MODELING AND CONTROL OF A HEATED TANK SYSTEM WITH VARIABLE LIQUID HOLD-UP

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MODELING AND CONTROL OF A HEATED TANK SYSTEM WITH VARIBALE LIQUID HOLD-UP

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ABSTRACT

MODELING AND CONTROL OF A HEATED TANK SYSTEM WITH VARIABLE LIQUID HOLD-UP

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July, 107 Pages

The aim of this thesis study was the modeling and controlling of a heated tank system with variable liquid hold-up. Mathematical model representation of the process was developed using mass and energy balances. Experiments were conducted to obtain the correct model parameters of the process. After obtaining the model parameters, linearized form of the model was developed. Proportional-Integral-Derivative (PID) controller parameters are found at different nominal conditions using a Simulink code that estimates optimum controller parameters by minimizing the time weighted absolute error of the response. Then, starting at different nominal conditions PID and Model Predictive Controller (MPC) performances were evaluated. MPC and PID controllers were implemented on the nonlinear process model using Simulink program. The results showed that MPC gives best response behavior for both set point tracking and disturbance rejection cases.

Keywords: Model predictive control, proportional-integral-derivative control, integral time absolute error

ÖZ

DEĞİŞKEN SIVI YÜKSEKLİKLİ ISITICILI TANK SİSTEMİNİN MODELLENMESİ VE KONTROLÜ

Tarık Yücel
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Temmuz, 107 Sayfa

Bu çalışmada , ısıtıcılı ve değişken sıvı yükeklikli tank sisteminin modellenmesi ve kontrolü amaçlanmıştır. Kütle ve enerji denklikleri yapılarak prosesin matematiksel modeli geliştirildi. Doğru model parametrelerini elde etmek için deneyler yapıldı. Model parametreleri elde edildikten sonra, sistemin lineerize edilmiş modeli oluşturuldu. Farklı olağan koşullarda mutlak toplam zamansal hatayı minimize eden Simulink kodu kullanılarak oransal-integral-türevsel (PID) kontrolcü parametreleri bulundu. Daha sonra farklı koşullarda PID kontrolcü performancı analiz edildi. Model öngörülü kontrol (MPC) ve PID kontrolcüleri Simulink programı kullanılarak lineer olmayan modele uygulandı. Sonuçlar hem set değeri değişimlerinde hem de sistemi bozan etkileri bertaraf etmede MPC'nin PID'den daha iyi bir performans ortaya koyduğunu gösterdi.

Anahtar Kelimeler: Model öngörülü kontrol, oransal-integral-türevsel kontrol, mutlak toplam zamansal hata

To my family

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Last and most special thanks to dear Burcu who will become my lifelong friend. She waited for me with her unlimited patience.

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NOMENCLATURE

ρ = Density of water, g/ml

C_p = Specific heat capacity of water, J/(g.^{0}C)

C_{p_h} = Specific heat capacity of heater, J/(g.^{0}C)

T_h = Temperature of heater, ^{0}C

T_1= Temperature of inlet water, ^{0}C

T= Temperature of water in the tank, ^{0}C

w_1= Inlet water flow rate, g/s

q_2 = Volumetric flow rate of discharge flow, ml/s

C_v = Valve flow coefficient

Q = Heat input, J/s

m_h= Mass of heater, g

h_h = Convective heat transfer coefficient, J/(s.m^2. ^{0}C)

A = Bottom area of tank, cm^2

A_h = Surface area of heater, cm^2

V = Volume of water in the tank, cm^3

a = Constant that relates water height to the flow through valve

θ = Time delay, s

τ = Process time constant, s

K= Gain

K_c = Controller gain

τ_I= Integral time constant, s

τ_d= Derivative time constant, s

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CHAPTER 1

INTRODUCTION

1.1. Objective and Thesis Overview

Processes with more than one input and more than one output are known as Multi input-Multi output (MIMO) systems. Controlling these MIMO processes is not an easy task. In these systems there are several controlled and manipulated variables and the numbers of these variables are not necessarily same. One of the manipulated variables can affect some or all controlled variables, due to process interaction. Also, one control loop can affect other control loops (control loop interaction) which may cause unexpected behaviors.

MIMO systems are encountered in various industries. Paper and pulp industries, petroleum refineries, chemical companies are a few examples of the industries in which MIMO processes exists. To avoid control loop interactions and provide a good control, proper control strategy must be selected.

In this study a heating water tank system with variable hold up was considered as a MIMO process since it has four inputs and two outputs. Temperature and level of the liquid in the tank are the output variables to be controlled. Four inputs are further categorized as manipulated and disturbance variables. The height in the tank depends on two inputs which are the inlet flow rate and discharge valve opening. Temperature depends on all the four inputs which are inlet flow rate, discharge valve opening, inlet water temperature and heat input to the system through the heater.

The aim is to evaluate different control strategies for this MIMO system. For this purpose, Model Predictive Control (MPC) is selected as the strategy of choice. Proportional-integral-derivative (PID) control was also evaluated as the baseline controller.

In the first chapter of this study a brief introduction is given about the importance of process control following the information about MPC and PID control. In the second chapter information on literature overview of MPC is given with history of MPC development and studies conducted. MPC formulation, PID controller tuning and reduction of high order models are also told about. In Chapter 3 mathematical model representation of the process is developed based on material and energy balances. The linearized form of the model equations are also represented here. Input and output relations are obtained in terms of transfer functions. In the fourth chapter information about the LabView code is given, which is required to monitor and control the experimental set up. Signal filtering and calibration activities are detailed. In Chapter 5, system identification studies are summarized to determine some physical unknowns stated in the mathematical model. After finding these unknowns complete nonlinear model is formulated and simulation results that validate the model predictions are outlined. Chapter 6 begins with proposed PID tuning approach the method that we used for estimation of proper controller parameters. Comparison of Smith's tuning relations with proposed method are done for first order plus time delay (FOPTD) and second order plus time delay systems (SOPTD). PID controller settings are found using our proposed PID tuning strategy. In Chapter 7, PID and MPC controllers' set point and disturbance tracking performances are evaluated and study is concluded with chapter eight.

1.2. Why Control?

Stronger competition, strict regulations on environment and safety policies and changing economic conditions have placed significant requirements on product quality. To meet these tight quality requirements and to increase energy and raw material efficiency, chemical process control has gained significant importance across all process industries.

During its operation a chemical plant should satisfy multiple requirements such as safety, product specification, environmental criteria, operational constraints and economics. Safe operation of a chemical plant is the most important factor that must be considered to prevent accidents and for continued support of economic progress. Desired quality requirements are also important for reliability and continued existence in the market.

Variables such as concentration of chemicals, flow rate of effluents should be within a certain range as specified by the regulatory agencies. Since different types of equipment are used in the plant, constraints of these equipments due to operational factors should also be considered. Moreover, utilization of raw material and energy must also be evaluated for economical operation of the plant .To satisfy all the requirements stated above, continuous monitoring and control of the plant is primary objective [1].

Process variables commonly controlled in the chemical process industries are concentration, level, flow, temperature and pressure. The principle behind the control of these variables is to provide a stable environment for the process. Only then, safe operations and product quality requirements can be ensured [2].

Up to the 1940's most chemical plants were operated manually with manual control elements such as valves. Operators tried to adjust process elements to satisfy the process variables (temperature, pressure, level etc.) at desired conditions. For this purpose operators were required to have a strong background and experience about the processes to avoid unwanted process responses in a way that best satisfy the requirements. However this caused increased labor cost. In addition, in early 1950s more advanced and complex equipment were taken into service. Therefore, it was uneconomical and almost not feasible to operate the plants of integrated equipments manually. At this point automatic control becomes a viable strategy [3].

Automatic control strategies are mainly based on feedback and feedforward control methods. In feedback control, controlled variable is measured and according to the deviation from set point, corrective action is taken to maintain the controlled variable at desired conditions. For this control strategy it is not needed to classify and measure the disturbances. Corrective action is taken without considering the source of disturbances. On the other hand, corrective action does not occur unless a deviation from set point is observed. This is a significant disadvantage of feedback control, meaning corrective action only takes place when a deviation from set point is observed. This causes the controlled variables to operate at undesired conditions for a while [4]. In the feedforward approach potential disturbances are monitored and corrective action is taken to eliminate effects of disturbances. Feedforward control requires a system model to predict impact of disturbances. But, since it is difficult to specify all disturbances in the model, feedforward control is not the proper strategy alone. In many situations combined feedback and feedforward methods are preferred [5].

1.3. Model Predictive Control

Model Predictive Control (MPC) refers to class of computer control algorithm that adjusts the manipulated variables to optimize future plant behavior [6, 7, 8, 9, 10]. MPC, also known as, moving horizon or receding horizon control, is an advanced feedback control strategy used in many areas. Useful reviews of MPC are given in [9, 11], where more than 2200 industrial applications are stated.

In the MPC approach process inputs are computed to optimize future plant behavior over a horizon which is known as the prediction horizon. This computation is done by using the mathematical representation of process model considering the input and output constraints. Process outputs are predicted and according to the deviations of predicted values from actual outputs, an objective function is used to calculate another input sequence. This objective function tries to minimize errors obtained due to the difference of actual and predicted outputs. For each measurement interval this procedure is repeated and a new input sequence is determined until process outputs follow the anticipated output trajectory. This is the idea which is called as *receding horizon* that is represented in Figure 1.1.

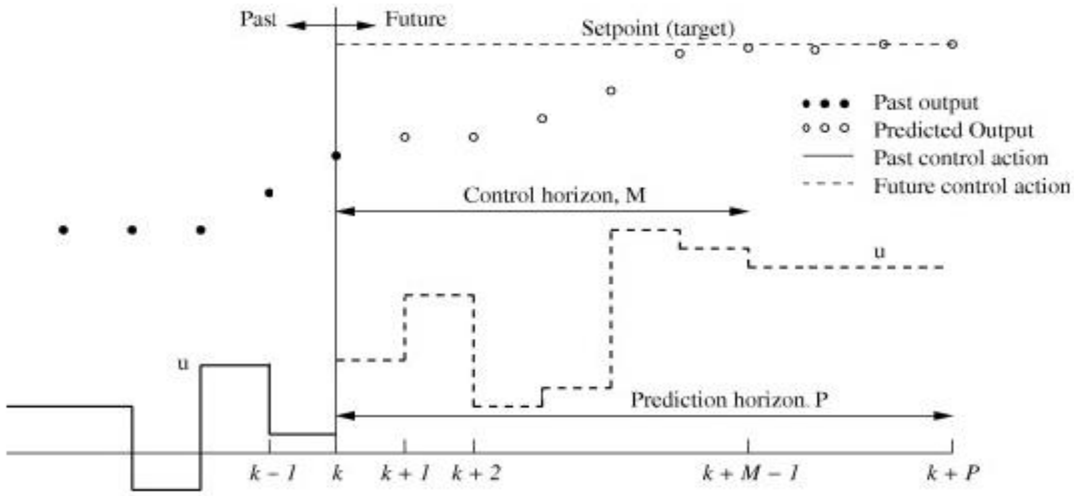

Figure 1.1 Principle of MPC

Since optimization is done using a process model, modeling is an important activity for the MPC strategy. But no model is perfect. If the model was perfect and there were no disturbances affecting on to the process, the first input sequence would yield desired output trajectory. Due to the model imperfections and disturbances affecting the process, predicted outputs and actual process response will be different. So manipulated input sequence will only be implemented until a new measurement becomes available. This advanced feedback mechanism can compensate the effects of disturbances and poor models [4, 12]. Further information about formulation of MPC was given in Chapter 2.

1.4. PID Control

Proportional-Integral-Derivative control is a classical feedback control method widely used in industry due to its simplicity, ease of implementation and convincing performance for different kind of processes [13]. PID Control consists of proportional, integral and derivative actions. Integral, proportional and derivative terms of PID control are based on the past, current and future values of the errors respectively. Application of PID control includes summation of these terms in a proper manner. Ideal form of PID control is stated as;

$$p(t) = \bar{p} + K_c \left[e(t) + \frac{1}{\tau_I} \int_0^t e(t)dt + \tau_d \frac{de(t)}{dt} \right] \quad \textbf{(1.1)}$$

The corresponding transfer function is;

$$\frac{P'(s)}{E(s)} = K_c \left[1 + \frac{1}{\tau_I s} + \tau_d s\right] \quad \textbf{(1.2)}$$

1.4.1. Proportional Action

Proportional term of PID structure provides a control action proportional to the current error signal. Expression of this action is stated below;

$$e(t) = r(t) - y(t) \quad \textbf{(1.3)}$$

$$p(t) = \bar{p} + K_c e(t) \quad \textbf{(1.4)}$$

Where, $e(t)$ is deviation of output from set point that is known as error signal, $r(t)$ is set point and $y(t)$ is measured output. The Controller output is represented as (t) , $\bar{p}$ is steady state controller output and K_c is the controller gain.

Since the proportional term provides control action proportional to the error signal, at small control errors, corresponding control action is also small, therefore excessive control attempt is avoided. Steady-state error (offset) produced is the main drawback of proportional only control.

In order to derive transfer function representation of proportional only control, deviation variables are required.

$$p^{'}(t) = p(t) - \bar{p} \quad \textbf{(1.5)}$$

$$p^{'}(t) = K_c e(t) \quad \textbf{(1.6)}$$

Rearranging the Eqn. 1.11 and taking the Laplace transform yields;

$$\frac{P^{'}(s)}{E(s)} = K_c \tag{1.7}$$

1.4.2. Integral Action

Integral action provides controller output, depends on the integral of control error over time.

$$p(t) = \bar{p} + \frac{1}{\tau_I}\int_0^t e(t)dt \tag{1.8}$$

Where, τ_I is integral time with units of time.

This action uses the accumulated error to calculate control output. Integral action ensures that the manipulated variable attains a value at which the steady state error is zero. However, integral action produces an oscillatory response. So, an improper value of integral time constant may cause stability problems. Since properly tuned integral term eliminates offset, it is widely preferred over P-only control.

Elimination of offset is an important factor that favors integral action. However, integral control cannot be used alone. This is because, corrective action occurs only after the error signal is sustained over time which causes controlled variable deviate from set point for a while [4]. Since proportional action acts immediately when a deviation is observed, integral action is used with combination of proportional term therefore offset is eliminated.

This combined proportional and integral action is named as proportional-integral (PI) control:

$$p(t) = \bar{p} + K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(t)dt\right) \tag{1.9}$$

After taking deviation variable, corresponding transfer function is;

$$\frac{P'(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s}\right) \tag{1.10}$$

1.4.3. Derivative Action

Derivative control action is based on the predicted future behavior of the errors. It predicts the future error signal regarding its rate of change and corrective action is taken without obtaining too much deviation from set point. With its predictive ability, derivative action is an important term and reduces settling time, but it is not sufficient alone. At constant errors derivative action has no action which means the existence of an output response with offset. At this point integral action comes in to the picture and takes corrective action.

There is a disadvantage of derivative term in PID controller. Any change in set point or noisy output signal which are the high frequency components obtained, causes an instantaneous change of error. Derivative of this change is infinity. This data is fed in to the PID controller which results an undesirable controller output that may cause actuator saturations. To decrease the sensitivity of these high frequency components, an approximation is done by adding a term called "derivative filter" to the derivative controller structure. The approximation acts as a derivative for low frequency signal components. The structure of a derivative controller with filter is shown in Eqn. 1.14. The term N is the filter coefficient which is taken as 0.1 for a select of choice [4].

Representation of controller output for derivative only action is stated as;

$$p(t) = \bar{p} + \tau_D \frac{de(t)}{dt} \quad \textbf{(1.11)}$$

Derivative action is not used alone for control purpose. It is used with proportional only or proportional integral control. Transfer functions of a PD controller with and without filter are;

$$\frac{P'(s)}{E(s)} = K_c(1 + \tau_D s) \quad \textbf{(1.12)}$$

$$\frac{P'(s)}{E(s)} = K_c(1 + \frac{\tau_D s}{N\tau_D s}) \quad \textbf{(1.13)}$$

For better control purposes combination of proportional, integral and derivative terms that yields a PID control structure should be used.

Final PID controller transfer function with derivative filter is;

$$\frac{P'(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{N\tau_D s}\right) \quad \textbf{(1.14)}$$

CHAPTER 2

LITERATURE SURVEY

2.1. MPC History

Wiener [14] and Hall [15] first proposed the idea of developing a feedback controller in which tracking square error is tried to be minimized. However this approach failed due to inadequate problem formulation and limited applicability in which only low order processes are considered.

Modern control concept was developed with the work of Kalman [16, 17]. He tried to determine when a control system can be said to be optimal. He worked on a Linear Quadratic Regulator (LQR). The aim was to minimize a quadratic objective function in which the process that is aimed to be controlled is described by discrete and state space form.

$$x_{k+1} = Ax_k + Bu_k \tag{2.1}$$

$$y_k = Cx_k \tag{2.2}$$

In the state space representations above, vectors u and y represent process inputs and outputs respectively. x is the vector that represents process state. According to the state space representations showed above, knowing the current state and inputs, future states of the process will be estimated. The idea that lies behind Kalman's work depends on the model described by equations 2.1 and 2.2. For the estimation of plant state from measured outputs, an approach termed Kalman filter is developed. LQR and Kalman filter together called the Linear Quadratic Gaussian (LQG) controller. Process constraints on inputs, outputs and states are not included in this control algorithm. Although LQG controllers were used many real world applications [18], its impact on control technology was not sufficient.

This development made the researchers and industrial people develop more advanced control strategy which makes online optimization of process at each control interval with considering process constraints. At this point MPC came in to the picture [9, 10].

Although model predictive control was developed and implemented by industrial people, the idea behind MPC principle was not a new concept. Propoi [19] first described moving horizon approach in which process outputs are measured at every time interval and a dynamic optimization problem is solved online to predict future behavior of the process. However implementation of this approach on real world scenarios was not easy enough at those times.

Application of MPC was first proposed by Richalet et al. [20]. The MPC algorithm developed by Richalet was named Model Predictive Heuristic Control (MPHC). Deterministic difference of this approach from Kalman's regulator was consideration of input and output constraints in the problem formulation. Process input and output relations are stated by discrete-time Finite Impulse Response (FIR) models. This model is stated below for single input single output (SISO) system:

$$y_{k+j} = \sum_{i=1}^{N} h_i u_{k+j-i} \qquad \textbf{(2.3)}$$

According to this representation, future outputs depend on past inputs. h_i is the input coefficients that makes correlations between past inputs and future outputs. The aim is providing predicted outputs close to the actual outputs. During this process, input and output constraints are considered at each measurement interval. For this purpose outputs and inputs are measured then corresponding coefficients are calculated. Then, with known outputs and coefficients, future inputs are estimated. At each measurement interval these coefficients are updated by small increment until plant and model outputs are equalized. Process outputs follow a reference trajectory while trying to reach set point. The speed of the response is determined by a time constant of reference trajectory. This time constant determines the robustness of the process. Long time constant means slow but more robust response while short time constant means fast but un-robust response. This methodology is valid only for stable processes.

Engineers from shell Oil Company developed their own MPC formulation which is named as Dynamic Matrix Control (DMC) [21]. Aim of dynamic matrix control is to provide outputs being as close as likely to the set points in least square sense. DMC algorithm has the ability of moving the system from one optimal steady state to another. Unlike quadratic linear programming (QLP), there are no optimal targets for input and outputs. To reach desired output targets, inputs may move far away from their optimal steady state values. An application of DMC to a fluid catalytic cracking unit was proposed by Prett and Gilette [22].

Although MPHC and DMC considered process constraints in their formulations, application of these methods on real systems with successful constraint consideration was still difficult. This problem was tried to be solved by a quadratic problem in which process constraints are considered explicitly. This approach is called as Quadratic Dynamic Matrix Control (QDMC) [23]. Garcia and Morshedi, [24] presented a paper in which application of QDMC to pyrolysis furnace. In this study they came up with applicable results showing smooth transition from one constrained operating point to another.

Prett and Garcia [25] proposed that including input and output constraints in only one objective function does not reflect the true process performance. Approach for the solution of this problem was names as IDCOM-M (identification and command-multi input multi output) and first explained by Grosdider [26]. Usage of two different objective functions is the main improvement of IDCOM-M approach. Minimizations of input and output objective functions are conducted separately by considering input and output constraints in a systematic manner.

Muske and Rawlings [6], provided an overview of Linear MPC approach. Linear state space, transfer function or convolution models are used by the controller. Development of the controller for infinite horizon provided the stability without controller tuning. Linear MPC considers the linear process model for optimization of inputs and outputs over infinite prediction horizon.

Hensen [8] and Findeisen [27] emphasized Nonlinear Model Predictive Control (NMPC) in which nonlinear dynamic models are considered. They focused on advantages of NMPC. Theoretical, implementation and computational aspects of NMPC were discussed for various processes. They come up with a result that NMPC gives good control performance for nonlinear multivariable processes in which linear control techniques are not sufficient.

Kwang and Jay H. Lee, [28] conducted studies on MPC of batch processes. They carried out experiments for a temperature control system. Their work was based on a time-varying linear system model which uses not only the current measurements coming into the batch but also, the information stored from the previous batches. This approach provided better performance despite disturbances and modeling errors.

Clarke [29], worked on Generalized Predictive Control (GPC) method which is effective self-tuning algorithm used for complex processes. He conducted studies on high speed control of robot arm. Difficulty stated for this control purpose was limited torque of the motor which causes saturation problem. Proposed method overcame this problem. GPC is based on minimization of long-range cost function. Clarke's results showed improved closed loop performance.

Kwon [30], presented receding horizon tracking control (RHTC) for time independent processes. This approach was derived from the receding horizon concept. RHTC is based on MIMO and time varying systems. Unlike classical predictive controllers which use output predictors, RHTC uses state observer. Control law defined for this concept is combined state feedback and feed forward control. In this study no disturbance was assumed and robustness properties were not considered.

Wang [31] conducted a study about application of GPC in the glass industry. Controllers which are used at different parts of the process were developed to improve the product quality. Good identification method and robustness of GPC with feed forward action produced successful results.

Ang Lee [32] conducted a study that compares model predictive control with classical PID control for a level control system. He developed process model equations and linearized these equations for simplicity. Some simulations based on MPC and PID were conducted on MATLAB environment. Developed controllers were implemented in experimental set-up under different process conditions. The results showed that MPC gives better and fast response than PID controller in presence of constraints.

Garcia [33], studied an approach called Unified Theory in which it is aimed to combine separate approaches like optimization and control. New techniques on optimization and control are given to develop instant technology. On the other hand, this concept required manipulation of setpoints beside load rejection. According to Garcia's work; the MPC developments up to that time were not translatable for every kind of processes. Therefore, many more developments are required in terms of constraint considerations, hardware capability etc. He named this systematic approach as "Unified Theory".

Badgwell [7], conducted a study in which a new methodology for target calculation of MPC was proposed. In this study, target calculation is considered explicitly to decrease effects of model uncertainties. Several simulations incluiding Shell control problem of heavy oil fractionator [34] have been provided to demonstrate the improvements possible with this approach.

C. Bonivento [35], examined the PID and predictive control performance of a heat exchanger. Experimental results showed better responses for predictive control in terms of set point tracking and disturbance rejection.

Henson [8] conducted studies about NMPC. Linear model predictive control (LMPC) was not applicable or difficult to incorporate for highly nonlinear process models. Concept of nonlinear model predictive control is almost same with linear model predictive control. Major difference comes from the usage of nonlinear dynamic models for process predictions and optimizations. Henson's work showed that NMPC gives good response for nonlinear multivariable processes with constraints. However, in many cases dynamic model of a process may be too complex to be useful for NMPC design. This is the major disadvantage of NMPC which makes it difficult to be applicable for highly complicated and nonlinear systems.

Lee [36] proposed a scheduled MPC approach for nonlinear processes. MPC controller designed based on process state by linearization of nonlinear model at every sampling interval is called scheduled MPC. The method that Lee used for this purpose is called

Hinging Hyperplanes .This method divides the system into several linear sections. According to the process conditions, corresponding linear model is used. In this study simulated results of CSTR and a batch fermenter were conducted to show performance of this methodology.

2.2. MPC Formulation

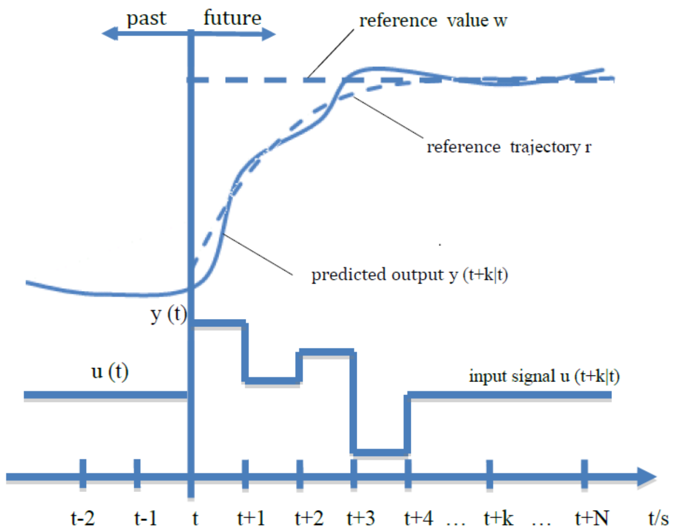

Figure 2.1 MPC idea following the reference trajectory [32]

Figure above shows the idea of MPC for a single input-single output system. Time t represents the current time. Plant output is stated as $y(t)$.Reference value w, reference trajectory r, and control signal $u(t+k|t)$ are also shown on the figure. Time from t to t+N is called the prediction horizon. Predicted outputs, $\hat{y}(t+k|t)$, are determined along that prediction horizon. Objective of MPC is providing the predicted outputs, $\hat{y}(t+k|t)$, being close to the reference value; w. Predicted outputs usually approach the reference value exponentially with following a reference trajectory, r.

The reference trajectory is defined to be as follows [37];

$$\in(t) = w(t) - y(t) \qquad \textbf{(2.4)}$$

in which $\in(t)$ is defined to be current error. Then, future error i steps later is represented by the formula;

$\in (t+i) = e^{-iT_s/T_{ref}} \in (t)$ **(2.5)**

In which T_s is the sampling interval, T_{ref} is speed of the response. Then;

$r(k+i|t) = w(t+i|t) - \in (t+i)$ **(2.6)**

$= w(t+i|t) - e^{-iT_s/T_{ref}} \in (t)$ **(2.7)**

To summarize the MPC methodology;

1. Predicted outputs, $\hat{y}(t+k|t)$, k=1… (N), are determined according to assumed input trajectory, $u(t+k|t)$, k=0…(N-1). Predicted outputs depend on the known values of past input and outputs obtained up to instant t.

2. A correction term for each element of predicted outputs are calculated according to the difference between predicted and actual outputs at each controller calculation interval. Model based value and correction terms are combined to produce estimated trajectory.

3. Predictive controller calculates new control sequence to reach desired trajectory. Control inputs are determined with minimization of square error between actual and estimated outputs.

4. Only first element, $u(t)$, of the sequence is implemented to the plant. Because at next sampling interval, corresponding output is measured and already known, this is probably different from predicted output. Therefore, step 1 is repeated in which a new output trajectory and input sequence is estimated again. This is the idea of receding horizon.

MPC is a very powerful control algorithm used in industry and studied in academia due to the reasons stated below [38];

- Structural changes of the process is being considered
- It optimizes over a trajectory
- Process constraints and limitation are considered in the formulation
- It is easy to tune
- It can be used for the control of multivariable processes

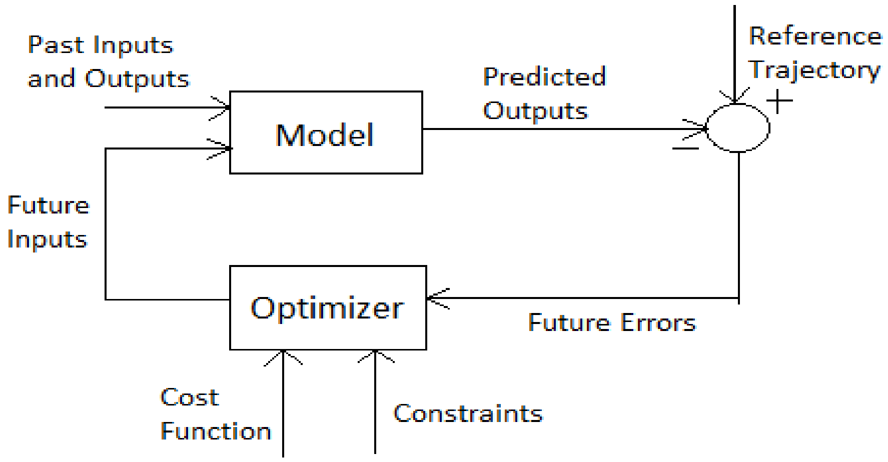

Figure 2.2 Structure of MPC

2.3. MPC Elements

2.3.1. Prediction Horizon (Tp)

The prediction horizon seen from Figure 1.1 is the number of samples in the future during which the MPC controller predicts the plant output. At each sampling interval prediction horizon shifts forward with new predicted output sequence. This horizon is fixed for the duration of the execution of the controller. Length of the prediction horizon determines the numerical effort required to solve the control problem. If prediction horizon is short, the controller receives only small amount of information about upcoming changes. This small amount of information reduces the predictive ability of controller. On the other hand long prediction horizon increases the predictive ability of the MPC controller. However, a long prediction horizon decreases the performance of the MPC controller by adding extra calculations to the control algorithm. Also, long distant estimations are not necessary due to inaccuracies as model is not perfect and disturbances change.

2.3.2. Control Horizon (T_c)

Control horizon is the number of samples in the future in which MPC calculates and only the present one is implemented. Like the prediction horizon, the control horizon also moves forward at each sampling interval. Control action does not change after the control horizon ends. A long control horizon provides more aggressive changes on control action. Short control horizon means more careful changes in the control action. So, optimum control horizon should be specified.

2.3.3. Cost Function

Cost function or objective function is the sum of squares of the predicted errors. Predicted errors are the difference between set points and predicted outputs. Cost function measures the performance of process model. The MPC controller calculates a sequence of future control action values such that a cost function is minimized. Type of cost function used in MPC is "quadratic" or "standard least squares". Weight matrices can be adjusted in this cost function. These weight matrices adjust the impact of each control action, rate of change in control action, and plant outputs. For a prediction horizon of T_p and a control horizon of T_c, the least squares objective function for single input-single output system is written in the form stated below;

$$J = \sum_{j=1}^{T_p} [w(t+j) - \hat{y}(t+j)]^2 + \lambda \sum_{j=1}^{T_c} \Delta u^2 (\mathrm{t} + \mathrm{j} - 1) \quad \textbf{(2.8)}$$

In the above equation T_p and T_c represent predictions and control horizons respectively, $w(t+j)$ is set point and $\hat{y}(t+j)$ is predicted output. Δu refers to change of manipulated input from one sample time to another. λ is weight for changes in the manipulated inputs.

2.3.4. Constraints

One of the most important characteristic of MPC is handling capability of process constraints. Constraints are the limitations of inputs and outputs. Input constraints may occur due to actuator limitations and operating conditions. For instance a valve that manipulates flow rate of a stream to a process can be adjusted around certain points due to the limiting valve opening.

$$u_{min} \leq u \leq u_{max}$$

$$\Delta u_{min} \leq \Delta u \leq \Delta u_{max}$$

u_{min} and u_{max} are minimum and maximum values of input while Δu_{min} and Δu_{max} are the minimum and maximum rate of change of inputs respectively.

Output constraints occur due to safety considerations or product quality requirements.

$$y_{min} \leq y \leq y_{max}$$

In its formulation MPC considers these input and output constraints in a systematic manner to ensure that these constraints are not violated.

2.4. PID Tuning

Controlling a process is primary objective of the process industry. For this purpose proper controller settings have to be adjusted. Controller tuning is the procedure to determine controller parameters that give best stable and robust response behavior with satisfying the required performance criteria. Performance criteria are usually evaluated considering the behaviors stated below; [4]

- Close loop stability of the process.
- Good disturbance rejection meaning disturbance effects are minimized.
- Good set point tracking, meaning high sensitivity and smooth responses to set point changes.
- Elimination of steady-state error.
- Providing a robust control system having sensitivity for wide range of process conditions and reasonable degree of model inaccuracy.

Controller settings can be adjusted based on process model (model based) as well as while the process is running (on-line). Model based controller settings provide good initial control. But it may not give sufficient control performance for complicated process models. Therefore, processes for which good controller performance is required, additional tunings may be necessary during the operation of process. At this point on-line control comes in to the picture. On-line controller tuning requires experimental tests and usually gives more exact results. Since on-line controlling requires much time and effort, it is usually preferred to use model based controllers for initial controller settings. While making on-line tuning, model based settings may be a good starting point to decrease time requirement and effort.

There are many model based controller tuning strategies for PID controller design. Model based methods provide controller parameters in the form of formula or algorithms. These methods differ in terms of complexity, applicability and amount of process information required. This information that depends on the method used is usually in transfer function form [39]. Limitations of computer capability such as speed and memory are another aspect that explains complexity and applicability of the methods that should be considered for tuning [40].

Some well-known PID controller tuning methods are Ziegler and Nichols [41], Internal model control (IMC) [42, 43], Direct Synthesis (DS) [44], and Integral time weight absolute error (ITAE) [4, 45, 46].

2.4.1. IMC Method

IMC is a well-known PID tuning method which is developed by Morari and co-workers [42]. The idea behind the IMC strategy is that the plant model and real plant are connected in parallel so that controller is provided to approach inverse plant dynamics [47]. With this strategy, process uncertainties are considered in a systematic manner so that robustness and performance requirements are satisfied [48].

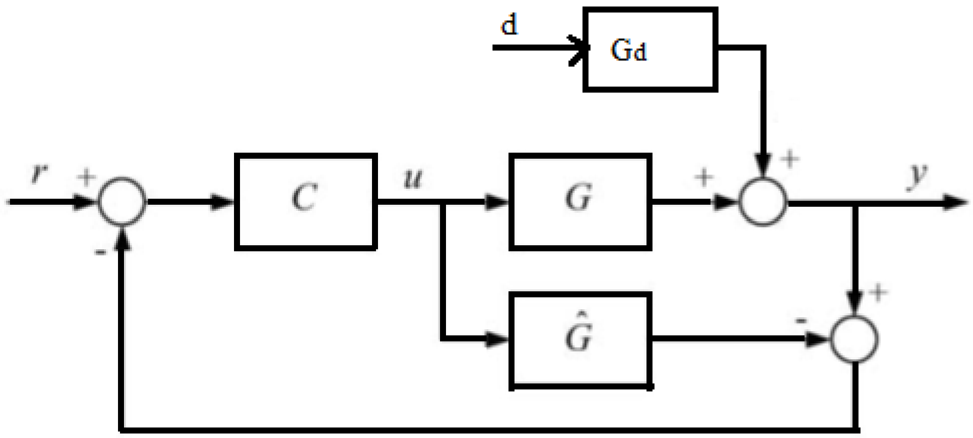

Figure 2.3 IMC Structure

G , $\hat{G}$, G_d and C refer to the actual plant, plant model, disturbance transfer function and controller respectively. Although IMC provides good set point tracking, it has poor disturbance rejection performance especially for the processes with small time delay- time constant ratio. However for many processes disturbance rejection is much more important than set point tracking. To improve disturbance rejection performance of IMC controllers, various filter structure for different types of processes were developed. [49, 50]

2.4.2. Direct Synthesis (DS) Method

Design methods for PID controllers are based on time domain or frequency domain performance criteria. There is not a straight relation between dynamic behavior of closed loop systems and these performance criteria. In direct synthesis, controller design is based on closed loop transfer function. This closed loop transfer function is usually based on set point changes. So, disturbance rejection performance of DS might not be satisfactory. Chen and Seborg [44], developed DS method to improve disturbance rejection performance of several types of process models.

2.4.3. ITAE Method

ITAE is a performance index used for designing of PID controllers [45]. The aim is minimization of time multiplied absolute error of the process response for disturbance or set point changes . This index is applicable for various types of process model. It is mathematically represented as;

$$ITAE = \int_0^t t|e(t)|dt \tag{2.9}$$

Where t is the time and $e\,(t)$ is the error which is calculated as the difference between the set point and the output. Time multiplication in the formula put low weighting on initial error but provides reduction of oscillation as the process proceeds. This causes response reaching to the desired value in short time interval that means short settling time. Aim of error minimization is providing the process running at desired conditions without actuator saturations.

This time domain controller tuning method does not require any model reduction for high order processes. The method is applicable for all kind of process models governed from physical laws. Use of integral performance criterion usually results in better closed loop response than other heuristic tuning methods. This tuning approach considers whole transient response of the system. [51]

In addition to ITAE, there are some other time domain performance indices such as Integral Square Error (ISE), Integral Time Square Error (ITSE) and Integral Absolute Error (IAE). Zhuang and Atherton analyzed set point tracking and disturbance rejection performances, using some of these different performance indices in their study. [52]

2.4.4. Ziegler and Nichols Method

Ziegler and Nichols tuning rule is one of the first tuning methods used for PID controller design. This rule is applied by firstly adjusting a proportional gain of P-only control which gives continuous oscillatory response. The gain that provides this continuos oscillatory response is called as ultimate gain (K_u) and this oscilllation period is is named as ultimate period (P_u). Then according to these ultimate gain and period values, and using Ziegler and Nichols tuning formulas corresponding PID settings are adjusted [53]. But, to determine ultimate gain and period experimentally is a time consuming process.

2.5. Model Reduction

In real life, most industrial processes are complicated and usually represented using high order mathematical expressions. These mathematical models lead to high order transfer functions which are difficult to use and require much time and effort for analysis and controller synthesis. Therefore, it is often necessary to find lower order transfer function representations of these higher order models which estimate behavior of the actual, higher order model.

In literature there are many studies based on different tuning approaches on tuning of the first and second order processes [54, 55, 56, 57]. According to these studies proper controller parameter correlations are set. These controller parameters correlations are usually based on low order models such as first order plus time delay (FOPTD) and second order plus time delay (SOPTD) transfer functions. In practice it is not an easy task to develop controller parameter correlations for every kind of process model. To simplify the method and use the controller parameters from one of these known methods as initial guesses for our code, transfer functions obtained from the input-output relations are reduced to FOPTD models in this study. Since we are trying to find control paramaters based on ITAE criteria, controller design relation of Smith and Corripio [58] was chosen to find initial controller parameters that will be implemented in code.

2.5.1. Skogestad's Model Reduction Technique

Skogestad's [57] model reduction technique called "half rule" was used to reduce the higher order models to the first order plus time delay models. Let the original model be in the form ;

$$\frac{\prod_j(-T_{jo}^{inv}s+1)}{\prod_j(\tau_{i0}s+1)}e^{-\theta_0 s}$$

Where the τ_{i0} are the lags ordered according to their magnitude at a descending manner. $T_{jo}^{inv} > 0$ expresses negative numerator time constant with a negative sign stated. According to half rule, to obtain a first order model in the form of $e^{-\theta s}/(\tau_1 s+1)$, it is used;

$$\tau_1 = \tau_{10} + \frac{\tau_{20}}{2} \quad \textbf{(2.10)}$$

$$\theta = \theta_0 + \frac{\tau_{20}}{2} + \sum_{i\geq 3}\tau_{i0} + \sum_j T_{j0}^{inv} + \frac{h}{2} \quad \textbf{(2.11)}$$

Skogestad's method states that largest denominator time constant is distributed to the dead time and smallest time constant. In practice, given a high order transfer function, each numerator term (T_0s+1) with $T_0 > 0$ is first simplified with a denominator term (τ_0s+1), $\tau_0 > 0$ using the rules stated below. If more than one positive numerator time constant exists, it can be approximated to start from largest T_0 as starting point. τ_0 is also selected as largest denominator time constant.

$\frac{T_0 s+1}{\tau_0 s+1}$ is equal ;

$$\frac{T_0}{\tau_0} \quad for \quad T_0 \geq \tau_0 \geq \theta \quad \text{(Rule T1)} \quad \textbf{(2.12)}$$

$$\frac{T_0}{\theta} \quad for \quad T_0 \geq \theta \geq \tau_0 \quad \text{(Rule T1a)} \quad \textbf{(2.13)}$$

$$1 \quad for \quad \theta \geq T_0 \geq \tau_0 \quad \text{(Rule T1b)} \quad \textbf{(2.14)}$$

$$\frac{T_0}{\tau_0} \quad for \quad \tau_0 \geq T_0 \geq 5\theta \quad \text{(Rule T2)} \quad \textbf{(2.15)}$$

$$\frac{(\tilde{\tau}_0/\tau_0)}{(\tilde{\tau}_0-T_0)s+1} \quad for \quad \tilde{\tau}_0 = \min(\tau_0, 5\theta) \geq T_0 \quad \text{(Rule T3)} \quad \textbf{(2.16)}$$

2.5.2 Simplification of High Order Models

We have higher order models which are required to be reduced for the estimation of initial controller parameters using Smith's relations that will be implemented in code. These models first were converted into a form of $\frac{T_0 s+1}{\tau_0 s+1}$ for which Skogestad's rule is applicible ;

$$\frac{T'(s)}{C_v'(s)} = \frac{\frac{K_3K_6}{(s-K_1)(s-K_4)}}{\left(1-\frac{K_8K_{11}}{(s-K_4)(s-K_9)}\right)} = \frac{-\frac{sK_3K_6}{K_1(K_4K_9-K_8K_{11})}+\frac{K_3K_6K_9}{K_1(K_4K_9-K_8K_{11})}}{\left(-\frac{s}{K_1}+1\right)\left(\frac{s^2}{(K_4K_9-K_8K_{11})}+s\frac{-(K_4+K_9)}{(K_4K_9-K_8K_{11})}+1\right)} =$$
$$\frac{\frac{K_3K_6K_9}{K_1(K_4K_9-K_8K_{11})}\left(-\frac{1}{K_9}s+1\right)}{(\tau_1 s+1)(\tau^2s^2+2\xi\tau s+1)} = \frac{\frac{K_3K_6K_9}{K_1(K_4K_9-K_8K_{11})}\left(-\frac{1}{K_9}s+1\right)}{(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)} \quad \textbf{(2.17)}$$

In which;

$$\tau_1 = -\frac{1}{K_1} \quad \textbf{(2.18)}$$

$$\tau = \frac{1}{\sqrt{K_4K_9-K_8K_{11}}} \quad \textbf{(2.19)}$$

$$\xi = -\frac{(K_4+K_9)}{2\sqrt{K_4K_9-K_8K_{11}}} \quad \textbf{(2.20)}$$

$$\tau_2 = \frac{\tau}{\xi-\sqrt{\xi^2-1}} \quad \textbf{(2.21)}$$

$$\tau_3 = \frac{\tau}{\xi+\sqrt{\xi^2-1}} \quad \textbf{(2.22)}$$

$$\frac{T'(s)}{Q'(s)} = \frac{\frac{K_8K_{10}}{(s-K_4)(s-K_9)}}{\left(1-\frac{K_8K_{11}}{(s-K_4)(s-K_9)}\right)} = \frac{\frac{K_8K_{10}}{K_4K_9-K_8K_{11}}}{\left(\frac{s^2}{K_4K_9-K_8K_{11}}-s\frac{K_4+K_9}{K_4K_9-K_8K_{11}}+1\right)} = \frac{\frac{K_8K_{10}}{K_4K_9-K_8K_{11}}}{(\tau^2s^2+2\xi\tau s+1)} =$$
$$\frac{\frac{K_8K_{10}}{K_4K_9-K_8K_{11}}}{(\tau_1 s+1)(\tau_2 s+1)} \quad \textbf{(2.23)}$$

In which;

$$\tau = \frac{1}{\sqrt{K_4K_9-K_8K_{11}}} \quad \textbf{(2.19)}$$

$$\xi = -\frac{(K_4+K_9)}{2\sqrt{K_4K_9-K_8K_{11}}} \tag{2.20}$$

$$\tau_1 = \frac{\tau}{\xi-\sqrt{\xi^2-1}} \tag{2.24}$$

$$\tau_2 = \frac{\tau}{\xi+\sqrt{\xi^2-1}} \tag{2.25}$$

$$\frac{T^{'}(s)}{T_1^{'}(s)} = \frac{\frac{K_7}{s-K_4}}{\left(1-\frac{K_8K_{11}}{(s-K_4)(s-K_9)}\right)} = \frac{s\frac{K_7}{K_4K_9-K_8K_{11}}-\frac{K_7K_9}{K_4K_9-K_8K_{11}}}{\left(\frac{s^2}{K_4K_9-K_8K_{11}}-s\frac{K_4+K_9}{K_4K_9-K_8K_{11}}+1\right)} = \frac{-\frac{K_7K_9}{K_4K_9-K_8K_{11}}\left(-\frac{1}{K_9}s+1\right)}{(\tau^2s^2+2\xi\tau s+1)} = \frac{-\frac{K_7K_9}{K_4K_9-K_8K_{11}}\left(-\frac{1}{K_9}s+1\right)}{(\tau_1 s+1)(\tau_2 s+1)} \tag{2.26}$$

In which;

$$\tau = \frac{1}{\sqrt{K_4K_9-K_8K_{11}}} \tag{2.19}$$

$$\xi = -\frac{(K_4+K_9)}{2\sqrt{K_4K_9-K_8K_{11}}} \tag{2.20}$$

$$\tau_1 = \frac{\tau}{\xi-\sqrt{\xi^2-1}} \tag{2.24}$$

$$\tau_2 = \frac{\tau}{\xi+\sqrt{\xi^2-1}} \tag{2.25}$$

After obtaining the simplified form of the models stated above, then Skogestad's appropriate reduction formula were applied to have first order plus time delay models. Reduction process is shown in Chapter 7. According to the values of steady state operating conditions we had different first order plus time delay models which are used to obtain initial guesses for controller tuning.

CHAPTER 3

PROCESS DESCRIPTION

3.1. System Introduction

The level and temperature control system under study consists of a tank with ten liters capacity and process elements such as a pump, valves, heating unit, piping, and sensors for measurement of level, temperature and flow rate. A simple schematic representation of the experimental set up is given below. E104 is heater that works in on-off mode. B104 and S111 are temperature and level sensors which read corresponding temperature and level values respectively. Level sensor is an ultrasonic sensor. It sends sound and this sound wave reflects from the surface of water in the tank. From the time that reflected sound wave reaches to the sensor again, corresponding water level is calculated. B114 and B113 are the sensors located for safety considerations which correspond to maximum and minimum operating level limits of the tank. P101 is the pump that adjusts the water flow inlet to the tank. Discharge from the tank is provided with a manual valve in which flow depends on the valve opening and water height in the tank.

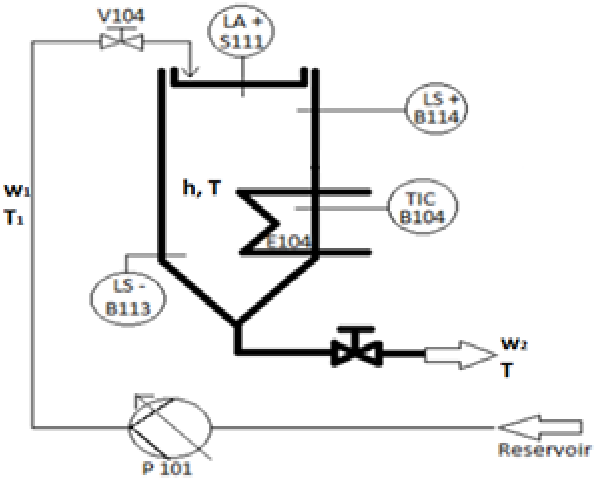

Figure 3.1 Schematic representation of experimental set-up

The level in the tank is maintained at desired conditions by the manipulation of inlet flow rate via the pump. Pump power, in other words inlet flow to the tank, can be adjusted accordingly. On the other hand, temperature is adjusted with on-off heater.

This system is MIMO process with four inputs and two outputs. Process inputs are inlet flow rate (w_1), heat input (Q), inlet temperature (T_1), discharge valve opening (C_v) whereas outputs are height (h) and temperature (T) of the water in the tank. Inlet flow rate and heat input are specified as manipulated variables. Manipulated variables provide controlled variables being at desired conditions. Inlet water temperature and discharge valve opening are specified as disturbance variables. These are the variables that affect the process and cause controlled variable deviations from desired condition. Height and temperature of the water in the tank are controlled variables that depend on four inputs mentioned above.

3.2. Mathematical Representation of Process

In the model development equations of the process, some assumptions are made. Physical properties such as convective heat transfer coefficient between water and heating element, density of water, specific heat capacity of heater and water are assumed to be constant.

Mass Balance;

$$\frac{d(V\rho)}{dt} = w_1 - w_2 \quad \textbf{(3.1)}$$

w_1 and w_2 are inlet and outlet water flow through tank respectively where w_2 depends on valve opening (C_v) and water height (h) in the tank. Density of water is represented as ρ which is assumed to be constant as 1 g/cm^3 . V is volume of water in the tank. Hence w_2 can be represented as below;

$$w_2 = \rho C_v h^a \quad \textbf{(3.2)}$$

Substituting Eqn. 3.2 in Eqn. 3.1;

$$\frac{d(V\rho)}{dt} = w_1 - \rho C_v h^a \quad \textbf{(3.3)}$$

Energy Balance for Water in the Tank;

$$\rho\frac{d(V\hat{H})}{dt} = w_1 C_p(T_1 - T_{ref}) - w_2 C_p(T - T_{ref}) + h_h A_h(T_h - T) \quad \textbf{(3.4)}$$

$$\rho V\frac{d\hat{H}}{dt} + \rho\hat{H}\frac{dV}{dt} = w_1 C_p(T_1 - T_{ref}) - w_2 C_p(T - T_{ref}) + h_h A_h(T_h - T) \quad \textbf{(3.5)}$$

$$\frac{d\hat{H}}{dt} = C_p\frac{dT}{dt} \quad \textbf{(3.6)}$$

$$H = C_p(T - T_{ref}) \quad \textbf{(3.7)}$$

T_1 , T and T_h are temperatures of inlet water, outlet water and heater respectively. $A_h, \hat{H}$ and h_h are area of the heater, specific enthalpy of water and convective heat transfer coefficient between heater and water respectively. C_p is specific heat capacity of water with a value of 4.18 $J/g.s$.

Water in the tank has uniform temperature distribution. To validate this we used external thermocouples and introduced them into the tank at three different locations which are bottom, middle and top. Temperature data were obtained and seen that there was only a difference of 1-2 ^{0}C between the thermocouples located at the bottom and top of the tank. This is not a significant difference, so that uniform temperature is assumed through the tank.

Substituting Eqn. 3.3, 3.6 and 3.7 in Eqn. 3.5 ;

$$w_1 C_p\left(T - T_{ref}\right) - w_2 C_p\left(T - T_{ref}\right) + \rho V C_p \frac{dT}{dt} = w_1 C_p(T_1 - T_{ref}) - w_2 C_p\left(T - T_{ref}\right) + h_h A_h (T_h - T) \quad \textbf{(3.8)}$$

$$\frac{dT}{dt} = \frac{w_1}{\rho V}(T_1 - T) + \frac{h_h A_h}{\rho C_p V}(T_h - T) \quad \textbf{(3.9)}$$

$$V = Ah \quad \textbf{(3.10)}$$

"A" and "h" are bottom area of the tank and water height in the tank respectively. "V" is volume of the water in the tank. Tank has the bottom area of, A, 305.6 cm^2.

$$\frac{dT}{dt} = \frac{w_1}{\rho A h}(T_1 - T) + \frac{h_h A_h}{\rho C_p A h}(T_h - T) \quad \textbf{(3.11)}$$

Energy Balance around Heater;

$$m_h C_{p_h} \frac{dT_h}{dt} = Q + h_h A_h (T - T_h) \quad \textbf{(3.12)}$$

3.3. Linear Model Representation

We need linearized model equations for tuning of PID controller. Linear controller tuning approaches are used by first estimating input-output transfer function relations.

Suppose a nonlinear model has been derived and given as below;

$$\frac{dy}{dt} = f(y,u) \quad \textbf{(3.13)}$$

where y is the output and u is the input. Linear approximation of this equation can be obtained by using Taylor series expansion and truncating after the first-order term. The reference point for linearization is the steady state operating point $(\bar{y},\bar{u})$.

$$f(y,u) = f(\bar{y},\bar{u}) + \left.\frac{\partial f}{\partial y}\right|_{\bar{y},\bar{u}} (y-\bar{y}) + \left.\frac{\partial f}{\partial u}\right|_{\bar{y},\bar{u}} (u-\bar{u}) \quad \textbf{(3.14)}$$

Steady state condition which is represented as $f(\bar{y},\bar{u})$ is equal to zero. Deviation variables from the steady state arise from Taylor series expansion where $y^{'} = y - \bar{y}$ and $u^{'} = u - \bar{u}$.

Hence linearized differential equation yields;

$$\frac{dy^{'}}{dt} = \left.\frac{\partial f}{\partial y}\right|_{s} y^{'} + \left.\frac{\partial f}{\partial u}\right|_{s} u^{'} \quad \textbf{(3.15)}$$

where $(\partial f/\partial y)|_{s}$ is equal to $(\partial f/\partial y)|_{\bar{y},\bar{u}}$.

From the theory stated about linearization of nonlinear models, now we can move on to the linearization of our process models.

Mass balance;

$$\frac{d(A.h.\rho)}{dt} = w_1 - \rho C_v h^a \qquad \textbf{(3.3)}$$

$\frac{dh}{dt}$ is function of $w_1,\ C_v,$ and h

$$\frac{dh}{dt} = f(w_1, C_v, h) \qquad \textbf{(3.16)}$$

$$\frac{dh'}{dt} = f(x) = \left(\frac{\partial f}{\partial w_1}\right)_s W_1{}' + \left(\frac{\partial f}{\partial C_v}\right)_s C_v{}' + \left(\frac{\partial f}{\partial h}\right)_s H' \qquad \textbf{(3.17)}$$

$$\left(\frac{\partial f}{\partial h}\right)_s = -\frac{a\bar{C}_v\bar{h}^{a-1}}{A} = K_1 \qquad \textbf{(3.18)}$$

$$\left(\frac{\partial f}{\partial w_1}\right)_s = \frac{1}{A\rho} = K_2 \qquad \textbf{(3.19)}$$

$$\left(\frac{\partial f}{\partial C_v}\right)_s = -\frac{\bar{h}^a}{A} = K_3 \qquad \textbf{(3.20)}$$

Taking Laplace transform of Eqn. 3.17 and rearranging;

$$sH'(s) = K_1H'(s) + K_2W_1{}'(s) + K_3C_v{}'(s) \qquad \textbf{(3.21)}$$

$$H'(s) = \frac{K_2}{(s-K_1)}W_1{}'(s) + \frac{K_3}{(s-K_1)}C_v{}'(s) \qquad \textbf{(3.22)}$$

Energy balance;

$$\frac{dT}{dt} = \frac{w_1}{\rho A h}(T_1 - T) + \frac{h_h A_h}{\rho C_p A h}(T_h - T) \quad \textbf{(3.11)}$$

$\frac{dT}{dt}$ is function of T, w_1, T_1, h and T_h. Hence;

$$\frac{dT}{dt} = f(T, w_1, T_1, h, T_h) \quad \textbf{(3.23)}$$

$$\frac{dT'}{dt} = f(x) = \left(\frac{\partial f}{\partial w_1}\right)_s W_1{}' + \left(\frac{\partial f}{\partial T_1}\right)_s T_1{}' + \left(\frac{\partial f}{\partial h}\right)_s H' + \left(\frac{\partial f}{\partial T}\right)_s T' + \left(\frac{\partial f}{\partial T_h}\right)_s T_h' \quad \textbf{(3.24)}$$

$$\left(\frac{\partial f}{\partial T}\right)_s = -\frac{\bar{w}_1}{\rho A \bar{h}} - \frac{h_h A_h}{\rho C_p A \bar{h}} = K_4 \quad \textbf{(3.25)}$$

$$\left(\frac{\partial f}{\partial w_1}\right)_s = \frac{\bar{T}_1 - \bar{T}}{\rho A \bar{h}} = K_5 \quad \textbf{(3.26)}$$

$$\left(\frac{\partial f}{\partial h}\right)_s = -\frac{\bar{w}_1}{\rho A \bar{h}^2}(\bar{T}_1 - \bar{T}) - \frac{h_h A_h}{\rho C_p A \bar{h}^2}(\bar{T}_h - \bar{T}) = K_6 \quad \textbf{(3.27)}$$

$$\left(\frac{\partial f}{\partial T_1}\right)_s = \frac{\bar{w}_1}{\rho A \bar{h}} = K_7 \quad \textbf{(3.28)}$$

$$\left(\frac{\partial f}{\partial T_h}\right)_s = \frac{h_h A_h}{\rho C_p A \bar{h}} = K_8 \quad \textbf{(3.29)}$$

Taking Laplace transform of Eqn 3.24 and rearranging;

$$sT^{'}(s) = K_4T^{'}(s) + K_5W_1{}^{'}(s) + K_6H^{'}(s) + K_7T_1^{'}(s) + K_8T_h^{'}(s) \quad \textbf{(3.30)}$$

$$T^{'}(s) = \frac{K_5}{s-K_4}W_1^{'}(s) + \frac{K_6}{s-K_4}H^{'}(s) + \frac{K_7}{s-K_4}T_1^{'}(s) + \frac{K_8}{s-K_4}T_h^{'}(s) \quad \textbf{(3.31)}$$

Energy balance around heater;

$$m_hC_{p_h}\frac{dT_h}{dt} = Q + h_hA_h(T - T_h) \quad \textbf{(3.12)}$$

$$\frac{dT_h}{dt} = f(T_h, T, Q) \quad \textbf{(3.32)}$$

$$\frac{dT_h{}'}{dT} = \left(\frac{\partial f}{\partial T}\right)_s T^{'} + \left(\frac{\partial f}{\partial T_h}\right)_s T_h^{'} + \left(\frac{\partial f}{\partial Q}\right)_s Q^{'} \quad \textbf{(3.33)}$$

$$\left(\frac{\partial f}{\partial T_h}\right)_s = -\frac{h_hA_h}{m_hC_{p_h}} = K_9 \quad \textbf{(3.34)}$$

$$\left(\frac{\partial f}{\partial Q}\right)_s = \frac{1}{m_hC_{p_h}} = K_{10} \quad \textbf{(3.35)}$$

$$\left(\frac{\partial f}{\partial T}\right)_s = \frac{h_hA_h}{m_hC_{p_h}} = K_{11} \quad \textbf{(3.36)}$$

Taking Laplace transform of Eqn. 3.33 and rearranging;

$$sT_h^{'}(s) = K_9T_h^{'}(s) + K_{10}Q^{'}(s) + K_{11}T^{'}(s) \quad \textbf{(3.37)}$$

$$T_h^{'}(s) = \frac{K_{10}}{s-K_9}Q^{'}(s) + \frac{K_{11}}{s-K_9}T^{'}(s) \quad \textbf{(3.38)}$$

Substituting Eqn. 3.22 and 3.38 in Eqn. 3.31;

$$T^{'}(s) = \frac{K_5}{s-K_4}W_1^{'}(s) + \frac{K_6}{s-K_4}\left[\frac{K_2}{(s-K_1)}W_1{}^{'}(s) + \frac{K_3}{(s-K_1)}C_v{}^{'}(s)\right] + \frac{K_7}{s-K_4}T_1^{'}(s) + \frac{K_8}{s-K_4}\left[\frac{K_{10}}{s-K_9}Q^{'}(s) + \frac{K_{11}}{s-K_9}T^{'}(s)\right] \quad \textbf{(3.39)}$$

$$T'(s)\left(1 - \frac{K_8K_{11}}{(s-K_4)(s-K_9)}\right) = W_1'(s)\left[\frac{K_5}{s-K_4} + \frac{K_6K_2}{(s-K_4)(s-K_1)}\right] + C_v'(s)\frac{K_3K_6}{(s-K_1)(s-K_4)} + T_1'(s)\frac{K_7}{(s-K_4} + \frac{K_8K_{10}}{(s-K_4)(s-K_9)}Q'(s) \quad \textbf{(3.40)}$$

3.4. Input-Output Relations

$$G_{21}(s) = \frac{H^{'}(s)}{W_1^{'}(s)} = \frac{K_2}{s-K_1} \quad \textbf{(3.41)}$$

$$G_{22}(s) = \frac{H^{'}(s)}{C_v^{'}(s)} = \frac{K_3}{s-K_1} \quad \textbf{(3.42)}$$

$$G_{11}(s) = \frac{T^{'}(s)}{W_1^{'}(s)} = \frac{\left[\frac{K_5}{s-K_4}+\frac{K_6K_2}{(s-K_4)(s-K_1)}\right]}{\left(1-\frac{K_8K_{11}}{(s-K_4)(s-K_9)}\right)} = \frac{s^2K_5+s(K_6K_2-K_5K_1-K_5K_9)+K_9K_5K_1-K_9K_6K_2}{s^3-s^2(K_4+K_9+K_1)+s(K_4K_9-K_8K_{11}+K_4K_1+K_1K_9)-K_1K_4K_9+K_1K_8K_{11}} \quad \textbf{(3.43)}$$

$$G_{12}(s) = \frac{T^{'}(s)}{T_1^{'}(s)} = \frac{\frac{K_7}{s-K_4}}{\left(1-\frac{K_8K_{11}}{(s-K_4)(s-K_9)}\right)} = \frac{sK_7-K_7K_9}{s^2-s(K_4+K_9)+K_4K_9-K_8K_{11}} \quad \textbf{(3.44)}$$

$$G_{13}(s) = \frac{T'(s)}{Q'(s)} = \frac{\frac{K_8K_{10}}{(s-K_4)(s-K_9)}}{\left(1-\frac{K_8K_{11}}{(s-K_4)(s-K_9)}\right)} = \frac{K_8K_{10}}{s^2-s(K_4+K_9)+K_4K_9-K_8K_{11}} \tag{3.45}$$

$$G_{14}(s) = \frac{T'(s)}{C_v'(s)} = \frac{\frac{K_3K_6}{(s-K_1)(s-K_4)}}{\left(1-\frac{K_8K_{11}}{(s-K_4)(s-K_9)}\right)} = \frac{sK_3K_6-K_3K_6K_9}{s^3-s^2(K_4+K_9+K_1)+s(-K_8K_{11}+K_4K_9+K_1K_4+K_1K_9)-K_1K_4K_9+K_1K_8K_{11}} \tag{3.46}$$

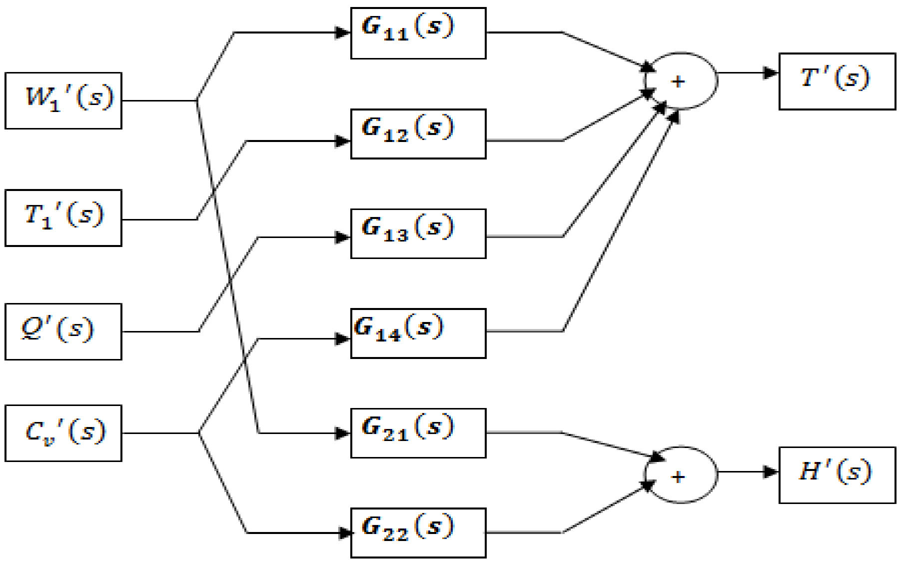

Figure 3.2 Input-Output transfer function relations

CHAPTER 4

HARDWARE AND SOFTWARE IMPLEMENTATION

4.1. Process Monitoring

After obtaining process model equations and input-output relations, experiments decided to be conducted. To capture dynamic data coming from the process measurement elements, software that provides connections between experimental set-up and computer is required.

For controlling and monitoring the process, LabVIEW is used. LabVIEW is a software package that provides tools needed to create and deploy measurement and control systems. [59]

It consists of two main parts called *user interface* and *block diagram*. In LabVIEW, you build a user interface, or front panel, with controls and indicators. Controls are knobs, push buttons, dials, and other input mechanisms. Indicators are graphs, LEDs, and other output displays. After you build the user interface, you add code using VIs and structures to control the front panel objects. The block diagram contains this code.

A LabVIEW control structure was created to provide connection between experimental system and computer. With this control structure pump power and heater can be adjusted. Temperature and level of the water in the tank can continuously be monitored. All the measured process variables are also recorded.

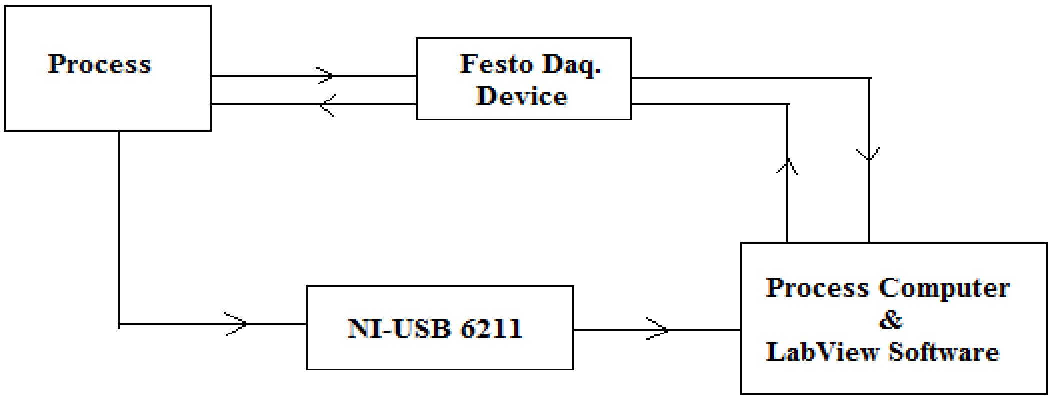

Figure 4.1 Data flow through process

4.2. Signal Conditioning

Signals coming from the level and temperature sensors have random fluctuations called noise. These noisy signals usually occur due to sensitivity of electronic devices. Since water is introduced from top of tank, noisy signal of level measurement also occurs because of fluctuation of water level in the tank at the very beginning of filling process. To avoid these unwanted random and noisy signals a threshold value of 0.2 V is determined to be used. That means signals coming below then this threshold will not be considered. This 0.2 value is determined by observing the level signal data while tank is being filled with water. Up to this signal, random variations of data coming from the level sensor were observed. For high level signals a moving average filter is implemented in to the LabVIEW control structure.

4.3. Moving Average Filter

A moving average is commonly used with time series data to smooth out short-term fluctuations. The moving average filter operates by averaging a number of points from the input signal to produce each point in the output signal. In equation form, this is written:

$$y(i) = \frac{1}{M}\sum_{j=0}^{M-1} x(i-j) \qquad \textbf{(4.1)}$$

Where x is the raw signal coming from the process measurement elements, y is the filtered signal. M is the number of points that average is taken.

Number of points in the average is determined according to the noise smoothness performance of filter. For our case five points moving average filter (M=5) where the performance can be seen from Figure 4.3 was sufficient for signal conditioning. To determine the number of points that average is taken, as an example of filtered signal for five points moving average, corresponding filtered signal is calculated as;

$$y[5] = \frac{x[1]+x[2]+x[3]+x[4]+x[5]}{5} \quad \textbf{(4.2)}$$

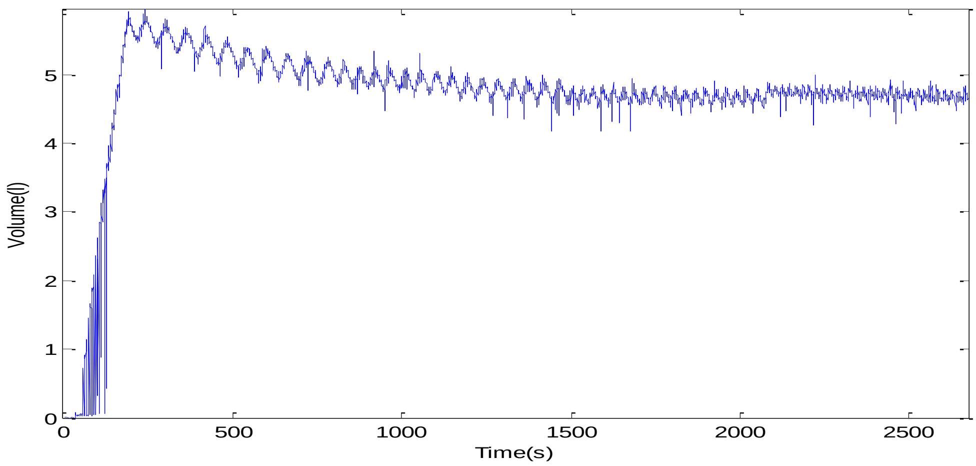

Figure 4.2 Raw signals coming from the measurement element

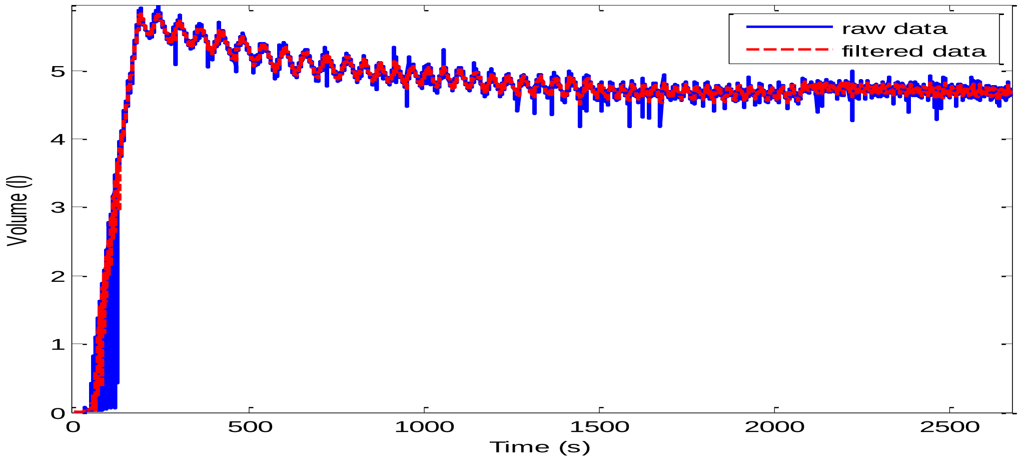

Figure 4.3 Raw and filtered signal responses

4.4. Calibration of Process Signals

Signals coming from the measurement elements are electrical signals. These signals must be converted to the actual physical values which are level in terms of centimeter (cm), temperature in terms of degree Celsius (^{0}C) and flow in terms of grams per second (g/s) for our case.

For temperature calibration, heat was applied to the water filled tank. As the water in the tank got hot, temperature signals coming from the sensor and corresponding actual temperature values, which were measured with an external thermometer, were recorded. A plot of actual temperature versus sensor signal output was obtained. Using this plot, an equation was fitted in the form of y = mx, in which y and x refer actual temperature and temperature signal respectively. The constant “m” was introduced in LabVIEW code that we created.

Same procedure was conducted for level signal calibration. At different water heights corresponding level signals were recorded and constant stated above were found for level signal conversion into actual physical value.

For flow rate calibration at different pump powers ranging from 3V to 9V, tank was filled with water and corresponding level change graphs were obtained. Average signals were also determined in the range of 3V-9V pump powers. From the level change responses and known area of tank, for every pump power (or flow signal) corresponding flow rate were calculated. From the plot of flow rate (g/s) versus flow signal, required constant that converts signal into physical value was found and implemented into the LabView code.

Values of calibration constants which are specified as “m” were found as 2.88, 9.03 and 11.61 for level, temperature and flow conversions respectively.

Plots for signal conversions are shown below;

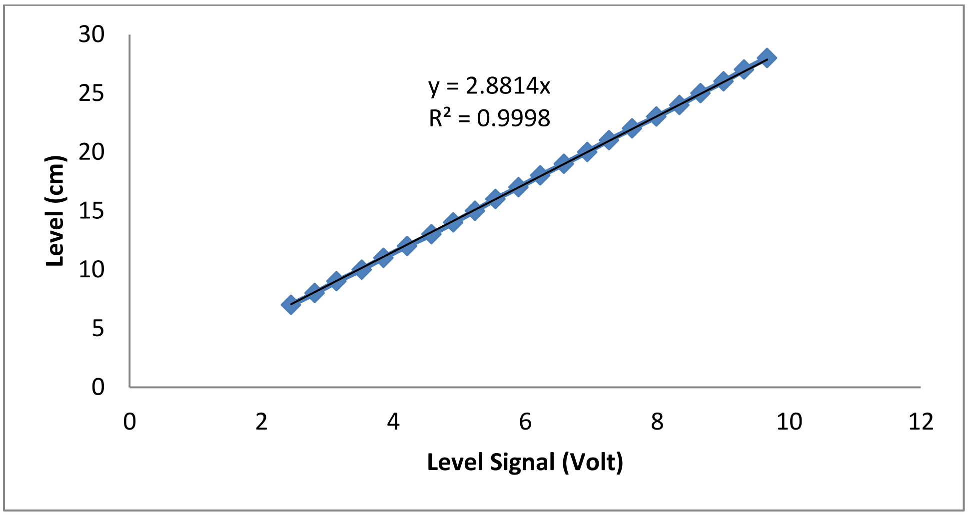

Figure 4.4 Calibration plot of level signal

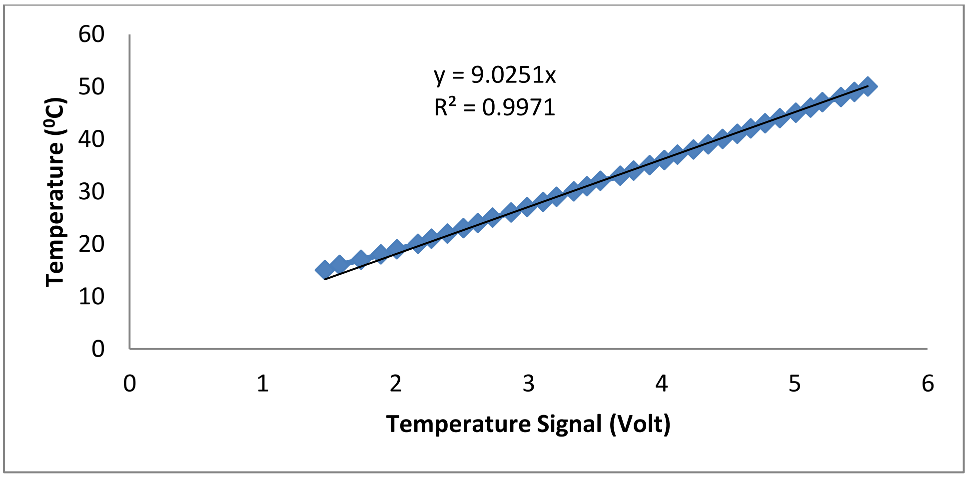

Figure 4.5 Calibration plot of temperature signal

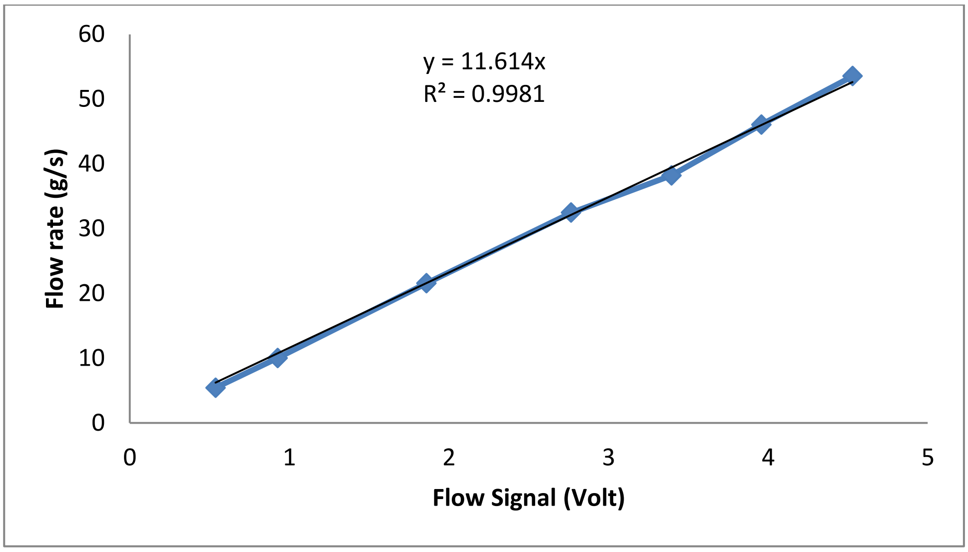

Figure 4.6 Calibration plot of flow signal

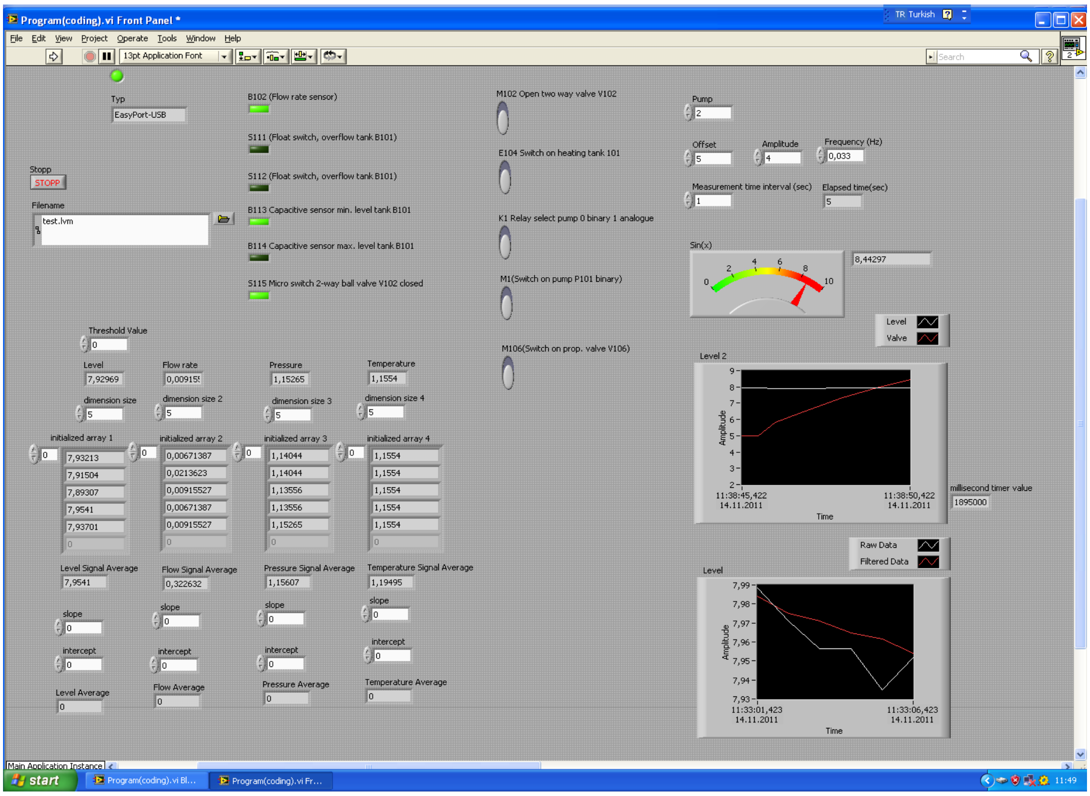

Figure 4.7 Front panel (user interface) view of LabView code

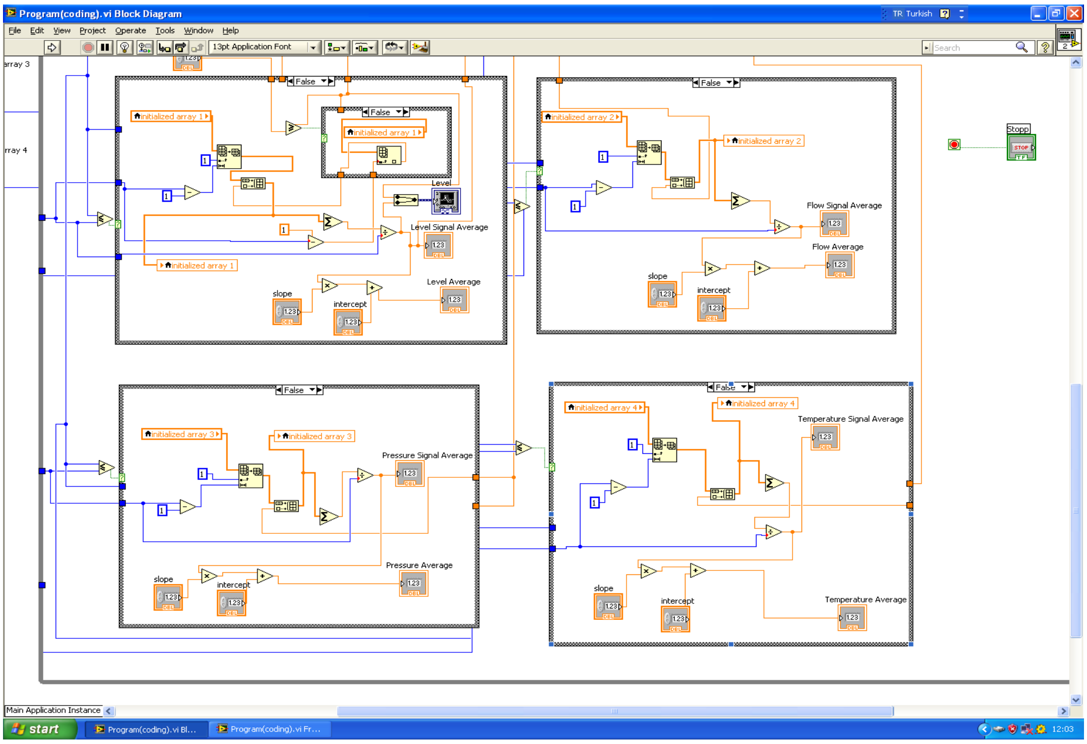

Figure 4.8 Block diagram view of LabView code

CHAPTER 5

SYSTEM IDENTIFICATION

5.1. Valve Opening Characterization

After installation of LabView and provide connection between experimental set-up and computer, experiments were conducted to find unknown process model parameters represented in model equations.

As it is mentioned at experimental description section, discharge flow from tank depends on the outlet valve opening and water height in the tank. For this reason experiments were designed to determine the coefficient of the valve.

$$q_2 = C_v h^a \quad \textbf{(5.1)}$$

Where, q_2 is the volumetric flow rate of water outlet from tank. C_v is a constant called valve flow coefficient that represents outlet valve opening. Since flow through each valve openings differ from each other, C_v must be different for each valve openings. And "a" is the constant that provides a relation between outlet flow and water height in the tank. Because, flow through outlet valve depends on the water level in the tank as it depends the valve openings.

For ideal case, liquid flowing from a tank through the pipe at the bottom without valve, "a" term yields to 0.5. However, since we have valve and do not know the flow characteristic through that valve, the term "a" must be determined experimentally.

At different valve openings fully filled tank was emptied and change of water level in the tank with respect to time were recorded. Data was captured at each half a second time interval. From the water level change data, corresponding flow rates are calculated at each sampling interval. Taking the logarithm of both sides of the Eqn. 5.1 provides;

$$\ln(q_2) = \ln(C_v) + a\ln(h) \quad \textbf{(5.2)}$$

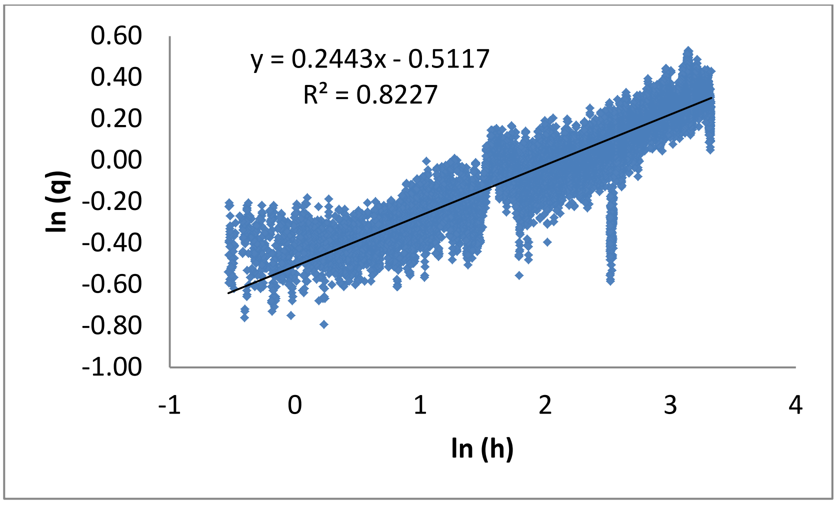

Figure 5.1 ln q vs. ln h plot for 1/4 valve opening

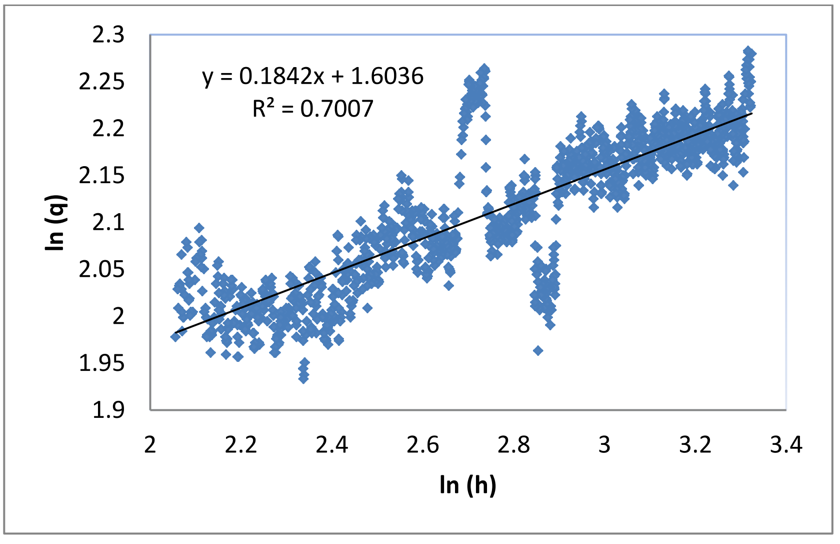

Figure 5.2 ln q vs. ln h plot for 2/4 valve opening

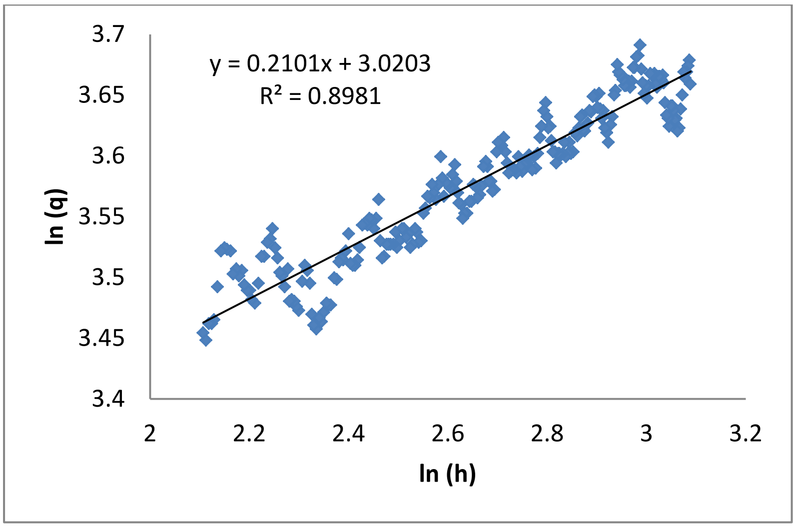

Figure 5.3 ln q vs. ln h plot for 3/4 valve opening

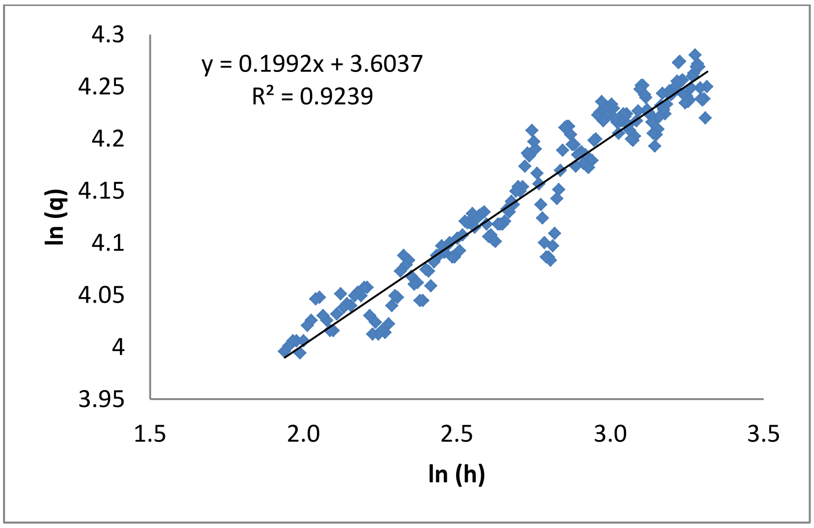

Figure 5.4 ln q vs. ln h plot for 4/4 valve opening

The graphs above are the only typical experimental result for each valve openings. These experiments are conducted three times for every valve openings. From the slopes and intercepts of the graphs, corresponding C_v and a values are found. Results are tabulated in the tables below.

Table 5.1 Tabulated values of "C_v" and "a" for 1/4 valve opening

1/4 Valve Opening			
C_v	0.5590	a	0.3935
C_v	0.5995	a	0.2443
C_v	0.4590	a	0.3442
Avg. C_v	0.5392	Avg. a	0.3272

Table 5.2 Tabulated values of "C_v" and "a" for 2/4 valve opening

2/4 Valve Opening			
C_v	5.090	a	0.1996
C_v	4.971	a	0.1842
C_v	4.995	a	0.2236
Avg. C_v	5.019	Avg. a	0.2025

Table 5.3 Tabulated values of "C_v" and "a" for 3/4 valve opening

3/4 Valve Opening			
C_v	23.338	a	0.1962
C_v	20.497	a	0.2102
C_v	19.563	a	0.2153
Avg. C_v	21.133	Avg. a	0.2072

Table 5.4 Tabulated values of “C_v” and “a” for 4/4 valve opening

4/4 Valve Opening			
C_v	36.734	a	0.1992
C_v	39.662	a	0.1800
C_v	36.825	a	0.1998
Avg. C_v	37.740	Avg. a	0.1930

Statistical analysis of the results was carried out. A type of interval estimate of a population parameter called confidence interval is used to indicate how reliable the results are. How frequently the observed interval contains the parameter is determined by the confidence level or confidence coefficient. The program JMP was used for this purpose. [60]

Obtained results were analyzed at 95% confidence level. Green lines at the top and bottom of data points shown in the Figure 5.5 below represent population interval for each data set. That means we have 95% confidence that our data will be in the range of these lines for each set of experiment. Since we cannot conduct these experiments unlimited number of times, this type of analysis is useful to determine the reliability of the results and population interval.

From the plot on the left hand side in Figure 5.5 it is seen that area between each line pairs do not intersect. This means, at 95% confidence level, C_v values for each valve openings differ from each other as it is expected. When the figure on the right hand side for the analysis of “a” value is evaluated, it is seen that area between line pairs for three valve openings intersects. As it is mentioned before “a” value is constant that represents outlet flow to the height of water in the tank. It does not depend valve opening. However, intersection cannot be seen at quarter valve opening which means, somehow “a” value for this valve opening different than others. This difference can be due to the changing flow characteristic at this very little valve opening or system dynamics are being forced. Flow regime through the pipe effects the value of “a”. That means flow characteristics are identical for other three valve openings. From analysis of C_v it is seen that there is linear relation between C_v values for the valve openings except quarter open. Linearity is lost between half and quarter open valves. This is also evident for changing flow characteristic or system dynamics at this valve opening. For half open, three quarters open and fully open valves “a” values are almost identical at 95% confidence level.

According to these statistical analysis results, it was decided to work for half, three quarter and fully open valve openings. Data of quarter valve opening were ignored. Average of “a”

values are taken and determined to be 0.2 which is decided to be used for further calculations.

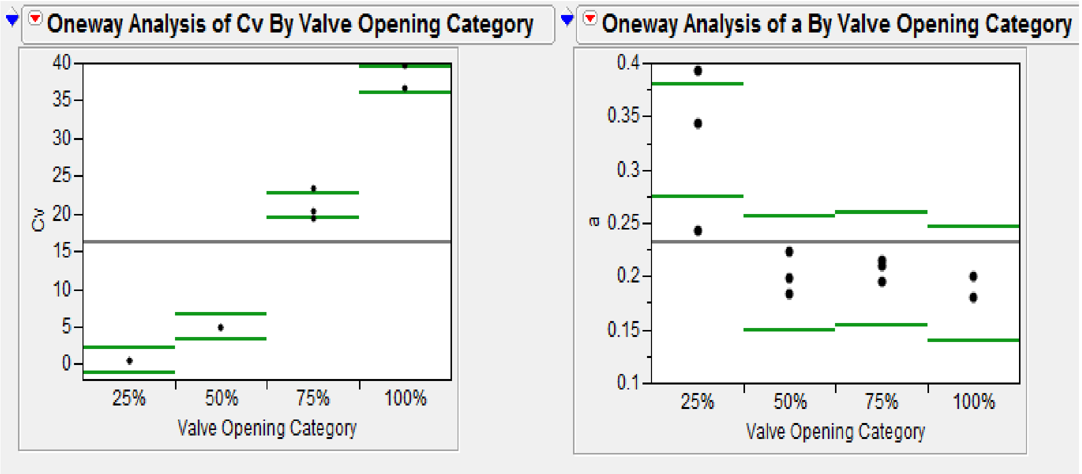

Figure 5.5 Analyses of obtained results at 95 % confidence interval

5.2. Experiments with Heat Input

There are some other unknowns in the model equations. These are m_h, C_{p_h}, h_h, A_h and related to the heater. C_{p_h} and m_h are specific heat capacity and mass of heater respectively whereas h_h and A_h are convective heat transfer coefficient between heater and water and area of heater respectively. For controller design all the unknowns in the mathematical representation must be determined. For this purpose experiments with heat inputs were conducted.

An experimental procedure was proposed without water flow through tank at specified water levels. Heat is introduced to the system and corresponding response behavior was observed. This experiment was done for three different water heights in the tank. Using these experimental response behaviors $m_h C_{p_h}$, $h_h A_h$, and Q values represented in the model equations are found.

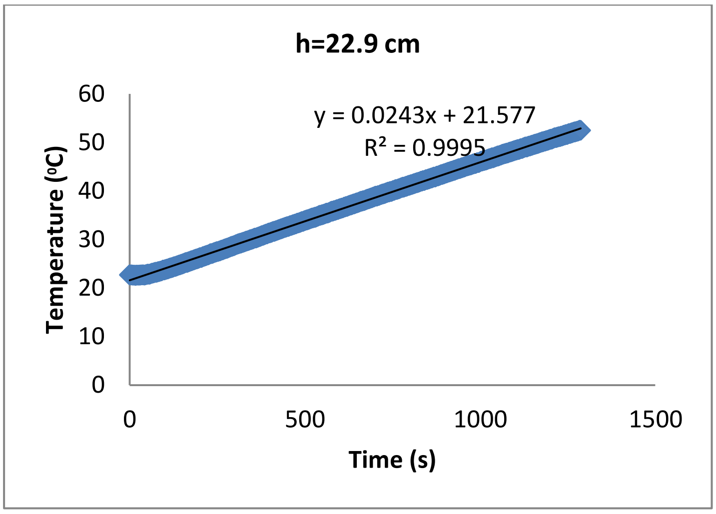

Figure 5.6 Temperature response at $h = 22.9$ cm

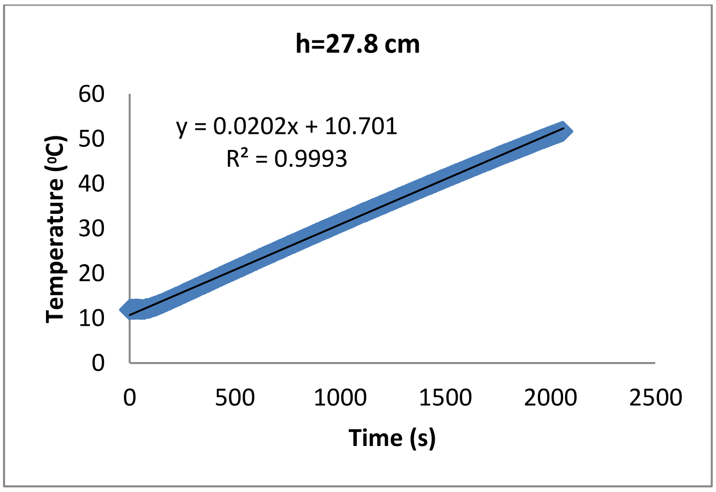

Figure 5.7 Temperature response at $h = 27.8$ cm

Mathematical representation of this experimental procedure is shown below.

Energy Balance for water in the tank;

$$\frac{dT}{dt} = \frac{w_1}{\rho Ah}(T_1 - T) + \frac{h_h A_h}{\rho C_p Ah}(T_h - T) \qquad \textbf{(3.11)}$$

Since there is no water flow ($w_1 = 0$) in to the tank, Eqn. 3.11 yields;

$$\frac{dT}{dt} = \frac{h_h A_h}{\rho C_p Ah}(T_h - T) \qquad \textbf{(5.3)}$$

Energy balance around heater;

$$m_h C_{p_h} \frac{dT_h}{dt} = Q + h_h A_h (T - T_h) \qquad \textbf{(3.12)}$$

Now solve Eqn. 5.3 for T_h;

$$\frac{\rho C_p Ah}{h_h A_h} \frac{dT}{dt} + T = T_h \qquad \textbf{(5.4)}$$

Now substitute Eqn. 5.4 in Eqn. 3.12;

$$m_h C_{p_h} \left(\frac{\rho C_p Ah}{h_h A_h} \frac{d^2T}{dt^2} + \frac{dT}{dt} \right) = Q + h_h A_h \left[T - \left(\frac{\rho C_p Ah}{h_h A_h} \frac{dT}{dt} + T \right) \right] \qquad \textbf{(5.5)}$$

$$m_h C_{p_h} \frac{\rho C_p Ah}{h_h A_h} \frac{d^2T}{dt^2} + m_h C_{p_h} \frac{dT}{dt} = Q - \rho C_p Ah \frac{dT}{dt} \qquad \textbf{(5.6)}$$

$$m_h C_{p_h} \frac{\rho C_p A h}{h_h A_h} \frac{d^2 T}{dt^2} + \frac{dT}{dt} (\rho C_p A h + m_h C_{p_h}) = Q \quad \textbf{(5.7)}$$

$$h_h A_h = C_1 \quad \textbf{(5.8)}$$

$$m_h C_{p_h} = C_2 \quad \textbf{(5.9)}$$

Resulting Equation;

$$\frac{C_2}{C_1} \rho C_p A h \frac{d^2 T}{dt^2} + \frac{dT}{dt} (\rho C_p A h + C_2) = Q \quad \textbf{(5.10)}$$

$$\frac{C_2}{C_1} \rho C_p A h = n \quad \textbf{(5.11)}$$

$$\left(\rho C_p A h + C_2\right) = b \quad \textbf{(5.12)}$$

$$Q = d \quad \textbf{(5.13)}$$

$$n \frac{d^2 T}{dt^2} + b \frac{dT}{dt} = d \quad \textbf{(5.14)}$$

Solving the linear constant coefficient ordinary differential equation yields;

$$T = \frac{R_1}{\exp\left(\frac{\mathrm{b.t}}{\mathrm{n}}\right)} - \frac{(R_2 + n.d - b.d.t)}{b^2} \quad \textbf{(5.15)}$$

When initial conditions considered;

For $h = 27.8$ cm

$t = 0\ , T = 11.8\ ℃$

$t = 0\ \ , \frac{dT}{dt} = 0$

$$T = \frac{(11.8\ b^2 + d.t.b - n.d)}{b^2} + \frac{n.d}{b^2 \exp(\frac{b.t}{n})} \quad \textbf{(5.16)}$$

Slope of the experimental response for h=27.8 cm, when time goes to infinity is;

$$\frac{dT}{dt} = \frac{d}{b} = \frac{Q}{(\rho C_p A h + m_h C_{p_h})} = \frac{Q}{(\rho C_p A h + C_2)} = 0.0202 \quad \textbf{(5.17)}$$

$$\frac{Q}{((1\frac{g}{cm^3}) \times \left(4.18\frac{J}{g.C}\right) \times (305.6\ cm^2) \times (27.8\ cm) + C_2)} = 0.0202 \quad \textbf{(5.18)}$$

$$717.27 + 0.0202 C_2 = Q \quad \textbf{(5.19)}$$

Slope of the experimental response for $h = 22.9$;

$$\frac{dT}{dt} = \frac{d}{b} = \frac{Q}{(\rho C_p A h + m_h C_{p_h})} = \frac{Q}{(\rho C_p A h + C_2)} = 0.0243 \quad \textbf{(5.20)}$$

$$\frac{Q}{(1\frac{g}{cm^3} \times \left(4.18\frac{J}{g.C}\right) \times (305.6\ cm^2) \times (22.9\ cm) + C_2)} = 0.0243 \quad \textbf{(5.21)}$$

$$710.77 + 0.0243C_2 = Q \quad \textbf{(5.22)}$$

Now, we have equations 5.19, 5.22 and two unknowns C_2 and Q that yields a unique solution. By solving these two equations C_2 and Q are found to be 1585.6 and 749.3 respectively. Since we know C_2 and Q, we can calculate b and d values from equation 5.12 and 5.13. With known C_2 and Q, by using heat response data of the tank at the water height of 27.8 cm, value of "n" can be found for a random time which is selected as 1000s that corresponds to T=30.6 ^{0}C, using equation 5.16. From the value of "n" C_1 can be calculated using Eqn. 5.11. Now, all the unknowns are found and their values are tabulated on the Table 5.5 below.

It must be noted that in our model equations, m_h, C_{p_h} and h_h, A_h pairs are at multiplication form. Therefore, individual values of these constants were not found. Values at multiplication forms are sufficient for complete nonlinear model.

Table 5.5 Values of process parameters

$m_h C_{p_h}$ *(J/K)*	1585.6
$h_h A_h$ *(J/s.K)*	22.14
Q *(J/s)*	749.3

5.3. Model Validation

We conducted further experiments to be sure that our model equations fit experimental behavior. For this purpose, heating experiments with water inlet and outlets are also conducted and obtained responses are compared with simulated responses. Simulink block diagram of complete nonlinear model that is shown in Figure 5.16 is created for this purpose. At two different valve openings and two different steady state heights in the tank, four experiments were conducted totally. Flow rates are adjusted with pump to maintain a steady state height in the tank. That means flow in to the tank was equalized to the discharge flow from outlet valve.

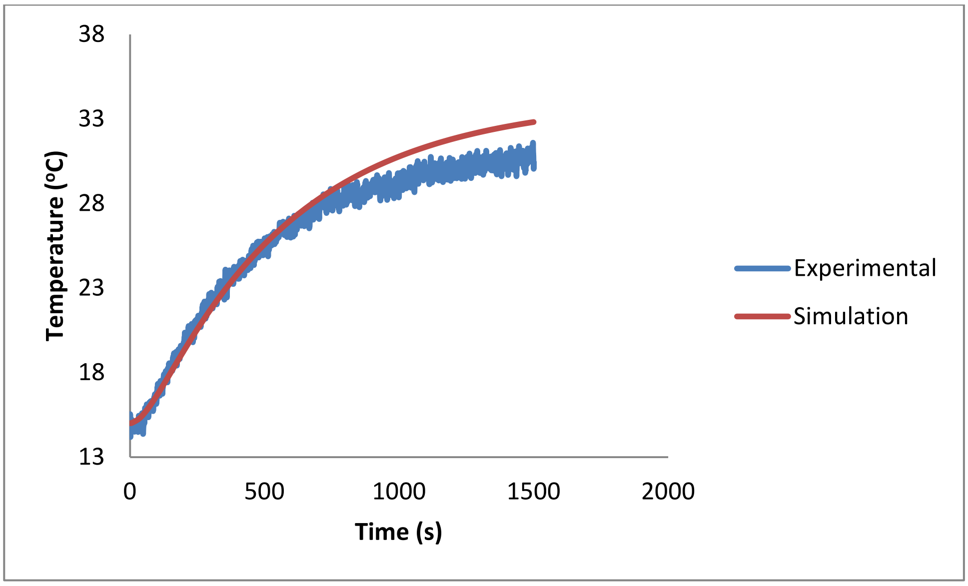

Figure 5.8 Experimental vs. Simulation plot $C_v = 5$, h =15 cm

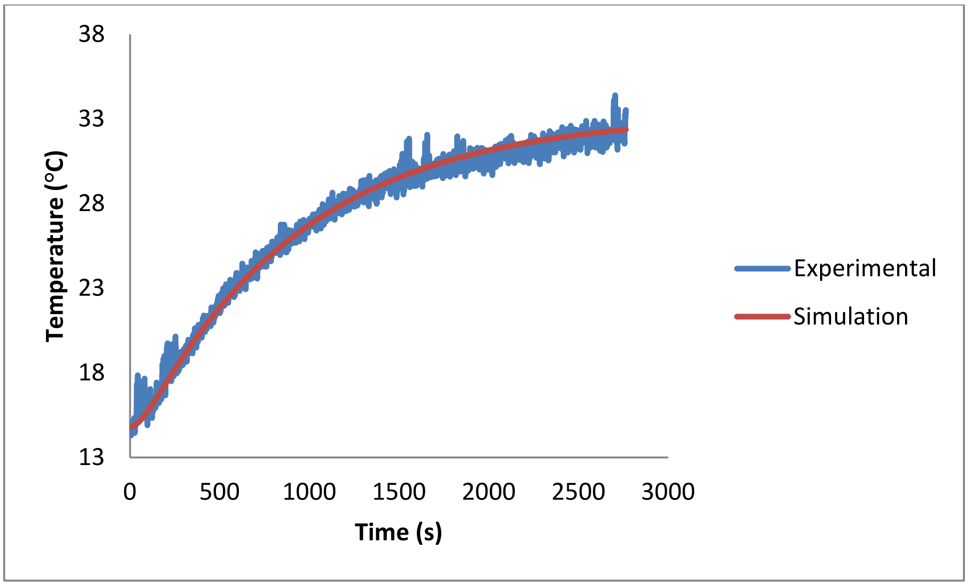

Figure 5.9 Experimental vs. Simulation plot C_v =5, h = 27 cm

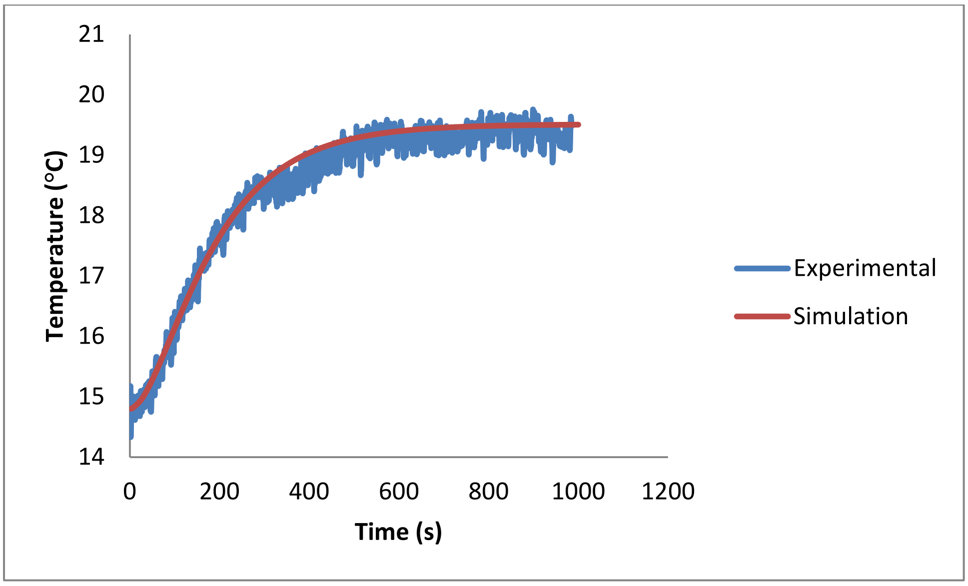

Figure 5.10 Experimental vs. Simulation plot C_v =22.1, h =15 cm

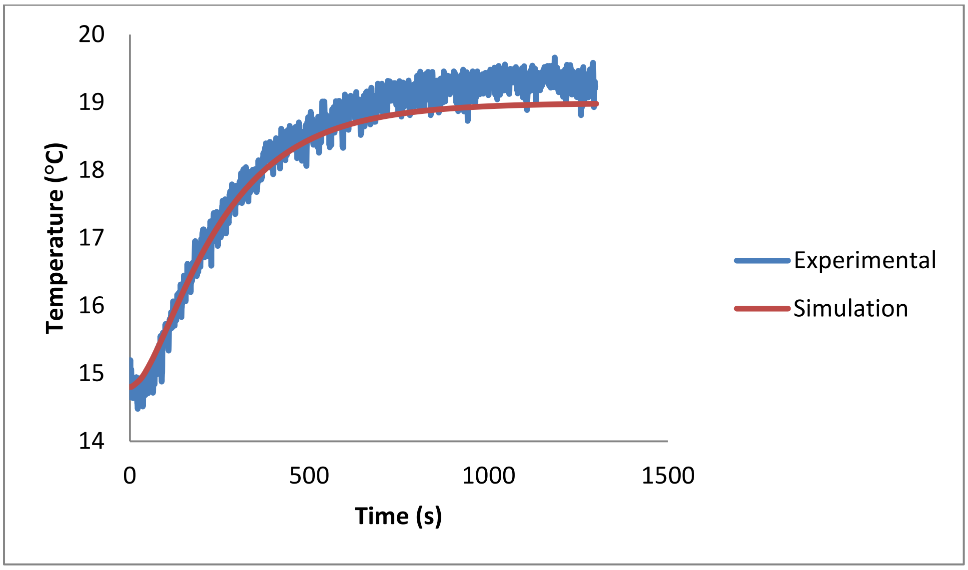

Figure 5.11 Experimental vs. Simulation plot C_v =22.1, h =27 cm

Table 5.6 Experimental and simulation conditions

	Height(cm)	C_v	w_1 (g/s)	T_1(°C)
Figure 5.13	15	5	8.6	14.8
Figure 5.14	27	5	9.7	14.8
Figure 5.15	15	22.1	38.1	14.8
Figure 5.16	27	22.1	42.8	14.8

Percentage errors between experimental and simulated responses are calculated for each data point, and figures are tabulated below for four experiments.

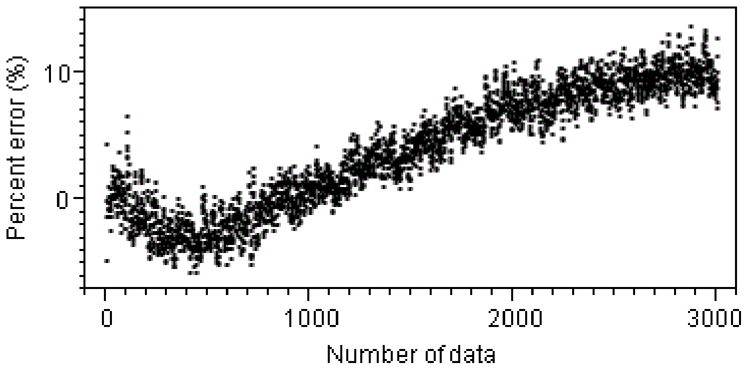

Figure 5.12 Distribution of error at C_v =5, h =15 cm

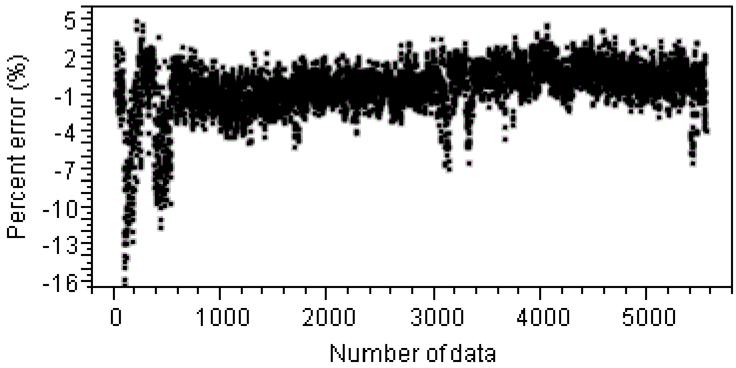

Figure 5.13 Distribution of error at C_v =5, h =27 cm

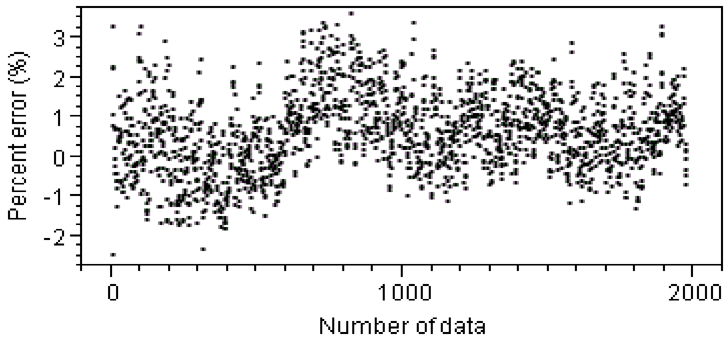

Figure 5.14 Distribution of error at C_v =22.1, h =15 cm

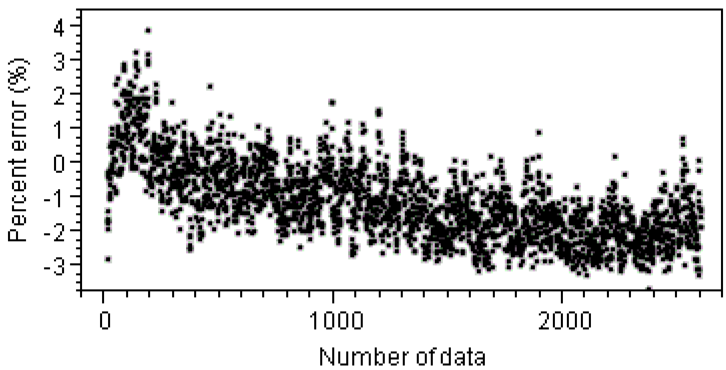

Figure 5.15 Distribution of error at C_v =22.1, h =27 cm

Experimental responses with water inlet-outlet and heat input fit the simulated responses as they are seen from the Figures 5.5, 5.6, 5.7 and 5.8 above. Percentage error distributions are also at acceptable levels. At this point we have the complete nonlinear model with all known parameters.

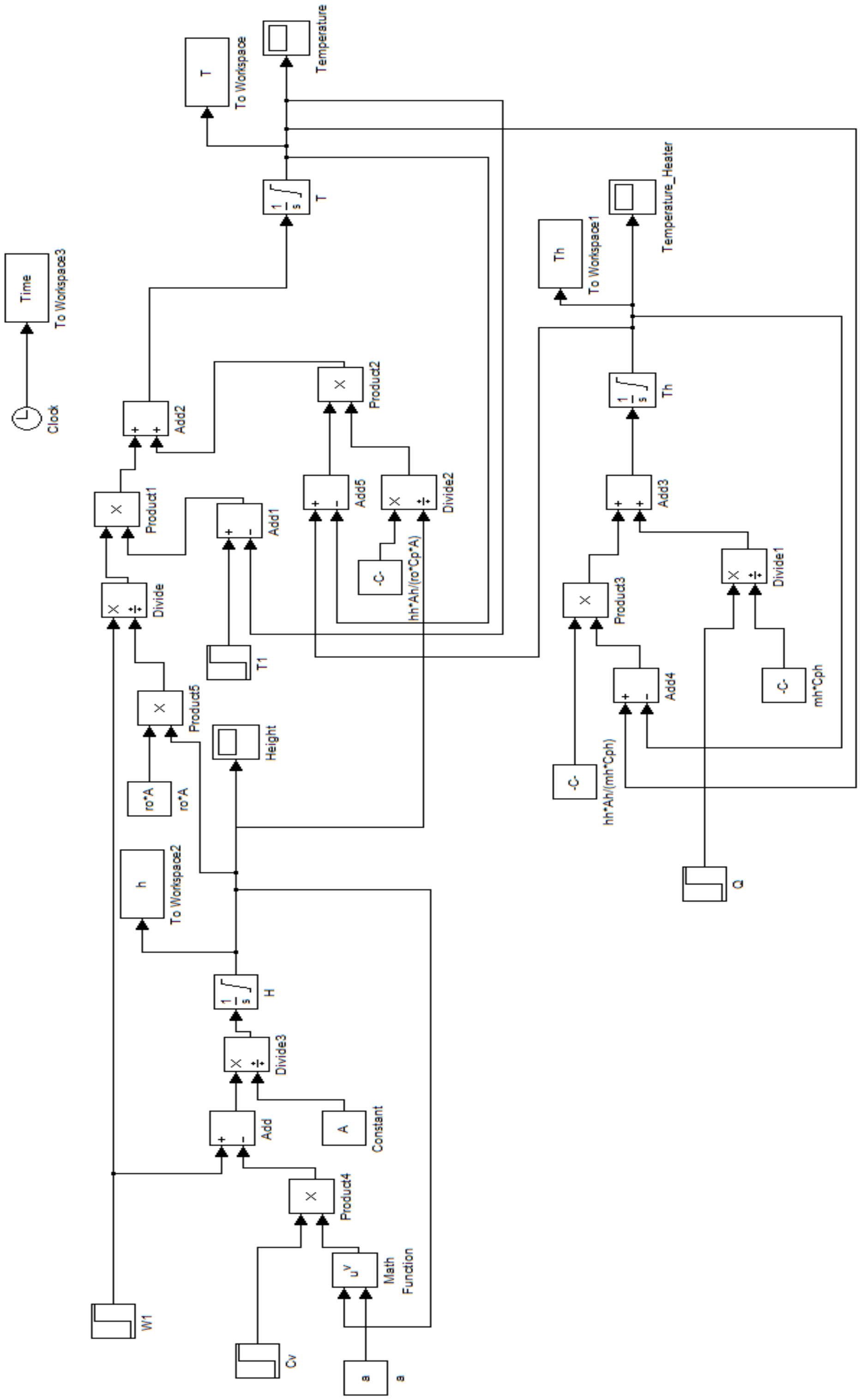

Figure 5.16 Simulink block diagram of complete nonlinear model

CHAPTER 6

CONTROLLER TUNING

6.1. Proposed PID Tuning Method

ITAE is selected as proposed tuning method due to its simplicity and applicibility for wide range of process models. Martin's [45] work was considered as a guideline for the application of ITAE to our process. For this purpose following steps are implemented ;

- Firstly, an m-file represented in Appendix B.1 which calculates ITAE index is defined using MATLAB.
- A loop of PID controller simulink block diagram shown in Figure 6.2 is created.
- MATLAB optimization toolbox calculates minimum of the objective function using the fminsearch command. Fminsearch finds minimum of a function started at an initial estimate. Objective function mentioned here is not same as the objective function defined in Chapter 2. However, the principle is same. Objective function here is the formula that calculates the time weighted absolute error of a closed loop response. Aim is minimization of this error by estimating suitable controller parameters.
- On each evaluation of objective function shown in Appendix B.2, the process model developed in the simulink is executed and ITAE performance index is calculated using Simpson's 1/3 rule.
- For this purpose initial guesses for controller settings have to be determined by using one of the existing tuning methods.
- Analytical tuning of Integral time absolute error method on Table 6.1 that is developed by Smith and Corripio [58] for first order plus time delay models were used to determine initial guesses to avoid the initial condition dependence of optimal solution.
- Since we have different type of process models, firstly these higher order models were reduced to first order plus time delay models using proposed model reduction techniques to use Smith and Corripio's FOPTD controller tuning relations as an initial estimate of our code.

Table 6.1 Controller Design Relation Based on the ITAE Performance Index and a First-Order-plus-Time-Delay Model [58]

Type of input	Type of Controller	Mode	A	B
Disturbance	PI	P	0.859	-0.977
		I	0.674	-0.680
Disturbance	PID	P	1.357	-0.947
		I	0.842	-0.738
		D	0.381	0.995
Set Point	PI	P	0.586	-0.916
		I	1.03[b]	-0.165[b]
Set Point	PID	P	0.965	-0.85
		I	0.796[b]	-0.1465[b]
		D	0.308	0.929

[a]Design relation: $Y = A(\theta/\tau)^{\ B}$ where $Y = KK_c$ for the proportional mode, τ/τ_I the integral mode, and τ_D/τ for the derivative mode.
[b]For set-point changes, the design relation for the integral mode is , $\frac{\tau}{\tau_I} = A + B(\theta/\tau)$.

6.2. FOPTD Model Analysis and Smith's Theory Confirmation

We conducted set of closed loop simulations to see how well our code find optimum controller setting of a process. To confirm the reliability of code, FOPTD models for which we have tuning relations represented on Table 6.1, are considered. That means we compared our controller parameters with the parameters found from Smith's analytical relations. Since Smith's relation is valid for the process where disturbance transfer function is equal to the process transfer function (Gd=Gp), we proposed a simulink model obeys this rule in the form of which is shown in Figure 6.1. Gp and Gd are process and disturbance tranfer functions respectively. G_{13}, G_{21} stated in Chapter 3.4 are specified as Gp whereas $G_{22},\ G_{12}$, G_{14} represents Gd for our process model.

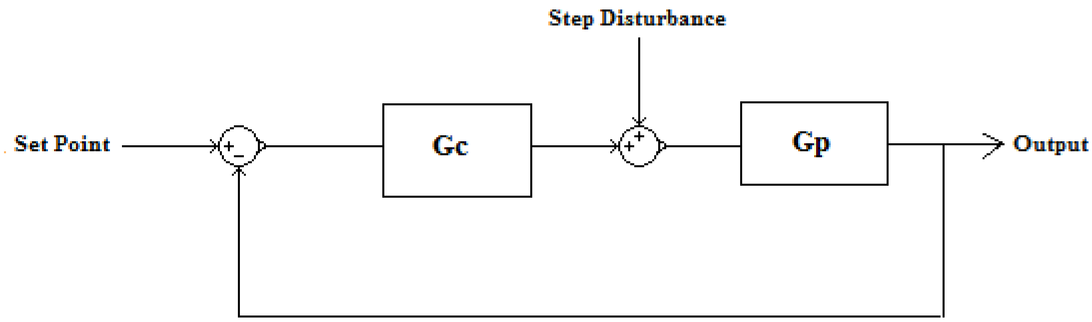

Figure 6.1 Closed loop block diagram of a process where Gp=Gd

First of all, controller parameters of different FOPTD models were found using Smith's correlations shown in Table 6.1. After using these correlations and finding controller parameters, corresponding closed loop responses of models were obtained using the Simulink code of Figure 6.2. Parameters found from tuning relation were used as initial guesses in our code to find optimum controller parameters. After finding controller parameters from the code given in Appendix B, a closed loop response was obtained for each FOPTD model. ITAE score of the responses of each FOPTD models were compared and results are shown in the Figure 6.6. From the Smith's tuning relation table it is seen that controller parameters are the function of θ/τ. Therefore, for different FOPTD models, that means different θ/τ values, plots showing controller parameter relations were obtained and analyzed. These analyses were first conducted for set point tracking performance. Hence, Smith's tuning relations for set point changes were used.

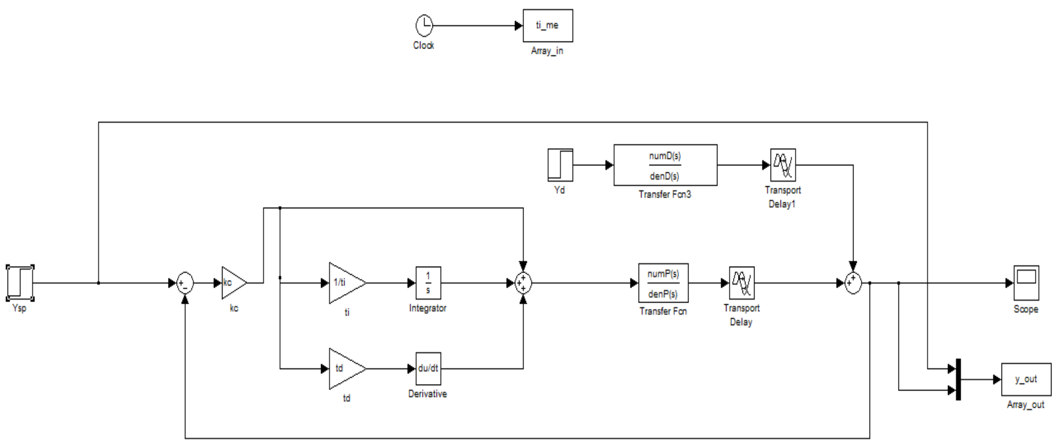

Figure 6.2 Closed loop block diagram of a PID controller

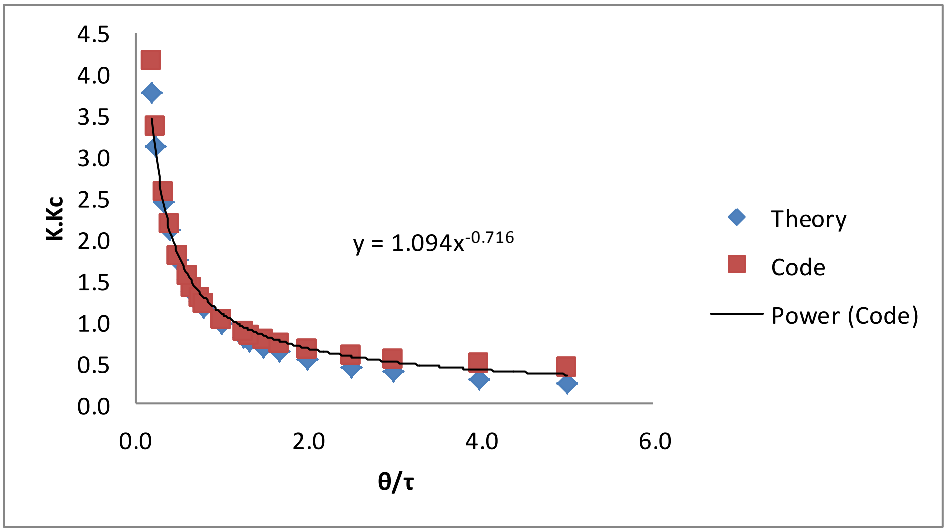

Figure 6.3 KK_c vs. θ/τ plot for set point change

From Figure 6.3 it can be seen that Kc values found from Smith's theory and our proposed code are close to each other.

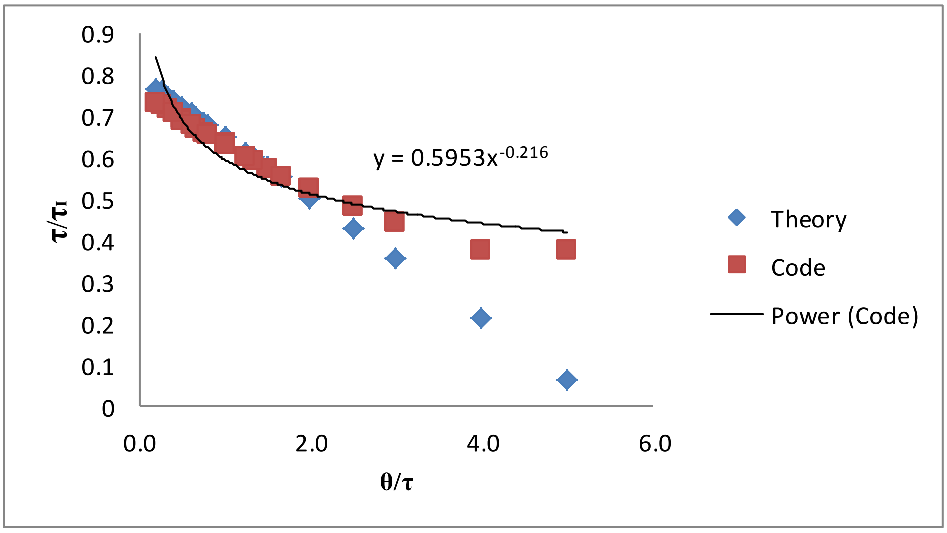

Figure 6.4 τ/τ_I vs. θ/τ plot for set point change

In Figure 6.3, τ_I values are close to each other up to θ/τ value of 3. After a certain point a deviation starts to occur between theory and code values.

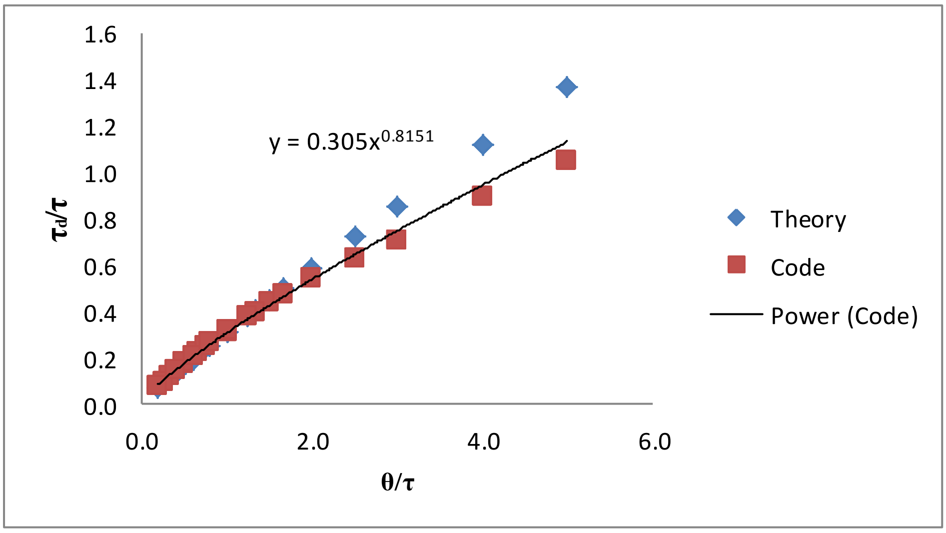

Figure 6.5 $\boldsymbol{\tau_d/\tau}$ vs. $\boldsymbol{\theta/\tau}$ plot for set point change

From Figure 6.5, it is seen that there is almost an identical τ_d trend with that occurs in Figure 6.4 for τ_I. As θ/τ ratio increases a deviation starts to occur between theoretical and programmatically found values.

From Figures 6.3, 6.4 and 6.5, it can be seen that Kc values found from the code and the theory are almost identical while τ_I and τ_d values differ in some regions. To determine, which approach gives optimum tuning relation, ITAE score of all FOPTD models are calculated and shown in the Figure 6.6.

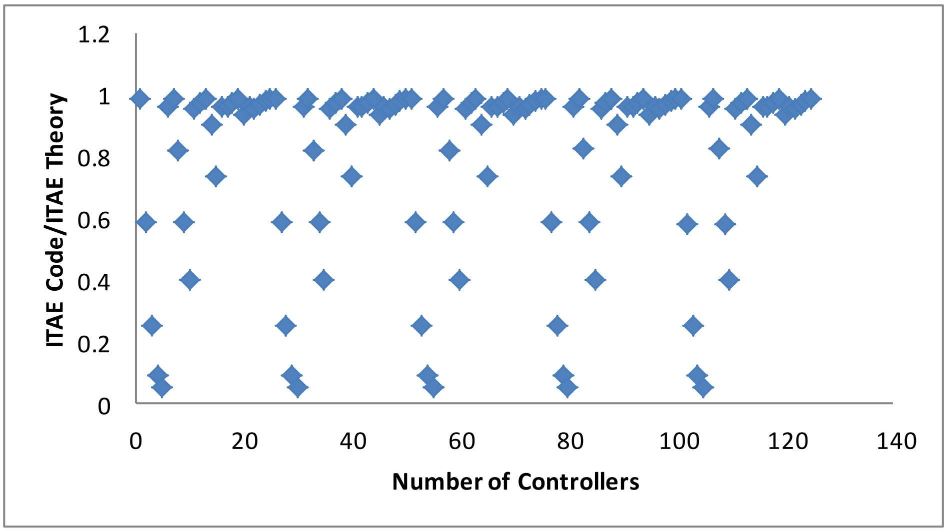

Figure 6.6 ITAE score ratios of various FOPTD models for set point changes

From the Figure 6.6 it can be seen that ITAE scores calculated from the codes are less than ITAE calculated from Smith's approach which proves the reliability of the proposed method over Smith's relation for FOPTD models in the case of Gp=Gd.

Same procedure was repeated for disturbance rejection performances and the results are splayed in Figures 6.7, 6.8, 6.9 and 6.10.

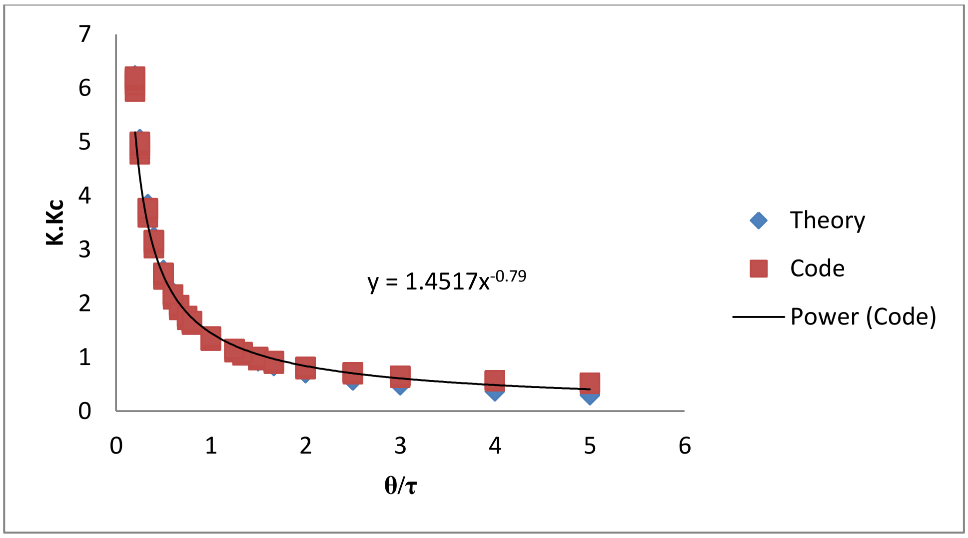

Figure 6.7 KK_c vs. θ/τ plot for disturbance input

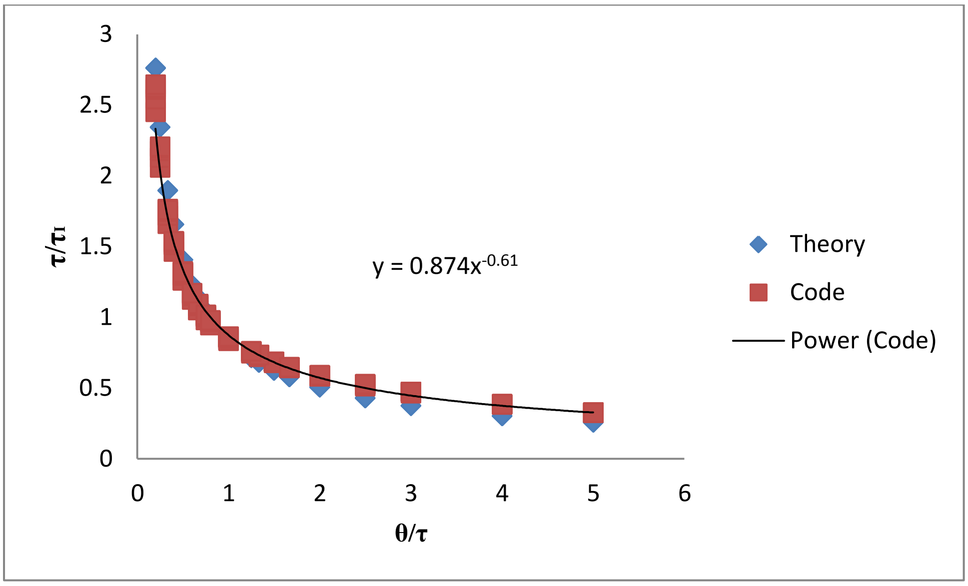

Figure 6.8 τ/τ_I vs. θ/τ plot for disturbance input

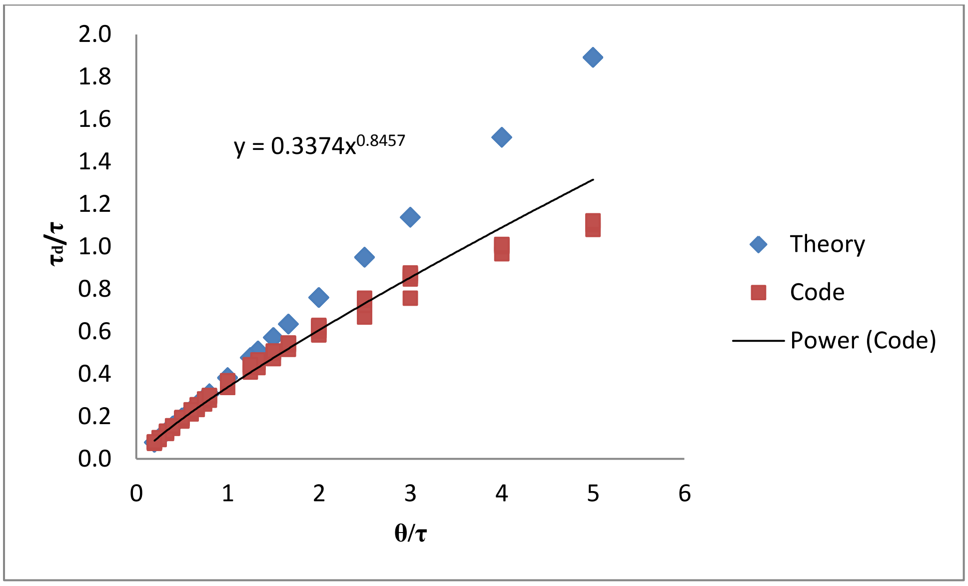

Figure 6.9 τ_d/τ vs. θ/τ plot for disturbance input

From the plots it is seen that Kc and τ_I values are almost identical whereas there is some difference between τ_d values obtained from theory and code.

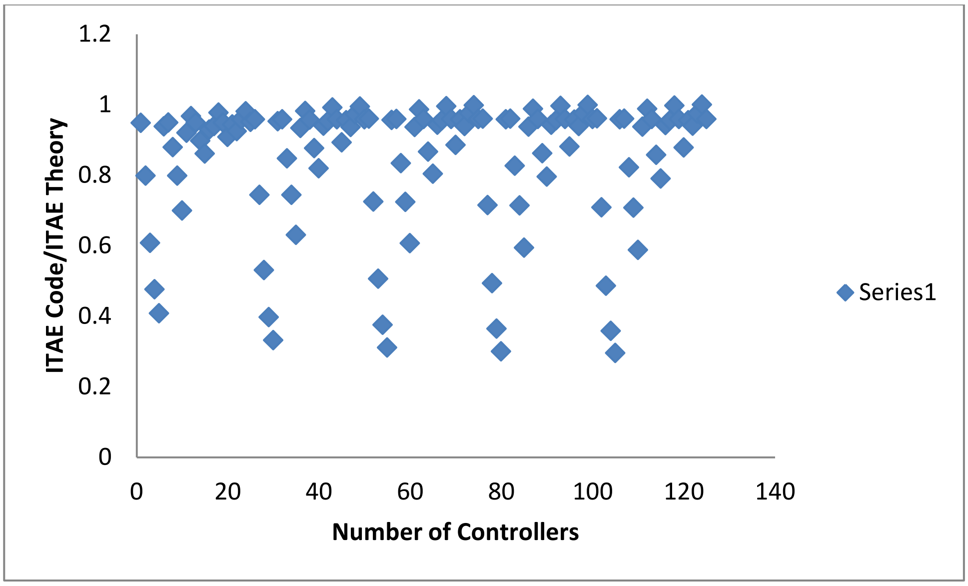

Figure 6.10 ITAE score ratios of various FOPTD models for disturbance input

From the ITAE score ratios it can be concluded that proposed method gives better response than the theoretical approach for FOPTD models with disturbance rejection performances.

6.3. SOPTD Model Analysis

We further applied our approach to SOPTD systems. FOPTD fits of these SOPTD models were obtained. Gp=Gd case was considered again to use Smith's relations. It must be noted that Smith's correlation is valid for FOPTD models. Therefore, FOPTD approximation of SOPTD models were obtained using step test method and Sundaresan & Krishnaswamy approach [4]. Using these approximated FOPTD models and Smith's tuning relations, corresponding controller settings were determined. These controller settings were used as baseline controller parameters of actual SOPTD models implemented on Simulink. These controller parameters were also used as initial guesses in our code to find optimum controller parameters. Closed loop performance of actual SOPTD models were evaluated for set point and disturbance rejection cases. Typical output responses of underdamped, critically damped and overdamped systems were analyzed.

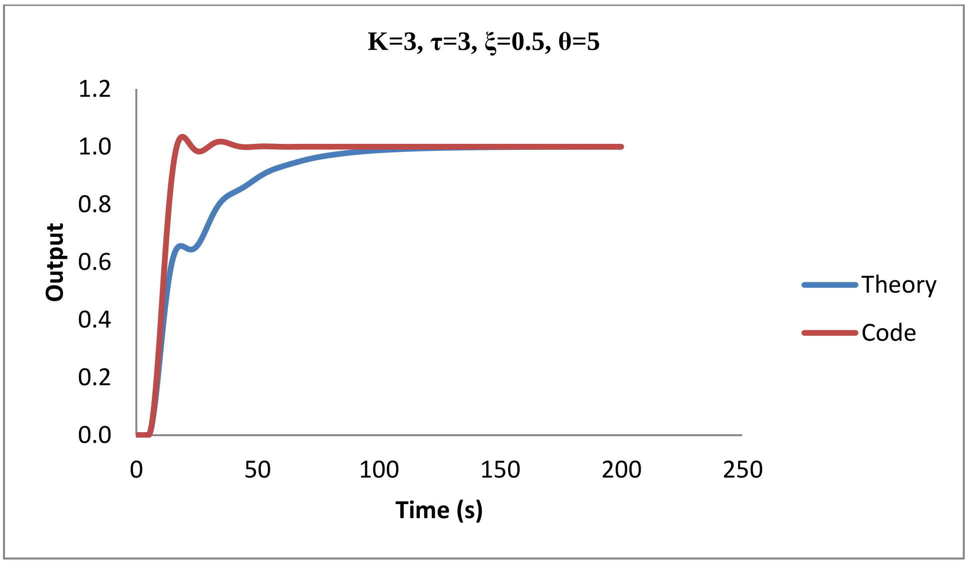

Figure 6.11 Set point tracking performance of the controllers for an underdamped SOPTD system

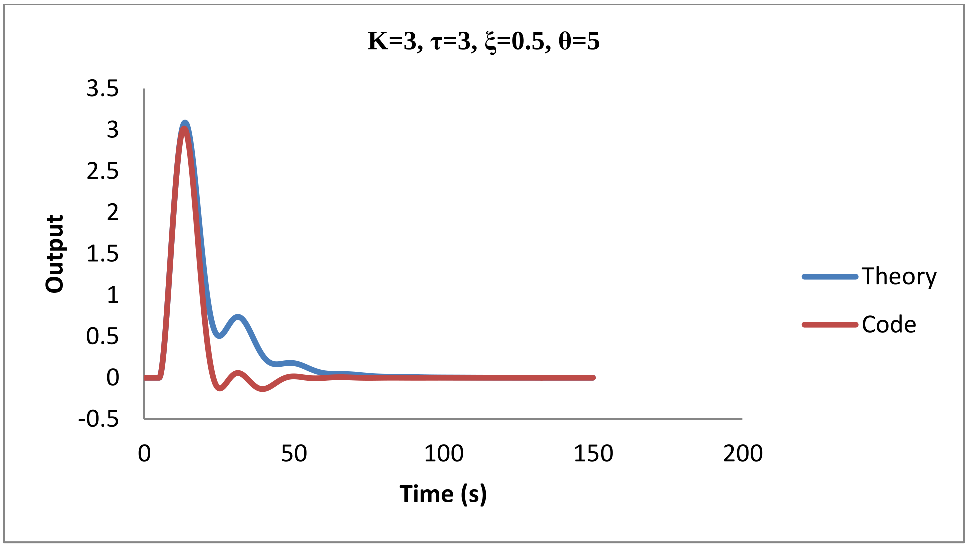

Figure 6.12 Disturbance rejection performance of the controllers for an underdamped SOPTD system

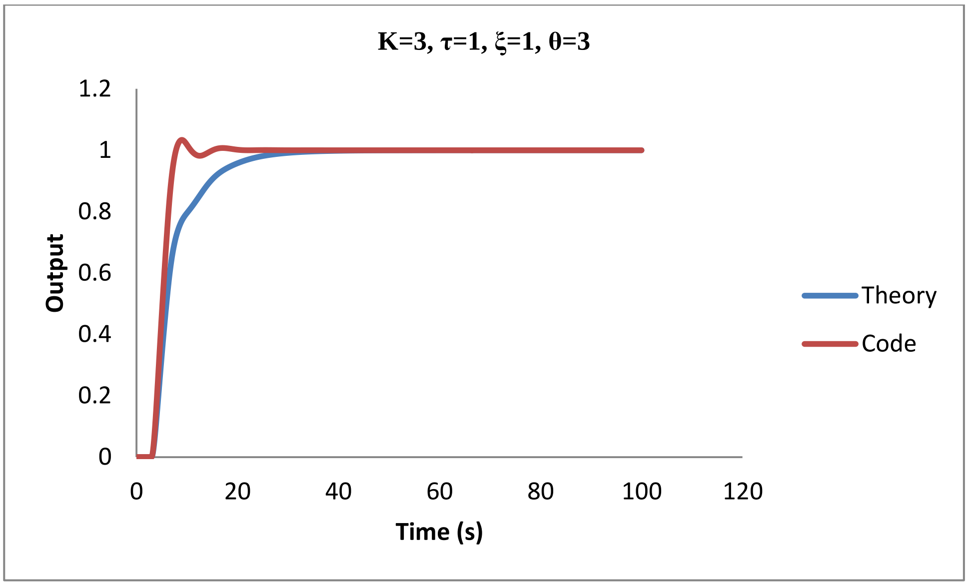

Figure 6.13 Set point tracking performance of the controllers for a critically damped SOPTD system

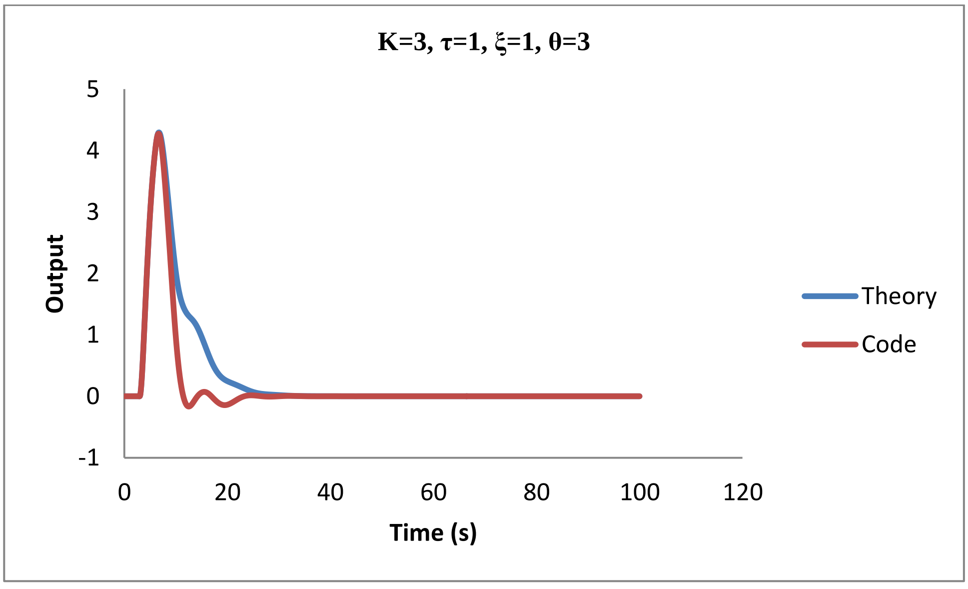

Figure 6.14 Disturbance rejection performance of the controllers for a critically damped SOPTD system

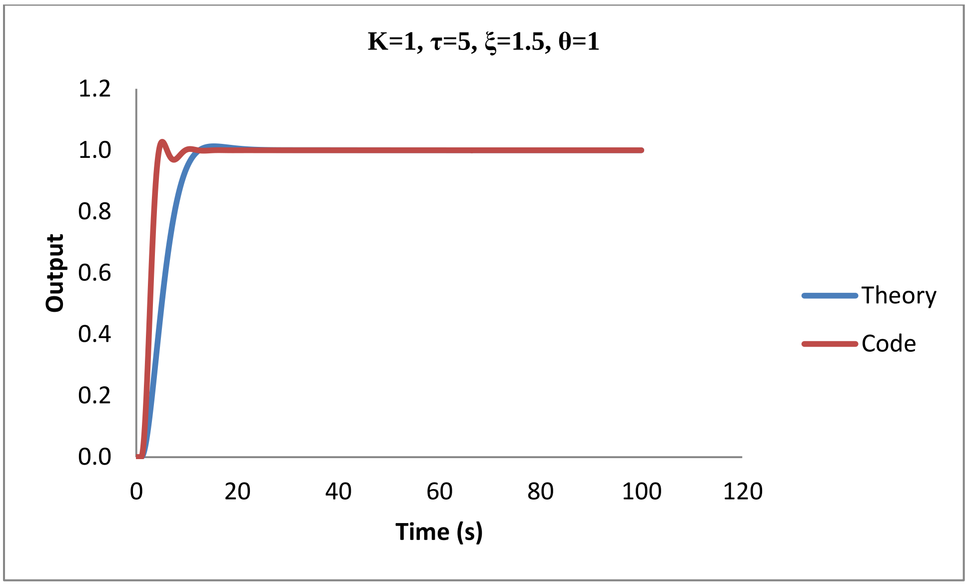

Figure 6.15 Set point tracking performance of the controllers for an overdamped SOPTD system

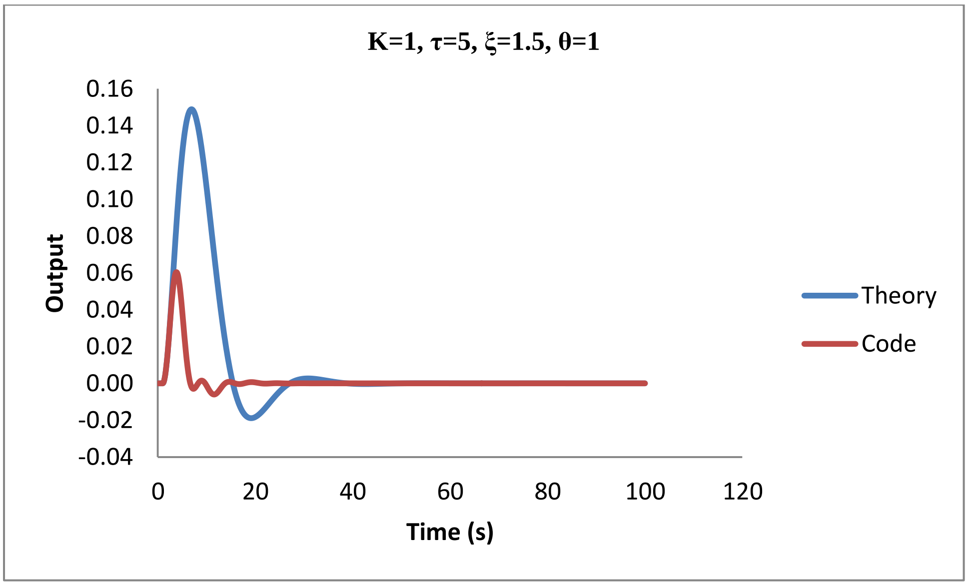

Figure 6.16 Disturbance rejection performance of the controllers for a overdamped SOPTD system

It is seen from the Figures through 6.11 to 6.16 for the case of Gp=Gd, performance of code is better than Smith's relation for the case of set point tracking and disturbance rejection when considering SOPTD models.

6.4. Analysis of Gp≠Gd case for FOPTD models

Finally we evaluated the case of Gp≠Gd where closed loop block diagram is represented on Figure 6.17. In our process model, we also have the case of Gp≠Gd. Smith's relation is not exact relation for this case. Anyway, we used Smith's relation as initial controller parameter and analyzed performance of Smith's tuning relation for disturbance rejection case. We considered it as baseline controller again and compared with the code performance.

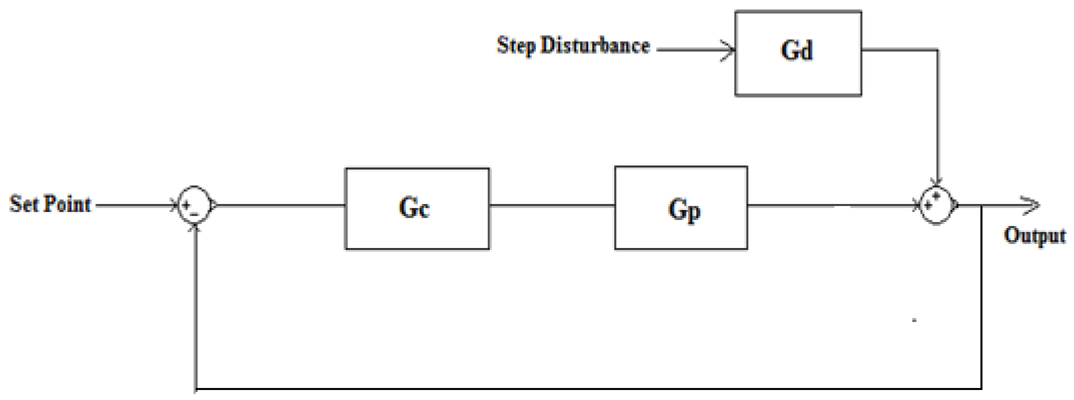

Figure 6.17 Closed loop block diagram of a process where Gp≠Gd

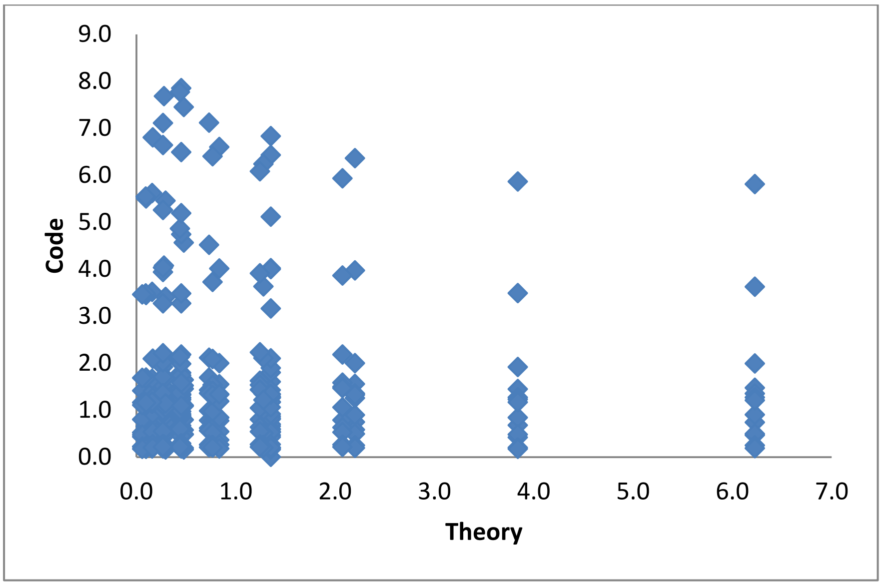

Figure 6.18 K_c comparison of various FOPTD models for the case of Gp≠Gd

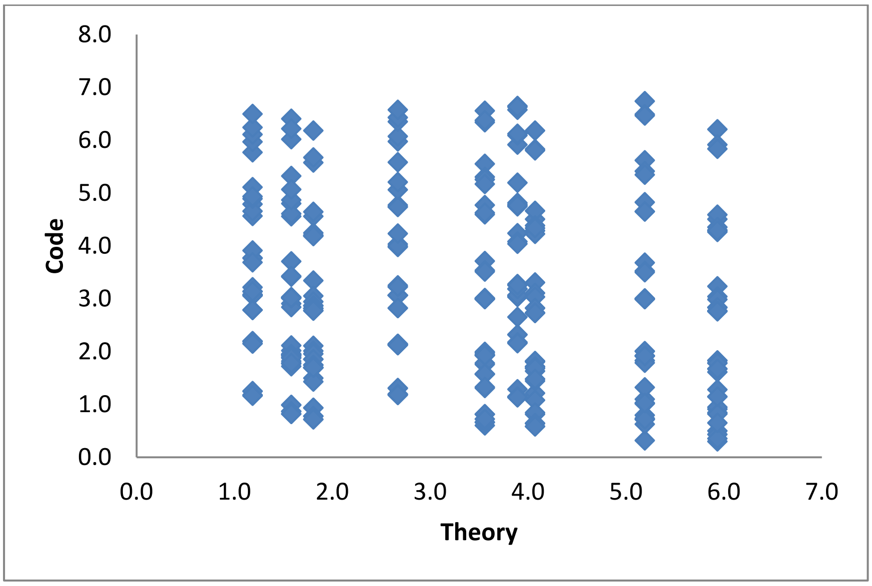

Figure 6.19 $\boldsymbol{\tau_I}$ comparison of various FOPTD models for the case of Gp≠Gd

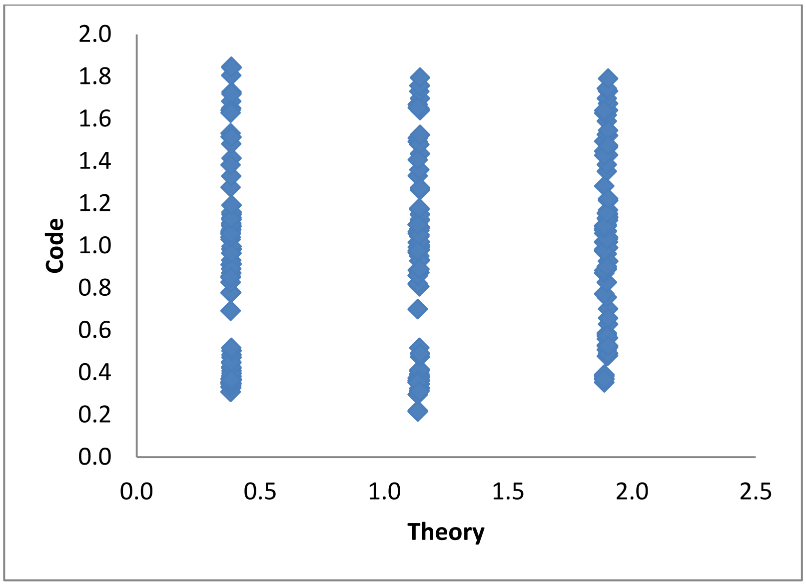

Figure 6.20 τ_d comparison of various FOPTD models for the case of Gp≠Gd

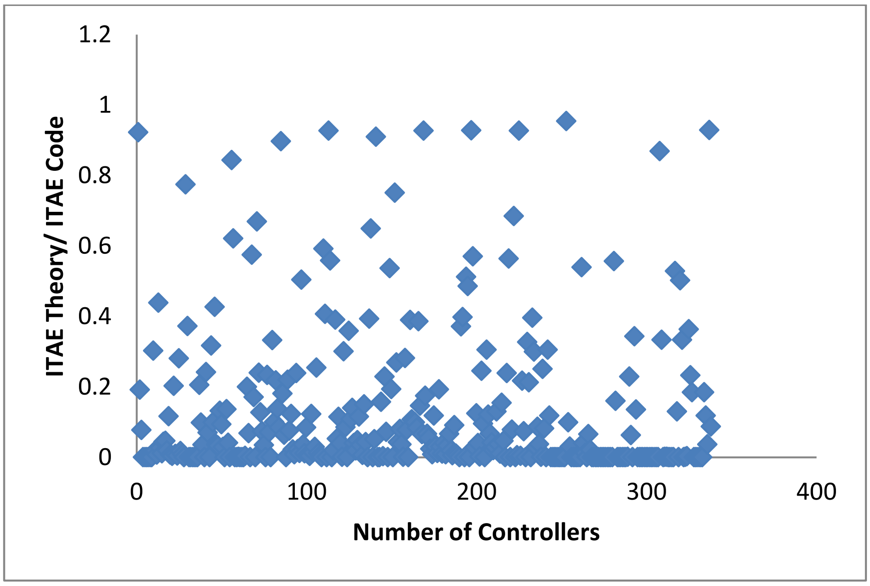

Figure 6.21 ITAE score ratios of various FOPTD models for the case of Gp≠Gd

From Figures 6.18 through 6.20 it is clearly seen that Smith's relation based controller parameters are not close to the parameters found by optimization code. Anyway, there is a correlation between Smith's tuning parameters and code output based controller parameters. Data points are not random for K_c , τ_I and τ_d that means Smith's method can be extended for the case of Gp≠Gd processes. Since Smith's relation is not the appropriate tuning technique for Gp≠Gd models, ITAE score of proposed model is quite far away from the ITAE score of Smith's method when compared with the Figures 6.6 and 6.10.

CHAPTER 7

CLOSED LOOP RESULTS AND DISCUSSION

7.1. Simulation Studies

In Chapter 3 we obtained transfer functions those represents input-output relations. These linearized forms of our process model include steady state values. According to these steady state values different transfer functions were obtained. Simulation conditions were determined and corresponding transfer functions were obtained at those conditions.

After obtaining input-output relations at specified conditions these relations were implemented in our simulink block diagram of Figure 6.2 to find proper PID controller parameters. Before this, from reduced models and using Table 6.1, corresponding initial guesses were determined. Then, these initial guesses were introduced to our code, in which an objective function tries to find optimum controller settings, by using the actual model input-output transfer function relations implemented in simulink block.

7.2. Test Cases

We have transfer functions that relate outputs (H, T) to the inputs (W_1, C_v, T_1, Q). $W_1$and Q are manipulated inputs whereas C_v and T_1 are disturbance variables. For set point tracking performance of the controller, transfer functions that relate H to W_1 and T to Q were used in the code developed to find controller settings. On the other hand, H-C_v, T-T_1 and T-C_v transfer function relations were used for disturbance rejection tuning.

Firstly set point change simulations were conducted. For this purpose at specified conditions, individual set point changes were applied for height and temperature. Then, set points of two outputs were changed simultaneously. Finally, disturbance rejection performances of controllers were analyzed. At different conditions, valve opening were changed and resulting controller effect on these changes were obtained. For temperature, T_1 was also changed as disturbance, and corresponding rejection performance was evaluated.

Model that relates $H^{'}$ to $W_1{}^{'}$ is first order without time delay. Transport delay for different nominal conditions is calculated that is the time required for water to fill in to the tank after pump is turned on. With known values of pipe length (72 cm), diameter (1.1 cm) and flow rate that satisfies specified steady state condition, corresponding transport delay can easily be calculated.

For; $\bar{h} = 15\ cm, \bar{C_v} = 5$

$$\frac{H^{'}(s)}{W_1^{'}(s)} = \frac{K_2}{s-K_1} = \frac{-\frac{K_2}{K_1}}{-\frac{1}{K_1}s+1} = \frac{8.72}{2666.8s+1} \qquad \textbf{(7.1)}$$

$$\frac{H^{'}}{W_1{}^{'}} = \frac{8.72}{2666.8s+1}e^{-31.8s} \qquad \textbf{(7.2)}$$

From the flow rate at specified steady state condition, velocity of inlet water can be calculated. Length of the piping divided to the velocity of water yields transport delay which is 31.8 s for the case above.

For; $\bar{h} = 15\ cm, \bar{C_v} = 21.13$

$$\frac{H^{'}}{W_1{}^{'}} = \frac{2.06}{631s+1}e^{-7.53s} \qquad \textbf{(7.3)}$$

Model that relates $T^{'}$ to $Q^{'}$ is a second order model. Therefore, this model should be reduced to first order plus time delay model using appropriate reduction technique.

For; $\bar{h} = 15\ cm,\ \bar{C_v} = 5, \bar{T}_1 = 15\ ℃, \bar{T} = 15\ ℃$

$$\frac{T^{'}(s)}{Q^{'}(s)} = \frac{\frac{K_8K_{10}}{(s-K_4)(s-K_9)}}{\left(1-\frac{K_8K_{11}}{(s-K_4)(s-K_9)}\right)} = \frac{K_8K_{10}}{s^2-s(K_4+K_9)+K_4K_9-K_8K_{11}} = \frac{\frac{K_8K_{10}}{K_4K_9-K_8K_{11}}}{(\tau_1s+1)(\tau_2s+1)} = \frac{0.0278}{(583.7s+1)(65.5s+1)} \qquad \textbf{(7.4)}$$

Using the half rule;

$$\frac{T^{'}(s)}{Q^{'}(s)} = \frac{0.0278}{616.4s+1} e^{-32.72} \quad \textbf{(7.5)}$$

For; $\bar{h} = 15\ cm, \bar{C_v} = 21.13, \bar{T_1} = 15$ ℃, $\bar{T} = 15$ ℃

$$\frac{T^{'}(s)}{Q^{'}(s)} = \frac{0.0066}{(146.63s+1)(61.65s+1)} = \frac{0.0066}{177.45s+1} e^{-30.82} \quad \textbf{(7.6)}$$

Model that relates $H^{'}$ to $C_v{}^{'}$ is a first order model without time delay. Since C_v depends on valve opening and valve is adjusted manually, a sudden step change can be applied to the C_v. Therefore a delay of 1 second is assumed to be used for this model.

For; $\bar{h} = 15\ cm, \bar{C_v} = 5$

$$\frac{H^{'}(s)}{C_v^{'}(s)} = \frac{K_3}{s-K_1} = \frac{-\frac{K_3}{K_1}}{-\frac{1}{K_1}s+1} = \frac{-15}{2666.76s+1} e^{-s} \quad \textbf{(7.7)}$$

For; $\bar{h} = 15\ cm, \bar{C_v} = 21.13$

$$\frac{H^{'}}{C_v{}^{'}} = \frac{-3.55}{631s+1} e^{-s} \quad \textbf{(7.8)}$$

Model that relates $T^{'}$ to $C_v{}^{'}$ is a third order model, represented as follow;

For; $\bar{h} = 15\ cm, \bar{T} = 19$ ℃, $\bar{T_1} = 15$ ℃, $\bar{C_v} = 5$

$$\frac{T^{'}(s)}{C_v^{'}(s)} = \frac{sK_3K_6 - K_3K_6K_9}{s^3 - s^2(K_4+K_9+K_1) + s(-K_8K_{11}+K_4K_9+K_1K_4+K_1K_9) - K_1K_4K_9 + K_1K_8K_{11}} =$$

$$\frac{\frac{K_3K_6K_9}{K_1(K_4K_9-K_8K_{11})}\left(-\frac{1}{K_9}s+1\right)}{(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)} = \frac{-0.0678(71.62s+1)}{(2666.76s+1)(583.67s+1)(65.45s+1)} \quad \textbf{(7.9)}$$

Using the Skogestad's reduction rule of T3 represented as Eqn. 2.16 and half rule;

$$\frac{T'(s)}{C_v'(s)} = \frac{-0.0454}{2006.6s+1} e^{-357.28s} \quad \textbf{(7.10)}$$

For; $\bar{h} = 15\ cm, \bar{T} = 19$ °C, $\bar{T}_1 = 15$ °C, $\bar{C}_v = 21.13$

$$\frac{T^{'}(s)}{C_v^{'}(s)} = \frac{-0.000108(71.62s+1)}{(631s+1)(146.6s+1)(61.6s+1)} = \frac{-0.000108}{632.73s+1} e^{-134.96s} \quad \textbf{(7.11)}$$

Model that relates $T^{'}$ to $T_1{}^{'}$ is a second order model represented below;

For; $\bar{h} = 15\ cm, \ \ \bar{T}_1 = 15$ °C, $\ \bar{C}_v = 5$

$$\frac{T^{'}(s)}{T_1^{'}(s)} = \frac{\frac{K_7}{s-K_4}}{\left(1-\frac{K_8K_{11}}{(s-K_4)(s-K_9)}\right)} = \frac{-\frac{K_7K_9}{K_4K_9-K_8K_{11}}\left(-\frac{1}{K_9}s+1\right)}{(\tau^2s^2+2\xi\tau s+1)} = \frac{-\frac{K_7K_9}{K_4K_9-K_8K_{11}}\left(-\frac{1}{K_9}s+1\right)}{(\tau_1s+1)(\tau_2s+1)} = \frac{(71.62s+1)}{(583.67s+1)(65.45s+1)} \quad \textbf{(7.12)}$$

Using the Skogestad's reduction rule of T3 represented as Eqn. 2.16 and half rule;

$$\frac{T^{'}(s)}{T_1^{'}(s)} = \frac{(71.62s+1)}{(583.67s+1)(65.45s+1)} = \frac{0.28}{(124.73s+1)} e^{-32.72s} \quad \textbf{(7.13)}$$

For; $\bar{h} = 15\ cm, \ \ \bar{T}_1 = 15$ °C, $\ \bar{C}_v = 21.13$

$$\frac{T^{'}(s)}{T_1^{'}(s)} = \frac{(71.62s+1)}{(146.63s+1)(61.65s+1)} = \frac{1}{(105.8s+1)} e^{-30.82s} \quad \textbf{(7.14)}$$

Initial guesses for the reduced models represented above are found using the Smith's tuning technique tabulated on Table 6.1. These initial guesses are inserted in to the MATLAB code of Appendix A in which an objective function tries to find optimum controller parameters for actual high order models by evaluating the code in Figure 6.2.
Controller parameters which are used as initial guesses for the code and the actual program output found from that MATLAB code are tabulated on the tables below.

Table 7.1 H-W_1 (h=15 cm, T=15 ^{0}C, C_v=5)

Controller Parameters	Initial Guess	Program output
K_c	4.8	5.1
τ_I	3358	3280.3
τ_d	13.4	13.6

Table 7.2 T-Q (h=15 cm, T=15 ^{0}C, C_v=5)

Controller Parameters	Initial Guess	Program output
K_c	420.4	455.67
τ_I	782	757.07
τ_d	12.4	12.68

Table 7.3 T-T_1 (h=15 cm, T=15 ^{0}C, C_v=5)

Controller Parameters	Initial Guess	Program output
K_c	17.19	18.90
τ_I	55.18	49.66
τ_d	12.55	13.80

Table 7.4 H-C_v (h=15 cm, T=15 ^{0}C, C_v=5)

Controller Parameters	Initial Guess	Program output
K_c	158.82	166.76
τ_I	9.38	9.38
τ_d	0.4	0.4

Table 7.5 T-C_v (h=15 cm, T=19 ^{0}C, C_v=5)

Controller Parameters	Initial Guess	Program output
K_c	153	160
τ_I	667	667
τ_d	137	137

Table 7.6 H-W_1 (h=15 cm, T=15 ^{0}C, C_v=21.13)

Controller Parameters	Initial Guess	Program output
K_c	20	22.07
τ_I	794	774.40
τ_d	3	3.09

Table 7.7 T-Q (h=15 cm, T=15 ^{0}C, C_v=21.13)

Controller Parameters	Initial Guess	Program output
K_c	648.61	1383.40
τ_I	230.29	400.70
τ_d	10.75	23.40

Table 7.8 T-T_1 (h=15 cm, T=15 ^{0}C, C_v=21.13)

Controller Parameters	Initial Guess	Program output
K_c	4.36	6.77
τ_I	50.57	51.30
τ_d	11.82	14.04

Table 7.9 H-C_v (h=15 cm, T=15 ^{0}C, C_v=21.13)

Controller Parameters	Initial Guess	Program output
K_c	171.42	179.99
τ_I	6.43	6.43
τ_d	0.39	0.39

Table 7.10 T-C_v (h=15 cm, T=15 ^{0}C, C_v=21.13)

Controller Parameters	Initial Guess	Program output
K_c	54438.86	56949
τ_I	240.27	241
τ_d	51.82	52

Figure 7.1, 7.2 and 7.3 represents response behavior of first order, second order and third order models on which performance of Smith's controller parameters (initial guesses) and controller outputs found by MATLAB program are compared respectively. We have input output relations which varies from first order to third order and are used for tuning approach. Only one of each order models are taken as sample to show whether our proposed method is better than the Smith's proposed approach or not.

From the figures it seems that the response behavior of two tuning methods are almost identical for various ordered models. For healthier comparison ITAE performance index of these responses are calculated and results are tabulated on Table 7.11. Although it seems there is no much difference between response curves, from the ITAE index, parameters obtained by the code gives better results for various ordered models.

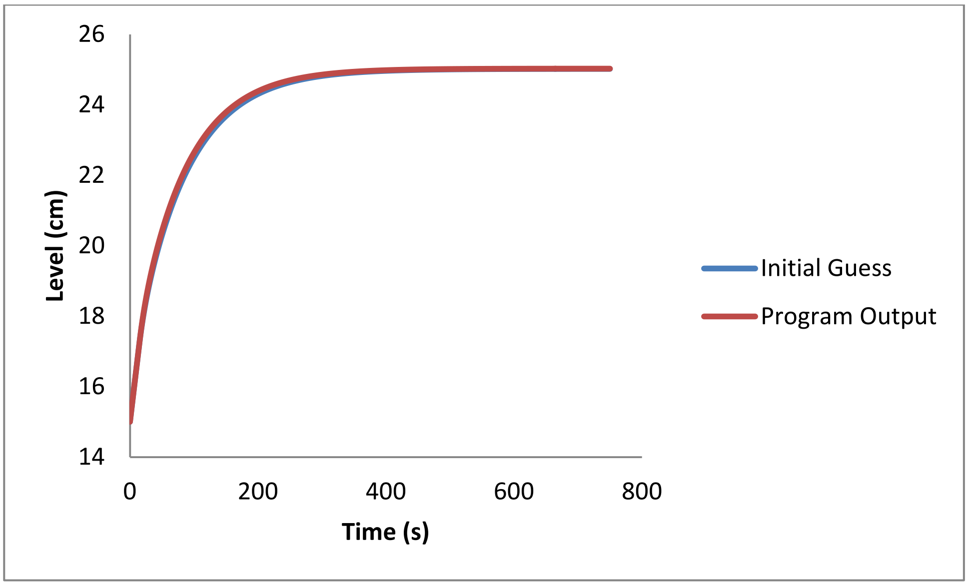

Figure 7.1 Set point change response of level ($C_v = 5$)

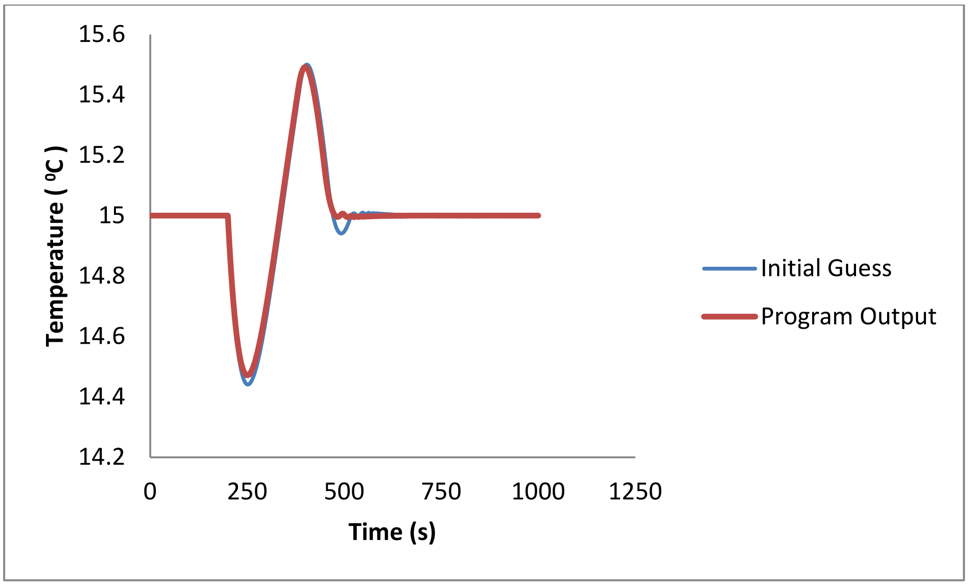

Figure 7.2 Disturbance rejection for inlet temperature change ($T_1 = 15$ °C−12 °C)

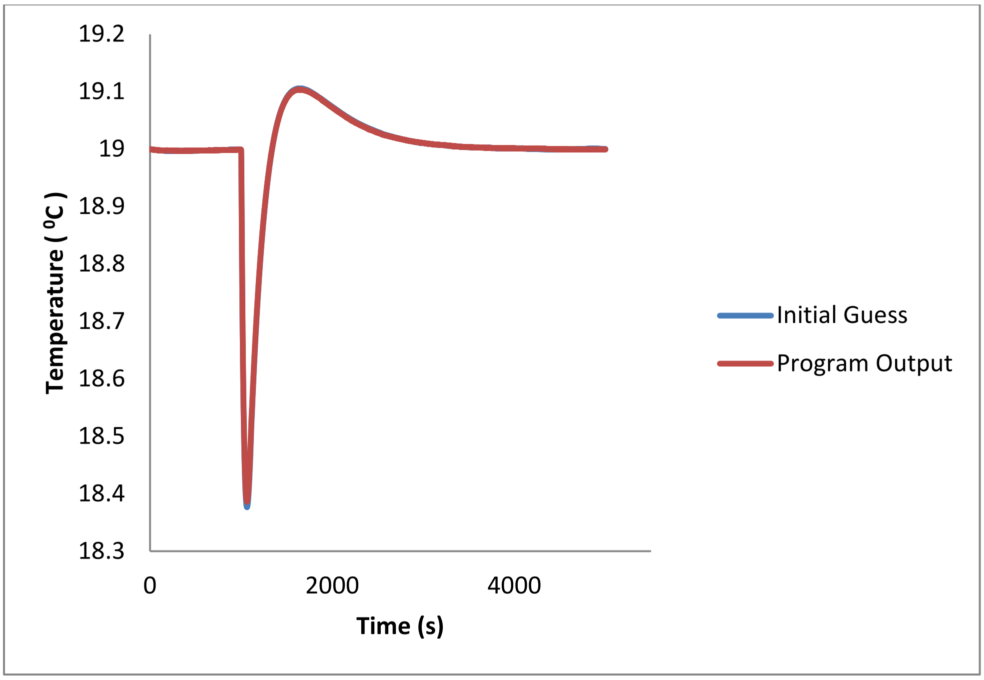

Figure 7.3 Disturbance rejection for C_v change (C_v = 21.13 – 5)

Table 7.11 ITAE Performance Index of Responses on the Figures

	ITAE (Initial guess)	ITAE (Program output)
Figure 7.1	55143	51593
Figure 7.2	30090	27531
Figure 7.3	300431	296724

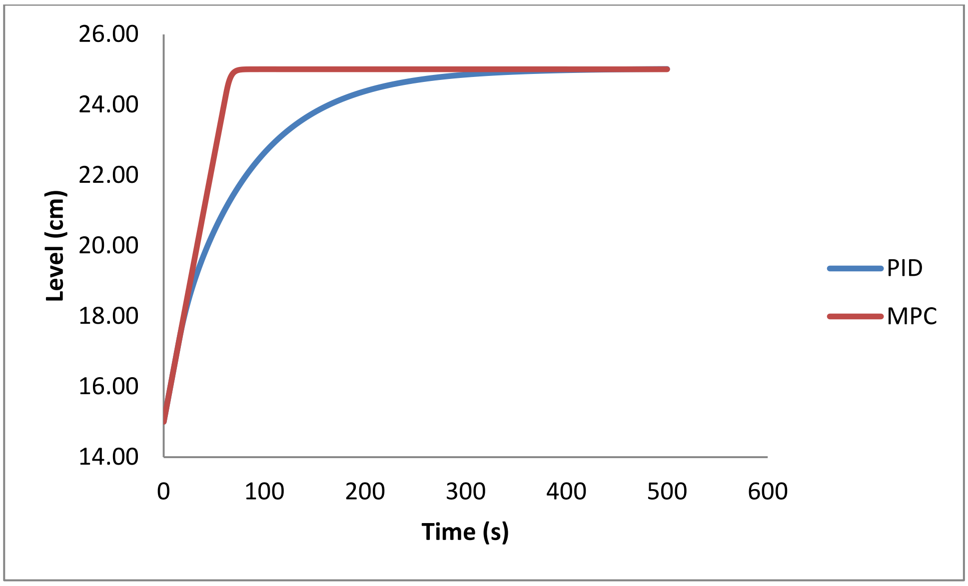

Figure 7.4 Level response (C_v = 5)

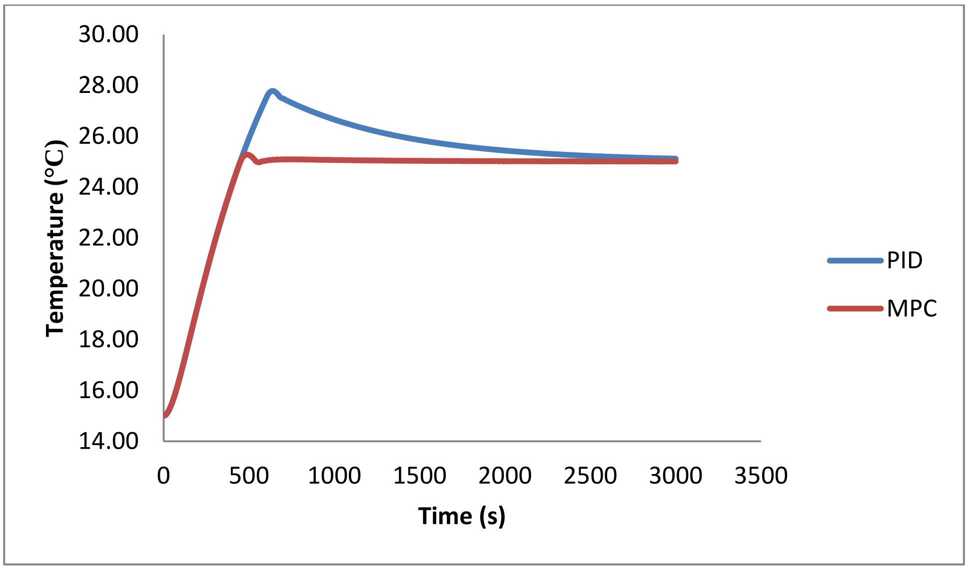

Figure 7.5 Temperature response ($C_v = 5$)

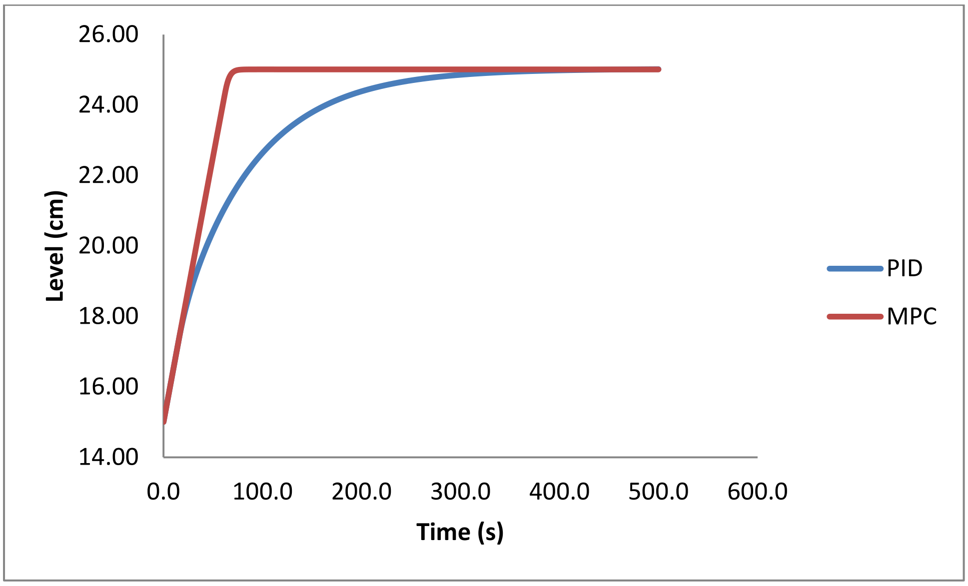

Figure 7.6 Level reponse ($C_v = 5$, and temperature change)

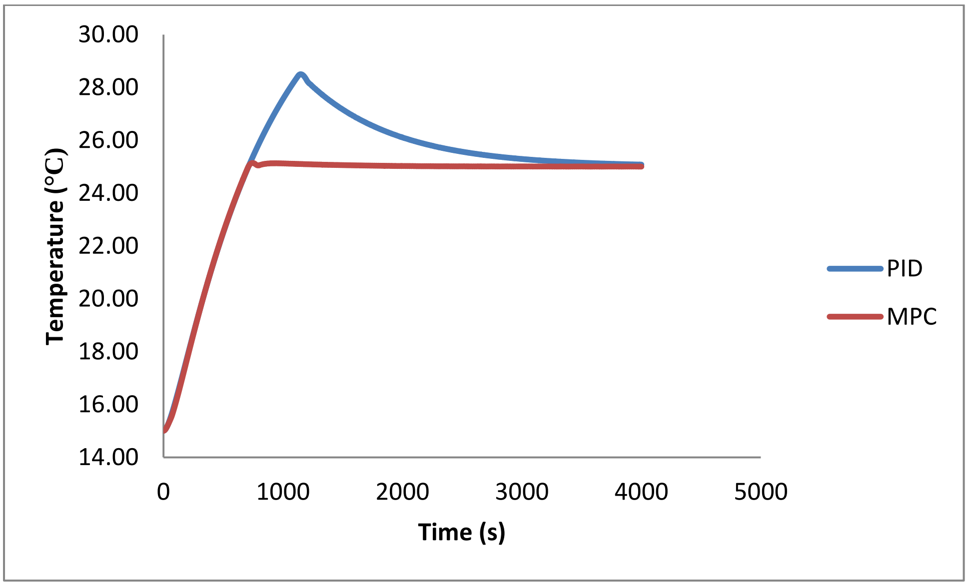

Figure 7.7 Temperature response ($\mathrm{C_v} = 5$, and level change)

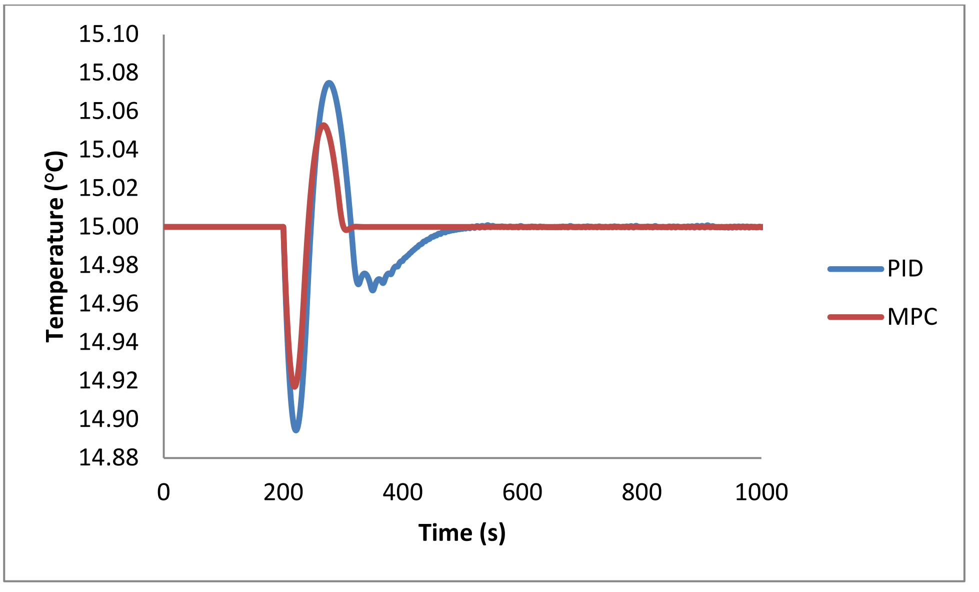

Figure 7.8 Temperature response for T_1 change (C_v =5, T_1= 15 °C – 10 °C)

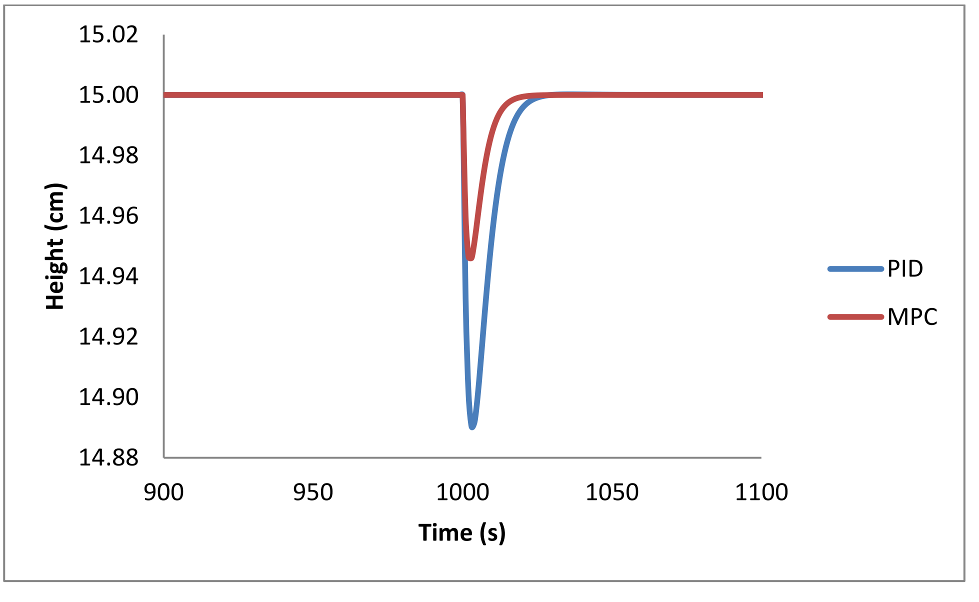

Figure 7.9 Level response for C_v change (C_v =5 – 21.13)

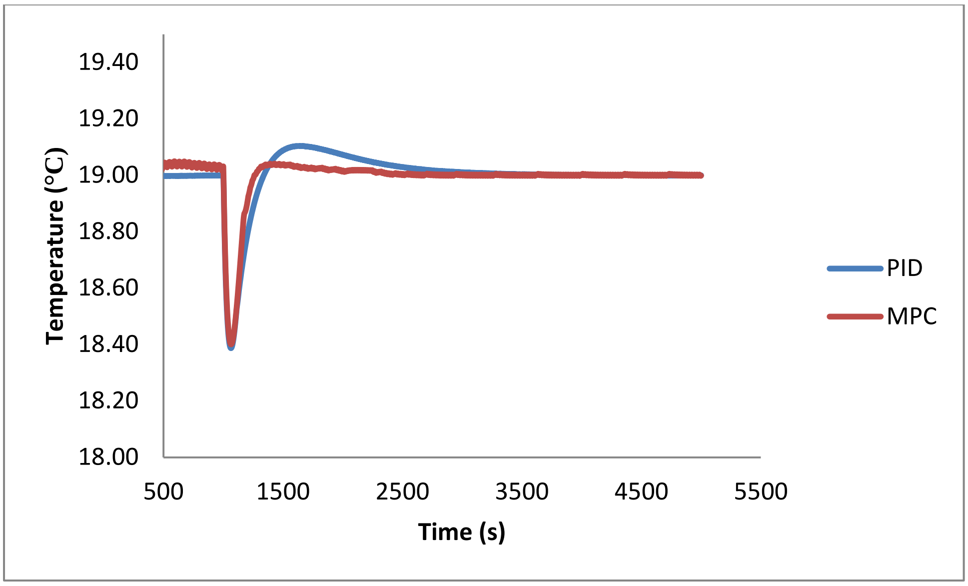

Figure 7.10 Temperature response for C_v change (C_v =5 – 21.13)

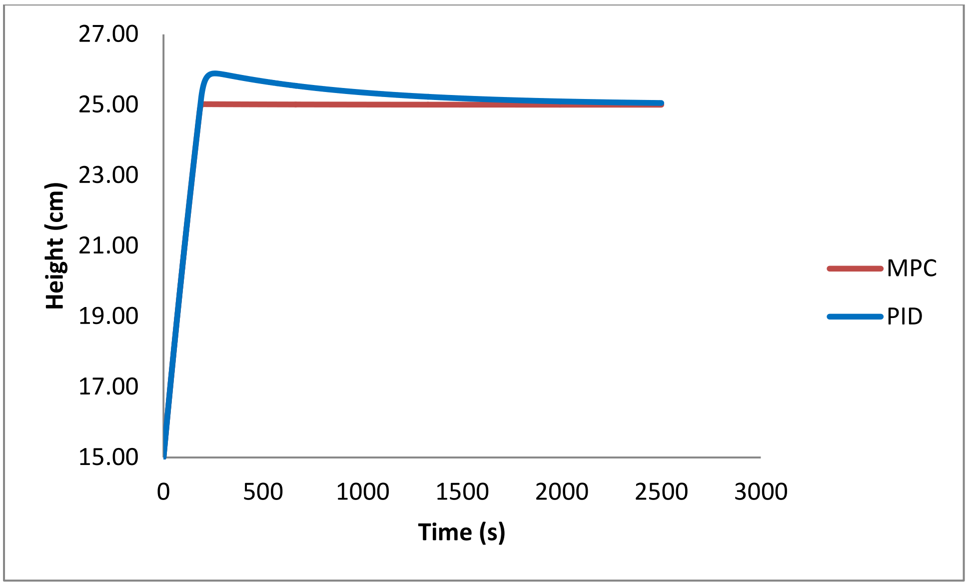

Figure 7.11 Level response (C_v =21.13)

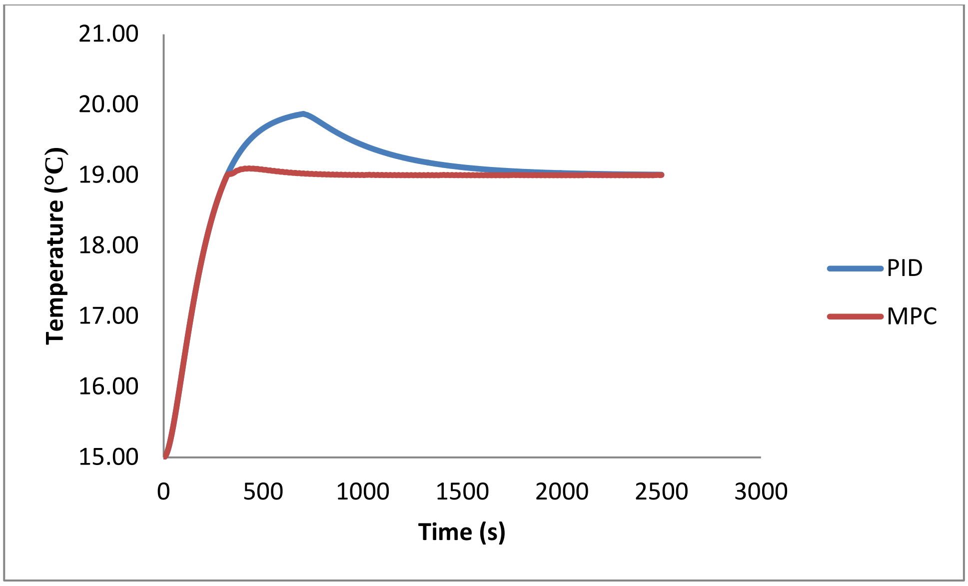

Figure 7.12 Temperature response (C_v =21.13)

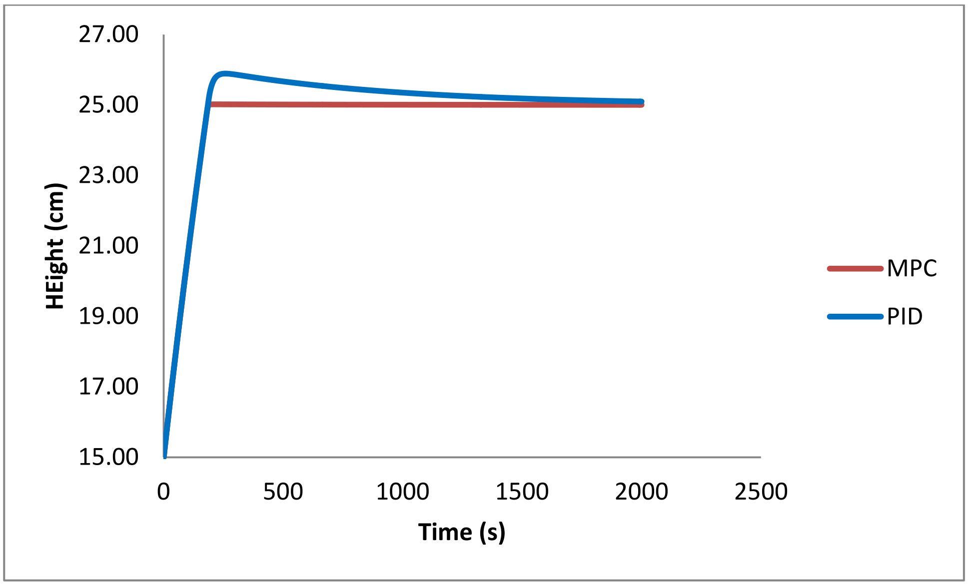

Figure 7.13 Level response (C_v =21.13 and temperature change)

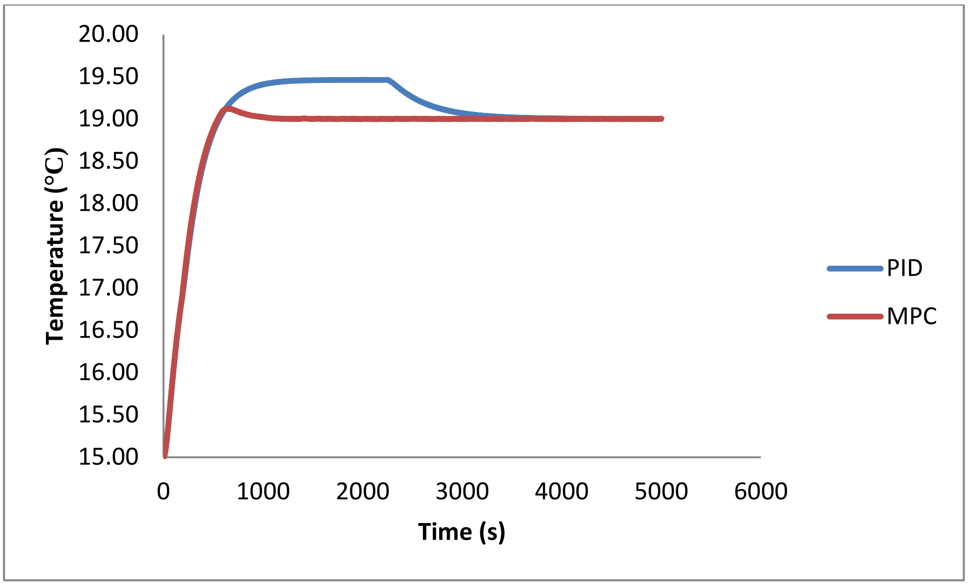

Figure 7.14 Temperature response (C_v =21.13 and level change)

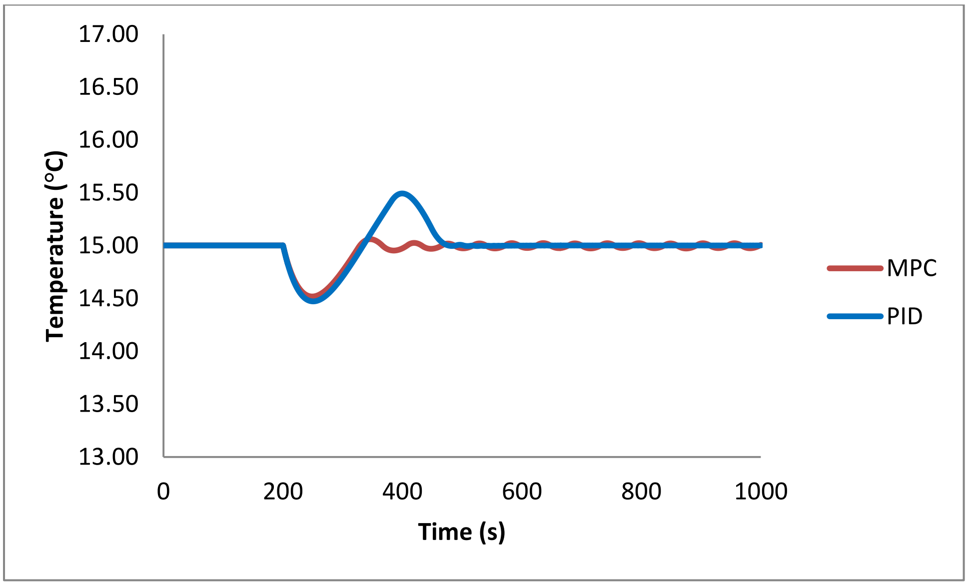

Figure 7.15 Temperature response for T_1change (C_v =21.13, T_1=15 °C – 12 °C)

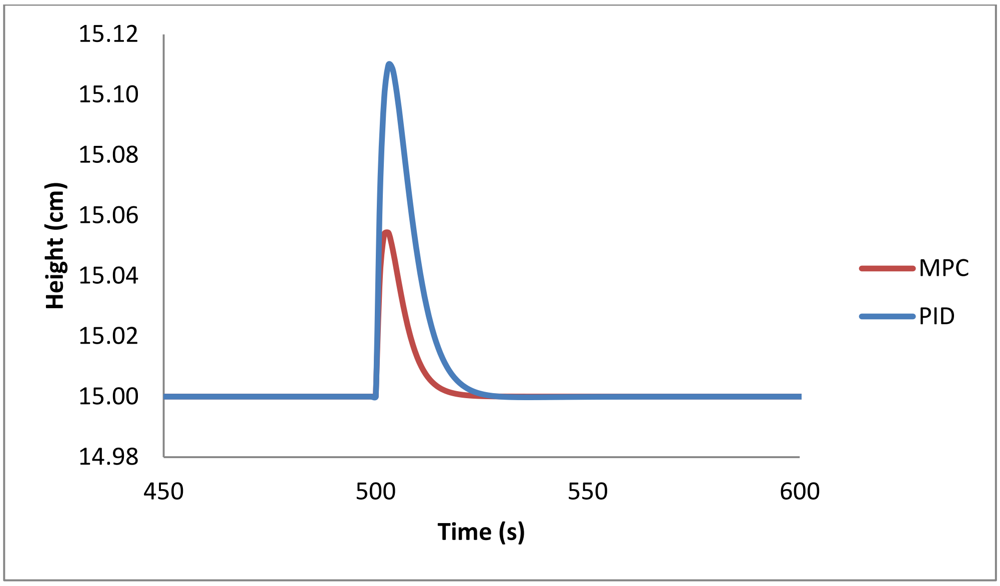

Figure 7.16 Level response for C_v change (C_v =21.13 - 5)

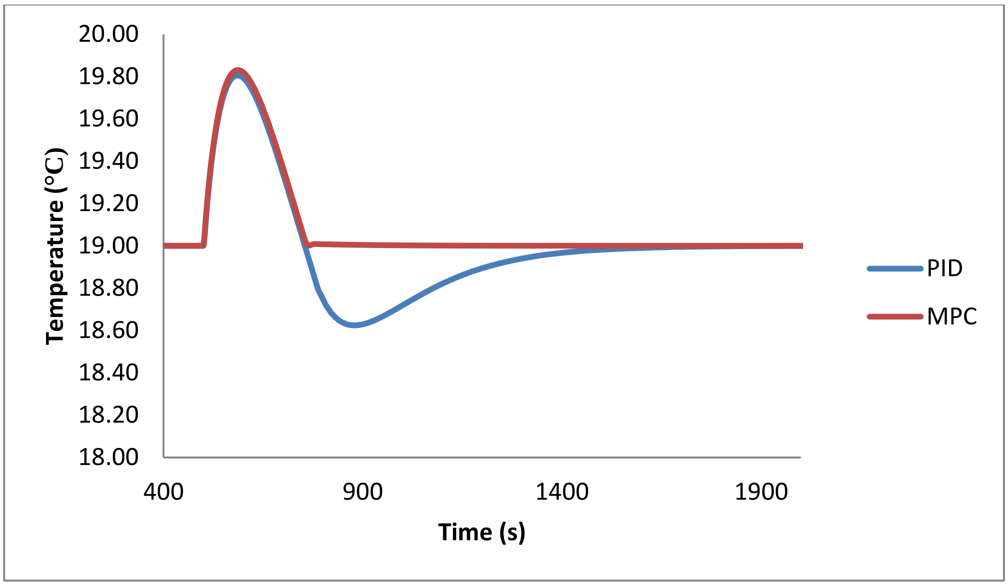

Figure 7.17 Temperature response for C_v change (C_v =21.13 – 5)

Table 7.12 ITAE Results of PID and MPC Closed Loop Responses

	ITAE (PID)	ITAE (MPC)	MPC Improvement (%)
Figure 7.4	48207	8236	82.9
Figure 7.5	2951794	421925	85.7
Figure 7.6	48208	8236	82.9
Figure 7.7	6234383	851548	86.3
Figure 7.8	3216	1027	68.1
Figure 7.9	1001	387	61.3
Figure 7.10	296048	157224	46.9
Figure 7.11	660840	61387	90.7
Figure 7.12	555212	73160	77.9
Figure 7.13	583605	60670	89.6
Figure 7.14	1647879	168574	89.8
Figure 7.15	27531	21485	21.9
Figure 7.16	504	208	58.7
Figure 7.17	209485	84402	59.7

Table 7.13 MPC Parameters Used for Closed Loop Simulations

Sampling time	1
Predicition horizon	10
Control horizon	2
Weight on height	1
Weight on temperature	1000
Weight on inlet flow rate	1
Weight on heat input	1

From the closed loop responses of PID and MPC controllers plotted on the figures through 7.4 to 7.17, it seems that MPC controller gives better response. From the ITAE index it can also be concluded that MPC has better performance The reason leads behind this is the predictive ability of MPC. Since MPC predicts process behavior according to an anticipated input trajectory, and does this at each sampling interval, it approaches best predicted response in short time by calculating an optimum input sequence. However, PID takes instantanous information from the process and correcitive action takes place after that. This means, response for PID control will take more time to reach desired set point when compared to the MPC. It must be noted that a weight of 1000 is specified for temperature, that means our MPC controller will be thousand times more sensitive to the temperature deviations than level deviations. In another term, controller will act faster to the temperature variations. Select of choice for this weight terms for inputs and outputs depend on the importance and deviation tolerance of inputs and outputs. These weights adjust the impact of each control action, rate of change in control action, and plant outputs. Since 1000 value for temperature weight give better response when compared to the PID this value is determined to be used.

CHAPTER 8

CONCLUSION

In the scope of this thesis, MPC and PID controllers were implemented on a heating tank system with variable hold-up at simulation environment using Simulink. At different outlet valve openings set point changes were applied to the system and corresponding set point tracking performances were evaluated. Disturbances were also applied to the system at different conditions to see disturbance rejection performance. From the output response plots obtained, it can be concluded that MPC gives better performance than PID controllers for all simulation cases conducted. This is because of predictive ability of MPC that makes it successful over PID control. On the other hand, with its applicability to all kind of process models, the code used for the optimization of PID controller parameters, showed better performance than the available analytical tuning relations.

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APPENDIX A

A.1 Simulink Block Diagrams

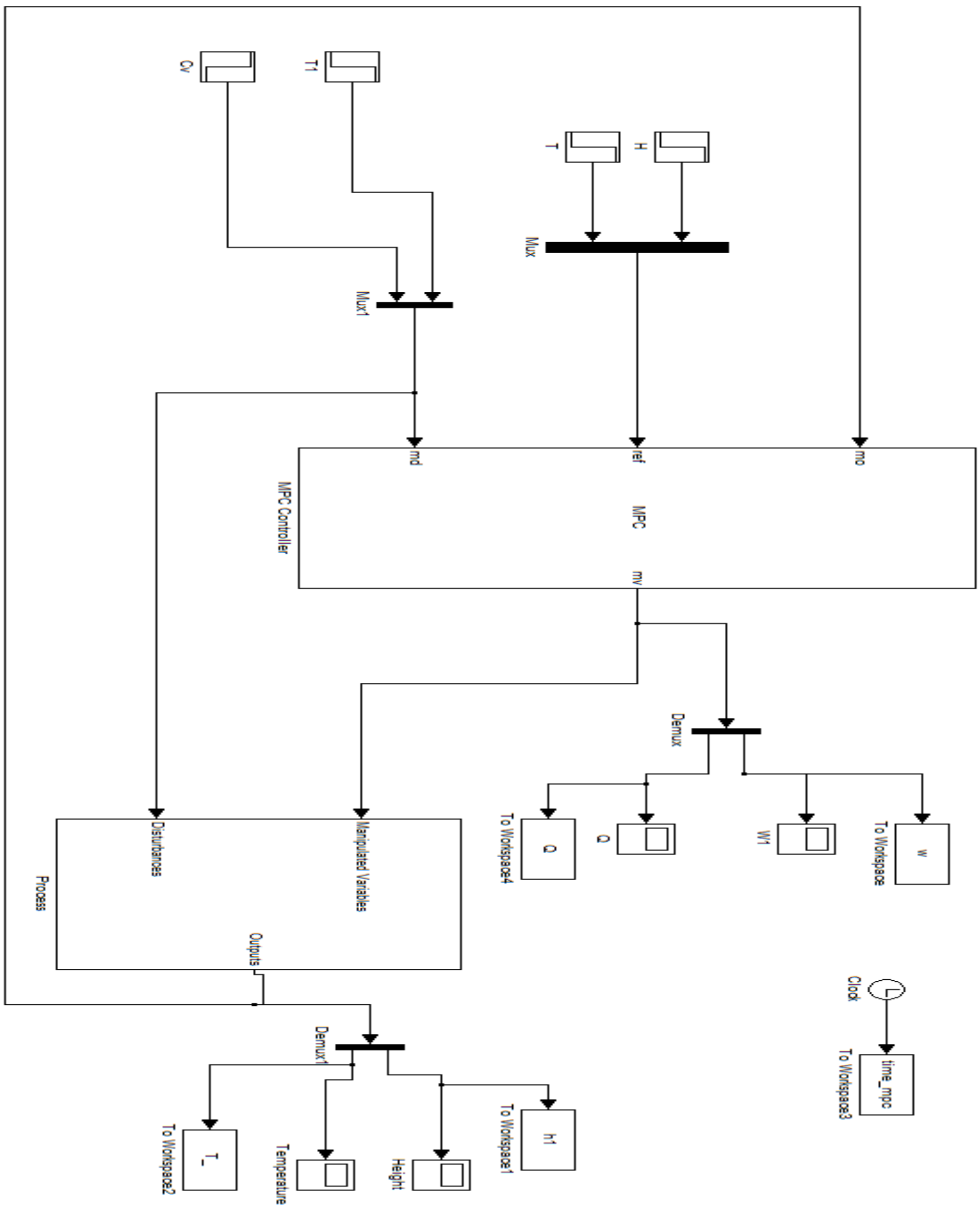

Figure A.1 MPC Block Diagram

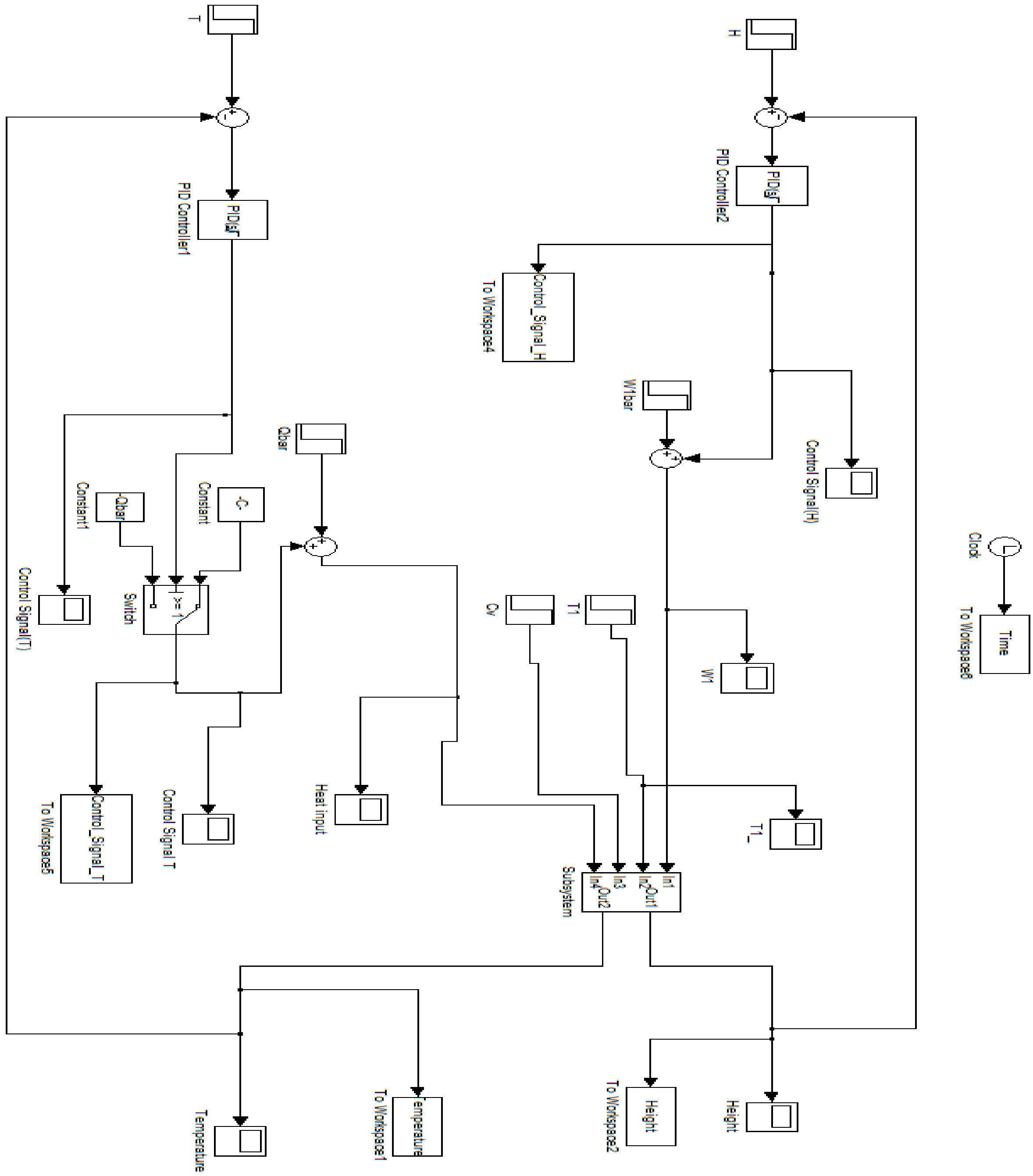

Figure A.2 PID Block Diagram

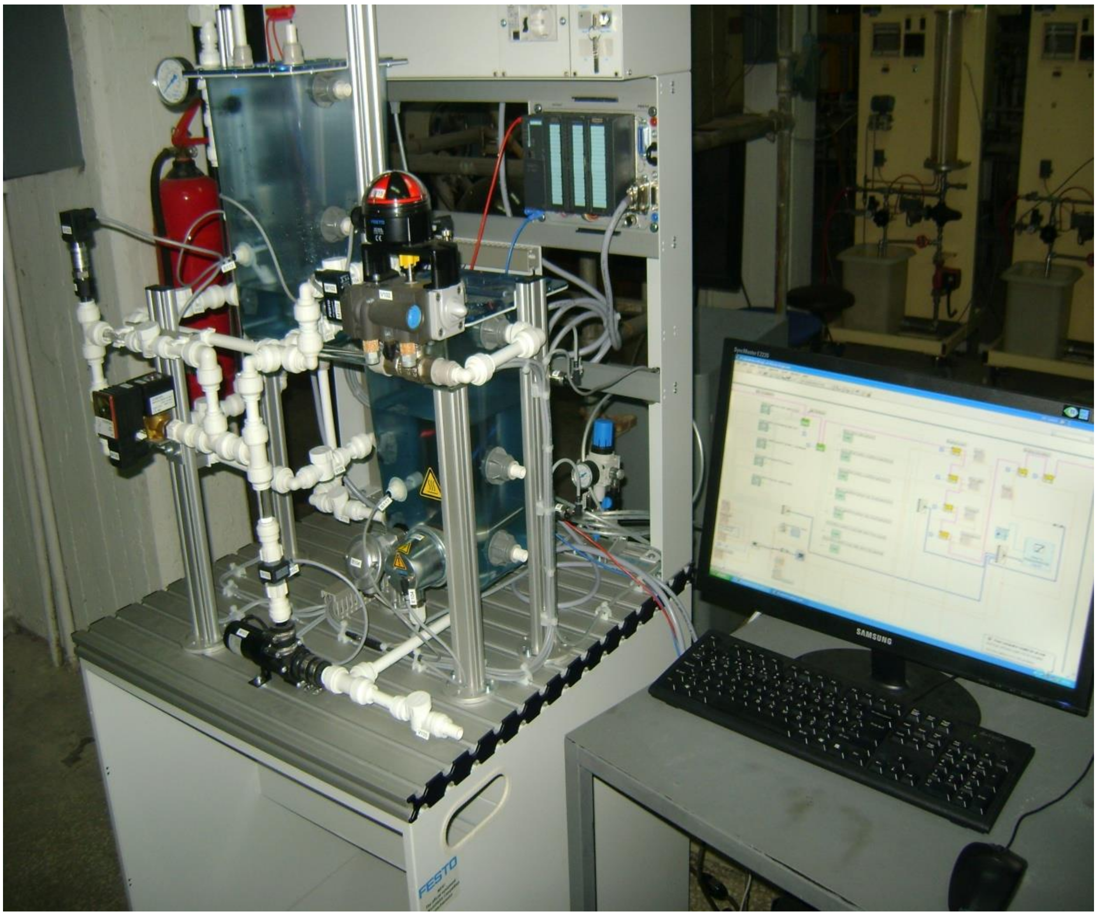

Figure A.3 Photograph of experimental set-up

APPENDIX B

B.1. Main Program

```
%Main program
global kc
global ti
global td
global y_out
global ti_me
global Msp Md


global numP denP thetaP
global numD denD thetaD

% Decision on which problem to test for - set-point tracking or
disturbance
% rejection
Msp = 0;
Md = 1;

% Define the system parameters
% Disturbance transfer function
numD = [1];
denD = [5^2 2*5*1.5 1];
thetaD = [1];
% Process transfer function
numP = [1];
denP = [5^2 2*5*1.5 1];
thetaP = [1];



x0=[5.18 5.4364 1.2125];
results = [];
error=[];

% Setting the initial guesses for the controller parameters

for m = 1:2:5 %k
    for n = 1:2:5 %tou
        for p = 1:2:5 %theta


for i = 1:2:5  %k
    for j = 1:2:5  %tou
        for k = 1:2:5   %theta
            numP = m;
            denP = [n 1];
```

```
            thetaP = p;
            numD = i;
            denD = [j 1];
            thetaD = k;


            %Identification of a FOPTD version of the system model
            [y,t] = step(tf(numP,denP));
            t_end=100;
            h=0.1;

            % Initial Guess for Set Point
            x0=[((0.965*(p/n)^(-0.85))/m) (n/(0.796+(-0.1465)*(p/n))) ...
                (n*(0.308*(p/n)^(0.929)))];

            % Initial Guess for Disturbance
            x0=[((1.357*(k/j)^(-0.947))/i) (j/(0.842*(k/j)^(-0.738))) ...
                (j*(0.381*(k/j)^(0.995)))];

            x=fminsearch(@fobs,x0,[],t_end,h);

            results = [results ; x(1) x(2) x(3)]

        end
    end
end

        end
    end
end
```

B.2.Code of Objective Function

```
% Objectivefunction
function f=fobs(x,t_end,h)
global kc
global ti
global td
global y_out
global ti_me

kc=x(1);
ti=x(2);
td=x(3);
tt=(0:h:t_end);
[t,y]=sim('Untitled_H',tt);
f=0;
l=length(ti_me);
f=ti_me(l)*abs(y_out(l,1)-y_out(l,2));
for i=2:2:l-1
    f=f+4*ti_me(i)*abs(y_out(i,1)-y_out(i,2));
end
for i=3:2:l-2
    f=f+2*ti_me(i)*abs(y_out(i,1)-y_out(i,2));
end
f=h/3*f;
```