

TRACKING OF MULTIPLE GROUND TARGETS IN CLUTTER WITH INTERACTING
MULTIPLE MODEL ESTIMATOR

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MULTIPLE MODEL ESTIMATOR**

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ABSTRACT

TRACKING OF MULTIPLE GROUND TARGETS IN CLUTTER WITH INTERACTING MULTIPLE MODEL ESTIMATOR

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In this thesis study, single target tracking algorithms including IMM-PDA and IMM-IPDA algorithms; Optimal approaches in multitarget tracking including IMM-JPDA, IMM-IJPDA and IMM-JIPDA algorithms and an example of Linear Multi-target approaches in multitarget tracking including IMM-LMIPDA algorithm have been studied and implemented in MATLAB for comparison. Simulations were carried out in various realistic test scenarios including single target tracking, tracking of multiple targets moving in convoy fashion, two targets merging in a junction, two targets merging-departing in junctions and multitarget tracking under isolated tracks situations. RMSE performance, track loss and computational load evaluations were done for these algorithms under the test scenarios dealing with these situations. Benchmarkings are presented relying on these outcomes.

Keywords: Target Tracking, Multitarget, Interacting Multiple Model, Data Association, Linear Multi-Target

ÖZ

ÇOKLU YER HEDEFLERİNİN YANKILI ORTAMDA ETKİLEŞİMLİ ÇOKLU MODEL KESTİRİCİSİYLE TAKİBİ

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Bu tez çalışmasında, EÇM-OVİ (IMM-PDA) ve EÇM-EOVİ (IMM-IPDA) algoritmalarını içeren tek hedef takibi algoritmaları; EÇM-BOVİ (IMM-JPDA), EÇM-EBOVİ (IMM-IJPDA) ve EÇM-BEOVİ (IMM-JIPDA) algoritmalarını içeren çoklu hedef takibinde Optimal yöntemler ve EÇM-DÇEOVİ (IMM-LMIPDA) algoritmasını içeren çoklu hedef takibinde Doğrusal Çoklu-Hedef yöntemlerinden bir örnek çalışılmakta ve MATLAB'ta karşılaştırma için gerçekleştirilmektedir. Tek hedef takibi, çoklu hedeflerin konvoy hareketi halindeki takibi, iki hedefin bir kavşakta karşılaşması, iki hedefin kavşaklarda karşılaşması-ayrılması ve izole izler altında çoklu hedef takibi durumlarını içeren çeşitli gerçekçi test senaryolarında simülasyonlar yürütülmüştür. KOKH (RMSE) performans, iz kaybı ve hesaplama yükü değerlendirmeleri yapılmıştır. Bu sonuçlara dayanılarak karşılaştırmalar sunulmaktadır.

Anahtar Kelimeler: Hedef Takibi , Çoklu Hedef, Etkileşimli Çoklu Model, Veri İlişkilendirme, Doğrusal Çoklu-Hedef

To My Family

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CHAPTER 1

INTRODUCTION

Since the Gulf War in 1991, Ground Moving Target Indicator (GMTI) radar has become an extremely useful sensor for military surveillance[20, 15] as well as civilian applications[4]. Tracking of multiple ground targets with airborne GMTI sensor measurements often suffers from high clutter density and low visibility of targets under track. Tracking of multiple ground targets is more challenging problem than tracking of underwater or aerial targets where heavy and dense false alarms and high target density problems are inevitably encountered in the surveillance scenes. Target originated measurements may be present in each scan with a certain probability of detection. Major difficulties of tracking of ground target(s) results from the target motion origin uncertainty[1] and the measurement origin uncertainty[1].

The *target motion origin uncertainty* appears in the situations where target(s) may undergo a known or unknown maneuver during an unknown time period[1]. In a fact, a nonmaneuver and different maneuvers can be described only with different dynamic motion models[3]. The use of an incorrect model at a specific time interval often causes unacceptable errors. When tracking maneuvering targets, it is very important to make a decision accurately on time the right model to use. So, instead of using a single model based filter, a bank of filters based on a set of multiple models should be considered which represent possible maneuvers under consideration[1]. Recommended approach[1] in target tracking under target motion origin uncertainty existence is to use Interacting Multiple Model (IMM) Estimator. The IMM is a recursive cost-effective and practical filter that shows elegant performance when targets being tracked undergo frequent maneuvers during unknown time periods[1].

The *measurement origin uncertainty* appears due to unreliable measurement(s) obtained by the sensor system. Unreliable measurements may have arisen from an irrelevant source, including clutter, false alarms, and neighboring targets, as well as the target under track. Target tracking under this kind of situation takes all measurements into account for track update in each scan. Validation of measurements is crucial in this situation in order to reduce further computation. A track quality measure should be employed to discriminate the target track(s) has been followed whether either of them true or false track(s). Under measurement origin uncertainty existence, to select the right measurement(s) to initiate true track(s) recommended approach in the literature[1] is to use Data Association.

An outstanding property of almost all target tracking algorithms is in the way of the data association probabilities are computed. Single Target Tracking algorithms assume that validated measurements are either target originated or formed by an interfering source, often termed clutter. In Multitarget tracking situation, in addition to Single target tracking situation false alarm measurements may have also originated from neighboring targets. In this situation, assignment of target measurements to right target tracks has a great importance. So, formation of all feasible measurement-to-track joint events and assignment of right joint event via calculation of a posteriori probabilities of each feasible joint event in each scan are required. Hence, optimal approaches in Multitarget tracking require too much computational load as the numbers of target track(s) and measurements grow linearly while the du-

ration of overall process rises exponentially. Consequently, using optimal approaches instead, more practical suboptimal approaches, as an example Linear Multi-Target approach is considered due to linear number of operations in the number of target tracks and measurements sense with a negligible performance loss compared to optimal approaches.

The most practical method for Multitarget tracking is basically to run a bank of Single Target Tracking algorithms, based on different dynamic motion models per track, where in the case of Linear Multi-Target approaches, i.e. IMM-LMIPDA. This method has rarely been proven satisfactory in practice[19] because almost all suboptimal algorithms in the literature[19, 27, 1] suffer from deficiencies in performance. Particularly, these algorithms have been shown in [43] to be more susceptible to "track loss" where heavy and dense false alarms are often encountered and targets are closely spaced[19] and the numbers of targets and measurements are considerably high[43].

In this study, simulations are carried out in various realistic test scenarios, where actual ground target(s)'s movement taken into consideration, dealing with single target tracking, tracking of multiple targets moving in convoy fashion, two targets merging in a junction, two targets merging-departing in junctions and multitarget tracking under isolated tracks situations in order to compare single target tracking algorithms including IMM-PDA and IMM-IPDA algorithms; Optimal approaches in multitarget tracking including IMM-JPDA, IMM-IJPDA and IMM-JIPDA algorithms and an example of Linear Multi-target approaches in multitarget tracking including IMM-LMIPDA algorithm. Comparison of these target tracking algorithms are done under RMSE performance, track loss and computational load perspective relying on evaluation results over related test scenarios.

The outline of the thesis is organized as follows:

In Chapter 2, target motion origin uncertainty problem is defined and the development of approaches for kinematic state estimation is step-by-step presented. Fundamental parts of the approaches are explained and kinematic state estimation mechanism via these approaches are demonstrated. IMM approach is mentioned in detail. Next, measurement origin uncertainty problem is defined and the development of approaches for data association is presented. General methodology for tracking in measurement origin uncertainty is briefly demonstrated. Approaches proposed to be used under this methodology and considered to be used in the study for tracking in the case of measurement origin uncertainty problem are presented. Fundamental parts of the approaches are explained in detail. As the following, tracking of single ground target in clutter problem is defined as the combination of the problems defined in previous chapters and the major solution to single target tracking in clutter problem is presented as the fusion of methodology and approaches given in previous chapters, respectively. IMM-PDA process is given as an example of solutions addressing to single target tracking in clutter problem. At the end of this chapter, problems encountered in tracking of multiple ground targets in clutter have been investigated. Detailed literature survey and the development of approaches for multitarget data association algorithms is step-by-step mentioned.

In Chapter 3, all target tracking algorithms used in comparison throughout the simulations are given in detail, which are: Single target tracking algorithms including IMM-PDA and IMM-IPDA algorithms; Optimal approaches in multitarget tracking including IMM-JPDA, IMM-IJPDA and IMM-JIPDA algorithms and an example of Linear Multi-target approaches in multitarget tracking including IMM-LMIPDA algorithm.

In Chapter 4, simulation studies are presented to demonstrate RMSE performance comparisons and computational load evaluations of the target tracking algorithms mentioned in the previous chapter. Simulations have been carried out under the problem of tracking of single/multiple ground target(s) in a dense clutter environment in various realistic test scenarios including tracking of single maneuvering target in clutter, tracking of multiple targets moving in convoy fashion, two targets merging in a junction, two targets merging-departing in junctions and multitarget tracking under isolated tracks situations. At the end of this chapter, relying on the RMSE performance, track loss and computational load evaluation results, benchmarkings of these algorithms under the test scenarios dealing with these situations have been presented.

In Chapter 5, all the results obtained throughout this study are concluded with remarks.

CHAPTER 2

BACKGROUND

2.1 Target Motion Origin Uncertainty and Kinematic State Estimation

Target tracking is defined in [2, 4] as a *hybrid estimation* problem which involves both *continuous* and *discrete uncertainties*. Challenging problem in tracking of maneuvering target(s) results from the target motion origin uncertainty[4]. The *target motion origin uncertainty* [1] appears in the situations where target(s) may undergo a known or unknown maneuver during an unknown time period. In a fact, a nonmaneuver motion and different maneuvers can only be described with different dynamic motion models[3]. In target tracking, for **Kinematic State** (e.g. position, velocity, acceleration) **Estimation** of target(s) under track, the mathematical modeling of all possible target motion dynamics / kinematics is essential[1, 3]. The use of incorrect models or insufficient number of models often causes undesired consequences[1]. Generally, a continuous-valued process noise is considered to cover the unknown modeling errors or deviations of the mathematical model from the exact behavior of the system. However, while tracking a maneuvering target, deciding accurately on time the right model to use constitutes vital importance. In order to handle this situation, all the models according to possible target motion dynamics should be formed and considered, the right model which fits to true target kinematics at that time should be selected. Hence, the major approach naturally is to consider a method where more than one model - multiple models are taken into account.

Major approach in target tracking under target motion origin uncertainty existence is to use **Multiple Model (MM)** method which is one of the most consented approaches to solve hybrid estimation[2, 4] problem. MM method recommends using a bank of filters based on a set of multiple models that represent/cover possible system behavior patterns (e.g. maneuvers) for the problem under consideration[1]. These system behavior patterns are discrete in nature and referred to as *system modes*. The **system mode** at specific instant has stair-case type trajectory which may stay unchanged or jump. For such a system the transition between system modes, shortly modal state is generally modelled with *Markov Chain* due to its nature and consistency in theory[4].

The early results of Static ("Non-interacting") Multiple Model (SMM) estimation were valid for targets with a *time-invariant* unknown or uncertain system mode while they are ineffective in frequent system mode transitions[5]. By the development of the highly cost-effective Interacting Multiple Model (IMM) estimator[9], the MM approach has become not only capable of handling frequent mode transitions (e.g. maneuvers) but also practical for maneuvering target tracking applications where in [1, 4, 5, 7, 10, 12] has been proven.

For target(s) under track, many different maneuver models are possible where all of them may not be represented sufficiently by a small set of models. To accomplish better performance, use of large filter banks based on different motion models may be necessary. Use of more model based filters in IMM estimator has been shown in [3, 13] that does not guarantee enhancement in performance. Because,

use of more models increases the computational complexity very considerably. In fact, increment in the number of model based filters in the IMM Estimator deteriorates estimator's performance significantly due to the fact that model likelihood difference between the models decreases[3]. Thus, using less and sufficient number of models in the IMM Estimator has been shown theoretically in [13, 14] yields better performance with less computation complexity which has also been discussed in [3, 1]. In our all tracking in clutter simulations, we attach more attention on this result, hence, instead of using more models in the IMM Estimator, for sake of better performance with less computation complexity, we consider to use 2 models which is sufficient and also recommended to use at [3]: *CV with Low Process Noise* for nonmaneuvering motion and *CV with High Process Noise* for any maneuvers including coordinated turns and acceleration modes.

2.1.1 Multiple Model (MM) Estimation

The basic idea of the Multiple Model Estimation approach is to assume a set of models which can be denoted as \mathbf{M} for the hybrid system; form a bank of filters based on each unique model in \mathbf{M} correspondingly; make them run cooperatively; combine the estimates from these filters and form the overall estimation with a certain combination of the estimates.

For a Markovian jump linear system[4],

$$x_{k+1} = F_k^{(i)} x_k + G_k^{(i)} w_k^{(i)} \quad (2.1)$$

$$z_k = H_k^{(i)} x_k + v_k^{(i)} \quad (2.2)$$

respectively, where superscript (i) denotes the quantities belong to model m_i and the jumps of the system mode are assumed to have the following transition probabilities

$$P\{m_{k+1}^{(j)} | m_k^{(i)}\} \triangleq P\{s_{k+1} = m_j | s_k = m_i\} = \pi_{ij} = \text{constant}, \quad \forall m_i, m_j, k \quad (2.3)$$

where $m_k^{(i)}$ denotes the event model m_i matches the system mode at time k :

$$m_k^{(i)} \triangleq \{s_k = m_i\} \quad (2.4)$$

Frequently used terms *mode* and *model* may sometimes be confused even in the literature. In order to make it clear, **mode** is referred to exact behavior pattern of a system and **model** is referred to a mathematical representation or description of the system behavior at a certain accuracy level.

Briefly, the recursive MM estimator[1, 4] involves the following:

2.1.1.1 Model-set determination

The performance of an MM estimator mostly depends on the set of models used. The major task in the application of MM estimation lies in the design of the set \mathbf{M} of multiple models [3]. Once the set \mathbf{M} is determined, the MM estimator implicitly assumes that each system modes in the set \mathbf{S} can be represented/covered "exactly" by the members of \mathbf{M} [4].

2.1.1.2 Filter selection

For sake of attaining the optimal solution to problem at hand, filter type may be chosen as Kalman Filter (KF) for a jump-linear system; Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF) or Particle Filter (PF) for nonlinear problems[15].

2.1.1.3 Filter reinitialization

Except in the first generation (static) MM estimators[4], the recursive filters do not operate independently. The input for a recursive cycle of such a filter depends on the other filters as well as the output of the same filter from the previous cycle. This is referred to as *reinitialization*. It is a natural and important way of “**interaction**” – using the information obtained by other filters.

2.1.1.4 Estimate Fusion

The overall estimate is obtained from *all* filter-obtained estimates $\hat{x}_{k|k}^{(i)}$; that is, no hard decision is made concerning the use of the filter estimates. If the conditional mean of the base state is used as the estimate, such as under the *Minimum Mean Square Error (MMSE)* criterion [4], then the overall estimate is the *probabilistically* weighted sum of all filter estimates:

$$\hat{x}_{k|k} = E[x_k|z^k] = \sum_i \hat{x}_{k|k}^{(i)} P\{m_k^{(i)}|z^k\} \quad (2.5)$$

and the overall covariance is determined accordingly.

The operation of most (single-scan) recursive MM estimators of M models is illustrated in Figure 2.1, where $\hat{x}_{k|k}^{(i)}$ is the estimate of x_k obtained from the filter based on model i at time k given the measurement sequence through time k ; $\bar{X}_{k-1}^{(i)}$ is the reinitialized estimate at time $k - 1$ as the input to filter i for k th time cycle; $\hat{x}_{k|k}$ is the overall estimate.

In the first generation (Static) MM algorithm[4], individual model based recursive filters operate independently without any interaction with one another because it is assumed that the mode does not jump, formally time-invariant.

To achieve target tracking under frequent mode jumps reliably on time Interacting Multiple Model (IMM) algorithm has been developed[9, 4].

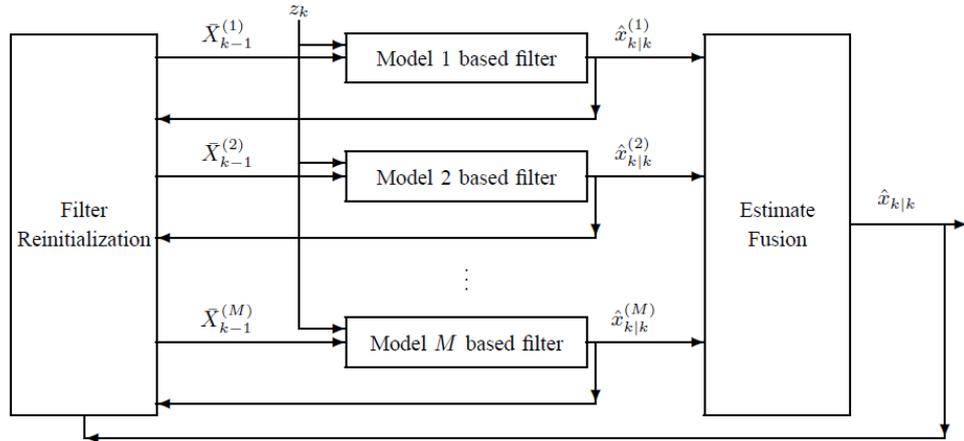


Figure 2.1: Structure of a Recursive MM Estimator (Figure is adapted from [4])

2.1.2 Interacting Multiple Model (IMM) Estimator

To decrease the complexity, many MM estimators utilize the mode-history-specific information about the state which is referred to as *filter reinitialization*.

In the Static Multiple Model (SMM) algorithm[4], the system mode is assumed time-invariant, therefore, there is no filter reinitialization. The Interacting Multiple Model (IMM) estimator uses the following smart reinitialization:

$$\bar{X}_{k-1}^{(i)} = E[x_{k-1}|z^{k-1}, m_k^{(i)}] = \sum_j \hat{x}_{k-1|k-1}^{(j)} P\{m_{k-1}^j | z^{k-1}, m_k^{(i)}\} \triangleq \bar{x}_{k-1|k-1}^{(i)} \quad (2.6)$$

and the covariance is determined accordingly (as illustrated in Figure 2.2).

Each filter i at scan k has its own input $\bar{x}_{k-1|k-1}^{(i)}$ and $\bar{P}_{k-1|k-1}^{(i)}$, which form the best possible *quasi-sufficient statistic*[1] of all old information and the *knowledge or assumption that model m_i matches the true mode at k* . This has been shown in Figure 2.2, where the reinitialized estimate as input to each model based recursive filter is a weighted sum of the most recent estimates from all model based recursive filters. The IMM Estimator also runs each model based recursive filter only once per cycle (as in Table 2.1).

The structure of the IMM Estimator is illustrated in Figure 2.2 for three models case. A complete cycle of the IMM Estimator with Kalman Filters as its model based recursive filters is summarized on Table 2.1 for the Markovian jump linear system described by (2.1) and (2.2), with MMSE optimality criteria [4] and the priori fundamental assumption that $w_k^{(i)}$ and $v_k^{(i)}$ are White Gaussian process and measurement noises, with means $\bar{w}_k^{(i)} \triangleq E[w_k^{(i)}]$, $\bar{v}_k^{(i)} \triangleq E[v_k^{(i)}]$ and covariances $Q_k^{(i)} \triangleq COV[w_k^{(i)}]$, $R_k^{(i)} \triangleq COV[v_k^{(i)}]$, respectively.

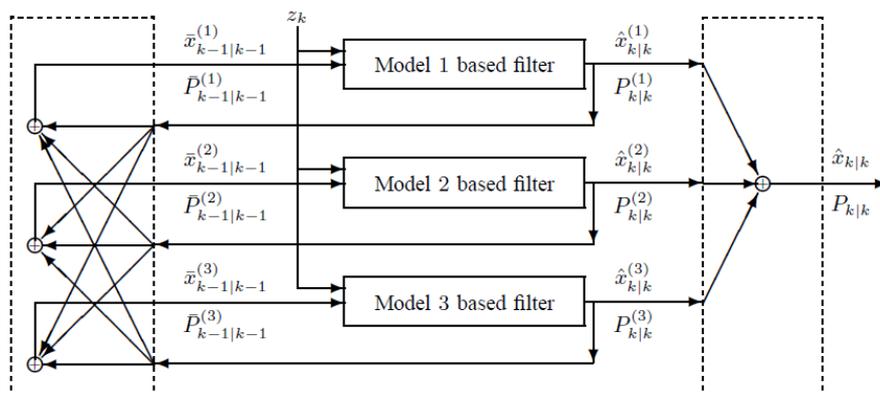


Figure 2.2: Structure of the IMM Estimator (with three models) (Figure is adapted from [4])

Table2.1: One Cycle of IMM Algorithm [4]

1. Model-conditioned reinitialization (for $i = 1, 2, \dots, M$):

Predicted model probability: $\mu_{k|k-1}^{(i)} \triangleq P\{m_k^{(i)} | z^{k-1}\} = \sum_j \pi_{ji} \mu_{k-1}^{(j)}$

Mixing weight: $\mu_{k-1}^{ji} \triangleq P\{m_{k-1}^{(j)} | m_k^{(i)}, z^{k-1}\} = \pi_{ji} \mu_{k-1}^{(j)} / \mu_{k|k-1}^{(i)}$

Mixing estimate: $\bar{x}_{k-1|k-1}^{(i)} \triangleq E[x_{k-1} | m_k^{(i)}, z^{k-1}] = \sum_j \hat{x}_{k-1|k-1}^{(j)} \mu_{k-1}^{ji}$

Mixing covariance: $\bar{P}_{k-1|k-1}^{(i)} = \sum_j [P_{k-1|k-1}^{(j)} + (\bar{x}_{k-1|k-1}^{(i)} - \hat{x}_{k-1|k-1}^{(j)})(\bar{x}_{k-1|k-1}^{(i)} - \hat{x}_{k-1|k-1}^{(j)})'] \mu_{k-1}^{ji}$

2. Model-conditioned filtering (for $i = 1, 2, \dots, M$):

Predicted state: $\hat{x}_{k|k-1}^{(i)} = F_{k-1}^{(i)} \bar{x}_{k-1|k-1}^{(i)} + G_{k-1}^{(i)} \bar{w}_{k-1}^{(i)}$

Predicted covariance: $P_{k|k-1}^{(i)} = F_{k-1}^{(i)} \bar{P}_{k-1|k-1}^{(i)} (F_{k-1}^{(i)})' + G_{k-1}^{(i)} Q_{k-1}^{(i)} (G_{k-1}^{(i)})'$

Measurement residual: $\bar{z}_k^{(i)} \triangleq z_k - H_k^{(i)} \hat{x}_{k|k-1}^{(i)} - \bar{v}_k^{(i)}$

Residual covariance: $S_k^{(i)} = H_k^{(i)} P_{k|k-1}^{(i)} (H_k^{(i)})' + R_k^{(i)}$

Filter gain: $K_k^{(i)} = P_{k|k-1}^{(i)} (H_k^{(i)})' (S_k^{(i)})^{-1}$

Updated state: $\hat{x}_{k|k}^{(i)} = \hat{x}_{k|k-1}^{(i)} + K_k^{(i)} \bar{z}_k^{(i)}$

Updated covariance: $P_{k|k}^{(i)} = P_{k|k-1}^{(i)} - K_k^{(i)} S_k^{(i)} (K_k^{(i)})'$

3. Model probability update (for $i = 1, 2, \dots, M$):

Model likelihood: $L_k^{(i)} \triangleq p[\bar{z}_k^{(i)} | m_k^{(i)}, z^{k-1}] = \frac{\exp[-(1/2)(\bar{z}_k^{(i)})' (S_k^{(i)})^{-1} \bar{z}_k^{(i)}]}{|2\pi S_k^{(i)}|^{1/2}}$

Model probability: $\mu_k^{(i)} \triangleq P\{m_k^{(i)} | z^k\} = \frac{\mu_{k|k-1}^{(i)} L_k^{(i)}}{\sum_j \mu_{k|k-1}^{(j)} L_k^{(j)}}$

4. Estimate fusion:

Overall estimate: $\hat{x}_{k|k} \triangleq E[x_k | z^k] = \sum_i \hat{x}_{k|k}^{(i)} \mu_k^{(i)}$

Overall covariance: $P_{k|k} = \sum_i [P_{k|k}^{(i)} + (\hat{x}_{k|k} - \hat{x}_{k|k}^{(i)})(\hat{x}_{k|k} - \hat{x}_{k|k}^{(i)})'] \mu_k^{(i)}$

2.2 Measurement Origin Uncertainty and Data Association

The *measurement origin uncertainty* arises from unreliable measurement(s) obtained by the sensor system. Unreliable measurement(s) may have arisen from an interfering source, including clutter, false alarms, and neighboring targets, as well as the target under track. This situation constitutes the “greatest” challenge for ground target tracking applications. Tracking targets in a measurement origin uncertainty with frequently high density makes the problem much more difficult to solve.

Data Association algorithms[10, 7, 16, 1, 17, 18, 19] are required in situations where target tracking is being attempted with unreliable measurements where measurements of uncertain origin situation (as illustrated in Figure 2.3) appears. Moreover, the target measurements are unreliable and are only present at each scan time with a certain “Probability of Detection (P_D)”. Reliable initiation, confirmation and deletion of tracks under such conditions will be greatly assisted if *data association probabilities* is computed.

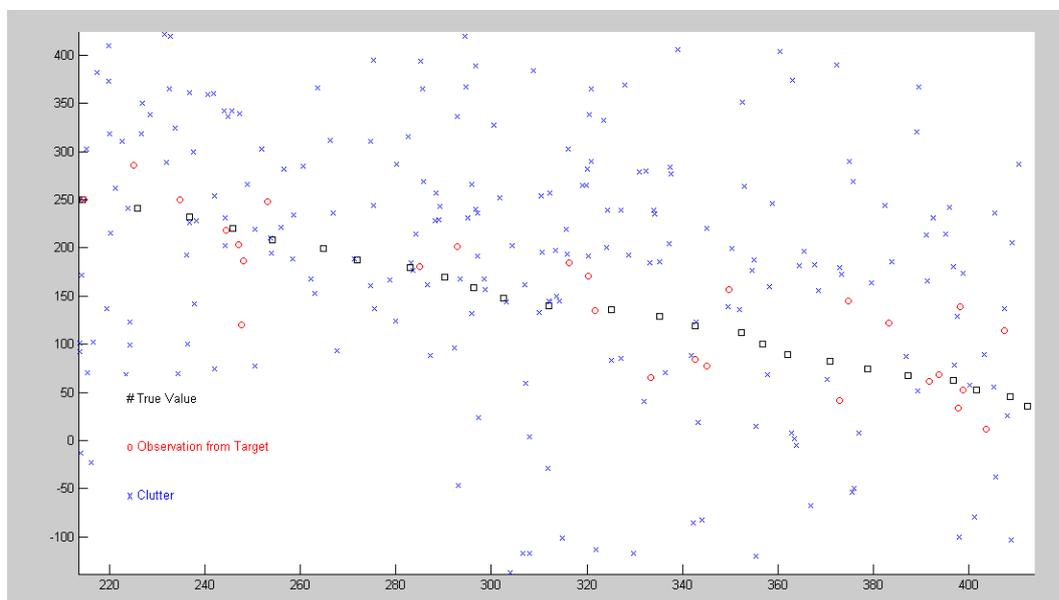


Figure 2.3: Measurements of uncertain origin

2.2.1 Track Management

A fundamental objective of any tracking system is to have one track number associated with each target under track. To achieve this goal, a tracking system must employ a *track management* process to perform a number of tasks related to managing the track database. The track management process and associated functionality can be viewed in numerous ways[1]. For the purposes of this discussion, track management is separated into two basic functions.

The first is concerned with the *initiation* of tracks (i.e. Forming the initial track state and associated covariance matrix). This process also determines the maturity of a track for reporting it to the other elements of the tracking system.

The second track management process relates to track number management and deals with the as-

signment and maintenance of track numbers as track enter and evolve in the database. This process involves a number of subtasks that must be performed to accommodate a wide variety of situations. These processes tend to be a rule based such as a *track score function*[21, 10] rather than algorithmic in nature. The first subtask is to assign new track numbers to tracks that have just been introduced to the system; it then follows sometimes naturally that the last subtask must be to delete the number upon loss of a track. Between *initiation* and *deletion* of track numbers, there are a multitude of number management activities that must be addressed. These include choosing one track number to use when two or more tracks are determined to represent the same target, detecting and resolving track number conflicts in the system; and regulating the recycling of previously used track numbers till *confirmation* as track(s).

2.2.1.1 Track Score Function

Following the approach first developed by Sittler[21], a likelihood ratio is defined for a given combination of data (including a-priori probability data) into a track to be

$$LR = \frac{p(D|H_1)P_0(H_1)}{p(D|H_0)P_0(H_0)} \triangleq \frac{P_T}{P_F} \quad (2.7)$$

Hypotheses H_1 and H_0 are the true and false alarm hypotheses with probabilities P_T and P_F , respectively and D is the data, so that $p(D|H_i)$ is the probability density function (pdf) evaluated with the received data under the assumption that H_i is correct and $P_0(H_i)$ a-priori probability of H_i .

It is convenient to use the *Log Likelihood Ratio (LLR)* [10] such that

$$LLR = \ln \left[\frac{P_T}{P_F} \right] \quad (2.8)$$

Assuming that the accuracy of the measurement process is independent of the target kinematics, the likelihood ratio LR (in Equation 2.7) can be partitioned into a product of two terms, LR_K and LR_S , which represent kinematic and signal-related contributions, respectively. Also, given K scans of data and assuming scan-to-scan independence of the measurement error, LR can be partitioned into a product of terms, $LR(k)$, for each of the K scans. Thus, defining $L_0 = P_0(H_1)/P_0(H_0)$

$$LR(K) = L_0 \prod_{k=1}^K LR_K(k)LR_S(k) \quad (2.9)$$

and the Log Likelihood Ratio (LLR), or score, for a given track is the sum of K kinematic and K signal-related terms. The *track score (L)* is thus defined to be

$$L(K) \triangleq \ln[LR(K)] = L_0 \sum_{k=1}^K [LLR_K(k) + LLR_S(k)] + \ln[L_0] \quad (2.10)$$

If the data received on scan k is considered to have two components which are kinematic as denoted $[D_K(k)]$ such as position measurement and signal related data denoted as $[D_S(k)]$ such as measured target SNR.

Considering the kinematic term, which is assumed to be *Gaussian* distribution for true target returns and a *Uniform* distribution over the *measurement volume* (V_C) for false alarm returns. Then,

$$LR_K = \frac{p(D_K|H_1)}{p(D_K|H_0)} = \frac{\exp\{-d^2/2\}/[(2\pi)^{M/2} \sqrt{|S|}]}{1/V_C} = \frac{\exp\{-d^2/2\}V_C}{[(2\pi)^{M/2} \sqrt{|S|}]} \quad (2.11)$$

where time index has been dropped due to recursion of these operation in each scan, M is the measurement dimension, V_C is the measurement volume element, S is the measurement residual covariance,

can be recalled on Table 2.1 in the previous section, d^2 is the normalized distance for the measurement can be recalled on Table 2.1 in the previous section which has been defined as $d^2 \triangleq (\hat{z}_k^{(i)})'(S_k^{(i)})^{-1}\hat{z}_k^{(i)}$ in model likelihood $L_k^{(i)}$ for model (i) at time instant k .

By considering Equation 2.10, the recursive form of computation of the track score is defined[10] as

$$L(k) = L(k-1) + \Delta L(k) \quad (2.12)$$

where

$$\Delta L(k) = \begin{cases} \ln[1 - P_D]; & \text{no track update on scan } k \\ \Delta L_U(k), & \text{no track update on scan } k \end{cases} \quad (2.13)$$

The increment ΔL_U , occurs upon update in each scans, is the sum of kinematic and signal-related terms

$$\Delta L_U = \Delta L_K + \Delta L_S \quad (2.14)$$

In our case where the only kinematic information related term survives due to that a detection or a miss occurred, so, signal related term become identically zero in Equation 2.14. Finally, the track score increment upon track update becomes

$$\Delta L_U = \ln \left[\frac{P_D V_C}{P_F \sqrt{|S|}} \right] - \frac{[M \ln[2\pi] + d^2]}{2} \quad (2.15)$$

where P_D, P_F are the probability of detection and false alarm respectively.

Defining *false target density* as $\beta_{FT} \triangleq P_F/V_C$ and placed into Equation 2.15, ΔL_U becomes

$$\Delta L_U = \ln \left[\frac{P_D}{(2\pi)^{M/2} \beta_{FT} \sqrt{|S|}} \right] - \frac{d^2}{2} \quad (2.16)$$

2.2.1.2 Track Initiation, Confirmation and Deletion

The initial track score is entirely based on the first observation in the track volume[10, 29, 28]. By considering the Equations 2.12, 2.13, 2.14 and 2.16 as $k = 1$, the initial track score becomes

$$L(1) = \ln \left[\frac{P_D \beta_{NT}}{\beta_{FT}} \right] \quad (2.17)$$

where β_{NT} is the *new target density* which may in general be a function of position in the measurement space. Use of track score for confirmation and deletion is an application of the classical *Sequential Probability Ratio Test (SPRT)*[21] where the LLR required for the SPRT is the track score. Using SPRT, the LLR (or score L) is tested (as illustrated in Figure 2.4) versus upper and lower thresholds T_2 and T_1 respectively. The alternatives to confirm the track, delete track, or continue test are defined to be

$$L \geq T_2; \text{ declare track confirmation}$$

$$T_1 < L < T_2; \text{ continue test}$$

$$L \leq T_1; \text{ delete track}$$

Following the standard SPRT formulation[21], the thresholds are defined as

$$T_2 = \ln \left[\frac{1-\beta}{\alpha} \right], \quad T_1 = \ln \left[\frac{\beta}{1-\alpha} \right]$$

where the specified false decision probabilities are defined as *false track confirmation probability* denoted with α and *true track deletion probability* denoted with β .

The allowable false track confirmation probability, α , can be defined from the system requirements on false track initiation. As an example in [10], by assuming that the system produces N_{FA} false alarms per second and that there are N_{FC} false track confirmations allowed per hour. Then, α , can be defined as

$$\alpha = \frac{N_{FC}}{3600N_{FA}} \quad (2.18)$$

Because β has less effect on the track confirmation threshold[10, 21], its choice is less important but a small value such as $\beta \leq 0.1$ can be used for computation of T_2 . Then, the deletion rule for low-score tracks is best determined based on system track maintenance capability.

The threshold values are chosen on the assumption that the initial track score is zero. Thus, if an initial score value other than zero, such as given by Equation 2.17, should be considered to add to confirmation threshold (T_2).

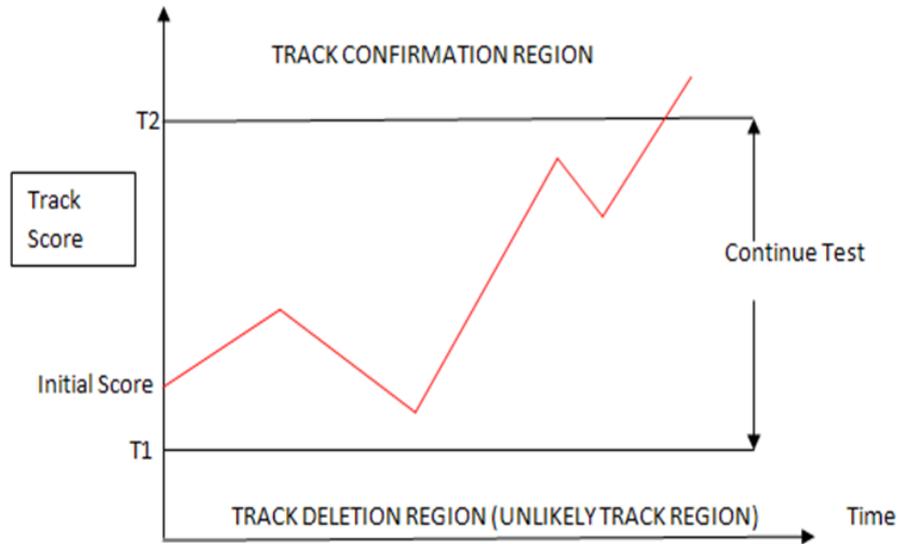


Figure 2.4: Score-Based Track Confirmation and Deletion (Figure is adapted from [10])

In our simulations, in order to compute T_1 and T_2 , we have considered N_{FC} parameter as 3 which is given as an example in [10] and N_{FA} parameter as the maximum number of measurements observed per second which is set to 11 for consistency with our simulations. Generally, these parameters (N_{FC} , N_{FA}) effecting decision threshold settings naturally depend on the true track deletion and false track confirmation statistics and sensor receiver operating characteristics of the real system (*i.e.* radar, sonar, IRST[10]) under concern. If there is no practical information or real data about the system behavior

under concern is available, series of Monte Carlo simulations can be carried out for the choice of these parameters to achieve optimal threshold settings which is also recommended in [10].

2.2.2 Validation of Measurements (Gating)

Gating is a technique for eliminating unlikely measurement-to-track pairings. A gate is formed about the predicted measurement and all observations that fall within the gate are considered for track update. Gate is often called with a name *validation region* in literature and gating is the *validation of measurements* falling inside the gate which are used for track update. The manner in which the observations are actually chosen to update the track depends on the data association technique which will be discussed later. However, most data association methods utilize gating in order to reduce further computation.

If it is recalled from Table 2.1, at $k - th$ time instant, the measurement residual, for simplicity let us omit superscript (i)'s and denote as \tilde{z}_k , is the difference between the *actual* measurement vector z_k and the *expected* measurement vector \hat{z}_k where

$$\hat{z}_k = H_k \hat{x}_{k|k-1} - \bar{v}_k \quad (2.19)$$

\bar{v}_k is assumed Zero-mean White Gaussian measurement noise with covariance matrix R_k . The residual covariance matrix is defined in Table 2.1 as

$$S_k = H_k P_{k|k-1} (H_k)' + R_k \quad (2.20)$$

where $P_{k|k-1}$ is the one-step prediction covariance matrix.

If the dimension of measurement is assumed to be M and the normalized distance for the measurement is defined as $d^2 \triangleq (\tilde{z}_k)' (S_k)^{-1} \tilde{z}_k$, model likelihood L_k can be rewritten as

$$L_k = \frac{\exp\{-d^2/2\}}{[(2\pi)^{M/2} \sqrt{|S_k|}]} \quad (2.21)$$

where $|S_k|$ is the determinant of S_k .

Gate is defined such that association is allowed if the following relationship is satisfied by the norm(d^2) of the residual vector

$$d^2 = (\tilde{z}_k)' (S_k)^{-1} \tilde{z}_k \leq G \quad (2.22)$$

where G is defined as *Gating Threshold* for ensuring that the target-originated measurement falls in the validation region with a probability P_G , *Gating Probability*.

d^2 is typically assumed[10] to have the chi-square (χ_M^2) distribution for M degrees of freedom. The threshold G is often referred to as the number of sigmas or standard deviations for the gate and G is determined from a χ_M^2 table[7, 10].

The *Volume* within the gate is given in [10] as

$$V_G(M) = C_M \sqrt{|S_k|} G^{M/2} \quad (2.23)$$

where M is the dimension of a valid observation satisfying the threshold G and

$$C_M = \begin{cases} \frac{\pi^{M/2}}{(\frac{M}{2})!}, & M \text{ even} \\ \frac{2^{M+1} (\frac{M+1}{2})! \pi^{\frac{M-1}{2}}}{(M+1)!}, & M \text{ odd} \end{cases} \quad (2.24)$$

In our simulations, we consider elliptical validation gate[1, 10] for a two-dimensional measurement. So, 2.23 turns into

$$V_G(2) = \pi \sqrt{|S_k|} G \quad (2.25)$$

G can be computed correspondingly via χ_M^2 table[7, 10] in terms of P_G as

$$G = -2 \ln(1 - P_G) \quad (2.26)$$

For other gating types (**i.e.** Rectangular[1, 10], Maneuvering[49, 10],...etc.) and detailed information about gating is given in [49, 10].

2.2.3 Optimal Assignment Algorithms

The assignment problem was originally considered for problems in economic theory such as assigning a personnel to jobs and delivery trucks to locations. The objective in these problems is to minimize the cost using the available resources[10]. This is done as a constrained optimization where in our case in target tracking the cost of associating the measurements to target tracks is minimized subject to certain feasibility constraints. This optimization problem can be solved using number of algorithms[1] (**i.e.** Auction Algorithm[11, 1, 10], Jonker-Volgenant-Castanon Algorithm(JVC)[1], N-Best Solutions[10],...etc.). In our simulations, we consider to use Auction Algorithm[11, 1, 10] due to its superiority to rest of the algorithms where the detailed information is given in [11], analysis and simulation results are given in [1].

The elements in the measurement-to-track assignment matrix are best chosen to be the score gains associated with the allowed assignment (validated measurements fall inside the gate). Alternatively, the elements can be chosen to be the gate value(G) minus the normalized distance(d^2) [10].

Outline of the steps involved in the Auction Algorithm is given in [10] where the detailed analysis is available at [11].

For Multitarget tracking, *Modified Auction Algorithm*[11, 1] is required to solve the generalized assignment problem. Modified Auction Algorithm is nothing more than a generalization of the classical Auction Algorithm mentioned above. In a Multitarget tracking situation, "each" track wants to be assigned to a measurement that minimizes its cost individually.

2.2.4 Data Association

Since the pioneering work of Sittler[21], who provided the term *data association* to literature, a number of algorithms have been developed[10, 16, 7, 1, 17, 18, 19] over the past three decades to solve the measurement origin uncertainty problem. Two simple solutions been proposed were the *Strongest Neighbor Filter (SNF)* and the *Nearest Neighbor Filter (NNF)*. In the SNF, the signal with the highest intensity among the validated measurements (in a gate) is used for track update and the others are discarded. In the NNF, the measurement closest to the predicted one is used to update the target states. While these simple data association techniques work reasonably well with benign targets in sparse scenarios[1], they begin to fail as the false alarm rate increases or with low probability of detection, or with low or partial observability (**e.g.** passive sensors that measure only lines of sight). Instead of using only one measurement among the received ones and discarding the others, an alternative approach is proposed which is known as *Probabilistic Data Association (PDA)*. PDA uses all the latest validated measurements with different weights[7, 22]. The standard PDA and its numerous improved versions have been shown in [7] to be effective in tracking a single target in clutter.

PDA is a widely used recommended[7] method for data association when tracking a single target in clutter, however, it is derived under the assumption that a track exist in the validation region(gate) at each scan with a certain gating probability P_G which is very close to 1 and consequently is unable to provide the probability of track existence information for unreliable target measurements. In [28, 29], PDA algorithm is rederived without an initial assumption of track existence and the resulting algorithm

is named *Integrated Probabilistic Data Association (IPDA)* which simultaneously and recursively provides expressions for both probability of track existence and data association.

Data association becomes more difficult with multiple targets where the targets compete for measurements. Hence, in addition to a track validating multiple measurements as in the single target tracking case, a measurement itself can be validated by multiple tracks (*i.e.* one faces contention among tracks for measurements). Many algorithms[1, 10, 18, 19] exist in the literature to handle this contention. To prevent excessive information, this topic will be fully covered and discussed in detail in Section 2.4.

2.3 Tracking Single Target in Clutter

In a target tracking problem, if more than one measurements are observed from the surveillance environment under concern at a current scan, which of the measurements should be used to update each track is a crucial problem. This problem appears especially when tracking target(s) with probability of detection less than unity and in the presence of false alarms.

Primary common approach was the nearest neighbor (NN) method using only one measurement (the nearest) among all observation and discard all the rest[10, 1]. Unfortunately, this simple solution results in undesired estimation errors[7]. Because of the fact that the target tracking system does not know a priori which is the correct measurement among all observation. Validation region(gating) approach[10], which reduces less likely hypotheses, is applied to use sufficient number of measurements instead of all observation per scan. As a solution to the measurement origin uncertainty problem, Probabilistic Data Association(PDA)[1, 16, 22], has been proposed.

The Interacting Multiple Model (IMM) Estimator is recommended as a powerful method[1, 4] to encounter target motion origin uncertainty. IMM is capable of dealing with target maneuvers by introducing a set of different state space models to describe the possible target behaviors via Markov switching between the models and reinitialization of estimates from recursive model based filters.

So far, the problems have been faced in both kinematic state estimation and data association for tracking a single ground target in a cluttered environment have been investigated separately. Although, the problems and proposed solutions to problems are seemed to be discrete naturally, in order to find a complete solution to tracking of a single ground target in a cluttered environment problem, the solutions should be logically fused to make them incorporate to achieve a complete solution. The most common known method has been proposed by Bar-Shalom et.al to achieve a complete solution to tracking a single target in clutter problem is given as a combination of IMM with PDA is called IMMPDFAF[7], that extends PDA to include a measure of track quality. Track quality measure is used for false track discrimination. It is shown[7] that IMMPDFAF and its variants solve tracking problems such as presence of a clutter and maneuvering nature of the target with disappearances and reappearances successfully. For convenience with other sections and chapters, IMMPDFAF will be denoted as IMM-PDA where IMM is the approach considered for kinematic state estimation as the solution for target motion origin uncertainty and PDA is the approach considered to achieve the right measurement-to-track assignment as the solution of measurement origin uncertainty problem.

2.3.1 Combining IMM with a Probabilistic Data Association Algorithm

One important feature of the PDA[22] approach or any Probabilistic Data Association based algorithms such as IPDA[28], JPDA[23], LIMPDA[19],...etc. is the relatively straightforward manner in which either of them can be combined with IMM filtering (as described in Section 2.1). These methods are discussed in more detail in [7, 18, 35], however, this Section only summarizes the pioneer method IMMPDFAF[7], denoted as IMM-PDA (illustrated on flowchart on Figure 2.5), as an example and for

the rest, in fact, the same process is in effect with Probabilistic Data Association algorithm differs only. If we assume that a track has been formed at $(k-1)$ th scan and there are M IMM Filter models (on Table 2.1). Given the data received through scan $k-1$, each IMM filter will have its model probability $\mu_{k-1}^{(i)}$, state prediction $\hat{x}_{k|k-1}^{(i)}$ and Kalman Filter Covariance matrix $P_{k|k-1}^{(i)}$ for use with the next data set (scan k). Then, the next step is to define a validation region (gate) in order to determine which observations are to be considered for track update.

Given the new data (at scan k), the IMM-PDA process is defined by following steps (also illustrated on flowchart on Figure 2.5):

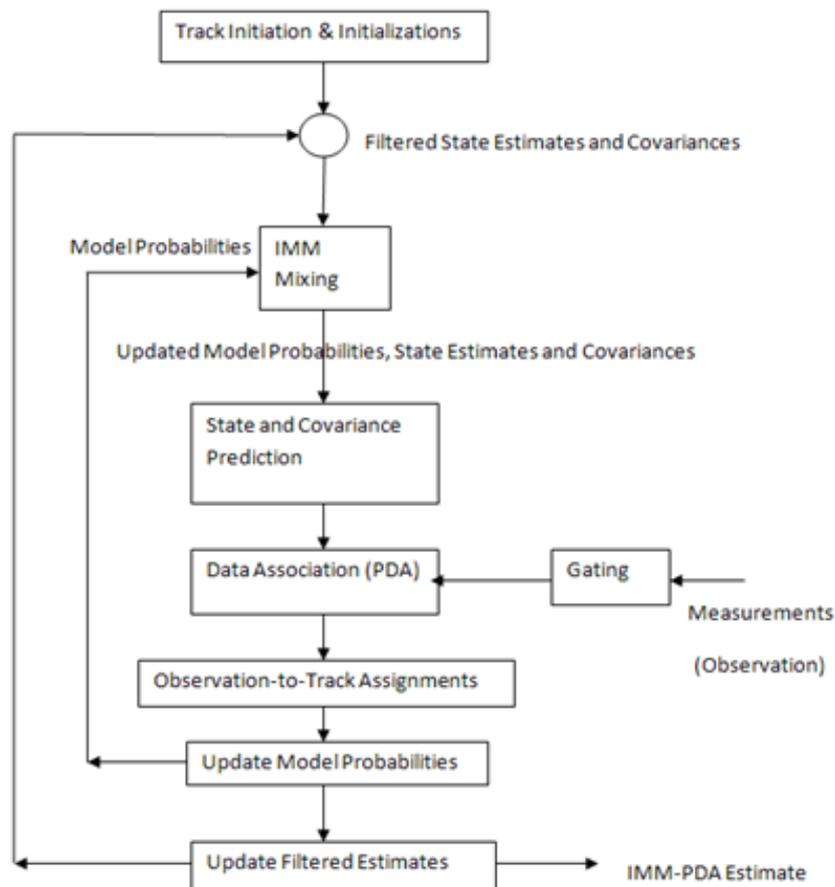


Figure 2.5: IMM-PDA Flowchart

1. Compute a likelihood function $L_k^{(i)}$ for each IMM model i according to an extension on Table 2.1 that includes the N data association hypotheses corresponding to each observation ($m = 1, 2, \dots, N$) in the gate and the hypothesis that none of the observations is valid:

$$L_k^{(i)} = (1 - P_D P_G) \lambda + \sum_{m=1}^N \frac{P_D \exp[-d_{im}^2/2]}{2\pi \sqrt{|S_k^{(i)}|}} \quad (2.27)$$

where λ is the spatial density of the clutter (assumed known), P_D is the probability of detection and P_G is the gating probability defined in Section 2.2.2 as the probability of the target-originated measurement falling inside the validation region (gate).

2. Given the prior probabilities $\mu_{k-1}^{(i)}$ and the likelihood functions $L_k^{(i)}$ from (2.27), compute the updated model probabilities $\mu_k^{(i)}$ on Table 2.1.
3. Update the state estimates and covariances matrices for each IMM filter model according to the PDA relationships given in [1], also will be given in Section 3.1.1.
4. Given the state estimates $\hat{x}_{k|k}^{(i)}$ and the Kalman Filter covariance matrices $P_{k|k}^{(i)}$, for each model i , the composite state and covariance matrix can be computed using the standard IMM relationships given on Table 2.1.

The method outlined above is for *parametric* case which requires a priori knowledge of λ , the spatial density of the clutter. If the spatial density of the clutter is not known a priori, where in *non-parameric* case, λ can be computed as $\lambda = N/V_G$ where V_G is the gate volume defined in Section 2.2.2 and N is the number of observations in the validation region.

2.3.2 Track Initiation and Deletion

The original PDA[22] method did not include explicit provisions for track initiation and deletion. It was implicitly assumed that tracks had been established and the main issue was track maintenance. Since then, IMM-PDA[7] method is modified to handle this issue via using "target" and "no target"[7] models and Integrated PDA (IPDA) is derived in [29, 28] under "track exists" and "track does not exist" possibilities taken into account via probability of track existence parameter computed recursively as an extra state.

[29, 28, 18] has shown that in IPDA-based algorithms[18], probability of track existence parameter is also considered in track initiation and deletion operations in where employed as a track score. However, in our simulations, we consider this parameter for track maintenance purpose in order to update data association probabilities recursively. Instead of using probability of track existence parameter as in the references [29, 28, 18], for track initiation, confirmation and deletion operations, LLR testing under SPRT procedure[21](also discussed previously in Section 2.1.1.1) is preferred in our simulation studies.

2.4 Multitarget Tracking in Clutter

2.4.1 Multitarget Background

Multitarget tracking is used in surveillance systems to provide both a unified and comprehensive picture of the environment using data reported by sensors. The goal of an Multitarget tracking system is to form and maintain tracks on targets of interest from scans of measurements provided by the sensors. The Multitarget tracking problem is made more difficult by maneuvering targets and by the presence of clutter. An estimator such as a Kalman Filter is often used to generate the state estimates contained in the track files. However, if precautions are not taken, the accuracy can degrade during maneuvers to the point where the state estimates are less accurate than the unfiltered measurements. Before the state estimator can be used to update the tracks, some form of data association is needed to assign the measurements to the tracks, or to declare certain measurements as being false (i.e. clutter detections). There is the potential for assigning clutter detections to target tracks, and for closely-spaced targets, there is the potential for assigning a target's measurement to another target's track. Merged (i.e. unresolved) measurements can also occur for closely-spaced targets. Assigning wrong measurements to tracks often results in lost tracks and track breaks. Moreover, clutter can produce false tracks, and if the clutter density is sufficiently large, especially in ground target tracking, the resulting number of false tracks can overwhelm the available computational resources of the Multitarget tracking system, as well as degrade the overall performance of the system. For these reasons, techniques dealing with maneuvering targets and techniques dealing with data association have received much attention in Multitarget tracking research where the recent publications referred[1, 10, 18, 19] are the obvious evidence.

2.4.2 Kinematic State Estimation Background

The Kalman Filter has some ability to adapt to maneuvers by tuning the Kalman Filter to the most stressing maneuver expected[1]. However, for targets that are not maneuvering or maneuvering at a level less than the most stressing maneuver, this approach results in less noise reduction than could be achieved with a Kalman Filter tuned to a less stressing maneuver. In many Multitarget tracking systems, the Kalman Filter is augmented with a maneuver detector[1, 4]. The Kalman Filter is designed for a relatively benign maneuver to give adequate noise reduction when targets are not maneuvering, and the maneuver detector is used to adapt the filter to maneuvers and provide improved tracking performance through maneuvers. A problem encountered in practice with this approach is effective and reliable maneuver detection. There is a time lag between the actual onset and the detection of the maneuver, and a time lag between the detection and the actual end of maneuver. These lags typically last for several scans and large state errors can arise during these lags. In addition, random noise can trigger the maneuver detector and there is a time lag before switching back to the nonmaneuver model. During this lag, the noise reduction will be less than could have been obtained with the filter used for the target's nonmaneuver mode. Also, the combination of maneuver detection with data association requires heuristics and results in lack of robustness[1, 4].

In the *Interacting Multiple Model (IMM)* Estimator, a probabilistic (Markov) switching between these models is assumed[1, 4]. During one sampling period, one of the models may describe concerned target's motion, but over another sampling period, a different model may describe the more appropriate one. Therefore, during maneuvers, the IMM Estimator typically produces smaller state errors and lags of smaller duration than Kalman Filter with maneuver detectors.

2.4.3 Multitarget Data Association Algorithms - Survey

Another and the most important part of the Multitarget tracking problem where a great deal of effort is required, the inherent problem of measurements of uncertain origin. In Multitarget tracking systems, besides tracking of single target in undesired measurements (i.e. clutter), the problem of inferring from which target, if any, a particular measurement originates is of most interest. In order to solve this problem, many different algorithms are available[1, 10, 18, 19] for Multitarget tracking systems, starting from the simpler nearest-neighbor approaching to the very complex *Multi Hypothesis Tracker (MHT)*[10]. The simpler techniques are commonly used in Multitarget tracking systems, but their performance degrades in clutter significantly[1, 10]. MHT[10] provides improved performance, but it is difficult to implement and in clutter environments a large number of hypotheses may have to be maintained, which requires extensive computational resources[1]. Because of these difficulties, recursive algorithms having smaller computational requirements has been developed[1, 18, 19]. These techniques are based on *Probabilistic Data Association (PDA)*, which uses a weighted average of all the measurements falling inside a track's validation region (gate) at the current time to update the track state.

2.4.3.1 Optimal Multitarget Data Association Algorithms

Probabilistic Data Association Filter (PDAF) has been developed[22] as the first PDA algorithm to track a single target in clutter. PDAF is derived under the assumption of a single target (single track). In multiple target situations, each measurement can be either clutter or a measurement of the target being followed. However, in real-life situations with multiple targets with crossing trajectories and/or closely-spaced movements, this assumption holds no longer true. It has been shown in [23, 24] that PDAF can get "confused" under these circumstances and start following a different target, or it can diverge altogether and stop following any target. To compensate this situation, the *Joint Probabilistic Data Association (JPDA)* algorithm has been developed[23, 24]. Then, the PDAF has been extended to multitarget case, resulting in *Joint Probabilistic Data Association Filter (JPDAF)*[1]. It generates all possible joint measurement to track assignments, which are called *joint events*, and calculates the a-posteriori probability of each joint event. From these probabilities, the data association probabilities of each track are calculated and then used to update the state estimates.

Although the JPDAF shows much better performance than the simpler data association techniques and requires less computational resources than the MHT, the JPDAF can be difficult to implement and still requires extensive computational resources in environments with more than two closely-spaced targets. In addition to this, a number of problems (e.g. track biases and coalescence) with JPDAF have been documented[25].

To circumvent some of the complexity and problems associated with the JPDAF, various approximations to the JPDAF has been developed. To reduce the computation requirements of the JPDAF, the *Cheap JPDA* has been developed[6], which approximates the association probability computations in the JPDAF. More accurate approximations for the association probability computations has then been presented in [26] as *Suboptimal JPDA*. To avoid some of the problems with the JPDAF, such as track coalescence, the *Nearest-Neighbor JPDA (NNJPDA)* has been developed[6]. The NNJPDA abandons the updating of a track with a weighted average of all measurements in its validation region (gate); in favor of one-to-one assignment of measurements to tracks. It is only the association probabilities that are used in making the one-to-one assignments of measurements to tracks. This approach greatly simplified the logic and improved the performance of the JPDAF[1, 6]. Results in [1] show that using the association probabilities rather than the likelihoods (or, equivalently, the normalized distances) to make the assignments dramatically improves the association performance provided that an appropriate

approximation to JPDAF such as the Suboptimal JPDA is used to compute the probabilities. While the Nearest-Neighbor (NN) and JPDA methods use different criteria to assign an “optimal” measurement to track association in a single scan, as mentioned in [27], most operating radar systems are still using the independent NN methods[10] such as SNN, GNN,...etc. due to some concerns. The main concern is the practicality of these methods since most of them are based on mathematical analysis and computer simulations under some idealistic assumptions. They are developed under various assumptions about statistical models of process and measurement noise, and clutter. They are often evaluated using simulated data generated under the same assumptions their development are based on. According to study at [27], using actual radar data in stressful conditions such as heavy clutter, closely spaced targets, targets crossings, maneuvers, missed target detection,...etc., their usefulness in real stressful radar tracking environments has proven to be questionable since their underlying theoretical assumptions are not always valid. Although the weighted sum approach of JPDA methods which has been shown in [1, 6] theoretically effective, results in [27] shows that in real stressful radar tracking environments they show poor performance in tracking closely spaced maneuvering targets. They are also observed susceptible to deviations from the assumed clutter model. In addition to these, it has been shown that JPDAF could not be implemented in a modern radar tracking system with current computing power. It has been concluded in [27] that, using one of the independent NN methods[10] such as SNN, GNN,...etc. with an optimal assignment technique such as Munkres algorithm[10] could be more robust to inaccuracies in clutter model than JPDA methods.

In addition to these deficiencies, all the JPDA methods are based on pre-assumption that the target(s) exist(s) in each scan like PDA. Tracks are not differentiated according to the probability of target existence and track maintenance is difficult without the probability of target existence information. JPDAF is also rather complex as mentioned in [27] because it creates a joint event for each possible combination of measurement to track assignments [1]. The number of joint events can grow exponentially in a dense clutter situation. Another problem is that the area of each cluster is assumed to be encompass the whole surveillance region. To improve upon JPDAF, the *Integrated JPDAF (IJPDAF)* algorithm has been proposed[38]. It builds upon IPDA algorithm proposed in [39] and also uses the probability of target perceivability to develop recursive expressions for the a-posteriori probability of target perceivability and data association for each track. In IJPDAF, the number of joint events is much higher than in case of JPDAF[38].

The *Joint IPDA (JIPDA)* algorithm[30, 34] (dealing with “joint” IPDA tracks) is developed in a similar fashion to the IPDA algorithm given in[28, 29]. It uses the probability of target existence and results in recursive expressions for the probability of target existence and data association probabilities. The number of joint events is the *same* as in the case of JPDAF. However, JIPDA still has the same complexity as JPDAF, which may preclude it from being used on all tracks in a dense clutter situation.

2.4.3.2 Linear Multitarget Data Association Algorithms

Optimal approaches, methods, or algorithms, has been mentioned so far[23, 24, 38, 30, 34], generate and evaluate all possible hypotheses of measurement origin in the current scan whereas the number of these hypotheses grows exponentially with the number of tracks and the number of measurements involved. As the number of such hypotheses grows exponentially with the number of scans, these approaches are not used in practice [27], especially in cases where a large number of targets are close to each other, or in a dense clutter situation with a large number of false tracks. Instead, various suboptimal data association algorithms such as in [6, 26] have been proposed with an inevitable performance penalty.

A notable exception to these algorithms, which has been proposed in [31], is the *Linear Joint Integrated Probabilistic Data Association (LJIPDA)* algorithm, which is basically a multitarget version of

IPDA[29, 28] with only a “linear” number of operations within the number of tracks and the number of measurements in present. LJIPDA uses *a-priori probabilities of measurement origin* to calculate (for each track and for each measurement) the probability that *the measurement belongs to some other track*. The a-priori probabilities of measurement origin are the major conduit for *inter-track information transfer*. These probabilities are then used to calculate the probability of track existence and data association probabilities. In this manner, multitarget tracking is achieved without exhaustive measurement-to-track hypothesis processing. Rather than forming joint events by creating all possible combinations of measurement-to-track assignments, only a single track is processed at a time. Therefore, the number of operations is linear in the number of tracks and the number of measurements. This important property permits target tracking in much denser clutter or closely-spaced target situations by using less computational resources than optimal approaches such as JPDA, IJPDA, or JIPDA.

By the following of this pioneering approach, a generic procedure has been proposed in [33], namely *Multi-target Linear Converter (MLC)*, which converts any single target data association algorithm belonging to a certain class, into an equivalent multitarget data association algorithm using a number of operations which is linear in the number of tracks and the number of measurements as in LJIPDA. MLC uses a similar approach to LJIPDA by using the probabilities of measurement origin as a conduit for *information exchange* between tracks. The difference is that MLC uses “*a-posteriori*” *probabilities of measurement origin* calculated by the single target data association algorithm considered as a core. It corrects these probabilities to allow for multitarget existence and uses them directly to calculate the probability of track existence and data association probabilities for each track. Thus, MLC simply converts single target data association algorithms into multitarget data association algorithms. The only requirement for MLC on the single target data association algorithm is that it must provide the a-posteriori probabilities of measurement origin information. Single target data association algorithms such as IPDA[29, 28], IMMPDFAF[22, 7], IPDA[39] can be considered.

Both MLC and LJIPDA algorithms achieves multitarget data association capability by *splitting* the measurements according to the a-posteriori or a-priori probabilities of measurement origin. In situations when a measurement is allocated to multiple tracks, it is “split”, and each track uses a “fragment” of the measurement[35]. The algorithm presented in [35, 32, 18, 19] is also a multitarget data association algorithm with a linear number of operations in the number of tracks and the number of measurements, with apparently negligible performance penalty compared to optimal approaches such as JIPDA[32]. However, the difference between both MLC and LJIPDA, it is derived by *modifying the clutter density with the foreign target measurement density*. The resulting new approach is called *Linear Multi-Target (LM)* procedure[35, 32, 18, 19]. It is a general procedure for converting certain class of single target data association algorithms into multitarget data association algorithms. In effect, the LM approach is to run a bank of “coupled” single target data association filters, where the coupling is achieved through modifying the clutter density for each tracking filter. The clutter density at each measurement point is modified by the pdf of measurements originating from neighboring tracks. Briefly, other tracks are treated as additional clutter sources[35]. This coupling eliminates most of the problems experienced when running single target data association filters in a multitarget tracking situation with very little additional computational cost. Use of the LM method means “each measurement” in the current scan may potentially be used to update more than one track which has been shown[32, 19] to be better performance than previously proposed algorithms, LJIPDA and MLC, using a “fragment” of the measurements in track coalescence situations. The differences in implementation complexity between a single target tracking data association algorithm and its LM equivalent are also very small. In LM method, single target tracking algorithms which can be converted are those which provide an *a-priori* probability that the target detection is selected such as IPDA[28]. When this method is applied to a single target data association algorithm, the resulting multitarget data association algorithm is recognized by the prefix “LM”, which stands for Linear Multi-Target such as LMIPDA[35, 18].

CHAPTER 3

ALGORITHMS IN COMPARISON

Even without data association problems, estimation of the kinematic state of the target(s) under track accurately is a crucial problem because target(s) can maneuver at unknown times. The Interacting Multiple Model (IMM) method is an efficient algorithm which deals with this difficulty.

Six single-scan algorithms are considered in comparison in our thesis study for automatic tracking of maneuvering ground target(s) in clutter situations under various test scenarios. They are termed: IMM-PDA, IMM-IPDA, IMM-JPDA, IMM-IJPDA, IMM-JIPDA and IMM-LMIPDA. All of them use the IMM method to estimate the kinematic state of target(s) under track. Target kinematic state estimation in a cluttered environment is achieved by combining IMM with a specified Probabilistic Data Association Algorithm incorporate which has been outlined briefly in Section 2.3.1 previously. By considering IMM-PDA process as an example in Section 2.3.1 (process is illustrated with a flowchart on Figure 2.5), the overall process is identical in all algorithms. The algorithms differ only in the calculation of the data association probabilities.

So, to prevent excessive information and mathematical equations burden, in this Chapter, it is sufficient to present only Data Association Algorithm part of algorithms(IMM-PDA, IMM-IPDA, IMM-JPDA, IMM-IJPDA, IMM-JIPDA and IMM-LMIPDA) considered which are basically: PDA, IPDA, JPDA, IJPDA, JIPDA and LMIPDA algorithms, respectively. For each data association algorithm, the update of related IMM parameters are also given at the end of each sections, respectively.

3.1 Single Target Data Association Algorithms

3.1.1 Probabilistic Data Association (PDA) Algorithm

The *Probabilistic Data Association (PDA)* is a Bayesian approach that computes the probability that each measurement in a track's validation region (gate) is correct measurement (or, target originated) and the probability that none of the validated measurements is target originated. These probabilities and all of the validated measurements are then used in a kinematic state estimator (*i.e.* Kalman Filter, Particle Filter, IMM,...etc.) to update the target state.

PDA assumes that a *single target is present* and a *track related to that target exists* for the target (track has been initialized), that at most one of the validated measurements is target originated and that rest of the validated measurements are clutter detections.

The clutter detections are modeled as independent, identically and *uniformly distributed* random interference in space, whereas, target originated measurement is assumed to have *Gaussian distribution*. By an inference, the discrimination capability of PDA arises from the statistical difference between the Gaussian and uniform distributions[7].

There are two versions of the PDA algorithm depending on the stochastic model used for the number of clutter detections in each scan. The *parametric* PDA assumes that the number of clutter detections in each scan is modeled with *Poisson* distribution, whereas, *nonparametric* PDA assumes that the number of clutter detections in each scan is modeled with *diffuse* distribution[5], which means that any number of clutter detections is equally likely. The parametric version of PDA requires prior knowledge of the *spatial density of the clutter*, which we denote with λ , whereas the nonparametric version does not. Although the number of clutter detections in each scan can be modeled differently with either of two stochastic models, it does not affect the fundamental assumption that *the spatial distribution of the clutter is assumed to be uniform*.

In our simulations, we consider using the parametric version of PDA because we model the number of clutter detections in each scan with a Poisson distribution with a rate $\lambda = 10$ where the clutter detections, themselves, are uniformly distributed with that rate in space (on two-dimensional surveillance region).

PDA algorithm from [16, 22, 7, 1] is briefly outlined below:

For m_k measurements falling inside the validation region (gate) at scan k , the probability that the j^{th} validated measurement $z_{k,j}$ is target originated, denoted with $\beta_{k,j}$, is

$$\beta_{k,j} = \frac{e_{k,j}}{b_k + \sum_{l=1}^{m_k} e_{k,l}} \quad (j = 1, \dots, m_k) \quad (3.1)$$

whereas the probability that none of the measurements is target originated, denoted with $\beta_{k,0}$, is

$$\beta_{k,0} = \frac{b_k}{b_k + \sum_{l=1}^{m_k} e_{k,l}} \quad (3.2)$$

The term $e_{k,j}$ in both (3.1) and (3.2) is given in [16] as

$$e_{k,j} = \exp \left\{ -\frac{1}{2} \nu_{k,j}^T (S_k)^{-1} \nu_{k,j} \right\} \quad (j = 1, \dots, m_k) \quad (3.3)$$

where $\nu_{k,j}$ is the residual for the j^{th} validated measurement and S_k is the residual covariance (on Table 2.1, let us omit (i) 's for simplicity) for the measurements. All measurement residuals are assumed to have the same covariance.

The term b_k in both (3.1) and (3.2), which accounts for the possibility that none of the validated measurements is target originated and that the target originated measurement was not detected (or fell outside of the gate), is given in [1] as

$$b_k = \lambda \sqrt{|2\pi S_k|} \frac{1 - P_D P_G}{P_D} \quad (3.4)$$

where $|2\pi S_k|$ is the determinant of $2\pi S_k$, λ is the spatial density of the clutter (assumed known), P_D is the probability of detection, P_G is the gating probability defined in Section 2.2.2 as the probability of the target-originated measurement falling inside the validation region (gate).

In PDA, the kinematic state of the target is updated using **all** of the validated measurements. The update is given by

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \nu_k \quad (3.5)$$

instead of $\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \tilde{z}_k$ line on Table 2.1, where $\hat{x}_{k|k}$ is the updated state, $\hat{x}_{k|k-1}$ is the predicted state K_k is the Kalman gain and ν_k is the *combined residual*, which is given by

$$\nu_k = \sum_{j=1}^{m_k} \beta_{k,j} \nu_{k,j} \quad (3.6)$$

where $\nu_{k,j}$ is the residual for the j^{th} validated measurement

$$\nu_{k,j} = z_{k,j} - H_k \hat{x}_{k|k-1} \quad (3.7)$$

The updated covariance is given by

$$P_{k|k} = \beta_{k,0}P_{k|k-1} + (1 - \beta_{k,0})P_k^c + \tilde{P}_k \quad (3.8)$$

where

$$P_k^c = P_{k|k-1} - K_k S_k K_k^T \quad (3.9)$$

and

$$\tilde{P}_k = K_k \left[\sum_{j=1}^{m_k} \beta_{k,j} \nu_{k,j} \nu_{k,j}^T - \nu_k \nu_k^T \right] K_k^T \quad (3.10)$$

3.1.2 Integrated Probabilistic Data Association (IPDA) Algorithm

PDA assumes that a target track exists in each scan, so it cannot be used for track quality measure concept. Because, in reality, target may become unobservable or target track may not exist depending on terrain conditions and obtained measurements in each scan.

Musicki et al.[29, 28] rederived the PDA algorithm but without the assumption that a target track exist and introduced a track quality measure, the *Probability of Track Existence*, integrated with PDA, which resulted in the *Integrated Probabilistic Data Association (IPDA)* algorithm. IPDA assumes that track existence is an event with a corresponding probability, the *Probability of Track Existence*, whereas PDA assumes track existence is certain which effectively removes the target existence information.

IMM implementations with the IPDA, have first appeared in [41, 42], resulting in IMM-IPDAF algorithms, where the IPDAs in the IMM structure were based on different motion models for the target. A different approach was proposed by the authors in [39, 40] where it is assumed that a target exists “behind” each track and the *Probability of Perceivability of the Target* is recursively calculated instead of the Probability of Track Existence as the track quality measure. Recently, Musicki et al. have proposed another IMM implementation with the IPDA, resulting in IMM-IPDA algorithm [36], differently from [41, 42], in which there is a single IPDA for all models in the IMM structure, as in the IMM-PDA structure discussed in the Section 2.3.1. In spite of the differences between these algorithms[41, 42, 39, 40, 36], all the authors “just to defy” have elected to use the same name for their algorithms. To differentiate between the algorithms we have tried giving references properly which algorithm belong to which author, which algorithm is used or considered in simulations, results and comparison throughout the study.

In [36], IMM-PDA[6], IMM-IPDAF[41] and IMM-IPDA[36] algorithms are compared and the results in [36] indicates that IMM-IPDA[36] outperforms both IMM-PDA[6] and IMM-IPDAF[41] both in terms of estimation accuracy and true/false track statistics.

By considering the result presented in [36], in our study, we have implemented IMM-IPDA algorithm[36], in which there is a single IPDA for all models in the IMM structure, as in the IMM-PDA structure discussed in the Section 2.3.1. IPDA algorithm used in [36] and also in our study is the version given in [29, 28] which is given below:

To differentiate between the true and false tracks, a track quality measure is defined in the IPDA[29, 28] structure which is named as *Probability of Track Existence*, denoted with $\psi_{k|k}$. This parameter is computed as an output of the IPDA algorithm recursively as an extra state in each scan k for track update. The propagation model of track existence is assumed in [29, 28] as a Markov Process with two possible cases:

The first model, which is defined as *Markov Chain One model* in [28], models the probability that the track exists at scan $k - 1$ with $\psi_{k-1|k-1}$ and the probability that the track does not exist at scan $k - 1$ with $1 - \psi_{k-1|k-1}$. The propagated values, which is defined as a *priori* probabilities of track existence,

are obtained at scan k from a *posteriori* probabilities of track existence from scan $k - 1$ as

$$\psi_{k|k-1} = \Pi_{11}\psi_{k-1|k-1} + \Pi_{21}(1 - \psi_{k-1|k-1}) \quad (3.11)$$

$$1 - \psi_{k|k-1} = \Pi_{12}\psi_{k-1|k-1} + \Pi_{22}(1 - \psi_{k-1|k-1}) \quad (3.12)$$

where track existence is assumed to evolve and recursively updated over time as a Markov Chain with a specified transition probability matrix Π with entries are defined as

$$\Pi_{11} = P\{\text{Track exists at scan } k | \text{Track exists at scan } k - 1\} \quad (3.13)$$

$$\Pi_{12} = P\{\text{Track does not at scan } k | \text{Track exists at scan } k - 1\} \quad (3.14)$$

$$\Pi_{21} = P\{\text{Track exists at scan } k | \text{Track does not exist at scan } k - 1\} \quad (3.15)$$

$$\Pi_{22} = P\{\text{Track does not at scan } k | \text{Track does not exist at scan } k - 1\} \quad (3.16)$$

The second model, which is defined as *Markov Chain Two model* in [28], which distinguishes three possibilities, models the probability that the track exists and observable at scan $k - 1$ with $\psi_{k-1|k-1}^0$, the probability that the track exists and unobservable at scan $k - 1$ with $\psi_{k-1|k-1}^n$ and the probability that the track does not exist at scan $k - 1$ with $1 - \psi_{k-1|k-1}$ where $\psi_{k-1|k-1} = \psi_{k-1|k-1}^0 + \psi_{k-1|k-1}^n$. It is sufficient to just mention about this model due to limited space, for further information about this model is given in [28].

In our study, in implementation of all IPDA-based algorithms (IMM-IPDA, IMM-IJPDA, IMM-JIPDA and IMM-LMIPDA) we considered using *Markov Chain One model* as a propagation model of track existence parameter which is sufficient to use because we do not consider unobservability situations in our simulations (because $P_D \neq 0$) and so *Markov Chain Two model* is out of our scope throughout the simulation studies.

As in PDA, both parametric and nonparametric versions are available also for IPDA algorithm. In our simulations, we consider using the parametric version of IPDA because we model the number of clutter detections in each scan with a Poisson distribution with a rate $\lambda = 10$ where the clutter detections, themselves, are uniformly distributed with that rate in space (on two-dimensional surveillance region). Data association probabilities and a *posteriori* probability of track existence for scan k is computed briefly as given below:

For m_k measurements falling inside the validation region (gate) at scan k , the probability that the j^{th} validated measurement $z_{k,j}$ is target originated, denoted with $\beta_{k,j}$, is

$$\beta_{k,j} = \frac{P_D P_G p_{k,j}}{(1 - \delta_k) \rho_{k,j}} \quad (j = 1, \dots, m_k) \quad (3.17)$$

whereas the probability that none of the measurements is target originated, denoted with $\beta_{k,0}$, is

$$\beta_{k,0} = \frac{1 - P_D P_G}{1 - \delta_k} \quad (3.18)$$

a *posteriori* probability of track existence is computed as

$$\psi_{k|k} = \frac{(1 - \delta_k) \psi_{k|k-1}}{1 - \delta_k \psi_{k|k-1}} \quad (3.19)$$

where

$$\delta_k = P_D P_G \left(1 - \sum_{j=1}^{m_k} \frac{p_{k,j}}{\rho_{k,j}} \right) \quad (3.20)$$

and

$$p_{k,j} = \frac{1}{P_G} e_{k,j} \quad (j = 1, \dots, m_k) \quad (3.21)$$

where denotes the a priori measurement pdf of each selected measurement j at scan k , and $e_{k,j}$ is given at (3.3), and

$$\rho_{k,j} = \frac{\lambda}{m_k} \quad (j = 1, \dots, m_k) \quad (3.22)$$

which denotes the a priori clutter measurement density for each measurement $z_{k,j}$ at scan k where $j = 1, \dots, m_k$.

Equations from (3.5) to (3.10) are identical with PDA.

3.2 Multitarget Data Association Algorithms

3.2.1 Joint Probabilistic Data Association (JPDA) Algorithm

IPDA and PDA are derived under the assumption of a single target (single track). Each measurement can be either clutter or a measurement target being followed. In tracking multiple targets with crossing trajectories, this assumption is incomplete. It has been shown in [23, 24] that PDA is incapable of tracking of multiple targets with crossing trajectories. To remedy this situation *Joint Probabilistic Data Association (JPDA)* algorithm has been proposed[23].

JPDA is the extension of PDA to multitarget case. The multitarget case must not only consider random interference caused by clutter but also must consider persistent interference caused by the measurements from neighboring targets consistently fall in the individual validation regions.

JPDA is the same as PDA except for the computation of the data association probabilities. The measurement-to-track association probabilities are computed jointly across all targets and all measurements.

JPDA assumes that there is a "known" number of targets with existing tracks (all of the tracks have been initialized). In addition to this, JPDA assumes that *a target can generate at most one measurement per scan and a measurement could have originated from at most one target (i.e. no unresolved measurements)*. Non-target originated measurements are modeled as in the PDA. Similar to PDA and IPDA, there are parametric and nonparametric versions of JPDA available. However, we will present the parametric version of JPDA which we have used in our study for consistency in comparison with the rest of algorithms.

Brief description of JPDA algorithm from [23, 24, 16, 7] is given below:

JPDA first enumerates all *feasible joint association events* θ_k in the current scan k . A feasible joint association event is a set of nonconflicting validated measurement-to-track pairings in which a measurement can originate from only one source, and at most one measurement can originate from a target. Any number of measurements can originate from clutter.

For formation of feasible joint association events, the same basic examples are given in [1, 38] with different ideas for three measurements competing for two target tracks in a conflicting validation region scenario. In our simulations, we have formed and checked our feasible joint association events considered for all optimal algorithms, throughout the study, by considering the example given in [38] because it is more clear to understand and easy to implement.

To automatically form all feasible joint events by considering the validation matrix(as in the example given in [38]) for T targets and m_k measurements for current scan k , the $\theta_{k,j}^t$'s are denoted as the single events making up a joint event θ_k . Each $\theta_{k,j}^t$ denotes the single event that measurement j ($j = 1, \dots, m_k$) originated from target t ($t = 0, 1, \dots, T$), where m_k is the *total* number of measurements in the current scan k and $t = 0$ denotes that the measurement is a clutter detection.

The binary *target detection indicator* denoted with $\sigma_{k,t}$ for target t ($t = 0, 1, \dots, T$) has a value of one if a measurement assigned to target t in θ_k and it is zero otherwise at current scan k . The binary *measurement association indicator* denoted with $\tau_{k,j}$ for measurement j ($j = 1, \dots, m_k$) has a value of one if the measurement j is assigned to a target t ($t = 0, 1, \dots, T$) in θ_k and it is zero otherwise at current scan k . The quantity ϕ_k is the number of measurements originating from clutter at current scan k in θ_k .

The *joint association event probabilities*, denoted with $P\{\theta_k|Z^k\}$, are given by

$$P\{\theta_k|Z^k\} = \frac{\gamma(\theta_k)}{C_k} \quad (3.23)$$

where the normalization constant C_k is

$$C_k = \sum_{\theta_k} \gamma(\theta_k) \quad (3.24)$$

and

$$\gamma(\theta_k) = \lambda^{\phi_k} \prod_{j=1}^{m_k} (\Lambda_{k,t_j,j})^{\tau_{k,j}} \prod_{t=1}^T [(P_D)^{\sigma_{k,t}} (1 - P_D)^{1-\sigma_{k,t}}] \quad (3.25)$$

where Z^k denotes the set of all measurements from the initial time to current scan k (all the measurements in all scans), λ is the spatial density of the clutter. The Gaussian likelihood $\Lambda_{k,t_j,j}$ of associating measurement j with the track t_j to which it is assigned in the joint event θ_k at current scan k is given by

$$\Lambda_{k,t_j,j} = \frac{1}{\sqrt{|2\pi S_{k,t_j,j}|}} \exp \left\{ -\frac{1}{2} \mathbf{v}_{k,t_j,j}^T (S_{k,t_j,j})^{-1} \mathbf{v}_{k,t_j,j} \right\} \quad (3.26)$$

where $|2\pi S_{k,t_j,j}|$ is the determinant of $2\pi S_{k,t_j,j}$, $\mathbf{v}_{k,t_j,j}$ is the residual for track t_j and measurement j and $S_{k,t_j,j}$ is the residual covariance (on Table 2.1).

The marginal data association probability of each target t ($t = 0, 1, \dots, T$) originating the measurement j at current scan k , denoted by $\beta_{k,j}^t$, where ($j = 1, \dots, m_k$), is obtained by summing over all feasible joint events θ_k in which the single event $\theta_{k,j}^t$ occurs. It is given by

$$\beta_{k,j}^t = P\{\theta_{k,j}^t | Z^k\} = \sum_{\theta_k: \theta_{k,j}^t \in \theta_k} P\{\theta_k | Z^k\} \quad (3.27)$$

the probability that none of the measurements is target originated, denoted with $\beta_{k,0}^t$, is computed easily by

$$\beta_{k,0}^t = 1 - \sum_{j=1}^{m_k} \beta_{k,j}^t \quad (3.28)$$

Once the marginal data association probabilities are computed they are used for each track t separately to update their kinematic state estimates individually. So, for each track t the rest of computation from (3.5) to (3.10) are identical with PDA.

3.2.2 Integrated Joint Probabilistic Data Association (IJPDA) Algorithm

IJPDA has the same problem as PDA, because it assumes that the target(s) exist. Tracks are not differentiated according to a track quality measure and track maintenance is the difficult without a track quality measure. *Integrated Joint Probabilistic Data Association (IJPDA)* algorithm provides a measure of track quality and handles multiple target measurement origin possibility by creating all possible joint events. The measure of track quality is calculated in a manner similar to IPDA[39, 40] with another point of view differently from [29, 28]. It is assumed in [39, 40] that a target exist "behind" each track and the *Probability of Perceivability of the Target* is recursively calculated as the track quality measure. The propagation model for the perceivability is equivalent to Markov Chain One model for track existence propagation of IPDA which has been discussed widely on Section 3.1.2 where the equations from (3.11) to (3.16) is identical for individual tracks in IJPDA also.

IJPDA is similar to JPDA, however the difference comes from the perceivability of targets being involved in the data association process. To take into account the perceivability and unperceivability of each target, feasible joint association events formed previously for JPDA must be modified to gather probability of target perceivability information from all feasible joint association events. This has been done in [38] by adding an extra row vector to all feasible event matrices created for JPDA previously with indice $j = 0$ corresponding to a dummy measurement which will describe the perceivability state of each target t .

Brief description of parametric IJPDA algorithm from [38] is given below:

IJPDA also enumerates all *feasible joint association events* θ_k in the current scan k . A feasible joint

association event is a set of nonconflicting validated measurement-to-track pairings in which a measurement can originate from only one source, and at most one measurement can originate from a target as in the JPDA. Any number of measurements can originate from clutter.

In IJPDA, we have formed and checked our feasible joint association events considered for all optimal algorithms, by considering the example given in [38] because it is more clear to understand and easy to implement.

To automatically form all feasible joint events by considering the validation matrix(as in the example given in [38]) for T targets and m_k measurements for current scan k , the $\theta_{k,j}^t$'s are denoted as the single events making up a joint event θ_k . Each $\theta_{k,j}^t$ denotes the single event that measurement j ($j = 1, \dots, m_k$) originated from target t ($t = 0, 1, \dots, T$), where m_k is the *total* number of measurements in the current scan k and $t = 0$ denotes that the measurement is a clutter detection.

The binary *target detection indicator* denoted with $\sigma_{k,t}$ for target t ($t = 0, 1, \dots, T$) and the binary *measurement association indicator* denoted with $\tau_{k,j}$ for measurement j ($j = 1, \dots, m_k$) and the quantity ϕ_k have been previously defined for JPDA, also they are considered to be used as in the same way in IJPDA also.

As a difference from JPDA, for IJPDA, the binary *target perceivability indicator*, denoted with $\pi_{k,t}$, is defined for target t ($t = 0, 1, \dots, T$), has a value of one if the target is perceivable and it is zero if it is unperceivable.

The *joint association event probabilities*, denoted with $P\{\theta_k|Z^k\}$, are given by

$$P\{\theta_k|Z^k\} = \frac{\gamma(\theta_k)}{C_k} \quad (3.29)$$

where the normalization constant C_k is

$$C_k = \sum_{\theta_k} \gamma(\theta_k) \quad (3.30)$$

and

$$\gamma(\theta_k) = \lambda^{\phi_k} \prod_{j=1}^{m_k} (\Lambda_{k,t_j,j})^{\tau_{k,j}} \prod_{t=1}^T [(P_D)^{\sigma_{k,t}} (1 - P_D)^{1-\sigma_{k,t}}] [(P_{k|k-1}^{O_t})^{\pi_{k,t}} (1 - P_{k|k-1}^{O_t})^{1-\pi_{k,t}}] \quad (3.31)$$

where Z^k denotes the set of all measurements from the initial time to current scan k (all the measurements in all scans), λ is the spatial density of the clutter. The Gaussian likelihood $\Lambda_{k,t_j,j}$ of associating measurement j with the track t_j to which it is assigned in the joint event θ_k at current scan k is given by

$$\Lambda_{k,t_j,j} = \frac{1}{\sqrt{|2\pi S_{k,t_j,j}|}} \exp \left\{ -\frac{1}{2} v_{k,t_j,j}^T (S_{k,t_j,j})^{-1} v_{k,t_j,j} \right\} \quad (3.32)$$

where $|2\pi S_{k,t_j,j}|$ is the determinant of $2\pi S_{k,t_j,j}$, $v_{k,t_j,j}$ is the residual for track t_j and measurement j and $S_{k,t_j,j}$ is the residual covariance (on Table 2.1).

If (3.31) is compared with (3.25), there is an extra term $P_{k|k-1}^{O_t}$ denotes a *priori* probability of perceivability of target t at current scan k . This is a propagated term of $P_{k-1|k-1}^{O_t}$, a *posteriori* probability of perceivability of target t , coming from previous scan $k-1$ which is computed by following the same process (for each aarget t this process is independent) as shown in equations from (3.11) to (3.16).

By considering an important remark in [38], integrated joint event probability evaluations has shown to form a huge number of integrated event matrices with an example in each scan k compared to JPDA due to target perceivability event integrated with feasible event matrices formed for JPDA. By following this[38] remark, it has been shown that IJPDA can be simplified in track maintenance applications. Because for track maintenace, it is stated in [38] that marginal data association probabilities are required which they do not depend on target perceivability parameter.

By following this result[38], the marginal data association probability of each target t ($t = 0, 1, \dots, T$)

originating the measurement j denoted with $\beta_{k,j}^t$ where ($j = 1, \dots, m_k$), the probability that none of the measurements is target originated, denoted with $\beta_{k,0}^t$ and the probability that target t is unperceivable, denoted with $\beta_{k,\bar{0}}^t$, at current scan k , are reduced in a form

$$\beta_{k,j}^t = \sum_{\theta_k: \theta_{k,j}^t \in \theta_k} P\{\theta_k | Z^k\} P_{k|k-1}^{O_t} \prod_{j \neq t} (P_{k|k-1}^{O_j})^{\sigma_{k,j}} \quad (3.33)$$

$$\beta_{k,0}^t = \sum_{\theta_k: \theta_{k,j}^t \in \theta_k} P\{\theta_k | Z^k\} P_{k|k-1}^{O_t} \prod_{j \neq t} (P_{k|k-1}^{O_j})^{\sigma_{k,j}} (1 - \sigma_{k,t}) \quad (3.34)$$

$$\beta_{k,\bar{0}}^t = \sum_{\theta_k: \theta_{k,j}^t \in \theta_k} P\{\theta_k | Z^k\} [1 - P_{k|k-1}^{O_t}] \prod_{j \neq t} (P_{k|k-1}^{O_j})^{\sigma_{k,j}} (1 - \sigma_{k,t}) \quad (3.35)$$

As in the JPDA, marginal data association probabilities are used for each track t separately to update their kinematic state estimates individually. So, by inclusion of (3.35) as an extra data association probability for each track t the rest of computation from (3.5) to (3.10) are identical with PDA.

Hence the computation cost involved in IJPDA has become almost the same (as the results at Chapter 4 indicates) compared to JPDA. Only the cost of computation of (3.35) is added to cost of computation of (3.35) JPDA.

There is also a fundamental remark which can be proved by setting the probability of target perceivability as $P_{k|k-1}^{O_t} = 1$ in equations (3.33), (3.34) and (3.35) where we get (3.27), (3.28) and $\beta_{k,\bar{0}}^t = 0$ consistently.

3.2.3 Joint Integrated Probabilistic Data Association (JIPDA) Algorithm

As in the IJPDA algorithm, *Joint Integrated Probabilistic Data Association (JIPDA)* algorithm also adds a track quality measure to JPDA, which is the concept of track existence. The main difference between JIPDA and IJPDA algorithms is that *JIPDA* [34] calculates the track state estimate pdf "conditioned" on target existence, whereas, in *IJPDA*, track state estimate "depends" on the probability of target perceivability. Due to nonperceivable target possibility, the number of feasible joint events is much larger than both JIPDA and JPDA. JIPDA enumerates the same feasible joint events as JPDA.

JIPDA is developed as a multitarget generalization of IPDA algorithm derived by the authors [28]. It uses the *Probability of Track Existence* and computes data association probabilities and the probability of track existence as an extra state for track update. It is stated in [34] that JIPDA becomes identical to IPDA when tracking single target in clutter situations.

The propagation model for track existence can be selected as either of Markov Chain One or Markov Chain Two models where we have discussed about these models and for consistency in implementation for overall IPDA-based algorithms, in JIPDA, Markov Chain One model is considered and will be presented as the propagation model for track existence.

As in the IPDA both parametric and nonparametric versions of JIPDA is available. However, we consider using parametric version in our simulations. So, brief description of parametric version of JIPDA from [34, 18] is presented below:

JIPDA enumerates all *feasible joint association events* θ_k in the current scan k as in the JPDA. Feasible joint association events are formed which is a set of nonconflicting validated measurement-to-track pairings in which a measurement can originate from only one source, and at most one measurement can originate from a target. Any number of measurements can originate from clutter.

For formation of feasible joint association events, the same process as in JPDA is accomplished.

To automatically form all feasible joint events by considering the validation matrix (as in the example given in [38]) for T targets and m_k measurements for current scan k , in [34, 18], χ_i is given as the joint event i , and X is given as the number of joint events in the cluster. T_0 and T_1 are defined as the set of

tracks allocated no measurements and the set of tracks allocated one measurement respectively in the joint event. The a posteriori probability of χ_i is given

$$P\{\chi_i|Z^k\} = \frac{1}{C_k} \prod_{i \in T_0} (1 - P_D P_G \psi_{k|k-1}^i) \prod_{i \in T_1} \left(P_D P_G \psi_{k|k-1}^i \frac{p_i^t}{P_i^t} \right) \quad (3.36)$$

where C_k is the normalization constant, rest of the parameters are nothing more than multitarget extensions of related parameters given in IPDA.

The joint events must form a complete set where the constant C_k is calculated by using

$$\sum_{i=1}^X P\{\chi_i|Z^k\} = 1 \quad (3.37)$$

The a posteriori probabilities of individual track events are obtained by summing the a posteriori probabilities of all joint events containing the event.

$\Xi(t, j)$ is denoted as the set of joint events in which track t has been allocated measurement j , with measurement 0 denoting no measurement. Set $\Xi(t, j)$ may be empty.

The a posteriori probability of no measurement originating from the track t is

$$P\{\chi_0^t|Z^k\} = \sum_{\chi_e \in \Xi(t,0)} P\{\chi_e|Z^k\} \quad (3.38)$$

and a posteriori probability that track t exists and that measurement j originated from the track t is

$$P\{\chi^t \chi_j^t|Z^k\} = \sum_{\chi_e \in \Xi(t,j)} P\{\chi_e|Z^k\} \quad (3.39)$$

The a posteriori probability that track t exists and that no measurements have originated from track t is

$$P\{\chi^t \chi_0^t|Z^k\} = \frac{(1 - P_D P_G) \psi_{k|k-1}^t}{1 - P_D P_G \psi_{k|k-1}^t} P\{\chi_0^t|Z^k\} \quad (3.40)$$

The a posteriori probability of track existence of track t can be computed as

$$\psi_{k|k}^t = P\{\chi^t \chi_0^t|Z^k\} + \sum_{j \in \{\mu(t,j) > 0\}} P\{\chi^t \chi_j^t|Z^k\} \quad (3.41)$$

where $\{\mu(t, j) > 0\}$ denotes the set of measurements falling in the validation region of track t .

The marginal data association probability of each target t ($t = 0, 1, \dots, T$) originating the measurement j denoted with $\beta_{k,j}^t$ where ($j = 1, \dots, m_k$), the probability that none of the measurements is target originated, denoted with $\beta_{k,0}^t$ are computed as

$$\beta_{k,j}^t = \frac{P\{\chi^t \chi_j^t|Z^k\}}{\psi_{k|k}^t} \quad (3.42)$$

$$\beta_{k,0}^t = \frac{P\{\chi^t \chi_0^t|Z^k\}}{\psi_{k|k}^t} \quad j \in \{\mu(t, j) > 0\} \quad (3.43)$$

As in the JPDA, marginal data association probabilities are used for each track t separately to update their kinematic state estimates individually. The rest of computation from (3.5) to (3.10) are identical with PDA.

3.2.4 Linear Multitarget Integrated Probabilistic Data Association (LMIPDA) Algorithm

Optimal approaches, mentioned so far, in multitarget tracking in clutter considers all feasible measurement-to-track allocations to achieve optimal data association performance. JIPDA is the multitarget generalization of IPDA[28], in which the number of operations grow exponentially with the number of tracks and measurements. *Linear Multitarget Integrated Probabilistic Data Association (LMIPDA)* algorithm has the number of operations which is linear in the number of tracks and the number of measurements, with apparently negligible performance penalty compared to JIPDA[32, 19].

LMIPDA is an IPDA filter to which Linear Multitarget (LM) procedure has been applied. LM reduces the computational complexity of multitarget tracking in clutter by eliminating the measurement-to-track assignment step entirely[32, 19]. Instead, the clutter density at each measurement point is modified by the pdf of measurements originating from the neighboring tracks. Other tracks are treated as additional clutter sources and LM achieves multitarget tracking capabilities using single target tracking computational resources.

In IPDA, $\rho_{k,j}^t$ (Equation (3.22)) is defined as the clutter density in validation region of track t for measurement $z_{k,j}$ at current scan k , then the a priori probability that j^{th} measurement is the true measurement for track t given single track t is

$$P_{k,j}^t = P_D P_G \psi_{k|k-1}^t \frac{\rho_{k,j}^t}{\sum_{j=1}^{m_k} \rho_{k,j}^t} \quad (3.44)$$

where $\rho_{k,j}^t$ has been defined in Equation (3.21).

The *modified clutter density* for track t at the point $z_{k,j}$ at current scan k is

$$\Omega_{k,j}^t = \rho_{k,j}^t + \sum_{\substack{s=1 \\ s \neq t}}^T P_{k,j}^s \frac{P_{k,j}^s}{1 - P_{k,j}^s} \quad (3.45)$$

where T is the number of tracks.

$\Omega_{k,j}^t$ is used for each track t separately and individually instead of clutter density $\rho_{k,j}^t$ (via substitution to Equations from (3.17) to (3.22) where the equations are given without upscript t) when calculating the data association probabilities for track t .

The marginal data association probabilities are computed individually from IPDA parts of the algorithm as in the classical IPDA fashion, however, in LMIPDA, T IPDA is used to compute the marginal data association probability related to each target t where $t = 0, 1, \dots, T$. So, the marginal data association probability of each target t ($t = 0, 1, \dots, T$) originating the measurement j denoted with $\beta_{k,j}^t$ where ($j = 1, \dots, m_k$), the probability that none of the measurements is target originated, denoted with $\beta_{k,0}^t$ are computed as

$$\beta_{k,j}^t = \frac{P_D P_G \rho_{k,j}^t}{(1 - \delta_k^t) \Omega_{k,j}^t} \quad (j = 1, \dots, m_k) \quad (t = 1, \dots, T) \quad (3.46)$$

$$\beta_{k,0}^t = \frac{1 - P_D P_G}{1 - \delta_k^t} \quad (j = 1, \dots, m_k) \quad (t = 1, \dots, T) \quad (3.47)$$

a *posteriori* probability of track existence is computed for each track t as

$$\psi_{k|k}^t = \frac{(1 - \delta_k^t) \psi_{k|k-1}^t}{1 - \delta_k^t \psi_{k|k-1}^t} \quad (t = 1, \dots, T) \quad (3.48)$$

where

$$\delta_k^t = P_D P_G \left(1 - \sum_{j=1}^{m_k} \frac{P_{k,j}^t}{\Omega_{k,j}^t} \right) \quad (t = 1, \dots, T) \quad (3.49)$$

As in the JPDA, once the marginal data association probabilities are computed they are used for each track t separately to update their kinematic state estimates individually. The rest of computation from (3.5) to (3.10) are identical with PDA.

An important remark is made in [32, 19], if the tracks are far apart, their validation regions do not intersect, $\Omega_{k,j}^t = \rho_{k,j}^t$ for all j and t at scan k where LMIPDA reverts to IPDA.

CHAPTER 4

SIMULATIONS

Tracking estimation accuracy of each target tracking algorithm under concern has been compared via RMSE performance evaluations. In our simulations, RMSE performance of each algorithm is computed by considering simulation runs, where the same number of confirmed true tracks has been established at each run for all target tracking algorithms, are taken into account.

Let us consider that K is achieved as the number of successful runs out of total L runs, so, K runs are taken into account for RMSE performance computation. Among L runs, if N total successful confirmed true tracks out of M overall total tracks in M scans over all runs have been established by the target tracking algorithm under concern, has $M - N$ track losses. These $L - K$ run periods are not taken into account for RMSE performance computation. However, these run periods are taken into account for the computation of *the percentage of track loss* which has been defined in [48]. To compare the reliability[48] of the target tracking algorithm under concern, *the percentage of track loss* is computed by averaging $M - N$ over total M scans in L runs. Then, overall formula for *the percentage of track loss* of the target tracking algorithm becomes

$$\text{the percentage of track loss} = \frac{M - N}{M} \times 100\% \quad (4.1)$$

In our simulations, for fair comparison of RMSE performances, the number of successful runs, K , is fixed as 100 which means the number of total runs, L , ($L \geq 100$) may vary for each target tracking algorithm to achieve K successful runs, ($K = 100$).

Some of the parameters of the algorithms can be tuned to optimal values to achieve optimal overall system performance. These parameters are often selected by a priori information about the system under concern. However, by regarding no a priori information about the system, these parameters can also be set optimally via detailed Monte Carlo simulations which has been recommended in [10]. The List of Tunable Parameters of algorithms is given in Appendix A.

We have carried out simulations in five consecutive test scenarios to compare the RMSE performance of Single Target Tracking algorithms including IMM-PDA, IMM-IPDA and Optimal approaches in multitarget tracking including IMM-JPDA, IMM-IJPDA and IMM-JIPDA with an example of Linear Multi-target approaches in multitarget tracking including IMM-LMIPDA algorithm. At the end of each section, we present the computation time of each algorithm has been used in these test scenarios to achieve the computational load evaluations.

We conduct the simulations in MATLAB 2008 platform in a CPU with specifications:

Intel®Core™i3 - 2100 CPU with 3.10 GHz with 4 GB RAM

in order to get the following RMSE Performance plots and the tables showing the percentage of track loss and execution time of the target tracking algorithms.

4.1 Single Target Tracking Scenario

A two-dimensional surveillance scenario is formed where a single point target is present which can move either on-road or off-road. It may accelerate or decelerate at some time instants, generally on-road in the course of straight movement. It may turn either some direction at junction points. It may break to off-road or enter the road at some points. The simulation scenario (as illustrated in Figure 4.1) is formed where a real ground target's movement taken into consideration. Hence, its movements in the simulation scenario is thought to be benign in a constraint fashion. The true target trajectories, measurement observed from the target is labeled and false alarm measurements considered as clutter and the resulting state estimates are all depicted in Figure 4.1. As illustrated in Figure 4.1, heavy and dense clutter measurements are always present as in actual ground target tracking case. Measurement noise is added inherently to the true state (position) of the target under track with a variance 30 m to simulate the target observations obtained from the system. Clutter measurements are generated per scan where the distribution of number of clutter measurements is Poisson with a rate $\lambda = 10$ at each scan where they are distributed uniformly on the surveillance region.

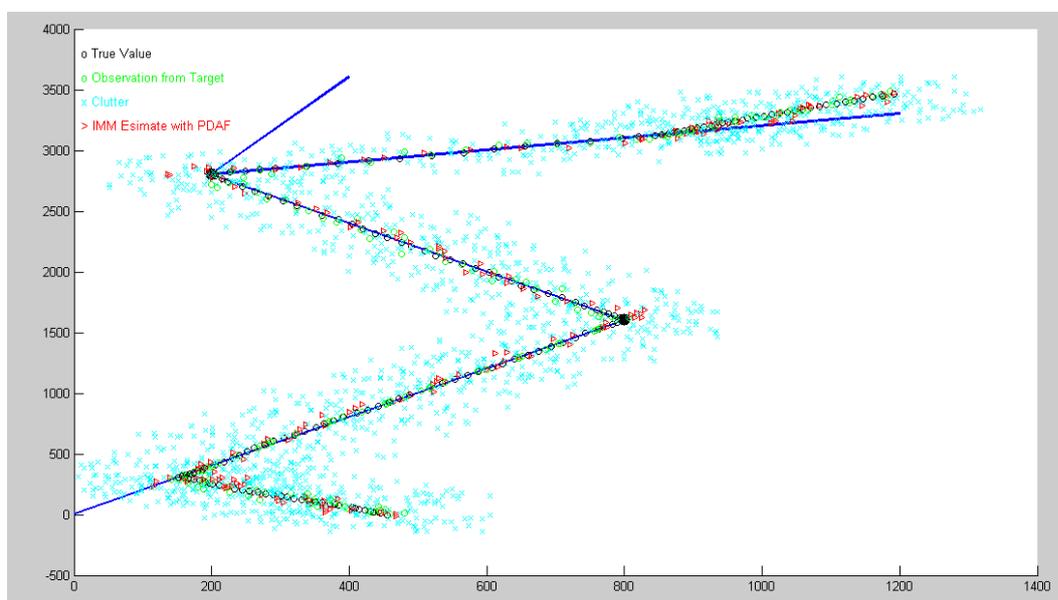


Figure 4.1: Tracking of single target in clutter

In the simulations, the track is initiated from a single measurement in a scan. New measurements will become the predicted positions of the new tracks in the next scan. In the next scan, around the predicted position an elliptical validation region (gate) is formed for the track and a new track is formed from each measurement that fall into the validation gate. Track is formed, maintained and deleted per scan independently.

For IMM-IPDA, the initial probability of track existence is considered as 0.2, in our simulations, where the references[29, 28] recommend that value due to sensitivity of the algorithms to this parameter. Markov Chain One model uses transition matrix Π with entries:

$$\Pi_{11} = 0.98; \quad \Pi_{21} = 0;$$

$$\Pi_{11} = 0.02; \quad \Pi_{21} = 1;$$

where also the references[29, 28] uses the same transition matrix, we used this matrix also for convenience.

For the kinematic state estimator, an IMM estimator with two model-based Kalman Filters is considered which are CV with Low Process Noise for nonmaneuvering motion and CV with High Process Noise for any maneuvers including coordinated turns and acceleration modes, with process noise values are considered as with standard deviations 2.5 m, 30 m respectively. As discussed at [3] and following the result presented in [3] that, it is sufficient to use because our targets move in a constraint fashion, not any evasive, high or different maneuvers are expected. The target and observation models used in Kalman Filters are taken from the references [3] and [1], respectively.

Simulation experiments consist of $K = 100$ runs. In each simulation run, target retraces the trajectory, however, the measurements obtained from the target as well as clutter measurements number and position are generated independently in each run and scan.

The resulting RMSE Performance plot is presented in Figure 4.2. The execution time comparison of these algorithms considered, shown with “STT” in paranthesis, are presented on Table 4.2, in Section 4.2.

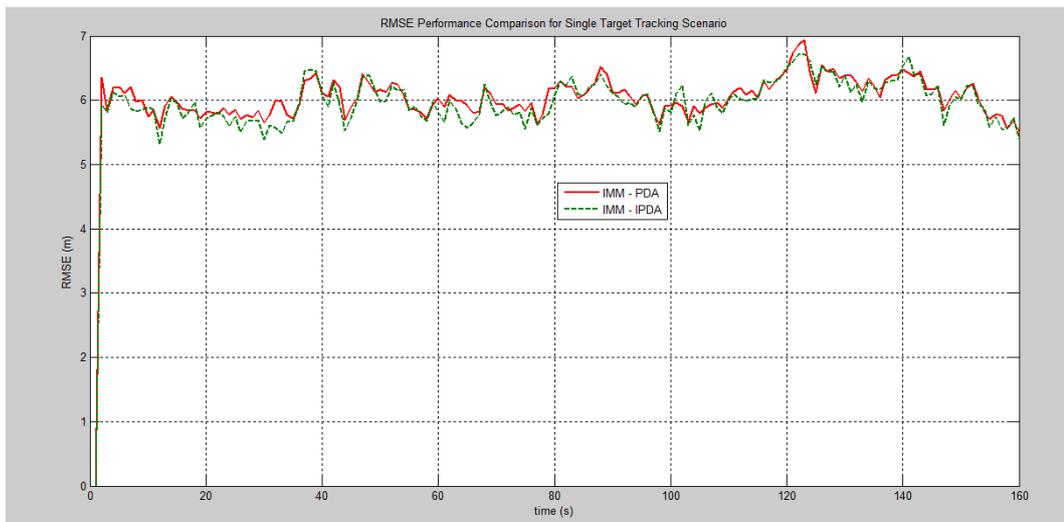


Figure 4.2: RMSE Performance Comparison of IMM-PDA with IMM-IPDA in Single Target Tracking Scenario

The result shown in Figure 4.2 demonstrates that, IMM-IPDA shows slightly better performance than IMM-PDA due to the probability of track existence parameter updated as an additional state. Performance improvements are observed over observation time period when the target undergoes maneuver where it may fall out of the validation region easily.

The result shown in Figure 4.2 is an important outcome which will supply us a “ground truth” to compare the multitarget tracking algorithms presented in consecutive sections.

4.2 Convoy Scenario

A two-dimensional surveillance scenario is formed where two point targets are present which can move either on-road or off-road. They move at the same time, may accelerate or decelerate independently at some time instants, generally on-road in the course of straight movement. So, their trajectories may intersect at some points. They may turn either some direction at junction points. They may break to off-road or enter the road at some points. The simulation scenarios are formed where real ground targets' movement taken into consideration. Hence, their movements are thought to be benign in a constraint fashion. The true target trajectories, measurements observed from targets are labeled individually and false alarm measurements considered as clutter and the resulting state estimates are all depicted in Figure 4.3. As illustrated in Figure 4.3, heavy and dense clutter measurements are always present as in real ground target tracking case. Measurement noise is added inherently to actual states (positions) of the targets under track with a variance 30 m to simulate the target observations obtained from the system. Clutter measurements are generated per scan for each target where the distribution of the number of clutter measurements is Poisson with a rate $\lambda = 10$ at each scan where they are distributed uniformly on the surveillance region.

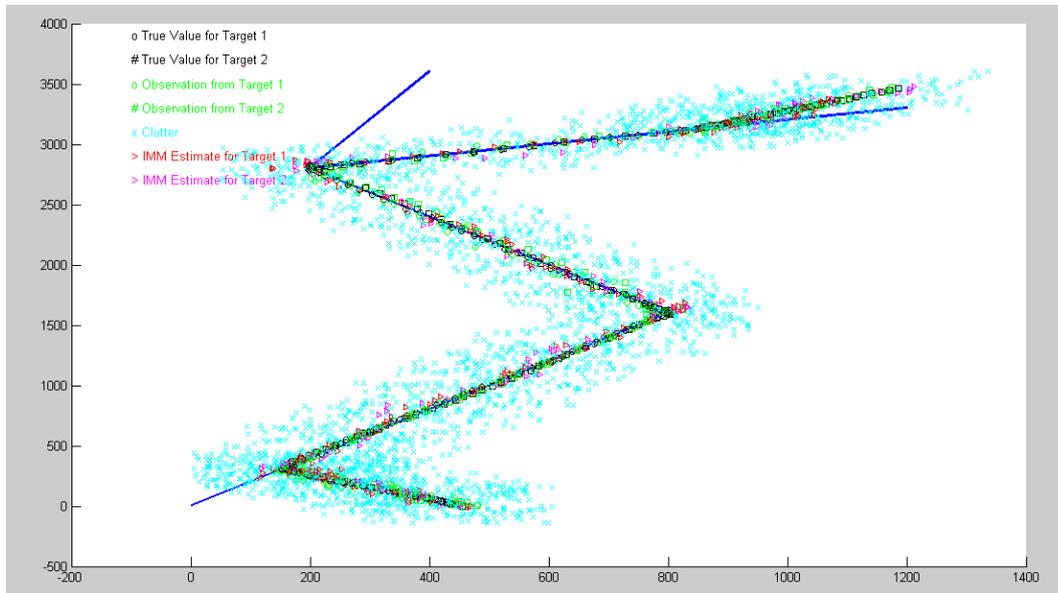


Figure 4.3: Tracking of two targets in clutter moving in a convoy fashion

As illustrated in Figure 4.3, two targets move in a convoy fashion. As mentioned above, they move independently, may accelerate or decelerate at some time instants independently. This results in the possibility that their trajectories may intersect at some points.

In all simulations, each track is initiated from a single measurement in a scan. New measurements will become the predicted positions of the new tracks in the next scan. In the next scan, around the predicted positions elliptical validation regions (gates) are formed individually and independently for each track and new tracks are formed from each measurement that fall into each validation gate individually. Each track is formed, maintained or deleted independently as in the single target tracking case. To control the number of tracks formed in each scan, measurements are divided into sets. All sets

consists of the measurements used to update an individual existing track. Each track has an independent validation gate which means that some measurements from the other validation gate may be used to update more than one track.

For all probability of track existence based target tracking algorithms (IMM-IPDA, IMM-IJPDA, IMM-JIPDA and IMM-LMIPDA), the initial probabilities of track existence are considered as 0.2 individually, in our simulations, where the references[29, 28] recommend that due to sensitivity of the algorithms to this parameter. Markov Chain One model uses transition matrix Π with entries:

$$\Pi_{11} = 0.98; \quad \Pi_{21} = 0;$$

$$\Pi_{11} = 0.02; \quad \Pi_{21} = 1;$$

where also the references[29, 28] uses the same transition matrix, we used this matrix also for convenience.

Pruning is considered in this scenario before the application of optimal approaches in multitarget data association algorithms including JPDA, IJPDA and JIPDA in IMM-JPDA, IMM-IJPDA and IMM-JIPDA, respectively, where the measurements at the intersection region of the gates are taken into consideration to reduce the computation requirements to a feasible level. In [30, 34], JIPDA and IJPDA are applied only on confirmed tracks, for unconfirmed tracks IPDA[29, 28] and IPDA[39] is applied respectively. In our simulations, JPDA, IJPDA and JIPDA are applied on all tracks and the measurements at the intersection region of the gates.

During tracking, tracks are confirmed or deleted as in the single target tracking case, but herein, in multitarget tracking case, all the operations are done individually and independently for each track. In the application of multitarget data association algorithms only data association operation is carried out jointly but at the end marginal data association probabilities are extracted from joint data association probabilities to update individual tracks under concern.

For the kinematic state estimator, an IMM estimator with two model-based Kalman Filters is considered consisting of CV(Constant Velocity) with Low Process Noise for nonmaneuvering motion and CV with High Process Noise for any maneuvers including coordinated turns and acceleration modes, with process noise values are considered with standard deviations 2.5 *m*, 30 *m* respectively, as in the single target tracking case. As discussed at [3] and following the result presented in [3] that, it is sufficient to use two model-based filters because our targets move in a constraint fashion, not any evasive, high or different maneuvers are expected. The target and observation models used in Kalman Filters are taken from the references [3] and [1], respectively.

The RMSE performance of single target tracking algorithms including IMM-PDA, IMM-IPDA and optimal approaches in multitarget tracking including IMM-JPDA, IMM-IJPDA and IMM-JIPDA with an example of Linear Multi-target Approaches in multitarget tracking including IMM-LMIPDA algorithm has been compared.

Each simulation experiment consists of $K = 100$ runs. In each simulation run, targets retrace the trajectory, however, for the measurements obtained from the targets as well as clutter measurements, the numbers and positions of all measurements are generated independently from a pre-specified distributions per each run and scan.

The resulting RMSE Performance plots for Target 1 and Target 2 are obtained individually and presented in Figures 4.4, 4.6, 4.8 and 4.5, 4.7, 4.9 respectively. The percentage of track loss and the computation time of algorithms in comparison are presented on Tables 4.1 and 4.2, respectively, in where IMM-PDA and IMM-IPDA considered in multitarget tracking scenarios, shown with "MTT" in paranthesis, in single target tracking scenario (in previous section), shown with "STT" in paranthesis.

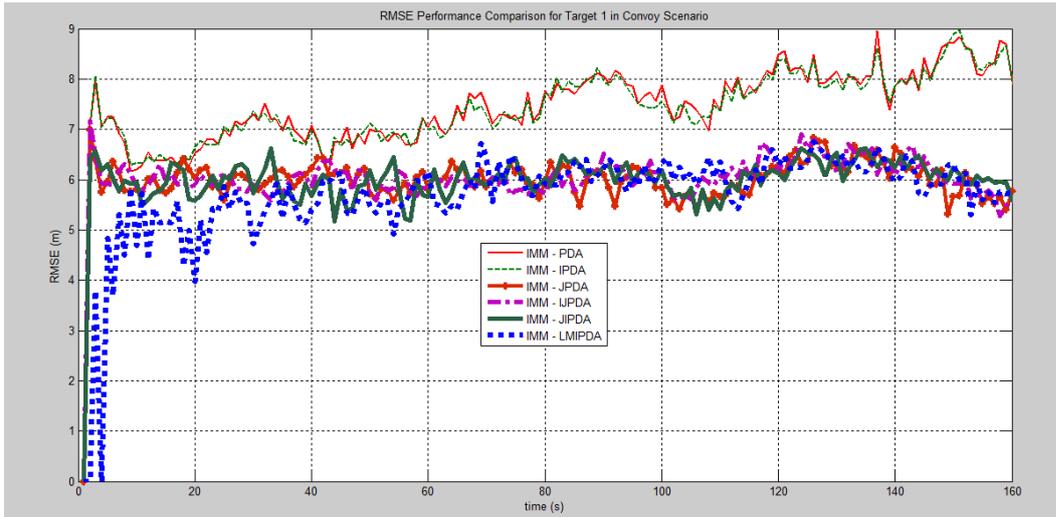


Figure 4.4: RMSE Performance Comparison of all present Target Tracking Algorithms in Convoy Scenario for Target 1

Before starting comparison of performances of target tracking algorithms, an important result is exposed via comparison of the Figure 4.2 with Figures 4.4, 4.6 and 4.5, 4.7, where the performance of single target tracking algorithms deteriorates significantly in multitarget tracking situations in convenience where also been shown in [23, 24].

The results in Figures 4.4, 4.6 and 4.5, 4.7 demonstrate that, IMM-IPDA shows slightly better performance, also in multitarget tracking situations, than IMM-PDA due to the probability of track existence parameter updated as an additional state and used for track update. However, in multitarget tracking

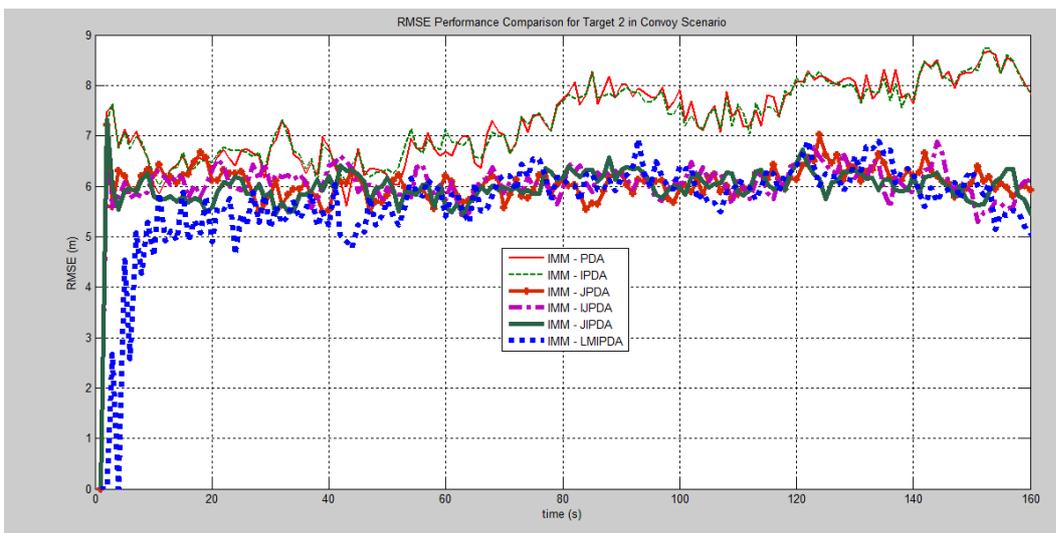


Figure 4.5: RMSE Performance Comparison of all present Target Tracking Algorithms in Convoy Scenario for Target 2

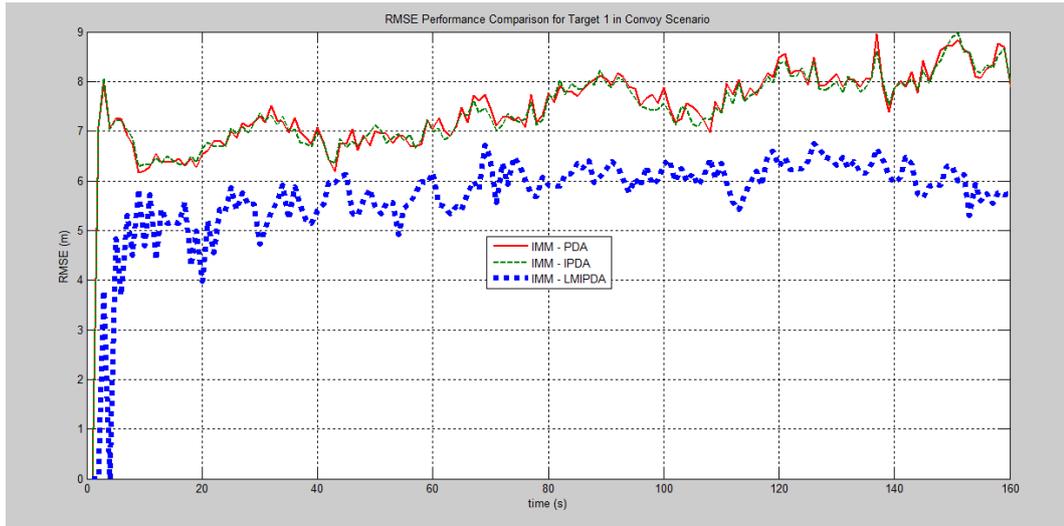


Figure 4.6: RMSE Performance Comparison of IMM-LMIPDA with Single Target Tracking Algorithms in Convoy Scenario for Target 1

situations, IMM-IPDA still consists a single target data association algorithm, which is basically IPDA for each target track. Hence, much performance improvement than achieved is not expected than any multitarget tracking algorithms as shown also in [35, 32, 18, 19].

A notable performance improvement has been accomplished with IMM-LMIPDA than both IMM-PDA and IMM-IPDA as shown in Figures 4.4, 4.6 and 4.5, 4.7 where it is expected due to multitarget tracking capability by modifying clutter density with a priori probability of measurement origin param-

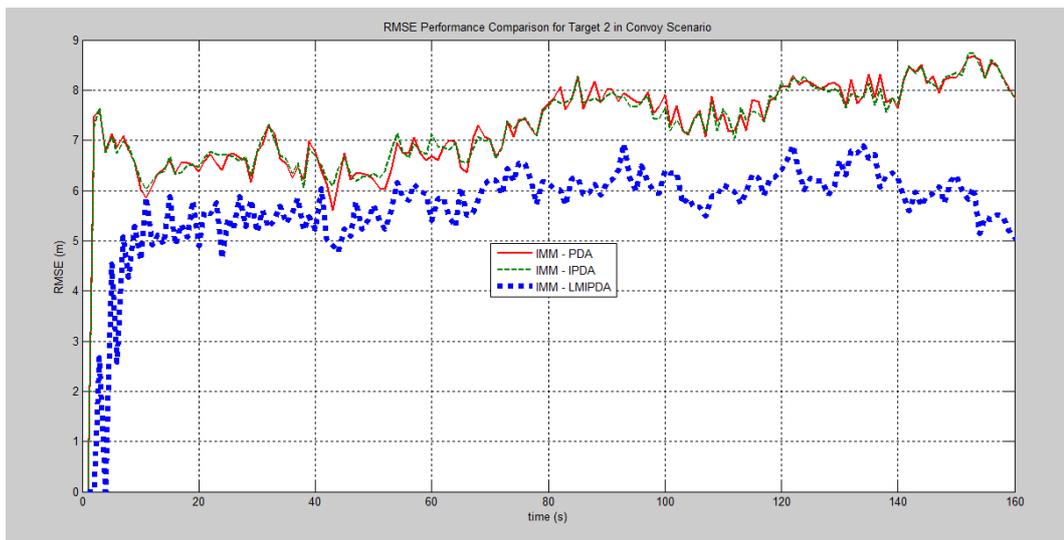


Figure 4.7: RMSE Performance Comparison of IMM-LMIPDA with Single Target Tracking Algorithms in Convoy Scenario for Target 2

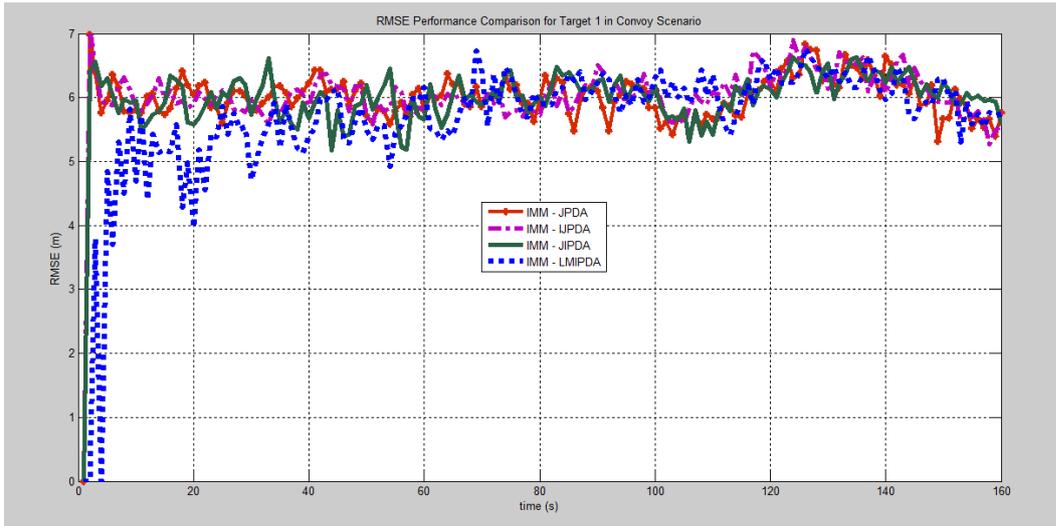


Figure 4.8: RMSE Performance Comparison of IMM-LMIPDA with Optimal Multitarget Tracking Algorithms in Convoy Scenario for Target 1

eter (recalled from Eq. (3.44)) interchanged between tracks which is also convenient with the results presented in [35, 32, 18, 19].

In the comparison of optimal approaches in multitarget tracking, the results shown in Figures 4.4, 4.8 and 4.5, 4.9 proves the theoretical enhancement of IJPDA algorithm[38] that IMM-IJPDA shows dramatically better performance than IMM-JPDA due to the probability of track perceivability parameter computed recursively to calculate track state estimates. IMM-IJPDA shows the “best” performance among all multitarget tracking algorithms due to the probability of track existence parameter updated

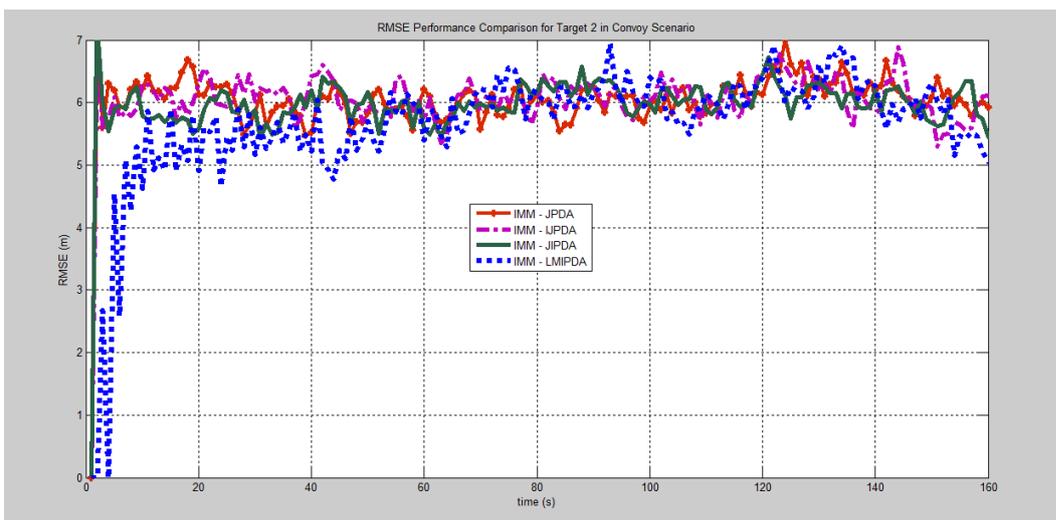


Figure 4.9: RMSE Performance Comparison of IMM-LMIPDA with Optimal Multitarget Tracking Algorithms in Convoy Scenario for Target 2

Table4.1: Track Loss Statistics in Convoy Scenario

Target Tracking Algorithm	The percentage of Track Loss (%)
IMM-PDA (STT)	0.243
IMM-IPDA (STT)	0.171
IMM-PDA (MTT)	0.545
IMM-IPDA (MTT)	0.371
IMM-LMIPDA	35.2517
IMM-JPDA	0.3886
IMM-IJPDA	0.57796
IMM-JIPDA	0.12931

Table4.2: Computation time Comparison in Convoy Scenario

Target Tracking Algorithm	Computation time (in seconds)
IMM-PDA (STT)	185.489790
IMM-IPDA (STT)	188.984625
IMM-PDA (MTT)	355.237801
IMM-IPDA (MTT)	359.318903
IMM-LMIPDA	391.983213
IMM-JPDA	14278.511871
IMM-IJPDA	14381.274496
IMM-JIPDA	14339.510438

as an additional state used for both track update and track state estimate calculations which is also convenient with the idea and results presented in [30, 34].

An important result can also be inferred from Figures 4.8 and 4.9 that IMM-LMIPDA follows the RMSE performance of optimal approaches closely with apparently negligible performance deterioration which is also convenient with the results presented in [35, 32, 18, 19].

Table 4.2 indicates that, IMM-LMIPDA takes negligibly small increment in execution time more than both IMM-PDA (MTT) and IMM-IPDA (MTT); where the increment is linear when compared with the execution time of both IMM-PDA (STT) and IMM-IPDA (STT) in the number of target tracks sense. Whereas all optimal multitarget tracking algorithms (IMM-JPDA, IMM-IJPDA and IMM-JIPDA) require bursts of computation time compared to IMM-LMIPDA and single target tracking algorithms IMM-PDA (MTT) and IMM-IPDA (MTT) due to regarding of all feasible joint events for measurement-to-track assignments and computation of probabilities and likelihoods of all joint events. Even pruning of distant gate measurements are applied before data association, the computation time requirement of optimal multitarget tracking algorithms (IMM-JPDA, IMM-IJPDA and IMM-JIPDA) remain more than 30 times of computation time requirement of IMM-LMIPDA and single target tracking algorithms IMM-PDA (MTT) and IMM-IPDA (MTT). This result proves also the information discussed in [35, 32, 18, 19] and detailed analysis given in [19] in convenience.

Although IMM-LMIPDA shows close RMSE performance with less computation time when compared with optimal approaches (IMM-JPDA, IMM-IJPDA and IMM-JIPDA), Table 4.1 indicates that IMM-

LMIPDA suffers from track loss inevitably which is one of common problems of general suboptimal approaches[43, 19, 44, 45]. The results are convenient with the results presented in [48, 47, 46], where in [48, 47, 46] results are computed for each target individually, whereas on Table 4.1, the averaged values are presented.

4.3 Merging in a junction Scenario

A two-dimensional surveillance scenario is formed where two point targets are present which can move either on-road or off-road. They move at the same time, may accelerate or decelerate independently at some time instants, generally on-road in the course of straight movement. So, their trajectories may intersect at some points. They may turn either some direction at junction points. They may break to off-road or enter the road at some points. The simulation scenarios are formed where real ground targets' movement taken into consideration. Hence, their movements are thought to be benign in a constraint fashion. The true target trajectories, measurements observed from targets are labeled individually and false alarm measurements considered as clutter and the resulting state estimates are all depicted in Figure 4.10. As illustrated in Figure 4.10, heavy and dense clutter measurements are always present as in real ground target tracking case. Measurement noise is added inherently to actual states (positions) of the targets under track with a variance 30 m to simulate the target observations obtained from the system. Clutter measurements are generated per scan for each target where the distribution of number of clutter measurements is Poisson with a rate $\lambda = 10$ at each scan where they are distributed uniformly on the surveillance region.

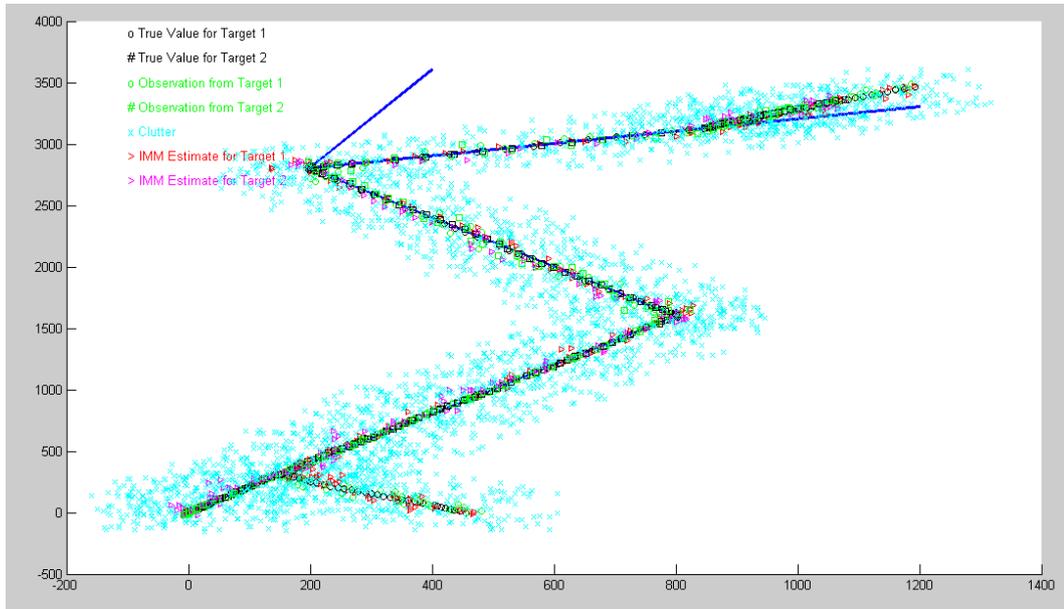


Figure 4.10: Tracking of two targets in clutter merging in a junction

In this scenario, in addition to previous scenario(as illustrated in Figure 4.3), they start movement at distant points and join in a junction point and continue moving in a convoy fashion. Like in the second scenario, they move independently, may accelerate or decelerate at some time instants independently which conveys the possibility that their trajectories may intersect or at some points or depart completely from each other after the time instant they merge together.

In all simulations, each track is initiated from a single measurement in a scan. New measurements will become the predicted positions of the new tracks in the next scan. In the next scan, around the predicted positions elliptical validation regions (gates) are formed individually and independently for each track and new tracks are formed from each measurement that fall into each validation gate in-

dividually. Each track is formed, maintained or deleted independently as in the single target tracking case. To control the number of tracks formed in each scan, measurements are divided into sets. All sets consists of the measurements used to update an individual existing track. Each track has an indepent validation gate which means that some measurements from the other validation gate may be used to update more than one track.

For all probability of track existence based target tracking algorithms (IMM-IPDA and IMM-LMIPDA), the initial probabilities of track existence are considered as 0.2 individually, in our simulations, where the references[29, 28] recommend that due to sensitivity of the algorithms to this parameter. Markov Chain One model uses transition matrix Π with entries:

$$\Pi_{11} = 0.98; \quad \Pi_{21} = 0;$$

$$\Pi_{11} = 0.02; \quad \Pi_{21} = 1;$$

where also the references[29, 28] uses the same transition matrix, we used this matrix also for convenience.

During tracking, tracks are confirmed or deleted as in the single target tracking case, but herein, in multitarget tracking case, all the operations are done individually and independently for each track. In the application of multitarget data association algorithms only data association operation is carried out jointly but at the end marginal data association probabilities are extracted from joint data association probabilities to update individual tracks under concern.

For the kinematic state estimator, an IMM estimator with two model-based Kalman Filters is considered consisting of CV(Constant Velocity) with Low Process Noise for nonmaneuvering motion and CV with High Process Noise for any maneuvers including coordinated turns and acceleration modes, with process noise values are considered with standard deviations 2.5 *m*, 30 *m* respectively, as in the single target tracking case. As discussed at [3] and following the result presented in [3] that, it is sufficient to use two model-based filters because our targets move in a constraint fashion, not any evasive, high or different maneuvers are expected. The target and observation models used in Kalman Filters are taken from the reference [3] and [1] respectively.

Each simulation experiment consists of $K = 100$ runs. In each simulation run, targets retrace the trajectory, however, for the measurements obtained from the targets as well as clutter measurements, the numbers and positions of all measurements are generated independently from a pre-specified distributions per each run and scan.

The RMSE performance of single target tracking algorithms including IMM-PDA, IMM-IPDA with an example of Linear Multi-target Approaches in multitarget tracking including IMM-LMIPDA algorithm has been compared. Opimal approaches in multitarget tracking including IMM-JPDA, IMM-IJPDA and IMM-JIPDA are not considered in comparison because pruning of distant gate measurements has been applied before data association, only the measurements at the intersection region of the gates are taken into consideration to reduce the computation requirements of algorithms to a feasible level. Hence, comparison of these algorithms under this situation becomes idle. In addition to this, using single target data association algorithms or Linear Multi-target Approaches in data association where data association is applied to all tracks and measurements, has also been shown[32, 37, 19] much more practical than optimal approaches in this situation.

The resulting RMSE Performance plots for Target 1 and Target 2 are obtained individually and presented in Figures 4.11 and 4.12, respectively. The percentage of track loss and the computation time of algorithms in comparison are presented on Tables 4.3 and 4.4, respectively, where single target tracking algorithms considered in multitarget tracking scenarios, shown with "MTT" in paranthesis.

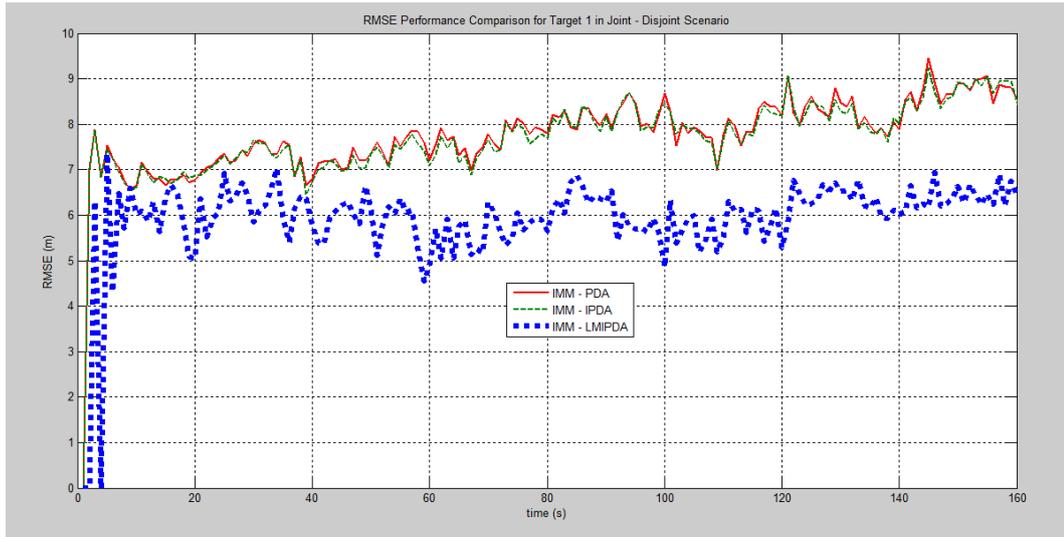


Figure 4.11: RMSE Performance Comparison of IMM-LMIPDA with Single Target Tracking Algorithms in Merging in a junction Scenario for Target 1

An important result can be inferred from Figures 4.11 and 4.12 that IMM-LMIPDA shows close performance with IMM-IPDA in isolated tracks situation where tracks are sufficiently far apart where their validation regions (gates) do not intersect. The enhancement in the RMSE performance of IMM-LMIPDA which is clearly observed after the first 50 seconds in Figures 4.11 and 4.12 where the gates of tracks do not intersect (as illustrated in Figure 4.10). This result proves the theoretical foundation and results given in [35, 32, 18, 19] in convenience. Table 4.4 shows again and proves that, IMM-LMIPDA takes negligibly small increment in computation time more than IMM-PDA (MTT) and

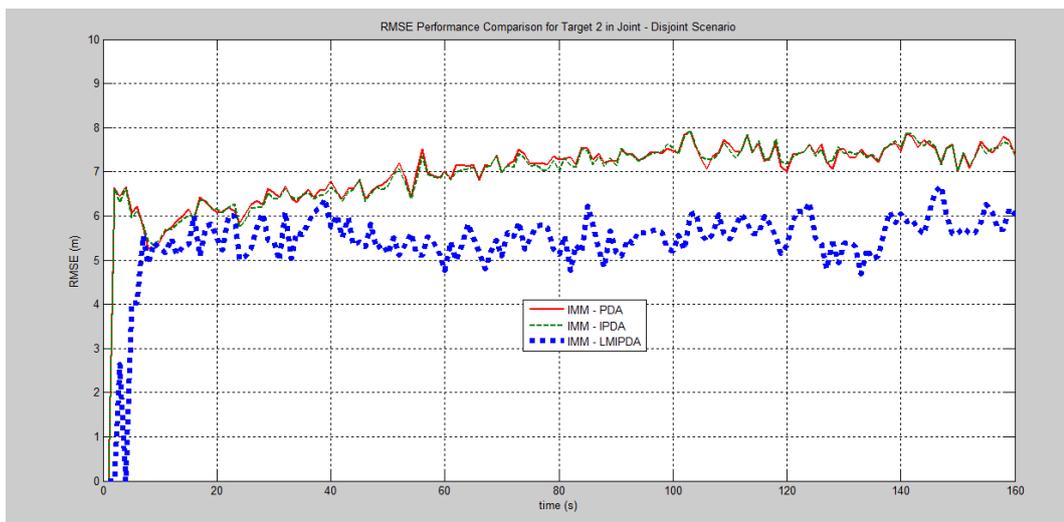


Figure 4.12: RMSE Performance Comparison of IMM-LMIPDA with Single Target Tracking Algorithms in Merging in a junction Scenario for Target 2

Table4.3: Track Loss Statistics in Merging in a junction Scenario

Target Tracking Algorithm	The percentage of Track Loss (%)
IMM-PDA (MTT)	0.733
IMM-IPDA (MTT)	0.446
IMM-LMIPDA	33.093

Table4.4: Computation time Comparison in Merging in a junction Scenario

Target Tracking Algorithm	Computation time (in seconds)
IMM-PDA (MTT)	463.995819
IMM-IPDA (MTT)	468.006861
IMM-LMIPDA	479.113278

IMM-IPDA (MTT) as inferred from Table 4.2 also.

Although IMM-LMIPDA shows better RMSE performance with when compared with Single target tracking algorithms IMM-PDA (MTT) and IMM-IPDA (MTT), Table 4.3 indicates that IMM-LMIPDA is more susceptible to track loss which is one of common problems of general suboptimal approaches[43, 19, 44, 45]. The results are convinient with the results presented in [48, 47, 46, 44], where in [48, 47, 46, 44] results are computed for each target individually, whereas on Table 4.3, the averaged values are presented.

4.4 Merging-Departing in junctions Scenario

A two-dimensional surveillance scenario is formed where two point targets are present which can move either on-road or off-road. They move at the same time, may accelerate or decelerate independently at some time instants, generally on-road in the course of straight movement. So, their trajectories may intersect at some points. They may turn either some direction at junction points. They may break to off-road or enter the road at some points. The simulation scenarios are formed where real ground targets' movement taken into consideration. Hence, their movements are thought to be benign in a constraint fashion. The true target trajectories, measurements observed from targets are labeled individually and false alarm measurements considered as clutter and the resulting state estimates are all depicted in Figure 4.13. As illustrated in Figure 4.13, heavy and dense clutter measurements are always present as in real ground target tracking case. Measurement noise is added inherently to actual states (positions) of the targets under track with a variance $30\ m$ to simulate the target observations obtained from the system. Clutter measurements are generated per scan for each target where the distribution of number of clutter measurements is Poisson with a rate $\lambda = 10$ at each scan where they are distributed uniformly on the surveillance region.

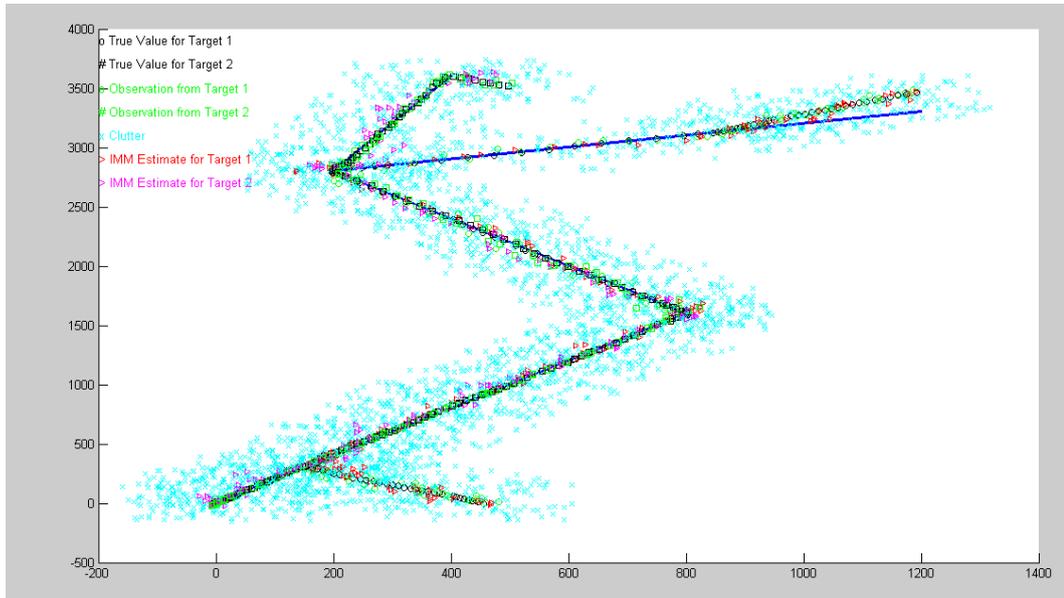


Figure 4.13: Tracking of two targets in clutter merging-departing in junctions

In this scenario, in addition to previous scenario(as illustrated in Figure 4.10), after 130 seconds later they start movement to distant directions in a junction point and depart completely. Like in the third scenario, they move independently, may accelerate or decelerate at some time instants independently which conveys the possibility that their trajectories may intersect or at some points or depart completely from each other after the time instant they merge together.

In all simulations, each track is initiated from a single measurement in a scan. New measurements will become the predicted positions of the new tracks in the next scan. In the next scan, around the predicted positions elliptical validation regions (gates) are formed individually and independently for each track and new tracks are formed from each measurement that fall into each validation gate in-

dividually. Each track is formed, maintained or deleted independently as in the single target tracking case. To control the number of tracks formed in each scan, measurements are divided into sets. All sets consists of the measurements used to update an individual existing track. Each track has an indepent validation gate which means that some measurements from the other validation gate may be used to update more than one track.

For all probability of track existence based target tracking algorithms (IMM-IPDA and IMM-LMIPDA), the initial probabilities of track existence are considered as 0.2 individually, in our simulations, where the references[29, 28] recommend that due to sensitivity of the algorithms to this parameter. Markov Chain One model uses transition matrix Π with entries:

$$\Pi_{11} = 0.98; \quad \Pi_{21} = 0;$$

$$\Pi_{11} = 0.02; \quad \Pi_{21} = 1;$$

where also the references[29, 28] uses the same transition matrix, we used this matrix also for convinience.

During tracking, tracks are confirmed or deleted as in the single target tracking case, but herein, in multitarget tracking case, all the operations are done individually and independently for each track. In the application of multitarget data association algorithms only data association operation is carried out jointly but at the end marginal data association probabilities are extracted from joint data association probabilities to update individual tracks under concern.

For the kinematic state estimator, an IMM estimator with two model-based Kalman Filters is considered consisting of CV(Constant Velocity) with Low Process Noise for nonmaneuvering motion and CV with High Process Noise for any maneuvers including coordinated turns and acceleration modes, with process noise values are considered with standard deviations 2.5 *m*, 30 *m* respectively, as in the single target tracking case. As discussed at [3] and following the result presented in [3] that, it is sufficient to use two model-based filters because our targets move in a constraint fashion, not any evasive, high or different maneuvers are expected. The target and observation models used in Kalman Filters are taken from the reference [3] and [1] respectively.

Each simulation experiment consists of $K = 100$ runs. In each simulation run, targets retrace the trajectory, however, for the measurements obtained from the targets as well as clutter measurements, the numbers and positions of all measurements are generated independently from a pre-specified distributions per each run and scan.

The RMSE performance of single target tracking algorithms including IMM-PDA, IMM-IPDA with an example of Linear Multi-target Approaches in multitarget tracking including IMM-LMIPDA algorithm has been compared. Opimal approaches in multitarget tracking including IMM-JPDA, IMM-IJPDA and IMM-JIPDA are not considered in comparison because pruning of distant gate measurements has been applied before data association, only the measurements at the intersection region of the gates are taken into consideration to reduce the computation requirements of algorithms to a feasible level. Hence, comparison of these algorithms under this situation becomes idle. In addition to this, using single target data association algorithms or Linear Multi-target Approaches in data association where data association is applied to all tracks and measurements, has also been shown[32, 37, 19] much more practical than optimal approaches in this situation.

The resulting RMSE Performance plots for Target 1 and Target 2 are obtained individually and presented in Figures 4.14 and 4.15, respectively. The percentage of track loss and the computation time of algorithms in comparison are presented on Tables 4.5 and 4.6, respectively, where single target tracking algorithms considered in multitarget tracking scenarios, shown with "MTT" in paranthesis.

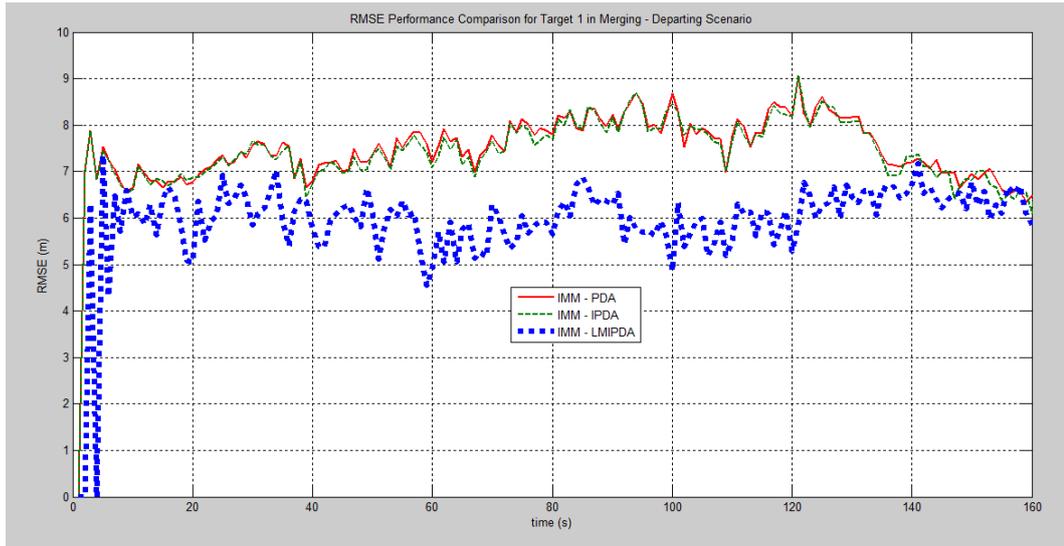


Figure 4.14: RMSE Performance Comparison of IMM-LMIPDA with Single Target Tracking Algorithms in Merging-Departing in junctions Scenario for Target 1

An important result can be inferred from Figures 4.14 and 4.15 that IMM-LMIPDA shows close performance with IMM-IPDA in isolated tracks situation where tracks are sufficiently far apart where their validation regions (gates) do not intersect. The enhancement in the RMSE performance of IMM-LMIPDA which is clearly observed after the first 50 and last 30 seconds in Figures 4.14 and 4.15 where the gates of tracks do not intersect (as illustrated in Figure 4.13). This result proves the theoretical foundation and results given in [35, 32, 18, 19] in convenience. Table 4.6 shows again and

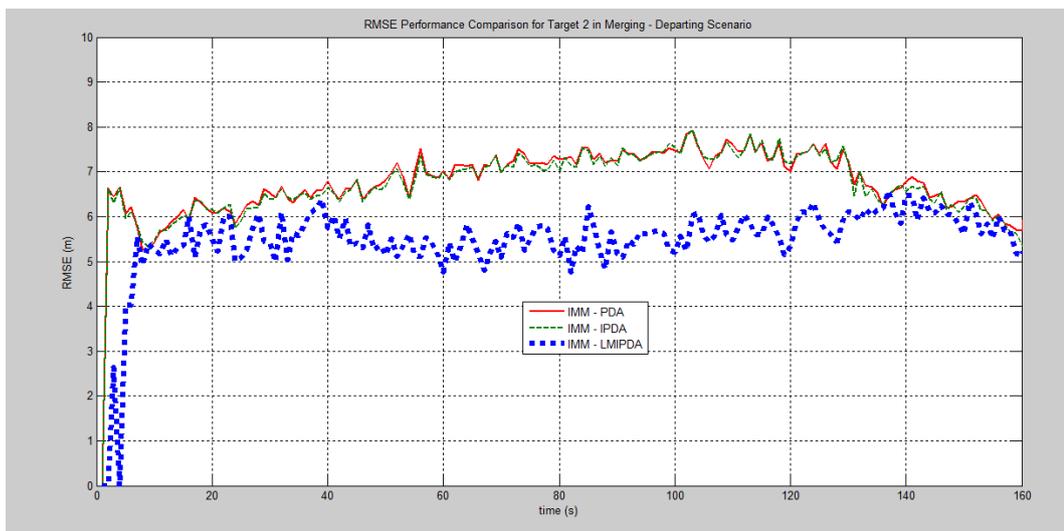


Figure 4.15: RMSE Performance Comparison of IMM-LMIPDA with Single Target Tracking Algorithms in Merging-Departing in junctions Scenario for Target 2

Table4.5: Track Loss Statistics in Merging-Departing in junctions Scenario

Target Tracking Algorithm	The percentage of Track Loss (%)
IMM-PDA (MTT)	0.843
IMM-IPDA (MTT)	0.719
IMM-LMIPDA	34.892

Table4.6: Computation time Comparison in Merging-Departing in junctions Scenario

Target Tracking Algorithm	Computation time (in seconds)
IMM-PDA (MTT)	491.3286
IMM-IPDA (MTT)	492.90469
IMM-LMIPDA	497.360067

proves that, IMM-LMIPDA takes negligibly small increment in computation time more than IMM-PDA (MTT) and IMM-IPDA (MTT) as inferred from Table 4.2 also.

Although IMM-LMIPDA shows better RMSE performance with when compared with Single target tracking algorithms IMM-PDA (MTT) and IMM-IPDA (MTT), Table 4.5 indicates that IMM-LMIPDA is more susceptible to track loss which is one of common problems of general suboptimal approaches[43, 19, 44, 45]. The results are convinient with the results presented in [48, 47, 46, 44], where in [48, 47, 46, 44] results are computed for each target individually, whereas on Table 4.5, the averaged values are presented.

4.5 Single Target Tracking (Isolated Tracks) Scenario with Multitarget Tracking Algorithms

A two-dimensional surveillance scenario is formed where two point targets are present which can move either on-road or off-road. They move at the same time, may accelerate or decelerate independently at some time instants, generally on-road in the course of straight movement. So, their trajectories may intersect at some points. They may turn either some direction at junction points. They may break to off-road or enter the road at some points. The simulation scenarios are formed where real ground targets' movement taken into consideration. Hence, their movements are thought to be benign in a constraint fashion. The true target trajectories, measurements observed from targets are labeled individually and false alarm measurements considered as clutter and the resulting state estimates are all depicted in Figure 4.16. As illustrated in Figure 4.16, heavy and dense clutter measurements are always present as in real ground target tracking case. Measurement noise is added inherently to actual states (positions) of the targets under track with a variance 30 m to simulate the target observations obtained from the system. Clutter measurements are generated per scan for each target where the distribution of number of clutter measurements is Poisson with a rate $\lambda = 10$ at each scan where they are distributed uniformly on the surveillance region.

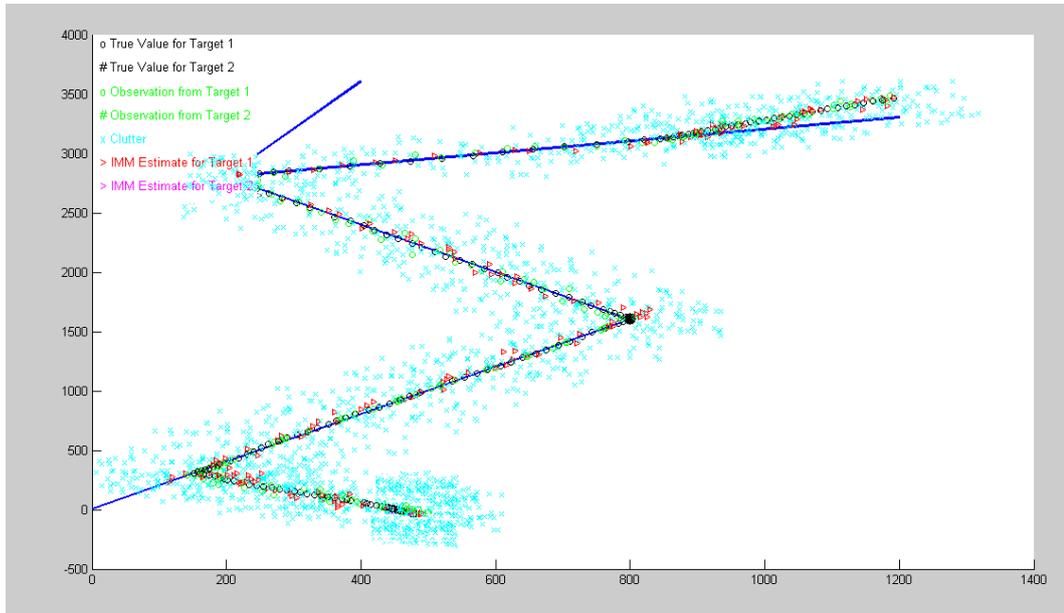


Figure 4.16: Tracking of two targets in clutter where one of the targets stops at initial point

In this scenario, in addition to second scenario (as illustrated in Figure 4.3), they start movement at the same point but one of the targets stops and continues not moving in a convoy fashion as in Figure 4.3. Like in the first scenario, as in Figure 4.1, moving target may accelerate or decelerate at some time instants independently. After they start movement at the same point at initial time, they depart completely from each other at each time instant.

In all simulations, each track is initiated from a single measurement in a scan. New measurements will become the predicted positions of the new tracks in the next scan. In the next scan, around the predicted positions elliptical validation regions (gates) are formed individually and independently for each track and new tracks are formed from each measurement that fall into each validation gate in-

dividually. Each track is formed, maintained or deleted independently as in the single target tracking case. To control the number of tracks formed in each scan, measurements are divided into sets. All sets consists of the measurements used to update an individual existing track. Each track has an indepent validation gate which means that some measurements from the other validation gate may be used to update more than one track.

For all probability of track existence based target tracking algorithms (IMM-IJPDA, IMM-JIPDA and IMM-LMIPDA), the initial probabilities of track existence are considered as 0.2 individually, in our simulations, where the references[29, 28] recommend that due to sensitivity of the algorithms to this parameter. Markov Chain One model uses transition matrix Π with entries:

$$\Pi_{11} = 0.98; \quad \Pi_{21} = 0;$$

$$\Pi_{11} = 0.02; \quad \Pi_{21} = 1;$$

where also the references[29, 28] uses the same transition matrix, we used this matrix also for convenience.

Prunning is not considered in this scenario (which has been considered in the second scenario) before the application of optimal multitarget data association algorithms including JPDA, IJPDA and JIPDA in implementation of optimal multitarget tracking algorithms IMM-JPDA, IMM-IJPDA and IMM-JIPDA, respectively.

During tracking, tracks are confirmed or deleted as in the single target tracking case, but herein, in multitarget tracking case, all the operations are done individually and independently for each track. In the application of multitarget data association algorithms only data association operation is carried out jointly but at the end marginal data association probabilities are extracted from joint data association probabilities to update individual tracks under concern.

For the kinematic state estimator, an IMM estimator with two model-based Kalman Filters is considered consisting of CV(Constant Velocity) with Low Process Noise for nonmaneuvering motion and CV with High Process Noise for any maneuvers including coordinated turns and acceleration modes, with process noise values are considered with standard deviations 2.5 m, 30 m respectively, as in the single target tracking case. As discussed at [3] and following the result presented in [3] that, it is sufficient to use two model-based filters because our targets move in a constraint fashion, not any evasive, high or different maneuvers are expected. The target and observation models used in Kalman Filters are taken from the reference [3] and [1], respectively.

Each simulation experiment consists of 100 runs. In each simulation run, targets retrace the trajectory, however, for the measurements obtained from the targets as well as clutter measurements, the numbers and positions of all measurements are generated independently from a pre-specified distributions per each run and scan.

The RMSE performance of optimal approaches in multitarget tracking including IMM-JPDA, IMM-IJPDA and IMM-JIPDA with an example of Linear Multi-target Approaches in multitarget tracking including IMM-LMIPDA algorithm has been compared. For the optimal approaches in multitarget tracking including IMM-JPDA, IMM-IJPDA and IMM-JIPDA, prunning of distant gate measurements is not considered before data association in this situation because in this scenario after the targets start movement at the same point at initial time, they depart completely from eachother at each time instant. Hence, prunning of distant gate measurements under this situation becomes idle. So, all data association algorithms are applied to all tracks and measurements in this situation for comparison.

The resulting RMSE Performance plot for Moving Target is obtained individually and presented in Figure 4.17. The percentage of track loss and the computation time of algorithms in comparison are

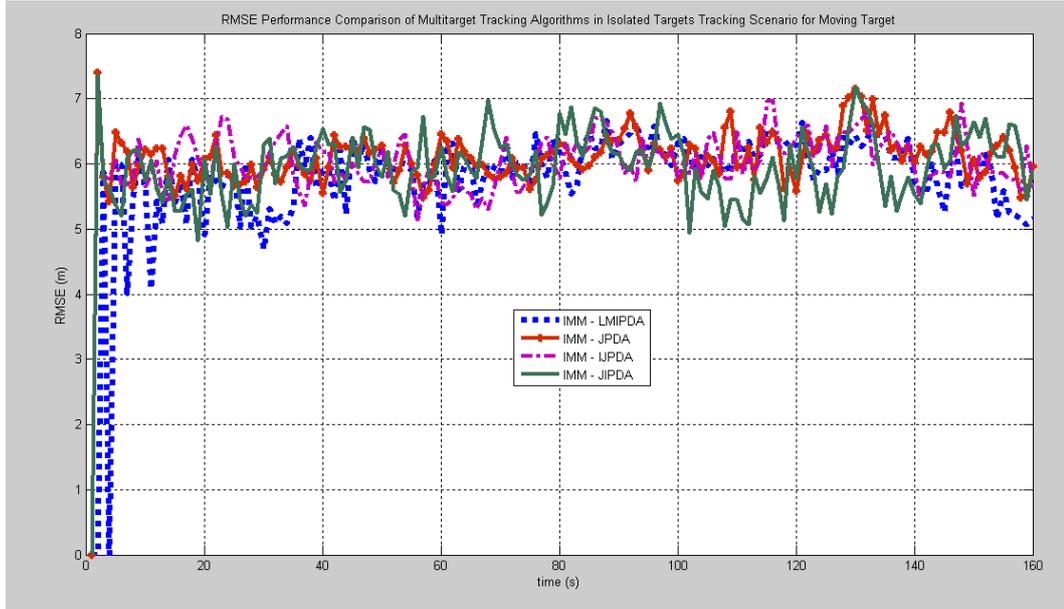


Figure 4.17: RMSE Performance Comparison of Multitarget Tracking Algorithms in Single Target (Isolated Targets) Tracking Scenario for Moving Target

presented on Tables 4.7 and 4.8, respectively.

Table4.7: Track Loss Statistics in Single Target (Isolated Targets) Tracking Scenario

Target Tracking Algorithm	The percentage of Track Loss (%)
IMM-LMIPDA	14.928
IMM-JPDA	0.1515
IMM-IJPDA	0.3085
IMM-JIPDA	0.24878

An important result can be inferred via comparison of Figure 4.17 with Figure 4.2 that all multitarget tracking algorithms can also achieve single target tracking via suitable measurement-to-track assignments in isolated tracks situation. In the literature[1, 38, 30, 34, 32, 19] all multitarget tracking algorithms are mentioned to be derived as “the generalization of single target tracking algorithms to track multiple targets”. Our results proves this information. IMM-JPDA which is the multitarget generalization of IMM-PDA obviously achieves the close performance with IMM-PDA shown in Figure 4.2. IMM-IJPDA which is the multitarget generalization of IMM-IPDA where IPDA[39] considered, achieves also good performance with negligible performance degradation compared with IMM-IPDA shown in Figure 4.2, although in Figure 4.2 IPDA[29, 28] is considered to be used. IMM-JIPDA which is the multitarget generalization of IMM-IPDA where IPDA[29, 28] is considered to be used, “integrates seamlessly”(as mentioned with these words in [30]) with IMM-IPDA shown in Figure 4.2.

Table4.8: Computation time Comparison in Single Target (Isolated Targets) Tracking Scenario

Target Tracking Algorithm	Computation time (in seconds)
IMM-LMIPDA	471.106861
IMM-JPDA	17978.511871
IMM-IJPDA	18281.274496
IMM-JIPDA	18339.510438

Obviously IMM-LMIPDA shows also a close performance, like IMM-JIPDA, with IMM-IPDA in Figure 4.2 in isolated tracks situation where tracks are sufficiently far apart where their validation regions (gates) do not intersect. This result also proves the theoretical information and results given in [35, 32, 18, 19] in convenience. Table 4.8 shows again and proves that, IMM-LMIPDA takes small increment in execution time whereas for optimal multitarget tracking algorithms execution time become more than 60 times of nominal values as inferred from Table 4.2.

Although IMM-LMIPDA shows close RMSE performance with significantly less computation time when compared with optimal approaches (IMM-JPDA, IMM-IJPDA and IMM-JIPDA), Table 4.7 indicates that IMM-LMIPDA still suffers from high track loss even in isolated tracks situation when compared with optimal approaches.

CHAPTER 5

CONCLUSION

In this thesis study, literature survey for various algorithms was done, RMSE performance, track loss and computational load evaluations of target tracking algorithms in interest have been carried out in various test scenarios, benchmarkings are presented.

Problems encountered in tracking of multiple ground targets in clutter have been investigated which are defined basically as the target motion origin uncertainty and the measurement origin uncertainty. General methodology and recommended approaches to these problems are studied and simulations are conducted.

The results show that use of the probability of track existence parameter has been proved to improve the performance modestly. Use of multitarget data association algorithms has been shown to improve the performance significantly in multitarget tracking situations. It has been shown that single target tracking can also be achieved by the use of any multitarget data association algorithms due to the fact that multitarget data association algorithms are nothing more than generalizations of single target data association algorithms under concern.

The use of optimal approaches in multitarget tracking proved the fact that they still offer a good solution in multitarget tracking situations, only in small numbers of measurements and target tracks cases. However, in a ground target tracking application, where heavy and dense measurements and target tracks are present optimal approaches require huge computation load.

Instead of optimal approaches, use of Linear Multi-Target (LM) approaches has been shown to be very efficient method to achieve multitarget tracking, with apparently negligible RMSE performance error compared to optimal approaches, in a dense clutter environment with linear number of operations in the number of tracks and the number of measurements which is comparable with single target tracking algorithms and much less than optimal approaches which require too much excessive computational resources.

Although LM approaches has been shown modestly better in RMSE performance with significantly less computation time than optimal approaches, these methods have rarely proved satisfactory in practice. It has been shown that, they are highly susceptible to track loss when the targets are closely spaced and the number of targets and measurements are considerably high.

Hence, use of LM approaches, i.e. IMM-LMIPDA, in ground target tracking applications, can be a very efficient method to achieve multitarget tracking in a heavy and dense clutter environment on condition that high track loss problem is taken into consideration.

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APPENDIX A

LIST OF TUNABLE PARAMETERS

A.1 Common Parameters

1. T : Sampling (scanning) period for each Kalman Filter model
2. π : Model probability transition matrix (on Table 2.1)
3. N_{FC} : Number of false track confirmations allowed per hour (Eq. (2.24))
4. N_{FA} : Maximum number of false alarms system produces per second (Eq. (2.24))
5. α : False track confirmation probability (Eq. (2.24))
6. β : True track deletion probability (Section 2.2.1.2)
7. P_D : Probability of detection
8. P_G : Gating probability (Section 2.2.2)
9. $\sigma_{CV,LPN}$: Standard deviation value for Constant Velocity (CV) model with Low Process Noise
10. $\sigma_{CV,HPN}$: Standard deviation value for Constant Velocity (CV) model with High Process Noise
11. σ^2 : Measurement noise variance

A.2 Parameters specific only to IMM-IPDA-Based Algorithms

1. Π : Markov Chain One model transition matrix (Eq. (3.13)-(3.16))
2. $\psi_{0|0}^t$: Initial probability of track existence for target track t (Section 3.1.2)
3. $P_{0|0}^O$: Initial probability of track perceivability for target track t (Section 3.2.2)