JOINT QUANTITY FLEXIBILITY UNDER MARKET INFORMATION UPDATE

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ABSTRACT

JOINT QUANTITY FLEXIBILITY UNDER MARKET INFORMATION UPDATE

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In this study, we consider a decentralized supply chain consisting of a single retailer and a single manufacturer who manufacture two products in a given single period. The retailer commits to purchase an aggregate order quantity for the products at the beginning of the period before information on the market condition is revealed. Accordingly, the manufacturer then determines its initial production quantities for the products. Once market condition is revealed the retailer allocates its initial order to individual products and the manufacturer has to fulfill retailer's orders fully, through expedited production if necessary.

In this setting, we analyze the expected profits of both parties and compare them with two different benchmark settings where all decisions have to be given before information on market condition is available. Through a detailed computational analysis, we aim to assess the effects of the flexibility of allocating orders after information update on the profitability of the retailer and the manufacturer.

Keywords: Quantity Flexibility Contract, Information Update, Decentralized Supply Chain

ÖZ

PİYASA BİLGİSİ GÜNCELLEMESİ ALTINDA BİRLEŞİK ORTAK ESNEKLİĞİ

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Bu çalışmada, bir dönemde iki ürün üretip satan bir perakendeci ve bir üreticiden oluşan merkezi olmayan bir tedarik zincirini inceliyoruz. Perakendeci ürünlerin piyasası hakkında bilgi elde etmeden önce ürünlerin toplamı için bir sipariş miktarını almayı taahhüt eder. Üretici daha sonra buna göre ürünlerin ilk üretim miktarlarını belirler. Perakendeci, piyasa durumu belli olduğunda, ilk siparişindeki miktarı münferit ürünlere böler ve Üretici, gerekirse hızlandırılmış üretim yöntemi kullanarak perakendecinin siparişini tümüyle karşılamak zorundadır.

Bu ortamda, her iki tarafın beklenen karlarını analiz ediyor ve bu karları piyasa durumu hakkında bilgiler erişilebilir olmadan önce tüm kararların verilmesi gerektiği iki farklı değerlendirmeli ortamla karşılaştırıyoruz. Detaylı sayısal değerlendirmeler vasıtasıyla amacımız, Perakendeci ve Üreticinin karlılıkları hakkındaki bilginin güncellenmesinden sonra sipariş verme esnekliğinin etkilerini değerlendirmeyi amaçlamaktayız.

Anahtar Kelimeler: Miktar Esneklik Anlaşması, Bilgi Güncellemesi, Merkezi Olmayan Tedarik Zinciri

To My Beloved Daughter Hüma,

for the hours I stole from her to complete my thesis.

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CHAPTER 1

INTRODUCTION

To create solutions for matching supply with demand is one of the core businesses of supply chain management. Although there are several developments, innovations and new approaches in the field which aid the companies to offer right products/services at the right place and time; managing the uncertainty is yet a big problem for all the parties in the supply chain. The companies, especially the ones in the decentralized supply chains struggle with the costs associated with highly volatile market conditions and uncertain demands. Rapid technological developments/changes as well as ever changing buying behaviors of the customers, which are driven by several factors like fashion and globalization, cause the customers demand a wider variety of products. Such fact forces the companies to adopt different supply chain models rather than classical order and purchase models. In decentralized supply chains, managing the costs are even more difficult due to the fact that the parties act in accordance with their individual benefits rather than the whole supply chain benefits.

One of the methods to deal with the uncertainty in supply chains is Quantity Flexibility contracts. A Quantity Flexibility (QF) contract is an agreement between a buyer and a seller such that the buyer commits to an initial order with a certain level of flexibility and the manufacturer promises to deliver the retailer's order in full. Hence, a QF contract can be considered as a supply chain management tool that allows the retailer to gain time for better observing the market demand before placing its final order to the manufacturer. The retailer takes the advantage of waiting for the market condition information to be revealed before placing its final orders thanks to the flexibility of revising its initially committed order up-to a predefined level called flexibility limit. With the presence of such a contract, the disadvantages due to a decentralized supply chain setting as well as uncertain market conditions are minimized (as long as defined and applied correctly by the parties).

A joint quantity flexibility contract, on the other hand, is a contract valid for multiple products where the flexibility is defined over the total commitment. That is; the buyer reserves an aggregate quantity at the beginning and has the flexibility to determine the composition of its order after some time and the seller has to satisfy the buyer's final individual orders. The power of quantity flexibility contracts come from the possibility of forecast/information update and order update by postponing final order decision to later time than the initial order placed.

The concept of information update becomes vital when the target group of products are particularly high-tech products and fashion items. It is difficult to make proper forecasts for these products due to their nature. The demand for fashion items has to be forecasted at least 6-8 months in advance however, the demand for products is highly uncertain at that time and therefore the forecast quality is poor [19]. As selling season approaches, more information about market condition and product demand become available to the company. Making accurate forecasts for high-tech products is difficult as the demand of the products is not known and no past sales/demand data is available. In addition, lead times are quite high compared to the duration of selling season of the products. For instance, the lead time for fashion items is 6-8 months while the selling season of such items is around 3 months at most. The difficulty in forecasting makes the supply chain costs such as leftovers, inventory holding and stock-out to increase and the profits of parties in the supply chain to decrease. Information update possibility becomes lifesaving especially for the retailer in order to better match the demand with supply. Along with information update, the retailer has an opportunity to place a second order to renew its initial order at a closer time to the selling season. This is possible, since the manufacturer shares the uncertainty with the retailer for a win-win situation. Namely, flexibility contracts have been used between parties for higher performance.

In this study, we consider a joint quantity flexibility model in a decentralized supply chain that consists of a single retailer and a single manufacturer. The retailer and the manufacturer sell two products, where the quantity flexibility contract enables the retailer to allocate an aggregate order to products in exchange of commitment of such aggregate order in advance. In this setting, the retailer commits an initial order for the total of two products before the selling season and then, after the market condition information revealed, allocates this total commitment (initial, aggregate order) to the products in second stage. Namely, this is observing the market and updating/improving its forecast.

In the scope of this study, the supply contract (flexibility contract) between parties enables the retailer to give its order in two steps. At first step, the retailer determines an overall total order quantity, called initial order quantity and then revises its initial order by allocating pre-determined initial order quantity into products at the second step. This way the retailer has the option to gather more information while the manufacturer procures raw materials, sets up the production line and etc. In other words, the manufacturer enables the retailer to use some of the lead time between retailer's order and delivery of goods for observation of market to gather more information about the market. As mentioned above, information update is particularly beneficial when long lead times of fashion items (orders are given almost 6-8 month in advance, far before the market conditions is predictable) and in high-tech products like computers, mobile phones, android devices, where the product life is short compared to long design and production phases are considered.

In the Joint Quantity Flexibility model, when the initial order is placed by the retailer, the manufacturer starts its operations based on the order of the retailer and also the uncertain market demand distributions of the products, which are known by both parties. After information collection and forecast update, the retailer allocates the total amount to the products in accordance with the latest information and updated demand distributions. Once the retailer gives its final orders, the manufacturer produces the products based on the first stage production quantities. Note that the manufacturer uses expedited production mode at the second stage since faster production is a must to catch up before the start of the selling season. The final order of the retailer is limited with its initial order quantity, which protects the manufacturer from unlimited uncertainty and ensures/fixes a commitment quantity for the manufacturer at the beginning of the season for its own arrangements.

Such a QF contract is particularly relevant in the sense of aggregate ordering when the products are similar to each other and differ in a limited number of features only. This differentiation can be considered as customizations, where the cores of the products are the same and the products are in the same product family. As example of such a product family iPhone 16, 32 and 64 GB can be considered. Besides, most of the technological products with small differentiations like computers, hi-fi systems, cars in different accessorizes are also examples of the "products from the same product family case".

In this setting, our objective is to analyze the joint quantity flexibility contract between the retailer and the manufacturer in detail and to determine the optimal behaviors of each party. The performance of the joint quantity flexibility contract model, where the risk of uncertainty is shared by the parties, is compared with two benchmark settings. One of them is a classical retailer/manufacturer relationship model, where there is no flexibility, no information update and risk sharing. This scenario is a version of the classical newsvendor model, where the retailer gives its only order at the beginning of the period without collecting information on market condition and having improved forecast. In the second benchmark setting, on the other hand, the retailer does not give an initial order but instead gives its single order after the market condition is observed. Note that first benchmark setting is considered as "no flexibility" and second benchmark is considered as "full flexibility" settings.

The study also includes a detailed numerical analysis for different parameter settings to figure out the performance of the joint quantity flexibility contract model (QF) compared to benchmark settings. We try to identify what percentage of the total possible improvement may be provided by the QF model in different parameter settings. We also aim to analyze the effects of various problem parameters on the expected profits and order quantities of the retailer and manufacturer. Furthermore, we would like to observe the cost and benefits of QF contract to the parties and to the whole supply chain. Besides, we try to reveal whether and under what conditions QF is beneficial to parties compared to the benchmark settings.

The rest of the study is organized as follows: In chapter 2, the related literature is presented. In chapter 3, we give the details of the problem setting, environment and problem formulation. In chapter 4, we give the analysis of the benchmark settings and discuss the optimal decisions of the parties in these settings. In chapter 5, we summarize the numerical analysis and discuss the behaviors of the parties. In Chapter 6, we conclude the study with a summary and directions to possible further studies.

CHAPTER 2

LITERATURE REVIEW

The relationship between the retailer and the manufacturer mentioned in this work is organized under a quantity flexibility (QF) contract. A quantity flexibility contract not only enables better information flow between the parties in a supply chain, but also helps to reduce costs in supply chains by preventing parties from allocating over capacity/inventory by enabling them to eliminate the bullwhip effect. Therefore, it is a tool that helps the companies -both manufacturer and buyer- cope with the uncertainty in demand and to better match the supply with demand. This is possible through risk sharing and providing flexibilities to handle the uncertainty.

Quantity flexibility is closely related to the concept of information update because in the presence of such a contract the retailer has the flexibility to postpone final ordering decision and the manufacturer enjoys the delayed product differentiation as long as it is possible due to the nature of products. Since the retailer has the option to update its initial order up to a certain level -flexibility limit- it has time to collect more information for making better forecasts. Information update is important when the lead time is relatively long in comparison to the length of the selling season of the products. Between the order and delivery of goods by the manufacturer, there is considerably long time that new information to make better forecasts regarding the market condition and demand of the products may become available.

Quantity flexibility contracts are tools of supply chain management, which aim to help to match the supply with demand for greater profit for buyer and manufacturer in highly volatile demand settings. The literature about the quantity flexibility contract can be considered relatively new, where most of the literature is as new as 90’s. On the other hand, the concept of information update takes its roots back to 60’s. For instance the study of Murray and Silver (1966) is one of the first studies that analyze the value of information update for style good inventory problem with Bayesian update scenario. The studies in the literature that consider the information update may be divided into two parts depending on whether they utilize the Bayesian information update or not. The work of Eppen and Iyer (1997a) is one of the attention-grabbing studies of literature that focuses on Bayesian information update. They focus on the problem of buying fashion goods for a catalogue merchandiser, who also owns outlet stores and thus has the opportunity, as the season evolves, to divert inventory to the outlet store. They analyze how much to order originally and how much to divert to the outlet store as actual demand is observed. The problem is modeled as an inventory control problem in a single period. In addition to Eppen and Iyer (1997a), Lovejoy (1990) studies Bayesian information update on uncertain demand distributions.

A few examples from the literature that use update model other than Bayesian update is the work of Wang et al. (2012) about the newsvendor model with dynamic forecast update using a martingale update model. In addition, Pinçe et al. (2008) discuss Bayesian demand update in a partial backorder newsvendor model, and Choi et al. (2006) focus on information update and Quick Response policy. We do not go into further details of the literature as information update is not the mean focus of this work.

The literature on Quantity Flexibility Contracts may be classified according to a number of characteristics of the problem setting and environment such as single or multi product, single or multi period and as flexibility definition and limitation. Related literature about this work is around quantity flexibility contract and information update as mentioned above and numerous related studies on these subjects are discussed below.

Bassok et. al. (1997) study a supply chain with a buyer and a supplier in a multi-period planning horizon for a single product. They consider two types of flexibility; flexibility to update previous commitments and flexibility to purchase different than previous commitments. That is; the buyer has a certain level of flexibility on the purchase commitments and also flexibility to update previously made commitments as time passes for the upcoming periods. In this setting, the buyer is offered different levels of flexibilities at different costs and it has to choose the level of flexibility he is willing to pay for additional amount. At the beginning of planning horizon the buyer makes purchasing commitments to the supplier for every period in the horizon. The actual purchasing quantities and the commitments may be modified as time passes and as more information about demand is collected. The problem is to minimize the total cost of the purchasing from the buyer's perspective. They develop a heuristic to find near-optimal purchasing quantities and period commitments easily. They also compare the performance of the heuristic to the original solutions and observe that the difference between optimal and heuristics is found to be around 2% in the worst case. The heuristic is also used to discuss the worth of flexibility by using iso-curves for flexibility and purchase cost. Such discussion and iso-curves are especially important for decision makers at negotiation for determining terms and conditions of such flexibility contracts. They conclude by indicating a study can be performed as extension of theirs to consider a family of components supplied from a single supplier, where the flexibility is stated for the family as a whole. Note that the scope of our study covers the problem setting where the flexibility of a family of products is considered although not the same approach is adopted.

Eppen and Iyer (1997b) study supply chain problem in fashion goods industry of a catalogue retailer and the manufacturer under Bayesian update. They develop a model to find the optimal solution to the problem and practical heuristics that help the buyer to decide the initial order quantity and how much to divert to outlet stores. They model a backup agreement, where the catalogue company makes an initial commitment in advance the selling season starts. The vendor in return agrees to hold a certain percentage of that amount in reserve and delivers only the rest. After the catalogue is mailed, the company has to opportunity to buy any or all of the items in reserve. According to their results, backup is an important practice that benefits both for the retailer and manufacturer. Besides, order commitment has a significant effect on expected profit.

Gurnani and Tang (1999) focus on a supply chain consisting of a manufacturer and a retailer, where the retailer orders twice and both orders arrive before the selling season. In between the orders, the retailer collects additional information about the uncertain market demand and condition and updates its demand forecast. Different than most of the studies in the literature, the unit cost that the retailer has to pay at the second instant is uncertain and could be higher (or lower) than the unit cost at the first instant. In addition, the information obtained from the market between periods may be worthless or may be perfect and these extreme cases are investigated in detail. They analyze the optimal order quantities of the retailer under such uncertain cost and market conditions. In conclusion, they observe that when the service level is high, the value of information about the demand increases, which increases the retailer’s profit and decrease manufacturer’s profit.

Tsay and Lovejoy (1999) consider a Quantity Flexibility Contract in a multi-echelon supply chain, which consists of a supplier (parts supplier to the manufacturer), a manufacturer and a retailer, for the coordination of information and material in a multi-period setting. In this setting, the retailer makes a period-by-period rolling horizon updating. In other words, it updates its desired replenishment quantity in accordance with its inventory position and demand forecast, as new information becomes available. This becomes the demand forecast of the manufacturer and reflected to the part supplier. In accordance with the proposed Quantity Flexibility Contract between each pair of parties, the revision of estimate for future period is limited with a certain fraction, (α, ω) for upward and downward. These flexibilities enable the supplier to guarantee the buyer a specific safety margin for excess ordering and the buyer to limit its order reductions by giving a minimum purchase commitment. This way, each party in the chain may benefit from Quantity Flexibility Contract and excess inventory in the chain is minimized by forcing the parties to share the risk. Tsay and Lovejoy mentioned that the QF contracts are applied in industry but with little guidance of academic literature. Therefore, the aim of the work of Tsay and Lovejoy is to introduce a formal framework of the QF contract benefits and help the firm’s decisions in real life applications and finally show the benefit of QF contracts in terms of cost saving due to inventory levels in the supply chain. They focus on the effects of Quantity Flexibility Contract on the inventory and service levels. They particularly, focus on the inventory level, which increases to cope with the inflexibility of supplier while meeting customer's demand for flexible response. Increasing a party's input flexibility reduces its costs where output flexibility increases inventory cost. Besides, they have found that QF contracts can decrease the transmission of order variability throughout the chain and this way retard the "bullwhip effect". They also provide guidance to choosing the level of flexibility that is worth to pay for and discuss briefly other considerations such as level of flexibility actually required, pricing and the conflicts between parties on the application of QF contracts.

Tsay (1999) studies a supply chain consisting of a single supplier and a single retailer. He focuses on the costs incurred from the individually rational but systematically inefficient behaviors of the parties and suggests remedies. Tsay considers specifically the Quantity Flexibility contract, in the form of $\{c,(\alpha,\omega)\}$, which forces the retailer purchase no less than a certain percentage below the forecast (a minimum purchase commitment), $(1+\omega)$ and the supplier guarantees to deliver up to a certain percentage above retailer' initial forecast $(1+\alpha)$. Tsay's model is very similar the one constructed in the scope of this study. One major difference of our study from the study of Tsay is that we consider two products simultaneously and flexibility is defined over aggregate quantity for two products. Besides, the demand structure of the retailer and the source of the information gathered after the market condition observation are different. Tsay assumed random market demand of the form $X = \mu + \varepsilon$ where μ and ε represent the mean demand and independent error respectively. The retailer observes μ, where in our work the retailer observes the market condition. Tsay discusses the inefficiencies in the classical relationship between parties in such a decentralized supply chain and concludes that a QF contract, to some extent, becomes the solution to most of the problems. Since a QF Contract does not guarantee efficiency by itself, Tsay discusses under what conditions the QF Contract is beneficial to each of the parties. The main outcome of this discussion pinpoints the trade-off between commitment flexibility of the retailer and the corresponding unit price it has to pay to the manufacturer for this flexibility.

Donohue (2000) studies a supply contract between one distributor and one manufacturer, who produces and sells a single product in a single period to the market. The aim of the study is to evaluate the effectiveness of the two mode production system by using information update. Although the focus is different, Donohue's study is constructed on a very similar setting of our study, where the distributor gives an initial order based on uncertain demand information in the first stage, and then it updates its order based on "new market information" in the second stage. There are two production modes that the manufacturer uses to fulfill the order of the distributor. One of the production modes is cheaper and requires longer lead time and the other one cost more but with a shorter lead time and the wholesale prices also differ according to the stage the order is given by the retailer. Donohue aims to provide guidance for wholesale pricing scheme so that the parties benefit from the two-mode production. The distributor's problem is to allocate the orders into stages and the manufacturer's problem is to arrange the manufacturing quantities for each period based on the distributor's orders and also determine the prices in each order. Donohue considers a benchmark setting; centralized system, where distributor and the manufacturer are owned by a single firm and all the decisions are made centrally to obtain a better channel performance rather than individual performances. Donohue offers a supply contract, in the form of (w_1, w_2, b), where w_i represents wholesale price according to production modes and b represents return price of left overs. Four attention-grabbing conclusions are derived from the work of Donohue; first one is that the manufacturer always orders less than channel optimal. Equal wholesale margin between production modes does not improve channel performance. Donohue even finds that batch production margin should be set higher than fast production margin to ensure better channel coordination. Thirdly, more efficient price schemes may be possible as the predictive power of information update increases. And last one is that the benefit of setting a second production mode depends on the initial wholesale price in addition whether this second production mode is slower or faster. Although the application of order update is different, the work of Donohue and our study intersects in the sense that information and order update is handled in the sense that new information comes as probability density function of the market demand. The work of Donohue differentiates from ours in its basic concentration which is the pricing scheme, whereas in our study we do not analyze the wholesale prices.

Milner and Kouvelis (2005) differentiate the products in accordance to their demand characteristics and mentioned about the innovative products becoming important for the companies with technological developments since they contribute more to profit. They study the effect of demand characteristics on the supply chain flexibilities where, quantity and timing flexibilities are under consideration based on total inventory cost. The aim of the work is to compare the value of such flexibilities for different product demand characteristics. They consider a buyer and a supplier that produce and sell a single product in single period. In the problem setting, there are two ordering opportunities; first order is placed before the season and manufacturer starts production accordingly and then second order is placed within the bounds of flexibility settings. They analyze total inventory cost for different demand characteristics that are defined for three different product types standard stationary demand is used for functional products (commodities; bread and butter of company), Bayesian demand case for fashion goods and Martingale demand case is for innovative products. They define 4 cases of flexibility as follows;

- Static case; two production modes, quantities and timing pre-specified,
- Quantity flexible case; timing of the second run is fixed but quantity is flexible,

- Timing Flexible case; quantity of the second run is fixed but timing is flexible,
- Fully dynamic; both timing and quantity of the second run flexible.

They find out that the value of quantity flexibility is high for Bayesian demand model and it is of moderate value for the standard demand model for all lead times. The value of quantity flexibility is high for the case of short lead times but low for the case of long lead times for Martingale demand model. They observe that timing flexibility has the highest value for the case of standard demand and has slightly lower value for Bayesian demand, each with higher value for increased holding cost.

Miltenburg and Pong (2007a) are the first to study multiple products with demand uncertainty and two order opportunities. Since their study focuses on product families, it is closely related to the context of our study. There is uncertain demand for family of style goods, which are produced and sold in a single period. There is no capacity constraint but note that Miltenburg and Pong have another work with similar setting where they focus on the capacity constraint scenario (Miltenburg and Pong (2007b)). In Miltenburg and Pong (2007a) there are 2 order opportunities; first one with long lead time and low cost, and the second one with short lead time but high cost. New demand information between two orders becomes available so that the order can be updated. Demand forecasts are revised by using Bayesian estimation process. They focus on 2 main problems; determination of best order quantities and determination the best demand forecast at time of each order. The study of Miltenburg and Pong are related to our study in the sense that it is on determining best order quantities and the usage of information update for better demand forecast quality.

Wang (2008) states that the flexibility concept is limited with manufacturing flexibility in previous literature. He focuses on supply chain flexibilities since they are promising for increasing the efficiencies of parties in a supply chain. Two supply chain flexibilities are under consideration in his study: order quantity flexibility and delivery lead time flexibility. Wang constructs a discrete-event simulation to evaluate the performance of the supply chain in different levels of quantity and order flexibilities. In the problem setting, there is a manufacturer and buyer, who itself is also a manufacturer that produces and stocks the goods until they are sold. Manufacturer and buyer (retailer sells to market under stochastic demand) check inventory periodically (R, s, S). Buyer takes orders directly from customers and orders from the manufacturer by choosing the quantity and delivery lead time. Three levels of quantity flexibilities are represented by three ordering rules that define the batch size constraints of order quantities. In addition to the order quantities, the order lead time can be shortened with a cost that is again predefined, which represents lead time flexibilities. In conclusion, he shows that order quantity flexibility provides cost savings when the holding cost dominates the total cost and lead time flexibility improves service level and reduces the possibility of shortages. Besides, he shows that when the cost per shortage is relatively high, lead time flexibility allows the buyer to improve its service level and reduce the shortage cost.

Choi et. al (2004) consider the inventory stocking problem of a retailer in a supply chain, where there are multiple delivery modes and information update. The optimum ordering policy of the retailer that orders a seasonal product with uncertain product demand is analyzed. The retailer has multiple delivery modes, where the delivery cost increase when the lead time decreases. The retailer can choose among delivery modes and it can also update its order based on market observation if it chooses a faster delivery mode to gain time for information update. For information update, Bayesian update model is used. In this setting, the retailer orders once and the aim of the study is to find an optimal single ordering policy for the retailer.

Sethi et al. (2004) consider a supply chain that consists of a buyer and manufacturer where there is a quantity flexibility contract between parties involving multi periods, rolling horizon demand and forecast update. The study of them differs with the possibility of spot market option where the buyer can also purchase goods.

Most of the literature mentioned so far is more closely related to the Quantity Flexibility Contracts. However, more literature is available for supply chain and demand uncertainty problems. Some of the literature considered related in this context but not directly focuses on Quantity Flexibility Contracts are summarized below;

Bassok et. al. (1997) consider a supply chain with multiple products in single-period with zero lead time. In this supply chain, downward demand substitution is allowed, which means that the demand for a lower class product can be satisfied by using higher class product is allowed but not vice versa. They demonstrate the benefits of using downward substitution while determining optimal ordering quantities and under which cost and parameters setting the benefits are higher. They conclude that salvage value, coefficient of variation of demand, substitution cost, price to cost ratio and similarity of products have significant effects on the performance of the model.

Garavelli (2003) studies on different degrees of flexibilities on the performance of a multi-product and multi-echelon supply chain, which is subject to stochastic market demand. He analyzes and compares the performance of supply chain that combines plants to markets via products under different supply chain flexibility types and degrees. The supply chain flexibility has two main aspects; the number of product types that can be manufactured in each production site and the different logistics strategies which can be adopted either to release a product to a market or to procure a component from a supplier. The degrees of flexibility; total flexibility, no flexibility and limited flexibility refer to the possibility of processing a product in different possible plants that in total define nine possible configurations. He performs simulation to analyze the performance of the supply chain under these combinations and concludes that limited flexibility of either supplier provides better performance.

Chen et. al (2006) consider a single-manufacturer and a single-retailer supply chain, where the retailer sells a perishable/fashionable product to the market. The selling season is relatively short compared to the long lead time of production and distribution. The retailer gives an initial order based on its initial knowledge about demand distribution and the manufacturer commits to a production quantity in the first period. Based on accumulated information, the retailer updates its order at the beginning of the second period. The leftovers of the manufacturer will be partially compensated by the retailer and the leftovers at the retailer will be returned to the manufacturer. They propose a contract- called return contract- of the form (w, p, b) where w is the wholesale price, p is the proportion of loss of manufacturer shared by the retailer and b is the return price for the leftovers. The parameter p, is used to allow allocation of total supply chain benefits among the parties. Thus, the aim of such a return contract is to reach channel optimal by eliminating double marginalization. The numerical analysis show that the proposed return contract is significantly advantageous compared to classical return contracts.

Boulaksil et. al. (2011) consider a supply chain with a single manufacturer and multiple customers. The manufacturer does not have its own products or inventory but operates a job shop with single capacitated production line that produces items for its customers on order basis and the parties have long-term relationships based on contracts. There are two levels of decision for the manufacturer: allocating the line capacity to the customers and production planning on the operational level. The allocation of capacity flexibility to the customers is decided when having contract negotiations and is the amount of demand that the manufacturer always accepts from those customers. This allocation provides input to the operational planning decisions of the manufacturer. The outsourcers share their advance demand information (before exact order), which is considered as capacity reservation. There is a trade-off between allocating more flexibility to outsourcers and earning more and paying penalty to outsourcers by not producing items due to over allocation. Their numerical study shows that capacity flexibility is very sensitive to unit penalty cost. Besides, more capacity flexibility is allocated to the customer with more uncertainty since it is willing to pay more for capacity flexibility.

There are also studies that are conceptually close to our study; imperfect information update in the sense the information update that only helps to learn the distribution of the demand. Chen et. al (2010) study Bayesian information update in the case of information asymmetry and imperfect information comes from different sources of information supply, which is in some cases information from only some of the sources can be achieved. The study of Zang et. al. (2013) is one of the examples in the literature that focus supply chain problems from a broader context by considering a coordination of postponed product differentiation and forecast update. They study a two-stage supply chain setting for multiple products with a common component. Their study also compromises a detailed numerical analysis that compares the proposed model with traditional systems as well as Quick Response models.

On top of all the related literature discussed above, our study is an extension of the work of Karakaya and Bakal (2013), in which they consider a decentralized supply chain with a single retailer and a single manufacturer, who produces and sells two products in a single period as in the case of our study. The period is divided into two stages, where at the beginning of second stage the retailer observes actual demand of products. In their work, the retailer places an initial aggregate order for products based on preliminary demand forecasts and has the opportunity to update its initial order at the beginning of second stage after receiving perfect demand information. The total of second stage order quantities of the retailer is limited with the initial aggregate order quantity, where the retailer can modify order of each product within this limitation freely. The initial order quantity of the retailer determines the initial commitment for manufacturer and it gives its component orders accordingly. The manufacturer has two options of procurement of components; regular delivery and expedited delivery, where the expedited delivery is more expensive with shorter lead time. The manufacturer gives its regular order after retailer's initial order at the beginning of the period and then gives its expedited order after retailer's final order in the second stage. Since the demand of common components are determined at the beginning of the period with the initial orders of the retailer, the manufacturer orders common components fully in the first stage. Therefore, the cost of common components to the manufacturer is assumed to be zero and the analysis is performed only for uncommon components. The aim of the study of Bakal and Karakaya is to determine the optimal order quantities of the parties under this joint flexibility agreement at each stage. Besides, they perform a detailed computational study to gain insights on the effects of the flexibility scheme on the expected profits and order quantities of the retailer and the manufacturer. They also analyze the effects of various system parameters on the problem setting. They compare their findings with a no flexibility benchmark setting, which is the classical newsvendor model the retailer has no order update opportunity. They find out that the retailer always benefits from such type of quantity flexibility contract. In addition, they conclude that the improvement in the retailer's expected profit increase in the uncertainty in the demand, and closely related with the discounted sales price of the products. According to their observation, the improvement under such quantity flexibility contract can be very significant (based on demand setting and parameters). They further find out that when the improvement is not impressive, it is not because the joint flexibility arrangement is not effective, but it because the retailer can perform quite well without any flexibility, in other words when there is not much possibility for improvement. Besides, they reveal that the manufacturer may also benefit from joint flexibility agreement especially when the total order quantity is increased.

As our study is an extension of the work of Bakal and Karakaya, the problem setting as well as the construction of the model and solution methods has major similarities. However, there also substantial differences that our study differ from their study, which are discussed below;

- Two products under consideration are very similar to each other and differ only in limited number of features in their study. However, in our study there is no such limitation and any two products produced and sold by the same manufacturer and retailer can be considered.
- At the beginning of the second stage, the retailer observes actual demand in their study whereas; in our study the retailer only learns the market condition information. Based on the market condition information the retailer knows the distribution of uncertain market demand.

Table 1 shows a very brief summary of the common keywords of the studies that are most closely related to our study. As seen from the table, together with the work of Bakal and Karakaya, our work is one of the rare studies in the literature that focus not only on multiple products but also joint ordering flexibility over multiple products.

Our study suggests a Joint Quantity Flexibility Contract with information update on uncertain demand distribution. It differentiates from the existing literature in the approach of aggregate ordering of two products, which can be extended to multiple products. This finds correspondence both in product families and also multiple products ordered from the same manufacturer without any interdependency. In addition, the form of the information update is also attention grabbing in the sense that it does not assume perfect demand information reveal and also it is performed over demand distributions rather than previous data. Therefore, it is more realistic compared to a lot of information update schemes in the literature and it may be referred in real case applications as well as literature extensions.

Table 1 Summary of Related Literature

Related Literature	**Single/ Multiple Product**	**Single/ Multiple Period**	**Information Update Method**	**Degree of Information Update**	**Flexibility**	**Notes/ Explanations/ Highlights**
Murray and Silver (1966)	Single	Single	Bayesian		Order Quantity	
Eppen and Iyer (1997a)	Single	Single	Bayesian		Order Quantity	Pure Demand Process
Eppen and Iyer (1997b)	Single	Single	Bayesian		Order Quantity	Backup Agreement Pure Demand Process
Bassok et. al. (1997)	Single	Multiple	Not Bayesian	Observes actual orders	Update previous commitment and order quantity	Inventory level Demand distributions of products
Gurnani and Tang (1999)	Single	Single	Not Bayesian	Worthless or Perfect Information	Twice Ordering	The cost in second order can be lower or higher
Tsay and Lovejoy (1999)	Single	Multiple	Not Bayesian	Exponentially Weighted Moving Average	Order Quantity	Cumulative Flexibility over Periods
Tsay (1999)	Single	Single	Not Bayesian	Observes μ of the market demand distribution	Order Quantity	Quantity Flexibility Contract
Donohue (2000)	Single	Single	Not Bayesian	Perfect Information	Order Quantity	
Miltenburg and Pong (2007a)	Multiple	Single	Bayesian		Order Quantity Delivery Lead Time	Family of Style Goods Products
Wang (2008)	Single	Single	N/A	N/A	Order Quantity (Batch Size) Delivery Lead Time	(R,s,S) Inventory Control Policy
Choi et. al (2004)	Single	Single	Bayesian		Delivery Lead Time	Quick Response Policy
Sethi et al. (2004)	Single	Multiple	Not Bayesian	Demand Distribution Update	Order Quantity	QF Contract Spot Market
Bakal and Karakaya (2011)	Multiple	Single	Not Bayesian	Perfect Information	Joint Order Quantity	Aggregate Order Quantity Flexibility
Our Study	Multiple	Single	Not Bayesian	Demand Distribution Update	Joint Order Quantity	Aggregate Order Quantity Flexibility Market Conditions

CHAPTER 3

PROBLEM DEFINITION AND FORMULATION

In this study, we analyze a decentralized supply chain consisting of a single manufacturer and a single retailer that offers two products to the market in a single period. The demands for the products are random and they depend on the market condition, which is uncertain when initial order decisions are made.
The retailer first gives an initial order to the manufacturer at the beginning of the period and then it updates this initial order after observing market condition information before the start of the selling season. For a better understanding of the setting, the decision making process may be separated into two stages through time. In the first stage, the retailer has little information on the market condition and demand of products, thus it makes the initial order on the total quantity of the products with its limited information and poor demand forecasts. Based on this, the manufacturer determines its regular production quantity for each product. After its first order, the retailer collects more information about the market condition and the position of the products in the market. Based on the information collected during this period, the retailer comes up with the information about the market condition and demand distributions of the products. This marks the start of the second stage of the problem, where the retailer updates its order by allocating total order quantity to the products and the manufacturer continues production to fulfill retailer's order fully based on final orders.

The market condition is modeled as a discrete random variable for n possible market conditions. We assume that both parties know in advance what the demand distribution for products will be according to market condition. To illustrate, consider the following example for two market conditions; with probability p, the market condition will be such that the demand of the products is uniformly distributed in [100,200] and [300,500], respectively, and with probability $(1-p)$, in [150,300] and [200,600]. This information is available to both parties in the first stage and the second stage is characterized by the market condition information. Once the retailer is aware of the market condition, it places a final order for each product. The retailer enjoys allocating the initial order quantity to the products without any limitation as long as the sum of the final orders is equal to the initial order. The manufacturer has to satisfy retailer's final order completely. Hence, after the retailer's final order, the manufacturer may use the expedited production mode, if necessary, when compared to the first stage production quantities and based on retailer's final order. Although it costs more, the expedited production mode enables the manufacturer to wait for the retailer's final order before completing its production.

The main objective of this study is to construct the model of the quantity flexibility contract in a defined problem setting and determine the optimal order quantities of the retailer and the optimal production quantities of the manufacturer that maximize their individual profits. In addition, the effects of information and order update in a decentralized supply chain are to be shown by comparing the quantity flexibility model setting with two benchmark settings. As mentioned in Chapter 1 briefly, the retailer in the first benchmark setting does not have the order update option and gives its order for the products separately at the beginning of the first stage. In the second benchmark setting, the retailer gives its single order after observing the market condition at the beginning of second stage and the manufacturer determines initial production quantities in the first stage without being aware of any decision of the retailer. The parameters, decision variables and notation used through the study are introduced in Table 2.

Table 2 Parameters and Decision Variables

Parameters	
p_i	Unit selling price of the retailer for product *i*
w_i	Wholesale price of product *i*
s_i	Discounted price of product *i* at the retailer after selling season
r_i	Discounted sales price of product *i* at the manufacturer
d_{i1}	Regular production cost of the manufacturer for product *i*
d_{i2}	Expedited production cost of the manufacturer for product *i*
Y	Random variable denoting the market condition
b_j	Probability of market condition *j*.
X_i	Random variable denoting the demand for product *i*
x_i	A realization of X_i
$f_{ij}(x)$, $F_{ij}(x)$	The pdf and cdf of demand for product *i* if market condition is given as $Y=j$
Decision Variables	
Q	Initial commitment of the retailer
Q_i	Retailer's final order quantity for product *i*
q_{i1}	Manufacturer's regular production quantity for product *i*
q_{i2}	Manufacturer's expedited production quantity for product *i*

The modeling assumptions are summarized as follows;

1. The problem is analyzed in a single period and the decision making process through time is divided into two stages.
2. The market condition is modeled as a discrete random variable; that is, there are possible realizations of the market condition.
3. Given the market condition, the demands for the products are independent and the demand distributions for each market condition are available to both parties.
4. The sum of final orders of the retailer may not differ from the initial commitment even if it is beneficial to both parties.
5. The manufacturer has two modes of production; regular, during stage 1 and expedited, during stage 2, where expedited production mode provides a shorter production time with a higher cost.
6. The manufacturer has to satisfy the final orders of the retailer completely.
7. The revenue of the retailer is assumed to be always greater than the wholesale price and the wholesale price is assumed to be always greater than discounted selling price, $p_i > w_i > s_i$. The same logic is also true for the costs and revenue of the manufacturer, $w_i > d_{i2} > d_{i2} > r_i$.
8. The cost of retailer includes only the unit procurement cost. The revenue of the retailer includes only the unit revenue acquired from the sales of the products during selling season.
9. The manufacturer's cost includes unit production cost of products. The revenue of the manufacturer includes only the unit revenue acquired from the sales of the products to the retailer at the wholesale price.
10. The transfer prices between the retailer and the manufacturer are exogenous.

The sequence of events in the above mentioned setting is as follows (refer to Figure 1 for illustration);

1. At the beginning of stage 1, the retailer determines a total initial order quantity, Q, for both products based on the available information of the uncertain market demand of products.
2. The manufacturer determines regular production quantities for each product; q_{i1}*'s* based on the retailer's initial order, Q, and the available information about the uncertain market demand. From this point, the first stage production costs of products, d_{i1}'s, are incurred to the manufacturer.
3. At the beginning of stage 2, market condition, Y, is revealed. Based on the market condition, the distribution of the demand for product i is characterized as $f_{ij}(x)$ (when $Y=j$).
4. The retailer determines the final order quantity for each of the products, Q_i where $Q_1 + Q_2 = Q$.

5. Based on retailer's final orders, and its initial orders, q_{i1}*'s,* the manufacturer determines the expedited production quantity for each product, q_{i2}. From this point, the second stage production costs of products, d_{i2}'s are incurred to the manufacturer.
6. The manufacturer delivers the goods to the retailer based on the final order of the retailer and sells the leftovers at a discounted price. The wholesale price of the goods is incurred to the retailer having supplied the products.
7. Market demand for each product is realized and filled as much as possible in accordance at hand products at the end of second stage. The items that are not sold at the regular selling season are cleared out at a discounted price by the retailer.

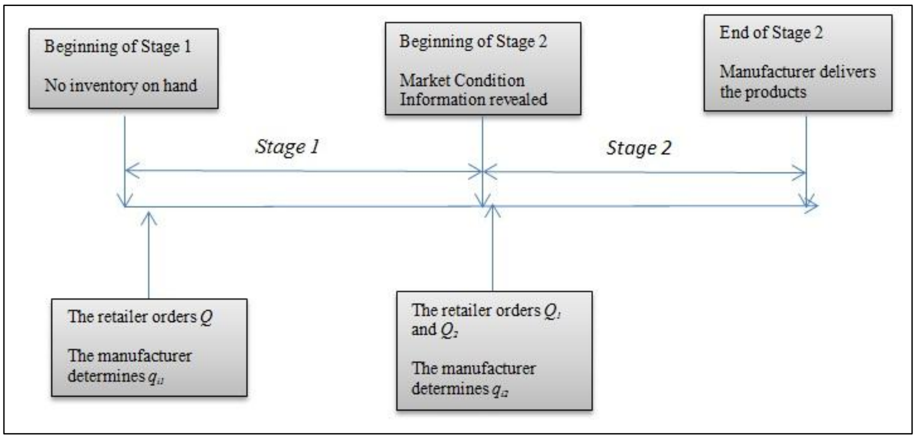

Figure 1 Sequence of Events in Quantity Flexibility Contract

3.1 ANALYSIS OF THE MODEL

In this section, we analyze the setting described in Section 3.1 in detail. The analysis will be performed for the retailer first as the retailer's decisions will also shape the manufacturer's actions.

3.1.1 Retailer's Problem

In this section, the retailer's problem for determining an optimum ordering policy under the problem setting will be analyzed. The retailer has a different problem at the beginning of each stage; firstly to determine the initial order quantity (aggregate commitment) and secondly to determine the final order quantities. We use backward induction to formulate and solve the retailer's problem. That is, we start with the second stage problem, where the market condition is known and initial commitment is fixed. As a result, we generate the optimal allocation of a given initial commitment to individual products for the realization of the market condition. Then, we incorporate this to the first stage problem, where the optimal initial commitment is to be determined.

In order to analyze the retailer's problem at the second stage, we assume that the market condition is revealed as $Y=j$, noting that the analysis will be the same for other market conditions. Given Q, x_1 and x_2 (realized demands for products); the retailer's realized profit is given as;

$$\begin{aligned}\pi_r(Q_1,Q_2 \mid Q,Y=j,X_1=x_1,X_2=x_2) = {} & (p_1-w_1)\min\{x_1,Q_1\} \\ & +(p_2-w_2)\min\{x_2,Q_2\} \\ & -(\mathrm{w}_1-s_1)\max\{0,Q_1-x_1\} \\ & -(\mathrm{w}_2-s_2)\max\{0,Q_2-x_2\}\end{aligned}$$

where $Q_1+Q_2=Q$. The expected profit of the retailer with respect to X_1 and X_2 is as follows;

$$\begin{aligned}\pi_r\left(Q_1,Q_2|Q,\ \mathrm{Y{=}j}\right) = {} & (p_1-w_1)\left[\int_0^{Q_1} x_1 f_{1j}(x_1)dx_1 + \int_{Q_1}^{\infty} Q_1 f_{1j}(x_1)dx_1\right] \\ & +(p_2-w_2)\left[\int_0^{Q_2} x_2 f_{2j}(x_2)dx_2 + \int_{Q_2}^{\infty} (Q_2)\, fj(x_2)dx_2\right] \\ & -(w_1-s_1)\left[\int_0^{Q_1} (Q_1-x_1)\, f_{1j}(x_1)dx_1\right] \\ & -(w_2-s_2)\left[\int_0^{Q_2} (Q_{2-}x_2)\, f_{2j}(x_2)dx_2\right] \\ s.t. \qquad & Q=Q_1+Q_2\end{aligned}$$

Since the total amount of the individual orders for product 1 and product 2, Q_1 and Q_2, cannot exceed the initial order of the retailer, the equation Q_2=Q-Q_1 always holds. Therefore, the problem of the retailer can be written so as it only depends on Q_1.

$$\begin{array}{l} \max \pi_r\left(Q_1,Q_2|Q\right) \\ \text{s.t} \\ \quad Q_1{+}Q_2{=}Q \\ \quad Q_1,\ Q_2 \ge 0 \end{array} \quad \xrightarrow[Q_2=Q-Q_1]{} \quad \begin{array}{l} \max \pi_r\left(Q_1|Q\right) \\ \text{s.t} \\ \quad 0 \le Q_1 \le Q \end{array}$$

Thus, the expected profit may be written such that it depends only on Q_1 as below;

$$\begin{aligned}\pi_r\left(Q_1|Q,Y=j\right)= {} & (p_1-w_1)\left[\int_0^{Q_1} x_1 f_{1j}(x_1)dx_1+\int_{Q_1}^{\infty} Q_1 f_{1j}(x_1)dx_1\right] \\ & +(p_2-w_2)\left[\int_0^{Q-Q_1} x_2 f_{2j}(x_2)dx_2+\int_{Q-Q_1}^{\infty}(Q-Q_1)f_{2j}(x_2)dx_2\right] \\ & -(w_1-s_1)\left[\int_0^{Q_1}(Q_1-x_1)f_{1j}(x_1)dx_1\right] \\ & -(w_2-s_2)\left[\int_0^{Q-Q_1}(Q-Q_1-x_2)f_{2j}(x_2)dx_2\right]\end{aligned} \tag{1}$$

Lemma 1: $\pi_r\left(Q_1|Q,Y=j\right)$ is strictly concave.

Proof: First and second derivatives of the retailer's profit $\pi_r\left(Q_1|Q,Y=j\right)$ are given in Equation (2) and Equation (3). The lemma directly follows from the second derivative.

$$\begin{aligned}\frac{d\pi_r(Q_1\,|\,Q,Y=j)}{dQ_1}= {} & (p_1-w_1)\left[1-F_{1j}(Q_1)\right]-(p_2-w_2)\left[1-F_{2j}(Q-Q_1)\right] \\ & -(w_1-s_1)F_{1j}(Q_1)-(w_2-s_2)F_{2j}(Q-Q_1)\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{d^2\pi_r(Q_1\,|\,Q,Y=j)}{d^2Q_1}= {} & -(p_1-w_1)f_{1j}(Q_1)-(p_2-w_2)f_{2j}(Q-Q_1) \\ & -(w_1-s_1)f_{1j}(Q_1)-(w_2-s_2)f_{2j}(Q-Q_1)<0\end{aligned} \tag{3}$$

□

Proposition 1: The global maximum of $\pi_r\left(Q_1|Q,Y=j\right)$ is given by the unique solution to $\frac{d\pi_r(Q_1\,|\,Q,Y=j)}{dQ_1}=0$.

Proof: By Lemma 1, $\pi_r\left(Q_1|Q,Y=j\right)$ is strictly concave.

Hence, all we need to show is that $\frac{d\pi_r(Q_1\,|\,Q,Y=j)}{dQ_1}=0$ has a solution, which follows since

$$\frac{d\pi_r(Q_1 \mid Q, Y=0)}{dQ_1}\Big|_{Q_1 \to \infty} = -(p_2 - w_2) - (w_1 - s_1) < 0 \tag{4}$$

and

$$\frac{d\pi_r(Q_1 \mid Q, Y=0)}{dQ_1}\Big|_{Q_1 \to -\infty} = (p_2 - w_2) + (w_1 - s_1) > 0 \tag{5}$$

Let $Q_1^{'}$ denote the global maximum of $\pi_r\left(Q_1 | Q, Y=0\right)$. That is, let $Q_1^{'}$ be the unique solution to $\frac{d\pi_r(Q_1 \mid Q, Y=0)}{dQ_1} = 0$. □

Proposition 2: The optimal allocation of a given initial order quantity, Q under the market condition realization $Y=j$ is characterized as follows:

$$Q_1^* = \begin{cases} 0 & if \quad Q < F_{2j}^{-1}\left(\dfrac{(p_2 - w_2) - (p_1 - w_1)}{(p_2 - s_2)}\right) \\ Q & if \quad Q > F_{1j}^{-1}\left(\dfrac{(p_2 - w_2) - (p_1 - w_1)}{-(p_1 - s_1)}\right) \\ Q_1^{'} & otherwise \end{cases} \tag{6}$$

and $Q_2^* = Q - Q_1^*$.

Proof: If $\frac{d\pi_r(Q_1 \mid Q, Y=j)}{dQ_1}\Big|_{Q_1 = 0} = (p_1 - w_1) - (p_2 - w_2) + (p_2 - s_2) F_{2j}(Q) < 0$, (7)

then, the unconstrained optimal solution, $Q_1^{'}$ is less than zero. Due to concavity, we have $Q_1^* = 0$.

If $\frac{d\pi_r(Q_1 \mid Q, Y=j)}{dQ_1}\Big|_{Q_1 = Q} = (p_1 - w_1) - (p_2 - w_2) - (p_1 - s_1) F_{1j}(Q) > 0$, (8)

Then, the optimal unconstrained solution is greater than Q and due to concavity we have $Q_1^* = Q$.

Finally, if $\frac{d\pi_r(Q_1 \mid Q, Y=j)}{dQ_1}\Big|_{Q_1=0} > 0$ and $\frac{d\pi_r(Q_1 \mid Q, Y=j)}{dQ_1}\Big|_{Q_1=Q} < 0$, then we can conclude that the unconstrained solution is in $[0, Q]$. That is, $Q_1^{'}$ is feasible and hence optimal, which completes the proof. □

Proposition 2 presents intuitive yet interesting results. If product 2 is more profitable and the initial commitment is less than a threshold value, the retailer allocates all of its initial order to the second product. This condition may hold true if the profit difference is substantial and/or the second product demand is large. A similar argument is also valid for product 1.

Corollary 1: If $(p_1 - w_1) > (p_2 - w_2)$, then, $Q_1^* = \min\{Q_1^{'}, Q\}$ and $Q_2^* = \max\{0, Q - Q_1^{'}\}$.

Otherwise, $Q_1^* = \max\{0, Q_1^{'}\}$ and $Q_2^* = \max\{Q, Q - Q_1^{'}\}$

Proof: If $(p_1 - w_1) > (p_2 - w_2)$, then,

$$\frac{d\pi_r(Q_1 \mid Q, Y=j)}{dQ_1}\Big|_{Q_1=0} = (p_1 - w_1) - (p_2 - w_2) + (p_2 - s_2)F_{2j}(Q) > 0$$

Since $(p_2 - s_2)F_{2j}(Q) \geq 0$.

If $p_1 - w_1 < p_2 - w_2$, then,

$$\frac{d\pi_r(Q_1 \mid Q, Y=j)}{dQ_1}\Big|_{Q_1=Q} = (p_1 - w_1) - (p_2 - w_2) - (p_1 - s_1)F_{1j}(Q_1) < 0$$

Since $(p_1 - s_1)F_{1j}(Q_1) > 0$. Then, the corollary follows from Proposition 2. □

An illustrative example is provided in Table 3 for the retailer's optimal allocation in different initial commitment, *Q*, for both $p_1 - w_1 < p_2 - w_2$ and $p_1 - w_1 > p_2 - w_2$. Note that in this setting the demand of products are assumed to be normally distributed and μ_1, μ_2 are mean of the normally distributed demand for Market Condition 1 for product 1 and 2 and μ_3, μ_4 are mean of the demand for Market Condition 2 for product 1 and 2 respectively.

In this example, when the product 2 is more profitable, the retailer allocates all of the initial order, *Q*, to the second product in the case of Market Condition 2 since the demand for product 2 is larger in Market Condition 2 for *Q=50, 100, 200, 300*. The retailer starts to allocate some of the initial order to product 1 only when initial order quantity increased to 400. The same is valid also for Market Condition 1 but since the demand of product 2 is lower in this case, the retailer starts to allocate the initial order to product 1 earlier. Likewise, when the product 1 is more profitable, the retailer allocate the entire initial to product 1 for $Q=50$, 100. The difference in the profitability of products also changes threshold value until which the retailer allocates all of the initial order to either of the product. Besides, the decision of the allocation is affected from the profitability of products, initial order quantity and the demand of products in related market condition.

Table 3 Parameter Setting and Optimal Allocation Example

Parameter Setting				
$\mu_1=120$	$\mu_2=80$	$\mu_3=100$	$\mu_4=300$	$b=0.8$
$w_1=60$	$w_2=60$	$s_1=0$	$s_2=0$	$r_1=5$
$d_{11}=10$	$d_{12}=10$	$d_{21}=20$	$d_{22}=20$	$r_2=5$
Optimal Allocations				
	$\boldsymbol{p_1=100, p_2=300}$ $p_1-w_1<p_2-w_2$		$\boldsymbol{p_1=160, p_2=100}$ $p_1-w_1>p_2-w_2$	
	MC1	MC2	MC1	MC2
	Q_1^*, Q_2^*	Q_1^*, Q_2^*	Q_1^*, Q_2^*	Q_1^*, Q_2^*
Q=50	0, 50	0, 50	50, 0	50, 0
Q=100	15.08, 84.92	100, 0	100, 0	84.38, 15.62
Q=200	111.5, 88.5	200, 0	124, 76	89.38,110.62
Q=300	178, 122	300, 0	180, 120	90.33, 209.67
Q=400	201, 199	72.95, 327.05	251, 149	110.7, 289.7

Having characterized the optimal second stage orders, we can now incorporate these into the first stage. At the beginning of first stage, the retailer has to decide the initial commitment quantity. At this stage, the retailer knows the individual demand distributions of the products corresponding to each possible market condition. Therefore, the expected profit of the retailer can be written as

$$\pi_r(Q) = \sum_{j=1}^{n} b_j \Pi_r(Q \mid Y=j) \tag{9}$$

Note that $\pi_r(Q \mid Y=j)$ can be obtained by plugging Q_1^* characterized in Proposition 2 into the expected profit function provided in Equation (1). Unfortunately, a closed form representation of $\pi_r(Q \mid Y=i)$ is not available. However, our extensive computational experiments suggest that $\pi_r(Q)$ is a concave function. Hence, we employ a basic search algorithm to find the optimal initial order.

For an initially assigned total order quantity, Q, optimal allocations, Q_1 and Q_2 and corresponding expected profits are calculated for each market condition as defined in Proposition 2 and Equation 1 respectively. Then initial Q value is increased by a step size and the procedure starts from the beginning for the second iteration. If the retailer's expected profit increase when Q increased, Q is increased by step size; else, Q is decreased by half of the step size. This procedure continues until changing Q does not bring any improvement in the profit. Table 4 presents an example of the iterations to find optimal Q.

Table 4 Example Optimization Steps of Finding Optimal Q

Iteration #	Q	Retailer's Expected Profit	Iteration #	Q	Retailer's Expected Profit
1	50	3871.3456564	20	206.25	10490.0572310
2	75	5361.5916905	21	207.8125	10488.8047271
3	100	6813.3894686	22	207.0313	10489.6765465
4	125	8146.1994471	23	206.25	10490.0572310
5	150	9259.9898196	24	205.4688	10489.9449207
6	175	10095.7835154	25	205.8594	10490.0628117
7	200	10475.2246488	26	206.25	10490.0572310
8	225	10350.5627858	27	206.0547	10490.0754418
9	212.5	10473.3791180	28	205.8594	10490.0628117
10	200	10475.2246488	29	205.957	10490.0729835
11	187.5	10348.9242687	30	206.0547	10490.0754418
12	193.75	10428.2210644	31	206.1523	10490.0701897
13	200	10475.2246488	32	206.1035	10490.0737792
14	206.25	10490.0572310	33	206.0547	10490.0754418
15	212.5	10473.3791180	34	206.0059	10490.0751766
16	209.375	10485.5957245	35	206.0303	10490.0755501
17	206.25	10490.0572310	36	206.0547	10490.0754418
18	203.125	10486.6332609	37	206.0425	10490.0755561
19	204.6875	10489.3378776	38	206.0303	10490.0755501

3.1.2 Manufacturer's Problem

The problem of the manufacturer is to determine the regular and expedited production quantities that maximize its expected profit, subject to market condition, market demand of the products and ordering decisions of the retailer at each stage. In this section, the manufacturer's problem of determining optimum production quantities will be analyzed in detail.

The manufacturer first stage problem is to decide the regular production quantities for each product, q_{11} *and* q_{21}, based on retailer's initial order, Q and the demands of the products, which depend on the uncertain market conditions. The manufacturer's second stage problem, on the other hand, is to decide the expedited production quantities for products, q_{12} *and* q_{22}, in accordance with the retailer's final order, and initial procurement quantities of manufacturer, q_{11} and q_{21}.

Similar to the approach that we utilize in the retailer's problem, the analysis of the manufacturer's problem starts with the second stage problem and then the first stage problem is constructed by incorporating the optimal decisions in the second stage problem.

At the beginning of the second stage, just after the retailer's final order, the manufacturer has to determine its expedited production quantities for each product. At this point, all of the information regarding the market condition and demand has already been gathered; all decisions have been taken by the retailer and manufacturer, except the expedited production quantities. Therefore, the manufacturer's second stage problem is straightforward. The manufacturer already knows exactly how many units from each product it has to deliver to the retailer regardless of the demand realization. Given the quantities it already produced in the first stage using regular production mode, the decision of determining the expedited production quantity for each product is nothing but restocking accordingly. Hence, we have $q_{12}^* = \max\{0, Q_1 - q_{11}\}$ and $q_{22}^* = \max\{0, Q_2 - q_{21}\}$, and the resulting realized profit of the manufacturer is given by the following equation, where Q_1^* and Q_2^* denote optimal allocation of the retailer under market condition j;

$$\begin{aligned}
&\pi_m(q_{11}, q_{21} \mid Y = j) = \pi_m^{(1)}(q_{11} \mid Y = j) + \pi_m^{(2)}(q_{21} \mid Y = j) \\
&\textit{where} \\
&\pi_m^{(1)}(q_{11} \mid Y = j) = Q_1^* w_1 - q_{11} d_{11} - d_{21} \max\{0, (Q_1^* - q_{11})\} + r_1 \max\{0, (q_{11} - Q_1^*)\} \\
&\textit{and} \\
&\pi_m^{(2)}(q_{21} \mid Y = j) = Q_2^* w_2 - q_{21} d_{21} - d_{22} \max\{0, (Q_2^* - q_{21})\} + r_2 \max\{0, (q_{21} - Q_2^*)\},
\end{aligned} \tag{10}$$

For $(p_1 - w_1) > (p_2 - w_2)$ the optimum expedited order quantities, q_{12} and q_{22}, are calculated as follows;

$$(q_{12}^*, q_{22}^*) = \begin{cases}
(0, 0) & q_{11} \geq \min\{Q_1', Q\} \\
 & q_{21} \geq \max\{0, (Q - Q_1')\} \\
(0, \max\{0, (Q - Q_1')\} - q_{21}) & q_{11} \geq \min\{Q_1', Q\} \\
 & q_{21} < \max\{0, (Q - Q_1')\} \\
((\min\{Q_1', Q\} - q_{11}), 0) & q_{11} < \min\{Q_1', Q\} \\
 & q_{21} \geq \max\{0, (Q - Q_1')\} \\
((\min\{Q_1', Q\} - q_{11}), & q_{11} \geq \min\{Q_1', Q\} \\
\max\{0, (Q - Q_1')\} - q_{21}) & q_{21} < \max\{0, (Q - Q_1')\}
\end{cases} \tag{11}$$

Similarly for $(p_1 - w_1) < (p_2 - w_2)$ the optimum allocation q_{12} and q_{22} values are calculated as follows;

$$(q_{12}^*, q_{22}^*) = \begin{cases} (0,0) & q_{11} \geq \max\{0, Q_1^{'}\} \\ & q_{21} \geq \min\{Q, (Q - Q_1^{'})\} \\ (0, \min\{Q, (Q - Q_1^{'})\} - q_{21}) & q_{11} \geq \max\{0, Q_1^{'}\} \\ & q_{21} < \min\{Q, (Q - Q_1^{'})\} \\ ((\max\{0, Q_1^{'}\} - q_{11}), 0) & q_{11} < \max\{0, Q_1^{'}\} \\ & q_{21} \geq \min\{Q, (Q - Q_1^{'})\} \\ ((\max\{0, Q_1^{'}\} - q_{11}), & q_{11} < \max\{0, Q_1^{'}\} \\ \min\{Q, (Q - Q_1^{'})\} - q_{21}) & q_{21} < \min\{Q, (Q - Q_1^{'})\} \end{cases} \tag{12}$$

First stage problem of determining the regular production quantities for each product starts when the retailer gives its initial order to the manufacturer. At this point, the manufacturer knows nothing else than the initial order of retailer, Q and the demand distributions of the products for each possible realization of market condition. That is, the market condition information is not revealed, the allocation of total order quantity to the products have not been completed, when the manufacturer is about to determine the production quantities for the regular production mode.
It should be noted that knowing the initial order quantity of the retailer, the manufacturer can derive the optimal allocation of the initial order to the individual products for a realization of the market condition, which effectively is the demand distribution that the manufacturer faces. Hence, we can conclude that the only source of uncertainty that the manufacturer faces is the market condition information. Thus, the demand of product i that manufacturer faces can be characterized as $D_i = Q_i^*(Y = j)$ if $Y = j$, where Q_i^* can be derived in accordance with Equation (2) and Proposition (2).

Then, the expected profit of the manufacturer in the first stage is given by

$$\pi_m(q_{11}, q_{21}) = \sum_{j=1}^{n} b_j \pi_m(q_{11}, q_{21} \mid Y = j) \tag{13}$$

Noting also that the manufacturer's profit function given in Equation (10) is separable with respect to product type, the expected profit can be rewritten as follows;

$$\pi_m(q_{11,}q_{21}) = \pi_m^{(1)}(q_{11}) + \pi_m^{(2)}(q_{21})$$

where, $\pi_m^{(1)}(q_{11}) = \sum_{j=1}^{n} b_j \pi_m^{(1)}(q_{11} \mid Y = j)$ and $\pi_m^{(2)}(q_{21}) = \sum_{j=1}^{n} b_j \pi_m^{(2)}(q_{21} \mid Y = j)$

Therefore, the manufacturer's problem can be solved for each product type separately.

Then, assuming without loss of generality that $Q_i^*(Y = j) < Q_i^*(Y = j+1)$, the following proposition characterizes the optimal procurement quantity for the manufacturer for product i.

Proposition 3: The optimal first stage procurement quantity of the manufacturer for product i is given by

$$q_{1i}^* = \min\left\{ Q_i^*(Y = j) : \sum_{k=1}^{j} b_k \geq \frac{d_2 - d_1}{d_2 - r_i} \right\} \tag{14}$$

Proof: A closer look at the manufacturer's problem reveals that it is basically a newsvendor problem with discrete demand distribution, where the cost of underage is $d_2 - d_1$ and the cost of overage is d_1-r_i. The proposition directly follows. □

CHAPTER 4

BENCHMARK SETTINGS

In order to show the effects of the joint quantity flexibility scheme with information update on the order quantities and expected profits of the retailer and manufacturer two benchmark settings are constructed; (i) no flexibility setting and (ii) full flexibility setting.

4.1. BENCHMARK I: NO FLEXIBILITY

The first benchmark setting (B1), the retailer gives its orders for each of the product separately at the beginning of first stage and does not have the option to make allocation between products or update its orders after the market condition information is revealed. The manufacturer, on the other hand, does not need to use expedited production mode since no change occurs in the initial orders of the retailer. For both the retailer and the manufacturer the initial orders are equal to the final orders.
Sequence of events in benchmark scenario, which is also given in Figure 2, is as follows;

1. At the beginning of stage 1, the retailer determines the order quantities for each product based on the available information about the uncertain market demand and orders separately for each product, Q_1 and Q_2.
2. The manufacturer determines regular production quantities for each product i, q_{i1} based on the retailer's initial orders.
3. The manufacturer delivers the goods to the retailers and sold out leftovers at a discounted price.
4. Market demand for each product is realized and filled as much as possible in accordance with on-hand products. The items that are not sold at the regular selling season are cleared out at a discounted price by the retailer.

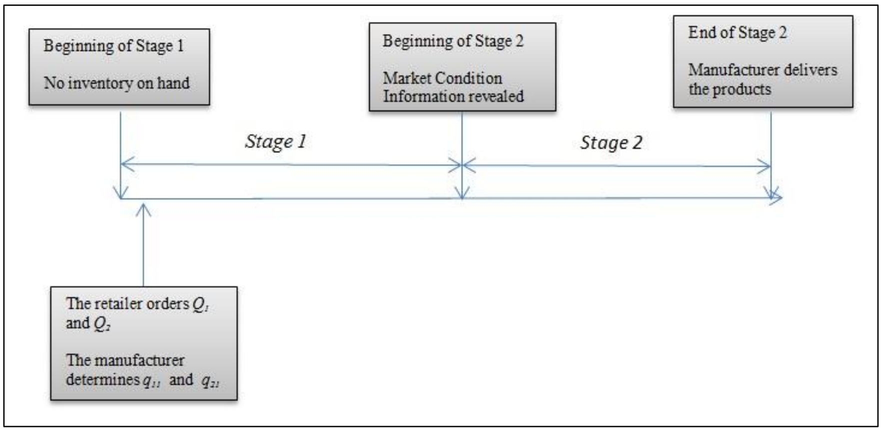

Figure 2 Sequence of Events in Benchmark 1

We start our analysis with the retailer's problem. The expected profit of the retailer is calculated by considering both market conditions. In such a setting, the problem is decomposable into two parts with respect to the product types since the demand of each product is independent of each other. The retailer has to solve the same problem twice for two products to determine the optimum order quantities at the beginning of first stage and the total profit of the retailer can be found by adding the profits earned for each product. That is, each part of the retailer's problem becomes a version of classical newsvendor model for two products.

$$\pi_r\left(Q_1,Q_2\right)=\pi_r(Q_1)+\pi_r(Q_2)$$

The same profit function is valid for both product 1 and product 2 except the demand distribution functions. The further analysis and discussions for B1 setting will be given only for product 1.

The expected profit function of the retailer for product 1 is given by the following equation.

$$\pi_r\left(Q_1\right)=\sum_{j=1}^{n} b_j \left[\begin{array}{l} p_1\left[\int_0^{Q_1} x_1 f_{1j}(x_1)dx_1+\int_{Q_1}^{\infty} Q_1 f_{1j}(x_1)dx_1\right]+ \\ s_1\left[\int_0^{Q_1}(Q_1-x_1)f_{1j}(x_1)dx_1\right]-w_1Q_1 \end{array}\right] \tag{15}$$

Note that, b_i denotes the probability that $Y=j$, probability of market condition as in the case of quantity flexibility setting.

Lemma 2: $\pi_r\left(Q_1\right)$ is strictly concave.

Proof: First and second derivatives of the retailer's profit $\pi_r\left(Q_1\right)$ are given in Equation (16) and Equation (17).

$$\begin{aligned}\frac{d\pi_r(Q_1)}{dQ_1} &= \sum_{j=1}^{n} b_j\left[p_1(1-F_{1j}(Q_1))+s_1F_{1j}(Q_1)-w_1\right] \\ &=(s_1-p_1)\sum_{j=1}^{n} b_j F_{1j}(Q_1)+(p_1-w_1)\end{aligned} \tag{16}$$

and

$$\frac{d^2\pi_r(Q_1)}{d^2Q_1} = \sum_{j=1}^{n} b_0\left[-p_1 f_{1j}(Q_1)+s_1 f_{1j}(Q_1)\right]<0 \tag{17}$$

The lemma directly follows from the second derivative. □

Proposition 4: The global maximum of $\pi_r\left(Q_1\right)$ is given by the unique solution to $\frac{d\pi_r(Q_1)}{dQ_1}=0.$

Proof: By Lemma 2, $\pi_r\left(Q_1\right)$ is strictly concave. Hence, all we need to show is that $\frac{d\pi_r(Q_1)}{dQ_1}=0$ has a finite solution, which follows since

$$\frac{d\pi_r(Q_1)}{dQ_1}\Big|_{Q_1\to\infty} = (s_1-p_1)(Q_1)+(p_1-w_1)<0 \tag{18}$$

and

$$\frac{d\pi_r(Q_1)}{dQ_1}\Big|_{Q_1\to-\infty} = (p_1-w_1)>0 \tag{19}$$

Let $Q_1^{'}$ denote the global maximum of $\pi_r\left(Q_1\right)$. That is, let $Q_1^{'}$ be the unique solution to $\frac{d\pi_r(Q_1)}{dQ_1}=0.$ □

Note that in this setting, the retailer seems obviously disadvantageous when compared to quantity flexibility contract with information update setting because it does not have the option to observe the market condition and to collect information. Consequently, it has to order before the market information is revealed, based on insufficient information.

The manufacturer's problem in this benchmark scenario case is straightforward because no order update is allowed. The retailer directly determines the final order quantities, Q_1 and Q_2 for both products. Therefore, the manufacturer does not have any uncertainty and does not use expedited production mode, produces in the first stage what is ordered by the retailer. The manufacturer's profit depends only on the values of retailer's order quantities Q_1 and Q_2 for each product regardless of market condition and other details.

4.2. BENCHMARK II: FULL FLEXIBILITY

In the second benchmark setting (B2), the retailer gives its order after the market condition information is revealed at the beginning of the second stage. Therefore, the retailer does not have an initial order quantity. The manufacturer, on the other hand, has to start production at the beginning of first stage based on its decision on production quantities. Once the retailer gives its order, the manufacturer determines the second stage production quantities based on its initial production quantities and retailer's order so that it can meet retailer's order fully.
Sequence of events in the second benchmark scenario, which is also given in Figure 3, is as follows;

1. At the beginning of stage 1, the manufacturer determines regular production quantities for each product, q_{i1} based on the available information about the uncertain market demand.
2. At the beginning of stage 2, the market condition information is revealed and then the retailer gives its order, Q_1 and Q_2.
3. The manufacturer determines its final production quantities for each product, q_{i2} based on retailer's order and its initial production quantities.
4. The manufacturer delivers the goods to the retailer and sold out leftovers at a discounted price.
5. Market demand for each product is realized and filled as much as possible. The items that are not sold at the regular selling season are cleared out at a discounted price by the retailer.

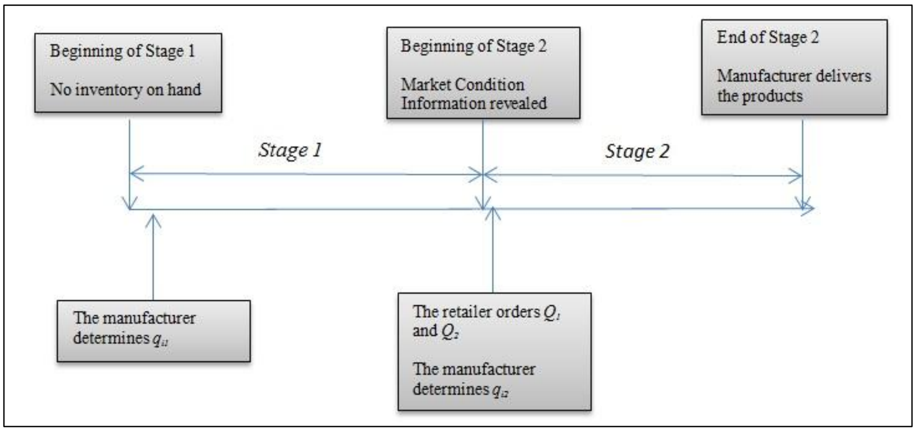

Figure 3 Sequence of Events in Benchmark 2

We start our analysis with the retailer's problem. As the retailer determines its orders after observing the market condition without any restriction, its problem reduces to two newsvendor problems, one for each product. Although the newsvendor problem is very well-known and quite straightforward we still produce technical analysis for the sake of completeness.
Given *Y=j*, the retailer's profit from product *i* can be expressed as follows;

$$\pi_r^{(i)}(Q_i \mid Y=j) = p_i\left[\int_0^{Q_i} x_i f_{ij}(x_i)dx_i + \int_{Q_i}^{\infty} Q_i f_{ij}(x_i)dx_i\right] + s_i\left[\int_0^{Q_i}(Q_i - x_i) f_{ij}(x_i)dx_i\right] - w_i Q_i \tag{20}$$

Proposition 5: The optimal order for product *i* is given by $(Q_i^* \mid Y=j) = F_{ij}^{-1}(\frac{p_i - w_i}{p_i - s_i})$.

Proof: First and second derivatives of the retailer's profit $\pi_r^{(i)}(Q_i \mid Y=j)$ are given in Equation (21) and Equation (22).

$$\frac{d\pi_r^{(i)}(Q_i)}{dQ_i} = \left[p_i(1-F_{ij}(Q_i)) + s_i F_{ij}(Q_i) - w_i\right] \tag{21}$$

$$\frac{d^2\pi_r^{(i)}(Q_i)}{d^2Q_i} = \left[-p_i f_{ij}(Q_i) + s_i f_{ij}(Q_i)\right] < 0 \tag{22}$$

Hence, $\frac{d}{dQ} = 0$ gives the solution. □

Having characterized the optimal order quantity at the retailer for each product given the market condition, we then calculate the optimal expected profit of the retailer before market condition is revealed as follows;

$$\pi_r = \sum_{j=1}^{n} b_j \sum_{i=1}^{2} \pi_r^{(i)}(Q_i^* \mid Y = j) \tag{23}$$

The manufacturer's problem in the second benchmark setting is very similar the one in QF setting. At the beginning of second stage the market condition information is revealed and the retailer gives its order. The manufacturer then decides how much more to produce, if any, according to its first stage production quantities and retailer's orders for each product. Therefore, second stage problem of manufacturer is nothing else than restocking again, as in the case of QF model. Hence, we have $q_{12}^* = \max\{0, Q_1 - q_{11}\}$ and $q_{22}^* = \max\{0, Q_2 - q_{21}\}$, where Q_1 and Q_2 denote order quantities of the retailer.

Since the retailer gives its order in the second stage, the manufacturer has to decide how much to produce in the first stage before retailer's order in order to benefit from the lower production cost of first stage as much as possible. To determine its production quantities the manufacturer has to calculate what retailer's order quantities could be. When the manufacturer calculates what the retailer would order according to the realized market condition, it can also derive the optimal allocation of the initial order to the individual products for a realization of the market condition, which effectively is the demand distribution that the manufacturer faces as in QF setting. Hence, we can conclude that the only source of uncertainty that the manufacturer faces is the market condition information. Thus, the demand of product *i* that manufacturer faces can be characterized as $D_i = Q_i^*(Y = j)$ if $Y = j$, where Q_i^* can be derived from Equation (21) in Proposition (5).

As in QF model, the manufacturer's problem can be solved for each product type separately and optimal production quantities, q_{1i}'s, for each product is given as follows;

$$q_{1i}^* = \min\left\{ Q_i^*(Y = j) : \sum_{k=1}^{j} b_k \geq \frac{d_2 - d_1}{d_2 - r_i} \right\} \tag{24}$$

And the expected profit of the manufacturer is given as follows;

$$\pi_m(q_{11,} q_{21}) = \pi_m^{(1)}(q_{11}) + \pi_m^{(2)}(q_{21})$$

where

$$\pi_m^{(1)}(q_{11}) = \sum_{j=1}^{n} b_j \pi_m^{(1)}(q_{11} \mid Y = j) \text{ and } \pi_m^{(2)}(q_{21}) = \sum_{j=1}^{n} b_j \pi_m^{(2)}(q_{21} \mid Y = j)$$

CHAPTER 5

NUMERICAL ANALYSIS

In this chapter, our aim is to analyze the effects of various problem parameters on the order quantities and expected profits of the retailer and manufacturer under the joint quantity flexibility contract model (QF). We also aim to observe the cost and benefits of QF model to the retailer and the manufacturer. Besides, we try to reveal whether and under what conditions QF is beneficial to parties compared to the benchmark settings; especially to B1, which represents the no flexibility setting and is more commonly used in practice.

Throughout the numerical analysis, market demands of products are assumed to be normally distributed with mean and standard deviation defined in Table 5 in two possible market conditions; Market Condition 1 (MC1) with probability *b* and Market Condition 2 (MC2) with probability *(1-b)*. Note that the demands of products are assumed to be independent of each other.

Table 5 Demand Parameters Definitions in Market Conditions

	Market Condition 1 (MC1)	Market Condition 2 (MC2)
Product 1	(σ_1, μ_1)	(σ_3, μ_3)
Product 2	(σ_2, μ_2)	(σ_4, μ_4)

In order to compare the performances of QF, B1 and B2, we first evaluate percentage improvement that QF provides over B1. Percentage improvement is calculated as in Equation (25).

$$\%imp = \frac{QF - B1}{B1} \times 100 \tag{25}$$

B1 may be considered as the worst setting for the retailer since it has to order at the beginning of stage 1 before market condition information is revealed and there is no order update flexibility. On the other hand, B1 seems to be the best setting for the manufacturer at first because it faces no uncertainty in B1 and directly produces and sells according to the retailer’s orders. However, this line of thought may be misleading as order update flexibility may lead to an aggregate increase in the retailer’s orders leading to a possible increase in the manufacturer’s profits as well. In fact, there are many cases, where QF is more profitable for the manufacturer compared to B1. Hence, it is also worthwhile to examine the cases under which the manufacturer benefits from QF.

B2 provides the best setting for the retailer since it allows the retailer to postpone its ordering decisions until after the market condition is revealed. Therefore, in this setting, the uncertainty is minimized from the retailer’s perspective.

The difference between the retailer’s expected profits in B2 and B1 represents the maximum available profit improvement for the retailer. In order to evaluate the efficiency of the QF, we define another performance measure; how much of the available improvement from B1 to B2 can be achieved by QF, which is presented in Equation (26).

$$\%imp2 = \frac{QF - B1}{B2 - B1} \times 100 \qquad (26)$$

Other than percentage improvements, we also examine the expected profit of the parties, expected profit of the supply chain, total order quantities and the difference in order quantities of parties in different settings.

In the numerical analysis, we generally started with a base setting and changed 1 or 2 parameters at a time in order to reveal their effects on the benefits in QF model. In total, we conducted over 100,000 experiments with different parameter settings. Mostly, we focused on the demand structure of products, profitability of the products at the retailer and the probability of market condition.

5.1. DEMAND STRUCTURE

The demand structure of products in market conditions is a major factor that affects the performance of the QF model as it directly determines the problem environment. The demand structures we consider in this study may be grouped into 2 categories; stationary demand, where total demand size is stationary in market conditions and non-stationary demand, where total demand size of the products change in market conditions.

For each demand structure, we keep some of the parameters fixed and perform a set of experiments for the rest of the parameters (Please see Table 6). Table 7 and Table 8 show mean demand values for products used in stationary and non-stationary demand structure cases.

Table 6 Common Parameters for Demand Structure Case Experiments

Fixed Parameters			
w_1=60 w_2=60	s_1=0 s_2=0	d_{11}= d_{12}=10 d_{21}= d_{22}=20	r_1=5 r_2=5
Changing Parameters			
p_1	100	200	300
p_2	100	200	300
b	0.2	0.5	0.8
σ_1 / μ_1	1/3	1/7	
σ_2 / μ_2	1/3	1/7	
σ_3 / μ_3	1/3	1/7	
σ_4 / μ_4	1/3	1/7	

Table 7 Mean Demand Values for Stationary Demand Structure Cases

Stationary Demand Structure				
(μ_1, μ_2) (μ_3, μ_4)	(120,80) (80,120)	(150,50) (50,150)	(120,80) (150,50)	(120,80) (50,150)
(μ_1, μ_2) (μ_3, μ_4)	(120,80) (100,150)	(150,50) (62.5,187.5)	(120,80) (187.5,62.5)	(120,80) (62.5,187.5)

Table 8 Mean Demand Values for Non-Stationary Demand Structure Cases

Non-Stationary Demand Structure			
(μ_1, μ_2) (μ_3, μ_4)	(120,80) (160,240)	(150,50) (100,300)	(120,80) (300,100)
(μ_1, μ_2) (μ_3, μ_4)	(120,80) (100,300)	(120,80) (240,160)	(120,80) (360,240)

In total, the parameter settings that we consider result in 3,456 cases for the stationary demand structure and 2,592 cases for the non-stationary demand structure.
Percentage improvements due to QF compared to B1 for these cases are summarized in Table 9.

Table 9 Summary of Results for Demand Structure Cases for the Parameter Setting given in Table 6, 7 and 8

	Retailer		
	Average % Improvement	Maximum % Improvement	Minimum % Improvement
Stationary	9.84	84.76	0.28
Non-Stationary	3.76	43.17	-1,5E-05
	Manufacturer		
Stationary	-3.2	65.7	-31.5
Non-Stationary	-5.6	52.3	-40.9

QF model provides as high as 84.76% improvement when the demand structure is stationary and this improvement is obtained in the parameter setting given in Table 10.

Table 10 Parameter Setting of Maximum Improvement Case in Stationary Demand

$\mu_1=150$	$\mu_2=50$	$b=0.5$
$\mu_3=50$	$\mu_4=150$	$p_1=100$
$\sigma_1 / \mu_1 = \sigma_4 / \mu_4$ =1/7	$\sigma_2 / \mu_2 = \sigma_3 / \mu_3$ =1/4	$p_2=100$
$w_1=60$	$w_2=60$	$d_{11}=d_{12}=10$
$r_1=r_2=5$	$s_1=s_2=0$	$d_{21}=d_{22}=20$

The reason why this particular parameter setting provided maximum improvement is that in this setting, the difference of product's demands in each market condition is higher compared to other cases in stationary demand examples. Having 150 unit of expected demands from product 1 while 50 from product 2 represent a quite extreme difference in demands of products.

Besides, the other parameters such as selling prices of products and market condition probability seem to effect the percentage improvement compatibly with our findings detailed observations. In the maximum percentage improvement case, the selling prices of the products are equal, which seem to help the retailer higher percentage improvement. When the market condition probability is closer 0.5, the more benefit the retailer takes in the QF model. The details of these findings will be given later on this chapter.

As a result, we conclude that the QF model is more advantageous in an environment, where the total demand can be more or less stationary but the division of total demand to the products differs. This conclusion is also compatible with the nature of the model in the sense that its most important characteristic is to enable the retailer to allocate the total order quantity to the products. This result is especially meaningful when the products are similar to each other as mentioned in Chapter 1.

In the non-stationary demand structure case, together with the uncertainty in the demands of the individual products, the total volume of the sale is uncertain as well. Therefore, having order allocation flexibility does not help QF much to be more beneficial than benchmark setting. If there was a model, which can enable the retailer to make better forecast on the total volume of the market in advance, it would definitely achieve higher percentage improvement in non-stationary demand structure cases.

Nevertheless, the maximum percentage improvement in the non-stationary demand case is significantly high. This is because basically one case that provides the maximum percentage improvement in non-stationary demand structure cases increases the average. The parameter setting of this case is provided in Table 11.

Table 11 Parameter Setting of Maximum Improvement in Non-Stationary Cases

$\mu_1=150$ $\mu_3=100$	$\mu_2=50$ $\mu_4=300$	$b=0.5$
$\sigma_1/\mu_1=\sigma_2/\mu_2=\sigma_3/\mu_3$ $=1/3$	σ_4/μ_4 $=1/7$	$p_1=100$ $p_2=100$
$w_1=60$	$w_2=60$	$d_{11}=d_{12}=10$
$r_1=r_2=5$	$s_1=s_2=0$	$d_{21}=d_{22}=20$

In this setting, the demand structure is non-stationary but there is negative change (not complete opposite as mentioned in oncoming discussions) of the product demand as market condition changes. That is, in MC1, the demand for product 1 is high, where in MC2 the demand for product 2 is high. Besides, the differences of products demand especially demand for product 2 (the difference is 250) is high between market conditions. This demand structure together with conditions the other parameters provides such as the selling $p_2=300$ turns out to be this case provides high percentage improvement. Although this case shows that in non-stationary demand structure significant improvement can be obtained, the average percentage improvement is not promising. Therefore, we do not further analyze the effects of other parameters in this setting. Instead, we focus on the stationary demand cases throughout the rest of the analysis in order to better show the parameter effects and the environment in which QF is more advantageous.

Yet an example for non-stationary demand structure cases is given below for the sake of completeness. In this example, market condition probability effect is generated for the parameter set given in Table 12 to show percentage improvements for the retailer, manufacturer and total supply chain with respect to *b*, which is provided in Figure 4.

Table 12 Parameter Setting for Non-Stationary Demand Example

$\mu_1=100$	$\mu_2=150$	$p_1=160$
$\mu_3=200$	$\mu_4=300$	$p_2=100$
$\sigma_i = \mu_i / 4$	$w_1=60$	$w_2=50$
$r_1=r_2=5$	$s_1=s_2=10$	$d_{11}=d_{12}=10$ $d_{21}=d_{22}=20$

As mentioned above, QF model does not provide flexibility on the aggregate order quantity and its benefits under such a demand setting are limited. The uncertainty in the market is mainly about the overall market size, not about how the aggregate demand is distributed over the products. That is why order allocating flexibility does not help the retailer for better ordering. Therefore, QF performs marginally better than B1 in such cases and the percentage improvement it provides is not significant. Also, the manufacturer's situation is not much different. Manufacturer's percentage improvement in QF is negative for different *b* values. The total of supply chain does not as well improve due to very small improvement in retailer.

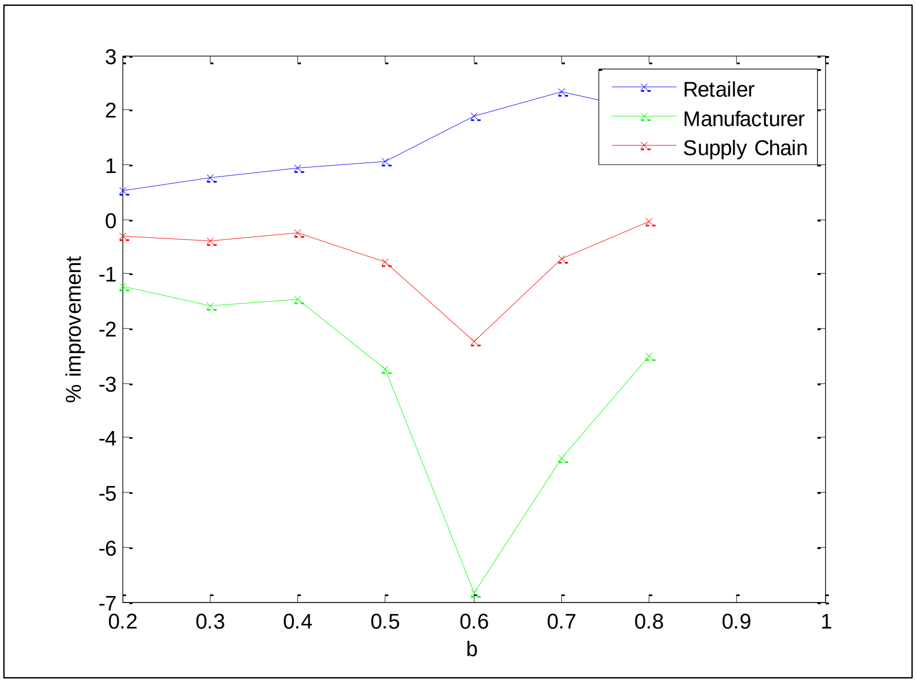

Figure 4 Percentage Improvement of Parties in Non-Stationary Demand for the Parameter Setting given in Table 12

5.1.1 Stationary Aggregate Demand

Through our extensive computations aforementioned, we observed that the quantity flexibility arrangement is significantly beneficial for the retailer when the total expected demand is stationary. The benefits become even larger if one of the products has larger demand in one market condition, and the other one has larger demand in the other market condition.

We change the way how the total market demand is distributed between the products by keeping total demand size fixed. In doing this, we construct the markets as complete opposites. That is, if μ_1=60 (μ_2=140) in MC1, then μ_3=140 (μ_4=60) in MC2. Together with demand parameter change, we perform a quite large set of experiment for stationary demand (complete opposite cases) by changing the selling prices and market condition probability by keeping other parameters fixed. The table shows in which range the parameters are changed is defined in Table A1 in Appendix A. The results for these experiment settings are provided in Table B1 in Appendix B.

To illustrate our finding, we work on the parameter setting presented in Table 13.

Table 13 Parameter Setting for Stationary Complete Opposite Demand Example

$\mu_1=60$	$\mu_2=140$	$b=0.4$
$\mu_3=140$	$\mu_4=60$	$p_1=160$
$\sigma_i = \mu_i / 4$	$w_1=60$ $w_2=50$	$p_2=100$
$r_1=r_2=5$	$s_1=s_2=10$	$d_{11}=d_{12}=10$ $d_{21}=d_{22}=20$

Figure 5 illustrates our findings from the retailer's perspective, where the horizontal axis is $\mu_{1,}$ mean demand of the product 1's demand distribution in MC1. Note that $\mu_1+\mu_2= \mu_3+\mu_4=200$. As the blue line in Figure 5 shows, the percentage improvement of the retailer increases as the difference in expected demand of the products in each market condition increases. The benefits when the difference between product's demands in market conditions is high, can be very substantial; the percentage improvement is 86.7% when μ_1=170 and 1.6% when μ_1=110.

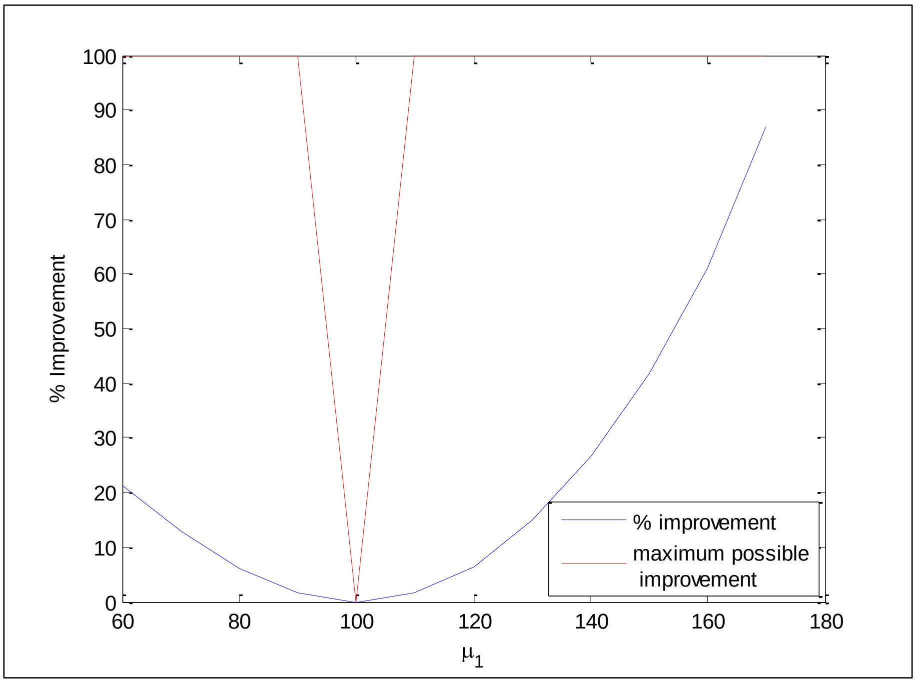

Figure 5 Percentage Improvement and Maximum Improvement for Stationary Demand Example for the Parameter Setting given in Table 13

These observations are quite intuitive for the following reasons. In B1, the retailer has to place firm orders before the market condition information reveals, and this may mean a very large uncertainty in product's demand. For instance, consider the difference in demands of product 1 in market conditions, for the case with μ_1=60 and μ_3= 140; the retailer's order quantities have to account for this difference. As a result, one of the products turns out to be over-ordered, whereas the other one is under-ordered in B1. However, in QF, the retailer has the flexibility to allocate the total order quantity to individual products after observing the market condition, which is a very valuable piece of information.

Figure 5 also illustrates the efficiency of the QF model. The red line in Figure 5 shows the maximum percentage improvement that can be possible between the difference in expected profits of the retailer in B2 and B1, as described in Equation (26). It is observed that for settings, where the individual demands of the products are even moderately different, the QF model can generate almost all the potential benefits for the retailer.

For the manufacturer, QF seems not beneficial for the setting provided in Table 13. The expected profit of the manufacturer in QF presented in Table B2 in Appendix B is lower than the expected profit in B1. This is an expected result in general since in B1, the manufacturer does not have any uncertainty unlike in QF and B2, whereas it has to share the uncertain market and demand risk with the retailer by letting it to update its order in QF and to give its order after market condition information revealed. In addition, particularly for this setting, total amount of sale of manufacturer, that is the total order of the retailer from product 1 and 2, is at highest almost equal and generally lower than the one in QF. For instance, the case when μ_1= 170 is the worst case for the manufacturer and we see that Q_1+Q_2 in B1 is 271.45 and Q is 214.05 in QF. This decrease in total sale cause the manufacturer has a -27.9% as also shown in Figure 6.

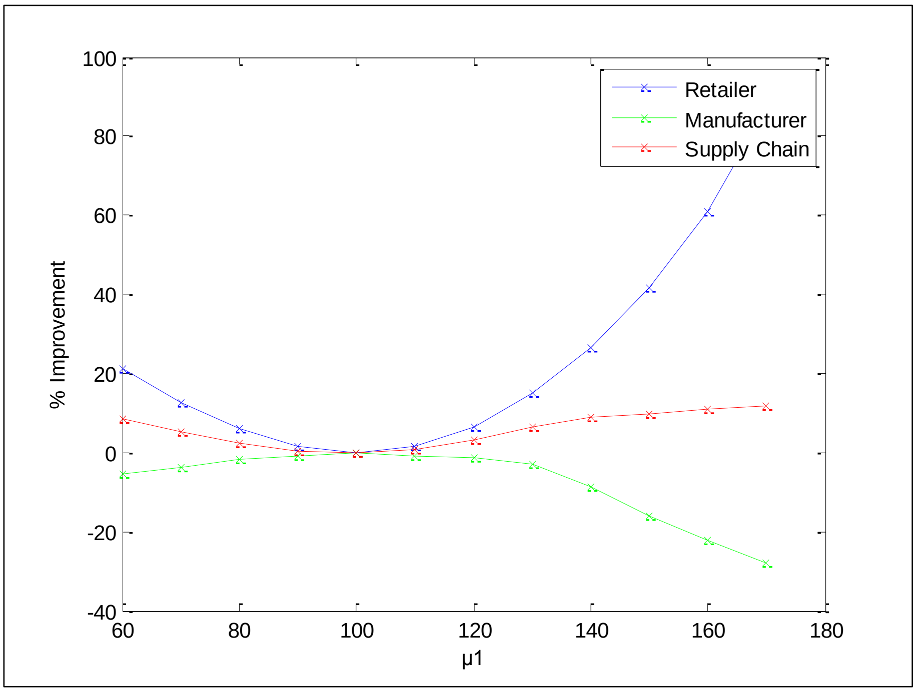

Figure 6 Percentage Improvement of the Parties in Stationary Demand Example for the Parameter Setting given in Table 13

Figure 6 shows the percentage improvement in the expected profits of the retailer, manufacturer and the supply chain. Although the manufacturer is disadvantageous in QF, the total of supply chain performance still improves in QF up to 11.95 %. Note that the run results are available in Table B3.

In summary, we can conclude that the improvement in the retailer's expected profit become more apparent and significant when the aggregate market size is stationary regardless of the market condition and the difference in product demands are high, especially when demand is complete opposite in market conditions. However, it should also be noted that, no totally symmetrical demand structure is required for this observation to be valid. For instance, in a stationary demand setting, where all the parameters other than demand parameters are equal to the setting given in Table 13 and when $\mu_1=170$, $\mu_2=30$ and $\mu_3=60$, $\mu_4=140$, the retailer's percentage improvement in QF is 30 %.

Although the manufacturer does not seem to benefit from the flexibility scheme in this setting, there are other cases, where the manufacturer is better off with QF. We next have a closer look at the effects of QF to the manufacturer.

5.1.2 Manufacturer's Perspective

As briefly mentioned above for the discussion for Figure 6, in QF and B2, the manufacturer has to share the risk of uncertain market demand and condition with the retailer by providing order update or later ordering flexibilities to the retailer. This way, the retailer has the advantage of using some of the production lead time of the manufacturer for market condition observation before it gives its final order. These models bring extra uncertainty to the manufacturer instead of producing directly what is ordered by the retailer and still has the risk of having unsold items at hand at the end of selling season.

At first, the manufacturer seems to be disadvantageous in QF and B2 due to the nature of the models mentioned above. However, it is not always the case because there are several cases, where the retailer increases the total order quantity by taking advantage of the QF model. An increase in the total order quantity of the retailer naturally increases the total sales of the manufacturer and at the end; both retailer's and manufacturer's expected profits may improve compared to B1.

In the detailed computational analysis that we performed for the opposite stationary demand structure defined in Table A1 in Appendix A, percentage improvement of the manufacturer can be significantly high as well.

To illustrate, a case the manufacturer benefits from QF, we can consider the setting given in Table 14.

In this setting, percentage improvement of the manufacturer in QF is 150%. This is basically caused by the total amount of products the manufacturer sells. In QF total order quantity, Q, is 174.98 in QF, where Q_1+Q_2 is 64.56 in B1.

In general, we can say that the percentage improvement of the manufacturer is high when the difference of the order quantities of retailer in QF compared to B1. The most important factor that affects the percentage improvement is the total amount of sale for manufacturer because the profitability of the products for the manufacturer is very close to each other. Furthermore, it is observed that the manufacturer benefits from the

QF model when the difference in demands (in complete opposite demand structures) in market conditions is high, which is also valid for the retailer. In addition, manufacturer is also advantageous when the difference in selling prices is low.

Table 14 Parameter Setting for Maximum Improvement Case of the Manufacturer

$\mu_1=170$	$\mu_2=30$	$b=0.5$
$\mu_3=30$	$\mu_4=170$	$p_1=80$
$\sigma_i = \mu_i / 4$	$w_1=60$ $w_2=50$	$p_2=70$
$r_1=r_2=5$	$s_1=s_2=10$	$d_{11}=d_{12}=10$ $d_{21}=d_{22}=20$

In total, there are over 500 instances out of 23943 stationary demand cases, in which the manufacturer has 10% or more improvement.

On the other hand, when the retailer's expected profit improves by taking advantage of pooling effect (provided by aggregate ordering flexibility in QF), in other words when it makes more profit with less amount of total order, the manufacturer becomes disadvantageous. That is basically because in such cases the manufacturer has to face more uncertainty along with the decrease in total sales.

5.2 Selling Prices

We next consider the effects of the selling prices on the effectiveness of QF. We start with the analysis from the retailer's perspective.

5.2.1 Retailer's Perspective

The analysis was generated from a part of the sets of experiments introduced in Table A1 in Appendix A. To illustrate, we analyze the effect of selling prices by keeping the parameters given in Table 15 fixed and changing p_1 and p_2 (per 10 units) between 80-260 and 70-210 respectively.

Table 15 Parameter Settings for Selling Price Analysis

$\mu_1=170$	$\mu_2=30$	$b=0.4$
$\mu_3=30$	$\mu_4=170$	$w_1=60$
$\sigma_i = \mu_i / 4$	$r_1=r_2=5$	$w_2=50$
$d_{11}=d_{12}=10$	$d_{21}=d_{22}=20$	$s_1=s_2=10$

These 285 run results are summarized in Figure 7 that shows the percentage improvement with respect to profitability of products.

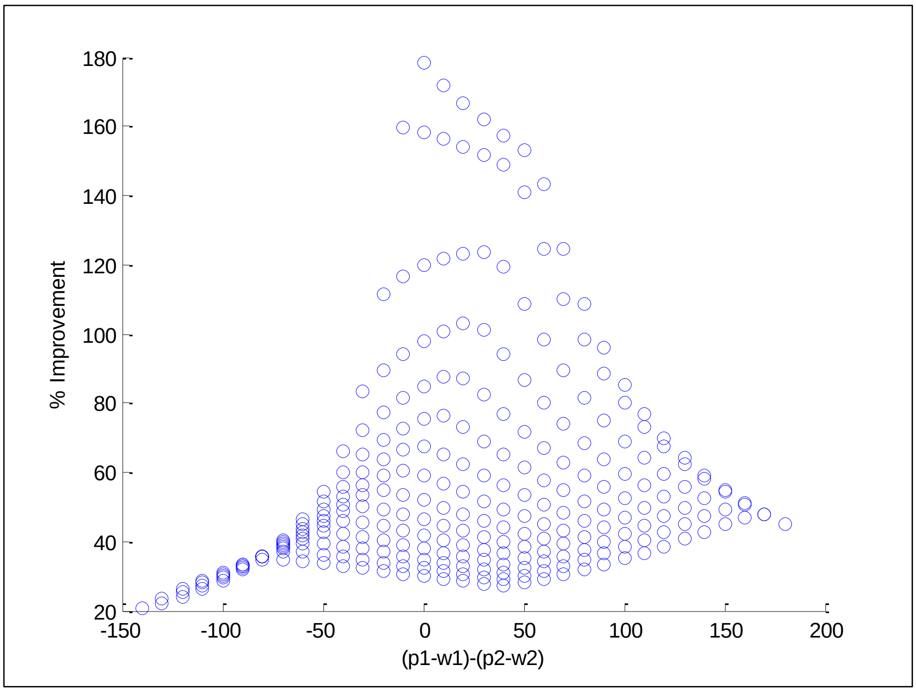

Figure 7 Percentage Improvement vs. Profitability of Products for the Parameter Setting given in Table 15

It should also be noted that due to the demand structure in this example (can be considered as an extreme case for complete opposite stationary demand) percentage improvement is not lower than 20%. We observe that highest percentage improvement is achieved when the difference in products profitability is zero. Besides, all the extremely high percentage improvements are always achieved when the difference is relatively low. When the profit of the retailer comprised of 2 products as in this study and the demand of products change significantly depending on the market condition, the behavior observed in profitability of the products makes more sense. Since, the equally profitable products increase the expected profit. When the difference between the profitability of products is high the profit of the retailer varies too much depending on the market condition. Namely, if the demand of the low profit product is high and of the high profit product is low then, the retailer acquires less profit.

To illustrate, consider the parameter setting example given in Table 15 for p_1-w_1=200 and p_2-w_2=20; in such case, the retailer's expected profit is 9,400 if MC2 is realized and 40,000 if MC1 is realized.

However, when the profitability of products is closer to each other, the retailer's expected profit will be closer to each other in each market condition. In QF, this characteristic is additionally advantageous combined with the order allocation flexibility. In fact, QF loses its advantage of order allocation flexibility if the profitability of products differ too much because in such a case, if the demand for profitable product is low, it does not make any difference whether or not the total order quantity optimally allocated since the product with low profitability cannot bring enough profit for the retailer to achieve a considerable improvement in QF. That is why higher percentage improvements can be achieved as the profitability of products gets closer.

Figure 8 is constructed for the parameter setting given in Table 15 to show the percentage improvement of the retailer with respect to p_1-w_1, for different values of p_2-w_2. According to Figure 8, percentage improvement decrease as p_2-w_2 increases. Besides, for greater values of p_1-w_1, percentage improvement decreases.

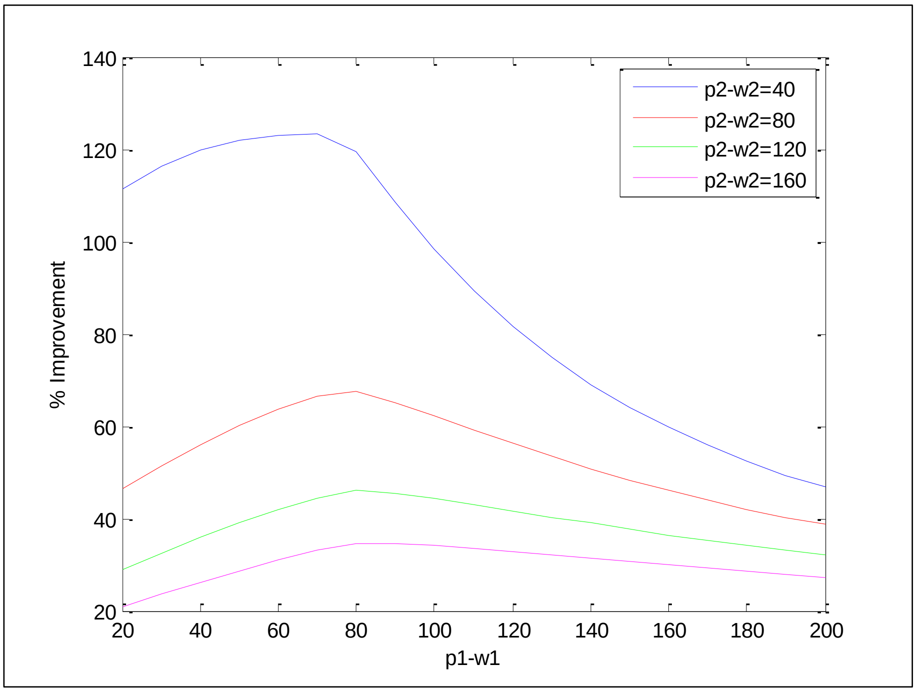

Figure 8 Retailer's Percentage Improvement vs. Profitability of Products for the Parameter Setting given in Table 15

Nevertheless, we cannot say that percentage improvement decreases while p_1-w_1 increases because for smaller values of p_1-w_1, percentage improvement increases first and then starts to decrease. This behavior cannot directly be explained by profitability of products but their effect on the total order quantities and the percentage improvement through which the retailer achieve implicitly. Table 16 shows the order quantities in accordance with QF and B1 models for the example setting for p_2-w_2=40 case only for simplicity.

Table 16 and Figure 8 indicate a breakpoint, at which the percentage improvement starts to decrease as p_1-w_1 increases. This point is p_1-w_1=*80* for this parameter setting. Since the cost of over-ordering is high compared to profit margin of product 1, Q_1 is low until p_1-w_1 increase a threshold value where product 1 becomes profitable enough to increase the order quantity. As a result, having a selling price or profitability worth to increase the order quantity for Q_1 corresponds to a threshold value for the retailer in B1. After that point, the QF model does not provide a significant benefit for the retailer and the percentage improvement starts to drop since B1 also performs well.

Table 16 Order Quantities of Retailer in QF and B1 for the Parameter Setting given in Table 15

Q1+Q2	***Q1***	***Q2***	***Q***	***Q1* MC1**	***Q2* MC1**	***Q1* MC2**	***Q2* MC2**	p_1-w_1
158.4	29.5	128.9	188.5	156.9	31.5	24.3	164.2	20.0
161.3	32.4	128.9	193.2	162.2	31.0	26.9	166.3	30.0
163.7	34.8	128.9	196.9	166.4	30.5	28.7	168.2	40.0
166.1	37.2	128.9	200.0	170.0	30.0	30.0	170.0	50.0
168.9	40.0	128.9	202.7	173.1	29.6	31.1	171.6	60.0
173.1	44.2	128.9	205.0	175.8	29.2	31.9	173.1	70.0
223.7	94.8	128.9	207.1	178.3	28.8	32.7	174.4	80.0
246.1	117.2	128.9	209.0	180.5	28.5	33.4	175.6	90.0
257.8	128.9	128.9	210.7	182.5	28.2	33.9	176.8	100.0
265.9	137.0	128.9	212.3	184.4	27.9	34.5	177.8	110.0
272.2	143.3	128.9	213.8	186.1	27.6	34.9	178.8	120.0
277.3	148.4	128.9	215.1	187.7	27.4	35.4	179.8	130.0
281.6	152.7	128.9	216.4	189.2	27.1	35.8	180.6	140.0
285.3	156.5	128.9	217.6	190.6	26.9	36.1	181.4	150.0
288.6	159.8	128.9	218.7	192.0	26.7	36.4	182.2	160.0
291.6	162.7	128.9	219.7	193.2	26.5	36.8	182.9	170.0
294.2	165.4	128.9	220.7	194.4	26.3	37.1	183.6	180.0
296.7	167.8	128.9	221.6	195.5	26.1	37.3	184.3	190.0
298.9	170.0	128.9	222.5	196.6	25.9	37.6	184.9	200.0

On the other hand, it can be observed from Table 16 that in QF, the total order quantity and its allocation to products seem not to be affected from the change in p_1-w_1. The reason why QF is not affected from the change in p_1-w_1 is the fact that the retailer determines an aggregate order quantity at the beginning and then makes the allocation according to demand of products. Aggregating the order quantity enables the retailer to minimize the risk and accordingly to order from product 1 without considering such threshold value.

In conclusion, when profitability of product 1 exceeds the threshold value, the retailer takes the risk of over-ordering in B1 and Q_1 rapidly increases. This causes the percentage improvement of the retailer makes a rapid decrease because after the threshold value, QF's aggregate order cannot provide additional advantage.

This behavior can be best explained by the change of $\overline{Q}$, which is defined as the difference of the total order quantities of retailer in QF and B1, $(Q_{QF}-(Q_1+Q_2)_{B1})$. The change of $\overline{Q}$ is illustrated in Figure 9, which is in parallel with the behavior discussed above for Figure 8.

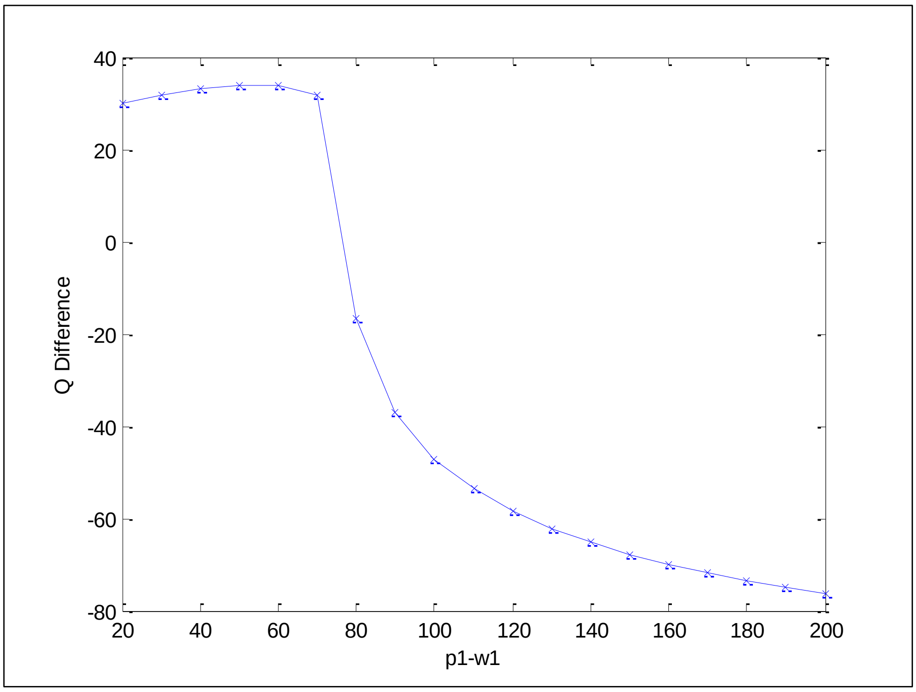

Figure 9 $\overline{Q}$ vs p_1-w_1 for the Parameter Setting given in Table 15

5.2.2 Manufacturer's Perspective

The percentage improvement of manufacturer's expected profit in QF seems to be higher when selling price of the products in retailer is lower. Besides, percentage improvement of manufacturer also tends to take higher values when the difference in p_1 and p_2 are closer to each other. To illustrate, we can focus on an example by changing p_1, where the other parameter settings are given in Table 17 as follows;

Table 17 Parameter Setting for Selling Price Analysis for the Manufacturer

μ_1=150	μ_2=50	p=0.5
μ_3=50	μ_4=150	p_2=100
$\sigma_i = \mu_i / 4$	w_1=60	w_2=50
r_1=r_2=5	d_{11}=d_{12}=10 d_{21}=d_{22}=20	s_1=s_2=10

In Figure 10, the retailer's behavior on the increase of p_1 is the same as discussed for Figure 8. The percentage improvement of the manufacturer is also in parallel with the change of $\overline{Q}$, when Figure 9 and Figure 10 are compared. The difference of the total order quantities of retailer in QF and B1, $\overline{Q}$, is one of the most critical factors that strongly affect the percentage improvement of manufacturer as it directly determines total sales of it.

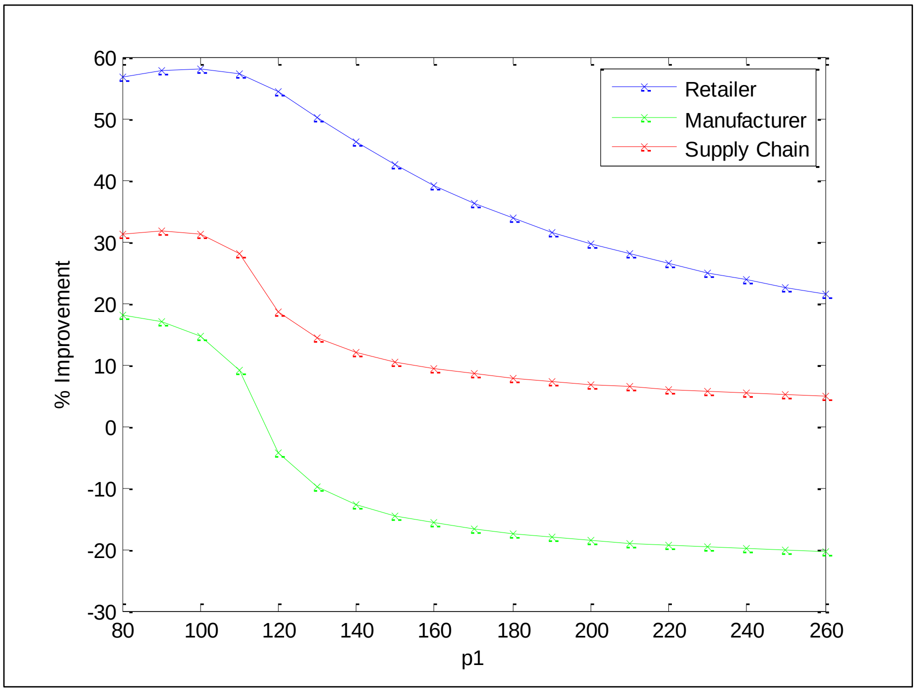

Figure 10 Percentage Improvement of Parties vs p_1 for the Parameter Setting given in Table 17

5.3 Probability of Market Conditions

Through our analysis, we observe that percentage improvement is higher when the probability of MC1, *b*, is between 0.4 and 0.6.; that is, when neither of the markets is more probable to observe. Figure 11 and Figure 13 show 2 different cases that support this observation in different demand parameters settings, where other parameters are common to both cases and are presented at Table 18 and Table 20, respectively.

For the setting provided in Table 18; in B1, the retailer tends to order closer to the mean demand of the demand distribution in each market condition for each product when probability of MC1, *b*, is close or equal to probability of MC2, *(1-b)*.

Table 18 Parameter Settings for the Analysis of *b* for the Supply Chain

$\mu_1=60$	$\mu_2=140$	$p_1=160$
$\mu_3=140$	$\mu_4=60$	$p_2=100$
$\sigma_i = \mu_i / 4$	$w_1=60$	$w_2=50$
$r_1=r_2=5$	$s_1=s_2=10$	$d_{11}=d_{12}=10$ $d_{21}=d_{22}=20$

On the other hand, in QF, *b* does not seem to be critical information for the retailer in respect of total order quantity. Table 19 shows that the total order quantity in QF is almost stable with respect to a change in *b* because estimating total order quantity at the beginning of first stage is critical for the retailer since it has the chance to allocate it later. However, in B1, the retailer's decision is affected by *b* because it orders based on *b* and demand of products in market conditions, at the beginning.

Table 19 Summary of Results for the Analysis of *b* for the Parameter Setting given in Table 18

QF							B1			
Q	***Q1* MC1**	***Q2* MC1**	***Q1* MC2**	***Q2* MC2**	**Expected Profit Retailer**	***b***	***Q1+Q2***	***Q1***	***Q2***	**Expected Profit Retailer**
216.2	67.5	148.7	154.5	61.7	13,831.6	0.2	214.8	147.4	67.4	12,578.2
215.6	67.4	148.2	154.2	61.5	13,468.8	0.3	213.8	142.1	71.7	11,657.7
215.1	67.3	147.8	153.8	61.3	13,106.3	0.4	214.2	135.1	79.1	10,819.2
214.5	67.2	147.4	153.5	61.0	12,744.0	0.5	223.2	124.9	98.3	10,146.0
213.9	67.0	146.9	153.1	60.8	12,382.1	0.6	223.8	106.4	117.4	9,782.8
213.3	66.9	146.4	152.8	60.6	12,020.4	0.7	210.2	82.3	127.9	9,819.4
212.7	66.8	145.9	152.4	60.3	11,659.1	0.8	209.2	74.1	135.1	10,102.4

Figure 11 shows the performance of all parties with respect to *b*. The retailer seems to be more advantageous although the total order quantity in QF is lower than the one in B1. The retailer takes the advantage of aggregate ordering (pooling effect) and decrease the total order quantity by using the advantage of allocating total order to individual products. In other words, in QF the retailer said to be more accurate in providing right product to the market with less amount of total product according to realized market condition. In B1, the retailer may be more likely to place wrong orders from products since it has to give its final order before the market condition information realize. This way the retailer in B1 turns out to be less profitable due to leftovers.

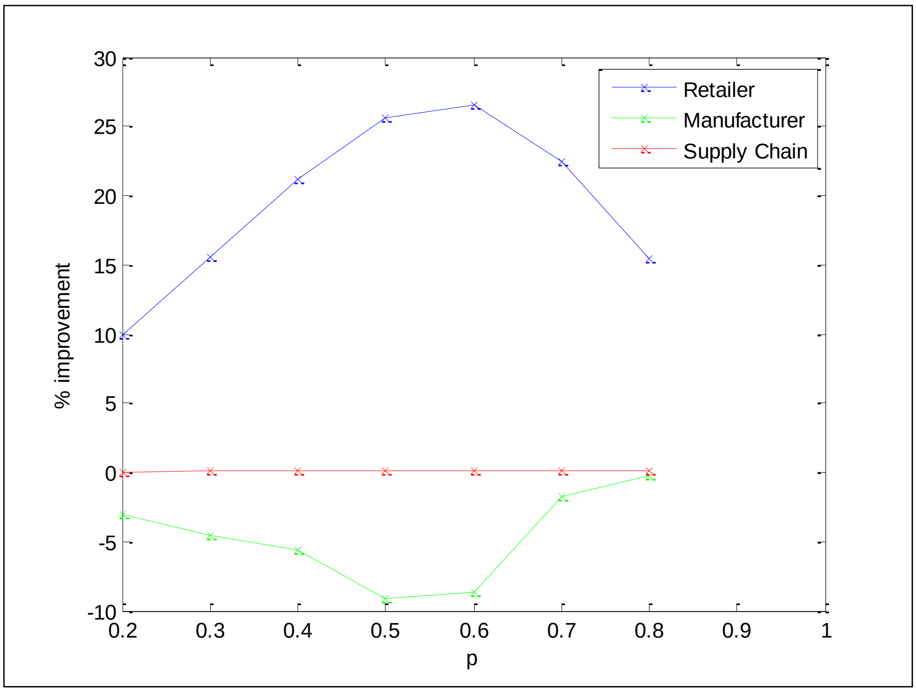

Figure 11 Percentage Improvement of Retailer, Manufacturer and Total Supply Chain for the Parameter Setting given in Table 18

Figure 11 reveals another result on the effect on probability of market condition, *b* on percentage improvement of the parties. For the retailer, the maximum improvements are achieved when *b* is 0.5 and 0.6 (for this parameter setting). This is because when the probability of either market condition to be realized closer to each other, the uncertainty in the demand of products increased compared to the cases where *b* is too small (favoring market condition 1) or too large (favoring market condition 2). In other words, when the occurrence of either market condition is equally likely, the demand of each product's realization in accordance with market condition is also equally likely and in such a case, retailer cannot prioritize a particular product. This in turns cause the order quantities of both product to increase although only one will turn to high demand realization. As a result, the amount of leftover at the retailer increases in B1. The benefit of QF become quite significant since such a case is very suitable to for the order allocation advantage of QF. The retailer can enjoy having right product at hand without increasing total amount of order, which increases the percentage improvement of the retailer and decrease the percentage improvement of the manufacturer due to the decrease in total amount of sale compared to B1.

For instance, in the example setting if MC1 is realized, which is more probable as $b=0.6$, the demand for second product will be higher. In B1, the retailer has to salvage some of the product 1 at the end, while it fails to fulfill the market demand of product 2. However, in QF, the retailer has the option to allocate more from Q to product 2 if MC1 is realized and obtain more profit compared to B1 by selling more product 2. In such a case percentage improvement is higher when $b=0.6$ since *QF* benefits the order allocation flexibility and sell more Product 2.

On the other hand, Figure 15 presents the effect of b on retailer's percentage improvement in the setting given in Table 20 and Table 21 shows the summary of results for this setting.

Table 20 Parameter Setting for the Analysis of *b* for the Retailer

$\mu_1=170$	$\mu_2=30$	$p_1=160$
$\mu_3=30$	$\mu_4=170$	$p_2=100$
$\sigma_i=\mu_i/4$	$w_1=60$	$s_1=s_2=10$
$r_1=r_2=5$	$w_2=50$	$d_{11}=d_{12}=10$ $d_{21}=d_{22}=20$

In Figure 12, percentage improvement of the retailer is graphed with respect to b in a very similar parameter setting given in Table 18 and used for Figure 11. The main difference is the demand of products in these two examples. The general behavior of the retailer is similar to each other in Figure 11 and Figure 12 and the main difference in Figure 12 is that percentage improvement reached to the highest level when $b=0.4$.

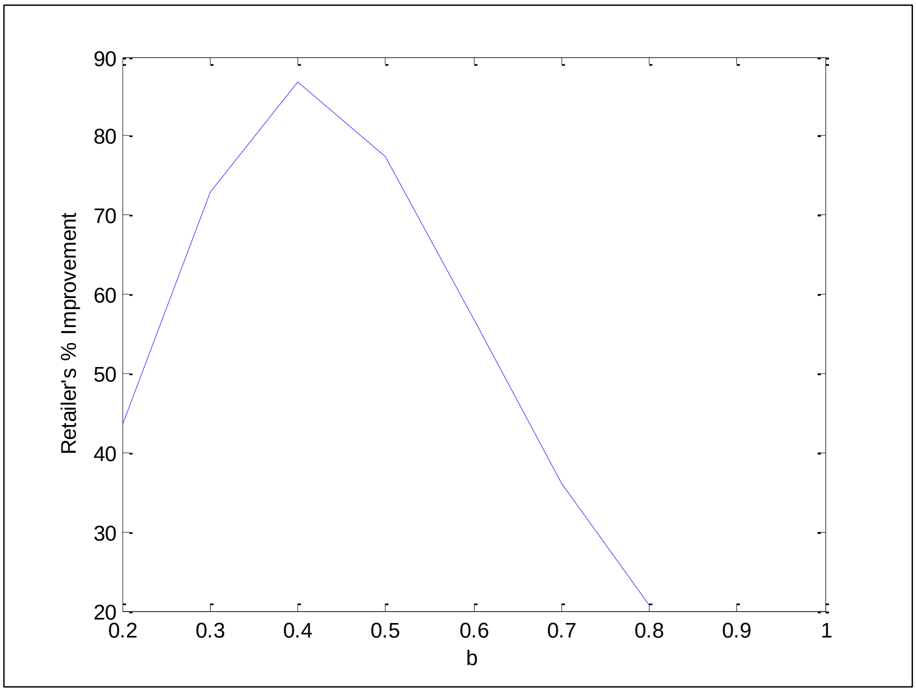

Figure 12 Effect of *b* on Percentage Improvement of the Retailer for the Parameter Setting given in Table 20

The reason of having maximum percentage improvement in Figure 12 at *b=0.4*, where in Figure 11 it is at *b=0.6* is because the demand for the first product is higher in the first stage in this setting and the first product is more profitable.

The summary of results is given in Table 21 and it seen from the table that the total order quantity for *b=0.4* in QF is not significantly greater than the one in B1, where the expected profit is significantly greater due to the fact that the retailer benefits from order allocation between products. Similar discussions for the performance comparison of the retailer, manufacturer and total supply chain are also valid for this example setting as discussed for Figure 11.

Table 21 Summary Results of the Analysis of b for Retailer for the Parameter Setting given in Table 20

QF							B1			
Q	***Q1* MC1**	***Q2* MC1**	***Q1* MC2**	***Q2* MC2**	**Expected Profit Retailer**	***b***	***Q1+Q2***	***Q1***	***Q2***	**Expected Profit Retailer**
211.8	182.4	29.4	33.5	178.3	10839.0	0.20	201.3	37.2	164.1	7545.9
213.0	183.3	29.7	33.6	179.3	11470.5	0.30	197.8	42.5	155.3	6634.8
214.1	184.1	29.9	33.7	180.3	12103.0	0.40	271.5	128.9	142.6	6479.5
215.1	185.0	30.1	33.8	181.2	12736.5	0.50	269.8	151.7	118.1	7178.8
216.0	185.7	30.3	33.9	182.1	13370.8	0.60	204.9	164.1	40.8	8531.6
216.9	186.4	30.5	34.0	182.9	14006.0	0.70	208.7	172.5	36.1	10287.9
217.8	187.1	30.7	34.1	183.7	14641.8	0.80	212.8	178.9	33.8	12120.6

Besides, in non-stationary demand structure settings, the behavior observed in the percentage improvement of parties with respect to the change in market condition probability, *b*, is compatible as well with the discussion in the stationary demand structure provided above.

5.4 Demand Variance

In this section, we analyze the effects of demand variance by considering σ / μ for each product in each market condition. For the analysis, we construct a set of parameter settings by keeping the means fixed and changing standard deviations by changing σ / μ in each case.

Furthermore, as discussed in detailed in Section 5.2, the difference of the profit margin and overage cost of the products play an important role in determining the order quantities. This fact has another role in the analysis of variance. In classical newsvendor systems with low profit margin compared to overage cost, the order quantity tends to decrease as variance in demand increases because the retailer tries to avoid excessive leftovers. For a product with low profit margin, it is not worth taking such risk of increasing order quantity, as the potential loss is greater than potential profit. Namely, the retailer focuses on minimizing its loss rather than increasing its income. On the contrary, if the profit margin is larger than the overage cost, an increase in variance indicates potential for more demand and the retailer increases the order quantity with the expectation of better profit.

Taking these into account, this section is divided into 2 sub-sections, where we discuss the effect of change in variance in low and high profit margin scenarios.

In order to analyze the effects of variance on the percentage improvement of the parties in the system, we conducted a series of experiments, where we keep the mean demand fixed and change standard deviation of the demand distribution by changing the coefficient of variation σ / μ. Please note for the analysis it is assumed that $\sigma_1 / \mu_1 = \sigma_3 / \mu_3$ and $\sigma_2 / \mu_2 = \sigma_4 / \mu_4$ and they are changed together in order to avoid the market condition uncertainty to effect the analysis of variance. Table C-1 and C-2 in Appendix C provide the details how the parameters are changed for the analysis for low and high margin scenarios.

5.4.1 Low Profit Margin

For the analysis of demand variance in low profit margin scenario, the experiments defined in Table C1 are conducted. Figure 13 shows the change in total order quantity in QF and B1 with respect to the change in standard deviation of product's demand distribution and Figure 14 shows how percentage improvement of the retailer changes as standard deviation of product's demand distribution changes.

As discussed at the beginning of section, it was expected that the order quantity of products decrease while standard deviation increases since the profit margin of the products are low compared to overage cost. In the observations percentage improvement behavior is matching with the decreasing behavior of the total order quantities, as expected. Figure 14 shows the percentage improvement of the retailer which decreases while $\sigma_1 / \mu_1 = \sigma_3 / \mu_3$ increases. Increase in variance leads the retailer to decrease the order quantity and any decrease in the order quantity turned out to decrease the percentage improvement of the retailer. The same is also true for the manufacturer. As Figure 15 suggests the percentage improvement of the manufacturer decreases while standard deviation increases and *Q* decreases.

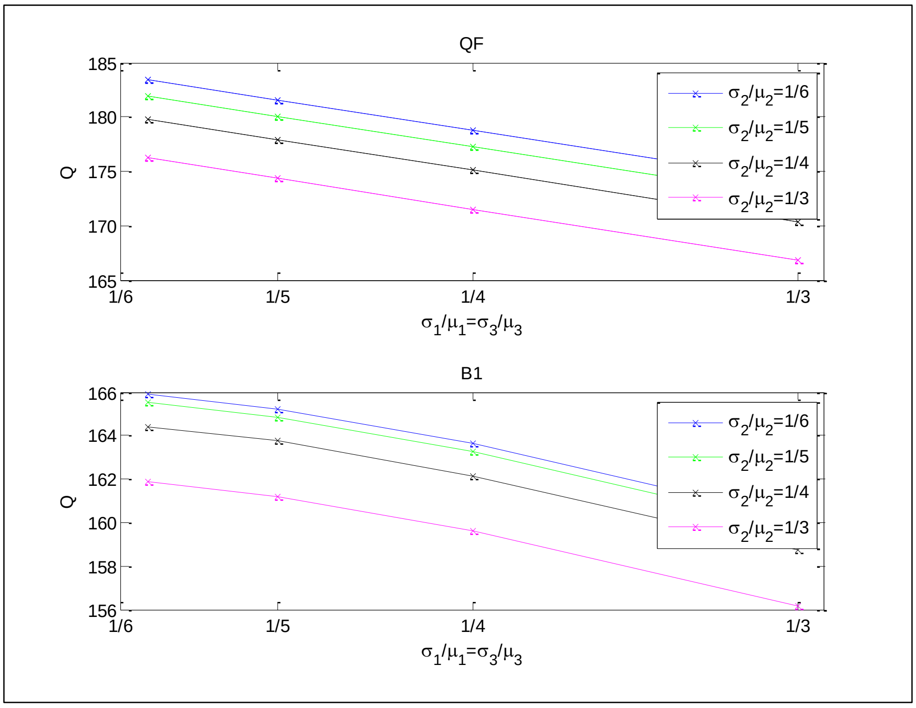

Figure 13 Q vs. Standard Deviation Change for the Parameter Setting given in Table C1 in Appendix C

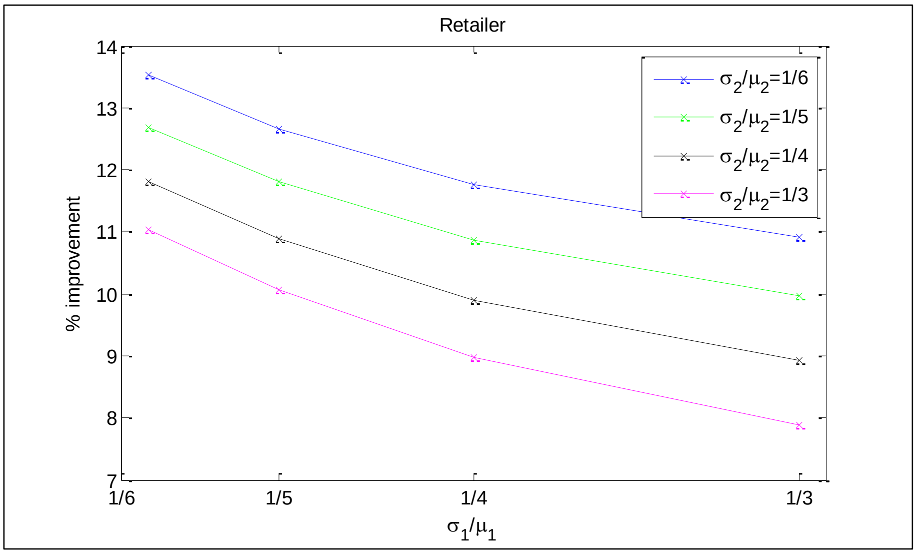

Figure 14 Retailer's Percentage Improvement vs. Standard Deviation Change for the Parameter Setting given in Table C1 in Appendix C

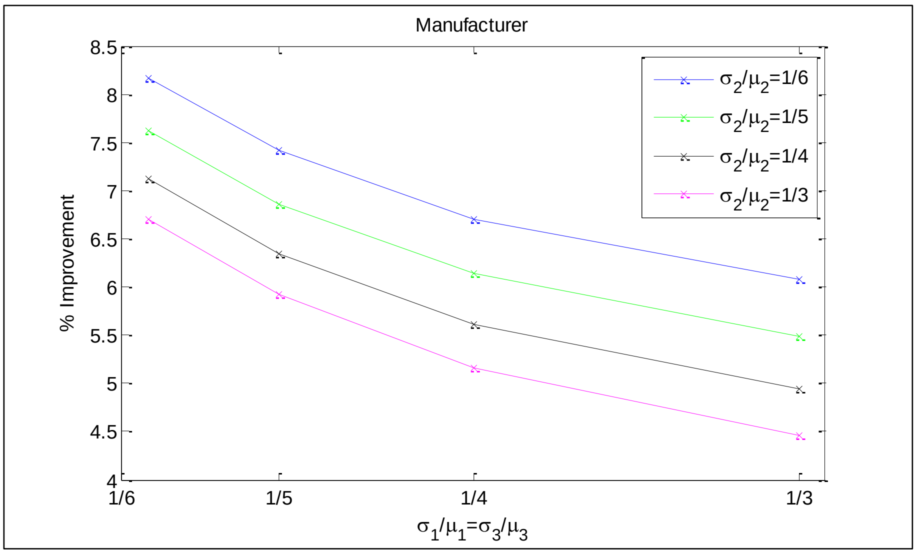

Figure 15 Manufacturer's Percentage Improvement vs. Standard Deviation Change for the Parameter Setting given in Table C1 in Appendix C

5.4.2 High Profit Margin

The experiments defined in Table C2 are constructed for the analysis of high profit margin case demand variance. Figure 16 shows the change of *Q* in QF and Table 22 show the results for *Q* and expected profit of the retailer in QF and B1. Figure 16 shows the percentage improvement with respect to the change in $\sigma_1/\mu_1=\sigma_3/\mu_3$ and $\sigma_2/\mu_2=\sigma_4/\mu_4$.

As Figure 16 suggests and as expected, *Q* increases when $\sigma_1/\mu_1=\sigma_3/\mu_3$ increases and also when $\sigma_2/\mu_2=\sigma_4/\mu_4$ increases. However, percentage improvement does not increase while *Q* increases due to variance increase decrease the profit. Table 22 shows that the expected profit of the retailer decreases as σ_1/μ_1 increases. This can indicate one thing; the retailer cannot sell as much as it orders. In QF, the increase in *Q* and therefore decrease in expected profit is higher, which at the end cause a decrease percentage improvement that QF achieves as σ_1/μ_1 increases.

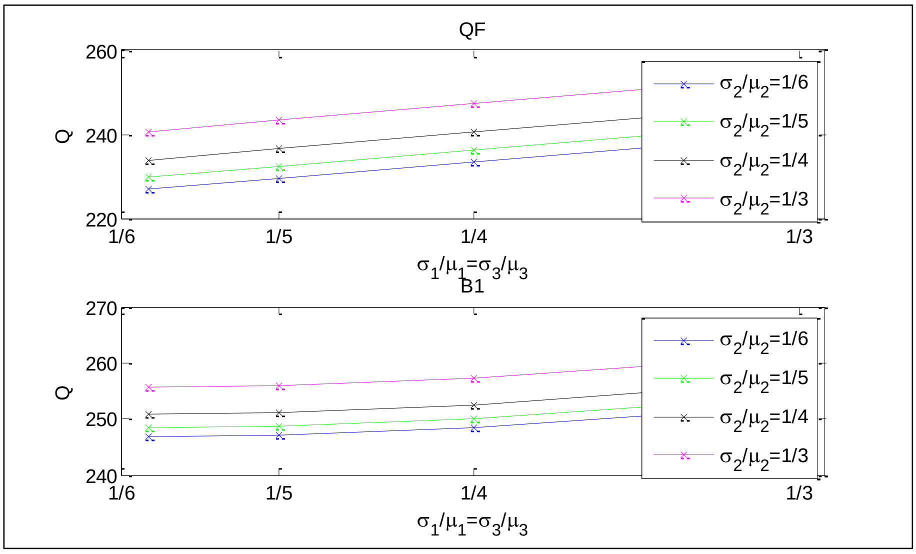

Figure 16 Changes in *Q* for the Parameter Setting given in Table C2 in Appendix C

Table 22 Order Quantity and Expected Profit of Retailer in QF & B1 for the Parameter Setting given in Table C2 in Appendix C

QF							**B1**			
Q	**Q_1 MC1**	**Q_2 MC1**	**Q_1 MC2**	**Q_2 MC2**	**Expected Profit Retailer**	**σ_1/μ_1**	**Q_1+Q_2**	**Q_1**	**Q_2**	**Expected Profit Retailer**
233,8	136,9	96,9	90,5	143,3	31.311,4	1/6	250,8	119,1	131,8	29.964,1
236,5	139,9	96,5	92,8	143,7	31.091,9	1/5	251,1	119,4	131,8	29.806,6
240,4	144,3	96,1	96,2	144,1	30.761,8	1/4	252,5	120,8	131,8	29.557,2
246,9	151,4	95,4	102,1	144,8	30.210,0	1/3	256,6	124,9	131,8	29.103,7

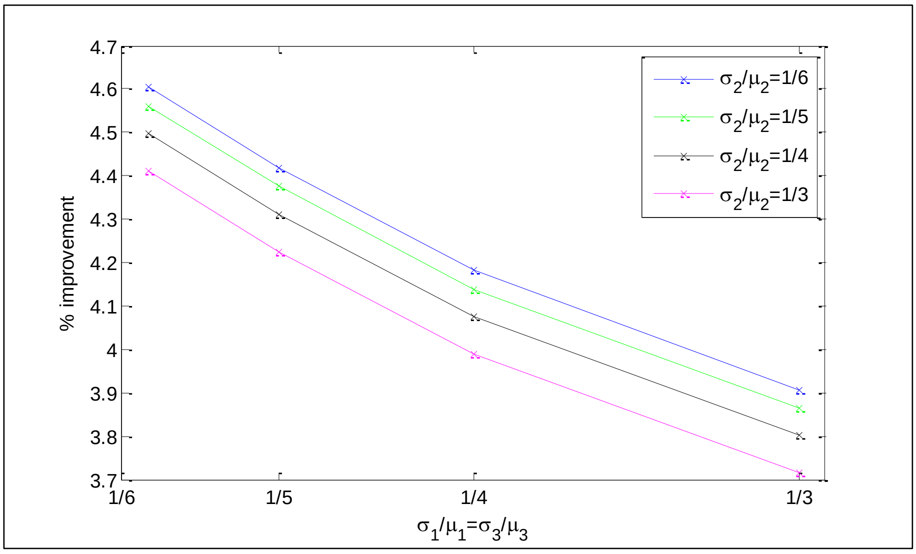

Figure 17 Graph on the effect of σ_1 / μ_1 and σ_2 / μ_2 for the Parameter Setting given in Table C2 in Appendix C

We can conclude that increase in variance in products demand distribution mean does not increase the percentage improvement QF provides to the retailer. However, the situation is different for the manufacturer, percentage improvement of manufacturer increases as standard deviation for either product increase because the total order quantity increases for high profit margin case. Although the percentage improvement of the manufacturer is still negative for QF setting, the behavior has increasing tendency as shown in Figure 18.

Moreover, percentage improvement is around 1% when the total supply chain performance is considered and increases while standard deviation of products increases.

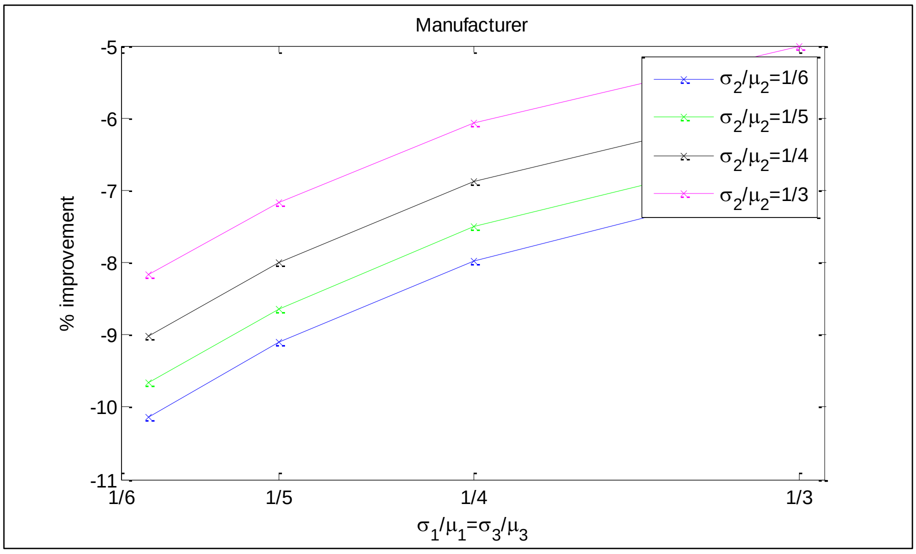

Figure 18 Manufacturer's Percentage Improvement vs. $\sigma_1 / \mu_1 = \sigma_3 / \mu_3$ for the Parameter Setting given in Table C2 in Appendix C

5.5 Total Performance of the Supply Chain

Throughout the numerical analysis, we try to identify the conditions in which QF model brings improvement to the expected profit of the parties. As mentioned at the beginning of Chapter 5, QF seems to be more advantageous for the retailer, as it provides aggregate ordering and order updating flexibilities to the retailer. On the other hand, the manufacturer has to share the risk of uncertainty with the retailer to provide this flexibility to the retailer. Therefore, the percentage improvement of the manufacturer was expected to be lower than the retailer. In most of the runs, indeed QF model is more advantageous to the retailer. However, as discussed in previous sections there are numerous cases the manufacturer may benefit from QF model as well.

For the overall performance comparison, the difference of the total order quantities of retailer in QF and B1, $\overline{Q}$ seems to be an indicative factor to consider since it affects the percentage improvements of the parties because Q directly specifies the profit. The reason of such difference in order quantities between QF and B1 is inherited by the nature of the models and type of relationship between parties in these models. The manufacturer's percentage improvement especially affected by the change of $\overline{Q}$ to a great extent. The order quantity of the retailer becomes the demand for manufacturer and how QF changes this factor reflects how QF affects manufacturer's profitability.

Figure 18 is constructed with the experiments defined in Table A1 and shows the percentage improvement of manufacturer in these mentioned 23.493 runs with respect to $\overline{Q}$. Figure 18 shows that percentage improvement of the manufacturer increases as $\overline{Q}$ increases. In addition, the figure shows also that there are numerous cases that the percentage improvement of the manufacturer is significantly high, as high as 150%.

Another interesting observation is that the percentage improvement of the manufacturer is greater than zero only if $\overline{Q}$ is greater than zero. This means that the manufacturer may benefit from QF model only when the retailer's total order quantity in QF increases compared to the total order quantity in B1.

Figure 19 shows the percentage improvement of the parties with respect to Q Difference. As discussed for several examples above, in almost all of the cases, the retailer benefits from QF model. Besides, Figure 19 shows that there are numerous cases where $\overline{Q}$ is negative, which means that the retailer in QF orders less than in B1 because of aggregate ordering advantage (pooling effect). In such cases, manufacturer's performance drops to negatives as well. Nevertheless, the performance of the total supply chain rarely drops to zero.

In addition, there are also several cases, where the retailer's percentage improvement is so high that it can share some of its profit with manufacturer in order to motivate the manufacturer for the QF setting being feasible for manufacturer as well. Figure 20 show how the percentage improvement of the retailer, manufacturer and the total of supply chain changes as difference in Q changes in QF and B1.

According to our observations in defined in A1 in Appendix A, in around %9 cases, the manufacturer obtains at least %1 improvement in QF. Besides, total supply chains performance is turned out to be at least %1 increased in %74 of the cases and at least %10 improves in obtained in almost %15 of the cases.

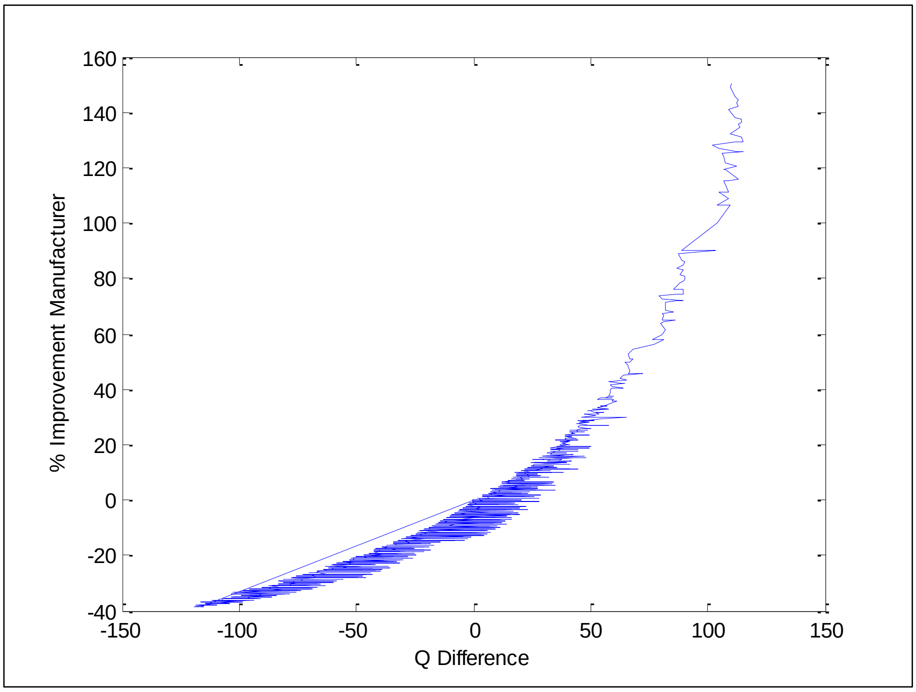

Figure 19 Manufacturer's Percentage Improvement in Stationary Demand

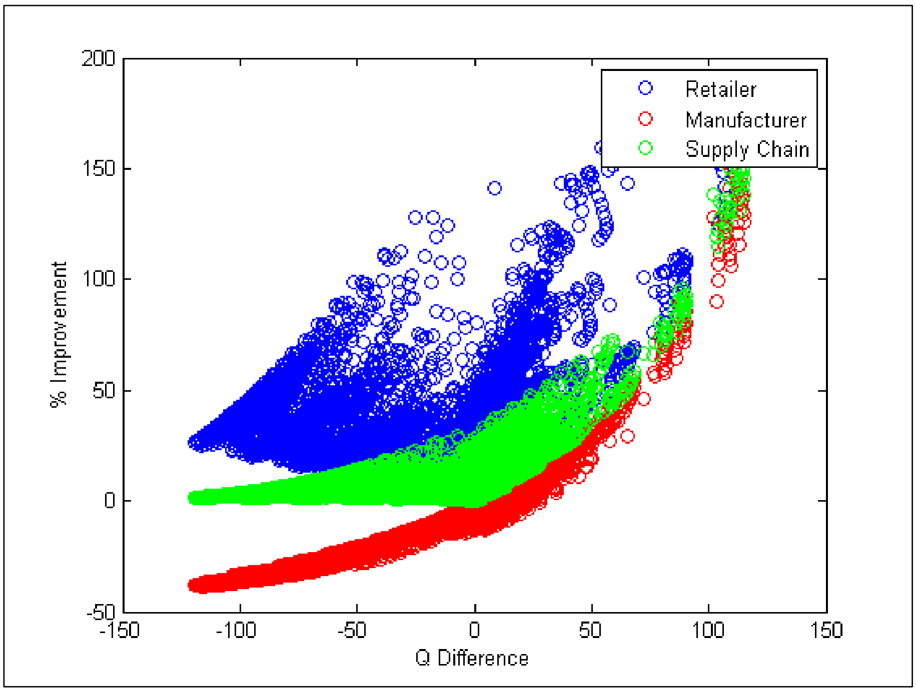

Figure 20 Percentage Improvement of Retailer, Manufacturer and Total Supply Chain in Stationary Demand

Having discussed the performance of parties and the supply chain for the stationary demand structure, we want to compare it to the non-stationary demand structure. Figure 21 provides the percentage improvements of the retailer, manufacturer and the supply chain in the non-stationary demand structure for the settings defined in Table 6 and Table 8.

Figure 21 shows that although there are some cases with as high as 40% improvement for the retailer in the non-stationary demand structure, the retailer's general percentage improvement is between 0 and 5. The situation is even worse for the manufacturer; the percentage improvements are mostly below 0% although there are cases with percentage improvement around 30%. The percentage improvement of the supply chain is not promising either, below or close to zero in most of the times.

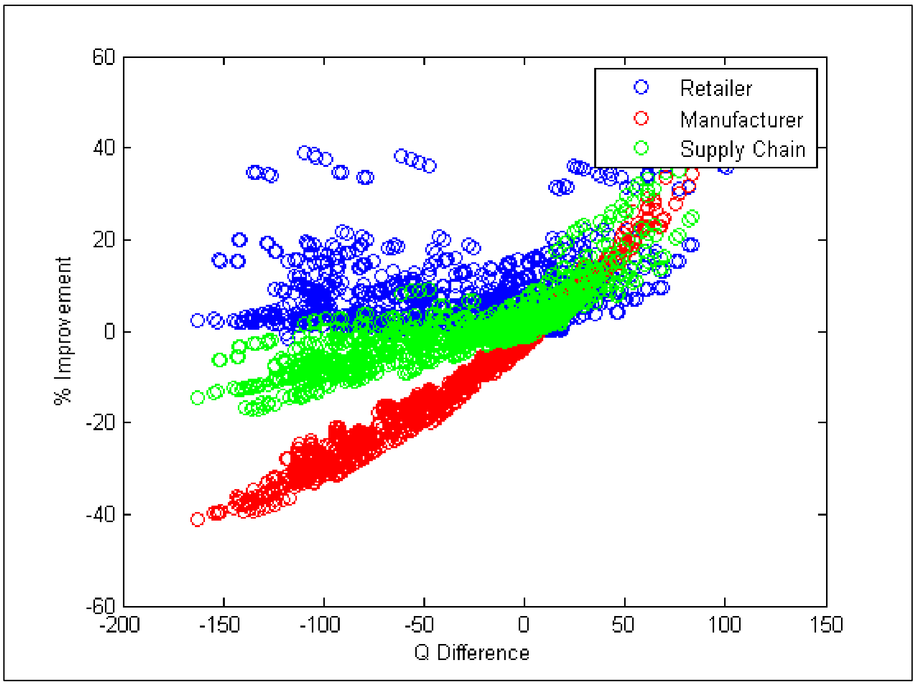

Figure 21 Percentage Improvements of Parties in Stationary Demand

Major Findings

- The retailer benefit from QF contract almost all the time.
- Although the manufacturer seems disadvantageous from QF contract, there are many cases where the manufacturer benefits from QF contract.

- The QF Contract is beneficial for the total of supply chain in terms of expected profit.
- The percentage improvement the parties achieve from implementing QF contract may change in accordance with the parameter setting and demand structure.
- The performance of QF contract is better when the total demand of the products in market conditions is stationary. Especially when the demand of one product is substitute the of the other market condition.

CHAPTER 6

CONCLUSION

In this study, we analyze a decentralized supply chain consisting of a single manufacturer and a single retailer that offers two products to the market in a single period. The model analyzed in this study is a joint quantity flexibility contract between manufacturer and retailer, where the risk of uncertainty is shared by the parties. The single period of the problem setting is divided into two stages for better understanding the behaviors of the retailer and the manufacturer. In the first stage, the retailer makes a commitment on the total quantity of the products to be ordered from the manufacturer. Based on this, the manufacturer determines its regular procurement quantity for each product. The second stage is characterized by the market condition information. Once the retailer knows the market condition, it places a final order for each product. The retailer has unlimited flexibility in determining the final orders except that the sum of the final orders should match the initial commitment. The manufacturer has to fulfill the retailer's order in full.

We construct an analytical model for the joint quantity flexibility contract between manufacturer and retailer to determine the optimal behaviors of each party so as to maximize their individual profits in defined problem setting. In the numerical analysis, the market condition is modeled as a Bernoulli random variable for ease of exposition. We only consider two possible market conditions; as high market with probability *(1-b)* and low with probability *b* and assume that the market condition characterizes the distribution of demand for each product.

In numerical analysis chapter, the effects of various parameters on the expected profits and order quantities of the retailer and manufacturer under the joint quantity flexibility contract model. We aim to reveal whether and under what conditions QF is beneficial to both parties compared to the benchmark settings.

We performed an extensive numerical analysis and observe that QF contract is profitable for retailer in all of the cases and benefits are especially high when the variability of product demand is high. Although the manufacturer has several cases with negative percentage improvement in its expected profit, there are also many cases that result better profit both for retailer and manufacturer. Besides, in almost all of the case, the percentage improvement of total supply chain is positive.

When we consider possible real life application, if risk sharing between parties also applied to outcome of the game, retailer can share some of the benefits earn from using QF contract with manufacturer. In such cases, the percentage improvement in the total of supply chain improved and a win-win situation for parties become possible. The example is especially applicable to the global markets especially offering new age products (which has no accumulated past data for forecasting), such as technological products and fashion items.

Extension for future works can be to change the structure of information update. As mentioned in literature review chapter often used information update models are possible. However, a change in structure of information update may necessitate also a change in the form of information gathered as a result of market observation.

One other possible extension can be a scenario, where the demands of two products being dependent rather than being independent. Such setting will definitely find real life applications on technological products.

Including multiple products and multiple periods into the analysis is always good extension possibilities for future studies if methods for handling the complexity of such models can be offered.

The area of work has been expanding due to necessity of the industry by changing market rules. Therefore, a possible future work will contribute to the area of study and worth to think over.

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Appendix A: Experiment Setting for Stationary Demand Analysis

Table A1 Experiments Settings for Stationary Demand Analysis

Fixed Parameters			
$w_1=60$ $w_2=50$	$s_1=0$ $s_2=0$	$d_{11}=d_{12}=10$ $d_{21}=d_{22}=20$	$r_1=5$ $r_2=5$
Changing Parameters			
p_1	80-260	Step Size	10
p_2	70-210	Step Size	10
b	0.2-0.8	Step Size	0.1
Demand Parameters			
$\mu_1+\mu_2=\mu_3+\mu_4=200$*			
μ_1	30-170	Step Size	10
μ_2	30-170	Step Size	10
μ_3	30-170	Step Size	10
μ_4	30-170	Step Size	10
σ_1	$\mu_1/4$		
σ_2	$\mu_2/4$		
σ_3	$\mu_3/4$		
σ_4	$\mu_4/4$		

* If μ_1=60, then μ_2=140 in *MC1* and μ_3=140, μ_4=60 in *MC2*.

Appendix B: Results for Stationary Demand Structure Analysis

Table B1 Order Quantities of B1 & QF

QF			**B1**		
Q	**Q1 MC1**	**Q1 MC2**	**Q1**	**Q2**	**μ1**
215.100	67.283	153.828	135.109	79.103	60
214.844	78.258	143.124	125.500	88.737	70
214.612	89.163	132.376	117.367	95.835	80
214.429	100.004	121.597	113.151	100.622	90
214.258	110.768	110.767	110.770	103.494	100
214.136	121.468	99.901	108.398	104.988	110
214.038	132.094	88.988	105.655	106.191	120
213.977	142.652	78.032	103.570	109.782	130
213.947	153.136	67.029	106.356	117.407	140
213.947	163.546	55.980	113.727	125.788	150
213.977	173.879	44.882	121.309	134.174	160
214.050	184.144	33.737	128.891	142.560	170

Table B2 Expected Profit of QF, B1 & B2

Expected Profit of Retailer				**Expected Profit of Manufacturer**		
QF	B1	B2	**μ1**	QF	B1	B2
13106.25	10819.23	13109.69	60	8748.97	12146.91	8727.43
13017.24	11543.70	13019.18	70	8878.90	11432.38	8863.26
12927.80	12193.89	12928.67	80	9010.09	10717.86	8999.09
12837.94	12641.61	12838.16	90	9142.05	10014.09	9134.93
12747.65	12747.65	12747.65	100	9363.38	9919.54	9364.55
12656.92	12457.13	12657.14	110	9274.78	9569.79	9270.76
12565.75	11796.99	12566.64	120	9441.20	9824.46	9442.98
12474.13	10859.19	12476.13	130	9408.30	9530.40	9406.60
12382.05	9782.76	12385.62	140	9519.32	9701.72	9521.41
12289.51	8681.90	12295.11	150	9542.87	9619.42	9542.43
12196.50	7580.69	12204.60	160	9598.78	9682.41	9599.84
12103.01	6479.48	12114.09	170	9677.98	9678.27	9622.88

Table B3 Percentage Improvement of the Parties

% IMP Retailer	% IMP Manufacturer	% IMP Supply Chain	μ1
21.14	-5.61	8.35	60
12.76	-3.90	5.10	70
6.02	-1.88	2.52	80
1.55	-0.86	0.50	90
0.00	0.00	0.00	100
1.60	-0.80	0.56	110
6.52	-1.28	3.03	120
14.87	-3.08	6.46	130
26.57	-8.71	8.72	140
41.55	-15.93	9.79	150
60.89	-22.34	10.85	160
86.79	-27.97	11.95	170

Appendix C: Experiment Setting for Variance Analysis

Table C1 Low Profit Margin Case

Fixed Parameters				
$\mu_1 = 120$	$\mu_2 = 80$	$\mu_3 = 80$		$\mu_3 = 120$
$p_1=80$	$P_2=70$	$b=0.4$		
$w_1=60$ $w_2=50$	$s_1=0$ $s_2=0$	$d_{11}= d_{12}=10$ $d_{21}= d_{22}=20$		$r_1=5$ $r_2=5$
Changing Parameters				
σ_1	$\mu_1 / 6$	$\mu_1 / 5$	$\mu_1 / 4$	$\mu_1 / 3$
σ_2	$\mu_2 / 6$	$\mu_2 / 5$	$\mu_2 / 4$	$\mu_2 / 3$
σ_3	$\mu_3 / 6$	$\mu_3 / 5$	$\mu_3 / 4$	$\mu_3 / 3$
σ_4	$\mu_4 / 6$	$\mu_4 / 5$	$\mu_4 / 4$	$\mu_4 / 3$

Table C2 High Profit Margin Case

Fixed Parameters				
$\mu_1 = 120$	$\mu_2 = 80$	$\mu_3 = 80$		$\mu_3 = 120$
$p_1=250$	$P_2=200$	$b=0.4$		
$w_1=60$ $w_2=50$	$s_1=0$ $s_2=0$	$d_{11}= d_{12}=10$ $d_{21}= d_{22}=20$		$r_1=5$ $r_2=5$
Changing Parameters				
σ_1	$\mu_1 / 6$	$\mu_1 / 5$	$\mu_1 / 4$	$\mu_1 / 3$
σ_2	$\mu_2 / 6$	$\mu_2 / 5$	$\mu_2 / 4$	$\mu_2 / 3$
$\sigma_3 = \sigma_1$	$\mu_3 / 6$	$\mu_3 / 5$	$\mu_3 / 4$	$\mu_3 / 3$
$\sigma_4 = \sigma_2$	$\mu_4 / 6$	$\mu_4 / 5$	$\mu_4 / 4$	$\mu_4 / 3$