A MULTI-PERIOD STOCHASTIC PORTFOLIO OPTIMIZATION AND HEDGING MODEL APPLIED FOR THE AVIATION SECTOR IN THE EU ETS

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submitted by **ERKAN KALAYCI** in partial fulfillment of the requirements for the **degree of Philosophy of Doctorate in Department of Financial Mathematics, Middle East Technical University** by,

Prof. Dr. Bülent Karasözen ________________
Director, Graduate School of **Applied Mathematics, METU**

Assoc. Prof. Dr. Sevtap Kestel ________________
Head of Department, **Financial Mathematics, METU**

Assist. Prof. Dr. Esma Gaygısız ________________
Supervisor, **Institute of Applied Mathematics, METU**

Prof. Dr. Gerhard Wilhelm Weber ________________
Co-Supervisor, **Institute of Applied Mathematics, METU**

Examining Committee Members:

Assist. Prof. Dr. Esma Gaygısız ________________
Institute of Applied Mathematics, METU

Assoc. Prof. Dr. C. Coşkun Küçüközmen ________________
Institute of Applied Mathematics, METU

Assist. Prof. Dr. Seza Danışoğlu ________________
Institute of Applied Mathematics, METU

Assist. Prof. Dr. Ayşe Özgür Pehlivan ________________
Department of Economics, Bilkent

Assist. Prof. Dr. Nil Şirikçi ________________
Department of Economics, METU

Date: ________________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: ERKAN KALAYCI

Signature

ABSTRACT

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IN THE EU ETS

Kalaycı, Erkan

Ph.D., Department of Financial Mathematics

Supervisor : Assist. Prof. Dr. Esma Gaygısız

Co-Supervisor : Prof. Dr. Gerhard-Wilhelm Weber

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In this thesis, we set up and solved a multi-period stochastic portfolio optimization and hedging model with futures from an airline company's point of view, by taking into account all the specific EU ETS (EU Emission Trading Scheme) regulatory and board-defined trading and risk constraints. That is, in order to hedge the natural physical short position in CO_2 emission allowances, we developed an optimal hedging strategy consisting of futures contracts. We thereby successively and comprehensively derived all the mathematical formulations for the system of equations with regard to the specific composition of the profit function and all the underlying real-world constraints in the model. In order to span the space of all possible states, in addition to the modeling of constraints, we also run Monte-Carlo (MC) simulations of correlated geometric Brownian motions (GBM) for traded EUA (EU Emission Allowance) and CER (Certified Emission Reduction) futures prices of different CO_2 delivery time periods. Based on the constructed scenario-trees of EUA and CER futures prices and space of feasible states, the optimal buy-hold-sell decision (i.e., futures trading strategy) were determined and the corresponding earnings calculated. Based on the distribution of the revenues, the Value-at-Risk (VaR) measure for the 95% and 99% confidence level was calculated, in order to measure the risk exposure of the portfolio manager.

Our contribution to existing academic literature is multiple. As the first ever case, we will apply the multi-stage stochastic programming technique to the aviation sector, which is a brand new included sector within the EU ETS. The methodology and mathematical formulation for the optimization problem including the MC simulated multi-correlated GBMs of EUA and CER financial futures of different CO_2 delivery time periods and the resulting system of equations have been self-developed. That is, the consideration of all the actually valid EU ETS regulatory and real-world oriented, managerial, trading constraints in the airline sector, makes our model to a real-life application, which in the constellation and idea, set up in this thesis, has not been applied in academic research before. Hence, the developed methodology in thesis could be widely used implemented, adapted and extended to other academic problems and practical applications.

The thesis ends with a conclusion and outlook to future studies.

Keywords: Multi-stage stochastic portfolio optimization, correlated geometric Brownian motion, Monte-Carlo simulation, futures prices, value-at-risk

ÖZ

EU ETS HAVACILIK SEKTÖRÜ İÇİN UYGULAMALI ÇOKLU SÜREÇLİ STOKASTİK PORTFÖY OPTİMİZASYON VE KORUMA MODELİ

Kalaycı, Erkan

Doktora, Finansal Matematik Bolümü

Tez Yöneticisi : Yrd. Doc. Dr. Esma Gaygısız

Ortak Tez Yöneticisi : Prof. Dr. Gerhard-Wilhelm Weber

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Bu tezde, bütün spesifik EU ETS (AB Emisyon Ticaret Şeması) düzenlemeleri ve kurul tanımlı ticaret ve risk kısıtlamaları hesaba katılarak bir hava yolu şirketinin bakış açısından vadeli işlem kontratları kullanarak çoklu süreçli stokastik portföy optimizasyon ve koruma modeli kuracak ve çözümleyeceğiz. Buna göre, CO_2 emisyon haklarındaki doğal fiziksel kısa posizyonları korumak için vadeli işlem kontratları içeren en uygun koruma stratejisini geliştireceğiz. Böylece modelde, kar fonksiyonu ve tüm temel gerçek dünya kısıtlamalarının spesifik bileşimine istinaden başarılı ve kapsamlı bir şekilde denklem sistemine yönelik tüm matematiksel formülasyonları elde edeceğiz. Olası tüm durumların uzayını kurabilmek için, kısıtlamaların modellemesine ek olarak, farklı CO_2 teslim periyodlarında işlem yapılan EUA (AB Emisyon Hakkı) ve CER (Sertifikalandırılmış Emisyon Azaltım) için ilişkili geometrik Brownian hareketlerinin (GBM) Monte-Carlo (MC) simülasyonlarını çalıştıracağız. Vadeli EUA ve CER fiyatları ve olası tüm durumların uzayına göre kurgulanmış senaryo ağacına dayalı olarak, en uygun satın alma, tutma ve satma kararları (örneğin, vadeli işlem stratejisi) belirlenecek ve buna karşılık gelen kazanımlar hesaplanacaktır. Portföy yöneticisinin maruz kaldığı riski ölçebilmek için gelir

dağılımına dayalı olarak, %95 ve %99 güven düzeyi için riske-maruz-değer (VaR) hesaplanacaktır.

Akademik literatüre çoklu katkıda bulunacağız. İlk olarak, EU ETS içine dahil edilen yepyeni bir sektör olan havacılığa çoklu süreçli stokastik programlama tekniğini uygulayacağız. MC simülasyonu yapılmış çoklu-ilişkili, farklı CO_2 teslim periyodlarında işlem yapılan EUA ve CER finansal vadeli işlem kontratların GBM'lerinin içinde bulunduğu optimizasyon problemi ve ortaya çıkan denklem sistemi için metodoloji ve matematik formülasyonu kendimiz geliştirireceğiz. Buna göre, gerçekten geçerli EU ETS düzenlemeleri ve havayolu sektöründeki gerçek dünya amaçlı, yönetsel ticari kısıtlamaların göz önünde bulundurulması bu tezde oluşturulan modelimizi daha önce akademik araştırmalarda uygulanmamış gerçek bir hayat uygulaması yapıyor. Sonuç olarak, bu tezde geliştirilen metodoloji yaygın olarak diğer akademik konular ve pratik uygulamalarda kullanilabilir, uyarlanabilir ve genişletilebilir.

Tez bir sonuç ve gelecekti çalışmalara görünüm.ile bitecektir.

Anahtar Kelimeler: Çoklu süreçli stokastik portföy optimizasyon, ilişkili geometrik Brownian hareketi (GBM), Monte-Carlo simülasyonu, vadeli fiyatlar, riske-maruz-değer

In memory of my brother Hayrettin Kalaycı

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CHAPTER 1

INTRODUCTION

At 1 January 2012, the global airline sector, being responsible for 2% of the global CO_2 emissions [57] and 3% of the EU's total greenhouse gas emissions [16], was included into the European Union Emission Trading Scheme (EU ETS), being forced by law to compensate all the CO_2 emissions resulting from their flights to and from Europe [13]. The legal practice until now has prescribed that, from 1 January 2005, all large-scale energy- and industrial-intensive EU installations and sectors such as power and heat, refineries, metals, minerals or pulp and paper has to be mandatorily included in the EU ETS [12]. With this new legal enforcement [13] in the airline sector, for the first time also non-EU companies are obliged to control their CO_2 emissions. According to this directive, apart from 1 January 2012, airline companies are obliged to cap their CO_2 emissions to 97% of their average 2004-2006 emission levels (baseline), and from 2013, to 95% of their average 2004-2006 emission levels (baseline), respectively. The EU ETS allocation plan for the airline sector prescribes that approximately 85% of the EU Emission Allowances (EUAs) are distributed for free to airline companies with respect to their average 2004-2006 baseline. Therefore, this regulatory feature results in an obligation for airline companies to purchase the remaining 15% of CO_2 emission allowances from the market to mandatorily offset their yearly CO_2 emissions and hold the regulatory cap. This means that an airline company initially faces a natural short position in CO_2 emission allowances, thereby suffering from an increase in CO_2 prices and gaining from their decrease, respectively. To fulfill their yearly regulatory obligation (i.e., CO_2 compliance period), airline companies are allowed to surrender EUAs, and up to a regulatory-defined limit, Certified Emission Reductions (CERs). If airline companies fail to fully compensate their occurred CO_2 emissions at the end of each CO_2 compliance year, they are forced to pay a penalty fee in the amount of 100 EUR for each missing ton of CO_2 to the EU administering authority. Another important feature of the EU ETS is the banking and borrowing possibility of EUAs between CO_2 compliance periods.

Empirical evidence shows a doubling of CO_2 emissions between 2005 and 2020 [19], forcing airline companies to buy an increasing amount of emission allowances [37]. Until 2050, the CO_2 emissions from the aviation industry are even estimated to grow by a further 300-700% according to 2005 levels [57]. Hence, despite its most recent inclusion in the EU ETS, the aviation sector is expected to face a large growth in CO_2 emissions, illustrating its current and future importance in the EU ETS.

This new regulatory obligation has the crucial implication that, in addition to the well-known existing cost factor kerosene, the new cost factor CO_2 has occurred, which has also to be considered in the business practices and operations of airline companies from now on. Today, in academic research [2, 7, 41, 49, 53, 60, 53] the concept of *risk management* and *hedging* of kerosene, as the major cost factor for airlines, is well understood and mainly operationally implemented. However, this empirical evidence does not yet hold for the new cost factor CO_2. In contrast to kerosene, where optimal hedging strategies through financial derivatives such as options, forwards/futures and swaps are implemented, the CO_2 emission allowances are mainly bought in spot markets for current prices, though being fully exposed to their *market price* and *volume risk*. The airline companies fully pass on the actual purchasing (spot) price of CO_2 emission allowances to their current ticket prices, resulting in an overall increase in their ticket prices. Thus, in order to increase their competitiveness in the airline market, airline companies could design and execute optimal hedging strategies through financial derivatives (e.g., futures strategies) to hedge the price and volume risk. In this way, the additional cost for CO_2 allowances and, therefore, the additional increase in ticket prices could be minimized.

There exists a well-researched academic field with regard to the *Kyoto Protocol* and emission sector with regard to game theory and mechanical design [21, 35, 40, 55]. However, all these works deal with *equilibrium price models* for emissions trading. That is, they consider agents being able to influence the emission price and analyze, simulate and optimize the emissions market from a macro perspective where prices are outcomes from different strategies of emission market participants. In our case, we consider the case for a single airline company, within the airline sector which is a specific sector in the EU ETS, and assume that the prices are given from the airline company's point of view and the company cannot influence the whole EU ETS by its strategy. Moreover, no cooperation with another company is allowed. Therefore, as compared to the widely academically applied game-theoretical approach for emissions trading, the objective and information of our work are of a much different nature.

In this thesis, we apply a *multi-period stochastic portfolio optimization model* for the derivation of an optimal hedging strategy for CO_2 emission allowances from an airline company's point of view. That is, rather than the optimization of the whole emission market and system, we focus on a constrained revenue maximization of one single airline company by taking into account realistic, airline sector-specific and given financial market restrictions.

Our model specifically focuses on the aviation sector due to its brand new status and increasing importance within the EU ETS, implying a large stimulation potential for academic and applied research in this area as well as its adoption potential by other sectors to be included in the EU ETS in future.

In academic research, multi-period stochastic portfolio optimization technique finds a broad application for the energy sector such as the determination of optimal running (i.e., dispatch) strategies of hydro power plants [1, 14, 18, 22, 25, 43] or the valuation and optimization of natural gas storage and value chains [23, 44, 57, 58, 65, 66]. With regard to the emissions sector, until now, the multi-period stochastic portfolio optimization models have been either applied for the derivation of optimal SO_2 compliance planning issues in the US, where mainly technical power plant / engineering constraints have been considered [6, 30, 38, 59] or for the combined heat and power (CHP) sector, which, in addition to the airline sector, is another sector included in the EU ETS [50, 54].

However, these academic works consider technical and physical rather than financial features, or they are set up for short-term planning issues. But, for the management of its cash flow streams from assets, a company should also consider the medium term perspective. Today, likewise other commodities, also CO_2 emission allowances can be traded at liquid energy exchanges such as ICE, Bluenext, Nordpool, etc., or over-the-counter (i.e., brokers) such as Spectron, GFI, ICAP, etc., in the form of day-ahead spot or for longer trading horizons in the form of derivatives. Therefore, optimal medium-term hedging strategies should be developed, which is also true for the CO_2 sector.

Additionally, previous academic works do not take into account the existence of different types of CO_2 emission allowances such as EUA or CER, nor do they address the potential of trading CO_2 emissions allowances as optimal (EUA, CER) portfolios. Furthermore, they do not consider any *stochasticity* of emissions allowances prices and any cross correlations between each other.

Therefore, our contribution to existing academic literature is multiple. As the first ever case, we apply the multi-stage stochastic programming technique to the aviation sector, which is a brand new included sector within the EU ETS. The methodology and mathematical formulation for the optimization problem and the resulting system of equations are self-developed according to the actually valid EU ETS regulation for the aviation sector and in line with real-world oriented managerial trading constraints, which in the constellation and idea, set up in this thesis, has not been applied in academic research before. This makes our model to a real-life application, which could easily be adapted and extended to other future sectors to be included in the EU ETS such as the shipping sector. Furthermore, more than only incorporating physical and technical ("engineering") features and focusing on short-term planning issues, we particularly address financial pricing features and focus on mid-term planning issues. That is, unlike the common feature of hedging and optimizing an open position in a physical asset against short-term oriented spot prices, we use mid-term oriented futures prices of different CO_2 delivery time periods, in order to take into account flexibility. Therefore, by the use of existing exchange-traded emission allowance types EUA and CER futures, we result in two main contributions to academic research with regard to emission allowance prices. First, we use not only one unspecified type of emission allowance, but two types of real-world emission allowance prices, namely EUAs and CERs. Secondly, EUA and CER futures for various CO_2 delivery periods are considered, implying an increase in the number of correlated emission allowances.

We note that the stochastic model input parameters EUA and CER are modeled by the stochastic price process geometric Brownian motion (GBM), whereas the stochastic model input parameter CO_2 emissions are modeled by given deterministic scenarios due to the fact that otherwise more specific fundamental airline data, such as type and the corresponding capacity of owned airplanes, current and future flight plans to and from the specific EU locations, sold flight tickets of the airplanes, weight of the transported luggage etc. would be required. This would necessitate a much more comprehensive, fundamental analysis, airplane engineering and detailed modeling of technical airplane parameters, and thus explode the scope of this thesis. Hence, in our model, the CO_2 emissions prices EUA and CER are considered as *endogenous* variables, whereas CO_2 emissions represent *exogenous* variables.

As a result, this model will contribute to the change in paradigm, by combining the "financial" with the "physical (engineering)" world, rather than considering them separately, and be applied to a completely new area within the emissions sector, incorporating a huge research potential. The developed methodology in this thesis could be widely used, adapted and extended to other academic problems with regard to hedging of physical assets against other financial derivatives than futures such as options or swaps. Moreover, it could be practically applied to other future sectors to be included in the EU ETS such as the shipping sector, or other sectors within the cap-and trade carbon market regimes such as the US RGGI.

This thesis is organized as follows: In Chapter 2, the functioning of EU ETS and the inclusion of the aviation sector to the EU ETS will be highlighted. In Chapter 3, we will explain the idea of the natural short position in CO_2 emission allowances and the design of an optimal hedging strategy with EUA and CER futures. Chapter 4 incorporates the mathematical derivation and formulation of the geometric Brownian motion (GBM) of correlated EUA and CER futures prices and MC simulation of them, as basis for the construction of the EUA and CER futures price scenario tree. Chapter 5 gives general mathematical foundations with regard to the multi-stage stochastic programming technique. In Chapter 6, the multi-stage stochastic programming technique will be concretely applied to an airline company in the EU ETS, where the decision (i.e., futures trading) algorithm will be methodologically developed and the optimization problem including the whole the system of equations successively derived. Chapter 7 contains the time-series properties of the applied GBM model and the input parameters in our model. In Chapter 8, the MC simulation for the GBMs of the correlated EUA and CER futures prices will be conducted and the optimization model according to the methodological and mathematical procedure, described in Chapter 6, solved and the resulting output including the trading strategy, earnings and value-at-risk measure for each scenario presented. The thesis will be terminated with Chapter 9, where conclusions and outlook for further research in this area will be made.

CHAPTER 2

THE EUROPEAN UNION EMISSION TRADING SCHEME (EU ETS) AND THE AVIATION SECTOR

2.1 Functioning of the European Union Emission Trading Scheme (EU ETS)

The Kyoto Protocol, agreed in 1997, as an immense and pioneering regulatory framework to combat climate change by reducing global greenhouse gas emissions (GHG)[1], initiated the launch of the global carbon market which consists of the regulated and the voluntary carbon market. Whereas the first implies to the mandatory obligation of companies being under CO_2 compliance to reduce their GHG to a pre-defined regulatory limit (i.e., cap), the latter one refers to the voluntary commitment of companies to reduce their GHG, seeking to manage their emission exposure for non-regulatory purposes such as for corporate social and climate responsibility issues.

With a global market value of 147.5 bn. US$ and share of 84.0 % [15, 63], the European Union Emission Trading Scheme (EU ETS), established in 2005 as the first international emission trading scheme, has worldwide become by far the most important, liquid and well-functioning cap-and-trade system to reduce industrial GHG. At the launch at 1 January 2005, the EU ETS covered all large-scale energy- and industrial-intensive EU installations and sectors such as power and heat, refineries, metals, minerals or pulp and paper.

At 1 January 2012, the airline sector has been included in the EU ETS as the most recent (infant) sector. As per January 2013, more than 11,000 installations with a net heat capacity above 20 MW were mandatorily included in the EU ETS [16]. The EU ETS covers 31

[1] Carbon dioxide (CO_2), Methane (CH_4), Nitrous oxide (N_2O), Hydrofluorocarbons (HFCs), Perfluorocarbons (PFCs) and Sulphur hexafluoride (SF_6).

countries, which refers to all 27 EU member countries, including Norway, Croatia, Iceland and Liechtenstein; they all are fully responsible for about 45% of the total GHG within the EU [17, 61].

The EU ETS consists of three trading phases. Phase I, which was a three-year pilot phase, lasted from 1 January 2005 to 31 December 2007, which acted as a market establishing period. Phase II, as the first “real” commitment phase, lasted from 1 January 2008 to 31 December 2012, where the market matured and liquidity increased. The actual phase III, as the longer trading and commitment period, is running from 1 January 2013 to 31 December 2020, where EU ETS market participants as well as financial institutions are performing CO_2 trading strategies, taking risk positions providing liquidity.

The EU ETS imposes a mandatory “cap” or limit on the total amount of the specified GHG that are allowed to be emitted by power facilities, factories and other installations, mandatorily included in the cap-and-trade system. By this means, a shortage in CO_2 emission allowances in the market is achieved to launch their trading. Within this regulatory cap, up to a certain amount of CO_2 emission allowances, called EU Allowances (EUAs[2]), are distributed for free to companies, dependent on the national target levels of the each country and National Allocation Plans (NAP), respectively. This implies the allocation of CO_2 emission allowances on a national basis according to the national promises of the EU burden sharing. The remaining and missing difference between the cap and the free distributed CO_2 emission allowances are to be bought from the market. Thus, the EU ETS ensures the cost-effective selling and buying of CO_2 emission allowances between companies up to the predefined regulatory cap. Hence, the regulatory limit on the total number of CO_2 emission allowances available put a real price on CO_2 emissions.

To fulfill the yearly regulatory CO_2 emission cap requirement (i.e., CO_2 compliance), at the end of each year, companies have to surrender enough EUAs to cover all their occurred and verified emissions. Otherwise, they are fined with a penalty of 100 EUR for each missing ton of CO_2, simultaneously being obligated to buy the missing emission allowances from the market at the then existing market price. Up to 1.5% of their yearly occurred CO_2 emissions, the companies are allowed to meet their yearly regulatory compliance with Certified Emission Reductions (CERs), generated through CDM projects in developing

[2] A European Union Allowance (EUA) is an assigned amount unit for the EU ETS. An EUA is a tradable unit of 1 tCO_2e.

countries.[3]

The market prices for CERs are fundamentally less than the market prices for EUAs. The economic explanation for this fundamental price-relationship is quite simple: EUAs are existing CO_2 emission allowances in the EU where abatement costs (production and technology-switching costs) are relatively higher than CERs, which are generated through emission reduction projects in developing countries and where production costs are significantly lower. Hence, the EUA price has to be fundamentally be higher than the CER prices. Or, in other words, if the market price for EUAs would be lower than the market price for CERs, there would be no economic incentive for EU countries to invest in emission reduction projects in developing countries.

The EU ETS cap-and-trade system with regard to trading of EUAs and CERs are illustrated in Figure 2.1. Here, we see that a company has to purchase the missing amount of CO_2 emission allowances, which in that case, is the difference between verified CO_2 emissions at the end of each CO_2 compliance year and the EUAs distributed for free to the company by the regulatory authority. This short position can be closed by trading of EUAs and CERs. Hence, the short position strongly depends on the verified total CO_2 emissions per year. If these increase, the short position increase, implying the purchase of more EUAs and CERs from the market, and vice versa.

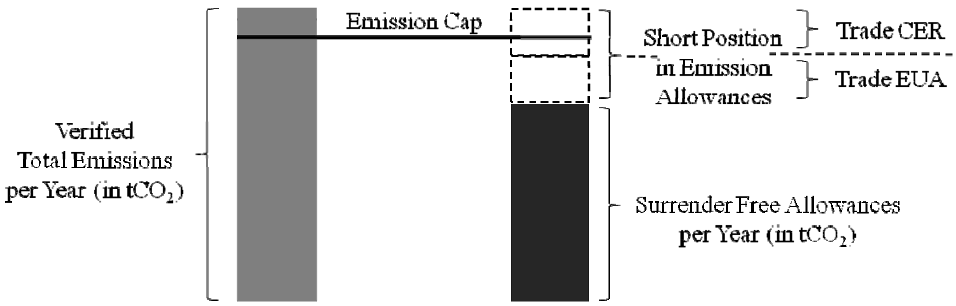

Figure 2.1. EU ETS cap-and-trade system with regard to EUA and CER trading.

[3] Certified Emissions Reductions (CERs) are generated through CDM (Clean Development Mechanism) which is a flexible Kyoto mechanism for project-based emission reduction activities in developing countries. Certificates will be generated through the CDM from projects financed by companies of industrial countries that lead to certifiable emissions reductions that would otherwise not occur. A CER is a standardized tradable unit in 1 t/CO_2e.

If a company achieves to reduce its CO_2 emissions, it can either up to the regulatory-defined 2.5% amount of its excess EUAs or CERs keep (i.e., transfer) in its CO_2 account for the subsequent year(s) to meet its future CO_2 compliance requirements, called *banking*[4] (i.e., going long), or sell them to the market or to another company, which is short in CO_2 emission allowances. If a company's CO_2 emissions exceed the regulatory cap, it can either use (i.e., transfer) up to the regulatory-defined 2.5% amount of the free distributed EUAs from the subsequent year for CO_2 compliance into the current year, called *borrowing*[5] (i.e., going short), or buy it from the market and from a company which is long in CO_2 emission allowances. Hence, the trading as well as banking or borrowing possibility of EUAs or CERs ensures the flexibility that emissions are cut where the emission abatement costs are at least to do so and to provide the companies in EU ETS compliance to balance their CO_2 accounts over some years. The principle of banking and borrowing of EUAs and CERs between CO_2 compliance periods is depicted in Figure 2.2.

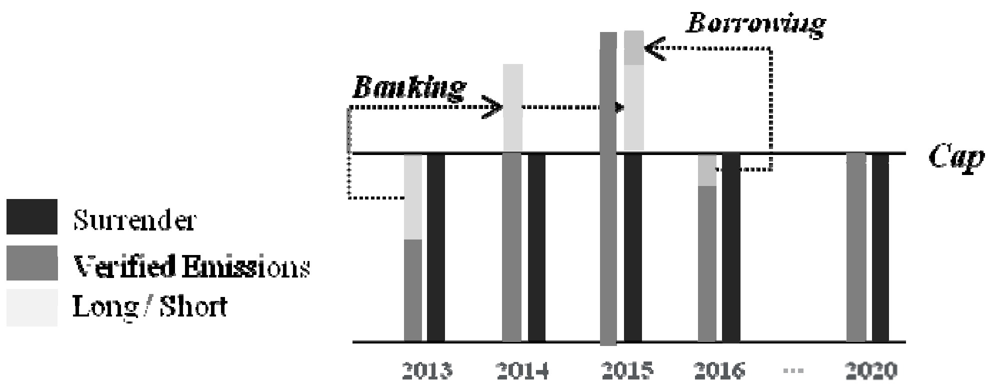

Figure 2.2. Banking and borrowing principle between CO_2 compliance periods.

According to EU ETS regulations, the number of allowances is reduced over time such that total emissions will fall, implying that in 2020, the emissions are estimated to be 21% lower than in 2005. From 2013, a progressive increase of auctioning of EUAs will occur, thereby improving its effectiveness in phase III (2013–2020). In 2013, installations included in the EU ETS have to buy 20% of their EUAs from the market, increasing to 70% in 2020.

[4] Banking is the possibility of transferring EUAs and CERs from one year to the following year.
[5] Borrowing is the possibility of using EUAs and CERs from the following year in the current year.

2.2 Inclusion of the Aviation Sector in the EU ETS

At 1 January 2012, the global airline sector was included into the EU ETS, being responsible for 2% of the global GHG and 3% of the EU's total GHG. Table 2.1 illustrates the percentage shares of classified sectors on the total CO_2 emissions within the EU ETS [16].

Sectors	Percentage Share (%)
Energy industries	31.9
Transport (excl. aviation)	21.3
Industry (energy and process related)	20.0
Household and solvents	12.4
Agriculture	8.6
Aviation	**3.0**
Waste	2.6
Solvents	0.2
Total	100.0

Table 2.1. Percentage shares of sectors on the total CO_2 emissions within the EU ETS.

At first glance, aviation sector's percentage share of 3% seems relatively low, as compared to energy industries, transport activities (excl. aviation) or industry. However, all these represent the sum of individual sectors together, i.e., energy industries involves power and heat production from various fossil fuel types such as gas or coal etc. or industry incorporates cement, lime, glass, pulp and paper industries etc., such that the aviation sector as one single sector within the classified sectors gains significant weight. Moreover, the extensive expansion plans and growth expectation of airline companies, and the resulting estimated growth of CO_2 emissions by 300-700% until 2050, as compared to 2005 levels, clearly shows the aviation sector's substantially increasing importance within EU ETS [57]. The global aviation sector is therefore considered as one of the fastest growing polluters.

Nevertheless, the actual small percentage share of the airline companies in total CO_2 emissions indicates that these companies are price takers rather than price setters. We take this fact into account in the modeling of behaviors of these companies in the carbon market.

According to Directive 2008/101/EC, all airline companies are forced by law to compensate all the CO_2 emissions resulting from their flights to and from Europe. Therefore, as compared to the legal practice until now, where only large-scale energy- and industrial-intensive EU installations were mandatorily included in the EU ETS, with this new legal enforcement in the airline sector, for the first time also non-EU companies are obliged to control their CO_2 emissions, which made the airline sector as a global sector with regard to the carbon market (i.e., EU ETS).

According to this directive, airline companies, in 2012, are obliged to cap their CO_2 emissions to 97% of their average 2004-2006 emission levels (baseline), and from 2013, to 95% of their 2004-2006 average emission levels (baseline), respectively. The EU ETS allocation plan for the airline sector prescribes that approximately 85% of the EUAs are distributed for free to airline companies with respect to their average 2004-2006 baseline, therefore resulting in an obligation for airline companies to purchase the remaining 15% of CO_2 emission allowances from the market to fully compensate their CO_2 emissions and hold the regulatory cap, respectively. If the yearly CO_2 emissions exceed the regulatory cap, airline companies have to purchase more than 15% of CO_2 allowances from the markets. Figure 2.3 shows the systematics of the EU ETS with regard to the aviation sector.

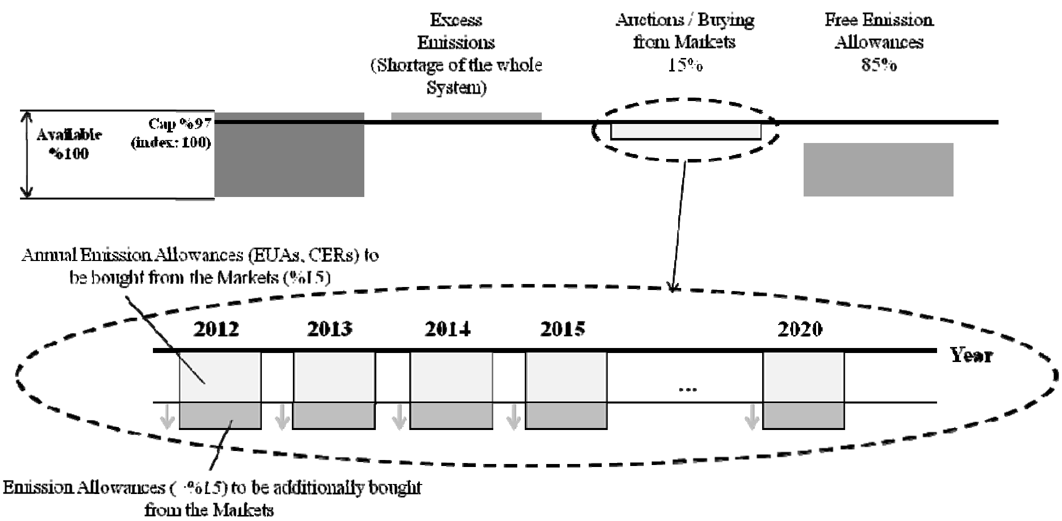

Figure 2.3. EU ETS and the aviation sector.

Here, the regulatory authority takes excess CO_2 emissions in the amount of 3% out of the system creating a shortage, such that the cap sets the new index of 100. From these, each year 85% are distributed for free to the airline companies and the remaining 15% has to be purchased by the airline company from the market. Thus, the airline company faces a natural short position in CO_2 emission allowances, which can even increase if the yearly occurred CO_2 emissions are higher than planned, such that in addition to these 15 % more CO_2 allowances have to be bought from the market (see Section 3.1).

CHAPTER 3

NATURAL SHORT POSITION AND HEDGING

3.1 Natural Short Position in CO_2 Emission Allowances

As in the case for kerosene, as the major cost factor for an airline company, an airline's initial physical position with regard to CO_2 is a natural short position, indicating that the 15% of the missing amount of CO_2 emission allowances has to be purchased from the market to mandatorily offset the remaining CO_2 emissions in their yearly CO_2 account.

Hence, by implementing pure spot trading strategies, an airline company would suffer from an increase in prices of CO_2 emission allowances and gain from their decrease, respectively. However, due to the various growth targets, expansion strategies and plans of the global airlines, the natural short position of the airlines is even expected to be further broadened. This implies that, in addition to market price risk, the airline companies are also fully exposed to volume risk of CO_2 emission allowances. Lufthansa reports, that for the year 2012, it has to purchase more than 40% of its CO_2 exposure from the market [37], which is much more than the regulatory set of 15%. Furthermore, due to the mature technological status of the airline sector, the aircrafts delivered to the global airlines until 2020 are not expected to have a fundamental impact on the improvement of fuel efficiency [5, 24, 42, 43, 52]. Empirical evidence shows a doubling of CO_2 emissions between 2005 and 2020, forcing airline companies to buy an increasing amount of CO_2 emission allowances [19]. Until 2050, the CO_2 emissions from the aviation industry are even estimated to grow by a further 300-700%, as compared to 2005 levels [57], implying a substantial widening of the natural short position in CO_2 allowances and thus increase of the amount of CO_2 allowances to be purchased from the market by an airline company.

Indeed, World Bank reports that global carbon trading transactions are 2% spot, 10% options and 88% futures transactions, showing the leading position of financial derivatives for hedging of long-term position in CO_2 emission allowances [64].

However, in contrast to kerosene, where optimal hedging strategies through financial derivatives such as options, forwards/futures and swaps are well-known and implemented in practice, airline companies mainly buy CO_2 emission allowances in spot markets for current prices, though being fully exposed to their market price and volume risk. As a consequence, the airline companies fully pass on the actual purchasing (spot) price of CO_2 emission allowances to their current ticket prices, resulting in an overall increase in their ticket prices, which may have a negative impact on their competitiveness in the global airline market. Thus, in order to increase their competitiveness and decrease their fully exposure to spot prices of CO_2 emission allowances, airline companies could design and execute optimal hedging strategies through financial derivatives (e.g., futures strategies) to hedge the price and volume risk. In this way, the additional cost for CO_2 emission allowances and, therefore, the additional increase in ticket prices could be minimized.

Figure 3.1 illustrates the physical natural short position in CO_2 emission allowances from an airline company's point of view, thereby suffering from a price increase and gaining from a price decrease. By taking an offsetting long position in EUA and CER futures, the company can hedge itself against price increases.

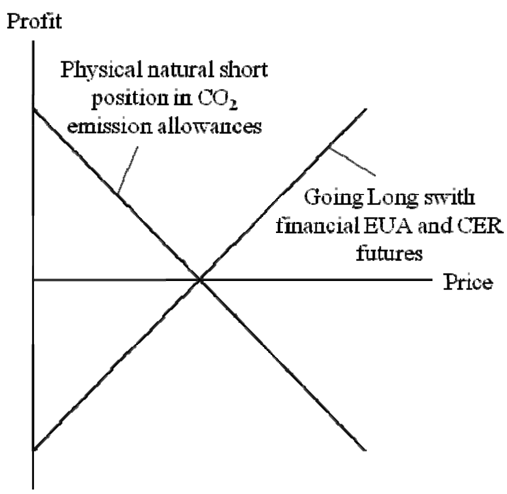

Figure 3.1. Physical natural short position in CO_2 emission allowances and offsetting long position with EUA and CER futures contracts.

The following Section 3.2 illustrates how an airline company can design and set up an optimal hedging strategy in practice for the closing of its natural short position in CO_2 emission allowances with EUA and CER futures.

3.2 Design of an Optimal Hedging Strategy with EUA and CER Futures

To avoid the variability in the price for CO_2 emission allowances, we in the following design and set an optimal hedging (i.e., reduction) strategy and hedging (i.e., reduction) optimization of the natural short position in CO_2 emission allowances consisting of EUA and CER futures of various delivery periods to be purchased from the market, over a trading period of *n*-years (i.e., compliance periods). That is, the physical short position of the airline company is hedged with financial EUA and CER futures, traded at liquid carbon exchanges (see Chapter 1). The traded futures thereby ensure an airline company to apply in tranches a net purchasing strategy in CO_2 emission allowances at a specified future time at a price agreed upon today, therefore serving as cash flow hedge. A net purchasing strategy in this sense means that, despite of the implementation of optimal buy-sell strategies, in total, more buying than selling strategies result in the amount of offsetting the natural short position in CO_2 emission allowances. The set up and execution of a net purchasing strategy is therefore dependent on optimal buy-sell decisions of the portfolio manager.

The management of an airline company defines the trading rules for the portfolio manager such as the hedging strategy and the upper purchasing and lower selling limits. To guarantee its portfolio manager a certain degree of hedging flexibility and optimization of his hedging position, the board also allows short selling, dependent on the portfolio manager's view(s) of decreasing (and according to his expectations favorable) market situations. Hence, within those trading rules, the portfolio manager has to apply an optimal trading strategy for the closing of the short position in CO_2 emission allowances, which we call in the following reduction tactics.

Figure 3.2 illustrates the systematics of the reference strategy, hedging strategy and hedging optimization from an airline company's point of view.

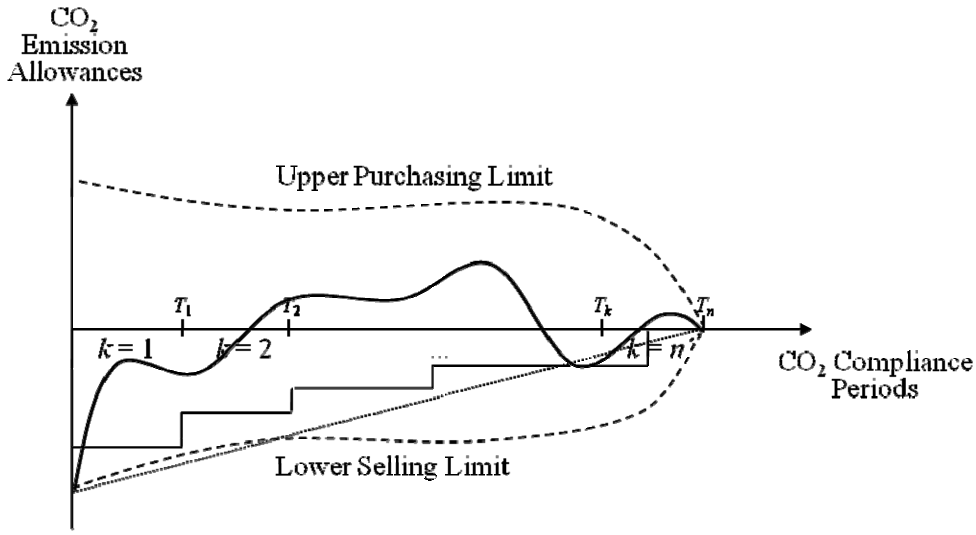

Figure 3.2. Reference strategy, hedging strategy and hedging optimization of the natural short position in CO_2 emission allowances (EUAs, CERs).

We denote $k = 0,1,...,n-1,$ as the index for the k+1th CO_2 compliance period, where, at the end of each T_{k+1}, as the last discrete time of the corresponding terminating CO_2 compliance period $k+1$, the missing amount of CO_2 emission allowances in the CO_2 account of the portfolio manager has to be mandatorily surrendered to the regulatory authority, and such that $T_{k+1} = T_1,...,T_n.$ Thus, the whole hedging period covers a total trading time horizon of $t = T_0(=0),1,2...,T_1,...,T_{k+1},...,T_n,$ which will be explained and specified in more detail in Subsection 6.4.1. The hedging system is defined in the following three parts.

Reference strategy

A reference strategy is a board-defined, passive and neutral strategy for the reduction of the short position in CO_2 emission allowances, which serves as a benchmark for the evaluation of the active trading strategies of the portfolio manager. Hence, it indicates the market index. Here, the reference strategy is represented as a linear increasing line, which means that for each trading time period t an equal amount of the natural short position in CO_2 emission allowances is closed. However, dependent on the executive board's decision, it could also be represented as piece-wise linear.

Hedging (i.e., reduction) strategy

A hedging strategy is an active strategy, conducted by an airline company's advisory committee, independent from the board, and which is based on market expectations for the reduction of the short position in CO_2 emission allowances. The hedging strategy, incorporates of a defined percentage amounts of CO_2 emission allowances to be closed by the portfolio manager through hedging (i.e., reduction) optimization over the periods $t = 1,...,T_n$. It also contains binding upper purchasing limits $u_{i,t}$ and binding lower selling limits $v_{i,t}$, for $i = 1, 2,$ which denotes the index for EUAs and CER, respectively. Thus, the upper and lower trading limits sets limits the possibilities of the reduction of the short position in CO_2 allowances (i.e., corridor). The trading limits are periodically determined by the advisory committee, and could be the same for all periods, i.e., $u_{i,1} = u_{i,2} = ... = u_{i,T_n}$ and $v_{i,1} = v_{i,2} = ... = v_{i,T_n}$, or vary for each period *t*, respectively. Here, the upper and lower trading limits are presented as corrugated, dashed lines, and the reduction strategy is represented as a piece-wise increasing line.

Hedging (i.e., reduction) optimization

The hedging (i.e., reduction) optimization, which we can also shortly denote reduction tactics refer to the trading actions taken by the portfolio manager for the implementation of the board-defined reduction strategy within the binding limits in operative business (trading business unit). The portfolio manager defines, conducts and is the only responsible for the reduction tactics, which consist of optimal futures-spot trading strategies over the whole trading period $t = 1,...,T_n$, based on his market expectations. That is, within the defined trading rules, the portfolio manager is allowed to implement any futures trading action which optimizes his value of the portfolio over the whole trading period $t = 1,...,T_n$. Here, the reduction tactics is illustrated as a wavy, bold line, beginning in a short (i.e., negative) position in CO_2 emission allowances and ending in a just offsetting position at the last point in time of the trading period T_n. We remark that in this illustrative Figure 5, the increase of the wavy, bold line means that the short is being closed by purchasing of CO_2 emission allowances, and the its decrease refers to their sales, implying an re-increase of the short position.

CHAPTER 4

GEOMETRIC BROWNIAN MOTION OF CORRELATED EUA AND CER FUTURES PRICES

Due to our multi-period hedging optimization problem of the short position in CO_2 emission allowances, i.e., the consideration of n-CO_2 compliance periods, we will use futures instead of spot prices. Thereby, we will use EUA and CER futures contracts of various delivery periods.

Empirical evidence [4, 11, 29] shows that unlike electricity or gas prices, EUA and CER prices do neither exhibit a mean-reversion (i.e. long-term trend path) nor any seasonal patterns nor any jumps. For that purpose, we model the stochasticity of the EUA and CER futures prices by correlated geometric Brownian motion (GBM) processes, rather than applying the traditional Ornstein-Uhlenbeck (O-U) mean-reversion process, as applied for energy prices [8, 20, 56, 32]. In Chapter 7, we will empirically justify the use of the GBM in our model.

Figure 4.1 and 4.2 illustrate the historical yearly EUA and CER futures prices for various CO_2 delivery periods for the period 23/03/2009–16/11/2012, traded at the Intercontinental Exchange (ICE), London. There is a relatively strong correlation between EUA and CER futures prices, whereas the futures prices for CERs are fundamentally less than the market prices for EUAs (see Section 2.1). Therefore, developments in these two markets are influencing each other significantly. For the first sub period from 3 March 2009 to 11 May 2011 both the EUA and CER market faced a slightly increasing growth with partially volatile periods. For the second period from 12 May 2011 to 16 November 2012, however, there exists another picture. Due to the worldwide actual discussions and uncertainty about the further continuation of the Kyoto Protocol or its adoption of another form in 2015, the market for EUA and CER futures has been decreasing since 12 May 2011. Especially, due to the unsatisfactory outcomes of the climate change conferences[6] in Cancun in November 2010 and in Durban in November 2011, pessimistic expectations regarding both markets have been resulting. For the whole period, both markets have been exhibiting neither any seasonal patterns nor jumps.

Due to the occurrence of both increasing and decreasing carbon market situations, in Chapter 7, we will build two scenarios, an *optimistic* and a *pessimistic* market scenario, for the modeling of correlated EUA and CER price futures prices through GBM, and solve our optimization model, based on these.

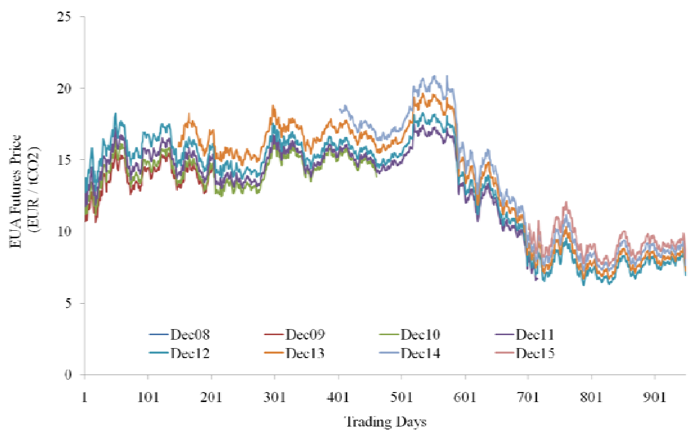

Figure 4.1. Historical EUA futures prices 03/03/2009–16/11/2012 (ICE data).

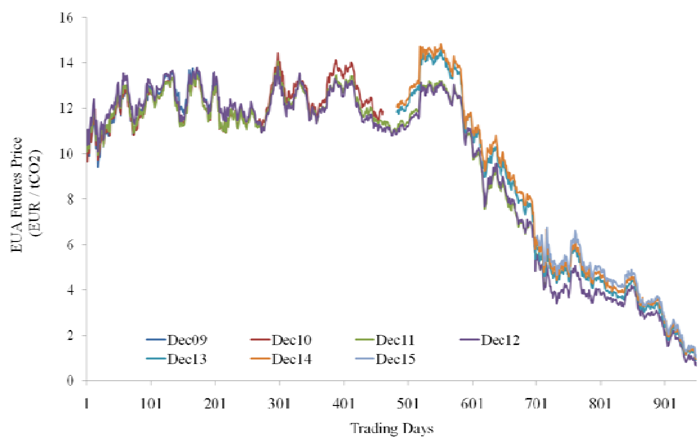

Figure 4.2. Historical CER futures prices 03/03/2009–16/11/2012 (ICE data).

[6] Conference of Parties (COP)

In the next sections, we will first give theoretical foundations for multi-correlated GBM for spot prices and determine the explicit solutions for the Stochastic Differential Equations (SDE's). Based on them, we will derive the explicit solutions for *n*-correlated futures prices, which have not found enough emphasis in academic finance literature until now. However, instead of using spot prices for MC simulation and facing the problem of modeling risk premia, for mid-term planning and hedging issues, taking directly observable futures prices (including risk premia) from a liquid exchange seems to be more purposeful. In the last part of this chapter, we will explain and set up the MC simulation method, used in the next chapters.

4.1 Multi-Correlated Brownian Motions: Theoretical Foundations

Since we will use a mid-term hedging optimization horizon, the evolution of the underlying EUA and CER futures prices, which incorporate strong correlations with each other, will be modeled and developed in the following. The model thereby used will be the geometric Brownian motion (GBM) process of correlated EUA and CER futures prices.

The GBM dynamics of *n*-correlated asset prices is given by the following stochastic differential equations (SDEs) [8, 20, 46, 56]:

$$\frac{dS_{j,t}}{S_{j,t}} = \mu_j dt + \sigma_j dW_{j,t}, \tag{4.1}$$

where $(W_t)_{t\geq 0}$ are *n*-dimensional correlated Brownian motion, with correlation matrix $\rho = (\rho_{jk})_{1\leq j,k\leq n}$, with $-1 \leq \rho_{jk} \leq 1$, $S_{j,t}$ represent spot prices for assets $j = 1,...,n$, at time $t \geq 0$, the parameters $\mu_j \in \mathbb{R}$ and $\sigma_j > 0$ are the drift and the volatility of asset *j*, respectively, both being constants. Thus, the returns of $S_{j,t}$ are correlated with ρ.

The correlation structure of $S_{j,t}$ will be analyzed in the following and prepared for Monte-Carlo simulation.

For that purpose, let us firstly give a general definition for an *n*-dimensional Brownian motion.

Definition 4.1.1. ***n-Dimensional Brownian Motion:*** *A standard Brownian motion (or a standard Wiener process) in* $\mathbb{R}^n$, *or a standard n-dimensional Brownian motion, is a stochastic process* $(Z_t)_{t\geq 0}$ *whose value at time t is simply a vector of n independent Brownian motions t, such that*

$$\mathbb{Z}_t = \left(Z_{1,t},...,Z_{n,t}\right).$$

Each $Z_{k,t}$ represents the value of one-dimensional Brownian motion at time $t \geq 0$. Additionally, the various elements $Z_{j,t}, Z_{k,t}\ (j \neq k)$ are independent for all times $t, t^* \geq 0$.

Now, for simplicity let us consider the case of two independent Brownian motions $Q_{1,t}$ and $Q_{2,t}$ with $-1 \leq \rho \leq 1$ as constant. We define for $0 \leq t \leq T$ a new process

$$Z_t = \rho Q_{1,t} + \sqrt{1-\rho^2}\,Q_{2,t}. \tag{4.2}$$

We will see later how equation (4.2) is derived and how its relation to our formulation above is. Equation (4.2) says, that at each time step *t*, Z_t is a linear combination of independent normals $\left(Q_{1,t}, Q_{2,t}\right)$. Thus, Z_t is normally distributed.

We have to show that Z_t is a Brownian motion such that $E[Z_t]=0, Var[Z_t]=t, Z_t \sim N(0,t),$ $Z_t - Z_s \sim N(0,t-s).$ We know that $E\left[Q_{1,t}\right]=0$ and $E\left[Q_{2,t}\right]=0.$ Thus, the expected value of Z_t is

$$\begin{aligned}
E\left[Z_t\right] &= E\left[\rho Q_{1,t} + \sqrt{1-\rho^2}\,Q_{2,t}\right] \\
&= \rho E\left[Q_{1,t}\right] + \sqrt{1-\rho^2}\,E\left[Q_{2,t}\right] \\
&= \rho \cdot 0 + \sqrt{1-\rho^2} \cdot 0 \\
&= 0.
\end{aligned}$$

The variance of Z_t is

$$Var\left[Z_t\right] = Var\left[\rho Q_{1,t} + \sqrt{1-\rho^2}\,Q_{2,t}\right]$$

$$= Var\left[\rho Q_{1,t}\right] + Var\left[\sqrt{1-\rho^2}\,Q_{2,t}\right].$$

Since both random variables $\rho Q_{1,t}$ and $\sqrt{1-\rho^2}\,Q_{2,t}$ are independent, we know that $Var\left[Q_{1,t}\right]=t$ and $Var\left[Q_{2,t}\right]=t$. Therefore,

$$\begin{aligned} Var\left[Z_t\right] &= \rho^2 Var\left[Q_{1,t}\right] + \left(\sqrt{1-\rho^2}\right)^2 Var\left[Q_{2,t}\right] \\ &= \rho^2 t + (1-\rho^2)t \\ &= t. \end{aligned}$$

As a next step, we consider the increment

$$\begin{aligned} Z_{t+s} - Z_t &= \left[\rho Q_{1,t+s} + \sqrt{1-\rho^2}\,Q_{2,t+s}\right] - \left[\rho Q_{1,t} + \sqrt{1-\rho^2}\,Q_{2,t}\right] \\ &= \rho\left[Q_{1,t+s} - Q_{1,t}\right] + \sqrt{1-\rho^2}\left[Q_{2,t+s} - Q_{2,t}\right]. \end{aligned}$$

The expressions $Q_{1,t+s} - Q_{1,t}$ and $Q_{2,t+s} - Q_{2,t}$ are the independent random increment of Brownian motions Q_1 and Q_2, respectively, over the time interval *s*. Since both random increments are independent, through multiplication by a constant, the variance of the sum gets the sum of the variance

$$\begin{aligned} Var\left[Z_{t+s} - Z_t\right] &= Var\left\{\rho\left[Q_{1,t+s} - Q_{1,s}\right] + \sqrt{1-\rho^2}\left[Q_{2,t+s} - Q_{2,s}\right]\right\} \\ &= Var\left\{\rho\left[Q_{1,t+s} - Q_{1,s}\right]\right\} + Var\left\{\sqrt{1-\rho^2}\left[Q_{2,t+s} - Q_{2,s}\right]\right\} \\ &= \rho^2 Var\left[Q_{1,t} - Q_{1,s} - Q_{1,t}\right] + \left(\sqrt{1-\rho^2}\right)^2 Var\left[Q_{2,t} - Q_{2,s} - Q_{2,t}\right] \\ &= \rho^2 Var\left[Q_{1,s}\right] + \left(\sqrt{1-\rho^2}\right)^2 Var\left[Q_{2,s}\right]. \end{aligned}$$

Since $Var\left[Q_{1,s}\right]=s$ and $Var\left[Q_{2,s}\right]=s$,

$$\begin{aligned} Var\left[Z_{t+s} - Z_t\right] &= \rho^2 s + (1-\rho^2)s \\ &= s. \end{aligned}$$

Consequently, the variance does not depend on the starting time t of the increment s, and is equal to the length of the interval. Therefore, Z_t follows a Brownian motion. The variance of the random increment $Q_{1,t+s} - Q_{1,t}$ is

$$\begin{aligned} Var\left[Q_{1,t+s} - Q_{1,t}\right] &= Var\left[Q_{1,t+s}\right] + Var\left[Q_{1,t}\right] - 2Cov\left[Q_{1,t+s}, Q_{1,t}\right] \\ &= (t+s) + t - 2\min(t+s,t) \\ &= (t+s) + t - 2t \\ &= s. \end{aligned}$$

The Brownian motions Q_1 and Z are correlated at time t. According to Itô's product rule,

$$\begin{aligned} d\left(Q_{1,t}Z_t\right) &= Q_{1,t}dZ_t + Z_t dQ_{1,t} + dQ_{1,t}dZ_t \\ &= Q_{1,t}dZ_t + Z_t dQ_{1,t} + \rho t. \end{aligned}$$

By integrating, we get

$$Q_{1,t}Z_t = \int_0^t Q_{1,u}dZ_u + \int_0^t Z_u dQ_{1,u} + \rho t. \tag{4.3}$$

By taking the expectation of expression (4.3), we come up with the covariance between $Q_{1,t}$ and Z_t, such that

$$\mathrm{E}\left[Q_{1,t}Z_t\right] = \mathrm{E}\left[\int_0^t Q_{1,u}dZ_u\right] + \mathrm{E}\left[\int_0^t Z_u dQ_{1,u}\right] + \rho t.$$

We know, that the expectation of an Itô integral is zero, i.e.,

$$\mathrm{E}\left[\int_0^t Q_{1,u}dZ_u\right] = 0 \quad \text{and} \quad \mathrm{E}\left[\int_0^t Z_u dQ_{1,u}\right] = 0.$$

Hence, the covariance between $Q_{1,t}$ and Z_t becomes

$$E\left[Q_{1,t}Z_t\right] = \rho t,$$

for $-1 \leq \rho \leq 1$. Let the covariance $E\left[Q_{1,t} Z_t\right]$ denote as $Cov\left[Q_{1,t}, Z_t\right]$ then correlation between $Q_{1,t}$ and Z_t is defined as

$$Corr\left[Q_{1,t}, Z_t\right] = \frac{Cov\left[Q_{1,t}, Z_t\right]}{\sqrt{Var\left[Q_{1,t}\right]}\sqrt{Var\left[Z_t\right]}}.$$

Since

$$E\left[dQ_{1,t} dZ_t\right] = Cov\left[Q_{1,t}, Z_t\right] = \rho t,\ Var\left[Q_{1,t}\right] = t \text{ and } Var\left[Q_t\right] = t,$$

we obtain

$$Corr\left[Q_{1,t}, Z_t\right] = \frac{\rho t}{\sqrt{t}\sqrt{t}} = \rho.$$

Consequently, at all times *t* the Brownian motions $Q_{1,t}$ and Z_t have correlation ρ.

Now, let us generalize our two-asset case to *n* correlated asset prices, which are based on *n* correlated Brownian motions. Their main components are correlated normal random variables, beginning with a vector of *n* uncorrelated standard normal variables for each $t \geq 0$, , i.e., $\mathbb{Z}_t = \left(Z_{1,t}, \ldots, Z_{n,t}\right)$. Through these, we create normal random variables which are correlated by pre-defined constant correlation coefficient by linear combinations of $Z_{j,t}$. We denote the weights as α_{jk}. Then

$$\begin{aligned}
X_{1,t} &= \alpha_{11} Z_{1,t} + \ldots + \alpha_{1k} Z_{k,t} + \ldots + \alpha_{1n} Z_{n,t}, \\
&\vdots \\
X_{j,t} &= \alpha_{j1} Z_{1,t} + \ldots + \alpha_{jk} Z_{k,t} + \ldots + \alpha_{jn} Z_{n,t}, \\
&\vdots \\
X_{n,t} &= \alpha_{n1} Z_{1,t} + \ldots + \alpha_{nk} Z_{k,t} + \ldots + \alpha_{nn} Z_{n,t},
\end{aligned}$$

such that

$$X_{j,t} = \sum_{k=1}^{n} \alpha_{jk} Z_{k,t},$$

or, written in matrix notation,

$$X_t = AZ_t, \tag{4.4}$$

where

$$A = \begin{bmatrix} \alpha_{11} & \dots & \alpha_{1k} & \dots & \alpha_{1n} \\ \dots & \ddots & \dots & \ddots & \dots \\ \alpha_{j1} & \dots & \alpha_{jk} & \dots & \alpha_{jn} \\ \dots & \ddots & \dots & \ddots & \dots \\ \alpha_{n1} & \dots & \alpha_{nk} & \dots & \alpha_{nn} \end{bmatrix}, \ Z_t = \begin{bmatrix} Z_{1,t} \\ \dots \\ Z_{j,t} \\ \dots \\ Z_{n,t} \end{bmatrix}, \ X_t = \begin{bmatrix} X_{1,t} \\ \dots \\ X_{j,t} \\ \dots \\ X_{n,t} \end{bmatrix}.$$

We know that the random variables $X_{j,t}$ and $X_{k,t}$ are correlated with $\rho_{jk,t}$. Then, for one single element, we have

$$E\left[X_{j,t}X_{k,t}\right] = \rho_{jk,t}. \tag{4.5}$$

Putting all the corresponding expected values together into a matrix, results in the correlation matrix ρ such that

$$\rho = E\begin{bmatrix} X_{1,t}X_{1,t} & \dots & X_{1,t}X_{k,t} & \dots & X_{1,t}X_{n,t} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ X_{j,t}X_{1,t} & \dots & X_{j,t}X_{k,t} & \dots & X_{j,t}X_{n,t} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ X_{n,t}X_{1,t} & \dots & X_{n,t}X_{k,t} & \dots & X_{n,t}X_{n,t} \end{bmatrix}$$

$$= \begin{bmatrix} E\left[X_{1,t}X_{1,t}\right] & \dots & E\left[X_{1,t}X_{k,t}\right] & \dots & E\left[X_{1,t}X_{n,t}\right] \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ E\left[X_{j,t}X_{1,t}\right] & \dots & E\left[X_{j,t}X_{k,t}\right] & \dots & E\left[X_{j,t}X_{n,t}\right] \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ E\left[X_{n,t}X_{1,t}\right] & \dots & E\left[X_{n,t}X_{k,t}\right] & \dots & E\left[X_{n,t}X_{n,t}\right] \end{bmatrix}$$

$$
= \begin{bmatrix} 1 & \cdots & \rho_{1k,t} & \cdots & \rho_{1n,t} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \rho_{j1,t} & \cdots & 1 & \cdots & \rho_{jn,t} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \rho_{n1,t} & \cdots & \rho_{nk,t} & \cdots & 1 \end{bmatrix}.
$$

We can also write the matrix

$$
\begin{bmatrix} X_{1,t}X_{1,t} & \cdots & X_{1,t}X_{k,t} & \cdots & X_{1,t}X_{n,t} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ X_{j,t}X_{1,t} & \cdots & X_{j,t}X_{k,t} & \cdots & X_{j,t}X_{n,t} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ X_{n,t}X_{1,t} & \cdots & X_{n,t}X_{k,t} & \cdots & X_{n,t}X_{n,t} \end{bmatrix}
$$

as the matrix product of X_t and its transpose X_t^T, i.e., $X_t X_t^T$. By using equation (4.4), we get

$$
X_t X_t^T = \left(A Z_t \right)\left(A Z_t \right)^T = \left(A Z_t \right)\left(Z_t^T A^T \right) = A \left(Z_t Z_t^T \right) A^T .
$$

Based on that, the expected value is

$$
E\left[X_t X_t^T \right] = E\left[A\left(Z_t Z_t^T \right) A^T \right] = AE\left[Z_t Z_t^T \right] A^T .
$$

The matrix $E\left[Z_t Z_t^T \right]$ is the correlation matrix of standard normal variables Z_t. Since all of these are uncorrelated, this results in the identity matrix I, implying

$$
E\left[X_t X_t^T \right] = AE\left[I \right] A^T = AA^T .
$$

Therefore, according equation (4.5), we have

$$
\rho = AA^T .
$$

The correlation matrix $\rho = \left(\rho_{jk} \right)_{1 \le j,k \le n}$ is symmetric, i.e., $\rho_{jk} = \rho_{kj} \in [-1,1]$, and positive definite, i.e.,

$$\sum_{j=1}^{n}\sum_{k=1}^{n}\rho_{ij,t}X_{j,t}X_{k,t}\geq 0,$$

for all $X_t=\left(X_{1,t},...,X_{1,t}\right)_{t\geq 0}$, implying that all eigenvalues of ρ are positive. Consequently, ρ can be decomposed into the product of a lower-triangular matrix *A* and its transpose A^T, which is called the *Cholesky* decomposition, i.e.,

$$\rho=\begin{bmatrix}1 & \cdots & \rho_{1k,t} & \cdots & \rho_{1n,t}\\ \vdots & \ddots & \vdots & \ddots & \vdots\\ \rho_{j1,t} & \cdots & 1 & \cdots & \rho_{jn,t}\\ \vdots & \ddots & \vdots & \ddots & \vdots\\ \rho_{n1,t} & \cdots & \rho_{nk,t} & \cdots & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\ \alpha_{21} & \alpha_{22} & 0 & 0 & 0\\ \alpha_{31} & \alpha_{32} & \alpha_{33} & 0 & 0\\ \vdots & & & & \\ \alpha_{n1} & \alpha_{n2} & \alpha_{nn} & & \alpha_{nn}\end{bmatrix}\times\begin{bmatrix}1 & \alpha_{21} & \alpha_{31} & \cdots & \alpha_{n1}\\ 0 & \alpha_{22} & \alpha_{32} & & \alpha_{n2}\\ 0 & 0 & \alpha_{33} & & \alpha_{nn}\\ \vdots & & & & \\ 0 & 0 & 0 & & \alpha_{nn}\end{bmatrix}.$$

If we use the elements of *A*, we get the correlated random variables X_i as

$$\begin{aligned}&X_{1,t}=1Z_{1,t}=Z_{1,t},\\ &X_{2,t}=\alpha_{21}Z_{1,t}+\alpha_{22}Z_{2,t},\\ &X_{3,t}=\alpha_{31}Z_{1,t}+\alpha_{32}Z_{2,t}+\alpha_{33}Z_{2,t},\\ &\vdots\\ &X_{n,t}=\alpha_{n1}Z_{1,t}+...+\alpha_{nj}Z_{j,t}+...+\alpha_{nn}Z_{n,t}.\end{aligned}$$

In academic literature, there are many numerical algorithms for conducting the Cholesky decomposition [9, 27, 33, 36, 45, 48]. We conducted it with the formula provided in MATLAB. For the two-asset case $n=2,$ we have

$$A=\begin{bmatrix}1 & 0\\ \rho & \sqrt{1-\rho^2}\end{bmatrix},\ A^T=\begin{bmatrix}1 & \rho\\ 0 & \sqrt{1-\rho^2}\end{bmatrix},\ AA^T=\begin{bmatrix}1 & \rho\\ \rho & 1\end{bmatrix},$$

$$\alpha_{21}=\rho,\ \alpha_{22}=\sqrt{1-\rho^2},$$

Therefore,

$$X_{1,t} = Z_{1,t},$$
$$X_{2,t} = \rho Z_{1,t} + \sqrt{1-\rho^2} Z_{2,t},$$

which just equals equation (4.1).

Since in financial mathematics $W_{i,t}$ is used for Brownian motion notation, we define and use from now on

$$X_{i,t} = W_{i,t}.$$

Now, denoting $\mathbb{Z}_t = \left(Z_{1,t},...,Z_{n,t}\right)$ as the standard n-dimensional Brownian motion and by use of the lower-triangular matrix $A = \left(\alpha_{jk}\right)_{1\le j,k\le n}$, which is the Cholesky decomposition of ρ, a vector consisting of correlated Brownian motions $W_t = \left(W_{1,t},...,W_{n,t}\right)_{t\ge 0}$ can be defined, such that

$$W_t = A\mathbb{Z}_t,$$

or, with regard to individual elements in the system,

$$W_{j,t} = \sum\nolimits_{k=1}^{n} \alpha_{jk} Z_{k,t}, \tag{4.6}$$

Now, turning back to equation (4.1), by rearranging we get,

$$dS_{j,t} = \mu_j S_{j,t} dt + \sigma_j S_{j,t} dW_{j,t} \tag{4.7}$$

By inserting the elements of equation (4.6) in equation (4.7) we get

$$dS_{j,t} = \mu_j S_{j,t} dt + \sigma_j S_j \sum\nolimits_{k=1}^{n} a_{jk} Z_{k,t} \tag{4.8}$$

resulting in the following system of SDE's,

$$
\begin{aligned}
dS_{1,t} &= \mu_1 S_{1,t} dt + \sigma_1 S_{1,t} \left(\alpha_{11} dZ_{1,t} + \ldots + \alpha_{1n} dZ_{n,t} \right), \\
&\vdots \\
dS_{n,t} &= \mu_n S_{n,t} dt + \sigma_n S_{n,t} \left(\alpha_{n1} dZ_{1,t} + \ldots + \alpha_{nn} dZ_{n,t} \right),
\end{aligned} \tag{4.9}
$$

In order to solve for $S_{j,t}$, we can rewrite equation (4.7) in integral form

$$
S_{j,t} = S_{j,0} + \mu_j \int_0^t S_{j,s} ds + \sigma_j \int_0^t S_{j,s} dW_{j,s}, \tag{4.10}
$$

We can solve equation (4.10) by the well-known Itô formula

$$
f\left(S_{j,t}\right) = \mu_j S_{j,t} \frac{\partial f}{\partial S_{j,t}} dt + \sigma_j S_{j,t} \frac{\partial f}{\partial S_{j,t}} dW_{j,t} + \sigma_j^2 S_{j,t}^2 \frac{1}{2} \frac{\partial^2 f}{\partial S_{j,t}^2} dt.
$$

By use $f\left(S_{j,t}\right) = \log S_{j,t}$, we get

$$
d \log S_{j,t} = \mu_j S_{j,t} \frac{\partial f}{\partial S_{j,t}} dt + \sigma_j S_{j,t} \frac{\partial f}{\partial S_{j,t}} dW_{j,t} + \sigma_j^2 S_{j,t}^2 \frac{1}{2} \frac{\partial^2 f}{\partial S_{j,t}^2} dt.
$$

By determining $\frac{\partial f}{\partial S_{j,t}} = \frac{1}{S_{j,t}}$ and $\frac{\partial f^2}{\partial S_{j,t}^2} = -\frac{1}{S_{j,t}^2}$, we result in

$$
\begin{aligned}
d \log S_{j,t} &= \mu_j S_{j,t} \left(\frac{1}{S_{j,t}} \right) dt + \sigma_j S_{j,t} \left(\frac{1}{S_{j,t}} \right) dW_{j,t} + \sigma_j^2 S_{j,t}^2 \frac{1}{2} \left(-\frac{1}{S_{j,t}^2} \right) dt \\
&= \mu_j dt + \sigma_j dW_{j,t} - \frac{1}{2} \sigma_j^2 dt \\
&= \left(\mu_j - \frac{1}{2} \sigma_j^2 \right) dt + \sigma_j dW_{j,t}.
\end{aligned}
$$

Thus, we get

$$
\log S_{j,t} - \log S_{j,0} = \int_0^t d \log S_{j,r}
$$

$$= \int_0^t \left(\mu_j - \frac{1}{2}\sigma_j^2 \right) dr + \int_0^t \sigma_j dW_{j,r}$$

$$= \left(\mu_j - \frac{1}{2}\sigma_j^2 \right) t + \sigma_j W_{j,t}.$$

Hence, we get the explicit solution of the SDE's

$$S_{j,t} = S_{j,0} \exp\left(\left(\mu_j - \frac{1}{2}\sigma_j \right) t + \sigma_j W_{j,t} \right). \tag{4.11}$$

By inserting our system in equation (4.9) into equation (4.11), we result in the following systems of equations

$$\begin{aligned} S_{1,t} &= S_{1,0} \exp\left(\left(\mu_1 - \frac{1}{2}\sigma_1 \right) t + \sigma_1 \left(\alpha_{11} Z_{1,t} + \ldots + \alpha_{1n} Z_{n,t} \right) \right), \\ &\vdots \\ S_{n,t} &= S_{n,0} \exp\left(\left(\mu_n - \frac{1}{2}\sigma_n \right) t + \sigma_n \left(\alpha_{n1} Z_{1,t} + \ldots + \alpha_{nn} Z_{n,t} \right) \right). \end{aligned}$$

Finally, in general form, we can write

$$S_{j,t} = S_{j,0} \exp\left(\left(\mu_j - \sigma_j \frac{1}{2} \right) t + \sigma_j \left(\sum_{k=1}^n \alpha_{jk} Z_{k,t} \right) \right). \tag{4.12}$$

4.2 Multi-Correlated Brownian Motions of Futures Prices

As explained several times before, in our model, due to mid-term planning and hedging issues of the natural physical short position in CO_2 allowances, we will use futures contracts and therefore, futures prices instead of spot prices.

Futures contracts allow the exchange of future unconditional obligations on terms which are defined in advance. Thus, they enable market participants to make planned transactions prematurely to smooth cash flows and thus to generate added value. Hence, futures are considered as instruments that are purely used to obtain an optimal risk allocation in the market. In addition to that, futures contracts meet the other two functions of financial markets: information processing and investment motive.

In academic research as well as in practice, carbon meets all the properties that are typical for commodities: It is a consumer good, which is standardized in terms of quality, place of delivery and delivery period. That is why, carbon is clearly considered as a commodity [31, 62]. It is important to note that in the case of commodities, since a delivery period is usually defined, at which physical delivery takes place, they are associated with embedded option(s). We refer on that issue in more detail in Section 6.1.

The well-known fundamental, non-arbitrage relation between spot and futures prices used for the pricing of commodities is

$$F_{t,T} = S_t \exp\left((r_T + u_T - y_{t,T} - q_{t,T})(T-t)\right), \tag{4.13}$$

where $F_{t,T}$ is the futures prices of a commodity at time $t = 1,\dots,T$ for delivery in period (i.e., maturity) T, S_t is the spot price at t, r_T is the risk-free for period T, u_T is the storage cost for T, $y_{t,T}$ is the convenience yield for holding the asset through the period t and T, and $q_{t,T}$ is the accrued dividend yield of the asset through the period t and T. The expression in the bracket at the right hand-side of equation (4.13) represents the risk premium of an investor of holding an asset through the period t until the maturity T.

There are traded futures of various delivery periods at liquid exchanges such as ICE (London), European Energy Exchange (Leipzig) or Nordpool (Scandinavia), such that for our further purpose, we do not have to care about the determination and consideration of any risk premiums since these are directly incorporated in the futures prices. Thus, we can directly make use of the liquidly traded futures prices $F_{t,T}$.

Therefore, the explicit solution for the SDE's in equation (4.12) can be reformulated to

$$F_{j,(t,T)} = F_{j,(0,T)} \exp\left(\left(\mu_j - \frac{1}{2}\sigma_j\right)t + \sigma_j\left(\sum_{k=1}^{n}\alpha_{jk}Z_{k,(t,T)}\right)\right) \tag{4.14}$$

In case of n-delivery periods of futures prices (i.e., maturities), we can set $T = T_n$ such that equation (4.14) becomes

$$F_{j,(t,T_n)} = F_{j,(0,T_n)} \exp\left(\left(\mu_j - \frac{1}{2}\sigma_j\right)t + \sigma_j\left(\sum_{k=1}^{n}\alpha_{jk}Z_{k,(t,T_n)}\right)\right) \tag{4.15}$$

In order, to set up explicit solutions for correlated GBM's of futures prices for different maturities between any observed time t and maturity T_n, we can adjust equation (4.15) to

$$F_{j,(t+\Delta t,T_n)} = F_{j,(t,T_n)} \exp\left(\left(\mu_j - \frac{1}{2}\sigma_j\right)\Delta t + \sigma_j\left(\sum_{k=1}^{n}\alpha_{jk}Z_{k,(t+\Delta t,T_n)}\right)\right),$$

(4.16)

where Δt is the discrete time step.

4.3 Monte-Carlo Simulation

According to equation (4.6), by substituting $W_{j,t} = \varepsilon_{j,t}$, we get

$$\varepsilon_{j,t} = \sum_{k=1}^{n}\alpha_{jk}Z_{k,t}.$$

Thus, after having created correlated normal random variables $\varepsilon_{j,t}$ by the calculated α_{jk} from the given correlation matrix ρ and the standard n-dimensional Brownian motion $Z_{k,t}$ (see Section 4.1), it is now possible to compute the correlated Brownian motions for any given time step Δt through multiplication of each correlated random variable $\varepsilon_{j,t}$ by $\sqrt{\Delta t}$.

Knowing this, to perform MC simulation of correlated EUA and CER futures prices for any t to maturity T_n (i.e., delivery period), we can construct equation (4.16) as a system of SDE's with a more notation convenient form as

$$F_{1,(t+\Delta t,T_n)} = F_{1,(t,T_n)} \exp\left(\left(\mu_1 - \sigma_1 \frac{1}{2}\right)\Delta t + \sigma_1 \varepsilon_{1,(t,T_n)} \sqrt{\Delta t}\right),$$
$$\vdots$$
$$F_{j,(t+\Delta t,T_n)} = F_{j,(t,T_n)} \exp\left(\left(\mu_j - \sigma_j \frac{1}{2}\right)\Delta t + \sigma_j \varepsilon_{j,(t,T_n)} \sqrt{\Delta t}\right), \quad (4.17)$$
$$\vdots$$
$$F_{n,(t+\Delta t,T_n)} = F_{n,(t,T_n)} \exp\left(\left(\mu_n - \sigma_n \frac{1}{2}\right)\Delta t + \sigma_n \varepsilon_{n,(t,T_n)} \sqrt{\Delta t}\right),$$
$$\forall\, t = 1,\ldots,n.$$

where $F_{1,(t+\Delta t,T_n)},\ldots,F_{n,(t+\Delta t,T_n)}$ are the corresponding futures prices to be obtained by MC simulation, Δt is the discrete time step, and $\varepsilon_{1,(t,T_n)},\ldots,\varepsilon_{n,(t,T_n)}$ are standard normal variables, i.e., $\varepsilon_{1,(t,T_n)},\ldots,\varepsilon_{n,(t,T_n)} \sim N(0,1)$.

Thus, we can state $E = \left(\varepsilon_{1,(t,T_n)}\sqrt{\Delta t},\ldots,\varepsilon_{n,(t,T_n)}\sqrt{\Delta t}\right) \sim N(0,\Sigma)$ with

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_{12} & \cdots & \sigma_1\sigma_k\rho_{1k} & \cdots & \sigma_1\sigma_n\rho_{1n} \\ \sigma_2\sigma_1\rho_{21} & \sigma_2^2 & \cdots & \sigma_2\sigma_k\rho_{2k} & \cdots & \sigma_2\sigma_n\rho_{2n} \\ \vdots & & \ddots & & & \vdots \\ \sigma_j\sigma_1\rho_{j1} & \cdot\cdot & & \sigma_j\sigma_k\rho_{jk} & \cdots & \sigma_j\sigma_n\rho_{jn} \\ \vdots & & \ddots & & \ddots & \vdots \\ \sigma_n\sigma_1\rho_{n1} & \cdots & \cdots & \sigma_n\sigma_k\rho_{nk} & \cdots & \sigma_n^2 \end{pmatrix},$$

being is the variance-covariance matrix between the *n*-futures prices. We note that for $j = k$, the terms $\sigma_j\sigma_k\rho_{jk}$ becomes $\sigma_j\sigma_j\rho_{jj} = \sigma_k\sigma_k\rho_{kk} = \sigma_j^2 = \sigma_k^2$, which is the variance of futures price *j* or *k*, respectively. For all $j \neq k$, the term $\sigma_j\sigma_k\rho_{jk}$ denotes the covariance between futures prices *j* and *k*.

Hence, in order to generate one sample value of $F_{1,(t+\Delta t,T_n)},\ldots,F_{n,(t+\Delta t,T_n)}$ we have to generate one sample value of $E = \left(\varepsilon_{1,(t,T_n)}\sqrt{\Delta t},\ldots,\varepsilon_{n,(t,T_n)}\sqrt{\Delta t}\right)$. Thus, by taking the Cholesky lower triangular matrix *A*, derived and explained in more detail in Section 4.1, and the generated random vector of independent unit normals $Z \sim N(0,I)$, where I is the unit matrix, we obtain a correlated random vector $E = AZ$.

For our further purpose, by use of the standard MC method, we will generate a finite number of s scenarios for $\left(F^{s}_{1,(t+\Delta t,T_n)},\ldots,F^{s}_{n,(t+\Delta t,T_n)}\right)$ with $s=1,\ldots,n,$ each considered with equal probability $p^s = n^{-1}$.

The concrete procedure for MC simulation, which will be performed in Chapter 8, looks as follows:

Step 1: *Collection of historical EUA and CER futures price data of relevant delivery periods (i.e., maturities).*

Step 2: *Estimation of the relevant GBM parameters* $\left(\mu_j,\sigma_j\right)$ *and determination of the variance-covariance matrix* Σ *from historical EUA and CER futures price data.*

Step 3: *Determination of the discretization size* $\Delta t,$ starting time point *t* and maturity T_n.

Step 4: *Generation of uniformly distributed random numbers between 0 and 1.*

Step 5: *Generation of correlated random vectors* $E=\left(\varepsilon_{1,(t,T_n)}\sqrt{\Delta t},\ldots,\varepsilon_{n,(t,T_n)}\sqrt{\Delta t}\right)$ *by using the variance-covariance matrix* Σ .

Step 6: *Determination of the scenario size n and generation of* $s=1,\ldots,n$ *scenarios for* $\left(F^{s}_{1,(t+\Delta t,T_n)},\ldots,F^{s}_{n,(t+\Delta t,T_n)}\right)$, *based on the use of the MC simulation parameters* $\left(\mu_j,\sigma_j,\Sigma\right)$ *and* $E=\left(\varepsilon_{1,(t,T_n)}\sqrt{\Delta t},\ldots,\varepsilon_{n,(t,T_n)}\sqrt{\Delta t}\right)$.

Step 7: *Determination of the weights* $p^s = n^{-1}$ *and weighting each scenario s by* p^s.

We note that the MC simulation in Chapter 8 will be done for concrete, liquidly traded EUA and CER futures contracts of specific delivery periods, which will defined in Chapter 7, such that our general theoretical framework, derived above, will be applied to a concrete optimization problem.

CHAPTER 5

MULTI-STAGE STOCHASTIC PROGRAMMING

There is a huge academic literature on multi-stage stochastic programming. We will only highlight the main issues and set up, which will be relevant for our purposes in next chapters.

The basic idea of a multi-stage stochastic programming model is that an agent makes optimal decisions for *T*-stages, given the uncertainty of events (i.e., random event). Let an agent make a decision in the first stage. After that decision a random event appears influencing the outcome of the first-stage decision. The agent can then make a recourse decision in the second stage which offsets any unfavorable outcomes that might have been resulted from the decision of the first-stage. Therefore, the optimal decision policy results in making the best decision in stage $t-1$, taken into account both the possible realizations of random outcomes and the best recourse decision in stage t for each random outcome. This procedure is then sequentially repeated for *T*-stages. All the possible random events are assigned with weights, called probability measure over the events.

In the following, we will only concentrate on the linear case of multi-stage stochastic programming, where the objective function is linear and set of feasible solutions are expressed by linear the constraints. For the generic formulation and all the other forms, cases and more detailed discussions of multi-stage stochastic programming, we refer to [3, 10, 34, 51].

5.1 Linear Multi-Stage Stochastic Programming

In multi-stage stochastic programming, we deal with optimized decision-making for several periods, given the uncertainty of events, which can be described by random vectors. Let us firstly give a definition for a random vector.

Definition 5.1.1. ***Random Vector****: Let us consider a finite space of T-stages (i.e., finite time horizon),* $t = 1,...,T$*, then a random vector* $\xi = \{\xi_1,...,\xi_T\}$, $\xi_t \in \mathbb{R}^{d_t}$ *(with* d_t *as a positive integer), is an underlying process of discrete-time stochastic data, defined on the filtered probability space* (Ω, F, P)*, and whose realizations are of d-dimensional data vectors.*

The set of all possible realizations of ξ_t for $t = 1,...,T$ is defined as the state space $\Omega := \Omega_1 \times \cdots \times \Omega_T$ with $\Omega_t \subseteq \mathbb{R}^{d_t}$. Thus, Ω limits the range of all possible outcomes of a random experiment. The σ-algebras incorporate the available ξ_t for the decision maker at time t, such that

$$F_1 \subseteq F_2 \subseteq ... \subseteq F_t \subseteq ... \subseteq F \quad (t = 1,...,T),$$

denoting that the set of information is increasing with time t. Hence, σ-algebras incorporate a sum of ξ_t and subset of Ω. Each ξ_t is associated with an occurrence probability $P : \xi_t \to [0,1]$.

The discrete-time stochastic data ξ_t have to be modeled and generated through a stochastic process. Now, according to each ξ_t optimal decisions should be undertaken by the decision maker. These could be represented by a vector of stochastic decision process, which we can define as follows.

Definition 5.1.2. ***Vector of Stochastic Decision Process:*** *A vector of stochastic decision process* $x = [x_1,...,x_T]$ *is an* F_t*-measurable function of* ξ*.*

This means, that x contains for all $t = 1,...,T$ the sequence of stochastic decisions x_t, which are assumed to be measurable with respect to the filtration

$$F_t := \sigma\{\xi_1,...,\xi_t\} \qquad (t=1,...,T).$$

A sequence of possible decisions x_t is commonly called a policy which responds conditionally to the random events ξ_t of the state space Ω. Therefore, a policy can be considered as a contingency plan and only incorporates the embedded flexibility in the system, which is a crucial feature in option pricing (see Section 6.1).

We assume that the probability distribution *P* of ξ is known and independent of *x*. That is, using the notation in [28], for $t = 1,...,T$, we can define the probability distribution of the random vector ξ.

Definition 5.1.3. ***Probability Distribution of the Random Vector*** ξ: *A Probability distribution of the random vector* ξ *can be defined as* $P = \mathrm{P}\xi^{-1}$ *and its tth marginal probability distribution by* P_t *such that*

$$P_t(B_t) = \mathrm{P}\xi^{-1}(\Xi_1 \times \cdots \times \Xi_{t-1} B_t \times \Xi_{t+1} \times \cdots \times \Xi_T), \;\; B_t \in \mathrm{B}(\Xi_t),$$

where $\Xi_t \in \mathbb{R}^d$ *stands for the support of* ξ_t *and* $\mathrm{B}(\Xi_t)$ *is the σ-field of its Borel subsets. Specifically,* $\Xi_1 \in \mathbb{R}^d$ *stands for* $\Xi_1 = \{\xi_1\}$.

We note that ξ_1 is deterministic, such that for $t = 1$, x_1, this defines the (deterministic, non-recursive) decision in the first stage. For all $t > 1$, x_t, which incorporates the corrective or recursive decisions in the following stages. Therefore, all the decisions and realizations can be represented as the sequence

$$x_1, \xi_1, x_2(x_1,\xi_1), \xi_2, x_2(x_1,\xi_1,\xi_2), ..., x_T(x_1,\xi_1,...,\xi_{T-1}),$$

meaning that the process of decisions incorporates nonanticipativity. Or, in other words, any decision x_t taken at time *t* depends only from the past information of the random values ξ_t and not from their future realizations, i.e.,

$$x_t = x_t(x_{t-1}, \xi_{t-1}, P) \quad (t = 1,...,T).$$

Let us define $\xi_{[t]} = (\xi_1,...,\xi_t)$ to indicate the history of the stochastic data process up to time t. Then, the general multi-stage stochastic programming model can be formulated as [51]:

$$\min_{x_1,...,x_T} \mathrm{E}\left[f_1(x_1) + f_2\left(x_1\left(\xi_{[2]}\right),\xi_2\right) + ... + f_T\left(x_T\left(\xi_{[T]}\right),\xi_T\right)\right], \tag{5.1}$$

subject to $\quad x_1 \in X_1,$

$$x_t\left(\xi_{[t]}\right) \in X_t\left(x_{t-1}\left(\xi_{[t-1]}\right),\xi_{[t]}\right) \quad (t = 2,...,T),$$

with

$$f_t(x_t,\xi_t) := c^T x_t,\ X_1 := \{x_1 : A_1 x_1 = b_1, x \geq 0\},$$

$$X_t(x_{t-1},\xi_t) := \{x_t : B_t x_{t-1} + A_t x_t = b_t, x_t \geq 0\} \quad (t = 2,...,T).$$

The data vector $\xi_1 := (c_1, A_1, b_1)$ is known at the first stage and thus is deterministic. We can define

$$\xi_t := (c_t, A_t, B_t, b_t) \in \mathbb{R}^{d_t} \quad (t = 2,...,T),$$

which implies that all or some elements of ξ_t can be random. We note that in formulation (5.1) $x_2,...,x_T$ are functions of the data process, and thus are suitable functional spaces, while $x_1 \in \mathbb{R}^{n_1}$ is a deterministic vector. For the whole sequence of policies (i.e., measurable mappings) $x_t : \mathbb{R}^{d_1} \times ... \times \mathbb{R}^{d_t} \to \mathbb{R}^{n_t}$. Likewise for the functions f_t we have $f_1 : \mathbb{R}^{n_1} \to \mathbb{R}$, which is deterministic, and $f_t : \mathbb{R}^{d_1} \times \mathbb{R}^{n_1} \to \mathbb{R}$, which are continuous. Since for our purposes, the discrete-time stochastic data process $\xi_1,...,\xi_T$ has a finite number of realizations, formulation (5.1) will result in a finite dimensional optimization problem. That is, all ξ_t have a finite distribution and Ω is the set of all possible combinations of realizations of ξ_t, which we call scenarios. Thus, we can replace ξ_t by ξ_t^s, such that $\xi^s = \{\xi_{1,...,}^s \xi_T^s\}$ for $s = 1,\ldots,n$. Therefore, with each scenario $s = 1,\ldots,n$ of ξ_t we can associate an occurrence probability

$$p^s = P\left(\xi^s\right),\ p^s \geq 0,\ \sum_{s=1}^{S} p^s = 1.$$

Based on the notations and terminology above and by the use of scenarios *s*, the general *linear multi-stage stochastic programming model with recourse and with a finite number of scenarios* can be formulated as [51]

$$\min_{x_{1,s},\ldots,x_{T,s}} \sum_{s=1}^{S} p^s \left[c_1^T x_1^s + (c_2^s)^T x_2^s + \ldots + (c_T^s)^T x_T^s \right], \tag{5.2}$$

subject to

$$\begin{aligned}
A_1 x_1^s \quad & & &= b_1, \\
B_2^s x_1^s + A_2^s x_s^s & & &= b_2^s, \\
& B_3^2 x_2^s + A_3^s x_3^s & &= b_3^s, \\
& \ddots & &\vdots \\
& & B_T^s x_{T-1}^s + A_{T-1}^s x_T^s &= b_T^s,
\end{aligned}$$

$$x_1 \ge 0,\ x_t^s \ge 0 \quad (t = 2,\ldots,T;\ s \in \Omega),$$

$$x_t^s - x_t^{s^*} = 0, \quad \forall s, s^* \in \Omega : \left(\xi_t^s,\ldots,\xi_t^s\right) = \left(\xi_t^{s^*},\ldots,\xi_t^{s^*}\right), \ (t = 2,\ldots,T),$$

where the latter constraint denotes the non-anticipativity condition, mentioned above, implying that the decisions made at $t > 1$ are equal for the whole set of scenarios that have the same history until stage $t > 1$. Accordingly to above, we replaced x_t by x_t^s, such that $x^s = \left\{x_{1,\ldots,}^s x_T^s\right\}$ for $s = 1,\ldots,n$. This model set up guarantees that all elements of the stochastic decision vector x^s may depend on all elements of the stochastic data vector ξ^s. Consequently, each element of x_t may only depend on the stochastic data known ξ_t until stage *t*.

5.2 Constructing a Scenario-Tree

Let us denote from now on the first stage as $t = 0$, where ξ_0 is deterministic, i.e., the initial state is given, and the decision x_0 is known. The random vector including scenarios $s = 1,\ldots,T$ can then be defined as $\xi_t^s = \left\{ \xi_0, \xi_t^s,\ldots,\xi_T^s \right\}$, and stochastic decision vector as $x_t^s = \left\{ x_0, x_t^s,\ldots,x_T^s \right\}$. Thus, all realizations of ξ_t^s in $t > 0$ incorporate recursive decisions x_t in the subsequent stages. Possible realizations of ξ_t^s for $t > 0$ can be represented by a scenario tree, which consists of nodes constructed in levels referring to decision stages $t = 1,\ldots,T$. Thus, a scenario can be defined as a generated (i.e., random) path from the root node at stage $t=0$ to a node at the last stage *T*, incorporating a history of the stochastic data process ξ_t^s. At level $t=0$ the value of ξ_0 is known, such that in the next level $t=1$ the root node is then connected with θ possible realizations of ξ_1^s, called θ nodes, with $\theta = 1,\ldots,n$ and $\theta \in O_t$, where O_t is the set of all nodes at level $t=1,...,T$. This procedure is then repeated until the association of the generated nodes in the level $T-1$ with the ones in level *T*. The connection from one node to the next node is called arc, where the stochastic decisions x^s are made. A conditional probability π_θ can then be related with each node θ at the *t*th level such that

$$\pi_\theta = p\left\{\xi_t \left|\xi_{t-1}\right|\ldots\left|\xi_2\right.\right\},\ \ \pi_\theta > 0,\ \sum\nolimits_{\theta\in O_t} \pi_\theta = 1 \quad (t = 1,\ldots,T),$$

Therefore, the arcs in the scenario tree illustrate the finite probability distribution of ξ_t^s. As *t* and *s* increases, the number of arcs and consequently, the scenario-tree increases.

For simplified illustration issues, let us only concentrate on the objective function of general linear multi-stage stochastic program

$$\min_{x\in X} f\left(x^s;\xi^s\right),$$

where $x^s = \left\{x_1^s,\ldots,x_T^s\right\}$ and $\xi^s = \left\{\xi_1^s,\ldots,\xi_T^s\right\}$ for $t = 1,\ldots,T$ and $s = 1,\ldots,n$. That is, in order to generate all the decision possibilities x_t^s with regard to scenarios *s*, all the possible *s* realizations of the random variables ξ_t^s must be modeled.

Therefore, to construct a scenario-tree we set an initial value *c* for ξ_0, i.e., $\xi_0 = c$ and then, for all scenarios $s=1,...,n,$ generate paths with ξ_0, taken as a root.

Thereby, for our stochastic optimization model, we use the Monte Carlo simulation technique, which requires historical data, number of stages $t = 1,\ldots,T.$ and scenarios $s=1,...,n.$ Concretely, we apply the GBM method of correlated futures prices of EUA and CER (i.e., random variables), explained in detail in Sections 4.2 and 4.3.

For more theoretical background and discussion with regard to the construction and handling of scenario trees, we refer to [28, 34, 51].

CHAPTER 6

APPLICATION TO AN AIRLINE COMPANY IN THE EU ETS

6.1 Optimized Decision-Making: Option Pricing

Classic option pricing refers to the valuation of financial contracts with option rights. Due to their abstract nature, this can be of any forms, whereas a market for standardized (plain-vanilla) and exotic products has been established.

We have seen in Section 6.2, that commodities incorporate implicit flexibilities (i.e., embedded option(s)). That is, through the proper disposition of assets, added value can be generated. Difficulty in valuation arises from the consideration of dependencies (contingencies).

Options, as derivatives (derivative transactions), require a special valuation methodology. This is based on the principle of arbitrage - the so-called risk-neutral valuation method (Black-Scholes-Merton approach). Traditional approaches are based on a stochastic influencing variable (state variable), the price. By the conditional payoff function of a contingent, the typical non-linear payout structure results. Hence, the option theoretical difficulties of multiple exercise before maturity (American style) and path dependence (Asian style) arise.

The resulting analytical problems (non-time additivity) can be solved, by adding a further dimension in the state space [26, 47]. This eliminates the dependence of previous decisions.

Option pricing methods imply the solution to a stochastic dynamic optimization problem. This means that, in the option pricing model, the value-maximizing decision is made for each state. Through the expansion of the action possibilities by a new action, the number of action possibilities per state increases significantly. Along the time axis, the correct action must be determined for each state. The introduction of restrictions (constraints) of various kinds, results in a reduction of the state space, i.e., certain states may (must) not be reached. This is a central assumption of option pricing theory.

6.2 Modeling of the State Space

As explained in Chapter 2, the main risk factors (i.e., random parameters) for an airline company within the EU ETS are the uncertain EUA futures and CER prices as well as its yearly CO_2 emissions from flights from and to EU countries.

Hence, according to our notation in Chapter 5, our stochastic data process can be represented as of $\xi_t^s = \left\{ p_{1,t}^s, p_{2,t}^s, c_t^s \right\}$, for each stage (i.e., trading time horizon) $t = 1,...,T$ and scenario $s = 1,...,n$, where $p_{1,t}^s$ denote the stochastic futures prices for EUA, $p_{2,t}^s$ denote the stochastic futures prices for CER, and c_t^s stands for the periodic stochastic CO_2 emissions. We note that in our model c_t^s will be represented as stochastic constraints parameter, explained later.

Thus, our modeling procedure consists of the set up of the optimization model including the constraints, the scenario generation of ξ_t^s, setting of the state space Ω and solving of the model through the CPLEX, which is available in MATLAB.

The scenarios for ξ_t^s and constraints are needed to specify all the possible and feasible stochastic decisions

$$x_t^s = \left[\left(x_{1,t}^s, x_{2,t}^s \right), ..., \left(x_{1,T}^s, x_{2,T}^s \right) \right] \quad (t = 1,...,T;\, s = 1,...,n),$$

to be taken within a two-dimensional state space, where $x_{1,t}^s$ denote the stochastic purchase, holding or selling decision of EUA futures contract and $x_{2,t}^s$ denote the stochastic purchase, holding or selling decision of a CER futures contract, respectively. That is, along the time axis t for each state ($p_{1,t}^s, p_{2,t}^s$ combinations) and s, the value-maximizing decision must be made out of the decision matrix D_t^s which consists of nine possible combinations of trading decisions, $x_{i,t}^s < 0$ denoting selling decision, $x_{i,t}^s > 0$ purchasing decision and $x_{i,t}^s = 0$ holding decision of the portfolio manager such hat

$$D_t^s = \begin{Bmatrix} x_{1,t}^s > 0, x_{2,t}^s > 0 & x_{1,t}^s > 0, x_{2,t}^s = 0 & x_{1,t}^s > 0, x_{2,t}^s < 0 \\ x_{1,t}^s = 0, x_{2,t}^s > 0 & x_{1,t}^s = 0, x_{2,t}^s = 0 & x_{1,t}^s = 0, x_{2,t}^s < 0 \\ x_{1,t}^s < 0, x_{2,t}^s > 0 & x_{1,t}^s < 0, x_{2,t}^s = 0 & x_{1,t}^s < 0, x_{2,t}^s < 0 \end{Bmatrix}.$$

For simplifying notation, for further purpose, let us denote, for $i = 1,2,$ $x_{i,t}^s < 0$ (i.e., selling decision) and $x_{i,t}^s > 0$ (i.e., purchase decision) as $x_{-,i,t}^s$ and $x_{+,i,t}^s$, respectively, such that

$$0 \le x_{+,i,t}^s \le x_{+_{\max},i,t}^s \qquad \forall t = 1,\ldots,T,\ \forall s = 1,\ldots,n,$$

$$0 \le x_{-,i,t}^s \le x_{-_{\max},i,t}^s \qquad \forall t = 1,\ldots,T,\ \forall s = 1,\ldots,n,$$

where $x_{+_{\max},i,t}^s$ denotes the maximum selling and $x_{-_{\max},i,t}^s$ the maximum purchasing amount of $i = 1,2,$ respectively.

Actually, the decision matrix D_t^s contains a strip of call and put options, with EUA and CER futures as underlying, giving the portfolio manager the right of purchasing and selling EUA and CER futures at each point in time *t*, based on his market expectations and the modeled state space Ω. The concrete decision algorithm is described in Section 6.3.

In fact, the state space Ω consists of a *product* and a *market* state. The product state refers to internal factors of the product that influence the income, e.g., regulatory, managerial and trading constraints including the periodic stochastic emissions c_t^s, whereas the market state refers to external factors, the product's underlying, that influence the income. In our case, these are the correlated EUA and CER futures prices for different delivery periods. The regulatory, managerial and trading constraints, which will be explained in more details in the Section 6.4, results in a diminishment of the state space, such that certain states may (must) not be attained. In general, the evolution of the product state is affected by the evolution of the market state, but vice-versa is not true. Therefore, the market state's evolution can be described independently of any product state. For our purpose, we assume the future realizations of different product states as independent, whereas future realizations of the market variables EUA and CER futures prices are considered correlated (see Chapter 4).

Figure 6.1 illustratively shows the two-dimensional state space consisting of the stochastic random variables EUA futures and CER futures prices and constraints including stochastic c_t^s, whereas on the horizontal axis time horizon (i.e., stage) t is shown. The two-dimensional state space has the form of a plan or grid, respectively.

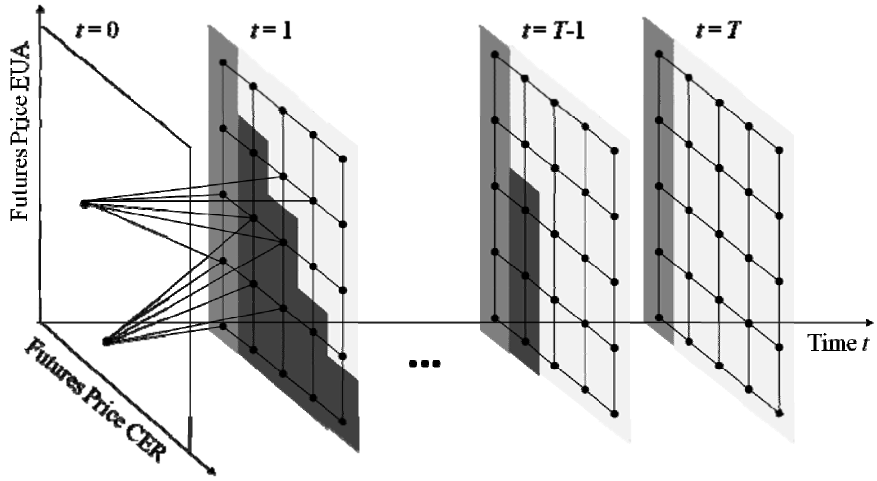

Figure 6.1. Illustrative example of the two-dimensional state space.

All the stages $t = 1,...,T$ contain θ nodes, $\theta = 1,...,n,\ \theta \in O_t$, representing the possible states within the space, where at each stage t the stochastic decision $\left(x_{1,t}^s, x_{2,t}^s\right)$ can be made. At stage $t = 0$, based on the initial prices for EUA and CER futures, s scenarios are created through MC simulation for $t = 1$ to $t = T$. In Figure 8, some illustrative simulation paths are depicted for EUA and CER prices between the stages $t = 0$ and $t = 1$. Illustratively, the states in gray are associated with non-feasible decisions and the states in dark gray represent feasible, but not valuable (i.e., loss) decisions. Therefore, only states in light gray are feasible and valuable decisions for our model.

The general methodology for the multi-stage stochastic optimization model is as follows:

Step 1: *Modeling the state space (see part above).*

Step 2: *Determination of the optimal decision (i.e., trading strategy) and earnings on the basis of the uncertain EUA and CER futures prices and in compliance with the constraints. This is done by backward induction such that starting from the last stage $t = T$ moving backward to stage $t = T - 1$ until $t = 0$, likewise the option pricing theory, valuation is done for each state. This procedure can be called as the* ***value perspective****.*

Step 3: *Given the valuation for each state, the uncertainty of the earnings is determined by the MC simulation. This is conducted by forward induction and be called as* ***the risk perspective****.*

Based on Step 3, the well-known risk measure V*alue-at-Risk* (VaR), which is commonly implemented in academic finance research as well as in practice, can be determined. VaR tells a portfolio manager how much money he is likely to loose over a specific holding period at a given confidence interval α. We can mathematically define VaR of a portfolio X as follows:

Definition 6.2.1. ***Value-at-Risk (VaR):*** *Given a confidence level* $\alpha \in (0,1)$, *the VaR at level* α *of a portfolio X with distribution* P, *is defined as the specified negative deviation (i.e., loss) j,* $j \in \mathbb{R}$, *from the expected value or return of X, such that the probability that a given loss J is greater than the critical loss value or return j is at least* α, *i.e.,*

$$VaR_{\alpha}(J) := \inf\left\{ j \in \mathbb{R} \middle| P\left[J > j\right] \geq \alpha \right\}.$$

Therefore, for our purpose, the $VaR_{X,\alpha}$ of a portfolio X at a specified confidence level α can be calculated as the expected \$-value of the portfolio X minus the product of \$-standard deviation σ_X of portfolio X and the given confidence level α and, i.e.,

$$VaR_{X,\alpha} = \$\mu_X - \left(\$\sigma_X \alpha\right),$$

where the term in brackets denotes the maximal at loss *j* at α confidence level according to Definition of 6.2.1.

We will use the widely used 95% and 99% *α*-confidence interval, which have *z*-values of 1.645 and 2.33, respectively. The \$-standard deviation σ_X of portfolio X can directly be derived after determination of the distribution of revenues for each single trading strategy and their corresponding final expected value through MC simulation. Hence, in addition to the optimal trading strategies and expected values, we will determine how much the portfolio manager at least will gain with 95% and 99% probability, respectively. VaR of the various trading strategies will be calculated in Chapter 8.

6.3 Decision Algorithm for a Portfolio Manager

The value of the CO_2 trading strategy V_t^s at $t = 1,...,T$ and for $s = 1,...,n$ is a function of the state variables $p_{1,t}^s$, $p_{2,t}^s$ and c_t^s, where $p_{1,t}^s$ and $p_{2,t}^s$ denote stochastic futures prices for EUA and CER, respectively, and c_t^s are the stochastic periodic CO_2 emissions, i.e.,

$$V_t^s = f\left(t, p_{1,t}^s, p_{2,t}^s, c_t^s\right)$$

$$(t = 1,...,T;\ s = 1,...,n),$$

The decision mechanism is based on the valuation methodology of American options. We remark that the Asian property "path dependence" is already incorporated by the extension of the state space. The concrete decision algorithm is as follows:

Step 1: *At a given CO_2 emission level c_t^s for scenario $s = 1,...,n$ the option value is calculated by numeric integration.*

Step 2: *The C_t^s stochastically change the constraints when switching to the next stage $t = 1,...,T$. Based on that, the transition probabilities between the states are derived. The $(n+1)\times(n+1)$ transition matrix M_t^s for each $t = 1,...,T$ and $s = 1,...,n$ can be represented as*

$$M_t^s = \begin{pmatrix} p_{\le 0} & p_1 & p_2 & \cdots & p_{n-1} & p_{\ge n} \\ p_{\le -1} & p_0 & p_1 & \cdots & p_{n-2} & p_{\ge n-1} \\ p_{\le -2} & p_{-1} & p_0 & & p_{n-3} & p_{\ge n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{\le -n+1} & p_{\le -n+2} & p_{\le -n+3} & \cdots & p_0 & p_{\ge 1} \\ p_{\le -n} & p_{\le -n+1} & p_{\le -n+2} & \cdots & p_{-1} & p_{\ge 0} \end{pmatrix},$$

where, for each scenario $s = 1,...,n$, the entry m_{ij}^s stands for the probability to migrate to state j given the state is equal to k, with $j,k = 1,...,n$, within the period from t from $t+1$, as a result of changing stochastic periodic CO_2 emissions c_t^s.

Step 3: *Let us denote $\overline{F}_{+,i,t}$ and $\overline{F}_{-,i,t}$ as price thresholds for sales and purchases of EUA and CERs futures for each t, respectively, with $\overline{F}_{+,i,t}, \overline{F}_{+,i,t} \in \mathbb{R}$. Hence, the decision* *algorithm for a portfolio manager in order to maximize net income can be described by*

$$\max\left\{p^s_{i,t} - \overline{F}_{+,i,t};0\right\}x_{i,t}, \qquad \text{in the case of a } \textit{Call option,}$$
$$\max\left\{\overline{F}_{+,i,t} - p^s_{i,t};0\right\}x_{i,t}, \qquad \text{in the case of a } \textit{Put option,} \tag{6.1}$$
$$\forall i = 1,2,\ \forall t = 1,\ldots,T,\ \forall s = 1,\ldots,n.$$

In expression (6.1), the first expression refers to a Call option and the second expression to a Put option, which means that at each point in time t, the portfolio manager has the right of purchasing or selling EUA and CER futures. Thus, summarized, the illustrated trading strategy allows both selling and purchasing of EUA and CER futures against a (board)-defined price thresholds $\overline{F}_{+,i,t}$ *and* $\overline{F}_{-,i,t}$.

For our further purpose, we assume that the (board)-defined price thresholds are equal for the call and put option, i.e.,

$$\overline{F}_{+,i,t} = \overline{F}_{-,i,t} = F_{i,t},$$
$$(i = 1,2;\ t = 1,\ldots,T),$$

Thus, according to expression (6.1), we can formulate for each time *t*,

$$Call_{i,t} - Put_{i,t} = e^{-r(T-t)}\left[\left(\left(p^s_{i,t} - \overline{F}_{i,t}\right)_+ x_{i,t}\right) - \left(\left(\overline{F}_{i,t} - p^s_{i,t}\right)_+ x_{i,t}\right)\right]$$
$$(i = 1,2;\ t = 1,\ldots,T;\ s = 1,\ldots,n),$$

where *r* is the constant risk-free rate. If at time *t* for each scenario *s*, $p^s_{i,t} > \overline{F}_{+,i,t}$, then the call option is exercised, and the put option is not exercised, such that we result in

$$Call_{i,t} - Put_{i,t} = e^{-r(T-t)}\left[\left(p^s_{i,t} - \overline{F}_{i,t}\right)x_{i,t}\right] \tag{6.2}$$
$$(i = 1,2;\ t = 1,\ldots,T;\ s = 1,\ldots,n),$$

If at time t for each scenario s, $p_{i,t}^{s} < \bar{F}_{+,i,t}$, then the put option is exercised, and the call option is not exercised, which implies

$$Call_{i,t} - Put_{i,t} = -e^{-r(T-t)}\left[\left(\bar{F}_{i,t} - p_{i,t}^{s}\right)x_{i,t}\right] = e^{-r(T-t)}\left[\left(p_{i,t}^{s} - \bar{F}_{i,t}\right)x_{i,t}\right] \tag{6.3}$$

$$(i = 1, 2;\ t = 1, \ldots, T;\ s = 1, \ldots, n),$$

which is equal to expression (6.2). Since we consider a portfolio view of EUAs and CERs over a time horizon $t = 1, \ldots, n,$, we can just build the sum of these, resulting in

$$\sum_{i=1}^{2}\sum_{t=1}^{T}\left(Call_{i,t} - Put_{i,t}\right) = e^{-r(T-t)}\left[\sum_{i=1}^{2}\sum_{t=1}^{T}\left(p_{i,t}^{s} - \bar{F}_{i,t}\right)x_{i,t}\right]. \tag{6.4}$$

We will make use the right hand side of expression (6.4) in Subsection 6.4.4, when deriving the portfolio manager's optimal trading budget for the for each CO_2 compliance period and the profit function in our optimization model.

6.4 Formulation of the Optimization Model

In the following, we will successively develop and set up our linear multi-stage stochastic portfolio optimization model for the closing of the natural short position in CO_2 emission allowances with EUA and CER futures.

6.4.1 The CO_2 Trading Period

The total trading period in our model consists of $n-1$ CO_2 compliance periods, where $k = 0, 1, \ldots, n-1$ stands for the (k+1)th CO_2 compliance period. The last discrete point in time of the corresponding terminating (k+1)th CO_2 compliance period is denoted by T_{k+1}, such that $T_{k+1} = T_1, \ldots, T_n$. Additionally, the board defines a percentage amount of the short position in CO_2 certificates, which should be mandatorily closed by the airline's portfolio manager until a defined discrete point in time τ_{k+1} within the CO_2 compliance period $k+1$, where $\tau_{k+1} = \tau_1, \ldots, \tau_n$. Therefore, the total trading time horizon (i.e., stages) is t, that can be represented as $t = T_0 (= 0), 1, 2 \ldots, \tau_1, \ldots, T_1, \ldots, \tau_{k+1}, \ldots, T_{k+1}, \ldots, \tau_n, \ldots, T_n$. We note that for $k = 0$, $T_k = T_0 = 0$, and τ_0 does not exist. For all $k > 0$, $\tau_{k+1}, T_{k+1} \in \mathbb{N}$. Hence, for $T_k < \tau_{k+1} < T_{k+1}$ for each $k = 0,1,...,n-1$. Figure 6.2 illustrates the systematics of the trading time horizon *t* which the portfolio manager faces.

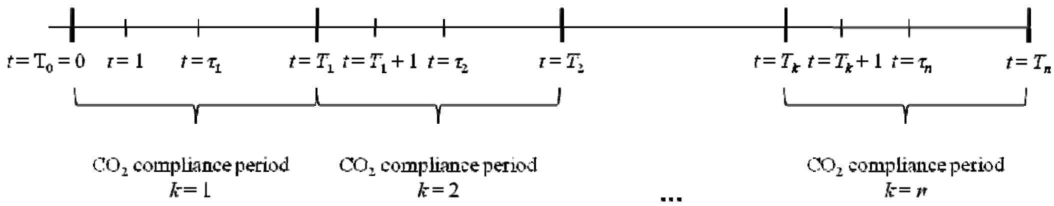

Figure 6.2. Systematics of the CO_2 compliance and trading periods.

6.4.2 CO_2 Trading Strategy

Now, let us consider a portfolio manager of an airline company who has to determine over a given trading time horizon *t*, his optimal total hedge portfolio

$$\sum_{k=0}^{n-1} \sum_{i=1}^{2} \sum_{t=T_k+1}^{T_{k+1}} x_{i,t},$$

where $x_{i,t}$ for $i = 1,2$ is the amount of EUAs and CERs, respectively, traded in each *t*.

Let $\bar{x}_{1,k+1}$ be the yearly fix amount of EUAs distributed for free to the airline company by the regulatory authority, valid for the whole CO_2 compliance period *n*. Let also denote c_t^d the total amount of stochastic CO_2 emissions in *t* for the emissions scenarios $d = 1,...,n$, which are set as deterministic (estimated) scenarios by the airline company for each *t*. Then we have

$$\begin{aligned} C_1^d &:= \sum_{t=1}^{T_1} c_1^d, \\ C_2^d &:= \sum_{t=T_1+1}^{T_2} c_2^d, \\ &\vdots \\ C_{k+1}^d &:= \sum_{t=T_k+1}^{T_{k+1}} c_t^d, \end{aligned} \tag{6.5}$$

$$\forall k = 0,1,...,n-1, \ \forall d = 1,...,n,$$

where C_{k+1}^{d} is the total amount of stochastic CO_2 emissions for each emissions scenario $d = 1,...,n,$ in the corresponding terminating CO_2 compliance period $k+1.$ We remark that, unlike Section 6.3, from now on, we use index d for the denotation of the CO_2 emissions scenarios in order to separate their notation and number from those of EUA and CER futures prices scenarios (see Section 6.3).

Thus, according to definition (6.5), the optimal CO_2 trading strategy of the portfolio manager for the (*k*+1)th CO_2 compliance period can be defined as

$$\begin{aligned} &\sum_{i=1}^{2}\sum_{t=1}^{T_1} x_{i,t} = C_1^d - \overline{x}_{1,1}, \\ &\sum_{i=1}^{2}\sum_{t=T_1+1}^{T_2} x_{i,t} = C_2^d - \overline{x}_{1,2}, \\ &\vdots \\ &\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} = C_{k+1}^d - \overline{x}_{1,k+1}, \end{aligned} \tag{6.6}$$

$$\forall k = 0,1,...,n-1,\ \forall d = 1,...,n,$$

where the right hand side of equation (6.6) stands for the natural short position in CO_2 emission allowances for each emissions scenario *d* to be closed by the portfolio manager in each $k+1.$ Let us define the natural short position in CO_2 emission allowances for each emissions scenario *d* and for each $k+1$ as Δ_{k+1}^{d} such that

$$\begin{aligned} &\Delta_1^d := C_1^d - \overline{x}_{1,1}, \\ &\Delta_2^d := C_2^d - \overline{x}_{1,2}, \\ &\vdots \\ &\Delta_{k+1}^d := C_{k+1}^d - \overline{x}_{1,k+1}, \end{aligned} \tag{6.7}$$

$$\forall k = 0,1,...,n-1,\ \forall d = 1,...,n.$$

Then, according to equation (6.6), the sum of all CO_2 compliance periods $k=0,1,...,n-1$ the total CO_2 trading strategy for each emissions scenario *d* be stated as

$$\sum_{k=0}^{n-1}\left[\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + (1+b)\overline{x}_{1,k+1}\right] = \sum_{k=0}^{n-1} C_{k+1}^{d} \tag{6.8}$$

$$(d = 1,...,n).$$

6.4.3 Regulatory, Managerial and Trading Constraints

Banking and borrowing possibility of CO_2 allowances

According to EU ETS regulation, an airline company is allowed to bank or borrow a percentage amount of b of the yearly fix amount of EUAs distributed for free $\bar{x}_{1,k+1}$ between CO_2 compliance periods. Whereas *banking* is the possibility of transferring of $\bar{x}_{1,k+1}$ from one year to the following year, i.e., from $k+1$ to $k+2$, *borrowing* is the possibility of using EUAs from the following year in the current year, i.e., from $k+2$ to $k+1$. Thus, let us denote the portfolio manager's possibility of borrowing with and banking of $\bar{x}_{1,k+1}$ between CO_2 compliance periods, with a regulatory-defined constant rate of b, $b \in [-1,1]$, whereas $b > 0$ represents borrowing and $b < 0$ banking, respectively.

Figure 6.3 shows the idea of closing of the natural short position in CO_2 emission allowances including the banking (going long) and borrowing (going short) possibilities between CO_2 compliance periods.

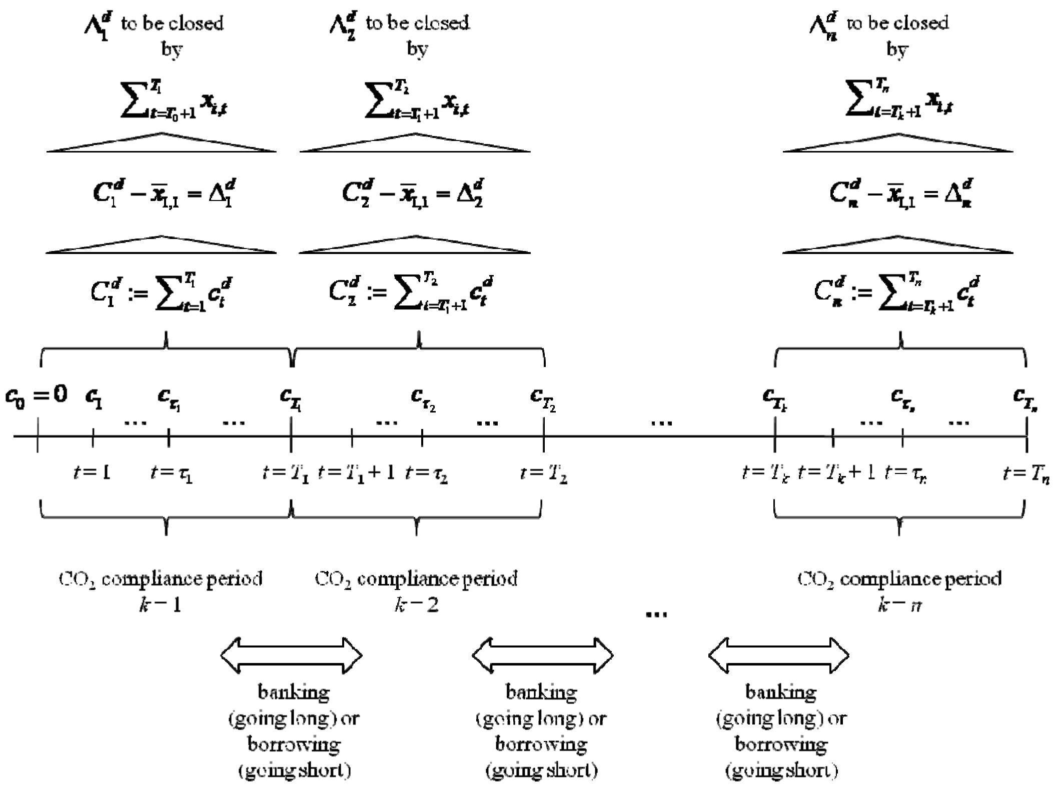

Figure 6.3. Idea of closing of the natural short position in CO_2 emission allowances.

For each CO_2 compliance period $k+1$, the regulatory CO_2 emission cap RC_{k+1} can be represented as

$$
\begin{aligned}
&RC_1 = \sum_{i=1}^{2}\sum_{t=1}^{T_1} x_{i,t} + \overline{x}_{1,1}, \\
&RC_2 = \sum_{i=1}^{2}\sum_{t=T_1+1}^{T_2} x_{i,t} + \overline{x}_{1,2}, \\
&\vdots \\
&RC_{k+1} = \sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + \overline{x}_{1,k+1},
\end{aligned}
\qquad (6.9)
$$

$$\forall k = 0,1,\ldots,n-1,\ \forall d = 1,\ldots,n,$$

due to the fact that the cap has been determined by the regulatory authority in such a way that the airline companies should face a yearly physical short position in CO_2 emission allowances, disciplining them to control their CO_2 emissions by compensating the missing amount of CO_2 emissions by

$$\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} \qquad (k = 0,1,\ldots,n-1).$$

By rearranging the system in (6.6), we get

$$
\begin{aligned}
&\sum_{i=1}^{2}\sum_{t=1}^{T_1} x_{i,t} + \overline{x}_{1,1} = C_1^d, \\
&\sum_{i=1}^{2}\sum_{t=T_1+1}^{T_2} x_{i,t} + \overline{x}_{1,2} = C_2^d, \\
&\vdots \\
&\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + \overline{x}_{1,k+1} = C_{k+1}^d,
\end{aligned}
\qquad (6.10)
$$

$$\forall k = 0,1,\ldots,n-1,\ \forall d = 1,\ldots,n.$$

Therefore, by including the possibility of banking and borrowing for each $k+1$, our equilibrium amount of CO_2 emissions for each emissions scenario d becomes

$$\sum_{i=1}^{2}\sum_{t=1}^{T_1} x_{i,t} + \overline{x}_{1,1} + b\overline{x}_{1,1} = C_1^d,$$
$$\sum_{i=1}^{2}\sum_{t=T_1+1}^{T_2} x_{i,t} + \overline{x}_{1,2} + b\overline{x}_{1,2} = C_2^d,$$
$$\vdots$$
$$\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + \overline{x}_{1,k+1} + b\overline{x}_{1,k+1} = C_{k+1}^d,$$

$$\forall k = 0,1,\ldots,n-1,\ \forall d = 1,\ldots,n,$$

and subsumed

$$\begin{aligned}
&\sum_{i=1}^{2}\sum_{t=1}^{T_1} x_{i,t} + (1+b)\overline{x}_{1,k+1} = C_1^d,\\
&\sum_{i=1}^{2}\sum_{t=T_1+1}^{T_2} x_{i,t} + (1+b)\overline{x}_{1,k+1} = C_2^d,\\
&\vdots\\
&\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + (1+b)\overline{x}_{1,k+1} = C_{k+1}^d,
\end{aligned} \tag{6.11}$$

$$\forall k = 0,1,\ldots,n-1,\ \forall d = 1,\ldots,n.$$

Depending on the portfolio manager's strategy, for each emissions scenario *d* the *banking* decision of the portfolio manager with $b < 0$ can be described as

$$\begin{cases}
\sum_{i=1}^{2}\sum_{t=1}^{T_1} x_{i,t} + \overline{x}_{1,1} > C_1^d, & b\overline{x}_{1,1} < 0,\\
\sum_{i=1}^{2}\sum_{t=T_1+1}^{T_2} x_{i,t} + \overline{x}_{1,2} > C_2^d, & b\overline{x}_{1,2} < 0,\\
\vdots\\
\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + \overline{x}_{1,k+1} > C_{k+1}^d, & b\overline{x}_{1,k+1} < 0,
\end{cases} \tag{6.12}$$

$$\forall k = 0,1,\ldots,n-1,\ \forall d = 1,\ldots,n,$$

and the *borrowing* decision of the portfolio manager with $b > 0$ can be stated as

$$\begin{cases}
\sum_{i=1}^{2}\sum_{t=1}^{T_1} x_{i,t} + \overline{x}_{1,1} < C_1^d, & b\overline{x}_{1,1} > 0,\\
\sum_{i=1}^{2}\sum_{t=T_1+1}^{T_2} x_{i,t} + \overline{x}_{1,2} < C_2^d, & b\overline{x}_{1,2} > 0,\\
\vdots\\
\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + \overline{x}_{1,k+1} < C_{k+1}^d, & b\overline{x}_{1,k+1} > 0,
\end{cases} \tag{6.13}$$

$$\forall k = 0,1,\ldots,n-1,\ \forall d = 1,\ldots,n,$$

We remark that for $b = 0$ the systems of equations in (6.11) just equal the systems of equations in (6.10).

In the special case, where

$$\bar{x}_{1,k+1} > C_{k+1}^{d} \qquad (k = 0,1,\dots,n-1;\ d = 1,\dots,n).$$

Then, we have

$$\begin{aligned}
&\sum_{i=1}^{2}\sum_{t=1}^{T_1} x_{i,t} + b\bar{x}_{1,1} < 0,\\
&\sum_{i=1}^{2}\sum_{t=T_1+1}^{T_2} x_{i,t} + b\bar{x}_{1,2} < 0,\\
&\vdots\\
&\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + b\bar{x}_{1,k+1} < 0,
\end{aligned} \tag{6.14}$$

$$\forall k = 0,1,\dots,n-1,\ \forall d = 1,\dots,n,$$

which illustrates the situation that in the CO_2 compliance period $k+1$ the amount of CO_2 emissions is less than the CO_2 emission allowances distributed for free, which implies a long position in CO_2 emission allowances and both sales of EUAs and CERs in $k+1$ or banking of b-amount of free distributed EUAs to $k+2$..

There is an important regulatory requirement that over the sum of all CO_2 compliance periods, the total amount of banked and borrowed free distributed EUAs $\bar{x}_{1,k+1}$ should be equal to zero, such that

$$\sum_{k=0}^{n-1} b\bar{x}_{1,k+1} = 0, \tag{6.15}$$

and therefore, implying that whole system should be in equilibrium over the sum of all trading and CO_2 compliance periods for each emissions scenario d,

$$\sum_{k=0}^{n-1}\left[\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + (1+b)\bar{x}_{1,k+1}\right] = \sum_{k=0}^{n-1} C_{k+1}^{d} \tag{6.16}$$

$$(d = 1,\dots,n).$$

EU ETS regulatory limit for CERs

The EU ETS imposes a regulatory trading (i.e., CO_2 compliance) limit for CERs. Let the regulatory CER limit of total amount the short position Δ_{k+1}^{d} be m, $m \in [0,1]$, which can be used for compliance by the airline company for each $k+1$. Hence, the regulatory CER limit constraint can be represented as

$$\begin{aligned}
&\sum\nolimits_{t=1}^{T_1} x_{2,t} \le m\left(C_1^d - \overline{x}_{1,1}\right),\\
&\sum\nolimits_{t=T_1+1}^{T_2} x_{2,t} \le m\left(C_2^d - \overline{x}_{1,2}\right),\\
&\vdots\\
&\sum\nolimits_{t=T_k+1}^{T_{k+1}} x_{2,t} \le m\left(C_{k+1}^d - \overline{x}_{1,k+1}\right),
\end{aligned} \qquad (6.17)$$

$$\forall k = 0,1,\ldots,n-1,\ \forall d = 1,\ldots,n.$$

Accordingly, the remaining amount of Δ_{k+1}^{d} has to be closed by EUAs such that

$$\begin{aligned}
&\sum\nolimits_{t=1}^{T_1} x_{1,t} \le (1-m)\left(C_1^d - \overline{x}_{1,1}\right),\\
&\sum\nolimits_{t=T_1+1}^{T_2} x_{1,t} \le (1-m)\left(C_2^d - \overline{x}_{1,2}\right),\\
&\vdots\\
&\sum\nolimits_{t=T_k+1}^{T_{k+1}} x_{1,t} \le (1-m)\left(C_{k+1}^d - \overline{x}_{1,k+1}\right),
\end{aligned} \qquad (6.18)$$

$$\forall k = 0,1,\ldots,n-1,\ \forall d = 1,\ldots,n.$$

Upper and lower trading limits

Although, the portfolio manager has to implement purchasing strategies to close the natural short position in CO_2 emission allowances Δ_{k+1}^{d}, to ensure him a certain trading flexibility, we assume that short selling of emission allowances is allowed. Therefore, the board of the airline company defines for each $t = 1,\ldots T_n$, $i = 1,2$ and $k = 0,1,\ldots,n-1$ both the binding upper purchasing limits

$$u_{i,t} \in [0,1],\ \sum\nolimits_{t=T_k+1}^{T_{k+1}} u_{i,t} = U,$$

and the binding lower selling limits

$$v_{i,t} \in [0,-1], \ \sum_{t=T_k+1}^{T_{k+1}} v_{i,t} = L,$$

respectively, where U and L are defined scalars, with $U, L \in \mathbb{R}$. That is, the portfolio manager is allowed to increase his long (short) position in CO_2 emission allowances to a factor of U (L) for each CO_2 compliance period $k+1$.

Therefore, the upper trading limits for EUAs and CERs can be represented as

$$\begin{aligned} x_{1,t} &\le u_{1,t}\left[m\left(C_{k+1}^{d} - \overline{x}_{1,k+1}\right)\right], \\ x_{2,t} &\le u_{2,t}\left[(1-m)\left(C_{k+1}^{d} - \overline{x}_{1,k+1}\right)\right], \end{aligned} \tag{6.19}$$

$$\forall t = 1,\ldots,T_n, \ \forall k = 0,1,\ldots,n-1, \ \forall d = 1,\ldots,n.$$

and their lower trading limits as

$$\begin{aligned} -x_{1,t} &\ge v_{1,t}\left[m\left(C_{k+1}^{d} - \overline{x}_{1,k+1}\right)\right], \\ -x_{2,t} &\ge v_{2,t}\left[(1-m)\left(C_{k+1}^{d} - \overline{x}_{1,k+1}\right)\right], \end{aligned} \tag{6.20}$$

$$\forall t = 1,\ldots,T_n, \ \forall k = 0,1,\ldots,n-1, \ \forall d = 1,\ldots,n.$$

Risk constraint

The board also predetermines a percentage amount of the natural open position in CO_2 emissions, and therefore in CO_2 allowances, that has to be closed until a specific point in time. In this way, the volume and liquidity risk, and therefore the exposure, can be reduced and the portfolio manager disciplined (i.e., controlled). Let us denote, for each $i = 1,2$, a percentage $q_{i,\tau_{k+1}}$, $q_{i,\tau_{k+1}} \in [0,1]$, of the total amount of the short position Δ_{k+1}, that has to be closed until a board-defined point in time τ_{k+1}, where $T_k < \tau_{k+1} < T_{k+1}$, for each $k = 0,1,\ldots,n-1$.

This implies for CERs that

$$\begin{aligned}
&\sum\nolimits_{t=1}^{\tau_1} x_{2,t} \le q_{2,\tau_1}\left[m\left(C_1^d - \overline{x}_{1,1}\right)\right] \\
&\sum\nolimits_{t=T_1+1}^{\tau_2} x_{2,t} \le q_{2,\tau_2}\left[m\left(C_2^d - \overline{x}_{1,2}\right)\right] \\
&\vdots \\
&\sum\nolimits_{t=T_k+1}^{\tau_{k+1}} x_{2,t} \le q_{2,\tau_{k+1}}\left[m\left(C_{k+1}^d - \overline{x}_{1,k+1}\right)\right]
\end{aligned} \tag{6.21}$$

$$\forall t = 1,\ldots,T_n,\ \forall k = 0,1,\ldots,n-1,\ \forall d = 1,\ldots,n.$$

and accordingly for EUAs

$$\begin{aligned}
&\sum\nolimits_{t=1}^{\tau_1} x_{1,t} \le q_{1,\tau_1}\left[(1-m)\left(C_1^d - \overline{x}_{1,1}\right)\right], \\
&\sum\nolimits_{t=T_1+1}^{\tau_2} x_{1,t} \le q_{1,\tau_2}\left[(1-m)\left(C_2^d - \overline{x}_{1,2}\right)\right], \\
&\vdots \\
&\sum\nolimits_{t=T_k+1}^{\tau_{k+1}} x_{1,t} \le q_{1,\tau_{k+1}}\left[(1-m)\left(C_{k+1}^d - \overline{x}_{1,k+1}\right)\right],
\end{aligned} \tag{6.22}$$

$$\forall t = 1,\ldots,T_n,\ \forall k = 0,1,\ldots,n-1,\ \forall d = 1,\ldots,n.$$

6.4.4 Derivation of the Profit Function

Now, we have to derive the total profit function for the portfolio manager's CO_2 trading strategy. By rearranging the system of equations (6.10), we get

$$\begin{aligned}
&\sum\nolimits_{i=1}^{2}\sum\nolimits_{t=1}^{T_1} x_{i,t} + (1+b)\overline{x}_{1,1} - C_1^d = 0, \\
&\sum\nolimits_{i=1}^{2}\sum\nolimits_{t=T_1+1}^{T_2} x_{i,t} + (1+b)\overline{x}_{1,2} - C_2^d = 0, \\
&\vdots \\
&\sum\nolimits_{i=1}^{2}\sum\nolimits_{t=T_k+1}^{T_{k+1}} x_{i,t} + (1+b)\overline{x}_{1,k+1} - C_{k+1}^d = 0,
\end{aligned} \tag{6.23}$$

$$\forall t = 1,\ldots,T_n,\ \forall k = 0,1,\ldots,n-1,\ \forall d = 1,\ldots,n,$$

which states that for each $k = 0,1,...,n-1$ the difference between the amount of traded CO_2 emission allowances (including banking and borrowing possibility) and the airline's verified CO_2 emissions should equal to zero. However, if the at the end of the (k+1)th CO_2 compliance period, i.e., at the point in time T_{k+1}, the verified CO_2 emissions exceed the amount of existing CO_2 emission allowances to be delivered to the regulatory authority such that

$$\begin{aligned} &\sum_{i=1}^{2}\sum_{t=1}^{T_1} x_{i,t} + (1+b)\overline{x}_{1,1} - C_1^d < 0, \\ &\sum_{i=1}^{2}\sum_{t=T_1+1}^{T_2} x_{i,t} + (1+b)\overline{x}_{1,2} - C_2^d < 0, \\ &\vdots \\ &\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + (1+b)\overline{x}_{1,k+1} - C_{k+1}^d < 0, \end{aligned} \tag{6.24}$$

$$\forall t = 1,...,T_n,\ \forall k = 0,1,...,n-1,\ \forall d = 1,...,n,$$

i.e., still a short position in CO_2 emission allowances exists, then the airline has to pay a penalty fee g to the authority in the amount of the missing CO_2 emission allowances. The penalty paid is then deducted as an additionally occurred cost from the profit of the portfolio manager. Let us define $[a]^- := \max\{0,-a\}$, for each $a \in \mathbb{R}$, then we can formulate the penalty term as

$$-g\left[\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + (1+b)\overline{x}_{1,k+1} - C_{k+1}^d\right]^- \tag{6.25}$$

$$(k = 0,1,...,n-1;\ d = 1,...,n).$$

In equation (6.41), this term will be introduced as a penalty term in the profit function of the portfolio manager.

In order to span a two-dimensional state space of EUA and CER futures prices (i.e., forward scenario-tree), described in detail in Section 5.2, through the use of the MC simulation method, we will generate price s scenarios for EUA and CER futures prices for each stage $t = 1,...,T_n$, denoted by $p_{i,t}^s$, with $s = 1,...,n$. The price scenarios $p_{i,t}^s$ will be weighted by their occurrence probabilities π^s with $\pi^s \in [0,1]$ and $\sum_{s=1}^{n} \pi^s = 1$. Therefore, the revenues can be described as

$$\sum_{i=1}^{2} \sum_{t=1}^{T_1} \pi^s p_{i,t}^s x_{i,t} \tag{6.26}$$

$$(k = 0,1,\ldots,n-1;\ s = 1,\ldots,n).$$

The board also determines the maximum trading budget $B_{k+1} \in \mathbb{N}^+$ to be spent by the portfolio manager for the net purchase of EUAs and CERs in order to compensate the short position Δ_{k+1} for each CO_2 compliance period $k+1$. This trading budget B_{k+1} occurs as a cost parameter in the profit function. Or, in other words, the portfolio manager has to generate income that exceeds B_{k+1} in order to result in successful trading strategies such that for each $k+1$

$$\begin{aligned}
&\sum_{i=1}^{2} \sum_{t=1}^{T_1} \pi^s p_{i,t}^s x_{i,t} > B_1, \\
&\sum_{i=1}^{2} \sum_{t=T_1+1}^{T_2} \pi^s p_{i,t}^s x_{i,t} > B_2, \\
&\vdots \\
&\sum_{i=1}^{2} \sum_{t=T_k+1}^{T_{k+1}} \pi^s p_{i,t}^s x_{i,t} > B_{k+1},
\end{aligned} \tag{6.27}$$

$$\forall t = 1,\ldots,T_n,\ \forall k = 0,1,\ldots,n-1,\ \forall d = 1,\ldots,n.$$

Or, by rearranging

$$\begin{aligned}
&\sum_{i=1}^{2} \sum_{t=1}^{T_1} \pi^s p_{i,t}^s x_{i,t} - B_1 > 0, \\
&\sum_{i=1}^{2} \sum_{t=T_1+1}^{T_2} \pi^s p_{i,t}^s x_{i,t} - B_2 > 0, \\
&\vdots \\
&\sum_{i=1}^{2} \sum_{t=T_k+1}^{T_{k+1}} \pi^s p_{i,t}^s x_{i,t} - B_{k+1} > 0,
\end{aligned} \tag{6.28}$$

$$\forall t = 1,\ldots,T_n,\ \forall k = 0,1,\ldots,n-1,\ \forall d = 1,\ldots,n.$$

The board usually determines the trading budget B_{k+1}, based on his market expectation about his expected futures prices and the corresponding amount of CO_2 allowances which is expected to be bought from the market in order to close the natural short position in CO_2 allowances (see Section 3.1). Thus, the board indirectly sets a threshold price $F_{i,t}$ against which EUAs and CERs could be bought or sold in the market to optimize the value of the portfolio (see Section 6.3). However, B_{k+1} varies for each CO_2 emissions scenario $d = 1,...,n,$ such that, from now on,

$$B_{k+1} = B_{k+1}^d.$$

Therefore, for each k+1th CO_2 compliance period B_{k+1}^d could be defined as the weighted product of the expected average yearly EUA and CER futures prices times the corresponding expected amount of EUAs and CERs for each CO_2 emissions scenario $d = 1,...,n,$ to be bought from the market by the portfolio manager, i.e.,

$$B_{k+1}^d = \bar{F}_{1,k+1}\left(1-m\right)\left(C_{k+1}^d - \bar{x}_{1,k+1}\right) + \bar{F}_{2,k+1}m\left(C_{k+1}^d - \bar{x}_{1,k+1}\right) \tag{6.29}$$

$$(t = 1,...,T;\ k = 0,1,...,n-1;\ d = 1,...,n).$$

where $\bar{F}_{1,t}$ and $\bar{F}_{1,t}$ are the threshold futures prices for EUAs and CERs, respectively, m is the import limit of CERs, C_{k+1}^d are CO_2 emissions scenarios and $\bar{x}_{1,k+1}$ are free distributed EUAs for each k+1th CO_2 compliance period.

Now, we remember the right hand side of expression from (6.4) as

$$e^{-r(T-t)}\left[\sum_{i=1}^{2}\sum_{t=1}^{n}\left(p_{i,t}^s - \bar{F}_{i,t}\right)x_{i,t}\right]$$

$$(s = 1,...,n),$$

which, adjusted for $t = 1,...,T$, and written out for $i = 1,2,$ gets

$$e^{-r(T_n-t)}\left[\sum_{t=1}^{T_n}\left(p_{1,t}^s - \bar{F}_{1,t}\right)x_{1,t} + \sum_{t=1}^{T_n}\left(p_{2,t}^s - \bar{F}_{2,t}\right)x_{2,t}\right] \tag{6.30}$$

$$(s = 1,...,n).$$

Building the sum for all k+1 CO_2 compliance periods, results in

$$e^{-r(T_{k+1}-(T_k+1))}\left[\sum_{t=T_k+1}^{T_{k+1}}\left(p_{1,t}^{s}-\overline{F}_{1,k+1}\right)x_{1,t}+\sum_{t=T_k+1}^{T_{k+1}}\left(p_{2,t}^{s}-\overline{F}_{2,k+1}\right)x_{2,t}\right] \qquad (6.31)$$

$$(k=0,1,\ldots,n-1;\ s=1,\ldots,n).$$

We can write out and rearrange expression (6.31) as

$$e^{-r(T_{k+1}-(T_k+1))}\left[\left(\sum_{t=T_k+1}^{T_{k+1}}p_{1,t}^{s}x_{1,t}+\sum_{t=T_k+1}^{T_{k+1}}p_{2,t}^{s}x_{2,t}\right)-\left(\sum_{t=T_k+1}^{T_{k+1}}\overline{F}_{1,t}x_{1,k+1}+\sum_{t=T_k+1}^{T_{k+1}}\overline{F}_{2,k+1}x_{2,t}\right)\right]$$

$$(k=0,1,\ldots,n-1;\ s=1,\ldots,n). \qquad (6.32)$$

Concentrating only on the second rounded bracket in (6.32) and adjusting for the regulatory import limit of CERs denoted by m, we have

$$\sum_{t=T_k+1}^{T_{k+1}}\overline{F}_{1,k+1}\left(1-m\right)x_{1,t}+\sum_{t=T_k+1}^{T_{k+1}}\overline{F}_{2,k+1}mx_{2,t} \qquad (6.33)$$

$$(k=0,1,\ldots,n-1).$$

Since we know that for each k+1 CO_2 compliance period, the total amount CO_2 emissions has to be compensated by the yearly total amount of EUAs and CERs, we can define

$$\sum_{t=T_k+1}^{T_{k+1}}\left(1-m\right)x_{1,t}+\sum_{t=T_k+1}^{T_{k+1}}mx_{2,t}:=\left(1-m\right)C_{k+1}^{d}+mC_{k+1}^{d} \qquad (6.34)$$

$$(k=0,1,\ldots,n-1;\ d=1,\ldots,n).$$

By using the right hand side of definition (6.34) and deducting the free distributed EUAs $\overline{x}_{1,k+1}$, we can adjust definition (6.34) to

$$\left(1-m\right)\left(C_{k+1}^{d}-\overline{x}_{1,k+1}\right)+m\left(C_{k+1}^{d}-\overline{x}_{1,k+1}\right) \qquad (6.35)$$

$$(k=0,1,\ldots,n-1;\ d=1,\ldots,n).$$

Thus, combining expression (6.35) and (6.33), results in

$$\overline{F}_{1,k+1}\left(1-m\right)\left(C_{k+1}^{d}-\overline{x}_{1,k+1}\right)+\overline{F}_{2,k+1}m\left(C_{k+1}^{d}-\overline{x}_{1,k+1}\right) \tag{6.36}$$

$$(k=0,1,...,n-1;\ d=1,...,n),$$

which is just the left hand side expression in (6.29), such that, as result,

$$B_{k+1}^{d}=\overline{F}_{1,k+1}\left(1-m\right)\left(C_{k+1}^{d}-\overline{x}_{1,k+1}\right)+\overline{F}_{2,k+1}m\left(C_{k+1}^{d}-\overline{x}_{1,k+1}\right) \tag{6.37}$$

$$(k=0,1,...,n-1;\ d=1,...,n).$$

Thus, the budget B_{k+1}^{d} implicitly contains the futures prices threshold values $\overline{F}_{1,t}$ and $\overline{F}_{2,t}$, the CER import limit *m*, the CO_2 emission scenarios C_{k+1}^{d} and the free distributed EUAs $\overline{x}_{1,k+1}$. Since these are all know scalars, B_{k+1}^{d} can easily be calculated and directly used in the formulation of our optimization model.

Accordingly, we can sum the two terms in the first rounded bracket in expression (6.32) to

$$\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} p_{i,t}^{s}x_{i,t} \tag{6.38}$$

$$(k=0,1,...,n-1;\ s=1,...,n),$$

which serves as the revenue term in our profit function, which according to (iii) gets

$$\sum_{i=1}^{2}\sum_{t=1}^{T_1} \pi^{s} p_{i,t}^{s}x_{i,t} \tag{6.39}$$

$$(k=0,1,...,n-1;\ s=1,...,n).$$

Then, according to expression (6.32), the profit of a trading strategy for each *k*+1 CO_2 compliance period, can be state as

$$e^{-r(T_{k+1}-(T_k+1))}\left[\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} \pi^{s} p_{i,t}^{s}x_{i,t}-B_{k+1}^{d}\right] \tag{6.40}$$

$$(k=0,1,...,n-1;\ s=1,...,n).$$

We remember that we defined above, that for $k = 0$, $T_k = T_0 = 0$, and τ_0 does not exist, and for each $k > 0$, $T_k < \tau_{k+1} < T_{k+1}$, $\tau_{k+1}, T_{k+1} \in \mathbb{N}$. We remember that for the penalty term in expression (6.25), we defined $[a]^- := \max\{0, -a\}$, for each $a \in \mathbb{R}$.

Then the total profit function z, including the penalty term, in our optimization model for the sum of all CO_2 compliance periods can be stated as

$$z = \sum_{k=0}^{n-1} \Big[e^{-r(T_{k+1}-(T_k+1))} \Big[\sum_{i=1}^{2} \sum_{t=T_k+1}^{T_{k+1}} \pi^s p_{i,t}^s x_{i,t} - B_{k+1} \Big] - g \Big[\sum_{i=1}^{2} \sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + (1+b)\overline{x}_{1,k+1} - C_{k+1}^d \Big]^- \Big], \tag{6.41}$$

where the expression in the sub bracket of the function z stands for the paid penalty by the airline company, if the expression is negative.

Now, let us denote the EUA and CER futures prices from equation (4.17) in Section 4.3 for each scenario s as $F_{i,(t+\Delta t,T_n)}^s$, then according to the notations above, $F_{i,(t+\Delta t,T_n)}^s$ has the same meaning as $p_{i,t}^s$, that is, the MC simulated EUA and CER futures prices over the trading time horizon $t = T_0(=0), 1, 2.., T_1, ..., T_{k+1}, ..., T_n$. Then, we can just set

$$F_{i,(t+\Delta t,T_n)}^s = p_{i,t}^s,$$

and thus

$$F_{i,(t,T_n)}^s = F_{i,t}^s,$$

then expression (6.40) gets

$$z = \sum_{k=0}^{n-1} \Bigg[e^{-r(T_{k+1}-(T_k+1))} \sum_{i=1}^{2} \sum_{t=T_k+1}^{T_{k+1}} \pi^s \left(F_{i,t}^s \left(\mu_i - \sigma_i \frac{1}{2} \right) \Delta t + \sigma_i \varepsilon_{i,t} \sqrt{\Delta t} \right) x_{i,t} - B_{k+1} - g \Big[\sum_{i=1}^{2} \sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + (1+b)\overline{x}_{1,k+1} - C_{k+1}^d \Big]^- \Bigg], \tag{6.42}$$

where is Δt discrete time-step used in the MC simulation.

6.4.5 Optimization Model

Given the formulations above, we can formulate our multi-period stochastic portfolio optimization model as the following profit maximization problem with regard to derived corresponding constraints:

maximize
$(x_{1,t}, x_{2,t})_{t=1,\dots,T_n}$

$$z = \sum_{k=0}^{n-1}\left[e^{-r(T_{k+1}-(T_k+1))} \sum_{i=1}^{2} \sum_{t=T_k+1}^{T_{k+1}} \pi^s \left(F_{i,t}^s \left(\mu_i - \sigma_i \frac{1}{2} \right) \Delta t + \sigma_i \varepsilon_{i,t} \sqrt{\Delta t} \right) x_{i,t} - B_{k+1} - g\left[\sum_{i=1}^{2} \sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + (1+b)\overline{x}_{1,k+1} - C_{k+1}^d \right]^- \right], \tag{6.42}$$

subject to

EU ETS regulatory limit for CERs:

$$\begin{aligned} &\sum_{t=1}^{T_1} x_{2,t} \le m\left(C_1^d - \overline{x}_{1,1}\right), \\ &\sum_{t=T_1+1}^{T_2} x_{2,t} \le m\left(C_2^d - \overline{x}_{1,2}\right), \\ &\vdots \\ &\sum_{t=T_k+1}^{T_{k+1}} x_{2,t} \le m\left(C_{k+1}^d - \overline{x}_{1,k+1}\right), \end{aligned} \tag{6.17}$$

$$\forall k = 0,1,\dots,n-1,\ \forall d = 1,\dots,n,$$

$$\begin{aligned} &\sum_{t=1}^{T_1} x_{1,t} \le (1-m)\left(C_1^d - \overline{x}_{1,1}\right), \\ &\sum_{t=T_1+1}^{T_2} x_{1,t} \le (1-m)\left(C_2^d - \overline{x}_{1,2}\right), \\ &\vdots \\ &\sum_{t=T_k+1}^{T_{k+1}} x_{1,t} \le (1-m)\left(C_{k+1}^d - \overline{x}_{1,k+1}\right), \end{aligned} \tag{6.18}$$

$$\forall k = 0,1,\dots,n-1,\ \forall d = 1,\dots,n,$$

Regulatory banking and borrowing constraints:

$$\begin{aligned}
&\sum_{i=1}^{2}\sum_{t=1}^{T_1} x_{i,t} + (1+b)\overline{x}_{1,k+1} = C_1^d, \\
&\sum_{i=1}^{2}\sum_{t=T_1+1}^{T_2} x_{i,t} + (1+b)\overline{x}_{1,k+1} = C_2^d, \\
&\vdots \\
&\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + (1+b)\overline{x}_{1,k+1} = C_{k+1}^d,
\end{aligned} \qquad (6.11)$$

$$\forall k = 0,1,\dots,n-1,\ \forall d = 1,\dots,n,$$

$$\sum_{k=0}^{n-1}\left[\sum_{i=1}^{2}\sum_{t=T_k+1}^{T_{k+1}} x_{i,t} + (1+b)\overline{x}_{1,k+1}\right] = \sum_{k=0}^{n-1} C^{k+1}, \qquad (6.16)$$

$$\sum_{k=0}^{n-1} b\overline{x}_{1,k+1} = 0, \qquad (6.15)$$

Upper (i.e,. purchasing) trading constraints:

$$\begin{aligned}
&x_{1,t} \le u_{1,t}\left[m\left(C_{k+1}^d - \overline{x}_{1,k+1}\right)\right], \\
&x_{2,t} \le u_{2,t}\left[(1-m)\left(C_{k+1}^d - \overline{x}_{1,k+1}\right)\right],
\end{aligned} \qquad (6.19)$$

$$\forall t = 1,\dots,T_n,\ \forall k = 0,1,\dots,n-1,\ \forall d = 1,\dots,n,$$

Lower (i.e., selling) trading constraints:

$$\begin{aligned}
&-x_{1,t} \ge v_{1,t}\left[m\left(C_{k+1}^d - \overline{x}_{1,k+1}\right)\right], \\
&-x_{2,t} \ge v_{2,t}\left[(1-m)\left(C_{k+1}^d - \overline{x}_{1,k+1}\right)\right],
\end{aligned} \qquad (6.20)$$

$$\forall t = 1,\dots,T_n,\ \forall k = 0,1,\dots,n-1,\ \forall d = 1,\dots,n,$$

Risk constraints:

$$\begin{aligned}
&\sum_{t=1}^{\tau_1} x_{2,t} \le q_{2,\tau_1}\left[m\left(C_1^d - \overline{x}_{1,1}\right)\right], \\
&\sum_{t=T_1+1}^{\tau_2} x_{2,t} \le q_{2,\tau_2}\left[m\left(C_2^d - \overline{x}_{1,2}\right)\right], \\
&\vdots \\
&\sum_{t=T_k+1}^{\tau_{k+1}} x_{2,t} \le q_{2,\tau_{k+1}}\left[m\left(C_{k+1}^d - \overline{x}_{1,k+1}\right)\right],
\end{aligned} \qquad (6.21)$$

$$\forall k = 0,1,\dots,n-1,\ \forall d = 1,\dots,n,$$

$$
\begin{aligned}
&\sum_{t=1}^{\tau_1} x_{1,t} \le q_{1,\tau_1}\left[(1-m)\left(C_1^d - \bar{x}_{1,1}\right)\right],\\
&\sum_{t=T_1+1}^{\tau_2} x_{1,t} \le q_{1,\tau_2}\left[(1-m)\left(C_2^d - \bar{x}_{1,2}\right)\right],\\
&\vdots\\
&\sum_{t=T_k+1}^{\tau_{k+1}} x_{1,t} \le q_{1,\tau_{k+1}}\left[(1-m)\left(C_{k+1}^d - \bar{x}_{1,k+1}\right)\right],
\end{aligned}
\tag{6.22}
$$

$$
\forall k = 0,1,\ldots,n-1,\ \forall d = 1,\ldots,n.
$$

Therefore, our optimization problem consists of a two-dimensional modeled state space for the stages $t=1,...,T_n$, consisting of the stochastic variables $F_{1,t}^{s}$ (EUA futures price) and $F_{2,t}^{s}$ (CER futures price), with $s=1,...,n$, and the stochastic variable c_t^d (CO_2 emissions), for $s=1,...,n$, representing possible states for each stage *t*. We remember that $p_{i,t}^{s}$ is influenced by random processes, whereas c_t^d occurs as stochastic constraint scalars, modeled through deterministic scenarios $d=1,...,n.$ by the airline company.
Hence, the optimization results will result in optimal futures hedging strategies, with the remaining part of the CO_2 short position to be closed by spot contracts.

The structure of the optimization model is illustrated in Figure 6.4. The model input parameters include market parameters for MC simulation such as expected return, volatilities, variance-covariance matrix and EUA and CER initial futures prices of various specified delivery periods (see Section 4.3). Furthermore, optimization parameters (scalars) such as regulatory usage limits, upper / lower trading limits, banking / borrowing limits, amount of free allowances, penalty fee and trading budget will be used, whereas the amount of yearly CO_2 emissions will be modeled by deterministic scenarios. The model algorithm consists as follows:

Step 1: *MC simulation of possible $s=1,...,n$ paths for correlated EUA and CER futures prices (see Section 4.3). As a consequence, the scenarios become tree-structured with nodes θ from a finite set O, i.e., forward scenario-tree. Each node θ therefore denotes a decision point (i.e., state), corresponding to the realization of $p_{i,t}^{s}$ up to θ, represented by the trading time ste $t=1,...,T_n$. Or, in other words, each state θ represents a combination ("couple") of simulated EUA and CER futures prices at the trading time instant t.*

Step 2: *Set up of the multi-stage stochastic optimization model (see above).*

Step 3: *Solve the stochastic program on the scenario tree via LP technique (CPLEX solver in MATLAB).*

The model outputs will include the value and distribution of earnings, the optimal trading strategies and the risk measure VaR to determine the risk exposure of the portfolio manager.

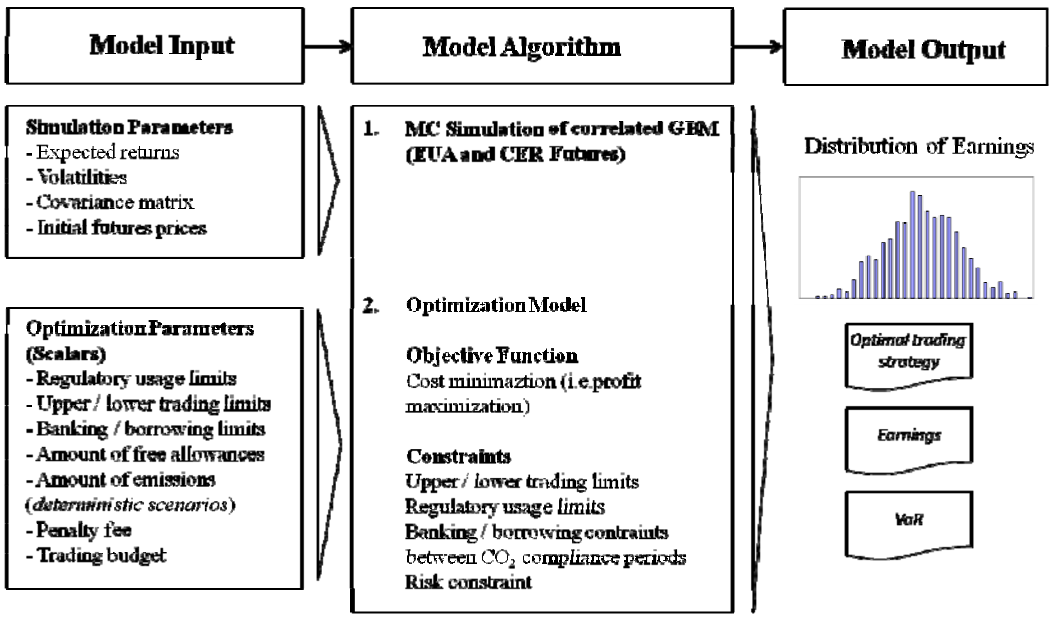

Figure 6.4. Structure of the optimization model.

CHAPTER 7

TIME-SERIES PROPERTIES AND M ODEL INPUT PARAMETERS

In our analysis, we use EUA and CER futures prices, traded at the *Intercontinental Exchange* (*ICE*) in London, which today, by far, is the most liquid electronic platform for CO_2 emissions trading, representing more than 80% of the exchange-traded volume of EUAs and CERs in the European carbon market [39].

7.1 Time-Series Properties

In the introduction part of Chapter 4, we mentioned that carbon prices follow a GBM process, whose fundamental assumption is the normal distribution. We will apply the well-known Jarque-Bera (JB) test for normality in returns of EUA and CER futures prices for various CO_2 compliance periods. Here, the null hypothesis is tested, that the skewness of the distribution together with its excess kurtosis (i.e., kurtosis minus 3) are both zero, and therefore follows a normal distribution, against the alternative hypothesis of non-normal distribution.

For all EUA and CER retruns, the p-values of the received JB test statistics are both larger than 0.01 and 0.05, such that all probability levels the null hypothesis of a normal distribution in returns can clearly not be rejected. The corresponding EViews values can be found in Appendix B.1.

The GBM process is nothing else than a random walk plus drift model. We will justify its application in our model, by the use of the Dickey-Fuller test for testing the unit root property in the EUA and CER futures prices, and therefore, their non-stationarity. This implies that they are perennially subject of random shocks and thus depend on their drift and volatilities. Hence, we will test the null hypothesis of EUA and CER futures prices having a root unit and therefore following random walk plus drift model (i.e. non-stationary), against the alternative hypothesis of not incorporating a unit root (i.e. stationary), such that the EUA and CER futures prices will converge at a long-term mean. In that case, the use of the Ornstein-Uhlenbeck (O-U) process would be appropriate.

Now, starting with a price model y_t with a constant drift

$$y_t = c + a y_{t-1} + \varepsilon_t,$$

where c denotes the constant drift term and ε_t denotes the error term, by differentiating both sides by y_{t-1} we get,

$$\Delta y_t = c + b y_{t-1} + \varepsilon_t,$$

where

$$b = a - 1.$$

Consequently, our hypothesis testing becomes

$$H_0 : b = 0,$$
$$H_1 : b < 0.$$

For all EUA and CER futures prices, the absolute value of the Augmented-Dickey-Fuller (ADF) unit root test statistics are less than the 1%, 5% and 10% critical values. Hence, the null hypothesis cannot be rejected, justifying the use of the random walk plus drift model, and therefore the GBM model, as the underlying price process in our model. Appendix B.2 contains the resulting EViews outputs for the ADF tests, verifying the non-rejection of the null hypothesis. Additionally, these results have been cross checked by also applying other unit root tests such as Philipps-Perron, Ng-Perron or Elliott-Rothenberg-Stock-Point

Optimal tests. All their outcomes support the ADF test results, and thus verify the use of the GBM model as our underlying price model..

7.2 Model Input Parameters

7.2.1 Input Parameters for MC Simulation

Our optimization model is assumed to have a three-year CO_2 compliance period for the years 2013, 2014, and 2015, i.e., $k=1,2,3$. We consider monthly trading steps $t=1,2,...,36$, i.e., the discrete time-step Δt used in the MC simulation is 1, resulting in a total trading time horizon of 36 months results. Hence, the last trading period within each CO_2 compliance period is $T_1=12$, $T_1=24$ and $T_1=36$.

For all of those CO_2 compliance periods there are yearly EUA and CER traded futures contracts available, implying Dec'13, Dec'14 and Dec'15 futures contracts for both CO_2 emission allowance types. Therefore, we can subdivide the traded amount of EUA futures and CER futures, which we denoted as $x_{i,t}$ in Subsection 6.4.5, specifically into EUA and CER Dec'13, Dec'14 and Dec'15 futures contracts with the following notation of the variables:

- EUA Dec'13 futures contracts: $x_{11,t}$, $\forall t=1,...,12$,
- EUA Dec'14 futures contracts: $x_{12,t}$, $\forall t=13,...,24$,
- EUA Dec'15 futures contracts: $x_{13,t}$, $\forall t=25,...,36$,
- CER Dec'13 futures contracts: $x_{21,t}$, $\forall t=1,...,12$,
- CER Dec'14 futures contracts: $x_{22,t}$, $\forall t=13,...,24$,
- CER Dec'15 futures contracts: $x_{23,t}$, $\forall t=25,...,36$,

Accordingly, we can do the same for the prices of the traded amount of EUA futures and CER futures, which we denoted as $F_{i,t}^s$ in Subsection 6.4.5, such that the following new variables for the MC simulated correlated futures prices for $n=10.000$ scenarios result:

- EUA Dec'13 futures prices: $F_{11,t}^s$, $\forall t=1,...,12$, $\forall s=1,...,250$,
- EUA Dec'14 futures prices: $F_{12,t}^s$, $\forall t=13,...,24$, $\forall s=1,...,250$,
- EUA Dec'15 futures prices: $F_{13,t}^s$, $\forall t=25,...,36$, $\forall s=1,...,250$,

- CER Dec'13 futures prices: $F_{21,t}^{s}$, $\forall t = 1,...,12,\ \forall s = 1,...,250,$
- CER Dec'14 futures prices: $F_{22,t}^{s}$, $\forall t = 13,...,24,\ \forall s = 1,...,250,$
- CER Dec'15 futures prices: $F_{23,t}^{s}$, $\forall t = 25,...,36,\ \forall s = 1,...,250,$

We assume that the MC scenarios for

$$F_t^s = \left\{ F_{11,t}^s, F_{12,t}^s, F_{13,t}^s; F_{21,t}^s, F_{22,t}^s, F_{23,t}^s \right\}_{t=1,...,36},$$

with $s = 1,...,250,$ are uniformly distributed, i.e.,

$$\pi^s = \frac{1}{n} = \frac{1}{250}, \quad \sum_{s=1}^{250} \pi^s = 1.$$

The market parameters for the modeling of the market state space (i.e., MC simulation) are correspondingly denoted as follows:

- Average return / volatility EUA Dec'13 futures prices: μ_{11} / σ_{11},
- Average return / volatility EUA Dec'14 futures prices: μ_{12} / σ_{12},
- Average return / volatility EUA Dec'15 futures prices: μ_{13} / σ_{13},
- Average return / volatility CER Dec'13 futures prices: μ_{21} / σ_{21},
- Average return / volatility CER Dec'14 futures prices: μ_{22} / σ_{22},
- Average return / volatility CER Dec'15 futures prices: μ_{23} / σ_{23},

The existence of 6 different futures prices data which are all cross-correlated with each other, thereby implying a 6×6 variance-covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_{11}^2 & \sigma_{11}\sigma_{12}\rho_{11,12} & \sigma_{11}\sigma_{13}\rho_{11,13} & \sigma_{11}\sigma_{21}\rho_{11,21} & \sigma_{11}\sigma_{22}\rho_{11,22} & \sigma_{11}\sigma_{23}\rho_{11,23} \\ \sigma_{11}\sigma_{12}\rho_{11,12} & \sigma_{12}^2 & \sigma_{12}\sigma_{13}\rho_{12,13} & \sigma_{12}\sigma_{21}\rho_{12,21} & \sigma_{12}\sigma_{22}\rho_{12,22} & \sigma_{12}\sigma_{23}\rho_{12,23} \\ \sigma_{11}\sigma_{13}\rho_{11,13} & \sigma_{12}\sigma_{13}\rho_{12,13} & \sigma_{13}^2 & \sigma_{13}\sigma_{21}\rho_{13,21} & \sigma_{13}\sigma_{22}\rho_{13,22} & \sigma_{13}\sigma_{23}\rho_{13,23} \\ \sigma_{11}\sigma_{21}\rho_{11,21} & \sigma_{12}\sigma_{21}\rho_{12,21} & \sigma_{13}\sigma_{21}\rho_{13,21} & \sigma_{21}^2 & \sigma_{21}\sigma_{22}\rho_{21,22} & \sigma_{21}\sigma_{23}\rho_{21,23} \\ \sigma_{11}\sigma_{22}\rho_{11,22} & \sigma_{12}\sigma_{22}\rho_{12,22} & \sigma_{13}\sigma_{22}\rho_{13,22} & \sigma_{21}\sigma_{22}\rho_{21,22} & \sigma_{22}^2 & \sigma_{22}\sigma_{23}\rho_{22,23} \\ \sigma_{11}\sigma_{23}\rho_{11,23} & \sigma_{12}\sigma_{23}\rho_{12,23} & \sigma_{13}\sigma_{23}\rho_{13,23} & \sigma_{21}\sigma_{23}\rho_{13,23} & \sigma_{22}\sigma_{23}\rho_{22,23} & \sigma_{23}^2 \end{pmatrix}.$$

According to the specified CO_2 trading and compliance periods and adjusted notation of the all the variables above, the detailed formulation of the optimization model in Subsection 6.4.5 could be written out (see Appendix A).

Two market scenarios, consisting of the optimistic and pessimistic scenario, have been defined. The *optimistic* market scenario incorporates the historical expected returns, volatilities, correlations and the resulting variance-covariance matrix of Dec'09, Dec'10 and Dec'11 EUA and CER futures for the period 03/03/2009–28/05/2009, a 60-trading day period where the market was rapidly increasing with a relative high level of volatility. The *pessimistic* market scenario contains the historical expected returns, volatilities, correlations and the variance-covariance matrix of Dec'13, Dec'14 and Dec'15 EUA and CER futures for the period 22/02/2010–19/05/2010, a 60-trading day period where the market was rapidly decreasing with a medium level of volatility. Hence, the underlying historical market data of the delivered Dec'09, Dec'10 and Dec'11 EUA and CER futures contracts as well as those for Dec'13, Dec'14 and Dec'15 EUA and CER futures contracts will be used as market input parameters for MC simulation of correlated Dec'13, Dec'14 and Dec'15 EUA and CER futures prices.

Through the log returns of the historical ICE ECX data for the corresponding time period of the optimistic and pessimistic scenario, the average historical returns, volatilities, correlations and the variance-covariance matrix as input for MC simulation of Dec'13, Dec'14 and Dec'15 EUA and CER futures prices have been determined. Those values as well as the initial values for EUA and CER prices are shown in Tables 7.1–7.9.

As it can be seen from Tables 7.1–7.9, the main characteristic difference between the optimistic and pessimistic scenario are that, in the one hand, the expected returns of the optimistic scenario, in absolute terms, are smaller than those of the pessimistic scenario, and in the other hand, the volatilities of the optimistic scenario are relatively higher than those of the pessimistic scenario. All the EUA futures prices among themselves as well as with CER futures prices exhibit a high correlation. Nevertheless, the variance-covariance matrix is positive definite and therefore, is appropriate for conducting the Cholesky decomposition.

Initial Values (EUR/tCO_2)	Dec'13	Dec'14	Dec'15
EUA price	7.52	7.91	8.47
CER price	0.51	0.61	0.69

Table 7.1. Initial values for EUA and CER futures prices for both the optimistic and pessimistic scenarios.

Returns	Dec'13	Dec'14	Dec'15
EUA price	0.12002	0.11909	0.11763
CER price	0.08846	0.09479	0.08449

Table 7.2. Optimistic scenario: Monthly historical returns of EUA and CER prices.

Volatilities	Dec'13	Dec'14	Dec'15
EUA price	0.15338	0.15625	0.15685
CER price	0.14811	0.18284	0.15717

Table 7.3. Optimistic scenario: Monthly historical volatilities of EUA and CER prices.

Correlations	EUA Dec'13	EUA Dec'14	EUA Dec'15	CER Dec'13	CER Dec'14	CER Dec'15
EUA Dec'13	1	0.96240	0.93721	0.88269	0.72905	0.67272
EUA Dec'14	0.96240	1	0.94731	0.89447	0.76678	0.68829
EUA Dec'15	0.93721	0.94731	1	0.87826	0.70615	0.67974
CER Dec'13	0.88269	0.89447	0.87826	1	0.84407	0.77595
CER Dec'14	0.72905	0.76678	0.70615	0.84407	1	0.84180
CER Dec'15	0.67272	0.68829	0.67974	0.77595	0.84180	1

Table 7.4. Optimistic scenario: Correlations of EUA and CER prices.

Covariance	EUA Dec'13	EUA Dec'14	EUA Dec'15	CER Dec'13	CER Dec'14	CER Dec'15
EUA Dec'13	0.023525	0.023065	0.022547	0.020052	0.020445	0.016217
EUA Dec'14	0.023065	0.024415	0.023265	0.020701	0.021906	0.016904
EUA Dec'15	0.022547	0.023265	0.024602	0.020403	0.020251	0.016757
CER Dec'13	0.020052	0.020701	0.020403	0.021937	0.022858	0.018063
CER Dec'14	0.020445	0.021906	0.020251	0.022858	0.033430	0.024191
CER Dec'15	0.016217	0.016904	0.016757	0.018063	0.024191	0.024703

Table 7.5. Optimistic scenario: Variance-covariance matrix.

Returns	Dec'13	Dec'14	Dec'15
EUA price	-0.16058	-0.16454	-0.16421
CER price	-0.17643	-0.18260	-0.17948

Table 7.6. Pessimistic scenario: Monthly historical returns of EUA and CER prices.

Volatilities	Dec'13	Dec'14	Dec'15
EUA price	0.09305	0.09696	0.09556
CER price	0.09901	0.10629	0.10632

Table 7.7. Pessimistic scenario: Monthly historical volatilities of EUA and CER prices.

Correlations	EUA Dec'13	EUA Dec'14	EUA Dec'15	CER Dec'13	CER Dec'14	CER Dec'15
EUA Dec'13	1	0.97623	0.94138	0.85855	0.84396	0.79106
EUA Dec'14	0.97623	1	0.85855	0.90275	0.85094	0.78401
EUA Dec'15	0.94138	0.85855	1	0.88199	0.80699	0.77599
CER Dec'13	0.89644	0.90275	0.88199	1	0.85660	0.77984
CER Dec'14	0.84396	0.85094	0.80699	0.85660	1	0.82439
CER Dec'15	0.79106	0.77599	0.77599	0.77984	0.82439	1

Table 7.8. Pessimistic scenario: Correlations of EUA and CER prices.

Covariance	EUA Dec'13	EUA Dec'14	EUA Dec'15	CER Dec'13	CER Dec'14	CER Dec'15
EUA Dec'13	0.008659	0.008898	0.008637	0.008259	0.009133	0.008562
EUA Dec'14	0.008898	0.009400	0.010923	0.008666	0.009594	0.008842
EUA Dec'15	0.008637	0.010923	0.009131	0.008345	0.008967	0.008625
CER Dec'13	0.008259	0.008666	0.008345	0.009804	0.010093	0.009212
CER Dec'14	0.009133	0.009594	0.008967	0.010093	0.013523	0.011287
CER Dec'15	0.008562	0.008842	0.008625	0.009212	0.011287	0.013530

Table 7.9. Pessimistic scenario: Variance-covariance matrix.

7.2.2 Optimization Parameters (Scalars)

We assume that the amount of free distributed EUAs by the EU ETS regulatory authority is constant for each CO_2 compliance period k as 800,000, i.e., $\overline{x}_{1,1} = \overline{x}_{1,2} = \overline{x}_{1,3} = 800,000$. Let the penalty fee $g = 100$ EUR for each missing ton of CO_2. We intentionally use the actually valid regulatory EUA banking and borrowing constraints $b = 0.025$ for the free distributed EUAs $\overline{x}_{1,k+1}$ for each (k+1)th CO_2 compliance and the CER import limit constraint $m = 0.01$ to reveal the true regulatory situation in the EU ETS.

The board-defined upper and lower trading limits for EUAs and CERS are assumed to be constant for all $t = 1,...,36$ as $u_{i,t} = \overline{u}_{i,t} = 0.15$ and $v_{i,t} = \overline{v}_{i,t} = -0.15$, respectively.

For three-year CO_2 compliance period the airline company estimates yearly CO_2 emissions C_1^d, C_2^d, C_3^d for $d = 1,2,3$ deterministic scenarios which are provided in Table 10. The board also provides the portfolio manager with a total trading budget for the three-year CO_2

compliance period $\sum_{k=0}^{2} B_{k+1}^{d}$, dependent of scenarios d for the yearly CO_2 emissions C_1^d, C_2^d, C_3^d. According to equation (6.27), taking the given scalars above, we result in the total budget values for $\sum_{k=0}^{2} B_{k+1}^{d}$ in Table 7.6.

Scenario d	C_1^d	C_2^d	C_2^d	$\sum_{k=0}^{2} B_{k+1}^{d}$
1	900.000	1.000.000	1.200.000	7.5 Mio. EUR
2	1.000.000	1.200.000	1.400.000	10.0 Mio. EUR
3	1.100.000	1.300.000	1.400.000	12.5 Mio. EUR

Table 7.10. Deterministic scenarios for CO_2 emissions and the resulting trading budget.

The board also predetermines the percentage amount of the natural open position in CO_2 emissions for EUAs as $q_{1,\tau_1}, q_{1,\tau_2}, q_{1,\tau_3}$ and for CERs as $q_{2,\tau_1}, q_{2,\tau_2}, q_{2,\tau_3}$ to be closed up to time τ_1 and τ_2 to limit the risk exposure of the open position of the portfolio manager. Various risk levels for the open position and board-defined point in time τ_1, τ_2, τ_3 are defined in Table 7.11. Here, $\tau_1 = 6$, $\tau_2 = 18$ and $\tau_3 = 27$ denotes that, until the half of each CO_2 compliance period, a percentage amount of the open position in EUAs and CERs, respectively, should have been closed by the portfolio manager. The indices $\tau_1 = 9$, $\tau_2 = 21$ and $\tau_3 = 33$ denote the end of the third quarter of each CO_2 compliance period.

%-amount to be closed	τ_1		τ_2		τ_3	
	$\tau_1 = 6$	$\tau_1 = 9$	$\tau_2 = 18$	$\tau_2 = 21$	$\tau_3 = 30$	$\tau_3 = 33$
$q_{1,\tau_1}, q_{1,\tau_2}, q_{1,\tau_3}$	0.5	0.5	0.5	0.5	0.5	0.5
$q_{1,\tau_1}, q_{1,\tau_2}, q_{1,\tau_3}$	0.75	0.75	0.75	0.75	0.75	0.75
$q_{2,\tau_1}, q_{2,\tau_2}, q_{2,\tau_3}$	0.5	0.5	0.5	0.5	0.5	0.5
$q_{2,\tau_1}, q_{2,\tau_2}, q_{2,\tau_3}$	0.75	0.75	0.75	0.75	0.75	0.75

Table 7.11. Various risk levels for the open position and board-defined points in time.

For our optimization model, we make following assumptions with regard to the EU ETS market:

- No transaction costs are considered.
- No liquidity constraints are considered. That is, all transactions are carried out without being able to influence the market price.
- No margins (and margin calls) are considered.

CHAPTER 8

OPTIMIZATION RESULTS

Figures 8.1–8.12 illustrate n = 250 MC simulated correlated price paths for EUA Dec'13, Dec'14 and Dec'15 futures as well as for CER Dec'13, Dec'14 and Dec'15 futures, for the optimistic and pessimistic market scenarios.

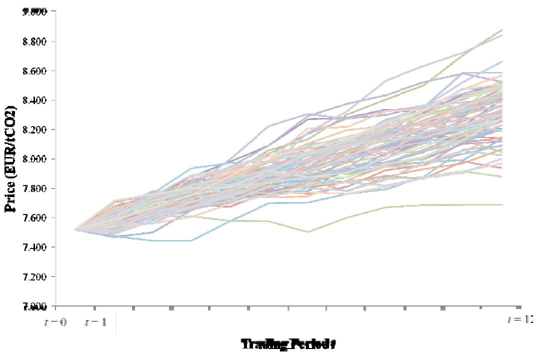

Figure 8.1. Optimistic scenario: MC simulated price paths for EUA Dec'13

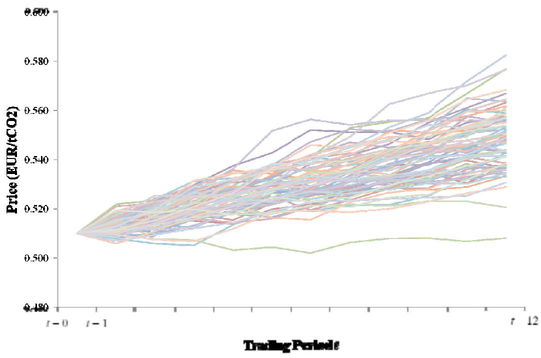

Figure 8.2. Optimistic scenario: MC simulated price paths for CER Dec'13

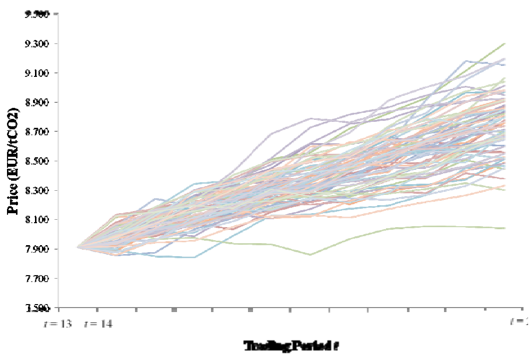

Figure 8.3. Optimistic scenario: MC simulated price paths for EUA Dec'14

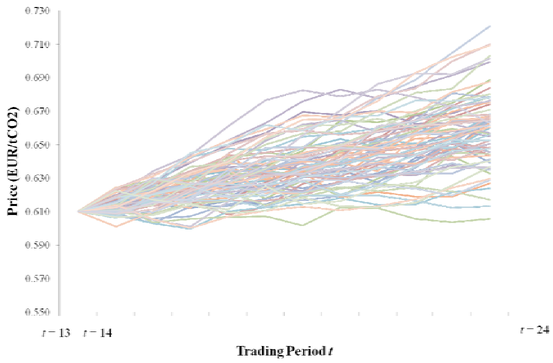

Figure 8.4. Optimistic scenario: MC simulated price paths for CER Dec'14

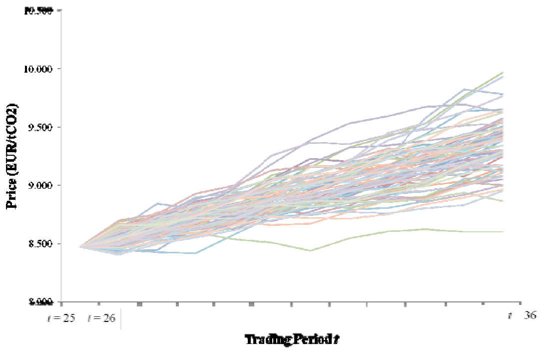

Figure 8.5. Optimistic scenario: MC simulated price paths for EUA Dec'15

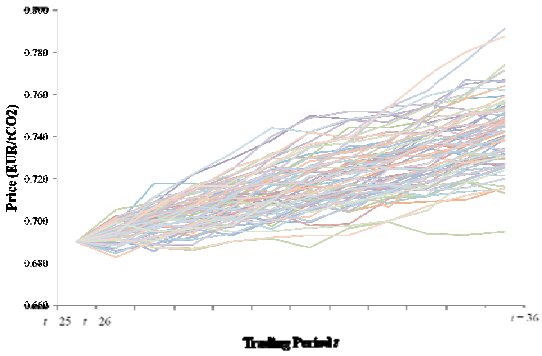

Figure 8.6. Optimistic scenario: MC simulated price paths for CER Dec'15

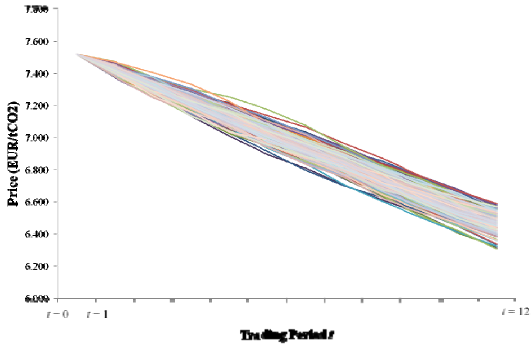

Figure 8.7. Pessimistic scenario: MC simulated price paths for EUA Dec'13

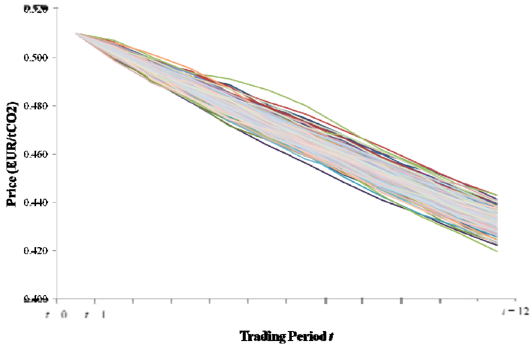

Figure 8.8. Pessimistic scenario: MC simulated price paths for CER Dec'13

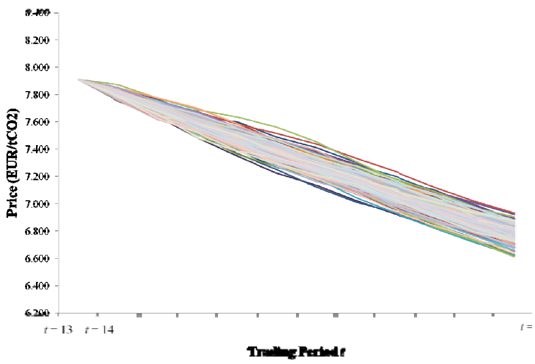

Figure 8.9. Pessimistic scenario: MC simulated price paths for EUA Dec'14

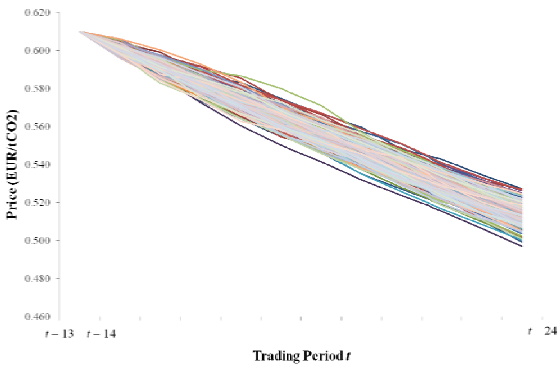

Figure 8.10. Pessimistic: MC simulated price paths for CER Dec'14 futures.

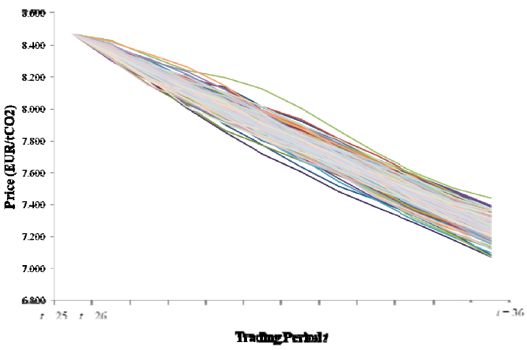

Figure 8.11. Pessimistic scenario: MC simulated price paths for EUA Dec'15

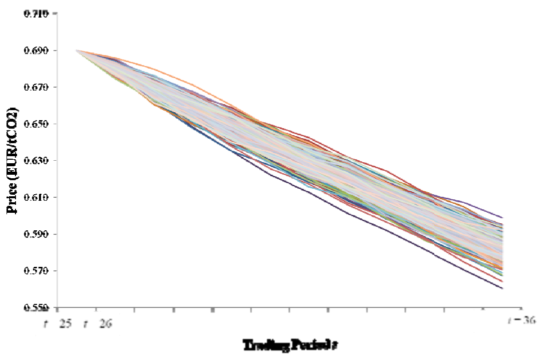

Figure 8.12. Pessimistic scenario: MC simulated price paths for CER Dec'15

The graphs for MC simulation of the optimistic and pessimistic scenario clearly reveal that, due to the existence of the relative higher volatilities, the scenario tree of the optimistic scenarios is more stretched, whereas the scenario tree of the pessimistic scenario, in absolute terms, has a relative larger slope due to the, in absolute terms, higher expected returns.

We modeled and solved our optimization problem through the CPLEX solver in MATLAB, based on each received MC simulated EUA and CER Dec'13, Dec'14 and Dec'15 futures price, and obtained a feasible solution.

Figures 8.13–8.16 show the resulting MC simulated trading strategies for EUAs and CER futures for the optimistic and pessimistic market scenario for a trading budget of 10 Mio. EUR, $\tau_1 = 9$, $\tau_2 = 21$, $\tau_2 = 33$, and $q_1, q_2 = 0.5$.

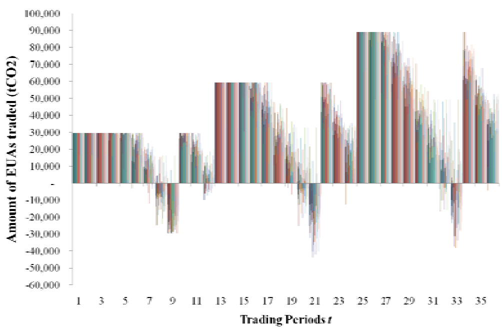

Figure 8.13. Optimistic scenario: MC simulated optimal trading strategies for EUA futures.

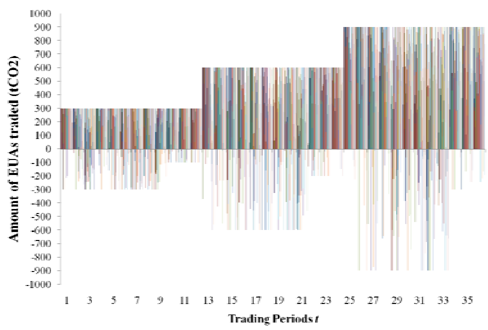

Figure 8.14. Optimistic scenario: MC simulated optimal trading strategies for CER futures.

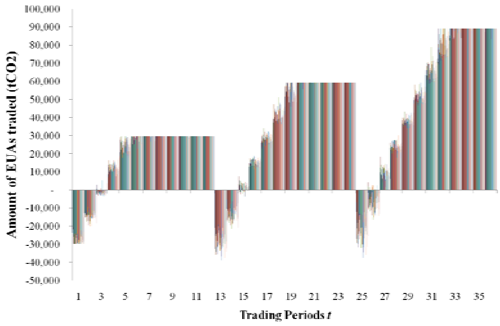

Figure 8.15. Pessimistic scenario: MC simulated optimal trading strategies for EUA futures.

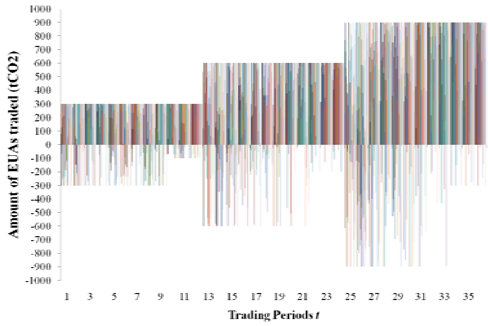

Figure 8.16. Pessimistic scenario: MC simulated optimal trading strategies for CER futures.

All figures clearly depict the increasing amount of EUA and CER futures traded in each subsequent CO_2 compliance period due to the increasing amount of the short position for each subsequent CO_2 compliance period. At the end of each CO_2 compliance period, the traded amount of CO_2 emission allowances equals the amount of verified CO_2 emissions of the airline company, such that the portfolio manager does not pay any penalty fee to the regulatory authority. In order to close the physical short position in CO_2 emission allowances, the portfolio manager primarily executes buy strategies of EUA and CER futures. However, these differ for the optimistic and pessimistic market scenario. Whereas in the optimistic scenario, where the prices are increasing, the portfolio manager mainly buys EUA and CER futures in the first half of a CO_2 compliance year, in the pessimistic market scenario, where the prices are decreasing, he mainly buys those in the second half of a CO_2 compliance year. In the optimistic scenario, shorting of EUA futures is more likely happening in the third each quarter, whereas in the pessimistic scenario, it is more likely happening in the each first quarter. To optimize his total hedge portfolio in CO_2 emission allowances, the portfolio manager relatively intensively implements buy-selling strategies for CER futures. Since the maximum difference in the price level is between T_1 and T_3, in the optimistic market scenario, the portfolio manager borrows banks (i.e., goes long) the maximum possible 2.5%-amount of free allowances from T_1 to the next period T_2, and borrows (i.e., goes short) the maximum possible 2.5%-amount of the free distributed EUAs from T_3 to the previous period T_2, and vice versa for the pessimistic market scenario.

Table 8.1 and Table 8.2 lists the expected values (i.e., revenues) z and their corresponding VaR values at the 95% and 99% confidence level, for the optimistic and pessimistic scenario, respectively, according to the given CO_2 emission scenarios d and the corresponding total trading budgets $\sum_{k=0}^{2} B_{k+1}^{d}$, provided in Table 7.10, and the various risk levels of the open position in CO_2 emission allowances and board-defined points in time, provided in Table 7.11.

Budget	τ_1	τ_2	τ_3	q_1,q_2	C_1^d	C_2^d	C_3^d	z	$VaR_{95\%}$	$VaR_{99\%}$
7.5 Mio. €	6	18	30	0.50	900,000	1,000,000	1,200,000	322,724	165,878	101,238
	6	18	30	0.75	900,000	1,000,000	1,200,000	269,149	132,716	76,489
	9	21	33	0.50	900,000	1,000,000	1,200,000	334,639	156,850	83,579
	9	21	33	0.75	900,000	1,000,000	1,200,000	307,871	182,900	131,397
10.0 Mio. €	6	18	30	0.50	1,000,000	1,200,000	1,400,000	550,573	371,959	298,348
	6	18	30	0.75	1,000,000	1,200,000	1,400,000	455,142	270,024	193,733
	9	21	33	0.50	1,000,000	1,200,000	1,400,000	605,740	353,672	249,789
	9	21	33	0.75	1,000,000	1,200,000	1,400,000	503,589	342,037	275,459
12.5 Mio. €	6	18	30	0.50	1,100,000	1,300,000	1,500,000	763,237	537,682	444,726
	6	18	30	0.75	1,100,000	1,300,000	1,500,000	635,230	394,677	295,540
	9	21	33	0.50	1,100,000	1,300,000	1,500,000	794,794	496,086	372,982
	9	21	33	0.75	1,100,000	1,300,000	1,500,000	730,794	526,340	442,080

Table 8.1. Optimistic scenario: Expected revenues and VaR values.

Budget	τ_1	τ_2	τ_3	q_1,q_2	C_1^d	C_2^d	C_3^d	z	$VaR_{95\%}$	$VaR_{99\%}$
7.5 Mio. €	6	18	30	0.50	900,000	1,000,000	1,200,000	255,698	170,962	136,041
	6	18	30	0.75	900,000	1,000,000	1,200,000	238,979	149,887	113,170
	9	21	33	0.50	900,000	1,000,000	1,200,000	267,189	196,385	167,205
	9	21	33	0.75	900,000	1,000,000	1,200,000	247,557	162,283	127,140
10.0 Mio. €	6	18	30	0.50	1,000,000	1,200,000	1,400,000	402,757	283,148	233,854
	6	18	30	0.75	1,000,000	1,200,000	1,400,000	395,803	292,649	250,137
	9	21	33	0.50	1,000,000	1,200,000	1,400,000	427,129	358,261	329,880
	9	21	33	0.75	1,000,000	1,200,000	1,400,000	396,216	276,580	227,276
12.5 Mio. €	6	18	30	0.50	1,100,000	1,300,000	1,500,000	601,153	454,555	394,138
	6	18	30	0.75	1,100,000	1,300,000	1,500,000	579,687	419,247	353,126
	9	21	33	0.50	1,100,000	1,300,000	1,500,000	662,089	562,459	521,400
	9	21	33	0.75	1,100,000	1,300,000	1,500,000	598,598	454,944	395,742

Table 8.2. Pessimistic scenario: Expected revenues and VaR values.

We see that, for both the optimistic and pessimistic market scenario, the revenues are higher, either, if all else equal, the lower $q_1,q_2 = 0.5$, or, if all else equal, the higher τ_1,τ_2,τ_3. Hence, the highest revenues for the portfolio manager results throughout all CO_2 emissions scenarios and trading budgets for $\tau_1 = 9$ $\tau_2 = 21$, $\tau_2 = 33$ and $q_1,q_2 = 0.5$. Accordingly, for all scenarios the corresponding VaR values are highest for $\tau_1 = 9$ $\tau_2 = 21$, $\tau_2 = 33$ and $q_1,q_2 = 0.5$. This result was to expect since, on the one hand, it reveals the situation where the portfolio manager faces the lowest board-defined amount of CO_2 emission allowances to be mandatorily traded from the market up to a board-defined in time, and on the other hand, that this board-defined point in time is the end of the third quarter and, thus near the end, of each CO_2 compliance period. Or, in other words, this combination of τ_1, τ_2, τ_3 and q_1, q_2 guarantees the portfolio manager the highest trading flexibility or highest possible risk position, respectively. Consequently, throughout all CO_2 emissions scenarios and trading budgets for $\tau_1 = 9$, $\tau_2 = 21$ and $\tau_2 = 33$, the lower q_1, q_2 are, the higher are the resulting revenues for the portfolio manager, and vice versa. Thus, from the board's point of view it does significantly matter if it limits the risk position of the portfolio manager to the end of the third quarter of each CO_2 compliance year or to the half of each CO_2 compliance year.

Due to the existence of higher volatilities in the optimistic market scenario, the underlying VaR values at the 95% and 99% confidence level are relatively higher than those for the pessimistic market scenario. For the same trading budget, the revenues are relatively higher for the optimistic market scenario than for the pessimistic market scenario. This is very likely due to the relatively higher volatilities in the optimistic scenario, implying a higher gain potential. That is, the portfolio manager can make use of the relatively higher difference between low and high prices.

Figures 8.17–8.18 illustratively shows, both for the optimistic and pessimistic market scenario, the distribution and the expected revenues of the EUA and CER trading strategies of the portfolio manager for a total trading budget of 10.0 Mio. EUR, $\tau_1 = 9$, $\tau_2 = 21$, $\tau_2 = 33$ and $q_1, q_2 = 0.5$.

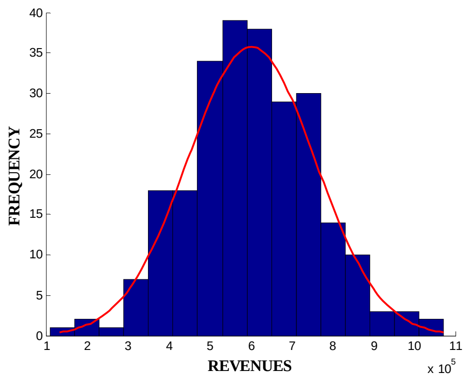

Figure 8.17. Optimistic scenario: Distribution and expected revenues of EUA and CER trading strategies, trading budget of 10.0 Mio. EUR, $\tau_1 = 9$, $\tau_2 = 21$, $\tau_2 = 33$, $q_1, q_2 = 0.5$.

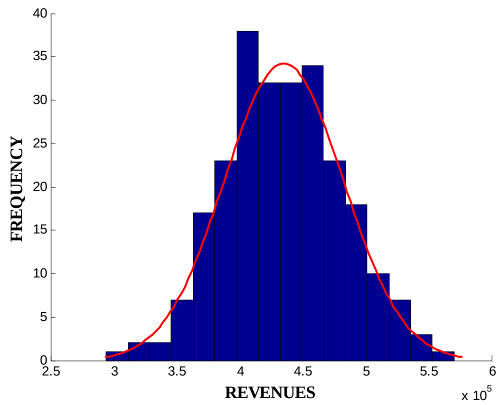

Figure 8.18. Pessimistic scenario: Distribution and expected revenues of EUA and CER trading strategies, trading budget of 10.0 Mio. EUR, $\tau_1 = 9$, $\tau_2 = 21$, $\tau_2 = 33$, $q_1, q_2 = 0.5$.

From both figures we can see that the distribution of revenues seem to incorporate a relatively low kurtosis with a more rounded peak and shorter, thinner tails. Moreover, neither any positive or negative skewness in the distribution can be detected. These attributes imply a bell-shaped distribution (red line). Hence, the distribution of the revenues can be considered as approximating a normal distribution, which justifies the application of the VaR measure. This feature has been tested by applying JB test in EViews for the existence of a normal distribution in the revenues. All p-values of the received JB test statistics are larger than 0.05 (and 0.01). Hence, for the optimistic market scenario as well as for the pessimistic scenario, the null hypothesis of normal distribution can clearly not be rejected. The corresponding EViews values can be found in Appendix B.3.

The revenues in the optimistic market scenario in Figure 8.17 range between 110,862 and 1,071,004 EUR for the whole trading period of three years. The corresponding values for the pessimistic market scenario in Figure 8.18 are 294,036 and 570,487 EUR. Consequently, depending on each budget-risk position combination of the portfolio manager, by implementing common futures buy-hold-sell strategies of EUA and CERs, he additionally generates revenues between 110,862 and 1,071,004 EUR in the optimistic market scenario and, between 294,036 and 570,487 EUR for the airline company in the pessimistic scenario, respectively, instead of only buying all the missing amount of CO_2 emission allowances in the spot. Or, in other words, the operative use of the portfolio manager was beneficial for the airline company.

CHAPTER 9

CONCLUSION AND OUTLOOK

In this thesis, we set up and solved a multi-period stochastic portfolio optimization model from an airline company's point of view, by considering all the existing EU ETS (EU Emission Trading Scheme) regulatory and board-defined trading and risk constraints. In order to hedge the natural physical short position in CO_2 emission allowances, we developed an optimal hedging strategy consisting of futures contracts.

After the comprehensive mathematical derivation of the whole system of equations consisting of the profit function and constraints, in order to model the whole space of feasible states, we run Monte-Carlo (MC) simulations of correlated geometric Brownian motions (GBMs) for traded EUA (EU Emission Allowance) and CER (Certified Emission Reduction) futures prices of different CO_2 delivery time periods (i.e., maturities). We modeled two market scenarios, an optimistic and a pessimistic market scenario, based on which the corresponding a forward-scenario trees were constructed. We thereby justified the use of the GBM as the appropriate price process in our model, by empirically showing that the returns are normally distributed and contains a unit root, implying their non-stationary. Based on the generated scenario-trees, we determined optimal buy-hold-sell decisions (i.e., futures trading strategy) and calculated the corresponding earnings. This procedure was conducted by backward induction, where according to the American option pricing methodology, starting from the last stage moving backward to the previous stage, valuation was conducted for each stage (i.e., value perspective). The Asian property "path dependence" thereby was already taken into account by the extension of the whole state space. Thereafter, given the valuation for each state, the uncertainty (i.e., distribution) of the revenues was determined by the MC simulation, which was conducted by forward induction (i.e., risk perspective). Based on the distribution of the revenues, the Value-at-Risk (VaR) measure for the 95% and 99% confidence level was then determined, in order to measure the risk exposure of the portfolio manager.

Concretely, in order to include the existence of various CO_2 emission allowance types, the existence of their futures prices and their stochasticity, we run n = 250 Monte-Carlo simulations for the optimistic and pessimistic market scenario, by considering all cross correlations (i.e., correlated GBM) and solved our linear multi-stage stochastic program based on the constructed forward-scenario tree, generated by simulated correlated price paths for EUA and CER futures. Therefore, our model algorithm was composed of a MC simulation of correlated GBM (EUA and CER futures) part and an optimization model part. We thereby used simulation (expected returns, volatilities, covariance matrix, initial futures prices) as well as optimization parameters (upper / lower trading limits, banking / borrowing limits, amount of free allowances, risk constraints, amount of stochastic CO_2 emissions, penalty fee, budget) as model input parameters. As model output, we received optimal futures trading strategies, distribution of revenues and their corresponding VaR. The normal distribution of the revenues has been empirically shown, and therefore, the justified application of the VaR values as suitable risk measure.

We solved our model with the CPLEX solver, which is available in MATLAB. For each EUA and CER futures price scenario, we found an optimal feasible solution, satisfying all the required constraints. Thus, our portfolio manager never ends paying penalties, and can therefore optimize his revenues from trading strategies. Due to the maximum difference in the price level between the first and third CO_2 compliance period, the portfolio manager, in the optimistic market scenario, banks free distributed EUAs in the first and borrows the corresponding amount in the third CO_2 compliance period, and vice versa for the pessimistic market scenario. The portfolio manager mainly uses EUA buy strategies to close his initial short position in CO_2 emission allowances, which significantly differ for the optimistic and pessimistic market scenario. However, to optimize his portfolio, the portfolio manager very actively buys and sells CERs. The higher the flexibility for the portfolio manager, that is, the closer the point in time to the end of each CO_2 compliance period, up to which he has to mandatorily close the board-defined percentage amount of his natural short position, and the lower this board-defined percentage amount, the higher the revenues he generates, and vice versa. As a result, since all scenarios and constellations implied positive revenues for the portfolio manager, the airline company benefited from the use of a portfolio manager implementing active futures trading strategies instead of applying simple spot buying strategies at the end of each CO_2 compliance period.

With this thesis, our contribution to the existing academic literature thereby was of various nature. Until now, the multi-period stochastic portfolio optimization technique has found a broad application for the energy sector (i.e., hydro power and gas value chain optimization) and for optimal SO_2 compliance issues in the US. As the first ever case, we specifically applied this technique to the airline sector, which is a brand new included sector within the European Union Emission Trading Scheme (EU ETS). Furthermore, more than mainly incorporating physical and technical ("engineering") features and focusing on short-term planning issues within the optimization model, especially we also addressed financial features and focused on mid-term planning issues. That is, by taking into account actually traded futures prices for CO_2 emission allowances for longer trading horizons (i.e., different CO_2 delivery periods) and the derivation of optimal trading strategies, based on futures rather than spot contracts, we particularly highlighted an airline company's need to plan and manage its cash flow streams from a medium term's perspective. In contribution to the existing academic literature, we thereby specifically referred to the two actually existing CO_2 emission allowances types, EUA (EU Emission Allowance) and CER (Certified Emission Reduction), and their traded futures prices for various CO_2 delivery time periods. Based on them, we run Monte-Carlo simulations, by considering all cross correlations between the EUA and CER futures prices, which is a further contributing feature to previous academic works, which mainly used one single, unspecified type of a CO_2 emission allowance for an undefined trading period. That is, unlike our separation of the total trading period to real-world oriented sub trading periods (i.e., CO_2 compliance periods), where specific exchange-traded futures of different maturities are available and various EU ETS regulatory, managerial and trading constraints have to be taken into account, the academic literature mainly focused on the assumption of unspecified CO_2 emission allowances and not detailed trading periods.

As a result, our model contributes both to the change in paradigm, by integrating the "financial" with the "physical" world, rather than considering them separately, and to the application of the multi-period stochastic portfolio optimization technique to a completely new area within the emissions sector, specifically within European Emission Trading Scheme (EU ETS).

This thesis may serve as stimulation for further research in this area, not only due its actuality and real-world orientation, but especially due to its openness and academic generalization and development potential. Firstly, the emissions trading horizon can be expanded to more CO_2 compliance years, depending on the board's decision and/or increasing future liquidity of the exchange-traded EUA and CER futures. In addition to that, other board-defined constraints such as upper and lower trading constraints could be stressed and varied for each trading period *t*. Furthermore, since implied volatilities of vanilla options are available for both EUAs and CERs and due to their sufficient and more and more increasing liquidity, the market data can be used for the calibration of the MC input parameters. Also GARCH models could be set up to model the volatility as an important model input parameter. A crucial assumption in the thesis was the amount of CO_2 emissions for CO_2 compliance, given as deterministic scenarios. These could also be modeled by a suitable CO_2 emission production function or on the basis of fundamental airline data, such as type and the corresponding capacity of owned airplanes, current and future flight plans to and from the specific EU locations, sold flight tickets of the airplanes, weight of the transported luggage etc. However, this procedure would require a much more comprehensive, fundamental analysis and detailed modeling of technical airplane parameters. Additionally, the time-series properties of the underlying price model could be changed such that stochastic drift and volatility parameters could be incorporated. For constructing a forward-scenario tree, EUA and CER futures price scenarios could also be generated through other scenario generation techniques such as ARMA, VAR, property matching methods, bootstrapping or Markov Chains.

As a consequence, the self-developed system of real-world oriented equations in this thesis, could be easily developed, adapted and extended to either other future sectors to be included in the EU ETS such as the shipping sector, or other sectors within the cap-and trade carbon market regimes such as the US RGGI. Based on the methodology derived in this thesis, the hedging procedure of physical assets could be further developed and implemented against other financial derivatives than futures such as options or swaps.

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APPENDIX A

Detailed formulation of the optimization model in Subsection 6.4.5

$$\underset{(x_{11,t},x_{12,t})_{t=1,\dots,12},(x_{12,t},x_{22,t})_{t=13,\dots,24},(x_{13,t},x_{23,t})_{t=25,\dots,36}}{\text{maximize}}$$

$$\begin{aligned}
z = \Bigg[& e^{-r(12-t)}\sum_{t=1}^{12}\pi^s\left(F_{11,t}^s\left(\mu_{11}-\sigma_{11}\frac{1}{2}\right)+\sigma_{11}\varepsilon_{11,t}^s\right)x_{11,t} \\
& + e^{-r(24-t)}\sum_{t=13}^{24}\pi^s\left(F_{12,t}^s\left(\mu_{12}-\sigma_{12}\frac{1}{2}\right)+\sigma_{12}\varepsilon_{12,t}^s\right)x_{12,t} \\
& + e^{-r(36-t)}\sum_{t=25}^{36}\pi^s\left(F_{13,t}^s\left(\mu_{13}-\sigma_{13}\frac{1}{2}\right)+\sigma_{13}\varepsilon_{13,t}^s\right)x_{13,t} \\
& + e^{-r(12-t)}\sum_{t=1}^{12_1}\pi^s\left(F_{21,t}^s\left(\mu_{21}-\sigma_{21}\frac{1}{2}\right)+\sigma_{21}\varepsilon_{21,t}^s\right)x_{21,t} \\
& + e^{-r(24-t)}\sum_{t=13}^{24}\pi^s\left(F_{22,t}^s\left(\mu_{22}-\sigma_{22}\frac{1}{2}\right)+\sigma_{22}\varepsilon_{22,t}^s\right)x_{22,t} \\
& + e^{-r(36-t)}\sum_{t=25}^{36}\pi^s\left(F_{23,t}^s\left(\mu_{23}-\sigma_{23}\frac{1}{2}\right)+\sigma_{23}\varepsilon_{23,t}^s\right)x_{23,t} \\
& - B_1 - B_2 - B_3 \\
& - g\left[\sum_{t=1}^{12_1}x_{11,t}+\sum_{t=1}^{12}x_{21,t}+(1+b)\overline{x}_{1,1}-C_1^d\right]^- \\
& - g\left[\sum_{t=13}^{24}x_{12,t}+\sum_{t=13}^{24}x_{22,t}+(1+b)\overline{x}_{1,1}-C_2^d\right]^- \\
& - g\left[\sum_{t=25}^{36}x_{13,t}+\sum_{t=25}^{36}x_{23,t}+(1+b)\overline{x}_{1,1}-C_3^d\right]^- \Bigg]
\end{aligned}$$

subject to

EU ETS regulatory limit for CERs:

$$\sum_{t=1}^{12} x_{11,t} \leq (1-m)\left(C_1^d - \overline{x}_{1,1}\right) \qquad (d=1,2,3),$$

$$\sum_{t=13}^{24} x_{12,t} \leq (1-m)\left(C_2^d - \overline{x}_{1,2}\right) \qquad (d=1,2,3),$$

$$\sum_{t=25}^{36} x_{13,t} \leq (1-m)\left(C_3^d - \overline{x}_{1,3}\right) \qquad (d=1,2,3),$$

$$\sum_{t=1}^{12} x_{21,t} \leq m\left(C_1^d - \overline{x}_{1,1}\right) \qquad (d=1,2,3),$$

$$\sum_{t=13}^{24} x_{22,t} \leq m\left(C_2^d - \overline{x}_{1,2}\right) \qquad (d=1,2,3),$$

$$\sum_{t=25}^{36} x_{23,t} \leq m\left(C_3^d - \overline{x}_{1,3}\right) \qquad (d=1,2,3),$$

Regulatory banking and borrowing constraint:

$$\sum_{t=1}^{12} x_{11,t} + \sum_{t=1}^{12} x_{21,t} + (1+b)\overline{x}_{1,1} = C_1^d \qquad (d=1,2,3),$$

$$\sum_{t=13}^{24} x_{12,t} + \sum_{t=13}^{24} x_{22,t} + (1+b)\overline{x}_{1,2} = C_1^d \qquad (d=1,2,3),$$

$$\sum_{t=25}^{36} x_{13,t} + \sum_{t=25}^{36} x_{23,t} + (1+b)\overline{x}_{1,3} = C_3^d \qquad (d=1,2,3),$$

$$\left.\begin{array}{l} \sum_{t=1}^{12} x_{11,t} + \sum_{t=1}^{12} x_{21,t} + (1+b)\overline{x}_{1,1} \\ + \sum_{t=13}^{24} x_{12,t} + \sum_{t=13}^{24} x_{22,t} + (1+b)\overline{x}_{1,2} \\ + \sum_{t=25}^{36} x_{13,t} + \sum_{t=25}^{36} x_{23,t} + (1+b)\overline{x}_{1,3} \end{array}\right\} = C_1^d + C_1^d + C_1^d$$

$$(d=1,2,3),$$

$$b\overline{x}_{1,1} + b\overline{x}_{1,2} + b\overline{x}_{1,3} = 0,$$

Upper trading limits:

$$x_{11,t} \leq u_{11,t}\left[\left(1-m\right)\left(C_1^d - \overline{x}_{1,1}\right)\right], \qquad (t=1,\ldots,12;\ d=1,2,3),$$

$$x_{12,t} \leq u_{12,t}\left[\left(1-m\right)\left(C_1^d - \overline{x}_{1,1}\right)\right], \qquad (t=13,\ldots,24;\ d=1,2,3),$$

$$x_{13,t} \leq u_{13,t}\left[\left(1-m\right)\left(C_1^d - \overline{x}_{1,1}\right)\right], \qquad (t=25,\ldots,36;\ d=1,2,3),$$

$$x_{21,t} \leq u_{21,t}\left[m\left(C_1^d - \overline{x}_{1,1}\right)\right], \qquad (t = 1,...,12; d = 1,2,3),$$

$$x_{22,t} \leq u_{22,t}\left[m\left(C_1^d - \overline{x}_{1,1}\right)\right], \qquad (t = 13,...,24; d = 1,2,3),$$

$$x_{23,t} \leq u_{23,t}\left[m\left(C_1^d - \overline{x}_{1,1}\right)\right], \qquad (t = 25,...,36; d = 1,2,3),$$

Lower trading limits:

$$-x_{11,t} \geq v_{11,t}\left[(1-m)\left(C_1^d - \overline{x}_{1,1}\right)\right] \qquad (t = 1,...,12; d = 1,2,3),$$

$$-x_{12,t} \geq v_{12,t}\left[(1-m)\left(C_2^d - \overline{x}_{1,2}\right)\right] \qquad (t = 13,...,24; d = 1,2,3),$$

$$-x_{13,t} \geq v_{13,t}\left[(1-m)\left(C_3^d - \overline{x}_{1,3}\right)\right] \qquad (t = 25,...,36; d = 1,2,3),$$

$$-x_{21,t} \geq v_{21,t}\left[m\left(C_1^d - \overline{x}_{1,1}\right)\right] \qquad (t = 1,...,12; d = 1,2,3),$$

$$-x_{22,t} \geq v_{22,t}\left[m\left(C_2^d - \overline{x}_{1,2}\right)\right] \qquad (t = 13,...,24; d = 1,2,3),$$

$$-x_{23,t} \geq v_{23,t}\left[m\left(C_3^d - \overline{x}_{1,3}\right)\right] \qquad (t = 25,...,36; d = 1,2,3),$$

Risk constraints:

$$\sum_{t=1}^{\tau_1} x_{11,t} \leq q_{11,\tau_1}\left[(1-m)\left(C_1^d - \overline{x}_{1,1}\right)\right], \qquad (d = 1,2,3),$$

$$\sum_{t=13}^{\tau_2} x_{12,t} \leq q_{12,\tau_1}\left[(1-m)\left(C_2^d - \overline{x}_{1,1}\right)\right], \qquad (d = 1,2,3),$$

$$\sum_{t=25}^{\tau_3} x_{13,t} \leq q_{13,\tau_1}\left[(1-m)\left(C_3^d - \overline{x}_{1,1}\right)\right], \qquad (d = 1,2,3),$$

$$\sum_{t=1}^{\tau_1} x_{21,t} \leq q_{21,\tau_1}\left[(1-m)\left(C_1^d - \overline{x}_{1,1}\right)\right], \qquad (d = 1,2,3),$$

$$\sum_{t=13}^{\tau_2} x_{22,t} \leq q_{22,\tau_1}\left[(1-m)\left(C_2^d - \overline{x}_{1,1}\right)\right], \qquad (d = 1,2,3),$$

$$\sum_{t=25}^{\tau_3} x_{23,t} \leq q_{23,\tau_1}\left[(1-m)\left(C_3^d - \overline{x}_{1,1}\right)\right], \qquad (d = 1,2,3),$$

APPENDIX B

Statistical Tests and EViews Outputs

B.1 Jarque-Bera Test for Normality of Returns of EUA and CER Futures Prices

H_0: Normal distribution, skewness and excess kurtosis (i.e., kurtosis minus 3) are jointly zero

H_1: No normal distribution

Returns EUA Dec'11 futures

Sample: 1 686

	R_EUA_DEC11
Mean	-0.000026
Median	-0.000694
Maximum	0.046762
Minimum	-0.046762
Std. Dev.	0.016500
Skewness	0.099686
Kurtosis	3.081921
Jarque-Bera	1.327997
Probability	0.514789
Sum	-0.018248
Sum Sq. Dev.	0.186490
Observations	686

Returns CER Dec'11 futures

Sample: 1 686

	R_CER_DEC11
Mean	-0.000146
Median	0.000000
Maximum	0.045876
Minimum	-0.045294
Std. Dev.	0.016910
Skewness	0.000740
Kurtosis	3.060593
Jarque-Bera	0.105312
Probability	0.948706
Sum	-0.100355
Sum Sq. Dev.	0.196448
Observations	686

Returns CER Dec'14 futures

Sample: 1 466

	R_EUA_DEC14
Mean	-0.001224
Median	-0.001259
Maximum	0.044452
Minimum	-0.044901
Std. Dev.	0.019603
Skewness	0.045591
Kurtosis	2.536536
Jarque-Bera	3.960247
Probability	0.138052
Sum	-0.521635
Sum Sq. Dev.	0.163313
Observations	466

Returns CER Dec'14 futures

Sample: 1 466

	R_CER_DEC14
Mean	-0.001470
Median	-0.000773
Maximum	0.045810
Minimum	-0.045359
Std. Dev.	0.020325
Skewness	0.104044
Kurtosis	2.616543
Jarque-Bera	3.013718
Probability	0.221605
Sum	-0.558638
Sum Sq. Dev.	0.156564
Observations	466

B.2 ADF Unit Root Tests

H_0: Unit root (i.e. non-stationary), prices follow a random walk plus drift model

H_1: No unit root (stationary), prices converge to a long-term mean unequal to zero

EUA Dec'13 futures

Null Hypothesis: EUA_DEC13 has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=17)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.954451	0.7703
Test critical values:	1% level	-3.444219	
	5% level	-2.867549	
	10% level	-2.570034	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(EUA_DEC13)
Method: Least Squares
Date: 08/14/13 Time: 18:22
Sample (adjusted): 3 466
Included observations: 464 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.

EUA_DEC13(-1)	-0.002798	0.002931	-0.954451	0.3404
D(EUA_DEC13(-1))	0.199849	0.045721	4.371040	0.0000
C	0.015514	0.033545	0.462486	0.6440

R-squared	0.041232	Mean dependent var	-0.017845
Adjusted R-squared	0.037072	S.D. dependent var	0.250537
S.E. of regression	0.245849	Akaike info criterion	0.038244
Sum squared resid	27.86357	Schwarz criterion	0.065010
Log likelihood	-5.872514	Hannan-Quinn criter.	0.048780
F-statistic	9.912682	Durbin-Watson stat	1.992059
Prob(F-statistic)	0.000061		

EUA Dec'14 futures

Null Hypothesis: EUA_DEC14 has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=17)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.894503	0.7897
Test critical values:	1% level	-3.444219	
	5% level	-2.867549	
	10% level	-2.570034	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(EUA_DEC14)
Method: Least Squares
Date: 08/14/13 Time: 18:23
Sample (adjusted): 3 466
Included observations: 464 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
EUA_DEC14(-1)	-0.002679	0.002995	-0.894503	0.3715
D(EUA_DEC14(-1))	0.166127	0.046022	3.609723	0.0003
C	0.014494	0.036644	0.395540	0.6926

R-squared	0.028791	Mean dependent var	-0.019224
Adjusted R-squared	0.024577	S.D. dependent var	0.274139
S.E. of regression	0.270749	Akaike info criterion	0.231199
Sum squared resid	33.79372	Schwarz criterion	0.257965
Log likelihood	-50.63809	Hannan-Quinn criter.	0.241735
F-statistic	6.833049	Durbin-Watson stat	2.004736
Prob(F-statistic)	0.001190		

EUA Dec'15 futures

Null Hypothesis: EUA_DEC15 has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=17)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-0.879275	0.7945
Test critical values: 1% level	-3.444219	
5% level	-2.867549	
10% level	-2.570034	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(EUA_DEC15)
Method: Least Squares
Date: 08/14/13 Time: 18:25
Sample (adjusted): 3 466
Included observations: 464 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
EUA_DEC15(-1)	-0.002841	0.003232	-0.879275	0.3797
D(EUA_DEC15(-1))	0.113788	0.046398	2.452427	0.0146
C	0.016597	0.041900	0.396106	0.6922

R-squared	0.014246	Mean dependent var	-0.020129
Adjusted R-squared	0.009970	S.D. dependent var	0.307817
S.E. of regression	0.306279	Akaike info criterion	0.477805
Sum squared resid	43.24501	Schwarz criterion	0.504572
Log likelihood	-107.8509	Hannan-Quinn criter.	0.488342
F-statistic	3.331204	Durbin-Watson stat	1.998596
Prob(F-statistic)	0.036613		

CER Dec'13 futures

Null Hypothesis: CER_DEC13 has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=17)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-0.220890	0.9330
Test critical values: 1% level	-3.444219	
5% level	-2.867549	
10% level	-2.570034	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(CER_DEC13)
Method: Least Squares
Date: 08/14/13 Time: 18:05
Sample (adjusted): 3 466
Included observations: 464 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CER_DEC13(-1)	-0.000458	0.002073	-0.220890	0.8253
D(CER_DEC13(-1))	0.188949	0.045802	4.125295	0.0000
C	-0.014778	0.015864	-0.931537	0.3521

R-squared	0.035602	Mean dependent var	-0.021940
Adjusted R-squared	0.031418	S.D. dependent var	0.174851
S.E. of regression	0.172082	Akaike info criterion	-0.675242
Sum squared resid	13.65130	Schwarz criterion	-0.648475
Log likelihood	159.6561	Hannan-Quinn criter.	-0.664705
F-statistic	8.509199	Durbin-Watson stat	1.989933
Prob(F-statistic)	0.000235		

CER Dec'14 futures

Null Hypothesis: CER_DEC14 has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=17)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.036950	0.9604
Test critical values:	1% level	-3.444189	
	5% level	-2.867536	
	10% level	-2.570027	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(CER_DEC14)
Method: Least Squares
Date: 08/14/13 Time: 18:26
Sample (adjusted): 2 466
Included observations: 465 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CER_DEC14(-1)	8.04E-05	0.002176	0.036950	0.9705
C	-0.023940	0.018224	-1.313681	0.1896

R-squared	0.000003	Mean dependent var	-0.023355
Adjusted R-squared	-0.002157	S.D. dependent var	0.194290
S.E. of regression	0.194499	Akaike info criterion	-0.432483
Sum squared resid	17.51531	Schwarz criterion	-0.414667
Log likelihood	102.5522	Hannan-Quinn criter.	-0.425470
F-statistic	0.001365	Durbin-Watson stat	1.847737
Prob(F-statistic)	0.970540		

CER Dec'15 futures

Null Hypothesis: CER_DEC15 has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=17)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.022281	0.9592
Test critical values:	1% level	-3.444189	
	5% level	-2.867536	

10% level	-2.570027

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(CER_DEC15)
Method: Least Squares
Date: 08/14/13 Time: 18:18
Sample (adjusted): 2 466
Included observations: 465 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CER_DEC15(-1)	5.24E-05	0.002352	0.022281	0.9822
C	-0.024179	0.020224	-1.195544	0.2325

R-squared	0.000001	Mean dependent var	-0.023785
Adjusted R-squared	-0.002159	S.D. dependent var	0.211934
S.E. of regression	0.212162	Akaike info criterion	-0.258640
Sum squared resid	20.84092	Schwarz criterion	-0.240825
Log likelihood	62.13380	Hannan-Quinn criter.	-0.251628
F-statistic	0.000496	Durbin-Watson stat	1.989906
Prob(F-statistic)	0.982234		

B.3 Jarque-Bera Test for Testing Normal Distribution of Revenues

Distribution of Revenues for the Optimistic Market Scenario

Sample: 1 250

	REVENUES
Mean	602019.8
Median	601228.2
Maximum	1071004.
Minimum	110862.3
Std. Dev.	156494.9
Skewness	0.095077
Kurtosis	3.194482
Jarque-Bera	0.767562
Probability	0.681280
Sum	1.50E+08
Sum Sq. Dev.	6.07E+12
Observations	250

Distribution of Revenues for the Pessimistic Market Scenario

Sample: 1 250

	REVENUES
Mean	435152.7
Median	432654.5
Maximum	570487.2
Minimum	294036.0
Std. Dev.	47212.71
Skewness	0.093690
Kurtosis	2.914110
Jarque-Bera	0.442584
Probability	0.801483
Sum	1.09E+08
Sum Sq. Dev.	5.55E+11
Observations	250

VITA

PERSONAL INFORMATION

Surname, Name	**:**	Kalaycı, Erkan
Nationality	**:**	Swiss, Turkish
Date and Place of Birth	**:**	1980, Basel, Switzerland
E-mail	**:**	erkan.klyc@gmail.com

EDUCATION

Ph.D.	Department of Financial Mathematics, September 2013 Institute of Applied Mathematics Middle East Technical University, Ankara
FRM	Financial Risk Manager, October 2008 Global Associates of Risk Professionals (GARP) Zurich, Switzerland
M.Sc.	Business and Economics, February 2005 Focus: Energy Finance and Modeling University Basel, Switzerland
Lic.rer.pol ("B.Sc.")	Business and Economics, February 2004 University Basel, Switzerland

WORK EXPERIENCE

December 2011–Present	Managing Partner Green Consult and Finance Ltd., Ankara and Basel
March 2011–December 2011	Senior Consultant AF-Mercados EMI Europe, Ankara and Madrid
June 2009–December 2010	Carbon Business Coordinator Axpo Group, Zurich
April 2007–May 2009	Energy Risk Manager Axpo Group, Baden
April 2005–March 2007	Quantitative and Research Analyst Axpo Group, Baden

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