# AN INVESTIGATION OF PRIVATE MIDDLE SCHOOL STUDENTS' COMMON ERRORS IN THE DOMAIN OF AREA AND PERIMETER AND THE RELATIONSHIP BETWEEN THEIR GEOMETRY SELF-EFFICACY BELIEFS AND BASIC PROCEDURAL AND CONCEPTUAL KNOWLEDGE OF AREA AND PERIMETER

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#### ABSTRACT

# AN INVESTIGATION OF PRIVATE MIDDLE SCHOOL STUDENTS' COMMON ERRORS IN THE DOMAIN OF AREA AND PERIMETER AND THE RELATIONSHIP BETWEEN THEIR GEOMETRY SELF-EFFICACY BELIEFS AND BASIC PROCEDURAL AND CONCEPTUAL KNOWLEDGE OF AREA AND PERIMETER

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The purposes of the present study were to investigate private middle school students' procedural and conceptual knowledge in the domain of area and perimeter of geometric figures and to examine the most common errors in their knowledge. Private middle school students' geometry self-efficacy throughout the grade levels was also investigated. The other specific interest of the study was to examine how students' conceptual and procedural knowledge aspects of area and perimeter of geometric figures changed with respect to their geometry self-efficacy.

The study was conducted during the second semester of the academic year 2011-2012. The sample was consisted of 111 private middle school students from a

private elementary school in Çayyolu district in Ankara. Data were collected through procedural and conceptual knowledge tests prepared by the researcher and geometry self-efficacy scale developed in a previous research. In order to examine the relationship between geometry self-efficacy beliefs of students and their procedural and conceptual knowledge, Pearson product-moment correlation analyses were run. One-way ANOVA was used to investigate how self-efficacy, procedural knowledge, and conceptual knowledge changed according to grade levels separately.

The results of data analysis indicated that private middle school students had common errors and misconceptions about area and perimeter concepts. ANOVA results revealed that there were no statistically significant differences in private middle school students' procedural knowledge and self-efficacy belief scores in terms of grade levels. However, there was a significant difference in conceptual knowledge scores of private middle school students. Moreover, according to Pearson Product Moments Correlation results, there was significant correlation among students' procedural knowledge, conceptual knowledge and self-efficacy belief scores.

Keywords: Procedural Knowledge, Conceptual Knowledge, Self-Efficacy, Area, Perimeter, Error

# ÖZEL ORTAOKUL ÖĞRENCİLERİNİN ALAN VE ÇEVRE KONUSUNDAKİ TİPİK HATALARININ VE GEOMETRİYE YÖNELİK ÖZ-YETERLİK İNANIŞLARI İLE ALAN VE ÇEVRE KONUSUNDAKİ İŞLEMSEL VE KAVRAMSAL BİLGİLERİ ARASINDAKİ İLİŞKİNİN İNCELENMESİ

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Bu çalışmanın temel amacı ortaokul 6., 7. ve 8. sınıf öğrencilerinin geometrik şekillerin alanı ve çevresi konusundaki işlemsel ve kavramsal bilgilerinin ne düzeyde olduğunu araştırmak ve bu bilgileri ölçen testlerdeki tipik hatalarını incelemektir. Ayrıca, öğrencilerin geometriye yönelik öz-yeterlikleri de sınıf seviyesine göre incelenmiştir. Bu çalışmanın diğer bir amacı da, öğrencilerin alan ve çevre konusundaki işlemsel ve kavramsal bilgilerinin geometriye yönelik öz-yeterliklerine göre nasıl değiştiğinin incelenmesidir.

Bu çalışma 2011-2012 öğretim yılının 2. döneminde gerçekleştirilmiştir. Çalışmaya Ankara ilinin Çayyolu semtinde bulunan özel bir ortaokuldan 111 ortaokul 6.,7. ve 8. sınıf öğrencisi katılmıştır. Veriler, alan ve çevre konusundaki işlemsel ve kavramsal bilgileri ölçen başarı testleri ve geometriye yönelik öz-yeterlik inanışlar testi ile toplanmıştır. Öğrencilerin geometriye yönelik öz-yeterlik inanışları ile işlemsel ve kavramsal testlerdeki başarıları arasındaki ilişkiyi incelemek amacıyla Pearson çarpım-moment korelasyon analizi kullanılmıştır. Öğrencilerin geometriye yönelik öz-yeterlik inanışlarını, alan ve çevre konusundaki işlemsel ve kavramsal bilgilerinin sınıf seviyesine göre nasıl değiştiğini incelemek amacıyla da tek yönlü varyans analizi kullanılmıştır.

Veri analizi sonuçlarına göre, 6., 7. ve 8. sınıf öğrencilerinin alan ve çevre konusunda bir takım hatalara ve kavram yanılgılarına sahip oldukları görülmüştür. Öğrencilerin işlemsel bilgipuanları ve geometriye yönelik öz-yeterlikinanış puanlarının sınıf seviyesine göre anlamlı bir şekilde değişmediği ancak kavramsal bilgi puanlarının değiştiği bulunmuştur. Bunun yanında yapılan korelasyon analiz sonuçları 6., 7. ve 8. sınıf öğrencilerinin geometriye yönelik öz-yeterlik inanış puanları, işlemsel bilgileri ve kavramsal bilgileri arasında anlamlı ve pozitif yönde bir ilişki bulunduğu görülmüştür.

Anahtar Kelimeler: İşlemsel Bilgi, Kavramsal Bilgi, Öz-Yeterlik, Alan, Çevre, Hata

To My Father, Mother, Brother and Husband

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# ANOVA: Analysis of Variance

CKT: Conceptual Knowledge Test

GSE: Geometry Self-efficacy

M: Mean

MAX: Maximum

MIN: Minimum

MoNE: Ministry of National Education

N: Number of participants

NAEP: The National Assessment of Educational Progress

NCTM: National Council of Teachers of Mathematics

PASW: Predictive Analytics SoftWare

PISA:Programme for International Student Assessment

PKT: Procedural Knowledge Test

SD: Standard Deviation

STG: Self-efficacy Towards Geometry

TIMMS: Trends in International Mathematics and Science Study

# **CHAPTER 1**

#### **INTRODUCTION**

Mathematics is important in the education of children since it forms a base for science and technology (MoNE, 2005) and a tool which gives opportunity and options for people's futures (NCTM,2000). It is important to learn mathematics with understanding (Hiebert & Carpenter, 1992).Understanding mathematics is not only gaining some basic skills or concepts, but also building relationships between them. This is a highly complex process which is possible when proper ideas exist and new connections are formed (Lehrer, 1999 as cited in Van De Walle, 2007). Therefore, when a new mathematical knowledge is understood, this knowledge provides a base for the new learning and can be implemented in different areas. In this respect, knowledge types are important in mathematics education.

Hiebert (1986) referred knowledge types in mathematics as conceptual and procedural knowledge. Hiebert and Lefevre (1986) described conceptual knowledge as "rich relationships and web of ideas", whereas procedural knowledge was described as "rules and step-by-step procedures used in doing mathematical tasks and symbolism used to represent mathematics" (p.3). Hiebert and Lefevre (1986) argued that these two types of knowledge cannot be totally separated from each other. Linking procedural and conceptual knowledge has been the main goal of mathematics education. As a result of this, several studies were conducted about types of knowledge.

Studies have addressed conceptual and procedural knowledge in different aspects such as the relationship between them, the interaction between them, their development, and their importance (Hapaasalo & Kadijevich, 2000; Rittle-Johnson & Siegler, 1998; Siegler, 1991; Siegler & Crowley, 1994; Star, 2005). Researchers have claimed that procedural and conceptual knowledge, which are important elements in

mathematical understanding, are positively correlated (Hiebert & Lefevre, 1986; Rittle-Johnson & Siegler, 1998). The knowledge of the mathematical concepts depends on the context and topics and the development of the knowledge types and the interaction between them (Johnson & Siegler, 1998). Therefore, it is more meaningful to investigate the development of knowledge types on basis of topics.

Considering the mathematical strands, "geometry" and "measurement" have an important place in mathematics curriculum (Tan, 2010). Geometric figures are mostly seen in peoples' life, in everywhere as separate or combined figures (Özsoy, 2003). Geometry is important in mathematics education because geometric objects are in real life and geometry is used in science and technology, in mathematical modeling, and problem solving (Aksu, 2006). Similarly, measurement is significant because of its well-known importance in quantifying the word (Hart, 1984), in science, and in everyday experience (Lehrer, 2003).

Besides its considerable involvement in the daily life, measurement supports learning opportunities for students in domains such as, operations, functions, statistics, fractions, and geometry (NCTM, 2000). Therefore it is seen as a bridge across mathematical strands (Clements & Battista, 2001). Most specifically, it connects two main areas of mathematics: geometry and numbers (Clements, 1999; Kilpatrick, et al., 2001).

Considering the relationship between measurement and geometry, Van De Walle (2012) claims that measurement is important for the development and understanding of perimeter, area, and volume formulas since they require an understanding of the shapes and relationships. The prosperities of geometric shapes and the relationships between them can be examined in two dimensions: (i) together with measure, called as "measurable geometry"; and (ii) without a measure, called as "non-measurable geometry" (Kültür & Kaplan, 2002). Since measure is used in the concepts of perimeter, area, and volume, these are accepted in the dimension of measurable geometry (Dağlı, 2010).For example; the proof of the area of the triangle is a non-measurable geometric activity although the application of this formula is a measurable geometric activity (Altun, 2008). Since measurement and geometry

involve several concepts, it is probable that students might have misunderstandings in these topics as shown in many studies.

In mathematics education, several terms have been used to express students' problems in mathematics learning. For example, "difficulty" was the most comprehensive one and it addressed the general problems in learning (Bingölbali & Özmantar, 2009). However, this term seems to be most general to understand students' learning problems. Therefore, the terms "error" and "misconception" was used to express students' difficulties in more detail.

According to Luneta and Makonye (2010), an error is a mistake or inaccuracy and a deviation from accuracy. There are unsystematic errors which are unintended and producing non-recurring incorrect answers (e.g. while calculating the perimeter of a square whose one side is 6 cm, finding the answer as 25)and there are systematic errors which produce current incorrect answers (Riccomini, 2005). It was pointed out that misconceptions are students' conceptions that produce systematic errors (Smith, diSessa & Roschelle, 1993). Misconception (e.g. the thought that when the two figures are combined, the perimeter of the combined figure is equal to the sum of the perimeters of figures separately) means "the perception of concepts by students in a different way than their scientifically accepted definitions" (Keşan & Kaya, 2007, p. 27). In this respect, error and misconception are related, but they are different. Errors can be seen from learners' products; however misconceptions can be hidden even in correct answers. Therefore error could be regarded as the conclusion of the misconception (Keşan & Kaya, 2007). According to Eryılmaz and Sürmeli (2002) all misconceptions could be accepted as errors; however not all errors could be accepted as misconceptions.

Students have common misconceptions and errors in certain mathematics topics and area and perimeter is one of them. Comparison and assessment studies such as TIMMS and NAEP showed that students performed poorer in understanding of area and perimeter concepts than in any other topic in the curriculum (Thamson & Preston, 2004). In the literature most studies investigated that area and perimeter of geometric figures are the most problematic topics in which students have more

difficulties and problems than any other topic(Chappell & Thompson, 1999; Woodward & Byrd, 1983).Specifically, students seemed to have difficulty in understanding and relating measurement to geometry concepts and utilizing them in solving non-routine problems since they memorized rules and the formulas for the area and perimeter of the geometric shapes (Dağlı, 2010; Kennedy & Lindquist, 2000).Middle school students, regardless of grade level, have been reported to have lack of conceptual and procedural knowledge about area and perimeter concepts (Kidman & Cooper, 1997). Seventh grade students have confused the perimeter and area concepts and have thought that there was a linear relationship between them (Moreira& Contente, 1997). Similarly, middle school students in Turkey have been deduced to have errors and misconceptions especially in perimeter and area concepts and their formulas (Emekli, 2001; Tan Şişman & Aksu, 2009).

It is important to support procedural knowledge with conceptual knowledge for a deeper understanding of mathematics (Aksu, 1994; Noss & Baki, 1996). There are several concepts in the study of measurement in geometric shapes (e.g. perimeter, surface area). Therefore, it is significant to gain both procedural and conceptual knowledge in the scope of measurement and geometry. Considering the previous findings, one of the aims of this study is to investigate students' procedural and conceptual knowledge in the domain of area and perimeter in order to have a better picture of their understanding of these concepts.

The other concern of this study is students' self-efficacy. Self-efficacy is defined as, "people's judgments of their capabilities to organize and execute courses of action required to attain designated types of performances" (Bandura, 1986, p. 391). In the literature, there are several studies concerning the relationship between self-efficacy and mathematical knowledge, self-efficacy and mathematics self-efficacy or geometry self-efficacy. In some studies, it is suggested that self-efficacy is primarily a domain-specific (which means performing specific tasks, e.g. a geometry task) construct (Pajares, 1996). As a result of this, Schunk (2008) reported that "it is important to mention about self-efficacy for concluding from solving fractions, equations, geometry problems and so on" (p. 106). Therefore, in the present study, self-efficacy will be investigated in terms of geometry with a specific

task in the domain of area and perimeter. There is no study in the accessible literature investigating the relationship between self-efficacy and mathematical knowledge or geometry knowledge in terms of conceptual knowledge and procedural knowledge. As stated previously, since mathematical knowledge consists of both procedural and conceptual understanding and since self-efficacy and mathematical knowledge is mostly related, it is important to examine students' procedural and conceptual knowledge in relation to their self-efficacy.

## **1.1.Purpose of the Study**

The main purpose of this study was to investigate the relationship between private middle school students' geometry self-efficacy and their basic procedural and conceptual knowledge in the domain of area and perimeter. Moreover, their common errors in area and perimeter tasks were also investigated. One specific interest of this study was to examine how students' performance in conceptual and procedural aspects of area and perimeter changes with respect to their geometry self-efficacy.

#### **1.2.Research Questions**

- **1.** What is private middle school students' (6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> grade) performance on the conceptual and procedural knowledge tests about area and perimeter?
- **2.** How does private middle school students' (6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> grades) basic procedural and conceptual knowledge in the domain of area and perimeter change through the grade levels?
- **3.** What are the most common errors of private middle school students (6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> grades) in the domain of area and perimeter?
  - **3.1.**What are the most common errors of private middle school students (6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> grades) about area and perimeter in procedural knowledge test?
    - **3.1.1.** What are the most common errors of private middle school (6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> grades) about perimeter of the square, rectangle and parallelogram?
    - **3.1.2.** What are the most common errors of private middle school ( $6^{th}$ ,  $7^{th}$  and  $8^{th}$  grades) about perimeter of the irregular figures?

- **3.1.3.** What are the most common errors of private middle school students (6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> grades) about area of square, rectangle and parallelogram?
- **3.1.4.** What are the most common errors of private middle school students (6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> grades) about area of the polygon which is the combination of regular geometric shapes?
- **3.2.**What are the most common errors of private middle school students (6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> grades) about area and perimeter in conceptual knowledge test?
  - **3.2.1.** What are the most common errors of private middle school students  $(6^{th}, 7^{th} \text{ and } 8^{th} \text{ grades})$  about area of irregular geometric shapes?
  - **3.2.2.** What are the most common errors of private middle school students  $(6^{th}, 7^{th} \text{ and } 8^{th} \text{ grades})$  about surface area?
  - **3.2.3.** What are the most common errors of private middle school students  $(6^{th}, 7^{th} \text{ and } 8^{th} \text{ grades})$  about area and perimeter of geometric shapes?
- 4. What is the geometry self-efficacy belief level of private middle school students?
- **5.** How do private middle school students' (6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> grades) geometry self-efficacy belief levels change through the grade level?

**6.** Is there a significant relationship between private middle school students' ( $6^{th}$ ,  $7^{th}$  and  $8^{th}$  grades) geometry self-efficacy beliefs, and their basic procedural and conceptual knowledge in the domain of area and perimeter?

**6.1.** Is there a significant relationship between private middle school ( $6^{th}$ ,  $7^{th}$  and  $8^{th}$  grades) students' geometry self-efficacy and their basic procedural knowledge in the domain of area and perimeter?

**6.2.** Is there a significant relationship between private middle school ( $6^{th}$ ,  $7^{th}$  and  $8^{th}$  grades) students' geometry self-efficacy and their basic conceptual knowledge in the domain of area and perimeter?

**6.3.** Is there a significant relationship between private middle school ( $6^{th}$ ,  $7^{th}$  and  $8^{th}$  grades) students' basic procedural knowledge and basic conceptual knowledge in the domain of area and perimeter?

#### **1.3.Significance of the Study**

It is important in mathematics education that students build relationships between the mathematical concepts and related arithmetic skills (MoNE, 2013). From this respect, there have been several studies conducted in terms of conceptual and procedural knowledge in different subject domains. However, not many studies examined the nature of conceptual and procedural knowledge on the same concept in different grade levels and its change through the grade levels in private middle school schools in Turkey. Therefore, this is the one of the main significance for the present study.

Geometry and measurement are one of the important learning areas in the elementary mathematics curriculum (Dağlı, 2010; Tan, 2010). It is important to pay more attention to geometry and measurement in schools (NCTM, 2000 as cited in Smith, Silver & Stein, 2005). Although there are several studies about geometry and measurement concepts such as perimeter, area, volume, angle, time in the literature(i.e. Tan, 2010), it is important to examine the extent to which students' procedural and conceptual knowledge are changed in the domain of area and perimeter as in the present study. Several studies have been conducted on students' understanding of area and perimeter; however, not many studies have examined students' basic conceptual and procedural knowledge in elementary grades at once. Therefore, the present study addresses these as main concerns.

Students' self-efficacy beliefs have been investigated in various academic areas, including mathematics (Hacket, 1985; Hackett & Betz, 1989; Pajares& Miller, 1994, 1995). Relationships between mathematics self-efficacy beliefs and mathematics performance, geometry performance, and attitudes toward mathematics, have been studied by several researchers. However, as in the present study, there has not been a specific study examining the relationship between self-efficacy and conceptual and procedural knowledge in the domain of area and perimeter separately in the accessible literature. Specifically, the extent of students' performance in conceptual and procedural aspects of area and perimeter concepts in relation to their self-efficacy beliefs has not been investigated much. Therefore, this is the one of the main concern of the present study. From this respect, carrying out the present study might be beneficial for teachers in order to design conceptually or procedurally oriented lessons in terms of different subject matters and according to their students' self-efficacy beliefs.

In the light of the mentioned gaps in the mathematics education research literature, investigation of private middle school students' main conceptual and procedural knowledge in area and perimeter concepts and the relationship between geometry self-efficacy beliefs and conceptual and procedural knowledge is likely to provide an initial attempt in understanding the nature of this aspect of mathematics education.

## **1.4.My Motivation for the Study**

I have been working as a private middle school mathematics teacher in a private school in Ankara. Although this is my third year as a mathematics teacher, I had a chance to see students in all grade levels from 4<sup>th</sup> grade to 8<sup>th</sup> grade. While I was teaching the concept "area and perimeter" in different grade levels, I realized that most students had difficulty in these concepts in line with the literature. They had problem with formulas. I have realized that they had memorized all formulas related to area and perimeter of geometric figures without constructing any meaning. Although they were aware that they knew only formulas but not meaning of the area and perimeter, they ignored this fact. Therefore, I wanted to examine how the knowledge of these common concepts changes in private middle school students as they progress through the grades. I think this study will help me and other teachers in designing lessons more carefully for each grade level based on the knowledge that should be strengthened. In my opinion it might also provide several viewpoints for curriculum developers.

Moreover, I have realized that students feel themselves good at mathematics if they know all the formulas about the concepts. For example, one of my students told me that she did not believe that she was good at mathematics although she got high points from mathematics examinations. When I asked her the reason, she said that when she knew the formulas and the other things that could be memorized; she got high points from the mathematics examinations. She thought that her mathematics was not good if she could not make connections between concepts. At that time I thought that a study which examined how students' performance in conceptual and procedural aspects of area and perimeter changed with respect to their geometry self-efficacy could help teachers to observe students more carefully and design more thorough lessons. In my opinion, understanding students' performance in conceptual and procedural aspects of area and perimeter with respect to their geometry self-efficacy may help in organizing well-communicated lessons and meaningful experiences for students.

## **1.5.Definitions of the Important Terms**

The constitutive and operational definitions of the important terms in research questions are given below.

*Procedural knowledge* was defined by Hiebert and Lefevre (1986) as "the knowledge which contains rules and step-by-step procedures used in doing mathematical tasks and symbolism used to represent mathematics" (p. 25). In this study, the term procedural knowledge was used as the knowledge type that helped in applying the formulas to calculate area and perimeter of given figures successfully.

*Conceptual knowledge* was defined by Hiebert and Lefevre (1986) as "the knowledge which contains rich relationships and web of ideas" (p. 25). In this study, the term *conceptual knowledge* was used as building relationships between area and perimeter of given figures and stating ideas related to the meaning of area and perimeter.

*Basic conceptual and procedural knowledge* was defined specific to the study as the procedural and conceptual knowledge in area and perimeter of regular and irregular shapes, rectangular, square, triangle, and parallelogram. This knowledge was measured by the Procedural Knowledge Test (PKT) and Conceptual Knowledge Test (CKT) developed by the researcher based on the literature and previously designed instruments. *Self-efficacy* was defined by Bandura (1986) as, "People's judgments of their capabilities to organize and execute courses of action required to attain designated types of performances" (p. 391). In this study, *geometry self-efficacy* has been the focus and it was defined as students' evaluations of their capabilities to organize and perform certain actions required to manage specific geometric tasks or problems successfully(Dursun,2010).In this study, geometry self-efficacy scores of participants were measured by Geometry Self-Efficacy (GSE) Scale which was developed by Cantürk-Günhan and Başer (2007).

*Private middle school students* were the students who were in  $6^{th}$ ,  $7^{th}$  and  $8^{th}$ grades in a private school.

Area was defined as "the two dimensional space inside a region" (Van De Walle, 2007, p. 383).

*Perimeter* was defined as the "the distance around a region" (Van De Walle, 2012, p. 380).

*Error* was defined as "a mistake or inaccuracy and a deviation from accuracy" (Luneta & Makonye, 2010). In this study the term error was used as the incorrect answers made as a result of having the lack of knowledge or probable misconceptions.

#### **CHAPTER 2**

#### LITERATURE REVIEW

The purpose of the present study was to investigate private middle school students' procedural and conceptual knowledge in the domain of area and perimeter and to examine most common errors in their knowledge. The other specific interest of the study was to examine how students' conceptual and procedural knowledge aspects of area and perimeter changes with respect to their geometry self-efficacy. Moreover, private middle school students' procedural knowledge, conceptual knowledge and geometry self-efficacy throughout the grade levels was also investigated.

In the following part, the literature review of the present study is presented. Based on the content and main objectives of the study, this chapter is classified into four sections: related studies on definitions about procedural and conceptual knowledge and relationships between them, area and perimeter and most common errors in these concepts, self- efficacy toward geometry, and a summary of the literature review.

#### 2.1. Procedural and Conceptual Knowledge

Procedural and conceptual knowledge of mathematics has been debated through the years among mathematics researchers (Hiebert, 1986). Several learning theories have shown the importance of understanding the role of knowledge in learning process (Rittle-Johnson, 1999). Schneider and Stern (2006) state that there are two types of knowledge which shape understanding. The first one is knowledge of concepts; the second one is knowledge of procedures in a domain. Linking and developing knowledge of procedures and concepts are required for gaining mathematical competence (Bisanz & Lefevre, 1992). Hiebert and Carpenter (1992, as cited in Tan, 2010) explain the importance of connection and relationship between kinds of knowledge as:

A mathematical idea or procedure or fact is understood if it is part of an internal network...The degree of understanding is determined by the number and the strength of the connections. A mathematical idea, procedure, or a fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections (p.67).

Similarly, Van De Walle (2007) addresses that conceptual and procedural knowledge should be taught in relation to each other since when one is missing, the rest would result in errors and lack of enjoyment for students.

# 2.1.1. Definitions of Procedural and Conceptual Knowledge

As stated in the previous chapter, knowledge types have become one of the important issues in learning process over the years and different terminologies have been used by several researchers. For example Resnick (1982) mentioned about "semantics" and "synax". Shulman (1986) named the knowledge as "knowing that" and "knowing why", whereas Anderson (1983) addressed "knowing that" as "declarative knowledge" and "knowing why" as "procedural knowledge". Piaget (1978) labeled the knowledge as "conceptual understanding" and "successful action". Gelman and Meck (1983) called knowledge as "conceptual competence (knowledge of principles)" and "procedural competence (procedure performance)". Finally, as used in the present study, Hiebert and Lefevre (1986) categorized as "conceptual knowledge" and "procedural knowledge".

Hiebert and Lefevre (1986) defined the conceptual knowledge as connected web of ideas which addressed relationships and connections between ideas. According to Hiebert and Lefevre (1986) the quality of the conceptual understanding depends on the creations of new connections and relationships of these connections with existing ideas. As a support to this idea, Broody et al. (2007) stated that when a new idea was assimilated into existing schemas, development of conceptual knowledge was enhanced. Moreover, Kilpatrick et al., (2001, as cited in Tan, 2010) argued that a conceptual mathematical understanding required knowing the importance of a mathematical idea, applying the idea in different contexts and making an organization between the existing and new ideas. Furthermore, Schneider and Stern (2004, p.2) addressed conceptual knowledge as "the knowledge of the core concepts and principles and their interrelations in a certain domain." They see conceptual knowledge as the knowledge which can be generalized for several types of problems in a domain.

Procedural knowledge, on the other hand, was defined as the knowledge of rules and step-by-step procedures used in performing mathematical tasks and the symbols used to represent mathematics by Hiebert and Levefre (1986). They categorized procedural knowledge in two parts. First one is the familiarity with the symbolic representations of mathematics or being aware of the mathematical symbols and the rules, whereas second one is the use of algorithms and rules. Furthermore, in contrast to Schneider and Stern's (2004) perception of conceptual knowledge mentioned above, they held procedural knowledge as "the knowledge of operators and the conditions under which these can be used to reach certain goals" (p.2). As a result of this, procedural knowledge is seen as tied to specific problem types. Although procedural and conceptual knowledge is defined separately, their importance for each other was discussed in most studies.

## 2.1.2. Connection between Procedural and Conceptual Knowledge

Star (2000) asserts that since the conceptual knowledge required a more meaningful and deep thinking process, it is accepted as more important in the field of mathematics education. On the other hand, Van de Walle (2007) stated the importance of procedures as "Computational procedures offer opportunity to see how understanding differs from one child to another" (p. 25). However, Star (2002, as cited in Tan, 2010) concluded that conceptual knowledge would not only address what was known but also the ways that concepts would be known. Similarly, procedural knowledge indicated not only what was known about procedures, but also the ways those procedures could be known (superficially, without rich connections). Moreover Baroody, et al., (2007) asserted that procedural knowledge could not be

separated from conceptual knowledge. Although Baroody, et al. and Star (2000) have a common opinion on the characterization of knowledge types, they contradicted with the definition of procedural knowledge. Baroody, et al. claimed that conceptual knowledge is necessary for the procedural knowledge whereas Star (2007) claimed that procedural understanding would not require conceptual knowledge.

Procedural knowledge of a principle together with its conceptual base enables students to transfer their knowledge in similar problems (Hiebert & Lefevre, 1986). To put it differently, integration of the conceptual and procedural knowledge results in better usage of procedures (Carpenter, 1986). If procedures are given with the conceptual base, students are able to give meaning to formulas and symbols, as a result they gain deep understanding about what they are doing (Hiebert & Lefevre, 1986). Furthermore, Kilpatrick, et al. (2001) stated that when students learn procedures without understanding, they are likely to have difficulty in understanding the reasons behind the procedures. Similarly, if students do not have sufficient procedural fluency, they may experience problems in understanding and solving problems.

Some studies have addressed that procedural skills may develop before conceptual understanding of the subjects (Briars & Siegler, 1987; Fuson, 1988; Wynn, 1990). Opposite to these studies, there are researches advocated that conceptual understanding of the subject is likely to develop before the procedural understanding of the subject (Byrnes & Wassik, 1991; Gelman & Meck, 1992; Greeno & Heller, 1983; Wynn, 1998). Regardless of which one is developed the first, researchers generally agree that connections between procedures and their conceptual bases increase understanding of mathematics (Hiebert & Lefevre, 1986; Silver 1986; Star, 2005). In the study conducted by Alibali and Rittle-Johnson (1999) procedural and conceptual knowledge of 4<sup>th</sup> and 5<sup>th</sup> graders' in the domain of equivalence were compared. As a result, it was seen that both kinds of knowledge developed iteratively and procedural knowledge has an effect on the development of conceptual knowledge and vice versa.

The relationship between conceptual and procedural skills has also been a concern of the field of mathematics education (Resnick & Ford, 1981, as cited in Hattikudur, 2011). Although most mathematics education researchers have common opinions on the idea that both conceptual and procedural knowledge is important in doing and learning mathematics (Hiebert & Lefevre, 1992; Hiebert & Carpenter, 1992; 1986; Star, 2000; Van De Walle, 2007), what would be the most beneficial among them for mathematics competency and in which order they are developed have been discussed among mathematics educator (Hiebert & Carpenter, 1992; Rittle-Johnson & Siegler, 1998; Star, 2000).

The relationship between students' conceptual and procedural knowledge is also investigated in terms of different variables such as the students' confidence level, cognitive styles, and influences of instruction (Engelbrecht et al., 2005; Jitendra, et al., 2002). In addition to this, Star (2000) claims that especially the topics of fractions, decimals, counting, addition, and multiplications have been the focus on the studies related to conceptual and procedural knowledge. For example, Bekdemir and Işık (2007) investigated the relationship between procedural and conceptual knowledge of 8<sup>th</sup> grade students in the domain of algebra and compared their knowledge in this area. Moreover, they examined most common errors and misconceptions. Although students had higher performance in procedural test than in the conceptual test, their performance in both of the tests was low in general. Furthermore, it was examined that students could solve the questions required procedural knowledge; however, they generated several errors in the procedures. In addition to this, they could not relate and interpret the questions which required conceptual knowledge.

Similarly, Akkuş et al. (2009) investigated the procedural and conceptual knowledge of 7<sup>th</sup> grade students in ratio and proportion. According to the purposive interviews conducted by students, students' lacked conceptual knowledge while solving the questions which required procedural knowledge without reasoning.

Since one of the purposes of this study is to investigate the private middle school students' procedural and conceptual knowledge in the domain of area and

perimeter and most common errors in this domain, the studies related to area and perimeter concept are given below.

#### **2.2. Misconception and Error**

Mathematics concepts are important since they build a base for procedural and conceptual knowledge. Kaptan (1999) defined concept as the common name that was given to the groups in which properties, ideas, and events are grouped. In terms of concepts in mathematics, according to Sfard (1991), concept was defined as the theoretical construct of a mathematics idea (as cited in Li, 2006). Senemoğlu(1997) stated that concepts are the mental tools which make comprehensive information usable. Moreover it was specified that concepts are the core of the knowledge and people organize and classify what they have learned in a logical sequence. Nevertheless, if students formed concepts in non-scientific and undesired ways, misconceptions appear (Demirel, 2003).

Bingölbali and Özmantar (2009) stated that the difficulties and misconceptions of mathematics concepts result from three main reasons. First one is epistemological reasons which occur because of the nature and feature of the knowledge. These obstacles are inevitable and the main part of the knowledge is learned. Second one is the psychological reasons which are related to personal development including biological, cognitive, and sensory dimensions. Students' background knowledge, readiness to learn, ability, and skill affect students' learning. Pedagogical reasons that are resulted from teaching models, application of these models, analogies used by the teacher, textbooks, and the concepts in these textbooks are the third reason of the misconceptions.

Reasons of misconceptions are addressed in different ways in other studies (Ayas & Demirbaş, 1997; Lawson & Thomson, 1988). In these studies it was stated that the students' wrong and deficient perception of the background knowledge, language problems, no suitable teaching environment, no relationship between concepts and real life, and no determination of students' misunderstandings were the other probable reasons of misconceptions.
As stated by Hiebert and Lefevre(1986) before, mathematical knowledge was gained by learners as either procedural or conceptual knowledge. They stated that while procedural knowledge was taught through practice and drill to be more efficient and rapid in mathematics tasks, this efficiency and speed caused misunderstandings for conceptual knowledge. Therefore, misconceptions and errors could occur while passing from procedures to conceptual knowledge (Luneta & Makonye, 2010).

In some studies errors were determined as procedurally and conceptually. A study focusing on conceptual errors rather than procedural errors was conducted by McGatha, Bush and Rakes (2010). In their study, they followed a group of  $7^{th}$  grade students whose teachers focused on their conceptual errors. When students came to  $8^{th}$  grade in which they were tested, teachers focused on their procedural errors instead of conceptual errors. It was examined that the growth of students' achievement in  $7^{th}$  grade was higher than the growth of students' achievement in  $8^{th}$  grade.

Students maintain concepts which do not correspond to scientific truths (Büyükkasap & Samancı, 1998). These concepts may be defined as the obstructive knowledge which comprise from personal experiences to avoid learning the scientifically accepted concepts (Çakır & Yürük, 1999). They can also be defined as misconceptions which address students' understanding of the concept different from its actual meaning accepted as scientific (Yağbasan & Gülçiçek, 2003). Baki (1998) defined misconceptions as the behaviors that occur as a result of wrong beliefs and experiences. Although these definitions are clear, the terms "misconception" and "error" are generally used interchangeably. Smith, diSessa and Roschelle (1993) pointed out that misconceptions could be accepted as errors, all errors could not be defined as misconception (Eryılmaz & Sürmeli, 2002).

In constructivism, current learning is affected by prior learning. That means while students are learning something, they give meaning to things they have learned before. Therefore, they construct their own knowledge (Van De Walle, 2007). In learning, they are the active participants and they connect new ideas with existing ideas (Olivier, 1989). Therefore, prior knowledge is important for students' new learning. The important thing is that misconceptions generally comprise prior learning since misconceptions avoid learning of expert contents (Smith, diSessa & Roschelle, 1993). Misconceptions are reflected in students' conceptual structure and they might affect the learning negatively (Olivier, 1989). Students do not come to school as empty minds (Resnick, 1983, as cited in Mestre, 1989). Most teachers try to fill students' empty minds with the disciplinary concepts which are called as expert concepts (Yağbasan & Gülçiçek, 2003). However, since students have prior knowledge when they come to class, their conceptions might be inconsistent with the expert concepts (Smith, diSessa & Roschelle, 1993). Therefore, misconceptions occur.

Concerning the studies about misconceptions and errors, it was seen that misconceptions and errors have important roles in students learning. Since one of the aims of the present study was to examine the most common errors about area and perimeter of geometric figures, the studies about them will be presented in the following part.

#### 2.3. Errors in Area and Perimeter

In the elementary mathematics program, it was aimed that students should be able to recognize the geometric figures which they met in their lives, comprehend their features, and gain the knowledge and skills to find their area and perimeter by measuring and calculating (Baykul, 2000). Most students memorize the formulas while learning the area and perimeter of geometric shapes (Maybery, 1983; as cited in Clements & Battista, 1992).

Related literature shows that most of the students have difficulty in understanding geometry and measurement concepts and particularly area and perimeter (Barret& Clement, 2003; Martin & Kenney, 2002; Walter, 1970). According to the Van Hiele (1985), students in the middle school identify and compare geometric figures; however they do not understand concepts underlying terms such as perimeter and area.

Most students try to solve the problems about perimeter and area without understanding the concepts underlying the formulas (Mulcahy, 2007). Sherman and Randolph (2003) conducted a study with twenty-seven 4<sup>th</sup> grade students to investigate the knowledge of students in the domain of area and perimeter. It was found that students could apply formulas; however they were not aware of why they used those formulas. Kidman and Cooper (1997) investigated how 4<sup>th</sup>, 6<sup>th</sup> and 8<sup>th</sup>grade students considered about the area of quadrilaterals in terms of length and width. About 50% of the students found the area of a quadrilateral by summing up the length of the edges.

According to NAEP (2007), 4<sup>th</sup> and 8<sup>th</sup> grade students could not distinguish area concept from the perimeter concept. They could not answer questions related to the usage of area or perimeter in a given situation. Similarly, Malloy (1999) stated that students tend to confuse perimeter and area concepts in his study. Furthermore, it was examined that length measure was generally considered by young children as area measure (Lehrer et al., 1998).

Seventh grade students were also reported to get confused the area and perimeter concepts as they believed that there was a linear relationship between area and perimeter of a figure (Moreira & Contente, 1997). In the study conducted by Kami and Kysh (2006), it was found that 8<sup>th</sup> grade students reckoned that when a figure divided into parts, and a new shape was constructed, the area of the figure changed.

Several studies have been conducted in order to investigate students' understanding of area and perimeter concepts in Turkey. Emekli (2001) designed a study to examine 7<sup>th</sup> and 8<sup>th</sup> grade students' misconceptions on measurement concepts. He found that students had significant difficulties about the concepts of area and perimeter such as conservation of area and perimeter, and the usage of formulas in these subjects. Moreover, Dağlı (2010) investigated the misconceptions

of 5<sup>th</sup> grade students in the domain of area and perimeter and found similar results. This study was conducted with 262 fifth grade students. According to the results of the study, students had difficulty especially in the area subject. They also confused area and perimeter concepts. They calculated the perimeter and area of the figure whose length of the edges was given; however, they could not find the edge which was not given. Besides these results, more specific results were pointed out in this study. For example, it was observed that students could find the perimeter and area of a square, rectangle or triangle easily. However they had difficulty in finding the perimeter or area of a parallelogram. In addition to these results, students had difficulty in the type of questions in which students were required to find the remaining area of a rectangle which had small squares in it.

Students have problems in understanding of that the same quality of objects can be compared (Zembat, 2009). Because of this misconception, students confuse the perimeter and area of the given figures and they cannot assimilate that perimeter is one dimensional and area is two dimensional measurements. Tan Şişman and Aksu (2009) conducted a study with 134 seventh grade students and had similar results. They found that students had difficulty with the formulas, they confused the perimeter and the area of a figure, and they had problems in understanding the unchanged area of a figure when that figure was divided into parts and then combined in different form. They also had difficulty in the changeability of perimeter of a figure.

In addition to those studies, Tan Şişman (2010) carried out another study which aimed to examine 6<sup>th</sup> grade students' procedural and conceptual understanding in area, length, and volume. That study was conducted with 445 sixth grade students attending public schools in Ankara. Students were more successful in the test which contained questions related to the length measurement than in the test which had questions related to area and volume measurement. Moreover, students were more successful in the procedural test rather than the conceptual test.

Küçük and Demir (2009) conducted a study in which 7<sup>th</sup> and 8<sup>th</sup> grade students' basic procedural knowledge and conceptual knowledge in mathematics

lesson was examined. In the study, when the questions belong to geometry learning area, it was investigated that only 33.5% of the students could answer the question related to parallelogram. This result corresponded to the results occurred in Dağlı's(2010) results that students had difficulty in solving the questions about parallelograms.

In the study of Sherman and Randolphe (2003) what students knew and how they knew related to area and perimeter concepts were investigated. In this study, 4<sup>th</sup> grade students had taken mathematics lessons in which the area and perimeter concepts were taught for about four weeks. It was examined that understanding the difference and the relationship between area and perimeter completely made students solve daily life problems related to area and perimeter. Moreover, it was observed that memorizing the rules revealed temporary solutions for the problems.

Concerning the studies related to area and perimeter, it was seen that most studies were about the misconceptions, errors, and achievement of students in the domain of area and perimeter. Present study is also concerned with the relationship between students' knowledge of area and perimeter and their self-efficacy beliefs towards geometry. Therefore, studies about self-efficacy are also presented below.

#### **2.4. Self-Efficacy Beliefs**

Self-efficacy has been studied by educational researchers for years (Hoy & Spero, 2005). Bandura (1986) defined self-efficacy as "People's judgments of their capabilities to organize and execute courses of action required to attain designated types of performances" (p. 391). Self-efficacy was also defined by Zimmerman (1995, as cited in Dursun, 2010) as judgments of an individual about his or her own ability to perform and accomplish a task. On the other hand, Schunk (1991) defined academic self-efficacy as people's beliefs about performing an academic task. Social cognitive theorists claimed that students' self-efficacy beliefs affected their attempts, decisions and motivation (Işıksal & Aşkar, 2005).

Bandura (1986) asserted the sources of self-efficacy as enactive mastery experiences, vicarious experiences, verbal persuasion and affective and

psychological states. First source of the self-efficacy is enactive mastery experiences which have the most effect on self-efficacy (Bandura, 1986). According to Bandura (1986) this source is related to peoples' past success and failure. Bandura (1986) claimed that while success supports self-efficacy beliefs, failure has a negative effect on self-efficacy.

Vicarious experiences is the second source which means after peoples observe other peoples' performances, they regulate their self-efficacy beliefs (Usher &Pajares, 2006). According to Bandura (1997) and Pajares (1997), if the observer has limited experiences and he is unsure about his performances, this source becomes most effective.

According to Usher and Pajares (2006), people sometimes want approvals of their parents or teachers unless they make self-evaluation of their performances on tasks which they engage in. That is called verbal persuasion. This source could have a long term effect on individuals as well as sometimes the effect could become short term.

The last source is affective and psychological states which encompass feelings such as anxiety, stress, emotions, and mood. Hodges and Murphy (2009) claimed that these feelings affected peoples' self-efficacy beliefs. Because of these feelings people might have misunderstandings and wrong interpretations about tasks (Hodges & Murphy, 2009). Pajares (2007) stated that people with high self-efficacy approached difficult tasks comfortably however, people with low self-efficacy felt stress towards the task. As a result of this, it is seen that self-efficacy beliefs affects peoples' motivation and achievement. Similarly, Gawith (1995) claimed that although people have the required ability to perform the task, unless they had self-efficacy beliefs, they could not be successful (as cited in Cantürk-Günhan & Başer, 2007).

The importance of self-efficacy beliefs on behavior of individuals has been widely stated. Self-efficacy beliefs have a significant influence on people's choices and people engage in the tasks in which they feel competent (Pajares, 1996). It was examined that achievement of students in performing a task was influenced by selfefficacy beliefs (Bandura, 1993; Pajares, 1997). For example, Schunk and Pintrich (2008) stated that while students with low self-efficacy beliefs were reluctant to performing tasks, the students with high self-efficacy beliefs were eager to complete tasks. In addition to this, Schunk (1995) addressed that students with high selfefficacy were more eager and consistent while carrying out an academic task.

Moreover, Schunk and Pajares (2002) asserted that self-efficacy beliefs could change according to the different situations. For example, characteristics of the task, physical conditions of the students, and physical conditions of the classrooms might have an influence on self-efficacy beliefs (Schunk, 2008).

Research has concluded that "Self-efficacy is a domain-specific construct" (Pajares, 1996; p. 106). Similarly self-efficacy was seen by Smith and Fouad (1999) as a distinctive to a subject area (as cited in Schunk, 2008). In the literature, research about self-efficacy focused on two points. One of them is the relationship between carrier and college choices specifically in the area of science and mathematics (e.g. Farmer, Wardrop, Anderssn & Risinger, 1995; as cited in Zarch & Kadivar, 2006). The other one is the relationship between the self-efficacy beliefs and academic constructs such as motivation and achievement (Zarch & Kadivar, 2006). Thorndike (1986) claimed that self-efficacy is the important predictor of academic performance. Similarly, it was investigated that mathematics self-efficacy is an important predictor of problem solving and mental capability (Pajares & Kranzler, 1995).

Establishing the importance of self-efficacy in mathematics education, NCTM stated that one of the goals for students is "that they become confident in their ability to do mathematics" (NCTM, 1989, p. 5, as cited in Kahle, 2008). Mathematics self-efficacy was defined as people's judgments of their capabilities to organize and perform a series action required to attain specific mathematics task or problem successfully (Hackett & Betz, 1989). When the tasks are related to geometry, this definition turns into people's judgments of their capabilities to organize and perform a series of actions required to attain specific geometry task or problem successfully (Dursun, 2010).

Several studies were conducted about the relationship between students' mathematics achievement and self-efficacy beliefs (Hackett & Betz, 1989; Isiksal & Askar, 2005; Pajares & Miller, 1995). One of them is an experimental study conducted by Işıksal and Aşkar (2005). The aim of that study was to examine the effect of dynamic geometry software and spreadsheet on achievement and mathematics self-efficacy of 7<sup>th</sup> grade students. In the study mathematics self-efficacy scale and mathematics achievement tests were used. According to the results it was seen that self-efficacy beliefs were not affected by treatments.

In some researches, it was mentioned that there was no relationship between students' academic performances and self-efficacy beliefs (Işıksal, 2002). For example, Cooper and Robinson (1991) conducted a study with the aim of examining the relationship between mathematics self-efficacy beliefs and performance. Their participants were 290 undergraduate students. The result of the study showed that there was not a significant relationship between mathematics performance and mathematics self-efficacy. Similarly, another study was conducted by 72 children of 9-10 ages. It was found that self-efficacy did not have an influence on mathematics performance (Norvich, 1987; as cited in Işıksal, 2002).

A recent study conducted by Tsamir (2012) investigated the relationship between young children's geometric knowledge and their self- efficacy beliefs. In the study, 141students were asked self-efficacy questions and geometry tasks were given them. Results indicated that students had high self-efficacy and their self-efficacy was not significantly related to their geometric knowledge.

#### 2.5. Summary of the Literature Review

The importance of procedural and conceptual knowledge in mathematical competence cannot be denied (Hiebert & Lefevre, 1992). Several studies were conducted about the different aspects of knowledge types such as, relationship between them, developmental order of them and their importance (Crowley & Siegler, 1994; Gelman & Williams, 1998; Hapaasalo & Kadijevich, 2000; Rittle-

Johnson & Siegler, 1998; Siegler, 1991; Sophian, 1997; Star, 2005).Van de Walle (2012) addresses that conceptual and procedural knowledge should be taught in relation to each other since when one is missing, the rest would result in errors and lack of enjoyment for students. To summarize the common results of the past research it can be said that students have both types of knowledge, the knowledge types are linearly correlated with each other, the improvements in one knowledge type can result in advances in other knowledge types (Rittle-Johnson, et al., 2001).However; since there are limited studies related to the conceptual and procedural knowledge changes of private middle school students in the basic objectives of a specific topic, this part of the topic will constitute as one of the main theme of the present study.

Moreover, as stated in the related literature, geometry and measurement are important in mathematics education curriculum. Especially in area and perimeter, which are addressed together, students in any grade level have difficulty. Related literature shows that most of the students have difficulty in understanding geometry and measurement concepts and particularly area and perimeter (Barret & Clement, 2003; Martin & Kenney, 2002; Walter, 1970). There are several studies about the students' perceptions, misconceptions, and errors about the domain of area and perimeter. According to NAEP (2007), private middle school students could not distinguish area concept from the perimeter concept. Similarly, Emekli (2001) stated that students had significant difficulties about the concepts of area and perimeter such as conservation of area and perimeter, and the usage of formulas in these subjects. However; studies in which basic conceptual and procedural knowledge of students are examined in upper grades have not been accessed.

Finally, as stated above, literature have several studies about the relationship between self-efficacy beliefs and mathematics performance. It was examined that achievement of students in performing a task was influenced by self-efficacy beliefs (Bandura, 1993; Pajares, 1997). For example, Schunk and Pintrich (2008) stated that while students with low self-efficacy beliefs were reluctant to performing tasks, students with high self-efficacy beliefs were eager to complete tasks. Moreover there were studies about the relationship between self-efficacy beliefs and mathematics achievements (Hackett & Betz, 1989; Isiksal & Askar, 2005; Pajares & Miller, 1995). However, there are limited studies about the performance in a geometry task and geometry self-efficacy. Moreover, it has not been investigated how students with high self-efficacy perform in a procedurally oriented or conceptually oriented task.

#### **CHAPTER 3**

#### **METHOD**

The purpose of the present study was to investigate private middle school students' procedural and conceptual knowledge in the domain of area and perimeter and to examine most common errors in their knowledge. The other specific interest of the study was to examine how students' conceptual and procedural knowledge aspects of area and perimeter changes with respect to their geometry self-efficacy. Moreover, private middle school students' procedural knowledge, conceptual knowledge and geometry self-efficacy throughout the grade levels were also investigated.

The focus of this chapter is the method employed in this study. This chapter is devoted to information about the research design, population and sample, data collection instruments, pilot study, validity and reliability of the study, data collection procedure, analyses of data, assumptions, limitations, and lastly internal and external validity of the study.

#### **3.1.** Design of the Study

In order to the investigate research questions, quantitative research methods were used. The main purpose of this study was to investigate the relationship between students' procedural and conceptual knowledge, and those with their self-efficacy beliefs towards geometry. Since this research question of the study intended to describe an existing relationship between variables which were procedural knowledge, conceptual knowledge, and self-efficacy towards geometry, correlational research design was preferred (Fraenkel & Wallen, 2006).

The other purpose of the study was to examine conceptual and procedural knowledge of private middle school students in the domain of area and perimeter and their self-efficacy beliefs towards geometry. Due to the fact that this research question aimed at describing some aspects and characteristics such as knowledge,

survey research design and particularly cross-sectional survey was preferred. The aim of this research type was collecting data at one point of time from similar but different populations, in terms of grade level groups in this study, selected to describe a population (Fraenkel & Wallen, 2006). Data were analyzed through mean, standard deviation, percentages, frequencies, Pearson Product-Moment Correlation, and one-way ANOVA.

#### **3.2.** Population and Sample

The target population of the study was all private middle school students in all private schools in Ankara. The private middle school students in private schools in Çayyolu district in Ankara was the accessible population for this study. The subjects of the study were the private middle school students (6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> grade) in one private school in Ankara. Specific characteristic of the sample was that they were educated in classrooms which included small number of students and which were highly equipped in terms of instructional materials such as technological devices and manipulative. Further, the school was placed in a high socio-economic level district; and parents were mostly university graduates, and some with graduate degree.

Convenience sampling methods was used in this study because the researcher was a mathematics teacher in that school and she was able to reach the sample conveniently. In convenience sampling method, researchers collect data from the individuals who are available; however, this sample cannot be accepted as the representative of any population because it is not randomly selected (Fraenkel & Wallen, 2006). There were 111private middle school students in the school at the time of the study. The number of participants of the study in terms of grade level is given in the Table 3.1 below.

Table 3.1: Numbers of students in terms of grade level

| Grade Level        | 6 <sup>th</sup> grade | 7 <sup>th</sup> grade | 8 <sup>th</sup> grade | Total |
|--------------------|-----------------------|-----------------------|-----------------------|-------|
| Number of Students | 31                    | 35                    | 45                    | 111   |

#### **3.3. Data Collection Methods and Instruments**

Data for this study were collected through conceptual and procedural knowledge tests and self-efficacy towards geometry scale. Data collection methods, instruments, and procedures are explained below in detail.

#### 3.3.1. Procedural and Conceptual Knowledge Tests

These tests were partially developed by the researcher. Some of the questions were developed by the researcher and some of them were taken or adapted from the literature. All questions in the PKT and CKT presented in the following pages were translated by the researcher. Before preparing questions for the tests, related literature was reviewed. It was found that conceptual tasks were generally characterized as non-routine and novel tasks. These required the use of understanding of underlying principles or concepts in a mathematics topic, not necessarily involving computations. Procedural tasks were the tasks which required the use of step-by-step solution methods, computations, algorithms, or formulas (Hiebert & Lefevre, 1986; Kulm, 1994). After establishing this distinction, Turkish Elementary Mathematics Curriculum (grades 1-8) was examined. Since the purpose of this study was to examine the basic conceptual and procedural knowledge in all upper grade levels, there was a need to develop a test suitable for all private middle school levels. Therefore, 6<sup>th</sup> grade mathematics objectives were taken into account to form conceptually and procedurally oriented tests in the domain of area and perimeter. For the final forms of the PKT and CKT, see Appendix A and B.

#### **3.3.1.1 Procedural Knowledge Test**

The main aim of the procedural knowledge test (PKT) was to measure students' knowledge of area and perimeter in terms of computational skills such as using formulas to find area and perimeter of given figures. The PKT consisted of 10 questions which were developed by the researcher through consulting the textbooks, curriculum documents, a variety of mathematics books, and the existing literature. Content domains included in the PKT were area and perimeter of regular and irregular geometric shapes, triangles, squares, parallelograms, and rectangles. Since the perimeter of the triangle was not in the private middle school mathematics program it was not included in the tests. Table 3.2 presents the questions for area and perimeter concepts and for content domains in the PKT.

Table 3.2: Questions for Area and Perimeter Concepts and Content Domains in the Procedural Knowledge Test (PKT)

|           | Square | Rectangle  | Parallelogram | Triangle | Irregular Shapes |
|-----------|--------|------------|---------------|----------|------------------|
| Area      | Q1, Q2 | Q1, Q2, Q6 | Q1, Q10       | Q2, Q3   | Q4               |
|           | Q6     | Q8, Q9     |               |          |                  |
| Perimeter | Q1, Q7 | Q1, Q9     | Q1            | -        | Q5               |

Objectives covered in the PKT in each question are as presented in Table 3.3.

| QUESTIONS | OBJECTIVES : The students will be able to  |
|-----------|--|
| Q1        | calculate perimeters and areas of the square, rectangle, and parallelogram   |
| Q2        | calculate the ratio of the area of a colored triangle located in a rectangle and the uncolored area in the rectangle               |
| Q3        | calculate the area of the acute, obtuse, and right angle triangles<br>whose one side and the height belong to given side are given |
| Q4        | calculate the area of a polygon which is a combination of the regular geometric shapes   |
| Q5        | calculate the perimeter of irregular shapes.   |
|           | 20   |

Table 3.3: Table of specifications for procedural knowledge test based on objectives

Table 3.3. (Continued)

| Q6  | calculate the area of the rectangle by using small and colored squares in the rectangle  |
|-----|--|
| Q7  | calculate the perimeter of a square whose area is given.   |
| Q8  | calculate the area of a rectangle whose perimeter and length of the one of the sides are given.                                      |
| Q9  | calculate the perimeter and area of a rectangle which is formed by squares.  |
| Q10 | calculate the height of the long side of a parallelogram whose short<br>side, height of the short side, and the long side are given. |

\*Translations are done by the researcher.

#### 3.3.1.2. Conceptual Knowledge Test

Conceptual knowledge test (CKT) contained 10 open-ended questions. For this test, two questions (Q1 and Q7) were taken from the PISA 2006 test. Four of the questions (Q3, Q4a, Q5, Q8, and Q10) were chosen from an instrument designed and used by Tan (2010). Two of the questions (Q6 and Q9) were taken from the past SBS examinations. One question was prepared by the researcher. Table 3.4 presents the questions for area and perimeter concepts and for content domains in the CKT.

Table 3.4: Questions for Area and Perimeter Concepts and Content Domains in the Conceptual Knowledge Test (CKT)

|           | Square     | Rectangle | Parallelog | gram       | Triangle | Irregular Shapes |
|-----------|------------|-----------|------------|------------|----------|------------------|
| Area      | Q2, Q3, Q4 | a Q3, Q5  | , Q6b      | Q4a        | -        | Q1, Q8a, Q9,     |
| Perimeter | Q4a        | Q6a, Q7   | (4) Q      | 24b, Q7 (2 | ) - Q7   | (1, 3), Q8b,Q10  |

Objectives covered in the CKT in each question are as presented in Table 3.5. Table 3.5: Table of specifications for conceptual knowledge test based on objectives

| QUESTIONS | OBJECTIVES: The students will be able to  |
|-----------|---|
| Q1        | estimate the area of an irregular geometric shape.  |
| Q2        | decide how surface area of a structure consisted of unit cubes<br>changes when one of the unit cubes is taken away.   |
| Q3        | decide for which kind of information will be used to calculate the area of paper to cover a given square prism.   |
| Q4        | interpret the change in the area and circumference of a figure<br>obtained by breaking an initial figure into pieces and then bringing<br>them together in a different combination. |
| Q5        | interpret the changes in the area of a square whose dimensions<br>change in a given specific ratio.   |
| Q6        | identify changes in area and perimeter of two rectangles in different combinations.   |
| Q7        | choose the figures with the same perimeter among different figures.   |
| Q8        | compare the perimeters and areas of two irregular shapes drawn on dot paper.  |
| Q9        | estimate the area of an irregular geometric shape drawn on a dot paper.   |
| Q10       | identify different irregular shapes with the same perimeter.  |

\*Translations are done by the researcher.

#### 3.3.2. Geometry Self - Efficacy Scale

The Geometry Self-Efficacy (GSE) Scale (see appendix C) was developed by Cantürk-Günhan and Başer (2007) in order to measure private middle school students' self-efficacy toward geometry. The scale consisted of 25 items on a 5-point Likert type items (1-Never, 2-Sometimes, 3-Undecided, 4-Most of the time, 5-Always). The scale had three sub-dimensions which were positive self-efficacy beliefs, negative self-efficacy beliefs, and beliefs on the use of geometry knowledge. A sample item for each sub-dimension is given in Table 3.6.

| Sub-dimension                            | Sample Item   |
|--|---|
| Positive self-efficacy beliefs           | When I see a geometrical shape, I can remember its properties.  |
| Negative self-efficacy beliefs           | I cannot explain the relationships between geometrical shapes.  |
| Beliefs on the use of geometry knowledge | I believe that if I select a job related to<br>the use of geometrical knowledge in the<br>future, I will be successful. |

Table 3.6: Sample Items of GSE Scale Sub-Dimensions

In this scale meant that the participants had high geometry self-efficacy. Reliability analysis and factor analysis of GSE were employed by Cantürk-Günhan and Başer (2007) and the scale was found to be highly reliable (r = .90) and valid. In the present study reliability coefficient was calculated as .77 which is an appropriate reliability coefficient. For the final form of the GSE scale, see Appendix C. Finally, the research questions and the data gathering instruments are given in the Table 3.7 below.

Table 3.7: The Research Questions and Data Gathering Instruments

| Re | esearch Questions  | Data Gathering Instruments    |
|----|--|-------------------------------|
| 1. | What is private middle school students' (6 <sup>th</sup> 7 <sup>th</sup> and 8 <sup>th</sup> and | Conceptual Knowledge Test     |
|    | students') performance on the  | Procedural Knowledge Test     |
|    | concentual and procedural  |                               |
|    | knowledge tests about area and   |                               |
|    | perimeter?   |                               |
|    |  |                               |
| 2. | How does private middle school   | Conceptual Knowledge Test     |
|    | students' (6 <sup>th</sup> , 7 <sup>th</sup> and 8 <sup>th</sup> grades)                         | Procedural Knowledge Test     |
|    | basic procedural and conceptual  |                               |
|    | and perimeter change through the   |                               |
|    | orade levels?  |                               |
|    | Sidde le tells.  |                               |
|    |  |                               |
| 3. | What are the most common errors  | Conceptual Knowledge Test     |
|    | of private middle school students  | Procedural Knowledge Test     |
|    | (6, / and 8 grades) in the   |                               |
| 1  | What is the geometry self officeory  | Geometry Self Efficiency Test |
| 7. | belief level of private middle school  | Geometry Sen-Efficacy Test    |
|    | students?  |                               |
| 5. | How do private middle school   | Geometry Self-Efficacy Test   |
|    | students' (6 <sup>th</sup> , 7 <sup>th</sup> and 8 <sup>th</sup> grades)                         |                               |
|    | geometry self-efficacy belief levels   |                               |
|    | change through the grade level?  |                               |
| 6. | Is there a significant relationship  | Conceptual Knowledge Test     |
|    | between private middle school  | Procedural Knowledge Test     |
|    | students' (6 <sup>th</sup> , 7 <sup>th</sup> and 8 <sup>th</sup> grades)                         | Geometry Self-Efficacy Test   |
|    | geometry self-efficacy beliefs, their  |                               |
|    | basic procedural and conceptual  |                               |
|    | knowledge in the domain of area and  |                               |
|    | perimeter?   |                               |

## 3.4. Pilot Study

After establishing the content validity of all of the instruments, they were piloted with  $6^{th}$ ,  $7^{th}$  and  $8^{th}$  grade public elementary school students. There were 70

private middle school students who were taught in one public school and one private school in Yenimahalle district in the pilot study. The aim of this implementation was to check the comprehensiveness of items, convenience of time duration, and difficulty of the items. For conceptual and procedural tasks, students were given 50 minutes for the tests and for the self-efficacy scale, they were given 15 minutes. Another aim of pilot study was to determine the possible difficulties that may occur in the actual study. After the pilot study, necessary analyses, stated in the validity and reliability section in more detail, were conducted with data in order to check the construct validity and reliability of instruments. In addition to this, any observed difficulties during administration were considered and revisions were done accordingly. In the pilot study, students could not complete the tests in 40 minutes. Therefore, in the actual study students were given 80minutes to complete questions in the tests and the geometry self-efficacy scale. Moreover, changes in the language of the questions were done.

#### **3.5 Data Collection Procedure**

After the pilot study, the tests were prepared for the actual study to implement. The study was conducted in a private school which has 111 private middle school students (31 sixth grade students, 35 seventh grade students and 45 eighth grade students). The tests were implemented to students in their mathematics lessons. The PKT and CKT were distributed at the same time. They had the chance to start from the test that they wanted. STG test was distributed after they completed the PKT and CKT. They were given two lessons time which was 80 minutes. They had 10 minutes break after the first 40 minutes. The same teacher was in their class for each application.

#### **3.6.** Data Analysis

In this study, data were analyzed according to the research questions which were designed to examine the  $6^{th}$ ,  $7^{th}$  and  $8^{th}$  grade students' procedural and conceptual knowledge. The aim was to obtain a wider and more detailed picture about the extent of this knowledge in the domain of area and perimeter through the

curriculum and how students' performance in conceptual and procedural aspects of area and perimeter changed with respect to their geometry self-efficacy.

First of all, all answers of students in conceptual and procedural tests were examined separately. The frequency of correct, incorrect, and empty answers was calculated for each question in each test. Moreover, incorrect answers were investigated in detail. The wrong responses of students were categorized according to the responses of the students. Then, the frequency of the most common answers was given in tables for questions.

Second, quantitative research methodologies were used to analyze data through a number of descriptive and inferential statistics by using PASW 18 software. A codebook was prepared and a number of data screening procedures were carried out to run the primary data analysis. There were no missing values. Then, in order to examine the relationship between geometry self-efficacy beliefs of students and their scores in procedural and conceptual tests, Pearson product-moment correlation analyses were run. In the geometry self-efficacy test, Likert type items were scored. The negative items were reversed for the scoring.

One-way ANOVA was used to investigate how self-efficacy, procedural knowledge, and conceptual knowledge changed according to grade levels. Prior to running this analysis, the assumptions were checked. Eta squared was calculated to investigate the practical significance of the results.

#### **3.7.** Validity and Reliability of the Instruments

Validity is the "appropriateness, correctness, meaningfulness and usefulness of the inferences" (Fraenkel & Wallen, 2006, p. 151) claimed based on the results of the study. For this reason, after questions were prepared, tests were given to the experts who were five elementary mathematics teachers and two graduate students and four academicians in the field of mathematics education to gain evidence for the content validity of instruments. The aims of the tests, and coverage, and objectives of the study were clearly explained to the experts. Table of specifications of each test were also presented. They were asked if test items corresponded for the grade levels and the aim of the tests, if the language of items was clear, and the order of the questions were adequate. After opinions were taken, items were examined once more and suitable changes were completed before the pilot study of the instruments. According to their comments, language of the some questions was corrected. Moreover, some questions were taken out from procedural test since they did not investigate the procedural skills. Also, the number of the questions was decreased from the tests by taking out the questions with similar objectives.

Internal consistency methods were utilized to examine the reliability of the instruments. For the statistical analyses of the internal consistency, Cronbach's alpha coefficient was obtained through PASW 18 program. The reliability estimated for scores on PKT was found as .90 and for scores on CKT as .78 in the pilot study. Since reliability coefficient of a scale should be at least .70 (Fraenkel & Wallen, 2006) to have a reliable instrument, it could be said that PKT and CKT were reliable instruments.

In addition to the internal consistency between items, there should be a scoring agreement which refers to inter-rater reliability since there were open-ended items and they were scored according to a rubric. In order to check inter-rater reliability, the correlation coefficient was calculated for two independent scorers of the same instrument. The correlation coefficient was calculated for PKT as .99 and for CKT was calculated as .99. These values indicated quite high reliability between scorers. In addition, the Pearson Product-Moment Coefficient was calculated between two scorers. This correlation coefficient was found as .99 which indicated a very high consistency between scorers for PKT and CKT. However, in order to compute the total scores of students for PKT and CKT, the different scores given by two raters (one rater was the researcher who is a mathematics teacher and the second rater was another mathematics teacher) were examined again and the exact scores of students were determined after both scorers agreed.

Geometry self-efficacy scale which was used in the study was developed by Cantürk-Günhan and Başer (2007). For the reliability of GSE scale, Cantürk-Günhan and Başer (2007) administered a pilot study on 385 elementary students. The Cronbach alpha values for each sub-dimension of the scale calculated by Cantürk-Günhan and Başer were summarized in Table 3.8:

| Sub-dimensions                 | Number of items | Cronbach alpha |
|--------------------------------|-----------------|----------------|
| Positive self-efficacy beliefs | 12              | .88            |
| Negative self-efficacy beliefs | 6               | .70            |
| Beliefs on the use of geometry | 7               | .70            |
| knowledge                      |                 |                |
| General                        | 25              | .90            |

Table 3. 8: Cronbach Alpha Values

According to the information given in Table 6, the reliability of the GSE scale was satisfactory (.90). Cantürk-Günhan and Başer (2007) conducted factor analysis to check the validity of the scale. They mentioned that the items loaded in three factors which explained the 42.4% of the variance. According to the factor analysis results conducted by the test developers, the first factor explained 27.41 percent of the variance with 12 items; the second factor explained 9.81 percent of the variance with 6 items; third factor explained 5.20 percent of the variance with 7 items. For the content-related validity of the scale, expert (two academicians, two graduate students, and two mathematics teachers) opinions were taken into consideration.

To establish reliability of the instrument in the present study, internal consistency methods were used. For the statistical analyses of the internal consistency, Cronbach alpha coefficient was computed as .77 which was an acceptable reliability.

#### **3.8.** Assumptions and Limitations

There are several assumptions of the present study as in other studies. First of all it was assumed that procedural knowledge, conceptual knowledge and selfefficacy of students can be measured adequately through the items in the in the instruments. Second, the participants of the study were assumed to answer the questions in the instruments sincerely and accurately. Third, it was assumed that the instruments were completed under similar conditions. Lastly, there was no interaction between the subjects in order not to affect the results of the study.

There were also the limitations of this study. One of them was the sampling method. Since the participants were not selected randomly, the results of this study could not be generalized to a larger population. The other one was the status of the researcher who was the teacher of 8<sup>th</sup> grade participants. Since the researcher was their teacher, they could answer the questions based on their feelings towards their teachers. For example, students who liked their teacher might have answered the questions more reluctantly whereas students who did not specifically like their teacher might not have given importance to questions. Another limitation of the study was that results were limited with the questions in the instruments.

#### **3.9. Internal and External Validity**

#### **3.9.1. Internal Validity**

If the difference on dependent variable is caused by only independent variables rather than unintended variables, this means that the internal validity is established (Fraenkel & Wallen, 2006). In general, for survey studies, the possible internal threats are location, instrumentation (instrument decay, data collector characteristics, and data collector bias), instrument decay and mortality (Fraenkel & Wallen, 2006). Moreover, location, testing, mortality, instrumentation and subject characteristics are the common threats for correlation studies (Fraenkel & Wallen, 2006).

Data collector bias might be an internal threat for research studies. Data collector might behave unconsciously to gather the desired results (Fraenken & Wallen, 2006). However, in the present study the researcher did not communicate with students during the implementation. Besides this, a detailed answer key was prepared and used while scoring the answers. Since the researcher remained unbiased

and non-directive during the data collection and data analysis, data collector bias did not become a threat for the present study. Moreover, since the data collector was the same in all classes in which the implementation occurred, data collector characteristics could not be a threat for this study. The data collection time and scoring were scheduled so as to prevent instrument decay.

In studies, when the aim was comparing the groups, the subjects in the groups might differ from each other in several variables. This threat was called as subjects characteristics. In this study there were three groups which changed in grade levels, in other words, in ages. Also, both genders were represented. However, groups were evaluated in their grade levels for some analysis and questions in the achievement tests were prepared from the common objectives. Also, students were assumed to have the same mathematical experiences as they were in the same school. Therefore, the subject characteristics threat was reduced.

In this study, location might have been a threat since the researcher would have carried out the study in different classrooms. To eliminate this threat, the researcher conducted the questionnaire in the mathematics classroom in the school so that she could eliminate the effect of this threat.

The other threat for the present study might have been mortality which was also called as loss of subjects. Some of the subjects might have been absent during the administration time. However, during the application of the test, this problem was not occurred.

Fraenkel and Wallen (2006) stated that testing which was referred as influences of instruments on each other could be a threat for studies. However, in this study this threat did not occurred since PKT, CKT and STG tests measured different constructs.

#### **3.9.2. External Validity**

Fraenkel and Wallen (2006) defined the external validity as the degree to which results can be generalized to the population. In order to generalize the results

of the study to the population, the sample should be representative of the population in terms of nature and environmental issues. Since the sampling method was convenience sampling and only one private school was used to collect data, the results of this study could not be generalized to a larger population. Nonetheless, at certain conditions, the results of this study could be generalized to a population. Fraenkel and Wallen (2006) defined this type of generalizability as ecological generalizability which meant "the extent to which the results of a study can be generalized to conditions or settings other than those that prevailed in particular study" (p. 108). There may be schools with students who have same academic and social characteristics in other districts. Thus, the results of this study may be generalized to a population who has the subjects having the mentioned characteristics.

#### **CHAPTER 4**

#### RESULTS

The purpose of the current study was to examine conceptual and procedural knowledge of private middle school students in the domain of area and perimeter of geometric figures and their self-efficacy beliefs towards geometry. Another purpose of the study was to investigate the relationship between students' procedural and conceptual knowledge, and their self-efficacy beliefs towards geometry. Moreover, the most common errors of students in the domain of area and perimeter concepts in procedural and conceptual tests were identified.

In this chapter, the results of the data analysis are presented in detail according to the research questions. First of all, descriptive statistics including minimum and maximum values mean, and standard deviation related to procedural and conceptual tests' scores and self-efficacy belief test scores were presented. Second, in order to examine whether there were significant differences between the procedural, conceptual and geometry self-efficacy test scores with respect to three grade levels (6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> grade) one way ANOVA was conducted. In addition to this, students' answers in the procedural and conceptual tests were analyzed to examine their performance according to the question types and their most common errors. For this purpose, questions were categorized for procedural and conceptual tests in terms of the domain area and perimeter. Also, sub-categorizations were done under the area and perimeter concepts and frequencies of correct, incorrect and empty answers were presented. Finally, in order to examine the relationship between students' self-efficacy beliefs towards geometry and their procedural and conceptual knowledge separately, Pearson-product moment correlation analysis was conducted.

## 4.1. Analysis of Private middle school Students' Procedural Knowledge about Area and Perimeter of Geometric Shapes

In this section descriptive statistics on scores of procedural knowledge test (PKT) related to area and perimeter of geometric shapes was given. Then, the mean score of procedural knowledge test were compared according to the grade levels. Finally, the frequencies of the correct answers and the most common incorrect answers were presented.

#### 4.1.1. Descriptive Statistics about Procedural Knowledge Test

In this section descriptive statistics about procedural knowledge test about area and perimeter of geometric shapes were presented. Table 4.1 illustrates descriptive statistics of three grade level students' scores of procedural knowledge test.

| Grade Level           | Ν   | Max | Min | Mean  | SD    |
|-----------------------|-----|-----|-----|-------|-------|
| 6 <sup>th</sup> grade | 31  | 100 | 2   | 45,26 | 28,25 |
| 7 <sup>th</sup> grade | 35  | 100 | 1   | 56,63 | 34,62 |
| 8 <sup>th</sup> grade | 45  | 100 | 3   | 62,13 | 25,87 |
| Total                 | 111 | 100 | 1   | 55,68 | 30,06 |

Table 4.1Descriptive Statistics of Private middle school Students' Scores of Procedural Knowledge Test

Table 4.1 is an overall summary of the descriptive statistics gathered from the procedural knowledge test scores from the  $6^{th}$ ,  $7^{th}$ , and  $8^{th}$  grade students. As shown in the table, the mean score of  $6^{th}$  grade students was 45, 26 (*SD*= 28,25), the mean score of  $7^{th}$  grade students was 56, 63 (*SD*= 34,62), and the mean score of  $8^{th}$  grade

students was 62, 13 (SD= 25, 87) while the mean score of all total was 55, 68 (SD= 30, 06) in the procedural knowledge test. As a results, the procedural knowledge test scores of the 6<sup>th</sup> grade students was under the mean of all students' scores, the procedural knowledge test scores of the 7<sup>th</sup> and 8<sup>th</sup> grade students was above the mean of all students' scores. Moreover, it can be seen that the procedural test scores of 8<sup>th</sup> grade students was higher than the procedural test scores of 7<sup>th</sup> grade students and the procedural test scores of 7<sup>th</sup> grade students was higher than the procedural test scores of 6<sup>th</sup> grade students.

# 4.1.2. The Difference in Procedural Knowledge of Private middle school Students in terms of Grade Level

One of the aims of the present study was to examine how procedural knowledge of private middle school students changed according to their grade level. Therefore, mean scores of students from each grade level were compared to see whether there was a significant mean difference between the scores of the groups. In order to compare the mean differences between the groups, ANOVA should be used (Pallant, 2007). Therefore, in order to examine the mean difference of procedural knowledge test scores in terms of grade levels, one-way ANOVA was used. Prior to running the analysis, the assumptions of one-way ANOVA were checked. In the next sections the assumptions and analysis of results were summarized.

### 4.1.2.1. Assumptions of one way ANOVA

The assumptions of one-way ANOVA were level of measurement, independence of observations, normality, and homogeneity of variance (Pallant, 2007).

The variables for one-way ANOVA were mean scores of procedural knowledge test of  $6^{th}$ ,  $7^{th}$ , and  $8^{th}$  grade students which were continuous variable. Hence, level of measurement assumption was ensured.

Each measurement of grade levels was not influenced by each other, since each grade level was taken the tests in different times. It was assumed that their scores did not influence one another. Therefore, level of measurement assumption was ensured.

The other assumption to be considered before conducting one-way ANOVA was normality. For parametric techniques, mean scores for each variable should be normally distributed (Pallant, 2007). Since the sample sizes for all groups was large enough (e.g. 30+), this assumption should not to cause major problems. In order to check normality of procedural knowledge test scores for each group, histograms, normal Q-Q plots, and skewness and kurtosis values were checked.

In addition to this, skewness and kurtosis values were checked. Since these values are between -2 and +2 (Pallant, 2007), the normality of three groups' scores was assured. The values are given in the table 4.2 below.

Table 4.2 Skewness and Kurtosis Values of Procedural Knowledge Test Scores for each grade level

| Grade level           | Skewness | Kurtosis | Number |
|-----------------------|----------|----------|--------|
|                       |          |          |        |
| 6 <sup>th</sup> grade | .291     | 975      | 31     |
| 7 <sup>th</sup> grade | 501      | -1.301   | 35     |
| 8 <sup>th</sup> grade | 775      | 380      | 45     |

For homogeneity of variance assumption, Pallant (2007) stated that the variability of all groups should be similar, that is, the samples were obtained from populations of equal variances. In this analysis, Levene's Test of Equality of Error Variances showed that the homogeneity of variance assumption was not assured (p = .030). However, according to Stevens (1996), "Analysis of variance is reasonably robust to violations of this assumption, provided the size of your groups is reasonably similar" (as cited in Pallant, p. 207). Since in the sample, largest/smallest < 1.5, the violation of this assumption was acceptable.

#### 4.1.2.2. Results of one - way ANOVA for Procedural Knowledge Test

According to one-way ANOVA results, there was no significant mean difference (F (2,108) = 3, 03, p>.05) among three grade levels in terms of procedural knowledge test scores. Although 7<sup>th</sup> and 8<sup>th</sup> grade students' scores were higher than the 6<sup>th</sup> grade students, this difference was not statistically significant. In Figure 4.1PKT scores are given according to grade levels.



Figure 4. 1 The PKT Scores of 6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup>Grade Students

# 4.1.3. The Analysis of the Correct and Most Common Errors in the Procedural Knowledge Test

Before the analysis, the questions were categorized in terms of the perimeter and area domains. Then, the perimeter part was also categorized in itself as the perimeter of regular geometric figures (questions 1, 7 and 9), perimeter of irregular shapes (questions 5). Similarly area part was classified as the area of regular shapes (questions 1, 2, 3, 6, 8, 9, 10), area of the combination of regular shapes (question 4).

#### 4.1.3.1. Perimeter of the square, rectangle and parallelogram

The objectives related to perimeter of the geometric figures were examined in  $1^{\text{st}}$ ,  $5^{\text{th}}$ ,  $7^{\text{th}}$  and the  $9^{\text{th}}$  questions in the procedural test.

The objective "to calculate perimeters of the square, rectangle and parallelogram" was examined in the 1<sup>st</sup> question. The perimeter part of the 1<sup>st</sup> question is given in the figure 4.2 below.



Figure 4.2 First Question in the Procedural Knowledge Test

In this question the students were expected to calculate the perimeter of the geometric figures in which the length of the edges and the heights were given. The answers were examined in terms of the frequency of correct answers, empty answers and incorrect answers with respect to grade levels. The frequency of correct, incorrect and empty answers is given in Table 4.3.

Table 4.3 The frequency of correct answers, empty answers and incorrect answers in the part related to perimeter in the 1<sup>st</sup> question with respect to grade levels

|                               | Square       |           |            | Rectangl    | e              |           | Parallelogram |             |             |
|-------------------------------|--------------|-----------|------------|-------------|----------------|-----------|---------------|-------------|-------------|
| Grac<br>evel                  | FRA          | FWA       | FEA        | FRA         | FWA            | FEA       | FRA           | FWA         | FEA         |
| E e C                         | (%)          | (%)       | (%)        | (%)         | (%)            | (%)       | (%)           | (%)         | (%)         |
| 6 <sup>th</sup> grade<br>(31) | 27<br>(87%)  | 1<br>(3%) | 3<br>(10%) | 26<br>(84%) | 3<br>(10%<br>) | 2<br>(6%) | 17<br>(55%)   | 8<br>(26%)  | 6<br>(81%)  |
| 7 <sup>th</sup> grade<br>(35) | 33<br>(94%)  | 0         | 2<br>(6%)  | 30<br>(86%) | 3<br>(9%)      | 2<br>(5%) | 21<br>(60%)   | 2<br>(6%)   | 12<br>(34%) |
| 8 <sup>th</sup> grade<br>45   | 44<br>(98%)  | 1<br>(2%) | 0          | 42<br>(93%) | 2<br>(4%)      | 1<br>(3%) | 28<br>(62%)   | 5<br>(11%)  | 12<br>(27%) |
| TOTAL<br>(111)                | 104<br>(94%) | 2<br>(2%) | 5<br>(4%)  | 98<br>(88%) | 8<br>(7%)      | 5<br>(5%) | 66<br>(60%)   | 15<br>(14%) | 30<br>(38%) |

FWA: Frequency of wrong answers

FRA: Frequency of right answers

FEA: Frequency of empty answers

According to the above table, large majority of the students gave correct answer for the square and rectangle part of this question. The most successful group was 8<sup>th</sup> grade students and the 7<sup>th</sup> grade students followed them. Sixth grade students had lower performance than the 7<sup>th</sup> and 8<sup>th</sup> grade students although the success rate of all grade levels were close to one another.

The analysis showed that students from all grade levels had lower performance in finding the perimeter of the parallelogram. Most of the students did not give any answers for parallelogram part noted that they forgot the perimeter formula of parallelogram.

The objective "to calculate perimeters of the square whose area was given" was examined in the 7<sup>th</sup> question. The 7<sup>th</sup> question is given in the figure 4.2.1.2 below.



#### Figure 4.3: Seventh Question in the PKT

In the 7<sup>th</sup>question, students were expected to find the length of the one side of the square and then calculate the area of the square. For this question, the answers were analyzed in terms of the frequency of correct answer, incorrect answers, and empty answers with respect to the grade levels. The frequency of correct, incorrect, and empty answers is given in Table 4.4.

Table 4.4: The frequency of correct answers, empty answers and incorrect answers in the 7thquestion with respect to grade levels

| Grade level                | The frequency of right answers (%) | The frequency of wrong answers (%) | The frequency of<br>empty answers<br>(%) |
|----------------------------|------------------------------------|------------------------------------|--|
| 6 <sup>th</sup> grade (31) | 10 (32%)                           | 19 (61%)                           | 2 (7%)                                   |
| 7 <sup>th</sup> grade (35) | 21(60%)                            | 6(17%)                             | 8(23%)                                   |
| 8 <sup>th</sup> grade (45) | 35(78%)                            | 6(13%)                             | 4(9%)                                    |
| <b>Total (111)</b>         | 66(59%)                            | 31(28%)                            | 14(13%)                                  |

According to the table, it was investigated that only about half of the students gave correct answers for this question. It was seen that  $8^{th}$  grade students performed higher than the  $6^{th}$  and  $7^{th}$  grade students in this task. The lowest performance was seen in  $6^{th}$  grade students. Only less than 50% of the  $6^{th}$  grade students gave correct answers. Moreover, it was seen that  $7^{th}$  grade students gave more empty answers compared to the  $6^{th}$  and  $8^{th}$  grade students.

When the most common errors were analyzed, it was seen that there was no problem in calculating the perimeter of the square. However, students had difficulty when they calculated the length of the one edge of the square whose area was given. It was seen that most students tended to divide the area of the square into four or two to find the length of the edge of the square. This showed that in this type of question, most common errors were resulted from this reason. The frequency of the most common incorrect answer for this question is in Table 4.5.

Table 4.5: The frequency of the most common incorrect answer for the 7th question in the procedural test

| Response type  | Frequency of the 6 <sup>th</sup> grade students for the response | Frequency of the 7 <sup>th</sup> grade students for the response | Frequency of the 8 <sup>th</sup> grade students for the response |
|--|--|--|--|
| Dividing the area<br>of the square into<br>2 to find the<br>length of the edge | 7  | 4  | 3  |
| Dividing the area<br>of the square into<br>4 to find the<br>length of the side | 8  | 1  | 0  |

The objective "to calculate the perimeter of a rectangle which is formed from squares" was investigated in the 9<sup>th</sup> question. The 9<sup>th</sup> question is given in the figure 4.4 below.



#### Figure 4.4: Ninth Question in the PKT

In this question students were expected to put the 3 square side to side and calculate the perimeter of the formed rectangle. For this question, the answers were analyzed in terms of the frequency of correct answer, incorrect answers, and empty answers for the perimeter part with respect to the grade levels. The frequency of correct, incorrect, and empty answers is given in Table 4.6.

Table 4.6: The frequency of correct answers, empty answers and incorrect answers in the part related to perimeter of the rectangle in the 9<sup>th</sup> question with respect to grade levels.

| Grade level                | The frequency of right answers (%) | The frequency of wrong answers (%) | The frequency of<br>empty answers<br>(%) |
|----------------------------|------------------------------------|------------------------------------|--|
| 6 <sup>th</sup> grade (31) | 22 (71%)                           | 8(26%)                             | 1 (3%)                                   |
| 7 <sup>th</sup> grade (35) | 22(63%)                            | 5(14%)                             | 8(23%)                                   |
| 8 <sup>th</sup> grade (45) | 26(58%)                            | 15(33%)                            | 4(9%)                                    |
| Total (111)                | 70(63%)                            | 28(25%)                            | 13(12%)                                  |

According to the table, about 63% of the students gave correct answers for the question. It was investigated that the most successful group was  $6^{th}$  grade students and the  $7^{th}$  grade students followed them. Eighth grade students had lower performance with respect to the  $6^{th}$  and  $7^{th}$  grade students. When the incorrect answers were examined, it was seen that two students from  $6^{th}$  grade and three students from  $8^{th}$  grade tended to calculate the perimeter of three squares separately.

#### 4.1.3.2. Perimeter of the irregular figures

The objective "to calculate the perimeter of irregular shapes" was investigated in the  $5^{th}$  question. The  $5^{th}$  question is given in the figure 4.5 below.



Figure 4.5: Fifth Question in the PKT

In the 5<sup>th</sup> question in the procedural test, the students were expected to find the other edges by the help of given edges and then calculate the perimeter of the polygon. The answers were analyzed in terms of the correct, incorrect, and empty answers. The frequency of the correct, incorrect, and empty answers is given in table 4.7 below.

| Grade level                | The frequency of right answers (%) | The frequency of wrong answers (%) | The frequency of<br>empty answers<br>(%) |
|----------------------------|------------------------------------|------------------------------------|--|
| 6 <sup>th</sup> grade (31) | 16(52%)                            | 12(39%)                            | 3(9%)                                    |
| 7 <sup>th</sup> grade (35) | 17(49%)                            | 8(23%)                             | 10(28%)                                  |
| 8 <sup>th</sup> grade (45) | 28(62%)                            | 10(22%)                            | 7(16%)                                   |
| Total (111)                | 61(55%)                            | 30(27%)                            | 20(18%)                                  |

Table 4.7: The frequency of correct answers, empty answers and incorrect answers in the 5th question with respect to grade levels
According to the table, only about half of the students gave the correct answers for the question although the objective corresponding to this question was even addressed in the 5<sup>th</sup>gradeand emphasized in the sixth grade. While the most successful group was 8<sup>th</sup> grade students (about 67% of the students gave correct answers for the question), the success rate of 6<sup>th</sup> and the 7<sup>th</sup> grade students followed 8<sup>th</sup> grade students and it corresponded to about 50% of the students for each. The most common errors were seen in only 6<sup>th</sup> and 8<sup>th</sup> grade students. The frequency of the incorrect answers is given in the table 4.8 below.

| Response type   | Frequency of the 6 <sup>th</sup><br>grade students for the<br>response | Frequency of the 8 <sup>th</sup><br>grade students for the<br>response |
|---|--|--|
| Adding the only given numbers                                 | 2  | 2  |
| Summing up 3, 4 and 2 to calculate the length of the edge AF. | 7  | 2  |

Table 4.8: The frequency of the most common incorrect answer for the 5th question in the procedural test

The objectives related to area of the geometric figures were examined in  $1^{st}$ ,  $2^{nd}$ ,  $3^{rd}$ ,  $4^{th}$ ,  $6^{th}$ ,  $8^{th}$ ,  $9^{th}$  and  $10^{th}$  questions in the procedural test.

## 4.1.3.3. Area of the square, rectangle and parallelogram

The objective "to calculate areas of the square, rectangle and parallelogram" were examined in the 1<sup>st</sup> question. The area part of the 1<sup>st</sup> question is given in the figure 4.6 below.



Figure 4.6: First Question in the PKT (Area Part)

In this question students were expected to calculate the area of the geometric figures in which the length of the sides and the heights were given. The answers were examined in terms of the frequency of correct answers, empty answers and incorrect answers with respect to grade levels. The frequency of correct, incorrect and empty answers is given in Table 4.9.

| Grade                            | Square      |       |     | Rectangle   |       | Parallelogram |             |       |     |
|----------------------------------|-------------|-------|-----|-------------|-------|---------------|-------------|-------|-----|
| lovol                            | FRA         | FWA   | FEA | FRA         | FWA   | FEA           | FRA         | FWA   | FEA |
| IEVEI                            | (%)         | (%)   | (%) | (%)         | (%)   | (%)           | (%)         | (%)   | (%) |
| 6 <sup>th</sup><br>grade<br>(31) | 20<br>(65%) | 9     | 2   | 20<br>(65%) | 9     | 2             | 14<br>(45%) | 11    | 6   |
| 7 <sup>th</sup><br>grade<br>(35) | 29<br>(83%) | 4     | 2   | 29<br>(83%) | 2     | 4             | 13<br>(37%) | 6     | 16  |
| 8 <sup>th</sup> grade<br>45      | 42<br>(93%) | 2     | 1   | 42<br>(93%) | 2     | 1             | 34<br>(76%) | 12    | 22  |
| TOTAL                            | 91          | 15    | 5   | 91          | 13    | 7             | 61          | 29    | 44  |
| (111)                            | (82%)       | (14%) | 5   | (82%)       | (12%) | ,             | (55%)       | (26%) |     |

Table 4.9: The frequency of correct answers, empty answers and incorrect answers in the part related to area in the 1th question with respect to grade levels

According to the above table, most students (82 % of the all students) gave correct answer for this question. When examined with respect to the grade level, it was found that the most successful group was 8<sup>th</sup> grade students and the 7<sup>th</sup> grade students followed them. Sixth grade students had the lowest performance.

Students from all grade levels showed lower performance in finding the area of the parallelogram. Students' most common procedural error was multiplying the two edges and multiplying the edge and the height and then dividing it by two to find the area of the parallelogram. The frequency of the most common incorrect answer for this question is in Table 4.10.

| Response type  | Frequency of the<br>6 <sup>th</sup> grade students<br>for the response | Frequency of the<br>7 <sup>th</sup> grade<br>students for the<br>response | Frequency of the<br>8 <sup>th</sup> grade students<br>for the response |
|--|--|---|--|
| Multiplying the two<br>edges (10x3=30)                               | 4  | 2   | 8  |
| Multiplying the edge<br>and the height and<br>then dividing it by 2. | 3  | 3   | 1  |

Table 4.10: The frequency of the most common incorrect answer for the 1<sup>st</sup> question in the procedural test

The objective "to calculate the area of the painted region by using area of rectangle and square" was investigated in the  $6^{th}$  question. The  $6^{th}$  question is given in the figure 4.7.



Figure 4.7: Sixth Question in the PKT

In this question, students were expected to calculate the painted area by subtracting the area of squares from the area of rectangle. The answers were examined in terms of the frequency of correct answers, empty answers and incorrect answers for both with respect to grade levels. The frequency of correct, incorrect, and empty answers is given in Table 4.11.

| Grade level                | The frequency of right answers (%) | The frequency of wrong answers (%) | The frequency of<br>empty answers<br>(%) |
|----------------------------|------------------------------------|------------------------------------|--|
| 6 <sup>th</sup> grade (31) | 11(35%)                            | 16(%)                              | 4(%)                                     |
| 7 <sup>th</sup> grade (35) | 17(49%)                            | 10(%)                              | 8(%)                                     |
| 8 <sup>th</sup> grade (45) | 30(67%)                            | 4(%)                               | 11(%)                                    |
| Total (111)                | 58(52%)                            | 30(%)                              | 23(%)                                    |

Table 4.11: The frequency of correct answers, empty answers and incorrect answers in the 6th question in the procedural test with respect to grade levels

According to the table, almost only half of the students gave correct answers for the question. However, some of these students could calculate the area of the rectangle or the squares but they could not complete the answer. The frequency of the correct and the incorrect answers for squares and rectangle separately with respect to the grade levels is given in the table 4.12.

Table 4.12: The frequency of correct answers and incorrect answers in the  $6^{th}$  question for squares and rectangles separately in the procedural test with respect to grade levels

| Grade level                | The frequenc    | y of the<br>ers | The frequency of the in correct answers |           |  |
|----------------------------|-----------------|-----------------|---|-----------|--|
|                            | Squares         | Rectangle       | Squares                                 | Rectangle |  |
| 6 <sup>th</sup> grade (31) | 12(39%)         | 19(61%)         | 15                                      | 8         |  |
| 7 <sup>th</sup> grade (35) | 18(51%)         | 20(57%)         | 9                                       | 7         |  |
| 8 <sup>th</sup> grade (45) | 30(67%)         | 30(67%)         | 4                                       | 4         |  |
| Total                      | 60(54%) 69(62%) |                 | 28                                      | 19        |  |

According to the table the rate of the students who could only the area of the rectangle is more than the rate of the students who could find only the area of the squares. It was investigated that although there are four squares in the rectangle some students calculated the area for only one square.

The objective "to calculate the area of the rectangle whose perimeter and length of the one of the edges are given" was examined in the 8<sup>th</sup> question. The 8<sup>th</sup> question is given in the figure 4.8 below.



Figure 4.8: Eighth Question in PKT

In this question, students were expected to find the length of the unknown edge of the rectangle and then calculate the area of the rectangle. The responses were examined in terms of the frequency of correct answers, empty answers and incorrect answers for each grade level. The frequency of correct, incorrect and empty answers is given in Table 4.13.

Table 4.13: The frequency of correct answers, empty answers and incorrect answers in the 8thquestion with respect to grade levels

| Grade level                | The frequency of right answers (%) | The frequency of wrong answers (%) | The frequency of empty answers (%) |
|----------------------------|------------------------------------|------------------------------------|------------------------------------|
| 6 <sup>th</sup> grade (31) | 17(55%)                            | 11(36%)                            | 3(9%)                              |
| 7 <sup>th</sup> grade (35) | 23(66%)                            | 5(14%)                             | 7(20%)                             |
| 8 <sup>th</sup> grade (45) | 32(71%)                            | 7(16%)                             | 6(13%)                             |
| Total (111)                | 72(65%)                            | 23(21%)                            | 16(14%)                            |

According to the table, it was examined that 65% of the students gave correct answers for the questions. As the most successful group was  $8^{th}$ grade students,  $6^{th}$ grade students had the lowest performance in this question. When the errors were examined, it was seen that four students from  $6^{th}$  grade and one student from  $8^{th}$ grade

divided the perimeter of the rectangle into four to find the length of the other edge as the square. Moreover, one 6<sup>th</sup> grade student and four 8<sup>th</sup>grade students had errors in arithmetic procedures.

The objective "to calculate the area of a rectangle which is formed from three squares" was investigated in the 9<sup>th</sup> question. The 9<sup>th</sup> question is given in the figure 4.9 below.

*Question 9:* Calculate the **area** of the rectangle which is formed from 3 squares with one side as 7 cm.

#### Figure 4.9: Ninth Question in the PKT (Area Part)

In this question students were expected to put the 3 square side to side and calculate the area of the formed rectangle. For this question, the answers were analyzed in terms of the frequency of correct answer, incorrect answers, and empty answers for the area part with respect to the grade levels. The frequency of correct, incorrect, and empty answers is given in Table 4.14.

Table 4.14: The frequency of correct answers, empty answers and incorrect answers in the part related to area of the rectangle in the 9<sup>th</sup> question with respect to grade levels

| Grade level                | The frequency of right answers (%) | The frequency of wrong answers (%) | The frequency of empty answers (%) |
|----------------------------|------------------------------------|------------------------------------|------------------------------------|
| 6 <sup>th</sup> grade (31) | 16(52%)                            | 3(10%)                             | 12(38%)                            |
| 7 <sup>th</sup> grade (35) | 22(63%)                            | 5(14%)                             | 8(23%)                             |
| 8 <sup>th</sup> grade (45) | 34(76%)                            | 9(20%)                             | 2(4%)                              |
| Total (111)                | 72(65%)                            | 17(15%)                            | 22(20%)                            |

According to the table, 65% of the students gave correct answers for the question. It was investigated that this rate was very close to the rate related to the perimeter part (63%) for the same question. The most successful group was  $8^{th}$  grade students while the  $6^{th}$  grade students had the lowest performance. The number of the students who gave empty answers was more than the number of students who gave incorrect answers. However, it was seen that three students from  $6^{th}$  grade and two students from  $8^{th}$  grade calculated the area of only one square.

The objective "to calculate the other height of parallelogram by the help of its area" was investigated in the  $10^{th}$  question. The  $10^{th}$ question is given figure 4.10 below.



Figure 4.10: Tenth Question in the PKT

In this question, students were expected to calculate the length of the other height of the parallelogram by the help of its area. For this question, the answers were analyzed in terms of the frequency of correct answer, incorrect answers, and empty answers with respect to the grade levels. The frequency of correct, incorrect, and empty answers is given in Table 4.15.

Table 4.15: The frequency of correct answers, empty answers and incorrect answers in the 10th question with respect to grade levels

| Grade level                | The frequency of right answers (%) | The frequency of wrong answers (%) | The frequency of empty answers (%) |
|----------------------------|------------------------------------|------------------------------------|------------------------------------|
| 6 <sup>th</sup> grade (31) | 5(16%)                             | 17(55%)                            | 9(29%)                             |
| 7 <sup>th</sup> grade (35) | 10(29%)                            | 9(26%)                             | 16(45%)                            |
| 8 <sup>th</sup> grade (45) | 4(9%)                              | 8(18%)                             | 33(77%)                            |
| Total (111)                | 19(17%)                            | 34(31%)                            | 58 (52%)                           |

According to the table, it was investigated that only 17% of the students gave correct answers and about half of them gave empty answers for this question. The most successful group was the 7<sup>th</sup> grade students and the 6<sup>th</sup> grade students followed them. However, only 9% of the 8<sup>th</sup> grade students gave correct answers for the question. The objective related to that question was in the 6<sup>th</sup> and 7<sup>th</sup> grade curriculum. Therefore, 8<sup>th</sup> grade students might have forgotten the procedures.

When the most common errors were examined it was investigated that the errors were changed among grade levels. For example,  $one6^{th}$  grade student multiplied the two edges of the parallelogram to find its area. Three students added the given numbers without any rationale. One student from  $6^{th}$  grade and one student from  $8^{th}$  grade claimed that the length of [AK] must be equal to the length of the other height. Moreover, six students from  $8^{th}$  grade tried to find the required height with Pythagorean Theorem.

The objective "to calculate the area of the acute, obtuse-angled and rightangled triangles whose side and the height belong to given side" was examined in the  $3^{rd}$  question. The  $3^{rd}$  question is given figure 4.11 below.



Figure 4.11: Third Question in the PKT

In this question students were expected to calculate the area of the rectangles by multiplying the length of the side and the length of the height and then dividing it by two. The answers were analyzed in terms of the frequency of correct, incorrect, empty answers and most common error. The frequency of correct, incorrect, empty answers is given table 4.16.

| Table 4.16: The frequency of correct answers, empty answers and incorrect answ | ers |
|--|-----|
| in the 3 <sup>rd</sup> question with respect to grade levels                   |     |

|                       | Acute t | riangle |     | Right tr | iangle |     | Obtuse | triangle |     |
|-----------------------|---------|---------|-----|----------|--------|-----|--------|----------|-----|
| Grade level           | FRA     | FWA     | FEA | FRA      | FWA    | FEA | FRA    | FWA      | FEA |
|                       | (%)     |         |     | (%)      |        |     | (%)    |          |     |
| 6 <sup>th</sup> grade | 17      |         | ~   | 15       | 11     | ~   | 11     | 10       | 0   |
| (31)                  | (55%)   | 9       | 5   | (35%)    | 11     | 5   | (35%)  | 12       | 8   |
| 7 <sup>th</sup> grade | 15      | 12      | 7   | 13       | 15     | 7   | 10     | 16       | 0   |
| (35)                  | (43%)   | 15      | /   | (37%)    | 15     | /   | (29%)  | 10       | 9   |
| 8 <sup>th</sup> grade | 30      | 11      | 4   | 29       | 10     | 4   | 18     | 12       | 1.4 |
| (45)                  | (67%)   | 11      | 4   | (64%)    | 12     | 4   | (40%)  | 13       | 14  |
| Total (111)           | 62      | 33      | 16  | 57       | 38     | 16  | 39     | 41       | 31  |
|                       | (57%)   |         |     | (51%)    | 20     |     | (35%)  |          | ~   |

#### FRA: Frequency of right answers

### FWA: Frequency of wrong answers

#### FEA: Frequency of empty

According to the table it was investigated that about 57% of the students gave correct answers for the area of the acute triangle, 51% of the students gave correct answers for the area of the right triangle, and about 35% of the students gave correct answers for the area of obtuse triangle. This showed that students had lower performance for the area of obtuse triangle. Moreover, for the area of acute and obtuse triangles, the most successful group was 8<sup>th</sup> grade students and 6<sup>th</sup> grade students followed them. For the area of right triangle, while the highest performance belonged to 8<sup>th</sup> grade students, 7<sup>th</sup> grade students followed them and 6<sup>th</sup> grade students had the lowest performance.

When the incorrect answers were examined, it was seen that most students did not divide the multiplication of the height and the length of the edge by two. The frequency of the most common error with respect to the grade levels is given below table 4.17.

| Response type  | Frequency of the 6 <sup>th</sup> grade students | Frequency of the 7 <sup>th</sup> grade students | Frequency of the 8 <sup>th</sup> grade students |
|----------------|---|---|---|
|                | for the response                                | for the response                                | for the response                                |
| Students did   |   |   |   |
| not divide the |   |   |   |
| multiplication |   |   |   |
| of the height  | 10  | 5   | 7   |
| and the length |   |   |   |
| of the side by |   |   |   |
| two.           |   |   |   |

Table 4.17: The frequency of the most common error in the  $3^{rd}$  question in the procedural test with respect to the grade levels

In the answers of 8<sup>th</sup> grade students, it was seen that two of them tried to apply Pythagorean Theorem. Furthermore, while one student from 6<sup>th</sup> grade divided the multiplication of the height and the length of the edge of the acute triangle by

two, he divided the multiplication of the height and the length of the side of the right triangle by three and obtuse triangle by four.

The objective "to calculate the ratio of painted area to not painted area which is formed by a rectangle and a triangle" was investigated in the  $2^{nd}$  question in the procedural test. The  $2^{nd}$  question is given in the figure 4.12 below.



Figure 4.12: Second Question in the PKT

In this question students were expected to calculate the area of rectangle and the triangle separately and then find the ration of painted area to not painted area. The answers were analyzed in terms of the frequency of the correct, incorrect and empty answers. The frequency of correct answers, empty answers and incorrect answers is given table 4.18 below with respect to grade levels.

| Grade level                | The frequency of right answers (%) | The frequency of wrong answers (%) | The frequency of empty answers (%) |
|----------------------------|------------------------------------|------------------------------------|------------------------------------|
| 6 <sup>th</sup> grade (31) | 3(10%)                             | 18                                 | 10                                 |
| 7 <sup>th</sup> grade (35) | 3(9%)                              | 16                                 | 16                                 |
| 8 <sup>th</sup> grade (45) | 7(16%)                             | 22                                 | 16                                 |
| Total (111)                | 13(12%)                            | 56(51%)                            | 42(37%)                            |

Table 4.18: The frequency of correct answers, empty answers and incorrect answers in the 2ndquestion with respect to grade levels

According to the table, it was seen that only 12% of the students gave correct answers for the question. The success rate was very low with respect to the other questions in the procedural test. Seventh grade students had the lowest performance. The most successful group was 8<sup>th</sup> grade students although their success rate was also very low. However, when the incorrect answers were analyzed, it was examined that most students calculated the ratio of area of triangle to the area of rectangle. The frequency of that type of answers is given with respect to the grade level in table 4.19 below.

| Response type   | Frequency of              | Frequency of              | Frequency of              |
|---|---------------------------|---------------------------|---------------------------|
|   | the 6 <sup>th</sup> grade | the 7 <sup>th</sup> grade | the 8 <sup>th</sup> grade |
|   | students for              | students for              | students for              |
|   | the response              | the response              | the response              |
| Students calculated the ratio of the area of the triangle to the rectangle (not to the empty area) therefore the ratio was found as $\frac{1}{2}$ . | 5                         | 9                         | 11                        |

Table 4.19: The frequency of the most common incorrect answer for the 2th question in the procedural test

Eleven students from  $6^{th}$  grade, 10 students from  $7^{th}$  grade and 14 students from  $8^{th}$  grade students were able to calculate the area of triangle correctly. Fifteen students from  $6^{th}$  grade, 12 students from  $7^{th}$  grade and 14 students from  $8^{th}$  grade students were able to calculate the area of rectangle correctly. However, they did not find the ratio.

# 4.1.3.4. Area of the polygon which is the combination of regular geometric shapes

The objective "to calculate the area of a polygon which is combination of the regular geometric shapes" was investigated in the 4<sup>th</sup> question in the procedural test. The 4<sup>th</sup> question is given in the figure 4.13 below.

*Question 4* :In the below figure, **ABGH** is a rectangle, **BCEF** is a square and **CED** is a right triangle. Calculate the area of the polygon ACDFGH.





In this question students were expected to calculate the area of rectangle, square, and triangle and then sum up them. The answers were analyzed in terms of the frequency of correct answers, empty answers and incorrect with respect to grade levels. The frequency of the correct answers, empty answers and incorrect is given in the table 4.20.

| Table 4.20   | : The f | frequency  | y of corre | ct answers  | , empty | answers | and ir | ncorrect | answers |
|--------------|---------|------------|------------|-------------|---------|---------|--------|----------|---------|
| in the 4th o | questic | on with re | espect to  | grade level | S       |         |        |          |         |

| Grade level                | The frequency of right answers (%) | The frequency of wrong answers (%) | The frequency of empty answers (%) |
|----------------------------|------------------------------------|------------------------------------|------------------------------------|
| 6 <sup>th</sup> grade (31) | 8(26%)                             | 14                                 | 9                                  |
| 7 <sup>th</sup> grade (35) | 18(51%)                            | 6                                  | 11                                 |
| 8 <sup>th</sup> grade (45) | 26(58%)                            | 10                                 | 9                                  |
| <b>Total (111)</b>         | 52(47%)                            | 30(27%)                            | 29(26%)                            |

According to the table, less than half of the students gave correct answers for the question. While the  $8^{th}$  grade students were most successful group, the success rate of the  $6^{th}$  grade students was very low although the related objective was in the  $6^{th}$  grade program. When the errors were examined, one student from  $7^{th}$  and  $8^{th}$  grade and 5 students from  $6^{th}$  grade tried to calculate the perimeter of the polygon rather than the area. Moreover, it was seen that 3 eighth grade students did not complete the answer since they could not find the area of triangle.

# 4.1.5. Summary of the results the perimeter concept in procedural knowledge test

In the procedural test, the students' procedural knowledge and most common errors of perimeter of geometric figures were examined through different types of questions.

In the question in which the side lengths of the figures were given, students were wanted to calculate perimeter. In this type of questions most of the students (about 94% of them) could give correct answers. However, it was examined that students' procedural knowledge was weak in calculating the perimeter of the parallelogram. It was seen that most students who gave empty answers for the parallelogram part said that "I forgot the formula of the perimeter of the parallelogram."

While the most successful group was 8<sup>th</sup> grade students, the success rate of 6<sup>th</sup> grade and 7<sup>th</sup> grade students were close to each other. When students were asked to calculate the perimeter of the square whose area was given, their success rate was decreased. Especially 6<sup>th</sup> grade students had very low performance compared to the 7<sup>th</sup> and 8<sup>th</sup> grade students while the success rate of 7<sup>th</sup> and 8<sup>th</sup> grade students was close to each other. However, when the errors were examined, it was seen that the procedural error was related to finding the length of the side of the square rather than calculating the perimeter of the square.

In the question where students were asked to calculate the perimeter of the rectangle which was formed from squares, 6<sup>th</sup> grade students were more successful

than the  $7^{th}$  and  $8^{th}$  grade students. Seventh grade students followed  $6^{th}$  grade students. It was investigated that  $8^{th}$  grade students had the lowest performance.

Finally, when students were required to calculate the perimeter of an irregular shape, only about half of the students gave correct answers. While the most successful group was  $8^{th}$  grade students,  $6^{th}$ grade students were the second most successful group. Seventh grade students had the lowest performance. However, the success rate of  $6^{th}$  grade and  $7^{th}$  grade students were close to each other.

#### 4.1.6. Summary of the results about the area concept in the procedural test

When the procedural test results were examined in terms of the area concept, it was analyzed that students had strong procedural knowledge in the area questions in which the edges and the heights of the regular geometric shapes were given directly. However, they had difficulty in calculating the area of parallelograms and obtuse-angled triangle. Furthermore, it was realized that most students had problems with the heights of these geometric figures therefore; the conception of height might have led procedurally wrong steps. In addition to these, it was analyzed that in questions which include parallelograms and triangles, 6<sup>th</sup> grade students had higher procedural performance than 7<sup>th</sup> grade and 8<sup>th</sup> grade students when compared to the other type of questions.

Students had low performance in questions in which the geometric shapes were given as a combination of different known shapes (such as questions 2 and 4). The figures might seem considerably confusing especially for the  $6^{th}$  grade students. However, in question 2, since the right answer was related to the ratio concept, students might have problem with the ratio rather than area.

Finally, in questions where the area of rectangle whose one side and the perimeter were given was asked, students had difficulty to find the other length of the edge. The same problem occurred in questions in which it was asked to find the perimeter of the square whose area was given. Therefore, students had problems to calculate the length of the side of the rectangle or square whose area or perimeter was given.

## 4.2. Analysis of Private middle school Students' Conceptual Knowledge about Area and Perimeter of Geometric Shapes

In this section descriptive statistics on scores of conceptual knowledge test (CKT) related to area and perimeter of geometric shapes was given. Then, the mean score of conceptual knowledge test were compared according to the grade levels. Finally, the frequencies of the correct answers and the most common incorrect answers were presented.

#### 4.2.1. Descriptive Statistics about Conceptual Knowledge Test

In this section descriptive statistics about conceptual knowledge test about are and perimeter of geometric shapes were presented. Table 4.21 illustrates descriptive statistics of three grade level students' scores of conceptual knowledge test.

| Grade Level           | Ν   | Max | Min | Mean  | SD    |
|-----------------------|-----|-----|-----|-------|-------|
| 6 <sup>th</sup> grade | 31  | 95  | 12  | 46,13 | 21,80 |
| 7 <sup>th</sup> grade | 35  | 100 | 1   | 53,43 | 24,65 |
| 8 <sup>th</sup> grade | 45  | 100 | 14  | 60,09 | 21,32 |
| Total                 | 111 | 100 | 1   | 54,09 | 23,07 |

Table 4.21: Descriptive Statistics of Private middle school Grade Students' Scores of Conceptual Knowledge Test

Table 4.21 is an overall summary of the descriptive statistics gathered from the conceptual knowledge test scores from the 6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> grade students. As shown in the table, the mean score of 6<sup>th</sup> grade students was 46, 13 (SD= 21, 80), the mean score of 7<sup>th</sup> grade students was 53, 43 (SD= 24, 65), and the mean score of 8<sup>th</sup> grade students was 60, 09 (SD= 21, 32) while the mean score of all total was 54, 09

(SD=23, 07) in the conceptual knowledge test. The conceptual knowledge test scores of the 6<sup>th</sup> and 7<sup>th</sup> grade students were under the mean of all students' scores whereas test scores of the 8<sup>th</sup> grade students was above the mean of all students' scores. Moreover, it can be seen that the conceptual test scores of 8<sup>th</sup> grade students was higher than the conceptual test scores of 7<sup>th</sup> grade students and the conceptual test scores of 7<sup>th</sup> grade students was higher than the conceptual test scores of 6<sup>th</sup> grade students.

# 4.2.2. The Difference in Conceptual Knowledge of Private middle school Students in terms of Grade Level

One of the aims of the present study was to examine how private middle school students' conceptual knowledge changed according to the grade level. This research question addressed a comparison of means for whether there was a significant mean difference between the scores of the groups. Therefore, in order to examine the mean difference of conceptual knowledge test scores in terms of grade levels as mentioned in analysis part related to PCK, one-way ANOVA was used. Prior to running the analysis, the assumptions of one-way ANOVA for CKT were checked also. In the next sections the assumptions and analysis of results were summarized.

#### 4.2.2.1. Assumptions of one way ANOVA for Conceptual Knowledge Test

The assumptions for ANOVA related to conceptual knowledge test were also checked as in the procedural knowledge test .The variables for one-way ANOVA were mean scores of conceptual knowledge test of  $6^{th}$ ,  $7^{th}$ , and  $8^{th}$  grade students which were continuous variable. Hence the level of measurement assumption was ensured. Since for PKT level of measurement assumption and the homogeneity of variance assumption (p= .63)were ensured, for CKT they were also ensured.

In order to check normality of conceptual knowledge test scores for each group, histograms and skewness and kurtosis values were checked. The values are given in the table 4.22 below.

| 6 <sup>th</sup> grade | .669 | 324 | 31 |
|-----------------------|------|-----|----|
| 7 <sup>th</sup> grade | 077  | 505 | 35 |
| 8 <sup>th</sup> grade | .194 | 605 | 45 |

Table 4.22: Skewness and Kurtosis Values of Conceptual Knowledge Test Scores for each grade level

### 4.2.2.2. Results of one-way ANOVA for Conceptual Knowledge Test

According to the one-way ANOVA results, there was statistically significant difference (F (2, 108) = 3.54, p < .05) among mean scores of students from three grade levels in conceptual knowledge test scores.

In order to reveal the difference among grade levels, the post-hoc analysis was performed and the results are presented in Table 4.23.

Table 4.23: Multiple Comparison for Post- Hoc Results for CKT

| Grade Level           | Grade Level           | Mean Difference | Sig.  |
|-----------------------|-----------------------|-----------------|-------|
| 6 <sup>th</sup> grade | 7 <sup>th</sup> grade | 7.30            | .391  |
| 8 <sup>th</sup> grade | 6 <sup>th</sup> grade | 13.96           | .025* |
| 7 <sup>th</sup> grade | 8 <sup>th</sup> grade | 6.66            | .392  |

Post Hoc comparison using the Turkey HSD test indicated that the mean score for  $6^{\text{th}}$  grade students (M = 46.13, SD = 21.80) was significantly different from  $8^{\text{th}}$  grade(M = 60.09, SD = 21.32). However, the mean score for  $7^{\text{th}}$  grade students was not significantly different from  $6^{\text{th}}$  grade students and  $8^{\text{th}}$  grade students.

In Figure 4.14 CKT scores are given according to grade levels.



Figure 4.14: The CKT scores of 6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup>grade students

As can be seen from the above figure, the mean score of  $8^{th}$  grade students was significantly higher than the mean score of  $6^{th}$  grade students. The mean score of  $7^{th}$  grade students was close to  $6^{th}$  grade students' and  $8^{th}$  grade students' mean scores.

The effect size, eta square value, was calculated by dividing the sum of squares between groups by the total sum of squares (Pallant, 2007). Eta squared was found as .06which corresponded to a small effect Cohen (1988). Therefore, only 6% of the variance in the CKT was explained by the grade level in this study.

## 4.2.3. The Analysis of the Correct, Incorrect and Empty Answers and Most Common Errors in the Conceptual Knowledge Test

As in the procedural test results, in this part the questions were analyzed by categorizing them as the area of irregular shapes (questions 1 and 9), surface area (questions 2 and 3) and area and perimeter of regular shapes (questions 4, 5, 6, 7, 8 and 10).

### 4.2.3.1. Area of Irregular Geometric Shapes

The objective "compare the area of given irregular geometric shapes" was evaluated in the  $1^{st}$  question and the objective "to predict the area of an irregular geometric shape" was evaluated in the 9<sup>th</sup> question in the conceptual test. The  $1^{st}$  and  $9^{th}$ questions are given in Figure 4.15.



Figure 4.15: The first and ninth questions in the conceptual test

The first question in which students were expected to predict and then order the given figures from the biggest figure to the smallest one according to their areas had one correct answer. Hence, the frequency of correct answers, empty answers, and possible combination of incorrect answers were calculated with respect to grade levels. The frequency of correct, incorrect and empty answers is given in Table 4.24 and Table 4.26 for these two questions.

|                 | The frequency of | The frequency of  | The frequency of |
|-----------------|------------------|-------------------|------------------|
| Grade level     | right answers    | incorrect answers | empty answers    |
|                 | (percentage)     | (percentage)      | (percentage)     |
| 6 <sup>th</sup> | 22 (71%)         | 8(26%)            | 1(3%)            |
| 7 <sup>th</sup> | 30 (86%)         | 514%)             | 0                |
| 8 <sup>th</sup> | 38(84%)          | 7(16%)            | 0                |
| Total           | 90(81%)          | 20(18%)           | 1(1%)            |

Table 4.24: The frequency of students' answers for the first question in the conceptual test with respect to the different grade levels

According to the above table, it was examined that 81% of the students gave the correct answer for this question. However, with respect to the grade levels, the most successful group was 7<sup>th</sup> grade students and the 8<sup>th</sup> grade students followed them. Sixth grade students had lower performance compared to the 7<sup>th</sup> and 8<sup>th</sup> grade students.

When the most common incorrect answers were examined, it was seen that the most common answer was "A>C>B". The frequency of the most common incorrect answer for this question is in Table 4.25.

Table 4.25: The frequency of the most common incorrect answer for the 9<sup>th</sup> question in the conceptual test

| Response type | Frequency of the               | Frequency of the               | Frequency of the               |
|---------------|--------------------------------|--------------------------------|--------------------------------|
|               | 6 <sup>th</sup> grade students | 7 <sup>th</sup> grade students | 8 <sup>th</sup> grade students |
|               | for the response               | for the response               | for the response               |
| A > C > B     | 5                              | 4                              | 2                              |

Similarly, in the 9<sup>th</sup> question students were expected to predict the given area. However, this time the area was given on the squared paper. For this question, the answer was 11, 5 units square. Hence, there were two alternative answers. The answers"11 and 12 unit squares" were accepted as the correct answer and the others were incorrect. The frequencies of correct, incorrect, and empty answers are given in Table 4.26.

Table 4.26: The frequency of students' answers for the ninth question in the conceptual test with respect to the different grade levels

| Grade level     | The frequency of | The frequency of  | The frequency of |
|-----------------|------------------|-------------------|------------------|
|                 | correct answers  | incorrect answers | empty answers    |
|                 | (percentage)     | (percentage)      | (percentage)     |
| 6 <sup>th</sup> | 14(45%)          | 17 (65%)          | 0                |
| 7 <sup>th</sup> | 25(71%)          | 9(26%)            | 1(3%)            |
| 8 <sup>th</sup> | 37(82%)          | 6(13%)            | 2(5%)            |
| Total           | 76(69%)          | 32(29%)           | 3(2%)            |

According to the above table,69% of the all students gave correct answer for this question. When examined with respect to the grade level, it was found that  $6^{th}$  grade students' performance were lower than the  $7^{th}$  and  $8^{th}$  grade students' performance. Moreover,  $6^{th}$ grade students who gave correct answer for this question were less than the 50% of the all  $6^{th}$  grade students.

To sum up,  $7^{th}$  and  $8^{th}$  grade students performed better than the  $6^{th}$ grade students in tasks including predicting and the comparing the area of geometric figures although related objectives were in the  $6^{th}$  grade program.

### 4.2.3.2. Surface Area

The objective "to decide how surface area of a structure consisted of unit cubes change when one of the stated unit cube taken away" was evaluated in the2<sup>nd</sup> question in the conceptual knowledge test. The  $2^{nd}$  question is given in Figure 4.16 below:

| Question 2: Which cube she | ould be taken away from the below structure so that the |
|----------------------------|---|
|                            | surface area of the structure will not change? Why?     |
| 1 2                        | Answer:   |
| 3 4                        | Because:  |
|                            |   |
|                            |   |

Figure 4.16: The 2<sup>nd</sup> question in the conceptual test

In the 2<sup>nd</sup> question in the conceptual test, students were expected to choose one of the cubes and explain the reasoning for their answer. Students' answers were analyzed in terms of the frequency of correct answer, explanation of the correct answer, the possible combination of incorrect answers, and empty answers with respect to the grade levels. The frequency of correct, incorrect, and empty answers is given in Table 4.27.

| Table 4.27: The frequency of stu    | udents' answe  | ers for the second | question in the | e |
|-------------------------------------|----------------|--------------------|-----------------|---|
| conceptual test with respect to the | ne different g | grade levels       |                 |   |

|                 | The frequer                                      | ncy of right   |   |                                     |
|-----------------|--|--|---|-------------------------------------|
|                 | answers (percentage)                             |  |   | The frequency                       |
| Grade level     | Right<br>answer and<br>sufficient<br>explanation | RightThe freeanswer butincorrednot(percsufficientexplanation | The frequency of<br>incorrect answers<br>(percentage) | of empty<br>answers<br>(percentage) |
| 6 <sup>th</sup> | 4(13%)   | 1  | 20 (65%)  | 6                                   |
| 7 <sup>th</sup> | 7(18%)   | 3  | 20(57%)   | 5                                   |
| 8 <sup>th</sup> | 12(27%)  | 1  | 29(64%)   | 3                                   |
| Total           | 23(21%)  | 5 (4%)   | <b>69 (62%)</b>                                       | 14(13%)                             |

According to the table above, it was seen that few students (only 21% of all students) gave correct answer for this question. Eighth grade students were more successful than 6<sup>th</sup> and 7<sup>th</sup> grade students. However, more than half of the students from all grade levels could not give correct answer although this objective was addressed in all grade levels.

The analysis of the most common incorrect answers showed that students tended to give the answer "the cube with number 3". All the students who gave this answer explained their reasoning as "the cube with number 3 is in the middle of the structure, so the cube with the number 3 does not affect the surface area of the structure". The frequency of the most common incorrect answer for this question is in the table 4.28.

Table 4.28: The frequency of the most common incorrect answer for the  $2^{nd}$  question in the conceptual test

| Response type   | Frequency of the<br>6 <sup>th</sup> grade students<br>for the response | Frequency of the<br>7 <sup>th</sup> grade students<br>for the response | Frequency of the 8 <sup>th</sup> grade students for the response |
|---|--|--|--|
| Explanation: Cube<br>3. Because, cube 3<br>is in the middle so<br>it does not affect<br>the surface area. | 15   | 12   | 24   |

Beside this most common incorrect response, eight of the 7<sup>th</sup> grade students and one of the 8<sup>th</sup> grade students claimed that taking away all of the cubes change the surface area of the structure since lack of a piece of surface affected the surface area.

The objective "to decide which kind of information will be used to cover a given box" was evaluated in the  $3^{rd}$  question. The  $3^{rd}$  question is given in Figure 4.17 below:

| Question 3: There is a present box below. It will be covered by present packet. |                             |                         |  |  |  |  |  |
|---|-----------------------------|-------------------------|--|--|--|--|--|
| Which kind of information   | should be known to find the | amount of packet?       |  |  |  |  |  |
|   |                             |                         |  |  |  |  |  |
| A.The sum of the sides of   | B.The surface area of the   | C.The volume of the box |  |  |  |  |  |
| the box   | box                         | D                       |  |  |  |  |  |
|   |                             | Because:                |  |  |  |  |  |
| Because;  | Because;                    |                         |  |  |  |  |  |
|   |                             |                         |  |  |  |  |  |
|   |                             |                         |  |  |  |  |  |

Figure 4.17: The 3<sup>rd</sup> question in the conceptual test

In the 3<sup>rd</sup> question in the conceptual test, the students were expected to choose the correct answer and explain the reasoning for the answer. Students' answers were analyzed in terms of the frequency of correct answer, explanation of the correct answer, the possible combination of incorrect answers, and empty answers with respect to the grade levels. The frequency of correct, incorrect, and empty answers is given in Table 4.29.

|                 | The frequer | ncy of right | The frequ | iency of |               |
|-----------------|-------------|--------------|-----------|----------|---------------|
|                 | answers (p  | ercentage)   | incorrect | answers  | The frequency |
| Grade level     | (choose su  | rface area)  | (perce    | ntage)   | of empty      |
|                 | With right  | No           | Chose     | Choose   | answers       |
|                 | explanation | explanation  | sum up    | volume   | (percentage)  |
|                 | 1           | 1            | the sides |          |               |
| 6 <sup>th</sup> | 19(61%)     | 1(3%)        | 5         | 5        | 1             |
| 7 <sup>th</sup> | 24(69%)     | 0            | 0         | 1        | 10            |
| 8 <sup>th</sup> | 32(71%)     | 1(2%)        | 7         | 4        | 1             |
| Total           | 74 (67%)    | 2 (2%)       | 12(11%)   | 10(9%)   | 12(11%)       |

Table 4.29: The frequency of students' answers for the third question in the conceptual test with respect to the different grade levels

According to the above table, about 69% of the students gave correct answer for the question. However, two of the students who gave the right answer could not explain their reasoning. When examined according to the grade levels, more  $8^{th}$  grade students gave the right answer compared to the  $6^{th}$  and  $7^{th}$  grade students. It was seen that the success rate of the  $6^{th}$  grade students was almost close to the  $7^{th}$  grade students.

#### 4.2.3.3. Area and Perimeter

The objective "to interpret the area and circumference of the figure which is obtained by breaking the first figure into pieces and then bringing them together in a different combination" was evaluated in the 4<sup>th</sup> question. The 4<sup>th</sup> question is given in Figure 4.18 below:



Figure 4.18: The 4<sup>th</sup> question in the conceptual knowledge test

In the 4<sup>th</sup> question, students were expected to provide the correct answers for area and perimeter of the figure and then explain their reasoning. Therefore, the frequency of correct answers, empty answers, and possible combination of incorrect answers were calculated with respect to grade levels. Moreover, the most common wrong explanations were emphasized. The frequency of correct, incorrect, and empty answers is given in Table 4.30.

|       | Grade<br>level  | The frequency of right<br>answers (percentage)<br>With<br>explanation(<br>A) No/wrong<br>explanation(A) |               | The frequency of<br>incorrect answers<br>(percentage)BC |         | The F<br>of<br>empty<br>answers<br>(%) |            |        |
|-------|-----------------|---|---------------|---|---------|--|------------|--------|
|       | 6 <sup>th</sup> | 4(13%)  | 4(13%) 6(19%) |   | 1(3%)   | 20(65%                                 | <b>b</b> ) |        |
| ter   | Total           | 10(32%)   |               | 21(68%)   | )       |  | 0          |        |
| ime   | 7 <sup>th</sup> | 8(23%)  | 2(            | 6%)   | 1(3%)   | 22(63%                                 | <b>5</b> ) |        |
| Per   | Total           | 10(29%)   |               | 23(66%)   |         | 2(6%)                                  |            |        |
|       | 8 <sup>th</sup> | 16(36%)   | 3(            | 7%)   | 4(9%)   | 20(44%                                 | <b>b</b> ) |        |
|       | Total           | 19(43%)   |               | 24(53%)   |         |  | 2(4%)      |        |
| Total | I               | 39(35%)   |               |   | 68(61%) | )                                      |            | 4(4%)  |
|       |                 | С   |               | С   | Α       | B                                      |            |        |
|       | 6 <sup>th</sup> | 15(48%)   | 0             |   | 8(26%)  | 6(19%)                                 | )          | 2(6%)  |
|       |                 | 15(48%)   | 1             |   | 14(45%) |  |            |        |
| Area  | 7 <sup>th</sup> | 21(60%)   |               | 1(3%)   | 7(20%)  | 1(3%                                   | )          | 5(14%) |
|       | _               | 22(63%)   |               | I   | 8(23%)  |  |            |        |
|       | 8 <sup>th</sup> | 33(73%)   |               | 0   | 5(11%)  | 4(9%                                   | )          | 3(7%)  |
|       |                 | 33(73%)   |               | 9(20%)  |         |  |            |        |
| Total | 1               | 70(63%)   |               |   | 31(28%) |  |            | 10(9%) |

Table 4.30: The frequency of students' answers for the fourth question in the conceptual test with respect to the different grade levels

F: Frequency

This question was examined in two parts. In the first part, the perimeter of the geometric figure and in the second part the area of the geometric figure was interpreted. According to the table, only 35% of the all students gave correct answer for interpreting the perimeter. The rate of success increased to 63% when the area was in question. This showed that students' knowledge about perimeter might be weaker than their knowledge about area. In both parts, 8<sup>th</sup> grade students performed better than the 7<sup>th</sup> and 6<sup>th</sup> grade students. However, while 7<sup>th</sup> graders performed better than 6<sup>th</sup> graders in the area part, they showed lower performance in the perimeter part. This might be a result of an emphasis in the perimeter concept in the 6<sup>th</sup> grade and perimeter of polygons." The perimeter concept was mentioned in the 7<sup>th</sup> grade mathematics curriculum as integrated to area concept.

When the explanations of right answers were examined, it was seen that there were students who gave right answer but they could not explain their reasoning correctly. For example, 5 students from 6<sup>th</sup> grade, 2 students from 7<sup>th</sup> grade, and 2 students from 8<sup>th</sup> grade explained their reasoning as "The perimeter of figure 4 is bigger than the perimeter of figure 1 because there are more geometric figures in figure 4". In addition to this explanation, 2 students from 7<sup>th</sup> grade and 3 students from 8<sup>th</sup> grade explained their reasoning as "The perimeter of parallelogram is always bigger than the perimeter of a square". Moreover, most 8<sup>th</sup> grade students explained their reasoning as "In a right triangle, hypotenuse is always longer than the other sides". It was not unexpected that 8<sup>th</sup> grade students explained their answers with the hypotenuse concept since they have learned this concept recently.

The most common wrong explanation for the incorrect answers was analyzed. The frequency of the most common incorrect answer for the perimeter part of the  $4^{th}$ question is in the table 4.31.

Table 4.31: The frequency of the most common incorrect answer for the perimeter part of the 4th question in the conceptual test

| Response type                        | Frequency of the               | Frequency of the               | Frequency of the               |
|--------------------------------------|--------------------------------|--------------------------------|--------------------------------|
|                                      | 6 <sup>th</sup> grade students | 7 <sup>th</sup> grade students | 8 <sup>th</sup> grade students |
|                                      | for the response               | for the response               | for the response               |
| "Since the shape<br>does not change" | 13(42%)                        | 12(34%)                        | 12(27%)                        |

When the area part was examined, it was realized that more students gave correct answers for area task compared to the perimeter task. Although there were not many students who gave the same incorrect explanation for the incorrect answer, it was seen that one student from each grade level explained their reasoning as "The area of parallelograms are bigger than the area of squares."

An objective similar to the objective in the question 4, "to identify the changes in area and perimeter of two figures in different combinations," was evaluated in the  $6^{th}$  question. The  $6^{th}$  question is given in Figure 4.19 below:



Figure 4.19: The sixth question in the conceptual test

In the sixth question, students were expected to compare the area and perimeter of the different combinations of the two identical figures and answer the questions related to this comparison. Students' answers were analyzed in terms of the frequency of correct answer, the possible combination of incorrect answers, and empty answers with respect to the grade levels. The frequency of correct, incorrect, and empty answers is given in Table 4.32.

|           | Grade<br>level  | The frequency of<br>right answers<br>(percentage) | The frequency of<br>wrong answers<br>(percentage) | The frequency of<br>empty answers<br>(percentage) |  |
|-----------|-----------------|---|---|---|--|
|           | $6^{th}$        | 20(65%)   | 7   | 3   |  |
| Perimeter | 7 <sup>th</sup> | 28(80%)   | 1   | 6   |  |
|           | 8 <sup>th</sup> | 8 <sup>th</sup> 34(76%) 11                        |   | 1   |  |
| Total     |                 | 82(74%)   | <b>19(17</b> %)                                   | 10(9%)  |  |
|           | 6 <sup>th</sup> | 9(29%)  | 15  | 7   |  |
| Area      | 7 <sup>th</sup> | 12(34%)   | 11  | 11  |  |
|           | 8 <sup>th</sup> | 26(58%)   | 11  | 8   |  |
| Total     |                 | 47(42%)   | 37(33%)   | 26(25%)   |  |

Table 4.32: The frequency of students' answers for the sixth question in the conceptual test with respect to the different grade levels

According to the given table above, similar result was seen as in the  $4^{th}$  question. Eighth graders performed better than the  $6^{th}$  and  $7^{th}$  graders. Seventh graders performed better than the  $6^{th}$  graders in both perimeter and area tasks. However, as in the fourth question, when examined totally, students showed lower

performance in the area part. Although there was a not common incorrect answer for this question, one interesting answer from  $6^{th}$  grade student was that he tried to complete the figures to a regular shape. Moreover, one of the  $8^{th}$  grade students claimed that this question could not be solved without any numerical information.

To sum up, similar objectives were examined in questions 4and 6. How perimeter and area changed when identical figures or the part of a figure reformed was asked. It was found that for this type of tasks, 8<sup>th</sup> graders were more successful than the other grade levels. In general, students from all grade levels showed low performance in the area concept.

The objective "to relate dimensions of a rectangle with its area" was evaluated in the  $5^{\text{th}}$  question. The  $5^{\text{th}}$  question is given in Figure 4.20.

*Question 5:* If you increase the edges of a rectangle to 4 times of its current length, how will its area change?

- A. The area becomes 4 times of the first rectangle. Because,....
- B. The area does not change.Because,.....
- C. The area changes, but it does not become 4 times of the first rectangle. Because,....

Figure 4.20: The fifth question in the conceptual test

In the 5<sup>th</sup> question, students were expected to realize in which rate the area of rectangle changed according to the changes in the length of its sides. They were expected to choose the right answer and then explain their reasoning. Students' answers were analyzed in terms of the frequency of correct answer, the possible

combination of incorrect answers and empty answers with respect to the grade levels. The frequency of correct, incorrect, and empty answers is given in Table 4.33.

|                 | The frequency of<br>right answers<br>(percentage) | The frequ<br>wrong a<br>(perce | uency of<br>unswers<br>ntage) | The frequency of empty answers |  |
|-----------------|---|--------------------------------|-------------------------------|--------------------------------|--|
|                 | (C)   | А                              | В                             | (percentage)                   |  |
| 6 <sup>th</sup> | 12(39%)   | 16                             | 1                             | 2                              |  |
|                 |   | 17(55%)                        |                               |                                |  |
| 7 <sup>th</sup> | 16(46%)   | 13                             | 2                             | 4                              |  |
|                 |   | 15( <b>43%</b> )               | l                             |                                |  |
| 8 <sup>th</sup> | 30(67%)   | 13                             | 1                             | 2                              |  |
|                 |   | 14(31%)                        |                               |                                |  |
| Total           | 58(52%)   | 42(38%)                        | 4(4%)                         | 8(6%)                          |  |

Table 4.33: The frequency of students' answers for the fifth question in the conceptual test with respect to the different grade levels

According to the table, only about half of the students gave correct answers for this question although this kind of questions could be seen commonly in elementary mathematics curriculum materials. Eighth grade students performed higher than the 6<sup>th</sup> and 7<sup>th</sup> grade students in this task. Moreover, the rate of correct answer for these questionsfor6<sup>th</sup> graders was close to the rate of correct answers for this question for 7<sup>th</sup> graders. When the correct answers were analyzed, it was seen that two 7<sup>th</sup> grade students and 8<sup>th</sup> grade students provided numbers to calculate the rate of change. In the answers of students who chose A (becomes 4 times of the first rectangle)as the right answer, the same common answer emerged. They explained their reasoning as "The rate of increase in the area is equal to the rate of increase in the length of sides."

The objective "to choose the suitable figure with the perimeter with a given length" was evaluated in the 7<sup>th</sup> question. The 7<sup>th</sup> question is given in Figure 4.21.

*Question 7:* A farmer has 32 -meter wire. He wants to enclose a garden with one row of wire. Decide whether the wire is sufficient or not for the below gardens which have different shapes.



Figure 4.21: The seventh question in the conceptual test

In the 7<sup>th</sup> question, the students were expected to calculate or predict (for B) the perimeter of the given figures and then decide for which ones the wire was sufficient. For this question, the answers were analyzed under the frequency of correct answer, incorrect answers, and empty answers for the parts separately with respect to the grade levels. The frequency of correct, incorrect, and empty answers is given in Table 4.34.

|                 | The fr<br>answer | The frequency of right<br>answers (percentage)The frequency of<br>wrong answers<br>(percentage)The frequency<br>empty<br> |             |             |    |    |    | The frequency of<br>wrong answers<br>(percentage) |    |    | ency<br>an | of<br>swers |
|-----------------|------------------|---|-------------|-------------|----|----|----|---|----|----|------------|-------------|
|                 | F1               | F2  | F3          | F4          | F1 | F2 | F3 | F4  | F1 | F2 | F3         | F4          |
| 6 <sup>th</sup> | 23(74<br>%)      | 8(26<br>%)  | 20(65<br>%) | 26(84<br>%) | 6  | 18 | 7  | 2   | 2  | 5  | 4          | 3           |
| 7 <sup>th</sup> | 15(43<br>%)      | 15(43<br>%)   | 11(32<br>%) | 24(69<br>%) | 9  | 15 | 16 | 5   | 11 | 5  | 8          | 6           |
| 8 <sup>th</sup> | 27(60<br>%)      | 19(42<br>%)   | 22(49<br>%) | 34(76<br>%) | 12 | 16 | 15 | 6   | 6  | 10 | 8          | 5           |

Table 4.34: The frequency of students' answers for the seventh question in the conceptual test

In the seventh question, students were expected to decide if the wire was sufficient for the perimeter of each geometric shape. For the first (irregular shape), third(irregular shape) and the fourth(rectangle)figure, the rate of correct answers of  $6^{th}$  grade students was higher than the rates of  $7^{th}$  and  $8^{th}$  grade students' correct answers. However, for the second figure (parallelogram), it was realized that even less than half of the students could not give correct answer. While the rate of correct answers of  $7^{th}$  grade students was close to the rate of correct answers of  $8^{th}$  grade students,  $6^{th}$  grade students showed lowest performance in this task.

49(42

%)

38(34

%)

65(59

%)

Total

42(38

%)

53(48

%)

84(76

%)

27(22

%)

13(11

%)

19(17

%)

20(18

%)

20(18

%)

14(13

%)

Certain common wrong explanations for students' answers were detected. Some students provided their answers based on the appearance of the figures. They said that since the figures (figure 1 and 3) were irregular, their perimeters were bigger than the others (figure 2 and 4) which were more regular. The frequency of the most common incorrect answer for this question is in the table 4.35.

| Response type                                    | Frequency of the 6 <sup>th</sup> grade students for the response | Frequency of the 7 <sup>th</sup> grade students for the response | Frequency of the 8 <sup>th</sup> grade students for the response |
|--|--|--|--|
| "Since the figures<br>are washboard or<br>plane" | 2  | 7  | 5  |

Table 4.35: The frequency of the most common incorrect answer for the 7<sup>th</sup> question in the conceptual test

The objective "to compare the perimeters and areas of two shapes drawn on dot paper" was evaluated in the 8<sup>th</sup> question. The 8<sup>th</sup> question is given in Figure 4.22.

Figure 4.22: The eighth question in the conceptual test
In the 8<sup>th</sup> question, students were expected to find the area and perimeter of given figures and to realize that different figures could have the same area, same perimeter, or not. Students' responses were analyzed under the frequency of correct answer, incorrect answers and empty answers for the parts (area and perimeter) separately with respect to the grade levels. Moreover, their explanations were examined. The frequency of correct, incorrect, and empty answers is given in Table 4.36.

|           | <b>a</b> 1      | The frequency of              | The frequency of           | The frequency       |
|-----------|-----------------|-------------------------------|----------------------------|---------------------|
|           | Grade<br>level  | right answers<br>(percentage) | wrong answers (percentage) | of empty<br>answers |
|           | 10,01           | (proceedings)                 |                            | (percentage)        |
|           | 6 <sup>th</sup> | 23(74%)                       | 7                          | 1                   |
| Area      | 7 <sup>th</sup> | 31(89%)                       | 1                          | 3                   |
|           | 8 <sup>th</sup> | 41(91%)                       | 4                          | 0                   |
| Total     |                 | 95(86%)                       | 12(11%)                    | 4(3%)               |
|           | 6 <sup>th</sup> | 14(45%)                       | 11                         | 5                   |
| Perimeter | 7 <sup>th</sup> | 17(49%)                       | 13                         | 5                   |
|           | 8 <sup>th</sup> | 31(69%)                       | 13                         | 1                   |
| Total     |                 | 62(56%)                       | 37(34%)                    | 11(10%)             |

Table 4.36: The frequency of students' answers for the eighth question in the conceptual test with respect to the different grade levels

In this question, it was investigated that students had strong conceptual knowledge to realize the area of a given figure on a dot paper. Most of the students (about 95%) could give correct answer for the area part. However, in the perimeter part, the rate of correct answer decreased to 56%. It was seen that the rate of  $8^{th}$  grade students' correct answers were more than  $7^{th}$  and  $6^{th}$  grade students' rate of correct answers were more than  $7^{th}$  and  $6^{th}$  grade students' rate of correct answers. When students' answers were examined in detail, certain conceptions of area and perimeter appeared. A  $6^{th}$  grade-student and an  $8^{th}$  grade-student said that "If the area of given figures are different, then their perimeters are

also different." Moreover, one student from each grade level said that "Since the figures are different, the area/perimeter must be different."

The objective "to identify the different shapes with the same perimeter" was evaluated in the  $10^{\text{th}}$  question. The  $10^{\text{th}}$  question is given in Figure 4.23.



Figure 4.23: The tenth question in the conceptual test

In the 10<sup>th</sup> question, students were expected to choose the correct answer from the given statements and then explain their reasoning. They were supposed to realize that different shapes may have the same perimeter. The answers were examined under the frequency of correct answers, incorrect answers and empty answers. The explanations from incorrect answers were also investigated. The frequency of correct, incorrect, and empty answers is given in Table 4.37.

| Total           | 44(4        | 0%)                    | 5  | 9(53%            | <b>)</b> | 8(7%)         |
|-----------------|-------------|------------------------|----|------------------|----------|---------------|
| Total           | 40(36%)     | 4(4%)                  | 21 | 9                | 29       | 8             |
| 8 <sup>th</sup> | 15(33%)     | 2(5%)                  | 6  | 5                | 14       | 3             |
| 7 <sup>th</sup> | 15(43%)     | 1(4%)                  | 4  | 2                | 9        | 4             |
| 6 <sup>th</sup> | 10(33%)     | 1(3%)                  | 11 | 2                | 6        | 1             |
|                 | explanation | explanation            |    |                  |          |               |
|                 | With right  | With right No          |    | C                | D        | (percentage)  |
|                 | (1          | (B)                    |    | (percentage)     |          | answers       |
|                 | answers (p  | answers (percentage)   |    | wrong answers    |          | of empty      |
| Grade level     | The freque  | The frequency of right |    | The frequency of |          | The frequency |

Table 4.37: The frequency of students' answers for the tenth question in the conceptual test with respect to the different grade levels

According to the table above, it was examined that only 40% of the students gave correct answers and sufficient explanation. In this question, the rate of correct answers of  $7^{\text{th}}$  grade students was higher than the rate of correct answers of the  $6^{\text{th}}$  and  $8^{\text{th}}$  grade students. Moreover, the incorrect answers were examined and common incorrect explanations were analyzed. The frequency and the response type of students who chose A (Garden D has the biggest perimeter) as the right answer is shown table 4.38 below:

Table 4.38: The frequency of the most common incorrect explanation for choice A in the 10th question in the conceptual test

| Response type  | Frequency of the               | Frequency of the               | Frequency of the               |  |
|--|--------------------------------|--------------------------------|--------------------------------|--|
|  | 6 <sup>th</sup> grade students | 7 <sup>th</sup> grade students | 8 <sup>th</sup> grade students |  |
|  | for the response               | for the response               | for the response               |  |
| "Because the<br>appearance of<br>garden D seems<br>big". | 8                              | 4                              | 12                             |  |

It seemed that the  $8^{th}$  grade students gave importance to the appearance of the figures much more than the  $6^{th}$  and  $8^{th}$  grade students.

Students who chose D (The perimeter of garden C is bigger than the perimeter of garden A) as the correct answer explained their reasoning as "C is more regular than the A and the others." Furthermore, one student from 7<sup>th</sup> grade and one student from 8<sup>th</sup> grade compared the shapes by completing all the shapes to a square or rectangle.

#### 4.3. Summary of the results about conceptual knowledge test

In the conceptual test, the students' conceptual knowledge of area and perimeter of geometric figures were examined through different objectives. According to the results; students had conceptual difficulties both in area and perimeter tasks. While they performed better in some tasks such as predicting and comparing the area of given figures, they had low performance in tasks related to analyzing different geometric figures with the same perimeter. Seventh and 8<sup>th</sup> grade students performed better than the 6<sup>th</sup> grade students in comparison and prediction tasks.

Students' conceptual knowledge was stronger in area when they were expected to interpret the area and perimeter of the shapes which are decomposed and formed differently. On the other hand, when they were expected to interpret combined figures, they performed better in interpreting the perimeter. Students' conceptual knowledge of area was also stronger compared to their conceptual knowledge of perimeter when they were analyzing a figure on a dot paper.

The analysis indicated that students interpreted and compared the area and the perimeter of the figures by considering their appearance. According to them, the perimeter of the irregular shapes figures was bigger than the plane ones. It seemed that students' conceptual knowledge was affected by the visual evidences they had about the geometric figures. Furthermore, it was examined that students from all grade levels had difficulties in tasks related to the perimeter of the parallelogram.

Students had lower performance in relating the length of the edges and the area of the given rectangle and realizing the different shapes with the same

perimeter. These kinds of tasks were completed correctly by only less than half of the students.

# 4.4. Analysis of Private middle school Students' Self-Efficacy towards Geometry **Test Scores**

In this section descriptive statistics on scores of self-efficacy towards geometry test (STG) was given. Then, the mean score of self-efficacy towards geometry test were compared according to the grade levels.

# 4.4.1. Descriptive Statistics about Self Efficacy towards Geometry Test

In this section, descriptive statistics about self-efficacy towards geometry test about area and perimeter of geometric shapes were presented. Table 4.39 illustrates descriptive statistics of three grade level students' scores of self-efficacy towards geometry test.

| Grade Level           | Ν   | Max | Min | Mean  | SD    |
|-----------------------|-----|-----|-----|-------|-------|
| 6 <sup>th</sup> grade | 31  | 119 | 41  | 97,90 | 17,18 |
| 7 <sup>th</sup> grade | 35  | 124 | 59  | 95,57 | 18,94 |
| 8 <sup>th</sup> grade | 45  | 120 | 63  | 89,27 | 15,80 |
| Total                 | 111 | 124 | 41  | 93,67 | 17,48 |

Table 4.39: Descriptive Statistics of Private middle school Students' Scores of Self Efficacy towards Geometry (Out of 125)

Table 4.39 is an overall summary of the descriptive statistics gathered from the self-efficacy towards geometry test scores from the  $6^{th}$ ,  $7^{th}$  and  $8^{th}$  grade students. As shown in the table, the mean score of  $6^{th}$  grade students was 97, 90 (SD= 17, 18), the mean score of  $7^{\text{th}}$  grade students was 95, 57 (SD= 18, 94), and the mean score of 93

 $8^{th}$ grade students was 89, 27 (*SD*= 15, 80) while the mean score of all total was 93, 67 (*SD*= 17, 47) in the self-efficacy towards geometry test. The self-efficacy towards geometry test scores of the  $6^{th}$  and  $7^{th}$  grade students was above the mean of all students' scores, the self-efficacy towards geometry test scores of the  $8^{th}$  grade students was under the mean of all students' scores. Moreover, it can be seen that the self-efficacy towards geometry test scores of  $6^{th}$  grade students was higher than the self-efficacy towards geometry test scores of  $7^{th}$  grade students and the self-efficacy towards geometry test scores of  $7^{th}$  grade students and the self-efficacy towards geometry test scores of  $7^{th}$  grade students was higher than the self-efficacy towards geometry test scores of  $7^{th}$  grade students was higher than the self-efficacy towards geometry test scores of  $7^{th}$  grade students was higher than the self-efficacy towards geometry test scores of  $7^{th}$  grade students.

# 4.4.2. The Difference in Geometry Self-Efficacy of Private middle school Students in terms of Grade Level

One of the aims of the present study was to examine how geometry selfefficacy of private middle school grade students changed according to the grade level. This research question also required a comparison of means and look for whether there was a significant mean difference between the scores of the groups as in the PCK and CKT scores. Therefore, in order to examine the mean difference of self-efficacy towards geometry test scores in terms of grade levels, one-way ANOVA was used. Prior to running the analysis, the assumptions of one-way ANOVA were checked for geometry test scores. In the next sections the assumptions and analysis of results were summarized.

# 4.4.2.1. Assumptions of one way ANOVA

The assumptions of one-way ANOVA were level of measurement, independence of observations, normality and homogeneity of variance Pallant (2007). According to the geometry self-efficacy test results, they were summarized below.

The variables for one-way ANOVA were mean scores of self-efficacy towards geometry test of  $6^{th}$ ,  $7^{th}$  and  $8^{th}$  grade students which were continuous variable. Hence level of measurement assumption was ensured.

Each measurement of grade levels was not influenced by each other, since students in each grade level took the tests in different places and their scores did not influence each other. Therefore, level of measurement assumption was ensured.

The other assumption to be considered before one-way ANOVA was normality. For parametric techniques, mean scores for each variable should be normally distributed (Pallant, 2007). Since the sample sizes for all groups was large enough (e.g. 30+), this assumption ought not to cause major problems. In order to check normality of self-efficacy towards geometry test scores for each group histograms were checked.

In addition to this, skewness and kurtosis values were checked. Since these values were between -2 and +2 (Pallant, 2007), the normality of three groups' scores was assured. Only the kurtosis value for  $6^{th}$  grade students' test scores was above the +2; however this was ignorable since the other values for normality was acceptable. The values are given in the table 4.40 below.

 Table 4.40: Skewness and Kurtosis Values of Self-Efficacy towards Geometry Test

 Scores for each grade level

| Grade level           | Skewness | Kurtosis | Number |  |
|-----------------------|----------|----------|--------|--|
| 6 <sup>th</sup> grade | -1.493   | 3.026    | 31     |  |
| 7 <sup>th</sup> grade | 284      | 891      | 35     |  |
| 8 <sup>th</sup> grade | 091      | -1.144   | 45     |  |

For homogeneity of variance assumption, the variability of all groups should be similar addressing that the samples were obtained from populations of equal variances. In this analysis, Levene's Test of Equality of Error Variances showed that the homogeneity of variance assumption was assured (p=.39).

#### 4.4.2.2. Results of one-way ANOVA for Self-Efficacy toward Geometry Test

According to the one-way ANOVA results for geometry self-efficacy test revealed that there was no statistically significant difference (F(2, 108) = 2.62, p > .05) among three grade level in terms of geometry self-efficacy test scores. Although there are mean difference in the scores, this mean difference is not statistically significant. The STG test scores for each grade level were given in the figure 4.24below.



Figure 4.24: STG Test Scores for the 6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> grade students

# 4.5. Correlation among Geometry Self-Efficacy, Procedural Knowledge and Conceptual Knowledge

One specific aim of the present study was to investigate if there was a relationship among the scores of PKT, CKT and STG test scores. In order to examine the strength and the direction of the relationship among these variables Pearson-Product Moment was calculated. Before conducting the correlation analysis, the assumptions for the analysis were checked.

### 4.5.1. Assumptions of Pearson-Product Moment Correlation

The assumptions of Pearson-Product Moment Correlation were level of measurement, related pairs, independence of observations, normality and homogeneity of variance (Pallant, 2007).

In the present study, the dependent variables were PKT, CKT and STG tests scores which were continuous variables. Therefore, level of measurement assumption was assured.

It was important to satisfy related pairs assumption that all subjects had a score on the variables. That assumption was also established since in the present study all participants had the scores of PCK, CKT and STG test scores.

Independence of observations assumptions were assured since the each measurement was not influenced by another. In addition, sample size (N= 111) was appropriate to assure normality assumption of the study.

In order to check normality of procedural test, conceptual test and geometry self-efficacy test scores, histograms, normal Q-Q plots, and skewness and kurtosis values were examined. The shape of the graphs showed that variables were normally distributed. Only the histogram for procedural test score did not seem to be normal, however the other values showed that the distribution was normal. The skewness and kurtosis values of PKT, CKT and STG test scores for each group is summarized in the Table 4.41.

|     | Skewness | Kurtosis | Ν   |     |
|-----|----------|----------|-----|-----|
| РКТ | . 15     | 62       | 111 |     |
| СКТ | 38       | -1.13    |     | 111 |
| STG | 44       | 40       | 111 |     |

Table 4.41: Skewness and Kurtosis Values of PKT, CKT and STG Test Scores

As indicated in the Table 4.41, the skewness and kurtosis values were placed in the acceptable range. Therefore, there was no violation for the normality assumption.

Another assumption for the correlation studies was linearity assumption that the relationship between variables should be linear (Pallant, 2007). In order to examine the linearity, scatter plots for PKT, CKT and STG test mean scores were constructed. In the figures 4.25, 4.26 and 4.27 the relationships between PKT and STG test scores, CKT and STG test scores and PKT and CKT scores were shown respectively.



Figure 4.25: Scatter plot of PKT Mean Scores and STG Test Mean Scores

The spread of the points in the scatter plot indicated that there was a reasonable correlation between PKT and STG Test mean scores. The fit line could be drawn, so the Pearson correlation could be used. Therefore, the assumption was satisfied. The direction of relationship was positive since the line drawn through points upward from left to right. That is, high scores in PKT scores were associated with high scores in STG test scores. As a result, when geometry self-efficacy increased, the scores in procedural knowledge test increased and the relationship seemed to be weak.



Figure 4.26: Scatter plot of CKT Mean Scores and STG Test Mean Scores

The spread of the points in the scatter plot indicated that there was a reasonable correlation between CKT and STG Test mean scores. The fit line could be drawn, so the Pearson correlation could be used. The direction of relationship was positive since the line drawn through points upward from left to right. That is, high scores in CKT scores were associated with high scores in STG test scores. As a result, when geometry self-efficacy increased, the scores in conceptual knowledge test increased and the relationship seemed to be weak.



Figure 4. 27: Scatter plot of PKT Mean Scores and CKT Mean Scores

The spread of the points in the scatter plot indicated that there was a reasonable correlation between PKT and CKT mean scores. The fit line could be drawn, so the Pearson correlation could be used. The direction of relationship was positive since the line drawn through points upward from left to right. Thus, the linearity assumption was assured with fit lines of scatter plots. That is, high scores in PKT scores were associated with high scores in CKT test scores. As a result, when procedural knowledge increased, the scores in conceptual knowledge increased and the relationship seemed to be moderate.

When scatter plots above were examined, the homoscedasticity assumption was assured in Figure 4.25, Figure 4.26andFigure 4.27 since there was a fairly cigar shape in these figures. Therefore, the Pearson product-moment coefficient was used to investigate the correlations between PKT scores and STG test scores, between CKT scores and STG test scores, and between PKT scores and CKT scores.

#### 4.5.2. Correlation between Geometry Self-Efficacy and Procedural Knowledge

Preliminary analyses were performed to ensure no violation of the assumptions of normality, linearity, and homoscedasticity. Results indicated that there was a significant positive relationship between geometry self-efficacy and procedural knowledge test scores, r = .435, p < .01. This means that higher levels of geometry self-efficacy of private middle school students were associated with higher scores in the PKT. The strength of the relationship was considered as medium (Cohen, 1988). The coefficient of determination was calculated as .19 which meant there were 19 percent shared variance between PKT and STG test scores.

#### 4.5.3. Correlation between Geometry Self-Efficacy and Conceptual Knowledge

Preliminary analyses were performed to ensure no violation of the assumptions of normality, linearity, and homoscedasticity. Results indicated that there was a significant positive relationship between geometry self-efficacy and conceptual knowledge test scores, r(109)= .366, p< .01. This meant that higher levels of geometry self-efficacy of private middle school students were associated with higher scores in the CKT. The strength of the relationship was considered as medium (Cohen, 1988). The coefficient of determination was calculated as .13 which meant there were 13 percent shared variance between CKT and STG test scores.

#### 4.5.4. Correlation between Procedural Knowledge and Conceptual Knowledge

Preliminary analyses were performed to ensure no violation of the assumptions of normality, linearity, and homoscedasticity. Results indicated that there was a significant positive relationship between conceptual knowledge test scores and procedural knowledge test scores, r(109)=.697, p<.01. This meant that higher scores in CKT were associated with higher scores in the PKT. The strength of the relationship was considered as large (Cohen, 1988). The coefficient of determination was calculated as .48 which meant there were 48 percent shared variance between PKT and CKT test scores.

## 4.5.5. Summary of the Inferential Statistics

One of the purposes of the study was to investigate how procedural knowledge and conceptual knowledge of private middle school students about area and perimeter of geometric figures and their geometry self-efficacies change in terms of grade level. For this aim, one- way ANOVAs were conducted. According to the results, there were no statistically significant difference in private middle school students' procedural knowledge and self-efficacies in terms of grade levels. However, there was a significant difference in conceptual knowledge of private middle school students.

The other specific aim of the study was to investigate the relationships among private middle school students' procedural knowledge, conceptual knowledge and their geometry self-efficacies. For this aim, Pearson-Product Moment Correlation was calculated. According to the results, there was significant correlation between students' procedural knowledge and conceptual knowledge and geometry selfefficacies.

## **CHAPTER 5**

## DISCUSSION

The purpose of the present study was to investigate private middle school students' procedural and conceptual knowledge in the domain of area and perimeter of geometric figures and to examine most common errors in procedural and conceptual knowledge tasks. The other specific interest of the study was to examine how students' conceptual and procedural knowledge aspects of area and perimeter of geometric figures changed with respect to their geometry self-efficacy.

This chapter consists of the summary of the results and the discussion of the findings of the study. Moreover, recommendations and implications for the future studies will be presented.

# 5.1. Students' Procedural Knowledge about Area and Perimeter of Geometric Shapes

The descriptive statistics for procedural knowledge test scores of elementary students showed that the mean of the test scores was not very high. Although 8<sup>th</sup> grade students' mean scores was higher than 7<sup>th</sup> grade students' mean scores and 7<sup>th</sup> grade students' mean scores on the test was higher than the 6<sup>th</sup> grade students' mean scores, when their mean scores were investigated separately, it was seen that the mean scores were below 65out of 100 for all grade levels. These results showed that students' procedural knowledge on area and perimeter was not sufficient. Area and perimeter are considered to be among the mathematics topics that students have difficulty. Related literature shows that most of the students have difficulty in understanding geometry and measurement concepts and particularly area and perimeter (Barret& Clement, 2003; Martin & Kenney, 2002; Walter, 1970).

The results of the one-way ANOVA have revealed that there was no significant mean difference among three grade levels ( $6^{th}$ ,  $7^{th}$  and  $8^{th}$  grades) in terms

of procedural knowledge test scores. Hiebert and Levefre (1986) defined procedural knowledge as being aware of the mathematical rules and usage of algorithms and rules. For this study it was aimed to examine the usage of the rules which helps to calculate area and perimeter of geometric shapes. The tests used in the present study were developed in line with the common objectives of 6<sup>th</sup> and 7<sup>th</sup> grade mathematics curriculum and 8<sup>th</sup> grades students were assumed to have already achieved the objectives. The results showed that all students from different grade levels might have almost the same procedural knowledge. Their procedural knowledge about area and perimeter concepts did not change according to grade level. Since the questions were related to the common objectives in three grade levels, students might have practiced with the similar algorithms and rules in all grades. Moreover, since these topics are related to several mathematics topics (Clements & Battista, 2001; NCTM, 2000), students might have familiarity with these algorithms and rules in all grade levels.

Students were successful when they calculated the area and perimeter of a square, rectangle, and parallelogram in general. Moreover, in both area and perimeter,  $6^{th}$  grade students had lower performance while  $8^{th}$  grade students had highest performance. As mentioned above, this result could be due to the situation that  $8^{th}$ grade students studied these topics more than  $6^{th}$  and  $7^{th}$  grade students. However, in certain kind of questions, for example area of a parallelogram and triangles,  $6^{th}$  grade students were more successful. This might be resulted from the fact that objectives related to these domains were in  $6^{th}$  and  $7^{th}$  grade mathematics curriculum.

The analysis showed that students from all grade levels had lower performance in finding the perimeter and area of the parallelogram. Most of the students did not give any answer for parallelogram part and noted that they forgot the perimeter and area formula of parallelogram. Their lack of experiences with parallelogram in-and-out of mathematics classes might not have enhanced their knowledge of parallelogram as strong as other geometric figures. Students' most common procedural error was multiplying the two sides and multiplying the sides and the height and then dividing it by two to find the area of the parallelogram. The reason might be students' confusion of the area of a parallelogram with the area of a rectangle or a triangle. Moreover, they might have problems with locating the heights of quadrilaterals. These results showed that students did not have strong procedural knowledge of calculating the perimeter and area of a parallelogram. The other reason might be resulted from the fact that students rarely came across parallelograms in their daily lives compared to squares and rectangles and they forgot the formula easily. These results were consistent with the other studies conducted about area and perimeter. For example, Dağlı (2010) found that students could find the perimeter and area of a square, rectangle, or triangle easily. However, they had difficulty in finding the perimeter or area of a parallelogram. All students had the lowest performance when they were asked to find the height of a parallelogram by using its area and one side. Seventh and 6<sup>th</sup> grade students were more successful than the 8<sup>th</sup> grade students. The objective related to parallelogram was in the 6<sup>th</sup> and 7<sup>th</sup> grade curriculum. Therefore, 8<sup>th</sup> grade students might have forgotten the procedures in the present study.

Findings showed that students did not have much difficulty in calculating the perimeter of a square or a rectangle. However, they experienced problems when they calculated the length of the one side of the square or rectangle whose area or perimeter were given. This showed that students did not have sufficient procedural knowledge of finding the length of the side of the rectangle or square rather than the lack of procedural knowledge in finding the area or perimeter of the square and rectangle. This kind of error might be a result of students' possible confusion of perimeter and area formulas. Kidman and Cooper (1997) also reported that 4<sup>th</sup>, 6<sup>th</sup> and 8<sup>th</sup> grade students tried to calculate the perimeter of given quadrilaterals by summing up length and width instead of using area formula. Moreover, Van de Walle stated that the biggest misconception of students was mixing the perimeter and area (2012).

Students also had problems when they calculated the perimeter and area of combined geometric shapes. For example, while they were able to calculate the area or perimeter of a square separately, they had difficulty in calculating the area or perimeter of a geometric figure which was the combination of squares. It is usually

assumed that calculating the perimeter and area of combined figures need high order skills. Since it requires analyzing the figures separately and then considering the figures as whole, students might have difficulty in dealing with combined figures. Nevertheless, an interesting result was that 6<sup>th</sup> grade students were more successful in calculating the perimeter of combined figures than the 7<sup>th</sup> and 8<sup>th</sup> grade students. This might be resulted from the fact that 6<sup>th</sup> grade curriculum includes objectives related to calculating area and perimeter of combined figures whereas that objectives are not given place obviously in 7<sup>th</sup> and 8<sup>th</sup> grade curriculum. On the other hand, results showed that 8<sup>th</sup> grade students were more successful in calculating the area of combined figures. The strategy used by 8<sup>th</sup> grade students was to see the figures separately and this strategy worked for calculating the area part. However, for calculating perimeter it did not work. Although these kinds of figures might seem considerably confusing especially for the 6<sup>th</sup> grade students, 7<sup>th</sup> and 8<sup>th</sup> grade students also had difficulty. This kind of procedural error might have been resulted from the misunderstanding in the conception of the problem rather than the procedural skills. Furthermore, some 8<sup>th</sup> grade students tried to solve some questions by the help of Pythagorean Theorem which was not known by 6<sup>th</sup> and 7<sup>th</sup> grade students. This might have been resulted from the condition that 8<sup>th</sup> grade students had learned the Pythagorean Theorem just before the test was implemented. When they saw the right triangles, they might have decided to apply the Pythagorean Theorem. It can also be speculated that students' lack of procedural knowledge for calculating the area and perimeter of combined figures might be affected by students' lack of knowledge in different domains.

Students also had problems in calculating the area of the acute-angled, obtuse-angled, and right-angled triangles whose one side and the height belonged to that side was given. Among them, students had problems when they tried to calculate the area of obtuse-angled triangle. This might be a problem related to the height of the obtuse triangle because it was out of the triangle. Besides, another reason could be that students were less familiar with obtuse-angled triangles than the other types of triangles in their daily lives. One another reason might have been that, teachers do not mention obtuse-angled triangles in their classes.

# 5.2. Students' Conceptual Knowledge about Area and Perimeter of Geometric Shapes

The descriptive statistics of conceptual knowledge test showed that elementary students' conceptual knowledge test results were not very high (below 65 out of 100) similar to the mean scores of procedural knowledge test. This shows that students' conceptual knowledge was not sufficient in the domain of area and perimeter. It was examined in most studies that students from all grades might have difficulty in area and perimeter concepts (Barret & Clement, 2003; Martin & Kenney, 2002; Walter, 1970). Moreover, van Hiele (1985) stated that students did not understand concepts underlying terms perimeter and area. Therefore, the results of present study were consistent with these studies.

The mean score of private middle school students and mean scores of each grade separately for procedural and conceptual knowledge tests were close to each other. As in the procedural knowledge test, 8<sup>th</sup> grade students were more successful than 6<sup>th</sup> and 7<sup>th</sup> grade students in conceptual knowledge test. As they were taught these topics, they might make sense of area and perimeter concepts more conceptually. However, when the mean scores were examined, it was realized that scores in all grade levels were not very high. This showed that students might lack conceptual knowledge aspects of area and perimeter of geometric figures. This might be because students ignored conceptual knowledge of area and perimeter and the conceptual meaning underlying procedures and algorithms. Besides, teachers might also emphasize procedural tasks rather than conceptual tasks in their lessons.

One specific aim of the present study was to examine how conceptual knowledge of elementary students changed according to grade levels. Although procedural knowledge scores of all grade students were higher than their conceptual knowledge scores, there was a significant increase in students' conceptual knowledge scores by means of grade levels. The ANOVA results showed that there was a statistically significant difference among the mean scores of students from three grade levels in conceptual knowledge test. The mean score of  $8^{th}$  grade students was significantly higher than the mean score of  $6^{th}$  grade students. However, the mean

score of 7<sup>th</sup> grade students was close to 6<sup>th</sup> grade students' and 8<sup>th</sup> grade students' mean scores. As mentioned above, since the topics were taught in all grade levels in elementary school, students' conceptual knowledge might be improved. As a result of this, there might be a difference between 6<sup>th</sup> and 8<sup>th</sup> grade students 'conceptual knowledge. Furthermore, this might stem from the fact that students develop a better sense of concepts when they come to 8<sup>th</sup> grade.

Students' most common errors were also analyzed in the CKT. The analysis revealed that students had conceptual difficulties both in area and perimeter tasks. In general, the most successful group was 8<sup>th</sup> grade, whereas 6<sup>th</sup> grade students had the lowest performance. However, students' success changed according to the question types. More specifically, it was examined that 7<sup>th</sup> and 8<sup>th</sup> grade students performed better than the 6<sup>th</sup> grade students in tasks including predicting and the comparing of the area of geometric figures although related objectives were in the 6<sup>th</sup> grade program. Students have been asked to predict and compare objects since their childhood period in their daily lives. Therefore, success in predicting and comparing area and perimeter in higher grades might be resulted from this reason. One another reason might be that students internalized and conceptualized the meaning of area and perimeter. When most common errors were examined, it was realized that students might have the conception that the geometric figures which had more curls were bigger than the plain ones. They might have the wrong conception that if a figure had curves, its perimeter or area is bigger than the others. Furthermore, students had weak conceptual knowledge the fact that different shapes might have the same perimeter. Students might have interpreted the perimeter of a figure according to the appearance and irregularity.

In prediction and comparison area questions, while  $7^{th}$  and  $8^{th}$  grade students' performance did not change when they predicted the area given on the squared paper, it was realized that  $6^{th}$  grade students' performance was decreased from 75% to 45%. Here, it might be concluded that  $6^{th}$  grade students might have difficulty in predicting the area of a shape with curves given in a squared paper. However, in a similar question in which figures were given on a dot paper and students were expected to count the unit squares to find the area of the shape,  $6^{th}$  grade students' performance

increased. This showed that when the figures had curves, students' predictions skills became weaker. Therefore, it can be said that their conceptual knowledge to predict the area of an irregular shape which did not include the unit squares were low. Besides this, it was seen that students tended to count square units to find its perimeter or they tended to count lines which encircle the shape to find its area. This showed that students confused area and perimeter as in the study conducted by Tan (2009).

Private middle school students had the lowest performance in tasks related to the surface area. Although this topic was covered in all grade levels in the elementary mathematics curriculum, only 21% of all participated students gave correct answer for questions about surface area. While related questions could be difficult for 6<sup>th</sup> grade students, it should have been answered correctly by students who understood surface area conceptually sufficient. Yet, 7<sup>th</sup> and 8<sup>th</sup> grade students' performances were also low in tasks about surface area. This showed that elementary students might lack conceptual knowledge of surface area. Moreover, based on the results, it might be concluded that when the surface area was divided into pieces, students had difficulty to make sense of the area concept while they had better performance in undivided surface areas.

It was seen that students had difficulty in the changeability of perimeter and conservation of area when figures were decomposed and combined in a different shape. Similar results were also indicated in past research studies (Emekli, 2001; Kamii & Kysh, 2006).Furthermore, it was investigated that only 35% of the all students gave correct answer for interpreting the perimeter in these tasks. The rate of success increased to 63% in tasks related to area. This showed that students' knowledge about perimeter might be weaker than their knowledge about area, as in Tan and Aksu's (2009) study conducted with 7<sup>th</sup> grade students. In area and perimeter parts, 8<sup>th</sup> grade students performed better than the 7<sup>th</sup> and 6<sup>th</sup> grade students. However, while 7<sup>th</sup> graders performed better than 6<sup>th</sup> graders in the area part, they showed lower performance in the perimeter part. This might be a result of an emphasis on the perimeter concept in the 6<sup>th</sup> grade mathematics through the objective "explain the relationship between sides and perimeter of polygons." On the

contrary, in the 7<sup>th</sup> grade mathematics curriculum the perimeter concept isnot the focus but the area concept is emphasized mostly as integrated to area concept. There were also inconsistent results with the present study in the study conducted by Tan and Aksu (2009). For example, in their study, it was examined that 7<sup>th</sup> grade students had more problems about the conservation of the area than the conservation of perimeter, unlike the findings in this study.

A relationship objective was also investigated with the different combination of identical figures and similar results were revealed. When the change in the perimeter and area was asked when identical figures or the part of a figure was reformed, 8<sup>th</sup> graders were more successful than the students in other grade levels. This might be resulted from the fact that 8<sup>th</sup> graders' conceptual knowledge were more utilized in comparing area and perimeter of different combinations of figures compared to the other grade levels. However, students from all grade levels showed low performance in the area concept. This might show that area concept was not learned completely. Students had lack of conceptual knowledge about the conservation of the area which might address that their conceptual knowledge of area might be incomplete or inaccurate. These results were inconsistent with in the study conducted by Kami and Kysh (2006) who found that 8<sup>th</sup> grade students realized that when a figure was divided into parts, and a new shape was constructed, the area of the figure changed.

Moreira and Contente (1997) reported that seventh grade students confused the area and perimeter concepts as they believed that there was a linear relationship between area and perimeter of a figure. Also, in the present study, for the type of tasks in which students were required to realize in which rate the area of the figure changed according to the changes in the length of its edges, only about half of the students could give correct answers, although this kind of questions could be seen commonly in elementary mathematics curriculum materials. Although 8<sup>th</sup> grade students performed better than 6<sup>th</sup> and 7<sup>th</sup> grade students, when all grades were examined separately, the rate of success was not high for each grade level. When the answers were investigated, it was seen that most common error that students have done was their thinking that the rate of increase in the area was equal to the rate of increase in the length of sides. These answers might be a result of the lack of knowledge of the relationship between the area and the length of sides of a geometric figure. Moreover, students might have lack of conceptual knowledge of ratio rather than area and perimeter.

Finally, in the present study, the relationship between procedural and conceptual knowledge of elementary students was investigated. Results indicated that there was a significant positive relationship between conceptual knowledge test scores and procedural knowledge test scores. This meant that the students who had a strong conceptual knowledge also had strong procedural knowledge and vice versa. This positive relationship might result from the fact that conceptual knowledge are so interrelated that both conceptual and procedural knowledge are necessary for complete learning of a topic. It might also be speculated that the development of both types of knowledge influenced each other in the area and perimeter concepts.

#### 5.3. Students' Self-Efficacy towards Geometry

In general, descriptive statistics about geometry self-efficacy of elementary students revealed that elementary students had high (94 out of 125) self-efficacy. The fact that students were studying at a private school and they had several learning opportunities might be a reason of their high self-efficacy. The one-way ANOVA results for geometry self-efficacy test revealed that there was no statistically significant difference among three grade level in terms of geometry self-efficacy test scores. Nevertheless, scores of 6<sup>th</sup> grade students was higher than 7<sup>th</sup> grade students' scores. Similarly, geometry self-efficacy scores of 7<sup>th</sup> grade students were higher than 8<sup>th</sup> grade students' scores. Erktin and Ader (2004) investigated that students who had high mathematics anxiety had low self-efficacy. Since 8<sup>th</sup> grade students were preparing for the high school entrance examination, they might have been anxious in general due to the examination. Because of this, 8<sup>th</sup> grade students might have high self-efficacy togenetry self-efficacy togenetry self-efficacy togenetry self-efficacy togenetry self-efficacy scores of this, 8<sup>th</sup> grade students might have high self-efficacy scores of this, 8<sup>th</sup> grade students been anxious in general due to the examination. Because of this, 8<sup>th</sup> grade students might have low self-efficacy whereas 6<sup>th</sup> grade students might have high self-efficacy togenetry. Also, 8<sup>th</sup> grade students learned more geometry with respect to 6<sup>th</sup>

and  $7^{th}$  grade students. Therefore,  $6^{th}$  grade students might not be aware of their real capability in geometry and they might overestimate their capabilities.

Results indicated that there was a significant positive relationship between geometry self-efficacy and both procedural knowledge and conceptual knowledge test scores. This meant that higher levels of geometry self-efficacy of private middle school students were associated with higher scores in the PKT and CKT. In other words, students who had high self-efficacy towards geometry performed better in conceptual and procedural tasks in the area and perimeter concepts, confirming the positive relationship between mathematics performance and self-efficacy found in another studies (Erktin& Ader, 2004; Pajares & Miller, 1994; Reçber, 2011). On the other hand, Tsamir (2012) investigated the relationship between young children's geometric knowledge and their self-efficacy beliefs. Results indicated that students had high self-efficacy and their self-efficacy was not significantly related to their geometric knowledge. Therefore, results of the present study might be considered as inconclusive considering the previous studies.

#### 5.4 Implications and Recommendations for Further Research

In the study the main focus was to investigate private middle school students' procedural and conceptual knowledge in the domain of area and perimeter of geometric figures and to examine the most common errors in these types of tests. The other specific interest of the study was to examine how students' conceptual and procedural knowledge aspects of area and perimeter of geometric figures changed with respect to their geometry self-efficacy. Based on the findings of this study, several implications for teacher educators, teachers and curriculum developers, and Ministry of National Education could be deduced.

First, mathematics teachers should be aware of students' background knowledge, possible misconceptions and errors before teaching a topic. If teachers plan their lessons by the existing and possible future of the errors, effective teaching could be conducted on the basis of students' needs and the curriculum. In the study, it was seen that students had both procedural and conceptual difficulties in the domain of area and perimeter. If teachers enter classrooms by knowing the problems

of students in procedures and concepts, they might prepare well-oriented lessons. In mathematics, both procedural and conceptual knowledge is important. Therefore, teachers could design activities in which students could improve their procedural skills and conceptual knowledge about the topic.

According to the specific results of the study, teachers could emphasize on certain points in teaching area and perimeter concepts. For example, in the study it was investigated that all students had problems with perimeter and area of the parallelogram. Teachers can provide more meaningful activities to make students' knowledge permanent both procedurally and conceptually on that concept. Besides this, students' evaluation of the figures by looking at its appearance was one of the remarkable results of the study. Therefore, teachers should make students be engaged with both realistic measurements and predictions. Furthermore, teachers could design activities which prove the changeability of the perimeter and the conservation of the area. Students should be provided with different questions which also included daily life examples. In this way, different viewpoints should be initiated for students. Moreover, informal activities focusing on the area and perimeter attributes of figures rather than the formulas could be included in schools mathematics beginning from the early years of the primary school.

Not only for area and perimeter topics, but also for other mathematics topics starting from primary grade levels, studies focusing on students' procedural and conceptual knowledge should be conducted. According to the procedural and conceptual errors, for students and teachers textbooks and teacher books could be designed. Teachers become aware of what and how they might teach and students become aware of the nature of the topics. As a result of these, more meaningful teaching and learning could occur.

Faculties of education could take the findings of this study into the consideration while training mathematics teachers. During the method courses, topics should be investigated in detail. Especially, possible procedural and conceptual errors should be examined for specific topics. As mentioned in most of the study, teachers' content knowledge is very important for teaching. As a result of this,

making pre-service teachers being aware of the topics and teaching of these topics is also significant. Especially the errors that students might have and how to overcome those difficulties should be discussed. It is recommended that method lessons should be designed to make pre-service teachers be knowledgeable about the procedural and conceptual errors in the mathematics topics. Articles about studies in this domain could be recommended to not only pre-service teachers but also in-service teachers. Moreover, factors (e.g. self-efficacy as in the study) that affect students' performance could be considered in pre-service and in-service training.

Curriculum developers, textbook authors, and researchers could consider the results of the present study while preparing guidebooks for teachers. They should design curriculum and the books by taking into consideration of students' possible errors in the area and perimeter domain. The density of the part of the topic for each grade level could be considered. Also, Ministry of National Education should use the findings of the present study. The detailed information related to students procedural and conceptual errors in the domain of perimeter in all grade levels might be included in the curriculum.

The present study has limitations for generalizability. In the study, the sampling method was convenience sampling which meant that the researcher collected data from the individuals who were available (Fraenkel & Wallen, 2006). In order to make generalization of the findings to the population, further research including the randomly selected sample from the all elementary schools in Turkey might be conducted. Moreover, 5<sup>th</sup> grade students also could be added to sample due to the new regulations in the education system in Turkey in order to understand the development of procedural and conceptual knowledge across the middle school period. Besides, a longitudinal study could be conducted to see the changes in procedural and conceptual knowledge of elementary students. In this way, the suitability and design of the objectives in different grade level mathematics curriculum related to area and perimeter for grade levels could be investigated. Also, teachers' content knowledge in area and perimeter and process of teaching these topics could be investigated in further research.

## 5.5. Last words

I started this study with a conversation with one of my students in my mind. Although she was very successful in her mathematics examinations, she thought that she was good at only memorized procedures and she thought that she sometimes was not aware of some concepts. After this conservation, I realized that since she had low self-efficacy at conceptual tasks, she felt this way. I understood that when students gave meaning to concepts and when they were aware of procedures which they solved, they relied on themselves and these students had high self-efficacy. Moreover, I saw that the students who had high self-efficacy were more successful in tasks.

I have realized that area and perimeter were problematic topics in all grade levels. I am teaching 5<sup>th</sup> grade students now and I saw similar errors also in their tasks. However, because of the results of the present study, I taught this topic to my 5<sup>th</sup> grade students more carefully. Because, I knew the errors that they might do and I oriented my worksheets according to special questions in which they might have done possible errors. Now, I am planning that as I get more experienced, I will analyze students' errors in all topics and I will use the results in designing my further lessons as a middle school mathematics teacher.

Finally, I understood that as mathematics teachers, our duty is not to teach mathematics but to teach quality mathematics. Knowing both procedural and conceptual errors about a topic may provide teachers with the opportunity to help their students discover the truths and errors about a topic on their own.

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### **APPENDICES**

## **APPENDIX** A

### ÇEVRE UZUNLUĞU VE ALAN

### İŞLEM TESTİ

Soru 1: Aşağıda verilen şekillerin <u>çevrelerini</u> ve <u>alanlarını</u> hesaplayınız.



<u>Soru 2:</u> Aşağıdaki şekilde ABCD dikdörtgensel bölgedir. |AB|=5 birim, |BD|=4 birim ise, <u>taralı</u> <u>bölgenin alanını taralı olmayan bölgenin alanına oranı</u> kaç birim karedir?



#### Soru3: Aşağıda verilen üçgenlerin<u>alanlarını</u> hesaplayınız.



<u>Soru 4:</u> Aşağıda verilen şekilde **ABGH bir dikdörtgen**, **BCEF bir kare** ve **CED bir dik üçgendir**. Verilenlere göre, ACDFGH çokgensel bölgesinin <u>alanın</u> hesaplayınız.



Çözüm:

<u>Soru 5:</u> Aşağıda verilen şekilde |BC |=3 cm, |CD |=4 cm, |DE | =2 cm ve |FE |=12 cm ise, ABCDEF çokgeninin <u>çevresini</u> hesaplaymız.



Çözüm :

<u>Soru 6:</u> Aşağıdaki şekilde ABCD dikdörtgenin içinde bir kenar uzunluğu 2 birim olan kareler bulunmaktadır. |AB| = 12 birim, |BD| = 8 birim ise, <u>boyalı (taralı) alan</u> kaç birim karedir?



Soru 7: Alanı 64 metrekare olan yandaki ABCD karesinin çevresi kaç metredir?



<u>Soru 8:</u> Aşağıdaki dikdörtgenin <u>cevresi</u> 48 birimdir. AC uzunluğu 4 birim ise, dikdörtgenin <u>alam</u> kaç birim karedir?



<u>Soru 9:</u> Bir kenar uzunluğu 7 cm olan 3 tane kare yan yana konularak bir dikdörtgen elde ediliyor. Oluşan dikdörtgenin <u>alanını</u> ve <u>çevre uzunluğunu</u> hesaplayınız.



Yanda verilen paralelkenarda, AK uzunluğu DB kenarma, AH uzunluğu CD kenarma diktir. |AB |= 12 cm, |AH|= 4 cm ve |AC|=8 cm ise, AK uzunluğu kaç cm dir?

## **APPENDIX B**

### ÇEVRE UZUNLUĞU VE ALAN

#### KAVRAMSAL BİLGİ TESTİ

Soru 1: Aşağıdaki verilen şekilleri alanlarına göre büyükten küçüğe sıralayınız.



Cevap:.....>......>......

<u>Soru 2:</u> Aşağıda verilen eş küplerden oluşturulmuş cisimden kaç numaralı küpü çıkarırsak cismin yüzey alanında <u>değişme olmaz</u>? Nedenini açıklayınız.

|   |   |   | Cevap: |
|---|---|---|--------|
| - | - | ∕ | Çünkü; |
| 1 | 2 | X |        |
| 3 | 4 |   | l      |
|   |   |   |        |

<u>Soru 3:</u> Aşağıda kare prizma şeklinde olan bir hediye kutusu hediye paketi ile kaplanacaktır. Kutuyu tamamen kaplamada kullanılacak kâğıt miktarını bulmak için aşağıdaki bilgilerden hangisini bilmek gerekir? Size göre doğru olan seçeneği işaretleyerek nedenini açıklayınız.

| / |
|---|
|   |
|   |
|   |

| A. Kutunun ayrıtları toplamını | B. Kutunun yüzey alanını | C. Kutunun hacmini bilmeliyiz. |
|--------------------------------|--------------------------|--------------------------------|
| Cünkü:                         | Cüpkü:                   | Çünkü;                         |
| ş                              | çunxu,                   |                                |
|                                |                          |                                |
|                                |                          |                                |
|                                |                          |                                |
|                                |                          |                                |

| ÷ |         | <br>   |
|---|---------|--|
|   | ŞEKİL 1 | KARE   |
|   | ŞEKİL 2 | Verilen kare iki eşit parçaya<br>bölünüyor.  |
|   | ŞEKİL 3 | Daha sonra, bu parçalardan bir<br>tanesi, şekilde gösterildiği gibi<br>tekrar ortadan ikiye ayrılıyor. |
|   | ŞEKİL 4 | Oluşan tüm parçalar bir araya<br>getirilerek yandaki şekil<br>oluşturuluyor.                           |

<u>Soru 4:</u> Aşağıda verilen tabloyu inceleyiniz. Tablonun altında yer alan 2 soruyu verilen tabloya göre cevaplandırınız.

Aşağıda verilen seçeneklerden, size göre doğru olanı işaretleyerek, <u>nedenini açıklayınız.</u>

⊁ Yeni oluşan şeklin <u>çevre uzunluğu</u> için ne söylenebilir?

| <ul> <li>Yeni oluşan şeklin(4. Şekil) çevre<br/>uzunluğu 1.şeklin çevre<br/>uzunluğundan daha <u>BÜYÜKTÜR.</u></li> </ul> | B. Yeni oluşan şeklin(4. Şekil) çevre<br>uzunluğu 1.şeklin çevre<br>uzunluğundan daha <u>KÜÇÜKTÜR.</u> | C. Yeni oluşan şeklin(4. Şekil) çevre<br>uzunluğu 1.şeklin çevre<br>uzunluğuna <u>EşiTTİR.</u> |
|---|--|--|
| Çünkü;  | Çünkü;   | Çünkü;   |
|   |  |  |
|   |  |  |

### ➤ Yeni oluşan şeklin <u>alanı</u> için ne söylenebilir?

| <ul> <li>Yeni oluşan şeklin alanı 1. Şeklin<br/>alanından daha <u>BÜYÜKTÜR.</u></li> <li>Çünkü;</li> </ul> | <ul> <li>Yeni oluşan şeklin alanı 1. Şeklin<br/>alanından daha <u>KÜÇÜKTÜR.</u><br/>Çünkü;</li> </ul> | C. Yeni oluşan şeklin alanı 1. Şeklin<br>alanına <u>EŞİTTİR.</u><br>Çünkü; |
|--|---|--|
|  |   |  |
|  |   |  |
|  |   |  |

| Soru 5: | Bir dikdörtgenin kenar uzunluklan 4 katına çıktığında <u>alanı</u> nasıl değişir? |
|---------|---|
| А.      | Alanı 4 katına çıkar.   |
|         | Çünkü;  |
|         |   |
| B.      | Alanında bir değişiklik olmaz.  |
|         | Çünkü;  |
|         |   |
| C.      | Alanı değişir ama 4 katına çıkmaz.  |
|         | Çünkü;  |
|         |   |



**a.** Elimizde iki eş dikdörtgensel bölge var. Bu iki eş dikdörtgensel bölge yukarıdakilerden hangisindeki gibi birleştirirsek oluşan şeklin <u>çevre uzunluğu en az</u> olur?

#### Cevap:

**b.** . İki eş dikdörtgensel bölge yukarıdakiler gibi birleştirildiğinde <u>alanları</u> hakkında ne söyleyebilirsiniz?

Cevap:

<u>Soru 7:</u> Bir çiftçinin elinde 32 metrelik tel bulunmaktadır. Bu çiftçi elindeki teli kullanarak bir tarlanın <u>çevresine</u> bir sıra tel çekmek istiyor. Elindeki tel aşağıda farklı şekillerde verilen tarlalardan hangileri için yeterli hangileri için yetersizdir? <u>Nedenini açıklayınız</u>.



Soru 8: Aşağıda iki şekil veriliyor. Bu şekilleri dikkate alarak aşağıdaki soruları cevaplayınız.



Yukarıda verilen iki şeklin <u>alanları</u> birbirine eşit midir? Size göre doğru olan seçeneği işaretleyerek, nedenini <u>açıklayınız.</u>

| A. Evet, iki şeklin <u>alanları</u> birbirine<br><u>EşitttiR.</u> | B. Hayır, iki şeklin <u>alanları</u> birbirine<br><u>EşiT DEĞİLDİR.</u> |
|---|---|
| Çünkü;  | Çünkü;  |
|   |   |
|   |   |
|   |   |
|   |   |

Yukarıda verilen iki şeklin <u>çevre uzunlukları</u> birbirine eşit midir? Size göre doğru olan seçeneği işaretleyerek, n<u>edenini açıklayınız.</u>

| Çünkü; | Α.  | Evet, iki şeklin <u>çevre uzunlukları</u><br>birbirine <u>E<b>ŞİTTİR.</b></u> |  |
|--------|-----|---|--|
| ······ | Çüı | nkü;  |  |
|        |     |   |  |
|        |     |   |  |
|        |     |   |  |
|        |     |   |  |

B. Hayır, iki şeklin <u>çevre uzunlukları</u> birbirine <u>EŞİT DEĞİLDİR.</u>

Çünkü;.....

Soru 9: Aşağıda verilen şekilde boyalı <u>alanın</u> tahmini olarak kaç birim kare olduğunu yazınız.



<u>Soru 10:</u> Bir çiftçinin aşağıdaki şekillerde verildiği gibi farklı şekillere sahip 4 bahçesi vardır. Çiftçi bu bahçelerin etrafını tellerle çevirecektir. Bunun için elinde bulunan teli 4 eşit parçaya ayırıyor ve her bir bahçe için eşit miktarlarda tel kullanıyor. <u>Hiç bir tel parçası eklenmemiş veya artmamış</u> olduğuna göre aşağıda verilenlerden hangisi <u>doğrudur?</u>



A.En büyük çevre uzunluğu D bahçesine aittir

B. Tüm bahçelerin çevre uzunlukları birbirlerine eşittir.

C.En kısa çevre uzunluğuna sahip bahçe B bahçesidir.

D. C bahçesinin çevre uzunluğu, A bahçesinin çevre uzunluğundan daha büyüktür.

İşaretlediğiniz seçenek neden doğrudur. Cevabınızı açıklayınız.

## **APPENDIX C**

### GEOMETRİYE YÖNELİK ÖZYETERLİK ÖLÇEĞİ

Cinsiyet:

Smif:

Bu ölçekte 5"li derecelendirme yapılmış olup 1 hiçbir zaman, 2 ara sıra, 3 kararsızım, 4 çoğu zaman ve 5 her zaman olarak düşünülmüştür. Lütfen verilen ifadeler için 1-5 arası size en uygun olan rakamı işaretleyiniz.

|  | Hiçbir<br>zaman | Ara sıra | Kararsızım | Çoğu<br>Zaman | Her zaman |
|--|-----------------|----------|------------|---------------|-----------|
| 1. Geometrideki kavramları rahatlıkla anlayabiliyorum.   | 1               | 2        | 3          | 4             | 5         |
| <ol> <li>Günlük yaşamda gördüğüm nesneleri geometrik şekillere<br/>benzetebilirim.</li> </ol>        | 1               | 2        | 3          | 4             | 5         |
| 3. Geometride arkadaşlarım kadar iyi olmadığımı düşünüyorum.   | 1               | 2        | 3          | 4             | 5         |
| <ol> <li>Bir geometrik şekil gördüğümde onun özelliklerini<br/>hatırlayabilirim.</li> </ol>          | 1               | 2        | 3          | 4             | 5         |
| <ol> <li>Bir geometri sorusu görünce ne yapılacağını bilemem.</li> </ol>                             | 1               | 2        | 3          | 4             | 5         |
| <ol> <li>Saatlerce çalışsam bile geometride başarılı olamayacağımı<br/>düşünüyorum.</li> </ol>       | 1               | 2        | 3          | 4             | 5         |
| <ol> <li>Geometri ile el becerilerimi arttırabileceğimi düşünüyorum.</li> </ol>                      | 1               | 2        | 3          | 4             | 5         |
| <ol> <li>Geometri bilgimi diğer derslerde kullanabilirim.</li> </ol>                                 | 1               | 2        | 3          | 4             | 5         |
| 9. Geometri konusunda yeterli bilgiye sahip değilim.   | 1               | 2        | 3          | 4             | 5         |
| <ol> <li>Geometri konusunda verilecek olan projelerde başarılı<br/>olacağımı düşünüyorum.</li> </ol> | 1               | 2        | 3          | 4             | 5         |
| <ol> <li>Geometri sorusu çözdükçe kendime olan güvenimin artacağını<br/>düşünüyorum.</li> </ol>      | 1               | 2        | 3          | 4             | 5         |
| 12. Geometrik şekiller ile ilgili materyal geliştiremem.   | 1               | 2        | 3          | 4             | 5         |
| 13. Geometrik şekilleri kafamda canlandırabilirim.   | 1               | 2        | 3          | 4             | 5         |
| 14. Geometri ile ilgili problemler yazabilirim.  | 1               | 2        | 3          | 4             | 5         |
| 15. Geometri konusunda kendimi başarılı görüyorum.   | 1               | 2        | 3          | 4             | 5         |
| 16. Bir geometri problemini çözmek için gereken işlem<br>basamaklarını çıkarabilirim.                | 1               | 2        | 3          | 4             | 5         |
| 17. Matematiksel problemleri çözerken geometrik şekillerden<br>yararlanırım.                         | 1               | 2        | 3          | 4             | 5         |
| 18. Geometrik şekiller arasındaki ilişkileri söyleyemem.   | 1               | 2        | 3          | 4             | 5         |
| 1  |                 | 1        | 1          | 1             |           |

| 19. Geometrik şekillerin sahip oldukları çevre uzunluklarını tahmin edebilirim.  | 1 | 2 | 3 | 4 | 5 |
|--|---|---|---|---|---|
| 20. Yabancı bir yerde yolumu kaybedersem geometri bilgim ile yolumu bulabilirim.   | 1 | 2 | 3 | 4 | 5 |
| <ol> <li>Geometri ile ilgili sorun yaşayan arkadaşlarıma yardımcı<br/>olabilirim.</li> </ol>                                 | 1 | 2 | 3 | 4 | 5 |
| 22. Bir geometrik şeklin özelliklerini duyduğumda şeklini<br>çizebilirim.  | 1 | 2 | 3 | 4 | 5 |
| 23. Geometrik şekilleri kullanarak yeni bir geometrik şekil oluşturabilirim.   | 1 | 2 | 3 | 4 | 5 |
| 24. Bir geometri sorusunda işlemleri yaparken telaşa kapılacağımı<br>düşünüyorum.  | 1 | 2 | 3 | 4 | 5 |
| <ol> <li>Žleriki yıllarda geometri bilgisinin kullanıldığı bir meslek<br/>seçersem başarılı olacağıma inanıyorum.</li> </ol> | 1 | 2 | 3 | 4 | 5 |

# **APPENDIX D**

## RUBRIC FOR PROCEDURAL KNOWLEDGE TEST

| Q1 | EXPLANATION  | POINTS                              |
|----|--|-------------------------------------|
|    | Finding the area and perimeter of square and rectangle correctly           | 1,5 points each (1,5 x<br>4= 6 pts) |
|    | Finding the area and perimeter of parallelogram correctly                  | 2 points each ( 2 x 2<br>= 4 pts)   |
|    | Each computational error   | -1 point                            |
| Q2 |  |                                     |
|    | Finding the area of the rectangle correctly                                | 3 points                            |
|    | Finding the area of the triangle correctly                                 | 3 points                            |
|    | Finding the ratio correctly  | 4 points                            |
|    | Each computational error   | -1 point                            |
| Q3 |  |                                     |
|    | Finding the areas of acute-angled and right-<br>angled triangles correctly | 3 points each (3x2= 6 pts )         |
|    | Finding the area of obtuse-angled and right-<br>angled triangles correctly | 4 points                            |
|    | Each computational error   | -1 point                            |
| 04 |  |                                     |
|    | Finding the area of the rectangle correctly                                | 3 points                            |
|    | Finding the area of the square correctly                                   | 3 points                            |
|    | Finding the area of the triangle correctly                                 | 3 points                            |
|    | Finding the sum  | 1 point                             |
|    | Each computational error   | -1 point                            |
| Q5 |  |                                     |
|    | Finding the perimeter of the shape correctly                               | 10 points                           |
|    | Each computational error   | -1 point                            |
|    |  |                                     |

| Q6       |  |            |
|----------|--|------------|
|          | Finding the area of the small squares correctly  | 3,5 points |
|          | Finding the area of the rectangle correctly      | 3,5 points |
|          | Finding the difference                           | 3 points   |
|          | Each computational error                         | -1 point   |
| Q7       |  |            |
|          | Finding the side length of the square correctly  | 5 points   |
|          | Finding the perimeter of the square correctly    | 5 points   |
|          | Each computational error                         | -1 point   |
| Q8       |  |            |
|          | Finding the sum of the two adjacent side lengths | 3 points   |
|          | conectly   | 3 points   |
|          | Finding the unknown side length correctly        | 4          |
|          | Finding the area of the rectangle correctly      | 4 points   |
|          |  | -1 point   |
|          | Each computational error                         |            |
| Q9       |  |            |
|          | Finding the area of the rectangle correctly      | 5 points   |
|          | Finding the perimeter of the rectangle correctly | 5 points   |
|          | Each computational error                         | -1 point   |
| Q10      |  |            |
| <u> </u> | Finding the area of the parallelogram correctly  | 5 points   |
|          | Finding the length correctly                     | 5 points   |
|          | Each computational error                         | -1 point   |
|          |  |            |

# **APPENDIX E**

# RUBRIC FOR CONCEPTUAL KNOWLEDGE TEST

| Q1                            | EXPLANATIONS  | POINTS              |
|-------------------------------|---|---------------------|
|                               | Correct range for three figure  | 10 points           |
|                               | Correct range for only two figure   | 3 points            |
|                               | No answer, completely irrelevant answer   | 0 point             |
| Q9                            |   |                     |
|                               | For The answers 11-11,5 and 12  | 10 points           |
|                               | For the answers 10 and 13   | 8 points            |
|                               | For the answers 9 and 14  | 3 points            |
| 00.00                         | No answer, completely irrelevant answer   | 0 point             |
| Q2, Q3,<br>Q4, Q5,<br>Q8, Q10 |   |                     |
|                               | Choosing the correct answer and sufficient explanation  | 10 points           |
|                               | Choosing the correct answer and the explanation<br>is close to the correct explanation but not given<br>exactly | 8 points            |
|                               | Choosing the correct answer but no explanation  | 5 point             |
|                               | No answer, completely irrelevant answer   | 0 point             |
| Q6                            | Giving correct answer for both perimeter and area part  | 5 + 5 =10 points    |
|                               | No answer, completely irrelevant answer   | 0 point             |
| Q7                            | Giving correct answer and correct explanation<br>for four figure separately                                     | ( 2,5 x4 )10 points |
|                               | Giving correct answer but incorrect explanation or no explanation   | 6 points            |
|                               | No answer, completely irrelevant answer   | 0 point             |

### **APPENDIX F**

# THE HISTOGRAMS SUPPORTING THE NORMALITY ASSUMPTION FOR TEST SCORES FOR EACH GRADE LEVELS



Figure 1: Histogram of procedural knowledge test scores for 6<sup>th</sup> grade students



Figure 2: Histogram of procedural knowledge test scores for 7<sup>th</sup> grade students



Figure 3: Histogram of procedural knowledge test scores for 8<sup>th</sup> grade students



Figure 4: Histogram of conceptual knowledge test scores for 6<sup>th</sup> grade students



Figure 5: Histogram of procedural knowledge test scores for 7<sup>th</sup> grade students



*Figure 6*: Histogram of procedural knowledge test scores for 8<sup>th</sup> grade students.



*Figure 7*: Histogram of self-efficacy towards geometry test scores for 6<sup>th</sup> grade students



Figure 8: Histogram of self-efficacy towards geometry test scores for 7<sup>th</sup> grade



Figure 9: Histogram of self-efficacy towards geometry test scores for 8<sup>th</sup> grade

student

### **APPENDIX G**

1956

Orta Doğu Teknik Üniversitesi

Middle East Technical University

Sayı: B.30.2.ODT.0.AH.00.00/126/87-854

4 Temmuz 2012

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Gönderilen: Yrd. Doç. Dr. Çiğdem Haser İlköğretim Bölümü Gönderen : Prof. Dr. Canan Özgen IAK Başkan Yardımcısı İlgi : Etik Onayı

lawanbyen

" İlköğretim 6., 7. ve 8. Sınıf Öğrencilerinin Geometriye Karşı Öz-Yeterlik İnanışları ile Alan ve Çevre Konusundaki İşlemsel ve Kavramsal Bilgileri Arasındaki İlişkinin İncelenmesi " isimli araştırmanız "İnsan Araştırmaları Komitesi" tarafından uygun görülerek gerekli onay verilmiştir.

Bilgilerinize saygılarımla sunarım.

Etik Komite Onayı

Uygundur

04/07/2012

man

Prof.Dr. Canan ÖZGEN Uygulamalı Etik Araştırma Merkezi ( UEAM ) Başkanı ODTÜ 06531 ANKARA

# **APPENDIX H**

# TEZ FOTOKOPİSİ İZİN FORMU

# <u>ENSTİTÜ</u>

| Fen Bilimleri Enstitüsü        |   |
|--------------------------------|---|
| Sosyal Bilimler Enstitüsü      | X |
| Uygulamalı Matematik Enstitüsü |   |
| Enformatik Enstitüsü           |   |
| Deniz Bilimleri Enstitüsü      |   |

## **YAZARIN**

Soyadı : ORHAN Adı : Nagehan Bölümü : İlköğretim Bölümü

<u>**TEZİN ADI</u>** (İngilizce) : An Investigation of Private Middle School Students' Common Errors in the Domain of Area and Perimeter and the Relationship Between Their Geometry Self-Efficacy Beliefs and Basic Procedural and Conceptual Knowledge</u>

|    | TEZİN TÜRÜ : Yüksek Lisans X Doktora  |   |
|----|---|---|
| 1. | Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir.   |   |
| 2. | . Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. |   |
| 3. | Tezimden bir (1) yıl süreyle fotokopi alınamaz.   | X |

<u>Yazarın İmzası:</u>

<u>Tarih:</u>