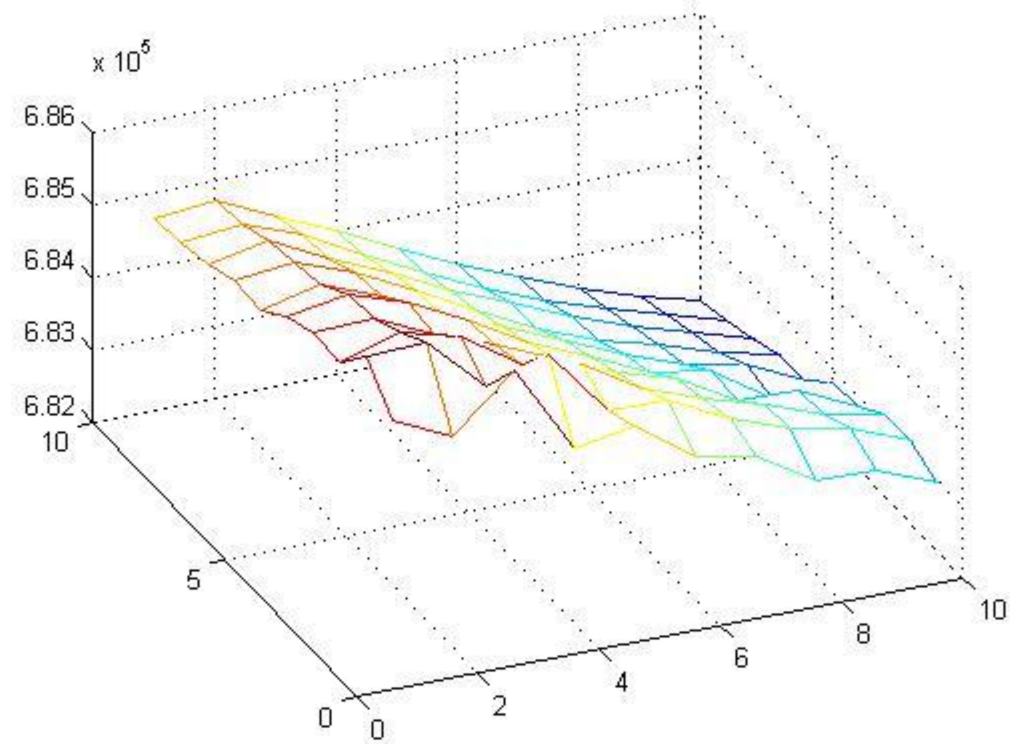

Figure 50 Supplier N1 and N2 (collaboration)

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