INVESTIGATING THE USE OF TECHNOLOGY ON PRE-SERVICE ELEMENTARY MATHEMATICS TEACHERS' PLANE GEOMETRY PROBLEM SOLVING STRATEGIES

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ABSTRACT

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The purpose of this study was to investigate plane geometry problem solving strategies of pre-service elementary mathematics teachers in technology and paper-and-pencil environments after receiving an instruction with GeoGebra. Qualitative research strategies were used to investigate teacher candidates' solution strategies. The data was collected and analyzed by means of a multiple case study design. The study was carried out with 7 pre-service elementary mathematics teachers. The main data sources were classroom observations and interviews. After receiving a three-week instructional period, the participants experienced data collection sessions during a week. The data was analyzed by using records of the interviews, answers to the instrument, and transcribing and examining observation records. Results revealed that the participants developed three solution strategies: algebraic, geometric and harmonic. They used mostly algebraic solutions in paper-and-pencil environment and geometric ones in technology environment. It means that different environments contribute separately pre-service teachers' mathematical problem solving abilities. Different from traditional environments, technology contributed students' mathematical understanding by means of dynamic features. In addition, pre-service teachers saved time, developed alternative strategies, constructed the figures precisely, visualized them easily, and measured accurately and quickly. The participants faced some technical difficulties in using the software at the beginning of the study but they overcome most of them at the end of instructional period. The results of this study has useful implications for mathematics teachers to use technology during their problem solving activities as educational community encourages to use technology in teaching and learning of mathematics.

Keywords: Dynamic geometry software, GeoGebra, mathematical problem solving, plane geometry, pre-service elementary mathematic teachers

ÖZ

TEKNOLOJİ KULLANIMININ İLKÖĞRETİM MATEMATİK ÖĞRETMENİ ADAYLARININ DÜZLEM GEOMETRİSİ PROBLEM ÇÖZME STRATEJİLERİ ÜZERİNDE İNCELENMESİ

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Bu çalışmanın amacı ilköğretim matematik öğretmeni adaylarının GeoGebra ile ilgili eğitimi aldıktan sonra teknoloji ve geleneksel kağıt&kalem ortamlarında düzlem geometrisi problem çözme stratejilerinin incelenmesidir. Öğretmen adaylarının çözüm stratejilerini incelemek amacıyla nitel araştırma yöntemleri kullanılmıştır. Veriler, çoklu durum çalışması kullanılarak toplanmış ve analiz edilmiştir. Çalışmanın katılımcıları ilköğretim matematik öğretmenliği bölümünden 7 öğretmen adayıdır. Sınıf gözlemleri ve görüşmeler çalışmanın veri kaynaklarını oluşturmaktadır. Üç haftalık uygulamadan sonra bir hafta boyunca veriler toplanmıştır. Veriler düzlem geometrisi ile ilgili dört açık uçlu soruya verilen cevaplar ile görüşme ve gözlem kayıtlarının incelenmesi ve yazıya aktarılması yoluyla analiz edilmiştir. Öğretmen adaylarından elde edilen veriler göre öğretmen adaylarının çözüm stratejileri üç kategoride incelenmiştir: Cebirsel, geometrik ve birleşik. Katılımcılar kağıt&kalem ortamında çoğunlukla cebirsel, teknoloji ortamında is geometrik çözümler geliştirmişlerdir. Bu sonuç, farklı ortamların öğretmen adaylarının problem çözme becerilerine ayrı ayrı katkıda bulunduklarını göstermektedir. Geleneksel kağıt&kalem ortamından farklı olarak, teknoloji dinamik yapısı sayesinde katılımcıların matematiksel anlayışlarına katkıda bulunmuştur. Ayrıca öğretmen adayları teknoloji ortamında zaman kazanmış, alternatif yöntemleri kolayca geliştirebilmiş, şekilleri eksiksiz çizerek ve kolayca görselleştirerek kesin ve hızlı hesaplamalar yapabilmişlerdir. Öğretmen adayları uygulama sürecinin başında GeoGebra kullanımı ile ilgili bazı teknik zorluklarla karşılaşmış; ancak eğitimlerinin sonunda bu zorlukların üstesinden gelebilmişlerdir. Eğitim çevrelerinin matematik öğrenme ve öğretme sürecinde teknoloji kullanımı ile ilgili teşvikleri doğrultusunda, bu çalışmanın matematik öğretmenlerinin problem çözme etkinlerinde teknoloji araçlarını kullanmaları konusunda faydalı olacağı düşünülmektedir.

Anahtar Kelimeler: Dinamik geometri yazılımı, GeoGebra, matematiksel problem çözme, düzlem geometrisi, ilköğretim matematik öğretmeni adayları

To my parents and my brother Hüseyin Koyuncu

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TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVIATIONS

DGS	Dynamic Geometry Software
CAS	Computer Algebra System
MoNE	Ministry of National Education
ICT	Information and Communication Technology
NCTM	National Council of Teachers of Mathematics
GGB	GeoGebra Based (Solutions)
PPB	Paper-and-pencil Based (Solutions)

CHAPTER 1

INTRODUCTION

In the last century, technology integration into mathematics education has brought about many innovations in the mathematics classroom in terms of development as well as accessibility (Preiner, 2008). Computers are one of the most important tools of technology-supported teaching environments. According to Baki (2001), computer-assisted instruction is a way of instruction that teaching and learning activities are carried out by using computers to acquire knowledge to the students more easily than traditional ways. By doing this, students use necessary software interactively, solve problems step by step, and learn their mistakes by taking instant feedback. If calculations, solutions, modeling activities and graphs are shown in such an electronic environment, it paves the way for new perceptions, estimations, generalizations and explorations (Baki, 2001).

According to National Council of Teachers of Mathematics (NCTM) which is one of the world's largest associations in mathematics education, technology is one of six principles for school mathematics. NCTM (2000) insists on technology-supported school mathematics and continues:

> "The effective use of technology in the mathematics classroom depends on the teacher. Technology is not a panacea. As with any teaching tool, it can be used well or poorly. Teachers should use technology to enhance their students' learning opportunities by selecting or creating mathematical tasks that take advantage of what technology can do efficiently and well — graphing, visualizing, and computing."(p.25)

Instead of exposing students to do long calculations and memorize many mathematical formulas and concepts, there are many suggestions about using technology tools to develop their mathematical thinking, problem solving and creativeness (Ersoy, 2003). Technology supported environments help students to avoid wasting time and exploring mathematical ideas and conjectures easily. Ersoy

(2003) states that since the benefits of using technology in mathematics education are supported by many researches, the teachers and educators should be aware of the opportunities of these technologies, and use them as an essential part of their activities and instruction.

NCTM (2008) argues that calculators, computer algebra systems, interactive geometry software, applets, spreadsheets, and interactive presentation devices are very important for a high quality mathematics education. The Council asserts that technology is essential in teaching and learning mathematics and hence all schools need to have necessary equipment for active use of technology. In addition, if technology is effectively used, all students will be able to learn mathematics.

However, Risser (2011) discusses the arguments against using technology in mathematics education. Although the advent in Computer Algebra Systems (CAS) and graphing software helps students solve many routine problems, some teachers and educator have arguments against using technology in mathematics education. For example, the integration of technology at early ages can cause ‘perceived neglect of basic skills. Calculators may weaken paper-and-pencil skills and deprive students from basic skills for higher level mathematics, and they may harm number sense and the skills for thinking abstractly. However, after listing all these arguments, Risser (2011) added that the letters and articles which were analyzed are an extremely small percentage of total publications in this field and there are also vast amounts of articles and letters which praise the benefits of technology use in mathematics education.

Altun (2011) also stresses that the biggest concern in the use of technology tools in mathematics education is the fact that it will decrease the quality of education by weakening calculation and operation skills. Hence, it will eliminate the need of understanding and comprehending some mathematical concepts. However, since computers and other technological tools save the time and make life easier they are inseparable part of human life in the 21. Century. In his book, Altun (2011) gave an example from real life. He exemplified that humans do not forget walking after the automobiles come into their life, on the contrary, they had the possibility to reach the environments that they could not go in the past and the opportunity to meet a great deal of new things. Using new technologies in mathematics education will provide similar benefits. Therefore, instead of using these tools untimely, teachers need to adjust the time and situations for using them, and utilize them to contribute teaching and learning activities (Altun, 2011).

Association of Mathematics Teacher Educators (AMTE), which is the largest professional organization devoted to mathematics teacher education with over 1000 members, defines technology tools as computers with appropriate mathematical software, internet and other digital resources, handheld computing tools and their extensions, and future and emerging forms of similar devices and applications. AMTE (2006) agrees with NCTM (2000) by supporting technology principle and the association proposed that with the use of technology, the process of mathematical discovery, understanding and complicated connections can be facilitated. Moreover, technology provides effective representation of mathematical ideas, processes, and activities that make mathematical exploration and sense making easier. With the use of technology, mathematical knowledge of students, the instruction, and mathematical resources could be empowered and enhanced (AMTE, 2006).

The integration of technology in mathematics is a process in which technological tools have both external and internal role. Chen (2011) discusses *instrumental* and *substantive* theory of technology in mathematics education. According to *instrumental* theory, human mind is separated from technology. Hence, while mathematical calculations, demonstrations and manipulations are done; technology tools play an external role. However, substantive theorists believe that while doing mathematical activities, technology is internalized by the students and it mediates their development of mathematical knowledge (Chen, 2011). In other words, as mathematical knowledge is built, technology becomes a part of their way of learning mathematics. Therefore, while teachers use technology as a tool, the social and cultural effects of technology on their learning styles should be taken into consideration.

Moreover, research suggests that for the successful integration of technology into classrooms, many teachers think that merely providing technology is not enough (Cuban, Kirkpatrick & Peck, 2001). The main reason for using technology in mathematics education is to increase teacher effectiveness and improve student learning; therefore, teacher should learn not only how to use technology but also how to incorporate it into their instruction (Doğan, 2012). In addition, Borwein and Bailey (2003) stated that computer is used in mathematics education for gaining insight and intuition, discovering new patterns and relationships, graphing to expose mathematical principles, testing and especially falsifying conjectures, exploring a possible result to see whether it merits formal proof, suggesting approaches for formal proof, replacing lengthy hand derivations with tool computations, and confirming analytically derived results.

Koehler and Mishler (2005) conceptualize teachers' knowledge as a combination of their content, pedagogy and technology knowledge. The instruction will be effective if it focuses on these knowledge bases in relation to each other. The framework, jointly called as Technological Pedagogical Content Knowledge (TPCK), requires technology integration that supports successful representation of new concepts and causes dynamic, interrelation between all three components (Koehler & Mishler, 2005). Lee and Hollebrands (2008) argue that by improving teachers' understanding of TPCK with a focus on students' mathematical skills, teacher will be able to explore what is needed. In addition, they will realize when using technology in mathematics teaching and be equipped with appropriate uses of technology.

From pre-service teachers' perspective, knowing to use technology during learning and teaching is necessary for an effective instruction (Bulut & Bulut, 2011). Pre-service teachers learn basic computer tools during graduate education. However, Kokol-Volj (2007) stated that training pre-service teachers about how to use technology during their teaching is an essential aspect of mathematics education programs. They need to know how to integrate appropriate mathematical software to their instructions. In Turkish mathematics education curriculum, the use of mathematical software, especially dynamic ones, is strongly emphasized (MoNE, 2006). Therefore, their content knowledge needs to be supported by using technology tools in teacher training programs. AMTE (2006) determined the technology competencies for mathematics teacher candidates. The Association stresses that mathematics teacher candidates should have sufficient conceptual understanding of K-12 mathematics to support it by using technology, understand how the students learn mathematics and the how technology influence this learning, know the effective use of technology in teaching and learning mathematics, experience the use of variety of technology tools to increase the students' and their own mathematical learning.

Technology tools provide powerful range of visual representations which help teachers to focus students' attention to mathematical concepts and techniques (Zbiek, Heid, Blume & Dick, 2007). There are two fundamental types of technology tools in mathematics education; Computer Algebra Systems (CAS) and Dynamic Geometry Software (DGS). CAS is used to solve mostly algebraic problems. Drijver (2003) stated that this software is effective in contributing students' higher level of algebra concepts. In addition to numerical and graphical calculators, CAS is also widely used in solving mathematics problems. There are many research projects that center the use of CAS during their activities (Artigue 2002; Cuoco 2002; Kutzler, 2000; Ruthven, 2002).

However, in teaching and learning geometry, particularly Euclidean geometry, and solving problems related to geometry concepts, DGS - a group of programs for doing "dynamic geometry" – is the most appropriate tool (group of tools) (Kokol-Voljc, 2007). The term 'dynamic' refers to adapting and changing figure to observe the differences. According to Kokol-Voljc (2007), three main characteristics of DGS are:

- It is a dynamic model of paper and pencil with the drag mode
- A sequence of commands are combined to form a macro
- Movements of geometrical objects are visualized like a locus

While the students dragged the points or figures on these dynamic tools they have different goals (Arzarello, Micheletti, Olivero, Robutti, Paola, & Gallino, 1998; Hollebrands, Laborde & Strasser, 2006; Rivera, 2005). The students prefer to three types of dynamic movements; wandering dragging, lieu muet dragging, and dragging test (Arzarello et al., 1998). In wandering dragging, students' aim is to observe the regularities and exploring interesting results while dragging. In lieu muet dragging, the students aim to preserve some regularity in the construction. They drag a point to observe the difference while other variables are invariant. The third type, dragging test, means observing changes to test a hypothesis during dragging.

However, since the use of DGS decreases the need for ruler and compass, it is advised that DGS should not replace them but improve and complement them. Although there are many advantages of constructions made with DGS, the construction activities with paper-and-pencil should not be lost because both DGS and paper-and-pencil environments make great contributions to students' concept development (Kokol-Voljc, 2007; Coşkun, 2011). Therefore,in the present study both paper-and-pencil and GeoGebra as a DGS will be used to benefit the advantages of both environments. GeoGebra is a dynamic software that combines both algebra and geometry tools. It is an open source and freely available software. In addition, it is multilingual and includes more than 50 language options in both menus and commands. It is constructed on a Cartesian coordinate and accepts both geometric and algebraic commands (Suzuki, 2006).

In recent studies, the researchers mostly prefer to use GeoGebra in their studies instead of other DGS such as Cabri, GSP, etc. (Chrysanthou, 2008; Hohenwarter & Fuchs, 2004; Iranzo-Domenech, 2009; Preiner, 2008; Coşkun, 2011). The preference for GeoGebra is derived from the fact it combine geometry and algebra. In addition, easy-to-use, user friendly interface, and being open source are other factors for choosing GeoGebra. Coşkun (2011) used the software to determine students' visual and non-visual problem solving methods. Iranzo-Domenech (2009) also developed problems related to plane geometry and utilized GeoGebra to observe the synergy of environments. Since the effect of technology on students' problem solving strategies was investigated in the present study, it is essential to work over mathematical problem solving and the use of technology in this process.

Problem is a situation that consists of exact open questions which will "challenge somebody intellectually who is not in immediate possession of direct methods/procedures/algorithms, etc. sufficient to answer the question" (Blum & Niss ,1991). Problem solving is a process of engaging in a task or situation for which there is no obvious or immediate solution (Booker & Bond, 2008). The students learn content of the area and explore different ideas during this process.

In the mathematics classroom, the aims of teaching with problem solving are developing operation skills; getting used to deal with numbers and figures; collecting and classifying data; drawing figures and schemas that are appropriate to the context of the problem; explaining the ideas with the language of mathematics; and understanding mathematical expressions that are used in various publications (Altun, 2008). In the Turkish mathematics teaching curriculum, problem solving is seen as a main role of the students while learning mathematics in the classroom (MoNE, 2009). The Ministry determined the main aims of mathematics teaching and stressed the importance of the problem solving. For example, as a general aim, the students will be able to state their mathematical ideas and reasoning during the problem solving process. Moreover, the students will be able to develop different problem solving strategies and use them in daily life problem situations.

It is significant to integrate technology into problem solving process in addition to importance of teaching mathematics with problem solving. The educational community has a general acceptance of the significant role of technology in mathematical problem solving (Kuzle, 2011). As stated by MoNE (2006), the students need to use mathematical software during their activities in the classroom. Moreover, during problem solving in technology environment, teachers will be able to realize students' difficulties in understanding mathematical thinking and their problem solving tendencies (Coşkun, 2011).

There is not much studies focus on the effect of technology on students' problem solving preferences (Coşkun, 2011; Harskamp, Suhre & Van Streun, 2000; Iranzo-Domenech, 2009; Yerushalmy, 2006). In technology environment, the students are able to develop alternative strategies and explore different strategies that could not be explored easily in paper-and-pencil environment (Coşkun, 2011).

Moreover, Iranzo-Domenech (2009) stressed that when students solve problems in technology environment, they tended to develop different competences based on their mathematical knowledge.

Krutetskii (1976) defines analytic, geometric and harmonic thinkers according to gifted children's thinking preferences for solving mathematical problems. These preferences were determined by looking at students' verbal-logical and visual-pictorial components of mathematical abilities. The students who solve the problems by thinking analytically use more verbal-logical components than visual-pictorial ones. Geometric thinkers solve the problems by using mostly visual-pictorial means, and harmonic thinkers have a relative equilibrium between verbal-logical and visual-pictorial components. Presmeg (1997) stressed that this classification of problem solvers is appropriate for the students at all levels. Hacıömeroğlu (2007) identifies Krutetskii's (1976) verbal-logical component of solution methods as interpreting visually presented concepts with mathematical symbols. The students who use mostly their visual-pictorial components tend to use visual schemas of figures.

While students solve the problems they use different types of mental activities such as verbal, numeric, mathematical symbols to form an image (Coşkun, 2011). Presmeg (1986) defines five types of visual imagery: concrete pictorial imagery, pattern imagery, memory images of formulae, kinesthetic imagery, and dynamic imagery. In concrete pictorial imagery the students memorize the objects in detail such as memorizing images of trigonometric functions. The students who use pattern imagery disregard concrete details and determine pure relationships such as chess masters‘ remembering the places of pieces on a chessboard for a given unfamiliar situation (Coşkun, 2011). The students who use memory images of formulae have abstract information such as remembering a formula written in a book. In kinesthetic imagery, the students use muscular activity such as doing calculations by using their fingers. The students who use dynamic imagery prefer the images of dynamic movements such as transforming mentally a rectangle into a parallelogram. Among these five types of imagery, dynamic imagery is the most efficient type in describing dynamic movements in GeoGebra environment.

1.1 The Purpose of the Study and the Research Problem

The purpose of the present study is to show and reveal how pre-service elementary mathematics teachers develop their strategies when solving plane geometry problems in GeoGebra and paper-and-pencil environments. After a three-week instructional period concerning GeoGebra use, teacher candidates have been observed and interviewed about their solution strategies for the plane geometry problems by using both GeoGebra and paper & pencil. In addition, prospective teachers were asked about the practicality of the software in terms of difficulty, time commitment and their comfort using GeoGebra. Therefore, it is also aimed to show how GeoGebra is a useful tool in teaching plane geometry concepts. The present study aims to address the following research problem:

How does GeoGebra play role in pre-service elementary mathematics teachers' plane geometry problem solving strategies?

Within this question, the researcher aims to determine pre-service teachers' solution strategies in technology and paper-and-pencil environments, role of both environments during problem solving process, benefits and drawbacks of using technology in mathematical problem solving, and difficulties that pre-service teachers experienced in using technology.

1.2 Significance of the Study

The use of technology tools in mathematics classroom has great benefits for teaching geometry concepts due to its various benefits such ease to use, availability, or visualization of mathematical relationships. Research suggests that using technology in classrooms facilitates classroom activities and enhances productivity and quality of lessons (Chrysanthou, 2008). Technology tools facilitate and develop students' skills to solve problems and give students the chance to think about problems and their solution strategies when used efficiently (Altun, 2011; Baki, 2001; Lee & Hollebrands, 2008; Risser, 2011).

Ministry of National Education (MoNE) in Turkey determined mathematics teachers' special area competencies for technology use. MoNE (2010) suggests that mathematics teachers should be able to use technology resources in mathematics education and know mathematics software. In addition to these competencies, mathematics teachers need to be technology literate and follow developments in Information and Communication Technologies (ICT) (MoNE, 2006). In addition, NCTM (2012) stresses the use of DGS in mathematics classroom and determined the following objectives:

- Exploring properties of rectangles and parallelograms using dynamic software.
- Learning about length, perimeter, area, and volume of similar objects using interactive figures.
- Learning about properties of vectors and vector sums using dynamic software.
- Understanding ratios of areas of inscribed figures using interactive diagrams.

It is clear that before using dynamic geometry software in solving geometry problems, teachers need to be well prepared concerning use of the software to benefit at the maximum level during their teaching experience.

Furthermore, Turkey has recently (in February 2012) initiated the FATIH (Movement of Enhancing Opportunities and Improving Technology) Project. The main aim of the project is to enable equal opportunities in education and improve technology use in the schools. The Ministry will equip all 620.000 schools including preschool, primary, and secondary institutions through providing tablets and LCD smart boards. It is aimed to achieve active use of ICT in every class in the country by the end of 2013 (MoNE, 2010). For this reason, prospective teachers need to be well prepared for using computer software in their lessons. Therefore, present study is expected to contribute to the use of technology in mathematics classroom, particularly in solving plane geometry problems.

Moreover, the students face problems in learning the distance and area in plane geometry because it requires coordination of both visualization and reasoning processes (Iranzo-Domenech, 2009). DGS helps students to visualize, explore, and understand the relation between the distance and area of plane geometry proof problems. During the process of proving any relationship on plane geometry problems, GeoGebra is expected to be a key element in facilitating both visualization and reasoning of prospective teachers at the same time. Therefore, in order to improve students' skills to deal with such problems, the integration of GeoGebra in geometry needs to be emphasized. The present study is expected to focus students and teachers' attention to the significance of using DGS by revealing benefits of GeoGebra in discovering different problem solving strategies for plane geometry proof problems.

In addition the main rationale for the present study was the lack of sufficient in-depth research on the effect of technology on students' preferences for problem solving strategies. There are some studies analyzing students' solution strategies in different environments including DGS and paper-and-pencil (Coşkun, 2011; Harskamp, Suhre & Van Streun, 2000; Iranzo-Domenech, 2009; Yerushalmy, 2006). For example, Coşkun (2011) used the software to determine students' visual and non-visual problem solving methods. Iranzo-Domenech (2009) also developed problems related to plane geometry and utilized GeoGebra to observe the synergy of environments. Harskamp et al. (2000) and Yerushalmy (2006) investigated how students differ in their solutions in different environments. However, although students' variations in developing different problem solving strategies were analyzed in the present study as it was in the past studies, the combination of the content area of study and the data analysis framework was the first in comparing and contrasting their solution strategies.

1.3 Definition of Important Terms

Problem: It is a situation that consists of exact open questions which will "challenge somebody intellectually who is not in immediate possession of direct methods/procedures/algorithms, etc. sufficient to answer the question" (Blum & Niss, 1991).

Problem solving: It is a process of engaging in a task or situation for which there is no obvious or immediate solution (Booker & Bond, 2008).

Dynamic Geometry Software: It is a kind of computer software that allows users to visualize geometric figures and shapes by multiple representations including dragging and moving them while mathematical relationships are still preserved (Goldenberg & Couco, 1998).

Plane Geometry: The geometry of planar figures with two-dimensional surface.

CHAPTER 2

LITERATURE REVIEW

In this chapter, the literature related to the content of the present study is outlined. The chapter begins with discussing the studies and viewpoints of education community about the integration of technology in mathematics education. Then, as a technology tool used in the present study, the inquiry about dynamic geometry software is narrated. Since there are different types of dynamic software such as GeoGebra, The Geometry's Sketchpad, the literature related to this software is summarized. Next, problem solving and students' problem solving preferences in technology environment is explained in light of the literature. Finally, this chapter was summarized in the last part to make a clear picture of all mentioned information.

2.1 The Use of Technology in Learning and Teaching of Mathematics

During the last quarter of a century, educators witnessed a great growth in the use of technology in mathematics classrooms. According to many researchers, teachers, and documents for the reform in mathematics education, technology supports students' understanding of mathematics, and they suggest the integration of technology into mathematics teaching and learning (Hollebrands, 2003). The research community has a parallel interest in the effect of technology on learning and teaching mathematics, and the curriculum (Zbiek et al., 2007).

Many studies were investigated to determine the effectiveness of technology in mathematics education (Baki, 2001; Borwein & Bailey, 2003; Doğan, 2012; Ersoy, 2003; Hollebrands, 2003; Koehler & Mishler, 2005; Lester, 1996; NCTM, 2000). In a study, Doğan (2012) investigated a study on pre-service teachers' views about the use of technology in mathematics education. The data was collected from 129 students at two universities in Turkey. He categorized the data comprising of written responses according to TPACK framework and analyzed data qualitatively. The results of the study showed that prospective mathematics teachers have positive views about computers and the use of technology in teaching mathematics. Most of the students thought that using technology in mathematics education will result in an effective teaching and learning of mathematics (Doğan, 2012).

In addition, in order to determine the effectiveness of instruction in a technology environment, Lester (1996) designed an experimental study. In the study, the participants were high school students and The Geometry's Sketchpad was used as a technological tool. The experimental group was taught in a cognitively guided technology environment. The control group experienced a course book based on traditional instruction. In addition to pre-test and post-test results, she interviewed the students in experimental group. The results of the study showed that the students who were taught with computer-assisted instruction scored over the students who were taught with traditional instruction (Lester, 1996).

Similarly, Hollebrands (2003) investigated on the use of the technological tool, The Geometry's Sketchpad, to examine the nature of students' understanding of geometric transformations including reflections, translations, dilations, and rotations. The case study approach and constant comparison method were used with 16 tenth grade students. The students experienced a seven-week instructional period. The data sources were students' worksheets, observations, and interview documents. The researcher analyzed data in-depth and used a research framework to characterize students' understanding of geometric concepts and their methods in interpreting of geometrical representations. Hollebrands (2003) suggested that with the use of technology, students' understanding of transformations were critical for promoting the improvement of deeper understanding of transformations as functions. The study was seen as a first step to see how technology affects students' understanding of geometry. The researcher suggested investigating more on understanding the complexities that students and teachers experienced in teaching and learning mathematics by the use of technology. Ersoy (2003) also investigated a study on the use of computers and calculators in teaching and learning mathematics to contribute in developing strategies and developments in mathematics teaching process. The results of his study showed that the students need to understand how to use technology tools in their learning experiences.

In addition, Van Voorst (1999) studied on the effectiveness of using computers in teaching and learning mathematics and argued that computers are effective tools in learning mathematics. That is, computers make students more active in forming solutions steps, producing new information, asking further questions, solving the problem, and exploring new ideas and strategies. Moreover, Van Voorst (1999) stressed that technology provides students the opportunity to visualize mathematical concepts easier than traditional ways, and overcome individual problems by providing personal activities.

The researchers has an obvious assumption that teachers with better mathematical content knowledge, pedagogy, and knowledge about the research results about technology integration would help them to integrate technology easily and effectively into their instruction (Preiner, 2008). Bulut and Bulut (2011) investigated a study to explore pre-service teachers' views about the use of technology in mathematics teaching and learning. The participants were 47 prospective teacher and qualitative data analysis methods were used. They designed an instructional period to teach basis software commands. Then, they analyzed students' responses to the mathematical problems and interviewed with them. They concluded that prospective teachers have positive views and they want to use technology in their future instruction. In addition, pre-service teachers want to offer mathematical interactive software to their students for better learning of mathematical concepts (Bulut & Bulut, 2011).

In a study, Güven (2007) designed an exploratory study with 40 pre-service teachers to observe the actions as they were working with minimal network problems. The students were taught in Teaching Mathematics with Computers course during a semester. In the introductory phase, the students gained basic technical knowledge about the use of the software, Cabri Geometry. In exploratory phase, the students experienced a problem based instruction about points and distances in plane geometry. The data were students' worksheets and classroom observations. The results of the study revealed that in computerize environments; the students are able to develop their skills in decision making, experimental verification, conjecturing, and even construction of proofs.

In addition, Habre and Grunmeier (2007) designed an exploratory study to determine the views of pre-service teachers' use of technology before, during and after phase of a set of classroom activities. The participants were 29 prospective teachers and the computer software that used was Excel, Fathom, and The Geometry's Sketchpad. The participants taught during a ten-week instructional period. The data included classroom observations, the results of a survey administered at the beginning of the semester, students' homework assignments, lesson plans, and article critiques. Before the activities, most of the students thought that technology is helpful in mathematics but they were not aware of its potential applications. When, they experienced the activities, they developed their abilities with the use of the software in mathematical activities. They developed their views on using technology and they realized that integration of technology in their learning and teaching can enhance their understanding of mathematical ideas and their instruction (Habre & Grunmeier, 2007).

In order to understand how teachers at different levels of technology usage and teaching abilities used technology, Pierson (2001) designed a qualitative case study. In addition, the researcher investigated on how technology use related to general teaching practice. For the data collection, 16 in-service teachers' teaching observed and three of them were interviewed. The participants, exemplary technology integrators, differed in the ways these teachers taught with technology, including the existence of teachers' personal definitions of technology integration, distinctive planning habits when planning for technology inclusion, strategies for teaching about technology that matched teacher learning strategies, management of student computer use, and perspectives on assessment (Pierson, 2001). Therefore, the technology integrators at different levels of technology usage and teaching abilities may lead to these differences.

Baki (2000) analyzed an undergraduate course about the use of technology in mathematics education. The course aimed to teach pre-service teachers and to investigate their perceptions on their preparation to use computers in their future teaching experience. Data was collected from questionnaires and students' ideas about the course. The students who felt themselves prepared for teaching mathematics were able to make connections between computer-based mathematics activities and school mathematics, and they had more potential for using technology tools in the course.

2.2 Dynamic Geometry Software in Mathematics Education

Dynamic environments allow users to change the appearance of the geometric figure while mathematical relationships on the figure are still preserved (Goldenberg & Couco, 1998). In this environment, the visual figures are enriched with dynamic movements to help students in developing their strategies and improving their mathematical understanding. Visualization is among the one of the most important aspects of geometric thinking (NCTM, 2000); therefore, it has vital importance. The students drag and move the points to observe changes in the relationships on the figures by using the software.

While the students use dragging options of the dynamic environments, they have different goals (Arzarello, Micheletti, Olivero, Robutti, Paola, & Gallino, 1998; Hollebrands, Laborde & Strasser, 2006; Rivera, 2005). The students mostly prefer three types of dynamic movements; *wandering dragging*, *lieu muet dragging*, and *dragging test* (Arzarello et al., 1998). In *wandering dragging*, students' aim is to observe the regularities and exploring interesting results while dragging (Zbiek et al., 2007). For example, in Figure 1, a student who drags the point C wonders about what happens to point F where CF is the altitude of the triangle CDE. The pink shape in Figure 1 represents the tracing area existed when the point C dragged.

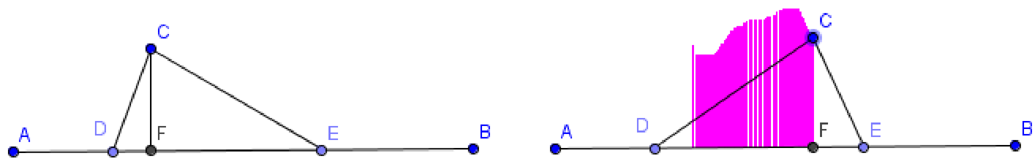


Figure 1. Wandering dragging

In *lieu muet dragging*, the students aim to preserve some regularity in the construction (Zbiek et al., 2007). They drag a point to observe the difference while other variables are invariant. For instance, in Figure 2, a student might drag point E along line segment AB, keeping angle Y in triangle DEC and the length of line segment DC constant. The pink shape in Figure 2 represents the tracing area existed when the point E dragged along line segment AB.

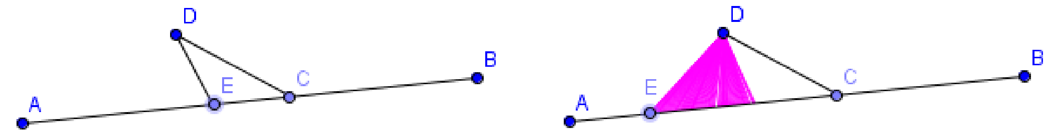


Figure 2. Lieu muet dragging

The third type, *dragging test*, means observing changes to test a hypothesis during dragging (Zbiek et al., 2007). For example, by dragging the point D, a student might test the conjecture that the angle DCE is obtuse when the perpendicular line segment DF is exterior of the triangle ABC. The pink shape in Figure 3 represents the tracing area existed when the point D dragged.

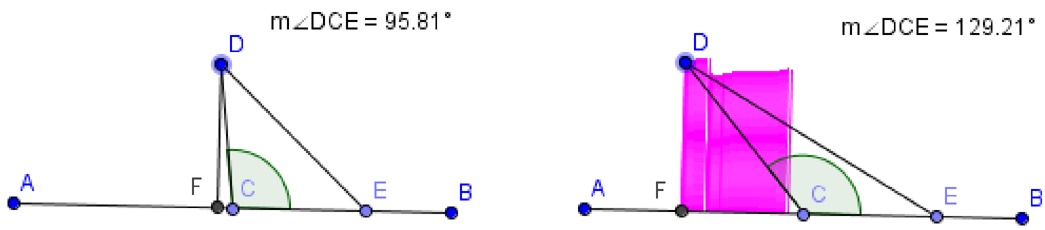


Figure 3. Dragging test

The effectiveness of Dynamic Geometry Software (DGS) in mathematics classroom is a widely researched area (Baki, Kosa & Güven, 2011; Christou, Mousoulides, Pittalis & Pitta-Pantazi, 2004; Güven, Baki & Çekmez, 2012; Habre, 2009; Pandiscio, 2010; Stols & Kriek, 2011). In order to compare the effects of using DGS and concrete materials, Baki, Kosa and Güven (2011) investigated a study with 96 prospective teachers by using a pre- and post-test quasi-experimental design. The Purdue Spatial Visualization Test was used as pre- and post-test. Intervention groups used Cabri 3D DGS and concrete materials while the control group used the traditional instruction method. The results of the study revealed that compared to the traditional method, the instruction with DGS and concrete materials is more effective. In addition, it is found that using DGS is a more powerful way of teaching than using materials (Baki et al., 2011). These findings contribute to the present study by observing pre-service teachers' spatial visualization performance using GeoGebra DGS in visualizing students' different problem solving strategies.

Güven, Baki and Çekmez (2012) investigated a study to observe different problem solving strategies of 34 undergraduate students by using DGS including *Logo*, *Coypu*, *Derive* and *Cabri*. They designed an explanatory study during problem solving sessions with geometric constructions worksheets. At the end of the study, the students developed five different approaches to the problem. However, they used three approaches that exemplify the contribution of DGS to the problem solving process; Solution with locus, solution with observations, solution with measurement. In solution with locus, the student used locus the feature of the software to observe the movement of any point. In solution with observations, the dragging feature of the software was used to observe the changes dynamically. In solution with measurement, the students solve the problem by measuring the lengths, angles, areas, and volumes. The tools and features of the software help students to explore various problem solving strategies that could not be solved in a paper & pencil environment. Moreover, since DGS allowed students to find accurate dynamic calculations and measurements, it has many advantages over a traditional setting (Güven et al., 2012).

In addition to exploring different problem solving strategies, DGS allows users to show and prove mathematical relationships on the geometric figures quickly and easily. DGS's dragging feature enables variation in the geometric configurations and allows one to explore whether geometric hypotheses are true or not (Christou, Mousoulides, Pittalis & Pitta-Pantazi, 2004). For example, Christou et al (2004) investigated a case study with three prospective primary teachers to observe how DGS could be improved to teach proof and make it meaningful to the students. The study revealed that DGS provides both the ways to confirm or reject a conjecture and ideas for explaining and verifying this conjecture (Christou et al, 2004).

Pandiscio (2010) also investigates a case study on perception of secondary pre-service mathematics teachers about the usefulness of using DGS in formal proof problems. The participants were four prospective teachers and high school students. They used *The Geometry's Sketchpad* to solve two proof problems. The surveys, observations, and interviews are conducted to collect data. According to survey results, high school students think that after using DGS, the need for formal proofs in geometric tasks decrease because the software allows users to see obviously geometric relationships and the rationale for solutions. In addition, pre-service teachers explored the great advantages of geometric software in understanding the relationships within geometric conjectures and DGS helped them think about solutions more deeply than traditional ways (Pandiscio, 2010). The results of Pandiscio's (2010) study overlapped with the study of Christou et al (2004) in terms of the effectiveness of DGS in proof problems.

However, there is an ongoing debate about the use of DGS in the mathematics classroom regarding enhancing students' learning and understanding of mathematical topics (Habre, 2009). The limitation of using DGS in classroom is another matter that needs to be handled in the present study. Habre (2009) investigated an experiential qualitative study with prospective mathematics teachers to determine whether DGS contribute to students' understanding of geometrical conjectures and concepts. During a course designed for this purpose, 29 students use *Geometry's Sketchpad*, *Fathom* and *Excel* while solving three tasks related to Euclidian geometry. Habre(2009) observed students while solving problems and then interviewed with them about their solutions. The results showed that dynamic movements of figures might sometimes be misleading and the technology used in solving a given geometrical problem need to be properly overlapped with the features of the problem. Moreover, the teachers' role is an important factor in the development of the solution approaches (Habre, 2009).

Indeed, teachers' beliefs and proficiencies are other factors that limit the usage of DGS in classrooms (Stols & Kriek, 2011). In order to examine why some teachers do not want to use DGS, Stols and Kriek (2011) designed a correlational research study. The participants were 22 high school teachers, and a Behavior Belief

Perceived Usefulness questionnaire was used. The results were analyzed according to correlational statistics and regression analysis. According to the results of the analysis, the researchers developed the following *Simplified Model* for dynamic software (Figure 4).

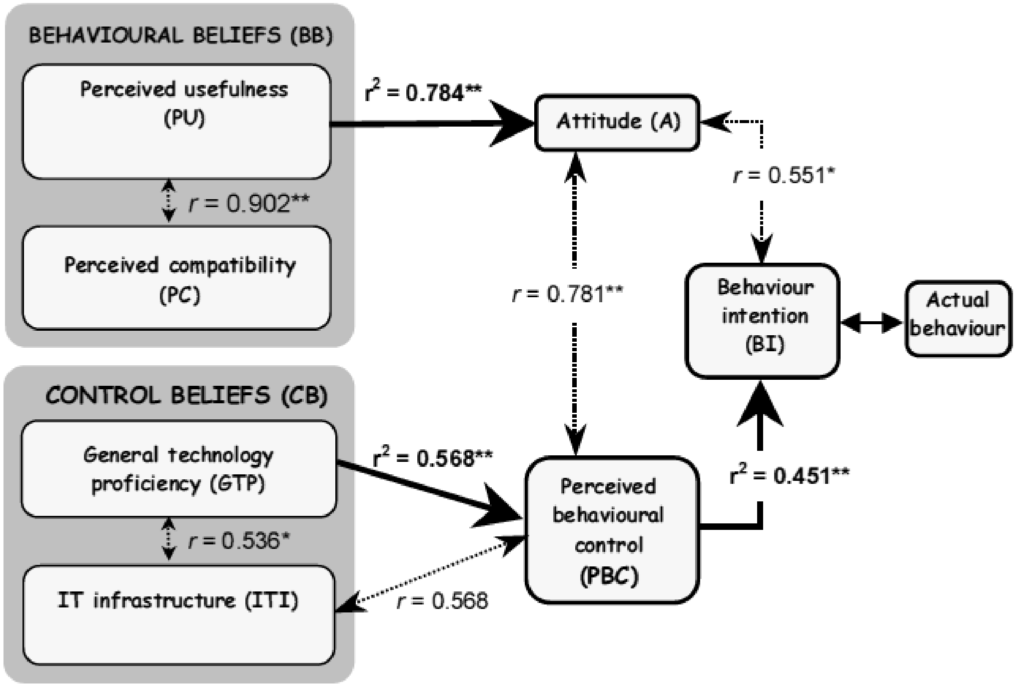


Figure 4.Stols and Kriek's simplified model of dynamic software

According to Figure 4, teachers' beliefs about perceived usefulness (PU) of the use of DGS determine their attitude. Their general technology proficiency (GTP) affects their perceived behavioral control (PBC). This means that if teachers' do not have enough GTP, they do not use technology in their classrooms. Although this study found a positive significant correlation between attitudes (A) and behavior intention (BI), only PBC, in terms of GTP, significantly determines their BI. Moreover, teachers' actual behavior is affected by the PU of technology. Stols and Kriek (2011) conclude that "A way to improve teachers' use of dynamic geometry software in their classrooms is therefore, firstly, to ensure that the teacher possess general computer proficiency and, secondly, to let them experience the advantage of using the software." Therefore, in the present study, the data collection is preceded by a three-week instructional period aimed to improve candidates' interaction with application of DGS.

Similar to the results mentioned above, Güven (2002) expressed that according to the findings of many studies, while students regard mathematics as a crowd of formulas that should be memorized in traditional learning environments, their ideas change in DGS environments and in this sense they regard mathematics as a whole of relationships which need to be investigated. Therefore, DGS is a great teaching and learning method that enhances students' skills of understanding mathematical relationships and justifications (Jiang, 2002).

2.3 GeoGebra in the Literature

Since CASs and DGSs are partially disconnected, GeoGebra is a newly developed software that includes both dynamic geometry and computer algebra tools (Hohenwarter & Fuchs, 2005). It integrates geometry and algebra in one tool. GeoGebra is one of the most popular DGS all around world. There are 300,000 visitors from 188 different countries (March, 2008). It is estimated that more than 100,000 teachers already use GeoGebra to construct both static and dynamic mathematics materials for improving their students' learning (Preiner, 2008). The software is freely available at www.GeoGebra.org and it is an open source under the GNU General Public License1. Since it is based on Java, it can be downloaded and installed on every operating system.

Moreover, GeoGebra is multilingual by having more than 50 language options in both its menu and commands (GeoGebra 4.2). This open source software is developed by Marcus Hohenwarter and Yves Kreis, and hosted at the University of Salzburg. It is constructed on a Cartesian coordinate and accepts both geometric and algebraic commands (Suzuki, 2006). Hohenwarter, Hohenwarter, Kreis and Lavicza (2008) stress the importance of having open-source software as:

> "Open-source packages do not only offer opportunities for teachers and students to use them both at home and in the classroom without any restriction, but they also provide a means for developing support and user communities reaching across borders. Such collaboration also contributes to the equal access to technological resources and democratization of mathematics learning and teaching" (p.8)

In addition to these advantages, in GeoGebra any constructions can be done with points, segments, vectors, lines, conic sections as well as functions and they can be changed dynamically afterwards. Moreover, equations and coordinates can be entered directly by means of an input tool (Rincon, 2009). Figure 5 shows the representation of GeoGebra window and tools.

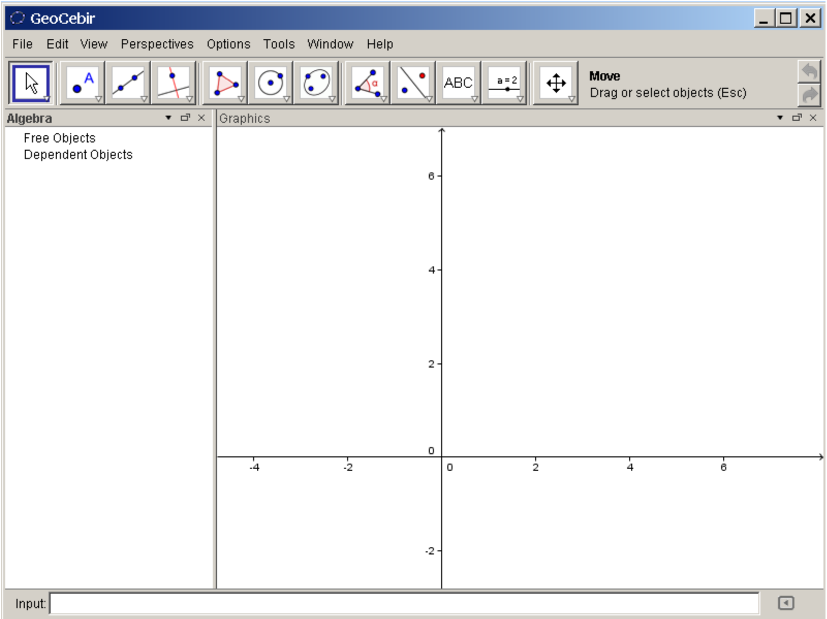


Figure 5.GeoGebra window

Moreover, like all DGS, GeoGebra also has a dragging tool called a 'slider'. Algebraically it is a variable that has a value for its interval. Graphically it is a segment that allows the user to change the value of the variable by dragging (Bu & Hacıömeroğlu, 2010). Figure 6 shows two representations of a slider and its properties in GeoGebra.

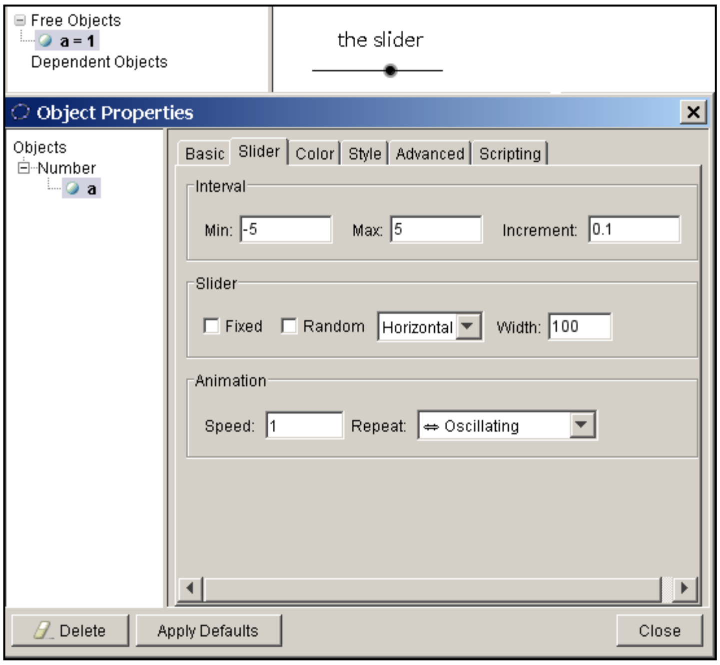


Figure 6. The two representations of a slider and its properties in GeoGebra

GeoGebra enables teachers and students to make strong connections between geometry and algebra (Hohenwarter & Jones, 2007). In other words, GeoGebra supports visualization skills of learners in a computerized dynamic environment (Hacıömeroğlu, 2011) as well as their understanding of algebraic operations and equations. In order to investigate the effect of using GeoGebra software on the students' achievement, Selçik and Bilgici (2011) conducted an experimental research with 32 seventh grade students. Data were collected at the end of a total of 11 hours by using GeoGebra worksheets were prepared by the researchers. The students in the experimental group showed higher performance in achievement test than the ones in the control group. Moreover, it is found that the classroom which experienced the technology integrated lessons had more permanent learning than the traditional classroom according to the permanence test that was done after a month of the investigation (Selçik & Bilgici, 2011).

Moreover, since GeoGebra provides the opportunity to construct and dynamically visualize geometric figures, Fahlberg-Stojanovska and Trifunov (2010) investigated a study to show how GeoGebra improved students' understanding of construction and geometric proof. They conducted a qualitative exploratory study by using tasks that include construction and proof problems for the relations on the triangles. The results showed that using GeoGebra in these tasks improves the percentage of students that are able to solve the triangle construction and proof problems (Fahlberg-Stojanovska & Trifunov, 2010). This result is consistent with that of Christou et al (2004) and Pandiscio (2010) in terms of DGS's effectiveness in justification and verification of both geometric and algebraic problems' solutions.

In addition to Selçik and Bilgici (2011), and Fahlberg-Stojanovska and Trifunov (2010), Dikoviç (2009) also investigated the effectiveness of GeoGebra in mathematics classrooms. Data was collected from 31 (gender: 19 female, 12 male) students of The Accredited Business-Technical School of the vocational studies in Uzice, Serbia. The researcher designed an experimental research by using special GeoGebra worksheets and an achievement test as a pre- and post-test. Statistical analysis showed that the experimental group who trained with GeoGebra tools significantly improved their achievement scores. Moreover, the results revealed that GeoGebra helps students to feel mathematical process intuitively and visualize it adequately. Additionally, GeoGebra tools allowed students to explore many function types and make the connection between symbolic and visual figures (Dikovic, 2009). In parallel with the findings of Dikoviç (2009), Velichová (2011) investigated an analysis on the use and applications of GeoGebra by giving examples and comparing it with other software. She concluded with the fact that simply drawing mathematical objects and figures is not enough for developing mathematical understanding. Therefore, visualizing these objects and figures dynamically will support a student's mathematical understanding. GeoGebra is a didactic tool that allows constructing, dynamically visualizing, and improving mathematical understanding in an easy, natural and user-friendly way (Velichová, 2011).

İçel (2011) conducted an experimental quantitative study with 40 eighth grade students to determine the effect of GeoGebra on their achievement. She designed a two-week instructional period and used a pre- and post-test control design. As a result, it is found that GeoGebra has positive effects on learning and achievement and learning with GeoGebra is more permanent according to permanence test results (İçel, 2011). In addition to İçel's study, Zengin (2011) also investigated an experimental quantitative study with 51 students at the high school level to determine the effect of GeoGebra on both achievement and attitude toward mathematics. The researcher designed GeoGebra workshops for the experimental group and used a pre- and post-test control design. Similar to İçel's study (2011), it is found that GeoGebra has positive effect on achievement (Zengin, 2011). However, there is no difference between experimental and control group in terms of attitudes towards mathematics (Zengin, 2011).

Furthermore, the ideas of teachers and educators about the feasibility of the software are another aspect that should be reviewed. For this purpose, Baydaş (2010) conducted a qualitative case study with pre-service teachers and educators. Data was collected through seminars and face-to-face interviews with prospective mathematics teachers, graduate and doctoral students at Erzurum University Faculty of Education in the 2009-2010 academic year. As a result, in keeping with the literature, construction protocol, and algebraic and geometric entries are found as advantages of GeoGebra in terms of usability (Baydaş, 2010). In addition, Kutluca and Zengin (2011) conducted a case study with 23 tenth grade students to gather their ideas concerning GeoGebra. Data was collected by using GeoGebra workshops and seven open-ended questions. The results revealed that the lessons with GeoGebra provide better quality learning in terms of being enjoyable, appealing, and supplying permanent learning by means of visual and dynamic figures (Kutluca & Zengin, 2011).

In addition, Turkey has already initiated the FATİH project and therefore all classrooms from primary to high school will be equipped with smart boards (MoNE, 2010). The question here is how GeoGebra can be integrated into environments equipped with interactive white boards. In order to find the answer to such a question, Lavicza and Papp-Varga (2010) conducted a study by using workshops with teachers and teacher educators. Data were collected by using an online questionnaire (67 participants) and doing eight interviews with secondary school mathematics teachers. The preliminary results of the ongoing project showed that the complexity of integrating GeoGebra into smart boards necessitates adding a further layer into the software (Lavicza & Papp-Varga, 2010).

The investigators were able to improve the software and develop workshops that are suitable with both GeoGebra and smart boards. Velichová (2011) stated that "Concerning the software development of GeoGebra, authors are always looking for talented Java programmers with good ideas for new features and extensions." The software is improved continuously and there is a high possibility of having a fully integrated GeoGebra version for interactive white boards in the future. The results of the present study seem to be more meaningful in computerized classes with the integration of GeoGebra software into smart boards.

2.3.1 Advantages of GeoGebra

GeoGebra is an open source software that includes both dynamic geometry and computer algebra tools (Hohenwarter & Fuchs, 2005). Therefore, it includes almost all features of DGS and CAS environments. According to Dikovic (2009), the advantages of using GeoGebra are:

- It is more user friendly than a graph calculator due to its easy-to-use interface, multilingual menus, commands and help.
- It supports guided discovery and experimental learning, projects and multiple presentations.
- By means of its adaptable interface, users can customize their works.
- GeoGebra is created to support students' mathematical understanding. By using slider and moving free objects property, they can drag objects to see how changes influence the other variables. By this way, the students are able to understand mathematical relations dynamically while solving a problem.
- It provides opportunity for cooperative learning.
- The users can create or modify the objects by the command line of algebra input. The worksheet files can easily be published as Web pages.
- It encourages teachers to use technology in their teaching.

In addition to these benefits, the software is freely available at www.GeoGebra.org and it is an open source under the GNU General Public License1. Since it is based on Java, it can be downloaded and installed on every operating system (GeoGebra 3.2).

2.3.2 Limitations of GeoGebra

Although GeoGebra has advantages (Dikovic, 2009; Hohenwarter & Fuchs, 2005), there are also some limitations (Dikovic, 2009);

- It will be hard to enter algebraic commands for the users who have not experienced programming before. The basic commands are not hard to learn and apply but the users will likely still feel uncomfortable with using them.
- Some methods such as experimenting or learning by discovery might not be appropriate for many students.
- Future layers that will be added to GeoGebra should make more symbolic features of CASs such as complex applications and 3D extensions.

2.4 Problem Solving and the Effectiveness of Dynamic Geometry Software in This Process

Problem is a situation that consists of exact open questions which will "challenge somebody intellectually who is not in immediate possession of direct methods/procedures/algorithms, etc. sufficient to answer the question" (Blum & Niss ,1991). Problem solving is a process of engaging in a task or situation for which there is no obvious or immediate solution (Booker & Bond, 2008). It is a powerful and effective way for learning. Therefore, it plays an important role in teaching and learning mathematics. NCTM (2000) underlines the importance of teaching with problem solving and as *Principles and Standards* states:

> “Solving problems is not only a goal for learning mathematics but also a major means of doing so. Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program. Problem solving in mathematics should involve all five content areas described in these Standards.”

Polya (1957) stressed that problem solving is to overcome a problem situation as well as find a result and a solution strategy. In daily life, when people encounter a problem situation, they develop a strategy to eliminate or solve it. Their strategies depend on how they understand the problem, their proficiency in the area of the problem, and their special skills about developing solution strategies. Problem solving is, relevant to the definition of the problem, “knowing how to solve the problem when the situation is not clear” (Dağlı, 2004).

Altun (2008) defined ‘problem solving skill’ as the ability to understand and comprehend the problem, determine appropriate solution strategy, apply this solution to the problem and evaluate the solution when confronting a problem. Similarly, Polya (1957), a famous mathematician, divided problem solving process into four steps that are accepted by many mathematicians (Altun, 2008; Van de Walle, Karp & Bay-Williams, 2013), namely, understanding the problem, devising a plan, carrying out the plan, looking back.

1. *Understanding the problem*: In order to understand the problem situation deeply and clearly, we need to answer; what the question is about, how to redefine or restate the question, what is given, what the conditions are and what is asked.

2. *Devising a plan*: In this step, the students look for the appropriate strategy to solve the problem. Problem solving strategies guide during the process; however, they do not guarantee a solution for the problem (Mayer, 1983). There are many strategies that are widely used in solving mathematical problems such as making an organized list, guessing and checking, drawing a diagram, writing an equation, using a simpler form of the problem, making a table or chart, looking for a pattern or relationship, drawing a picture, working backward, etc.. In order to solve a problem, one or more strategies are sometimes used together. Indeed, for solving a problem, there might be different strategies to solve the same problem.

3. *Carrying out the plan*: The strategy or approach that determined in the previous step is applied. If the strategy does not end with a solution, we turn back to the second step and look for any other strategies.

4. *Looking back*: At the final step, it is time to evaluate the whole problem solving process. We check whether our solution is correct, look for any other solution strategies, restate the problem in a different way and solve the new problem by using the strategy that we have already used.

Implementing the strategy and checking the solution are two important aspect of problem solving process because the students are able to see a whole picture of the problem. Polya (1957) argued that by checking the result and solution path, the students develop their ability to solve problems and strengthen their knowledge. In this way, problem solver reflects on their knowledge and skills based on checking problem solving process, resolving and extending the problem situation. The selected strategy and solution path give ideas about students' mathematical knowledge and problem solving skills.

2.4.1 Students' Problem Solving Preferences

Krutetskii (1976) proposed students' preferences in mathematical thinking rather than abilities. For example, the students might prefer to solve a problem in an algebraic method but their abilities might be sufficient for using a geometric strategy for the same problem. He emphasized students' verbal-logical and visual-pictorial modes of mathematical abilities in their strategy preferences. Verbal logical component of mathematical skills is related to the use of verbal messages during the process of problem solving. On the other hand, visual-pictorial component focus on students visual representations during their solutions. Therefore, Krutetskii (1963, 1976) categorized students as analytic, geometric and harmonic thinkers according to their verbal-logical and visual-pictorial components of mathematical abilities. Although Krutetskii (1976) made this classification for the case of gifted students, the research supported that it can be utilized for the student at all levels (Hacıömeroğlu, 2007). The first one, analytic thinkers, have a clear predominance of well developed verbal-logical components over a visual pictorial one. This type of problem solvers prefers algebraic and numeric representations justified by verbal messages. They prefer to use a less efficient and much complicated solutions method even a much simpler and more efficient visual solution is possible.

Geometric thinkers have a clear predominance of well developed visual pictorial components over a verbal-logical one. This type of students interprets the solutions by relying on visual representations. They solve the problems and tasks by using visual schemas even if the problem could be easily solved reasoning (Coşkun, 2009). The third type is harmonic thinkers. They have a relative equilibrium between verbal-logical and visual-pictorial components of mathematical ability. In addition, Krutetskii (1976) categorizes harmonic thinkers into abstract-harmonic and pictorial-harmonic ones. Abstract-harmonic students have a relative equilibrium between verbal-logical and visual-pictorial components but they prefer less pictorial components during their mental operations. On the other hand, pictorial-harmonic ones have also a relative equilibrium between verbal-logical and visual-pictorial components but they have an inclination for using more visual-pictorial representations in their mental operations.

In addition to Krutetskii (1976), Presmeg (1986) classifies the use of students' visual imageries during problem solving. She identifies concrete pictorial imagery, pattern imagery, memory images of formulae, kinesthetic imagery, and dynamic imagery according to their use of different images such as models, shapes, and pictures in mind. In concrete pictorial imagery the students memorize the objects in detail such as memorizing images of trigonometric functions. For example, the students who have a picture of the sign of sin, cos, tan, and cot functions on the coordinate axes, tend to use this picture during their solutions. The students who use pattern imagery disregard concrete details and determine pure relationships such as chess masters' remembering the places of pieces on a chessboard for a given unfamiliar situation (Coşkun, 2011). For instance, the students who use the pattern of the ratios of the sides of special triangles have an inclination for using the images of these patterns.

The students who use memory images of formula have abstract information such as remembering a formula written in a book. In kinesthetic imagery, the students use muscular activity such as doing calculations by using their fingers. The students who use dynamic imagery prefer the images of dynamic movements such as transforming mentally a rectangle into a parallelogram. The students might use all these visual imageries but dynamic imagery is the most efficient type in describing dynamic movements in GeoGebra environment. Presmeg (1986) studied on the following problem to analyze students' dynamic imagery: "Given the area of the square ABCD is 4 square units, and that E and F are midpoints. Find the area of AECF, which had been proved a parallelogram." (Figure 7). The student in the experiment explained that the answer is 2 square units. The student ‘slid’ the parallelogram up into the rectangle by using a moving image to explain his result.

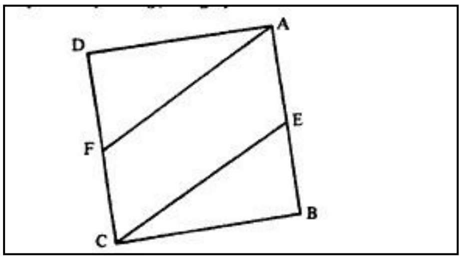


Figure 7. Dynamic imagery

In the mathematics classroom, the aims of teaching with problem solving are developing operation skills; getting used to deal with numbers and figures; collecting and classifying data; drawing figures and schemas that are appropriate to the context of the problem; explaining the ideas with the language of mathematics; and understanding mathematical expressions that are used in various publications (Altun, 2008). In the Turkish mathematics teaching curriculum, problem solving is seen as a main role of the students while learning mathematics in the classroom (MoNE, 2009). The Ministry determined the main aims of mathematics teaching and stressed the importance of the problem solving. For example, as a general aim, the students will be able to state their mathematical ideas and reasoning during the problem solving process. Moreover, the students will be able to develop different problem solving strategies and use them in daily life problem situations.

In addition to significance of teaching mathematics with problem solving, it is important to integrate technology into this process. Kuzle (2011) argued that educators, researchers and educational associations stress on the use of technology in mathematical problem solving. In addition, in Turkish mathematics curriculum, there is a great emphasis on teachers’ use of mathematical software during their activities in the classroom (MoNE, 2006). The students are able to develop different strategies than ones in traditional environments, alternative strategies to the same problem, and they can evaluate mathematical content in the problem to explore different aspects of the problem. Moreover, teachers will be able to realize students’ difficulties in understanding mathematical thinking and their problem solving tendencies during the process in technology environment (Coşkun, 2011).

The studies on the effect of technology in problem solving mostly focus on the students' strategy preferences (Coşkun, 2011; Harskamp, Suhre & Van Streun, 2000; Iranzo-Domenech, 2009; Yerushalmy, 2006). The comparison of different environments revealed that technology environment has great influence on the process of learning and teaching mathematics during problem solving.

In her study, Iranzo-Domenech (2009) focused on the synergy of environments during the process. The participants of the study were twelve high school students and a qualitative multiple case study research design was used. An instructional period was prepared to attain to the students basic GeoGebra tools and commands. After this period, the students were given plane geometry problems to solve in both GeoGebra and paper-and-pencil environments. The students developed visualization, structural, instrumental and deductive competencies in the process of solving plane geometry problems. Iranzo-Domenech (2009) analyzed each student by keeping in mind these competencies. The results of the study showed that the students were able to develop their understanding of mathematical concepts in the problems, and they overcome their difficulties in displaying different competencies by using the software. In addition, since the students could solve the problems with GeoGebra and paper-and-pencil, the synergy of these environments helped them to develop different thinking styles and solution strategies.

Similarly, Coşkun (2011) studied on students' visual and non-visual problem solving preferences in different environments. In the investigation, a qualitative multiple case study research design was used. The researcher administered two Mathematical Processing Tests, and three cases were chosen out of eight volunteer participants according to the results of these questionnaires. The results showed that the students were able to develop different strategies in different environments. In addition, they could look for alternative strategies in GeoGebra environment easier than paper-and-pencil environment. However, it was concluded that each environment had different contributions to the students' problem solving skills hence both of them could be used when needed during the process.

In a study, Yerushalmy (2006) investigated on the influence of graphing software on the less successful students' mathematical problem solving preferences. Graphing software that is similar to basic graphing calculators was used to compare their solutions in paper-and-pencil and technology environments. The students had already learnt software tools and they learned algebra for three years. Most of the students used numeric and graphic representations during their solutions. Although they usually did not prefer to use the software, the students used this environment in order to gain a broader view to verify conjectures and complete difficult operations.

The students who preferred to use technology environment perform better when compared to the students who used traditional settings in solving problem tasks (Harskamp, Suhre & Van Streun, 2000). However, the students tend to display different competencies in different environments (Iranzo-Domenech, 2009). The concrete results of Coşkun's (2011) study also support this argument. Therefore, the researcher in the present study preferred to observe students' problem solving preferences in both technology and traditional environments.

2.5 Euclidean Geometry

As stated by Kokol-Voljc (2007), in teaching and learning geometry, particularly Euclidean geometry, and solving problems related to geometry concepts, DGS is the most appropriate tool. Since the mathematical content in the present study is plane geometry, the researcher preferred to use GeoGebra as a DGS. However, in addition to mentioning about technology environment and problem solving in the previous part of the present study, it is also essential to talk about Euclidean geometry.

The Alexandrian Greek mathematician Euclid identifies Euclidean geometry in his book, the Elements and it consists of assuming a small set of axioms and deducing theorems from these axioms. Although Euclid's propositions are discovered by different mathematicians in the past, he was the first to state them in a logical system. The Elements begin with plane geometry. The propositions and axioms stated in this part are still taught in schools. They are the first examples of axiomatic system and formal proof. In other parts, there are axioms and theorems related to number theory and solid geometry of three dimensions.

In his book, the Elements, Euclid proposes five postulates for plane geometry. These postulates are;

1. A straight line segment can be drawn joining any two points.

2. Any straight line segment can be extended indefinitely in a straight line.

3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.

4. All right angles are congruent.

5. If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. This postulate is equivalent to what is known as the parallel postulate.

These postulates are unique and assert the existence of the geometric constructions. Hilbert (1904) made the moderns formalization of Euclidean geometry. Since fifth postulate is controversial, Hilbert was the first who present crucial axioms on this postulate (Iranzo-Domenech, 2009). He proposed the postulate of continuity which is the fact that a line can be identified with the completion of the field of rational numbers.

Euclidean geometry consists of plane geometry, number theory and solid geometry. Since the subject of the present study is plane geometry, the other parts were not included in the content of the study. The nature of the problems used in the present study is based on Euclid's identification of plane geometry (Iranzo-Domenech, 2009). That is, the postulates stated in the Elements constitute the basis for the plane geometry problems.

2.6 Summary of the Literature

Technology tools are integral part of learning and teaching mathematics in this century. Educational community has a general acceptance of using technology in mathematics education (Kuzle, 2011). The study results and documents for the reform in mathematics classrooms justify this argument (AMTE, 2006; Baki, 2001; Borwein & Bailey, 2003; Ersoy, 2003; Hollebrands, 2003; Koehler & Mishler, 2005; Lester, 1996; MoNE, 2009; NCTM, 2000). The main focus is on the effective use of technology during learning and teaching of mathematics.

Research suggests that DGS, as a technology tool, facilitates students' understanding of mathematical concepts and their relationships due to its symbolic and dynamic visualization property (Baki et al., 2011). During problem solving process DGS tools allows users to develop the strategies that might not be possible in a paper & pencil environment (Güven et al., 2012). In addition to help problems solve easily and quickly, DGS is a powerful tool in verification and justification of solutions. Therefore, this software provides opportunities for users to understand mathematical relationships and think more deeply about them than traditional ways (Pandiscio, 2010).

GeoGebra includes almost all properties of DGS. The users are able to make strong connections between geometry and algebra (Hohenwarter & Jones, 2007). The research on the effectiveness of GeoGebra in the mathematics classroom showed that it enhances students' performance in solving geometric and algebraic problems and helping them to gain permanent learning (İçel, 2011; Selçik & Bilgici, 2011; Zengin, 2011). Moreover, it enables users to justify and verify mathematical relationships on the geometric figures by means of dynamic visualization (Fahlberg-Stojanovska & Trifunov, 2010). Additionally, the students, teachers, and educators have positive attitudes toward usage of GeoGebra in mathematics classroom (Baydaş, 2010; Kutluca & Zengin, 2011).

According to Dikovic (2009), some of the advantages of using GeoGebra are; being more user-friendly, promoting different teaching methods, customizing the works, dragging the figures, writing commands easily and being able to publish the works as Web pages. These benefits of GeoGebra paved the way for using this DGS in order to find answers to the problem in the present study. However, for new users entering commands correctly and using 3D extensions could be two limitations of GeoGebra. However, they did not constitute problems for the present study because the students did not need use command functions and 3D extensions.

Mathematical problem solving as a step by step process that the students attempt find solutions to the mathematical problems (Polya, 1957). In teaching and learning mathematics, problem solving is an effective way of using mathematical knowledge (Van de Walle et al., 2013).The integration of technology in this process has vital importance. Kuzle (2011) stressed that educators, researchers and educational associations agree about the use of technology in mathematical problem solving. In many studies, it has been concluded that technology is effective in problem solving because it helps students to understand logical structure of the problem and develop different thinking styles and solution strategies (Coşkun, 2011; Harskamp, Suhre & Van Streun, 2000; Iranzo-Domenech, 2009; Yerushalmy, 2006).

Euclidean geometry is taught in many schools to form a base for further geometry concepts hence it is so significant to learn it effectively. DGS is a powerful technology tool in teaching and learning Euclidean geometry because graphic window, Cartesian coordinates, and grids of this software are the most appropriate tools for plane geometry (Kokol-Voljc, 2007). Since the content of this study is plane geometry, DGS was chosen as the most suitable technology tool for the present study.

In the light of all information mentioned above, the aim of the study could be accomplished by observing students' problem solving preferences in GeoGebra environment and paper-and-pencil environments. The related literature provide significant data for choosing the most appropriate software, data collection, and data analysis methods for the aims of the study.

CHAPTER 3

METHODOLOGY

In this chapter, first of all, the design of the study is discussed. Then, the sampling method for selection of cases is outlined. In the instrument part, the background and possible solution strategies for the problems are mentioned. Next, whole process is summarized in the procedures part. Finally, the method of analyzing data is explained in detail.

3.1 Research Design

In the present study, a qualitative research design will be used to analyze the current situation in depth. Qualitative research is known as the research studies that examine the quality of relationships, activities, situations, or materials (Fraenkel & Wallen, 2006).Creswell (2007) suggest beginning with assumptions, the possible use of theoretical views, and the study investigating the meaning attributed by individuals or groups to social and human problem. In their studies, the researchers using qualitative methods benefits from a new qualitative way to inquiry, the set of data in a natural environment that is sensitive to the people and places studied in order to study this problem. In addition, the data analysis conducted by qualitative researchers is inductive and creates patterns and themes. The subjects' voices, the reflexivity of the researcher, and a complex definition and interpretation of the problem are covered in the final form of the written report or presentation. This written report or presentation also expands the literature or triggers a call for action.

It can be understood that a qualitative study should focus on specific cases, be investigated in a natural setting, take personal or environmental characteristics into account, establish themes, explain the problem in detail, and include different sources of data. LeCompte and Schensul (1999), Marshall and Rossman (2006) and Hatch (2002) suggest that a qualitative study need to be investigated in a natural setting for close interaction and the data should be analyzed inductively, recursively, and interactively. In addition, Marshall and Rossman (2006), and Hatch (2002) asserted that the research design needs to emerge according to the characteristics of the study instead of tightly pre-figures designs and there should be a holistic view of social phenomena. Moreover, LeCompte and Schensul (1999), and Marshall and Rossman (2006) stressed that multiple source of data in word or images is an important characteristic of qualitative inquiry. Furthermore, the researcher should focus on participants' perspectives, their meanings, and their subjective views (Hatch, 2002; LeCompte & Schensul, 1999). By taking into consideration all these features of qualitative research, it was chosen as the appropriate research design for subject structure and research problems in the present study.

Among five qualitative approaches that were determined by Creswell (2007), case study methodology is appropriate to the characteristics of the present study. This research strategy covers the study of a problem situation that examines one or more cases within a 'bounded system' (Creswell, 2007). Since the researcher in the present study will study a group of pre-service teachers in the classroom setting by observing their solution strategies in depth, it is better to prefer this approach. In addition, Stake (1997) has determined three types of case studies; *intrinsic*, *instrumental*, and *multiple*. In the multiple cases method, the researcher wants to study two or more cases at the same time as part of one overall study (Fraenkel & Wallen, 2006). This type of case study will be appropriate for the present study, because the researcher prefer to analyze the solution strategies of multiple cases at the same time. Another reason for selecting a multiple case study is to maximize having information from students solution processes.

3.2 Participants

In the present study, the participants of the study were selected from 33 sophomore students who took Computer Supported Mathematics Education course in spring semester. The reason for selecting sophomore students was the fact that they took Geometry course in the second semester and they were assumed to have sufficient capability for developing different problem solving strategies for Euclidean geometry problems. In addition, they took Basic Computer course and hence they were capable of using computers at least at an average level according to their grades taken from this course.

Among nonrandom sampling methods, convenient sampling was appropriate for selection of cases because the researcher was able to use an available sample (Fraenkel & Wallen, 2006). Seven sophomore students were selected according to their availability for a three-week instructional and a one-week data collection period. Participation was voluntary. The students were from a middle socio-economic level and they are from different cities in Turkey. The researcher preferred communicative students who were interested in Euclidean geometry to facilitate data extraction process. Therefore, he contacted and interacted with these students during instructional period. He chose two students for the pilot study and five of them for the actual study. These students were selected not only for their performance but also for their willingness and well communication with the instructor. In addition, the researcher analyzed their grades from Computer Supported Mathematics Education course. The students chosen for the pilot and actual study had average grades when compared to other students. By doing this, the researcher was able to observe effectiveness of the instructional period.

3.2.1 Selection of the Case of Merve

Merve solved all tasks that are included in the GeoGebra Booklet during the instructional period. She had some technical difficulties in the use of the software. One of the biggest problems for her was selecting appropriate tools in order to draw necessary figures. For example, while constructing a rectangle she preferred to select polygon tool. However, when she drew the rectangle, she realized that moving vertices of the figure disrupted the figure and it would not be dynamic. Hence, she used perpendicular line and parallel line tools for constructing a dynamic rectangle. Moreover, constructing process was time consuming for her because she could not use the tools comfortably. Fortunately, she studied to make the process faster and she got rid of this problem at the end of the instructional period.

Merve had difficulties in connecting her knowledge of geometry and DGS tools. For instance, when the researcher asked her to learn whether she knew how to construct an isosceles with a compass or not, the answer was ‘Yes’. However, although the steps that she needed to follow for construction were the same as using a compass, she could not use the tools effectively in GGB environment. The researcher gave the clues for using compass tool and encouraged her to follow the same instructions. Then, she was able to draw the triangle successfully. During the instructional period another important thing about Merve was that when she learned how to overcome a problem in the technology environment, she effectively used her previous knowledge in further problems. For example, after she learned how to construct an isosceles triangle in GGB environment, she was able to construct an equilateral triangle and determine the elements of a triangle such as medians, perpendicular bisectors, angle bisectors, and heights.

In order to make a clear picture of Merve’s geometry and computer use background knowledge, the researcher investigated her success in the Basic Geometry and Basic Computer courses from previous semesters. The exam grades for these courses were above average and, especially, her computer grades were better than her geometry ones. This situation helped the researcher to observe the effectiveness of instructional period more meaningfully. Moreover, the researcher could compare how technology could affect a student having good geometry background knowledge. There might be some important differences between the solution strategies in GeoGebra based (GGB) and paper-and-pencil based (PPB) environments for the same problems. That is, the researcher could assess whether the student who was successful to explain her result in PPB environment will also have the same success in GGB environment or not. Since one of the aims of the instructional period was to provide basic GGB knowledge for solving plane geometry problems, the researcher expected Merve to combine her knowledge of plane geometry and GGB gathered from this period.

In addition, during the instructional period, she usually asked critical questions related to constructions and it makes her a communicative participant of the present study. As it is mentioned before, communicative and exclusive students are extremely significant for the data extraction process because it is the best way to understand how the student understand and react in a problem situation. For example, if the student gets a result without any logical explanations, it will not be possible to assess the student's solution process and make interpretations. However, Merve explained every step of her constructions during the instructional period. Since she frequently asked question and communicate well, examining her solution strategy for plane geometry problems in the instrument will help the researcher to get meaningful data.

3.2.1 Selection of the Case of Kübra

During the instructional period in October 2012, the researcher observed sophomore students that took Basic Computer, Elementary Geometry and Technology Supported Mathematics Education courses. Therefore, all students participated in this period were assumed to be prepared for the GeoGebra introduction sessions in terms of computer and geometry knowledge. In addition, they learnt about some computer algebra and dynamic geometry software in Technology Supported Mathematics Education course. Since this course was elective, there were 33 students and only 7 of them accepted to participate in the present study. During GeoGebra introduction period, the researcher introduced GeoGebra and prepared 9 activities. Therefore, the selection of the cases was mostly based on students' performance in this introductory period, characteristics and background knowledge. Therefore, the reason for selecting Kübra will be discussed in terms of these topics before starting to analyze the data.

The students who are communicative are important for the data extraction process as it was mentioned before. Kübra was one of the most communicative students in the class during the instructional period. She asked questions about mathematical concepts, relationships and construction processes of the figures and get instant feedback. She interacted with the tutor well when compared to many of other students. Although she had some problems with construction activities in the GeoGebra booklet, she overcame them by asking to the tutor and following the instructions in the activities. In essence, the success of Kübra in the activities was not the unique reason for the selection. She usually attempted to develop different strategies for her constructions. Trying to construct a figure different from usual forms was a good indicator of critical and creative thinking. She generally preferred to search for distinct way of solutions for the problems during the instructional period. Since, the researcher in the present study is investigating for the effect of the dynamic geometry software on the students' solutions of plane geometry problems; creative students who have an inclination for different solution strategies will contribute much more to obtain meaningful data.

In addition, the researcher obtained the grades of Kübra from her transcript by getting university administration and her permission. Contrary to Merve's grades, Kübra was more successful in geometry than basic computer. Actually, the researcher also experienced this situation during the instructional period. She was successful to explain and use geometric knowledge but she had some problems with technology use. According to my observations and conversation with Kübra, she used computers little in the past and hence she had some biases for the use of technology in teaching. However, when the researcher showed how dynamic geometry software could be used in teaching plane geometry, she was interested more in technology. Moreover, she learned the use of such dynamic geometry tools in the problem situations. Kübra provide the researcher the opportunity to observe how a student biased against the use of technology and low level student performed in learning and problem solving process.

3.3 Data Collection Tools

In the present study, data were collected through four plane geometry problems developed by Iranzo-Domenech (2009); namely, *the root problem*, *the scaled triangles problem*, *the median problem*, and *the quadrilateral problem*. In her study, she expected from students to solve the root and quadrilateral problem with paper-and-pencil, and the scaled triangles and the median problems with both paper-and-pencil and GeoGebra. However, since it is aimed to observe the effect of technology on problem solving strategies in the present study, the students were allowed to use paper-and-pencil and GeoGebra for all problems. Moreover, this way of solutions gives the researcher the opportunity to observe all solutions by comparing them to each environment. In addition, the problems are used in the order of the complexity to see how students develop different strategies in more complex situations.

The problems, some informative explanations about problems, and some solution strategies are given below.

3.3.1 The Root Problem

Let E be any point on the diagonal of a rectangle ABCD such that AB = 8 units and AD= 6 units. The parallel line to the line (AB) through the point E intersects the segment [AD] at the point M and the segment [BC] at the point O. The parallel line to the line (AD) through the point E intersects the segment [AB] at the point N and the segment [DC] at the point P. What relation is there between the areas of the rectangles NEOB and MEPD in the figure below?

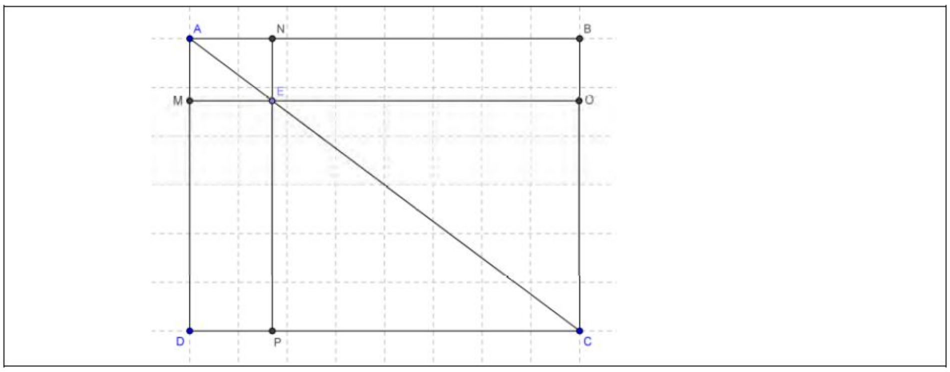


Figure 8. The root problem

The root problem's logical structure is shared with other problems and it has a medium complexity (Iranzo-Domenech, 2009). Theoretically, the selected root problem corresponds to Euclid's 43rd proposition of the *Elements* (about 300 B.C., Euclid of Alexandria wrote the treatise in thirteen books called the *Elements)*. Some solution strategies for this problem are;

- ***Diagonal property of the rectangles:*** The diagonal splits the rectangle into two congruent triangles; thus we get following equalities: A(ADC)=A(ABC), A(AME)=A(ANE) and A(EPC)=A(EOC). Then, A(MEDP) = A(ADC) - A(AME) - A(EPC) = A(ABC) - A(ANE) - A(EOC) = A(NEOB).

- ***Thales theorem:*** The right-angles triangles ANE and CPE are similar. Thus, we have the equalityAN/PC = NE/EP, then AN x EP = PC x NE. Hence, the areas are equal.

- ***Trigonometry:*** If we express the angle A angle of the triangle AME as α then, tanα = 8/6 = ME/AM = 4k/3k where k is an unknown. Then, A(MEDP) = MD x ME = (6-3k) x 4k = $24k-12k^2$. Also, A(NEOB) = AM x NB = 3k x (8-4k) = $24k-12k^2$. Thus, the areas are equal.

- ***Auxiliary parallel lines:*** When the auxiliary lines AD and FG have drawn, there exist similar triangles CFG and CAD because AD and FG are parallel lines and the angle A is common (Figure 9). Therefore, by using Thales theorem, we have CF/FA = CG/GD. Thus, we have the equality of areas.

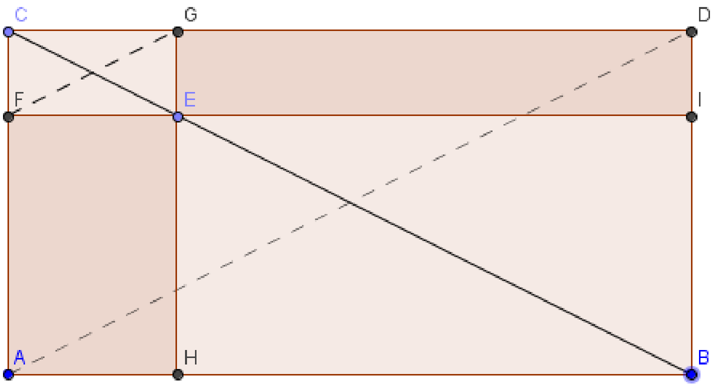


Figure 9. The equivalent problem

- ***Analytic geometry:*** The equation of diagonal is y=3/4x. Hence, the coordinates of E is E(x, 3/4x).

  A(MEDP) = x. (6-3/4 x)=-3/4 x^2+6x

  A(NEOB) = (8-x).3/4 x=-3/4 x^2+6x

  Thus, we have the equality areas.

Iranzo-Domenech (2009) determined mathematical content of the problem since it includes definitions and elements of figures, the diagonal of the rectangle splits the rectangle in two congruent triangles, congruence criteria of triangles, formula for the area of triangle and the area of a rectangle, decomposition of areas, Thales' theorem, trigonometry of right-angled triangle, similarity of triangles and ratio between homolog sides, coordinate axes, straight line equation, and distance between points.

3.3.2 The Scaled Triangles Problem

Let P be any point on the median [AM] of a triangle ABC. Let m and n parallel lines through P to the sides (AB) and (AC) of the triangle.
a) What relation is there between the segments EM and MF?
b) Where must the point P be positioned such that BE = EF = FC?

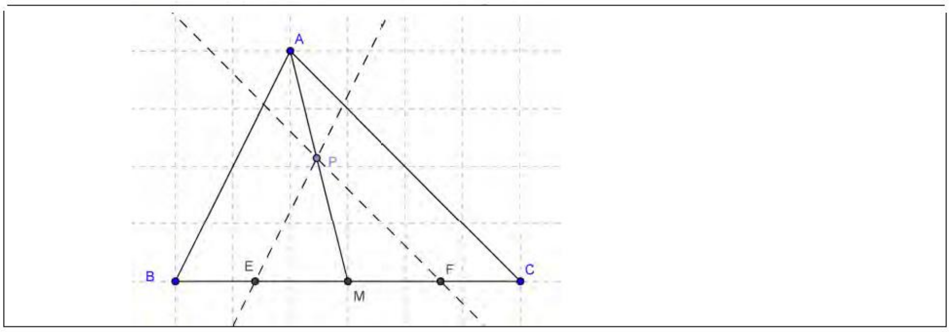


Figure 10. The scaled triangles problem

This problem is more complex than the root problem because the students need to understand the logical dependence of the problems' elements (Iranzo-Domenech, 2009). Some solution strategies for this problem are;

- ***Thales theorem:*** For the first question; since we have similar triangles AMB and PME, and similar triangles AMC and PMF, we obtain the ratios; MA/MP=MB/ME and MC/MF=MA/MP. Then, we have MC/MF=MB/ME. Since MC=MB, we obtain MF=ME.

  For the second question; by applying Thales theorem, we have DE/DC=DG/DA = 1/2. Thus, the point E is positioned (Figure 11).

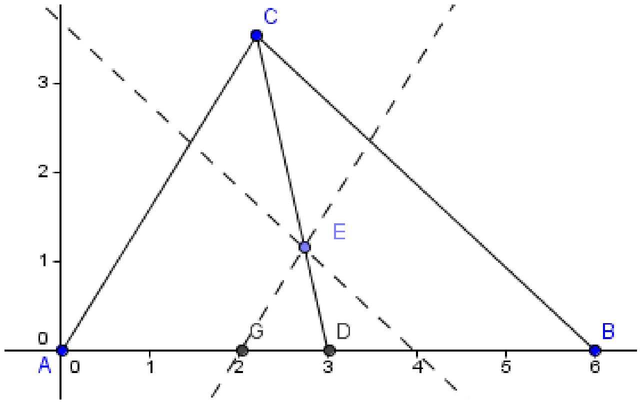


Figure 11. Trisection of the segment AD

Another strategy is using vectors. We have $\overrightarrow{DE}=k\overrightarrow{DC} \leftrightarrow \overrightarrow{DH}=\overrightarrow{DE}+\overrightarrow{HE}=k\overrightarrow{DC}+k\overrightarrow{BC}=k\overrightarrow{DB}$. By using Thales theorem and applying k= 1/3, we found the position for E(Figure 12).

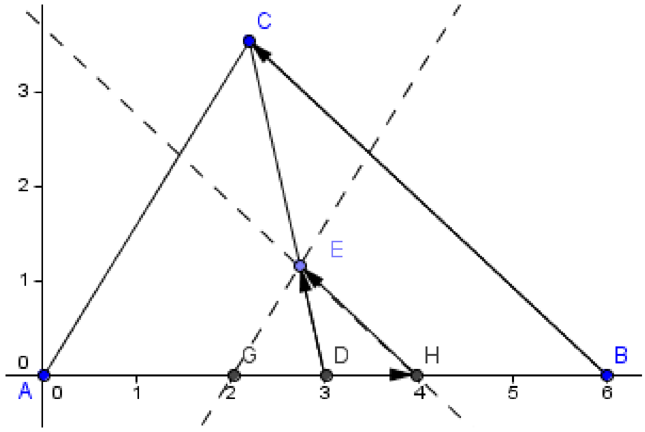


Figure 12. Using vectors based on Thales theorem

When we draw the median of the triangle ABC, we can solve this problem (Figure 13). Since the centroid is on the median and trisects it, and by applying Thales theorem, we have AI/AE=AB/AD. Thus, AB=3HB.

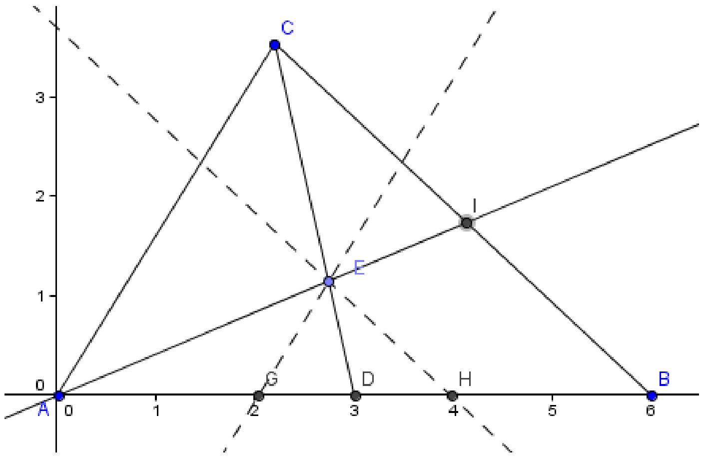


Figure 13. Using the median based on Thales theorem

- ***Analytic geometry:*** This strategy could be applied as it is shown for analytic geometry solution of the root problem. It bases on finding the coordinates of E, finding distance between two points, find the equation of a line, and solve a linear system of two equations.

- ***Particularization:*** By assuming the triangles as an isosceles triangle and a right-angled triangle, the point could be positioned and dragged to generalize the solution. In isosceles triangle case, for the first question, the median splits the triangles into two triangles with the same area. Since their areas and heights are equal the bases should be equal. For the second question, the height of the triangle ABC is also the median. Thus, the area of this triangle is 9 time the area of EGH (by Thales theorem). Since the height GH is one third of BA, the height ED is one third of CD. In order generalize the solution; an auxiliary line through E parallel to BA has drawn. Then, if the point E is dragged along this line, the ratios will remain the same.

For the scaled triangle problem, necessary mathematical content includes definitions of elements of a figure, triangles congruence criteria, formula for the area of a triangle, relation between the angles determined on parallel lines by a secant line, decomposition of areas, Thales theorem, similarity of triangles, triangles similarity criteria, relation between the areas of similar triangles, dilatations and its properties, coordinate axes, straight line equations, and vectors and operations with vectors (Iranzo-Domenech, 2009). In addition to above strategies, by using dragging and moving properties of the software, the solutions could be justified.

3.3.3 The Median Problem

Let P be any point on the median [AM] of a triangle ABC. What relation is there between the areas of the triangles APB and APC?

This problems' logical structure is the same as the first question of the scaled triangle problem. The main difference is to compare the areas of triangles instead of the line segments. Iranzo-Domenech (2009), the developer of the instrument, proposed following solutions strategies for the median problem.

- ***The median property of the triangle:*** Since the median of the triangle splits it into the triangles with the same areas, we have the following equalities: A(ABM)=A(BMC) and A(APM)=A(PMC). Thus, A(APB)=A(ABM)-A(APM)=A(BMC)-A(PMC)=A(BPC). The use of the GeoGebra can help the students to measure the areas and compare them based on this strategy.

- ***Auxiliary parallel line:*** As shown in the figure below, the line through E parallel to the side AB divides the triangles APB and APC in two triangles respectively. The median splits CFG into two the triangles CFE and CGE which have the same areas. Since FAE and EGB have bases with equal lengths and equal heights of these bases respectively, they have equal areas. Thus, we have A(CAE)=A(CFE)+A(FAE)=A(CGE)+A(EGB)=A(CEB).

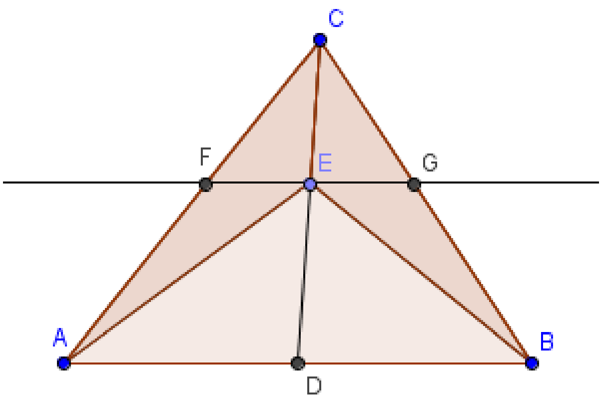


Figure 14. Auxiliary parallel line

In addition, by using GeoGebra, one can drag and move the points to observe that the equality of area is invariant.

- ***Comparing the common base and the respective heights of the triangles:*** By using this strategy, the students can prove the equality of areas. Since they have equal bases CE, they need to show the equality of heights. Therefore, they can draw the respective heights of the triangle CAE and the triangle CBE and find the length of these heights. After observing the equality of areas, the students can drag the point E and move the sides to justify their solutions.

- ***Extending the triangles by using auxiliary parallel lines:*** In order to get equivalent triangles FCE and GCE of the triangles CAE and CBE respectively, parallel lines to the median through A and B, and parallel line to AB through C could be constructed (Figure 15). The triangles FCE and CAE have common bases and equal respective heights. Therefore, they have equal area. The triangles GCE and CBE have also the same areas to the same reason. The use of GeoGebra helps to have such a configuration by using measuring and construction tools.

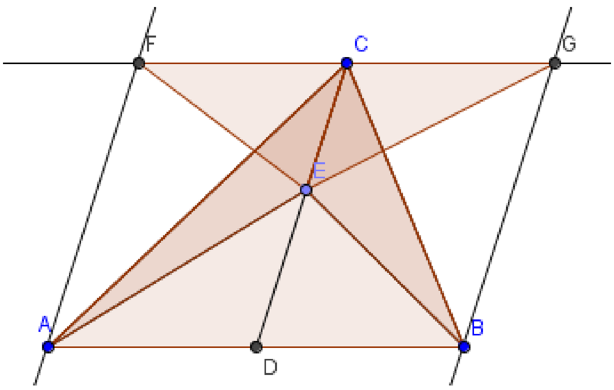


Figure 15. Equivalent problem

- ***Particularization:*** Using particular cases can help the students to find the relationship for the general case. By using dragging tool, various particular cases can be observed. For example, the degenerate cases E=D and E=C shows that the triangles have equal areas. By dragging the point along the median, it can be observed that the equality is invariant at every point.

The mathematical content of the median problems covers definitions and elements of figures, congruence of triangles criteria, formula for the area of a triangle, congruence of inner angles formed by a secant to parallel lines, congruence of parallel segments comprised between parallel, decomposition of areas, the median splits the triangle in two triangles that have the same area, Thales' theorem, identification of triangles in Thales theorem, configuration, and concept of height as distance from a point to a line (Iranzo-Domenech, 2009).

3.3.4 The Quadrilateral Problem

Let ABC a triangle and let P any point of the side BC, N and M be the midpoints of the sides AB and AC respectively. What relation is there between the area of the quadrilateral ANPM and the addition of the areas of the triangles BNP and PMC?

The logical structure of the quadrilateral problem is shared with the median problem. The difference is the fact that the area of a quadrilateral is included into this problem. Some suggested solutions are;

- ***Comparison of areas:*** If an auxiliary line segment connecting the points A and P is drawn, then the quadrilateral is divided into two triangles. Since the triangles ANP and BNP have equal bases and common respective heights, their areas are equal. Similarly, since the triangles APM and PMC have equal bases and common respective heights, they have equal areas. Thus, we have following equation: A(ANPM)=A(ANP) + A(APM) = A(BNP)+A(PMC)

  The use of GeoGebra can help the students to explore the variations in the magnitude of the areas by using dragging and moving tools. Moreover, they are able to observe the fact that while dragging and moving, the equality is invariant.

- ***Compare the base and height of the triangles:*** If an auxiliary line segment DE is drawn, the quadrilateral is divided in two triangles ADE and DEG (Figure 16). By using Thales theorem, the line segments DE and BC are parallel and hence the heights DF and EH have equal lengths. Then, by applying Thales theorem, the length of the height AI is half of the height of the triangle ABC. Therefore, sum of the areas of triangles DBG and EGC is half of the area of triangle ABC. Following equalities emerged:

A(DBG)+A(EGC)=1/2A(ABC),A(ADGE)= 1/2A(ABC),hence

A(DBG)+A(EGC)= A(ADGE).

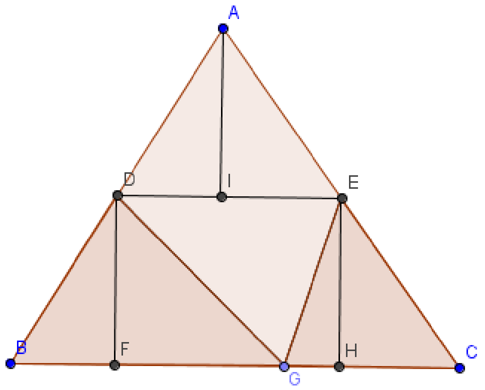


Figure 16. Comparing heights and bases of the triangles

- ***Considering particular cases:*** For the particular case, F is the midpoint of BC, there are four congruent triangles in the triangle ABC. The reason is the fact that all line segments has equal lengths.

For the particular case, F is at the point B, the median splits the triangle in two triangles with equal areas (Figure 17). Hence, the sum of the areas of NBF and EFC is equal to the half of the area of the triangle ABC. That is, the area of quadrilateral ADFE is equal to the sum of the areas of NBF and EFC.

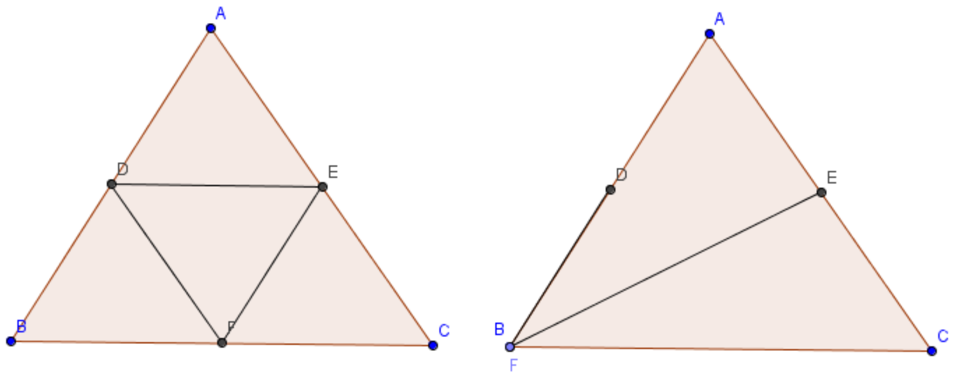


Figure 17. Particular cases

The mathematical content included in the quadrilateral problem covers definitions and elements of figures, congruent triangles, criteria of congruence of triangles, similar triangles, criteria of similarity of triangles, congruence of parallel segments comprised between parallel lines, Thales theorem, formula for the area of a triangle, ratio between homolog sides of similar triangles, and squared ratio of areas of similar triangles (Iranzo-Domenech, 2009).

3.4 The Role of the Researcher

The researcher in the present study has all information about plane geometry problems and solution strategies. He is the designer and applicant of both the instructional and experimental periods. The students were expected to solve all problems on their own, and if needed the researcher gave minimal explanation for the solution phases. Since the researcher is a research staff and gives courses in the class that the investigation was done, he knows everything about the context of the study. In addition, he is an active user of GeoGebra and has taught the software in the Computer Based Mathematics Education courses for a year.

3.5 The Instructional Period

In this qualitative research, there were 7 pre-service elementary mathematics teachers who had a four-week treatment period. The researcher was also the instructor during this period. Although participants had experience on GeoGebra from the Computer-Supported Mathematics Education Course, they were also given a four-hour training program on the use of GeoGebra at the first week of the treatment period. The students were trained about how to use GeoGebra and they carried out all the tasks during the instructional period. The content of the instructional period was prepared by analyzing GeoGebra manuals, online tutoring videos, and the content of plane geometry taught at elementary level in Turkish mathematics curriculum. The researcher prepared 9 activities by using objectives related to plane geometry problems in the curriculum. All of the GeoGebra menus that could be used in plane geometry tasks were introduced to the students. Except relation between two objects, probability calculator and function inspector, all construction tools were introduced.

The instructional period began with giving general information about the use of GeoGebra and introducing example GeoGebra tutorials. Then, all menus that could be used in carrying out plane geometry tasks were taught. The instructor showed the use of basic tasks such as constructing polygons, drawing lines, moving front, moving back, etc. and gave students time to experience on their own. Then, he introduced the activities by working on each activity with the students simultaneously. At the end of each activity, the students were expected to accomplish the task related to measurement and assessment given at the end of the activity. After the students finished all the tasks, data were collected during the last week of treatment period.

The main aim of this period was to introduce functions basic GeoGebra tools and train the students on the use of this software in carrying out plane geometry tasks. The students gained required experience so that they feel comfortable in using the software during data collection period.

3.6 Procedures

Initially, plane geometry problems were translated to Turkish and adapted to Turkish students' understanding of geometric problems. In order to address the validity and reliability of the instrument, expert opinions were taken and a pilot study has designed. The experts were mathematics educators from METU and Amasya University. In addition, the theoretic background and resolution processes given by Iranzo-Domenech (2009), and results obtained from her study are also evidences for the reliability and validity of the instrument.

Then, the researcher designed the content of a three-week instructional period for the actual study. It was prepared by taking into consideration the curriculum initiated by MoNE (2005). For that purpose, the objectives in Table 1 that are related to plane geometry problems were determined. Lesson plans and GeoGebra worksheets were prepared according to these objectives.

Table 1. Objectives for 9 GeoGebra activities

Statement	Grade Level
Construct polygons	6
Draw the triangle whose measures of sufficient components are given	8
Construct medians, perpendicular bisectors of the sides, angle bisectors and height of a triangle	8
Solve and pose problems related to area of planar regions	6
Explain conditions for equality of triangles	8
Explain conditions for similarity of triangles	8
Apply conditions for similarity of triangles to problems	8
Explain the relationship between area and length of sides	7
Determine and construct reflection of a polygon according to coordinate axes, translation along any line, rotation around the origin	7
Construct the graph of linear equations	7

Nine activity sheets were prepared in the light of these objectives. The main purpose of preparing these activities to teach students the use of basic GeoGebra tools that could be necessary for solution of the problems in the instrument. As a sub-goal, the researcher also aimed to give the students the opportunity to have practice with plane geometry activities. The activity sheets were examined during pilot study. According to the feedback obtained from this period and an expert at METU, they were revised and prepared for the actual study. An example activity sheet was taken from GeoGebra booklet, translated in English and given in Table 2. In this activity, it was aimed to teach how the students can use GeoGebra to show equality and similarity of triangles on picture of an Egyptian Pyramid and Ephesus Library.

Table 2. A sample GeoGebra activity

Activity 5 – Similar and Identical Triangles

Lesson: Mathematics

Class: 8

Area of Learning: Geometry

Sub-Area of Learning: Triangles

Skills: Computer use, geometric thinking, reasoning, mathematical correlation, problem solving

Objectives: Explain conditions for equality of triangles

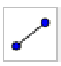
Explain conditions for similarity of triangles

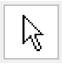
Materials: Computer, GeoGebra software

Geometry in Egyptian Pyramid and Ephesus Library

1. Hide algebra window and coordinate axes since you will not need to use them. To hide algebra window click on Algebra in the View menu. To hide coordinate axes click on Graphics in View menu.
2. Show identical polygons on the picture of Ephesus Library.

3.  First of all, insert the file efes_kutuphanesi.ggb in your computer into the GeoGebra window by using Insert Image button.

4.  Determine the triangles and polygons in the library by using Segment between Two Points.

5.  If the labels of the points are not displayed, click on the Move button. Then, right click on every point and click on the Show Label option in the menu. Do not forget that GeoGebra names the points in an alphabetic order.

6. As shown in figure below, compare the identical polygons and determine theproperties of them by using GeoGebra tools (Figure 44).

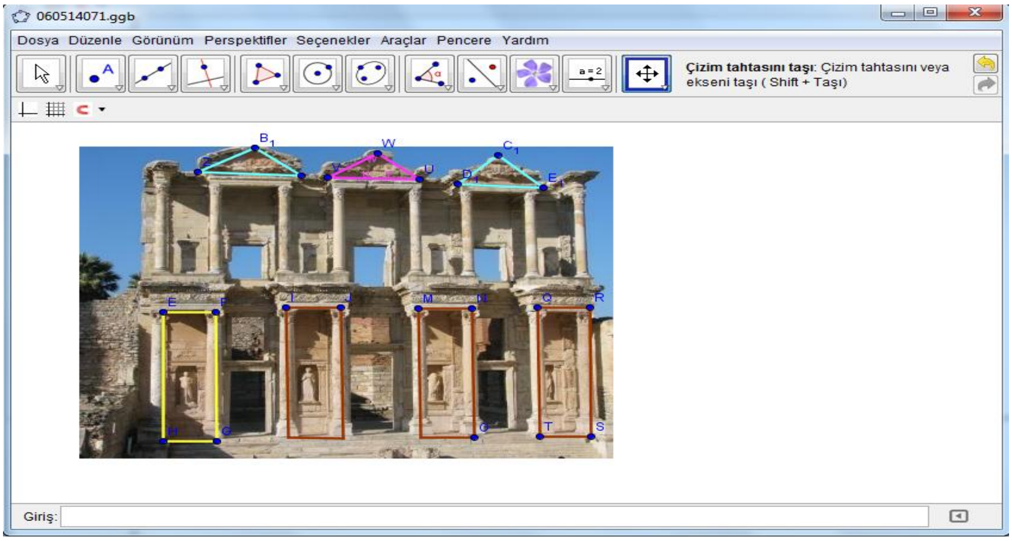


Figure 44. Identical triangles

7. Show similar triangles on the picture of Egyptian Pyramid.

8.  First of all, insert the file misir_piramidi.png in your computer into the GeoGebra window by using Insert Image button.

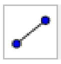
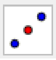
9.  Determine the biggest triangle on the image by using Segment between Two Points.

Table 2. A sample GeoGebra activity (Continued)

10.  Find the midpoints D and G of the side AB and the side AC respectively by using Midpoint or Center tool. Use the same tool and find the midpoints of line segments AD, DB, AD, and GC.

11. As shown in figure below, compare similar triangles and determine the properties of them by using GeoGebra tools (Figure 45).

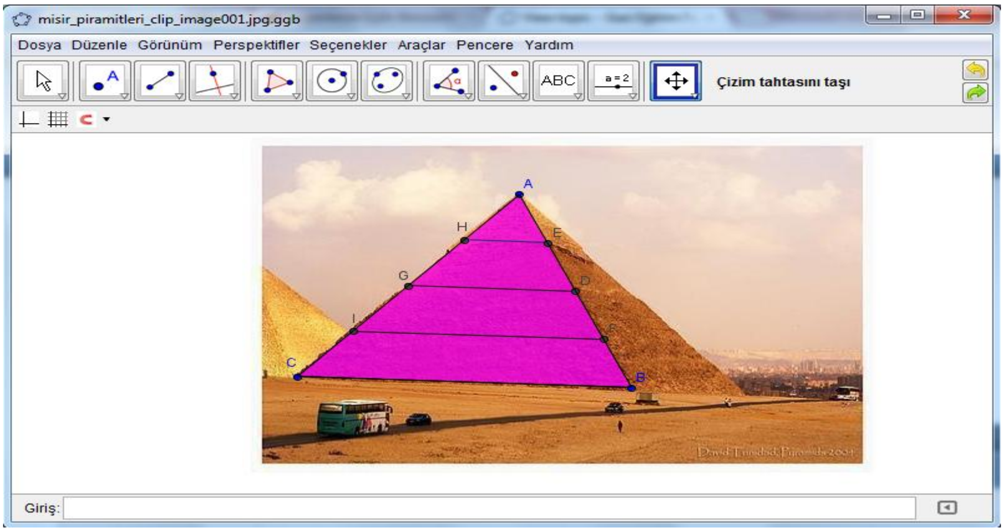


Figure 45. Similar triangles

Teaching and Learning Process

1. What are sufficient conditions for equality of polygons that you determined on Ephesus Library?

2. What are sufficient conditions for similarity of triangles that you determined on Egyptian Pyramid?

Measurement and Assesment

1. Give examples from constructions around you that include identical and similar triangles.

2. Which points on the figure must be connected with the edge points of the line segment KL in order to have triangle equal to the triangle ABC (MoNE, 2010)?

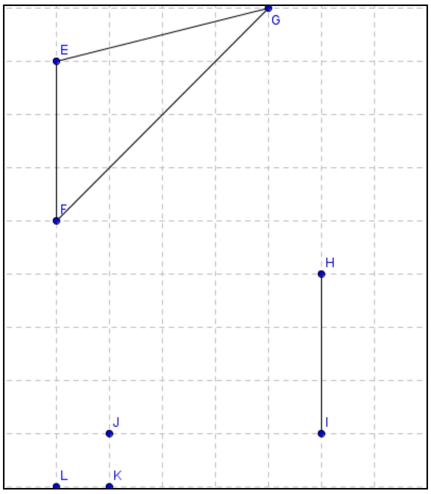


Figure 46. Equal triangles problem

In addition, GeoGebra and paper-and-pencil worksheets were prepared to give clear directions to the students. Table 3 shows the worksheet for paper-and-pencil environment.

Table 3. Paper-and-pencil worksheet

Problem: Paper-and-Pencil	**Name:**	**Date:**

Instruction: The tutor can help you about;

- Understanding the problem statement
- Solving the problem
- Checking out the solution

Solution: It is important for solving the problems to know how to do and what to do. After solving the problem, you can want the tutor to help you about giving clues or propose another way when you suspect about your solution.

Table 4 shows the worksheet for GeoGebra environment.

Table 4. GeoGebra worksheet

Problem: GeoGebra	**Name:**	**Date:**

Instruction : The tutor can help you about;

- Understanding the problem statement and using GeoGebra
- Solving the problem
- Checking out the solution

Solution: It is important for solving the problems to know how to do and what to do since you will not use paper&pencil. After solving the problem, you can want the tutor to help you about giving clues or propose another way when you suspect about your solution.

Read given problems carefully and construct the figures by using GeoGebra.

After you finish your work, please save GeoGebra file as name_prob_num.ggb

The students solved all problems with both GeoGebra and paper & pencil. In addition if they have trouble with the software, they were allowed to solve only with paper-and-pencil. Table 5 shows the time schedule of the present study.

Table 5. Time schedule of the present study

Week	Time	Activity	Duration
1	09-28 April & 7-11 May 2012	The Pilot Study	2 hours
2	1-5 October 2012	Instruction (Basic GeoGebra tools)	4 hours
3	8-12 October 2012	Instruction (GeoGebra Activities)	4 hours
4	15-19 October 2012	Instruction (GeoGebra Activities)	4 hours
5	22-27 October 2012	Data Collection	4 hours

3.6.1 The Pilot Study for the Instrument and GeoGebra activities

The main purpose of designing a pilot study was to examine whether the activities and the instrument were appropriate for the present study or not. The whole process expected to be carried out in actual study was assessed by means of this pilot study. The researcher was giving Computer Based Teaching in Mathematics Education course to pre-service mathematics teachers at Amasya University in 2011-2012 Spring Semester. Two sophomore students who are taking this course were selected by taking account of their performance observed by the instructor during the course period, voluntariness and being communicative for obtaining plausible feedback about the instrument. The researcher designed a three-week instructional period for teaching basic GeoGebra tools and the tools that could be necessary while solving the problems as a part of the course syllabus. The activity sheets which were prepared for the actual study were also examined during this period.

GeoGebra and paper-and-pencil were given to the students. They were allowed to use them whenever they want during problem solving process. While they solved the problems, the process recorded with a video camera. The researcher gave clues if they needed during their solutions. He mostly asked ‘why?’ and ‘how?’ questions in order to understand the reasons for selection of their solutions strategies. After finishing data collection period, the researcher transcribed data obtained from video records. He analyzed worksheets, video records and transcribed data.

In order to interpret students' solutions strategies, the researcher used the framework of Krutetskii (1976) and Presmeg (1986). Their solution strategies consisted of algebraic, geometric and harmonic methods as argued by Krutetskii (1976). In addition, during using the software, the students preferred dynamic solutions. These solution strategies were determined based on Presmeg's (1986) idea of dynamic imagery. The researcher deduced that the students were able to solve the problems in the instrument in both environments and develop specific solution strategies for each problem separately. In addition, the researcher reviewed his position during whole process, revised the activity tasks and determined the usability of the instrument in the light of the results of the pilot study.

3.6.2 Data collection

The data was collected in the 2012-2013 Fall Semester from two sophomore pre-service teachers attending Department of Elementary Mathematics Education at Amasya University. The students were given half an hour for solution of each problem. The data collection period was recorded with a video camera. The researcher gave worksheets to the students for solving problems on them. For data triangulation, it is important to have plausible data from video records, GeoGebra files and worksheets. Hence, the students showed all of their works on these documents. Moreover, the researcher mostly preferred to ask questions as follows;

- What do you mean …?
- Why do you think so?
- How can you make sure that your solution is correct?
- Why did you choose this strategy?
- Is it possible to use another strategy? Think about it.
- Would you like to try it with Geogebra/paper-and-pencil?

3.7 Analysis of Data

The researcher analyzed GeoGebra and paper-and-pencil worksheets, GeoGebra files and video records. The students usually did not prefer GeoGebra worksheets while solving the problems in GeoGebra. The researcher watched all video records and transcribed it into the dialogues. In order to compare students' solutions, their strategies are grouped according to their dominant characteristics. Krutetskii's (1976) framework was used in order to categorize the solutions strategies. Krutetskii (1976) suggested that there are analytic, geometric and harmonic thinkers according to their relative predominance of using verbal-logical and visual pictorial components of mathematical skills during problem solving process.

Analytic ones mostly preferred to use verbal-logical justifications when compared to visual pictorial ones. Analytic problems solver mostly use algebraic, numeric and verbal representations (Coşkun, 2011). However, the instrument in the present study consists of plane geometry problems; there are not any alternative numeric solutions, hence the researcher preferred to use algebraic solution methods instead of analytic ones. Since verbal representations are in both PPB and GGB environments, they are preferred to be used as sub-categories. In the present study, algebraic solutions include calculating or proving the result by solving equations that are derived from geometric relationships. Since students' solutions comprised of either verbal-logical justifications or logical verifications without verbal explanations, the researcher divided this category in two sub-categories, namely, logical and verbal-logical ones. In logical solutions, the students mostly preferred to use paper-and-pencil without using verbal messages.

Geometric solutions consists of mostly visual-pictorial components when compared to verbal-pictorial ones. In the present study, these solutions covered finding the result by extracting logical statements from common approaches. The students who preferred this method solved problems used either verbal explanations or dynamic representations. Therefore, the researcher preferred to divide geometric solutions in verbal-pictorial and dynamic ones. The idea of using dynamic solutions emerged from Presmeg's (1986) imagery framework. Among five imagery types, the students who preferred dynamic imagery use moving images (Presmeg, 1976). That is, they move or drag a figure and deduce the result from particular cases. Since GeoGebra is a dynamic geometry software, it was better to use such a classification.

In harmonic solutions, there is a relative equilibrium between verbal-logical and visual components of mathematical skills. Ktuteskii (1976) divides this category in two sub-categories; abstract-harmonic and pictorial-harmonic. The students who used abstract harmonic solutions prefer less pictorial components for their mental operations than pictorial harmonic ones. In the present study, the students used both algebraic and geometric methods in different solution steps of the problem during their harmonic solutions. However, the equilibrium between verbal-logical and visual components was preserved.

All in all, the framework for analyzing the data in the present study has shown in figure below. The data was analyzed in the light of this frame by comparing the solutions in each environment separately.

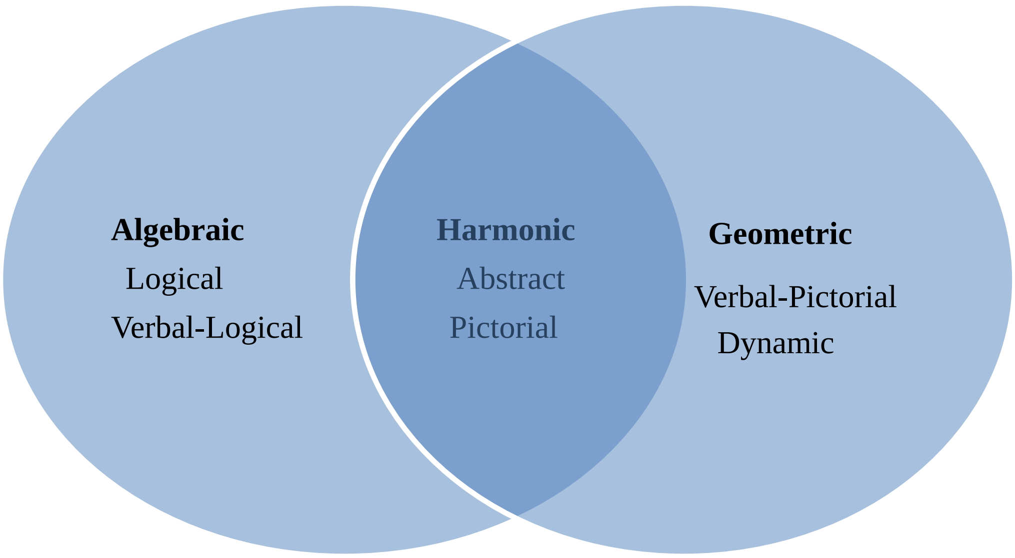


Figure 18. Classification of solution strategies

3.8 Validity and Reliability

In the present study, a multiple case study approach was used hence validity and reliability be evaluated in the context of the case studies. For validity of the study, internal and external validity need to be examined. Merriam (1988) stated that there are six principles for interval validity of a study:

1. *Data triangulation**:*** Interviews, student worksheets and classroom observations was used for the data triangulation process in the present study.

2. *The control of the data by the subjects*: After analyzing data, the researcher asked the participants whether the data is correctly stated or not.

3. *Having long time for observations*: The researcher observed the participants and the process for about a one-month period hence it was sufficient time duration for observations.

4. *Using participant research methods*: The participants ideas about the use of technology and the content of the study were taken hence they are allowed to be involved in whole process.

5. *Examining the study by other researchers*: After collecting data, they are discussed by the researchers at different universities who experienced similar studies.

6. *Bias of the researcher*: The role of the researcher was determined in the beginning of the study hence the threats to internal validity were minimized.

Moreover, Fraenkel & Wallen (2006) stressed that there is a rare methodological justification for generalizing the findings of a particular case study. Therefore, this situation constitutes a limitation for the qualitative studies. However, Çepni (2009) thinks that comparing the results obtained from similar studies that have the same methods can be a method for increasing the external validity of a study. Therefore, the results of the present study were compared with similar studies with the same research design.

For the accuracy of the findings of the present study, they are compared with the findings obtained from a multiple case study in which the same instrument was used. It was found that the results of two studies were consistent. In addition, the findings of the pilot and actual studies were also consistent. Moreover, the consistency between the results from different data resources such as interviews, observations and worksheets increased the accuracy of the results.

3.9 Assumptions and Limitations

3.9.1 Assumptions

First of all, it was assumed that all participants provided honest and accurate information during the interviews. In addition, they were counted as the fact that they answered all problems accurately and honestly. Moreover, it is assumed that the researcher correctly recorded the data gathered from the interviews and classroom observations.

3.9.2 Limitations

The limitations of the present study are related to generalizability of the results as it was a qualitative multiple case study. First of all, there are two particular cases and the results might not be valid for other cases since each individual has different characteristics. Secondly, the data were collected in a particular university. In addition, the subject was plane geometry. The results might vary with other subjects such as solid geometry, trigonometry, or functions. Moreover, the instrument was limited to four problems. There could be any other problems that help students' to develop different strategies. The complexity of problems might be another issue because they are all related to each other and it might negatively affect the variation in solutions methods.

CHAPTER 4

THE CASE OF MERVE

In this chapter, Merve's solution strategies were analyzed in detail. First of all, her solutions in PPB environment were introduced, and then they were categorized according to their basic characteristics. Then, her solutions for the same problems in GGB environment were mentioned, and grouped according to their properties. Finally, Merve's solutions were summarized for each environment.

4.1 Merve's PPB Solutions

In this section, the data gathered from paper & pencil environment and from video tapes will be analyzed. The solutions for each question will be grouped according to students' algebraic, geometric and harmonic solution strategies.

4.1.1 The Root Problem

For the root problem, Merve summarized the problem and determined what is expected by the solution of the problem. Firstly, Merve attempted to solve the problem by using Pythagoras theorem. She named the lengths and tried to find hypotenuses (Figure 19). She tried to find a relationship between the sides of the rectangles NEOB and MEPD. When she calculated the hypotenuses by using the theorem, she realized that it is hard to find the relationship in this way. This solution path was an algebraic solution because Merve used Pythagoras theorem and tended to calculate algebraic equations and find a relationship between two unknown variables.

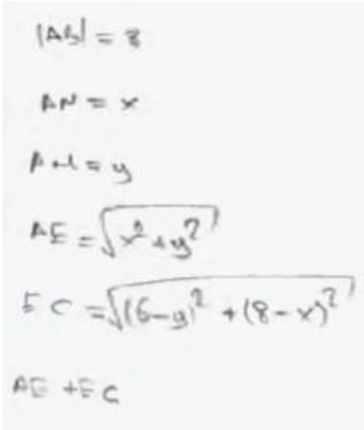


Figure 19.Merve’s use of Pythagoras theorem

This algebraic way of thinking made her insist on developing a strategy based on the relationships between the sides of the rectangle. Then, Merve figure out that there should be another way to find a relationship between the sides of rectangles NEOB and MEPD. She explored the similarity between triangles AME and ADC. Next, she found that if AM=3k, then ME=4k (Figure 2).

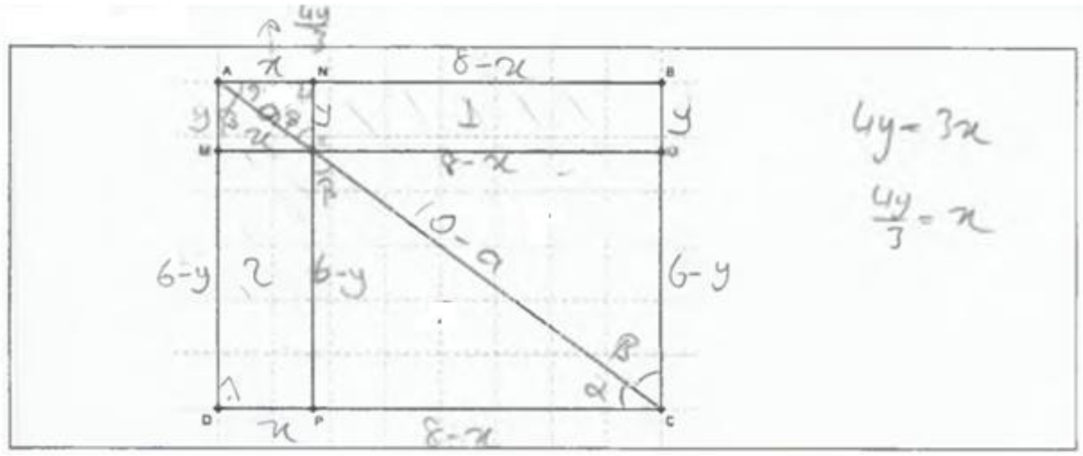


Figure 20. Expression of the sides in terms of unknowns

She verbally explained how she found the ratio between two sides of the right angle triangle AME as shown in the following dialogue.

Tutor: How will you solve the problem?

Merve: If I find a relationship between the sides of the rectangles NEOB and MEPD, then I can find the relationship between the areas of these rectangles. For this purpose, I can use the similarity between triangles inside the rectangle ABCD. The triangles AME and ADC are similar. The ratio between the sides of the right triangle ADC is

(6,8). Therefore, the ratio between the sides of the right triangle AME is (6k, 8k), i.e. (3k, 4k), where k is a constant variable.

Tutor: Well, why these triangles are similar?

Merve: Because, the angle A is common and the other angles are equal due to the the fact that the side ME is parallel to the side DC.

This paragraph showed that she understood the logical structure of the problem. As shown in Figure 21, Merve calculated that the rectangles DPNA and MOBA have the same area. Since the rectangle ANME is common in two rectangles, she subtracted this rectangle from other rectangles and found the area equality of the rectangles NEOB and MEPD. This strategy based on finding equality of areas of rectangles and subtracting common rectangle.

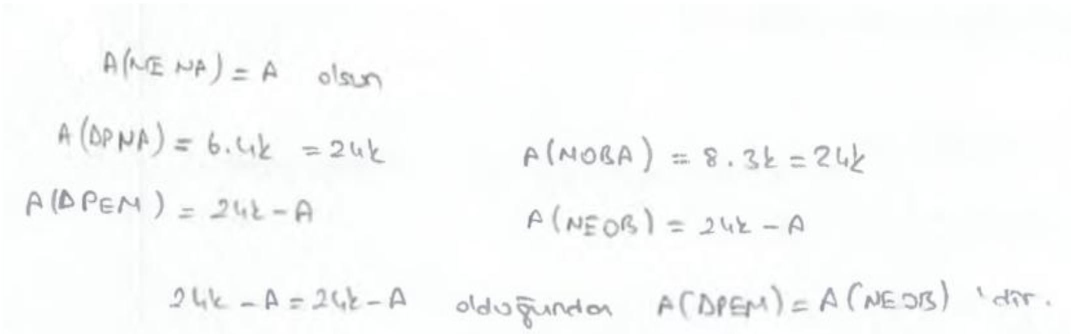


Figure 21.Merve’s solution of the root problem

This solution strategy was classified as a harmonic solution because she attempted to solve the problem by using both geometric and algebraic approaches in different steps of the solution. In the first step, Merve found the relationship between the sides of rectangles based on the geometric approaches. She did not use any algebraic equation. However, in the next step, while calculating the area of the rectangles, she set equations and found the equality of them. Therefore, a relative equilibrium in the use of algebraic and geometric approaches makes the strategy a harmonic solution. Moreover, Merve preferred to use less visual-pictorial components than algebraic ones during her solution. Therefore, the solution is also an abstract-harmonic solution method.

4.1.2 The Scaled Triangles Problem

For the first part of the problem, Merve named the sides in terms of unknowns (Figure 22). While summarizing the basic structure of the problem, she pointed out that Thales theorem could be used to solve the problem. First of all, she showed similar triangles on the figure. Then, she verbally stated that the triangles EFM and ABM are similar. Accordingly, the triangles MPF and MAC are also similar.

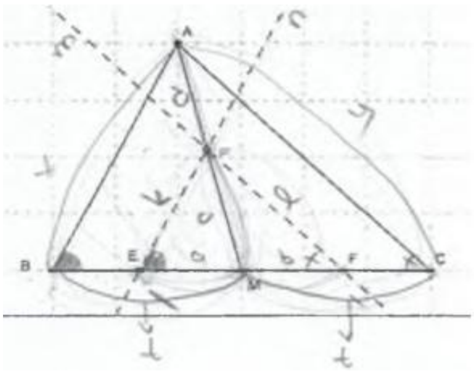


Figure 22. Expression of the sides in terms of unknowns

However, she did not know how she can use this information for the solution. At this point, after considering the common side AM for the triangles, she preferred to write the ratios for similar triangles. Then, she explored that the ratio of PM and AM are common for similar triangles. Finally, after writing all equations, she found that the length of line segment EM and MF are equal (Figure 23). This strategy was called as equality of ratios based on similarity theory.

$$\frac{a}{t} = \frac{k}{x} \qquad \frac{a}{t} = \frac{c}{c+d}$$

$$\frac{b}{t} = \frac{l}{y} \qquad \frac{b}{t} = \frac{c}{c+d}$$

$$\frac{a}{t} = \frac{b}{t} \Rightarrow a = b$$

Figure 23.Merve’s solution of the scaled triangles problem (a)

However, she exactly did not know which ratio will be useful for the solution of the problem. She realized fortunately the equality between the ratio of the sides:

(EM/BM)=(MF/MC) (Figure 22 & Figure 23). Following dialogue shows how she realized the equality EM=MF.

Tutor: You expressed the sides in terms of unknowns. How do you use these unknowns?

Merve: I will write the ratio of similarities in terms of unknowns.

Tutor: Well, what do you expect to obtain by using these ratios with unknowns?

Merve: Actually, I exactly do not know, but I consider having a relationship between the equalities.

Tutor: You expect to have a relationship?

Merve: Yes. (After writing the equalities) I found the equality of the sides. In fact, I could show this equality on the figure, but this way is much easier.

This algebraic way of representation helped her to solve the problem. She could not explore the relationship on the figure. Therefore, this solution strategy could be classified as an algebraic solution. Merve used Thales theorem and set equations for the solution. Then, she derived the result from the equations. This algebraic way of solution gave the opportunity to the student to understand geometric relationships easily. In addition, she justifies her drawing and algebraic expressions (Figure 5) by verbal explanations in above dialogues. Therefore, the strategy is also a verbal-logical algebraic solution.

For the second part of the problem, Merve drew the figure again such that BE = EF = FC. She used the information EM=MF from previous part of the problem. Therefore, she easily expressed the lengths in terms of coefficient k. If EM=MF=2k, then BE=EF=FC=4k. By using similarity between the triangles EFM and ABM, she found (MP/PA) =1/2. That is, the distance between the point P and M is half of the distance between the point P and A. Then, she realized that this point is at the centroid of the triangle (Figure 24). Since this solution was related to previous algebraic solution and here also she used algebraic expressions, it was categorized as an algebraic solution. In addition, she justifies her drawing and algebraic expressions by verbal explanations. Therefore, the strategy is also a verbal-logical algebraic solution.

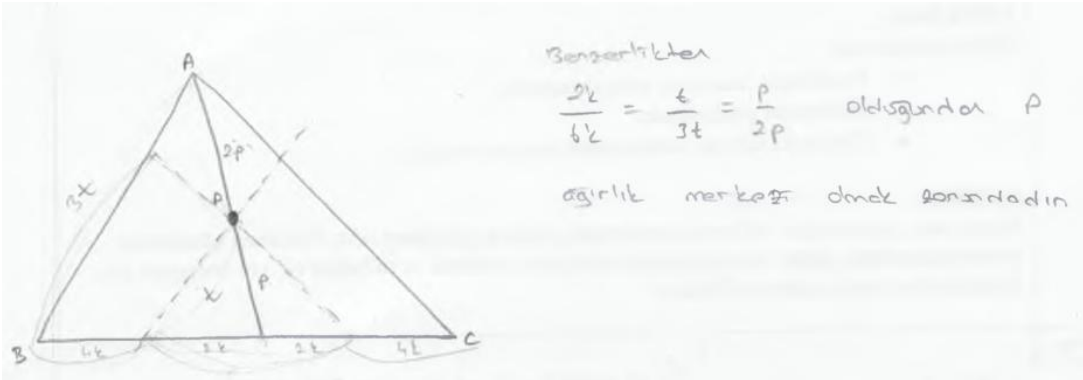


Figure 24.Merve’s solution of the scaled triangles problem (b)

4.1.3. The Median Problem

Merve tried to show the relation between the area of the triangle APB and the triangle APC by comparing common base and the respective heights. First of all, she focused on the side AB and the side AC. Following dialogue shows the exploration phase for the solution of the median problem.

Tutor: Well, how do you plan to solve the problem?

Merve: I will draw the heights of the sides AB and AC. Then, I will compare the areas of the triangles APB and APC.

Tutor: Is this enough for showing the relationship between the areas of these triangles?

Merve: I will draw and show my justification on the figure.

Tutor: Ok.

Merve: (After drawing the figure) I cannot compare the areas in this way because the bases AB and AC are not equal.

Tutor: So, you need the bases with same lengths in addition to the equality of heights?

Merve: Yes.

Tutor: Is this enough for your justification?

Merve: If I find the ratios of the heights of these sides, it will be enough. If the bases are equal, then there is a relationship between the areas as much as the ratio of heights.

She realized that the bases should be equal in order to compare them. These are the side BM and MC which are constructed by the division of the side BC by the median of the triangle. She explored that respective heights of the bases are also equal because of the median of the triangle. Then, she stated that since common bases and the respective heights are equal, the areas of the triangle AMC and ABM are equal. The situation is the same for the triangle PBM and the triangle PMC. Therefore, the areas of the triangle APB and the triangle APC are equal (Figure 25).

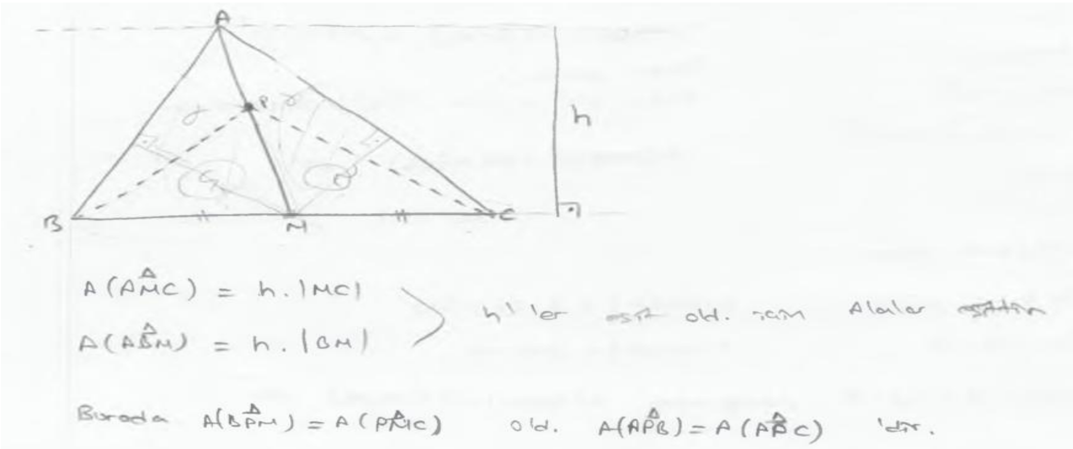


Figure 25.Merve’s solution of the median problem

She used geometrical ideas and constructed algebraic expressions to show her solution. Therefore, this strategy, comparing common base and the respective heights of the triangle APB and the triangle APC, was an algebraic solution. Since she has logical explanations and did not need any verbal justifications, this solution was also an algebraic-logical solution. In essence, in our grouping of solution strategies for algebraic ones, it can be easily seen that the strategies are supported by mental operations. However, some of them are expressed verbally; others are in the written form. The distinction made by the researcher was exactly based on either the student used verbal explanations or not. More specifically, if a student wrote her solution and did not need any verbal explanations, the solution is logical. Otherwise, if the student used both written and verbal explanation, in this case, it was a verbal-logical strategy.

4.1.4 The Quadrilateral Problem

For the last problem, Merve drew the figure and, then explained it verbally in the light of her geometry knowledge.

Tutor: How did you solve the problem?

Merve: Firstly, I drew the auxiliary line AP. Then, I had 4 triangles. Since the line segment NP is the median of the triangle ABP, it divided the area in two equal areas. Then, if I expressed the area one of this triangles as S, then another also has the area S. The situation is the same for the triangle APC. I wrote the letter A for the areas of each triangle. Then, The rectangle ANMP has the area A+S. The sum of the areas of triangles BNP and PMC is also A+S. Hence, I have A(ANPM)=A(BNP)+A(PMC) (Figure 26).

Tutor: Well, you used the median property of the triangle ABC. Well then, why the median divided the area of the triangle ABC in two equal areas?

Merve: Because, the median divides the triangle in two triangles with equal bases and equal respective heights. Therefore, the areas of two triangles are equal.

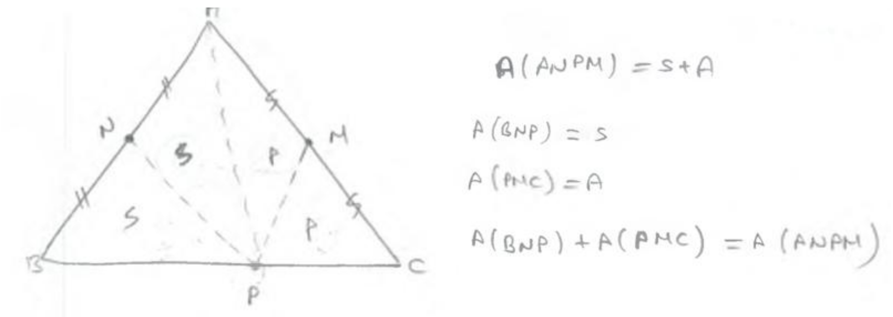


Figure 26.Merve's solution of the quadrilateral problem

Merve solved the problem easier and quicker than previous problems. The reason might be that all problems are related to each other and the mathematical concepts that are asked in the last problem are included in previous problems. The mathematical ideas and solution methods in previous problems are similar. Merve gained experience and did practice until the last problem. In this case, Merve preferred to solve the problem by comparing the areas and expressed the triangle ABC with decomposition of areas. She justified this solution method with verbal explanations. Therefore, this solution was classified as a verbal-pictorial geometric solution. She used geometric relationships based on verbal explanations instead of algebraic ones.

4.1.5 Summary of Merve's PPB Solutions

In the PPB solutions, Merve mostly solved the problem by using Krutetskii's (1976) verbal-logical framework. Even visual solution for the quadrilateral problem was also supported by verbal explanations (Table 6). In paper-pencil environment, Merve had an inclination for algebraic solutions based on geometric figures.

Table 6. Classification of Merve's PPB solution methods.

The problem		Solution method	Solution category
The root problem		Equality of area of rectangles and subtracting common rectangle	Harmonic-Abstract
The scaled triangles problem	a)	Equality of ratios based on similarity theory	Algebraic Verbal-Logical
	b)	Equality of ratios based on similarity theory	Algebraic Verbal - Logical
The median problem		Comparing common base and the respective heights of the triangle APB and the triangle APC	Algebraic-Logical
The quadrilateral problem		Decomposition and comparison of areas	Geometric-Verbal-Pictorial

4.2 Merve's GGB Solutions

In this section, the data obtained from GGB environment and video tapes will be analyzed. The solutions for each question will be grouped according to students' algebraic, geometric and harmonic solution strategies.

4.2.1 The Root Problem

Merve confused about drawing either on the grid or using a blank graphic window. The main reason for this confusion was whether the lengths given in the problem (AB=8 and AD=6) are important or not. Merve thought that she must be careful about using accurate lengths in her construction. For this reason, she used grid and placed the points on the sides of the figures with respect to given lengths

AB=8 and AD=6. Following dialogue shows how she thought about the use of given lengths.

Tutor: How will you the lengths of the sides?

Merve: I will use grid view or measure the lengths on the graphic window. I prefer to use grid view.

Tutor: Well, why do you measure the lengths? Will you use this information in your solution?

Merve: Because, they are given in the problem statement. I used them in my paper-and-pencil solution. I will also use them in GGB solution.

Tutor: Ok.

The researcher did not give clues about the use of the length of the sides because such a help might constitute a thread for the students' pure solutions. After deciding the issue she drew the figure. Then, she thought for a while and decided that measuring the areas will help her to understand the relationship. Since she had solved the problem in paper-and-pencil environment, she expected that the areas should be equal. Then, she measured and found the equality of areas (Figure 27). In order to justify her solution, she dragged the point E and showed that the equality of areas is satisfied along the diagonal of the rectangle.

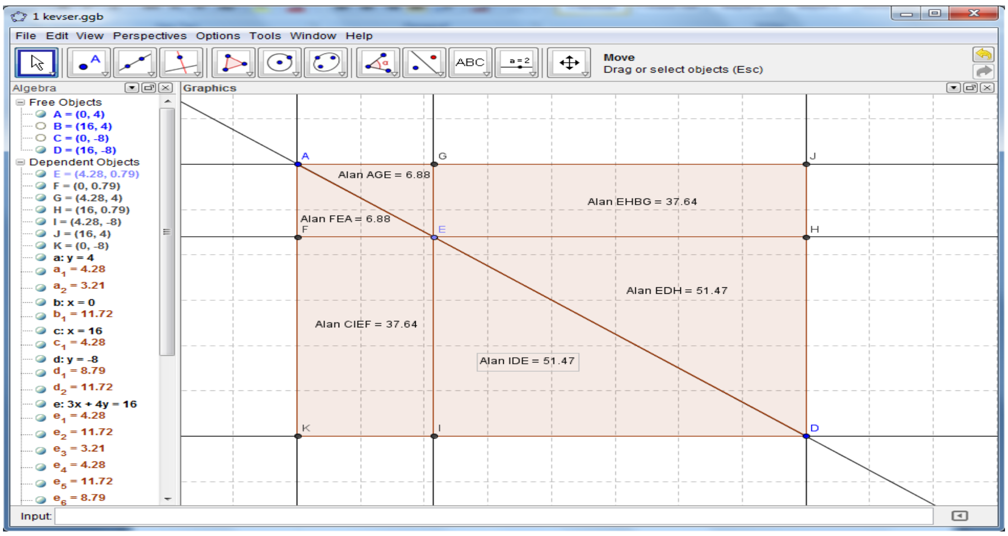


Figure 27.Merve's GGB solution of the root problem

Then, in the following dialogues, it can be seen how she explained and justified her solution:

> **Tutor**: You measured the areas and found the equality of areas. So, why they are equal?
>
> **Merve**: Because, when we drag the point E, the equality remain the same along the diagonal of the rectangle AJDK (She showed this situation on GeoGebra file).
>
> **Tutor**: What is the main reason for this situation?
>
> **Merve**: The point E is on the diagonal.
>
> **Tutor**: So, what is the function of diagonal?
>
> **Merve**: It divides the rectangle in two right triangles with equal areas. When we applied this rule on this figure, we can see the equality.
>
> **Tutor**: Well, another important point is that you did not use the lengths of the sides. I considered the necessity of expressing these lengths at the beginning of the problem.
>
> **Merve**: If we measure the sides, we can see that the software used the ratio of the sides.
>
> **Tutor**: So, the ratio (6,8) is necessary or not? Do any other ratios satisfy this equality?
>
> **Merve**: I think that the ratio will remain the same because the point A is on the diagonal and the lines EI and EH are parallel to the sides. The equality is true for all rectangles that satisfy this condition. We can show this by dragging feature at the same time.

According to dialogue, she explored the equality from diagonal property of the rectangles. She thought that the diagonal divides all rectangles into equal parts. Another important exploration for her was that the lengths of sides were not important in this case. It can be clearly understand from the dialogue that she understood the logic behind the equality of areas of the rectangle NEOB and the rectangle MEPD. She summarized her solution and identified the reasons for the equality as diagonal property of rectangles, the place of the point E which is on the diagonal, and the line segments passing through the point E and parallel to the side AB and AD. She preferred confirming her solution with verbal explanations as shown in the dialogue and summarized above. Since, she drew the figure and used geometric verifications; this solution is a geometric solution. Moreover, she verbally and visually justified her solution hence it was classified as a verbal-pictorial geometric one. She used her geometric knowledge supported by verbal explanations in addition to knowledge of technology tools usage in plane geometry problems.

4.2.2 The Scaled Triangles Problem

For the first part of the scaled triangle problem, Merve again thought of using measuring tool. However, she did not consider her solution in the PPB environment. She just focused on the measuring the lengths of the line segments. The use of similarity theory and algebraic equations was disregarded in this environment. Nevertheless, during the instructional period, there was an activity (Thales theorem) about the use of similarity theory in GeoGebra environment. Most likely, she had forgotten either how to apply the similarities of triangles with GeoGebra or thought that following such a solution path will be much more confusing. Therefore, she focused on another way of solution for this problem. The solution process could be observed in following dialogue:

> **Tutor**: How did you solve the problem?
>
> **Merve**: I measured the side FD and the side DG. They are equal.
>
> **Tutor**: How can you verify this equality?
>
> **Merve**: While dragging point E, the equality remains the same. Moreover, moving the sides and vertices of the triangle did not affect this equality.
>
> **Tutor**: What are main reasons for this equality?
>
> **Merve**: AD is the median of the triangle, FE and GE are parallel to side AB and the side AC respectively, and E is at the median and the intersection of parallel line FE and the line GE.
>
> **Tutor**: In other words, you summarized information given in the problem.
>
> **Merve**: Yes.

According to the dialogue, Merve measured and found the equality of the side FD and the side DG (Figure 28). She preferred to use the dynamic feature of the software. She justified her solution by moving the vertices and sides of the triangle. However, that was just a verification of the equality. At this point, after thinking a while, she claimed that the length of line segments FD and DG are equal because the point E is on the median and the line FE and EG are parallel to the sides of the triangles. She did not use the similarity theory during her explanation. Therefore, she experienced little difficulty in justifying her solution. Her explanations consisted of repeating the information given in the problem and movement of the figure. That solution was grouped as a geometric solution. Moreover, since she used measuring tool and justify her solution by dragging tool, it was a dynamic geometric solution. When the researcher asked to explain her result, she visually demonstrated the relationship and verified it by using the features of the software as shown in above dialogue.

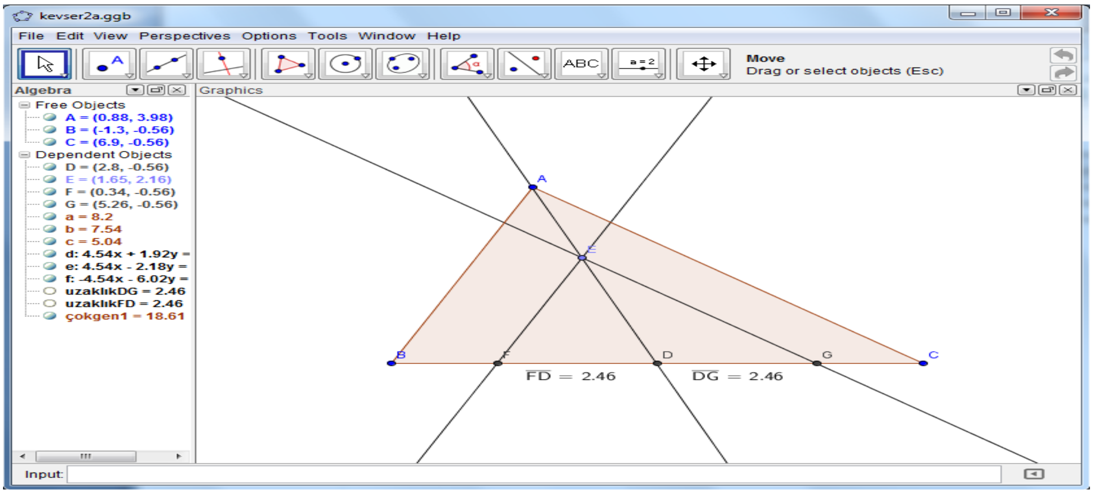


Figure 28.Merve’s GGB solution of the scaled triangles problem (a)

For the second part of the problem, she used an inductive method in order to explain where the point E is. In this case, an inductive method is that one solves a plane geometry problem by using basic properties of the figure and then explores a general solution for the problem. In the following dialogue, the solution process is identified.

Tutor: What will you do?

Merve: (After drawing the figure) I measure the lengths. Then, by dragging the point P, I can find the point where the equality of three line segments is satisfied.

Tutor: Ok.

Merve: (After measuring) The point P is here.

Tutor: So, what are the distances of that point to the sides?

Merve: I draw the lines from the vertices B and C passing through the point E. This lines bisects the sides AB and AC (Shows by measuring).

Tutor: Ok.

Merve: These two lines are also medians of the other sides of the triangle so the point E is the intersection point of all medians of the triangle ABC.

She preferred to draw lines that connect the vertices and the point E. After drawing lines and measuring the line segments that divide the sides of the triangle, she found out that these lines are medians of the triangle. Therefore, the point E is the centroid of the triangle (Figure 29). She verified this inductive method with verbal explanations shown in above dialogue enriched with geometric ideas. Visual demonstrations and geometric reasoning made the solution process clear. Therefore, this strategy was as a verbal-pictorial geometric solution.

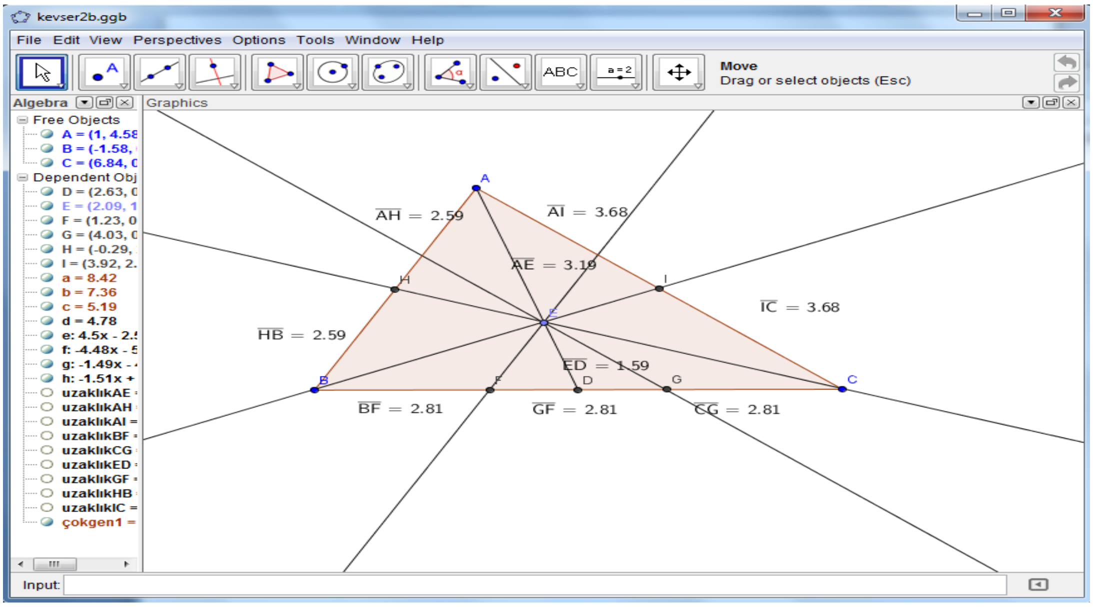


Figure 29.Merve's GGB solution of the scaled triangles problem (b)

4.2.3 The Median Problem

Merve used dynamic properties of the software in the solution of the median problem. She knew that the areas must be equal from her PPB solution of this problem. Therefore, she measured the areas of the triangle APB and the triangle APC and showed that the areas are equal. In this case, her PPB solution constituted a thread for her solution in GGB environment. It can be easily derived from the following paragraph.

Tutor: How do you plan to solve the problem?

Merve: I solved the problem by using paper-and-pencil before. I knew that the areas of the triangles APB and APC are equal. I can show this equality by using measuring tool.

Tutor: Well, assume that you did not know the result. How would you solve the problem?

Merve: (After thinking for a while) Again, I would use measuring tool because I cannot think of any other way. After measuring the areas, I can show that the equality is the same while the point E is dragged along the median.

Tutor: Is this enough for showing that your solution is correct?

Merve: Yes. If I show that the equality remains the same when the point E is on the points A and D, then I can generalize my solution.

She thought that she have already known that the areas are equal and she had showed it with paper and pencil. Therefore, she preferred to measure the angles. Then, she dragged the point E along the median. Then, she justified her solution based on particular cases E=A and E=D (Figure 30). Merve thought that these particular cases are enough to generalize the solution. Although it is not a totally inductive method, it was named as a particular case strategy. Moving the point E and observing the stability of equal areas at particular points made the solution a dynamic one. This particularization method was also a geometric approach because Merve extracted the solution from common geometric approaches and did not use any algebraic equations.

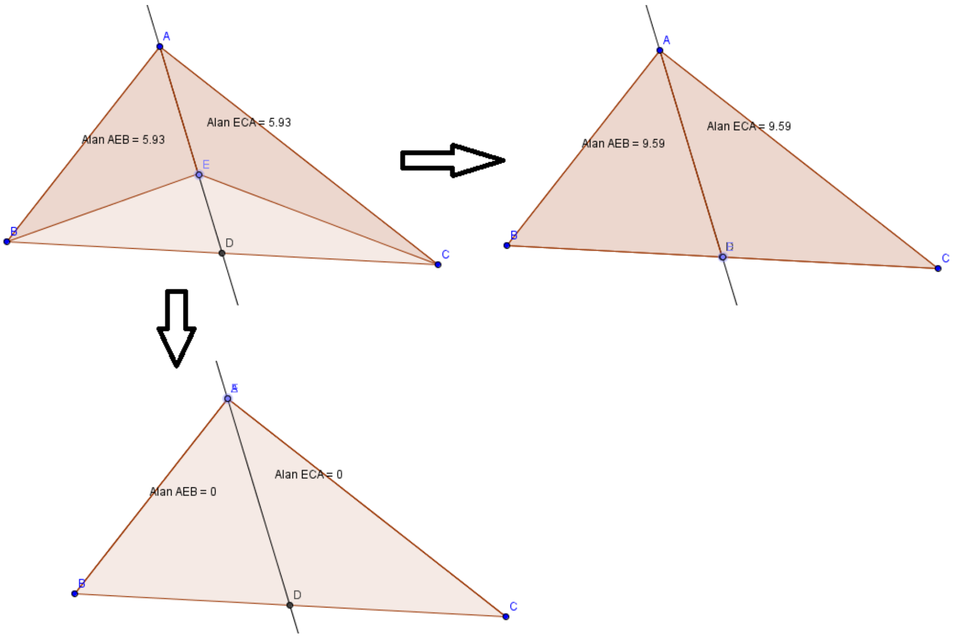


Figure 30.Merve’s GGB solution of the median problem

4.2.4 The Quadrilateral Problem

Merve solved the last problem by using again the idea of particular and degenerate cases. After drawing the figure, she measured the area of the quadrilateral and triangles. She found that sum of the areas of the triangle BDF and the triangle FEC is equal to the area of quadrilateral ADFE. Then, she again thought of dragging the point F and obtaining particular and degenerate cases to verify this equality. For the particular cases that F was the midpoint of the side BC, the equality was satisfied (Figure 31). When she dragged the point F, she again showed that the equality was true for the particular cases F=B and F=C. During her solution, Merve used dynamic properties of the software. Moreover, she easily measured the lengths and save time during whole solution process. Similar to the solution of the median problem, Merve used an inductive method based on particular and degenerate cases. She used less verbal explanations when compared to the dynamic demonstrations. Instead of algebraic expressions she used geometric relationships on the figure. For example, she stated that "If the point F is the midpoint of the side BC, then, there will be four triangles with equal areas because the side DE is parallel to the side BC, the point D and the point E are midpoints of the side AB and the side AC respectively, and DEFB and DECF are parallelograms with equal areas." She concluded that these four triangles with the same area also justified her solution.

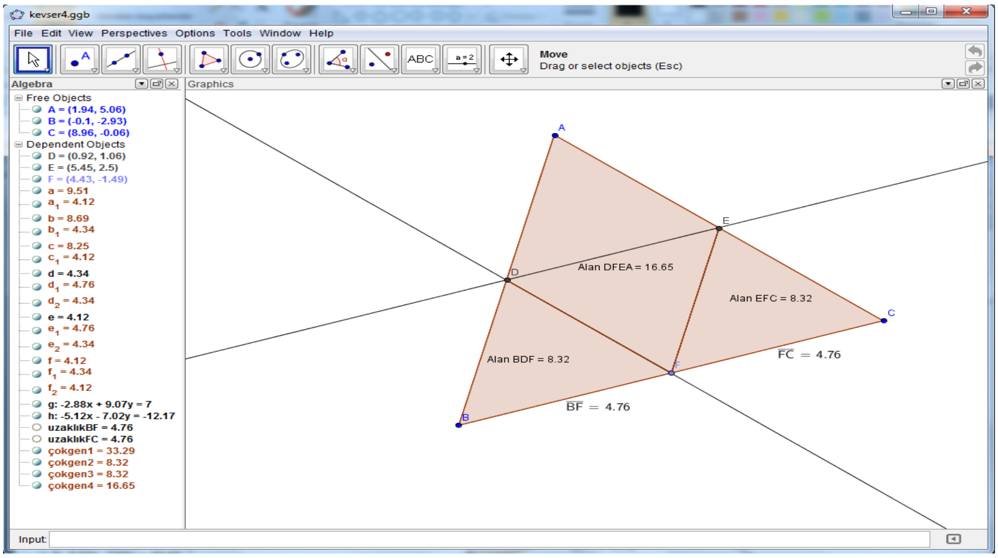


Figure 31.Merve's GGB solution of the quadrilateral problem

4.2.5 Summary of Merve's GGB Solutions

For GGB solutions of all problems, Merve used geometric solution strategies (Table 7). She applied dynamic properties of the software to the solution of the problems and used verbal explanations to justify her solutions. Table 8 shows the classification of Merve's GGB solution methods.

Table 7. Classification of Merve's GGB solution methods

The problem		Solution method	Solution category
The root problem		Properties of diagonal of the rectangles	Geometric Verbal - Pictorial
The scaled triangles problem	a)	Measuring lengths and justifying by dragging tool	Geometric Dynamic
	b)	Reasoning by exploring properties of the centroid of the triangle	Geometric Verbal - pictorial
The median problem		Particularization	Geometric Dynamic
The quadrilateral problem		Particular and degenerate cases	Geometric - Dynamic

When Merve attempted to solve the same problems with GeoGebra, she tended to use geometric approaches verified verbally or dynamically. There were not any algebraic solutions even if the software has features such as algebra window, and inserting text and functions. She thought that solving these problems with the software is easier and more time efficient than with paper & pencil.

CHAPTER 5

THE CASE OF KÜBRA

In this chapter, Kübra's solution strategies were analyzed in detail. First of all, her solutions in PPB environment were introduced, and then they were categorized according to their basic characteristics. Then, her solutions for the same problems in GGB environment were mentioned, and grouped according to their properties. Finally, Merve's solutions were summarized for each environment.

5.1 Kübra's PPB Solutions

In this section, the data gathered from paper & pencil environment and from video records will be analyzed. The solutions for each question will be grouped according to Krutetskii's (1976) framework of students' algebraic, geometric and harmonic solution strategies. In order to be more specific,their solutions will be grouped according to subcategories which were formed based on Krutetskii's (1976) verbal-logical and visual-pictorial components of solutions methods. In addition, Presmeg's (1986) dynamic imagery will also be included into these subcategories.

5.1.1 The Root Problem

Kübra developed a different strategy from other students in the experiment. She preferred to use a trigonometric approach. Before starting to solve the problem, she thought that in order to compare the areas of rectangles, she needed to calculate the areas and found the areas in terms of the same unknowns. In the following dialogue, this thinking process could be observed.

> **Tutor**: What is your plan to solve the problem?
>
> **Kübra**: I will calculate the areas of the rectangle NEOB and MEPD. However, I need to express the areas in terms of the same unknowns in order to find a relationship between them.

Tutor: So, what will you do?

Kübra: First of all, I need to find a relationship between the sides of the rectangles. Actually, I can use trigonometry to find a relationship.

Tutor: How will you use trigonometry?

Kübra: The angle A is common in the triangle AME and ADC. We know the tangent value of the triangle ADC because the sides of rectangle ABCD were given. Hence, If I expressed the sides of the triangle AME in terms of unknowns x and y, I can find a relationship between x and y (Figure 32).

First of all, she expressed the sides of the rectangle NEOB and MEPD in terms of the unknowns x and y (Figure 32). Before calculating the areas of these rectangles, she needed to find the relationship between x and y. Therefore, she looked for the triangles that she could use her trigonometric approach. Then, she realized that the angle A is common in the triangle AME and ADC. Therefore, she could find tangent value of this angle for the triangle AME. By using the tangent value 6/8 in the triangle ADC, she found the relationship between the unknowns x and y (Figure 32 and Figure 33). However, as shown in Figure 32, she also expressed the hypotenuses of these triangles in terms of the unknown a. However, she did not use this knowledge. She explained the reason for this expression and stated that "I thought that I might use Pythagoras theorem but I realized that I could easily find the relationship by using the tangent value of the triangle AME according to some mental operations that I quickly did in my mind. Then, I abandoned this strategy and used the trigonometric approach." This explanation showed that she was able to develop different approaches quickly in order to solve problem.

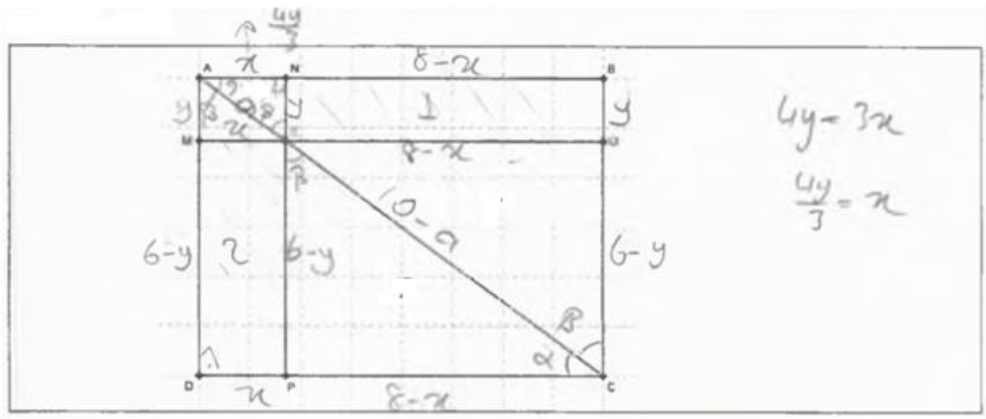


Figure 32. Expression of the sides in terms of unknowns

Her solution process could be observed in Figure 33. After finding the relationship between x and y, she calculated the areas separately. Then, she computed the areas in terms of x and found the equality of the rectangle NEOB and the rectangle MEPD.

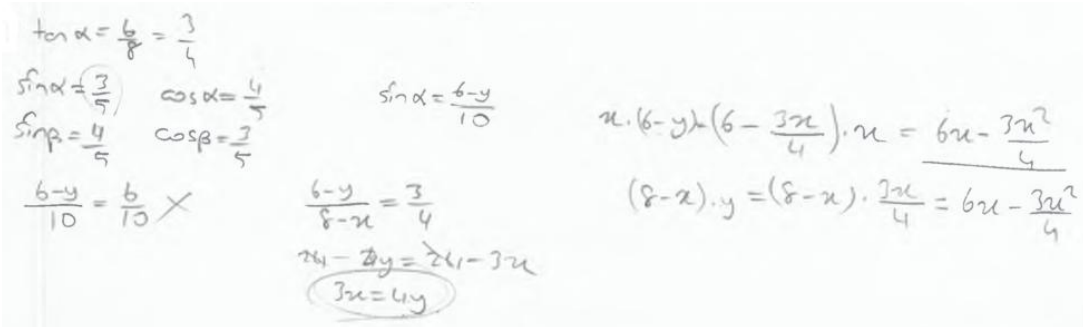


Figure 33.Kübra’s solution of the root problem

Kübra totally used algebraic expressions to find the relationship between the sides and, eventually the rectangles. Therefore, her solution was an algebraic one according to the framework of the present study. During the solution process, she used little verbal explanations. She supported her mental operations with logical explanations and justifications on the paper. Therefore, the solution strategy was a logical algebraic solution according to our subcategories in the framework.

5.1.2 The Scaled Triangles Problem

For the first part (a) of scaled triangles problem, Kübra expressed the sides of the given triangle in terms of unknowns (Figure 34). She thought solving problem by using Thales theorem. However, there were different triangles that are similar and she could not decide how to use the unknowns at first glance.

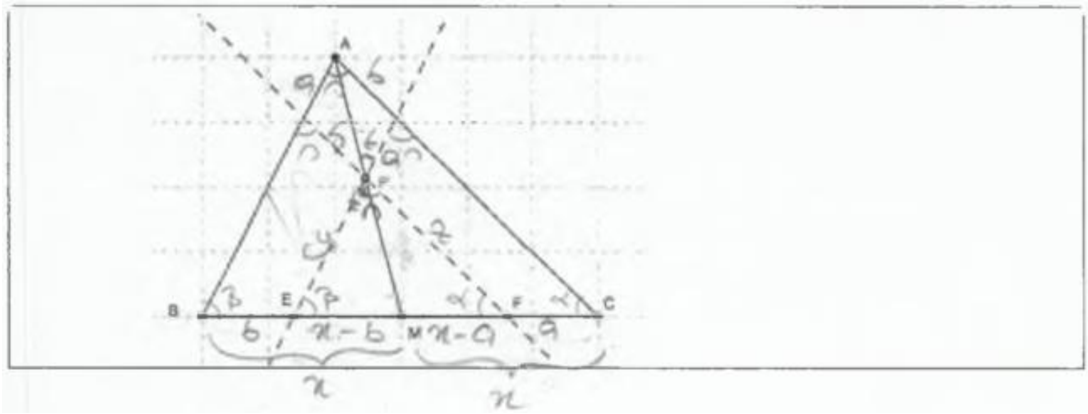


Figure 34. Expression of the sides in terms of unknowns

In the following dialogue, Kübra's process of developing her strategy could be observed based on her verbal explanations:

> **Tutor**: You expressed the sides in terms of unknowns. How will you use this expression?
>
> **Kübra**: There are similar triangles in this figure. For example, EMP and the triangle that existed between the lines EP, EC and AC are similar. When I applied similarity theory to these triangles, I obtained the equalityy/(y+a)=(2x-a-b)/(2x-b) (Figure 4). However, this equality will not help to find a solution.
>
> **Tutor**: Why?
>
> **Kübra**: Because, If I calculate this equality, there will be an equation with three unknowns. Maybe, I will get a solution in this way but there other similar triangles. I will look for more simple similarities, then I will decide about which will be helpful for my solution.
>
> **Tutor**: So which triangles are similar?
>
> **Kübra**: The triangle MFP and the triangle MCA are similar. In addition, the triangle MEP and the triangle MBA are also similar. The median AM of the triangle ABC is a common side for all these triangles. By using this knowledge, I get ME/MB=MP/MA andMF/MC=MP/MA. Since MP/MA is common in two equalities, I obtain the equality ME/MB=MF/MC . Moreover, we know that MB=MC due to the median, I have ME=MF.

While Kübra was explaining her strategy verbally, she was writing the equalities based on the similar triangles at the same time (Figure 35). Although she began with the similarity shown in Figure 4, she continued with using the similarities on the triangle MFP ~ the triangle MCA and the triangle MEP ~ the triangle MBA. According to the dialogue above, she stated that "Since ME/MB=MP/MA andMF/MC=MP/MA, then ME/MB=MF/MC. Using MB=MC (due to the median) and ME/MB=MF/MC, I obtained ME=MF.”

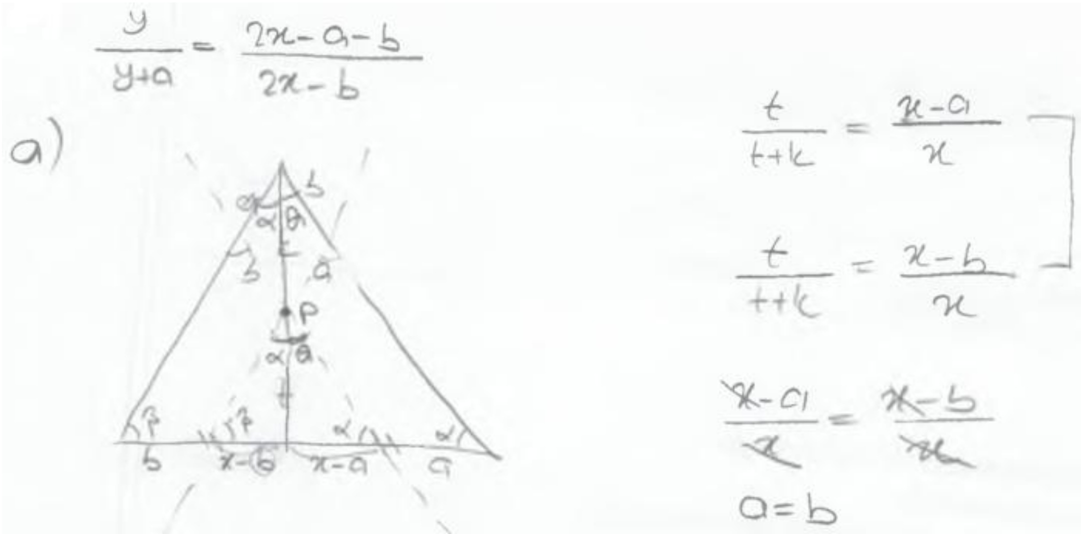


Figure 35.Kübra’s solution of the scaled triangles problem (a)

Kübra realized that the side AM is common for the triangle AMB and AMC and if she wrote the equalities, there would be a common ratio, which is MP/MA, in the equations. However, Merve explored this situation after writing the equations. It showed that Kübra again thought differently and she was much more aware of what she was doing while writing the equations. She was able to solve the problem by using algebraic equations and support them with verbal explanations as shown in above dialogue and summarized after the dialogue. Therefore, that was a verbal-logical algebraic solution strategy.

For the second part (b) of the scaled triangle problem, Kübra solved the problem based on the result of the first part of the problem. She again used similarity theory and did not need to show her solution by setting up any equations. She summarized and justified her solution as shown in the dialogue below.

Tutor: How do you plan to solve the problem?

Kübra: I can solve this problem by using the similarities that used in the first part of scaled triangle problem. In the previous part, I found that EM=MF. If I expressed EM=MF= a/2 in terms of unknown a, I will have BE=FC=A. By applying one of the similaritiesthat I used in the first part into the triangle AMB or the triangle AMC, I will have the equalityEM/EB= (a/2)/a=MP/PA=1/2. Hence, if I state MP=h where h is an unknown, then I will obtain PA=2h by using this equality (Figure 36).

Tutor: So, where the point P must be?

Kübra: It will be on the median with a ratio of (1,2) to the point M and the point A respectively. That is also centroid of the triangle ABC.

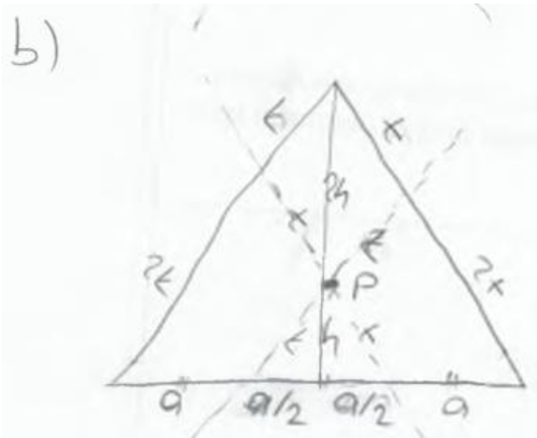


Figure 36.Kübra’s solution of the scaled triangles problem (b)

According to the dialogue, Kübra used the information EM=MF in the previous part of the problem and the logic behind it help her to solve the second part easily. She expressed the sides in terms of unknowns a and h. Then, she thought if EM=MP=a/2, then EB=PA=a. Hence, If MP=h, thenPA=2h by using similarity theory. This means that point P is at the centroid of the triangle ABC.

Kübra preferred to use verbal explanations in order to describe and justify her solution as shown in above dialogue and summarized after the dialogue. Since she had experience about the basic structure of the problem from the previous part, she did not think too much to develop this solution. The solution was a geometric solution because she used her geometry knowledge and did not need to set up any equations. She just extracted necessary knowledge from common approaches (Thales Theorem) in geometry. Her visual solution justified by verbal explanations helps us to categorize this solution as a verbal-pictorial geometric strategy.

5.1.3 The Median Problem

For the median problem, Kübra drew the figure and expressed the sides and the areas in terms of unknowns as shown in Figure 37. After thinking for a while she realized that the median splits the triangle PBC into two triangles with equal areas. She expressed the area of each with the letter A. Similarly, the median splits the triangle ABC into two triangles with the same areas and she expressed them with S. She explained the reason for these equalities as having common bases and equal respective heights (Figure 37). In order to show the relationship between the areas of the triangle APC and the triangle APB, she expressed each with the letter B and showed that S-A=B. That is, if the area of ABC is divided in two equal parts and A(PMC)=A(PMB), then it is trivial that A(APC)=A(APB).

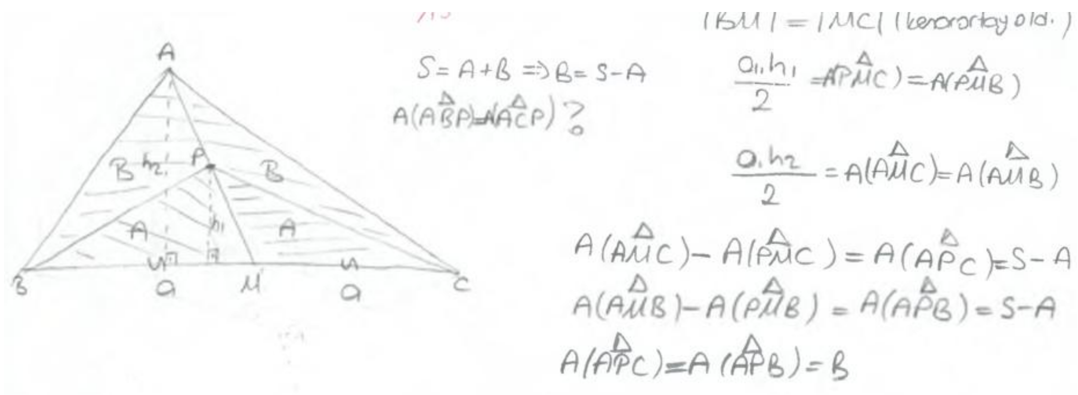


Figure 37.Kübra’s solution of the median problem

The researcher named this solution path as comparing common base and the respective heights of the triangle APB and the triangle APC. More generally, it was an algebraic solution because Kübra used algebraic equations derived from geometric approaches during her solution. In addition, she did not need any verbal explanations and she showed the logic of her solution on the paper. Therefore, she developed a logical algebraic strategy during her solution process. In fact, she used visual components during constructing the figure. However, she had an inclination for setting up algebraic equations during whole process and she found the answer by using this equations.

5.1.4 The Quadrilateral Problem

For the last problem, Kübra developed a strategy based on particular cases. She assumed that the point P was the midpoint of the side BC. Then, she used the similarity theory in finding the ratio of the areas. The process of Kübra's solution could be observed in the light of following dialogue:

> **Tutor**: What is your plan for the solution?
> **Kübra**: I will assume that the point P bisects the side BC. If I find a solution, I can generalize it to the result.
> **Tutor**: Why do you plan to solve the problem in such a way?
> **Kübra**: It will be easier and more meaningful for me.
> **Tutor**: So, how do you use this knowledge?
> **Kübra**: First of all, I find the equality NM/BC=1/2 by using similarity. According to the similarity theory, square of the ratio of the sides is equal to the ratio of the areas. Therefore, if A(ANM)=S, then I obtain A(ABC)=4S. Since NMBP and NMCP are parallelograms, the triangle NPM is common in them and its area is half of the parallelograms, I find A(BNP)+A(PMC)= 2S=A(ANPM).

First of all, Kübra found the ratio of the areas NM/BC=1/2 based on Thales theorem. According to the dialogue, she thought that based similarity theory, square of the ratio of the sides is equal to the ratio of the areas. Next, she calculated the ratio of areas as 1/4 which is the square of the ratio of the sides (Figure 38). That is, if area of the triangle ANM=S, then area of the triangle ABC=4S where S is the unknown for the real values. Next, since NMBP and NMCP are parallelograms and the triangle NPM is common in them A(BNP)= A(PMC)= S. Finally, she found that A(BNP)+A(PMC)= 2S=A(ANPM).

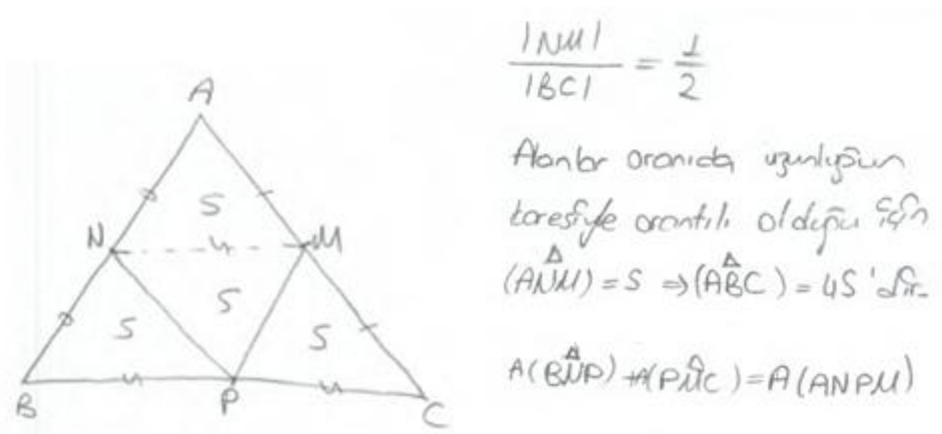


Figure 38.Kübra's solution of the quadrilateral problem

Kübra preferred to use both geometric and algebraic approaches to solve the problem. For example, while finding the ratio of similarity, she used Thales theorem and find ratio of similarity 1/2 without the use of algebra on the paper. However, she expressed the areas in terms of equations and that was and algebraic approach. Therefore, the whole solution process could be categorized as harmonic. Moreover, although there was a relative equilibrium between the use of verbal-logical and visual-pictorial components, she had an inclination for the use of pictorial means more than verbal logical ones.

5.1.5 Summary of Kübra’s PPB Solutions

Kübra’s solution methods are summarized in Table 9. They were algebraic or harmonic that included algebraic expressions except second part of the scaled triangle problem (Table 8). She preferred to use less verbal explanations than Merve. Kübra supported her solutions by logical operations as much as verbal descriptions. In addition, she solved second part of the scaled triangle problem with a geometric approach based on verbal and pictorial explanations. She usually thought both geometrically and algebraically, and developed different strategies during the process in PPB.

Table 8. Classification of Kübra’s PPB solution methods

The problem		Solution method	Solution category
The root problem		Calculating area of rectangles based on a trigonometric approach	Algebraic Logical
The scaled triangles problem	a)	Equality of ratios based on similarity theory	Algebraic Verbal-Logical
	b)	Equality of ratios based on similarity theory	Geometric Verbal - Pictorial
The median problem		Comparing common base and the respective heights of the triangle APB and the triangle APC	Algebraic Logical
The quadrilateral problem		Particular and degenerate cases	Harmonic Pictorial

5.2 Kübra's GGB Solutions

In this section, the data obtained from GGB environment and video tapes will be analyzed. The solutions for each question will be grouped according to students' algebraic, geometric and harmonic solution strategies.

5.2.1 The Root Problem

Kübra solved this problem by calculating and comparing the areas of the rectangle NEOB and the rectangle MEPD. However, when she drew the figure in GeoGebra, she realized her previous knowledge about rectangles. In below dialogue, this exploration phase could be observed:

> **Tutor**: How do you solve the problem?
>
> **Kübra**: When I solve this problem in paper-and-pencil environment, I did not think of using the diagonal property of the rectangle. The diagonal divides the rectangle into two equal parts. I realized it during the constructions in GGB environment.
>
> **Tutor**: So, how do you use this knowledge?
>
> **Kübra**: I will measure the areas. They are equal because the diagonal divides the rectangle EHDF and the rectangle AGEI into two triangles with equal areas. Therefore, the areas of rectangle GCHE and the rectangle IEFB will also be equal because this diagonal divides the rectangle ABCD into two triangles with equal areas at the same time.
>
> **Tutor**: In your construction, you did not use the grid and measured the lengths of the sides. Why?
>
> **Kübra**: Yes, because I used the diagonal property, the lengths of the sides are not important if the figure yields the conditions given in the problem statement.

Kübra explained her solution by verbal justifications. She did not know how to show the equality based on the diagonal property with the software. She just measured the areas of the triangle and colored the triangles and rectangles with the same color (Figure 39). She explained her solution with the idea that the diagonal divides the rectangles into two equal triangles and hence the areas of them are equal. In addition, she thought that lengths of the sides are not important if construction was drawn according to the given conditions. However, she could not use the dynamic property of the software until the tutor gave the clues for using this feature as shown in below dialogue.

> **Tutor**: You measured the areas. You stated that they are equal because of the

diagonal. So, how will you show that this equality is always satisfied?

Kübra: Actually, I do not know how to show it.

Tutor: So, you will give you a clue. We have already talked that GeoGebra is a dynamic software. How does the dynamic property of the software contribute to justify your solution?

Kübra: Ok, I have already remembered. We can move the points and the sides. If we drag the point E along the diagonal, the result will not change. This is also an evidence for the solution.

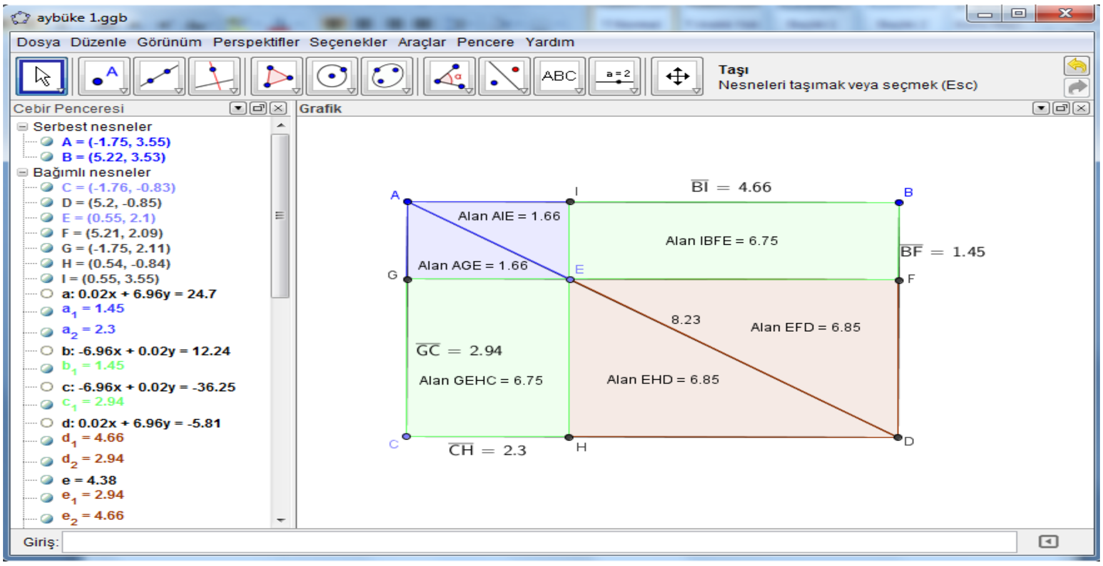


Figure 39.Kübra's GGB solution of the root problem

After the tutor gave the clues she remembered to drag the points and sides of the rectangle to verify her solution. Dragging feature was an important factor in determining the category because it helps students to understand logical structure of the problem (Iranzo-Domenech, 2009). However, the researcher categorized this solution as a verbal pictorial one because she did not use dragging feature of the software at first glance.She used geometric approaches supported by verbal-pictorial explanations as shown in above dialogues.

5.2.2 The Scaled Triangles Problem

For the first part of the problem, Kübra thought of measuring the lengths directly. However, her justification for the equality was different when compared to Merve's solution. She used the areas in order to verify her solution. In the following dialogue, Kübra summarized her solution strategy:

Tutor: How do you solve this problem?

Kübra: By using measuring tool, I can find the lengths. For this reason, I will measure the areas of the triangle EFD and the triangle EDG and try to find e relationship between the areas. For this two triangles, the height belong to the bases are equal. Therefore, the ratio of the areas is equal to the ratio of the bases.

Tutor: (After Kübra measured the areas) The areas are equal. And what about the bases?

Kübra: The bases are also equal.

Tutor: So, how do you show your solution is always true?

Kübra: Of course. If I drag the point E, the sides and vertices, the equality is always satisfied as was in the first question.

According to the dialogue, she measured the areas of triangle EFD and the triangle EDG, and found that the areas were equal. Since the respective heights of these triangles for the side FD and the side DG were common, these sides must be equal (Figure 40). By solving the problem in this way, she did not need to explain the function of the median in this equality. However, the main reason for the equality of areas was the median ED of the triangle EFG. She also justified her solution by moving the point E and the sides of the triangle ABC. However, this solution was considered as a geometric solution verified by verbal explanations in the dialogue. She used dynamic features as a secondary verification method by using the clues given by tutor. For example, the tutor wanted her to show that her result is always true. As it was previously mentioned in the instructional period, by dragging the points, the changes in the results could be observed at every point along the dragging line. Therefore, she probably remembered this feature and used it in her alternative solution.

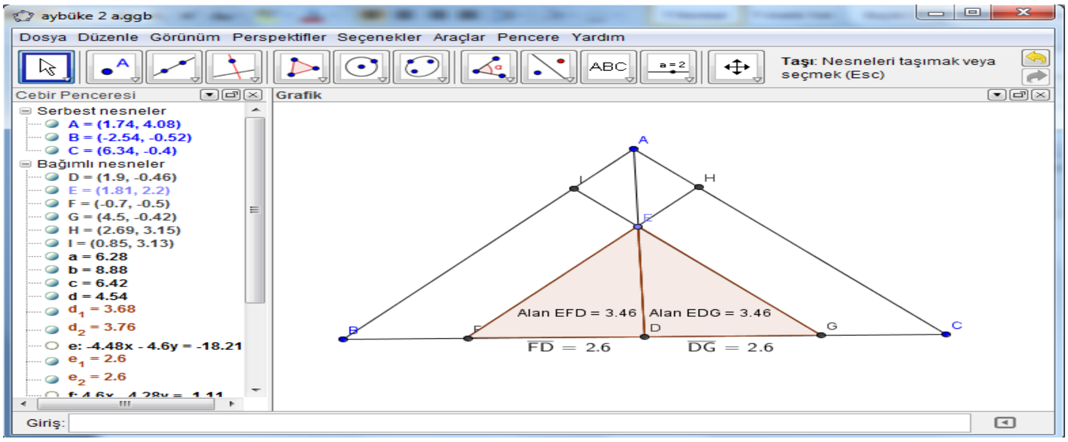


Figure 40.Kübra’s GGB solution of the scaled triangles problem (a)

For the second part (b) of the scaled triangle problem, Kübra developed a strategy based on the dynamic properties of the software. After constructing the figure, she dragged the point P until she got the equality BE=EF=FC given in the problem statement(Figure 41). Then, she used the measurement tool to calculate the length AE and the length ED. In order to understand where the point E is, she attempted to find the relationship between AE and ED. Finally, she found that the length AE is two times the length ED (Figure 10). She also moved the vertices of the triangle ABC in show that equality was satisfied for every kind of triangles ABC. She also stated that “the logic behind this ratio was due to the fact that AD is the median of the triangle, FH and GI are parallel lines two the side AC and the side AB respectively, and the point E is at the intersection of two lines, and the equality BI=IH=HC.”

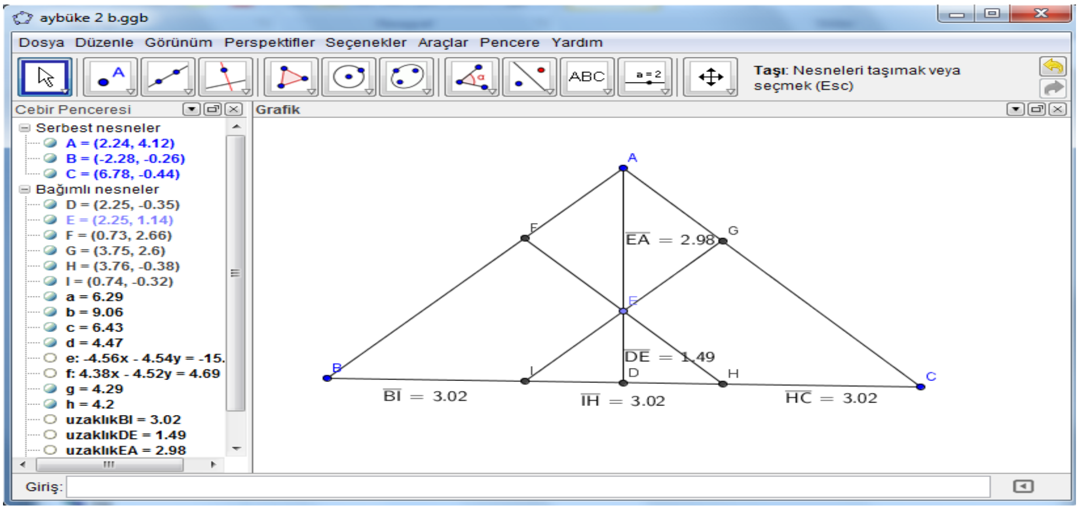


Figure 41.Kübra’s GGB solution of the scaled triangles problem (b)

However, although Kübra used similarity theory, she did not talk about it and its applications for this problem during her solution. The software made solution easier than PPB environment and the student did not need to use this theory to explain her result. She solved the problem by using dragging and moving feature of the software.These dynamic movements help students to understand logical structure of the problem (Iranzo-Domenech, 2009) and hence they did not need to use other strategies. Therefore, this solution was grouped as a dynamic geometric solution.

5.2.3 The Median Problem

Kübra used an unusual solution method for the median problem. After drawing the figure, she measured the areas and observed the equality. However, she justified her solution by showing the common base and equal heights of the triangle APB and the triangle APC given in the problem statement. In the following dialogue the solution process could be observed:

> **Tutor**: You measured the areas and find the equality. How do you verify your solution?
>
> **Kübra**: If two triangles with the same areas have common bases, their respective heights are also equal. In order to show the equality of heights, I can use again measuring tool.
>
> **Tutor**: So, why are the heights equal?
>
> **Kübra**: Because the triangle BFD and the triangle CGD are equal as seen in Figure 11. The line segments BD and DC are equal due to the median. The angle F and the

angle G are 90°. Therefore, we obtain BF=GC.

After showing BF=GC, Kübra explained this equality as the fact that the triangle BFD and CGD are congruent because the angle D is common, BD=DC (due to the median), and F=G=90° (the heights of APB and APC)(Figure 42). She also justified her solution by dragging the point E, moving the sides and vertices. It can be observed in the following dialogue:

Tutor: Is there any other justifications for your solution?

Kübra: Yes, for example, by using dynamic property of the software, when we drag the point E, the equality will remain the same. Similarly, moving the sides and vertices will not affect the result.

Although she used dynamic justifications, this solution was a verbal-pictorial geometric one because she verified her solution mostly by verbal and visual explanations. She used dynamic justifications after explaining her result by verbal-pictorial ones. It can be considered as a second way of solution.

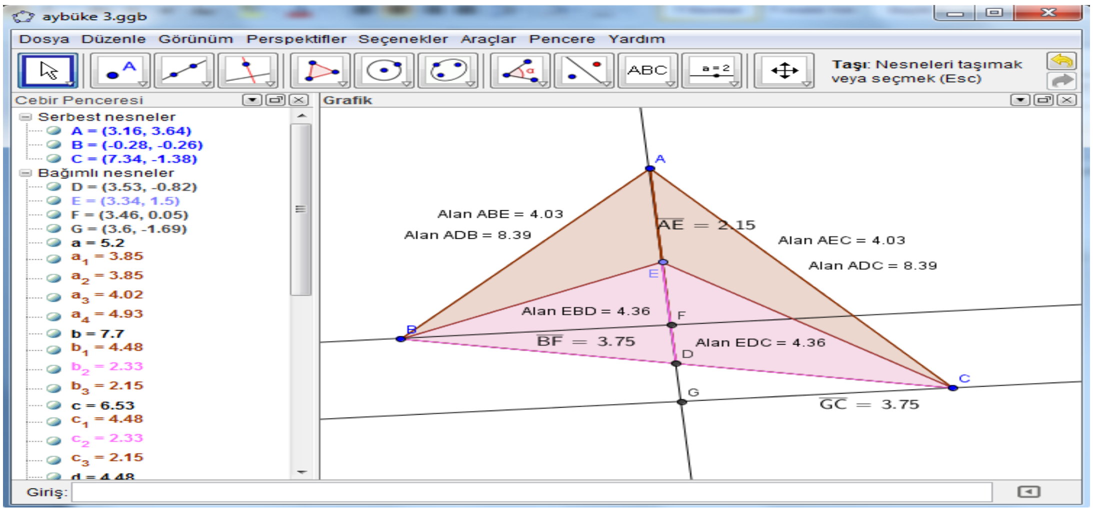


Figure 42.Kübra’s GGB solution of the median problem

5.2.4 The Quadrilateral Problem

In GGB environment, Kübra usually used different strategies from her PPB solutions. As a matter of fact, she compared equal bases and respective heights of the triangles. After drawing the figure, she measured the areas and observed the equality of areas. In order to show her solution, she drew auxiliary lines that were the heights of ADB and ADC (Figure 43). Then, she verbally explained her solution as shown in the following dialogue:

> **Tutor**: You measured the heights of the triangle ABD and the triangle ADC. How will you use this information?
> **Kübra**: I showed that the bases and respective heights of the triangle BDE and the triangle EDA are equal. Similarly, the triangle ADF and the triangle FDC also have bases and respective heights with equal lengths. Therefore, the area of quadrilateral AEDF is equal to sum of the areas of the triangle BDE and the triangle FDC.
> **Tutor**: Ok.
> **Kübra**: In addition, when we drag the point D along the side BC, we also justify the solution. For example, if the point is on the middle of the side and the vertices B and C, that was also an evidence for the solution.

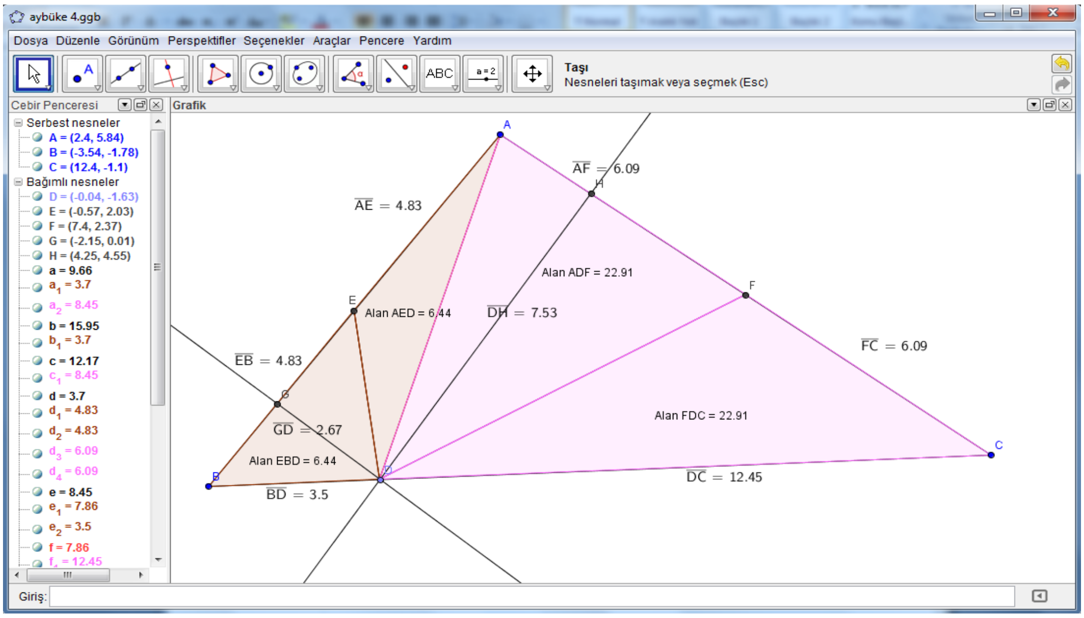


Figure 43.Kübra's GGB solution of the quadrilateral problem

Kübra used dynamic justifications as a second way of solution. By dragging the point D, she obtained particular cases such as D=B, D=C, and D as a midpoint at BC. However, first of all, she used a visual-pictorial geometric solution. She supported her solution by verbal explanations as shown in above dialogue. Visual demonstrations and geometric ideas made the solution meaningful. That is, constructing the figure and showing the equality of heights of the triangles with the same bases helped her to explain the relationship between areas.

5.2.5 Summary of Kübra's GGB Solutions

During GGB solutions, Kübra preferred to use geometric approaches in solving the problems (Table 9). She mostly used verbal and visual explanations. However, she supported her verbal-pictorial solutions by using software's dynamic features such as moving, dragging as a second way. She also used measurement tools several times to verify her solution. For the second part of the scaled triangle problem, she preferred to use a different solution strategy according to the framework established in this study based on Krutetskii's (1976) and Presmeg's (1986) frameworks. In other solutions, she preferred to use verbal-pictorial geometric solutions. The effect of DGS environment would be assessed based on these different solution categories.

Table 9. Classification of Kübra's GGB solution methods

The problem		Solution method	Solution category
The root problem		Properties of diagonal of the rectangles	Geometric Verbal - Pictorial
The scaled triangles problem	a)	Equality of areas by using measuring tool	Geometric Verbal - Pictorial
	b)	Measuring lengths and justifying the relationship	Geometric Dynamic
The median problem		Comparing common base and the respective heights of the triangle APB and the triangle APC	Geometric Verbal-Pictorial
The quadrilateral problem		Comparing the base and height of triangles	Geometric Verbal-Pictorial

CHAPTER 6

CONCLUSION AND DISCUSSION

In this qualitative study, the researcher investigated two pre-service elementary mathematics teachers' solution methods for four plane geometry problems and analyzed the role of technology use in their solution strategies. The analysis consists of examining the data gathered from interviews, paper & pencil solution sheets and GeoGebra files. The data triangulation process was carried out according to the framework that categorizes students' solutions into algebraic, geometric and harmonic ones. This framework was formed in the light of Krutetskii's (1976), and Presmeg's (1986) studies on students' preferences of problem solving strategies. In addition, the researcher split three main categories into subcategories, according to students' verbal, logical, visual and dynamic explanations.

An instructional period was designed to introduce the basic GGB tools. In addition, nine activities were prepared to show how the software could be used in solving plane geometry problems. After this period, a pilot study was done with two volunteer students in order to have an idea about the usability of the instrument for the actual study. Then, the other selected students participated in data collection period. The data were collected during a week. In this period, the instrument that consisted of four plane geometry problems was given to the students and they solved the problem both in PPB and GGB environments. While they were solving the problems, the researcher conducted interviews with the students. Whole data collection process was recorded with a video camera. Finally, the researcher transcribed video records and analyzed PPB and GGB worksheets. In the previous chapter, the data were analyzed and interpreted in detail. In this chapter, students' solutions will be compared and discussed. In addition, the limitations and further implications will be stated in the following parts of this chapter.

6.1 Interpretation of the Students' Solutions Based on the Framework

According to Krutetskii (1976), there are three types of problem solvers, namely; analytic, geometric and harmonic. However, in the present study, the researcher preferred to use the term "algebraic" instead of the term "analytic" in reference to Coşkun's (2011) study of students' representation methods. The reason was that since the instrument consists of plane geometry problems, students' solutions in the PPB environment comprised mostly algebraic representations. Krutetskii's (1976) verbal-logical and visual-pictorial components of problem solving process and Presmeg's (1986) dynamic imagery were keystones for sub-categories in the framework of the present study. In this part, students' solutions will be summarized according to their solution methods and preferences for justifying their solutions.

6.1.1 The Case of Merve

In PPB environment, Merve developed algebraic, geometric and harmonic solutions for the problems. However, she mostly used algebraic representations in this environment. When she started to solve the problems, she had an inclination for setting up algebraic equations although the problems could easily be solved with geometric methods. The reason might be the fact that she learned to use mostly algebraic methods in this environment. In addition, she usually justified her solutions with verbal explanations even in her geometric and harmonic solutions. Krutestkii's (1976) verbal-logical component of problem solving process could be observed in this case. She used visual demonstrations to set her algebraic equations during the solutions. However, she solved the quadrilateral problem in a geometric way. It might be due to the fact that the problems are similar to each other and she practiced until the last problem. Therefore, she did not need to use algebraic equations; she preferred to verbally explain her result on the figure. In addition, the information given in the problem might push her to use some usual methods. For example, for the root problem, she tried to solve the problem by using Pythagoras theorem at first because she observed some right triangles with given sides. Then, she realized that it was not possible to find an exact relationship and altered her method.

However, when she solved the problems in GGB environment, she preferred to solve the problems with geometric methods. Although GeoGebra has algebra window, Cartesian coordinates and spreadsheet functions and she know how to use them, she used graphics window. For example, she used the strategy comparing common base and the respective heights of the triangle APB and the triangle APC for the median problem in PPB environment. However, when she solved the same problem in GGB environment, she preferred to use particularization strategy with dynamic property of the software instead of setting up equations or using coordinates. It might be because of the fact that the software enforced her to look for geometric solutions. However, she again used many verbal explanations in order to verify her solutions. That was an indication for the fact that she was not able to explain her solution only using the software. In this case, it helped her to explore geometric relationships easier than PPB environment but it was not enough for justifying her solution.

However, in particular case strategies, the software exactly helped her to solve the problem without using any other justifications methods. For this reason, she showed the characteristics of Presmeg's dynamic imagery in problem solving by means of the dynamic features of the software. Moreover, although the logic behind solutions was almost the same, her solutions in GeoGebra environment were different from those in PPB environment. It shows that different environments let her develop different strategies and helped her to think more about mathematical and geometric relationships in the problem situations. For example, for the root problem, she solved with an algebraic method in PPB but she used a geometric approach in GGB environment. She explored that whatever the lengths of rectangles' sides are; the equality of inner rectangles is satisfied in the software environment. Moreover, she thought that GGB is an effective tool in solving such kind of plane geometry problems because it gives them the opportunity to solve the problems easier and quicker than PPB environment.

6.1.2 The Case of Kübra

Kübra also developed geometric and harmonic solutions in addition to algebraic ones in PPB environment. Nevertheless, she preferred mostly to use algebraic representations in her solutions. Only for the second part of the scaled triangles problem, she selected a geometric approach. The reason for such a method was that she had already solved the first part of the problem which was a primary step for the second part hence she did not need to use algebraic expressions. She just solved the problem on the figure and verified her solution by verbal explanations. In addition, she verbally explained most of her solutions even the harmonic and algebraic ones. Her verbal solutions fit to Krutetskii's verbal-logical approach during problem solving. For the quadrilateral problem, she chose a harmonic solution having more visual-pictorial components than verbal messages. The nature of the PPB environment might be the reason for selecting mostly algebraic way of solutions. For instance, for the first problem, she used a trigonometric approach and set up equations based on this idea. However, if she would not have an inclination for such a method, she might easily consider the diagonal property and realized that even the lengths of the rectangles were not given; the problem could be solved by using the other conditions given in the problem.

When Kübra solved the problems in GGB environment, she completely used geometric approaches while it has algebra window, Cartesian coordinates and spreadsheet functions. Her GGB solutions consisted of verbal-pictorial and dynamic strategies. Therefore, Kübra showed the characteristics of Krutetskii's idea of using verbal messages in problem solving and Presmeg's dynamic imagery category of visual solution methods. She used a dynamic method for the second part of scaled triangles problem and verbal-pictorial methods for all other solutions. Although she could mostly develop dynamic property of the software, she preferred mostly verbal explanations to justify her solutions. The reason for this situation might be the fact that she still had an inclination for using algebraic methods. Since she did not prefer algebra tools of the software, she verbally justified her solutions. It means that the software pushed her to use geometric methods. However, she preferred to use dynamic properties of GeoGebra as a second way of solutions. During interviews, Kübra also stressed that GGB helps them to solve plane geometry problems easier and quicker than PPB environment. For example, for the second part of the scaled triangle problem, she just dragged the point P and found the equality BE=EF=FC. Then, she measured the distance between the point P and the points E and M. Finally, she found the point P as the centroid of the triangle ABC. Compared to the solution in PPB environment, she thought that this way of solution is simple and time efficient.

6.2 Discussion

Before starting to state and discuss the results of the present study, it will be useful to mention the main goal. The main purpose of this study was to understand how technology affects students' solution strategies while solving plane geometry problems. For this reason, the researcher analyzed students' solutions in both PPB and GGB environments in previous chapters. By comparing solution strategies in both environments, the researcher had the opportunity to draw conclusions about the effectiveness of them. In this section, the results concerning the main goal of the study will be discussed based on related literature.

First of all, the students usually preferred to justify their solutions by verbal explanations in PPB environment. When they used the software, they again verified their solutions with verbal messages in addition to dynamic movements. Hacıömeroğlu (2007) identifies Krutetskii's verbal-logical component of solution methods as interpreting visually presented concepts with mathematical symbols. The students used these verbal messages even with their algebraic solution methods. It shows that they have difficulties in stating some of their mental operations. When the research about the use of different environments for problem solving (Coşkun, 2011; Harskamp, Suhre & Van Streun, 2000; Iranzo-Domenech, 2009; Yerushalmy, 2006) is analyzed, it can be clearly understood that verbal justifications are integral part of problem solving process in PPB and technology environments.

The characteristics of the environments affect students' strategy preferences. For example, Merve solved the problems in PPB environment without considering particular and degenerate cases. She tried to solve problems by using conventional methods such as using similarity theory in PPB environment. However, she used this particularization method by using dragging and moving free objects in GGB environment. Iranzo-Domenech (2009) stated that the students are able to encounter with deep information about the logical structure of the problem in such a dynamic environment. In other words, dynamic solutions helped students to understand logical structure of the problem and made the solution more meaningful (Christou et al, 2004; Iranzo-Domenech, 2009). Therefore, this result supported Sağlam, Altun and Aşkar's (2009) findings of their study that the students were able to develop the strategies that could not be developed in traditional environments by using the software.

However, the students solved some of the problems without using verbal or dynamic components in PPB environment. In this logical solution method, the students used different mathematical contents such as similarity theory, trigonometry and areas of polygons during their solutions. It shows that each environment had different contributions to the students' mathematical thinking and problem solving skills. This result supported Coşkun's (2011) study about the effectiveness of technology in developing visual and non-visual solution methods in different environments since she had found that the use of each environment has different influences on students' thinking styles. In addition, Iranzo-Domenech (2009) stressed that different environments helps students to develop different competencies as a results of her study about the synergy of environments.

In addition, the students preferred to use algebraic and harmonic methods for PPB; and geometric ones for GGB environment. Contrary to Coşkun's (2011) study, the students participating in problem solving process preferred to use geometric solutions in GGB environment; they did not use algebra window while the tutor taught how to use it in instructional period. The reason for this situation might be the fact that the nature of problems pushed them for geometric solutions in GGB environment. In other words, all problems were related to plane geometry and the students need to construct the figures; therefore they found the software effective in solving these problems without the use of algebra window. However, they used algebraic equations based on some mathematical subjects such as similarity theory, trigonometry, the area of polygons, etc. in PPB environment. They sometimes did not use these topics in their GGB solutions but they developed dynamic solutions in this environment.

Although the problems are related to plane geometry and geometric approaches are much more available, they preferred to use algebraic ones in PPB. The students focused on algebraic representations and they were not able to use different ones while using paper-and-pencil. For example, Merve tried Pythagoras theorem for the root problem then she realized it was not a proper way. Then, she changed her strategy and used again an algebraic one. Probably, she thought that there are right triangles on the figure and Pythagoras theorem might help her to find the solution. In fact, she used Presmeg's (1986) concrete imagery which means remembering the image of the right triangle and its relationship with Pythagoras theorem. However, she used visual and dynamic features of the software and did not face such problems in technology environment. This result supported the fact that the students can easily access visual images of mathematical ideas, organize data; compute efficiently and accurately in the technology environment (NCTM, 2000).

There are not enough studies comparing students' solutions in these two different environments (Coşkun, 2011). Many studies focused on the effect of technology on learning and performances (Filiz, 2009; İçel, 2011; Kepçeoğlu, 2010; Lester, 1996; Li & Ma, 2010; Zengin, 2011). However, there are less investigations examining that student' solutions changed when using technology (Coşkun, 2011; Harskamp et al., 2000; Iranzo-Domenech, 2009; Yerushalmy, 2006). The findings of these studies supported the finding of the present study in terms of variation of students' solutions with the use of technology. It can be inferred from the study Iranzo-Domenech (2009) that when students solve the problems they tended to develop different competences based on their mathematical knowledge. The findings of the present study also overlap with the findings of Iranzo-Domenech (2009) because the students developed different solution methods with technology based on their knowledge of mathematical ideas.

According to findings of the present study, technology gives the opportunity to develop alternative strategies. After solving each problem, the students attempted to find alternatives strategies and they usually found alternative solutions. For example, Kübra moved free objects in graphics view and generated alternative solutions for the root, the median and quadrilateral problems. As an alternative solution for the quadrilateral problem, she stated that "if the point E bisects the side BC, or it was at the vertex B or at the vertex C, the equality will again be satisfied; therefore, this particular cases method was also an evidence for verification." This method was also a known strategy as given in the resolution of this problem in Iranzo-Domenech's (2009) study. The result, students were able to develop alternative strategies in technology environment, is consistent with related literature (Cai& Hwang, 2002; Christou et al, 2004: Coşkun, 2011). Nevertheless, Mehdiyev (2009) found that the students with little conceptual understanding are not able to develop new strategies. In other words, the students with little knowledge of concepts face difficulties in developing additional solutions. In the present study, Merve and Kübra have different conceptual understanding because they developed different solutions for the same problems. In addition they did not encounter with any problems related to concepts and they were able to develop alternative solutions. For example, they developed dynamic solutions in addition to their verbal-pictorial solutions in technology environment.

The students explored geometric ideas in GGB environment easier than PPB environment**.** For example, for the root problem, Merve thought that the lengths of the sides are important factors in order to find the equalities of areas in PPB environment. However, when she solved the same problem in GGB environment, she realized that it is not actually necessary if the conditions given in the problem are satisfied. In addition, although Merve and Kübra would be able to solve the problems by using particularization method in GGB environment, they did not think of this method most of the time in PPB environment. When they used the software, they have inclinations for using this particular cases method. Coşkun (2011) and Iranzo-Domenech (2009) also found that the students preferred more visual methods in GGB environment and the dynamic feature of the software helped the students to discover geometric relationships. Therefore, this result is consistent with their studies about the effects of technology on students' problem solving strategies.

According to the results of the present study, it is important to state the benefits and difficulties of using technology in problem solving. Dikoviç (2009) found that GeoGebra has many advantages such as being user friendly, having opportunity for multiple representations, supporting different learning styles, customizing opportunities, supporting mathematical understanding, having opportunity for cooperative learning, entering commands, publishing files as Web pages, and encouraging teachers to use technology. In concordance with these advantages reported by Dikoviç (2009), the participants of the present study were able to save time, make correct and accurate constructions, calculate quickly and easily, use multiple representations, and understand the logical structure of mathematical content by means of dragging and moving features of GeoGebra.

In addition, GeoGebra allows students to drag points, lines, figures, and shapes. This dynamic feature helped students to understand the logical structure of the problem (Iranzo-Domenech, 2009). That is, they can observe the changes in the figures and algebraic relationships while some elements of figure are dragged (Bu & Hacıömeroğlu, 2010; Velichová, 2011). Moreover, GeoGebra helps students to visualize the figures and shapes in this dynamic environment (Fahlberg-Stojanovska & Trifunov, 2010; Hacıömeroğlu, 2011; Kutluca & Zengin, 2011; Velichová, 2011). The results of the present study are also consistent with these results. That is to say, the participants of the study experienced great benefits of dragging and easy visualization features of the software in understanding the problems, understanding the mathematical content of the problems, and developing and changing their strategies.

Technology tools motive students for learning and provide them the opportunity to participate actively in classroom activities (Kaplan, 2010, Özgün-Koca, 2009). In concordance with this assumption, the participants of the present study were willing for learning how to use GeoGebra in plane geometry tasks and the instructor observed active participation of the students during the treatment period. In a study, Kaplan (2010) reported the views of elementary mathematics teachers' views on the use of technology tools and found that using technology tools in mathematics classroom enhance motivation of the students for active participation. In addition to this finding of Kaplan's (2010) study, the participants of the present study stressed that using GeoGebra in mathematics activities helps them to save time, make drawings and calculations accurately and quickly, enhance participation, and feel comfortable during carrying out plane geometry tasks.

Moreover, the students are able to connect algebra and geometry by using algebra and graphic views of GeoGebra (Hohenwarter & Fuchs, 2005; Hohenwarter & Jones, 2007). However, the participants of the present study have difficulty in using algebra tools. The participants thought that they did not need to use algebra window since the subject is plane geometry. They added that dynamic and visualization features of the software were sufficient for finding and verifying their solutions. Similarly, they did not use spreadsheets during their solutions. However, they preferred to use verbal messages for justifying their results. It shows that although they are aware of algebra window and spreadsheet, they prefer to use the graphic options of GeoGebra for plane geometry problems.

All in all, each of PPB and GGB environments contributed different aspects of students' problem solving strategies. Depending of the problems' characteristics, students justified their solutions with dynamic movements in GGB, logical explanations in PPB and verbal explanations in both environments. In addition, the students mostly preferred algebraic and harmonic solutions in PPB environment; and geometric ones in GGB environment. However, the students have an inclination for using algebraic equations in PPB environment although the problems are related to plane geometry and geometric approaches are much more available. Moreover, technology gives the opportunity to develop alternative strategies and they explored geometric ideas easily in this environment. In addition, the students experienced many advantages of using GeoGebra such as being time saver, opportunity for easy, accurate and quick drawings and calculations, having multiple representations, enhancing motivation, feeling comfortable with user-friendly interface, dragging feature, and visualization capabilities. However, the participants have difficulties in using algebra and spreadsheet views of GeoGebra because they thought that they did not need to use these windows in carrying out plane geometry tasks.

6.3. Recommendations, Implications and Further Research

Technology provides students the opportunity to develop the strategies that could not be developed in traditional environments. They used dynamic methods to justify her solutions. The software provides a flexible environment in which they can easily develop alternative strategies and explore geometric ideas. Therefore, the teachers can use dynamic geometry software during their instruction to contribute students' understanding of geometrical concepts and problem solving skills.

In addition, the synergy of environments helps students to display different competencies. That's why; they used different strategies in different environments. In other words, the characteristics of environments affect students' strategy preferences. For example, they usually preferred to use algebraic and harmonic methods for PPB; and geometric ones for GGB environment. For this reason, teachers can benefit from both environments to reveal students' different skills and promote them to think in another ways during problem solving process.

Moreover, as it was mentioned before, although the problems are related to plane geometry and geometric approaches are much more available, they preferred to use algebraic ones in PPB environment. The reason might be the fact that the students are mostly exposed of setting up algebraic equations during their learning. In some cases they were not even aware of why they are setting up them. In addition, the geometric aspect of the problem was sometimes disregarded in this case. Therefore, using geometric methods as well as algebraic ones might encourage students to develop different strategies in their solutions. For this purpose, supporting problem solving process with technology tools might also be helpful.

In addition, by using activity sheets during the use of the software as it was done in the present study, the students first can develop strategies on paper-pencil and then develop strategies during the use of the software. Students' different solution strategies and their understanding of mathematical ideas and concepts could be observed in this way.

In the present study, the researcher analyzed plane geometry problem solving strategies of two particular cases at a particular university to understand how technology affects their methods. In future studies, it might be better to work with different cases at different universities and with other topics in the mathematics curriculum to have more generalizable results. In addition, the number of problems in the instrument and their complexity could be increased.

Moreover, the students were allowed to use the software whenever they want during problem solving process. However, they solved the problem in PPB environment at first, and then they used the software. The solutions with paper-and-pencil probably affected their solutions with the software. This situation constituted a thread to internal validity. If the period between the environments was longer, the solution methods might differ. Therefore, this study might be replicated with a longer data collection period. In this period, after solving the problem with paper-and-pencil, it might be better to have a time span to use the software.

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APPENDIX

TEZ FOTOKOPİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü ☐

Sosyal Bilimler Enstitüsü ☐

Uygulamalı Matematik Enstitüsü ☐

Enformatik Enstitüsü ☐

Deniz Bilimleri Enstitüsü ☐

YAZARIN

Soyadı : Koyuncu
Adı : İlhan
Bölümü : İlköğretim Bölümü

TEZİN ADI (İngilizce) : Investigating the Use of Technology on Pre-Service Elementary Mathematics Teachers' Plane Geometry Problem Solving Strategies

TEZİN TÜRÜ : Yüksek Lisans ☐ Doktora ☐

1. Tezimin tamamı dünya çapında erişime açılsın ve kaynak gösterilmek şartıyla tezimin bir kısmı veya tamamının fotokopisi alınsın. ☐

2. Tezimin tamamı yalnızca Orta Doğu Teknik Üniversitesi kullanıcılarının erişimine açılsın. (Bu seçenekle tezinizin fotokopisi ya da elektronik kopyası Kütüphane aracılığı ile ODTÜ dışına dağıtılmayacaktır.) ☐

3. Tezim bir (1) yıl süreyle erişime kapalı olsun. (Bu seçenekle tezinizin fotokopisi ya da elektronik kopyası Kütüphane aracılığı ile ODTÜ dışına dağıtılmayacaktır.) ☒

Yazarın imzası .... Tarih ....