

DETERMINATION OF STRESS CONCENTRATION FACTOR IN STONE
COLUMNS BY NUMERICAL MODELLING

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
CIVIL ENGINEERING

AUGUST 2013

Approval of the thesis:

**DETERMINATION OF STRESS CONCENTRATION FACTOR IN STONE
COLUMNS BY NUMERICAL MODELLING**

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ABSTRACT

DETERMINATION OF STRESS CONCENTRATION FACTOR IN STONE COLUMNS BY NUMERICAL MODELLING

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August 2013, 81 pages

The behaviour of stone columns in soft cohesive soil is investigated by finite element analyses. Conventional design methods and settlement calculations of improved ground require the knowledge of stress concentration factor which is in stone column design practice either determined by field tests or estimated from recommendations given in literature. The former is not economical for small to medium scale projects. This study focuses on the determination of stress concentration factor in stone columns by numerical modelling. Numerical analyses are carried out by using Plaxis 2D software. The stress concentration factor is defined to represent the load sharing between the column and the surrounding soil. A parametric study is carried out to define the change of stress concentration factor with modulus of elasticity of clay, the column length and applied foundation pressure. The study includes assessment of settlement reduction ratio with the same parameters. The rigid foundation analyses show that the stress concentration factor changes between 2.5 and 5.0. The ratio decreases by the increasing rigid foundation pressure and linearly decreases with increasing modulus of elasticity of soil. The floating columns give values close to each other while end bearing columns give higher stress concentration ratios. The flexible foundation analyses are carried out to compare the stress concentration factors with those of the rigid foundations. The ratio is found to change between 1.8 and 3.0. The behaviour of change of the ratio with modulus of elasticity of soil in floating and end bearing columns is similar to the rigid foundations. The stress concentration factors are almost constant at different flexible foundation pressures. The stress concentration factor in flexible foundation analyses is determined to be approximately 30% smaller than in rigid foundation analyses.

Keywords: Stone columns, stress concentration factor, finite element analysis, settlement reduction ratio, numerical modelling, rigid foundations, flexible foundations.

ÖZ

NÜMERİK MODELLEME İLE TAŞ KOLONLARDA GERİLME DAĞILIM FAKTÖRÜNÜN BELİRLENMESİ

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Tez Yöneticisi: Prof. Dr. A. Orhan Erol

Ağustos 2013, 81 sayfa

Sonlu elemanlar analizleri ile taş kolonların yumuşak kohezyonlu zeminlerdeki davranışı incelenmiştir. Gerilme dağılım faktörü kolon ile çevre zemin arasındaki yük paylaşımını ifade etmek için tanımlanır. Konvansiyonel tasarım metotları ve iyileşme sonrası oturma hesapları gerilme dağılım oranı bilgisini gerektirir. Taş kolon tasarım uygulamalarında, gerilme dağılım faktörü ya saha deneyleriyle belirlenir ya da literatürde verilen önerilerden tahmin edilir. Bunlardan birincisi, küçük ve orta ölçekli projeler için ekonomik değildir. Bu çalışma nümerik modelleme ile taş kolonlardaki gerilme dağılım faktörünü belirlemek üzerinde yoğunlaşmıştır. Nümerik modellemeler, Plaxis iki boyutlu yazılım programı ile gerçekleştirilmiştir. Gerilme dağılım faktörünün kilin elastisite modülü, taş kolon boyu ve temele uygulanan basınç ile değişimini belirlemek için parametrik bir çalışma yapılmıştır. Çalışma, aynı parametrelere bağlı olarak oturma azaltma oranının belirlenmesini de kapsamaktadır. Rijit temel analizleri gerilme dağılım faktörünün 2.5 ile 5.0 arasında değiştiğini göstermiştir. Oran artan rijit temel basıncı ile azalmakta ve zeminin artan elastisite modülü ile lineer olarak azalmaktadır. Yüzen kolonlar birbirleri ile yakın değerler verirken, uç direnci kolonları daha yüksek gerilme dağılım oranları vermektedir. Gerilme dağılım faktörlerini rijit temellerdekiler ile karşılaştırmak için bükülebilir temel analizleri yapılmıştır. Oranın 1.8 ile 3.0 arasında değiştiği görülmüştür. Yüzen ve uç direnci kolonlarında zeminin değişen elastisite modülüne göre orandaki değişim davranışı rijit temellerdekine benzerdir. Farklı bükülebilir temel basınçları altındaki gerilme dağılım faktörleri neredeyse sabittir. Bükülebilir temel analizlerindeki gerilme dağılım faktörlerinin rijit temellerdekine göre yaklaşık olarak %30 az olduğu tespit edilmiştir.

Anahtar kelimeler: Taş kolonlar, gerilme dağılım faktörü, sonlu elemanlar analizleri, oturma azaltma oranı, nümerik modelleme, rijit temeller, bükülebilir temeller.

To The Chapullers

ACKNOWLEDGEMENTS

I would like to express my sincere thanks to my advisor Prof. Dr. Orhan EROL without whom I could not complete my thesis successfully. I am grateful to him for sharing his greatest academic experience with me, guidance, motivations throughout the study and never letting me give up when I was ready to do.

My next sincere gratitude is for Aslı ÖZKESKİN ÇEVİK and Muzaffer ÇEVİK the owner of “Sonar Drilling and Geological Research Center” and every member of my company. I am deeply thankful to Aslı ÖZKESKİN ÇEVİK for motivating me to be a geotechnical engineer when I was a bachelor student, shaping my career just after my graduation, providing me consultancy when I need any help, motivating me not just for this study, but for every obstacle I have in my life. I could not achieve to “work and study” together without tolerances of Muzaffer ÇEVİK and I am really thankful to him.

I also would like to present my endless thanks to Assoc. Prof. Dr. Yalın ARICI whom helped me anytime when I needed. He is the unique professor I have ever had that I can share things other than the courses and he also never complained about this. His genius always helped me in my career and academic life.

My next appreciation goes to Assistant Zeynep ÇEKİNMEZ. She never gave up motivating, guiding and helping me from the beginning to the end of this study.

My dear colleague M. Erdem İSPİR is one of the best motivators of me about the completion of my thesis, he always supported me, shared his knowledge and listened to my complaining about the difficulty of doing master’s degree while hardly working in the field. I am also thankful to my ex colleague Osman YILMAZ for his motivations.

I would like to thank to my “sidekick” Melike YAKA for her endless moral support, friendship, hospitality during the sleepless nights while studying and everything I cannot express now. I also feel very lucky to have the friendship of my classmates Melek YILMAZTÜRK, Ali Utku TOPAK, Buse TOPÇUOĞLU KURT, Açelya Ecem YILDIZ, Alper TURAN, Burhan ALAM, Yaprak SERVİ, Mebrure İtir ÖZKOÇAK, Adem YEŞİLYURT and Emir SAYIT. They all have special place in my heart for the times we lived in METU together.

My best friend, İlkem TURHAN has always made me feel like she is still my desk mate as she was for twelve years. I am grateful to her for supports, motivations and best friendship not just for this study but for all of my life. Doubtlessly, I could not be successful in my academic and non-academic life without the encourages, fellowship and loves of my dear friends, Gökay Çağlar MEMİŞOĞLU, Ayşegül GÜLTEPE, Emre YÜKSEL, Elif YÜKSEL and Ayşe Merve YÜNEY in nearby or kilometres away.

I wish to acknowledge every member of ÇOSEV since they are very valuable, warm-hearted, helpful and merciful people and I am really thankful for the value they bring to my life.

My special thanks go to Erdi YILMAZ whom is very special for me from the beginning he came into my life. He always trusted, encouraged, helped me and tried to make everything in my life better and easier like my thesis. I am also grateful to his elder-sisters, Şenay YILMAZ HAYIRLIOĞLU and Altınay YILMAZ ÖZDEMİR for their listening to my complains and moral support.

My last but not least thanks are for my dear family members, my father Adnan, my mother Ezhar, my elder sister Neslihan YILDIZ for their patience, love, encouragement and trust. They always believed in me that I would be successful in all areas of my life; they even believed that I would complete my thesis before I started to it.

I cannot forget to present my deepest gratitude to Mustafa Kemal ATATÜRK. If he were not alive, we would not be alive, either.

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CHAPTER 1

INTRODUCTION

1.1 Statement of the Problem

As the need for land has increased worldwide, the land has become more valuable and the need to construct on unsuitable soils has become a major issue especially in urban areas. Over the last sixty years stone column technique has become popular among other ground improvement techniques and it has been applied successfully around the world.

Stone columns have four major improvement effects on soils: increasing the bearing capacity and slope stability of the soils, lowering the total and differential settlements, working as a drainage system to accelerate the consolidation settlement of cohesive soils and reducing the liquefaction potential of loose saturated cohesionless soils. (Barksdale and Bachus, 1983). Thus, stone columns are generally applied on soft clays and silts, silty clays and liquefiable loose sands.

Stone columns have a great range of applicable structures. They are applied on flexible foundations such as embankments, highway, railroad and bridge approach fills; also rigid foundations such as tanks, bridge abutments, parking garage and other building foundations.

Generally, the area replacement ratio of the stone columns varies between 15 to 35 percent (Barksdale and Bachus, 1983). They also give the typical stone column design load as 20-50 tons. The stone column decreases the overall compressibility and increases the shear strength of the native soil by forming a composite material. The lateral stresses within the weak soil confine the stone column and this confinement limit the stiffness of the column. A vertical load application results in concentration of stress on the stone column and stress reduction in the surrounding weak soil. The most important reason for this stress concentration is the column material being stiffer than the surrounding soil.

There have been many studies on the behaviour of the stone column-improved soils and some design aspects have been clarified with unique assumptions. It is highly important to choose adequate design parameters, one of which is the stress concentration factor. To estimate the improved soil settlement or defining the stone column design load and the group pattern (diameter and spacing of the stone column in a group), the stress concentration factor is needed to be known by either being specified for the project or being chosen from the literature. Some full-scale load tests, laboratory model tests and finite element analyses were carried out to determine the stress concentration on the stone columns. However, it is necessary to support those finding with further research to guide for later design parameter selections.

1.2 Purpose and Scope of the Study

In the scope of this study, two dimensional numerical analysis of a stone column-supported rigid foundation on soft clay layer is carried out to define the stress concentration on the columns and the settlement reduction after the improvement of the soil. Moreover, flexible foundations supported by stone columns are also analysed in the same soil conditions to compare the stress distribution between the stone columns and the surrounding soil.

In order to define the improvement after stone column construction in weak soil, the unimproved rigid foundation loading is needed to be analysed at first. The settlements of the models with and without improvement are compared and then the settlement reduction ratio is defined by the ratio of those settlement values. Moreover, the stress distribution at the top of the stone columns is measured to determine the stress concentration factor.

Parametric study is done by changing three key parameters of the stone column improved soil: modulus of elasticity of the soil, stone column length and foundation pressure. Three modulus of elasticity values of soft silty clay are defined while the modulus of elasticity of the stone column is kept constant. Three floating columns and one end bearing column are analysed for each set of rigid foundation analyses. In flexible foundation analyses, one floating and one end bearing columns are modelled. The maximum stress on the foundation is determined as 120 kPa.

In Chapter II, a literature review on stone columns focused on stress concentration is presented. Modelling and details of the finite element analyses are described in Chapter III while the results of the analyses of parametric studies are given in Chapter IV. Finally, the summary and results of the study is presented in Chapter V.

CHAPTER 2

LITERATURE REVIEW ON STRESS CONCENTRATION IN STONE COLUMNS

2.1 General

Stone columns are one of the most widely used ground improvement techniques in soft clays, silts and loose silty sands for the last 60 years in Europe and 40 years in the U.S.A. The technique is also used as a remedy to liquefaction in loose liquefiable cohesionless soils. Stone columns are very effective to increase the bearing capacity and slope stability of embankments and slopes, decrease the total and differential settlements, increase the time rate of consolidation settlement by working as a drainage system in cohesive soils and reducing the liquefaction potential of liquefiable cohesionless soils (Barksdale and Bachus, 1983).

Alonso and Jimenez (2012) clearly explained the mechanism of these improvements as: bearing capacity due to shear strength increase, settlement reduction due to stiffness improvement, acceleration of the consolidation of cohesive soils and reduction of the susceptibility to liquefaction of cohesionless soils due to increase in the soil mass permeability.

After vibro-compaction method was invented, stone column construction has been developed similarly by vibro-replacement or vibro-displacement techniques. The techniques are based on the machine used to vibrate the weak soil and they have been clearly documented so far in many studies, so they will not be presented in this study.

Stone columns are type of rigid inclusions driven into the weak soil layer and they also change the properties of the native soil. The improvement degree in the bearing capacity of surrounding cohesive soil by those granular inclusions depends on the confinement of the soil around the column and the diameter and the compaction degree of the column (Greenwood, 1970). As the column is stiffer than the surrounding weak soil, when the load is applied on a footing, the load concentrates on the column and this concentration causes lateral expansion to the surrounding soil. Passive pressures resist the lateral expansion of the columns. This situation results in the column behaving as if it were in a triaxial chamber.

Hughes and Withers (1974) reported the ultimate bearing capacity of stone columns with the triaxial stress state approach. They explained the situation as “the column expands, the radial resistance of the soil reaches a limiting value at which indefinite expansion occurs.” Hence, the stone columns in soft soils behave like they were in a triaxial chamber where a limited cell pressure occurs. They gave the ultimate bearing capacity of the stone columns as in the following equation and they concluded that the critical state of stress is reached by the top gravel/sand area with this relation.

$$q_{ult} = \frac{(1+\sin \varphi')}{(1-\sin \varphi')} (\sigma_{r0} - u + 4c) \quad (2.1)$$

where q_{ult} is the maximum vertical stress that the column can carry, φ' is the angle of shearing resistance of the column material, σ_{r0} is the initial total radial stress of the untreated soil, c is the undrained shear strength of the soil and u is the pore water pressure. Hughes and Withers (1974) also reported that no increase in the carrying capacity of the column is gained by increase in the length of the column beyond depth to diameter ratio greater than 6.3.

The unit cell concept is introduced by Barksdale and Bachus (1983) for the purposes of stability and settlement analyses. The tributary area of soil surrounding each column is related to the unit cell concept. For different patterns such as triangular and square the unit cell idealization to a circle is also changed as the diameter of the circle (D_e) representing the square patterns equals to $1.13s$ while it is $1.05s$ for triangular patterns where s is the spacing between the centers of two columns. This resulting equivalent cylinder including one column and enclosing tributary soil having diameter D_e is defined as *unit cell*.

The area of the column is divided by the tributary area of the soil to determine the area replacement ratio, a_s . The soil improvement is highly dependent on the amount of soil replaced in terms of settlement and bearing capacity improvement. Most of the applications of stone columns give an average value for area replacement ratio (a_s) between 15 – 35 percent. The definition of a_s is given as the following:

$$a_s = \frac{A_s}{A} \quad (2.2)$$

where A_s is the stone column area, A is the total unit cell area. The ratio could also be defined in terms of the equivalent unit cell diameter as:

$$a_s = 0.907 \left(\frac{D}{S}\right)^2 \quad (\text{triangular pattern}) \quad (2.3)$$

$$a_s = 0.783 \left(\frac{D}{S}\right)^2 \quad (\text{square pattern}) \quad (2.4)$$

The unit cell idealization could be extended for uniformly loaded infinitely large group of stone columns (Barksdale and Bachus, 1983). In such a group of columns, each interior column can be considered as a unit cell. The assumptions of this idealization consist of no lateral deformation on the boundaries and no shear stresses outside the boundaries of the unit cell because of symmetry of geometry and load. Thus, the physical modelling of the unit cell includes a cylindrical-shaped container with a frictionless, rigid exterior wall symmetrically located around the stone column (Figure 2.1).

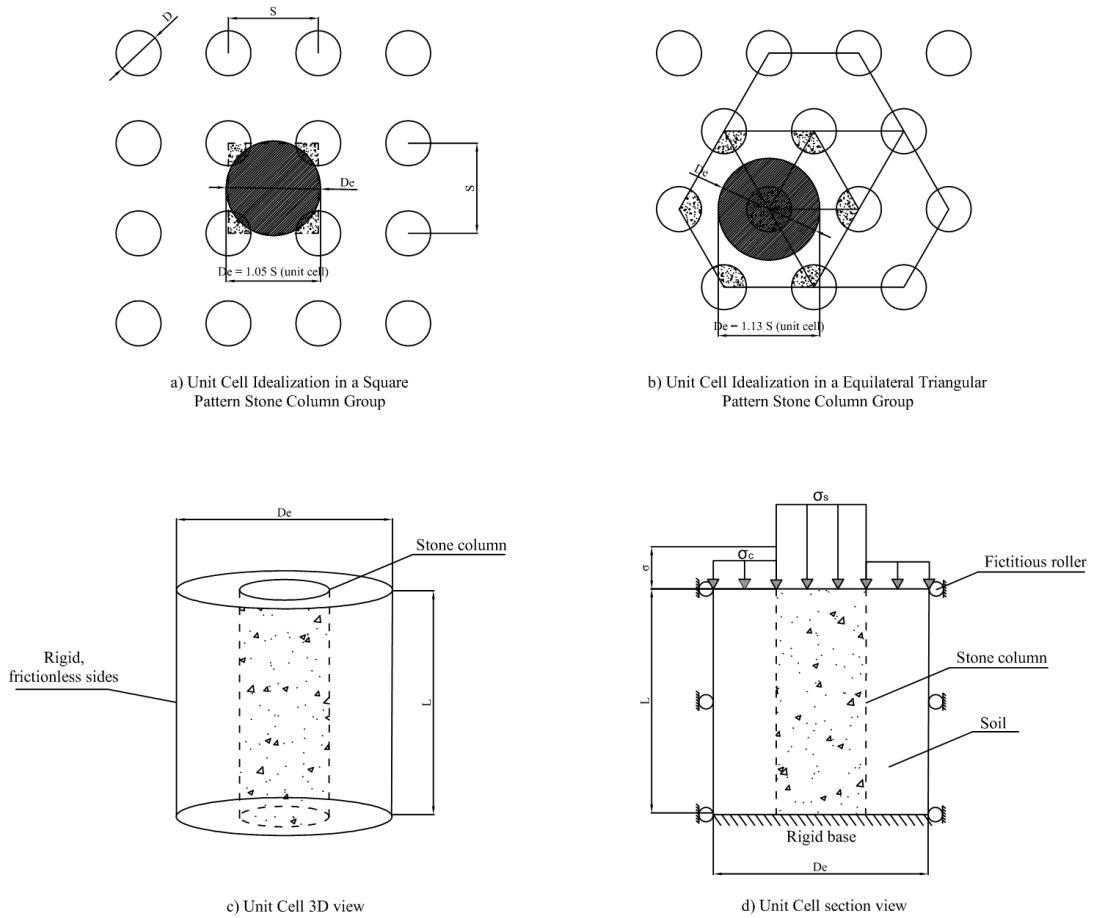


Figure 2.1 Unit Cell Idealization of Stone Columns (Barksdale and Bachus, 1983)

Aboshi et al. (1979) stated that the column diameter generally changes between 60 cm to 80 cm while it may be up to 200 cm. They gave the composite foundation characteristics supported by the stone columns by introducing the stress concentration. When a load is applied to the composite foundation, the stress concentrates on the column since the deformation characteristics of surrounding soil and the stiffer column material are different. Then, the stress concentration factor for stone and sand columns in a cohesive soil matrix is defined as follows:

$$n = \frac{\sigma_s}{\sigma_c} \quad (2.5)$$

$$\sigma = \sigma_s * a_s + \sigma_c * (1 - a_s) \quad (2.6)$$

$$\sigma_c = \frac{\sigma}{1+(n-1)a_s} = \mu_c \sigma \quad (2.7)$$

$$\sigma_s = \frac{n\sigma}{1+(n-1)a_s} = \mu_s \sigma \quad (2.8)$$

where n is the stress concentration factor, σ is the total applied stress, σ_s is the stress in the stone/sand column, σ_c is the stress in the cohesive soil, μ_c and μ_s are the ratio of stresses in clay and sand/stone to the average stress, respectively.

As Barksdale and Bachus (1983) stated that the above equations giving the stresses in terms of the applied stress in the column and the surrounding soil are essential in both settlement and stability analyses. In the derivation of those equations the following assumptions were made:

1. The extended unit cell concept is valid.
2. Statistics is satisfied.
3. The stress concentration value is known or it can be estimated.

Although in the cases where the unit cell concept is not valid, the equations still give satisfactory results for settlement calculations since the vertical stress does not change a lot with horizontal distance. However, as the column number in the group increases, the accuracy of the results also increases.

The ultimate bearing capacity of the column is dependent on the compressibility of the surrounding cohesive soil. Thus, it is convenient to express the ultimate bearing capacity of a single isolated stone column or a stone column located within a group in terms of the shear strength of the surrounding cohesive soil as given in the following equation (Barksdale and Bachus, 1983):

$$\tilde{q}_{ult} = c\tilde{N}_c \quad (2.9)$$

where \tilde{q}_{ult} = ultimate stress which the stone column can carry
 c = undrained shear strength of the surrounding cohesive soil
 \tilde{N}_c = bearing capacity factor for the stone column ($18 \lesssim \tilde{N}_c \lesssim 22$)

The ultimate capacity of the surrounding cohesive soil should also be considered while determining the composite soil carrying capacity. For this purpose, the limiting value for stress on the cohesive tributary area could be taken as $5c$ while $\mu_c \sigma$ is the upper limit. The stress concentration factor given in equation 2.5 should be taken into account while determining \tilde{N}_c from field test results.

Mitchell and Katti (1981) recommend using $\tilde{N}_c = 25$ for stone columns constructed by vibroreplacement method while Datye et al. (1982) recommend using 25 to 30 for such columns. They also give recommendations for other column types such as 45 to 50 for cased rammed stone columns and 40 for uncased rammed stone columns.

There are other studies on determining N_c value from back analysis of field test results. The following figures summarize two different studies:

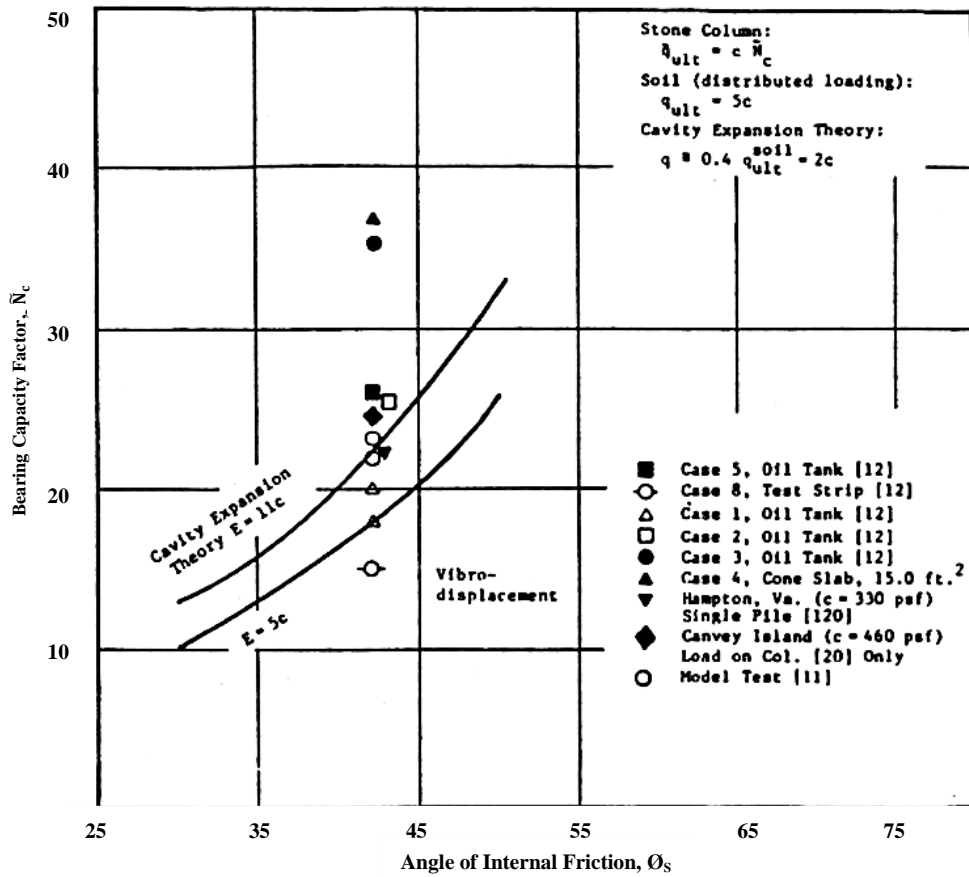


Figure 2.2 Bearing Capacity Factors (Barksdale and Bachus, 1983)

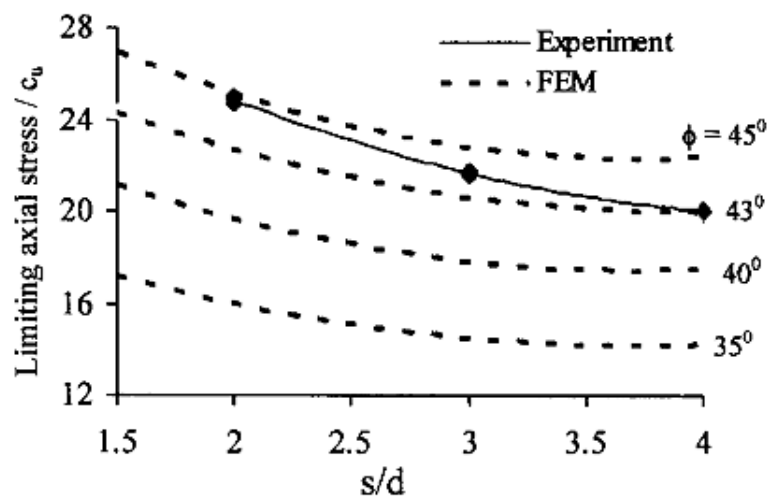


Figure 2.3 Effect of "column spacing/column diameter" on Bearing Capacity Factor (Ambily and Gandhi, 2007)

Barksdale and Bachus (1983) summarize the design aspects of stone columns as the following:

1. The design load of stone columns is usually between 15 – 60 tons for each column.
2. The effective treated soil layer thickness is between 6 to 9 meters for economic reasons. However, they also added that in Europe and America there are applications of stone columns as long as 21 meters.
3. In cohesive soils of shear strength less than 7 kPa the design of stone columns is not recommended while from 7 to 19 kPa shear strength values are to be treated carefully. In soils having sensitivity greater than 5, the stone column construction is also not recommended.
4. The soils including peat layers thicker than 1 meter are reported not to be suitable for the conventional stone column construction.
5. Stress concentration factor is generally used as 2 to 2.5 and the internal friction angle of the column material 38 to 45° in theoretical analyses.

2.2 Stress Concentration Factor

When a load is applied over stone columns, it causes shear strength increase in stone columns and reduction of settlements in surrounding soft soil; thus, the concentration of stress in the columns due to the stone column being considerably stiffer than the surrounding soil. The deflection of the two materials, stone and soil, is approximately the same, so the stress in the stiffer stone column material should be greater than in the soft surrounding soil for equilibrium considerations. The effect of stress concentration factor on the stress taken by the surrounding soil is given in Figure 2.4 (Bachus and Barksdale, 1989).

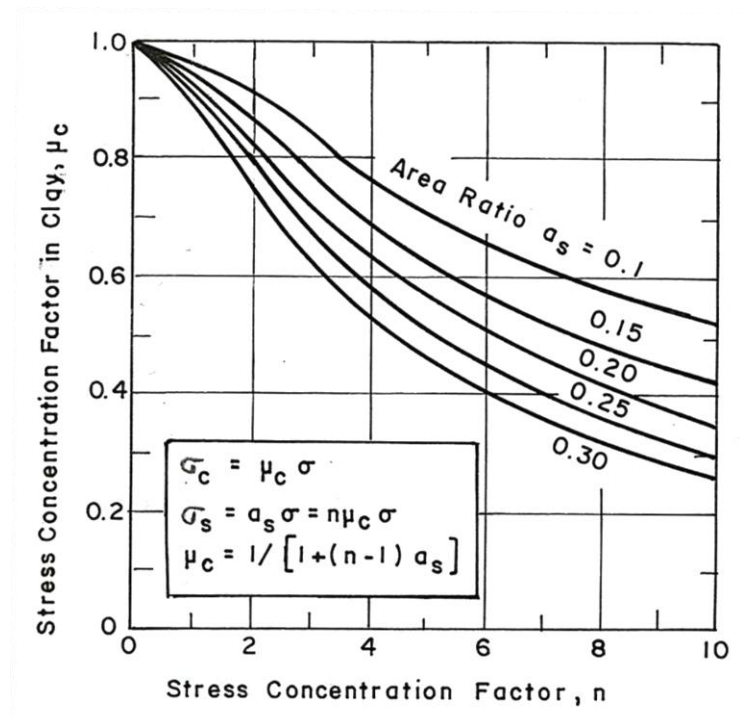


Figure 2.4 Variation of stress concentration factor (Bachus and Barksdale, 1989)

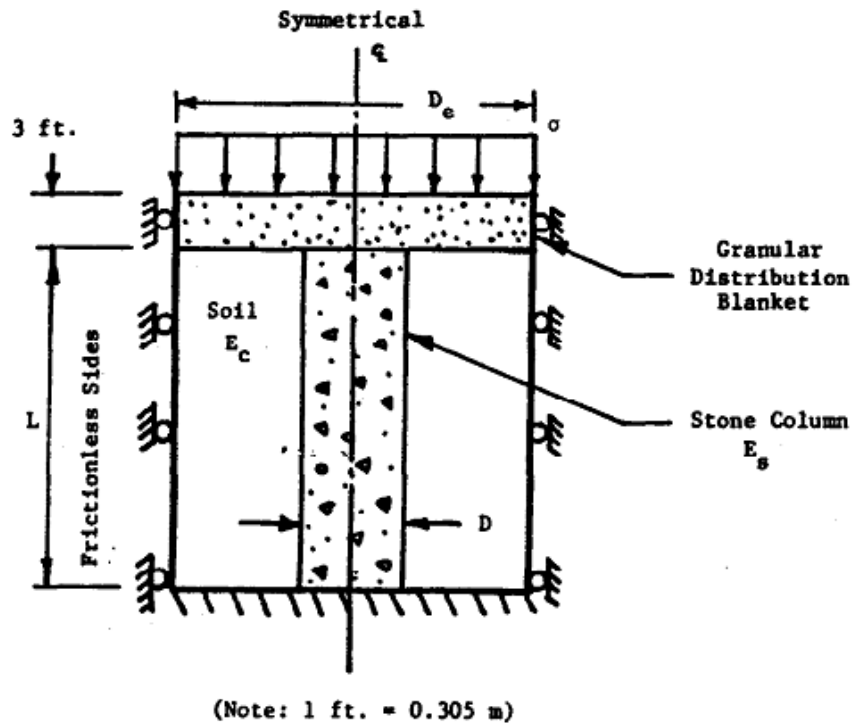
The stress concentration factor n changes with some factors including the relative stiffness between the column and soil materials, stone column length, area ratio and the characteristics of the granular blanket placed over the stone column. Values of stress concentration measured in field and laboratory studies are summarized in Table 2.1 (Barksdale and Bachus, 1983). The stress concentration factors were measured as generally between 2.5 and 5.0. The stress concentration factor measured in four of the five researches was either approximately constant or increased as the consolidation time passes. Balaam (1978) indicated in his theory that the stress concentration factor should increase with time. As the secondary settlements in cohesive soils improved with stone columns are larger than in the stone column alone, the long-term stress concentration factor should be greater than that at the end of primary settlement (Barksdale and Bachus, 1983). Aboshi et al. (1979) carried out field measurements on sand compaction piles at four sites in Japan and stated that stress concentration likely decreased with depth, but stayed larger than 3.0 at the sites studied.

Table 2.1 Observed Stress Concentration Factors in Stone Columns (Barksdale and Bachus, 1983)*

Type Test	Design	Location	Stress Concentration (n)	Time variation of n	Stone column length (m)	Subsurface Conditions
Embankment	Square Grid, s=1.7m, D=0.9m a _s =0.25	Rouen,France Vautrain, 1977	2.8 (avg)	Approx. Constant	6.6-7.8	Soft clay C=19-29 kN/m ²
Load Test; 45 stone columns (91cmx127cm)	Triangular Grid, s=1.74m, D=1.2m a _s =0.43	Hampton, Virginia Goughnour and Bayuk (1979)	3.0 (initial) 2.6 (final)	Decreasing	6.15	Very soft and soft silt and clay with sand C=9.6-38 kN/m ²
Test Fill 14 stone columns	Triangular grid s=2.1m, D=1.125m a _s =0.26	Jourdan Road Terminal, New Orleans,	2.6-2.4 (initial) 4.0-4.5 (final)	Increasing	19.5	Very soft clay with organics, silt and sand lenses; loose clayey sand; soft sandy clay
Embankments	a _s = 0.1-0.3	Japanese Studies-Sand compaction piles Aboshi et.al.(1979)	2.5-8.5 4.9 (average)	Increases	Variable	Very soft and soft sediments
Model Test	a _s = 0.07-0.4 D=2.9cm	GaTech Model Tests; Unit cell; Sand column	1.5-5.0	Constant to slightly increasing	Variable	Soft clay; n appears to increase with a _s

*Vertical stress measured just below load except where indicated otherwise

Barksdale and Bachus (1983) carried out elastic finite element study on a unit cell model to predict the settlement of low compressibility soils reinforced with stone columns such as sands, silty sands and some silts. They defined the low compressible soils as having modular ratios (E_s/E_c , which are modulus of elasticity values of stone column material and surrounding soil, respectively) smaller than 10. The unit cell model properties are given in Figure 2.5. The Poisson's ratio of the soil was taken as 0.30 while it was taken as 0.35 for the stone. The stress concentration values are given for area ratios of 0.1, 0.15 and 0.25 and E_s/E_c ratios up to 40 in stone columns having length to diameter ratios between 4 and 20. Variation of stress concentration factor with modular ratio showed very nearly linear behaviour as the modular ratio increased the stress concentration factor n also increased for different values of area ratios and L/D ratios (Figure 2.6). The study indicated the stress concentration values changed between 2 and 10.



Definitions: $a_s = A_s/A$ where A_s = area of stone and A = total area
 Vertical Settlement, $S = I_s \left(\frac{P}{E_s L} \right)$ where $P = \sigma \cdot A$

Figure 2.5 Unit Cell Linear Elastic Finite Element Model (Barksdale and Bachus, 1983)

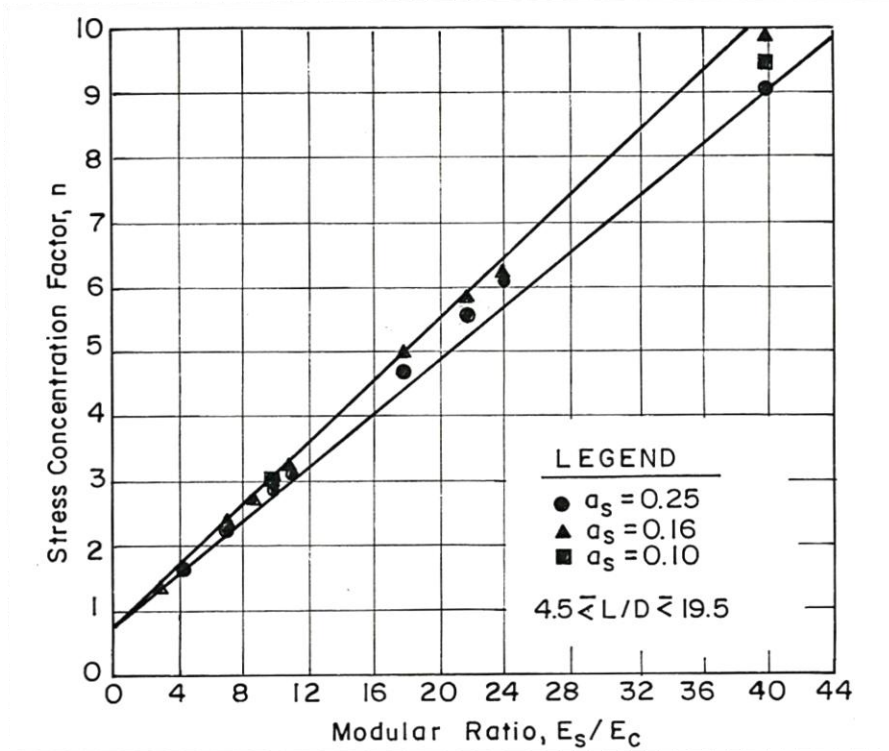


Figure 2.6 Variation of stress concentration factor with modular ratio - Linear elastic analysis (Barksdale and Bachus, 1983)

Han and Ye (2001) developed an analytical simplified solution for computing the rate of consolidation of reinforced foundations with stone columns. They also presented a formulation for stress concentration ratio for the condition that the consolidation of the surrounding soil is complete; thus the effective stresses of the stone column and the surrounding soil reach to the steady state and equal to the total stresses. Then, the following formulation was given:

$$n_s = \frac{\sigma_{cs}}{\sigma_{ss}} = \frac{m_{v,s}}{m_{v,c}} = \xi \frac{E_c}{E_s} \quad (2.10)$$

where n_s is the steady-stress concentration ratio as the consolidation is complete, σ_{cs} and σ_{ss} are the steady vertical stresses within the stone column and the surrounding soil, respectively and

$$\xi = \frac{(1+v_s)(1-2v_s)(1-v_c)}{(1+v_c)(1-2v_c)(1-v_s)} \quad (2.11)$$

where v_c and v_s are Poisson's ratio of stone column and surrounding soil.

They compared the solutions for Poisson's ratio values of 0.15 and 0.45 for column material and surrounding soil, respectively, to the existing theoretical and empirical study conducted

by Barksdale and Bachus (1983). As shown in Figure 2.7 for a typical modular ratio of 10 to 20, the results obtained for the study carried out by Han and Ye (2001) gave similar results with Barksdale and Bachus (1983).

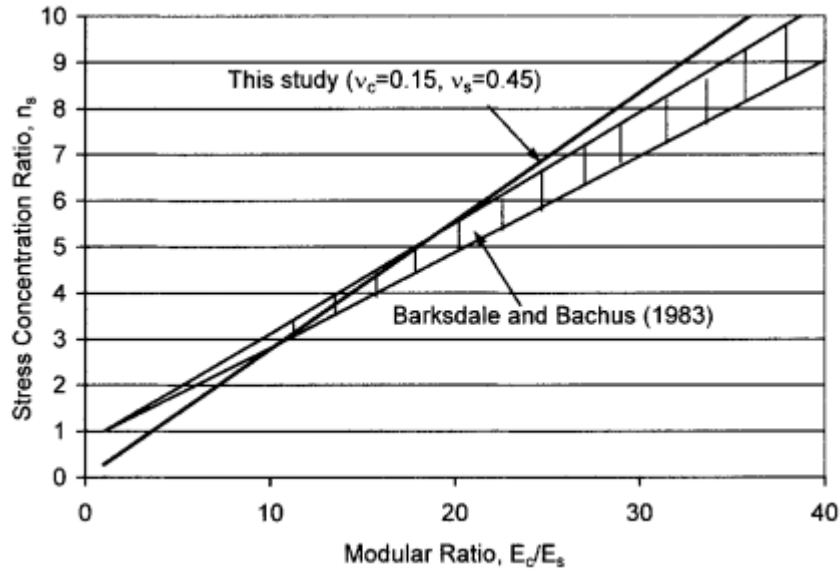


Figure 2.7 Relationship between stress concentration ratio and modular ratio (after Han and Ye, 2001)

The stress concentration values were studied in nonlinear analysis by Barksdale and Bachus (1983). Model geometric properties and the parameters used in the non-linear analyses are given in Figure 2.8. The theoretical variation of the stress concentration factor n with the modulus of elasticity of the soil and length to diameter ratio, L/D is shown in Figure 2.9. Stress concentration factor ranged between 5 and 10 for short to moderate length columns treating very compressible clays ($E_c < 1380$ to 2070 kN/m²). These results showed that the nonlinear theory may under predict settlements (Barksdale and Bachus, 1983).

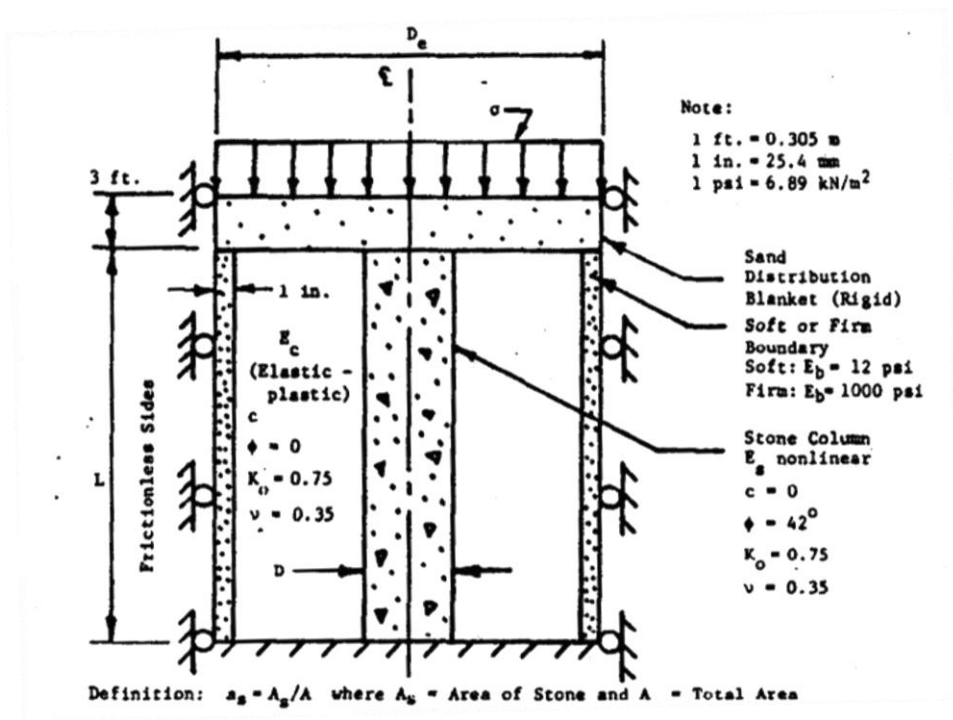


Figure 2.8 Unit Cell Non-linear Finite Element Model (Barksdale and Bachus, 1983)

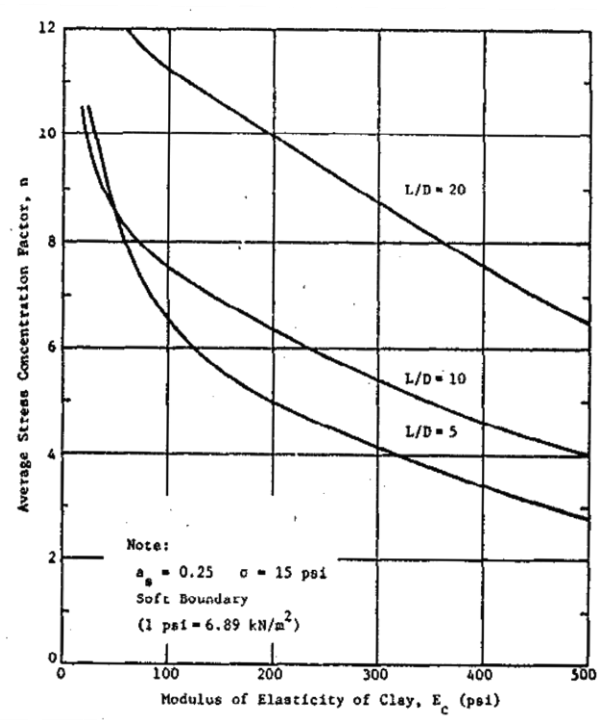


Figure 2.9 Variation of Stress Concentration Factor with Modulus of Elasticity of Soil – Nonlinear Analysis (Barksdale and Bachus, 1983)

Greenwood (1991) carried out field tests in St Helen's site on a dummy footing fitted with 150 mm stress cells resting on unusual soil consisting of siliceous particles with a content of Jewellers rouge. This fill material existed in huge depths; thus the stone columns were floating. The columns were constructed by wet vibrofloatation method. Ground water table was low and the soil behaved as drained during construction and testing. The ratio of the stress in the column to the surrounding soil, stress concentration ratio, was measured to be around 3.5 at low stress levels and it was decreasing to 2.5 as the stress increases. Figure 2.10 illustrates the model of the field test and the resulting stress concentration values under increased ground pressure.

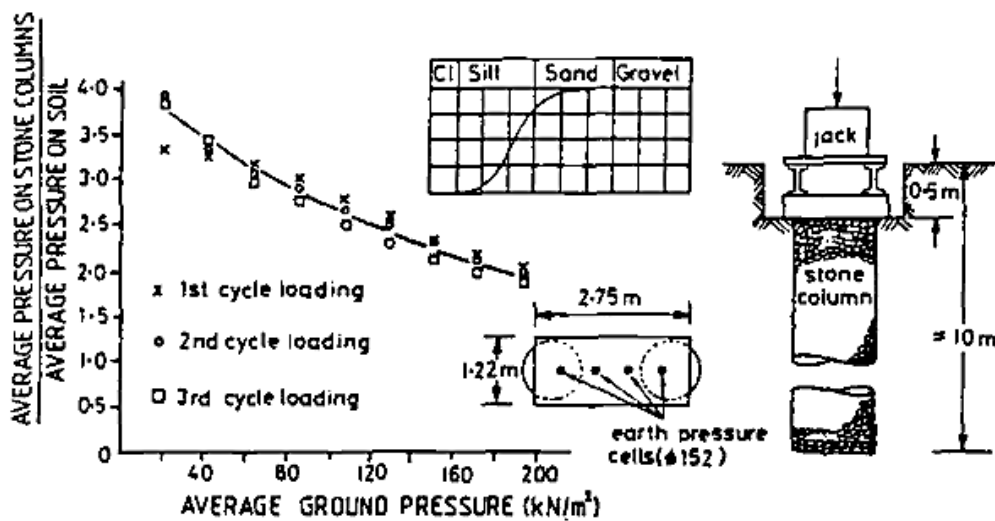


Figure 2.10 St. Helen's - Loading a strip footing on stone columns (Greenwood, 1991)

A second test site was Humber Bridge South Approach to construct stone columns limiting the settlements under a rolled chalk fill embankment and carry out field tests on the columns (Greenwood, 1991). The stone columns were constructed by wet vibrofloatation technique and the columns were treating soft organic silty clays. The stone columns were in 9 meter length to rest in stiff boulder clay. The loading was applied in two cycles, in the first cycle there was no bulging observed while in the second one bulging effect was obvious from stress measurements (Figure 2.11). The stress in the soil was nearly constant at all loading stages while it raised a little upon the end of the test when the maximum settlement is reached. In Figure 2.12 the stress concentration ratio reached 5.0 after bulging in the second cycle of loading (Greenwood, 1991). The author concluded his studies that the stress in the soil scarcely changed with loading due to pre-stressing effect on the soil. With increasing loading, the stress concentration on the columns also increased since the extra load went into the columns and this effect was opposite of the situation in St. Helen's.

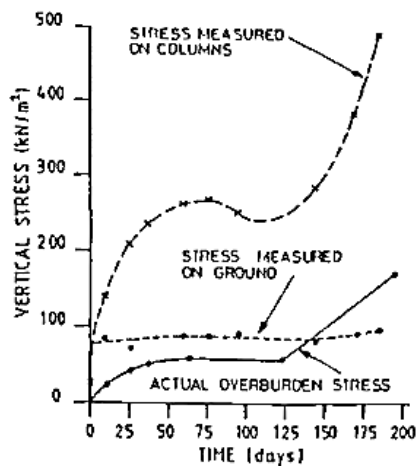


Figure 2.11 Humber Bridge – Measured stresses (Greenwood, 1991)

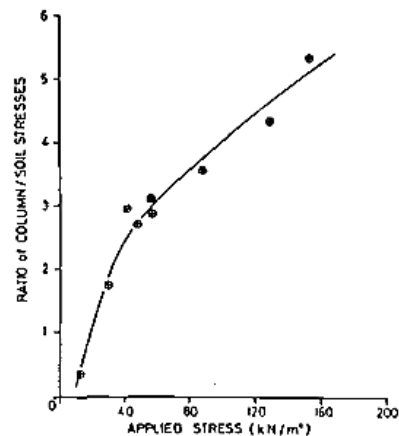


Figure 2.12 Humber Bridge – Measured stress ratios (Greenwood, 1991)

Wood et al. (2000) studied model tests of stone columns and they compared the results with Humber Bridge field tests conducted by Greenwood (1991). The area ratio of the model test was 0.24 while it was 0.21 in the field measurements. The important difference between those studies was that the model data came from a rigid footing loading while the field measurements were done for a flexible foundation loading. In Figure 2.13 stress concentration ratios were plotted as a function of the ratio of applied pressure to initial undrained strength of the soil. As it could be seen from the figure, the results of stress concentration ratios gave similar results for both of the studies, model tests and field tests.

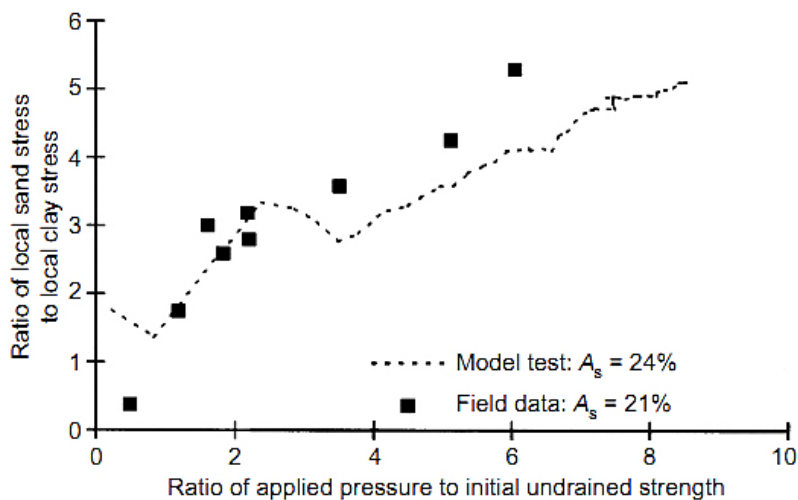


Figure 2.13 Stress concentration ratio in model test and field test data from Greenwood (1991) (after Wood et al., 2000)

Three full scale load tests were conducted by Özkeskin (2004) on three different lengths of stone columns by 3, 5 and 8 meters in an 8-m-thick silty clay layer resting on a clayey very dense sand layer. In the scope of that study, the stress concentration factor varied between 2.1 and 5.6 with an average of 3.5. The stress concentration ratio n was observed to decrease with increasing applied vertical stress in a linear trend (Figure 2.14). Özkeskin also observed that the shortest (3m –long) column behaved like floating pile and gave the highest values for stress concentration factor while the other two test results for 5 and 8m-long-columns were similar to each other since they behaved like end-bearing columns due to no significant stress transformation under 5 m depth. She also gave an analytical solution of three test results in terms of stress concentration ratio as follows:

$$n = (0.93 - 0.35) \frac{E_s}{E_c} \quad (2.12)$$

where E_s and E_c are the modulus of elasticity values of stone column material and surrounding cohesive soil, respectively.

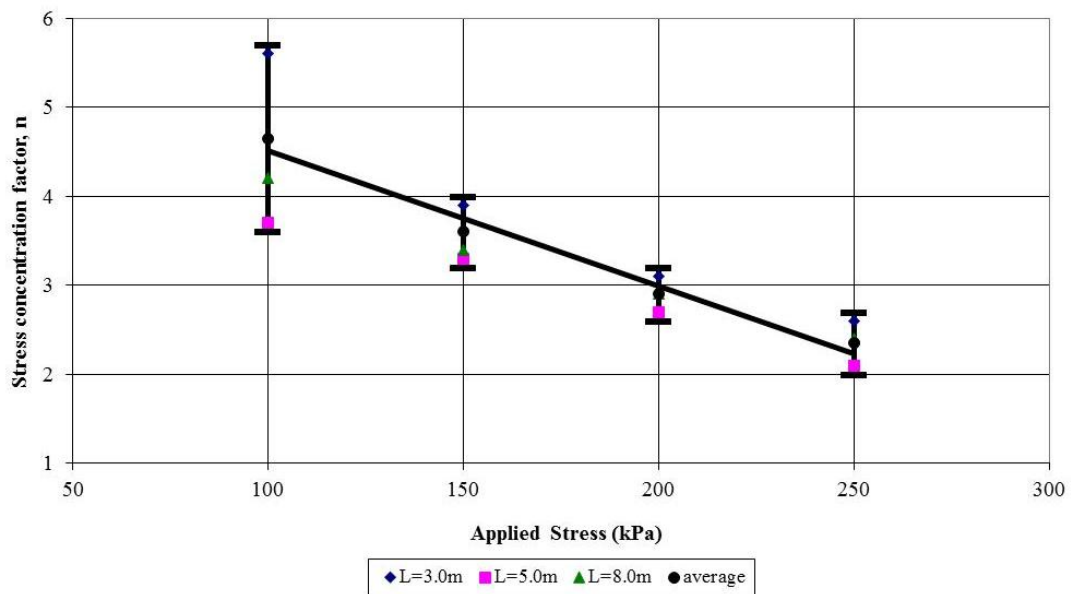


Figure 2.14 Variation of stress concentration factor n with applied surface pressure (Özkeskin, 2004)

Ambily and Gandhi (2007) carried out laboratory model tests on stone column groups and finite element analyses on single column and column groups. They summarized the stress concentration ratios obtained in both laboratory and numerical studies as given in Figure 2.15. The charts were given for changing stone column spacing to diameter ratio (s/d) with

different undrained shear strength values of clay surrounding the stone columns. The single stone column analyses gave almost same results with the stone column group analyses as the unit cell concept represents the interior column behaviour when a large group of stone column is loaded. The stress concentration ratios varied between 3.5 and 6.0. They showed that the stress concentration ratio increases as the shear strength of the clay decreases. Similar results were obtained for other s/d ratios between 1.5 and 4.0 in single and 7-stone column-group loadings. They concluded that for smaller s/d ratios, the unit cell concept is questionable and it should be further studied.

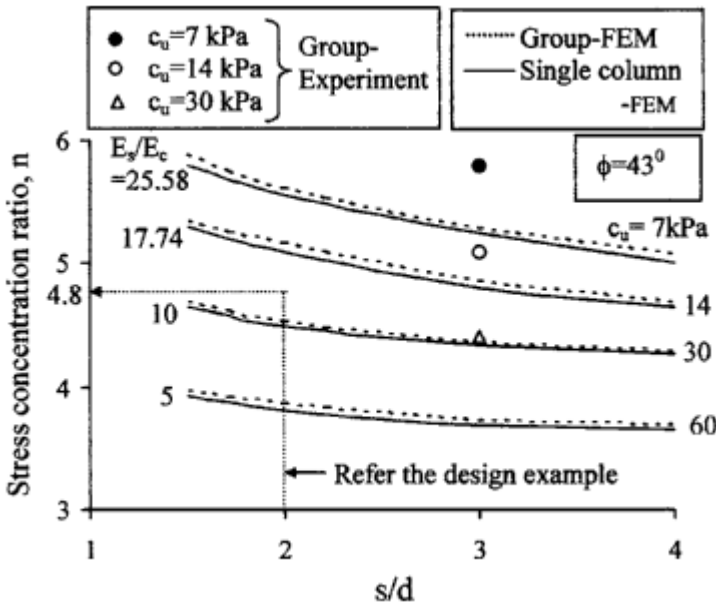


Figure 2.15 Effect of s/d and c_u on stress concentration ratio (Ambily and Gandhi, 2007)

2.3 Settlement Reduction Ratio

Priebe (1995) presented a method for estimating the settlements of the improved soil by stone columns, also known as Priebe Method in the literature. He made some assumptions in his theory. The column material was assumed to be incompressible and based on a rigid layer. The soil and stone column bulk density were ignored. Moreover, the soil around the stone columns was assumed to be displaced while the stone columns were constructed until the initial resistance of the soil reached to the liquid state; i.e. the coefficient of earth pressure, K , was equal to 1.0. Then, the improvement factor, the ratio of settlement before to after stone column installation, was given as a function of area improvement (1/area replacement ratio). The angle of internal friction was chosen between $35^\circ - 45^\circ$ for a Poisson's ratio of 1/3. The design chart proposed by Priebe (1995) is given in Figure 2.16.

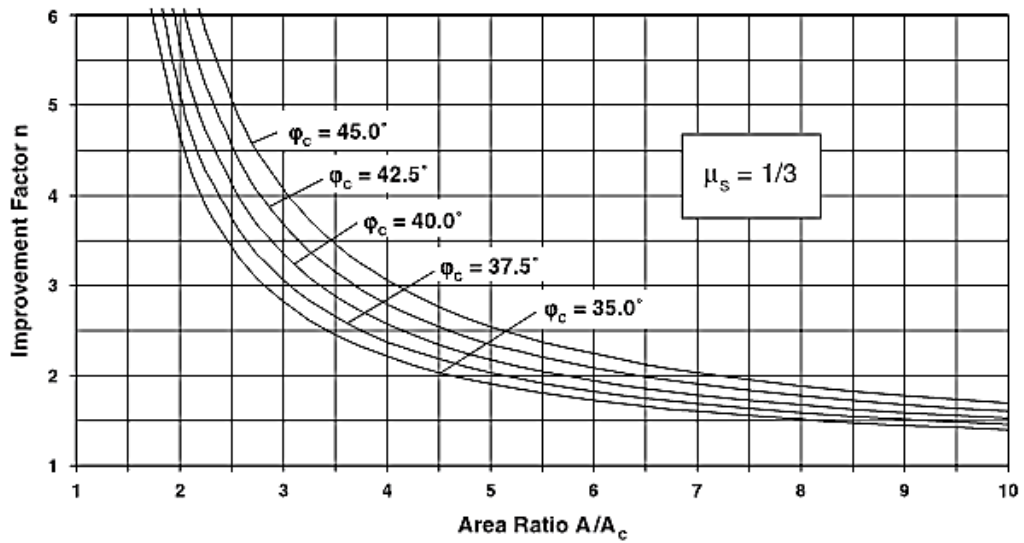


Figure 2.16 Design chart for vibroreplacement (Priebe, 1995)

Aboshi et al. (1979) and Barksdale (1981) described a method called Equilibrium Method used in Japanese practice to determine the settlement of the improved ground after sand compaction pile installation. This method can be applied to the stone columns and the settlement reduction of the improved soil with stone columns can be estimated by this simple, but realistic solution. For the application of Equilibrium Method, the stress concentration factor should be estimated by field tests or past experience. The charts were given for the stress concentration factors equal to 3, 5 and 10 (Barksdale and Bachus, 1983). To stand on the safe side, low stress concentration factor could be chosen to estimate the reduction in settlements. The assumptions of long stone column and very small applied stresses, the following relation is obtained and the graphical presentation is given in Figure 2.17. Equilibrium Method gives slightly higher values than expected ones at field applications, so it is convenient to use it for the preliminary estimations.

$$\frac{S_t}{S} = \frac{1}{1+(n-1)a_s} = \mu_c \quad (2.13)$$

where S_t is the preliminary consolidation settlement of stone column treated ground

S is the untreated soil settlement

n is the stress concentration factor

a_s is the area replacement ratio

μ_c is the stress ratio of the cohesive soil after treatment

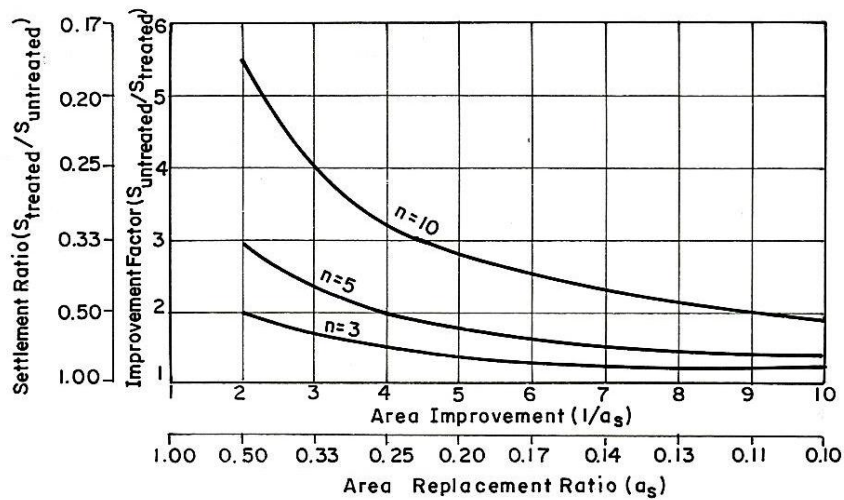


Figure 2.17 Maximum reductions in settlement that can be obtained using stone columns-equilibrium method of analysis (Barksdale and Bachus, 1983)

Greenwood (1975) also presented empirical curves to estimate the settlement reduction after stone column installation. He gave the settlement reduction ratio as a function of undrained shear strength of soil and the column spacing. Barksdale and Bachus (1983) replotted those curves to compare the results with Priebe and Equilibrium Methods, so they gave the settlement improvement ratio instead of settlement reduction ratio as a function of area replacement ratio instead of stone column spacing. The upper and lower boundaries were plotted by assuming the diameter of the column as 0.9 m and the undrained shear strength of soil as 40 kN/m² for the upper bound while they were taken as 1.07 m and 20 kN/m² for the lower bound curves, respectively.

In the following figures those presented three methods are compared with each other. Priebe Method appears between the upper bound equilibrium curves for stress concentration factor of 5 and 10. The Priebe improvement factors are majorly greater than the observed values of equilibrium curves in the range of 3 to 5 for the stress concentration factor. There are two measurements from Greenwood and Jordan Road sites which give similar results with the upper bound of the equilibrium method curves for n in the range of 3 to slightly less than 5 (Barksdale and Bachus, 1983).

As it could be seen from Figure 2.19, for the area ratios less than 0.15, Greenwood suggestion gives similar results with the equilibrium curves of n between 3 and 5. However, for the area ratio values range between 0.15 and 0.35, Greenwood results fall outside the upper boundaries of equilibrium curves (Bachus and Barksdale, 1989)

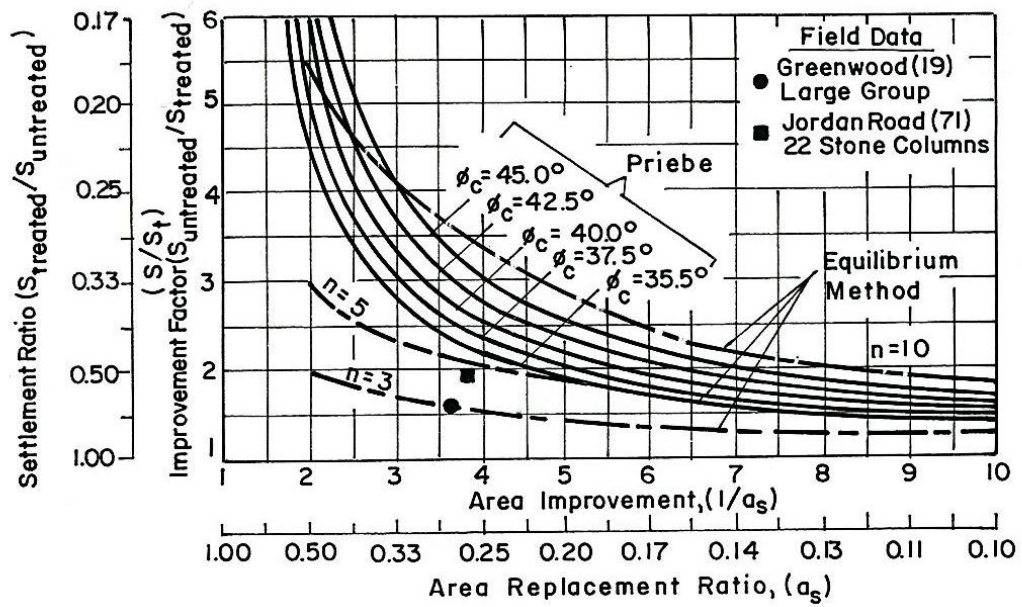


Figure 2.18 Settlement reduction due to stone column- Priebe and Equilibrium Methods (Barksdale and Bachus, 1983)

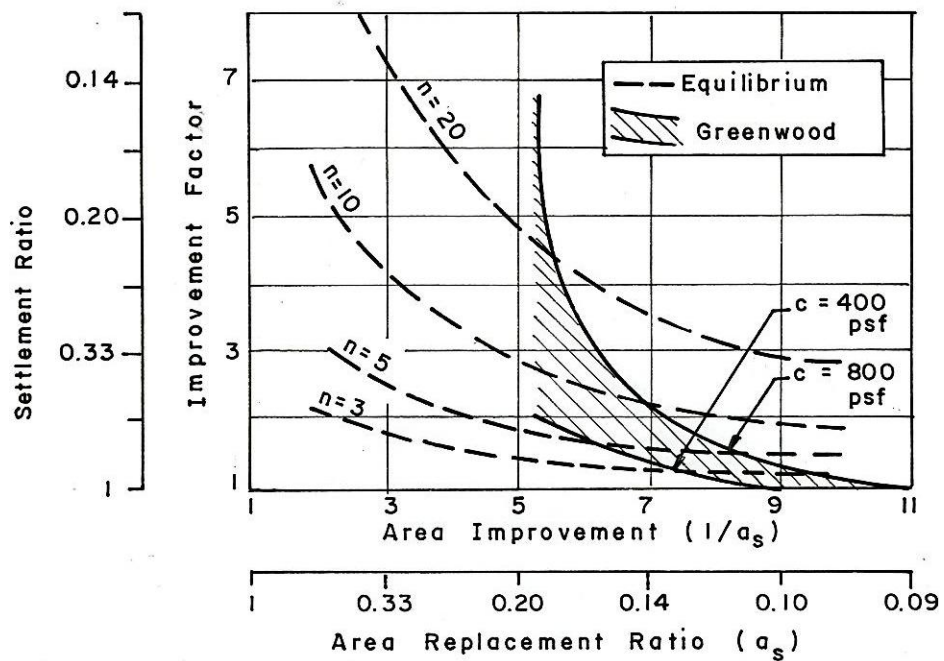


Figure 2.19 Comparison of Greenwood and Equilibrium Methods for predicting settlement of stone column reinforced soil (Barksdale and Bachus, 1989)

Özkeskin (2004) reported from three full scale load tests on rammed stone columns that the settlement reduction ratio showed a decreasing trend against increasing pressure applied in the staged loading. From 50 kPa to 250 kPa vertical loading, the reduction ratio ranged from 0.6 to 0.2, which meant that as the vertical stress on the footing increased the efficiency of settlement reduction. Moreover, the length of the columns on the settlement reduction ratio was also examined. As the rammed stone column length increased, the efficiency to improve the settlement of the improved ground also increased. The results are shown in Figure 2.20.

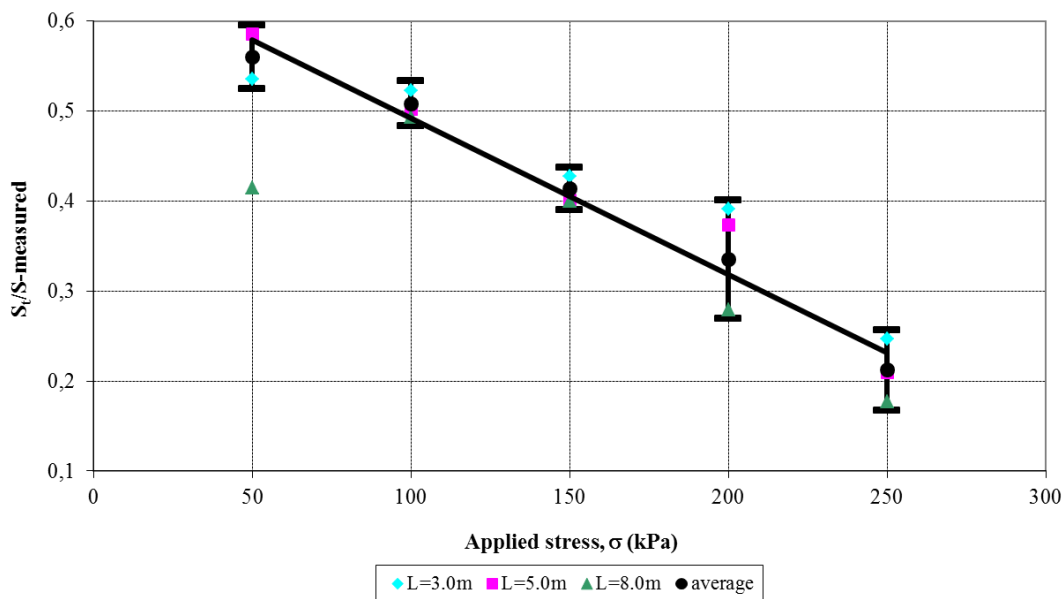


Figure 2.20 Settlement reduction ratio variation with applied pressure (Özkeskin, 2004)

2.4 Finite Element Method

Stone columns behave essentially as rigid inclusions with higher stiffness, shear strength and permeability values than the native soil and these three improvement effects of stone columns on natural soil were independently studied by different solutions. The analytical solutions of radial consolidation (Barron, 1948), the settlement improvement (Priebe, 1995), the bearing capacity and the stability of embankments or foundations (Hughes and Withers 1974; Vesic 1972; Bergado et al. 1996) were presented in the literature. In fact, these characteristics of the improvement system are inter-related and should be given in the same studies (Canizal et al., 2012).

Numerical analyses of stone columns generally need a complex modelling of the soil reinforcement system. As Canizal et al. (2012) stated that there are five main approaches for modelling stone columns numerically:

1. Unit cell approach: In axisymmetric model only one column and its surrounding soil consisting of a “unit cell” is modelled (Balaam and Booker, 1981).
2. Plane strain method: The cylindrical columns are modelled as stone trenches, which is usually used under long loads, such as embankments (Van Impe & De Beer 1983).
3. Axial symmetry technique: Stone rings are modelled instead of cylindrical columns to represent the columns under circular loads such as tanks (Elshazly, 2008).
4. Homogenization technique: The composite soil parameters are used to model the improved homogeneous soil with stone columns (Schweiger, 1989).
5. Full 3D Model: The most complex representation of the system of stone column improved soil numerical models is 3D modelling (Weber et al., 2008).

There are a number of numerical studies on stone column systems in the literature. The soil and stone parameters and the behaviour of the whole system may guide further studies. Therefore, the basic properties of the numerical models are given from selected studies in this part of this study.

As given in Part 2.2 of this study, Barksdale and Bachus (1983) have conducted finite element analyses to give non-linear solutions to settlement and stress concentration of stone columns reinforcing soft cohesive soils. In that study, the parameters of the stone column were selected as the angle of internal friction, $\phi_s = 42^\circ$ and a coefficient of at-rest earth pressure $K_0 = 0.75$ for both the stone and soil. From the analysis results, they reported that soil shear strength did not influence the settlement since the soils which have modulus of elasticity greater than 1100 kN/m^2 did not fail by an interface or soil failure. The soil having smaller stiffness than 1100 kPa had been assumed to have a shear strength value of 19 kN/m^2 . In non-linear analyses, the effect of interface was found to be very little as increasing the settlements; therefore, the effects of interface elements, having able to model the no slip, slip or separation using Mohr-Coulomb failure criteria, were ignored.

Balaam, Brown and Poulos (1977) have carried out finite element analyses on stone columns using the unit cell concept. They chose the parameters of stones by the available data and compared the results of the finite element analyses with the existing analytical solutions and the full-scale load tests. They reported that the modulus of elasticity of the gravel was given as between $40000\text{-}70000 \text{ kN/m}^2$ within the limited data available. In their analyses, the ratio of the modulus of elasticity of stone to soil varied from 10 to 40 while the Poisson's ratio was selected as 0.3 for both materials. The at-rest earth pressure was assumed to be equal to 1.0. Both elastic and elasto-plastic solutions were given and it was concluded that there was not a significant change in two solutions in terms of settlements with a 6 percent difference. They reported that numerical studies showed a little difference of settlements in terms of varied modular ratio of stone column to soil, but the diameter of the column and the penetration depth had major effect on settlement behaviour of the reinforced system.

The modulus of elasticity of the stone column material is required for both incremental and elastic methods (Barkasdale and Bachus, 1983). Vautrain (1977) back calculated the the field measurements of load tests in Rouen and determined the modulus of elasticity of stone column material constructed by vibroreplacement method as 30000 kN/m^2 while Balaam

(1978) estimated a value of 50000 kN/m² for Canvey Island field measurements obtained from the linear part of the undrained load-settlement curve. Engelhart and Kirsh (1977) reported a value of 58000 for the modulus elasticity of the stone column material. Datye (1982) also back calculated from measured settlements and concluded a value of 48000 for rammed stone columns. Özkeskin (2004) carried out full-scale load tests on silty soft clay reinforced by rammed stone columns and back calculated modulus of elasticity value of the stone material as around 39000 kN/m² by finite element analysis of a single stone column.

Kuruoğlu (2008) modelled the plate loading tests previously completed by Özkeskin (2004) and calibrated the parameters of the soil. He has completed his modelling in Plaxis 3D finite element software. After calibrating the untreated soil parameters by Mohr-Coulomb soil model, he carried out three rigid plate loading tests on rammed stone column improved soil. He conducted parametric study by using elastic composite soil model which matched the load-settlement behaviour with 3D stone column analysis. He analysed the silty clayey soil in undrained conditions and found the parameters from back analyses as; the undrained shear strength, $c_u = 22$ kN/m²; modulus of elasticity, $E_c = 4500$ kPa and the Poisson's ratio, $\mu = 0.35$. The stone columns were modelled with linear elastic material model and its modulus of elasticity was selected as 39 MPa, recommended by Özkeskin (2008) resulted from the back analysis of the single rammed stone column loading at the site.

Elshazly et al. (2008) modelled stone column and improved soft clayey soil by the unit cell idealization in two-dimensional axisymmetric modelling in Plaxis software programme. They calibrated the load-settlement behaviour of the improved ground by stone column installation obtained by full-scale field load tests. The stress-strain behaviour of all soils was modelled by Hardening Soil model satisfying also Mohr-Coulomb's failure criterion. The area ratio of the calibrated model was equal to 0.29 and the column length was 10.8 end-bearing to harder stratum as in the field load tests. The modular ratio of the stone to soft clay was 8.5 and the loading was applied in three stages as 30 – 90 and 150 kPa. They also regarded the vibration effect of the stone column installation by changing the lateral earth pressure from initial at rest values (*Jacky's formula* : $k_0 = 1 - \sin \varphi$) to 1.50 under the foundation for the composite ground while the unimproved soil out of the foundation not effected by installation of column remained constant. Furthermore, they summarized the lateral earth pressure values after stone column installation reported in past studies and they are given in Table 2.2

Zahmatkesh and Choobbasti (2010) carried out finite element analyses to estimate the settlement of the treated ground with stone columns by using Plaxis 2D software with plane strain modelling. Mohr Coulomb's failure criterion was utilized in drained analyses for all of the soil models including stones, soft clay and sand stress distribution layer. The selected parameters of the stone column and soft clay were reported as the modulus of elasticity was 4000 kPa for the soft clay while it was equal to 55000 for the stone column material and 20000 for sand layer. The Poisson's ratio was given as 0.35 for soft clay whilst 0.30 for stones and sand. The internal friction angle was defined for soft clay 23°, for stone column material 43° and sand layer 30°. Angle of dilatancy was also defined for stones and sand as 10° and 4°, respectively. Drained cohesion was zero for all of the soil types defined. Finally, the interface strength values were given as 0.7 to soft clay, 0.9 to stone column material and

1.0 (rigid) for sand layer. The column installation effect was given by the stress state change in the improved soil due to compaction. They explained that the post-installation coefficient of earth pressure, k^* , was greater than the coefficient of lateral earth pressure at rest, k_0 , as supported by many researches before. They performed an axisymmetric analysis for the determination of post lateral coefficient of earth pressure of the soil displaced by the vibration or compaction effect of the column installation. They also indicated that this coefficient was highly dependent on the soil type, stone column spacing and stone column installation method. In their analysis, the hole made by the vibroprobe was modelled as cylinder having 0.25 m radius and subjected to radial displacement until the radius reached 0.5 m. The column length was given as 10 meters and the stresses were measured at the mid-thickness of the soft soil layer. Finally, they reported that the initial stresses (*Jacky's formula* : $k_0 = 1 - \sin \varphi = 0.609$) increased to the values exceeding 1.0 at the periphery of the column. The resulting measurements of the coefficient of lateral earth pressures (the ratio of horizontal to vertical stresses) after column installation are given in Figure 2.21. They also added that those values should be twice the reported ones as the installation effect was twice from both sides compaction.

Table 2.2 Published K^* (Ratio of post-installation horizontal to vertical stresses) Values (Elshazly et al., 2008)

References	K^* value	Method of determination
Elshazly et al. (2006)	Between 1.1 and 2.5, with best estimate of 1.5	Back calculations from full-scale load test performed on a stone column within an extended array of column.
Elkasabgy (2005)	Between 0.7 and 2.0, with average of 1.2	Back calculations from 3 full-scale load tests performed on stone columns within three extended arrays of columns.
Pitt et al. (2003)	Between 0.4 and 2.2, with average of 1.2	Full-scale load tests on vibro-displacement stone columns in compressible clays and silts underlain by highly weathered shale.
Watts et al. (2000)	Between K_0 and K_p	Full-scale load tests on vibro-displacement stone columns in variable fill.
Priebe (1995)	1.0	Analytical solution of end-bearing incompressible columns, neglecting the geo-field stress effect.
Goughnour (1983)	Between K_0 and $1/K_0$	Analytical solution based on elastic and rigid-plastic behaviour using the unit cell concept.

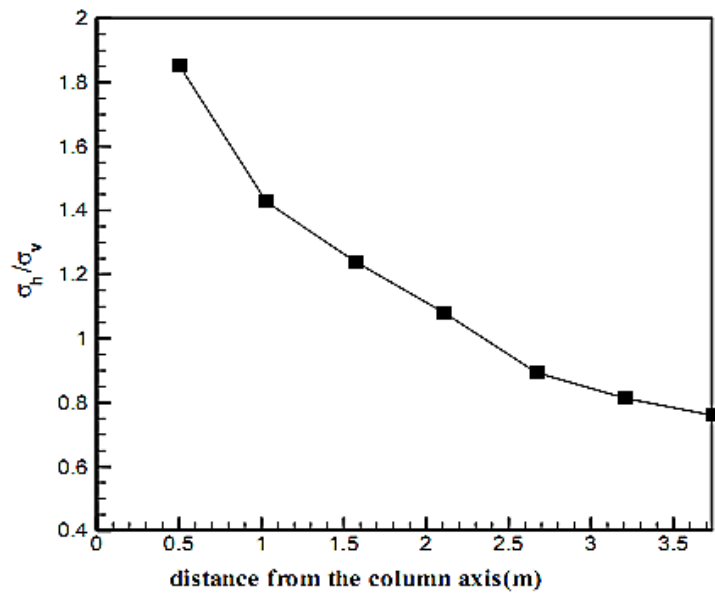


Figure 2.21 Variation of stresses in soft clay with distance from the column (Zahmatkesh and Choobbasti, 2010)

CHAPTER 3

GENERATION OF FINITE ELEMENT MODEL

3.1 Introduction

Finite element analysis is the most widely used method to analyse the stone column improved soil behaviour. Three-dimensional modelling is the best way to match the real conditions; however, for practical purposes either axisymmetric or plane strain model is generally used. Many studies have utilized axisymmetric model for the unit cell model of uniformly loaded stone column groups or plane strain models (Barksdale and Bachus, 1983).

In the scope of this study, two dimensional plane strain models will be utilized to specify the concentration of stress in the stone columns and the reduction in settlement by improving the soil with stone columns by the finite element software Plaxis 2D V9.02.

3.2 Details of Finite Element Analyses

Weak silty clayey layer with 20 m thickness is modelled to be improved by stone columns. A 5m-thick-clayey sand layer underlies the silty clay layer. The ground water table is at the ground level. In the reference analysis, a plate load test is applied on the unimproved soil with a 10 m width and 10 cm thick steel plate. For the analyses of the improved cases, stone columns are modelled to improve the weak soil. Under the steel rigid plate 20 cm-thick sand layer is defined to distribute the load on the stone columns homogeneously and to behave as a drainage layer. Floating and end bearing columns in weak silty clay layer having altered modulus of elasticity comprise the parametric study. The behaviour of the stone columns of different lengths and in soils with different stiffness's was analysed under changing stresses. By the results of the parametric studies the settlement reduction ratio and stress concentration factor in stone columns are determined for each case.

For an embankment loading condition, flexible foundation analyses are carried out under the same soil conditions. The embankment width is given as 10 m and the same sand stress distribution layer is defined in 20 cm thickness. The stress concentration in the stone columns is compared with the rigid foundation analyses with the same key parameters.

3.2.1 Geometric Modelling

In both of the rigid and flexible foundation analyses, stone columns are designed to have 80 cm diameter. The spacing of the columns is 1.6 m in a square pattern, which makes the area ratio equal to approximately 0.20.

Fifteen-noded triangular elements are used in the finite element model. The medium mesh is generated as the global coarseness; however, it is refined in the improvement area as the stresses and the deformations are higher in this area. The finite element mesh and the boundary conditions of the model are shown in Figure 3.1 and 3.2 for rigid and flexible foundations, respectively. In the model, distributed load is defined to represent the foundation pressure; i.e plate and embankment loadings as shown by “A...A” in the figures.

There are no interface elements between the rigid steel plate and the sand layer because no slippage occurs under the plate (Zahmatkesh and Choobbasti, 2010). No interface elements are defined around the stone columns, either; because no significant shear occurs between the stone column and the soft clay, the deformation and the failure of the column is generally by bulging (Ambily and Gandhi, 2007).

Full fixity is assigned at the base of the geometry while roller conditions at the vertical sides where the displacement of the model is restricted in horizontal direction whilst it is free in vertical direction are generated as the boundary conditions of the model.

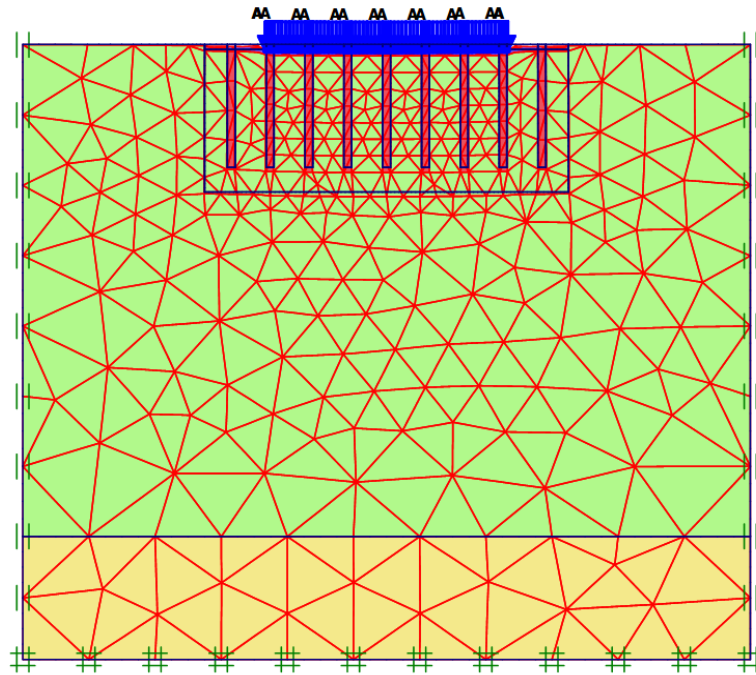


Figure 3.1 Finite Element Mesh of Rigid Foundation Analysis Supported by Stone Columns Having $L/H=0.25$

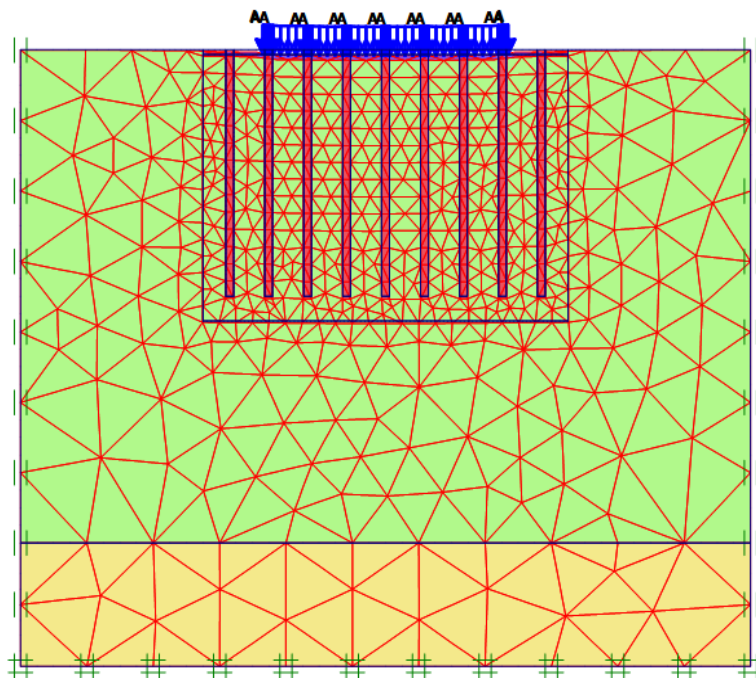


Figure 3.2 Finite Element Mesh of Flexible Foundation Analysis Supported by Stone Columns Having $L/H=0.50$

In plane strain analysis the geometry of stone column should be converted from axisymmetric unit cell model to plane strain model. As stated by Tan et al. (2008), there are two alternatives to do this conversion. The first method is based on the preservation of the composite stiffness of the improved soil same in both of the models - axisymmetric and plane strain.

The following formula is used to match the stiffness values of the composite models in axisymmetric and plane strain conditions.

$$E_{c,2D}a_{s,2D} + E_{s,2D}(1 - a_{s,2D}) = E_{c,3D}a_{s,3D} + E_{s,3D}(1 - a_{s,3D}) \quad (3.1)$$

where E_c and E_s are the modulus of elasticity of the column material and the surrounding soil, respectively. The plane strain 2D and the axisymmetric 3D (real) cases are denoted by the subscripts, 2D and 3D respectively. a_s is the area ratio defined as $a_s = A_c / (A_c + A_s)$ where A_c is the column and A_s is the surrounding soil cross-sectional areas.

If the plane strain column (wall) width is taken equal to the diameter of the column, the modulus of elasticity values should be modified to provide the same column-soil composite stiffness's. For the sake of simplicity, the column stiffness could be taken equal in both models, axisymmetric and plane strain; therefore, the elastic moduli of the soil can be determined accordingly.

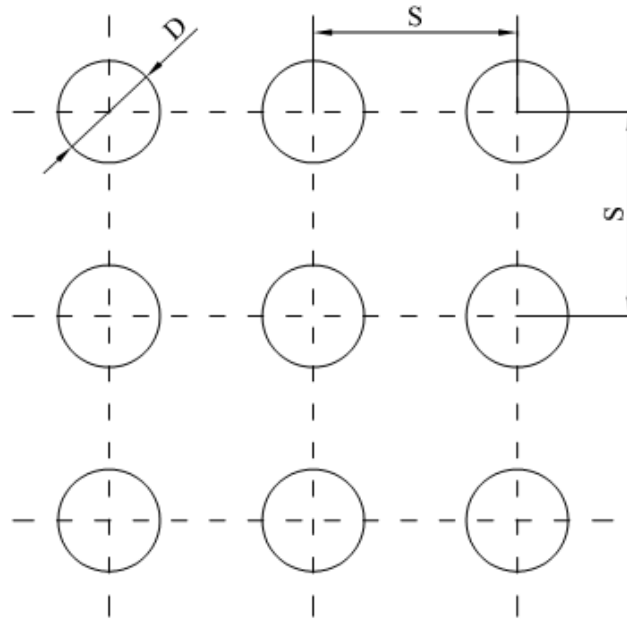
In the second alternative, the material properties of the column and the surrounding soil are kept constant; hence, the geometry of the model is adjusted such that the area ratio of the stone column surrounded by the soil in plane strain model is equal to the area ratio in the axisymmetric model.

In this study the second method is utilized. The following relation is used to determine the stone trench width.

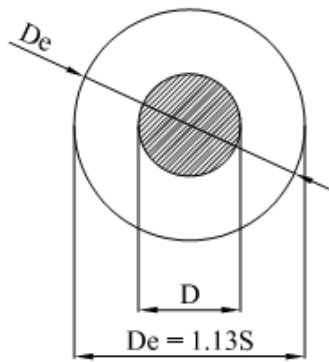
$$\frac{\pi D^2}{4S^2} = \frac{t}{S} \quad \rightarrow \quad t = \frac{\pi D^2}{4S} = \frac{\pi 0.8^2}{4 \cdot 1.6} \cong 0.32 \text{ m (stone wall width)} \quad (3.2)$$

0.32 m stone wall width makes the area ratio equal to exactly 0.20.

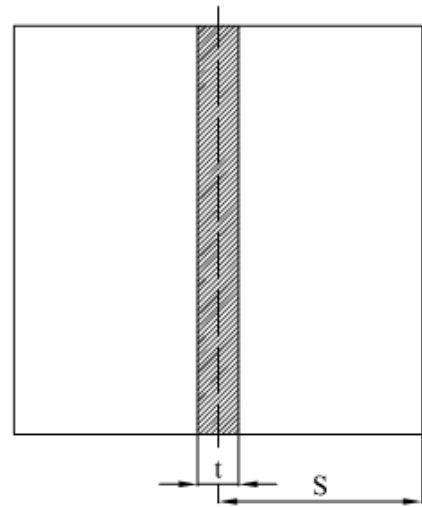
In the following figures the idealization of stone column groups in plane strain modelling and the geometric properties of the numerical models studied are given.



a) Stone Column Group in a Square Pattern



b) Axisymmetric Model



c) Plane Strain Model

Figure 3.3 Idealization of Stone Column Group in Plain Strain Model

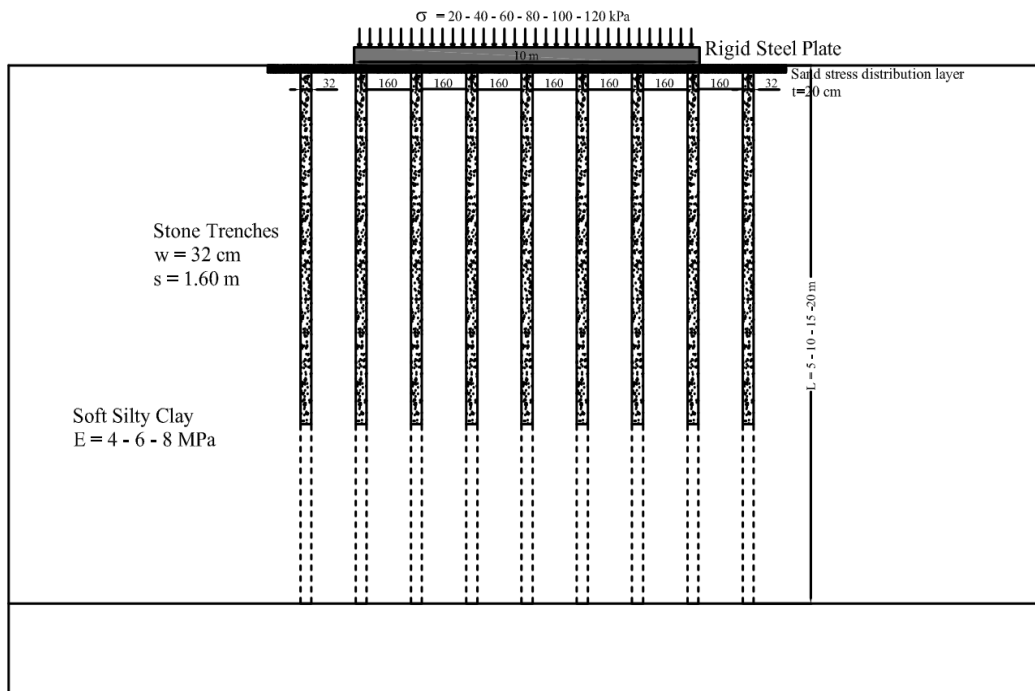


Figure 3.4 Geometric model properties of rigid foundation finite element analyses

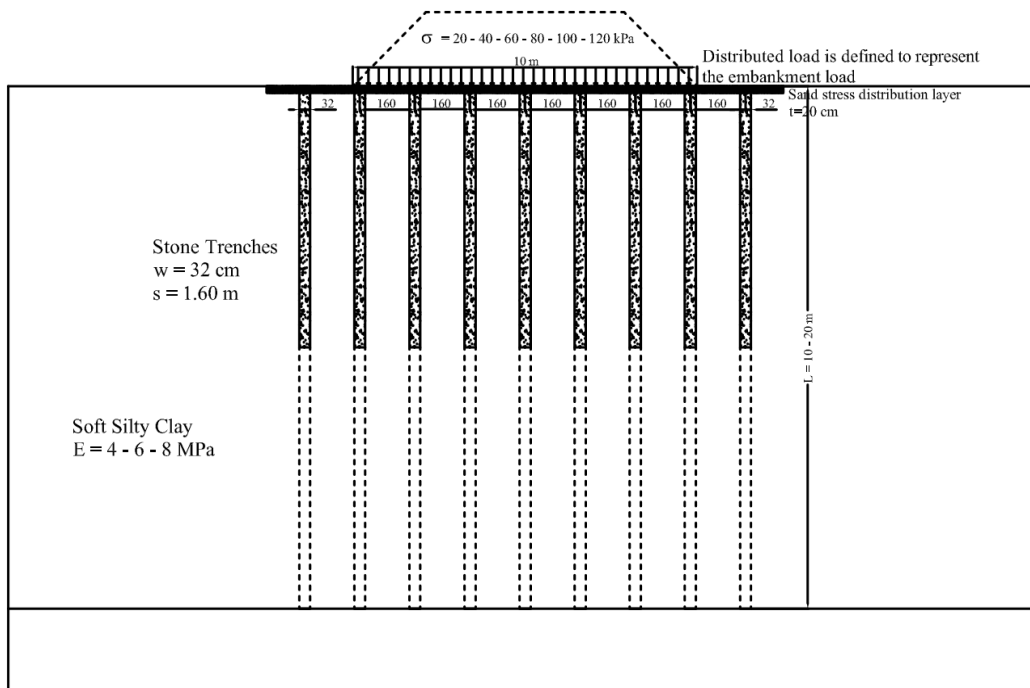


Figure 3.5 Geometric model properties of flexible foundation finite element analyses

3.2.2 Material Modelling

The choice of the material properties is essential in a numerical analysis to represent the system accurately. In the scope of this study, drained case for all materials including the soft silty clay, clayey sand, stones and the sand drainage-stress distribution layer is used for the generation of the finite element model. Mohr-Coulomb's failure criterion considering the elasto-plastic behaviour is utilized for all materials. The input parameters of the materials are found from the literature for soft silty clay, sand, stones and clayey sand layer (Özkeskin 2004, Kuruoğlu 2008; Zahmatkesh and Choobbasti 2010; Ambily and Gandhi 2007; Barksdale and Bachus 1983). Angle of dilatancy of the cohesionless soils are defined as " $\Psi = \emptyset - 30^\circ$ " as given in Brinkgreve (2008). The parameters of the Mohr-Coulomb model are given in Table 3.1.

In finite element analysis the initial stresses of the unimproved soil is calculated by the Jacky (1944) formula;

$$k_0 = 1 - \sin \emptyset \quad (3.3)$$

Initial stresses play an essential role in the reinforced ground numerical model since installation of stone columns into soft ground increases lateral stresses to higher values. As stated before, there are some investigations on the coefficient of lateral earth pressure increased due to column installation (Barksdale and Bachus 1983; Priebe 1995; Elshazly et al. 2008; Zahmatkesh and Choobbasti, 2010). In the scope of this study, k_0 values of the soft silty clay layer around the stone columns are taken as 1.0 due to improvement effect of column installation.

Table 3.1 Mohr-Coulomb Model Parameters Used in the FEM Analysis

<i>Soil Type</i>	γ_{unsat} (<i>kN/m³</i>)	γ_{sat} (<i>kN/m³</i>)	E' (<i>MPa</i>)	ν'	c' (<i>kPa</i>)	\emptyset' ($^\circ$)	Ψ ($^\circ$)	R_{inter}
Silty clay	17	18	4-6-8	0.35	5	24	0	0.67
Stone column	20	20	40	0.3	1	42	12	0.90
Clayey sand	20	20	50	0.3	1	40	10	0.80
Sand	18	18	20	0.3	1	32	2	1.00

CHAPTER 4

RESULTS OF THE PARAMETRIC STUDY

4.1 Introduction

Key parameters are defined to show the change of the stress distribution and settlement behaviour of the stone column improved soil with those parameters; i.e. the modulus of elasticity of the clay surrounding the stone columns, the stone column length and the pressure on the stone column system. The first set of numerical analyses is carried out on rigid foundation to define the settlement reduction ratio and the stress concentration factor. Three floating columns are analysed with lengths of 5, 10 and 15 meters in 20-m-thick soft silty clay and one end bearing column analyses are completed in three different modulus of elasticity values of clay; i.e. 4000, 6000 and 8000 kPa at six foundation pressure values increasing from 20 to 120 kPa. Then, the study focuses on the stress concentration ratio values. An embankment loading is defined to determine the behaviour of stone column system in soft clays to compare the stress concentration factor of rigid and flexible foundation analyses. The flexible loading parametric study are carried out for one floating and one end bearing stone column alternatives with 10 and 20 m length, respectively at the same stress levels and in soils having modulus of elasticity values as 4000-6000-8000 kPa.

Table 4.1 Key Parameters of the parametric study for rigid and flexible foundation analyses

Key Parameters of Rigid Foundation Analyses						
Modulus of Elasticity of Soil (kPa)	4000	6000	8000			
Rigid Foundation Pressure (kPa)	20	40	60	80	100	120
Ratio of Stone Column Length to Clay Layer Thickness (L/H)	0.25	0.50	0.75	1.00		

Key Parameters of Flexible Foundation Analyses						
Modulus of Elasticity of Soil (kPa)	4000	6000	8000			
Flexible Foundation Pressure (kPa)	20	40	60	80	100	120
Ratio of Stone Column Length to Clay Layer Thickness (L/H)	0.50	1.00				

4.2 Results of Rigid Foundation Analyses

2-D finite element analyses are carried out to define the improvement of 20 m-thick-soft silty clay with stone columns under a rigid plate staged loading. At first unimproved soil is loaded with a distributed load of 20-40-60-80-100-120 kPa stages on a 10-m-width rigid steel plate having 10 cm thickness and 2×10^8 kPa modulus of elasticity. The deformation behaviour of the unimproved soil is analysed by changing the modulus of elasticity values as 4000, 6000 and 8000 kPa. The geometry of the numerical model is given in Figure 4.1.

A set of reinforced ground with stone column analyses are carried out to define both deformation and stress concentration values. Group 1 loading consists of 5-m-length stone column analyses. The modulus of elasticity of soft silty clay is changed as 4000-6000-8000 kPa; hence three analyses are completed for 5m-length column. Also 6 loading stages are defined to investigate the behaviour of improved soil under changing stresses (20-40-60-80-100-120 kPa). Therefore, 18 results of analyses are obtained for the first group loading.

Similarly, 3 sets of analyses are completed with 10 m, 15 m and 20 m column installation. The first three sets represent floating stone column behaviour, while the last set, 20 m-length-stone column analyses, represents the end bearing condition. In total, results of 144 analyses are presented in terms of settlement reduction ratio and stress concentration factor.

For the selected cases including the rigid and flexible foundation analyses which L/H ratio equals to 0.50, modulus of elasticity of clay layer equals to 6000 kPa at foundation pressure of 120 kPa; the results of the settlements, effective stresses and pore pressures are given in Appendix-A.

The change of stress concentration ratio is given under changing column length, foundation pressures and modulus of elasticity of clay. Stresses are measured just below the 20 cm-thick-sand layer. Settlement reduction ratio is also presented under similar conditions. The stress concentration and settlement reduction ratios are calculated by the following formulas:

$$\text{Stress concentration factor} = n = \frac{\sigma_c}{\sigma_s} = \frac{\text{Stress on stone column}}{\text{Stress on surrounding soil}} \quad (4.1)$$

$$\text{Settlement reduction ratio} = SRR = \frac{\text{Settlement of the soil after improvement}}{\text{Settlement of unimproved soil under the same stress}} \quad (4.2)$$

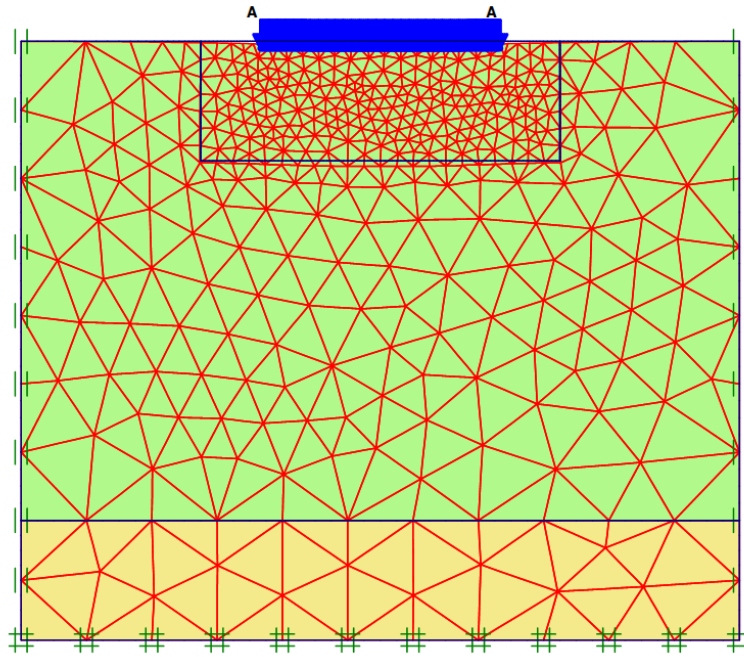


Figure 4.1 Finite Element Mesh of Unimproved Soil by Plaxis 2-D Software

The stress concentration factor is derived from the results of the effective normal stress distribution graph obtained in finite element analysis by taking a cross section just under the sand distribution layer; i.e. 20 cm below the ground surface. The average stresses carried by the columns and the surrounding soil are calculated by dividing the area under the plot of effective normal stress vs. width of the columns to the total column width and dividing the area under the same plot for soil between the columns to the total soil width, respectively.

A representative calculation is given for the numerical model of stone column having ratio of stone column length to the soft clay layer thickness (L/H) as 0.50 improving the soil having the modulus of elasticity as 6000 kPa at foundation pressure 120 kPa for both rigid and foundation analyses.

For rigid foundation analyses, the area under the effective normal stress vs. width of the model is calculated for stone columns and clay surrounding the columns separately. The areas for stone columns are 45.094, 94.503, 92.727, 92.556, 94.407, 91.330 and 45.945 kN/m from left edge to the right as given in Figure 4.2 and the sum of the values are divided to the total width of the columns, (7x0.32m), resulting in 248.46 kPa stress in the columns. The similar calculations are done for the soil between the columns. The total area is the sum of 116.499, 106.693, 109.610, 108.607, 103.166 and 115.256 kN/m. By dividing the total area under the soil to the total soil width of (6x1.28m) the average calculated soil stress is 85.92 kPa. Thus, the stress concentration factor;

$$n = \frac{\sigma_c}{\sigma_s} = \frac{248.46}{85.92} = 2.89 \quad (4.3)$$

Similarly, flexible foundation analyses are examined for the stress concentration factor calculation. The area under the effective normal stress vs. width of the model is calculated for stone columns and soil surrounding the columns separately. The calculated trapezoidal areas for stone columns are 51.957, 82.680, 85.256, 84.416, 85.490, 83.341 and 45.191 kN/m from left edge to the right as given in Figure 4.3 and the sum of the values are divided to the total width of the columns, (7x0.32m), resulting in 231.40 kPa stress in the columns. The similar calculations are done for the soil between the columns. The total area is the sum of 141.270, 128.509, 130.049, 129.981, 130.774 and 144.719 kN/m. By dividing the total area under the soil to the total soil width of (6x1.28m) the average calculated soil stress is 104.86 kPa. Thus, the stress concentration factor;

$$n = \frac{\sigma_c}{\sigma_s} = \frac{231.40}{104.86} = 2.21 \tag{4.4}$$

Detailed calculations are given in Appendix B for these two cases. In the following figures red lines show the effective normal stress distribution graph obtained by Plaxis software while the black arrows represent the calculated average values.

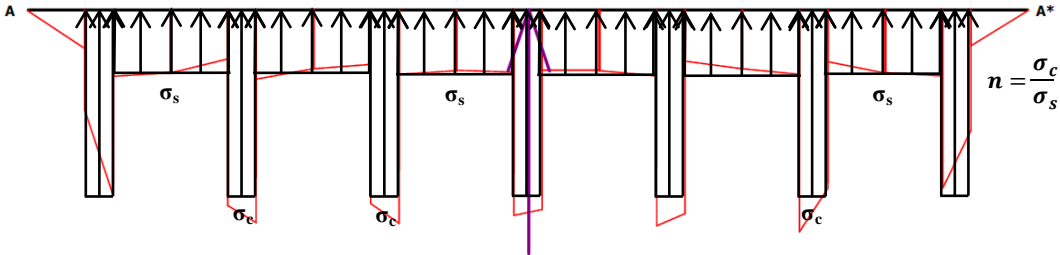


Figure 4.2 Distribution of stresses between stone column and soil (For the model L/H=0.50 stone column improving the soil having E=6000 kPa at 120 kPa rigid foundation pressure)

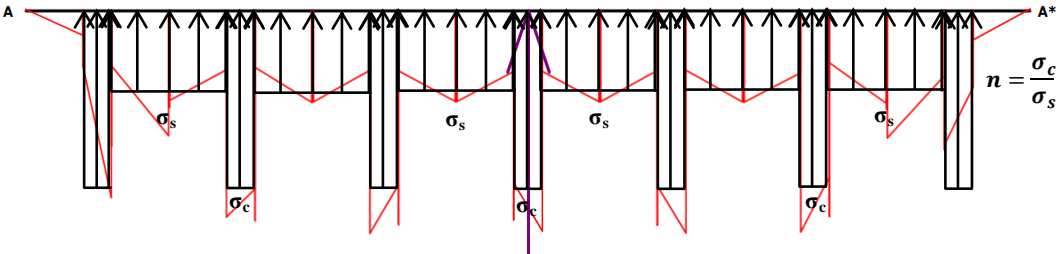


Figure 4.3 Distribution of stresses between stone column and soil (For the model L/H=0.50 stone column improving the soil having E=6000 kPa at 120 kPa flexible foundation pressure)

It is important to note that the edge stone columns generally carry smaller loads. It is dependent on the stress level, so variable behaviour could be observed at different foundation pressures. However, as a general trend it could be stated that the edge columns under rigid and flexible foundations are loaded less than the center and intermediate columns; thus the stress concentration factor is higher when the edge columns are ignored. In the given example above for the rigid foundation analyses, the stress concentration factor is determined as 2.89 when the average stresses are calculated from all of the seven columns. However, it is calculated as 3.39 when the edge columns are ignored in column stress computations while the factor is found as 3.37 in the center column. In flexible foundation analyses, the similar findings are derived in terms of the stresses carried by the center and edge columns. The stress concentration factor is determined as 2.21 by taking the average of all column stresses while it increases to 2.52 in center columns. The following table summarizes the comparison of stress concentration factors obtained from the average stresses on the columns, center columns and ignoring the edge columns for one case of rigid and flexible foundation analyses. It should be noted that, in this study the average stresses on the columns are considered to calculate the stress concentration factor.

Table 4.2 Comparison of Stress Concentration Factors for the Numerical Models of Stone Columns Having L/H=0.50 in Clay Having Modulus of Elasticity as 6000 kPa at 120 kPa Pressure

Stress on the Stone Column	Stress Concentration Factor (n)	
	Rigid Foundation Analyses	Flexible Foundation Analyses
Average	2.89	2.21
Average except edge columns	3.39	2.51
Center	3.37	2.52

4.2.1 Effect of Modulus of Elasticity of Soil on the Stress Concentration Factor

The stress concentration factor decreases with increasing modulus of elasticity of the soft silty clay layer, E_{soil} , as it is shown in Figure 4.4. The graphs are given for different L/H ratios where “L” represents the column length and “H” represents the soft clay layer thickness. The decreasing effect is more pronounced in lower stress levels.

For the analyses of improved soft clay with stone columns having the ratio L/H as 0.25, the upper boundary is composed of the change of stress concentration factor, n, with the modulus of elasticity of soil at 20 kPa foundation pressure. The stress concentration factor decreases linearly from 4.05 to 3.00 as the modulus of elasticity of clay is increased from 4000 kPa to 8000 kPa. The lower boundary belongs to the line obtained at 120 kPa

foundation pressure. The stress concentration ratio decreases linearly from 3.34 to 2.64 as the modulus of elasticity of clay is increased from 4000 kPa to 8000 kPa. Between the stress levels of 20 and 120 kPa, the lines of n vs. E_{soil} fall between the upper and lower boundaries showing similar behaviour. The slope of the lines decreases from 0.263 to 0.175 MPa^{-1} as the foundation pressure increases from 20 to 120 kPa.

In the analyses having L/H ratio as 0.50, the stress concentration factor decreases linearly from 4.30 to 3.06 while the modulus of elasticity of clay is increasing from 4000 kPa to 8000 kPa at 20 kPa foundation pressure. It changes between and 3.28 to 2.57 as the modulus of elasticity of clay is increasing from 4000 kPa to 8000 kPa at 120 kPa foundation pressure. Between the stress levels of 20 and 120 kPa, the lines of n vs. E_{soil} show similar behaviour with the upper and lower boundaries. The slope of the lines decreases from 0.310 to 0.178 MPa^{-1} .

Stone column analyses having the ratio L/H equal to 0.75, the upper boundary is composed by the line obtained at 20 kPa foundation pressure. The stress concentration factor decreases linearly from 4.51 to 2.98 as the modulus of elasticity of clay increases from 4000 kPa to 8000 kPa. The lower boundary is composed by the line obtained at 120 kPa foundation pressure. The stress concentration ratio decreases linearly from 3.33 to 2.53 while the modulus of elasticity of clay increases from 4000 kPa to 8000 kPa. Between the stress levels of 20 and 120 kPa, the lines of n vs. E_{soil} fall between the upper and lower boundaries showing similar behaviour. The slope of the lines decreases from 0.383 to 0.200 MPa^{-1} .

In end bearing column analyses, i.e. L/H equals to 1.00, the stress concentration factor decreases linearly from 4.57 to 2.95 as the modulus of elasticity of clay increases from 4000 kPa to 8000 kPa at 20 kPa pressure. It should be noted that the lowest value of the stress concentration factor, 2.95, of the upper boundary falls under the lowest value obtained as 3.04 at 40 kPa. At 120 kPa foundation pressure, the stress concentration ratio decreases linearly from 3.54 to 2.74 while the modulus of elasticity of clay is increasing from 4000 kPa to 8000 kPa. Between the stress levels of 20 and 120 kPa, the lines show similar behaviour and the slope of the lines is decreasing from 0.405 to 0.200 MPa^{-1} .

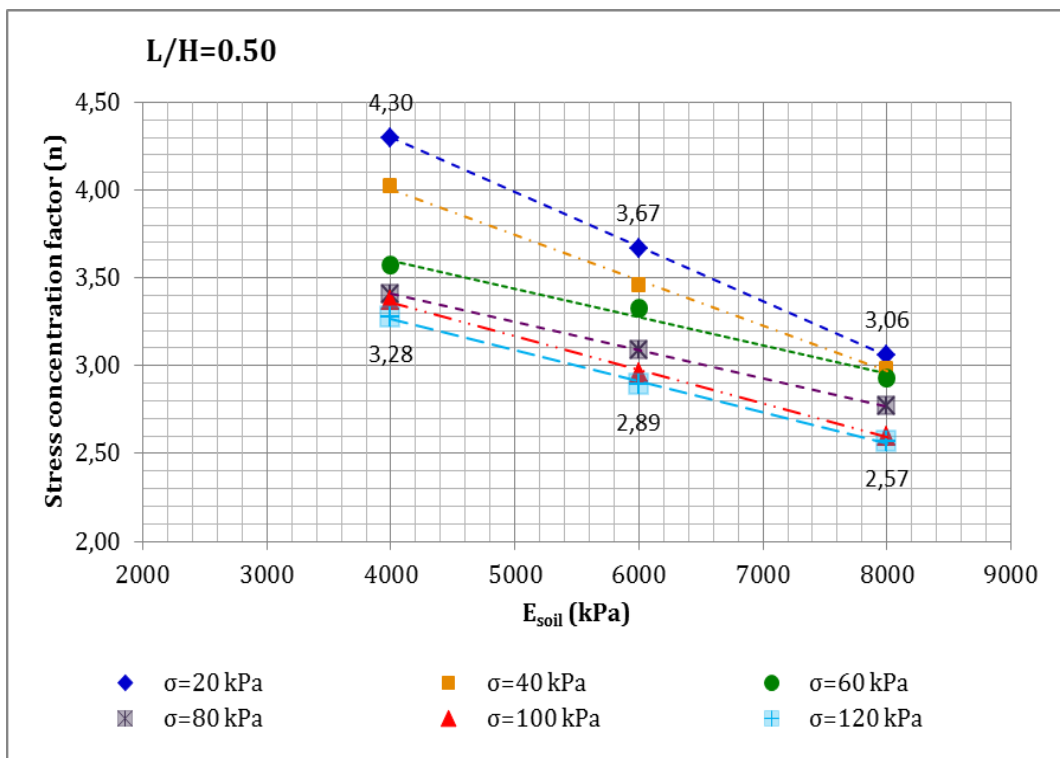
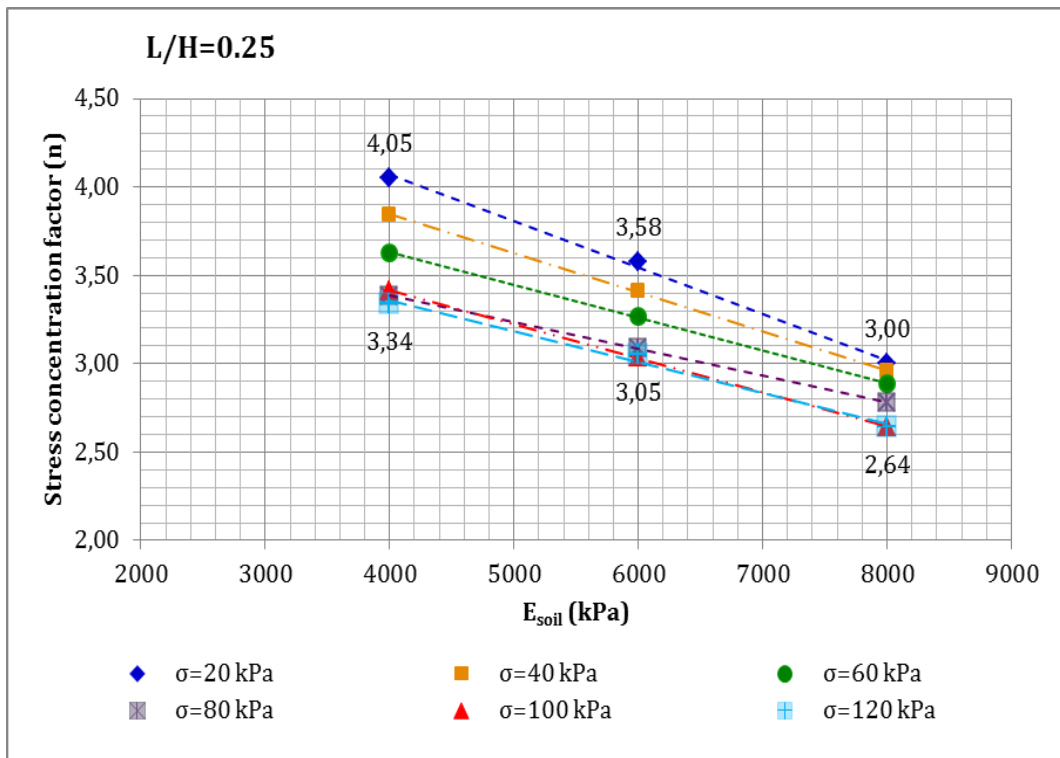


Figure 4.4 Stress Concentration Factor vs. Modulus of Elasticity of Soft Silty Clay Layer Charts at $\sigma = 20\sim 120$ kPa Foundation Pressure

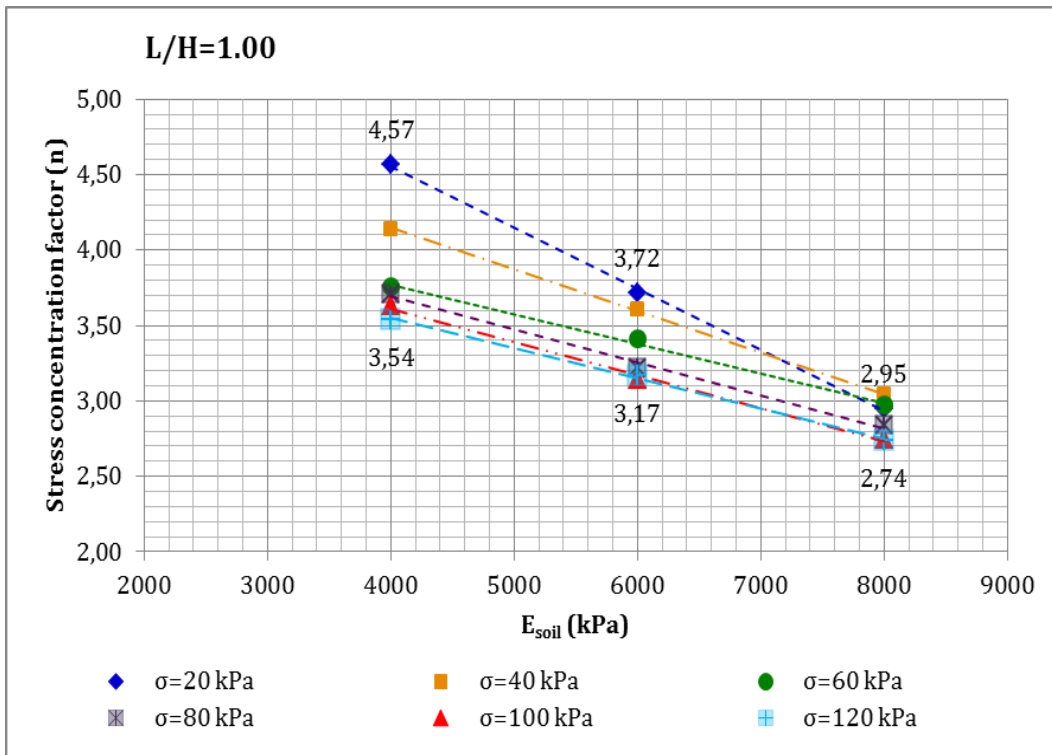
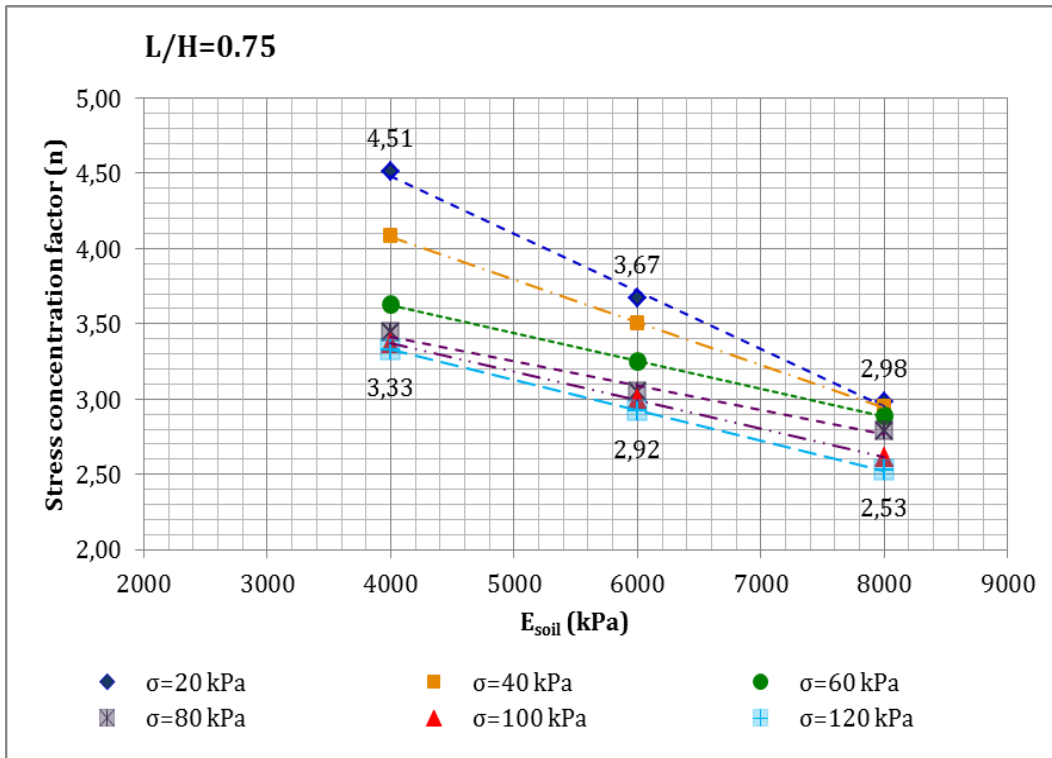


Figure 4.4 (Cont'd)

4.2.2 Effect of Stone Column Length on the Stress Concentration Factor

Floating columns give similar results for stress concentration ratio while the ratio increases in end bearing columns as it is shown in Figure 4.5. The difference is more significant at higher stress levels.

At 40 kPa foundation pressure the difference between the floating and end bearing columns is not significant for the analyses of stone columns improving the clay having 4000 kPa modulus of elasticity value. For $E_s=4$ MPa, the stress concentration ratio changes from 3.84 to 4.08 in floating columns while it increases to 4.14 in end bearing columns. For $E_s=6$ MPa, the stress concentration factor changes between 3.41 and 3.50 whilst it increases to 3.61 in end bearing columns. For $E_s=8$ MPa, the stress concentration factor is almost the same for three results obtained from floating column analyses as it is changing between 2.98 and 2.95. The ratio increases to 3.04 for $L/H=1.00$ analysis; i.e. end bearing columns.

Stress concentration factors are nearly constant in three analyses of different modulus of elasticity values of soil for floating columns at 60 kPa foundation pressure. The ratio changes between 3.57 and 3.63; 3.25 and 3.33; 2.89 and 2.93 for the analyses obtained in improved clay having the modulus of elasticity value as 4000, 6000 and 8000 kPa, respectively. In the same order, the end bearing columns have the stress concentration factors as 3.76, 3.41 and 2.97.

At 80 kPa foundation pressure the stress concentration factors of floating columns; i.e. L/H equals to 0.25, 0.50 and 0.75, range between 3.38 and 3.44; 3.05 and 3.09; 2.77 and 2.79 for the analyses obtained in improved soft clay having the modulus of elasticity value as 4000, 6000 and 8000 kPa, respectively. The stress concentration ratios of end bearing columns increase to 3.71, 3.22 and 2.84, respectively.

The stress concentration factors of floating columns at 100 kPa pressure range between 3.37 and 3.41; 2.96 and 3.04; 2.60 and 2.64 for clay having the modulus of elasticity value equal to 4000, 6000 and 8000 kPa, respectively. In the same order, the end bearing columns have the stress concentration factor as 3.63, 3.14 and 2.75.

The stress concentration factors change between 3.28 and 3.34; 2.89 and 3.05; 2.53 and 2.64 in floating columns at 120 kPa foundation pressure and in clay having the modulus of elasticity value as 4000, 6000 and 8000 kPa, respectively. The stress concentration factor increases to 3.54, 3.17 and 2.74 values in end bearing columns, respectively.

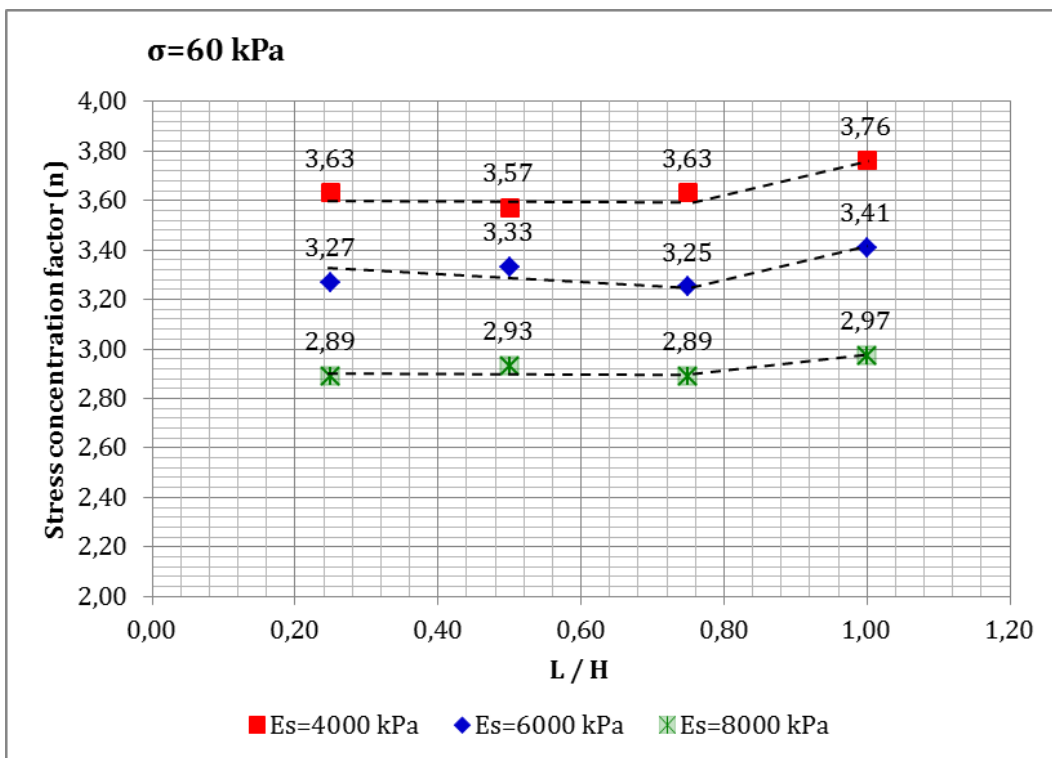
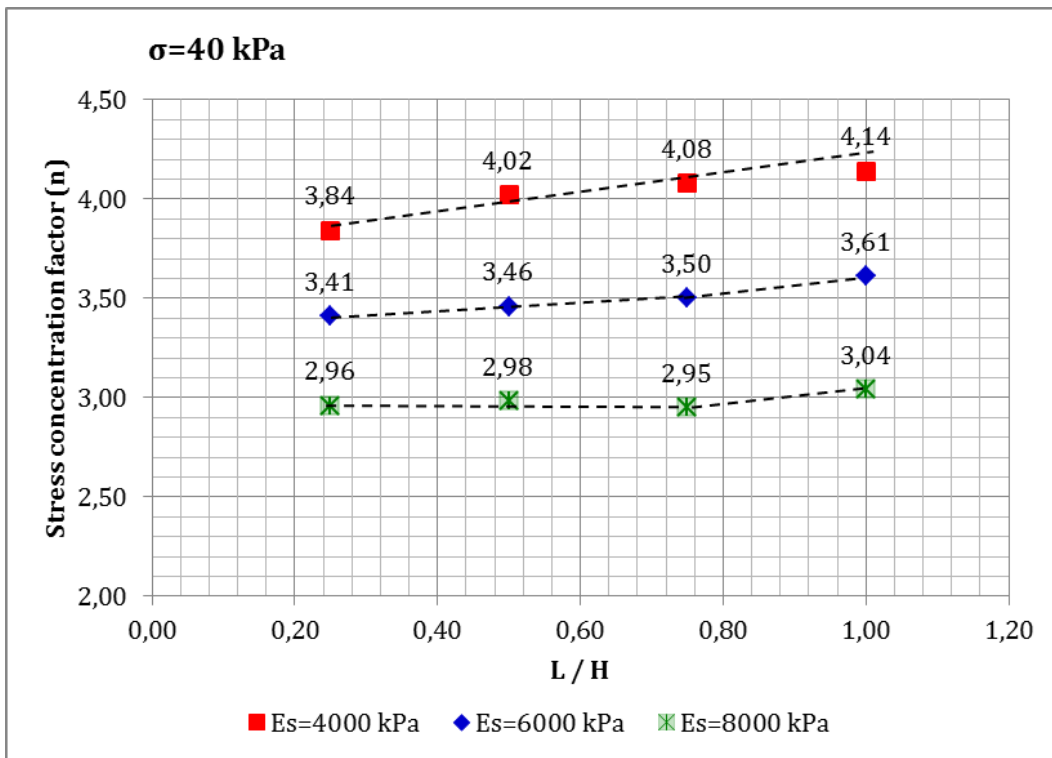


Figure 4.5 Stress Concentration Factor vs. Ratio of Stone Column Length to Clay Layer Thickness (L/H) Charts with $E_s = 4000\sim 8000$ kPa

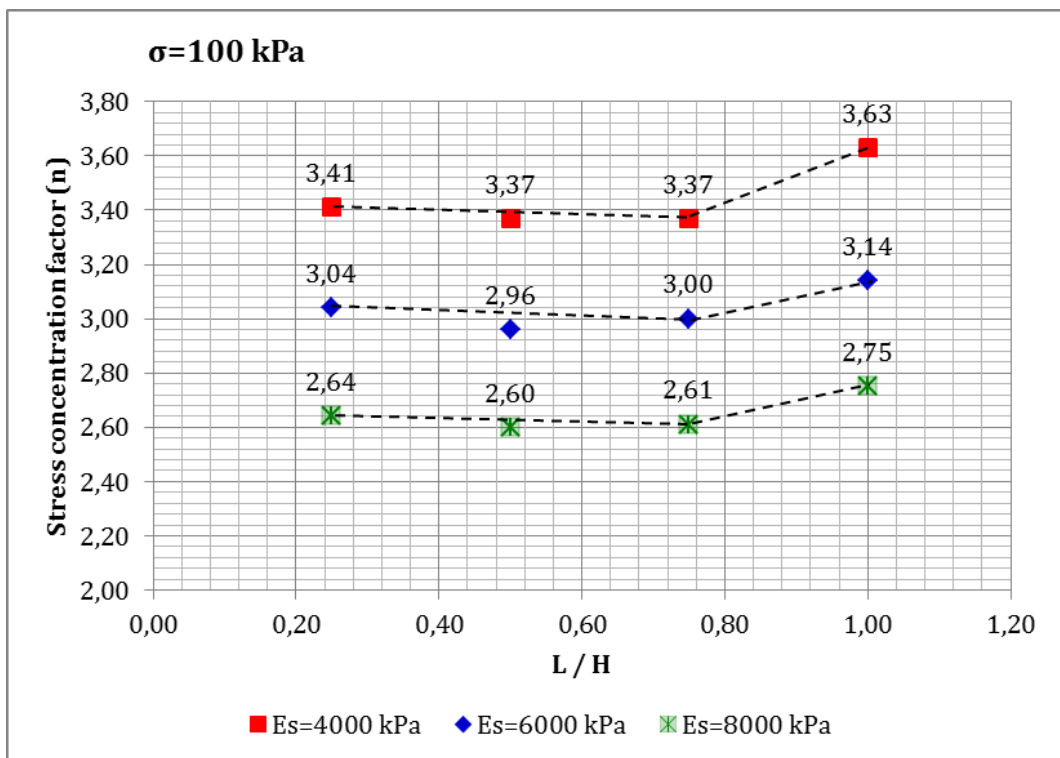
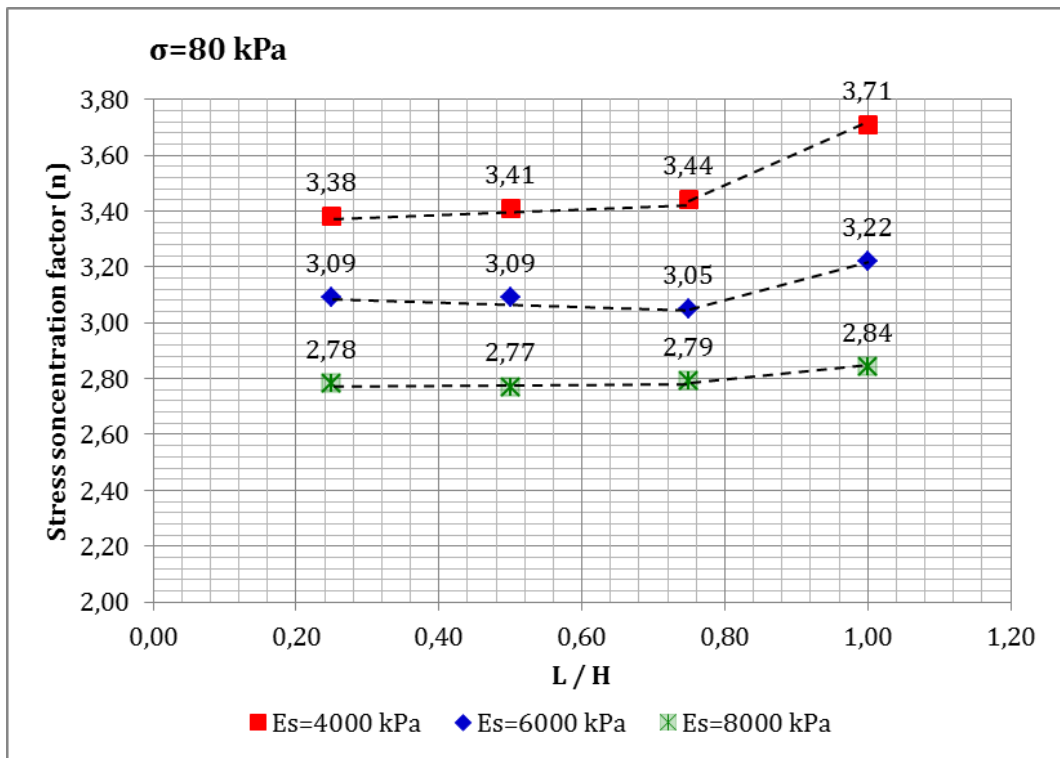


Figure 4.5 (Cont'd)

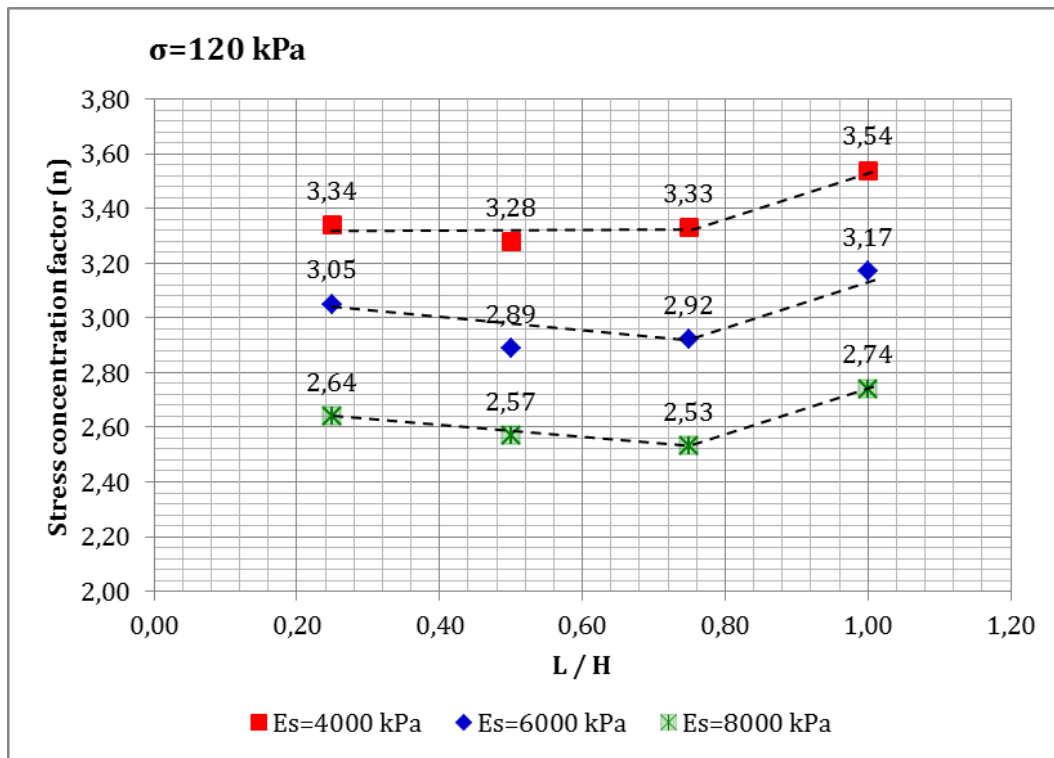


Figure 4.5 (Cont'd)

4.2.3 Effect of Foundation Pressure on the Stress Concentration Factor

The stress concentration factor decreases with increasing foundation pressure as it is shown in Figure 4.6. The effect is more pronounced in soils having smaller modulus of elasticity values.

In the analyses having L/H ratio as 0.25, the stress concentration factor decreases from 4.05 to 3.34; 3.58 to 3.05; 3.00 to 2.64 at 20 and 120 kPa stress levels in the improved soil with modulus of elasticity value 4000 kPa, 6000 kPa and 8000 kPa, respectively. The stress concentration factor decreases from 4.30 to 3.28; 3.67 to 2.89; 3.06 to 2.57 at foundation pressures changing from 20 to 120 kPa in stone columns having L/H ratio equal to 0.50. For the stone column analyses having L/H ratio as 0.75, the stress concentration factor decreases from 4.51 to 3.33; 3.67 to 2.92; 2.98 to 2.53 as the foundation pressure increases from 20 to 120 kPa. The decreasing effect is less significant for higher modulus of elasticity of soil values.

In the end bearing column analyses; i.e. L/H equals to 1.00, the stress concentration factor decreases from 4.57 to 3.54; 3.72 to 3.17; 2.95 to 2.74 at 20 and 120 kPa foundation pressure for soil modulus of elasticity of 4000, 6000 and 8000 kPa, respectively. Decreasing effect is less significant for higher soil modulus of elasticity values; i.e. the difference decreases from 1.03 to 0.21 as the modulus of elasticity of clay increases from 4000 to 8000 kPa.

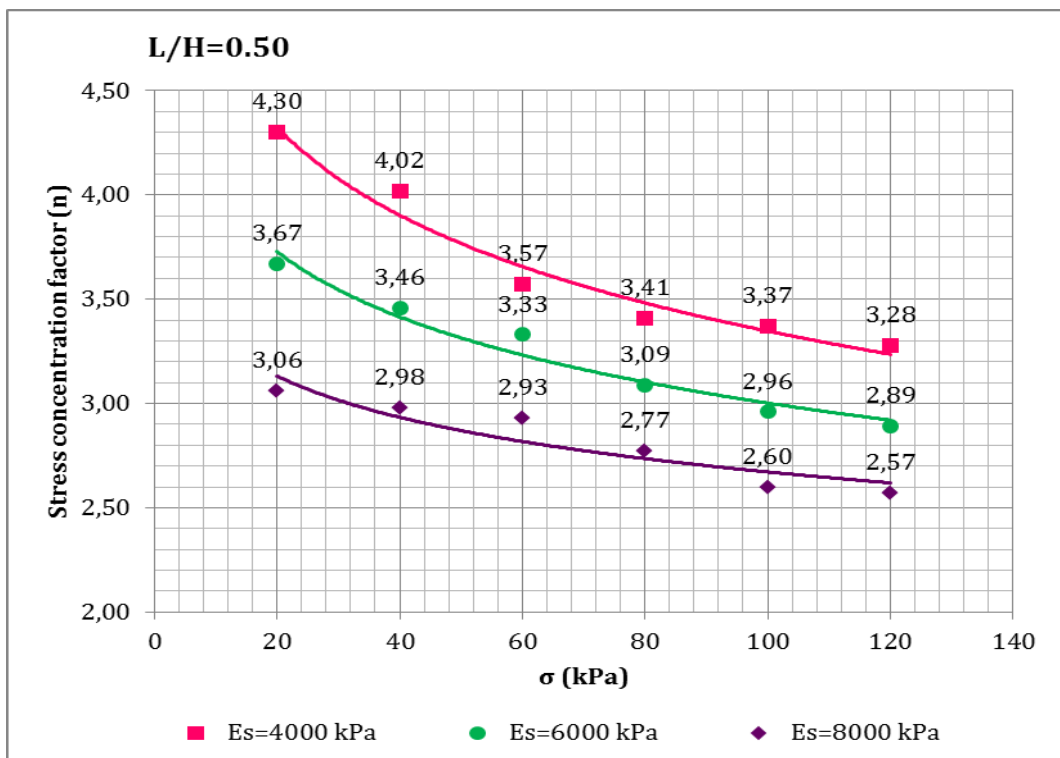
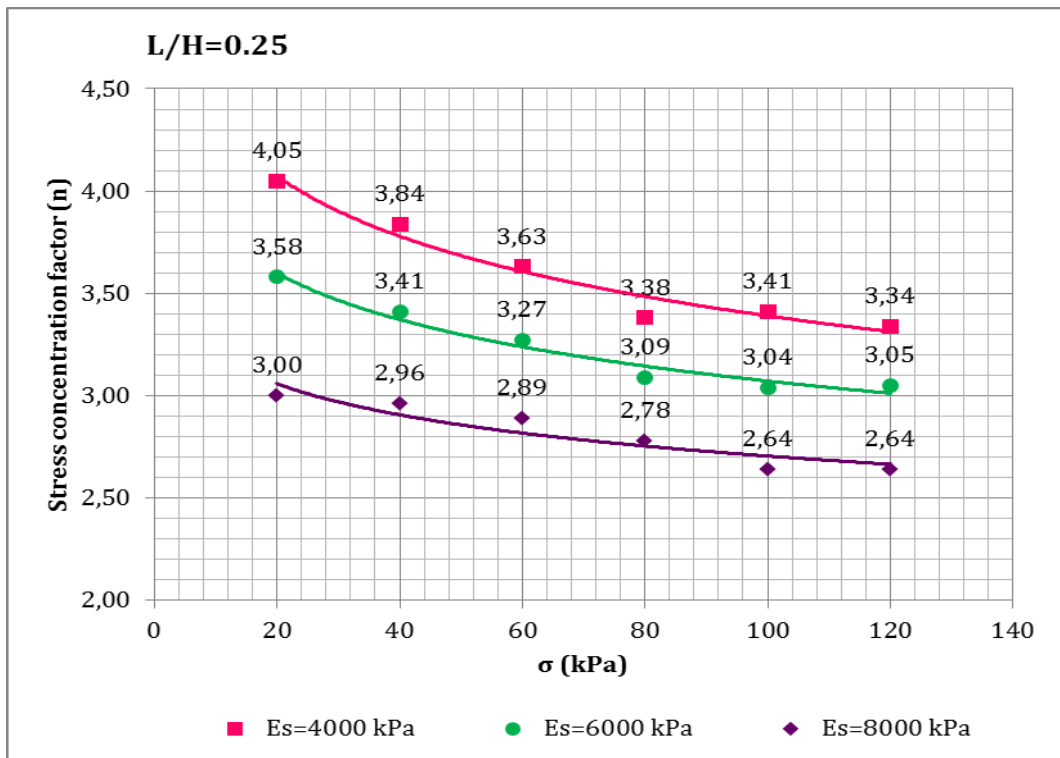


Figure 4.6 Stress Concentration Factor vs. Foundation Pressure Charts with $E_s = 4000\sim 8000$ kPa

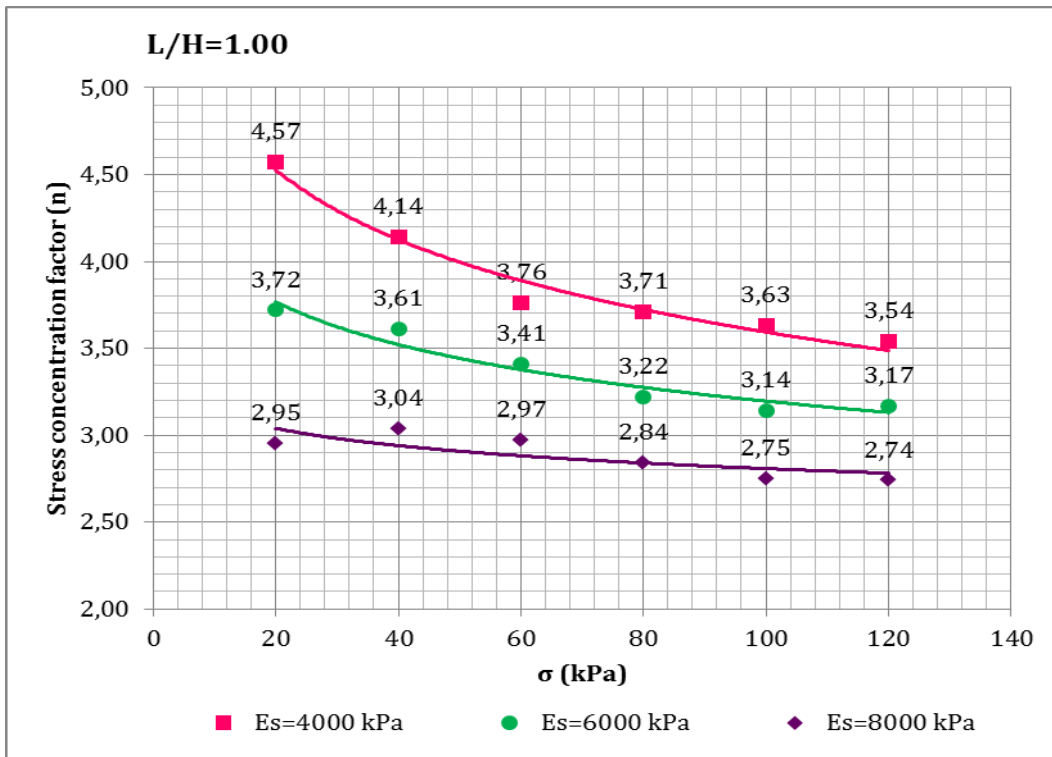
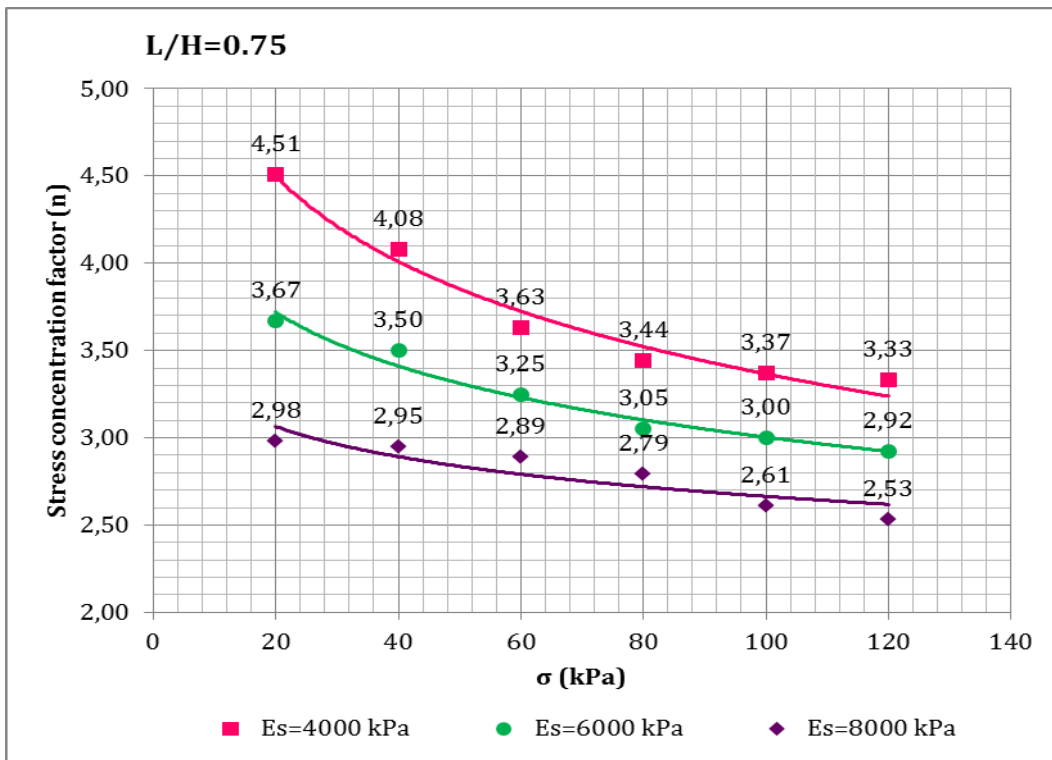


Figure 4.6 (Cont'd)

4.2.4 Effect of Modulus of Elasticity of Soil on the Settlement Reduction Ratio

The settlement reduction ratio slightly increases as the modulus of elasticity of the soft silty clay increases as it is shown in Figure 4.7. The increasing effect is more significant in end bearing columns than the floating columns. Nearly constant values are obtained for the shortest column analyses, i.e. stone columns having L/H ratio equal to 0.25.

At foundation pressure of 20 kPa, in the upper boundary the settlement reduction ratio remains nearly constant for L/H=0.25 analyses with an average value of 0.85. In the lower boundary for the analyses of end bearing columns, the settlement reduction ratio increases from 0.51 to 0.68 as the modulus of elasticity of clay increases from 4000 to 8000 kPa.

At 40 kPa foundation pressure, the upper boundary is composed of the L/H=0.25 analyses with an average value of the settlement reduction ratio equal to 0.86. The end bearing column analyses comprise the lower boundary that the settlement reduction ratio increases from 0.55 to 0.70 as the modulus of elasticity of clay increases from 4000 to 8000 kPa.

At 60 kPa foundation pressure, the settlement reduction ratio is averagely 0.87 in stone columns having L/H ratio as 0.25. In the analyses of end bearing columns, the settlement reduction ratio increases from 0.58 to 0.70 while the modulus of elasticity of clay ranges between 4000 and 8000 kPa.

The average value of the settlement reduction ratio is 0.86 at 80 kPa foundation pressure in L/H=0.25 analyses. In the lower boundary for the analyses of end bearing columns, the settlement reduction ratio increases linearly from 0.60 to 0.70 as the modulus of elasticity of clay increases from 4000 to 8000 kPa.

At the foundation pressure of 100 kPa, the average value of settlement reduction ratio is 0.86 for L/H=0.25 analyses. The ratio increases linearly from 0.60 to 0.68 in end bearing columns while the modulus of elasticity of clay is increasing from 4000 to 8000 kPa.

At 120 kPa foundation pressure, in the upper boundary composed of the L/H=0.25 analyses the settlement reduction ratio is averagely 0.85. In the lower boundary for the analyses of end bearing columns, the settlement reduction ratio increases linearly from 0.60 to 0.66 as the modulus of elasticity of clay increases from 4000 to 8000 kPa.

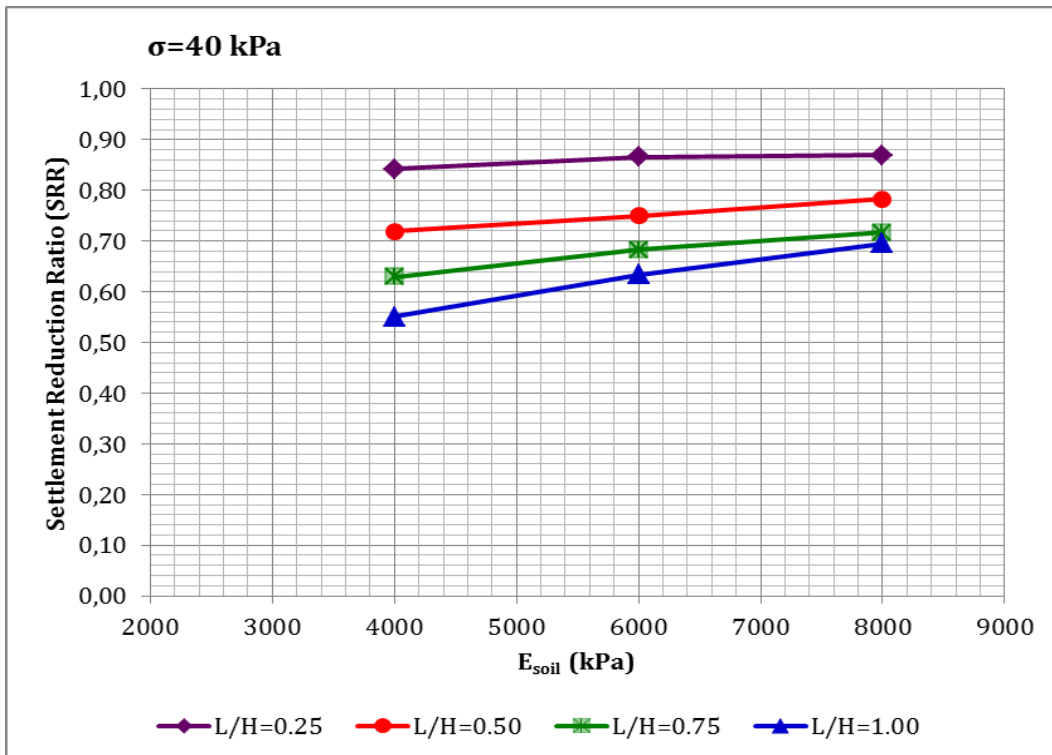
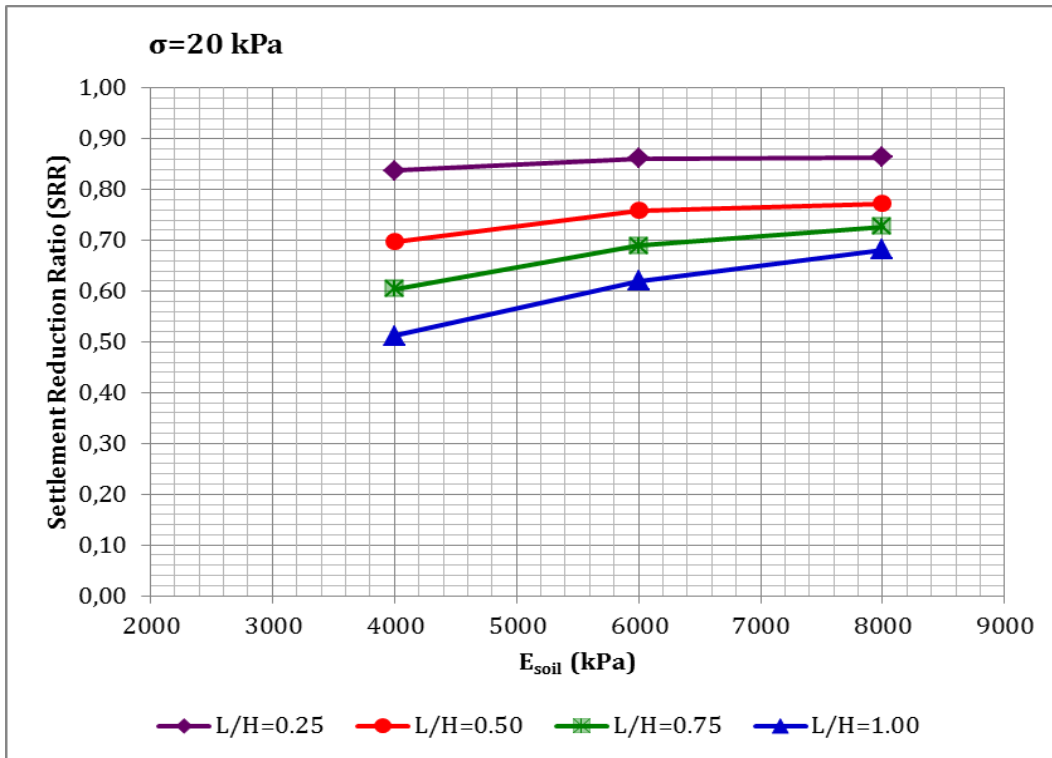


Figure 4.7 Settlement Reduction Ratio vs. Modulus of Elasticity of Soft Silty Clay Layer Charts with L/H = 0.25 ~ 1.00

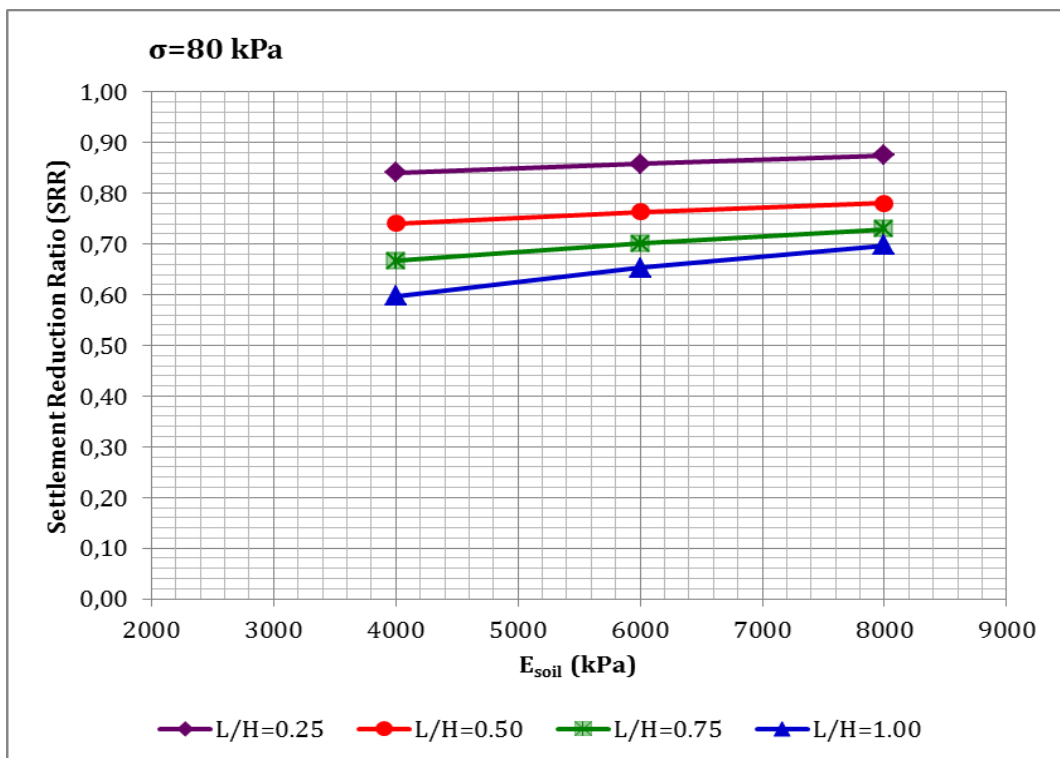
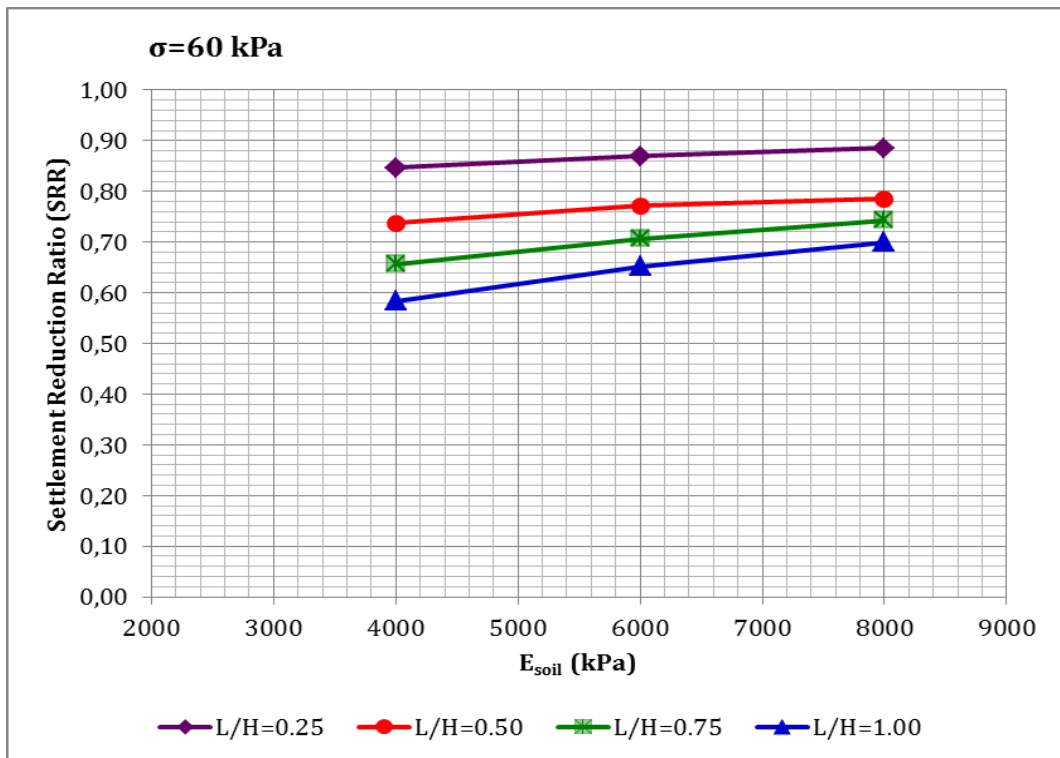


Figure 4.7 (Cont'd)

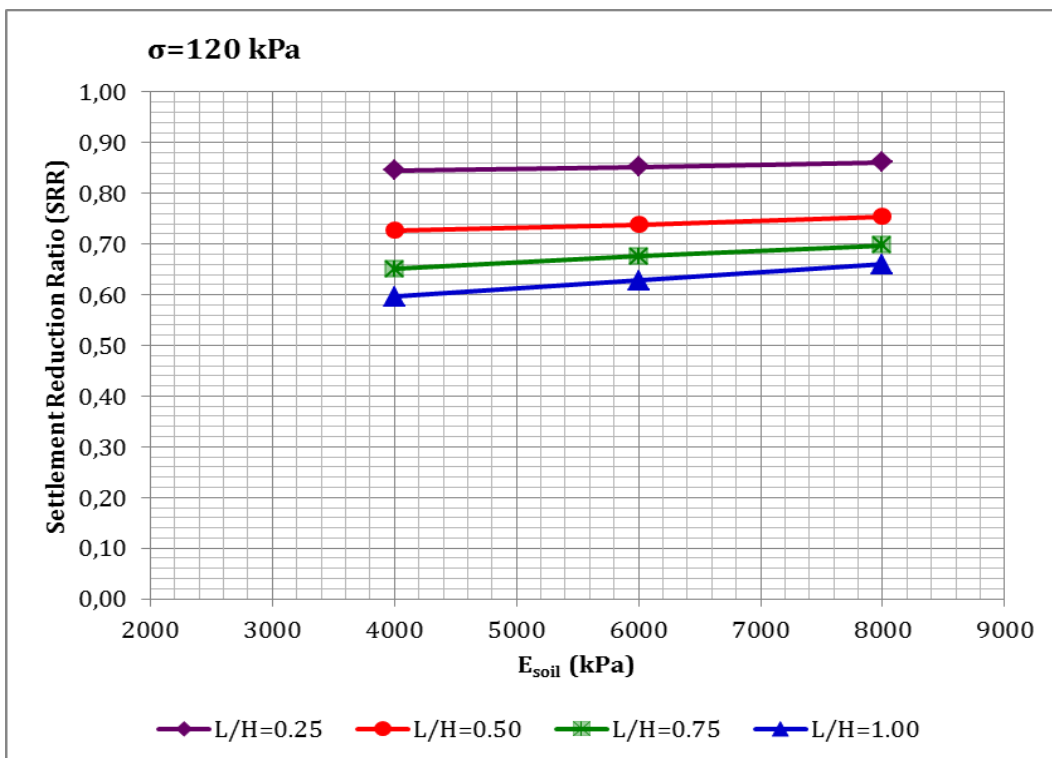
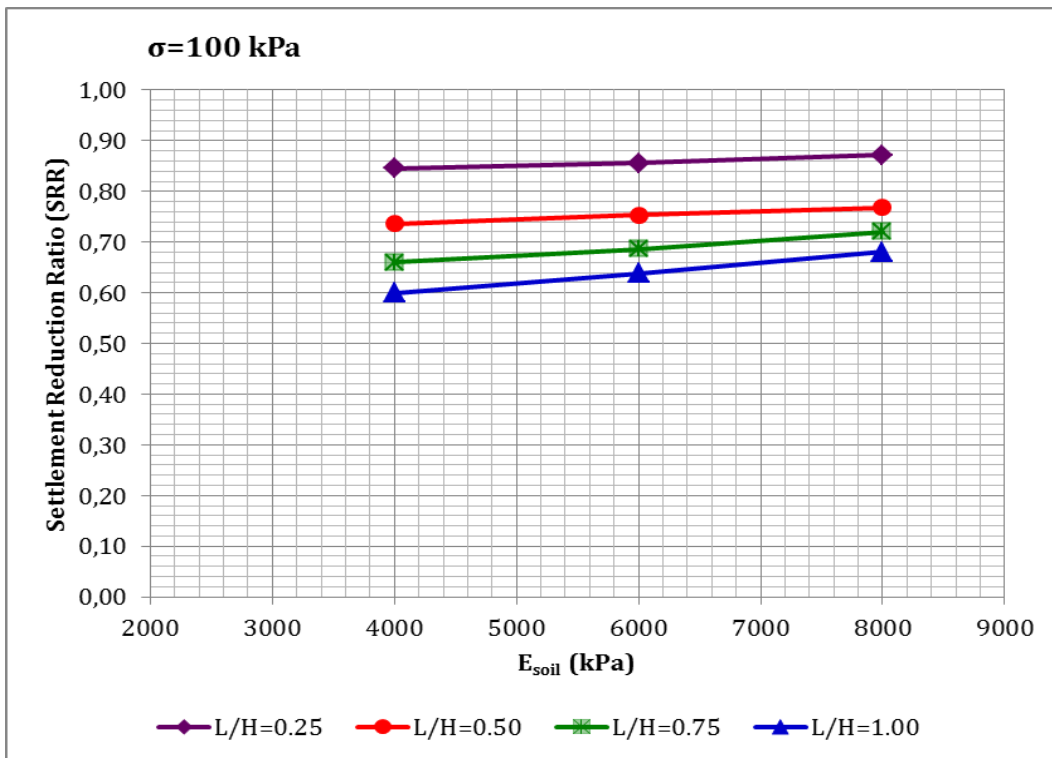


Figure 4.7 (Cont'd)

4.2.5 Effect of Stone Column Length on the Settlement Reduction Ratio

The settlement reduction ratio decreases with increasing stone column length as it is shown in Figure 4.8. The settlement reduction ratio reaches to a constant value as L/H ratio increases for soils having higher modulus of elasticity. There is no significant change in settlement reduction ratio at different pressures.

In the analyses of improved clay having modulus of elasticity equal to 4000 kPa, the settlement reduction ratio decreases from 0.84 to 0.51 at 20 kPa foundation pressure for L/H ratio as 0.25 and 1.00, respectively. There is no significant difference between the other stress levels. At 120 kPa pressure, the ratio decreases form 0.85 to 0.60 in the same order.

In clays having modulus of elasticity as 6000 kPa, the settlement reduction ratio shows nearly the same behaviour at different foundation pressures from 20 to 120 kPa. The maximum value of the ratio observed is 0.87 while the minimum is 0.62 as the L/H ratio is increasing from 0.25 to 1.00.

The change in the settlement reduction ratio is almost the same at different foundation pressures ranging between 20 and 120 kPa. The maximum value of the ratio is 0.86 while the minimum is 0.66.

Decreasing effect is less pronounced for higher modulus of elasticity values of soil.

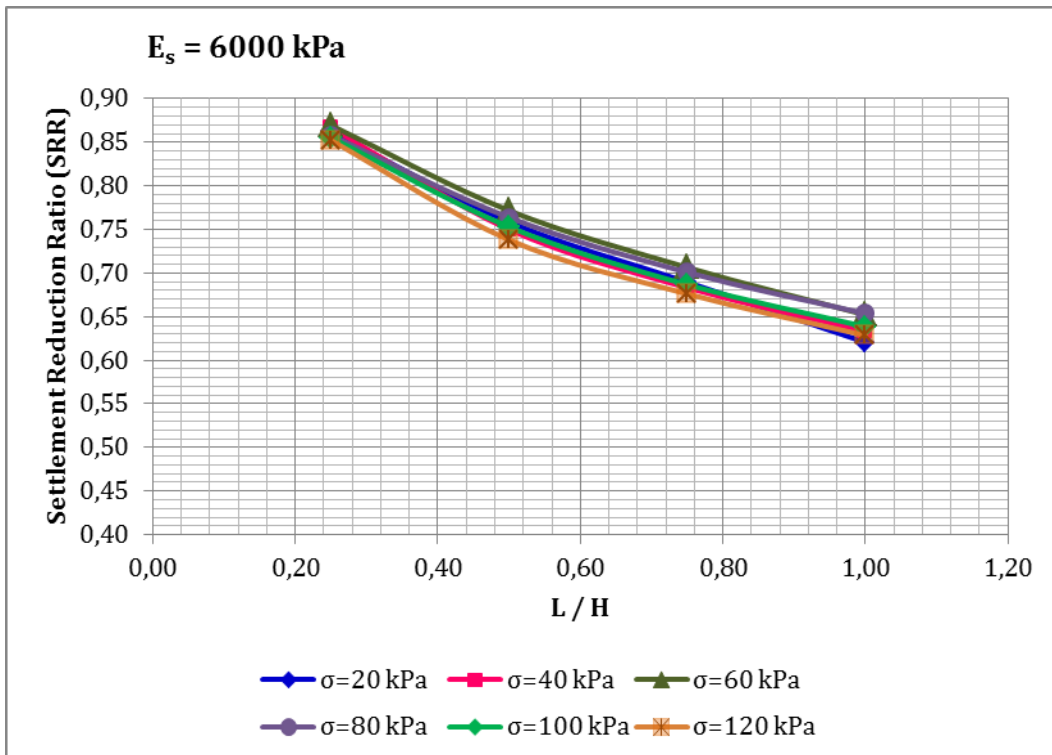
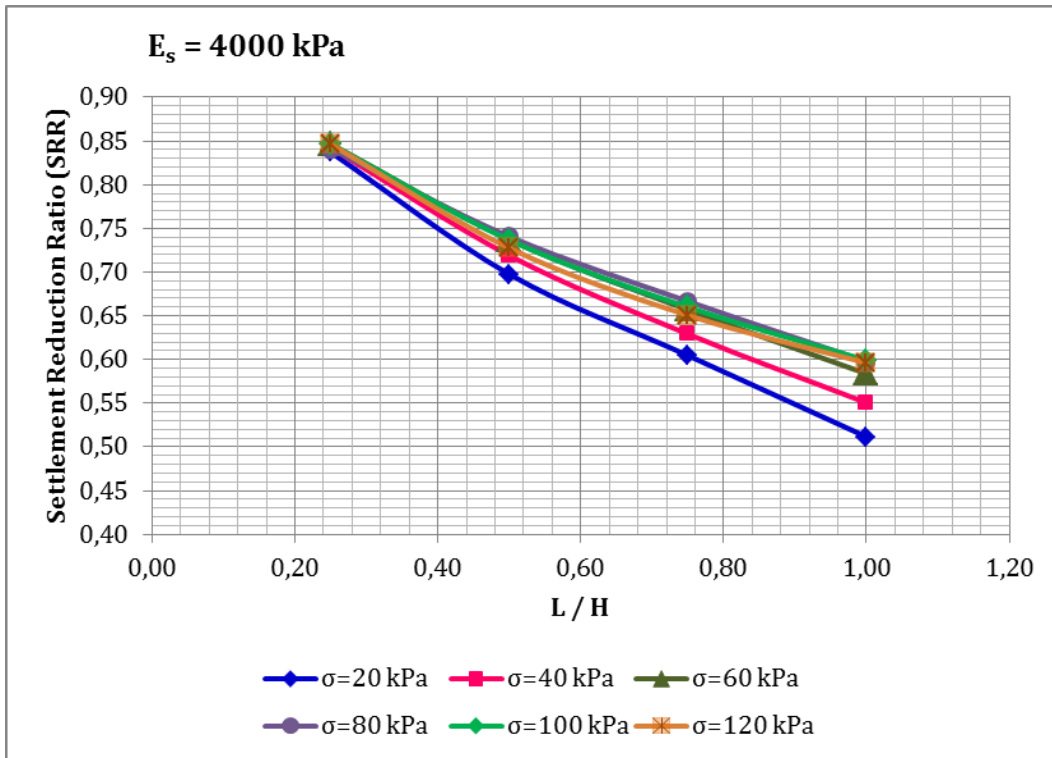


Figure 4.8 Settlement Reduction Ratio vs. Ratio of Stone Column Length to Clay Layer Thickness (L/H) Charts with $\sigma = 20\text{-}120 \text{ kPa}$

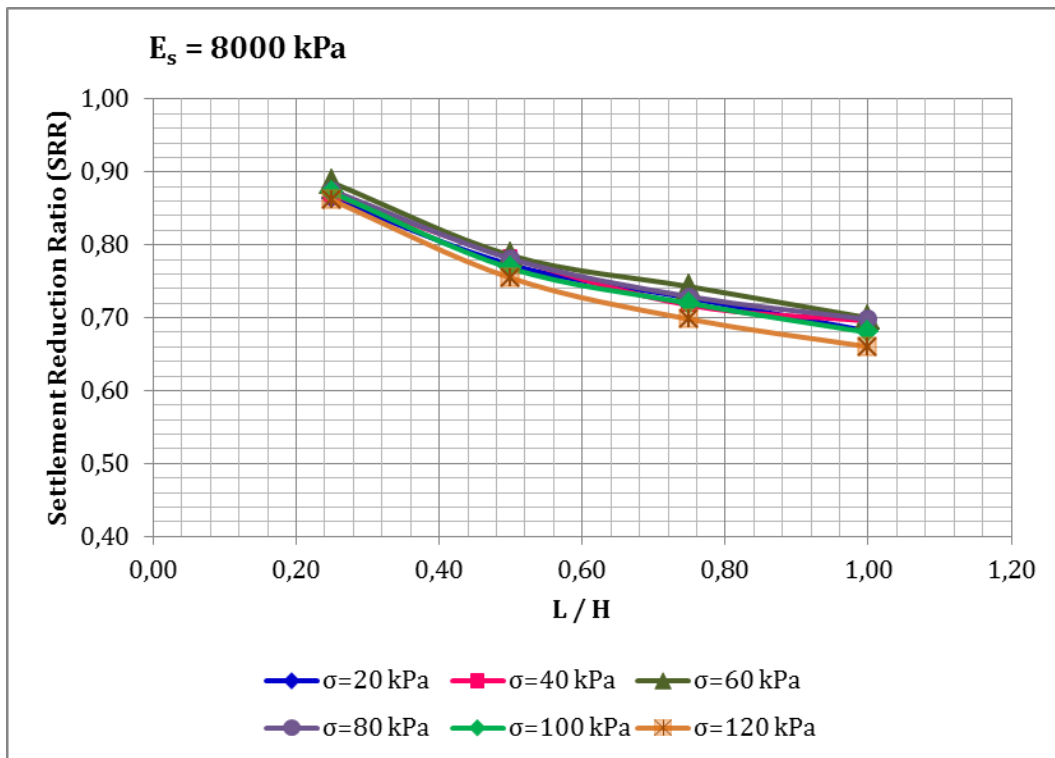


Figure 4.8 (Cont'd)

4.2.6 Effect of Foundation Pressure on the Settlement Reduction Ratio

As the stress on the foundation increases the settlement reduction ratio slightly increases until the stress reaches 80 kPa, then the ratio begins to decrease slightly up to the final stress level, 120 kPa as it is shown in Figure 4.9. The shortest stone column analyses; i.e. L/H equals to 0.25, give nearly constant and the highest ratios meaning the settlement improvement after the column installation is not significant. The results for the rest of three sets of loading get closer as the modulus of elasticity of the soil increases.

For the soils having modulus of elasticity value as 4000 kPa, the settlement reduction ratio is almost constant with an average value of 0.85 at increasing foundation pressure from 20 to 120 kPa at the upper boundary composed of the analyses of $L/H=0.25$. The lower boundary is composed of the end bearing column analyses with the ratio increasing from 0.51 to 0.60 at 20 and 120 kPa stress levels, respectively. Other curves fall between the boundaries with similar behaviour. The settlement reduction ratio ranges between 0.70 and 0.74; 0.68 and 0.71 for $L/H=0.50$ and 0.75 analyses, respectively.

In the analyses of soils having modulus of elasticity value as 6000 kPa, the upper boundary comprises the results obtained for the ratio of L/H equal to 0.25 and the settlement reduction ratio is almost constant with an average value of 0.86 at the foundation pressures ranging from 20 to 120 kPa. The lower boundary is composed of the end bearing column analyses

with the ratio increasing from 0.62 to 0.65 at 20 and 80 kPa stress levels, respectively. Then, the ratio decreases to 0.63 at 120 kPa pressure. Other curves fall between the boundaries and they are getting closer for the increased modulus of elasticity values. The average value of the settlement reduction ratio is 0.76 and 0.69 for L/H equal to 0.50 and 0.75 analyses, respectively.

The settlement reduction ratio is almost constant with an average value of 0.87 at increasing foundation pressure from 20 to 120 kPa in soils having modulus of elasticity value as 8000 kPa and in stone columns having L/H ratio as 0.25. In the end bearing column analyses, the ratio increases from 0.68 to 0.70 at 20 and 80 kPa pressures, respectively. Then, the ratio decreases to 0.66 at 120 kPa pressure. Other curves fall between the boundaries and they are getting closer compared to lower modulus of elasticity values. The average value of the settlement reduction ratio is 0.77 and 0.72 for L/H ratios equal to 0.50 and 0.75 analyses, respectively.

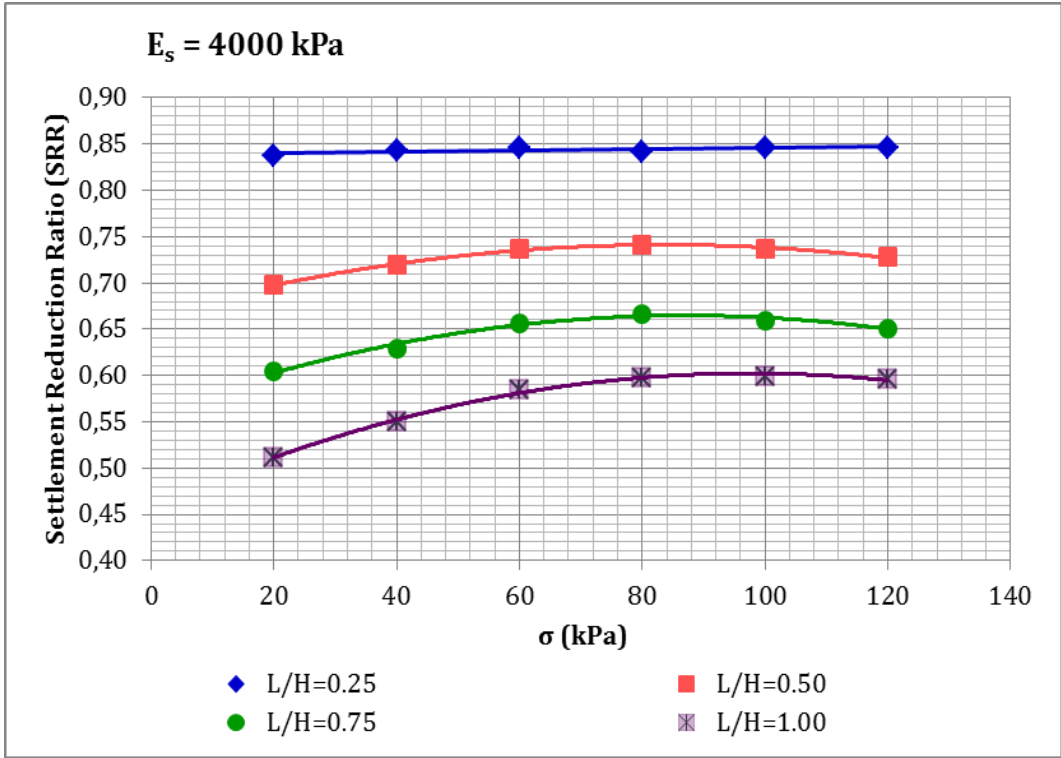


Figure 4.9 Settlement Reduction Ratio vs. Foundation Pressure Charts with L/H = 0.25 ~ 1.00

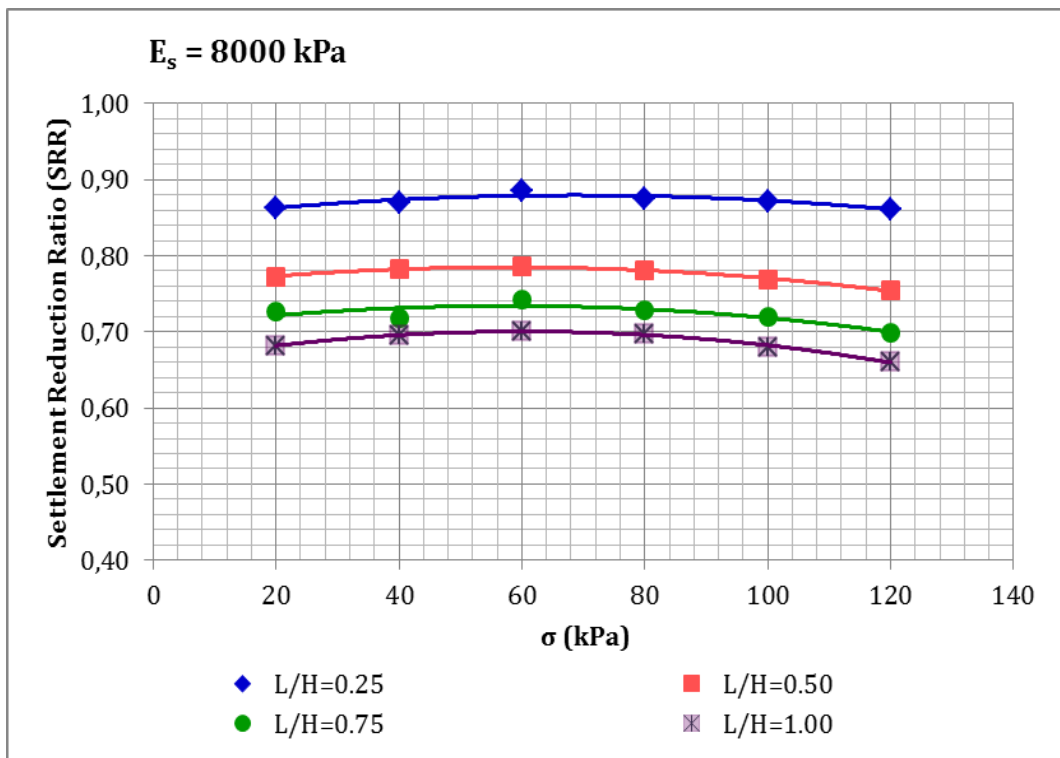
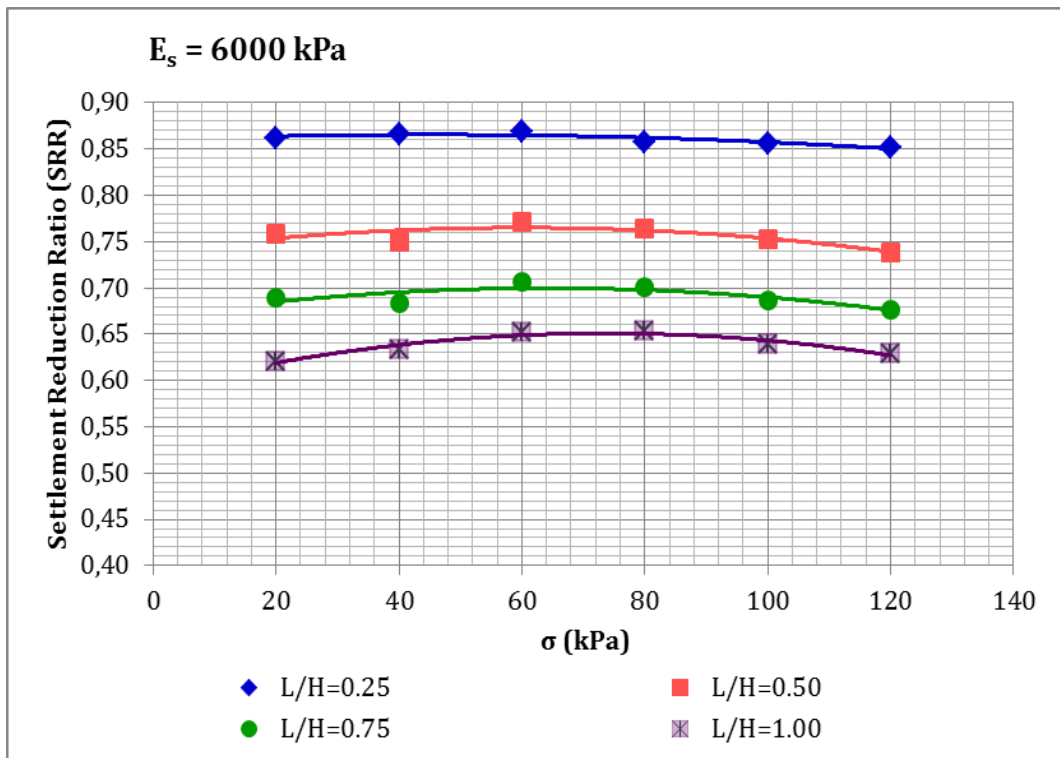


Figure 4.9 (Cont'd)

4.3 Results of Flexible Foundation Analyses

In order to compare the rigid foundation and flexible foundation behaviour reinforced by stone columns in terms of stress distribution between the stone columns and surrounding soft clay, 2-D finite element analyses of an embankment are carried out. At the top of the stone column system a 20-cm-thick sand stress distribution layer is defined and distributed load at the top of this layer is introduced to represent the embankment loading (from 20 to 120 kPa with 20 kPa increments) in the numerical model. The modulus of elasticity of the clay is changed between 4000, 6000 and 8000 kPa for the parametric study. Floating columns are modelled with 10-m-length stone columns and a set of end bearing columns in 20-m-length is defined. Therefore, results of 18 analyses are obtained for the floating columns and 18 results for end bearing columns in terms of stress concentration factor.

Similar to the rigid foundation analyses, the stress concentration factor is derived from the results of the effective normal stress distribution graph obtained in finite element analysis by taking a cross section just under the sand distribution layer; i.e., 20 cm below the ground surface. The averages of the effective normal stresses carried by the columns and the surrounding soil are calculated by dividing the area under the columns to the total column width and dividing the area under the soil to the total soil width, respectively. The ratio is computed by the equation 4.1 given before.

4.3.1 Effect of Modulus of Elasticity of Soil on Stress Concentration Factor

In flexible foundation analyses, the stress concentration factor decreases with increasing modulus of elasticity of soft clay as shown in Figure 4.10 and 4.11 for floating and end bearing columns, respectively. The decreasing effect is slightly more pronounced in end bearing columns than the floating columns.

For the analyses carried out in floating columns, i.e. L/H ratio equals to 0.5, at 20 kPa flexible foundation pressure the stress concentration factor decreases from 2.62 to 1.85 as the modulus of elasticity of soil increases from 4000 to 8000 kPa. At different embankment pressures the ratio does not show significant change. It decreases from 2.62 to 1.86, 2.60 to 1.89, 2.50 to 1.95, 2.49 to 1.96 and 2.55 to 1.95 at 40-60-80-100-120 kPa pressures, respectively.

The stress concentration factor decreases from 2.74 to 1.82 while the modulus of elasticity of soil is increasing from 4000 to 8000 kPa at 20 kPa flexible foundation pressure for the end bearing column analyses. The results for the stress concentration ratio are similar at different pressures; i.e. it decreases from 2.75 to 1.90, 2.65 to 1.94, 2.65 to 2.02, 2.75 to 2.02 and 2.80 to 2.00 at 40-60-80-100-120 kPa pressures, respectively.

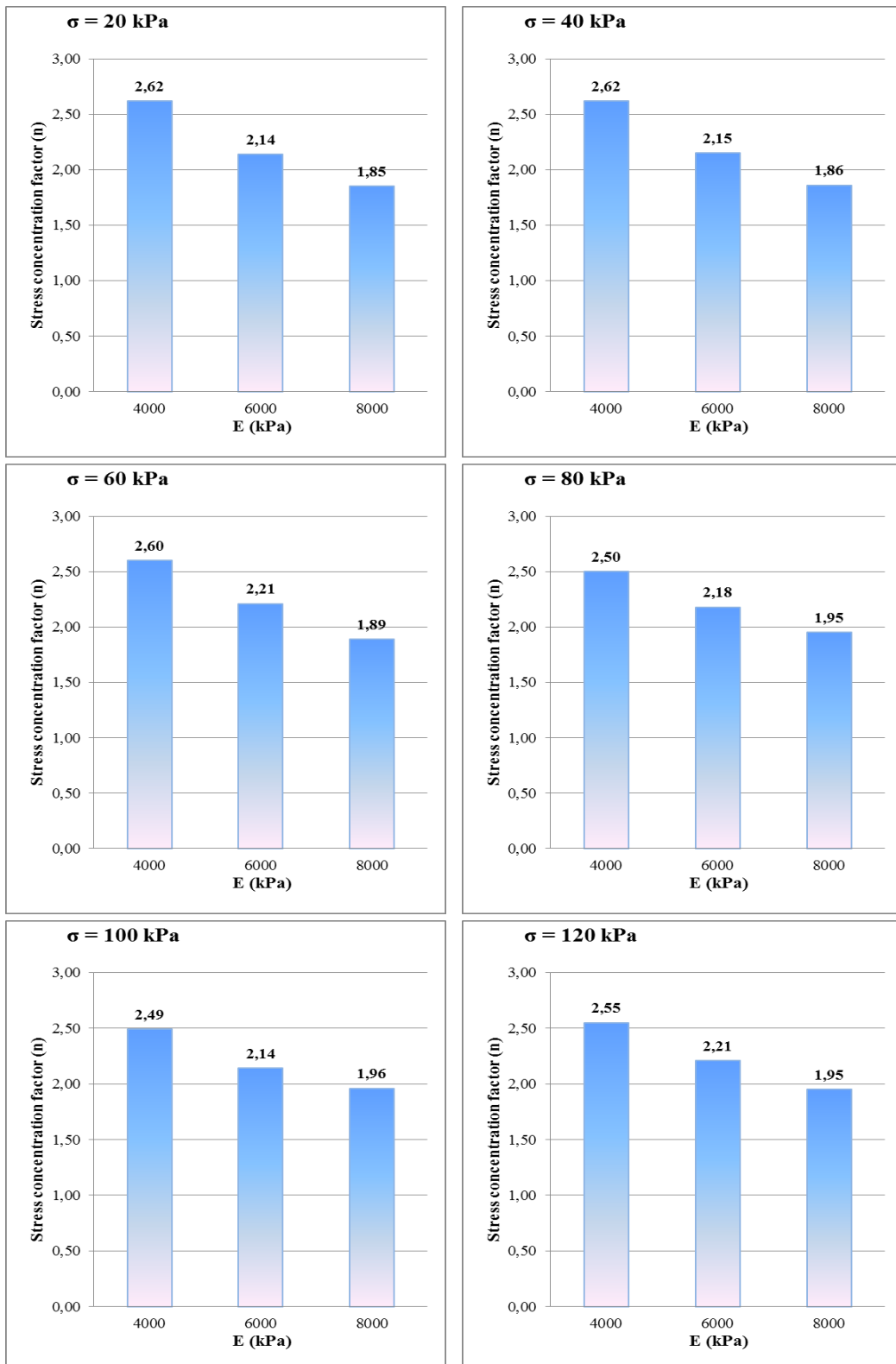


Figure 4.10 Stress Concentration Factors for Changing Modulus of Elasticity of Soil for Floating Columns (L/H=0.5) at 20~120 kPa Flexible Foundation Pressure

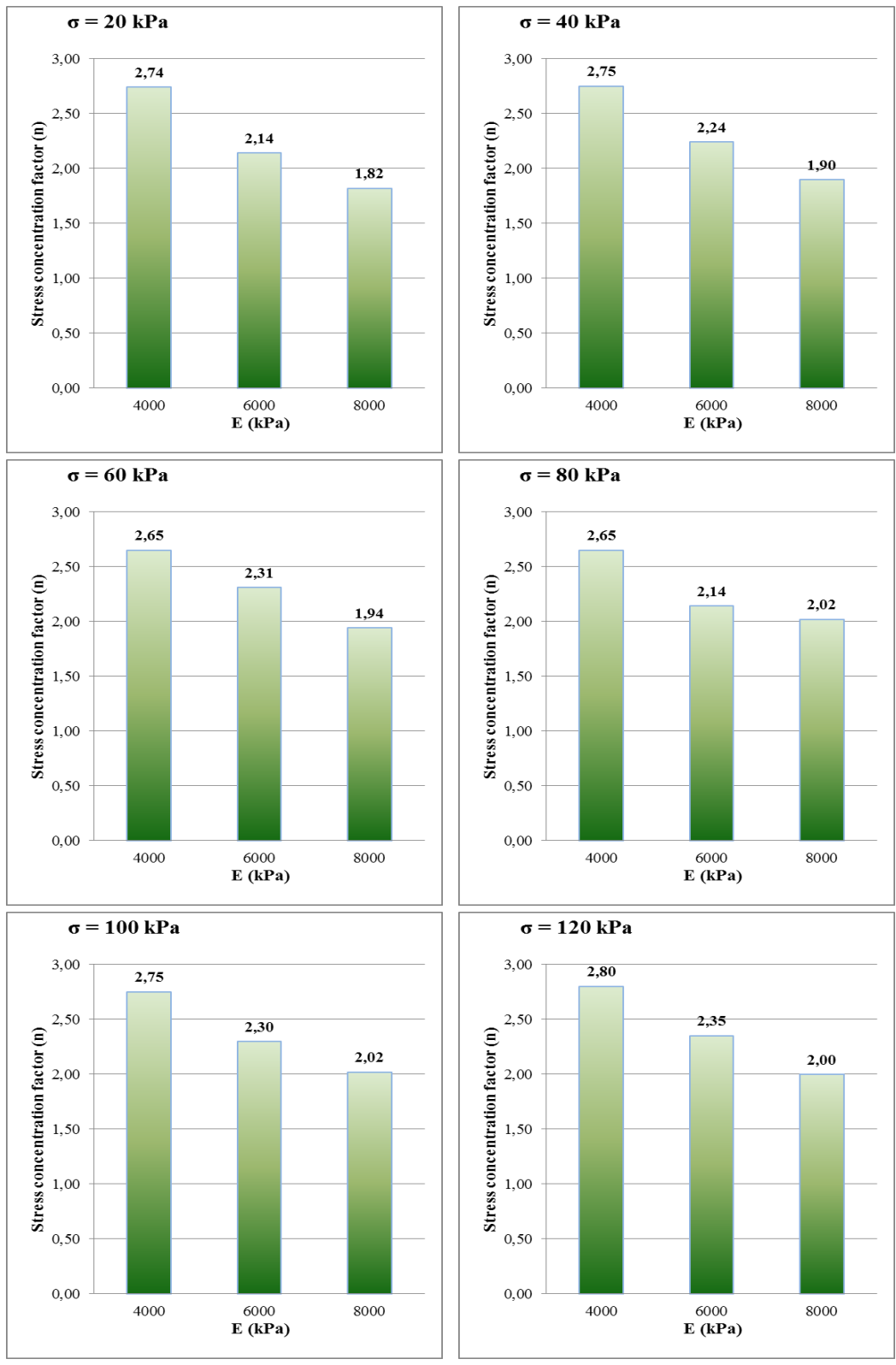


Figure 4.11 Stress Concentration Factors for Changing Modulus of Elasticity of Soil for End Bearing Columns (L/H=1.0) at 20-120 kPa Flexible Foundation Pressure

4.3.2 Effect of Embankment Pressure on Stress Concentration Factor

As the embankment loading increases, there is no significant change in stress concentration factor as shown in Figure 4.12, 4.13 and 4.14 for different modulus of elasticity values of soil from 4000 to 8000 kPa.

The average value of stress concentration ratio defined from the analyses of floating columns in 4000 kPa-modulus of elasticity-soil is 2.56 while it is 2.72 in end bearing columns as the flexible foundation pressure is altering from 20 to 120 kPa.

In the analyses carried out in clays having modulus of elasticity value equal to 6000 kPa, the average value of stress concentration ratio is 2.17 in floating columns; i.e. $L/H=0.50$ while it is 2.25 in end bearing columns as the flexible foundation pressure is increasing from 20 to 120 kPa.

Floating columns in clays having 8000 kPa modulus of elasticity value gives an average value of 1.91 for the stress concentration factor while in end bearing columns the average value is 1.95 as the flexible foundation pressure increases from 20 to 120 kPa.

The stress concentration factor is found slightly higher in end bearing columns than the floating columns, i.e. L/H equals to 0.5. The difference is getting smaller for higher modulus of elasticity values of clay with 6%, 4% and 2% increase in 4000, 6000 and 8000 kPa modulus of elasticity values of clay, respectively.

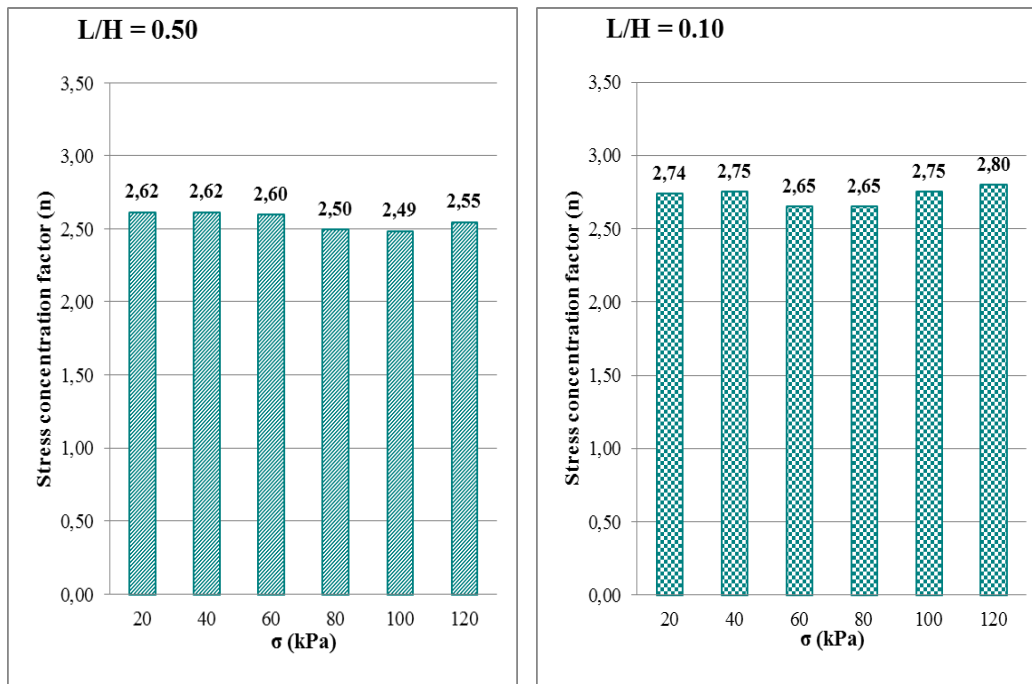


Figure 4.12 Stress Concentration Factors for Changing Flexible Foundation Pressure from 20 to 120 kPa in Clays Having Modulus of Elasticity Value Equal to 4000 kPa

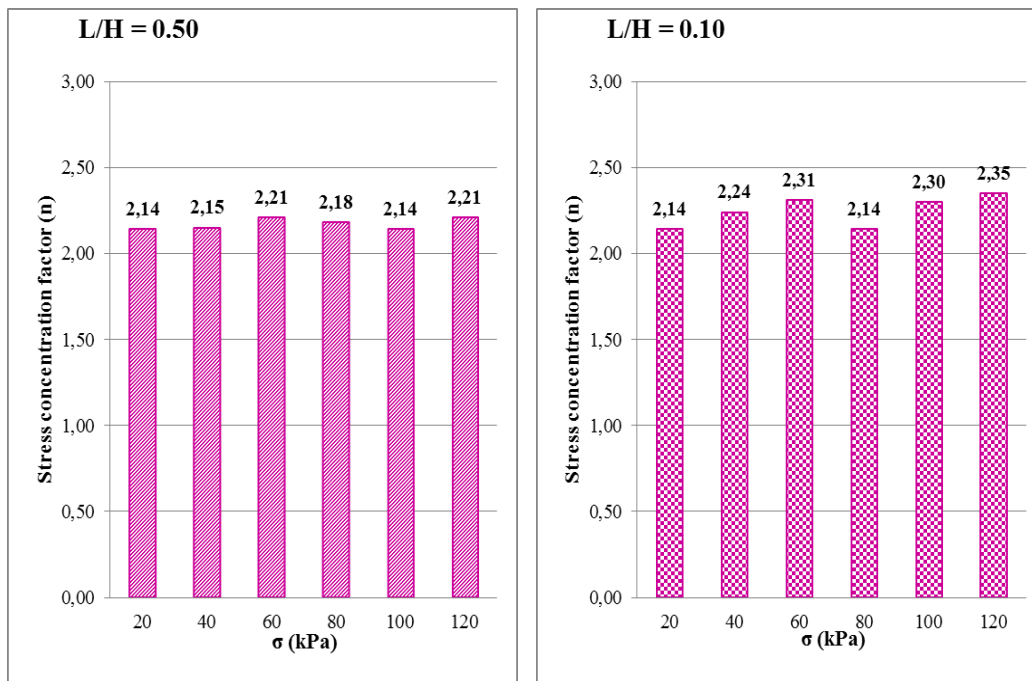


Figure 4.13 Stress Concentration Factors for Changing Flexible Foundation Pressure from 20 to 120 kPa in Clays Having Modulus of Elasticity Value Equal to 6000 kPa

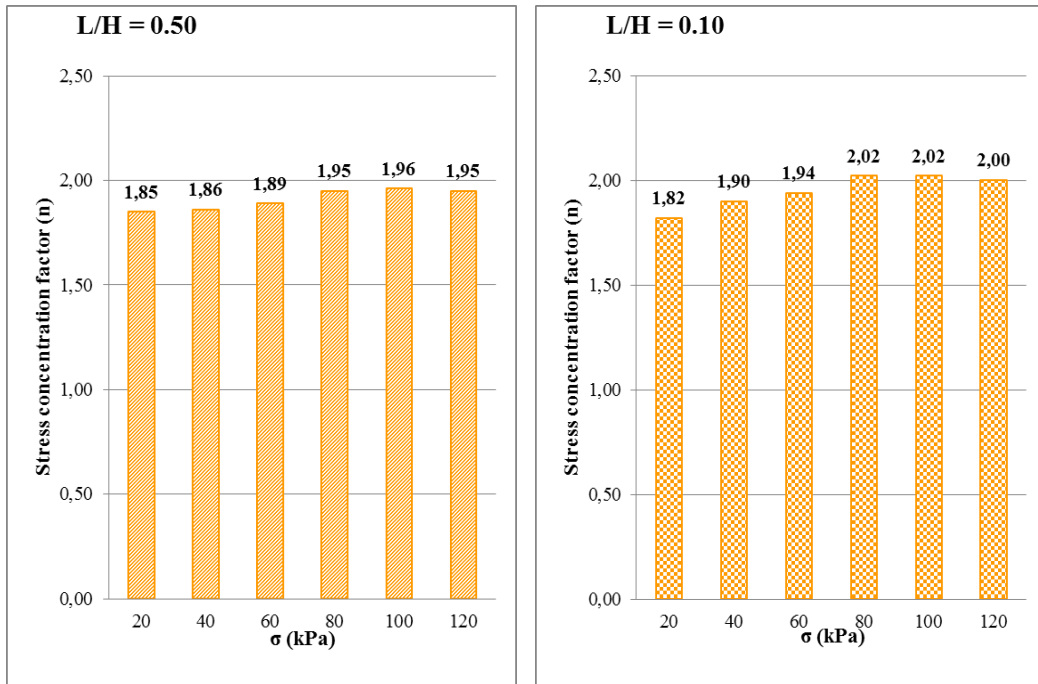


Figure 4.14 Stress Concentration Factors for Changing Flexible Foundation Pressure from 20 to 120 kPa in Clays Having Modulus of Elasticity Value Equal to 8000 kPa

4.4 Comparison of the Results of Rigid and Flexible Foundations

The results of the analyses carried out on rigid and flexible foundations are compared in terms of the stress concentration factors. Floating columns are represented by the columns having L/H ratio as 0.5 and the stress concentration factor is given with changing modulus of elasticity values of soil at different foundation pressures (Figure 4.15). The similar charts are also given for the end bearing columns (Figure 4.16). The stone column systems supporting the flexible foundations are found to have smaller stress concentration ratios than the rigid foundations. The difference is changing with the key parameters, i.e. the foundation pressure and the modulus of elasticity value of surrounding soil, but as a general trend the difference between the ratios obtained from the flexible and rigid foundation analyses is averagely 31% in floating columns and 32% in end bearing columns.

In the numerical analyses of floating columns which L/H ratio equals to 0.5, at 20 kPa foundation pressure the stress concentration factors are defined as 4.30 and 2.62 in clays having modulus of elasticity as 4000 kPa on rigid and flexible foundations, respectively. As the modulus of elasticity of the soil is increasing from 6000 to 8000 kPa, the ratio is decreasing from 3.67 to 2.14 and 3.06 to 1.85, respectively.

At 40 kPa pressure, the stress concentration factor is 4.02 in rigid foundation analyses while it is 2.62 in flexible foundation analyses for clays having modulus of elasticity as 4000 kPa

in stone columns having the ratio of L/H equal to 0.50. The ratio is decreasing from 3.46 to 2.15 and 2.98 to 1.86 in rigid and flexible foundation analyses and in clays having modulus of elasticity value equal to 6000 and 8000 kPa, respectively.

The stress concentration ratio is decreasing from 3.57 to 2.60, 3.33 to 2.21 and 2.93 to 1.89 at 60 kPa foundation pressure in rigid and flexible foundation analyses as the modulus of elasticity value of soil increases from 4000 to 8000 kPa, respectively.

In floating column analyses at 80 kPa foundation pressure the stress concentration factors are defined as 3.41 and 2.50, 3.09 and 2.18, 2.77 and 1.95 in rigid and flexible foundation analyses for clays having modulus of elasticity values as 4000, 6000 and 8000 kPa, respectively.

In floating columns analyses; i.e. L/H ratio equals to 0.5, at 100 kPa foundation pressure the stress concentration factors are defined as 3.37 and 2.49 in clays having modulus of elasticity as 4000 kPa on rigid and flexible foundations, respectively. As the modulus of elasticity of the soil is defined as 6000 and 8000 kPa, the ratio is decreasing from 2.96 to 2.14 and 2.60 to 1.96, respectively.

At 120 kPa foundation pressure the stress concentration factors are decreasing from 3.28 to 2.55, from 2.89 to 2.21 and from 2.57 to 1.95 in rigid and flexible foundation analyses for clays having modulus of elasticity values as 4000, 6000 and 8000 kPa, respectively.

The decreasing effect is generally more pronounced in higher modulus of elasticity values of clay and the difference is getting smaller as the foundation pressure increases. The percentage of decrease in the stress concentration ratio is averagely 40, 37, 32, 29, 26 and 23% from 20 to 120 kPa pressures.

The end bearing column analyses on rigid and flexible foundations are also compared in terms of stress concentration factor. At the foundation pressure of 20 kPa, the stress concentration factor is decreasing from 4.57 to 2.74, from 3.72 to 2.14 and from 2.95 to 1.82 as the modulus of elasticity of clay is increasing from 4000 to 8000 kPa.

At 40 kPa foundation pressure, the stress concentration factor decreases from 4.14 to 2.75, from 3.61 to 2.24 and from 3.04 to 1.90 in rigid and flexible foundation analyses as the modulus of elasticity of clay is increasing from 4000 to 8000 kPa.

In rigid and flexible foundation analyses, the stress concentration factor is decreasing from 3.76 to 2.65, from 3.41 to 2.31 and from 2.97 to 1.94 at 60 kPa foundation pressure while the modulus of elasticity of soil is changing as 4000, 6000 and 8000 kPa, respectively.

At the foundation pressure of 80 kPa, the stress concentration ratio decreases from 3.71 to 2.65, from 3.22 to 2.14 and from 2.84 to 2.02 in rigid and flexible foundation analyses as the modulus of elasticity of clay changes between 4000 and 8000 kPa.

At 100 kPa foundation pressure, in rigid and flexible foundation analyses the stress concentration ratio decreases from 3.63 to 2.75, from 3.14 to 2.30 and from 2.75 to 2.02 in clays having the modulus of elasticity values as 4000, 6000 and 8000 kPa, respectively.

The stress concentration factor decreases from 3.54 to 2.80, from 3.17 to 2.35 and from 2.74 to 2.00 at 120 kPa foundation pressure in rigid and flexible foundation analyses and in soils having the modulus of elasticity values ranging between 4000 and 8000 kPa.

The difference between the stress concentration factors are getting less significant as the foundation pressure is increasing from 20 to 120 kPa. The difference is found to be averagely 40, 37, 32, 31, 26 and 25 percent at 20, 40, 60, 80, 100 and 120 kPa pressures, respectively. The results of the difference are almost the same with the floating columns. Therefore, it could be concluded that the stress concentration ratio is smaller in flexible foundations than in rigid foundations. The difference is getting less significant with increasing foundation pressure but the length of the column has almost no effect in the difference between the ratios.

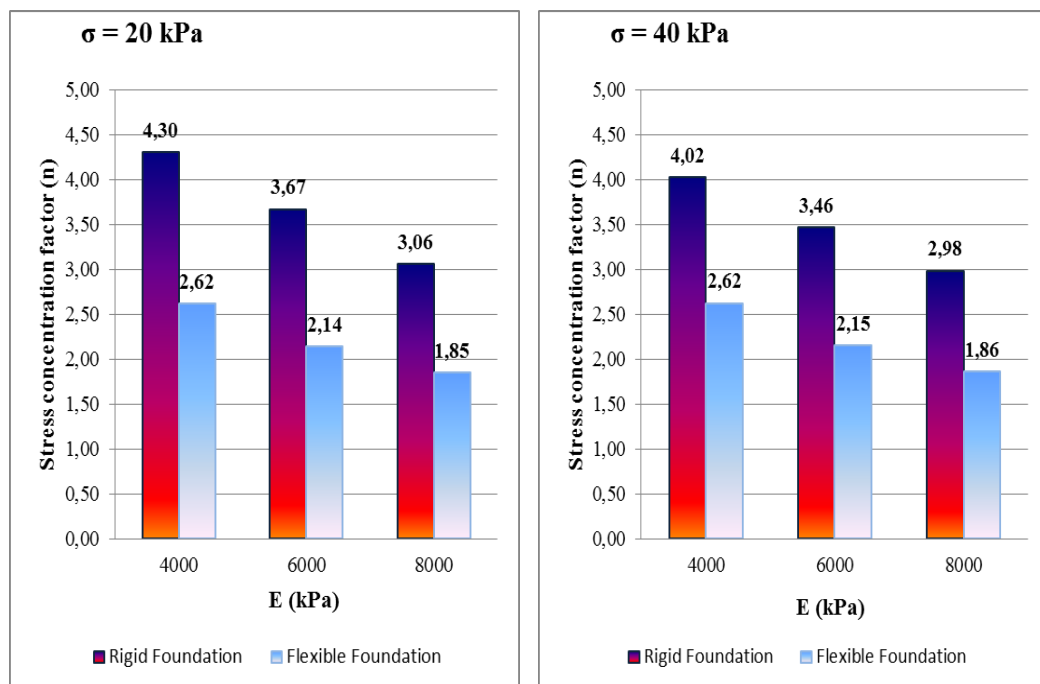


Figure 4.15 Comparison of the Stress Concentration Factors Obtained from the Rigid and Flexible Foundaiton Analysis for Floating Columns

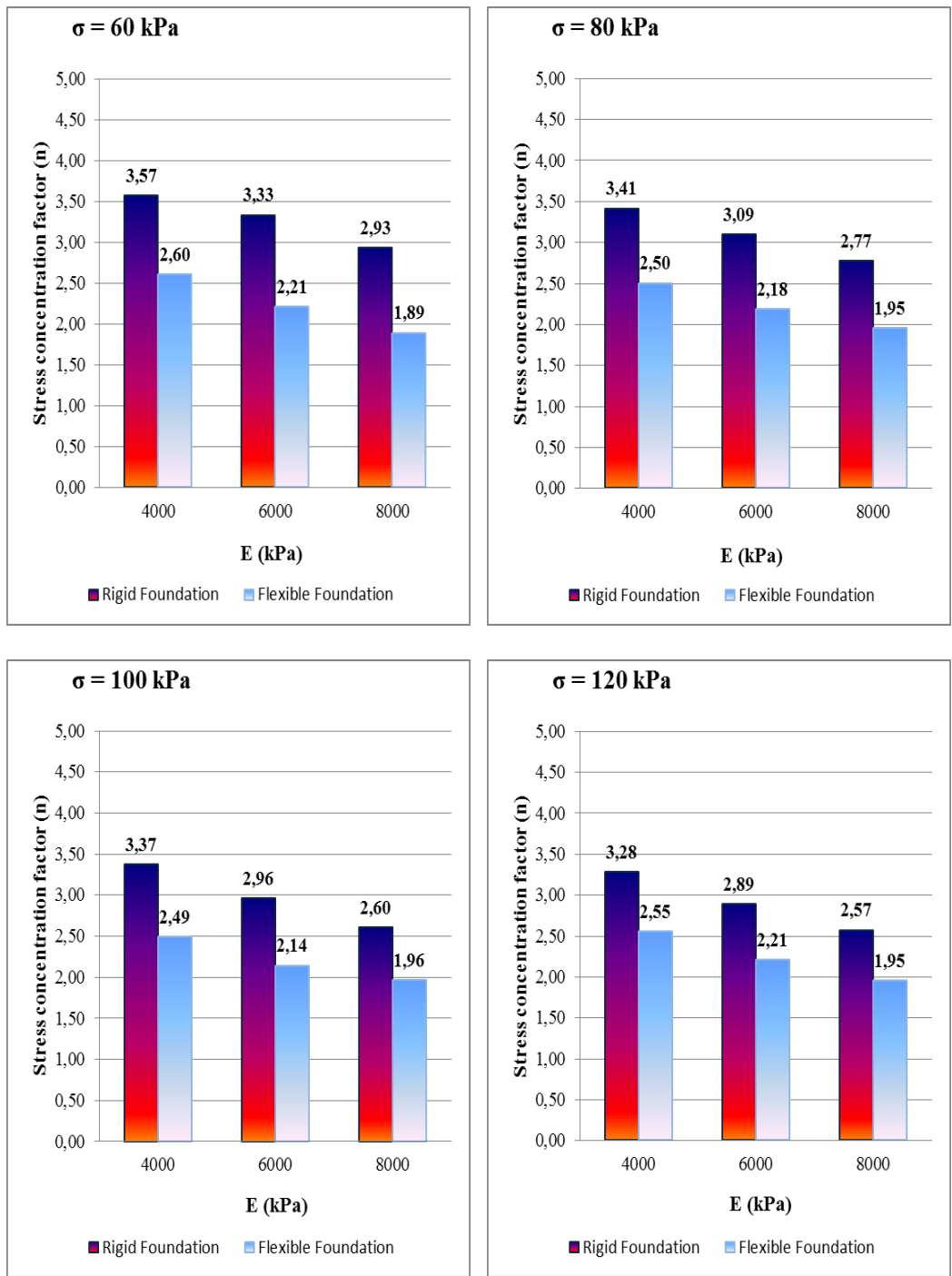


Figure 4.15 (Cont'd)

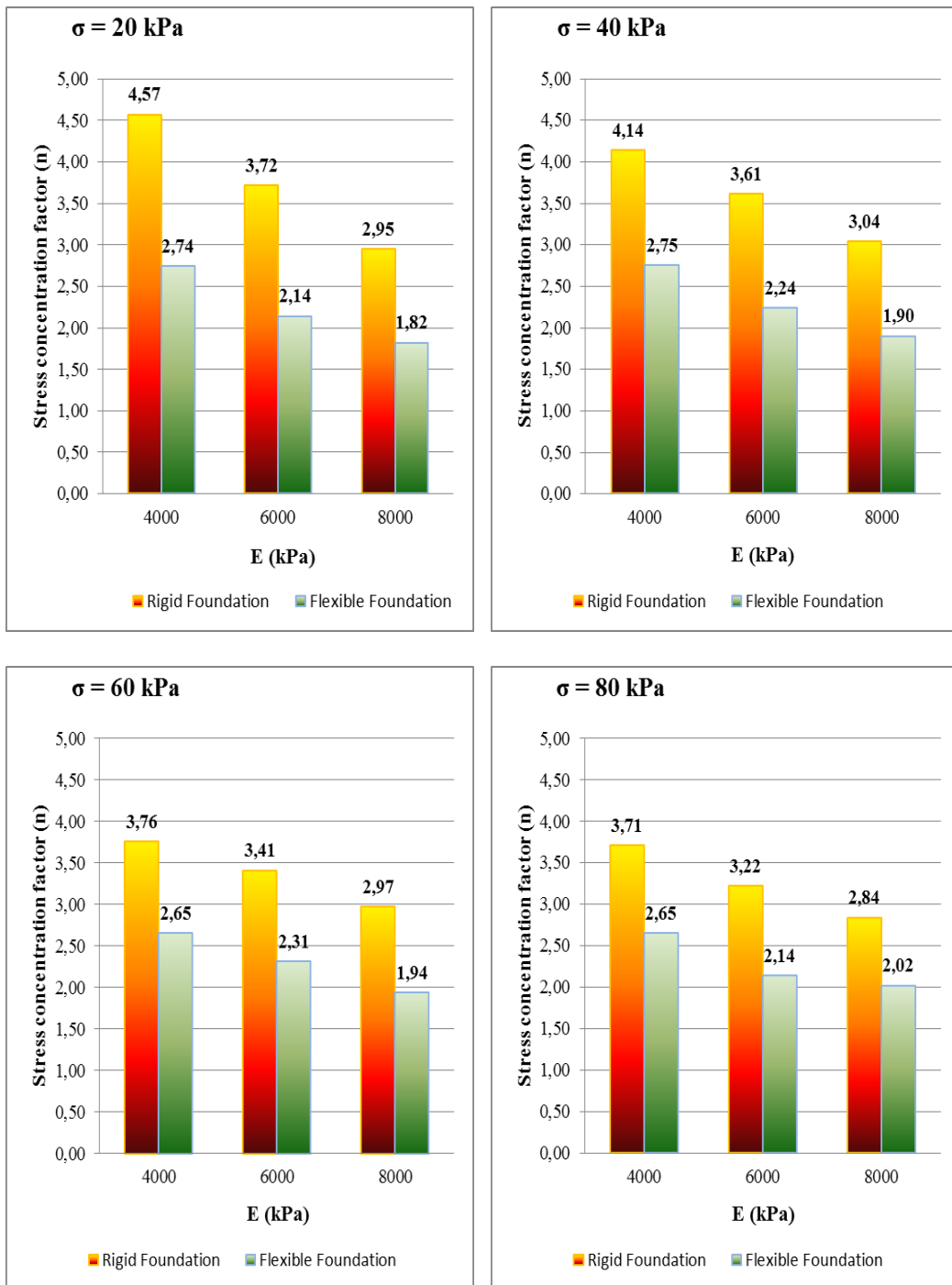


Figure 4.16 Comparison of the Stress Concentration Factors Obtained from the Rigid and Flexible Foundation Analysis for End Bearing Columns

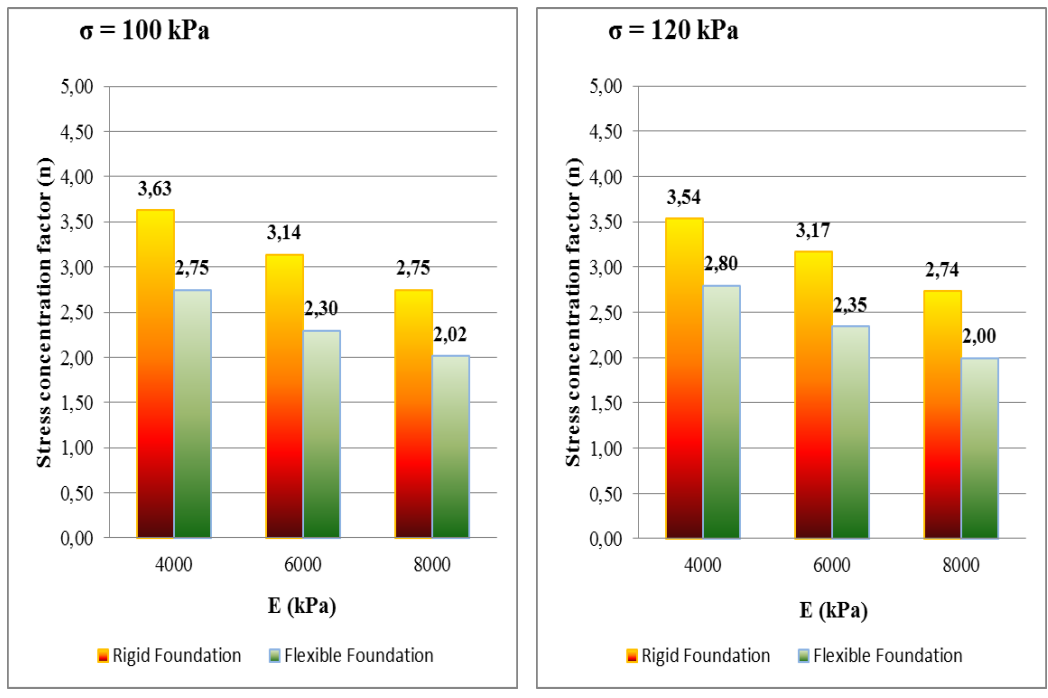


Figure 4.16 (Cont'd)

CHAPTER 5

SUMMARY AND CONCLUSIONS

Finite element analyses of rigid and flexible foundations were carried out on clay reinforced with stone columns to determine the stress concentration on the columns. Furthermore, the settlements after stone column improvement were compared with the settlements of the unimproved ground by means of the settlement reduction ratio for rigid foundation loading. Plane strain analyses were performed with 0.32 m-width stone trenches having 1.6 m spacing; therefore, the area ratio of the stone column model was equal to 20%. Mohr-Coulomb's failure criterion was utilized for all of the soil models including silty clay, clayey sand, sand and stone column material. Parametric study was conducted by using the key parameters of modulus of elasticity of soft silty clay, the length of the stone column and foundation pressure. The modulus of elasticity of the clay varied between 4000 and 8000 kPa while the stone column modulus was selected as 40000 kPa; therefore, the modular ratio was as much as 10 in the scope of the parametric study. Soft silty clay layer thickness was 20 m and the stone column length to clay layer thickness ratio (L/H) varied from 0.25 to 1.00. In rigid foundation analyses, a 20-cm-thick steel plate was loaded in stages from 20 to 120 kPa. Three sets of analyses represented the floating column behaviour, while one set was for end bearing columns. The stress concentration on the stone columns was determined at every stress level while changing modulus of elasticity of soil in different column lengths. Furthermore, the settlement reduction ratios were reported for all cases. Embankment loading was represented by flexible foundation analyses. The floating and end bearing columns were defined with $L/H=0.50$ and $L/H=1.00$ models. Other key parameters such as the modulus of elasticity of soil and the foundation pressure were taken same as those taken in the rigid foundation analyses to compare the results for stress concentration ratio.

The following conclusions are derived from the findings of the study for rigid foundations:

- The stress concentration factor ranges between 2.5 and 5.0, which is in agreement with the previously reported values in the literature.
- The stress concentration factor decreases with increasing modulus of elasticity of the soft clay layer, E_{soil} , and the decreasing effect is more pronounced in lower stress levels. Upper boundaries are composed of the analyses at 20 kPa foundation pressure while the lower boundaries are obtained in 120 kPa pressure analyses. In stone columns having L/H ratio equal to 0.25, the maximum value is 4.05 whilst the minimum is 2.64. For the other floating columns the ratio is changing between 4.30 and 2.57; 4.51 and 2.53 at 20 and 120 kPa stresses in $L/H=0.50$ and 0.75 analyses, respectively. In end bearing column analyses, the maximum value of the stress concentration factor is obtained as 4.57 at 20 kPa pressure and 4000 kPa modulus of elasticity of soil while the minimum value equals to 2.74 and at 120 kPa pressure and 8000 kPa modulus of elasticity of soil. The change is linear in n vs. E_{soil} graphs at different foundation pressures.

- Floating columns with L/H ratio ranging from 0.25 to 0.75 give similar stress concentration ratio values, while the ratio increases in end bearing columns. The difference is more significant at higher stress levels and lower soil modulus of elasticity values. At 40 kPa foundation pressure and in clays having 8000 kPa modulus of elasticity value, the ratio increases 2% in end bearing columns. For the analyses carried out in clays having 4000 kPa modulus of elasticity at the foundation pressure of 120 kPa, the value of the stress concentration factor is 7% greater in end bearing columns than the average value of the floating columns.
- The stress concentration factor decreases with increasing foundation pressure and the effect is more pronounced in soils having lower modulus of elasticity values. For the floating columns with L/H ratio equal to 0.25, the stress concentration factor decreases 12% at 20 and 120 kPa foundation pressures in clays having modulus of elasticity value as 8000 kPa. In the end bearing column analyses the ratio decreases 23% in the analyses of clays having 4000 kPa modulus of elasticity value from 20 to 120 kPa pressure.
- The settlement reduction ratio ranges between 0.50 and 0.87 which means that the settlements could be improved as much as twice by the modelled stone column reinforcement.
- The settlement reduction ratio slightly increases as the modulus of elasticity of clay increases. The increase is more significant in end bearing columns than the floating columns at lower foundation pressures. The shortest stone column analyses, i.e. L/H=0.25, show almost no change in settlement reduction ratio with increasing modulus of elasticity of soil and the highest ratios belong to those analyses with an average value of 0.86 at different foundation pressures from 20 to 120 kPa. At 20 kPa stress level, the settlement reduction ratio in end bearing columns increases from 0.51 to 0.68 as the modulus of elasticity of clay increases from 4000 to 8000 kPa.
- The settlement reduction ratio decreases with increasing stone column length. Decrease is not linear and the settlement reduction ratio resumes a constant value in end-bearing columns. At different foundation pressure values, there is no significant change in the ratio. The settlement reduction ratio decreases from 0.85 to 0.51, 0.87 to 0.62 and 0.86 to 0.66 at different stress levels from 20 to 120 kPa in clays having modulus of elasticity values of 4000, 6000 and 8000 kPa, respectively.
- As the stress on the foundation increases the settlement reduction ratio slightly increases until the stress is 80 kPa, then the ratio begins to decrease slightly up to the highest stress level, 120 kPa. The shortest (L/H=0.25) stone column analyses give nearly constant and the highest ratios meaning the settlement improvement after the column installation is not significant. The results for the rest of three sets of loading get closer as the modulus of elasticity of the soil increases. In the shortest column analyses; i.e. L/H=0.25, the settlement reduction ratio equals on average 0.86 for different values of modulus of elasticity of clay while the ratio increases from 0.51 to 0.60 at 20 and 120 kPa foundation pressures in clays having modulus of elasticity of 4000 kPa.

The following conclusions are derived from the findings of the study for flexible foundations:

- The stress concentration ratio is found to vary between 1.8 and 3.0.
- The stress concentration factor decreases with increasing modulus of elasticity of clay for floating and end bearing columns. The decreasing effect is slightly more pronounced in end bearing columns than the floating columns. At different foundation pressures, the ratio at average decreases 25% in floating and 28% in end bearing column analyses.
- As the embankment loading increases, there is no significant change in stress concentration factor. The average value of the ratio increases from 2.56 to 2.72, 2.17 to 2.25 and 1.91 to 1.95 at the foundation pressures ranging between 20 and 120 kPa in floating and end bearing columns and in clays having modulus of elasticity values of 4000, 6000 and 8000 kPa, respectively.
- The stress concentration factor is found to be slightly higher in end bearing columns than the floating columns, i.e. L/H equals to 0.5. The difference is getting smaller for higher modulus of elasticity values of clay with 6%, 4% and 2% increase in 4000, 6000 and 8000 kPa values, respectively.

The summary of the comparison of rigid and flexible foundation analyses results is as follows:

- The stone column systems supporting the flexible foundations are found to have smaller stress concentration ratios than the rigid foundations.
- The difference between the stress concentration factors of rigid and flexible foundation analyses is getting less significant with increasing foundation pressure from 20 to 120 kPa. The ratio decreases 40% and 24% at 20 and 120 kPa pressures, respectively in both floating ($L/H=0.5$) and end bearing columns.
- The difference between the ratios is dependent on the modulus of elasticity of soil and foundation pressure. The length of the column has almost no effect in the difference between the stress concentration factors obtained in rigid and flexible foundation analyses. As a general trend the difference between the ratios obtained from the flexible and rigid foundation analyses is averagely 31~32%.
- The stress concentration factors presented in the study are calculated by considering the average stresses on the columns. It is shown that for both of the rigid and flexible foundation analyses the edge columns carry lower loads decreasing the average value of the stress concentration factor %15 and 12%, respectively. Under large areas of loading, such as large embankments it is more convenient to consider the center column stresses or ignore the edge column stresses for computing the stress concentration factor.

Further study in means of the finite element analyses of reinforced soft soils by stone columns is needed in different area ratios and in different soil conditions. Three dimensional analysis is the most realistic representation of the real case especially for the stone column groups. Therefore, three dimensional numerical modelling could be utilized for further study on stone columns.

REFERENCES

- ABOSHI, H., ICHIMOTO, E., ENOKI, M., HARADA, K., 1979. "The Compozer- a Method to Improve the Characteristics of Soft Clays by Inclusion of Large Diameter Sand Columns", *Proceedings of International Conference on Soil Reinforcement: Reinforced Earth and Other Techniques*, Paris Vol.1, pp.211-216
- ALONSO, J.A., and JIMENEZ, R., 2012. "Reliability-based Design of Stone Columns for Ground Improvement Considering Settlement and Bulging as Failure Modes", *Proceedings of ISSMGE-TC211 International Symposium on Ground Improvement IS-GI Brussels*, Vol.III, pp.319-328
- AMBILY, A.P. and GANDHI, S.R., 2007. "Behavior of Stone Columns Based on Experimental and FEM Analysis", *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, Vol.133, No.4, pp.405-415
- BACHUS, R.C. and BARKSDALE, R.D., 1989. "Design Methodology for Foundations on Stone Columns", *Vertical and Horizontal Deformations of Foundations and Embankments, Proceedings of Settlement'1994*, Texas, ASCE Geotechnical Special Publication No.40, pp.244-257
- BALAAM, N.P., 1978. "Load-Settlement Behavior of Granular Piles", *Thesis presented to the University of Sydney in partial fulfillment of the requirements for the degree of Doctor of Philosophy*.
- BALAAM, N.P. and BOOKER, J.R., 1981. "Analysis of Rigid Rafts Supported by Granular Piles", *International Journal for Numerical and Analytical Methods in Geomechanics*, Vol.5, pp.379-403
- BALAAM, N.P., BROWN, P.T., and POULOS, H.G., 1977. "Settlement Analysis of soft Clays Reinforced with Granular Piles", *Proceedings of 5th Southeast Asian Conference on Soil Engineering*, Bangkok, pp.81-92
- BARKSDALE, R.D., 1981. "Site Improvement in Japan Using Sand Compaction Piles", Georgia Institute of Technology, Atlanta, July.
- BARKSDALE, R.D., and BACHUS, R.C., 1983. "Design and Construction of Stone Columns", *Report No. FHWA/RD-83/026*, National Technical Information Service, Virginia, USA.
- BARRON, R.A., 1948. "Consolidation of fine-grained soils by drain wells", *Transactions ASCE*, Vol.113, pp.718-742
- BERGADO, D.T., ANDERSON, L.R., MIURA, N. and BALASUBRAMANIAM, A.S., 1996. "Soft Ground Improvement in Lowland and Other Environments", ASCE, New York.
- BRINKGREVE, R.B.J., BROERE, W. and WATERMAN, D., 2008. "Manuals of Plaxis 2D-Version 9", *Plaxis by Delft*, the Netherlands.
- CANIZAL, J., CASTRO, J., CIMENTADA, A., DA COSTA, A., MIRANDA, M. and SAGASETA, C., 2012. "Theoretical analyses of laboratory tests of kaolin clay improved

with stone columns”, *Proceedings of ISSMGE - TC 211 International Symposium on Ground Improvement IS-GI Brussels*, Vol.III, pp.373-381

DATYE, K.R., 1982. “Settlement and Bearing Capacity of Foundation System”, *Symposium on Recent Developments in Ground Improvement Techniques*, Bangkok, pp.85-103

ENGELHART, K. and KIRSH, K., 1977. “Soil Improvement by Deep Vibration Technique”, *Proceedings of 5th Southeast Asian Conf. on Soil Engineering*, Bangkok.

ELKASABGY, M.A., 2005. “Performance of Stone Columns Reinforced Grounds”, *M.Sc. Thesis, Zagazig University, Faculty of Engineering at Shobra, Cairo*.

ELSHAZLY, H., ELKASABGY, M. and ELLEBOUDY, A., 2008. “Effect of Inter-column Spacing on Soil Stresses due to Vibro-installed Stone Columns: interesting findings”, *Geotechnical and Geological Engineering*, Vol.26, pp.225-236

ELSHAZLY, H.A., HAFEZ, D.H. and MOSSAAD, M.E., (2006). “Back calculating vibro-installation stresses in stone columns reinforced grounds”, *Ground Improvement*, Vol.10, No.2, pp.47-53

ELSHAZLY, H.A., HAFEZ, D.H. and MOSSAAD, M.E., 2008. “Reliability of Conventional Settlement Evaluation for Circular Foundations on Stone Columns”, *Geotec. Geol. Eng.*, Vol.26, pp.323-334

GOUGHNOUR, R.R., 1983. “Settlement of Vertically Loaded Stone Columns in Soft Ground”, *Proceedings of 8th ECSMFE*, Helsinki, Vol.1, pp.23-25.

GREENWOOD, D.A., 1970. “Mechanical Improvement of Soils below Ground Surface”, *Proceedings of the Conf. on Ground Engineering*, London, ICE.

GREENWOOD, D.A., 1975. “Vibrofloatation: Rationale for Design and Practice”, *Methods of Treatment of Unstable Ground*, London, pp. 189-209

GREENWOOD, D.A., 1991. “Load Tests on Stone Columns”, *Deep Foundation Improvements: design, construction and testing*, ASTM STP 1089, Philadelphia, pp.148-171

HAN, J. and YE, S.L., 2001. “Simplified Method for Consolidation Rate of Stone column Reinforced Foundation”, *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, Vol.127, pp.597-603

HUGHES, J.M.O., and WITHERS, N.J., 1974. “Reinforcing of Soft Cohesive Soils with Stone Columns”, *Ground Engineering*, Vol.7, No.3, pp.42-49

JACKY, J., 1944. "The coefficient of earth pressure at rest," *Journal for Society of Hungarian Architects and Engineers*, pp.355-358

KURUOĞLU, Ö., 2008. “A New Approach to Estimate Settlements Under Footings on Rammed Aggregate Pier Groups”, *Thesis presented to the Middle East Technical University in partial fulfillment of the requirements for the degree of Doctor of Philosophy*.

MITCHELL, J.K., and KATTI, R.K., 1981. “Soil Improvement – State-of-the-Art Report”, *Proceedings of 10th International Conference on Soil Mechanics and Foundation Engineering*, Stockholm, pp.261-313

- ÖZKESKİN, A., 2004. "Settlement Reduction and Stress Concentration Factors in Rammed Aggregate Piers Determined from Full Scale Load Tests", *Thesis presented to the Middle East Technical University in partial fulfillment of the requirements for the degree of Doctor of Philosophy*.
- PITT, J.M., WHITE, D.J., GAUL, A., HOEVELKAMP, K., 2003. "Highway Applications for Rammed Aggregate Piers in Iowa soils", *Iowa DOT Project TR-443, CTRE Project 00-60, USA*.
- PRIEBE, H., 1976. "Estimating Settlements in a Gravel Column Consolidated Soil", *Die Bautechnik*, 53, pp.160-162
- PRIEBE, H., 1995. "The Design of Vibro Replacement", *Journal of Ground Engineering*, Vol.28, No.10
- SCHWEIGER, H.F., 1989. "Finite Element Analysis of Stone Column Reinforced Foundations", *Thesis presented to the University of Wales, Swansea in partial fulfillment of the requirements for the degree of Doctor of Philosophy*.
- TAN, S.A., TJAHYONO, S. and OO, K.K., 2008. "Simplified Plane-Strain Modeling of Stone-Column Reinforced Ground", *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, Vol.134, No.2, pp.185-194
- VAN IMPE, W.F, DE BEER, E., 1983. "Improvement of Settlement Behavior of Soft Layers by Means of Stone Columns", *Proceedings of 8th International Conference on Soil Mechanics and Foundation Engineering*, pp.309-312, Rotterdam, Balkema.
- VAUTRAIN, J., 1977. "Mur en Terre Armee Sur Colonnes Ballastees", *Proceedings of Int. Symposium on Soft Clay*, Bangkok
- VESIC, A.S., 1972. "Expansion of Cavities in Infinite Soil Mass", *Journal of the Soil Mechanics and Foundation Division*, ASCE, Vol.98(SM3), pp.265–290
- WATTS, K.S., JOHNSON, D., WOOD, L.A., SAADI, A., 2000. "An In-situ-mental Trial of Vibro Ground Treatment Supporting Strip Foundations in a Variable Fill", *Geotechnique*, Vol.50, No.6, pp.699–708
- WEBER, T.M., SPRINGMAN, S.M., GÄB, M., RACANSKY, V. and SCHWEIGER, H.F., 2008. "Numerical Modelling of Stone Columns in Soft Clay under an Embankment", *Geotechnics of Soft Soils-Focus on Ground Improvement*, pp.305-311, London, Taylor & Francis.
- WOOD, D.M., HU, W. and NASH, D.F.T., 2000. "Group Effects in stone column Foundations: model tests", *Geotechnique*, Vol.50, No.6, pp.689-698
- ZAHMATKESH, A. and CHOBBASTI, A.J., 2010. "Settlement Evaluation of Soft Clay Reinforced by Stone Columns, Considering the Effect of Soil Compaction", *IJRAS*, Vol.3, No.2, pp.159-166

APPENDIX A

PLAXIS OUTPUTS FOR SELECTED CASES

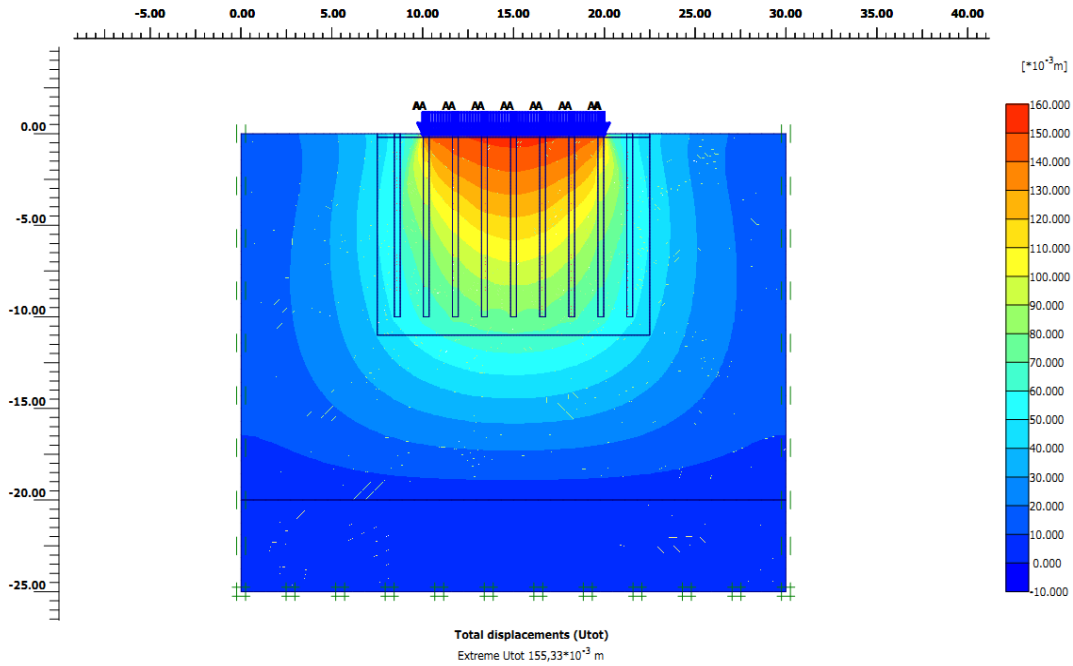


Figure A.1 Total displacement of the numerical model composed of stone columns with $L/H=0.5$ in clay having $E= 6000$ kPa at 120 kPa rigid foundation pressure

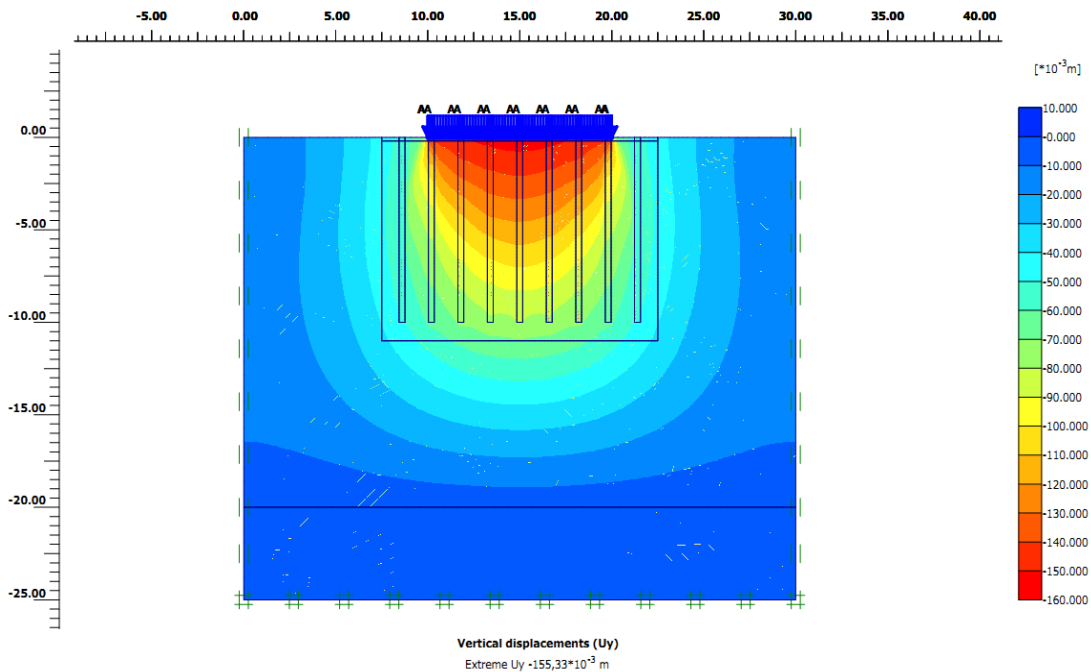


Figure A.2 Vertical displacement of the numerical model composed of stone columns with $L/H=0.5$ in clay having $E= 6000$ kPa at 120 kPa rigid foundation pressure

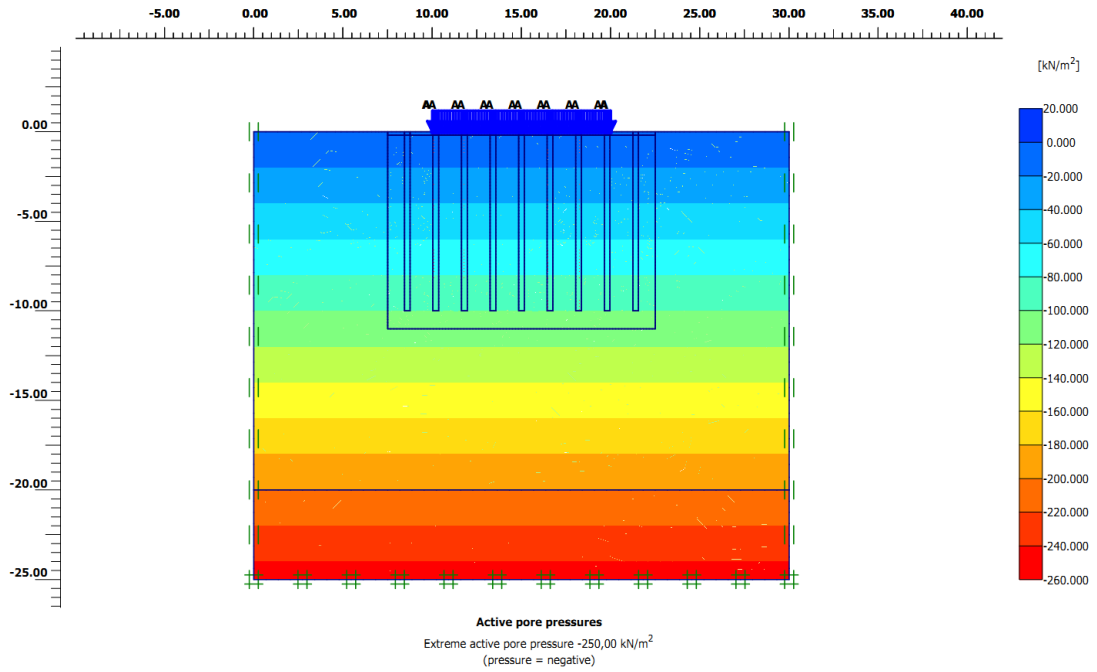


Figure A.3 Active pore pressures of the numerical model composed of stone columns with $L/H=0.5$ in clay having $E= 6000$ kPa at 120 kPa rigid foundation pressure

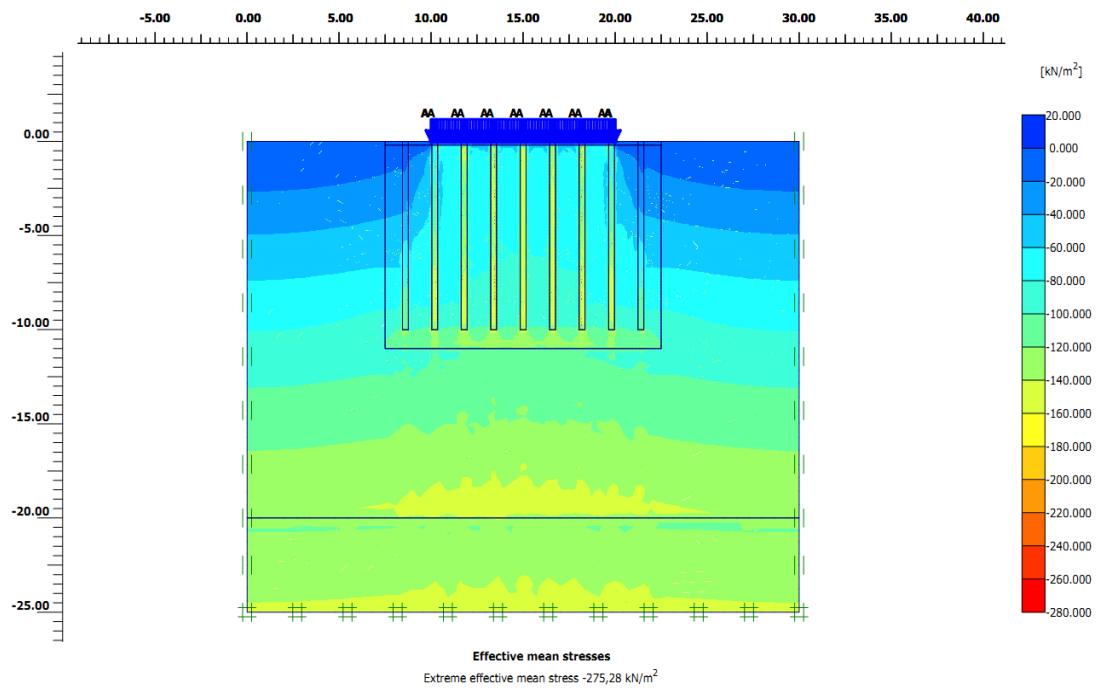


Figure A.4 Effective mean stresses of the numerical model composed of stone columns with $L/H=0.5$ in clay having $E= 6000$ kPa at 120 kPa rigid foundation pressure

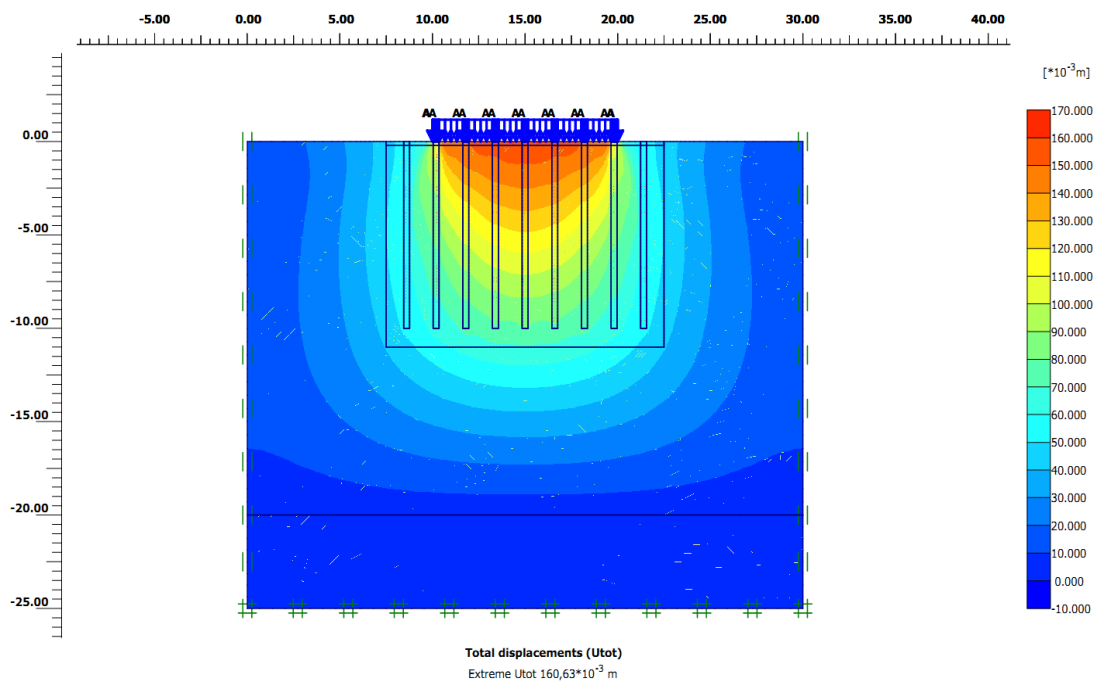


Figure A.5 Total displacement of the numerical model composed of stone columns with $L/H=0.5$ in clay having $E= 6000$ kPa at 120 kPa flexible foundation pressure

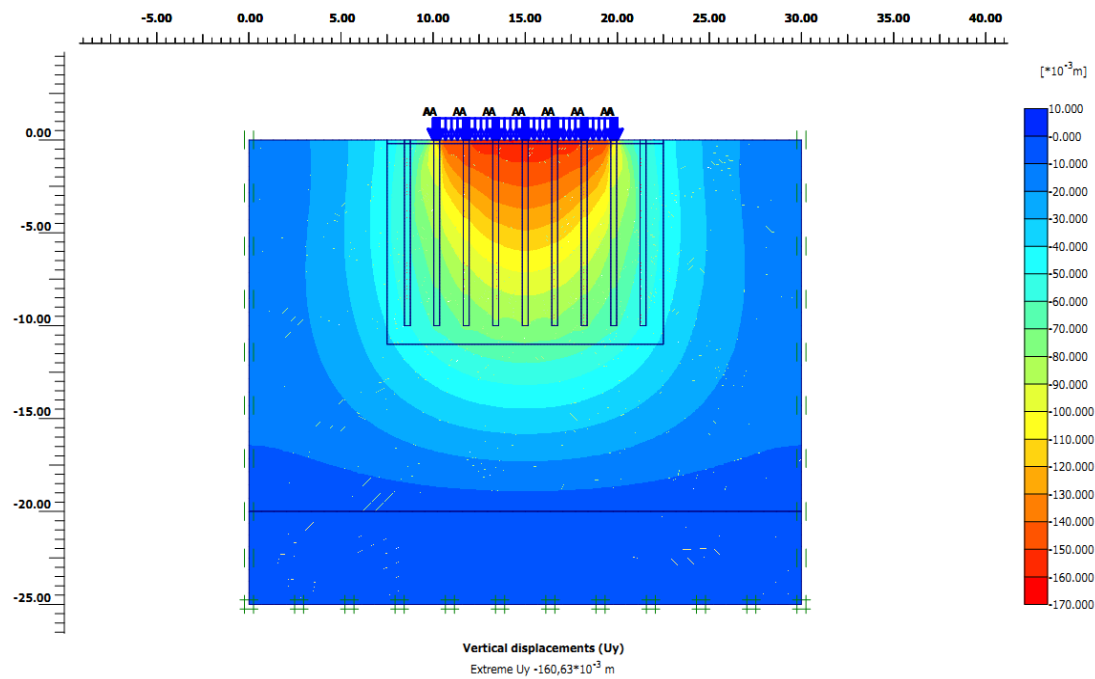


Figure A.6 Vertical displacement of the numerical model composed of stone columns with $L/H=0.5$ in clay having $E= 6000$ kPa at 120 kPa flexible foundation pressure

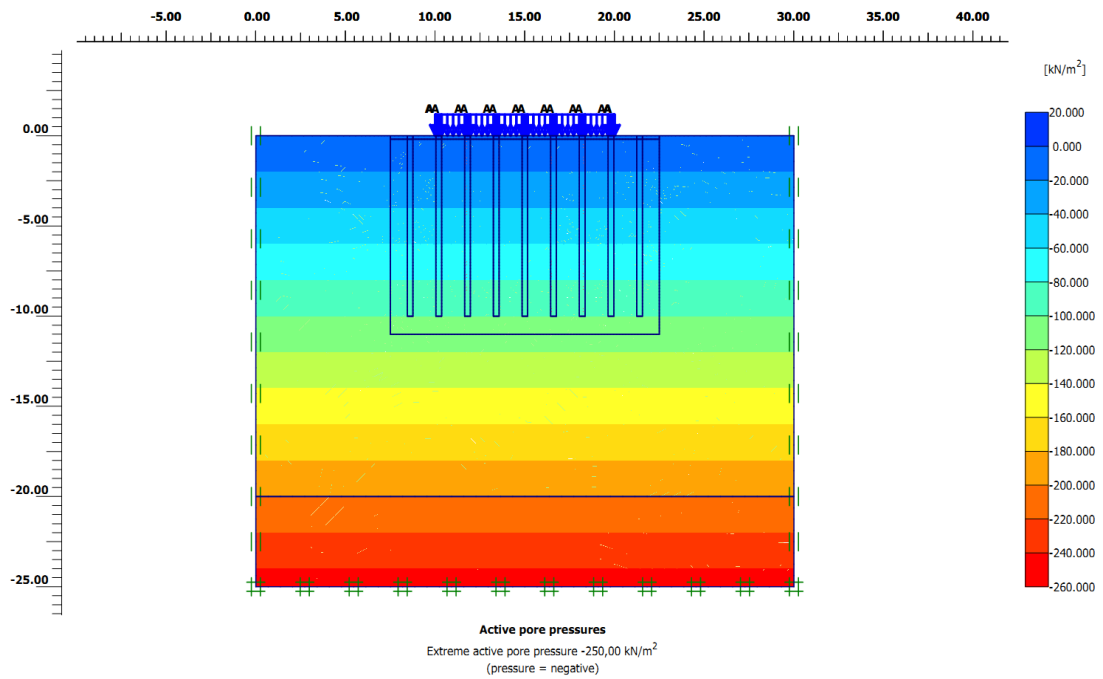


Figure A.7 Active pore pressures of the numerical model composed of stone columns with $L/H=0.5$ in clay having $E= 6000$ kPa at 120 kPa flexible foundation pressure

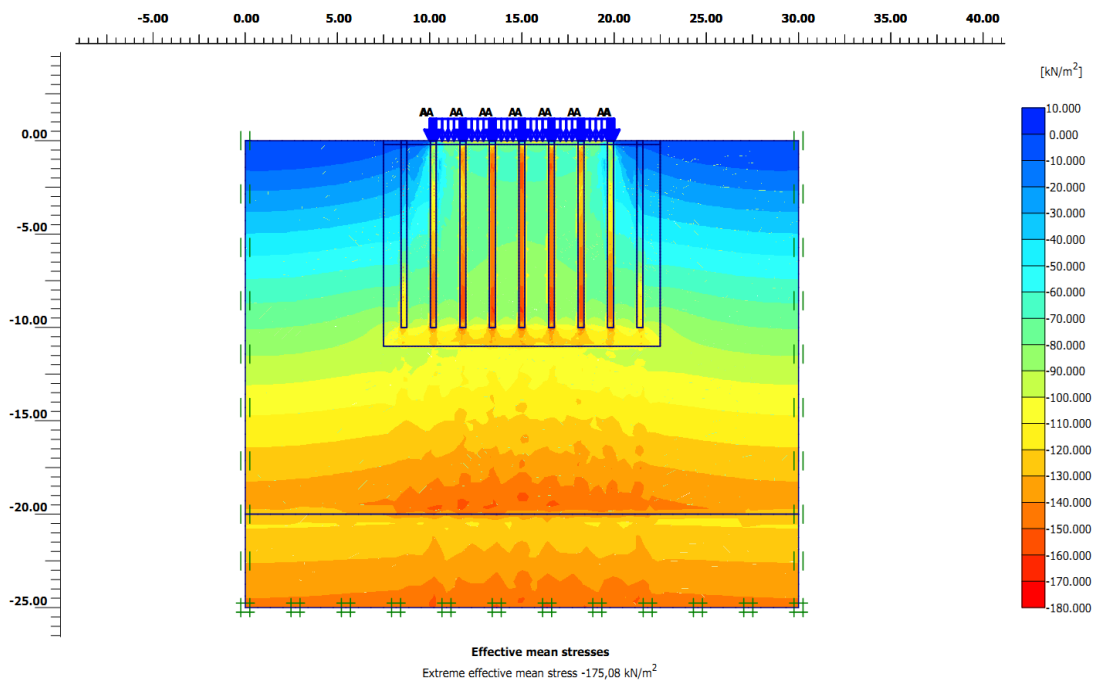


Figure A.8 Effective mean stresses of the numerical model composed of stone columns with $L/H=0.5$ in clay having $E= 6000$ kPa at 120 kPa flexible foundation pressure

APPENDIX B

AVERAGE STRESS CALCULATIONS FOR SELECTED CASES

	X [m]	Y [m]	σ'_N [kN/m ²]	X_2-X_1	$(\sigma_1+\sigma_2)/2$	Area $((X_2-X_1) * (\sigma_1+\sigma_2)/2)$ [kN/m]	Total Area [kN/m]	ΔX [m]	σ'_{avg} [kN/m ²]
Stone Column	10.040	-0.204	-35.230						
	10.040	-0.204	-40.076	0.000	-37.653	0.000			
	10.358	-0.204	-239.528	0.318	-139.802	-44.508	-45,094	0.320	-140.92
	10.358	-0.204	-356.374	0.000	-297.951	0.000			
	10.360	-0.204	-358.994	0.002	-357.684	-0.586			
10.360	-0.204	-94.623	0.000	-226.808	0.000				
10.997	-0.204	-95.394	0.637	-95.008	-60.494				
Silty Clay	10.997	-0.204	-98.551	0.000	-96.972	0.000	-116,499	1.280	-91.01
	11.003	-0.204	-98.416	0.007	-98.483	-0.645			
	11.003	-0.204	-96.830	0.000	-97.623	0.000			
	11.640	-0.204	-77.058	0.637	-86.944	-55.359			
	11.640	-0.204	-308.293	0.000	-192.675	0.000			
Stone Column	11.958	-0.204	-282.872	0.318	-295.582	-94.102	-94,503	0.320	-295.32
	11.958	-0.204	-244.732	0.000	-263.802	0.000			
	11.960	-0.204	-244.281	0.002	-244.507	-0.401			
	11.960	-0.204	-88.642	0.000	-166.462	0.000			
Silty Clay	12.597	-0.204	-82.722	0.637	-85.682	-54.556	-106,693	1.280	-83.35
	12.597	-0.204	-83.454	0.000	-83.088	0.000			
	12.603	-0.204	-83.453	0.007	-83.453	-0.547			
	12.603	-0.204	-82.680	0.000	-83.066	0.000			
	13.240	-0.204	-79.371	0.637	-81.026	-51.591			
Stone Column	13.240	-0.204	-284.352	0.000	-181.861	0.000	-92,727	0.320	-289.77
	13.558	-0.204	-295.778	0.318	-290.065	-92.346			
	13.558	-0.204	-233.345	0.000	-264.561	0.000			
	13.560	-0.204	-232.775	0.002	-233.060	-0.382			
Silty Clay	13.560	-0.204	-87.669	0.000	-160.222	0.000	-109,610	1.280	-85.63
	14.197	-0.204	-83.586	0.637	-85.628	-54.521			
	14.197	-0.204	-84.217	0.000	-83.902	0.000			
	14.203	-0.204	-84.209	0.007	-84.213	-0.552			
	14.203	-0.204	-83.243	0.000	-83.726	0.000			
Stone Column	14.840	-0.204	-88.061	0.637	-85.652	-54.537	-92,556	0.320	-289.24
	14.840	-0.204	-236.013	0.000	-162.037	0.000			
	14.842	-0.204	-236.662	0.002	-236.338	-0.387			
	14.842	-0.204	-290.906	0.000	-263.784	0.000			
Silty Clay	15.160	-0.204	-288.113	0.318	-289.509	-92.169	-108,607	1.280	-84.85
	15.160	-0.204	-84.107	0.000	-186.110	0.000			
	15.797	-0.204	-83.327	0.637	-83.717	-53.305			
	15.797	-0.204	-84.455	0.000	-83.891	0.000			
	15.803	-0.204	-84.462	0.007	-84.459	-0.553			
Stone Column	15.803	-0.204	-83.878	0.000	-84.170	0.000	-94,407	0.320	-295.02
	16.440	-0.204	-88.092	0.637	-85.985	-54.749			
	16.440	-0.204	-226.948	0.000	-157.520	0.000			
	16.442	-0.204	-227.504	0.002	-227.226	-0.372			
Silty Clay	16.442	-0.204	-294.059	0.000	-260.782	0.000	-103,166	1.280	-80.60
	16.760	-0.204	-296.680	0.318	-295.369	-94.034			
	16.760	-0.204	-74.601	0.000	-185.640	0.000			
	17.397	-0.204	-82.897	0.637	-78.749	-50.142			
	17.397	-0.204	-83.830	0.000	-83.364	0.000			
Stone Column	17.403	-0.204	-83.827	0.007	-83.828	-0.549	-91,330	0.320	-285.41
	17.403	-0.204	-83.030	0.000	-83.428	0.000			
	18.040	-0.204	-81.799	0.637	-82.415	-52.475			
	18.040	-0.204	-297.084	0.000	-189.442	0.000			
Silty Clay	18.358	-0.204	-273.729	0.318	-285.407	-90.863	-115,256	1.280	-90.04
	18.358	-0.204	-285.392	0.000	-279.560	0.000			
	18.360	-0.204	-285.442	0.002	-285.417	-0.468			
	18.360	-0.204	-79.140	0.000	-182.291	0.000			
	18.997	-0.204	-97.865	0.637	-88.502	-56.352			
Stone Column	18.997	-0.204	-98.804	0.000	-98.335	0.000	-45,945	0.320	-143.58
	19.003	-0.204	-98.971	0.007	-98.888	-0.648			
	19.003	-0.204	-94.311	0.000	-96.641	0.000			
	19.640	-0.204	-88.676	0.637	-91.494	-58.256			
Silty Clay	19.640	-0.204	-411.067	0.000	-249.872	0.000	-115,256	1.280	-90.04
	19.642	-0.204	-409.809	0.002	-410.438	-0.672			
	19.642	-0.204	-207.154	0.000	-308.481	0.000			
	19.960	-0.204	-77.258	0.318	-142.206	-45.273			
	19.960	-0.204	-35.529	0.000	-56.394	0.000			

Figure B.1 Stresses on the stone columns and surrounding clay for the model composed of stone columns with L/H=0.5 in clay having E= 6000 kPa at 120 kPa rigid foundation pressure

	X	Y	σ'_N	$X_2 \cdot X_1$	$(\sigma_1 + \sigma_2)/2$	Area $((X_2 \cdot X_1) \cdot (\sigma_1 + \sigma_2)/2)$	Total Area	ΔX	σ'_{avg}
	[m]	[m]	[kN/m ²]			[kN/m]	[kN/m]	[m]	[kN/m ²]
	10,040	-0,207	-32,773						
Stone Column	10,040	-0,207	-73,373	0,000	-53,073	0,000	-51,957	0,320	-162,36
	10,357	-0,207	-250,978	0,317	-162,175	-51,373			
	10,357	-0,207	-179,716	0,000	-215,347	0,000			
	10,360	-0,207	-182,122	0,003	-180,919	-0,584			
Silty Clay	10,360	-0,207	-73,761	0,000	-127,941	0,000	-141,270	1,280	-110,37
	10,994	-0,207	-167,625	0,634	-120,693	-76,464			
	10,994	-0,207	-129,866	0,000	-148,745	0,000			
	11,006	-0,207	-129,166	0,013	-129,516	-1,673			
	11,006	-0,207	-120,117	0,000	-124,641	0,000			
	11,640	-0,207	-79,185	0,634	-99,651	-63,133			
Stone Column	11,640	-0,207	-277,015	0,000	-178,100	0,000	-82,680	0,320	-258,37
	11,957	-0,207	-239,265	0,317	-258,140	-81,772			
	11,957	-0,207	-280,521	0,000	-259,893	0,000			
	11,960	-0,207	-282,149	0,003	-281,335	-0,908			
Silty Clay	11,960	-0,207	-75,758	0,000	-178,954	0,000	-128,509	1,280	-100,40
	12,594	-0,207	-122,968	0,634	-99,363	-62,951			
	12,594	-0,207	-120,538	0,000	-121,753	0,000			
	12,606	-0,207	-120,528	0,013	-120,533	-1,557			
	12,606	-0,207	-122,386	0,000	-121,457	0,000			
	13,240	-0,207	-79,657	0,634	-101,021	-64,001			
Stone Column	13,240	-0,207	-298,780	0,000	-189,218	0,000	-85,256	0,320	-266,43
	13,557	-0,207	-233,664	0,317	-266,222	-84,332			
	13,557	-0,207	-285,764	0,000	-259,714	0,000			
	13,560	-0,207	-287,210	0,003	-286,487	-0,925			
Silty Clay	13,560	-0,207	-80,585	0,000	-183,897	0,000	-130,049	1,280	-101,60
	14,194	-0,207	-121,893	0,634	-101,239	-64,139			
	14,194	-0,207	-119,891	0,000	-120,892	0,000			
	14,206	-0,207	-119,886	0,013	-119,888	-1,548			
	14,206	-0,207	-121,711	0,000	-120,799	0,000			
	14,840	-0,207	-81,469	0,634	-101,590	-64,361			
Stone Column	14,840	-0,207	-280,722	0,000	-181,095	0,000	-84,416	0,320	-263,80
	14,843	-0,207	-279,404	0,003	-280,063	-0,904			
	14,843	-0,207	-231,451	0,000	-255,427	0,000			
	15,160	-0,207	-295,820	0,317	-263,635	-83,512			
Silty Clay	15,160	-0,207	-81,146	0,000	-188,483	0,000	-129,981	1,280	-101,55
	15,794	-0,207	-121,972	0,634	-101,559	-64,342			
	15,794	-0,207	-120,056	0,000	-121,014	0,000			
	15,806	-0,207	-120,064	0,013	-120,060	-1,550			
	15,806	-0,207	-122,203	0,000	-121,134	0,000			
	16,440	-0,207	-80,113	0,634	-101,158	-64,088			
Stone Column	16,440	-0,207	-288,168	0,000	-184,141	0,000	-85,490	0,320	-267,16
	16,443	-0,207	-286,702	0,003	-287,435	-0,928			
	16,443	-0,207	-235,572	0,000	-261,137	0,000			
	16,760	-0,207	-298,325	0,317	-266,949	-84,562			
Silty Clay	16,760	-0,207	-79,766	0,000	-189,046	0,000	-130,774	1,280	-102,17
	17,394	-0,207	-122,339	0,634	-101,052	-64,021			
	17,394	-0,207	-120,509	0,000	-121,424	0,000			
	17,406	-0,207	-120,512	0,013	-120,510	-1,556			
	17,406	-0,207	-122,741	0,000	-121,627	0,000			
	18,040	-0,207	-83,074	0,634	-102,907	-65,196			
Stone Column	18,040	-0,207	-296,867	0,000	-189,970	0,000	-83,341	0,320	-260,44
	18,357	-0,207	-223,706	0,317	-260,287	-82,452			
	18,357	-0,207	-274,895	0,000	-249,301	0,000			
	18,360	-0,207	-276,322	0,003	-275,608	-0,890			
Silty Clay	18,360	-0,207	-69,827	0,000	-173,074	0,000	-144,719	1,280	-113,06
	18,994	-0,207	-124,104	0,634	-96,965	-61,432			
	18,994	-0,207	-131,219	0,000	-127,661	0,000			
	19,006	-0,207	-131,995	0,013	-131,607	-1,700			
	19,006	-0,207	-170,590	0,000	-151,293	0,000			
	19,640	-0,207	-86,969	0,634	-128,779	-81,587			
Stone Column	19,640	-0,207	-186,248	0,000	-136,608	0,000	-45,191	0,320	-141,22
	19,643	-0,207	-184,523	0,003	-185,386	-0,598			
	19,643	-0,207	-179,341	0,000	-181,932	0,000			
	19,960	-0,207	-102,204	0,317	-140,773	-44,593			
	19,960	-0,207	-38,812	0,000	-70,508	0,000			

Figure B.2 Stresses on the stone columns and surrounding clay for the model composed of stone columns with L/H=0.5 in clay having E= 6000 kPa at 120 kPa flexible foundation pressure