AN INVESTIGATION OF PROSPECTIVE TEACHERS' MATHEMATICAL MODELLING PROCESSES AND THEIR VIEWS ABOUT FACTORS AFFECTING THESE PROCESSES

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ABSTRACT

AN INVESTIGATION OF PROSPECTIVE TEACHERS' MATHEMATICAL MODELLING PROCESSES AND THEIR VIEWS ABOUT FACTORS AFFECTING THESE PROCESSES

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Researchers have proposed various models to explain mathematical modelling process for different purposes by using different kinds of modelling problems. There has been agreement mainly on that the process basically includes the interplay between the mathematics and reality. Review of the literature shows that there is not enough information about how prospective teachers develop mathematical models while they were engaging in modelling activities by describing the nature of the processes in each stages of modeling in detail or explaining the possible factors affecting these processes. The main aim of this study is to understand how prospective teachers develop models while they were engaging in modelling activities, and their views about the factors that might influence their modelling process. The study was conducted as a part of fourteen week course. Five modelling activities were selected for further analysis. The participants were six prospective teachers who were selected by purposive sampling. The study was designed as a case study. The results showed that PTs modelling consisted of four main stages: understanding the modeling problem, devising a solution plan, performing the plan, and interpreting and verifying the model. The results indicated that several factors impede the modelling processes: exam-oriented education system and insufficient practice with modelling activities, inadequate conceptual understanding, time limitations, grade considerations, and disorganized and unsystematic work. These factors hinder an atmosphere conducive for exploration of modelling activities and cause the prevalence of result-oriented approach with a single modelling cycle.

Keywords: Mathematics Education, Prospective Teacher Education, Mathematical Modeling Process, Mathematical Modelling Activities

ÖZ

ÖĞRETMEN ADAYLARININ MATEMATİKSEL MODELLEME SÜREÇLERİNİN VE BU SÜRECE ETKİ EDEN FAKTÖRLERE İLİŞKİN GÖRÜŞLERİNİN İNCELENMESİ

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Araştırmacılar, değişik özellikte modelleme aktivitelerini kullanarak, değişik araştırma amaçlarına hizmet eden ve modelleme sürecini yansıtan farklı modeller geliştirmişlerdir. Geliştirilen bu modellerde, sürecin temel olarak matematik ve gerçeklik arasındaki etkileşimi içerdiği konusunda bir uzlaşma vardır. Literatür, öğretmen adaylarının modelleme süreç aşamalarını detaylı bir şekilde tanımlayan çalışmaların ve sürece etki eden faktörlere ilişkin görüşlerinin yansıtıldığı yeterli düzeyde bilgi olmadığını göstermektedir. Bu çalışmanın esas amacı, öğretmen adaylarının modelleme etkinlikleri üzerinde çalışırken modelleri nasıl oluşturduklarını anlamak ve modelleme süreçlerine etki eden faktörlerin neler olabileceği konusundaki görüşlerini ortaya koymaktır. Çalışma ondört hafta süren bir dersin parçası olarak beş modelleme etkinliği çerçevesinde yürütülmüştür. Çalışmanın katılımcıları amaçlı örneklem yöntemiyle seçilmiş altı öğretmen adayıdır. Bu çalışmada durum çalışması kullanılmıştır. Sonuçlar, genel olarak öğretmen adaylarının modelleme sürecinin dört ana aşamadan oluştuğunu göstermiştir. Bunlar, modelleme problemini anlama, plan geliştirme, planı uygulama ve oluşturulan modeli yorumlama ve test etme aşamalarıdır. Sonuçlar göstermiştir ki öğretmen adaylarının modelleme ile ilgili deneyim eksiklikleri, yetersiz kavramsal anlayışları, zaman sınırlılıkları ve değerlendirme kaygıları gibi birçok faktör modelleme sürecinin başarılı bir şekilde uygulanmasını engellemektedir. Çalışma, tüm bu faktörlerin modeleme etkinliklerinin başarılı bir şekilde uygulanmasını sağlayacak ortamın oluşmasını engelleyebileceğini ve öğretmen adaylarının modelleme sürecinde sonuç odakli tek bir modelleme döngüsü sergilemelerine sebep olabileceğini göstermiştir.

Anahtar Kelimeler: Matematik Eğitimi, Hizmet Öncesi Öğretmen Eğitimi, Matematiksel Modelleme Süreci, Matematiksel Modelleme Etkinlikleri

To my mother Sündüs Şen

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LIST OF ABBREVIATIONS

MoNE: Ministery of National Education
NCTM: National Council of Teachers of Mathematics
PT: Prospective Teacher
PTAn: n-th Prospective Teacher in Group A
PTBn: n-th Prospective Teacher in Group B

CHAPTER 1

INTRODUCTION

The skills aimed to be gained and developed by students, and emphasized by new secondary school mathematics curriculum in Turkey are “mathematical modelling and problem solving, using mathematical language and terminology correctly and effectively, mathematical reasoning and proving, making connections between the mathematical topics and concepts and other disciplines, valuing mathematics and learning of mathematics, developing psychomotor skills, operating effectively in information and communication technologies” (Ministry of National Education [MoNE]-Board of Education, 2013a, p. 4). The program developed as a part of reform efforts, emphasizes the mathematical modelling and problem solving skills, and draws attention to the importance of these skills by emphasizing the need for working on real-life problems, and thus using mathematics beyond school education. The new proposed elementary mathematics curriculum also states that one of the aims of the program is to develop students’ skills to understand mathematical concepts and transfer these concepts to real-life situations and other learning areas (MoNE-Board of Education, 2013b, p. 2). The curriculum refers to non-routine problem solving as one of the skills aimed to be gained by the students. In addition to this, it has a goal of improving these skills to be likely to solve real-life problems by modelling. In this context, the curriculum states that the problems should be selected to help students see the applications of mathematics clearly in real-life problem situations and to enable them to offer different solution approaches to the problems.

In addition to the reform efforts in Turkey, modelling also takes part in international documents related to the reform oriented mathematics education (Blomhøj & Kjeldsen, 2006; Hodgson, 1995; Lesh & Doerr, 2003; Mousoulides, Pittalis, Christou, & Sriraman, 2010; NCTM 1989). In these documents, emphasize is given to the students’ work with the entire modelling process (Blomhøj & Kjeldsen, 2006). These documents also serve as evidence that one of the important components of the mathematics curricula is mathematical modelling (Lingefjärd, 2006; Blomhøj & Kjeldsen, 2006).

The PISA study (Program for International Student Assesment) done by OECD (Organization of Economic Co-operation and Development) defines mathematical literacy as “capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to engage in mathematics, in ways that meet the needs of that individual’s life as a constructive, concerned, and reflective citizen.” (OECD, 2000, p. 50). Thus, related to the mathematical literacy, the emphasis in PISA is on the functional usage of mathematics in variety of situations and contexts and the development of abilities to meet the challenges of real-life (Turner, 2007). With the tasks involving modelling, PISA emphasizes the importance of the ability to translate everyday problems into mathematical context and applying mathematical knowledge and procedures to solve them (Turner, 2007).

Following the announcement of the resulting data that measure the mathematical literacy of students, several countries began to debate on the aim and structure of the mathematical instruction given at schools and especially on the role of mathematical modelling providing the link between the school mathematics and mathematics required for today's knowledge societies (Blum et al., 2002; Turner, 2007).

In this context, related literature emphasizes the development of modeling skills as an outcome of programs at schools and universities (Berry, 2002; Lesh & Doerr, 2003; Usiskin, 1991). The rationale for the mathematical modeling in curricula is stated as the inadequacy of current teaching approaches in teaching mathematics and mathematical problem solving (Verschaffel, Greer & De Corte, 2002) and the inadequacy to prepare students to be able to use their knowledge to solve the problems in their real-life (Niss, Blum, Galbraith, 2007, Usiskin, 1991). As a supporting argument to the inadequacies, the studies showed the students' struggle within the both routine and non-routine word problems in school textbooks and tests (English & Sriraman, 2010). In routine problems, although students are required to apply only standard and apparent procedures, they face with difficulties. Furthermore, in non-routine problems, students have difficulty in finding a way from the givens to a stated goal (English & Sriraman, 2010). The studies also showed that the students faced with difficulties while working on real-life problems, nonroutine in nature, by trying to use school mathematics (Lesh & Doerr, 2003; Kaiser, Blomhøj, & Sriraman, 2006; Niss, et al., 2007). Besides students, studies showed that so many people saw no relevance of mathematics to their life (Verschaffel, et al., 2002, p.273) as they could not convey mathematical ideas beyond school (Arora & Rogerson, 1991; English, 2009).

In the study covering the problem solving literature, Schoenfeld (1992), and a decade after, Lester and Kehle (2003) concluded that although Polya's contribution to problem solving literature is important, teaching of problem solving strategies suggested by Polya has not made significant improvements on student's problem solving abilities. One reason of ineffectiveness of such strategies is that the students have difficulties in selecting the correct procedures or appropriate tools to be able to solve the problem. English and Sriraman (2010) stated that students need to know when, where, why, and how to use these strategies. On the other hand, current teaching practices of problem solving are barrier to develop these skills. Problem solving has been taught at classrooms as an isolated topic and problem solving activities are implemented after the teaching of basic mathematical concepts (English & Sriraman, 2010). Thus, by directing students' attention toward particular concepts problem solving activities do not enable students to transfer their learning and develop problem solving skills (Lobato, 2003).

An increasing number of recent research studies focused their efforts on mathematical modelling and documented the importance of implementing mathematical modeling activities at every grade even including the primary school (English, 2006; English, 2009; English & Watters, 2005; English & Sriraman, 2010; Mousoulides, Christou, Sriraman, 2008). However, the meanings of the terms, "mathematical models" and "modelling" are defined differently based on the different research purposes and different perspectives (Arora & Rogerson, 1991; Berry, 2002; Kaiser & Sriraman, 2006; English, 2009). In the broadest sense, mathematical modeling could be defined as a complex process including the application of mathematics to solve realistic problems (Berry & Houston, 1995; Brown, 2002; Edwards & Hamson, 1990; Maaß & Gurlitt, 2010; Schaap, Vos, Goedharhart, 2011; Verschaffel, et al., 2002). Researchers argued that modeling activities provide an ideal environment to develop problem solving abilities needed for success in today's world (English & Sriraman, 2010; Lesh & Doerr, 2003). While engaging in modeling activities, students try to mathematize meaningful situations which draw upon several disciplines (English & Sriraman, 2010; Lesh & Doerr, 2003). In these situations, students need to organize, select, sort, or quantify information to be able to reach a solution of a realistically complex problems (Lesh & Doerr, 2003). Hence, rather than simply adding, subtracting, dividing, or multiplying to reach simple closed form algebraic equations or short numeric answers, students try to find the patterns and regularities and try to generate important mathematical ideas (English & Sriraman, 2010; Lesh & Doerr, 2003). Modelling problems do not direct students by keywords or sentences in the problem text like in the traditional problems in which the problem is already mathematized for the students. Instead, modelling problems allow students to create their own mathematical constructs which could then be applied to other related situations, and thus allow generalizations (English & Sriraman, 2010). Contrary to the traditional problem solving approach, in modelling activities, students work collaboratively. Group works provide opportunities for students to work independently from the teacher and supply environment for students both to develop communication skills and learn mathematics themselves and in a meaningful way (Lesh & Doerr, 2003). Thus, modelling problems go beyond traditional school mathematics problems by leading students to significant learning.

1.1. Modelling Process

Especially for the last two decades, modelling has been viewed as being crucial component of students' mathematics education. In principle, as every mathematical model is a product of a mathematical modelling process, one of the major issues in the literature of the teaching and learning mathematical modelling is the mathematical modelling process (Kaiser et al., 2006). The literature suggests so many different models for modelling process that have been developed and used for different research purposes (Kaiser, et al., 2006; Borromeo-Ferri, 2006; Galbraith, 2012). Modelling process defined in the Standards (NCTM, 1989) as involving number of interrelated stages: identifying and simplifying the given situation, building a mathematical model, transforming and solving the model, interpreting and validating the model (p. 138). If the solution is found unsatisfactory in the validation stage, or cannot yield an answer to the initial problem, the process starts over. This feature represents the cyclic nature of the mathematical modelling process. This process of modelling also forms the theoretical basis for this research study.

Galbraith (2012) mentioned the two main functions of modelling process diagrams. One is the representation of problem solving process as students work on the modelling problems. It is because of this reason that the diagrams emerged in the literature to understand how students explore the modelling tasks and go through the process. In this sense, Blum and Borromeo-Ferri (2009) explained that in addition to representing the essential stages in modeling, the diagrams could also help to identify the cognitive barriers of students while working on the modelling problems. Other function of the framework is that it provides a supportive mental structure for students' learning of modelling process, so that students could use these diagrams as a referent both to see their progress and to monitor their learning (Galbraith, 2012). The studies in the literature also revealed that knowledge about the modelling process positively influence the acquisition of modelling competencies (Maaß, 2006).

1.2. Modelling in Mathematics Education

Although modelling provides an environment that allows students to test, revise, refine and extent their thoughts, the studies show the difficulties of students while engaging in modelling activities (Blum & Borromeo-Ferri, 2009; Galbraith & Stillman, 2006; Maaß, 2006; Mousoulides & English, (in press)). The studies show that students sometimes even could not provide any mathematically robust models while engaging in modelling activities (Mousoulides & English, (in press)). In terms of modelling process, especially, understanding and constructing, simplifying, and validating stages are particularly found problematic for students in the studies (Berry, 2002; Blum & Borromeo-Ferri, 2009; Galbraith & Stillman, 2006; Sol, Gimenez, & Rosich, 2011). These studies show that students have difficulty in understanding and structuring the correct problem situation by identifying key elements, then making the related assumptions to simplify the situations to be able to make it workable. Accordingly, studies reveal the difficulties of students' in moving from the real-world to mathematical world. Researches also indicate that in general, the students do not check for the reasonableness and the appropriateness of their solutions.

In this context, the essential role of teacher should be recognized. In modelling activities, although students experience themselves as being the active participant of the modelling process, when students tackle somewhere in the modelling process and could not pursue, the teacher should identify their difficulties and ask them to justify their conjectures or solution approaches (Galbraith, 2012). While doing these, it is also important and hard to maintain the balance between the students' independence, as emphasized in the modelling perspective, and the teacher's support and guidance (Borromeo-Ferri & Blum, 2010). However, studies show both in-service and pre-service teachers' difficulties in modelling process (Biembengut & Faria, 2011; Cross & Moscardini, 1985; Julie & Mudaly, 2007; Kaiser, 2007) and their difficulties in understanding the several approaches to modelling (Borromeo-Ferri & Blum, 2010). Studies also show that prospective teachers (PTs) found teaching and learning modelling difficult and complex (Lingefjärd & Holmquist, 2005). Therefore, PTs should be more knowledgeable in the field of modelling and have an actual firsthand experience with modelling activities (Niss et al., 2007; Swetz & Hartzler, 1991). The acquisition of PTs' adequate knowledge and skills on modelling is the prerequisite for the development of students' modelling skills (Niss et al., 2007). Therefore, in order to use modeling in classes, it is essential in teacher education programs to prepare PTs who are knowledgeable in modelling and skilled to teach modelling (Blum et al., 2002; Niss et al., 2007). Furthermore, developing mathematical modelling skills is generally agreed to be more difficult to teach (Blum &Ferri, 2009). Stillman and Brown (2011) reported the part of the results of an international study, conducted at sites in Germany, Australia, Hong Kong, the Chinese mainland and Taiwan by the goal of investigating and evaluating the professional knowledge and competencies of PTs in modelling. They found that most of the PTs could not diagnose students' difficulties in modelling tasks and analyze students' responses to these tasks although they had related school experience allowing them to acquire related pedagogical content knowledge. Although these reported difficulties of PTs, the researchers claimed that the courses received during their training had power to shape the PTs' dispositions toward the aspects of modelling (Verschaffel, et al., 2002). The studies showed that long-term engagement with modelling activities and actually solving real world problems offer significant opportunities for professional development of PTs (Berry & Houston, 1995; Borromeo-Ferri, 2011; Burkhardt, 2011; Maaß, 2006).

In line with the mentioned shortcomings above, the questions of how PTs deal with modelling activities as they go through the modelling process and what are their thoughts about the factors that might influence their modelling process are two important questions open to investigation. This study tries to respond to the needs in the literature by trying to answer these two research questions. The research questions addressed in the study are as follows:

1) How do prospective teachers construct mathematical models when they engage in modelling activities?
   - What are the basic processes that prospective teachers go through when they attempt to understand the given modelling problems?
   - What are the basic processes that prospective teachers perform when they devise a plan for the given modelling activities?
   - What are the basic processes that prospective teachers perform when they work on the problem?
   - What are the basic processes that prospective teachers perform when they interpret and verify the model constructed for the given modelling activities?
   - What are the basic processes that prospective teachers perform when they present their models?
2) What are the prospective teachers' views about the factors affecting their modelling processes?

1.3. Significance of the Study

Modeling activities help significant forms of learning that differ from traditional problem solving at least in three important ways. First, modeling problems provide opportunities for students to elicit their thought process as they work on the problems designed to help them make sense the realistic situations (Lesh et. al., 2000; Lesh & Doerr, 2003). Second, while engaging on modelling activities, students see the need for mathematization and need to use mathematical concepts together (Lesh & Doerr, 2003; Lesh & Zawojewski, 2007). Third, while engaging on modeling activities, students try to create models that are also applicable to other similarly structured situations. Thus, instead of finding only an answer, they make generalizations and extend their solutions (English, 2006; Lesh, et al., 2000). Consequently, modeling process could be considered as a significant learning process (Lesh & Doerr, 2003; Biembengut & Faria, 2011). Besides these cognitive superiorities, modeling activities also address the affective aspects of learning (Lesh & Doerr, 2003). Since modeling problems involve the examples of real-life problem situations, students could find the answer of the question of "where do we use this mathematics?" and hence they would be more motivated to learn mathematics (Doruk, 2010; Özalp, 2006; Swetz & Hartzler, 1991). As a result, while going through the modelling process, they both improve their problem solving skills and learn to use mathematics in their real-life. Based on these superiorities of applications of modelling activities, researchers and curriculum developers in mathematics education agree that mathematics education should enable students to learn mathematics in a meaningful way and learn to apply mathematics in their everyday-life (Doruk, 2010; English, 2009; Maaß & Gurlitt; 2010; Verschaffel, et al., 2002). For that reason, researchers agree that modelling should be the important part of elementary, secondary, tertiary, and teacher education curricula.

Researchers, for different purposes, have proposed many different models for modelling process while students engaging in modelling tasks. However, there has been agreement mainly on that the process basically includes the interplay between the mathematics and rest of the world. Moreover, the nature of the modeling tasks used in these studies also shows differences. Review of the literature demonstrates that there is a lack of research on describing the stages of modelling process more in detail (Borromeo-Ferri, 2006).

On the other hand, most of the international studies on modeling focused on how students engage in modelling tasks or how modelling could be taught in schools (Borromeo-Ferri & Blum, 2009). Studies conducted in Turkey highlighting the importance of modelling based on problem solving approaches and strategies of students' on the non-routine word problems (Kılıç, 2011; Arslan & Altun, 2007; Olkun, Şahin, Akkurt, Dikkartın & Gülbağcı, 2009), prospective and in-service teachers' knowledge and views on modelling (Aydoğan-Yenmez, 2012; Çiltaş & Işık, 2013; Eraslan, 2011; Özer-Keskin, 2008; Türker, Sağlam & Umay, 2010; Tekin, Kula, Hıdıroğlu, Bukova-Güzel, & Uğurel, 2012), the relationship between PTs' academic achievement and modelling approaches (Bukova-Güzel & Uğurel, 2010), and searching the effects of mathematical modelling method on students' academic achievement (Özturan-Sağırlı, Kırmacı & Bulut, 2010). The studies conducted in Turkey to understand the prospective elementary or secondary mathematics teachers' modelling process are new, and therefore very limited (Bukova-Güzel, 2011; Bukova-Güzel & Uğurel, 2010; Çiltaş & Işık; 2013; Eraslan, 2012; Kertil, 2008; Özdemir & Üzel, 2013; Özer-Keskin, 2008; Türker et al., 2010) Furthermore, none of these studies discuss how PTs construct models while they are engaging in modelling activities in detail by describing the nature of the processes in each stages of modeling or explaining the factors affecting the process they go through depending on their views. Most of these studies are based on the participants' solution papers or interviews based on their views on modelling. More detailed analysis supported by video and audio tape is needed to better understand the modelling process (Hodgson, 1997). This is another reason to make a more detailed research supported by audio and videotaped observations of all stages of modelling process, as well as supported by PTs' solution papers and in-depth interviews.

To be able to improve the modelling instruction, as emphasized in the new curricula, in mathematics education standards, and in teacher competencies, modelling courses should take part in teacher education programs as a part of their professional development (Antonius, Haines, Jensen, Niss, & Burkhardt, 2007; Çiltaş & Işık, 2013; Lingefjärd, 2007; Bukova-Güzel, 2011; Özturan-Sağırlı et al., 2010). This supports the claim of Cohen and Ball (1990) "how teachers can teach mathematics that they never learned, in ways they never experienced?" (p.233). Although mathematical modelling has became an important part of mathematics curricula in elementary and secondary school levels both in Turkey and in other countries, modelling is not integrated explicitly in the curriculum in the education of mathematics teachers (Biembengut & Faria, 2011; Oliveira & Barbosa, 2009), so most teachers have not had any experience or received training in mathematical modelling in their education (Blomhøj & Kjeldsen, 2006; Biembengut & Faria, 2011). For instance, in their study, Biembengut and Faria (2011) reported that only thirty percent of the mathematics teacher education programs in Brazil include modelling in the curriculum. The studies reveal that the situation is not different also in Europe (Maaß & Gurlitt, 2011; Blum, et al. 2002). The studies note the teachers' challenges in and concerns about the applications of modelling in their classrooms rooted from the inexperience in modelling (Burkhardt, 2011; Biembengut & Faria, 2011), because their roles changed from teaching mathematics as set of separate subjects to well-connected concepts, from assessing short numeric answers to their reasoning, and from being the source of the answer to being an adviser (Blomhøj & Kjeldsen, 2006; Burkhardt, 2011).

The studies reveal that most of the teachers have inadequate background to apply alternative approaches or change their current practices in their classrooms (Biembengut & Faria, 2011). Researchers argued that teachers' lack of knowledge on mathematical modelling and inexperience in teaching modelling is the main shortcoming of the applications of modelling activities in the classrooms (Antonius, et al., 2007). Recent research studies in Turkey indicate that although both secondary and elementary curricula in Turkey emphasize the importance of modelling, there are PTs having inadequate knowledge on modelling and even could not describe it properly (Özer-Keskin, 2008; Çiltaş & Işık, 2013; Tekin et al., 2012).

On the other hand, effective applications of modelling activities in classes depend heavily on the teachers (Blum et al. 2002; Niss, et al., 2007, Schorr & Lesh, 2003). Therefore, it is necessary to include modelling courses in prospective teacher education programs to provide opportunities for future teachers' learning various dimensions of modelling (Blum et al. 2002; Schorr & Lesh, 2003; Niss et al., 2007; Lingefjärd, 2002) to be able to create an appropriate modelling environment in classes and to teach students modelling effectively (Borromeo-Ferri & Blum, 2010; Burkhardt, 2011; Biembengut & Faria, 2011; Kuntze 2011; Maaß, 2006). Additionally, as the principles for students' learning is also true for teacher learning, this learning should be through meaningful problem solving experiences, namely by exploring modelling activities (Borromeo-Ferri & Blum, 2010; Lingefjärd, 2002; Schorr & Lesh, 2003). Researchers explained that good modelling skills could be developed either by "actually solving real problems, or working through the formulation of well known models" (Berry & Houston, 1995, p.25). Similarly, Blomhøj and Kjeldsen (2006), Borromeo-Ferri (2006), Cross and Moscardini (1985), Edwards and Hamson (1990) argued that modelling was learned and developed through practice.

It is important for teacher educators to have an idea about how PTs work on modelling activities to provide support and design appropriate courses. Review of the literature also shows the lack of research involving the essential components of a modelling course for PTs including the appropriate and well-prepared modeling tasks and also methodology of how to integrate these tasks into this course (Antonius, Haines, Jensen, Niss, & Burkhardt, 2007; Borromeo-Ferri & Blum, 2010). This poses a challenge for both PTs and teacher educators (Spainer, 1980). To be able to develop courses to teach mathematical modelling skills, there is a need for research to investigate how PTs work on the modelling problems (Berry, 2002). By giving the details of PTs' modelling process and providing information about the factors affecting their process, this study is believed to help the improvement of the appropriate course on modelling in teacher education programs.

Inherently, the study is intended to improve PTs’ knowledge about basic ideas in mathematical modelling in education and improve their experiences with modelling activities by providing an environment to solve modelling tasks. In this way, PTs' awareness about modeling, as well as their motivations for implementing modelling activities with their students in their future classes are attempted to be increased.

Definition of Terms:

Some important terms associated with this study are as the following.

Mathematical Models and Modeling:

In this study, mathematical modelling is defined as a complex mathematical process that involves observing a phenomenon, conjecturing the relationships among variables/ parameters, applying appropriate mathematical analysis (equations, formulas, symbolic structures etc.), obtaining results, and reinterpreting them in the context of the phenomena under investigation (Swetz & Hartzler; 1991). According to Swetz and Hartzler (1991), mathematical model is a “mathematical structure that approximates the features of a phenomenon” (p.1) and accordingly mathematical modelling is “the process of devising a mathematical model” (p.1).

Modelling activity:

Modeling activity is used in this study as the term “model-eliciting activity” which is defined as the tools that are thought-revealing in nature, thus help reveal the students’ ways of thinking, and their learning process (Lesh, et al., 2000). In addition to revealing students’ thought and learning processes, main characteristic of model-eliciting activities is that they help the development of students’ learning by providing meaningful real-life situations, and providing an environment for students to test, revise, and refine repeatedly their thoughts to construct more powerful models (Lesh, et al., 2000; Lesh & Doerr, 2003).

CHAPTER 2

LITERATURE REVIEW

The purpose of this study is to understand how PTs construct mathematical models while dealing with modelling activities and their thoughts on factors that might influence their modelling process. This chapter was divided into four sections that provide a review of the literature on particular topics that address the aim of the study: Approaches to mathematical modelling, mathematical modelling process, modeling problems, and modelling process difficulties and possible reasons. The first section contains a review of the literature related to different approaches to modelling. The second section includes the review of the literature related to the purposes to use modelling process and differences in the offered modelling processes. The third section of the literature review contains the features of modelling problems and principles behind their development. Finally, the last section includes the modelling process difficulties of students in different grade levels and different departments including the teacher education programs, and also in-service teachers and covers possible reasons of these difficulties.

2.1. Approaches to Mathematical Modelling

Mathematical modelling has gained substantial attention for mathematics education research especially over the last two decades (Galbraith, 2012). International Conference on Teaching of Mathematical Modelling and Applications (ICTMA) held since 1983 reflects the growing interest over the world. Modelling also took part in 1989 in National Council of Teachers' of Mathematics (NCTM) Standards document. The interests in modelling for the education area also lead the fourteenth International Commission on Mathematical Instruction (ICMI) to be on this theme in 2004. Despite this growing interest, the studies on this research area show that there is not the same understanding of modelling, partially because of the fact that there is a very large variation in approaches to modelling in international perspectives (Kaiser & Sriraman, 2006; Sriraman, Kaiser, & Blomhøj, 2006). Moreover, the meaning of the terms, the mathematical models and modelling, is defined differently in research studies in teaching, learning, and doing mathematics, based on the different research purposes and different perspectives (Niss et al., 2007). In general, modelling is used as an umbrella term referring the solution process of the problems set in the real-world (Berry, 2002). In their study, Kaiser and Sriraman (2006) classified the modelling perspectives into six categories: realistic or applied modelling, contextual modelling, educational modelling, socio-critical modelling epistemological modelling, and cognitive modelling. By focusing on the central purposes of these perspectives, referring the study of Julie and Mudaly (2007), Galbraith (2012) stated that all these approaches to the modelling in education actually falls into two categories: "modelling as vehicle approach" and “modelling as content approach”. Modelling as vehicle approach aims to provide settings to teach curricular mathematics to students, without aiming primarily the development of modelling skills. Contrary to this approach, modelling as content approach enables students to develop their modelling skills to solve real-life problems without prescribing certain mathematical concepts or procedures (Julie & Mudaly, 2007; Galbraith, 2012). Julie and Mudaly (2007) expressed that in modelling as vehicle approach, the mathematics applied to solve the problem is uncovered by cues given in the problem. On the contrary, in modelling as content approach structuring of the mathematical problem from real world problem situation based entirely on the modeler. Galbraith (2012) expressed the dual nature and power of “modelling as content approach”. He explained that instead of providing students experience for solving real-life problems, this approach, also helps students to develop their ability to use mathematical knowledge to solve problems and thus develop their modelling abilities over time. Similar to this distinction, Stillman (in press) mentioned the three perspectives of modelling. According to Stillman (in press) first of these perspectives regards mathematical modelling as a tool for motivating and developing mathematical learning, and second perspective considers teaching of modelling having some educational purposes without aiming to achieve development in mathematical learning mentioned in the first perspective. Moreover, the third perspective is models and modelling perspective offered by Lesh and Doerr (2003), while basically including the goal of the first perspective, encompassing the dimensions of the second perspective (Stillman, in press). Stillman (in press) defined her approach belonging to the second perspective involving the first perspective. By defining these perspectives Stillman (in press) also considered the intersection of the two approaches explained by Galbraith (2012).

2.2. Mathematical Modelling Process

Although there are some differences in approaches of modelling, the common view in these approaches is that the modelling is the implementation of mathematics in real-life situations to develop solutions to real-life problems (Brown, 2002; Berry & Houston, 1995; Cross & Moscardini, 1985; Houston, 2007; Lingefjärd, 2000; Maaß & Gurlitt, 2011; Özalp, 2006) and involves a process including basically the formulating, mathematizing, solving, interpreting and evaluating stages (Stillman, in press). By “real-life” (real-world) it is referred to the rest of the world outside mathematics including everyday life and other disciplines different from mathematics (Niss et al., 2007). Although it is defined in this way, mathematical modelling could not be considered as just taking a situation from the real world, selecting variables, using some functions to represent the situation, and then reaching a conclusion that could be interpreted in given real-life situation (Lingefjärd, 2006). Mathematical modelling is a complex mathematical process that involves observing a phenomenon, conjecturing the relationships among variables/parameters, applying appropriate mathematical analysis (equations, formulas, symbolic structures etc.), obtaining results, and reinterpreting them in the context of the phenomena under investigation (Swetz & Hartzler; 1991). According to Swetz and Hartzler (1991), mathematical model is a “mathematical structure that approximates the features of a phenomenon” (p. 1) and accordingly mathematical modelling is “the process of devising a mathematical model” (p.1). Like the relationship between the terms product and process, behind every mathematical model there is a corresponding modelling process (Kaiser et al., 2006). In broadest sense Berry and Houston (1995) defined mathematical models as “mathematical representation of relationships” (p. 1). In this regard, mathematical models could be equations (formulas), tables, graphs, diagrams (Swetz & Hartzler, 1991; Edwards & Hamson, 1990) that are produced as products of the modelling process.

In the literature, there are so many different models for modelling process developed and used for different purposes (Galbraith, 2012; Stillman, in press; Kaiser et al., 2006; Maaß & Gurlitt, 2011). Kaiser et al. (2006) offered six different purposes for using a general modelling cycle. They listed following purposes to use the modelling cycle as a tool for: a) analyzing the modelling process to select and design appropriate modelling problems and to understand and validate the process, b) identifying the major elements of modelling competency, c) analyzing the students’ modelling work in general in order to identify their difficulties, d) supporting the students’ work in modelling courses and related metacognition, e) planning modelling courses, and f) defining and analyzing the curricular elements in mathematics teaching. Galbraith (2012) mentioned two main functions of modelling frameworks. One is that they properly reflect problem solving process of students or other problem solvers as they engage in modelling activities, and other is that they provide a supportive framework for teaching modelling process especially for the beginner modelers.

Galbraith (2012) also expressed that the Figure 1 below describes the modelling process as the summary of the steps, basing the description of Pedley (2005), the president of the Institute of Mathematics and its Applications, and explained that this process represents what the applied mathematicians do to solve problems. By making distinction between the modelling process, and modelling diagram, Galbraith (2012) explained that the given modelling process below represents the core structure and should be kept distinct from the modelling diagrams developed for different research purposes.

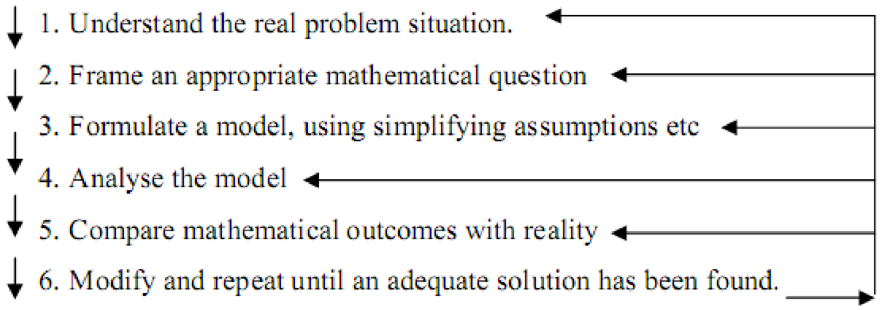


Figure 1 Modelling process (after Pedley 2005, Galbraith, 2012, p. 7)

Galbraith (2012) expressed that based on different purposes the research studies suggest so many modelling diagrams involving the embedded modelling cycle. Based on the study of Galbraith and Stillman (2006), Galbraith (2012) offered modelling process given in Figure 2 which involves both the basic modelling cycle and also the research focus on understanding the mental activities of students and also their blockages during transitions in the modelling process.

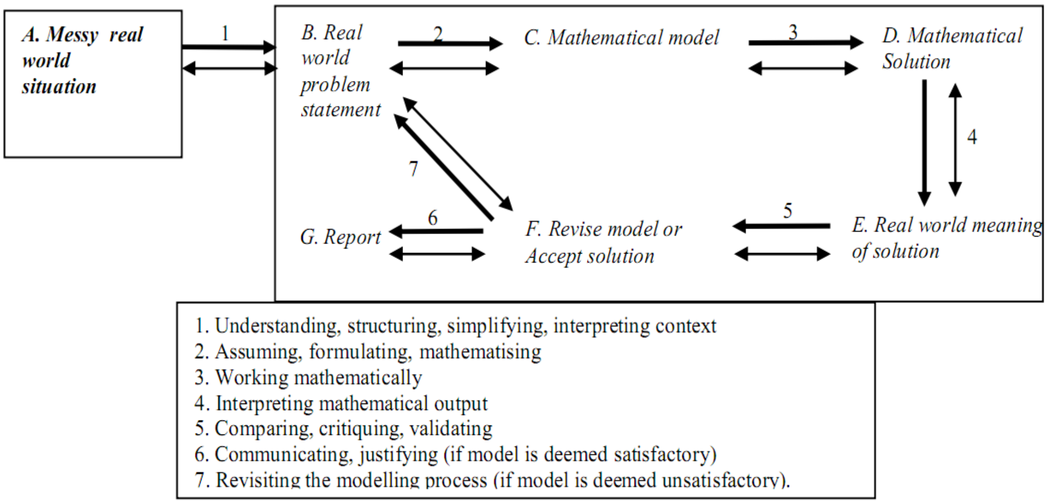


Figure 2 Modelling process (Galbraith, 2012, p. 10)

In the diagram above, the heavy arrows represents the problem solving process that ends with a report of modelling outcome including the cases of further cycles when the solution deemed unsatisfactory. On the other hand, the double-headed arrows represents the metacognitive activities including the reflections on the back and forth stages represented as A to G.

Blum and Leiß (2007), in the DISUM (Didactical intervention modes for mathematics teaching oriented towards self-regulation and directed by tasks) project, focusing on the modelling process, introduced the "situation model" as an intermediate stage. The term "situation model", also called as the problem model, is described as "a representation of the content of a text, independent of how the text was formulated and integrated with other relevant experiences" (Kintsch & Greeno, 1985, p. 110). Borromeo-Ferri (2006) described the situation model as the "mental representation of the situation". Seeing the most important stage of the modelling process, Blum and Leiß (2007) described the transition between the real-situation and situation model as the stage of understanding. According to this process, the next step involves the simplifying and structuring the situation model into a "real model". Then, real model is converted to the "mathematical model" by mathematizing. After working mathematically on this model, obtained mathematical results are interpreted in the context of the initial problem. If these results cannot be found satisfactory to answer the original problem, the modelling cycle should be run through. The modelling diagram developed by Blum and Leiß (2007) is given in Figure 3.

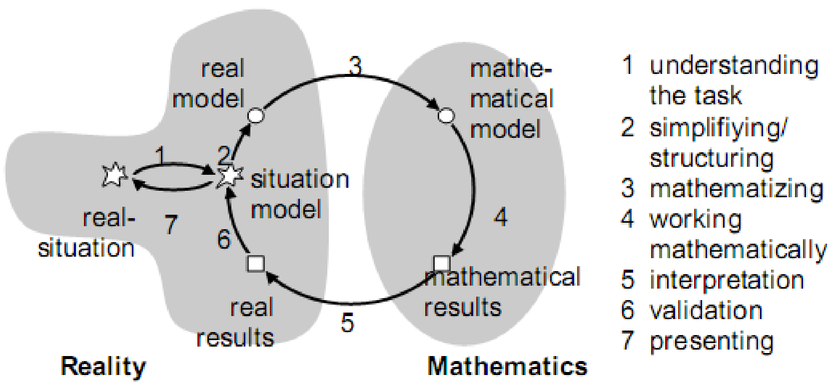


Figure 3 Modelling cycle (Blum & Leiß, 2007, p. 225)

Analyzing various modelling cycles in the COM[2] project (Cognitive psychological analysis of modelling processes in mathematics lessons), and focusing on the first three stages of modelling cycle developed by Blum and Leiß (2007), Borromeo-Ferri (2006) tried to show the differences between the terms "real situation" (RS), "situation model" (SM) or having similar meaning in the study the term "mental representation of the situation" (MRS), "real model" (RM) and "mathematical model" (MM) given in the modelling process charts. Borromeo-Ferri (2006) divided the different perspectives concerning the first three stages of the modelling process given in Figure 3 into four groups. The first group included the distinction between the situation model and real-model like in the Figure 3. The second group included the mixed type of SM and RM including only the direct transition from SM plus RM to MM. The third group included no distinction between the SM and RM, and the fourth group included no distinction between the SM and RM involving the direct transition from RS to MM.

As putting herself into the first group, Borromeo-Ferri (2006) concluded that the researchers in the first group focused on the cognitive processes during the modelling process and this is the reason of SM included in the process. Related to the second group (involves for instance the studies of Verschaffel, et al., 2002), Borromeo-Ferri (2006) expressed that not inclusion of RS stage in the modelling process might be the result of the nature of the task and specifically the formulation of the task. As word problems have already presented the problem situation in a simplified version, this brings no need for the development of the SM. Related to the third group, Borromeo-Ferri (2006) expressed that the researchers in this group did not differentiate SM and RM from each other. Therefore, SM does not appear as a stage in the modelling process. The modelling cycle offered by Kaiser (1995) and Blum (1996) could be classified in this group.

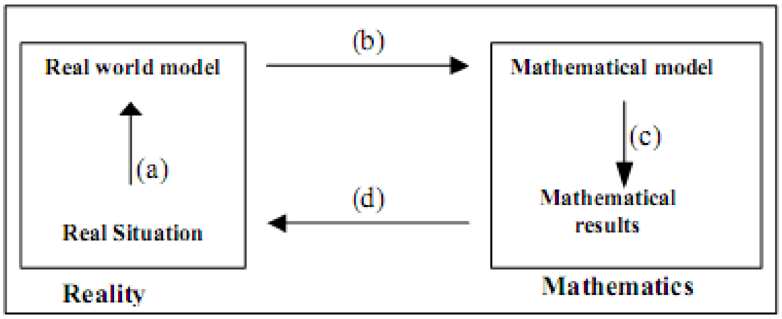


Figure 4 Modelling cycle from Kaiser (1995) and Blum (1996) (as cited in Borromeo-Ferri 2006, p. 88)

In Figure 4, the process begins with the real-world situation. In the first stage, the situation is simplified and thus structured to get a real-world model. In the next stage, the real-world model was converted into mathematical model by using mathematics, in the following stage, mathematical results were obtained by studying mathematically on the model. Then the results were interpreted considering real situation. And finally the results were checked for validity and if it is found unsatisfactory the process starts over (Kaiser, 2005, as cited in Ferri, 2006, p. 91).

According to Borromeo-Ferri (2006), the researchers in the fourth group included the direct transition from RS to MM in the modelling process, and accordingly no distinction between the RM and SM. Borromeo-Ferri (2006) explained that this could be the result of the kind of the modelling problems used. She described the nature of these problems as realistic and complex and added that the level of mathematics required to solve these kinds of problems is different than the more simple problems. Lesh and Doerr (2003), approaching modelling with more pedagogical and psychological concerns, presented a cycle without including RM or SM into the process. Lesh and Doerr (2003) described the modelling process including four stages: description, manipulation, prediction, and verification. This process is given in Figure 5 below. They explained that the first stage involves establishing a map from real world to model world. The second stage involves the manipulation of the model. The third stage involves making predictions about the real-world problem situation and the last one involves verification of the made predictions in terms of usefulness.

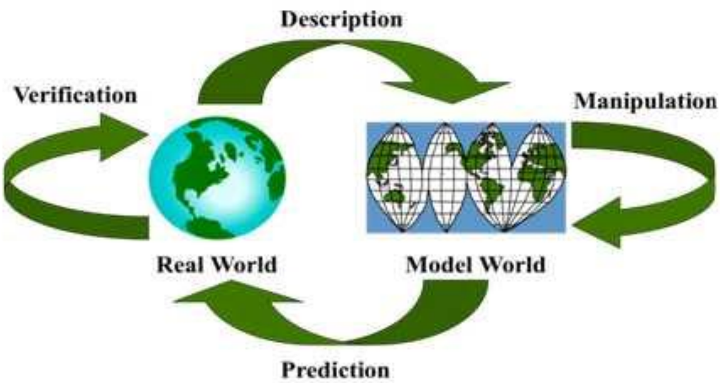


Figure 5 Modelling cycle (Lesh & Doerr, 2003, p. 17)

Being one of the main components of the modelling perspectives, as given in the examples from the literature, modelling process was defined differently based on the different research purposes and different modelling views and also different modelling tasks that were used (Kaiser et al., 2006; Borromeo-Ferri, 2006). Although the researchers used different number of stages in their cycles, in general, they conceived the modelling process as cyclic process involving some basic stages. As literature suggests the modelling process begins with the specification of the problem by construction of the simplified version of the problem. Then, by using mathematical terms, the problem is translated into the mathematical model consisting of variables and mathematical expressions depicting the relationships between them. The problem is then analyzed and solved to obtain mathematical results. Then, the results are interpreted in terms of the simplified real-world situation. Finally, solution generated for the simplified version of the problem is verified in the context of the original problem situation by the aim of answering the problem posed originally. If the solution found unsatisfactory, or could not yield an answer to the initial problem, the process starts over.

In the literature, while defining and explaining the modeling process, researchers also tried to present the process by listing corresponding set of actions (Berry & Houston, 1995, Galbraith & Stillman, 2006; Greefrath & Riess, in press; Sol et al., 2011). In their study, Berry and Houston (1995) summarized the key activities related to the modelling cycles in the modelling process as given in Table 1.

Table 1 The key activities in the modelling process (Berry & Houston, 1995, p. 39-40)

1. Understand the problem • decide what aspect of the problem to investigate • collect and analyze some data appropriate to the problem
2. Choosing variables • 'brainstorm' the situation or problem to form a feature list • Refine and sort your list into the key features • For the key features define variables to be used in your model
3. Set up a mathematical model • try to describe the situation or problem as a word model • write your word model in symbols using the defined variables • state word model and mathematical model, remember that a simple model is easier to work with initially and a simple model can often bring an insight into the situation or problem that may help in your later improvements.
4. Formulate and solve the mathematical problem • often a mathematical modelling activity leads to setting up and solution of a mathematical problem, at this stage you should keep to familiar mathematical territory.
5. Interpret the solution • describe the solution in words, is there qualitative agreement between the outcome of your model and the situation being considered

Table 1 (continued)

• decide what data you need to validate your model and collect them.
6. Compare with reality • test the outcomes of your model with appropriate data. • criticize your model, in particular, look back at your simplifying assumptions.
7. Improve the model • revise your simplifying assumptions. • formulate a revised model. • repeat the process of solving, interpreting, and validating.
8. Report on your modelling activity • prepare a report describing the problem and its outcomes, this might be in the form of a poster, a written report or an oral presentation.

Berry and Houston (1995) stated that the actions given in the table are neither the final list that one should follow while working on the modelling problems nor the ones that should be followed in order.

2.3. Nature of Modelling Problems and Importance of These Problems in Mathematics Instruction

Based on the different views of modelling, the definition of the nature of the modelling problems also varies in the literature. Referring the study of Galbraith (1999), Brown (2002) stated that there are two types of mathematical modelling: (a) structured modeling, and (b) open modeling. He explained that in structured modeling tasks, problem statement is asked to find a particular unknown, thus eliminates some challenges of the modelling process in which mathematics and context inseparable. On the other hand, open modeling tasks, including no apparent links to mathematics, require problem solver to formulate the mathematical problem and investigate the given information. Therefore, the solution process involves integration of the real data, and correspondingly checking of the results necessitates the using the real data again (Brown, 2002). In his study, including the categorization of mathematical tasks, Berry (2002) defined modelling tasks as “typically real problems that require mathematical principles and formulas in order to be solved” (p. 215). He added that the modelling tasks are open in terms of choice of procedures and outcomes. Berry (2002) explained that in most of the textbooks modelling problems are actually not involves modelling, or aim to teach modelling, instead, they aim to “teach mathematics in context” (p. 214).

In his study, Berry (2002) stated that “the role and nature of the task are important ingredients that can be used to elicit the development of modelling skills” (p. 214). Considering shortcomings of the traditional problem-solving activities which appeared in the traditional text books and standardized achievement tests, Lesh, et al., (2000) developed six principles that are intended to guide the productive formation of modeling activities, or "model-eliciting activities": a) model construction principle, b) the reality principle, c) the self assessment principle, d) the construct documentation principle, e) the construct shareability and reusability principle, f) the effective prototype principle. These principles were developed for instruction to support students' modelling abilities and go through the desired modelling process (Lesh et al., 2000). According to researchers (Lesh et al., 2000) "model construction principle", requires the construction of the explicit description or explanation of a structurally significant system as a solution and requires the manipulations, predictions and control of this system, "the reality principle" requires the task to be meaningful for students by enabling them to use their own existing knowledge and experience, "the self-assessment principle" ensures that students can assess themselves if their interpretations or ideas are good enough or need improvement, "the construct-documentation principle" requires the students to create documentation that would explicitly reveal their thoughts and solution approaches throughout the activity, "the construct shareability and reusability principle" challenge students to develop solutions to be applicable in other situations and sharable with interested groups. Finally, "the effective prototype principle", entails the situation described in the problem to be as simple as possible, yet at the same time requires students to develop mathematically significant solutions.

In a similar manner, Blomhøj and Kjeldsen (2006) gave the details of the necessary qualifications of the modelling tasks in their study. They emphasized that the tasks should be understandable for students so that they could build the situation model of the problem. Blomhøj and Kjeldsen (2006) added that the modelling tasks should challenge students in appropriate level that enabled them to work independently without teachers' guidance and also to work with the concepts relevant to their learning. Moreover, the task should be authentic or should include authentic data from students' own experiences, and it should reveal interesting results so that students saw the need and the role of modelling (Blomhøj & Kjeldsen, 2006).

In their study, Lesh and Doerr (2003) gave the details about the features of the model-eliciting activities. They explained that model-eliciting activities were tools that were thought-revealing in nature, thus helped revealing the students' ways of thinking and their learning process. In these activities, students tried to make sense the situation, besides revealing their learning process, these activities also helped the development of students' learning by providing meaningful real-life situations in which students needed to discover the mathematics behind it (Lesh & Doerr, 2003; English & Sriraman, 2010). While engaging in these activities, students needed to identify and collect the relevant data, convert them into the same form, identify operations to generate new information, and use variety of representations (e.g. tables, graphs, equations) like in the actual real-life problem situations where the information was distributed in different forms and needed to be interpreted (English & Sriraman, 2010; Lesh & Doerr, 2003). Students externalize their results by using these representations which in the end led them change, test and refine their thoughts and solutions to construct more powerful models (Lesh et al., 2000; Johnson & Lesh, 2003). Thus, students engage in multiple modelling cycles involving testing, refining, revising current ways of thinking (Johnson & Lesh, 2003) and also develop or rebuild their current understanding of mathematical concepts (English, 2006). Modelling tasks, involving complex real-world problem situations, benefit from several other disciplines such as biology, physics, or economics and necessitate the small group work, where the members share their ideas, and discuss on numerous issues (Lesh et al., 2000; English & Sriraman, 2010). Thus, in attempts to solve modelling problems, students develop both their communication skills and gain insight on the function and power of mathematics to understand and formulate the problems from different subject areas (English & Sriraman, 2010; Lesh & Doerr, 2003; Lesh & Zawojewski, 2007). Therefore, they learn to improve their competencies to solve real-life problems in addition to developing their understanding of mathematical concepts (Blomhøj & Kjeldsen; 2006; English & Sriraman, 2010; Lesh & Doerr, 2003; Lesh & Zawojewski, 2007).

Several recent researches showed the role and importance of modelling tasks in students' interest and attitude towards the mathematics instead of problem solving skills. For instance, involving the applications of challenging real-world modelling tasks, Bracke and Geiger (2011) reported the long term teaching experience with 14-15 year old students in their study. They concluded that the students were interested in this kind of modeling activities and also changed their attitude towards mathematics. The study of Kaiser, Schwarz and Buchholtz (2011) involving the applications of challenging authentic modelling problems was conducted with students from upper secondary level. Instead of showing students' success in these kinds of problems, this study showed the students' impressions on that kind of problems and interest in these activities. In their study, Carlson, Larsen, and Lesh (2003) found that modelling activities helped the development of undergraduate students' reasoning, communication, and problem solving abilities.

2.4. Difficulties in Modelling Process and Possible Reasons

"The process of modelling constitutes the bridge between mathematics as a set of tools for describing aspects of the real-world on the one-hand and mathematics as the analysis of abstract structures on the other" (Verschaffel et al., 2002, p. 272). The studies showed that there exist some challenges and difficulties of this process for both students and teachers, and researchers tried to identify the possible reasons of these difficulties. Aiming to identify and classify the important aspects of modelling problem through the stages in the modelling cycle and aiming to identify the pedagogical issues related to task design and organization of learning, Stillman (in press) argued that most of the students had difficulty in formulating adequate mathematical representations of the tasks or in other words switching from the real world problem statement to mathematical model. She also stated that it is the most challenging part of the entire modelling process and it actually constitutes the core of modeling (Edwards & Hamson, 1990). Stillman (in press) stated that one of the reasons of these difficulties was insufficient knowledge to comprehend the situation and strategies which ended up with selecting inappropriate mathematical tools and construction of inappropriate problem model. In the same study Stillman (in press) explained that as maintained by most of the modelling studies, main aim of both teaching and doing research should be helping students to achieve this transition. Stillman (in press) added the role of "well-developed strategic store which could be developed by rich repertoire of metacognitive strategies, together with an appropriate knowledge base" (p. 7) in overcoming difficulties arouse in the process. She explained that students should have adequate knowledge to be able to ensure complete understanding of the situation and strategies to be able to specify the mathematical model. She further explained the role of the task context in construction of the situational model. She stated that the construction of situational model based on students' prior knowledge of task context which was derived from different kinds of knowledge, namely students' academic knowledge, encyclopedic knowledge, and episodic knowledge (knowledge derived from experience).

Van-den Heuvel-Panhuizen (1999) explained in her study that task context could create both favorable and unfavorable environment for students' problem solving process. She explained that although task context provided students with solution strategies as students could "image themselves in the situations" (p.136), students sometimes attempted to solve the problem non-mathematically as they could produce plausible alternative solutions rooted from the realistic context of the task. Van-den Heuvel-Panhuizen (1999) stated that in some other cases students neglected the realistic considerations and real world knowledge while solving problems. Thus, in some problems they showed no tendency to incorporate real-world knowledge into their solutions (Verschaffel et al., 2002). The possible reason of this situation was explained by Verschaffel et al. (2002) as the students' conceptions related to the mathematics word problems which are shaped by the current education system. Studying the effects of traditional mathematics education on PTs' beliefs about word problems, they found that PTs showed strong tendency to exclude realistic considerations when trying to solve the problematic word problems. Moreover, they listed some of the students' assumptions as the following:

"- Assume that the task can be achieved using the mathematics one has access to as student, in fact, in most cases, by applying the mathematical concepts, formulas, algorithms, etc. recently encountered in mathematics lessons.

- Assume that the final solution, and even the intermediate results, involve "clean" numbers (generally, small whole numbers).

- Assume that the word problem itself contains all the information needed to find the correct mathematical interpretation and solution of the problem, and that no information extraneous to the problem may be sought.

- Assume that people, objects, places, plots, etc. are different in a school word problem than in a real-world situation, and don't worry (too much) if your knowledge or intuitions about the everyday world are violated in the situation described in the problem situation." (p. 265)

Stillman (in press) mentioned the role of task complexity as an impeding condition on the modelling process. She explained that the students' need to make assumption to formulate a model, need to find the mathematical tools to solve the problem or need to set the aim of the task which was not obvious in the problem, increased the complexity of the task.

In their study, Verschaffel, et al., (2002) stated that many students could not pass through all stages of the modelling process while attempting to solve word problems. They explained that students were generally guided by problem text immediately, and directly applied an operation which was automatically derived from the text, and they did not check the meaningfulness of the answer of the original problem. Verschaffel et al. (2002) also tried to understand the reasons of students' tendency to exclude realistic considerations in the solution process. To understand this fact, researchers gave accompanying instruction to students to make them consider alternative answers. However, similar to the findings of Reusser and Stebler (1997), and Van-den Heuvel-Panhuizen (1999), results showed that their beliefs in word problems were very strong that this instruction was not powerful to overrule their beliefs in problems and to make them consider alternative approaches to the problem.

Analyzing the full PISA 2003 problem sets, Turner (2007) stated that the context of the problem also had an effect on problem difficulty. Turner (2007) concluded that the problems required constructing a model or manipulating a given model identified by students as the most difficult ones, and problems involving only the interpretation and reflection stages of modelling process were marked as having intermediate level difficulty.

In her study conducted with thirty five seventh grade students and lasted fifteen months, Maaß (2006) used various data collection tools such as modelling tests, interviews, concept maps. She found that all of the students were qualified for model problems given in different contexts at the end of the study. On the other hand, she also reported the students' mistakes in the process. Maaß (2006) explained that at the beginning of the process some students could not describe the real model, others made erroneous assumptions. She explained that at the stage of constructing the mathematical model, some students could not use correct symbols and others applied incorrect formulas. She reported that at the solution stage, some students made computational mistakes, some others could find the appropriate strategy, and at the interpretation stage, misinterpretations were made by some of the students. Lastly, the findings showed that some of the students did not validate the adequacy of their model. Related to the whole modelling process, Maaß (2006) explained that there were times that some students "lost track of their own proceeding" (p. 130). She also stated that modelling process, in some cases, was stopped because of the fact that the students selected a way that they could not proceed. Maaß (2006) concluded that the lack of experience for demanding modelling tasks which were new for students and the poor mathematical knowledge had a negative impact on the modelling process.

The study of Berry (2002) supports the claim of aforementioned studies that in general, the model formulation to describe the real-world situation was found to be difficult for students. Analyzing the related literature, and based on their experience with undergraduate students in modelling courses for many years, Berry (2002) reported that students are working in a haphazard way, and instead of reflecting on the situation, students begin immediately to collect data. He explained that students' mathematical competence and knowledge level were also factors that might obstruct the modelling process.

In the study conducted with grade eleven students by making task based interviews, Schaap et al. (2011) reported the blockages and opportunities that students' encountered during their experience in the mathematical modelling process. At the beginning of their study, researchers paired six students into three groups and asked them to work on the different mathematical modelling problems. After the applications of the activities, researchers made task based interviews. Referring the modelling cycle of Blum and Leiß (2007), Schaap et al. (2011) found that in the understanding stage students overlooked important parts of the problem, and looked for cues in the problem text to guide them through the answer. In the structuring stage, students made erroneous assumptions that hindered their progress. Related to this stage they reported that students could not recognize relevant variables. Researchers found that lacking algebraic skills, and not being able to specify the relationships between variables caused to the difficulties in correctly formulating the problem situation. On the other hand, besides these blockages, researchers also observed some opportunities. For the understanding stage they observed the discussions of the students on the issues related to the problem and identified this as opportunity for exploring the problem. For the structuring stage, researchers identified the problem situation by drawing, subconsciously simplifying the problem situation, and writing down the results systematically as opportunities. Finally, researchers identified working with concrete examples, and verifying the model by estimation as opportunities in the modelling process.

In their long term study lasted from 2005 to 2008, Sol et al., (2011) asked students to work on the modelling projects each of which would include about four weeks study and required them to work in groups of two to four. In their study conducted with 12-16 year old students, based on the written reports on realistic mathematics project works, Sol et al. (2011) concluded that although there were some differences between 12-14 and 14-16 year old students, in general students have difficulty in stating the assumptions and recognizing the mathematical concepts needed to solve the problems. They reported that students participated in the study also did not check the coherence of the assumptions and mathematical relationships with respect to the real situation. They added that, students participated to the study neither validated their model nor attempted to improve it. Researchers explained that especially 12-14 year old pupils faced with difficulties in expressing the relationships between the real objects and mathematical counterparts partly because of the fact that the problem asks for the operations with abstract quantities like distance. Sol, et al. (2011) reported the students' difficulties in formulating hypothesis mathematically. Researchers concluded that the solution process of the students showed the classical structure of the problem solving process. They stated that the pupils could not realize the entire modelling process, and argued that students' approaches might be the result of their view that the problems asked in the projects is a set of small problems rather than one major problem. Sol et al. (2011) claimed that this might be the result of their in-experience with the open-ended problems.

Besides the studies conducted with elementary or secondary school students, there are research studies conducted with undergraduate students that aim to understand these students' difficulties and develop their modelling competencies. Soon, Lioe and McInnes (2011), tried to understand 1500 Singaporean engineering students' abilities in modelling problems who did not have any modelling experience before in their research. Based on the survey results, they concluded that the students have difficulty in realizing the relationship between the "real-life contexts and mathematical representations". They concluded that students did not have even basic knowledge about the classification of the variables as known and unknown or dependent and independent. They also stated that students did not show the conceptual understanding of the mathematical and physics concepts necessary to solve the problems. Similar results were reported by the Huang's (2012) study conducted with engineering students attending calculus courses in Taiwan.

Nyman and Berry (2002) in their study performed with undergraduate mathematics students mentioned about a change that the modeling course produced. They concluded that while working with modelling problems especially at the start of the process, students feel frustrated and sometimes discouraged and showed emotional struggles. Although the modelling process was overwhelming for students, researchers concluded that students challenged to think differently and learned to approach and look at problems from different angles.

The studies conducted with in-service and pre-service teachers revealed that the difficulties faced in the modelling process show similarities with the studies conducted with students. For instance, Julie and Mudaly (2007) focused on the both teachers' and tenth grade students' behaviors in their research project while engaging in modelling activities, they used various data collection tools such as; observations, video recordings of teachers at work, final reports on the model, interviews and questionnaires. They found that actually there were similarities between the students' and teachers' modelling approaches. Researchers stated that instead of discussing on the issues related to the problem situation, both of the groups tried to reach mathematical expressions immediately. Julie and Mudaly (2007) also reported the absence of the processes of returning to the real problem situation and checking for the appropriateness of the reached solution. They argued that these behaviors are the product of current school mathematics programs applying the modelling as vehicle approach. Julie and Mudaly (2007) concluded that modelling as content approach could provide environment for successful modelling.

In their research, Blomhøj and Kjeldsen (2006) conducted a study with in-service mathematics teachers, which aimed to support teachers to develop, practice, and evaluate projects on mathematical modelling. Researchers reported that during the course for teachers although participated students were positive about the whole experience in modelling problems, most of the students found the problems so difficult that teachers provided the entire solution to them.

In his quantitative study, conducted with 230 prospective and 79 secondary teachers, Kuntze (2011) aimed to understand the views of PTs' about modelling. He found that PTs contrary to in-service teachers preferred the tasks that require low modelling activities which ask for one correct solution, instead of higher modelling activities allowing different solutions and requiring at least one translation from real model to mathematical model. He attributed this result to the fear that the tasks allowing different solutions might not be in congruence with the aim of mathematical exactness. The results of the Kuntze's (2011) study also suggested that although in-service teachers might be more aware of the power of modelling activities in students' learning, some of them have difficulty in providing appropriate modelling environment in classrooms. Similar findings reported by Doğan Temur's study (2012). She reported that most of the PTs, who were trained in problem solving including both routine and non routine problems for six hours as a part of "Teaching Experience" course, preferred to teach routine problems in their practicum schools as the PTs find it difficult to form and apply the non-routine problems. The findings of Doğan Temur's (2012) study also showed that the difficulty level of the problems formed and applied by PTs was not appropriate and PTs had difficulty in explaining the solution procedures to the students such that in some cases the students could not understand what PTs were trying to explain. Moreover, the study showed that some PTs perceived modelling different than its aim and therefore instead of students' own discovery in the activities and they saw themselves in the center of the course.

Lingefjärd and Holmquist (2005) conducted a study with prospective secondary mathematics teachers and tried to assess these PTs' attitudes and skills in mathematical modelling. Researchers explained that PTs in the study found the teaching and learning mathematical modeling much difficult and more complex compared to their initial views at the beginning of the course. Lingefjärd and Holmquist (2005) concluded that the change was hard for many PTs as they came from traditional way of instruction involving regular lectures where the content of the textbook was generally presented. Because the PTs as a student spend many of their years with this type of teaching methodology, the findings showed that it was very difficult for them to change their views of mathematics, mathematical modeling, and teaching and learning of mathematics.

Aiming to identify the difficulties and advances during the distance course in mathematical modelling for teachers and prospective mathematics teachers, Biembengut and Faria (2011) reported that most of the participants faced with difficulties not only in reading and understanding the task, but also in remembering the mathematical knowledge needed to be able to solve the problem. Biembengut and Faria (2011) mentioned that these difficulties decreased the motivation of the participants, and therefore nearly half of the teachers gave up the work by showing lack of time as a reason. They also noted that although the participant teachers and future teachers had good understanding of Calculus, they faced with difficulty in deciding how to use their knowledge.

In their qualitative case study research on PTs' professional knowledge on modelling competencies, Kaiser, Schwarz, and Tiedemann (2010) found that PTs needed to develop their mathematical knowledge and competencies. They reported that although students had knowledge of modelling in different degrees, most of them did not consider the phase of validation. They concluded that there were some differences between the student teachers' knowledge about the modelling process and their competencies to conduct the process. Of the three PTs, they reported that especially one of the PTs faced with difficulties in many of the stages of modelling. They reported that this student as the others could not understand the real-world situation, and thus made inappropriate and oversimplified assumptions to construct the real model. This result is consisted with the studies of Blum and Leiß (2007). Jonassen (1997) reported that the students' major difficulties in modelling process were the stage of understanding the problem situation and constructing the idealized version of the problem. The findings of their study implied that good understanding of real world situations was strongly connected to having substantial knowledge of modelling process. Kaiser et al. (2010) also explained that after finishing the mathematical work, the students faced with difficulties in reinterpreting results in the real-world situation, and in some cases they did not test the results. Similar findings were reported by Hodgson (1997) study in which the selected teachers administered the open-ended real-world problems to their students and it was found that the students did rarely test or revise their models.

In her study Kaiser (2007) reported the findings of the study conducted with PTs and students in upper secondary level. Based on the evaluation by means of a test applied at the beginning and at the end of the modelling seminar and based on the evaluation of participants attempts on modelling problems Kaiser (2007) reported that although there was generally an improvement in modelling competencies, there was not sufficient improvement in the partial competencies of each modelling stage due to the relatively short-time of the given instruction and training.

The studies conducted in Turkey revealed that PTs faced with somewhat similar difficulties in the modelling process. In their study carried out with thirty five PTs, Çiltaş and Işık (2013) found that modelling instruction created significant change in prospective elementary mathematics teachers' knowledge, skills, and opinions. Researchers gave instruction with the mathematical modelling method for four weeks in a total of 42 hours to the research group using the course book prepared according to the modelling method. Based on their findings from the pre-MMT (Mathematical Modelling Test), which was composed of questions including the concepts taught an Analysis III course, they observed that the majority of the students already passed the stage of understanding the problem, but faced with difficulty in the stage of forming the model. When comparing the pre-MMT and post-MMT (Mathematical Modelling Test), they found that there was an improvement in the success of the research group. However, the research group also faced with difficulty in interpreting the attained solution in daily life. This supports the findings of the study of Bukova-Güzel and Uğurel (2010). Results of the Bukova-Güzel and Uğurel's study (2010), conducted with 12 prospective secondary mathematics teachers, showed that some of the students even could not construct a mathematical model to many of the asked modelling problems, or they provided insufficient models. Findings of their study also revealed that the students with poor academic performance in calculus faced with difficulty in the third stage of the modelling, translating the problem into a mathematical form. Researchers also reported that students with higher academic performance faced with difficulty in the fourth stage of the modelling process in terms of solving the problem in mathematical terms, and reaching a model. Using the modelling cycle developed by Borromeo-Ferri (2006), Bukova-Güzel (2011) in her study conducted with 35 PTs who were taking the mathematical modelling course first time, found that these teacher candidates were successful in understanding and simplifying stages of the modelling process, however faced with most difficulties in interpreting and validating stages. Bukova-Güzel (2011) reported that PTs, in some problems, showed no approach to test the validity of the model.

Eraslan (2012) in his study, examining the prospective elementary mathematics teachers' thought processes on a model-eliciting activity and the blockages in the process, found that PTs faced with difficulty in determining the variables as a result of lack of data in the problem related to these variables. He reported that PTs faced with difficulties in transforming the real-world problem into a mathematical model. He explained that PTs reduced the number of variables to overcome this difficulty, and thus they built a simpler and more limited model.

Similar to their findings, the study of Özer-Keskin (2008), conducted with 21 prospective secondary mathematics teachers, showed that PTs were mostly successful in the understanding stage of the modelling process. However, Özer-Keskin's (2008) study revealed these teacher candidates' difficulties in the remaining stages of modelling: formulating the mathematical model, working on the mathematical model and interpreting the model in terms of real-world problem situation. Despite these difficulties, Özer-Keskin (2008) concluded that based on the modelling instruction, prospective mathematics teachers showed success in the post-Mathematical Modelling Skill Test. She also reported that instead of going through the modelling cycle, PTs tried to solve the problem by using the problem solving approach. Özturan-Sağırlı, et al.'s (2010) findings also supports this finding of Özer-Keskin's (2008) study.

In his study conducted with prospective secondary mathematics teachers, Kertil (2008) stated that although these teacher candidates showed some improvement in their modelling skills, they faced with difficulties in particular stages of modelling process: clarifying the goal, identifying the variables, choosing, and applying a mathematical expression or formula, and using the graphic representations.

To summarize, all these studies showed that students in elementary and secondary school levels, or students in the departments of mathematics, science or engineering, or in teacher education programs, or in-service teachers had difficulties in different degrees in the certain stages of the modelling process: understanding the problem, structuring and simplifying the problem, making appropriate assumptions, exploring relationships between variables, making connections between real world problem situation and mathematical representation, interpreting the model in terms of real-life problem situation, and validating the model obtained.

2.5. Developments in Turkish Mathematics Education Curriculum and Barriers to its Implementation

Over the past three decades there are ongoing reform efforts in Turkish educational system (Grossman, Önkol, & Sands, 2007; Erbaş & Ulubay, 2008). Changes in the content and vision of the primary and secondary school education curriculum forms the important part of this reform efforts (Erbaş & Ulubay, 2008; Bulut, 2007). Among other skills, the new curriculum emphasize the importance of learning mathematics with understanding (Çetinkaya, 2012) and aims to raise individuals who are independent thinkers, can make connections among ideas, use mathematical knowledge and ideas to solve real-life problems, and apply them other disciplines different than mathematics (Erbaş & Ulubay, 2008). These cause changes in the students' role during the learning process. In broadest sense, the students' roles in the classrooms changes from passive to active learning, and from product oriented to process oriented working (Bulut, 2007; Çetinkaya, 2012). In addition to the changing roles of the students, there are significant changes in the roles of the teachers. The changes in the curriculum require teachers to implement new instructional methods which go beyond simply adopting curricular guides (Erbaş & Ulubay, 2008). Moreover, the new curriculum requires teachers to take the role of facilitator, instead of information giver (Bulut, 2007). Besides other factors like content and clarity of the curriculum, degree of the complexity of the change, government support etc. (Güneş & Baki, 2012), the successful implementation of the curriculum depends heavily on the teachers (Erbaş & Ulubay, 2008; Güneş & Baki, 2012). Therefore, it is important to provide sufficient support for professional development of teachers' to make them have necessary knowledge, skills, and qualifications (Erbaş & Ulubay, 2008; Güneş & Baki, 2012; Işıksal, Koç, Bulut, & Atay-Turhan, 2007). Moreover, in addition to paying attention to the development of their knowledge and skills (Erbaş & Ulubay, 2008), paying attention to teachers' beliefs, feelings, and difficulties is important and necessary as their beliefs about teaching and learning mathematics and their concerns may prevent them process of change that they expected to undergo (Çetinkaya, 2012).

On the other hand, research studies showed that the approach of the new curriculum could not be adopted by teachers (Kartallıoğlu, 2005; Orbeyi, 2007) partly because of the fact that they have not provided with sufficient training and professional development activities (Akşit, 2007; Kartallıoğlu, 2005; Orbeyi, 2007). Lack of guidance about how to implement the activities, and alternative assessment methods influenced negatively the effective implementation of the new curriculum (Bulut, 2007; Duru & Korkmaz, 2010; Erbaş & Ulubay, 2008; Kartallıoğlu, 2005). Erbaş and Ulubay (2008) explained that the teachers who believed that they are implementing the new curriculum properly, may be in practice could not achieve this actually. They exemplified that teachers who perceived themselves applying the curriculum appropriately, might assigning some classroom activities as homework (Erbaş & Ulubay, 2008).

Similarly, the findings of Güneş and Baki's (2012) study showed that the teachers' opinions about the applicability of the mathematics curriculum did not reflect to their classroom applications directly. Güneş and Baki (2012) stated those teachers' positive views about effectiveness of the mathematics curriculum on student learning or in general about the curriculum had little effects on the settlement of the student-centered classroom environment. In spite of the change in curriculum and philosophy behind, their study showed that there are teachers solving multiple choice tests in the classrooms and use traditional way of instruction because of the central examination pressures. Similarly, based on the findings of their study Küçük and Demir (2009) concluded that it is possible that mathematics instruction in Turkey is still teacher-centered where students are passive recipients of the knowledge, just listening the teacher and unquestioning or discussing the ideas on the books. This is a matter of concern in terms of the development of the mathematical ideas and relating mathematics with real-life (Toluk & Olkun, 2002).

The parents' concerns which also shared by the teachers regarding the national examinations seen as barriers on the effective implementation of the new curriculum (Erbaş & Ulubay, 2008; Kartallıoğlu, 2005; Orbeyi, 2007). As the national examinations to enter secondary schools and universities based on multiple-choice tests, application of other assessment methods such as projects, peer and self evaluations and portfolios raised the concerns on the success of the students' on these exams and therefore reduced the possibility of implementations of them. Another challenge explained by the teachers in the adaptation of the curriculum is the students' difficulties in adapting their new roles required by the new curriculum (Bulut, 2007; Erbaş & Ulubay, 2008).

In addition to the above mentioned factors, the time allocated for mathematics in weekly schedule is found unrealistic by teachers to implement the activities recommended by the curriculum (Bulut, 2007; Erbaş & Ulubay, 2008; Orbeyi, 2007; Soycan, 2006). Similarly, in his study Özar (2012) explained that with such a limited time frame it seems implausible for students to learn the content of the mathematics course. Studies also showed that the crowdedness of the classrooms seen as a barrier by the teachers in the implementation of the problem solving activities offered by the new curriculum (Duru & Korkmaz, 2010; Erbaş & Ulubay, 2008; Güneş & Baki, 2012; Küçük & Demir, 2009). Lack of infrastructure of schools required to implement the activities recommended by the curriculum and provide learner-centered environment cause other obstacles to implementation of the activities (Akşit, 2007; Bulut, 2007; Erbaş & Ulubay, 2008; Güneş & Baki, 2012; Kartallıoğlu, 2005; Orbeyi, 2007).

Moreover, the studies showed that besides course book, the teachers rely on books which involve the multiple choice format problems that aim students to prepare for the tests (Özgeldi & Çakıroğlu, 2011). The study of Bayazıt (2013) related to the quality of the tasks in the new Turkish elementary school textbooks showed that textbooks comprised of tasks at all cognitive levels. However, he stated that in the implementation, the common is that the high-demanding activities declined in less-demanding forms of activities because teachers tend to proceduralize these tasks due to the pressures posed by students to reduce the ambiguity of the tasks and due to the pressures posed by the national exam systems. Thus, the most challenging parts of the high cognitive level tasks suggested by the mathematics curricula, take over by the teachers and thus these tasks turns into less-demanding ones (Bayazıt, 2013).

As claimed by researchers, the strengths of the curriculum should be considered together with the weaknesses of the other components of the education system that make it difficult to implement new curriculum (Akşit, 2007; Özar, 2012). It is also stated by the researchers that there is a need for time to overcome the difficulties and see whether the changes will create the intended outcomes (Olkun & Babadoğan, 2009; Güneş & Baki, 2012).

CHAPTER 3

METHODOLOGY

The goal of this study was to understand how PTs develop mathematical models and the factors affecting their modelling process. In this chapter, the method of research was described in detail. The related issues concerning the context in which the study took place, the participants of the study, the role of the researcher, the procedures of data collection and data analysis were described.

3.1. The Design of the Study

This research study aimed to "gain an in-depth understanding of the situation and meaning for those involved" (Merriam 1998, p. 19) and particularly interested in gaining an understanding the basic process that PTs perform in each stages of modelling process and understanding the factors affecting this process depending on their views. Therefore, qualitative research approach of case study was used in this study as a strategy of inquiry. Creswell (2003) stated that the case studies allow researcher to "explore in depth a program, an event, an activity, a process, or one or more individuals" (p. 15) The case in this study was the six PTs. The basic processes that PTs go through when they engage in modelling activities and their views about the factors affecting their processes were the two dimensions of the study that were analyzed deeply.

3.2. Setting and the Procedures

This research study was conducted as a part of a fourteen-week long "mathematical modelling" course conducted as four course hours each week, by the researcher during the fall semester of 2010-2011 academic years at a public university in Ankara, Turkey. The course was offered as an elective course in the elementary mathematics teacher education program. Of the nineteen PTs taking the course, fifteen were female, four were male. Eighteen of the PTs were from the department of elementary mathematics education and one of the PTs was from the computer education and instructional technology program. The PTs from the department of elementary education had previously followed courses on calculus with analytic geometry, calculus for functions of several variables, discrete mathematics, basic algebraic structures, basic physics, and introduction to probability and statistics. PT from the department of computer education and instructional technology previously followed the courses titled Basics Mathematics I and Basic Mathematics II covering mathematical concepts such as sets, functions, matrices, probability, limit, continuity, and differentiation and integration. The contents of the mathematical modelling activities engaged during the courses were also selected by taking into account the contents of those courses. Therefore, PTs were thought as capable of exploring the modelling tasks.

The course contents involved the examples of mathematical modelling activities including the mathematical concepts ranging from elementary school mathematics to the undergraduate mathematics. Before the semester, to be able to form the modelling activities, relevant literature and sources were searched extensively, and eighteen modelling activities were prepared in total and were adopted by the researcher. During the selection and adaptation of the activities, five criteria were considered as important:

a) the complexity of the activities,

b) the mathematical content of the activities (tried to be diverse),

c) the time required to explore each problem (as intended duration of each task was two lessons),

d) the degree of openness in terms of procedures and outcomes (Berry, 2002; Bracke & Geiger, 2011),

e) the suitability of the activities to students' real-life experiences (Bracke & Geiger, 2011; Lesh et al. 2000).

Then, as well as to improve the design of the main study, pilot study was conducted in order to test the adequacy of some of the modelling activities (see Appendix A for pilot study). The pilot study allowed the researcher to see the nature of the students' answers and their approaches to the activities, and this provided information about the preparation of the appropriate modelling activities. After making some modifications (see Appendix C for the initial version of the problems) based on the analysis of the data came from the pilot study, the problems examined by a mathematics education researcher according to its content and appropriateness of language, and revised with his suggestions. Moreover, each problem was asked another researcher in mathematics education to check any kind of unclear points that would likely to cause misunderstanding among participants during the implementation. His suggestions in relation to the content and appropriateness of language were taken into account. Accordingly, all these revisions ensured the clarity of the meaning. After the preparation of the activities the course syllabus was prepared (see Appendix F for syllabus).

In all activities, students were expected to develop a solution to the given real-life problem situation without evident mathematical character. To work on the modelling activities, the students worked in groups of three throughout the semester while only one group of PTs worked in groups of four. Each group was formed according to participants' willingness. The sitting plan is given in Figure 6.

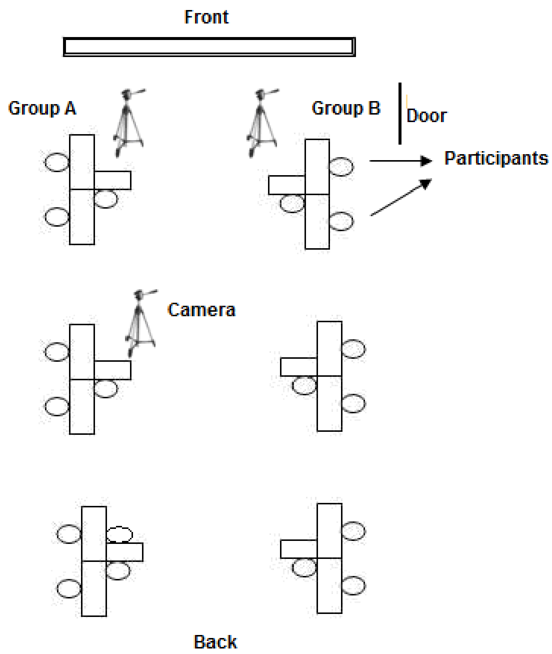


Figure 6 The sitting plan during the data collection

The rectangular desks were decided to be arranged like in Figure 6 since such a way would maximize the group interaction. The desks were also placed to ensure the works of groups' independency from each other. Thus, this type of arrangement was expected to both maximize the group interaction in itself and minimize the audibility and the influence of other groups' ideas.

Prior to the each of the activity, voice recorders and video cameras were set up for each group to capture their dialogues with each other and with the researcher. The researcher, a research assistant, and the instructor of the course were present during the each activity. The research assistant undertook the task of video recording. He followed the groups' works and tried to capture the group members' dialogs and the details of their works on the solution sheets to help them understand the process in detail for later analysis. Similarly, the instructor of the course sometimes helped the management of the video cameras to capture the dialogs, and sometimes he walked around the classroom. To make the PTs think aloud, by asking questions such as "What are you doing?", "What do you find?" "Why are you doing it?", and thus facilitated the discussions.

3.2.1. The structure and the schedule of the course

In the first week of the course, PTs were informed that the data that would be collected throughout the semester for research purposes. Moreover, all subjects were assured that any data collection about them would be kept confidential, and the names of the subjects would never be used in any publication. They were also informed that no one else other than the researcher had access to data. They were also notified that they had the right to withdraw from the study or to request the data collected about them. Thus, PTs informed about the researcher's dissertation study and what they were required as a participant of the present study. At the beginning of the lesson of the second week, informed consent was obtained from each participant (see Appendix G for the consent form).

In the first week of the semester, course expectations and objectives were also stated. In the following weeks the modeling activities listed in Table 2 were implemented. Each week, only one modelling activity was implemented. Table 2 lists the modelling activities and the mathematical concepts that can be used in the solution of these activities.

Although the activities and their implementation order was formed as in the syllabus given in the Appendix F, during the semester, some changes were made in the schedule according to PTs' level of engagement of the activities, in other words according to their level of difficulties. The initial problems implemented during the semester were open in terms of both procedure and the outcomes (Berry, 2002), and thus not restricted to the specific mathematical concepts. Instead, these initial activities were formed in such a way that every PT could produce a solution to the problem by using elementary mathematics and get familiar with the nature of the activities.

Table 2 Implementation schedule of the course activities and mathematical concepts that can be used in the solution of the activities

Weeks	**Course content**	**Mathematical concepts that can be used in the solution of the activities**
Week 1 (Sept.28, 2010)	First meeting of course (The course was introduced to the students-Information Forms)	-
Week 2 (Oct. 5, 2010)	"The Postman" activity	Algebra (manipulating equations and inequalities)
Week 3 (Oct. 12, 2010)	"Bus stop" activity	Algebra (constructions of functions and inequalities)
Week 4 (Oct. 19, 2010)	Presentation on models and modelling perspective – "Gardenning" activity "A Manufacturing" activity	Construction of equations, derivative,
Week 5 (Oct. 26, 2010)	"How to store the containers?" activity	Pythagorean Theorem
Week 6 (Nov. 2, 2010)	"Let's organize a volleyball tournament!" activity	Ranking, scoring, re-scaling weighted averages
Week 7 (Nov. 9, 2010)	"Forest management" activity	Derivative, integral, limit, sequences (convergence), power functions
Week 8 (Nov. 23, 2010)	"Who wants 500 billion?" activity	Expected value, probability permutation.
Week 9 (Nov. 30, 2010)	"Drug theraphy" activity	Power and logarithmic functions, derivative (rate of change), limit, integral.

Table 2 (continued)

Week 10 (Dec. 7, 2010)	"The cashier" activity	Elementary probability, random number generation (simulation)
Week 11 (Dec. 14, 2010)	"Dentist appointment" activity	Elementary probability, random number generation (simulation)
Week 12 (Dec. 21, 2010)	"Traffic lights" activity	Elementary probability, simulation,
Week 13 (Dec. 28, 2010)	Students' presentations of modelling activities they developed	-
Week 14 (Jan. 4, 2010)	Students' presentations of modelling activities they developed Overall evaluation of the course	-

During the implementations of the first two activities, PTs were not given any instruction either about modeling process or modelling approaches in education. In the fourth week of the study, the PTs were given instruction by the researcher about the models, mathematical models, modelling process, and modelling in Turkish elementary and secondary school curricula. The PTs were also given sample solutions about the activities of the first two activities based on the introduced modelling cycle described in Standards (NCTM, 1989, p. 138). Starting from the fourth week, at the beginning of the each lesson, the students were given sample solution ways of the activities solved at previous week. The explanations related to the sample solutions given through the stages of modelling: understanding the problem, choosing variables, making assumptions, solving the equations, interpreting the model, verifying the model and criticizing and improving the model. After presenting sample solution of the problem based on these stages, identified strengths and weaknesses of PTs' models were shared by the researcher and discussed together. Thus, the role of the researcher in this study was also the instruction. Course content and course schedule were designed by the researcher. Furthermore, all the modelling activities were implemented by the guidance of the researcher.

During the applications of the activities researcher listened PTs' ways of thinking and tried not to intervene during all the stages of the modelling process in order to make them construct their own models and make them evaluate their own strategies. Possible mathematical concepts behind the activities were not given or implied in the explanations in order not to limit the students' solution approaches. However, when a student or a group requested assistance about what to do in the next step, or when they got really stuck and clearly ask for help, they were asked some probing questions by the researcher to help them refocus on their thinking. For example, "why do you say that?", "how do you know it?", "what is another way to look at it?". In some other cases, they were asked to use sketches.

In general, at the beginning of the each activity the expectations and time limitations were explained briefly while handing out the problem sheets to all PTs. They were told that they had about 20 - 30 minutes to work on the activities individually, and then worked with the group about one and half hour until the presentations. They were also told that the presentations would take place in the last half hour of the course. Along with the modelling task sheet, each group was given empty sheets to be used as solution papers. PTs were asked to write in detail how they approached and got their answer to the given modelling tasks. After each modelling activity, their solution papers were collected for further analysis.

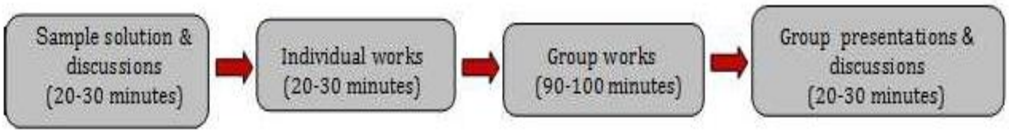


Figure 7 General structure of the lesson

PTs' attendance and participation to the discussions to solve modelling activities were used for assessment purposes. They were acknowledged that their attendance and participations would be considered as an important part of the course grading. On the other hand, the class atmosphere was tolerant towards any idea or mistakes.

3.3. Participants of the Study

In this study, one of the aims was to understand deeply and gain insight into how PTs developed mathematical models. Therefore, it was necessary to select cases most of which could be learned. Merriam (1998) stated that "purposive sampling based on the assumption that the researcher wants to understand and gain insight, and thus must select a sample from which deep analysis can be performed (p.61). Patton (2002) argues that "the logic and power of purposeful sampling lie in selecting information-rich cases for study in depth" (p. 230). Accordingly, after the first modelling activity, groups who were relatively more participative to the discussions were selected as a case. Patton (2002) also stated that findings from even a small sample of great diversity will yield "important shared patterns that cut across cases and derived their significance from having emerged out of heterogeneity" (p. 235). Moreover, PTs in the different grade levels and from different program (elementary mathematics education and computer education and instructional technology) were preferred in order to represent the diversity of the approaches.

In this regard, two groups of three PTs were selected from six groups of nineteen PTs' in total by purposive sampling for in-depth analysis. Four of the participants of this study were female and two of them were male. These students acknowledged that they did not take any course similar to the mathematical modelling before. In the information forms (see Appendix E for information form) handed to the students in the first week, PTB2 explained that she had seen some examples of the modelling problems before while searching for some information, but did not work on them. PTB3 explained that she had worked on a modelling problem ("Let's organize a volleyball tournament!" problem asked in this study) for a part of research study, but she stated that neither she saw any sample solution of the problem nor she informed about the modelling process. Thus, the modelling activities were given to the students with no special modelling experience.

Since it was offered as an elective course, PTs from different programs and different levels took the modelling course. In the academic semester, this study was carried out, three of the participants (PTA1, PTA2 and PTA3) were in their third year of study in the department of elementary mathematics education, while the other two participants were continuing their master programs (PTB2 and PTB3) in the same program, and one of the participant was in his fourth year of the study in the computer education and instructional technology program. Participants' program, year in the program and gender are provided in Table 3.

Table 3 Characteristics of the Participants

Groups	**Participants**	**Program**	**Year in the Program**	**Cumulative GPA (over 4)**	**Type of School Graduated**	**Gender**
Group A	PTA1	B. S. Program in Elementary Mathematics Education	Third	2.51	Anatolian Teacher Training High School	Male
	PTA2	B. S. Program in Elementary Mathematics Education	Third	2.32	Anatolian Teacher Training High School	Female
	PTA3	B. S. Program in Elementary Mathematics Education	Third	2.47	High School	Female
Group B	PTB1	B. S. Program in Computer Education and Instructional Technology	Fourth	2.77	Anatolian Vocational High School	Male
	PTB2	M. S. Program in Elementary Mathematics Education	Third Semester	2.88	Anatolian Teacher Training High School	Female
	PTB3	M. S. Program in Elementary Mathematics Education	Third Semester	3.14	Anatolian Teacher Training High School	Female

In general, all PTs stated in the information forms that they were interested in mathematics and in general they evaluated themselves as good at mathematics. However, they reported that they have difficulty with some concepts in calculus and geometry. When they were asked about their ability to use computers, in general participants evaluated their ability to use computer as good. This information was summarized in Table 4.

Table 4 Participants evaluation of their mathematical knowledge and computer skills

Participants	*The areas of mathematics identified as difficult*	*The level of computer skills*
PTA1	Geometry	Basic
PTA2	Three dimensional systems	Good
PTA3	Proof and abstract thinking	Basic
PTB1	-	Good
PTB2	Abstract mathematics, proof and calculus	Good
PTB3	Derivation and integration	Good

3.5. Data Collection

Due to the density of the data, PTs' modelling processes in five modelling activities were decided to be examined. While deciding the activities following considerations were taken into account. PTs were given instruction about the modelling process in the fourth week. The selected activities were chosen from both before and after the instruction. The activities implemented in the fourth week, seventh week and after the eight week required PTs to use knowledge of specific higher level mathematical concepts like derivative and integral or procedures like simulation which might prevent PTs from fully engaging in modelling activities. Therefore, remaining activities were selected for the analysis. These activities allowed PTs to produce a solution by using elementary mathematics. Therefore, first two activities (applied in second and third week) were selected as the activities implemented before the instruction, and the activities implemented in the fifth, sixth and eight weeks were selected as the activities implemented after the instruction.

In this study, following data collection procedures were employed: a) The audio and videotaped classroom observations; b) In-depth semi structured interviews; c) Content analysis of PTs' solution papers; d) Information Forms

a) Audio and videotaped classroom observations

In this study, each of the two groups was audio and videotaped during the course so that their dialogue with each other and with the instructor could be monitored. As explained in the setting and procedures section, the research assistant of the course undertook the task of video recording. He followed the works of the groups and tried to capture the group members' dialogues with each other. He also tried to capture the details of the PTs' works on the solution papers that were then used in data analysis. With a third camera, the whole class was also videotaped to get the details of the whole class activities. This camera especially used to capture the details of the group presentations. Prior to the each of the activity, voice recorders were put on to the desks for each group to capture their dialogues with each other. The participants were also asked to talk loudly about what they were thinking while they were working on the problems.

b) In-depth semi structured interviews

Task based interviews eased data collection procedure of this study by allowing face to face interaction which was required to be able to thoroughly understand the solution process of the participants. For the present study in-depth semi structured interviews were conducted with six PTs by the researcher. Interviews were conducted right after the applications of the each modelling activity at the time suitable for both the researcher and the participants. As the modelling course ended in the evenings, three of the PTs interviews were conducted on the first day after the implementation, and the remaining three of the interviews were conducted on the following day after the implementation. The interviews were conducted in the rooms or the classrooms that were suitable at that time in the department. Each interview took about 30 - 45 minutes. During the each interview session, PTs' both individual and group solution papers were provided. At the beginning of the each interview session, PTs were told that they were expected to rethink about their experience with modeling activities and provide as much detail and example as possible while describing their modeling process and while explaining their works on the solution papers.

In this study, semi-structured interview protocol containing both open ended questions and certain set of questions were used (see interview questions in Appendix D). The interview questions included the processes of the modelling stages offered by NCTM (1989, p. 38). These stages were also used as a starting point of the data analysis to identify and document the nature of the PTs' modelling processes. These stages were; identifying and simplifying the real world problem situation; building a mathematical model; transforming and solving the model; interpreting the model; validating and using the model. Examples of interview questions are as follows: What was the problem situation that you focused on?, What was your aim in this problem?, What was the first solution method that came to your mind to solve the problem?, How did you begin to solve the problem, after understanding what the problem was about?, While formulating the problem what were the assumptions and situations about the problem that you took into consideration?, How did you determine these assumptions? What were the ease and difficulties you encountered while dealing with the problem?

During the interviews, participants were encouraged to explain their solution approaches and the reasons behind their processes or difficulties. The questions were prepared by the researcher and a researcher in mathematics education, and examined by another researcher in mathematics education field in relation to its content and appropriateness of language, and revised with his suggestions. The revisions were made in order to give the details of the process as much as possible and to ensure the clarity of the meaning.

c) Content analysis of the solution papers

In addition to the interviews, students' solution papers were collected after each of the modelling activity. The students were told to write their arguments and whatever they thought on the papers while working through the problem. The aim of this writing was to understand the details about their thoughts and solution approaches. After the individual works, they were also told to use empty papers provided and not to write anything on their individual solution papers. In order to understand the details of the modelling process in the later analysis, PTs also told not to erase anything they wrote on their solution papers while working on the problem.

The students' audio and videotaped transcripts, interview transcripts, and solution papers were analyzed together to assist in understanding how PTs constructed models while they were engaging in modelling activities.

d) Information Forms

Before the beginning of the study, the information form was prepared with the researcher and a researcher in mathematics education in order to get some demographic information from participants. In general, this form included the questions related to their academic background, and their previous knowledge and experience with modelling activities. The information form was given in the Appendix E. In the first week of the semester PTs were handed with Information Forms and asked to answer each question. After they completed the forms, they gave back to the researcher.

3.6. Data Analysis

The data were analyzed using qualitative data analysis techniques. All the data from the videotaped classroom sessions and the interview sessions were transcribed. During the transcription of the videotaped classroom sessions, audio recordings were also used for unclear points. After the transcription, these transcripts were analyzed together with the participants' solution papers. During the first analysis, the aim was to understand the participants' and groups' solution approaches to each of the problem. To understand the general solution approach of each participant and group to every single problem, the researcher read through all the transcribed data including videotaped classroom sessions and interview sessions sentence by sentence. While reading the data from the transcriptions of videotaped classroom observations, the participants' responses to the corresponding interview questions and their corresponding parts of the solution papers were analyzed together to achieve the correct understanding. After observing general solution ways of each of the participant both in individual work and in the group work for activities, the main points of the participants' solution approaches on the same activity were summarized.

Based on the review of the related literature, it was decided to make the initial analysis of the PTs' modelling process under the framework suggested by Standards (NCTM, 1989). Their modeling cycle was decided to be used as a starting point to identify and document the nature of the PTs' modelling processes. Figure 8 portrays the modeling process described in the Standards (NCTM, 1989, p. 138).

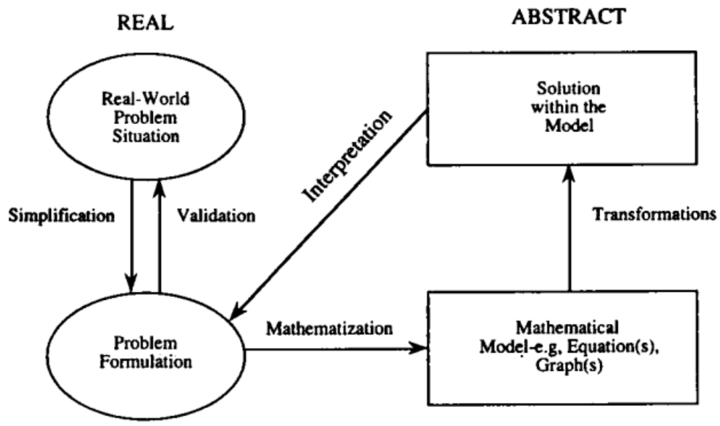


Figure 8 Standards' Model of the Modelling Process

Thus, initial categories (stages) were identified as simplification, mathematization, transformation, interpretation, and validation. Moreover, as the PTs presented their solution approaches on the board at the end of the each activity, presentation stage was added for additional category. Then, each of the transcribed videotape and interview sessions were coded together with the solution papers to assist in identifying the subprocesses of each stage of the modelling process. This process was done for each of the participant one by one across the problems, including both for the individual works and group works. The next step in the data analysis involved identifying recurring patterns in the coded data by making comparison between the patterns emerged for the same category across the problems. To generate the name of the subprocesses, researcher read again and again the coded data in terms of similarities and differences. As certain codes/phrases about students' ways of thinking were repeated, the names of the sub-processes were begun to emerge, and accordingly initial modelling process was developed. Then, the researcher revised categories (stages) and subcategories (processes) by using transcribed data again until the final categories and subcategories were formed. Continuing analysis of the activities led to further refinements. There were three main refinements. First one was that individual works and group works were considered interrelated and combined together. Second one was that mathematization stage which represents the construction of a mathematical model of the problem (Hodgson, 1995), and transformation stage which represents the identification of solution by working with the mathematical entity were considered as interrelated, and combined under the stage of "working". As PTs approached towards the problems without using mathematics in some cases, instead of identifying the name of the stage as working mathematically, the name of the category was defined as "working". The third one was that interpretation and validation stages were considered as interrelated and grouped together as in the study of Dunne (1998). On the other hand, "identification and simplification of the problem situation" stage considered under two different stages: "Understanding" and "planning". Understanding was simply consisted of processes of reading, summarizing and evaluating in terms of first noted factors whereas planning included more deliberate processes to be able to get a solution.

Consequently, data analysis suggested five categories of modelling process: Understanding, planning, working, interpreting and verifying, and presenting. These final categories and subcategories were also reviewed with the transcribed data for the last time. This approach helped the researcher finally develop a model that describes each stage (categories) of PTs' modelling process.

Another aim in this study was to understand the PTs' thoughts about the factors affecting their modelling processes. To determine the PTs' views about these factors, transcription of the interview data together with the transcriptions of videotaped classroom observations were coded. While reading through the data, it was seen that certain words and phrases were repeated and stood out. While coding the data, the researcher focused on PTs' own explanations that directly or indirectly refer to the factors affecting their modelling process. While reading the data from the transcriptions of interviews and videotaped classroom observations, the corresponding parts of the solution papers were analyzed to confirm the codes and thus to achieve the correct understanding. Finally, these codes were grouped under the categories that helped the understanding of PTs' views about the factors affecting their modelling process. For instance, the expressions like "as I didn't know mathematical modelling", "I didn't see any problem that asked for the real life experience", "everything arises from the lack of modelling lessons" were categorized as lack of experience with modelling activities. The expressions like "I have difficulty in using my knowledge", "we can't make any connection among subjects"," it's because of extreme memorization", "conceptual understanding hasn't been developed" were categorized as lack of conceptual understanding.

3.7. Validating the Accuracy of Findings

Creswell and Miller (2000) stated that in a qualitative study, "what is the most important is that the credibility of the account be conveyed" (p. 129). To increase the accuracy of the findings of the study, following five procedures were used.

a) Data triangulation: Different data sources (transcriptions of audio and videotaped observations, transcription of interviews, students' solution papers, and information forms) were evaluated together in order to build consistent justification for themes (Creswell, 2003; Merriam, 1998; Miles & Huberman, 1994).

b) Rich and thick description: To convey the findings, researcher used rich and thick description. One to one transcriptions of both audio and videotaped classroom observations and interviews, supported by participants' solution papers were used to describe the themes in the study such a way that "statements that produce for the readers the feeling that they have experienced, or could experience, the events being described in the study" (Creswell & Miller, 2000, p. 128)

c) Prolonged engagement in the field: The study constitutes a part of a semester-long study. During the semester, by making repeated observations, researcher gained in-depth understanding of the participants modelling processes and their thoughts (Creswell, 2003; Merriam, 1998).

d) Peer debriefing: To enhance the accuracy of the findings, three mathematics education researchers familiar with the modelling approach reviewed the data and asked questions about interpretations to challenge the researcher's assumptions (Creswell & Miller, 2000). This procedure provided feedback to the researcher.

e) Audit trail: A researcher with a doctorate degree, who is new to the present research study, examined the document with the following specific questions in mind: Are inferences logical? Is the category structure appropriate? What is the degree of researcher bias? (Schwandt & Halpern; 1988, as cited in Creswell & Miller, 2000, p. 128),

CHAPTER 4

RESULTS

This chapter reports the results of the study that were conducted to answer the research questions. The chapter describes how PTs went through a modelling process in line with the stages of understanding, planning, working, interpreting and verifying, and presenting. The chapter presents PTs' model of modelling process developed within the present study and ends with PTs' views about the factors affecting their modelling process.

4.1. Description of the Stages of Prospective Teachers' Went Through During the Modelling Processes

The results related to description of the stages of PTs modelling process were arranged according to the following stages based on the data analysis: understanding, devising a plan, working, verifying, and presenting. In the first part of the each stage, main processes are stated, and then the details of the processes are described. The processes under the stages do not follow a specific order with respect to the time of occurrence.

4.1.1. Stage: Understanding

Understanding means to grasp the meaning of the problem text and identify the problem situation. During the analysis of the data basically three processes of the understanding was observed: Reading, rereading the problem, evaluating the problem and summarizing the given information. PTs began to engage in the modeling activities by reading and rereading the problems to ensure correct understanding. Afterwards, they evaluated the problems in terms of the mathematical structure or content which, in some cases, influenced them to narrow the problem into a routine problem type. They also evaluated the problem in terms of the difficulty level and the contextual setting. In order to facilitate their understanding, they summarized the given information in the longer problems part by writing key points or using drawings. Their drawings, during this stage, included the shape of the givens, not the situation model. These helped them understand the problems better. This processes summarized in Table 5.

Table 5 Processes that PTs perform in the "understanding" stage

Understanding Stage
A. Reading/rereading the problem to ensure correct understanding
B. Evaluating the problem in terms of the

Table 5 (continued)

a) Mathematical structure **b)** Mathematical content **c)** Difficulty level **d)** Contextual setting of the problem
C. Summarizing the given information **a)** by writing key points **b)** by using drawings

The following explanations give the details of their processes in the understanding stage.

A. Reading/rereading the problem to ensure correct understanding

PTs started the modelling process by reading silently for the given problem. In this stage of modelling process, they generally reread the problem to ensure correct understanding. For the "Postman" problem, PTA1 stated that he reread problem as he had found the problem interesting. Since this problem was their first experience with the modelling activities, he mentioned that he had reread to explore the nature of the problem.

PTA1: Actually, in my first reading, I understood the problem, but I reread it. The reason of rereading the problem was not failing to understand. I reread it because I found it interesting. (Postman)

For the other problems, PTs explained during the interviews that, in general, they had read twice the problem to be able to fully understand. They stated that, in some cases, they had reread the problem as they could not concentrate on it at first reading and sometimes they reread and scanned for the numeric values or the numbers given in the problems.

R(Researcher): How many times did you read the problem? (How to store the containers?)
PTA1: I've read it one more time, because in one time you can not understand what it asks.

R: How many times did you read the problem? (Who wants 500 billion?)
PTA1: In general, I read it two times. Firstly, I read it and then recheck it like taking a look, such as what the numerical values were.

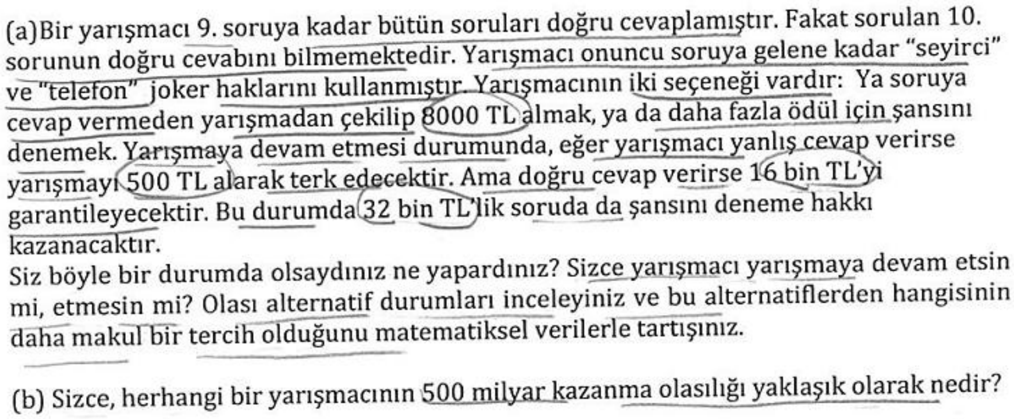
PTA2: I thought I could solve the problem when I read it. You know, at least I thought I could do something or generate an idea. (How to store the containers?)
R: Do you mean when you read the problem once again?
PTA2: Yes. I mean, I read the problem once again. Because I need to read it by focusing on it. When I read the problem by giving my mind somewhere else, I mean when I read just to read it, I thought that I could not understand and solve it.

PTA2: For the previous activities, I had begun solving problems incorrect way. Therefore, I read it a few times to ensure correct understanding (Let's organize a volleyball tournament!)

While reading the problem, PTs generally underlined the words or sentences. The data revealed that while reading the problem, except PTA1, PTs generally underlined the sentences to better understand the problem and to remember the details (numbers) given in the problem while working on the problem. While, in the "Postman" and "Let's organize a volleyball tournament" activities, only PTA2 and PTA3 underlined the sentences, respectively, in the other problems PTs generally underlined the question sentences, numerical values, mathematical words (short term, minimum cost, right circular cylinder, minimum distance, dividing into fair teams) or some words given in the problems (products, customer, firm).

The data indicated that PTs did not underline any words or phrases in the problem text including no outstanding mathematical information such as numbers, except PTA2. PTA2 explained in the interview that she underlined every single sentence to better understand the problem. A part of her solution paper of the "Who wants 500 billion?" activity is below. English translation of this problem is provided in Appendix B.

PTA2: Then, I started to read the problem. For better understanding, I underlined the problem pretty thoroughly. (Who wants 500 billion?)


(a)Bir yarışmacı 9. soruya kadar bütün soruları doğru cevaplamıştır. Fakat sorulan 10. sorunun doğru cevabını bilmemektedir. Yarışmacı onuncu soruya gelene kadar "seyirci" ve "telefon" joker haklarını kullanmıştır. Yarışmacının iki seçeneği vardır: Ya soruya cevap vermeden yarışmadan çekilip 8000 TL almak, ya da daha fazla ödül için şansını denemek. Yarışmaya devam etmesi durumunda, eğer yarışmacı yanlış cevap verirse yarışmayı 500 TL alarak terk edecektir. Ama doğru cevap verirse 16 bin TL'yi garantileyecektir. Bu durumda 32 bin TL'lik soruda da şansını deneme hakkı kazanacaktır.
Siz böyle bir durumda olsaydınız ne yapardınız? Sizce yarışmacı yarışmaya devam etsin mi, etmesin mi? Olası alternatif durumları inceleyiniz ve bu alternatiflerden hangisinin daha makul bir tercih olduğunu matematiksel verilerle tartışınız.

(b) Sizce, herhangi bir yarışmacının 500 milyar kazanma olasılığı yaklaşık olarak nedir?

Figure 9 A part of PTA2's solution paper on the "Who wants 500 billion?" problem

In the interview, PTB3 explained that she underlined the words or sentences that she considered as important. She explained that this would help her when she felt the need to look back at the given information.

PTB3: I decided to underline more important things in order to see easily when I need to look back again later, and to remember and keep in mind them. (Bus stop)

B. Evaluating the problem

During the understanding stage, PTs evaluated the problems in terms of the mathematical structure, mathematical content, difficulty level, and the contextual setting.

a) Mathematical structure

Related to the first one, mathematical structure, PTs categorized the problems as mathematical or non-mathematical (logic) problems. Although they did not state explicitly for every problem, they gave the details of this process in some parts in the interviews. For the “Postman” task, PTA2 stated that she could not see any mathematical information given in the problem. Her explanations showed that she was searching for mathematical information in the problem while reading it.

PTA2: At my first look to the problem, I have not seen anything mathematical. (Postman)

In the interviews of the “Let’s organize a volleyball tournament!” activity, PTA3 explained that she categorized the problem as the one that could be solved through logical reasoning and it did not necessitate any mathematical work.

PTA3: When I first read the problem, I said it was again a logic problem. I mean, the problem seemed to me that it could be solved again by logic. (Let’s organize a volleyball tournament!)

PTB3: In such problems (referring to the Let’s organize a volleyball tournament! problem), they (referring to another group after seeing their presentation) faced with difficulties. Because they could reach an answer in mathematical problems, but they could not get an exact result for such problems. (Let’s organize a volleyball tournament!)

Some of the PTs also evaluated the problems as requiring a single correct answer or open to alternative solution approaches. Interviews revealed that, PTB3 categorized the “Let’s organize a volleyball tournament!” problem as open to alternative solution approaches after reading the problem. PTB3 also explained her preference of engaging with these kinds of problems.

PTB3: I think there should be flexibility for the answer; that is, using problems open to alternative solution approaches is better… In the problems open to alternative solution approaches like this one, I think the students feel more comfortable for the solution. (Let’s organize a volleyball tournament!)

Accordingly, PTs did not state explicitly for every problem, but their explanations indicated that they had read the question to see if it included any mathematical information or not, or required one or alternative answers.

b) Mathematical content

PTs also evaluated the problems in terms of the mathematical concepts involved. They attempted to fit the problem into a mathematical concept for “How to store the containers?” and “Who wants 500 billion?” problems. For “How to store the containers?” problem, PTs’ solution papers’ analysis showed that five of the PTs thought that the problem was related to the volume concept. Below excerpts from the interviews exemplified the evaluation according to the mathematical concepts involved.

R: In your first reading, did you consider a method different from volume? (How to store the containers?)
PTA2: In my first reading, the volume came to my mind directly. When I see such things, the calculation of volume comes to my mind.

R: Why did you consider the volume? (How to store the containers?)
PTA1: We call the place covered by something as volume. I don't know, eventually when we put the cylinder there, it will cover a place. Since three dimensions were also given in our problem, I thought that the volume calculation should be made.

PTB1: Here, it was quite obvious that the volume calculation would be made in a sort of way. (How to store the containers?)

For the "Who wants 500 billion?" activity, after quick skimming the task, PTB3 maintained that the problem was related to the probability concept; thus, she classified the problem type.

PTB3: This problem was definitely a probability problem, I understand it before reading the problem. (Who wants 500 billion?)

For the same problem, PTA1 also explained that the words in the question also evoked the probability concept.

PTA1: The second part of the problem says the probable alternatives, the probability of winning 500 billion. Because of that, the concept of probability came to my mind. First, I read the three parts of the problem, then I thought about part a. (Who wants 500 billion?)

PTB1: I think a little tip should be given in the question; for example, the problem of "who wants 500 billion" apparently says probability. (Who wants 500 billion?)

These explanations indicated that PTs looked for certain kinds of clues or expressions to decide on the concept involved in the tasks. Since they thought that these problems would be solved using particular concepts, PTs narrowed the problem into a routine problem type that asked for only making related calculations on these concepts. In the "How to store the containers?" activity, the students were expected to find out the maximum number of cylindrical containers that fit into each storage area. Four of the students, PTA1, PTA2, PTA3, and PTB3 did not consider the rearrangement of the containers to provide the lowest cost. Accordingly, the data showed that PTs sometimes picked something that appeared to be the problem. For this problem, they did not show the signs of understanding about what they were exactly supposed to do.

For this problem, only PTB1 and PTB2 considered the different arrangements. PTB1 did not continue to work on the idea of rearrangement after trying for it. On the contrary, in the interview, PTB2 talked about her experience with similar kind of problem in an educational exhibition, and because of that, she considered that the idea of different arrangements would be used in the solution process after reading the problem.

R: When you read the question, what did you think? (How to store the containers?)
PTB2: First I understood that it was related to the arrangement types.
R: How did you understand?
PTB2: Because I had attended to an exhibition at Bilkent. There were different activities in different tables. There was a wooden box, we tried to insert the cylinders into the box, but we could not since we were trying to insert them in upright position. Then we decided to use the empty areas under the box and placed the cylinders in a disorganized way, so that all of them were

placed. Therefore, when I read the problem, I recognized that we should solve the problem by rearranging the cylinders.

Thus, only PTB2 realized what the “How to store the containers?” problem asked actually. During the group work, she tried to persuade the other group members that the problem actually asked to consider the different arrangement.

The analysis of the PTs’ solution papers showed that all of the PTs, except PTA1 and PTB2, responded to the probability of a contestant winning five hundred billion which was asked in the second part of the “Who wants to be a millionaire?” as $(1/4)^{15}$. They did not make the calculations taking into account the difficulty levels of the questions asked in the contest. This evidence showed that they considered the problem as the problems asking for routine probability calculations.

c) Difficulty level

PTs’ explanations in the interviews showed that they also evaluated the problem in terms of the difficulty level. For the “Postman” and the “Bus stop” problems, PTA2, PTA3, and PTB3 stated that they perceived the problems as easy ones. For the “Let’s organize a volleyball tournament!” activity, in the interview, PTA3 explained that as a group they thought the problem as an easy problem.

PTA3: When I read the problem, I thought that it was very easy (Postman)

PTB3: The problem was already easy, I read it quickly. (Postman)
... [Throughout the transcription three dots represent the unrelated parts of the conversations which are omitted from the excerpts]
PTB3: The problem is not hard; you are asked to imagine the daily life.

R: Was the problem easier than the other one? What did you think? (Bus stop)
PTA1: No, actually I became glad since the problem was not so hard [smiling]

PTA3: ... We thought that the problem was easy. (Bus stop)

On the contrary, during the interviews, PTA2 and PTA3 evaluated “How to store the containers” problem as difficult one in accordance with their first impressions about the problem. PTA2 explained that as she saw the numbers in the problem text, she considered that the problem was a complex one. Similarly, after reading the problem, since PTA3 thought that the solution would require a mathematical knowledge that she had not, she perceived it difficult.

PTA2: When I took the paper in front of me before reading the problem I thought that it would definitely be a difficult problem and I would be confused and could not solve it. (How to store the containers?)
R: Why does it happen?
PTA2: I don’t know, maybe because of seeing the numbers at first sight. Probably, since the previous problems were easier I did not expect the next problems to be such a complex ones.

PTA3: I had already approached toward the problem so desperately, I said that I could not solve it since I had lots of lack of knowledge. (How to store the containers?)

For the "Who wants 500 billion?" activity, although PTB2 did not indicate any difficulty with the problem during the class session, she explained the perceived difficulty in the interview. She explained that they needed more time to solve the problem in the individual sessions. Similarly, PTB3 evaluated the problem as difficult after reading the problem.

PTB2: It was impossible that the solution of this problem was to be finished in the individual session. (Who wants 500 billion?)

PTB3: Ahh, it is hard (after reading the Who wants 500 billion?)

These data indicated that some of the PTs categorized the "How to store the containers?" and the "Who wants 500 billion?" problems as difficult ones. Their explanations showed that they categorized the problems including numbers as difficult ones. Although they categorized the problems as difficult ones according to their first impressions, they engaged in these activities as well as the other PTs.

d) Contextual setting of the problem

PTs also evaluated the problems in terms of the contextual setting. For the "Bus stop" activity, PTA1, PTA2, and PTB2 explained in the interview that after reading the problem, they found similarities between the "Postman" and the "Bus stop" problems in terms of the context.

R: What did you think when you first read the problem? (Bus stop)
PTA1: When I first read this, I thought that there was a similar way we had last week. Last week, the postman was distributing mails along the street, this time the bus will go from there.

R: What did you think when you first saw this problem? (Bus stop)
PTA2: I thought it was quite similar to the postman problem.

R: How did you feel when you read first? (Bus stop)
PTB2: Oh, actually I recognized similar things with this (showing the postman problem). I thought that I had to pay attention again to the distance between the houses, streets, etc.

R: Did you understand the problem and what it asked when you read it first time? (Bus stop)
PTB2: Yes, I understood, I guess. I likened this problem to our problem of the first class.
R: In what way?
PTB2: In the postman problem the postman was to circulate the homes for distributing the posts. This time a bus would circulate for the students. Therefore, I likened the circulation of the bus by collecting the students to the postman problem.

For the "Let's organize a volleyball tournament!" some of the PTs explained that they liked the context of this problem as it seemed enjoyable to them.

PTA1: Oh very spectacular [smiling], this problem is very nice [referring to the "Let's organize a volleyball tournament!"]

PTA2: This is a more funny problem. The other problems, for example the arrangement of cans, are not interested me. But in this problem, the names are from everyday life, we separate them into groups. The other problems were boring. (Let's organize a volleyball tournament!)

PTB3: Frankly, I liked the problem a lot. (Let's organize a volleyball tournament!)

For the "Bus stop" and "Who wants 500 billion?" activities, PTB2 explained in the interviews that they liked the context of the problem as it seemed more realistic and meaningful for them. For the "Bus stop" activity, PTB3 explained that when compared to the "Postman", the problem was closer to their daily-life.

PTB2: I thought this problem was very relevant in everyday life. I mean, it's more meaningful and useful than the case of postman (Bus stop)
R: Why do you think?
PTB2: Maybe because of being a student, being familiar with bus stations [smiling]. I mean it is something from our life.

PTB3: When I first read the problem, it seemed more familiar to me, maybe because of the student district service. The postman problem seemed to me a bit utopian. I don't know, maybe since the case of the postman may not be seen in my life, this problem (referring the Bus stop problem) seemed to me more familiar. (Bus stop)

PTB2: Firstly, I thought that this problem was a good one, I mean, I thought that it was good to discuss this problem in such a platform. Because we always watch the competition, no one wins lots of money. I thought that there might be something wrong the people made. (Who wants 500 billion?)

For the "Let's organize a volleyball tournament!" activity, PTB1 explained that he felt comfortable with the context as he knew the terms of the volleyball play. For the "Who wants 500 billion?" activity, PTB1 implied that he liked the context.

PTB1: When I looked at the problem, I see that it was volleyball. Then, actually, I felt comfortable, because I know the terms of it. (Let's organize a volleyball tournament!)

PTB1: Firstly, I approached the problem positively, because the contest was my favorite one.(Who wants 500 billion?)

On the other hand, for the same problem, PTB2 stated that as she did not know the rules of the volleyball game, she thought that she could not solve the problem. Experts below both from the interviews and the class observations showed that she perceived the context of the problem unfamiliar and thought that she could not solve the problem.

PTB2: At first, I thought that I could not solve this problem since I don't have knowledge about volleyball. (Let's organize a volleyball tournament!)

PTB2: I cannot solve this problem because I do not know the terms. (during the class work of "Let's organize a volleyball tournament!")
...

PTB2: I do not know them (including text showing the problems of the explanations of the spikes)
R: Okay, we'll go through soon. The important thing is the method. How do you solve?
PTB2: hı hı

On the other hand, when PTs began group work and shared their ideas, it was observed that PTB2 knew about the terms as well as the other group members.

PTB1: There are something unclear to me here around. Do you know the meaning of dunk returned and spike returned? (Let's organize a volleyball tournament!)
PTB2: No one gains from these, I think.
PTB1: No one gains but there is something like the following: For example, I hit the spike and you returned, who got the ball, you or me? It is not clear.
PTB2: Yes, it depends on whether you returned or not.
PTB1: But, who has the right to attack?
PTB2: That team has the right to use it

To provide understanding about the terms given in the problems and make them more familiar with the situation in the applications of "Let's organize a volleyball tournament!" and "Who wants 500 billion?" activities, PTs were shown informative videos. For the "Let's organize a volleyball tournament!" activity, after the individual work, a video of five minutes volleyball play was shown and the types of the spikes were discussed. For the "Who wants 500 billion?" activity, sample contest was shown to PTs to make them familiar with the situation after they read the text. It was seen that most of the PTs were familiar with the task as they watched the series of the contest from TV.

C. Summarizing the problem

In the stage of understanding, after finishing the reading of "How to store the containers?", "Let's organize a volleyball tournament!" and "Who wants 500 billion?" problems, which are longer when compared to the other problems, PTs summarized the information given in these problems to see and understand the problems. They summarized the information in two ways; either writing the key points or using drawings.

a) Using drawings

In the "How to store the containers?" activity, PTA1, PTA2, PTA3, and PTB1 summarized the information by using drawings. PTB1 drew a picture of the container introducing radius and height of the container like in Figure 10.

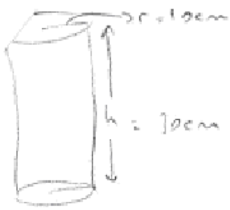


Figure 10 A part of the work of PTB1 on "How to store the containers?" problem

The features of their drawings for the “How to store the containers?” activity showed that they did not consider the situation asked in the question; instead, they considered the givens unconnected with the problem situation. For this problem, the drawings indicated that most of the students did not consider the shape of the placement of the containers into the storage units both for this stage and the following stages. The data showed that they did not change this shape throughout their studies. Figure 11 is a part of PTA2’s work on the problem. The figure also showed that as she did not label the value of the radius on the circle which is the base of the cylindrical shape, she took the value of the radius as the value of the diameter while working on the problem. English translations are provided in brackets as in the other figures.

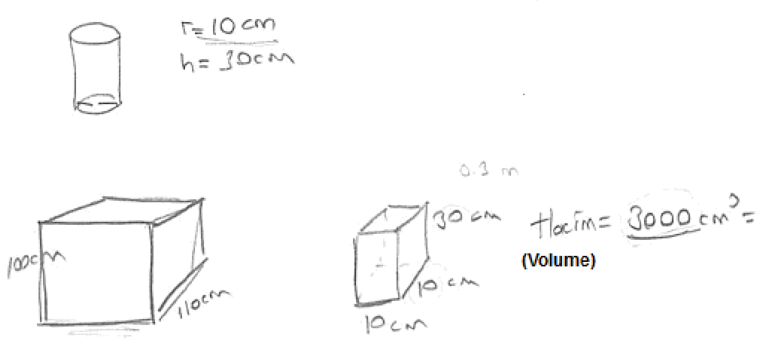


Figure 11 A part of the work of PTA2 on the “How to store the containers?” problem

Related to the “How to store the containers?” problem, PTs, except PTB2, did not draw the situation model and began to work on the problem mathematically after reading the problem. PTA1 explained in the interview that he should have drawn the situation to better understand the problem. In the interview, he explained that he could not see the gaps between the sides of the storage unit as he did not draw any figure and began to work on the problem algebraically after reading it.

PTA1: Even if we fill the storage, some place at the top still remains empty, this did not come to my mind beforehand. (How to store the containers?)
R: What do you think about the reason?
PTA1: I think it could have been seen if I had drawn. That means we need to draw, I directly worked algebraically without drawing. Drawing could be useful to see the situation. Therefore, in my solution, drawing was probably missing, if I had done it, I would have seen it better.

b) Writing key points

In the problem of “How to store the containers?”, PTB3 summarized the information given by using mathematical notations. Figure 12 shows this. As PTB2 had an idea about the solution way, she began directly to solve the problem.

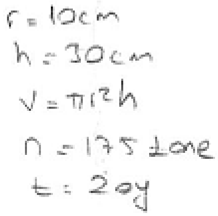
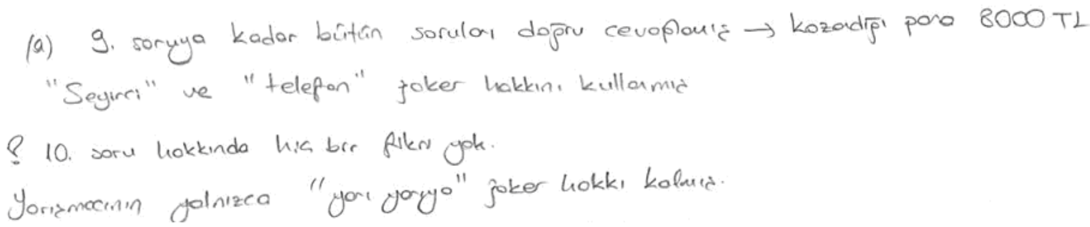


Figure 12 A part of the work of PTB3 on “How to store the containers?” problem

For the “Who wants 500 billion?” activity, PTA3 and PTB1 summarized the problem by writing the key points. Other PTs began to work on the problem without summarizing. English translations are provided in brackets.



((a) He correctly answered all the questions up to 9th, the earned money is 8000 TL.
He used "seek help from the audience" and phone a friend" lifelines.
He has no idea about the answer of 10th question. The contestant has only the "50/50" lifeline)

Figure 13 A first part of the work of PTA3’s solution paper on “Who wants 500 billion?” problem.

4.1.2. Stage: Planning

Planning means to see the essential features of the problem and accordingly devise solution approaches and decide a solution way. During the analysis of the data basically three processes of the planning was observed: imaging and representing the situation in mind, devising a plan, and deciding a solution way. In general, at the beginning of this stage, PTs imagined and represented the problem situation in their mind. While devising their plan, they connected the problem with previous experiences. They identified the variables and set a criterion to be able to decide a solution. They made assumptions and draw the situation model. They also searched for mathematical concepts to use to solve the problems. Their aim was to construct a plan that would enable them to prove their intuitive answer or end up with a formula or mathematical expressions. When deciding a solution way, in some cases, they made real-life based intuitive decisions. In general, they selected convenient or practical solution ways. In some cases, they accepted the first solution idea that occurred to them instead of developing multiple alternatives. In the group work, after sharing their ideas, in general, they decided to use one’s individual solution approach. The summary of the planning stage description is given in Table 6.

Table 6 Processes that PTs perform in the "planning" stage

Planning Stage
A. Imagining and representing the problem situation in mind
a) Imagining themselves in the problem situation
b) Context dependent/independent visualizations
B. Devising a plan
a) Connecting the problem with previous experience
b) Identifying the variables and the criteria
c) Making assumptions i. simplifying assumptions to solve the problem easily, ii. supportive assumptions to strengthen the correctness of their intuitive answer, iii. assumptions based on their knowledge about the problem context.
d) Drawing the situation model
e) Searching for the mathematical concepts to use
f) Aiming to construct a plan that will enable them to i. prove their intuitive answer ii. end up with a formula/mathematical expression
C. Deciding a solution way
a) Real-life based intuitive decision making
b) Selecting convenient/practical solution ways
c) Deciding a general solution approach and start working immediately
d) Deciding to use one of the group members' solution

A. Imagining and representing the problem situation in mind.

At the very beginning of the planning stage, PTs imagined and represented the problem situation in mind. This process was twofold: by imagining themselves in the problem situation or by context dependent or independent visualizations.

a) Imagining themselves in the problem situation

In the "Bus stop" and "Who wants 500 billion?" activities, PTs imagined themselves in the problem situation. In the interview, PTA3 stated that after reading the "Bus stop" problem, she thought her experience related to the asked problem situation. Similarly, for the "Who wants 500 billion?" activity, PTA1 stated that he imagined himself in the contest and put himself in the place of the contestant.

PTB1: I consider as possible as actual. For example, in this problem I am thinking about which way I would choose if I were the postman. (Postman)

PTA3: ... When I read the following paragraph, my own situation immediately came to my mind. Because of being a man using the buses too much, I thought that if there was a solution of this problem, I should forward it to the relevant authorities [smiling] (Bus stop)

PTA1: ... then I thought myself in the competition. I thought that I would have used the joker immediately. (Who wants 500 billion?)

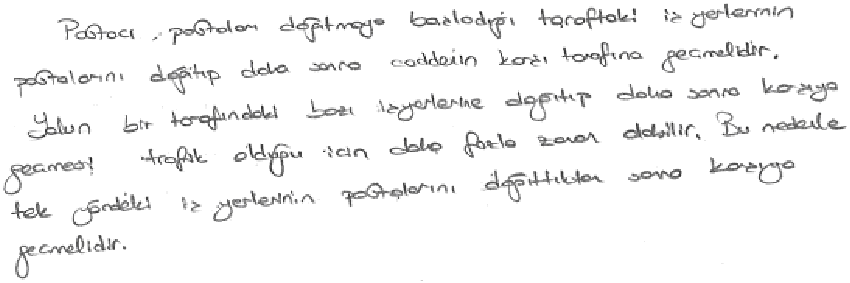
b) Context dependent and context independent visualizations

For the "Postman", PTs' explanations and drawings showed that they represented the problem situation in mind by context dependent or context independent visualizations. For this problem, in the interview, PTB1 explained that he visualized a specific context based on his experiences and began to think that context.

PTB1: there is a particular street settled in my subconscious, when I read the question somehow it figures out in my mind and I cannot push it to back. (Postman)
R: I get it. Do you think yourself how to cross the street?
PTB1: Yes. When you say a street, for example, the road of Eskişehir is one you cannot cross. In Cepa and in Armada, there are overpasses. Especially I use them a lot. Well, it is stated as street in the question, then it is difficult to cross the street because of the traffic flow. If you choose the traffic lights, it may take more time than the time of crossing via overpass, because I saw a red light lasting more than 2 minutes, therefore the street settled in my subconscious comes to my mind.
R: You mean, this street bears in one side of your mind.
PTB1: Because the problem leads you in that way. For example, a postman came to you, explained situation and requested from you to develop a method. Since you see the problem as a real-life one, then you imagine the real-life. At least I do it.

R: Do you imagine a wide street with high traffic density?
PTB1: Yes, all the time that particular street came to my mind. I could not suppress it.

For this problem, other PTs did not state such kind of context dependent examples from their daily-life experiences, but when their explanations and drawings on their written works were analyzed, it was seen that all of the PTs thought a lengthy street with traffic density. For instance, PTA3 wrote on her solution paper that as there would be traffic density, the postman should follow the U method.



(The postman should deliver all the posts of the side of the road where he began delivering. Then he should cross the road and deliver the posts of the other side of the road. Since there is traffic, delivering some of the posts belongs to any side of the road and then crossing the road cause loose of time. Therefore, he should cross the road after finishing the delivery of the posts of any side of the road.)

Figure 14 PTA3's answer on the solution paper related to the "Postman" problem

For the "Postman", contrary to PTB1's context dependent visualizations, PTA2 and PTB3 explained their context independent visualizations in the interviews.

PTA2: I had just imagined a normal street and thought how a postman delivered the posts on this street?(Postman)

In the interview, PTB3 did not explain specific kind of a street, but she stated the following explanations.

PTB3:.. when it was asked a street, it already visualized a shape in my mind (Postman)
R: Is that shape? (pointing the drawing in her solution paper)
PTB3: Yes,

As it was seen from Figure 15, in the individual work, PTB3 crossed two parallel lines and drew the direction of the U method. The videotaped analysis showed that the zig-zag drawings on the road were drawn in the group work.

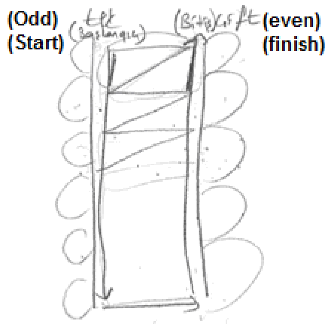


Figure 15 PTB3's drawing related to the "Postman" problem

B. Devising a Plan

a) Connecting the problem with previous experience

During the planning stage, to find a solution way and devise a plan, PTs tried to connect the problem with their previous experiences. This was done by thinking previously solved similar problems or more accessible similar experience. For the "Bus stop" activity, PTB2 explained in the interviews that after reading the problem, she recalled her previous experience in an exhibition in which she had seen an example of a similar problem. This previous experience also helped her understand the problem situation. Contrary to other PTs, she realized that the problem was actually asking rearrangement of the containers because, in that experience, she had seen that more containers could be fit by rearranging them.

PTB2: First of all, I understood that the solution was related with the arrangement of the containers (How to store the containers?)
R: How could you comprehend?
PTB2: Because I had participated in an exhibition in Bilkent. Activities were being done at different tables. There was a wooden box, we tried to place the cylinder into the box, but could not do in any way; of course we were trying to set in a straight way because of which it did not

become. Afterwards, while we decided to also use the spaces under the box we put them disorderly, and thus the remaining cylinders fit, too.
Because of that reason, when I read this question, I understood that we should already solve the problem by compressing the boxes.

For the same problem, PTA2 stated in the interview that she remembered a similar problem they solved in high school and considered the method used to solve that problem. Her explanations showed that she perceived the task similar to the problems asked for volume computations, and decided to use the same method.

PTA2: ...there were such questions at high school. For instance, we were finding how many ones fit into the inside by dividing the total volume to something's volume. Exactly those questions came into my mind. Even, we solved a question with round ball. To think of airspace while solving this question resulted from that question at high school. Because of that, I benefitted from the volume. (How to store the containers?)

During this process, PTs also compared the problem with the previous weeks' problems and tried to approach the problem like that. For the "Bus stop" activity, PTA2 explained that she considered and applied the method used in the previous week to solve the problem. In the "Postman" problem, as they were required to make a comparison, they tried to use the same method to solve the "Bus stop" problem.

R: What reason made you use mathematics here? (Bus stop)
PTA2: I started to think that I should use mathematical expression in the postman question. There (postman problem) we had compared two situations. I thought that I should initially compare two situations for this problem as well. Afterwards, I thought about how to do it, and decided to compare two situations by taking constants like x and a. However, it was postman problem that originally affected me.

PTB3: In the second problem (Bus stop), I understood better that we should deal with it by making comparison between the two situations; it was at least a possibility that came to my mind. (Bus stop)

In this process of planning, while comparing the problem with the previous week's problem PTs tried to find the similarities and differences between them by comparing with respect to the contextual setting of the problem and the density of numerical information. In the understanding stage of the "Bus stop" problem, as PTA1 and PTA2 evaluated the problem in terms of the context and realized that both problems had similar contextual setting, they devised a plan similar to the plan that was developed for the "Postman" problem. They explained this situation in the interview sessions. When other PTs' solution papers and videotaped group discussions were analyzed together, it was seen that they also considered the methods used to solve the "Postman" problem to solve the "Bus stop" problem, namely writing equations depicting the walked distance. Videotaped analysis of the "Let's organize a volleyball tournament!" activity also revealed that PTB2 also tried to find connections between the previous weeks' problems.

R: What did you think when you saw the problem first? (Bus stop)

PTA2: I thought it was quite similar to postman problem.

PTB2: To which of the activities that we did before is this problem similar. (Let's organize a volleyball tournament!-class work)
R: What is your opinion?
PTB2: I do not know, I'm trying to find it.[smiling]
R: Are you trying to find similarities?
PTB2: Certainly.
R: What are the similarities?
PTB2: So, the numbers are just given in the form of data over there. However, I guess we won't find any equation for this question. It seems as if this question is a lot more like the two activities that we did before, but we have certain variables for this. Yet to solve this question, I'm not sure about some kind of stuff such as whether I should take set or put the players into order.
R: Discuss it together.

Similarly, in the interview of the "How to store the containers?" activity, PTA1 reported that he generally considered the solution approaches used in the previously solved problems in the solution of the problems. In that regard, he emphasized the role of the order of the problems for the selection of the strategy to solve the problems.

PTA1: ...if we solve such a problem once more, I will directly start to rearrange the containers. I will do what I observed before; the previous solution can somewhat affect you. (How to store the containers?)

During the interviews of the "Who wants 500 billion?" activity, PTA2 explained that she remembered a previously taken course and a learned method at that course to solve those kinds of probability problems. On the other hand, PTB3 gave the details about her thoughts and predictions about the answer of the "Who wants 500 billion?" activity in the interview. She made over generalization based on her previous experience. She explained that she had seen a movie depicting a similar problem and thought that the answer of the problem could be the same as the answer reached in the movie in which the contestant should go on for winning more money.

PTA2: In the first year of the school, there was a class coded with 112, and we had subjects like discrete probability. I remembered such a representation from that class, so I solved it by that way here. You have 8 thousands; it would be 16 thousands with the probability 1/2, or 5 hundreds with the probability 1/2. (Who wants 500 billion?)

PTB3: I thought that contestant would continue to the contest since I had watched a film the name of which was 21 about probability. Like this question, a film began with a contest in which whether the contestant would continue or walk away by taking the money. What remained in my mind from that film is that it was statistically advantageous for the contestant to continue the contest. (Who wants 500 billion?)

b) Identifying the variables and the criteria

To be able to solve the problems, during the planning stage, PTs decided variables to be used in the solution. During their attempts to solve the "Postman" problem, they identified the traffic density and the arrangement of the shops on both sides of the street as variables.

For the “Postman” problem, PTB1, PTB2, and PTB3 considered the arrangement of the shops as closed or far to each others while the other PTs did not take into account this consideration. Although PTs did not directly explain that they took into account these considerations as variables, when their solution papers were analyzed, it was seen that PTB2 and PTB3 considered the number of shops, the arrangement of shops on both sides of the street, and the size of the each shop as variables.

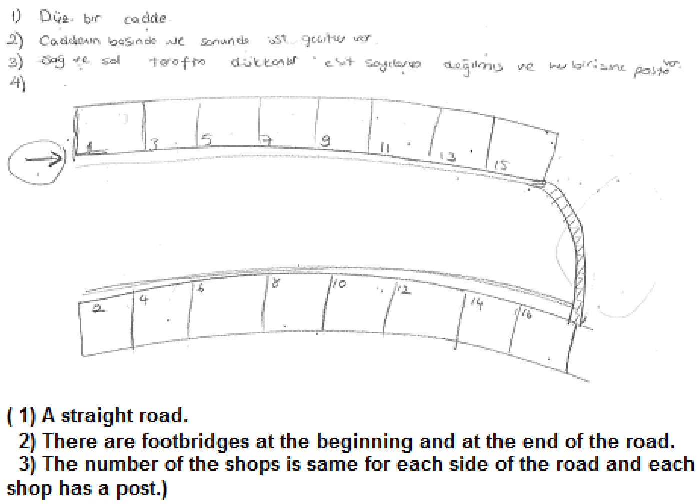


Figure 16 A drawing of PTB2 related to the “Postman” problem

The distance between the shops was an important variable to decide a way, but it was not considered as a variable by PTA1, PTA2, and PTA3 in their works. The analysis of the data showed that dependency to a visualized real-life situation and developing intuitive answers might prevent them to consider the related variables.

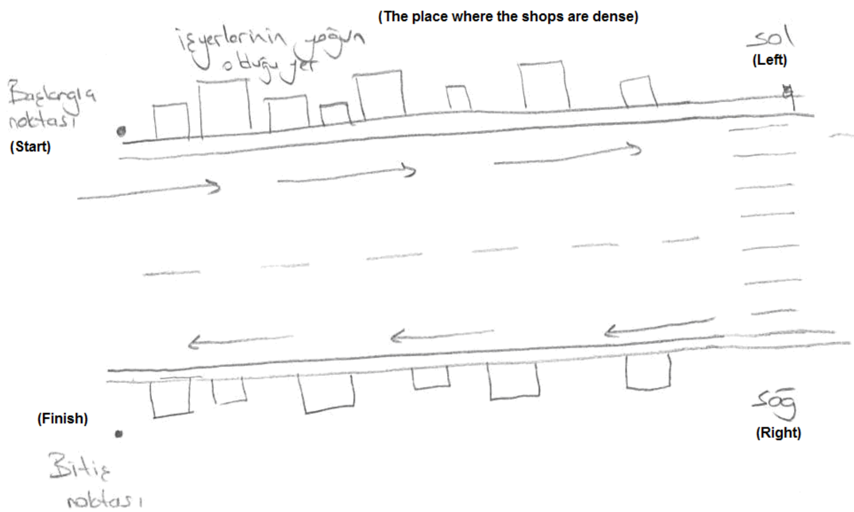


Figure 17 A drawing of Group A related to the “Postman” problem

Similar to the “Postman” activity, in the “Bus stop” activity, they did not clearly verbalized that the distance between the houses and the width of the houses was selected as variables, but the analysis of their solution papers showed that all of the PTs considered them as variables both in their individual and group work.

For the “Lets’ organize a volleyball tournament”, except PTA3, PTs tried to use all of the quantitative information (height of the player, vertical leap in inches, 40 meter dash in seconds, serve and spike results) given in the problem situation. While explaining their plans, they wrote on their solution papers that they would consider the data from volleyball try outs and would deal with quantitative information. Thus, except PTA3, they also considered to quantify the qualitative information given as spike results. PTA2 and PTB1 did not consider the coach’s comments which were given in the qualitative form in their solution attempts. PTB1 explained that he would consider the coach’s comments in the end if there was a need to use them since it included subjective information. PTA1, PTB2, and PTB3 also considered the coach’s comments. Contrary to the other PTs, PTA3 considered only the qualitative information to solve the problem. This student decided to solve the problem by using qualitative data. She did not attend to use any mathematical data in her solution.

In this stage of modelling process, PTs also selected a real-life based criterion to be able to find a solution. For the “Postman” problem, PTs had to define what “effective” should mean in the problem text. The videotape group discussion analysis showed that in their solution attempts, they took into account both criteria at the same time: “the time need to deliver all posts” and “the walking distance”. During the group work, for a moment, PTA1 also considered “the tiredness of the postman” as a criterion.

In problem two, they had a given criteria which was to minimize the total walking distance traveled by all of the students. For this problem, after writing the assumptions, PTA2 calculated only the walking distance of the student living on the farthest side. The analysis of her solution paper showed that she correctly considered the criteria given in the problem while writing the assumptions. She wrote as an assumption that “only one student from each house uses the ring”. This indicated that she at first took into account the travelling distance of each student. However, during the process, she worked with other criteria: to minimize the travelling distance of the student live at the furthest. She realized her mistake when they began group work and share their solution. In the interview, she explained that she considered the most profitable (convenient) way for all students when to decide a shelter location, so she took into account the student live on the far. This data revealed that while working on the problem, she might consider the real-life dimension of the problem and could not keep in their aim in the problem in mind and keep track of their proceeding.

PTA2: Now here, as the problem asks for the minimum walking distance in total, I’ve made a mistake in that problem. I sought for the most profitable way for all of the students, so I also wanted it to be profitable for the farthest ones. (Bus stop)
R: What do you mean by saying the most profitable? Is it less walking?
PTA2: I mean walking less, but the farthest one (referring student living on the end) will walk the least.
R: I see.

PTA2: According to my calculation, all of the students would walk less. Instead of calculating the total walking distance, I calculated it according to the farthest student's walking distance.

For the "How to store the containers?" activity, although the criterion was given in the problem, only PTB3 worked with different criteria. The criterion was given in the problem as to minimize the total cost, but she considered to minimize the cost per storage unit which caused her to make additional calculations. In the interview, she explained that she worked with different criteria to make the calculations easier for the second part of the problem.

PTB3: ... I looked at the cost here because there were 175 cans. However, in the second part of the problem, it asks what happens for more cans. I thought that if I found the cost for per can, I would better understand the cost as the cans increase the cost also increase. If I made the calculation over these 175 cans, then I couldn't make the exact calculation while generalizing it as the number of the cans would increase. Thus, I made the calculation for per can. (How to store the containers?)

c) Making assumptions

PTs also made assumptions while devising a plan. This was done in three ways: making simplifying assumptions to solve the problem easily, making supportive assumptions to strengthen the correctness of their intuitive answer, and making assumptions based on their knowledge about the problem context. Explanations given below exemplify these processes.

i) Making simplifying assumptions to solve the problem easily: Especially to be able to solve the "Postman" (after the intervention to make them use mathematics in their solutions) and the "Bus stop" problems, PTs made some simplifying assumptions. PTB3 explained in the interview of the "Postman" activity that she made some simplifying assumptions to be able to solve the problem easily.

PTB3: There was so much uncertainty in the problem. I thought directly that I had to imagine such street that it would ease the problem; there would be equal number of houses on both sides of the street. In other words, I want to simplify it as much as possible. Then, I focused on how to find the answer simply by regarding the requirement of the problem. (Postman)

For the "Postman" problem, in order to simplify the situation, PTB2 and PTB3 took the width of the houses as equal to each other and also took the houses on each side of the street as symmetric to each other. She also assumed that each house had only one post. By these assumptions, they created a case that could be easily worked on. For the "Postman" problem, PTA2 and PTB2 set the type of the street as straight. In the group work, after sharing their ideas about what assumptions might be made, the group B also considered the walking speed of the postman as constant.

R: So, you took all the houses here side by side and symmetrical. Is there any reason for that? I mean, you didn't use a situation like a house here and a house there. Did you take them side by side and symmetrical just for imagining the situation? (Postman)
PTB2:I did it to simplify the situation. I also thought that he needed to go as many houses as possible. I restricted the situation to make it easy.

Group B tried to reach n agreement on the assumptions especially in the group work of the "Postman" activity. While PTB3 tried to simplify the assumptions, PTB1 tried to make the assumptions by thinking real-life considerations.

PTB3: He (PTB1) is really mentioning lots of details, and he was incredible last week, he is thinking all aspects [smiling]; on the other hand, I'm making up the easiest generalization as much as possible without getting into much details. (Postman)

PTB3: How many details do you think?[smiling] (to PTB1)(Bus stop)

PTB3 also explained in the group work of the "Postman" activity that she always believed the simpler the solution was, the better the solution was. Thus, she explained that they needed to keep simplifying the situation further.

PTB3: The simplest one is always the best one. (in the group work- Postman)

Group A did not make simplifying assumptions during their work. After they finished their work, the instructor asked them to show the correctness of their work mathematically. By this, it was aimed to understand how PTs would utilize mathematics to solve the real-world problems. Then, they made some simplifying assumptions.

For the "Bus stop" problem, although PTs did not state clearly the assumptions they set, PTs' solution papers indicated that all of the PTs made similar assumptions to the "Postman" problem. As they were asked to show the correctness of their solutions using some mathematics at the end of the group work of the "Postman" activity, and as they worked on the "Postman" activity by using mathematics, PTs tried to make similar simplifying assumptions in the "Bus stop" activity. They took the number of houses equal on both sides of the street and assumed that only one student used the ring from each house. Related to this problem, PTB1 stated that to make the calculations easier, he assumed there were $2n$ houses on the street and assumed that the width of the houses and the width of the street were constant. Similarly, PTB3 took the width of the houses as equal to each other and explained that she tried to structure the problem by making assumptions to be able to solve easily. PTB2 and PTB3 assumed that the houses were close to each other. An example drawing of PTB2 was given in Figure 18. Similarly, PTA1, PTA2, and PTA3 assumed the distance between the houses and width of the houses as equal to each other.

R: You took the number of the houses as equal. (Bus stop)
PTB1: yes, I represented the half of the houses on one side of the street by n, in a sense, there are 2n on one side of the street. I represented them by n to simplify the operations. So, it means one fourth of the whole street. The width of the houses, x, and y are again constants here.
PTB1:... From each house just one person at most use the bus.

PTB3: Again, to simplify it as much as possible and to illustrate it simply I took only one side of the street. Therefore, that comes to my mind directly; I took that side into consideration. Then, as the students come from every direction, I thought the same distance for each of them to restrict the problem as much as possible. I tried to form more narrow extent to solve it easily. (Bus stop)

PTB3: I want to deal with one side of the street as it's always the easiest one. (Bus stop-in group work)

PTB2: In the first case, we assumed that there was only one bus stop there and also supposed that all of them were crossing across from there. (Bus stop)

R: Well, did you think of the case where the distance among the houses were different from each other? (Bus stop)
PTA3: Firstly, I thought so; I regarded the distance as different from each other and added them. Then for the sake of easiness, I took distance as fixed and same.

For the same problem, to simplify the situation, PTA1, PTA2, PTA3, and PTB3 considered only one side of the street, thus solved the problem only for one side of the street. When the PTs' solution papers were analyzed, it was seen that PTB1 assumed that there were eleven houses on each side of the street while PTB2 considered nine houses on each side of the street and assumed that there was a pedestrian crossing at the beginning of the street. Moreover, PTB3 assumed that there were nine houses on the street, while PTA3 considered only five houses to be able to solve the problem. In the group work, group B worked with nine houses considering only one side of the street. However, it is not clear that why they selected this number of houses. For the "Bus stop" problem, all of the PTs took the number of the houses on both sides of the street, and the distance between the houses were equal. In the interview, PTA2 explained that she thought that considering different distances would make the solution difficult.

R: Well, have you ever thought of taking different distances among them? (Bus stop- interview)
PTA2: No I haven't. I thought that if I took different distances, it would be difficult to make comparison. Therefore, I did not try it.

The analysis of the solution paper of PTB2 also showed that she assumed that the arrangement of the houses on both sides was symmetric. PTB2 also made the same assumptions for the "Postman" problem. A drawing of PTB2 for the "Bus stop" problem was below.

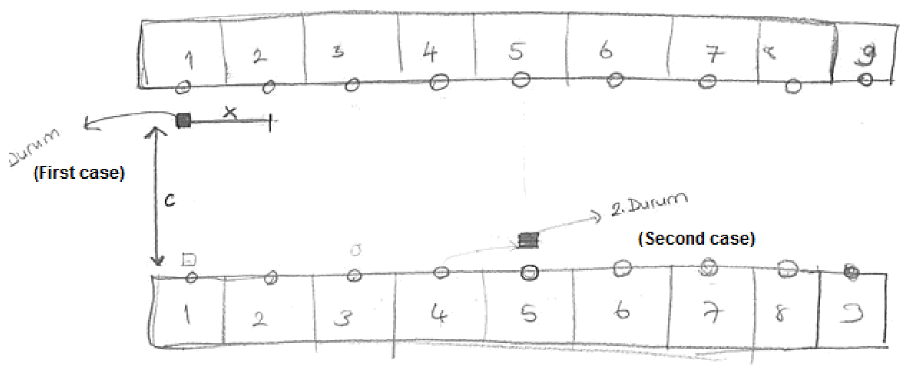


Figure 18 A drawing of PTB2 related to the "Bus stop" problem

For the first part of the “Who wants 500 billion?” problem, they assumed that the contestant used the “50-50” lifeline at the tenth question instead of the eleventh. Although this assumption was not stated clearly in the interviews or in the solution papers, all PTs assumed that the contestant would use the “50-50” lifeline to increase the probability of correctly answering the tenth question to 0.5. For the second part of the “Who wants 500 billion?” problem, the probability of winning the five hundred thousand Turkish Liras, PTB2 and PTB3 assumed that the contestant did not use any of their lifelines in any of the questions. PTA1, PTA2, and PTB1 assumed that the contestant used all of their lifelines. Moreover, PTA1 and PTB1 assumed that two of the lifelines, help from the audience and phone a friend, would not change the probability of choosing the correct answer. On the contrary, PTA2 assumed the probability of these two lifelines was equal to one. Related to the second part, PTA1 wrote on his solution paper that the contestant answered the questions randomly, thus took the probability of correctly answering each question as $(1/4)$. He solved the problem according to this assumption. Similarly, PTB2 wrote on her solution paper that she assumed that the contestant did not know the answer of any question.

During the group discussions, PTB1 and PTB3 decided to make an assumption that the contestant used all of his lifelines. PTB3 made this assumption to solve the problem easily, but PTB1 made this assumption as he thought that use of these two lifelines would not change the probability.

PTB1: Write an assumption here assuming that all lifelines were used. (Who wants 500 billion?- group work)
PTB3: No, just use the fifty-fifty lifeline as the other lifelines do not change the probability, actually it’s possible for them to change it or not. For instance, the person on the phone told that one was the correct answer. Then there would be confusion. I know what to get only for using fifty-fifty lifeline. For example if he/she (referring the contestant) asked to audience and they eliminated c and d options, two options would be left. He/she already vacillated between a and b, then, how such kind of things would be taken into consideration.
PTB1: OK, whether he/she uses lifeline, it will not affect the result.
...
PTB3: Let’s think that way, 99 percent of the audience chose option a, then, would he/she doubt about the response? Of course not. How will you represent this situation mathematically? Making an assumption, I will say audience and phone lifelines can not affect the contestant in any way. Otherwise, several things will derive from that situation.
PTB1: What does the problem tell?
PTB3: I think, we should take the probability as1/4. Do you know why? Suppose that the lifelines are just standing there.
PTB1: I’ve just said so. They are only giving idea, but we can’t say the audience or the person on the phone knows the correct answer or has no idea about the answer.
PTB3: OK. We let him/her use all lifelines.
PTB1: Right. It doesn’t matter where he/she used fifty-fifty lifeline.

On the other hand, during the group discussions, PTA1 explained that the probability of winning five hundred billion was $(1/4)^{15}$ when the contestant did not use any of the lifelines. In the group work, they considered only the lifeline that reduced the number of choices from four to two as effective. Thus, they took the probability of the correctly answering the question as $(1/4)$, even when the contestant used the lifelines of seeking for help from the audience and phone a friend.

For this problem, none of the PTs considered the difficulty level of the questions. PTA3 did not give any answer to this question.

ii. Making supportive assumptions to strengthen the correctness of their intuitive answer: PTs also made some supportive assumptions to strengthen the correctness of their intuitive answers. For the "Postman" problem, PTB2 assumed that the pedestrian crossing was at the beginning and at the end of the street which support the usage of U method for the postman. Similarly, PTA1 wrote that the postman should use the pedestrian crossing and drew the pedestrian crossing at the end of the street. When their solution papers analyzed, it was seen that PTA1 and PTA3 also took the road with traffic density to support their answer, U method. By this assumption, they tried to prevent the postman crossing the road, thus made him use the U method. The excerpt below taken from the interviews exemplifies the supportive assumptions. To be able to support the U method, PTA2 made an assumption that the postman should cross the road at the end of the street. This case is supported by content analysis of the solution papers.

PTA2:… I made an assumption about wherever the postman crossed the street, it wouldn't make any difference. (Postman)

In the group discussions, group A also considered a wider street to prevent the crossings. PTB2 wrote that the postman should classify the posts as evens and odds to minimize the crossings. PTB3 also made this assumption. All these data indicated that the assumptions which were made supported their intuitive answer: the postman should follow the U method.

iii. Making assumptions based on their knowledge about the problem context: For the "Let's organize a volleyball tournament!" activity, PTs were told that they could use the internet if they felt the need for searching some information about the volleyball play. However, none of the PTs used the internet. For this problem, they made assumptions based on their knowledge about the problem context. Related to this problem, PTA1 explained that every player should play in every position in volleyball, so they should take the weight of the categories derived from the tryouts equal to each other. Thus, during the planning stage, they took the weight of the categories equal to each other.

PTA2: We are giving equal weight to these categories, but is it right to do so? (Let's organize a volleyball tournament!-group discussions)
PTA1: Do you want me to explain why we take equal weight? Each player plays in every site in volleyball. That is why we take equal weight.

PTA2: As he said (referring PTA1) that each of the players will play in every site in volleyball, we want all of them to have equal weight. (Let's organize a volleyball tournament!)

On the contrary, none of the PTs in group B had the knowledge about how to make the weighting. When their excel sheets analyzed, it was seen that they assumed the categories having equal weights.

PTB2: ..Is it logical to take equal weight for all these categories? (Let's organize a volleyball tournament!)
PTB1: Right. For example I thought that there would be 5 categories, and each of them would have %20 percent weight. However, I think, there is a certain order of importance.
PTB2: Maybe, the 40 meter dash of the player is more important than the vertical leap of the player or height of her.

In general, during the interviews, PTB1 explained his difficulty in making assumptions. He told that he preferred working on the structured problems that did not require him to make any assumption to be able to solve the problems. The interview data revealed that he took into account the real-life dimension of the problems more than the other group members and made context dependent visualizations about the problem situation.

PTB1: I think, this is a good modelling problem [referring the Let's organize a volleyball tournament?] because there is no assumption. When we need to make many assumptions, we think about current issues more. We think whether it would be this way or another. (Let's organize a volleyball tournament!)

d) Drawing the situation model or real model

When devising a plan, PTs drew a situation model or real model in line with their assumptions. PTs drew a situation model for the "Postman" and the "Bus stop" problems. For the "Postman" problem, PTA1, PTB1, PTB2, and PTB3 drew a situation model in their individual work. PTB3 explained in the interview that she generally began to solve the problem by drawing a figure.

PTB3: Firstly, I drew the situation. I usually start solving the problem in this way. (Postman)

Before the activities, PTs did not tell to use mathematical or any other solution approaches to deduce a specific solution approach. For the "Postman" activity, after finishing their group work based on intuitive decision (see Figure 19), PTs were told to show the appropriateness of their answer and convince the instructor about the correctness of it. As they insisted on the correctness of their answer and told it was obvious, the instructor asked them to show it by using mathematics. After that time, they drew a real model as given in Figure 20 by making some simplifying assumptions to solve the problem mathematically easily. Working in group, they took the number of the shops in each side of the street as equal, and the width of the shops as equal to each others. They also drew the shops symmetric to each other whereas in their first attempts they drew the shops disordered.

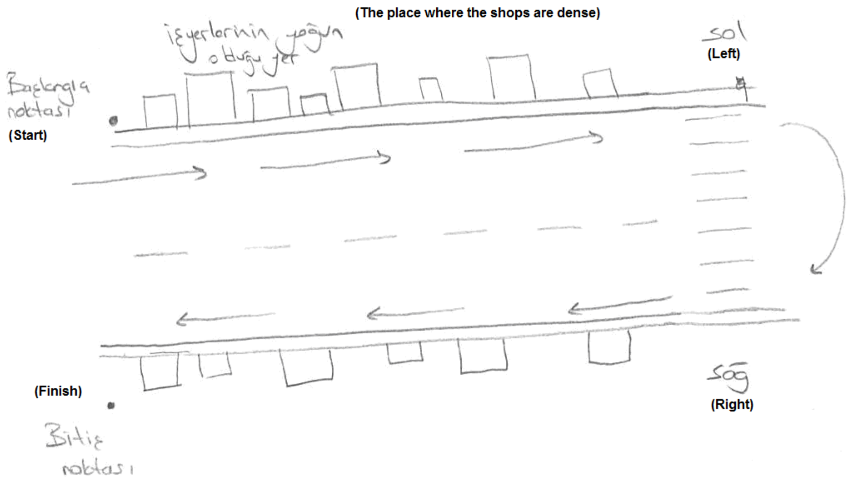


Figure 19 First attempts of Group A on the “Postman” problem

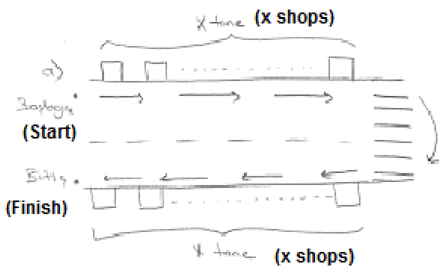


Figure 20 A part of Group A’s work after considering mathematics in their solution

Similarly, as seen from Figure 19, in group work, PTA1, PTA2, and PTA3 considered the width of the street nearly seven times bigger than the width of the shops which support their intuitive answer that the postman should follow a U method. Similarly, the analysis of PTB2’s solution paper indicated that she drew a pedestrian bridge based on their assumptions that made to support their intuitive answer.

In a similar vein, the written works of the PTs indicated that PTA1, PTB1, PTB2, and PTB3 drew a situation model compatible with their supportive assumptions for the “Postman” problem. Figure 21 was an example of PTB2’s drawing according to supportive assumptions on the place of the pedestrian bridge, the number of the shops, and the arrangement of the shops.

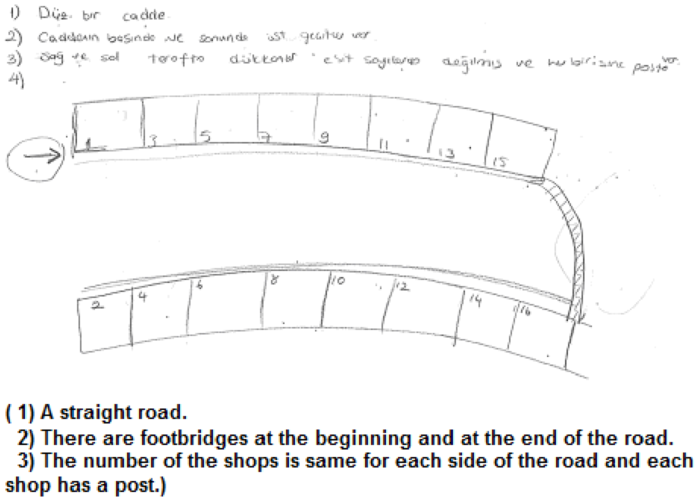


Figure 21 A part of PTB2’s work on the “Postman” problem

When compared to the “Postman” problem in which they strongly believed the correctness of their model and thus drew in line with supportive assumptions, in the “Bus stop” problem, all of the PTs drew a situation model in line with their simplifying assumptions. For instance, the analysis of the PTB3’s solution paper showed that she took the width of the houses equal to each other and considered the entrance of the buildings at the end (corner) of the building to make the solutions easier.

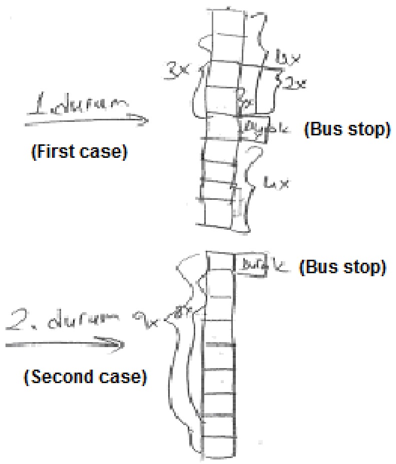


Figure 22 A part of PTB3’s work on the “Bus stop” problem

For the “How to store the containers?” problem, in general, they did not draw the situation model. Drawings made during this stage did not include the exact situation model. In the individual studies, only PTB2, as she had an experience with similar situation, draw the situation model.

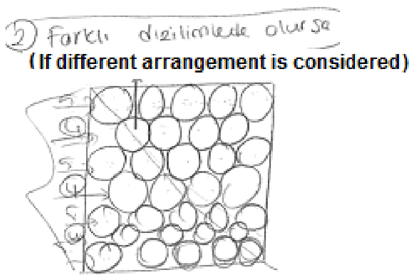


Figure 23 A drawing of PTB2 related to the rearrangement in “How to store the containers?” problem

PTB3 tried to draw the situation model, but not in detail. The diagram that she used was shown in Figure 24. PTB3 also considered different arrangement to put more containers, but not drew a detailed situation model.

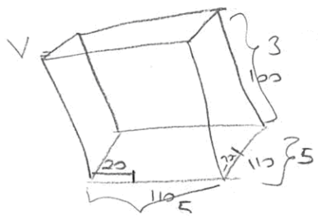


Figure 24 A part of PTB3’s work related to the problem situation of “How to store the containers?” problem

Figure 25 A part of PTB1’s work related to the problem situation of “How to store the containers?” problem

For the “How to store the containers?” problem, since group A did not draw a situation model, the researcher asked them to draw the situation after they stated that they finished their work. Only then, they realized the gaps from the sides of the storage units. During the interviews, they explained the role of the drawing to be able to correctly answer the problem.

PTA3: It is really better understood when you draw the shape properly. When we drew the shape roughly, we did not pay attention to space. It seemed as if it is not so important. You cannot understand from the rough drawing that the space would make such a difference. (How to store the containers?)

e) Searching for mathematical concepts to use

When devising a plan, PTs searched for mathematical concepts that might be used to solve the problem. For the “Postman” problem, after they finished their work that based on an intuitive answer, the instructor told them to show the correctness of their answers mathematically. Therefore, they tried to devise a new plan as a group. They examined their mathematical knowledge and wrote some formulas that could be used to solve the problem. They tried to recall their prior knowledge not only in mathematics and sometimes in physics. To identify the mathematical concept to use, they tried to remember all the formulas related to the concept. PTB3 wrote down the formula of work, and PTB1 wrote down the formula of total distance. The excerpt below shows that they were also identifying variables and the criteria while searching for mathematical or physics concepts.

PTB3: The work is equal to F.x, isn’t it guys?

...

PTB3: What does affect F; I mean, the force? We regarded the force as standard, so does it mean that the less you go the less work you do? (Postman)

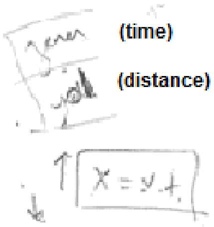


Figure 26 A part of PTB1’s work on the “Postman” problem

Related to the “Postman” activity, in the interview, PTA1 explained his difficulty in searching for mathematical concepts to use.

PTA1: Firstly, I solve the problem taking the real life into consideration, and I didn’t actually have any difficulty; I wrote my thought immediately. However, I got confused when I tried to find mathematical expressions. (Postman)

For the “Bus stop” problem, PTs had a goal to find the location that gives the minimum travelling distance amount. PTA1, PTA2, and PTB1 decided to write a general mathematical expression that gives the total travelling distance by using variables. PTA3, PTB2, and PTB3 decided to try to solve the problem for a specific number of houses.

For the “How to store the containers?” problem, except PTA3 and PTB2, all of the PTs first considered to apply their prior knowledge in concept of the volume. They wrote down the formula of the volume as

$$V = \pi . r^2$$

Related to this, in the interview, PTA2 explained that as she saw the dimensions of the storage unit given as width, length, and height, she considered computing the volume of the containers and storage units to solve the problem. Considering the use of volume concept in the solution could be evaluated together with the perceptual simplification of the problem into a routine problem type in the understanding stage. As PTA2 perceived the task as a routine problem that asks for volume calculations, she considered the mathematical concept "volume" to solve the problem.

PTA2: As it's stated in the problem that the radius is 10 cm and the height is 30 cm, when I see them, I directly think of calculating the volume. (How to store the containers?)

While thinking on the solution ways, some of the PTs considering the volume calculations for the solution abandoned their plan. They stated that the problem did not give the value of the π in the problem text, so they thought that they would not solve the problem using the π value. Then, they considered finding the number of containers that could be fit over and over, and side-to-side. PTB3 explained in the interview that although she considered firstly applying the volume concept at first but then she did not use that method to be able to solve the problem.

PTB3: As I stated before, since there is no π in the problem, I thought that I needed to solve the problem by calculating the number of items (How to store the containers?)

Similarly, PTA3 firstly decided to find the number of containers that could be fit over and over, and side to side. To be able to find that number, she considered the division of the height of the containers to the height of the storage units. However, after that, she considered using the formula of circumference of the circle which is $C = 2 \cdot \pi \cdot r$. Interview data revealed that she was affected from the ideas of the other PTs as they were asking loudly the value of the π to the instructor in the class after reading the problem. During the interview, she explained this situation.

PTA3: As everybody spoke of pi (π) in the class, I said that I would use pi, and I decided to use circumference formula. (How to store the containers?)

PTA3 tried to decide how to use the formulas to be able to reach a solution. She explained that as she could not remember the formula of the volume of the cylinder, she tried to apply incorrect formula, the formula for circumference. In the interview, she appeared to be aware of her weak knowledge base.

PTA3: The formula of the volume of the cylindrical shapes didn't come to my mind. (How to store the containers?)
R: Well, if I ask it now, can you say anything about how to calculate the volume?
PTA3: I can say height times the area of the base. In fact, I know that formula, but I couldn't exactly make it out as volume formula at that time. Otherwise, it is something I know, $\pi \cdot r^2$. I didn't remember that volume formula and then I thought of circumference; that is, $2 \cdot \pi \cdot r$. I

focused on how much place the cans occupy on the base by taking into account the circumference of cans.

Other PTs divided the width, length, and height of the storage units to the diameter of the containers respectively, to find the number of the containers that could be fit into each row. PTB2 made the same division calculations. Moreover, PTB2 thought about the rearrangement of the boxes by the help of her previous experience with the similar problem (see Figure 23). During the group work, after sharing her idea of rearrangement, they thought together about the appropriate mathematical concept to be able to solve the problem, but they had difficulty in finding that mathematical tool. They referred to the mathematical concepts such as finding an area of the geometrical regions and proportional relationship. They also considered using the diameter of the circles for progressing on this solution way for a moment. However, in the end, they decided to use the proportional relationship. They proportioned the place earned by using the manipulatives which were circle based discs 6cm in diameter to the asked value in the original problem which was 20 cm in diameter.

PTB3: I understand what you sad (to PTB2), but how can I show it mathematically? (referring to the case of rearrangement of the boxes) How will we find how to place the cylinders into the storage units; how can we show how much space remain between them? (in group work of How to store the containers?).

...

PTB3: Firstly, we have to calculate the total area in order to find and compare the space among cylinders.

...

PTB3:..Could you explain how we can calculate the space that remains among them? How can I show that the area over there is smaller than the earlier state of the shape. Will we subtract the circles' area from the rectangle's area?

...

PTB1: Let's find the area that we get when we put the disks in 3 to 3 arrangement, and then make a proportion.

PTB3: Yet, how can I proportion it without finding the amount of increment?

PTB1: We can find it, here it is.

...

PTB2: Guys, that question is relevant with diameter. I think, we can solve it with diameter calculation. As we say 'how many' that shows the solution related to diameter.

For the "Let's organize a volleyball tournament!" activity, except PTA1, PTs decided to convert all of the raw scores of the volleyball players got from the try outs into ranked scores and added these scores to get a final score for each of the player. Then, they listed these scores of the players from the biggest to the smallest or vice a versa, and distributed the players to the groups respectively. By ranking, they tried to put the data in a form so that it is sensible to combine the results. On the other hand, PTA1 considered rescaling the given data. He decided first to combine the scores for each player and thus get a final score for each player. However, he had difficulty in rescaling while working on it.

For the "Who wants 500 billion?" activity, although they, at first, produced intuitive answers that the contestant took a guess on the answer and risked loosing, or intuitive answer that the contestant chose not to respond to the question and take the amount won, to be able to reach a robust solution, PTs tried to find a proportional relationship between the amount of money won and the probability to win that amount.

PTB3: I think we should use that: the money we will earn goes up to 4 times of it, but the possibility of getting the money goes down 1/8 times. (Who wants 500 billion?)
...
PTB3: I just said so, the money will become 4 times, but as a probability is it 6 times of it?
PTB1: The possibility of getting money will become 8 times less, but the amount of the money will become 4 times more.

The group B also considered finding an interval for the probabilities in which loosing would represent the lowest probability value of the interval and winning would represent the highest probability value of the interval. They faced with difficulty at this stage. The following dialogs taken from the group discussions showed their difficulty.

PTB2: At worst, I will gain with the probability of %3, (Who wants 500 billion?)
PTB3: It's also a data, has it necessarily to be mathematical? I think we should calculate it using proportion; while the probability of getting it increases that much ... I can't make connection.(disappointed)

In the interview of the "Who wants 500 billion?" activity, PTA1 explained that he, in general, employed a mathematical concept that he felt comfortable and carried him through the solution.

PTA1: I generally use the concept that I know better; for example, derivative. (Who wants 500 billion?)
R: You mean, you don't use other methods as you thought you can't solve the problem by using them?
PTA1: Right. But, if the formula was given, I could solve the problem then.

f) Setting the aim

PTs aimed to construct a plan that would enable them to prove or support their intuitive answer or end up with a formula or mathematical expression or a result. These are explained below.

i) Aiming to construct a plan that will enable them to prove their intuitive answer: The content analysis of the solution papers showed that they first developed an intuitive answer for "Postman", "Bus stop", and partially for the "Who wants 500 billion?" problems, then wrote down some assumptions that supported or proved their answer. For the "Postman" activity, after reading the problem, all of the PTs developed an intuitive answer and stated that the postman should follow a way similar to U shape. To be able to support the correctness of their answer, they made assumptions to make the postman follow the U method. The examples of these assumptions were explained in the "making assumptions" process in the devising a plan stage. Similar aim was also case for the "Bus stop" problem. The content analysis of the solution sheets combined with the interview data indicated that PTs aimed to use mathematics as a tool to show the correctness of their intuitive answers. In the interviews, they explained that they aimed to prove their intuitive answer and make supportive work. The analysis of the group discussions also showed that, during the group work of the first two activities (Postman and Bus stop), PTs aimed to create a situation in which they could prove their believed solution by using algebra. (For the "Postman" problem, after finishing their group work, they were told to show their answer mathematically, and after that, they began to use algebra). As they believed their intuitive answer, they thought a counter case to compare and show the correctness of their answer.

R: In the beginning, you were saying that was already my model. (Postman)
PTB3: you're completely right. When you asked how you knew that, we necessarily put that model there (referring cross road method), and said we know it mathematically from here.

PTA1: We were defending that it was a good idea (referring their method), and we had to put something against it to show that our idea was better. For that reason, we thought of the other idea (referring the cross road method). (Postman)

PTA1: I supposed that the solution would be like that. Afterwards, I said I had to prove that mathematically. (Bus stop)

PTB3: The model that was used first is always the model that is the easiest, the most practical and the best. I just drew the second one (the station in the end case) to compare them. If someone comes and asks how you know that is the solution, I drew the second model to show from here I know the solution is that. … However, I directly say the solution is the middle. I neither thought of any other alternatives nor tried them because middle is the shortest one. Frankly, I considered that it would be either the beginning or the middle. I didn't think of other points; I've just realized it. (Bus stop)

PTA3: I planned that firstly write about what you think about the solution, then, improve something to support it. (Bus stop)

For the "Bus stop" problem, PTs only considered the end and the mid points of the street. In the group work of the "Bus stop" activity, only PTB2 suggested considering all points in the solution, but then, as a group, they did not try for this solution way. An excerpt from the videotaped group discussions is below. After the given dialogs below, PTB2 did not insist on her approach and they went on considering only the end and the mid-point of the street.

PTB2: I put the bus stop both at the beginning and middle of the street. I compared these two bus stops, and found that the walking distance for the middle on was shorter. What about doing like that; let's say something about the whole street (they're talking over the figure that PTB3 drew), and name this point as 0 (pointing the end of the street). Imagine that the length of the street is 10. Then, if the distance to the bus stop were x, the rest of it would be 10 – x. So, we can change the location of the bus stop whenever we want. (Bus stop)
PTB3: You mean to do more general.

ii) Aiming to construct a plan that will enable them to end up with a formula/ mathematical expression: PTs also aimed to construct a plan that would enable them to end up with a formula or mathematical expression. For the "Bus stop" activity, at the end of the individual work, PTB1 explained that he could not reach a formula. For the same problem, analysis of the videotaped group discussions showed that PTA1, PTB1, and PTB2 aimed to reach a formula although the answer offered as a solution of the problem was not a formula.

PTB1: I couldn't bring it to the exact formula. Even if I work on it more, I may not reach the exact formula. Then, the group work will fall behind, therefore, I stop working individually. (Bus stop)

PTB2: What about starting to create the formula from here? Let's say x here (pointing the distance between the entrance of the houses). (Bus stop)

PTA1: I'm wondering whether we think wrong or not. Is it necessary to find a formula for the solution of the problem? [smiling] If it's not, don't beat the air. (Bus stop-interview)
R: It changes according to what the question requires.
PTA1: For the sake of finding a formula, we fix everything to reach the formula easily.

R: Have you asked them why they deal with the question by giving values like a and x instead of numeric values. (referring other group members) (Bus stop-interview)
PTA3: I think so [smiling] . Then, I see that I need to make generalization.
R: How did you notice that you need to make generalization?
PTA3: Because, whenever I solve the question by giving numeric values, they always say to me it's OK, but how we will make generalization. I think, it's more logical to solve the question by giving numeric values in the individual work instead of giving the values like a and x. PTA1 has already tried to find a formula in his individual work; that surprised me.

PTA1: Now, if we make comparison, we can find the formula. (Bus stop)

Contrary to "Postman" and the "Bus stop" activities, in the group work of the "How to store the containers?", "Let's organize a volleyball tournament!", and second part of the "Who wants 500 billion?" activities, PTs did not develop an intuitive answer as the content of the problem required them to make calculations to reach a correct answer. Therefore, they tried to find an answer to the asked question without aiming to prove their answer; they aimed to find a mathematical expression or a result.

To summarize, while devising a plan, PTs tried to recall their previous knowledge and experience about the problem situation with the aim of finding a similar method to solve the problem. They evaluated the problems in terms of the similarities and differences between the previously solved problems or previous experiences. In general, they made simplifying assumptions to be able to solve the problem easily, and sometimes, as in the "Postman" activity, they made supportive assumptions with the aim of proving their intuitive answers. They made drawings in line with their assumptions. They tried to draw the situation model for the "Postman" and "Bus stop" problems; however, the drawings for the "How to store the containers?" problem exactly did not reflect the exact situation asked in the problem. They searched for relevant directions from the problem text about which mathematical concept would be employed. In the problems, as the mathematics behind the problems was not obvious, they searched for mathematical tools/concepts through which they could solve the problem. During the interviews, probing questions about the solution process showed that, in some cases actually, they possessed the essential prior knowledge and abilities; however, they could not decide which mathematical concept to implement at those problems. The results indicated that they chose the mathematical concepts through which they believed to be able to solve the problem. Although, in some cases, they realized other mathematical tools/approaches that would help the solution process, in general, they did not choose that method as they thought it would be difficult to reach an answer when they used that method. They stated that they chose mathematical concepts that they felt comfortable to carry the computations or related procedures, thus to carry them to the solution. Their explanations indicated that, to able to reach an answer (immediately), they used the methods that required them to use basic mathematical concepts.

C. Deciding a Solution Way

a) Real-life based intuitive decision making

The interview data confirmed that PTs developed real-life based intuitive answers for the “Postman”, “Bus stop” and “Who wants 500 billion?” problems after reading and understanding the problems. For the “Postman” problem, as the mathematics is not obvious in this problem, PTs did not use mathematics in their first attempts. Without any mathematical work, they thought a real-life situation and created logical arguments for this situation.

PTA1: Firstly, it comes to my mind that as there would be traffic on the street, delivering the posts by crossing the street would be problem for the postman. It’s more rational to deliver the posts of the shops on one side of the street and then the others. I thought about how to deliver them in real life and then related it with mathematics. (Postman)

PTA2: I gave the most reasonable answer after reading it. It didn’t come to my mind to use x or y. (Postman)

PTB1: When I first read the question, I directly think of U method. (Postman)

R: Why did you eliminate that way (referring to the cross road method)? (Postman)
PTB3: Well, I don’t know what to say. It was really clear that would be in that way like 2+2 =4 (referring to the U method as a correct answer). I deduced from observations in real life. To come from one way then cross the street and then come back again, so it’s quite clear that U method is more advantageous.

As seen from these explanations, PTs developed a solution alternative in mind (cross road method) for somewhat specific real-life situation, and compared the two ways intuitively based on a selected real-life criterion (time or walking distance etc). Similarly, for the “Bus stop” problem, all of the PTs used intuition to solve the problem. Except the PTA1, all of the PTs maintained that the bus shelter should be located at the middle of the street without any work and calculation.

PTA2: When I looked at the question, I thought that the bus stop should be in the middle. [smiling] For me the bus stop was already in the middle. (Bus stop)

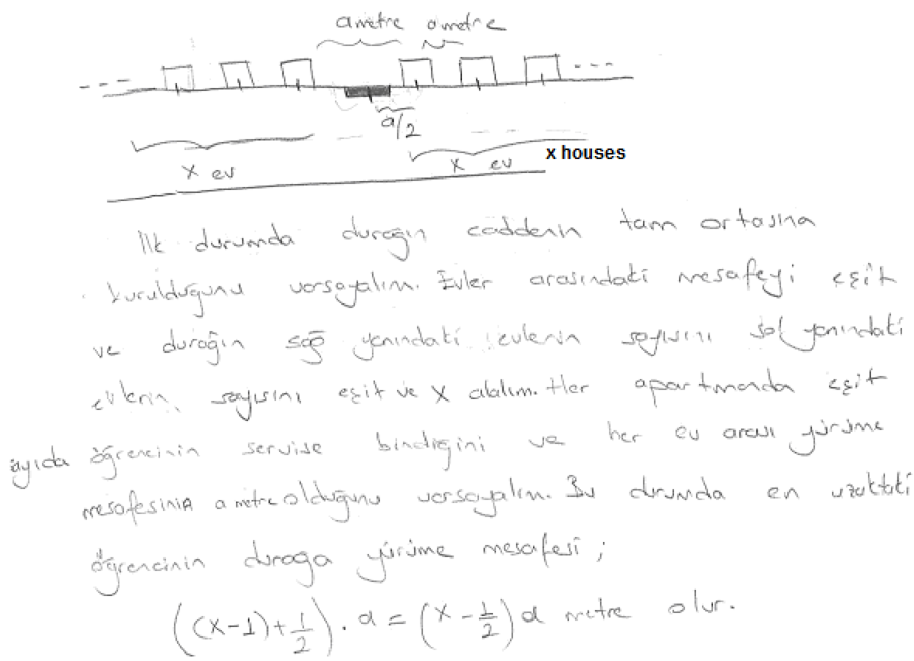
PTB3: … I neither thought of any other alternative nor tried any. I said that middle is the shortest. In fact I regarded that either middle or the beginning is thought. I didn’t focus on any place between them. (Bus stop)

PTB2: There was a bus stop in the beginning and I put one more in the middle. Then, I compared two of them. Surely, when the bus stop is in middle, the walking distance is a bit shorter. But, at this time I wonder if we do the following. What about attributing a value to all. (Bus stop)

For the same problem, the interview data of PTB3 indicated that while producing an intuitive answer, they also thought the reasonableness of the answer in real-life. In the interview, PTB3 explained that the student live at the furthest also should walk less.

PTB3: The reason why the bus stop needs to be on the middle is that I know the difficulties from the real life when it is at the beginning of the street. Some people get on the bus infront of their houses, but I have to walk a long distance to get on it. Now that there will be only one bus stop, I have to think of the people walking a long distance. Not only that man but also that one can walk less (showing the both end of the street). For example, the man can come in a short time if it is put here (showing the end of the street), but the man at the end of the street have to walk more distance. (Bus stop)
R: Do you want to be fair?
PTB3: Right. Maybe, it's because of our experiences: I wish that bus stop would be a bit closer.[smiling] I remembered that that one and that one (the people on both end of the street) would walk equal distance because they would be the people on the farthest.

Similar situation was also case for the PTA2. During the planning stage, after deciding to solve the problem for two cases, the shelter located at the middle of the street and at the end of the street, and after making some simplifying assumptions related to these two cases, she began to solve the problem only for the student live at the furthest, namely at the end of the street, instead of calculating the total walking distance of all of the students and which was actually asked in the problem. A part of her work is below.



In the first case let's assume that the bus stop is at the middle of the street. Let's take the distance between the houses as equal, and take the number of houses on the left and right hand side of the street as equal to each other and as x. Let's assume that equal number of students get on the bus from each apartment, and assume that the walking distance between each house is a meters. In this case the walking distance of the student at the furthest is
$((x-1)+\frac{1}{2})\cdot a=(x-\frac{1}{2})a$ meters.

Figure 27 A part of PTA2's work on "Bus stop" problem

For this problem, PTA1 intuitively thought that the total walking distance would be same wherever the shelter is placed. During the interview, he explained this case.

R: What was the first shape visualized in your mind like? For example, was it a two sided street? (Bus stop)
PTA1: In fact, the first thing that came to my mind was that nothing would change wherever I put the bus stop.
R: Wherever we put it?
PTA1: Yes, I thought so. I even said that why they asked such a question.

For the part (a) of the "Who wants 500 billion?" problem, "what they would do if they correctly answered 9 questions, but they had no idea what the correct answer to the 10 th question was", PTs made decisions about the answer of the problem based on their daily-life considerations. In the individual work of the "Who wants 500 billion?" activity, except PTA2 and PTB3 who made some logical arguments, all of the PTs answered this problem based on their daily-life considerations. They thought that what they would do if they were in a situation like that. They thought about risking 8000TL or not, without making any calculation.

PTA1: Suppose that I didn't know that question, and used fifty-fifty lifeline. As it were fifty-fifty probability, it's beyond my control, but if I withdrew from the contest, it would be 100% my own choice. So taking the money seems more logical. Therefore, there is nothing to say mathematically for that situation. (Who wants 500 billion?)

PTA3: One more, I believe in luck for such situations. If I had no idea about the question, I would most probably choose the wrong option even if there were only two options. Thus, I think I would withdraw from the contest. Even if there were a high possibility, I think, luck is really important. So, I would withdraw. (Who wants 500 billion?)

PTA3: If I were a contestant, I would give up without looking tenth question. (Who wants 500 billion?-group work)
PTA2: Don't you look at it?
PTA3: If I had some idea about the question, after using fifty-fifty lifeline, luck would come into play, as I am unlucky person, most probably; I would choose the wrong one and be eliminated from the contest.
PTA1: However, we have to find something related with mathematics.

PTB1: Actually, I began to think about the human psychology in part (a) of the problem before thinking mathematically. I thought more directly about it in individual work; "if I were in his/her shoes, what would I do…" There is a 7500 liras difference between losing the contest and leaving it. I think, I couldn't take that risk, so I would give up the contest. (Who wants 500 billion?)

PTB2: Now that there is an option like if it were you, I would give up. Since the risk of losing the contest is 50%, it's a great risk. I mean, I wouldn't go on. (Who wants 500 billion?)

b) Selecting convenient/practical solution way

For the "Postman" and the "Bus stop" problems, although PTs developed an intuitive answer after they were asked to show the correctness of their solutions in the "Postman" activity, they tried to show the correctness of their intuitive answers by selecting the convenient solution way. During the interviews conducted with PTA2, referring to the "Postman" and the "Bus stop" activities, she explained that she solved the problems by considering the convenient solution way that came to her mind at first. Similarly, related to the "Postman" problem, the interview data revealed that group B selected the convenient solution way in their solution attempts. Similarly, PTB3 explained that she generally simplified the problems to be able to solve easily.

PTA2: To tell the truth, I just look at the question, and start to write the easiest solution for example the one like that comes to my mind. (Bus stop)

PTB1: The reason behind it is that we really adore the assumption. I mean we put so much emphasis on it. In other words, we took the easy way…

PTB3: .. There is no information here about any variables. For that reason, making it really difficult, you can need a lot of time. However, I can't focus on something for a long time. I try to make it as easy as possible, and then try to solve it as soon as possible. (Postman)

PTs made some simplifying assumptions to be able to solve the problem easily. In the interviews, when they were asked about whether they considered changing the assumptions to provide other solutions, thus to develop their models, PTA1 explained their difficulty in getting the equations. As they faced with difficulty, he explained that they did not consider the other situations that probably would make the solution process difficult.

R: You assumed that the number of students coming from each house are equal (researcher reading the written on the solution paper). Well, have you ever thought about what happens if the number of the students coming from each house would be different? (Bus stop)
PTA1: No we haven't thought. [smiling]We have already found these equations barely. We haven't even tried it.

In the "How to store the containers?" activity, after reading and understanding the task, PTB1 considered the rearrangement of the boxes which was actually the question asked for. After drawing the shape of the rearrangement of boxes in staggered alignment and after thinking on it about two minutes, PTB1 explained to the instructor his idea by showing the diagram given in Figure 28. An excerpt from the videotaped classroom discussion is below.

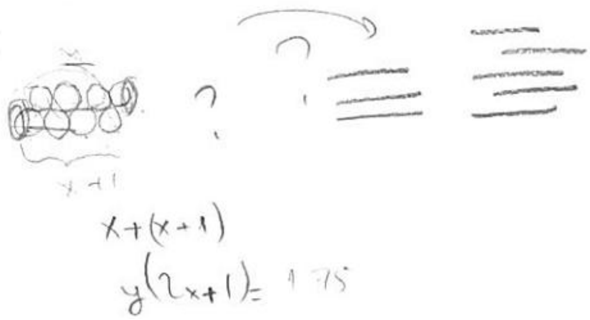


Figure 28 A part of PTB1's work on "How to store the containers" problem

PTB1:..Is it possible to put these cylindrical containers to that storage unit in that way; there isn't such an expression in the problem (referring the rearrangement) [at this time there was no mathematical work in his solution paper including the x and y values given in Figure 28] (How to store the containers?)
Instructor: Why not?
PTB1: It seems nothing will change about volume; the surplus from here and the surplus from there will balance each other. Namely, that is. (drawing and showing the lines on the right hand side of Figure 28.) Is it necessary to focus on such a calculation? I think, no.
Instructor : What is your aim?
PTB1: If it were done in that way, it's possible to put more cans.
...
Instructor: I see. Let's set in this way and see.

The videotaped discussions showed that after this dialog, PTB1 tried to find the number of the containers by assigning x and y values to the number of containers and the number of rows respectively. He thought about how to write a mathematical expression including x and y for about two minutes than abandoned this idea and decided that there would be no advantage of the rearrangement. Then, he considered the identical alignment and began to solve the problem for identical alignment of the containers in which it was easier to reach a solution when compared to the rearrangement of the containers to manage to put more containers. For the same problem, PTB3 explained that she considered the practical solution way in her solution attempt.

R: Well, have you thought about while setting them straightly that if it would be in real life, I would compress them instead of setting them straightly? (How to store the containers?)
PTB3: Not really because in real life you don't pay attention to the place which is the most minimum here, you put them the way that would be the most practical.
R: What do you mean with the most practical?
PTB3: I mean that while solving the problem, it seems to me that it should necessarily be in single line. It can be because of the fact that when the containers sandwich in the gaps, I couldn't know how to calculate it, so I did it in this way. For that reason, I didn't think any other situation.
R: OK.
PTB3: In real life you don't do any calculation. You put them without thinking.
R: However, when you need to use mathematics, did you think that there is no need to make the problem too complex like the one here?
PTB3: I don't want to put myself into difficulty; all in all, that setting fill the bill [smiling].

For the same problem, PTA1 explained in the interview that they decided to choose the convenient way to solve the problem. He explained that, in their school life, they used to solve routine problems in which they needed to apply the easiest way without looking from different perspectives and thinking different alternatives. Similar to PTB3, he also added that he chose a way to solve it. Similarly, PTA2 explained that she chose the convenient way.

PTA1: I don't know how to express it, but there is something like background. We were grown up in such a way that we always think about the easiest one. You see that the containers will be set and there is storage unit. Find the volume and divide it and get the result. As we were grown in that way, to place and rearrange them is very difficult for us. You will set it and labor on it; it's really hard. (How to store the containers?)
...
PTA1: As I said before while solving the problem, I use the method that I know well, so I used that one which is the easiest for me.
R: Is it volume, which comes to your mind?
PTA1: It's easy, I don't want it to be complex.

PTA2: When rearrange the containers, it would be difficult to calculate it. Furthermore, I thought that I could not get enough space from that arrangement. So, I didn't even need to try it. (referring to rearrangement).

For the "Let's organize a volleyball tournament!" activity, after finishing his individual solution, PTB1 considered whether the concept of median or standard deviation could be used in the solution of the problem. He explained his idea to the researcher. An excerpt from their dialogs is below. After the dialog, he tried for some values on the computer to remember the function of the median and standard deviation. He thought about these concepts for a while, and then they began group work. Although PTB2 listened this conversation, the group did not consider these concepts in their solution attempts.

PTB1: Does the median do it?
R: What was median?
PTB1: Let's try median, take the mid range of those and their mean and find the distance of each value to the mean. Let's try median and see what happens by giving different values.
PTB2: Aren't they the ones here?
PTB1: No, there are some values in my mind; I will try them to find out what the median does.
PTB2: Maybe, it can be q1s and q2s; let's divide them into 4.
...(PTB1 were trying)
PTB1: Hıımmm I doubt whether it was standard deviation?
...
PTB2: What did you do (to PTB1); what are you thinking about?

Related to this dialog, interview data showed that PTB1 considered a practical solution approach in his answer to the "Let's organize a volleyball tournament!" problem. He explained in the interview that he actually considered another weighting by using standard deviation, but chose the practical way. He also did not share this idea with other group members at the beginning of the group work.

PTB1: Here you evaluate 1.70 based on 1.85, but 53.34 based on 63.5; the weights are different. Therefore, those should be fixed and proportioned at a point before working on them. In fact, I thought it to be like that at the beginning. However, putting them into this order seemed easier [that is ordering without taking into account the amount of difference in the scores], and we solved the problem by using that. (Let's organize Volleyball Tournament?)

The analysis of the excel sheets indicated that PTs selected the practical solution way when they needed to make a decision as part of their work. During the group work of the "Let's organize a volleyball tournament!" activity, the group B got same ranked score for the serves and spike results for some of the players. When they encountered with the same score of the players, they used the alphabetical order to be able to rank them. Accordingly, they gave the better score to the player whose first letter of the name came first in the rank list. During the presentations, a student asked this case to the presenter of the group B; PTB1, and she answered that they ranked in alphabetical order when they got same rank number for different players.

A student: How did you deal with the same scores while ranking them? For instance, the spike scores are same there as 1; how did you do the ranking in such a case? (Let's organize a volleyball tournament!)
PTB1: We ranked them by their name. We didn't do anything for that.

The analysis of the solution papers indicated that, during the individual work of the same problem, PTA2 considered the players with the same scores. PTA2 planned to rank the players according to each category based on the scores from try-outs beginning from the one for the worst score and eighteen for the best score (as there were eighteen players). After deciding a general solution approach to be used, she wrote on her solution paper her solution approach. After explaining her solution way, she wrote at the end of her solution paper that

“I realized the same scores”. In the interview, PTA2 explained that she realized this case during her work, but concluded that the scores did not have to go up to eighteen.

PTA2: I noticed that there are same scores, and then said to myself that it is not necessary to rise up to 18 for all categories; it’s just my own idea.(Let’s organize a volleyball tournament!)

In the group work, this group chose to apply the idea of PTA2. Therefore, for the categories of serves and spikes, they gave the same ranking scores to the players with the same scores. After aggregating the ranked scores for each player and getting same ranked scores, they considered the coach’s comments to decide the better player and to be able to divide the teams.

Related to this problem, during the interview, PTA1 explained that considering another way instead of increasing the ranks one by one for different score intervals would yield better ranking, but it would require a standardization and thus an extra work. Therefore, they did not try for it, and thus they applied the easier way.

PTA1: Actually, I thought that… for example, giving three points instead of two points for the player with a height of 1. 68.
R: Why did not you do that?
PTA1: We decided to do the other way. We talked and agreed, there was no distinctive difference between two ways. I did not concern so much by thinking that it was not crucial.
R: well, did you find it as a correct way?
PTA1: We would also have given more points since there were much more increase for the others, but how much was it more, we would have had to decide a standard for it.

For the “Let’s organize a volleyball tournament!” activity, during the individual work, PTA1 considered first combining the scores and then ranking approach. However, while combining different kinds of quantitative information got from the try-outs for an individual player, he did not use a systematic way. He tried rescaling to make the effects of the variables close to the each other. Although he referred to an important point by this idea, he considered his approach unreasonable as he did it unsystematically. He chose the practical way and made weighting in an unsystematic way by giving random numbers to each variable to weight them. He did not share this idea in the group work. An excerpt from the interview was below.

R: The score of the service is 0.8, how did you decide? (Let’s organize a volleyball tournament!)
PTA1: I said that, I said I would do an arbitrary thing.
R: Did you make a guess for 0.8?
PTA1: yes, that is anyway bullshit. It cannot be applied. I thought that if closer values were to exist, I might do something.
...
PTA1: .. I firstly thought of such things, I did not even share these in the group. Actually, my purpose was how to somehow compare these, and to establish a ranking.

The interview data revealed that PTB3 considered the easier solution ways in her solution approaches. She explained that she preferred to work on problems allowing different solutions instead of working on modelling problems asking for single solution. She stated that, in open ended problems allowing different solutions, she felt herself comfortable, as they had an opportunity to select and decided a solution way (probably the easier one).

PTB3: To tell the truth, if especially the computer had not been here, I could not have done even if standard deviation idea came into my mind at that moment because I may have actually said why I should deal with so much. One more point, it can be stated either asking the problem in a way for finding the only team meeting the standards or asking to establish a team from different options. However, when I am asked to divide the team in such a way, and that especially these players will be in one team, I am becoming more prejudiced. (Let's organize a volleyball tournament!)
R: Since the solution is single by this way (referring close ended problems)?
PTB3: Because you have to do and find it. To me, offering alternative solutions to the problems is better. The difficulty or simplicity comes into everyone's mind for these questions (referring open-ended problems) for the sake of following the easy way in case I should not make it more difficult. However, before everything, knowing different solutions for the problem is that it is already obvious when you look at the problem; for instance, it is evident that there is an equation. However, from my point of view, the students are more likely to find the solution and feel comfortable when the problems asking for alternative approaches (referring open-ended problems) like this one. At least, for me, it is so.

For the part b of the "Who wants 500 billion?" problem, interview data confirmed with the analysis of group discussions showed that PTs did not consider some cases to be able easily reach a solution. For instance, PTA1 considered the case that the contestant did not know the answer of any of the questions. Similarly, PTB2 explained that, instead of making mathematical analysis in the individual work, she considered answering the question by putting herself in place of the contestant by ignoring some points of the contest. She explained that she did not consider the situation of winning the sixteen thousand TL which is the amount of money guaranteed regardless of what happens later as she could not represent it mathematically in the time given for the individual work.

PTA1: For the other item as well (referring part b of the problem) he finds the correct one with probability 1/4. If he uses the fifty-fifty lifeline, the probability becomes1/2, but it is valid when nothing is known (Who wants 500 billion?)
...
PTA1:..here, I just thought the random response of questions like making up the alternatives in the multiple-choice test.

PTB2: The critical point will be 16.000, I responded the question without taking that into consideration, but in later stages, the contestant will guarantee 16.000, then everything will change. For that guarantee, it seems possible to continue the contest by taking a risk, but here (referring individual work) since I cannot state mathematically and in addition to the time limitation, I did not think that case. Most of our individual works were already so. (Who wants 500 billion?)

During the presentations, PTs were asked about the place of the bus stop if the distance between the houses was different. It was seen from the dialogs below taken from the videotaped group presentations that PTB1 and PTB3 even did not try for that situation as they thought that it would be difficult to solve it in that case.

R: What if we had taken the distance between houses not equal?(Bus stop)
A student: We would never have generalized them
Instructor: the group who has taken not equal can present (no answer)
PTA2: we cannot make comparison if we take them not equal.
PTB3: oohooo, we cannot find a solution.
PTA2: Is there anyone who has taken not equal?
PTB1: I am just curious about the assumptions of the ones who have taken the distances not equal. Then, it is necessary to limit the number of houses on the street.

c) Deciding a general solution approach and start working immediately

The videotaped class observations showed that, in general, especially for the individual work, all PTs decided a way to solve the problem after the initial reading of the problem. This is not different for the group work except for the group B in the "How to store the containers?" and the "Who wants 500 billion?" activities as they discussed on the solution approach to decide a solution way. PTs showed disposition to accept the first solution idea that occurred to them instead of developing multiple alternatives.

The videotaped classroom observations showed that, for the "How to store the containers?", "Let's organize a volleyball tournament!" and "Who wants 500 billion?" activities especially, PTA1 desired to immediately reach the answer. In general, after reading the problem, he immediately developed a general solution approach, and began to make calculations or the offered solution. In group work, he also continued this approach and wanted to reach a result of the problem swiftly. The videotaped observations indicated that he thought that they needed to start writing the answer directly. In problem three, although other group member, PTA2, criticized his approach, he continued to rush.

PTA1: Let's write now? Ok, let's start. Shall we start like this, then we can use x (How to store the containers?)
...
PTA1: Come on, let's be quick.
...
PTA2: Just a minute, do not make a mistake here, (to PTA1)
PTA1: Do not stop, write please. Come on.
PTA2: You are in a hurry; first of all, let's think for a while (to PTA1)[smiling]

PTA1: Write something (to PTA2), we did not get an answer in hand (Who wants 500 billion?)
PTA2: There is no point to write something without finding the right way?
PTA1: We are thinking, but we should write something.

Similarly, for the "Let's organize a volleyball tournament!" and "Who wants 500 billion?" activities after reading the problem, PTA1 immediately began to work on the solution. The excerpt below taken from the videotaped class observations exemplifies this situation.

Instructor: You have already started to write? (Who wants 500 billion?)
PTA1: I am the one who writes without thinking.
Instructor: Or, the ones who think while writing?
PTA1: Actually, I am thinking a little bit at first. The rest is coming after you have started; otherwise, how we can think so long. I am thinking while writing.

R: Well, do you say that this part of the problem could have been better thought? (item b) (Who wants 500 billion?)
PTA1: I had not thought the lifeline at first, then it came into my mind while writing .

In general, during the planning stage, after deciding a general solution approach to be used, PTs left the related details about the solution approach undecided at the beginning.

R: Did you say that I could solve it without volume calculation? Or else, had the first one coming into your mind been the volume, then did you decide to start with the volume? (How to store the containers?)
PTA1: I could not think any different idea, immediately began with the volume calculation.

Related to the "Let's organize a volleyball tournament!" activity, PTA3 explained in the interview that as they focused to reach a result and as they believed the correctness of their solution way, they wanted to make related calculations as soon as possible.

R: How did it happen? Forgetting into transform centimeter to meter? (Let's organize a volleyball tournament!)
PTA3: That is totally related with carelessness. Nobody said to transform. Since we just focused on the solution, it might be because of that. We were thinking that the solution way was correct and we wanted to do it instantly.

PTB3 explained that, in general, she began working without thinking on the alternative ideas or approaches to the problem. In an interview, she explained that she believed the first thought about any problem to be generally the best and the most correct approach.

PTB3: I am using the way that comes into my mind firstly, well… I am not so much focusing on the 2nd, 3rd alternatives (How to store the containers?)

PTB3: Actually, it is mostly the option that has been firstly thought (referring method), from my perspective, it is the most advantageous and the correct method for people (Postman)

In the interview of the "Postman" activity, PTB3 also explained her personality structure that she was not a kind of person thinking long time on the details of the solution approach.

PTB3: ..I am not a person like the one who thinks so much, writes the details, then finds a solution. Namely, I prefer drawing instantly, writing whatever it is. In other words, I did not think so much (Postman)

PTs were required to submit the solution papers to the researcher at the end of the each course. Although other PTs did not state explicitly, videotaped classroom observations combined with interview data showed that especially PTA1 had some considerations about managing to fulfill the work since he assumed the written works would be part of their assessment. In the interview of the "Who wants 500 billion?" activity, PTA2 explained that PTA1 generally wanted to have a written solution at hand.

PTA1: Let's write them, you (to PTA2) write the things on the paper that we will submit to the teacher [referring researcher]. We will give it to the teacher. Look! (to PTA2) we will not be able to write and become late. Be quick, be quick (Let's organize a volleyball tournament!)

PTA1: We would not have any solution in hand in case the teacher [referring to researcher] terminated the time. (Who wants 500 billion?)

PTA2: For instance, during the class PTA1 always says let's write, let's write. I say, let's think on the problem, but he all the time wants something written at hand. He wants us to write something, as everybody is writing. Because of that, looking at things from different angles is somewhat difficult for us (Who wants 500 billion?)

Interview responses and solution paper analysis combined with class work analysis confirmed that, after the initial reading of the problem, PTs, in general, devised a general plan to solve the problem and they followed the initial plans that they created. The analysis showed that especially PTA1 and PTB3 focused on getting one answer and got the task done fast.

The explanations of PTA2 showed that she began to try to look from different perspectives to develop their models although she could not produce different solution ways for the problems as they continued working on the problems,

PTA2: I am solving through simple logic. I am trying to look for different options to solve, but cannot find anything. But at least, I am thinking about whether I could look from different perspectives or not any more. I was not even questioning this, and I was following my own solution way by thinking that it was certainly the correct one. (Let's organize a volleyball tournament!)

d) Deciding to use one of the group members' solution approach

After working on the problem individually, PTs shared their ideas at the beginning of the group work. The content analysis of the solution papers indicated that PTs chose one of the group members' individual solution way and made some little improvements on this solution in the group work.

When the solution papers analyzed, it was seen that, for group A, the solution of the "Postman" problem was nearly the same as the solution of the individual work of PTA1, and for the "Bus stop" problem it was nearly the same as for PTA2. Interview data also supported this case. During the interviews of "Let's organize a volleyball tournament!" and "Who wants 500 billion?" activities PTA2 explained that she dominated the group work on these activities. The interview data supported this group member's control over the group in these problems.

PTA2: in individual work, I decided my own solution way like so, then in group work, I made them accept it. [smiling] (Let's organize a volleyball tournament!)
...
PTA2: Mostly, one of us in the group is better at the problem, (Let's organize a volleyball tournament!)
R: Among them, one's idea is standing out?
PTA2: yes
R: Well, what are the factors that lead the idea to come into prominence?

PTA2: The way to present and impose your idea…, I just talk. I state that I find by this way, and if it seems illogical, then you (friends) can state your own ideas. They also accept it as logical; namely the others should believe your ideas. If they also convince me, we apply theirs.

PTA2: Here (in this question) we wrote my idea since he [referring PTA1] also liked it (Who wants 500 billion?)

For the "Postman" problem, PTB1, PTB2, and PTB3 intuitively decided the answer. As their answer to the "Postman" problem was the same, they combined the individual ideas and assumptions to conclude an answer in the group work of the "Postman" activity. In the group work of the "Bus stop" activity, group B took a joint decision on the answer and they tried to reach a mathematical conclusion together. For the "How to store the containers" problem, because any of them did not find the correct mathematical concept to solve the problem, they tried to find together. For the "Let's organize a volleyball tournament!" activity, after deciding a solution way based on the individual work of PTB1, they made little improvements on the method. For the undecided points in the individual work of PTB1, they shared and tried to combine the individual ideas again. The interview conducted with the PTB1 after the application of the "Let's organize a volleyball tournament!" activity exemplifies this case.

PTB1: Well, I am thinking about whether there is any difference between my own solution way and the group's way. Probably, no. (Let's organize a volleyball tournament!)
R: Then, you ranked the players, and by this way classified them into groups, didn't you? (looking at the solution papers)
PTB1: For example, we assigned players a different rank score taken by these trials. For instance, this idea was given by PTA3. In my individual work, I wrote that four different lists could be established, but how the four-different list could be established did not seem very clear in my mind. It was PTA2's idea to rank them by giving number. We applied it.

During the interview conducted with PTB3 after the "Let's organize a volleyball tournament!" activity, she implied that the ideas discussed in the group work could not be different from the ideas in the individual work.

R: They (referring to another group) said that they firstly tried your method; then tried another method when this one did not work. (Let's organize a volleyball tournament!)
PTB3: Maybe, at the very beginning, if they think more thoroughly, they will realize to apply that method (referring the final method). Because, if the idea of using standard deviation comes to one's mind in group work, it is certain that they realized it in individual work. Does anything come to our mind like something which we all have not thought in our individual work?

In the part (a) of the "Who wants 500 billion?" problem, although group B developed intuitive answers, two of them concluded that the contestant should take the earned money and one of them (PTB3) should not, when they tried to see the probabilities of two situations mathematically, they faced with difficulty. In that case, similar to the "How to store the containers?" activity, they tried together to be able to reach an answer.

The data analysis showed that PTs generally developed similar approaches to the problems in individual work. When they could not developed an idea about how to form a mathematical model (group B in “How to store the containers?”and “Who want 500 billion?”), they decided together. In general, in group work, they did not think on the other possible strategies. For instance, in the “How to store the containers?”, PTA1 and PTA2 considered the “volume approach” in their individual work. They did not consider different arrangement for minimum costs in the group work. Interview data indicated that their approaches to the problem were nearly the same and they considered applying one of the group members’ individual solution way. An excerpt from the interview conducted with PTA2 was below

PTA2: When I first read it, I directly thought about the volume. In group, PTA1 also followed the volume although the representation was different. (How to store the containers?)

To summarize, while deciding a solution approach, for the “Postman”, “Bus stop”, and part (a) of the “Who wants 500 billion?” problems, they developed intuitive solutions. After the “Postman”, when they were asked to show their answer mathematically, they selected the convenient solution ways that they believed to carry them to an answer. After they decided a general solution approach to be used, they did not think or work on the possible alternatives that have potential to yield better models. In general, PTs produced similar approaches to the problems, except some cases (both groups in “How to store the containers?” and group A in “Let’s organize a volleyball tournament!” activities) in individual works. When they had different approaches to the problems, they tried to persuade each other. As a group, they selected the approach of the group member who insisted on his/her approach. If they had no idea about the mathematical concept to be used they tried to find together (group B in “How to store the containers?” and “Let’s organize a volleyball tournament!” activities).

4.1.3. Stage: Working

Working means to perform the plan in order to formulate an appropriate model and reach a solution. During the analysis of the data basically two processes of the planning was observed: performing the plan based on an intuitive decision, performing the plan using mathematics. During this stage, PTs performed their plans devised in the previous stage. Depending on the nature of their solution approaches, they employed the process of mathematization or not. During the mathematization process, PTs mathematically represented the givens and the problem situation. They used specific cases/values to be able to reach a solution or to put forward an idea. They also used some other strategies such as using manipulative and making guesses. To be able to reach a solution, they ignored some of the variables. In some cases, they worked on the ideas developed by other students. During this stage, when they faced with confusions, they re-examined their purpose. By using some strategies, they also made some computations to be able to reach a solution. They also tried to combine the mathematical expressions to be able to reach a solution. This was done in three ways: by making assumptions, by using logical arguments, and by writing the expressions in a comparable form. The summary of this stage is given in Table 7.

Table 7 Processes that PTs perform in the “working” stage

Working Stage
A. Performing the plan based on an intuitive decision (no mathematization)
B. Performing the plan using mathematics
a) Representing mathematically
b) Applying strategies to progress mathematically (analyze mathematically)
i. Use of specific cases to be able to reach a solution or to put forward an idea
ii. Using manipulatives or manipulating the givens to solve the problem easier
iii. Guessing about the unknown variables and relationships
iv. Ignoring some (important) variables to be able to reach a solution.
v. Working on ideas of developed by other students
vi. Re-examining the purpose
vii. Using different computing strategies
c) Combining the expressions
i. By making assumptions
ii. By using logical arguments
iii. By writing the expressions in a comparable form

PT’s modelling processes in this stage is presented below.

A. Performing the plan based on an intuitive decision

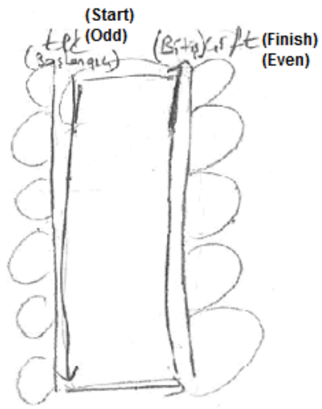
During this process, PTs wrote real-life based intuitive decisions constructed in the previous stage with superficial drawings and explanations. For the “Postman” problem, as they developed intuitive answers in the individual work, all of the PTs wrote down their answers by supporting it with the assumptions. Figure 29 was the individual work of the PTA2 on the “Postman” problem. As she decided to answer the problem based on intuition, she began with writing supportive assumptions and concluded that the postman should begin from the nearest part of the street, deliver the mails, cross the road, and deliver the other mails.

Caddenin, trafiğe açık ve trafik akışının iki yönlü olduğunu varsayalım. Yolun düz, birim sürede geçen araba sayısı ve arabaların hızının sabit olduğunu düşünürsek, bu durumda postacının yolun neresinde karşıdan karşıya geçtiği postaların dağıtım süresini etkilemeyecektir. Postacı eğer caddenin kendisine en yakın ucundan başlayıp önce o taraftaki işyerlerine sırasıyla postaları dağıtıp caddenin sonuna gelince karşıya geçip tekrar sırasıyla o uçtan itibaren postaları dağıtmaya devam ederse, postaları en kısa sürede dağıtmış olur.

(Assume that the road is open to the traffic and the flow of the traffic is two-way. If the road is straightforward and the number of automobiles passing in unit of time and the velocity of them is stable, then the place where the postman crosses the road will not be important. If the postman begins to deliver the posts from the place nearest to him and delivers the posts at that side and then at the end of the street crosses the road and continues to deliver the posts in order, he delivers the posts in the minimum amount of time).

Figure 29 PTA2' work on the "Postman problem"

Similar answer was given by other PTs. An example of the PTB3's work on the "Postman" problem was below.



* Önce elindeki postaları tek ve çift numaralı evlerin posta numarasına göre sıralar
* Sonra tek numaralı postaları dağıtır ardından çift numaralı postaları dağıtır. Böylece zaman kazanmış olur.
* Sadece 1 kere karşıya geçtiği için ve postalar sıralı olduğu için zamandan kazanç sağlanır aynı zamanda daha az yol gittiği için yoldan da kazanç sağlanır. Az emekle çok iş yapılmış olur.
* Ev sayıları eşit caddelerdeki

* Firstly, he orders the posts as odds and evens according to numbers of houses.
* Then, he delivers the odd numbered posts, and subsequently delivers even numbered posts. Thus he saves time.
* As he crosses the road once, the gains achieved both from time and from road as he walk less. A lot of work with less effort would have been made.
* The number of houses on the road is equal to each other.

Figure 30 PTB3's work on "Postman" problem.

For the part (a) of the "Who wants 500 billion?" problem, except PTA2 and PTB3, PTs chose to answer the problem intuitively without producing a mathematical model. Four of the PTs explained that even they used their lifelines, the probability of loosing was %50, so they would leave the contest without risking their money.

PTB2: Here, the probability of loss is fifty-fifty. But the probability is % 100 when withdrawing. I think I would withdraw since the probability of losing is high. I could not have continued.

B. Performing the plan using mathematics

In the process of working, PTs used various strategies to be able to reach a solution. Following explanations give the details of this process.

a) Representing mathematically

In general, PTs began mathematization process by using mathematical notations. For the "Postman" problem, after the individual work in which they produced intuitive answers, they were told to show the correctness of their answers mathematically. After that, they began to use mathematical symbols in their solution attempts. They represented the variables selected in the previous stage by using mathematical notations. An excerpt from the group work of "Postman" problem is below.

PTA1: I wonder what would happen if the number of shops was equal? For example, suppose that there are x shops here and x shops there. For each shop, delivering period is thought as t. Be careful, we decided the delivering period as t. (Postman-group work)
PTA2: in xt time, for here; in yt time, for there, the delivery is performed. For the crossing period of the street, we may name it as z.

Similarly, in group work, PTB3, who wrote the reports in the group, represented the width of the houses by x and the number of houses as n. They also represented the time spent in each shop by t on the situation model. Thus, they assigned each variable as a mathematical symbol.

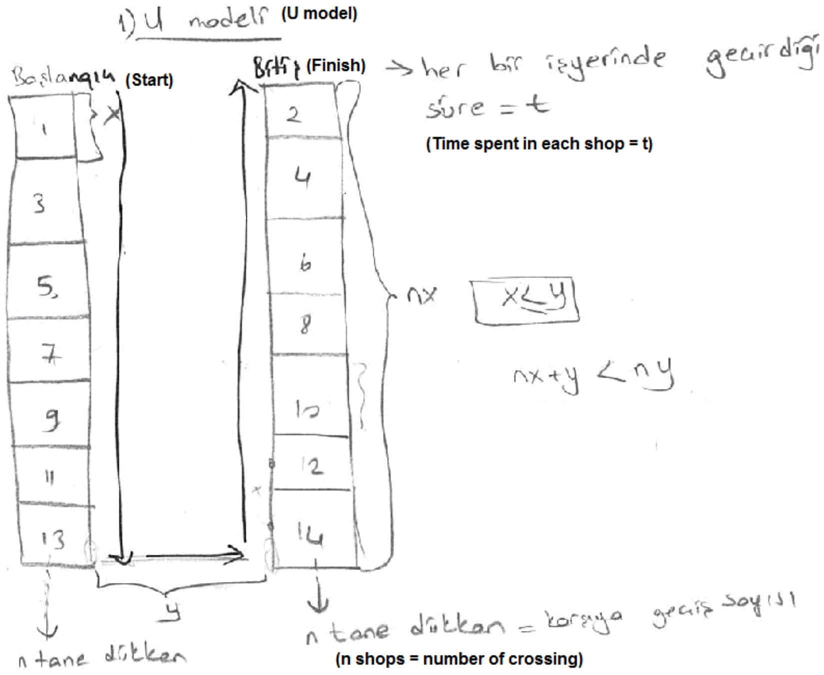


Figure 31 A part of Group B's work on the "Postman" problem

Below is an excerpt from their discussions.

PTB3: Then, the number of shops with even numbers, let x represent it. If so, the others (referring odd number of shops) will certainly be x, too. Because, we took them as equal in quantity. (Postman-group work)

...

PTB3. We will generate a formula now…yes, the number of shops with even numbers is x, the number of shops with odd numbers is y

PTB3: The duration of delivery by the postman to each shop t .ok? Let's be t.

For the "Bus stop" activity, with the experience of the "Postman" activity, all of the PTs began to mathematize the situation in their individual studies. They mathematically represented the variables identified in the previous stage. Below is a part of their individual work.

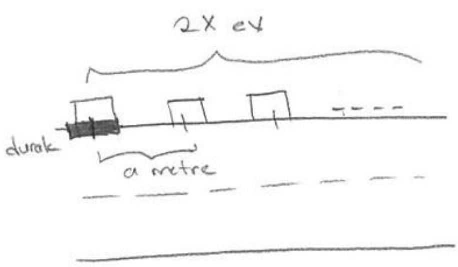


Figure 32 A part of PTA2's work on the "Bus stop" problem

For the “How to store the containers?”, in general, PTs wrote down the givens using mathematical notations. They represented the radius of the cylindrical containers as r, the height of the containers as h, and they wrote down the numerical values of those givens.

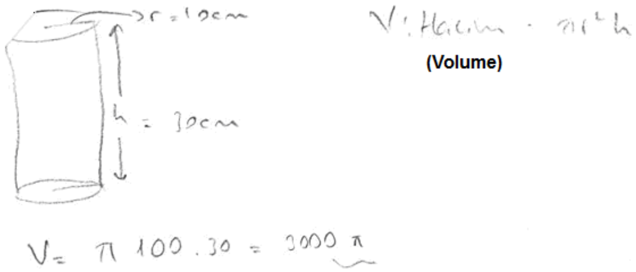


Figure 33 A part of PTB1’s work on the “How to store the containers?” problem

All of the PTs, except the PTA3, transferred verbal data into the quantitative one in the “Let’s organize a volleyball tournament!” activity. They assessed the effect of the variables and assigned positive or negative values to them. They assigned numbers such as 0, 1, 2, or -1, 0, 1 for the spike results: dink returned, dink unreturned, and kill, in the net and out of bonds. PTA1 also evaluated the coach’s comments and gave negative values for poor evaluations.

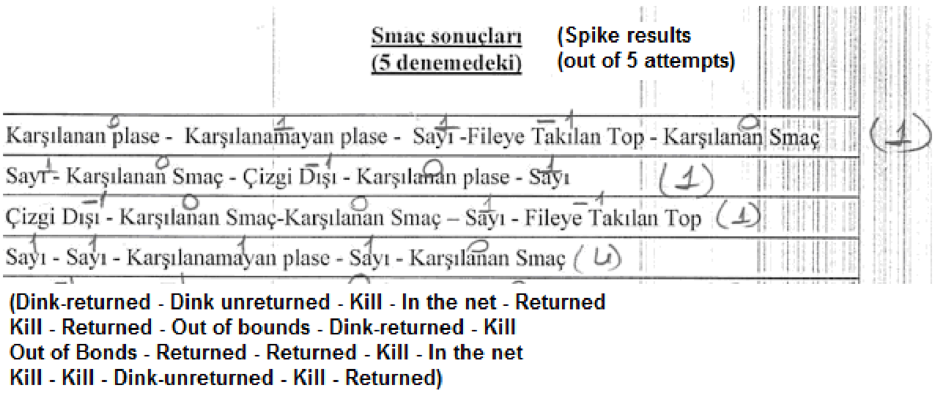


Figure 34 A part of PTB3’ work on “Let’s organize a volleyball tournament!” problem

b) Applying strategies to progress mathematically (analyze mathematically)

i. Use of specific cases to be able to reach a solution or to put forward an idea: To be able to solve the problem and put forward an idea, PTs assigned specific numbers to the variables for the “Bus stop” problem. For this problem, PTA3 took the distance between the houses as 100 m and made the calculations for this number. She calculated for specific cases to put forward an idea. Figure 35 is a part of her work on her solution paper. PTB2 and PTB3 considered a situation including a certain number of houses. They drew nine houses and made the calculations according to this number.

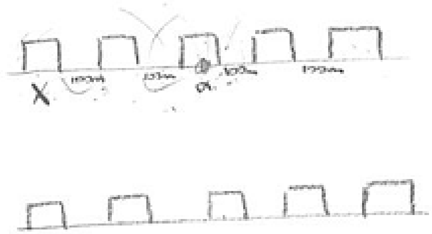


Figure 35 A part of PTA3's work on the "Bus stop" problem

During the interviews, PTB2 also explained that she compared the two points for the shelter location to be able to have an idea about the situation: the middle of the street and the end of the street. Except PTA1 and PTA2 who produced different approaches, all of the PTs tried only for these two cases to reach a conclusion.

PTB2: To have an idea, I tried the specific points, well I expected that specific points would help me in formulating an idea…I looked for something to compare in the question in order to understand which one is more logical. For that reason, I counted these distances, the distances to be walked (Bus stop)

PTB1: In our individual work, we all tried [referring the end and mid points]. Here (referring to the end of the street) it was found that all (the students) should walk much more.

For the second part of the "How to store the containers?" problem, PTs tried some specific values of the variables to be able to see the pattern and reach a solution. In the group work, as group A found the number of the containers that could be placed to the first, second and third storage area as 75, 165, and 240, respectively, PTA1 tried the numbers 250, 350 (250+100), 320, and 315 (240+75), respectively. By doing this, they tried to find a pattern. An excerpt from their group discussions was below.

PTA1:..I wonder if we'll continue with 2 and 3 [referring the storage unit] after 240? We will make some trial, let's do it
PTA2: I think we should not
PTA1: Are you serious!?Ok I will do it,
PTA2: I think we can try it on this sheet
PTA1: Well, we can say the first storage unit is more suitable for 75 and less. For the ones between 75 and 165, it is the 2nd storage unit; between 165 and 240, the 3rd storage unit seems suitable (writing these values on a sheet). Let's check for 250. How many first one do we need for 250. 75 plus 75 equal 150, it is four, isn't it? Four times one hundred is 400. If it is such a storage unit, 165 plus 165 equals? It passes 250, isn't it? We need two ones, for each piece it is 150, then two ones are 300. If it were other cabinet, we would need two again. For one piece with 200, the total is 400. Well, it seems that the second one is appropriate for 250. For example, let's add 100 to the 250, then it becomes 350. For 75, how many does it make??

PTA3: 5?
PTA1: well, how much money? 500 TL.
PTA3:.. How can we generalize this?

ii. Using manipulatives or manipulating the givens to solve the problem easier: During the mathematization stage of the "How to store the containers?" activity, group B used some circular objects to better visualize the problem and to see whether rearranging the boxes would enable to fit more containers. Because of the fact that, PTB2 had similar experience before, she tried to convince other members that they could fit the remaining ten more containers (175 (given)-165 (found)) into the second storage unit by rearranging. After questioning approximately five minutes about how they could show it mathematically, they could not find an idea. Then, in order to check roughly whether they could fit the containers into the second storage unit, PTB1 first intended to use the glasses on the desk. Afterwards, PTB3 asked for objects with bases as circles to the course instructor. Thus, instead of using algebraic approaches, they worked with the manipulatives to be able to make a decision whether they could fit ten more containers. At this point, they aimed to find approximate results instead of exact solution and to see the reasonableness of the rearrangement. After seeing that they could fit more containers by rearranging shown in Figure 36, they went on working with the manipulatives. They took the measure of the length earned by rearranging and tried to find a proportional relationship between the data obtained from the manipulatives and the data given in the actual problem. An excerpt from the group discussions was below.

PTB1: Look, is there any different cup? There is no different cup (How to store the containers?)
PTB2: I am sure that it can be solved by this way, but I do not know how.
PTB3: Do you know what PTA2 actually says? She says that do not place it in this way, but the other
PTB2: Yes, put the cup here that you will secondly put.
PTB3: here it says that we should not put this one here, but to the other place (referring rearranging); do not put the one that we should put near it here, but put it there and we can put many more boxes by reducing the spaces to minimum
PTB1: To me, those spaces seem useless; that is, I cannot imagine it
PTB3: I wish there would be cans and try them; or else cylindrical objects can also be better
I: In fact, there should be in those cupboards, let's look.

Figure 36 A snapshot from Group B's work while working with manipulatives on "How to store the containers?" problem

After this idea, they began arithmetic calculations by using the values of manipulatives. By using proportional relationship, they found that they could get one more row for extra containers and found that second storage unit could be used for storage. However, they could not get the exact number of containers. PTB2 worked on the problem to find the exact number while PTB1 and PTB3 was writing the report and doing related calculations. Then, PTB1 and PTB3 also joined her to find the exact earning from rearrangement by mathematical computations. However, while they were working mathematically, they used the values of the manipulatives and could not switch to the values of the asked cylindrical containers. The numbers written on Figure 37 were attached after they were told toward the end of the lesson that they could try for the given values of the containers.

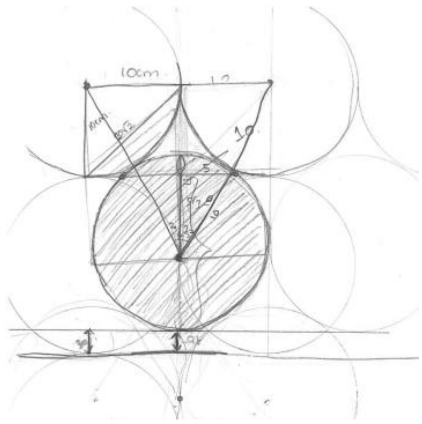


Figure 37 A drawing of Group B on the “How to store the containers?” problem

PTB2: … I thought of rearranging the boxes into the gaps, but did not think by writing these real numbers given in the question on those shapes we had drawn. It would have been possible if I had written the values given in the problem, we would have solved the problem; however, I could not be sure since I had not written the values on the circle. For example; if I had written this one’s diameter which was 20, on this shape, I could have solved it (How to store the containers?)

…

PTB2: .. we all three were stuck in that point. We should use the value of the diameter given in the problem. On what we studied? 6 cm disks. We always thought that the cans with 6 cm were so. Because the objects that we use were 6 cm.

For the same problem, to be able to solve the problem easier, PTA2 rendered the shape of the given containers in different geometric form. While working mathematically, she considered the shape of the cylindrical containers as the square prisms. Thus, she eliminated the computational problems aroused by the gaps between the containers. In the group work, they also based their solution to this approach.

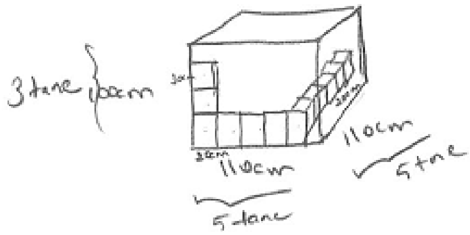
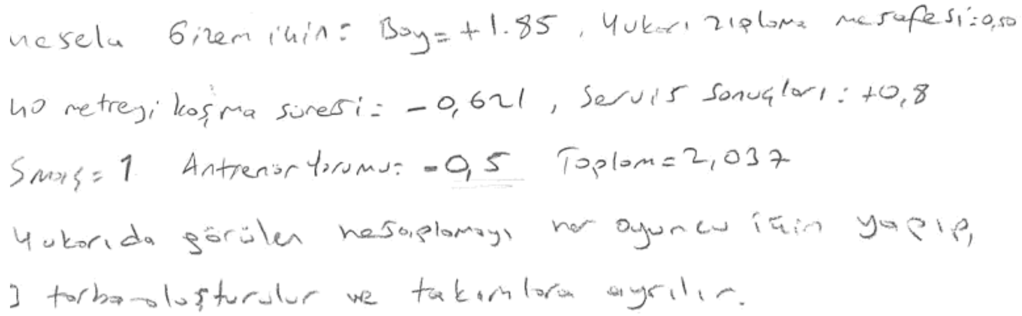


Figure 38 A part of the work of Group A on the “How to store the containers?” problem

The data analysis showed that they did not think about different arrangements. The fact that they used prisms instead of cylinders might hinder them to see the other types of arrangements.

For the “Let’s organize a volleyball tournament!” activity, PTA1 firstly decided to combine the scores of each player, thus get a final score for each player. While doing this, he made informal weighting (rescaling) to make the effects of the variables closer. He changed the values of the variables in an unsystematic way. He wrote on his solution paper that he would change the scores got from the try outs for the volleyball players. He exemplified this case on his solution paper just like in Figure 39 for the first player on the list.



(For instance, for Gizem = Height = 1.85, Vertical leap = 0.50, 40 meter dash = - 0.621, Serves Results = 0.8, Spike = 1, Coach's Comment = - 0.5 Total = 2. 037
Above calculations is done for every player and 3 bags is formed and teams are divided)

Figure 39 A part of PTA1’s work on the “Let’s organize a volleyball tournament!” problem

In the interview, he explained that although he changed the weighting to make the variables closed to each other, as he did not do it in a systematic way, he found his approach unreasonable and did not share this idea in the group work. It was a method that could have yielded a better model if they had thought about it to make in a systematic way.

R: Here, you found the score of serve as 0.8, how did you find it? (Let’s organize a volleyball tournament!)

PTA1: I decided to randomly select something, it was random [8 was divided by 10]
R: Did you make a guess for 0.8?
PTA1: yes so[smiling] the way already seems absurd. It cannot be applied, but I tried to provide values closer to each other…(silence).
...
R: Did you share this idea in your group work?
PTA1: No, because It did not sound logical for me, too; when PTA2 offered to list according to ranked scores it was more reasonable... I divided serve results by ten in order to make the values closer [smiling], some others can divide them by 20, there was no reason behind my operation, but there was for hers. At least, hers was more logical.

The videotaped observations showed that he thought about his approach for few minutes to do it in a systematic way, but then they began group work. In the group work, as he found this approach unreasonable, he did not share his idea with the other group members.

iii. Guessing about the unknown variables and relationships: PTs also made a guess about an unknown variable by comparing it with a known one. In the "Postman" activity, when comparing the "U method" and "cross road method", group A guessed that the postman would walk $2l$ (l represents the length of the street) distance for the cross road method plus walking distances for crossing the road. As the postman would walk on two sides of the street which is $2l$ for the U method, they guessed that the postman would also walk the same distance for the "cross road method" addition to the walking distances for crossing the road. The videotaped observations showed that, according to group A, cross road method always produced greater walking distance than the "U method".

PTA1: In every respect, he will walk more way for coming and going (referring cross road method) (Postman)
I: In every respect, do you say more?
PTA1: Right, he will walk more in all respects.

The analysis of their solution papers also supported this case. They wrote on their solution papers the corresponding mathematical expressions for these two ways as:
$2xt + z < 2xt + (2x-1)z$
where x was the number of houses, t was the time for delivering a post, and z was the time for crossing the road.

Similar situation was also the case for the other group. They thought that the cross road method would produce greater distance than the U method.

PTB1: We represented the length of the street by y, here is k. In any way, the postman will walk the 2y+k, but if he goes in a way of zig-zag, it will become (2y plus something times k) (Postman)
PTB3: He will cross the street 7 times, 7k
PTB1: Therefore, it is more distance.

Only after they were told to show the correctness of their answer, they realized that the postman would walk only one side of the street in cross road method instead of two sides.

PTB1: Accordingly, this man will have gone one street, namely, will have gone from just one side of the street
PTB3: OK, but he will have crossed the street several times (Postman)

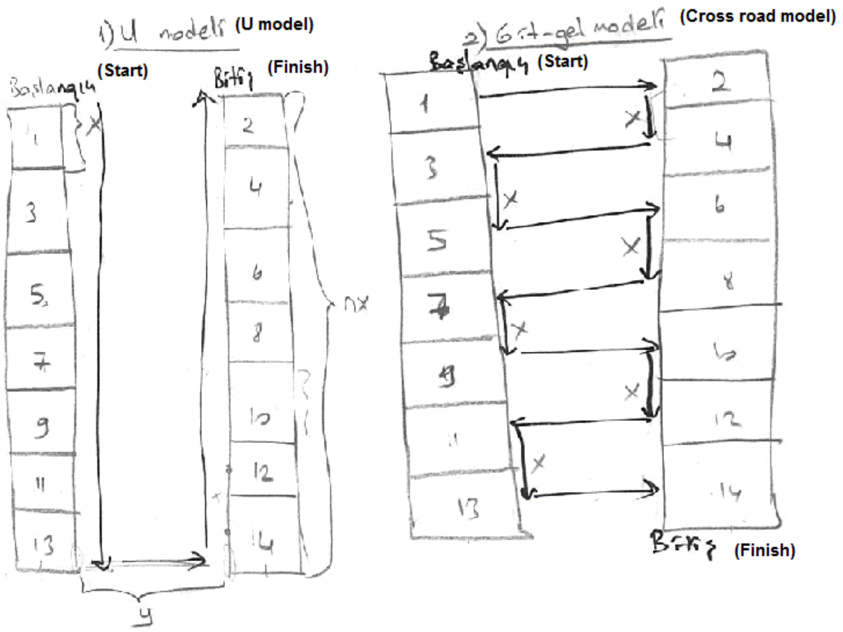


Figure 40 A part of Group B's work on the "Postman" problem (U model and cross road model)

PTs also made a guess about the number of passing the road. In a zig-zag way (see Figure 41) which they considered roughly as an alternative to the U method, PTA2 calculated the number of passing the road as $2n-1$ in mind and PTA1 agreed to her. However, they guessed that this calculation would be same also for cross road method. Thus, they took the number of passing the road as $2n-1$ for cross road method and made the related computations by using this value.

For the "How to store the containers?" activity, PTB2 made a conjecture based on her prior experience. As she had previous experience similar to the problem, without any calculation, PTB2 guessed that all the containers could be fit into the second storage unit by rearrangement. In the individual study, she could not find the correct mathematical concept that would help her to reach a solution.

While searching for mathematical concepts that bring them to the solution, PTs also made some guesses. PTB2 and PTB3 guessed that, for the second storage unit, the number of rows including five containers would be the same as the number of rows including four containers.

PTB2: To me, it seems that we will align four times fivefold rows and four times fourfold rows. (How to store the containers?)
PTB3: But, how can we show that?

PTB2: I am also thinking about it. Now we will do it. Because we all found how many we could fit into the storage in straight alignment.
PTB3: How can we find how this one will be placed there? How will we show the amount of space under the storage unit when we rearrange (compress)?Is it half and half?

While working on this problem, PTs in group B made a conjecture about a relationship in order to be able to progress on the solution. During the activity, they worked with the circle based objects with the diameter of 6 cm. As the problem asked for the case of containers 20 cm in diameter, they used direct proportional relationship between them. The videotaped group discussions showed that they guessed the type of the relationship as direct proportional relationship.

PTB1: Can we find such a proportional relationship? When the radius is 6, it was 0.8 at the second line, then what happens for 10? [carelessly they took radius of the circle instead of diameter] [using direct proportional relationship] (How to store the containers?)
PTB3: I am now constructing direct proportional relationship
PTB1: Here, does the direct proportional relationship worth?
PTB2: It worth.
..
PTB3: we are using direct proportional relationship, but we do not know how much it is correct.

As seen from the excerpt belonging to the group discussions above, they questioned the correctness of the type of the relationship, but went on without testing it.

iv. Ignoring some (important) variables to be able to reach a solution: In the solution process, to be able to easily obtain a result, in some cases, PTs ignored some of the variables. In the “Postman” activity, when they were told to show the correctness of their intuitive answer after finishing their work, they began to use mathematics. During the mathematization process, they tried to find an expression that gave the time required to deliver all posts. As they set the time for crossing the road in the first way (U method) as z which is for straight crossing, in the second way (zig-zag way), they faced with difficulty to find the time required to cross the road in terms of z, as it was changed into a zig-zag shape (see Figure 41). An excerpt from the class work was below.

PTA1: Yet, it is difficult to find how many times he will cross the other side, x (Postman)
PTA2: it is 2x-1. (trying for zig-zag way)
PTA1: it is a bit problematic because of being crosswise. Namely, z will become much more, but we may directly take it as z second. Such a way seems more logical.

The videotaped observations showed their drawings related to the zig-zag way. Because of the fact that they faced with difficulty, PTA1 offered to ignore the distance between the houses. An excerpt from the class work was below.

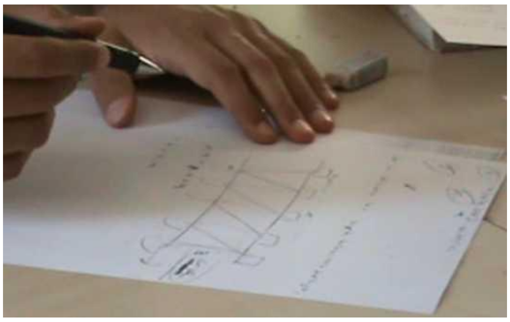


Figure 41 A snapshot of zig-zag drawing of PTA1 for the “Postman” problem

PTA1: Write one item here. Write that the distances among shops are equal and it can be ignored (Postman)
PTA2: Ok [smiling] it is becoming when we ignore it. Because I had problem with that, the problem has been solved thanks to ignorance, it has become then.
…
PTA2: Now it is fairly rational, it has become direct since we have ignored it.

They decided to ignore the relevant variable to be able to solve the problem mathematically easily. Thus, the zig-zag crossing turned into the shape shown in Figure 42 on the right side.

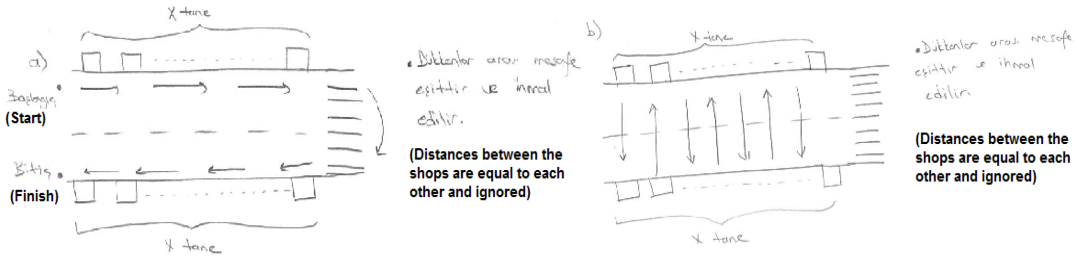


Figure 42 A part of work of Group A on “Postman” problem

An excerpt from the group discussions was below. During the presentations, he also explained that the time required to deliver all posts would be same for two paths, so they ignored the distance between the houses.

PTA1: We ignored the distances among shops. Since the distances will enter in the same way to the equations, there will not be any difference. (Postman)

PTA2: We did it [referring ignorance] in order to show with mathematical terms …. In order to defend the first way [referring the U method] we ignored. (during presentation, Postman)

During the presentations, a student realized a mistake of group B. This student explained that the postman needed not to walk to the end of the street because the mailboxes placed at the middle of the shops. Therefore, this student stated that the walking distance of the postman should be equal to $(n-1)\cdot x$ instead of $n\cdot x$ for cross road method. A part of group B's work related to the "Postman" activity was below.

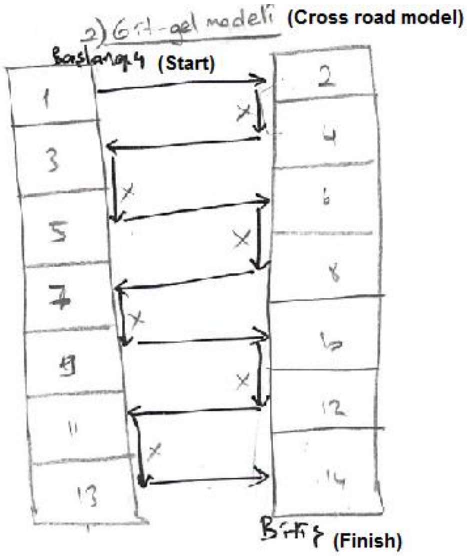


Figure 43 A part of Group B's work on the "Postman" problem

Although they did not consider this point in the group work, PTB1 explained that the amount could be ignored because of their assumption of nx>>y. An excerpt from the group presentations was below.

A student: The walking distance should be equal to (n-1) x. (Postman-during presentations)
R: Does he walk the whole street? Where do you locate the starting point?
PTB1: well, you mean here or there? (showing the at the very beginning of the street and showing the mid of the shops) it can be ignored because of the assumption of (pointing the written assumption nx>>y).

Similar situation occurred in the "How to store the containers?" activity. In group work, while sharing ideas, PTA2 offered to render the shape of the cylindrical containers as prisms to take into account the gaps between the containers. Although they needed to take into account the gaps between the containers due to the method they used, PTA1 offered ignoring the gaps between the containers. He explained that it would not cause significant change in the results as they made the same ignoring for all storage units. The excerpt below taken from the group work showed that he ignored the gaps as he thought it would be difficult to calculate the amount of gaps between the containers.

PTA2: … Why don't you take the gaps into account?(How to store the containers?)
PTA1: I ignore it.

PTA2: How can you do that?
PTA1: Well, what can I do, there is such a condition for every storage unit. You cannot calculate it?

In the interview, PTA1 also explained that they did not need to find exact numbers as they aimed to make only comparison among the storage units.

PTA1: I thought that since there would be the same amount of ignorance for all three [referring storage units], we could find correct result (How to store the containers?)
...
PTA1: I thought as. Here, my purpose was not to find a clear-cut result, but just compare them, so I ignored for all. In other words, the ignorance would be for each one.
R: However, while the amount of ignorance may be little for any one, the amount may be much for another one?
PTA1: That came into my mind for that moment. Well, I could not think much more
..
PTA1: I had stated that I ignored since it had been just a comparison. I thought that there might be some mistakes, but those would not affect the result.

Similar explanations were made for the "Let's organize a volleyball tournament!" activity. In "Let's organize a volleyball tournament!" activity, while considering the combining the scores for each player, PTB3 considered to add height of the player given in centimeter units and vertical leap given in meter units. In the interview, she explained that as she ignored the type of the measurements for all of the volleyball players, she thought that this would not create a difference for the result.

PTB3: Since I will follow the same procedures for every player, there will be more or less similar results (How to store the containers?)
R: Well, do you think that it is correct?
PTB3: To me, yes, because I want to establish something average. If I were to apply for just Aliye and not to do for others, ok…it would be irrational, but I am applying the same thing for everyone.
…
R: …When you stated a short time ago, you said that you would sum the numbers; does it seem correct for you?
PTB3: I still have the same idea. Ok, perhaps in all reason, the apples and pears will be summed, but I will find an average thing since I have done the same for all. Actually, it may be wrong, but I will have the wrong result of all.

Later in the same interview, while probing her ideas about the solution approach, she explained that they ignored the amount of differences between the scores of the players while ranking. She added that, without ignoring some points, they could not reach a mathematical conclusion, they generally needed to ignore some of the variables.

PTB3 : .. As you stated, for instance; we did not include the difference between 3rd and 4th players or else 4th and 5th players. What if we had included? I do not think that we will find a solution by this way, we should ignore some points. In mathematics, we should ignore something in order to reach another thing. We would ignore them. (How to store the containers?)

v. Working on the ideas of developed by other students: During the mathematization process, in some cases, PTs worked on the ideas developed by other PTs. In the “How to store the containers?” activity, PTA3 explained that she was influenced by other PTs’ ideas in her solution approach. The content analysis of the solution papers combined with her explanations during the interviews showed that she considered another approach at first; however, she was distracted by other PTs’ questions to the instructor. She stated that, as some of the PTs asked the value of pi, she thought they needed to use the pi value in their solutions. Thus, she jumped to another method to enable her to use pi value.

PTA3: … One more thing. At class, everybody was asking something about the pi value and asking whether to take pi value or not. From that point, I remembered the circumference. (How to store the containers?)

…

PTA3: .. When a word of pi was talked about at class, I decided to use pi number and moved toward the class

..

PTA3: In any way, there sometimes may be misleading information because of the sayings from not only environment but also friends; for example, if I had not heard the pi, I perhaps could have used r [referring radius].

Additionally, in the interview of the “Let’s organize a volleyball tournament!” activity, PTB2 explained that she worked on the idea developed by PTB3.

R: How did this idea come into being?[referring the transferring verbal data into the quantitative one] (Let’s organize a volleyball tournament!)
PTB2: PTA3 said, in fact I saw it from PTA3. [smiling]

Instead of these interview data, solution paper data analysis also showed that PTs were affected by each others’ ideas while working on the problems. Although PTs did not mention in the interviews, content analysis of their solution papers revealed some similarities in their solutions. For example, for the “Bus stop” activity, the analysis indicated that there was a similarity between the drawings of PTB1 and PTB2, and similarity between the drawings of PTA1 and PTA2.

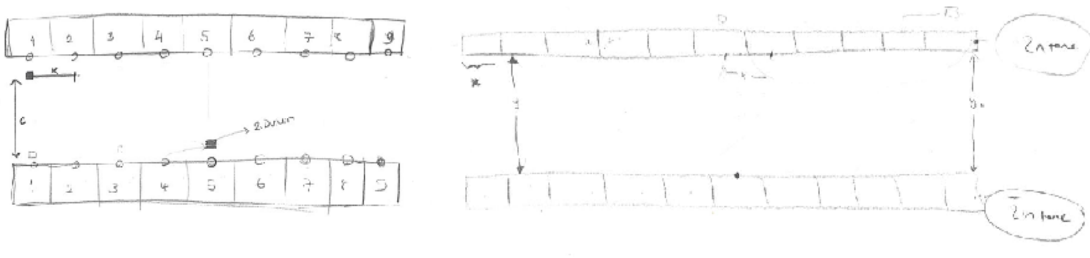


Figure 44 PTB1’s and PTB2's drawings (respectively) for the “Bus stop” problem

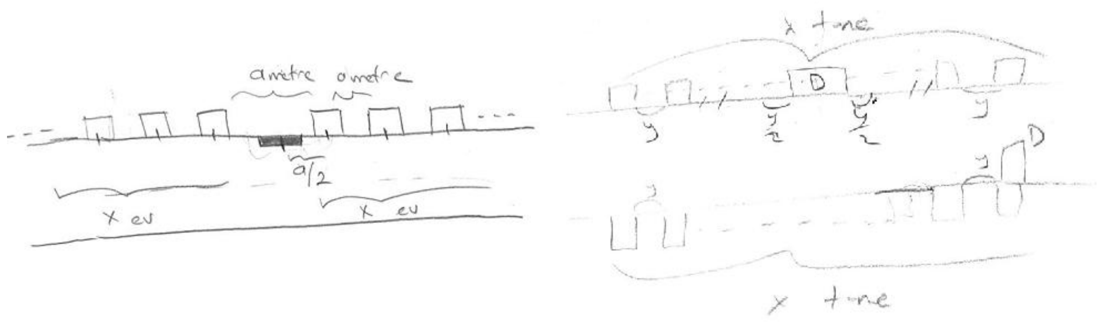


Figure 45 PTA1’s and PTA2’s drawings (respectively) for the “Bus stop” problem

vi. Re-examining the purpose: During the mathematization process in some cases, PTs needed to re-examine the purpose. For the “Postman” activity, while working on the problem together, PTA1 reflected on the strategy and checked for the aim by rereading the problem.

PTA1: Now, let’s decide whether it is by bicycle or on foot? [writing the second item of the conditions] (Postman)
PTA1:..[rereading the problem] it asks what kind of a way do you follow .But we do get into details. Anyway it does not matter.

In the interview, he explained that he realized his confusion about the aim while setting the conditions. Although he stated that he could not understand the problem, the content analysis of the solution papers combined with interview data showed that he actually understood the problem correctly, but while working on the problem, he somewhat deviated from the aim. The fact that he answered the problem intuitively as “the postman should follow the U method” showed that he understood the asked question indeed.

PTA1:.. First of all, I thought as daily life, how it happened in daily life, I did not make any connection with mathematics. Afterwards, I thought...I realized, at the very beginning, that I could not understand the question very well. Because we would not have any concern about which car was used. In fact, we should not consider them (Postman)
R: Why should we not consider?
PTA1: Because we would generate an equation, whether the man used bicycle or walked would not be important in this equation.

For the “Postman” activity, during the group work to be able to progress, PTB3 questioned their aim in the problem and the related criteria. An excerpt from the class work was below.

PTB3: Then, what is our purpose here? Do we benefit from the time or the way? Which one? (Postman)
PTB1: [Reading loudly the problem]
PTB3: The question does not give anything. That is all, there is nothing else.

This excerpt showed that they searched for cues in the problem text to guide them through the answer. These dialogs also showed that because they considered two criteria, the "time" needed to deliver all posts" and the "walking distance", in the planning stage, they needed to decide to one of them to be able to progress toward solution in the working stage.

In the interview of the "Postman" activity, PTB2 also explained that she questioned the aim of the problem towards the end of the work. She thought about what they were required to do in this open-ended nonroutine problem. An excerpt from the interview conducted with her was below. The data showed that the environment might be the determinant of how they would engage in the activity.

PTB2: ..in my opinion, the problem was very very easy. Namely, I was thinking about why they asked such a problem. (Postman)
R: then, was it easy?
PTB2: Actually, I could not understand what was required. Well, if it asks this, then the condition is so simple, but of course it does not ask it. In other words, I thought that they did not want such simple answer while they allocated such time period.

During the group work of the "Bus stop" activity, PTB1 questioned the aim of the problem and asked other group members with regard to their aim in those calculations. An excerpt about their discussion about the aim of the calculations is below.

PTB1: Now by writing this equation will we represent the gap in terms of n? (Bus stop)
PTB3: Now, we are looking for the evens, we will do the same for the case when the bus stop was at the beginning of the street.
PTB2: we do not need this?
PTB3: Will not we compare?
PTB2: We told teacher [referring researcher] that we would find formula, we have already found in our studies whether the bus stop is to be at the beginning or middle of the street.
PTB3: mm...You mean that we needn't show it here. Ok, we are now doing for the evens, for k=0.
PTB1: We will write the difference in terms of n and finish, don't we?
PTB2: No, what difference? We are just striving to make generalization, are not we?
PTB3: there is no difference, how can we show any difference between odd and even?
PTB2: we are not comparing this one (odd) with that one (even). We are only trying to adjust to both situations
...
PTB3: Now we made the model of the bus stop in the middle. Shall we make the bus stop located at the beginning model instead of at the middle model?
PTB2: we did it individually; after all, we started this one by accepting it as true
PTB3: but it is individual. Even so, I think let's ask it to the teacher [referring researcher]; to me they probably want us to do it.

For the "How to store the containers?", while working on the problem, PTB1 asked other group members to check for their aim. They discussed on the aim and re-checked their purpose.

PTB1: Here, we calculated the number of containers that could be fit into the storage, we did it for the first part of the problem, did not we? (How to store the containers?)

PTB3: no, it was for the second part. Because we all agreed that the third storage unit was more advantageous for the first part of the problem.
PTB1: this is the first case
PTB2: no, we are trying to say that the third storage unit is advantageous.
PTB3: uuu right.

For the same problem, while working on the problem, group A needed to reexamine the purpose to be able to progress. They reread the problem and thus reexamined the purpose. Accordingly, the group A realized that they were not required to use the same type of storage units for the rent.

PTA2: well, just a minute, is such a thing possible? Should we rent only the same kind of storages? It is now ok. [smiling] Come on now, our main purpose was to be able to fill all the storages. (How to store the containers?)
PTA1: Look!, the problem tells that is it suitable for the company to use the same kind of storages for that purpose? [reading the part of the problem] well then, you will not use the same ones?
PTA2: uuu [smiling] we had not read the problem. [smiling] At last, we realized it.

During the interview, PTA2 explained that they reexamined the purpose and thus realized the false perception of the problem.

PTA2: We read the problem again. Here, it asks that is it suitable for the company to use the same kind of storages? We could said something after we had read here, uuu we were asking this (How to store the containers?)

In the interview of the "How to store the containers?" activity, PTA2 also explained that she questioned the aim of the problem towards the end of the working. She reported that as she considered that she had reached the solution easily, she thought she had made a mistake.

PTA2: I easily reached this solution, so I said that I must have made a mistake here around (How to store the containers?)

For the "Let's organize a volleyball tournament!" activities, the videotaped group discussions showed that PTs lost track of their proceeding while sometimes working on the problem. For this problem, the analysis showed that PTA1 and PTA2 assigned different meanings to some part of the problem. An excerpt from their talks was below.

PTA2: He [referring to PTA1] says "do not give me to the worst team" [smiling] I laughed (Let's organize a volleyball tournament!)
PTA3: I think you seem not to understand it, PTA1 [smiling]
PTA2: PTA1 has not understood [smiling]
PTA2: we have been trying for a long time to divide the players into equal teams, PTA1 says " the worst" [smiling] but do you know that I also thought similarly at the beginning? It was as if the first team had been the best, the second one had been average, the third one had been the worst; however, all is equal.
PTA1: I thought that the third team was the worst one.
…
PTA1: They will give me the weakest team [smiling] there is no weak team, all are equal.

The solution paper analysis combined with interview data indicated that they actually understood the problem. However, while working on it, they could not keep in mind the final goal, and in some cases, differently interpreted the aim of the problem out of its actual meaning.

For the “Bus stop” problem, the analysis of the solution papers showed that PTA2 could not keep the aim of the problem in mind. Although she understood the problem correctly as seen by the set assumptions, she calculated the walking distance only for the student live on the far.

In the interview, she explained that she considered the most profitable (convenient) way for all students when to decide a shelter location, so she took into account the student live on the far.

PTA2: ... but I calculated the walking distance according to the student living in the farthest place but not to the total walking distance. I suppose, I thought that the student at the farthest place could walk the least while walking to the bus stop. Namely, all the students will walk less, but the student at the farthest will walk the least. (Bus stop)

In the “Who wants 500 billion?” activity, Group B re-examined the purpose when they did not reach a consensus on a part of their work.

PTB1: Since the question does not say anything about the eleventh question of the contest [rereading the problem], it says to discuss it with mathematical data. In my opinion, we have to accept all the alternatives equal (Who wants 500 billion?)
PTB2: to me, we have to accept the probability between 12,5% and 50%

vii. Using different computing strategies: In this stage of modelling, PTs made computations to reach a final result. While making computations, they used some strategies: performing computations in mind, using situation model, using specific known formulas, making simplifying assumptions, finding a pattern, and using technology. However, they made some computational mistakes.

Performing calculations in mind: In the “Postman” activity, in some cases, PTs performed the calculations in mind. While calculating the walking distance of the postman for the cross road method, they took it as $n.x$ instead of taking the distance as $(n-1)\cdot x$. They realized this point while presenting their results to the class.

In the “How to store the containers?” activity, as they performed mental calculations, they forgot adding 10 cm, and also did a mistake in the addition. Instead of adding $5\sqrt{3}$, she should have added 10 cm.

For the “Let’s organize a volleyball tournament!” activity, videotaped class observations showed that PTA1 also performed the calculations in mind and carried numbers incorrectly in the individual work.

Using situation model to be able to make the calculations easier: In the “How to store the containers?” activity, to compute the number of containers that could be fit into each storage unit, PTs used the situation model to be able to compute easily.

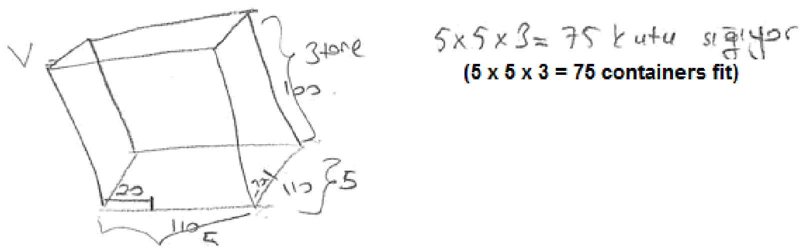


Figure 46 A part of PTB3’s work on the “How to store the containers?” problem

Using specific known mathematical formula to solve easily: For the “Bus stop” problem, they used the summation formula to make the calculations easier. For calculating the value of the summation of the numbers from 1 to 10, they used the formula of $\dfrac{n(n+1)}{2}$

Making simplifying assumptions to be able to compute easily: In the “Bus stop” activity, to make the calculations easier, PTB3 took the place of the mailboxes at the beginning of the houses. Although she made simplifying assumptions to solve easily, she took into account the width of the bus stop incorrectly while computing. Although she should have aggregated the distance of the bus stop for every house, she only added it into the equation once.

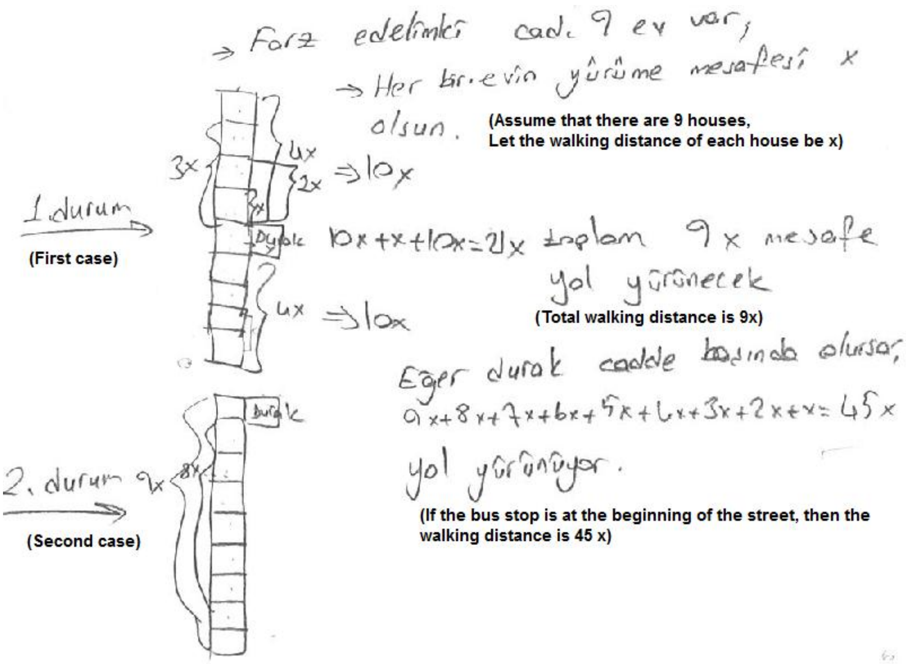


Figure 47 A part of PTB3’s work on the “Bus stop” problem

During the group work of the same activity, they again made some simplifying assumptions to make the calculations easier.

PTB2: For this house, he was walking the distance that was the amount of the half of the house; but to me, he should not walk half. We should calculate it by putting the bus stop here. (Bus stop)
PT3: ok, no need to tackle with the halves.

Finding a pattern: Related to the "How to store the containers?" activity, analysis of the videotaped classroom observations showed that to make the calculations easier PTB3 formed a pattern. She formed this pattern by using the values of the distance earned by rearrangement for two and three rows although they did not calculate the length gained by rearrangement for the fourth, and fifth rows,. Although the increment was 1.3 centimeter for each row, they took the increment as it was two-fold. Thus, they made overgeneralization by taking the pattern like that.

Figure 48 A part of group B's work on the "How to store the containers?" problem

Performing computations by using technology: For the "Let's organize a volleyball tournament!" activity, PTs decided to use excel as a tool to work on the problem and make related computations. PTA2 explained in the interview that she realized the need for using Excel for the computations when she saw the data given in the problem.

PTA2: After I had seen the data, I decided to certainly use excel. (Let's organize a volleyball tournament!)

However, the analysis of videotaped classroom observations showed their difficulties in using excel. When their excel sheets were analyzed, it was seen that they faced with difficulty in sorting the players according to their ranked scores.

PTB2: Please listen to me, this (Excel) does not take 50. (Let's organize a volleyball tournament!)
PTB3: no, it takes
PTB2: why then are the places of values is not changing?

PTB3: give it to PTB1 to do.
PTB3: no need, I can do it manually.
..
PTB3: … anyway, let's sum manually.

For the same problem, the analysis of the excel sheets showed that PTs did not convert all the numerical values into comparable units of measurement. In the problem, height of the players was given as meter, and the leap was given as centimeter. They did not attach the units of measures to the variables while entering the data into the Excel. Later in the activity, they did not convert the numerical values into comparable units. PTA3 attributed this mistake to the low concentration and did not fill the tables properly without putting the units of measures. It was seen from the excerpt below that, while working on the problem, they focused on the results, and they forgot what the variables represented while concentrating on the calculations. The analysis of the excel sheets also showed that they made the calculations by mind, instead of using the functions of excel.

PTA3: It seems that we all three could not pay attention to it at that moment. We missed out transforming centimeter into meter and reversing the running time for 40 meter from smallest to biggest as well. I accept that it missed out for this one [referring transforming centimeter into meter], but I think that is a concentration problem. Or we have such a deficiency; while writing the name of columns, we should add the units, for the players' height whether it is centimeter or meter. (Let's organize a volleyball tournament!)

c) Combining the expressions

To be able to combine the reached mathematical expressions and thus to be able to reach a final answer, they used several strategies. By combining them, they tried to make a comparison among the found mathematical expressions. This was done in three ways: by making assumptions, by using logical arguments, and by writing the expressions in a comparable form.

i. Combining the mathematical expressions by making assumptions: For the "Postman" problem, PTs tried to conjoin the algebraic expressions found for the "U method" and the cross road method. Group A conjoined these expressions by using inequality and concluded their solution by comparing the terms in those equations.

To be able to conclude, they tried to solve the reached model for a specific situation. They analyzed the inequality for cases $x = 1$ and $x > 1$.

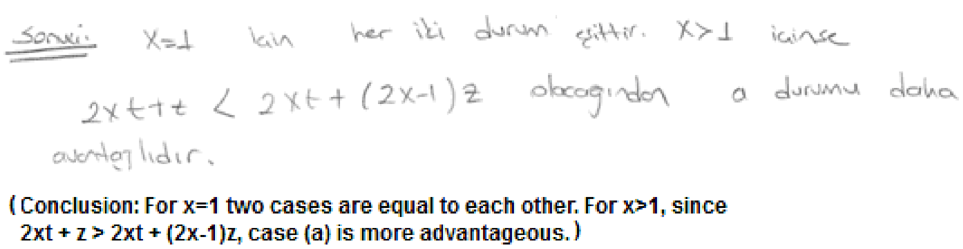


Figure 49 Group A's final answer to the "Postman" problem

In the interview, PTA2 explained that she made an assumption to be able to solve an inequality. The excerpt below from the interview exemplifies this situation.

PTA2:..It's as if the reasoning you mentioned didn't make sense to me. [referring solving inequality] (Postman)
R: Which one?
PTA2: By solving the inequality. I mean, if you were to give me such a question, I would do similarly, I think. I use cases for the situation; for example, if it becomes x>y
R: You say that you put this assumption at the beginning
PTA2: Right. For instance, we have x number of houses, here [referring the width of the street] is y meter. At first, I state that if it is x>y.
R: Well, you make an assumption at first, don't you?
PTA2: Of course, I make an assumption at the beginning, then if this side of the inequality becomes bigger, this side is more advantageous or vice versa.

As seen from the excerpt above, she explained that she firstly made a specific assumption instead of manipulating the equation or inequality to find the relationship among the x, t, and z.

Similar situation was also case for the other group. After reaching the mathematical expressions denoting the paths of the postman, they combined expressions by using inequality. As they believed that "U method" would produce shorter walking distance, they set the direction of inequality. Then, they tried to show the validity of the inequality below

$$2nx + y < nx + ny$$

where n is the number of shops, x is the width of the shops, and y is the width of the street. By eliminating the same terms in the equation, they reached the following inequality. Thus, they tried to show the following inequality

$$nx + y < ny$$

To be able to make this inequality true, they thought about the assumption that yielded it. PTB3 offered them making assumption of x was smaller than y and wrote the following inequality

$$x < y$$

An excerpt from their dialogs is below.

PTB2: Then, we have to show 7x [height] is greater than y [width], (showing the drawing) (Postman)
PTB3: Right, we will accept it. It will be the assumption.
...
PTB3: Guys, I have to show nx + y is smaller than ny. [all three are thinking individually]
PTB2: One of the assumptions is missing.
PTB3: Yes, we have to make a connection with x [the width of apartment] and y [the width of street]. Let's accept x is smaller than y ([writing x<y]
PTB2: that's exactly what I'm saying, ...

PTB1 explained in the interview that as he visualized a specific road in his mind after reading the problem, he made assumption based on that visualization while solving the problem.

R: Why did you consider x as greater than y? (Postman)
PTB1: That is because of the kind of the street that I cannot get rid of while thinking about the problem.
…
PTB1: ..Actually, our missing point may also be that… We did not solve the problem without putting one expression on this side of inequality and the other expression to the other side of inequality [referring solving the inequality]. Our mistake is probably to start solution by saying that this expression is bigger than that expression. Solving the inequality is just came to my mind.
…
PTB1: ….Since we solve the problem by thinking real-life context, we based the solution to the assumption.

As seen from the excerpts above, to be able to reach their intuitive answer set at the beginning, they also thought about the assumption that y (the width of the street) is much greater than the x (the width of the shop) to ensure the correctness of their intuitive answer. The data indicated that they thought that they needed to make an assumption to be able to solve the inequality.

ii. Combining the mathematical expressions by using logical arguments: PTs combined the mathematical expressions by making logical mathematical statements in the "Who wants 500 billion?" activity. In her individual study, PTA2 analyzed the options of risking 7500 for walking away with 16.000 TL, thus winning 8000 TL more, or not. She explained that, in case of using the lifeline(joker) reducing the number of choices from four to two, the probabilities of correctly or incorrectly answering the tenth question would be the same, %50, but the money that could be won would be greater than the money that could be lost (8000>7500). Therefore, she concluded that answering the tenth question by using the remaining lifeline (joker) would be logical for the contestant. In the group work, they also used this argument as their final answer.

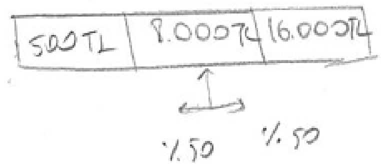


Figure 50 A part of PTA2's individual work on the "Who wants 500 billion?" problem

An excerpt from the group discussions was below. As seen from these dialogs, PTA1 also considered her approach logical and thus decided this solution as a group answer.

PTA1: When solving the problem with mathematical way, why will the answer change, why will the contestant continue? (Who wants 500 billion?)

PTA2: Because, in case of continuation you will win the right of continuation in addition to eight thousand, but if you cannot know the question on the other side, you will lose seven thousand and five hundred liras. The probability of them is equal, and then continuing seems more rational. I said so.
PTA3: I would take it and go away.
PTA1: It makes sense, write it down (to PTA2)
PTA1: ok, mathematically, we can say that it is more reasonable to continue the contest.

Similarly, in the interview, PTB3 also provided justification for her individual answer. The interviews conducted with PTB3 showed that PTB3 conceived the logical argument as legitimate problem solution.

PTB3: Either he will continue by winning sixteen thousand liras with a probability of 50 % or he will withdraw by taking eight thousand liras with a probability of 100 %. Now, if he withdraws by taking eight thousand, the probability of winning thirty thousand is 0 %; however, if he knows the sixteen thousand liras-question with a probability of 50 %, he will has a chance of winning thirty two thousand liras with a probability of 25 %. Therefore, I thought it was more advantageous for the contestant to continue the contest. (Who wants 500 billion?)

For this problem, while Group A answered the problem by using logical arguments, Group B faced with difficulty in combining the probability of correctly answering the problems and the corresponding amount of money won. They needed to consider the amount of money and the probability together, but they could not combine these two values. They tried to use the proportional relationship between the values. An excerpt from the group discussions showing their attempts to find a relationship between these values is below.

PTB3: How many times is the probability bigger than this, six? (Who wants 500 billion?)
....
PTB3: well, the earned money becomes fourfold, but is this one sixfold of that one? Eight fold. For probability, there is eightfold.
PTB1: the probability of winning is decreasing eightfold, but the earned money will be fourfold.

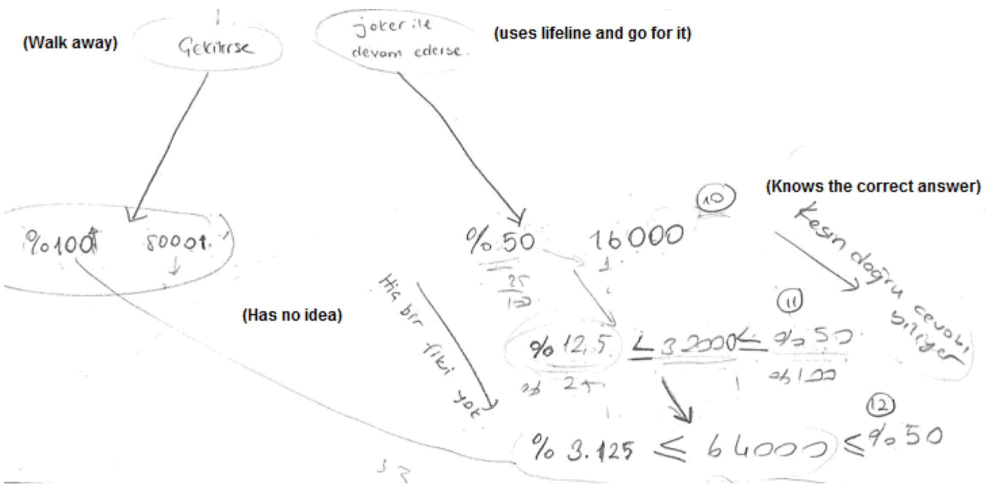


Figure 51 A part of group B’s work on the “Who wants 500 billion?” problem

In the interview, PTB2 explained that they mainly focused on the probabilities, and so they could not combine the amount of money earned and the corresponding probabilities.

PTB2: Here, we focused on probability too much. We ignored the increasing amount of the money; we always went on with the probability calculation. (Who wants 500 billion?)

iii**. Combining the mathematical expressions by writing them in a comparable form:** To be able to combine the mathematical expressions and so reach a final answer, PTs tried to write the expressions in a comparable form. For the "Bus stop" activity, after writing the mathematical expression for the bus station located at the middle of the street for the case of odd number of houses, they tried to liken the general term of the second sequence, which was for the even number of houses, to the first one.

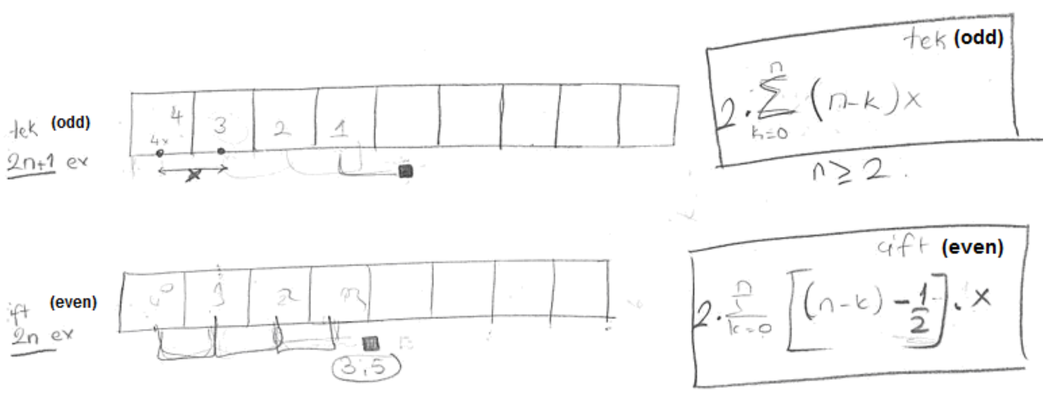


Figure 52 A part of Group B's work on the "Who wants 500 billion?" problem

Similarly, after forming the sequences, they tried to do the same for the bus stop located at the beginning of the street case. They tried to form the general terms of the sequences for shelter located at the beginning of the street by simulating the first sequences obtained for the shelter located at the middle of the street. In this problem, to be able to write the general terms of the sequences for odd and the even number of houses in a comparable form PTB1 deviated from the reality. He took the index of the sequence as infinity to be able to get rid of the term 1/2 appearing in Figure 52. An excerpt from the group discussions was below.

PTB2: yaa…it is becoming one over two (½). All in all, the student on this side will have to walk there, too (Bus stop)
PTB1: there will not be any half (½) in my total formula. If we approach the powers to infinity, there will not be any one over two (½) since it is an even number.

Group A also tried to form the equations in a comparable form for the "Bus stop" problem. The students formed the equations depicting the travelling distance of the students when bus stop located at the middle of the street as

$$2ax^2 - ax$$

PTA2 tried to write a mathematical expression for the shelter located at the beginning of the street so as to yield similar terms. The mathematical expression formed for the second case was written as

$$\left[\left(\frac{(x-1)x}{2}\right)a + (x-1)\frac{1}{2}a\right]2 = ax^2 - ax + a(x-1)$$

PTA2: Now, what will I do in order to be able to compare these cases? It is $2ax^2$ *minus [he is making distribution on the equations again since he could not end up with the desired cancelation; that is, since he will not be able to make comparison] (Bus stop-group work)*
PTA2: ...In both equations, there is ax.
…
PTA2: Ok, guys, there is something missing or awkward here (referring to expression). There is again an "a". Now it seems good for the comparison, both of these are equal, these are our ax (making necessary cancelations. What we will say; since it is $x^2 > x-1$*, the total walking distance of the first case is bigger than that of the second total case.*

In the equation above, instead of writing $x \cdot \frac{1}{2}a$, PTA2 wrote $(x-1)\frac{1}{2}a$.

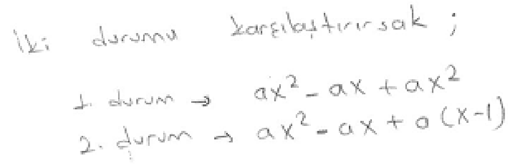


Figure 53 A part of Group A's work on the "Bus stop" problem

They used trial-error method to form the mathematical expressions in a comparable form. After deriving the general term of the first sequence, they tried to derive the second expression by using the trial-error method. An excerpt from their trials in the group work was below.

PTB3: I know that n is three, what should I get? ..is it ok if I say (n +k)/2?… hımmm. aa I found. (Bus stop)
PTB2: It should be just a division
PTB1: or, is it (2n +k)/2?
PTB3: there should also be minus one. Because you should put minus near it in order to meet for the first house. It meet all, but how will ½ be met?
PTB2: what did you find?
PTB3: (n+k)x - ½
PTB1: I think we should put something like two in front of k. Because, k should increase 2 by 2
PTB2: Why?

PTB1: Then, it becomes 4 /2 if k does not increase 2 by 2.
PTB3; Sure?..Let's try.

Afterwards, PTB2 realized the correct expression, while thinking on the problem for several minutes individually. After PTB2's explanations, PTs realized that they worked unnecessarily to derive the expression. The excerpt from videotaped group discussions below also supported that PTs tried to combine the mathematical expressions by writing them in a comparable form.

PTB2: What do you think why we cannot liken this to the first formula? (Bus stop)
PTB2: Maybe, we may control the first formula by this way? Why do not we liken to it?
PTB1: I will add + ½ to this. Because + half is always being walked, I mean ½ x
PTB1: Look, when we add ½ x to that formula we get the formula we first found.
PTB3: It is good idea. Let's try.
PTB3: Yesss, it becomes. There was no need, why did we strive so much? Take a look, it has been found.

4.1.4. Stage: Interpreting and Verifying

Interpreting means to evaluate the reasonableness of the solution in terms of the real-life situation and verifying means to check the appropriateness of the model for the original problem situation and the correctness of the computations. In general, PTs did not verify their solution ways. In some cases they interpreted their findings. They verified the reached solution by using some strategies only in some cases. To be able to verify the correctness of their answer, they interpreted it in real-life. They used some specific values of the variables to check the correctness of their solution. They tested the correctness of their solution approach in mind by simulating mathematical truths. In some cases, PTs also cross-checked their answers whether they satisfy their aim. Additionally, in some cases, they asked the authority to check the correctness of their method.

Table 8 Processes that PTs perform in the "interpreting and verifying" stage

Interpreting and Verifying Stage
A. No interpretation or verification
B. Interpretation/misinterpretation
C. Verification
a) Verifying the reached solution by interpreting in real-life.
b) Using some specific values of the variables or mathematical truths to check the correctness of their solution.
c) Cross checking whether the answer satisfies their aim
d) Asking authority to check the correctness of their model

A. No interpretation or verification

In general, PTs did not verify and interpret the correctness of their solution ways or of the computations both in individual work and in group discussions. In group work, although they tried to work on the problems together a PT writing the report, this is PTA2 in the group A and PTB3 in group B, generally conducted the computations, and other group members mostly did not check for the correctness. The interview data showed that they did not feel the need to interpret the reached results in some of the problems. In the interview conducted with PTB3 related to the "Bus stop" activity, she explained that she did not feel the need to interpret the found results.

R: well, you have found the walking distance as 20x and 16x (Bus stop)
PTB3: It is very normal because here is 2n, and there are 2n+1houses. In other words, the distance to be walked increases since the number of houses increases.
R: Did you think it at that moment?
PTB3: I did not think, but I did not feel any need of thinking about it because I had a purpose to compare. It was the comparison of two conditions when the bus stop was placed at the front and the middle of the street. However, I never thought about why this one was 20x and the other was 16x.

PTs' content analysis of the solution papers showed that, for the "Bus stop" activity, PTA2 did not write the correct mathematical explanation for the case of shelter located at the middle of the street. She needed to add an x into the equation. She did not check carefully whether the expression was constructed correctly or not. Similarly, PTB3 calculated the total travelling distance incorrectly, but could not realize her mistake as she did not check for the calculations.

For the "How to store the containers?" activity, when their individual solution papers were analyzed, it was seen that PTB3 calculated the (number of containers that could be fit into the each storage area/the number of containers) by giving the number of storage unit that would be rent for storage. However, she needed to divide the value of the denominator (the number of the containers) into the value of the numerator (the number of containers that could be fit into the each storage area) to find the number of storage unit. This indicated that she did not check for this.

PTB3: I have just noticed now that I would divide the number of cans I have into p in order to find the number of storages, but had done it reverse (How to store the containers?)

In the "How to store the containers?" activity, they did not check the reasonableness of their calculations. After doing the calculations, they found that they would earn extra 20.8 centimeter when they rearranged the containers that were 10 centimeter in diameter. This means that they can fit two more lines of containers. This also means that, by doing the same calculations and by using the proportion, they will reach 41.6 centimeter which is 4 times bigger than the needed. Although, at the beginning of the calculations, they argued that it would be hard to get extra line by rearranging, they found that they could fit extra four lines of containers with those calculations. They did not interpret the value found and thus did not check the pattern they found.

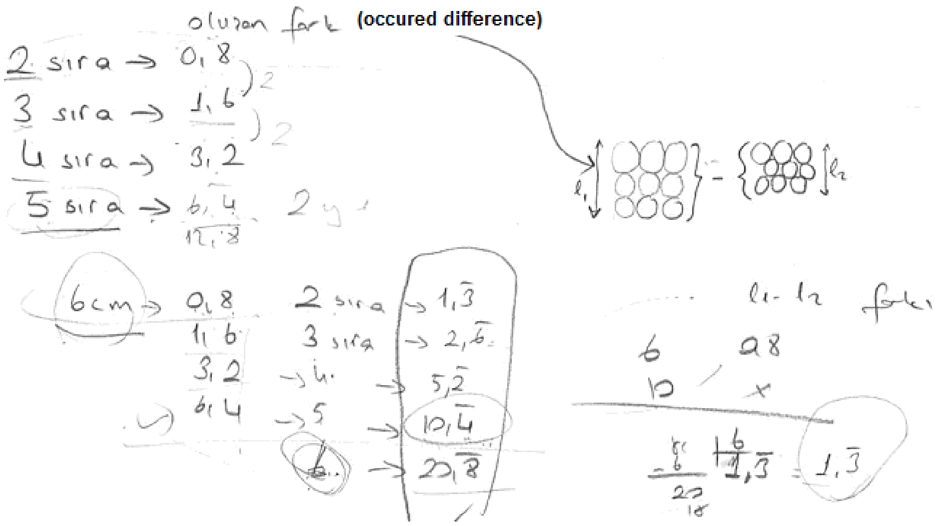


Figure 54 A part of group B’s work on the “How to store the containers?” problem

For the “Let’s organize a volleyball tournament!” activity, during the presentations, while presenting their solutions, PTB1 explained that they did not check for the calculations. The videotaped class observations combined with Excel sheet analysis showed that they did not check for the final score of the each player. When their Excel sheets analyzed it was seen that while aggregating the ranks of the players, instead of adding the values of the five columns, as expected because of the fact that there were five variables, they aggregated the values of the six columns. As seen from Figure 55, although they need to aggregate the values in columns A, D, G, J, M, they also aggregated with the column O. The values shown in the second figure also implied this result. Other group members did not check the correctness of the calculations.

PTB1: We did not control at that moment, but we cached later. After the final list was established, there might be some minor mistakes, but they are not the mistakes that will affect the course of our method (during the presentations of Let’s organize a volleyball tournament! activity)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1		Oyuncunun Adı	Oyuncunun		Oyuncunun	Yukarı Zıplama		Oyuncunun			Oyuncunun			Oyuncunun	
2			Boyu (m)		Adı	Mesafesi		Adı			Adı			Adı	
3						(cm)									
4	3	Aliye	1.78	1	Aliye	68.58	3	Aliye	6.01	4	Aliye	9	1	Aliye	4
5	13	Aslı	1.65	6	Aslı	60.96	6	Aslı	6.32	5	Aslı	9	18	Aslı	-4
6	11	Ayla	1.68	4	Ayla	63.5	14	Ayla	6.95	1	Ayla	10	14	Ayla	-2

16	Aliye
34	Candan
34	Gizem
42	Ayla

Figure 55 A part of work of Group B on Excel sheet for the “Let’s organize a volleyball tournament!” problem

For the same problem, “Let’s organize a volleyball tournament”, PTs developed a model including the ranking of the players according to each category and then by aggregating the ranked scores. However, they did not validate this model’s reasonableness. After devising a plan, PTs generally worked on it without questioning the details. An excerpt from the interviews and videotaped group presentations was below.

R: Did you test it in the end? (Let’s organize a volleyball tournament!)
PTA1: She said something during the presentations. She said that a team you build could have either all short ones or tall ones in your team. We did not think that.

A student: You have made them equal, but too short players may be included in one team (Let’s organize a volleyball tournament!)
PTA2: A player can be short, but he/she may compensate the shortness with good spikes.
A student: It should be considered after the team has been made.

For this problem, while working mathematically on the problem, they did not test the intermediate steps. While adding the ranked scores, PTA2 aggregated the centimeter and meter together. Videotaped classroom observations showed that other group members did not check for the calculations of PTA2 while doing them. They realized this mistake during the presentations of their models on the board. A student in the classroom realized this mistake and asked for it. The data below showed that they did not test for the units of measurement.

A student: Here, should it be two-meter [to PTA2, the presenter of the group A)]? (Let’s organize a volleyball tournament!-presentations)
[Other students became aware]
PTA1: [silently] we have done wrong. You have done 67 centimeter yaa [to PTA2]. You have added centimeter with meter. You have not transformed. It is wrong yaa.

After the presentation, they began to talk with the each other about this mistake. An excerpt from their dialogs was below.

PTA1: .. they did not like the centimeter issue [smiling] We did not transform centimeter into meter,…ya (Let’s organize a volleyball tournament!)
PTA2: what happened then?
PTA1: then, a guy with 1.80 height has an advantage of it.
PTA2: I could not understand that point while presenting, ooo what did we do.

The analysis of the videotaped classroom observations combined with interviews also revealed why PTs did not test their model. PTs either believed to the correctness of their model, or saw their solution as a proof. PTs did not verify the results as they believed in the correctness of their model. For example, for the “Postman” and the “Bus stop” activities, they strongly believed in the correctness of their intuitive answers. For the “Postman” problem, they believed that the most efficient way of the delivery would be the “U method”. They did not intent to consider different ways of delivery.

PTB3: Firstly, I thought of U method. The other way [referring cross road method] is impossible at all, but this [U method] is absolutely more advantageous, but we just tried that one [smiling] (Postman)

…

PTB3: First of all, I thought and said that he delivered the posts by crossing the street and also U, but the U method was more advantageous, draw the U method and neglect the other, I said.

Instructor: This group seems excited, they can present after yours. (Postman)
PTB1: We are not excited.
PTB3: We are sure about what we do [smiling]
Instructor: Sure? [smiling]

Similarly, as explained and exemplified in the planning stage, most of the PTs believed in the correctness of their intuitive models for the "Bus stop" activity.

PTA2: I was quite sure about my solution way that I did not feel the need for testing. (Bus stop)

During the presentation stage of the same activity, after seeing other PTs' approaches, group B evaluated their models indicating their belief to the correctness of their solution.

PTB1: We were challenged whether it was odd or even (Bus stop)
PTB3: We thought professionally.
PTB2: Ours is nice, isn't it?
PTB1: Of course it became nice.

In the interview, PTB3 also explained her ideas related to the correctness of her model.

PTB3: Well, I am quite sure that this model is the correct one, but I am also drawing it in order to show an evidence for the correctness of that… However, I did not think on it or try, but just said that it was apparent since the middle one [referring the location of the bus stop] was in any case the shortest one. (Bus stop)

While working on the "How to store the containers?" activity, PTA1 considered the model offered by PTA2 as successful. PTA1 focused on the method used by PTA2 and found it correct without questioning the computations. PTA1, PTA2, and PTB1 also explained in the interviews that they, in general, were sure about the correctness of their models.

PTA1: Ok I see. It is quite nice [referring the work of PTA2] (How to store the containers?- group discussions)

PTA1: It seems that we did not ponder enough, if we had thought adequately; it would have come to my mind. However, you cannot do if you are not sure. On the other hand, I was rather sure about what I did [referring the method]. (How to store the containers?)

PTA2: I don't know why we are so sure about the results we find (How to store the containers?)

PTB1: ..While working on that problem, it seemed as if we had done a good job [referring to the How to store the containers?], When we saw the solution from you, we understood that we missed a trivial point. (Let's organize a volleyball tournament!)

For the "Let's organize a volleyball tournament!" activity, PTA2 thought that all steps were done right, and therefore she did not feel the need for testing. She also added that after deciding the model and entering data into the Excel, she thought that they finished the work, and therefore did not pay attention to the intermediate steps and features of the variables such as the units of the measurements and the adjustment of the sort because higher scores did not indicate the better performance in some cases (e.g. run in seconds).

PTA2: Well, since I thought that I had followed the correct way, I went on. I never controlled. Everything seemed ok. (Let's organize a volleyball tournament!).

PTA2: When we sorted the data in Excel, then I felt that I finished my work. Afterwards, I decided to rank and give scores to players. I did not question much more. Because of that, I could not catch the point. (Let's organize a volleyball tournament!).

In the interview of the "Let's organize a volleyball tournament!" activity, PTA2 explained that other group members did not check out the computations, either, since they relied on each other's computations.

R: Other friends also missed this point? (Let's organize a volleyball tournament!)
PTA2: Maybe, they trusted me; they may have thought that I must have made right without any need of questioning it.
…
PTA2: They must have trusted my ideas. To me, if you treat something absolutely right, then you cannot see the simplest. In other words, when you look at something without questioning, you cannot catch the mistake.

R: Then, you trust your group friends (Postman)
PTB1: I speak as a member of group; for instance if one offers to use Pythagorean Theorem or something else, it seems clear that the person has already done it on his/her paper [referring individual work]. Because of that, we don't focus on that issue. Therefore, this makes our job easier since we did not question that approach any more.

For the same activity, PTA1 explained that he did not think that other group members would make computational mistakes.

PTA1: … at contests I am aware that how much less the finishing time is, it is better. I think that anyone can think such a thing (Let's organize a volleyball tournament!)
…
PTA1: ... but there are mistakes; for example, the units of vertical leaps are stated with centimeter
R: Did not you notice in Excel?
PTA1: No, I never looked at the values; I never considered that the values could be entered wrongly.

The videotaped class observations and interviews showed their belief in the correctness of their work. An excerpt from their dialogs and interview is below.

PTB1: After our presentation, stand in front of the class. You cannot stand with being courageous [smiling] (Let's organize a volleyball tournament!).

PTB3: They cannot criticize our solution since it is correct (Let's organize a volleyball tournament!).

R: Well, what do you think about the method in which standard deviation was used? (Let's organize a volleyball tournament!).
PTB3: maybe, the approach seems good since the differences between scores are taken into consideration. However; we still think that our model is more rational, useful, and correct. (Let's organize a volleyball tournament!).

For the "Who wants 500 billion?" activity, the interview data indicated that they considered their method as logical and had a belief in the correctness of their solution. They thought that it is impossible to make any mistake because the way of the solution is correct. An excerpt from the interviews conducted with the PTB1 and PTB3 is below.

R: Well, do you find your solution logical? Or do you have some questions in your mind ? (Who wants 500 billion?)
PTB3: There is a calculation mistake, when we passed from ninth problem to the tenth problem, we skipped a multiplication with ½, but our logic seems true. The most distinctive reason behind it was that we considered the condition of contestant's knowing the answer of the problem or not knowing it.

R: Ok, do you find all logical [referring to the solution way]? Or do you say that this point does not make sense completely? (Who wants 500 billion?)
PTB1: It had been logical until the presentation time yesterday.

In some cases, PTs considered their solution as a proof and therefore did not check out the correctness of their solution. For the "Postman" problem, PTA1 and PTA2 explained in the interviews that they offered the "cross road method" to prove the correctness of their intuitive answer.

PTA2:, when we looked at the problem, we said that the result was apparent. We thought that the postman would walk, then crossed the street, and then walked again to the starting point. We never thought such a different solution way, we just made it for justification; otherwise we had believed the U method since the beginning (Postman)
R: I see. Did you ever say why the teacher [referring researcher]asked "why not for a different way"? [smiling]
PTA2: Right [smiling] I thought. Maybe, the format of mathematical modeling is something like that. You know, I thought that you should prove certain things even if it was simple. Therefore, I proved.

R: well, when you draw these two shapes [referring U method and cross road method], did you say that this one might be correct or the shortest walking distance was the U method? (Postman)

PTA1: Since we believed the U method, we did it [referring cross road method]to prove. Our purpose was to prove.

For the "Bus stop" activity, PTs also considered their solution as a proof. After presenting their solution, the instructor asked whether there was a need to check out other points instead of the final point and the mid-point. PTA2 explained that those two points were enough to look for the points to give the minimum values.

I: You mean that we do not need to look at other points. Then, is this a proof ?(Bus stop)
PTA2: yes

PTA1: After we did these examples [referring the trial of the specific values], I decided to prove this mathematically (Bus stop)

These data indicated that, in some cases ("Postman" and "Bus stop" problems), as they believed strongly in their intuitive answer and thought that they proved their intuitive answer, they did not feel the need for testing or verifying.

When PTs' solution papers were analyzed, it was seen that there were not any computational mistakes in some parts of the solution. Although they did not describe their validation process, or mention the checking out some of the intermediate calculations, as all PTs correctly calculated the number of the containers that could be fit into the storage units for the "How to store the containers?" problem, it is probable that they might have done the checking in their mind.

B. Interpretation-Misinterpretation

PTs interpreted their findings in terms of real-world situation in some problems. Regarding the "How to store the containers?" activity the interview data showed that PTA1 and PTA2 interpreted the results in terms of the real-life counterparts.

PTA2: I divided by this number. This showed that the volume of the storage was bigger than the cans, I said (How to store the containers?)

PTA1: …here I found it 0,68, I found 0,45 for the third box. Here, I reached different results. I said that the cans might not fill the storage units. (How to store the containers?)

PTs sometimes misinterpreted the findings. In the interview conducted for the "How to store the containers?" activity, PTB3 interpreted the fractions in the results as gaps.

PTB3: When that fraction became, I should not include it, I think. It should be an integer, that fraction shows the spaces in the storage (How to store the containers?)
R: For example, assumed that it is 17.2?
PTB3: It means that 17 cans go into. Could we think that part of 0.2 as space?
R: Do you think we do so?
PTB3: It does not seem logical, but it is not too illogical to think so.

For the second part of the "Who wants 500 billion?" problem, PTB2 interpreted the solution found by other group members in terms of real-life. In the group work, she explained that they should not take into account the probability of correctly answering any of the questions as (1/4), as it represents the case in which the contestant did not know anything. Thus, PTB2 provided justification for the incorrectness of the other group members' answer.

PTB2: You decreased the contestant's probability of knowing the question to 1/4 in the eleventh problem, [PTB1 and PTB3 took the probability of each question of correctly answered by the contestant as 1/4]; namely, you behave like you do not know the issue towards the man. Did you understand? (Who wants 500 billion?)
PTB1: Because he does not know.
PTB2: No, the problem says that he does not know just for the tenth problem.

For the "Who wants 500 billion?" activity, in the interview PTA2 also gave the details of her thought about how she interpreted the results.

R: Have you ever thought that your result that is 1/2 to 30 is so big; I mean did you interpret it?
PTA2: I thought that this value was so low at that moment. Accordingly, I even thought that nobody could win five hundred-thousand (Who wants 500 billion?)

C. Verifying

a) Verifying the reached solution by interpreting it into real-life: PTs verified the reached solution by interpreting it into real-life. During the group work of the "Postman" activity, PTs compared the effectiveness of the two paths ("U method" and "cross road" method) in terms of real-life. Thus, they informally tested their answer and supposed the correctness. As seen from the below dialogs, their discussions based on the reasonableness of their answers' in real-life.

PTA1: .. Ya, it is always nonsense to cross the street. Why do you cross? (Postman)
PTA2: However, when these houses are so close to each other, using this way [referring zig zag way] seems more logical instead of using the U method?
PTA1: How do they become so close?
PTA3: First of all, you become confused to which shops I deliver and do not deliver the posts while crossing.

Similar interpretations were made by PTB1. He explained that using U method would result in the shorter walking distance in real-life.

PTB1: You deliver along the way. Does the job of going on for the next shop to deliver the posts take less time or do the job of crossing the street and giving the posts of the opposite shop take more time? of course, the former takes less time. (Postman)

After making related calculations for the "How to store the containers?" problem and answering the problem as the third storage unit, PTA1 verified this finding in terms of real-life context. The videotaped classroom observations showed that they did not question the possibility of the other solutions. They verify the reasonableness of their answer by considering the real-life situation, not using mathematical ways.

PTA1: It is always better to take something big. I mean the total (How to Store the Containers)

R: One more point, after you saw that the first storage unit seemed fine, you did not try second and third storage units. Is it because of cost? (How to Store the Containers)
PTA2: Right, because of that reason. I mean, I thought that 100 was the least of the renting cost. Thus, I never felt any need of looking for others.

For the "Let's organize a volleyball tournament!" activity, after aggregating the ranked scores of the players of each group to check the efficiency of their model, they interpreted the results. By interpreting them, they provided rationale for why their answer is reasonable. An excerpt from their dialogs in group work is below.

PTB1: 274 297 295. It did not become very good. (Let's organize a volleyball tournament!)
PTB3: Why did so many gaps become?
PTB2: First group became different
PTB3: Ha, because Aliye was so strong.
PTB2: 74?
PTB3: No problem, there can always be extreme scores.
PTB2: 274?
PTB3: 297 295
PTB2: To me, they are not so big differences because we deal with numbers like 200 and 300. If we think such an interval, it seems normal
PTB1: Then, because of Aliye.

b) Using some specific values of the variables to check the correctness of their solution/equation: For the "Postman" problem, after writing an inequality ($nx + y < ny$) under an assumption, PTs wanted to be sure about the correctness of their answer. PTB1 checked the equation by using specific values of the variables. An excerpt from their dialogs during the group work is below.

PTB3: Now I say that I am taking n multiple of y and n multiple of x. Is this correct? That y will not be, ok? Is $n.y$ certainly bigger than $n.x$ (Postman)
PTB1: Look, if you give values to them, it better visualizes in your mind. Give 100 for y, 1 for x, 5 for n; so this side becomes 500, and the other becomes 105. Why is this side big [referring right hand side of the inequality], because we took this (pointing out y>>x),if we take this assumption then the result becomes so [PTB1 is writing all these on the paper at the same time]

In the "Bus stop" activity, they tried different values of the index to check the correctness of the general term of the sequence they found the following

$$\sum_{k=0}^{n}(n-k)x .$$

After reaching the sequence, representing the total walking distance for odd number of houses and for the shelter located at the mid-point of the street as the following, (where x represents the distance between the houses and n represents the number of houses) group B checked the correctness for the values of k for 0, 1, 2, and 3.

They did the same testing for even number of houses and for the bus station located at the end point. A part of Group B's work on the "Bus stop" problem is below.

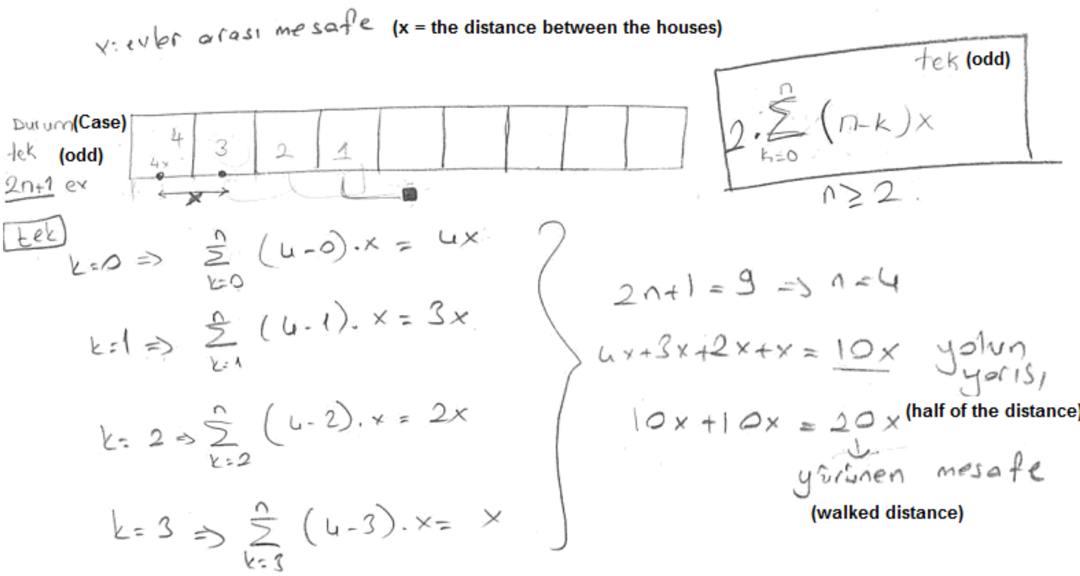


Figure 56 A part of work of Group B on the "Bus stop" problem

As seen from Figure 56, they checked their solution for first four terms of the sequence by plugging in 0, 1, 2, and 3 for k, respectively. In the interview, PTB3 explained this case.

PTB3: To become sure about the correctness of the solution we added up the terms placed in the summation function. (Bus stop)

Although group B checked the correctness of the general term by substituting each value of k between 0 and 3 for the expression $(n-k)x$, the analysis of the solution papers revealed that they did not check out the other values of the index k. When they create the expression in such a way, upper limit of the index of the summation should be $n-1$ instead of n. As seen from Figure 56, they represented the number of houses as $n=4$. However, they substitute for up to $k=3$.

Moreover, group B, found the sequence as $2\sum_{k=0}^{n}((n-k)-\frac{1}{2})x$ for the even number of houses when the shelter located at the mid-point. If they had substituted the value of k for n into the expression, $k=n$, they could have seen that the sequence would yield negative walking distance. This data proved that they did not test it carefully.

On the other hand, while checking the expressions for the values of k, PTB2 questioned the properties of the summation function to write it correctly.

PTB2: Now we are writing so, is this proper to write the sum symbol here? Otherwise, I remember something wrong? (Bus stop)
PTB1: Sorry?.

PTB2: We do not need to write sum since we work for k=0. What else, probably it does not matter.

All these data indicated that they checked the attained mathematical expression for $n = 4$ which was the special case of the explanation found. However, the data showed that sometimes they did not carefully check out the calculations or expressions.

In the same problem, “Bus stop”, PTA1 checked the correctness of his intuitive answer by solving simpler relevant problem. At the beginning of the group work, while sharing his ideas developed in the individual work, he tried to persuade other members about the correctness of his individual idea. In the interview, he also made similar explanations to explain his idea.

PTA1: It does not change at all, if you put the bus stop to the beginning of the street, one of the students will walk less while one of them will more. If you put it to the middle, both of them will walk normal. I mean 0 plus 5 equals 5; similarly 2.5 plus 2.5 equals five. (Bus stop)
PTA3: The sum becomes 600 for one while it becomes 1000 for the other
PTA1: However, it says that the walking distance of the student should be the least. The walking distance becomes equal; it will not change in sum.

In the “Bus stop” activity, PTA3 also asked other group members to check other possible points that could give the shortest walking distance. PTA2 explained the correctness of their answer by giving the example of a mathematical truth.

PTA3: Then, do we have to put the bus stop to the middle? Can not we find the shortest if we put it to another place? (Bus stop)
PTA2: PTA3, look ...I am thinking that the closer two numbers become to each other, the bigger their multiplication, or vice versa. Therefore, I think that comparing two conditions [Bus stop at the beginning of the street and middle] will yield the most accurate result

c) Checking/cross checking the findings: Interview data combined with the solution paper analysis showed that after finishing her work on the “How to store the containers?” activity, PTA2 thought that she might have made a mistake since she reached the solution easily. Then, she checked her solution and realized her mistake. She wrote her mistake on her solution paper as well.

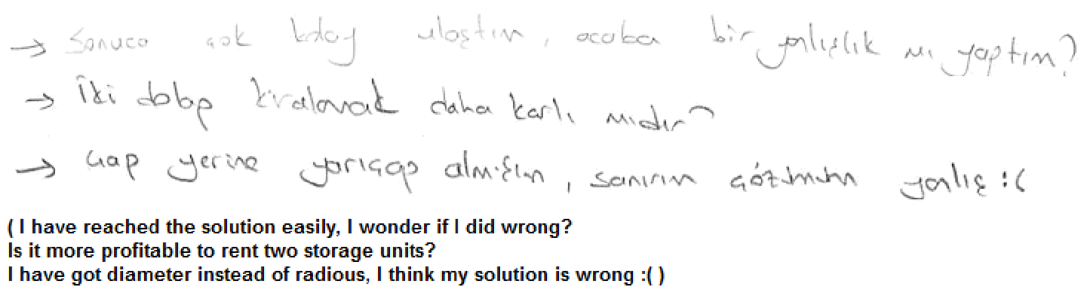


Figure 57 A part of PTA2’s work on “How to store the containers?” problem

For the “Let’s organize a volleyball tournament!” activity, as they were told to think on the solution approaches in individual work, PTs formed the teams in the group work. After forming each team, both groups tested whether they divided the players into fair teams. PTA1 offered to look for the total scores of each team. In order to achieve this, they aggregated the ranked scores of each player for three teams.

PTA3: However, not all the players have the same strength. Shall we sum and look. We can give the weakest one to you, PTA1. All in all, it will give different result. (How to store the containers?)
PTA1: Ok, let’s sum the scores and look.
PTA3: It will result in differently.
PTA1: But, should not it be that the sum of the scores became equal
PTA3: it cannot.
PTA1: Let’s see.

d) Asking the authority to check the correctness of the method/model: During or after their work, PTs expected the approval of the correctness of their solution in some cases from the instructor and in some other cases from the other group members.

PTA1: We have done, but are not sure that is it enough. (Postman-class observations)

PTB2: Have we thought so simple, otherwise should we do something more complex? (Bus stop-class observations)

PTA1: May I ask a question? In my first study [referring individual works], I had found the volume of cylinders, and then my friend said something. When we consider cylindrical boxes, there are air spaces; if we use rectangular prisms; do we take the spaces into consideration, don’t we? I mean, it is logical, isn’t it? (How to store the containers? class observations)

PTA2: Have we done the first question correct or wrong? [smiling] (How to store the containers?-class observations)

PTA2: Now, I want to ask this to the teacher [referring researcher] .(Let’s organize a volleyball tournament!- class observations)
PTA1: We want to ask.

PTB2: Hey, can one of you say this to the teacher [referring researcher] (Who wants 500 billion?- class observations)
PTB1: We will ask a question.
PTB2: It becomes eight fold; should we ask to the teacher [referring researcher] whether we are on the right way or not; otherwise, we find something nonsense that is not related with the result.

During the group work, the same PT generally wrote the group solution papers and did the mathematical computations. Other PTs, only in limited cases, checked the computations of the PT who wrote the group solution paper. However, while writing the report, PT asked other group members to check for the correctness of the equations or the solution methods. For instance, in group A PTA2 mainly wrote the group solution paper and thus made the computations. In general, she asked for other group members to check for the computations and also check for the reasonableness of the offered method, while working on the problem.

PTA2: Does my solution way seem logical, doesn't it? (Bus stop)
PTA1: Right, ok. [PTA2 began writing on the solution paper])
PTA2: Then, the total walking distance…. am I writing correctly; guys? Confirm me!
PTA1: Correct correct

PTA2: There should not be any mistake in idea… yaaa, (Let's organize a volleyball tournament!)
PTA3: How deep you think yaa,

In group B, PTB3 mostly wrote the report and performed the calculations. In the "How to store the containers? and "Let's organize a volleyball tournament!" activities which include more mathematical calculations, PTB1 tried to check the correctness of the calculations of PTB3. In the other problems they discussed and worked together on the problems. An excerpt from their dialogs in group work is below.

PTB3: Let's divide 80 by 6 (How to store the containers?)
PTB3: 10.333
PTB1: Why do we divide 80 by 6?
PTB3: We are making direct proportion, I will divide 8 by 6,
PTB1: 1.0333, no, 1.333, then 10. 333 no, result becomes 1.333, no it becomes 13.333.
PTB3: No, we will divide 8 by 6.
PTB1: When I have divided 80 by 6, I have just put the comma to this side
PTB3: Well, ok.

4.1.5. Stage: Presenting

Presenting means to describe the solution processes and the reasons behind them orally. After the group work, all student groups were asked to present their models on the blackboard. Presentations were about five minutes long for each group. While deciding the presentation order, only the groups who used different approaches were tried to be selected for the presentations in order to show the different approaches to all groups. Moreover, presentations took place from simple approaches to the complex ones. When two or more groups had similar approaches, to decrease the number of presentation of the similar approaches, the selection of the groups based on their wishes or the participation rates: the groups showed less participation in previous presentations or the discussions were especially selected for the presentations. To increase the PTs' participations to the discussions, all the groups were encouraged to present their models. The presenter of the group was decided by the group members. Throughout the presentations, PTs were encouraged not only to listen to the other groups' models but also to communicate and ask questions to each other. The students' presentations in front of the class gave clear information about their modelling process.

In general, PTs explained the notations and the assumptions used in their models. Then they presented their models by focusing on either the results or the procedures and the reasons. Table 9 summarizes this stage.

Table 9 Processes that PTs perform in the "presenting" stage

Presenting stage
A. Explaining the notations and assumptions
B. Presenting the solution by focusing on **a)** the results **b)** the procedures and the reasons

The presenters were selected according to participants' willingness. In general, same PTs presented their group's model. In most of the problems, for group A, PTA2 presented their models, and in group B, PTB3 presented their models.

Table 10 The presenters of the groups in each modelling activity

The number of the activity	The presenters of the groups Group A	Group B
Activity 1	PTA1	PTB1
Activity 2	PTA2	PTB3
Activity 3	-	PTB3
Activity 4	PTA2	PTB1
Activity 5	-	PTB3

Group A did not present their results for the "How to store the containers?" and "Who wants 500 billion?" activities, because their models were very similar to the groups presented their approaches before them. The excerpt below taken from videotaped observations exemplifies this type of situations.

R: PTA1, you can present (Who wants 500 billion?)
PTB2: They said similar things.
..
PTA1: Can we do part b of the problem? (Who wants 500 billion?)
R: First of all, is there anyone who thinks differently? [referring to the solution way of the group just finished their presentations]

A. Explaining the notations and assumptions

PTs usually began their presentations by drawing the situation model on their solution papers. They also put the mathematical symbols they used for the variables on their drawings. For the "Postman", both of the presenters (PTA1 and PTB1) firstly drew the situation model, and then mentioned the notations. For instance, PTA1 explained that x represented the number of houses and z represented the time needed to cross the street. Similarly, PTB1 explained the meanings of the symbols at the beginning of his presentation. For the "Postman" activity, while explaining the symbols they also stated the related assumptions. For instance, PTB1 explained that the number of shops where the postman should deliver the posts was equal for both sides of the street and represented by n. PTA1 also explained the assumptions they made during their work as written on their solution paper.

For the “Bus stop” problem, PTA2 draw the situation model including the mathematical notations as in the group solution paper. However, she neither mentioned about the assumptions nor the meanings of the symbols verbally. Similar to the PTA2, PTB3- presenter of the group B - stated the meanings of the notations only after a student asked about it. Contrary to PTA2, after drawing the situation model PTB3 mentioned about the assumptions made during their work. She explained that they took into account only one side of the street. She added that they regarded the width of the houses as equal to each other and the number of students that use the ring as one for each house.

Related to the “Let’s organize a volleyball tournament!” activity both of the groups’ presenters - PTA2 and PTB1- explained that they assumed each variable driven from the try-outs as in equal weight. For the solution of the “Let’s organize a volleyball tournament!” and “Who wants 500 billion?” problems, they did not use any notations. During the presentation of the “Who wants 500 billion?” activity, PTB3 did not mention about the assumptions. Although they assumed that the contestant would use the “50-50” lifeline to make the probability of correctly answering the tenth question 0.5, she mentioned it only after a student asked for that case.

B. Presenting the solution

PTs presented their solutions by focusing either on the result or the procedures and the reasons.

a) Presenting the solution by focusing on the results

The videotaped classroom observations showed that PTs gave result oriented presentations for the “Bus stop” and the “How to store the containers?” activities. For the “Bus stop” problem, after drawing the situation model as in their solution papers, PTA2 wrote on the board the final form of the mathematical expressions just like in their solution papers. After writing the mathematical expressions for each case (shelter located at the middle and at the end of the street), she concluded that the walking distance of the first case (U method) would be greater than the second case (cross road method) for every x, as $x^2 \succ x-1$, She only presented the result without explaining the operations or how they reached those mathematical expressions. Similarly, PTB3 -presenter of the group B- did not mention the computational procedures. She tried to keep the presentation simple. She presented a part of the solution which was produced for the case of even numbered of the houses. After drawing the situation model, she directly wrote the reached mathematical expression. Because of the fact that PTB3 focused to the results, she ignored using the related symbols. She did not write the summation function while comparing the general terms of the sequences of the two cases: “U method” and the cross road method. She added the sigma notation into the expressions after it was asked by the instructor. In general, the type of the presentation of PTA2 and PTB3 was very similar to each other.

In the “How to store the containers?” activity, after sketching the situation model roughly, she explained the attained solution very quickly without giving the details of the calculations. She did not explain how they reached those calculations. The relevant solution of the PTB3 is below. After sketching the situation roughly, she began her presentation with the following explanations.

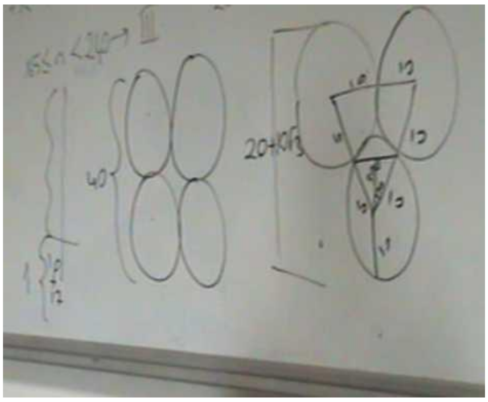


Figure 58 A snapshot from the presentation of PTB3 on "How to store the containers?" problem

PTB3: Here is becoming equilateral triangle [showing the triangle in the right hand side of the figure]. Then here, $5\sqrt{3}$ *comes from 30-60-90 triangle. Here is 10, this distance is [by looking at the solution paper] becoming* $20+10\sqrt{3}$. *We thought this alignment for two lines. If it becomes for 5 lines [by looking at the solution paper, this one is already 110, 10 increases. From that difference, 17 comes. When we add 17 here, another line occurs.*
R: How did 17 become?
PTB3: Here is not $20+10\sqrt{3}$ *, but it is* $40+25\sqrt{3}$.
A student who used the similar method: We found a different result. We did not find as $20+10\sqrt{3}$.
PTB3: $10\sqrt{3}$ *makes 17.* $20+17$ *becomes thirty seven. Even, it is 37.2. I did not write all here. If it becomes 27 cm, the diameter is 20 cm, and then I can add another line. Then, I can arrange extra six ones for each line.*
R: Did you take the width 110?
PTB3: We thought this for the second storage. Six from the first line, six from the second line, six from the third line; I can arrange 18 ones. I needed to arrange 10 ones. All of them easily fit into the storage unit. Therefore, the second storage becomes more advantageous.

As seen from the data given above, although she mentioned the main points of the procedure, it could be concluded that she focused on the result and did not give the details and the reasons of the procedures clearly.

b) Presenting the solution by focusing on the procedures and the reasons: During the presentations of the "Postman" activity, both groups explained the procedures and the computations in detail. After drawing the situation model and explaining the meanings of the notations and the assumptions, PTA1 explained how they found the mathematical expressions step by step. He explained the general approaches used, gave the details of the calculations/operations, and explained the reasons. An excerpt from his presentation is below.

PTA1: We stated that there was x shops here, x shops there [showing the other side of the street], we took the delivery period of posts as t. If we had calculated the crossing period as z second, then the period of giving posts to x number of shops equals xt here [showing the one side of the street], and it is x.t for there. Also, we stated the crossing period to there as z second and plus z, this is the first case. Of course, the distance between shops is ignored here.

Similarly, after explaining the assumptions, PTB1 explained the procedures by explaining the reasons and giving the details of the calculations. However, while combining the equations, after stating the assumption, $nx >> y$, to be able to compare the two situations he just concluded that the first method was logical without showing how the assumption was used in the inequality. Then, discussion was begun since other PTs could not understand how they came to that conclusion.

PTB1: If you write this in this way, n.x' es disappear. If we think of a street in daily life and we already assume that n.x is much bigger than y. If so, this method seems more sensible. Accordingly, the other way [showing cross road method] does not work.

While working in group, they talked about the assumption $y >> x$, but while presenting, PTB1 changed this assumption and wrote on the board as $nx > y$. Because of this change, he could not make the comparison clearly.

After the "Let's organize a volleyball tournament!" activity, PTA2 presented the procedures step by step. She also gave the details and mentioned the reasons. She presented the results in a clearly outlined fashion.

An excerpt from her presentation is below.

PTA2: We thought that it would be a bit nonsense to conceive differently the height of the players and their vertical leap because each player leap differently and each of them has different height. For that reason we took into consideration the total vertical leap distance, and we found it by adding these two values. Then, we wrote down the players' 40 meter dash, their successful serves and spikes. Additionally, we transferred them to an excel table and rank them from the highest one to the lowest one.

…

PTA2: Then, starting from the lowest one we gave points as 1,2, 3, 4. The lowest one got 1 and the highest one got 18, but if two of the players had same scores, they got the same points. So, in some categories the points reached to 17 not 18. Each player got his own score by this way. After we added the points, we got the players who had same scores, for example three players with the score of 24 and 2 players with 25. As their scores became same, we applied to the coach's comments, for example, we looked at the coach's comment for İpek and Seda.

From the other group, PTB1 tried to give the details of their procedures to solve the problem. He began his presentation by telling that they entered the given data into the Excel. Afterwards, he continued his presentation with the explanations below.

PTB1: These five qualities of the students were taken into consideration as equal importance. For instance, if we regard it over 100%, each of them would be 20%. I mean, before beginning our solution we thought that they have similar features. Then, we sorted them from the smallest one to the biggest one among themselves. We ranked the 40 meter running duration of a player reversely as it was inverse proportion. Apart from that, we gave values to the spike results such as 1, 0 and -1. We arranged them in the way that changes the course of the game (explaining how they gave these scores). We gave values to each list from 1 to 18, so we got scores ranging from 16 to 64. Then, we ranged that list in itself, and divided the players into teams by transverse method like the first group did. Lastly, we added the scores of the teams and got 274, 297, and 295. When we considered the general range, the difference, which was 74 versus 95 between the teams, was normal. You can say that one of the players is very tall, and the tree players who are higher than others can be in any team, of course it's possible. However, as we thought of 20%, here it provides such a benefit for us. After forming the teams, we just looked at the comments of the coaches.

A student: What did you say for the players who got the same scores? For example, there are three players who got 1 at the last row. How did you regard them?

PTB1: We did it according to their name. [silence]

R: The one whose name comes before gets nine and the one after it gets ten.

PTB1: Right.

PTA1: We had checked the coaches' comments and when the players had got same scores, we brought the player whose comment was positive to forward.

Although he tried to give the details of the procedures while explaining the ranking procedure, he did not explain that they ordered the players according to their names when they got the same scores for different players. Only after it was asked by a student, he said that they did it in accordance with the names of the players without explaining the details. In the interview, he also mentioned the ordering only after it was asked. He also wanted to switch to another point without explaining the details of this process. Although he tried to give the main steps of their approach, he did not mention about the type of the ordering. This data combined with the videotaped observations revealed that he did not want to explain that point. The questions asked by the researcher were answered directly but not confidently, which suggested that he was not satisfied with their answer.

R: You ranked the students from 1 to 18 for each category. What did you say for the players who got the same score? According to name order?

PTB1: Right.

R: Well, could you do a better ranking? For example, it's said in the presentation that there were players with the height of 1.69, 1.70, 1.80, but these students got points like 16, 17, 18. Have you thought about that?

PTB1: No, we haven't.

In the presentation related to the case of the three of the taller players' appearance in one group, his explanations suggested that he considered the other students' questions to the presenters in the previous presentations. Although they did not consider this case in the group work, he mentioned this case after it was asked by a student in the presentation of the previous groups. Thus, he offered different explanations rather than the group work.

For the "Who wants 500 billion?" activity, PTB3 firstly drew the situation on the board and wrote the related calculations as it can be seen in Figure 59. The videotaped observations showed that PTB3 explained the procedures and related calculations step by step. The excerpt below from her presentation exemplifies how they used their method.

> *PTB3: We proceeded from ninth question to tenth question. We asked about what would happen if the contestant knew the problem. For the case where he didn't know the problem he got 4000 TL, we multiplied the probability and money. For the case where he knew it we multiplied 16.000 and ½, and he got 8000. If the contestant would take 8000 and leave the contest, multiplication of 8000 and 1, it's again 8000. For 100% probability, 1 is coming from there. Then, we supposed that he would know the tenth question, we wanted to see what would be happen for the eleventh question, when he know the answer and not know. When he didn't know the answer, he would get 16.000, it was fixed. What would happen if he knew? We regarded the probability as ½ instead of 1/4, 3/4….*

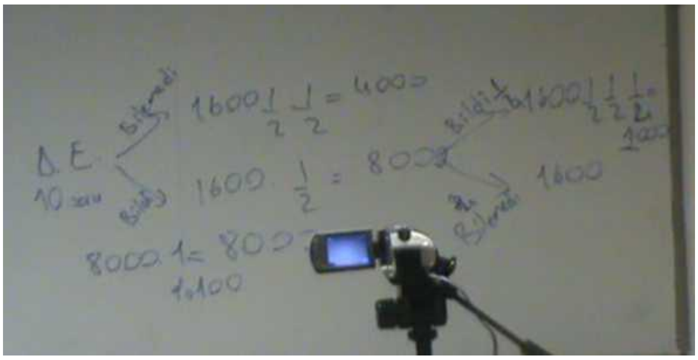


Figure 59 A snapshot of the presentation of PTB3 towards the end of the "Who wants 500 billion?" problem

4.2. PTs' Views about the Factors Affecting Their Modelling Processes

In the previous section, taking the purpose of the study into account, the results of the data analysis related to the PTs' modelling processes in each stage of modelling were described. In this section, PTs' own explanations in the interviews, which directly or indirectly refer to the factors affecting their modelling process are described under five headings:

- Lack of experience with modelling activities
- Lack of conceptual understanding of mathematics
- Focusing on reaching a result
- Difficulty to link mathematics and real-life
- Disorganized/unsystematic writing/work
- Time limitations

While presenting the results, the data came from PTs' solution papers, audio and video recorded observations, and information forms used to support the categories.

4.2.1. Lack of experience with modelling activities

All of the PTs stated in the interviews that the lack of experience with modelling activities was a significant factor on the developing models in the activities. In general, they stated that modelling activities were not carried out in their undergraduate courses or before. This situation is also supported by their explanations in the information forms handed in the first week of the semester to get some demographic information from participants.

In the interview of the "Postman" activity, PTA1 expressed that as he did not know the mathematical modelling, he attempted solving the problem by thinking the situation in real-life without any mathematical work.

PTA1: When I first read the problem, I thought that as I didn't know mathematical modelling, I deal with the problem by thinking the difficulties that the postman had. I thought it something in real life. I didn't relate it with mathematics (Postman)

Related to the same problem, PTB1 explained that the reason of their difficulty was the lack of experience with the real-life problems. He stated that they did not solve these kinds of problems or receive training on these problems in their previous education.

PTB1: Neither in our education nor in the exams, I didn't see any problem that asked for the real life experience, at least in my years of education. (Postman)

…

PTB1: As we moved away from the mathematics that we were taught in primary school and high school, and were asked for relating the problem with real life, we got confused.

In the interview of the "How to store the containers?" activity, PTA1, PTA2, PTA3, and PTB3 explained that they did not use to solve the problems requiring them to consider alternative ways. PTA1 reported that they used to solve problems that enabled them to reach an answer immediately and out of touch with real-life. Similarly, PTA2 expressed her conceptions about the problem and problem solving. She explained that they used to solve problems that situated in a structured context like the identical alignment of the containers instead of contexts requiring them to consider rearrangement to put more containers. She explained her conceptions about problem solving. An excerpt from the interviews conducted with her is below.

PTA1: …when we were at school, it was the same. We didn't think about the different way for the solution. For example we didn't think that the containers should be set into order; we would find the result right away. Rearranging them is really a far away thought for us. As our system of thought were built up in this way, setting the containers tightly seems us something that shouldn't be done. (How to store the containers?)

PTA2 also talked about the role of the educational system in the interview of the "Let's organize a volleyball tournament!" activity. Giving the "How to store the containers?" problem as an example, she explained that they were accustomed to solving problems that were constructed in such a way that they only needed to consider standard situations. She added that they were not asked to think about the problems in terms of real-life in the exams.

R: Have you ever thought that if we rearrange the containers, we can use the space on the corner of the containers better? (in the interview of the Let's organize a volleyball tournament!)
PTA2: No, we haven't. Because we thought that the best way is setting that. We didn't develop a different point of view ... We didn't think about rearranging it really. Surely, we really think formally. We think that the boxes should be fit only in one way.
R: What do you think about the reason?
PTA2: I think it's because of the sense of OSS. Because I consider that I shouldn't add any personal contribution in my solving. While solving the problem, I thought that the containers should be set in a neat way. In real life, if I were required to fill a box, most probably I would compress them. We didn't see any decimal point even in OSS, they gave integers. For example not 99, but 100. For that reason, we always expect to see round numbers. We never think that if it would be in real life, I would do in other way, so I would apply it here. I have never thought so.

PTA : It exactly derives from our educational system because we haven't question anything up to now. For instance, we didn't see any decimal number even in the options of the problem. I don't know how to express, but when I get the result, and if it has fraction, I thought that it is surely wrong. [smiling] So, it's really hard to break taboos. (Who wants 500 billion?)

PTA3: I think, everything arises from the lack of modelling lessons. I couldn't find any other reason because we always see the problems in a routinized way. I can't say it's because of my laziness since we are given instruction about these concepts over years. I don't learn probability in just a year; I always come it across in some way, but not in this way [referring to the real-life context]. I think, it's wrong over there. Maybe I also have it. I couldn't focus on it, while solving the problem. I didn't teach to think like that, either. As it wasn't taught, it's the fault of the system. Maybe, if I were in America, I could do it [smiling] (Who wants 500 billion?)
..
PTA3: .. And one more, when you mention the word mathematics in our system, the only thing that comes to your mind is numbers and operations; it's like there is no relationship with real life.

PTB3 reported in the interview of the same problem that they were taught to reach an answer immediately while solving problems in their school life. Therefore, she explained that thinking on the other solution ways was somewhat unusual for them. Similarly, PTA1 expressed that because of the fact that they were always assessed by tests in which they were expected to reach an answer immediately, they developed a habit of solving such problems in short time. In a similar vein, PTA2 explained that they did not use to consider alternative solution ways after deciding a solution way. Additionally, she expressed that they did not test the correctness of the solution if they got an integer number. She also added the role of experience in the modelling activities on this process.

PTB3: ... one more the education we have had until now influences the way of solution. As we came from ÖSS system, I think about the shortest way to reach the result. We were accustomed to solving problem in a practical way. When you know the practical way, you can't deal with deciding these things then making equation. I know that from myself. (How to store the containers?)

PTA1: We get used to tests. This is results from the habit of finding the result immediately and writing it down as soon as possible (Let's organize a volleyball tournament!)

PTA2: You said that analyze the questions with a different perspective, but when we deciding on the way of its solution, we rush into solving it. Then, if we get an exact number, we really get relax. We believe that we did it correctly (Who wants 500 billion?)

4.2.2. Lack of conceptual understanding of mathematics

In general, PTs tried to answer the questions intuitively or using basic mathematical concepts they were familiar with, otherwise they had difficulty in that. In the interview, PTA1 explained that they mostly did not conceptualize the mathematical idea behind the problems.

PTA1: You know, they say for the problem that it's really easy, it has no contribution to us. It's because of the fact that we couldn't understand the basic aim of the problem [referring the mathematical idea behind]. If we conceive that, we will see how beautiful concepts the problem includes, but when you don't understand, you can't see. (Who wants 500 billion?)

In the interviews, PTs mentioned their lack of conceptual understanding as a factor affecting their modelling process. They explained their difficulty in finding the correct mathematical concept to use in the problem. They attributed this difficulty to the lack of conceptual understanding about some mathematical concepts. In the interview of the "How to store the containers?" activity, PTA3 reported that in general speaking they mainly knew so many mathematical concepts, but they had difficulty in selecting the correct mathematical tool to solve these kinds of problems with no evident mathematical character. Similarly, PTB2 explained their difficulty in finding the mathematical concepts to use in the problem.

PTA3: I noticed that I have related knowledge, but I have difficulty in using that knowledge. (How to store the containers?)
…
PTA3: I know many many things as a concept or a formula, but I don't know how to use them because I have never confronted such a thing.

PTA3:.. you could have learnt function while preparing for an exam, and after learning that it is seen in the exams. In that case you have already known that you would use function concept in the problems. However, in the final exams you could think about what to use for the solution. Is it function or fraction? As the subjects increased, you could get confused. These activities involve that; I can use everything in such problems (Who wants 500 billion?)
…
R: Why could not you make a connection between the probability of getting the money and the money you would get? What is the reason of that?
PTA3: We have that knowledge, but we don't know how to use it,

PTB2: If it were asked after we just learned the concept in the lesson, it would be easier. You can do it at that time, but here [referring modelling problems] it's difficult to decide which one to apply. [referring the mathematical concept use] (Who wants 500 billion?)

PTs also explained their difficulty in realizing the relationships between the mathematical concepts. Generally speaking, PTA1 stated that they memorized the concepts without understanding the links between them. He explained that they could not achieve conceptual understanding.

PTA1: If I had seen such a problem in a test, I would have multiplied these two [referring money and probability] to see what was the result; but I couldn't know why I did this multiplication. For example the last presenter group multiplied it, but they couldn't explain why they did so. The reason is that we learned the subjects separately; we can't make any connection among them. (Who wants 500 billion?)
R: uh huh
PTA1: one more, something has been memorized, how? I think it's because of extreme memorization such as the center of gravity, multiplication, division. We didn't know why but conceptual understanding hasn't been developed yet.
…
PTA1: .. additionally, not thinking about the relation among the subjects can be another reason. As you know, the more relationship is created between the major idea, and following ideas the more the concept will be understood. I think the relationship we have is not much.

In the interview, PTB3 also explained that lack of conceptual understanding of some mathematical concepts was a barrier in their solution process.

PTB3: That is a problem which can be solved with arithmetic mean that we already know. I'm not sure but I think I learnt this subject in sixth or seventh grade, but I didn't realize that it is used it in this way. (Who wants 500 billion?)
…
PTB3: I can't think that I have to use it. [referring weighted averages]. I have never seen that the subject in mathematics are so related to each other like one within the other. (Who wants 500 billion?)

4.2.3. Focusing on reaching a result

The interview responses showed that some of the PTs did not consider developing their models because they focused on getting a result. PTB3 explained in the interviews that they did not want to consider the alternative ways to get a better approach to the problem as they wanted to reach a result. She expressed that working on alternative answers instead of reaching a quick solution is a waste of time. She added that she was result-oriented and also explained that while tutoring, she was teaching her students by showing the practical solution ways since they were assessed by the tests requiring them to solve the problems immediately.

PTB3: While solving the problem, we should focus on how to solve it rather than the result. However, it's not so easy; it's not something to be learnt at once. Not reaching to result at once seems very strange to us. Dealing with other things sounds very nonsense. I seek for the direct result. Instead of thinking about them, I think about leaving much time to solve the problem, get the result. I'm asking myself that why I'm dealing with such things. (How to store the containers?)

PTB3: If we take into consideration such a lot of thought, relations, and such things, we get stuck. The reason behind it is that I learn really late that I need to think, make connections, and also many of the subjects in mathematics are related with each other. We are the people interested in mathematics since a certain age and think ourselves as if we were mathematician. However, we didn't think of it. I think it's our upbringing style. We haven't been growth to think of something. In one of the mathematics exams at high school, the reasoning that I wrote for the problem is *really true, but as I did calculation error in somewhere, I got 0. So from now on, I haven't pay attention to solution, I think that let me go to the result, it doesn't matter how. (Who wants 500 billion?)*

…

PTB3: The important point is to decide the method. However, we only dealt with the result, they did not teach how to make the decision.

…

PTB3: …as I said before, thinking so is a cornerstone that should be acquired basically. However, that thought got deep-seated into us, and it's difficult to break such habits. I also teach result-oriented way to my SBS students. I tell the practical points.

In the class discussions, a student explained that they focused on reaching a result, and thus they did not want to engage more on the problem after finding a result. After these explanations, PTA1 and some other PTs also agreed with her. Another student in the class added that the problems asked in the exams during their education mostly did not require them to think on the alternative ways, besides they required them to focus on getting a result. PTA1 explained that similar state was also case for them and added that they were not used to "think" on the problems.

R: For instance, the duration of running was not intended to be reversed in the solution. What do you think about its reason? (at the beginning of the lesson of the Who wants 500 billion?-class observations)

A student: When we get some of the solution, it serves our purpose not to think anymore [referring to the solution approach]. I mean we directly focus on the solution, and we don't question the correctness whether we did it right or wrong.

PTA1: Yes.

…

Another student: Extreme cases aren't given in such situations as we grew up with the sense of OSS, and they don't want us to think about different perspectives. They ask for result-oriented answers. For that reason, we didn't think of rearranging the cans.

Instructor: While you are saying these, your friends are nodding their heads. Is that so? [referring to PTA1 and PTA2]

PTA1: Yes, it's true. We didn't get accustomed to thinking.

R: How do you think then?

PTA2: As my friend said, we are approaching problems with the sense of OSS. We have simple logic. I mean, I thought about how I can thought differently, but I can't find anything.

In the interview of the "Who wants 500 billion?" activity, PTA2 explained their considerations about getting an answer in hand. She explained that they predominantly felt uncomfortable when they could not reach any written answer. She explained that as their assessments based on reaching an answer instead of basing on how they got an answer, the solution was everything for them. She also explained that, in group especially, PTA1 wanted too much to get something for written responses. These explanations showed that the reasons behind the result-oriented approaches aroused from assessment considerations considerably.

PTA2: ... for example, we always think over something at class. However, PTA1 says: Let's write it down, write it down. I say that we should think over; PTA1. PTA1 wants to have something ready very much. As everybody does something, he also wants to do. [smiling] I said him to wait and think over it a bit. For that reason looking from different perspective is difficult. (Let's organize a volleyball tournament!)

PTA1: ... It resulted from the fact that I am accustomed to solving the problem and write it down at once. (Let's organize a volleyball tournament!)

R: I think, there is a desire to get the solution and tendency to reach a number as soon as possible even if there were mistakes. (Who wants 500 billion?)
PTA2: Yes you're right. As we are coming from a system that based on the result, his behavior is related with it. Everything is result for us. When we don't get a written solution, we feel really uncomfortable. We think that everybody does something, why don't we do anything?
R: However, we express that the important one is thought not the result.
PTA2: Unfortunately, we can't go beyond it.

4.2.4. Difficulty to link mathematics and real-life

PTs also explained that the fact that could not switch between the mathematics and real-life, also had an effect on their modelling process. For the "Postman" problem in which they tried to find a path that the postman should follow, PTB1 explained that they might wander away from the mathematical thinking since they thought real-life dimension of the problem. Similarly, PTA1 explained his difficulty in beginning to solve the problem using mathematical terms after considering the solution in real-life. The excerpts below were examples of this situation.

R: Well, What do you think about the reason behind it? When you are asked to solve this equation system, you manage to do it, but when it is given in a different context, solving the equation doesn't come to your mind. (Postman)
PTB1: While I was thinking about whether nx+y or ny is smaller, the thing in my mind was that the postman should walk less. When you just look at the equation, x means only x here, nothing else. On the other side, x means the way taken, y is the coefficient of it, and n is the number of shops. When focus on real life a bit, we really moved away from mathematics. Thus, we found our solution grounding it on an assumption.

PTA1: In fact, I didn't have difficulty at the beginning when I found the solution by relating it with the real life. I wrote my thought as soon as possible. When I began to find mathematical expression, I started to get confused. (Postman)

Related to the "Who wants 500 billion?" activity, in which PTs focused on the probability calculations, PTA2 explained that focusing on mathematical computations and getting mathematical results might be a factor for ignoring the real-life dimension of the problem.

PTA2: I always suppose that I understand it, but it is as if my mind was stuck into somewhere, and I can't see it. [referring real-life interpretation] (Who wants 500 billion?)
R: What is the reason in your opinion?

PTA2: I don't know but maybe habits. When we think real life, the logical one is the solution itself [referring rearrangement]. However, I'm not sure whether we think more academic or not [referring to use of mathematics] I don't know maybe that take me away from the daily life.

4.2.5. Disorganized and unsystematic work

Especially in the group work of the "Bus stop", "How to store the containers?, and the "Who wants 500 billion?" activities, PTs worked in a such a disorganized way that it made it hard to carry on solution steps. An excerpt taken from the interview conducted with PTA1 is below. He reported that since they did not work in an organized manner on the problem, they had difficulty in finding the mathematical expressions and they could not think about the other cases.

R: You took the equal number of students coming from each house again in the second case [researcher reading their writing]. Did you think about what would happen if we took different number of students during the study? (Bus stop)
PTA1: No, we didn't [smiling]. We hardly found that equation. We couldn't find the one you asked about. Moreover, the reasons why we have so much difficulty is that all papers were mixed, and we couldn't work on it properly.

The analysis of the videotaped group discussions supports this factor. A snapshot from their work related to the "Bus stop" activity is below. Owing to the fact that PTA2 used small place for computations on the solution paper, she forgot adding x near the minus a.

PTA2: What's that below? ax2-a+ax-a (group discussions for the "Bus stop" activity)
PTA1: Yes.
PTA2: what's happening here; everything gets confused again.

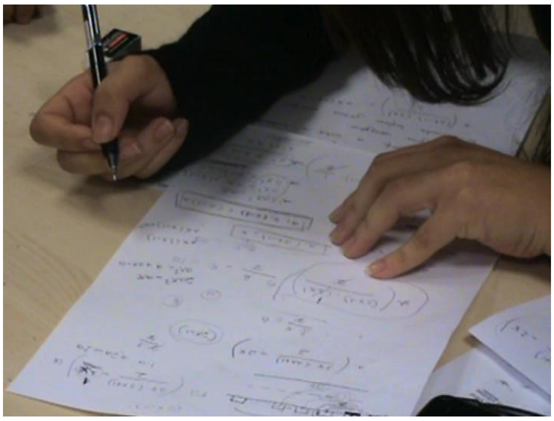


Figure 60 A snapshot of group A's work on the "Bus stop" problem

For the "How to store the containers?" activity, PTB3 explained in the interview that if they had worked on the problem systematically, they could have easily found a correct answer. In the interview, she explained that they could not achieve the systematic work.

PTB3: When I saw the sample solution from you yesterday, I thought what an easy problem that was. However, I can't understand why we can't think of it or on systematically at that time. I mean, if we start solving the problem in the way you do, and set the equation like first, second and third, then follow a systematic way, I believe that we can reach the solution; we can also do it in a short time. After all, we can't do it properly; we can't continue in a systematic way. (How to store the containers?)

For the "Let's organize a volleyball tournament!" activity, PTA3 explained that they did not enter the given data properly into the Excel. They did not attach the dimensions of the variables next to them. Therefore, they made mistake of aggregating the units of centimeter and meter together.

PTA3: .. we did incompletely. It would be better if we attached the height of the players like centimeter or meter while writing the name of the columns. (Let's organize a volleyball tournament!)

	A	B	C	D	E
1	Column1	Oyuncunun Boyu	Yukarı Zıplama Mesafesi	Toplam Zıplama Mesafesi	40 Metreyi Koşma Süresi
2	Gamze	1.6	66.04	67.64	7.01
3	Ayla	1.68	63.5	65.18	6.95
4	Canan	1.65	58.42	60.07	7.34
5	Nilay	1.7	48.26	49.96	8.18

Figure 61 A part of work of Group A on Excel sheet on the "Let's organize a volleyball tournament!" problem

Similarly, the analysis of the solution papers supports this factor. The analysis showed that, especially in the "Who wants 500 billion?" activity, Group B worked in such a disorganized way that the condition made it hard for them to follow the procedures. Figure 62 shows their attempts to solve the problem.

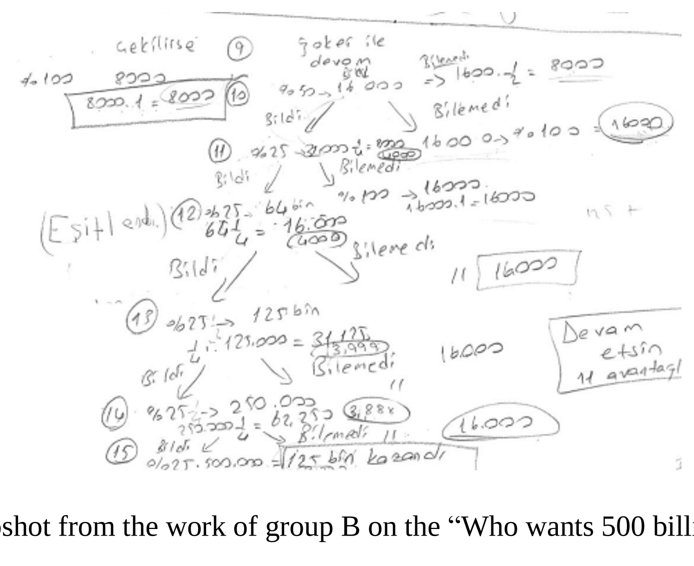


Figure 62 A snapshot from the work of group B on the "Who wants 500 billion?" problem

4.2.6. Time Limitations

Interview data showed that PTs also considered the time limitations during their work. In the interviews, they mentioned the role of time limitations on the modelling process.

PTA3: As you said that complete it until four o'clock, we rushed up. Then it turned to half past four, though. It was done in a hurry; we did it to complete in this week. (Let's organize a volleyball tournament!)

PTB3: ... while telling it on the board, I noticed that we found how many cans we can fit into the second storage by rearranging them, but we didn't do it for the first and third storages. Actually we couldn't do it because of the lack of the time. Maybe third storage unit could be more advantageous. (How to store the containers?)

With regard to this, the analysis of the solution papers combined with videotaped classroom observations supported that group B could not have time to answer some parts of the problems for the "How to store the containers?" and the "Who wants 500 billion?". The analysis showed that they could not answer the second part of the "How to store the containers?" problem, and thus they could not have time to answer the third part of the same problem after writing the second part of the "Who wants 500 billion?" problem immediately.

In the interviews, even though PTs did not state explicitly, some explanations of the PTs also supported the effect of time considerations on their solution process. An excerpt from the interviews conducted with PTB1 for the "Let's organize a volleyball tournament!" activity is below.

PTB1: Some don't draw the shape. They visualize the situation in their mind, and they think that they can continue solving without drawing the shape. Time is also important while solving the problems. You sometimes think that you can lose time while drawing the shape. (Let's organize a volleyball tournament!)

The analysis of the videotaped classroom observations also supports PTs' time considerations. The excerpt below exemplifies this consideration.

PTB2: Let's write it; be quick. We will rank them according to the height of the students. PTB3, start to rank them as 1,2,3. Hurry up! (Let's organize a volleyball tournament!)

CHAPTER 5

DISCUSSIONS, CONCLUSIONS AND IMPLICATIONS

The purpose of this study was to examine the modelling processes of PTs when they engage in modelling activities and to understand the factors affecting their modelling processes depending on their views. In the previous chapter, taking the purpose of the study into account, the results of data analysis were organized as PTs' processes in each stage of the modelling process and their thoughts about the factors affecting their modelling processes. In this respect, the discussion part was given under two headings: the nature of PTs' modelling process and the factors affecting this process based on their views. Following the discussion section, conclusions were given on the overall nature of the study, followed by the implications, limitations and suggestions for teacher education programs, for the design of the mathematics courses and for further research.

5. 1. The Nature of Prospective Teachers' Modelling Process

The models for each modelling stages presented in the previous chapter provides a representation of the modelling processes based on the subprocesses carried out by the PTs engaging in modelling activities. Based on the analysis of the results, it is evident that both groups showed similar approaches to the modelling tasks presented during the activities. In general, PTs acted on the tasks in a result-oriented way consisting a single cycle of the modelling process. This main result supports the findings of the studies of Eisentraut and Günter (1997) and Berry (2002) where the students showed result-oriented simple solution approaches and did not spend much time in analyzing and clarifying the given tasks. This result was also evident in their presentations on the board, where PTs focused on presenting their results instead of giving the details related to the solution processes they used and making explanations during and after their work. Especially during the presentations of the "Bus stop" and "How to store the containers?" activities, they neither mentioned the procedures in detail nor gave the reasons about how they came to those conclusions. PTs in general did not spend enough time to reflect on the problem situation and the way they approached the various parts of the problem. Therefore, in some cases they spent time fruitlessly on inappropriate ways of solution and therefore provided insufficient models. This finding supports the findings of Bukova-Güzel and Uğurel (2010) who found that PTs especially low achievement group could not build an appropriate model to the problem situation in given amount of time or they could not provide appropriate models. Schoenfeld (1985) explained that good control is needed to overcome deflections of time and energy caused by incorrect perceptions during the solution process, and resulted in lower performance.

In the current study, upon reading and understanding the problem situation, for the "Postman", "Bus stop", and part (a) of the "Who wants 500 billion?" problems, PTs imagined themselves in the problem situation. Van den Heuvel-Panhuizen (1999) explains that the context could provide environment to students to "imagine themselves in the situation" (p.136) and use solution strategies inspired by this imagination. In those problems, PTs produced an intuitive solution based on their knowledge or experience in the context. This supports the findings of Van den Heuvel-Panhuizen (1999), who suggested that as plausible scenarios could be produced especially in familiar contexts, the students could prefer to work on the problem based on those scenarios. On the other hand, the analysis of the results indicated that PTs sometimes visualized a specific context based on their experiences and began to think based on that context. As a result of dependency on the real-life context, particularly in the first activity, "Postman", PTB1 made context-dependent visualizations. Thus, PTs spent time on making effective assumptions that would lead them to the solution. This finding supports the findings by Busse and Kaiser (2003) who concluded that the context, by providing environment in some cases, could be motivational for students. On the other hand, by activating broad range of real-life knowledge on the context, it might also cause confusions. Although PTs mostly identified the variables correctly and developed appropriate assumptions, in some cases they faced difficulty in developing appropriate assumptions to structure the problem due to focusing on the context. Especially during the first weeks of the study, and especially for PTB1, it took a great deal of time to simplify and identify the task because of working on the details of the context, rather than working on the problem. Similar difficulties regarding making assumptions were mentioned by several other researchers (Blum & Borromeo-Ferri, 2009; Blum & Leiß, 2007; Ikeda, 1997; Kaiser et al., 2010; Sol et al., 2011). Blum and Leiß (2007) drew attention to the cognitively demanding feature of this process of simplifying and constructing the idealized version of the problems. Similarly, Jonassen (1997) and Spandaw (2011) explained that this process is both an important and a difficult one in solving ill structured problems as it requires identifying the constraints and the possible states of the problem. In some cases, PTs oversimplified the assumptions to be able to easily reach a result, which, in the end, produced simple models. This result agrees with the results found by Kaiser et al. (2010), Maaß (2006) and Schaap et al. (2011), whom also reported the students' difficulties causing to construction of simple models.

While trying to develop a solution plan, PTs tried to remember a previously solved problem and tried to apply the solution method used in that problem to a current problem. Polya (1973) and Schoenfeld (1985) explained that this is usually the first method that people use when trying to solve the problems. The results showed that PTs attempted to solve the "Bus stop" problem like the "Postman" problem as they find similarities between the contexts of the problems. After reading the "How to store the containers?" and part b of the "Who wants 500 billion?" problems which are closed in terms of outcomes, PTs evaluated the problems and fit the problems into a mathematical content/structure. Then, in the planning stage, they directly employed the solution method of the routine problems on that specific content. This supports also the arguments of Schoenfeld (1985), Dunbar (1998) and Olkun et al. (2009) that problem solvers use the previously used strategies allowing them to move through the solution. For the other problems in the current study, namely the "Let's organize a volleyball tournament!" and part (a) of the "Who wants 500 billion" problems, the results indicated that as the mathematics behind the problems is not obvious, and as it does not direct the PTs to a specific mathematical concept through keywords given in the text, PTs searched for mathematical concepts with the help of which they could solve the problem. They tried to recall their previous knowledge and experiences. For the "How to store the containers?" and "Let's organize a volleyball tournament!" problems, the findings of the present study suggest that PTs chose the mathematical concepts/tools through which they believe they could solve the problem. The analysis of the results showed that, although in some cases they realized other mathematical tools that would help the solution process and might yield better models, in general PTs did not even try those tools as they believed it would be difficult to reach an answer using those tools. They stated that they choose mathematical concepts with which they feel comfortable to carry out the computations and which lead them to the solution.

PTs' explanations indicated that to be able to quickly reach an answer, they did not want to engage in problems dictating a specific model or requiring them to use complex mathematical concepts. The analysis of the data revealed that PTs preferred to work on the open-ended tasks allowing them freedom to set the conditions and decide on a model. This is contrary to Kuntze's (2011) findings where PTs explained their preferences for the tasks asking for one correct solution instead of higher modelling activities allowing different solutions. Kuntze (2011) attributed this result to the PT's views and fear of incompatibility of the modelling activities with the mathematical exactness. During the implementation of the activities, PTs did not attempt to build a model when they thought that the mathematics is beyond their level of mathematical knowledge. Therefore, the solution process of PTs resulted in the selection of the simple models based on the selected mathematics. These results are consistent with Eraslan's study (2012), whereas they are contrary to Berry's (2002) finding that the problem solvers attempt to develop a model where the selected mathematics is beyond the modelers' knowledge or competence level.

Before offering mathematical solutions to the "Postman", "Bus stop" and part (a) of the "Who wants 500 billion?" problems, PTs developed intuitive answers to the problems. This might be because of the context of the problem which includes a familiar situation and which allows them to produce an answer intuitively without making any calculations. In the first and second problems, where they used their intuition, they could not consider some of the relevant variables. Tyre, Eppinger and Csizinszky (1995) explained that when problem solvers adopt intuitive approaches instead of systematic approach, they are more likely to fail to consider the role of the relevant information or they ignore it. Particularly for the first two activities, "Postman" and "Bus stop" PTs used intuition, which resulted in lower quality solutions. This is consistent with Tyre et al. (1995) and Borromeo-Ferri (2011)'s studies in which they stated the role of intuition in the occurance of blockages in the process. Tyre et al. (1995) and Spandow (2011) mentioned that solving open-ended real problems, most of the time, is a fuzzy process including multiple goals and interests which are sometimes conflicting and which make the problem hard to solve. Tyre et al. (1995) argued that everyday problem solving involves intuition instead of including some formal processes. One of the PTs explanations in the interview (PTB3) also revealed the drawback of solving the daily-life problems by taking into account mathematical knowledge. For instance, considering the "Postman" problem, PTB3 explained that she considered solving the problem as in daily-life and exemplified her point by asking whether a postman would use mathematical reasoning to select the best available path for the delivery of the posts, or just choose any convenient one. Galbraith and Stillman (2006) mentioned this dilemma as a blockage which occurs during the transition from the mathematical solution to the real-world meaning of the solution.

After deciding on a general solution approach to be used, PTs intended to start quantitative work swiftly. They simply accepted the first solution idea that occurred to them instead of developing multiple alternatives. Especially some of the PTs (PTA1 and PTB3) were eager to get the task done faster. Instead of thinking on the situation in detail, they showed willingness to begin to write the solution and reach an answer immediately. This finding is consistent with the findings of several other researchers (Verschaffel et al., 2002; Berry 2002; July & Mudaly, 2007). Furthermore, PTs generally did not draw the accurate and appropriate situation model. In general, they immediately started with the quantitative work without making some drawings. Thus, they could not get the insight that comes from accurate drawings (Schoenfeld, 1985).

While working on the activity, might be the result of cognitively demanding feature of the modelling tasks, PTs realized in some cases that they lost track of their proceeding and felt the need to re-examine their purpose in the problem. This result is consistent with Maaß's (2006) study. She suggested that the learners of modelling should keep their aim in the problem in mind and keep track of their proceeding. Although PTs correctly understand the problem and decide on a solution method, in some cases, while working on the problem, they unconsciously shift their thoughts to the real life context. This might be the result of being distracted by knowledge of the real-life context of the problem (Busse & Kaiser, 2003).

Following the individual work, PTs shared their ideas about the solution of the problem. However, upon sharing ideas, they decided to use one PT's individual solution approach in the group work. In other words, they adopted one of the methods and began to work on it with the aim of reaching a solution. For the undecided points in the selected solution approach of any PT, they tried to combine the individual ideas. Although the solution planning should be an interactive and social process in which the group members discuss and brainstorm as well as assessing the solution options available (Tyre et al., 1995), in our case PTs selected one individual solution approach and began to write a report. In general, similarity between the developed approaches and the desire to reach a solution might be the factors leading to the adoption of such an approach. As a result, it can be stated for the present study that group problem solving was not successful except for group B on the "How to store the containers?" and "Who wants 500 billion?" problems. For the "How to store the containers?" problem, PTB2 offered a different solution approach based on her similar experience, while for the "Who wants 500 billion?" problem, they could not produce an appropriate mathematical solution approach at first and therefore they worked together to find an approach. This supports the argument of Dunbar (1998) that students from the same background in a group would increase the probability of similarity between the individual solution approaches, and hence decrease the success of the group problem solving.

While working on the solution, PTs made guesses about the unknown variables and the relationships. Instead of calculating the correct value or testing the correctness of the relationships, sometimes they continued working using their guesses in their solution processes. In a similar vein, when they faced with difficulty at a point in the solution process, they ignored the relevant variables to be able to reach a solution. Especially in the "Postman", "How to store the containers?" and "Let's organize a volleyball tournament!" problems, they decided to ignore relevant variables to simplify the situation and to be able to reach a solution. This is an important finding of the present study which is not mentioned in the literature.

In the “How to store the containers?” problem, group B worked with manipulatives instead of using formal mathematical tools. It might be due to the fact that group members became more experienced in using manipulatives. Results indicated the positive role of mathematical methods courses in PTs’ use of manipulatives (Yetkin-Özdemir, 2008; Quinn, 1998). Although this provided environment for them to explore the problem in a more interested manner (Moyer, 2001), they faced some difficulty after deciding to use formal mathematical tools. PTs drew circles on their solution papers by using these manipulatives to demonstrate the containers. However, they continued working formally with the values of the manipulatives instead of making use of the actual values given in the problem.

While working on the problems, in some cases although PTs had the sufficient knowledge to be able to solve the problem, they were affected by each others’ ideas and worked on the ideas developed by others. This also caused them to adopt inappropriate approaches. For some other cases, although they made logical arguments to solve the problem and reach a final result, while working on the problems PTs faced difficulty in applying mathematical rules to reach a final answer. Instead of applying correct mathematical rules to conjoin the mathematical expressions, PTs made some assumptions or tried to form the expressions in a comparable form in order to overcome this difficulty and to be able reach a solution. Moreover, while forming these equations, instead of reflecting on the situation, they used trial-error method to find the correct mathematical expression representing the situation. Additionally, while working on the problem, they did not pay attention to the intermediate steps and features of the variables. Therefore, in some cases, they also made some computational mistakes. Overall, PTs had difficulty in translating the problem into a mathematical form and working on it. This result agrees with the result found by Bukova-Güzel and Uğurel (2010).

Although it is not observed in every problem, PTs also interpreted the intermediate or final results. In some cases, they used their knowledge of the task context to verify their results. This finding supports the findings of Busse and Kaiser (2003). On the other hand, in some cases, they did not interpret the reached solution in real-life, which supports the findings of Çiltaş and Işık (2013), Bukova-Güzel and Uğurel (2010), and Özer-Keskin (2008). On the other hand, Zbiek and Conner (2006) stated that interpreting may also not be explicitly stated by modelers in the solution process as it could be considered as a subliminal act.

In addition to acting on the tasks in a result-oriented way, the analysis of the results also indicated that PTs went through one single cycle. Rather than being cyclic, there was a linear progression toward a solution. PTs did not seek an improved solution by revising and refining their models. The results indicated that, except for some cases, PTs believed in the correctness of their solution and even did not feel the need to test it. In some cases where they had an intuitive answer, they tried to show the correctness of their intuitive answers by making supportive assumptions at the beginning of their solution process. In some other cases, they considered their solution as a proof. The results indicated that the verifying stage was particularly absent in the modelling process. In fact, PTs generally neither checked for the computational mistakes, nor they validated their solutions reasonableness for the given problem situation. This is consistent with the findings of several research studies conducted with students from different grades or backgrounds (Blum &Ferri, 2009; Blum & Leiß, 2007; Bukova-Güzel, 2011; Galbraith & Stillman, 2006; Hodgson, 1997, Kaiser et al., 2010; July & Mudaly, 2007; Maaß 2006; Sol et al., 2011). PTs believed strongly in the correctness of their model, and therefore they did not make use of all checking procedures. Thus, instead of revising the model for unsatisfactory conditions or computational mistakes, PTs accepted the model developed as a solution for the problem. In their study conducted with students and teachers, Blum and Leiß (2007) reported that even the best PTs did not reflect further on the solution process. The findings also showed that the PTs expected the instructor to check for the correctness of their solutions. This is supported by the studies of Blum and Borromeo-Ferri (2009), where students considered the instructor to be responsible for the correctness of the solutions.

PTs in the current study could not realize the entire modelling process. As they proceeded in a straightforward manner, in general they produced simple models by using basic mathematics. This finding is partially consistent with the findings of Özer-Keskin (2008), Özturan-Sağırlı et al. (2010), and Sol et al. (2011), who concluded that the solution process of the students showed the classical structure of the problem solving process.

5.2. The factors affecting the modelling process

Based on the views of PTs' on the factors affecting their modelling process and based on the data analysis of the PTs' modelling processes, it seems that the exam-oriented education system and insufficient learning opportunities through modelling activities, lack of conceptual understanding on some mathematical concepts, grade considerations, time limitations, and disorganized and unsystematic work are the major factors that determined the type of the modelling process.

5.2.1. Exam-oriented education system and insufficient learning opportunities through modeling activities

The data showed that the PTs' low performance in the modelling activities was mainly caused by lack of experience with the modelling problems. Several researchers stated that the current education system and little involvement with demanding modeling tasks cause difficulties in the modeling process (Eraslan, 2012; Kaiser, 2007; Kertil, 2008; Maaß, 2006; Sol et al., 2011).

The results showed that PTs mainly hold the present exam-oriented education system responsible for their low performance with the modelling problems. PTs' explanations revealed that several high-stakes tests taken throughout their education life and the resultant study habits inhibit a favorable environment for exploring mathematical modelling tasks. In these test, as they are heavily engaged with problems that usually require straightforward thinking and procedural understanding to produce quick solutions, they attempted to solve the modelling problems in a similar way. As, in these kinds of exams, they are expected to reach a result immediately without considering the alternative solutions, PTs tried to solve the problems as is they are routine short-items. In the interviews, they expressed that they are used to solving traditional problems that require a single answer and solution. Thus, they explained that they have difficulties with problems that require them to determine different variables, make assumptions themselves, and consider different alternatives to better models. That is, they reported that they have difficulties with problems like the ones in our study. Lingefjärd and Holmquist (2005) stated that the modelling process is generally hard for many PTs. Similar to the findings of the present study, in their study Lingefjärd and Holmquist (2005) maintained that one reason why PTs find building mathematical models difficult is the traditional instruction involving regular lectures. In a similar vein, Eraslan (2012) maintained that teacher centered systems telling students directly what to do and assessing performance based on the final product instead of the processes cause difficulties for students in the modeling process. Similarly Schaap et al. (2011) attributed the difficulties in modelling process to the students' inexperience with modelling activities during their education. They added that most of the time students were provided already with mathematical models (formulas) in tests. Similarly, Julie and Mudaly (2007) explained that the absence of testing stage in modelling process is the product of current school mathematics programs.

PT's also reported that they had difficulty with switching between the mathematics and real-life because they think that they did not get sufficient learning experience with modelling problems. Although the present study was conducted with PTs, the results were consistent with the results of the study of Ikeda (1997) who emphasized that, when students depended more on the real-life situation in the solution process, it became more difficult for students to reach a mathematical solution. Similarly, when students depended more on mathematics to be able to solve the problem, it became more difficult to provide consistency with a given real-world situation. In our case, it resulted in the building of simple models.
PTs' conceptions of problem and problem solving might hinder modelling process due to their being inexperienced. Their conceptions of problem and problem solving are consistent with the conceptions explained by Verschaffel, et al. (2002). Some of the PTs' conceptions were as follows: problems contain all the required knowledge to solve them, so no need to seek extra information; there is only one correct answer to the problem; and the answer could be isolated from real-life and it could be violated in the given real-world problem situation. As argued by Verschaffel, et al. (2002) and Reusser and Stebler (1997), most of the problems in presented in textbooks, or employed in classrooms did not challange students to use their everyday knowledge or experiences. Moreover, most of the school problems gave the impression that all problems are solvable, and every information that is relevant to its solution is included in the problem text. Jonassen (1997) explained that this view of a well-structured problem solving develops over time, and jeopardizes students' correct disposition towards ill-structured problems like modelling problems. Verschaffel et al. (2002) explained that courses given in teacher education programs are not conducive to the development of disposition towards realistic mathematical modelling and do not emphasize the importance of the courses that foster real disposition towards mathematical modelling, i.e. "treating the text as a description of some real-world situation to be modeled mathematically" (p.258).

The results indicated that although PTs were given instruction about modelling in education and modelling process on the fourth week and after each activity they were provided with a sample solution approach including explanations about the modelling process according to the modelling stages, this instruction was not powerful enough to overrule students' tendency to solve the problems in superficial ways, without taking into account the real-life context of the problem. This finding is consistent with above mentioned findings of the study of Verschaffel, et al. (2002), in which they explained the role of students' beliefs or conceptions on the solution process.

5.2.2. Lack of conceptual understanding

PTs' explanations about the factors affecting their modelling processes which were also supported by data analysis of their modelling processes showed that the participants are deficient in conceptual understanding of some mathematical concepts. Similarly, Bukova-Güzel and Uğurel (2010), and Soon, et al. (2011) reported in their studies that students did not even possess basic knowledge and show conceptual understanding of certain mathematical concepts. Several researchers described the role and importance of mathematical knowledge and competencies in a successful modeling process and emphasized that it heavily depends on mathematical knowledge and competencies (Berry, 2002; Bukova-Güzel & Uğurel, 2010; Jonassen, 1997; Maaß, 2006; Niss, 2007; Schaap et al., 2011; Soon, et al., 2011). These researchers explained that inadequate mathematical knowledge and skills causes difficulties in correctly formulating the problem situation, and realizing the relationships between the "real-life contexts and mathematical representations".

Although the participating teachers entered their undergraduate programs by showing high performance in high-stakes tests (University Entrance Examination), which is composed of well-structured problems, they faced difficulty in engaging with modeling activities, indicating poor conceptual understanding. Researchers stated that performance in solving ill-structured problems, including modeling problems, is independent of performance in well-structured problems (Schraw, Dunkle, & Bendixen, 1995). Bukova-Güzel and Uğurel (2010) also focused on this problem. They reported that even students with higher academic performance faced difficulty in solving the problem in mathematical terms and concluded that academic success is necessary but not sufficient for success in the modeling process. On the other hand, the findings of this study are contrary to those of Bukova-Güzel (2011), wherein PTs displayed successful approaches to formulating the problem mathematically and using their previous mathematical knowledge.

In the first and second problems, which require them to form the mathematical expressions for two situations and make a comparison to make a decision, instead of applying the correct mathematical rules to conjoin the mathematical expressions and reach the final solution, they made some assumptions or tried to write the expressions in a comparable form. Moreover, instead of using the correct mathematical rules or ways, they sometimes ignored the appropriate variables to eliminate the difficulty. Their responses also revealed that they chose to integrate basic mathematical concepts in the solution process. Indeed, they aimed to make the problem easier, which they believe would be too difficult otherwise. The analysis of the results showed that PTs are aware of their lack of conceptual understanding of some mathematical concepts and explained that this was a barrier in their solution process.

Although the data revealed the inadequate conceptual understanding for certain problems, in some other problems, it is seen that PTs have the sufficient mathematical background to solve the problem and to construct better models. However, they could not use their mathematical knowledge effectively to solve real-life problems, and this hindered their success in these problems. This finding is in agreement with Biembengut and Faria (2011), Kertil, (2008), Lingefjärd (2002) who emphasized the importance of learning when and how to use the knowledge and skills in modeling activities. Actually, the mathematics behind the problems is not obvious in the problem texts, which could be a reason why PTs could not decide on the correct mathematical tool to solve the problem. On the other hand, this important result concerns knowledge transfer. Several studies related with the situated cognition maintain that knowledge is situated in context, and learning depends on and develops in specific learning context, and hence there is no simple transfer from one situation to others (Brown, Collins, & Duguid, 1989; Niss, 1999). About the situated cognition theory, Blum (2011) explained that this is also true for the learning of mathematical modeling and argued that modelling should be learned specifically. He further explained that the transfers between real-world situations and mathematical concepts should be explicitly given to the students by providing them broad range of contexts of both real-life situations and mathematical concepts.

5.2.3. Grade considerations

The results also indicated that grade considerations could cause PTs to consider producing the simple models by using basic mathematics. Indeed, because of the fact that they are expected to orally present their models at the end of the lesson, and required to submit their solution papers to the instructor, they had some considerations about managing the task, time and building a model. Therefore, for PTs, it is more important to arrive at a solution quickly than develop alternative solutions. Because of this consideration, they tended to select a solution without analyzing the problem thoroughly. Although it was emphasized during the courses that the thought processes are much more valuable than the product (models), and attempts were made to create an environment less grade pressures, it seems that these considerations created pressure on them, and thus interfere with the modeling process. Schoenfeld (1982) explained that behaviors of the subjects could be affected by expectations, and context.

5.2.4. Time limitations

Another reason for the difficulties as stated by the PTs is time limitation. PT's were given approximately two hours to work on each problem. However, the analysis of the results showed that the entire process of mathematical modelling was somewhat slow and time consuming for PTs so that they could not find enough time to answer some parts of the problems. Although the tasks and participants were different to some degree, these findings corroborates with the findings of (Anaya, Cavallaro, Dominquez, 2005; Galbraith & Stillman, 2006; Kaiser et al., 2011). The findings showed that in some cases PTs felt they did not have enough time to reach a solution and to analyze the problem thoroughly to get a better answer. The data analysis showed that, in some cases, time constraints might prevented them from going through more than a single cycle of the modelling process. Therefore, in the present study, the evaluation of PTs was dependent on the works of PTs' on the given time.

Kaiser et al. (2011), and Biembengut and Faria (2011) explained that time pressure was generally the most commonly reported factor hindering the implementation of modelling activities in classrooms. Eisentraut and Günter (1997) explained that in situations where time pressures are high, students might consider it more important to reach a solution quickly than think on alternative solutions. Eisentraut and Günter (1997) added that in situations where there is high time pressure, students might encounter difficulties in concentrating their attention to important points and thus get average results. These researchers also reported that students have individual study styles, and whereas for some students it may not create a problem, for some others (e.g. analytical style), high time pressure could be an obstacle to dealing with complexity and uncertainty and organizing solution approaches.

5.2.5. Disorganized and unsystematic work

The results showed that, especially in some of the problems (e.g. first, third and fifth problems), PTs could not achieve systematic work. Systematic problem solving refers to "following a set of logically connected steps that lead the problem solver from problem identification through devising and testing a preferred solution" (Tyre, et al., 1995, p. 6). Although systematic problem solving encourages problem solvers to develop multiple alternatives, or better ways rather than simply accepting the first solution idea that comes to their mind (Baron, 1988), in general PTs in this study stuck to the first solution plan and began to work on it, which resulted in simple models. Moreover, PTs usually seemed satisfied with the constructed model. Only in some of the problems (e.g. fourth problem), they tested or revised some parts of the constructed model. Accepting the correctness of the inadequate model without carrying out some or all checking procedures reduced the possibility of developing better models (Galbraith & Stillman, 2006). Finally, PTs worked unsystematically in some problems (third and fifth problems); that is, they performed the computations in a disorganized way, which made it hard to carry on the solution steps correctly.

5.3. Conclusions

It can be concluded that the PTs' approaches to different modelling problems showed similarities. Basically, the nature of their modelling process is somewhat result-oriented with a single modeling cycle. Although there are some differences in terms of the problems used in the study and the participants, this finding is in concordance with those of Eisentraut and Günter (1999), and Berry (2002).

In general, PTs modelling processes consisted of four main stages: understanding the modeling problem, devising a solution plan, performing the plan, and interpreting and verifying the solution. However, the verification stage was not performed properly. They did not perform all the checking procedures to see whether the created model fulfills the requirements set by the task. For some of the problems (e.g. first, second, and partially the fifth problem), PTs developed an intuitive answer, which might be the result of the familiar context of the problem, allowing them to produce an answer intuitively without making any calculations. Going through these stages, they aimed at showing the appropriateness of their intuitive answer. In the third, fourth, and part (b) of the fifth problems, they needed to make some calculations and ultimately selected the convenient solution to solve the problem. When confronted with a problem during the working process, they made simplifying assumptions or disregarded a variable. In the planning stage, they tended to adopt the first solution that occurred to them rather than questioning the problem situation from different dimensions. Furthermore, PTs generally seemed satisfied with the constructed model. Instead of revising the model for possible imperfect conditions, they automatically accepted the model and acknowledged it as an effective solution to the problem. To conclude, PTs attach greater importance to arriving at a solution than to developing alternative, and maybe better, solutions. For this very reason, they chose to use a solution method without reflecting on the problem well.

The present exam-oriented education system which fails to provide students with substantial modelling activities was the leading factor preventing a favorable environment for exploring mathematical modelling tasks. Because of the fact that PTs got used to being engaged with problems that usually require straightforward thinking without considering alternative solutions, the participants attempted to solve the modelling problems in a similar way. Seeking an instant result, they approached the modelling problems as single answer routine problems.

In a similar vein, grade considerations might lead them to the existing result-oriented approaches with one single modelling cycle. Although the class atmosphere was tolerant towards any idea or mistakes, PTs were expected to present their models on the board at the end of the lesson. Their solution papers were gathered and examined for further analysis. Therefore, it became evident again that, for students, arriving at a solution is more important than developing alternative approaches.

The lack of conceptual understanding pertaining to some mathematical concepts was also evident for PTs. Moreover, PTs faced difficulty in transferring and linking their knowledge to solve real-life problems. Because of this reason, they might also have chosen the safe way and produced basic (simple) models instead of trying to search for more complex models, which could result in unproductive or weak models.

The results showed that, doing some of the problems, PTs could not find sufficient time to attempt at all the parts of the problem. Therefore, time pressure is presumably another factor, causing PTs to fail to notice the essential parts of the problem and concentrating to the reaching a result. It could be concluded that the time limitation is a factor generating blockages against a favorable environment.

To summarize, it was seen that several factors impedes exploration of modelling activities: exam-oriented education system and insufficient practice with modelling activities, inadequate conceptual understanding, time limitations, grade considerations, and disorganized and unsystematic work. These factors hinder an atmosphere conducive for exploration of modelling activities and causes the prevalence of result-oriented approach with a single modelling cycle.

5.4. Implications, limitations, suggestions

This part was organized under the two headings: suggestions for mathematics teacher education programs and teacher educators, and suggestions for the design of the mathematical modelling courses.

5.4.1. Suggestions for mathematics teacher education programs and teacher educators

Lingefjärd and Holmquist (2005) argued that "Applied mathematics as a field and the process of mathematical modeling in particular are parts of the mathematical curriculum for PTs that may be broadened, enhanced and become even more important in the future because of the continuous technological revolution."(p. 128). Similarly, Arora and Rogerson (1991) stated that "more and more emphasize will be placed on problem solving and modelling in mathematics, to bring out its utility in day to day life"(p.114). Consistent with these arguments, the findings suggest that teacher education programs should integrate modelling courses into the curriculum aiming to improve both the PTs' modelling competencies and to develop PTs' pedagogic knowledge on the application of modelling activities in the classrooms (Bukova-Güzel, 2011; Çiltaş & Işık, 2013; Kertil, 2008; Özer-Keskin, 2008; Lingefjärd, 2000; Maaß, 2006).

At the university level, mathematics courses such as calculus and geometry that are offered to the PTs should be developed to cover the examples of mathematical modelling activities to improve PTs' mathematics comprehension. While doing this, curriculum should be arranged by enabling enough time for the exploration of modelling activities. Although it needs more time when compared to the traditional word problems, long-term benefits of the modelling activities should be taken into consideration (Bracke & Geiger, 2011).

Although this research was conducted as a part of semester-long study, the findings support the need for longer engagement of PTs in the modelling activities (Kaiser, 2007). It is known that "the modelling competency has to be built up in long term learning processes" (Blum & Ferri, 2009, p.56). In this context, integrating modelling activities into mathematics courses beginning from the elementary school would be a beneficial approach to implement mathematical modelling instruction. Swetz and Hartzler (1991) stated that mathematical modelling could be incorporated in elementary or secondary school curricula in different ways. They emphasized that modelling approach should be incorporated gradually into the curricula but should not be given as a separate course, which might cause perception that mathematical modelling is difficult and disconnected from the other mathematical lessons.

This study indicates that mathematical modeling activities may provide PTs with opportunities to develop conceptual understanding of particular mathematical concepts such as manipulating inequalities, volume of the cylinders, and weighted averages. In the present study, although the improvement of PTs' conceptual understanding of some mathematical concepts was not planned, the results suggest that developing and implementing modeling activities covering various mathematical topics could help the development of conceptual understanding. Accordingly, a course could be designed to develop PTs' mathematical knowledge and skills.

It is also necessary to include tasks suggested by curriculum, modelling tasks in particular, in assessment activities, especially in high-stakes assessment, to provide environment for both teachers and students in order to consider modelling more importantly (Turner, 2007; Küçük & Demir, 2009). As the national examinations to enter secondary and tertiary education institutions based on multiple-choice tests, application of other assessment methods seen as a barrier on the success of the students' on these exams (Erbaş & Ulubay, 2008). Therefore, there should be related changes or developments in the assessment system of our education system regarding the support for the creation of more productive and powerful model(s) emphasizing the modelling process instead of the product itself. While doing this it is also important to provide support for professional development of teachers (Erbaş & Ulubay, 2008; Küçük & Demir, 2009). The research studies related to the implementation of the new mathematics curricula showed that there is a need for guidance for teachers about how to implement the activities and alternative assessment methods (Bulut, 2007; Erbaş & Ulubay, 2008). Although this study was conducted with PTs, the findings also suggest the need for offering training on the modelling for in-service teachers in terms of intensive seminars, courses, or workshops (Dindyal & Kaur, 2010). Teachers also need to learn more about modelling perspective and how to use the modelling activities in the classrooms to be able to use effectively modelling in their teaching (Bukova-Güzel, 2011). Studies showed that insufficient training of teachers is resulted in incomplete applications of approaches given in the new mathematics curricula (Kartallıoğlu, 2005)

As the literature suggests the role of the textbooks in serving an import source for instructional activities (Ball & Cohen, 1996; Freeman & Porter, 1989; Mayer, Sims & Tajika, 1995), as well as the curriculum development, the textbooks for elementary and secondary school level should also be arranged including the examples of the modelling activities and informing about the applications in the classrooms. Modelling problems should take part in the textbooks with the appropriate implementation guidelines enabling teachers to use and adapt in their classrooms (Dindyal & Kaur, 2010).

Although this study was conducted with PTs, the findings also suggest the need for offering training on the modelling for in-service teachers in terms of intensive seminars, courses, or workshops (Dindyal & Kaur, 2010). Teachers also need to learn more about modelling perspective and how to use the modelling activities in the classrooms to be able to use modelling in their teaching (Bukova-Güzel, 2011).

5.4.2. Suggestions for the Modelling Courses Offered to Teacher Candidates

As mentioned above, the findings suggest the need for developing mathematical modelling course for PTs. This study especially revealed the need for learning mathematics by its real-world applications. In this context, engaging in the applications of modelling activities covering different real-life contexts and various mathematical concepts could provide opportunities for PTs to use knowledge of mathematics more effectively in real-life situations.

While achieving this, students should be provided with modelling activities beginning from elementary mathematics and then they should be introduced to the more complex ones. Focus should be placed on "teaching students Complex Thinking using Simple Mathematics" (Iverson & Larson, 2006, p. 281)Tasks which are open in terms of procedures, but rather closed in terms of outcomes (Berry, 2002) could be better for the beginners. After internalization of these tasks (e.g. "How to store the containers?" problem, the problems which are also open in terms of outcomes should be introduced to the students.

To develop PTs' skills in different stages of modelling, students could have experience in problems that require the specific stages of modelling process (Turner, 2007) as well as experience in problems requiring all stages of modelling. For instance, the findings suggest problems requiring them to draw a picture to represent the given scenario, or problems that require them making assumptions. Additionally, while solving the problems, the appropriate use of technology should be carefully taught to students so that it will not be used improperly (Berry, 2002).

PTs specifically, students in general, should be encouraged to produce different approaches. Rather than finding exact numerical answers, students should be encouraged to reflect on the problem situation in detail before attempting to solve the problem by using the approach developed after the first reading of the problem (Berry, 2002). To help them achieve this, after each group's reflection on the problem, groups share their solution approaches with the reasons behind. Accordingly, the importance given to the ideas or solution approaches rather than product itself can be emphasized. Moreover, the time allowed for the presentations can be increased to allow students to discuss in detail how the proposed model performs, and decide if it is satisfactory when taking into account the original problem situation. PTs should have opportunities to discuss several possible solution approaches, “each of which offers advantages and disadvantages to different people and situations in the context of their application” (Jonassen, 1997, p. 78). On the other hand, the question of “what counts as a good solution procedure” (Schoenfeld, 1991; Verschaffel et al., 2002) for mathematical word problems, is also case for modelling problems. Therefore, there should be some criteria to help students understand “what counts as a better model among the offered models?”. The assesment of the performance in modelling tasks is not easy to achieve (Lingefjärd, 2002). However, peer assessments could be used to achieve this. The students could be provided with a similar rubric one of which was offered by students in the study of (Lingefjärd & Holmquist, 2005), or they could develop a rubric themselves. This kind of attempts could motivate students to reflect on the process and possible solution approaches.

The important point to be considered here was the potentiality of cognitive load during the solution process on acquisition of related problem schemas (Sweller, 1988). To achieve this, at the end of the activity, PTs should be provided with clear representations of the modelling process that belongs to the problem. The students should reflect on the problem situation and the offered solution ways to develop appropriate problem schemas (Jonassen, 1997). Therefore, PTs should be given enough time not only to develop better mathematical models (Williams & Wake, 1997) but also to reflect on the overall solution process. The present study suggests that the duration of the application of the activities should be three to six lessons. On the other hand, studies showed that the limited time allocated for mathematics in weekly schedule is making it hard to do the suggested activities in the curriculum (Bulut, 2007; Erbaş & Ulubay, 2008; Özar, 2012; Soycan, 2006). It seems that narrowing the mathematical content could partly increase the time allocated for modelling activities in which students could engage deeply with the important mathematical concepts (Erbaş & Ulubay, 2008; Özar, 2012).

5.4.3. Limitations and suggestions for further research

In this study, understanding the modelling problem, planning, working, interpreting and verifying stages are considered as the main stages of the modelling process. Applications of different modelling activities with different PTs would help the further development of the proposed model for identifying PTs’ modelling process.

This study was conducted as a part of a semester-long study. Long-term studies could be conducted to see the development of the modelling abilities of PTs’. Further investigations with in-service teachers and students of different grades are also needed.

In this study, the role of the individual and group working styles and the role of the brainstorming in group working process were not analyzed (Eisentraut & Günter, 1997; Lesh & Zawojevski, 2007). The role of these factors on the group solution process could also need further attention for future researches.

Future research questions are needed to address the role of the factors such as beliefs, feelings, and metacognitive behaviors while engaging in the modelling activities (Lesh & Zawojewski, 2007; Schaap et al., 2011). Especially the role of the beliefs related to the problem solving and self-efficacy beliefs could be investigated further. The instructional approaches dealing with these types of beliefs could also need further research (Lesh & Zawojevski, 2007).

In the present study, during the applications of the activities, the researcher as an instructor walked around the classroom for asking some probing questions: What are you doing? What do you find? Why are you doing it? The researcher did not use different teaching styles. Related to the modelling instruction, the role of different teaching styles or strategies (Berry, 2002) and the role of the instructor to execute the modelling activities for different class settings need also further attention (Ikeda, 1997). The research on this area could help the development of correct modelling instruction. Furthermore, these kind of methodological choices used in this study might have an impact on the results of the study and therefore might not be appropriate to identify certain processes.

Modelling activities explored in this study enabled PTs to reveal their thoughts during the group works. As they externalized their thoughts while engaging in these activities, modelling activities could be used for research purposes in order to understand the details of students' thought processes (Lesh & Doerr, 2003).

For future research studies, this study also has a methodological implication. During the interviews, PTs were provided with their solution papers and they were told that they were expected to rethink their experience and provide as many details and examples as possible while describing their modeling process. Although this method helped the understanding of their modelling process, it was dependent to the PTs' solution papers that did not reflect all the details of the process. In this study, because of the limited time before the interviews, the interview sessions were conducted without analyzing the videotaped recordings. The cases including important episodes from video recordings of the PTs' modelling attempts could be analyzed and asked for unclear points in the interviews. Therefore, first of all, the video-recordings of the PTs' work should be analyzed while they are engaging in modelling activities, then conducting the interviews will be more beneficial for better understanding of the process.

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APPENDIX A

PILOT STUDY

In order to improve the design of the study, a pilot study was conducted before the main research. In general, by this pilot study, potential problems in the main research procedure were identified and the quality and efficiency of the main study were tried to be improved. The pilot study was conducted during the spring semester of 2009-2010 academic years at a university in Ankara, Turkey. This university was different from the university at which the main research was conducted. The participants were tried to be as similar as possible to the participants of the main research. The participants of this pilot study were the prospective mathematics teachers who were in their fourth year of the studies in the secondary mathematics education program. The reason to choose these subjects was the common features of these subjects with the participants of the main research: they were all from mathematics education program and they were not given any modeling course before.

The pilot study was conducted with two classes which were the branches of the same class. Courses were hold in three different days. Each course lasted for approximately two hours. Pilot study included the data collected from six class hours. Of these six hours, two class hours were conducted with one group, and four class hours with the other group. Each class was consisting of twenty students. However, in the first and in the second applications, a few students were absent. All the courses were in the same place. In the classroom, there were eight round-tables, and students were sitting around them with three to four groups. The data were collected through audio recorded semi-structured interviews, class observations, and students' solution papers.

Process

On the first day of the pilot study, to support both the PTs' understanding the role of the mathematical modelling in education, and also the modelling process, the researcher made a presentation before implementing the activities including these topics. Then, the researcher handed out the "Postman" activity sheet (see Appendix C), and asked them to try to answer the problem. The PTs were solicited to think aloud while solving the problem. They tried to answer the problem, but they had difficulties in understanding and simplifying the problem. After waiting them about 15 minutes to finish their work, the researcher showed them sample solution of the problem by explaining each step of the modelling process. Then, PTs' solution papers' were gathered. Then, they went for a break about fifteen minutes. When, all the students came back, the researcher handed out another problem, "Bus stop" (see Appendix C), and asked them to try to answer. After waiting them about 15 minutes to finish their work, the researcher again showed them sample solution of the problem by explaining each step of the modelling process. Then, PTs solution papers' were collected.

Four days after the first study, another study was conducted with different groups of PTs. The same procedures were applied in this group like the procedures done in the first group. However, the researcher firstly asked the different version of the “Bus stop” problem for this time.

Three days after the second study, another study was conducted with the first group again. For this one, “Forest management” problem was asked, and again students were solicited to think aloud while solving the problem. Firstly, the students were told to read and try to understand the problem on their own. Some students tried to understand on their own, and some others tried to understand with the help of group members. Nevertheless, between fifteen to twenty minutes later, all the students tried to answer the question with the group members. The students tried to explore the problem about an hour. Meanwhile, the researcher got around the tables, and asked the students “Did you understand what the problem was about? Could you tell me?”, and then the researcher attempted to clarify the vague conditions with regard to their answers. After these explanations, when they sought to identify the variables and conditions, the researcher asked questions to the class such as “Does the variable affect the solution? To what degree does the variable affect the solution?”. Most of the time, students took into account so many variables which made the solution of the problem difficult. Hence, at those times, the researcher asked that “Is it easy to solve the mathematical problem under the present conditions?” “How can we set up conditions in order to solve the mathematical problem easily?” Although some of the students understood what the problem was about, most of the students had difficulty in understanding the problem and mathematizing the problem situation. Therefore, for this activity, the modelling problem was tried to be solved with interaction between the students and the researcher. After the activity, the interviews were conducted with two students.

Table 11 The structure of the pilot study

Applications	Groups	Modelling activities	Duration
First (14 May 2010)	Group1	Instruction on modelling	Fifty minutes (including break)
		“Postman” activity version 1 Presenting sample solution of the task	Half an hour
		“Bus stop” activity version 1 Presenting sample solution of the task	Half an hour
Second (18 may 2010)	Group2	Instruction on modelling	Fifty minutes (including break)
		“Bus stop” activity version 2 Presenting sample solution of the task	Half an hour
		“Postman” activity version 2 Presenting sample solution of the task	Half an hour

Table 11 (continued)

Third (21 may 2010)	Group1	"Forest management" activity	Two class hours After the individual work on half an hour on the task, class work by the guidance of the researcher

Interviews

Interviews were conducted by the researcher with the two students of Group 2, PT1 and PT2, even after the second application, and were performed with a student of Group 1, PT3, right after the third application of the pilot study. All of these three PTs voluntarily participated in individual semi-structured interviews after the application of related modeling activities.

The interviews took approximately forty-five minutes. Thus, in addition to the students' solution papers related to the modeling activities, data from the interviews provided information about the PTs' thought processes. All the interviews were audio taped and then listened.

Since the researcher was the instructor of the lesson, it was hard to observe all the details of the students' modelling processes. Therefore, interviews were conducted to understand the details of PTs' modelling processes, to understand in which steps they encounter with obstacles, and understand the reasons of these obstacles from the participants' point of view. The aim of the interviews was to obtain a more complete picture of the modeling process, so interviews also based on the PTs' solution papers. The interviews consisted of the questions used in the main study.

Findings

The reasons of the pilot study could be given as follows:

a) Developing and testing adequacy of some of the modelling activities,

b) Assessing the problems which might occur by using proposed methods,

c) Determining the sample size,

d) Determining what resources (time, staff) are needed for a planned study,

e) Collecting preliminary data,

f) Training the researcher in as many elements of the research process as possible (Van Teijlingen & Hundley, 2002). The findings of this study were given according to these topics.

a) One of the reasons for conducting the pilot study was to develop and test the adequacy of modelling activities. The pilot study allowed the researcher to see the nature of the students' answers and their approaches to the activities, and this provided information about the preparation of the appropriate modelling activities.

Related to the first version of the "Postman" problem, some of the PTs in the class stated that the paths (or models) that the postman follows should not be given in the problem text after the end of the first class application. An excerpt from the audio-taped classroom discussions at the end of the activity is below.

PT3: You know, here you [referring postman version 1] had given us two options, here whether he will proceed two sides of the streets along the street or will go one by one, you know, normally If just the problems were given here and we thought the proceeding ways, would not it be better? You gave two models here, but I think that making more different models and testing them could have been better…That is because I felt that we had been made to analyze the two models. (towards the end of the application of the Postman activity)

In line with the comments coming from the PTs, the newer version of the "postman" problem was prepared and applied to the other class. After the application, the views of a PT were asked in the interview and similar explanations were made by this PT. She explained that the paths that the postman should follow should be asked to the students, instead of giving in the problem text.

R: Do you think that firstly giving the kinds of ways the postman will follow, and then which way is suitable should be asked? (interview)
PT2: I do not think it should be so.
R: So if I said there were two methods?
PT2: No, I think the student should be made to find it.

Similarly, for the second version of the "Postman" problem, PT1 explained in the interview that the problems should be open-ended.

PT1: I think the problems should be open-ended in order not to reach the results directly.
… ready information should not be always given to the students like those formulas. Their discovery should be provided.

For the second version of the "Postman" problem and the second version of the "Bus stop" problem, when their solution papers were analyzed, it was seen that PTs answered the problems intuitively without any mathematical attempts. In the "Postman" activity, they created some cases and answered intuitively according to those cases. It was also seen that approaches of PTs in both groups to the first and the second versions of the postman problem did not change. They used intuition. As a result, based on these and considerations above, it was decided to apply the second version of the "Postman" problem in the main study.

PT2: First of all, from where I should start was a problem because I had never seen such an problem, there was nothing numerical, then how should I start?(interview-Bus stop problem version 2)

R: Well, if we said that it made the walking distance the least in the problem? (Postman problem version2)
PT2: Yes, but then you were limiting it. The walking distance already comes into everyone's mind, but if it was said, it would seem to me more mathematical. Then I would say x and y [smiling], but we did it like more verbal.

R: What did you think in your first reading? (Postman problem version 2)

PT2: In my first reading, I started to draw with paper and pencil [smiling] I said that he basically goes straight ahead, crosses to the other side, and gives one by one.

Related to the first version of the “Bus stop” problem, when PTs’ solution papers were analyzed, it was seen that almost every student used the trial and error method to solve the problem. They tried the points especially B, C, D, and E to find the point that minimizes the total travelling distance. Most of the students could not answer all the questions asked by the problem because of the expiry time of the lesson. It was seen that none of the students tried it with the mentioned modelling process. To eliminate this approach, newer version of the problem was formed more open ended without giving the numerical values. It was seen during the application that the newer version of the problem increased students’ participation to the solution process. Related to this, PT1 explained in the interview that PTs shared their ideas and continued to discuss on the problem during the lesson although other modelling problem, “the postman”, was handed out to the students. Moreover, PT1 explained in the interview that the order of the activities should be changed by deciding the “Postman” first and the “Bus stop” next.

PT1: After this activity [referring Postman version 2] solving this one [referring Bus stop version2], personally, this one seems simpler [referring postman version 2], perhaps it [referring postman version2] should be given firstly since the ways seem to be apparent. You will go on, but a lot became a part of it, the distance to the bus stop, where to start; evens, odds were discussed much more if you were aware. Maybe it should have been given beforehand. Even you gave this one, everyone was discussing the other.

After the application of the second version of the “Bus stop” activity, it was observed that students were more motivated to solve the problem so that the two of the groups could solve the problem correctly. Then, the researcher asked the newer version of the “Postman” problem. This time, only a few students tried to simplify the problem. It was also observed that some of the students did not attempt to solve the problem.

After the analysis of the students’ answers and approaches to the problems, and the interviews conducted with three students, it was decided to design the problems as little bit more open ended. The analysis of the students’ answers, and their motivation during the activities, and comments related to the activities in the interviews showed that the last versions of the problems were more appropriate and well-matched with our purpose.

b) The reason for the pilot study was also to evaluated for the possible problems which might occur using the proposed methods. The class observations combined with the solution papers’ analysis showed that the time given for PTs to explore the modelling problems were very limited. They were given approximately twenty minutes to work on the problems (“Postman” and “Bus stop”). It was decided that the PTs should be given approximately two class hours to work on the problem in the main study.

c) In the classroom, there were eight round-tables, and students were sitting around the six of them with three to four groups. The observations showed that the students working in group of four could not work effectively on the activities. In general, two of them worked together and other two of them together, and, there were times that some of these students did not work with their groups and share the responsibility. Thus, the group cooperation and division of the group work could not be achieved in some of these groups. Based on these findings, to increase the individual responsibilities, it was decided that the students should work on the activities first individually and then in group in the main study.

During the analysis, it was realized that voice recorders on the desks could not capture the voices in most of the cases. It was seen that the students did not think aloud although they were said to speak aloud and tell what they were thinking while trying to solve the problems. Thus, the pilot study offered the selection of the subjects with good verbalization skills for deeper analysis in the main study.

Moreover, when the voice recordings were listened, it was seen that there were many times that the recorders could not capture the voices of the students because of the noises of the other groups. So, based on these findings, it was decided that, in the main study, maximum three students would be grouped, and the tables would be set apart from each other. Related to this, Student2 explained her difficulty during the process. She stated that they were mostly eager to work on the problem, but she explained that the groups should be placed far from each other to provide the independency and the effective work.

PT2: We were talking with the nearby groups, but asking to each other. They had done too long [referring second version of the "Bus stop" problem]. At that moment, I thought that we did it wrong

... Actually it would be better if the groups were far away because one was saying something like "if it had been so" while we completely focused on. That is, everything went away because you were focusing on at that moment, you were being asked how you could find. Then you forgot your own way at that time, followed the way of other, and so you became confused. In short, the groups should not be influenced.

d) Another reason for this pilot study was to determine what resources (time, staff) were needed for a planned study. By taking into account that point, the researcher was also the instructor of the lesson, and it was decided that one more person (staff) who was informed about the modelling process and aim of the study, and also who could help the control of the cameras and voice recorders would be needed in the main study.

During the applications of modelling activities, the researcher tried to observe the classroom for each group. In the classroom, besides the researcher, there was also an observer who was a doctorate student in mathematics education. Additionally, she was knowledgeable about the modelling process. She made an effort to observe the two groups and also took notes. However, she explained that it was hard to take notes and grasp their ideas because the expressions and ideas of the students developed very fast. It was seen that to be able to get the details related to the students' modelling attempt, and understand their modelling process, the researcher should be focused on maximum three groups. Furthermore, the researcher should monitor only these groups carefully during the lessons. These also revealed the need for stuff and video-recordings. Taking into all these considerations, it was decided that the main study should be conducted with maximum twenty students.

e) Collected preliminary data showed the details of the students' initial attempts in modelling activities. This study demonstrated that although the students understood what the problems were asking for, they could not understand what they were expected to do as a solution for the "Postman" and for the "Forest management" problems. The data indicated that as the problems were open-ended and did not include any numerical information to directly answer the asked problem, PTs did not go beyond the intuitive answer and show any mathematical work. Although the process could not be observed in detail in the pilot study, it was seen that most of the students created cases and intuitively decided the way the postman should follow for the postman problem. During the interview, PT2 explained that the nature of the task also had an effect on their solution approaches. She explained that as the question did not refer to using any mathematics, they did not use mathematics, she also stated that if the question inferred the mathematical calculations such as asking for "minimizing the total travelling distance" they worked mathematically on it.

R: If we had not said total walking distance, what would you think? (for the "Bus stop" version 2)
PT2: If you had not said, we would not have write "a" [referring the using notation], but we draw when it was said. If you had also said the postman, I would have said the same.

R: Did the presentation [instruction about modeling] have an effect on your solution? (for the postman version2)
PT2: Yes for simplifying, but we did not transform into mathematical formula, the simplifying came to my mind. However, it seems that simplifying took precedence over the other. Maybe, simplifying is easier, but formulization sounds unpleasant [smiling]. We should have done that, but I could not remember at that time.

For the "Forest management" problem, students tried for some values to decide the time to cut the trees. As the limitations in their solution approaches were observed, after giving fifteen minutes to the students to work on the "Forest management" problem, the problem was explored through the interaction between the PTs and the instructor (Ikeda, 1997, p.54). For the "Postman and the "Bus stop" problems, students worked approximately half an hour on the problem. It was seen that the time given to the PTs was very limited when considering their inexperience with the modelling activities.

R: well, was that [referring the problems] nice or useful?
PT1: Yes, but if one does it himself/herself… because our time is limited [smiling].
R: Ok, if we do it again, should we give more time? For example, should it be two, two, four hours?
PT1: I think so, because they are not so much easy to do at once. Even, I don't think that four-hour will be enough. Perhaps, by giving homeworks, projects, they may be encouraged to think in that way [referring all the questions]

PT3: This solution is not the one that can be explored within two hours. We need more time. [referring "Forest management"]… I wish more time were given; this cannot be solved in two hours at all. If I worked more on that point, I could catch that point, too. However, assistance is crucial in some steps; otherwise I could not have continued.

Based on these findings, it was decided to begin the lesson with modelling activities which allowed students to produce answers and give chance to work on it such as the "Postman" and the "Bus stop" problems. In the light of the findings from the pilot study, among the eighteen modelling activities, some of them were revised and some of them were prepared by taking into account the mentioned considerations above.

The detailed lesson plan was prepared according to these considerations. In total, eighteen modelling problems were developed by the researcher before the implementation. This was six more than the required. This also helped the researcher arrange the lesson according to the phase of the class and the motivation level of the students.

The results showed that some of the students could not attempt to solve the problem, and most of them could not reach possible answers. Seeing their difficulties arising from inexperience in modelling process, for instance, in simplifying the problem, and then mathematizing the problem situation, it can be concluded that there is a need for developing PTs' modelling competencies, and thus showed that this research is worth studying and justified the significance of the main research study.

f) In her previous research studies, the researcher had tried to investigate the pre-service and in-service teachers modelling competencies in modelling activities, and saw their difficulties and tried to understand possible reasons for their difficulties in solving real-world problems. Thus, the researcher had some experience with similar research investigations. This piloting experience was also significant, and thus beneficial for the researcher, because previous studies were conducted with small groups of pre-service and in-service teachers, and this was the first time that she had ever tried. This also made her competent and knowledgeable in as many elements of the research process as possible.

The pilot study helped to see what was going on in the classroom, and provided information about the class environment during the modelling activities. This study helped to see possible shortcomings, and designed a well planned study.

APPENDIX B

MODELLING PROBLEMS

Name- Surname: …………………………………………..
Date :……………………………………………

Postman

A postman needs to deliver the mail to both sides of a street. To do this, the postman could use different methods in terms of order of delivery. The postman needs to make a decision from where to begin and which way to follow. For you, which method would be better?

The "Postman" problem adapted from Swetz, F. and Hartzler, J. S. (1991) Mathematical modeling in the secondary school curriculum: A resource guide of classroom exercises. Reston, VA: NCTM. (pp. 38)

Name- Surname: ………………………………………..
Date :…………………………………………

Bus stop

Many factors must be considered when a company wants to open a new store or a plant. One of the most important considerations is where to locate the facility so that the distances travelled by suppliers and customers, or the distances its product must be shipped, are kept to a minimum. A company can save millions of Turkish Lira every year by properly locating its facility.

Similarly, when we consider a school bus, there is a need to decide a place of the school-bus shelter for a group of students living along a road. Determine where the shelter should be located so that the total distance the students have to walk is the minimum amount.

The "Bus stop" problem adapted from Swetz, F. and Hartzler, J. S. (1991) Mathematical modeling in the secondary school curriculum: A resource guide of classroom exercises. Reston, VA: NCTM. (pp. 29-30)

Name- Surname: ………………………………………..
Date   :……………………………………………

How to store the containers?

A small company in the production of canned goods needs to find short-term storage for some cylindrical containers. The company wants to do this in minimum expense. The containers are right circular cylinders with a radius of 10 cm and a height of 30 cm. The company plans to store 175 containers for two months.
Storage units are available for rent in three sizes. The unit sizes of the storage units, each of which has a height of 100 cm, and the rental costs are given in the table below.

Width (cm)	Height (cm)	Rent Cost for a month (TL)
110	110	100
110	220	150
110	330	200

1. If you were owner of the company, which storage unit do you choose in order to minimize the cost?

2. In later productions, the company may need to store a large number of containers.

Therefore, is it appropriate for the firm to place the cans to the same storage unit? What would you suggest to the company?

Note: The cans must be stored in an upright position, it is important for the security.

The "How to store the containers" problem adapted from Swetz, F. and Hartzler, J. S. (1991) Mathematical modeling in the secondary school curriculum: A resource guide of classroom exercises. Reston, VA: NCTM. (pp. 12)

Name- Surname: …………………………………………..
Date :……………………………………………

Let's organize a volleyball tournament!

The organizers of the volleyball camp want to have more competition in the camp's tournament. Thus, they need a way to fairly divide the campers into teams. They have compiled information, given in the table below, about some of the players from tryouts and from the coaches.

Your task is to split the players into three equal teams and to write a report explaining how you created your teams. Organizers will use your process for the next camp when they need to split a large number of players into equal teams. Thus, you need to make sure that your process for creating teams will also work for a very large number of players.

VOLLEYBALL SPIKES! In volleyball, spikes are often classified as follows:
Kill: The other team was unable to return the ball. **Out of Bounds:** The hitter spiked the ball out of bounds so the other team gets the serve. **Returned:** The other team returned the spike **Dink-unreturned:** The hitter faked the spike and only tipped the ball over the net. The other team failed to return the dink. **Dink-returned:** The hitter faked the spike and only tipped the ball over the net. The other team returned the dink. **In the Net:** The hitter failed to hit the ball over the net

Coach's Comments

Gizem	She is tall, but slow getting to the ball.
Betül	She is very agile on her feet.
Candan	Her height could prove to be an asset for any team.
Aliye	She is an awesome leaper, but she needs to know when to use it.
Ayla	She comes from teams that have not been successful.
Demet	She has great quickness to get to the ball after serves.
Rana	She plays best when the team is playing well.
Canan	Her family life has negatively impacted her ability to play well.
Aslı	She is exceptionally strong for her age.
Nilay	She does many things well. In particular she serves well.
Yeliz	She is a great blocker.
Elif	She is the hardest worker we've ever had at the high school.
Nalan	She is a girl that others want to be with because whatever event she's in, she seems to always find a way to win.
Seda	She does not always get her serve over the net.
Naz	She is one of the most intense players we have ever seen.
Eda	Her father coaches at a local school.
Gamze	Her sister is a very good volleyball player at Ankara University.
İpek	She is very coachable.

Data from Volleyball tryouts

Name of the player	Height of the player (m)	Vertical leap (cm)	40 meter dash (sn)	Number of serves successfully completed out of 10	Spike results (out of 5 attempts)
Gizem	1.85	50.8	6.21	8	Dink-Returned , Dink-Unreturned, Kill, In the Net, Returned
Betül	1.57	63.5	5.98	7	Kill Returned Out of Bounds Dink-Returned Kill
Candan	1.78	60.96	6.44	8	Out of Bounds Returned Returned Kill In the Net
Aliye	1.78	68.58	6.01	9	Kill Kill Dink-Unreturned Kill Returned
Ayla	1.68	63.5	6.95	10	Out of Bounds In the Net Returned Returned Dink-Returned
Demet	1.73	43.18	7.12	6	Kill Dink-Unreturned Kill Returned Kill
Rana	1.60	53.34	6.34	5	Out of Bounds Kill In the Net In the Net Dink-Returned
Canan	1.65	58.42	7.34	8	In the Net Kill Kill Kill Dink-Unreturned
Aslı	1.65	60.96	6.32	9	In the Net Out of Bounds In the Net Out of Bounds Returned
Nilay	1.70	48.26	8.18	10	Dink-Unreturned Kill Kill Out of Bounds Returned
Yeliz	1.75	58.42	6.75	7	Dink-Returned Kill Returned Out of Bounds Kill
Elif	1.73	38.1	5.87	8	Kill Kill Kill Dink-Unreturned In the Net
Nalan	1.63	53.34	6.72	8	Kill - Returned - Out of Bounds - In the Net Dink-Returned
Seda	1.70	48.26	6.88	9	Out of Bounds In the Net In the Net Kill Returned
Naz	1.55	60.96	6.27	6	Dink-Unreturned Dink-Returned Dink-Returned Kill Out of Bounds
Eda	1.78	58.42	6.54	8	Out of Bounds Kill Out of Bounds Out of Bounds Dink-Returned
Gamze	1.60	66.04	7.01	9	Dink-Unreturned In the Net Kill Kill Kill
İpek	1.75	45.72	6.78	10	In the Net Out of Bounds Kill Dink-Returned Kill

The "Let's organize a volleyball tournament" problem adapted from Lesh, R., Yoon, C. Zawojevski, J. (2007). John Dewey Revisited- Making mathematics practical VERSUS

making practice mathematical. In Lesh, R. A., Hamilton, E. & Kaput, J. (Eds.) *Foundations for the future in mathematics education*. (pp. 315-348). Lawrence Erlbaum Associates, New Jersey.

Name- Surname: ………………………………………..
Date   :……………………………………………

Who wants 500 billion?

The hit television game show "who wants to win 500 billions?" or with its new name "who wants to win 500 thousands?", which has been on screens in many countries and has taken place in our country since 2000, continues to be on the screens in some periods.

While watching the game show we might thought that if I were the contestant I would take the risk and continue in the competition or I would withdraw from the competition and receive the amount earned.

In this quiz show, ten contestants compete for the right to play the game. The contestants are presented with a question and a list of four answers. Then, they are asked to put those answers in a particular order. The contestant who does it correctly in the shortest amount of time is awarded for playing the game. The rules of the quiz show are as follows:

Question	Money Won
1	50 TL
2	100 TL
3	200 TL
4	300 TL
5	500 TL (guaranteed)
6	1000 TL
7	2000 TL
8	4000 TL
9	8000 TL
10	16.000 TL (guaranteed)
11	32.000 TL
12	64.000 TL
13	125.000 TL
14	250.000 TL
15	500.000 TL

(1) The contestant must answer 15 multiple-choice consecutive questions correctly in order to win 500 thousand TL. The cumulative amount of money won after answering each question correctly is given in the table on the right side. If the contestant answers a question incorrectly, no further questions are asked and the amount of the winnings reverts to 0 TL, 500 TL or 16.000 TL depending on which question is missed. When a contestant correctly answers the 5th question, the 500 TL accrued at that point cannot be lost regardless of what happens in later questions. Similarly, if the 10th question is answered correctly, the contestant is assured of leaving with 16.000 TL.

(2) At any point, the contestant can choose not to respond after seeing the question. In this case, the contestant keeps the amount of money earned at that point, but cannot continue.

(3) The contestant has 3 lifelines that can be used at his or her discretion. These lifelines allow the contestant to seek help from the audience, phone a friend, or reduce the number of choices from 4 to 2.

Ask the audience: Audience members answer the question based on what they believe the correct answer to be.

Phone a friend: The contestant calls one of the friends who provided their phone numbers in advance and ask the question to answer.

50/50: The computer eliminated two incorrect answers, leaving one incorrect answer and the correct answer.

(4) The contestant can use as many lifelines as desired per question, but each lifeline can only be used once per game.

Consider what you would do in the scenarios given below.

(a) A contestant has correctly answered the first nine questions, but has no idea what the correct answer to the 10th question is. The contestant has already used the ‘ask the audience’ lifeline and the ‘phone a friend’ lifeline. Therefore, his options are to leave away with 8.000 TL or to make a guess among the answers and risk leaving him with 500 TL. If, however, the contestant guesses correctly, he cannot finish with less than 16.000 TL and he has a risk-free chance of increasing their earnings to 32.000 TL.
If you were in such a condition, what would you do? What do you think about whether the contestant should continue to compete or not? Examine possible alternative situations and discuss on the alternatives that are more reasonable with mathematical data.

(b) What is the probability of correctly answering the 15 questions?

(c) To be able to have the right to play the game in each episode, the contestant must win the fastest finger competition. In this competition, 10 contestants are asked to place four items in order. The order might be chronological, east to west, largest to smallest, etc. The contestant who correctly orders the choices in the shortest period of time is the winner. Therefore, in order to be successful in the ‘fastest finger’ competition, the contestant should use some strategies. When the contestant sees the question and has no idea about the order of the two item what should he do: immediately entering the answer based on a random guess, or thinking on the correct order and wasting valuable time?

As a part modeling task development stage of a project supported by TUBITAK (Grant no 110K250), the “Who wants 500 billion” problem adapted from Quinn, R. J., (2003). Exploring the Probabilities of ‘Who Wants to be a Millionaire? *Teaching Statistics 25*(3), 81-84.

APPENDIX C

INITIAL VERSION OF THE PROBLEMS

Name- Surname: …………………………………………..
Date   :……………………………………………

Postman

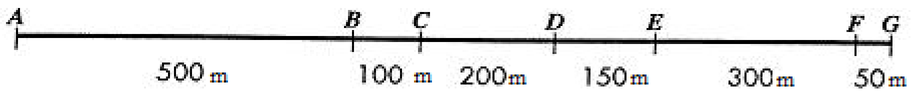
A postman needs to deliver the mail to both sides of a street. To do this, the postman could use different methods in terms of order of delivery. He can deliver to all the boxes on one side, cross the street, and deliver to all the boxes on the other side. Or he can deliver to one box, cross the street, deliver to two boxes, cross again and deliver to two boxes, and so forth, until all the mail has been delivered. Which method is better?

Name- Surname: …………………………………………..
Date :……………………………………………

Bus stop

Many factors must be considered when a company wants to open a new store or a plant. One of the most important considerations is where to locate the facility so that the distances travelled by suppliers and customers, or the distances its product must be shipped, are kept to a minimum. A company can save millions of Turkish Lira every year by properly locating its facility.

Similarly, when we consider a school bus, there is a need to decide a place of the school-bus shelter for a group of seven students living along a road. The distances between their houses are given below. Determine where the shelter should be located so that the total distance the seven students have to walk is the minimum amount.



a) Suppose that more than one student lives in each house. Determine where the shelter should be located?

b) Suppose that six houses were located as illustrated below. Determine where the shelter should be located?

c) Write a function that would determine the total distance travelled if the shelter were located at any point on the road between the houses.

APPENDIX D

INTERVIEW QUESTIONS

In this interview, you are expected to recall your experience during the activity, and gave as many details and examples as possible to explain the solution process in detail. The questions to be asked consist of the proposed questions only. Without limiting yourself with these questions, you could evaluate and criticize anything about the activity.

1. What was the problem situation that you focused on ? What was your aim in this problem?

2. Thoughts before attempting to solve the problem.

a) What did you think about the problem situation?
b) Did you fully understand the problem situation? If you did not understand, what did you do?
c) What was the first solution method that came to your mind to solve the problem (please state even it was wrong)? etc.
d) After reading the problem, did you think whether you could solve it or not? What made you think so?

3. Please define your thought processes from the beginning to the end (please explain even if it is wrong)

a. How did you begin to solve the problem, after understanding what the problem was about?

b. What were the ease and difficulties you encountered while dealing with the problem? What were the blockages? To overcome the blockages, what did you do? Are there any points that you thought that you understand, but during the solution you faced with difficulty and read the problem again? What were they, please give the details.

c. While formulating the problem what were the assumptions and situations about the problem that you took into consideration? How did you determine these assumptions? What factors affected this process (group discussions, previous knowledge, etc.)? How did you use these assumptions in the solution process? What were the mathematical ideas, concepts or strategies you used in the solution process?

d. How did you use different kinds of representations (graphs, tables, drawings, etc.) in different stages of the process: understanding, solving, verifying?

4. During the solution process, did you use solution methods that you used previously? Please explain.

5. When you could not arrive at any solution, did you change your solution method? Did you check the solution steps? Did you ask yourself what I was doing or how I was doing? Please explain.

6. During the solution process, did you ask yourself "Is there a way to solve this problem easier". If yes, please explain at which points and how you thought.

7. How do you interpret your findings? How do you show the validity and usability of your solution,?

8. How do you generalize your results? How do you show the accuracy of your generalization?

9. Did you use the learned technologies/software while solving the problem? If yes, how did you use? What can you say about the contribution or limitations of technology?

10. While attempting to solve the problem, you might have tried ways that you could not get a solution. These ways could also help you understand the problem and reach a solution. Explain if there are such interesting ways.

11. What did you learn after you solved this problem?

12. How did you rate your performance in the solution process?

13. Do you think that the group work was efficient for you? How?

14. What kind of problems did you observe during the application of the activity? How could we develop this activity to be more meaningful and useful?

15. If you implemented this activity in the classroom, what would the results be? Explain the reasons.

APPENDIX E

INFORMATION FORM

Name, Surname: Date:

1. Introduce yourself briefly. Provide brief information about your academic background.(high school, courses taken at university, CGPA-Cumulative Grade Point Average-, interests)

2. How would you rate your knowledge and skills in mathematics lesson? Are there courses/subjects that you think you are good at or slightly weaker than others? Explain.

3. Evaluate your skills in computer use.

4. Have you ever heard the phrase “Mathematical modeling” before? Explain what you understand from this statement with examples.

5. Have you taken a course on mathematical modeling before? Please write in short.

6. Have you ever solved modeling activity? Can you give examples?

7. If you were given a real-life problem situation, what way could you follow to find a solution to the problem?

8. Do you think that university education you have received during your studies is useful in solving real-life problems? Explain briefly.

9. What could be the ease and the challenges in the use of real-life problems in math class?

APPENDIX F

SYLLABUS OF THE LESSON

SSME 402
Technology Supported Mathematics Teaching
Fall 2010

Instructors:
Assist. Prof. Dr. Bülent Çetinkaya
Office: EF-215, Telephone: 210 3651, e-mail: bcetinka@metu.edu.tr
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Time: Tuesdays, 13:40 – 17:30

Place: EF-A38 - Mathematics Education Research and Technology Lab.

Office Hours: Mondays, 9:30-10:30, or by appointment

Catalog Description: Models and modeling approach on mathematics problem solving, learning and teaching. Use of technological tools through modeling activities. Applications of mathematics to model real-world problem situations of different branches of mathematics. Training teachers in modeling process: understanding the real-world problem situations, making assumptions, formulating mathematical problem, solving the mathematical problem, interpreting the solution, verifying the model, reporting, explaining and making predictions based on the model. Implications of modeling in the classrooms, training PTs for teaching modeling in the classrooms.

Course Objectives: Upon successful completion of the course, students should be able to

- ✓ Develop their modeling competencies such as, understanding the real-world problem, setting up a model based on the reality, solving mathematical questions within the mathematical model, interpreting mathematical results in a real situation and making decisions about the results whether they need revisions or extensions, and whether they satisfy the conditions and assumptions given in the problem.
- ✓ Learn to apply their mathematical knowledge and skills to solve real-world problems.
- ✓ Develop their reasoning and communication skills using mathematical language, notation, diagrams, and graphs.

- ✓ Improve their knowledge about the use of technology in teaching and learning mathematics.
- ✓ Understand the characteristics of modeling activities.
- ✓ Learn how to use modeling activities in mathematics teaching

Reference Resources

Hodgson, T. (1995). Secondary mathematics modeling: Issues and challenging. *School Science and Mathematics, 95*(7), 351-358.

Doerr, H. M. & Tripp, J. S. (1999). Understanding how students develop mathematical models. *Mathematical Thinking and Learning, 1*, 231-254.

Lesh, R. Hole, B., Hoover, M., Kelly, E., & Post, T. (2000). Principles for developing thought-revealing activities for students and teachers. *Handbook of research design in mathematics and science education*. (pp. 591- 645). Mahwah, NJ: Lawrence Erlbaum.

Maaß, K. (2006). What are modelling competencies? *Zentralblatt für Didaktik der Mathematik*, *38*(2), 113-142.

Maaß, K. (2005). Barriers and Opportunities for the Integration of Modelling in Mathematics Classes: Results of an Empirical Study. *Teaching Mathematics and Its Applications: An International Journal of the IMA*, *24*, 61-74.

Content Outline

Schedule		**Course Content**
Week 1	(Sept. 28)	Overview and organization of the course
Week 2	(Oct. 5)	Discussion on the importance of mathematical models and modeling in teaching and learning mathematics. “Postman" Problem & “Bus stop” Problem
Week 3	(Oct. 12)	"Icing Cake" Problem & "Gardening" Problem
Weeks 4	(Oct. 19)	“A Manufacturing" Problem
Week 5	(Oct. 26)	“How to store the containers?” Problem
Week 6	(Nov. 2)	“Birthday Party" Problem
Week 7	(Nov. 9)	"Let’s organize Volleyball Tournament" Problem
Week 8	(Nov. 23)	"An Irrigation" Problem
Week 9	(Nov. 30)	"Managing the Population" Problem
Week 10	(Dec. 7)	"Drug Therapy" Problem

Week 11	(Dec. 14)	"Forest management" Problem
Week 12	(Dec. 21)	"The Cashier" Problem & "Dentist Appointment" Problem
Week 13	(Dec. 28)	"Simple Pendulum" Problem & "Baby Bouncer" Problem
Week 14	(Jan. 4)	"Traffic Lights" Problem and Overall evaluation of the course

Assignments
More details will be given about each assignment later during the course.

Attendance and Participation:
Attendance is mandatory because of the nature of this course. Students will be expected to attend all the class and arrive on time unless there is a valid and legitimate reason that can be documented. Furthermore, class discussions of modeling activities for each week will be a significant part of this course. Students are expected to join the group works, and contribute to the group's studies and class discussions as speaking, listening, observing, sharing ideas and reflecting on the assigned readings and related materials. We expect everyone to engage actively in activities.

Activity: Students will complete a modeling activity per week. Each activity will be introduced and explained by the Instructors. Students will be expected to work collaboratively on the activities and complete activity sheet(s). Be prepared to discuss the activity for the next class.

Reflections after the activity (individual):
Students will write a reflection paper on each modeling activity that will have been presented throughout the semester. Students should use the set of questions that will be provided in the second week as a guide when developing their reflection paper. Reflection papers will be evaluated based on the degree of detail and examples provided in paper.

Project: Constructing a modeling problem and writing a reflective report: Students are expected to develop a new modeling problem from a real-world situation. After constructing the modeling problem, students are expected to

- find a solution to this problem,
- analyze and report their findings.
- describe their modeling and learning process
- discuss implications of modeling in teaching and learning mathematics

Grading

Attendance & Participation	:	10%
Modeling Problem	:	35%
Reflections after the activity	:	35%
Project	:	20%

Submission of assignments:

Late assignments will be accepted within seven days. However, a 10% per calendar day penalty will be enforced to late submissions unless a valid and legitimate reason (such as serious illness or family emergency) can be documented. In the case of a legitimate excuse, an official certificate must accompany the late submission. Assignments received more than seven calendar days after the due date will not be graded.

APPENDIX G

CONSENT FORM

This study will be conducted within the doctoral dissertation of Aysel Şen Zeytun under the supervision of Assist. Prof. Dr. Bülent ÇETİNKAYA. To reveal knowledge and skills of elementary school prospective mathematics teachers in relation to the use of mathematical modelling in teaching mathematics and to investigate the nature and development of this knowledge and skills through the designed pre-service training program constitute the subject of this doctoral dissertation. It is thought that the study will develop PTs' modelling skills and professional knowledge and skills about how the modeling activities should be applied at classes within the framework of the activities applied during the study.

During the study designed to achieve these objectives at the undergraduate level and planned to last for 12-14 weeks, the data will be collected through the following ways: (i) modelling test, (ii) survey questionnaire, (iii) solution papers PTs use in modelling activities, (vi) observations supported by audio and video recorders, (v) interviews, (vi) reflection papers after the applications, (vii) modeling problems prepared by PTs.

The collected data during the study will be kept completely confidential, and only be evaluated by the researchers. The findings will be used in dissertation and scientific publications. Participation in the study is entirely based on voluntariness. It is not expected a potential risk for the participants during the study period. However, for any reason during the study, you may want the data, which will be collected or received for the course requirements, not to be used for the purposes of thesis. This condition will not constitute a definitely negative situation for the evaluation of your course performance.

For more information about the study, you can contact with Assist. Prof. Dr. Bülent ÇETİNKAYA, Faculty of Education, Secondary Science and Mathematics Education, METU (Phone: 210 3651; e-mail: bcetinka@metu.edu.tr) and Phd. student Aysel Şen Zeytun (e-mail: sen.aysel@gmail.com.tr). Thank you for your participation in this study in advance.

I know that I participate in the study completely voluntarily, and I can cut in half whenever I want. I agree to the use of the information given in scientific publications. (Please fill out and sign the form and then give back to the practitioner).

Name, Surname	Date	Signature	Course
	----/----/-----		

CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Şen Zeytun, Aysel
Nationality: Turkish
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EDUCATION

Degree	Institution	Year of Graduation
MS Gazi University	SSME	2007
BS Ankara University	Mathematics Dept.	2005

WORK EXPERIENCE

Year	Place	Enrollment
2010-2011	TÜBİTAK - SOBAG 110K250 Project,	Fellowship Researcher

FOREIGN LANGUAGES

English

PUBLICATIONS

Şen, A., Çetinkaya, B., & Erbaş, A. K. (2010). Mathematics Teachers' Covariational Reasoning Levels and Predictions about Students' Covariational Reasoning Abilities, *Educational Sciences: Theory & Practice, 10*(3), 1601-1612.

Çetinkaya, B., **Şen, A.** & Baş, S. (2008). Integrating Mathematical Modelling and Applications in Teaching and Learning Mathematics. *VIII. International Educational Technology Conference, IETC 2008 Proceedings,* 241-246.

Aydoğan-Yenmez, A., Erbaş, A. K., Çetinkaya, B., & Şen-Zeytun, A. (2011). Gerçekten Kanser mi, Değil mi?, *8. Eğitimde İyi Örnekler Konferansı*, p. 38-39.

Şen-Zeytun, A., Çetinkaya, B., Yıldırım, U., Erbaş, A. K. (2009). Pre-service Physics Teachers' Difficulties in Constructing Mathematical Models of Simple Harmonic Motion. *14th International Conference on the Teaching of Mathematical Modelling and Applications, University of Hamburg, Germany*.

http://www.ictma14.de/media/files/ICTMA14_Abstracts_FINAL.pdf

Şen-Zeytun, A, Çetinkaya, B. (2011). Understanding Pre-Service Mathematics Teachers' Difficulties in Constructing A Mathematical Model, 10-15 July, *35th Conference of the International Group for the Psychology of Mathematics Education*, Middle East Technical University, Ankara, Turkey, p. 387.

HOBBIES
Jogging, travelling, reading books.