A PHENOMENOLOGICAL STUDY ON INCORPORATING THE HISTORY OF MATHEMATICS INTO TEACHING FROM THE PERSPECTIVE OF PRIMARY AND MATHEMATICS TEACHERS

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ABSTRACT

A PHENOMENOLOGICAL STUDY ON INCORPORATING THE HISTORY OF MATHEMATICS INTO TEACHING FROM THE PERSPECTIVE OF PRIMARY AND MATHEMATICS TEACHERS

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The aim of the present study is to describe the incorporating the history of mathematics into teaching approach from the perspectives of primary and middle grade mathematics teachers in the classroom environment by means of phenomenological research design. Two primary and two mathematics teachers participated in the study. Data collected from three interviews and two lesson hours observation were analysed based on phenomenological research design. The teachers' background information and the essence of incorporating the history of mathematics into teaching were provided. Findings of the study showed that the teachers described and perceived the approach under eight core themes: professional development, attractiveness, the environment for speaking up, the more the practice there is, the better it becomes, adaptability of historical materials, problem solving versus the history of mathematics, lack of knowledge and resources and time constraint. This structure of meaning contributes to the mathematics education field to understand the approach from the perspective of teachers who are encouraged incorporating the history of mathematics by the authorities in education.

Keywords: History of mathematics, Primary teachers, Mathematics teachers

ÖZ

SINIF VE MATEMATİK ÖĞRETMENLERİNE GÖRE MATEMATİK TARİHİNİN MATEMATİK ÖĞRETİMİNE KATILIMASI ÜZERİNE BİR OLGUBİLİM ÇALIŞMASI

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Bu çalışmanın amacı, sınıf ve ortaokul matematik öğretmenlerine göre sınıf ortamında matematik tarihinin matematik öğretimine katılması yaklaşımını olgubilim çalışması yoluyla betimlemektir. İki sınıf öğretmeni ve bir matematik öğretmeni çalışmaya katılmıştır. Üç görüşme ve gözlemlerde toplanan veriler olgubilim yaklaşımıyla incelenmiştir. Öğretmenlerin arkaplan bilgisi ve matematik tarihinin öğretime katılımı olgusunun öz bilgisi sunulmuştur. Çalışmanın bulguları göstermiştir ki öğretmenler yaklaşımı sekiz ana temayla betimlemişlerdir: mesleki gelişim, dikkat çekicilik, özgür konuşma ortamı, daha çok uygulama daha başarılı bir ders, tarih materyallerinin uyarlanabilirliği, problem çözme matematik tarihine karşı, bilgi ve kaynak eksikliği ve zaman sorunu. Bu temalar, eğitim uzmanları tarafından teşvik edilen öğretmenlerin bakış açısıyla matematik tarihinin öğretime katılması yaklaşımını anlamada eğitim alanına katkı sağlamaktadır.

Anahtar Kelimeler: Matematik Tarihi, Sınıf Öğretmeni, Matematik Öğretmeni, Olgubilim

To My Family, Best Friends and My Fiancée

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LIST OF ABBREVIATIONS

NCTM	National Council of Teachers of Mathematics
MoNE	Ministry of National Education of Turkish Republic
HEC	Higher Education Council
ICMI	International Congress on Mathematical Education
NCATE	National Council for Accreditation of Teachers Education

CHAPTER 1

INTRODUCTION

"The primary concern is that students shall be neither afraid of mathematics nor slaves to its supposed absolute authority, but shall become masters of this essential and central tool for understanding the world and how it works."

(Seltman & Seltman, 1977, p.28)

Mathematics is a collection of human efforts from each civilization and is still *in the making*, evolving (Jardine, 2007; Siu & Siu, 1979). Being so connected to everyday life, inevitably most civilizations left a collection of mathematical ideas in their manuscripts, papyruses or old handbooks. One of the written sources, the approximately 4000 year old Rhind papyrus is a concrete example of those collections. Humanity needed mathematics and still needs it.

Ancient papyruses, manuscripts, drafts or handbooks have shown the role of mathematics in society (Fauvel, 1991). By means of written historical sources, teachers and students can witness the various applications of mathematics in daily life and they can find opportunities to investigate the mathematics of different cultures (Fauvel, 1991; Grugnetti, 2000; Jardine, 2007). Mathematics has been affected by the sociocultural and economic conditions of the countries (Jardine, 2007). The teachers and students may gain multicultural perspectives towards mathematics by including biographies of many mathematicians and the sociocultural conditions of that country. Would knowing a part of this development process of mathematics as a learner or a teacher have some positive effects on mathematics education?

The number of researchers emphasizing the importance of history of mathematics in mathematics education cannot be underestimated. These researchers agree upon the idea that there is a need for the incorporation of the history of mathematics into the mathematics education program (e.g., Albayrak, 2008; Alpaslan, 2011; Bagni, 2008;

Ellington, 1998; Gazit, 2013; Grattan- Guinness, 1978; Siu & Siu, 1979). Apart from researchers there are also educational organizations which encourage the applications of history of mathematics into mathematics teaching. The National Council of Teachers of Mathematics (NCTM) published a book which is a source of mathematical background knowledge for teachers (NCTM, 2004). Furthermore, the NCTM has frequently highlighted the necessity of the history of mathematics in mathematics courses from teachers' and learners' perspectives (NCTM, 1989, 2000). In the latest news, it has been announced that 6-12 grade mathematics teachers will be funded by the NCTM for the 2014-2015 academic year after "…completing credited course work in the history of mathematics, creating and field-testing appropriate classroom activities incorporating the history of mathematics, and preparing and delivering a professional development presentation to colleagues." (NCTM, 2013). It shows that in-service teacher training was supported by the educational authorities as well. The International Commission on Mathematical Instruction (ICMI) is another worldwide foundation for mathematics teaching and learning which organizes international conferences and presents ICMI studies based on contemporary issues in mathematics education. One of the ICMI studies aimed to introduce the history of mathematics thoroughly by covering issues such as to what extent was the history of mathematics a necessity or how to incorporate the history of mathematics into mathematics lessons (Fauvel & van Maanen, 2000). Apart from learners, main focus of the ICMI Study was mathematics teachers and prospective mathematics teachers.

Apart from theoretical studies, empirical research can be considered limited for teachers to evaluate the method properly. Therefore, it is a necessity to draw a framework for incorporating the history of mathematics into teaching to be better prepared for the future applications of the approach. Teachers have the most of the responsibility to transfer the historical knowledge. Therefore, their perspectives toward this approach may provide some hints to understand what is to be done for the next step for the teachers about how to make them incorporate it into their instruction.

1.1 Significance of the Study

Current mathematical knowledge is the accumulation of products of previous studies, efforts or experiences (Bolinger-Horton, 2011). With respect to this, the history of

mathematics may provide deep insight into mathematics education and can be regarded as the only tool to relate modern mathematics to its past. Therefore, not only mentioning a set of arbitrary rules and procedures, but also the knowledge related to the origins of mathematics should be emphasized in the classroom to evaluate current mathematical knowledge better and to foresee the future of mathematics as well (Siu & Siu, 1979). However, the absence of knowledge related to the origin of mathematics has been one of the criticized issues among researchers (Fasanelli et al., 2000; Haile, 2008; Siu & Siu, 1979; Smestad, 2009). From elementary to secondary school mathematics education, formal school mathematics including curricula, program outcomes and lesson plans have been deprived of knowledge related to the origin of mathematics even if the regulations have highlighted the necessity of it (Fasanelli et. al, 2000; Smestad, 2009). For this reason, the nature of school mathematics has been called *ahistorical* (Haile, 2008). What would "being historical" bring into the classroom? From the perspective of teachers, the research reports that pre-service teachers improved their content and pedagogical knowledge (Clark, 2011; Huntley & Flores, 2010, 2011). They felt more satisfied with school mathematics and became more enthusiastic about teaching (Bellomo & Wertheimer, 2010).

Primary teachers and mathematics teachers can incorporate history of mathematics into their repertoire of instructional strategies in order to strengthen students' learning and comprehension of mathematics and consequently, realizing something valuable in mathematics makes teachers confident to teach and learn (Driscoll, 1994). To achieve this, teachers who are not used to incorporating the history of mathematics into teaching should be involved some activities. Teachers who did not attend any course related to the history of mathematics in their initial training are encouraged to involve in in-service teacher training (Heide, 1992).

There are standards prepared by the international organizations for teachers related to the history of mathematics integration (NCATE, 2003; NCTM, 1989, 2000). It is not clear how to integrate history of mathematics with teaching: as a separate subject or inclusion in mathematics courses. The studies conducted with teachers do not show an example of incorporating the history of mathematics into teaching (Fraser & Koop, 2006). The research conducted with teachers has demonstrated that teachers had positive attitudes towards incorporating the history of mathematics into teaching (Siu,

2004) however, they did not prefer to use it in the classroom due to lack of time. This situation arose from radical separation (Fried, 2001) which can be exemplified as teachers' leaving at least one mathematics lesson hour per week to teach the history of mathematics. History of mathematics as a separate subject has been studied by many researchers (Baki & Güven, 2009; Fraser & Koop, 1978; Karakuş, 2009). According to Siu and Tzanakis (2004), the history of mathematics is an educational tool which should not be perceived as a separate subject; it should be infused or permeated through the whole of mathematics. There is a limited research on history of mathematics incorporated into teaching. Therefore, the researchers indicated that empirical evidence was not sufficient to understand incorporating the history of mathematics into teaching (Clark, 2011; Liu, 2003). Therefore, teachers are the key to understand the approach. Their views or perspectives on this approach are significant for the future research such as whether it is included in the mathematics curriculum or not by means of teachers' descriptions and perceptions of the experience.

The studies conducted in Turkey were related to students and pre-service mathematics teachers (Albayrak, 2008; Alpaslan, 2011; Baki & Yıldız, 2012; Bütüner, 2008; Karakuş, 2008; Tözlüyurt, 2008; Yenilmez, 2011). Studies related to incorporating the history of mathematics into teaching have not been found in the accessible literature yet, which constitutes a gap for the future applications. New mathematics program for the middle grades includes a title which mentions possible benefits of incorporating the history of mathematics approach (MoNE, 2013b). This change can be considered first among other curricula innovations and revisions in Turkey. As a result, the present study may be the first to describe incorporating the history of mathematics from primary and mathematics teachers' perspectives in the accessible literature. The problem addressed by this phenomenology is a lack of research addressing exactly how primary and mathematics teachers perceives and describes incorporating the history of mathematics into teaching approach. It contributes to integration history of mathematics with teaching process.

1.2 The Purpose of the Study and Research Questions

This study aspires to describe the incorporation of the history of mathematics from the perspectives of four primary and middle grade mathematics teachers in the classroom

environment by means of phenomenological research design. Incorporating the history of mathematics will be generally defined as an approach which refers to the teaching of mathematics with its history. The teachers reflected on their experiences by means of interviews after they discussed the historical materials and taught lessons with this approach. From this perspective, the researcher aimed to understand:

How do primary and middle grade mathematics teachers perceive and describe incorporating the history of mathematics into teaching approach in mathematics classrooms?

Within this question, the researcher aims to understand the meaning of incorporating the history of mathematics for teachers.

1.3 Definition of the Terms

Incorporating the history of mathematics into teaching approach is defined as "…where history is not part of the existing curriculum, and therefore in some way or another must play the role of a tool in teaching the students topics (in-issues) within this curriculum" (Jankvist & Kjeldsen, 2011, p.4). Sometimes the words "integrating" and "incorporating" are used interchangeably in the literature. Integrating was defined by Swetz as "…giving background knowledge related to the inventor of the theory, sign, era, culture, country in short information that constitutes mathematics taught in the classroom to make mathematics challenging and more interesting." (1994, p.2). The difference between the two terms generally comes from whether extra historical knowledge permeates throughout the lesson (Seltman & Seltman, 1977) or the teacher allocates one or more lesson to teach the history of mathematics separately. This research was designed according to the incorporating the history of mathematics approach. Broadly, incorporating approach means providing the relevant history of core concepts as part of the instruction in order to help students understand mathematics and in order to support mathematics teaching.

History of mathematics is historical information directly related to the origins of mathematical concepts, topics, axioms, definitions or problems and their development process.

Primary teachers refer to teachers who are responsible for students from first grade to fourth grade in primary schools of the Turkish education system. They are responsible for teaching many disciplines from mathematics to science and technology.

Middle grade mathematics teachers are teachers working in middle schools which begins from fifth grade and continues to eighth grade in Turkish education system.

CHAPTER 2

REVIEW OF THE LITERATURE

In this chapter, previous studies are presented to gain insight about incorporating the history of mathematics. It started with how history of mathematics incorporation idea into mathematics education emerged. Afterwards, its contributions to teachers and learning; and ways of presenting historical information are described. Finally, relevant studies conducted in Turkey are briefly summarized to give information about the place of incorporating the history of mathematics in Turkish literature.

2.1 Emergence of History of Mathematics in Mathematics Education

When exactly mathematics began to exist in human minds is still in question, however, its existence spans at least thousands of years. Berlinghoff and Gouvéa (2004) reported an example of 37,000 year-old artifact from Africa. Civilizations similar to Africa were reported to have number systems as well as writing systems (Berlinghoff and Gouvéa, 2004). Based on their writing systems, they were enabled to write and keep their collection of mathematical knowledge on the manuscripts, artifacts, papyrus or handbooks and this tradition still continues. Knowing something about this process could make mathematics more valuable. As a matter of fact, NCTM (1989) associated learning to value mathematics with incorporating the history of mathematics.

Increase in the degree of importance given to mathematics necessitated emergence of a need for history of mathematics within education system, which went back to the *social transition* from agricultural to industrial age by the end of 19th century (Swetz, 1994). Those were the times that mathematics needed to be understood and to be valued by the learner. This issue frequently was articulated by the authorities to solve problems of mathematics education and it was spread by means of internationally coordinated activities. *International Congress for Comparative Historical Research (*Paris, 1900), *International History Congress (*Rome, 1903) and *International*

Mathematical Congress (Heidelberg, 1904) were platforms which gathered mathematics educators, mathematicians and historians of mathematics (Schubring et al., 2000). Inclusion of history into the mathematics curriculum was among the recommended ideas to be investigated (Schubring et al., 2000). Interactions between the authorities leaded to new attempts and further developments in mathematics education. The idea of inclusion history of mathematics drew attention of teachers and textbook writers. They began to add historical information within mathematics textbooks such as biographies of a mathematician (Mitchell, 1938).

As historical information was demanded, there was a need to standardize its incorporation in mathematics classroom. For this reason, National Council of Teachers of Mathematics published its Thirty-first Yearbook *Historical Topics for the Mathematics Classroom* in1969 which includes historical materials for mathematics instruction (NCTM, 2004). The book was prepared for teachers as "capsules" in order to provide basic historical materials immediately available for the classroom use. Furthermore, NCTM added a new goal for students to the book *Curriculum and Evaluation Standards*: learning to value mathematics (NCTM, 1989). This commitment to the history of mathematics progressed afterwards and it was directly associated with daily life applications and mathematics as a cultural heritage (NCTM, 2000). New goals had some implications on teacher education as well. National Council for Accreditation of Teachers Education (NCATE) cooperated with NCTM and regulated programs for elementary mathematics specialists. Within the program there are fourteen standards divided into four areas: process standards, pedagogy, content and field based experiences. Prospective mathematics teachers are expected to demonstrate the knowledge of historical development of content areas such as algebra or geometry (NCATE, 2003). All in all, history of mathematics has been embedded into mathematics discipline for both teaching and learning. In order to spread out the issue and to investigate its aspects deeply, International Commission on Mathematical Instruction (ICMI) studied worldwide and organized international conferences and presented an ICMI Study. This study aimed to introduce history of mathematics thoroughly such as to what extent history of mathematics was a necessity or how to incorporate history of mathematics into mathematics lesson (Fauvel & van Maanen, 2000). Contemporary issues reported in ICMI Study related to history of mathematics were informative about current situation of different education systems. When policies

of education systems, classroom applications and teacher training programs were considered, from United States of America to Japan, many countries have been investigated and it showed countries which performed some reforms to regulate incorporating the history of mathematics issues in education (Fasanelli, et al., 2000; Schubring, et al., 2000)

2.2 Contributions of History of Mathematics

There is a variety of approaches accompanying with mathematics education in order to improve mathematics teaching and learning. Incorporation of history of mathematics was acknowledged as one of those approaches. Benefits of incorporating the history of mathematics into mathematics discipline have been discussed by researchers and authorities in mathematics education.

2.2.1 History of Mathematics from Learners' Perspectives

Mathematics learners were reported to have some perceptions about mathematics. To give an example, they believed that algebra, geometry or arithmetic was branches independent from mathematics and mathematical terms, concepts, definitions or formulas would not change (Dematte & Furinghetti, 1999). Students were ready to grasp what was written in the textbooks or what was explained by the teachers without knowing that it was the latest version of a mathematical topic (Tall & Vinner, 1981). Mathematics is still an evolving discipline, however, students barely realize that mathematics is not an *end product*, *fixed* and *definite* (Bagni, 2004; Grugnetti, 2000; Siu & Siu, 1979; Yılmazer, 2011). Thus, the students may be mistaken that there is nothing they can do for the future of mathematics as a learner (Grattan- Guinness, 1978; Siu & Siu, 1979; Tözlüyurt, 2008).

Geometry, algebra and arithmetic are parts of the mathematics. If any interrelation between them is not noted, students may think that they are separate subjects. History of mathematics may show the overall picture of mathematics (Fauvel, 1991; Siu & Siu, 1979). Furthermore, it enables learners not only to see the relationship between parts of the mathematics and but also the relationship between other disciplines such as music, art, physics, economics or geography (Grugnetti & Rogers, 2000; Jardine,

2007). Most of the students admitted that if they would have been taught with the history of mathematics, they would have learnt better (Albayrak, 2008; İdikut, 2007).

While actively engaging with historical materials, learners got motivated; their appreciation of mathematics was increased (Fauvel, 1991; Jardine, 2007). After actively engaged in the classroom, it was also reported that their attitude toward mathematics changed. McBride and Rollins (1977) conducted an experimental research with 67 college students. 25 items from sources of mathematics history was incorporated into mathematical discussions. Aiken's Attitudes towards Mathematics Scale was used to determine changes in attitudes towards mathematics before and after the treatment. Pre-test and post-test results were reported statistically significant difference in attitudes toward mathematics (McBride & Rollins, 1977). All in all, the development of affective side was much more emphasized than development of cognitive side of the students (Siu, 2004).

Fauvel (1991) stated that history of mathematics provided mathematics a human face. It was because source of mathematical knowledge is human (Grattan-Guiness, 1978). Mathematics is not a product of only geniuses and heroes but a product of hard work (Siu & Siu, 1979; Byers, 1982). Mathematicians once upon a time experienced various difficulties, mistakes, challenges and personal conflicts as well. It was even possible to find different explanations from different mathematicians for the same mathematical problem (Berlinghoff & Gouvéa, 2004; Gulikers & Blom, 2001). Knowing something about the procedure will contribute to significant appreciation of learners to the concepts and processes.

2.2.2 History of Mathematics from Teachers' Perspectives

Teachers may come across different questions from their students related to mathematics during mathematics lessons. Those questions generally begin with "why….?". NCTM (1989) divided those questions into three parts: The chronological whys were more apparent questions that are related to the origins of the terms, definitions, formula or concepts such as "why there are sixty minutes in an hour?". Logical ones and pedagogical whys are less likely to occur in the classroom environment. They were questions about the nature of a theorem or axiomatic system

and they were questions about the order mathematical topics according to its development process respectively. NCTM (1989) also emphasized that historical approach was not the only way to explain those questions.

Mathematics history has full of epic and didactic stories as much as disappointing studies of mathematicians. The increase of teachers' interest and enthusiasm towards mathematics was reflected by teachers who developed positive attitude towards mathematics to students (Bellomo & Wertheimer, 2010; Siu & Siu, 1979; Fauvel, 1991; McBride & Rollins, 1977).

Huntley and Flores (2010) referred benefits of history of mathematics on two aspects: mathematical understanding and mathematics content knowledge. More specifically, in his qualitative research, Clark (2011) investigated 80 prospective middle grades and secondary mathematics teachers' on-line reflections toward using Al Khwarizmi's geometric method for solving quadratic equations. It was found that prospective teachers were very fond of the method and it would be helpful to develop mathematical understanding for any other topic (Clark, 2011). Similarly, Panasuk and Bolinger-Horton (2012) showed the strengthening effect of history of mathematics on both mathematical content knowledge and on confidence of teachers. Furinghetti (2000) as a professor in mathematics department and mathematics education department instructed prospective secondary mathematics teachers. These pre-service teachers were reported as unsatisfied and inadequate about teaching mathematics since training in universities were not in parallel with teaching in schools, therefore, they had a tendency to replicate the way of teaching of their secondary school teachers. The author claimed that history of mathematics could solve this problem if it was used in teacher training program (Furinghetti, 2000).

Didactic skills of teachers were claimed to be improved by reading old sources (Gulikers & Blom, 2001). From historical perspectives, teachers understood why a certain concept was difficult for the student and this would lead them develop their didactical point of views. (Fauvel, 1991; Frunghetti, 2000; Grugnetti, 2000). It was from genesis approach that human followed the steps of their ancestors and how people learnt mathematics now was similar to their ancestors did (Byers, 1982; Gulikers & Blom, 2001; Jankvist, 2009; Mitchell, 1938; Schubring et. al, 1977; Siu &

Siu, 1979). Furthermore the ordering of the topics in the curriculum was determined mostly according to historical development of the mathematical ideas (Katz, 1993; NCTM, 1989). To exemplify, ordering of the number systems throughout all education levels from natural numbers to complex numbers is directly related to their development throughout the ages (Jankvist, 2009) or a calculus course for students based on its historical development designed by Katz (1993). Therefore, teachers may get better understanding of the origins and connections within mathematics and between other disciplines, cultures (Byers, 1982; Fauvel, 1991; Furinghetti, 2000; Seltman & Seltman, 1977; Siu &Siu, 1979).

History of mathematics utilization in mathematics education brought about some points to consider carefully. One of them is related to the policies of the countries. Historical topics are open to be expressed from different perspectives. From this point of view, the authorities may want to control over the past and may add provocative issues in mathematics education (Fauvel, 1991) which will harm the objectivity of mathematics as a science discipline. Therefore, primary sources needed to be analysed by the authorities very carefully. Apart from policies, history of mathematics is open to evaluation of teachers during the lesson. Any change in the context of the anecdotes may lead students to think about whether the story is true or not (Berlinghoff & Gouvéa, 2004).

2.3 Ways of Incorporating The History of Mathematics

History of mathematics drew attention importance in education by educational organizations and researchers, how to present it in education leaded to different results for both learners and teachers. Therefore, in order to better identify the place of history of mathematics in education some researchers prepared categorizations related to the utilization of mathematics history. One of these categorizations was prepared according to the studies presented at Sixth International Congress on Mathematical Education (ICMI) in 2000. There are two parts: *history as a tool* and *history as a goal* (Jankvist, 2009). History as a tool was defined as teaching history for meanings and understandings of mathematics (Fasanelli et al., 2000). The role of the process may change as affective tool, cognitive tool or tool for evolutionary arguments (Jankvist, 2009). Respectively, affective tool includes motivating factors towards mathematics.

Cognitive tool, on the other hand, was for improvement in teaching and learning process (Jankvist, 2009). The last one, tool for evolutionary arguments was based on arguing issues of recapitulation, which comes from the idea in biology 'ontogeny recapitulates phylogeny' (Jankvist, 2009; Mitchell, 1938; Schubring et al., 1977; Siu & Siu, 1979). Its effect on mathematics education is that "children follow their ancestors while learning mathematics" (Mitchell, 1938, p.27). Thereby, it may help to understand learning steps of mathematical understanding in case learners miss any step their ancestors experienced before (Gulikers & Blom, 2001) or it might be concluded as learners should follow the steps of historical development of the subject area (Byers, 1982). Second utilization of history of mathematics described by Jankvist (2009), history as a goal, if the aim is to learn history of mathematics but not mathematics then history of mathematics should be considered separately from mathematics course. It was explained history as a goal in his further article as:

> It is considered a goal to show students that mathematics exists and evolves in time and space, that is a discipline which has undergone an evolution over millennia, that human beings have taken part in the evolution, that evolution of mathematics is due to many different cultures throughout history, and that these cultures have had an influence on the shaping of mathematics as well as the other way around. (Jankvist, 2010, p.54)

Jankvist's (2009, 2010) categorization is helpful to understand difference between teaching history and teaching mathematics. From history as a tool perspective teachers have been trained and applications of history of mathematics in the classrooms still continue.

2.3.1 History of Mathematics in Elementary and Secondary School Level

Elementary and secondary education could be considered as poor based on history of mathematics applications within curricula, didactics or policies of the countries (Fasanelli et al., 2000). Previous studies showed that history of mathematics were generally integrated with mathematics lessons. This idea was supported by the researchers (e.g. Bagni, 2008. Siu & Siu, 1979). History of mathematics should not be as a separate subject in the school mathematics program because it was a tool for teaching and learning in use of teachers and students at the right moment (Siu & Siu, 1979). There are different forms of presenting historical information within

mathematics teaching. Those forms would help teachers to incorporate them at the right moment. Sometimes they were jointly incorporated within the classroom which also enriched the classroom environment.

Direct historical information from course books or textbooks: Tzanakis , Arcavi and colleagues (2000) offered this type of information in order to hear mathematical topics from the history. To mention development of a mathematical topic will be helpful for students to realize some changes occurred on mathematical signs, constructs and formulas. Quadratic equations came from the statement of squares and rectangles, which was a good example of this type of utilization of history (Katz, 2004).

Anecdotes: This kind of historical materials include a short story from the past mathematicians' lives (Fauvel, 1991). Katz (2004) qualified this kind of materials as *spice* which brings classroom entertainment. For example, old mathematicians were also having hard times to overcome mathematical problems like students even they failed, sometimes they lost one of their eyes while studying hard on mathematical issues without giving up. This sort of anecdotal knowledge may be helpful to show human side of the mathematics.

Visual Aids: Films, documentaries, cartoons, posters, manuscripts, shapes, photographs or drawings of famous mathematicians are in the category of visual aids. Kar and İpek (2009) highlighted the significance of aiding visual materials while solving old problems from the history.

Activities: These are sources to make mathematics more interesting and enjoyable. It generally aims to motivate and provide historical information to learn topics in mathematics better. Swetz (1994) published a book generally based on this kind of activities.

*Biography of a mathematician: C*omparing two mathematicians' solution to a problem or developing a concept helps to see mathematics from different cultural perspectives. Small historical information about the political, social and economic context the mathematician lived in are also provided to enrich the development of a mathematical idea under what conditions.

Outdoor experiences: By preparing outdoor organizations such as visit a museum that includes mathematicians or primary sources from different civilizations are also helpful to understand connections between mathematics and real life situation. Tzanakis, Arcavi and colleagues (2000) gave an example from Japan. Some high school mathematics clubs organized tours to the places which included bulletin boards to solve the problems differently from the mathematics they learn today.

Old problems: It is useful to show old problems and their answers if they were within the context of mathematical topics. Learners might try to solve problem with their current knowledge. At this point teacher may supply a little background information of the mathematician or culture that the old problems belong then may answer the question as the same way mathematician solved. Therefore, it may be a chance for students to see mathematics is a developing science and once upon a time mathematicians do not have some mathematical knowledge. Grugnetti (2000) gave an example from the book Liber Abaci of Leonardo Pisano. There was a problem related to Pythagorean Theorem, however, Fibonacci implicitly used it and wrote the problem arithmetically without a fashion of writing operations only. It was claimed that while utilizing this procedure students would conclude that mathematics was not an unchanged discipline (Swetz, 1994).

Projects for students: It is an extra-curriculum activity (Siu & Siu, 1979) in order learners to further develop their knowledge about history of mathematics. For example, students may be required to collect studies of a famous mathematician.

Plays: Tzanakis, Arcavi and colleagues (2000) represented plays as enacting a brief life of famous mathematicians or enacting a mathematical situation. Fraser and Koop (1978) provided teachers some historical materials which were in the form of play and article. The play was regarded by the teachers more favourable than article. Based on this, plays may be helpful to increase participation in the mathematics lesson.

2.3.2 History of Mathematics in Teacher Education Programs

The contributions of incorporating the history of mathematics for teachers put in mind of the question about teacher training programs. There were authors who were in

favour of such courses which provided historical background about mathematics (Furinghetti, 2000; Huntley & Flores, 2011). Furinghetti (2000) as a professor in mathematics department and mathematics education department mentioned her prospective secondary mathematics teachers. They were reported as unsatisfied and inadequate about teaching mathematics since training in universities were not in parallel with teaching in schools, therefore, they had a tendency to replicate the way of teaching of their secondary school teachers. The author claimed that history of mathematics could solve this problem if it was used in teacher training program (Furinghetti, 2000).

The literature showed that historical knowledge could be supplied under different forms. Firstly, history of mathematics as a must course in teacher training programs was to provide prospective mathematics teachers gain insight about mathematics for their future practice *Using History* course was one of those special courses within secondary mathematics teacher education programs in USA (Clark, 2011). It was a special course for middle and high school level that included the study of the history of mathematical ideas (Clark, 2011, p.6.). Another university in USA also reported to include a course on the history of mathematics for prospective secondary mathematics teachers to improve their mathematical content knowledge and ways of thinking about this content. (Huntley & Flores, 2010, 2011). Another choice is to provide historical knowledge as an elective course. Yılmazer (2011) conducted a study based on the perceptions of prospective teachers towards the course *History of Mathematics* which took place as an elective course in mathematics education programs in Turkey. Examples from universities were quite limited because history of mathematics is nowadays considered as an important issue in education by the authorities (Schubring et al., 2000). Bagni (2004) associated this situation with to what extent different universities gave importance to history of mathematics.

2.4 Research in Turkey about the History of Mathematics in Education

Inclusion of history in mathematics education has some limitations; however, the beneficial aspects for learners and teachers showed that it is one of the useful approaches to overcome some problems in mathematics education all around the world. Fasanelli and his friends (2000) conducted a study to highlight experiences of

the countries whose political regulations were in favour of inclusion of mathematics history in the school mathematics curriculum. There were sixteen countries: Argentina, Austria, Brazil, China, Denmark, France, Greece, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Poland, United Kingdom and United States of America. Although most of them were not ready to incorporate history of mathematics there were successful examples such as China (Fasanelli et al., 2000).

It was a new issue for Turkey to discuss the place of history of mathematics within mathematics teaching approach. Historical view in Turkish mathematics education started to take part in new curriculum in 2004 (Baki & Yıldız, 2012). Mathematics education was constituted on a more concrete basis; in fact, many instructional materials and teaching activities with daily life problems and events were enrolled in the classrooms (Argün, Arıkan & Bulut, 2010). Elementary mathematics curriculum was modified in 2009 and included many new goals and outcomes for students one of which was directly highlighting the necessity of the historical parts of mathematics. It was stated as "[Students will be able to] comprehend the historical development of mathematics, correspondingly its role on and value of development of human thoughts, [and] importance of its utilization in other disciplines" (MoNE, 2009, p.9). As a result, teacher guide books, student workbook and elementary mathematics curricula have examples of historical sources related to biographies of famous mathematicians, project based homework that included famous scientists and old problem solving methods. New education system was adopted in 2012-2013 academic year in Turkey, "4+4+4". In this system, compulsory education becomes 12 years and students start compulsory education when they are 66 month old. Fifth grade level was transferred to middle school education; therefore, primary education period decreased to four years from five years. Afterwards, there was news related to the elementary mathematics curricula programs renewal in 2013 due to the new education system in Turkey. Its name was changed to mathematics curricula program for middle grades. This reviewed curriculum will be adopted in 2013-2014 academic year for fifth grade and will be applicable for sixth grade, seventh and eighth grade gradually (MoNE, 2013a). In this curriculum change, possible benefits of incorporation of mathematics history were summarized under a title which had never mentioned previous curricula in mathematics education (MoNE, 2013b). In order to support those modifications and take advantage of benefits of incorporating mathematics history, historical sources

should be included in mathematics classroom not only in the books but also in the repertoire of teachers.

The choice of using history of mathematics within mathematics teaching is related to teachers' background knowledge which is mostly affected by their teacher training program. Research showed that teachers did not feel themselves sufficient enough to mention historical knowledge in classroom (Alparslan, 2011; Yenilmez, 2011). Thus, it was convincing that adding history of mathematics into teacher training programs may be helpful to improve their content knowledge. Higher Education Council (HEC), the institution responsible for higher education in Turkey, stated that there should be an increase with the number of common courses and new courses such as History of Science or Philosophy of Science must be added to the teacher education programs (HEC, 2007). Continually, new course added to the mathematics teacher education curriculum, *History of Mathematics*, whose content was about development of mathematics in 5000 years based on learning areas and biographies of mathematicians who contributed to this development and it started to take place in Turkish Universities in academic year 2009-2010 (as cited in Yenilmez, 2011). Those changes in education system of Turkey related to history of mathematics have been encouraging researchers to study this phenomenon.

Learners are the main focus of this approach. If they get benefit from this approach, the aim of incorporation will be meaningful. Based on this assumption, effects of the approach on students were carried out in Turkey. Middle and secondary grade level students were investigated. There were two researches with eighth grade participants. Firstly, self-efficacy and mathematics achievement level of 131 elementary eighth grade students from two schools were inspected by means of quasi-experimental method (Albayrak, 2008). Findings of the research emphasized that there were no statistically significant effect for self-efficacy level of both control and experimental group, however there was a statistically significant increase in students' achievement level for experimental group in mathematics education only for one school. Bütüner (2008) developed a lesson plan for eighth grade students in the subject of algebraic problems incorporated history of mathematics. Famous mathematicians were introduced in the lesson plan in addition to the old methods in Ancient Egypt, Babel

and Ancient China. The study focused on learning of history of mathematics but not the learning of mathematics.

Karakuş (2009) was also planned a lesson for eighth grade students describing calculation of square root of a number by means of Babel method. He emphasized lack of classroom applications of incorporating the history of mathematics into mathematics teaching.

Apart from affective and cognitive effectiveness, how learners view history of mathematics is also another interest area. A qualitative research was conducted based on the views of history of mathematics of eight senior high school students (Tözlüyurt, 2008). The research described students' perceptions of history of mathematics integration within number learning area. Students had positive change toward history of mathematics integration with the curriculum. However, the research demonstrated that they associated history of mathematics with teaching only development process of mathematics, in other words teaching only history (Tözlüyurt, 2008). The idea of integration history of mathematics causes participants to think that integration history of mathematics means history. If the situation was like mentioned there would occur a threat that a person may misjudge this approach. As a result, the person who dislikes history has a tendency to dislike mathematics (Jankvist, 2009). All in all, although there is a need to develop research on learners, significant results were shown that learners' attitude were positive towards this approach.

Primary and mathematics teachers may utilize history of mathematics as a repertoire for instructional strategies in order to strengthen students' learning and comprehension of the mathematics. Historical knowledge is also a part of mathematical knowledge that a teacher should know (Bagni, 2008). Although history of mathematics is not the only way to recover some problems in mathematics education (Frunghetti, 2000; Siu & Siu, 1979), incorporation approach may help how to fill the gap between teachers' and students' ideas about the place of mathematics in daily life. Therefore, perspectives of pre-service teachers towards this approach in Turkey were investigated as well.

Attitudes of pre-service mathematics teachers towards using history of mathematics were one of the main concerns in Turkey. A research conducted in 2004 with 14 senior mathematics teacher education students revealed that there was an increase in their attitudes towards using history of mathematics but results were not statistically significant (Oprukçu-Gönülateş, 2004). This was due to the high attitude at the beginning of the research towards utilization of this approach. However, participants improved high quality teaching methods with historical materials, which were helpful to figure out the ways of incorporation. Another research with pre-service mathematics teachers investigated their knowledge of mathematics history, beliefs and attitudes towards history of mathematics (Alpaslan, 2011). Attitude levels towards using history of mathematics were reported as high; pre-service mathematics teachers' knowledge of mathematics history was also reported as not adequate and they admitted that there was a need for history of mathematics as a course and believed that history of mathematics would increase students' interest towards mathematics. It was also enlightening to investigate pre-service elementary mathematics teachers' perspectives towards necessity of history of mathematics course. 121 pre-service teachers who took *History of Mathematics* course were attended for the survey. Their opinions were in favour of necessity of history of mathematics course especially for teaching some mathematical topics such as numbers, geometry and solving equations (Yenilmez, 2011).

There were also other research giving examples of integration mathematics history. Baki and Güven (2009) conducted a teaching experiment for third year 41 pre-service secondary school mathematics teachers in order to integrate history of mathematics in mathematics education. Omar Khayyam's method for solving cubic equations was cooperated with dynamic geometry software in teaching experiment. It was concluded that students recognized the difference between modern mathematics and Khayyam's method; moreover, they were amazed by the ability of Omar Khayyam's drawing geometric shapes without using algebraic forms (Baki & Güven, 2009). There was also another concern about how teachers use history of mathematics in their mathematics courses. As a result, pre-service teachers had a tendency to teach with this approach but they did not trust their knowledge of history of mathematics (Siu & Siu, 2004; Tzanakis & Arcavi et al. 2000). However, research on teachers about

incorporating the history of mathematics into mathematics teaching was not found in the Turkish accessible literature yet.

2.5 Summary of the Reviewed Literature

In this chapter, reviewed literature was presented according to the categorizations that helped the researcher the history of mathematics thoroughly. Although the history of mathematics dated back to the thousands of years back (Berlinghoff and Gouvéa, 2004), it was not known when exactly their incorporation started. However, the idea of learning to value mathematics was considered as a reason to incorporate the history of mathematics into teaching (NCTM, 1989, 2000; Swetz, 1994).

There are international organization such as NCATE, NCTM and ICMI which encourage incorporating the history of mathematics within teaching approach since it was found beneficial for both learners and teachers as an aid for learning and teaching mathematics (e.g. Albayrak, 2008; Alpaslan, 2011; Bagni, 2008; Ellington, 1998; Fraser & Koop, 2006; Gazit, 2013; Grattan- Guinness, 1978; Siu & Siu, 1979). The main arguments about this issue in the literature were gathered around the need that teachers and learners gain mathematical understanding. Other benefits for the learners, specifically, were reported to be mostly based on affective and cognitive domain development (Jankvist, 2009). It was considered as a process students were reportedly attracted by the historical knowledge. They became motivated, enthusiastic and interested in mathematics (Fauvel, 1991; Jardine, 2007). Therefore, they learnt to value mathematics and there was a positive change in their attitude towards mathematics (McBride & Rollins, 1977). It has some effects on teachers as well based on their content and pedagogical content knowledge (Clark, 2011; Huntley & Flores, 2010, 2011). Moreover, according to Bellomo and Wertheimer (2010) teachers felt more satisfied with school mathematics and became more enthusiastic about teaching (Bellomo & Wertheimer, 2010). When the studies conducted in Turkey was considered, preservice teachers and students were investigated, however, studies related to teachers have not been found in the accessible literature yet.

The history of mathematics can be integrated within mathematics courses in elementary and secondary education by means of various ways: direct historical

information from course books, anecdotes, visual aids, activities, plays, biographies, outdoor experiences, old problems and projects. For teacher education, there are examples of institutions who added a special history of mathematics courses in their teacher training programs (Schubring et al., 2000; Yılmazer, 2011).

CHAPTER 3

METHODOLOGY

The aim of this study is to describe incorporating the history of mathematics into teaching from the perspectives of primary and mathematics teachers. This research was designed based on phenomenology. Basic characteristics of the qualitative approach and rationale for the selection of phenomenology are discussed in this chapter. Participant selection, data collection tools, pilot study, data collection procedures, field and ethical issue, trustworthiness of the study and assumptions and limitations are presented to provide detailed information about the method this study applied. Lastly, the role of the researcher is described to inform the reader about the personal assumptions that the researcher brought to the study.

3.1 The Design of the Study

Qualitative and quantitative research designs are different in their natures (Creswell, 2012; Flick, 2007; Fraenkel & Wallen, 2006) and they have distinct characteristics. The present study was designed to be a qualitative research which leads the researcher to gather data about a phenomenon from a more holistic perspective for teaching (Fraenkel & Wallen, 2006). This phenomenon was incorporating the history of mathematics into teaching approach. Qualitative research method was appropriate to investigate this complex phenomenon since it provides opportunity to investigate the phenomenon in its natural settings, schools. Moreover, it allowed collecting data from multiple sources and gives detailed information from the participants' point of view (Creswell, 2009; Yin, 2011).

There are various paths to conduct qualitative research. Phenomenological study is one of the qualitative research methods described as "...investigates various reactions to, or perceptions of a particular phenomenon…" (Fraenkel & Wallen, 2006, p. 436). Van Manen described the phenomenological point of view "…to do research is always to question the way we experience the world, to want to know the world in which we

live as human beings" (1990, p.5). Phenomenological view was used to describe how primary teachers and middle grade mathematics teachers experience the incorporating the history of mathematics into teaching. It is the study of lived experience and asks directly what this is or what that kind of experience is (Moustakas, 1994; van Manen, 1990). Therefore, phenomenological study enabled the teachers to share their experiences and enabled the researcher to describe meanings of these experiences thoroughly. Additionally, the participants gave voice about the phenomenon (Lester, 1999). Besides, the present study has the characteristics of phenomenology. Under those characteristics, firstly, teachers' experiences were eliminated from the biases that they were told true. Secondly, different perspectives of teachers constituted a unified whole picture of the essences of phenomenon. Next, descriptions of the experiences are the ways of presenting the phenomenon rather than explanations or illustrations. They are the core facets of human science research defined by Moustakas (1994), which have important roles for the selection of the research design.

3.2 Selection of the Participants

The participants were purposefully selected from schools in Çankaya which is a district of Ankara. There were 71 public primary schools and 52 public middle schools in Çankaya, in 2013. The researcher conveniently selected the schools of the participants near her office and home to have easy contact with the teachers for the further investigations of the approach. While purposefully selecting the participants, they were required to work in public primary and middle schools. The teachers did not have the experience of incorporating the history of mathematics (Englander, 2012, p.19) as it was expected from the studies conducted in Turkey (Alpaslan, 2011; Yenilmez, 2011) however they were at least six-year experienced to be able to manage the classroom experience.

The research question of the study was directly related to the primary teachers and middle grade mathematics teachers. Teachers from different grades contributed to the variation of the data since different grade levels had different objectives related to learning areas in mathematics. For example, in May 2013, while third grade teacher was teaching fractions, focus of the fifth grade was multiplication of natural numbers with three digits. It was to show that history of mathematics was not constrained to

some topics such as geometry and was to enrich the data gathered from different grade level teachers. As a result, different materials were prepared for each teacher according to their flow of the program. This variation among the curricula allows the researcher to select participants according to maximal variation sampling. It was helpful to demonstrate existence of common or different variations among participations. Therefore it provided wide range of possibilities for readers before coming to a conclusion (Seidman, 2006). There is not a certain answer for how many participants satisfy the research question of this study. Many experts leave the answer to the researcher (Creswell, 2007; Seidman, 2006). Seidman gave a criterion to maintain the study until "…to hear the same information reported." (2006, p. 55). The researcher considered this criterion.

Each school was visited respectively till the researcher conducted six teachers from third grade to eighth grade. First grade and second grade teachers were excluded from the study because they were getting used to mathematics discipline. The historical materials covered not only mathematical terms but information related to the culture. As a result, historical materials for them have to be designed in a different manner.

The teachers needed to be volunteer to apply the approach in the classroom in order to reflect their subjectivity toward the approach without biases or prejudgements. They were all open to learn something new in mathematics. Ayşe, Bilge, Canan and Derya were the participants from four different schools in Çankaya district. They were third, fourth, fifth and sixth grade teachers respectively. Table 3.1 provides a summary of the profiles of the teachers.

Table 3.1. The Teacher Profiles

Name	Sex	Experience	Graduation	Pedagogical Formation	Job Title
Ayşe	Female	21	Two-year Upper Secondary Education	Yes	Primary teacher (Third grade)
Bilge	Female	16	French Language Teaching	Yes	Primary teacher (Fourth grade)
Canan	Female	15	Mathematics	Yes	Middle grade mathematics teacher (Fifth grade)
Derya	Female	6	Elementary Mathematics Education	No	Middle grade mathematics teacher (Sixth grade)

They were at least six-year experienced female teachers. Three of them began teaching profession after completing pedagogical formation. Their school and classroom environment was different as well. The school of Ayşe was small sized primary school and it had at most 200 students. The socioeconomic level of the school was regarded by Ayşe as low. The number of students she had was 21 and she was pleased with that. The school of Bilge was considered bigger than the school of Ayşe relatively and it was also a primary school as well. Bilge had 24 students in the classroom and the design of the classroom was in the form of "U" shape. She preferred this type of classroom design because it made the environment more learners centred. Canan, on the other hand, works in a school including both primary and middle grade school. She said that the families worked hard to provide the needs of the school. There were 35 students in her fifth grade classroom. She was new at teaching fifth grade students. It was due to change in the education system in Turkey. Lastly, the school of Derya was in the centre of the city and it included both primary and middle school. It was a crowded school and Derya had a crowded classroom. She had 41 sixth grade students. However, she was pleased with that the school had enough technological equipment to incorporate into her instruction. All in all, they had

different conditions based on school or classroom environment; however, this situation had contributions to the findings of the study.

3.3 The Role of the Researcher

A researcher is claimed to be *a key instrument* in qualitative research (Bogdan & Biklen, 2007; Fraenkel & Wallen, 2006; Yıldırım & Şimşek, 2011). Therefore, it is crucial to report the role of the researcher to guide readers about the data collection procedure (Yıldırım & Şimşek, 2011). Creswell (2009) also recommended involving the researchers' experiences. Therefore, the readers would know about the researcher's point of view. The researcher was a curious student who used to ask the origin of mathematical concepts, formulas and definitions to their teachers during her elementary and secondary education. She was not satisfied with the answers of teachers since they were reluctant or had inadequate information to share with. As a result, she was determined to be a teacher who would teach her students mathematics with its *whys*. In one of her undergraduate courses, the researcher was introduced to this approach and her interest towards the issue has started. She believed that mathematics needed to be provided with its whys. The researcher, from the beginning of the study, was aware of this tendency and dedicated herself to bracketing her personal biases or prejudgements. It was a necessity to conduct phenomenological research (Moustakas, 1994). In order to achieve this, except explanation of incorporating the history of mathematics into teaching approach, she did not prefer speaking with the participant about the approach. However, during the interviews, the researcher encouraged the participants by using "little handling emotional outbursts, and using icebreakers" as Creswell remarked (2012, p. 218) in order to make participants feel comfortable and make them share information from their lives. The researcher left the choice of historical materials to the teacher for the classroom practice provided that they incorporated at least one of them when they need. The researcher informed them that she could prepare extra materials if given historical materials were not found sufficient or appropriate for the classroom practice. The teachers were not told what to do with the historical materials in the classroom. It depended on their creativity where, when and how they would incorporate the approach. However, when they had some problems to understand the historical knowledge or how to incorporate them, the researcher explained the materials and

gave examples about the way to incorporate the materials. The researcher involved in the classroom practice of the teachers as a nonparticipant observer which enabled the researcher to watch and record the classroom experience as an outsider at the back of the classroom (Creswell, 2012). She did not involve in the discussions or instructions.

3.4 Data Collection Tools

3.4.1 Interviews

Interview is a way of "…getting insight into the experiences, concerns, interests, beliefs values, knowledge and ways of seeing, thinking and acting of the other." (Schostak, 2006, p. 10). Interviews are important tools to gather in-depth information from participants in phenomenological studies (Creswell, 2007; Seidman, 2006). Open-ended interview questions lead participants to share in-depth information with researchers (Creswell, 2012; Seidman, 2006). The design of the interviews of the present study was identified according to this criterion. Face to face and semi-structured interviews were suitable for the aim of the study. Although it was rather time-consuming and costly approach, face to face interviews were comfortable both for participants and the interviewer. It allowed each participant respond to interview questions in detail. The aim of the selection of the interviews as one of the data collection tool was in order to present descriptions of the teachers' experiences of incorporating the history of mathematics into teaching and each interview had a meaning in itself (See Appendix B). Semi-structured interview questions were paid attention to be broad and general in order participants to construct the meaning of the situation (Creswell, 2009).

First interview was to learn about the background of the participants before incorporating the history of mathematics into teaching. Phenomenological studies aim to understand the meaning structure of the experience. This gives idea about the person who has had the same experience. Since it investigates socially constructed issues, there may be some changes about the structure of the meaning among different cultures. This situation was considered as the problem of all scientific research (Englander, 2012). Therefore, to provide background information of participants is common among phenomenological studies and considered as one of the ways to

reflect these social constructs (e.g. Kay, 2007; Miller, 2011). Since the present study aimed to reflect teachers' perspectives about incorporating the history of mathematics, background information of the participants had to be provided in order to understand the constructs that may have had effect under the topics related to mathematics, mathematics teaching, their classroom environment and past experiences with regard to the approach. First interview consists of two parts. In the first part (Interview questions 1-11), there are general questions related to their personal views and experiences. Four of the interview questions in this part were inspired by the studies demonstrating relationships between the approach and variables (Bolinger-Horton, 2011; Jardine, 2007). Interview questions four and five are asking personal ideas about mathematics and asking personal ideas about school mathematics. Bolinger-Horton (2011) demonstrated that teachers' views about mathematics and school mathematics influenced the choice of incorporating the approach. On the other hand, Jardine (2007) emphasized the importance of active learning process on teaching with this approach, therefore, interview questions seven and eight were about how the teachers instruct mathematics. These were the first part of the questions. Second part was about teachers past experiences related to the history of mathematics. Interview questions were ordered from general to specific subjects so that participants would tell about them as much as possible. Especially, they reflected on their initial meaning of the history of mathematics in classroom.

In second interview, participants' were investigated after studied on the historical materials. It was aimed teachers to reflect on historical materials and about incorporating the history of mathematics. The teachers evaluated the historical materials and when it was necessary it was offered to prepare extra historical materials. Second interview was basically to transfer the challenges of the teachers had while understanding the historical materials. It was a necessity to transfer the challenges of the teachers because their description of the experience affected from those challenges. Furthermore, second interview was designed to inform readers about the process the teachers had.

Lastly, participants described their experiences in the third interview. After teaching two lesson hours, they were asked to share their experiences through third interview in detail. First and main question was about how they describe their experience of

incorporating the history of mathematics into teaching. It was offered by the researchers to make participants transfer their experiences (Moustakas, 1994; Englander, 2012).

All in all, each interview lasted between 30 and 60 minutes. Participants were informed about required time that the present study would last. There was at least two day break between two interviews and days were identified according to the availability of the participant.

3.4.2 Observation

Observation is another tool to enrich the information gathered from the participants by means of interviews. It provides "…first-hand information by observing people and places at a research site." (Creswell, 2012, p.213). Therefore, the teachers were observed to give verbatim examples from the classroom they referred while transferring their experiences.

Teachers introduced the researcher as an "outsider" (Creswell, 2012). The researcher as an observer did not interfere with the flow of the instruction. The role of the researcher as a nonparticipant observer limited the study because she did not participate actively in the classroom activities which may cause some problems related to gaining rapport (Creswell, 2009). It was not a threatening factor for the quality of observation since the main focus was to observe classroom practices of teachers. Each teacher was observed for two lesson hours. They were required to incorporate at least one of the historical materials within their teaching. Primary teachers offered to divide two-hour classroom observation into two parts due to their routine practices. They avoided teaching two successive lesson hours mathematics teaching because concentration of students decreased and four independent hours of mathematics in a week would make mathematical knowledge more enduring. Teachers used parts of the historical sources given by the researcher according to their preferences. Classroom practices of teachers were recorded by videotapes while the researcher was taking notes during two lesson hours.

3.5 Field and Ethical Issues

Bogdan and Biklen (2007) commented on field and ethical issues as how to conduct the research from entering to leaving the field in order to be careful about what to do next in the schools and to be prepared for unexpected events to happen. Therefore some precautions were followed. Field issues were handled according to the way Creswell (2012) listed. Necessary permissions from institutions were provided. School administrators were the first to be informed about the research process. Afterwards, with the cooperation of administrative staff, teachers were contacted for whether they were volunteers to take part in the research. All participants were asked to sign voluntary participation form.

On the other hand, ethical issues are important points including consent, harm, privacy and deception that a researcher must consider (Fraenkel & Wallen, 2006; Moustakas, 1994). Before starting to collect data, participants were informed about that it was not an obligatory to participate in the research, which was also written in the voluntary participation form. Moreover, they had right for the withdrawal anytime. There was no physical and psychological harm predicted for the participants during the research (Fraenkel & Wallen, 2006). Confidentiality or privacy was another concern of the researcher. Each participant was given names as T3, T4, T5 and T6 according to their teaching grades; therefore interview protocols did not involve any name and surname, except for pseudonyms. The access for the raw data was not possible for third party and only findings were presented provided that the participants approved it. Last ethical issue was deception. Primary and middle school mathematics teachers were given detail of the study after the taken permission from the council of ethics. In order to further inform teachers, the e-mail addresses were asked to return the findings of the study.

3.6 Pilot Study

The aim of the pilot study was to evaluate appropriateness of the instruments and design of the study for the research; and was to have an insight about the research findings. For this purpose, one pilot study was conducted before beginning interviews and observations with teachers. One volunteer middle school mathematics teacher,

Gökçe, participated in three interviews and two lesson hours' observation. Gökçe was informed about participating in the pilot study so that it may have affected her willingness (Yin, 2011). She had graduated from elementary mathematics education program and she was three year experienced mathematics teacher. Gökçe selected her fifth grade students to apply the historical materials. For the three interviews, there needed to be done some clarifications for the questions. Therefore, each question was revised at the end of the interviews and they were rephrased by the researcher. Moreover, first question in the third interview was extended with subheadings to collect in-depth information about the experience because the pilot study showed that first question was not comprehensible for the volunteer teacher to reflect on incorporating the history of mathematics. Additionally, Gökçe said that she had no problems to understand the historical materials and to incorporate it into teaching. It was important because similar study could not be found in the accessible literature. Gökçe said that her next topic was fractions. Therefore, the researcher prepared historical materials related to fractions: historical development of fraction concept, brief biographical information about Fibonacci and fractions in Liber Abaci. The time limit for each interview was decided according to the pilot study. A brief report of the pilot study was also shared with both the administrator of the school and the teacher.

3.7 Data Collection Procedures

In order to collect data in its natural settings, from schools, there was a need to gain access to participants. There are two review boards for the applicability of the research. Ministry of National Education of the Turkish Republic (MoNE) and Applied Ethics Research Centre at Middle East Technical University approved for the convenience of the study. Before starting interviews, administrative staff and teachers were asked for their permissions respectively. It was the first time the researcher met the participants. The participants of this study were teachers who were working one of the public primary or middle schools in Çankaya as mathematics or primary teachers. The research was designed to involve third and fourth grade primary teachers and fifth, sixth, seventh and eighth grade middle school mathematics teachers. Firstly, the researcher met eight teachers and invited to participate in the study. Six of the teachers from five different public schools were volunteers: third, fourth, fifth, sixth, seventh and eighth grade teachers. They were informed about the aim of the research and data

collection procedure. However, two of the teachers from seventh and eighth grade withdrew from the research after their second interview was finished due to the concerns of keeping up with the curriculum before the academic year ended. In conclusion, there were four teachers from public schools participated in the study. Two of them were third and fourth grade primary teachers and other two teachers were middle school mathematics teachers. Willingness of the participants was important, therefore, they were asked to fill in voluntary participation form (See Appendix A) to declare their participations. Teachers' contact information was collected after the approval of teachers and first appointments for the interviews were arranged.

The researcher interviewed each participant three times in their schools. Before interviewing, the researcher scheduled the weeks according to the teachers' available days. The teachers were asked for permission for the second time to record the interviews with the audiotape at the beginning of the each interview. They had already been informed about interview recording by means of volunteer participation form. Additionally, the researcher took some brief notes on the interview protocols in case of tape-recorder malfunctions and in order to better follow what the participant had said during the interview At the end of the first interview, each teacher decided for the topic she would teach according to the flow of the curriculum so that the researcher would prepare history of mathematics materials. In the second interview, teachers were informed about the materials and were given suggestions about how to use them in mathematics classroom. They are the teachers who decided to select among the historical materials to incorporate into their instruction. Each teacher had been observed for two mathematics lesson hours. During the observation the researcher took notes about the classroom environment to enrich the data. The researcher involved in the classroom as a nonparticipant observer. The researcher followed the schedule given in the Table 3.2:

Table 3.2. Chronology of the Study Events for Each Teacher

Pseudonym		Ayşe	Bilge	Canan	Derya
1st Interview		26 April	25 April	7 May	3 May
2nd Interview		8 May	30 April	13 May	10 May
Observation	1st Hour	10 May	3 May	17 May	13 May
	2nd Hour	15 May	9 May		
3rd Interview		15 May	9 May	17 May	16 May

3.8 Analysis of Data

Accessible literature on teachers' incorporating the history of mathematics into teaching approach was not sufficient to frame this research theoretically. It is possible to find phenomenological studies which had no theoretical orientation as Creswell (2009) stated. Therefore, the researcher aimed to provide rich description of the approach by means of providing a synthesis of the experience from the perspectives of the participants.

Collected data analysed according to the phenomenological view defined by Moustakas (1994) and van Manen (1990). It requires to satisfy fundamental phases for data analysis procedure: *Epoche*, *Phenomenological Reduction*, *Imaginative Variation* and *Synthesis of Meanings and Essences* (Moustakas, 1994). Epoche is a fresh start for phenomenological study in which the researcher is aware of the personal assumptions and biases toward the phenomenon in order not to affect the subjectivity of the participant (Moustakas, 1994). Thus, the researcher bracketed her assumptions and ideas toward incorporating the history of mathematics into teaching approach. For the analysis of the transcriptions, horizons or meanings of the experiences are defined. Repetitive or overlapping segments are set aside. This process is defined as Phenomenological Reduction (Moustakas, 1994). After meanings or horizons are recorded they constitute the description of the experience. At this point, Imaginative Variation enables the researcher to grasp the structural essences of the experience

(Moustakas, 1994). Structures of experience can be regarded as phenomenological themes (van Manen 1990). Essence means "a concept which is common or universal, the condition or quality without which a thing would not be what it is." (as cited in Moustakas, 1994, p. 100). Synthesis of Meanings and Essences is the last phase to construct a universal structure of the phenomenon. Based on these steps Moustakas (1994) modified a method for analysis called The Stevick-Colaizzi-Keen Method for phenomenological data. The order of analysis was as given below:

1. Obtain full description of the phenomenon, transcription of the interviews and observations.
2. a. Give each statement relevant to the experience equal value, horizonalization.

   b. Record all statements

   c. Set aside repetitive, overlapping statements. The rest constitutes the meaning units or invariant horizons of the experience.

   d. Cluster relating units or horizons into themes

   e. Describe the experiences as textures by using themes and give quotes from the participants.

   f. Reflect on the structure of the textural descriptions, structural analysis.

   g. Construct a *textural-structural description* of the essences.
3. Repeat the process from 2.a to 2.g for each participant.
4. Construct a *composite textural-structural description* of the essences which means synthesizing experiences of all participants to describe the phenomenon universally (Moustakas, 1994).

Data collected from interviews and observations were transcribed respectively. While transcribing, interview questions were highlighted; some expressions or behaviours of participants were shown in brackets (Creswell, 2012). Transcriptions were checked for the accuracy from the audio and video recordings repeatedly and were read and reread until there was no misunderstood part.

First and second interviews analysed different from the third interview because they were related to the background information of the teachers and challenges of the historical materials according to the teachers for which content analysis was used.

Transcriptions of the observations were not completely reflected in the findings sessions. The parts from the classroom practice were provided since the teachers referred at those parts in the third interview. They are the lived experiences of teachers and included verbatim examples from the classrooms to describe their practices in the classroom environment.

3.9 Trustworthiness of the Study

Nature of qualitative and quantitative studies is different from each other. Therefore, trustworthiness of qualitative studies cannot be explained thoroughly by quantitative concepts, validity and reliability. Shenton (2004) summarized the strategies for the trustworthiness under four headings: credibility, transferability, dependability and confirmability. Credibility corresponds to internal validity for quantitative research. It is to show how credible the inferences were for qualitative research (Creswell & Miller, 2011), in other words, the trustworthiness of the inferences (Lincoln & Guba, 1985). Some strategies were followed to ensure the credibility of the inferences that the researcher made (Creswell, 2009; Creswell & Miller; 2011; Yıldırım & Şimşek, 2011). Creswell (2009) recommended discussing multiple strategies. Firstly, A methodological triangulation was carried out between interviews and classroom observations, main data collection tools of the study. They allowed researcher to cross-check and to form themes among data sources so that it enhanced credibility of the study (Creswell, 2009; Shenton, 2004). Second strategy was for participants to have a right to withdraw from the study. For example, seventh and eighth grade mathematics teachers quitted participating due to the concerns about rushing for the mathematics program. After each interview and observation was transcribed participants were asked to collaborate to check the accuracy of their described experiences and themes. The researcher contacted with participants through their e-mail addresses. Not only member checking but also two graduate students who did not involve in the study reviewed accuracy of methodology. Another strategy is to describe the researcher bias (Creswell, 2009). The researcher reflected her interest toward the subject of the study under the heading, “The Role of the Researcher”. Last strategy is about inadequacy of the time spent for the research question as Creswell stated “The more experience that a researcher has with participants in their actual setting, the more accurate or valid will be the findings” (2009, p. 192). The researcher

spent at least 6 hours within at least two weeks at each setting which was considered by the researcher sufficient time to gather data for this study.

For the confirmability of the research, objectivity of the research findings should be provided (Shenton, 2004). The researcher did not have the experience of teaching mathematics with its history. However, the literature she dealt with may cause some biased effect on her while conducting the research. Bogdan and Biklen (2007) advised that using this subjectivity as a reflective and strengthening tool would enrich the research. Thus, she aimed to reflect all ideas of teachers toward including history of mathematics in teaching. Triangulation was also an important factor to reduce the investigator biases. (Shenton, 2004).

Dependability corresponding to reliability of the research in quantitative studies demonstrated that if the researcher provides detailed information for the study, similar findings may be obtained when similar conditions are prepared. To achieve this, Shenton (2004) offered three parts to be included in the study: describing of the methodology and implementation process in detail, addressing what was done in the field and evaluating the effectiveness of the process of inquiry. These three parts were distributed to the present research. This is due to the assumption that the present research was constructed on the socially changing conditions (Marshall & Rossman, 2006). Therefore, the researcher tried to present these changing conditions by giving detailed information about the present study.

The last consideration was about the generalizability of the study. Due to the selection of the participants and qualitative nature of the research, it was not appropriate to generalize the results to the entire population (Englander, 2012). Instead of generalizability, there is another concept for qualitative research, transferability. It defines the responsibility of a qualitative researcher as discussing the possibility of transferring findings of the research to similar settings (Yıldırım & Şimşek, 2011). Therefore, the researcher described whole process in detail (Yıldırım & Şimşek, 2011) in order readers to transfer the descriptions to similar settings. Moreover, findings of the research were compared and contrasted with literature whether it was representative of previous findings (Bogdan & Biklen, 2007).

3.10 The Assumptions and Limitations of the Research

There were some assumptions based on the research design which allowed presenting the experiences of teachers. Their expressions about incorporating the history of mathematics into mathematics teaching approach were based on subjective meanings of individual experiences (Creswell, 2009). The nature of the phenomenology requires to rely on these meanings.

The historical materials provided at the beginning of the second interviews were prepared by the researcher and they were limited to the accessible literature. Researchers encourage incorporating the history of mathematics into each mathematical topic (Clark, 2011; Fraser & Koop, 1978). Thus, it was assumed in the present study that each particular topic in mathematics could be taught by incorporating the history of mathematics.

Participants were primary teachers and middle school mathematics teachers from third, fourth, fifth and sixth grades in Çankaya, Ankara. It was conducted in the spring term of 2012-2013 academic year. Two-hour classroom observation and three interviews with each participant were the sources of data. Therefore, the study was restricted with time, places and data collection tools.

CHAPTER 4

FINDINGS

In this chapter, the meaning of incorporating the history of mathematics into teaching from the perspective of primary and middle grade mathematics teachers is presented based on the themes that appeared after the analysis. Representation of the findings is demonstrated under two parts: individual background and a synthesis of the meanings of incorporating the history of mathematics into teaching.

4. 1 Individual Backgrounds

In this part of the chapter, there are two sections: background information about teachers and information about their teaching experiences. Teachers' background information is introduced first to understand their perspectives toward mathematics, mathematics teaching, the classroom environment and past experiences related to incorporating the history of mathematics into teaching based on first interview findings. Secondly, classroom practices are shared to inform readers about historical materials (See Appendix C) and their classroom practices through second and third interview findings. Observation findings of classroom practice are included to present a verbatim example from the classroom and to triangulate interview findings.

4. 1. 1 Third Grade Teacher (Ayşe)

Profile of Ayşe

Ayşe was a primary teacher in a public primary school. She had graduated from her two-year upper secondary education and had finished an undergraduate primary education programme in a university in Turkey. As a primary teacher, she was also responsible for teaching mathematics. Based on 21 years of teaching experience, she commented on the meaning of mathematics as:

> Mathematics is life itself for me. We use it in every field of life. For shopping, in our daily life, at home… It is something that we need to use; it develops our brain or keeps our brain alive… [thinks] We can call it an instrument.

After reflecting on her ideas about mathematics, Ayşe talked about school mathematics. She directly compared the new mathematics curriculum and old mathematics curriculum. She said that mathematics in the old education system was harder and more abstract and therefore, students did not like mathematics and were alienated from it, however, now mathematics was more related to everyday life since there were more concrete examples. She mentioned that they discussed topics in a simpler and more enjoyable manner so the students became more attracted. Ayşe concluded the comparison in favour of the new system but with a small criticism.

> At the beginning of the academic year we begin to follow the curriculum; there is no going back to the topic for the second time when the academic year ends. This is the only part I am criticizing. There is no repetition. One topic is delivered in one month; another topic is delivered in another month. Afterwards, they are all forgotten [by students]. The same topic is repeated one year later. As a result, one year just passes.

Ayşe compared the new and old education system and she mentioned how she generally taught mathematics. She articulated that she instructed topics directly, solved various problems; let the students solve the problems as much as possible and sometimes they studied collaboratively. Ayşe also added that she was trying to be a guide for students and cared about students actively participating in classroom activities. The teacher's book, student textbook and student workbook were regarded as the main resources. When she was asked about the constraints of mathematics teaching, she responded that the students' achievement level was low. Ayşe associated it with careless families. She added that mathematics was not a discipline a student achieved just by listening to the teacher. She said "Students should at least repeat the current topic at home and solve some problems; however there are families that cannot even read and write." Therefore, families could not help their children so she pointed out that she repeated the topic in the classroom and solved the rest of the problems from previous homework. Apart from the low achievement level of students, she talked about some constraints of teaching "abstract" mathematics such as "multiplication, division, operational problems". According to her, geometry was less abstract and more visual. She explained that students understood better when the

subject was more visual. Another constraint was the students' different levels of mathematical understanding, and also she added:

> …In particular, there are two newcomers. One of them was illiterate. How could it be possible to teach an illiterate student mathematics? Now, they have learnt to read and write, for mathematics he can do simpler things… but his progress is behind that of his peers… Therefore it affects us. Your workload increases. We are getting slower. We have to show more effort. It is completely related to the student profile.

After indicating the importance of student profile, Ayşe termed the constraints on her as health issues and the daily change in her "energy". She had some health problems and needed frequent check-ups. Therefore, she added that she could not show the same performance in each lesson.

Ayşe said that the constraints on her teaching could be partially solved by new approaches or methods. However, she reported that incorporating the history of mathematics into teaching was not an approach she used or knew about. She explained this approach as measurement units to build houses or to tailor clothes depending on their lifestyles. She described herself as being indifferent to the approach; however, she counted two possible benefits: "knowing about the past will make students learn mathematics better" and "by taking lessons from the past we can avoid making the same mistakes again". Setting aside the benefits, Ayşe thought that it was not an appropriate approach for third grade students. She stated that "The higher the students' grade, the more their comprehension rises."

About Ayşe's Classroom Practice

Ayşe was observed for two hours while she was teaching fractions. The objectives of the two-hour lesson were about identifying unit fractions and exemplifying proper functions. Based on these objectives, different historical materials were prepared for Ayşe: direct historical information from course books or textbooks (the Meaning of Fractions, Broken Numbers, The Development Process of Fractions, Examples from Civilizations) visual aids (Rhind Papyrus, Table of Greek Numbers and Table of Egyptian Numbers), old problems (A Problem from Papyrus) and a biography of a mathematician (Fibonacci). Ayşe briefly incorporated these historical materials except

the biography of the mathematician since she reported that the students were not familiar with the mathematician Fibonacci.

Ayşe started the lesson with the meaning of fractions and briefly continued with its development process. She showed the visual materials from time to time. One excerpt from the classroom she concluded as an important outcome is given below:

> Teacher Ayşe: See, historical studies have been conducted and it was seen that Egyptians wrote something about fractions on papyrus. [She showed the picture of an old problem from Egypt and asked students about the fractions on the papyrus one by one] Well, I would like you [to answer my question], they used unusual animals to express numbers, how would you express them if you were them?
>
> Mehmet: I would use shapes.
>
> Murat: Ma'am, I would use lines.
>
> Mine: I would use numbers.
>
> Münevver: With Roman numerals.
>
> Teacher Ayşe: Well, guys! Which one would be easy to understand?
>
> Mürşide: Ma'am, what we use today, for me, is the best.
>
> Teacher Ayşe: Here it is guys! In fact, mathematics has become easier for people as it has developed. Perhaps it will become easier in the upcoming days.

Ayşe said that students understood what we have taught in the context of mathematics may be considered as better to learn mathematics. Dialogues similar to this were frequently observed in the classroom. Ayşe evaluated her classroom practice as "positive". She also mentioned her nervousness while teaching because it was her first teaching of fractions incorporated history of mathematics. Apart from anxiety, Ayşe said that she behaved similar to her everyday teaching.

> I tried to behave in an unaffected manner as far as possible, not to do anything different but naturally I was a little bit nervous…

Ayşe talked about her classroom environment as well. She reported that there was nothing to affect the students negatively and they had all behaved the same as how

they normally would. According to her, the students were most affected by the visual aids, especially the Rhind Papyrus.

4. 1. 2 Fourth Grade Teacher (Bilge)

Profile of Bilge

Bilge was a primary teacher in a public primary school. She had graduated from a French language teaching programme and became a primary teacher after completing a pedagogical formation programme. Based on 16 years of teaching experience, she commented on the meaning of mathematics for her.

> I think that mathematics is a numerical concept which we use in every field of our lives. For example, in shopping…

After reflecting on her ideas about mathematics, Bilge talked about school mathematics. She mentioned that school mathematics was abstract, therefore, students had some difficulties but by repeating the topics regularly they could handle these difficulties. Within this process, she said that games and drama were tools to teach mathematics and gave an example of a lesson instructed using drama: she asked a student to become a metre, afterwards she called on three students first to make them decimal multiples of the metre then she called on three students to make submultiples of the metre and continued discussing the topic based on this drama. Bilge explained that the reason for selecting drama was to enable the active participation of students. While teaching, she said that she followed the teacher's book. Bilge defined her role in the classroom as a leader and continued:

> …For example, I teach fractions. Children are not familiar with fractions; I need to instruct the topic first so I need to dramatize and then to instruct the topic, therefore I am in a position of the leader… I need to push the button.

Bilge had to instruct the lesson first and she associated this situation with the fact that most of the students had learnt the topic in a different way from their parents. Bilge was also having difficulties answering some of the students' questions including open–ended questions such as "why, how". Bilge indicated that most of the time she answered those kinds of question with "let's search and learn" instead of "I do not

know". Therefore, she stated that it was important to be prepared for the lesson and especially to be prepared for the questions students may ask.

> ...on a day when you come to class unprepared, the lesson does not end successfully. It is necessary to answer questions clearly because primary grade students want clear answers; they want concrete things, not imaginary things.

Bilge stated that being prepared for the lesson was important. However, she added that four hours of lessons a week were not sufficient for mathematics. Within this time period, Bilge emphasized that if one topic had not been understood, she tried different forms of teaching, for example making the best student teach the topic, using materials or preparing drama. Unlike drama and play, Bilge said that she was not actively incorporating the history of mathematics. She remembered one of her experiences of teaching triangles through the Egyptian Pyramids and said that she could not get her students' attention. She thought that teaching triangles by means of the Pyramids in Egypt was directly related to incorporating the history of mathematics into teaching. Therefore, she gave some useful advice for incorporating the history of mathematics:

> It should be appropriate to the level of the students. For example, it should be something that they see in parks. I haven't even seen the Pyramids in Egypt so what can I say? Pictures are not enough. In conclusion, students need to see, hold, touch and live it, just imagining is not enough.

Bilge was asked whether she discussed mathematicians in the lessons. She gave an example from a different discipline; Social Sciences. Bilge requested presentations about Pythagoras' life from the students and asked "If you [students] had been Pythagoras, what would you have done?" She stated that:

> I want students to learn and I also want to learn. Mathematics is an abstract concept, why did it show up? In our daily life how has it affected us? ...To know the origin of a topic means to learn better, of course.

If the history of mathematics provided such an environment, Bilge both wanted to learn about history of mathematics and also wanted her students to learn it. According to her, different forms of teaching would attract students and make learning more enduring.

About Bilge's Classroom Practice

Bilge was observed for two hours while she was teaching measuring circumferences. The objectives of the two lesson hours were about measuring the circumferences of geometric shapes and identifying the relationship between the circumference and the side length of squares and rectangles. Based on these objectives, different historical materials were provided for Bilge: direct historical information from course books or textbooks (the Meaning of Geometry, The Development of Measurement and Ancient Egypt), visual aids (An Example of Papyrus, A Video about the River Nile), old problems (A Problem from Papyrus), and the biography of a mathematician (The Contributions of Atatürk to Geometry). Bilge said that she was in favour of visual materials and expected more visual materials to be prepared by the researcher. Therefore, Bilge incorporated historical information at the beginning of each lesson hour.

Bilge started the lesson by mentioning the historical information. She had been teaching numbers previously. She passed from numbers to the measurement of geometric shapes. She began with the meaning of geometry and talked about an example from the Egyptians' measurement method. Afterwards, she continued with the meaning of the word "square". Before discussing the measurement, the importance of standardized measurement units was emphasized and old measurement terms were interactively discussed by Bilge and students in the classroom. Historical knowledge was incorporated at the beginning of the lesson. Then the students took some measurements of the objects in the classroom with their foot and hand spans and summarized the properties of the square concept, and at the end of the lesson, they solved the problems.

The second lesson hour observation occurred six days later. They recalled what they had done in the previous lesson. Bilge asked about the historical information she had mentioned. Most of the students correctly remembered the historical information, which surprised Teacher Bilge. This lesson was about measuring the circumference of the rectangles. Students freely measured the objects around them with rulers. After the properties of rectangles were discussed, the students solved two problems.

After the lesson, Bilge said that there was nothing unordinary from her everyday class to affect her classroom practice negatively. She stated that the historical materials were understandable and parallel with the current mathematics programme. Bilge also concluded her classroom experience as:

> …In the first lesson hour I discussed the subject, to be honest, superficially but it remained in the back of the students' minds. In the second thing [lesson] when I started to teach I asked questions like "what we did do in the previous lesson?". Since I got the answers from them it means that I was successful in the first lesson as well, I concluded.

Bilge did not find herself to be as successful in the first lesson hour as she was in the second hour. She superficially talked about historical knowledge to attract students' attention to the lesson. She also expressed her surprise when she heard the students' answers of the students to the questions concerning the history of mathematics.

4. 1. 3 Fifth Grade Teacher (Canan)

Profile of Canan

A teacher with fifteen years of experience, Canan graduated from the mathematics department and became a middle grade mathematics teacher after completing a pedagogical formation programme. The education system changed in 2012 and fifth grade students became the first year of the middle grade. She said that it was a difficult year but she had got used to it. Based on her experience, Canan explained the meaning of mathematics for her:

> Mathematics is everything within life for me and I also make my students approach mathematics like that. There is mathematics everywhere.

When the meaning of "everything" was asked, she gave the example of "fractals, card games and cooking". According to her, mathematics was everywhere in life, however, she added that fifth grade school mathematics was not suitable for students. She indicated that the first six years' topics of mathematics were to be regulated based on topics of everyday life such as shopping or interests. She said that too many topics

were loaded into the curriculum and they needed to repeat the same topic every year and students did not feel interested enough in the same topic because unknown topics were interesting for them. Therefore, she liked the eighth grade mathematics curriculum and she claimed that her students also liked it since there were many newly introduced topics such as factorizing or trigonometry.

> When I introduce a topic, students enjoy it very much because they have so many questions to ask. What is the meaning of learning? It means to wonder. You can learn what you worry about but if you know even a little bit, you can say "Aa, I know this", therefore you can miss out on learning new things, unfortunately… We need to make it interesting or we should not make it too routine… Since a child is bored with hearing the same thing, she/he has difficulty in the details you try to teach.

Canan pointed out that in order to learn something new, you needed to worry about it. She added that while repeating some topics, students felt bored. In order to decrease the boredom, Canan said that she generally used educational websites. She stated that these websites attracted students. She indicated that after the topic had been understood, students were given problems or quizzes. The routine of mathematics lessons was described by Canan. Additionally, before beginning the lesson, Canan reported that to make contact with the students was important because mathematics was regarded as frightening. However, she described her role as "I am teaching, the students are listening" even if it was not a desirable situation for her. She described her experience of teaching fifth grade students for the first time. Both Canan and the students had problems with adaptation. These adaptation problems also affected her teaching. Canan indicated that concentration on the lesson was the main problem. Fifth grade students were slow learners according to her and they easily lost their concentration.

While teaching mathematics, Canan said that she may use different forms of teaching. However, incorporating the history of mathematics was not one of the forms she used. Canan recalled an example related to the history of mathematics where she asked students to prepare one part of the journal of a famous mathematician. She concluded that students were more interested in the "magazine" part of their lives than what they had achieved. Based on this conclusion, she suggested that the lives of mathematicians could be animated so that it would attract fifth grade students.

Canan had not thought about incorporating the history of mathematics approach to overcome the constraints on teaching. She added that a little historical information was available in the teacher'sbooks, however there are no objectives of the topics related to the history of mathematics. She indicated that time was rather limited, therefore most of the time Canan preferred not to share this historical information. She also added that even if she did, it was not because incorporating the history of mathematics was helpful, it was because students should have general knowledge.

Canan said that she did not like history and was not interested in historical knowledge and she was even afraid of it. Since "history of mathematics" includes the word "history", she expressed that her feelings were as the same as what she felt for history.

About Canan's Classroom Practice

Canan was observed for two hours while she was teaching multiplication of the four operations. The objectives of the two-hour lesson were about doing multiplications that resulted in most seven-digit products. Based on these objectives, different historical materials were provided for Canan: direct historical information from course books or textbooks (The Development of Multiplication and Division, Where did the four different multiplication symbols come from?), visual aids (An Example of Old Mathematics Books), activities (Italian Multiplication Method, Gelosia Method and Rabdologia) and the biography of a mathematician (John Napier). Canan started the lesson by mentioning the historical information. Afterwards, she chose to incorporate activities from the Italian multiplication method, the Gelosia Method and Rabdologia to make the topic "multiplication" interesting. Before doing the activities, Canan gave information about the three multiplication methods to introduce them. She ordered the three methods from simple to complex. The Italian multiplication method was for multiplying two-digit factors only. The Gelosia method enabled the multiplication of more than two-digit factors. Finally, Rabdologia was a primitive calculator, a basic algorithm of the first computers which had the structure of the Gelosia method but which was more practical. Two lesson hours were allocated to discuss these activities. At the end of the second hour, there was a dialogue between the students and the teacher:

Ali: Teacher, why are we learning this?

Teacher Canan: Let's guess.

Ali: … [thinks]

Berke: It is because the topic is multiplication.

Can: Since it is a mathematics lesson [students laugh].

Didem: It is because we can encounter different ideas.

Esin: It is because we can also learn different multiplication methods.

Feride: It is because we can grasp knowledge.

Gökhan: Which one is the correct answer, teacher?

The students were waiting for the approval of the teacher for the one correct answer, she did not respond though. Canan reflected on this experience as "…fifth grade students grasp what you give and they perceive everything within the framework of the mathematics lesson, therefore some of the students said 'since it is a mathematics lesson'. When she was asked about the teaching experience, she reported that it was good except for the fact that it was Friday so there was a disorder caused by the classroom change.

> It was good but I did not expect that much participation, in fact I was quite prejudiced; I never thought that they would participate but as you observed, there was nearly ninety per cent participation. All in all, it was a nice lesson, to me I liked it.

Canan indicated that classroom participation was nearly ninety per cent and she had not expected that much participation. At the beginning of the first interview she said that she was afraid of history, therefore she concerned about knowledge related to the history of mathematics but after her classroom experience Canan confessed that she liked it.

4. 1. 4 Sixth Grade Teacher (Derya)

Profile of Derya

Derya was in her sixth year of teaching mathematics. She had graduated from an elementary mathematics education programme. It was her first year in Ankara. She considered herself to be pleased with her new school and it was more equipped than her old school. Derya agreed to participate in this study in order to learn new ideas. Based on her experience, she explained the meaning of mathematics as:

> When I define mathematics, I perceive it as a body of rules and I believe that it is a discipline, obviously it brings order. For me, it is fun but it depends.

Derya defined mathematics as a discipline of order and a hard discipline for students who did not have mathematical intelligence. She explained mathematical intelligence as being one of the intelligences mentioned in the multiple intelligence theory. She claimed that without having mathematical intelligence, students could only achieve basic mathematical knowledge and they could not go any further. Derya added that:

> …the education provided by your family when you were young is the fundamental. Even while eating cracker sticks, "How many crackers do you have after you have eaten one?" this is where it begins. The more the families care about their children, the more they can help to develop their mathematical intelligence in the family environment…

Derya suggested that one of the methods to improve this intelligence was to get support from the family. While teaching mathematics, she mentioned other constraints. She stated that she always had difficulty managing the sixth grade mathematics topics; it was not possible to slow down since there was a time problem. It was a constraint for sixth grade students. She also added that the content of the sixth grade curriculum was not applicable to daily life. This was what her students complained about.

Derya emphasized that the "question and answer" was her favourite teaching method. She said that she was sad about the times she had to teach the topics by direct instruction due to time constraints. However, she insisted on staying back during the lesson "I should not dictate, they should search for the answer, and learn". She emphasized that in her early years of teaching she did not frequently teach the topics interactively.

Derya explained that the main constraint on her teaching was health problems. The rest of the constraints resulted from students who were not mathematically intelligent or who did not like his/her previous mathematics teacher.

> The frequently asked question is "Where will we use this [a mathematical topic], ma'am?" Actually, there cannot always be directly applicable things. For example, we say that you will use these in architecture, "Ma'am, I won't become an architect"… This is what they are bothered about. If everything is transferred to daily life such as money or grocery calculations, it will become from daily life much more. To associate with everyday life is very important for them.

Teacher Derya stated that she regarded the students to be in need of the relationship between daily life and mathematics. She admitted that there were some topics which were not directly relevant to daily life. As a result, Derya expressed that students became bored when mathematics seemed to be irrelevant to daily life. To solve this kind of problem, she suggested using a projector and mathematical games. However, she did not mention incorporating the history of mathematics. Derya explained this approach:

> Here, when you said the history of mathematics, I understood that we shall tell them [historical information] and infuse it within the classroom...

Derya deduced that incorporating the history of mathematics approach was to infuse historical information. After this deduction, she gave an example of tangrams and the story of the number symbols. She said that students liked them and it was relaxing to go away from the topics. Additionally, she reported that it was not enough to attract the students since they felt unsuccessful when they returned to the topic again.

Derya did not have a teacher who incorporated history of mathematics into teaching but she admitted that she would have liked to have a primary or mathematics teacher who incorporated history of mathematics. Derya stated that she did not ever feel the need to incorporate history of mathematics and she was weak in terms of historical knowledge. She pointed out that she sometimes mentioned mathematicians in the classroom:

> Mathematicians are one of the things we generally present to make students happy. In particular, we give examples of Turkish mathematicians, sometimes of foreigners. They search them.

Derya incorporated historical information via biographies of mathematicians. She added that it was in order to make her students happy. She preferred Turkish mathematicians but there were times she mentioned foreign mathematicians.

About Derya's Classroom Practice

Derya was observed for two hours while she was solving problems about guessing and measuring the circumference of geometric shapes. The objectives of the two-hour lesson were about guessing and measuring the circumferences of geometric shapes. Based on these objectives, different historical materials were provided for Derya: direct historical information from course books or textbooks (The Meaning of Geometry, The Development Process of Standardized Measurement Units, Ancient Egypt), visual aids (An Example of Papyrus), anecdotes (King Henry I.), old problems (A Problem from China) and biographies of mathematicians (The Contributions of Atatürk to Geometry, Euclid). Derya blended historical materials within two lesson hours except for the old problems. Since Derya was in the middle of the topic, these two hours were devoted to solving problems related to the topic. Derya started by connecting the previous lesson and current lesson with a 5 minute discussion of the meaning of geometry, as given below.

> Teacher Derya: Okay, guys. We have been trying to measure the circumferences of the geometrical shapes since previous lessons but have we ever talked about the meaning of geometry? For example, square, rectangle, pentagon or decagon. What do we call them?
>
> Students: Polygons [Altogether]
>
> Teacher Derya: Polygons. Okay, what do we call them in general?
>
> Students: Geometric shapes [Altogether].
>
> Teacher Derya: Even though we have talked about them a lot, we have never discussed the meaning of geometry. Did you ever discuss it in primary school?
>
> Students: No! [Altogether]

Teacher Derya: Do you have any idea about that? Try to find it by yourself. What is coming to your minds?

Students:…[No answer]

Teacher Derya: Geo- metry.

Students:… [No answer]

Hatice: Teacher, while solving geometry problems it is written as geometric shapes in general. Geometry may mean the common name of the polygons.

Teacher Derya: You are right but what may be the meaning of the word, in your opinion?

Students: Shapes [Altogether].

Derya: Is there anyone who has got a different idea?

Students: … [No answer]

[Teacher Derya writes a hint on the white board geo-metry]

Görkem: Metry looks like metre.

Furkan: Is it the first name of the founder? [Everybody laughs]

Teacher Derya: Your friend does not talk nonsense since in general the first name or the last name of the mathematician was used as the name of the theorem in general. What will you say Nilay?

Nilay: Metre of Edge

Teacher Derya: I think everybody found the similarity between metry and metre. What does metry mean here?

Sanem: It means the length.

Teacher Derya: Any other guesses?

Fırat: If geo means shapes, and metry means the circumference.

Teacher Derya: All right, let me give you another hint and say a documentary channel?

Anıl: National Geographic.

Teacher Derya: What does geographic mean to you? Do you know the Turkish translation?

Anıl: Ulusal coğrafya.

Teacher Derya: What can it be then?

Murat: Measuring the geography

Derya: Yes, maybe this is the time I need to say bingo. Geo means Earth, earth, geography and metry means measurement. As a result, it means measuring the earth, measuring the geography. Do we not measure the sides, circumference or area on geometric shapes, already?

This example from classroom practice was a discussion about the meaning of the word "geometry". There were two more discussions about the standardization of measurement units. Derya incorporated those discussions into problem solving sessions in her lesson plan. According to her, the classroom practice was good; however, she said that she did not solve enough problems to feel satisfied. Because of this feeling, she asked students about their feedbacks for the lesson. She said that students were affected so much by the camera that they could not see any difference from other lessons therefore they did not mention any distinct characteristic of the lesson. Additionally, she confessed that "students were not as amused by the historical knowledge as I was".

4.2 A Synthesis: Incorporating the history of mathematics into teaching

A synthesis of the meanings based on phenomenological research is presented by providing textural and structural composite descriptions of primary and middle grade mathematics teachers. The experience of incorporating the history of mathematics into teaching was described by the teachers under eight core themes which constituted the essence of the experience.

4.2.1 Professional improvement

The teachers mentioned some benefits of history of mathematics for themselves and they stated that they had not been aware of the benefits before. The participants evaluated two benefits considering content knowledge and insights into mathematics. Historical knowledge of mathematics was evaluated by the teachers under mathematical content knowledge. The teachers articulated that while studying the

historical materials, the process provided general knowledge about mathematics which was regarded by the teachers as knowledge that a teacher or student should know. Additionally, one of the teachers regarded it as the "magazine part of mathematics" since the history of mathematics included the life of mathematicians and anecdotes.

Secondly, the teachers agreed that the history of mathematics prepared an environment to make students think about mathematical issues. They gave examples of mathematical issues saying that "mathematics is a product of human efforts", "it is not something that emerged on its own", "it exists throughout human life" and "we need mathematics all the time". They said that while studying the history of mathematics, they pushed themselves to think about the existence of mathematics. Ayşe commented that:

> I realized that there existed great effort and surmised that it would be very beneficial for us in the future or for the next generations. I gained insight that next generations would benefit from us, add something onto it.

The teachers concluded that "now" was not an end for mathematics, mathematics still continues to develop. They said that it was possible to find simpler or easier ways to understand mathematics better. They deduced that methods, concepts, formulas or definitions may change as long as people study mathematics, just like anything in life. Accordingly, one of the teachers gave an example:

> You use the best car model but as I said three years later your car will be out of fashion. However, it is in our power to change it.

After the participants described the process of thinking about mathematical issues, they concluded that they had gained insight into the development of mathematics.

4.2.2 Attractiveness

The teachers explained that each mathematics lesson may not be equally attractive for both students and teachers. They attributed it to three reasons: uninterested student profiles, uninteresting subjects and uninterested teachers. Ayşe summarized it:

> We cannot always draw the same level of interest in every mathematics lesson. That is, sometimes their interest might be at a low level or the subject may be boring or I might make it boring. I don't know, maybe I can make it enjoyable but sometimes it is related to the subject. If it draws attention, the subject becomes more productive.

Drawing attention was an important issue for teachers, for this reason, the teachers frequently emphasized it during their interviews. According to them, the indifference of the students resulted from the "information age". They explained that it was easy for them to reach information willingly or unwillingly. The teachers added that students became uninterested in subjects since they had already heard or knew the knowledge presented in the classroom. Moreover, they were taught the same subject with different objectives nearly every year. At that point, teachers complained about not drawing enough attention to make them listen to the lesson. Despite all these complaints, the teachers evaluated incorporating the history of mathematics into teaching as interesting and that it drew the attention of students. The teachers also expressed that their students were interested in the historical materials.

> …When talking with my students, I would understand from their facial expressions if they were bored, however, this lesson was never found to be boring.

> I would rather try to solve the problems myself directly when I see them. It also drew my attention. Solving those questions immediately, I feel like "let's do it, what might be the answer?"

Moreover, Ayşe said that "although the age group of the children is quite young, talking about papyrus or the Egyptians made them interested in the topic". Such examples relevant to the attractiveness of the historical materials were given by teachers. In addition, enjoyment was another factor that makes the history of mathematics more attractive. Teachers agreed that they had enjoyed it as much as students had. Canan commented that:

> I don't think that they will use it in daily life, however, the method [the Gelosia method for multiplication] was enjoyable.

Although Canan gave importance to the mathematics of everyday life, she found the historical materials enjoyable. To summarize, attractiveness was described by the teachers as an aspect of incorporating the history of mathematics into teaching and

there were times in the classroom that both the students and the teachers enjoyed themselves while discussing historical materials.

4.2.3 The environment for speaking up

The teachers mentioned another aspect of incorporating the history of mathematics: the environment for speaking up. To begin with, they pointed out that the history of mathematics incorporated parts within the mathematics lesson that allowed students to speaking up. Within this frame, two aspects of the speaking up environment were mentioned by the teachers: students discussed their ideas about mathematics and actively participated in the class. Firstly, the teachers commented about that different students found the chance to express themselves in a discussion environment. Derya expressed her experience as:

> In the lesson, again students participated; however, it [the lesson] was different than usual since different students participated. I mean, a discussion environment does not happen all the time; it was the first time that a further discussion environment was created.

The teachers stated that this discussion environment was triggered by the challenging elements provided by the history of mathematics. According to them, students were thinking deeply about mathematics. Derya exemplified it with a classroom anecdote:

> Today, for example, I asked the students about the submerging field problem. A student gave an answer that in my opinion there was a better solution than tightening the rope [the Egyptian method to measure the circumference of fields]. He said "I would plant some strong trees in each corner of the field". It was very logical. They said "Ma'am, is not the rope also damaged?". The trees were more logical for them and for me as well.

Derya's anecdote was to give an example about challenging discussions between students and teachers. The teachers emphasized the importance of questioning mathematics using such examples from the classroom. They said that a challenging discussion environment was a way of questioning mathematics. It was mentioned by the teachers that questioning was necessary for the classroom environment. Canan added that:

> Students may not question, it was again up to me to provide them the stimulation for questioning but first I should wonder so that I can make my students wonder. It was my insufficiency.

Canan added the teacher's role is to make students question mathematics. The teachers commented that students actively participated in the classroom activities while all challenging discussions went on. It was the teachers' complaint that the direct instruction of mathematics did not provide an environment to make students participate, therefore it was important for students to speaking up, according to them. The teachers explained that while incorporating the history of mathematics, the amount of participation in the classroom environment was surprising and even surpassed teachers' expectations. It was explained as:

> …for example, normally, if 7-8 students participate in solving the problems because the problem is hard, of course the number of raised hands to open-ended questions will be greater. Therefore, we enable more students to participate.

The active participation of the students and a challenging discussion environment made students and teachers question mathematics. The teachers indicated that students expressed their ideas freely in a speaking up environment, and as a result they became motivated to participate actively in the classroom.

4.2.4 The more practice there is, the better it becomes

The teachers claimed that incorporating the history of mathematics was an approach which developed as long as you practised. They described themselves as nervous and unsuccessful in the first hour of the observation. In the second hour, when they became more experienced, they felt that the students were also more interested. The teachers discussed the idea that students and teachers needed practice to "adapt" to incorporating the history of mathematics and to perform better. Bilge commented that:

> In the first lesson I was more stressed "What can I do? How can I do it?" but in the second lesson I was much more flexible.

The teachers were more flexible in the second lesson than the first lesson. Three of the teachers incorporated the history of mathematics more than once to test the claim "the more practice there is, the greater the success" before classroom observation with

other classes. Canan and Derya found a chance to observe different classes while Ayşe experienced different and interesting answers for the same questions from her students. Since the teachers were satisfied with the results, they concluded that they would continue to incorporate the historical material given.

> We didn't use this method within the lesson but I am sure that I will again talk about them [the historical information provided in the historical material] in order to draw students' attention. I am sure that I will mention them.

> It is a method [Gelosia method] that I will add to my next fifth grade programme. Although it will not cover two hours, in one lesson hour I can make students practice.

> As long as we mentioned about these topics [history of mathematics materials about fractions], maybe as long as we incorporate the history of mathematics not only with fractions, but with every topic, it will be more beneficial I think and I want to use it in my lessons.

The teachers said that they would continue to incorporate the historical materials given. One of the teachers even added that she would incorporate it into every topic of mathematics. In conclusion, the teachers were in agreement with regards to using the given historical materials during lessons and they incorporated them more than once. Surprisingly, Bilge offered this approach to pre-service primary teachers and shared the historical materials given to her.

4.2.5 The adaptability of historical materials

The teachers said that they had some concerns about how to present historical knowledge in the classroom. They concerned about the objectives of the topic in the mathematics programme and the age group in order to adapt the given historical materials to the classroom. The teachers were asked whether incorporating the history of mathematics left them behind in terms of the programme. They evaluated their classroom practice according to the objectives of the topic. All of the teachers agreed that historical information within teaching was not a challenge to teach the topic. One of the teachers stated that:

> I think the objectives were achieved. There are some inadequacies but expecting all of them to understand one hundred per cent, in other words, expecting them to accommodate the objectives was a flight of imagination.

Another concern was based on the appropriateness of historical materials for the students' age group. The teachers frequently mentioned the adaptability of the historical materials to the age group. Additionally, they stated age group as a condition for students to understand historical information. Two phases were described by the teachers to determine the appropriateness of historical materials according to the students' age group. The first phase was the "prior knowledge of the students". They said that it was important to decide which historical materials were going to be presented in the classroom. Prior knowledge was relevant to the knowledge of other disciplines or relevant to students' mathematical knowledge. Three examples from the teachers were:

> Students do not know Archimedes from Science lessons, I suppose, therefore I cannot mention Archimedes in the lesson…
>
> We had mentioned this ancient Greek alphabet already when we finished Roman numerals. I specifically mentioned about this [Greek numerals].
>
> I considered the age group. For example, types of multiplication symbol. We will talk about them when the time comes. I did not want to tire them. We just simply use x now. Therefore, I did not mention other types.

The teachers used the word "eliminate" when deciding appropriate historical materials according to the age group and objectives of the topic. According to them, they eliminated inappropriate materials by considering the age group and objectives of the topic. After elimination, the rest of the materials which were considered to be appropriate for the classroom were selected according to some properties. Teachers insisted on incorporating "simple" information. They described simplicity with different words such as not complex, not comprehensive, not very detailed information, concrete information, shallow information and not boring information. One of the teachers gave an example of simplicity:

> I tried not to say too many numbers, not to say exact historical dates. I avoided saying 1100s or 1500s…I rounded up the speed of light to make it remarkable.

Furthermore, the teachers selected "remarkable" information from the historical materials, not for "memorisable" ones. For them, memorisable information consisted

of different civilisations, the meaning of mathematical terms, different methods, answers to the questions "Who did it? What did they do? When did they do it?". One of the teachers also added that there should not be all the biographical information of the mathematicians. There should be less but important and relevant information. They confirmed this "remarkableness" by asking students questions about what they had taught in the previous lesson hour. It was stated that

> I am having a hard time with numerical information. Even I was surprised that one of my students said that it was related to the speed of light when I talked about metres which have been accepted for thirty years. I said that I may not keep it in mind so there was something that the students kept in their minds.

The adaptability of historical materials was reported in the interviews by the teachers. Generally, they found it possible to incorporate the history of mathematics into their teaching by considering the age group, goals and objectives.

4.2.6 Problem solving vs. incorporating the history of mathematics

The teachers considered that they necessarily spared time for problem solving. They were quite confused about this issue. Especially, mathematics teachers defined problem solving as a tool to understand mathematics, as Canan stated "I think students understand better when we solve problems". Interestingly, the teachers also stated that their main aim was to make students achieve the objectives, not solving the problems. However, they added that in their lesson plans there were problems "absolutely" to be solved.

> After discussing a subject, I needed to solve three problems related to the subject and accordingly, I prepared my plan.

Problem solving confusion led them to a dilemma in the lesson: problem solving or the history of mathematics. There were different explanations by the teachers: the students may force them to solve the problems or they felt themselves to be insufficient. Moreover, they perceived problem solving as "the more problems that are solved, the more successful the lesson becomes". A teacher described her experience of problem solving:

> For me, on the one hand, I was constantly worried that I needed to teach the subject and after that solve the problems as well. I cannot help it but I tend to solve problems because if I do not, I feel like students will not be able to solve problems in the examination. On the other hand, we referred to history: this aspect is what I enjoyed. I am just saying, does achieving the objective mean solving problems? No. However, if we do not, we feel one side of us is insufficient.

One of the teachers also implied that problem solving was not a skill that everyone had and without problem solving, a student could not improve herself. For her, it was easier to answer open-ended questions within the boundary of the history of mathematics. She concluded that questions related to the history of mathematics and problem solving were not replaceable. All in all, they all agreed that solving fewer problems could not be "repaired" by teaching the history of mathematics according to teachers.

4.2.7 Lack of knowledge and resources

When the teachers were given historical materials, they shared their first thought as "the need to do research". Their explanation was that they needed to question the accuracy of what was written in the historical materials, they felt inadequate to do that, and they asked "where can I get information". The second question of the teachers was "how could they decide which historical information and where to incorporate them?". Ayşe stated that:

> Indeed, I thought that I wished I could have more information. I searched on the Internet but I could not reach much more information.

This was a complaint of the lack of resources stated by Ayşe and it was reflected by each teacher. One of the teachers pointed out that confining themselves to the given historical materials naturally made her not question further what was written in the materials. They felt uncomfortable because of "not having a grasp of the subject". Two of the teachers said that they had to know students' many conceptions and misconceptions. Therefore, the teachers said that they were new to the approach and they could not "go any further than the historical materials". As a result, the historical materials did not satisfy them; on the contrary, their feeling of insufficiency about historical knowledge continued:

> Perhaps, if I had a grasp of the [historical] subjects, it would be more beneficial. I cannot say it was useless. It was beneficial but it was not a one-hundred per cent efficient lesson. If I had a grasp of the subject, maybe, time management would be much better. The hints I gave for the questions could be much better, maybe.

Lack of knowledge seemed to make them feel insufficient in terms of historical subjects. Canan even associated the reason of not incorporating them before with her lack of knowledge. She added that her lack of knowledge originated from both her secondary and undergraduate education. Indeed, the other teachers had not been educated with a historical approach in mathematics either. Therefore, they did not have a clue about the history of mathematics incorporation approach. Canan expressed her feelings as:

> If I knew the studies related to history of mathematics, when appropriate I would talk to students about those studies… Even if I did not show any method [like the Gelosia method], just giving a piece of information would be great, I think.

Canan thought that if she knew the historical information, she would share it with the students. The teacher continued to feel the lack of historical knowledge not only while studying the historical materials but also when classroom practice ended. One of the teachers asked the researcher for other historical materials, books or websites about the history of mathematics. One of the teachers gave an example for the lack of sources that the teacher's book provided biographical information of mathematicians, however, the content was not found to be "interesting to read". The teachers agreed that some visual materials should be produced for the sake of mathematics education such as presentations" or "cartoons" related to the history of mathematics. All in all, they needed to be trained about incorporating the history of mathematics in order to fill the gap concerning historical knowledge.

4.2.8 Time constraints

Four lesson hours per week are allocated for mathematics education at both the primary and middle grade level. All of the teachers did not find the time sufficient to complete the mathematics programme. They added that they could not even do the activities or complete the exercises in the students' textbook. Therefore, the teachers always felt like they were constrained by time.

> Four hours per week is very limited. For this reason, I do not think we teach mathematics adequately. It is not productive due to the lack of time.

Due to time constraints teachers may approach innovations within teaching cautiously, just as what happened for the incorporating the history of mathematics approach. Bilge commented that:

> …let me say I was opposed to the history of mathematics before but if we manage the time well, we can incorporate it within the lessons, I thought.

Time management was important for them in order to handle historical materials, problem solving and discussing the subject. They emphasized that when they practiced the approach, they began to manage time well.

4.3 Summary

The analysis of data was presented to the readers in order to understand the meaning of incorporating the history of mathematics from the perspective of primary and middle grade mathematics teachers. Teacher profiles and classroom practices with brief examples were provided before introducing the synthesis of the lived experiences of the teachers.

Teachers had at least six years of experience in their teaching profession. They did not consciously incorporate the history of mathematics into their teaching. They had been having some problems with mathematics due to teaching, students and students' parents. They all believed there was a difference in intelligence between students who could do mathematics and those who could not do mathematics.

Before classroom practice, at least seven historical materials were prepared for the teachers to discuss in the lesson and the teachers incorporated at least five materials into the classroom. Teachers regarded the materials as suitable for the classroom environment. In general, they evaluated their practices as good.

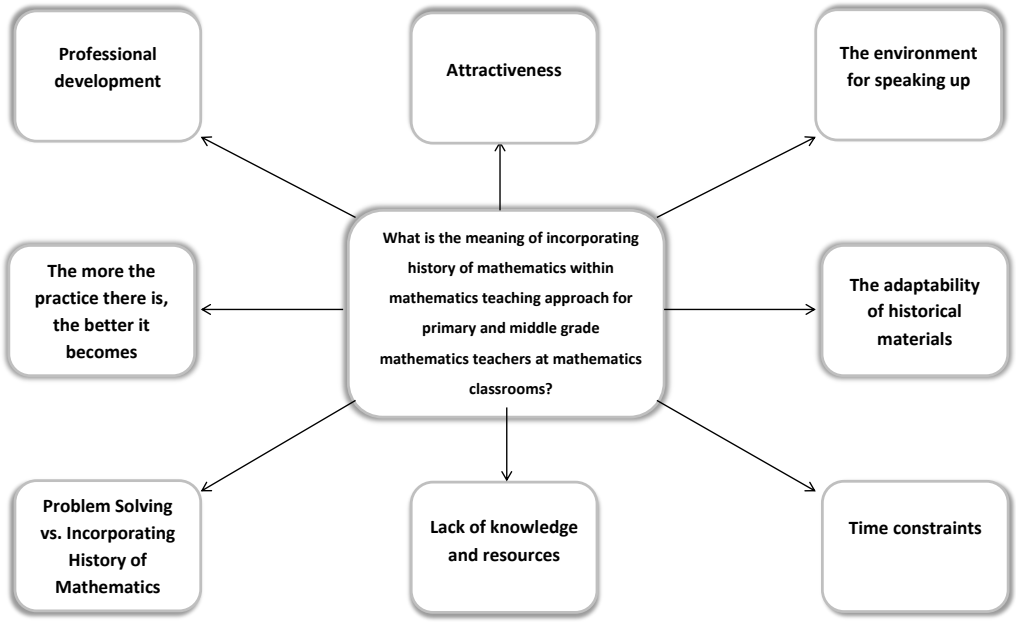

Figure 5.1. Themes revealed by the lived experiences of the teachers

Lastly, the lived experience of the teachers were analysed and presented in the form of synthesis. Eight core themes represented by Figure 5.1 helped to describe incorporating the history of mathematics through teachers' lived experiences. Those themes were professional development, attractiveness, the environment for speaking up, the more practice there is, the better it becomes, the adaptability of historical materials, problem solving vs. history of mathematics, lack of knowledge and documents and time constraints.

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CHAPTER 5

DISCUSSIONS, IMPLICATIONS, RECOMMENDATIONS

In this final chapter, findings of the study are presented as interpretations related to the previous literature in the discussion part. The researcher was impressed by Lester (2009) about saying implications instead of conclusions since it recalls "finality and surety" (Lester, 2009, p. 3). Implications part includes arguments drawn from the discussion part and recommendations present the ideas developed for the further investigations.

5.1 Discussion

The purpose of this study was to investigate how primary teachers and middle grade mathematics teachers perceive and describe incorporating the history of mathematics into teaching. The study was conducted with the participation of 2 primary teachers and 2 middle grade mathematics teachers from Ankara. Each teacher participated in three interviews and they were observed two lesson hours for their classroom practice. The data gathered from interviews and observations were analysed based on phenomenology. Teacher profiles and synthesis of the experience were provided.

The teachers described incorporating the history of mathematics into teaching as a means for professional improvement, attractiveness, the environment for speaking up. On the other side, they added three constraints: lack of knowledge and resources, time constraints and the dilemma of problems solving versus incorporating the history of mathematics. Lastly, the adaptability of the historical materials and more practicing were perceived as ways to improve classroom practice of incorporating the history of mathematics. According to these descriptions and perceptions, there were found some similarities and differences were found within the literature.

At the beginning of the study, the teachers said that they did not incorporate history of mathematics within their teaching. However, each teacher gave example from their

classroom practices such as presentation of mathematicians or writing a journal with students about a biography of the mathematician, which was considered as *to incorporate the history of mathematics*. That is, they were applying the approach however they were not aware of it. This is because they had not attended in a training or mathematics course which includes history of mathematics in itself. Incorporating history of mathematics was not an approach that the teachers were unfamiliar with though. As a result, all teachers may be informed about the approach to be conscious about the historical information such as biography of a mathematician or different problem solving methods in the textbooks.

The challenges about mathematics teaching and learning were not directly related to the problems that history of mathematics may solve according to the teachers. They were challenges caused by parents, mathematical intelligence of the students and health issues of the teachers. It may be resulted from that the teachers regarded themselves adequate for mathematics teaching and said that they memorized the teacher's book and even had "two books in case of forgetting". Teachers' books were defined as the main source of the mathematics teaching by the teachers. Within teacher's book, it was reported that there was small information about biography of mathematicians which found ineffective by the teachers. Teacher's book can be considered as an important source for teachers. Therefore, historical information may be permeated entire book. Lack of historical knowledge was not a characteristic of teacher's book. Neither did teacher training programs include history of mathematics nor did the participants have teachers who incorporated the history of mathematics into teaching. The point is that, the participants did not have background information enough to incorporate history of mathematics, they described themselves as inadequate. Confining themselves to given historical materials compelled them to question the accuracy of the materials. Therefore, the teachers said that in need of finding new resources they researched and realized the lack of resources about history of mathematics. Similar to the present research, lack of knowledge and resources were reported by the previous studies as one of the main challenges for teachers to incorporate history of mathematics (Bolinger-Horton, 2011; Haile, 2008; Siu, 2004; Tzanakis & Arcavi, 2000). As a result, lack of knowledge and sources may have significant role among the challenges to prevent teachers from incorporating.

Based on previous studies another challenge for the incorporating approach was time constraints (Blanco & Ginevard, 2009; Fried, 2001; Oprukçu-Gönülateş, 2004; Siu, 2004; Tzanakis & Arcavi, 2000). For this reason, Fried (2001) implied that incorporating the history of mathematics was a challenge to maintain regular mathematics program. In the present study, teachers were complained about lack of time for the general mathematics program. In other words, incorporating the history of mathematics was not regarded by the teachers as a time limiting factor. They said that time was always a problem for mathematics programme to include innovations within teaching. After classroom practice session ended, the teachers were asked whether they had problems about achieving the objectives while incorporating the history of mathematics. They said that they covered nearly all the objectives and they both discussed the topic and incorporated history of mathematics, which was what this study longed for. The teachers associated time constraint with time management issue. Therefore, the issue may be about time management rather than lack of time. The teachers described themselves inexperienced since it was their "first experience". They emphasized the importance of practicing, "the more the practice is the more it becomes better". They may solve time management problem by practicing a lot since there is no time limitation for this approach. It depends on teachers' need.

Lack of time, knowledge and sources were the expected findings of the present study. However, problem solving vs. incorporating the history of mathematics was a thought provoking issue which was not found in the accessible literature. Teachers mentioned that they could not solve sufficient number of problems that was planned. It was reflected as history of mathematics was a challenge for problem solving. Moreover, they referred problem solving as a necessity and a tool for understanding mathematics meanwhile they questioned the relationship between the meaning of achieving objectives and problem solving. For this description, problem solving vs. the history of mathematics, teacher's book may have some role. It is a recommendation that teachers are to complete the book at the end of the academic year and the book includes mostly content knowledge and problems related to the topic.

Teachers said that they would use the historical materials prepared for them. There were contributions not only to teachers but also students. Professional improvement, attractiveness and the environment for speaking up can be evaluated under positive

aspects of the approach according to the descriptions of teachers. To begin with professional improvement, the literature has already shown the effect of incorporating the history of mathematics on teachers' content knowledge (Clark, 2011; Huntley & Flores, 2010, 2011). The teachers regarded historical information as general knowledge and they found it beneficial to answer "why questions" of students. This kind of questions was defined by NCTM (2004) as chronological questions that could be best explained by the history of mathematics. They felt the need to do more research since they were insufficient about historical information. To do research may affect teachers' understanding of mathematics and their point of view toward mathematics, which directly reflects to students. The teachers were open to the training. After the data collection process ended, they requested other historical materials for her students or invitation for some training to improve themselves.

Attractiveness of the approach was the most repeated aspect of history of mathematics. Classroom environment was regarded as sometimes bored due to teachers, students and topics. At that time the teachers were in need of something to keep the flow of learning. History of mathematics could be used as a tool for affective issues (Jankvist, 2009). Teachers said that most of the students actively participated while incorporating the history of mathematics. There may be correlation between environment to "speak up" and attractiveness. Jardine (2007) stated that active participation of students was important while incorporating the history of mathematics. According to the participants, different students found a chance to express themselves while incorporating the history of mathematics. Their experiences can be summarized that attractiveness and environment to "speak up" allowed the students to involve actively in the discussions and activities in the classroom.

5.2. Implications

The present study contributes to the growing body of research about incorporating the history of mathematics into teaching since studies conducted with teachers were inadequate to understand the situation better in Turkey and other countries. It enabled to investigate different aspects lying behind the phenomenon. The essence of the experience can give an idea about issues to consider in the classroom.

Although there is an increase in research on mathematics education incorporating the history of mathematics lately, teachers can be concluded as not ready to incorporate history of mathematics within teaching. There were some studies resulted that even if the teachers were aware of historical knowledge, they would not prefer to incorporate history of mathematics (Fraser & Koop, 1978; Siu, 2004). Siu asked the critical question "…why do most teachers still hesitate to integrate history of mathematics with their classroom teaching?" (2012, p.11). The reason may be lack of knowledge and source, time constraint and problem solving versus history of mathematics issues based on teachers' descriptions of the experience in Turkish case.

Lack of knowledge, sources and time constraint were frequently encountered within the accessible literature. However, problem solving versus history of mathematics was a new issue in the accessible literature. Problem solving and history of mathematics were different approaches in mathematics, they cannot be reversible. However, solving old problems from different civilizations was one of the methods to incorporate history of mathematics. Moreover, Meavilla and Flores (2007) provided a course design that included old problems so that students found chance to compare their answers with old answers. That kind of courses may be helpful sometimes to satisfy teachers toward the dilemma they had.

The present study showed that historical information provided to teachers was limited to the teacher guide book which was not sufficient to incorporate within mathematics teaching. Mathematics education from primary education to teacher training do not include historical environment enough to collect information for sharing. Mathematics is alienated from its origin. It was referred from the study that teacher guide book was important to transfer the mathematics content knowledge to students. While preparing the historical materials, teachers mentioned criteria to incorporate historical materials. Adaptability of the historical materials that the teachers mentioned in the study can be considered by the textbooks related to the history of mathematics.

5.3 Recommendations

The study can be enlarged by including teachers from seventh grade and eighth grade to enrich the meaning of the phenomenon. For the transferability of the study, teachers

from different regions in Turkey can be involved in the activities concerning incorporating the history of mathematics which may provide different aspects of the experience. The time interval of incorporating the history of mathematics into teaching can be extended to one semester or one academic year and both teachers and students can be observed and investigated by the researchers. It may enable to gain insight about benefits and pitfalls of the incorporating approach for teachers and students.

Teachers had a tendency to replicate the way of teaching of their previous teachers (Furinghetti, 2000). The present study also demonstrated the similar tendency of the teachers. Furinghetti (2000) claimed that history of mathematics could solve this problem if history of mathematics was replaced as a special course in teacher training program. Therefore, teachers will be encouraged to renew their historical perspectives in mathematics education . In-service teacher training programs can be prepared by teacher educators to inform teachers about its contributions to the teaching and learning and the ways of incorporation in the classroom. For the teacher training programs and elementary and secondary mathematics education, the number of sources concerning history of mathematics may be increased. Visual and hands on materials may be prepared to direct use in the classroom. What the teachers described in the present study can be considered as a base for the adaptability of the historical materials. “Simplicity” and “remarkableness” of the historical information presented in the classroom can be further investigated and historical textbooks can be written according to the findings. Additionally, special courses can be designed in teacher training programs to show origins of the mathematics and how to incorporate history of mathematics in the classroom in order to provide historical perspectives of mathematics.

Reflecting teachers’ experiences in a phenomenological framework will allow educators and policy makers to be sensitive to the points that teachers have expressed. Moreover, the findings may lead researchers to investigate new aspects of the phenomenon and its effects on students as well.

All in all, primary teachers and middle grade mathematics teachers’ descriptions of incorporating the history of mathematics into teaching approach through their personal experiences may enlighten the ideas and perceptions concerning this phenomenon.

Descriptions of incorporating the history of mathematics approach from the perspectives of the teachers will help to develop common courses in teacher training programs and give an idea about the contributions to teaching with this approach.

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APPENDICES

Appendix A: The Voluntary Participation Form

GÖNÜLLÜ KATILIM FORMU

Bu çalışma, "Sınıf ve Matematik Öğretmenlerine göre Matematik Tarihinin Matematik Öğretimine Katılması Üzerine Bir Olgubilim Çalışması" başlığıyla ODTÜ İlköğretim Fen ve Matematik Eğitimi yüksek lisans programında okuyan Sinem SÖZEN tarafından yüksek lisans tezi olarak hazırlanmıştır. Tez danışmanları Dr. Didem AKYÜZ ve Dr. Erdinç Çakıroğlu'nun katkılarıyla hazırlanan bu çalışmada öğretmenlerin gönüllü katılımı esastır. Katılımın gönüllü olması gereği, katılmamaktan ötürü ya da katılımdan vazgeçme sonucunda katılımcı olumsuz hiçbir sonuçla karşılaşmayacaktır.

Matematik tarihi son yıllarda araştırmacılar tarafından incelenen ve birçok ülke tarafından etkin bir şekilde uygulanmaya başlanan eski bir yöntemdir. Çalışmanın, Matematik Tarihi'nin alan yazında belirtilmiş bir anlatım yöntemi olarak faydalarından yola çıkarak, öğretmenlerin ders içinde kullanması sonucu yaşadıkları süreci tanımlamaları ve algılarının belirlenmesiyle bir sonraki adımda araştırmacılar tarafından hazırlanacak olan matematik tarihi ile ilgili ders materyalleri ve planlar hazırlamada fikir verebileceği öngörülmektedir.

Katılımcıların bu süreçte, sırasıyla iki tane görüşme, bir ders ve ardından son görüşmeye katılmaları beklenmektedir. Her bir görüşmenin en fazla 60 dakika, ders anlatımının ise iki ders saati sürmesi planlanmaktadır. Ders anlatımında video kaydının, görüşmelerde ise ses kaydının tutulması verilerin ivedilikle incelenmesi açısından gerekli görülmektedir.

Araştırma sonuçlarının, etkin kullanılan e-posta adresi iletildiği takdirde bir dökümü araştırmacı tarafından iletilecektir. Çankaya İlçe Milli Eğitim Müdürlüğü ve Orta Doğu Teknik Üniversitesi tarafından onaylanan izinler istendiği takdirde katılımcıyla paylaşılabilir. Görüşmelerden elde edilen bilgiler hiçbir şekilde üçüncü şahıslarla paylaşılmayacaktır.

Çalışmamıza katkıda bulunduğunuz için çok teşekkür ederiz.

Gönüllü Katılımcının İmzası

…/…/2013

Araştırmacının İletişim Bilgileri

e-posta: sinem.sozen@metu.edu.tr
Telefon: 0505 6154090
Adres: TED Üniversitesi, Kolej/ Ankara

Appendix B: Interview Protocols

GÖRÜŞME PROTOKOLÜ 1

Görüşülen: *Görüşmeci:*
Gün-saat: *Yer:*
Süre:

Değerli Meslektaşım,

Bu çalışma, "Sınıf ve Matematik Öğretmenlerine göre Matematik Tarihinin Matematik Öğretimine Katılması" ile ilgili tez çalışması kapsamında sınıf öğretmenleri ve ortaokul matematik öğretmenlerinin görüş ve deneyimlerini aktarmak amacıyla yapılmaktadır. 3 aşamalı görüşme sürecinin ilk aşamasında bulunmaktasınız. Bu görüşmenin 60 dakika sürmesi planlanmaktadır. Hem daha fazla zamanınızı almamak, hem de aktardığınız bilgileri tam ve doğru olarak kaydedebilmek için, eğer izniniz olursa ses kayıt cihazı kullanmak istiyorum.

1. Kaç yıllık öğretmensiniz?
2. Çalıştığınız okulda kaç yaş grubuna giriyorsunuz?
3. Hangi okul ve hangi bölümden mezunsunuz?

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4. Deneyimlerinize dayanarak matematiği nasıl ifade edersiniz?

5. Okul matematiği hakkında düşünceleriniz nelerdir?

6. Matematik derslerini nasıl işliyorsunuz? Örnek verebilir misiniz? Kılavuz kitaptan faydalanıyor musunuz? Ek kaynaklar kullanıyor musunuz?

7. Matematik Öğretmeni olarak sınıftaki rolünüz nasıl tanımlarsınız?

8. Matematik derslerini işlerken ne gibi sorunlarla yaşıyorsunuz?

9. Öğrencileriniz size göre matematik dersi ile ilgili ne gibi sorunlar yaşıyor?

10. Bu sorunların öğretmenden kaynaklanan yönleri nelerdir sizce?

11. Mesela ders anlatımından kaynaklanan bahsettiğiniz sıkıntılar başka yöntemler, yaklaşımlar ya da uygulamalar kullanılarak çözülebilir mi? Nasıl?

12. Derslerinizde Matematik Tarihi kullandınız mı? Nasıl bir yöntem olabilir sizce?

13. Önceden bahsettiğiniz ders anlatımıyla ilgili sorunları çözebilmek için Matematik Tarihinin yardımcı olacağını düşünür müsünüz? Nasıl? Neden?

14. Öğrenim hayatınızda Matematik Tarihinden bahseden öğretmenleriniz oldu mu? Ya da matematik tarihiyle birlikte işlediğiniz dersler oldu mu? İşlemesini ister miydiniz?

15. Derslerinizde bahsettiğiniz matematikçiler oldu mu? Hangileri? Nasıl bahsettiniz? Öğrencilerden nasıl bir tepki aldınız?

16. Derslerinizde Matematik Tarihi'nin matematikçilerin biyografileri gibi derste kullanım şekillerinden faydalanma hakkında ne düşünürsünüz?

17. Uygulamanızda hangi konu içerisinde matematik tarihi yer alsın? Bu konu hangi kazanımları içeriyor?

GÖRÜŞME PROTOKOLÜ 2

Görüşülen: *Görüşmeci:*
Gün-saat: *Yer:*
Süre:

Değerli Meslektaşım,

Bu çalışma, "Sınıf ve Matematik Öğretmenlerine göre Matematik Tarihinin Matematik Öğretimine Katılması" ile ilgili tez çalışması kapsamında sınıf öğretmenleri ve ortaokul matematik öğretmenlerinin görüş ve deneyimlerini aktarmak amacıyla yapılmaktadır. 3 aşamalı görüşme sürecinin ikinci aşamasında bulunmaktasınız. Bu görüşmenin 60 dakika sürmesi planlanmaktadır. Görüşmeci tarafından size Matematik Tarihi'ni uygulamak istediğiniz konuyla ilgili hikâyecik, biyografi, çeşitli görseller gibi matematik tarihi kaynakları iletilecektir. Bu materyalleri gözden geçirmeniz için araştırmacı tarafından yeterli süre tanınacaktır. Hem daha fazla zamanınızı almamak, hem de aktardığınız bilgileri tam ve doğru olarak kaydedebilmek için, eğer izniniz olursa ses kayıt cihazı kullanmak istiyorum.

1. Matematik Tarihi'yle ilgili kaynakları inceleyin. Derste uygulanabilirliği konusunda düşünceleriniz nelerdir?

2. Verilen materyaller size nasıl fikirler veriyor derste kullanımı ile ilgili?

3. Peki derste nasıl kullanmayı planlıyorsunuz?

4. Hangi materyalleri kullanmak istemezsiniz? Neden?

5. Ders esnasında zorlanacağınızı düşünüyor musunuz? Ne zaman?

6. Bu yöntemi kullanırsanız ne gibi yardıma ihtiyaç duyacağınızı düşünüyorsunuz?

7. Matematik Tarihi ile ilgili kaynaklar hangi beklentilerinizi karşıladı? Hangisini karşılamadı?

GÖRÜŞME PROTOKOLÜ 3

Görüşülen: *Görüşmeci:*
Gün-saat: *Yer:*
Süre:

Değerli Meslektaşım,

Bu çalışma, "Sınıf Ve Matematik Öğretmenlerine Göre Matematik Tarihinin Matematik Öğretimine Katılması" ile ilgili tez çalışması kapsamında sınıf öğretmenleri ve ortaokul matematik öğretmenlerinin görüş ve deneyimlerini anlamak amacıyla yapılmaktadır. 3 aşamalı görüşme sürecinin üçüncü aşamasında bulunmaktasınız. Bu görüşmenin 60 dakika sürmesi planlanmaktadır. Hem daha fazla zamanınızı almamak, hem de aktardığınız bilgileri tam ve doğru olarak kaydedebilmek için, eğer izniniz olursa ses kayıt cihazı kullanmak istiyorum.

1. Ders anlatma deneyimi nasıldı?

   a) Öğrenci kazanımları gerçekleştirdi mi? Kimler gerçekleştirdi kimler gerçekleştirmedi?
   b) Bu konuyu bir kez daha anlatsaydınız neler yapardınız neler yapmazdınız?
   c) Ters giden durumlar oldu mu? Neden?
   d) İyi giden kısımlar nelerdi?
   e) Sizce öğrenciler açısından faydalı bir ders miydi?
   f) Ders ne kadar başarılıydı?

2. Her zamanki ders anlatımınızdan ne gibi farklılıkları vardı?

3. Derse girmeden önce ne gibi hazırlıklar yaptınız?

4. Derse hazırlanırken ne gibi sorunlarla karşılaştınız?

5. Verilen materyallerde herhangi bir sorunla karşılaştınız mı?

6. Bu zorluklar nasıl aşılabilir?

7. Hangi kaynaklardan faydalandınız? Neden?

8. Ders esnasında ne gibi sorunlarla karşılaştınız? Neler iyi gitti neler kötü?

9. Deneyiminize dayanarak, bu yöntemi kullanmanızın sebepleri neler olabilir? Bu yöntemi kullanmamanızın sebepleri neler olabilir?

10. Hangi beklentileriniz karşılanmadı?

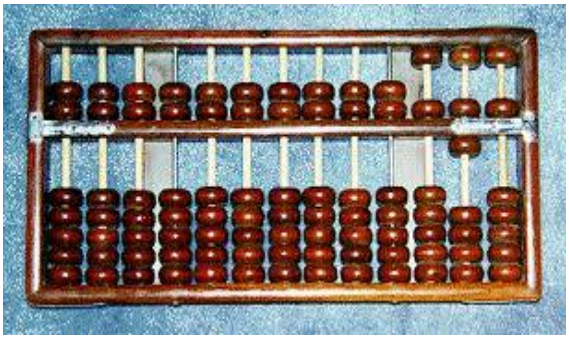
11. Hazırlanacak ders planlarında Matematik Tarihi'nden faydalanırken öğretmenlerin nelere dikkat etmesi gerekiyor? Nasıl sonuçlar çıkardınız?

12. Özetleyecek olursak, bu yöntemle neler fark ettiniz? Neler keşfettiniz?

13. Son olarak, Matematik Tarihi'nin sizce ilkokul ve ortaokul matematik derslerindeki rolü ne olmalıdır?

Appendix C: The part of the historical materials: Fifth Grade

ÇARPMA ve BÖLME



Aritmetik işlemlerin en eskisi toplamadır. Oldukça eski olan toplama işlemi hemen hemen her uygarlıkta vardır. Kolay toplama yapabilmek için hesap makinesi gibi değişik araçlar kullanılmaya başlanmıştır. En çok kullanılanı ise Abaküstür. İkinci aritmetik işlem ise benzer olarak çıkarma olmuştur. Bu işlemler de her ülkenin kendine has kuralları olmuştur. Aslında toplama ve çıkarma işlemleri insanların en yakın gereksinimleri için bir araç olmuştur. Bu iki işlem her ulus tarafından başarılı bir şekilde oturtulmuştur. Oysa çarpma ve bölme işlemlerinin bugünkü kurallara oturtulması o kadar kısa ve kolay olmamıştır. Başlangıçta çarpma için bazı kurallar getirilmiştir ama bölme özellikle çok zaman almıştır.

ARİTMETİĞİ OKUMA ve YAZMA

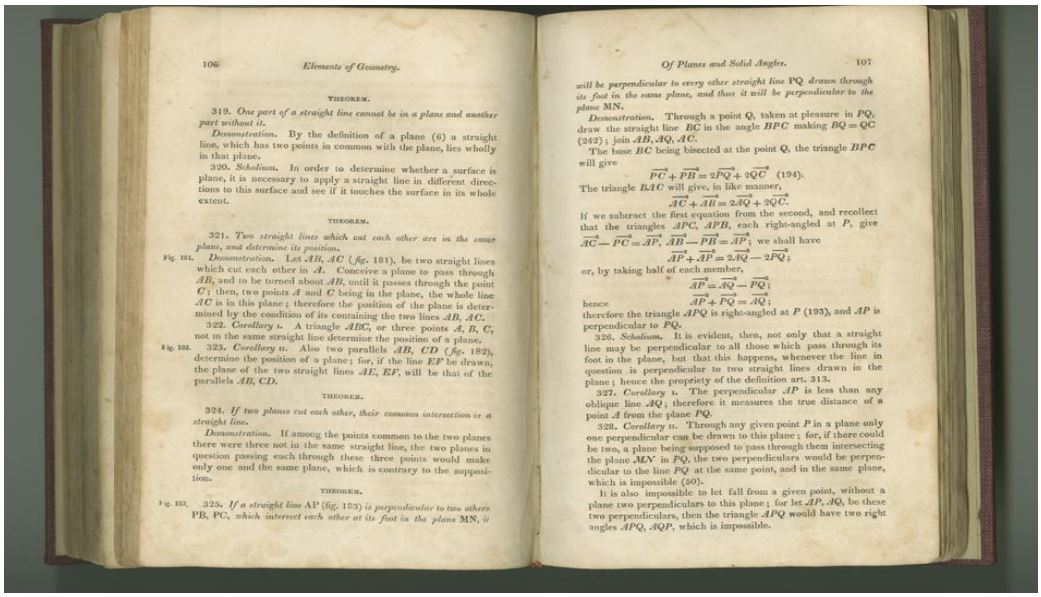
SEMBOLLER NEREDEN GELDİ?

Size sorulsa "5 ile 6'nın toplamından 7 çıkarıldığında sonuç 4'tür" ü aritmetik semboller kullanarak nasıl yazardınız? (5+6)-7=4 mü yazardınız? Muhtemelen. Eğer öyle yaptıysanız, şöyle ifade etmişsinizdir bu şekilde yazmak daha açık, daha anlaşılır, okuması daha kolay hangi ülkede olursanız olun ya da hangi dili konuşursanız konuşun.

Bu semboller zamanla evrenselleşmiştir. Herhangi bir dilin kısaltmaları ya da alfabenin harflerinden çok daha anlaşılır bir yapıya sahiptir. Fakat her zaman böyle değildi. Eski Yunanlılar ve Araplar hiç sembol kullanmadılar. Sadece problemleri kelimelerle ifade ettiler. Orta Çağlarda gerçekten bu semboller olmaksızın bütün matematiksel ifadeler kelimelerle yazıldı. Fakat 15i yüzyıldan beri bir takım semboller ülkeler tarafından oluşturulmaya başladı. Biraz çarpmadan bahsedecek olursak, çarpma işleminin şu anda dört şekilde ifade edilir.

1. 3(4 + 5) Bu ifade 9. Ve 10. Yüzyıllarda Hint yazıtlarında ve 15. Yüzyılda bazı Avrupa yazıtlarında yer almaktadır ve günümüze kadar gelmiştir.

2. X sembolü: Bu ifade ise 17. Yüzyılda Avrupa'da yazılmış metinlerde görülebilir. Legendre tarafından 1794'te basılan bu kitapta örneği görülebilir.



106 *Elements of Geometry.*

THEOREM.

319. *One part of a straight line cannot be in a plane and another part without it.*

Demonstration. By the definition of a plane (6) a straight line, which has two points in common with the plane, lies wholly in that plane.

320. *Scholium.* In order to determine whether a surface is plane, it is necessary to apply a straight line in different directions to this surface and see if it touches the surface in its whole extent.

THEOREM.

321. *Two straight lines which cut each other are in the same plane, and determine its position.*

Fig. 181. *Demonstration.* Let *AB*, *AC* (*fig.* 181), be two straight lines which cut each other in *A*. Conceive a plane to pass through *AB*, and to be turned about *AB*, until it passes through the point *C*; then, two points *A* and *C* being in the plane, the whole line *AC* is in this plane; therefore the position of the plane is determined by the condition of its containing the two lines *AB*, *AC*.

322. *Corollary* I. A triangle *ABC*, or three points *A*, *B*, *C*, not in the same straight line determine the position of a plane.

Fig. 182. 323. *Corollary* II. Also two parallels *AB*, *CD* (*fig.* 182), determine the position of a plane; for, if the line *EF* be drawn, the plane of the two straight lines *AE*, *EF*, will be that of the parallels *AB*, *CD*.

THEOREM.

324. *If two planes cut each other, their common intersection is a straight line.*

Demonstration. If among the points common to the two planes there were three not in the same straight line, the two planes in question passing each through these three points would make only one and the same plane, which is contrary to the supposition.

THEOREM.

Fig. 183. 325. *If a straight line* AP (*fig.* 183) *is perpendicular to two others* PB, PC, *which intersect each other at its foot in the plane* MN, *it*

Of Planes and Solid Angles. 107

will be perpendicular to every other straight line PQ *drawn through its foot in the same plane, and thus it will be perpendicular to the plane* MN.

Demonstration. Through a point *Q*, taken at pleasure in *PQ*, draw the straight line *BC* in the angle *BPC* making $BQ = QC$ (242); join *AB*, *AQ*, *AC*.

The base *BC* being bisected at the point *Q*, the triangle *BPC* will give

$$\overrightarrow{PC} + \overrightarrow{PB} = 2\overrightarrow{PQ} + 2\overrightarrow{QC} \quad (194).$$

The triangle *BAC* will give, in like manner,

$$\overrightarrow{AC} + \overrightarrow{AB} = 2\overrightarrow{AQ} + 2\overrightarrow{QC}.$$

If we subtract the first equation from the second, and recollect that the triangles *APC*, *APB*, each right-angled at *P*, give $\overrightarrow{AC} - \overrightarrow{PC} = \overrightarrow{AP}$, $\overrightarrow{AB} - \overrightarrow{PB} = \overrightarrow{AP}$; we shall have

$$\overrightarrow{AP} + \overrightarrow{AP} = 2\overrightarrow{AQ} - 2\overrightarrow{PQ};$$

or, by taking half of each member,

$$\overrightarrow{AP} = \overrightarrow{AQ} - \overrightarrow{PQ};$$

hence

$$\overrightarrow{AP} + \overrightarrow{PQ} = \overrightarrow{AQ};$$

therefore the triangle *APQ* is right-angled at *P* (193), and *AP* is perpendicular to *PQ*.

326. *Scholium.* It is evident, then, not only that a straight line may be perpendicular to all those which pass through its foot in the plane, but that this happens, whenever the line in question is perpendicular to two straight lines drawn in the plane; hence the propriety of the definition art. 313.

327. *Corollary* I. The perpendicular *AP* is less than any oblique line *AQ*; therefore it measures the true distance of a point *A* from the plane *PQ*.

328. *Corollary* II. Through any given point *P* in a plane only one perpendicular can be drawn to this plane; for, if there could be two, a plane being supposed to pass through them intersecting the plane *MN* in *PQ*, the two perpendiculars would be perpendicular to the line *PQ* at the same point, and in the same plane, which is impossible (50).

It is also impossible to let fall from a given point, without a plane two perpendiculars to this plane; for let *AP*, *AQ*, be these two perpendiculars, then the triangle *APQ* would have two right angles *APQ*, *AQP*, which is impossible.

http://mathtreasures.blogspot.com/2012/02/legendres-elements-of-geometry.html

3. 4.5 Bu sembol Leibniz adlı matematikçinin cebirsel ifadeleri yazarken x ile X in sıklıkla karıştırılmasını önlemek amacıyla kullandığı bir yöntemdir. 18. Yüzyılda bu kullanım yaygınlaşmaya başladı.

4. 4 * 5 Modern hesap makineleri ve bazı bilgisayar programları çarpma işlemini bu işaretle gösterirler. 17. Yüzyılda Almanlar tarafından yaygın bir şekilde kullanılırdı. Elektronik çağına gelene kadar da bir süre ortalıktan kayboldu.

ORTAÇAĞ'DA ÇARPMA İŞLEMİ

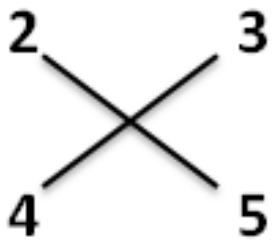
(Ortaçağ Kavimler Göçü ile başlar İstanbul'un Fethine kadar sürer)

Eski zamanlardan beri, matematikçiler hesaplamalar için daha kolay yollar aradılar. Ortaçağ'da Avrupa'nın hesaplama aracı abaküs ve abaküsü kullanmak için konulan kuralları bilmek gerekirdi. Bu bilgileri çok az insan bilirdi ve onlara Abaküsün Efendisi ya da Hesap Efendisi denilirdi. Bu insanlar ya uzman matematikçi ya da tüccardı. Bir Abaküs Efendisi oldukça iyi kazanırdı bu yüzden sahip olduğu bu özel hesaplama bilgilerini kimseyle paylaşmak istemedikleri durumlar da olurdu. Eğer karşılığında para öderlerse, bu hesaplama bilgisini başkalarıyla paylaşırlardı.

Bugünlerde varolan sayı sistemimizle hesaplamayı herkes öğrenebilir. Bir de 15. Yüzyılda insanların çarpma işlemini öğrendikleri için yaşadıkları sevinci biz de gözlemleyelim.

İlk önce basit çarpma işlemi kurallarını öğrendiler. Şu şekilde

1. İki basamaklı çarpma işlemlerinde İtalyanlar bir yöntem kullanırlardı siz ne düşünürsünüz? Mesela 23 ile 45 i çarpalım.



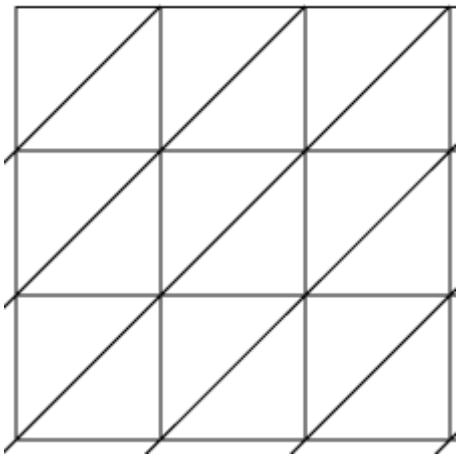
* Önce 5 ile 3'ü çarpın. 5X3=15. 15 'in 5 ini en sağa yazın 1 de elde var unutmayın. …….5

* Sonra 4 ile 3 ü, 5 ile 2'yi çapın ve toplayın. 4X3=12, 5X2=10, toplayınca 10+12=22. Bir de elde vardı onu da ekleyin. 22+1=23. 3'ü 5 in hemen yanına yazın elde de şimdi 2 var unutmayın. …..35

* 4 ile 2yi çarpın. 4X2=8. Bu sayıya 2 eldeyi ekleyelim . 8+2=10 ve en soluna yazalım. 1035. Bu çarpma işleminin sonucu 1035'tir.

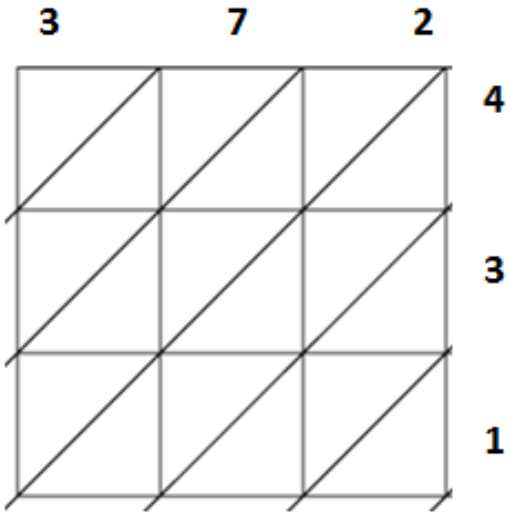
Birkaç alıştırma da siz yapın. 56X82 25X34 81X43

2. Bir de Gelosia yöntemini unutmamak gerek. Bu yöntem 14. Yüzyılda Avrupalılar tarafından yaygın olarak kullanılmıştır. Bu yöntemi Hinduların bulduğu belirtilir. Avrupa' ya nasıl gelmiş? Tabii ki ticaret. Arap tüccarlar bu yöntemi Avrupa'ya taşımışlardır. Bu örnek 5. sınıf çalışma kitabında da mevcut.

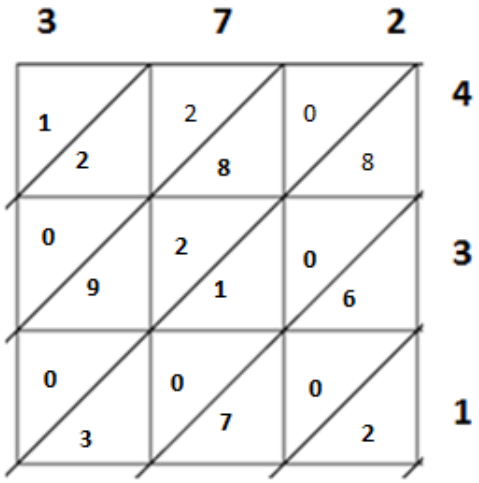


Yukarıdaki şekil 15. Yüzyıl aritmetik kitaplarında çarpma işlemini gösterir. Her biri karedir ve kareler köşelerinden bölünmüştür. Sütun sayısı bir çarpanın basamak sayısı kadardır. Satır sayısı da diğer çarpanın basamak sayısı kadardır.

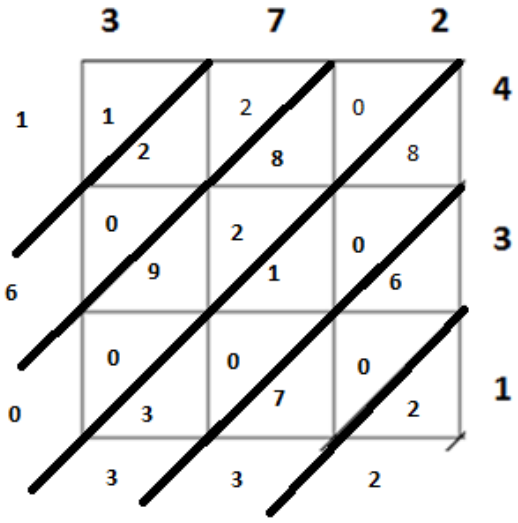
Önce 372 ile 431'i çarpalım.



Her bir karenin iki parçadan oluşur. İkisinin de farklı özelliği vardır. Üst kısımlara onlar alt kısımlara birler basamağı yazılır. Sırasıyla her bir sayıyı bir diğer sayıyla çarpıp kutuları doldurduğumuzda:



Daha sonra çapraz olarak düşünüp sayılar toplanır ve yazılır.



Sayı da yukarıdan başlayıp sağa doğru bir yol takip eder ve en son değeri 160, 332 olarak okunur.

İstenirse bununla ilgili birkaç çarpma alıştırması yapılır.

Rabdologia

http://www.17centurymaths.com/contents/napier/jimsnewstuff/Napiers%20Bones/NapiersBones.html

İnsanlar uzun yıllardan beri işlemleri kolaylaştırmanın yollarını aramaktadırlar. Bu amaçla gerçekleştirilen gelosia yöntemi başarılı olmuştur. Fakat bu yöntemler hala yetersiz kabul edilmektedir. Bu amaçla John Napier adlı matematikçi (1550-1617) gelosia yönteminden yola çıkarak sopalarla mekanik bir hesaplama aracı geliştirmiştir. Rabdologia Yunanca bir kelime ve "sopalar dizisi" anlamına gelmektedir. Daha sonra "Napier'in Kemikleri" olarak da anılmıştır. Çünkü fildişlerinden yapılmıştı ve oldukça pahalılardı. Bu fikir şimdiki bilgisayar programlarının temelini oluşturmuştur.

Şimdi bu basit bilgisayar programının nasıl çalıştığını görelim. Mesela 354 X 8 işlemini yapalım.

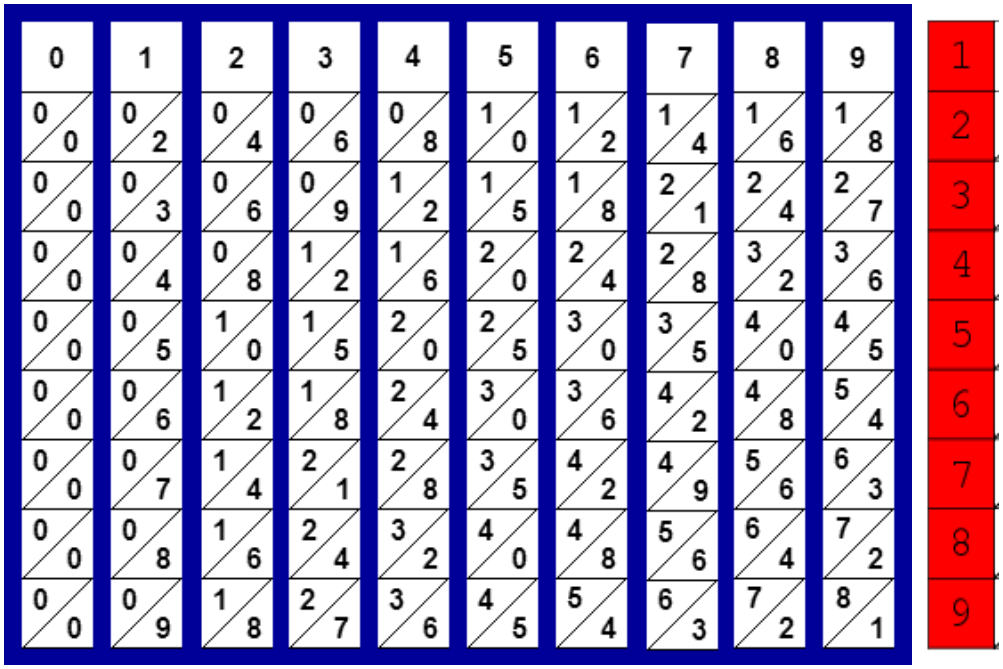
a. Önce size verilen kağıtlardan (eskiden sopa ya da kemiktendi) 3 ve 5 ve 4 ü alın sonra da 1-den dokuza kadarki sopayı alın. Yan yana koyduğunuzda 8. Sıraya bakın. Gelosia yöntemindeki gibi toplayın. Cevap 2,832

b. Peki üç basamakla üç basamak çarpılacaksa, örnek: 354 X 628. O zaman hesaplama parçalara ayrılarak yapılmalıdır.

354 X 8 =
354 X 20 =
354 X 600 =

Aşağıdaki örnekleri size verilen kağıtları bir araya getirerek siz yapın.

127 X 83 = 329 X 566 =


0 1 2 3 4 5 6 7 8 9
0 0 0 2 0 4 0 6 0 8 1 0 1 2 1 4 1 6 1 8
0 0 0 3 0 6 0 9 1 2 1 5 1 8 2 1 2 4 2 7
0 0 0 4 0 8 1 2 1 6 2 0 2 4 2 8 3 2 3 6
0 0 0 5 1 0 1 5 2 0 2 5 3 0 3 5 4 0 4 5
0 0 0 6 1 2 1 8 2 4 3 0 3 6 4 2 4 8 5 4
0 0 0 7 1 4 2 1 2 8 3 5 4 2 4 9 5 6 6 3
0 0 0 8 1 6 2 4 3 2 4 0 4 8 5 6 6 4 7 2
0 0 0 9 1 8 2 7 3 6 4 5 5 4 6 3 7 2 8 1
1 2 3 4 5 6 7 8 9

John Napier Kimdir?

:http://www.famous-mathematicians.com/john-napier/

John Napier 1550 yılında doğmuştur. Hem matematikçi hem de bir gökbilimciydi. Gökyüzü ile ilgili hesaplamalarında oldukça fazla zaman ayırması onu daha kısa yollara aramaya yöneltmiştir. Bu kısayolu bulabilmek için 20 yıl boyunca uğraşmıştır. Daha sonra bütün büyük sayıların üslü sayılar şeklinde yazılabileceği fikrinden yola çıkarak fizik, matematik, gök bilimi ve hatta astrolojide bile kullanılabilecek bir yolun kapılarını açtı. Sadece çarpmada değil ayrıca ondalık kesirlerin yazımında nokta işaretinin kullanımını yaygınlaştırmıştır.

Appendix D: Thesis Copy Permission Form

TEZ FOTOKOPİSİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü ☐

Sosyal Bilimler Enstitüsü ■

Uygulamalı Matematik Enstitüsü ☐

Enformatik Enstitüsü ☐

Deniz Bilimleri Enstitüsü ☐

YAZARIN

Soyadı : Sözen
Adı : Sinem
Bölümü : İlköğretim Fen ve Matematik Eğitimi

TEZİN ADI (İngilizce) : A Phenomenological Study on Incorporating The History of Mathematics into Teaching from The Perspective of Primary and Mathematics Teachers

TEZİN TÜRÜ : Yüksek Lisans ■ Doktora ☐

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
3. Tezimden bir (1) yıl süreyle fotokopi alınamaz. ■

TEZİN KÜTÜPHANEYE TESLİM TARİHİ: